Spatial organisation of low-frequency unsteadiness in oblique shock wave – boundary layer interactions

Paige Rabey
Imperial College London, Department of Aeronautics

Submitted for the PhD degree of Imperial College London and the Diploma of Imperial College
1st April 2020
This page is intentionally left blank.
I declare that the contents of this thesis is my own work and is appropriately referenced where not.

The copyright of this thesis rests with the author. Unless otherwise indicated, its contents are licensed under a Creative Commons Attribution-Non Commercial 4.0 International Licence (CC BY-NC). Under this licence, you may copy and redistribute the material in any medium or format. You may also create and distribute modified versions of the work. This is on the condition that: you credit the author and do not use it, or any derivative works, for a commercial purpose. When reusing or sharing this work, ensure you make the licence terms clear to others by naming the licence and linking to the licence text. Where a work has been adapted, you should indicate that the work has been changed and describe those changes. Please seek permission from the copyright holder for uses of this work that are not included in this licence or permitted under UK Copyright Law.
This page is intentionally left blank.
Acknowledgements

I would firstly like to express my genuine and deep gratitude to my supervisor, Paul Bruce, for providing me with such sound academic and personal support. I always appreciated his patience, the (probably too much) faith that he had in me, and his concern for my development.

Secondly, I would like to thank my grandparents, Jim and Peggy Paget, for instilling good East End values in me, such as working hard and always being lucky. Also my grandad, Tony Rabey, for always taking a keen interest in my career path and making me feel that I was on a special one. I will always be sorry I did not finish soon enough to share this with all of you.

I cannot look back over my PhD without also including the MRes year with my colleagues on the Fluids CDT. They made this foundation year immensely enjoyable and I would not have continued onto the PhD had it not been for this. Thank you Laura, Emma, Raquel, Marco, Alejo, Owen, Aran, David and Steven.

Also during this time and since I am incredibly grateful to Davie, for being there unconditionally, for culturing me by showing me the artistic side of London, and for reminding me of the perception of my work from outside of the Imperial bubble.

During the PhD itself, I am primarily thankful for all of the help and social engagement from the Department of Aeronautics technical staff. To Ian James, for always saying ‘funny machine, this’ before fixing the compressor for the wind tunnel; to Ian Pardew, for always engaging in chats about football in addition to fitting small jobs for me into his busy schedule; to Roland Hutchins, for doing his best to accommodate my incompetence; to Mark Grant, for taking the time on many occasions to help me with slightly unconventional requests; and to Franco Giammaria, for teaching me about electronics in the proper way.

Lastly I would like to thank Henrique, for his support and encouragement during a drawn-out and challenging write-up stage. Without this I would have been over-whelmed and unable to remain positive, thanks for being a true partner.
This page is intentionally left blank.
Abstract

This thesis examines the spatial organisation of low-frequency unsteadiness of incident – reflected shock – boundary layer interactions (SBLIs). Such SBLI unsteadiness continues to be an active topic of research due to its detrimental effects on supersonic engine intakes, for example. Recent studies have investigated the influence of duct geometry on the topology of the shock-induced separation bubble. This thesis aims to unite these two topics in order to investigate the interdependence of the low-frequency unsteadiness and the three-dimensional separation bubble shape and to contribute to the understanding of the low-frequency dynamics.

In this study, experiments have been undertaken on eight test cases in the Imperial College blow-down supersonic wind tunnel with a freestream Mach number of two. Three different interaction strengths were achieved by turning the flow $12^\circ$, $10^\circ$ and $8^\circ$. Each strength interaction was examined at three different effective aspect ratios $AR = 1$, $AR = 1.3$ and $AR = 2$, with the exception of the $12^\circ$ for which the latter was not examined. The separation regions were measured by oil flow visualisation and the unsteadiness was inferred from pressure fluctuation measurements under the separation shock across the full span of the wind tunnel floor.

The obtained measurements provide the first experimentally-measured two-dimensional maps that capture the unsteady characteristics beneath an SBLI. The maps show that low-frequency motion exists across the entire span of the separation shock and is more prevalent where earlier separation occurs. Similar low-frequency unsteadiness is observed in the corner separations and it appears that the interaction, including the central and corner separation bubbles and the separation shock, moves as a whole entity. This might be driven by the pressure gradient across the incident shock rather than by the upstream boundary layer or separation bubble. The driving force of the incident shock strength appears to dictate the characteristic frequency and the product of the amplitude of shock motion and separation volume.
Contents

List of Figures 10

List of Tables 14

1 Introduction 15

2 Literature Review 17
   2.1 Shock – Boundary Layer Interactions 18
   2.2 Unsteadiness in SBLIs 20
      2.2.1 Introduction 20
      2.2.2 Upstream 21
      2.2.3 Downstream 27
      2.2.4 Both 30
      2.2.5 Three-dimensional studies 32
      2.2.6 Three-dimensionality in the mean 33
      2.2.7 Three-dimensionality and unsteadiness 41
   2.3 Resulting Aims 45

3 Experimental facility 47
   3.1 Imperial College Supersonic Wind Tunnel 48
   3.2 Experimental methods 50
      3.2.1 Oil flow visualisation 50
      3.2.2 High-frequency pressure measurements 50
   3.3 Data processing and error estimation 55
      3.3.1 Oil flow visualisation 55
      3.3.2 High-frequency pressure measurements 55
      3.3.3 Uncertainty estimations 57

4 Global characteristic flow organisation 61
   4.1 Separation regions 62
      4.1.1 Introduction 62
      4.1.2 Characterisation of central separations 64
      4.1.3 Characterisation of corner separations 66
      4.1.4 Characterisation of core separations 67
   4.2 Centreline PSDs 71
4.2.1 Introduction ................................... 71
4.2.2 Characterisation of most powerful PSDs ................... 74
4.2.3 Centreline Strouhal number estimates .......................... 78
4.3 Proposed model ..................................... 82
4.4 Summary ........................................ 85

5 Spatial variation of low-frequency unsteadiness 87
5.1 Introduction ....................................... 88
5.2 Spatial variation in PSDs .................................. 91
  5.2.1 Variation of characteristic frequency and power ............... 91
  5.2.2 Variation of statistical moments ............................. 101
5.3 Spatial variation in Strouhal number ............................ 107
5.4 Temporal correlations .................................... 113
  5.4.1 Autocorrelations ........................................ 113
  5.4.2 Cross-correlations ...................................... 118
5.5 Summary ........................................ 123

6 Conclusions ........................................ 125
  6.1 Relationship between separation length and low-frequency unsteadiness ...... 125
  6.2 Overall comparison of low-frequency PSDs of interactions with different sepa-
      ration regions ............................................. 126
  6.3 Inter-dependence of separation and low-frequency unsteadiness ............ 126
  6.4 Three-dimensional unsteadiness characteristics ........................... 127
  6.5 Temporal flow physics .................................... 127
  6.6 Future work ....................................... 128

Bibliography ........................................ 129
List of Figures

2.1 Boundary layer division into triple deck theory. .......................... 18
2.2 Effect of viscosity on pressure distribution under an incident – reflected SBLI. 18
2.3 Change in flow properties and shock behaviour with Mach number. 19
2.4 Schematic of separated incident – reflected SBLI. ...................... 19
2.5 Conditional upstream boundary layer properties. .......................... 22
2.6 Examples of reconstructed spatial data from PLS at $y/\delta = 0.2$. 23
2.7 Velocity fields conditionally averaged on the centreline separation point location. 23
2.8 Weighted PSDs of wall-pressure near reflected shock foot from LES models and experiments. .................................................. 24
2.9 Instantaneous volumetric ($0.1\delta < y < 0.6\delta$) representations of the velocity isosurfaces for $0.9U_\infty$ (red), $0.75U_\infty$ (green) and $0.55U_\infty$ (blue) near $M = 2.1$ incident – reflected $8^\circ$ interaction. 25
2.10 Cross-correlations between boundary layer momentum, shock position and bubble size of $M = 2$ incident – reflected $8^\circ$ interaction. .......................... 26
2.11 Streamlines in separation region of $M = 2.3$ $9.5^\circ$ interaction. 27
2.12 Pressure signals of adjacent transducers $9.5\text{mm}$ apart in the streamwise direction. 28
2.13 Conditional RMS (root mean square) of fluctuations of vertical velocity at four distances from the floor for a $9.5^\circ$ incident – reflected SBLI. 28
2.14 Distribution of characteristic frequency with different normalisations. 29
2.15 Comparison of PSD and modes from DMD. ................................. 30
2.16 Real and imaginary parts of DMD modes $\phi_1$ (top) and $\phi_2$ (bottom) with pressure fluctuations. .................................................. 30
2.17 Range and amplitude of separation shock for compression corners of varying strengths studied by Dolling & Or. .............................. 31
2.18 Proposed variation in separation region length with tunnel ‘viscous aspect ratio’. 34
2.19 Normalised centreline separation as a function of viscous aspect ratio. 35
2.20 Time-averaged density gradient fields of a $M = 2.7$ $9^\circ$ incident – reflected SBLI. 36
2.21 Pressure contours of $A_R = 2$ case at $y = 20\text{mm}$. .................... 37
2.22 Pressure distribution through different aspect ratio interactions showing separation and reattachment points defined as $c_f = 0$. ............... 37
2.23 Variation of separation length with $L = x_{\text{exp}} - x_{\text{imp,inv}}$. .............. 37
2.24 Schlieren photographs of Mach 2 $12^\circ$ reflected interactions with varying sidewall gaps. .................................................. 38
2.25 Previous studies of separation length as a function of viscous aspect ratio $\delta/W$. 39
2.26 Proposed model of incident oblique shock three-dimensionality. 39
2.27 Example of potential impact of confinement on reduced unsteadiness. 40
2.28 Mean skin friction evolution (black) with local minima and maxima shown in grey for long integration time $380\delta/U_\infty$ and black dots for $446\delta/U_\infty$. 42
2.29 DMD modes of skin friction fluctuations in $x-z$ plane. 43
2.30 Weighted power spectral density across span of separation shock for different Strouhal numbers for a Mach 2.7 $9^\circ$ incident – reflected SBLI. 44
2.31 Examples of asymmetry in separation regions. 44
2.32 Photographs of wind tunnel and real-world geometry. 46
3.1 Schematic diagram of wind tunnel. 48
3.2 CAD representation of experimental set-up with nearest sidewall cutaway. 49
3.3 Example pressure history over ten channels recorded simultaneously at different spanwise positions $Z^*$. 51
3.4 Sensor mount. 52
3.5 Diagram showing measurement locations (dashed lines) relative to predicted mean reflected shock location (solid lines) and expected range (arrows) of separation shock motion. 53
3.6 Sensor mounting arrangement in ‘A’ location. 53
3.7 Effect of varying aspect ratio on $10^\circ$ $\mathcal{A} = 1$ interaction. Full schlieren image. 54
3.8 Effect of varying aspect ratio on $10^\circ$ $\mathcal{A} = 1$ interaction. Schlieren image zoomed-in on SBLI. 54
3.9 Effect of varying window size on PSDs (50% overlap). 56
3.10 Effect of varying window overlap on PSDs ($2^{14}$ window size). 57
4.1 Oil flow images. Reattachment lines are superimposed and sensor measurement locations are shown by to-scale circles. Crosses indicate critical points: magenta - foci, cyan - saddle points, green - dispersion nodes. 63
4.2 Calculation of p.d.f.s of separation lengths in central separation regions. Leftmost plots show the separation lengths used. Centre plots show the variation in separation length across the span, white bars show the variation due to the reattachment line. Rightmost plots show the resulting histograms. 65
4.3 P.d.f.s of length scales across separation regions. 66
4.4 Schlieren images with pressure measurement points and $h_{sep,max}$ estimates in mm. 68
4.5 Schlieren images with pressure measurement points and $h_{sep,max}$ estimates in mm. 69
4.6 Example of $V_{sep}$ calculation for $12^\circ$ $\mathcal{A} = 1.3$ test case. 70
4.7 Variation of length measures for different test cases. 70
4.8 Centreline PSDs. Solid lines are at $Z^* = 0.02$, dashed lines at $Z^* = -0.02$. Inset figures show corresponding streamwise development of mean centreline pressure intermittency. Error bars show values at $Z^* = 0.02$ and $Z^* = -0.02$. 71
4.9 Above: pre-multiplied PSD distribution in $X^*$ averaged over $Z^* = \pm 0.02$. Contours scale logarithmically from $10^{-4.5}$ to $10^{-1.2}$. Below: corresponding streamwise variation in standard deviation of pressure fluctuations averaged over $Z^* = \pm 0.02$. Error bars show values at $Z^* = 0.02$ and $Z^* = -0.02$. ...................................... 73
4.10 PSDs with peak low-frequency power recorded for each interaction pre-multiplied by absolute frequency. ........................................... 74
4.11 PSDs with peak low-frequency power recorded for each interaction pre-multiplied by reduced frequency. ........................................... 75
4.12 Most energetic pre-multiplied PSDs with statistical moments. .................. 76
4.13 Comparison of statistical moments of $L_{sep}$ and $p'$. ............................ 77
4.14 Proposed relation of separation region shape (on left) and resulting p.d.f. of separation lengths (centre) to PSD shape (on right). ........................... 78
4.15 Centreline PSDs normalised by centreline separation length. .................. 79
4.16 Effect of different length scales on Strouhal numbers. ............................ 81
4.17 Proposed spring – mass model of incident – reflected SBLIs. ................... 82

5.1 Standard deviations of pressure fluctuations normalised by mean pressure. . . . 88
5.2 Temporal mean pressure distributions overlaid onto oil flow images. ........... 90
5.3 Swept SBLI upstream influence ............................................................. 91
5.4 PSDs at different spanwise locations for $12^\circ \, R = 1$ interaction. ............ 92
5.5 Spatial variation in peak power for $12^\circ \, R = 1$ case. ............................ 93
5.6 Spatial variation in power in $St = 0.03$. Black boxes highlight corner separation unsteadiness. ......................................................... 94
5.7 Spatial variation in power in $St = 0.03$ with mean separation regions. ........ 95
5.8 Spatial variation in WPSD value for $St = 0.03$, $St = 0.07$ and $St = 0.09$ along streamwise location of central peak power. .............................. 96
5.9 Spatial variation in ‘area-weighted’ median frequencies and associated power for $12^\circ \, R = 1$ interaction. ................................. 97
5.10 Left: spatial variation in ‘area-weighted’ median frequencies. Right: power associated to ‘area-weighted’ median low frequencies of PSDs. .......... 98
5.11 Proposed typical distribution of unsteadiness of incident – reflected SBLIs at floor plane. ............................................................ 99
5.12 Distribution of power associated with area-weighted median frequency overlaid onto oil flow images. ............................................... 100
5.13 Area-weighted higher-order statistical moments of $12^\circ \, R = 1$ PSD. ........ 102
5.14 Area-weighted standard deviation $\sigma$ of PSDs. .................................... 103
5.15 Area-weighted skewness $\gamma [-]$ of PSDs. ........................................ 105
5.16 Area-weighted kurtosis $\kappa [-]$ of PSDs. ........................................ 106
5.17 Spanwise local variation in peak power and minimum frequency for $12^\circ \, R = 1$ case. ................................................................. 107
5.18 Energy map at $X^* = -1.20$ and separation region for $12^\circ \, R = 1$ case. .... 108
5.19 Variation in maximum low-frequency power and its position of occurrence across span of interactions. ........................................... 109
5.20 Spanwise variation in minimum Strouhal number $St_{\text{min}}$ (black) and normalised by local separation length $L(z)$ (red). ........................................... 111
5.21 Spanwise variation in maximum power (black) and normalised by square of local separation length $L(z)^2$ (red). ............................. 112
5.22 Examples of pressure history for $12^\circ \mathcal{R} = 1$ case. ........................................... 113
5.23 Examples of p.d.f.s of pressure history for $12^\circ \mathcal{R} = 1$ case. ............................. 114
5.24 Autocorrelation function of $12^\circ \mathcal{R} = 1$ interaction at $Z^* = -0.02$. ............. 115
5.25 Autocorrelation function of $12^\circ \mathcal{R} = 1$ interaction at $X^* = -1.20$. ............. 116
5.26 Spatial variation in integral time scale $T_{xx}$ of pressure signals .............................. 117
5.27 Joint p.d.f.s between pressure measurements near wall, in attached channel and at edge of central separation for $12^\circ \mathcal{R} = 1$ test case. ............................. 118
5.28 Cross-correlation function of $12^\circ \mathcal{R} = 1$ interaction at $X^* = -1.20$, $Z^* = -0.02$. 120
5.29 Two-point correlations from centreline pressure along same spanwise axis. ............ 121
5.30 Two-point correlations from near wall pressure along same spanwise axis. ............ 122
5.31 Two-point correlations from left: $Z^* = 0.02$ and right: $Z^* = 0.94$. ............. 122
# List of Tables

2.1 Numerical studies considering low-frequency plateau in SBLI unsteadiness. ..... 42

3.1 Compressible boundary layer parameters in Imperial College supersonic wind tunnel ahead of 10° reflected shock interaction. .................. 48

3.2 Distance between incident shock and upstream Prandtl-Meyer expansion wave inviscid impingement points on tunnel floor [mm]. ..................... 49

3.3 Sources of uncertainty and estimates of standard error. .................... 59

3.4 Estimated measurement uncertainties. .................................. 60

4.1 Centreline separation lengths $L_{sep,Q}$ [mm]. .......................... 64

4.2 Statistical moments for separation length variation. Mean $\mu$ [mm], normalised standard deviation $\sigma/\mu [-]$, skewness $\gamma [-]$ and kurtosis $\kappa [-]$. ............... 66

4.3 Separation heights $h_{sep,max}$ [mm] and ratios to centreline separation lengths $h_{sep,max}/L_{sep,Q}$ [-]. ............................................. 68

4.4 Estimated volumes of central separation regions $V_{sep} \times 10^4$mm$^3$. ............... 69

4.5 Maximum pre-multiplied PSD [-]. ....................................... 73

4.6 Variation in median low frequency [Hz]. ................................. 75

4.7 Statistical moments of most energetic PSDs. ............................ 76

4.8 Frequencies with absolute maximum pre-multiplied power and associated Strouhal numbers. .................................................. 79

4.9 Median frequencies weighted by PSD area and associated Strouhal numbers ... 80

4.10 Product of maximum power and central separation region volume ........... 84

5.1 Location ($X^*$) of maximum upstream influence of sidewall interactions based on a conically similar swept SBLI [-]. ........................................ 91
Chapter 1

Introduction

Shock – boundary layer interactions (SBLIs) unavoidably occur in a range of aerospace applications. Normal shock interactions occur commonly on the wings of transonic passenger jets; incident – reflected shock interactions occur in internal geometries of fully supersonic aircraft; and stand-off shock interactions form on the external geometry. A physical description of each can be found in Babinsky & Harvey (2011).

In all interaction types, there is a large adverse pressure gradient across the shock which can cause the boundary layer to separate. It seems that when separation is present, the shock always exhibits an unsteady motion. Currently, the origin and drivers of the range of detected frequencies of the shock motion are not well understood (Dolling, 2001; Dussauge et al., 2006; Clemens & Narayanaswamy, 2014; Ganapathisubramani et al., 2007a; Bruce et al., 2010; Touber & Sandham, 2011b). In particular, the separation shock in each case typically exhibits a motion with a frequency one or two orders of magnitude below the upstream boundary layer fluctuations.

This low-frequency motion can cause severe damage if it leads to self-sustained oscillations. For example, buffeting can occur on transonic aerofoils and engine unstart can be caused in supersonic engine intakes (Bruce & Babinsky, 2010). Air intake buzz or fluctuating side loads in nozzles can occur as a consequence of unsteadiness associated with large-scale reflected shock-induced separation (Babinsky & Harvey, 2011).

Designers must aim to minimise shock-induced separation, which creates drag and hence aerodynamic inefficiency, while avoiding such undesirable flow field unsteadiness. However, to date there has been no focus on the inter-dependencies of global separation and unsteadiness traits of interactions as a whole. Furthermore, little attention has been paid to characteristics of the low-frequency unsteadiness other than the frequency at which the largest energy content is observed, despite a clear relevance of the total energy content and its distribution across a range of frequencies to designers, for example.

Previous academic studies have normalised this peak low-frequency unsteadiness that is inherent to SBLIs by the centreline shock-induced separation length. However, this yields a large spread in the resulting Strouhal numbers and it is evident that this approach is no longer furthering our understanding. Many researchers are reaching the conclusion that we need to look to three-dimensionality for a complete understanding of the drivers of low-frequency motion (Dussauge et al., 2006; Clemens & Narayanaswamy, 2014).
Separation regions have been shown to have a large spanwise variability by many researchers (Reda & Murphy, 1973; Settles et al., 1979; Bruce et al., 2011). Nonetheless, little consideration has been afforded to how representative the centreline is of the entire interaction for a given test case. Moreover, interaction two-dimensionality is often stated as an assumption without evidence and despite the existence of corner effects in real-world applications of interest, such as supersonic engine intakes.

Recent studies conducted in the Imperial College supersonic wind tunnel have found that interactions of the same strength can exhibit vastly different separation regions (Grossman & Bruce, 2017, 2018). This includes a large variation in the centreline shock-induced separation length, with up to a two-fold increase observed across two Mach 2 $12^\circ$ incident – reflected SBLIs with different test article geometries.

In addition, many numerical studies have found a changing separation length dependent upon the span of the domain, even with spanwise-periodic boundary conditions (Touber & Sandham, 2009; Adler & Gaitonde, 2018). This further illustrates that reduced frequencies across different interactions may not be fairly normalised. In addition, it demonstrates that even the most two-dimensional representations of SBLIs exhibit some inherent three-dimensionality.

This work aims to investigate the impact of the previously-documented variation in separation length (Grossman, 2017) on the low-frequency unsteadiness of incident – reflected SBLIs. Furthermore, a global study of incident – reflected SBLIs in rectangular ducts will be undertaken, consisting of a comprehensive characterisation of all features of the interactions: especially the spatial organisation of unsteadiness and separation. Very few studies have documented the unsteadiness away from the centre of the duct and it is hoped that measurements of three-dimensional interactions might further our understanding of the sources of low-frequency unsteadiness.
Chapter 2

Literature Review

This chapter presents a review of relevant literature, with a particular focus on three-dimensional effects on SBLI unsteadiness. Section 2.1 gives a brief physical introduction to SBLIs, focussing on the physics of incident – reflected SBLIs, as this will be the configuration studied experimentally in this work. Section 2.2 discusses relevant previous results, largely the recent developments from studies of three-dimensionality. Finally, this leads onto the aims of the study in Section 2.3.

2.1 Shock – Boundary Layer Interactions .......................... 18
2.2 Unsteadiness in SBLIs ............................................. 20
   2.2.1 Introduction .................................................. 20
   2.2.2 Upstream ....................................................... 21
   2.2.3 Downstream .................................................... 27
   2.2.4 Both ............................................................. 30
   2.2.5 Three-dimensional studies .................................... 32
   2.2.6 Three-dimensionality in the mean ........................... 33
   2.2.7 Three-dimensionality and unsteadiness ..................... 41
2.3 Resulting Aims ......................................................... 45
CHAPTER 2. LITERATURE REVIEW

2.1 Shock – Boundary Layer Interactions

Inviscid flow theory is highly applicable in supersonic flowfields due to the high Reynolds numbers and low turbulence intensities. The accuracy breaks down near boundaries but still closely follows experimental data in isentropic regions as the boundary layer tends to be small due to the high speed outer edge.

![Figure 2.1: Boundary layer division into triple deck theory, from Babinsky & Harvey (2011).](image)

It is in the viscous sublayer within the boundary layer (see Figure 2.1) that the flow is affected by viscosity and from which the shock – boundary layer interaction (SBLI) stems. Across a shock, the flow is highly non-isentropic and there is a strong adverse pressure gradient; this acts to thicken the boundary layer and increase the size of the subsonic and viscous region. In terms of aircraft performance, this can be viewed as a detriment to efficiency.

Figure 2.2 shows the deviation from inviscid theory in the pressure distribution under an incident – reflected SBLI. Chapman (1957) also compares a laminar and transitional case for the same flow conditions and configuration, showing that the laminar case is further from inviscid theory. This demonstrates that the discrepancy is due to thickening of the viscous region as a laminar boundary layer is less resistant to separation in the presence of the adverse pressure gradient across the shock.

![Figure 2.2: Effect of viscosity on pressure distribution under an incident – reflected SBLI (Chapman, 1957).](image)
In the bulk freestream and for the upper part of the boundary layer, shocks can propagate through the effectively inviscid flow at orientations determined only by the flow conditions and boundary geometry. Within a compressible boundary layer, flow properties change significantly with local Mach number $M$, demonstrated in Figure 2.3a. The variation in speed of sound $a$ and pressure $p$ modify the shock strength and direction. As the Mach number decreases towards the wall, the transmitted shock steepens until it reaches the sonic line, beyond which it can no longer propagate (Figure 2.3b).

This study focusses on separated incident – reflected SBLIs whereby an oblique (incident) shock intersects a boundary layer, leading to the formation of a separation bubble. Consequently, the subsonic region dilates to pass over the bubble as shown in Figure 2.4. The deflection of the oncoming flow over the separation region and back to be parallel with the boundary causes compression waves to form ahead and downstream of the bubble. The compression waves ahead of the bubble coalesce above the boundary layer to form the reflected (or separation) shock. It is at the foot of this shock (the point nearest the wall) that low-frequency unsteadiness exists.
2.2 Unsteadiness in SBLIs

2.2.1 Introduction

"Even though there are sound technological reasons for seeking an understanding of the source of the low-frequency unsteadiness, there is no question that a major motivating factor for researchers is that we simply should be able to understand the supersonic flow over a two-dimensional compression ramp, for example."

– N. T. Clemens & V. Narayanaswamy (2014)

SBLI unsteadiness has remained an enticing mystery to researchers since the mid-1960s, since perhaps the first high-frequency measurements by Kistler (1964) (Muck et al., 1985; Dolling, 2001; Pasquariello et al., 2017). Kistler (1964) proposes a simple explanation for the observed bi-stability: due to the presence of the shock a pressure jump is expected along the wall; if this jump location can move then a switching between the two pressures will be observed when measuring at a fixed location.

However the research community has further sought to ascertain why this switching occurs at the observed period; particularly as the motion cannot be readily explained by disturbances in the incoming boundary layer, as it occurs at frequencies around two orders of magnitude lower than those in the incoming high-speed flow (Touber & Sandham, 2011b; Bermejo-moreno et al., 2014; Guiho et al., 2016; Waindim et al., 2016; Threadgill & Bruce, 2017). Compared to the expected characteristic boundary layer frequencies characterised by the boundary layer height $\delta$ and the freestream velocity $U_\infty$, the low-frequency motion occurs at $f \sim 0.01U_\infty/\delta$. When expressed in terms of shock-induced separation length $L$, the motion is typically characterised by a non-dimensional frequency of $St_L = fL/U_\infty = 0.03$.

Moreover, this long-period motion seems to be universal as it is documented not just for forward-facing step interactions, as Kistler studied, but for compression ramp (Dolling & Murphy, 1983; Dolling & Or, 1985; Erengil & Dolling, 1991b); swept compression ramp (Erengil & Dolling, 1993; Gonsalez & Dolling, 1993; Wu & Martín, 2004, 2007); blunt fin (Robertson, 1971; Dolling & Bogdonoff, 1981); sharp fin (Tan & Bogdonoff, 1985; Tran et al., 1985; Schmisseur & Dolling, 1992); reattaching shear layer (Poggie & Smits, 2001); incident – reflected (Dupont et al., 2006; Pirozzoli et al., 2010); and normal (Bernardini et al., 2011) shock – boundary layer interactions, suggesting the existence of a compact explanation.

"Doubtlessly, when the explanations emerge, these results will fall neatly into a logical framework"

– D. S. Dolling (2001)

Arguably the most successful attempt to model the shock motion came ten years later from Plotkin (1975). Plotkin modelled the shock as a linearly damped system subjected to random (turbulent) velocity fluctuations which seems to accurately describe, if not fully explain, its low-frequency motion. The source of damping suggested by Plotkin was the restoring response of the shock to changes in the pressure jump across it. Many studies have subsequently published evidence in support of the model (Beresh et al., 2002; Poggie & Smits, 2005; Pirozzoli et al., 2010; Bisek, 2015; Poggie et al., 2015; Adler & Gaitonde, 2016; Sandham, 2016; Nichols et al., 2017; Adler & Gaitonde, 2018). Touber & Sandham (2011a) later went on to show that such
a model can be derived from the momentum integral equation. They showed that a low-pass
filter behaviour is observed with different forcing functions, and therefore propose that it is an
intrinsic property of the coupled system (Touber & Sandham, 2011b).

