CDS Pricing with Counterparty Risk

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Abstract

This thesis focuses on the impact of counterparty-risk in CDS (Credit Default Swap) pricing. The exponential growth of the Credit Derivatives Market in the last decade demands an upsurge in the fair valuation of various credit derivatives such as the Credit Default Swap (CDS), the Collateralized Debt Obligation (CDO). Financial institutions suffered great losses from Credit Derivatives in the sub-prime mortgage market during the credit crunch period. Counterparty risk in CDS contracts has been intensively studied with a focus on losses for protection buyers due to joint defaults of counterparty and reference entity. Using a contagion framework introduced by Jarrow and Yu (2001)\cite{JarrowYu2001}, we calculate the swap premium rate based on the change of measure technique, and further extend both the two-firm and three-firm model (with defaultable protection buyer) with continuous premium payment. The results show more explanatory power than the discrete case. We improve the continuous contagion model by relaxing the constant intensity rate assumption and found close results without loss of generality. Empirically this thesis studies the behaviour of the historical credit spread of 55 sample corporates/financial institutions, a Cox–Ingersoll–Ross model is applied to calibrate spread parameters. A proxy for counterparty spread is introduced as the difference between the spread over benchmark rate and spread over swap rate for 5 year maturity CDS. We then investigate counterparty risk during the crisis and study the shape of term structure for the counterparty spread, where Rebonato's framework is deployed to model the dynamics of the term structure using a regime-switching framework.
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The paper is written using \LaTeX based on \TeX, and the programming language is MATLAB and VBA in Excel. All mistakes are mine.
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Glossary of Acronyms

BBA  British Bankers’ Association
CDO  Collateralized Debt Obligation
CDS  Credit Default Swap
CGH  Collin-Dufresne, Goldstein, and Hugonnier (2002)[11]
CIR  Cox- Ingersoll-Ross Model
HJM  Heath, Jarrow, and Morton (1992)[37]
IIC  International Index Company
IRS  Interest Rate Swaps
ISDA  International Swaps and Derivatives Association
JY   Jarrow & Yu (2001)[48]
LIBOR London Inter-Bank Borrow Rate
LMM LIBOR Market Model
MBS  Mortgage-Backed Security
MTM  Mark to Market
OTC  Over-the-Counter
P&L  Profit & Loss
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<tr>
<td>RMSE</td>
<td>Root-Mean-Squared-Error</td>
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<td>SEC</td>
<td>Securities and Exchange Commission</td>
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<td>SFT</td>
<td>Security Financing Transactions</td>
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<td>SP</td>
<td>Spread Over Benchmark Curve</td>
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<td>VW</td>
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Chapter 1

Introduction

“It was the best of times, it was the worst of times, it was the age of wisdom, it was the age of foolishness, it was the epoch of belief, it was the epoch of incredulity, it was the season of Light, it was the season of Darkness, it was the spring of hope, it was the winter of despair, we had everything before us, we had nothing before us, we were all going direct to heaven, we were all going direct the other way - in short, the period was so far like the present period, that some of its noisiest authorities insisted on its being received, for good or for evil, in the superlative degree of comparison only.”

— “A Tale of Two Cities”, by Charles Dickens (1812 - 1870)

It has indeed been the best and worst of times for the financial market and world economy in the past decade, especially for the newly developed credit derivatives market.

The valuation of credit derivatives has for a long time been based on default-free counterparties, as this allows a risk-free valuation of the payments made under credit derivatives. Even though financial institutions own subsidiaries,
which could act as counterparties in OTC derivatives, and reach strong ratings of “AAA”, less than half of the market participants have a rating of “A” or above. Moreover, no credit derivatives are exchange-traded up to now. Following these arguments, the consideration of counterparty risk is essential for a correct and consistent valuation of credit derivatives.

Counterparty credit risk is the risk that the counterparty to a financial contract will default prior to the expiration of the contract and will not be able to make the full payments required by the contract. Contracts privately negotiated between counterparties, this includes over-the-counter (OTC) derivatives such as Credit Default Swaps and Interest Rate Swaps, and security financing transactions (SFT), are subject to counterparty risk. Exchange-traded derivatives are not affected by counterparty risk, because the exchange guarantees the cash flows promised by the derivative to the counterparties. Counterparty risk is theoretically present in derivatives of all asset classes; FX derivatives, IR swaps, OTC equity derivatives and credit derivatives. In this thesis we focus on credit derivatives, on Credit Default Swaps (CDSs) to be more specific.

Counterparty risk is similar to other forms of credit risk in that the cause of economic loss is obligor’s default. However, there are two features that set counterparty risk apart from more traditional forms of credit risk: the uncertainty of exposure and bilateral nature of credit risk. Canabarro and Duffie (2003)[8] provide an excellent introduction to this subject. Unless derivative contracts are collateralised or guaranteed, their ultimate value is dependent on the creditworthiness of the counterparties to them. In the meantime, before a contract is settled, the counterparties record profits and losses in their current earnings statements.

Here we focus on two main issues: modelling the default process and pricing counterparty risk, more precisely, the effect on premium rate changes due to the possibility of defaultable counterparties, this involves both the protection seller and the protection buyer.

The most important aspect of modelling counterparty risk is the treatment
of correlations between the credit risk of the underlying asset and the credit risk of the counterparty. If perfect correlation is assumed, one can easily see the importance of correlation for valuation purposes. For example, if a financial institution with an AAA-rating is selling CDSs as a protection seller and the reference entity is its own bond, then the swap with an assumed recovery rate of zero is worthless. This is due to the fact that in the event of default of the reference entity, which triggers the credit event under the CDS, the protection seller, or say, risk buyer, default as well. If, in contrast, the protection seller is solvent, i.e., no credit event occurs, the protection seller will not be drawn on.

1.1 Credit Derivatives

1.1.1 Credit Default Swaps

Credit Default Swap is the most popular type of credit derivatives, according to Credit Derivatives Survey by BBA (British Bankers’ Association), it accounts for 45% of the global credit derivatives market on average, and an even higher 73% according to Credit Derivatives Survey by Risk Magazine. Due to the crisis in 2007 and 2008, the notional amount outstanding of credit derivatives has decreased by 12 percent on Mid-Year of 2008 to $54.6 trillion from $62.2 trillion, but the annual growth for credit derivatives was 20 percent from $45.5 trillion at mid-year 2007\footnote{For the purposes of the Survey, credit derivatives comprise credit default swaps referencing single names, indices, baskets, and portfolios.}, this is from the market survey provided by ISDA (the International Swaps and Derivatives Association).

As mentioned in earlier chapters, a credit default swap is a contract that protects the holder of an underlying obligation from the losses caused by the credit event to the obligation’s issuer, referred to as the reference entity. Credit events that trigger a default swap payment includes: bankruptcy, failure to make a principle or interest payment, obligation acceleration, obligation default, repudiation/moratorium (for sovereign borrowers) and structuring; these events
are referred to as default. A credit default swap, only pays out once the reference entity defaults. The protection buyer either pays an up-front amount or makes periodic payments to the protection seller, typically a percentage of the notional amount, which is called the spread if the percentage gives the contract zero value at initiation stage.

1.1.2 Recovery Rate

Should default occurs, the default swap can be settled in either:

- a cash settlement - the buyer keeps the underlying asset, but is compensated by the seller for the loss incurred by the credit event;
- a physical settlement - the buyer delivers the reference entity to the seller, and receives the full notional amount in return.

Either way, the value of the buyer’s portfolio is restored to the initial notional amount, and the percentage the protection seller is obligated to compensation is called the Recovery Rate.

\[
Recovery\ Rate = 1 - Loss\ Rate
\]  

(1.1)

1.1.3 Counterparty Risk

CDS contracts have been around since the mid 90s, they have grown explosively since 2002. Credit default swaps are the most widely traded credit derivatives, with an estimated $62.2 trillion notional amount outstanding at the end of 2007, up from $2.1 trillion five years earlier, according to the International Swaps and Derivatives Association. The growth is shown in Figure 1.1.

A CDS is an agreement between two counterparties, typically for five years, in which the buyer makes periodic payments to the seller in return for a promised payoff if a third party defaults. The cost of such protection against default has risen sharply as a result of the global credit crunch, as well as the growing risk of corporate defaults in a weakening economy. Banks are the primary sellers of
Figure 1.1: Notional Amount of Outstanding Credit Derivatives

Notice that the notional amount outstanding is $45 trillion, while the economic exposures of derivatives are a fraction of the notional amount.

CDS, contributing up to 40% of all written CDS and representing a notional exposure of $18.2 trillion. As should be evident from the events in subprime crisis, even the most sophisticated systems are often unable to fully hedge risks of this size and degree of complexity. Hedge funds share a major proportion of the CDS market, by taking up to 32% of all CDS contracts, insuring an exposure of $14.5 trillion. Recent estimates indicate that the entire hedge fund market has approximately $2.5 trillion in net assets under management. Thus hedge funds are bearing risk in excess of their ability to pay off the contractual amount if anything goes wrong.\(^2\)

When a default occurs, the seller of the protection must take possession of the defaulted bond at par value or pay the buyer the difference between the par value and the recovery value of the bond. CDS contracts often change hands many times, however, there is no guarantee that the current holder, such as a

\(^2\)By theory, these positions are fully margined and collateralised on a daily basis, which implies that all daily MTM P&L is locked in and immune from risk from counterparty default, however, it also implies that all the investors, in the money and out of the money protection sellers and buyers, are exposed to the same gap risk due to counterparty defaults, which is the risk of spreads jumping significantly from levels at which posting of collateral took place and levels at which the contract is terminated. If a major counterparty defaults, credit spreads are believed to be re-priced significantly and immediately.
CHAPTER 1. INTRODUCTION

hedge fund, will have the assets to pay out in the event of a default.

Moody's Investors Service said in a report released in late May 2008 that counterparty risks in the CDS market pose greater potential threats to banks and dealers than other OTC derivatives, such as interest rate swaps. While banks can spread risk around the globe with CDS trades, it is the interconnected nature of the market that could pose a systematic risk to the financial system, according to Moody's.

"The most significant systemic risk posed by CDS is the effect the failure of a major counterparty, such as a large universal bank or securities firm, would have on the operational integrity and pricing in the markets for CDS and, potentially, the underlying securities, such as corporate bonds", the rating agency said in its report as quoted in Platt, G. (2008). Such counterparty risk was highlighted by the collapse of Bear Stearns in 2008. The firm was a major participant in the CDS market. In May 2005 price swings in the credit derivatives market following the downgrades of Ford Motor and General Motors debt caused hedge funds to lose between $1 billion and $2 billion and raised fears of a wider financial meltdown.

Moody's says the sheer size of the market does not pose an undue concern in and of itself. The real danger, according to the rating agency, is that if a large counterparty failed, this would cause a major re-pricing throughout the CDS market and depress the prices of the underlying bonds of the companies on which the credit protection is purchased. For actively traded issues, the contractual CDS notional amounts are substantially greater than the outstanding corporate debt.

Figure 1.2 below shows the maximum potential loss to both the buyer and seller of a CDS contract. The maximum potential loss to the seller of protection is the contract premium for the rest of the contract duration, whereas the buyer of protection could arguably lose the full notional amount of the contract (in case of a joint default of the counterparty and the reference entity with zero

---

3 Although we could add the caveat that netting agreements and collateral posting could significantly reduce estimated losses, data on netted exposures is very hard to obtain, therefore the figures above are un-netted.
Figure 1.2: Maximum potential loss to either party of a CDS contract referencing a corporate credit (based on 25 January 2008 data). As shown, counterparty risk is highly skewed towards the buyer of CDS protection.

Therefore, counterparty risk is more of a concern for the buyer of protection.

At a basic level and assuming no collateral has been exchanged, in the event of a failure of a counterparty the protection buyer may face one of the following scenarios:

- *The original contract is out-of-money*, in which case the survivor closes out the position with the default party by paying off its obligations and then enters into a new contract with a different counterparty. No profit or loss incurs in this case by the survivor due to the default of the counterparty.

- *The original contract is in-the-money*, in which case the survivor closes out the position with the default party but does not receive its dues. The survivor incurs a loss equal to the difference between the market value of the old CDS contract and the new one, this is also called replacement cost.
1.2 Credit Risk

There are currently two main streams in Credit Risk Modelling: Reduced-form model and Structural model.

1.2.1 Structural Model

The Structural Model is also called the Merton Model. Merton (1974)\textsuperscript{[62]} introduces the original model, and this leads to the extensive research on structural models. Merton models a firm’s asset value as a lognormal process and assumes that the firm would go bankrupt once the asset value, $A$, falls below the firm’s debt value, $D$. The equity value, $E$, is given as the difference between the asset value $A$, and debt value, $D$.

\begin{equation}
E = A - D
\end{equation}

Default is only allowed at one point in time, $T$. The equity value of the firm, $E$, was modelled as a call option on the underlying assets and is given as:

\begin{equation}
E = A\Phi(d_1) - De^{-rT}\Phi(d_2)
\end{equation}

where

\begin{align*}
d_1 &= \frac{\ln(A/D) + (\mu + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} \\
d_2 &= d_1 - \sigma\sqrt{T}
\end{align*}

where $A$ is the initial value of the firm, $D$ is the default barrier for the firm, which means the firm will default once the asset value $A$ falls below $D$, or say, a firm’s equity value $E$ is negative, at terminal date $T$. $\Phi$ represents the cumulative normal distribution function. $\mu$ is the drift of the asset return, and $\sigma$ is the volatility of the asset returns.
The structural model is particularly useful for practitioners in the credit portfolio and credit risk management fields. The intuitive interpretation of the model facilitates consistent discussion regarding a variety of credit risk exposures. It is also possible to apply the structural model to corporate transaction analysis. In general, the Merton model provides a more realistic concept of a firm’s financial structure to represent the relationship among asset value, equity and debt.

One of the principal strengths of the Merton approach is that it makes clear links between the price of a firm’s debt and observables, which provides specific testable predictions. However, the empirical evidence for the model and its various extensions is mixed, Sarig and Warga (1989)\cite{68} demonstrate that yield spreads vary with maturity in a manner that is roughly consistent with Merton’s model, whereas Helwege and Turner (1999)\cite{32} argue that this result is an artifact of the method used to construct the corporate term structure. Moreover, while direct tests of Merton-style models find that the model underpredicts the level of long-term corporate bond spreads, whereas Lyden and Saraniti (2000)\cite{60} and Eom, Helwege, and Huang (2004)\cite{28} find that some extensions of the Merton model overpredict spreads for poorly capitalised firms, but continue to underpredict spreads for large, well-capitalised firms. It is also well known that structural models severely underpredict yield spreads for bonds maturing in less than 1 year, besides, the parameters of Merton model are hard to estimate, because assets’ market value and volatility are difficult to observe.

