Aerodynamic and thermal characterization of turbocharger turbines: experimental and computational evaluation

by

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I declare that the work in this thesis is my own and relevant citations are included to acknowledge
the work of others

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Alessandro Romagnoli
“Non puo’ esistere separazione definitiva
finche’ esiste il ricordo”
ABSTRACT

Turbochargers are widely used in both passenger and commercial vehicle applications to increase power density, improved fuel economy leading to significant emissions reductions. In recent years, car manufacturers have introduced turbochargers widely in the diesel market in response to the stricter regulations in exhaust emissions. Although investment in turbocharger technology has made it possible to overcome issues related to reliability and cost, research is much needed in the area of design, testing methodologies and model development. This is particularly the case when considering unsteady flow effects.

Computational codes are used by engine manufacturers to predict its performance and size components; prediction accuracy is crucial in this process. This thesis contributes to this process in several ways: steady modelling and heat transfer predictions. Furthermore, most aero-thermal design and analysis codes need data for validation; often the data available falls outside the range of conditions the engine experiences in reality leading to the need to interpolate and extrapolate excessively. The current work also contributes to this area by providing extensive experimental data in a large range of conditions. A further contribution of this work is the understanding of the turbine performance under pulsating flow; it shows that this performance deviates from the commonly used quasi-steady assumption in turbocharger/engine matching. A turbocharger is subjected to high temperature conditions; heat transfer within the turbine and the compressor severely affects the compressor performance at low rotational speeds and mass flow rates. Compressor maps provided by turbocharger manufacturers do not usually take into account the effects of heat transfer; this causes a mismatch when fitting the maps into engine codes which is detrimental to the overall engine performance prediction.

The experimental investigation was conducted on three different turbine designs for an automotive turbocharger. The design progression was based on a commercial nozzleless unit modified into a variable geometry single as well as a twin-entry turbine configuration. The main geometrical parameters of these turbines were kept constant to allow equivalent performance assessment. The mixed-flow rotor used in this study consists of 12 blades with a constant inlet blade angle of +20°, a cone angle of 50° and a tip diameter of 95.2mm. The variable geometry stator consists of 15 vanes fitted into a ring mechanism, capable of pivoting in the range of 40° and 80° (with reference to the radial direction). The design progression into twin-entry turbine was completed by fitting a divider (accounting for only ≈6% of the overall internal volume) within volute. The turbine response for different vane angles (40° to 70°) and mass flow ratios between the two entries of the turbine was
assessed. The turbine was tested under steady and pulsating flow conditions for two rotational speeds, 27.9 rev/s·√K and 43.0 rev/s·√K, a velocity ratio (U2/Cₐ) of 0.3 - 1.1 and a pulse frequency of 40 - 80Hz under both in-phase and out-of-phase conditions.

A meanline aerodynamic model capable of predicting the performance parameters was developed for the nozzleless and the variable geometry single-entry turbine. The former was validated against experimental results spanning an equivalent speed range of 27.9 rev/s·√K and 53.8 rev/s·√K while the latter validated against one single speed (43.0 rev/s·√K) and three different vane angle settings (40°, 60° and 70°). The wide range of tests data from the Imperial College High Speed Dynamometer enabled the evaluation of the model in areas of the maps where currently no data exists. Based on the model prediction, a breakdown aerodynamic loss analysis was performed. As for the twin-entry turbine, the interaction between the two entries was investigated. Based on experimental evidence, a map-based method was developed to uniquely correlate the flow capacity within the entries for both partial and unequal admission.

An investigation into the effects of heat transfer on a turbocharger was performed using a commercial turbocharger mounted on a 2.0 litre diesel engine. The global objective of these tests was to improve the understanding of heat transfer in turbochargers under realistic engine conditions. Measurements were obtained for engine speeds between 1000 and 3000 rpm at a step of 500 rpm – for each engine speed the load applied was varied from 16 to 250 Nm. In addition to the standard set of measurements needed to define the turbo operating point, the turbocharger was equipped with 17 thermocouples positioned in different locations in order to quantify the temperatures of the components constituting the turbocharger. A simplified 1-D heat transfer model was also developed and compared with experimental measurements. The algorithms calculate the heat transferred through the turbocharger, from the hot to the cold end by means of lump capacitances. The compressor performance deterioration from the adiabatic map was then predicted and based on the data generated by the model a multiple regression analysis was developed in order to assess the main parameters affecting the compressor non-adiabatic performance.
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NOMENCLATURE

ENGLISH

A  Area  [m$^2$]
a  Acoustic velocity  [m/s]
b  Radial chord length  [m]
C  Absolute velocity  [m/s]
c_p  Specific heat of air  [J/kg·K]
D  Diameter  [m]
E  Voltage  [V]
h  Enthalpy, height  [J],[m]
L  Length  [m]
k  Thermal conductivity  [W/mK]
K  Loss coefficient
M  Mach number
M_u  Peripheral Mach number
m  Mass flow rate  [kg/s]
N  Rotational speed  [rev/s]
P  Pressure  [Pa]
q  Specific amount of heat transfer  [J/kg]
Q  Heat flux  [J/s]
R  Specific gas constant  [J/kg/K]
r  Radius  [m]
S  Entropy, Shaft  [J/K]
T  Temperature  [K]
t  Thickness  [m]
U  Blade speed  [m/s]
u  Velocity  [m/s]
x  Axial direction, Explanatory variable
W  Power  [W]
W  Relative flow velocity  [m/s]
y  Direction normal to the turbine axis
Z  Blade number

GREEK

α  Absolute flow angle, Angular acceleration  [Degree, Rad/s$^2$]
β  Relative flow angle  [Degree]
γ  Ratio of specific heats
ε  Emissivity
η  Efficiency
λ  Work input
μ  Dynamic viscosity, slip factor  [Pas]
ν  Kinematic viscosity  [St]
ξ  Heat number, Kinetic energy loss coefficient
ρ  Density  [kg/m$^3$]
σ  Stefan Boltzmann constant, Entropy gain  [m$^{-2}$·kg/(K·s$^2$)]
τ  Torque  [Nm]
Φ  Flow coefficient
Ψ  Azimuthal angle
θ  Tangential component
**SUBSCRIPTS**

adi  Adiabatic
amb  Ambient
act  Actual
after State after compression or expansion
ax  Axial
b  Blade
before State before compression or expansion
bs  Back swept
C  Compressor
calib  Calibrated
const  Constant
conv  Convection
cl  Clearance
dia  Non-adiabatic
E  Isenthalpic process
eff  Effective
exh  Exhaust
fc  Forced convection
fluc  Fluctuating
free flow Limb free to flow (no pressure set)
G  Number of joint cylinders
i  Inner
Id  Ideal
inc  Incidence
inl  Inlet
inner  Inner limb entry
inst  Instantaneous
is  Isentropic
m  Meridional component
meas  Measured
MeaPl  Measurement Plane
N  Nozzle
o  Outer
outer  Outer limb entry
opt  Optimum
orif  Orifice plate
p  Passage
phshift  Phase shifted
PL  Pressure loss
r  Radial
rad  Radiation
rot  Rotor
s  Static
surf  Surface
T  Turbine
tot  Total
ts  Total to static
un  Unequal

**NUMBERS**

0  Total condition
1  Inlet to the turbine
2  Inlet to the rotor
3 Station upstream exit to the rotor
4 Exit to the turbine

**Chapter 5 only**
0 Total condition
1 Inlet to the compressor
1* Inlet to the impeller
2 Exit to the compressor
2* Exit to the compressor impeller
3 Inlet to the turbine
3* Inlet to the rotor
4* Exit to the rotor
4 Exit to the turbine

**Abbreviations**

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>Nu</td>
<td>Nusselt number</td>
</tr>
<tr>
<td>CTA</td>
<td>Constant temperature anemometer</td>
</tr>
<tr>
<td>BH</td>
<td>Bearing housing</td>
</tr>
<tr>
<td>Gr</td>
<td>Grashof number</td>
</tr>
<tr>
<td>Pr</td>
<td>Prandtl number</td>
</tr>
<tr>
<td>BP</td>
<td>Back plate</td>
</tr>
<tr>
<td>BH</td>
<td>Bearing housing</td>
</tr>
<tr>
<td>Ta</td>
<td>Taylor number</td>
</tr>
<tr>
<td>Re</td>
<td>Reynolds number</td>
</tr>
<tr>
<td>TP</td>
<td>Temperature parameter</td>
</tr>
<tr>
<td>BHL</td>
<td>Bearing housing length</td>
</tr>
<tr>
<td>ER_u</td>
<td>Unequal expansion ratio</td>
</tr>
<tr>
<td>MFP_u</td>
<td>Mass flow parameter ratio</td>
</tr>
<tr>
<td>MFP</td>
<td>Mass flow parameter</td>
</tr>
<tr>
<td>QS</td>
<td>Quasi-steady</td>
</tr>
</tbody>
</table>

\[
\frac{hL}{k} = \frac{\beta g (T_{\infty} - T_o) L^3 \rho^2}{\mu^2}
\]

\[
\frac{\mu c_p}{k} = \frac{\rho \omega \sqrt{r_1 (r_o - r_1) 1.5}}{\mu}
\]

\[
\frac{u L \rho}{\mu} = \frac{p R_{un}}{p R_{umb}}
\]

\[
\frac{MFP_{un}}{MFP_{umb}} = \frac{\sqrt{T_{01}}}{P_{01}}
\]
1.1 Perspective

To meet the global greenhouse gas emission reduction targets it is necessary to reduce the emissions from the transport sector, the second largest emitter. The dependence of the transport sector on oil results in a significant challenge of how to reduce greenhouse gas without decreasing human mobility. Land transportation is responsible for 11% of global greenhouse gas emissions and a number of changes to current methods can offer a reduction of greenhouse gas emissions. In the short term, turbo-charging and downsizing in combination with weight reduction of current vehicle technology provide the best compromise towards a reduction of greenhouse gas emissions. Improved internal combustion engines drive trains such as hybrid electric vehicles and aggressive downsizing could significantly reduce emissions. However hybrid electric vehicles only unfold their full potential when they are operated in the environment they were designed for, routes with dense traffic such as an urban area. When the main use pattern is long distance travel in areas with light traffic, these cars are at a disadvantage. Here, a sensible choice is an efficient turbocharged gasoline or diesel drive trains in downsized vehicles. Turbocharging is already proven to help smaller engines deliver the performance of larger ones but with the added advantage of improvements in fuel consumption of up to 40% in diesel engines and up to 20% in gasoline engines. Additionally turbocharged vehicles are considerably cheaper than hybrid-electric vehicles and their manufacturing is less emission intensive.

Overall, the current status of technology seems to suggest that road transport will continue to rely on the internal combustion engine in an optimised classic (turbocharged) set-up. Electric vehicles will remain a small percentage of the overall fleet composition up to the medium- to long-term. When combined with a drastic downscale of both vehicle size and weight, a significant reduction of emissions from road transport could be achieved, especially in combination with low carbon fuels.\(^1\)

\(^1\) The information of this paragraph have been gathered from the “Future of mobility road map”, edited by University of Oxford, 2010.
1.2 Supercharging

The principal aim of supercharging an internal combustion engine is to improve the power density. Supercharging can be defined as the introduction of air (or air/fuel mixture) into an engine cylinder at a density greater than ambient. In doing this, a greater quantity of fuel can be burned in one engine cycle with a consequent rise in the power output. Although the cycle efficiency is not improved by supercharging, the overall engine efficiency may benefit as the friction losses remain constant while the power output rises. Supercharging is distinguished in mainly two different methods, mechanical supercharging and turbocharging.

Mechanical supercharging systems (or positive displacement supercharger) deliver an almost constant mass flow to the cylinder at any pressure. This means that the mass flow rate of the air moved depends on the speed of operation and on the volume of the initial chamber while it is relatively independent of pressure ratio. Most are driven by an accessory belt, which wraps around a pulley that is connected to a drive gear. In addition to the frictional losses the power required to the drive the compressor must be debited from the power output of the engine. This is not the case in turbochargers. In fact, unlike superchargers (which draw their power directly from the crankshaft) turbochargers use the exhaust gases generated by combustion to power the compressor. By doing this it is no longer necessary to debit the power requirement from that of the engine and the energy of the exhaust gases which would be wasted is then recovered. Given the aim of the current research to provide a contribution towards the understanding and the improvement of energy recovery systems, turbochargers will be treated more in detail.

1.2.1 Turbochargers

Turbocharging is the most common way of supercharging an internal combustion engine since turbochargers are smaller in size, lighter and cheaper than a mechanical supercharging device. The working scheme of a turbocharger is shown in Figs. 1.1 and 1.2. As the exhaust gases quickly move out of the cylinder and flow into the exhaust manifold, they are directed into the turbine housing’s scroll. As the gases try to find an airflow path through the turbine housing, they come in contact with the turbine wheel on their way to the centre outlet of the housing. As they flow through this airflow path and into the exhaust down pipe, they spin the turbine wheel, imparting a portion of their kinetic energy to the turbocharger. By the connecting shaft the power gained in the expansion process is transferred to the compressor which compresses the incoming air which will be squeezed into the engine cylinders.

Although the increase in power is advantageous to the car, a turbocharger has its drawbacks. Firstly, a turbocharged engine must have a lower compression ratio than a normally aspirated engine; a lower compression ratio means the engine will run less efficiently at low power. Another drawback of a turbocharger is the phenomenon known as turbo lag. Because the turbocharger runs on exhaust
INTRODUCTION

The turbine requires a build-up of exhaust before it can power the compressor. Due to rotational inertia of the shaft and turbine and compressor wheels, the turbocharger cannot deliver a high boost pressure instantaneously when speeding up the engine; this means that the engine must gain speed before the turbocharger can start to operate. Depending on the size of the turbocharger it can take up to a second to speed up the turbocharger and produce significant boost pressure. Another drawback occurs when the mass flow rate through the compressor is progressively reduced, at constant impeller speed, as the overall pressure ratio increases until the surge condition is reached. Surge is an operating point at which the forward flow through the compressor can no longer be maintained, due to an increase in pressure across the compressor, and a momentary flow reversal occurs. At this point the compressor becomes unstable and this can damage or even destroy the compressor wheel. On the other hand, when the mass flow rate through the compressor or turbine is progressively increased, the so called choking condition can be reached. This occurs when the sonic condition in a certain cross-section in the turbine or in the compressor is met, and no further increase of mass flow is possible.

Figure 1.1: Schematic principle of a turbocharged engine

Figure 1.2: Inside the turbocharger (Holset)
1.3 Turbocharger components

A typical turbocharger design is shown in Fig. 1.3. The main components of a turbocharger are the compressor, the bearing housing and the turbine.

In turbocharger applications centrifugal compressors are used. The main advantage of centrifugal compressors is that, due to the high-speed operation, they can move a lot of air relatively to their physical size, which makes them easier to package in an engine compartment. Additionally a small number of moving parts reduces manufacturing costs and enhances durability. Unfortunately, while lag is not an issue, dynamic characteristics remain a problem. The mass flow rate of a centrifugal compressor is proportional to the square of the compressor rotational speed. This means that boost rises nonlinearly with rpm, and power is biased strongly towards the top end. Due to this power extraction issue, the overall engine efficiency decreases as the cycle efficiency remains unchanged.

The bearing housing connects both the compressor and the turbine side. Inside the bearing housing is the main shaft. The main shaft undergoes a great amount of pressure, rotational speed and heat originated from the bearings and from the presence of the engine and the turbine housing. The most common and basic bearing housings use ‘thrust bearings’ to keep the shaft spinning, and oil flow from the engine to both lubricate and cool the unit. However ‘ball bearings’ units and water cooling are becoming more common and affordable. The main advantage of the ‘ball bearings’ is in the improved durability and in the more efficient at transmitting power to the compressor wheel, making it better for performance and longevity. The water cooling is more for reliability than anything else, helping to stabilize temperatures and prevent oil coking in the housing.

![Figure 1.3: Typical turbocharger design (Shaaban, 2004)](image-url)
The last component constituting turbocharger is the turbine which embeds the turbine housing and the turbine wheel. The turbine housing can be either be single or multiple-entry and include nozzle vanes, either fixed or movable. The turbine wheel is usually a radial even though in the last few years mixed-flow turbines started to be increasingly more appealing to turbocharger manufacturers.

### 1.3.1 Turbine Casing

The function of the volute consists in converting the energy of the engine exhaust gases into kinetic energy and in accelerating the flow towards the rotor inlet at an appropriate flow angle. The design for better performance of the spiral housing volute used commonly in radial and mixed-flow turbine is of prime importance as it affects the stage performance. The turbine casing can be divided into two main categories defined as single and multiple-entry. The former is the simplest solution and couples compactness and good performance together with lower manufacturing costs. However, since all the turbochargers operate under conditions of unsteady flow and as in multi-cylinder engines the separate pulses tend to overlap, the latter is often required. In particular, in large cylinders engines, the exhaust manifold is usually divided into two units, and each of these is connected to an individual entry to the turbine. These are generally adjusted in a way that in each section, the pressure pulsations are out-of-phase. This enables to avoid the overlapping of pulses and consequently to avoid that the total mass flow through the turbine never falls to zero. Prior to any consideration on multiple-entry turbines, a fundamental distinction must be made between multiple-entry turbines depending on the method of flow division (Fig. 1.4):

- **Circumferentially divided turbine**: this configuration corresponds to a double-entry turbine where the scroll is divided such that each entry feed a separate section of the rotor;
- **Meridionally divided turbine**: this configuration is usually referred as twin-entry turbine. The scroll has a single divider around the entire perimeter of the housing.

In a double-entry turbine each entry feeds a separate section of the rotor. The advantage of this design is that there is little interaction between the flows leaving the two entries thus reducing the losses due to mixing effects. In addition to this a circumferentially divided turbine also allows for more than two entries which makes this configuration suitable for large multi cylinder engines. In a twin-entry turbine instead, the flow interaction between the entries is significant; this leads to a penalty in efficiency due to flow mixing. The problem arises in the interspace between entries 11 and 12 to the rotor, as shown in Fig. 1.5 since the velocity and pressure field are highly non uniform due the pulsating nature of the engine operation and to the phase shift of the pressure waves from either of the turbine entries.
Previous studies conducted by Pishinger and Wunsche (1977) on twin and double-entry turbines showed that, under steady state condition, the double entry turbine can achieve higher efficiency than the twin-entry\(^2\). However as we move far from the peak efficiency region, the double-entry turbine exhibits a significant deterioration in efficiency in respect the twin-entry. Hence it is hard to establish which configuration is better performing since both turbines own weaknesses which make the choice of a turbine dependent only on the purpose of the application. Nevertheless twin-entry turbines seem to be preferred to double-entry turbines given their simple and cost effective design.

### 1.3.2 Turbine Wheel

Radial inflow turbines are the most common in turbochargers. However large engines can also use axial and mixed-flow turbines. The main advantages of radial/mixed-flow turbines in comparison to axial flow turbines can be found in a lower manufacturing cost since the radial/mixed-flow turbines are a single piece casting and in a more appropriate flow characteristic as the flow does not have to pass through many stages. However, a distinction between radial and mixed-flow turbines must also be made.

\(^2\) More details are provided in Chapter 2
In order to provide a good engine-turbocharger match a well performing turbine would have to have a relatively flat efficiency and mass flow rate curves. This would allow a better utilization of the available exhaust gas energy and it is for this reason that mixed-flow turbines seem to provide a better response than a radial turbine. Mixed-flow turbines have gained attention in the last years due to higher flow capacity and their capability to achieve peak efficiencies at lower velocity ratio compared to a radial turbine. A mixed-flow turbine has an inclined leading edge which can accept flows with both axial and radial component. The zero blade angle limitation typical of radial turbine is overcome but still maintaining structural stability with radial fibers. In fact, for both a radial and mixed-flow turbine at any position along the blade, by following a path radially towards the central bore, the material follows a radial direction. This is sought in order to avoid bending stresses due to the high centrifugal forces imposed on the blades during their rotation. However, this results in geometric inflexibility and for a radial turbine dictates that the blade inlet angle is zero, as shown on the right hand side of Fig. 1.6. This geometric limitation sets the optimum velocity ratio of the turbine to be equal to approximately 1/√2. This is not the case in mixed-flow turbines since the radially swept leading edge, besides providing structural stability due to its radial fibre, also provides an extra degree of freedom to optimize turbine performance. A positive blade angle lowers the optimum velocity ratio at the peak efficiency thus increasing the optimum pressure.

![Figure 1.6: Schematic description of radial and mixed-flow turbine](Karamanis, 2000)

The velocity ratio is a non-dimensional parameter defined as the ratio between the blade tip speed ($U$) and the isentropic velocity ($C_{is}$). The isentropic velocity corresponds to the velocity which would be attained to the flow if it could be brought isentropically to rest. The expression for the velocity ratio is given in Eq. (1.1).

$$\frac{U}{C_{is}} = \frac{U}{\sqrt{\frac{2yRT_{0,init}}{\gamma - 1}\left[1 - \left(\frac{P_{exit}}{P_{0,init}}\right)^{\frac{\gamma - 1}{\gamma}}\right]}} = \sqrt{\frac{\eta_{ts}}{2\psi}}$$ (1.1)
\[
\psi = \frac{1}{1 - \left( \frac{\tan \beta_{\text{blade}}}{\tan \alpha_{\text{flow}}} \right)}
\]  

where \( \psi \) is the blade loading factor which is proportional to the ratio between the inlet absolute flow angle (\( \alpha_{\text{flow}} \)) and the leading edge of the wheel (\( \beta_{\text{blade}} \)). From Eq. (1.1) and Eq. (1.2) it can be inferred that a positive blade angle leads to a lower value of the optimum velocity ratio thus increasing the optimum pressure ratio. This is particularly beneficial in pulsating flow conditions since most of the energy available from the exhaust gases is held within the high pressure pulse. In fact, in the exhaust manifold, high pressure pulses correspond with high values of instantaneous mass flow. Therefore an increase in the swallowing capacity leads to a better energy extraction out of the exhaust gases and hence to an improved turbine efficiency.

### 1.3.3 Variable geometry turbine

Variable geometry turbines are of particular interest to advanced diesel powertrains, since they can improve system transient response to changes in speed and load. In fact turbocharged engines are usually slow to respond to load or demand speed changes which cause of *turbo lag*, previously outlined. This issue can be minimized with the development of methods to actively control the turbine geometry, namely variable geometry turbines.

[Figure 1.7: Variable geometry turbine: pivoting vanes]

The most common variable geometry turbines are those constituted by pivoting vanes even though other methods are currently available (refer to Chapter 2). The working principle is shown in Fig. 1.7. Each of the vane angles is able to change throughout the engine's revolutions per minute range to optimize the overall efficiency of the turbines. As the vane angles increase with an actuator, in correlation with the engine’s increasing rpm, the exhaust flow is better directed onto the turbine blades. Controlling the vane angle allows the exhaust flow gases, at low engine speeds, to pass over narrow, almost closed vanes. Gases accelerate as they move through the narrow passage towards the turbine blades, which in turn accelerates the turbine blades.
Besides minimizing the *turbo lag*, variable geometry turbines enable the improvement of the turbine performance in off-design conditions since the stator throat area is adjusted according to the mass flow rate. In a turbine, the rotor incidence angle is a function of the stator exit flow angle which in turn is controlled by the stator flow area. The departure of the flow angle away from the optimum determines incidence loss and thereby a drop in efficiency. Variable geometry turbines seem to provide an adequate response to this issue.

### 1.4 Heat transfer

Usually most processes in turbo machinery applications are considered as adiabatic since the influence of heat transfer in the performance calculation is often negligible. In turbocharger applications, the performance is usually assessed in adiabatic conditions. Adiabatic performance occurs in the absence of heat transfer between the components of the turbocharger and such a condition can be achieved in the laboratory by supplying the turbine with compressed air in place of the hot gases. The maps generated with this type of tests are usually referred as *cold maps*. However this is not always correct, since in some cases heat transfer can have an influence on performance. The high temperature of the exhaust gases entering turbine makes the turbochargers operate under non-adiabatic conditions. Due to the close proximity of the compressor to the turbine casing, heat from the turbine will be transferred to the compressor by mean of convection, radiation and conduction. All of these three processes occur at the same time and are interrelated.

![Figure 1.8: Heat fluxes in a turbocharger (Shaaban, 2004)](image)

The complex turbocharger geometry introduces many possible heat transfer paths inside the turbocharger as well as from the turbocharger to surroundings. The high temperature gradient existing between the turbine and the other components of the turbocharger and also between the turbine and
the surroundings is the main cause of heat transfer phenomenon. In particular heat transfer between the components of the turbocharger as well as between the turbocharger and the surroundings can be classified into (Fig. 1.8):

- heat transfer from the hot turbine to the lubrication oil by means of forced convection in the clearance between shaft and bearing;
- heat transfer from the turbine to the compressor through the bearing housing even if the cooling oil reduces the amount of heat that is transferred by conduction from the turbine to the compressor;
- heat transfer from the turbine to the surroundings by means of radiation and free convection;
- heat transfer from the compressor to the surroundings by means of radiation and free convection\(^3\).

The heat transfer from the turbine to the compressor causes a temperature rise of the air leaving the compressor and therefore it results in an underestimation of the compressor isentropic efficiency\(^4\). This should be taken into account when fitting turbocharger maps into engine software since they heavily rely on turbocharger maps as boundary conditions.

### 1.5 Research motivation

Nowadays turbochargers technology can count on several years of research and technical improvements. Common problems like reliability, costs and manufacturing issues have been tackled with good success thus leading to a product with a well consolidated market. However, the stringent emission regulations coupled with the continuous higher demand of turbocharger performance (above all away from design conditions) make it necessary to evaluate the turbocharger performance in regions of the maps where little or no data are currently available. Aerodynamic and heat transfer investigation are the two areas where most of research in turbochargers is currently focusing on and it is within these two areas that the current work is based.

**Aerodynamics:** the need for engines to work more and more in off-design conditions requires the evaluation of turbine performance in the extreme regions of the efficiency maps. Nevertheless the lack of experimental data in these regions usually constrains software developers to rely excessively on map extrapolation. Although this is common practice, it leads to the detriment of overall engine performance. In fact in a turbocharger turbine, the impact of losses far from the design condition is substantial and this must be carefully taken into account if a good matching to

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\(^3\) However the effects of radiation are very small because of the low emissivity of the compressor casing and the low difference temperature between the casing and the ambient.

\(^4\) More details are given in Chapter 5.
the engine wants to be achieved. A great contribution in this direction is provided by turbine modelling; meanline models are amongst the most powerful tools thanks to their immediate and simple application. However the effectiveness of these models can only be proven in the range of data used for validation; such a range is usually narrow which makes the prediction of these models subject to high degrees of uncertainty when applied outside this range. This is an issue still open which needs to be addressed. Similar considerations can be made on pulse flow performance. Despite several studies have been conducted in this direction, the complexity of testing procedures together with complexity of flow mechanisms limits the understanding of turbine performance under pulsating flow. Most of the studies and measurements primarily concern the validation of the quasi-steady assumption but still with no appreciable results on the design process. Therefore the need to extend the investigations to a wider range of turbine settings is much needed in order to acquire a deeper understanding of the flow conditions and hence of its performance. For the purpose of the current thesis, twin-entry variable geometry turbine was investigated. No data are currently available for such a turbine configuration which makes its study a mandatory step to fulfil a gap in current research.

Heat transfer: the effects of heat transfer on turbocharger performance are usually neglected. However experimental investigations have proven that heat transfer has a large impact on the deterioration of turbocharger efficiency. In fact turbocharger maps are usually generated with more convenient and easier adiabatic testing. As a consequence of this a proper understanding of turbocharger performance under non-adiabatic conditions still has to be gained and this leads to poor performance prediction of engine simulation programs which rely heavily on adiabatic maps. Calibration of engine models is altered by an incorrect evaluation of turbocharger performance which causes a high degree of uncertainty when applied to the real engine. Therefore the demand for methods able to quantify the effects of heat transfer on turbocharger performance is high. Implementation of turbocharger models is certainly the most feasible way even though the complexity of heat transfer mechanisms requires following different approaches (network models, statistical analysis). A contribution in this direction is proposed in the current thesis with an experimental and computational investigation.

**Thesis Objectives**

The main objectives of this thesis are as follows:

1. To assess the correlation existing between the turbine performance under full, partial and unequal admission. Interaction between turbine entries is analysed.
2. To evaluate the capability of meanline models to accurately predict turbine efficiency in those regions of the maps where no data is currently available.

3. To evaluate the performance of a variable geometry twin-entry mixed flow turbine in comparison with equivalent geometry single entry turbines. The variable geometry and the second gas inlet introduce two extra degrees of freedom that need to be fully tested in order to cover all the states that the turbine might experience. Using a wide selection of performance tests, steady-state (under full, partial and unequal admission) and unsteady-state (in-phase and out-of-phase) conditions need to be analysed in order to investigate the influence of turbine configuration on performance.

4. To quantify the effects of heat transfer on a turbocharger under non-adiabatic conditions. The presence of the engine and the heat from the turbine to the compressor need to be evaluated in order to quantify the penalty in efficiency.

5. To assess the possibility to treat turbocharger performance under non-adiabatic conditions by means of statistical analysis. The possibility to correlate the adiabatic and non-adiabatic performance parameters with multiple regression analysis is investigated.

**Thesis outline**

The current thesis is divided into 7 chapters followed by references and appendices. The description of each chapter is given below:

**Chapter 1: Introduction**

A general overview on turbochargers technology is provided in this chapter. The working principles and the features of each component constituting turbochargers are outlined together with a description of the issues related to heat transfer. This chapter also includes the objectives and the motivation of the current thesis.

**Chapter 2: Literature review**

This chapter provides a discussion of the published literature that is most pertinent to the current work. The topics include: mixed-flow turbines, variable geometry turbines, twin-entry turbines and heat transfer on turbochargers.
Chapter 3: Experimental facility
In this chapter, an outline of the test-rig with all the associated instruments is given. A detailed description of the measuring techniques (including the procedures to derive steady and unsteady performance of a turbine) is discussed. Finally an assessment of the uncertainty analysis adopted in the current thesis is also given.

Chapter 4: Steady state performance
This chapter contains the results of the analysis conducted on steady state performance. Firstly an evaluation on the performance for the three turbine configurations (nozzleless single-entry and variable geometry single and twin-entry) is done on an equivalent geometry basis. Then, a discussion of the analysis carried out on the twin-entry turbine is provided: a map-based method to predict the mass flow parameter under unequal and partial admission is proposed and validated against the experimental results. Finally an evaluation of the prediction of a meanline model developed for a nozzleless and variable geometry single-entry turbine is reported in this chapter. The validation with experimental results is discussed.

Chapter 5: Heat transfer analysis on turbocharger performance
This chapter discusses the results of a computational and experimental evaluation of heat transfer on a commercial turbocharger. The experimental results are reviewed and the non-adiabatic efficiency is calculated. Then a discussion on the prediction of a 1-D for the turbocharger under non-adiabatic conditions is provided and, based on the model prediction, a statistical analysis of the turbocharger performance is performed. The outcomes of the analysis are evaluated on the basis of experimental evidence.

Chapter 6: Unsteady state performance
This chapter discusses the outcomes of the experimental analysis conducted on a variable geometry twin-entry mixed-flow turbine under pulsating flow conditions. The turbine performance is assessed for different frequencies, speeds and vane angles under in-phase and out-of-phase flow conditions; based on the experimental results an evaluation on the quasi-steady assumption is carried out. Finally a comparison between the performance of the twin-entry turbine and an equivalent geometry single-entry is preformed for different vane and flow conditions.

Chapter 7: Conclusions
In the final chapter are discussed the unique findings of the thesis together with recommendations for future research.
CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

As already reported in the introductory section, the aim of this thesis is to contribute to the characterization of turbochargers performance in those areas where partial or no research is currently available.

The aerodynamic investigation looked into the steady and unsteady evaluation of the efficiency of a twin-entry variable geometry mixed-flow turbine. Although a number of researches on twin-entry turbines have been proposed over the years, these have been mainly focusing on twin-entry nozzleless radial turbines. Furthermore the application of mixed-flow turbines is still limited to few cases, even though mixed-flow turbines have been proven to provide high efficiency over a wider range of velocity ratios than the radial counterpart. Variable geometry turbocharger has a similar history, where earlier applications suffered the lack of technological knowledge which caused it to be highly unreliable and not cost effective. However the findings obtained in the last few decades in different technical areas (in particular the control strategies and materials) made it possible for the wide application of variable geometry turbines. However the use of variable geometry twin-entry turbines is still insufficient. Based on this, the following discussion will provide a review of the past research conducted on mixed-flow, variable geometry and twin-entry turbines. The three groups will be treated separately and for each of these a distinction between experimental and computational analysis (either steady or unsteady) will be made where possible.

Additionally as a part of the current research the effects of heat transfer on turbocharger performance have also been investigated. Research under non-adiabatic conditions represents a very small fraction of all the works conducted on turbochargers. The role of heat transfer on turbochargers has only given attention in the last decade with solid and significant research. Thus, the amount of literature available in this area is limited to few works which will be extensively reviewed in this chapter.
2.2 Mixed Flow Turbines

The pulsating nature of the exhaust gases in an internal combustion engine requires a turbocharger to operate better at higher pressure ratios which means better energy extraction at the peak of the pulses. A mixed-flow turbine seems to accomplish these requirements and for these reasons the interest towards this design concept has been rising in the last years.

Figure 2.1: a - Velocity ratio variation with absolute ($\alpha$) and inlet relative flow angle ($\beta_4$), (Rajoo, 2007); b - Efficiency comparison of an axial, radial and mixed-flow turbine (Whitfield and Baines, 1990)

The first studies on a mixed-flow turbine were described by Hamerick et al. (1950) and Stewart (1951) who showed some improvements in the flow field when compared to a radial counterpart. Wallace and Pasha (1972) addressed their studies on mixed-flow turbines particularly in the automotive field providing the first parameterization for the design procedure. The pulsating nature of the exhaust flow in an internal combustion engine requires a turbocharger turbine to operate efficiently at higher pressure ratio conditions which leads to better energy extraction. A radial turbine was found to be less flexible than a mixed-flow turbine in the lower velocity ratio region (corresponding to higher pressure of the pulse) due to the radial inlet requirement. In a radial turbine the peak efficiency point can be achieved for velocity ratio of about 0.7. This is not the case in a mixed-flow turbine which enables to achieve the peak efficiency point for lower velocities, refer to Fig. 2.1-a. Watson and Janota (1982) and Whitfield and Baines (1990) superimposed the performance characteristics of an axial, radial and mixed-flow turbine, as shown in Fig. 2.1-b. From this figure it is evident that both radial and mixed-flow turbines are suitable for automotive applications because of their high efficiency at a range of pressure ratios. Additionally the mixed-flow turbine also shows the benefit of lower velocity ratios in comparison with the radial turbine. On the design side, the first parametric study of a mixed-flow turbine was done by Wallace and Pasha (1972). They succeeded in improving the flow capacity of the mixed-flow turbine by 25% by modifying a radial turbine with swept leading edge and a mean cone angle of 45° (Fig. 2.2-a). The improvement was found to be more significant in the high pressure regions where the radial turbines usually show a
dramatic drop in the flow capacity. Further studies made by Baines et al. (1979) and Abidat et al. (1992), continued the work done by Wallace and Pasha (1972) including loss factors and blade incidence effects to the mixed-flow design. They succeeded in showing analytically and experimentally an improved flow capacity, at various pressure ratios, of the mixed-flow turbine in comparison to the radial counterpart (Fig. 2.2-b).

Figure 2.2: a- Improvement in mass flow for a mixed-flow turbine at various pressure ratios (Wallace and Pasha, 1972)

b- Mixed-flow turbine efficiency (Baines et al., 1979)

The main efforts to fit a mixed-flow turbine into an internal combustion engine were made by Yamaguchi et al. (1984), Chou and Gibbs (1989), Nagub (1986), and Minegashi (1995), who tried to exploit the higher flow capacity in order to reduce the turbine size and consequently minimize the turbo lag. Yamaguchi et al. (1984) designed a mixed-flow turbine tested under steady state conditions and experimentally showed an increase in efficiency in respect to a radial counterpart. However the mixed-flow turbine developed by Yamaguchi et al. (1984) was aimed at big engines.

Figure 2.3: Comparison in performance of a diesel engine equipped with radial and mixed-flow turbine (Chou and Gibbs, 1989)
Ikeya et al. (1992) developed a small mixed-flow turbine for passenger vehicles. The tests conducted on the newly designed mixed-flow turbine showed an increase of 14% in the flow capacity and 8% in efficiency in respect to a radial counterpart. In the early 90’s Chen and Baines (1992) introduced a design optimization methodology for mixed-flow turbines. The design method concentrated on loading factor and exit loss. The optimization method was based on the reduction of losses by reducing the velocity components at the inlet to the rotor and assuming a positive exit swirl angle at the exit. As such this method was found to reduce the overall turbine losses at the expense of the reduced specific power.

**Experimental investigation**

From an experimental point of view, the first investigations on a mixed-flow turbine were reported by Wallace and Pasha (1972). These tests were carried out in a cold test stand facility and they were usually run under steady state conditions. The cold condition is in many ways not representative of the actual condition occurring on a turbocharger mounted on a real engine, for which very hot pulsating gasses are experienced. The technical limitations at that time, did not allow of doing any better. Nevertheless the work done by Wallace and Pasha (1972) was relevant since it
demonstrated that in a mixed-flow turbine higher peak efficiency can be achieved at lower velocity ratio in respect to a radial turbine. Baines et al. (1979) and Abidat et al. (1992) did similar tests in order to validate their optimization methodology for mixed-flow turbine design. The test results are shown in Fig. 2.2-b where it can be seen that a good agreement was found between the theoretical and experimental efficiency. Given the proven capability of a mixed-flow turbine to perform better than a radial turbine, research took a step further to optimize the mixed-flow design. Abidat et al. (1992), and Chen et al. (1992, 1997), designed and tested a series of rotors to assess the effects of incidence and exit flow on the mixed-flow turbine performance. The optimization process resulted in the improvement of the turbine efficiency. The promising results obtained by stand-alone testing had yet to be demonstrated by an adequate benefit in engine performance. Chou and Gibb (1989) installed a mixed-flow turbine on a diesel engine and proved an improvement in engine power and specific fuel consumption, as shown in Fig. 2.3. The advantage of a mixed-flow turbine is noticeable especially in the lower speed region, which is one of the critical improvement factors sought by engine and turbocharger manufacturers. This is due to the improvement of the pressure ratio and flow capacity with a mixed-flow turbine, which leads to a better turbocharger boost as shown in Fig. 2.4. (Minegashi et al. 1995). Furthermore, the lower inertia of a mixed-flow rotor enables better turbocharger response (Minegashi et al., 1995) and the desired performance is achieved at lower specific speed (Naguib, 1986).

It is evident that investigation under steady state conditions is crucial in design-phase, however the advancement of the computational capacity soon enabled the prediction of the unsteady effects in a turbine flow, which requires more challenging experimental results. In fact a turbocharger turbine, when installed in an engine, constantly operates under unsteady conditions due to the pulsating nature of the exhaust gases exiting the valve of mechanism. Measurements in unsteady state require higher response rates of the measuring devices and, although unsteady state testing have been documented since the 60s (Wallace and Blair, 1965, and Benson and Scrimshaw, 1965), it was not until the 90s that the first experimental works on mixed-flow turbines were reported (Arcoumanis et al., 1995, and Szymko et al., 2005). Experiments conducted by Winterbone et al., (1991), and Baines et al. (1994) on a radial turbine showed that the unsteady flow capacity and efficiency of the turbine produced a hysteresis loop around the steady state profile. A similar trend was also observed by Arcoumanis et al. (1995), Karamanis and Martinez-Botas (2002) and Szymko et al. (2005) on a mixed-flow turbine. The mass flow looping behaviour can be attributed to the filling and emptying effects due to flow pulsation and the finite volume of the turbine stage. Szymko et al. (2005) measured the instantaneous turbine efficiency for a mixed-flow turbine over an entire pulse cycle. This is shown in Fig. 2.5 and it can be observed that the efficiency in some instances is higher than unity. Szymko et al. (2005) attributed this to the inertial effect of the turbine rotor, which causes its rotation to continue even when the mass flow rate is low. In addition to this it must be noticed that the unsteady efficiency is also affected by the phase shifting methodology which can lead to unrealistic
values. In fact one of the main difficulties when testing in unsteady conditions is the time difference between the measured isentropic conditions (at the volute entry) and the actual turbine output (at the inlet to the rotor). The different measuring locations require all the measurements to be phase shifted in order to relate the actual and isentropic conditions at the same instant. Many methods have been proposed for phase shifting - the sonic velocity (Dale and Watson, 1986), the bulk flow velocity (Baines et al., 1994) or a combination of both. Arcoumanis et al. (1995) and Karamanis and Martinez-Botas (2002), employed sonic velocity phase shift, and recorded instances of efficiency higher than unity in all pulsating frequencies. Arcoumanis et al. (1999) employed both the sonic and bulk flow velocity phase shift and also recorded instances of efficiency higher than unity in both cases. Szymko (2005) showed that good agreement can be obtained by assuming the pulse travelling at a speed equal to the sum of the sonic and bulk flow velocities. He recorded efficiencies higher than unity only at the higher frequency cases (60Hz and 80Hz) which indicates a better choice of phase shifting method. As the trend of the performance parameter under unsteady conditions is not useful in practical terms, it is common practice to compare the unsteady performance to a quasi-steady one. The latter assumes that at any instance in time during pulsating flow the turbine behaves in an identical manner to the steady flow, thus the instantaneous turbine efficiency can be obtained from the steady curve with reference to the equivalent velocity ratio. Arcoumanis et al. (1995), showed the quasi-steady efficiency to deviate significantly from the steady assumption hence questioning the credibility of using a quasi-steady method in the design of a turbine. Szymko (2005) obtained some better agreement proposing a new method aimed to relate the quasi-steady and unsteady performance with an isentropic power averaging technique. It introduces the isentropic power as a weighting factor in the turbine efficiency and velocity ratio evaluation in an unsteady cycle; a better correlation was achieved with the quasi-steady performance.

**Computational analysis**

The earliest computational studies done on a mixed-flow turbine were based on steady state conditions since the unsteady conditions required computational efforts and hardware equipments which were not available in the past. However, the importance of understanding the unsteady flow pattern became inevitable after Wallace (1971) introduced the possibility of using a mixed-flow turbine in automotive turbochargers. In order to optimize the computational resources, Baines et al. (1979) treated the unsteady conditions as a summation of steady points and carried out a 3-D quasi-steady flow analysis aiming to describe the flow pattern through the blades. This approach led to good results that were further developed with Euler code (Chou and Gibbs, 1989, and Okapu, 1987) to complex Navier-Stokes methods (Kim and Civinskas, 1994, Kirtley et al., 1993, and Palfreyman and Martinez-Botas, 2004). As such the computational analysis enabled the capture of flow separation in the suction surface of the turbine rotor. The combination of the reducing radius with the pressure difference between the rotor surfaces generates the Coriolis force in the rotor flow channel. This
induces a counter flow opposite to the direction of rotation of the rotor which moves the separation from pressure to suction surface as the incidence angle goes from high negative to zero and to positive. Computational analysis done by Okapu (1987), Chou and Gibbs (1989), Kim and Civinskas (1994), found that the optimum incidence lies in between \(-20^\circ\) and \(-40^\circ\).

The work done by Kirtley et al. (1993) showed that the large radial velocities of a mixed-flow turbine generate a Coriolis force larger than an axial turbine. Kirtley et al. (1993) found the accumulation of high loss flow in the shroud suction-side and the flow pattern of a mixed-flow turbine was predicted to be more similar to radial turbine even though the lack of experimental data did not enable to validate the computational analysis. This problem was solved by Kim and Civinskas (1994), who carried out a similar work to Kirtley et al. (1993), with the addition of experimental investigation. The test results showed some significant difference between the mixed-flow and radial turbine; the flow angle and the exit pressure agreed better with the computational analysis and showed discrepancies with design intent (based on radial turbines). Similarly to Kirtley et al. (1993), and Kim and Civinskas (1994), Palfreyman and Martinez-Botas (2004) simulated the flow field of a mixed-flow turbine with commercial CFD software. They found that the interaction between the tip clearance flow and the swept flow from the relative motion caused a highly disturbed flow region associated with secondary flow. This was in agreement with Kirtley et al. (1993) and Kim and Civinskas (1994) (refer to Fig. 2.6).

The studies on unsteady flow for mixed-flow turbines were not conducted until the 90s. Chen et al. (1996) modelled the flow in a volute and a mixed-flow rotor using a modified 1-D code originally developed for a radial turbine (Chen and Winterbone, 1990). The rotor was considered to behave in a quasi-steady manner and the prediction agreed well with experimental results. By mean of the Runge-Kutta scheme, Abidat et al. (1998) managed to reproduce the hysteresis loop observed experimentally. Abidat et al. (1998) found that the Runge-Kutta scheme to be more accurate than the method of characteristics used by Chen et al. (1996). The latest computational work on unsteady
analysis was conducted by Costall et al. (2006), who developed a bespoke 1-D code (called ONDAS) for mixed-flow turbines. Central to the use of ONDAS there is a transmissive and APL (adiabatic pressure loss) rotor boundary conditions which permit appropriate 1-D wave action simulations. The implementation of these two elements of the code proved to be capable of predicting the instantaneous turbine mass flow with good results at low frequencies while the prediction was found to deteriorate at higher frequencies.

It is worth noting that the turbine design procedure for an automotive turbocharger is still primarily confined to the steady state consideration due to the huge computational resources and design time needed for a full unsteady analysis. In addition to this, the need for engine to work under off-design conditions makes the importance of steady state models still crucial to the overall design process. The more complex 1-D codes still rely on steady state maps for accurate calibration. Meanline models are usually adopted to generate turbine maps under steady state conditions. These models can be considered zero dimensional since the flow path is considered to go through a single streamline. Despite their apparent simplicity, an extensive research has been carried out over the years on meanline models. Given that a part of the current thesis is looking into the analysis of steady state models, it is worth to provide a short overview of the work done so far in this direction.

**Meanline models.** The first studies toward mean line models were made in NASA by Futral and Wasserbauer (1965), Wasserbauer and Glassman (1975), and later on by Meitner and Glassman (1983) who developed steady state models for nozzled radial turbines. The main structure of their studies is still valid even though, the limited number of equations describing the effects of losses inevitably produces poor performance prediction. The model proposed by Futral and Wasserbauer (1965) was a mean kinetic energy loss which was considered to occur through the rotor blades, taking into account all of the dissipative effects. Although this approach provided a good qualitative understanding of the phenomena, it needed to be developed since it tend to predict too low a value for the mass flow in the region of the rotor choke. An attempt made by Glassman and Meitner (1983) proposed a model where the disk friction, clearance and vane-less space losses were added to the previous loss model. Furthermore, a more detailed analysis of the stator and trailing edge flow pattern was introduced by considering the state of the flow just upstream and downstream the inlet and exit to the rotor. By applying the conservation of the angular momentum and of the mass flow rate into the model, a more accurate prediction of the performance could be obtained. However, at high pressure ratios the calculated performance was still deviating from the actual one. In 1998, Abidat et al. proposed a method to predict the performance in a mixed-flow turbine under both steady and unsteady conditions. Several losses affecting the path of the flow as the blading loss and the skin friction loss were taken into account. Similar to the other model, the code proposed by Abidat et al. (1998) was based on solving the one-dimensional fluid dynamic equations at a series of key stations along the turbine path. Although at 100% speed the model seemed to show a good prediction of peak efficiency,
the validation against experimental data had to rely on a narrow range of available test data. Finally Baines (2007) proposed a method for radial turbines taking into account the throat area at the inlet to the rotor. The main assumption here is that when the pressure ratio is adequate to choke the blade passage, the mass flow rate is controlled directly by the throat area. However, the geometric throat area tends to over predict the flow capacity showing that the effective flow area is reduced caused by the aerodynamic blockage. This problem was recognized to exist but no longer faced in the previous models. Baines proposed to satisfy the continuity equation within the plane passage by adjusting the deviation flow angle; in this way he could take into account the effects of blockage. Although this approach seems to provide a good performance prediction, it provided a poor prediction of the exit flow angle in comparison to the experimental values. A key aspect to consider in all of the computational work conducted in the past is that they had to be validated against data obtained in conventional test facilities using a compressor as a loading device. However, such a way of testing only enables data in a very limited range of velocity ratios, between 0.6 and 0.75 at 100% speed. The limitations are caused by the compressor surge and choke. The permanent magnet eddy current dynamometer in Imperial College test-rig is capable of testing the turbine over a range of velocity ratio of 0.37 to 1.1. Such a range is well beyond other testing methodologies commonly used for turbochargers and enlarges the maps available to the researcher by about three fold. The mean line model described in this thesis is validated over a wide range of velocity ratios thus reducing the extrapolation of the maps.

Turbine loss modelling plays an important role in a mean line analysis. During the years, an improved understanding of the fluid dynamics and thermodynamic laws of turbo machines together with an improvement in the test methods, made possible to achieve a better understanding of the flow pattern. However it was not since the second half of the 20th century that the first studies on performance prediction methods were made. Howell (1945), Ainley (1948) and Ainley and Mathieson (1951) categorized the turbine losses as profile loss, secondary loss and tip leakage loss. These models were based on experimental test data and the physics behind this was not fully understood. The methods used provided very little accuracy and each manufacturer tended to tune the loss coefficients to obtain agreement with existing machines and extrapolate the performance maps to be adapted in the design of new machines. During the 60s and the 70s the loss models development continued at NASA with Futral and Wasserbauer (1965), Benson (1970) and Denton (1987) looking into the physics of the loss mechanisms. It was only in the 80s and 90s with the development of new test techniques such as laser anemometry and hot wires, that the flow field could be investigated showing that the flow pattern is highly unstable and three-dimensional. This also made clear that a high degree of accuracy in the analysis could be achieved only with more complex methods such as CFD techniques. As a result of these developments, the designers and researchers assimilated an improved understanding of the flow but most practical performance predictions continued to be based on correlations. Loss correlations are usually categorized in three main groups (Moustapha et al., 2003): gross correlations,
correlated coefficients and fundamental coefficients. It is within these groups that the loss models rely on, depending on what degree of accuracy need to be achieved.

![Variable geometry nozzleless casing with open and close wall settings](image1.png)

Figure 2.7: Variable geometry nozzleless casing with open and close wall settings (Chapple et al., 1980)

![Volute exit and tongue area control methods with relevant engine performance](image2.png)

Figure 2.8: Volute exit and tongue area control methods with relevant engine performance (Flaxington and Szczupak, 1982)

2.3 Variable geometry turbines (VGT)

The first research on variable geometry turbines was conducted by Chapple et al. (1980), who performed an aerodynamic investigation into designing series of radial turbine volutes. These included a variable geometry nozzleless casing where the area available to the flow was varied by mean of a moving wall in order to create fluid passage with consistent energy and momentum conversion. The experimental investigation was limited only to two wall settings (open and closed) as
shown in Fig. 2.7. Only a partial benefit in efficiency was measured for low mass flow rates even though the work done by Chapple et al. (1980), mainly focused on establishing a systematic design approach for a radial turbine casing. In 1986 Wallace et al. presented a series of engine testing with a variable geometry turbocharger to establish its benefits. The improvement in engine transient response was accompanied by a penalty in efficiency. This was attributed by the lack of an optimal control strategy for the variable geometry turbine. Further investigation by Flaxington and Szczupak (1982) was done on variable geometry controlling devices (exit volute area and volute tongue area) showed an improvement over the engine performance (Fig. 2.8). The test results showed an improvement in engine torque and transient performance.

Flaxington and Szczupak (1982) also suggested that amongst the different control strategy for the VGT, namely boost control (engine speed control and optimum full/part load control), no one proved to be superior for all application. Each of these devices exhibits benefits in certain operating conditions and therefore an accurate analysis should be made prior the selection of control device. In the same year, Watson and Janota (1982) reported a work showing the improvement in brake power, specific fuel consumption and reduced emission in respect to a fixed geometry turbine (Fig. 2.9). Nevertheless they also stated that one of the main drawbacks for VGT devices is their cost and reliability which are not corresponded by an adequate benefit in efficiency. Walsham (1990) conducted similar research to Flaxington and Szczupak (1982) who compared the benefits in transient torque for a waste-gated turbine, variable geometry turbine and turbo compounding. Walsham (1990) suggested technically that the variable geometry turbines are better for transient response as shown in Fig. 2.10, as well as the transient emission. Nevertheless, Walsham (1990) demonstrated significant efficiency drop in the VGT at closed positions. In addition, the complexity of the VGT system is another factor which needs to be weighed upon for an engine application.
Figure 2.10: Transient response of an engine with alternative turbocharging techniques (Walsham, 1990)

Hawley et al. (1999) equipped a direct injection diesel engine with pivoting nozzle vane turbocharger turbine. An improvement up to 45% in the NOx emission and 10% in the limiting torque was achieved in comparison with a standard fixed geometry turbocharger (Fig. 2.11). However the VGT position was controlled manually which made the validity of the results limited to research applications. Capobianco and Gambarotta (1992) presented an aerodynamic investigation under steady and pulsating flow conditions on variable geometry turbines, namely variable area turbine and variable nozzle turbine. The test results showed that the latter was marginally better than the former. Nevertheless the main features of the variable geometry turbines like wider operating range and peak efficiency drop were maintained. Additionally the variable geometry turbine efficiency was concluded to be better than the equivalent fixed geometry for the overall operating range. So far only two methods proved to be reliable and since used in most VG turbochargers; these are the variable area through moving wall and the variable nozzle through pivoting methods. However different methods were proposed by Kawaguchi et al. (1999) and Pesiridis and Martinez-Botas (2007). Kawaguchi et al. (1999) proposed a technique called VFT (Variable Flow Turbocharger) which consists of an additional scroll outside the normal fixed normal scroll. The working principle is shown in Fig. 2.12 for low and high-flow rate. A control valve is used to control the air-flow into the outer scroll within the nozzle vane ring placed in the intermediate region between both scrolls. The test results showed an improvement of 10kPa boost pressure in respect to an equivalent VGT under similar inlet conditions. In more recent years, Pesiridis and Martinez-Botas (2007) developed a new method called ACT (Active flow Control Turbocharger) for actively regulating the nozzle vane ring in order to adapt to the exhaust gas pulsation. This is suggested as an advance technique where a variable geometry turbine operation extended to consider the pulsating nature of the inlet exhaust flow. Marginal power improvement was documented but with efficiency penalty largely caused by the aerodynamically poor sliding nozzle employed.
2.4 Twin-entry turbines

Multiple-entry turbines are usually adopted to preserve the flow features of the multiple engine exhaust pipes. This is the case for turbochargers of multi-cylinder engines where the turbine often works under off-design conditions. The use of multiple-entry turbines hence gives the advantage of using the exhaust pulse energy of gases and therefore turbine performance characteristics at such conditions are important. As already reported in Chapter 1, a fundamental distinction must be made depending on the method of flow division: meridionally divided (or twin-entry) and circumferentially...
divided (or double-entry), refer to Fig. 2.13. Both these turbine designs serve a similar purpose, however the twin-entry turbines are more appealing to turbine manufacturers due to their inexpensive and simple design. The following discussion will focus mainly on meridionally divided turbines as this is the subject of the current thesis.

![Diagram showing meridionally divided turbine and circumferentially divided turbine](image)

Figure 2.13: a - Meridionally divided turbine; b – Circumferentially divided turbine (Katrasnik, 2007)

![Graph showing twin vs. double-entry turbine](image)

Figure 2.14: Twin vs. double-entry turbine (Pischinger and Wunsche, 1977)

The very first studies on twin-entry turbines were done by Pischinger and and Wunsche, (1977) who performed a direct comparison between double entry and twin-entry turbines by retaining the same admission effective area ($A_s$). The results of their investigation are given in Fig. 2.14, showed that the efficiency loss under unequal admission is dependent on the volute geometry. For the high rate of unequal admission conditions, the twin-entry turbine seems to perform better than the double entry turbine even though the twin-entry turbine shows a penalty in the maximum efficiency achievable. As part of their research, Pischinger and Wunsche (1977) also measured the instantaneous
flow angles under unsteady conditions. The test results showed that the unsteady flow characteristics were similar to those measured under steady conditions at a similar mass flow. This led them to suggest that the fluid in the rotor can be considered as quasi-steady (such an assumption is supported by the observation that the pulse frequency is much lower than the rotor blade passing frequency).