Historically there has been a focus on debating whether the unsteadiness originates
from upstream or downstream sources, as highlighted by the review paper of Clemens &
Narayanaswamy (2014). Although Plotkin (1975) suggests that upstream random turbulent
fluctuations are forcing the system, the same response would be observed as a result of
downstream disturbances. Indeed subsequent papers have interpreted the results as evidence
for both downstream and upstream causes of shock motion. It appears that the main advocates
of the upstream mechanism are Clemens’ group at UT Austin while perhaps the majority of
downstream arguments have been proposed by the Aix-Marseille Université group in the early
2000s.

2.2.2 Upstream

Clemens & Narayanaswamy (2014) draw on earlier results from UT Austin when listing support
for the upstream mechanism. Erengil & Dolling (1990, 1991b) studied compression ramp
interactions and found that, while there was a correlation between pressure fluctuations in
the incoming flow and changes in the direction of shock motion, this was a weak correlation
with large downstream shock motions and no correlation with large upstream motions. They
found that changes in shock motion direction correlated with pressure fluctuations immediately
upstream and therefore concluded that the turbulent fluctuations are only responsible for high-
frequency ‘jittering’ shock motions. The authors later found that motions in the separation
bubble were correlated with subsequent motions of the shock foot and argued that large-scale
bubble pulsations drive the large-scale shock motions (Erengil, 1993).

Beresh et al. (2002) further investigated the thickening/thinning mechanism (boundary
layer height variations) suggested by Ünalmis & Dolling (1994), using more recently developed
particle image velocimetry (PIV) to record the upstream boundary layer. Their first finding
was that the incoming boundary layer velocity profile, conditionally sampled on shock position
(Figure 2.5a), was fuller when the shock was downstream, as measured by pressure transducers
in the intermittent region at a time delay corresponding to a convection velocity of \(0.9U_\infty\). This
is expected as a fuller boundary layer has greater resistance to separation. However the authors
noted that this only occurred in the lower part of the boundary layer so the low-frequency
thickening/thinning mechanism was not found. Further, the fluctuations are less than \(0.01U_\infty\)
and so perhaps these small correlations below the sonic line might be expected, regardless of
low-frequency motion.

They then considered the conditionally sampled shock motion (rather than position) for
frequencies of 10 kHz, 4 kHz and 2 kHz. The largest correlation was found for motions with
frequencies of 4 kHz, again suggesting that the boundary layer fluctuations may correlate with
the high-frequency jitter of the shock. The upstream boundary layer velocity fluctuations
conditioned on the frequency closest to the dominant frequencies found in a previous study of
the same configuration by Erengil & Dolling (1993), ranging 0.3 – 0.5 kHz, are shown in Figure 2.5b.
It can be seen that the correlations are weak and likely to be statistically insignificant at a quarter of the conditioned frequency (2kHz).

(a) Velocity profile conditioned on shock position ($\delta \approx 26$ mm).

(b) Velocity fluctuations conditioned on shock motion at 2kHz.

Figure 2.5: Conditional upstream boundary layer properties (Beresh et al., 2002).

Arguably the most convincing argument from the group came from the results of Ganapathisubramani et al. (2007a, 2009) for a $20^\circ$ compression ramp interaction at Mach 2. They first used planar laser scattering (PLS) in the upstream boundary layer to find long coherent ($\sim 40\delta$) structures (Ganapathisubramani et al., 2007a) and then high-speed PIV to find $\sim 30\delta$ structures (Ganapathisubramani et al., 2009). The long structures found from the PLS images were captured at 100$\mu$s intervals, during which time the structures travelled around $3.5\delta$ which was $60\%$ of the frame. Therefore utilising Taylor’s frozen turbulence hypothesis, the image sequences could be reconstructed to find the equivalent spatial distribution, examples of which are shown in Figure 2.6. It is unclear why the figures stop at $x/\delta = -40$, it is possible that this was deemed to be the limit of validity of Taylor’s frozen turbulence hypothesis. Assuming that a high or low-momentum region $40\delta$ in length represented half of the wavelength of a full structure, the authors concluded that elongated coherent regions (‘superstructures’) existed in the upstream boundary layer with wavelengths of $80\delta$, which could account for the lower frequency range of shock motion.

Furthermore, Ganapathisubramani et al. (2007a) found the correlation coefficient between upstream velocity fluctuations and the shock location to be 0.4. This was found by considering each spanwise location and taking the streamwise-averaged velocity fluctuation in the upstream boundary layer at that location and the ‘surrogate for the separation point’, the streamwise location at which the velocity fell below 4 standard deviations below the mean at $y/\delta = 0.2$. They also performed conditional sampling, based on the separation location at the spanwise centre being upstream of $-2.2\delta$ from the ramp or downstream of $-1.4\delta$ from the ramp, the mean velocity fields for which are shown in Figure 2.7a and Figure 2.7b, respectively. They found that there were high and low-speed momentum regions ahead of the centreline for the shock being far downstream and upstream respectively, with average velocity fluctuations of $\pm 0.025U_\infty$.

The authors caution that the data shown in Figure 2.7 is not statistically converged. Further, they observe the elongated structures to undergo a spanwise meandering as can be seen in Figure 2.6 which they argue reduces the magnitude of the calculated conditionally averaged
velocity fluctuations. However, the spanwise meandering brings into question whether a high or low-speed momentum region is impacting one spanwise ‘surrogate of the separation point’ for a time equivalent to $40\delta/U_\infty$.

Shortly after the initial results were published, Wu & Martín (2008) published results from a direct numerical simulation (DNS) of a $24^\circ$ compression ramp interaction at Mach 2.9. They found a correlation coefficient of 0.5 when performing an analysis in the same method as Ganapathisubramani et al. (2007b); however when using the location of zero skin friction coefficient to define separation point and upstream average momentum $\rho u$ they found a coefficient of 0.23. This highlights the difficulty in selecting representative indicators of shock motion. Wu & Martín (2008) further found the correlation between the mass flux in the incoming boundary layer and the spanwise mean streamwise location of pressure rise to $1.3p_\infty$ was 0.35. They sampled at a height of 0.75$\delta$ as, due to the comparatively low Reynolds number $Re_\theta = \rho u \theta / \mu = 2300$, the shock did not penetrate the boundary layer as deeply. They therefore claimed that low-momentum fluid in the boundary layer is responsible for high-frequency spanwise shock wrinkling and does not significantly affect the streamwise shock motion.
Subsequently Touber & Sandham (2011b) sought to confirm this mechanism by performing a large-eddy simulation (LES) of an $8^\circ$ incident – reflected SBLI at Mach 2.3. They computed the power spectral density (PSD) under the reflected shock foot, modelling the experiment of Dupont et al. (2006). They also derived a stochastic ordinary differential equation (ODE) for the low-frequency motions and found the PSD that would have been produced if the incoming boundary layer were white noise turbulence (no ‘superstructures’). These PSDs are shown in Figure 2.8.

![Figure 2.8: Weighted PSDs of wall-pressure near reflected shock foot from LES models and experiments of Dupont et al. (2006) (Touber & Sandham, 2011b).](image)

As can be seen from Figure 2.8, both the LES and model with white noise turbulence closely match the PSD found in the experiments of Dupont et al. (2006). As noted by the authors, the model does not require any particular forcing as there is no transfer of energy between frequencies. Rather, the system acts to damp high frequencies and amplify low ones, regardless of their respective energies. They further assert that therefore the broadband spectrum that is typically observed about a characteristic Strouhal number is a property of the system itself rather than the forcing.

"the origin of the low-frequency oscillations is not in the forcing but in the dynamics of the system formed by the shock/boundary-layer interaction"
– Touber & Sandham (2011)

Although not discussed in the review paper (Clemens & Narayanaswamy, 2014), subsequent work from UT Austin has emerged in favour of the upstream argument. Vanstone et al. (2017) used 50 kHz PIV in the $x-z$ plane (parallel to the floor) to investigate a Mach 2 SBLI of a $22.5^\circ$ compression ramp with a $30^\circ$ sweep. Although they do not give the expected characteristic frequency of shock motion, they bandpassed and low-pass filtered the data at intervals of 1 kHz, 10 kHz and 50 kHz and found the joint probability density functions (p.d.f.s) of the boundary layer streamwise velocity fluctuations and the separation line.
Although joint p.d.f.s only indicate correlation and not causation, by looking at their magnitude and the films from the PIV the authors concluded that the mid-frequency, large-scale motion of the separation line ($1 - 10\,\text{kHz}$) is associated with boundary layer superstructures. However it is unclear if the characteristic low-frequency shock motion is in this band; Adler & Gaitonde (2017) replicated the flow with LES and found that it lacked the low-frequency unsteadiness at $St_L \sim 0.03$ that is prominent in unswept ramp interactions. Lastly, it would have been interesting to find the correlation between frequency bands, as other studies have proposed that mid-range frequency instabilities such as those found in the shear layer fuel low-frequency motions (Pirozzoli & Grasso, 2006; Wu & Martin, 2008; Priebe & Martin, 2012; Priebe et al., 2016; Pasquariello et al., 2017).

Finally Clemens & Narayanaswamy (2014) draw on results from two other sources in support of upstream causes of unsteadiness. Firstly, Poggie & Smits (2001) found that the wall-pressure fluctuations at the reattachment point of a $M = 2.9$ shear layer generated from a backwards-facing step had more energy at lower frequencies when the upstream boundary layer was injected with air. The authors concluded that the shock motion in the reattaching shear layer was caused by the incoming turbulent structures; however, in contrast to all other examples this was observed at a reattachment point rather than a separation point, so it is unknown if a similar behaviour would be observed in separation shocks. Moreover, Dussauge & Piponniau (2008) pointed out that this set-up had no separation region downstream and so the shock was free to develop its own frequencies.

Secondly, they draw on experimental results from Delft University of Technology where probably still the most extensive three-dimensional data of an SBLI was collected for a Mach 2.1 8° incident – reflected interaction. They used tomographic PIV at 10 Hz to obtain 200 velocity vector volumes of each of two lower and upper regions of the interaction ($0.1\delta < y < 0.6\delta$ and $0.6\delta < y < 1.0\delta$) (Humble et al., 2009a). They found high and low-speed regions in the incoming boundary layer, an example is shown in Figure 2.9, similar to those found by Ganapathisubramani et al. (2007a). Note that in the figure $y$ corresponds to the spanwise dimension, in the text it refers to height for consistency with other papers and $z$ denotes the spanwise distance.

While the PIV domain was not large enough to confirm the length of the high and low-speed velocity structures, they were able to observe the influence of the regions on the spanwise rippling of the separation region. They also computed the correlation coefficient between velocity fluctuations in the incoming boundary layer and the surrogate instantaneous shock position in the same manner as Ganapathisubramani et al. (2007a). They also found that positive velocity fluctuations meant that the reflected shock was more likely to be located downstream of its median position and vice versa. At $y/\delta = 0.12$ and $y/\delta = 0.82$, they found a coefficient of about 0.5 and 0.4, respectively, in good agreement with Ganapathisubramani et al. (2007a).

They later performed a statistical analysis on the same flow with 1000 velocity vector fields and found that the boundary layer fullness weakly correlated with the shock wave position and separation bubble size (Humble et al., 2009b). They concluded their results could be consistent with the large-scale unsteadiness being caused by either boundary layer structures or a feedback loop between the separation bubble, separated shear layer and shock.
A later publication from the group conducted time-resolved PIV on the interaction, on the central $x-y$ plane at 20kHz, although much of the analysis was conducted from 10kHz measurements which were deemed more reliable (van Oudheusden et al., 2011). Using the 10kHz PIV images they were able to compute the cross-correlations between boundary layer momentum, shock position and bubble size to 50µs precision, shown in Figure 2.10.

The peak correlation coefficient between the boundary layer momentum and the shock was 0.26 after 100µs. However a larger coefficient of $-0.28$ was recorded between the shock foot and the bubble size with no delay, showing that a downstream shock position is associated with a smaller bubble. Furthermore, a larger coefficient still of $-0.50$ was recorded after a 150µs delay between the boundary layer momentum and the bubble size.

This means that the strongest correlation was between a higher boundary layer momentum and a smaller bubble size and \textit{vice versa}. This and the observation of the bubble exhibiting a dominant frequency between that of the boundary layer and reflected shock led them to support the concept of the bubble dynamics low-pass filtering excitations to determine the characteristic interaction and shock frequency.
2.2.3 Downstream

On the other hand, much support of a downstream mechanism for unsteadiness has emerged from the experimental results of the Marseille group since 2005. Indeed this makes up most of the examples listed in Clemens & Narayanaswamy (2014), alongside DNS results from Princeton (Wu & Martín (2008) and Priebe & Martín (2012)).

An early Marseille paper found the existence of a pair of off-centre ‘tornado vortices’ above the separation region of a $M = 2.3\, 9.5^\circ$ interaction using PIV (Dussauge et al., 2006), shown in Figure 2.11. They found that the rotational frequency of these vortices matched the low-frequency shock motion. However, as later noted by Dussauge & Piponniau (2008), the same case with a deflection angle of $8^\circ$ had no vortices in the separation but still exhibited low-frequency unsteadiness. Therefore the tornado vortices may not have been the cause of the shock motion.

![Figure 2.11: Streamlines in separation region of $M = 2.3\, 9.5^\circ$ interaction (Dussauge et al., 2006).](image)

Otherwise they employed similar experimental techniques to those at UT Austin, such as correlating pressure signals at different streamwise locations and taking conditional samples. The first such work concerning unsteadiness was by Dupont et al. (2006) for which two transducers were placed separated by 9.5 mm in the streamwise direction ‘in the vicinity of the shock foot’ of a $9.5^\circ$ incident – reflected SBLI. Figure 2.12 shows that the downstream probe followed by the upstream probe experienced increases in pressure, as if the bubble had undergone an upstream excursion. They simply tested for low frequencies in the incoming boundary layer to conclude that no low-frequency energy was originating from there.

Similarly, Piponniau et al. (2009) collected 5000 PIV images of $8^\circ$ and $9.5^\circ$ incident – reflected shock interactions in the $x - y$ plane. They found the maximum height of the recirculation bubble in each instance and performed a conditional analysis on the minimum, median and maximum 10% of these height events (‘shallow’, ‘medium’ and ‘thick’). They inferred the median reflected shock location under each condition as the point with maximum $\sqrt{v'^2}$, which is plotted in Figure 2.13. They concluded that while the bubble is ‘shallow’ the shock displacement downstream from its position at the median bubble height (— vs. - - -) is limited. However, while the bubble is ‘thick’ the upstream shock displacement is more than double this distance from its mean ‘medium’ bubble state (||| vs. - - -). This potentially conforms with the results of Erengil & Dolling (1990), discussed in Section 2.2.2, to show that downstream shock
motions are caused by bubble collapse while upstream motions are caused by falling upstream pressure.

Priebe & Martín (2012) also observed bubble collapse events in their DNS study of a compression ramp interaction, finding that bubble collapse events produced a secondary separation location. They found that the low-frequency of the system involved breathing of the separation bubble, an associated ‘flapping’ motion of the shear layer and the shock. They also observed a weak statistical relation of the motion to the flow in the incoming boundary layer, in agreement with Piponniau et al. (2009).

Piponniau et al. (2009) argue that fluid entrainment from the separation bubble into the outer, inviscid flow is an important factor in frequency scales. They define a Strouhal number based on the frequency of the breathing of the separation bubble due to this (2.1), incorporating alternative length and velocity scales based on the height of the separation bubble $h$ and the velocity over the top of the mixing layer $u_1$, respectively. They further show that this can be related to the mixing layer formed by the flow above the separation bubble and the flow within it. This can be reduced to a dependence upon the normalised spreading rate of the mixing layer $\Phi(M_c)$, where $M_c$ is the convective Mach number $(u_1 - u_2)/(a_1 + a_2)$, and a function $g$ of the velocity and density ratios below to above the mixing layer, $u_2/u_1$ and $\rho_2/\rho_1$. 

Figure 2.12: Pressure signals of adjacent transducers 9.5 mm apart in the streamwise direction (Dupont et al., 2006).

Figure 2.13: Conditional RMS (root mean square) of fluctuations of vertical velocity at four distances from the floor for a 9.5° incident – reflected SBLI (Piponniau et al., 2009).
\[ S_h = \frac{f h}{u_1} = \Phi(M_c)g(r, s) = \Phi(M_c)g\left(\frac{u_2}{u_1}, \frac{\rho_2}{\rho_1}\right) \]  

(2.1)

(a) \( S_L \): ♦ compression ramp cases; + over-expanded nozzle (restricted shock separation); ★ blunt fin; ◎ estimated superstructures upstream influence for the 8° IUSTI case.

(b) \( S_h \): ◆ Thomas et al. (1994); ▼ Dolling & Brusniak (1989); ▲ Erengil & Dolling (1991a); ▼ Wu & Martin (2008) (DNS) and Ringuette & Smits (2007) (experiment).

Figure 2.14: Distribution of characteristic frequency with different normalisations (Piponniau et al., 2009). Common symbols: ■: subsonic separation from Kiya & Sasaki (1983); * IUSTI reflection cases; • Touber & Sandham (2008).

Figure 2.14 compares the peak frequencies across multiple interactions conventionally normalised by interaction length and upstream velocity \( S_L \) (Figure 2.14a) to those normalised by mixing layer scales \( S_h \) (Figure 2.14b). On the one hand, \( \Phi(M_c) \) is seen to bring the subsonic turbulent separated bubble case (■) in line; however, it is questionable whether this is helpful in finding the mechanism of low-frequency unsteadiness in SBLIs. Indeed Dupont et al. (2006) note the observation of Erengil & Dolling (1991a) that subsonic scaling may not be relevant in characterising shock oscillations, particularly for compression ramp cases (triangles in Figure 2.14b) as \( S_L = fL/U \) was developed for low-velocity detached flows.

The factor \( g(r, s) \) is similar across all cases and does little to collapse different supersonic interactions. Similarly, Threadgill & Bruce (2016) later found little variation in these parameters across different configurations and strengths in the same facility and thus the characteristic frequencies did not collapse.

Recently, there have been developments in analysis techniques such dynamic mode decomposition (DMD) (see Schmid (2010)) which enable spatially coherent dynamic motions to be extracted from a series of data of a fluid flow. For example, DMD can be applied to a sequence of velocity or pressure fields evolving in time in order to find significant instabilities driving the flow field and which areas of the field they involve.

This technique was employed by Pasquariello et al. (2017) on their LES data of a Mach 3 19.6° incident – reflected SBLI to extract coherent frequencies across the domain. The similarity of the PSD at the separation location and the DMD modes is shown in Figure 2.15 which the authors say shows that the low-frequency unsteadiness is related to a global flow feature. They identify the modes \( \phi_1 \) and \( \phi_2 \) at frequencies of \( f_1 = 0.039U_\infty/L_{sep} \) and \( f_2 = 0.114U_\infty/L_{sep} \) to
further investigate as representative of the region I modes in Figure 2.15b which describe a movement related to the shock system and separation bubble (modes in region II represent shedding motions of the detached shear layer).

![Figure 2.15: Comparison of PSD and modes from DMD (Pasquariello et al., 2017).](image)

Figure 2.16 shows the real and imaginary parts of these modes for pressure fluctuations, representing the magnitude and phase, respectively. It can be seen that the separation and reattachment shocks oscillate coherently out of phase to each other at $f_1$, representing an expansion and contraction of the separation region at these low frequencies. There is no associated low-frequency motion in the incoming boundary layer; however there are pressure fluctuations at the bubble apex which seem to be related to motions of the transmitted part of the incident shock at $f_2$.

![Figure 2.16: Real and imaginary parts of DMD modes $\phi_1$ (top) and $\phi_2$ (bottom) with pressure fluctuations (Pasquariello et al., 2017).](image)

### 2.2.4 Both

It can be seen from Sections 2.2.2 and 2.2.3 that historically a lot of energy has been spent trying to find a single cause of low-frequency shock unsteadiness. However neither upstream boundary layer fluctuations nor downstream separation bubble motions have been fully proven to be the single source of the motions.

In 2009, Souverein et al. (2009) and Clemens & Narayanaswamy (2009) proposed the idea that upstream and downstream sources have more influence on the interaction depending on how much separation occurs. Interactions can be classed as fully, incipiently or weakly
2.2. UNSTEADINESS IN SBLIS

separated depending on how often they exhibit reversed flow. There is largely agreement that for weakly separated interactions the upstream boundary layer influence is more important while for strongly separated interactions the downstream separation region has a stronger influence. For example, Priebe & Martín (2012) suggest their results can be reconciled with Ganapathisubramani et al. (2007a, 2009) for which the upstream forcing would have had a greater influence due to the interactions being incipiently separated.

Following the beginnings of the dimensional analysis by Dussauge & Piponniau (2008), boundary layer superstructures of length $L_{sup}$ would have a characteristic frequency of $U_\infty / L_{sup}$. This means that the interaction Strouhal number can be reduced to $S_L = L / L_{sup}$. For the structures to be driving the interaction motion $S_L = 0.03$, their length would have to be $L_{sup} \sim 33L$. This seems plausible for weakly separated interactions when $L \sim \delta$ but less so for strongly separations where $L > 4\delta$. As tabulated by Clemens & Narayanaswamy (2009), most interactions which meet this latter criterion are dominated by downstream influences.

Dussauge & Piponniau (2008) also note that there is a large shift in the shock oscillation range after the incipient separation criterion is passed in Figure 2.17, which shows data adapted by Selig et al. (1988) from Dolling & Or (1983). The data is of Mach 2.9 compression corner interactions with different angles; the limiting incipiently separated case was found to be $16^\circ$ by Settles (1976) and in this case there is no separation before this point. This serves to highlight the definition of incipient separation typically used to categorise compression ramp interactions. Unlike the proportion of time that there is reversed flow as with incident – reflected interactions, it is when the shock stands off from the ramp, something which likely brings an abrupt change in physics.

An exception to this agreement can be found by considering the investigation of Thomas et al. (1994) of a weak compression ramp interaction. The shock excursions were of the order of

![Figure 2.17: Range and amplitude of separation shock for compression corners of varying strengths studied by Dolling & Or (1983) (Selig et al., 1988).](image)
one boundary layer thickness such that the shock motion due to upstream burst events would be clearly identifiable. However they found that there was no relationship between the burst events and the large scale shock motion; while there was coherence between the separation bubble and shock motion.

However it seems that more recently authors are converging on the idea expressed by Touber & Sandham (2011b) that the low-frequency unsteadiness is intrinsic to the system as a whole, for example due to a global instability (Robinet, 2007; Pirozzoli et al., 2010; Hadjadj, 2012; Grilli et al., 2012; Nichols et al., 2017); or via interaction of the shock with the shear layer or acoustic waves in the bubble (Pirozzoli & Grasso, 2006; Wu & Martín, 2008; Priebe & Martín, 2012; Aubard et al., 2013; Huang & Estruch-samper, 2018). It also seems reasonable that the unsteadiness might be some innate property of the shock system as such low frequencies are not observed in subsonic separations. While the system may require continuous forcing, it could come from upstream or downstream or both.

Sansica et al. (2014) performed a DNS of a $M = 1.5$ laminar incident – reflected SBLI. They applied white noise forcing upstream of the interaction and inside of the bubble. Without any upstream forcing, the low-frequency response at the separation point was observed due to the forcing in the bubble. With upstream forcing, the low-frequency unsteadiness was driven by non-linearities from the downstream shedding.

In a two-dimensional sense, the shock is an instantaneous representation of where there is an abrupt change in flow velocity and pressure, such that the Rankine-Hugoniot conditions are satisfied. The shock fulfils these criteria to the extent that a normal shock subjected to a change in back pressure will move upstream or downstream, changing its relative velocity and hence its strength (Bruce & Babinsky, 2008). In this manner the shock changes location at a slower rate than changes in upstream or downstream conditions. In the context of three-dimensionality, small-scale boundary layer fluctuations will affect the span of the shock front differently, at the scale of the disturbances. This may explain the spanwise rippling with wavelengths of the order $\delta$ observed by Muck et al. (1988) and Wu & Martín (2008), for example. However, if the shock were to instantaneously respond to these fluctuations across its span, there would be a pressure mismatch in the spanwise direction. This reinforces the idea of a damping intrinsic to the three-dimensional shock itself.

2.2.5 Three-dimensional studies

There is an entire field considering three-dimensionality in SBLIs in the context of micro-vortex generators (MVGs), see for example the review of Lu et al. (2011). However this is because the MVGs induce a highly three-dimensional upstream flow which then interacts with the shock and separation bubble.

In studies not considering the control of SBLIs, three-dimensionality is largely neglected. In 1973, Reda & Murphy (1973) obtained incident – reflected interaction test surface oil flow patterns showing large vortices near the corners and a highly spanwise-varied reattachment line. They warned that previous assumptions of two-dimensionality may not be very reasonable. However, it appears that the first study concerning the effects of sidewalls on SBLIs was a detached eddy simulation (DES) by Garnier (2009).
"corner separations induced by the presence of lateral walls reduce the effective section of the wind tunnel and strengthen the interaction, making periodic computations irrelevant for strongly separated cases."

– E. Garnier (2009)

To the author’s knowledge, all simulations of SBLIs have employed periodic boundary conditions with the exception of Garnier (2009); Hadjadj (2012); Benek et al. (2013, 2014); Bermejo-moreno et al. (2014); Morgan et al. (2014) (in the context of shock trains in an isolator); Bisik (2014, 2015); Wang et al. (2015); Poggie & Porter (2018) and Lusher & Sandham (2019). These will be discussed in more detail below.

A handful of early experimental studies have also looked at small spanwise correlations in addition to streamwise ones. Muck et al. (1985, 1988) found the separation shock over a Mach 2.9 compression corner exhibited a spanwise ‘rippling’ in addition to the streamwise ‘flapping’ using a spanwise array of pressure transducers with a centre-to-centre spacing of 5.08 mm. Marshall & Dolling (1992) used pressure transducer configurations to span 10.25 – 30.65 mm from the tunnel centreline (6 – 17% of the tunnel width). They inferred a spanwise ripple wavelength ranging from 0.35δ – 2δ. Also Ünalmis & Dolling (1994) found small but non-zero spanwise correlations over the range ΔZ = 0.2δ – 2.3δ.

In the last few years there have been some studies conducted considering spanwise effects on SBLIs. Clemens & Narayanaswamy (2014) listed it as the first topic for which future research is needed in 2014. They highlighted that an understanding of three-dimensional effects is crucial for real-world geometries where SBLIs occur (rectangular supersonic engine intakes being the principal example).

Earlier studies of three-dimensionality focussed on so-called ‘corner effects’ in SBLIs, including the stimulated detached eddy simulation (SDES) of Garnier (2009) and the experimental work carried out at the University of Cambridge (Bruce et al., 2011; Babinsky et al., 2013; Xiang & Babinsky, 2017). There were also experiments motivated by understanding engine unstart at the University of Michigan (Morajkar et al., 2014, 2016; Eagle & Driscoll, 2014) and by Funderburk & Narayanaswamy (2016).

2.2.6 Three-dimensionality in the mean

Morajkar et al. (2014) showed that the corner separations in a Mach 2.75 6° incident – reflected SBLI with the shock generator spanning the full test section were very significant. They later found that the interaction in the low-aspect-ratio duct (AR = W/H = 0.83) consisted of a corner vortex pair, a swept-shock sidewall vortex and a horseshoe-like vortex connecting the central and corner interactions (Morajkar et al., 2016).

Bruce et al. (2011) showed that the size of the corner separations influenced the extent of the main separation for normal SBLIs. This was expanded upon by Babinsky et al. (2013) who proposed that interactions can be categorised into three types depending on the wind tunnel ‘viscous aspect ratio’ (see Figure 2.18). This is based upon the hypothesis that glancing shocks are created at the corner separations which impose an additional pressure rise on the central interaction. The two-dimensionality of the main interaction therefore depends on where it is met by these shocks.
Later, the impact of confinement was considered in a joint numerical/experimental investigation between the Air Force Research laboratory and the University of Cambridge (Benek et al., 2013, 2014, 2016) and later Saint Louis University (Pizzella et al., 2017); wall-modelled large-eddy simulations (WLES) of Bermejo-moreno et al. (2014); the large-eddy simulation (LES) of Wang et al. (2015); and the experimental investigations of Grossman & Bruce (2016, 2017, 2018).

Figure 2.19a shows the effect of viscous aspect ratio on centreline separation length (here denoted $\Delta x$), based on LES data of a Mach 2.9 13° incident – reflected SBLI by Benek et al. (2013). Following from this, Poggie & Porter (2018) investigated the behaviour for a Mach 2.25 24° compression interaction, matching the conditions of Bisek (2015), shown in Figure 2.19b. Figure 2.19 shows that both types of interaction exhibit a maximum separation length for a viscous aspect ratio of $\delta/w \sim 0.05$. This demonstrates that, as proposed by Babinsky et al. (2013) in Figure 2.18, there is a complex relationship between interaction aspect ratio and separation length. It is unknown if the different regimes seen here for interaction length can be expected to reduce the low-frequency unsteadiness to a common Strouhal number.

