Efficient Nonlinear Analysis of Elasto-Plastic 3D R/C Frames
Using Adaptive Techniques

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ABSTRACT

This paper presents a new nonlinear analysis method for 3D reinforced concrete (R/C) frames employing adaptive analysis concepts. The first two components of the proposed adaptive method, namely the elastic and elasto-plastic beam-column formulations, are described. The details of automatic mesh refinement, which is the third component of the adaptive method, are then presented. These include the construction of an effective interaction surface representing generalised stress states at the elastic limit, the determination and checking of generalised stresses for exceeding the elastic limit along an elastic element, and the refinement of an elastic element into an appropriate number of elastic and elasto-plastic elements. Examples are finally presented to illustrate the accuracy and efficiency of the proposed analysis method.

KEYWORDS

Reinforced concrete frames; Nonlinear structural analysis; Adaptive nonlinear analysis.

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1. INTRODUCTION

Nonlinear analysis utilising one-dimensional beam-column elements has been widely recognised as the most effective and realistic method for modelling the nonlinear response of reinforced concrete (R/C) frames [1,2,3,4,5,6]. However, despite considerable advances in the formulation of applicable one-dimensional elements and the steady increase in computing power, the nonlinear analysis of R/C framed structures using conventional methods can still be prohibitively expensive.

In order to address the modelling and computational shortcomings of conventional nonlinear analysis, an adaptive analysis method was previously proposed by the first author [2,4,7,8,9], which has been shown to achieve computational savings often in excess of 80% for steel and R/C framed structures. In the proposed adaptive method, analysis is always started using only one elastic element per member; subsequently, automatic refinement (or subdivision) of elastic elements into expensive elasto-plastic elements is performed only when and where necessary, during analysis and within the structure, respectively. This method maximises computational efficiency on two accounts; firstly, elastic elements are considerably more efficient than elasto-plastic elements, which typically involve cross-sectional discretisation into monitoring areas; secondly, one elastic element can accurately model a whole elastic member, whereas several elasto-plastic elements are needed to represent an elasto-plastic member. Therefore, by maximising the extent to which inexpensive elastic elements are utilised within the structure and during analysis, considerable computational benefits arise without compromising the solution accuracy.

The first application of the proposed adaptive method was to the nonlinear analysis of 2D and 3D steel frames, where consideration was given to static and dynamic loading [7,8] as well as to fire conditions [9]. This required the development and utilisation of an advanced elastic formulation [10], which is capable of representing the elastic geometrically nonlinear response of steel beam-columns using only one element per member. Given the linear nature of the generalised response of elastic steel cross-sections, the criteria governing the automatic refinement of elastic elements into expensive elasto-plastic elements were simply based on checking the extreme fibre stresses against
the elastic limit (i.e. yield strength) [7,8], modified by elevated temperature in the presence of fire loading [9].

The extension of the adaptive method to the nonlinear analysis of R/C frames proved to be considerably more challenging than for the case of steel frames. This is primarily due to the difficulty of developing an elastic formulation capable of representing accurately the geometrically nonlinear response of a whole R/C member with only one element. In order to be applicable within the proposed adaptive framework, such a formulation must be capable of modelling various typical R/C cross-sections, the tensile cracking and the nonlinear compressive response of concrete as well as the variation in the reinforcement scheme along the member length. These issues were first addressed in a novel elastic beam-column formulation for 2D R/C frames [3], paving the way for the application of adaptive methods to the nonlinear analysis of such structures [4]. However, because of the highly nonlinear nature of the generalised response of R/C cross-sections, the criteria governing the automatic refinement of elastic elements into expensive elasto-plastic elements proved to be more demanding than for steel frames. In this respect, an efficient approach, based on utilising an interaction curve between the axial force and the bending moment, was proposed to check whether a generalised cross-section stress state exceeds the elastic limit.

The application of adaptive nonlinear analysis to 3D R/C frames has been facilitated by the availability of a 3D fibre-type elasto-plastic formulation [7,8] and the recent development of an appropriate 3D elastic beam-column formulation [11], which represents a major extension of the 2D R/C beam-column formulation [3]. In the new 3D R/C beam-column formulation [11], the explicit derivation of the generalised response of typical R/C cross-sections is undertaken using a novel approach based on analytical integration over triangular sub-domains. This paper discusses the development of the remaining component of adaptive analysis for 3D R/C frames, namely the automatic refinement of elastic elements into elasto-plastic elements.

The paper begins with a description of the previously developed elastic and elasto-plastic formulations which are utilised by the proposed adaptive method for 3D R/C frames. Both formulations are derived in a local convected system, which isolates strain inducing states from rigid
body states, and utilise generic transformations between the local and global reference systems which account for the effects of large displacements and rotations in 3D space [12], thus enabling the modelling of geometric nonlinearitys. The details of the automatic refinement of elastic elements into elasto-plastic element are then discussed. This includes 1) the construction of an effective interaction surface between the axial force and biaxial moments which defines the elastic limit, 2) the determination of the generalised cross-section stress state along an elastic element, 3) the checking of a cross-section stress state for exceeding the elastic limit, and 4) the automatic subdivision of an elastic element into an appropriate number of elastic and elasto-plastic elements. Examples are finally presented, using the nonlinear analysis program ADAPTIC [7], to illustrate the accuracy and simultaneous efficiency of the proposed nonlinear adaptive analysis method for 3D R/C frames.

2. ELASTIC FORMULATION

A principal component of the proposed adaptive nonlinear analysis method is an accurate elastic formulation capable of representing a whole member using only one element. In the context of nonlinear analysis of 3D R/C frames, such a formulation must be capable of:

i. modelling various typical R/C cross-sections;

ii. representing varying reinforcement schemes along the member length;

iii. accounting for tensile cracking and the nonlinear compressive response of concrete, assuming of course that such phenomena are fully recoverable; and

iv. modelling geometric nonlinearities, including large displacements and the beam-column effect.

A new elastic formulation has been recently proposed by the authors [11] for 3D R/C frames, which extends a previous 2D formulation [3]. The proposed formulation is derived in a convected local system [12] which isolates rigid body states from strain-inducing states, and which provides a convenient reference system for the application of automatic mesh refinement. In the local system, the proposed formulation [11] uses quartic shape functions for the transverse displacements v(x) and
w(x), with the eight associated local freedoms \( \{ \theta_{y1}, \theta_{z1}, \theta_{y2}, \theta_{z2}, t_y, t_z, \Delta, \theta_T \} \) shown in Figure 1, where \( \{ \theta_{y1}, \theta_{z1}, \theta_{y2}, \theta_{z2} \} \) are local rotations at the two element nodes, \( \{ t_y, t_z \} \) are transverse displacements at mid-length relative to the element chord, and \( \{ \Delta, \theta_T \} \) are the relative axial displacement and twist rotation, respectively, of node 2 with respect to node 1. A shape function is not required for the axial displacement \( u(x) \), due to the utilisation of a constant axial force criterion, which is greatly responsible for the accuracy of the elastic formulation. This criterion is an essential equilibrium condition, which is satisfied by means of an iterative procedure on the element level [11]. Numerical integration is used for determining the element resistance forces and tangent stiffness matrix, where any number of Gauss points can be specified over the element length, although 6 Gauss points have been shown to provide sufficient accuracy.

