Transient thermal modelling of an Axial Flux Permanent Magnet (AFPM) machine with model parameter optimisation using a Monte Carlo method

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ABSTRACT

This paper presents the development of a transient thermal model of the EVO Electric AFM 140 Axial Flux Permanent Magnet (AFPM) machine based on a hybrid finite difference and lumped parameter method. A maximum deviation between simulated and measured temperature of 2.4°C is recorded after using a Monte Carlo simulation to optimise model parameters representing a 53% reduction in temperature deviation. The simulated temperature deviations are lower than the measurement error on average and the thermal model is computationally simple to solve. It is thus suitable for transient temperature prediction and can be integrated with the system control loop for feed forward temperature prediction to achieve active thermal management of the system.

Keyword: Axial Flux Permanent Magnet (AFPM) machine, Monte Carlo simulation, transient thermal modelling

<table>
<thead>
<tr>
<th>Nomenclature</th>
<th>Description</th>
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<tbody>
<tr>
<td>A</td>
<td>Area of heat transfer (m²)</td>
</tr>
<tr>
<td>C</td>
<td>Thermal capacitance (J/°C)</td>
</tr>
<tr>
<td>dev₉₆</td>
<td>Deviation (%)</td>
</tr>
<tr>
<td>I</td>
<td>Current (A)</td>
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<tr>
<td>k</td>
<td>Thermal conductivity (W/m)</td>
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<tr>
<td>k'</td>
<td>Thermal conductance (W/m²)</td>
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<tr>
<td>K</td>
<td>Thermal conductance (W/°C)</td>
</tr>
<tr>
<td>N</td>
<td>Number of samples</td>
</tr>
<tr>
<td>p</td>
<td>Spread in fraction</td>
</tr>
<tr>
<td>q</td>
<td>Heat flow rate (W)</td>
</tr>
<tr>
<td>Q</td>
<td>Heat generation rate (W)</td>
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<tr>
<td>R</td>
<td>Thermal resistance (°C/W)</td>
</tr>
<tr>
<td>t</td>
<td>Thickness (m)</td>
</tr>
<tr>
<td>T</td>
<td>Temperature (°C)</td>
</tr>
<tr>
<td>δt</td>
<td>Simulation time step (s)</td>
</tr>
<tr>
<td>Ω</td>
<td>Electrical resistance (Ω)</td>
</tr>
</tbody>
</table>

Subscript

amb Ambient
bdry Boundary
cd Conduction
cv Convection
Cu Copper
exp Experimental
link Interface
max Maximum
min Minimum
o Central value
sim Simulated
tot Total
1 INTRODUCTION

Axial flux permanent magnet (AFPM) machines can have high torque densities and high efficiencies in compact packages [1]. In general, the thermal aspects of electrical machine design have been less thoroughly researched to date compared to the electromagnetic aspects [2] and this is also the case for AFPM machines. However, torque density is usually limited by maximum temperature and therefore internal temperatures achieved during machine operation must be predicted in order to determine the machine’s maximum continuous torque rating without avoiding overheating. Excessive temperatures can cause breakdown of the insulation around the stator windings, and demagnetisation of permanent magnets. The machine efficiency is also affected by temperature due to the change in resistivity of copper with temperature.

Lumped parameter thermal networks have been used to predict both fluid and solid domain temperatures in AFPM machines [3-5]. These are fast to solve and widely used in electrical machine design. However, in order to achieve accurate predictions, the model parameters such as convective heat transfer coefficients, thermal contact resistances, and component geometries and properties must be determined accurately. These parameters can be determined by calculation, from the machine geometry, and by experimental measurements, such as the stator convective heat transfer coefficients recently measured by Howey et al. [6]. However in practice there will always be uncertainty surrounding the exact model parameterisation and therefore empirical tuning is required. Such parameterisation is the subject of the current paper.