Bisek conducted three LES investigations of a Mach 2.25 compression ramp interaction: one with spanwise periodic boundary conditions (Bisek et al., 2013); one for which one sidewall was modelled and a symmetry condition imposed at the centreline (Bisek, 2014) and one simulating both sidewalls and the full spanwise domain (Bisek, 2015). The simulation of the full domain had a small viscous aspect ratio $\delta/w = 0.025$ such that the flow near the centreline could be considered unaffected by the sidewall shocks and nominally two-dimensional (it could be classified as ‘quasi-2D’ according to the earlier discussed relation proposed by Babinsky et al. (2013), shown in Figure 2.18). However the behaviour at the midspan exhibited a three-
2.2. UNSTEADINESS IN SBLIS

(a) Incident – reflected interactions (Benek et al., 2013).
(b) Compression ramp interactions (Poggie & Porter, 2018).

Figure 2.19: Normalised centreline separation as a function of viscous aspect ratio.

Dimensionality not predicted by the case with spanwise-periodic boundary conditions (Bisek, 2015).

"the domain was sufficiently wide to de-correlate the midspan flow from the core-corner, but the current results also show that the flow was still highly three-dimensional, even near the midspan so the resultant shock structure and its behavior in that region could not be replicated with a spanwise-periodic boundary condition"


Furthermore, the separation length at the centreline was greater in the full-span results compared to the half-span, as the boundary layer was around 15% thicker in the latter case. However it is difficult to detect if this had any effect on the characteristic low-frequency motion as the frequency resolution in the study is low. One reason that Bisek proposed for the differences is that only ‘even spanwise modes’ could exist in the imposed-symmetry case while both ‘odd and even spanwise modes’ could exist in the full-span case.

Bermejo-moreno et al. (2014) performed LES simulations of a Mach 2 incident – reflected SBLI with both spanwise-periodic and no-slip boundary conditions at \( A = 9.5 \). Even at this high aspect ratio, they found that the inclusion of sidewalls gave a separation bubble that was approximately 20% longer and twice as high compared to the spanwise-periodic case and highlight that this is consistent with other studies which have compared spanwise-periodic simulations to experiments, such as Priebe et al. (2009); Pirozzoli et al. (2010) and Morgan et al. (2013).

Wang et al. (2015) published LES results in support of the theory proposed by Babinsky et al. (2013), that the central separation size is largely determined by the glancing shock interactions emanating from the corner separation regions. They considered a Mach 2.7 9° incident – reflected SBLI with channel aspect ratios \( \mathcal{A} = W/H \) (duct width to height ratio) of 1, 2 and 4. They found that in the most three-dimensional case (\( \mathcal{A} = 1 \)), a weak shock from the sidewall interaction propagated to the tunnel centreline and acted to increase the angle and hence strength of the main incident shock to effectively that of an 11.5° interaction. This is shown in Figure 2.20a, alongside a case without sidewalls in Figure 2.20b for comparison. The origin of the weak shock can be seen when viewing the \( x – z \) plane, shown in Figure 2.20c, with an \( \mathcal{A} = 2 \) interaction (where the weak shocks do not reach the central \( z \)-plane) shown in Figure 2.20d.
The authors explained this by considering the pressure distribution taken from a slice of Figure 2.20d, shown in detail in Figure 2.21. It can be seen that upon passing through the separation shock of the ‘lambda’ shock structure (Wave-1) caused by the corner separation and the main incident shock, the pressure is higher behind the incident shock towards the sidewall than in the core flow. The incident shock therefore curves at points C and S and across Wave-2 the pressure field returns to a uniform value of that in the core flow. As a result, the separation length increases by 30% from 21.1 mm for the quasi-two-dimensional (baseline) case without sidewalls and the $AR = 4$ case to 27.9 and 28.5 mm for the $AR = 1$ and 2 cases. It can be seen from Figure 2.22 that this largely results from the separation point shifting forward and that the reattachment point shifts upstream by a relatively small distance.

On the other hand, Grossman & Bruce (2017) performed oil flow visualisation for Mach 2 $12^\circ$ incident – reflected SBLIs at $AR = 1.00$ and 1.35 and found that for the higher aspect ratio cases the separation length was significantly greater (although Wang et al. (2015) did record a small increase in this range). However they also found that the length increase was mainly due to the separation point moving upstream. They also tested SBLIs with $AR = 1.25$, using shock generators with different thicknesses ($T = 15, 19$ and $23$ mm) and one with a rounded corner (RC) in order to control where the expansion fan met the interaction. They showed that the interaction separation length linearly scaled with the distance from the inviscid incident shock.
2.2. Unsteadiness in SBLIS

Figure 2.21: Pressure contours of $AR = 2$ case at $y = 20$ mm (Wang et al., 2015).

Figure 2.22: Pressure distribution through different aspect ratio interactions showing separation and reattachment points defined as $c_f = 0$ (Wang et al., 2015).

The impingement point $x_{imp,inv}$ to the impingement point of the first expansion wave $x_{exp}$ (shown in Figure 2.23).

Figure 2.23: Variation of separation length with $L = x_{exp} - x_{imp,inv}$ (Grossman & Bruce, 2017).

Grossman & Bruce (2017) also investigated the effect of the gap between the shock generator and the tunnel sidewalls (‘sidewall gap’) on separation length and their results seem to be largely in agreement with those of Wang et al. (2015). They found that for the sidewall gap increasing from 0 to 3 to 10 mm, the separation length decreased from 44 to 39 to 30 mm. The sidewall gap appears to affect how ‘two-dimensional’ the interaction is. An increase in sidewall gap can be
associated with increased two-dimensionality (see Figure 2.24 which demonstrates a decreasing thickening of the sidewall boundary due to the incident shock glancing the sidewalls). This shows that the separation length decreased with increased two-dimensionality. In contrast, Wang et al. (2015) found that it increased and plateaued for decreasing two-dimensionality. This demonstrates that the study of Wang et al. (2015) may have covered more regimes of the dependence of separation length on aspect ratio than was possible in the experiments of Grossman & Bruce (2017).

![Figure 2.24](image)

(a) 0 mm sidewall gap.  (b) 3 mm sidewall gap.  (c) 10 mm sidewall gap.

**Figure 2.24:** Schlieren photographs of Mach 2 12° reflected interactions with varying sidewall gaps (Grossman & Bruce, 2017).

Grossman & Bruce (2017) increased their aspect ratio by decreasing the \( y \)-dimension of the domain; whereas Wang et al. (2015) increased their aspect ratio by increasing the domain size (in both absolute size and number of grid points) in the \( z \)-direction. Benek et al. (2016) and Bisek (2015) also effectively increased the aspect ratio by increasing the domain size in the \( z \)-direction by employing spanwise boundary conditions for the \( \delta/w = 0 \) cases. It is possible that decreasing the effective duct height over the range of \( AR \) tested by Grossman & Bruce (2017) would not affect whether the weak shock discussed above reaches the central plane or not. Varying the aspect ratio by moving the shock generator itself affects the expansion fan placement. This is the manner in which Grossman & Bruce (2017) and most other experimental arrangements vary their aspect ratio. In numerical simulations, the aspect ratio is typically varied by making the domain wider which only affects how far into the core flow the swept SBLIs from the corners reach.

Benek et al. (2014) noted that their tests at \( W/H = 0.5 \) are not directly comparable to experiments conducted at the same aspect ratio as \( \delta/H \) will be greater in experiments. As such, they proposed there would be less low-energy flow induced by the sidewall shocks swept into the corner flows. This may mean that the experimental higher aspect ratio cases of Grossman & Bruce (2016, 2017) were less subject to three-dimensional effects and were more in the two-dimensional (low \( \delta/W \)) regime of Figure 2.18. Figure 2.25 shows that the results of Wang et al. (2015) exhibit similarity to those of Benek et al. (2014) while those of Grossman & Bruce (2017) cannot be compared in the same trend.

Aside from this, the investigation of Benek et al. (2014) into the variation of separation length with the viscous aspect ratio demonstrates that the separation length varies unpredictably and non-linearly with different shock strengths and Mach numbers. Figures 2.25a and 2.25b show the large variation in separation length (here denoted by \( \Delta x \)) across single and multiple configurations.
2.2. UNSTEADINESS IN SBLIS

(a) Mach 2.9 (Benek et al., 2014).

(b) Mach 2.5 (Benek et al., 2014).


Figure 2.25: Previous studies of separation length as a function of viscous aspect ratio $\delta/W$.

Wang et al. (2015) proposed a classification of the three-dimensionality of reflected SBLIs based on the virtual conical origin (VCO) of the shock generator leading edge. This is shown in Figure 2.26. Point F defines how far the shock penetrates the sidewall boundary layer, which the authors showed to be at the limit of the penetration Mach number $M_p = 1/\sin \beta$, where $\beta$ is the shock angle. Points C and S are the locations of the incident shock distortion due to the corner separation, discussed previously.

For $\mathcal{A} > 2 \tan \beta_S$, the sidewall shock discussed earlier does not affect the centre of the domain and hence the interaction could be considered quasi-two-dimensional. If the sidewall shock (S) meets the centre of the domain, the interaction will be highly three-dimensional as the incident shock is distorted and strengthened towards the centre, causing the separation point to move upstream. If the compression waves (C) meet the centre of the domain, the incident shock is weakened and the separation length decreases. Lastly, if $\mathcal{A} < 2 \tan \beta_F$, the sidewall interactions will occupy the entire channel.

Figure 2.26: Proposed model of incident oblique shock three-dimensionality (Wang et al., 2015).

As demonstrated above, we are only beginning to understand the drivers in separation length change due to changes in geometry. It appears that there may be different regimes dependent
upon flow deflection angle (Benek et al., 2014) or dependent upon the shock generator height (Grossman & Bruce, 2017). This brings into question whether taking the centreline separation length to reduce the low-frequency unsteadiness is a valid approach. The changing length may only be representative of a linear change in dynamics when other variables, such as Mach number, flow deflection angle or duct geometry, are held constant. On the other hand, it may be the case that separation length is a good proxy for the separated dynamics of the interactions and therefore we might expect the characteristic frequency to scale accordingly.

An example to illustrate this is the experiments conducted in the IUSTI wind tunnel at $M = 2.3$ on incident – reflected SBLIs with different flow deflection angles. Figure 2.27a shows the changing Strouhal number based on the centreline separation length for these cases (the four circles at $M = 2.3$). The Strouhal number increases with increasing flow deflection angle. Figure 2.27b shows that an increased flow deflection angle was achieved whilst holding the incident shock impingement location constant, thus leading to a small decrease in the confinement (decrease in $R$).

![Variation in Strouhal number for different configurations and strengths (Dussauge et al., 2006).](image1)

![IUSTI shock generator set-up (Haddad, 2005).](image2)

**Figure 2.27:** Example of potential impact of confinement on reduced unsteadiness.
2.2. UNSTEADINESS IN SBLIS

It might be assumed that the behaviour here follows the same trends as in the experiments of Grossman & Bruce (2017), as the width of the duct is not changing, for example. Therefore a decrease in $A$ might be expected to be accompanied by an decrease in separation length. Therefore it is possible that without the change in test article geometry the reduced frequency would have collapsed for these experiments. Conversely the reason that they did not collapse could be indicative of a change in the unsteady behaviour due to the change in geometry.

Aside from specific studies, there is evidence in other studies of the importance of three-dimensionality. Humble et al. (2007) found a change in flow properties at distances further than 30% of the tunnel span from the centreline despite a quoted viscous aspect ratio of $1/14 \approx 0.07$.

"...slight three-dimensional effects exist. However, they seem characteristic of the fluid dynamic processes present and not due to the sidewall boundary layers." – Humble et al. (2007)

2.2.7 Three-dimensionality and unsteadiness

Numerical studies of SBLIs (two- or three-dimensional) are computationally expensive, as the number of grid points scales with Reynolds number $Re_\theta$ which is typically high in supersonic flows. For DNS, computational cost increases with $Re_\theta^3$ (Pope, 2000). For studying SBLI unsteadiness, solving Reynolds-averaged Navier-Stokes (RANS) or unsteady RANS (URANS) is inadequate (Wang et al., 2015). This leaves DNS and LES and variants thereof, with the former being the most computationally expensive.

"Numerical investigations (DNS, LES) for impinging SWBLI that reached sufficiently long integration times, suitable for addressing the low-frequency unsteadiness, are rare in the literature." – Pasquariello et al. (2017)

It is difficult to compare the temporal length of numerical and experimental studies as the practice in numerical studies is to normalise time-scales by the incoming boundary layer height $\delta$; whereas experimental studies usually quote in terms of separation length $L_{sep}$ such that one low-frequency period at $St_L = 0.03$ can be expected to be around $T_{char} = 33L/U_\infty$. The relation $\delta/L_{sep}$ varies unpredictably across interactions; however as pointed out by Clemens & Narayanaswamy (2014) upstream boundary layer fluctuations are of order $U_\infty/\delta$ so under the shock foot motions are of the order $0.01U_\infty/\delta$, giving a time-scale for the period of approximately $100\delta/U_\infty$. A collection of numerical studies investigating the low-frequency unsteadiness is given in Table 2.1, from which it can be seen that numerical studies can be expected to capture at most 90 low-frequency cycles, as quoted by the authors in the small-span case of Touber & Sandham (2009).

As noted by Touber & Sandham (2011b), there is a seemingly inherent broadband nature of the low-frequency fluctuations. Therefore, in order to perform reliable statistical analyses or well-resolved spectra, capturing the shape of the PSD, many samples of low-frequency motion are required.

Table 2.1 shows that Pasquariello et al. (2017) ran simulations at a significantly higher Reynolds number than other simulations and match the experimental conditions of Daub et al. (2016). This enabled them to capture the intermittent behaviour of the low-frequency separation...
Table 2.1: Numerical studies considering low-frequency plateau in SBLI unsteadiness. The symbols † and ‡ denote the same codes. In some instances runtime was only specified in $T_{\text{char}} \approx 100\delta/U_{\infty}$.

<table>
<thead>
<tr>
<th>Study</th>
<th>Type</th>
<th>$M$ [-]</th>
<th>$Re_{\theta}$ [-]</th>
<th>Time [$\delta/U_{\infty}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adams (2000)</td>
<td>compression ramp</td>
<td>3.0</td>
<td>1685</td>
<td>385</td>
</tr>
<tr>
<td>Pirozzoli &amp; Grasso (2006)</td>
<td>reflected</td>
<td>2.25</td>
<td>3725</td>
<td>25</td>
</tr>
<tr>
<td>Priebe et al. (2009) †</td>
<td>reflected</td>
<td>2.9</td>
<td>2300</td>
<td>975</td>
</tr>
<tr>
<td>Touber &amp; Sandham (2009)</td>
<td>reflected</td>
<td>2.3</td>
<td>5100</td>
<td>10,000</td>
</tr>
<tr>
<td>Pirozzoli et al. (2010) ‡</td>
<td>reflected</td>
<td>2.28</td>
<td>2280</td>
<td>1203.4 - 2354.8</td>
</tr>
<tr>
<td>Priebe &amp; Martín (2012) †</td>
<td>compression ramp</td>
<td>2.9</td>
<td>2900</td>
<td>1000</td>
</tr>
<tr>
<td>Agostini et al. (2012)</td>
<td>reflected</td>
<td>2.3</td>
<td>5000</td>
<td>150$T_{\text{char}}$</td>
</tr>
<tr>
<td>Hadjadj (2012)</td>
<td>reflected</td>
<td>2.28</td>
<td>5350</td>
<td>$\sim$ 850</td>
</tr>
<tr>
<td>Aubard et al. (2013)</td>
<td>reflected</td>
<td>2.25</td>
<td>2725</td>
<td>900</td>
</tr>
<tr>
<td>Bermejo-moreno et al. (2014)</td>
<td>reflected</td>
<td>2</td>
<td>14,000</td>
<td>2000</td>
</tr>
<tr>
<td>Nichols et al. (2017) ‡</td>
<td>reflected</td>
<td>2.28</td>
<td>2300</td>
<td>2037 - 4660</td>
</tr>
<tr>
<td>Pasquariello et al. (2017)</td>
<td>reflected</td>
<td>3.0</td>
<td>14,000</td>
<td>3805</td>
</tr>
<tr>
<td>Jammy &amp; Sandham (2017)</td>
<td>reflected</td>
<td>2.7</td>
<td>4300</td>
<td>23$T_{\text{char}}$</td>
</tr>
</tbody>
</table>

Shock motion, which is not seen in simulations at lower $Re_{\theta}$ as the shock is diffused by the greater viscous action of the boundary layer and hence does not penetrate as far as it would in experiments.

They employed spanwise-periodic boundary conditions and considered the spanwise variation in the time-averaged (over $3805\delta/U_{\infty}$) skin friction coefficient $\langle C_f \rangle$ (Figure 2.28). They showed that the mean skin friction did not converge over the long-time integration near separation and reattachment, indicating that phenomena with long periods existed here. They also found high levels of spanwise-normal Reynolds stress $\langle w'w' \rangle$ along the detached shear layer from the reflected shock foot to the separation bubble apex and near the reattachment location, with a maximum at the wall $3\delta$ downstream of the reattachment location.

![Figure 2.28](image)

Figure 2.28: Mean skin friction evolution (black) with local minima and maxima shown in grey for long integration time $3805\delta/U_{\infty}$ and black dots for $446\delta/U_{\infty}$ (Pasquariello et al., 2017).

Using DMD, the authors found dynamically important low-frequency modes in the flow (see Figure 2.15b) and considered three-dimensional dynamics, shown in Figure 2.29. They found that the separation line (at $(x - x_{\text{imp}})/\delta \approx -12$) moved two-dimensionally (which was also found
by Wu & Martín (2008)); while at reattachment (at \((x - x_{\text{imp}})/\delta \approx 4\)) there were ‘Görtler-like’ vortices \(2\delta\) in width which caused a spanwise wave in the reattachment line which appeared to be connected to overall bubble breathing motion. They also confirmed that the vortices were of width \(2\delta\) in a simulation with a greater span of \(4.5\delta\).

Wu & Martín (2008) observed a large-amplitude, large-wavelength spanwise wrinkling of the shock with a wavelength greater than \(4\delta\) correlated with the incoming boundary layer momentum. However, they found that this spanwise ‘wrinkling’ was smaller than the large-scale streamwise shock motion.

There are even fewer numerical studies with sidewalls considering unsteadiness, as the combined computational expense of additional no-slip boundaries (and hence more grid refinement) and a long runtime is very high. In addition to this, Wang et al. (2015) note that in an incident – reflected shock interaction, the sidewall interaction is effectively a glancing fin interaction which is highly dependent on its virtual conical origin, the shock generator tip. Therefore to create an accurate simulation the domain must include the shock generator as well as the two sidewalls (Wang et al., 2015). Their study considered temporal-mean features to confer on the theory that where the sidewall shocks meet the main shock determines the two-dimensionality of an incident – reflected interaction, as proposed by Babinsky et al. (2013).

Using this dataset, Jammy & Sandham (2017) studied SBLI unsteadiness with sidewalls over about 23 low-frequency periods.

Jammy & Sandham (2017) found that the most energetic instance of \(St = 0.03\) occurred at one side of the centreline asymmetrically (see Figure 2.30). They attributed this to the short run time of the simulation due to the higher Strouhal numbers exhibiting more symmetric distributions. However it is also possible that some configurations exhibit an asymmetry or that the system is bi-stable, similarly to there being a difference between the half-span and full-span cases of Bisek (2014) and Bisek (2015), for example. Some previous experimental examples of SBLIs which are asymmetric about the spanwise centre in the mean are shown in Figure 2.31. This gives further credit to the idea that taking the centreline separation length or frequency alone may not be characteristic of the whole interaction.
CHAPTER 2. LITERATURE REVIEW

Figure 2.30: Weighted power spectral density across span of separation shock for different Strouhal numbers for a Mach 2.7 \( 9^\circ \) incident – reflected SBLI (Jammy & Sandham, 2017).

(a) PIV of Dussauge et al. (2006) (flow is left to right).

(b) Surface streak patterns of Settles et al. (1979) (flow is bottom to top).

Figure 2.31: Examples of asymmetry in separation regions.

Jammy & Sandham (2017) also looked at two-point correlations between pressure fluctuations in the corner separations and those at more central locations across the span. They did not find significant correlations that extended into the central separation and therefore concluded that the low-frequency dynamics of the two regions were independent.

Experimentally, Funderburk & Narayanaswamy (2016) considered the differences in the unsteady behaviour of the central and corner separations in a compression ramp interaction, motivated by a better understanding of engine unstart. Noting the differences in the cross-coherences of the power spectra under the most energetic locations at the centre and corner with locations upstream and downstream of them, they concluded that the separations behave independently of each other. This is consistent with the conclusion of Jammy & Sandham (2017). However, they were not able to simultaneously measure the pressure in the two regions due to the constraints of the experimental arrangement.

There were also some early experimental studies concerning unsteadiness over small streamwise and spanwise regions. Ünalms & Dolling (1994) investigated how far in the spanwise
direction the pressure fluctuations decay and found small but non-zero spanwise correlations. Similarly, Beresh et al. (2013) considered spanwise correlations after low-pass filtering their pressure data and found weak correlations in the spanwise direction at low frequencies.

Muck et al. (1985) inferred from streamwise correlations that a streamwise fanning of the shock did not exist and therefore the apparent thickening of the shock in shadowgraph images must have been due to spanwise rippling. Moreover, Muck et al. (1985, 1988) considered streamwise correlations and spanwise pressure histories to conclude that the shock motion comprises of a streamwise ‘flapping’ and a spanwise ‘rippling’, again showing that a single frequency at a single location does not characterise the shock unsteadiness.

"no single frequency seems to play a dominant role and the typical frequency quoted by previous workers to characterize the shock appears to be some average measure of the frequency range in which the dominant gross unsteadiness occurs."

– Muck et al. (1985)

In agreement with this, Threadgill & Bruce (2016) argued that taking the frequency corresponding to the maximum power may not be representative of a ‘characteristic frequency’, in particular if the power spectrum has a poor frequency resolution, as is often the case. This is also true of spectra that have broadband low-frequency peaks spanning a range of near-maximum powers. While Threadgill & Bruce (2016) argued that a more representative frequency could be found by taking an area-weighted peak frequency, the question remains as to how much this approach is furthering our understanding of the low-frequency unsteadiness. Furthermore, as found by Touber & Sandham (2011b), the broadband nature of SBLI unsteadiness appears to be an inherent feature. However characteristics other than the frequency at which the maximum power occurs have been largely neglected.

2.3 Resulting Aims

There is a large scatter in the non-dimensionalised characteristic low-frequency unsteadiness exhibited across different SBLIs, and various attempts to incorporate correction factors have yielded little collapse.

Importantly, the entire shape of the low-frequency peak in PSDs is relevant from a design perspective, not just the frequency corresponding to the maximum power, which has been the focus historically. Little or no attention has been paid to characteristics such as the energy content, skewness and kurtosis which would determine how prevalent these frequencies are. For example, if an interaction were known to exhibit a steep peak at one characteristic frequency, it may be possible to design a damper around this. On the other hand, designers could steer towards an interaction which has a shallow peak in which a range of frequencies with comparatively little energy exist.

The similarities between the wind tunnel working section to be used in this study and a supersonic engine intake can be noted from Figure 2.32. Whilst many previous experimental studies have assumed two-dimensionality, the real test geometry is a good model of the reality of a full engine intake. Therefore in considering the entire geometry and interaction, there is an
opportunity to conduct tests on changing the intake geometry by changing the shock generator angle and height.

Finally, there appears to be some evidence that recent work exploring variations in separation region lengths in SBLIs with different geometry test cases (Benek et al., 2013; Grossman & Bruce, 2017; Poggie & Porter, 2018) might offer some insight into the spread in the reduced low frequencies; or the cause of why a reduction should not be expected. With all of this in mind, this leads onto the below aims for the study.

1. Explore the relationship between the separation lengths and characteristic low frequencies of $M = 2$ incident – reflected SBLIs by testing interactions of the same strength but at different duct aspect ratios. This is currently assumed to be linear for a constant freestream velocity by $St_L = 0.03$.

2. Perform an overall qualitative and quantitative comparison of the variation in the low-frequency part of the PSDs for interactions with differently shaped separation regions. Previous characterisations have typically been attempted by taking only the reduced frequency.

3. Identify the dominant flow features in SBLIs and their inter-dependence. For example, find the relationship between the amount of separation and the severity of the unsteady behaviour in order to find an optimum for intake geometries.

4. Produce the first experimental two-dimensional map of SBLI unsteadiness in the $x−z$ plane in order to characterise the spatial organisation of the different traits of the unsteadiness.

5. Use simultaneously measured pressure histories to evaluate the significance of any motions and correlations in the spanwise direction to explore the unsteady flow physics of SBLIs in real three-dimensional geometries.
Chapter 3

Experimental facility

This chapter gives information on three key aspects of how the results to be presented in Chapter 4 and Chapter 5 were obtained. Firstly Section 3.1 gives information about the wind tunnel facility in which the experiments were conducted; secondly Section 3.2 details the methods used to acquire data and lastly Section 3.3 explains the post-processing of the obtained data and contains a table of estimated uncertainties.
3.1 Imperial College Supersonic Wind Tunnel

Experiments were conducted in the Imperial College London blow-down supersonic wind tunnel, fitted with a converging-diverging nozzle to give a nominal freestream Mach number $M_\infty = 2$. The tunnel was supplied with dried air stored at 400 psi in tanks totalling $48 \, \text{m}^3$. Air from the tanks entered the settling chamber through a pneumatic valve which was operated using a PID (proportional-integral-derivative) controller in LabVIEW to control the stagnation pressure.

The working section of the tunnel had a constant cross-section measuring $150 \times 150 \times 727 \, \text{mm}$ (height $\times$ span $\times$ streamwise length) before the flow entered a diverging diffuser which exhausted to atmosphere, as shown in Figure 3.1. The set-up resulted in a unit Reynolds number of approximately $2 \times 10^7 \, \text{m}^{-1}$. For this configuration, Threadgill & Bruce (2016) characterised the boundary layer in the tunnel, obtaining the values listed in Table 3.1.

![Schematic diagram of wind tunnel](Threadgill & Bruce, 2014).

Table 3.1: Compressible boundary layer parameters in Imperial College supersonic wind tunnel ahead of $10^\circ$ reflected shock interaction (Threadgill & Bruce, 2016).

<table>
<thead>
<tr>
<th>$\delta_0$ [mm]</th>
<th>$\delta^*$ [mm]</th>
<th>$\theta$ [mm]</th>
<th>$H$ [-]</th>
<th>$C_f$ [-]</th>
<th>$Re_\theta$ [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.71</td>
<td>0.87</td>
<td>0.29</td>
<td>2.98</td>
<td>0.0023</td>
<td>7,900</td>
</tr>
</tbody>
</table>

Tests were conducted with the shock generators and instrumentation installed in the more upstream of the two floor and ceiling liners of the constant area part of the test section (see Figure 3.2). The shock generators were steel wedges with their upper sides parallel to the ceiling and the more upstream part of their undersides angled into the flow to create the shocks.

The shock generators were mounted to the ceiling liner by two columns which screwed into the top of the wedge. The streamwise position could be varied by installing the columns at different ceiling ports (at 45 mm increments) and by screwing the columns into different positions along the wedge. The shock generators could be positioned at different distances from the ceiling by a traverse motor connected to the two mounting columns. This enabled the aspect ratio $A_r$, defined here as the ratio of shock generator leading edge height from the floor to the (fixed) duct width, to be varied.

Tests were conducted on three incident - reflected SBLIs, spanning the range of viable strengths in the configuration. The weakest was generated by turning the flow $8^\circ$; this was
shown to exhibit no mean reversed flow by Threadgill & Bruce (2015) and thus can be considered incipiently separated. The strongest was generated by turning the flow $12^\circ$ which approaches the limit of blockage with which the tunnel can run: Grossman (2017) ascertained that this strength can be sustained for an aspect ratio $\mathcal{A} \leq 1.38$. The intermediate strength was generated by turning the flow $10^\circ$, Threadgill & Bruce (2016) classed this SBLI as fully separated.

Experiments were conducted on all three geometries at $\mathcal{A} = 1$ and $\mathcal{A} = 1.3$. Additional tests at $\mathcal{A} = 2$ were conducted for the $8^\circ$ and $10^\circ$ strength cases. There was a small gap between the shock generators and each of the sidewalls (1 mm for the $8^\circ$ and $10^\circ$ and 3 mm for the $12^\circ$) such that they spanned 99% or 96% of the width of the test section.