Figure 2.15: Flow deviation from radial plane at full admission (Baines and Yeo, 1994)

Figure 2.16: Flow deviation from radial plane at extreme partial admission flow (Baines and Yeo, 1994)

Dale and Watson (1986) continued the work done by Pischinger and Wunsche (1977) by developing the Imperial College test facility to measure the efficiency for a twin-entry turbine. A series of tests was carried out in steady state conditions under partial and unequal admission. Most interestingly it was found that even though the turbine housing was symmetrical in axial direction and the measured mass flow characteristics for the two entries were almost coincident, their influence on the turbine efficiency was such that the peak efficiency point occurred when the mass flow of the shroud side entry was more than the hub side (not at full admission). In addition to this the minimum efficiency was obtained under partial admission conditions when the entry on the shroud side is fully closed. No explanation for this phenomenon was put forward by the authors, but in order to provide an answer to
this issue, Baines and Yeo (1990, 1994) directly measured the performance and the flow field of a vanless twin-entry radial turbine under full and partial admission conditions. The outcomes of their work showed that under equal admission conditions the flow angle is unaffected by changes in turbine operating conditions (Fig. 2.15). On the contrary they found that under unequal admission conditions the variation of flow velocity is much greater in spanwise direction. At extreme partial admission (one entry blanked off) a large evidence of flow recirculating from one limb to the other was observed with consequent large penalty in efficiency (Fig. 2.16). This would seem to indicate that a significant amount of unequal entry loss is attributable to incidence losses in the twin-entry flow housing. Steady and unsteady flow performance of a twin-entry automotive turbocharger turbine was also measured under full and partial admission by Capobianco and Gambarotta (1993). They found the two entries appeared to be significantly different, both in terms of mass flow rate and efficiency characteristics. Full and partial admission tests showed that flow capacity and efficiency were always higher for outer entry from the centre housing (shroud). They explained this dissimilar behaviour by taking into account the housing and rotor geometry, which showed an apparent asymmetry with reference to the meridional dividing plane. Highest efficiency was reached in partial admission conditions with very high values of mass flow ratio in between the two turbine entries. This was later confirmed by Aghaali and Hajilouy-Benisi (2007).

The first computational model on twin-entry turbines belongs to Benson (1982) who proposed to represent the two entries of the turbine as separate turbines. Due to the simplicity of the model, such an approach does not take into account the interaction of flows between different entries and it was found to be inappropriate for the modeling of the twin-entry radial turbines. Much of Benson’s work was later summarized by Winterbone and Pearson (1983). They proposed to introduce a short pipe between the manifolds in order to simulate the passage in the turbine casing. As Winterbone and Pearson (1983) confirmed, this was done in favour of better validation with experimental results than the exact representation of the fluid dynamics processes in the twin-entry turbine. Winterbone and Pearson (1983), also stated that a twin-entry turbine cannot be treated as a simple sum of two half turbines, because the flow through one entry is affected by the flow in the other. A more comprehensive approach for simulating meridionally divided twin-entry turbines was proposed by Hribernik (1994) and Baines et al. (1994). The model proposed by Hribernik (1994) is the most sophisticated, since it simulates one-dimensional flow through both turbine inlets and the interaction of both flows in the inter-space; it also considers relative rotor flow angle as well as its angular variation. Baines et al. (1994) described a turbine model for unsteady flow predictions in which the volute is represented as a volume between the pipe and the turbine entries. They showed that, the model which is effectively zero-dimensional, predicted some of the measured features of the unsteady flow; however they did not compare measured and predicted pressure traces. Katrasnik (2007) generated a code of the twin-entry turbine intended to be used as a boundary condition within an engine wave action simulation. Even though the model did not consider the pulsating nature of the
flow, the authors claim good agreement with experimental engine transient data. Ghasemi et al. (2005) proposed a meanline model that enables to replicate the steady state characteristics with some success. The work was based on the introduction of a loss coefficient reflecting the entropy generated as an effect of flow mixing. This work was further developed by Aghaali and Hajilouy-Benisi (2007) who complemented the previous work with an experimental and computational investigation on an unsymmetrical twin-entry turbine volute. The test results confirmed the findings of Dale and Watson (1986) and Capobianco and Gambarotta (1993) showing a strong difference in the mass flow rate and efficiency characteristics depending on the mass flow ratio between the entries. The developed code appeared to be able to capture the turbine performance under partial admission with good degree of approximation. The work started by Ghasemi et al. (2005) was finally completed by Shahosseini et al. (2008) who performed a full 3-D CFD model of a twin-entry turbine. The features of flow field within twin-entry turbine stage were modeled numerically at both full and extremes of partial admission conditions. The predicted absolute flow angle and the flow velocity at the turbine exit were found to be in agreement with five-hole probe measurements data. Numerical results showed that at under equal admission conditions, the flow is complex and varies in three-dimensional space, especially at volute tongue. Due to distortion near this region, the lowest entropy gain factor is obtained. One of the most complete and promising attempts at a fully unsteady 1-D, twin-entry turbine model is that of Costall et al. (2009). Their model was validated against steady and unsteady experimental results obtained at Imperial College. The outcomes of their simulation showed a very good agreement with experimental results when the flow in the turbine entries are in-phase to each other. However the prediction was found to deteriorate when it was applied in out-of-phase flow conditions. The mass flow and power from the out-of phase experiments were largely underestimated and the authors suggest that this is due to an incorrectly calibrated rotor loss coefficient attributed to insufficient steady, unequal and partial admission data.

2.5 Heat Transfer in Turbochargers

In the past the compression and expansion processes in a turbocharger were considered to be adiabatic. In reality there is certain degree of heat transfer within the turbocharger installed on an operating engine. This need to considered as correct values for the compressor and turbine efficiencies are needed in engine simulations to accurately predict the pressures and temperatures in the intake and exhaust manifold that are boundary conditions for the calculation of the engine cycle. Rautenberg et al. (1983) first analyzed the influence of heat transfer from the hot turbine to the compressor. They found that the additional heat transferred to the compressor rises the compressor outlet temperature to a higher value than it would be in an adiabatic compression process. The basic equations for a non adiabatic expansion and compression processes were later developed by Rautenberg and Krammer (1984). As the isentropic compressor efficiency is calculated by measuring the pressure and
temperature ratio between inlet and outlet, the higher outlet temperature results in underestimated compressor efficiency. The same approach cannot be used for a turbine since the high thermal inertia of the exhaust gases require longer travelling period between the measurements of different operating points as a steady state has to be reached (Jung et al., 2002). Shaaban and Seume (2006), argue that the high turbine temperature causes a large amount of heat transferred away from the turbine which results in a heavily deteriorated turbine outlet temperature and therefore a highly overestimated turbine efficiency. For this reason the turbine efficiency should be calculated from a power balance while, as the compressor is less affected by heat transfer compared to the turbine, this method results in more accurate results. The experimental results conducted by Jung et al. (2002), Shaaban (2004) and Cormerais et al. (2006) showed that the influence of heat transfers increases for smaller mass flows, Fig. 2.17.

![Figure 2.17: Effects of heat transfer on compressor performance for different rotational speed and total temperatures at the turbine entry (Shaaban, 2004)](image)

Beside the usual paths of heat transfer such as radiation, convection and conduction some heat is taken away by the lubrication oil. Therefore the estimation of the occurring forms and amounts of heat transfer becomes crucial. As a consequence, many approaches have been developed to calculate
the heat fluxes in a turbocharger. Jung et al. (2002) proposed a model that splits the temperature rise in the compressor in two parts. The first part consists of the temperature rise due to compression while the second part describes the influence of heat transfer. With this approach the overall compressor efficiency is split into two parts: one due to the aerodynamic efficiency and another due to heat transfer. Hagelstein et al. (2002) assumed that the heat transferred during the compression and expansion process can be neglected and does not affect the global result. Chapman et al. (2002) developed a FEA of a turbocharger to determine the heat fluxes going through the main bodies. The results of this analysis showed that the external heat transfer from the turbine is two orders of magnitude larger than that occurring in the compressor. Abdelhamid et al. (2003) measured the turbocharger performance at low rotational speeds, developing a method to predict the turbine and compressor performance in non-adiabatic conditions.

Bohn et al. (2003) and Heuer et al. (2005) carried out an experimental and computational analysis on a turbocharger at different operating points. Beyond the standard measurements to determine the main performance parameters, the surface temperature of the turbine and the compressor casings were measured. These results were set as boundary conditions for a numerical calculation. A parametric study was carried out for different turbine inlet temperatures and mass flow rates. The calculation used a one dimensional Nusselt number that enabled the prediction of heat transfer within the compressor. Although the heat transfer calculation through the proposed Nusselt number proved to be satisfactory for different operating conditions, the analysis did not lead to good agreement with experimental results when applied to different turbochargers. In order to get a good prediction, the Nusselt number had to be fitted with experimental results for each turbocharger. As Cormerais et al. (2006) report, the main problem is that the axial distance between the turbine and the compressor is not a parameter in the Nusselt correlation. Rautenberg et al. (1983) found that axial distance has a great influence on the heat transfer between turbine and compressor.

Shaaban (2004) derived an analytical solution for the temperature distribution in the bearing housing taking into account free convection to the ambient, forced convection to the lubrication oil and heat conduction in the axial direction. A correlation was used as boundary conditions on both sides of the bearing housing, which had to be fitted with experimental results. By applying this procedure it is possible to predict the total turbine outlet temperature within an error no greater than 2%. Cormerais et al. (2006) proposed a model based on the difference between exhaust and intake manifold temperature in the same way as Jung et al. (2002). However, instead of an exponential function for the factor of proportionality, they used the convective heat transfer coefficients of the turbine and compressor and the thermal conductivity of the bearing housing. The heat transfer coefficients are calculated from a Nusselt correlation proposed by Dittus and Boelter (1930) for flat plates of uniform thickness. However, unlike Bohn’s method, the model proposed by Cormerais does not need to be fitted with experimental results and hence is not limited to a specific turbocharger. For this reason the Cormerais’ model seems be the most promising compared to others. Baines et al.
(2009) proposed a heat transfer network model of a turbocharger based on tests conducted on three different turbochargers. A set of heat transfer coefficient values was found using convectional convective heat transfer correlations. These coefficients showed to be independent of the turbocharger model and the heat transfer prediction within the turbocharger could be performed with good approximation. The only exceptions were found to be in prediction of heat transfer by free convection and heat transfer to the oil.
CHAPTER 3

EXPERIMENTAL FACILITY

SYNOPSIS

This chapter describes the test facility available at Imperial College to conduct turbine performance experiments. The test cell is accommodated in the cell 180A of the Internal Combustion Engine Laboratory at the Mechanical Engineering Department. The core of the test rig is a novel eddy current dynamometer that extends the loading capability compared with conventional compressor based turbine tests stands. The test facility is designed to run experiments under steady and unsteady flow conditions replicating the exhaust pulsation of an engine operating at different operating points. The measured raw parameters will need to be post-processed to evaluate the turbine performance. The procedures for data analysis and the experimental setup will be described hereafter.
3.1 Dimensionless analysis

The parameters used to evaluate the turbine performance are the mass flow parameter and the total to static efficiency. By means of Buckingham π theorem, both the mass flow parameter and the total to static efficiency can be reduced to a set of non-dimensional parameters as given in Eq. (3.1) with reference to Glassman (1972) and Cohen et al., (1993). Watson and Janota (1982) also suggested the use of velocity ratio to the evaluate turbine performance. Velocity ratio is defined as the ratio between the rotor tip speed and the isentropic velocity. The isentropic velocity corresponds to the velocity that would be attained by the flow if it was to go through an ideal expansion at the measured pressure ratio.

\[
\text{mass flow parameter, } \frac{\dot{m} \cdot \sqrt{RT_0}}{D^2 \cdot P_{01}} = f\left(\frac{N \cdot D_4}{\sqrt{R \cdot T_0}}, \frac{P_{01}}{P_{exit}}, \frac{\dot{m}}{\mu D_{exit}}, \gamma\right)
\]

The independent parameters influencing the velocity ratio can be reduced to non-dimensional parameters as listed in Eq. (3.2).

\[
\text{velocity ratio, } \frac{U_{exit}}{C_{is}} = f\left(\frac{N \cdot D_4}{\sqrt{R \cdot T_0}}, \frac{P_{01}}{P_{exit}}, \gamma\right)
\]

As reported by Dale (1990) the effect of gas constant \(R\) is generally small while the effect of the specific heat ratio \(\gamma\) is included in the Reynolds number \((\dot{m}/\mu D_{exit})\). Furthermore, as suggested by Hiett and Johnston (1964) the Reynolds number is of secondary importance to turbine performance.

On the basis of these considerations, the parameters of Eqs. (3.1) and Eq. (3.2) further reduced into Eq. (3.3). The parameters in Eq. (3.3) are of the final form used to represent the turbine performance and will be used throughout this research. These will be reported in terms of mass flow parameter vs. pressure ratio and efficiency vs. velocity ratio.

\[
\text{mass flow parameter, } \frac{\dot{m} \cdot \sqrt{T_0}}{P_{01}} = f\left(\frac{N}{\sqrt{T_0}}, \frac{P_{01}}{P_{exit}}\right)
\]

\[
\text{velocity ratio, } \frac{U_{exit}}{C_{is}} = f\left(\frac{N}{\sqrt{T_0}}, \frac{P_{01}}{P_{exit}}\right)
\]

**Equivalent conditions.** The rig used for the current research is a cold flow test facility originally developed by Dale and Watson (1986). The cold test conditions established during testing are far from the actual conditions experienced by turbochargers in its normal engine operation. Glassman (1972)
developed a similarity approach of the performance parameters between the actual and test rig conditions. The equation for mass flow and speed correlating the test-rig and actual conditions are summarized in Eq. (3.4) and Eq. (3.5).

$$\left( \frac{m_0 \sqrt{T_{01}}}{P_{01}} \right)_{test-rig} = \left( \frac{m_0 \sqrt{T_{01}}}{P_{01}} \right)_{actual} \tag{3.4}$$

$$\left( \frac{N}{\sqrt{T_{01}}} \right)_{test-rig} = \left( \frac{N}{\sqrt{T_{01}}} \right)_{actual} \tag{3.5}$$

### 3.2 Test rig layout

The test facility used for the current research is a cold flow rig that enables the running of experiments under adiabatic conditions. A schematic representation of the test rig is given in Fig. 3.1 where the layout and the main components are shown. The test-rig is supplied by three Ingersoll Rand screw-type compressors, capable to deliver up to 1 kg/s compressed air at maximum pressure of 5 bars (absolute). The air is filtered through a three-stage cyclone. Downstream of the filter system, there are two motorised valves: one acts as a safety valve while the other is the main valve for regulating the mass flow rate of air into the turbine. The safety valve is connected to the ‘guillotine valve’ which shuts down the air supply in case of emergency. During testing the main valve is turned open and the cold air flows through a stack of heating elements, where its temperature is raised in order to avoid water vapour condensation due to the expansion process across the turbine stage. The temperature of the heaters is regulated by PID controller of the series West 4200 enabling the user to specify the flow temperature which will be maintained throughout the testing period.

Downstream of the heater stack, the airflow is branched into two 81.40-mm diameter pipes. These pipes are called ‘inner’ and ‘outer’ limb, referring to its relative position shown in Figure 3.1. Each of these limbs incorporates a 59.85mm diameter orifice plate for measuring the air mass flow rate according to the standard, BS 5167-1:1997. The two limbs stay independent of each other right to the ‘measurement plane’, before a connecting duct. The connecting duct can be changed according to the type of test conducted. For instance, when testing a twin-entry turbine, the connecting duct will have two passages in order to maintain two separate air passages into the turbine. Under unsteady state conditions, the flow goes through a rotary air pulse generator referred as ‘chopper plates’ (Fig. 3.1). The chopper plates are used to experimentally simulate the exhaust gas pulsation of an engine (the chopper plates have been designed by Dale and Watson, 1986). Two D.C stepper motors control the frequency of rotation which corresponds to the frequency of the pulsation. The entry pulses in both limbs can be set either to be in-phase or out-of-phase by shifting one of the two plates by 180° in
respect to the other. Under steady state conditions, the two plates are locked in the fully open position. Downstream of the pulse generator (at 800 mm) the warm airflow is monitored through an instrumented test-section, called the ‘measurement plane’. The ‘measurement plane’ is instrumented with high response pressure transducers, static pressure tapping, thermocouples and hotwire traversing system. After the ‘measurement plane’ the warm airflow goes through the connecting duct and into the turbine stage. The turbine is coupled to an eddy current permanent magnet dynamometer with maximum power absorption of 60 kW. The eddy current dynamometer is cooled with high flow rate of water, thus the absorbed energy is turn into heat and dissipated. The dynamometer is instrumented with a load cell and optical speed sensor for direct torque and speed measurement respectively. The details of each instruments and its associated analysis will be discussed in the section 3.3.

![Figure 3.1: Test rig layout](image)

### 3.3 Test-rig instrumentation

In order to evaluate the performance parameters of Eq. (3.3) the following measurements need to be taken during testing.

- *Air mass flow rate*

---

1 In the current study both in-phase and out-of-phase testing were carried out and hence the chopper plates had to be set up accordingly.
- Pressure
- Temperature
- Rotational speed
- Power

The location of each instrument used in the current tests is shown in Fig. 3.1. A detailed description of each instrument used here is provided hereafter.

3.3.1 Air Mass Flow Rate

Steady Flow Condition

The mass flow rate of air ($\dot{m}_{\text{air}}$) under steady state conditions was measured by mean of two sharp-edged orifice plates (two limbs), with a diameter of $\varnothing 59.85\text{mm}$. The measurement was done in accordance to the British Standards (BS 5167-1:1997) with $D$ and $D/2$ tappings. The mass flow rate was calculated with Eq. (3.6), where it involves iterations to determine the discharge coefficient ($C_d$).

$$\dot{m}_{\text{air}} = C_d E \varepsilon A_{orif} \sqrt{2\Delta P \rho_{s1}}$$ (3.6)

where $C_d$ is the discharge coefficient given by Stolz’s equation
$E$ is the velocity of approach factor
$\varepsilon$ is the expansion factor
$A_{orif}$ is the orifice plate area
$\Delta P$ is the pressure drop across the orifice plate
$\rho_{s1}$ is the static density of air upstream of the orifice plate

In order to evaluate Eq. (3.6), the direct measurements of the orifice upstream static temperature ($T_{s,orif}$), orifice upstream static pressure ($P_{s,orif}$) and the pressure drop across the orifice plate ($\Delta P$) were required. These measurements are discussed in the pressure and temperature sub-sections.

Calibration. A small amount of leakage exists within the test-rig, especially at the pulse generator and the hotwire traversing mechanism. This has to be evaluated and a leakage calibration has to be performed throughout. Calibration was conducted to measure the leakage mass flow rate at different pressure conditions in the limbs. This involved sealing the duct-end without the turbine attached and carefully pressurizing the duct. The duct was fed with compressed air until the pressure reaches the maximum value that would be experienced during testing. The valves were then shut and the temperature and pressure in the duct was continuously recorded until the compressed air was completely leaked. Given that the volume of the duct is known ($0.1712\text{m}^3$) the amount of air in the
duct at a known interval of recording time could be calculated and consequently the leakage deduced (Fig. 3.2-a). The measured mass flow rate during testing was corrected for the corresponding pressure in the duct.

**Pulsating Flow Condition**

The instantaneous mass flow rate measured under unsteady conditions was measured by a constant-temperature type hot-wire anemometer (CTA) located at the ‘measurement plane’, Fig. 3.1. The probe used for the hotwire is of platinum plated tungsten type with 10μm diameter and coupled to the StreamLine CTA system by Dantec Dynamic. The total resistance (sensor + lead + cable + support) is 2.129 Ω with over heat ratio of 0.85 and over temperature of 239°C. The response rate of the hotwire was indirectly measured with the ‘square-wave’ test and it yields 7.7 kHz.

A CTA setup is effectively a Wheatstone bridge, where the hotwire temperature is kept constant by appropriate voltage balance. A servo amplifier keeps the bridge in balance by controlling the current to the sensor so that the resistance, and hence temperature, is kept constant, independent of the cooling imposed by the fluid. The combination of the sensor's low thermal inertia and the high gain of the servo loop amplifier give a very fast response to fluctuations in the flow. Thus, the measuring principle of a CTA is that the voltage required to balance the electronic bridge is related to the velocity of the flow. The relation between the voltage and velocity is given by the King’s Law (Lomas, 1986 and Brunn, 1995) which correlates the Nusselt number (heat transfer dependant) and the Reynolds
number (flow velocity dependant) in an infinite cylinder of incompressible low Reynolds number flow, as given in Eq. (3.7) - \( a \) and \( b \) are two constants.

\[
Nu = a + bRe^{0.5}
\]  

(3.7)

In order to measure the through flow velocity and mass flow rate during testing, the hotwire is traversed in the plane perpendicular to the flow direction. The traversing procedure is in accordance to British Standards (BS1042, 1983), where 36 points measurements are achieved through an in-house built traversing mechanism.

**Calibration.** The unsteady mass flow measurement goes through two-stage calibrations. First, the hotwire CTA was calibrated through the calibrator unit of the Streamline system. This unit is able to set velocities from 0.02 m/s to more than 300 m/s and it is primarily designed to provide multi-points calibrations of standard wire probe. As for the current research, the hotwire CTA was calibrated in 15-points to establish the heat transfer function from the recorded voltage to the flow velocity and the mass flux. The calibration was conducted for air flow velocity between 3 m/s – 300 m/s at an air temperature of about \( \approx 24^\circ C \), thus the static temperature at the hotwire sensor varies with Mach number (the calibration curve of the CTA for the inner and outer limb is shown in Fig. 3.2-b). To compensate for this the flow unit was corrected for the averaged room temperature and it was used in the second stage as the reference temperature. The transfer function from the calibration points is established by curve fitting in accordance to *King’s Law* and given in Eq. (3.8).

\[
E^2 = A_{\text{calib}} + B_{\text{calib}}(\rho U)^n
\]  

(3.8)

where \( E \) is the CTA voltage (volts) \( A, B, n \) are the power law coefficients (curve-fit) \( \rho U \) is the mass flux (kg/m\(^2\).s)

The typical value for the calibration constant is 4.61, 2.06 and 0.46 for \( A_{\text{calib}}, B_{\text{calib}} \) and \( n \) respectively. In the second phase of the calibration process, the hotwire was traversed for 36 grid points in the ‘measurement plane’ and the voltage reading taken under steady flow for a range of mass flow rates that will be experienced by the turbine in the actual testing process. The voltage readings are converted into mass flux with Eq. (3.8) from the first stage. Consequently, the 36 points readings are integrated as specified in the standard *BS:1042* (1983). The integrated mass flux is then multiplied with duct area, \( A_{\text{meas,plane}} \) in order to calculate the instantaneous mass flow, as given in Eq. (3.9).

\[
\dot{m} = (\rho U)_i \cdot A_{\text{meas,plane}}
\]  

(3.9)
In pulsating flow conditions, the flow temperature is known to fluctuate over a wide range away from the calibration temperature of the hotwire (room temperature in our case). For this reason it is essential to conduct the second phase of calibration for the same temperatures that are experienced during actual testing conditions. The StreamLine CTA system was setup to run on constant over-heat ratio during both the calibration process and the actual testing. Given that the hotwire measurement is very sensitive to the temperature changes in the flow, the measured raw data of the hotwire were corrected for the temperature. The fluid properties which affect the hotwire reading are the Prandtl number \( (Pr) \), thermal conductivity \( (k) \), dynamic viscosity \( (\mu) \) density \( (\rho) \) and the Mach number \( (M) \). The effect of these properties was assessed and used to correct the calibration factors of the hotwire \( (A_{calib} \text{ and } B_{calib}) \) during unsteady testing.

![Figure 3.3: a - Orifice plate vs. CTA mass flow comparison; b - Procedure followed to derive the instantaneous mass flow (Rajoo, 2006)](image)

The corrected calibration factors \( (A_{corr} \text{ and } B_{corr}) \) were then used in Eq. (3.8) and Eq. (3.9) to solve for the mass flow rate. The mass flow rate measured with the traversing hotwire is compared to the orifice plate reading in order to establish a consistent transfer function for the whole range of conditions that would be experienced by the turbine in an actual testing. This involves iteration of matching process to determine the suitable value of temperature loading factor \( (m) \) in Eq. (6.13) and Eq. (6.14). For the current study, the \( m \) value of 2 was chosen, where all the hotwire readings fall within the orifice plate reading as shown in Fig. 3.3-a. The procedure followed to derive the instantaneous mass flow rate is summarized in the flow chart of Fig. 3.3-b. It is worth noting that based on the calibration points, the uncertainty in CTA measurement is \( \approx 5.0\% \) to a 95% confidence level.
3.3.2 Pressure

The pressure transducers used within this research are described in this section. The different experimental conditions, steady and unsteady, made it necessary to use different types of transducers.

**Steady Flow Condition**

The static pressure of the flow were measured at the orifice plates and the ‘measurement plane’, (refer to Fig. 3.1). The pressure was measured through static-hole tappings on each limb which were pneumatically connected to a rotary-switch, Scanivalve. The rotary switch of the Scanivalve is controlled via National Instruments FieldPoint digital output module NI FP-DO-401 and its channels are monitored via digital input module NI FP-DI-330. The Scanivalve was coupled with two strain gauge pressure transducers, for low and high pressure range. Druck PDCR 23D type pressure transducer was used for the high-pressure measurements with a range of ±3.5 bar (gauge) and uncertainty of ±0.02% FS (±90 Pa). Meanwhile, Druck PDCR 22 transducer is used for the low-pressure measurements with a range of ±0.35 bar (gauge) and uncertainty of ±0.008% FS (±36 Pa). Each of these transducers is connected to a signal conditioning module, Fylde FE-492-BBS Mini-Bal and a signal amplifier, Fylde FE-351-UA Uni-Amp. The output of the transducer-conditioner-amplifier modules are connected to the National Instruments FieldPoint analogue input module NI FP-AI-110 on the data acquisition system. The analogue and digital channels were remotely connected to the control computer via Ethernet connection. An in-house built LabView program switches the Scanivalve channels during the test-logging period to record all the relevant static pressures of the flow.

**Pulsating Flow Condition**

The instantaneous pressures measured at the measurement plane and at the exit of the volute, were used to calculate the instantaneous performance parameters. Two high-response Schaevitz type P704-0001 strain gauge pressure transducers, rated for a range of 0 – 3.45 bars (gauge) with 0.059% FS maximum deviation, were used to measure the instantaneous static pressure at the measurement plane. These transducers were mounted close to the duct wall in order to reduce the pulsation effects (Winterbone et al., 1991). The length of the air passage from the flush-inner face of the duct to the surface of the sensor is 33mm. The corresponding Helmholtz resonant frequency for the passage length is approximately 2800Hz. Given that the maximum frequency experienced during testing is below 1/5 fraction of the resonant frequency, the Helmholtz effect on the pressure reading can be assumed to be negligible. The exit static pressure is measured with a SensorTechnics high response strain gauge transducer model 19C 50P G7K , with range of 0 – 3.45 bars (gauge) and maximum deviation of 0.009% FS. The pressure transducers were connected to signal conditioners and amplifier modules of the series Fylde FE-492-BBS Mini-Bal conditioner and Fylde FE-351-UA Uni-Amp.
amplifier. The output of the conditioner amplifier module was directly connected to the high-speed analogue-to-digital PCI card, *NI 6034E* by *National Instruments*. An in-house built *LabView* program was used to record the transducer-conditioner-amplifier output with reference to a trigger pulse from the pulse generator (via the shaft encoder). This is to ensure that all pressure reading as well as other relevant parameters was recorded from a consistent reference point in a pulse.

*Calibration*. The calibration procedure for the pressure transducers is straightforward. A pressure calibrator unit *Druck DPI 610* was used for all the pressure transducers in the test-rig. The calibration established the transfer function for each transducer from voltage to gauge pressure. This is a linear correlation and the day-to-day atmospheric pressure was used to calculate the absolute pressure corresponding to test conditions. The maximum deviation of the calibrator is < 0.025% *FS*.

### 3.3.3 Temperature

**Steady Flow Condition**

The temperatures required for steady testing, were monitored upstream of the orifice plates and at the measurement plane. The temperatures were also monitored at the heater in order to ensure constant temperature of the flow throughout the experiments and avoid condensation due to expansion in the turbine. Three different types of thermocouples were used during testing: E-type, K-type and T-type. In the measurement of temperature in a moving fluid, the compressibility effects must be taken into account. Compressibility is a function of the Mach number and it causes the measured temperature to fall in between the static (*T*<sub>s</sub>) and total (*T*<sub>o</sub>) temperature. However Yahya (1982) demonstrated that the compressibility effect needs to be corrected only for Mach number of the flow above 0.3. The thermocouples located upstream of the turbine orifice plates are exposed to low velocities and hence the static temperatures at these locations were measured directly without the use of a recovery factor. At measurement plane instead, where the flow velocity corresponds to a Mach number of order 0.3 and greater, the effects of compressibility and convective heat transfer could not be neglected, thus making necessary to use of the recovery factor. The recovery factor is defined in Eq. (3.10) and the corresponding static temperature is given in Eq. (3.11).

\[
r = \frac{T_{\text{meas}} - T_s}{T_0 - T_s}
\]

\[
T_s = \frac{T_{\text{meas}}}{1 + r \left(\frac{Y - 1}{2}\right) M^2}
\]

(3.10)  
(3.11)
where \( r \) is the recovery factor of the two thermocouples used at the measurement plane and \( T_{\text{mean}} \) is the temperature sensed by the thermocouple. In order to calculate the recovery factor, the thermocouple is immersed in a wind-tunnel where the flow is accelerated gradually and the total temperature and pressure are measured at the plenum before the test section while the corresponding static values are measured at the test section. The recovery factor can therefore be calculated with Eq. (3.10).

**Pulsating Flow Condition**

The monitoring of temperatures for the pulsating flow test conditions is similar as those in the steady state flow testing. However, the temperatures needed for the performance calculation were evaluated in a different way than in the steady flow testing. The instantaneous static temperature of the pulsating flow is deduced by assuming isentropic compression between the pressure and temperature at the measurement plane. At the measurement plane, the T-type thermocouples provide the time-mean static temperature \( (T_{\text{mean}}) \) while the time-mean pressure \( (P_{\text{mean}}) \) and instantaneous static pressure \( (P_{s,\text{inst}}) \) are measured through static tapping and high-response pressure transducers. These parameters are related and consequently the instantaneous static temperature \( (T_{s,\text{inst}}) \) is calculated based on Eq. (3.12).

\[
T_{s,\text{inst}} \approx T_{\text{mean}} \left( \frac{P_{s,\text{inst}}}{P_{\text{mean}}} \right)^{(\gamma-1)/\gamma}
\]

**Calibration.** The thermocouple calibration procedure is described in Hakeem and Khezzar (1994). All the thermocouples were calibrated at the freezing (273.15 K) and boiling point (373.15 K) of water with an additional room temperature point (293 K). Local pressure variations were taken into consideration in cases where it is necessary. A mercury bulb thermometer with a resolution of 0.1 K was used for the room temperature point calibration. Repeated calibration test shows temperature reading repeatability within ±0.4 K.

**3.3.4 Rotational Speed**

In order to measure the rotational speed of the turbine, a reflective optical-switch of type Omron EE-SX4101 with integrated amplifier was used. This is an infra-red optical sensor triggered by a 10–toothed wheel mounted on the turbine shaft.
**Steady Flow Condition**

During steady state testing, 10 pulses per revolution produced by the optical sensor are de-rated to a single pulse. Each pulse is then used as a digital gate for a 16MHz clock in a 16-bit counter which measures the time for one revolution of the turbine. This is then converted into DC voltage, such that increasing turbine speed results in decreasing output voltage. Fig. 3.4-a shows the transfer function between the circuit output voltage and the corresponding turbine rotational speed (RPS). The output voltage of the counter is connected to the National Instruments FieldPoint analogue input module *NI FP-AI-110*, which is then recorded during the testing.

**Calibration.** In order to calibrate the transfer function between the counter’s output voltage and the rotational speed of the turbine a 5 kHz square wave signal generator was used in replacement of the optical sensor output. The square wave signal represents a turbine speed of 500RPS since the circuit de-rates 10 pulses into 1. The accuracy of the speed reading was found to be 500 ± 0.017RPS. In addition to this, an external tachometer was used to directly measure the rotor speed in order to qualitatively verify the speed measurement.

**Pulsating Flow Condition**

The measurement of the instantaneous speed of the turbine is crucial in evaluating its performance under pulsating flow. In order to enhance the accuracy of the reading all 10 pulses per revolution from the speed sensor is directly connected to a National Instruments PCI counter card, *NI 6602*. The pulses from the optical sensor are used as a gate for an internal reference 20MHz clock. These pulses are recorded during testing with reference to a trigger mark from the pulse generator (via
the shaft encoder). Hence the time required for the turbine shaft to rotate for a given angle can be deduced and the rotational speed consequently evaluated.

**Calibration.** An indirect calibration was made necessary due to the manufacturing inconsistency of the angular distance between each tooth of the 10-toothed wheel. In order to correct this inconsistency, Szymko (2006) recorded the steady output of 10 pulses per revolution from the optical sensor via the PCI counter card and consequently calculated the instantaneous rotational speed. A distinct repetitive pattern was observed for every 10 pulses as shown in Fig. 3.4-b. In this way a dynamic angle \( \theta_n \) between each tooth could be calculated based on the \( i \)-th segment’s angular speed \( \omega_i \) and the per-revolution averaged angular speed \( \omega_{\text{mean}} \) as given in Eq. (3.13). The described speed correction improves the measurement uncertainty from ±7.85RPS to ±0.16RPS.

\[
\theta_n = \frac{\omega_i \cdot 2\pi}{\omega_{\text{mean}} \cdot 10}
\]

(3.13)

### 3.3.5 Power Measurement

Compressors are usually used to load a turbine and its power is calculated indirectly through an energy balance procedure. Unfortunately the range of the turbine map that is possible to obtain by using such a method is limited by the surge and choke margins of the compressor. In order to overcome this limitation, a permanent magnet eddy current dynamometer was used as the loading device. Besides providing a much broader range of testing (without the associated aerodynamic limitations of a compressor) the inertial problem compared to a hydraulic dynamometer are also eliminated. A permanent magnet dynamometer designed and developed at Imperial College by Szymko (2006) was utilised in the test-facility to load the turbine. The dynamometer works on the principle of eddy-current braking by incorporating 14 ground Neodymium-Iron-Boron magnets of 12mm depth onto a rotor. The rotor spins co-axially to a set of stationary plates known as the stators which are located on either side of the rotor (refer to Fig. 3.5-a). These plates are water-cooled and are connected to the main body of the dynamometer which is attached to a ‘gimble’ bearing system held by a load cell coupling. The rotating shaft produces a torque (transferred to the main body) which is measured by the load cell reaction.

**Steady Flow Condition**

In steady state conditions, the torque \( \tau \) of the rotating shaft is given by the direct reaction of the dynamometer. The torque is measured with a cantilever beam load cell of type Tedea Huntleigh 1040-1-20 and the output of the load cell is connected to Fylde FE-492-BBS Mini-Bal bridge conditioner and Flyde FE-351-UA Uni-Amp amplifier modules. The signal is then connected to a
National Instruments FieldPoint analogue input module NI FP-AI-110. The combination of the measured torque and the turbine speed (N) enable us to calculate the turbine power or the actual work, \( W_{\text{act}} \) as in Eq. (3.14).

\[
\dot{W}_{\text{act}} = 2\pi N \tau
\]  

(3.14)

Pulsating Flow Condition

Under pulsating flow conditions, the instantaneous torque of the turbine shaft (\( \tau_{\text{inst}} \)) is the sum of its fluctuating (\( \tau_{\text{fluc}} \)) and mean (\( \tau_{\text{mean}} \)) components as given in Eq. (3.15)

\[
\tau_{\text{inst}} = \tau_{\text{fluc}} + \tau_{\text{mean}}
\]  

(3.15)

The fluctuating torque of the rotating shaft is the product of acceleration of the rotor (\( \alpha \)) and its polar moment of inertia (\( I \)), as given in Eq. (3.16), while the mean torque is obtained by the reading of the load cell recorded for the duration of the data logging (as described in the ‘steady flow condition”).

\[
\tau_{\text{fluc}} = I \cdot \alpha = I \cdot \left( \frac{d\omega}{dt} \right)
\]  

(3.16)

The first derivative of the angular speed \((d\omega/dt)\) in Eq. (3.16) corresponds to the acceleration of the rotating component and it is calculated with the first central difference numerical technique.
**Calibration.** In order to establish the load cell-reaction transfer function (Fig. 3.5-b), static torque calibration was conducted. A calibration arm is attached radially on the periphery of the dynamometer with its loading direction parallel to the load cell. The length from the loading point of the arm to the center of dynamometer is 0.599 m. During calibration the load cell is supplied with water, oil and air running at operational level. This is made necessary in order to reduce the uncertainty in the torque measurement given by the external connection of pipes exerting a pre-load on the free floating ‘gimble’ bearing. In addition to this, the calibration is repeated on a regular basis to ensure its consistency. The combined calibration points suggest an uncertainty in the torque measurement of ±0.025 Nm.

### 3.4 Uncertainty Analysis

Each instrument of the measurement system in the test facility carries a degree of error which will propagate into the final result. Thus, the uncertainty in the measured performance parameter is the combination of all the variable uncertainties involved in its deduction. The uncertainty methodology was proposed by Kline and McClintock (1953) and developed by Moffat (1982). A good description of the uncertainty methodology based on the ASME and AIAA standards is given in Stern et al., (1999). The uncertainty analysis specific to the current test facility was done by Szymko (2006) who utilized the Root-Sum-Square (RSS) method to evaluate the propagated uncertainties, as shown in Eq. (3.17). This is used as the basis throughout the current thesis.

The RSS uncertainty calculation for the turbine performance parameters are given in Eq. (3.18) to Eq. (3.22).

\[
\pm Par_{RSS} = \left( \sum_{i=1}^{n} \left( \pm var_i \cdot \frac{\partial Par}{\partial var_i} \right)^2 \right)^{1/2} \tag{3.17}
\]

where

\[\pm Par_{RSS}\] is the RSS uncertainty of parameter

\[\pm var_i\] is the individual variable uncertainty

\[\frac{\partial Par}{\partial var_i}\] is the sensitivity coefficient

The equations for the \( P_{aRSS} \) of the performance parameters used in the current thesis are given in Appendix A1. In the appendix is also provided a table with the sensitivity coefficients associated to each variable and the dependant parameters. The evaluation of the uncertainty associated with the performance parameters is instead given in the relevant chapters.
CHAPTER 4

STEADY STATE PERFORMANCE: EXPERIMENTAL AND COMPUTATIONAL EVALUATION

SYNOPSIS

This chapter reports the experimental results obtained for a variable geometry twin-entry mixed-flow turbine. The performance parameters were calculated and compared with those of a single-entry turbine (nozzled and nozzleless) previously characterised at Imperial College (Rajoo, 2007, Szymko, 2006). The turbines performance was assessed on an equivalent geometry basis. The design progression of the volute was aimed to maintain the $A/r$, the exit flow angle and the shape of the cross-section similar to the base line turbine (HOLSET H3B). In this manner, a comprehensive comparison of different configurations could be carried out.

Based on the test results a meanline model for the nozzled and nozzleless single-entry turbine was developed. The turbine performance parameters (efficiency, pseudo-dimensional mass flow) were then calculated and validated against experimental results. For the twin-entry turbine a map-based method was developed to evaluate the mass flow under partial and unequal admission conditions and compared with experimental results.
4.1 Turbine settings

Turbine configurations can be divided into two main groups: single and twin-entry turbines. Within these two groups we can distinguish between nozzleless and nozzled configurations that can either be fixed or variable geometry (VGT). The combination of these four possible turbine settings covers most automotive turbocharger applications.

Despite extensive research in the past on turbine performance, they mainly have focused on assessing individual features of the turbines relevant to each group in Fig. 4.1. One could say that research has been mainly in a vertical characterization of turbine performance while our intention here is to go through a horizontal analysis.

Baines and Lavy (1990) firstly conducted a direct comparison of nozzled and nozzleless turbine with same rotor. However the different surface polishing of the vanes in respect to that of the stator made it difficult to provide an appropriate evaluation of the turbine efficiency. These issues were lately solved by Spence et al. (2007), who performed a direct comparison of three different single-entry turbines with vaneless and vaned stators. The stators and the vanes used for the investigation were machined in order to obtain a comparable degree of surface finishing and the mass flow rates were matched within 1%. The outcomes of the experimental investigation showed that (for any operating condition) the vaneless turbine always provides an improvement in performance in respect to the vaned turbine.

The present research aims to contribute to the understanding of the differences between turbine configurations. The turbine performance was assessed across the two main groups, single and twin Entry. Wastegated turbines should also be included in the classification but these would fall within one of the groups included in Figure 4.1.

---

*Wastegated turbines should also be included in the classification but these would fall within one of the groups included in Figure 4.1.*
twin-entry (horizontal comparison in Fig. 4.1.). Unlike Spence et al. (2007) who based their analysis on equivalent flow capacity conditions, the current study the performance was assessed on an equivalent geometry basis. The design progression from single to variable geometry twin-entry turbine was experimentally investigated by retaining the size of the turbine housing (same A/r) and keeping the same rotor throughout the whole tests.

Additionally, the current study also contributes to the partial and unequal admission performance prediction of twin-turbines. The meanline models that can be found in the literature (Ghasemi et al., 2005, Hajilouy-Benisi et al., 2009) are usually validated against full admission conditions for which they provide a good prediction. However the effectiveness of these models as predictive tools deteriorates when applied under partial and unequal admission conditions. In fact the complexity of the interaction between the flows leaving the entries can hardly be captured by the simple assumptions behind a meanline model. In addition to this, the narrow range of experimental data usually available for validation limits the suitability of these models in the extreme regions of the maps. Unlike meanline models, the current study will mainly focus on a map-based method to predict the partial and the unequal admission mass flow curves starting from a given full admission curve.

The main difference between an off-design performance and a map-based method is that the former needs to identify every single loss affecting the performance in order to generate the maps while the latter relies more heavily on the analysis of the experimental results. In other words, off-design analysis looks at the effects of any specific loss within any single stage of the turbine while a map-based method looks at the overall effects of the losses on the turbine performance.

4.2 Turbine design

The turbine used in the current research is based on a commercial nozzleless unit (HOLSET H3B) that was tested at Imperial College by Karamanis (2002) and Szymko (2006). The HOLSET H3B unit was consequently modified by Rajoo (2007) who designed a new volute to allocate a pivoting nozzle-ring (VGT). The geometrical parameters of the HOLSET H3B were left unaltered and the turbine volute was manufactured in two halves for increased flexibility of the turbine. As for the purpose of this research, a divider was inserted within the volute and clamped between the two halves of the volute. Such an operation enabled us to switch turbine configuration from single to twin-entry by leaving the main turbine features unaltered.

4.2.1 Volute

The function of the casing is to increase the kinetic energy of the exhaust gases available for the turbine. The flow is redirected and accelerated towards the rotor in order to maximize the energy output by matching the rotor blades at an optimum incidence angle that reduces the dissipative effects at the rotor inlet. The casing can be either vaneless or include nozzle guide vanes. Vaneless stators are
often preferred compared to the nozzle geometries except in the case of very high pressure ratio applications. This is because nozzleless casings are cheaper, smaller and enable the flow angle to vary to some extent with the mass flow, giving a higher efficiency over a wider flow range.

During the design-phase a distinction should be made as to whether or not the turbine housing is going to include nozzles. If nozzles are present in the turbine, the volute is designed to provide uniform flow within the nozzles while, in the nozzleless case, the volute design will have to guarantee an optimum incidence angle for the rotor. The turbine tested in the current study includes nozzles and therefore the following discussion will describe the steps taken for the volute design.

The volute design was carried out using the well established meanline analysis method introduced by Watson and Janota (1982) and Whitfield and Baines (1990). The main assumptions of this method are the free vortex conditions in the volute and the uniform flow distribution around the volute periphery. Assuming a free vortex condition, the angular momentum of the flow is conserved and given in Eq. (4.1).

\[ r_\psi \cdot C_\psi = S \quad \text{where } S \text{ is a constant} \tag{4.1} \]

By considering the continuity equation within the volute (incompressible fluid) and the assumption of uniform mass flow distribution in the volute periphery, the fractional mass flow rate can be expressed as a function of the azimuth angle as in Eq. (4.2) and Eq. (4.3).

\[ m_\psi = \dot{m} \left(1 - \frac{\psi}{\pi}\right) \tag{4.2} \]
\[ m_\psi = \rho_\psi A_\psi C_\psi \tag{4.3} \]

The \( A/r \) as a function of the azimuth angle \( \psi \) is then readily given by the combination of Eq. (4.1), Eq. (4.2) and Eq. (4.3), as expressed in Eq. (4.4).

\[ \frac{A_\psi}{r_\psi} = \frac{\dot{m}}{K \rho_\psi} \left(1 - \frac{\psi}{2\pi}\right) = f(\psi) \tag{4.4} \]

Eq. (4.4)\(^7\) sets the physical guideline for the dimensioning of the volute. In order to distribute the mass uniformly around the circumference of the rotor, the ratio between the area and the radius must be a linear function of the azimuth angle. Eq. (4.4) shows that the critical parameters that have to be taken into account in the design of the volute are the cross sectional area \( (A) \) and the correspondent centroid radius \( (r) \). At the exit to the volute, the exit flow angle \( (\alpha_2) \) is given by the combination of the radial

\(^7\) For incompressible flow, the changes in density can be neglected. The \( A/r \) is therefore largely a function of the azimuth angle \( \psi \).
and tangential component of the absolute velocity. The $A_2/r_2$ at the exit to the volute depends on the geometry of the next component downstream that could be either the nozzle ring or the rotor. In a fixed geometry stator the $A_2/r_2$ is constant. This implies that the radial component of the velocity going into the rotor is mainly dependent on the area ratio $A_1/r_1$. The volute exit flow angle can then be expressed as in Eq. (4.5).

$$\tan \alpha_2 = \frac{C_{\psi_2}}{C_{r_2}} = \frac{\rho_2}{\rho_1} \left( \frac{A_2}{r_2} \right) \left( \frac{A_1}{r_1} \right)$$  \hspace{1cm} (4.5)

As the new volute was designed for a variable geometry nozzle vane ring, the area at the exit to the volute slightly changes depending on the nozzle positions. However the exit-area ratio at the exit to the volute $A_2/r_2$ is only a function of the downstream width regardless of the radius, Eq. (4.6). Therefore even for the new volute design the exit flow angle was determined as a function of the $A_1/r_1$ at the inlet to the volute.

$$\frac{A_2}{r_2} = \frac{2\pi r_2 b_2}{r_2} = 2\pi b_2$$  \hspace{1cm} (4.6)

In order to determine the exit flow angle, a simple analysis was carried out using Eq. (4.1) to Eq. (4.5). A similar analysis of the HOLSET H3B turbine provided an exit flow angle of $\approx 68^\circ$ in respect to the radial direction. For the new turbine, a range of exit condition as a function of inlet area-radius ratio was obtained and given in Fig. 4.2-a. Given the proven commercial application of the HOLSET H3B a target range of $67^\circ$ to $72^\circ$ exit flow angle was chosen and the main geometrical parameters (area and radius) calculated accordingly (Table 4.1).
The free vortex assumption was applied from the volute. The volute tongue in the **HOLSET H3B** is located at 50° from the inlet. Due to constraints associated with the physical dimension of the test rig, the volute tongue in the new turbine had to be shifted by 20° upstream in respect to the **HOLSET H3B**. However, despite the constraints due to the test rig assembly, the A/r of the newly designed volute (A/r = 33) was maintained similar to that of the **HOLSET H3B** (A/r = 34.7).

The cross sectional area of the single-entry turbine is not symmetrical in respect to a meridional plane. The choice of cross-sectional area of the volute is important to provide a smooth flow passage through the volute. The conventional cross sectional shapes for turbine volutes can vary substantially from each other. As for the current application, the cross sectional shape of the **HOLSET H3B** was maintained, considering the proven commercial application. A picture of the final machined volute is given in Fig. 4.2-b.

<table>
<thead>
<tr>
<th>Table 4.1: Nozzleless vs. nozzled turbine volute dimensions</th>
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</thead>
<tbody>
<tr>
<td><strong>Volute dimensions</strong></td>
</tr>
<tr>
<td><strong>Nozzleless</strong></td>
</tr>
<tr>
<td>Volute tongue position</td>
</tr>
<tr>
<td>Centroid throat radius at ψ=0 (mm)</td>
</tr>
<tr>
<td>Stator throat area (mm²)</td>
</tr>
<tr>
<td>A/r</td>
</tr>
</tbody>
</table>

**Design of the divider**

The twin-entry configuration was obtained by modifying the single-entry turbine designed by Rajoo (2007). A meridional divider was inserted within the two halves of the volute and clamped. Given that the main geometrical parameters were already fixed, the divider design focussed in finding the best compromise between area available to the flow and its structural strength.

The first aspect that had to be considered when designing the divider was to maintain the same area for both the entries across each section of the volute. This was made necessary in order to guarantee that the analysis on the interaction between the two entries of the turbine could be performed on an equivalent geometry basis. In order to determine the profile of the divider the original CAD model of the nozzled single-entry turbine was used. The non-symmetrical shape of the cross sectional area forced the divider to be positioned slightly offset with respect to the meridional plane. The design procedure for the divider is given in Fig. 4.3. Firstly the internal volume of the volute (from the inlet to the tongue) was modelled as a solid body (Fig. 4.3-a). Secondly the cross sectional profile was determined in 15 stations and the area of each of these sections was calculated (Fig. 4.3-b). Then an initial profile of the divider was fitted into the volute (Fig. 4.3-c) and it was tapered and chamfered in the edges in order to give the final model (Fig. 4.3-d). Fig. 4.3 also provides the section profile for each of the 15 stations considered during the design. It is noted that the divider
accounts for less than \( \approx 7\% \) of the area of each section. In section \( S1 \) the area variation due to the divider is \( \Delta A \approx 7\% \) that remains fairly constant for every section. Critical to the design of the divider was the last section (\( S15 \)) at the exit to the volute. Here the area is smallest and a large decrease of the area available would cause the flow to enter more tangentially into the rotor with the consequence of reducing the flow capacity of the turbine. The total volume of the divider corresponds to \( \approx 6\% \) of the total volume of the volute. This can be considered as a good compromise in terms of area available to the flow.

![Cross sections considered for design of the divider (refer to figure B)](image)

Figure 4.3: Design progression of the divider

![Figure 4.4: Volute divider](image)

Given the high pressure difference occurring between the two entries of the turbine when operating under out-of-phase conditions, a material with high flexural strength had to be selected.
Low thermal expansion for continuous working temperatures up to 100°C and high machinability had also to be guaranteed. The material chosen was the high quality epoxy resin bonded glass fabric, TUFNOL 10G/40. This material has a mechanical strength with good dimensional stability and resistance that is suitable for continuous use temperatures up to approximately 130°C (Class B). Technical specifications are provided in Appendix A1 while the finished divider inserted in the turbine volute is shown in Fig. 4.4.

![Volute and wheel dimensions](image)

**Figure 4.5: Volute and wheel dimensions**

### 4.2.2 Nozzles and rotor wheel

The nozzle vane ring consists of 15 straight vanes designed by Rajoo (2007). In Table 4.2 the main geometrical parameters for the nozzles are provided. The pivoting range varies from 40° to 80° (with respect to the radial direction) that correspond to a fully open and fully closed position respectively.

The volute dimension constrains the upper limit of the nozzle ring at ø=140 mm. The lower limit instead must take into account that the leading edge of the mixed-flow turbine is swept radially. The downstream limitation varies from the hub-side to the shroud-side end wall. The limitations are given by the rotor leading edge (Fig. 4.5) and are respectively equal to ø=72.02 mm (hub-side) and ø=95.16 mm (shroud-side). However these maximum limitations would not be achieved since there is an interspace between the rotor and the nozzle vane.

<table>
<thead>
<tr>
<th>Nozzle vane geometry</th>
<th>Number of nozzles</th>
<th>Clearance on each side (mm)</th>
<th>Chord (mm)</th>
<th>Pitch (mm)</th>
<th>Axial height (mm)</th>
<th>Pivoting range</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>15</td>
<td>0.015</td>
<td>17</td>
<td>21.75</td>
<td>13.79</td>
<td>40°-80°</td>
</tr>
</tbody>
</table>
The turbine wheel used for all the tests is of a mixed-flow nature previously designed at Imperial College by Abidat (1991). For consistency with previously reported results, the wheel is referred to as rotor “A”. The main geometrical parameters are given in Table 4.3 and shown in Fig. 4.5. More details can be found in available literature (Abidat, 1991, Abidat et al. 1992).

<table>
<thead>
<tr>
<th>Rotor Type A</th>
<th>Number of blades</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Inlet mean diameter (mm)</td>
<td>83.58</td>
</tr>
<tr>
<td></td>
<td>Blade span angle</td>
<td>40°</td>
</tr>
<tr>
<td></td>
<td>Inlet blade height (mm)</td>
<td>18.0</td>
</tr>
<tr>
<td></td>
<td>Trailing blade curvature</td>
<td>varied</td>
</tr>
<tr>
<td></td>
<td>Rotor blade length (mm)</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>Exit mean blade angle</td>
<td>varied</td>
</tr>
<tr>
<td></td>
<td>Exducer tip diameter (mm)</td>
<td>78.65</td>
</tr>
<tr>
<td></td>
<td>Inlet blade angle</td>
<td>-40°</td>
</tr>
<tr>
<td></td>
<td>Exducer hub diameter (mm)</td>
<td>27.07</td>
</tr>
</tbody>
</table>

### 4.3 Performance parameters

The parameters used to define the performance of a turbine are the total-to-static efficiency and the mass flow parameter.