2 METHODOLOGY

2.1 Test machine geometry and specification

The EVO AFM140 machine is used as a case study for the thermal modelling work presented in this paper. The main components of the machine are the rotor disc with embedded permanent magnets and the stator with copper wires wound around a steel core as illustrated in Figure 1(a). The copper windings are bundles of individual wires of 0.5mm diameter coated a layer of mica insulation and the bundles are lined with insulation paper as shown in Figure 1(b). The EVO AFM140 machine is used in Hybrid Electric Vehicles as a motor and generator. The high power density and efficiency of this machine make it a suitable choice for vehicular applications. The machine specification and operating parameters are tabulated in Table 1.

<table>
<thead>
<tr>
<th>Table 1 EVO AFM140 specifications and operating parameters</th>
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<tbody>
<tr>
<td>Motor Parameters</td>
</tr>
<tr>
<td>Nominal Continuous Torque</td>
</tr>
<tr>
<td>Nominal Power @ 3250 rpm</td>
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<tr>
<td>Peak Torque Density</td>
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<tr>
<td>Peak Power Density</td>
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Only half a machine is used in this investigation as illustrated in Figure 1(c) reducing the complexity of the thermal model. The majority of the loss is due to the resistive heating of the stator coils and therefore the temperature response depends strongly on this. Removing the rotor and magnets allowed for an in depth analysis of the heat
transfer mechanisms due to stator resistive heating. The thermal model covers a 7.2 degree sector of half the machine due to the machine’s axi-symmetry.

Figure 1 (a) machine components, (b) sectional view of the stator with copper wires and (c) one half of the EVO AFM140 machine without rotor and magnets

2.2 Transient thermal model

2.2.1 Model development
The approach used for thermal modelling is based on a hybrid finite difference and lumped parameter method. This involves discretising motor components into small elements and applying an energy balance on each element. The formulation of the governing equation has been presented in previous publication such as the one by Mellor et al. in [7]. An outline of the formulation process would be presented here. The rate of temperature change of each element is described by the balance of the heat stored, generated and net heat flow in or out of the element. Conservation of energy applied at each element gives rise to equation (1) where \([C]\) and \([K]\) are \(m \times m\) matrices of heat capacitance and thermal conductance with \(m\) being the number of elements; \([Q]\) the heat generation vector and \([T]\) the temperature vector which has dimensions equal to \(m\). The thermal capacitance matrix \([C]\) is a diagonal matrix of elemental heat capacitance given by the product of its mass and specific heat capacity. Derivation of the thermal conductance matrix \([K]\) will be detailed following the discussion below.

\[
[C][T] = [K][T] + [Q]
\] (1)

It is assumed that heat transfer across each element in the 3 Cartesian axes is decoupled for the selected element sizes. Heat transfer across elements is sufficiently described by the linear conduction and convection equations given by (2) and (3) respectively.

\[
q = \frac{1}{R_{cd}} (T_i - T_{i+1})
\] (2)

\[
q = \frac{1}{R_{cv}} (T_i - T_{amb})
\] (3)

\(R_{cd}\) and \(R_{cv}\) are the conduction and convection thermal resistances respectively (see [8] for derivations); \(q\) is the rate of heat flow; \(T_i\) is the temperature of a particular element; \(T_{i+1}\) the temperature of the adjacent element and \(T_{amb}\) the ambient temperature of the surrounding air. Direct modelling of radiation heat transfer is neglected in this model. Each diagonal element of the thermal conductance matrix \([K]\) represents the quantitative sum of the thermal conductance of all the paths connected to the \(i^{th}\) element.
The off diagonal elements represent the thermal conductance of the path between the $i^{th}$ and $j^{th}$ element as represented by the row and column of the matrix respectively. The general form of the thermal conductivity matrix is shown in equation (4).

$$[K] = \begin{bmatrix}
\sum_{i=0}^{m} -\frac{1}{R_{1,i}} & \cdots & \frac{1}{R_{1,m}} \\
\vdots & \ddots & \vdots \\
\frac{1}{R_{m,1}} & \cdots & \sum_{i=0}^{m} -\frac{1}{R_{m,i}}
\end{bmatrix}$$

(4)