The $12^\circ$, $10^\circ$ and $8^\circ$ shock generators were 19.4 mm, 12 mm and 9.5 mm thick, respectively. Therefore the distances between the inviscid impingement point of the incident shock and the first expansion wave generated by the convex corner on the underside of the wedge impinging on the floor in each case were those shown in Table 3.2. Grossman & Bruce (2018) showed that the centreline separation length of a $12^\circ$ reflected SBLI grew linearly with the distance between these two points.

<table>
<thead>
<tr>
<th>$\mathcal{A}$</th>
<th>$\mathcal{A} = 1$</th>
<th>$\mathcal{A} = 1.3$</th>
<th>$\mathcal{A} = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$12^\circ$</td>
<td>26.0</td>
<td>37.5</td>
<td></td>
</tr>
<tr>
<td>$10^\circ$</td>
<td>10.9</td>
<td>21.6</td>
<td>34.0</td>
</tr>
<tr>
<td>$8^\circ$</td>
<td>16.5</td>
<td>26.0</td>
<td>37.1</td>
</tr>
</tbody>
</table>
3.2 Experimental methods

Two primary experimental techniques were employed in this investigation to fulfil the aims of the study. The mean separation regions were found and characterised using oil flow visualisation. The frequency spectra and unsteady separation shock behaviour were analysed from high-frequency recordings of the pressure at the floor. It was not feasible to conduct oil flow and pressure measurements simultaneously due to the risk of oil entering the diaphragms of the pressure sensors. However, as care was taken to place the interaction at the same location in each instance and the oil flow gives a temporal mean image of the separation region anyway, this is deemed unimportant.

3.2.1 Oil flow visualisation

An oil mixture comprising titanium dioxide powder, kerosene and oleic acid was injected into the working section once the wind tunnel was started and interactions established. The oil entered the working section via holes in the floor plate. A copper rod lined the inside of the oil injection ports and led to plastic tubes connected to oil-filled syringes. Through careful control of the oil flow mixture, the low pressure of the working section drew the oil through the injection ports with as little momentum as possible. In this manner, as little disturbance to the incoming boundary layer as possible was caused. Images taken shortly after a large injection were discarded.

The injection ports were located at least 40 mm upstream of the separation shock for all cases. There were five floor ports: one 1 mm in diameter located on the spanwise centre, two at ±17 mm and a further two at ±65.5 mm from the tunnel centreline, all 3 mm in diameter.

Where possible, the final separation region images were constructed using different injection ports. A mixture of the three central holes was used to build up a composite image of the central separation regions, taking the parts of the images where the separation regions appeared to be least affected by the injection. The two outer holes were used to visualise the corner separations.

The tunnel floor was sprayed with matt black paint to maximise the contrast with the white titanium dioxide. The working section was illuminated using lights with diffusers to minimise reflections from the window glass and over-exposure of parts of the oil.

Stills were taken with a Nikon D7000 DSLR (digital single-lens reflex) camera through the side window. The camera was mounted to a tripod with a view as planar to the tunnel floor as possible so as to minimise the amount of de-warping necessary during post-processing (see Section 3.3). Stills were also taken of a calibration image (square graph paper) with the wind tunnel off and the same camera position was used during the runs for each test case.

3.2.2 High-frequency pressure measurements

The pressure histories during runs were recorded using Kulite XCQ-093-15PSIA miniature pressure transducers. The manufacturer-quoted resonant natural frequency of the sensors was 190 kHz. The sensor heads were 2.4 mm in diameter and mounted flush with the wind tunnel floor. Samples were taken at 1 MHz over 14 s, giving $1.4 \times 10^7$ data points per run. An example pressure history is shown in Figure 3.3. It shows that the dynamics at each channel appear to
be captured very well by the high sampling frequency and 50kHz filtering (described later), as the plots are very smooth. One of the lines has markers showing that there are 100 samples in the 100 μs window. The minor grid lines are at the frequency of the low-pass filter.

![Figure 3.3: Example pressure history over ten channels recorded simultaneously at different spanwise positions $Z^*$](image)

A sensor mount that enabled the pressure history to be found under the interactions with as fine a spatial resolution as possible within machining tolerances was designed. Tests were conducted with 10 Kulite sensors placed in the same streamwise axis and spaced 3.6 mm apart in the spanwise direction (giving a gap between the edges of neighbouring sensors of 1.8 mm). This enabled the pressure to be measured at quarter-spans of the tunnel floor. Preliminary experiments revealed that significant cross-correlations existed across spanwise distances of 12 mm (Rabey & Bruce, 2017). This mount is shown in Figure 3.4a.

As can be noted from Figure 3.4b, the mount was placed inside a larger mount which could be reflected in its spanwise axis. This was so that the sensors could be shifted in order to measure 6.8 mm further upstream or downstream, without moving the shock generator. This ensured that the interaction was not significantly changed in aspect ratio or by impinging on a different location along the boundary layer. The measurement locations are represented by the circles in Figure 3.5 which are to the scale of the sensor heads. This range was chosen to cover the estimated intermittency length of the 12° interaction (blue arrow), which is expected to exhibit the largest shock motion. The solid coloured lines show the predicted mean reflected shock foot locations.

With the mounting arrangement shown in Figure 3.6, the sensors were measuring at position ‘A’ shown on the upper x-axis in Figure 3.5. With the sensors in the downstream measurement position (mount reflected in spanwise axis such as in Figure 3.4b), the sensors were measuring at position ‘B’ shown in Figure 3.5. The streamwise resolution was increased five-fold by also shifting the shock generator up and down for each of these ‘large scale’ Kulite placements. These additional measurement points are shown by the subscripts ‘+’ and ‘−’ in Figure 3.5. For the 8°, 10° and 12° interactions, the shock generator was moved up or down by 1.03, 1.11 and
1.21 mm respectively in order to effectively shift the interactions by $\pm 1.36$ mm in the streamwise direction, giving a total of 10 equally spaced measurement locations. These are represented by dashed lines in Figure 3.5.

In shifting the shock generator by such small vertical increments, the aspect ratio was at worst $A = 1.016 \pm 0.016$, $A = 1.300 \pm 0.027$ or $A = 2.000 \pm 0.061$ which causes a minimal change in the constriction and in the interaction boundary layer thickness at the shock impingement. For each of these movements, the plug could also be placed at four spanwise positions across the wind tunnel. This gave a total of 40 runs for each of the $8^\circ$, $10^\circ$ and $12^\circ A = 1$ test cases, giving $5.6 \times 10^9$ pressure samples per test case. It was found that the full range of shock motion...
3.2. EXPERIMENTAL METHODS

Figure 3.5: Diagram showing measurement locations (dashed lines) relative to predicted mean reflected shock location (solid lines) and expected range (arrows) of separation shock motion. Distance on x-axis is from upstream edge of the floor plate.

Figure 3.6: Sensor mounting arrangement in ‘A’ location.

was not captured with this streamwise range for the $12^\circ \mathcal{R} = 1.3$ case. In this instance a further four measurement positions further downstream were taken, giving a total of 56 runs.

An example series of schlieren images of the five small changes in aspect ratio around $\mathcal{R} = 1$ is shown in Figure 3.7. A close-up series of the interaction regions is also shown in Figure 3.8. It can be seen that there is no discernible difference in the interactions across the range of shock generator positions used in this test case.

All schlieren still images in the study were taken using a Z-type configuration consisting of a point light source, two 200 mm parabolic mirrors, a vertical knife-edge and a high-speed camera.
CHAPTER 3. EXPERIMENTAL FACILITY

The camera was a Phantom v641 focussed at infinity with an exposure time of 30 μs and the knife-edge was oriented such that regions of large positive streamwise density gradients (shock waves) appeared clearly as shadows.

![Figure 3.7](image1.png)  
(a) $R = 1.000$. (b) $R = 1.008$. (c) $R = 1.015$. (d) $R = 1.023$. (e) $R = 1.031$.

**Figure 3.7:** Effect of varying aspect ratio on $10^\circ$ $R = 1$ interaction. Full schlieren image.

![Figure 3.8](image2.png)  
(a) $R = 1.000$. (b) $R = 1.008$. (c) $R = 1.015$. (d) $R = 1.023$. (e) $R = 1.031$.

**Figure 3.8:** Effect of varying aspect ratio on $10^\circ$ $R = 1$ interaction. Schlieren image zoomed-in on SBLI.

Figure 3.6b also shows the origin of the axis system to be used henceforth. It is at the incident shock inviscid impingement point on the spanwise centre of floor of the duct $(x_0, 0, 0)$. Measurements in the $x$-direction are to be normalised by the centreline separation length $X^* = (x - x_0)/L_{sep}$. Measurements will not be normalised in the $y$-direction and in the $z$-direction are to be normalised by the tunnel half-span $Z^* = z/(w/2)$.

Each Kulite sensor comprises a diaphragm which becomes stressed due to a pressure differential across its two surfaces. This diaphragm is connected to a four-arm Wheatstone bridge such that the resistance varies according to how much stress the diaphragm is under. The sensors were excited with 10 V, supplied by a Fylde FE-579-TA Bridge Amplifier. The output from the sensors was of the order of mV and was also passed through the amplifier, with each channel applying a gain of 30 to maximise the precision of the measurements. At this stage the signals were also low-pass filtered with a cut-off frequency of 50 kHz by resistors in the channels. This cut-off is well above the expected low-frequency dynamics of interest which are at $f \leq 10$ kHz. It is also well below the bandpass filter cut-off required to prevent signal aliasing (500 kHz).

The output of the amplifier was connected via BNC (Bayonet Neill–Concelman) cables to two BNC-2110 differential terminal blocks with the channels set to floating source. This was found to give less noise than ground source. The terminal blocks were connected to two PXI-6366 DAQ (Data Acquisition) modules. These were able to sample up to 2 Ms/s/channel, twice the sampling frequency, simultaneously. Each signal was sampled differentially to minimise earthing errors. The entire system was powered by a direct current battery to ensure that the signals were not contaminated with the alternating mains current frequency.
3.3 Data processing and error estimation

3.3.1 Oil flow visualisation

A MATLAB program was written to de-warp the oil flow images such that accurate estimates of separation lengths and fair comparisons between different test cases could be made. The program prompts the user for the maximum vertical and horizontal lengths available from the graph paper in the reference image in order to minimise the precision error. Then, the user clicks on the four corners around these lengths. This two-dimensional mapping given by (3.1) has the most degrees of freedom possible as it is likely that the images were not taken completely parallel to the floor (and therefore not just skewed). The program solves for the projective transform matrix $T$ (3.2) and applies the projection to the oil flow images.

$$
\begin{bmatrix}
  x' \\
  y' \\
  \text{norm}
\end{bmatrix}
= 
\begin{bmatrix}
  A & B & C \\
  D & E & F \\
  G & H & 1
\end{bmatrix}
\begin{bmatrix}
  x \\
  y \\
  1
\end{bmatrix}
$$

(3.1)

$$
T = 
\begin{bmatrix}
  A & D & G \\
  B & E & H \\
  C & F & 1
\end{bmatrix}
$$

(3.2)

The images were converted into greyscale (as the tunnel floor was black and oil mixture white, any colour was a reflection and unnecessary), sharpened and the contrast increased. They were aligned to a common frame of reference using the central oil injection port. The inviscid impingement point could then be marked on the images and separation lengths extracted and used as the reference length.

Image resolution was typically at least 10 pixels per mm and therefore any separation length is quoted to the nearest mm. Furthermore, sometimes the oil settled over an area covering several pixels. In these cases care was taken to be consistent in selecting the outermost pixel.

3.3.2 High-frequency pressure measurements

The gain from the amplifier and sensitivity of each sensor were accounted for to obtain data in gauge pressure. The mean sensitivity of the sensors (quoted by the manufacturer) was 95.8 mV/Bar, with a standard deviation of 3.54 mV/Bar. The gauge pressures were then added to the atmospheric pressure recorded from a barometer in the laboratory to obtain absolute pressures and finally the sample mean was subtracted from the dataset of each sensor.

Obtaining the correct absolute pressure is not essential for the characterisation of the unsteadiness of pressure measurements (frequencies contained in the signals and their energy content). These are extracted from the pressure histories by expressing them as a power spectral density (PSD). The PSD in frequency space $\hat{x}(\omega)$ of a signal in time $x(t)$ is defined as in (3.3). It is the expression of a signal as the sum of sine waves $\mathfrak{I}(e^{-2\pi i\omega t})$ with frequencies $\omega$ and amplitudes $\hat{x}(\omega)$. 

55
\[ \hat{x}(\omega) = \int_0^T x(t)e^{-2\pi i \omega t} dt \] (3.3)

In reality the signal is discrete and therefore cannot be integrated exactly over all time \( T \) but rather summed over the individual sample points. The in-built discrete Fourier transform in MATLAB was used to perform this calculation. The discrete or fast Fourier transform is an algorithm that quickly computes the Fourier transform on a signal that has \( 2^n \) samples.

A sensitivity study was conducted to identify optimal parameters for producing the final averaged PSDs using segments of the full sample. Figures 3.9 and 3.10 show the effect of using different segment lengths and different window overlaps (the percentage of a segment that overlaps the previous one), respectively. The frequency resolutions (one period of the time of the window) resulting from the different window sizes are 30.5 Hz, 61 Hz and 122 Hz for Figures 3.9a, 3.9b and 3.9c, respectively. Figure 3.9 shows that all of these frequency resolutions are low enough to give confidence that the peak has been captured (the power decreases towards zero at lower frequencies).

Figure 3.9a exhibits several peaks on the main peak of each PSD which suggests that this window length does not provide enough averaging to mitigate the noise of the signal. It could be that these peaks are somewhat physical but would still cause difficulty in selecting a representative frequency. On the other hand, Figure 3.9b captures much of the same energy distribution without such abrupt spikes. Figure 3.9c has the smallest window length and therefore the final spectra are composed of the average of more segments. It can be seen that this has caused the peaks to become unclear with the peak often appearing as a plateau between two frequency resolutions. A segment length of \( 2^{14} \) samples was selected as a good compromise between smoothness and containing lots of low-frequency information for calculating the area-weighted median frequency, the procedure for which is described below.

Figure 3.10 shows that the PSDs are less sensitive to the overlap of the windows. However the signals become slightly more noisy with a 25\% overlap when fewer windows enter the sample and slightly more smooth with a 75\% overlap when more windows enter the sample to be averaged. Therefore a window length of \( 2^{14} \) and 50\% overlap was selected to produce the final PSDs with a frequency resolution of 61 Hz.

![Figure 3.9: Effect of varying window size on PSDs (50\% overlap).](image)

Each 14 \times 10^6 channel sample history was divided into 1705 50\% overlapping segments of 16384 (\( 2^{14} \)) data points. The mean was subtracted again to mitigate low-frequency noise from
any wandering of the total pressure $p_0$ during the run and the segments were multiplied by a Hanning window function. This reduced the spectral leakage that would otherwise occur from effectively using a boxcar window (which is a summation of sine waves of infinite frequencies in Fourier space) in dividing the sample into segments.

Even with the PSDs being produced with carefully chosen averaging parameters, selecting the characteristic frequency is not trivial. Historically, this is defined as the frequency at which the maximum power occurs; however this would restrict the precision to the nearest frequency resolution ($\pm 31.5\,Hz$). Threadgill & Bruce (2016) found that taking an ‘area-weighted’ frequency rather than the frequency at which the power peaked gave the expected $St = 0.03$ collapse across different SBLI configurations. This approach was adopted using (3.4) to calculate the weights of each trapezium on the pre-multiplied, semi-logarithmic scale. The trapeziums that make up the area of the PSD have sides the height of the pre-multiplied power $p_i f_i$ and widths of the difference in the logarithm of the frequencies $\log(f_{i+1}) - \log(f_i) = \log(f_{i+1}/f_i)$.

$$w_i = 0.5(p_i f_i + p_{i+1} f_{i+1}) \log(f_{i+1}/f_i)$$  \hspace{1cm} (3.4)

The spectra were linearly extrapolated to $f \times PSD = 0$ using the gradient of the spectrum between the second and third resolved frequencies. At the upper end, a cut-off frequency of $f = 7kHz$ was selected to capture the low-frequency hump in the spectra. The total areas of the PSDs were calculated as the sum of the weights in (3.4) and the area of the triangle above the extrapolated zero crossing to the second resolved frequency. The area-weighted median frequency is that at which half of the total PSD area is above and below. This was found by linearly interpolating the PSD at the frequency resolutions between which the median exists. The area-weighted median power was similarly found as the interpolated height of the PSD at this frequency.

### 3.3.3 Uncertainty estimations

Threadgill (2017) characterised the variation in total temperature over runs in the Imperial College supersonic wind tunnel. On average he found $dT_0/dt = -0.9\,K/s$, causing a negligible variation in the boundary layer thickness of $d\delta_0/dt = 0.001\,s$.

Table 3.4 lists the total estimated measurement uncertainties for the experimental data. For the high-frequency pressure measurements, the measurement position uncertainty in the $z$-
direction was estimated as being the sensor radius (machining tolerances put the outer radius to 1.200 – 1.205 mm from the expected centre). In the $x$-direction, there is also some uncertainty in the predicted incident shock inviscid impingement point $x_0$ which is used as a reference point for the final quoted position $X^*$, in addition to the separation length $L_{\text{sep}, \text{CL}}$ which was used to normalise it.

As explained in Section 3.2.2, the shock generator was positioned at slightly different heights for each test case in order to increase the streamwise measurement resolution. This gives an uncertainty in the inviscid impingement point of $\pm 2.71$ mm for each case. The centreline separation lengths can be estimated from the oil flow images by the distance between the separation and reattachment lines. The lowest resolution photograph was 10 pixels per mm, which would give an uncertainty of $\pm 0.2$ mm. There are, however, uncertainties in the de-warping process which are harder to quantify. Given the range of separation lengths of the different interaction test cases, this gives percentage uncertainties in the range $0.3 – 2.9\%$. The combined uncertainty of $x_0$ and $L_{\text{sep}, \text{CL}}$ in $X^*$ leads to a worst case absolute uncertainty of $\pm 0.6$ for $X^* = -5.13$ in the $8^\circ \mathcal{R} = 1$ test case. The worst case relative uncertainty in $X^*$ is $\pm 16\%$ and on average it is $\pm 7\%$.

The estimation of the uncertainty associated with the experimental measurements entering into the PSDs is based on the law of propagation of uncertainty (see Fraden (2004), for example). The combined standard uncertainty $\sigma$ over the quantity of interest (for example, the absolute pressure or its PSD) is calculated by summing the uncertainties associated with each source of error, as shown in (3.5), where $\sigma_i$ is the standard deviation of the $i$th of $n$ error sources.

$$\sigma^2 = \sigma_1^2 + \sigma_2^2 + \ldots + \sigma_n^2$$

(3.5)

The experimental apparatus contains four large sources of uncertainty, as per the description in Section 3.2. First, the pressure transducer converts the pressure into an electrical signal; then the signal is amplified by the bridge amplifier; this is then digitised by the DAQ module; lastly this is added to the atmospheric pressure reading from the barometer in the laboratory. There is an uncertainty associated with each measurement or conditioning operation performed by the pieces of hardware.

It is assumed that these uncertainties are a combination of several independent sources of error. For example, the pressure transducer measurement is affected by different uncertainties, including those in the operating temperature and the non-ideal transfer function. All sources of uncertainty are considered to be random and to follow a Gaussian probability distribution with zero mean and a specified standard deviation or standard uncertainty. The latter is estimated based on the information found in the device specifications provided by the manufacturers. Where there is insufficient data, a worst-case scenario approach is taken. The so-called uncertainty budget (Fraden, 2004) is summarised in Table 3.3.

The effects of time jitter and other timing errors are negligible. Typical measurements of the differential pressure during the course of the experimental campaign ranged from 0.15 to 0.65 Bar, which equates to relatives errors of 5.13% and 1.18% for one standard deviation, respectively, based on the combined uncertainty in Table 3.3.
Table 3.3: Sources of uncertainty and estimates of standard error.

<table>
<thead>
<tr>
<th>Source of uncertainty</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Pressure transducer</strong></td>
<td></td>
</tr>
<tr>
<td>Hysteresis, non-linearities and repeatability</td>
<td>0.1667 mV</td>
</tr>
<tr>
<td>Thermal zero shift</td>
<td>0.6176 mV</td>
</tr>
<tr>
<td>Thermal sensitivity shift</td>
<td>0.3550 mV</td>
</tr>
<tr>
<td><strong>Bridge amplifier</strong></td>
<td></td>
</tr>
<tr>
<td>Input voltage drift</td>
<td>0.0033 mV</td>
</tr>
<tr>
<td>Input voltage noise</td>
<td>0.0025 mV</td>
</tr>
<tr>
<td>Gain error</td>
<td>0.0192 mV</td>
</tr>
<tr>
<td>Gain stability</td>
<td>0.0192 mV</td>
</tr>
<tr>
<td>Output noise</td>
<td>0.0083 mV</td>
</tr>
<tr>
<td>Output offset</td>
<td>0.0556 mV</td>
</tr>
<tr>
<td><strong>DAQ module</strong></td>
<td></td>
</tr>
<tr>
<td>Absolute accuracy</td>
<td>0.0083 mV</td>
</tr>
<tr>
<td>Total combined standard uncertainty (differential pressure)</td>
<td>0.7344 mV (7.7 mBar)</td>
</tr>
<tr>
<td><strong>Atmospheric pressure</strong></td>
<td></td>
</tr>
<tr>
<td>Precision of barometer</td>
<td>$3.33 \times 10^{-2}$ mBar</td>
</tr>
<tr>
<td>Total combined standard uncertainty (absolute pressure)</td>
<td>7.7 mBar</td>
</tr>
</tbody>
</table>

It can be assumed that the measured pressure signal which enters the PSD calculation can be decomposed as $p' = p'_{\text{true}} + e_n$, where $p'_{\text{true}}$ is the ‘true’ fluctuation of the pressure value and $e_n$ represents the uncertainty as a random variable following a Gaussian probability distribution with zero mean and a standard deviation of $\sigma = 7.7$ mBar. The Fourier transform of this pressure signal will be that shown in (3.6).

$$\mathcal{F}(p') = \mathcal{F}(p'_{\text{true}} + e_n) = \mathcal{F}(p'_{\text{true}}) + \mathcal{F}(e_n)$$ (3.6)

Therefore, the uncertainty in the Fourier transform of the signal can be modelled as the Fourier transform of the error term. Since this term is considered to be white noise, we know that its discrete Fourier transform has a constant value of $\sigma^2 / (2N)$, where $N$ is the number of samples in the time signal. Hence this corresponds to the absolute uncertainty of the Fourier transform at any frequency. This analysis is also shown in the theoretical error propagation calculations in Betta et al. (2000).

Accounting for the use of windowing and the definition of the PSD, the standard deviation associated with the amplitude of the PSD of $p'$ is shown in (3.7). In (3.7), $N_w = 2^{14}$ is the number of samples in each window; $k_h = 0.3536$ is a factor accounting for the scaling introduced by using the Hanning window; $W_n = 1705$ is the number of windows and $N = 14 \times 10^6$ is the total number of samples in the time signal. This demonstrates that the use of the Hanning window, a high total number of samples and a large number of windows decrease the uncertainty in the PSD.

$$\sigma_{\text{PSD}} = \sqrt{2} \sigma^2 N_w k_h^2 / N \sqrt{W_n} = 2.97 \times 10^{-10}$$ (3.7)

Typical values of the maximum of the PSD without the pre-multiplication are of order $\mathcal{O}(10^{-4})$ (see Figure 3.9b, for example). This indicates that the estimated uncertainty is typically
six orders of magnitude lower than that at the peak of the PSDs, and therefore negligible. This also means that calculating the peak frequency, particularly using the weighted average technique described in Section 3.3, will have a low uncertainty.

However, a number of additional sources of error could not be included in this analysis. Firstly, the error term potentially does not follow a Gaussian probability distribution function and therefore its Fourier transform will not be flat. Secondly, the spectral leakage in the calculation of the discrete Fourier transform of the ‘true’ signal will contribute to the error term. Also, it is difficult to fully account for the effects of windowing over both the ‘true’ and error terms in the error propagation analysis. Lastly, the measured ‘true’ pressure signal may be erroneous, due to wind tunnel noise for example.

A second approach to estimate a worst-case uncertainty in the PSDs was also adopted. The manufacturer-quoted combined non-linearity, hysteresis and repeatability of the Kulite sensors is ±0.1% of the full-scale output of 100 mV. This gives an uncertainty in the output of ±0.1 mV which on average corresponds to ±1.04 mBar. A test scenario was run in which all of the values above the mean pressure output were given an offset of +1.04 mBar and all of the values below were given an offset of −1.04 mBar. This led to differences in the power in the peak of the PSD of up to 10%. It is very unlikely that the error will combine in this worst case scenario described above and it is more realistic that some will cancel out. This conservative uncertainty estimate is given alongside those of the other measurements in Table 3.4.

Table 3.4: Estimated measurement uncertainties.

<table>
<thead>
<tr>
<th>$x_0$ [mm]</th>
<th>$L_{sep, L}$ [mm]</th>
<th>$X^*$ [-]</th>
<th>$Z^*$ [-]</th>
<th>$p$ [mBar]</th>
<th>PSD [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>±2.71</td>
<td>±0.2</td>
<td>±0.6</td>
<td>±0.016</td>
<td>±7.7</td>
<td>10</td>
</tr>
</tbody>
</table>
Chapter 4

Global characteristic flow organisation

The separation regions for each test case are characterised from the oil flow visualisation and schlieren photography in Section 4.1. In Section 4.2, the power spectral densities (PSDs) at $Z^* = \pm 0.02$ (near the spanwise centreline) are compared. A resulting model of the interdependence of the dynamics and separation of incident – reflected SBLIs is proposed in Section 4.3 (building on that proposed in Rabey & Bruce (2019)) and a summary is given in Section 4.4.
4.1 Separation regions

4.1.1 Introduction

The oil flow images processed as described in Section 3.3 are shown in Figure 4.1. The figure shows that the separation regions are different across the different interaction strengths. They are also different for interactions of the same strength but different aspect ratio. This latter observation is particularly noteworthy as such interactions might have been reported as the same in previous literature.
4.1. SEPARATION REGIONS

Figure 4.1: Oil flow images. Reattachment lines are superimposed and sensor measurement locations are shown by to-scale circles. Crosses indicate critical points: magenta - foci, cyan - saddle points, green - dispersion nodes.
4.1.2 Characterisation of central separations

Figure 4.1 highlights how differences in the central separation regions cannot be quantified by consideration of the centreline separation length \( L_{sep,C} \) alone. Nonetheless, this conventional characteristic is given in Table 4.1 and shows that the trend is for centreline separation length to increase with increasing aspect ratio and shock strength. Interestingly, the separation lengths of the \( 12^\circ \), \( A = 1 \) and \( 10^\circ \), \( A = 2 \) are very close. This brings into question whether shock strength and separation length can be used as interchangeable terms to describe interaction strength.

**Table 4.1:** Centreline separation lengths \( L_{sep,C} \) [mm].

<table>
<thead>
<tr>
<th>( A )</th>
<th>( A = 1 )</th>
<th>( A = 1.3 )</th>
<th>( A = 2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 12^\circ )</td>
<td>34</td>
<td>65</td>
<td></td>
</tr>
<tr>
<td>( 10^\circ )</td>
<td>20</td>
<td>22</td>
<td>36</td>
</tr>
<tr>
<td>( 8^\circ )</td>
<td>7</td>
<td>9</td>
<td>13</td>
</tr>
</tbody>
</table>

Figure 4.1 also shows how the curvature of the reattachment and separation lines differs across the test cases. The \( 12^\circ \), \( A = 1.3 \) case has the straightest separation line and a large continuous curvature in its reattachment line. The \( 12^\circ \), \( A = 1 \) case also has a fairly straight separation line, albeit with an anomalous wander of the separation line in the positive z-plane. However its reattachment line is much straighter than that of the \( 12^\circ \), \( A = 1.3 \) case. All of the \( 10^\circ \) and \( 8^\circ \) test cases exhibit a localised downstream curvature of the separation line near \( Z^* = 0 \). The \( 8^\circ \) cases also exhibit an upstream curving of the reattachment line at \( Z^* = 0 \), resulting in a necking of the separation regions at the centreline.

Overall it appears that the separation regions fall on a spectrum between two types. The first type is the more classical separation region with a fairly straight separation line and a continuous upstream curvature of the reattachment line from the centre. In terms of the critical points marked on Figure 4.1, these interactions have two clear vortices and a dispersion node between two saddle points on the reattachment line: the ‘owl-face of the second kind’ as described by Perry & Chong (1987). The \( 12^\circ \), \( A = 1.3 \) is a clear example of this and the \( 12^\circ \), \( A = 1 \) and \( 10^\circ \), \( A = 2 \) are close but with small deviations.