1.2.2 Reduced-form model

The other major approach of credit risk modelling is called “reduced-form” models of default. Having less assumptions than structural models, this approach assumes a firm’s default time is unpredictable or inaccessible and is driven by a default intensity that is a function of latent state variables. Jarrow, Lando, and Turnbull (1995)\cite{17}, Duffee and Singleton (1999)\cite{29}, and Hull and White (2000)\cite{42} present detailed explanation of several well-known reduced-form mod-
eling approaches. More and more practitioners in the credit risk trading world have tendency to apply this model due to its mathematical tractability. Jarrow and Protter (2004)\cite{46} argue further that reduced-form models are more appropriate in an information theoretic context given that it’s very unlikely to have complete information about the default point and expected recovery rate.

Theoretically, most structural models assume complete information, however, in practice, the complete information assumption in structural models is only an approximate way of capturing various economics of a firm. On the other hand, a reduced-form model has other weaknesses including lack of clear economic rationale to define the nature of default process, however, it does not compromise the theoretical issue of complete information.

The empirical validation of reduced-form models is still lacking. The reason related to lack of theoretical guidance on characterising the default intensity process. Duffie (1999)\cite{22} found that the parameter estimation using a square-root approximation can be fairly unstable. Another reason is the bond data, on which the models are commonly calibrated, typically creates problems as information slowly leaks into the price, which results misleading results. The recent study in credit default swap data provides a new opportunity to understand the power of both structural and reduced-form model framework.

In reduced form models, the direct reference to the firm’s asset value process is abandoned. Instead, credit risk is determined by the occurrence of default and the recovered amount at default. Default is represented by a random stopping time with a stochastic or deterministic arrival intensity, also known as the hazard rate, while the recovery rate is usually assumed to be constant, there is no need to model the hazard rate and recovery components of credit risk separately, however, it does suffice the process to model the spread.

Arora et al. (2005)\cite{2} from KMV does a empirical comparison study on structural model, reduced-form model and KMV’s own structural based model.

This paper follows the concept of reduced-form models and will explain reduced-form models in more details. We extend recent studies by Jarrow and
Yu (2001)[48] and Longstaff et al. (2005)[58] and refine their approaches by taking counterparty risk into consideration. We also improve the completeness of bond information by gathering information from more than one bond of the underlying defaultable entity.

1.3 Counterparty Risk

1.3.1 Collateralisation

It is standard practice for financial institutions to enter derivative contracts documented on Master Agreements as recommended by the ISDA (International Swaps and Derivative Association). All ISDA contract holders are ranked pari passu to senior debt, in terms of claims on a defaulting counterparty.

Margin agreements require banks to post different levels of collateral on their outstanding contracts depending on the current mark-to-market of the contract. Typical acceptable collateral is either cash or highly-rated (AA or higher) securities. Margin thresholds are usually the previous day’s mark-to-market for all outstanding contracts, but exceptions might be made for highly-rated corporates.

Given their higher risk profile, margining for hedge funds tends to be somewhat more stringent. They typically post collateral at 100% of their current exposure, and furthermore might also be asked to post collateral to cover close-out risk on their contracts for a certain number of days going forward. The estimation of forward exposure is done through forecasting future scenarios.

1.3.2 Netting Agreements

Another advantage of trading within the ISDA framework is the provision of netting. Netting agreements come into action in the case of actual counterparty default. Without such agreements, a surviving counterparty would legally have to fully meet its obligations to the defaulting counterparty, while only being left with a claim on its dues from the same. However, the provision for netting allows
a bank to calculate its dues to a defaulter by netting out-of-the-money and in-
the-money contracts and to arrive at a single figure for dues. In fact netting
agreements are typically applicable across all derivatives that are traded on
ISDA contracts, effectively building in a natural hedge to counterparty default
risk on a firm-wide level.

Even in the absence of reference entity default, a failure of a major coun-
terparty could lead to losses across the financial system. Upon the default of
the counterparty, OTC derivatives would be immediately and significantly re-
priced, with credit spreads likely to widen dramatically. This means that CDS
contracts would be terminated at a spread significantly higher than the spread
at which collateral was last posted, leading to the crystallisation of significant
losses. Our analysis uses un-netted data, as data on netted exposures is very
hard to obtain.

There are two factors which could cause the realised losses to be larger than
our estimates. The first is the fact that, while we assumed full collateralisation,
in reality, collateralisation is imperfect. This would mean that at the point of
last posting of collateral, there would be some mark-to-market positions which
are not backed by collateral and any losses on these positions would increase
the loss from gap risk. The second is forward margining, any collateral posted
by hedge funds with the defaulting counterparty as part of forward margining
would be subject to a loss. This loss would amount to the value of collateral
less recovery.

If no recovery is possible, the total credit loss will not consist simply of the
amount of the next payment due under the terms of the swap contract but will
equal the present value of the net interest payments over the remaining life of the
contract. This amount is termed the replacement cost of the derivative contract.
The replacement cost of a derivative contract is the appropriate measure of the
credit loss resulting from the default of one’s counterparty. If the derivative
contract has a positive mark-to-market value to the non-defaulting counterparty,
then the replacement cost of the amount that counterparty would have to pay
in the market to obtain a derivative contract with the same terms. However, if default occurs on derivative contracts with negative mark-to-market values to the non-defaulting counterparty, then that counterparty is typically not free to walk away from these transactions and reap a windfall gain. This condition implies that the replacement cost of a derivative contract is equal to the greater of zero and the current mark-to-market value of the contract.

In our analysis we have concentrated solely on credit derivatives. However, in terms of amounts outstanding, credit derivatives constitute only 8% of all the OTC derivatives, with interest rate derivatives constituting the largest proportion of 67%. We believe that a default of a major counterparty would cause a significant re-pricing in all OTC derivatives. Given the enormous amounts outstanding of these derivatives, netted exposures could be large and therefore gap risk losses on other OTC derivatives could be significant.

1.4 Contribution

In this thesis, an intensity-based approach to the modelling of correlated default is presented. Under the framework of the reduced-form model, default is an unpredictable jump with an exogenously specified hazard rate process, also called intensity. We adapt the contagious default framework proposed by Jarrow and Yu (2001)[48] and apply the change of measure technique introduced in Collin-Dufresne (2002)[11] in their valuation procedures. However, instead of discrete premium payment which represents the fraction of time interval between successive payment dates as in Leung and Kwok (2005)[55], we simplify the calculation by extending it to continuous premium payment, the results derived are much more indicative in terms of explanatory power for individual variables.

Firstly, a two-firm model is investigated in Chapter 3, which means only the reference entity and the protection seller are due to default. We derive the joint density of default times as well as the marginal survival probability. We then calculate the swap premium in continuous payment environment by relaxing
CHAPTER 1. INTRODUCTION

the assumptions about the protection buyer being obliged to make continuous
premium payment at the swap rate till maturity of the contract.

In Chapter 4 we extend the contagion model into three-firm model, which
involves the impact of default induced by the protection buyer. We found that
the default of the protection buyer does not place a big impact on the premium
rate, which agrees with the nature of the CDS contract. At last, instead of as-
suming the intensities are constant before jump caused by other party’s default,
we relax the assumption by making it time-variant, we present the up turn and
down turn of the economic cycle by making the intensity rate follow a wave
shape.

In Chapter 5, an empirical study is carried out based on the time series of
credit spread on 55 companies between July 2004 - June 2007, we compare the
spreads derived from the bond market and the one from CDS market, we found
that credit market provides a better indication of credit risk in terms of liquidity
and faster response to market information, we also found the credit spreads de-
2
rived from both of the market bear counterparty risk, which means their “real”
spread should be even higher than the one from CDS market. A counterparty
spread proxy is then constructed, by taking the difference between the 5-year
swap rates and the 5-year risk-free rate (Euro Benchmark curve in this case)
and apply the counterparty spread on top of credit spread from CDS or Bond
market, to reveal the true default intensity of the underlying entity.

In Chapter 6, we study the counterparty risk during the credit crunch period
from July 2007 until June 2009, using the same framework for counterparty
spread, and find much higher counterparty risk during the crisis time due to
the lack of confidence from investors on default or other credit events of major
financial institutions. We then extend our observation from 5 year maturity
spread onto the term structure of CDS spread from 1 year to 10 years maturity,
and investigate the shape of term structure for counterparty spread, we found
great similarity of the counterparty spread shape to the shape of term structure
of instantaneous volatility for Libor Market Model, whose calibration was first introduced by Rebonato (2006) and then improved by adding a regime-switching, stochastic volatility feature to the model. We found out that the term structure for counterparty spread shows a flat or a humped hockey stick shape, until when the market comes to an “excited” stage, where the term structure shows a downward decreasing shape.

1.5 Structure of the Thesis

The content of this thesis is organised as follow:

Chapter 1 states background of credit risk and motivation to study counterparty risk based on credit derivatives, it briefly outlines the contribution of the thesis. Chapter 2 is the literature review. Chapter 3 studies the contagion model introduced by Jarrow & Yu (2001) and extends the framework into continuous-time using the change of measure technique. Chapter 4 extends the contagion model from 2-firm to 3-firm, which involves a defaultable protection buyer. Chapter 5&6 carries out an empirical study, where the credit spread is modelled as a mean-reverting, strictly positive process, as in Cox–Ingersoll–Ross. A proxy is constructed for counterparty spread in order to add on top of credit spread from the CDS market. Chapter 7 carries out the empirical study using the Cox–Ingersoll–Ross framework and investigates the counterparty spread during credit crunch period. The term structure of counterparty spread is then modelled in Chapter 8 using the regime-switching, stochastic volatility Libor Market Model introduced by Rebonato & Kainth (2003). Chapter 9 concludes.
Chapter 2

Literature Review

In this chapter we go through the literature for each substantial topic involved in this thesis, in order to provide an introduction and deeper insight of the models applied in the thesis.

2.1 Contagion

A number of articles have been done to study analytically on counterparty risk. Brigo & Chourdakis (2008)\cite{6} model credit spread volatility and take default correlation into account, this is considered important when the underlying reference contract is a CDS, as the counterparty credit valuation adjustment involves CDS options, and modelling options without volatility in the underlying asset is undesirable. They investigate the impact of the reference volatility on the counterparty adjustment as a fundamental feature that is not studied in other approaches.

Differently from the intensity based approach, Hull and White (2001)\cite{43} characterise credit risk by constructing a credit index for each company and model the default by the event that the credit index hits a certain barrier. This approach generalises the structural model originally proposed by Merton (1974)\cite{62} to consider all credit information of a firm including its asset value and
its credit rating. In order to avoid the complicated calculation of the default probability, they assume that generally a credit index process can be transformed to a Wiener process, which is in doubt since it implies that the credit risk is independent of the risk-free security market. They also assume a flat term structure of interest rates which does not reflect the market risk and its association with other factors.

Kim & Kim (2003) \[50\] develop a methodology for valuing CDSs that takes account of counterparty risk as well as market and credit risk. It incorporates market risk into determining default correlations between multiple firms using the first-passage-time approach, based on structural approach with stochastic interest rate (market risk). Their model under market and credit risk correlation is applied to the valuation of vanilla credit default swaps with counterparty default risk, and to the valuation of basket credit default swap.

Distinct from all these methods, Chen and Filipovic (2003) \[9\] model the risk-free rates, credit indices and default events altogether by a multi-dimensional affine process. In this way, not only can the dynamics of a credit index be extended from a Wiener process to any affine process (including affine jump-diffusion processes), but also this generalised affine model provides us an analytical framework to consider all the essential concerns mentioned before because of the analytical tractability and the rich structure of affine Markov processes. Moreover, they have demonstrated that this model can produce explicit formulas for the prices of default swaps and other credit derivatives.

Walker (2006) \[72\] approximates an analytical solution to a Markov model describing transitions between the states of a basket of two obligors, this allows easy calibration to the market prices of the bonds for individual obligors. It determines two of the model parameters which describe the dependence between the two obligors. This analytical formula for the spread of a CDS with counterparty risk can in principle be used to determine one of the two dependency parameters of the model from market data on the price of the CDS. He also compares results of a time-dependent default-correlation coefficient with
results obtained using the market-standard Gaussian copula model, it is con-
cluded that the market-standard model is very limited in its ability to price
correlation-sensitive derivatives, these limitations could be largely overcome by
using a time-dependent Gaussian copula correlation coefficient.

Jarrow & Yu (2001)[48] propose a contagion default model, which considers
the credit risk induced by the interdependence among firms by generalising
the intensity based model to allow a firm to be exposed to some firm-specific
default risk, as well as to common risk factors. However, due to the complexity
of the analysis, they confine the discussion to the situation where the default
intensity follows a simple primary-secondary framework process and only price
the idealised default swaps by assuming that each party within the contract is
obligated to pay until its own default, regardless of whether the other party has
defaulted or not.

Later on, Yu (2004)[74] uses the “total hazard” approach to construct the
default processes from independent and identically distributed exponential ran-
dom variables. An analytical expression of the joint distribution of default times
is obtained in his two-firm and three-firm contagion models. In a copula-based
approach that combines ingredients of a reduced-form model, Schonbucher and
Schubert (2001)[69] propose a modification of the default time with $E^i$ following
a joint distribution $C(U^1, \ldots, U^I)$, where $U^i = \exp(-E^i)$ and $C$ is a copula
function. Yu shows that his approach can be considered as an extension of the
Schonbucher and Schubert construction where the copula function is indexed by
the sample path of $X$. In other words, the Schonbucher and Schubert construc-
tion is a special case of Yu’s approach where the copula function is invariant to
the history of $X$.

However, despite the intuition of the framework, Yu places several assump-
tions on the payment structure in order to simplify his calculations. Firstly,
the protection buyer is assumed to make continuous premium payments at the
swap rate till expiration, provided that the buyer does not default prior to expiry,
which does not perfectly represent the market practice that the swap terminates
upon default of any one of the three parties. Secondly, the protection seller is assumed to make the contingent compensation payment on the expiration date, provided that the protection seller survives beyond the expiration date of the swap.

Collin-Dufresne, Goldstein and Hugonnier (2002) show that the Duffie and Singleton (1999) pricing formula can be preserved even when the no-jump condition is violated, if one changes to a measure that places zero probability on default prior to the maturity of the claim. This formula has been applied to derive analytic solutions for defaultable bond prices in a setting with two issuers. However, it is not clear whether it can be applied to more general settings such as a large number of issuers or where the defaultable claim is of the basket type.

An analytic solution for the CDS premium framework introduced by Jarrow and Yu (2001) is provided by Leung and Kwok (2005). They employ the change of measure introduced in Collin-Dufresne (2004) in their valuation procedure. Using the change of measure, the counterparty risk model is reduced to the standard reduced form model. This approach provides less tedious analytical derivations compared to the total hazard construction Yu proposed. Yu (2007) himself also presents a procedure where the CGH formula can be applied to derive the marginal distribution of default times in the presence of an arbitrary number of correlated defaults.

Apart from all the studies above, Bai, Hu and Ye (2007) argue it is unrealistic to assume that one firm’s default intensity keeps a constant jump after the other firm defaults. They introduce a geometric function to reflect the attenuation speed of impact of one firm’s default to its counterparty. They argue that if two firms are co-partners, the default intensity of one firm will increase abruptly when the other firm defaults, as time goes on, the impact will decrease gradually until extinct as time goes on.
2.2 Cox–Ingersoll–Ross Model

In order to discover the dynamic of CDS spread in Chapter 5 and 6, a number of papers have been studied to spot the determinants of corporate yield spreads, this section also provide the literature that has been done in the area of liquidity risk.