**-Total-to-static efficiency:** this is defined as the ratio between the actual work output and the isentropic work output in an ideal process between two defined states. This can be expressed as the ratio between the actual and the isentropic enthalpy change across the stage as given in Eq. (4.7).

\[ \eta = \frac{\text{actual work output}}{\text{work output in an ideal process}} = \frac{\dot{W}_{\text{act}}}{\dot{W}_{\text{is}}} \]  \hspace{1cm} (4.7)

Equation (4.7) can be rewritten as in Eq. (4.8).

\[ \eta_{\text{ts}} = \frac{h_{01} - h_{04}}{h_{01} - h_{4\text{is}}} \]  \hspace{1cm} (4.8)

The efficiency as defined in Eq. (4.8) is usually referred as total-to-static efficiency. Such a definition is introduced to point out that the exit kinetic energy is not recovered. In the current test facility the isentropic power is calculated as given in Eq. (4.9).

\[ \dot{W}_{\text{is}} = \dot{m}(h_{01} - h_{4\text{is}}) = \dot{m}\left[\left(\int_{4\text{is}}^{1} c_p dT\right) + (0.5C_i^2)\right] = \dot{m}\frac{C_i^2}{2} \]  \hspace{1cm} (4.9)

From Eq. (4.9) it can be seen that the calculation of \( \dot{W}_{\text{is}} \) requires the knowledge of the mass flow rate \( \dot{m} \), the temperatures at the inlet \( T_{01} \) and the exit \( T_{4\text{is}} \) to the volute, the specific heat capacity \( c_p \) and the inlet flow velocity \( C_i \):
- mass flow rate ($\dot{m}$): this is directly measured during experiments;
- total inlet temperature ($T_{01}$): the temperature measured at the ‘measurement plane’ is used to calculate the static temperature by means of Eq. (4.10) and (3.11). The inlet total temperature is then calculated as in Eq. (4.10):

$$T_{01} = T_1 \left(1 + \frac{\gamma_1 - 1}{2} M_i^2\right)$$  \hspace{1cm} (4.10)

In order to determine the Mach number Eq. (3.10), Eq. (3.11) and Eq. (4.10) require iteration to solve for temperatures;
- isentropic exit static temperature ($T_{4is}$): the measured static pressure at the inlet ($P_1$) and the calculated temperatures are used to calculate the total inlet pressure ($P_{01}$) as given in Eq. (4.11):

$$P_{01} = P_1 \left(\frac{T_{01}}{T_1}\right)^{\gamma_1/\gamma_1-1}$$  \hspace{1cm} (4.11)

The isentropic exit static temperature ($T_{4is}$) can then be calculated with the isentropic flow relationship as given in Eq. (4.12) below:

$$T_{4is} = T_{01} \left(\frac{P_1}{P_{01}}\right)^{\gamma_{14}-1/\gamma_{14}}$$  \hspace{1cm} (4.12)

- specific heat capacity ($c_p$): the calculation of the isentropic exit temperature requires the iteration to evaluate the mean specific heat ratio ($\gamma_{14}$) within the inlet and exit state. The specific heat ratio can be written as a function of the universal gas constant ($R$) and the specific heat capacity ($c_p$). Zucrow and Hoffmann (1977) provided an expression for the specific heat capacity of the air as a function of temperature:

$$c_p = A_0 + A_1 T_1 + A_2 T_1^2 + A_3 T_1^3 + A_4 T_1^4 + A_5 T_1^5$$  \hspace{1cm} (4.13)

where: \hspace{1cm} $A_0 = 0.10831165 \cdot 10^4$ \hspace{1cm} $A_3 = -0.11323656 \cdot 10^{-5}$

$A_1 = -0.68388122$ \hspace{1cm} $A_4 = -0.82943324 \cdot 10^{-9}$

$A_2 = 0.17875137 \cdot 10^{-2}$ \hspace{1cm} $A_5 = 0.11100191 \cdot 10^{-11}$

- inlet flow velocity ($C_1$): $C_1 = \dot{m}/\rho A$  \hspace{1cm} (4.14)
$C_o$ is the *isentropic velocity* that represents the velocity attained to an isentropic expansion over the total-to-static pressure ratio of the turbine as per Eq. (4.9). The efficiency is generally plotted against the *Velocity Ratio (VR)*. This is a dimensionless parameter defined as the ratio between the rotor blade tip speed and the isentropic velocity, as given in Eq. (4.15).

$$VR = \frac{U}{C_{is}}$$  

(4.15)

The velocity ratio can then be expressed as in Eq. (4.16).

$$\frac{U}{C_{is}} = \frac{\pi D_{tip} \cdot \rho \cdot N}{\sqrt{2 \left( \int_{0}^{1} c_{p} \cdot dT + 0.5c_{i}^{2} \right)}}$$  

(4.16)

![Figure 4.6: Dynamometer power absorption capacity for range of stator gaps and turbine speeds (Szymko, 2006)](image)

- **Mass Flow Parameter**: this is a pseudo non-dimensional parameter that is used to calculate the flow capacity of the turbine, given in Eq. (4.17).

$$MFP = \frac{\dot{m}\sqrt{T_{01}}}{P_{01}}$$  

(4.17)

This is plotted against the *Pressure Ratio (PR)*, defined as the ratio between the exit static pressure and the total pressure at the inlet to the volute, given in Eq. (4.18).
For each equivalent speed line, the following diagrams are typically plotted: Mass flow parameter vs. Pressure ratio and Total-to-static efficiency vs. Velocity ratio.

### 4.4 Test setup

The mixed-flow turbine under study was tested for a range of speeds and pressure ratios. The pressure ratio was controlled by the stator gap (corresponding to the loading of the dynamometer) and the inlet mass flow rate was calculated in order to match a fixed speed\(^8\). Within a fixed speed the stator gap was varied from 0.3 mm to 10 mm in order to sweep the maps over the widest range possible of pressure ratios. In order to avoid condensation due to expansion within the rotor, the temperature of the airflow was set and maintained during testing. The inlet mass flow rate was adjusted in order to match the desired speed and all the relevant properties were recorded once the condition stabilizes.

<table>
<thead>
<tr>
<th>Equivalent speed</th>
<th>Vane angle</th>
<th>Inlet temperature [K]</th>
<th>Turbine speed [rev/s]</th>
<th>(N/\sqrt{T_01}) [rev/s·√K]</th>
<th>PR in each limb</th>
</tr>
</thead>
<tbody>
<tr>
<td>80%</td>
<td>--</td>
<td>343</td>
<td>793</td>
<td>43.0</td>
<td>--</td>
</tr>
<tr>
<td>50%</td>
<td>--</td>
<td>323</td>
<td>488</td>
<td>27.9</td>
<td>--</td>
</tr>
</tbody>
</table>

**Nozzled single-entry**

<table>
<thead>
<tr>
<th>Equivalent speed</th>
<th>Vane angle</th>
<th>Inlet temperature [K]</th>
<th>Turbine speed [rev/s]</th>
<th>(N/\sqrt{T_01}) [rev/s·√K]</th>
<th>PR in each limb</th>
</tr>
</thead>
<tbody>
<tr>
<td>80%</td>
<td>40°, 60°, 70°</td>
<td>343</td>
<td>793</td>
<td>43.0</td>
<td>--</td>
</tr>
<tr>
<td>50%</td>
<td>40°, 60°, 70°</td>
<td>323</td>
<td>488</td>
<td>27.9</td>
<td>--</td>
</tr>
</tbody>
</table>

**Twin-entry**

<table>
<thead>
<tr>
<th>Equivalent speed</th>
<th>Vane angle</th>
<th>Inlet temperature [K]</th>
<th>Turbine speed [rev/s]</th>
<th>(N/\sqrt{T_01}) [rev/s·√K]</th>
<th>PR in each limb</th>
</tr>
</thead>
<tbody>
<tr>
<td>80%</td>
<td>40°, 60°, 70°</td>
<td>343</td>
<td>793</td>
<td>43.0</td>
<td>--</td>
</tr>
<tr>
<td>50%</td>
<td>40°, 60°, 70°</td>
<td>323</td>
<td>488</td>
<td>27.9</td>
<td>--</td>
</tr>
</tbody>
</table>

**Partial admission (either the inner and the outer entry blanked off)**

<table>
<thead>
<tr>
<th>Equivalent speed</th>
<th>Vane angle</th>
<th>Inlet temperature [K]</th>
<th>Turbine speed [rev/s]</th>
<th>(N/\sqrt{T_01}) [rev/s·√K]</th>
<th>PR in each limb</th>
</tr>
</thead>
<tbody>
<tr>
<td>80%</td>
<td>60°</td>
<td>343</td>
<td>793</td>
<td>43.0</td>
<td>--</td>
</tr>
<tr>
<td>50%</td>
<td>60°</td>
<td>323</td>
<td>488</td>
<td>27.9</td>
<td>--</td>
</tr>
</tbody>
</table>

**Unequal admission (either the inner and the outer entry kept at constant pressure)**

<table>
<thead>
<tr>
<th>Equivalent speed</th>
<th>Vane angle</th>
<th>Inlet temperature [K]</th>
<th>Turbine speed [rev/s]</th>
<th>(N/\sqrt{T_01}) [rev/s·√K]</th>
<th>PR in each limb</th>
</tr>
</thead>
<tbody>
<tr>
<td>80%</td>
<td>60°</td>
<td>343</td>
<td>793</td>
<td>43.0</td>
<td>1.3/1.4/1.5</td>
</tr>
<tr>
<td>50%</td>
<td>60°</td>
<td>323</td>
<td>488</td>
<td>27.9</td>
<td>1.5/1.6/1.7/1.9</td>
</tr>
</tbody>
</table>

In Table 4.4 the test conditions for the three configurations under study were reported. The results available for the single-entry turbine encompass test points for two different speeds (27.9 rev/s·√K and 43.0 rev/s·√K) and three different vane angles setting: 40°, 60° and 70° (with

\(^8\) As the stator gap increases, the power absorption by the dynamometer decays exponentially but it remains fairly linear with the speed changes, as shown in Fig. 4.6 (Szymko, 2006).
respect to the radial direction). Similar test conditions were chosen for the twin-entry turbine with the addition of partial and unequal admission conditions\(^5\). At the present no generally accepted procedure exists for tests conducted under unequal admission. In the current study the pressure ratio \(\frac{P_{\text{limb}}}{P_{\text{exit}}}\) in one limb was fixed at a given value and the other limb allowed free to vary (referred as free flow limb).

4.5 Experimental results and discussion

In this section the discussion of the experimental results is reported. The discussion will be based on the comparison of the performance parameters between the three turbine configurations (nozzleless single-entry, variable geometry single and twin-entry). Firstly a comparison between the single-entry configurations will be made and then an overall comparison between the single and twin-entry configurations will be performed for different vane settings and flow capacities. Finally the partial and unequal admission conditions will be analyzed. The progression of the discussion is outlined below:

- Single-entry: nozzle vs. nozzleless
- Single vs. twin-entry
- Twin-entry: partial and unequal admission

Table 4.5: Performance parameters for the single (nozzled, nozzleless) and twin-entry turbine configurations under full admission

<table>
<thead>
<tr>
<th>Speed [rev/s√K]</th>
<th>Vane angle</th>
<th>η\text{peak}</th>
<th>U/C\text{peak}</th>
<th>IMP parameter ([\text{kg/s}^2\cdot\sqrt{\text{K/Pa}}\cdot10^{-3}])</th>
<th>PR\text{min-max}</th>
</tr>
</thead>
<tbody>
<tr>
<td>43.0</td>
<td>--</td>
<td>0.77</td>
<td>0.69</td>
<td>8.02</td>
<td>1.2-2.2</td>
</tr>
<tr>
<td>27.9</td>
<td>--</td>
<td>0.76</td>
<td>0.76</td>
<td>8.83</td>
<td>1.1-2.0</td>
</tr>
<tr>
<td>43.0</td>
<td>40°</td>
<td>0.61</td>
<td>0.63</td>
<td>8.40</td>
<td>1.1-2.0</td>
</tr>
<tr>
<td></td>
<td>50°</td>
<td>0.63</td>
<td>0.64</td>
<td>8.21</td>
<td>1.1-2.0</td>
</tr>
<tr>
<td></td>
<td>60°</td>
<td>0.80</td>
<td>0.72</td>
<td>7.12</td>
<td>1.2-2.2</td>
</tr>
<tr>
<td></td>
<td>70°</td>
<td>0.77</td>
<td>0.54</td>
<td>7.76</td>
<td>1.4-2.5</td>
</tr>
<tr>
<td>27.9</td>
<td>60°</td>
<td>0.76</td>
<td>0.64</td>
<td>8.10</td>
<td>1.1-2.0</td>
</tr>
<tr>
<td>43.0</td>
<td>40°</td>
<td>0.56</td>
<td>0.61</td>
<td>8.10</td>
<td>1.1-2.0</td>
</tr>
<tr>
<td></td>
<td>60°</td>
<td>0.79</td>
<td>0.68</td>
<td>6.77</td>
<td>1.2-2.2</td>
</tr>
<tr>
<td></td>
<td>70°</td>
<td>0.76</td>
<td>0.48</td>
<td>7.23</td>
<td>1.4-2.5</td>
</tr>
<tr>
<td>27.9</td>
<td>60°</td>
<td>0.76</td>
<td>0.66</td>
<td>8.10</td>
<td>1.1-2.0</td>
</tr>
</tbody>
</table>

In Table 4.5, the peak efficiencies with the corresponding velocity ratios are shown for comparison. As for the flow capacity the IMP parameter (Integral Mass flow Parameter) was

\(^5\) Partial admission is usually used to indicate the condition when one entry is blanked off and the flow only goes through the other limb. Unequal admission instead refers to the condition when flow is still going through both entries but with different pressure ratio within the two.
introduced. This corresponds to the area of the region in the \( MFP-PR \) plane bounded by the mass flow curve within the minimum and maximum pressure ratio into which the mass flow curve was generated\(^{10} \). The \textit{IMP parameter} is defined as given in Eq. (4.19).

\[
IMP = \int_{PR_{min}}^{PR_{max}} MFP(PR) dPR
\]  
(4.19)

### 4.5.1 Single-entry: nozzle vs. nozzleless

Figures 4.7 and 4.8 report the performance parameters for the nozzled and the nozzleless turbine (base line) for 27.9 \( \text{rev/s} \cdot \sqrt{\text{K}} \) and 43.0 \( \text{rev/s} \cdot \sqrt{\text{K}} \). For the nozzled turbine, the vane angle was fixed at 60° which corresponds to the vane angle set at the \textit{design-point}.

For the nozzleless turbine, a peak efficiency of 0.77 was measured at 43.0 \( \text{rev/s} \cdot \sqrt{\text{K}} \) showing a drop of \( \approx 1 \) percentage points at low speed condition (27.9 \( \text{rev/s} \cdot \sqrt{\text{K}} \)). In the nozzleled configuration at 43.0 \( \text{rev/s} \cdot \sqrt{\text{K}} \), an efficiency improvement of \( \approx 3 \) percentage points at low velocity ratios was found in respect with the nozzleless case. The same improvement was not measured at 27.9 \( \text{rev/s} \cdot \sqrt{\text{K}} \) for which the same peak efficiency of 0.76 was found. In the nozzleless configuration the efficiency curve shows a shift towards higher velocity ratio areas of the map; at the peak efficiency point, the velocity

---

\(^{10}\) For each operating condition, the same minimum and maximum pressure ratios were considered in order to evaluate the \textit{IMP parameter} within the same boundaries. In the regions of the maps where no test data were available, an extrapolation of the mass flow curve was performed and the \textit{IMP parameter} was successively calculated.
ratio shifts from 0.76 to 0.64. Such a shift is significant for energy extraction, as in the real pulsating condition of the turbine high efficiency at high pressure ratio (low velocity ratio) is desired. One would thus presume that such a performance curve would lead to greater pulse flow performance. A comparison between the pulsating flow performance for single-entry nozzled and nozzleless turbine is provided in Chapter 6.

![Graphs showing Efficiency vs. U/C and MFP vs. PR for single-entry mixed-flow turbine performance at 27.9 rev/s\sqrt{\text{K}}](image)

Although the peak efficiency point between the nozzled and the nozzleless configurations did not vary substantially, the addition of nozzles revealed to be detrimental to the flow capacity. At 43.0 \text{rev/s}\sqrt{\text{K}} the IMP parameter for the nozzled turbine is \(7.12 \times 10^{-3}\) (kg/s)\sqrt{\text{K/Pa}} which is \(\approx 12\%\) lower than the nozzleless turbine (=\(8.02 \times 10^{-3}\) (kg/s)\sqrt{\text{K/Pa}})\(^{11}\). As we reduce the speed to 27.9 \text{rev/s}\sqrt{\text{K}} the IMP parameter variation between the two turbine configurations is \(\approx 8\%\) (\(8.83 \times 10^{-3}\) (kg/s)\sqrt{\text{K/Pa}} and \(8.10 \times 10^{-3}\) (kg/s)\sqrt{\text{K/Pa} for the nozzled and nozzleless turbine respectively) which is less than that measured at 43.0 \text{rev/s}\sqrt{\text{K}}. This can be attributed to a reduced effect of the centrifugal head at lower rotational speeds that enables the turbine wheel to swallow more mass flow.

The equivalent geometry between the two configurations did not reveal neither particularly detrimental nor beneficial effect to the turbine performance. Despite a slight improvement on the efficiency for high rotational speed, Figs. 4.7 and 4.8 show that that the nozzled configuration is less performing than an equivalent geometry nozzleless turbine. In applications where the turbine is expected to work for most its life at high speeds and within a tiny range of pressure ratios the choice of a nozzled configuration is beneficial. On the other hand for applications where the turbine is

\[^{11}\Delta\text{IMP} = |1 - (\text{IMP}_{\text{prediction}}/\text{IMP}_{\text{exp}})|\]
expected to work at low speeds or with speed varying substantially within high and low ranges, the nozzleless configuration would be best.

4.5.2 Single vs. twin-entry in full admission

Twin-entry turbines are usually adopted to isolate the gas flow from each separate bank of manifolds. In twin-entry configuration the turbine works under unequal and/or partial admission conditions for most of its operation; consequently, full admission conditions do not replicate the working conditions of the turbine under normal engine operation. Nevertheless, turbine maps are usually available only for full admission conditions.

Figures 4.9 and 4.10 report the turbine efficiency under full admission at 43.0 rev/s·√K and 27.9 rev/s·√K. Similar to the single-entry, the peak efficiency point was found to occur at 43.0 rev/s·√K for the 60° vane angle. Nevertheless, it can be noticed that in the high velocity ratio areas of the map, the twin-entry turbine performs better than the nozzled single-entry. This occurs at 27.9 rev/s·√K and 43.0 rev/s·√K.

A previous study by Baines and Yeo (1994), using LDV on a nozzleless twin-entry turbine, showed that under full admission conditions, the flow field at the exit to the volute remains unaltered across different operating conditions. The flow angle leaving the volute follows closely the conservation of the angular momentum in inviscid conditions, according to which the angle is a function of the geometry only. However, it must be noted that the measurements by Baines and Yeo (1994) only covered operating conditions around the design-point (0.56 – 0.74) and no measurements were carried out for high velocity ratios, where significant mixing effects might be expected. The test results obtained within this research support the experimental results obtained by Baines and Yeo (1994). At low velocity ratios, the peak efficiency of the twin-entry turbine was found to be 0.79; this is only ≈1% lower than that measured in the nozzled single-entry configuration. Such a difference tends to be smaller at 27.9 rev/s·√K, for which no discrepancy between the efficiencies was measured in the low velocity ratios area of the map (U/C_w ≈ 0.4 to ≈ 0.7). This seems to suggest that the presence of the divider has very little effect on turbine efficiency. However, as we move towards high velocity ratio regions of the maps (U/C_w=0.8-1.0), the efficiency of the single-entry configuration was found to be 12% and 14% percentage points lower than the twin-entry at 27.9 rev/s·√K and 43.0 rev/s·√K respectively (refer to Figs. 4.9 and 4.10). This can be attributed to the flow mixing process occurring in the nozzle and rotor passage.

A mixing process is associated with entropy generation and hence with loss of efficiency. The experimental evidence of Figs. 4.9 and 4.10 indicates that the performance of the twin-entry turbine is higher than the single-entry in the high velocity ratio of the maps. The mixing loss that generates at the exit to the volute seems to be overcome by a favorable flow field that reduces the efficiency drop.
associated with energy dissipation. This could probably be attributed to a reduced impact of incidence loss\(^{12}\) (that accounts for a large portion of the energy lost) in respect to the single-entry case.

\(^{12}\) Also secondary flows and blockage due to growth of boundary layers should be taken into account.
In order to further assess the performance of single and twin-entry turbines, an additional set of tests were carried out for different vane angles. The turbine performance in the 70° and 40° vane angle range was measured at 43.0 rev/s·√K for both the single and the twin-entry turbine. Figures 4.11 and 4.12 report the results for 70° and 40° vane angles while Fig. 4.13 merges together all the cases (40°, 60° and 70°) to help comparison. From the test measurements, it can be noticed that the flow capacity for the twin-entry configuration is not heavily affected by the presence of the divider.

Figure 4.11: Single vs. Twin-entry at 43.0 rev/s·√K - 70° vane angle

Figure 4.12: Single vs. Twin-entry at 43.0 rev/s·√K - 40° vane angle

Figure 4.13: Single vs. Twin-entry at 43.0 rev/s·√K - 40°, 60°, and 70° vane angles

(SE = Single-entry, TE = Twin-entry)
The IMP parameter for the twin-entry turbine at 70° and 40° vane angles is \(7.23 \times 10^{-3}\) (kg/s) \(\sqrt{K/Pa}\) and \(8.10 \times 10^{-3}\) (kg/s) \(\sqrt{K/Pa}\) respectively that is \(\approx 6.6\%\) and \(\approx 4.0\%\) lower than that measured in single-entry. Regarding the efficiency, at 70° vane angles for both single and twin-entry configurations showed no difference for low velocity ratios while at higher velocity ratios an improved efficiency was measured for the twin-entry turbine. At 70° vanes angle the flow is still well guided by the nozzles and the discrepancy with the efficiency measured at 60° vane angle is no larger than few percentage points (\(\approx 3.0\%\)). The same pattern was not observed at 40° vane angle where the twin-entry turbine exhibits an efficiency drop of almost 5% over the entire range of velocity ratios in respect to the single-entry. This can be attributed to both the departure of the vane angle from the design-point and to the effects of flow mixing. In fact, by looking at the efficiency measured for the single-entry turbine (refer to Fig. 4.13), we can see that as the vane angle departs from the design-point (60° in our case) the efficiency drops as a consequence of the not optimum incidence conditions.

In order to complete the analysis, the turbine performance based on an equivalent flow capacity was also carried for the 43.0 rev/s \(\sqrt{K}\) conditions. The mass flow measured for the nozzleless turbine was taken as a reference value and therefore the vane angle had to be adjusted accordingly. The mass flow rate between the nozzleless and the twin-entry configuration was matched for 40° vane angle with a discrepancy of 1%. On the single-entry case, the vane angle was set at 50° and the IMP parameter was \(\approx 3.0\%\) larger than that calculated for the nozzleless configuration, close enough to help the comparison. The data plot for the efficiency and mass flow is given in Fig. 4.14.
The efficiency diagram confirms the considerations previously made about the role of incidence loss and flow mixing. As the vanes tend towards the fully open position, the flow deviates from the optimum incidence angle with consequent increase in incidence loss. The penalty in efficiency between the three configurations is significant; the peak efficiency drops from 0.77 (for the nozzleless turbine) down to 0.63 and 0.56 for the nozzled single and twin-entry respectively. However it is worth noting that in order to match the same flow capacity conditions, the operating point have moved away from the design-point. The design progression from single to twin-entry aimed to maintain the same geometrical parameters for the turbine while no assumption was made on the mass flow. No equivalence between the mass flows was sought for the three turbine configurations and this is confirmed by the data of Fig. 4.9 where the measured mass flow for the three turbines do not match each other even though they are operating at design conditions. Therefore the performance comparison based on equivalent flow capacity is not truly representative of the features owning to each turbine configuration because this would have implied a bespoke design of three different turbine configurations with the same flow capacity.

### 4.5.3 Twin-entry turbine analysis

**Partial admission**

Turbocharger turbines operate under unsteady conditions due to the pulsating nature of the exhaust gases. In consequence, twin-entry turbines are generally designed and used for better energy
extraction from the pulsating exhaust gases. As it was previously stated, twin-entry turbochargers always exhibit an imbalance of flow conditions between the two entries, caused by the manifold arrangement. A partial admission condition is usually associated with no flow through one entry while unequal admission is used to refer to a condition when the flow is not equally shared within the two entries. However it must be taken into account that, under normal engine operating conditions, the partial admission condition is not likely to occur. In fact given the high pulsating nature of the flow, even when the flow in one entry drops to zero and reverse flow might occur, the remaining flow will cover the rotor around the complete periphery. Nevertheless the importance of partial admission conditions becomes apparent when evaluating aerodynamic losses and translating these into the real pulsating operation of the turbocharger. This reveals to be particularly useful in software development where the models are usually calibrated against full and partial admission conditions (Fig. 4.15). For the study of twin-entry turbines, a new definition for the performance parameters as given in Eq. (4.17) and Eq. (4.18) must be provided. The mass flow parameter is calculated considering the contribution of each limb on the overall flow capacity. From the “energy equation” the stagnation temperature is calculated as a mass weighted average value while for the total pressure an area average value is considered. The final equation for the mass flow parameter in twin-entry is given in Eq. (4.20).

$$MFP_{\text{twin-entry}} = \frac{\sqrt{\frac{m_{\text{inner}}}{m_{\text{tot}}} T_{0,\text{inner}} + \frac{m_{\text{outer}}}{m_{\text{tot}}} T_{0,\text{outer}}}}{\frac{P_{0,\text{inner}}}{2} + \frac{P_{0,\text{outer}}}{2}}$$

$$= (m_{\text{inner}} + m_{\text{outer}}) \frac{\sqrt{\frac{m_{\text{inner}}}{m_{\text{inner}} + m_{\text{outer}}} T_{0,\text{inner}} + \frac{m_{\text{outer}}}{m_{\text{inner}} + m_{\text{outer}}} T_{0,\text{outer}}}}{\frac{P_{0,\text{inner}}}{2} + \frac{P_{0,\text{outer}}}{2}}$$

(4.20)
The equation above is of a general form and it includes the full admission condition. In fact for a given stagnation temperature \(T_{inner}=T_{outer}\) and equal mass flow rates within the two entries \(m_{inner}=m_{outer}\), Eq. (4.20) simplifies to Eq. (4.17). The mass flow parameter is usually plotted against the pressure ratio that in a twin-entry study corresponds to an area average pressure ratio between the two limbs, as given in Eq. (4.21).

\[
PR = \frac{A_{inner}}{A_{tot}} PR_{inner} + \frac{A_{outer}}{A_{tot}} PR_{outer} = \frac{1}{2} (PR_{inner} + PR_{outer})
\]  

(4.21)

The outcomes of the experimental results are reported in Figs. 4.16 and 4.17 where the turbine performance under full and partial admission conditions is shown for 27.9 rev/s·√K and 43.0 rev/s·√K. The test results showed that a large fall in efficiency occurs for the partial admission in respect to the full admission (Table 4.6); the drop is as large as ≈18 percentage points at 43.0 rev/s·√K and ≈23 percentage points at 27.9 rev/s·√K. Within the two limbs it was found that the inner and outer limb has the same flow capacity. Nevertheless the two limbs perform differently to one another.

Table 4.6: Turbine efficiency under partial admission condition

<table>
<thead>
<tr>
<th>Condition</th>
<th>(N/\sqrt{T_{0i}}=43.0) [rev/s·√K]</th>
<th>(N/\sqrt{T_{0i}}=27.9) [rev/s·√K]</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\eta_{full\ admission})</td>
<td>0.79</td>
<td>0.76</td>
</tr>
<tr>
<td>(\eta_{partial\ adm\ - outer\ closed})</td>
<td>0.61</td>
<td>0.53</td>
</tr>
<tr>
<td>(\eta_{partial\ adm\ - inner\ closed})</td>
<td>0.61</td>
<td>0.53</td>
</tr>
</tbody>
</table>

The equation above is of a general form and it includes the full admission condition. In fact for a given stagnation temperature \(T_{inner}=T_{outer}\) and equal mass flow rates within the two entries \(m_{inner}=m_{outer}\), Eq. (4.20) simplifies to Eq. (4.17). The mass flow parameter is usually plotted against the pressure ratio that in a twin-entry study corresponds to an area average pressure ratio between the two limbs, as given in Eq. (4.21).

\[
PR = \frac{A_{inner}}{A_{tot}} PR_{inner} + \frac{A_{outer}}{A_{tot}} PR_{outer} = \frac{1}{2} (PR_{inner} + PR_{outer})
\]  

(4.21)

The outcomes of the experimental results are reported in Figs. 4.16 and 4.17 where the turbine performance under full and partial admission conditions is shown for 27.9 rev/s·√K and 43.0 rev/s·√K. The test results showed that a large fall in efficiency occurs for the partial admission in respect to the full admission (Table 4.6); the drop is as large as ≈18 percentage points at 43.0 rev/s·√K and ≈23 percentage points at 27.9 rev/s·√K. Within the two limbs it was found that the inner and outer limb has the same flow capacity. Nevertheless the two limbs perform differently to one another.

![Figure 4.16: Partial admission twin-entry at 43.0 rev/s·√K - 60° Vane angle a- \(\eta_{ts}\) vs. \(U/C_{is}\) and b-MFP vs. PR](image-url)
The peak efficiency point for the inner and outer limb is the same for both speeds, which is 61\% and 53\% at 43.0 rev/s·√K and 27.9 rev/s·√K respectively. However as soon as one moves away from the peak efficiency point, the turbine performance when the outer limb is open shows a penalty compared with the inner limb. This can be attributed to the different paths taken by the flow for a given shroud curvature. As reported by Baines and Yeo (1994) when the outer limb is closed, the flow shifts significantly towards the shroud side of the turbine. On the other hand when the inner limb is closed, the axial component of the velocity is heavily reduced but still directed towards the shroud side even though the flow would be expected to move towards the lower pressure region. This demonstrates that, even though the turbine geometry is symmetric in respect to a radial plane, no symmetry exists in the flow path and hence in the turbine performance.

Under partial admission conditions only half of the volute is operating and this corresponds to the case when one entry is blanked off. At the present no method is available in literature to address how the partial and full admission conditions are correlated to each other. The general expression for the mass flow parameter given in Eq. (4.20) seems to work well under full and unequal admission condition but it fails to provide a satisfactory prediction of the mass flow parameter when used for the partial admission condition. This is shown in Figs. 4.18 and 4.19 where the flow capacity of twin-entry turbine under full and partial admission conditions are given at 27.9 rev/s·√K and 43.0 rev/s·√K. By simply halving the mass flow curve obtained in full admission, one would be treating the turbine as a single-entry turbine of half size without taking into account the interaction existing between the two limbs.
The resulting mass flow calculated by means of such an approach is given in Figs. 4.18 and 4.19. Here it can be seen that the mass flow calculated with Eq. (4.22) falls far from that measured experimentally (blue circles). The discrepancy between the measured and calculated mass flow
remains very large for both 27.9 rev/s√K and 43.0 rev/s√K over the entire range of pressure ratios considered in the maps. The weakness of the approach used in Eq. (4.22) lies in the fact that, even though in partial admission conditions no air flows at the inlet of one of the entries, stagnant air at near atmospheric pressure is still present within the no-flow limb. A flow leakage into the no-flowing entry from the flowing side can occur. This implies that atmospheric pressure must be included in Eq. (4.22) that hence it assumes the form given in Eq. (4.23). By blanking off one entry (for instance the inner limb) the total mass flow rate \( m_{\text{tot}} = m_{\text{outer}} \) and therefore Eq. (4.20) reduces to Eq. (4.23) given below.

\[
MFP_{\text{outer}} = \frac{\sqrt{T_{0,\text{outer}}}}{\frac{P_{0,\text{outer}}}{2} + \frac{P_{0,\text{inner}}}{2}} \approx \frac{m_{\text{outer}}}{\frac{P_{0,\text{outer}}}{2} + \frac{P_{\text{atm}}}{2}} \quad (4.23)
\]

As already stated, the total pressure \( P_{0,\text{inner}} \) has to be near atmospheric (\( P_{\text{atm}} \)) as there is no flow dynamic head. The centrifugal head imposed by the rotor rotation can be deemed to be small enough to consider a pressure equal to atmospheric conditions in the analysis. The mass flow parameter calculated with Eq. (4.23) is given in Figs. 4.18 and 4.19 by the black solid line. From the figures it can be noticed that the mass flow calculated from the full entry maps is much improved for both speeds (refer to Table 4.7). At high pressure ratios the newly computed mass flow matches that measured experimentally within few percentage points. At 43.0 rev/s√K and pressure ratio \( PR=2.1 \), the predicted mass flow deviates from that measured experimentally by only \( \approx 2\% \) while at 27.9 rev/s√K and pressure ratio \( PR=1.77 \) the discrepancy is about \( 5\% \). However at low pressure ratios the prediction is less accurate; the deviation from the test results goes from 17% at 43.0 rev/s√K to 21% at 27.9 rev/s√K. This can be explained if we consider that at low pressure ratios, the total pressure in the flowing limb is similar to atmospheric and hence the denominator of Eq. (4.23) does not change significantly when the term \( \frac{P_{0,\text{inner}}}{2} \) is added. However if we look at the IMP parameter in Table 4.7 we can see that the in overall, the mass flow calculated with Eq. (4.23) is three times more accurate than that provided by simple application of Eq. (4.22). At 27.9 rev/s√K the IMP parameter calculated with Eq. (4.23) is \( 10.4 \cdot 10^{-3} \) (kg/s)√K/Pa that is \( 22\% \) more accurate than that calculated with Eq. (4.22). The same occurs at 43.0 rev/s√K where the IMP parameter moves from \( 6.2 \cdot 10^{-3} \) (kg/s)√K/Pa of Eq. (4.22) to \( 7.9 \cdot 10^{-3} \) (kg/s)√K/Pa of Eq. (4.23), showing \( \approx 20\% \) improvement in the prediction.

**Unequal admission**

The previous discussion centred on what is commonly called partial admission where one port is completely closed. In this section, we report the results for cases between full and partial admission. These are labelled as unequal admission cases and they are very useful for assessing the
turbine performance on engine behaviour; no much data is available in literature for these unequal conditions.

Table 4.7: MFP and IMP parameter for the partial admission condition

<table>
<thead>
<tr>
<th>PR</th>
<th>MFP (x10^5) (Kg/s)^0.5/Pa for 27.9 rev/s/√K</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental results</td>
<td>1.63</td>
</tr>
<tr>
<td>Eq. (4.22)</td>
<td>1.25</td>
</tr>
<tr>
<td>ΔMFP = 30%</td>
<td>1.44</td>
</tr>
<tr>
<td>ΔMFP = 18%</td>
<td>1.44</td>
</tr>
<tr>
<td>ΔMFP = 7%</td>
<td>1.44</td>
</tr>
<tr>
<td>ΔMFP = 5%</td>
<td>1.44</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>PR</th>
<th>MFP (x10^5) (Kg/s)^0.5/Pa for 43.0 rev/s/√K</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental results</td>
<td>1.40</td>
</tr>
<tr>
<td>Eq. (4.22)</td>
<td>1.71</td>
</tr>
<tr>
<td>ΔMFP = 33%</td>
<td>2.11</td>
</tr>
<tr>
<td>ΔMFP = 30%</td>
<td>2.11</td>
</tr>
<tr>
<td>ΔMFP = 31%</td>
<td>2.11</td>
</tr>
<tr>
<td>ΔMFP = 32%</td>
<td>2.11</td>
</tr>
</tbody>
</table>

Comparison IMP (x10^-3) (Kg/s)^0.5/Pa

<table>
<thead>
<tr>
<th>N/√T01</th>
<th>IMP (x10^-3) (Kg/s)^0.5/Pa</th>
</tr>
</thead>
<tbody>
<tr>
<td>27.9 rev/s/√K</td>
<td>ΔIMP = 11.7</td>
</tr>
<tr>
<td>Test values</td>
<td>ΔIMP = 33%</td>
</tr>
<tr>
<td>Eq. (4.22)</td>
<td>ΔIMP = 11%</td>
</tr>
<tr>
<td>Eq. (4.23)</td>
<td>ΔIMP = 11%</td>
</tr>
</tbody>
</table>

One of the concerns in unequal admission is the procedure to conduct the steady tests. The most recent data available on unequal admission conditions for a twin-entry turbine were provided by Capobianco and Gambarotta (1993). Measurements under unequal admission were conducted for constant speed lines with flow ranging from zero to full flow in one limb (and vice-versa in the other limb) with unequal admission in between. The test results showed that the equivalent flow area\(^{15}\) of each entry strongly depends on the inlet pressure ratio even though the asymmetry of the housing used for the experiments made it difficult to perform a direct comparison between the two limbs\(^{16}\). This is not the case here since, as already shown in section 4.2.1, the area and the volume of the two limbs were maintained equally. Given that no standard procedure exists for testing under unequal admission conditions, for the purpose of this research, the tests were conducted by keeping constant the pressure ratio in one limb and let the other free to vary in order to match a given operating condition (in the following discussion, such a limb will be referred as free flow limb). The test conditions are reported in Table 4.4 while in Figs. 4.20 to 4.23 the efficiencies under unequal admission are given together with those in full and partial admission to aid comparison\(^{17}\).

\(^{14}\) ΔMFP = |MFP\(_{\text{prediction}}\) - MFP\(_{\text{test}}\)|

\(^{15}\) The test results were presented in terms of equivalent area (defined as a function of inlet pressure ratio for each sector) and mean expansion ratio.

\(^{16}\) The calculated area ratio between the cross sectional areas of the two limbs was \(0.96\) while the volume ratio was \(0.92\).

\(^{17}\) In Figs. 4.20 to 4.23, Outer and Inner Open refer to the partial admission conditions when either the Inner and Outer limb is closed.
From Figs. 4.20 and 4.21, it can be gathered that the full and partial admission efficiency curves are on either side of those measured under unequal admission. This shows that the maximum penalty in efficiency occurs for the partial admission conditions and this can be attributed to the flow recirculation from one limb to another. As soon as some mass flow flows through the non-flow limb, a significant change in turbine efficiency can be observed. For instance, at 27.9 rev/s√K and PR=1.3,
the turbine efficiency rises from ≈0.52 (partial admission) to ≈0.64 (unequal admission). As more pressure is made available in the constant pressure limb, the turbine efficiency under unequal admission tends to be equivalent to that measured in full admission. This can be seen in Fig. 4.20 where for PR=1.9 a peak efficiency of 0.79 was measured; however a sudden drop can be observed at lower velocity ratios.
The mass flow measured under unequal admission condition (Figs. 4.22 and 4.23) was plotted together with the corresponding mass flow measured under full and partial admission. The test results showed that unlike the partial admission condition, for which the mass flow curves depart substantially from those in full admission, the unequal admission condition presents similar flow capacity to that in full admission. For a given speed, the mass flow curves superimpose to one another for any pressure ratio within the constant pressure limb. As shown in Fig. 4.22, at 43.0 rev/s\(\sqrt{K}\) and constant pressure ratio of 1.9 in one limb, the mass flow under full and unequal admission is well matched. This suggests that under unequal admission condition the flows leaving the limbs are in favourable conditions for expanding in the turbine stage in a similar manner as the full admission case. In fact when one entry is blanked-off, the incoming flow from the other side will expand in a low pressure region (almost atmospheric) that causes an axial mal-distribution of the absolute velocity and of the flow angle at the rotor inlet. The flow tends to migrate from one limb to another with strong evidence of recirculation and reverse flow which cause the mass flow to drop (Baines and Yeo, 1994). This does not occur in unequal admission since even when a little amount of mass flow goes through one limb, this seems to be large enough to prevent any mechanism of recirculation and reverse flow affecting the overall flow capacity. However a certain departure between the mass flow curves can still be observed at high pressure ratios. The experimental results show that under unequal admission the turbine chokes at lower pressure ratios than those measured under full admission. This is clear in Fig. 4.22 where the turbine under full admission conditions chokes at a pressure ratio \(\approx 2.2\) while the choking point under unequal admission moves from \(\approx 1.86, \approx 2.0\) and \(\approx 2.07\) for pressure values of 1.5, 1.6 and 1.7 in the constant pressure limb. A similar pattern was observed at 27.9 rev/s\(\sqrt{K}\) as shown in Fig. 4.23.

Given the similarity exhibited by the mass flow curves, an attempt is made to understand whether or not a common pattern could be found between the overall mass flow and the individual limb. In order to do this, the mass flow within the free flow limb was plotted together with the corresponding mass flow under unequal admission conditions, Eq. (4.20). The mass flow curves are shown in Figs. 4.24 and 4.25. From Figs. 4.24 and 4.25, it can be seen that the mass flow in the free flow limb closely follows the same trend as for the corresponding unequal admission situation. The main difference is that the mass flow curve in the free flow limb covers a wider range of pressure ratios. This can be explained by looking at Eq. (4.24), which is used to evaluate the pressure ratio under unequal admission conditions.

\[
PR_{un} = \frac{1}{2} (PR_{inner} + PR_{outer}) = \frac{1}{2} (PR_{free\ \text{flow}} + PR_{\text{const}}) \tag{4.24}
\]

From Eq. (4.24) it is clear that by retaining the pressure in one limb, the overall pressure ratio shifts towards higher or lower values depending on whether the pressure ratio in the free flow limb is lower.
or higher than that in the constant pressure limb respectively. This constrains the mass flow parameter under unequal admission to vary in a narrow range of pressure ratios but still maintaining the same trend and magnitude of the mass flow measured under full admission.

Figure 4.24: Mass flow parameter under unequal admission within the free flow limb at 43.0 rev/s·√K: a- Constant pressure ratio in the outer limb; b- Constant pressure ratio in the inner limb

Figure 4.25: Mass flow parameter under unequal admission within the free flow limb at 27.9 rev/s·√K: a- Constant pressure ratio in the outer limb; b- Constant pressure ratio in the inner limb
In order to verify whether or not a correlation exists between the mass flow under unequal admission and that in the free flow limb, the ratio between the mass flow parameters and pressure ratios was calculated. Two non-dimensional parameters are introduced to help explanation: the Mass Flow Parameter ratio\(^{18}\) (MFP\(_R\)) and the Unequal Expansion Ratio (ER\(_U\)), see Eq. (4.25) and Eq. (4.26).

- **Mass Flow Parameter ratio:**

\[
MFP_R = \frac{MFP_{un}}{MFP_{free\ flow}} = \frac{m_{free\ flow} T_{0, free\ flow}}{m_{tot} T_{0, const}} + \frac{m_{const} T_{0, const}}{m_{tot} T_{0, const}}
\]

\[
= \frac{m_{free\ flow} \sqrt{P_{0, free\ flow}}}{m_{free\ flow} \sqrt{P_{0, free\ flow}}} + \frac{P_{0, const}}{2} + \frac{P_{0, const}}{2}
\]

\[\text{(4.25)}\]

- **Unequal Expansion Ratio:**

\[
ER_U = \frac{PR_{un}}{PR_{free\ flow}} = \frac{PR_{free\ flow}}{2} + \frac{PR_{const}}{2}
\]

\[\text{(4.26)}\]

The mass flow parameter ratio obtained with Eq. (4.25) was calculated for both limbs and plotted against the unequal expansion ratio. The data points for each limb were grouped together\(^{19}\) and are given in Fig. 4.26 with the x axis in reverse in order to aid comparison. The data plot reveals that both the inner and outer limbs follow a similar trend for any operating conditions. All the points collapse onto a unique curve independent of the speed and pressure ratio. Hence it can be inferred that a unique correlation exists between the mass flows under unequal admission and that in the free flow limb. Such a correlation can be expressed by a power trend line with good degree of approximation (\(R^2 \approx 0.85\)). The data plot of Fig. 4.26 also shows that the geometrical symmetry of the turbine (given by the meridional divider) is reflected in the two groups of the mass flow parameter ratios which are equivalent with respect the unequal expansion ratio. Such an assumption is further supported by the coefficients of the power trend line which present similar values for the two limbs (\(A_{outer} = 2.23, B_{outer} = 4.42, A_{inner} = 2.28, B_{inner} = 4.95\)). Therefore we can state that under unequal admission conditions, the two limbs interact in a similar manner and also that for a given operating condition in one limb (mass flow and pressure ratio), the operating conditions in the other are uniquely defined by a power trend correlation given in Eq. (4.28).

\[\text{The mass flow within the free flow limb was calculated as: } MFP_{free\ flow} = \frac{m_{free\ flow} \sqrt{T_{0, free\ flow}}}{P_{0, free\ flow}}\]

\[\text{(4.27)}\]

\(^{18}\)Each group was labelled as “Inner limb” and “Outer limb”.

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where $A$ and $B$ are the two constants by the power trend line. The similarity existing between the unequal and full admission mass flow curves, also seems to suggest that the ratio of Eq. (4.28) is equal to the ratio between the full admission mass flow and that going through the free flow limb, as given in Eq. (4.29).

$$\frac{MFP_{un}}{MFP_{free\ flow}} = A(ER_U)^B$$

(4.28)

If such an assumption was true it would be possible to uniquely correlate the unequal to the full admission conditions and hence determine each point of the flow capacity under unequal admission given a point in the full admission curve. By expanding Eq. (4.29), we can see that the mass flow ratio is a function of the ratio between mass flows and pressure ratios as given in Eq. (4.30).

$$\frac{MFP_{un}}{MFP_{free\ flow}} \approx \frac{MFP_{full}}{MFP_{free\ flow}}$$

(4.29)

$$\frac{\dot{m}_{un}}{\dot{m}_{free\ flow}} \approx \frac{\dot{m}_{full}}{\dot{m}_{free\ flow}} \frac{\sqrt{T_{0,full}}}{\sqrt{T_{0,full}}} = \dot{m}_{full} \frac{P_{0,free\ flow}}{P_{0,full}}$$

(4.30)

The term $P_{0,free\ flow}/P_{0,full}$ corresponds to the unequal expansion ratio, as shown in Eq. (4.34). Therefore Eq. (4.30) becomes:

$$\frac{MFP_{full}}{MFP_{free\ flow}} = \frac{\dot{m}_{full}}{\dot{m}_{inner}} (ER_U)^{-1}$$

(4.31)

By combining Eq. (4.27) and Eq. (4.31), the final correlation for the mass flow parameter ratio is given in Eq. (4.32).

$$\frac{\dot{m}_{full}}{\dot{m}_{free\ flow}} = A(ER_U)^{(B+1)}$$

(4.32)

---

20 The temperatures $T_{0,full}$ and $T_{0,free\ flow}$ simplify since similar total temperatures were set during testing.

21 $\rho_{0,full} = \frac{P_{0,full}}{\dot{m}_{free\ flow}} + \rho_{const}$...
The general expression of Eq. (4.32) is provided in Eq. (4.35).

\[
\dot{m}_{\text{limb}} = \frac{\dot{m}_{\text{full}}}{A} \left( \frac{P_{\text{limb}}}{P_{\text{full}}} \right)^{(B+1)}
\]  

(4.35)

Eq. (4.35) correlates the mass flow in full admission with that in one limb for the unequal admission conditions. For a given point in the full admission curve the mass flow going through each limb is uniquely defined for a given point of the total pressure.
In order to validate Eq. (4.35) a simple procedure was developed and applied against the experimental results. For a fixed pressure ratio in one limb, the mass flow rate in the free flow limb was calculated for each point of the full admission curve. The calculation procedure is shown in Fig. 4.27. The input parameters are the mass flow and the total pressure and temperature in full admission \( (\dot{m}_{\text{full}}, P_{0,\text{full}}, T_{0,\text{full}}) \) and the total pressure in the constant pressure limb \( (P_{0,\text{const}}) \). The total pressure in the free flow limb is calculated using Eq. (4.33). The unequal expansion ratio is then calculated with Eq. (4.26) and finally the mass flow in the free flow limb is determined with Eq. (4.35).

The outcome of the procedure described above is given in Figs. 4.28 to 4.34. Each figure contains the mass flow in the free flow limb measured for a given speed and pressure ratio in the constant pressure limb, either inner or outer. The mass flow in the constant pressure limb were also included and plotted against the pressure ratio of the free flow limb. In this manner it is possible to visualize how the mass flows in the two limbs add up in order to give the overall mass flow under unequal admission. The predicted mass flow in the free flow limb is given by a dashed black line for each of the test cases considered in the analysis.

![Figure 4.28: Unequal admission: mass flow calculation procedure, PR=1.9 – 43.0 rev/s√K](image)
Figure 4.29: Unequal admission: mass flow calculation procedure, PR=1.7 – 43.0 rev/s·√K

Figure 4.30: Unequal admission: mass flow calculation procedure, PR=1.6 – 43.0 rev/s·√K
Figure 4.31: Unequal admission: mass flow calculation procedure, PR = 1.5 – 43.0 rev/s√K

Figure 4.32: Unequal admission: mass flow calculation procedure, PR = 1.5 – 27.9 rev/s√K
The results of Figs. 4.28 to 4.34 show that the prediction of the mass flow in the free flow limb is good over the whole range of operating conditions considered here. The prediction curve
follows consistently with the experimental results over the entire range of pressure ratios with good agreement between the IMP parameters, refer to Table 4.8. For PR=1.9 in the constant pressure limb, the mass flow calculated with Eq. (4.35) is almost fully matched with the value measured experimentally; the IMP parameters differs only by 1.3%. The same level of agreement is maintained for lower pressure ratios: PR=1.7, 1.6, 1.5 at 43.0 rev/s√K. For these cases, the discrepancy between the IMP parameters of Eq. (4.35) and the experimental results is 3.3%, 1.3%, 3.5% respectively. The discrepancy slightly increases to 4.5%, 2.3%, 4.2% for PR=1.5, 1.4, 1.3 respectively at the lower speed of 27.9 rev/s√K. This can mainly be attributed to the disagreement between experimental and computed mass flows in the low pressure region of the maps. In these regions it can be noticed that Eq. (4.35) does not completely capture the experimental mass flow. This is particularly evident at 27.9 rev/s√K and low pressure ratio in the constant pressure limb (PR=1.4 and 1.3); this is due to the inability of the power trend function of Eq. (4.35) to fit the MFP ratio data points for high values of the unequal expansion ratio. In fact, from Fig. 4.26 it can be seen that for high values of ERU the power trend line fails to provide an adequate fit of the data points. High values of ERU are associated with low values of pressure in the free flow limb that explains the lack of agreement for Eq. (4.35) in the low pressure ratio regions of the maps.

<table>
<thead>
<tr>
<th>PR</th>
<th>IMP –Test [kg/s]·√K/Pa-10&lt;sup&gt;3&lt;/sup&gt;</th>
<th>IMP – Eq. (4.35) [kg/s]·√K/Pa-10&lt;sup&gt;3&lt;/sup&gt;</th>
<th>ΔIMP [%]</th>
</tr>
</thead>
<tbody>
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<td>1.9</td>
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<td>5.72</td>
<td>≈1.3</td>
</tr>
<tr>
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<td>7.04</td>
<td>≈3.3</td>
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<tr>
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<tr>
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<td>7.17</td>
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<tr>
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<td>6.28</td>
<td>6.58</td>
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<td>6.08</td>
<td>≈2.3</td>
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<tr>
<td>1.3</td>
<td>6.03</td>
<td>6.29</td>
<td>≈4.2</td>
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</tbody>
</table>

### 4.6 Meanline analysis

This section reports a method for the off-design performance prediction in a nozzled and nozzleless mixed-flow turbine. The method is termed as a mean line analysis procedure as it does not take into account away from the centre blade/volute conditions. As such, it is suitable for turbine map generation and preliminary design evaluation.

The continuous demand for engines to operate in off-design conditions makes meanline models crucial to determine accurately the engine performance. Current engine software relies extensively on steady state maps that need to be extrapolated beyond the conventional range of velocity ratios. In fact, the lack of test data in the lower and higher velocity ratios, forces designers to extend turbine performance characteristics relying on polynomial extrapolation. However, the lack of data to validate these models makes this work particularly important as the dynamometer at Imperial
College can obtain data well away from the area where the design point lies; the rig available at Imperial College enables to extend the performance maps up to five times the conventional test facility. In a meanline model, the main difficulty arises from the fact that the flow within the turbine is three-dimensional. This behaviour cannot be easily captured by a meanline model and makes it challenging for software developers to generate a valid prediction. A meanline model in steady state conditions considers the flow path through the main stream line, neglecting the flow patterns from blade to blade and along the radius. Obviously this leads to some inevitable assumptions that arise from considering the properties of the gas to be constant with respect to a plane normal to the turbine axis. The meanline analysis relies on the solution of the turbomachinery equations and in order to take into account the effect of energy dissipation which occurs throughout the turbine, a set of relations describing the losses is considered. The loss equations used in the current study have been taken from those available in the literature; their effectiveness when applied over a wide range of pressure ratio was investigated and the outcomes are reported in the sections below.

![Figure 4.35: a- Inlet velocity triangle; b- Exit velocity triangle](image)

1. Inlet to the turbine
2. Inlet to the rotor
3. Station upstream exit to the rotor
4. Exit to the turbine

![Figure 4.36: Stations included in the model – a: Nozzleless – b: Nozzled](image)

1. Inlet to the turbine
2. Inlet to the nozzles
3. Station upstream exit to the nozzles
4. Station downstream exit to the nozzles
5. Inlet to the rotor
6. Station upstream exit to the rotor
7. Station downstream exit to the rotor
4.7 Flow model

The starting point of the analysis considers the velocity triangles entering and leaving each stage of the turbine (Fig. 4.35). Due to the large change in radius, the velocity triangles should be considered at the hub and at the tip, however in meanline analysis the gas properties will refer to the mean radius. The extraction of power will then be a direct calculation of the energy associated with the flow and consequently the performance parameters will be predicted.

Three main elements must be included in the model in order to fully characterize the conditions of the flow going through the turbine. These are: the stator, the nozzle ring and the rotor. For each of these elements the main geometrical parameters must be specified together with the losses associated with the flow conditions. This step is crucial to the effectiveness of the model to accurately predict the turbine performance. The stations where the flow is solved are given in Fig. 4.36. The geometrical specifications for each element included in the model can be found in paragraphs 4.2.1 & 4.2.2, while the losses included in the model are described in the following sections.

4.7.1 Stator

In order to distribute the mass uniformly around the circumference of the rotor, the ratio between the cross sectional area and the corresponding centroid radius must be a linear function of the azimuth angle (refer to Eq. (4.4)) For a given mass flow rate and density, the radial component of the velocity going into the rotor is fixed by the cross sectional area at the rotor inlet. This means that by varying the area at the inlet, the angle is consequently changed according to Eq. (4.5). However the effect of mixing, recirculation and secondary flow causes the flow pattern to deviate from the ideal one. Some correlations can be used to describe the effects of these losses on the turbine performance:

\[ K_{PL} = \frac{P_{01} - P_{02}}{P_{02} - P_2} \]  

Typical values for this coefficient are between 0.1 and 0.3 (Japikse and Baines, 1994). In the current study a value of 0.1 was found to fit with the turbine.
Swirl coefficient ($S$): under ideal conditions, the angular momentum of the flow is conserved. In actual conditions, the friction existing between the wall end of the volute and the flow produces a loss. A swirl coefficient $S$ in the angular momentum equation is introduced to model this loss, as in Eq. (4.37).

$$C_{\theta 1}r_1 = SC_{\theta 2}r_2$$  \hspace{1cm} (4.37)

Typical values for the swirl coefficient are in a range between 0.85 and 0.95 (Japikse and Baines, 1994) and $S=0.9$ was considered in the model.

Blockage factor ($B$): additional losses occurring near the leading edge are generally defined as secondary losses. These losses are due to the growth of boundary layers, secondary flows with the influence of tip leakage and recirculation occurring at the tongue. In a mixed-flow turbine, the inlet flow angle lies between the axial and radial design; the flow path is thus decreased and as a consequence, the formation of secondary losses is also reduced. However, like radial inflow turbines, the secondary losses are strongly affected by the angle of approach of the flow to the blades. Conditions of radial (zero degree) or negative large incidence angles cause the flow to separate at the suction or pressure surface thus creating a region of recirculation (Moustapha et al., 2003). It is clear that the sum of all these effects will lead to a complex flow pattern that can only be fully described with more complex two or three dimensional techniques. For a meanline model the introduction of a factor in the continuity equation is generally used to lump all these losses together:

$$\dot{m} = \rho A(1 - B)C$$  \hspace{1cm} (4.38)

The blockage factor $B$ is given by the ratio between the geometrical area of the throat and the effective area. It depends on the design of the wheel and usually lies in a range between 0.95 and 0.85 (Japikse and Baines, 1994), shown in Eq. (4.39).

$$B = 1 - A_{ratio} = 1 - \frac{A_{eff}}{A_{geo}}$$  \hspace{1cm} (4.39)

For the current wheel, the blockage factor $B$ was found to be approximately equal to 0.9.

4.7.2 Nozzles

The function of nozzles in a turbine is to accelerate and direct the fluid flow at the required design angle to the rotor with as minimum loss as possible. The total enthalpy within the nozzles
remains constant since no work transfer or heat transfer is present. However the total pressure drops as a result of the combination of friction and incidence losses.

-Kinetic energy loss: the losses going through the nozzles heavily rely on empirical data and are usually assumed to be a function of the mean kinetic energy of the flow. As for the current study, a static kinetic energy loss, $\zeta_N$, was included in the model. For each vane angle, different values for $\zeta_N$ were included in the model in order to calibrate the model. More details on the evaluation of this loss are given in section 4.9.

-Exit flow angle (nozzles): the flow leaving the nozzles does not follow the vanes completely. The growth of boundary layers and the sudden expansion due to the finite trailing edge thickness, cause the flow to turn towards the meridional direction. Hiett and Johnston (1963) evaluated the flow angle at the exit to the nozzles by mean of a cosine rule given in Eq. (4.40).

$$\cos \alpha_3 = \frac{l_{th}}{l_{sp}}$$

where $l_{th}$ is the length of the throat formed by two adjacent nozzle blades and $l_{sp}$ is the circumferential spacing at the trailing edge. Eq. (4.40) was used in the model and applied in station 3 of Fig. 4.36-b.