The second type exhibits tortuous separation and reattachment lines. In particular the separation regions narrow near the centre and tend towards two distinct separation ‘cells’. These regions suggest the presence of another pair of vortices at the centre of the region which may be the new opposite pairs to those at the edges of the regions. These separation regions occupy less of the spanwise width of the duct and are quite distinct from the corner separations. The \( 8^\circ \), \( A = 1 \) is the clearest example of this type; the other \( 8^\circ \) interactions are also very similar and the \( 10^\circ \), \( A = 1 \) and \( A = 1.3 \) cases show traits from both types as their reattachment lines are still fairly regular.

The separation lengths were extracted from the oil flow at the spanwise locations of the sensors described in Section 3.2, giving between 27 and 34 measurements in the central separation for each test case. These lengths are shown in the left-hand plots in each of the sub-figures in Figure 4.2. The spanwise distribution of the separation lengths are shown by the corresponding coloured bars in the centre plots in each of the sub-figures in Figure 4.2. The white bars in these plots show the curvature of the reattachment lines and confirm that the separation lengths of
4.1. SEPARATION REGIONS

the $12^\circ \mathcal{A} = 1.3$ are almost entirely governed by the shape of the reattachment line while the separation lengths of the $8^\circ$ interactions are a function of both the separation and reattachment line curvature, for example.

The separation lengths were distributed into a histogram with equally spaced bin edges between $L_{\text{sep}} = 0$ and $L_{\text{sep}} = 2L_{\text{sep}}$, shown in the right-hand plots in the sub-figures of Figure 4.2. The corresponding probability density functions (p.d.f.s) of these measurements are shown together in Figure 4.3 and the statistical moments are given in Table 4.2.

Figure 4.2: Calculation of p.d.f.s of separation lengths in central separation regions. Leftmost plots show the separation lengths used. Centre plots show the variation in separation length across the span, white bars show the variation due to the reattachment line. Rightmost plots show the resulting histograms.

Figure 4.3 shows that the distribution of length scales in the $12^\circ \mathcal{A} = 1$ and $\mathcal{A} = 1.3$ and $10^\circ \mathcal{A} = 1$ and $\mathcal{A} = 2$ cases are very similar about their mean values. All these cases have a negative skewness $\gamma$ towards lengths greater than the mean, as quantified in Table 4.2, and a high kurtosis $\kappa$. This is because the separation lengths remain high for large central portions of the separations with little tapering of the regions towards the edges, as can be seen in Figures 4.1a, 4.1b, 4.1c and 4.1e.

The $10^\circ \mathcal{A} = 1.3$ and $8^\circ \mathcal{A} = 1.3$ cases are both more centred about the mean although still negatively skewed, reflecting the gradual tapering of the separation lengths towards the edges and the narrowing near the centreline. These cases exhibit a kurtosis near the expected value for a normal distribution $\kappa \approx 3$. 
CHAPTER 4. GLOBAL CHARACTERISTIC FLOW ORGANISATION

The $8^\circ A = 1$ and $A = 2$ cases exhibit near zero skewness, which is consistent with the more broadband length distribution and low kurtosis. The $8^\circ A = 1$ case has a double-peaked p.d.f. due to its ‘moustache’ shape, with shorter length scales existing at the centre as well as at the edges. It therefore has a much higher standard deviation $\sigma = 0.41$ than any of the other cases.

![Figure 4.3: P.d.f.s of length scales across separation regions.](image)

It is interesting to note from Table 4.2 that for the $10^\circ A = 1$ and $A = 1.3$ and all $8^\circ$ cases the centreline separation lengths $Q_L$ are less than the mean separation lengths across the entire span of the interactions $\mu$. This is in contrast to the more ‘typical’ separation regions of the $12^\circ A = 1$ and $A = 1.3$ and $10^\circ A = 2$ interactions where the separation length decreases almost monotonically towards the edges of the region. This may have consequences for using $L_{sep}Q_L$ to normalise the low-frequency unsteadiness, as the centreline separation length is normally assumed to be the longest that exists in the central separation and therefore appropriate for reducing the lowest frequencies.

<table>
<thead>
<tr>
<th>$A$</th>
<th>$\mu$ [mm]</th>
<th>$\sigma$</th>
<th>$\gamma$</th>
<th>$\kappa$</th>
<th>$\mu$</th>
<th>$\sigma$</th>
<th>$\gamma$</th>
<th>$\kappa$</th>
<th>$\mu$</th>
<th>$\sigma$</th>
<th>$\gamma$</th>
<th>$\kappa$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$12^\circ$</td>
<td>34, 29, 0.21, -1.18, 4.41</td>
<td>65, 53, 0.25, -1.17, 3.54</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$10^\circ$</td>
<td>20, 23, 0.25, -1.57, 5.17</td>
<td>22, 24, 0.21, -0.48, 2.94</td>
<td>36, 25, 0.21, -1.04, 3.50</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$8^\circ$</td>
<td>7, 11, 0.41, -0.02, 2.12</td>
<td>9, 14, 0.34, -0.34, 2.94</td>
<td>13, 19, 0.27, +0.03, 2.08</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4.1.3 Characterisation of corner separations

Some information on the corner separations can be discerned from Figure 4.1. In some cases streaklines can be observed passing around the corner separation. From these it can be deduced
that in general, as the aspect ratio or shock strength is increased the size of the corner separations also increases. The size of the corner separations increases with the size of the central separations, meaning that the total increase in separation severity is very large between the cases. For the $8^\circ \mathcal{AR} = 1$ and $\mathcal{AR} = 1.3$ cases there are large attached channels between the central and corner separations. On the other hand, for the $12^\circ \mathcal{AR} = 1.3$ and $10^\circ \mathcal{AR} = 2$ cases the corner separations appear to have some influence on the curvature of the central separation reattachment line.

In order to quantify the observations of the corner separations two measures are proposed. The first is the location of the initial deflection of the corner boundary layer away from the wall. The second is the furthest into the central flow the corner boundary layer separation reaches and the streamwise location of this occurrence. The turning of the streaklines in the corner separations in Figure 4.1 are used as proxies to estimate these points. The locations relative to the inviscid impingement points are labelled on the figure for the corner at $Z^* = -1$.

The streamwise distance between these two marked flow features is typically of the order of one centreline separation length. This means, as can be inferred from Figure 4.1, that the central separation regions of the weaker interactions are less affected by the corner separations. In this sense they might be considered more ‘two-dimensional’ although the large spanwise variation in their central separation discussed above might qualify them as more ‘three-dimensional’. On the other hand the corner separations of the $12^\circ \mathcal{AR} = 1.3$ and $10^\circ \mathcal{AR} = 2$ cases reach their maximum distance from the sidewall relatively far downstream and possibly interact with the central separation.

The points are further upstream of the inviscid impingement point ($X^* = 0$) for smaller shock strengths or aspect ratios, as is the central separation region. However, the location of the initial inward deflection of the corner separations occurs a consistent distance upstream of the central separation line in all cases. This shows that the foot of the incident shock on the sidewall always extends slightly upstream of the reflected shock foot on the floor.

### 4.1.4 Characterisation of core separations

Representative schlieren images for the cases presented in Figure 4.1 are shown in Figure 4.4. Like Figure 4.1, the coloured markings indicate the locations of the pressure measurements to be discussed in Section 4.2. Due to the plug mounting in the floor, three Mach waves are generated which make it difficult to distinguish the separation shock foot; this is shown more clearly in the enlargements of the SBLIs in Figure 4.5.

Figure 4.4 gives some idea of the relative shock strengths by the apparent thickening of the incident shocks towards the floor. This is in fact due to the swept SBLI between the incident shock and the sidewall boundary layer, as described in Wang et al. (2015). This interaction grows conically along the sidewall with distance from the leading edge of the shock generator. For the weaker interactions the incident and the rear shocks stemming from the glancing shock are quite distinct from each other whereas for the $12^\circ$ and $10^\circ \mathcal{AR} = 1$ interactions there is a continuous compression through the sidewall interactions.

Figure 4.4 also gives some idea of how much separation into the core flow above the floor each interaction causes. As expected, this is greater for larger shock strengths. An estimate of the maximum height of the separation region $h_{sep,\text{max}}$ is given on the $y$-axis of each of the figures.
(a) $12^\circ$ $\mathcal{A} = 1$.

(b) $12^\circ$ $\mathcal{A} = 1.3$.

(c) $10^\circ$ $\mathcal{A} = 1$.

(d) $10^\circ$ $\mathcal{A} = 1.3$.

(e) $10^\circ$ $\mathcal{A} = 2$.

(f) $8^\circ$ $\mathcal{A} = 1$.

(g) $8^\circ$ $\mathcal{A} = 1.3$.

(h) $8^\circ$ $\mathcal{A} = 2$.

**Figure 4.4:** Schlieren images with pressure measurement points and $h_{\text{sep, max}}$ estimates in mm.

This is taken to be the maximum height of the separation region as this is what is observable from the spanwise integration of the entire interaction by the schlieren photography. This height does not scale trivially with the centreline separation length, demonstrated by Table 4.3.

**Table 4.3:** Separation heights $h_{\text{sep, max}}$ [mm] and ratios to centreline separation lengths $h_{\text{sep, max}}/L_{\text{sep, C}}$ [-].

<table>
<thead>
<tr>
<th>$\mathcal{A}$</th>
<th>$\mathcal{A} = 1$</th>
<th>$\mathcal{A} = 1.3$</th>
<th>$\mathcal{A} = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$12^\circ$</td>
<td>14.4, 0.42</td>
<td>15.6, 0.24</td>
<td></td>
</tr>
<tr>
<td>$10^\circ$</td>
<td>11.5, 0.58</td>
<td>12.0, 0.55</td>
<td>13.1, 0.36</td>
</tr>
<tr>
<td>$8^\circ$</td>
<td>8.1, 1.15</td>
<td>9.8, 1.09</td>
<td>8.1, 0.63</td>
</tr>
</tbody>
</table>

Grossman & Bruce (2018) conducted PIV in the same facility on $M = 2$ $\mathcal{A} = 1$ and $\mathcal{A} = 1.38$ interactions with a $12^\circ$ 23 mm thick shock generator (meaning that the expansion fan impinged further downstream than in the present case). From the contour of $M_x = 1$ for the test cases of Grossman & Bruce (2018), the maximum centreline separation region heights were approximately
4.1. SEPARATION REGIONS

Figure 4.5: Schlieren images with pressure measurement points and $h_{sep,max}$ estimates in mm.

12.2 mm and 13.3 mm for the $AR = 1$ and $AR = 1.38$ cases, respectively. Although these are slightly smaller than the estimates from the schlieren images for the present case (despite the closer expansion fan), the increase in height of the larger aspect ratio case is approximately 9% in both estimates. This gives confidence that the correct trends for the relative heights have been identified from the schlieren images.

Table 4.3 shows that the height generally increases with increasing aspect ratio and shock strength. The absolute heights grow nearly linearly first with aspect ratio and then with shock strength from the smallest $8^\circ \ AR = 1$ case to the largest $12^\circ \ AR = 1.3$ case.

The volume of fluid in each separation region can be approximated from the estimates of the central separation heights and spanwise lengths. In each case, the volume $V_{sep}$ was estimated by summing the areas under the triangles shown in the example in Figure 4.6 and multiplying by the spanwise distance between the measurement points for $L_{sep}$ (3.6 mm). The height of the separation bubble was assumed to decrease linearly from $h_{sep,max}$ at the centre to zero at the edges of the separation region in each case. The totals are listed in Table 4.4.

Table 4.4: Estimated volumes of central separation regions $V_{sep}$ [$\times 10^4$ mm$^3$].

<table>
<thead>
<tr>
<th></th>
<th>$AR = 1$</th>
<th>$AR = 1.3$</th>
<th>$AR = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$12^\circ$</td>
<td>1.42</td>
<td>2.86</td>
<td></td>
</tr>
<tr>
<td>$10^\circ$</td>
<td>0.85</td>
<td>1.00</td>
<td>1.59</td>
</tr>
<tr>
<td>$8^\circ$</td>
<td>0.26</td>
<td>0.44</td>
<td>0.51</td>
</tr>
</tbody>
</table>

Table 4.4 shows that, in terms of volume of separation, the $12^\circ \ AR = 1.3$ case is very distinct from the others, having a volume about twice that of the $12^\circ \ AR = 1$ and $10^\circ \ AR = 2$ cases.
This might be because it has transitioned from a regular-reflected SBLI to a Mach-reflected SBLI (see Grossman & Bruce (2018), for example), as suggested by the multiple slip lines and normal shock visible just above the interaction in Figure 4.5b. As with the other measures of separation, the volume generally increases from the $8^\circ \mathcal{A} = 1$ case with both increasing aspect ratio and increasing shock strength and is more sensitive to the latter. This is summarised in Figure 4.7.

Figure 4.6: Example of $V_{sep}$ calculation for $12^\circ \mathcal{A} = 1.3$ test case.

Figure 4.7: Variation of length measures for different test cases. – $12^\circ$, — $10^\circ$, ·– $8^\circ$. 
4.2 Centreline PSDs

4.2.1 Introduction

The power spectral densities (PSDs) of the centreline wall-pressure fluctuations were obtained following the method outlined in Section 3.2 and are presented in Figure 4.8. Results are presented for the two spanwise measurement locations closest to the duct centreline ($Z^* = \pm 0.02$): the solid lines represent measurements at $Z^* = 0.02$ and the dashed lines represent those at $Z^* = -0.02$.

Figure 4.8: Centreline PSDs. Solid lines are at $Z^* = 0.02$, dashed lines at $Z^* = -0.02$. Inset figures show corresponding streamwise development of mean centreline pressure intermittency. Error bars show values at $Z^* = 0.02$ and $Z^* = -0.02$. 
Inset in Figure 4.8 is the streamwise development of the intermittency factor $\gamma$, taken from the definition given by Muck et al. (1985) as in (4.1). $p_1$ is taken to be the freestream pressure upstream of the interaction and $p_2$ is the corresponding inviscid post-shock pressure assuming that the deflection angle of the separation shock is equal to that of the incident (and reflected) shock. The intermittency factor $\gamma$ should therefore be close to zero when the separation shock is always downstream of the sensor (it will be slightly higher due to measuring in the boundary layer rather than in the freestream) and near unity when always measuring downstream of the separation shock, under the separation bubble.

$$\gamma = \frac{p_w - p_1}{p_2 - p_1} \quad (4.1)$$

Figure 4.8 shows that the measurement points span most of this range and that the most powerful low-frequency peak was captured in all of the test cases, typically when $\gamma \approx 0.6$. The $8^\circ \mathcal{AR} = 1$ case in Figure 4.8f provides a good example. From the most upstream points, the mean pressure remains nearly constant and the low-frequency power increases slowly. When the mean pressure begins to rise there is a large increase in low-frequency power. This then gradually decreases downstream and the mid-frequency power increases as the sensor is under the separation region for a greater proportion of time and $\gamma$ tends to unity. The other cases are similar but capture varying distances upstream and downstream of the reflected shock.

The first point to note from Figure 4.8 is that the data is not perfectly symmetric across the positive and negative $z$-planes. The error bars in the figures of intermittency variation show that the differences are greater when the low-frequency power is greatest. This is not unexpected, as the mean and unsteady pressures are highly sensitive to position in the region of the separation shock. The greatest discrepancies in the PSDs are also at the locations with the largest power.

However in some cases the discrepancies may be attributable to the asymmetry of the interaction itself. Figure 4.1 shows that even in cases where the interaction is largely symmetric, the symmetry plane is not the exact duct centre. For example, Figure 4.1c shows that the sensors in the positive $z$-plane of the $10^\circ \mathcal{AR} = 1$ case might be expected to exhibit the pressure rise earlier than those in the negative plane and this is what is seen in Figure 4.8c.

This information is confirmed in Figure 4.9 which shows the shifting dominant frequencies along the centreline and the streamwise development of the standard deviation of the pressure fluctuations. It is expected (see Dolling & Or (1983) and Muck et al. (1985), for example) that the separation point occurs after a spike in pressure fluctuations and is followed by a plateau in $\sigma_p$ at a larger value than upstream of the interaction. The figures show that regions of low energy were recorded upstream and downstream of the coherent bands of low-frequency energy, confirming that the entire region of low-frequency unsteadiness was captured in all cases.

Figure 4.9a shows that in the $12^\circ \mathcal{AR} = 1$ case there was a low-frequency, high energy band at $-1.25 \leq X^* \leq -1.10$ due to the separation shock. Little of the upstream boundary layer was captured but several recordings were taken under the separation region where $\sigma_p$ was roughly constant and there was a shift to medium-energy, high frequencies. On the other hand, in Figure 4.9d it can be seen that for the $10^\circ \mathcal{AR} = 1.3$ case more of the upstream boundary layer was captured (with low-energy high-frequency fluctuations) with fewer measurements under the separation region. Figures 4.9f, 4.9g and 4.9h show that large regions both upstream and
4.2. CENTRELINE PSDS

(a) $12^\circ \mathcal{A} = 1$

(b) $12^\circ \mathcal{A} = 1.3$

(c) $10^\circ \mathcal{A} = 1$

(d) $10^\circ \mathcal{A} = 1.3$

(e) $10^\circ \mathcal{A} = 2$

(f) $8^\circ \mathcal{A} = 1$

(g) $8^\circ \mathcal{A} = 1.3$

(h) $8^\circ \mathcal{A} = 2$

Figure 4.9: Above: pre-multiplied PSD distribution in $X^*$ averaged over $Z^* = \pm 0.02$, contours scale logarithmically from $10^{-4.5}$ to $10^{-1.2}$. Below: corresponding streamwise variation in standard deviation of pressure fluctuations averaged over $Z^* = \pm 0.02$. Error bars show values at $Z^* = 0$ and $Z^* = -0.02$.

downstream of the separation shocks were captured in the $8^\circ$ cases as weaker interactions are expected to have the smallest intermittency lengths.

It can also be seen from both the energy in the PSD distributions and the magnitude of the pressure fluctuations about their means in Figure 4.9 that the $12^\circ \mathcal{A} = 1$ case had the most prevalent low-frequency motions, followed by the $10^\circ \mathcal{A} = 2$ case. The peak powers are shown in Table 4.5 and illustrate that there is no general trend for pressure fluctuation magnitude with shock strength or aspect ratio, demonstrating that a complex relationship exists in which several factors most likely have varying degrees of influence on the different interactions.

Table 4.5: Maximum pre-multiplied PSD $[-]$.

<table>
<thead>
<tr>
<th></th>
<th>$\mathcal{A} = 1$</th>
<th>$\mathcal{A} = 1.3$</th>
<th>$\mathcal{A} = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$12^\circ$</td>
<td>0.210</td>
<td>0.060</td>
<td>0.029</td>
</tr>
<tr>
<td>$10^\circ$</td>
<td>0.029</td>
<td>0.027</td>
<td>0.079</td>
</tr>
<tr>
<td>$8^\circ$</td>
<td>0.010</td>
<td>0.021</td>
<td>0.018</td>
</tr>
</tbody>
</table>
CHAPTER 4. GLOBAL CHARACTERISTIC FLOW ORGANISATION

4.2.2 Characterisation of most powerful PSDs

In this section the PSDs with the greatest low-frequency power for each test case are compared (one streamwise and spanwise position is selected for each case). These are deemed to be representative of the low-frequency behaviour of each case relative to each other.

Historically the frequency associated to the highest peak of the pre-multiplied PSD has been used as the defining characteristic of the unsteadiness of an interaction. However, Figure 4.8 shows that some of the interactions exhibit quite broadband peaks which a single frequency may not appropriately capture. This could also be an important consideration when characterising unsteadiness as it reveals how typical the characteristic frequency is and also if the unsteadiness will be particularly energetic over a small band of frequencies or less severe over a large band, for example.

The most powerful PSDs of each case over all streamwise measurement points at $Z^* = \pm 0.02$ are plotted together in Figure 4.10. Plotting the cases this way (on the same axis) illustrates how the interactions can be split into two categories: the ‘type 1’ $12^\circ A = 1$, $12^\circ A = 1.3$ and $10^\circ A = 2$ have a high-energy single-peaked PSD while the remaining ‘type 2’ test cases have a less powerful, more broadband distribution which may be tending towards two peaks.

Figure 4.11 shows the same PSDs as in Figure 4.10 but plotted in terms of reduced frequency $St$ based on the centreline separation lengths for each test case. With this normalisation, the PSDs are drawn into the opposite trend for peak frequency whereby the $8^\circ$ interactions present the lowest characteristic reduced frequencies. The $10^\circ A = 1$ and $A = 1.3$ PSDs are brought more in line with each other while the $8^\circ$ interactions become more staggered in terms of their prevalent frequencies and the power contained therein.

Threadgill & Bruce (2016) observed that the frequency corresponding to the maximum energy content for incident – reflected SBLIs with deflection angles of $7^\circ$, $8^\circ$, $9^\circ$ and $10^\circ$ remained constant to within one frequency resolution ($234Hz$) but that the peak became more skewed towards low frequencies with increasing shock strength. They therefore proposed that taking a ‘weighted average’ was more representative of the characteristic frequency of an interaction.
4.2. CENTRELINE PSDS

Figure 4.11: PSDs with peak low-frequency power recorded for each interaction pre-multiplied by reduced frequency.

Figure 4.12 shows the area-weighted median and mean frequencies for each PSD in red and blue lines, respectively. These are found by weighting the mid-points between each resolved frequency with the apparent area of the trapeziums (in semi-logarithmic space) below them. The PSDs were extrapolated to zero from 61 Hz along the same gradient as the first segment. In order to only consider low-frequency behaviour, frequencies up to a cut-off of 7 kHz were considered. The figure shows that the median values give the best representation of the characteristic low-frequencies that might be selected ‘by eye’ and these are listed in Table 4.6.

Table 4.6: Variation in median low frequency [Hz].

<table>
<thead>
<tr>
<th>A R = 1</th>
<th>A R = 1.3</th>
<th>A R = 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>12°</td>
<td>375</td>
<td>374</td>
</tr>
<tr>
<td>10°</td>
<td>500</td>
<td>545</td>
</tr>
<tr>
<td>8°</td>
<td>674</td>
<td>699</td>
</tr>
</tbody>
</table>

The variation in the skewness of the PSDs observed by Threadgill & Bruce (2016), as well as their other statistical moments, could also be used to describe the changing low-frequency behaviour across the interactions. The statistical moments of the PSDs as if they were p.d.f.s are also shown in Figure 4.12 and presented in Table 4.7. This assigns the product of the pre-multiplied power associated with each frequency and the apparent width of the frequency band as the prevalence of its occurrence. The values of $2\sigma - \mu$, $4\sigma - \mu$ and $6\sigma - \mu$ (the expected bands for 68.2%, 95.4% and 99.6% of the data in a normal distribution) are shown by the green lines in Figure 4.12.

Table 4.7 shows that the 12° A R = 1, 12° A R = 1.3 and 10° A R = 2 cases have a large positive skewness $\gamma \approx 3.3$. This quantifies the expectation that these stronger interactions have a stronger tendency towards very low-frequency motion, in addition to the absolute most prevalent low frequency being known to be lower. On the other hand, the 10° A R = 1 and 10° A R = 1.3 interactions have a smaller positive skewness $\gamma \approx 2.7$ and the 8° A R = 1 and 8° A R = 1.3 have a smaller skewness still $\gamma \approx 2.25$. The 8° A R = 2 case is between these two cases.
Figure 4.12: Most energetic pre-multiplied PSDs with statistical moments. Vertical black lines show the cut-off frequency below which statistics were calculated, black dashed lines show the extrapolation of the PSDs to $f = 0$ Hz. The line styles and symbols correspond to those in Figure 4.3 and the symbols are not shown at every data point nor equally spaced.

Table 4.7: Statistical moments of most energetic PSDs. Mean $\mu$ [Hz], normalised standard deviation $\sigma/\mu$ [−], skewness $\gamma$ [−] and kurtosis $\kappa$ [-].

| $\mathcal{R}$ | $\mu$ | $\sigma$ | $\gamma$ | $\kappa$ | $\mathcal{R}$ | $\mu$ | $\sigma$ | $\gamma$ | $\kappa$ | $\mathcal{R}$ | $\mu$ | $\sigma$ | $\gamma$ | $\kappa$ |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 12° | 679, 1.43, 3.23, 15.43 | 696, 1.50, 3.22, 14.81 | 679, 1.43, 3.23, 15.43 | 696, 1.50, 3.22, 14.81 | 12° | 679, 1.43, 3.23, 15.43 | 696, 1.50, 3.22, 14.81 | 12° | 679, 1.43, 3.23, 15.43 | 696, 1.50, 3.22, 14.81 | 12° | 679, 1.43, 3.23, 15.43 | 696, 1.50, 3.22, 14.81 | 12° | 679, 1.43, 3.23, 15.43 | 696, 1.50, 3.22, 14.81 | 12° | 679, 1.43, 3.23, 15.43 | 696, 1.50, 3.22, 14.81 |
| 10° | 876, 1.31, 2.76, 11.46 | 903, 1.30, 2.69, 11.01 | 614, 1.45, 3.44, 17.82 | 614, 1.45, 3.44, 17.82 | 10° | 876, 1.31, 2.76, 11.46 | 903, 1.30, 2.69, 11.01 | 10° | 876, 1.31, 2.76, 11.46 | 903, 1.30, 2.69, 11.01 | 10° | 876, 1.31, 2.76, 11.46 | 903, 1.30, 2.69, 11.01 | 10° | 876, 1.31, 2.76, 11.46 | 903, 1.30, 2.69, 11.01 | 10° | 876, 1.31, 2.76, 11.46 | 903, 1.30, 2.69, 11.01 |
| 8°  | 1111, 1.19, 2.24, 8.17 | 1070, 1.18, 2.28, 8.60 | 976, 1.30, 2.41, 9.19 | 976, 1.30, 2.41, 9.19 | 8°  | 1111, 1.19, 2.24, 8.17 | 1070, 1.18, 2.28, 8.60 | 8°  | 1111, 1.19, 2.24, 8.17 | 1070, 1.18, 2.28, 8.60 | 8°  | 1111, 1.19, 2.24, 8.17 | 1070, 1.18, 2.28, 8.60 | 8°  | 1111, 1.19, 2.24, 8.17 | 1070, 1.18, 2.28, 8.60 |

There is a similar trend for the kurtosis. All cases show a large kurtosis much greater than $\kappa = 3$ (the value for a normal distribution), indicating that the energy is concentrated in relatively narrow bands of low frequencies. The $12° \mathcal{R} = 1$, $12° \mathcal{R} = 1.3$ and $10° \mathcal{R} = 2$ exhibit very large values showing that there is a narrow band of dominant frequencies. Conversely, the $8°$ interactions show the smallest kurtosis which reflects the more broadband distributions of energetic frequencies observed in these interactions.
4.2. CENTRELINE PSDS

Overall this shows that stronger interactions cause more severe unsteadiness for multiple reasons. Firstly, as previously established, the shock motion generally exhibits lower frequencies for stronger interactions. This is the reason that it is widely accepted that $St = fL/U$ is constant across different interactions. In addition to this, the statistical moments extracted from the PSDs show that there tends to be a tighter grouping of the energy in the lower frequency bands so the motion is also more concentrated at these lower frequencies.

The trends observed here for varying shock strength and aspect ratio are very similar to those presented in Section 4.1.2 on the shapes of the central separation regions. The statistical moments of the spanwise variations in separation lengths given in Table 4.2 show very similar patterns to the moments of the PSDs in Table 4.7. This is represented graphically in Figure 4.13. Likewise, the p.d.f.s of the separation lengths in Figure 4.3 show some relation to the shape of the corresponding PSDs in Figure 4.12.

Figure 4.13c shows that a negative skewness in separation length distribution towards greater lengths appears to correlate with a positive skewness of the PSDs towards lower frequencies, as might be expected if low-frequency motion is associated with large separation (similarly the mean values in Figure 4.13a are approximately mirrored).
Figures 4.13b and 4.13d show similar trends between the standard deviation and kurtosis, respectively, of the separation lengths and low-frequency pressure fluctuations. Furthermore, a more broadband, double peak is observed in both the p.d.f. of separation region lengths and the PSD of $p'$ of the $8^\circ \mathcal{AR} = 1$ case. It is therefore plausible that the duality of the low-frequency behaviour of this case arises due to its separation region having two dominant length scales.

Hence, it is proposed that the low-frequency behaviour of a separated SBLI is directly related to the shape of the shock-induced separation region. For example, the reflected shock of an interaction with near-constant separation length across its span can be expected to oscillate with a narrow band of dominant frequencies and a rapid drop-off at mid-frequencies (high kurtosis and positive skewness). Similarly, if there are a range of separation lengths across the span of an interaction, its PSD might be expected to be more broadband (low kurtosis and skewness). This would mean that the shock motion would exhibit a range of frequencies. This proposed inter-dependency is summarised in Figure 4.14.

Figure 4.14: Proposed relation of separation region shape (on left) and resulting p.d.f. of separation lengths (centre) to PSD shape (on right).