Firstly, the thermal conductance network of each component (e.g. stator) is developed independently. The interface thermal conductance network is then built up based on the interface conditions. The elements at the boundary of the components are treated with the appropriate boundary conditions such as prescribing a heat transfer coefficient if it is exposed to air. The individual thermal conductances are added together to obtain the overall thermal conductance network. The approach is illustrated in equation (5) where $[K_i]$ is the thermal conductance matrix of the $i^{th}$ component. The dimension of this matrix is equal to the number of elements present in the component. $[K_{link,i}]$ is the interface thermal conductance matrix which may overlap with $[K_i]$ depending on how the elements between components are connected. $[K_{bdry,i}]$ is the thermal conductance at the boundaries which are exposed to air and it will similarly overlap with $[K_i]$.

$$[K] = \begin{bmatrix}
K_1 & \cdots & \cdots \\
\vdots & \ddots & \vdots \\
\vdots & \vdots & \ddots
\end{bmatrix} + \begin{bmatrix}
\vdots & \cdots & \cdots \\
\vdots & \ddots & \vdots \\
\vdots & \vdots & \ddots
\end{bmatrix} + \cdots
$$

$$+ [K_{link,1}] + [K_{link,2}] + \cdots + [K_{bdry,1}] + [K_{bdry,2}] + \cdots$$

(5)

The conductance coefficient ($k'$) is a measure of the effectiveness of an interface between two solid surfaces to transmit heat. This depends on the presence of any interface materials and/or the contact condition between the surfaces. If an interface material is present, $k'$ can be derived based on its thickness ($t$) and thermal conductivity ($k$) as shown in equation (6). However, the actual $k'$ is still dependent on the surface roughness and contact pressure at the interface. If no interface material is present, average values of $k'$ are used based on empirical data published in previous work such as in [7].

$$k' = \frac{k}{t}$$

(6)

$$R_{link} = \frac{1}{k'tA}$$

(7)

The conductance coefficient ($k'$) is used to calculate interface thermal resistance ($R_{link}$) across a boundary with an area $A$ using equation (7). At boundaries which have multiple layers of material, the thermal resistances are added in series. The total interface thermal resistance is used to calculate the interface thermal conductance matrix ($K_{link}$) and added to the overall thermal conductance matrix ($K$) according to equation (5). The calculated interface thermal resistance carries with it uncertainty because of the nature of heat transfer across such a boundary as discussed earlier.
Heat generation at each element is derived from the loss model that was developed for similar electric machines as described by Gieras et al. [1]. The resistive heating in the copper windings can be calculated from the current ($I$) and the resistance of copper based on equation (8), where $\Omega_{Cu}$ is the winding resistance at 20°C while $T_{Cu}$ is the operating temperature. With all the model input predetermined, the first order differential equation (1) can be discretised with a forward difference method to obtain the difference equation (9) which gives the temperature at each time step ($\delta t$).

$$Q_{Cu} = \Omega_{Cu}(1 + 0.00393(T_{Cu} - 20))I^2 \quad (8)$$

$$\{T^{n+1}\} = \{T^n\} + \delta t[C]\{\{K\}[T^n] + \{Q\}\} \quad (9)$$

### 2.2.2 Model uncertainty and Monte Carlo simulation

As mentioned earlier, there is uncertainty in determining heat transfer parameters at the boundaries. Heat transfer to the surrounding air depends largely on the fluid flow condition and geometry. Heat transfer across solid interfaces depends on the contact pressure, surface roughness, interface material dimension and its property. All these parameters contribute to the uncertainty in the thermal conductance matrix $[K]$. Moreover, radiation heat transfer is not explicitly included in the current method of modelling because of its limited effect at low temperatures and modelling complexity. However, radiation heat transfer cannot be entirely ignored if the model accuracy is to be optimized. Radiation heat transfer can be linearized to yield an equivalent thermal resistance [9] which contributes a component to the overall thermal conductance matrix.