-Exit flow angle (clearance): the flow going through the clearance does not affect the main stream for low pressure ratios and/or large opening vanes angle. However, as the vanes close, the area available to the flow decreases and as a consequence of this the flow going through the clearance becomes significant. As such, the deviation of the flow from the main stream increases and the effects of mixing on the flow leaving the nozzles cannot be neglected.

In the current model the total temperature and pressure in the clearance were considered to be constant (similar to those of the mainstream). The flow angle and the mass flow rate were determined iteratively from the conservation of the tangential momentum (Meitner and Glassman, 1983) as given in Eq. (4.41) and Eq. (4.42).

$$r_3 c_{\theta 3,cl} = r_2 c_{\theta 2}$$  \hspace{1cm} (4.41)

$$\alpha_{3,cl} = \sin^{-1} \left( \frac{c_{\theta 3,cl}}{c_{3,cl}} \right)$$  \hspace{1cm} (4.42)

The mass flow going through the clearance was then calculated as in Eq. (4.43).
The flow leaving the nozzle will mix with the clearance flow. Here the same total temperature was considered for stations 3 and 4, \( T_{03} = T_{04} \). The conservation of the tangential momentum was applied in order to calculate the tangential component of the velocity, as shown in Eq. (4.44).

\[
C_{\theta 4} = (1 - \zeta)C_{\theta 3} + \zeta C_{\theta 3,cl} \quad (4.44)
\]

where \( \zeta = \dot{m}/\dot{m}_{cl} \) is the flow fraction. Japikse (1985) provides a thorough derivation of Eq. (4.44) that was firstly used to describe the mixing process within an imaginary duct with infinitesimal radius. It is worth noting that Eq. (4.44) does not really represent a loss (since no energy loss is considered) but it enables us to evaluate the state of the flow due to the mixing.

In the interspace region between the nozzle (station 4) and the rotor leading edge (station 5), the total pressure loss was calculated with the turbulent equations between parallel plates in order to calculate an average flow angle as given in Eq. (4.45). The tangential momentum between these two stations is conserved and the change in total pressure loss is given in terms of \( \Delta p \) as given in Eq. (4.46).

\[
\alpha_{4-5} = \frac{\alpha_4 + \alpha_5}{2} \quad (4.45)
\]

\[
\Delta p = \frac{4f \cdot l_{4-5} \cdot \rho_4 \cdot C_4^2}{2D_h} \quad (4.46)
\]

where \( l_{4-5} \) is a flow path length, \( D_h \) is an hydraulic diameter and \( f \) is a friction factor that changes depending on the values of Reynolds number in station 4 (Meitner and Glassman, 1983).

### 4.7.3 Rotor

A mixed-flow rotor differs from the radial design due to the forward sweep of the leading edge, allowing the flow to enter into the rotor at an angle in between the axial and the radial direction. This feature permits the zero blade limitation of radial turbines to be overcome giving one extra degree of freedom for designers. The mixed-flow turbine permits peak efficiencies to be achieved at lower velocity ratios and thus increases the turbine loading capacity. This is due to the cone angle at

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The stations used for this section refer to those of the nozzleless turbine, Fig. 4.36-a. Hence station 2=inlet to the rotor, station 3= exit to the rotor.
the leading edge which allows positive blade angles to be obtained at the inlet. The main geometrical dimensions of the rotor are fixed as reported in Table 4.3.

Four main losses were assumed to take place in the rotor:

- **Clearance flow loss** ($L_{cl}$): the gap between the rotor and its shroud produces a leakage which is a source of pressure loss. This is primarily associated with the mixing processes taking place between the leakage flow and the main stream. For a shrouded blade, a loss of work clearly occurs because the mass flow that passes through the rotor is less than it would be if there was no leakage. The flow exiting from the gap mixes with the main stream creating a turbulent kinetic energy. This energy is not associated with work extraction but increases the entropy that is dissipated into heat. Experimental investigations (Moustapha et al., 2003) showed the formation of a vortex on the end wall between the tip gap and the flows circulating between the blade together with the formation of small bubbles at the blade tip due to separation. The mixing of the main stream with the flow behind the blade row leads to a higher swirl velocity which is a source of loss. In a mixed-flow turbine, the clearance loss is a function of both radial and axial clearances. An expression of this loss is given in Eq. (4.47) and considers the energy loss to be proportional to the ratio of clearance to passage height at the rotor exit (Wasserbauer and Glassman, 1975):

$$L_{cl} = \frac{2\Delta h_{23}(h_{cl}/2r_{3,tip})}{1 - (r_{3,tip}/r_{3,tip})}$$  \hspace{1cm} (4.47)

- **Incidence loss** ($L_{inc}$): incidence occurs when the direction of the relative velocity approaching the blades is not aligned to the direction of the blades at the inlet to the rotor. The incidence loss is defined as the energy lost from the fluid when turning from its direction of approach to the direction determined by the rotor passage. Negative values for incidence angle were shown to give a smooth turning of the flow into the blade passage. The incidence loss is calculated as the kinetic energy loss tangential to the blade (Futral and Wasserbauer, 1965). The correlation used in the current model is given in Eq. (4.48).

$$L_{inc} = 0.5[W_2\sin(\beta_2 - \beta_{b2} - \beta_{opt})]^2$$  \hspace{1cm} (4.48)

Eq. (4.48) is not satisfactory because of the limitation imposed by the sinusoidal function. The maximum value of $\sin^2 [(\beta_2 - \beta_{b2} - \beta_{opt})]$ is 1, and this contradicts cascade tests that show how a higher level of loss occurs when $\beta_2 - \beta_{b2} - \beta_{opt}$ is greater than $\pi/4$. Mizumachi et al. (1979),
overcame this weakness by proposing a new relation that was not proportional to the sine function, as shown in Eq. (4.49) and Eq. (4.50).

\[ L_{\text{inc}} = K_{\text{inc}} \left[ W_2 \sin(\beta_2 - \beta_{b2} - i_{\text{opt}}) \right]^2 \quad \text{for} \quad |\beta_2 - \beta_{b2} - i_{\text{opt}}| < \frac{\pi}{4} \]  

\[ L_{\text{inc}} = K_{\text{inc}} W_2^2 \left[ 0.5 + |\beta_2 - \beta_{b2} - i_{\text{opt}}| - \frac{\pi}{4} \right]^2 \quad \text{for} \quad |\beta_2 - \beta_{b2} - i_{\text{opt}}| > \frac{\pi}{4} \]

where \( K_{\text{inc}} \) is a coefficient that was introduced in order to calibrate the model. In the equations above, \( i_{\text{opt}} \) is the optimum incidence angle that is defined as the angle of approach at which the smallest loss occurs once the flow goes into the rotor. It is calculated based on the Stanitz (1952) relation for slip factor. This leads to the following expression for the optimum incidence angle:

\[ i_{\text{opt}} = \tan^{-1} \left[ -1.98 \cdot \tan \alpha_2 / Z (1 - 1.98/Z) \right] \]  

-Rotor passage loss \((L_p)\): this loss refers to all the losses occurring internally in the blade passage and is assumed to be proportional to the mean kinetic energy of the rotor flow, as expressed in Eq. (4.52).

\[ L_p = K_p \left[ W_2^2 \cos^2(|\beta_2 - i_{\text{opt}}|) + W_3^2 \right] / 2 \]

The loss coefficient \( K_p \) is an empirical coefficient and must be determined by calibrating with experimental results. In the current study a value of 0.2 was found to fit well in the model.

-Disk friction loss \((L_{df})\): this loss is related to a frictional loss occurring in the back face of the turbine disk as fluid leaks between the rotor and the back plate. It is given in terms of power loss and it is generally small (Meitner and Glassman, 1983), as expressed in Eq. (4.53).

\[ L_{df} = \frac{0.02125 U_2^2 \rho_2^2}{\bar{m} \left( \rho_2 U_2 \tau_2 / \mu \right)^{0.2}} \]

4.8 Model structure

In this section, the flow chart of the code is presented. A distinction was made between the nozzleless and nozzled turbine.
**Nozzleless turbine**

Figure 4.37 shows the calculation flow chart for the nozzleless turbine model. The boundary conditions are set at the inlet to the volute with the total temperature \((T_{01})\) and the total pressure \((P_{01})\) for each rotational speed. The values of the boundary conditions at each rotational speed are given in Table 4.9. Since no work transfer occurs in the stator, it is assumed that the heat transfer is negligible. As a direct consequence of the first law of thermodynamics, the total temperature at the inlet to the volute (station 1) and at the inlet to the rotor (station 2) is assumed to be the same: \(T_{01}=T_{02}\). Since, at the start of the code, it is difficult to know the range of mass flows with which the turbine operates at a given speed, it is simpler to set the Mach number as a loop parameter. In the nozzleless case the Mach number at the exit to the volute \((M_2)\) was chosen as a loop parameter and, for a given rotational speed, the performance maps were swept for constants increments of \(M_2\) varying from \(\approx 0.2\) to \(\approx 1.1\).

Once the boundary conditions and the loop parameter have been imposed, it is possible to solve the stator by matching the continuity equation between the inlet and the exit to the volute. A guess on the Mach number \((M_1)\) must be made in order to fully define the flow conditions at the inlet to the volute; the mass flow \((m_1)\) entering the turbine can be determined and the calculation begin. In this stage the blockage, pressure and friction loss must be included.

![Figure 4.37: Model flow chart – Nozzleless turbine](image-url)

Before matching the turbine wheel, the flow leaving the volute will further expand in the annular region between the tongue and the rotor leading edge. Here the flow angle will slightly depart...
from that given by the free vortex condition in Eq. (4.37). However here it was assumed that the flow conditions at the inlet rotor are the same as those at the exit to the volute. In the rotor, the upstream conditions are known and hence an initial guess on the exit pressure $P_3$ is made. Similarly to the stator, the rotor is solved by matching the continuity equation. Here the incidence, passage, clearance and disc friction loss are taken into account. The flow leaving the rotor is considered to be perfectly guided by the rotor. The deviation angle $\delta$ (refer to Fig. 4.35), defined as an indication of the level of flow guidance in the blades, was set at $0^\circ$ (station 3). However the same does not occur at choking conditions.

The choking condition corresponds to a value of the relative Mach number ($M_{3r}$) greater than unity and sets a limitation on the maximum mass flow rate that can be swallowed by the rotor. As the pressure ratio across the turbine increases, the blade passage chokes when the flow reaches a sonic condition at the throat. The choking condition is associated with shock waves that originate near the trailing edge. By keeping on adding more mass, the losses within the rotor will increase dramatically: flow separation and blockage were identified amongst the two main causes affecting the flow passage. The combination of these two losses, leads the fluid flow to substantially deviate from the blade angle ($\delta \neq 0^\circ$). Once choking is reached, the code runs a subroutine that keeps account of changed flow conditions. The mass flow is fixed at the value measured at choking conditions and, in order to complete the calculation of the performance parameters, constant decrements of the exit pressure ($P_{3,\text{choking}}$ as calculated by the model at choking condition) is then performed by the model. At the exit to the rotor (station 4), the tangential momentum and the total temperature are considered to be the same as those at station 3. The total pressure loss is calculated by considering a sudden expansion from the rotor trailing edge to the plane just downstream the trailing edge. From an initial guess of the exit flow angle $\alpha_4$, the flow conditions at station 4 are then calculated.

**Nozzled turbine**

The main structure of the model for the nozzled turbine remains similar to that described for the nozzleless turbine. However the boundary conditions are set at the inlet to the nozzles (station 2) instead of the inlet to the volute. Similar the nozzleless case, the heat transfer is assumed to be negligible and the Mach number at station 3 is used as loop parameter (Fig. 4.38).

In order to start the calculation, an initial guess of the Mach number at the inlet to the nozzle is made and the mass flow is calculated by mean of the continuity equation. Once the mass flow is calculated, the model solves for stator between stations 0 and 1. The total pressure at the inlet to the turbine can then be calculated and so the pressure ratio of the whole turbine stage. In station 4 the flow leaving the nozzles mixes with the clearance flow and in station 5 the total pressure drop was obtained from the laminar and turbulent equations for flow between parallel plates (an average flow angle between stations 4 and 5 was considered here). The flow entering and leaving the rotor (stations 5, 6 and 7) was solved in the same way as the nozzleless case.
This section reports the outcomes of the model prediction against the experimental results. In order to fully characterize the turbine performance, the turbine was tested over six different speeds varying from 27.9 rev/s·√K to 53.8 rev/s·√K as shown in Table 4.9. Each operating point of the turbine was obtained by varying the turbine load through the dynamometer while the inlet pressure was adjusted to maintain the desired speed. The performance diagrams are shown in Figs. 4.39 to 4.44. A comparison between the computed and the experimentally measured peak efficiency and choked mass flow is given in Table 4.10. The percentage deviation is also provided in the table.

The prediction from 53.8 rev/s·√K speed to 37.6 rev/s·√K speed is very good for both efficiency and mass flow. At 53.8 rev/s·√K speed, the model yields an accurate prediction for both efficiency and mass flow parameter at any velocity and pressure ratio. The maximum efficiency computed is 0.733 at a velocity ratio of 0.671 compared to experimental values of 0.724 and 0.681.
respectively. Also, from Table 4.10, the subroutine used for choking shows a good prediction once the choking is reached. At low velocity ratios the deviation at 53.8 rev/s·√K speed remains in a range around 1% and, also the computed mass flow seems to compare well with the experimental value, predicting the choking point to be 0.582 at a pressure ratio of 2.3. This is in line with the tests where choking occurs at 0.58. For lower speeds, 32.2 rev/s·√K and 27.9 rev/s·√K, the predicted data shows some disagreement with experiments for efficiencies at high velocity ratios while the mass flow parameter is still very well predicted.

Table 4.9: Turbine steady state boundary conditions

<table>
<thead>
<tr>
<th>Equivalent Speed (%)</th>
<th>( \frac{N}{\sqrt{T}} ) [rev/s·√K]</th>
<th>Inlet Total Temperature [K]</th>
<th>Inlet Total Pressure [Pa]</th>
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<td>135000</td>
</tr>
<tr>
<td>50</td>
<td>27.9</td>
<td>333</td>
<td>119000</td>
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**Nozzled turbine - 40°/60°/70° vane angle**

<table>
<thead>
<tr>
<th>Equivalent Speed (%)</th>
<th>( \frac{N}{\sqrt{T}} ) [rev/s·√K]</th>
<th>Inlet Total Temperature [K]</th>
<th>Inlet Total Pressure [Pa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>43.0</td>
<td>339</td>
<td>170000</td>
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</table>

Table 4.10: Computed and measured comparison

<table>
<thead>
<tr>
<th>Speed [rev/s·√K]</th>
<th>Efficiency</th>
<th>Deviation</th>
<th>Mass Flow at choking point [kg/s·√K/Pa]</th>
<th>Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>53.8</td>
<td>Computed</td>
<td>0.733</td>
<td>1.1%</td>
<td>0.582</td>
</tr>
<tr>
<td></td>
<td>Measured</td>
<td>0.724</td>
<td></td>
<td>0.581</td>
</tr>
<tr>
<td>47.5</td>
<td>Computed</td>
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<td>1.4%</td>
<td>0.600</td>
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<tr>
<td></td>
<td>Measured</td>
<td>0.722</td>
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<td>0.599</td>
</tr>
<tr>
<td>43.0</td>
<td>Computed</td>
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<td>2.15%</td>
<td>0.606</td>
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<tr>
<td></td>
<td>Measured</td>
<td>0.741</td>
<td></td>
<td>0.603</td>
</tr>
<tr>
<td>37.6</td>
<td>Computed</td>
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<td></td>
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<td>0.614</td>
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<td>Computed</td>
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<td>0.5%</td>
<td>--</td>
</tr>
<tr>
<td></td>
<td>Measured</td>
<td>0.712</td>
<td></td>
<td>--</td>
</tr>
<tr>
<td>27.9</td>
<td>Computed</td>
<td>0.699</td>
<td>2.0%</td>
<td>--</td>
</tr>
<tr>
<td></td>
<td>Measured</td>
<td>0.681</td>
<td></td>
<td>--</td>
</tr>
</tbody>
</table>

Table 4.11: RMSD for Efficiency and Mass Flow (Nozzleless turbine)

<table>
<thead>
<tr>
<th>Equivalent speed</th>
<th>100%</th>
<th>90%</th>
<th>80%</th>
<th>70%</th>
<th>60%</th>
<th>50%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Efficiency - RMSD [%]</td>
<td>1.03</td>
<td>1.83</td>
<td>2.44</td>
<td>1.58</td>
<td>3.22</td>
<td>7.97</td>
</tr>
<tr>
<td>MFP - RMSD [%]</td>
<td>1.59</td>
<td>2.10</td>
<td>3.56</td>
<td>3.78</td>
<td>5.67</td>
<td>6.26</td>
</tr>
</tbody>
</table>
Root Mean Square Deviation (RMSD) can be used in order to give a sense of how well the predicted performance values compare to the experimental results. The RMSD of an estimator \( Y_1 \) with respect to an estimated parameter \( Y_2 \) is defined as the square root of the mean squared error. This is given in Eq. (4.55). For \( Y_1 = [x_{1,1}, \ldots, x_{1,n}] \) and \( Y_2 = [x_{2,1}, \ldots, x_{2,n}] \) the \( n \) deviations can be summarized with statistics of the overall deviation. In the case under study, \( Y_1 \) represents the actual data, \( Y_2 \) the predicted data and the RMSD for both the efficiency and mass flow parameter is shown in Table 4.11 for each equivalent rotational speed\(^2\).

\[
RMSD(Y_1, Y_2) = \sqrt{\frac{\sum_{i=1}^{n} (x_{1,i} - x_{2,i})^2}{n}} \quad (4.54)
\]

RMSD value for the efficiency and the mass flow parameter confirms that from 53.8 rev/s\(\sqrt{K}\) to 37.6 rev/s\(\sqrt{K}\), the prediction is quite good over the whole range of velocity and pressure ratios. The disagreement between the data at 32.2 rev/s\(\sqrt{K}\) and above all at 27.9 rev/s\(\sqrt{K}\) speed is considerable. This could be due to some uncertainty in the experimental results during these conditions. This stems from inaccuracies in recording the torque when testing the turbine at lower turbine powers.

Table 4.12 compares the torque at peak efficiency (U/C\(_n\) \(\approx 0.68\)) and a high velocity ratio (U/C\(_n\) \(\approx 1.0+\)) at 27.9 rev/s\(\sqrt{K}\) and 53.8 rev/s\(\sqrt{K}\) turbine speeds. It is clear that the combination of a low speed and a high velocity ratio produces a very low torque value which will be much more susceptible to uncertainties in the measurement.

<table>
<thead>
<tr>
<th>Speed [rev/s(\sqrt{K})]</th>
<th>Power peak efficiency [W]</th>
<th>Power (\approx 1.0) velocity ratio [W]</th>
</tr>
</thead>
<tbody>
<tr>
<td>27.9</td>
<td>4300</td>
<td>300</td>
</tr>
<tr>
<td>53.8</td>
<td>38000</td>
<td>1742</td>
</tr>
</tbody>
</table>

\(^2\)Since the efficiency of a turbine is given in percentage, the RMSD value is clearly also in percentage. However, since the mass flow parameter is pseudo non-dimensional, the RMSD value is somewhat ambiguous. Thus to provide the reader with a clearer understanding on the accuracy of the prediction of the mass flow parameter, an RMSD value which compares the deviation with the actual mass parameter is calculated as given below. Here, the RMSD of the mass flow parameter is evaluated dividing the \(n\)th value of the deviation by the correspondent value of the mass flow parameter. In this way, a percentage estimation of the overall deviation of the predicted mass flow parameter can be supplied.

\[
RMSD = \sqrt{\frac{\sum_{i=1}^{n} \left( \frac{MFP_{act,i} - MFP_{pred,i}}{MFP_{act,i}} \right)^2}{n}} \quad (4.55)
\]

where \( \pi_{act} = [MFP_{act,1}, \ldots, MFP_{act,n}] \) \( \pi_{pred} = [MFP_{pred,1}, \ldots, MFP_{pred,n}] \).
Figure 4.39: Performance parameters for $N/\sqrt{T_{01}} = 53.8$ rev/s·√K - a: $\eta_{ts}$ vs. $U/C_{is}$ - b: MFP vs. PR

Figure 4.40: Performance parameters for $N/\sqrt{T_{01}} = 47.5$ rev/s·√K - a: $\eta_{ts}$ vs. $U/C_{is}$ - b: MFP vs. PR
Figure 4.41: Performance parameters for $N/\sqrt{T_{01}} = 43.0 \text{ rev/s·√K}$ - a: $\eta_{ts}$ vs. U/C\text{is} - b: MFP vs. PR

Figure 4.42: Performance parameters for $N/\sqrt{T_{01}} = 37.6 \text{ rev/s·√K}$ - a: $\eta_{ts}$ vs. U/C\text{is} - b: MFP vs. PR
Figure 4.43: Performance parameters for \( \frac{N}{\sqrt{T_{01}}} = 32.2 \text{ rev/s} \cdot \sqrt{\text{K}} \):
- a: \( \eta_{ts} \) vs. \( \frac{U}{C_{is}} \)
- b: MFP vs. PR

Figure 4.44: Performance parameters for \( \frac{N}{\sqrt{T_{01}}} = 27.9 \text{ rev/s} \cdot \sqrt{\text{K}} \):
- a: \( \eta_{ts} \) vs. \( \frac{U}{C_{is}} \)
- b: MFP vs. PR
Table 4.13 shows the uncertainties associated with the experimental performance parameters measured by the Imperial College test facility. The uncertainty is given in terms of the root sum square (RSS). This shows that at high velocity ratio the uncertainty of the efficiency values at 27.9 rev/s √K is approximately 7% which is significantly more than the uncertainty associated with 53.8 rev/s √K. Thus, since the uncertainty in the experimental data was higher under these conditions, it was expected that a greater discrepancy between computational and experimental values would be noted.

<table>
<thead>
<tr>
<th>Uncertainty (±)</th>
<th>100%</th>
<th>90%</th>
<th>80%</th>
<th>70%</th>
<th>60%</th>
<th>50%</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Efficiency</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Higher U/Cₚₙ</td>
<td>≈2.1%</td>
<td>≈2.6%</td>
<td>≈3.3%</td>
<td>≈4.1%</td>
<td>≈5.2%</td>
<td>≈7%</td>
</tr>
<tr>
<td>Lower U/Cₚₙ</td>
<td>≈1%</td>
<td>≈1%</td>
<td>≈1%</td>
<td>≈1%</td>
<td>≈1%</td>
<td>≈1%</td>
</tr>
<tr>
<td><strong>Mass Flow Parameter</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Higher U/Cₚₙ</td>
<td>≈0.9%</td>
<td>≈0.9%</td>
<td>≈0.09%</td>
<td>≈0.9%</td>
<td>≈0.9%</td>
<td>≈0.9%</td>
</tr>
<tr>
<td>Lower U/Cₚₙ</td>
<td>≈1.2%</td>
<td>≈1.4%</td>
<td>≈1.6%</td>
<td>≈1.7%</td>
<td>≈1.9%</td>
<td>≈2%</td>
</tr>
</tbody>
</table>

It is worth noting that outcomes of the prediction are heavily affected by the incidence loss. In the current model the incidence losses were included as given in Eq. (4.49) and Eq. (4.50). The original expressions of these equations over-estimate the performance prediction for high velocity ratios. This can be explained by considering that the range of experimental data usually available did not allow of validating the loss models over a wide range of experimental data. This is not the case here since the performance maps are very wide indeed given the availability of a high speed dynamometer. In order to match the experimental data a factor K_{inc} = 1.4 was included in Eq. (4.49) and Eq. (4.50). By calibrating the K_{inc} to the mixed-flow rotor under study, a more accurate estimation of the turbine characteristic is obtained. This can be seen in Fig. 4.45 where the prediction is given for different values of the incidence factor.

A value of the incidence factor greater than 1.4 corresponds to an over-prediction of the turbine efficiency and vice-versa. For K_{inc} = 1, Eq. (4.49) and Eq. (4.50) coincide with the original equations proposed by Mizumachi (1979) where no incidence factor is included; here it can be noticed that the model fails to provide an adequate prediction in the high velocity ratio region of the maps even though at the peak efficiency the prediction is still good. In the zoom-in on the right hand side of Fig. 4.45 it can be seen that the incidence factor K_{inc} has little effect on the turbine efficiency in the velocity ratio region near the peak. However as the velocity ratio increases the incidence factor becomes crucial to a good prediction. This made necessary to include an incidence factor in the model even though it must be take into account that K_{inc} is an empirical factor which needs to be calibrated for different rotors. The use of K_{inc} provides a satisfactory result that might be taken as a starting point for the development of a more general loss correlation.
One of the main benefits associated with meanline modelling is the fact that it is possible to evaluate and quantify the losses going through the turbine separately. The distinction between different types of losses is a simplification that does not exist in reality. In fact the loss mechanisms within the flow occur concurrently and are strictly interrelated; this makes practically impossible to fully isolate them. Nevertheless if a physical basis were determined for a particular loss, then the response obtained by meanline analysis would contain an insight of the physics of the flow. Hence an assessment of the contribution of each loss included in the model could be made with good degree of confidence.

In order to compare different losses there is a need for a common thermodynamic expression to which all of the common definitions can be related. As for most of the problems in turbomachinery, the loss of an actual process can be defined as the difference between the work transfer in ideal and actual conditions. For a process between states 1 and 2, the entropy difference is given as in Eq. (4.56).

\[
s_{01} - s_{02} = c_p\ln\left(\frac{T_{01}}{T_{02}}\right) + R\ln\left(\frac{P_{01}}{P_{02}}\right) \tag{4.56}
\]

If we consider an isenthalpic process within the same two states \(2_e\), Eq. (4.56) would be rewritten as in Eq. (4.57).
By subtracting Eq. (4.56) from the Eq. (4.57) and re-arranging one would obtain a final expression as given in Eq. (4.58).

\[
\sigma = \frac{P_{01}}{P_{02,E}} = e^{-(\Delta S/R)}
\]  

(4.58)

The \( \Delta S \) represents the entropy difference between the isentropic and actual states and the \textit{entropy gain} \( \sigma \) represents a compact and useful way to represent losses (Whitfield and Baines, 1990). In fact the \textit{entropy gain} can be easily connected to the loss formulation given in the previous paragraphs by mean of Eq. (4.59).

\[
\sigma_I = \left( 1 - \frac{L_I}{c_p T_{01}} \right)^{\frac{\gamma}{\gamma-1}}
\]  

(4.59)

Eq. (4.59) gives a base reference to evaluate all the losses going through the turbine. For the stator a different expression of \( \sigma \) was proposed by Horlock (1960) who transformed the total pressure loss coefficient of Eq. (4.36) into Eq. (4.60).
A typical breakdown of these losses was proposed by Rohlik (1968) who reported each loss against the specific speed. As for the current research, a loss analysis similar to that proposed by Rohlik (1968) was included for the turbine under study. The good prediction of the model compared to the extended range of experimental results made it possible to track the impact of each loss on the turbine efficiency. The result of the loss analysis is shown in Figs. 4.47 to 4.52 for all the equivalent speeds considered in the model. The losses were calculated in terms of entropy gain ($\sigma$) and plotted as a fraction of the overall isentropic energy available. For any given speed the turbine efficiency predicted by the model (solid black line in the diagrams) was used as reference condition from which the loss contribution could be added up. By doing this it is possible to quantify the contribution of every single loss for any given velocity ratio of the maps.

The stator, clearance and disc friction loss account for a small portion of the efficiency drop while the incidence and the passage loss are those contributing to the higher rate of energy loss. The stator loss shows a decreasing trend with increasing velocity ratio. In the high velocity ratio region of the map the stator loss is almost negligible while it becomes significant ($\approx$5% of the overall energy available) as the velocity ratio decreases. This is consistent with the physics of the flow going into the
stator. The pressure loss between the inlet and the exit to the volute is mainly associated with viscous losses that originate between the flow and the turbine wall and it varies with the square of the flow velocity. In a turbine map, low velocity ratios are associated with high flow velocity and hence with high values of the viscous loss. This justifies the trend for the stator loss computed by the model.

![Figure 4.48: Loss analysis at 47.5 rev/s·√K](image)

Similarly to the stator loss, the entropy gain due to the clearance and disc friction were found to contribute only a small amount of the overall efficiency loss. This is shown in Fig. 4.46 where the clearance and disc friction loss are shown against the velocity ratio. It can be noticed that the impact of these losses decreases with increasing the velocity ratio and decreasing the rotational speed. At 53.8 rev/s·√K and high velocity ratios the decrement of efficiency associated to the clearance is significant and equal to \( \approx 3.5\% \). As the velocity ratio decreases the efficiency loss becomes small and no greater than 0.5%. In the low velocity ratio regions of the map the contribution of the rotational speed on the clearance loss becomes negligible since all the curves tend to converge towards zero independent of the turbine operating conditions. The effect of speed on the clearance loss is well captured by the model which computes a decrease in the efficiency with decreasing speed. This is consistent with experimental results shown by Kammeyer et al. (2010). Also the efficiency loss due to clearance decreases with decreasing the velocity ratio. This seems to be in agreement with the physics of the flow going through the clearance since as one moves far from design point, the energy extraction out of the flow is less efficient and hence a higher amount of loss is generated. The disc friction (Fig. 4.46) exhibits a similar trend as that of the clearance. The impact of disc friction on the
turbine performance is a function of the rotational speed and flow velocity even though its contribution on the overall efficiency loss is no greater than 1%. The maximum penalty due to disc friction is only \( \approx 0.3\% \) (at \( 53.8 \text{ rev/s} \cdot \sqrt{K} \) and high velocity ratios) and it goes down to \( \approx 0.05\% \) with decreasing velocity ratio.

![Figure 4.49: Loss analysis at 43.0 \( \text{rev/s} \cdot \sqrt{K} \)](image)

The losses that account for most of the energy dissipation are the passage and incidence loss. These losses must be analyzed together as they are interrelated. In fact according to the definitions provided for these two losses, Eq. (4.48) and Eq. (4.52), it can be noticed that the passage loss increases with decreasing incidence loss; for inlet relative flow angles \( \beta_2 = \beta_{2,\text{opt}} \) the computed incidence loss is equal to zero which means that no kinetic energy of the flow is destroyed when turning into the blade rows. As a consequence, the kinetic energy of the flow is maximum and so is the rotor passage loss. The rotor passage loss is a function of the relative kinetic energy at the entry and the exit to the rotor and therefore a large value for the inlet flow velocity corresponds to a large impact of the passage loss on the overall energy dissipation. This is well evident at \( 53.8 \text{ rev/s} \cdot \sqrt{K} \) and \( 47.5 \text{ rev/s} \cdot \sqrt{K} \) (Figs. 4.47 and 4.48 respectively) where in the velocity ratio regions near the peak efficiency, the incidence loss does not contribute to the overall turbine efficiency loss. The model (and also the experimental data) showed that the turbine efficiency peaks near the transition region from subsonic to supersonic. In the supersonic region, the model retains the upstream conditions to the inlet to the rotor and completes the calculation by constant increments of the pressure ratio. This causes the
incidence loss to remain within negligible values while the rotor passage loss becomes dominant (shaded area labelled as maximum passage loss region). However far from the peak efficiency area, the incidence loss becomes increasingly significant with respect to the passage loss. The departure of the turbine operating conditions from the design-point cause the deviation in the flow angle entering the rotor from the optimum incidence angle, thus increasing the incidence loss. This can be inferred from the values of the inlet relative flow angle computed by the model (dashed red line in the model, \( \beta_{\text{inl,rot}} \)). At 53.8 rev/s\(\sqrt{K}\) and 47.5 rev/s\(\sqrt{K}\) the predicted values for \( \beta_{\text{inl,rot}} \) is between -10° and -30°. This is similar to the range -20° to -40° corresponding to a smooth passage of the flow through the blade rows. At high velocity ratios instead, the inlet relative flow angle goes through a dramatic turn with values going up to \( \approx -77° \) (\( U/C_{\text{is}} \approx 1.1 \)) which cause a large increase in incidence loss.

At lower rotational speeds, the effects of incidence on the overall turbine efficiency tend to become more and more significant over the entire range of velocity ratios. As the rotational speed decreases, the turbine chokes for velocity ratios far from the peak. As consequence of this the maximum passage loss region becomes smaller and limited to the area near the peak efficiency point. This can be noticed for rotational speeds going from 43.0 rev/s\(\sqrt{K}\) to 27.9 rev/s\(\sqrt{K}\) where the maximum passage loss region covers a small portion of the turbine maps with a consequent increase in the incidence loss. Additionally, it can be noticed that the transition of inlet relative flow angle

![Figure 4.50: Loss analysis at 37.6 rev/s\(\sqrt{K}\)](image-url)
from negative to positive values corresponds with an increase of the incidence loss. Again this can be explained as a consequence of the flow separation which causes an increase in the incidence loss.

Such a trend for the incidence loss is well evident at 32.0 rev/s·√K and 27.9 rev/s·√K. In these cases...
the turning of the flow from negative to positive values is maximum with $\beta_{\text{int, rot}}$ going from $\approx -70^\circ$ up to $\approx 40^\circ$. However as the flow crosses to the supersonic region, the incidence loss is overcome by the passage loss that becomes predominant.

**Nozzled turbine**

The data used for the validation of the nozzled single-entry turbine model are available for one single speed line (53.8 rev/s $\sqrt{\text{K}}$) and three different vane angles (40°, 60°, and 70°), refer to Table 4.9. The performance parameters were computed for the turbine and a comparison with the experimental results is given in Figs. 4.53 to 4.55.

The results for the 60° vane angle are shown in Fig. 4.53. Similar to the nozzleless turbine configuration, the prediction for the efficiency is satisfactory over the entire range of velocity ratios of the map. The experimental results showed that for 60° vane angle the turbine works at optimum operating conditions. For such a vane angle the model succeeds in predicting the turbine performance with good accuracy. This can be seen in Table 4.14 where the $RMSD$ for the efficiency and the mass flow are reported. At 60° vane angle the $RMSD$ is 2.6% and 1.1% for the efficiency and the mass flow respectively.
Figure 4.54: Performance parameters at 70° vane angle for \( N/\sqrt{T_{01}} = 43.0 \text{ rev/s} \cdot \sqrt{K} \):
- (a) \( \eta_{ts} \) vs. \( U/C_{is} \)
- (b) \( \text{MFP} \) vs. \( \text{PR} \)

Figure 4.55: Performance parameters at 40° vane angle for \( N/\sqrt{T_{01}} = 43.0 \text{ rev/s} \cdot \sqrt{K} \):
- (a) \( \eta_{ts} \) vs. \( U/C_{is} \)
- (b) \( \text{MFP} \) vs. \( \text{PR} \)
However the accuracy of the prediction tends to deteriorate for vane angles other than 60°. In Figs. 4.54 and 4.55 the performance prediction for both 70° and 40° vane angle is provided.

<table>
<thead>
<tr>
<th>Vane angle</th>
<th>40°</th>
<th>60°</th>
<th>70°</th>
</tr>
</thead>
<tbody>
<tr>
<td>Efficiency - RMSD [%]</td>
<td>1.31</td>
<td>2.6</td>
<td>3.3</td>
</tr>
<tr>
<td>MFP - RMSD [%]</td>
<td>5.0</td>
<td>1.1</td>
<td>5.5</td>
</tr>
</tbody>
</table>

At 70° vane angle, the flow entering the nozzle will dissipate kinetic energy as a consequence of the flow turning when entering into the nozzles. In addition to this, at 70° vane angle the area available to the flow is reduced which causes a drop in the flow capacity and also a significant influence of the mass flow going through the clearance on the main stream. The outcome of the model show that such a variation in the flow field is only partly captured by the model. The RMSD provided in Table 4.14 shows that the degree of accuracy of the efficiency is as good as that measured at 60° vane angle. The RMSD is 3.3% that is above to that obtained for 60° vane angle. Nevertheless, such a good prediction for the efficiency is not reflected in the mass flow prediction. The computed mass flow is largely under predicted with the RMSD as large as 5.5% and this can mainly be attributed to the incapability of the model to take into account the area variation due to the different vane setting.

A similar discrepancy in the mass flow was also calculated for 40° vane angle even though the model tends to over predict the mass flow rather than under predict it. The RMSD is ≈ 5% which makes the mass flow to fall far from the experimental data. On the other hand, the predicted efficiency is still satisfactory over the entire range of velocity ratios with RMSD as low as 1.31%.

The conventional set of equations proposed for nozzled turbine seems to work well for a given vane angle but fail to provide a good response when the vane setting changes. In fact the loss models proposed for the nozzle row did not fit in the current model; the correlations proposed by Balje (1952) and Rodgers (1987) provided to be unsatisfactory when applied to variable geometry under study. At 60° vane angle, the loss model proposed by Rodgers (1987) seemed to provide a good prediction with experimental results even though, at 40° and 70° vane angle the efficiency calculated by the model was far from that measured experimentally. This could probably be attributed to the large variation in the vane angle that makes Rodgers’ model unable to capture the real amount of loss. The empirical nature of these models together with the limited amount of data available could be amongst the reasons for the not satisfactory response of these loss models. The model was calibrated at the peak efficiency point for each vane angle included in the analysis. Three different coefficients, $\zeta_N$, equal to 0.16, 0.15 and 0.12 were introduced in the model for 40°, 60° and 70° vane angle respectively. These were applied to the absolute velocity of the flow leaving the nozzle ring as given in Eq. (4.61). This was included in the model in terms of kinetic energy loss coefficient.
However it must be noted that, as reported by Benson (1965), the impact of $\zeta_N$ on the overall turbine performance is not significant and the calibration was mainly affected by an appropriate choice of the passage loss coefficient. The model prediction is given in Figs. 4.56 to 4.58. Although the mass flow prediction is not fully satisfactory, the good prediction of the efficiency makes the loss analysis still viable. This is shown in Figs. 4.56 to 4.58 for the 60°, 70° and 40° vane angle respectively. At 60° the trend of loss mechanisms is similar to the nozzleless configuration. The disc friction and the clearance loss still remain negligible while the stator loss is no greater than few percentage points. At 70° vane angle the incidence loss becomes dominant over the entire range of velocity ratios. This loss still goes through a minimum value even though such a value does not correspond to the peak efficiency point of the turbine. The region where the efficiency is higher is computed for flow conditions where the passage loss is minimal ($U/C_{is}$ between 0.46 and 0.71) and accounts for only 3% - 4% of the overall efficiency loss. At 40° vane angle, a similar trend with the 70° vane angle was found for the incidence loss. The incidence loss is dominant in respect to the other losses over the entire range of the map. However unlike the other cases analyzed so far, at 40° vane angle the incidence loss does not exhibit a minimum value and this means that the relative incidence flow angle never matches the optimum flow condition.

\[
\zeta_N = \frac{h_{3} - h_{3,\text{is}}}{0.5c_{3}^2}
\] (4.61)
Figure 4.57: Loss analysis at 43.0 rev/s√K for 70° vane angle

Figure 4.58: Loss analysis at 43.0 rev/s√K for 40° vane angle

4.10 Uncertainty Evaluation

- Mass flow parameter. The Root-Sum-Square (RSS) uncertainty in the mass flow parameter can mainly be attributed to the mass flow rate measurement. This varies between ± 0.9% – 2.3% for the
test points range and it shows little effect with equivalent speeds. The overall uncertainty in the steady pressure measurement is $\pm 470 \text{ Pa}$ and $\pm 90 \text{ Pa}$ for the high and low pressure transducer respectively.

The Root-Sum-Square ($RSS$) uncertainty in the pressure ratio is between $\pm 0.1\% - \pm 0.3\%$ for the test points range. In general, the uncertainty in pressure ratio increases for high pressure ratio region, mainly contributed by the higher uncertainty in the high pressure transducer.

- Efficiency. The uncertainty in efficiency varies substantially with both velocity ratio and equivalent speed. These in turn depend on the uncertainties in the torque and mass flow rate measurements. At high velocity ratios the uncertainty is dominated by the torque measurement since the turbine power in these regions is very low. The Root-Sum-Square ($RSS$) uncertainty in the turbine efficiency is between $\pm 1\% - \pm 7\%$ efficiency points for the range of test points. The highest uncertainty is measured for a combination of high velocity ratio and low equivalent speed while it remains within low values for high power absorption. The uncertainty stays below $\pm 1.5\%$ efficiency points for velocity ratio up to 0.7, above which the uncertainty increases. The Root-Sum-Square ($RSS$) uncertainty in the velocity ratio is between $\pm 0.2\% - \pm 0.6\%$ for the range of test points.

4.11 Summary

The results of the steady state performance analysis are presented in this chapter. A variable geometry twin-entry mixed-flow turbine was tested under full admission for two non-dimensional speeds of $27.9 \text{ rev/s} \cdot \sqrt{\text{K}}$ and $43.0 \text{ rev/s} \cdot \sqrt{\text{K}}$ and three different vane angle setting of $40^\circ$, $60^\circ$ and $70^\circ$. Tests under partial and unequal admission conditions were also performed for the same two speeds and $60^\circ$ vane angle. The tests were conducted with an eddy current dynamometer over a velocity ratio range of $\approx 0.4$ to $\approx 1.1$ and the performance parameters were compared with those previously obtained from an equivalent geometry single-entry mixed-flow turbine, nozzleless and nozzleled. The comparison was conducted on an equivalent geometry basis; the design progression from single to twin-entry was done maintaining the same wheel, same $A/r$ and same exit flow angle to the volute for all three configurations. The test results showed that at $43.0 \text{ rev/s} \cdot \sqrt{\text{K}}$ and $60^\circ$ vane angle (corresponding to the optimum vane angle), the nozzle single-entry turbine performs better in respect the others, achieving the highest efficiency of 0.80. At $70^\circ$ similar peak efficiency was measured for both the single and twin-entry turbine, 0.76 and 0.77 respectively, while a penalty of $\approx 5$ percentage points was found between the single and twin-entry turbine at $40^\circ$ vane angle. Using the flow capacity of the nozzleless turbine as a reference value, the performance comparison based on equivalent flow capacity was performed for the three configurations ($50^\circ$ and $40^\circ$ vane angles had to be set for the single and twin-entry turbine respectively). A significant drop in efficiency from 0.77 to 0.63 was measured between the single-entry turbines; for the twin-entry turbines an extra efficiency drop of 10 percentage points was measured.
An analysis of the performance parameters for the twin-entry turbine under partial and unequal admission was also carried out. Under unequal admission the tests were conducted by retaining the pressure ratio (inlet to exit to the turbine) of either the inner or the outer limb. Different range of pressure ratios were set, going from 1.3 to 1.9. Based on the full admission maps an approach to determine the partial admission flow capacity was proposed with an improvement of \( \approx 22\% \) in respect the standard methods. The test results also showed that the flow capacity under unequal admission is uniquely correlated to that under full admission. A map-based method to predict the mass flow in each limb was proposed; the prediction was found to agree with experimental results within \( \approx 3\% \) for any operating conditions.

On the single-entry side a meanline model was developed for both the nozzleless and nozzled mixed-flow turbine. The nozzleless turbine model was validated against the experimental results obtained for five different rotational speeds \((23.0 \text{ rev/s}\cdot\sqrt{K} \text{ to } 53.8 \text{ rev/s}\cdot\sqrt{K})\) while for the nozzled turbine three different vane angles \((40^\circ, 60^\circ \text{ and } 70^\circ)\) and one single speed line \((43.0 \text{ rev/s}\cdot\sqrt{K})\) were considered. The model prediction for the nozzleless turbine showed to be in good agreement with the experimental results; the RMSD is within \(1\% \text{ to } 7\% \) for all the rotational speed. For the nozzled turbine an accurate prediction of the efficiency was obtained for \(40^\circ\) and \(60^\circ\) vane angle. At \(70^\circ\) vane angle the deviation from the experimental results is significant, above all in the prediction of the flow capacity prediction \((\text{RMSD} \approx 5.5\% \text{)}\). The extended range of the maps available showed that the current loss models fail to provide an accurate prediction in the high velocity ratio regions of the maps. An incidence factor had to be included and calibrated accordingly. Based on the data generated by the model, a break-down loss analysis was also performed. The incidence and the passage loss were found to account for most of the energy loss (more than 90\%) while the clearance and passage loss correspond to no more than \(\approx 3\% \) percentage points of the overall loss.
CHAPTER 5

HEAT TRANSFER ANALYSIS ON TURBOCHARGER PERFORMANCE

SYNOPSIS

This chapter presents the performance of a turbocharger under non-adiabatic conditions by means of both experimental methods and a reduced order numerical method. A commercial turbocharger was installed on a 2.0 liter diesel engine and measurements were done at engine speeds ranging from 1000 to 3000 rpm. For each engine speed the load applied varied from 16 to 250 Nm. The test results enabled the assessment of the heat fluxes through the turbocharger and their impact of the engine on the compressor performance. The engine tests were performed at the Mechanical Engineering Department of Imperial College London with the assistance of Mr. K. Spyridon (who was supervised by Professors A. Taylor and Y. Hardalupas).

A 1-D heat transfer model was also developed and validated against the experimental measurements. The algorithms calculate the heat transferred through the turbocharger by means of lump capacitances. Compressor maps were then generated for a range of speeds and temperatures of the exhaust gases and the efficiency drop associated with heat transfer was quantified. Based on the data generated by the model, a new correlation for the compressor non-adiabatic efficiency was found by means of a multiple regression analysis; the work is based on a statistical description of the different parameters that affect the heat transfer model.
5.1 Heat transfer in a turbocharger

Most analysis of performance in turbomachinery is treated as adiabatic since the influence of heat transfer can be considered as small. However in some cases, such as the one treated in this thesis, heat transfer can have a significant influence, thus making a non-adiabatic treatment more appropriate. The turbine of the turbocharger is positioned in close proximity to the “cold” compressor, some heat exchange will inevitably occur between the turbine and the compressor. Heat transfer analysis usually involves quantifying the heat transfer rate for some known temperature difference. It is recognized that heat can be transferred by one or a combination of three separate modes known as conduction, convection and radiation. Although it is useful to look at each one of these processes separately, they often occur together. In a turbocharger in particular, all of these three processes occur at the same time and are strictly interrelated. The complex turbocharger geometry introduces many possible heat transfer mechanisms inside the turbocharger itself as well as from the turbocharger to ambient. The heat transfer between the components of the turbocharger as well as between the turbocharger and the surroundings can be classified into:

- heat transfer from the hot turbine to the lubrication oil by means of forced convection in the clearance between shaft and bearing
- heat transfer from the turbine to the compressor through the bearing housing (even though, the cooling oil reduces to a large amount the amount of heat that is transferred by conduction from the turbine to the compressor)
- heat transfer from the turbine, bearing housing and compressor to the ambient by means of radiation and free convection

The heat transfer process in the compressor (compression) or turbine (expansion) can be separated into three stages: heat transferred before the component, temperature changes due to expansion (turbine) or compression (compressor) and heat transferred after the component. Such a simplified process, which will be explained more in detail in the next sections, is at the basis of the analysis that was conducted in this work.

5.2 Performance parameters in hot conditions

In order to quantify the performance of a turbocharger turbine and compressor, the standard parameters usually adopted are the efficiency and the pseudo-dimensional mass flow rate. These parameters were discussed in Chapter 4 and defined in Eq. (4.8) and Eq. (4.17). This formulation comes from the Euler turbomachinery equations, it has a general validity, but it is suitable for adiabatic analysis (or “cold” conditions). As reported by Casey and Schlegel (2007) and Casey and Fesich (2009) the use of the isentropic enthalpy rise is not justified when dealing with a heat transfer
process. A reversible non-adiabatic flow is no longer isentropic and therefore it is inappropriate to use the isentropic process as the reference of the ideal work required by a perfect non-adiabatic compressor. Hagelstein et al. (2002), assume that the amount of heat transferred during the process does not have a significant impact on the global results and hence this part is neglected. Therefore only the heat transferred before and after the process is considered within this work. In figure 5.1 is given an h-s diagram of the change of state of a non-adiabatic and an adiabatic compression process\textsuperscript{24}. If the compression process were to be adiabatic, the change of state would follow the line \( I \) to \( 2_{ad} \), the corresponding isentropic process would follow the vertical line \( I \) to \( 2_s \). State 2 in the diagram corresponds to the end state when the compression process is non-adiabatic. As there is a heat addition, the final state of the compression 2 corresponds to a higher temperature and therefore higher enthalpy. The path of the compression process under non-adiabatic conditions can therefore be described as \( I \rightarrow I^* \rightarrow 2^* \rightarrow 2 \). The heat added to the overall compression is split into two parts, one before \((q_{C,\text{before}}: I \rightarrow I^*)\) and one after compression \((q_{C,\text{after}}: 2^* \rightarrow 2)\), while the compression process \((I^* \rightarrow 2^*)\) from inlet \((P_01)\) to exit \((P_02)\) is considered to be adiabatic.

The heat transfer process as described in Fig. 5.1 was initially introduced by Shaaban and Seume (2006) who defined the so called compressor diabatic efficiency. This represents the apparent compressor efficiency measured under non-adiabatic operating conditions and it is defined as the ratio between the isentropic and non-adiabatic enthalpy rise associated with the compression process:

\textsuperscript{24} The values of enthalpy and temperature are referred to as total values.
**Compressor diabatic efficiency:**

\[ \eta_{di,c} = \frac{\Delta h_{adi,is}}{\Delta h_{dia}} = \frac{T_{2,is} - T_1}{T_2 - T_1} \]  

(5.1)

The compressor adiabatic efficiency differs from Eq. (5.1) in that the exit temperature \( T_{2,adi} \) is used in place of \( T_2 \). This represents the efficiency in absence of heat transfer:

**Compressor adiabatic efficiency:**

\[ \eta_{adi,c} = \frac{\Delta h_{adi,is}}{\Delta h_{adi}} = \frac{T_{2,is} - T_1}{T_{2,adi} - T_1} \]  

(5.2)

The same procedure as described above can be applied to a turbine process, shown in Fig. 5.2.

The non-adiabatic effect results in a lower outlet temperature due to heat dissipation (line 3 to 4) and the overall expansion process follows the path 3→3*→4*→4. In a similar manner to the compressor, the heat transfer is split into two parts: the heat transferred before expansion \( (q_{T,\text{before}}: 3\rightarrow3^*) \) and after expansion \( (q_{T,\text{after}}: 4\rightarrow4^*) \). The expansion process \( (3^*\rightarrow4^*) \) from \( P_{03} \) to \( P_{04} \) is instead assumed to be adiabatic.

![Figure 5.2: Turbine diagram](image)

The turbine **diabatic and adiabatic efficiency** are defined as follows:

**Turbine diabatic efficiency:**

\[ \eta_{dia,T} = \frac{\Delta h_{dia}}{\Delta h_{adi,is}} = \frac{T_2 - T_4}{T_3 - T_{4,\text{is}}} \]  

(5.3)
Turbine adiabatic efficiency: \[ \eta_{adi,T} = \frac{\Delta h_{adi}}{\Delta h_{adi,is}} = \frac{T_3 - T_{4,adi}}{T_3 - T_{4,is}} \] (5.4)

From Fig. 5.2 it can be noticed that the actual non-adiabatic work of the turbine is lower than the adiabatic work. The larger the heat fraction is before expansion, the more the expansion shifts towards lower entropies, where the enthalpy difference between the pressure lines is smaller than for higher entropies. This causes the non-adiabatic efficiency to be greater than unity as \( \Delta h_{adi} > \Delta h_{adi,is} \). Obviously this is not physically possible and it comes as a consequence of the fact that the non-adiabatic efficiency as it is defined in Eq. (5.3) does not take into account the work done against the mechanical friction losses. In order to evaluate the non-adiabatic efficiency of the turbine, the shaft power should be directly measured on the turbine itself.

![Test rig layout](image)

Figure 5.3: Test rig layout (Kyartos 2006)

### 5.3 Experimental Investigation

The following discussion provides essential information on the experimental facility layout. A schematic diagram of engine test rig is shown in Fig. 5.3 while in Fig. 5.4 a general overview of the engine is also given. An eddy current dynamometer (Borghi & Saveri FE260-S) was used to keep the
engine load constant at a desired value. The engine was connected to it via a Universal-Joint (U-J) shaft (Clarke Transmissions TRSZV131001). This type of coupling is tolerant to small misalignments and ideal for that kind of applications. The dynamometer was water cooled with water passing inside the stator to dissipate the generated heat. The engine was air cooled by a 30 kW water-to-air heat exchanger (AKG T4). This was necessary since the standard internal water cooling of the engine was not enough. Moreover, a standard air-to-air intercooler with minor modifications was used to cool the air just before the inlet manifold. Additional modifications were made so that the intercooler was supplied with air from the departmental compressed air system.

The engine was operated via an instrumentation rack (Test Automation Ltd series 2000). The rack consisted of controls to operate the dynamometer, to crank and run the engine and to stop the operation in case of an emergency. A speed dial regulated the dynamometer resistance applied to the engine crankshaft while the load dial was employed to operate the throttle of the engine via an actuator. A graphical user interface (Lab View Virtual Instrument) program was built in order to monitor the engine during the operations as well as to acquire all the signals from the installed pressure transducers and thermocouples.

**5.4 Turbocharger Instrumentation**

The turbocharger used for this research was supplied by Garrett, series GT 17V. This turbo is small in size: the compressor wheel is 59 mm in diameter (turbine 45 mm), and weight is no more than 5 kilos. The turbine is constituted by nine vanes. The actuating gear includes nine levers that are welded to the shaft of the corresponding nozzle guide vane (NGV). The control ring is moved by the
main lever, located in the bearing housing of the turbo. The levers are driven by an adjustable connecting rod from the NGV vacuum actuator to the outer main lever. An adjustable throw limiting screw is located just above the centre of the connecting rod. In order to achieve the project objectives, a number of pressure transducers and thermocouples were installed at various turbochargers locations. In the following paragraphs, a detailed description of the measurements and of the instrumentation used is provided.

5.4.1 Pressure and temperature flow measurements

In order to determine the turbocharger operating point, the temperature and the pressure of the flow were measured at the inlet and the exit to the turbine and the compressor. Four pressure transducers (GE Druck, 1400 series) were installed in order to measure the pressure. These transducers have a gauge pressure range -1 to 4 bar, an output of 4 to 20mA, an accuracy ±0.15% and an operating temperature falling in a range between -20°C to 80°C. Such a narrow range for the temperatures did not permit to place them directly in the position next to the pressure measurement as the temperature of the exhaust can easily exceed 500°C. For this reason all the pressure transducers were attached to a panel nearby the engine and connected to it through stainless steel pipes.

In order to measure the temperatures, four thermocouples were installed in the same location as the pressure transducers. Three T-type thermocouples (copper, -200°C to 350°C) were chosen. On the turbine side and the compressor side of the exhaust manifold two K-type thermocouples were installed (nickel-chromium alloy, -200°C to +1350°C). The thermocouple accuracy, at the temperature range relevant to our application temperature, is ±1.5°C.

5.4.2 Surface temperatures of the turbine and compressor casing

The surface temperatures of the turbine and compressor casing were measured in three different locations (referred as Engine, Top and External side) with six thermocouples inserted along the turbine and compressor scroll at an angle of 90 degrees with respect to each other, Fig. 5.6-A and Fig. 5.6-B. For each position, the outer and inner temperatures of the casing were measured by mean of ceramic twin bore insulated thermocouples. For the turbine casing, where the temperatures are high, R-type thermocouples (platinum vs. platinum - 13% rhodium, 0°C to 1600°C) were chosen. For the compressor casing, instead, where the temperatures experienced are much lower than the turbine side, K-type thermocouples (nickel-chromium vs. nickel-aluminum, -200°C to 1350°C) were installed. The thermocouple insulator is aluminum porcelain (rated to 1350°C) for K-type and re-crystallized alumina (rated to 1750°C) for R-type thermocouples. The cement used to attach the thermocouples must guarantee heat conductivity, electricity insulation and good strength to stress. For this reason a high temperature Omega Bond Cement was chosen as it embeds all these requirements (maximum
service temperature up to 1450°C, thermal conductivity 1.58 W/mK and shear stress 375 psi). In order to support the thermocouples along the turbine and compressor casings, two semi-circular plates were produced. The frames were fixed onto the turbocharger using the existing bolts that attach the compressor and the turbine casing to the main body. In order to reduce the effects of heat dissipation due to radiation and conduction towards the plates, these were covered with a ceramic felt whereas ceramic washers were used to avoid conductive effects. On these plates some rectangular brackets, into which to fit the thermocouples, were bolted. The whole system (plates, thermocouples and cement) had to be rigid to avoid the risk of failing of the cement due to high vibrations, at which the turbocharger was subjected while the engine was running. Once the turbocharger was installed on the engine, two additional frames were fixed on top of the ones already existing. These were required in order to support the isothermal connector strips used to connect the thermocouples with the cable leading to the field point module. A schematic layout of the installation is given in Fig. 5.5.

