Power Processing for Electrostatic Microgenerators

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Abstract

Microgenerators are electro-mechanical devices which harvest energy from local environmental from such sources as light, heat and vibrations. These devices are used to extend the life-time of wireless sensor network nodes. Vibration-based microgenerators for biomedical applications are investigated in this thesis.

In order to optimise the microgenerator system design, a combined electro-mechanical system simulation model of the complete system is required. In this work, a simulation toolkit (known as ICES) has been developed utilising SPICE. The objective is to accurately model end-to-end microgenerator systems. Case-study simulations of electromagnetic and electrostatic microgenerator systems are presented to verify the operation of the toolkit models. Custom semiconductor devices, previously designed for microgenerator use, have also been modelled so that system design and optimisation of complete microgenerator can be accomplished.

An analytical framework has been developed to estimate the maximum system effectiveness of an electrostatic microgenerator operating in constant-charge and constant-voltage modes. The calculated system effectiveness values are plotted with respect to microgenerator sizes for different input excitations. Trends in effectiveness are identified and discussed in detail. It was found that when the electrostatic transducer is interfaced with power processing circuit, the parasitic elements of the circuit are reducing the energy generation ability of the transducer by sharing the charge during separation of the capacitor plates. Also, found that in constant-voltage mode the electrostatic microgenerator has a better effectiveness over a large operating range than constant-charge devices. The ICES toolkit was used to perform time-domain simulation of a range of operating points and the simulation results provide verification of the analytical results.
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List of Acronyms

BSN  Body Sensor Network
CDRG Coulomb-Damped Resonant Generator
CFPG Coulomb-Force Parametric Generator
EM  Electro-Magnetic
ES  Electro-Static
HDL Hardware Description Language
ICES Imperial College Energy-Harvesting Simulator
PZ  Piezo-Electric
MEMS Micro-Electro-Mechanical System
MSD Mass-Spring-Damper
SOC System-On-Chip
SOI Silicon-On-Insulator
WSN Wireless Sensor Network
## List of Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$A$</td>
<td>area [$m^2$]</td>
</tr>
<tr>
<td>$Acc$</td>
<td>acceleration [$ms^{-2}$]</td>
</tr>
<tr>
<td>$A_{semi}$</td>
<td>area of a semiconductor junction [$m^2$]</td>
</tr>
<tr>
<td>$B_m$</td>
<td>flux density [$T$]</td>
</tr>
<tr>
<td>$C$</td>
<td>capacitance [$F$]</td>
</tr>
<tr>
<td>$C_{var}$</td>
<td>capacitance of a variable capacitor [$F$]</td>
</tr>
<tr>
<td>$C_{fixed}$</td>
<td>fixed capacitance [$F$]</td>
</tr>
<tr>
<td>$d$</td>
<td>distance between comb fingers [$m$]</td>
</tr>
<tr>
<td>$D_a$</td>
<td>ambipolar diffusion coefficient [$m^2s^{-1}$]</td>
</tr>
<tr>
<td>$D_{ESD}$</td>
<td>end-stop impact damping coefficient</td>
</tr>
<tr>
<td>$D_n$</td>
<td>diffusion coefficient of electrons [$m^2s^{-1}$]</td>
</tr>
<tr>
<td>$d_o$</td>
<td>initial separation between variable capacitor plates</td>
</tr>
<tr>
<td>$D_p$</td>
<td>diffusion coefficient of holes [$m^2s^{-1}$]</td>
</tr>
<tr>
<td>$D_t$</td>
<td>total damping coefficient [$Nm^{-1}s$]</td>
</tr>
<tr>
<td>$E_{coupled}$</td>
<td>energy per cycle coupled into a microgenerator from the driving source [$J$]</td>
</tr>
<tr>
<td>$E_{eff,e}$</td>
<td>microgenerator system effectiveness</td>
</tr>
<tr>
<td>$E_{harvested}$</td>
<td>net harvested energy [$J$]</td>
</tr>
<tr>
<td>$E_{jloss}$</td>
<td>energy loss in parasitic junction capacitance [$J$]</td>
</tr>
<tr>
<td>$E_{leakage loss}$</td>
<td>energy loss due to parasitic leakage current [$J$]</td>
</tr>
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<td>$E_{mosfet loss}$</td>
<td>energy loss in MOSFET on-state resistance [$J$]</td>
</tr>
<tr>
<td>$E_{open}$</td>
<td>energy on the closed capacitor plates of an electrostatic microgenerator [$J$]</td>
</tr>
<tr>
<td>$E_{opp}$</td>
<td>opportunity energy per cycle of a microgenerator [$J$]</td>
</tr>
<tr>
<td>$E_{out}$</td>
<td>useful power output from a microgenerator system [$J$]</td>
</tr>
</tbody>
</table>
$E_{open}$: energy on the open capacitor plates of an electrostatic microgenerator [J]

$E_{o-head}$: operational overhead energy of a microgenerator system [J]

$E_{pre-ch}$: energy required to pre-charge an electrostatic microgenerator [J]

$F_e$: electrical damping force [N]

$F_{ESD}$: end-stop dynamic force [N]

$F_{optcz}$: the displacement constrained optimal force for the CDRG in terms of power generation [N]

$G_c$: position-dependent capacitance gain factor

$h$: comb finger height [m]

$I_{KF}$: high-level injection knee current [A]

$I_{leakage}$: parasitic leakage current [A]

$I_{coil}$: coil length [m]

$L_c$: side length of a cube [m]

$L_{ind}$: inductance [H]

$L_f$: overlap length of comb fingers [m]

$L_{p}$: ambipolar diffusion length [m]

$m$: mass [kg]

$N_d$: doping concentration of p-type semiconductor [$m^{-3}$]

$N_c$: number of coil turns

$N_{cells}$: Number of devices in parallel

$N_d$: doping concentration of n-type semiconductor [$m^{-3}$]

$N_f$: Number of comb fingers

$n_i$: equilibrium carrier concentration of silicon [$m^{-3}$]

$Q$: charge [C]

$q$: electron charge [C]

$Q_j$: charge stored in $pn$ junction [C]

$Q_{final}$: charge on variable capacitor at the end of generation stroke [C]

$Q_{precharge}$: charge on variable capacitor at the beginning of generation stroke [C]

$Q_{leakage}$: leakage current charge [C]

$R_{mos f et}$: on-state resistance of MOSFET [Ω]

$t_{flight}$: time of the flight of generation stroke [s]
$V_B$ blocking voltage of a semiconductor junction [V]

$V_{\text{final}}$ voltage across variable capacitor at the end of generation stroke [V]

$V_0$ diode voltage drop [V]

$V_{\text{precharge}}$ voltage across variable capacitor at the beginning of generation stroke [V]

$Y_0$ the amplitude of generation frame motion [m]

$Z_I$ the maximum allowed relative generator proof-mass amplitude [m]

$\omega$ angular frequency of the generator frame motion [radss$^{-1}$]

$\omega_n$ generator resonant frequency [radss$^{-1}$]

$\omega_c$ defined as $\frac{\omega}{\omega_n}$

$\varepsilon_0$ permittivity of free space [Fm$^{-1}$]

$\eta_{\text{conv}}$ efficiency of power conversion circuit

$\eta_{\text{coupling}}$ coupling effectiveness of a microgenerator

$\eta_{\text{gen}}$ generation efficiency of an electrostatic microgenerator

$\eta_{\text{mech}}$ mechanical coupling effectiveness of a microgenerator

$\eta_{\text{pre-ch}}$ pre-charge efficiency of an electrostatic microgenerator

$\eta_{\text{system}}$ effectiveness of a microgenerator system

$\rho_{au}$ density of gold [kg/m$^3$]

$\tau_{hl}$ carrier life-time [s]
Declaration of Originality

I Kondalarao Gandu declare that this thesis is my own work and has not been submitted in any form for another degree or diploma at any university or other institute of tertiary education. Information derived from the published and unpublished work of others has been acknowledged in the text and a list of references is given in the bibliography.

Kondalarao Gandu
18/01/2011
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Chapter 1

Introduction

Wireless power supplies have the same advantages for electronic devices as do wireless communications: they allow portability, and even for non-portable applications they can avoid costly wired installations, particularly where sources of wired power are not locally available. For this reason, improved wireless power supplies are increasingly sought after as electronic systems proliferate. Batteries in their various forms have so far been the primary solution; however, they frequently dominate the size, and sometimes the cost, of the devices in question, and introduce an unwanted maintenance burden of replacement or recharging. Alternative power sources that overcome these limitations are thus highly desirable. The possible approaches to this challenge are to use local energy supplies with higher capacity, to deliver power wirelessly from an active source introduced for this purpose, or to extract power from ambient sources in some way.

Extracting power from ambient sources is generally known as energy harvesting, or energy scavenging. This approach has recently attracted a great deal of interest within both the academic community and industry, as a potential inexhaustible source for low power devices. Generally energy harvesting suffers from low, variable and unpredictable levels of available power. However, the large reductions in power consumption achieved in electronics, along with the increasing numbers of mobile and other autonomous devices, are continuously increasing the attractiveness of harvesting techniques. Consequently the amount of research in the field, and the number of publications appearing, have risen greatly [1] since the field emerged in around 1998 to 2000.
The sources of energy available for harvesting are essentially of four forms: light, radio frequency (RF) electromagnetic radiation, thermal gradients, and motion, including fluid flow. All have received attention, in varying degrees. Solar cells are the most mature and commercially established energy harvesting solution [2, 3, 4], and are of course exploited across a wide range of size scales and power levels. While cost is a key parameter for large scale photovoltaic generation, at the small scale of portable electronic devices this is less of an issue, and light availability is instead the key limitation. A wide range of work has also been presented on small scale thermo-electric generation [5, 6, 7, 8]. Temperature differences tend to be small over the miniature size scale associated with most harvesting applications, which leads to poor thermodynamic efficiency, but useful power levels can be captured from differences as little as a few degrees C. Ambient RF has also received some attention [9, 10, 11], although availability of significant power levels is again an issue [12], and efficient extraction using devices much smaller than the radiation wavelengths is another key challenge. As an adjunct to the four main sources for harvesting, fuel based generation using ambient fluids as fuel, specifically glucose in the human body has also been reported [13].

The relative advantages and disadvantages of the different sources for energy harvesting have been discussed thoroughly by various authors [14, 15, 16, 17, 18, 19] and consequently the arguments will not be repeated here in detail. The general opinion from the literature is that whilst each application should be evaluated individually with regards to finding the best energy harvesting method, kinetic energy in the form of motion or vibration is generally the most versatile and ubiquitous ambient energy source available. Consequently, this thesis is concerned with study of vibration based energy harvesters. Operating principles and their possible applications in real world are discussed in the following sections.
1.1 Motion-Driven Energy Harvesters: Operating Principles

Motion driven microgenerators fall into two categories; those which utilise direct application of force, and those which make use of inertial forces acting on a proof mass. The operating principle of a direct-force generator is shown in Figure 1.1. In this case, the driving force $f_{dr}(t)$ acts on a proof mass $m$, supported on a suspension with spring constant $k$, with a damping element present to provide a force, $f(z)$, opposing the motion. If the damper is implemented using a suitable transduction mechanism then in opposing the motion, energy is converted from a mechanical to an electrical form. There are limits of $\pm Z_l$ on the displacement of the mass, imposed by device size or transducer limitations. Direct force generators must make mechanical contact with two structures which move relative to each other, and can thus apply a force on the damper.

![Figure 1.1: Generic model of direct-force generator](image)

The operating principle of inertial microgenerators is shown in Figure 1.2. Again a proof mass is supported on a suspension, and its inertia results in a relative displacement $z(t)$ when the frame, with absolute displacement $y(t)$, experiences acceleration. The maximum range of $z(t)$ is again $\pm Z_l$. Energy is converted when work is done against the damping force $f(z)$ which opposes the relative motion. Inertial generators require only one point of attachment to a moving structure, which gives much more flexibility in mounting than direct-force devices, and allows a greater degree of miniaturisation.
In order to generate power, the damper must be implemented by a suitable electro-mechanical transducer. This can be done using one of the methods described below.

1.1.1 Transduction Methods

Electromagnetic

In conventional, macro-scale engineering, electrical generators are overwhelmingly based on electromagnetic transduction. In small scale energy harvesting, two main additional techniques are also common. Electrostatic transduction, which is both impractical and inefficient for large machines, becomes much more practical at small size scales, and is well suited to micro-electro-mechanical (MEMS) implementation. Piezoelectric transduction is generally impractical for rotating systems, but is well suited to the reciprocating nature of the driving motion typically used for harvesting (e.g. vibration).

Rotating electromagnetic generators are in common use from power levels of a few watts (brushless DC domestic wind turbine systems), to several hundred megawatts (synchronous machines in power plants). It is possible to implement the damper of a microgenerator using the same principle, i.e. that described by Faraday’s law of induction, as illustrated in Figure 1.3. A change of magnetic flux linkage with a coil induces a voltage $v(t)$ in the coil, driving a current $i(t)$ in the circuit. The combined force $f(t)$ on the
moving charges in the magnetic field acts to oppose the relative motion, as described by Lenz’s law. The mechanical work done against the opposing force is converted to heat in the resistance of the circuit and to stored energy in the magnetic field associated with the circuit inductance. Some key practical issues for electromagnetic energy harvesters are as follows: strong damping forces require rapid flux changes, which are difficult to achieve in small geometries or at low frequency; the number of coil turns achievable in a MEMS or other micro-scale device will be limited, resulting in low output voltages (typically millivolts); and integration of permanent magnets, and ferromagnetic materials for the flux path, is likely to be required [20] and [21].

\[
\frac{\dot{z}(t)}{\dot{z}(t)}(t) \quad f(t) \quad i(t) \quad B \quad v(t) \quad R
\]

Figure 1.3: Principle of operation of the electromagnetic transducer

**Electrostatic**

In *electrostatic* generators, mechanical forces are employed to do work against the attraction of oppositely charged parts; in effect, such devices are mechanically variable capacitors whose plates are separated by the movement of the source. They have two fundamental modes of operation: switched and continuous [22]. In the switched type, the transducer and the circuitry is reconfigured, through the operation of switches, at different parts of the generation cycle. Switched transducers can further be split into 2 main types: constant-charge and constant-voltage. The first is illustrated in Figure 1.4(a). For a parallel plate structure with a variable separation and constant overlap (*i.e.* the horizontal component of $\dot{z}(t)$ is zero), and with a negligible fringing field, the field strength is proportional to the (constant) charge and thus the energy density of the electric field
is independent of plate separation. As the electrode separation increases (by doing mechanical work against the attractive force $f(t)$) additional potential energy is stored in the increased volume of electric field. If instead the plates are moved relative to each other laterally (i.e. the vertical component of $\dot{z}(t)$ is zero), mechanical work is done against the fringing field, and there is an increase in stored electrical energy because the electric field strength increases with the reduction in plate overlap, and the energy density of the field (proportional to the square of field strength) increases faster than its volume decreases.

![Figure 1.4: Principle of operation of the electrostatic transducer.](image)

Constant voltage operation is illustrated in Figure 1.4(b). If the plate separation is increased with a fixed overlap, the electric field strength falls, causing charge to be pushed off the plates into an external circuit as a current $i(t)$. If the plates are moved with constant separation and changing overlap, the field strength stays constant, but current is again forced to flow into the source because the volume of the field decreases. In both cases, the mechanical work done is converted into additional electrical potential energy in the voltage source.

For both modes, since the charge equals the capacitance times the potential ($Q = CV$), and stored energy is $\frac{1}{2}CV^2$, the electrostatic force is found to be half the voltage squared times the rate of change of capacitance, i.e.
\begin{equation}
F = \frac{1}{2}V^2 \frac{dC}{dz}
\end{equation}

for motion in the z direction. Thus a constant force is obtained for normal motion in the constant charge case, and for lateral motion in the constant voltage case. Because of practical constraints, such as non-zero conductance (for constant charge) and non-ideal voltage sources (for constant voltage), real electrostatic transducers work somewhere between these two extremes, although in many cases very close to one or the other, and both types have been reported in the literature for implementations of microgenerators.

A practical restriction of electrostatic transducers is that they require a pre-charge (or priming) voltage in order to operate. This can be avoided by use of an electret, i.e. a permanent charge buried in a dielectric layer and these types of electrostatic transducer tend to be of the continuous type [23]. On the other hand, since the damping force depends on the initial voltage, an active precharge system offers the possibility of dynamically optimising the generator to the applied motion.

Three types of micro scale electrostatic transducers are commonly referred in the literature; In-plane gap closing, In-plane Overlap and Out-of-plane gap closing. Figure 1.5 shows all the three types of transducers. It can be noted that depending on the structure movement the capacitance between comb fingers varies. Advantages and disadvantages of each device are discussed in [15]. Modelling of these transducers is discussed in Chapter 3 of this thesis.

**Piezoelectric**

The *piezoelectric* effect is a phenomenon whereby a strain in a material produces an electric field in that material, and conversely an applied electric field produces a mechanical strain [24]. The former can be used to realise microgenerators. When an external force is applied, some of the mechanical work done is stored as elastic strain energy, and some in the electric field associated with the induced polarisation of the material. If an external conduction path through a load is provided, a current which neutralises the net charge results (Figure 1.6). Piezoelectric materials with high electro-mechanical coupling coeffi-
Figure 1.5: Three possible topologies for micromachined electrostatic transducers from [15]

Figure 1.6: Principle of operation of the piezoelectric transducer.

Ceramics are generally ceramics, with PZT (lead zirconate titanate) being the most common. Such materials do not tolerate high strain levels, so some form of lever is required to combine them with devices of significant relative displacement. The most common geometry is to apply the piezoelectric as a thin layer on a cantilever beam from which the proof mass is suspended.

Although the three transduction methods discussed above dominate the literature on en-
ergy harvesting, others are possible, such as the magnetostrictive effect [25].

1.2 Applications for Motion-Based Energy Harvesting

Energy harvesters have main applications in wireless sensor networks and body sensor networks, which will now be discussed.

1.2.1 Wireless Sensor Networks

Traditionally, healthcare has concentrated upon short-term treatment, rather than long term monitoring and prevention of illnesses [26]. However, many chronically ill patients could have a significant increase in quality of life and life expectancy if certain biological signs could be continually monitored and controlled during their daily lives. Three examples illustrate the potential of this approach: continually monitoring blood pressure in patients with hypertension can significantly increase medication compliance [27]; real-time processing of electrocardiograph traces can be very effective at revealing early stages of heart disease [28]; and closed loop control of insulin administration for diabetic patients would significantly reduce the risk of hypoglycemia [29]. Monitoring can also allow better targeting of medicines, reducing costs and unwanted side effects. In order to achieve these benefits, many types of body mounted or implanted medical devices are desired [30].

Implantable or wearable devices will only significantly increase quality of life if they are unobtrusive to the patient [31, 32] in terms of both use and maintenance. It is especially important to eliminate maintenance for implantable devices, for which replacement of the power source in particular must be avoided [33]. Whilst some implanted sensors can be totally passive and used in conjunction with active equipment when a measurement is needed [34], and some active devices could be powered up occasionally by wireless energy transfer, many require a continuous source of electrical power [30]. Ideally, all implantable medical devices would have a power supply lifetime as long as the required
operational lifetime, thus keeping surgery, and cost, to a minimum. This vision of unobtrusive, automated health care [35] using wearable and implanted wireless medical devices is the main focus of a new and fast-growing multi-disciplinary research area, that of the body sensor network, or BSN [36, 37]. In general, the tiny size of information processing and radio frequency integrated circuits means that batteries dominate the size of devices which require long operating times [15, 29, 38] such as BSN nodes. However, the continual evolution of solid-state electronics, combined with new circuit design techniques, has led to vast reductions in power consumption, as well as size, for circuits required to perform given functions. This combination of low power requirements, tight size constraints, and the need to eliminate maintenance, makes BSN a particularly attractive application for energy harvesting.

The BSN is a specific instance of a more general topic, the wireless sensor network, or WSN [39, 40]. The general wireless sensor network concept is that of deploying many small, inconspicuous, self-contained sensor nodes, often referred to as motes, into an environment to collect and transmit information, and possibly provide localised actuation. Other than medical applications, potential uses for WSNs include: structural monitoring of buildings [41]; status monitoring of machinery; environmental monitoring of domestic environments to make them more comfortable [42, 43]; military tracking [43]; security; wearable computing; aircraft engine monitoring [44]; and personal tracking and recovery systems [45]. As with BSNs, many application areas will only be attractive for WSN use if motes can be powered by an inexhaustible energy source, such as harvested energy.

Figure 1.7 shows a block diagram of the signal and processing elements of a wireless sensor mote capable of sending the data to a remote location for processing. The minimum power requirements of such a device can be estimated using a mixture of currently available ‘off-the-shelf’ technology and devices which are the current state of the art in research. As an example, consider the following three elements:

- Sensor: the STLM20 temperature sensor from ST Micro [47] draws typically 12 µW quiescent power at 2.4 V supply voltage.
- ADC: an ADC reported by Sauerbrey et al [48] has power dissipation below 1 µW
for 8 bit sampling at 4 kS/s.