Finally, the absolute values of the characteristic frequencies about which these traits vary are expected to scale with the inverse of the separation length, if $St = 0.03$ holds. In the next section, we explore whether the centreline separation length or the modal length (analogous to the area-weighted median frequency) from the p.d.f. of the separation lengths provides a better collapse.

### 4.2.3 Centreline Strouhal number estimates

Initially the conventional definition of characteristic frequency will be discussed. This is the low frequency at which the maximum pre-multiplied power occurs, shown by the dashed black lines in Figure 4.15. As shown in Section 3.2, this can be estimated from the PSDs to the nearest 61 Hz.

Figure 4.15 shows that, although the associated power varies greatly with the streamwise measurement locations, the characteristic frequencies are consistent to within one frequency resolution for measurement points under the separation shocks, with the exception of the $10^\circ \mathcal{AR} = 2$ case. These characteristic frequencies and associated Strouhal numbers based on the centreline separation lengths (listed in Table 4.1) are taken from the most powerful PSD in each case and are given in Table 4.8.

Table 4.8 shows that the absolute maximum frequencies of all streamwise measurements were found at $f = 244\text{ Hz}$ for both $12^\circ$ interactions. Figures 4.15a and 4.15b show that for the
4.2. CENTRELINE PSDS

Figure 4.15: Centreline PSDs normalised by centreline separation length. Solid lines are at $Z^*$ = 0.02, dashed lines at $Z^*$ = −0.02. Dashed black line shows the frequency associated with the absolute maximum pre-multiplied power and solid line shows the median frequency weighted on the area of the spectrum.

Table 4.8: Frequencies with absolute maximum pre-multiplied power and associated Strouhal numbers $St_{L_{sep}}$.

<table>
<thead>
<tr>
<th></th>
<th>$AR = 1$</th>
<th>$AR = 1.3$</th>
<th>$AR = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$12^\circ$</td>
<td>$f = 244$ Hz, $St = 0.016$</td>
<td>$f = 244$ Hz, $St = 0.031$</td>
<td></td>
</tr>
<tr>
<td>$10^\circ$</td>
<td>$f = 305$ Hz, $St = 0.011$</td>
<td>$f = 916$ Hz, $St = 0.039$</td>
<td>$f = 427$ Hz, $St = 0.030$</td>
</tr>
<tr>
<td>$8^\circ$</td>
<td>$f = 916$ Hz, $St = 0.013$</td>
<td>$f = 916$ Hz, $St = 0.016$</td>
<td>$f = 916$ Hz, $St = 0.023$</td>
</tr>
</tbody>
</table>

different streamwise measurement points, maxima were recorded at adjacent frequency resolution increments. These were $f = 244$ Hz and $f = 305$ Hz. Therefore the true characteristic shock motion based on maximum power is probably between these frequencies for both cases. However, even if the upper bound of the characteristic frequency is taken for the $AR = 1$ case, given that its centreline separation length is almost half that of the $AR = 1.3$ case, this would still give a large discrepancy in the characteristic Strouhal numbers of $St = 0.020$ compared to $St = 0.031$. 79
CHAPTER 4. GLOBAL CHARACTERISTIC FLOW ORGANISATION

The $10^\circ \mathcal{A} = 1$ case exhibits an unexpectedly low characteristic frequency just one frequency resolution increment above those of the $12^\circ$ interaction. This may be an artefact of the low-frequency peaks having a fairly even power distribution for $0.009 \leq St \leq 0.032$ ($244 \text{ Hz} \leq f \leq 916 \text{ Hz}$).

Taking the characteristic frequency as that associated with the absolute maxima of the spectra gives $f = 916 \text{ Hz}$ for the $10^\circ \mathcal{A} = 1$, and all $8^\circ$ interactions. This is due to the spike possibly caused by noise interference at this frequency and means that the relative associated Strouhal numbers are solely dictated by the centreline separation lengths.

In order to more fairly assess the relative frequencies, the median characteristic frequency weighted by PSD area (shown by the solid black line in Figure 4.15 and given in Table 4.9) will be taken from now on. This overcomes some of the issues discussed above by not being restricted to the discrete frequency resolution and not taking the frequency at a noise spike.

Table 4.9: Median frequencies weighted by PSD area $\tilde{f}$ and associated Strouhal numbers $\tilde{St}_{Lsep,C_L}$.

<table>
<thead>
<tr>
<th>$\mathcal{A}$</th>
<th>$\tilde{f}=375 \text{ Hz}, \tilde{St}=0.025$</th>
<th>$\tilde{f}=374 \text{ Hz}, \tilde{St}=0.048$</th>
<th>$\tilde{f}=362 \text{ Hz}, \tilde{St}=0.026$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$12^\circ$</td>
<td>$f=500 \text{ Hz}, St=0.018$</td>
<td>$f=545 \text{ Hz}, St=0.024$</td>
<td>$f=578 \text{ Hz}, St=0.015$</td>
</tr>
<tr>
<td>$10^\circ$</td>
<td>$f=674 \text{ Hz}, St=0.009$</td>
<td>$f=699 \text{ Hz}, St=0.012$</td>
<td></td>
</tr>
<tr>
<td>$8^\circ$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.9 shows that taking this frequency brings the values of $\tilde{St}_{Lsep,C_L}$ closer together for both the $10^\circ$ and $8^\circ$ interactions, as might be expected for interactions of the same strength. However the $10^\circ \mathcal{A} = 2$ interaction still shows behaviour most similar to the $12^\circ \mathcal{A} = 1$ interaction. Further, the $12^\circ \mathcal{A} = 1.3 \tilde{St}_{Lsep,C_L}$ is still around twice that of the $12^\circ \mathcal{A} = 1$ case (with $L_{sep,C_L}$ being different by a factor of 2) as their $\tilde{f}$ was almost exactly the same.

Next, an attempt is made to normalise the median frequencies by a fairer representation of the separation length. Figure 4.16a shows the variation of Strouhal number (based on the median frequency and centreline separation length) with aspect ratio for each interaction strength. This is compared with Strouhal numbers scaled with the median, modal, mean and weighted separation lengths in Figures 4.16b, 4.16c, 4.16d and 4.16e, respectively. The weighted separation length is found considering the 8 data points nearest the centre (this will be shown to be the distance over which cross-communication is significant in Chapter 5). The lengths are then weighted with a cosine distribution (an approximate to the drop-off rate of correlation) centred about $Z^*=0$ such that data points nearer the centreline have a larger influence.

Figure 4.16b shows that taking the median separation length (as well as median frequency) to be characteristic brings the Strouhal numbers of different interaction strengths closer together (relative to taking $L_{sep,C_L}$). However the ratio of $St$ at different aspect ratios for one interaction strength remains almost the same as when taking centreline separation length. Taking the modal separation lengths from the bin centres in Figure 4.3 brings the Strouhal numbers of the $10^\circ \mathcal{A} = 1$ and $10^\circ \mathcal{A} = 1.3$ cases in line, possibly due to the necking at the centreline making it an unrepresentative length.

Like the median and modal normalisation, the mean normalisation also brings the $12^\circ \mathcal{A} = 1$ case in line with the $10^\circ \mathcal{A} = 1$ case. Taking the mean of all the lengths also brings the $\mathcal{A} = 1.3$ and $\mathcal{A} = 2$ cases in line for the $10^\circ$ and $8^\circ$ interactions. This might indicate that the location
at which these regions narrow is unimportant, rather the entire separation region affects the frequency of motion. Taking the weighted separation length as described above has a similar effect but with a greater spread in Strouhal numbers across the interaction strengths.

It seems that no single Strouhal number can describe these interactions, even though they all occurred in the same facility. Moreover, none of the interactions had a characteristic Strouhal number particularly close to $0.03$. It is probably more appropriate that the $12^\circ \ AR = 1.3$ interaction is considered to be quite distinct from the others; while the $12^\circ \ AR = 1$, $10^\circ \ AR = 1.3$ and $10^\circ \ AR = 2$ are similar in terms of their unsteadiness. The $8^\circ$ cases are also quite similar to each other although the $A = 2$ case might be closer to the $10^\circ \ AR = 1$ case. This suggests that the incident shock strength is not a good indicator of the interaction characteristics and cannot be used as a proxy for separation length.

Consistently, with both separation region and PSD characteristics, the interactions with different shock strengths have not behaved as distinct groups. Rather, the behaviour has been a continuous trend from the $12^\circ \ AR = 1.3$ to the $8^\circ \ AR = 1$ case. In some traits the $10^\circ \ AR = 2$ case is more similar to the $12^\circ \ AR = 1.3$ than the $12^\circ \ AR = 1$. This suggests that the incident shock strength is not a good indicator of the interaction characteristics and cannot be used as a proxy for separation length.

Returning to the raw frequencies in Tables 4.6 and 4.9, it is striking that the median frequencies of the $12^\circ$ interactions are so close, $\tilde{f} \approx 375 \text{Hz}$, for both aspect ratios. Therefore, for these two $12^\circ \ M = 2$ interactions, the characteristic frequency of the reflected shock motion seems to be independent of separation length. Grossman & Bruce (2018) postulated that the upstream movement of the separation line was due to the expansion fan from the underside of the shock generator meeting the floor further downstream for the higher aspect ratio cases.

Therefore it seems that the expansion fan placement does not influence the dominant frequency of the interaction nor the power distribution across all the low frequencies, as shown by the similarity of the statistical moments of the PSDs in Table 4.7. However, the absolute power
of each frequency is much greater for the smaller aspect ratio case (with the smaller separation bubble), as demonstrated by the peak powers listed in Table 4.5. With all else being near equal, it can be said that the $12^\circ \mathcal{A} = 1.3$ case has more severe separation but the $12^\circ \mathcal{A} = 1$ case exhibits more energetic (larger amplitude) low-frequency centreline separation shock motion.

### 4.3 Proposed model

Systems with a fixed frequency are analogous to a spring – mass balance for which frequency is determined as $\omega = \sqrt{k/m}$ by (4.2). In this analogy the separation bubble would be the mass $m$ moving with damping coefficient $k$ and the reflected shock would respond with displacement $x$, as shown in Figure 4.17.

$$F_0 = m \frac{d^2x}{dt^2} + kx \quad (4.2)$$

Plotkin (1975) previously successfully modelled low-frequency shock motion as a first-order linearly damped response to boundary layer perturbations. This seems to be a reasonable model given that a fluctuation in velocity upstream or downstream of the shock does not cause the shock to instantaneously change position. Rather, it results in the shock accelerating and therefore reducing the relative velocity fluctuation such that its displacement $x \propto \frac{d^2x}{dt^2}$, similarly to (4.2).

![Figure 4.17: Proposed spring – mass model of incident – reflected SBLIs.](image)

In this scenario, a cycle of shock motion would consist of an initial perturbation, for example due to an increased velocity in the boundary layer upstream of the shock. This would cause the shock foot to tend downstream, thereby reducing the relative increase in velocity upstream of it, in a frame of reference fixed to the shock. However the shock foot cannot move independently of the separation bubble as it represents the flow changing direction due to the presence of the bubble.

The bubble, as a viscous body, moves subject to friction and the inertia of the fluid mass trapped within it. Therefore it responds to the velocity fluctuation and resulting strengthening of the reflected shock at a damped rate. This damping might be expected to be greater for a larger bubble, as shown in Figure 4.17b. The shock excursion will then push the separation line downstream subject to the resistance of the mass until there is an opposite perturbation in the boundary layer large enough to overcome the bubble momentum.
Although the bubble mass probably varies in time, it fluctuates about a modal value which can be assumed to be associated to the characteristic frequency, otherwise the PSDs would exhibit a sharp spike at a single frequency. The temporal modal bubble dimensions are also what is measured from oil flow visualisation which cannot capture the relatively fast fluctuations in the separation and reattachment lines. For the remainder of the analysis these fluctuations will be assumed to be small compared to the total mass of the separated fluid.

The two $12^\circ$ interactions both have $f = 375\text{Hz}$ and are subjected to the same constant forcing $F_0$, which is the pressure difference across a $12^\circ$ oblique shock at $M = 2$. Table 4.4 shows that the ratio between the volume of fluid in the separated regions for the two cases was $V_{\mathcal{R}=1.3}/V_{\mathcal{R}=1} = 2$. Assuming that the densities in the subsonic separated regions were constant across the two cases, the mass ratio was also 2.

Given that both systems have approximately the same peak frequency $\omega = \sqrt{k/m}$, the relative damping of the motions would also be expected to be $k_{\mathcal{R}=1.3}/k_{\mathcal{R}=1} = 2$. The ‘stiffness’ $k$ of the bubble response would be inversely proportional to its amplitude of motion $A$, by (4.3). Hence it might be expected that the power in the characteristic frequency of the higher aspect ratio case will be about two times smaller than the lower aspect ratio case.

\[ A = \frac{F_0}{k} \quad (4.3) \]

Table 4.5 shows that the ratio of peak powers was measured to be around three times smaller $(A_{\mathcal{R}=1.3}/A_{\mathcal{R}=1} = 0.28)$. It may initially seem counter-intuitive that the case with the greater separation has a smaller amplitude of low-frequency motion. However, it seems reasonable that the larger bubble would experience a greater inertia due to its larger mass (manifesting as a greater damping coefficient $k$) and therefore have a proportionally smaller amplitude of shock displacement.

However, in this case the amplitude is not smaller according to the ratio of the volumes. This could be because the assumption of a constant $F_0$ in (4.3) does not hold. Grossman & Bruce (2018) showed that the shock polars changed significantly between $12^\circ$ $\mathcal{R} = 1$ and $\mathcal{R} = 1.38$ interactions at $M = 2$, such that the separation and transmitted shock strengths were different across the two interactions. Firstly, this would mean that the density increase through the separation shock would have been greater in the higher aspect ratio case, giving a slightly smaller expected mass ratio $m_{\mathcal{R}=1}/m_{\mathcal{R}=1.3} = 0.47$. Secondly, the relative strength of the transmitted shock was weaker for the higher aspect ratio case, giving a pressure difference (proportional to $F_0$) of 85% of that of the $\mathcal{R} = 1$ case. From (4.3), this would give an expected relative amplitude of motion $A_{\mathcal{R}=1}/A_{\mathcal{R}=1.3} = 0.4$, which is closer to the measured ratio of 0.28.

Although many assumptions were made in this analysis it has been somewhat successful in finding similarities between the two interactions even when the Strouhal numbers did not match. This shows that it remains possible that a common mechanism governs the unsteady behaviour of two interactions of the same strength but with different separation regions.

The same analysis can be applied to the $10^\circ$ $\mathcal{R} = 1$ and $\mathcal{R} = 1.3$ cases (Table 4.9 shows the two cases have close characteristic frequencies $f = 500\text{Hz}$ and $f = 545\text{Hz}$ but different reduced frequencies $St = 0.018$ and $St = 0.024$). Table 4.4 shows a volume ratio of $V_{\mathcal{R}=1}/V_{\mathcal{R}=1.3} = 0.85$
so the power ratio should be around 0.85. From Table 4.5, $A_{R=1.3}/A_{R=1} = 0.92$ which is reasonably close considering the characteristic frequencies are slightly different.

Performing the analysis again for the $8^\circ$ $\mathcal{R} = 1.3$ and $\mathcal{R} = 2$ cases yields an inverse volume ratio of $V_{R=1.3}/V_{R=2} = 0.86$ compared to a power ratio of $A_{R=2}/A_{R=1.3} = 0.87$, in spite of the characteristic frequencies being further apart. Overall the inverse proportionality of the volume and power holds better for weaker interactions.

For all the $8^\circ$ and $10^\circ$ interactions the transmitted shock strength $F_0$ is expected be constant for each shock strength (Figure 4.4 shows no transition to Mach reflection). However, Table 4.10 shows that the analogy does not hold for the $8^\circ$ $\mathcal{R} = 1$ and $10^\circ$ $\mathcal{R} = 2$ cases, which show significant differences to the other cases of their respective shock strengths.

It might have been expected that the $10^\circ$ $\mathcal{R} = 2$ case would not have the same product $AV_{\text{sep}}$ as its counterparts at lower aspect ratio as it has a different characteristic frequency. The different behaviour may be due to a transitioning to Mach reflection that is unobservable from the snapshot in Figure 4.4e. The $8^\circ$ $\mathcal{R} = 1$ case has the smallest separation volume (almost half of the $8^\circ$ $\mathcal{R} = 1.3$ case). Therefore the assumption of the fluctuations in the bubble mass being small compared to its total mass (for example due to the bubble collapse and refill events proposed by Piponnier et al. (2009)) is least likely to hold for this case.

Across different interaction strengths, the $12^\circ$ cases had characteristic frequencies just $13$ Hz higher than the $10^\circ$ $\mathcal{R} = 2$ case. Accounting for the different transmitted shock strengths, the ratios of $F_0/k$ for the $10^\circ$ $\mathcal{R} = 2$ case compared to the $12^\circ$ $\mathcal{R} = 1$ and $\mathcal{R} = 1.3$ were estimated to be 0.66 and 1.32, respectively. Table 4.5 shows that the amplitude ratios were 0.37 and 1.32. In this sense the $10^\circ$ $\mathcal{R} = 2$ has a similar behaviour to the $12^\circ$ $\mathcal{R} = 1.3$ case. However between the $8^\circ$ $\mathcal{R} = 2$ case and the $10^\circ$ $\mathcal{R} = 1$ and $\mathcal{R} = 1.3$ cases, which also had close $\bar{f}$, the relation does not hold.

This suggests that for certain conditions, the incident shock strength determines the product of the central separated mass and the amplitude of the reflected shock motion ahead of it $m_A \propto m/k$, given in Table 4.10. It shows that this was true for the $10^\circ$ $\mathcal{R} = 1$ and $\mathcal{R} = 1.3$ cases and $8^\circ$ $\mathcal{R} = 1.3$ and $\mathcal{R} = 2$ cases. This product is inversely proportional to the square of the characteristic shock frequency.

**Table 4.10:** Product of maximum power and central separation region volume $AV_{\text{sep}}$ [mm$^3$].

<table>
<thead>
<tr>
<th></th>
<th>$\mathcal{R} = 1$</th>
<th>$\mathcal{R} = 1.3$</th>
<th>$\mathcal{R} = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$12^\circ$</td>
<td>2985</td>
<td>1703</td>
<td></td>
</tr>
<tr>
<td>$10^\circ$</td>
<td>244</td>
<td>265</td>
<td>1251</td>
</tr>
<tr>
<td>$8^\circ$</td>
<td>27</td>
<td>91</td>
<td>92</td>
</tr>
</tbody>
</table>

By this measure (putting equal importance on volume of separation and amplitude of shock motion), the $12^\circ$ $\mathcal{R} = 1$ interaction was the most severe case tested. The severity was around half for the $12^\circ$ $\mathcal{R} = 1.3$ and $10^\circ$ $\mathcal{R} = 2$ interactions. For lower aspect ratios at $10^\circ$, there was a shift in behaviour towards higher characteristic frequencies $\bar{f}$ and similarly low severities $AV_{\text{sep}}$ of the interactions. The weakest interactions showed less severe behaviour still, with a shift to the weakest behaviour observed between $\mathcal{R} = 1$ and $\mathcal{R} = 1.3$. 

84
4.4 Summary

In this chapter, different incident – reflected SBLIs spanning a range of separation lengths, from near zero at the centreline of the $8^\circ \mathcal{A} = 1$ interaction to 65 mm at the centreline of the $12^\circ \mathcal{A} = 1.3$ interaction, have been characterised in terms of their separation regions and the low-frequency unsteadiness at the centre of the duct.

It is proposed that the characteristics of the PSDs are directly related to those of the distributions of separation lengths within the central separation regions. Conversely, this could ultimately enable design of ‘bespoke’ PSDs by affecting the separation regions caused by the SBLIs. For example, an interaction with greater variation in separation length across the span might cause a more broadband low-frequency peak in the PSDs which may be desirable from the perspective of fatiguing structures.

In general, the separation region and PSD characteristics fit on a spectrum increasing or decreasing first with aspect ratio and then with shock strength. The mean separation length and standard deviation of the PSDs increased continuously from $8^\circ \mathcal{A} = 1$ to $12^\circ \mathcal{A} = 1.3$. In contrast, the characteristic frequencies were very similar for some of the interactions and the peak power in the PSDs generally increased with shock strength but decreased with aspect ratio.

The cases with similar characteristic frequencies at different shock strengths ($12^\circ \mathcal{A} = 1$ with $10^\circ \mathcal{A} = 2$ and $10^\circ \mathcal{A} = 1$ with $8^\circ \mathcal{A} = 2$) also had similar centreline separation lengths and therefore similar reduced frequencies $St_{L_{\text{sep.}}}$ However for interactions with the same strength and frequency, the increase of separation length with aspect ratio meant that they had different $St_{L_{\text{sep.}}}$ In this sense, the separation length is a good indicator of the low-frequency unsteadiness and interaction strength cannot be used as a proxy for this.

Under certain conditions the parameter $AV_{\text{sep}}$ collapsed the opposite trends observed in separation region volumes and amplitudes of pressure fluctuations for the interactions with similar frequencies of shock motion. This was particularly effective where the reduced frequency $St$ did not collapse (across interactions of the same strength but different aspect ratio).

This approximate inverse relationship between separation volume and shock motion amplitude for interactions with similar shock frequencies means that there is no ideal solution when choosing SBLI characteristics, as both large separation regions and large-amplitude, low-frequency shock motions are undesirable. The parameter $AV_{\text{sep}}$ can therefore be used as a measure of the relative severities of the interactions in terms of separation and unsteadiness.

Table 4.10 shows that the product $AV_{\text{sep}}$ was greater for the stronger shocks, as might be expected. However, it is interesting to note that there seems to be a point where a different behaviour comes into effect. This is $1.3 < \mathcal{A} < 2$ for the $10^\circ$ interactions and $1 < \mathcal{A} < 1.3$ for the $8^\circ$ interactions. By this measure, the interactions behave categorically distinct from each other rather than on a spectrum. This might signify the existence of a sudden switch in behaviour similar to the regular – irregular (Mach) transition of the $12^\circ$ cases. In the $8^\circ \mathcal{A} = 1$ case, this might be due to the separation bubble becoming too small to be considered alongside the other separated SBLIs.

Although the estimated total central separation volume has been taken into account, only the centreline unsteadiness has been considered. This might mean that the global overall severity of unsteadiness follows different trends and that focussing on the centreline alone is misleading.
Furthermore, taking a measure of overall unsteadiness might yield a better still collapse of the data, in the same manner that taking the median separation lengths and frequencies did. The next chapter therefore explores the low-frequency behaviour of the interactions away from the centreline.
Chapter 5

Spatial variation of low-frequency unsteadiness

The spatial variation in the characteristics of the interaction PSDs in the floor plane is presented in Section 5.2, the variation in reduced frequency is presented in Section 5.3 and the relation between pressure histories across the span is evaluated in Section 5.4. Finally there is a summary in Section 5.5.
CHAPTER 5. SPATIAL VARIATION OF LOW-FREQUENCY UNSTEADINESS

5.1 Introduction

![Image of spatial distribution of time mean pressures and standard deviations]

Figure 5.1: Left: spatial distribution of time mean pressures normalised by expected pressure upstream of interaction $p(x, z)/p_\infty$. Coloured dots show centre of measurement locations. Right: spatial distribution of standard deviations of pressure fluctuations normalised by mean pressure $\sigma(p(x, z))/\bar{p}(x, z)$. Same contour scaling used for all plots but only shown on $12^\circ AR = 1.3$ plot for clarity.

The streamwise and spanwise distributions of the mean and standard deviation of the pressure recorded over the full 14 s runs are presented in Figure 5.1. The measurement locations
of the Kulite sensors are shown by coloured dots (not to scale). The plots illustrate the high spatial resolution of the measurements. Each sub-figure contains two plots: the left-hand plots show the mean pressure distribution and the right-hand plots show the variation in the standard deviation of the pressure. In this figure the pressures are normalised by the expected upstream pressure and plotted on the same colour scale, shown in Figure 5.1b.

The mean pressure plots show that for all test cases the pressure at the more upstream measurement points was close to what is expected for the upstream boundary layer \( \frac{p}{p_\infty} \approx 1 \). Further downstream, the mean pressure rises to near its post-shock value. For example, for the \( 12^\circ \) \( A = 1 \) case in Figure 5.1a there is a step-change in \( p \) at around \( X^* = -1.08 \). The mean pressure rise occurs fairly uniformly across the span of the interactions, including towards the sidewalls. However, the pressure increases to greater values near the centreline, where the mean flow velocity is greater and therefore the separation shock is likely to be the strongest.

The right-hand plots of the sub-figures in Figure 5.1 show the standard deviation of pressure fluctuations, normalised by the local mean and plotted on the same colour scale for all cases (shown in Figure 5.1b). These plots reveal different regions of high levels compared to the absolute mean pressures. The largest magnitudes of pressure fluctuations are observed slightly upstream of the regions of highest pressure. This is because the regions of high pressure are under the separation regions while the regions of high fluctuations are under the separation shocks upstream of this. The pressure fluctuations decrease under the separation region but remain slightly higher than those in the upstream boundary layer. The streamwise extent of the band of high pressure fluctuations is a good indicator of intermittent length and is seen to be longer for greater shock strengths and aspect ratios.

With the exception of the \( 12^\circ \) \( A = 1 \) case, the regions of high fluctuations in pressure are curved to be more upstream near the centre of the duct. Increased fluctuations are also observable for a large part of the corner separations in all cases. The fluctuations compared to the mean are greatest for the \( 12^\circ \) \( A = 1 \) case and generally decrease in magnitude with both increasing aspect ratio and decreasing shock strength.

Figure 5.2 shows the mean pressure distributions for each case overlaid onto the corresponding oil flow images. It shows that the pressure rise is well-correlated with the separation shock position inferred from the separation line in oil flow images. Furthermore, regions where the pressure is high compared to other locations at the same streamwise position correlate with a localised earlier separation. In all test cases a pressure rise is also observable in the corner separations. This streamwise pressure increase occurs at a similar location to that in the central separation, shortly downstream of the initial growth in the corner separations.

Lu et al. (1990) proposed the semi-empirical relation for the angle of upstream influence \( \beta_U \) in a conically similar swept SBLI in (5.1), where \( \beta \) is the shock angle and \( \mu_\infty \) is the freestream Mach angle. A diagram showing the angles \( \beta_U \) and \( \beta \) is shown in Figure 5.3. This relation suggests that this angle is therefore independent of aspect ratio and is \( 51.9^\circ, 48.1^\circ \) and \( 44.4^\circ \) for the \( 12^\circ \), \( 10^\circ \) and \( 8^\circ \) interactions respectively. From this, the location of the earliest expected pressure rise in the corner relative to the inviscid impingement point is shown in Table 5.1. Interestingly, this depends on the incident shock angle and the absolute duct heights in each case, not the aspect ratio.
Figure 5.2: Temporal mean pressure distributions overlaid onto oil flow images.
\[
\beta_U - \mu_\infty = 2.2(\beta - \mu_\infty) - 0.027(\beta - \mu_\infty)^2 \tag{5.1}
\]

Table 5.1: Location (\(X^*\)) of maximum upstream influence of sidewall interactions based on a conically similar swept SBLI [\(\text{-}\)].

<table>
<thead>
<tr>
<th>(\alpha)</th>
<th>(A = 1)</th>
<th>(A = 1.3)</th>
<th>(A = 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12°</td>
<td>-1.5</td>
<td>-0.6</td>
<td></td>
</tr>
<tr>
<td>10°</td>
<td>-2.4</td>
<td>-1.7</td>
<td>-0.7</td>
</tr>
<tr>
<td>8°</td>
<td>-6.4</td>
<td>-3.8</td>
<td>-2.0</td>
</tr>
</tbody>
</table>

The values in Table 5.1 do not correlate with where the corner pressure rise is seen in Figure 5.2. There is some correlation with the central separation pressure rise which means that the swept SBLIs might be part of the mechanism of where the central separation occurs.

![Swept SBLI upstream influence, from Lu et al. (1990).](image)

**Figure 5.3:** Swept SBLI upstream influence, from Lu et al. (1990).

### 5.2 Spatial variation in PSDs

#### 5.2.1 Variation of characteristic frequency and power

A selection of the PSDs computed for the 12° \(A = 1\) interaction are shown in Figure 5.4. Figures 5.4a, 5.4c, 5.4e and 5.4g each show ten streamwise measurements taken at \(Z^* = -0.02\), \(Z^* = -0.46\), \(Z^* = -0.70\) and \(Z^* = -0.94\), respectively. Based on the oil flow visualisation in Figure 4.1a, these locations represent the centreline; the midpoint between the centreline and edge of the central separation region; the attached channel between the central and corner separation; and the corner separation. Figures 5.4b, 5.4d, 5.4f and 5.4h show the PSDs at the corresponding locations in the positive \(z\)-plane.