The Duffie & Singleton (1999)\[26\] framework is used to specify that the instantaneous credit spread equals the product of the risk-neutral jump intensity and the loss rate. Each firm’s default intensity is modelled as a function of latent common factors and a latent firm-specific factor. In line with the results from Longstaff and Schwartz (1995)\[59\] and Duffie (1999)\[22\], the model allows for correlation between credit spreads and default-free interest rates, which are modelled using a two-factor “essentially affine” model. The model also corrects for tax difference between corporate and government bonds, and a liquidity factor that is based on the corporate bond age. Yu (2002)\[73\] also provides a decomposition of corporate bond returns using the intensity-based framework of Duffie and Singleton, however, he did not estimate the size of the components.

The first analysis of conditional diversification is performed by Jarrow et al. (2001)\[45\], however, they do not estimate the default event risk premium. They use the estimate of Duffie’s (1999)\[22\] model to compare the model-implied default probabilities with the default probabilities that are implied by an annual Markov migration model. It is shown that the use of the Markov migration model leads to downward biased estimates of the default jump risk premium.

In order to fit structural firm value models to historical rates and the equity premium, and compare model-implied and observed credit spreads, Huang and Huang (2002)\[40\] find that the implied credit spreads by all the models are too low for investment grade firms. The risk premia generated by these models, mostly based on a diffusion process for changes in the firm value, can be interpreted as risk premia on common changes in firm values, or equivalently, credit spreads. They find about 30% explanatory power for the risk premia
of the BBB-rated credit spread. They also consider a model that incorporates systematic jumps in the firm value.

A firm value framework is employed by Delianedis and Geske (2001)\cite{delianedis2001}, to assess the influence of several factors on the level of credit spread, including jump, tax, and liquidity factors, however, they study only on the level of credit spreads, and do not use historical default rates. They argue that priced jumps in firm value cannot solely explain the high level of credit spreads, although they do not empirically decompose the credit spread into all the factors.

Collin-Dufresne et al. (2003)\cite{collin2003} propose a theoretical model where a large upward jump in a firm’s credit spread causes a moderate market-wide jump in the credit spreads of other firms, known as the contagion effect. They distinguish the direct jump risk premium from the contagion risk premium. Collin-Dufresne et al. perform a calibration to obtain an indication of the size of the jump and contagion risk premium.

A new dataset of bid and offer quotes for credit swap spreads is used by Ericsson et al. (2005)\cite{ericsson2005} to investigate the relationship between theoretical determinants of default risk, such as firm leverage, volatility and the riskless interest rate, and actual market premia using linear regression. They find that estimated coefficients for these variables are consistent with theory and that the estimates are highly significant both statistically and economically. They conclude that leverage, volatility and the risk-free rate are important determinants of credit default swap premia as predicted.

### 2.3 Liquidity Risk

Recent research has shown that default risk accounts for only a part of the total yield spread on risky corporate bonds relative to their risk-less benchmarks. One candidate for the unexplained portion of the spread is a premium for liquidity. A few researchers have studied liquidity risk, or joint liquidity risk together with default risk.
Liu, Longstaff, and Mandell (2006) study how the market prices the default and liquidity risks incorporated into an interest rate swap spread. Treasury, repo and swap term structures are jointly modelled using a five-factor affine framework and estimate the model by maximum likelihood. In their paper, the credit spread is driven by a persistent liquidity process and a rapidly mean-reverting default intensity process. They found that the credit premium for all but the shortest maturities is primarily compensation for liquidity risk, and the term structure of liquidity premia increases steeply while that of default premia is almost flat. Both liquidity and default premia vary significantly over time.

As shown in Figure 2.1, they found that the credit spread in swaps consists of both a liquidity component and a default component. On average, the default component of the credit spread is larger, but the liquidity component is slightly more volatile. Both components vary significantly over time. The liquidity component display a high level of persistence. In contrast, the default component is rapidly mean reverting. In addition, the default component exhibits a number of large but temporary spikes in its level over time.

In order to explore the role of liquidity risk in the pricing of corporate bonds, De Jong and Driessen (2005) employ bid-ask spread of long-term US treasury bonds to measure liquidity. They show that liquidity is a priced factor in a multifactor model for the expected returns on corporate bonds. The corporate bond returns have significant exposures to fluctuations in treasury bond liquidity and equity market liquidity. Furthermore, the associated liquidity risk premia help to explain the credit spread puzzle. They discovered for the US market, the total estimated liquidity premium is around 45 basis points for long-maturity investment grade bonds; whereas for speculative grade bonds, which are exposed to higher liquidity risk, the liquidity premium is around 100 basis points. Similar evidence is found for the liquidity risk exposure of corporate bonds using a sample of European corporate bond prices.

An empirical study of market prices of credit default swaps in comparison
Figure 2.1: Liquidity and Default Components of the Credit Spread
The top panel plots the liquidity component of the spread. The middle panel plots the default component of the spread. The bottom panel plots the sum of the liquidity and default components which equals the credit spread. All time series are measured in basis points. The sample period is January 1988 to February 2002. Source: Liu, Longstaff, and Mandell (2006)
with model prices is carried out by Houweling and Vorst (2005)[39]. They show that a simple reduced-form model gives more accurate estimates of default swap premiums than the bond’s yield spreads. Moreover, they shed light on the choice of the default-free term structure of interest rates. Their model yields unbiased premium estimates for default swaps on investment grade issuers, but only if swap or repo rates are used, as they state that swap and repo curves significantly outperform the government curve as proxy for default-free interest rates. Their paper confirms that financial markets no longer see Treasury bonds as the default-free benchmark empirically.

Feldhütter and Lando (2008)[31] also attempt to decompose the swap spreads through a joint pricing model for treasury securities, corporate bonds and swap rates using six latent factors, and decompose swap spreads into three components: a convenience yield (liquidity) from holding Treasury securities; a credit spread arising from the credit risk element in LIBOR rates, which define the floating-rate payments of interest rate swaps; and a factor specific to the swap market. The convenience yield, which separates the Treasury yield from the riskless rate, is by far the largest component of the swap spreads. The other two components separate the swap rate from the riskless rate. The credit risk component does not contribute much to the time variation of spreads.

A discernible contribution from credit risk exists as well as from a swap-specific factor with higher variability which in certain periods is related to hedging activity in the MBS (mortgage-backed security) market. Their model also sheds lights on the relation between hazard rates and the spread between LIBOR rates and General Collateral repo rates and on the level of the riskless rate compared to swap and treasury rates.

A more structural approach is adapted by Shin (2008)[70] to explore the pricing of debt in a financial system, where the assets that borrowers hold to meet their obligations include claims against other borrowers. Accessing financial claims in a system context captures features that are missing in a partial equilibrium setting, such as liquidity spillovers across financial institutions re-
sulting from expansions and contractions of balance sheets. Aggregate liquidity can be seen as the rate of growth of a financial sector’s balance sheets. The focus of Shin’s paper is on the liquidity of the financial system as a whole, where “liquidity” refers to the funding conditions for current and potential borrowers. For existing borrowers, rising asset prices strengthen their balance sheets and make them more credit worthy. For potential borrowers, the stronger balance sheets of financial intermediaries play to their advantage. His framework is easily extended to deal with claims of differing seniority classes, and is well-suited to pricing complex debt instruments such as CDOs (collateralised debt obligations), since CDOs are obligations that are backed by claims on others. His model is also well-suited to quantitative analysis of risks such as value-at-risk calculations that take account of endogenous changes in asset prices and the feedback effects that result.

In contrast to previous evidence from corporate bond data, Ericsson and Reneby (2007)\cite{29} evaluate the price of default protection for a sample of US corporations using a set of structural models. They found that CDS premia are not systematically underestimated. They perform the same exercise for bond spreads by the same issuer on the same trading date for a robustness test, which shows that bond spreads relative to the Treasury curve are systematically underestimated, which is not the case when the swap curve is used as a benchmark, suggesting the previous documented underestimation results may be sensitive to the choice of risk free rate. They explain the reason why the swap curve outperforms the treasury curve is that the swap curve lies closer to the cost of funding for traders in the bond market.

2.4 Corporate Bond Yield Spreads

Corporate Bond yield spread has been used as an alternative method in order to observe default risk, liquidity risk together with other risk factors from the bond market.
Nashikkar et al. (2007)\textsuperscript{[63]} investigate the possibility of the existence of liquidity risk by relating the liquidity of corporate bond prices to the basis between the credit default swap spreads of the issuer and the par-equivalent corporate bond yield spread. A measure to assess a bond called latent liquidity is defined as the weighted average turnover of funds holding the bond, where the weights are their fractional holdings of the bond. They find that bonds with higher latent liquidity are more expensive relative to their CDS contracts, after controlling for other realised measures of liquidity. By documenting the positive effects of liquidity in the CDS market on the CDS-bond basis, they also find that several firm-level variables related to credit risk negatively affect the basis, which indicates that the CDS price does not fully capture the credit risk of the bond. Furthermore, when default risk of a firm is high, its illiquid bonds are more expensive. Their findings are consistent with limits to arbitrage between the CDS and bond markets, due to the cost of shorting bonds.

They have shown that the liquidity of the CDS contract, as measured both by the bid-ask spread, and the riskiness of the CDS market (as measured by the CDS market volatility), both of which have explanatory power for the basis of bonds over and above bond-specific liquidity variables, which shows evidence that bond market participants account for the liquidity of the CDS market when they price corporate bonds, due to the ease to hedge their positions. As the CDS market becomes more and more liquid, the CDS-bond basis is expected to decrease over time. They have also shown that the CDS price does not fully account for the effect of credit risk on bond prices, several firm-level effects that are related to credit risk seem to have explanatory power over the CDS-bond basis.

The components of corporate credit spreads are analysed by Delianedis and Geske (2001)\textsuperscript{[19]} using a structural model, which offers a framework to understand the decomposition. They believe that default risk may correctly represent only a small portion of corporate credit spreads. They conclude that credit risk and credit spreads are not primarily explained by default and recovery risk, but
are mainly attributable to taxes, jumps, liquidity, and market risk factors.

The determinants of very short-term corporate yield spreads is studied by Covitz and Downing (2007)\[^{13}\], who employ a comprehensive database on transactions of commercial paper issued by domestic U.S. non-financial corporations. They find that liquidity plays a role in the determination of spreads but credit quality is more important as a determinant of spreads, even at horizons of less than 1 month. Their results are sound across a variety range of proxies for liquidity and credit risk, and show important implications for the literature on the modelling of corporate bond prices.

Elton et al. (2001)\[^{27}\] show that expected default losses and tax effect cannot explain the observed level of credit spread. They explain part of this fitting error by relating corporate bond returns to equity market factors. However, Driessen (2005)\[^{20}\] uses a latent factor model instead of the Fama-French factor model as in Elton et al. (2001)\[^{27}\]. Collin-Dufresne et al. (2001)\[^{11}\] show that observable economic and financial variables (including equity returns) cannot explain the correlation of credit spread changes across firms. Moreover, Elton et al. (2001)\[^{27}\] do not incorporate a default event risk premium or a liquidity factor.

A similar study is done by Chen et al. (2007)\[^{10}\] based on corporate yield spreads and bond liquidity using a battery of liquidity measures covering over 4,000 corporate bonds spanning both investment grade and speculative categories. They find that more illiquid bonds earn higher yield spreads, and an improvement in liquidity causes a significant reduction in yield spreads. Their results hold after controlling for common bond-specific, firm-specific, and macroeconomic variables, and are robust to issuer’s fixed effect and potential endogeneity bias. Their findings justify the concern as in Collin-Dufresne et al. (2002)\[^{11}\] and Huang and Huang (2003)\[^{40}\], that neither the level nor the change in the yield spread of corporate bonds over Treasury bonds can be fully explained by the credit risk determinants proposed by structural form models. They also find inconsistency in statistical evidence of a tax effect, in line with
CHAPTER 2. LITERATURE REVIEW

Longstaff et al. (2005)[58], or an equity volatility effect for investment grade bonds, and little evidence of these effects for speculative grade bonds.

Chen et al. (2007)[10] contribute to the growing debate over liquidity’s influence on asset pricing and corporate finance decisions, with implications for both domestic and equity markets. Specifically, the issue of a liquidity premium on returns may now find common ground in both bond and equity markets. Furthermore, the evidence of a liquidity effect on corporate yield spreads may shed light on sovereign debt yield spread determinants.

2.5 Comparison between Bond Market and CDS market spread

Credit default swap spreads are an interesting alternative to bond prices in empirical research on credit ratings for two reasons:

a. The CDS spread data provided via a broker consists of firm bid and offer quotes from dealers, once a quote has been made, the dealer is committed to trading a minimum principal ($10m) at the quoted price. On the other hand, the bond yield data usually consist of indications from dealers. There is no commitment from the dealer to trade at the specified price.

b. The second attraction of CDS spreads is that no adjustment is required - they are already credit spreads, whereas bond yields require an assumption about the appropriate benchmark risk-free rate before they can be converted into credit spreads, the usual practice of calculating the credit spread as the excess of the bond yield over a chosen risk-free benchmark.

Hull, Predescu, and White (2004)[41] examine the relationship between credit default swap spreads and bond yields and reach conclusions on the benchmark risk-free rate by using a risk-free rate about 10 basis points less than the swap rate. They then carried out a series of tests to explore the extent to which credit rating announcements by Moody’s are anticipated by participants in the credit swap market.
An empirical comparison of credit spreads between the bond market and the CDS market is done by Zhu (2004) on how the two markets interact with each other. He confirms the theoretical prediction that the two spreads should be on average equal to each other, however, in the short run, there are significant price discrepancies, which is largely due to their different responses to change in the credit quality of reference entities. He also finds that market participants seem to use swap rates rather than treasury rates as the proxy for risk-free rates, he shows the failure of treasury rates to be the proxy for risk-free rates could be largely attributed to tax considerations. Overall, the derivatives market seems to lead the cash market in anticipating rating events and in price adjustment. His empirical study also suggests that the relative importance of the two markets in price discovery can vary substantially across entity’s liquidity matters. There is also evidence of market segmentation in that U.S. entities behave very differently from those in other regions. Finally, he finds that the existence of a delivery option in CDS contracts and the short-sale restriction in the cash market only have minor impacts on credit risk pricing.

The swap rate is taken as the risk-free rate in Blance et al. (2003), and they find credit default swap spreads to be quite close to bond yield spreads. They also find that the default swap market leads the bond market so that most price discovery occurs in the credit swap market. On the other hand, Houweling and Vorst (2005) argue that market participants no longer see the treasury curve as the risk-free curve and instead use the swap curve and/or the repo curve, they confirm that the credit default swap market appears to use the swap rate rather than the treasury rate as the risk-free rate. The result of Hull et al. (2004) is consistent with Vorst’s findings, they estimate that the market is using a risk-free rate about 10 basis points less than the swap rate to allow for the fact that the payoff does not reimburse the buyer of protection for accrued interest on bonds.