- Transmitter: IMEC recently announced an IEEE 802.15.4a standard-compliant ultra wideband transmitter [49], with a power consumption of only 0.65 nJ per 16 chip burst, operating at a low duty cycle.

The required data rates for bio-monitoring applications tend to be quite low due to the relatively low rates of change of the variables [38]. One of the highest rates required is for heart-beat monitoring, at around 100 samples/s. If this is combined with a resolution of 10 bits, then the data rate is 1 kbps which, if the transmitter power quoted can be scaled to such low data rates, requires only 0.65 $\mu$W. This suggests a total power consumption for the sensor node may be only 10 - 20 $\mu$W, or even 1 - 2 $\mu$W or less if the other components are also duty cycled. There would be some extra overhead for power-processing, interface and timing circuitry, but it is reasonable to estimate that the total device power consumption could ultimately be reduced to a few $\mu$W, at least for this biosensor application. This is within achievable levels for energy harvesters of modest (below 1 cc) size, even when harvesting low frequency body motion [46].
1.2.2 Other Applications

Limited battery life is a significant inconvenience for most portable electronic devices, so target applications for energy harvesting are primarily limited by the feasibility of harvesting in each case. This feasibility depends mainly on four factors: the typical power consumption of the device; the usage pattern; the device size (and thus the acceptable harvester size); and the motion to which the device is subjected (for motion harvesting specifically). For example, laptop computers are poor candidates for harvesting: although they are relatively large, they have high power consumption (10-40 W), and their typical usage patterns comprise long periods (tens of minutes to hours) of continuous use, with idle periods mostly spent in a low motion environment. Even if harvesting is used to supplement rather than replace batteries, the added battery life is likely to be marginal at best for most users.

Mobile telephones (cell-phones) are a somewhat more attractive target, as they tend to be carried on the body for much of the time, thus experiencing regular motion, while only being used (other than in low power monitoring mode) for relatively short periods. Of course the relative amounts of motion and usage are highly user dependent. The power levels during calls are typically a few W, and this is likely to reduce to some extent with advances in the relevant technologies. However, space is very much at a premium in hand-sets, and energy harvesting power densities reported to date for body motion sources, as reviewed below, are well below the levels at which this application becomes feasible. For other handheld devices, such as mp3 players and personal organisers, the considerations are similar to those for phones, with some differences in power requirements and usage patterns.

Thus, wireless sensors would appear to be the primary application area for motion harvesting and this thesis will investigate energy harvesters for biomedical sensing applications.
1.3 Thesis Outline

This thesis is organized and presented in the following chapters:

- Chapter 2: Contains a literature review of energy harvesters particularly focusing on inertial microgenerators.

- Chapter 3: Describes the modelling of inertial microgenerator systems and discusses case studies and model verification simulations.

- Chapter 4: Discusses PSpice modelling of custom-semiconductor devices.

- Chapter 5: Define and discusses the system effectiveness of an electrostatic microgenerator operating in constant-charge mode.

- Chapter 6: Define and discusses the system effectiveness of an electrostatic microgenerator operating in constant-voltage mode.

- Chapter 7: Is the conclusion and discusses author’s contribution and suggestions for further work.
Chapter 2

Literature Review

In this work, inertial type microgenerators only are considered since these generators can function simply by being attached to a source of motion at a single point, rather than relying on the relative motion of different parts of the host structure, and allows a greater degree of miniaturisation. Only recent literature (from year 2006 to date) on inertial microgenerators is reviewed in this chapter as literature prior to year 2006 has already been critically reviewed in a PhD thesis [50] and in journal review article [20]. However, the key points of these reviews are summarised here:

- The energy harvesting field is getting more research attention and continues to grow, particularly the field of motion harvesting microgenerators. All the three types of inertial transduction mechanisms have been widely investigated in the literature.

- The electromagnetic transduction mechanism offers a well-established technique of electrical power generation and there is wide variety of spring/mass configurations that can be used with various types of materials that are well suited and proven for cyclically stressed applications.

- Several companies are offering macro-scale electromagnetic microgenerators [46].

- With the electromagnetic transducer, comparatively very high current levels are achievable at the expense of low voltages. Various power processing schemes have been developed to amplify the transducer’s output voltage. Recently, research
groups are focusing more on active-rectification schemes with which very small voltages (as low as of 20 mV) can be rectified.

- The typical size of the electromagnetic microgenerator has been shrinking over the last decade. However, wafer-scale microgenerators are quite difficult to achieve owing to the relatively poor properties of planar magnets (limitations on the number of turns achievable with planar coils and the restricted amplitude of vibration, hence magnet/coil velocity).

- Piezoelectric transducers are simplest to fabricate and also this mechanism has been stated to be the best transduction mechanism for microgenerators by Roundy et al. because they can have the highest energy storage density of the three transduction mechanisms (Note that this statement was argued in [50] that if this value is not a limiting factor in the design of the generator, it is unclear which transduction mechanism is the best choice).

- Most of the implementations of piezoelectric transducers are meso-scale and they produced relatively high voltages. Various power electronic circuits have been proposed to process and regulate the piezoelectric transducer output. One of the significant contributions is the synchronous charge extraction scheme proposed by Guyomar et al. which effectively improves the piezoelectric material electromechanical coupling coefficient through the use of power electronics. Other research groups are also actively researching to exploit the advantages of this scheme.

- Piezoelectric materials are required to be strained directly and therefore their mechanical properties will limit overall performance and lifetime. It is difficult to make micro-scale piezoelectric transducers and is evident from the literature that there are very few reported micro-scale piezoelectric transducers.

- In contrast to the two transduction mechanisms mentioned above, the electrostatic transduction mechanism is easily realisable as a MEMS device as MEMS comb-drive actuators are commonly used. Thereby implementation of complete micro-generator system on a single chip is possible. Also, due to the nature of the scaling of the electrostatic force, at smaller dimensions the electrostatic microgenerators
have the potential to achieve greater power densities compared to the other transduction mechanisms [51].

This work will focus on electrostatic microgenerators as they potentially offer many advantages over the other types. Therefore, recent literature on electrostatic transduction mechanism will be reviewed here. Section 2.1 and 2.2 respectively discuss articles on electrostatic transducers and associated power electronic circuits. Some of this review work was published as [46]. Performance trends of the reported microgenerators are compared in Section 2.3. At the end of this chapter the research objectives of this thesis are discussed.

2.1 Electrostatic Inertial Microgenerators

Electrostatic microgenerators can be classified as either resonant or non-resonant devices. In the former case, mechanical parts of device such as spring and proof mass resonate with input excitation and the motion of the variable capacitor plates will be harmonic with the input excitation. Whereas, in the non-resonant devices, the mechanical parts will not resonate and the motion of the capacitor plates will be non-harmonic with the input excitation. Articles on both types of microgenerators are reviewed in the following paragraphs.

Resonant Electrostatic Microgenerators:

The first reported electrostatic device was by Meninger et al. [52] and development has continued on these devices. More recently, an in-plane gap closing comb structure type electrostatic microgenerator has been designed and fabricated by Chiu et al. in [53]. By constraining the output voltage to 3V and the device area to 1cm², various design parameters have been calculated and verified through simulations. An external steel mass of 7.2g was attached to match the device resonant frequency to the input excitation of 120Hz. The authors were unable to test the device because the variable capacitor is getting short.
circuited due to the residual particles.

Most of electrostatic devices rely on semiconductor switches or continuous operation into resistive loads, however [54] are one of the few research groups using integrated mechanical switches. The mechanical switches have advantages of zero charge leakage and synchronous operation to the input external vibrations. Figure 2.1 shows the fabricated device with external mass, comb fingers and mechanical switches arrangement. In this design authors used tungsten mass of 4g to adjust the resonant frequency of the device. The advantage of using tungsten material is that it has higher mass density compared to steel. Voltage and power measurements could not be made with the fabricated device due to various parasitic capacitances. No further work found in the literature by these authors.

![fabricated device](image)

Figure 2.1: Fabricated device from [54] (a) Center hole for positioning the external mass (b) Integrated mechanical switches (c) Comb fingers (d) External mass attached for testing

Despesse et al. reported an in-plane gap closing type electrostatic microgenerator [55, 56]. A fabricated macroscale structure with a volume of $18cm^3$, delivered $1mW$ at 50Hz and 1g acceleration, with a proof mass of 104g. And a similar structure with volume of $32.4mm^3$ produced $70\mu W$ of output power with same conditions.

A hybrid low frequency, low intensity energy harvester that couples both electrostatic and piezoelectric transduction mechanisms is proposed in [57]. It is an electrostatic oscillator suspended by piezoelectric springs as shown in Figure 2.2. The voltage developed
by spring elongation is used to prime the electrostatic transduction. The piezoelectric springs also provide signals for charge and discharge cycles. A block diagram is shown in Figure 2.3. The authors analysed the hybrid system by assuming in-plane gap closing
structure type electrostatic transducer. However, due to the complexity of fabrication, a simple out-of-plane type variable capacitor is considered in the later stage [58] and the modified hybrid harvester block diagram is shown in Figure 2.4. A working prototype has been characterised and it was found that the piezoelectric cantilever beam was working as expected. However, the desired change of capacitance could not be achieved with the variable capacitor due to the poor alignment of parallel plates of the capacitor.

To achieve a System-on-Chip (SoC) design of micro system, an in-plane overlap electrostatic device has been described by Sheu et al. in [59]. The device size is of $3000 \times 3000 \times 500 \mu m^3$ and delivered $0.09 \mu W$ at $105$Hz with $10 \mu m$ amplitude of the motion. Figure 2.5 shows the structure of the device with various dimensions marked. The gap between the comb-fingers is $4 \mu m$ which is dictated by the minimum line-width of DRIE process. The authors state that the device is most area-efficient monolithic solution for realising system-on-chip micro system. It is to be noted that the device power output of this level is hardly useful for most of the electronics. A further improvement of output power requires increasing either the proof mass or the number of comb fingers, thereby increased device dimensions.

![In-plane overlap electrostatic device with CMOS process from [59]](image)

Basset et al. [60] described a translational in-plane overlap electrostatic harvester. The proof mass is micro machined from silicon wafer and anodically bonded to a glass substrate. The electrodes of the variable capacitor, which are made of aluminum, are on the
The silicon mass is designed to have an in-plane translational degree of freedom. The device, 11mm long and 6.5mm wide produced 61nW at 250Hz after pre-charging the transducer at 6V. The authors described various methods of reducing the parasitic capacitance to achieve the designed change in transducer capacitance and also presented design of a power conditioning circuit which will make the device work autonomously i.e. no need of precharging. However, while testing the prototype, the authors used a separate voltage source to start the harvesting process.

![3D schematic view of in-plane translation electrostatic device from [60]](image)

One disadvantage of the electrostatic microgenerator is the need for a precharge voltage. Usually this is achieved with a voltage source and power conditioning circuit or with an electret. The electret is an electrostatic equivalent to a permanent magnet and it is a dielectric with quasi-permanent charge trapped inside which generate a strong electric field. In [23, 61], Halvorsen et al. presented design, fabrication and characterisation of an electrostatic device with electret as internal bias. The basic structure of this device is shown in Figure 2.7. Two important findings can be observed from the results. Firstly, when the device output power was measured for two different cavity pressures (atmospheric and 2.5mbar), it was found that for low cavity pressure, one to two orders more power was achieved. This demonstrates need for vacuum packaging of electrostatic devices. The second finding is about the method of studying and modelling the behaviour of the resonant devices under broadband input excitations.
The research at University of Tokyo is focused on electret based electrostatic devices. Different electret materials and various device structures have been studied and results are reported in [62, 63].

This completes the review of resonant electrostatic devices and the following section reviews the literature on non-resonant devices.

**Non-Resonant Electrostatic Microgenerators:**

Motions in biomedical applications are random without resonant frequencies. The above discussed resonant devices performance will be poor with non-resonant excitations, therefore, non-resonant or broadband devices are more suitable for biomedical applications. Early work on non-resonant device was reported by Miao et al. in [64]. The device measured output energy per cycle is 120nJ at 220V at frequency 30Hz with acceleration of 10ms$^{-2}$. The authors state that the power obtained is significantly below theoretically achievable values and they believe that the motion of proof mass in unwanted degrees of freedom is reducing the designed change in capacitance thereby reduction in the power output.

Recently, Kiziroglu et al. proposed a new non-resonant type electrostatic device [65]. In this device a series of strip electrodes (covered with thin dielectric layer) are used to form fixed plates of the variable capacitor. A stainless steel rod is used as proof mass. The device structure is shown in Figure 2.8(a). The key advantages with this structure are: more
proof mass can be used, no suspension structure is needed (although a guided structure will be required to constrain the motion); the travel range (to which achievable power is also proportional) is greatly increased and the output is provided in several pulses per motion rather than one. The later characteristics are valuable because parasitic capacitances typically make it difficult to benefit from a large motion range in single-pulse system. Characterisation of the prototype device and difficulties with measuring the output voltages are discussed in [66].

![Figure 2.8: (a) Schematic of the device structure. (b) Schematic circuit showing the electrical operation from [66]](image)

A new electrostatic microgenerator is developed in [67] to harvest power from low frequency vibrations such as from human motion. Microball bearings were used to control the gap between electrodes as shown in Figure 2.9. This structure is not only suitable for good gap control but also suitable for long-range movement at low frequency with a spring that has low-spring constant. The problem with low spring constant is that spring can not retain the moving part well and also can not keep a narrow gap between the electrodes. The fabricated device harvested a power of 40µW at 2Hz and 80µW at 6Hz with an input acceleration of 0.4g, which is a typical vibration at the waist.

In a review article [22], the authors summarised working principles of various electrostatic transduction mechanisms. Advantages and disadvantages of four different type of designs of electrostatic transducers are discussed. In the author’s opinion, the electrostatic microgenerators will achieve high power densities at micro scale levels compared to electromagnetic microgenerators, therefore making them very attractive to medical applications.
This completes the review of the literature on electrostatic transduction mechanism. Now, power processing circuits literature for electrostatic harvesters will be discussed in the next section.

### 2.2 Power Processing Circuits

There are two key reasons why conditioning of the output power of a microgenerator is called for. First, it is very unlikely that the unprocessed output of the transducer will be directly compatible with the load electronics and, second, in most cases it is desirable to maximise the power transfer from the transducer by optimising the apparent impedance of the load presented to it. It may also be necessary to provide energy storage for sources that are intermittent or for relatively high power loads that run in burst mode.

It is clear from the literature to date that much more attention has been paid to the transducer itself than to the power conditioning. Most researchers have used a simple resistive load to determine the electrical power output of their transducers. Only a small number of publications describe more sophisticated power processing stages with voltage regulation or power transfer optimisation. These articles will now be reviewed.
In the case of electrostatic microgenerators, power processing circuits are used to charge a variable capacitor through an external precharge power supply (battery) and to discharge the variable capacitor through a load (or to recharge the battery). A charge pump circuit is used to charge and discharge a variable capacitor as shown in Figure 2.10. Diode D1 will be on when the variable capacitor is at its minimum position i.e. capacitance is maximum. Diode D2 will be on when the voltage at node A is more than the load voltage. Both the diodes will be off during rest of the vibration cycle period. Diodes with low reverse leakage current are suitable for this application to reduce the leakage power loss. In [15], the author states that JFETs working in a diode mode have been used to reduce the reverse leakage current, but does not explain why. It may be that these diodes are low-leakage compared to off the shelf devices because of their size and the use of JFETs in diode mode was a convenient way of obtaining diodes of this size.

![Figure 2.10: Basic charge pump circuit](image)

Figure 2.10 shows an active switch based power processing circuit proposed in [52]. During the precharge condition, switch SW2 will be turned on to store energy in inductor L. The stored inductor energy will be used to charge the variable capacitor $C_{var}$ by turning on and off the switches SW1 and SW2 respectively. During the discharge period, the reverse switching sequence of precharging condition will be followed to discharge $C_{var}$. An external control circuit is used to generate the precharge and the discharge gate signals. Generating the synchronous gate signals is very difficult with this circuit.

To mitigate this problem an asynchronous power processing circuit is proposed [68]. This asynchronous circuit consists of a basic charge pump circuit and a flyback converter. The block diagram and the circuit details of the asynchronous power processing circuit are shown in Figure 2.12. The charge pump in the forward path converts vibration energy into electrical energy, which is then delivered to a temporary storage. The flyback circuit
Figure 2.11: Modified basic charge pump circuit from [52]

is used to transfer the energy from the temporary storage to a reservoir that powers the attached load and priming of the charge pump. Details of selection of various components and experimental verification of the circuit are discussed in the paper.

Figure 2.12: Block diagram and circuit details of asynchronous energy harvester from [68]

A new power processing circuit for electrostatic microgenerators has been proposed in [56]. A flyback transformer and two active switches are used to precharge and discharge the variable capacitor as shown in Figure 2.13. The primary winding $L_p$ will be switched
across the battery to store the energy. The switches SW2 and SW1 are respectively turned on and off to precharge the variable capacitor $C_{\text{var}}$. Both the switches are kept open until the variable capacitor reaches its minimum capacitance. Now the switch SW2 will be turned on to discharge the variable capacitor $C_{\text{var}}$. The switches SW1 and SW2 are respectively turned on and off to transfer the energy stored in the secondary winding, $L_s$, to the battery.

In an analysis of power processing circuits for the CFPG developed at Imperial, it has been shown that the power converter attached to this device needs to have an off-state impedance of more than $10^{12} \ \Omega$ and less than 1 pF of input capacitance to maintain 80% of the generated energy [69]. To achieve this high level of impedance, thin layer silicon-
on-insulator technology based semiconductors have been designed. Using these devices, a simple buck converter circuit shown in Figure 2.14 has been simulated in F.E. software, so that the physical effects such as electron hole pair generation and impact ionization, substrate currents and charge storage can be modelled in detail. Detailed simulation studies are carried out to optimize the MOSFET and diode device areas to optimise the energy generated from the system, taking into account conduction loss and charge sharing effects. It has been found that the on-state voltage drop of the MOSFET predominantly affects the conversion efficiency because of high peak currents, which are due to the low inductance used in the circuit in order that the inductor could be integratable on chip. It has also been found that when the MOSFET is replaced by an IGBT, the size of the inductor can be reduced by a factor of 2 whilst maintaining the same conversion efficiency.

The power processing circuit for a voltage constrained electrostatic microgenerator is shown in Figure 2.15. During the precharge condition, SW2 and SW5 will be switched on to store energy in the inductor L. Switches SW3 and SW4 will be turned on by simultaneously turning off SW2 and SW5 to charge the variable capacitor $C_{var}$. The unidirectional switch SW1 will be turned on to allow the current to flow from variable capacitor $C_{var}$ to the battery. When the variable capacitor reaches its minimum value, SW1 will be turned off. In order to completely recover the charge across the variable capacitor, reverse switching sequence of the precharge condition is used. A complete description of the circuit with waveforms has been discussed in [70].

![Circuit Diagram](image)

**Figure 2.15:** Voltage constrained electrostatic microgenerator for battery charging application from [70]

It can be concluded that the interface electronics for electrostatic devices must have low
leakage and low parasitic capacitance for efficient power conversion and require special semiconductor devices to process high voltage and low charge output of the transducer. It can also be concluded that there is no best practice available for power processing circuit design in the literature. The following section discusses the comparison of the reported microgenerators.

2.3 Energy Harvester Performance Metrics

A key issue in the discussion of energy harvesters is what performance metrics, or figures of merit, are appropriate to compare different devices or design approaches. Power efficiency could be defined for a harvester as the ratio of electrical power out to mechanical power in, but while this would give some indication of the effectiveness of the transduction, it misses a key aspect, namely that the input mechanical power itself strongly depends on the device design. On the other hand we cannot easily define the efficiency in terms of the potential mechanical power available from the source, since typically this is effectively limitless, i.e. loading by the harvester has a negligible effect on the source. Instead, the maximum output of the harvester is normally a function of its own properties, particularly its size.

Various metrics other than efficiency have been proposed, including power density [71], normalised power density [72], and two proposed measures of effectiveness [73, 74]. Power density is attractive because this measure is very important to the end user; however, it only provides a meaningful comparison for fixed vibration source characteristics, since attainable output is so dependent on these (as given by $P_{\text{max}} = \frac{2}{\pi} Y_a Z_l \omega^3 m$ [46]). Also, if specific source characteristics are used to compare two devices, they should each have been optimised with such a source in mind.

To reach a more universal metric, a possible normalised power density (with respect to source characteristics) is given in [72], in which the power density is divided by source acceleration amplitude squared. There are three difficulties with this approach. Firstly, it is desirable to have performance metrics with a maximum value of unity, so that it is clear
how close the design is to optimality. Secondly, maximum power is both proportional to acceleration squared divided by frequency, so the source dependence has not been fully removed. Thirdly, since attainable power is proportional to mass times internal displacement range, or to volume\(^{4/3}\), dividing by volume does not remove the size dependence completely and thus favours larger devices.