Figure 5.4 shows that at most spanwise locations the most upstream measurement point, at \(X^* = -1.28\), recorded a mean pressure close to the freestream and little low-frequency power. Figures 5.4g and 5.4b are an exception to this and show that at \(Z^* = -0.94\) and \(Z^* = 0.02\) the mean pressure at \(X^* = -1.28\) was already above that of the freestream and there is a considerable amount of low-frequency power in the PSDs here. The most powerful PSDs at all spanwise locations occur just before the pressure rise, at \(X^* = -1.12\), after which the power greatly decreases. For \(X^* > -1.12\) there is some mid-frequency power in the PSDs at all spanwise
locations, confirming that these measurement points are under the separation region at some times.
5.2. SPATIAL VARIATION IN PSDS

Figure 5.5: Spatial variation in peak power for $12^\circ \mathit{AR} = 1$ case.

The surface in Figure 5.5 shows the spatial variation in the power associated with the area-weighted median frequency, which was computed only at the centreline in Section 4.2.2. Alongside Figure 5.4 it also shows that the power in the low frequencies rose steeply at all streamwise locations from near zero at $X^* = -1.28$ (red line). This is with the exception that there is already some power at this streamwise position at the anomalous separation line curvature at $Z^* \approx 0.31$ and in the corner near $Z^* = -1$.

Many spanwise locations exhibit a small dip followed by a second peak. As the power associated with low frequencies is rarely reported in the literature, it is not possible to compare this to previous results. The dip in power is at the location of the small drop in mean pressure seen in the inset figures in Figure 5.4. It is also observed in the $12^\circ \mathit{AR} = 1.3$ case.

Finally there is a sharp decrease in power for the final five streamwise measurement locations. Figure 5.4 shows that there are still some low-frequency pressure fluctuations at these locations, as well as the additional spectrum peak at higher frequencies, of the order of $St \sim 1.5$, that is characteristic of locations under the separation region. However, Figure 5.5 emphasises how unenergetic the low-frequency power is here compared to the more upstream points.

Figure 5.5 shows that the peak power occurs in the region $-1.24 < X^* < -1.12$ at all spanwise positions. This is somewhat unexpected in the region where the mean separation line is strongly distorted and shown by the oil flow to vary in streamwise location by $0.29L_{sep,C}$. 

93
over 15% of the half-span. This is also not trivial with regards to the measurement in the corner separation, for which there is no immediately obvious reason for it to be in line with the central separation shock. In fact, this result is in contrast to that of Jammy & Sandham (2017) who saw little energy in the corner around $X^* = -1$ but found that the most energetic motion at $St = 0.03$ was 20 mm or $0.7L_{sep,G}$ downstream of that at the centre. A comparison of contour plots for the power at $St = 0.03$ is shown in Figure 5.6, with the peaks in the unsteadiness in the corner separations highlighted by the black boxes.

It should be noted that the LES of Jammy & Sandham (2017) was for a $M = 2.79^\circ$ reflected SBLI which gives an expected inviscid pressure rise of $p_2/p_1 = 1.82$, close to the expected value for the Mach 2 $12^\circ$ interaction considered here ($p_2/p_1 = 1.88$). However, the duct dimensions were different to those in the current study: width $\times$ height $= 100\text{mm} \times 50\text{mm}$ giving an aspect ratio $AR = 2$. The simulation is described further in Wang et al. (2015) from which Jammy & Sandham (2017) took the WH2 ($AR = 2$) case and ran it for 23 expected low-frequency periods in order to obtain statistics on low-frequency behaviour.

It is probable that the low-frequency peaks in the corner separations, seen in Figure 5.6, are associated with the separation shock of the swept SBLIs between the incident shock and the sidewall, in a manner similar to the oblique SBLI on the floor. As discussed in Section 5.1, the conical angle of upstream influence of the swept SBLI $\beta_U$ is $51.9^\circ$ for the experiment, and Figure 5.3 shows that the separation shock angle will be smaller than this. For the LES, (5.1) gives $\beta_U = 36.1^\circ$. This means that the distance between the upstream influence point and the inviscid impingement point of the main shock at the floor was 22 mm, compared with 51 mm for the experiment.

As the conical angle of upstream influence marks the beginning of the pressure rise due to the SBLI, it is expected that the location of high low-frequency energy due to the separation shock will be downstream of this. This means that peaks in the corners would be expected to be at

---

**Figure 5.6:** Spatial variation in power in $St = 0.03$. Black boxes highlight corner separation unsteadiness.
5.2. SPATIAL VARIATION IN PSDS

$X^* > -0.77L_{sep,C}$ and $X^* > -1.51L_{sep,C}$ for the LES and experiment, respectively. Figure 5.6 shows that the peaks are actually observed at $X^* \approx -0.4$ and $X^* \approx -1.2$, respectively.

Furthermore, extending the upstream influence line to a cone about the incident shock along the sidewall, the maximum extent of the influence of the swept SBLI into the centre can be estimated. This is $6.3\,\text{mm}$ from the walls or $|Z^*| > 0.87$ in the LES and $27.3\,\text{mm}$ from the walls or $|Z^*| > 0.64$ in the experiments.

Figure 5.7 shows the contour maps of the power in $St = 0.03$ overlaid with the mean separation regions. In Figure 5.7a, the separation regions are shown by the white lines along which $(\partial u/\partial y)_w = 0$. It shows that the increased power in the corners aligns well with the location of greatest separation in the corners. This gives further confidence that a second peak does not exist in the present case and the region of significant low-frequency corner behaviour has been captured spatially.

![Figure 5.7: Spatial variation in power in $St = 0.03$ with mean separation regions. Taken from Rabey et al. (2019).](image)

The corner separations in Figure 5.7a are much smaller than those in Figure 5.7b, corresponding with the proposed conical relations. This reduced separation might also be because the incident shocks penetrate different depths into the sidewall boundary layer. Wang et al. (2015) proposed that the sweepback effect at the sidewalls means that the penetration Mach number $M_p$ (the effective sonic line for the sidewalls) is where the normal component of the Mach number is equal to unity. This depends on the shock angle $\beta$ by (5.2).

$$M_p = \frac{1}{\sin \beta} \quad (5.2)$$

For the LES, $M_p = 2.07$ (Wang et al., 2015); while for the $12^\circ$ interactions in the present study, $M_p = 1.51$. This means that the incident shock can reach closer to the sidewall boundary layers in the experiments and therefore cause the larger corner separation. It might also be the reason the corner boundary layers appear to separate earlier in the experiments.
In further contrast, Figures 5.4g and 5.6b show that there is a large peak in power near the wall in the negative $z$-plane, upstream of that at the centre. As this peak is the first measurement position ($X^* = -1.28$) in this case, it is not possible to say if this is the exact peak location or if it occurs further upstream. The mean pressure at this location is $p/p_\infty = 1.4$ (shown in Figure 5.4g); therefore it is probable that this measurement is just after the initial pressure rise through the corner separation shock, seen in the opposite corner at $X^* = -1.12$ in Figure 5.4h.

This peak in power in the corner separation has a greater magnitude than those at the centre and in general the power in the corner is comparable to that in the central separation. Compared to the peak power in the central separation at $Z^* = 0.17$, Figure 5.6b shows that the energy in the corner was approximately 1.5 times smaller. On the other hand, Jammy & Sandham (2017) found that the power was approximately 8 times lower in the corner separations than at the most powerful central peak (see Figure 5.6a). This might be attributable to the lower experimental duct aspect ratio giving the swept sidewall SBLIs greater influence; however even higher ratios of corner to centre power were observed for the $12^\circ$ $AR = 1.3$ case.

Figure 5.5 shows that the peak power in the spanwise direction does not occur at the centreline. Instead, there are two off-centre peaks at $Z^* \approx \pm 0.20$. The peak in the positive $z$-plane is quite steep and seems to be associated with the strong curvature of the separation line here. In the negative $z$-plane the peak is more gradual over a large spanwise distance $-0.50 < Z^* < -0.02$. However, this can again be associated with the gradual upstream curvature of the separation line here.

This unexpected result is further explored in Figure 5.8 which shows a comparison of the spanwise variation in the weighted power spectral density (WPSD) contained in different Strouhal numbers along the streamwise line with the greatest power at the centre. Figure 5.8a shows that Jammy & Sandham (2017) also found off-centre peaks in the power at $St = 0.07$ and $St = 0.09$ at $Z^* \approx \pm 0.35$ in their LES simulation of an incident – reflected SBLI with sidewalls. Figure 5.7a shows that this does not correlate with earlier separation.

![Figure 5.8: Spanwise variation in WPSD value for $St = 0.03$, $St = 0.07$ and $St = 0.09$ along streamwise location of central peak power.](image-url)
 Furthermore, Figure 5.8a shows that Jammy & Sandham (2017) found asymmetric peaks in the WPSD at $St = 0.03$, similar to the experimental results in Figure 5.8b. This is interesting as it cannot be readily explained by differences in the incoming boundary layer across the span of the duct. However, as such asymmetry is not seen in the higher Strouhal numbers, it may be due to a lack of convergence due to the WPSD being computed with 23 low-frequency cycles (Jammy & Sandham, 2017). In the experiment, all of the peaks are not quite symmetric about the centreline. However, the ratio of the power across the peaks for the different Strouhal numbers remains fairly constant (at approximately 1.2), implying convergence.

The full, experimentally measured distributions of characteristic frequency $\tilde{f}$ and associated power $\tilde{p}$ under the $12^\circ$ $AR = 1$ interaction are shown in Figures 5.9a and 5.9b, respectively. They show that regions of high power are associated with low characteristic frequencies, and vice versa. Figure 5.9b shows that the peaks observed in Figure 5.8b extend in the streamwise direction, over approximately $0.15L_{sep}$.  

![Figure 5.9: Spatial variation in ‘area-weighted’ median frequencies and associated power for $12^\circ$ $AR = 1$ interaction. Black dashed lines are separation lines inferred from oil flow. Plots have been stretched by a factor of 2 in the x-direction.](image)

Figure 5.10 shows the same plots for all test cases. The left-hand plots in the subfigures in Figure 5.10 show the spatial distribution of the area-weighted median frequency (the computation of which was discussed in Section 4.2.2) and the right-hand plots show the
associated power at these frequencies. As expected, the WPSD contour plots in the right-hand figures of Figure 5.10 closely resemble the plots of pressure fluctuations in the right-hand figures of Figure 5.1. Similarly to Figure 5.1, in Figure 5.10 there are bands of high-frequency, low-power motion upstream of the separation shock; low-frequency, high-power motion along the separation shock and in the corner separations; and mid-frequency, mid-power motion under the separation region. This proposed typical two-dimensional distribution of unsteadiness under incident – reflected SBLIs is summarised in Figure 5.11.

![Figure 5.10](image)

**Figure 5.10:** Left: spatial variation in ‘area-weighted’ median frequencies. Right: power associated to ‘area-weighted’ median low frequencies of PSDs. Black dashed lines are separation lines inferred from oil flow. Plots have been stretched by a factor of 2 in the x-direction.

For each case, the median frequency remains fairly constant along the separation shocks (inferred to be just upstream of the separation lines marked with the black dashed lines) and into the corner separations. This might mean that the entire shock – separation bubble system oscillates as one entity. This possibility is explored further by means of cross-correlations in Section 5.4. Otherwise, it might mean that the same local phenomenon is occurring across
the span of the interactions. This would be equivalent to a series of distinct two-dimensional interactions with no spanwise interaction.

![Proposed typical distribution of unsteadiness of incident – reflected SBLIs at floor plane.](image)

**Figure 5.11:** Proposed typical distribution of unsteadiness of incident – reflected SBLIs at floor plane. Red: high frequency, low power; blue: low frequency, high power; green: mid frequency, mid power; yellow: high frequency, low power.

The contour maps of the power contained in the characteristic frequencies for all of the interactions from Figure 5.10 are overlaid onto the oil flow images in Figure 5.12. These composite images show a strong correlation in all cases between earlier separation and more powerful low-frequency pressure fluctuations. A decrease in the power exists in the regions of the attached channels between the corner and central separations in those test cases where such a channel exists.

Figure 5.12 also shows that there is some low-frequency power in the corners for all $12^\circ$ and $10^\circ$ test cases; however this is not symmetric. In most cases there is more power in the corner in the negative $z$-plane which can most likely be attributed to an asymmetry in the two corner boundary layers. Generally the low-frequency motions in the corner separations are stronger for interactions with greater shock strengths and aspect ratios.

Interactions of the same strength are expected to have the same shock and Mach angles and therefore the same upstream influence angle according to (5.1). In this set-up, when the aspect ratio increases the duct height decreases. Therefore the distance between the inviscid impingement point of the incident shock and the point of its upstream influence at the corner is expected to decrease by a factor equal to the ratio of the duct heights (in this case also equal to the ratio of the aspect ratios).

The separation lengths $L_{\text{sep},C}$ increase non-linearly with aspect ratio. This means that where the central low-frequency motions are the most prevalent might be expected to be at different streamwise locations relative to the occurrences in the corners. For example, the $12^\circ$ $\mathcal{R} = 1$ upstream influence is expected to extend to $X^* = -1.5$ and low-frequency power is observed in the corners slightly downstream of this in line with the central separation at around $X^* = -1.2$. For the $12^\circ$ $\mathcal{R} = 1.3$ case, the upstream influence is expected to have a maximum
Figure 5.12: Distribution of power associated with area-weighted median frequency overlaid onto oil flow images.
5.2. SPATIAL VARIATION IN PSDS

at \( X^* = -0.6 \) and therefore not cause corner separation upstream of the central interaction at the floor. However, both oil flow and power maps show the corners separate just upstream of the central separation at \( X^* = -1.0 \). Therefore the actual corner separation point may be whichever is more upstream out of the point of upstream influence and the location of the central separation.

Table 5.1 shows that, for the \( 8^\circ \) interactions, the swept SBLIs are expected to have an upstream influence at \( X^* > -6.4, X^* > -3.8 \) and \( X^* > -2.0 \) for the \( AR = 1, AR = 1.3 \) and \( AR = 2 \) cases, respectively. This is outside of the measurement range and may explain why low-frequency power is observed in the region near the sidewalls of the \( AR = 1 \) and \( AR = 1.3 \) interactions. The shallower shock angle of the \( 8^\circ \) cases means that the swept SBLIs on the sidewalls are much weaker than those of the stronger interactions and therefore any low-frequency motions might be expected to be less energetic.

The left-hand plots in the sub-figures of Figure 5.10 show the distribution of characteristic low frequencies. For all test cases, the characteristic frequency shows little variation along the separation line. As proposed in Section 4.2.2, taking one value to characterise the severity of the low-frequency behaviour may not give a fair comparison across different positions. It was therefore proposed that considering higher-order statistical moments could reveal more information on the low-frequency behaviour. Section 5.2.2 compares the higher-order statistical moments in the \( x - z \) plane to assess where the low-frequency behaviour is the most prevalent.

5.2.2 Variation of statistical moments

The characteristic frequencies and associated power discussed in Section 5.2.1 are good overall indicators of the spatial variation in low-frequency behaviour. It was proposed in Section 4.2.2 that taking the area-weighted median frequency is more representative of the characteristic frequency of the PSDs. Taking this frequency will also show more spatial variation due to not being constrained to multiples of the frequency resolution.

As suggested in Section 4.2.2, considering the higher-order area-weighted statistical moments of the PSDs provides further information on how representative the median frequencies are. For example, if a PSD has a low standard deviation and a high kurtosis, the median frequency can be expected to be dominant over other frequencies in the pressure signal. This section assesses whether the trends observed in Section 4.2.2 for \( Z^* = \pm 0.02 \) hold at off-centre positions.

For the \( 12^\circ \ AR = 1 \) test case, Figure 5.4 appears to show more skewed and peaky PSDs near the centre and near the sidewalls (Figures 5.4a, 5.4b, 5.4g and 5.4h). It was established in Section 4.2.2 that PSDs with a stronger skewness and kurtosis were indicative of a stronger interaction at the centreline. Figures 5.4c, 5.4d, 5.4e and 5.4f show that the PSDs of the \( 12^\circ \ AR = 1 \) case at \( Z^* = \pm 0.46 \) and \( Z^* = \pm 0.70 \) are less skewed towards lower frequencies and more broadband. This suggests that the low-frequency behaviour is strongest near the centreline and weaker in the outer central separation. It is interesting that the PSDs at \( Z^* = \pm 0.94 \) are more similar to those near the centreline than those in the central separation away from the centreline (\( Z^* = \pm 0.46 \)).

The full spatial distributions of the area-weighted standard deviation, skewness and kurtosis of the PSDs of the \( 12^\circ \ AR = 1 \) interaction are shown in Figure 5.13. The mean separation line inferred from oil flow is shown by the black dashed line. This shows clearly that the separation
line bounds an upstream region of low standard deviation, high skewness and high kurtosis and a downstream region of high standard deviation, low skewness and low kurtosis. This shows that the entire separation shock exhibits low-frequency motions that are tightly centred around a dominant frequency with a gradual drop-off in power towards higher frequencies. There is some decrease in this behaviour at $Z^* = \pm 0.7$, outside of the central separation region. Near the corners, particularly at $Z^* = -1$, these characteristics are strongest.

This is explored further alongside the other test cases in Figures 5.14, 5.15 and 5.16. From the PSD at each measurement location, the area-weighted statistical moments are computed by weighting each frequency by the apparent area of the PSD below it, in the same manner as in Section 4.2.2. The resulting spatial distributions of the standard deviation, skewness and kurtosis are shown in Figures 5.14, 5.15 and 5.16, respectively.

Figure 5.14 shows the spatial variation in the standard deviations of the PSDs with the separation lines, inferred from the oil flow visualisation, shown as black dashed lines. In all test cases the separation line bounds regions of higher and lower standard deviation. This shows that the mean separation position affects the spatial distribution of unsteady behaviour.

The same spatial distribution of the standard deviations of the PSDs relative to the mean separation line is observed for all interactions. The magnitudes of the standard deviations under the separation shock region are similar for all interactions ($1000 \text{ Hz} < \sigma < 1400 \text{ Hz}$). They also increase to approximately $2000 \text{ Hz}$ under the separation regions in all cases.

It can be inferred that the separation shock is associated with the band of low standard deviations in the frequency of pressure fluctuations immediately upstream of the mean separation line. Further, the separation region is downstream of the mean separation line, with higher standard deviations of fluctuations in pressure, close to the values upstream of the interactions. This shows that under the separation shock the frequencies associated with the pressure fluctuations are closely grouped around the characteristic lower frequencies, which are shown in the left-hand plots in the sub-figures of Figure 5.10.
5.2. SPATIAL VARIATION IN PSDS

Figure 5.14: Area-weighted standard deviation $\sigma$ of PSDs [Hz]. Black dashed lines are separation lines inferred from oil flow. Plots have been stretched by a factor of 5 in the $x$-direction.
In the $12^\circ$ and $10^\circ$ interactions, the regions of lower standard deviations upstream of the mean separation line are approximately $0.2L_{sep,Q}$ in streamwise length across the span of the interactions. This is indicative of the intermittent length scale. The $8^\circ$ interactions have larger intermittent lengths in comparison to the centreline separation length: approximately $0.5L_{sep,Q}$ for the $AR = 2$ case and approximately $L_{sep,Q}$ for the $AR = 1$ and $AR = 1.3$ cases.

Figure 5.15 shows the spatial variation in the area-weighted skewness of the PSDs under the interactions. A normal distribution has zero skewness while positive skewness indicates that the PSDs exhibit a steep rise in power towards the characteristic low frequency and a long tail at higher frequencies. Figure 5.15 shows that the skewness $\gamma$ of the PSDs approaches zero in the incoming boundary layers but is positive everywhere else. This measure exhibits a higher band under the separation shock, in almost the exact inverted trend of Figure 5.14. While all measurement locations exhibit a positive skewness, with a low-power tail at higher frequencies, the magnitude of the skewness along the separation shock is several times that of the upstream boundary layer and under the separation bubble in all cases.

In the corner at $Z^* = -1$ for the $12^\circ$ cases, the PSDs have a greater skewness still, 4 or 5 times that of the upstream boundary layer or under the separation region. In the corners of the $10^\circ$ interactions, the skewness of the PSDs are approximately the same as under the separation shock in the central region. In the $8^\circ$ interactions, the corner separations exhibit a lower skewness than at the centre, possibly because there is less separation here (see Figure 5.12, for example). However, it could be that the transducers were not close enough to the sidewalls for these interactions to capture the unsteady behaviour in the smaller corner separations.

Figure 5.16 shows the streamwise and spanwise variation in the area-weighted kurtosis of the PSDs for all test cases. It can be seen that all of the PSDs exhibit a kurtosis above the expected value of 3 for a normal distribution except in the upstream boundary layer. The high values are indicative that the PSDs exhibit steep peaks about the characteristic frequencies. There is a similar trend to the skewness distributions in that there are bands of higher kurtosis upstream of the mean separation line. The bands are narrower than the bands of increased skewness, showing that there is a small region where the frequencies become tightly grouped around the characteristic frequency. This possibly highlights the mean separation shock position.

In both Figures 5.15 and 5.16, there are locally higher moments along the separation line where the separation line is displaced upstream (inferred from the oil flow visualisation in Figure 4.1). This shows that more typical low-frequency PSD characteristics (strong positive skewness, very high kurtosis) are seen where there is locally stronger separation. This can also be inferred from the magnitudes of both moments increasing with increasing shock strength.
Figure 5.15: Area-weighted skewness $\gamma$ [-] of PSDs. Black dashed lines are separation lines inferred from oil flow. Plots have been stretched by a factor of 5 in the $x$-direction.
Figure 5.16: Area-weighted kurtosis $\kappa [-]$ of PSDs. Black dashed lines are separation lines inferred from oil flow. Plots have been stretched by a factor of 5 in the $x$-direction.
5.3 Spatial variation in Strouhal number

This section explores the spanwise variation in the reduced low-frequency motions. At each spanwise measurement location, a streamwise position is selected from which to take the representative Strouhal number using the $12^\circ \ AR = 1$ case as an illustration. Figure 5.12 shows that the power in the area-weighted median frequency is a good indicator of the separation line. The location of the maximum of this quantity at each spanwise measurement position is indicated by the solid red line on the floor plane in Figure 5.17.

It can be seen that the streamwise position of maximum power follows the curvature of the separation line fairly closely. Moreover, the variation in the power (shown by the solid red line in the $z$-axis in Figure 5.17) shows peaks and troughs that appear to correlate with instances when the separation line is more upstream or downstream, respectively.

The streamwise location of the lowest frequency at each spanwise position is shown in Figure 5.17 by the dashed red line on the floor plane. It also approximately follows the separation line curvature. The magnitude of the minimum frequency (shown by the dashed red line in the $z$-axis in Figure 5.17) shows a fairly smooth increase in both directions away from $Z^* = 0$. This shows that the frequency of the central separation shock motion is not greatly affected by small streamwise variations in the position of the separation line: the frequency only increases for $|Z^*| > 0.3$ as the separation line gradually curves downstream. In the corner separation near...
CHAPTER 5. SPATIAL VARIATION OF LOW-FREQUENCY UNSTEADINESS

\( Z^* = -1 \) there is a sharp decrease in the lowest frequency accompanied by an increase in the maximum power.

The greatest power at the centreline occurs at \( X^* = -1.20 \). The variation in the power in the PSDs at this streamwise location is shown in Figure 5.18. It can be seen that there are two large bands of low-frequency energy in the central separation region. In the negative \( z \)-plane, this is in line with the slight upstream curvature of the separation line; while in the positive \( z \)-plane this is in line with the abrupt curving of the separation line. The energy decreases away from the centre as the separation line moves further downstream of \( X^* = -1.20 \). There are large low-frequency energy bands again in the corner separations.

![Energy map at \( X^* = -1.20 \) and separation region for 12\(^\circ\) A = 1 case.](image)

**Figure 5.18:** Energy map at \( X^* = -1.20 \) and separation region for 12\(^\circ\) \( \mathcal{A} = 1 \) case.

The streamwise distributions of the most powerful low-frequency occurrence at each spanwise measuring position and its associated power are shown in Figure 5.19. Figure 5.19a shows that the spanwise variation of the streamwise position with the most powerful low frequency is similar across all test cases. The absolute spread of the most energetic frequencies, which is indicative of the separation shock radius of curvature, is around 6 mm in all cases.

The most powerful streamwise point tends downstream (\( x - \bar{x} \) increases) towards the sidewalls. This is expected due to the general curvature of the separation shocks. Figure 5.19b shows that there is little spanwise variation in the streamwise position of the highest low-frequency energy across the span of the 12\(^\circ\) and 10\(^\circ\) interactions, relative to \( L_{sep} \bar{q}_v \). The variation in \( X^* \) for the 8\(^\circ\) cases shows similar underlying trends but with larger fluctuations, particularly relative to the centreline separation length. These cases also show the most fluctuations in this position across the span, particularly in the positive \( z \)-plane, possibly due to the separation being incipient.

Figure 5.19c shows that the absolute spanwise variation in power was greatest for the 12\(^\circ\) \( \mathcal{A} = 1 \) interaction, followed by the 12\(^\circ\) \( \mathcal{A} = 1.3 \) and 10\(^\circ\) \( \mathcal{A} = 2 \) interactions. Thereafter it
5.3. Spatial Variation in Strouhal Number

Figure 5.19: Variation in maximum low-frequency power and its position of occurrence across span of interactions. Black - 12°, red - 10°, blue - 8°. ■ - \( AR = 1 \), ♦ - \( AR = 1.3 \), ▲ - \( AR = 2 \).

Generally decreases with decreasing aspect ratio and decreasing shock strength. Figure 5.19d shows that no test cases exhibit the maximum power in low-frequency motions at the centreline. Most cases show maxima in the positive \( z \)-plane. Conversely there are equally or more powerful motions compared with those near \( Z^* = 0 \) in the corner near \( Z^* = -1 \).
Figure 5.17 showed that the spanwise variation in peak power for the $12^\circ$ $AR = 1$ case followed the curvature of the separation line. Figure 5.19d shows that this is also true for the other test cases. For example, those for which the separation region necks down near the centre (the $8^\circ$ and $10^\circ$ cases) show a decrease in power near the centreline.

As is evident from Figure 5.10, the characteristic frequency and associated power do not scale in a straightforward way with distance from the centreline ($Z^*$) and therefore cannot be normalised by a measure of this. The spanwise variation in the characteristic Strouhal number based on the centreline separation length is shown by the black lines in Figure 5.20. For all cases this is fairly uniform across the span for $-0.5 < Z^* < 0.5$. It typically increases further from the centreline due to the increase in characteristic frequency.

A scaling of the characteristic frequency at each location based on the local separation length is attempted, as shown by the red lines in Figure 5.20. It can be seen that for the $12^\circ$ and $10^\circ$ interactions the Strouhal number based on the local separation length varies in a similar manner to the conventionally defined Strouhal number. This might have been expected based on the left-hand figures in the sub-plots of Figure 5.10 showing a fairly constant characteristic frequency along the separation shock.

The Strouhal number based on the local separation length for the $8^\circ$ cases strongly resembles the variation in separation length across the span. Again, the left-hand figures in the sub-plots of Figure 5.10 show that the characteristic frequency varies little in comparison to separation line position. This might mean that slightly off-centre, where the local Strouhal number increases, the $8^\circ$ $AR = 1$ and $AR = 1.3$ interactions behave more similarly to the $10^\circ$ $AR = 1$ interaction, for example. On the other hand, where the necking occurs at the centreline the very low local Strouhal number is indicative of a different behaviour.

The right-hand plots in the sub-figures of Figure 5.10 appear to show that the power associated with the characteristic frequencies is greater where there is earlier separation. As the power in the PSDs is proportional to $f^2$, Figure 5.21 shows the distribution of power normalised by the centreline separation length squared (in black) and the local separation length squared (in red). It can be seen that for all cases taking the local separation length makes the power distribution more uniform.
Figure 5.20: Spanwise variation in minimum Strouhal number $S_{\text{min}}$ (black) and normalised by local separation length $L(z)$ (red).
Figure 5.21: Spanwise variation in maximum power (black) and normalised by square of local separation length $L(z)^2$ (red).
5.4 Temporal correlations

5.4.1 Autocorrelations

Figure 5.22 shows examples of the pressure variation in time measured for the \(12^\circ \mathcal{A} = 1\) test case. Figure 5.22a shows how the pressure history changes in the streamwise direction near the centre of the wind tunnel. The most upstream measurement (at the top) generally exhibits very small pressure fluctuations which is characteristic of measurements in the upstream boundary layer. There are a few spikes in pressure which are likely due to a large upstream shock movement reaching this point.

It is evident that the measurements taken slightly further downstream are under the intermittent reflected shock region. This is because the measured pressure has a large variance as there are times when the sensor is either upstream or downstream of the shock. It can be seen that the pressure is most often at a state representative of the upstream boundary layer and that the low-frequency shock motion is a short, upstream excursion causing a sharp pressure rise and fall. Further downstream, for \(X^* \geq -1.08\), the pressure has a smaller range than under the shock but greater than that of the upstream boundary layer. These measurements are under the separation region.