Typical R/C cross-sections are modelled by means of dividing the cross-section into coarse rectangular areas, as illustrated in Figure 2, allowing for different concrete confinement factors (\( k_i, k_o \)) over the cross-section, where such factors are used to scale the basic stress-strain relationship of concrete in order to simulate the effects of various levels of confinement [11]. For each of these rectangular areas, explicit relationships between the three generalised cross-section stresses (axial force and two biaxial bending moments) and the three corresponding generalised strains (centroidal axial strain and two biaxial curvatures) are derived using a novel procedure based on analytical integration over triangular sub-domains [11]. The derivation is based on a linear elastic response for steel and a nonlinear compressive response with no tensile resistance for concrete, as shown in Figure 3, assuming a fully recoverable concrete response. Since the intended limit of applicability of the elastic quartic formulation is well before the crushing strain of concrete, \( \varepsilon_{ccon} \) is reached, the assumption of a constant post-crushing response is made only to facilitate the derivation and to avoid convergence problems in the iterative procedure which enforces the constant axial force criterion.
3. ELASTO-PLASTIC FORMULATION

An elasto-plastic formulation represents the second principal component of adaptive analysis. Such a formulation must account accurately for geometric nonlinearities due to large displacements and the beam-column effect, but more significantly for material nonlinearities due to the yielding of steel and the post-crushing of concrete, including the effects of strain reversal. A 3D beam-column formulation based on the fibre approach has been developed by the first author [7,8], which meets the aforementioned requirements. This formulation assumes cubic shape functions for the transverse displacements \( v(x) \) and \( w(x) \), and employs a constant centroidal axial strain criterion, with the six associated local freedoms \( \{ \theta_y1, \theta_z1, \theta_y2, \theta_z2, \Delta, \theta_T \} \) shown in Figure 4. Furthermore, this formulation utilises two Gauss points for the integration of the virtual work equation, at each of which the cross-section is divided into a number of monitoring areas where strains and stresses are evaluated, as depicted in Figure 5. It is worth noting that the elasto-plastic formulation does not utilise the internal transverse freedoms \( \{ t_y, t_z \} \) used by the previous elastic formulation, since the adopted approach for modelling the elasto-plastic structural response is based on representing a specific member with a number of basic elements instead of one higher order element. This choice is primarily in view of the fact that the latter approach would demand the use of very high order polynomial functions for all the displacement fields in order to represent the complexity of the member deformation in the presence of material plasticity effects.

The elasto-plastic cubic formulation allows for different materials within one cross-section, which is essential for modelling R/C cross-sections. Furthermore, various uniaxial material models are incorporated for steel and concrete, such as the bilinear steel model with kinematic hardening (Figure 6), and the concrete model of Karsan and Jirsa [13] (Figure 7).

As discussed in previous work by the first author [2,7,8,9], the elasto-plastic cubic formulation can be applied to the nonlinear analysis of framed structures using the conventional (non-adaptive) method, whereby a fine mesh of elements is required from the start of analysis for all structural members, so that the effects of the spread of material plasticity along the member length can be accurately represented. Consequently, the application of this conventional method to the nonlinear
analysis of 3D R/C frames can require a considerable modelling effort and can pose huge computational demands. As discussed in the following section, the nonlinear analysis method proposed here utilises the elastic and elasto-plastic formulations in an adaptive manner in order to address both modelling and computational shortcomings of the conventional approach.

4. AUTOMATIC MESH REFINEMENT

This section discusses in detail the automatic mesh refinement procedure, which is responsible for the adaptive application of the elastic and elasto-plastic elements within the structure and during analysis. This concept of nonlinear adaptive analysis is illustrated for a framed structure in Figure 8, where analysis is always started using only one elastic element per member. At the end of each equilibrium step of nonlinear analysis, elastic elements are checked for exceeding the elastic limit in pre-defined zones representing potential elasto-plastic elements. If the elastic limit is exceeded at the Gauss points of these potential elasto-plastic elements, the affected ‘parent’ elastic elements exceed their range of validity and are consequently subdivided into an appropriate number of elastic and elasto-plastic elements. This automatic process of conditional element refinement is illustrated in Figure 9 for a typical elastic element. If any of the elastic elements is subdivided, the solution corresponding to the current equilibrium step is re-established with the new mesh before continuing with the remainder of the incremental analysis procedure.

The checking of R/C cross-sections for exceeding the elastic limit poses some difficulty, since the evaluation of the generalised strains corresponding to given generalised stresses requires an iterative procedure due to the nonlinear response of the concrete material. As the check for exceeding the elastic limit must be continuously performed during analysis and at numerous locations within the elastic elements of the structure, such an iterative procedure can lead to excessive computational demands. Therefore, in line with the previous checking procedure developed for 2D R/C elastic members [2,4], an efficient approach is adopted, which utilises an interaction surface between the axial force and the two biaxial moments, that defines the elastic limit. Accordingly, interaction surfaces are determined for all employed cross-sections before the start of analysis, enabling the
check for exceeding the elastic limit to be performed simply through establishing whether the generalised stress state lies outside the governing interaction surface. It is emphasised that, in this work, the only use for such an interaction surface is in checking the applicability of an elastic element, and therefore it plays no role in representing the elasto-plastic behaviour; such behaviour is realistically modelled through the refinement of elastic elements into elasto-plastic elements which are based on uniaxial elasto-plastic material models instead of direct interaction relationships between the generalised stresses and strains.

Hereafter, the construction of an effective interaction surface defining the elastic limit for a general R/C cross-section, the determination of the cross-sectional generalised stresses along an elastic element, the check for exceeding the elastic limit, and the subdivision of elastic elements are discussed in detail.

4.1 Interaction Surface

As mentioned previously, an effective interaction surface is required which can be used to check whether a state of generalised stresses, consisting of an axial force (F) and two biaxial moments (M_y, M_z) about the cross-section geometric centroid, exceeds the elastic limit. An efficient approximate representation is proposed here, where (n_f) slices are considered in detail along the F-axis, which are equally spaced between the compressive (F^c, M_y^c, M_z^c) and tensile (F^t, M_y^t, M_z^t) limiting axial states, as illustrated in Figure 10 for (n_f=9). For each slice (i), corresponding to a specific level of axial force (F_i), (n_m) stress states are determined on the interaction surface along uniformly distributed radial directions which emanate from a common internal state, as depicted in Figure 11 for (n_m=12). This common state, (F_i^i, M_y^i, M_z^i), is conveniently chosen as the intersection of the straight line joining the two limiting axial states with the plane of slice (i) (Figure 10). Accordingly, the approximation of the interaction surface is fully defined by the two parameters (n_f) and (n_m), the two limiting axial states (F^c, M_y^c, M_z^c) and (F^t, M_y^t, M_z^t), and the (n_f*n_m) magnitudes of radial vectors (M^{i,j}). Intermediate states on the interaction surfaces are obtained by means of linear interpolation, as discussed in a later section. It is noted that the values of (n_f) and (n_m) required for an accurate representation of the interaction surface obviously depend on the configuration of the R/C cross-
section, although values of \((n_c=9)\) and \((n_m=12)\) have been found to provide a reasonable approximation for various typical cross-sections.