In [74], Roundy proposes a dimensionless figure of merit called ‘effectiveness’ to compare power output of various transduction mechanisms designs:

\[
e = (k^2) Q^2 \left( \frac{\rho}{\rho_o} \right) \left( \frac{\lambda}{\lambda_{max}} \right)
\]  

(2.1)

where \(e\) is the effectiveness, \(k^2\) is a coupling coefficient of the transduction mechanism, \(Q\) is the quality factor of the design, \(\rho_o\) is a baseline material density, \(\rho\) is the actual density of the device, \(\lambda\) is the transmission coefficient and \(\lambda_{max}\) the maximum transmission coefficient of the transduction mechanism. Broad comparison of harvester designs is possible with this metric, but it does not have a defined maximum value, since \(Q\) has no fundamental limit, and so it does not directly indicate how close a device is to optimal performance.

An alternative definition of effectiveness is introduced in [73], which is labelled here as Harvester Effectiveness:

\[
E_H = \frac{\text{Useful Power Output}}{\text{Maximum Possible Output}}
\]

(2.2)

\[
= \frac{\text{Useful Power Output}}{\frac{1}{2} Y_0 Z_l \omega^2 m}
\]

(2.3)

The harvester effectiveness as defined above has a theoretical maximum of 100%, and is mainly a measure of how closely a specific design approaches its ideal performance; it does not distinguish between designs of different proof mass density or geometry. For this reason a variant of this metric is introduced, which is termed as Volume Figure of Merit, FoM\(_V\), which aims to compare the performance of devices as a function of their overall size. This is done by substituting the actual \(m\) and \(Z_l\) of the devices with values for an
equivalent device of cubic geometry, having the same total package volume $Vol$, but with a proof mass, with the density of gold ($\rho_{Au}$), occupying half this volume, and space for displacement occupying the other half. This gives:

$$FoM_V = \frac{\text{Useful Power Output}}{\frac{1}{16}V_0\rho_{Au}Vol^\frac{2}{3}\omega^3}$$ (2.4)

A real device of cubic geometry could not reach an $FoM_V$ of 100%, since some space must be taken up by the frame, suspension and transducer components. However, since elongation of the device along the motion axis can increase the power density, the value for a non-cubic device can in principle exceed 100%.

Tables 2.1 to 2.3 present a summary of the important parameters of reported inertial energy harvesters. The research team is identified by the first author on the corresponding paper(s). Only papers reporting experimental results are included in the tables. Several observations can be made from the reported data:

- There has been significantly more work presented on electromagnetic generators than on the other two types.

- Around half of the reported work contains information regarding models of micro-generators, the other half giving measured results of prototypes. There are six cases where results of a model and a prototype are presented; of these, the piezoelectric generator by Roundy et al. achieves the closest match between the model and measurements.

- The designed operating frequency of most devices, independent of transducer type, is 50 Hz - 200 Hz. Only three groups, Tashiro et al., Kulah et al., and our own, have attempted to design inertial microgenerators to operate at frequencies below 5 Hz.

- There is a large variation in the amplitudes of the motion used to drive the generators, ranging from less than 1 nm to several mm. Generally, generators designed to work at higher frequencies are driven by lower displacement amplitude sources.
Table 2.1: Comparison of effectiveness of published electromagnetic motion harvesters

<table>
<thead>
<tr>
<th>Author</th>
<th>Reference</th>
<th>Generator Volume [cm³]</th>
<th>Proof Mass [g]</th>
<th>Input Amplitude [µm]</th>
<th>Input Frequency [Hz]</th>
<th>Zl [µm]</th>
<th>Power (un-processed) [µW]</th>
<th>Power (processed) [µW]</th>
<th>Power Density [µW/cm³]</th>
<th>Harvester Effectiveness [%]</th>
<th>Volume Figure of Merit [%]</th>
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<tbody>
<tr>
<td>Li</td>
<td>[75]</td>
<td>1.00</td>
<td>0.22</td>
<td>200</td>
<td>60</td>
<td>5000</td>
<td>100</td>
<td>100</td>
<td>1.70</td>
<td>0.08</td>
<td>0.01</td>
</tr>
<tr>
<td>Li</td>
<td>[75]</td>
<td>1.00</td>
<td>0.22</td>
<td>200</td>
<td>120</td>
<td>1000</td>
<td>100</td>
<td>100</td>
<td>1.07</td>
<td>0.01</td>
<td>0.2 × 10⁻³</td>
</tr>
<tr>
<td>Ching</td>
<td>[76]</td>
<td>1.00</td>
<td>210</td>
<td>107</td>
<td>1.50</td>
<td>1.50</td>
<td>680</td>
<td>680</td>
<td>0.2 × 10⁻³</td>
<td>0.8 × 10⁻³</td>
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<tr>
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<td>104</td>
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<td>2.4 × 10⁻³</td>
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Table 2.3: Comparison of effectiveness of published piezoelectric motion harvesters

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Figure 2.16: Harvester effectiveness of reported devices vs. device volume.

Figure 2.16 shows that harvester effectiveness values are mostly in the 1 - 10% range, with the best value over 30%. It can also be seen that the smallest devices have poor effectiveness, indicating the difficulty involved in achieving micro-engineered implementations. In general, no obvious trends can be discerned about the relative merits of the different transducer types. Figure 2.17 gives harvester effectiveness replotted against operating frequency, and shows the reduced values at high frequency, which is probably a result of the need for higher mechanical Q in these devices and the stronger influence of parasitic damping.
A crucial factor that is not captured by the metrics used above is bandwidth of operation. The frequency range over which a device can extract power effectively is an important consideration for most applications. A figure of merit which considers the bandwidth has been proposed in [46]. This metric is not included in the tables or figures, because information on frequency range is rarely available in published reports, but presentation of such data in future publications would be considerable value to the research and user community.

### 2.4 Research Objectives

As it can be seen from the literature that has been reviewed, many researchers are exploring microscale implementations of electrostatic microgenerators as these generators are thought to scale well with decreasing dimensions [51] and are also relatively easy to integrate with electronic circuits. Integration is important for developing self-sustaining on-chip microsystems.

The broad aim of the research to be described in this thesis is to aid design of optimised
microgenerator systems. The specific research objectives are to:

- Create a whole-system simulation model of a complete microgenerator so that interaction between various sub-systems can be studied and also optimisation of system design can be achieved;

- Identify the best operating region of electrostatic microgenerators since it is not clear what are the dimensions at which these microgenerators are effective for energy generation;

- Identify the best operating regime, power processing circuits and circuit components so as to improve whole-system effectiveness of electrostatic microgenerator and

- Identify factors that limit the performance in practice since it is clear from the literature that the effectiveness of the reported electrostatic microgenerators is very low compared to expectations from the present analysis.
Chapter 3

Imperial College Energy-Harvesting Simulator (ICES)

3.1 Introduction

A critical issue for the optimization of a microgenerator is the electromechanical link between the transducer and power processing circuitry because the performance of each subsystem depends on the behavior of the other. Therefore, in order to accurately simulate and optimize a microgenerator system, a combined electromechanical simulation is required. An equivalent circuit approach is commonly used for combined electromechanical simulation by representing the mechanical and electrical parts in electrical circuit elements [110], [111]. Figure 3.1. shows an equivalent circuit representation of a piezoelectric bimorph [15] and this can be simulated using any standard circuit simulator. However, the representation of mechanical parts by electrical elements is often not intuitive and in the case of inertial microgenerators, modeling of non-linear effects such as the proof mass hitting an end-stop and losing some momentum is very difficult using the mechanical to electrical analogy. A PSpice based simulation model which considers the non-linear effects and electromechanical interactions was developed in [50] for the electrostatic parametric generator. In this work, the model has been modified, extended and modularised to develop a complete simulation toolkit for inertial microgenerators that
includes Electromagnetic, Electrostatic and Piezoelectric transducer technologies. This work was published as [112].

![Diagram of equivalent circuit representation of piezoelectric bimorph from [15]](image)

Figure 3.1: Equivalent circuit representation of piezoelectric bimorph from [15]

The toolkit is called the Imperial College Energy-Harvesting Simulator (ICES) and is distributed under the conditions of GNU general public license to the research community [113]. It is currently in use by (or has been used by) research teams of various universities including Katholieke Universiteit Leuven [114, 115], University of Southampton [116] and University of Bristol [117].

At the start of this research, no unified simulation platform for energy harvester simulations existed. SPICE based modelling has been used here because of its advantages such as its familiarity for the community and its ease of use. Also as SPICE was developed to simulate low voltage integrated circuits, it includes semiconductor device models which are useful to model the power processing circuits in detail. However, it should be noted here that SPICE based simulation can be slow, the simulation run times mainly depend on the complexity and the number of nodes in the circuit that is being simulated, and on rates of change of voltage, current, charge etc. Whilst the work in this thesis was being done, researchers at University of Southampton have developed simulation platform using HDL (Hardware Description Language) [118]. This platform simulate energy harvesting systems more quickly than in SPICE. However, it is based on writing equations and it is not based on the schematic capture, making the system more difficult to use than schematic capture for many users. We have observed that many undergraduates and researchers new to energy harvesting are finding it easy to simulate the inertial microgenerator systems using our ICES toolkit.
A block diagram of the generic inertial microgenerator modelling setup, including electromechanical behaviour, is shown in Figure 3.2. It consists of a generic Mass-Spring-Damper (MSD) model with end-stop dynamics included. The electrical and parasitic damping directly arising from the transduction mechanism is also present. Modelling of the generic MSD model is discussed in the next section and modelling of electrical and parasitic damping for three different microgenerator systems is discussed in Section 3.3. These models are then verified using case study simulations in Section 3.4.

![Block diagram of the generic electromechanical behavioural model of the inertial microgenerator](image)

Figure 3.2: Block diagram of the generic electromechanical behavioural model of the inertial microgenerator

### 3.2 Generic Mass-Spring-Damper (MSD) Model

Figure 3.3 shows the representation of an inertial microgenerator system. The internal mass displacement is $z(t)$ and the frame excitation is $y(t)$. The generic MSD model is required to predict accurately the position of the proof mass with reference to the frame and end-stop limits. A differential equation 3.1, whose solution represents the proof mass displacement [119], is implemented by using the PSpice Analog Behavioral Model (ABM) library.
Figure 3.3: Generic model of inertial generator from [50]

\[
m\ddot{z}(t) = -m\ddot{y}(t) - K_s z(t) - F_e(\dot{z}(t)) - F_p(\dot{z}(t)) - F_{ESD}
\]  

(3.1)

The terms in equation 3.1 represent the linear and non-linear forces due to the mass and spring and the transducer damping forces (both electrical and parasitic), which are calculated in their respective models and fed to the generic MSD model as inputs. The MSD simulation model as implemented in SPICE is shown in Figure 3.4.

\[
F_{ESD} = \begin{cases} 
(z(t) - z_l) \times K_T + \dot{z}(t) \times D_T & \text{if } z(t) > Z_l \\
(z(t) + z_l) \times K_B + \dot{z}(t) \times D_B & \text{if } z(t) < Z_l \\
0 & \text{Otherwise}
\end{cases}
\]  

(3.2)

Figure 3.4: Generic MSD simulation model

\(F_{ESD}\) represents the non-linear end-stop dynamics. This end-stop dynamic force is modelled in such a way that whenever the proof mass hits either of end limits (i.e. \(+Z_l\) or \(-Z_l\)) some kinetic energy will be dissipated as heat. The \(F_{ESD}\) force is assumed as a damped spring system and is given as:
where

\( Z_l \) is the end-stop displacement limit which can be varied according to desired specification,

\( K_T \) and \( K_B \) are the impact spring coefficients respectively of top and bottom end-stops and

\( D_T \) and \( D_B \) are the impact damping coefficients respectively of top and bottom end-stops.

Figure 3.5. shows the PSpice symbol for the generic MSD model. It is to be noted that for a given input vibration displacement \((y_t)\) and transduction electrical \((F_e)\) and parasitic \((F_p)\) damping forces, the proof mass displacement \((z_t)\) and velocity \((z_{dott})\) are calculated internally and are given as outputs (which can be used as inputs to the transduction models). It is also to be noted that the model has a flexibility of easily changing attributes such as the end-stop displacement limit, \( Z_l \), as model parameters.

![Figure 3.5: PSpice symbol for generic MSD model](image)

**3.3 Modelling of Electrical and Parasitic Damping Forces**

In order to obtain the proof mass displacement and motion of the mass (and transducer) of an inertial system for a given driving motion, it is important to know the damping forces associated with the inertial system. In case of inertial microgenerators, there exist two
kinds of damping forces, namely electrical and parasitic. The electrical damping force is due to the transduction mechanism, i.e. conversion of mechanical energy into electrical energy through the transducer, whereas parasitic damping is due to the mechanical friction and hysteresis losses in the spring and air damping. In this section, modelling of these damping forces for the three transducer types (Electromagnetic, Electrostatic and Piezoelectric) are discussed.

### 3.3.1 Electromagnetic (EM) Transduction Mechanism

The Electromagnetic transduction mechanism (shown in Figure 3.6(a)) can be modelled as velocity dependent voltage source in series with a coil inductance and resistance. The magnitude of the voltage source is given by:

\[
V_{\text{coil}}(t) = (N_c \times B_m \times l_{\text{coil}}) \dot{z}(t)
\]

where \(l_{\text{coil}}\) is active length of the coil in the magnetic flux cutting region, \(N_c\) is number of the coil turns, \(B_m\) is flux density and \(\dot{z}(t)\) is velocity of the proof mass. The inductance and resistance of the coil are then added in series with this voltage source.

The PSpice model of the electromagnetic transduction mechanism is shown in Figure 3.6(b). The relative mass velocity is multiplied with the factor \(N_c \times B_m \times l_{\text{coil}}\) using a gain block to represent the rate of change of flux linkage and a voltage dependent voltage source (E) is used to represent velocity dependent voltage source. The electrical damping force (BIL force) is calculated by multiplying the current through the coil \(L_m\) with \(N_c \times B_m \times l_{\text{coil}}\). This force is then used as the electrical damping input to the generic MSD model. The two terminals \(L^+\) and \(L^-\) are used to connect the electrical load. This concludes the modelling of electrical damping force of the EM transduction mechanism.

There are two types of parasitic damping that exist in the case of EM microgenerators depending on their mechanical structure. Microgenerators with a flexible membrane which act as springs will have hysteresis damping loss due to cycling of mechanical stresses. Alternatively, the parasitic damping in cantilever beam type microgenerators may be dom-
Figure 3.6: (a) Principle of operation of EM transducer from [50] and (b) EM transduction PSpice simulation model
inated by viscous fluid damping loss due to the movement of magnet/mass in air if the device is not vacuum packaged. Most of the fabricated micro generator’s parasitic damping ratios have been measured, and varied from 0.0014 [120] to 0.011 [121]. A model to represent viscous parasitic damping is necessary to model either forms of parasitic damping as there is a method to find an equivalent viscous damping for a hysteresis damping system [122]. Therefore, a simple gain block to multiply the velocity with a constant to represent the parasitic viscous damping has been provided in this toolkit.

A verification and case study simulation results of an electromagnetic microgenerator are discussed in Section 3.4.

### 3.3.2 Electrostatic Transduction Mechanism

A capacitance which varies with proof-mass position is key to the electrostatic transduction mechanism. As discussed in Chapter 1, there are three distinct ways of implementing the variable capacitor at the microscale; in-plane gap-closing, in-plane overlap and out-of-plane gap closing type capacitive structures and they are considered here for modelling. Only one side of the comb drive structure variable capacitor is modelled here for simplicity in the case of in-plane gap closing and in-plane overlap type variable capacitors.

A capacitor can be thought of as a charge controlled voltage source. A time varying (non-constant) capacitor is a charge controlled voltage source with variable gain. To represent this kind of variable gain voltage sources, a generic variable capacitor is realised using a fixed capacitor and a multiplier as shown in Figure 3.7. It is to be noted that a non-constant capacitor is not readily available in PSpice (although it is present as part of its pn junction models).

The fixed capacitor voltage is multiplied with a position dependent factor, $G_c$, therefore the variable capacitor voltage shown in Figure 3.7 is given by:

$$V_{vvar} = (1 + G_c)V_{fixed}$$

(3.4)
where $V_{c\text{var}}$ and $V_{\text{fixed}}$ are voltages across the variable capacitor and the fixed capacitor respectively, which are referenced to $C_{\text{var}-}$, so that the variable capacitor can be realised as a fully floating component (i.e. it is not ground referenced).

The following derivation shows how the schematic shown in Figure 3.7 models a variable capacitor:

The current through the fixed capacitor is given by:

$$I = C_{\text{fixed}} \frac{dV_{\text{fixed}}}{dt} \quad (3.5)$$

from Eq. 3.4, the above equation can be written as:

$$I = \left( \frac{C_{\text{fixed}}}{1 + G_c} \right) \frac{dV_{\text{cvar}}}{dt} \quad (3.6)$$

Therefore, Eq. 3.6 represents a capacitor whose value is $\frac{C_{\text{fixed}}}{1 + G_c}$.

Therefore, any time varying capacitor can be simulated using the generic variable capacitor model shown in Figure 3.7, if its capacitance can be written of the form $\frac{C_{\text{fixed}}}{1 + G_c}$. The modelling of different variable capacitor structures using this generic variable capacitor will now be discussed.
In-plane gap closing comb structure:

The device capacitance of an in-plane gap closing comb structure shown in Figure 3.8 is given by:

\[
C(z(t)) = N_g \varepsilon L_f h \left( \frac{2d}{d^2 - z(t)^2} \right)
\]  

(3.7)

where

- \( N_g \) is no of comb fingers
- \( \varepsilon \) is dielectric constant of air
- \( L_f \) is length of the comb
- \( h \) is height of the comb finger
- \( d \) is distance between the comb fingers and
- \( z(t) \) is comb finger displacement as shown in Figure 3.8

Figure 3.8: In-plane gap closing comb structure

In order to model the device capacitance given in Eq. 3.7 by using the generic variable capacitor, the position dependent factor, \( G_c \), needs to be obtained, which can be derived as follows:
\[
\frac{C_{\text{fixed}}}{1 + G_c} = \frac{N_g \varepsilon L_f h}{d^2 - z(t)^2} \]  
(3.8)

by re-arranging the above equation, we obtain:

\[
\frac{1 + G_c}{C_{\text{fixed}}} = \frac{1}{N_g \varepsilon L_f h} \left( \frac{d^2 - z(t)^2}{2d} \right) \]  
(3.9)

if we set \( C_{\text{fixed}} = \frac{2N_g \varepsilon L_f h}{d} \), we have:

\[
\frac{1 + G_c}{C_{\text{fixed}}} = \frac{1}{C_{\text{fixed}}} \left( \frac{d^2 - z(t)^2}{d^2} \right) \]  
(3.10)

\[ G_c = \frac{-z(t)^2}{d^2} \]  
(3.11)

Equation 3.11 gives the position dependent factor for the in-plane gap closing type structure. The PSpice implementation of the in-plane gap closing type structure using the generic variable capacitor model is shown in Figure 3.9. It can be noticed from the figure that the PSpice model also includes computation of the electrostatic forces in the structure. This force can be used as the electrical damping force input to the generic MSD model, ensuring a fully coupled electromechanical model.

Figure 3.9: PSpice model of the in-plane gap closing type variable capacitor
The electrostatic force due to the electrical charge on the comb fingers is calculated based on the principle of virtual work:

\[ F = \frac{d\text{Energy}}{dz} \]  

(3.12)

The energy stored in the device capacitance is given by:

\[ \text{Energy} = \frac{1}{2} \frac{Q^2}{C_{\text{var}}} \]  

(3.13)

where \( Q \) is the amount of charge on the comb fingers (i.e. charge deposited during the precharge time).

Substituting the device capacitance from Eq. 3.7, we obtain:

\[ F = \frac{1}{2} \frac{dQ^2}{dzC_{\text{var}}} \left( \frac{Q^2}{2N_g\varepsilon L_f h} \right) \left( \frac{d^2 - z(t)^2}{2d} \right) \]  

(3.14)

\[ F = -\frac{Q^2 z(t)}{2N_g\varepsilon L_f h d} \]  

(3.15)

where \( Q = C_{\text{fixed}} \times V_{\text{fixed}} \)

Using this model case study simulations are discussed in Section 3.4.

**In-plane overlap comb structure:**

Figure 3.10 shows an in-plane overlap comb structure whose capacitance is given by:

\[ C(z(t)) = 2N_g\varepsilon h \left( \frac{L_f + z(t)}{d} \right) \]  

(3.16)

All the variables in this equation are as defined in the previous section.

