Experimental Test of Contextuality in Quantum and Classical Systems

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Contextuality is considered as an intrinsic signature of nonclassicality and a crucial resource for achieving unique advantages of quantum information processing. However, recently, there have been debates on whether classical fields may also demonstrate contextuality. Here, we experimentally configure a contextuality test for optical fields, adopting various definitions of measurement events, and analyze how the definitions affect the emergence of nonclassical correlations. The heralded single-photon state, which is a typical nonclassical light field, manifests contextuality in our setup; whereas contextuality for classical coherent fields strongly depends on the specific definition of measurement events, which is equivalent to filtering the nonclassical component of the input state. Our results highlight the importance of the definition of measurement events to demonstrate contextuality, and they link the contextual correlations to nonclassicality defined by quasiprobabilities in phase space.

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The probabilistic nature of measurements in quantum theory, in which observables do not have predetermined values, is one of the most distinguishing features in contrast to classical physics. This property is accentuated when considering measurements of compatible observables and can be formulated as contextuality [1,2], which means that the outcome of a measurement will depend on the context of the other measurements being performed. Contextuality can be demonstrated through the violation of noncontextual (NC) inequalities usually involving dichotomic measurements with binary outcomes. Similar to the nonlocality originated from quantum entanglement, contextuality is often considered as a signature of nonclassical behavior [3–7], as well as a critical resource of quantum computation [8–12] and quantum communication [13].

Recently, there has been a new type of investigation called classical entanglement, which draws the analogy between nonseparability among different degrees of freedom of classical light fields and entanglement among different particles, and suggests that features such as entanglement are not unique to quantum theory [14–18]. As a natural extension of classical entanglement, contextuality in classical systems has also drawn much interest. In particular, there have been debates on whether classical fields (i.e., coherent or stochastic light beams) can reproduce the correlations to violate NC inequalities [19,20] or not [21] and whether such correlations may enhance the performance of certain applications [22–24].

To date, tests of contextuality and entanglement in classical systems have used measurement devices different from those used for quantum systems [17–19], and the binary outcomes are not the direct response of the devices but rather defined artificially with certain selection rules and through proper considerations of measurement events [19]. Although these devices are elaborately designed to ensure the classical systems reproduce probability distributions given by the quantum theory, they have, nevertheless, different working mechanisms other than those used for quantum systems. This difference, together with the specific selection rules on measurement outcomes, makes the comparison between the behaviors of quantum and classical systems obscure; and the relation between the contextuality and the nonclassicality of the system remains unspecified.

To resolve the obscurity, in this work, we examine the violations of a fundamental NC inequality for both a single-photon state and coherent states with the same setup and different measurement events. Through this unified comparison, our configuration of the test confirms contextuality of a single-photon state, and it shows how contextual correlations emerge for coherent states under proper consideration of measurement events. Our results establish the relation between the violation of the NC bound for a linear-optical setup and the nonclassicality of the measured field.

The scenario of testing contextuality.—As shown in Fig. 1, a test of contextuality consists of a physical system

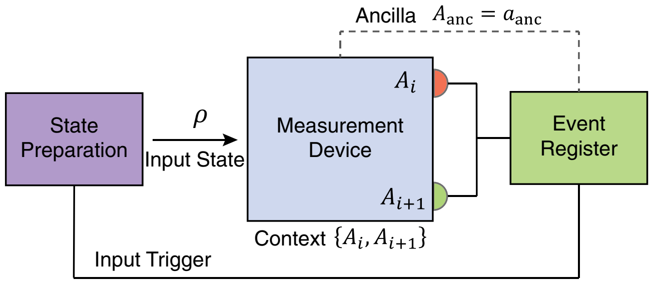

FIG. 1. The scenario of testing the Klyachko-Can-Binicioğlu-Shumovsky (KCBS) inequality. A physical system is prepared in state ρ and sent to a measurement device, which performs measurements of compatible observables $\{A_i, A_{i+1}\}$ and produces outcomes $\{a_i = \pm 1, a_{i+1} = \pm 1\}$. The measurement device can be set to different configurations to realize distinct contexts $\{A_i, A_{i+1}\}$. An event register records the statistics of outcomes (including potential ancillary outcomes $A_{anc} = a_{anc}$).