The construction of the proposed representation of the interaction surface for a general R/C cross-section is based on the availability of the following procedures:

- **P1:** determines the three generalised stresses \((F, M_y, M_z)\) for a given state of generalised strains \((\varepsilon_a, \kappa_y, \kappa_z)\), consisting of a centroidal axial strain and two biaxial curvatures;
- **P2:** establishes a \((3 \times 3)\) tangent modulus matrix \((k_t)\) reflecting the variation of the generalised stresses with a variation in the generalised strains;
- **P3:** determines the increment of generalised strains \((\Delta \varepsilon_a, \Delta \kappa_y, \Delta \kappa_z)\) corresponding to an increment in the uniaxial strain \((\Delta \varepsilon)\) at a specific location within the cross-section with the uniaxial strains at two other specific locations remaining constant; and
- **P4:** determines the minimum positive scaling factor \((r)\) which can be applied to an increment in the generalised strains to activate a new elastic limit constraint on the uniaxial strain. The procedure also determines the location \((y_m, z_m)\) within the cross-section where the new elastic limit constraint is activated as well as the value of the uniaxial strain \((\varepsilon_m)\) at this location.

The first two procedures (P1, P2) are implemented here by means of explicit relationships presented in a recent paper by the authors [11]. The last two procedures (P3, P4) are easily implemented through utilising the following relationship between the uniaxial strain and the generalised strains:

\[
\varepsilon = \varepsilon_a - \kappa_y y - \kappa_z z
\]

(1)

Procedure (P4) is in addition based on the evaluation and checking of the uniaxial strains \((\varepsilon)\) at the four corners of all rectangular concrete areas and in the steel reinforcement bars.

For concrete, only a compressive elastic limit constraint is employed:
\[ \varepsilon \geq -\varepsilon_{cm} \quad (2) \]

whereas for steel, tensile and compressive elastic limit constraints are necessary:

\[ -\varepsilon_{sm} \leq \varepsilon \leq \varepsilon_{sm} \quad (3) \]

In this work, the elastic limit for concrete is normally assumed to be at the crushing strain \((\varepsilon_{cm}=\varepsilon_{co})\), whereas for steel it is assumed to be at the yield strain \((\varepsilon_{sm}=\varepsilon_y)\). However, the proposed procedure can readily utilise smaller values for \(\varepsilon_{cm}\) and \(\varepsilon_{sm}\) enabling the construction of more conservative interaction surfaces, as illustrated in Section 5.1. This may be desirable particularly for concrete, since it exhibits some inelastic behaviour in the compressive range, the extent of which reduces steadily as the limit on the attained compressive strains is reduced.

Hereafter, the determination of the limiting axial states and the radial vectors for axial slice \((i)\) is discussed.

### 4.1.1 Limiting axial states

Since concrete is assumed by the elastic formulation [11] to be incapable of sustaining tensile stresses, the limiting tensile axial state is achieved when all the steel bars are at the tensile elastic limit strain of steel \((\varepsilon_{sm})\). Consequently, \((F^t, M_y^t, M_z^t)\) is determined from procedure (P1) using a uniform strain field \((\varepsilon_{sm})\) without curvatures:

\[
\begin{align*}
\varepsilon_a &= \varepsilon_{sm}, \kappa_y = 0, \kappa_z = 0 \\
&\xrightarrow{P1} (F^t, M_y^t, M_z^t)
\end{align*}
\]  

(4)

A similar situation arises in determining the limiting compressive axial state \((F^c, M_y^c, M_z^c)\) when the compressive elastic limit strain for concrete is less than or equal to that of steel \((i.e. \varepsilon_{cm} \leq \varepsilon_{sm})\). Given that reinforcement steel bars are always surrounded by concrete, \((F^c, M_y^c, M_z^c)\) in such a case is determined from a uniform strain field \((-\varepsilon_{cm})\) without curvatures:

\[
\begin{align*}
\varepsilon_a &= -\varepsilon_{cm}, \kappa_y = 0, \kappa_z = 0 \\
&\xrightarrow{P1} (F^c, M_y^c, M_z^c), \quad \text{for } (\varepsilon_{cm} \leq \varepsilon_{sm})
\end{align*}
\]  

(5)
However, the determination of \((F^c,M^c_y,M^c_z)\) becomes more involved if \((\varepsilon_{cm}>\varepsilon_{sm})\), in which case the maximum compressive axial force may be associated with biaxial curvatures. The following iterative procedure is adopted for this case:

1. Choose three initial locations \((y_k,z_k: k=1,3)\) as the corners of one of the concrete rectangular areas (Figure 2). Initialise the uniaxial strains at these locations \((\varepsilon_k=-\varepsilon_{sm})\), and set the generalised strains to \((\varepsilon_a=-\varepsilon_{sm}, \kappa_y=0, \kappa_z=0)\).

2. Establish a generalised strain state for which at least three elastic limit constraints are active, and no constraint is violated over the cross-section:

2.1. For each location \((y_k,z_k: k=1,3)\):

2.1.1. calculate the increment in generalised strains required to activate a constraint at location \((k)\) without changing the uniaxial strains at the other two locations:

\[
\Delta \varepsilon_k = \varepsilon_{sm} - \varepsilon_{cm}
\]

2.1.2. establish the first constraint to be activated due to the increment of generalised strains:

\[
\begin{pmatrix}
\Delta \varepsilon_a, \Delta \kappa_y, \Delta \kappa_z
\end{pmatrix}
\rightarrow
\begin{pmatrix}
r, y_m, z_m, \varepsilon_m
\end{pmatrix}
\]

2.1.3. store the current active constraint, and update the generalised strains:

\[
\begin{align*}
y_k &= y_m, \quad z_k = z_m, \quad \varepsilon_k = \varepsilon_m \\
\varepsilon_a &= \varepsilon_a + \Delta \varepsilon_a \\
\kappa_y &= \kappa_y + \Delta \kappa_y \\
\kappa_z &= \kappa_z + \Delta \kappa_z
\end{align*}
\]

3. Calculate the current limiting compressive axial state:
4. Establish if the relaxation of any of the current three constraints to the benefit of activating a new constraint leads to an increase in the compressive axial force:

4.1. Consider the current active constraints (k=1,3):

4.1.1. establish an increment of generalised strains associated with the relaxation of constraint (k):

\[ \Delta \varepsilon_k = -\varepsilon_k \]