Again, the device capacitance can be modelled by using generic variable capacitor shown in Figure 3.7 if it can be written of the form \( \frac{C_{\text{fixed}}}{1+\alpha_c} \):
Figure 3.10: In-plane overlap comb structure

\[
\therefore \frac{C_{\text{fixed}}}{1 + G_c} = 2N_g \varepsilon h \left( \frac{L_f + z(t)}{d} \right) 
\]

by re-arranging the above equation, we obtain:

\[
\frac{1 + G_c}{C_{\text{fixed}}} = \left( \frac{d}{2 N_g \varepsilon h L_f} \right) \left( \frac{1}{1 + \frac{z(t)}{L_f}} \right) 
\]

if we set \( C_{\text{fixed}} = \frac{2 N_g \varepsilon h L_f}{d} \), we have:

\[
\frac{1 + G_c}{C_{\text{fixed}}} = \frac{1}{C_{\text{fixed}}} \left( \frac{1}{1 + \frac{z(t)}{L_f}} \right) 
\]

\[
\therefore G_c = -\frac{z(t)}{L_f + z(t)} 
\]

Equation 3.20 gives the position dependent factor for the in-plane overlap comb structure. The PSpice implementation of an in-plane gap closing type structure using generic variable capacitor model is shown in Figure 3.11. Again, the PSpice model also includes computation of the electrostatic forces in the structure.
As discussed previously, the electrostatic force between the comb fingers can be calculated based on the principle of virtual work and in this case the force is given by:

\[
F = \frac{1}{2} \frac{d}{dz} \frac{Q^2}{C_{\text{var}}} = \frac{Q^2 d}{2N_g \epsilon h} \frac{d}{dz} \left( \frac{1}{L_f + z(t)} \right) = -\frac{Q^2 d}{2N_g \epsilon h \left( L_f + z(t) \right)^2}
\]

(3.21)

where \( Q = C_{\text{fixed}} \times V_{\text{fixed}} \)

The PSpice model shown in Figure 3.11 is verified in Section 3.4.

**Out-of-plane gap closing structure:**

In this structure, two parallel plates move relative to each other as shown in Figure 3.12 and the capacitance between them is given by:

\[
C(z(t)) = \frac{\epsilon A}{\text{MIN}_\text{SEP} + Z_l + z(t)}
\]

(3.22)
where

$A$ is area of the parallel plate

$MIN\_SEP$ is minimum separation between the plates and

$Z_l$ is end-stop displacement limit

Figure 3.12: Out-of-plane gap closing structure

The device capacitance is again modelled using the generic variable capacitor shown in Figure 3.7 by equating the capacitance to $C_{fixed}$:

\[
C_{fixed} = \frac{\varepsilon A}{1 + G_c} = MIN\_SEP + Z_l + z(t) \tag{3.23}
\]

by re-arranging the above equation, we obtain:

\[
\frac{1 + G_c}{C_{fixed}} = \frac{MIN\_SEP + Z_l + z(t)}{\varepsilon A} \tag{3.24}
\]

if we set $C_{fixed} = \frac{\varepsilon A}{MIN\_SEP}$, we have:

\[
\frac{1 + G_c}{C_{fixed}} = \frac{1}{C_{fixed}} \left( 1 + \frac{Z_l + z(t)}{MIN\_SEP} \right) \tag{3.25}
\]

\[
\therefore G_c = \frac{Z_l + z(t)}{MIN\_SEP} \tag{3.26}
\]
Equation 3.26 gives the position dependent factor for out-of-plane gap closing structure. The PSpice implementation of the out-of-plane gap closing type structure using the generic variable capacitor model is shown in Figure 3.13. The electrostatic force computation is again included.

Figure 3.13: PSpice model of Out-of-plane gap closing type variable capacitor

As discussed previously, the electrostatic force of the variable capacitor can be obtained by principle of virtual work. The electrostatic force between the plates of out-of-plane gap closing structure is given by:

\[ F = \frac{1}{2} \frac{dQ^2}{dz} C_{device} \]

\[ = \frac{Q^2 d}{2\varepsilon A \frac{d}{dz}} (MIN\_SEP + Z_d + z(t)) \]

\[ = \frac{Q^2}{2\varepsilon A} \]

(3.27)

This model is verified through PSpice simulations and are discussed in Section 3.4.

Position dependent factors of all three variable capacitors are summarised in Table 3.1.

Expressions for the parasitic damping force associated with each variable capacitor type is given in [15]. These expressions are also realised in PSpice, as their respective parasitic
Table 3.1: Parameters of various variable capacitors

damping models and can be interfaced with the generic MSD model during simulation studies.

### 3.3.3 Piezoelectric Transduction Mechanism

The Piezoelectric transduction mechanism is modelled by assuming a piezoelectric plate of cross sectional area $A (m^2)$ and thickness $t (m)$ which is fixed at one end. The free end will experience a force from the inertial mass. The constitutive equations associated with this type of piezoelectric transduction mechanism are given as [123]:

\[
F = K_{PE} \times Z(t) + \alpha \times V_p(t) \\
I = \alpha \times \dot{Z}(t) - C_p \times \dot{V}_p(t)
\]

where $F$ is force exerted by the piezoelectric material, $I$ is the piezoelectric terminal current, $V_p(t)$ is the piezoelectric plate voltage and $C_p$ is piezoelectric plate capacitance. The constants $\alpha$ and $K_{PE}$ depend on the piezoelectric material and shape of the piezoelectric plate and these can be calculated using $\alpha = \frac{e_{33}A}{t}$, $K_{PE} = \frac{C_{33}A}{t}$ where $e_{33}$ and $C_{33}$ are piezoelectric coefficient and compliance of the plate respectively.

Electrically, the piezoelectric material looks like a velocity controlled current source with a shunt capacitance (Eq. 3.29). These constitutive equations are realised in PSpice as shown in Figure 3.14. The force due to the piezoelectric transduction mechanism is the sum of a spring force and a force depending on the device terminal voltage. This force is calculated in the model and can be used as the electric damping force input to the generic
3.4 Case Studies: Simulations and Model Verification

In this section electromagnetic and electrostatic microgenerator sub-system toolkit models are verified using case study simulations. First, electromagnetic microgenerator system simulation results are discussed and then verification of three electrostatic transducer models is presented.

3.4.1 Electromagnetic Transducer

Figure 3.15 shows the simulation schematic of an EM transducer system. The following system parameters are considered for this case study simulation. They are representative of a typical millimeter scale microgenerator system.

Input amplitude $Y_0=22.3\mu m$, Frequency $f=50$ Hz, Coil self inductance $L_m=2mH$, No of turns $N_c=1000$, Active coil length $l_{coil}=2$ mm, Flux density $B_m=0.8T$, Coil resistance $R_c=25\Omega$, Load resistance $R_l=231\Omega$.

The displacement of the inertial mass under harmonic excitation is well known and given as:
Figure 3.15: Simulation schematic of EM microgenerator using ICES toolkit

\[
\frac{Z_l}{Y_0} = \frac{1}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_c}\right)^2\right)^2 + \left(2\zeta\frac{\omega}{\omega_c}\right)^2}} \tag{3.30}
\]

where \(Z_l\) is inertial mass peak displacement, \(Y_0\) is harmonic excitation peak amplitude, \(\omega_c\) is natural frequency and \(\omega\) is excitation frequency and \(\zeta\) is damping factor.

At resonance \textit{i.e.} \(\omega = \omega_c\),

\[
Z_l = \frac{Y_0}{2\zeta} \tag{3.31}
\]

The displacement of the inertial mass for a given excitation input can be calculated if the damping factor is known. The damping factor can be calculated as follows:

\[
\zeta = \frac{D_t}{2m\omega_c} \tag{3.32}
\]
where damping coefficient $D_t$ is sum of the electrical damping coefficient, $D_e$, and parasitic damping coefficient, $D_m$. The electrical damping coefficient in the case of EM transduction mechanism with a resistive load is given by:

$$D_e = \frac{(N_c \times B_m \times I_{coil})^2}{R_c + R_l}$$  \hspace{1cm} (3.33)

It is to be noted that the above equation was derived by assuming the reactance of the coil is much smaller than the load resistance [124].

The proof mass displacement is calculated by assuming a parasitic damping coefficient $D_m=0.001$ and mass $m=1g$ ($K = m(2\pi f)^2$):

$$Z_l = \frac{Y_o \times m \times (2 \times \pi \times 50)}{D_e + D_m} = 637\mu m$$  \hspace{1cm} (3.34)

$$Velocity = Z_l \times (2 \times \pi \times f) = 0.2ms^{-1}$$  \hspace{1cm} (3.35)

$$V_{emf} = (N_c \times B_m \times I_{coil}) \times velocity = 320mV$$  \hspace{1cm} (3.36)

$$V_{load} = V_{emf} \times \frac{R_l}{(R_l + R_c)} = 288mV$$  \hspace{1cm} (3.37)

The simulation results of proof mass displacement and velocity are shown in Figure 3.16. The electrical damping force coefficient is also shown in the figure which is matching with the calculations ($D_e$(simulation)=0.01). The load voltage waveform is also shown in Figure 3.16. As can be seen the graphs agree with the analytical results ($V_{load}$(simulation)=287mV).

### 3.4.2 Validation of variable capacitor model simulation results

In order to validate the In-plane gap closing variable capacitor PSpice model, the model has been interfaced with the generic MSD PSpice model and also with external circuit
Figure 3.16: PSpice simulation results of EM microgenerator

elements, which are used to charge and discharge the variable capacitor at specific points of mass travel. The variable capacitor is charged when it is in the maximum capacitance position i.e. at $z(t) = \pm Z_l$ and is discharged when it is in the minimum capacitance position i.e. at $z(t)=0$.

For given

No of comb fingers $N_g = 200$
Comb finger length $L_f = 200\mu m$
Comb finger height $h = 500\mu m$
Distance between comb fingers $d = 100\mu m$
Dielectric constant $\varepsilon_0 = 8.854 \times 10^{-12}$
Precharge voltage $V_{precharge} = 12V$ and
End-stop displacement limit $Z_l = 80\mu m$

The device maximum capacitance is
\[ C_{\text{device-maximum}} = (N_g \times \varepsilon \times L_f \times h) \times \left( \frac{2d}{(d^2 - Z_l^2)} \right) = 9.84 \text{pF} \tag{3.38} \]

and the device minimum capacitance is

\[ C_{\text{device-minimum}} = \frac{2 \times N_g \times \varepsilon \times L_f \times h}{d} = 3.54 \text{pF} \tag{3.39} \]

The charge placed on the variable capacitor during charging is \( C_{\text{device-maximum}} \times V_{\text{precharge}} \). The voltage across the variable capacitor at its minimum capacitance position is given by

\[ \frac{C_{\text{device-maximum}} \times V_{\text{precharge}}}{C_{\text{device-minimum}}} = 33.33 \text{V}. \]

Figure 3.17 shows the simulation results of the comb drive displacement and the voltage across the variable capacitor. The flattened portion at the peak of the displacement waveform is due to the end-stop dynamics of the generic MSD model as the mass strikes the end-stops and the flattened portion is used to generate charging and discharging control signals for the variable capacitor. The voltage across the variable capacitor simulation results match with the calculations. Note that discharge occurs at minimum capacitance, when \( z(t) = 0 \).

From equation 3.15, the electrostatic force peak can be calculated as:

\[ F_{\text{electrostatic}} = \left( \frac{C_{\text{device-maximum}} \times V_{\text{precharge}}}{2 \times N_g \times \varepsilon_0 \times L_f \times h} \right)^2 \times Z_l = 31.5 \mu \text{N} \tag{3.40} \]

It can be noticed from Figure 3.17 that the simulation result is matching with the analytical force value.

From [15], the parasitic damping (sum of couette damping and squeeze film damping) force associated with In-plane gap closing variable capacitor is given by:

\[ F_{\text{parasitic}} = \left( \frac{\mu_{\text{vis}} A_{cp}}{d_v} + 16 \mu N_g L_f h \right) \left( \frac{1}{(d-z)^3} + \frac{1}{(d+z)^3} \right) \dot{z} \tag{3.41} \]

where \( \mu_{\text{vis}} \) is the viscosity of air, \( A_{cp} \) is the area of the centre plate, \( d_v \) is the vertical distance between the centre plate and the substrate underneath. For given \( \mu_{\text{vis}} = 16 \times 10^{-6} \text{Pa.s}, \) \( A = 10^{-4} \text{m}^2, \) \( d_v = 500 \mu \text{m}, \) the parasitic damping force associated with the in-plane gap closing comb structure is also shown in Figure 3.17.

In the case of the in-plane overlap comb structure, the maximum capacitance occurs at \( z(t) = Z_l \), therefore
Figure 3.17: In-plane gap closing comb drive displacement and its variable capacitor voltage

\[
C_{device\text{-}maximum} = \frac{N_g \times \varepsilon \times h \times (L_f + Z_l)}{d} = 2.48 \text{pF} \quad (3.42)
\]

and the minimum device capacitance occurs when the \( z(t) = -Z_l \) if \( L_f >= 2Z_l \)

\[
C_{device\text{-}minimum} = \frac{N_g \times \varepsilon \times h \times (L_f - Z_l)}{d} = 1.06 \text{pF} \quad (3.43)
\]

The peak voltage across the variable capacitor at minimum capacitance position is given by

\[
V_{precharge} \times \frac{C_{device\text{-}maximum}}{C_{device\text{-}minimum}} = 28 \text{V}.
\]

From equation 3.21, the electrostatic force peak can be calculated as:

\[
F_{electrostatic} = \frac{(C_{device\text{-}maximum} \times V_{precharge})^2 \times d}{2 \times N_g \times \varepsilon \times h} \times \frac{1}{(L_f - Z_l)^2} = 3.47 \mu\text{N} \quad (3.44)
\]
Figure 3.18: In-plane overlap comb finger displacement and its variable capacitor voltage

The simulation results are shown in Figure 3.18. For this structure also the simulation results are closely matching with the analytical values.

From [15], the parasitic viscous damping force associated with the In-plane overlap comb structure is given by:

\[ F_{\text{parasitic}} = \mu N g L f h \ddot{z} \]  

(3.45)

Again, this force is also shown in Figure 3.18.

In the case of out-of-plane gap closing variable capacitor, the maximum capacitance between the plates depends on the minimum separation allowed between them. In this simulation it is assumed that the minimum separation allowed is \( MIN_{SEP} = 10 \mu m \) and for given plate area \( A = 10^{-4} m^2 \), the device maximum capacitance is given by:

\[ C_{\text{device-maximum}} = \frac{\varepsilon \times A}{MIN_{SEP}} = 88.54 \text{pF} \]  

(3.46)
The device minimum capacitance occurs at \( z(t) = Z_l \) and the separation between the two plates is \((2Z_l + MIN\_SEP)\). Therefore, the device minimum capacitance is given by:

\[
C_{device\_minimum} = \frac{\varepsilon \times A}{(MIN\_SEP + 2 \times Z_l)} = 5.2 \text{pF}
\]  

(3.47)

The peak voltage across the variable capacitor is given by \( \frac{C_{device\_maximum} \times V_{precharge}}{C_{device\_minimum}} = 204.32 \text{V} \)

\[\text{Figure 3.19: Simulation results of Out-of-plane gap closing variable capacitor}\]

The peak electrostatic force between the plates can be calculated from Eq. 3.27:

\[
F_{electrostatic} = \left( \frac{C_{device\_maximum} \times V_{precharge}}{2 \times \varepsilon_0 \times A} \right)^2 = 637.68 \mu\text{N}
\]

(3.48)

Simulation results of the parallel plate displacement, separation between the plates, voltage across the variable capacitor and electrostatic force are shown in Figure 3.19. Again, the simulation results are closely matching with the analytical values.
From [15], the parasitic fluid damping force associated with the out-of-plane gap closing variable capacitor is given by:

$$F_{\text{parasitic}} = \frac{16\mu W^3 L}{(\text{MIN}_\text{SEP} + z)^3} \ddot{z}$$ (3.49)

It is found that the parasitic damping force magnitude at atmospheric pressure is very high compared to the input acceleration force and also to the electrical damping force, resulting in no proof mass displacement. The parasitic damping force was therefore made a thousand times smaller to verify the variable capacitor model. Simulation results of the reduced parasitic damping force is also shown in Figure 3.19.

### 3.5 Conclusions

In this chapter, the development of a SPICE based simulation toolkit (ICES) to model inertial microgenerators has been described. Modelling of various components of the toolkit has been discussed and case study simulation results of electromagnetic and electrostatic microgenerator systems have been presented to verify the operation of the models. The simulation results closely match with the analytical values.

In the work presented so far, microgenerator systems were simulated with the load being a simple resistor. However, designing and interfacing of suitable power conversion circuits needs to be addressed, and the flexibility of ICES toolkit allows such simulations to be achieved by simply drawing the circuit schematics in SPICE. Because of its flexibility and ease of use, many universities are currently using (or have been previously used) this toolkit. It has been observed that researchers new to energy harvesting find it easy to simulate the inertial microgenerator systems using this toolkit among the methods described in the literature.

It was been found that ICES based system simulations can become unstable if they need to be simulated for very long run times. For example, charging a battery or super-capacitor from an inertial microgenerator system (typical charging times may be several hours).
The cause for instability is due to the inherent nature of the SPICE simulation as there is a maximum number of steps allowed in each simulation run. Therefore, as run times increase, the minimum time-step increases meaning that non-convergence may occur more easily. However, the ICES toolkit offers many advantages such as quickly studying the electromechanical interaction between various sub-systems and designing power processing circuits with accurate semiconductor device models. Such advantages have been exploited in Chapter 5 and 6 to verify the microgenerator system effectiveness analysis.
Chapter 4

Modelling of Custom-Semiconductor Devices

4.1 Introduction

As discussed in Section 2.2 of Chapter 2 on literature, in order to process the electrostatic microgenerator output, semiconductor devices were specially designed to meet the stringent leakage and low parasitic capacitance requirement for efficient power generation [69]. These devices were previously designed and simulated using Silvaco Finite Element simulator [125]. Since the ICES toolkit has been developed in PSpice, it is thus necessary to also have these custom devices modelled in PSpice as this will allow system level simulations to be performed in PSpice. This chapter discusses PSpice modelling of the custom designed diode and MOSFET in detail.

4.2 Custom-Diode Behavioural Modelling

The previously designed Silicon-On-Insulator (SOI) diode is shown in Figure 4.1 and its forward characteristics are shown in Figure 4.2 [69]. It can be noticed from the diode figure that it has an extra long $n^-$ region in addition to traditional $p,n$ regions. This $n^-$
region is required to block high reverse voltages. This region is lightly doped compared to other regions to block high voltages and therefore has high resistance. Diodes with this kind of structure are called “PiN diodes” [126].

In order to model the custom diode in SPICE based simulation platforms (per example PSpice), the implementation and limitations of the standard SPICE diode model must first be understood. In SPICE, the standard diode model is implemented based on Shockley equations [127]. The Shockley equations are derived based on assumption that the minority carrier concentration under forward bias is a small fraction of the equilibrium concentration of majority carriers of the $p, n$ type semiconductor regions. This is termed as low-level injection and its pictorial representation is shown in Figure 4.3. It can noted from the figure that the minority carrier concentrations, $n_p$ and $p_n$, in neutral regions of $p$
and n type semiconductors are respectively small fraction of equilibrium majority carrier concentrations, $N_A$ and $N_D$.

![Depletion Layer Diagram](image)

Figure 4.3: Carrier concentrations in a forward-biased p-n junction in low-level injection from [50]

Whereas in power semiconductor diode, as discussed previously an extra region which is low doped is required to block the high voltages. Because of low doping, under forward bias the minority carrier concentration in this region can exceed the zero-bias concentration of majority carriers. This operation is called as high-level injection. Figure 4.4 shows the carrier concentration of $p^+ - n$ diode under forward bias. It can be noted from the figure that the slope of the electron concentration and hole concentration in each side of the junction is the same. This again causes diffusion of majority carriers away from the junction. In order to maintain the requirement for zero net majority carrier current on the n-side (electrons), part of the applied voltage bias now appears across the n-type material, so that a drift current of majority carriers towards the junction (electrons) cancels the diffusion of electrons away from the junction. In the low-level injection case, this voltage
was assumed to have almost no effect on device operation because of the huge difference in the number of holes and the number of electrons and thus the negligible value of the voltage compared to the voltage across the depletion layer. In the high-level injection case, the concentration of holes and electrons is of the same order, and consequently the voltage across the neutral region cannot be ignored.

![Diagram showing carrier concentration in High-level injection from [50]](image)

Figure 4.4: Carrier concentration in High-level injection from [50]

However, the custom-diode is modelled in SPICE by specifying the parameters of standard diode model in SPICE (parameters values are taken from the previous work [50]). The diode forward characteristics of SPICE model along with Silvaco model are shown in Figure 4.5. It can be noted that the SPICE model is unable to model the forward bias dynamics of the custom-diode.

Whereas in PSpice, a parameter $I_{KF}$, high-level injection knee current is present. Which is not present in the standard SPICE model. After investigation we came to conclusion that this parameter will identify the current above which the high-level injection begins (as shown in Figure 4.6). However, it will not model the dynamics of the intrinsic region.
Figure 4.5: SPICE simulation results of custom diode of the custom-diode. The PSpice model results are presented in Figure 4.7(a). It can be noticed from the figure that PSpice model is not modelling the trend of voltage versus current. The results are shown in Figure 4.7(b) with current axis being logarithmic. It can be noted that at small currents, both the models have a similar trend.