Elton et al. (2001) decompose spot rates on corporate bonds into expected loss, taxes and residual. They examine how much of the variation over
time in the residual spread can be explained by systematic risk factors, and calculate a risk premium based on these contributions. The more recent paper by Driessen (2005)\cite{20} employs different methods and data to further decompose spreads into taxes, risk premium and liquidity premium.

Attempting to explain the precise relationship between credit default spread and credit risk, Amato and Remolona (2003)\cite{1} did a comparison study with Elton et al. (2001)\cite{27} and Driessen (2005)\cite{20}, they argue that the answer to the credit spread puzzle might lie in the difficulty of diversifying default risk. They review existing evidence on the determinants of credit spreads, including the role of taxes, risk premia and liquidity premia. In the end they suggest that the spreads are largely a compensation for the risk of unexpected losses from default that are invariably present in corporate bond portfolios.

In Chapter 5, the main argument/development is based on the framework Longstaff et al. (2005)\cite{58}, who study default risk over instruments such as interest rate swaps or credit default swaps, and find that using repo curve or swap curve provides a better fit than treasury curves, however, they also ignore the impact of counterparty risk, i.e. insurance company, investment banks, or even hedge funds, whose own default is at risk. The reason swap rates, or say, LIBOR curve provides a good fit on CDS spreads fitting models, is that they both ignored the underlying counterparty default risk, which is embedded in both LIBOR rates and CDS prices. Longstaff et al. (2005)\cite{58} also find there is a big difference between the spread obtained from the CDS market and spread generated by using bracketing bonds from the corporate bond market for the same underlying reference entity. They then state that counterparty risk cannot fully explain the difference between the spreads.

In Chapter 5, our model is developed to add counterparty spread onto CDS spreads from the market, by keeping the government bond rates as risk-free rate, we brought the counterparty-risk adjusted CDS spread onto the same platform as the spread generated by the bond market, and find much less difference between the two, which makes default risk a bigger component than previous
studies and liquidity risk count not as a big proportion of the total spread.

2.6 Term Structure of Counterparty Spread

In this section we introduce the literature for the final chapter, which studies counterparty risk effect during the credit crunch period.

Firstly we compared our counterparty risk proxy with the one Mercurio (2009) [61] introduces in his paper. Mercurio (2009) [61] describes the major changes that occurred in the quotes of market rates after 2007 sub-prime mortgage crisis and comments on the missing analogies and consistencies of those rates, and hints on a possible, simple way to formally reconcile them, part of which mentioned the discrepancy of “last” one-month EONIA rates and one-month deposit rates, from November 14th, 2005 to November 12, 2008, it can be observed that the basis was well below 10 basis points until August 2007, however, since then started to move erratically around different levels, which is believed to be due to counterparty risk.

Now the FRA cannot be priced as a trivial forward on a LIBOR rate, as it used to be, at least approximately. One possible explanation is the increased perception of bank-vs-bank counterparty/liquidity risk after the burst of sub-prime crisis. The presence of counterparty/liquidity risk in LIBOR market quotes is often estimated based on the difference between LIBOR rates and rates theoretically free of such counterparty/liquidity risk, such as EONIA. Thus now the risk-free rate is taken to be EONIA, whereas LIBOR would be a different default risky rate. All those rates, which were very closely interconnected, suddenly became different objects, each one incorporating their own liquidity or credit premium.

In second half of Chapter 6, we extend our counterparty risk proxy from spot 5-year maturity to a 1-to-10 year time horizon, which forms a term structure of the counterparty risk, we find the shape of the term structure of counterparty
spread shares a great similarity to what Joshi & Rebonato (2001) introduce for the instantaneous volatility term structure for Libor Market Model (LMM).

Joshi & Rebonato (2001) present an extension of the Libor Market Model, which allows the stochastic instantaneous volatility of the forward rates in a displaced diffusion setting. They successfully extend the deterministic volatility case to the stochastic volatility case while keeping all the powerful and important approximations. They also show that the market caplet surface across strikes and maturities can be well recovered even after reducing the number of the possible fitting parameters.

The model is later improved in Rebonato & Kainth (2003) to become a two-regime, Markov chain extension of the LMM model, where the unobservable instantaneous-volatility process migrates between two states, one of which is associated with the parameters that give a monotonically-decreasing term structure of the instantaneous volatility, and the other with the parameters associate with a humped shape. Rebonato (2006) suggests that a two-regime Markov chain approach may be more successful and better financially motivated, more generally, his study highlights the shortcomings of purely time-dependent or time-homogeneous approaches. He concludes that neither time-homogeneity nor time-dependence constitute a desirable modelling approach, and that the possibility of the instantaneous volatility migrating between a normal to an excited state is likely to be a necessary ingredient for a convincing description of the dynamics of the swaption surface.

2.7 Summary of Literature Review

This chapter summarises and review the studies that have been done related to the main subjects within this thesis. We first go through the papers working on the analytical solution of the counterparty risk, as discussed in Chapter 3 and 4; Reviews are done on subjects such as Cox–Ingersoll–Ross Model, liquidity risk, and comparison between spreads in CDS market and bond market, they
are intensively studied and developed in Chapter 5; The final section of this chapter introduces the term structure of the instantaneous volatility for Libor Market Model, a building block for our study in Chapter 6 and 7.
Chapter 3

Contagion Model

Notation of Variables

\( \tau \): stopping time in reduced-form model
\( \lambda \): default intensity
\( X \): exogenous state variable
\( E \): a unit exponential random variable assumed to be independent of \( X \)
\( N_i \): a Cox process presenting default, with intensity \( \lambda \)
\( P \): equivalent martingale (risk-neutral) measure
\( N^i \): default processes of the firm \( i \) in the economy
\( \mathcal{F}_t \): filtration process generated collectively by the information contained in the state variables and the default processes
\( p(t,T) \): time-\( t \) price of a default-free zero-coupon bond
\( B(t) \): bond price
\( V^i(t,T) \): defaultable bond price
\( \mathbb{E}_t (\cdot) \)  
the expectation conditional on time-\( t \) information \( \mathcal{F}_t \)

\( V^i (t, T) \)  
the defaultable bond price

\( \delta^i \)  
recovery rate

\( r_t \)  
denote the spot interest rate process adapted to \( \mathcal{F}_t \)

\( \lambda_P^i \)  
default intensity of protection seller

\( \lambda_C^i \)  
default intensity of reference entity

\( b_0, c_0 \)  
positive constants, default intensity for B/C without jump

\( b_1, c_1 \)  
positive constants, jump intensity when firm C/B defaults

\( S_i^T (T) \)  
conditional survival probabilities

\( Z_i^T \)  
a uniformly integrable \( P \)-martingale with respect to \( \mathcal{F}^i \)

\( E^C \)  
the expectation taken under the measure \( P^C \)

\( p \)  
swap premium
In this chapter, we study and extend the contagion two-firm model introduced by Jarrow and Yu (2001) [48], explain the original intuition of their framework as well as the shortcomings in the process of deriving an analytical solution for the CDS premium, which compensates the original two-direction contagion by introducing a primary-secondary framework instead. We simplify the derivation to an analytical solution using the change of measure technique by Collins-Dufresne et al. (2002) [11]. This differs from other approaches up to date, since instead of following the discrete payment setup, we integrate the swap premium calculation up to a continuous payment environment, which, as we can see at the end of this chapter, provides a much simpler and meaningful premium result.

3.1 Contagion Model

The model introduced by Jarrow and Yu (2001) [48] generalises Lando (1994, 1998) [51, 53] to include counterparty default risk. Under the framework of contagious defaults, the default risk is determined by an exogenously specific instantaneous default intensity. The contagious defaults are triggered by inter-dependent default risk structure between the parties, where the default intensity of one party jumps up when the default of another party occurs.

In order to discuss the contagion model, we begin with the description of a typical reduced-form model, which it is customary to associate with a stopping time $\tau$:

$$\tau = \inf \left\{ t : \int_0^t \lambda_s ds \geq E \right\}. \quad (3.1)$$

where $\lambda$ is called the default intensity and assumed to be dependent on exogenous state variables $X$, and $E$ is a unit exponential random variable assumed to be independent of $X$. $N_t = 1_{\{t \geq \tau\}}$ is defined as a Cox process presenting default, with intensity $\lambda$. In a setting with multiple default times $\{\tau_i\}_{i=1}^I$, one can extend the above definition to a collection of independent unit exponential random variables $\{E_i\}_{i=1}^I$, which implies that the default times are indepen-
dent conditional on the information contained in $X$, and default correlations arise because of the correlations of the intensities.

### 3.2 Definition of Default Process and Change of measure technique

#### 3.2.1 Construction of Default Process

Consider that uncertainty in an infinite horizon economy is represented by a filtered probability space $\left(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t=0}^T, \mathbb{P}\right)$ where $\mathcal{F} = \mathcal{F}_T$, $\mathbb{P}$ is an equivalent martingale (risk-neutral) measure under which all security prices discounted by the risk-free interest rate process are martingales, so that bond markets are complete and priced by arbitrage, as shown in discrete time by Harrison and Kreps (1979)\[35\] and in continuous time by Harrison and Pliska (1981)\[36\].

On this probability space there is an $\mathbb{R}^d$-valued process $X_t$, which represents $d$ economy-wide state variables. There are also $I$ point processes, $N^i$, $i = 1, ..., I$, initialised at 0. These represent the default processes of the $I$ firms in the economy such that the default of the $i$th firm occurs when $N^i$ jumps from 0 to 1.

The filtration is generated collectively by the information contained in the state variables and the default processes:

$$
\mathcal{F}_t = \mathcal{F}_t^X \vee \mathcal{F}_t^1 \vee \cdots \vee \mathcal{F}_t^I,
$$

where

$$
\mathcal{F}_t^X = \sigma(X_s, 0 \leq s \leq t) \text{ and } \mathcal{F}_t^i = \sigma(N^i_s, 0 \leq s \leq t).
$$

are the filtrations generated by $X_t$ and $N^i_t$, respectively.

We use the Cox framework (doubly stochastic Poisson processes) to specify the random default times:
\[ \tau^i = \inf \left\{ t : \int_0^t \lambda^i_s \, ds \geq E^i \right\}. \] (3.4)

where \( \{ E^i \}_{i \in I} \) is a set of independent unit exponential random variables. Further assumptions are made that \( \tau^i \) possesses a strictly positive \( \mathcal{F}_t \)-predictable intensity \( \lambda^i_t \) with right continuous sample paths such that

\[ M^i_t = N_t - \int_0^{t \wedge \tau^i} \lambda^i_s \, ds = N_t - \int_0^t 1_{\{ \tau^i > s \}} \lambda^i_s \, ds. \] (3.5)

is a \( (P, \mathcal{F}_t) \)-martingale for some strictly positive, \( \mathcal{F}_t^X \)-adapted process \( \lambda^i_t \) where \( t \wedge \tau^i = \min \{ t, \tau^i \} \), and \( "1" \) is the indicator function.

Given the above definition, the conditional survival probability of firm \( i \) is given by

\[ P \left[ \tau^i > T \mid \mathcal{F}_t \right] = \exp \left( - \int_t^T \lambda^i_s \, ds \right), \quad t \in [0, T], \] (3.6)

and

\[ P \left[ \tau^i > T \right] = E \left[ \exp \left( - \int_0^T \lambda^i_s \, ds \right) \right], \quad t \in [0, T]. \] (3.7)

This framework can be used in deriving a pricing formula for defaultable bonds, as described by Jarrow & Yu (2001) [48], which extends Lando (1994) [51].

### 3.2.2 Recovery Rate

The second ingredient in the modeling of defaultable securities is the recovery rate. There are two main definitions of recovery rate.

Let \( p(t, T) \) denote the time-\( t \) price of a default-free zero-coupon bond that pays one dollar at time \( T \) where \( 0 \leq t \leq T \leq T^* \), denote \( p^i(t, T) \) the time-\( t \) price of a zero-coupon bond maturing at time \( T \), issued by firm \( i \) where \( i = 1, ..., I \).

These corporate bonds are subject to default. If firm \( i \) defaults, a unit of its bond will pay an exogenously specified constant fraction \( \phi^i \in [0, 1) \) of a dollar at maturity, therefore the value of the bond after default is \( \phi^i \) times the price
of a default-free bond. This is called the “recovery of treasury” assumption in Jarrow and Turnbull (1995) [47] and Jarrow, Lando, and Turnbull (1997) [44].

An alternative to the above definition of recovery rate is the “recovery of market value” where a fraction of pre-default market value is recovered immediately upon default. With this assumption, only the loss rate can be recovered from the zero-coupon bond price. Therefore in order to estimate the default intensity and the recovery rate simultaneously, one would have to resort to either the price of credit derivatives, as suggested by Duffie and Singleton (1999) [26].

Using this recovery assumption, Duffie and Singleton (1999) [26] show that the defaultable bond price can be expressed as the discounted expected value of a dollar, where the discount factor is the spot rate plus the loss rate.

### 3.2.3 Valuation of Defaultable Bonds

Given the setup of default process and recovery rate, we now derive a pricing formula for defaultable bonds. First, let \( r_t \) denote the spot rate process adapted to \( \mathcal{F}_t^X \). The spot rate process could come from any arbitrage-free default-free term structure model, such as HJM (Heath, Jarrow, and Morton, 1992) [37].

Given \( \mathbb{P} \) as an equivalent martingale measure, the money market account and bond price is given by

\[
B(t) = \exp \left( \int_0^t r_s \, ds \right), \tag{3.8}
\]

\[
p(t, T) = \mathbb{E}_t \left( \frac{B(t)}{B(T)} \right), \tag{3.9}
\]

and

\[
V^i(t, T) = \mathbb{E}_t \left( \frac{B(t)}{B(T)} \left( \delta^i \mathbf{1}_{\{\tau^i \leq T\}} + \mathbf{1}_{\{\tau^i > T\}} \right) \right), \tag{3.10}
\]

where \( \mathbb{E}_t (\cdot) \) is the expectation conditional on time-\( t \) information \( \mathcal{F}_t \). Therefore, the defaultable bond price is, as proved in Jarrow and Yu (2001) [48]:
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\[ V^i(t, T) = \delta^i p(t, T) + \mathbf{1}_{\{\tau^i > T\}} (1 - \delta^i) \mathbb{E}_t \left[ \exp \left( - \int_t^T (r_s + \lambda^i_s) \, ds \right) \right], \quad T \geq t. \]  

Equation (3.11) is intuitive in a way that it states the risky bond’s price can be divided into two components. The first component is the recovery rate \( \delta^i \) that is received for sure, discounted to time \( t \). The second component is the residual \( 1 - \delta^i \) in the event of no default, discounted to time \( t \) with an adjusted spot rate which reflects the default risk. Therefore, pricing under the equivalent martingale measure depends critically on the evaluation of the expectation in eq. (3.11).