in a given state ρ as input and a measurement device including a set of observables $\{A_i\}$, where i labels a particular observable. In each trial, the device performs measurements of several compatible observables constituting a measurement context (e.g., $\{A_i, A_{i+1}\}$ [25]) and produces the well-defined measurement outcomes $\{a_i, a_{i+1}\}$. From the operational point of view, two observables are compatible if they can be jointly measured in a single device [1,26]. In actual implementations, there may be outcomes besides A_i and A_{i+1}, which can be encapsulated in an ancillary observable $A_{anc} = a_{anc}$ ($a_{anc} = 0, 1, \ldots, d$).

An NC inequality is usually associated with a parameter β involving the linear combination of the probabilities P of a set of events E [27]. The Klyachko-Can-Binicioğlu-Shumovsky inequality [28] is the most fundamental NC inequality satisfied by all noncontextual hidden variable models [29]. Consider five dichotomic observables $A_i (i = 1, 2, \ldots, 5)$, with each taking a value $+1$ or -1. Assuming the outcomes of observables reveal a predetermined joint probability distribution, we arrive at the KCBS inequality [28]

$$\beta = \langle A_1 A_2 \rangle + \langle A_2 A_3 \rangle + \langle A_3 A_4 \rangle + \langle A_4 A_5 \rangle + \langle A_5 A_1 \rangle \geq -3. \tag{1}$$

Violation of this inequality implies that the tested system is contextual. Quantum theory predicts the maximum violation of the inequality to be $5 - 4\sqrt{5} \approx -3.944$ [28–32].

In the KCBS inequality, the correlation between A_i and A_{i+1}, of the form $\langle A_i A_{i+1} \rangle$, can be measured by the joint probabilities of all possible outcomes

$$\begin{aligned} \langle A_i A_{i+1} \rangle &= P(A_i = 1, A_{i+1} = 1) + P(A_i = -1, A_{i+1} = -1) \\ &\quad - P(A_i = -1, A_{i+1} = 1) - P(A_i = 1, A_{i+1} = -1) \\ &= 1 + 4P(A_i = -1, A_{i+1} = -1) \\ &\quad - 2P(A_i = -1) - 2P(A_{i+1} = -1). \end{aligned} \tag{2}$$

Here, $P(A_i = a_i, A_{i+1} = a_{i+1})$ denotes the probability of the event in which "the outcomes a_i, a_{i+1} are acquired when the measurements A_i, A_{i+1} are performed."

Experiment.—Figure 2 illustrates the experimental setup to test the KCBS inequality using heralded single photons or classical coherent light fields. The former are generated through spontaneous parametric down-conversion (SPDC) within a potassium dihydrogen phosphate (KDP) crystal pumped by a frequency doubled pulsed Ti:sapphire laser (830 nm, 76 MHz) [33], whereas the latter is implemented with a set of coherent states generated by a Ti:sapphire laser with a reduced repetition rate (1.9 MHz) using a pulse picker and under different attenuation. The SPDC is generated at a low pump regime, and an avalanche