4.1.2. establish the first new constraint to be activated by this increment of generalised strains:

\[ \Delta \varepsilon_a, \Delta \kappa_y, \Delta \kappa_z \]

4.1.3. if a new constraint is activated (i.e. r>0):

4.1.3.1. determine the corresponding generalised stresses:

\[ \begin{align*}
\{ \varepsilon'_a \} & = \{ \varepsilon_a \} + \{ \Delta \varepsilon_a \} \\
\{ \kappa'_y \} & = \{ \kappa_y \} + r \{ \Delta \kappa_y \} \\
\{ \kappa'_z \} & = \{ \kappa_z \} + \{ \Delta \kappa_z \}
\end{align*} \]

\[ \left\{ \begin{array}{c}
\varepsilon'_a \\
\kappa'_y \\
\kappa'_z
\end{array} \right\} \stackrel{p_1}{\longrightarrow} \left\{ \begin{array}{c}
F'_c, M'_y, M'_z
\end{array} \right\} \]

4.1.3.2. if the current compressive force is greater than the stored value (i.e. \(-F' > -F''\)), update the current limiting compressive axial state:

\[ \begin{align*}
\{ y_k \} & = \{ y_m \} \\
\{ z_k \} & = \{ z_m \}, \quad \varepsilon_k = \varepsilon_m
\end{align*} \]
and repeat from (4.1).

4.1.3.3. if the condition of (4.1.3.2) is not satisfied, continue from (4.1.1) for the next active constraint.

4.1.4. if no new constraint is activated, continue from (4.1.1) for the next active constraint.

4.2. Exit with the stored limiting compressive axial state \((F^c, M^c_y, M^c_z)\).

Clearly, the above procedure searches for the maximum compressive axial force by traversing adjacent states of generalised strains for which at least three elastic limit constraints are simultaneously active.

4.1.2 Radial vectors

For each slice \(i\) at axial force \((F^i)\), \((n_m)\) radial vectors are established, which emanate from a common internal state \((F^i, M^i_y, M^i_z)\) at a uniform incremental angle \((\alpha)\), as shown in Figure 11. The common internal state is determined from the limiting axial states according to the expression:

\[
\begin{bmatrix}
F^i \\
M^i_y \\
M^i_z
\end{bmatrix} = \begin{bmatrix}
F^c \\
M^c_y \\
M^c_z
\end{bmatrix} + \begin{bmatrix}
\frac{1-i}{n_f+1} \\
\frac{i}{n_f+1}
\end{bmatrix} \begin{bmatrix}
F^i \\
M^c_y \\
M^c_z
\end{bmatrix}
\]

(6)

The determination of the magnitude \((\overline{M}^{i,j})\) of radial vector \((j)\) associated with axial slice \((i)\) is performed according to the following procedure:
1. Establish the current generalised strains \((\varepsilon_a, \kappa_y, \kappa_z)\) and tangent modulus matrix \((k_t)\) associated with the internal state \((F^i, M^y_i, M^z_i)\) through an iterative procedure using (P1) and (P2).

2. Determine the nominal increment of biaxial moments in radial direction \((j)\):
\[
\alpha = \frac{2\pi}{n_m}, \quad \begin{bmatrix} \bar{m}_y^j \\ \bar{m}_z^j \end{bmatrix} = \begin{bmatrix} \cos(j\alpha) \\ \sin(j\alpha) \end{bmatrix}
\]

3. Calculate the corresponding nominal increment of generalised strains from the common internal state:
\[
\begin{bmatrix} \Delta \varepsilon_a \\ \Delta \kappa_y \\ \Delta \kappa_z \end{bmatrix} = k_t^{-1} \begin{bmatrix} \bar{m}_y^j \\ \bar{m}_z^j \end{bmatrix}
\]

4. Establish the first constraint to be activated due to the increment of generalised strains. Obtain a first estimate of \((\bar{M}^{i,j})\) and store the location of the active constraint:
\[
\begin{bmatrix} \Delta \varepsilon_a, \Delta \kappa_y, \Delta \kappa_z \end{bmatrix} \xrightarrow{p^4} \begin{bmatrix} \rho, y_m, z_m, \varepsilon_m \end{bmatrix}
\]
\[
\bar{M}^{i,j} = r, \quad \begin{bmatrix} y_c \\ z_c \end{bmatrix} = \begin{bmatrix} y_m \\ z_m \end{bmatrix}
\]

5. Update the generalised strains:
\[
\begin{bmatrix} \varepsilon_a \\ \kappa_y \\ \kappa_z \end{bmatrix} = \begin{bmatrix} \varepsilon_a \\ \kappa_y \\ \kappa_z \end{bmatrix} + r \begin{bmatrix} \Delta \varepsilon_a \\ \Delta \kappa_y \\ \Delta \kappa_z \end{bmatrix}
\]

6. Establish the corresponding generalised stresses \((F, M_y, M_z)\) and tangent modulus matrix \((k_t)\) using (P1) and (P2).
7. Determine the out-of-balance between the required and calculated generalised stresses:

\[
\begin{align*}
\{\Delta F\} &= \{F_i\} - \{0\} - \{F\} \\
\{\Delta M_y\} &= \{M^i_y\} + \{\overline{M^{i,j}}\} - \{M_y\} \\
\{\Delta M_z\} &= \{M^i_z\} - \{\overline{m^j_y}\} - \{M_z\}
\end{align*}
\]

8. Exit with the last value of \((\overline{M^{i,j}})\) if the out-of-balance generalised stresses are within an acceptable tolerance.

9. Establish corrective increments for \((\overline{M^{i,j}})\) and the generalised strains, ensuring that the current constraint remains active. Solve for \((\Delta \varepsilon_a, \Delta \kappa_y, \Delta \kappa_z)\) and \((\Delta \overline{M^{i,j}})\) using the following appropriate equations:

\[
\begin{align*}
\begin{bmatrix}
\Delta \varepsilon_a \\
\Delta \kappa_y \\
\Delta \kappa_z
\end{bmatrix}
&= \frac{1}{k_t}
\begin{bmatrix}
0 \\
\Delta \overline{M^{i,j}} \\
\Delta \overline{m^j_y}
\end{bmatrix}
\begin{bmatrix}
\Delta F \\
\Delta M_y \\
\Delta M_z
\end{bmatrix}
\end{align*}
\]

\[
\Delta \varepsilon_a - \Delta \kappa_y y_c - \Delta \kappa_z z_c = 0
\]

10. Establish the first new constraint to be activated by \((\Delta \varepsilon_a, \Delta \kappa_y, \Delta \kappa_z)\):

\[
\begin{bmatrix}
\Delta \varepsilon_a \\
\Delta \kappa_y \\
\Delta \kappa_z
\end{bmatrix}
\xrightarrow{p^4}
\begin{bmatrix}
r \\
y_m \\
z_m \\
\varepsilon_m
\end{bmatrix}
\]