It is the uncertainty in the thermal conductance matrix that affects the accuracy of the temperature prediction. A systematic method has to be employed to deal with this uncertainty. The approach is based on the Monte Carlo simulation which has been widely used in predicting output from non deterministic systems [10]. These systems are usually characterized by uncertainty in the inputs or system parameters. The deviation ($dev_i$) between the simulated temperature ($T_{sim}$) and measured temperature ($T_{exp}$) is given by their absolute difference. A Monte Carlo simulation is used to investigate how the deviation is affected by changes in the thermal conductance matrix. The deviation can be expressed as a percentage of the measured temperature as calculated using equation (10). The total deviation ($dev_{tot}$) at each temperature measurement point can be calculated with equation (11) where $N$ is the total number of measurement taken in time and $dev_{%i}$ is the deviation at each time instant.

$$dev_{%i} = \frac{|T_{sim,i} - T_{exp,i}|}{T_{exp,i}} \times 100\% \quad (10)$$

$$dev_{tot} = \sum_{i=1}^{N} \left(\frac{dev_{%i}}{100}\right)^2 \quad (11)$$

The method works by feeding a large set of pseudo random inputs to the system and investigating the resultant output from the system. The most probable outcome or mean value from such a simulation would give an indication of the output from the system with a certain level of confidence measured by the standard deviation of the distribution. The pseudo random input that is introduced into the thermal conductance matrix $[K]$ acts on the separate matrices: the interface thermal conductance matrix $[K_{link}]$ and
boundary thermal conductance matrix \([K_{bdry}]\) as defined earlier in equation (5) which are representative of physical interfaces and boundaries.

A uniform probability distribution is applied to the sets of thermal conductance matrices within a selected percentage spread about a central value. The mathematical operation is represented in equation (12) where \(p\) is the spread in fraction about the central value \([K_o]\). The percentage spread applied to the thermal conductance matrix representing the mica insulation, insulation paper and heat transfer to the surrounding air is set from 10% to 100%. It is assumed that the thermal conductance of the interface material can only be lower than the central value due to the imperfect contact at the interfaces. The maximum possible value of heat transfer to the surrounding air is chosen as the central value. The limits of the percentage spread are thus constrained by the range of possible values of the actual parameters.

\[
[K_{link/bdry}] = ([p_{min}] + (\text{random number } (0, 1)) \cdot (p_{max} - p_{min})) \cdot [K_o]
\] (12)

The Monte Carlo simulation process can be described by a flow chart as shown in Figure 2. The whole process would be repeated at least 30 times to ensure a statistically significant result. In each run of the simulation, a particular set of thermal conductance matrix would result in a certain level of model accuracy. It is possible to adapt the simulation to search for the set of thermal conductance matrix that produces the best agreement. It is then stored as the optimal set to produce the final temperature prediction. This method is adopted to optimise the model accuracy.

![Figure 2 Monte Carlo simulation process flow chart](image)

3 EXPERIMENTAL SET UP

The stator coils are powered by a bank of 4 12 V DC batteries connected in series. The voltage fed into the stator coils is controlled by a DC motor controller. A current clamp meter measures the current that is fed into the stator coils. The stator coil is supplied with a constant current of 60A for a duration of 60 seconds and then the current is turned off; this gives a step input of current. The torque produced by the actual machine is proportional to the current supplied to the stator coils. This set up replicates the demand for current in the stator coil from a step input in torque for a period of time.
The temperature of the machine is measured using K type thermocouples attached at five points of interest at the (1) stator surface, (2) inner casing, (3) outer casing, (4) inner end winding surface and (5) outer end winding surface as shown in Figure 3(a). A FLIR SC640 thermal camera was used to obtain the infrared image of the machine as shown in Figure 3(c). The accuracy of the K type thermocouple is ± 1.5 °C while the FLIR SC640 has an accuracy of ± 2% of the measurement. The thermocouple data was recorded using a National Instrument Data Acquisition System at a sample rate of 10Hz.

![Figure 3 (a) location of the thermocouples, (b) physical set up and (c) FLIR SC640](image)

### 4 RESULTS AND DISCUSSIONS

Figure 4 shows the plot of the temperature with time at the stator, casing and end winding. A maximum temperature 95°C is recorded at the end winding. The top surface of the stator shows the second highest temperature followed by the inner casing then the outer casing. The end windings recorded the most rapid increase in temperature with time while the rate of temperature rise is reduced further away from the heating source. The maximum temperature reached and rate of temperature rise gives an indication of the underlying mechanism of heat transfer and the path it takes.