3.2.4 Contagion Two-firm Model

Given the default process defined above, assuming the existence of a collection of doubly stochastic Poisson processes, whose intensities satisfy a special measurability condition. The bond price of a firm whose default probability is affected only by macroeconomic conditions and not by the default of other firms can be calculated using eq. (3.11).

For a firm, B, whose default time distribution is strongly affected by the default of firm C, the calculation of its bond price entails knowing the distribution of the default time for C. In return, if C holds a significant amount of debt issued by B, the distribution of the default time for C would then depend on that of B. In which case, the relationship described above forms a loop. This makes it a difficult task in the process of bond price calculation, and then CDS premium valuation.

In the following, we first introduce the contagion model proposed by Jarrow & Yu (2001)\[^{[48]}\] and illustrate the complexity involved when looping default is introduced to the model.

Imagine under a CDS contract, party A holds defaultable bonds issued by party C, (which will be the reference entity in the CDS contract), with the
concern that C might default before the maturity of the bond $T^*$. To hedge this risk, party A enters into a CDS contract with party B to make a stream of payments to party B at a fixed rate (also called the “swap rate”) from $t=0$ to the maturity of CDS $T$, ($T < T^*$), in exchange for B’s promise to compensate A for its loss up to a certain amount in the event of C’s default. B’s payment is contingent on credit events occurring to C, such as missed interest payments or a credit downgrade or default and is payable at the expiration of the default swap. Not only can the reference entity default, but the two counterparties, A and B, can default as well. Thus the pricing of a credit default swap, or the determination of the swap rate, has to take into account the credit risk from all three sources.

In our two-firm model, A is assumed to be default-free\(^1\) as A’s default before reference entity C will not generate any loss to counterparty B or C.

If the protection seller B defaults prior to the default of reference entity, the contract terminates. The protection buyer enters in to a new CDS contract with another counterparty for the remaining life of the original CDS, the loss generated due to replacement of CDS contracts is called the replacement cost. If reference entity C defaulted first, and B failed to deliver the promised payment to A during the settlement period, then it will have defaulted at the same time as C or soon afterwards, A will lose the protection promised by B, and will be exposed to a large loss due to B and C’s joint default, this is called settlement risk.

Consider the case where firm B and C hold each other’s debt, so that when C defaults, B’s default probability will jump, and vice verse. In this two-firm contagion model, the inter-dependent default risk structure between firm B and C is characterised by the correlated default intensities:

\(^1\) Also as concluded in Leung and Kwok (2005)[55], the expression for the swap premium has little dependence on the default intensity of the protection buyer.
\[ \lambda_t^B = b_0 + b_1 \mathbf{1}_{\{\tau^C \leq t\}}, \]
\[ \text{and} \quad \lambda_t^C = c_0 + c_1 \mathbf{1}_{\{\tau^B \leq t\}}. \] (3.12)

Where \(b_0, b_1, c_0\) and \(c_1\) are positive constants, therefore, the default intensity \(\lambda_t^B (\lambda_t^C)\) for default time \(\tau^B (\tau^C)\) jumps by the amount \(b_1 (c_1)\) when firm \(C(B)\) defaults.

### 3.2.5 Primary-Secondary Framework

As one can see from equation (3.12) above, these distributions are defined recursively through each other which makes the conditional survival probabilities

\[ S_i^t(T) := \mathbb{P} \left\{ \{\tau^i > T \mid \mathcal{F}_t\} \right\}. \quad i \in \{B, C\} \] (3.13)

difficult to compute. Therefore (JY) modify their original model by introducing a primary-secondary framework. In a typical application, if bank \(B\) holds a significant amount of production firm \(C\)’s debt, it is unlikely that \(C\) is also holding bank \(B\)’s debt or equity, let alone amounts large enough to influence \(B\)’s default probability. Thus (JY) proposed primary firm, whose default process depends only on a macro variable, whereas secondary firm’s default process depends on a macro variable and the default process of the primary firm.

Under the CDS contract environment, in which, for example, reference entity \(C\)’s default arrival time is independent of \(B\)’s default and has constant intensity \(c_1\) only; on the other side, reference entity \(C\)’s default will place an impact on protection seller \(B\)’s default intensity \(\lambda_t^B\).

\[ \lambda_t^B = b_0 + b_1 \mathbf{1}_{\{\tau^C \leq t\}}, \] (3.14)
\[ \text{and} \quad \lambda_t^C = c_0. \] (3.15)
This primary-secondary framework makes it easy to construct the doubly stochastic Poisson (Cox) processes used in Section 3.2.1, it breaks the recursiveness in the definition of the default times and thus belong to the class of models that satisfy the no-jump condition. However, it only works around the problem by reducing the complexity of the model, and it still encounters tedious integrations as shown in (JY), and in some cases, the counterparty’s default plays an impact (or indicate an economy-wide turbulence indirectly) back onto the reference entity, which cannot simply be ignored.

### 3.2.6 Change of Measure

In order to overcome the recursive default intensity with no loss of generation, we adopt the change of measure introduced by Collins-Dufresne et al. (2002) in the valuation procedure of the swap rate, to demonstrate that the conditional survival probabilities possess simple analytic solutions. We define firm-specific probability measures \( \mathbb{P}^i |_{i=B,C} \) by

\[
Z^i_T \equiv \frac{d\mathbb{P}^i}{d\mathbb{P}} |_{\mathcal{F}^*_T} = 1_{\{\tau^i > T\}} \exp \left[ \int_0^T \lambda^i_s \, ds \right].
\]

and let \( \mathcal{F}^i = (\mathcal{F}^i_t |_{i=B,C})_{t \geq 0} \) denote the corresponding enlarged completed filtrations to proceed the calculation under measure \( \mathbb{P}^i \). One can show that \( \mathbb{P}^i \) is absolutely continuous w.r.t. \( \mathbb{P} \) on the stochastic interval \( [\tau^i, +\infty) \), it follows that \( Z^i_T \) is a uniformly integrable \( \mathbb{P} \)-martingale with respect to \( \mathcal{F}^i \), and \( \{\tau^C \leq T\} \) is a null set of the probability measure \( \mathbb{P}^C \). This implies that the intensity of firm B is almost surely constant \( (\lambda^B_T = b_0 + b_1 1_{\{\tau^C \leq T\}} = b_0) \) under probability measure \( \mathbb{P}^C \). As a result, when computing the survival probability of firm C conditional on the event \( \{\tau^B > T\} \), the potential impact of a jump in the intensity of firm C on B is effectively ignored. The intuition is that we are only interested in those paths where firm C does not default, we can ignore those paths where the intensity of firm B jumps before the survival horizon. An analogous argument also holds under the measure \( \mathbb{P}^B \).
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3.3 Continuous-Time Framework

In this section, instead of carrying on using the discrete-time premium rates from Jarrow & Yu (2001) [48], we extend it to a continuous-time framework. A much more meaningful result is derived at the end of this chapter.

We denote $E^C$ as the expectation taken under the measure $P^C$. For $t_1 < t_2$, the joint distribution for the pair of default times is

$$
E^C \left[ 1_{\{\tau^B > t_1, \tau^C > t_2\}} \right] = e^{-c_0 t_2} E^C \left[ 1_{\{\tau^B > t_1, \tau^C > t_2, \tau^B < \tau^C\}} \right]
$$

Equation (3.17) follows from the fact that $\lambda^B_t = b_1$ for $t \leq t_2$ under $P^C$

$$
E^C \left[ 1_{\{\tau^B > t_2\}} \right] = E \left[ \tau^B > t_2 \right] = E^C \left[ \exp \left( - \int_0^{t_2} \lambda^B_s ds \right) \right] = e^{-b_0 t_2} E^C \left[ \exp \left( - \int_0^{t_2} (c_0 + c_1 1_{\{\tau^B \leq s\}}) ds \right) \right]
$$

Similarly, for $t_2 \leq t_1$, the joint distribution of default times is given by

$$
P \left[ \tau^B > t_1, \tau^C > t_2 \right] = c_0 e^{-(b_0 + b_1) t_1} \left[ \frac{e^{(b_1 - c_0) t_2} - e^{(b_1 - c_0) t_1}}{c_0 - b_1} \right] + e^{-(b_0 + c_0) t_2}.
$$

Therefore, by setting $t_2 = 0$ (when $t_2 < t_1$) and $t_1 = 0$ (when $t_1 < t_2$) respectfully, the marginal survival probabilities are obtained:
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\[
P[\tau^B > t_1] = \frac{c_0 e^{-(b_0 + b_1)t_1} - b_1 e^{-(b_0 + c_0)t_1}}{c_0 - b_1}, \quad t_1 \leq T, \quad (3.20)
\]

\[
P[\tau^C > t_2] = \frac{b_0 e^{-(c_0 + c_1)t_2} - c_1 e^{-(b_0 + c_0)t_2}}{b_0 - c_1}, \quad t_2 \leq T. \quad (3.21)
\]

Differentiating the joint default probability with respect to \( t_1 \) and \( t_2 \) gives

\[
f(t_1, t_2) = \frac{\partial^2 P[\tau^B > t_1, \tau^C > t_2]}{\partial t_1 \partial t_2} = \begin{cases} 
  c_0 (b_0 + b_1) e^{-(b_0 + b_1)t_1} - (c_0 - b_1) e^{-(c_0 + c_1)t_2}, & t_2 \leq t_1, \\
  b_0 (c_0 + c_1) e^{-(c_0 + c_1)t_2} - (b_0 - c_1) e^{-(b_0 + c_0)t_1}, & t_2 > t_1.
\end{cases} \quad (3.22)
\]

Differentiating the marginal survival probabilities gives the marginal density of the default times \( \tau^B \) and \( \tau^C \):

\[
f(t_1) = \frac{d P[\tau^B > t_1]}{dt_1} = \frac{(b_0 + c_0) b_1 e^{-(b_0 + c_0)t_1} - (b_0 + b_1) c_0 e^{-(b_0 + b_1)t_1}}{c_0 - b_1}, \quad (3.23)
\]

and

\[
f(t_2) = \frac{d P[\tau^C > t_2]}{dt_2} = \frac{(b_0 + c_0) c_1 e^{-(b_0 + c_0)t_2} - (c_0 + c_1) b_0 e^{-(c_0 + c_1)t_1}}{b_0 - c_1}. \quad (3.24)
\]

### 3.4 Pricing Credit Derivatives Swaps

To simplify our CDS valuation, without loss of generality, we assume a flat term structure of risk-free rate \( r \) and furthermore, a zero recovery rate with the notional of $1. We modified the model introduced by Jarrow & Yu and relaxed the assumption that each party is obligated to pay until its own default, regardless of whether the other party has defaulted or not.

\footnote{Without loss of analytical tractability, JY’s framework can be extended to stochastic interest rate within the class of affine structure.}
The market value of A’s fixed rate premium leg at time 0 is
\[
\mathbb{E} \left[ \int_0^T \exp \left( - \int_0^s r_u du \right) p \mathbf{1}_{\{\tau_B \wedge \tau_C > s\}} ds \right].
\] (3.25)
in which \( p \) is the swap premium rate fixed at the beginning of the contract. Given either B or C’s default at certain point, party A is still obligated to pay off the residual amount that accumulates from the last payment date till the default date (when the contract terminates) in proportion. Using the continuous premium rate other than its discrete counterpart simplifies this process caused between quarterly payment dates.

The time-0 market value of B’s promised payment (protection leg) in the event of C’s default is
\[
\mathbb{E} \left[ \mathbf{1}_{\{\tau_C \leq T\}} \exp \left( - \int_0^{\tau_C + \delta} r_u du \right) \mathbf{1}_{\{\tau_B > \tau_C + \delta\}} \right].
\] (3.26)
in which \( \delta \) is the length of the settlement period, hence \( \tau_C + \delta \) represents the settlement date at the end of the settlement period.

Since it takes no cost to enter a CDS contract at the deal date, by equating the premium leg and protection leg, as in equation (3.25) and (3.26), we get the swap premium:
\[
p = \frac{\mathbb{E} \left[ \mathbf{1}_{\{\tau_C \leq T\}} \exp \left( - \int_0^{\tau_C + \delta} r_u du \right) \mathbf{1}_{\{\tau_B > \tau_C + \delta\}} \right]}{\mathbb{E} \left[ \int_0^T \exp \left( - \int_0^s r_u du \right) \mathbf{1}_{\{\tau_B \wedge \tau_C > s\}} ds \right]}.
\] (3.27)

The buyer may face potential replacement cost when party B defaults prior to the reference asset C, \( \tau_B < \min(\tau_C, T) \), however, since \( p \) represents the fair premium charged by the protection seller B, the replacement cost should not be included in the calculation of the swap premium.

To simplify the calculation, and without loss of generality a flat risk-free rate \( r \) is applied, with zero recovery and a notional of $1.

By applying the result of eq. (3.19) and setting \( t_1 = t_2 = s \), the premium leg is found to be:
where \( P \left[ \tau^B > s, \tau^C > s \right] = \mathbb{E} \left[ 1_{\{\tau^B > s\}} 1_{\{\tau^C > s\}} \right] = e^{- (b_0 + c_0) s} = \mathbb{E} \left[ 1_{\{\tau^B > s\}} \right] \mathbb{E} \left[ 1_{\{\tau^C > s\}} \right] \). 

which means if party B and C default at the same time, then there will be no jump parameter \((b_1 \text{ and } c_1)\) involved, and the intensity rates are almost surely constant \((b_0 \text{ and } c_0)\) under this special case, given the unique nature of the (JY) model.

Therefore, 

\[
\begin{align*}
\mathbb{E} \left[ \int_0^T e^{-rs} p 1_{\{\tau^B \wedge \tau^C > s\}} ds \right] &= p \int_0^T \mathbb{E} \left[ e^{-rs} 1_{\{\tau^B \wedge \tau^C > s\}} \right] ds \\
&= p \int_0^T \mathbb{E} \left[ e^{-rs} 1_{\{\tau^B > s\}} 1_{\{\tau^C > s\}} \right] ds \\
&= p \int_0^T \left[ e^{- (b_0 + c_0 + r) s} \right] ds \\
&= p \cdot \frac{1 - e^{- (b_0 + c_0 + r) T}}{b_0 + c_0 + r}.
\end{align*}
\]

The default leg is found out to be, using the same change of measure introduced above:
\[
E \left[ 1_{\{\tau C \leq T\}} e^{-r(\tau C + \delta)} 1_{\{\tau B > \tau C + \delta\}} \right] \\
= E^B \left[ 1_{\{\tau C \leq T\}} e^{-r(\tau C + \delta)} \exp \left( -\int_0^{\tau C + \delta} (b_0 + b_1 1_{\{\tau C \leq u\}}) du \right) \right] \\
= E^B \left[ 1_{\{\tau C \leq T\}} e^{-(r+b_0)(\tau C + \delta)} \exp \left( -b_1 1_{\{\tau C \leq \tau C + \delta\}} (\tau C + \delta - \tau C) \right) \right] \\
= E^B \left[ 1_{\{\tau C \leq T\}} e^{-(r+b_0)(\tau C + \delta)} \exp \left( -b_1 \delta \right) \right] \\
= e^{-(r+b_0+b_1)\delta} \int_0^T e^{-(b_0 + c_0 + r)s} ds \\
= e^{-(r+b_0+b_1)\delta} c_0 \left[ 1 - e^{-(b_0 + c_0 + r)T} \right]. \\
(3.30)
\]

Setting eq. (3.29) equal to eq. (3.30) gives the premium rate

\[
p = e^{-(r+b_0+b_1)\delta} c_0. \\
(3.31)
\]

### 3.5 Interpretation of the Results

The result we obtained in eq. (3.31) is an interesting one, imagine the settlement period equals to 0, which means party B and C default at the same time, the premium rate depends only on the intensity rate \(c_0\). In general, the jump factor based on party B’s default \((c_1)\) disappears in the calculation, whereas the default intensity factor \(b_0\) and \(b_1\) (based on C’s default) both matter in the swap premium derivation. However, one cannot tell whether the default intensity of party B \((\tau B)\) comes from the constant \(b_0\) only or the combination of \(b_0\) and \(b_1\), however, the higher the default intensity of B itself \((b_0)\), the lower the premium, and given \(b_0\) fixed, the higher the \(b_1\), the lower the premium rate.