11. If a new constraint is activated before the full application of \((\Delta \varepsilon_a, \Delta \kappa_y, \Delta \kappa_z)\) (i.e. \(r<1\)), scale down the increment and store the new constraint location:

\[
\begin{align*}
\{\Delta \varepsilon_a\} &= r \{\Delta \varepsilon_a\} \\
\{\Delta \kappa_y\} &= r \{\Delta \kappa_y\}, \quad \overline{M^{i,j}} = \overline{M^{i,j}} + r \Delta \overline{M^{i,j}} \\
\{\Delta \kappa_z\} &= \{\Delta \kappa_z\} \\
\{y_c\} &= \{y_m\} \\
\{z_c\} &= \{z_m\}
\end{align*}
\]

then repeat from (5).
12. If the condition in (11) is not satisfied, update \( \mathbf{M}^{i,j} \):

\[
r = 1, \quad \mathbf{M}^{i,j} = \mathbf{M}^{i,j} + \Delta \mathbf{M}^{i,j}
\]

and repeat from (5).

The above procedure is applied until the \((n_f \times n_m)\) magnitudes of radial vectors are determined, thus completing the representation of the interaction surface.

### 4.2 Generalised Stresses

The validity of the elastic quartic element is governed by whether the generalised stresses along the element length are within the elastic limit. For an element subjected to uniformly distributed loads, as shown in Figure 12, the determination of the generalised stresses along the element length must account for the beam-column effect, which is established here from equilibrium considerations.

Firstly, the influence of the distributed loading \((p_x, p_y, p_z)\) on the resultant axial force and biaxial moments at node (1) \((F_e^c, M_{y1}^c, M_{z1}^c)\) is evaluated in accordance with the approach proposed in [10]:

\[
M_{y1}^c = M_{y1}^o - \frac{p_y L^2}{60} \quad (7.a)
\]

\[
M_{z1}^c = M_{z1}^o - \frac{p_z L^2}{60} \quad (7.b)
\]

\[
F_{1e}^c = F^o + \frac{p_x L}{2} \quad (7.c)
\]

Rotational equilibrium about node (2) is then used to establish the resultant transverse forces at node (1):

\[
Q_{y1}^c = \frac{M_{y1}^o + M_{y2}^o}{L} - \frac{p_y L}{2} - \frac{p_x}{L} \int_{-L/2}^{L/2} v(x) \, dx \quad (8.a)
\]

\[
Q_{z1}^c = \frac{M_{z1}^o + M_{z2}^o}{L} - \frac{p_z L}{2} - \frac{p_x}{L} \int_{-L/2}^{L/2} w(x) \, dx \quad (8.b)
\]
In the above expressions, \((M_{y1}^o, M_{z1}^o, M_{y2}^o, M_{z2}^o, F^o)\) are the local element resistance forces representing the variation with respect to the associated element freedoms \((\theta_{y1}, \theta_{z1}, \theta_{y2}, \theta_{z2}, \Delta)\) of only the element strain energy (i.e. excluding the load potential energy).

Equilibrium considerations are finally employed to establish the generalised stresses along the element length:

\[
F = F_1^e - p_x \left( \frac{L}{2} + x \right) \quad \text{(9.a)}
\]

\[
M_{y1} = -M_{y1}^e + \left[ \frac{Q_{y1}^e}{2} \left( \frac{L}{2} + x \right) \left( \frac{L}{2} + x \right) + F v(x) + p_x \int_{-L/2}^{x} v(x_1) \, dx_1 \right] \quad \text{(9.b)}
\]

\[
M_{z1} = -M_{z1}^e + \left[ \frac{Q_{z1}^e}{2} \left( \frac{L}{2} + x \right) \left( \frac{L}{2} + x \right) + F w(x) + p_x \int_{-L/2}^{x} w(x_1) \, dx_1 \right] \quad \text{(9.c)}
\]

It is noted that the integrals in (8) and (9) can be expressed explicitly in terms of the local element freedoms, since the displacement integrands \(v(x)\) and \(w(x)\) are quartic polynomial functions of \(x\) scaled by these freedoms.

### 4.3 Check Against Elastic Limit

The validity of the elastic element at a specific location along its length can be established through determining the corresponding generalised stress state, as discussed in Section 4.2, and relating it to the interaction surface representing the elastic limit, as described in Section 4.1. The chosen representation of the interaction surface has particular advantages over the interaction curve previously employed in 2D adaptive analysis of R/C frames [4], in the sense that no iterations are required for determining the relative location of the stress state with respect to the interaction boundary.
The proposed procedure for checking a generalised stress state against the elastic limit is based on identifying the relevant biaxial moment wedge of the interaction surface, and determining the relative location of the stress state, as illustrated in Figure 13.

The aforementioned procedure consists of the following steps:

1. Establish the factor \( (r_f) \) representing the position of the current state along the \( F \)-axis of the interaction surface:
   \[
   r_f = \frac{F - F^c}{F_t - F^c}
   \]

1.1. If \( r_f \leq 0 \) or \( r_f \geq 1 \), indicate elastic limit exceeded; exit.

2. Determine the two slices \( (i) \) and \( (j) \) bounding the current state:
   \[
   i = \text{int} (r_f), \quad j = i + 1
   \]

3. Determine the magnitude of the radial vector to the current state \( (\overline{M}^f) \) and the corresponding interaction wedge defined by radial position indexes \( (a) \) and \( (b) \):
   \[
   \begin{align*}
   \left\{ \begin{array}{l}
   M_y^f \\
   M_z^f
   \end{array} \right\} &= \left\{ \begin{array}{l}
   M_y^i \\
   M_z^i
   \end{array} \right\} + (r_f - i) \left\{ \begin{array}{l}
   M_y^i - M_y^i \\
   M_z^i - M_z^i
   \end{array} \right\} \\
   \left\{ \begin{array}{l}
   \overline{M}_y^f \\
   \overline{M}_z^f
   \end{array} \right\} &= \left\{ \begin{array}{l}
   M_y - M_y^f \\
   M_z - M_z^f
   \end{array} \right\}, \quad \overline{M}^f = \sqrt{\overline{M}_y^f \overline{M}_y^f + \overline{M}_z^f \overline{M}_z^f}
   \end{align*}
   \]
   \[
   \gamma = \tan^{-1} \left( \frac{\overline{M}_z^f}{\overline{M}_y^f} \right) \pm \pi, \quad \alpha = \frac{2\pi}{n_m}
   \]
   \[
   a = \text{int} (\gamma / \alpha), \quad b = a + 1, \quad \beta = \gamma - a \alpha
   \]
   and finally if \( a=0 \) set \( a=n_m \).