Figure 4.6: I-V characteristics of a pn junction from [128]
Figure 4.7: PSpice simulation results of custom diode in forward bias (a) Linear current scale and (b) Logarithmic current scale
It can be concluded that the in-built SPICE diode model is not suitable to model the custom-diode with sufficient accuracy. Therefore, other modelling techniques need to be explored to accurately model the custom-diode behaviour. In this work, two different modelling techniques have been investigated, physics based modelling and look-up table based behavioural modelling and are discussed in Section 4.2.1 and 4.2.2 respectively.

4.2.1 PiN diode model

The flow of carriers in PiN diode is represented by ambipolar diffusion equation [126] and is given by:

$$\frac{\partial^2 p}{\partial x^2} = \frac{p}{L_a^2} + \frac{1}{D_a} \times \frac{\partial p}{\partial t}$$  \hspace{1cm} (4.1)

where

- $D_a$ is am bipolar diffusion coefficient, $\frac{2D_n D_p}{D_n + D_p}$,
- $L_a$ is ambipolar diffusion length, $\sqrt{D_a \times \tau_{hl}}$,
- $D_n$ and $D_p$ are electron and hole diffusion coefficients respectively and
- $\tau_{hl}$ is carrier life time.

Physical behaviour of PiN diodes can be modelled by solving this differential equation. Various solutions are reported in the literature [129]. One of the solution is using Finite Difference Method, which is easy to implement and has better numerical efficiency with good accuracy compared to other reported solutions [130, 131]. Using this method, PiN diode base region is discretised into finite number of nodes leading to set of differential equations. By electrical analogy, the obtained differential equations are then implemented as non-linear RC networks (as shown in Figure 4.8), providing the dynamic behaviour of the carriers in the base region. The carrier concentration in the base is then related to the diode voltage by the junction and ohmic laws under forward bias and the poission equation under reverse bias operation. Thus, diode equivalent circuit appears as a current-controller voltage source. Based on this method, PiN diode model has been created and
simulations are carried out by inputting the custom diode design parameters. Figure 4.9 shows the simulation results of forward characteristics of the custom diode using a PiN model. The Silvaco simulation results are also given in the figure. It can be observed that the PSpice simulation results closely match the Silvaco results.

Figure 4.8: Equivalent circuit model of PiN diode base region from [131]

Figure 4.9: Simulation result of PiN diode model

However, although this model closely predicts the behaviour of the custom diode, the model requires many subcircuit components and is thus slow and suffers difficulties converging on fast edges. Therefore, trade-offs have to be made between simulation run time and accuracy of the model.
4.2.2 Look-up table based diode model

Another way of modelling the PiN diode (or any component) is by using the look-up tables method [129]. In this method, data representing the characteristics of the diode will be stored in the look-up tables and will be used during the simulations. Figure 4.10. shows block diagram of the PiN diode model with look-up tables. For instance, the I-V characteristic data points of custom diode are stored in the table. By sensing the voltage between anode and cathode, the corresponding current value from the table will be passed onto the current source block. Thereby simulating the diode forward characteristics.

It is to be noted here that the PSpice uses a linear interpolation for in-between values of data points of the look-up table. However, PSpice will not extrapolate the characteristic values beyond the stored table data points. In order to maintain the continuity of the characteristics, extra components are added to the model. These are a voltage source $V_x$ and a resistor $R_{\text{forward}}$ (As the series resistance of the diode is dominant after certain forward voltage). The forward characteristics obtain using this model are shown in Figure 4.11. It can be noticed from the figure that look-up table based behavioural model closely represents the custom-diode characteristics. Figure 4.12 gives summary of forward characteristics of the custom-diode using different modelling techniques.
Modelling of parasitic capacitance of the custom-diode must also be included in the model. The parasitic capacitance of the diode is due to the charge storage in the depletion and diffusion regions. The C-V characteristic shown in Figure 4.13 shows the depletion region capacitance as the diode is reverse biased. As the diffusion region capacitance characteristics are not available for the custom-diode, simulation data of carrier concentration has been used to derive the diffusion capacitance characteristics of the custom-diode. Figure 4.14 shows the profile of the carrier distribution in the intrinsic
region of the custom-diode, which is where the majority of the stored charge occurs.

\[ Q = q \times \text{number of carriers} \times \text{device volume} \]  \hspace{1cm} (4.2)

For the given device volume of the i region \((15 \ \mu m^2 \times 0.5 \mu m)\), and number of carriers \(\approx 4 \times 10^{17}\) (area under the curve shown in Figure 4.14), \(Q = 4.8 \times 10^{-13} C\). The diffusion capacitance can be calculated if the voltage across the diode is known. From the steady state forward characteristics, the voltage corresponding to a 10mA current is \(\approx 2.19V\).
Therefore, \( C_{\text{diffusion}} = \frac{(4.8 \times 10^{-13}/2)}{2.25} = 0.1066 \) pF. Assuming the diffusion capacitance exponentially varies with the voltage, the diffusion capacitance characteristics of the custom diode are derived and are shown in Figure 4.15.

The diffusion and depletion capacitances characteristics (shown in Figure 4.13) of the custom diode are stored in the look-up tables and are used to calculate the current source value by using \( i = C(V) \times \frac{dV}{dt} \) formula. This current source therefore represents the parasitic capacitance of the custom diode. Complete behavioural modelling of the custom diode has now been achieved. Similarly, behaviour model to represent the custom-MOSFET will be investigated in the following section.

### 4.3 MOSFET behavioural modelling

As previously discussed, a behavioural model based on look-up tables can accurately represent the custom-diode. Similarly, the custom-MOSFET can be modelled using look-up tables as shown in Figure 4.16. The output I-V (drain current as a function of drain-source voltage) characteristics of MOSFET for range of gate source voltages are stored in a look-up table. For a particular drain-source voltage (at a given gate-source voltage), the corresponding drain current is found from the tables and applied to the controlled current source, thereby simulating forward characteristics. Figure 4.17 shows the forward characteristics.
characteristics obtain using this model. It can be observed that the simulation values match closely with data obtained from FE simulation in Silvaco. There is a discrepancy at high voltages (above 200V) which arises from the mechanism used to represent the parasitic diode (body diode). The real device (as simulated in Silvaco) has a much softer avalanche characteristics at high voltages. This discrepancy can be minimised by storing more data points in the look-up tables. However, this will lead to usage of more look-up table data and there is a limit (132 characters) on data that can be stored in one table in SPICE. Usage of more look-up tables will increase the complexity of the model and advantage of using look-up tables will be lost. Assuming a better design of custom-MOSFET will eliminate the soft avalanche breakdown, this model fairly represents the forward characteristics of the custom-MOSFET. Transient behaviour of the MOSFET could not be modelled because there was not sufficient data available on capacitances as a function of voltage.

![C-V Look Up Table](image)

**Figure 4.16: Custom MOSFET model with look-up tables**

Transient behaviour of the semiconductor devices is important to study the switching performance of the power processing circuits and their interaction with the transducer operation.
4.4 Conclusions

In this chapter, accurate modelling of custom semiconductor devices in PSpice was investigated so that a complete end-to-end system simulation could be performed. It was found that the in-built PSpice semiconductor device models cannot model power semiconductor devices in this context with sufficient accuracy. It was also found that subcircuits-based PSpice models can accurately model the custom diode characteristics, but the models were computationally inefficient. It was considered that such a model could not be used in end-to-end system simulation as the entire simulation will become slow (or unstable if the time step is extended).

A behavioural model using look-up tables was developed for the custom diode. This model accurately represented the custom diode characteristics in both dc and ac analysis and was quick enough to be practically useful. A similar behavioural model for the custom-MOSFET has been investigated. The behavioural model accurately represented
the dc characteristics except at high voltages (> 200V). However, sufficient data was not available to model the ac characteristics. A further investigation is necessary to establish a method to obtain the required data from a 3D finite-element MOSFET model. After having obtained custom-semiconductor device (Diode and MOSFET) models, a complete end-to-end system simulation of an electrostatic microgenerator can be achieved and thereby the interactions between various sub-systems can be fully studied and the whole microgenerator system optimised.
Chapter 5

Effectiveness of an Electrostatic Microgenerator - Constant Charge

5.1 Introduction

In previous chapters the modelling of microgenerator sub-systems has been discussed. These sub-system models are useful in studying the electromechanical interaction between the various sub-systems and therefore optimising the overall microgenerator system performance. However, even though this work provides a simulation platform to design and optimise a microgenerator, finding the optimal design of an electrostatic microgenerator at a particular operating point \( \left( \frac{Z_i}{Y_0} \text{ and } \omega_0 \right) \) using this toolkit would be a labour intensive task due to the large multi-parameter search required. In addition, using this toolkit to find operating regions for which electrostatic microgenerators can work effectively (over a range of sizes, frequencies and vibration amplitudes) is even more time consuming due to the very large search space involved. Therefore in this chapter an analytical approach is presented to find the useful operating regions of an electrostatic microgenerator with respect to its size for different input excitations. The results are then verified against the ICES toolkit for a range of operating points.

The analytical approach presented in this chapter is based on calculating the overall microgenerator system effectiveness, which is a metric for the performance of microgener-
ators. The system effectiveness is defined as a ratio of the useful energy output from the generator and the maximum amount of energy that could have been generated as a useful output if the generator had been operating optimally at the given operating point. More details on effectiveness can be found in [50] and [69]. From these references, the system effectiveness can be written as product of mechanical coupling effectiveness and generation efficiency. The coupling effectiveness indicates how the input mechanical energy is coupled to the transducer whereas the generation efficiency gives the efficiency of the power processing circuit that is attached to the transducer. There is a trade-off between these quantities; efforts to maximising the one quantity need not result in increased system effectiveness. For example, in the case of electrostatic transducer interfaced with a power processing circuit, as shown in Figure 5.1, by increasing the semiconductor area of the MOSFETs, the circuit efficiency of the power processing circuit will improve as conduction losses reduce. However, it will reduce the mechanical coupling effectiveness because as the transducer plates separate, some of the charge will be shared with junction parasitic capacitance and some of the charge discharged through the leakage conductance path. This shared charge will be lost when the MOSFET is on. As these parasitic elements are proportional to the semiconductor area, increasing the MOSFET device area, more charge will be shared and more charge will be leak off through the finite conductance path, which will reduce the energy stored in the transducer capacitor, thereby reducing mechanical coupling effectiveness. This chapter discusses how to calculate the system effectiveness of a CDRG type electrostatic microgenerator with an out-of-plane type transducer operating in constant charge mode. First, the transducer’s variable capacitor voltage expressions, with and without interfacing to a power processing circuit are derived. These expressions are then used to calculate the system effectiveness. Finally, the system effectiveness values are verified through PSpice simulations using the toolkit described in Chapter 3.
The optimal coulomb force of the CDRG type generator operating in displacement con-
strained mode (as is the case at resonance) is given in [132] by:

\[ F_{\text{opt}} = \frac{mY_0\omega^2\omega_c}{|U|} \sqrt{\frac{1}{(1 - \omega_c^2)^2} - \frac{1}{\omega_c^4}} \left(\frac{Z_l}{Y_0}\right)^2 \]  

(5.1)

where \( U = \frac{\sin(\frac{\pi}{\omega_c})}{1 + \cos(\frac{\pi}{\omega_c})} \), \( Y_0 \) is input excitation amplitude, \( m \) is mass of the generator proof-
mass, \( Z_l \) is peak internal displacement, \( \omega_c = \frac{\omega}{\omega_n} \), \( \omega \) is angular frequency of the generator frame motion and \( \omega_n \) is generator natural frequency.

At resonance (\( \omega_c = 1 \)), the optimal damping force that should be set is very insensitive to
the ratio of \( \frac{Z_l}{Y_0} \) and can thus be approximated for all \( \frac{Z_l}{Y_0} \) values as:

\[ F_{\text{opt}} = 0.785mY_0\omega^2 \]  

(5.2)

Note that for a CDRG at resonance, the optimal damping force is independent of the
constraint \( Z_l \).

From [15], the electrostatic force between parallel plates of a capacitor for a given sepa-
ration of \( z(t) \) is given by:
\[ F_{\text{electrostatic}}(t) = \frac{1}{2} \varepsilon A_{\text{plate}} \left( \frac{V_{\text{cap}}}{z(t)} \right)^2 \]  

(5.3)

where \( V_{\text{cap}} \) is capacitor voltage and \( A_{\text{plate}} \) is area of the parallel plates.

The electrostatic microgenerator will operate under optimal conditions if the electrostatic force is equal to the coulomb force given in Eq. 5.2. Therefore, equating Eq. 5.2 and Eq. 5.3, we see that the maximum voltage on the transducer capacitor is:

\[ V_{\text{cap max}} = k_{\text{cap max}} \times (2Z_l) \times \sqrt{\frac{mY_0\omega^2}{A_{\text{plate}}}} \]  

(5.4)

where

\[ k_{\text{cap max}} = 4.212 \times 10^4 \, [mF^{-1}]^{0.5} \]

Figure 5.2: Microgenerator with proof mass occupying half the volume

The above equation can be further simplified by assuming that the microgenerator is cube shaped with side length of \( L_c \) and that the proof mass occupies half of the cube volume (as shown in Figure 5.2). This trade-off between mass volume and swept volume is optimal at \( \omega_c = 1 \) [46]. The same assumption is used in Chapter 2 to compare the performance of different microgenerators. Therefore, \( m = \rho_{\text{Au}} L_c^3 \, [kg] \), \( \rho_{\text{Au}} = 19300 \, [kg/m^3] \) (assuming a gold proof mass), \( 2Z_l = \frac{L_c}{2} \, [m] \), \( Acc = Y_0 \times \omega^2 \, [ms^{-2}] \) and \( A_{\text{plate}} = L_c^2 \, [m^2] \). By substituting these values, the maximum capacitor voltage for optimal operation is simplified to:

\[ V_{\text{cap max}} = k_{\text{cap max}} \times \sqrt{L_c^3 Acc} \]  

(5.5)
where
\[ k_{\text{capmax}2} = 2.07 \times 10^7 \left[ m^{-1} k g^{0.5} F^{-0.5} \right] \]

Eq. 5.5 therefore gives the electrostatic transducer maximum voltage as a function of microgenerator size and input acceleration, assuming there are no parasitic elements (conductance or capacitance) connected to the capacitor as the plates separate.

### 5.3 Electrostatic Transducer Voltage with Parasitic Loading

The perfect operation of a constant charge microgenerator requires the capacitor plates to be completely isolated during their relative motion. However, in practice this is not possible because any semiconductor switch attached to the generator will exhibit some parasitic capacitance and conductance in the off-state. Therefore, in this section, analytical expressions of the electrostatic transducer voltage when interfaced with power processing circuit are derived. To simplify the analysis it is assumed that the transducer is loaded with one junction capacitance and associated off-state leakage current, as shown in Figure 5.3. The parasitic elements were first expressed in terms of microgenerator size and input acceleration and then by solving the charge balance equation the transducer voltage expressions are derived.

![Figure 5.3: Block diagram representation of an electrostatic microgenerator](image_url)
5.3.1 Semiconductor Device Parasitic Elements

The parasitic capacitance and off-state leakage current of a $p^+n^-n^+$ junction (commonly used for high voltage blocking and $N_a >> N_d$) can be expressed as [128, 133]:

\[
C_j = \frac{A_j}{2} \sqrt{\frac{2q\varepsilon}{(V_0 - V_{operation})} N_d}
\]  \hspace{1cm} (5.6)

\[
I_{leakage} = \frac{q n_i}{\tau} \vartheta_{dep}
\]  \hspace{1cm} (5.7)

where $C_j$ is junction capacitance, $A_j$ is area of the junction, $V_0$ is built in potential and $V_{operation}$ is junction operating voltage, $N_d$ is carrier doping density, $n_i$ is intrinsic semiconductor carrier density, $\vartheta_{dep}$ is depletion region volume, $q$ is the charge on the electron and $\tau$ is excess carrier life time.

![Depletion Layer Width](image)

Figure 5.4: $p^+n^-n^+$ diode structure

It can be noted from Figure 5.4 that the $p^+n^-$ junction blocks the applied voltage through the action of the depletion layer. The volume of the depletion layer of this junction can be calculated as follows:

\[
\vartheta_{dep} = A_{semi} \times \text{Width of the depletion layer}
\]  \hspace{1cm} (5.8)

Substituting the standard formula for the depletion layer width [128], we obtain:

\[
\vartheta_{dep} = A_{semi} \times \sqrt{\frac{2\varepsilon(V_0 - V_{operation})}{q} \left(\frac{N_a + N_d}{N_a \times N_d}\right)}
\]  \hspace{1cm} (5.9)
Because the junction exhibits asymmetric doping in order to block large voltages (as is common in power devices), \( N_a \gg N_d \) and therefore:

\[
\theta_{dep} = A_{semi} \times \sqrt{\frac{2\varepsilon(V_0 - V_{operation})}{q N_d}} \quad (5.10)
\]

From the analysis presented in [134], the uniform doping density of an \( n^- \) layer which minimises the on-state resistance of the semiconductor device and the depletion layer extends whole length of \( n^- \) layer for a given blocking voltage is given by:

\[
N_d = \frac{k_{nd}}{V_B} \quad (5.11)
\]

where

\( V_B \) is the required blocking voltage and

\[
k_{nd} = 14 \times 10^{22} [m^{-3} V]
\]

It is noted that the electric field in the \( n^- \) layer is constant as the region is lightly doped and the value considered in this analysis is \( 2.2 \times 10^7 V/m \).

Conduction loss for a given semiconductor area will therefore be a minimum if the semiconductor devices are designed by choosing this doping density. Substituting \( N_d \) in Eq. 5.6, the expression for the junction capacitance can be simplified to:

\[
C_j = \frac{k_{cj} \times A_{semi}}{\sqrt{(V_0 - V_{operation})V_B}} \quad (5.12)
\]

where

\[
k_{cj} = 1.1 \times 10^{-3} [m^{-2} C]
\]

Similarly, the off-state leakage current can be simplified to:

\[
I_{leakage} = k_{cCLEAKAGE} \times A_{semi} \sqrt{(V_0 - V_{operation})V_B} \quad (5.13)
\]

where
\[ k_{\text{ccleakage}} = 3.9 \times 10^{-4} \left[ m^{-2} s^{-1} V^{-1} C \right] \]

The above equation is derived by assuming \( q = 1.6 \times 10^{-19} C, \tau = 10^{-6} s \) (typical carrier lifetimes in Power MOSFETs [69]) and \( n_i = 2.53 \times 10^{16} m^{-3} \) (for silicon).

Assuming that the semiconductors are designed to block the maximum voltage of the transducer (given in Eq. 5.5), the semiconductor junction capacitance and off state leakage current can now be expressed in terms of the mechanical parameters of the device:

\[
\begin{align*}
C_j &= \frac{2.377 \times 10^{-7} A_{\text{semi}}}{\sqrt{(V_0 - V_{\text{operation}}) \left[ L_c^3 A_{\text{cc}} \right]}} \quad (5.14) \\
I_{\text{leakage}} &= 1.78 A_{\text{semi}} \sqrt{(V_0 - V_{\text{operation}}) \left[ L_c^3 A_{\text{cc}} \right]^{3/2}} \quad (5.15)
\end{align*}
\]

These equations can now be used to derive analytical expressions for the transducer voltage in the presence of parasitic loading (i.e. a power converter).

### 5.3.2 Charge Balance Equation

As it was previously assumed that the transducer is operating in constant-charge mode, a charge balance equation will give the relation between initial and final voltages of the transducer’s variable capacitor. Therefore, the transducer voltage when parasitic loading is present can be obtained by solving a charge balance equation for the system before and after the capacitor plates separate. The charge balance equation is given by:

\[
C_{g1} \times V_{\text{precharge}} + Q_{j\text{precharge}} = C_{g2} \times V_{\text{final}} + Q_{j\text{final}} + I_{\text{leakage}} \times t_{\text{flight}} \quad (5.16)
\]

where

- \( C_{g1} \) is initial capacitance \( C_{g1} = \frac{eA_{\text{plate}}}{d_0} \),
- \( C_{g2} \) is transducer final capacitance \( C_{g2} = \frac{eA_{\text{plate}}}{d_0 + 2Z_f} \),
- \( V_{\text{precharge}} \) is precharge voltage,
- \( V_{\text{final}} \) is transducer final voltage,
- \( Q_{j\text{precharge}} \) is initial stored charge in the junction,
$Q_{j\text{final}}$ is final stored charge in the junction, 
$I_{\text{leakage}}$ is the off-state leakage current, 
$t_{\text{flight}}$ is the transition time between initial to final capacitances and 
$d_0$ is initial separation of the transducer plates.

**Precharge Voltage:**

When the precharge voltage is applied, the electrostatic force between the plates is given by:

$$F_e = \frac{1}{2} \varepsilon A_{\text{plate}} \left( \frac{V_{\text{precharge}}}{d_0} \right)^2$$  \hspace{1cm} (5.17)

Equating this expression with that of mechanical force given in Eq. 5.2 and substituting

$$m = \frac{\rho au L_c^3}{2}, \quad Acc = Y_0 \times \omega^2,$$

we obtain:

$$V_{\text{precharge}} = 4.13 \times 10^7 d_0 \sqrt{L_c \times Acc}$$  \hspace{1cm} (5.18)

As expected, $V_{\text{precharge}}$ is proportional to the initial plate separation.