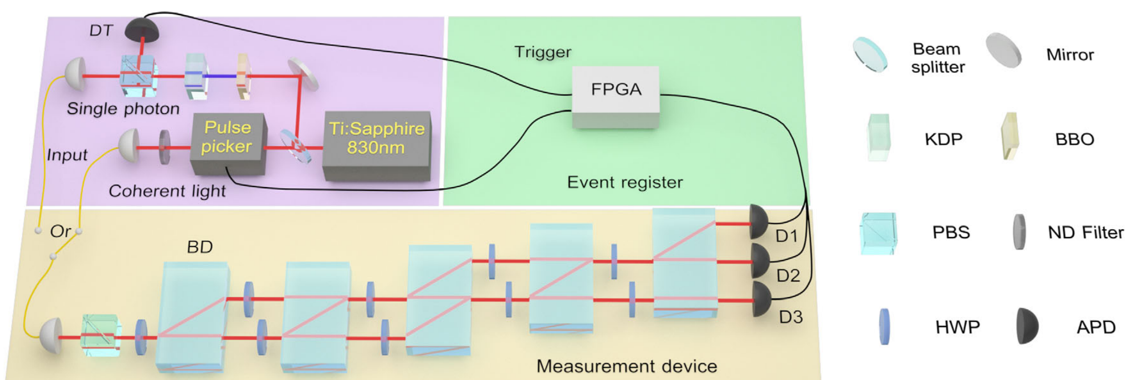

FIG. 2. Experimental setup for testing the KCBS inequality with single photons and coherent light. Single photons are heralded using spontaneous parametric down-conversion; the classical light is a weak coherent state generated by a Ti:Sapphire laser and attenuated by neutral density filters (purple area). The repetition rate of the pulses is reduced by a pulse picker (1.9 MHz). The measurement device (yellow area) is realized by a linear-optical network comprising birefringent beam displacers, half-wave plates (HWPs), and avalanche photodiodes. A FPGA registers relevant events (green area) with a coincidence window of 4.5 ns. BBO denotes β-barium borate crystal, and PBS is the polarizing beam splitter.

photodiode (APD) in the trigger arm heralds the generation of a single-photon state in the other arm because the multiphoton components are negligible. This can be confirmed by the value of the second order correlation function for heralded single photons $g^{(2)}(0) = 0.0397(38)$. The detected count rate of single photons is ~57 000 counts/series (in 1.4 s) under a heralding efficiency of $\eta_H = 16.2\%$ accounting for all the losses.

The test of the KCBS inequality involves three modes of the light field. In our setup, the three modes are defined as one spatial mode of the light field and the vertical polarization and horizontal polarization in another spatial mode. The light field is injected in the horizontal polarization in the second spatial mode. The measurement devices are composed of an optical network to perform transformations on polarization and spatial modes of the input optical field and three APDs to produce outcomes. We change the settings of wave plates to realize the measurements of a different pair of observables, and thereby distinct contexts. The transformation from one context to the other leaves one of the two measurements unaffected, ensuring the measurement is physically the same in the two contexts. The two measurements in each context are realized in a single device and defined by different detectors; thus they are comeasurable and compatible. We assign -1 for the measurement outcome when the corresponding APD gives a click and $+1$ when it does not. Two APDs give the outcomes of measurements A_i and A_{i+1}, and the third APD plays the role of the ancilla, which is used to identify the measurement event, i.e., only one detector clicks, at least one detector clicks, etc. The measurement of A_i corresponds to a transformation on the optical modes from the input configuration $(0,0,1)^T$ to $c_i(\cos(4\pi i/5), \sin(4\pi i/5), \sqrt{\cos(\pi/5)})^T$ (c_i is the normalization coefficient) [34]. All the relevant twofold, threefold, and fourfold coincidences between the three APDs and the trigger signal are registered using a field-programmable gate array (FPGA) to give the joint probabilities in $\langle A_i A_{i+1} \rangle$ [34]. We record clicks of each context in 150 series, at 1.4 s each, to acquire the expectation value and standard uncertainty.