Figure (3.1) below shows the premium rate based on eq. (3.31), by varying the settlement period and the counterparty B’s total intensity \((b_0 + b_1)\). Setting \(r = 0.05, c_0 = 0.1,\) and settlement period \(\delta\) varies from 0 to 1 year, and the total intensity rate of B \((b_0 + b_1)\) varies from 0 to 1. As one can observe in the figure,
Figure 3.1: Effect of settlement period and intensity rates on swap premium. Setting $r = 0.05, c_0 = 0.1$, and settlement period $\delta$ varies from 0 to 1 year, and the total intensity rate of B $(b_0 + b_1)$ varies from 0 to 1. As one can observe, once the intensity of party B goes higher, the premium rate drops exponentially with the increase of settlement date.

If the settlement period for B is short, the premium stays relatively close to C’s default intensity $c_0$, the total intensity of B does not have a big impact, however, if the settlement period is longer, the premium rate drops exponentially with the increase of total intensity. And vice versa, if the total intensity for B is low, the premium stays relatively close to C’s default intensity $c_0$, the settlement period can be ignored. However, if the total intensity of party B goes higher, the premium rate drops exponentially with the length of the settlement period.

This result makes practical sense, as if a protection buyer A gets into a CDS contract with a counterparty that has a poor credit rating instead of one with a good rating, on the same reference entity, A would expect to pay less premium, as there is a higher chance that the protection seller B goes bust either before the reference entity C defaults or within the settlement period once C defaults, which leaves A end up paying all the premium over the years and not being covered for
the potential default of C. Notice here $b_1$ represents the correlation between the reference entity C and the counterparty B. This means if two different protection sellers are similar rated (same default intensity $b_0$), a higher correlation between the reference entity and protection seller ($b_1$) indicates a higher chance for B to default together with C, therefore, a lower premium rate.

Given the same default intensity of the counterparty and reference entity, if it takes longer settlement period for the protection seller to pay back the loss agreed in the contract, it indicates that the counterparty is having a problem re-financing the payment, or in other words, is more likely to default due to the loss caused by the default of the reference entity, and hence lower the swap premium.

Notice here $c_1$, the jump on C’s intensity rate due to B’s default, disappears out of the equation due to the change of measure. This shows that once the counterparty default, it will not influence the credit rating of the underlying. This is similar to the case when an option seller defaults, it does not matter for the company issuing the underlying stock, as there is no direct loss in terms of contractual payment, the contract terminates automatically with no transaction needed. Whereas if the reference entity defaults, counterparty B is obliged a large contractual amount of the underlying notional to A, and it matters to B’s credibility how fast it can gather the contractual amount internally to repay A due to C’s default. Another way to explain why $b_1$ plays an important role here.

3.6 Conclusion

This chapter extends the contagion model originally introduced by Jarrow & Yu (2001) by applying the change of measure technique derived by Collin-Dufresne et al. (2002) for pricing defaultable securities. This solves the problem of violation of jump conditions under Cox processes for defaultable credit derivatives, instead of assuming a primary-secondary framework to overcome the tedious integrations. The author found a more simple and meaningful
result to the swap premium calculation by introducing continuous swap pre-
mium rate instead of discrete premium payment (commonly found in market
practice) without loss of generality. We found that given a fixed settlement pe-
riod after the reference entity’s default, a higher default intensity of protection
seller leads to a lower swap premium. On the other hand, by fixing the inten-
sity rate constant, a longer the settlement period indicates a more likely event
that the counterparty defaults within the settlement period, this also means
a protection buyer would like to pay less for protection against the default of
reference entity.
Chapter 4

Three-firm Model

Notation of Variables

\[\lambda_t^A\] default intensity of protection buyer

\[\lambda_t^B\] default intensity of protection seller

\[\lambda_t^C\] default intensity of reference entity

\[a_0, b_0,\text{ and } c_0\] default intensity of firm A, B, and C without the jump intensities

\[a_1, a_2\] jump intensity on A when B, or C defaults

\[b_1, b_2\] jump intensity on B when A, or C defaults

\[c_1, c_2\] jump intensity on C when A, or B defaults

\[f(u_1, u_2)\] joint density of \((\tau^A, \tau^B)\)

\[p\] swap premium

\[P[\tau^A > t_1, \tau^B > t_2, \tau^C > t_3]\] joint survival probability

\[f(t_1, t_2, t_3)\] joint density function

\[v\] the time variant component on top of constant intensities

\[h\] the volume of the flow
In order to study the effect of correlated default times among all parties in a CDS on the swap premium, this chapter extends the contagion counterparty risk model to a three-firm model. This means the protection buyer, party A, is defaultable as well as the protection seller B and reference entity C. We found that protection seller A’s possibility to default does not play an as important role as counterparty B and reference entity C’s default, in fact, some of the parameters linking A’s default impact to B or C disappear during the derivation to the swap rate, and by assuming B’s jump intensity due to A’s default to be relatively small, we get the same results as the two-firm model.

Finally, in order to relax some of the assumptions of the contagion framework to make the findings more general, we make the intensity rates time-variant by experiencing an economy upturn and downturn every decade, and demonstrated in a two-firm defaultable environment, and find that the general findings still hold. In fact if the settlement period is small enough, we find that the swap premium is equal to the one in the constant intensity case.

4.1 Joint Default Probability

The default risk structure is specified by the inter-dependent default intensities:

\[
\begin{align*}
\lambda^A_t &= a_0 + a_1 \mathbf{1}_{(\tau^A \leq t)} + a_2 \mathbf{1}_{(\tau^C \leq t)}, \\
\lambda^B_t &= b_0 + b_1 \mathbf{1}_{(\tau^A \leq t)} + b_2 \mathbf{1}_{(\tau^C \leq t)}, \\
\lambda^C_t &= c_0 + c_1 \mathbf{1}_{(\tau^A \leq t)} + c_2 \mathbf{1}_{(\tau^A \leq t)}. 
\end{align*}
\] (4.1) (4.2) (4.3)

The setting above shows that the default probability of each party in the CDS contract depends on the default status of other firms. It can be reduced to the two-firm model mentioned above by setting \(a_0 = a_1 = a_2 = 0\). If we take \(c_1 = c_2 = 0\), the default intensities of both the counterparties in the CDS do not affect the credit rating of the reference entity. This case can be financially
interpreted as the reference asset, say a risky bond, is issued by a large firm C whose default has an economy-wide impact. A small firm A holds this bond and wants to be protected from the credit risk upon C’s default, therefore A enters into a CDS with protection seller B, who has correlated default risk with A upon each other’s default as well as A’s default. Similarly, if we take $b_1 = b_2 = 0$, we can interpret party B as a large financial institution that has an economy-wide impact on default. Therefore, reference entity C and protection buyer A’s default intensity will jump upon B’s default.

Given the setting above for three defaultable parties in the CDS contract, one can derive the joint distribution of the three default times. For $t_1 < t_2 < t_3$:

$$\mathbb{P} \left[ \tau^A > t_1, \tau^B > t_2, \tau^C > t_3 \right]$$

$$= \mathbb{E} \left[ 1_{\{\tau^A > t_1\}} 1_{\{\tau^C > t_3\}} \right]$$

$$= \mathbb{E}^C \left[ 1_{\{\tau^A > t_1\}} 1_{\{\tau^B > t_2\}} \exp \left( - \int_0^{t_3} \lambda^C ds \right) \right]$$

$$= \mathbb{E}^C \left[ 1_{\{\tau^A > t_1\}} 1_{\{\tau^B > t_2\}} \exp \left( - \int_0^{t_3} \left( c_0 + c_1 1_{\{\tau^A \leq t\}} + c_2 1_{\{\tau^B \leq t\}} \right) ds \right) \right]$$

$$= e^{-c_{0t_3}} \mathbb{E}^C \left[ 1_{\{\tau^A > t_1\}} 1_{\{\tau^B > t_2\}} \exp \left( - \left( t_3 - \tau^A \right) c_1 1_{\{\tau^A \leq t_3\}} - \left( t_3 - \tau^B \right) c_2 1_{\{\tau^B \leq t_3\}} \right) \right].$$

(4.4)

Note that

$$1_{\{\tau^A > t_1\}} 1_{\{\tau^B > t_2\}}$$

$$= 1_{\{t_1 < \tau^A \leq t_2\}} 1_{\{t_2 < \tau^B \leq t_3\}} + 1_{\{t_2 < \tau^A \leq t_3\}} 1_{\{t_2 < \tau^B \leq t_3\}} + 1_{\{\tau^A > t_2\}} 1_{\{\tau^B > t_3\}}$$

$$+ 1_{\{t_1 < \tau^A \leq t_2\}} 1_{\{\tau^B > t_3\}} + 1_{\{t_2 < \tau^A \leq t_3\}} 1_{\{\tau^B > t_3\}} + 1_{\{\tau^A > t_2\}} 1_{\{\tau^B > t_3\}}.$$  

(4.5)

Therefore, we could derive:
\[ \mathbb{P} \left[ \tau^A > t_1, \tau^B > t_2, \tau^C > t_3 \right] \]

\[= e^{-\left(c_0 + c_1 + c_2\right) t_3} \mathbb{P}^C \left[ \mathbf{1}_{(t_1 < \tau^A \leq t_2)} \mathbf{1}_{(t_2 < \tau^B \leq t_3)} \mathbf{e}^{c_1 \tau^A + c_2 \tau^B} \right] \]

\[+ e^{-\left(c_0 + c_1 + c_2\right) t_3} \mathbb{P}^C \left[ \mathbf{1}_{(t_2 < \tau^A \leq t_3)} \mathbf{1}_{(t_2 < \tau^B \leq t_3)} \mathbf{e}^{c_1 \tau^A + c_2 \tau^B} \right] \]

\[+ e^{-\left(c_0 + c_1 + c_2\right) t_3} \mathbb{P}^C \left[ \mathbf{1}_{(\tau^A > t_2)} \mathbf{1}_{(t_2 < \tau^B \leq t_3)} \mathbf{e}^{c_2 \tau^B} \right] \]

\[+ e^{-\left(c_0 + c_1 + c_2\right) t_3} \mathbb{P}^C \left[ \mathbf{1}_{(t_1 < \tau^A \leq t_2)} \mathbf{1}_{(\tau^B > t_2)} \mathbf{e}^{c_1 \tau^A} \right] \]

\[+ e^{-\left(c_0 + c_1 + c_2\right) t_3} \mathbb{P}^C \left[ \mathbf{1}_{(t_2 < \tau^A \leq t_3)} \mathbf{1}_{(\tau^B > t_2)} \mathbf{e}^{c_1 \tau^A} \right] \]

\[+ e^{-\left(c_0 + c_1 + c_2\right) t_3} \mathbb{P}^C \left[ \mathbf{1}_{(\tau^A > t_3)} \mathbf{1}_{(\tau^B > t_3)} \right]. \quad (4.6) \]

Under the measure \( \mathbb{P}^C \) and for \( t < t_3 \), the default intensities \( \lambda_t^A \) and \( \lambda_t^B \) are given by

\[ \lambda_t^A = a_0 + a_1 \mathbf{1}_{(\tau^A \leq t)}, \quad (4.7) \]

\[ \lambda_t^C = c_0 + c_2 \mathbf{1}_{(\tau^C \leq t)}. \quad (4.8) \]

Using the joint density of \( (\tau^A, \tau^B) \)

\[ f(u_1, u_2) = a_0 (b_0 + b_1) e^{-(b_0 + b_1)u_2 - (a_0 - b_1)u_1}, \quad u_1 < u_2. \quad (4.9) \]

one can compute \( \mathbb{P}^C \left[ \mathbf{1}_{(t_1 < \tau^A \leq t_2)} \mathbf{1}_{(\tau^B > t_3)} \mathbf{e}^{c_1 \tau^A} \right] \) and other similar terms.

Once we have obtained \( \mathbb{P} \left[ \tau^A > t_1, \tau^B > t_2, \tau^C > t_3 \right] \), we differentiate the distribution function w.r.t. \( t_1, t_2 \) and \( t_3 \) to give the joint density function

\[ f(t_1, t_2, t_3) = a_0 (b_0 + b_1) (c_0 + c_1 + c_2) e^{-(a_0 - b_1 - c_1) t_1 - (b_0 + b_1 - c_2) t_2 - (c_0 + c_1 + c_2) t_3} \]

for \( t_1 < t_2 < t_3 \). \quad (4.10)

The rest sequence of \( t_1, t_2 \) and \( t_3 \) can be obtained similarly as:
\[ f(t_1, t_2, t_3) = \begin{cases} 
  a_0 (b_0 + b_1) (c_0 + c_1 + c_2) \\
  \times e^{-(a_0-a_1-c_1)t_1-(b_0+b_1-c_2)t_2-(c_0+c_1+c_2)t_3}, & t_1 < t_2 < t_3, \\
  a_0 (c_0 + c_1) (b_0 + b_1 + b_2) \\
  \times e^{-(a_0-b_1-c_1)t_1-(b_0+b_1+b_2)t_2-(c_0+c_1-b_2)t_3}, & t_1 < t_3 < t_2, \\
  b_0 (a_0 + a_1) (c_0 + c_1 + c_2) \\
  \times e^{-(a_0+a_1-c_1)t_1-(b_0-a_1-c_2)t_2-(c_0+c_1+c_2)t_3}, & t_2 < t_1 < t_3, \\
  b_0 (c_0 + c_2) (a_0 + a_1 + a_2) \\
  \times e^{-(a_0+a_1+a_2)t_1-(b_0-a_1-c_2)t_2-(c_0+c_2-a_2)t_3}, & t_2 < t_3 < t_1, \\
  c_0 (a_0 + a_2) (b_0 + b_1 + b_2) \\
  \times e^{-(a_0+a_2-b_1)t_1-(b_0+b_1+b_2)t_2-(c_0-a_2-b_2)t_3}, & t_3 < t_1 < t_2, \\
  c_0 (b_0 + b_2) (a_0 + a_1 + a_2) \\
  \times e^{-(a_0+a_1+a_2)t_1-(b_0+b_2-a_1)t_2-(c_0-a_2-b_2)t_3}, & t_3 < t_2 < t_1. 
\end{cases} \] (4.11)

4.2 Swap Premium

Similar to the swap premium rate calculation in section 3.4, we assume a flat term structure of risk-free rate \( r \), and a zero recovery rate with the notional of $1. The market value of A’s fixed rate premium leg at time 0 is

\[
\mathbb{E} \left[ \int_0^T \exp \left( - \int_0^s r_u du \right) p1_{\{\tau_A < \tau_B \land \tau_C > s\}} ds \right] \tag{4.12}
\]

in which \( p \) is the swap premium rate fixed at the beginning of the contract. Here the contract terminates if any of the counterparties or the reference entity default before the maturity. Again, we favour using the continuous function other than discrete type, so we do not need to worry about the premium payment between the recent premium payment date (usually quarterly) and default date.