**Junction Stored Charge:**

The charge shared with the pn junction parasitic capacitance during the transducer capacitor plates movement can be calculated as follows:

$$Q_{j\text{final}} = \int_{V_{\text{precharge}}}^{V_{\text{final}}} C_j \times dV_{\text{operation}}$$  \hspace{1cm} (5.19)

Substituting the junction capacitance from Eq. 5.14 and simplifying the integration we obtain:

$$Q_{j\text{final}} = \frac{4.75 \times 10^{-7} A_{\text{semi}}}{[L_c^3 \times Acc]^\frac{1}{4}} \left( \sqrt{V_0 + V_{\text{final}}} - \sqrt{V_0 + V_{\text{precharge}}} \right)$$  \hspace{1cm} (5.20)

where $V_0$ is junction built-in voltage.
Similarly, the initial stored charge is given by:

\[ Q_{precharge} = 4.75 \times 10^{-7} A_{semi} \left( \frac{\sqrt{V_0 + V_{precharge}} - \sqrt{V_0}}{L_c^3 \times Acc} \right)^{\frac{1}{2}} \]  

(5.21)

If we first make a simplification by assuming that the parasitic pn junction capacitance is connected to the generator, but that the junction has no leakage conductance path (i.e. \( I_{leakage} = 0 \)) and substituting Eq. 5.20 and Eq. 5.21 into Eq. 5.16 and simplifying, we obtain:

\[ V_{final} = \frac{1}{k_1} \left( 11.3 \times 10^{-3} \left( -\frac{1}{k_1} \left( 1291 A_{semi} \left( -2.3 \times 10^7 \times A_{semi} + \sqrt{k_3 + k_4 + k_5} \right) + k_6 \right) \right) \right) \]

(5.22)

where

\[ k_1 = L_c \times (L_c^3 \times Acc)^{0.25} \]

(5.23)
\[ k_2 = A_{semi} \times \sqrt{70 + 100 \times V_{precharge}} \]

(5.24)
\[ k_3 = 5.22 \times 10^{14} \times A_{semi}^2 + 2.19 \times 10^6 \times k_1^2 \]

(5.25)
\[ k_4 = 8.1 \times 10^{10} \times k_1 \times A_{semi} \times \sqrt{V_{precharge} - 6.76 \times 10^{10} \times k_1 \times A_{semi}} \]

(5.26)
\[ k_5 = 8.1 \times 10^9 \times k_1 \times k_2 + 1.57 \times 10^{12} \times k_1^2 \times L_c \times V_{precharge} \]

(5.27)
\[ k_6 = 2.3 \times 10^6 \times A_{semi} \times \sqrt{V_{precharge} - 1.91 \times 10^6 \times A_{semi} + 2.28 \times 10^5 \times k_2} + 4.43 \times 10^7 \times k_1 \times L_c \times V_{precharge} \]

(5.28)

If we now assume that both the parasitic elements (conductance and capacitance) are present and the time of the flight is half of the input vibration cycle i.e. transition time from initial capacitance to final capacitance as shown in Figure 5.5 as would be expected in a resonant system. The analytical expression for the transducer voltage is more complex and is given by:
Figure 5.5: Top plate position of out-of-plane gap closing variable capacitor

![Diagram](image)

\[
V_{final} = \frac{56.5}{k_1 \times \omega} \left( -\frac{1}{k_1} \left( 258.2 \times A_{semi} \left( k_7 + \sqrt{k_8 + k_9} \right) \right) \right) \\
- \frac{1}{L_c \times \omega} \left( 3.16 \times 10^9 \times A_{semi} (k_7 + \sqrt{k_8 + k_9}) \times (L_c^{-3} \times Acc)^{0.25} \right) + k_{10}
\]

(5.29)

where

\[
k_7 = -A_{semi} \left( 22.8 \times 10^3 \times \omega + 2.796 \times 10^{11} \sqrt{L_c^{-3} \times Acc} \right)
\]

(5.30)

\[
k_8 = A_{semi} \times 10^8 \left( 5.22 \omega^2 + 1.28 \times 10^8 \omega \sqrt{L_c^{-3} \times Acc + 7.82 \times 10^{14} \times L_c^{-3} Acc} \right)
\]

(5.31)

\[
k_9 = k_1 \omega \times 10^5 \left( 1.62 \times A_{semi} \times \sqrt{V_{precharge}} + 15.67 k_1 \times L_c \times \omega \times V_{precharge} \right)
\]

(5.32)

\[
k_{10} = \omega \left( 8854 \times L_c \times k_1 \times V_{precharge} + 914 \times A_{semi} \times \sqrt{V_{precharge}} \right)
\]

(5.33)

It can be noted that the transducer voltage expressions are functions of microgenerator size, input acceleration and semiconductor device area. These expressions are crucial in estimating the useful operating regions (i.e. microgenerator effectiveness versus input
excitation and generator size) of an electrostatic microgenerator operating in constant charge mode.

5.4 Microgenerator System Effectiveness

In this section, microgenerator system effectiveness is calculated and plotted for different generator sizes and for different input excitations. Firstly, the definition of the system effectiveness is explained, the material is reproduced from [50] and effectiveness equations are derived. By using these equations, a method of calculating the system effectiveness is described. Finally, the effectiveness trends of the electrostatic microgenerators are discussed.

5.4.1 Definition of System Effectiveness

The overall system effectiveness, $\eta_{\text{system}}$, is a measure of the useful output of the generator as a fraction of the maximum possible theoretical output for a generator of particular volume driven from a particular source, and can be expressed as:

$$
\eta_{\text{system}} = \eta_{\text{mech}} \times \eta
$$

(5.34)

$$
\eta_{\text{system}} = \frac{E_{\text{coupled}}}{E_{\text{opp}}} \times \frac{E_{\text{out}}}{E_{\text{coupled}}}
$$

(5.35)

where

- $E_{\text{out}}$ is the useful energy output per cycle after processing,
- $E_{\text{coupled}}$ is the mechanical energy coupled into the generator per cycle,
- $E_{\text{opp}}$ is the opportunity energy, or the maximum energy that could have been coupled per cycle using a generator operating optimally,
- $\eta_{\text{mech}}$ is the mechanical coupling effectiveness and
- $\eta$ is the generator efficiency

Whilst the above terms are generic to all types of inertial microgenerator, additional terms,
specific to particular generator implementations, can be defined and will now be defined for the constant charge mode electrostatic microgenerator. These terms allow the useful output of the generator to be determined in terms of the product of several ratios which describe the effectiveness or efficiency of different parts of the microgenerator based upon the separate stages of the generation cycle. These ratios can be used to consider trade-offs in the implementation of the electrostatic microgenerator system and set specifications for the power processing and the semiconductor devices used in that processing.

The system effectiveness is defined as a ratio of the useful energy output from the generator and the maximum amount of energy that could have been generated as useful output if the generator had been operating optimally. The useful energy output from the system (which can be used to power a load) is the energy extracted from the step-down converter power processing circuitry, $E_{\text{out}}$, less the energy required to pre-charge the capacitor on the next cycle, $E_{\text{pre-ch}}$, and any additional overhead, such as generator control, $E_{\text{o-head}}$. The system effectiveness is therefore:

$$\eta_{\text{system}} = \frac{E_{\text{out}} - E_{\text{pre-ch}} - E_{\text{o-head}}}{\hat{W}_{\text{field}}}$$

where $\hat{W}_{\text{field}}$ is the maximum work that could be done against the electric field under the specific operating conditions.

Effectiveness and efficiency ratios can now be defined for the various parts of the generation cycle. In order for the variable capacitor to be able to generate energy, a certain amount of charge must be pre-loaded onto the capacitor before the inertia of the proof mass causes the plates to separate. The amount of energy stored on the closed capacitor plates, $E_{\text{closed}}$, is a fraction of the energy required from the pre-charge energy, $E_{\text{pre-ch}}$, taken from the generator output. Losses may be due to parasitic capacitance in parallel with the moving plate capacitor and ohmic losses associated with charging the capacitor:

$$\eta_{\text{pre-ch}} = \frac{E_{\text{closed}}}{E_{\text{pre-ch}}}$$

When the plates separate, a maximum amount of work, $\hat{W}_{\text{field}}$, can be done against the
electric field. If the generator has been pre-charged to a non-optimal value, or if charge leaks off the plates during plate separation, only $W_{field}$ work will be done against the field, allowing a mechanical effectiveness to be defined as:

$$\eta_{mech} = \frac{W_{field}}{W_{field}}$$  \hspace{1cm} (5.38)

This is the direct equivalent of the coupling effectiveness previously defined.

As the plates separate, some charge may leak off the plates through a finite conductance path between the plates, or alternatively be shared with the parasitic capacitance of the power converter attached to the plates. Most of the energy lost from the plates during the plate separation cannot be recovered meaning that the only available electrical energy (for driving a load) when the plates have reached their full separation, is the energy stored on the variable capacitor itself. This is justified because almost all the charge which leaks off the generator during the plate separation is stored on the junction capacitance of the drain-body diode in the MOSFET. When the MOSFET is turned on to discharge the generator capacitor, this energy is lost as the drain body capacitance is internally short circuited within the MOSFET. This allows a generation efficiency, $\eta_{gen}$ to be defined as:

$$\eta_{gen} = \frac{E_{gen}}{W_{field}} = \frac{E_{open} - E_{closed}}{W_{field}}$$  \hspace{1cm} (5.39)

where $E_{gen}$ is the increase in stored energy in the generator.

The high voltage on the open generator capacitor then has to be down-converted to low voltage for either passing to a storage element or powering the load. The efficiency of this down-conversion can then be defined as:

$$\eta_{conv} = \frac{E_{out}}{E_{open}}$$  \hspace{1cm} (5.40)

Therefore the system effectiveness of an electrostatic microgenerator, assuming there is no precharge loss and no overhead loss is:
\begin{equation}
\eta_{system} = \eta_{mech} \times \eta_{gen} \times \eta_{conv}
\end{equation}

Since, the parasitic capacitance and conductance leakage losses reflect in the transducer voltage calculations, the product of \(\eta_{mech}\) and \(\eta_{gen}\) can be written as coupling effectiveness, \(\eta_{coupling}\), which is defined in the following section. Therefore, the system effectiveness of an electrostatic microgenerator is simplified to:

\begin{equation}
\eta_{system} = \eta_{coupling} \times \eta_{conv}
\end{equation}

### 5.4.2 Effectiveness Analysis

Having defined the microgenerator system effectiveness in the previous section as a product of coupling effectiveness and circuit conversion efficiency, we are now concerned with maximising this effectiveness through optimising the generator. As we have previously seen, the semiconductor switch area, \(A_{semi}\), is vital to optimising this effectiveness as increasing \(A_{semi}\) decreases coupling effectiveness but increases the power processing circuit efficiency. The optimal value for \(A_{semi}\) and in turn the best case system effectiveness are derived in this section.

**Coupling Effectiveness:**

The coupling effectiveness, \(\eta_{coupling}\), is a ratio of the energy that is stored in the transducer capacitor to the maximum possible energy that can be harvested for a given microgenerator size and input excitation:

\begin{equation}
\eta_{coupling} = \frac{\text{Energy stored in the transducer capacitor at maximum plate separation}}{\text{Maximum possible energy output per half cycle}} = \frac{1}{2} \times C_{final} \times V_{final}^2
\end{equation}

\begin{equation}
= \frac{1}{2} \times C_{final} \times L_{c}^4 \times Acc
\end{equation}

The maximum possible energy output derivation for the CDRG is given in Appendix A.
After substituting $V_{\text{final}}$ from Eq. 5.22 in Eq. 5.43, the coupling effectiveness of the transducer with parasitic capacitive loading from the attached semiconductor is given by:

$$\eta_{\text{coupling}} = \frac{1}{(L_c^{13} \times Acc^3)^{0.5}} \left( 2.98 \times 10^{-19} \left( -\frac{1}{k_1} \left( 1291 \times A_{\text{semi}} \left( -2.28 \times 10^7 \times A_{\text{semi}} 
+ \sqrt{k_3 + k_4 + k_5} \right) \right) + k_6 \right)^2 \right)$$

(5.44)

Similarly, substituting $V_{\text{final}}$ from Eq. 5.29 in Eq. 5.43, the coupling effectiveness of the transducer with both parasitic capacitance and conductance is given by:

$$\eta_{\text{coupling}} = \frac{1}{(L_c^{13} \times Acc^3)^{0.5} \omega^2} \left( 7.45 \times 10^{-12} \left( -\frac{1}{k_1} \left( 258.2 A_{\text{semi}} \left( k_7 + \sqrt{k_8 + k_9} \right) \right) \right) \right. \\
- \left. \frac{1}{L_c \times \omega} \left( 3.16 \times 10^9 \times A_{\text{semi}} \left( k_7 + \sqrt{k_8 + k_9} \right) \times \left( L_c^{-3} \times Acc \right)^{0.25} \right) + k_{10} \right)^2 \right)$$

(5.45)

These two equations are derived because although Eq. 5.45 represents the general solution, Eq. 5.44 allows us to more easily calculate the performance of a CDRG operating at high frequency where conductance effects become negligible.

**Circuit Efficiency:**

The circuit efficiency is a conventional efficiency. In a typical standard power processing circuit, in order to losslessly extract energy from a capacitor to another storage element which could be another capacitor or a battery, the energy is usually first transferred to an inductor. The efficiency of this stage is therefore limited by the conduction losses of the switching element and series resistance of the inductor. The inductor series resistance is neglected in this analysis as it is assumed the resistance of the MOSFET dominates. Here, only this stage efficiency is estimated by assuming that the variable capacitor energy is transferred to the inductor through a resistor (note that we also assume the MOSFET
operates in the linear region). Therefore, the analysis presented here provides a best case circuit efficiency for this type of generator.

Figure 5.6: First Stage of a power conversion circuit

Figure 5.7: Vertical MOSFET structure from [134]

Figure 5.7 shows the structure of a typical high voltage MOSFET. Assuming that the $n^{-}$ epilayer of the MOSFET is uniformly doped with a doping density given in Eq. 5.11. The relation between the on-state resistance of this layer and the voltage blocking ability of the MOSFET is given in [134] as:

$$R_{epilayer} = \frac{k_{epilayer} \times V_B^2}{A_{semi}}$$  \hspace{1cm} (5.46)
where \( R_{\text{epilayer}} \) is on-state resistance of the \( n^- \) epilayer, \( V_B \) is blocking voltage, \( A_{\text{semi}} \) is area of the epilayer and \( k_{\text{epilayer}} = 2 \times 10^{-11} \text{[m}^2\text{sV}^{-2}\text{F}^{-1}] \).

As the \( n^- \) epilayer resistance is the dominant part of on-state resistance of high voltage MOSFETs, this layer resistance is considered as the only contribution by MOSFET on-state resistance for the circuit efficiency in the analysis which follows:

The efficiency of the circuit shown in Figure 5.6 is given by:

\[
\eta_{\text{conv}} = \frac{\text{Energy stored in the transducer capacitor} - \text{Energy loss in the resistor}}{\text{Energy stored in the transducer capacitor}}
\]

which can be written as:

\[
\eta_{\text{conv}} = \frac{E_{\text{inductor-end}}}{E_{\text{cap-start}}}
\]

where

\( E_{\text{cap-start}} \) is the energy stored in capacitor at the start of discharge period and

\( E_{\text{inductor-end}} \) is the energy stored in inductor at the end of discharge period

The energy stored in the capacitor can be calculated by \( \frac{1}{2}C_{\text{final}}V_{\text{final}}^2 \). Calculating the energy loss in the resistor is not straightforward and requires some analysis, which will now be described:

Applying Kirchoff’s voltage law for the circuit shown in Figure 5.6, we obtain:

\[
V_{\text{cap}}(t) = i(t) \times R_{\text{mosfet}} + L_{\text{ind}} \times \frac{di(t)}{dt}
\]

where

\( V_{\text{cap}}(t) \) is the capacitor voltage,

\( C_{\text{final}} \) is the capacitance value at the end of the generation stroke,

\( L_{\text{ind}} \) is the inductance value and

\( R_{\text{mosfet}} \) is MOSFET on-state resistance.
By substituting, \( i(t) = -C_{final} \times \frac{dV_{cap}(t)}{dt} \), the equation governing the operation of the circuit shown in Figure 5.6 is given by:

\[
\frac{d^2V_{cap}(t)}{dt^2} + \left( \frac{R_{mosfet}}{L_{ind}} \right) \frac{dV_{cap}(t)}{dt} = \frac{-V_{cap}(t)}{L_{ind}C_{final}} \tag{5.50}
\]

Solving this differential equation with initial conditions, \( V_{cap}(0) = V_{final} \), \( \frac{dV_{cap}(0)}{dt} = 0 \) (i.e. the rate of change of capacitor voltage is zero at the instant of switch turn on); we obtain:

\[
V_{cap}(t) = V_{final} \times e^{-\left( \frac{R_{mosfet}}{2L_{ind}} \right) t} \times \\
\left[ \cosh\left( \frac{1}{2L_{ind}} \sqrt{\frac{k_{11}}{C_{final}}} \right) + R_{mosfet} \sqrt{\frac{C_{final}}{k_{11}}} \times \sinh\left( \frac{1}{2L_{ind}} \sqrt{\frac{k_{11}}{C_{final}}} \right) \right] \tag{5.51}
\]

where

\[
k_{11} = C_{final} \times R_{mosfet}^2 - 4L_{ind} \tag{5.52}
\]

The current flowing through \( R_{mosfet} \) can be obtained by using:

\[
i_{mosfet}(t) = -C_{final} \times \frac{dV_{cap}(t)}{dt} \tag{5.53}
\]

The energy loss in the MOSFET resistance is given by:

\[
E_{lossmosfet} = \int_{0}^{t_{period}} i_{mosfet}(t)^2 \times R_{mosfet} \times dt \tag{5.54}
\]

It is assumed that the variable capacitor voltage will be discharged to zero in quarter time period of the resonant cycle i.e. \( t = \frac{\pi}{2} \sqrt{L_{ind}C_{final}} \) (where the variable capacitor and the inductor forms a resonant tank). Therefore, by substituting this time period and current value (from Eq. 5.53) in Eq. 5.54, the energy loss in the MOSFET resistance is given by:
\[ E_{\text{lossmosfet}} = \frac{1}{k_{11}} \left( 0.25 \times V_{\text{final}}^2 \times C_{\text{final}} \left( 2k_{11} + 8 \times L_{\text{ind}} \times e^{-\left(1.57R_{\text{mosfet}} \frac{C_{\text{final}}}{L_{\text{ind}}} \right)} \right) ight. \]
\[ - R_{\text{mosfet}} \times e^{k_{12}} \left( \sqrt{C_{\text{final}} \times k_{11} + C_{\text{final}} \times R_{\text{mosfet}}} \right) \]
\[ + R_{\text{mosfet}} \times e^{k_{13}} \left( \sqrt{C_{\text{final}} \times k_{11} - C_{\text{final}} \times R_{\text{mosfet}}} \right) \right) \]

(5.55)

where

\[ k_{12} = \frac{1.57}{\sqrt{L_{\text{ind}} \times C_{\text{final}}}} \left( \sqrt{C_{\text{final}} \times k_{11} - R_{\text{mosfet}} \times C_{\text{final}}} \right) \]

(5.56)

\[ k_{13} = -\frac{1.57}{\sqrt{L_{\text{ind}} \times C_{\text{final}}}} \left( \sqrt{C_{\text{final}} \times k_{11} + R_{\text{mosfet}} \times C_{\text{final}}} \right) \]

(5.57)

In [69], it has been observed that the efficiency of the power processing circuit that is shown in Figure 5.1 also depends on the inductance value. The higher the inductance used the better the conversion efficiency. This is because with more inductance the peak currents in the circuit will be less therefore, less MOSFET conduction losses. However, in order to integrate the inductor into the microgenerator, it is desirable to use an integrated inductor. The practical range of integrated inductance values appears to be achievable in the range of 1-10 \( \mu \)H. In this analysis, an inductance value of 100 \( \mu \)H is considered which is on the high side of what is practically achievable. But again, this analysis is intended to show the upper limits for electrostatic generators.