![Figure 5.5: Thermocouples installation](image)

All the thermocouples were connected to a field point module FP-TC-120 from National Instruments. A source of error is the so called cold junction due to the connection between the thermocouples and the terminal base. This affects the measured readings and must be compensated for. The field point module has a built in cold junction temperature measuring element and the compensation is made by default. However, the non linear nature of the thermocouples makes it difficult to define the errors of the temperature measurements in a simple way. For both K and R-type thermocouples, their accuracy at the temperature range relevant to our application is ±1.5°C.

### 5.4.3 Exhaust manifold and bearing housing temperature

The temperatures of the exhaust manifold and of the bearing housing were also measured. Two thermocouples were positioned on two of the four pipes of the exhaust manifold (refer to Fig. 5.6-C). For the bearing housing one thermocouple was installed between the compressor and the
turbine (Fig. 5.6-D). For all of these measurements, only the surface temperatures were monitored by means of R-type thermocouples.

### 5.4.4 Oil Flow rate and oil temperature

The oil temperature measurements are very important to the entire study because the oil accomplishes the double function of lubricating and cooling the bearings. As the main part of heat transfer to the compressor occurs through the shaft and the bearing housing, it is very important to assess the role played by the oil in an accurate way. The oil temperatures were measured with two stainless steel mineral insulated K-type thermocouples. The tip junction is insulated. The thermocouples were fitted in the inlet and exit oil pipes attached to the bearing housing as shown in Fig. 5.6.-E. The oil flow rate was monitored with a 316 Stainless Steel Body flow sensor. The oil flow rate is expected to be in the range of 2-3 l/min. The output frequency of the pulses is directly proportional to the flow rate. The supply voltage varies between 4.5 and 24 Vdc and the maximum flow rate is in between 1.5 ÷ 6 l/min with an operating range that goes from -25°C to 125°C. The transfer function between the circuit output frequency and the corresponding oil flow rate is given in Appendix A4. In Fig. 5.6-F the oil flow sensor as installed on the engine is shown.

### 5.4.5 Rotational speed

The system is based around the non contacting transducer TQ401 and its matching signal conditioner IQS 451 that together form a calibrated proximity system. The output is the distance between the transducer tip and the target. The active part of the transducer is a coil of wire while the transducer body is made of stainless steel. The measuring range of the TQ 401 is 2 mm, with a sensitivity of 8 mV/μm and a frequency response ranging from DC to 20 KHz. This last characteristic makes it suitable for compressor wheels for which the speeds can go from few thousands rpm up to well beyond the current speed range of the turbine. The proximity transducer as installed in the compressor is shown in Fig. 5.6-G.

Table 5.1 is gives a summary of all the measurements taken.

<table>
<thead>
<tr>
<th></th>
<th>Turbine</th>
<th>Compressor</th>
<th>Bearing housing</th>
<th>Exhaust manifold</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total pressure</td>
<td>Inlet / Exit</td>
<td>Inlet / Exit</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total temperatures</td>
<td>Inlet / Exit</td>
<td>Inlet / Exit</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flow rate</td>
<td>-</td>
<td>Air</td>
<td>Oil</td>
<td></td>
</tr>
<tr>
<td>Surface temperatures</td>
<td>Engine, Top, External</td>
<td>Engine, Top, External</td>
<td>Engine, Top, External</td>
<td>Housing</td>
</tr>
<tr>
<td>- Inner</td>
<td>Engine, Top, External</td>
<td>Engine, Top, External</td>
<td>Engine, Top, External</td>
<td>Exhaust pipes</td>
</tr>
<tr>
<td>- Outer</td>
<td>Engine, Top, External</td>
<td>Engine, Top, External</td>
<td>Engine, Top, External</td>
<td></td>
</tr>
<tr>
<td>Speed</td>
<td>Shaft</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.1: Turbocharger test measurements
5.5 Test conditions setup

The turbocharger was tested at constant load points for a range of engine speeds. Measurements were obtained for engine speeds between 1000 and 3000 rpm at a step of 500 rpm; for each engine speed the load applied was varied from 16 to 250 Nm. The test conditions are summarized in Table 5.2.

<table>
<thead>
<tr>
<th>Speed/Load</th>
<th>16 Nm</th>
<th>50 Nm</th>
<th>100 Nm</th>
<th>150 Nm</th>
<th>200 Nm</th>
<th>250 Nm</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000 rpm</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1500 rpm</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>2000 rpm</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>2500 rpm</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>3000 rpm</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

Table 5.2: Test conditions
5.6 Experimental results

In this section the outcomes of the experimental investigation are reported. The discussion of the test results is structured as follows:

- surface temperature of the compressor and turbine casing
- cooling oil, bearing housing and exhaust manifold surface temperatures
- non-adiabatic efficiency and exit temperature to the compressor

5.6.1 Surface temperature of the compressor and turbine casing

The inner and outer wall temperatures of the turbine and compressor casings were measured at the Engine, Top and External side for each engine speed and load. Table 5.3 summarises the results for each engine speed\(^25\).

From Table 5.3 it can be seen that the surface temperatures of the compressor and turbine casings are not uniform; they tend to decrease substantially as one moves from the Engine side towards the External side. This can be attributed to the proximity of the turbocharger to the engine. This is shown in Table 5.3 by the temperature difference ($\Delta T_{\text{Eng-Ext}}$) between the Engine and External positions. This temperature difference goes from a minimum of 10 K (for low engine speeds and load) to a maximum of $\approx 68$ K, measured at 2000 rpm and 250 Nm.

Figure 5.7: Turbine casing; inner - outer wall temperature difference in the three locations Engine, Top and External side

\(^{25}\) Only the minimum and maximum load was included but a full table with all the test measurements can be found in Appendix A6
In Table 5.3 are also reported the wall temperatures difference $\Delta T_w$ across the turbine and compressor wall for every given reference position of the thermocouples. The temperature across the turbine wall decreases from the inner to the outer wall while the opposite occurs for the compressor. This can be explained if we consider that in a turbine, the incoming hot gases tend to heat up the inner surface of the casing by mean of forced convection. A temperature gradient between the inner and the outer surface of the casing is therefore created leading to the generation of a heat flux towards the external wall where, by means of radiation and natural convection to the environment, the turbine is cooled down. Conversely the inner wall of a compressor is at lower temperature than that of the outer wall. This can be attributed to the air flowing in the compressor which tends to cool down the casing which is subjected to radiation and conduction coming from the turbine and the bearing housing. In Fig. 5.7 and 5.8 the wall temperature difference $\Delta T_w$ was plotted against the temperature of the exhaust gases for both the turbine and the compressor. From Fig. 5.7 it can be seen that the wall temperature difference on the *External side* (labelled as $\Delta T_{WT,External}$) is greater than that on the *Engine side* ($\Delta T_{WT,Engine}$). As the temperature of the exhaust gases increases ($\approx 950$ K) the discrepancy between $\Delta T_{WT,Engine}$ and $\Delta T_{WT,External}$ can go up to 40 K. On the compressor side instead the temperature difference between the inner and the outer wall is greater on the *Engine side* than on the *External side*. The measured $\Delta T_{WC,Engine}$ can be as much as ten times larger than $\Delta T_{WC,External}$. For instance at 3000 rpm and 200 Nm, $\Delta T_{WC,Engine}$ is $\approx 29$ K while the corresponding $\Delta T_{WC,External}$ is $\approx 3$ K.
The wall temperature trends ($\Delta T_w$) shown in Figs. 5.7 and 5.8 show the impact of the engine on the overall temperature distribution. On the Engine side of the turbine casing, where the heat dissipation towards the ambient is minimized by the presence of the engine, $\Delta T_{WT,\text{Engine}}$ is relatively small in respect with $\Delta T_{WT,\text{External}}$. The opposite occurs on the compressor where the engine behaves like a heat source that tends to heat up the compressor casing with the consequence that the outer wall temperature is greater than the inner wall. As we move towards the External side, the effects of the engine become negligible and this is confirmed by $\Delta T_{WC,\text{Top}}$ and $\Delta T_{WC,\text{External}}$ that vary within few degrees.
Figure 5.9: Compressor casing: occurring heat fluxes

Figure 5.10: Turbine casing: occurring heat fluxes
A schematic diagram of the heat transfer process occurring within the turbine and the compressor casing is given in Figs. 5.9 and 5.10. The high temperature of the turbine casing causes the heat fluxes to be directed towards the surrounding environment while the opposite occurs on the compressor side where heat from the surrounding environment flows into the compressor\textsuperscript{26}.

A comparison between the heat fluxes through the turbine and the compressor casing was also carried out by averaging the heat conducted per unit area\textsuperscript{27} in each of the three locations (Engine, Top, and External). An accurate evaluation of the heat dissipated through the turbine wall is crucial to the overall turbocharger performance. In fact since the compressor efficiency is mainly affected by the heat transferred through the bearing housing, a correct calculation of the heat dissipated within the turbine casing before expansion occurs enables us to quantify the remaining portion of heat available to the bearing housing.

The data reported in Table 5.4 show that the heat flow conducted through the turbine and compressor casing is similar for every engine operating point. This is particular evident for high temperature of the exhaust gases where the heat conducted vary within few percentage points. At an exhaust temperature of 933 K, the heat conducted for both the turbine and compressor casing is matched, with a value of 44.3 W/m\textsuperscript{2}. Such a finding seems to suggest that all the heat transfer process within the turbocharger mainly occurs between the turbine and compressor. In this process, the bearing housing plays a small role that remains limited to the removal of the heat generated by the bearings. This action is the primary function of the lubricating oil. The similar magnitudes of the heat fluxes shows that the bearing housing analysis could be limited to a minimum detail or bypassed altogether.

### Table 5.4: Heat flux through the turbine and compressor casing

<table>
<thead>
<tr>
<th></th>
<th>1000 rpm</th>
<th>1500 rpm</th>
<th>2000 rpm</th>
<th>2500 rpm</th>
<th>3000 rpm</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Exhaust gases [K]</strong></td>
<td>388</td>
<td>506</td>
<td>417</td>
<td>949</td>
<td>430</td>
</tr>
<tr>
<td><strong>Turbine Q/A [W/m\textsuperscript{2}]</strong></td>
<td>8.5</td>
<td>15.3</td>
<td>9.7</td>
<td>49.6</td>
<td>11.8</td>
</tr>
<tr>
<td><strong>Compressor Q/A [W/m\textsuperscript{2}]</strong></td>
<td>9.9</td>
<td>12.0</td>
<td>10.5</td>
<td>48.0</td>
<td>14.9</td>
</tr>
</tbody>
</table>

5.6.2 Cooling oil, bearing housing and exhaust manifold temperatures

The inlet and exit oil temperatures for the bearing housing were measured together with the bearing housing surface temperature. The test results are reported in Table 5.5.

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\textsuperscript{26} In the compressor side this is not always true. In fact it might occur that the air after compression achieve higher temperatures than those of the compressor casing. Such a case was not measured during testing and was not shown in Fig. 5.10 where the heat flux from the outer wall moves towards the inner side of the scroll.

\textsuperscript{27}The heat conducted per unit area was calculated as $\frac{\Delta Q}{A} = k\Delta T_{\text{wall}}/\Delta x$, where $k$ is the thermal conductivity, $\Delta x$ the wall thickness and $\Delta T_{\text{wall}}$ the temperature wall difference.
The oil temperature varies from a minimum of 321 K at the inlet at 1000 rpm to a maximum of 394 K at the exit at 3000 rpm. The bearing housing test results highlighted that its surface temperature closely follows that of the cooling oil temperature (Fig. 5.11). The temperature difference ($\Delta T_{BH-oil}$) between the surface temperature of the bearing housing and the mean oil temperature (inlet to exit) is proportional with the temperature of the exhaust gases; for exhaust gas temperature $T_{Exh}=373$ K the temperature difference $\Delta T_{BH-oil}$ ≈ 5 K while as the exhaust gas temperature increases, $T_{Exh}=823$ K, the temperature difference $\Delta T_{BH-oil}$ goes up ≈ 33 K.

It is worth noting that the temperature of the bearing housing remains well above that of the oil. This can be explained by considering that the temperature of the bearing housing comes as the sum of the cooling effects due the oil and the convective, radiative and conductive heat fluxes due to its proximity to the engine and the turbine casing.

The surface temperature of the exhaust manifold was also measured. Two surface thermocouples were placed on the pipes located underneath the compressor and the turbine. The measured temperatures are reported in Table 5.5 and shown in Fig. 5.12 together with those of the exhaust gases. From Fig. 5.12 it can be seen that the difference between the surface temperature of the pipe on the turbine side and that of the exhaust gases varies from a few degrees at low loads up to around 130 K at higher loads.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Engine speed: 1000 rpm</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8 Nm</td>
<td>373</td>
<td>330</td>
<td>328</td>
<td>321</td>
<td>≈ 5</td>
<td>343</td>
<td>333</td>
</tr>
<tr>
<td>25 Nm</td>
<td>407</td>
<td>327</td>
<td>334</td>
<td>325</td>
<td>≈ 2</td>
<td>352</td>
<td>339</td>
</tr>
<tr>
<td>50 Nm</td>
<td>472</td>
<td>335</td>
<td>340</td>
<td>330</td>
<td>≈ 0</td>
<td>385</td>
<td>366</td>
</tr>
<tr>
<td><strong>Engine speed: 2000 rpm</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8 Nm</td>
<td>394</td>
<td>345</td>
<td>341</td>
<td>340</td>
<td>≈ 4</td>
<td>361</td>
<td>341</td>
</tr>
<tr>
<td>50 Nm</td>
<td>503</td>
<td>356</td>
<td>347</td>
<td>344</td>
<td>≈ 10</td>
<td>429</td>
<td>398</td>
</tr>
<tr>
<td>100 Nm</td>
<td>630</td>
<td>392</td>
<td>376</td>
<td>375</td>
<td>≈ 16</td>
<td>535</td>
<td>482</td>
</tr>
<tr>
<td>150 Nm</td>
<td>689</td>
<td>386</td>
<td>362</td>
<td>360</td>
<td>≈ 25</td>
<td>562</td>
<td>502</td>
</tr>
<tr>
<td>200 Nm</td>
<td>750</td>
<td>404</td>
<td>378</td>
<td>377</td>
<td>≈ 26</td>
<td>623</td>
<td>559</td>
</tr>
<tr>
<td>250 Nm</td>
<td>823</td>
<td>422</td>
<td>393</td>
<td>384</td>
<td>≈ 33</td>
<td>683</td>
<td>613</td>
</tr>
<tr>
<td><strong>Engine speed: 3000 rpm</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16 Nm</td>
<td>453</td>
<td>366</td>
<td>362</td>
<td>357</td>
<td>≈ 6</td>
<td>437</td>
<td>414</td>
</tr>
<tr>
<td>50 Nm</td>
<td>551</td>
<td>379</td>
<td>368</td>
<td>366</td>
<td>≈ 12</td>
<td>488</td>
<td>459</td>
</tr>
<tr>
<td>100 Nm</td>
<td>706</td>
<td>399</td>
<td>385</td>
<td>383</td>
<td>≈ 15</td>
<td>603</td>
<td>546</td>
</tr>
<tr>
<td>150 Nm</td>
<td>809</td>
<td>410</td>
<td>394</td>
<td>391</td>
<td>≈ 18</td>
<td>686</td>
<td>609</td>
</tr>
<tr>
<td>200 Nm</td>
<td>828</td>
<td>406</td>
<td>389</td>
<td>387</td>
<td>≈ 18</td>
<td>698</td>
<td>614</td>
</tr>
</tbody>
</table>
Such a temperature difference is even more severe for the pipe on the compressor side, where a maximum temperature drop of almost 200 K was measured. This is unexpected, since the compactness of the turbocharger suggests that the rate of heat exchange in both the turbine and compressor side should be the same. However such a discrepancy can be attributed to an increased ability of the pipe on the compressor side to exchange heat with the ambient environment, since no
heat sources are present in its proximity. This is not the case on the turbine side, where the turbine casing significantly reduces the chances for the exhaust pipe of cooling down.

5.6.3 Compressor non-adiabatic efficiency and exit flow temperature

The turbocharger used in this research was supplied with performance maps generated in a low temperature gas test stand (turbine inlet temperature ≈ 300 K). The maps generated with such a test facility are usually referred to as cold maps (near adiabatic conditions). However in a turbocharger under normal operations, the temperature of the exhaust gases is the result of a combustion process and its temperature goes up several hundred degrees leading to highly non-adiabatic conditions. The compressor process can be visualized in an enthalpy-entropy (h-s) diagram as given in Fig. 5.1. The non-adiabatic influence on the compressor is represented by an increase in the exit temperature (from $T_2^*$ to $T_2$) that leads to a drop in the compressor efficiency. In order to evaluate the effects of heat transfer on compressor efficiency, a comparison between the non-adiabatic and adiabatic efficiencies was carried out in terms of relative efficiency. The relative efficiency is defined as the ratio between the compressor peak efficiency as per the cold map and the efficiencies measured under hot conditions.

$$\eta_{rel} = \frac{\eta_{measured,c}}{\eta_{max,c}} \quad (5.5)$$

The compressor non-adiabatic efficiency was calculated by rearranging Eq. (5.1). From the test measurements the total pressure ratio ($P_{o2}/P_{o1}$), the total inlet ($T_{o1}$) and exit ($T_{o2}$) temperatures were measured and therefore the compressor non-adiabatic efficiency was calculated as given in Eq. (5.6).

$$\eta_{dia,c} = \left( \frac{P_{o2}}{P_{o1}} \right)^{(\gamma-1)/\gamma} \frac{(T_{o2}/T_{o1})^{(\gamma-1)/\gamma} - 1}{(T_{o2}/T_{o1}) - 1} \quad (5.6)$$

The compressor adiabatic efficiency instead was extrapolated from the cold compressor maps provided by the turbocharger manufacturers. The comparison between the adiabatic and non-adiabatic efficiency is shown in Fig. 5.13.

From Fig. 5.13 it can be noticed that the compressor efficiency drop in hot conditions is severe over the whole range of temperatures of the exhaust gases. This is well shown in Table 5.6 where the compressor efficiency in adiabatic and non-adiabatic conditions is given. The absolute
relative deviation\(^{28}\) \(\Delta \eta\) between the efficiencies goes from a minimum of \(\approx 17\%\) to a maximum of \(\approx 30\%\). The scatter of the compressor non-adiabatic efficiencies as the exhaust gas temperature increases seems to suggest that there is no direct correlation between the two. In fact one would expect that the deterioration of the efficiency increased with an increase of the exhaust gas temperature at the turbine entry. This is not always the case since the compressor efficiency in non-adiabatic conditions, besides being dependent on the exhaust gas temperature, is also affected by other physical properties like the mass flow rate and the rotational speed.

The importance of mass flow and speed on the compressor non-adiabatic efficiency was firstly assessed by Shaaban and Seume. (2006) who succeeded in quantifying the correlation between adiabatic and non-adiabatic efficiency in terms of fundamental turbomachinery parameters. The ratio between the compressor efficiencies in adiabatic and non-adiabatic conditions was found to be of the form given in Eq. (5.7).

\[
\frac{\eta_{\text{dia},C}}{\eta_{\text{adi},C}} = \left(1 + q_{C,\text{before}}\xi_{h,C}\right)^{-1} \left(1 + \frac{\xi_{h,C}}{(\gamma_{\text{air}} - 1)M u_{2}^{2} \mu} \left(\frac{1}{1 - \phi_{2}/\tan \beta_{b,2}}\right)\right)^{-1}
\]

where \(\mu\) is the slip factor, \(q_{C,\text{before}}\) the heat transferred before compression and \(\xi_{h,C}\)\(^{29}\) the compressor heat number. The shaft speed and the mass flow rate come into Eq. (5.7) in terms of peripheral Mach number \((Mu)\) and flow coefficient \((\phi)\). These two parameters are defined in Eq. (5.8) and Eq. (5.9).

\[
M_{2u} = \frac{U_{2}}{\sqrt{\gamma RT_{01}}}
\]

\[
\phi_{2} = \frac{C_{2m}}{U_{2}}
\]

From Eq. (5.7) it can be gathered that the peripheral Mach number is the most important parameter affecting the compressor non-adiabatic performance as it depends on the square of the compressor peripheral Mach number. The compressor can be assumed to work in adiabatic conditions for low values of the heat number and high peripheral Mach numbers. Conversely, increasing the flow coefficient is associated with decreasing the aerodynamic work. In fact the term \((1 - \phi_{2}/\tan \beta_{b,2})\) in Eq. (5.7) decreases with increasing the flow coefficient thus contributing to a larger deviation of the compressor non-adiabatic efficiency from that adiabatic. Therefore we can

\(^{28}\)Relative deviation defined as: \(\Delta \eta = \left[1 - \left(\frac{\eta_{\text{adi}}}{\eta_{\text{dia}}}\right)\right]\)

\(^{29}\)The compressor heat number represents the amount of heat transfer in non-dimensional form and it is defined as: \(\xi_{h,C} = q_{c}/\epsilon_{p,\text{air}T_{01}}\).

Typical values for the heat number are \(\xi_{h,C}: 0 \pm 0.01;\)
infer that the efficiency ratio of Eq. (5.7) tends towards unity with increasing peripheral Mach number, since the aerodynamic work increases with the rotational speed.

In order to validate the effectiveness of the finding of Eq. (5.7), the compressor adiabatic and non-adiabatic efficiency together with their ratio were plotted against the peripheral Mach number and

### Table 5.6: Adiabatic and non-adiabatic compressor efficiency and exit temperature

<table>
<thead>
<tr>
<th>Temperature exhaust gases [K]</th>
<th>PR [1 x 10^5 (kg/s)^-1 km/Pa]</th>
<th>MFP [1 x 10^5 m^2/s]</th>
<th>Speed [1 x 10^5 rev/s]</th>
<th>Adiabatic η</th>
<th>Non-adiabatic η</th>
<th>ΔT [K]</th>
<th>Adiabatic η</th>
<th>Non-adiabatic η</th>
<th>Δη [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>561</td>
<td>1.35</td>
<td>0.88</td>
<td>0.96</td>
<td>326</td>
<td>335</td>
<td>9</td>
<td>0.90</td>
<td>0.71</td>
<td>20%</td>
</tr>
<tr>
<td>596</td>
<td>1.40</td>
<td>1.00</td>
<td>1.24</td>
<td>330</td>
<td>343</td>
<td>13</td>
<td>0.91</td>
<td>0.69</td>
<td>24%</td>
</tr>
<tr>
<td>672</td>
<td>1.40</td>
<td>0.86</td>
<td>1.22</td>
<td>330</td>
<td>347</td>
<td>17</td>
<td>0.9</td>
<td>0.65</td>
<td>27%</td>
</tr>
<tr>
<td>698</td>
<td>1.40</td>
<td>0.87</td>
<td>1.05</td>
<td>330</td>
<td>344</td>
<td>14</td>
<td>0.91</td>
<td>0.66</td>
<td>26%</td>
</tr>
<tr>
<td>751</td>
<td>1.47</td>
<td>1.04</td>
<td>1.12</td>
<td>335</td>
<td>353</td>
<td>18</td>
<td>0.93</td>
<td>0.66</td>
<td>29%</td>
</tr>
<tr>
<td>755</td>
<td>1.57</td>
<td>0.87</td>
<td>1.13</td>
<td>345</td>
<td>363</td>
<td>18</td>
<td>0.91</td>
<td>0.68</td>
<td>25%</td>
</tr>
<tr>
<td>816</td>
<td>1.64</td>
<td>0.8</td>
<td>1.21</td>
<td>350</td>
<td>363</td>
<td>13</td>
<td>0.92</td>
<td>0.74</td>
<td>19%</td>
</tr>
<tr>
<td>868</td>
<td>1.73</td>
<td>0.72</td>
<td>1.24</td>
<td>364</td>
<td>383</td>
<td>19</td>
<td>0.82</td>
<td>0.65</td>
<td>20%</td>
</tr>
<tr>
<td>834</td>
<td>1.82</td>
<td>0.97</td>
<td>1.3</td>
<td>363</td>
<td>394</td>
<td>31</td>
<td>0.92</td>
<td>0.64</td>
<td>30%</td>
</tr>
<tr>
<td>949</td>
<td>1.85</td>
<td>0.71</td>
<td>1.32</td>
<td>374</td>
<td>395</td>
<td>21</td>
<td>0.82</td>
<td>0.66</td>
<td>19%</td>
</tr>
<tr>
<td>876</td>
<td>1.92</td>
<td>1.16</td>
<td>1.36</td>
<td>368</td>
<td>383</td>
<td>15</td>
<td>0.95</td>
<td>0.78</td>
<td>17%</td>
</tr>
<tr>
<td>928</td>
<td>2.04</td>
<td>1.04</td>
<td>1.48</td>
<td>384</td>
<td>404</td>
<td>20</td>
<td>0.90</td>
<td>0.73</td>
<td>18%</td>
</tr>
</tbody>
</table>
the flow coefficient. By using interpolation routines\textsuperscript{30}, three data surfaces could be generated as shown in Fig. 5.14. The 3-D mesh surface is useful as it shows the effect of the peripheral Mach number and flow coefficient on the compressor non-adiabatic efficiency separately by projection of the efficiency curves onto constant \(Mu\) and \(\phi_2\) planes. In Fig. 5.14-a, the efficiency ratio was plotted against the peripheral Mach number for constant values of the flow coefficients. The efficiency contour plot of Fig. 5.14-a seems to validate the findings of Eq. (5.7).

![Figure 5.14: Adiabatic vs. non-adiabatic efficiency – a: Efficiency ratio vs. peripheral Mach number b: Efficiency ratio vs. impeller flow coefficient](image)

In fact the efficiency ratio increases with increasing \(Mu\) while it decreases with increasing \(\phi\). The maximum compressor non-adiabatic efficiency was measured for low values of the impeller flow coefficient (\(\phi=0.03\text{-}0.05\)) and relatively large values of the peripheral Mach number (\(Mu\approx0.8\text{-}1\)), as reported in Table 5.7. The role of the flow coefficient on the compressor performance can be better gathered from Fig. 5.14-b where the efficiency ratio was plotted against the flow coefficient in place of the peripheral Mach number. Here it can be noticed that as the flow coefficient increases (\(\phi\approx0.03\text{-}0.085\)) the efficiency ratio drops dramatically from \(\approx0.8\) to \(\approx0.68\) for values of the peripheral Mach number going from 1.2 to 0.7.

\textsuperscript{30}A third order polynomial function was used to generate the efficiency surfaces.
The outcomes of Fig. 5.14 showed that the compressor non-adiabatic efficiency can be successfully described by means of the peripheral Mach number and flow coefficient. This is significant and it will be further considered when dealing with the results of the computational simulation later in this chapter.

One of the main challenges for engine calculations is the need to find a correlation for the compressor exit temperature for different operating conditions. This temperature the represents a boundary condition for the combustion analysis in the engine cylinders. In fact if it was possible to establish a unique correlation between the exhaust gases and the compressor exit temperatures, it would then be possible to calculate the compressor non-adiabatic efficiency by mean of Eq. (5.6). A solution to this issue is proposed here.

In the standard turbochargers configuration, the bearing housing is directly coupled to the compressor casing through a plate bolted on to the so called compressor back-plate, see Fig. 5.9. By means of conduction, the heat from the bearing housing will flow through the compressor back-plate that in turn will lead to heat up the compressed air as a result of forced convection. The general equation for forced convection is given in Eq. (5.10).

\[
\dot{Q}_{fc} = h \cdot A \cdot \Delta T_{fc}
\]  

where \( h \) is the heat transfer coefficient of the fluid involved in the heat transfer process (in this case air), \( A \) the surface area subjected to forced convection and \( \Delta T_{fc} \) the temperature difference between the fluid and the wall. If one assumes that all of the heat transferred to the air after compression is transferred through the compressor back-plate, then the plate temperature is equal to the surface temperature of the bearing housing and hence Eq. (5.10) becomes:

\[
\dot{Q}_{fc} = h_{air} A_{BP}(T_{BH} - T_{2*})
\]  

(5.11)

For a steady flow conditions, \( \dot{Q}_{fc} = \dot{m} \cdot c_p \cdot \Delta T \), leading to:

\[
\dot{Q}_{fc} = \dot{m}_{air} c_p (T_{2*} - T_2) = h_{air} A_{BP}(T_{BH} - T_{2*})
\]  

(5.12)

By solving for \( T_2 \), in the above equation, one obtains Eq. (5.13), giving the exit temperature to the compressor under non-adiabatic conditions.
\[ T_2 = T_{2*} + \frac{q_f}{m_{air}c_p} = T_{2*} + \frac{h_{air}A_{BP}}{m_{air}c_p}(T_{BH} - T_{2*}) \tag{5.13} \]

All the terms of Eq. 5.13 are known except \( T_{2*} \) and \( T_{BH} \). A derivation of these temperatures is proposed as follows.

- **\( T_{2*} \) calculation:** In Fig. 5.1 the non-adiabatic compression process was simplified into three paths: heat addition before compression \( (q_{C:before}: 1 \rightarrow 1*) \), adiabatic compression from \( 1* \rightarrow 2* \) and heat addition after compression \( (q_{C:after}: 2* \rightarrow 2) \). In reality only a small amount of heat is transferred before compression, since the incoming air goes through a very short passage (inducer inlet pipe). Hence the temperature \( T_{2*} \) can be assumed to be similar to the temperature \( T_{2,adi} \) that would occur if the compression process was fully adiabatic. Therefore we can assume that the compression process follows the path \( 1 \rightarrow 2, \text{adi} \rightarrow 2 \). Based on this assumption, the temperature \( T_{2*} \) can be considered to be similar to \( T_{2,adi} \) and its derivation reduces to Eq. (5.14).

\[ T_{2*} \cong (T_{2,adi}) = T_1 \left[ 1 + \left( \frac{PR^{\gamma-1/\gamma}}{\eta_{C,adi}} \right) \left( \frac{PR^{\gamma-1/\gamma}}{\eta_{C,adi}} \right) \right] \tag{5.14} \]

- **\( T_{BH} \) calculation:** Unlike the compressor efficiency, for which no correlation with exhaust gas temperatures could be observed, the exit temperature to the compressor seems to exhibit a linear trend with the temperature of the exhaust gases (dashed red line in Fig. 5.13). Such a trend was also observed for the surface temperature of the bearing housing for which the test measurements showed that \( T_{BH} \) and the temperature of the exhaust gases are linearly related (dashed red line in Fig. 5.15). Therefore since the temperature of the exhaust gases is known, it is possible to correlate the temperature of the bearing housing to the temperature of the exhaust gases as follows:

\[ T_{BH} = (T_{amb} + b) + g_{rad}T_{exh} \tag{5.15} \]

where \( g_{rad} \) is the gradient of the trend line, \( T_{amb} \) is the ambient temperature and \( b \) is a constant that takes into account the possible heat that would be transferred within the bearing housing at ambient temperature. By including Eq. (5.15) and Eq. (5.14) into Eq. (5.13), the exit temperature to the compressor \( (T_2) \) under non-adiabatic conditions can be calculated as given in Eq. (5.16).

\[ T_2 = T_1 \left[ 1 + \left( \frac{PR^{\gamma-1/\gamma}}{\eta_{C,adi}} \right) \left( \frac{PR^{\gamma-1/\gamma}}{\eta_{C,adi}} \right) \right] + \frac{h_{air}A_{BP}}{m_{air}c_p} \left[ T_1 \left[ 1 + \left( \frac{PR^{\gamma-1/\gamma}}{\eta_{C,adi}} \right) \left( \frac{PR^{\gamma-1/\gamma}}{\eta_{C,adi}} \right) \right] - [(T_{amb} + b) + g_{rad}T_{exh}] \right] \tag{5.16} \]
Table 5.8: Predicted exit temperature to the compressor

<table>
<thead>
<tr>
<th>T_{exh,gas} [K]</th>
<th>462</th>
<th>561</th>
<th>596</th>
<th>698</th>
<th>755</th>
<th>816</th>
<th>834</th>
<th>853</th>
<th>876</th>
<th>928</th>
<th>949</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bearing housing</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>surface temperature [K]</td>
<td>350</td>
<td>367</td>
<td>379</td>
<td>397</td>
<td>386</td>
<td>385</td>
<td>410</td>
<td>404</td>
<td>412</td>
<td>411</td>
<td>422</td>
</tr>
<tr>
<td>Predicted exit</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>temperature [K]</td>
<td>326</td>
<td>335</td>
<td>334</td>
<td>344</td>
<td>363</td>
<td>339</td>
<td>363</td>
<td>388</td>
<td>380</td>
<td>383</td>
<td>405</td>
</tr>
<tr>
<td>Experimental exit</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>temperature [K]</td>
<td>334</td>
<td>336</td>
<td>341</td>
<td>341</td>
<td>353</td>
<td>332</td>
<td>362</td>
<td>382</td>
<td>372</td>
<td>395</td>
<td>405</td>
</tr>
<tr>
<td>Deviation $\Delta T$ [%]</td>
<td>2.2</td>
<td>0.1</td>
<td>1.7</td>
<td>0.8</td>
<td>2.5</td>
<td>2.0</td>
<td>0.3</td>
<td>2.3</td>
<td>2.1</td>
<td>3.2</td>
<td>0.3</td>
</tr>
</tbody>
</table>

The outcomes of Eq. (5.16) are shown in Fig. 5.15, in which the measured and the calculated compressor exit temperatures are reported; their absolute deviation is also shown. The overall agreement of the computed temperatures is good over the whole range of test conditions. The absolute deviation between the predicted and measured temperatures is no bigger than 2%-3% (refer to Table 5.8). Such a deviation remains slightly over the uncertainty range associated with thermocouples, indicated by a dashed black line in the same figure.

![Figure 5.15: Compressor exit temperature](image)

It is worth noting that the choice of the surface temperature of the bearing housing $T_{BH}$ as reference temperature for the calculation of the exit temperature $T_2$ was made deliberately. In fact, even though the surface temperature of the compressor casing exhibited a linear trend with the temperature of the exhaust gases (refer to Fig. 5.8), the non-uniform temperature distribution due to the presence of the engine, makes it difficult to define a reference temperature. This is not the case in the bearing housing where the surface temperature was found to be highly dependent on that of the cooling oil (refer to Fig. 5.11). The range of temperatures for the cooling oil in turbocharger
applications is strictly related to that of the bearings for which the minimum and maximum operating temperatures are usually provided by the manufacturer. Given the limited number of bearing types used in turbochargers, this suggests that by extending the current testing to different turbocharger configurations; it could then be possible to map the trends for the oil and surface temperatures in order to ascertain the applicability of the correlation here proposed.

5.7 Heat transfer model

This section describes the outcomes of a 1-D heat transfer model of the turbocharger under study. As already mentioned before, the implementation of heat transfer models for turbochargers involves the quantification of a large number of parameters that complicates the analysis. Here we tried to simplify the approach. Having said this, a detailed quantification of the heat transfer process within the turbocharger would require a full 3-D conjugate heat transfer analysis.

In the model described below, the heat fluxes through the turbocharger were evaluated by means of well known correlations available for heat conduction, radiation and convection. The process was validated against experimental data on an engine.

5.7.1 Turbocharger Model

A reduced order turbocharger model developed consisting of an assembly of bodies of known geometry parameters. Such simplified model was obtained by means of progressive steps shown in Fig. 5.16. Firstly a full 3D-CAD model of the turbocharger was developed and then by analysis of the overall turbocharger configuration, the geometry was simplified to an assembly of three cylindrical bodies representing the turbine, the bearing housing and the compressor scroll (refer to Fig. 5.16-a.

Figure 5.16: Physical Model - a: Real model of the turbocharger - b: Simplified geometry included in the 1-D model
and b). It is worth noting that the compressor scroll in the heat transfer model will be simplified to a round plate, here referred as compressor *back-plate*. Such a simplification is based on the assumption that the temperature of the air flowing in the compressor is mainly affected by the heat exchanged with the back-plate. Although it might seem a crude assumption, this was demonstrated to provide good results (paragraph 5.6.3) and will be therefore maintained here.

Fig. 5.17: Reduced order heat transfer model

Fig. 5.17 shows the cross section of the 1-D model together with the main heat transfer paths. The exhaust gases coming from the combustion flow into the turbine, exchange heat by forced convection to the turbine casing and to the bearing housing (\(Q_{T\rightarrow BH}\)). Due to the gradient existing between the inner and outer surface of the turbine casing, heat is conducted through the wall and then dissipated by radiation (\(Q_{T\rightarrow rad}\)) and free convection (\(Q_{T\rightarrow conv}\)) to the surrounding environment. At the same time, the air that flows through the rotor expands and, as a consequence the pressure drops and the temperature decreases; heat transfer occurs to the blades and subsequently to the shaft. The turbine exit temperature is therefore calculated as the sum of the temperature drop due to the expansion and the heat transferred to the shaft (\(Q_{T\rightarrow s}\)). In the bearing housing the heat is dissipated by forced convection to the oil (\(Q_{oil\rightarrow BH}\)), and through free convection (\(Q_{BH\rightarrow conv}\)) and radiation (\(Q_{BH\rightarrow rad}\)) to the environment. In the shaft, the heat is dissipated only by forced convection to the oil (\(Q_{S\rightarrow oil}\)); note that the heat generated by friction within the bearing housing is not considered here. While the gases expand in the turbine, cold air flows into the compressor. The inlet air is heated up by the shaft (\(Q_{S\rightarrow air}\)) and compressed in the impeller with a consequent rise in temperature and pressure. After the compression, the air flows into the diffuser, where the gas will exchange heat by forced convection to the *back-plate* (\(Q_{C\rightarrow air}\)), the bearing housing (\(Q_{BH\rightarrow air}\)) natural convection (\(Q_{C\rightarrow conv}\)) and radiation (\(Q_{C\rightarrow rad}\)).
5.8 Heat Fluxes

In this section the heat fluxes through the elements of the turbocharger are evaluated (Kumm, 2007). Due to the high temperature of the exhaust gases flowing into the turbine volute, a higher surface temperature is expected on the turbine side. The compressor is expected to experience a lower temperature as heat is dissipated through bearing housing where the coolant plays a fundamental role to keep down the overall temperature.

5.8.1 Convective heat transfer coefficients

Convective heat transfer processes take place within the bodies constituting the turbocharger. Heat transfer correlations need to be applied to evaluate the heat transferred coefficients caused by natural and forced convection. The properties of the fluid must be evaluated at the operating temperature; reference tabular values provided by Turms (2000) are used\(^{31}\).

5.8.2 Turbine casing

The turbine casing was modeled as a horizontal cylinder given that most turbochargers are installed in horizontal position in order to ensure sufficient lubrication. Due to the hot exhaust gases entering the turbine, the inner surface is heated by mean of forced convection while heat is dissipated to the ambient by radiation and free convection. Considering the turbine model shown in Fig. 5.17, there are three surfaces that transfer heat:

- both ends of the cylinder. These surfaces correspond to two round face plates for which a correlation for heat transfer coefficient\(^{32}\) as proposed by Lewandowski and Radziemska (2001) was used:

\[
Nu = 0.667(Gr Pr)^{0.25}
\]

\[
h_{T, surf} = 0.667 \frac{K_T}{D_T} (Gr Pr)^{0.25}
\]

- the cylindrical surface joining the two vertical plates. A correlation for free convection was proposed by Bayley et al. (1972). Such a correlation is used to calculate the heat transfer coefficient on the cylindrical surface of the turbine casing and, similarly to the round face plates of the turbine, a laminar flow regime can be considered to occur and hence the following correlations were used:

\(^{31}\) The assumption that the specific heat \(c_p\), the dynamic viscosity \(\mu\) and the thermal conductivity \(k\) are independent of pressure was made in the model development; this is a valid assumption due to the small range of pressures experienced within a turbocharger.

\(^{32}\) The expression for the heat transfer coefficient given in Eq. (5.18) is valid for laminar flow defined as \(G_{\mu P} < 10^4\).
The bearing housing was modeled as a cylindrical surface. However, differently from the turbine volute where the surface was assumed to be as isothermal, here the surface temperature of the bearing housing varies in axial direction. This is due to the effects of the cooling oil going through the oil flow channel that leads to a variation of the heat transfer coefficient in axial direction. The bearing housing was hence sliced in a number of sections and it was assumed that the flow fields in two neighbouring regions do not affect each other\textsuperscript{33}. The heat transfer correlation for a horizontal cylinder can therefore be applied by taking into account that the heat transfer coefficient is only valid for a small portion of the bearing housing (for which the temperature can be considered to be constant). Under this assumption, the Prandtl number and the Grashof number vary in the axial direction and the air properties are calculated for the free stream and the mean temperature of the surface. The correlations for the Grashof number and Prandtl number are given by Bailey et al. (1972) and reported in Eqs. (5.21) and (5.22).

\[ Nu = 0.530(Gr Pr)^{0.35} \quad (5.19) \]

\[ h_{T,\text{surf}2} = 0.530 \frac{K_T}{D_T} (Gr_d Pr)^{0.35} \quad (5.20) \]

\textbf{5.8.3 Bearing housing heat transfer}

Besides proving the necessary lubrication to the bearings, the oil going through the oil flow channel also removes heat from the bearing housing. Due to the high rotational speed of the turbine shaft, the flow field of the oil within the oil channel becomes highly unstable. Given the axial-through direction of the cooling oil, the oil flow will form helical vortices thus making the heat transfer from the bearing housing and the shaft very difficult to be captured. Such a flow field can be approximated to a Taylor-Couette flow (1923) and its effect on heat transfer was investigated by Jakoby and Kim (1999). Their research showed that a critical rotational speed exists after which the flow field breaks up into these vortices and the heat transfer is significantly increased.

\[ Nu(x) = 0.530(Gr(x)Pr(x))^{0.25} \quad (5.21) \]

\[ h_{BH}(x) = 0.530 \frac{K_{BH}}{D_{BH}} (Gr(x)Pr(x))^{0.25} \quad (5.22) \]

\textbf{5.8.4 Cooling oil}

As the viscosity of air is very low and the flow velocities are laminar this is a valid assumption.
The non-dimensional number which provides information on the stability of the flow is the Taylor number, given in Eq. (5.23).

\[
T_a = \frac{\rho_{oill} \omega_s \sqrt{r_i (r_o - r_i)^{1.5}}}{\mu_{oill}} = \frac{2\pi n_s \rho_{oill} \sqrt{D_{BH,i} (D_{BH,i} - D_s)^{1.5}}}{\mu_{oill}}
\]  \hspace{1cm} (5.23)

where \( \omega_s = 2\pi n_s \) is the rotational speed of the shaft, \( D_{BH,i} \) the internal diameter of the bearing housing and \( D_s \) the shaft diameter. Experimental investigations showed that a critical Taylor number of about \( Ta_c \approx 1.76 \times 10^7 \) exists, after which the flow becomes unstable. In order to evaluate the Reynolds number of the oil in heat transfer calculation, an effective flow velocity has to be considered. This is given by the combination of the axial-through flow velocity \((u_{ax})\) and the radial component of rotational speed imposed by the shaft \((\pi n_s D_s)\). The axial flow velocity can be estimated through the continuity equation and the final expression for the effective velocity is given in Eq. (5.24).

\[
u_{eff} = \sqrt{u_{ax}^2 + (\pi n_s D_s)^2}
\]  \hspace{1cm} (5.24)

where

\[
u_{ax} = \frac{4\dot{m}_{oill}}{\pi \rho_{oill} (D_{BH,i}^2 - D_s^2)}
\]  \hspace{1cm} (5.25)

A final expression for the Nusselt number of the oil is then given in Eq. (5.26). This depends upon the effective Reynolds number and two constants \( c \) and \( n \). The constants \( c \) and \( n \) above depend on the flow regime and the ratio of the channel length \( L_s \) and the channel height \( s \). The relation is given by Eq. (5.27).

\[
Nu_{oill} = \frac{h_{oill} L_s}{k_{oill}} = c Re_{eff}^n
\]  \hspace{1cm} (5.26)

\[
c \left( \frac{L_s}{s} \right) ; n \left( \frac{L_s}{s} \right) = \frac{a_0 e^{t[(L_s/s) - b]} + a_1 e^{-t[(L_s/s) - b]}}{e^{t[(L_s/s) - b]} + e^{-t[(L_s/s) - b]}}
\]  \hspace{1cm} (5.27)

for which the constants \( a_0, a_1, t \) and \( b \) can be found in Table 5.9. In contrast to the surface of the bearing housing for which the heat transfer coefficient was highly dependent upon the air properties, the heat transfer to the lubricating oil is determined by the axial-through flow and the rotational speed of the shaft. Therefore it seems reasonable to use a mean value for the heat transfer coefficient.

---

\( ^{34} \) Here the assumption is introduced that the lubricating oil enters the oil channel on one side and leaves it on the opposite side. Furthermore the channel length is assumed to be the same length as the shaft. This does not reflect every possible configuration of the lubrication system but can be easily modified to do so.
in order to calculate the heat fluxes. Hence the heat transfer coefficient is treated constant along the oil channel, Eq. (5.28).

\[ h_{oil} = \frac{Nu_{oil} k_{oil}}{L_s} \]  

(5.28)

<table>
<thead>
<tr>
<th>Table 5.9: Constants for the flow regime equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steady flow (Ta &lt; 1.76 \times 10^7)</td>
</tr>
<tr>
<td>n</td>
</tr>
<tr>
<td>---</td>
</tr>
<tr>
<td>a_0</td>
</tr>
<tr>
<td>a_c</td>
</tr>
<tr>
<td>t</td>
</tr>
<tr>
<td>b</td>
</tr>
</tbody>
</table>

**5.8.5 Compressor casing**

The compressor casing was modeled as a round plate with the internal diameter equal to the external diameter of the bearing housing. Given that the thickness of the back-plate is small as compared to its diameter, the temperature was assumed to vary only in radial direction. The heat transfer coefficient is expected to vary in the radial direction and therefore a correlation for the Nusselt number and the heat transfer coefficient similar to that of the bearing housing was included in the model\(^{35}\), as given in Eq. (5.29) and Eq. (5.30).

\[ Nu = 0.667 \left( Gr(y)Pr(y) \right)^{0.25} \]  

(5.29)

\[ h_{BP} = 0.667 \frac{k_{BP}}{D_{BP}} \left( Gr(y)Pr(y) \right)^{0.25} \]  

(5.30)

**5.8.6 Gas flow within the compressor and turbine volute**

The volute is modeled as a straight pipe with varying diameter and the Nusselt correlation, as given in Eq. (5.31).

\[ Nu = 0.023 Re_d^{0.8} Pr^{0.4} \]  

(5.31)

\(^{35}\) Please note that the y direction in the Eq. (5.29) and Eq. (5.30) was introduced to show that the compressor back-plate is normal to the axial direction of the turbocharger (Figs. 5.16 and 5.17). The addition of an extra frame of reference might lead the reader to consider the model solution as a 2-D problem. This is not the case as the heat transfer model remains unaltered and it lies on the assumption of 1-D analysis.
The problem hereby lies in the calculation of the Reynolds for which the velocity and the diameter are dependent upon the position of the flow along the volute (azimuth angle $\psi$). The Reynolds number for a pipe is given in Eq. (5.32).

$$
Re = \frac{\rho u(\psi)d(\psi)}{\mu} = \frac{4\dot{m}(\psi)}{\pi \mu d(\psi)}
$$

(5.32)

where $\dot{m}(\psi) = \frac{\dot{m}_0}{2\pi} \psi$ & $d(\psi) = \frac{D_0}{2\pi} \psi$

(5.33)

By combination of Eq. (5.32) and (5.33), the Reynolds number reduces to Eq. (5.34)\textsuperscript{16}.

$$
Re = \frac{4\dot{m}_0}{\pi \mu D_0}
$$

(5.34)

From Eq. (5.34) the heat transfer coefficient can be calculated by mean of Eq. (5.35), where $D_0/2$ is chosen as characteristic length and the Nusselt number for a straight pipe is considered.

$$
h_{T,C} = Nu \frac{2k}{D_0} = 0.046 \frac{k}{D_0} Re^{0.8} Pr^{0.4}
$$

(5.35)

The estimation of the heat transfer coefficients for the gas flow in the compressor and the turbine volute completes the set of heat transfer coefficients used in the model. Next step is to evaluate the temperature distributions in the main bodies constituting the turbocharger model.

### 5.9 Temperature distribution and heat fluxes

In this section the surface temperature distributions and the corresponding heat fluxes were evaluated. In order to accomplish to this task, it is crucial to evaluate the heat transferred by convection, radiation and convection through the turbocharger components. The turbine casing was treated as isothermal surface while for the bearing housing and compressor back-plate numerical procedures were applied to solve the governing differential equations.

#### 5.9.1 Turbine casing

Due to the small thickness of the turbine casing, the turbine wall was assumed to be a flat plate. The temperature distribution occurring across the turbine wall is shown in Fig. 5.18. The inner

\textsuperscript{16} The dynamic viscosity in the Reynolds number depends on the dynamic viscosity of the air and the exhaust gases for the compressor and the turbine respectively.
wall temperature of the turbine was calculated as the mean temperature between the exhaust gases ($T_{Exh}$) and the flow temperature at the rotor inlet ($T_{3*}$). From the inner wall, heat flows towards the outer wall with temperature ($T_{surf}$). Here heat is exchanged by mean of radiation ($Q_{T,rad}$) and natural convection ($Q_{T,conv}$) to the surrounding environment. A heat balance across this infinitesimal element yields to the equation (5.36).

$$\frac{k_T}{t_T} \left( \frac{T_{Exh} + T_{3*}}{2} - T_{surf} \right) = h_{surf}(T_{surf} - T_{amb}) + \varepsilon(\sigma(T_{surf}^4 - T_{amb}^4))$$  \hspace{1cm} (5.36)

where $t_T$ is the turbine wall thickness.

By solution of Eq. (5.36) it is possible to calculate the surface temperature of the turbine volute. The heat fluxes that are of direct interest for the evaluation of the compressor non-adiabatic efficiency are those leaving the turbine towards the shaft and the bearing housing. These are given in Eq. (5.37) and Eq. (5.38).

$$\dot{Q}_{T\rightarrow BH} = -\frac{1}{4}\pi(D_{BH,o}^2 - D_{BH,l}^2)k_T \frac{dT_{BH}}{dx} \bigg|_{x=L_s}$$  \hspace{1cm} (5.37)

$$\dot{Q}_{T\rightarrow S} = -\frac{1}{4}\pi D_{S}^2 k_T \frac{dT_{S}}{dx} \bigg|_{x=L_s}$$  \hspace{1cm} (5.38)

In addition to this, the heat fluxes to the surrounding must also be calculated in order to have a full assessment of the heat balance across the turbocharger. It is worth noting that the heat fluxes calculated by means of Eq. (5.37) to Eq. (5.44) are negative. This was purposely sought as they illustrate a heat loss.
\[
\dot{Q}_{T,rad1} = -A_1 \sigma \varepsilon (T^{4}_{T,surf1} - T^{4}_{amb}) \\
\dot{Q}_{T,rad2} = -A_2 \sigma \varepsilon (T^{4}_{T,surf2} - T^{4}_{amb}) \\
\dot{Q}_{T,rad3} = -A_3 \sigma \varepsilon (T^{4}_{T,surf3} - T^{4}_{amb}) \\
\dot{Q}_{T,conv1} = -A_1 h_{T,surf1} (T_{T,surf1} - T_{amb}) \\
\dot{Q}_{T,conv2} = -A_2 h_{T,surf2} (T_{T,surf2} - T_{amb}) \\
\dot{Q}_{T,conv3} = -A_3 h_{T,surf3} (T_{T,surf3} - T_{amb})
\]

where \(A_1, A_2, \) and \(A_3\) are the surface areas of the two round plates and the cylindrical body considered for the turbine volute:

\[
A_1 = \frac{\pi}{4} (D^2_H - D^2_H,o) \\
A_2 = \frac{\pi}{4} D^2_f \\
A_3 = \pi D_T L_T
\]

### 5.9.2 Shaft

The heat flux from the turbine to the shaft is mainly flowing in the axial direction given the small diameter of the shaft. Hence the radial variation of the temperature was not included in the analysis (in Fig. 5.19 an infinitesimal element of the shaft is shown). As for the purpose of this research, only the steady state was considered and therefore no change in internal energy was taken into account. Hence the energy balance simplifies to Eq. (5.48).

\[
\dot{Q}_X = \dot{Q}_X + dx + \dot{Q}_{s{oil}}
\]

The conductive heat flux term can be rewritten using a Taylor series as:

\[
\dot{Q}_X + dx = \dot{Q}_X + \frac{d\dot{Q}_X}{dx} dx
\]

The heat flux to the oil and the axial derivative of the conductive heat flux\(^{37}\) are given in Eq. (5.50) and Eq. (5.51).

\[
\dot{Q}_{s{oil}} = \pi D_s h_{oil} (T_s(x) - T_{oil}) dx
\]

---

\(^{37}\) The Fourier’s law was used for the axial derivative.
Substituting these correlations into Eq. (5.48) yields the final differential equation of the temperature along the shaft.

\[ \frac{dQ_x}{dx} = -\frac{1}{4} k_s \pi D_s^2 \frac{d^2 T_s(x)}{dx^2} \]  \hspace{1cm} (5.51)

In order to solve the differential equations of second order, two boundary conditions are necessary. The main issue is the fact that the temperatures of the gases vary as they go through compression and expansion. Therefore two main assumptions were made:

1. the effective temperature of the gas corresponds to the average temperature at the inlet and the exit to the rotor; this enabled us to evaluate an initial (turbine side, \(x=L_s\)) and final (compressor side, \(x=0\)) temperature for the shaft
2. the heat transfer coefficients in both ends of the shaft is assumed to be equal to those of the turbine (\(x= L_s\)) and the compressor (\(x=0\))

Two boundary conditions are then calculated by assuming the condition of constant heat fluxes at both ends of the shaft (\(x=0\) and \(x=L_s\)):

\[ -k_s \frac{dT_s(x)}{dx} \bigg|_{x=0} = h_c \left( \frac{T_{1*} + T_{2*}}{2} - T_{s,(x=0)} \right) \]  \hspace{1cm} (5.53)
In reality the heat transferred to the shaft originates in the gases that flow through the turbine rotor. The problem of the above boundary condition is that the compressor rotor inlet temperature depends on the heat flux from the shaft. Therefore an iterative approach needs to be implemented in order to calculate all temperatures.