4. Establish the magnitudes of the interaction radial vectors \( (\overline{M}^{f,a}, \overline{M}^{f,b}) \) from the corresponding values \( (\overline{M}^{i,a}, \overline{M}^{i,b}) \) and \( (\overline{M}^{j,a}, \overline{M}^{j,b}) \) for slices \( (i) \) and \( (j) \), respectively, using
linear interpolation. If \(i=0\) of \(j=n\) use zero magnitudes for the corresponding radial vectors.

5. Use the following inequality to establish whether the elastic limit is exceeded (Figure 13):

\[
\left[ \overline{M}_f s^\beta (\overline{M}_f^{b,c} - \overline{M}_f^{a}) - \overline{M}_f^{b,s} (\overline{M}_f^{c} - \overline{M}_f^{f,a}) \right] \leq 0 \Rightarrow \text{Elastic limit exceeded}
\]

where,

\[ s^\alpha = \sin(\alpha), \quad c^\alpha = \cos(\alpha), \quad s^\beta = \sin(\beta), \quad c^\beta = \cos(\beta) \]

### 4.4 Element Refinement

Upon detection of generalised stress states exceeding the elastic limit along affected elastic elements, these elements are automatically refined into elastic and elasto-plastic elements, as appropriate, before resuming the nonlinear analysis procedure. In performing this refinement, several characteristics of a ‘parent’ elastic element must be inherited by its new replacement elements. These include element connectivity, cross-section details, material modelling, current values of nodal displacements, and applied distributed loading. Clearly, the refinement of an elastic element into elasto-plastic elements of a different type poses logistic programming issues which require careful consideration. More detail on the main requirements in this regard can be found in other related works by the first author [2,4,7,8,9].

### 5. EXAMPLES

The proposed adaptive method for 3D R/C frames has been implemented in the nonlinear structural analysis program ADAPTIC v2.7.3 [7]. Examples are provided hereafter, using ADAPTIC on a Silicon Graphics Indigo (R4000) workstation with 112 Mb of physical memory, which verify the accuracy of the proposed adaptive method, and which illustrate its superiority over conventional nonlinear analysis of 3D R/C frames.
5.1 Interaction Surfaces

The two R/C cross-sections, shown in Figure 14, are utilised in the 3D multi-storey frame example presented in Section 5.2. Interaction surfaces representing the elastic limit for the two cross-sections are determined according to the procedure proposed in Section 4.1. Whereas the elastic limit for steel is taken as \( \epsilon_{\text{sm}} = \epsilon_y = \frac{\sigma_y}{E} \), two alternative values for the elastic limit of concrete are considered, \( \epsilon_{\text{cm}} = \epsilon_{\text{co}} \) and \( \epsilon_{\text{cm}} = 0.3 \epsilon_{\text{co}} \), assuming uniform confinement factors \( k_i = k_o = 1 \) (Figure 2). As noted previously, conservative interaction surfaces based on a reduced elastic limit for concrete can be utilised by the proposed adaptive method to reduce inaccuracies related to the inelastic response of concrete at high compressive strains. Such inaccuracies can be significant when concrete is subjected to strain reversals in the range of compressive strains close to \( \epsilon_{\text{co}} \), for example due to severe dynamic loading. The influence of using conservative interaction surfaces on the adaptive procedure, however, would be to initiate the automatic mesh refinement of elastic elements at an earlier stage, thus leading to an increase in the computational demand.

The components of the interaction surface for the column cross-section are shown in Figures 15.a and 15.b for the two concrete elastic limits \( \epsilon_{\text{cm}} = \epsilon_{\text{co}} \) and \( \epsilon_{\text{cm}} = 0.3 \epsilon_{\text{co}} \), respectively, assuming \( n_i = 9 \) component slices and \( n_m = 12 \) radial vectors per slice. Each component slice \( i = 1, n_i \) is plotted with the origin taken at the common internal state \( (F_i, M_{yi}, M_{zi}) \), given in terms of the limiting axial states by (6). The results for the two limiting axial states, \( (F^c, M_{yc}, M_{zc}) \) and \( (F^t, M_{yt}, M_{zt}) \), indicate that both do not involve bending moment components, as expected due to the biaxial symmetry of the column cross-section. Furthermore, while the limiting compressive axial state and the size of the interaction slices are affected by the concrete elastic limit, the limiting tensile axial state is independent of such a limit. It is also worth noting that, for both concrete elastic limits, the interaction slices in the vicinity of the limiting axial states, \( i = 1 \) and \( i = 9 \), are almost linear; this is expected, since the cross-section response over the small range of allowable increments of generalised strains in these regions is adequately represented with first order approximation of the concrete stress-strain relationship. Finally, for both concrete elastic limits, the interaction surfaces are symmetric about the two biaxial moment axes as well as about the diagonal, which is due to corresponding geometric symmetries in the column cross-section. However, it is observed that some
interaction slices for the smaller limit ($\epsilon_{ci}=0.3\epsilon_{co}$) are not convex; this is attributed to the nonlinear material response of concrete, particularly in connection with the piecewise response from the tensile to the compressive range.

The components of the interaction surface for the beam cross-section are shown in Figures 16.a and 16.b for the two alternative concrete elastic limits. In contrast to the case of the column cross-section, the limiting axial states now include non-zero bending moment terms about the major axis. Furthermore, the mono-symmetry of the cross-section is reflected in a single axis of symmetry for the interaction slices. However, apart from these differences, similar comments to the case of the column cross-section can be made for the beam cross-section regarding the shape of the interaction slices.

5.2 Multi-Storey 3D R/C Frame

The multi-storey 3D R/C frame, depicted in Figure 17, is symmetric about its central X-Z and Y-Z planes as well as the two bisecting planes. The cross-sectional details of the columns are given in Table 1 with reference to the grid position and level, where three square cross-sections are used with the steel reinforcement uniformly distributed around the perimeter at a cover distance of 25 mm (e.g. Figure 14). On the other hand, only two different cross-sections are used for the beams, as given in Table 2, with a varying reinforcement scheme along the length according to Figure 18. The positioning of the steel reinforcement over the cross-section is illustrated for one beam cross-section in Figure 14. The bilinear model for steel (Figure 6) and the nonlinear compressive model for concrete (Figure 7) are used, where the material properties are as given in Figure 14.

<table>
<thead>
<tr>
<th>Grid reference</th>
<th>Level</th>
<th>Dimensions (mm × mm)</th>
<th>Reinforcement</th>
</tr>
</thead>
<tbody>
<tr>
<td>B2</td>
<td>1</td>
<td>350 × 350</td>
<td>16Φ18</td>
</tr>
<tr>
<td>B2</td>
<td>2, 3</td>
<td>300 × 300</td>
<td>16Φ14</td>
</tr>
<tr>
<td>All other columns</td>
<td></td>
<td>300 × 300</td>
<td>8Φ14</td>
</tr>
</tbody>
</table>

Table 1. Dimensions and reinforcement for columns
The 3D frame is provided with full translational and rotational supports, and is subjected to initial gravity loading which is uniformly distributed on the beams, as given in Table 2. Two cases of proportional horizontal loading, varying according to a single load factor ($\lambda$), are considered, as shown in Figure 19, both of which are applied in the presence of the initial gravity loading. Load case (A) consists of horizontal forces applied in the X direction at levels 3 to 6, whereas load case (B) consists of a further set of horizontal forces in the Y direction; any two forces applied at a particular level in a specific direction are equal, with the sum as depicted in Figure 19.