After substituting the \( C_{\text{final}} = C_{g2} \), \( L_{\text{ind}} = 100 \mu \)H and \( R_{\text{mosfet}} = \frac{2 \times 10^{-11} \times V_{\text{final}}^2}{A_{\text{semi}}} \) (from Eq.5.46), the energy loss in the MOSFET on-resistance is given by:
\[
E_{\text{lossmosfet}} = \frac{1}{k_{14}} \left( 4.43 \times 10^{-12} \times V_{\text{final}}^2 \times L_c \left( 2 \times L_c \times V_{\text{final}}^4 \right) \right. \\
-1.13 \times 10^{29} \times A_{\text{semi}}^2 \left( 1 - e^{-\left( \frac{1.32 \times 10^{-14} \times V_{\text{final}}^2 \times \sqrt{L_c}}{A_{\text{semi}}} \right)} \right) \\
+ V_{\text{final}}^2 \sqrt{L_c \times k_{14}} \left( e^{k_{15}} - e^{-k_{16}} \right) - V_{\text{final}}^4 \times L_c \left( e^{k_{15}} + e^{-k_{16}} \right) \left) \right)
\]

(5.58)

where

\[
k_{14} = V_{\text{final}}^4 \times L_c - 5.65 \times 10^{28} A_{\text{semi}}^2 \\
k_{15} = \frac{1.32 \times 10^{-14} \left( \sqrt{k_{14} L_c} - V_{\text{final}}^2 \times L_c \right)}{A_{\text{semi}} \sqrt{L_c}} \\
k_{16} = \frac{1.32 \times 10^{-14} \left( \sqrt{k_{14} L_c} + V_{\text{final}}^2 \times L_c \right)}{A_{\text{semi}} \sqrt{L_c}}
\]

(5.59) – (5.61)

Substituting the Eq. 5.58 in Eq. 5.47, the efficiency of the power processing circuit is given by:

\[
\eta_{\text{conv}} = \frac{2}{C_{\text{final}} \times V_{\text{final}}^2} \left( 0.5 \times C_{\text{final}} \times V_{\text{final}}^2 - \frac{1}{k_{11}} \left( 0.25 \times C_{\text{final}} \times V_{\text{final}}^2 \times (2k_{11} \\
+ 8 \times L_{\text{ind}} \times e^{-\left( \frac{1.57 R_{\text{mosfet}} \times \sqrt{C_{\text{final}} \times V_{\text{final}}^2}}{L_{\text{ind}}} \right)} - R_{\text{mosfet}} \times \sqrt{C_{\text{final}} \times k_{11}} \left( e^{k_{12}} - e^{-k_{13}} \right) - C_{\text{final}} \times R_{\text{mosfet}}^2 \left( e^{k_{12}} + e^{-k_{13}} \right) \right) \right) \]

(5.62)

If we multiply Eq. 5.44 (or Eq. 5.45) and Eq. 5.62, we will calculate the microgenerator system effectiveness, and our target is to maximising this function through choosing the optimal semiconductor area. This calculation is explained in the following section.
5.4.3 Effectiveness Calculation

From the previous section, it can be noticed that the coupling effectiveness and the circuit efficiency are both functions of semiconductor area for a given input acceleration and microgenerator size. As the semiconductor area is increased, the circuit efficiency will increase, but the coupling effectiveness will reduce. Therefore, there exists an optimal semiconductor area which will maximise the product of the coupling effectiveness and the circuit efficiency. In other words, the circuit efficiency will be higher for large semiconductor areas due to a reduction in the on-state resistance with area. However, the final transducer voltage value will decrease with increase in the semiconductor area because the parasitic elements are quantitatively proportional to the semiconductor area and thus they share more charge and leak more charge from the transducer capacitor and reduce the voltage generation ability of the transducer. This reduction in the transducer voltage will result in fall of the transducer coupling effectiveness. Therefore, there will be an optimal semiconductor area which will maximise the system effectiveness.

The following assumptions are made while choosing an optimal semiconductor area and calculating corresponding system effectiveness:

- The semiconductor area is increased by paralleling the semiconductor devices with an unit cell area of $110 \times 10^{-12} m^2$. A parameter $N_{cells}$ is used to define the number of devices in parallel.

- The transducer voltage, $V_{\text{final}}$, is limited to 1500V because commercially available MOSFETs have a maximum blocking voltage of 1.5kV [135]. Note that there are very few actual MOSFETs at this rating and so again, this analysis presents an upper bound of effectiveness for the CDRG.

- Precharge voltage is adjusted to limit the transducer voltage to 1.5kV.

At first, the final transducer voltage is computed for a given acceleration and microgenerator size, assuming $N_{cells} = 1$. If the transducer voltage is less than or equal to 1.5kV, the voltage is calculated as a function of $N_{cells}$ (by resetting the parameter $N_{cells}$). This voltage is then used to calculate the coupling effectiveness and the circuit efficiency, which
will also be functions of number of cells. An optimal value of $N_{\text{cells}}$ is chosen which maximises the product of coupling effectiveness and the circuit efficiency. The system effectiveness is then calculated for the corresponding optimal number of cells.

If the voltage is greater than 1500V, the precharge voltage is adjusted in such way that the transducer voltage will be 1.5kV for $N_{\text{cells}} = 1$. From then onwards the same procedure is followed to calculate the system effectiveness for optimised semiconductor area. The overall procedure is detailed in the flow chart of Figure 5.8.

Figure 5.8: Flow chart for system effectiveness calculation
5.4.4 Effectiveness Plots

In this section, microgenerator system maximum effectiveness values have been plotted by choosing an optimal semiconductor area for each combination of various microgenerator sizes and for various input excitations at different frequencies. Figure 5.9 shows the maximum achievable system effectiveness when the transducer is loaded with parasitic capacitance only, i.e. there is no leakage conduction path. This corresponds to a microgenerator operating with a very high frequency input acceleration. It can be noticed that:

- There is a clear region running diagonally through the center of the plot when the system effectiveness is quite high (above 50%) for various combinations of microgenerator size and input acceleration. To the right and left of this region as shown in Figure 5.9, the effectiveness falls. This corresponds to small generators operating at low accelerations and large generators operating at high accelerations respectively.

Figure 5.9: Maximum effectiveness of an electrostatic microgenerator (parasitic capacitance only)
Figure 5.10: (a) Transducer Voltage and (b) Optimal semiconductor area for maximum system effectiveness

- On the right side of the graph, the reduction in system effectiveness is primarily because as the microgenerator size decreases, the distance traveled by the proof mass will also reduce. This will result in reduction in the ratio of initial to final capacitance of the transducer and a lower available final value of transducer voltage. At low voltages, the parasitic capacitance of the blocking pn junction in the MOSFET is large and thus will share more charge from the transducer capacitor (the semiconductor junction has an inverse square root C-V characteristic). Therefore, due to significant charge sharing the voltage generation ability of the transducer will be reduced. This will result in a decreased coupling effectiveness and thus the system effectiveness drops rapidly. This trend contradicts the previously assumed perception that the electrostatic microgenerators are most suitable for small scale dimensions (as is the case of actuators).

- To the left of this region, when the product of length and acceleration is high, high voltages are required to achieve a high enough force for a maximum coupling effectiveness. However, a limit of 1.5kV was set for the blocking capacity of the devices. Therefore, for large values of $L^3 \times Acc$ the transducer voltage becomes saturated, as shown in Figure 5.10(a), and an optimal force is not achieved by the transducer. Therefore, the coupling effectiveness and overall system effectiveness is dramatically reduced.
Figure 5.11: Maximum effectiveness of an electrostatic microgenerator for f=1kHz

Figure 5.12: Maximum effectiveness of an electrostatic microgenerator for f=100Hz
By considering the presence of both parasitic capacitance and leakage current of the semiconductor junction, the microgenerator system effectiveness is calculated for different input excitation frequencies. Figure 5.11, 5.12, 5.13 and 5.14 show the effectiveness for input excitation frequencies of 1kHz, 100Hz, 10Hz and 1Hz respectively.

The general observations are:

- The system effectiveness graphs which include the leakage conductance loss have similar trends to that of parasitic capacitance only (shown in Figure 5.9). However, the effectiveness values are lower when compared with parasitic capacitance only.

- As the excitation frequency is reduced, the maximum system effectiveness values reduce. This is because leakage loss becomes dominant with a reduction in the frequency as the time of flight of the moving electrode is inversely proportional to the frequency.

![Graph showing effectiveness vs. frequency and charge for different input excitations](image-url)

Figure 5.13: Maximum effectiveness of an electrostatic microgenerator for f=10Hz
5.5 Verification

In order to verify the effectiveness analysis presented in the previous sections, the results of the analytical optimisation were tested against time-domain simulations in PSpice using the ICES toolkit. These time-domain simulations will provide information about displacement of the proof mass and its flight time. Also provide information about shape of the variable capacitor voltage. Per example in the analysis, the voltage across the variable capacitor is calculated by assuming that the proof mass flight time is half of the vibration period. But in the simulations the flight time depends on the damping force due to the variable capacitor. Again, this damping force depends on the charge that is present on the variable capacitor (part of the charge will be shared by parasitic elements of power processing circuit). The ICES toolkit models were used to model mechanical and electrical parts of the electrostatic microgenerator system (as shown in Figure 6.11). The verification was achieved in two parts. The final voltage on the transducer capacitor was first verified under different operating conditions before the system effectiveness was verified. The transducer voltage was verified under different loading conditions in Sections 5.5.1 and 5.5.2. The system effectiveness is verified in Section 5.5.3.
Figure 5.15: PSpice simulation circuit model of electrostatic microgenerator
5.5.1 Transducer Voltage - No Load

With a perfectly open circuit load condition, the electrostatic transducer voltages were obtained for different microgenerator sizes and for different input excitations. Figure 5.16 shows simulated and analytical values of the transducer voltage under no load conditions. It can be noticed from the graph that both the simulation and the analytical values match closely.

![Graph showing comparison between simulation and analytical data for transducer voltage under no load conditions.]

Figure 5.16: Comparison of microgenerator ideal voltage: Acc=50mg, f=100Hz (bottom line) and for Acc=1g, f=100Hz (top line)

5.5.2 Transducer Voltage - Parasitic Loading

Transducer with Parasitic Capacitance:

A voltage-dependent current source was used to model the parasitic capacitance loading on the transducer (as shown in Figure 5.17) to simulate the behaviour of the attached semiconductor switch, although to start with the leakage conductance was neglected. The current value is given by $C_j \frac{dV(t)}{dt}$, where $C_j$ is the voltage dependent parasitic capacitance (from Eq. 5.14).
Figure 5.17: PSpice simulation model for parasitic capacitance loading

\[ I_{\text{sharing}} = C_{\text{par}} \frac{d(V_{\text{in}})}{dt} \]

Figure 5.18: Microgenerator voltage with parasitic capacitor loading: Acc=50mg, f=100Hz (bottom line) and for Acc=1g, f=100Hz (top line)

Figure 5.18 shows simulated and analytical transducer voltage values with parasitic capacitance loading. It can be noticed that both the simulated and the analytical values match closely. The small discrepancy in the voltages at smaller sizes is because the voltage levels are very small (a few volts) and even a small difference in absolute voltage value is magnified on the logarithmic axis scale.
Transducer with Parasitic Capacitance and Leakage Current:

In order to check the results of the optimisation with realistic semiconductor models, the leakage conductance was also included in the SPICE models. Similar to that of modelling of parasitic capacitance loading, a voltage-dependent current source is used to represent the leakage current loading on the transducer. The simulation model of leakage current loading on the transducer is shown in Figure 5.19 and the current source value for leakage current is given by Eq. 5.15.

\[ I_{\text{leakage}} = f(V_{\text{in}}) \]

Figure 5.19: PSpice simulation model for parasitic leakage current loading

Figure 5.20, 5.21, 5.22 show the transducer voltage for three different excitation frequencies of 1kHz, 100Hz, 10Hz respectively. It can be noticed from the figures that the simulated values become larger than the analytical values as the excitation frequency is reduced. This is especially noticeable in Figure 5.22 for the 10Hz system. This is because in the analysis it is assumed that the flight time of the electrode will be the half of the input vibration time period as is the case for the resonant system if the proof-mass is damped sufficiently so as not to strike the end-stops. However, in the simulations, the flight times are less than the assumed values because of the reduction in force as charge sharing and leakage occurs, meaning the system is under-damped and the charge leakage actually occurs over a shorter time period than predicted by the earlier analysis. Therefore, the leakage losses are overestimated in the analysis.
Figure 5.20: Microgenerator voltage for $f=1$kHz: Top line: $Acc=1g$, Bottom line: $Acc=50mg$

Figure 5.21: Microgenerator voltage for $f=100$Hz: Top line: $Acc=1g$, Bottom line: $Acc=50mg$

### 5.5.3 System Effectiveness

Voltage values from the simulations were used in Eq. 5.43 to calculate the coupling effectiveness of the transducer. Figure 5.23 shows the simulated and the analytical values of
Figure 5.22: Microgenerator voltage for f=10Hz: Top line: Acc=1g, Bottom line: Acc=50mg

the coupling effectiveness.

Figure 5.23: Coupling Effectiveness with parasitic capacitance
Circuit efficiency is calculated in separate simulations. A capacitor with initial voltage is connected to a inductor (of value $100 \, \mu H$) through a resistor (of value given in Eq. 5.46). Figure 5.24 shows the efficiency plots of both simulated and analytical values. Here, the efficiency is given by the energy in the inductor when the capacitor voltage is zero divided by initial energy in the capacitor. It can be noticed that the efficiency falls with length. This is because high voltage MOSFETs need to be used for larger generators. These devices have higher specific on-state resistance.

![Figure 5.24: Circuit Efficiency](image)

As can be seen, the simulated values of the system effectiveness follow the trends of the analytical data. However, there is a difference between the values at both low frequency and high acceleration excitations. This is due to the difference between the simulated and analytical values of the transducer voltage which is due to different flight times between the analysis and the time domain simulation as described in Section. 5.5.2.
Figure 5.25: Microgenerator Effectiveness with parasitic capacitance loading only

Figure 5.26: Microgenerator Effectiveness for f=1kHz
Figure 5.27: Microgenerator Effectiveness for f=100Hz

Figure 5.28: Microgenerator Effectiveness for f=10Hz
5.6 Conclusions

In this chapter, an analytical approach is presented to estimate the effectiveness of an electrostatic microgenerator operating in constant charge mode across their entire possible operating regime. The effectiveness values are plotted for different microgenerator sizes and input excitations, from these plots useful operating regions of electrostatic microgenerator can be easily identified.

It has been found that the electrostatic transducer interfaced with power electronic circuit is not effective at small dimensions. This is contrary to the fact that electrostatic transduction mechanism is good at small dimensions as an actuator. The semiconductor device parasitic elements are reducing the energy generation ability of the transducer by sharing the charge during plates separation. It was also found that the effectiveness degraded with a decrease in excitation frequency. This is because leakage loss becomes dominant with a reduction in the frequency as the time of flight of moving electrode is inversely proportional to the frequency.

The analytical effectiveness values are verified through PSpice simulations and both are matching closely except when the excitation frequency is low and at higher accelerations. This is because of the assumption in the analysis that the time of flight is half of the vibration period. But in the simulations the flight times are less than what has been assumed.

In the next chapter, maximum effectiveness of an electrostatic microgenerator operating in constant-voltage mode is discussed.
Chapter 6

Effectiveness of an Electrostatic Microgenerator - Constant Voltage

6.1 Introduction

In the previous chapter, the effectiveness of an electrostatic microgenerator operating in constant charge mode has been presented. The other possible operation of electrostatic microgenerator is constant-voltage mode. This chapter discusses calculation of the system effectiveness of a CDRG type electrostatic microgenerator with an in-plane overlap type transducer [15] operating in constant-voltage mode. First, the voltage required for the optimal operation of the transducer is derived and then detailed description of the operation of the power processing circuit is presented. Finally, the expression for system effectiveness is derived and values are calculated for various generator sizes and input excitations. These results are then verified through PSpice simulations using the toolkit described in Chapter 3.

6.2 Optimal Source Voltage

The electrostatic force of a variable capacitor connected across a fixed voltage source is proportional to rate of change of capacitance with displacement and is given by:
\[
F_e = \frac{V_{source}^2}{2} \frac{dc}{dz}
\]  

(6.1)

where

\(V_{source}\) is voltage across the capacitor and

\[\frac{dc}{dz}\] is rate of change in capacitance over a distance. It is assumed that the capacitance changes linearly and therefore,

\[\frac{dc}{dz} = \frac{C_{max} - C_{min}}{2Z_t}.
\]

The electrostatic microgenerator will operate under optimal conditions if the electrostatic force is equal to the coulomb force given in 5.2. Therefore, equating Eq. 5.2 and Eq. 6.1, we obtain the voltage value that needs to be connected across the variable capacitor:

\[
V_{source} = \sqrt{1.57 \times m \times Y_0 \times \omega^2 \times (2Z_t)}
\]  

(6.2)

Figure 6.1: Microgenerator with mass occupying half the volume

Equation 6.2 can be simplified by assuming that the microgenerator is cube shaped and proof mass occupies the half of the volume. It has been assumed that the transducer occupies negligible space compared to the generator size and comb fingers are placed along the proof mass and have same displacement as that of proof mass as shown in Figure 6.1. Practical implementation of comb fingers with these assumptions is difficult, however to have one to one comparison of effectiveness of both operating modes (constant-charge and constant-voltage), these assumptions are thus necessary. Therefore, substituting \(m = \rho_{au}L_c^3\), \(\rho_{au} = 19300\text{ Kg/m}^3\) (assuming a gold proof mass), \(2Z_t \approx \frac{L_c}{2}\),

135
\( Acc = Y_o \times \omega^2 \) and \( C_{\text{max}} >> C_{\text{min}} \), we obtain:

\[
V_{\text{source}} = k_{cv} \times L_c^2 \times \sqrt{\frac{\text{Acc}}{C_{\text{max}}}}
\]

(6.3)

where \( k_{cv} = 87 \left[ \text{kgm}^{-3} \right]^{0.5} \).

This equation gives the voltage required for optimal operation of the transducer in constant-voltage mode. It is to be noted that the parameter \( C_{\text{max}} \) is a technology dependent and it is therefore best to derive expressions as a function of this parameter. The \( C_{\text{max}} \) for the in-plane overlap comb drive structure is given by:

\[
C_{\text{max}} = \varepsilon \times \frac{L_c}{d} \times N_{\text{gaps}} \times h
\]

(6.4)

where

- \( L_c \) is length of the cube
- \( N_{\text{gaps}} \) is number of gaps, \( \frac{L_c}{W_{\text{finger}} + d} \)
- \( W_{\text{finger}} \) is comb finger width,
- \( h \) is comb finger height and
- \( d \) is distance between fingers.

Figure 6.2: In-plane overlap comb structure
It is to be noted that the number of comb fingers and the $C_{\text{max}}$ depend on the length of the cube \textit{i.e.} microgenerator size. In order to estimate the maximum capacitance that can be achieved for a given microgenerator size, dimensions such as height and width of comb finger and distance between the fingers are considered as $50\mu m$, $50\mu m$ and $5\mu m$ respectively which are representative of a typical microscale comb fingers dimensions as given in [59].

### 6.3 Power Processing Circuit

The requirements for the operation of the constant-voltage generator are:

- Charging the transducer capacitor at its maximum capacitance;
- Holding the voltage constant across the transducer capacitor whilst extracting energy and
- Disconnecting the voltage source whilst the capacitor reset to maximum position.

The simplest circuit for doing this is shown in Figure 6.3. It is a buck power conversion circuit and the operation of this circuit is similar to the circuit proposed in [70]. There are three distinct phases of operation, they are precharging, harvesting and recovery. The precharging phase starts when the variable capacitor, $C_{\text{var}}$, is at maximum value. During this phase switch, $S1$, will be turned on for a short time and then diode, $D1$, will free-wheel the energy stored in the inductor, $L_{\text{ind}}$. Once the voltage on the variable capacitor reaches the input voltage, diode $D2$ will be on and the variable capacitor will get connected to the source voltage, current direction in this phase is shown in the sub circuits of Figure 6.4(a)-(c)). During the harvesting phase, \textit{i.e.} during the transition of maximum capacitance value to the minimum capacitance value, the current will flow through diode, $D2$, from the variable capacitor to input source voltage. Operation of this phase is shown in Figure 6.4(d). The harvesting phase ends when the variable capacitor reaches its minimum value. There will be still energy left in the variable capacitor. This is very small
energy and it may not be possible to recover it efficiently. This energy will naturally converted back to mechanical energy due to the increase in the capacitance value as the plates close.

![Buck converter diagram](image)

**Figure 6.3: Buck converter**

![Different operating phases of the circuit](image)

**Figure 6.4: Different operating phases of the circuit**

During the harvesting phase, the current from the variable capacitor not only flows to the input source but also through $D_1$ due to the reverse bias leakage current and junction capacitance charging current. These parasitic elements of the diode $D_1$ will reduce the effectiveness of the system. In other words, part of the stored energy in the variable
capacitor will be lost in the parasitic elements of $D_1$. This will reduce the efficiency of the harvesting phase and thereby the overall system effectiveness. It is to be noted that the parasitic elements of $D_1$ can be minimized by reducing the semiconductor area of the device. However, this will reduce the efficiency of the charging phase. Therefore, there is trade-off between the efficiency of the charging phase and efficiency of the harvesting phase. In order to identify the tradeoffs between efficiencies of these phases, quantification of energy loss in $D_1$ parasitic elements and the on-state resistance of $S_1$ is necessary. The following section discusses the quantification of energy loss in the parasitic elements.