The amount of heat delivered by the shaft to the air is given in Eq. (5.55). This was included in the model and corresponds to \( q_{c,before} \) in Fig. 5.1.

\[
\dot{Q}_{S\rightarrow air} = \frac{1}{4} \pi D_s^2 k_s \left. \frac{dT_s}{dx} \right|_{x=0} (5.55)
\]

### 5.9.3 Bearing housing

Similarly to the shaft, the derivation of the differential equation for the bearing housing was done by considering an infinitesimal element. In addition to the heat flux to the oil, the convective and radiative heat transfer and to the surrounding was also considered. The heat balance for an infinitesimal element is shown in Fig. 5.20:

\[
\dot{Q}_x = \dot{Q}_{x+d\dot{x}} + \dot{Q}_{BH\rightarrow oil} + \dot{Q}_{BH,conv} + \dot{Q}_{BH,rad} (5.56)
\]

Similarly to the calculation of the temperature distribution in the shaft, the Taylor series was used here to determine the differential element for the heat flux, as shown in Eq. (5.57).

\[
\frac{d\dot{Q}_x}{dx} = \frac{1}{4} k_{BH}(D_{BH,o}^2 - D_{BH,i}^2) \frac{d^2T_{BH}(x)}{dx^2} (5.57)
\]

The other terms of Eq. (5.56) will also depend on the axial position and will therefore be dependent on the axial direction.

\[
\dot{Q}_{BH\rightarrow oil} = \pi D_{BH,i} h_{oil}(T_{BH}(x) - T_{oil})dx (5.58)
\]

\[
\dot{Q}_{BH,rad} = \pi D_{BH,o} \varepsilon \sigma (T_{BH}(x) - T_{amb})dx (5.59)
\]

\[
\dot{Q}_{BH,conv} = \pi D_{BH,o} h(x)_{BH}(T_{BH}(x) - T_{amb})dx (5.60)
\]
By substituting all the equations above into Eq. (5.56), we obtain the differential equation for the temperature distribution in the bearing housing, as given in Eq. (5.61). A numerical procedure was used to solve Eq. (5.61).

\[
\frac{1}{4} k_{BH} \pi \left( D_{BH,0}^2 - D_{BH,i}^2 \right) \frac{d^2 T_{BH}(x)}{dx^2} = \pi D_{BH,i} h_{oil}(T_{BH}(x) - T_{oil}) + \pi D_{BH,o} \varepsilon \sigma (T_{BH}^4(x) - T_{amb}^4) + \pi D_{BH,o} h_{BH}(x)(T_{BH}(x) - T_{amb})
\]

Figure 5.20: Heat balance on an infinitesimal element of the bearing housing

The heat affecting the compressor performance is that transferred from the bearing housing to the air into the compressor. The expression for the heat transfer is given in Eq. (5.62).

\[
\dot{Q}_{BH-air} = \frac{1}{4} \pi k_{BH} \left( D_{BH,0}^2 - D_{BH,i}^2 \right) \left. \frac{dT_{BH}}{dx} \right|_{x=0}
\]

### 5.9.4 Compressor casing

Similarly to the bearing housing, the temperature of the compressor back-plate was calculated by considering the heat fluxes taking place through an infinitesimal element. The heat fluxes are shown in Fig. 5.21. Besides the heat going through the compressor wall, the compressor back-plate exchanges heat with the air within the compressor (by forced convection) and with the surrounding environment (by natural convection and radiation). Given the relatively small wall thickness of the compressor back-plate in respect with its diameter, the variations are assumed to change only in radial direction. The energy balance in the infinitesimal element is given in Eq. (5.63).
\[ \dot{Q}_y = \dot{Q}_{y+dy} + \dot{Q}_{BP-air} + \dot{Q}_{BP,conv} + \dot{Q}_{BP,rad} \]  

(5.63)

The terms of Eq. (5.63) were determined in a similar manner as the bearing housing.

\[ \frac{d\dot{Q}_y}{dy} = -\frac{1}{4} t_{BP} k_{BP} \pi (D_{BP,y+dy}^2 - D_{BP,y}^2) \frac{d^2T_{BP}(y)}{dy^2} \]  

(5.64)

\[ \dot{Q}_{C,rad} = \pi \varepsilon \sigma (D_{BP,y+dy}^2 - D_{BP,y}^2)(T_{BP}(y) - T_{amb})dy \]  

(5.65)

\[ \dot{Q}_{C,conv} = \pi h_{BP} (D_{BP,y+dy}^2 - D_{BP,y}^2)(T_{BP}(y) - T_{amb})dy \]  

(5.66)

\[ \dot{Q}_{C-air} = \pi h_{C} (D_{BP,y+dy}^2 - D_{BP,y}^2)(T_{BP}(y) - T_{air})dy \]  

(5.67)

Introducing these equations into the heat balance it yields to the final differential equation for the temperature distribution:

\[ \frac{1}{4} t_{BP} k_{BP} \frac{d^2T_{BP}(y)}{dy^2} = h_{BP}(T_{BP}(y) - T_{air})dy + h_{C}(T_{BP}(y) - T_{air})dy + \varepsilon \sigma (4_4 T_{BP}(y) - T_{amb}) \]  

(5.68)

Figure 5.21: Heat balance on an infinitesimal element of the compressor back-plate

The heat flux that is directly contributing to the temperature rise of the air is given in Eq. (5.67).

5.10 Model Flow Chart

Fig. 5.22 provides the flow chart of the model. The input parameters for the turbocharger model are the performance parameters extrapolated by the “cold” maps, the oil flow rate and the
temperature of the exhaust gases entering the turbine. The control parameter for the whole calculation is the temperature of the exhaust gases leaving the turbine ($T_{04}$). In order to start the calculation an initial assumption on the exit temperature of the exhaust gases ($T_{04}$) and the heat added before compression ($q_{C,\text{before}}$) and expansion ($q_{T,\text{before}}$) was made. With the initial estimated values of $q_{C,\text{before}}$ and $q_{T,\text{before}}$ the heat fluxes going through the turbocharger can be evaluated. On the basis of the calculation, a new evaluation for $q_{C,\text{before}}$ and $q_{T,\text{before}}$ is made according to the newly computed temperatures. These two new values for the heat transfer are compared with those calculated initially and if the convergence is not satisfied, a new estimation for $q_{C,\text{before}}$ and $q_{T,\text{before}}$ will be made until the convergence is satisfied.

**HEAT FLUX CALCULATION**

- **Turbine Casing → Surroundings**:
  - Free Convection
  - Radiation

- **Turbine Casing → Shaft**:
  - Forced Convection

- **Turbine Casing → Bearing Housing**:
  - Forced Convection

- **Bearing Housing → Surrounding**:
  - Free Convection
  - Radiation

- **Bearing Housing → Oil**:
  - Forced Convection

- **Shaft → Oil**:
  - Forced Convection

- **Shaft → $C_{ar}$**:
  - Forced Convection

- **Bearing Housing → $C_{ar}$**:
  - Forced Convection

- **Back-plate → $C_{ar}$**:
  - Forced Convection

**Computation:** $q_{C,\text{before new}}, q_{T,\text{before new}}$

**Heat fluxes going through the turbocharger can be evaluated.**

**Fixed:**
- Adiabatic performance parameters
- Geometrical Dimensions

**Set:**
- Turbine temperature exhaust gases

**Assume:** Turbine Total Exit Temperature $T_{04}$

**First computation of:** $q_{C,\text{before}}, q_{T,\text{before}}$

**Computation:** $q_{C,\text{before new}}, q_{T,\text{before new}}$

**Computation:** $T_{04}, T_{02}$

**Non-adiabatic efficiencies:**
- Turbine
- Compressor

**Repeat until converge**

**END**

Figure 5.22: Model flow chart
As with the heat, the exit temperature to the turbine ($T_{04}$) and the compressor ($T_{02}$) are calculated and a comparison is then made between two consecutive values until convergence is satisfied. Once the calculation is converged, the non-adiabatic efficiencies are finally computed.

### 5.11 Model setup

The input parameters for the turbocharger model were reported in Table 5.10; the shaft speed, the pressure ratios and mass flow rate for both the turbine and the compressor were gathered by the experimental results. Based on these values, the corresponding adiabatic efficiencies ($\eta_{C,\text{adi}}$ & $\eta_{T,\text{adi}}$) were then found from the cold maps provided manufacturers. Additionally the measured oil flow rate and oil temperature were also included in the model together with the temperature of the exhaust gases. Details are given in Table 5.11.

<table>
<thead>
<tr>
<th>$N_{shaft}$ [rev/s]</th>
<th>PR$_C$</th>
<th>$m_C$ [kg/s]</th>
<th>$\eta_C$</th>
<th>PR$_T$</th>
<th>$m_T$ [kg/s]</th>
<th>$\eta_T$</th>
<th>m$_{oil}$ [kg/s]</th>
<th>$T_{oil,\text{inlet}}$ [K]</th>
<th>$T_{oil,\text{outlet}}$ [K]</th>
<th>$T_{exh,\text{gas}}$ [K]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1650</td>
<td>1.35</td>
<td>0.052</td>
<td>0.9</td>
<td>1.19</td>
<td>0.054</td>
<td>0.33</td>
<td>0.049</td>
<td>377</td>
<td>378</td>
<td>561</td>
</tr>
<tr>
<td>1800</td>
<td>1.40</td>
<td>0.052</td>
<td>0.9</td>
<td>1.22</td>
<td>0.054</td>
<td>0.67</td>
<td>0.051</td>
<td>384</td>
<td>393</td>
<td>698</td>
</tr>
<tr>
<td>1800</td>
<td>1.40</td>
<td>0.059</td>
<td>0.91</td>
<td>1.19</td>
<td>0.062</td>
<td>0.47</td>
<td>0.059</td>
<td>368</td>
<td>368</td>
<td>596</td>
</tr>
<tr>
<td>1915</td>
<td>1.47</td>
<td>0.061</td>
<td>0.93</td>
<td>1.23</td>
<td>0.064</td>
<td>0.47</td>
<td>0.061</td>
<td>384</td>
<td>385</td>
<td>751</td>
</tr>
<tr>
<td>1940</td>
<td>1.57</td>
<td>0.051</td>
<td>0.91</td>
<td>1.40</td>
<td>0.053</td>
<td>0.63</td>
<td>0.039</td>
<td>360</td>
<td>362</td>
<td>755</td>
</tr>
<tr>
<td>2062</td>
<td>1.64</td>
<td>0.060</td>
<td>0.92</td>
<td>1.38</td>
<td>0.062</td>
<td>0.66</td>
<td>0.054</td>
<td>387</td>
<td>388</td>
<td>816</td>
</tr>
<tr>
<td>2117</td>
<td>1.73</td>
<td>0.043</td>
<td>0.82</td>
<td>1.61</td>
<td>0.045</td>
<td>0.78</td>
<td>0.034</td>
<td>380</td>
<td>386</td>
<td>836</td>
</tr>
<tr>
<td>2230</td>
<td>1.82</td>
<td>0.057</td>
<td>0.92</td>
<td>1.60</td>
<td>0.059</td>
<td>0.73</td>
<td>0.041</td>
<td>377</td>
<td>378</td>
<td>834</td>
</tr>
<tr>
<td>2238</td>
<td>1.72</td>
<td>0.070</td>
<td>0.92</td>
<td>1.40</td>
<td>0.073</td>
<td>0.64</td>
<td>0.064</td>
<td>389</td>
<td>392</td>
<td>882</td>
</tr>
<tr>
<td>2261</td>
<td>1.85</td>
<td>0.042</td>
<td>0.82</td>
<td>1.70</td>
<td>0.044</td>
<td>0.85</td>
<td>0.036</td>
<td>384</td>
<td>390</td>
<td>949</td>
</tr>
<tr>
<td>2325</td>
<td>1.92</td>
<td>0.068</td>
<td>0.95</td>
<td>1.59</td>
<td>0.071</td>
<td>0.78</td>
<td>0.056</td>
<td>384</td>
<td>387</td>
<td>876</td>
</tr>
<tr>
<td>2525</td>
<td>2.04</td>
<td>0.061</td>
<td>0.9</td>
<td>1.79</td>
<td>0.064</td>
<td>0.75</td>
<td>0.044</td>
<td>384</td>
<td>393</td>
<td>928</td>
</tr>
</tbody>
</table>

The oil used during testing is standard lubricating oil available in the market, *Shell Rimula X-SAE 30 W*. The oil specific heat, kinematic viscosity and density change with temperature and their values were calculated through interpolation of tabular values. The heat transfer to the environment strongly depends on the ambient temperature and pressure, these were also recorded in the test cell during testing. The emissivity and thermal conductivity of the turbocharger material are also very important. The thermal conductivity is taken to be equal to the value of common cast iron with a value of 60.5 W/mK. For the compressor casing instead a value of 150.7 W/mK was used that corresponds to the thermal conductivity for the aluminium alloy *LM27*. Bohn et al. (2003) found that the

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18 For the turbine side five different maps were made available according to five different vane positions (from fully closed to fully open). During experiments the vane position was monitored for each operating condition and the correct turbine map could then be selected in order to determine $\eta_{T,\text{adi}}$. The turbine efficiencies reported in Table 5.10 are relative efficiencies (calculated similarly to those of the compressor).

19 The oil density and the kinematic viscosity were determined with tests conducted at the Imperial College Tribology Department.
emissivity of the casing is about 0.6. This is might not be appropriate for all the materials but it seems to be a good mean for common materials and hence it was used here.

### Table 5.11: Geometrical parameters

<table>
<thead>
<tr>
<th>COMPONENT</th>
<th>Symbol</th>
<th>Value [m]</th>
<th>COMPONENT</th>
<th>Symbol</th>
<th>Value [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outer bearing housing diameter</td>
<td>( D_{BH,o} )</td>
<td>0.038</td>
<td>Compressor back-plate thickness</td>
<td>( T_{back-plate} )</td>
<td>0.005</td>
</tr>
<tr>
<td>Inner bearing housing diameter</td>
<td>( D_{BH,i} )</td>
<td>0.0139</td>
<td>Turbine inlet diameter</td>
<td>( D_{T,inl} )</td>
<td>0.033</td>
</tr>
<tr>
<td>Shaft length</td>
<td>( L_S )</td>
<td>0.045</td>
<td>Turbine outlet diameter</td>
<td>( D_{T,out} )</td>
<td>0.036</td>
</tr>
<tr>
<td>Shaft diameter</td>
<td>( D_S )</td>
<td>0.0071</td>
<td>Compressor inlet diameter</td>
<td>( D_{C,inl} )</td>
<td>0.0402</td>
</tr>
<tr>
<td>Turbine housing length</td>
<td>( L_T )</td>
<td>0.06</td>
<td>Compressor outlet diameter</td>
<td>( D_{C,out} )</td>
<td>0.036</td>
</tr>
<tr>
<td>Turbine housing wall thickness</td>
<td>( t )</td>
<td>0.0065</td>
<td>Outer bearing housing diameter</td>
<td>( D_{BH,o} )</td>
<td>0.038</td>
</tr>
<tr>
<td>Compressor back-plate diameter</td>
<td>( D_{BP} )</td>
<td>0.12</td>
<td>Inner bearing housing diameter</td>
<td>( D_{BH,i} )</td>
<td>0.0139</td>
</tr>
</tbody>
</table>

### 5.12 Model Validation

In this section a discussion over the validation of the heat transfer model is provided. The simulation results are compared with those obtained through experimental results and three main parameters will be used for validation:

- heat conducted through the turbine casing
- exit temperature to compressor
- compressor non-adiabatic efficiency

Additionally, the capability of the model to capture the effects of heat transfer for different rotational speeds and temperatures of the exhaust gases was also evaluated. Four rotational speeds were selected from the compressor cold maps and input into the model together with five different temperatures of the exhaust gases. The non-adiabatic efficiency maps could then be generated and the data used for statistical analysis.

It is worth noting that the turbocharger used in this research was supplied with performance maps generated in a low temperature gas test stand. Therefore, prior to any calculation, the model was firstly run in cold conditions and calibrated against the peak efficiency point extrapolated by the compressor cold maps. A calibration factor was then calculated and applied to the non-adiabatic efficiency for the entire simulation.

### 5.12.1 Heat conducted through the turbine casing

The heat flux through the turbine casing represents the amount of heat that is dissipated by the gas before expanding in the rotor. A good evaluation of the heat conducted is therefore important to the overall success of the simulation.
The heat conducted through the turbine casing in the three measuring locations (Engine, Top and External side) is given in Fig. 5.23. Due to the large scatter of the calculated points, a zone of actual heat conduction was drawn to aid comparison. The averaged values for the heat conducted in the three measuring locations was then calculated (blue diamond) together with the best fit line (solid blue curve). Despite the simplicity of the turbocharger model, the computed heat conduction (solid red line) falls well within the actual heat conduction area. The model prediction follows the measured trend line with reasonable accuracy. Although the discrepancy between calculated and measured values can go up to $\pm 18\%$, the overall averaged deviation over the entire range of exhaust gas temperatures remains low, (refer to Table 5.12). This seems to confirm the effectiveness of the assumptions made on the set up of the turbocharger model, particularly if one considers the large range of temperatures evaluated ($\approx 450$ K to $\approx 950$ K) and also the significant assumptions made on the geometry of the turbocharger. The calculated deviation can be attributed to several factors. First of all, the simplified geometry of the model does not take into account the fact that the turbine casing comes as a whole die cast body with the exhaust manifold. This leads to a temperature distribution difficult to predict locally. In addition to this, given the 1-D nature of the model, only a single value for the inner and outer wall temperature could be calculated for the whole turbine casing. This is not the case because the experimental results showed that the temperature distribution along the turbine casing can vary substantially when moving from the Engine towards the External side. Another factor contributing to the overall deviation is that, within the model the wall thickness of the turbine was
assumed to be uniform. This is not the case since the wall thickness of turbine housing varies as a consequence of the manufacture process and design requirements.

<table>
<thead>
<tr>
<th>Exhaust Gases [K]</th>
<th>836</th>
<th>949</th>
<th>755</th>
<th>834</th>
<th>928</th>
<th>561</th>
<th>698</th>
<th>816</th>
<th>876</th>
<th>596</th>
<th>751</th>
<th>882</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comp. heat [1x10⁵ W/m²]</td>
<td>≈4.0</td>
<td>≈5.3</td>
<td>≈3.5</td>
<td>≈4.5</td>
<td>≈1.2</td>
<td>≈2.3</td>
<td>≈3.2</td>
<td>≈3.6</td>
<td>≈1.3</td>
<td>≈1.5</td>
<td>≈3.6</td>
<td></td>
</tr>
<tr>
<td>Exp. heat [1x10⁵ W/m²]</td>
<td>≈3.6</td>
<td>≈4.5</td>
<td>≈3.3</td>
<td>≈4.4</td>
<td>≈1.4</td>
<td>≈2.5</td>
<td>≈3.3</td>
<td>≈3.8</td>
<td>≈1.6</td>
<td>≈2.6</td>
<td>≈3.4</td>
<td></td>
</tr>
<tr>
<td>Deviation ∆Q [%]</td>
<td>-10</td>
<td>-17</td>
<td>13</td>
<td>1.6</td>
<td>-2.3</td>
<td>14</td>
<td>10</td>
<td>1.5</td>
<td>4.5</td>
<td>18</td>
<td>2.2</td>
<td>-4.1</td>
</tr>
</tbody>
</table>

It is worth noting that the heat conduction lines follow an exponential trend. Such a trend can be explained considering that the wide range of temperatures experienced by the exhaust gases leads to a significant change in the Reynolds number and, hence, in the heat transfer coefficient. In Fig. 5.23 the calculated trend of the heat transfer coefficient was plotted against the temperature of the exhaust gases. It can be seen that heat transfer coefficient varies exponentially with temperature. This can be explained by considering that the inner wall temperature of the compressor casing is mainly affected by forced convection which in turn is proportional to the heat transfer coefficient. This is at the basis of the non linear variation of the heat conducted through the turbine casing.

| PR | 1.72 | 1.85 | 1.57 | 1.82 | 2.04 | 1.35 | 1.40 | 1.64 | 1.92 | 1.40 | 1.47 | 1.73 |
| N [1x10² rev/s·√K] | 1.24 | 1.32 | 1.13 | 1.30 | 1.48 | 0.96 | 1.05 | 1.21 | 1.36 | 1.05 | 1.12 | 1.29 |
| MFP [1x10⁻⁵ (kg/s)·√K/Pa] | 0.72 | 0.71 | 0.87 | 0.97 | 1.04 | 0.88 | 0.87 | 0.8 | 1.16 | 1.00 | 1.04 | 0.81 |
| Experimental Exit temp [K] | 383 | 395 | 363 | 394 | 404 | 335 | 344 | 363 | 383 | 343 | 353 | 373 |
| ∆T [K] | 0 | 4 | 3 | 1 | 5 | 2 | 2 | 3 | 5 | 1 | 3 | 0 |
| Experimental - ηc | 0.65 | 0.66 | 0.68 | 0.65 | 0.72 | 0.72 | 0.68 | 0.75 | 0.80 | 0.69 | 0.67 | 0.73 |
| Model- ηc | 0.65 | 0.64 | 0.71 | 0.68 | 0.69 | 0.69 | 0.65 | 0.72 | 0.75 | 0.70 | 0.70 | 0.73 |
| ∆η [%] | 0 | 2 | 3 | 3 | 3 | 3 | 3 | 3 | 5 | 1 | 3 | 0 |

### 5.12.2 Compressor exit temperature and non-adiabatic efficiency

The outcomes of the model prediction for the compressor exit temperature and compressor non-adiabatic efficiency is reported here. Table 5.13 shows the computed values for the compressor
efficiency and exit temperature compared with the experimental results. The simulation results are plotted in Fig. 5.24 where the compressor adiabatic efficiency and the corresponding exit temperature are also included for comparison.

Figure 5.24: Model validation: exit temperature and compressor non-adiabatic efficiency

From Fig. 5.24, it can be noticed that the model prediction for the compressor exit temperature is very good. The predicted exit temperatures closely follow those measured experimentally, with a difference no larger than few degrees. The absolute difference (refer to Table 5.13) for the predicted exit temperatures is not larger than 5 K and on the overall the averaged deviation from the experimental data is ≈2.5 K. This is only slightly above the uncertainty range associated with experimental measurements and it shows the effectiveness of the assumption made on the model geometry and the occurring heat fluxes. Nevertheless, such a good prediction does not correspond to an equally good prediction for the compressor efficiency. On the efficiency side the model prediction seems to be less accurate than that exhibited for the temperature. The scatter of data of the computed efficiency from that measured experimentally remains within ≈3 percentage points for most of the operating conditions considered here. This can be mainly be attributed to the error propagation associated with the computed exit temperature that makes the predicted non-adiabatic efficiency to deviate more from that measured experimentally. However, on the overall, the prediction for the compressor non-adiabatic efficiency remains within an acceptable range and it enables to extend our simulation to different operating conditions with good degree of confidence.
5.12.3 Model qualitative validation

As the experimental data was obtained on a turbocharger installed on a real engine, it was not possible to control all the turbocharger parameters so as to obtain a wide range of pressure. In order to overcome such a limitation, the performance parameters from the turbocharger cold maps were extrapolated for four different rotational speeds. For each of these, the non-adiabatic efficiencies were calculated for five different temperatures of the exhaust gases as given in Table 5.14.

<table>
<thead>
<tr>
<th>Turbine Speed [rev/s√K]</th>
<th>Temperature exhaust gases</th>
</tr>
</thead>
<tbody>
<tr>
<td>88.0 [90000 rpm]</td>
<td>550 K 650 K 750 K 850 K 950 K</td>
</tr>
<tr>
<td>107.6 [110000 rpm]</td>
<td>550 K 650 K 750 K 850 K 950 K</td>
</tr>
<tr>
<td>146.8 [150000 rpm]</td>
<td>550 K 650 K 750 K 850 K 950 K</td>
</tr>
<tr>
<td>163.3 [170000 rpm]</td>
<td>550 K 650 K 750 K 850 K 950 K</td>
</tr>
</tbody>
</table>

The outcomes of the model calculation are given in Figs. 5.25 to 5.28. The compressor non-adiabatic efficiency and the corresponding exit temperatures are reported against the mass flow rate for each condition of Table 5.14.
From Figs. 5.25 to 5.28, it can be seen that the efficiency drop associated with increasing heat transfer is very well captured by the model. At high rotational speed the predicted compressor efficiency does not deviate substantially from that measured in cold conditions. This is clearly seen in Fig. 5.25 where for $N=163.3$ rev/s√K and $T_{Exh}=550$ K, the efficiency drop is only $\approx 3\%$ while it goes up to $\approx 10\%$ for
This is fully consistent with the experimental findings, for which it was found that at high rotational speeds the effects of heat transfer on compressor performance is negligible. The experimental evidence also showed that as the rotational speed drops to low values, the temperature effect becomes dominant. This is also well captured by the model. In fact, as the temperature increases, the compressor performance decreases consistently with experimental evidence. At high rotational speeds for which the temperature effect on the compressor performance is not important, the predicted non-adiabatic efficiencies do not vary substantially to one another (Figs. 5.25 and 5.26). At 550 K the computed exit temperature to the compressor is almost equivalent to that calculated in adiabatic conditions. As the temperature of the exhaust gases increases (550 K to 950 K), the temperature rise to the compressor varies by only ≈10 K. On the contrary, at lower rotational speeds (Figs. 5.27 and 5.28) the effect of temperature on efficiency is more relevant and this corresponds to large variation in the compressor performance as the temperature increases.

![Compressor efficiency and exit temperature: N = 88.0 rev/s√K](image)

Figure 5.28: Compressor relative efficiency vs. mass flow rate for different temperatures of the exhaust gases (950 K, 850 K, 750 K, 650 K and 550 K) at 88.0 rev/s√K

5.13 Sensitivity analysis

As already discussed in the previous paragraphs, the heat transfer in a turbocharger is very complex. There are many parameters (either geometrical or physical) which affect the heat exchange between the different bodies of the turbocharger. In the development the model, many assumptions had to be made in order to reduce its complexity and to make it more adaptable to different turbocharger settings.
All of the assumptions and the simplifications introduce a certain error which needs to be evaluated. Hence a sensitivity analysis of all the parameters which are considered to play an important role in the overall heat transfer has been performed. The comparison base line for the analysis is an operating point with a standard rotational speed, temperature of the exhaust gases and mass flow parameter. As a consequence of such a choice, the outcomes of the sensitivity analysis are neither attenuated nor amplified. Details of the chosen operating point together with the compressor and turbine efficiencies are given in Table 5.15.

Table 5.15: Operating point for the sensitivity analysis

<table>
<thead>
<tr>
<th></th>
<th>Compressor</th>
<th>Turbine</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shaft speed [rev/s\cdot K]</td>
<td>130.9</td>
<td>130.9</td>
</tr>
<tr>
<td>MFP [1x10^-5 (kg/s)\cdot K/Pa]</td>
<td>0.97</td>
<td>1.00</td>
</tr>
<tr>
<td>Pressure ratio</td>
<td>1.82</td>
<td>1.60</td>
</tr>
<tr>
<td>Adiabatic efficiency(^{40})</td>
<td>0.92</td>
<td>0.73</td>
</tr>
<tr>
<td>Non-adiabatic efficiency</td>
<td>0.64</td>
<td>--</td>
</tr>
<tr>
<td>Temperature exhaust gases [K]</td>
<td>--</td>
<td>834</td>
</tr>
</tbody>
</table>

The sensitivity analysis was for the above operating point by varying the input parameters by ±10%; the influence of the parameters on the compressor non-adiabatic efficiency was then recorded and the results are reported in Table 5.16. The values reported in table are given in terms of deviation of the efficiencies as a relative error, given in Eq. (5.69):

$$\Delta \eta_{dia,c} = \frac{\eta_{dia,c} - \eta_{dia,c}^{\prime}}{\eta_{dia,c}}$$

(5.69)

where $\eta_{dia,c}^{\prime}$ is the calculated value with the deviated input parameter (Kumm, 2007). The sensitivity analysis showed that many parameters have a small influence on the efficiencies and therefore only the parameters with the largest influence were reported in Table 5.16. For example the ambient conditions seem to be very important for the calculations due to the fact that the overall heat transferred to the surrounding by convection and radiation is highly dependent on the ambient temperature. A variation of about ±1% was calculated for ±10% variation in the ambient temperature. In addition to this, Table 5.16 also shows that a relatively strong effect on the compressor non-adiabatic efficiency can be attributed to the compressor back-plate diameter, while its thickness does not seem to have a great effect. The compressor back-plate accounts for ±0.25% of the efficiency variation while its thickness has almost a negligible effect, ±0.10%.

\(^{40}\) The adiabatic efficiencies have to be intended as relative efficiencies.
Table 5.16: Sensitivity analysis for the most important parameters: input parameter ± 10%

<table>
<thead>
<tr>
<th>Parameter variation</th>
<th>$\Delta \eta_{\text{t, c}}$ (+10 %)</th>
<th>$\Delta \eta_{\text{t, c}}$ (-10 %)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shaft length</td>
<td>-0.20 %</td>
<td>-0.05 %</td>
</tr>
<tr>
<td>Shaft diameter</td>
<td>-0.01 %</td>
<td>-0.01 %</td>
</tr>
<tr>
<td>Bearing housing outer diameter</td>
<td>+0.08 %</td>
<td>-0.36 %</td>
</tr>
<tr>
<td>Turbine volute diameter</td>
<td>-0.03 %</td>
<td>-0.23 %</td>
</tr>
<tr>
<td>Turbine length</td>
<td>-0.01 %</td>
<td>-0.24 %</td>
</tr>
<tr>
<td>Thickness compressor back-plate</td>
<td>+0.09 %</td>
<td>-0.10 %</td>
</tr>
<tr>
<td>Diameter compressor back-plate</td>
<td>-0.25 %</td>
<td>+0.29 %</td>
</tr>
<tr>
<td>Turbine case emissivity</td>
<td>-0.32 %</td>
<td>+0.07 %</td>
</tr>
<tr>
<td>Ambient temperature</td>
<td>-1.07 %</td>
<td>+1.22 %</td>
</tr>
</tbody>
</table>

Other parameters that influence the overall calculation are the bearing housing diameter, the turbine diameter and the turbine length. These have a comparatively high influence given that this is the surface area that transfers heat in convection and radiation to the surroundings. Finally, as the radiative heat transfer parameter also has a significant influence since it is largely affected by the emissivity.

### 5.14 Statistical analysis

In order to complete the analysis on heat transfer, the data generated by the model was used to assess the compressor non-adiabatic performance by means of a regression analysis. Regression analysis is used to predict a continuous dependent variable from a number of independent variables. The main benefit of regression analysis is that the contribution of each parameter can rapidly be quantified. Potentially this could be very useful to turbine designers and software developers in the selection of turbochargers providing the best compromise in relation to the input parameters. In order to perform the regression analysis, the following steps were taken:

1. Generate a statistically significant amount of data to be used in the regression analysis,
2. Identify the minimum number of parameters that better describe the compressor efficiency in non-adiabatic conditions,
3. Perform the multiple regression analysis, determine the regression coefficients and evaluate the goodness of fit through the evaluation of conventional fit parameters,
4. Discuss the consistency of the regression response with the experimental findings and assess the capability of the statistical approach for the calculation of the compressor efficiency.

In order to do this a number of significant parameters, responding to Eq. (5.70), must be identified.
\[ \eta = f(x_i) \] (5.70)

where \( x_i \) are the explanatory variables (independent parameters) and \( \eta \) is the response variable.

In order to identify the parameters influencing the compressor efficiency, it is useful to refer to the conventional compressor maps. In a compressor map, a given point is uniquely defined by a pair of non-dimensional parameters selected amongst efficiency, pressure ratio, mass flow and speed. An example is given in Fig. 5.29 where the compressor operating point (red dot) is identified by the unique combination of mass flow and pressure ratio. A different choice of parameters could be made, leading to the same result with no ambiguity. Hence the minimum number of two parameters is enough to characterize the compressor efficiency. For the purpose of this research the pressure ratio \( PR \) and the rotational speed given in terms of Mach number were selected. However, since we are dealing with heat transfer, the choice of two parameters from the cold maps would not be fully representative of the actual conditions experienced by the compressor in non-adiabatic conditions. Hence a third parameter accounting for the heat transferred to the compressor must be included in the analysis. This was identified in the form of “temperature parameter”. A more detailed description of the parameters can be found below.

1. Pressure ratio (PR): in order to determine a point in a conventional compressor map, at least two parameters must be known. The pressure ratio was identified as an independent parameter. It is defined as the ratio between the stagnation pressures at the inlet and exit to the compressor:
2. **Mach number (M):** this is the local Mach number at the exit to the impeller blade row. The compressor impeller used for the current research has backward swept blades (blade angle $\beta_{B2} = -50^\circ$) and the velocity triangle is given in Fig. 5.30. The ideal case of a perfectly guided flow for radial blades was also considered here in order to quantify the effects of blade geometry and slip on the overall compressor performance. The absolute velocities for both the backward swept and radial blade impeller can be calculated as given in Eq. (5.72) and Eq. (5.73)\(^{41}\).

\[
C_{2,\text{adi}} = \sqrt{(\mu U_2 + C_{2m,\text{adi}} \tan \beta_{B2})^2 + C_{2m,\text{adi}}^2} \\
C_{2,\text{adi}} = \sqrt{U_2^2 + C_{2m,\text{adi}}^2}
\] (5.72) (5.73)

where $U_2$ is the tangential velocity, $C_{2m,\text{adi}}$ the meridional component of the absolute velocity calculated through the continuity equation, $m_2 = \rho_2 C_{2m,\text{adi}} A_2$ and $\mu$ the slip factor calculated with the Stanitz correlation (1952).

\[
\mu = 1 - \left(0.63\pi/Z_B\right) \quad \text{with } Z_B = \text{blades number}
\] (5.74)

The Mach number is then given as:

\[PR = \frac{P_{02}}{P_{01}}\] (5.71)

\(\text{Figure 5.30: Heat transfer process within the turbocharger and exit velocity triangles}\)
As already reported earlier in this chapter, Shaaban and Seume (2006) demonstrated that the parameters that are relevant to the heat transfer process within a compressor are the peripheral Mach number \( M_{2u} = U_2/\sqrt{\gamma R T_{01}} \), the flow coefficient \( \Phi = C_{2m, adi}/U_2 \), the slip factor \( \mu \) and the blade angle at the impeller outlet \( \beta_{B2} \). It can be demonstrated that the local Mach number \( M_{2, adi} \) is a function of all the parameters indicated above and it can be developed to yield to\(^4\):

\[
M_{2, adi} = \frac{C_{2, adi}}{\sqrt{\gamma R T_{2, adi}}} \quad (5.75)
\]

\[
M_{2, adi} = \frac{2M_{2u}^2(\Phi^2 + \lambda^2)}{2 + [(\gamma - 1)M_{2u}^2(2\lambda - \Phi^2 - \lambda^2)]} \quad (5.76)
\]

where

\[
\lambda = \frac{C_{\theta 2}}{U_2} = \frac{\mu}{1 - (\tan \beta_{B2}/\tan \alpha_2)} \quad (5.77)
\]

\( \lambda \) is defined as the work input factor (Rodgers, 1978) and for \( \lambda = 1 \), \( C_{\theta 2} = U_2 \), and Eq.(5.76) reduces to the particular case of a perfectly guided radial blade impeller:

\[
M_{2, adi} = \frac{2M_{2u}^2(\Phi^2 + 1)}{2 + [(\gamma - 1)M_{2u}^2(1 - \Phi^2)]} \quad (5.78)
\]

Eq. (5.76) and Eq. (5.78) were obtained for backward swept (with \( \mu \neq 0 \)) and radial blade impeller (with \( \mu = 0 \)). From Eq. (5.76) it can be seen as \( M_{2, adi} \) contains the effects of the main parameters involved in the heat transfer process. The benefit of using the local Mach number is that it reduces the number of variables to be used in a parametric analysis, thus simplifying the calculation of the compressor non-adiabatic performance.

3. **Temperature parameter (TP):** this parameter was introduced in order to take into account the effects of heat transfer on the compressor efficiency. The heat transferred within a turbocharger is mainly generated by the exhaust gases entering the turbine. After exchanging heat with the turbine housing, the flow will then expand in the rotor leaving the turbine with lower temperature \( T_{04} \). On the compressor side instead, heat from the turbine to the compressor is mainly transferred through the bearing housing that in turn will cause a rise in the compressor exit

\(^4\) A full derivation for Eq. (5.76) and Eq. (5.78) is given in Appendix A5.
temperature \((T_{02})\). We can then infer that the exit temperatures to both the compressor and the turbine are associated with the heat exchanged within the turbocharger and therefore the ratio between \(T_{02}\) and \(T_{04}\) was included in the regression analysis as a non-dimensional parameter, Eq. (5.79).

\[
TP = \frac{T_{04}}{T_{02}}
\]  

(5.79)

5.15 Multiple regression analysis

After having defined the \(x\), explanatory variables, Eq. (5.80) assumes the form:

\[
\eta = f(PR, M, TP)
\]  

(5.80)

The general computational problem that needs to be solved in multiple regression analysis is to fit a line to a number of points. In the multivariate case, when there is more than one independent variable, the multiple regression procedure will estimate a linear equation of the form:

\[
Y = A_0 + A_1X_1 + A_2X_2 + A_3X_3 + \cdots + A_iX_i
\]  

(5.81)

For this particular case Eq. (5.81) can be transformed as in Eq. (5.82).

\[
\eta = A_0 + A_1PR + A_2M + A_3TP
\]  

(5.82)

The expression in Eq. (5.82) is ready for multiple regression operation. The statistical software called “Origin©” was used in this study to determine the coefficients (Koonlaya, 2004). The assumptions behind any multiple regression analysis are that the residuals of the explanatory variables must be normal and independently distributed with a mean of 0 and some constant variance. The overall pattern of the residuals should be similar to the bell-shaped pattern observed when plotting a histogram of normally distributed data. In addition to this, the plot of residuals versus the predicted values should produce a distribution of points scattered randomly about 0. The selected explanatory variables \((PR, M, \text{and } TP)\) satisfied both the assumptions although the pressure ratio parameter \((PR)\) required a logarithmic transformation since a certain skewness was observed in the plot of residuals.

Prior to the multiple regression analysis, the single linear regression analysis for each explanatory variable was performed. The results of regression are reported in Table 5.17 which contains the essential statistics that help explain the obtained coefficient. The squared value of the Mach number was also included as an additional parameter in the regression analysis. This was done
since the regression response ($\eta$) is expected to have some curvature given that the compressor efficiency curve usually shows a parabolic trend.

### Table 5.17: Univariate > Outcome: $\eta$

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>95% CI</th>
<th>p-value</th>
<th>R value</th>
</tr>
</thead>
<tbody>
<tr>
<td>TP</td>
<td>-0.12834</td>
<td>-0.15781</td>
<td>&lt;0.0001</td>
<td>0.75</td>
</tr>
<tr>
<td>logPR</td>
<td>0.76078</td>
<td>0.70256</td>
<td>&lt;0.0001</td>
<td>0.760</td>
</tr>
<tr>
<td>$M_{bs}$</td>
<td>0.5105</td>
<td>0.46696</td>
<td>&lt;0.0001</td>
<td>0.718</td>
</tr>
<tr>
<td>$M_{bs}^2$</td>
<td>0.356</td>
<td>0.32432</td>
<td>&lt;0.0001</td>
<td>0.700</td>
</tr>
</tbody>
</table>

#### Backward swept blades - $\mu\neq0$

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>95% CI</th>
<th>p-value</th>
<th>R value</th>
</tr>
</thead>
<tbody>
<tr>
<td>TP</td>
<td>-0.12834</td>
<td>-0.15781</td>
<td>&lt;0.0001</td>
<td>0.75</td>
</tr>
<tr>
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<td>0.76078</td>
<td>0.70256</td>
<td>&lt;0.0001</td>
<td>0.760</td>
</tr>
<tr>
<td>$M_{bs}$</td>
<td>0.5105</td>
<td>0.46696</td>
<td>&lt;0.0001</td>
<td>0.718</td>
</tr>
<tr>
<td>$M_{bs}^2$</td>
<td>0.356</td>
<td>0.32432</td>
<td>&lt;0.0001</td>
<td>0.700</td>
</tr>
</tbody>
</table>

#### Radial blades - $\mu=0$

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>95% CI</th>
<th>p-value</th>
<th>R value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_r$</td>
<td>0.34557</td>
<td>0.3142</td>
<td>&lt;0.0001</td>
<td>0.692</td>
</tr>
<tr>
<td>$M_r^2$</td>
<td>0.17716</td>
<td>0.16031</td>
<td>&lt;0.0001</td>
<td>0.672</td>
</tr>
</tbody>
</table>

The results of the multiple regression analysis for both backward swept and radial blade impeller are provided in Table 5.18. The final expression for Eq. (5.82) is given in Eq. (5.83) and Eq. (5.84):

$\eta_{bs} = -0.04158TP + 0.12336 \log(PR) + 1.15921M_{bs} - 0.43205M_{bs}^2$  \hspace{1cm} (5.83)

$\eta_r = -0.04397TP + 0.19304 \log(PR) + 0.89422M_r - 0.28415M_r^2$ \hspace{1cm} (5.84)

It is worth noting that no constant is present in Eq. (5.83) and Eq. (5.84), $C_0=0$. This was purposely set equal to 0 since no efficiency is expected to exist if no flow is going through the compressor ($PR=1, M=0$).

### 5.16 Discussion of results

The linear regression analysis for the single *explanatory variables* shows that these are strongly correlated with the efficiency ($p<0.0001$). The confidence interval of the regression coefficient shows that null hypothesis is rejected as 0 is not included in the intervals. This supports the assumption of significant relationship existing between $\eta$ and the *explanatory variables*.

Proven the statistical significance of the *explanatory variables*, the goodness of fit of Eq. (5.83) and Eq. (5.84) with $\eta$ must also be checked by looking at the adjusted $R^2$ value. The adjusted $R^2$ value adjusts for the number of response variables and it is a statistic that gives
information about the goodness of fit. The adjusted $R^2$ is $\approx 0.9$ for both Eq. (5.83) and Eq. (5.84). Such a high value is in favour of the goodness of fit and seems to support the assumptions made on the compressor non-adiabatic efficiency and on the effectiveness of the regression analysis as a predictive tool.

Table 5.18: Multivariate > Outcome: $\eta$

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>95% CI</th>
<th>p-value</th>
<th>Adj. $R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Backward swept blades - $\mu\neq 0$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TP</td>
<td>-0.04158</td>
<td>-0.05761</td>
<td>-0.02555</td>
<td>&lt;0.0001</td>
</tr>
<tr>
<td>logPR</td>
<td>0.12336</td>
<td>0.03067</td>
<td>0.27739</td>
<td>&lt;0.0001</td>
</tr>
<tr>
<td>$M_{bs43}$</td>
<td>1.15921</td>
<td>1.03487</td>
<td>1.28355</td>
<td>&lt;0.0001</td>
</tr>
<tr>
<td>$M_{bs}^2$</td>
<td>-0.43205</td>
<td>-0.55297</td>
<td>-0.31112</td>
<td>&lt;0.0001</td>
</tr>
<tr>
<td>Radial blades - $\mu=0$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TP</td>
<td>-0.04397</td>
<td>-0.06004</td>
<td>-0.0279</td>
<td>&lt;0.0001</td>
</tr>
<tr>
<td>logPR</td>
<td>0.19304</td>
<td>0.05870</td>
<td>0.32738</td>
<td>&lt;0.0001</td>
</tr>
<tr>
<td>$M_r$</td>
<td>0.89422</td>
<td>0.80245</td>
<td>0.98599</td>
<td>&lt;0.0001</td>
</tr>
<tr>
<td>$M_r^2$</td>
<td>-0.28415</td>
<td>-0.34211</td>
<td>-0.22618</td>
<td>&lt;0.0001</td>
</tr>
</tbody>
</table>

After having verified that the selected explanatory variables are statistically significant, their significance under a physical point of view must also be assessed:

- **Temperature parameter (TP)**: the temperature parameter shows the smallest coefficient in respect to the other explanatory variables, $PR$ and $M$. This means that the effects of the temperature rise within the turbocharger are overcome by the aerodynamic effects as the pressure ratio and the Mach number increase. In addition to this, the regression coefficient also exhibits a negative value. This suggests that, for a fixed $PR$ and $M$, an increase in $TP$ corresponds to a decrease in efficiency. This is fully consistent with the experimental evidence (Romagnoli and Martinez-Botas, 2009). In fact, the main assumption behind heat transfer within a turbocharger is that the heat addition causes the compressor efficiency to drop. Hence a negative value for the temperature parameter coefficient supports this assumption thus making the explanatory variable $TP$ to be fully defined.

$43 \ M_{bs}=\text{Mach number back-swept}, \ M_{r}=\text{Mach number radial}.$

$44 \ \text{From now on when referring to the TP contribution to the compressor performance, this must be considered as a negative contribution leading to efficiency drop.}$
• **Pressure ratio (logPR):** the pressure ratio is one of the two independent parameters extrapolated from the compressor map. The pressure ratio appears in logarithmic form in both Eq. (5.83) and Eq. (5.84). This was made necessary because of the skewness exhibited by the histogram of residuals although this choice also owns a physical reason. In the case where the pressure ratio is equal to 1 this implies that no flow is going through the machine and that no work is produced. Hence the efficiency is equal to 0. The logarithm of PR takes into account this feature. In fact, as the pressure ratio increases the efficiency tends to increase while as soon as the pressure ratio drops to 1 the logPR is equal to 0.

• **Mach number (M):** this parameter is present both in linear and quadratic form. The regression coefficient for M is the biggest in both Eq. (5.83) and Eq. (5.84). This means that the compressor performance is strongly dependent on the Mach number and that the effects of the heat transfer diminish as the rotational speed increases. Again this is consistent with the experimental findings (Shaaban, 2004, Romagnoli and Martinez-Botas, 2009) that showed that there is no significant difference between the adiabatic and the non-adiabatic compressor performance at high rotational speeds.

Table 5.19: Typical values of the explanatory variables

<table>
<thead>
<tr>
<th>Value</th>
<th>M</th>
<th>log(PR)</th>
<th>TP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum</td>
<td>≈ 0.5</td>
<td>≈ 0.1</td>
<td>≈ 1.03</td>
</tr>
<tr>
<td>Maximum</td>
<td>≈ 1.2</td>
<td>≈ 0.38</td>
<td>≈ 2.73</td>
</tr>
</tbody>
</table>

It must be mentioned that looking at the magnitude of the regression coefficients does not help to give their real contribution to the overall compressor performance. Each term of Eq. (5.83) and Eq. (5.84) is given by the combination of the regression coefficients and the explanatory variables. Therefore, it is necessary to look at the range into which the explanatory variables vary in order to assess their contribution to the compressor efficiency. Typical values for M, logPR and TP are given in Table 5.19. The explanatory variable M, despite showing a large value of the regression coefficient, varies within a small range (0.5 to 1.2). The opposite occurs for the temperature parameter for which the small contribution given by its regression coefficient is in somehow compensated by a larger value of TP.

In Fig. 5.31, a 3-D plot of the contribution (in percentage points) of each parameter to the compressor non-adiabatic efficiency is given for both the backward swept and radial blade impeller. The Mach number is by far the most significant parameter and its value remains above 70% for the whole range of speeds and temperatures. The temperature parameter TP decreases with speed while the opposite occurs for PR. In Fig. 5.32 this is even more evident. The explanatory variables were plotted against the rotational speed and each point in the plot represents the average value of the
corresponding parameter over the whole range of temperatures of the exhaust gases (550 K to 950 K). Although the average is not entirely representative of the TP values (for which the temperature change has a large impact) this does not largely affect $M$ and $PR$.

![Figure 5.31: Contribution of $M$, $PR$, and $TP$ to the compressor performance, 3-D diagram](image1)

![Figure 5.32: Contribution of $M$, $PR$, and $TP$ to the compressor performance, 2-D plot with averaged values of the variables](image2)
From Fig. 5.32 it can be gathered that the Mach number accounts for the largest portion of the compressor non-adiabatic efficiency. Its trend remains fairly constant across the entire speed range meaning that its weight on the overall compressor efficiency is almost independent on speed (see Table 5.20). *TP* and *PR* instead represent a smaller portion of the efficiency. *TP* exhibits a decreasing trend with speed and at high speeds it is no larger than \(\approx 10\%\). This is more evident in Fig. 5.33 where *TP* was plotted against temperature for constant speed lines.

![Figure 5.33: Contribution of *TP* to the compressor performance: *TP* vs. temperature for constant speed lines](image)

In Figs. 5.31 and 5.32 the effect of geometry on the explanatory variables is also given. For the backward swept blade impeller, the contribution of the Mach number \(M_{bs}\) remains above that of the radial impeller \(M_r\). The reason is found in the deviation of the absolute velocity \((C_{2,adi})\) from the perfectly guided flow conditions and hence in a lower value of the Mach number (refer to Fig. 5.30). On the other hand, the pressure ratio \(PR\), for the backward swept blade impeller compensates for such a deficit of the Mach number \((M_r)\) and therefore the calculated values of \(PR\) are bigger than those calculated for the back swept \((PR_{bs})\). In Table 5.20 a quantification of the impact of geometry on the explanatory variables is provided.

The Mach number difference \(\Delta M\) goes from \(\approx 3\%\) to \(\approx 6\%\) (in absolute value) as the speed increases. Such a difference is partly compensated by the pressure ratio for which values of \(\Delta PR\) not larger than \(\approx 4\%\) were calculated (in absolute value). No variation instead was observed for the temperature parameter *TP* that remains unchanged over the whole range of speeds. This suggests that *TP* is insensitive to the geometry and it can be readily explained by looking at the definition of *TP* in which no geometry parameter is present.
Table 5.2: Contribution of the explanatory variables to the compressor efficiency and comparison with slip factor case

<table>
<thead>
<tr>
<th>Variable</th>
<th>( N \approx 88.0 ) [rev/s √K]</th>
<th>( N \approx 107.6 ) [rev/s √K]</th>
<th>( N \approx 146.8 ) [rev/s √K]</th>
<th>( N \approx 163.3 ) [rev/s √K]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_r )</td>
<td>≈ 72%</td>
<td>≈ 74%</td>
<td>≈ 78%</td>
<td>≈ 79%</td>
</tr>
<tr>
<td>( M_{bs} )</td>
<td>≈ 75%</td>
<td>≈ 77%</td>
<td>≈ 82%</td>
<td>≈ 85%</td>
</tr>
<tr>
<td>( \Delta M (=M_r-M_{bs}) )</td>
<td>( \Delta M = -3% )</td>
<td>( \Delta M = -3% )</td>
<td>( \Delta M = -4% )</td>
<td>( \Delta M = -6% )</td>
</tr>
<tr>
<td>( PR_r )</td>
<td>≈ 5%</td>
<td>≈ 6%</td>
<td>≈ 8%</td>
<td>≈ 10%</td>
</tr>
<tr>
<td>( PR_{bs} )</td>
<td>≈ 3%</td>
<td>≈ 4%</td>
<td>≈ 5%</td>
<td>≈ 6%</td>
</tr>
<tr>
<td>( \Delta PR (=PR_r-PR_{bs}) )</td>
<td>( \Delta PR = 2% )</td>
<td>( \Delta PR = 2% )</td>
<td>( \Delta PR = 3% )</td>
<td>( \Delta PR = 4% )</td>
</tr>
<tr>
<td>( TP_r )</td>
<td>≈ 22%</td>
<td>≈ 26%</td>
<td>≈ 13%</td>
<td>≈ 10%</td>
</tr>
<tr>
<td>( TP_{bs} )</td>
<td>≈ 21%</td>
<td>≈ 19%</td>
<td>≈ 12%</td>
<td>≈ 10%</td>
</tr>
<tr>
<td>( \Delta TP (=TP_r-TP_{bs}) )</td>
<td>( \Delta TP = 1% )</td>
<td>( \Delta TP = 1% )</td>
<td>( \Delta TP = 1% )</td>
<td>( \Delta TP = 0% )</td>
</tr>
</tbody>
</table>

5.17 Geometry effects on heat transfer

The current research was based on the test data available for a single turbocharger. Even though the validity of the analysis remains unaltered, this would imply that this analysis is insensitive to the turbocharger size. This is not the case in reality as the heat transfer occurring within a turbocharger strongly depends on the geometry of the bodies involved in it. In order to quantify what is the impact of geometry on the compressor performance, two parameters were identified as the most significant on heat transfer: the compressor casing diameter \((D)\) and the distance between the compressor and the turbine, here defined as bearing housing length \((BHL)\):

1. **Bearing housing length \((BHL)\)**: heat from the turbine side travels through the bearing housing towards the compressor. This parameter is mainly denoting the space available for the heat transfer dissipation by convection in the oil bearing assembly.

2. **Compressor casing diameter \((D)\)**: the air within the compressor is mainly heated up by forced convection with the casing. The size of the compressor casing is therefore crucial to determine the surface area available to the flow for heat exchange.

A non-dimensional parameter, defined as the ratio between \(BHL\) and \(D\), was then introduced in the regression analysis:

$$LD = \frac{BHL}{D} \quad (5.85)$$

Since no experimental data were available, \(BHL\) and \(D\) were scaled by \(±15\%, ±30\%\) from those of the turbocharger previously studied. Different values of \(LD\) were calculated by scaling alternatively \(BHL\) and \(D\). These are reported in Table 5.21.
The first row was obtained by fixing $D$ to its original value and scaling $BHL$. The opposite was done in the second row. The model was then run for the same range of speeds and temperatures as in Table 5.14 and the impact of $LD$ on the compressor efficiency was then assessed. The regression equation including $LD$ is given in Eq. (5.86). By the analysis of Eq. (5.86), it can be inferred that the impact of geometry on the overall compressor performance is not small. The regression coefficient for $BHL$ is significant, if compared to the others, and its contribution to the compressor efficiency ranges between $\approx 1\%$ and $\approx 2.4\%$, as reported in Table 5.21.

$$
\eta = +0.01756LD - 0.05208TP + 0.40201 \log(PR) + 1.5556M - 0.96984M^2 \quad (5.86)
$$

In Fig. 5.34 the variation of $LD$ is plotted against its contribution to the overall efficiency. It can be noticed as the impact of $LD$ on the compressor efficiency increases with $LD$. This suggests that $BHL$ and $D$ are inversely related to each other. An increase in $BHL$ is beneficial to the compressor efficiency since a lower amount of heat is transferred to the compressor. On the other hand, the role played by $D$ on the compressor efficiency is less clear. In fact, a decrease in $D$ corresponds to a reduced amount of surface area available to the flow for heat exchange.
5.18 Summary

This chapter reported the outcomes of the investigation on heat transfer in turbochargers. The turbocharger under study was tested at constant load points for a range of engine speeds. Measurements were obtained for engine speeds between 1000 and 3000 rpm in steps of 500 rpm; for each engine speed the load applied was varied from 16 to 250 Nm. The surface temperatures of three main bodies constituting the turbocharger (turbine and compressor casing, bearing housing) were measured at 17 stations. The test results showed that the engine has a large impact on surface temperature of the turbine and compressor casing and also that the surface temperatures of both the turbine and the compressor vary linearly with the temperature of the exhaust gases. A temperature difference up to \( \approx 66 \) K and \( \approx 68 \) K was measured between the Engine and the External side for the turbine and compressor casing respectively. A temperature gradient was also measured between the inner and the outer wall: on the turbine side this moves outward while the opposite occurs for the compressor. The surface temperature of the bearing housing was found to vary consistently with that of the cooling oil. The oil temperature remains well below that of the bearing housing with a temperature difference of about \( \approx 30 \) K. Similar trend to that of the bearing housing and the oil was found for the surface temperature of the exhaust manifold. Due to the presence of the turbine, the temperature of the duct on the turbine side remains well above to that of the compressor side; a temperature difference of 130 K was measured at high loads and speeds. The compressor non-adiabatic efficiency was also evaluated; the deviation from that measured under adiabatic conditions goes from 17% to 30% as the rotational speed and air flow rate decreases. Based on the experimental results, a correlation linking the compressor exit temperature with the exhaust gas temperature is proposed. The calculated temperature was found to agree well with the experimental results with a discrepancy no larger than 3%.

A 1-D model of the turbocharger was developed and validated against the experimental results. The model considers the turbocharger as constituted by an assembly of flat plates and uses the cold performance maps as boundary conditions. The validation against test results showed that the trend of the heat transferred through the turbine casing is well captured; the compressor exit temperature could be predicted with an uncertainty no greater than 5 K while an averaged deviation of about 3% was found for the compressor non-adiabatic efficiency. Based on the maps generated by the model, a multiple regression analysis was carried for the compressor non-adiabatic efficiency. In this analysis, the following explanatory variables were chosen: absolute compressor exit Mach number \( (M_{2,\text{ad}}) \), the compression ratio \( (PR) \) and the temperature parameter \( (TP) \), defined as the ratio between the temperature of the gas entering the turbine and that leaving the compressor). The high values of the adjusted \( R^2 \approx 0.9 \) showed that the compressor non-adiabatic efficiency can be fitted with good degree of approximation by means of the selected parameters. The Mach number was found to contribute for \( \approx 80\% \) of the overall efficiency, the temperature parameter \( \approx 20\% \) while the pressure
ratio only few percentage points. The impact of the geometry on the compressor non-adiabatic efficiency was also assessed; this was found to account for about ≈2% of the overall compressor efficiency.
CHAPTER 6

UNSTEADY FLOW ANALYSIS:
EXPERIMENTAL RESULTS

SYNOPSIS

This chapter reports the results of the experimental investigation conducted under pulsating flow conditions on a variable geometry twin-entry mixed-flow turbine. The data reduction, processing and the consequent instantaneous parameter derivation are discussed in detail. The presented results were acquired under various flow conditions and vane angles and compared with those for an equivalent single-entry variable geometry turbine.
6.1 Performance Parameters

Similarly to the steady state analysis, the parameters used to describe the performance of the twin-entry turbine under unsteady state conditions are the instantaneous efficiency and mass flow parameter. These were plotted against the instantaneous velocity and pressure ratio. The measuring methods of the required individual components to derive these parameters were described in Chapter 3 while a description of the performance parameters is provided below.