The 3D R/C frame is analysed for both load cases using the conventional nonlinear analysis method and the proposed adaptive method. The conventional structural model is based on a mesh of 10 elasto-plastic cubic elements per member, each utilising around 300 monitoring areas per Gauss integration cross-section. The adaptive structural model is based on an initial mesh of 1 elastic quartic element per member, each employing 6 Gauss integration points. In order to maintain equivalence between the conventional and adaptive models, all elastic elements are checked for exceeding the elastic limit in 10 pre-defined zones, each of which is replaced by an elasto-plastic element when the elastic limit is exceeded. The interaction surfaces representing the elastic limit for

### Table 2. Dimensions, reinforcement and initial gravity loading for beams

<table>
<thead>
<tr>
<th>Grid reference</th>
<th>Level</th>
<th>Dimensions (mm-mm)</th>
<th>Reinforcement D (mm)</th>
<th>Loading (kN/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1→B1, A1→A2</td>
<td>1</td>
<td>300×600</td>
<td>14</td>
<td>15</td>
</tr>
<tr>
<td>B1→B1, A2→B2, A2→A2, B1→B2</td>
<td>3</td>
<td>300×600</td>
<td>14</td>
<td>15</td>
</tr>
<tr>
<td>B2→B2</td>
<td>6</td>
<td>300×600</td>
<td>14</td>
<td>15</td>
</tr>
<tr>
<td>B1→B1, A2→A2</td>
<td>1, 2</td>
<td>300×600</td>
<td>14</td>
<td>30</td>
</tr>
<tr>
<td>A2→B2, B1→B2</td>
<td>2</td>
<td>300×600</td>
<td>14</td>
<td>30</td>
</tr>
<tr>
<td>B2→B2</td>
<td>4, 5</td>
<td>300×600</td>
<td>14</td>
<td>30</td>
</tr>
<tr>
<td>A2→B2, B1→B2</td>
<td>1</td>
<td>300×600</td>
<td>18</td>
<td>45</td>
</tr>
<tr>
<td>B2→B2</td>
<td>3</td>
<td>300×600</td>
<td>18</td>
<td>45</td>
</tr>
<tr>
<td>B2→B2</td>
<td>1, 2</td>
<td>300×600</td>
<td>18</td>
<td>60</td>
</tr>
</tbody>
</table>
all cross-sections featuring in this example are determined prior to the nonlinear analysis using the default elastic limits for steel ($\varepsilon_{sm}=\varepsilon_y$) and concrete ($\varepsilon_{cm}=\varepsilon_{co}$). It is important to note that the pre-processing computational demand of the adaptive model, which includes the cost of constructing the interaction surfaces, is less than 5sec of CPU time, which is negligible in comparison with the computational costs of nonlinear analysis.

5.2.1 Load case (A)

In order to trace the equilibrium path for the 3D frame under load case (A) in the pre- and post-ultimate ranges, load and displacement control strategies are successively applied. A load factor increment ($\Delta \lambda=0.5$) is first applied in 10 equal steps, which is then followed by an increment of 200 mm in the top floor X displacement applied in 20 equal steps, thus resulting in 30 overall incremental steps.

The load-displacement response of the 3D frame obtained using the conventional and adaptive methods is shown in Figure 20, where excellent agreement is observed. The very slight discrepancy which arises after a top floor displacement of 200 mm is attributed to strain reversals in the concrete which are assumed to be fully recoverable by the elastic elements. The axial force and bending moments at the lower ends of two columns on level (4) are shown in Figures 21.a-b; column 1 is on the same side of the applied loading, whereas column 2 is on the opposite side. Again, excellent agreement is observed between the results of conventional and adaptive analysis, with a very slight discrepancy in the bending moment of column 1 arising after a top floor displacement of 200 mm.

The deflected shapes and the progress of mesh refinement into elasto-plastic elements, depicted by means of thicker lines, are shown in Figures 22.a-c for incremental steps 10, 20 and 30, respectively. As expected, the deflected shapes and the refined meshes are all symmetric about the central X-Z plane due to the structural and loading symmetry about that plane. Furthermore, it is observed that elasto-plastic elements are required only in very few locations, particularly in floors 4 and 5. This implies that conventional nonlinear analysis based on an initially fine mesh of expensive elasto-plastic elements for all the frame members would pose a huge but unnecessary computational demand.
Consideration of the CPU time demand, shown in Figure 23.a, demonstrates the superiority of the proposed adaptive analysis method over the conventional method. Whereas conventional analysis requires 1hr 40min 46sec of CPU time, adaptive analysis is undertaken with a CPU time of only 9min 59sec, representing an overall saving with adaptive analysis of 90%. In fact, even greater computational savings, extending to 97%, can be achieved with the adaptive method if the nonlinear analysis is terminated at an earlier stage, as illustrated in Figure 23.b; this is due to the lower rate of CPU demand by the adaptive method over the early steps for which fewer elasto-plastic elements are needed. It is also worth noting that conventional analysis poses considerable memory demands, which affects adversely the real elapsed time for computations when the available physical memory is exceeded. For this example, the memory requirements of conventional analysis exceed the amount afforded by a workstation with 112 Mb of physical memory, thus leading to a substantial elapsed time of 6hr 52min 44sec. On the other hand, the elapsed time for adaptive analysis is only 10min 3sec, which represents an overall saving over conventional analysis of 97.6% in the required elapsed time.

Finally, the very slight discrepancy between the results of adaptive and conventional analysis, which arises after a top floor displacement of 200 mm, can be readily addressed with the adaptive method through using a more conservative elastic limit for concrete. Identical results between adaptive and conventional analysis are obtained when a reduced elastic limit for concrete ($\varepsilon_{cm}=0.3 \varepsilon_{co}$) is used. Since this implies an earlier initiation of automatic mesh refinement in comparison with adaptive analysis based on the original elastic limit ($\varepsilon_{cm}=\varepsilon_{co}$), the CPU and elapsed times of adaptive analysis increase to 28min 7sec and 28min 17sec, respectively. This increase, however, still represents considerable overall savings of 72.1% and 93.1% in CPU and real elapsed times, respectively, over the conventional analysis method.

5.2.2 Load case (B)

As for load case (A), load and displacement control strategies are successively applied for load case (B) in order to trace the equilibrium path in the pre- and post-ultimate ranges. A load factor increment ($\Delta \lambda=0.45$) is first applied in 9 equal steps, which is then followed by an increment of 150
mm in the top floor X displacement applied in 15 equal steps, thus resulting in 19 overall incremental steps.