The time-0 market value of B’s promised payment (protection leg) in the
event of C’s default is
\[
E \left[ 1_{\{\tau_C \leq T\}} \exp \left( - \int_0^{\tau_C + \delta} r_u du \right) 1_{\{\tau_A > \tau_C\}} 1_{\{\tau_B > \tau_C + \delta\}} \right]
\]
\hspace{1cm} (4.13)

in which \( \delta \) is the length of the settlement period, hence \( \tau_C + \delta \) represents the settlement date at the end of the settlement period. One more condition is added compared to the two-firm case, that protection buyer A does not default before the reference entity. Same conditions applies for protection seller B that will survive until the contract default payment is cleared off.

Since it takes no cost to enter a CDS contract at the deal date, by equalling the premium leg and protection leg, as in equation (4.12) and (4.13), we get the swap premium:

\[
p = \frac{E \left[ 1_{\{\tau_C \leq T\}} \exp \left( - \int_0^{\tau_C + \delta} r_u du \right) 1_{\{\tau_A > \tau_C\}} 1_{\{\tau_B > \tau_C + \delta\}} \right]}{E \left[ \int_0^T \exp \left( - \int_0^T r_u du \right) 1_{\{\tau_A \land \tau_B \land \tau_C > s\}} ds \right]}
\]
\hspace{1cm} (4.14)

Again we assume constant risk-free rate \( r \) and zero recovery on a notional of $1.

Using the joint density function \( f(t_1, t_2, t_3) \) for \( t_3 < t_1 < t_2 \), we obtain the default leg:

\[
E \left[ 1_{\{\tau_C \leq T\}} \exp \left( - \int_0^{\tau_C + \delta} r_u du \right) 1_{\{\tau_A > \tau_C\}} 1_{\{\tau_B > \tau_C + \delta\}} \right]
\]
\hspace{1cm} = \int_0^T \int_{\tau_C + \delta}^\infty \int_{\tau_C}^\infty e^{-r(t_C + \delta)} f(t_A, t_B, t_C) dt_A dt_B dt_C | t_3 < t_1 < t_2
\]
\hspace{1cm} = c_0 (a_0 + a_2) (b_0 + b_1 + b_2) \int_0^T \int_{\tau_C + \delta}^\infty \int_{\tau_C}^\infty e^{-r(t_C + \delta)}
\]
\times \exp \left( - (a_0 + a_2 - b_1) t_A - (b_0 + b_1 + b_2) t_B - (c_0 - a_2 - b_2) t_C \right) dt_A dt_B dt_C
\]
\hspace{1cm} = \frac{c_0 (a_0 + a_2) e^{-(b_0 + b_1 + b_2 + r) \delta}}{a_0 + a_2 - b_1} \cdot \frac{1 - e^{-(a_0 + b_0 + c_0 + r) T}}{a_0 + b_0 + c_0 + r}.
\]
\hspace{1cm} (4.15)
CHAPTER 4. THREE-FIRM MODEL

The premium leg can be handled in a similar fashion as that in the two-firm model:

\[
E \left[ \int_0^T \exp \left( - \int_0^s r_u du \right) p_1 \mathbf{1}_{\{\tau_A < \tau_B \wedge \tau_C > s\}} ds \right] \\
= p \int_0^T E \left[ e^{-rs} \mathbf{1}_{\{\tau_A > s\}} \mathbf{1}_{\{\tau_B > s\}} \mathbf{1}_{\{\tau_C > s\}} \right] ds \\
= p \int_0^T e^{-rs} E^A \left[ \mathbf{1}_{\{\tau_B > s\}} \mathbf{1}_{\{\tau_C > s\}} \exp \left( - \int_0^T (a_0 + a_1 \mathbf{1}_{\{\tau_B \leq s\}} + a_2 \mathbf{1}_{\{\tau_C \leq s\}}) ds \right) \right] \\
= p \int_0^T e^{-(a_0 + r)T} e^{-(b_0 + c_0)T} ds \\
= p \frac{1 - e^{-(a_0 + b_0 + c_0 + r)T}}{a_0 + b_0 + c_0 + r}. \tag{4.16}
\]

using equation \[4.2\], the premium rate is easily derived:

\[
p = \frac{E \left[ \mathbf{1}_{\{\tau_C \leq T\}} \exp \left( - \int_0^{\tau_C + \delta} r_u du \right) \mathbf{1}_{\{\tau_A > \tau_C\}} \mathbf{1}_{\{\tau_B > \tau_C + \delta\}} \right]}{E \left[ \int_0^T \exp \left( - \int_0^s r_u du \right) \mathbf{1}_{\{\tau_A \wedge \tau_B \wedge \tau_C > s\}} ds \right]} = \frac{c_0 (a_0 + a_2) e^{-(b_0 + b_1 + b_2 + r)\delta}}{a_0 + a_2 - b_1}. \tag{4.17}
\]

4.3 Interpretation of Three-firm Model Premium

Similar to the results of the two-firm model in eq.\[3.31\], in the premium for three-firm model in eq.\[4.17\], the correlation parameter due to the protection seller B’s default on C (c_2) does not count as part of the premium calculation, as a matter of fact, B’s default impact for A does not matter either as a1 goes out of the picture as well. In reality, once B defaults as a protection seller, neither C or A should be influenced unless they are holding a large amount of debt or defaultable bond from B, as there are no contractual payments in the CDS contract once B defaults. The only possible loss is the replacement cost for A. In order to protect itself from underlying C’s default, A needs to go to the market to enter a new CDS contract. The loss generated from the difference
of the old and new premium rates (if the mark-to-market CDS price is higher) will be the replacement cost to A.

Similar case for A, as no contractual payments are generated either by B or C once A defaults. Note the impact of A’s default on reference entity C, namely $c_1$, went out of the equation; and furthermore, by setting $b_1$, which is represented as the impact of A’s default on protection seller B, equal to zero, the premium for the three firm model equals to $c_0 e^{-(b_0+b_2+r)\delta}$, which is the same premium rate for the two-firm model, as stated in eq. (3.31). Which means the default of counterparty B and protection buyer A do not play a big role on the swap premium, even given that A’s defaultable. This is further supported by the discovery that both $c_1$ and $c_2$ disappeared, which leaves only the default intensity of C before either A or B’s default ($c_0$), to be essential for the swap premium.

Now let us have a look at the jump parameters for B’s intensity rate, which represents the counterparty risk, the main topic of this thesis. Both $b_1$ and $b_2$ stay in the premium calculation, although given $b_1$ relatively small, the premium rates gets close to the one in the two-firm case. Again, we cannot tell apart $b_0$ from $b_2$, as given all the other parameters fixed, a lower premium rate can be generated from either a higher $b_0$, or a higher combination of $b_0$ and $b_2$.

By setting the default intensity of the protection seller $a_0$, protection buyer $b_0$ and reference entity $c_0$ all equal to 0.05, interest rate $r = 0.05$, length of settlement period $\delta = 0.25$, and intensity parameters due to other parties’ default: $b_1 = 0.02$, $b_2 = 0.02$, $a_2 = 0.02$, the swap premium is $p = 0.0676$. Figure 4.1 below shows the dynamic of CDS premium by varying each individual parameter given others fixed.

As we can see from the graph $c_0$, $a_0$, and $a_2$ are more influential as a driving factor than that of $b_0$, $b_1$, and $b_2$, it is obvious that $c_0$ as the default intensity of reference entity itself plays an important role due to its direct relationship with the default probability of C. If the default intensity of reference entity is high,
Figure 4.1: CDS premium changes by varying each individual parameter in eq. (4.17), by setting $c_0 = 0.05$, $r = 0.05$, $\delta = 0.25$, $b_0 = 0.05$, $b_1 = 0.02$, $b_2 = 0.02$, $a_0 = 0.05$, $a_2 = 0.02$, the swap premium $p = 0.0676$. 
then the protection seller B should charge A for large premium. The reason why \( a_0 \) and \( a_2 \) produce such a steep slope is that, given \( b_1 \) is relatively small, \( \frac{a_0 + a_2}{a_0 + a_2 - b_1} \) increases more dramatically than that of \( b_0, b_1, \) and \( b_2 \) in an exponential speed. In reality, \( b_1 \) as the jump factor on B conditional on A’s default, should be relatively small. This is because in the CDS contract, only B is potentially obliged to pay A a notional amount, not the other way round, so theoretically A’s default should not disturb B’s credit quality.

The other factors are in line with practical sense, if the risk-free rate goes up, that means the future payments are further discounted back to today, hence, a smaller premium. The longer the settlement period, the more likely B will be facing financial trouble to fulfill the contract requirement once C defaults, and the lower the premium. The higher the combination of \( b_0, b_1, \) and \( b_2 \), the more likely the counterparty B is going to default due to either the credit quality of its own, or external influence caused by either A or B’s default, note the curves for them should be exponentially shaped, although the graph does not show significantly due to the small quantity of the intensity rates.

4.4 Time Variant Intensity Rate

To extend the model even further and relax the constant intensity rate assumption. We assume the intensity rate is time variant and following a flow as follows:

This could be interpreted as the upturn and downturn of the status of economy.

Applying the results on the two-firm model, we get the framework as:

\[
\begin{align*}
\lambda_t^B &= b_0 + v + b_1 1_{(\tau^C \leq t)} \\
\lambda_t^C &= c_0 + v + c_1 1_{(\tau^B \leq t)}.
\end{align*}
\]  

(4.18)

(4.19)

in which \( v \) represents the time variant component, if set \( v = \frac{\sin(2t/\pi)}{h}, h \)
Define default intensity $\lambda_t^B = b_0 + v + b_1 \mathbf{1}_{\{\tau_C \leq t\}}$ in which $V = \frac{\sin(2t/\pi)}{h}$. By setting $b_0 = 0.05$, $b_1 = 0.15$ and $h = 100$, we obtain the process of $\lambda_t^B$ as above, this is to represent the economic cycle every 10 years.

Figure 4.2: Time variant jump on intensity rate

defines the volume of the flow, the smaller the $h$, the more volatile the wave and vice versa. For example, by setting $b_0 = 0.05$, $b_1 = 0.15$ and $h = 100$, and we assume the business cycle experiences an upturn and downturn every decade, we have a plot as shown in figure (4.2). Say the reference entity defaults at the end of 6 year, which generates a jump in the counterparty intensity rate, therefore the grey line after 6 years shows the process of the intensity when no default happens, the light blue curve after 6 years shows the process once the reference entity defaults, which is shifted by 1.5 percent above the grey line.

Therefore taking the two-firm model, for $t_1 < t_2$, the joint default probability is:
\[ P [ \tau^B > t_1, \tau^C > t_2 ] = \mathbb{E} \left[ 1_{(\tau^B > t_1)} 1_{(\tau^C > t_2)} \right] \]

\[ = \mathbb{E}^C \left[ 1_{(\tau^C > s)} \exp \left( - \int_0^{t_2} \left( c_0 + v + c_1 1_{(\tau^N \leq s)} \right) ds \right) \right] \]

\[ = e^{-c_0 t_2 + k} \mathbb{E}^C \left[ 1_{(t_1 < \tau^B < t_2)} \exp \left( - c_1 (t_2 - \tau^B) \right) + 1_{(\tau^N > t_2)} \right] \]

\[ = e^{-c_0 t_2 + k} \int_{t_1}^{t_2} b_0 e^{-b_0 u - c_1 (t_2 - u)} du + \int_{t_2}^{t_1} b_0 e^{-b_0 u} du \]

\[ = b_0 e^{-(c_0 + c_1) t_2 + k} \left[ e^{(c_1 - b_0) t_1} - e^{(c_1 - b_0) t_2} \right] b_0 - c_1 + e^{-(b_0 + c_0) t_2 + k}. \] (4.20)

in which

\[ k = \int_0^{t_2} \sin \left( \frac{2t}{\pi} \right) h ds = \frac{\pi}{2h} \cos \left( \frac{2t_2}{\pi} \right) - 1. \]

Similarly, for \( t_2 \leq t_1 \), the joint distribution of default times is given by

\[ P [ \tau^B > t_1, \tau^C > t_2 ] = c_0 e^{-(b_0 + b_1) t_1 + k} \left[ e^{(b_1 - c_0) t_2} - e^{(b_1 - c_0) t_1} \right] \frac{c_0 - b_1}{c_0 - b_1} + e^{-(b_0 + c_0) t_2 + k}. \] (4.21)

By applying the result of eq. (3.19) and setting \( t_1 = t_2 = s \), the premium leg is found to be:

\[ P [ \tau^B > s, \tau^C > s ] = \mathbb{E} \left[ 1_{(\tau^B > s)} 1_{(\tau^C > s)} \right] = e^{-(b_0 + c_0) s + k}. \] (4.22)

Therefore, following eq. (3.27), we can derive the premium leg as:
CHAPTER 4. THREE-FIRM MODEL

\[
E \left[ \int_0^T e^{-rs} p \mathbf{1}_{\{\tau_B \land \tau_C > s\}} ds \right]
\]

\[
= p \int_0^T E \left[ e^{-rs} \mathbf{1}_{\{\tau_B \land \tau_C > s\}} \right] ds
\]

\[
= p \int_0^T E \left[ e^{-rs} \mathbf{1}_{\{\tau_B > s\}} \mathbf{1}_{\{\tau_C > s\}} \right] ds
\]

\[
= p \int_0^T e^{-(b_0 + c_0 + r)s + k} ds.
\] (4.23)

The default leg is found out to be, using the same change of measure introduced above:

\[
E \left[ \mathbf{1}_{\{\tau_C \leq T\}} e^{-r(\tau_C + \delta)} \mathbf{1}_{\{\tau_B > \tau_C + \delta\}} \right]
\]

\[
= E_B \left[ \mathbf{1}_{\{\tau_C \leq T\}} e^{-r(\tau_C + \delta)} \exp \left( - \int_0^{\tau_C + \delta} (b_0 + v + b_1 \mathbf{1}_{\{\tau_C \leq u\}}) du \right) \right]
\]

\[
= E_B \left[ \mathbf{1}_{\{\tau_C \leq T\}} e^{-(r + b_0)(\tau_C + \delta) + k'} \exp \left( -b_1 \mathbf{1}_{\{\tau_C \leq \tau_C + \delta\}} (\tau_C + \delta - \tau_C) \right) \right]
\]

\[
= e^{-(r + b_0 + b_1)\delta} E_B \left[ \mathbf{1}_{\{\tau_C \leq T\}} e^{-(r + b_0)\tau_C + k'} \right]
\]

\[
= e^{-(r + b_0 + b_1)\delta} \int_0^T c_0 e^{-(b_0 + c_0 + r)\tau_C + k'} d\tau_C
\] (4.24)

in which

\[
k' = - \int_0^{\tau_C + \delta} \frac{\sin (2t/\pi)}{h} ds = \frac{\pi}{2h} \left[ \cos \left( \frac{2(\tau_C + \delta)}{\pi} \right) - 1 \right].
\]

As the \( k \) and \( k' \) in eq. (4.23) and (4.24) both have time variables inside, therefore it is difficult to analytically integrate the last equations in both the fee leg and pay leg, to simplify the equation, we could assume the settlement period \( \delta \) to be relatively small, therefore \( k = k' \), and we get the same result as the constant intensity rate for the two-firm model:

\[
p = e^{-(r + b_0 + b_1)\delta} c_0.
\] (4.25)
Alternatively, numerical integration can be applied, or Monte-Carlo simulation based on the process of the intensity rate.