### 6.3.1 Energy loss in parasitic elements:

By assuming that the semiconductor devices in the circuit shown in Figure 6.3 are designed to block the input source voltage (given in Eq. 6.3), the parasitic elements can be expressed (following detailed derivation discussed in Section 5.3.1) as:

$$C_j = \frac{k_{cj} \times A_{semi}}{\sqrt{(V_o - V_{operation}) \times V_{source}}} \quad (6.5)$$

where

$$k_{cj} = 1.1 \times 10^{-3} \text{[m}^{-2}\text{C}]$$

Similarly, the off-state leakage current is given by:

$$I_{leakage} = k_{cvleakage} \times L_c^2 \times A_{semi} \times \sqrt{\frac{Acc}{C_{max}}} \quad (6.6)$$

where

$$k_{cvleakage} = 339 \times 10^{-4} \text{[m}^{-7}\text{kgF}^{0.5}] \text{s}^{-1}$$

These equations can now be used to derive analytical expressions for energy loss in the parasitic elements.
Loss due to junction capacitance:

It can be noticed from the Figure 6.4(d) that the voltage across diode D1 will be the source voltage and its junction capacitance is charged from the source voltage. This stored charge will be lost during the mechanical system operation as shown in Figure 6.4(e). Therefore, the energy loss because of the junction capacitance is given by:

\[ E_{jloss} = Q_j \times V_{source} \]  

(6.7)

where \(Q_j\) is charge stored in the pn junction capacitance and is calculated as follows:

\[ Q_j = \int_0^{V_{source}} C_j \times dV_{operation} \]  

(6.8)

Substituting the junction capacitance from Eq. 6.5 and simplifying the integration we obtain:

\[ Q_j = 2.2 \times 10^{-3} A_{semi} \left[ 1 - \sqrt{\frac{V_{diode}}{V_{source}}} \right] \]  

(6.9)

Therefore, substituting Eq. 6.9 in Eq. 6.7, we obtain:

\[ E_{jloss} = 2.2 \times 10^{-3} \times A_{semi} \times V_{source} \times \left[ 1 - \sqrt{\frac{V_{diode}}{V_{source}}} \right] \]  

(6.10)

As expected, the junction capacitance loss depends on source voltage and area of the semiconductor.

Leakage current loss:

Part of the generation current during harvesting phase will be leaked through diode off-state leakage current. Therefore, the leakage charge loss is given by:
\[ E_{\text{leakageloss}} = Q_{\text{leakage}} \times V_{\text{source}} \quad (6.11) \]

where \( Q_{\text{leakage}} \) is leaked charge and can be estimated as below:

\[ Q_{\text{leakage}} = I_{\text{leakage}} \times t_{\text{flight}} \quad (6.12) \]

where

\( I_{\text{leakage}} \) is the leakage current and is given in Eq. 6.6 and
\( t_{\text{flight}} \) is flight time of the transducer capacitor.

It has been assumed that the flight time will be half of the vibration period \( \frac{\pi}{\omega} \) as is the case in resonant microgenerators.

Substituting these values, the charge leaked off through diode is given by:

\[ Q_{\text{leakage}} = \frac{339 \times 10^{-4} \times L_c^2 \times \pi \times A_{\text{semi}}}{\omega} \sqrt{\frac{A_{\text{cc}}}{C_{\text{max}}}} \quad (6.13) \]

Therefore, the leakage charge loss in terms of \( V_{\text{source}} \) is given by:

\[ E_{\text{leakageloss}} = \frac{339 \times 10^{-4} \times L_c^2 \times \pi \times A_{\text{semi}} \times V_{\text{source}}}{\omega} \sqrt{\frac{A_{\text{cc}}}{C_{\text{max}}}} \quad (6.14) \]

Again, leakage current loss also depends on source voltage and area of the semiconductor.

**Loss in MOSFET on-state resistance:**

The charging phase is similar to that of the scenario in the previous chapter i.e. energy transfer through a basic RLC circuit. Therefore, the same expression can be used for the energy loss in the resistor (i.e. the on-state resistance of the MOSFET).
\[
\dot{E}_{\text{mosfet loss}} = \frac{1}{\sqrt{k_{20}}} \left( 0.25 \times V_{\text{source}}^2 \times C_{\text{max}} \left( 2k_{20} + 8 \times L_{\text{ind}} \times e^{-(1.57R_{\text{mosfet}} \sqrt{C_{\text{max}} L_{\text{ind}}})} \right) \right.
\]
\[
- R_{\text{mosfet}} \times e^{k_{21}} \left( \sqrt{C_{\text{max}} \times k_{20} + C_{\text{max}} \times R_{\text{mosfet}}} \right)
\]
\[
+ R_{\text{mosfet}} \times e^{k_{22}} \left( \sqrt{C_{\text{max}} \times k_{20} - C_{\text{max}} \times R_{\text{mosfet}}} \right) \right)
\]

(6.15)

where

\[
k_{20} = C_{\text{max}} R_{\text{mosfet}}^2 - 4 \times L_{\text{ind}} \quad \text{(6.16)}
\]
\[
k_{21} = \frac{1.57}{\sqrt{L_{\text{ind}} \times C_{\text{max}}} \left( \sqrt{C_{\text{max}} \times k_{20} - R_{\text{mosfet}} \times C_{\text{max}}} \right)} \quad \text{(6.17)}
\]
\[
k_{22} = -\frac{1.57}{\sqrt{L_{\text{ind}} \times C_{\text{max}}} \left( \sqrt{C_{\text{max}} \times k_{20} + R_{\text{mosfet}} \times C_{\text{max}}} \right)} \quad \text{(6.18)}
\]

It is to be noted that during the harvesting phase, charges from variable capacitor flow from variable capacitor to source voltage via parasitic diode of MOSFET. The energy loss in the diode is neglected because the rate of charge flow is very low during that operation.

### 6.4 Microgenerator System Effectiveness

In this section, microgenerator system effectiveness is calculated and plotted for different generator sizes and for different input excitations. Firstly, definition of the system effectiveness is explained and effectiveness equations are derived. By using these equations, a method of calculating the system effectiveness is described. Finally, the effectiveness trends of the electrostatic microgenerators operating in constant-voltage mode are discussed.
6.4.1 Definition of System Effectiveness

In this mode of operation, it is not possible to separate the coupling effectiveness and the circuit efficiency as the same power processing circuit is used to precharge the variable capacitor and to harvest energy. Therefore, the system effectiveness is given by:

\[
E_{\text{eff}} = \frac{E_{\text{harvested}}}{\text{Maximum Possible Energy Output}} = \frac{E_{\text{harvested}}}{3789.5 \times Lc^4 \times Acc}
\] (6.19) (6.20)

The maximum possible energy output derivation for the CDRG is given in Appendix A. \(E_{\text{harvested}}\) is harvested energy and is given by:

\[
E_{\text{harvested}} = E_{\text{varcap}} - [E_{\text{precharge}} + E_{\text{parasitic loss}}]
\] (6.21)

where
\(E_{\text{varcap}}\) is energy that can be extracted from the transducer capacitor, \(C_{\text{max}}V_{\text{source}}^2\)
\(E_{\text{precharge}}\) is initial stored energy in the transducer capacitor, \(\frac{1}{2}C_{\text{max}}V_{\text{source}}^2\)
\(E_{\text{parasitic loss}}\) is energy loss in the parasitic elements, \(E_{\text{Jloss}} + E_{\text{leakageloss}} + E_{\text{mosfetloss}}\)

6.4.2 Effectiveness Calculation

As previously discussed the energy losses in parasitic elements of the power processing circuit are functions of semiconductor area for a given input acceleration and microgenerator size. As the semiconductor area is increased, the efficiency of the charging phase increases. However, efficiency of the harvesting phase will reduce. Therefore, there exists an optimal semiconductor area which will maximise the overall system effectiveness.

The following assumptions are made while choosing an optimal semiconductor area and calculating corresponding system effectiveness:
The semiconductor area is increased by paralleling the semiconductor devices with an unit cell area of $110 \times 10^{-12} m^2$. A parameter $N_{\text{cells}}$ is used to define the number of devices in parallel.

The transducer voltage, $V_{\text{final}}$, is limited to 1500V because commercially available MOSFETs have a maximum blocking voltage of 1.5kV [135]. Note that there are very few actual MOSFETs at this rating and so again, this analysis presents an upper bound of effectiveness for the CDRG.

At first, the optimal source voltage is computed for a given microgenerator size and input acceleration. If the source voltage is less than or equal to 1.5kV, the system effectiveness is calculated as a function of $N_{\text{cells}}$. An optimal value of $N_{\text{cells}}$ is chosen which maximises the system effectiveness. Otherwise, $V_{\text{source}}$ is set to 1.5kV and same procedure is followed to calculate the system effectiveness for optimised semiconductor area. The overall procedure is detailed in the flow chart of Figure 6.5.
6.4.3 Effectiveness Plots

The system effectiveness values are calculated for various microgenerator sizes and input accelerations. These values are plotted and are shown in Figure 6.6, 6.8, 6.9 and 6.10 for input excitation frequencies of 1kHz, 100Hz, 10Hz and 1Hz respectively. The following observations can be made from the graphs:

- There is a region in which the effectiveness is constant for wide combinations of mi-
crogenerator size and input accelerations. This is because the parasitic elements are not having such a detrimental effect on the harvesting ability of the microgenerator, which was the case in constant-charge mode. In other words, harvesting in constant-voltage device always occurs at constant voltage as required but in constant-charge devices generation does not happen at constant charge because of parasitic charge leakage and charge sharing with parasitic capacitance.

- To the left of this region *i.e.* for high acceleration and higher sizes, the effectiveness falls rapidly due to limiting of source voltage to 1.5kV. The voltage plot is shown in Figure 6.7(a).

![Figure 6.6: Constant Voltage operation Effectiveness for f=1kHz](image)

- As the excitation frequency is reduced, the maximum system effectiveness values reduce. This is because leakage loss becomes dominant with a reduction in the frequency as the time of flight of the moving electrode is inversely proportional to the frequency.

- No useful energy can be harvested for operation at higher microgenerator sizes and...
Figure 6.7: (a) Optimal source voltage and (b) Optimal semiconductor area for maximum system effectiveness

high accelerations with frequency of 1Hz. This is again because of the dominant leakage loss.

Figure 6.8: Constant Voltage Operation Effectiveness for f=100Hz
Figure 6.9: Constant Voltage Operation Effectiveness for f=10Hz

Figure 6.10: Constant Voltage Operation Effectiveness for f=1Hz
Figure 6.11: PSpice simulation circuit model of electrostatic microgenerator in constant-voltage mode
6.5 Verification

In order to verify the effectiveness analysis presented in the previous sections, the results of the analytical optimisation were tested against time-domain simulation in PSpice using the ICES toolkit. The ICES toolkit models were used to model the mechanical and electrical part of the electrostatic microgenerator system. Ideal switches and diodes were used to model the power processing circuit so that parasitic elements such as junction capacitance and off-state leakage currents are separately added as calculated. A voltage dependent current source is used to model the junction capacitance and the off-state leakage current is modelled by a current source. Figure 6.11 shows the complete end-to-end system simulation model of electrostatic microgenerator operating in constant-voltage mode.

Loss in the parasitic elements were individually calculated from the simulation data (data such as flight time, charge stored in the junction, etc). After obtaining the losses, system effectiveness values were calculated. Both analytical and simulation effectiveness values are plotted in Figure 6.12, 6.13 and 6.14 for excitation frequencies 1kHz, 100Hz and 10Hz respectively.

It can be noticed from the plots that there are some discrepancies between simulation and analytical system effectiveness values. For example at high frequency, the effectiveness values calculated using simulation data are less than the analytical values. This is because of parasitic capacitance charge loss is higher in the simulation than the analytical value, which is due to in-built finite parasitic capacitances of the ideal diodes that were used to model the power processing circuit. These parasitic capacitances become dominant particularly at smaller dimensions. It is to be noted that parasitic capacitances of the ideal diodes can not be set to zero because the simulations will have convergence errors.

The other discrepancy that can be noticed is at low frequency (or as the excitation frequency is reduced), where the analytical system effectiveness values are less than simulation values. It was observed that the leakage charge loss that is estimated in the analysis is higher than the simulation values. This is because of the differences in flight time values.
Figure 6.12: Constant-Voltage Effectiveness for f=1kHz

Figure 6.13: Constant-Voltage Effectiveness for f=100Hz
between analytical and simulation data. Ideally, there should not be any difference in the flight time values (At 1kHz, there is no difference in flight time values). However, for low frequencies, the diode $D_2$ is getting turned off at the start of harvesting phase due to fall in the voltage across the variable capacitor. This is because of charging of parasitic elements of the diode $D_1$ by variable capacitor thereby reduction in its voltage. This voltage fall across the variable capacitor will result in decrease in electrical damping force and thereby decrement in proof mass travel time i.e. flight time. As the excitation frequency is reduced, charge of variable capacitor will be lost for longer time thereby more reduction in damping force and proof mass reaches end-stops more quickly.

![Graph](image.png)

Figure 6.14: Constant-Voltage Effectiveness for $f=10\text{Hz}$

### 6.6 Conclusions

In this chapter, an analytical approach was presented to estimate the effectiveness of an electrostatic microgenerator operating in constant-voltage mode across their entire possible operating regime. The effectiveness values are plotted for different microgenerator
sizes and input excitations, from these plots useful operating regions of electrostatic microgenerator can be easily identified.

It has been found that electrostatic microgenerator operating in constant-voltage is still favourable at small dimensions. It has also been found that in constant-voltage mode the electrostatic microgenerator has a better effectiveness over a large operating range than constant-charge devices. This is because the parasitic elements of power processing circuit are not having a detrimental effect on ability of energy generation, which is not the case in constant-charge mode. In other words, harvesting in constant-voltage devices always occurs at constant voltage as required but constant-charge devices do not generate at constant charge because of parasitic leakage and charging sharing with parasitic capacitance. It was also found that the effectiveness degraded with a decrease in excitation frequency. This is because leakage loss becomes dominant with a reduction in the frequency as the time of flight of the moving electrode is inversely proportional to the frequency.

The analytical derived effectiveness values were verified through PSpice simulations and a close match was found except for smaller generator sizes. This discrepancy arises because of the finite parasitic capacitances of the ideal components that were used to model the power processing circuit. Also due to the difference in flight times between simulation and analytical values.
Chapter 7

Conclusions and Further Work

7.1 Overview and Main Findings

A critical issue for the optimization of a microgenerator is the electromechanical link between the transducer and power processing circuitry because the performance of each subsystem depends on the behaviour of other. Therefore, in order to accurately simulate and optimize a microgenerator system, a combined electromechanical system simulation is required. In this thesis, the creation of a SPICE based simulation toolkit (ICES) that models inertial microgenerators has been reported. This toolkit offers many advantages such as rapid creation of system-wide models that reveal the electromechanical interaction between various subsystems and allow design of the power processing circuits with accurate semiconductor device models. Case study simulation results of electromagnetic and electrostatic microgenerator systems have been presented to verify the operation of the toolkit models. The simulation results closely match with the analytical values. Unified simulation solutions of this kind for inertial microgenerators have now started to appear in the literature [118] and [136].

The analytical approach presented in this work is useful to estimate the maximum end-to-end system effectiveness of an electrostatic microgenerator operating in constant-charge and constant-voltage modes across their entire operating regime covering different sizes, vibration amplitudes and excitation frequencies. It was found that the electrostatic trans-
ducer operating in constant-charge mode when interfaced with a power electronic circuit is not effective at small dimensions for energy generation. This is contrary to the fact that the electrostatic transduction mechanism is considered good at small dimensions as an actuator. The circuit parasitic elements were found to reduce the energy generation ability of the transducer by sharing the charge during separation of the plates. However, in constant-voltage operating mode the transducer has better effectiveness over a larger operating range (still favourable at small dimensions). Even though the parasitic effects are present in this mode, they have much less detrimental effect on the energy generation. In other words, harvesting in constant-voltage devices always occurs at constant voltage as required but constant-charge devices do not generate at constant charge because of parasitic leakage and charging sharing with parasitic capacitance.

The voltage magnitude required for optimal operation of the microgenerator as the size increases is very high (hundreds of volts) and requires specialised power device/electronic circuits such as proposed in [69]. However, the very high voltage source for pre-charging for constant-voltage operation is not practical and thus developing an optimised micro-generator with large dimensions is not considered to be feasible.

It was also found that in both operating modes, the effectiveness degraded with a decrease in excitation frequency. This is because leakage loss becomes dominant with a reduction in the frequency as the time of flight of the moving electrode is inversely proportional to the frequency.

The analytically derived effectiveness values were verified through PSpice simulations and a close match was found except when the excitation frequency is low or the acceleration is high. The discrepancy arises because of the assumption that the time of flight is half of the vibration period whereas in the simulations reveal that the flight times are sometimes significantly less.

Accurate modelling of custom semiconductor devices in PSpice was investigated so that a complete end-to-end system simulation could be performed. It was found that the in-built PSpice semiconductor devices models cannot model power semiconductor devices in this context with sufficient accuracy. It was also found that subcircuits based PSpice models
accurately modelled the custom diode characteristics, but the model was computationally inefficient. It was considered that such a model could not be used in end-to-end system simulation as the entire simulation will become slow (or unstable if the maximum allowed time step is extended).

A behavioural model using look-up tables was developed for a custom-diode. This model accurately represented the custom diode characteristics in both DC and AC analysis and was quick enough to be practically useful. A similar behavioural model for a custom-MOSFET has also been investigated. The behavioural model accurately represented the dc characteristics except at high voltages (> 200V). However, sufficient data was not available to model the ac characteristics accurately.

7.2 Author’s Contribution

At the start of this research no unified simulation platform for energy harvesting simulations existed. The first major contribution of this thesis has been the development of a SPICE based simulation toolkit to model inertial microgenerators. This toolkit allows design of power processing circuits for microgenerators with proper consideration of the interaction with other sub-systems. It also allows the simulation of complete end-to-end electromechanical microgenerator systems with accurate semiconductor device models. Researchers new to energy harvesting finding it easy to simulate microgenerator systems using this toolkit because of its flexibility and because SPICE is a familiar tool to the research community.

The second major and important contribution of this thesis has been the development of an analytical framework to estimate the effectiveness of an electrostatic microgenerator operating in constant-charge and constant-voltage modes. This framework helped develop an understanding of the effects of the parasitic elements of interfacing electronics on the performance of a microgenerator. It also enabled the plotting of the useful operating regions of electrostatic microgenerators. Conclusions drawn from the use of this analytical framework demonstrated that end-to end system analysis considering circuit parasitics is
required before designing microgenerator systems.

The reported work is part of an ongoing effort to develop microgenerators that generate energy from vibrations for biomedical implantable sensors. The introduction chapter and tables in Chapter 2 are built upon previous works [50, 46].

7.3 Suggestions for Further Work

It was found that a look-up table based diode behavioural model accurately modelled the custom-diode characteristics. However, sufficient data was not available to model the custom-MOSFET. A further investigation is necessary to establish a method to obtain the required data from a 3D finite-element MOSFET model. After having obtained custom-semiconductor device (Diode and MOSFET) models, a complete end-to-end system simulation of an electrostatic microgenerator can be achieved and thereby the interactions between various sub-systems can be fully studied and the whole microgenerator system optimised.

Because of the use of low charge at high voltages, special power electronic circuits are required to efficiently process the electrostatic microgenerator output. No significant amount of work on circuits has been found in the literature and investigation of more advanced topologies will improve the performance of the microgenerator.

In the analytical framework for determining effectiveness, a complete power processing circuit has not been considered and the aim was to estimate a maximum theoretical value. It is possible to estimate an accurate system effectiveness by considering complete power processing circuit including control electronics overhead.

In this work, only silicon semiconductor devices based circuits were considered. Circuits based on advanced semiconductor materials such as Silicon Carbide (SiC) and Gallium Nitride (GaN) need to be investigated as these devices may have smaller parasitics elements thereby achieving better system effectiveness.
7.4 Publications arising from this Work

7.4.1 Journals


7.4.2 Conferences


7.4.3 Software

Appendix A

Maximum Possible Energy Output

Maximum possible energy output derivation:

In an inertial system, the power at resonance is given by [46]:

\[
\text{Power at resonance} = \frac{1}{2} m Y_o^2 \omega^3 \left( \frac{Z_l}{Y_o} \right)
\]

By assuming energy will be generated during only one side of the motion, maximum possible energy is given by:

\[
\text{Maximum Possible Energy} = \text{Power at resonance} \times 2 \times \text{Frequency} = \frac{1}{4} m Y_o \omega^2 Z_l \times 2\pi
\]

(A.1)

Assuming the microgenerator is cube shaped and proof mass occupies half the volume and substituting \( m = \frac{\rho \pi L^3}{2}, Acc = Y_o \times \omega^2, Z_l = \frac{L^4}{4} \) we get:

\[
\text{Maximum Possible Energy} = 3789.5 \times L_c^4 \times Acc
\]

(A.2)

This equation gives the maximum possible energy that can be harvested for a given microgenerator size and input acceleration.
Bibliography


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