The data are registered with respect to the measurement event, i.e., a successful trial of the experiment, which can be defined with the aid of ancillary outcomes and the input trigger. For one single-photon input, ideally, one and only one detector can "click," neglecting dark counts, and thus provides a well-defined "measurement event." In contrast, when the coherent light field, which is considered to be classical, is input to the measurement device, there is a non-negligible probability for more than one detector to click due to the multiphoton component within the coherent state $|\alpha\rangle$. Moreover, there is a probability that no detector clicks even with the presence of an input state, due to the inherent vacuum component. This fact leads to an inconclusive definition of the measurement event for classical inputs if we only take the measurement outcomes without referring to the input. In the following analyses, we consider three conditions to define the measurement event: (i) E_1, in which only a single detector clicks; (ii) E_2, in which at least one detector clicks; and (iii) E_3, in which the full click statistics are registered, i.e., no postselection on detectors clicking or not clicking. Generally speaking, $E_1 \subset E_2 \subset E_3$. For a perfect single-photon input without any loss, $E_1 = E_2 = E_3$, and thus the measurement event is defined unambiguously.

In the experiment, the presence of single photons is heralded by the detection of photons in the trigger detector D_T. However, inevitable experimental imperfections (in particular, optical losses and finite detection efficiencies) result in a reduced heralding efficiency and add the vacuum component to the setup, rendering the statistics of measurement outcomes different from those with the perfect single-photon input. Because we are interested in the contextuality within different input states, we resort to a fair sampling assumption in the sense that the lost photons would have the same behavior as the registered photons. This assumption essentially removes the additional vacuum component and restores the result with perfect single photons, resulting in $E_1 \approx E_2 = E_3$ in our experiment. Then, a measurement event happens when at least one of the three detectors clicks conditioned on the trigger detector registering photons. By collecting statistics under such a definition, we obtained the KCBS value of $\beta = -3.9176(60)$, showing a clear violation of the KCBS inequality and confirming that the indivisible single-photon system is undoubtedly contextual. The results without fair sampling, which take into account the vacuum component caused by losses as part of the state, are given in the Supplemental Material [34]. In this case, the three definitions of the measurement event do not necessarily give the same result ($E_1 \approx E_2 \neq E_3$ for our source), and a heralding efficiency above 89.65% is required for an unambiguous violation of the KCBS inequality.

For the situation with the classical light field as the input, a fair sampling cannot be applied in the same way because it is hard to distinguish between the vacuum component inherent to the input state and those that appear due to loss. Yet, given that the coherent state remains a coherent state after loss, we can alternatively take all the vacuum components as part of the state. In this way, the losses are lumped together in the state preparation and backed out from the measurement devices. The experimental results for the classical light field are shown in Fig. 3(a), in which $\bar{n} = |\alpha|^2$ represents the average photon number per pulse of the coherent state. The violation of the NC inequality under E_1 is similar to that of single photons, which reaches the quantum bound. Note that, due to the saturation of the APD, the violations can even be stronger than the quantum bound [34]. E_2 takes a simultaneous response of different detectors into consideration, but it still filters out the