6.1.1 Instantaneous mass flow and pressure ratio

Under pulsating flow condition, the definition of the mass flow parameter and pressure ratio is similar as in the steady conditions. The main difference is that in pulsating flow, the instantaneous time varying individual components are used in place of the steady ones. Mass flow parameter is a pseudo-dimensionless parameter which is calculated by relating the inlet flow velocity and the inlet Mach number, resulting in a form as given in Eq. (6.1).

\[
(MFP)_{inst} = \frac{(m)_{inst}\sqrt{(T_{01})_{inst}}}{(P_{01})_{inst}}
\]

\[
(PR)_{inst} = \frac{(P_{01})_{inst}}{(P_{ext})_{inst}}
\]

Eq. (6.1) is no longer valid when applied to a twin-entry turbine. As already discussed in Chapter 4, the mass weighted average mass flow parameter must be used when considering a twin-entry turbine. In this way the contribution of each limb to the overall flow capacity is taken into account. This is provided in Eq. (6.3). Same considerations are valid for the pressure ratio, this parameter is area averaged, as given in Eq. (6.4).

\[
MFP_{twin-entry} = \frac{\sqrt{m_{inner,inst} T_{0,inner,inst} + m_{outer,inst} T_{0,outer,inst}}}{m_{tot,inst}}
\]

\[
(PR)_{inst} = \frac{A_{inner}}{A_{tot}} PR_{inner,inst} + \frac{A_{outer}}{A_{tot}} PR_{outer,inst} = \frac{1}{2} (PR_{inner,inst} + PR_{outer,inst})
\]

Equation (6.1) applies well for the calculation of the mass flow parameter for each entry of the turbine. Here the mass averaged is no longer needed since each limb can be considered a single
passage itself. This is particularly useful in the evaluation of the mass flow parameter under out-of-phase flow conditions.

6.1.2 Instantaneous efficiency and velocity ratio

The expression of the turbine instantaneous total-to-static efficiency with adiabatic assumption is given in Eq. (6.5).

\[
(\eta_{ts})_{\text{inst}} = \left( \frac{W_{\text{act, inst}}}{W_{ts, is, \text{inst}}} \right)
\]  

(6.5)

In Chapter 3 the derivation of the instantaneous isentropic power \( (W_{ts, is})_{\text{inst}} \) and instantaneous actual power \( (W_{\text{actual, inst}}) \) were described. In pulsating flow conditions, the instantaneous isentropic power is obtained as the sums of the isentropic powers owning to each limb, refer to Eq. (6.6).

\[
(W_{ts, is})_{\text{inst}} = (\dot{m} \cdot c_p \cdot T_{01})_{\text{inner, inst}} \left[ 1 - \left( \frac{P_{\text{exit}}}{P_{01}} \right)_{\text{inner}} \right]^{\frac{\gamma - 1}{\gamma}} \\
+ (\dot{m} \cdot c_p \cdot T_{01})_{\text{outer, inst}} \left[ 1 - \left( \frac{P_{\text{exit}}}{P_{01}} \right)_{\text{outer}} \right]^{\frac{\gamma - 1}{\gamma}}
\]  

(6.6)

The instantaneous velocity ratio is the ratio between the rotor tip speed and the inlet isentropic velocity as given in Eq. (6.7).

\[
\left( \frac{U}{C_{is}} \right)_{\text{inst}} = \frac{\pi D_{\text{rot}} N_{\text{inst}}}{\sqrt{2 \left( W_{is,\text{inner}} + W_{is,\text{outer}} \right) / m_{\text{tot}}}}
\]  

(6.7)

6.2 Data Reduction and Processing

The data acquisition system used for the data logging during the pulsating flow test conditions was described in Chapter 3. The measured raw values produced by the data acquisition system need to be processed and refined before they can be used to assess the pulsating flow performance of the turbine. In addition to the instantaneous properties, the measured time-mean values of inlet mass flow rate, \( \dot{m}_{\text{inner}} \) and \( \dot{m}_{\text{outer}} \), and inlet static temperature, \( T_{\text{inner,s}} \) and \( T_{\text{outer,s}} \), are also used in the data processing. The following subsections discuss the procedure followed for the data reduction, this is summarised in Fig. 6.1.
6.2.1 Resampling and Ensemble averaging

The rotational speed of the turbine is measured by monitoring the time taken for the rotating assembly to move across a fixed angular distance (refer to the toothed gate in Chapter 3). As a consequence of this, its sampling rate is not constant and it is in contrast to the fixed sampling rate of the analogue properties. In order to resolve this issue the speed data was resampled to match the constant sample rate of the other measured quantities. A cubic spline was fitted through each of the non-uniformly spaced data. The resultant spline was then interpolated at the constant sample rate of 20000 samples per second. Eq. (6.8) gives the general form of the cubic polynomial, where $t$ is the time and $a, b, c, d$ are the curve coefficients.

$$s_i(t) = a_i + b_i(t - t_i) + b_i(t - t_i)^1 + c_i(t - t_i)2 + d_i(t - t_i)^3$$ (6.8)
where $t \in [t_i, t_{i+1}]$ and $i=0,1,\ldots,n$

$$y = Ay_i + By_{i+1} + C \frac{d^2 y_i}{d t^2} + D \frac{d^2 y_{i+1}}{d t^2}$$

(6.9)

where $A = (t_{i+1} - t)/(t_{i+1} - t_i)$; $B = 1 - A$;

$C = \frac{1}{6} (A^3 - A) (t_{i+1} - t_i)^2$; $D = \frac{1}{6} (B^3 - B) (t_{i+1} - t_i)^2$

A natural cubic spline was chosen for interpolating and, in order to meet the interpolating conditions, the second derivatives of the end points are constrained to be zero. The interpolation value ($y$) at any time ($t$) in the interval of $[t_i, t_{i+1}]$ is defined as in Eq. (6.9).

$$\bar{y}(t_i) = \frac{1}{n} \sum_{i=1}^{n} y(t_{i+1})$$

(6.10)

Ensemble averaging was then applied for all properties recorded during pulsating flow tests; ensemble average is a powerful technique that enables to attenuate the noise without losing the cyclic information of the recorded properties. Each property was recorded for a total of 50 cycles for each of the 36 traversing points, resulting in 1800 cycles ($=36 \times 50$) for each test condition. For the subsequent processing and analysis these cycles are ensemble averaged to a single cycle. Unlike the other properties, the 36 traversing points for the hotwire were integrated using British Standard method (discussed in the Chapter 3); the 50 cycles for each traversing point are individually ensemble averaged. The general form of the equation defining ensemble averaging is given in Eq. (6.10). At each instant in time ($t$) the ensemble averaged value of a property $\bar{y}(t)$ is given as the result of averaging all the equivalent individual property, $y(t)$, for $n$ cycles.

6.2.2 Filtering

The use of ensemble averaging does not completely reduce all the noise in the data signal since the part of the remaining noises has cyclic features which are not associated to the physical property intended to be measured. In the following text are described the noise sources and the affected properties.

**Pressure Traces**

Hjelmgren (2002) showed that the dynamic pressure measured with a transducer mounted on a duct is affected by both the vibration of the duct and the resonance of the air passage at the inlet of
the transducer. These effects are visible in Fig. 6.2-a where the outcome of the FFT analysis of the ensemble averaged pressure signals is shown. Here the secondary peaks are at much higher frequency of the pulsating flow which shows the presence of cyclic noise. Thus, the cyclic noise is eliminated through filtering.

**Speed Signal**

The manufacturing tolerances of the speed sensor's encoder wheel and the out-of-balance of the rotating assembly produce fallacious high frequency components which occur at various harmonics of the turbine speed. The FFT analysis of the ensemble averaged speed signal shows secondary peak mainly at the first harmonic of the speed. Figure 6.2-b shows the speed signal for 20 Hz frequency together with the reduction of the raw data through ensemble averaging (red line) and filtering (blue line). The raw signal corresponds to one of the 50 cycles logged during the pulsating flow experiments. From Fig. 6.2-b it can be noticed that the FFT analysis of the single cycle raw signal shows the peaks at the harmonic of the rotor speed. These peaks partially are attenuated through ensemble averaging and completely eliminated through filtering.

**Choice of Filter**

In order to smooth all the traces without losing their primary features, a low pass *Finite Impulse Response (FIR)* filter is used for all the data signals.\(^\text{45}\) The *FIR* filter is defined as in Eq. (6.11) and it requires the specification of cut-off frequency and the ‘*taps*’, which refer to the number of *FIR* filter coefficients \((h)\) in Eq. (6.11):

\(^\text{45}\) LabVIEW Manual, 2004 and the LabVIEW 7.1 built-in routine called Filter.vi
where \( x \) is the input sequence, \( y \) the filtered sequence and \( h \) the FIR filter coefficients. Besides being used for eliminating the cyclic noise, FIR is also used to smooth-out the fluctuations in the mass flux and inlet pressure traces at frequencies beyond the response limit of the rotor. The upper limit is estimated as a fraction of the bandwidth of the travelling pressure wave around the periphery of the volute and it was found that the use of 10 times flow frequency as the cut-off value in the FIR low pass filter as suitable for all the current experimental results.

### 6.2.3 Hotwire Correction

The setup and calibration of the hotwire system is discussed in Chapter 3. Hence the following discussion will describe the steps taken for correction done before the instantaneous mass flow rate is derived. In fact under pulsating flow conditions, the temperature fluctuation away from the calibration temperature, requires further correction to the measured mass flux with a constant temperature hotwire. A common method\(^\text{46}\) used for temperature compensation in the hotwire measurement \((T_w)\) is that of correcting the measured voltage for the temperature difference between the measurement \((T_{meas})\) and calibration \((T_{calib})\). This is shown in Eq. (6.12) where the corrected voltage is given as function of the measured voltage \((E_{meas})\) and temperatures.

\[
E_{corr} = E_{meas} \left( \frac{T_w - T_{calib}}{T_w - T_{meas}} \right)^{0.5}
\]

\(\text{(6.12)}\)

---

\(^{46}\) Dantec Dynamics, 2002

---

![Figure 6.3: Temperature correction for the mass flow rate (Rajoo, 2006)](image-url)
However Eq. (6.12) does not take into account the temperature influence on the sensor’s calibration constants as well as the fluid property changes. The pulsating flow condition causes a fluctuation in the air pressure and temperature which makes it necessary to consider the changes in the fluid properties into the hotwire correction procedure. The fluid properties affecting the hotwire reading are the Mach number ($M$), density ($\rho$), thermal conductivity ($k$), Prandtl number ($Pr$), and dynamic viscosity ($\mu$). By considering these properties the calibration factors of the hotwire ($A_{calib}$ and $B_{calib}$) are corrected as given in Eq. (6.13) and Eq. (6.14). The corrected calibration factors are then used in Eq. (3.8) to solve for the mass flux ($\rho U$).

$$A_{corr} = A_{calib} \cdot \left( \frac{T_w - T_{calib}}{T_w - T_{meas}} \right)^{1.0m} \cdot \left( \frac{k_f - meas}{k_f - calib} \right)^{0.2} \cdot \left( \frac{MD_f - meas}{MD_f - calib} \right)$$ \hspace{1cm} (6.13)$$

$$B_{corr} = B_{calib} \cdot \left( \frac{T_w - T_{calib}}{T_w - T_{meas}} \right)^{1.33} \cdot \left( \frac{k_f - meas}{k_f - calib} \right)^{0.2} \cdot \left( \frac{MD_f - meas}{MD_f - calib} \right)$$ \hspace{1cm} (6.14)$$

where $T_{f - meas} = \frac{1}{2} (T_w + T_{meas})$ and $T_{f - calib} = \frac{1}{2} (T_w + T_{calib})$

In Eq. (6.13) and Eq. (6.14) the value of the temperature loading ($m$) was evaluated through the experimental calibration in-situ described in Chapter 3. The expression of the calibration factors $A_{corr}$ and $B_{corr}$ given above is consistent with the reference from Dantec Dynamics (2002) except for the last term. This term takes into account the departure of the test conditions from the Mach number independent region ($M < 1$) as suggested by Dewey (1965). The last term in Eq. (6.13) and Eq. (6.14) is defined in Eq. (6.15).

$$MD_l = 1 + \left\{ \frac{0.6039}{M_l} + 0.5701 \left( \frac{M_l^{1.222}}{1 + M_l^{1.222}} \right)^{1.569} - 1 \right\} \cdot \left\{ 1.834 - 1.634 \left( \frac{Re_l^{1.109}}{2.765 + Re_l^{1.109}} \right) \right\} \cdot \left[ 1 + \left( \frac{0.0650}{M_l^{1.670}} \right) \cdot \left( \frac{Re_l}{4 + Re_l} \right) \right]$$ \hspace{1cm} (6.15)$$

In Eq. (6.15) the Mach number ($M$) needs to be solved iteratively since it depends on the flow velocity which in turn corresponds to the solution for the hotwire correction process. The results of hotwire correction are shown in Fig. 6.3. Here the effect of the temperature correction can be seen clearly. The mean value of the mass flow rate without temperature correction is approximately 6% lower than the
actual condition. The mass flow rate is underestimated in the region where the flow temperature is higher than the calibration condition and vice versa.

![Figure 6.4: Location of the measurements taken under pulsating flow conditions (Rajoo, 2006)](image)

### 6.2.4 Phase Shifting

In pulsating flow experiments a time-lag\(^{47}\) occurs between the different instantaneous measurements at the various locations on the turbine; this fact must be considered. The critical phase difference occurs between the measuring location of the isentropic inlet properties\(^{48}\) and the actual power output. For the purpose of the current research, the reference time frame point used to phase all the signals was chosen to be at the exit of the pulse generator, refer to Fig. 6.4. A simple correlation was used for estimating the phase shifting in terms of the time-lag from the common time frame, Eq. (6.16). In order to apply Eq. (6.16), the length of each measuring location with respect to the reference point needs to be known. This can be done in a straightforward manner for all the stations on the inlet pipe of the turbine; the measuring plane is this section of the pipe. For the turbine volute the matter is more complex as there is a continuous release of mass flow around the turbine wheel. The length was chosen to extend up to the 180° azimuth angle around the volute, this has been shown to work well in previous research and it makes some sense in so far as a large proportion of the mass flow has left the volute by then. Besides the characteristic length, phase shifting also requires a propagation velocity. A number of approximations have been employed by various researchers, such as sonic velocity method (Dale and Watson, 1986), bulk flow method (Baines et al., 1994 and Winterbone et al., 1991) and sonic plus bulk flow method (Szymko, 2006). In the current study the method used by Szymko (2006)

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\(^{47}\) Time-lag due to the finite time required for the flow to propagate through the system.

\(^{48}\) The inlet isentropic properties are measured upstream of the volute while the actual turbine output properties are measured at the exit to the rotor.
was considered; the velocity at which the flow information propagates taken as equal to the total of bulk and sonic flow speeds. Once again, this speed was seen to work very well in the previous investigation and it is well founded on the propagation properties along the C+ or C+α characteristic lines in gas dynamic theory. It is worth noting that properties related to the bulk flow (such as enthalpy or mass flow) travel at the mean flow velocity.

\[ \Delta t_{pshift} = \frac{l_{pshift}}{C_{pshift}} \]  

(6.16)

The bulk and sonic flow velocity are derived on the basis of the properties measured at the ‘measurement plane’. This involves a certain degree of uncertainty since the bulk and sonic flow speeds vary at different locations. The mean velocities are calculated as given in Eq. (6.17) and (6.18) on the measuring plane.

\[ C_{bulkflow} = \frac{1}{n_{cycle}} \sum_{i=1}^{1=n_{cycle}} \frac{\dot{m}_{inst}(i)}{\rho_{inst}(i)A_{MeaPl}} \]  

(6.17)

\[ C_{sonic} = \frac{1}{n_{cycle}} \sum_{i=1}^{1=n_{cycle}} \sqrt{\gamma R T_{inst}(i)} \]  

(6.18)

6.3 Test setup

The current twin-entry volute and pulse generator is able to simulate a twin-entry turbocharged 6-cylinder 4-stroke diesel engine. The exhaust gas pulsation is produced in a cold flow environment and measured instantaneously. The frequency of the flow pulsation can be related to the engine speed with Eq. (6.19), where \( n_{stroke} \) is either 2 or 4 strokes and \( n_{cylinders} \) is the number of cylinders per manifold group.

\[ N_{engine} = 60 \cdot \frac{n_{stroke}}{2} \cdot \frac{f_{pulse}}{G \cdot n_{cylinders}} \]  

(6.19)

The pulsating flow frequencies of 20Hz, 40Hz, 60Hz and 80Hz translate into an engine speed of 800RPM, 1600RPM, 2400RPM and 3200RPM respectively. In the current study, three frequencies have been considered, 40Hz, 60Hz, 80Hz, two rotational speeds, 27.9 rev/s \( \sqrt{\text{K}} \) and 43.0 rev/s \( \sqrt{\text{K}} \), and three different vane angles, 40°, 60° and 70°. The tests conditions are shown in Table 6.1.
Table 6.1: Test conditions

<table>
<thead>
<tr>
<th>Vane Angle</th>
<th>60° vane angle</th>
<th>40° vane angle</th>
<th>70° vane angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>40Hz</td>
<td>27.9 rev/s√K</td>
<td>43.0 rev/s√K</td>
<td>27.9 rev/s√K</td>
</tr>
<tr>
<td>60Hz</td>
<td>27.9 rev/s√K</td>
<td>43.0 rev/s√K</td>
<td>27.9 rev/s√K</td>
</tr>
<tr>
<td>80Hz</td>
<td>27.9 rev/s√K</td>
<td>43.0 rev/s√K</td>
<td>27.9 rev/s√K</td>
</tr>
</tbody>
</table>

Under pulsating operating conditions, the tests are setup by considering the velocity ratio in steady state conditions as the base operating point. This is usually set to correspond with the peak steady efficiency. The time averaged non-dimensional speed is set to correspond to the equivalent steady condition and the mean temperature of the flow is held constant to the steady condition.

Discussion over the test results are provided in the coming sections. Firstly, the outcomes of the individual properties are discussed with focus on the typical features of each property. The instantaneous performance parameters of mass flow and isentropic efficiency are then analyzed and compared with those obtained for an equivalent variable geometry single-entry turbine.

6.4 Results and Discussion: Individual Properties

The following section will cover the discussion of the experimental results, focusing on the individual properties acquired during the testing. The individual properties in a pulsating flow analysis (speed, pressure, temperature and mass flow rate) are discussed for 27.9 rev/s√K and 43.9 rev/s√K equivalent speed conditions, 40Hz - 80Hz flow frequencies and at a nozzle vane angle of 60° (unless otherwise specified).

6.4.1 Turbine Speed

Figure 6.5 shows the typical trend for the instantaneous speed of the turbine in a pulse cycle for 40Hz and 60Hz when running in-phase and out-of-phase flow condition respectively. The ensemble averaged and filtered signal from the speed sensor shows that the turbine experiences a small change in speed due to the different flow conditions. Under in-phase conditions the pulse produces a speed change of approximately 4.0RPS and 2.0RPS for 40Hz and 60Hz respectively. In out-of-phase condition this change is much less since the pulse drives the rotor at staggered intervals. The speed change is about 1.7RPS and 0.7RPS for 40Hz and 60Hz respectively. This translates into 0.23%, 0.20% and 0.09% and 0.05% of the mean speed for the in-phase and out-of-phase pulse.
Uncertainty. The uncertainty associated with turbine speed is dependent on two variables which are: the known angle ($\theta_n$) and the time ($t$). The discrete rotational angle of the rotor is measured as described in Chapter 3 and its associated error is $\pm1.18 \times 10^{-4}$ radius. The time measurement is directly related to the reference 20 MHz clock and its uncertainty is $\pm5 \times 10^{-8}$ seconds. The Root Sum Square (RSS) for the speed is about 0.004RPS and 0.009RPS for the 27.9 rev/s√K and 43.0 rev/s√K.

6.4.2 Inlet static pressure

The inlet static pressure is measured in the ‘measurement plane’ which is positioned 316 mm upstream the inlet to the volute. As for the turbine speed, typical values for two different rotational speeds, 27.9 rev/s√K and 43.0 rev/s√K, two flow frequencies, 40Hz and 60Hz, and three different vane angles are reported. In Figs. 6.6 and 6.7 the pressure profiles for the in-phase pulse flow are given for 60Hz and 60° vane angle at 27.9 rev/s√K and 43.0 rev/s√K respectively. The main feature of these figures is that the inner limb experiences a higher pressure than the outer limb. This occurs in a similar manner for both 27.9 rev/s√K and 43.0 rev/s√K speed where a difference of about 15KPa and 20KPa was measured between the peak pressures of the two limbs respectively. For the 27.9 rev/s√K speed the inner limb and outer limb exhibit a peak pressure at approximately 155KPa and 140KPa respectively while for the 47.0 rev/s√K speed the peak pressures are much higher, 230KPa and 201KPa in the inner and outer limbs. In the out-of-phase conditions, the pressure traces seem to be fairly symmetrical and well shifted by 180°. Unlike the in-phase condition, the outer limb is more pressurized than the inner limb; for the 27.9 rev/s√K speed the peak pressure in the two limbs is approximately 180KPa and 172KPa for the outer and inner limb respectively while at 43.0 rev/s√K speed the peak pressures of about 250KPa and 230KPa are measured in the inner and the outer limb.
In both the 27.9 rev/s·√K and 43.0 rev/s·√K speeds, there are secondary peaks in the pressure traces (refer to Figs. 6.6 and 6.7); these are more evident for the out-of-phase flow condition even though for the in-phase flow, a secondary peak at approximately 210KPa and 200KPa was measured in the inner and outer limb respectively at 43.0 rev/s·√K. The secondary peaks can be attributed to the reflection and interference of the pressure waves. The static pressure measured at the ‘measurement plane’ is the product of the forward travelling waves and pressure wave reflections (backward travelling waves). In Fig. 6.8 the pressure pulses are shown for three different frequencies (40Hz, 60Hz, and 80Hz) at 27.9 rev/s·√K speed. At 60Hz the pressure trace fluctuations over the cycle is more prominent than the other two frequencies. At 40Hz flow, the pressure waveform is smooth with a clear primary peak, a similar behavior has been seen in other testing programs with different turbine although occurring at lower frequency (see Szymko, 2006, Copeland, 2010, Rajoo, 2007). This behavior represents a filling and emptying event on the pipe/turbine system. At the frequency conditions (80Hz), the level of fluctuation also reduces from the 60Hz condition it appears that the turbine at high frequencies damps the fluctuation in pressure which are arriving to the turbine in a much shorter time.

For the out-of-phase admission (Fig. 6.9), the onset of the peak pressure in the outer limb corresponds to no pressure in the inner passage. At this instant in time, some of the pressure wave going through the full flow limb interacts with the empty limb by travelling upstream towards the measurement plane. Here it will mix with the incoming pulse and give rise to secondary peaks as shown in Figs. 6.6 and 6.7.

Figures 6.10 and 6.11 shows the pressure traces for three different vane angle settings, 40°, 60° and 70°; results are presented for the in-phase flow at 27.9 rev/s·√K and 43.0 rev/s·√K for 60Hz flow frequency in the inner and outer limb respectively. At 27.9 rev/s·√K the overall pressure was observed to be higher in the closed nozzle position (corresponding to 70°), this might be expected as there is large blockage effect and the consequently a mass accumulation on the upstream plane. However it is worth noting that the pressure peak at 40° is greater than that measured at 60° vane angle and similar in magnitude to that measured at 70°; the difference in the peak pressure between 40° and 70° is small and equal to ≈ 1% and 3% for the inner and outer limb respectively.

Uncertainty. The deviation in the static pressure transducers is an average of ±0.65% of the full scale, which is approximately ±215 Pa. The uncertainty in the calibration of the transducers is ±90 Pa while the uncertainty due to the voltage drift of the bridge is ±60 Pa. All of these uncertainties combine to give an overall uncertainty of ±365 Pa.
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Figure 6.6: Typical trend of the instantaneous inlet pressure in the inner and outer limb under in-phase and out-of-phase flow condition at 27.9 rev/s√K at 60° vane angle at 60Hz

Figure 6.7: Typical trend of the instantaneous inlet pressure in the inner and outer limb under in-phase and out-of-phase flow condition at 43.0 rev/s√K at 60° vane angle at 60Hz

Figure 6.8: Typical trend of the instantaneous inlet pressure in the inner and outer limb for 40Hz, 60Hz, 80Hz under in-phase flow condition at 60° vane angle and 27.9 rev/s√K
Figure 6.9: Typical trend of the instantaneous inlet pressure in the inner and outer limb for 40Hz, 60Hz, 80Hz in out-of-phase flow condition at 60° vane angle and 27.9 rev/s√K.

Figure 6.10: Typical trend of the instantaneous inlet pressure in the inner and outer limb for 40°, 60° and 70° vane angle under in-phase flow condition at 27.9 rev/s√K and 60Hz.

Figure 6.11: Typical trend of the instantaneous inlet pressure in the inner and outer limb for 40°, 60° and 70° vane angle under out-of-phase flow condition at 27.9 rev/s√K and 60Hz.
6.4.3 Inlet static temperature

The fluctuating temperature of a pulsating flow is deduced by means of isentropic relation between pressure and temperature as discussed in Section 3.3.3. In order to determine the instantaneous temperature, the transient pressure and the time mean average pressure and temperature need to be evaluated; the resulting temperatures profile obtained with such an approach are given in Fig. 6.12. The curves shown have been obtained for 60Hz, 60° vane angle and 43.0 rev/s·√K speed; for each pulse flow condition, the inner and outer limb temperature traces are shown. The main feature of these curves is that they exhibit a strong similarity with the pressure traces corresponding to the same operating condition, which in this case are given in Fig. 6.7. This can be attributed to the method used to calculate the static temperature which is based on the measured instantaneous pressure. One might argue that this is not entirely correct since direct measurements would be more feasible. However the validity of this method was proven by Szymko (2006) who measured the instantaneous temperature of the pulsating flow using a dual hotwire probe. The findings of measurements showed that the assumption of isentropic compression agrees well with the main features of instantaneously measured temperature.

Uncertainty. The uncertainty associated with instantaneous temperature is dependent on mean static pressure, temperature and instantaneous pressure. The uncertainty associated with measurements, calculated as Root Sum Square (RSS) uncertainty is ±3K with 95% confidence level (Szymko, 2006).

6.4.4 Fluctuating torque

The instantaneous torque is derived from the instantaneous speed and it is shown in Figs. 6.13 and 6.14 for 27.9 rev/s·√K and 43.0 rev/s·√K speeds respectively under the in-phase and out-of-phase
pulse flow. The traces of the fluctuating torque are important to reflect the accuracy of the instantaneous speed measurement. The fluctuating torque is calculated by differentiating the instantaneous speed in order to obtain the rotor acceleration, as given in Eq. (3.16). This is added to the mean torque value obtained by the load cell and the result corresponds to the actual torque. At 27.9 rev/s√K under in-phase flow and 40Hz, the fluctuating torque peaks at approximately 3.3 Nm while at 60Hz it drops down to -0.1 Nm; at 43.0 rev/s√K and 40Hz the peak measured was measured at about 9.3 Nm with significant negative value of about -1.7 Nm at 60Hz. The negative value suggests that part of the power produced by the in-phase flow is lost as the turbine wheel freewhirls in a largely empty volume. As the speed increases this effect is more prominent since the negative torque is measured for a large portion of the latent pulse period. The same does not happen for the out-of-phase conditions since at any instant in time there is no combined latent period where the turbine volume is left empty. This is well shown in the torque traces on the right hand side of Figs. 6.13 and 6.14. Figure 6.15 shows the fluctuating torque for three different vane angles (40°, 60°, 70°) at 27.9 rev/s√K and 60Hz for the in-phase and out-of-phase flow. The main feature that can be gathered from this figure is that the fluctuating torque decreases as the vane angle moves towards the closed position (70° vane angle). The peak fluctuating torque decreases from 3.2 Nm at 40° down to 2.2 Nm at 70° vane angle with a drop of almost 32%. However such a large drop is not entirely reflected in the average fluctuating torque of the trough region which is only ≈ 11.0%. The possible reason to that can be found in the mass accumulation at the upstream of the nozzle vane ring as it closes (blockage effect). This causes a reduced amount of flow imparting momentum to the rotor during the peak of the pulse and hence a lower value for the fluctuating torque. On the contrary, mass accumulation corresponds to higher flow going through the nozzle during the trough period of the pulse and hence in an increased value of the fluctuating torque in this region.

Figure 6.13: Instantaneous torque for the in-phase and out-of-phase flow at 27.9 rev/s√K and two different frequencies, 40Hz and 60Hz
Uncertainty. The uncertainty associated with the fluctuating torque is given by the product of error in the angular acceleration ($\alpha$) and the polar moment of inertia ($I$). The tri-filar suspension method is used to determine polar moment of inertia. The polar moment of inertia of the whole rotating assembly (disc radius, suspension length, oscillation time, oscillation mass) is $4.563 \times 10^{-4}$ kgm$^2$ while that of the turbine wheel is $9.858 \times 10^{-5}$ kgm$^2$. The Root Sum Square (RSS) uncertainty in the calculation of polar moment of inertia is $\pm 1.55 \times 10^{-5}$ kgm$^2$. The angular acceleration is derived by the calculation of the instantaneous angular speed. The error associated with the angular acceleration $\alpha$ is $\pm 50$ rad/s and $\pm 225$ rad/s for 27.9 rev/s$\cdot\sqrt{K}$ and 43.0 rev/s$\cdot\sqrt{K}$ speed respectively. The combination of these
errors leads to an uncertainty in the fluctuating torque of about ±0.064 Nm and ±0.160 Nm for 27.9 rev/s√K and 43.0 rev/s√K speed respectively.

6.4.5 Mass flow rate

The instantaneous mass flow rate traces for both the inner and outer limb are given in Figs. 6.16 and 6.17 for 60Hz, 60° vane angle and 27.9 rev/s√K and 43.0 rev/s√K speed respectively. At a glance, the mass flow traces exhibit similar trend to that of the inlet static pressures even though the secondary peaks which are present in the pressure traces are not so evident. For the 43.0 rev/s√K speed, the secondary peaks are recorded for both the inner and outer limbs in the in-phase and out-of-phase flow conditions. Similar test results were reported by Rajoo (2006), Szymko (2004), and Karamanis (2000) for testing conducted on a single-entry turbine. In Fig. 6.17 this is partly confirmed since it can be seen that the additional peak is measured at approximately 170° for both the in-phase and out-of-phase conditions.

Figure 6.18 shows the effect of the vane angles on the mass flow rate under in-phase flow. The results are shown for 60Hz and 27.9 rev/s√K and three different vane angles (40°, 60° and 70°). Apart for the 60° vane angle which shows similar trend for both the inner and outer limbs, the mass flow traces for 40° and 70° vane angle exhibit different features. In the inner limb the fluctuation of the mass flow is amplified compared to the outer limb and the inner limb seems to swallow more mass than the outer limb. At 40° the peak mass flow rate is 0.28 kg/s and 0.20 kg/s in the inner and outer limb respectively; at 70° the respective peak mass flows are 0.23 kg/s and 0.18 kg/s. At 60° vane angle, no relevant fluctuation was observed which might be attributed to its being the optimum setting for the current design. In out-of-phase conditions (Fig. 6.19), the fluctuations exhibited for the in-phase flow are not significant. The mass flow rate for the 60° vane angle is substantially larger than that measured for 40° and 70° vane angle; the peak mass flow in the inner limb is 0.45 kg/s, 0.3 kg/s and 0.28 kg/s for 60°, 40° and 70° vane angle respectively. Similar values have been measured for the outer limb.

The effects of frequency on mass flow rate are given in Figs. 6.20 and 6.21 for 60° vane angle an 27.9 rev/s√K under in-phase and out-of-phase flow respectively. Under in-phase flow the peak of the mass flow rate reduces with increasing flow frequency even though a certain discrepancy is observed between the inner and the outer limb. The outer limb seems to be able to accept more mass flow in respect the inner limb. In the out-of-phase flow (Fig. 6.21) condition, the effects of frequency on mass flow are less clear. At 40 Hz the fluctuation in the mass flow rate traces is more significant than the 60Hz and 80Hz. However the variation of the mass flow rate in the inner and the outer limb seems to vary consistently at lower frequencies even though at 80Hz the mass flow rate the inner limb accepts twice as much the mass flow of the outer limb (∼0.21 kg/s against 0.1 kg/s for the outer limb).
Figure 6.16: Instantaneous mass flow rate for 60° vane angle at 27.9 rev/s·√K for the in-phase and out-of-phase flow at 60Hz

Figure 6.17: Instantaneous mass flow rate for 60° vane angle at 43.0 rev/s·√K for the in-phase and out-of-phase flow at 60Hz

Figure 6.18: Instantaneous mass flow rate for the inner and outer limb in-phase flow for 27.9 rev/s·√K and 60Hz at three different vane angles, 40°, 60° and 70°
Figure 6.19: Instantaneous mass flow rate for the inner and outer limb in out-of-phase flow for 27.9 rev/s·√K and 60 Hz at three different vane angles, 40°, 60° and 70°.

Figure 6.20: Instantaneous mass flow rate for the inner and outer limb under in-phase flow for 60° vane angle at three different frequencies, 40 Hz, 60 Hz and 80 Hz at 27.9 rev/s·√K.

Figure 6.21: Instantaneous mass flow rate for the inner and outer limb in out-of-phase flow condition for 60° vane angle at three different frequencies, 40 Hz, 60 Hz and 80 Hz at 27.9 rev/s·√K.
Uncertainty. The uncertainty associated with mass flow measurements is based on the multiple calibration points described in Chapter 3. The Root Sum Square (RSS) uncertainty of the mass flow rate measurement is approximately ±4.7%, with a confidence level of 95%.

6.4.6 Phase shifting

As reported in the section 6.2.6, the phase shifting of the flow properties posses difficulties when applied to a twin-entry turbine. As already explained, the shifting velocity used in the current research considers the velocity is the equal to sum of the bulk and sonic flow speeds. As for this study the range of bulk flow and sonic velocity was found to fall in between 56 m/s – 130m/s and 358m/s – 364m/s. In order to verify the suitability of the current phase shifting method, the instantaneous isentropic power is calculated and compared. The isentropic power comes as a combination of pressure, temperature and mass flow measurements, Eq. (6.6), and therefore a good agreement in-phase with the actual power serves as a validation of the effectiveness of the applied method. Nevertheless, unlike Szymko (2006) who considered a single-entry turbine, here a distinction must be made in whether it is an in-phase or out-of-phase flow condition. In Figs. 6.22 and 6.23 are given typical values of the isentropic (dark blue line) and actual power (red line) for 60° vane angle, two frequencies, 40Hz and 60Hz, and 27.9 rev/s√K and 43.0 rev/s√K speeds for the in-phase flow condition. From the figures it can be seen that the phase shifting results in a fairly good agreement with the actual power. The peak for the isentropic power occurs at 120° phase angle which is expected since the chopper plate cut-out is made to cover the first 1/3 of the full rotation. In some cases the isentropic power is lower than the actual power which leads to an efficiency greater than unity. This is physically meaningless even though it can be explained as wind-milling effect of the rotor wheel (refer to section 6.4.4). As further confirmation to the effectiveness of the phase shifting method, typical values for pressure and mass flow rate before and after phase shifting are given in Figs. 6.24 and 6.25 for two frequencies, 40Hz and 60Hz, at 27.9 rev/s√K and 60° vane angle. Mass flow rate and pressure are the two properties which mainly contribute to the calculation of the isentropic power, refer to Eq. (6.6), and therefore a good agreement of the phase shifted properties is an indication of the validity of the applied method. The figures show that the pressures undergo a marginal amount of phase shifting which is expected because of the high velocity of propagation of the pressure pulse (sonic + bulk velocity). This is not the case for the mass flow rate which travels at bulk speed along the volume (pipe + volute). The bulk speed is low compared to the sonic velocity and this retards the propagation of the mass flow with a consequent significant shifting in the mass flow trace. However from Figs. 6.24 and 6.25 it can be noticed that after phase shifting the mass flow trace is accurately positioned in the adopted time frame; the peak occurs approximately at 120°.

For the out-of-phase flow condition, unlike the in-phase flow conditions, the response of the phase shifting method is rather uncertain. In Figs. 6.26 and 6.27 the isentropic power for the out-of-
phase flow conditions are shown for 27.9 rev/s√K and 43.0 rev/s√K speeds at 40Hz and 60Hz. The figures show that there is no clear pattern in the outcomes of the phase shifted properties. The sonic plus bulk flow velocity assumption does not seem to apply to all cases. The out-of-phase between the isentropic and actual power is significant, especially in some cases (Figs. 6.27, 40Hz and 60 Hz at 43.0 rev/s√K). Nevertheless the shifting of the individual properties (pressure and mass flow rate) shows that the shifting method is appropriate since the out-of-phase condition is maintained and both pressure and mass flow peak at approximately 120° and 240° chopper plate phase angle (refer to Figs. 6.28 and 6.29). This seems to raise some issue on the validity of the phase-shifting adopted here when working in out-of-phase flow condition. Pressure wave reflection and back-flow could be amongst the causes of such a mismatch which need computational evaluation to identify the mechanisms occurring within the stage.

Figure 6.22: Isentropic and actual power for the in-phase flow for 40Hz, 60Hz at 27.9 rev/s√K

Figure 6.23: Isentropic and actual power for the in-phase flow for 40Hz, 60Hz at 43.0 rev/s√K
Figure 6.24: Phase shifting for pressure and mass flow rate for the in-phase flow condition at 40 Hz and 27.9 rev/s√K

Figure 6.25: Phase shifting for pressure and mass flow rate for the in-phase flow condition at 60 Hz and 27.9 rev/s√K

Figure 6.26: Isentropic and actual power for the out-of-phase flow condition at 40 Hz, 60 Hz at 27.9 rev/s√K
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Figure 6.27: Isentropic and actual power for the out-of-phase flow condition for 40Hz, 60Hz at 43.0 rev/s√K

Figure 6.28: Phase shifting for pressure and mass flow rate for the out-of-phase flow at 40Hz and 43.0 rev/s√K

Figure 6.29: Phase shifting for pressure and mass flow rate for the out-of-phase flow at 60Hz and 43.0 rev/s√K
As a further analysis on the phase shifting method, the isentropic power calculated using the constant atmospheric exit pressure has also been included. In a real engine where no exhaust energy recovery system is present downstream the turbine, the exhaust discharges to the atmosphere. The isentropic power thus calculated is plotted (blue line) in Figs. 6.22, 6.23 (in-phase flow), 6.26 and 6.27 (out-of-phase flow) and superimposed to that calculated using the instantaneous exit pressure (dark blue line). The figures show that the newly calculated isentropic power follows consistently with that using the non-instantaneous exit pressure, with no change in-phase shifting. Nevertheless the use of constant atmospheric exit pressure causes a drop in isentropic power, above all in the high power region of the flow. In the in-phase flow condition, the difference between the isentropic peak power calculated using the two methods is approximately 3.0 kW, 17.0 kW at 40 Hz and 27.9 rev/s·√K and 43.0 rev/s·√K respectively. At 60 Hz, such a difference decreases substantially to about 2.0 kW and 1.0 kW for 27.9 rev/s·√K and 43.0 rev/s·√K respectively. In the out-of-phase flow condition, a higher difference in power was measured for 60 Hz; this is approximately 5.0 kW and 3.0 kW for 27.9 rev/s·√K and 43.0 rev/s·√K respectively. However, it is worth noting, that the importance of phase shifting is only in the evaluation of unsteady efficiency, a concept itself questionable by the very need to phase shift.

6.5 Performance parameters

6.5.1 Mass flow parameter vs. pressure ratio

This section reports the mass flow parameter vs. pressure ratio for different speeds, frequencies and vane angles. The test conditions are given in Table 6.1. Given the large amount of plots available, in order to proceed with a more logical discussion, the plots have been divided in three main groups depending on the vane angles. For each vane angle the in-phase and out-phase plots have been grouped together for constant speeds lines. In order to evaluate the response of each limb in the pulsating flow conditions, the flow capacity of the individual limbs are also shown together with that calculated with mass weighted average given in Eq. (6.3). The equivalent quasi-steady curves for both the full and partial admission conditions have also been included in the graphs in order to aid comparison.

**Mass flow parameter at 60° vane angle**

The diagrams on the left handside of Figs. 6.30 to 6.35 show the flow capacity of the twin-entry turbine for the in-phase flow at 27.9 rev/s·√K and 43.0 rev/s·√K speeds and frequency spanning from 40 Hz to 80 Hz. All the comparisons are at 60° vane angle which corresponds to the optimum vane angle for the current turbine. At a glance one can notice that a hysteresis loop is observed in all the cases; this is in agreement with the findings of previous researches conducted by Dale and Watson (1986), Baines et al. (1994), Karamanis and Martinez-Botas (2002), Szymko et al. (2006), and Rajoo (2007). The loop is due to continuous filling and emptying of the volume (volute + pipe) during
pulsating flow conditions. This is more evident at low frequencies, 40Hz and 60Hz, where a wider hysteresis loop could be observed with low impact of the pressure wave changes in the volume. As the frequency increases, the wave action effect and the consequent variation in flow velocity influences the hysteresis loop which tends to reduce in size with consequent variation in its range and amplitude. This is valid for both 43.0 rev/s·√K and 27.9 rev/s·√K speed cases.

Another observation is that the loop encircles the steady curve only in limited cases; the flow capacity under full admission remains well above the equivalent quasi-steady curve. During the 27.9 rev/s·√K speed and 40Hz and 60Hz, the hysteresis loop tends towards the steady curve line in the low pressure region of the map. However as the pressure increases the hysteresis loop largely departs from the steady condition. At 43.0 rev/s·√K the flow capacity under unsteady condition is never similar to the equivalent quasi-steady. This is consistent with the observed characteristics of the nozzled single-entry turbine reported by Rajoo, (2007). Comparing the individual limbs it can be seen that the inner limb flow capacity tends toward steady characteristics for both 27.9 rev/s·√K and 43.0 rev/s·√K speeds. The flow capacity of the inner limb is consistently at the lower end of pressure ratio, which is more apparent in the 27.9 rev/s·√K speed conditions. This implies that the outer limb has a different response than the inner limb since, that is, for a given mass flow rate, a higher pressure ratio is needed to let the mass through. The geometry of the stator can be considered as one of the reasons causing an unequal flow capacity. The partition of the volute was designed to maintain the same area within the inner and outer limb in order to ensure an equivalent A/r. However the non-symmetrical shape (in respect to the meridional plane) of the turbine volute constrained the design of the divider to be slightly offset and inclined towards the shroud side of the volute (offset of approximately 8 mm with an inclination of 6 degrees). This might cause the mass flow to follow different paths when travelling in the volume (volute + pipe). In addition to this, a centrifugal pressure gradient exists along the leading edge of the mixed-flow rotor due to the non-zero cone angle, thus causes a different path of the flow when entering the rotor.

The diagrams on the right hand side of Figs. 6.30 to 6.35 report the flow capacity of the twin-entry turbine under 180° out-of-phase flow condition, 40Hz, 60Hz and 80Hz pulsations and 27.9 rev/s·√K and 43.0 rev/s·√K speeds. The mass averaged mass flow parameter of the two entries exhibit a reduced range covered during a cycle, compared to the individual entry. This is particularly true in the out-of-phase flow conditions and it can be seen in Figs. 6.30 to 6.35 where the typical hysteresis loop of the overall flow capacity (blue line) presents a double looping of the mass representing both entries. As for the in-phase flow conditions the flow capacity of the individual entries show clear filling and emptying, above all at lower rotational speed, 27.9 rev/s·√K. The pressure ratio range during a cycle emulates the steady curves more at lower frequencies (40Hz, 60Hz) than higher frequencies. In fact at higher frequency the departure of the hysteresis loop from the quasi-steady curves becomes significant, above all at 43.0 rev/s·√K. At 80Hz, the inner and outer flow capacities remain far from the steady curves and the pressure ratio range of a cycle reduces significantly.
Figure 6.30: Instantaneous mass flow at 40Hz under in-phase and out-of-phase flow for 60° vane angle and 27.9 rev/s√K

Figure 6.31: Instantaneous mass flow at 60Hz under in-phase and out-of-phase flow for 60° vane angle and 27.9 rev/s√K

Figure 6.32: Instantaneous mass flow at 80Hz under in-phase and out-of-phase flow for 60° vane angle and 27.9 rev/s√K
Figure 6.33: Instantaneous mass flow at 40Hz under in-phase and out-of-phase flow for 60° vane angle and 43.0 rev/s·√K

Figure 6.34: Instantaneous mass flow at 60Hz under in-phase and out-of-phase flow for 60° vane angle and 43.0 rev/s·√K

Figure 6.35: Instantaneous mass flow at 80Hz under in-phase and out-of-phase flow for 60° vane angle and 43.0 rev/s·√K
Additionally it can be noticed that at 43.0 rev/s·√K the flow capacity of the two limbs seems to behave similarly to the in-phase flow conditions (refer to Fig. 6.35 left hand side). This could be explained by considering that at high frequencies and speeds, the amount of time in which no flow is present in one limb is minimal. Therefore the turbine wheel seems to experience the same flow condition and hence similar flow capacity to the in-phase flow.

**Vane angle effect, 40° and 70°**

Figure 6.36 to 6.41 show the flow capacity for two different vane angle settings, 40° and 70°, three flow frequencies, 40Hz, 60Hz and 80Hz, at 27.9 rev/s·√K speed.

At 40° and for the in-phase flow condition (Figs. 6.36 to 6.38) the mass flow parameter for the two limbs deviates most from the steady curve. At 40Hz the two limbs seem to be equally pressurized even though the flow capacity does not increase with pressure and it remains constant at about 2·10^{-5} (kg/s)·√K/Pa. As a consequence of this the flow capacity of the overall turbine stage is fairly constant at a value of approximately 4·10^{-5} (kg/s)·√K/Pa (Fig. 6.36). At 60Hz the flow capacities for the two limbs are slightly shifted, with a fair degree of superimposition. The flow capacity increases with pressure; the hysteresis loop, typical of filling and emptying effect is no longer visible. The hysteresis curve collapses into a single line thus suggesting that the rate of filling and emptying of the volume during the pulse period is similar and follows progressively with the increase and decrease in pressure. Meanwhile for the 80Hz case, the flow capacities between the inner and outer limb depart from one another with the inner limb exhibiting less flow capacity. The amplitude of the hysteresis loop is found to be wider than the 60Hz case. In the out-of-phase flow conditions both the inner and outer limbs are pressurized at the same rate and the flow capacities superimpose well on one another. Interestingly the flow capacity for the inner limb at some instance goes down to zero, especially at 40° vane angle cases. This occurs at approximately a pressure ratio of 1.2 and 1.1 for 40Hz and 60Hz respectively. This could probably be attributed to the faster rate of volume emptying (volute + pipes) compared to the incoming pulses in the out-of-phase conditions.

In Figs. 6.39 to 6.41 are given the mass flow parameters for 70° vane angle under in-phase flow condition. Unlike the 40° vane angle, the mass flow parameters exhibit hysteresis loops which are wider and well defined. The reason to that is mainly associated with the fact that the closed nozzle position leads to a higher degree of mass accumulation and hence to an increase in the back-pressure. As a consequence of this the flow capacity of the overall turbine stage is fairly constant at a value of approximately 4·10^{-5} (kg/s)·√K/Pa for the entire range of pressure ratios. This is almost half of that measured at 40° and it can be seen as a direct consequence of the limited flow capacity due to the reduced amount of area available for the flow when the vanes are closed. Similar findings as for the in-phase flow apply to the out-of-phase flow (diagrams on the right hand side of Figs. 6.39 to 6.41).
Figure 6.36: Instantaneous mass flow at 40Hz under in-phase and out-of-phase flow for 40° vane angle and 27.9 rev/s√K

Figure 6.37: Instantaneous mass flow at 60Hz under in-phase and out-of-phase flow for 40° vane angle and 27.9 rev/s√K

Figure 6.38: Instantaneous mass flow at 80Hz under in-phase and out-of-phase flow for 40° vane angle and 27.9 rev/s√K
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Figure 6.39: Instantaneous mass flow at 40Hz under in-phase and out-of-phase flow for 70° vane angle and 27.9 rev/s·√K

Figure 6.40: Instantaneous mass flow at 60Hz under in-phase and out-of-phase flow for 70° vane angle and 27.9 rev/s·√K

Figure 6.41: Instantaneous mass flow at 80Hz under in-phase and out-of-phase flow for 70° vane angle and 27.9 rev/s·√K
The amplitude of the hysteresis loop is high and the range of pressure ratio covered by the hysteresis loops during a cycle remains in a low pressure region of the map is similar to that of the corresponding quasi-steady curves. This is particularly true at frequencies 40Hz and 60Hz, where the pressure ratio ranges within ≈1.25 to ≈1.65.

Uncertainty. The uncertainty associated with the mass flow parameter depends on the mass flow rate, pressure and temperature measurements. Amongst these three properties, the most important is the mass flow rate which contributes to a large portion of the uncertainty in the mass flow parameter. The mass flow parameter obtained with the mass weighted average of Eq. (6.3) varies between $0.1\cdot10^{-5}$ (kg/s)√K/Pa and $7.3\cdot10^{-5}$ (kg/s)√K/Pa; the corresponding Root Sum Square (RSS) uncertainty varies between ±0.02·10^{-5} and ±0.28·10^{-5}.

6.5.2 Efficiency vs. velocity ratio

This section discusses the turbine instantaneous isentropic efficiency, calculated based on measurements. Prior to any discussion about the test results there are some considerations to be made clear. Firstly, the instantaneous turbine efficiency is calculated as given in Eq. (6.5) and it relies on the phase shifting of the measured parameters. The phase shifting method (sonic + bulk velocity) showed good agreement between measurements at different locations. However there is still some degree of uncertainty which is reflected in the out-of-phase flow condition. Secondly, the isentropic waveform seen at the measurement plane should remain unchanged until the point where it imparts momentum to the rotor. However the interaction between the two divisions of the volute and the combination of travelling and reflecting waves seems to generate a flow field which affects the phase shifting of the measured parameters. For the in-phase flow conditions instead the twin-entry turbine is more likely to act as a single-entry turbine since the gas velocity in each limb is in-phase and travels at the same speed. In a single-entry turbine phase-shifting was found to be more consistent since there is only a single volume (volute + pipe) into which the pulse is travelling.

Additionally to the phase shifting method and the interaction between the entries, the presence of the nozzle vane ring must also be taken into account. As reported by Rajoo (2006) the nozzle ring acts as a restrictor which prevents the rotor from being entirely exposed to the unsteadiness of the flow thus contributing to the change of isentropic waveform from the point of measurement to the point where it imparts momentum. To summarize, based on the considerations above, some may argue the use of instantaneous efficiency should be treated with care.

In Figs. 6.42 to 6.47 the turbine instantaneous efficiency is shown for 60° vane angle at two different speeds (27.9 rev/s·√K and 43.0 rev/s·√K) and three pulse frequencies (40Hz, 60Hz and 80Hz) under the in-phase and out-of-phase flow conditions. The equivalent quasi-steady curve is also shown on each plot to aid comparison. At a glance, it can be seen that turbine efficiency shows hysteresis loop similar to the mass flow parameter. For the in-phase flow at 27.9 rev/s·√K, the range
of velocity ratios is 0.41-0.9, 0.42-0.93, 0.42-1.1 for 40Hz, 60Hz and 80Hz respectively. Some degree of encapsulation is observed, however, it is not consistently maintained over the entire range of velocity ratios since the turbine instantaneous efficiency exhibits points where it goes to negative or also above unity. The negative values occur in the high velocity ratio region of the map, which corresponds to the state of the flow with very low isentropic power, where the deceleration of the rotor leads to negative torque and hence negative efficiency. Meanwhile, an instantaneous efficiency of greater than unity occurs when the isentropic power is lower than the actual power. Besides being attributed to the phase shifting method, efficiency greater than unity can also be attributed to the inertial effect of the turbine rotor, which causes its rotation to continue even when the mass flow rate is low.

In the out-of-phase flow, the effect of the pulsed flow entering the rotor at staggered intervals imparts momentum to the rotor in different times and this is reflected in the point-to-point instantaneous efficiency. At 27.9 rev/s√K and 40Hz, the efficiency largely departs from the equivalent quasi-steady efficiency. The velocity ratio ranges from 0.4 to 1.6 and the instantaneous turbine efficiency largely goes beyond unity (Fig. 6.45). At 60Hz the instantaneous efficiency remains below the equivalent quasi-steady over the whole range of velocity ratios while at 80Hz, a certain degree of encapsulation can be observed. Although for the in-phase flow the effects of speed were not so evident, the same is not the case for the out-of-phase flow. At 43.0 rev/s√K the instantaneous turbine efficiency does no significantly deviates from the quasi-steady efficiency. The drop of the isentropic power is less apparent since no efficiency greater than unity is recorded (except for the 40Hz frequency). However the velocity ratio region of the map into which the instantaneous efficiency varies is almost five times smaller than that measured at 27.9 rev/s√K; the velocity ratio ranges from 0.55 to 0.68 for every frequency. Another important aspect in the trend of the instantaneous efficiency is that the efficiency loop is centered in the low velocity ratio region of the map which suggests that the setting is operating at higher pressure ratios.
Figure 6.43: Instantaneous efficiency at 60Hz under in-phase flow for 60° vane angle and 27.9 rev/s√K and 43.0 rev/s√K

Figure 6.44: Instantaneous efficiency at 80Hz under in-phase flow for 60° vane angle and 27.9 rev/s√K and 43.0 rev/s√K

Figure 6.45: Instantaneous efficiency at 40Hz in out-of-phase flow for 60° vane angle and 27.9 rev/s√K and 43.0 rev/s√K
Figure 6.46: Instantaneous efficiency at 60Hz in out-of-phase flow for 60° vane angle and 27.9 rev/s·√K and 43.0 rev/s·√K

Figure 6.47: Instantaneous efficiency at 80Hz in out-of-phase flow for 60° vane angle and 27.9 rev/s·√K and 43.0 rev/s·√K

Given the uncertainty associated with the meaning of the instantaneous turbine efficiency, an alternative method to evaluate energy conversion is proposed. This will use a cycle average of the parameters. The main advantage of the cycle-averaging method is that it is not affected by phase shifting.

The cycle-averaged efficiency represents the ratio of energy extracted by the turbine per pulse cycle divided by isentropic energy flowing into the system, as given in Eq. (6.20).

\[
(\eta_{ts})_{energy-avg} = \frac{\sum_{i=1}^{i=n_{cycle}} [\eta_{ts} \cdot \dot{W}(i)_{is}]}{\sum_{i=1}^{i=n_{cycle}} [\dot{W}(i)_{is}]} = \frac{\sum_{i=1}^{i=n_{cycle}} [\dot{W}(i)_{act}]}{\sum_{i=1}^{i=n_{cycle}} [\dot{W}(i)_{is}]} \tag{6.20}
\]

For the velocity ratio calculation, we follow an energy weighted average quantity, Eq. (6.21).
The calculated energy weighted cycle-averaged velocity ratio is then used to read the corresponding efficiency from the steady map for that given speed. The cycle-averaged efficiency is then compared with the equivalent quasi-steady efficiency. By doing this an appropriate comparison between parameters obtained at different conditions can therefore be performed.

Table 6.2 reports a comparison between the cycle-averaged and the quasi-steady efficiency obtained for 60° vane angle at 27.9 rev/s·√K and 43.0 rev/s·√K under in-phase and out-of-phase flow. For each of these conditions, three frequencies (40Hz, 60Hz and 80Hz) have been included. It is worth noting that the full admission assumption has been considered here for the quasi-steady value. This is appropriate when dealing with in-phase flow (since both entries flow at the same time) while it is not so clear when working on out-of-phase flow condition (since each limb is pressurized at staggered intervals).