The load-displacement response of the 3D frame obtained from conventional and adaptive analysis is shown in Figure 24 for the top floor X and Y displacements, where very good agreement is observed. A small discrepancy arises after a top floor X displacement of 150 mm, which is again attributed to strain reversals in the concrete at high compressive strains.

The deflected shapes and the progress of mesh refinement into elasto-plastic elements are shown in Figures 25.a-b for incremental steps 9 and 19, respectively. In this case, the deflected shapes and the refined meshes are no longer symmetric about the central X-Z plane, due to the absence of such symmetry from the loading. However, as with load case (A), it is observed that elasto-plastic elements are required only in very few locations.

The CPU time demand, shown in Figure 26.a, demonstrates again the superiority of the proposed adaptive analysis method over the conventional method. Whereas conventional analysis is undertaken with 1hr 52min 1sec of CPU time, adaptive analysis requires a CPU time of only 10min 18sec, representing an overall saving with adaptive analysis of 90.8%. As with load case (A), greater computational savings, extending to 97%, can be achieved with the adaptive method if the nonlinear analysis is terminated at an earlier stage, as illustrated in Figure 26.b. In terms of real elapsed time, conventional analysis requires 7hr 49min 17sec, whereas adaptive analysis requires only 10min 20sec, representing an overall saving over conventional analysis of 97.8%.

Finally, the small discrepancy between the results of adaptive and conventional analysis, which arises after a top floor X displacement of 150 mm, is completely eliminated when a reduced elastic limit for concrete ($\varepsilon_{cm}=0.3\varepsilon_{cm}$) is used. While this effects an increase in the CPU and elapsed times of adaptive analysis to 29min 41sec and 29min 53sec, respectively, this increase still represents respective overall savings of 73.5% and 93.6% over conventional analysis.
6. CONCLUSIONS

This paper extends the applicability of adaptive nonlinear analysis to 3D R/C frames, incorporating the effects of geometric and material nonlinearities. The proposed adaptive analysis method utilises two component formulations and an automatic mesh refinement procedure. The first component formulation is an inexpensive elastic element capable of modelling the recoverable response of R/C beam-columns with a varying reinforcement scheme using only one element per member. The second component formulation is an elasto-plastic element of the fibre type capable of modelling the elasto-plastic response of R/C beam-columns, including the effect of strain reversal. This type of element can be utilised in conventional nonlinear analysis, albeit at the cost of a fine mesh of elements per member which can render the nonlinear analysis prohibitively expensive. The automatic mesh refinement procedure is based on practical, rather than error estimation, considerations, and is applied to every elastic element when the elastic limit is exceeded in certain pre-defined zones along the element length. The proposed adaptive analysis method addresses the modelling and computational disadvantages of conventional analysis by starting the analysis with only one elastic element per member; after each equilibrium step, elastic elements are checked for exceeding the elastic limit and are refined as necessary into elasto-plastic elements, before proceeding to the next incremental step.

The paper provides a brief description of the two component, elastic and elasto-plastic, formulations of adaptive analysis. The details of automatic mesh refinement are then presented, including 1) the construction of an effective interaction surface for the elastic limit, 2) the determination of generalised stress states along the length of an elastic element, 3) the checking of a generalised stress state for exceeding the elastic limit, and 4) the subdivision of an elastic element into an appropriate number of elastic and elasto-plastic elements. A key feature of the proposed adaptive method is related to the checking of generalised stress states for exceeding the elastic limit by means of an interaction surface which can be used efficiently without the need for iteration. Furthermore, it is suggested that conservative interaction surfaces, based on a reduced elastic limit for concrete, can be used to limit any inaccuracies arising from the necessary assumption for the elastic formulation of a
fully recoverable concrete response in the compressive range. Such a device, however, should be expected to increase the computational demand of adaptive analysis, although the superiority over conventional analysis should be maintained.

The paper finally presents a multi-storey 3D R/C frame example, using the nonlinear analysis program ADAPTIC, which illustrates the benefits of the adaptive analysis method. With obvious modelling advantages, arising from a minimal initial mesh of only 1 element per member, the adaptive method is shown to provide substantial computational savings over the conventional method, exceeding 90% in CPU time, with only a very minor loss in accuracy. The use of more conservative interaction surfaces for the elastic limit of concrete eliminates the small loss in accuracy while still achieving savings in CPU time which exceed 70%. However, in both cases, adaptive analysis is shown to provide savings in excess of 90% in the real elapsed time over conventional analysis for which the memory requirements are excessive.

REFERENCES


Figure 1. Local freedoms of 3D elastic quartic formulation
Figure 2. Elastic R/C section decomposed into rectangular concrete areas and reinforcement
Figure 3. Recoverable stress-strain relationship for concrete
Figure 4. Local freedoms of elastoplastic cubic formulation
Figure 5. Monitoring areas for elasto-plastic R/C section
Figure 6. Bilinear model for steel with kinematic hardening
Figure 7. Stress-strain relationship for concrete
Figure 8. Automatic mesh refinement applied to a frame
Before refinement:
1 elastic element

After refinement:
2 elastic elements
2 elasto-plastic elements

Figure 9. Refinement of a typical elastic element
Figure 10. Representation of interaction surface (n_i=9)
Figure 11. Representation of slice (i) ($n_m=12$)
Figure 12. Elastic element subject to distributed loading
Figure 13. Biaxial moment interaction wedge
Steel
\[ E = 210 \times 10^3 \text{ N} / \text{mm}^2 \]
\[ \sigma_y = 275 \text{ N} / \text{mm}^2 \]
\[ \mu = 1\% \]

Concrete
\[ f_c = 30 \text{ N} / \text{mm}^2 \]
\[ \varepsilon_{co} = 0.002 \]
\[ k_1 = k_o = 1 \]

Figure 14. Geometric and material properties of column and beam cross-sections
$F^c = -3056, \ F^i = 678 \ (\text{kN})$

$M^c_y = M^i_y = 0 \ (\text{kN.m})$

$M^c_z = M^i_z = 0 \ (\text{kN.m})$

Figure 15.a. Interaction surface components for column cross-section: $e_{cm} = e_{co}$
Figure 15.b. Interaction surface components for column cross-section: $e_{cm} = 0.3 \cdot e_{co}$
Figure 16.a. Interaction surface components for beam cross-section: \( \varepsilon_{cm} = \varepsilon_{co} \)
Figure 16.b. Interaction surface components for beam cross-section: $e_{cm} = 0.3 \, e_{co}$
Figure 17. Geometric configuration of multi-storey 3D R/C frame
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Figure 22.e. Deflected shape and mesh refinement: Load case (A) - Step 30; (Scale = 5)
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Figure 23.b. CPU time savings of adaptive analysis: Load case (A)
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Figure 25.b. Deflected shape and mesh refinement: Load case (B) - Step 19; (Scale = 5)
Figure 26.a. CPU time demand: Load case (B)
Figure 26.b. CPU time savings of adaptive analysis: Load case (B)