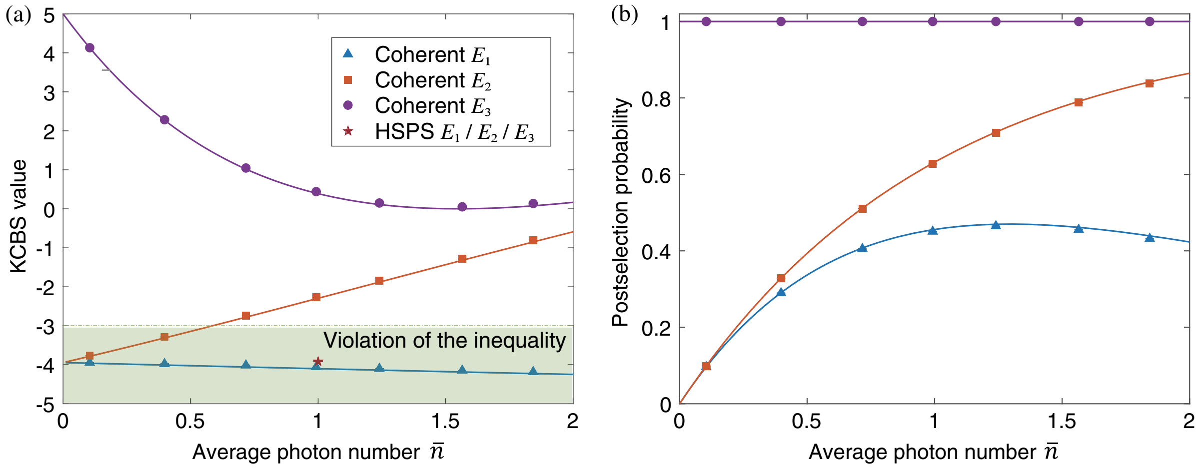

FIG. 3. (a) Experimental KCBS values (markers) and theoretical predictions (solid lines) for coherent light fields under different measurement events E_j: E_1 (triangle, blue), E_2 (square, red), and E_3 (dot, purple). The experimental result for a heralded single-photon source (HSPS) with fair sampling (pentagram, red) is also plotted for comparison. (b) Postselection probabilities for coherent light fields under different E_j. The error bars are too small [34] to see in the two figures.

contribution of the vacuum component, which does not fire the detectors. As a result, a significant violation is observed when the zero-click probability is considerable; and the violation disappears with the increase of $\bar{n}$ because the zero-click probability gets negligible. For E_3, we record all events with respect to the repetition rate of the input state. Under this condition, coherent states cannot violate the KCBS inequality. The experimental results are in agreement with the theoretical predictions [34]. The result for the heralded single photons with fair sampling is also shown in the figure for comparison. It can be seen that the results with coherent states are in strong contrast to those with single photons, where E_1, E_2, and E_3 give nearly the same violation of the inequality.

The definition of the measurement event is essential for testing contextuality with classical systems. Actually, we assign conditional probabilities $P(A_i = a_i, A_{i+1} = a_{i+1}|E_j)$ under E_j. Because, in general, $E_1 \subset E_2 \subset E_3$, E_1 and E_2 would register events conditionally, resulting in postselection in measurement outcomes. With such postselection, the probabilities of events are observed and renormalized in a subspace of the complete outcomes. The postselection probability under E_j can be calculated by

$$P(E_j) = \frac{P(\{A_i = a_i, A_{i+1} = a_{i+1}\} \cap E_j)}{P(\{A_i = a_i, A_{i+1} = a_{i+1}\}|E_j)}, \quad \forall \ a_i, a_{i+1}. \tag{3}$$

Figure 3(b) demonstrates the postselection probabilities $P(E_j)$ for different E_j [34]. E_1 always violates the inequality, but maintains a small postselection probability. The value of β under E_2 increases with the increase of the postselection probability $P(E_2)$, whereas $P(E_2)$ approaches one at large $\bar{n}$ due to the vacuum component of the input field becoming negligible. These results demonstrate the quantum-to-classical transition in the contextuality test, from single photons to coherent light fields. For the full statistics E_3, which poses no postselection, we observe no violation of the KCBS inequality.

Discussion.—The violation of the NC inequality is determined by the joint photon number distributions before the detectors in our setup. According to Eq. (2), the violation is related to the correlation function of the outcomes i and $i+1$ defined by

$$g_{i,i+1} = \frac{P(A_i = -1, A_{i+1} = -1)}{P(A_i = -1)P(A_{i+1} = -1)}. \tag{4}$$