Figure 5.22b shows some example pressure histories across the span of the \(12^\circ \mathcal{A} = 1\) test case at the streamwise location of the maximum low-frequency power at the centreline. The black lines show the pressure history near the centreline where it can be seen that there is little time at which the sensor is measuring the state upstream of the shock and there are many shock excursions passing over the sensor. Similar behaviour is generally observed across the span at this streamwise location. For \(Z^* = \pm 0.46\) and \(Z^* = \pm 0.70\) there is more time at which the pressure is near that of the upstream boundary layer. This is expected due to the shock curvature.

![Streamwise variation at \(Z^* = -0.02\).](image-a)

![Spanwise variation at \(X^* = -1.20\).](image-b)

**Figure 5.22:** Examples of pressure history for \(12^\circ \mathcal{A} = 1\) case (not recorded simultaneously). Not shown on the same scale. Minor grid lines are spaced at one characteristic low frequency period \(\tilde{T}\).

Figure 5.23 gives an indication of the overall pressure variation in time at the same measurement locations for the \(12^\circ \mathcal{A} = 1\) test case. It shows p.d.f.s of pressure deviations from the mean at each measurement location. Figure 5.23a shows the progression of the p.d.f.s
in the streamwise direction near the centreline. It shows that at the furthest upstream points the pressures are fairly tightly grouped about the mean value. At the locations of high low-frequency energy \((−1.24 \leq X^* \leq −1.16)\), the modal pressure values are below the mean and there is a positive skewness to the distributions. Further downstream, the pressure values once again become more symmetric about the mean but with a much larger variance than at the upstream points.

Figure 5.23b shows the p.d.f.s at the same spanwise locations as the PSDs in Figure 5.4 at the streamwise location of maximum low-frequency power at the centre. It shows that the p.d.f.s of the pressure fluctuations near the centreline are similar to those near the walls. Figure 5.4 shows that these locations have the largest low-frequency peaks in the PSDs of the pressure fluctuations. At the locations in between, the pressure fluctuations are more tightly grouped and have a positively skewed distribution. Based on Figure 5.22b, this is because at the locations of high low-frequency power, the shock is passing back and forth over the sensor; whereas at the middle locations the sensor spends more time measuring upstream of the shock and records occasional pressure rises as the shock travels upstream and back.

These characteristics of the pressure history are compatible with the shock usually being located downstream of its mean position and being disturbed into short upstream excursions. It is possible that the cause of these excursions are events in which the separation bubble decreases in height and increases in streamwise length, with the separation point moving upstream. The observed pressure history could also be due to the motion of the SBLI causing compression waves to fan out upstream of the shock.

![Figure 5.23](image)

**Figure 5.23:** Examples of p.d.f.s of pressure history for 12° \(A = 1\) case.

Figure 5.24 shows example autocorrelation functions \(R_{xx}\) recorded near the centreline for the 12° \(A = 1\) test case. It should be noted that, according to Box et al (1994), the autocorrelation value is significant if it is greater than the magnitude of the standard deviation of the autocorrelation function of white noise signals with the same variance and number of
samples. Here this magnitude is infinitesimal for all measurement locations. Therefore even small magnitudes of the autocorrelation may be considered significant. Furthermore, there are consistent, relevant trends in the relative values of $R_{xx}$, discussed below.

(a) Over ten low-frequency periods. (b) Over two low-frequency periods.

Figure 5.24: Autocorrelation function of $12^\circ \mathcal{A} = 1$ interaction at $Z^* = -0.02$.

Figure 5.24 shows that, at the locations recording under the separation region ($X^* \geq -1.04$), the function decreases fairly quickly to zero at a time delay of approximately a quarter of the characteristic low-frequency period. There are very small (almost insignificant) negative correlations at a time delay $0.25\tilde{T} \leq \tau \leq 2\tilde{T}$. This demonstrates that in the region downstream of the separation shock, the signal can be correlated with itself over time periods much less than the characteristic time of the shock motion.

In the locations more upstream ($-1.24 \leq X^* \leq -1.12$), the autocorrelation functions remain significant for longer time delays, with the first zero crossings at approximately $\tau = 0.4\tilde{T}$. This means that data separated by temporal lags of $\tau \leq 0.4\tilde{T}$ are typically both above or below the mean. Additionally, the anti-correlation is more significant, indicating that data points separated by $0.4\tilde{T} \leq \tau \leq 2\tilde{T}$ are typically on opposite sides of the signal mean. Lastly, for $\tau > 2\tilde{T}$, some correlation is observed, signalling that there is some periodicity to the signals. The measurement in the upstream boundary layer ($X^* = -1.28$) and just upstream of the separation region ($X^* = -1.08$) exhibit behaviour intermediate to these two states.

Figure 5.25 shows example autocorrelation functions for the $12^\circ \mathcal{A} = 1$ test case at the same spanwise locations as the PSDs in Figure 5.4 and at the streamwise location of maximum low-frequency power ($X^* = -1.20$). Figure 5.25a shows that, across the span of the interaction at $X^* = -1.20$, the correlation of the pressure signals with their own history becomes insignificant after approximately two of the characteristic low-frequency periods. This shows that the low-frequency motions are independent of each other as the autocorrelation function is not highly periodic, this is in agreement with the PSDs being broadband.

Figure 5.25b shows that there is little variation in the shape of the autocorrelation function across the span of the interaction. However, the points in the corner separations show a slightly slower drop-off in their autocorrelation function $R_{xx}$ which indicates that the signals are related
CHAPTER 5. SPATIAL VARIATION OF LOW-FREQUENCY UNSTEADINESS

(a) Over ten low-frequency periods.

(b) Over two low-frequency periods.

Figure 5.25: Autocorrelation function of $12^\circ \mathcal{AR} = 1$ interaction at $X^* = -1.20$.

to themselves for a longer time period. This might be due to the mean velocity being smaller here and therefore downstream convection being slower.

The first zero crossing for all measurement locations is close to $\tau \approx 0.3\tilde{T}$. This is followed by a period of anti-correlation and a second zero crossing at a time lag of $\tau \approx 2\tilde{T}$. This might be indicative that, on average, the SBLI goes through global unsteady cycles over a period of two low-frequency shock motions. This would consist of two low-frequency motions whose pressure signals are anti-correlated. It can be seen in Figure 5.22 that the shock motions are not evenly distributed in time.

The area under the autocorrelation function until the first zero crossing at $\tau \approx 0.3\tilde{T}$ is 80% of that under the negatively correlated part between the first and second zero crossings. This is consistent for all spatial points, to within a standard deviation of 1%. This is an expected artefact of the low-frequency motions manifesting as an increase in pressure for a short time followed by a return to a lower mean pressure, as shown in Figure 5.22.

By integrating the area under the autocorrelation functions until the zero-crossing points, an integral time scale can be calculated. This is representative of the ‘memory’ of a pressure signal at a particular location: for how long the pressure signal has some correlation with itself at an average time instant. This measure is shown for all test cases normalised by the median low-frequency periods from Table 4.9 in Figure 5.26.

Figure 5.26 shows that the pressure history at one point in space typically correlates with itself over a time period one order of magnitude smaller than the characteristic low-frequency period. In the central separations, the time scale is greater for weaker interactions, relative to the characteristic low-frequency period. It is observed to be $T_{xx} > 0.2\tilde{T}$ in Figures 5.26f, 5.26g and 5.26h for the $8^\circ$ interactions. This might show that the time scales in the separation regions are fairly constant across interactions of different strength rather than being proportional to the shock motion period. Most of the interactions show a high integral time scale in the corner separation. This might be expected as the flow is being convected downstream more slowly here.
Figure 5.26: Spatial variation in integral time scale $T_{xx}$ of pressure signals normalised by one low-frequency period based on the median frequencies in Table 4.9. Plots have been stretched by a factor of 5 in the $x$-direction.
5.4.2 Cross-correlations

(a) $X^* = -0.92$.

(b) $X^* = -0.92$.

(c) $X^* = -0.92$.

(d) $X^* = -1.00$.

(e) $X^* = -1.00$.

(f) $X^* = -1.00$.

(g) $X^* = -1.20$.

(h) $X^* = -1.20$.

(i) $X^* = -1.20$.

(j) $X^* = -1.28$.

(k) $X^* = -1.28$.

(l) $X^* = -1.28$.

Figure 5.27: Joint p.d.f.s between pressure measurements near wall, in attached channel and at edge of central separation for 12° $\mathcal{A} = 1$ test case. Black lines show the most probable pressure on opposite axis.

The evolution of the joint p.d.f.s at different streamwise measurement positions for the 12° $\mathcal{A} = 1$ test case are shown in Figure 5.27. The figures go from downstream to upstream from
top to bottom. Joint p.d.f.s can be computed for measurements taken simultaneously and can therefore show linear correlations. They are indicative of the correlation of the instantaneous pressure distributions but cannot show causation.

The joint p.d.f.s in Figure 5.27 are evaluated between three spanwise measurement locations: \( Z^* = 0.50 \), \( Z^* = 0.74 \) and \( Z^* = 0.94 \). These were chosen to represent the central separation region, the attached channel and the corner separation, respectively. The left-hand sub-figures show the joint p.d.f. between the centre-most measurement location and the attached channel, the central sub-figures show the joint p.d.f. between centre-most measurement locations and the near-wall and the right-hand sub-figures show the joint p.d.f.s between the measurements in the attached channel and the corner. Generally at a given streamwise position the joint p.d.f.s between these three locations exhibit similar shapes.

In each sub-figure, there is a box in the top right-hand corner which gives the sample correlation coefficient (or sample Pearson correlation coefficient) \( r_{xy} \), which is the covariance of the two signals normalised by the product of their standard deviations, shown in (5.3). The coefficient can vary between \(-1\), for negatively linearly correlated signals, and \(+1\), for two positively linearly correlated signals.

\[
 r_{xy} = \frac{\sum (p'_{Z^*=z_1} \cdot p'_{Z^*=z_2})}{\sqrt{\sum p'^2_{Z^*=z_1}} \sqrt{\sum p'^2_{Z^*=z_2}}} \tag{5.3}
\]

Figure 5.27 shows that all of the signals showed some positive correlation, that is when the pressure at one location is above or below its average, the pressure at the other location is also above or below its local mean. It can be seen that the central sub-figures have smaller sample correlation coefficients as these are joint p.d.f.s of locations 0.44 of the tunnel half-span from each other. Nonetheless, it is of interest to note that despite the two measurement locations being inside separate separation regions, there is still some correlation between them. This reinforces the idea presented in Chapter 4 that the entire SBLI, including the central and corner separations, moves as a global entity.

The joint p.d.f.s at the most downstream measurement location, \( X^* = -0.92 \), (Figures 5.27a, 5.27b and 5.27c) are close to the shapes that might be expected for two uncorrelated signals. That is the probability is evenly distributed in all radial directions, with the lines of the most probable value of the pressure at the other location for the range of pressures at a given location aligning closely to the mean. This is particularly true for the signals that were recorded furthest away from each other (Figure 5.27b).

Slightly further upstream, at \( X^* = -1.00 \), the joint p.d.f.s (Figures 5.27d, 5.27e and 5.27f) exhibit slight positive correlations. Again, there are stronger positive correlations between the closer locations. A positive correlation indicates that the variables are simultaneously above or below their mean values. This suggests that under the separation region, there is little correlation between points at a given streamwise position. The correlation becomes stronger closer to the separation shock.

Under the separation shock, at \( X^* = -1.20 \), Figures 5.27g, 5.27h and 5.27i show that the joint p.d.f.s develop into an interesting shape. Firstly, for a given pressure at any location, the most likely pressure at the other location is usually below its mean value. Beyond this, the
probability decreases quickly in the negative quarter and slowly in the positive quarter. This is probably an artefact of the positive skewness of the p.d.f.s under the shock shown in Figure 5.23. Here and at $X^* = -1.00$, the sample correlation coefficient is greater between the two locations nearest the corner than between the two most central locations.

At the most upstream location, at $X^* = -1.28$, Figures 5.27j, 5.27k and 5.27l are more tightly grouped. The most prevalent pressures at each location over the range of pressures at the other locations are near the means. All variables show infrequent positive deviations from their means. There are small positive sample correlation coefficients, of similar values to those at $X^* = -0.92$, showing that there are weak spanwise correlations in these regions.

Figure 5.28 shows the cross-correlation functions from near the centreline ($X^* = -1.20$, $Z^* = -0.02$) to the more outwards measurement locations recorded simultaneously. In this figure and for the remainder of the section, a correlation at a positive lag $\tau$ corresponds to the dependent variable occurring after the independent variable. Conversely, a peak at a negative lag $-\tau$ shows that the pressure signal at the location shown in the legend generally occurs before that shown in the caption. Figure 5.28 also includes the cross-correlation of the signal with itself (the autocorrelation) in black for reference.

It can be seen that, similarly to the autocorrelation functions in Figure 5.25, the cross-correlations first cross zero at approximately $\tau = \pm 0.3\tilde{T}$. This is generally at slightly greater delays for the locations nearest to $Z^* = -0.02$. This is followed by a period of anti-correlation until another zero crossing at approximately $\tau = \pm 2\tilde{T}$. The anti-correlation is greatest for the locations closer to the centreline. It is to be expected that the signals recorded nearest the centreline have the largest correlation to the centreline signal at a given time delay.

It might be expected that motions generally travel outwards from the centre along the separation shock due to the separation flow topology. Figure 5.28 shows that there is a slight tendency for similar motions to occur first at $Z^* = -0.02$ and then at less central positions. This is inferred from the peak $R_{xy}$ occurring at a positive time lag and the function being slightly skewed towards positive time delays.

Figure 5.29a presents the same information as in Figure 5.28 as a contour plot and with the contour levels $R_{pp}$ defined as in (5.4). Figure 5.29b shows the equivalent data obtained
by Jammy & Sandham (2017) from the centreline of their LES of a $M = 2.7$ $9^\circ$ incident – reflected SBLI. From the experimental data, it is only possible to find two-point correlations over the 32.4 mm ($\Delta Z^* = 0.43$) separating the furthest sensors in a given run. The LES has the advantage of all locations being accessible due to being recorded simultaneously.

$$R_{pp}(z; r_2, \tau') = \frac{\langle p'(z, \tau)p'(z + r_2, \tau') \rangle}{p_{RMS}(z)p_{RMS}(z + r_2)} \quad (5.4)$$

Figure 5.29 shows agreement between the experimental and LES test cases, in that significant correlations with the centreline behaviour diminish beyond about 20% of the half-span. This shows that there is some coherence across the separation shock, as is to be expected, but that, purely in the spanwise direction, this diminishes fairly rapidly. The plots are fairly symmetric about $\tau'/T_0 = 0$ which demonstrates that there is not a strong preference for pressure signals either to travel away from or towards the centreline.

![Two-point correlations from centreline pressure along same spanwise axis for $12^\circ$ $AR = 1$ test case and LES of Jammy & Sandham (2017).](image)

(a) Experiment from $Z^* = 0.02$. (b) LES from $Z^* = 0.00$.

Figure 5.29: Two-point correlations from centreline pressure along same spanwise axis for $12^\circ$ $AR = 1$ test case and LES of Jammy & Sandham (2017).

There have been previous attempts by Funderburk & Narayanaswamy (2016) and Jammy & Sandham (2017) to test for correlations between the central and corner separations in order to find if one area of unsteady motion drives the other. Figure 5.30 shows the two-point correlations (defined in (5.4)) between near-wall points and more central locations along the same streamwise plane of the $12^\circ$ $AR = 1$ test case and the LES of Jammy & Sandham (2017).

In contrast to Figure 5.29, Figure 5.30 shows that significant correlations extend far from the near-wall points into the central separations. Again, there is no strong preference for the pressure near the corners to precede or follow that at more central locations. Therefore, along the same streamwise plane, the pressure in the two separation regions are correlated over small time delays but there is not a significant causation of pressure fluctuations from one separation region to another.

Figure 5.30a suggests that the signal at $Z^* = 0.94$ tends to relate to those within the corner separation ($Z^* \geq 0.8$) for long time separations up to $|\tau'/T_0| = 0.5$. In the region of the attached flow channel and in the outer edge of the central separation where the vortex exists ($0.65 \leq Z^* \leq 0.8$), the pressure still correlates to that at $Z^* = 0.94$ but up to shorter temporal lags $|\tau'/T_0| = 0.3$. Interestingly, the time delays over which the pressure can be correlated to that near the wall increases again towards the centreline of the central separation.

The two-point correlations for the rest of the experimental test cases in the present study are shown in Figure 5.31. The left-hand plot in each sub-figure shows the two-point correlations from
Z* = 0.02 to the more outward measurement points, up to Z* = 0.46. The right-hand plot in each sub-figure shows the two-point correlations from Z* = 0.94 to the more central measurement points, up to Z* = 0.50. For these figures, a moving average over 0.1 s was subtracted from the pressure fluctuations. The streamwise location of each case was that containing the point with the greatest power in the PSD.

Figure 5.30: Two-point correlations from near wall pressure along same spanwise axis.

Figure 5.31: Two-point correlations from left: Z* = 0.02 and right: Z* = 0.94. Note the change in contour scale from Figures 5.29 and 5.30, shown in Figure 5.31b.
On the whole, the plots of the two-point correlations are similar for each test case. The correlations diminish by time delays of approximately $\tau' = \pm 0.2T_0$, one-fifth of the characteristic low-frequency period of each case. Generally, from the centre, the correlations become insignificant before the outer-most measurement point at $Z^* = 0.46$. This may be due to the shock curvature which means that when moving outwards, the measurements will be more in the upstream boundary layer or separation region, rather than directly under the separation shock. This is particularly true for the $8^\circ$ cases which have more tortuous separation lines.

On the other hand, the correlations from the near-wall measurement locations remain significant further away. In the $10^\circ \lambda R = 2$ case it seems that correlations both from the near-wall and from the centre might extend further than the duct quarter-span plotted. The plots of the correlations from the near-wall position (right-hand sub-plots) also show small signs of a temporal directional preference, with correlations $R_{xy} \leq 0.2$ showing a tendency towards the negative plane. This only occurs at a certain distance from $Z^* = 0.94$ in each test case, roughly corresponding to where the central separation is.

This means that, more frequently, what happens in the centre then happens in the corner. This could occur through the lambda shock structure of the swept SBLI whose feet extend from the central channel towards the sidewall. However, due to the small magnitude of the correlations, caution should be taken before drawing strong conclusions.

5.5 Summary

In this chapter, the low-frequency motions of incident – reflected SBLIs away from the centreline were investigated. Typically, previous studies have conducted measurements only at the centreline and assumed that interactions are two-dimensional. Both the spatial variation in unsteady behaviour across the floor plane of an interaction and the trends across interactions of different geometries were investigated.

A model of the organisation of the unsteadiness on the floor plane of incident – reflected SBLIs was proposed. All test cases were found to follow this arrangement fairly closely. It was found that the low-frequency behaviour was similar to that previously documented at the centreline across the span of the interactions, following the curvature of the separation shock. This demonstrates that the separation shock exhibits similar motions across its span and the severity of unsteadiness at the centreline is a good indicator of that of the whole interaction.

As the characteristic frequency was constant across the span, a common Strouhal number, based on the typical normalisation of centreline separation length, can reduce the motion across an interaction. In this sense the interaction can be thought of as a globally moving entity with a single characterising frequency. On the other hand, the interaction could be considered to be independent across its span. If the local separation length is taken as the representative length scale, the reduced frequency scales with the spanwise separation length and is very varied. In this sense, the motion across the span exhibits very different normalised frequencies.

The power in the motions decreases with local separation length and hence could be made uniform with a scaling based on this length scale. This shows that locally earlier separation is indicative of more severe unsteadiness, despite a larger total separation being shown to be
 CHAPTER 5. SPATIAL VARIATION OF LOW-FREQUENCY UNSTEADINESS

inversely proportional to shock amplitude in Chapter 4. In all cases, the most powerful shock motions were not observed at the centreline, further demonstrating that measurements here alone may not be representative of an interaction.

In the corner separations, there are regions of unsteadiness that have the same characteristics as those observed at the centre. This shows that similar phenomena are occurring here, possibly due to the interaction between the swept sidewall shock and the corner separation. It was found that the central and corner separation regions were linearly correlated in time, showing that the interaction moves with coherence.

Both autocorrelation functions at one point in space and cross-correlations between different locations under the reflected shock were significant for two of the characteristic low-frequency periods. In the upstream boundary layer and under the separation region the signals correlated with themselves for much shorter time delays. While correlations exist over time scales significant in comparison to the low-frequency motion, there was not a significant causation observed from one region of separation to another.
Chapter 6

Conclusions

6.1 Relationship between separation length and low-frequency unsteadiness

The linear relationship, implied by the commonly accepted universality of $St_L \approx 0.03$, between the centreline separation length and the characteristic low frequency of separation shock motion in $M = 2$ incident–reflected SBLIs was investigated. Unlike previous studies, the unsteadiness was measured against variations in both interaction strength (flow deflection angle) and duct aspect ratio.

It was found that the interactions produced significantly different separation regions, including interactions with incident shocks of the same strength causing different separation lengths. Despite this, at $A = 1$ and $A = 1.3$, the characteristic low frequency remained almost constant across interactions of the same strength. This enabled the hypothesis that interactions of the same strength with different separation lengths would also have different characteristic frequencies to be rejected. It shows that the separation length is affected by test article geometry and other parameters which are typically not held constant in parametric studies and therefore is not a pure indicator of the low-frequency behaviour.

The first consequence of this is that the reduced frequency ($St_L$) scaled almost directly with separation length and therefore was not constant across interactions of the same strength. This serves to strengthen speculation that $St \approx 0.03$ should not necessarily be expected for all SBLIs. Historically, SBLIs of the same strength in the same facility are not typically compared and therefore previous claims of a collapse in $St$ were made for different strengths or configurations. This demonstrates that a better collapse could be achieved for one interaction strength by normalising by shock strength or flow deflection angle rather than separation length. Moreover, this shows that interaction strength and separation length are not interchangeable.

The second consequence is that an interaction with more severe separation (a larger separation bubble) will not necessarily cause lower frequency shock motions. As discussed in Section 6.2, other characteristics of the unsteadiness, such as the energy distribution across the frequencies, are affected by the overall shape of the separation region. Further, as discussed in Section 6.3, while a reduction in separation length might not affect the frequency of motion, it can affect the amplitude.
6.2 Overall comparison of low-frequency PSDs of interactions with different separation regions

In Chapter 4, strong correlations were found between the variation in separation length across the span of the central separation regions and the characteristics of the PSDs at the centreline. It was therefore proposed that the shape of the separation region determines the low-frequency behaviour of the separation shock.

It was found that for stronger interactions, the PSDs typically had low characteristic frequencies, high positive skewness values and high kurtosis values. These traits are indicative of the energy in the shock motion being concentrated across relatively narrow bands of very low frequencies, with a steep drop-off towards mid and high frequencies. This shows that in addition to the characteristic motion of the separation shock being at a low frequency, the other frequencies of motion are grouped towards lower frequencies with relatively high energy content, which is potentially further damaging in real-world applications.

Several of the interactions tested had similar characteristic frequencies, for example, the $12^\circ A = 1$, $12^\circ A = 1.3$ and $10^\circ A = 2$ interactions or the $10^\circ A = 1$ and $10^\circ A = 1.3$ interactions. Therefore it was only possible to distinguish the interactions in terms of unsteady behaviour by considering other properties of the PSDs. Generally higher-order moments of the PSDs were close amongst cases with close characteristic frequencies but the absolute values of the PSDs varied.

6.3 Inter-dependence of separation and low-frequency unsteadiness

It was established in Section 6.1 that the $A = 1$ and $A = 1.3$ interactions of each of the tested flow deflection angles had similar characteristic frequencies. Due to the different centreline separation lengths across the interactions, this shows that characteristic frequency is not linearly related to centreline separation length. This meant that the hypothesis that an optimum would exist between the frequency of shock motion and the amount of shock-induced separation is not true, rather the frequency is largely independent of separation length.

Due to the SBLIs having the same characteristic frequencies and the same input forcing (the pressure gradient across the incident shock), a model for the system that is analogous to a spring-mass balance is proposed. This followed the observed trends in that, for the interactions of equal strength and characteristic low frequency, the estimated separation bubble mass was inversely proportional to the amplitude of shock motion, inferred from the peak value of the PSD. This physical model seems to be a realistic representation of the SBLI as a larger separation bubble has greater inertia and susceptibility to viscosity (greater stiffness) such that it might be expected to translate over smaller distances, with the shock following its motion.

Therefore, there is a direct trade-off between the separation volume and the amplitude of low-frequency shock motions. These are both likely to be undesirable characteristics of real-world SBLIs. It was proposed that a measure of the severity of an interaction could be the product of these parameters. This product was larger for interactions of higher strengths.
It seems that for the $10^\circ$ and $8^\circ$ interactions, the $AR = 2$ cases are in different behavioural regimes. In terms of characteristic frequency and higher order statistical moments of the PSDs, they are closer to the cases of higher flow deflection angle. This might be due to a transition occurring at the high aspect ratio from to a regular incident – reflected interaction to a Mach reflection type interaction.

6.4 Three-dimensional unsteadiness characteristics

In Chapter 5, it was shown that the characteristics of the PSDs of the low-frequency energy under each test case were relatively similar along the entire (curved) front of the separation shocks. The same unsteadiness characteristics were also observed in the corner separations for all cases, showing that similar motions exist here. In Section 6.5 possible causality across the central and corner separation regions is explored. As expected, the measurement points under the upstream boundary layer were characterised with low-power, high frequencies. Those under the separation region recorded mid-power, mid frequencies.

The PSDs across the span of the separation shock exhibited low frequencies, with distributions characterised by strong negative skewness and high kurtosis. Due to the near constant characteristic frequency and spanwise variation in local separation length, the reduced frequency based on the local separation length varied approximately according to the spanwise variation in separation length alone. It was therefore proposed that taking the centreline separation length to give a near uniform reduced frequency across the span was more representative of the coherent shock motion. It was also found that the power associated with the characteristic frequency could be reduced to being fairly uniform across the span when normalised by the square of the local separation length.

Thus the hypothesis that the off-centre frequencies would scale with local separation length was rejected. This hypothesis was based on the idea that measurements conducted at the centreline did not reflect the whole interaction and were likely to be affected by the spanwise variation in frequencies which would generally increase towards the outer edges of the interactions. Instead it was found that the centreline was not necessarily the location of the most powerful low-frequency motions and that the power varied with local separation length.

6.5 Temporal flow physics

The central and corner separations exhibited similar traits, such as characteristic frequency, associated energy and higher-order moments of the low-frequency part of the PSDs. However, along the same streamwise plane there was not a strong directionality to the pressure fluctuations. This is in agreement with previously discussed results that the entire SBLI system oscillates as a whole, including the corner separations. It suggests that the low-frequency unsteadiness observed at the centre is not caused by that in the corners or by the tornado vortices in the central separation region. Therefore it appears that the system is forced externally by the pressure gradient across the impinging shock. This is as opposed to being forced by spanwise
motions within the central separation bubble, due to the counter-rotating vortices, for example, or by the corner separations, as hypothesised.

The correlations were more significant and existed over greater time delays in the locations where the most powerful low-frequency shock motion was observed. The former is likely because the scale of pressure fluctuations were greatest in these regions. The latter is because the time scales under the separation shock are known to be greater than those in the upstream boundary layer or separation region.

6.6 Future work

Many points of interest have arisen from this investigation but were beyond the scope of the present study. A couple of suggestions for future work are detailed below.

Firstly, it would be of interest to take the trends found between separation region shape and separation shock unsteadiness forward into a ‘cause and effect’ study. This would add strength to the proposal that ‘bespoke’ unsteadiness can be achieved with knowledge of the separation region shape. This could be achieved by modifying the separation region and measuring the resulting unsteadiness.

The separation region could be affected by other geometry changes, such as increasing the gap between the shock generator and the sidewalls or changing the thickness of the shock generator. Perhaps more interestingly, the separation region could be changed by affecting the incoming boundary layer with roughness elements, for example. It could then be investigated if the same effects on the unsteadiness can be achieved by modifying the separation length in this manner.

Secondly, this study was restricted to measurements in the same streamwise plane. The results seem to suggest that correlations might be observed over greater distances if simultaneous measurements were taken in a streamwise and spanwise array, following the curvature of the separation shock. Furthermore, measurements could be taken simultaneously at the location of the most powerful unsteadiness in the corner separations at the same time as measurements along the central separation shock to see if a stronger directionality can be observed.
Bibliography


ROBERTSON, J. E. 1971 Prediction of in-flight fluctuating pressure environments including protuberance induced flow.