![Figure 4 Temperature variation as measured by thermocouples](image)
In this set up, the only source of heating is at the stator coils. Some of the heat generated goes into heating the copper windings and some is transmitted to the surrounding medium as seen from the thermal images in Figure 5 (a) – (c). The large temperature difference between the end windings and the stator surface indicates that there is high thermal resistance between these two points. This is reasonable given that the mica insulation and the insulation paper are poor conductors of heat. It took more than 60 seconds after heating is initiated before there is some appreciable rise in temperature on the other side of the casing as shown in Figure 5 (d) – (f). This means that the heat extracted to the coolant or surrounding would be limited initially because of this delay.

![Figure 5 Temperature of stator coils / casing at (a) 0s, (b) 60s, (c) 300s and temperature of the heat sink at (d) 0s, (e) 60s, (f) 300s](image)

The simulated and experimental temperatures are compared at each of the measurement points as shown in Figure 6. The Monte Carlo method outlined earlier is use as an optimisation tool to reduce the uncertainty in the model. The total deviation between the simulated and measured temperature is calculated using equation (11) before and after optimisation. The total deviation is lower than the total experimental error except at the end winding. As a general trend, the largest deviation would be recorded at the points nearest to the source of heating such as at the end winding. Any uncertainty in the thermal resistances would amplify the deviation in temperature with time.

A significant improvement in model accuracy is observed after applying the Monte Carlo method. The largest improvement in temperature agreement is at the end winding with a final total deviation of 41.8 reduced from the initial 67.9 representing an improvement of about 38%. The summary of the error and deviation with the associated reduction after optimisation is tabulated in Table 2. It is clear that the optimisation process has improved the model accuracy across all four points. Thus, the Monte Carlo simulation can be used as a tool for optimising model accuracy.
The temperature variation of the machine is most sensitive to the thermal resistances of materials that are limiting heat transfer. One would expect the thermal barriers in this machine be the mica insulation or insulation paper. The maximum temperature is reduced by 9.1% (8.3°C) at the end windings with a corresponding increase in temperature at the stator and casing when the thermal resistance of the insulation paper is halved. However, the change in temperature response is much less when the mica insulation thermal resistance is halved. This observation points to the fact that the final machine temperature is dependent on the limiting thermal barrier which is the insulation paper in this case.

A Monte Carlo simulation can be use to perform a sensitivity analysis of the temperature response to different sets of interface and boundary thermal conductance matrices. Equation (12) is used to generate 200 pseudo random thermal conductance matrices within a 90% variation from the maximum value corresponding to the upper physical bounds for each set of heat transfer parameter to be investigated as described earlier. Figure 7 shows the total deviation for 200 variations of each set of thermal conductance matrix representing the (a) mica insulation, (b) insulation paper and (c) heat transfer to the surrounding with the associated standard deviation.
The temperature response is affected differently for each set of thermal conductance matrix as measured by the standard deviation for each distribution. The variation in thermal conductance of the insulation paper produces the largest effect on temperature followed by the heat transfer coefficient at the boundary, then the mica insulation. The observations from the sensitivity analysis using the Monte Carlo simulation are in agreement with the ones made earlier. The thermal response of the machine is most sensitive to the limiting thermal barrier.

5 CONCLUSIONS

The hybrid thermal model predicted the transient temperature response of the EVO AFM140 machine with a maximum deviation of 2.4°C. The deviation between simulated and measured temperature is reduced by using a Monte Carlo method to optimize model parameters. An improvement in model accuracy of 53% is achieved resulting in temperature deviations that are lower than the measurement error on average. The temperature response is most sensitive to the thermal resistance of the limiting thermal barrier which is the insulation paper in this case and this agrees with the outcome from the sensitivity analysis done using a Monte Carlo method.

Reducing the thickness of the insulation paper would be a possible improvement which might lead to significant reduction in maximum temperature. The simplification used in the development of the model did not result in a significant loss of model accuracy. However, it led to the development of a thermal model which is computationally simple to solve and thus suitable for transient temperature prediction. The transient thermal model could be integrated within a system control loop for feed forward temperature prediction to ensure safe and optimal machine performance.

REFERENCE LIST