The results have been plotted in Figs. 6.48 and 6.49 for the in-phase and out-of-phase flow respectively. Observing the comparison, one can notice that the cycle-averaged unsteady efficiency drops substantially, especially at the in-phase flow conditions lower speeds. The cycle-averaged efficiency remains below the quasi-steady efficiency with a difference of 26.2%, 26.4% and 0.79% for 40Hz, 60Hz and 80Hz respectively. At higher speed instead, the discrepancy is less significant and in some cases the quasi-steady efficiency is higher than the cycle-averaged efficiency. This occurs at 80Hz and 40Hz where the cycle-averaged efficiency is 5.1% and 6.1% higher than the quasi-steady. At high frequency the similarity between the cycle averaged and the quasi-steady efficiency can be explained considering that at higher frequency, the pulse amplitude is smaller which makes the rotor exposed to more continuous flow. At low frequency instead, the flow follows a filling and emptying behaviour, thus significant swallowing capacity changes are experienced leading to the differences between the cycle averaged and quasi-steady efficiencies. Similar results were also recorded by Karamanis (2000), Szymko (2005), and Rajoo (2006) who measured higher cycle-averaged efficiency in a single-entry turbine for higher speeds and frequencies. However, it must be noted that in the current study, the cycle averaged efficiency variation with speed does not have a consistent trend as in their case. In the out-of-phase flow conditions, the cycle-averaged efficiency shows large difference from the quasi-steady; at 27.9 rev/s·√K a difference of 23.1%, 30.3% and 18.0% was measured at 40Hz, 60Hz and 80Hz respectively. A similar drop was also found at 43.0 rev/s·√K; the cycle-averaged efficiency deviates from the quasi-steady by 18.2%, 32.2% and 27.1% for the 40Hz, 60Hz and 80Hz cases respectively.
In Phase: Comparison of cycle averaged vs. Quasi-steady efficiency - 60° vane angle

Figure 6.48: Comparison cycle averaged vs. quasi-steady efficiency for the in-phase flow at 27.9 rev/s·√K and 43.0 rev/s·√K for 60° vane angle

Out of Phase: Comparison Cycle averaged vs. Quasi-steady efficiency - 60° vane angle

Figure 6.49: Comparison cycle-averaged vs. quasi-steady efficiency for the out-of-phase flow at 27.9 rev/s·√K and 43.0 rev/s·√K for 60° vane angle
Table 6.2: Comparison Energy weighted cycle-averaged and Quasi-steady efficiency

<table>
<thead>
<tr>
<th>60° vane angle</th>
<th>In-Phase</th>
<th>27.9 rev/s·√K</th>
<th>43.0 rev/s·√K</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>U/Cis</td>
<td>η_{cycle-avg}</td>
<td>η_{quasi-steady}</td>
</tr>
<tr>
<td>40Hz</td>
<td>0.542</td>
<td>0.472</td>
<td>0.631</td>
</tr>
<tr>
<td>60Hz</td>
<td>0.503</td>
<td>0.456</td>
<td>0.611</td>
</tr>
<tr>
<td>80Hz</td>
<td>0.533</td>
<td>0.631</td>
<td>0.626</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>60° vane angle</th>
<th>Out-of-Phase</th>
<th>27.9 rev/s·√K</th>
<th>43.0 rev/s·√K</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>U/Cis</td>
<td>η_{cycle-avg}</td>
<td>η_{quasi-steady}</td>
</tr>
<tr>
<td>40Hz</td>
<td>0.587</td>
<td>0.434</td>
<td>0.647</td>
</tr>
<tr>
<td>60Hz</td>
<td>0.464</td>
<td>0.415</td>
<td>0.586</td>
</tr>
<tr>
<td>80Hz</td>
<td>0.567</td>
<td>0.526</td>
<td>0.641</td>
</tr>
</tbody>
</table>

Another important aspect to consider is the effect of the pulsed flow phases (in-phase or out-of-phase) on the efficiency. From Table 6.2 it can be seen that the out-of-phase condition is detrimental to the overall efficiency above all at high speed. At 43.0 rev/s·√K the cycle-averaged efficiency drops from 0.835, 0.616, and 0.823 down to 0.631, 0.520, and 0.560 for 40Hz, 60Hz and 80Hz respectively, when going from in-phase to out-of-phase flow condition; this corresponds to a drop of 32.2%, 18.4%, and 46.9%. However, the same deficit is not observed at lower speed where the efficiency drop is much lesser. At 27.9 rev/s·√K the efficiency drops from 0.472, 0.456 and 0.631 down to 0.434, 0.415 and 0.526 (corresponding to a drop of 8.75%, 9.8% and 19.96%) for 40Hz, 60Hz and 80Hz respectively, when going from in-phase and out-of-phase flow condition.
In order to further understand the correlation between the cycle-averaged and the quasi-steady assumption, the ratio between these efficiencies has been calculated for both in-phase and out-phase flow at 27.9 rev/s\(\sqrt{\text{K}}\) and 43.0 rev/s\(\sqrt{\text{K}}\). The parameter called efficiency ratio\(^{49}\) is plotted in Fig. 6.50 against pulse frequency (40Hz, 60Hz, and 80Hz). As noted from Fig. 6.50, at 43.0 rev/s\(\sqrt{\text{K}}\) the efficiency ratio for the in-phase flow does not vary substantially with frequency which goes in favour of the quasi-steady assumption. The dip observed at 60Hz (efficiency ratio \(\approx 0.8\)) can be considered as the result of the transition through a region where the quasi-steady assumption no longer applies. In the out-of-phase flow conditions instead the efficiency ratio remains below unity for both 27.9 and 43.0 rev/s\(\sqrt{\text{K}}\).

In order to understand the efficiency trends, the unsteady time-averaged power and mass flow were calculated and compared to the quasi-steady values – these are shown in Figs. 6.51 and 6.52. The quasi-steady values are obtained from the steady curves at the equivalent unsteady isentropic energy averaged velocity ratio. These figures show that the quasi-steady average values generally over predict the corresponding unsteady values since the power and mass flow ratio remain below unity. The mass flow and power vary in a consistent manner for both in-phase and out-of-phase flow conditions – a decreasing value in the ratio observed with increasing frequency. At 43.0 rev/s\(\sqrt{\text{K}}\) and in out-of-phase flow condition the mass flow ratio is approximately equal to unity for almost any frequencies, which shows that the quasi-steady assumption is adequate for a full unsteady calculation. On the efficiency side, the trend observed for the ratios between the cycle-averaged and the quasi-steady efficiencies (Figs. 6.48 and 6.49) is reflected in the actual and isentropic power ratios – the isentropic power ratios remains consistently above the actual power ratio.

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\(^{49}\) Efficiency ratio \(\eta_{\text{ratio}} = \frac{\eta_{\text{cycle-avg}}}{\eta_{\text{quasi-steady}}}\)

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Figure 6.51: Ratio cycle-averaged and quasi-steady power and mass flow rate for the in-phase and out-of-phase flow at 27.9 rev/s\(\sqrt{\text{K}}\) and 60° vane angle
Comparing the cycle-averaged efficiency with the quasi-steady assumption based on full admission conditions is certainly appropriate when dealing with single-entry turbines. However such an approach does not entirely apply when dealing with twin-entry turbines. As already discussed, the twin-entry turbine is meant to work in out-of-phase flow conditions in most cases. The incoming pulses from each bank of manifolds occur at staggered intervals, thus the turbine will be working in more partial admission conditions than full admission. Therefore, in order to evaluate the quasi-steady assumption in the out-of-phase flow conditions, it might be more appropriate to refer to partial admission maps instead of the full admission. Table 6.3 reports the data for the quasi-steady efficiency calculated using the full and partial admission conditions. The cycle-averaged efficiencies are also reported and the ratio between the cycle-averaged and the quasi-steady efficiencies are shown in Fig. 6.53 in a similar way as shown in Fig. 6.50. Figure 6.53 shows that the ratio between the cycle-averaged and the quasi-steady assumption based on the partial admission condition is much closer to unity compared to the cases where the full admission map is used (see Figure 6.50). At 43.0 rev/s·√K the quasi-steady efficiency goes from 0.768, 0.763 and 0.757 to 0.530, 0.533 and 0.534 for the 40Hz, 60Hz and 80Hz cases respectively, when moving from full to partial admission condition; such a drop leads to values of the efficiency ratio (in partial admission) to be 1.03, 0.85 and 0.92 for 40Hz, 60Hz and 80Hz respectively. At lower speeds (27.9 rev/s·√K) the ratio between the cycle-averaged and quasi-steady efficiency is approximately 10% higher than that calculated considering the full admission curve, which results in an improvement in the evaluation of the quasi-steady assumption. In summary, the quasi-steady efficiency shifts from 0.647, 0.586 and 0.641 to 0.607, 0.607 and 6.08 for the 40Hz, 60Hz and 80Hz cases respectively, when moving from the full admission to the partial admission case.
Table 6.3: Comparison between Cycle-averaged and Quasi-steady efficiency considering full and partial admission assumption for the quasi-steady value

<table>
<thead>
<tr>
<th>Out-of-phase</th>
<th>27.9 rev/s·√K</th>
<th>43.0 rev/s·√K</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>U/Ciş</td>
<td>η_cycle_avg</td>
</tr>
<tr>
<td>40Hz</td>
<td>0.587</td>
<td>0.434</td>
</tr>
<tr>
<td>60Hz</td>
<td>0.464</td>
<td>0.415</td>
</tr>
<tr>
<td>80Hz</td>
<td>0.567</td>
<td>0.526</td>
</tr>
</tbody>
</table>

Table 6.4 shows the velocity ratio, cycle-averaged efficiency at 27.9 rev/s·√K for different vane angle settings (40°, 60°, 70°) and flow frequencies (40Hz, 60Hz and 80Hz). The values are energy weighted average as shown in Eq. (6.20) and Eq. (6.21). The data are also shown in Fig. 6.54 for the in-phase and out-of-phase flow conditions. It can be noticed that the trend of the cycle-averaged efficiency for different vane angles does not seem to follow a well defined pattern. For the in-phase flow, the cycle-averaged efficiency shows about 12 percentage points drop from 40° to 60° vane angle for 40Hz and 60Hz. As reported by Rajoo (2007) this can be directly linked to the high fluctuation of the torque exhibited by the turbine for open vane angles compared to the closed vane settings which leads to higher cycle-averaged efficiency. For 70° vane angle and in-phase flow 40Hz and 60Hz, the cycle-averaged efficiency is similar to that calculated for the 60° vane angle even though a large departure from one another could be observed at 80Hz.

Figure 6.53: Ratio cycle-averaged and quasi-steady efficiency for the out-of-phase flow at 27.9 rev/s·√K and 43.0 rev/s·√K obtained using the full and partial admission for the quasi-steady value
Table 6.4: Energy weighted cycle-averaged efficiency for different vane angles

<table>
<thead>
<tr>
<th>N/√T₀₁</th>
<th>40° vane angle</th>
<th>60° vane angle</th>
<th>70° vane angle</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>U/Cₐ</td>
<td>η_cycle-avg</td>
<td>U/Cₐ</td>
</tr>
<tr>
<td>40Hz</td>
<td>0.593</td>
<td>0.593</td>
<td>0.542</td>
</tr>
<tr>
<td>60Hz</td>
<td>0.548</td>
<td>0.590</td>
<td>0.503</td>
</tr>
<tr>
<td>80Hz</td>
<td>0.519</td>
<td>0.347</td>
<td>0.533</td>
</tr>
</tbody>
</table>

A different scenario can be observed for the out-of-phase flow. The cycle-averaged efficiency increases with increasing frequency. At 60° vane angle the efficiency is higher than those measured at 40° and 70°, while the 40° vane angle setting seems to perform better than the 70°. The efficiency deficit for the 70° vane angle compared to the optimum vane angle is approximately 17.3%, 10.1% and 17.7% for 40Hz, 60Hz and 80Hz respectively. Such a penalty in efficiency could be attributed to the blockage effects due to closed nozzle settings. The mass accumulation in the volume (volute + pipe) reduces the possible momentum imparted to the rotor with consequent lower power output.

Figure 6.54: Comparison cycle-averaged efficiency for different vane angles (40°, 60° and 70°) for the in-phase and out-of-phase flow at 27.9 rev/s·√K
6.6 Comparison with single-entry

Similarly to the steady state analysis, the pulsating performance of the twin-entry turbine is compared to that of the single-entry nozzled turbine. The design progression explained in Chapter 3, makes it possible to evaluate the effects of the two configurations on equivalent geometry basis. In the following analysis, the in-phase flow condition only has been considered for comparison. Figures 6.55 to 6.62 show the comparison between the single and twin-entry configuration for 60° vane angle at 27.9 rev/s·√K and 43.0 rev/s·√K for 40Hz, 60Hz and 80Hz. The equivalent quasi-steady curves are also given for comparison. The outcomes of the analysis carried out under steady state conditions showed that at 60° vane angle the addition of the divider did not cause a detrimental effect to the overall flow capacity for both 27.9 rev/s·√K and 43.0 rev/s·√K.

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**Figure 6.55: Comparison MFP between twin and single-entry turbine at 40Hz for 60° vane angle and 27.9 rev/s·√K**

**Figure 6.56: Comparison MFP between twin and single-entry turbine at 60Hz for 60° vane angle and 27.9 rev/s·√K**

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Figure 6.57: Comparison MFP between twin and single-entry turbine at 40Hz for 60° vane angle and 43.0 rev/s√K

Figure 6.58: Comparison MFP between twin and single-entry turbine at 60Hz for 60° vane angle and 43.0 rev/s√K

Figure 6.59: Comparison MFP between twin and single-entry turbine at 40Hz for 40° vane angle and 27.9 rev/s√K
Figure 6.60: Comparison MFP between twin and single-entry turbine at 40Hz for 40° vane angle and 27.9 rev/s√K

Figure 6.61: Comparison MFP between twin and single-entry turbine at 40Hz for 70° vane angle and 27.9 rev/s√K

Figure 6.62: Comparison MFP between twin and single-entry turbine at 60Hz for 70° vane angle and 27.9 rev/s√K
Similar outcomes were found for the pulsating flow condition even though some minor difference occurs. At 43.0 rev/s·√K the hysteresis loop traces for the single and twin-entry turbine follow a similar trend to each other; the amplitude of the loops is comparable and this seems to show that the turbines undergo similar filling and emptying processes (Figs. 6.57 and 6.58). At 40Hz and 60Hz the pressure ratio range for the twin-entry turbine is 1.3-2.5, 1.5-2.2 which is similar to 1.2-2.3, 1.3-2.0 of the single-entry turbine. However the hysteresis loop for the single-entry turbine seems to encapsulate the equivalent quasi-steady map in some portion of the map while the same does not occur for the twin-entry turbine which remains well above the steady line. The similarity between the hysteresis loops of the single and twin-entry turbines is not maintained at 27.9 rev/s·√K. In the single-entry turbine the rate of filling and emptying is more significant than that of the twin-entry turbine. For the single-entry turbine the pressure ratio range compares well with that of the equivalent quasi-steady. The same does not occur for the twin-entry turbine where the hysteresis loop seems to be slightly shifted towards the lower end of pressure ratio in the map. Unlike the single-entry configuration the hysteresis loop collapses into single line suggesting similar rate of filling and emptying of the volume during the pulse period (Figs. 6.55 and 6.56). The similarity observed at 60° vane angle between the hysteresis loops of the single and twin-entry configuration was not found at 40° vane angle. Here the mass flow traces follow completely different pattern. Figures 6.59 and 6.60 show the flow capacity of the twin-entry turbine is significantly lower than the single-entry turbine.

At 40Hz the maximum mass flow parameter for the twin-entry turbine is approximately 4.0·10⁻⁵ (kg/s)·√K/Pa which is almost half the corresponding single-entry. At 60Hz instead, the maximum flow capacity for the twin-entry is 5.2·10⁻⁵ (kg/s)·√K/Pa which is comparable to the 6.2·10⁻⁵ (kg/s)·√K/Pa measured for the single-entry turbine. For the 70° vane angle, the hysteresis loops of the twin and single-entry turbines seem to have a similar shape to that of the twin-entry turbine even though the flow capacity of the single-entry turbine is higher than the equivalent quasi-steady curve (Figs. 6.61, 6.62). The same does not happen for the twin-entry turbine which exhibits a lower flow capacity than the equivalent quasi-steady over the whole range of pressure ratios.

<table>
<thead>
<tr>
<th>N/T₀₁ ≈ 27.9 rev/s·√K</th>
<th>40° vane angle</th>
<th>60° vane angle</th>
<th>70° vane angle</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>U/Cₚ</td>
<td>η_cycle-avg</td>
<td>∆η</td>
</tr>
<tr>
<td>40Hz_single</td>
<td>0.461</td>
<td>0.514</td>
<td>-13.3%</td>
</tr>
<tr>
<td>40Hz_twin</td>
<td>0.592</td>
<td>0.593</td>
<td>-4.0%</td>
</tr>
<tr>
<td>60Hz_single</td>
<td>0.501</td>
<td>0.614</td>
<td>-37.5%</td>
</tr>
<tr>
<td>60Hz_twin</td>
<td>0.541</td>
<td>0.590</td>
<td>-4.0%</td>
</tr>
</tbody>
</table>

Table 6.5 shows the velocity ratio, cycle-averaged efficiency for different angle settings of the twin and single-entry turbine at 40Hz and 60Hz flow conditions at 27.9 rev/s·√K – the data is also
plotted in Fig. 6.63. The values shown are energy weighted cycle-averaged (Eq. (6.20) and Eq. (6.21)). The largest penalty in efficiency has been found for the optimum vane angle of 60°; the deviation from single to twin-entry is 18.0% and 37.5% at 40Hz and 60Hz respectively. At 40° vane angle the efficiency variation from single to twin-entry is 13.3% and 4.0% for 40Hz and 60Hz respectively. At 70° vane angle the cycle-averaged efficiency for the two turbine configurations varies within small values for both 40Hz and 60Hz frequencies; the efficiency variation for the respective frequency is 4.1% and 4.3%. Overall, the cycle-averaged efficiency for the single-entry turbine is higher than the twin-entry. The largest penalty drop for the 60° vane angle does not seem to fit well with the findings of the flow capacity where a good similarity was found between the single and twin-entry turbine. In addition to this Fig. 6.63 shows that the cycle-averaged velocity ratio for the single-entry turbine is lower than the twin-entry for all the vane angles and frequencies. This means that the single-entry turbine operates in a region where the energy of the pressure pulse is high, nevertheless the benefit in efficiency is not be observed compared to the twin-entry turbine.

![Comparison Single vs. Twin-entry cycle averaged efficiency](image)

Figure 6.63: Comparison Single vs. Twin-entry cycle-averaged efficiencies for different vane angle (40°, 60° and 70°) at 27.9 rev/s√K

**Uncertainty.** As for the steady state, the uncertainty in the efficiency is dependent on the velocity ratio. The uncertainty is higher at higher velocity ratio since it corresponds to the condition where the power absorption of the dynamometer is minimal. In this region the uncertainty is mainly due to the high fractional error of the mass flow rate and torque reading. The velocity ratio varies approximately between 0.4-2.6 and 0.56-1.2 at 27.9 rev/s√K and 43.0 rev/s√K. The corresponding Root Sum Square (RSS) for the efficiency varies between ±0.05 - ±1.98 and ±0.03 - ±0.34.
6.7 Summary

This chapter discussed the performance analysis of a variable geometry twin-entry mixed-flow turbine. The turbine was tested for a range of flow frequencies (40Hz, 60Hz, and 80Hz) and vane angles (40°, 60° and 70°) for two different speeds, 27.9 rev/s√K and 43.0 rev/s√K. For each of the operating condition, the turbine was tested under in-phase and out-of-phase flow. The presented results are compared with those obtained for an equivalent geometry single-entry turbine.

Under the in-phase flow condition and for the optimum vane angle (60°) the turbine is observed to swallow more mass flow than the equivalent quasi-steady. The encapsulation of the quasi-steady curve is not seen and the departure of the overall flow capacity from the quasi-steady curve becomes more significant at higher speeds. On the contrary, the flow capacity of the turbine entries (calculated independently) compare better with the quasi-steady assumption. For the in-phase flow the outer limb was found to be more pressurized than the inner limb even though the opposite could be found in the out-of-phase flow condition. As for the in-phase flow, the hysteresis loop obtained in out-of-phase flow condition for the two limbs (separately) shows good encapsulation with the equivalent quasi-steady curve.

The comparison between the energy weighted cycle-averaged and quasi-steady efficiency (using the full admission steady state map) shows that for both in-phase and out-of-phase flow there is a large penalty in efficiency which can go up to ≈32%. Additionally the quasi-steady efficiency in out-of-phase flow was calculated by using the partial admission. The use of the partial admission efficiency revealed to be beneficial to quasi-steady assumption with an improved agreement between the cycle-averaged and the quasi-steady efficiency. The comparison with the single-entry turbine was conducted at 27.9 rev/s√K, for three different vane angles (40°, 60° and 70°) and two frequencies (40Hz and 60Hz). For 60° vane angle the hysteresis loops of the two configurations showed to be fairly in good agreement. At 40° and 70° instead the variation between flow capacities is significant, with the single-entry turbine showing higher flow capacity than the twin-entry. Comparing the cycle-averaged efficiencies for the two configurations shows that single-entry turbine exhibits higher efficiency than the twin-entry. Nevertheless a shift towards lower velocity ratios could be observed for the single-entry in respect the twin-entry turbine, which suggests that the single-entry works in a region where the energy of the pressure pulse is higher than that available to the twin-entry.
CHAPTER 7

CONCLUSIONS

7.1 Thesis summary

The current thesis presents the outcomes of an aerodynamic and thermal investigation conducted on turbochargers. The work required an experimental analysis through which test data was generated in order to validate the computational models developed. The core of the thesis can be divided into three main sections: aerodynamic steady flow analysis, non-adiabatic performance assessment and aerodynamic unsteady analysis.

The steady flow analysis involved an experimental and computational investigation. A variable geometry twin-entry mixed-flow turbine was tested for a range of speeds and vane angle under full, partial and unequal admission. The results were then compared to an equivalent geometry (base line) single-entry turbine, nozzleless and nozzled. The turbine performance could then be assessed on equivalent geometry and flow capacity basis. The test results were also used to evaluate the correlation of the full admission condition with the partial and unequal admission conditions. The interaction between limbs was assessed and, based on experimental results, a map-based method to predict the flow capacity was proposed. As a part of the steady flow analysis, a meanline model was also developed for a nozzleless and nozzled single-entry turbine. The test results obtained from previous investigations were used for model validation.

The heat transfer experiments and analysis was conducted on a commercial turbocharger. Firstly the turbocharger was tested for a range of different engine speeds and loads. An extensive number of thermocouples were used to monitor the surface temperatures of the bodies constituting the turbocharger. The compressor non-adiabatic performance parameters were evaluated and used to validate a bespoke 1-D model for the turbocharger under study. Based on the model prediction, a multiple regression analysis was performed in order to evaluate the non-adiabatic performance of a compressor using an adiabatic compressor map. A set of three independent parameters was identified and the impact of each of these parameters on performance was then evaluated.

The third part of this research looked at the assessment of the turbine performance of a variable geometry twin-entry mixed-flow turbine under pulsating flow. As for the steady analysis, the turbine was tested for a range of vane angle and speeds. The in-phase and out-of-phase flow conditions were investigated and a performance comparison with a variable geometry single-entry turbine was done on equivalent geometry basis.
7.2 Steady flow analysis

Experimental analysis. The investigation focussed on the assessment of turbine performance for three different configurations (nozzleless single-entry and variable geometry single and twin-entry). Based on an existing nozzleless commercial turbine, a variable geometry single-entry turbine previously designed at Imperial College was modified into a twin-entry turbine. The variable geometry single-entry turbine volute comes as two halves; its design aimed to maintain the same $A/r$, same absolute flow angle and same wheel of the nozzleless turbine. The twin-entry configuration was obtained by inserting a meridional divider accounting for just 6% of the overall volume of the volute; the cross section area of the two entries was maintained the same along the whole turbine casing thus succeeding in having the same $A/r$.

The twin-entry turbine was tested under full admission conditions for two non-dimensional speeds, 27.9 rev/s·√K and 43.0 rev/s·√K, and three different vane angle settings, 40°, 60° and 70°. Tests under partial and unequal admission conditions were also performed for the same two speeds and 60° vane angle (corresponding to the optimum vane angle). The test results under full admission conditions conducted for 60° vane angle showed that the penalty in efficiency due to the addition of the divider is not significant. At 43.0 rev/s·√K a peak efficiency of 80% was measured for the nozzled single-entry turbine whereas the nozzleless and twin-entry turbine exhibited a slightly lower efficiency of about 77% and 79% respectively. At 27.9 rev/s·√K no difference in efficiency was measured between the three configurations which presented the same peak efficiency equal to 76%.

The effect of vane angle instead revealed different features depending on the vane setting. At 70° vane angle no difference was measured between the efficiencies of the single-entry and twin-entry turbines: flow mixing and incidence loss did not seem to play an important part on turbine performance. A peak efficiency of 77% and 76% was measured for the single and twin-entry turbine respectively. At 40° vane angle instead the addition of the divider revealed to go to the detriment of the efficiency; a significant drop of 5% in efficiency was measured in respect the nozzled single-entry. Using the flow capacity of the nozzleless turbine as a reference value, the performance comparison for the three turbine configurations was performed on an equivalent flow capacity basis (50° and 40° vane angles were set for the single and twin-entry turbine respectively). A significant drop in efficiency from 77% to 63% was measured for the single-entry turbines when going from the nozzleless to the nozzled configuration whereas for the twin-entry turbine an additional drop of 10 percentage points was measured.

An analysis of the twin-entry turbine performance under partial and unequal admission was also carried out. Based on the full admission maps, an approach was developed to determine the flow capacity under partial admission. Instead of considering the turbine as constituted by half volute, the pressure owing to the closed limb was included in the calculation of the flow capacity with an improvement of about 22% in the flow capacity prediction in respect to the case when only half volute
is considered. For the unequal admission case a map-based method to predict the flow capacity of each entry of the turbine was developed. A set of tests was conducted for two different speeds (27.9 rev/s√K and 43.0 rev/s√K) and pressure ratios spanning from 1.3 to 1.9. The test results showed that the flow capacity under unequal admission superimpose fairly well to that under full admission independently from the turbine operating conditions. In addition to this it was also found that the ratio between the flow capacity in the non-constant pressure limb and that under unequal admission follows a unique trend which is independent from speed and pressure ratio. The mass flow going through each limb of the turbine under partial admission condition can be calculated by mean of the correlation found for the mass flow within the limbs. The deviation between the measured and calculated mass flows was found to be no greater than 4.7%.

**Computational analysis.** A meanline model for a single-entry mixed-flow turbine was developed for two turbine configurations, nozzleless and nozzled; the former was validated against experimental data obtained for five different rotational speeds (27.9 rev/s√K to 53.8 rev/s√K) while for the latter three different vane angles (40°, 60° and 70°) and one single speed line (43.0 rev/s√K) was considered; the main feature of the experimental data lies in the unconventional range of velocity ratios available (=0.3 to 1.1). The model was calibrated for the peak efficiency point only; blockage and swirl were included in the volute while passage, incidence, clearance and disc friction loss were considered for the solution of the rotor. The model prediction for the nozzleless turbine was found to be in good agreement with the experimental results. For 53.8 rev/s√K and 47.5 rev/s√K the RMSD for the efficiency is not greater than 2% while it goes up to 7.9% at lower speeds. For the nozzled configuration efficiency the RMSD increases with closed nozzle vane setting going from 1.31% at 40° vane angle to 3.3% at 70°.

A breakdown loss analysis based on the model prediction, showed that the incidence loss accounts for the largest portion of the energy dissipated except for the peak efficiency point where incidence effect is negligible; on the contrary, the passage loss is higher at the peak efficiency given the large amount of kinetic energy available. The clearance and disc friction instead account for no more than 2%-3% of the overall energy dissipated. Based on this, an incidence factor was included to the incidence loss equation. An analysis for different values of this coefficient showed that at high velocity ratios the turbine efficiency is very sensitive to any small variation of such a coefficient.

### 7.3 Thermal analysis

**Experimental analysis.** The global objective of the experimental work was to improve the understanding of the heat transfer taking place in a turbocharger when installed on a real engine. In order to do this, beyond the standard set of measurements needed to define the operating point of the turbocharger, a set of seventeen thermocouples was installed. The inner and external wall temperature
of the turbine and compressor casing, and the temperature of the bearing housing and of the exhaust manifold were measured. The tests were carried out for engine speeds varying from 1000RPM to 3000RPM and, for each of these speeds a load from 8Nm to 250Nm was applied. The test results showed that the proximity of the engine has a large impact on the surface temperature of both the turbine and the compressor casing whose temperature varies linearly with that of the exhaust gases. A surface temperature difference up to ≈ 66 K and ≈ 68 K was measured between the Engine and the External side for the turbine and compressor casing respectively. A temperature gradient was also measured between the inner and the outer wall: on the turbine side this moves outward while the opposite occurs for the compressor; a maximum temperature difference of 27 K and 57 K was measured between the inner and outer wall for the compressor and the turbine casing respectively. The test results also showed that the surface temperature of the exhaust manifold varies linearly with that of the exhaust gases. Due to the presence of the turbine, the temperature of the pipe on the turbine side remains well above that of the compressor side with a temperature difference up to 130 K at high loads and speeds. The surface temperature of the bearing housing was found to vary consistently with that of the cooling oil. The oil temperature remains well below that of the bearing housing with a temperature difference of about ≈30 K at high engine speed and loads.

Based on the test results, an approach to calculate the exit temperature to the compressor was proposed. The test results showed that a linear correlation exists between the surface temperature of the bearing housing and that of the exhaust gases. On the assumption that most of the heat transferred to the air after compression occurs through the compressor back-plate and that its temperature is the same as that of the bearing housing, the compressor exit temperature was calculated. The results of this approach were in very good agreement with most of the test points, with an absolute deviation no greater than 3%.

A comparison between the non-adiabatic and adiabatic compressor efficiencies was also carried out. The efficiencies were plotted in a 3-D diagram as a function of engine load and speed. This showed that the deterioration of the compressor efficiency in non-adiabatic conditions is severe over the whole range of test conditions; the difference between the compressor adiabatic and non-adiabatic efficiency goes from a minimum of 17% to a maximum of 30% as the rotational speed and air flow rate decrease. This outcome is significant since it demonstrates that the effects of the heat transfer due to the engine account for a large part of the overall performance deterioration occurring in the turbocharger and therefore should be carefully taken into account when generating performance maps.

Computational analysis. The data generated from the tests were then used to validate a simplified 1-D heat transfer model. The turbocharger geometry was simplified to an assembly of three cylindrical bodies representing the compressor, the bearing housing and the turbine casing. For each
of these bodies the heat fluxes were calculated considering the three main heat transfer mechanisms: radiation, convection, conduction.

The heat conduction per unit area through the turbine casing was calculated. The comparison with the experimental data showed that the model manages to capture the general trend of heat conducted, although a significant scatter was obtained for some operating points. The discrepancy between calculated and measured values can go up to $\pm \approx 18\%$ even though the overall averaged deviation over the entire range of exhaust gas temperatures, remains low. The large scatter can be attributed to the simplified turbine geometry and to assumptions made on thermal properties.

The compressor exit temperatures and efficiencies were also calculated. The predicted exit temperatures were found to vary consistently with the experimental ones with a minimal scatter no larger than 5K over the whole range of experimental conditions. On the efficiency side the model prediction seems to be less accurate than that exhibited by the temperature although the deviation from the experimental results still remains within an acceptable range; an averaged deviation of 3% was calculated. The scatter can be attributed to the inability of the model to account for the aerodynamic effects occurring within the compressor. In fact as the turbocharger speed increases, the effects of heat transfer on the compressor efficiency become less significant.

The predictive capability of the model in generating non-adiabatic efficiency curves was also investigated. The compressor adiabatic efficiencies were extrapolated from the compressor cold maps for two different rotational speeds (163.3 rev/s·$\sqrt{K}$, 146.8 rev/s·$\sqrt{K}$, 107.6 rev/s·$\sqrt{K}$ and 88.0 rev/s·$\sqrt{K}$), and introduced into the model as boundary conditions. By changing the temperature of the exhaust gases from 950K to 650K at steps of 100K, the corresponding non-adiabatic efficiencies were then calculated by the model. The model managed to generate efficiency curves varying consistently with the test results, succeeding in taking into account the role played by the exhaust gasses and rotational speed on the overall deterioration of the compressor performance.

Finally a multiple variable regression analysis was used to look at factors associated with efficiency. The analysis was based on the data generated by a validated 1-D turbocharger model including the effects of heat transfer. The regression analysis revealed to be a valid way for determining the compressor efficiency in non-adiabatic conditions. The compressor efficiency could be fitted with good degree of confidence (adjusted $R^2$ value $\approx 0.9$) by means of three independent parameters ($Mach$ number, $Pressure$ ratio and $Temperature$ parameter defined as the ratio between the exit temperature to the turbine and the compressor). The Mach number was found to account for the largest portion of the compressor non-adiabatic efficiency independently from speed and temperature. Its impact on the efficiency was found to vary between $\approx 70\%$ and $80\%$ for any operating point. The contribution of pressure ratio was not larger than $10\%$ while the contribution of the temperature parameter was found to vary substantially with speed. At low speeds it could go up to $\approx 30\%$ while at high speeds its impact on overall efficiency was not significant. The impeller geometry showed to affect the contribution of the Mach number and the pressure ratio on the overall
compressor performance. The length of the bearing housing and the diameter of the compressor casing were identified as the two critical parameters. The regression analysis showed that the geometry parameter accounts for ≈2% of the compressor efficiency with a trend increasing with geometry.

7.4 Pulsating flow analysis

The response of a twin-entry turbine under pulsating flow conditions was investigated. Tests were conducted for two speeds, 27.9 rev/s·√K and 43.0 rev/s·√K, three frequencies (40Hz, 60Hz and 80Hz) and three vane angles (40°, 60° and 70°).

For the optimum vane angle setting (60°) and in-phase flow condition, the overall flow capacity is larger than the equivalent quasi-steady. The encapsulation of the quasi-steady curve is only partially achieved and it occurs mainly in the low pressure region of the maps for low speeds and frequencies. A more appreciable level of encapsulation with the quasi-steady curve is achieved for the flow capacity owning to each limb; a similar shape for the hysteresis loop could be found for the two limbs even though the outer limb was found to operate at higher pressure ratios than the inner limb. The opposite occurs for the in-out of phase flow condition where the inner limb is more pressurized than the outer limb. The pulsating nature of the flow at staggered intervals, leads to a higher rate of filling and empty which could be observed by the large amplitude of the hysteresis loop. For both the in-phase and out-of-phase flow conditions, the opening and closing of the vane angle (40° and 70° respectively) shows to go to the detriment of the flow capacity with the hysteresis loop which remains consistently lower than the equivalent quasi-steady trace.

Under in-phase flow condition the quasi-steady assumption seems to be only partially true. At 27.9 rev/s·√K and low frequency the cycle averaged efficiency is approximately 25% lower than the quasi-steady whereas at high frequency the cycle averaged and the quasi-steady efficiency are almost coincident (0.79% difference); at 43.0 rev/s·√K the quasi-steady assumption is satisfied at low and high frequencies (40Hz and 80Hz) whereas a transition region from quasi-steady to fully unsteady was observed at 60Hz.

In out-of-phase flow condition instead, the quasi-steady assumption is not satisfied; a difference going from 18% to more than 30% was measured for both 27.9 rev/s·√K and 43.0 rev/s·√K. The full admission quasi-steady efficiency is not fully representative of the out-of-phase flow condition in the turbine which, at each instant in time, is more likely to act as in partial admission conditions. At 43.0 rev/s·√K the ratio between the cycle averaged and the quasi-steady efficiency deviates only by few percentage points (1.03, 0.85 and 0.92 at 40Hz, 60Hz and 80Hz respectively) while at 27.9 rev/s·√K an improvement of almost 10 percentage points could be measured. The ratio between the cycle averaged and the quasi-steady assumption passed from 0.67, 0.70 and 0.82 for the full admission to 0.81, 0.77 and 0.98 at 40Hz, 60Hz and 80Hz respectively. The effects of vane angle
on the cycle averaged efficiency provided different response depending on the pulse flow. In-phase flow showed that at lower frequencies the close vane position (60° and 70° vane angle) is detrimental to the efficiency in respect the fully open position (40° vane angle). The same does not occur in out-of-phase flow where higher efficiency was found for the optimum vane angle (60° vane angle). This agrees with the findings found in the steady state testing which show a higher efficiency for the optimum vane angle.

Finally a performance comparison was conducted between the single-entry and the twin-entry mixed-flow turbine. The test results showed that for the optimum vane angle (60°) the presence of the divider does not influence the flow capacity of the twin-entry turbine. The hysteresis loop of the single-entry turbine does not deviate substantially in shape and magnitude from that of the twin-entry turbine. The same does not occur at 40° and 70° vane angle for which the hysteresis loops showed a large departure between single and twin-entry. The cycle averaged efficiency for the single-entry turbine was found to be consistently at lower velocity ratio than the twin-entry. This suggests that the single-entry turbine can work with pressure pulses with high energy content. However this does not reflect in a substantial benefit in performance. Apart for the 60° vane angle where the a benefit of almost 10% points could be measured, at 40° and 70° vane angle a similar efficiency could be measured for the two turbine configurations.

7.5 Future work

The analysis of the aerodynamic performance mainly focussed on a variable geometry twin-entry turbine which was tested under steady and pulsating flow conditions. The additional complexity brought in by the presence of the divider and of the movable nozzle vanes, showed that the performance parameters do not follow a well defined pattern. In order to capture the flow mechanisms going through the turbine, future work will have inevitably to focus on computational analysis by mean of 1-D and CFD analysis.

The thermal investigation focussed on the evaluation of the heat transfer going through a turbocharger with an experimental and computational analysis. Based on the sensitivity analysis the accuracy of the developed model can be increased by reducing the level of simplification on the turbine side. This includes the development of more appropriate heat transfer correlations for the turbine volute and the turbine wheel. Besides these basic improvements, in order to estimate the occurring heat fluxes more accurately, further experimental work is needed to gain a better insight into the temperature distributions of the surfaces of the turbine and the bearing housing. Future work will investigate the heat transfer in transient conditions. This, together with the test data generated within research, will form a benchmark for the implementation of a 1-D model in transient conditions and for a CFD analysis.
REFERENCES


- DANTEC DYNAMICS, 2002. MiniCTA Anemometer package: how to get started, a basic guide.


APPENDIX

A1

- Uncertainty performance parameters

\[
\pm(\eta_{ts})_{RSS} = \sqrt{\left(\pm m \cdot \frac{\partial \eta_{ts}}{\partial m}\right)^2 + \left(\pm T_{1} \cdot \frac{\partial \eta_{ts}}{\partial T_{1}}\right)^2 + \left(\pm P_{1} \cdot \frac{\partial \eta_{ts}}{\partial P_{1}}\right)^2 + \left(\pm \eta_{ts} \cdot \frac{\partial \eta_{ts}}{\partial \eta_{ts}}\right)^2}
\]

\[
\pm(VR)_{RSS} = \sqrt{\left(\pm m \cdot \frac{\partial VR}{\partial m}\right)^2 + \left(\pm T_{1} \cdot \frac{\partial VR}{\partial T_{1}}\right)^2 + \left(\pm P_{1} \cdot \frac{\partial VR}{\partial P_{1}}\right)^2 + \left(\pm VR \cdot \frac{\partial VR}{\partial VR}\right)^2}
\]

\[
\pm(MFP)_{RSS} = \sqrt{\left(\pm m \cdot \frac{\partial MFP}{\partial m}\right)^2 + \left(\pm T_{1} \cdot \frac{\partial MFP}{\partial T_{1}}\right)^2 + \left(\pm P_{1} \cdot \frac{\partial MFP}{\partial P_{1}}\right)^2}
\]

\[
\pm(PR)_{RSS} = \sqrt{\left(\pm m \cdot \frac{\partial PR}{\partial m}\right)^2 + \left(\pm T_{1} \cdot \frac{\partial PR}{\partial T_{1}}\right)^2 + \left(\pm P_{1} \cdot \frac{\partial PR}{\partial P_{1}}\right)^2 + \left(\pm PR \cdot \frac{\partial PR}{\partial PR}\right)^2}
\]

\[
\pm(\text{Neq})_{RSS} = \sqrt{\left(\pm m \cdot \frac{\partial \text{Neq}}{\partial m}\right)^2 + \left(\pm T_{1} \cdot \frac{\partial \text{Neq}}{\partial T_{1}}\right)^2 + \left(\pm P_{1} \cdot \frac{\partial \text{Neq}}{\partial P_{1}}\right)^2 + \left(\pm \text{Neq} \cdot \frac{\partial \text{Neq}}{\partial \text{Neq}}\right)^2}
\]

- Sensitivity analysis of the fractional importance of each propagated error (Rajoo, 2006)

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<th>( \pm m )</th>
<th>( \pm T_{1} )</th>
<th>( \pm P_{1} )</th>
<th>( \pm \alpha )</th>
<th>( \pm N )</th>
<th>( \pm P_{5} )</th>
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<td>15%</td>
<td>30%</td>
<td>0%</td>
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<td>0%</td>
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<td>6%</td>
<td>21%</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( \partial PR )</td>
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<td>3%</td>
<td>44%</td>
<td>-</td>
<td>-</td>
<td>13%</td>
</tr>
<tr>
<td>( \partial \text{Neq} )</td>
<td>21%</td>
<td>72%</td>
<td>6%</td>
<td>-</td>
<td>0%</td>
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**TUFNOL Grade 10G/40**

**SPECIFICATIONS**

**BRITISH STANDARDS**

Sheet  BS3953 Type EP-3
Rod    BS6128 Part 2 Type EP GC 21

**NEMA**

Sheet    Nema LI-1-1983 Type G10

*Certification to this standard is subject to special enquiry. Standard quality testing is to British Standards.*

**APPENDIX**

**APPROXIMATE WEIGHTS**

**Sheets**

Sheet size 1220 x 1220 approx.
Approx. weight in kg = 2.84 x thickness in mm

Sheet size 1600 x 1220 approx.
Approx. weight in kg = 3.86 x thickness in mm

Due to slight variations in density and nominal dimensions, weight cannot be calculated precisely.

**Weight Formulas**

**Cut pieces:**

\[ \text{Weight in kg} = \frac{1.96 \times \text{Length} \times \text{Width} \times \text{Thickness}}{1,000,000} \]

**Rod**

\[ \text{Weight in kg} = \frac{1.56 \times \text{Dia} \times \text{Length} \times \text{Wall in mm}}{1,000,000} \]

**PHYSICAL PROPERTIES OF GRADE 10G/40**

<table>
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<tr>
<th>PROPERTY</th>
<th>TYPICAL RESULT</th>
<th>UNITS</th>
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<tbody>
<tr>
<td>Cross breaking strength</td>
<td>490</td>
<td>MPa</td>
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<tr>
<td>Impact strength, notched, Charpy</td>
<td>60</td>
<td>kJ/m²</td>
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<tr>
<td>Compressive strength, flatwise</td>
<td>415</td>
<td>MPa</td>
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<td>Compressive strength, edgewise</td>
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<td>MPa</td>
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<td>Water Absorption 1.6mm thk.</td>
<td>5</td>
<td>mg</td>
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<tr>
<td>3mm thk.</td>
<td>7</td>
<td>mg</td>
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<td>6mm thk.</td>
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<td>mg</td>
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<tr>
<td>12mm thk.</td>
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<td>mg</td>
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<td>MV/m</td>
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<tr>
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<td>MV/m</td>
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<td>6mm thk.</td>
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<td>12mm thk.</td>
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<td>kV</td>
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<tr>
<td>Permittivity at 1 MHz</td>
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<td>-</td>
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<tr>
<td>Comparative tracking index</td>
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<tr>
<td>Relative density</td>
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<td>Maximum working temperature **</td>
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<tr>
<td>Continuous</td>
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<td>°C</td>
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<tr>
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<td>°C</td>
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**Sheets**

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<thead>
<tr>
<th>PROPERTY</th>
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<tr>
<td>Flexural strength</td>
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<td>Water absorption after immersion in water</td>
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<td>Insulation resistance</td>
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<tr>
<td>Axial electric strength at 90°C</td>
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<td>Relative density</td>
<td>1.90</td>
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Test methods as BS 6128

Test methods as BS 3953, where applicable.

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**APPENDIX**

The information in this leaflet is believed to be correct, but completeness and accuracy are not guaranteed. The user shall be fully responsible for determining the suitability of products for their intended use. TUFNOL is a Registered Trade Mark.
A3

- Mass flux calculation, British Standard BS:104:

\[(\rho U)_i = v_1 \left(\frac{m_w}{1 + m_w} j_1 + \left(\frac{1}{12 \cdot m_w} j_2^2\right) + \left(\frac{7}{12} \cdot j_2\right) - \left(\frac{1}{12} \cdot j_3\right)\right) + \right.

\left.v_2 \left[\left(\frac{1}{2} \cdot j_2\right) + \left(\frac{7}{12} \cdot j_3\right) - \left(\frac{1}{12} \cdot j_4\right)\right] + v_3 \left[\left(\frac{7}{12} \cdot (j_3 + j_4)\right) - \left(\frac{1}{12} \cdot (j_2 + j_5)\right)\right] + \right.

\left.v_4 \left[\left(\frac{7}{12} \cdot (j_4 + j_5)\right) - \left(\frac{1}{12} \cdot (j_3 + j_6)\right)\right] + v_5 \left[\left(\frac{1}{2} \cdot j_6\right) + \left(\frac{7}{12} \cdot j_5\right) - \left(\frac{1}{12} \cdot j_4\right)\right] + \right.

\left.v_6 \left[\left(\frac{m_w}{1 + m_w} j_7^2\right) + \left(\frac{1}{12 \cdot m_w} j_7^2\right) + \left(\frac{7}{12} \cdot j_6\right) - \left(\frac{1}{12} \cdot j_5\right)\right] \right)

where \( j_1 \ldots j_6 = \frac{l_i - l_{i-1}}{L} \); \( j_7 = \frac{L - l_6}{L} \)

\((\rho U)_i\) = integrated mass flux (Kg/m²s)

\(v_i\) = single point mass flux (Kg/m²s)

\(m_w\) = wall roughness factor

\(l_i\) = distance of point to the reference wall (m)

\(L\) = distance between the walls (m)
**A4**

- **Specific heat of the oil Shell Rimula X SAE 30 W:**

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<thead>
<tr>
<th>$T$ [K]</th>
<th>273</th>
<th>300</th>
<th>330</th>
<th>350</th>
<th>380</th>
<th>410</th>
<th>420</th>
<th>430</th>
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<tbody>
<tr>
<td>Specific heat (J g$^{-1}$/K$^{-1}$)</td>
<td>1.79</td>
<td>1.90</td>
<td>2.03</td>
<td>2.118</td>
<td>2.25</td>
<td>2.38</td>
<td>2.427</td>
<td>2.47</td>
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- **Kinematic viscosity of the Shell Rimula X SAE 30 W:**

<table>
<thead>
<tr>
<th>$T$ [K]</th>
<th>293</th>
<th>303</th>
<th>313</th>
<th>323</th>
<th>333</th>
<th>343</th>
<th>353</th>
<th>363</th>
<th>373</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kinematic viscosity [mm$^2$/s]</td>
<td>311.4</td>
<td>164.2</td>
<td>94.2</td>
<td>58.1</td>
<td>38.1</td>
<td>26.4</td>
<td>18.9</td>
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- **Oil density of the Shell Rimula X SAE 30 W:**

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<th>$T$ [K]</th>
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<th>313</th>
<th>323</th>
<th>333</th>
<th>343</th>
<th>353</th>
<th>363</th>
<th>373</th>
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</thead>
<tbody>
<tr>
<td>Density [g/cm$^3$]</td>
<td>0.885</td>
<td>0.879</td>
<td>0.873</td>
<td>0.866</td>
<td>0.860</td>
<td>0.854</td>
<td>0.848</td>
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<td>0.837</td>
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- **Oil flow rate of the 316 Stainless Steel Body** *(transfer function can be derived):*

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<th>Frequency</th>
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<th>47</th>
<th>80</th>
<th>120</th>
<th>152</th>
<th>191</th>
<th>225</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flow rate [l/min]</td>
<td>0.2</td>
<td>1.3</td>
<td>3.2</td>
<td>4.3</td>
<td>5.3</td>
<td>6.3</td>
<td></td>
</tr>
</tbody>
</table>
A5

• Mach number $M_{2,adi}$:

- Backward swept blade - ($\mu \neq 0$):

$$M_{2,adi} = \frac{C_{2,adi}}{\sqrt{\gamma R T_2}} = \frac{\sqrt{C_{2m,adi}^2 + C_{2\theta}^2}}{\sqrt{\gamma R T_2}} = U_2 \frac{\sqrt{C_{2m,adi}^2 / U_2^2 + C_{2\theta}^2 / U_2^2}}{\sqrt{\gamma R T_2}}$$

$$= \frac{U_2}{\sqrt{\gamma R T_0}} \left( \frac{T_0}{T_2} \right) \sqrt{\Theta^2 + \lambda^2}$$

(A.6)

The ratio $\frac{T_0}{T_2} = \sqrt{\frac{T_0}{T_0} \frac{T_2}{T_2}}$ where:

$$\frac{T_0}{T_0} = \frac{1}{1 + (\gamma - 1)\lambda M_{2u}^2}$$

(A.7)

$$\frac{T_2}{T_2} = 1 + \frac{\gamma - 1}{2} M_{2,adi}^2$$

(A.8)

By including Eq. (2) and (3) into either Eq. (1), the expressions for the Mach number $M_{2,adi}$ is:

$$M_{2,adi} = M_{2u} \sqrt{\frac{T_0}{T_2}} \left[ \frac{T_0}{T_0} \frac{T_2}{T_2} \right] (\Theta^2 + \lambda^2) = M_{2u} \left[ \Theta^2 + \lambda^2 \right] \left[ \frac{1}{1 + (\gamma - 1)\lambda M_{2u}^2} \right] \left( 1 + \frac{\gamma - 1}{2} M_{2,adi}^2 \right)$$

$$= M_{2u} \left[ \Theta^2 + \lambda^2 \right] \left( \frac{2 + (\gamma - 1)M_{2,adi}^2}{2(\gamma - 1)\lambda M_{2u}^2} \right)$$

(A.9)

By raising to the second power both members we have that:

$$2M_{2,adi}^2 + 2M_{2u}^2(\gamma - 1)\lambda M_{2u}^2 = (M_{2u}^2\Theta^2 + M_{2u}^2\lambda^2)(2 + (\gamma - 1)M_{2,adi}^2)$$

$$2M_{2u}^2\Theta^2 + 2M_{2u}^2\lambda^2 + M_{2u}^2 M_{2,adi}^2 (\Theta^2 (\gamma - 1) + M_{2u}^2 M_{2,adi}^2 \lambda^2 (\gamma - 1))$$

$$M_{2,adi}^2 \left[ 2 + 2M_{2u}^2 (\gamma - 1) - M_{2u}^2 \Theta^2 (\gamma - 1) - M_{2u}^2 \lambda^2 (\gamma - 1) \right] = 2M_{2u}^2 \Theta^2 + 2M_{2u}^2 \lambda^2$$

$$M_{2,adi}^2 = \frac{2M_{2u}^2\Theta^2 + 2M_{2u}^2\lambda^2}{2 + 2M_{2u}^2 (\gamma - 1) - M_{2u}^2 \Theta^2 (\gamma - 1) - M_{2u}^2 \lambda^2 (\gamma - 1)}$$

$$= \frac{2(M_{2u}^2 \Theta^2 + \lambda^2)}{2 + M_{2u}^2 (\gamma - 1)(2\lambda - \Theta^2 - \lambda^2)}$$

$$M_{2,adi} = \sqrt{\frac{2(M_{2u}^2 \Theta^2 + \lambda^2)}{2 + M_{2u}^2 (\gamma - 1)(2\lambda - \Theta^2 - \lambda^2)}}$$

(A.10)

- Radial blade - perfectly guided flow ($\mu = 0 \rightarrow C_{2\theta} = U_2$):
\[ M_{2,\text{adi}} = \frac{C_{2,\text{adi}}}{\sqrt{\gamma RT_2}} = \sqrt{\frac{C_{2m,\text{adi}}^2 + U_2^2}{\gamma RT_2}} = \frac{U_2}{\sqrt{\gamma RT_2}} \sqrt{\frac{C_{2m,\text{adi}}^2}{U_2^2} + 1} = \frac{U_2}{\sqrt{\gamma RT_2}} \sqrt{\phi^2 + 1} = M_{2\mu} \sqrt{\phi^2 + 1} \]

\[ \frac{T_{01}}{T_2} = \frac{T_{01}}{T_2} \]  

(A.11)
APPENDIX

A6

Inner and outer wall temperature of the compressor casing, inlet and outlet temperature of the oil through the bearing housing

<table>
<thead>
<tr>
<th>RPM</th>
<th>8 Nm</th>
<th>25 Nm</th>
<th>50 Nm</th>
<th>1050 RPM</th>
<th>1500 RPM</th>
<th>2000 RPM</th>
<th>2500 RPM</th>
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<td>Inner</td>
<td>Outer</td>
<td>Inner</td>
<td>Outer</td>
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<td>358.6</td>
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</table>
Compressor: Inner and outer temperature along the compressor casing on the Engine side of the turbine for different loads

Compressor: Inner and outer temperature along the turbine casing on the Top of the compressor for different loads

Compressor: Inner and outer temperature along the compressor casing on the External side for different loads

- **Inner wall**
- **Outer wall**
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Compressor: Inner and outer temperature along the compressor casing on the External side for different loads

- Inner wall
- Outer wall
Surface temperatures of the exhaust manifold, bearing housing and inner and outer wall temperature of the turbine casing

<table>
<thead>
<tr>
<th>RPM</th>
<th>Exhaust Compressor</th>
<th>Exhaust Housing</th>
<th>Exhaust Turbine</th>
<th>Engine Outer</th>
<th>Engine Inner</th>
<th>Top Outer</th>
<th>Top Inner</th>
<th>External Outer</th>
<th>External Inner</th>
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</table>
Turbine: Inner and outer temperature along the compressor casing on the Engine side of the turbine for different loads

Turbine: Inner and outer temperature along the turbine casing on the Top of the compressor for different loads

Turbine: Inner and outer temperature along the compressor casing on the External side for different loads

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- Inner wall
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Oil Temperature in the bearing housing at different engine speeds

- **1000 RPM**: Oil temperature increases with load, showing a steady rise.
- **1500 RPM**: Similar trend to 1000 RPM, with a slightly steeper increase.
- **2000 RPM**: More pronounced increase in temperature with load, especially at higher loads.
- **2500 RPM**: Temperature continues to rise with load, but the increase is not as steep as at lower RPMs.
- **3000 RPM**: Moderate increase in temperature with load, but the rise is less compared to lower RPMs.

Legend:
- **■** Oil temperature at the Exit to the bearing housing
- **♦** Oil temperature at the Inlet to the bearing housing
Surface temperature of the exhaust manifold and bearing housing at different speeds

- Surface temperature of the exhaust manifold - Turbine side
- Surface temperature of the exhaust manifold - Compressor side
- Surface temperature of the bearing housing
Instantaneous efficiency at 40Hz under in-phase and out-of-phase flow for 40° vane angle and 27.9 rev/s√K

Instantaneous efficiency at 60Hz under in-phase and out-of-phase flow for 40° vane angle and 27.9 rev/s√K

Instantaneous efficiency at 80Hz under in-phase and out-of-phase flow for 40° vane angle and 27.9 rev/s√K
Instantaneous efficiency at 40Hz under in phase and out-of-phase flow for 70° vane angle and 27.9 rev/s√K

Instantaneous efficiency at 60Hz under in-phase and out-of-phase flow for 70° vane angle and 27.9 rev/s√K

Instantaneous efficiency at 80Hz under in-phase and out-of-phase flow for 70° vane angle and 27.9 rev/s√K