This quantity is similar to the second order correlation function $g^{(2)}$ often used to assess the nonclassicality of the light field. For a linear-optical network with a coherent light field as the input, the output state is uncorrelated between different modes [38,39], i.e., $g_{i,i+1} = 1$. It follows that $\langle A_i A_{i+1}\rangle = [1-2P(A_i=-1)][1-2P(A_{i+1}=-1)] = \langle A_i\rangle\langle A_{i+1}\rangle$, and thus the KCBS inequality cannot be violated. Moreover, if the input field is not perfectly coherent (e.g., the thermal light), the output modes are classically correlated [40] but the result still does not violate the NC inequality [34]. Violating the KCBS inequality requires at least $g_{i,i+1} < 1$, implying nonclassical correlations of outcomes that put the requirement on input states. To elucidate this requirement, we start with the Glauber–Sudarshan P representation of the input optical field $\hat{\rho} = \int d^2\alpha P(\alpha)|\alpha\rangle\langle\alpha|$ [41,42]. Then, the value of β can be represented as

$$\beta_{E_j} = \int d^2\alpha P(\alpha)\beta(\alpha|E_j), \tag{5}$$

where $\beta(\alpha|E_j)$ denotes the value of β for the coherent state $|\alpha\rangle$ under E_j. If one aims at testing the contextuality of the input system, the measurement event should be correspondingly defined in accordance with the input (E_3) instead of the help of ancillary outcomes (see Fig. 1). For our setup $\beta(\alpha|E_3) = 5[1 - 2\exp(-|\alpha|^2/\sqrt{5})]^2 \geq 0$ [34], we have

$$\beta_{E_3} = 5\int d^2\alpha P(\alpha)[1 - 2\exp(-|\alpha|^2/\sqrt{5})]^2, \quad (6)$$

where $P(\alpha)$ uniquely determines the photon number distribution [34]. $\beta_{E_3} < -3$ requires $P(\alpha)$ to be negative for certain α or more singular than the Dirac-delta function. In this sense, violation of the KCBS inequality without postselection provides a sufficient criteria for the nonclassicality of the input optical state. As an example, we provide a numerical simulation on the violation of the KCBS inequality with different optical states (see Fig. S4 of the Supplemental Material [34]).

On the other hand, the measurement event definition procedure in our experiment can be understood as a postselection on certain components of the input optical state. For example, E_1 effectively selects the single-photon component when the intensity is low, whereas E_2 removes the vacuum component of the state [43]. Such postselection can be understood as the time reversal between input and measured states [44], and it can be confirmed by comparing β_{E_1} and β_{E_2} of coherent states with β_{E_3} of the postselected components, respectively [34]. Both the postselected states possess nonclassical superposition in the P function, allowing a possible violation of the KCBS inequality. Indeed, the postselected correlation $g_{i,i+1}$ for E_1 always equals zero, whereas $g_{i,i+1} = 1 - \exp(-|\alpha|^2) < 1$ for E_2. From this perspective, the violation of the inequality should be attributed to the postselected nonclassical components, which are nondeterministically generated with a probability of $P(E_j) < 1$. In particular, for the coherent state input, $P(E_1) = \exp(-2|\alpha|^2/\sqrt{5}) + 2\exp[-(\sqrt{5}-1)|\alpha|^2/\sqrt{5}] - 3\exp(-|\alpha|^2)$ and $P(E_2) = 1 - \exp(-|\alpha|^2)$. Accordingly, the resource for this nonclassical behavior should be carefully evaluated [45]. One can demonstrate quantum (even beyond quantum) contextual statistics with only classical inputs but at a cost of more resources (i.e., more copies of states) due to the reduced postselection probability. The relation between this resource and the memory cost needed to demonstrate contextuality [46,47] is an interesting issue to be further investigated.

Conclusion.—We show how the definition of measurement events affects the emergence of the violation of the KCBS inequality for both single photons and coherent states with a linear-optical setup. It is shown that the violation of the noncontextual inequality requires nonclassicality in the quasidistribution of the optical states in phase space. Quantum correlations can emerge by filtering the nonclassical component of the optical field through an appropriate definition of measurement events, but at a cost of reduced postselection probability. Our results shed new light on the boundary between the quantum and classical domains, and they foreshadow a new means of resource evaluation for classical simulation of quantum contextuality.

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