4.5 Conclusion

To summarise, this chapter extends the two-firm model of the previous chapter by assuming the defaultability of the protection buyer A. We find that making A defaultable does not make a big difference in terms of the swap premium calculation. A’s default only has a small impact on B’s default, and none of A or B’s default changes the underlying reference entity C’s default intensity. If \( b_1 \), B’s intensity jump volume due to A’s default is small enough, we get the same result as in the two-firm model. The settlement period and counterparty B’s total intensity rate still play a crucial role in the swap premium calculation, this is in line with the results from the two-firm model: a longer settlement period increases the counterparty B’s intensity rate, and lower the continuous premium rate protection buyer A would like to pay.

To make the results more general, we also relax the assumptions by extending the intensity rate from constant to time-variant. We design the default intensity of a wave shape with an upturn and downturn every 10 years to represent the macro economy circle. We find that the premium rate shows similar results to the constant intensity rate case. If settlement period is short enough, the time-variant intensity generates the same results to the premium rate with constant intensity. By making the protection buyer defaultable and default intensity time-variant, we obtain some generalised results with a more practical setup, whereas at the same time, conclude some meaningful and close results to the one from Chapter 3.
Chapter 5

Cox–Ingersoll–Ross Model

In this chapter we carry out an empirical Study on Counterparty Risk within CDS spread, using the default spread information extracted from corporate bond data and credit swap premia to provide direct measures on the size of default and non-default components in corporate yield spreads. To explain counterparty risk, a proxy is introduced to represent the average counterparty spread across the financial industry, by taking the difference between 5-year credit spread over benchmark rate, and 5-year credit spread over swap rate. We compare our results to Longstaff et al. (2005)[58] and argue that government bonds remain a good candidate for risk-free rates, the reason swap rate outperforms government bond rate in Longstaff et al. (2005)[58] is because counterparty risk is not properly adapted into the credit spread calculation.

In this chapter we challenge the results of Houweling and Vorst (2005)[39] and many other papers’ conclusion that swap and repo curves outperform the government curve as default-free interest rates. We find that counterparty risk is ignored in most of the studies on default risk study in CDS pricing, where the difference between the swap yield and the government curve can be employed as a good candidate for counterparty premium in CDS pricing. The results of Houweling and Vorst (2005)[39] are relatively insensitive to the value of the assumed recovery rate, which agrees with our findings.
5.1 Pricing Default Swaps

Several features of the default swaps are worth mentioning. If the contract is based on periodic payments, and default occurs, the buyer is required to pay fraction of the premium payment up to date since the last payment, this is called the accrual payment. The credit event may apply to a single reference obligation, but more commonly the events can refer to a much broader class of debt securities, including bonds and loans.

Counterparty risk is generally not taken into account in determining deal prices; if a party is unwilling to take on credit risk to its counterparty, i.e. protection seller in the default swap, it can either cancel the trade or alleviate the exposure, e.g. by demanding a collateral provided from the counterparty, or that the premium is paid up-front instead of periodically.

An important application of default swaps is shorting credit risk. The lack of a market for repurchase agreements (repos) for most corporates makes shorting bonds unfeasible. Therefore, credit derivatives take on the role to short corporate credit risk in a viable way. Even if a bond can be shorted in the repo market, investors can only do so for a relative short period of time, exposing them to changes in the repo rate. On the other hand, default swaps allow investors to go short credit risk at a known cost for longer time spans from 1 year up to 10 years, liquidity rapidly decrease for even longer terms. Typically 5 year credit default swaps are most highly traded credit derivatives, hence least liquidity risk.

O’Kane and McAdie (2001)\textsuperscript{[64]} mention Counterparty Risk and its potential impact in their paper with a comprehensive look at the differences between the cash swap and default swap market. The aim is to explore the differences between the cash and default swap markets for a given credit name and develop a framework to look at these differences.

Several fundamental factors due to the differences between the natures of the contracts cause the spreads of cash swap and default swap to differ. Fun-
fundamental reasons such as the delivery option, risk of technical default, coupon step-ups in the bond, P&L realisation, default swap spreads must be positive and assets trading below par, will increase the default swap basis, whereas factors such as funding, off balance sheet, leverage, accrued interest, counterparty risk and assets trading above par will cause the default swap basis to decrease.

\[
\text{DefaultSwapBasis} = \text{DefaultSwapSpread} - \text{ParFloaterSpread}
\]  

(5.1)

A cash bond is a straightforward transaction between an issuer and a bondholder involving no other credit risk, whereas a default swap is a bilateral over-the-counter derivative transaction which is entered into with a counterparty, protection buyer and seller in this case. This adds the new dimension of counterparty risk to the default swap. The protection buyer will therefore have the tendency to pay a lower spread as compensation against the risk of counterparty - protection seller default, which reduces the default swap basis.

For the protection seller, the exposure is to a loss of mark-to-market caused by a narrowing of spread, due to protection buyer’s default. The size of the loss depends on the level of the spreads at the initial stage of the contract;

For the protection buyer, the risk is when counterparty is unable to make the payment on the default of underlying reference entity. For a physically settled default swap this corresponds to a payment of par when the default asset is delivered. The protection buyer’s exposure can therefore be considerable. Due to the fact that it is an exposure contingent on default, it is highly dependent upon the degree of default correlation between the reference entity and the counterparty. A high default correlation means that it is likely for the default of reference entity to be associated with default of the counterparty, and vice-versa.

For example, buying protection on a German company from a German bank may be considered as a highly correlated trade in which the protection buyer should consider the likelihood that the German bank may be unable to compensate the buyer for the loss generated due to the default of the German company.
The protection buyer will be more willing to pay for this “protection” to a less correlated bank with the German company.

At trade level, the market does not generally price the counterparty risk into default swap spread. The counterparty risk is alleviated in the case that a) either counterparty in the swap contract decides that it is not willing to take on the counterparty risk of another - therefore the trade does not occur; or b) some other means such as dynamic collateral posting agreement is reached.

At market level, counterparty risk is mostly a concern to protection buyers, who have the tendency short the cash bond, although it is unrealistic to short corporate and emerging market sovereign bonds. Counterparty risk is a concern at trade level, it does not play a significant role at a market level. Its effect could cause default swap spreads to narrow as protection buyers require a compensation for their counterparty exposure, or rather pay a higher premium to a higher-rated counterparty.

5.2 Model measuring the default component

In this section, the CIR framework is used to measure the size of the default component in corporate yield spreads. The definition of corporate bond yield spreads is the yield on a corporate bond over the yield on a riskless bond with identical coupon rate and maturity date. This means the yields on risky and riskless bonds with identical promised cashflows are obtained for comparison.

In the sections below, we follow industry practice of assuming that the credit default swaps premium equals to the default component for the firms' bonds. We hence compare the default swap premium for a 5-year contract directly with the corporate spread for a 5-year bond. This approach provides a simple model-independent measure of the percentage size of the default component.
5.3 The Data

The choice of default-free interest rate has been debatable over recent years, interest rate swap curves such as LIBOR/EURIBOR has become favoured by practitioners over the years above the treasury curve/government bond approach. Some argue that it has a relatively greater supply to the market, this provides a better liquidity than government bond, which has a limited amount of supply to the market. However, LIBOR (London Inter-bank Borrow Rate), has become questionable for its credit quality, especially over the crisis period, market participants realise there is a chance that bank who lends the money can default, counterparty risk cannot be ignored, especially in interest rate swaps or credit default swaps deals.

Many papers, such as Longstaff et al. (2005) study credit risk over instruments such as interest rate swaps or credit default swaps, and they find that using the repo curve or the swap curve provides a better fit as a risk-free benchmark than treasury curves, however, they also ignore the impact of counterparty risk, i.e. insurance company, investment banks, or even hedge funds, whose own default risk is at risk. The reason swap rates, or say, LIBOR curve provides a good fit on CDS spreads fitting models, is that they ignore the underlying counterparty default risk, which is embedded in both LIBOR rates and CDS prices.

In this thesis we believe that the old-fashioned government bond curve is still the best proxy for risk-free rate, despite its liquidity disadvantage to swap curve, which is relatively a small difference. In the corporate bonds data we later explain in detail, we look for both bond yield over benchmark government bond rate and bond yield over the swap rate. We then take the difference between the two yields and use the difference as the proxy for average counterparty proxy across the market. We then add the difference due to counterparty risk over government bonds onto CDS data from the market, whose issuers are also issuers of the interest rate swaps, which makes the new adjusted CDS spreads
CHAPTER 5. COX–INGERSOLL–ROSS MODEL

counterparty-risk-free. We could then apply CIR model and calibrate the default intensity component, which only represents the default risk of the underlying reference entity.

On the data collected from the corporate bond market, we carry on using the spreads over benchmark curves to represent the default component of the underlying reference entity, this is blended with the liquidity risk factor of the corporate bond on top, and then calibrate a total intensity to represent the sum of default component of the entity and liquidity factor of the bracketing bonds. The difference between the default intensity from the adjusted CDS spread and the intensity from the bond market is then chosen into account to represent liquidity risk.

We assume there is no liquidity risk in the CDS spreads, as we pick up the 5-year CDS data which is recognised to be the most liquid out of all the CDS spreads for the same underlyings, and we only pick up the major underlyings that show up frequently within the iTraxx series 1-11. Another important reason is that the contractual nature of credit default swaps, rather than being a security, this makes them far less sensitive to liquidity risk (also called convenience yield effects). This is because for securities supply is fixed, which causes a bigger liquidity issue; whereas the types of supply and demand pressure are much less likely to influence credit default swaps. Since new credit default swaps can always be created rather than being “squeezed” like the underlying corporate bonds, which means even if an investor wants to liquidate a credit default swap position, it may be less costly to enter into a new swap in the opposite direction rather than trying to sell the current position. Thus, the liquidity of the current position is less relevant given one’s ability to replicate swap cash flows through other contracts. Finally to support our assumption, Blance, Brennan, and Marsh (2003) find that credit derivative market is more liquid than corporate bond market in the sense that new information is included into CDS premia more rapidly than into corporate bond prices.

\footnote{Only reference entities that appears more than 5 series within the past 11 series are selected to reduce the possible effect of liquidity.}
Previous papers such as Longstaff (2005)[58] find there is a big difference between the spread obtained from the CDS market and the spread generated by using bracketing bonds from the corporate bond market for the same underlying reference entity. They explained that counterparty risk cannot fully explain the difference between the spreads. Our model adds counterparty spread to CDS spreads from the market, by doing so, we brought the CDS spread onto the same platform as the spread generated by the bond market, we therefore found much less difference between the spreads, which makes default risk a bigger component than previous studies and liquidity risk does not count as big a proportion of the total spread.

5.3.1 Risk-free Rate

To estimate the default component, we firstly need to identify a risk-free benchmark curve for the discount factor generation. Three risk-free curves have been introduced in previous studies: The Treasury (Government Bond), Refcorp, and swap curves.

1. Treasury Bill/ Government Bond curve is the original riskless benchmark since it is the standard in most empirical tests in finance, it assumes the issuers, who are usually government banks such as the US Federal Reserve or Bank of England, do not default as sovereign issuers. Unless the entire country goes bust, these counter-parties will not default. However, the Treasury/ Government Bond curve has been challenged for its limited supply to the public, which restricts its liquidity compared to swap curves, which is widely accepted as a risk-free benchmark among the practitioners.

Here, because our study is based on European issued CDS and corporate bonds, we use a generic European benchmark curve from Bloomberg as the risk-free benchmark. The Euro Benchmarks Curve is comprised of Euro-denominated fixed-rate government bonds from France and Germany. Bonds and bills are selected based on the closest current nominal maturity to the indicated term. We combine the liquidity spread of government bond together with the liquidity
spread of corporate bond and model them together as one liquidity factor. This is due to the fact that the European Benchmark shares the same liquidity issue across the market so it only causes generic shift to the market if there is any liquidity issue of the government bond.

2. The swap curve is widely accepted by practitioners to discount cashflows in fixed income and its derivatives markets, however, the swap curve includes both credit and default components. This makes it an ideal candidate to represent counterparty risk across the market. The complete list for all the 16 banks that currently contribute to the fixing of Euro bbalibor, which was last reviewed in May 2009, is: Bank of America, Bank of Tokyo-Mitsubishi UFJ Ltd, Barclays Bank plc, Citibank NA, Credit Suisse, Deutsche Bank AG, HSBC, JP Morgan Chase, Lloyd’s Banking Group, Mizuho Corporate Bank, Rabobank, Royal Bank of Canada, Société Générale, The Royal Bank of Scotland Group, UBS AG, West LB AG.

With considerations due to current economic situation, bbalibor submits from panel members the lowest inter-bank unsecured loan providers within the money market. The banks listed above are also major CDS issuers in the credit derivatives market, we use the banks above to represent the protection sellers of CDSs, by doing this the credit risk within swap rates are representable as the counterparty risk in the CDS premia.

During the credit crunch, the market realises the banks above cannot provide a default-free guarantee to products they provide, scandals from Société Générale and the huge loss and later nationalisation of The Royal Bank of Scotland Group indicates the importance of counterparty exposure within the pricing of the derivatives they provide. Hence in our paper we do not believe swap rates is a reliable default-free benchmark.

EURIBOR is the rate of interest at which panel banks borrow funds from other panel banks, in marketable size, in the EU inter-bank market. In other words, this is the rate at which participant banks within the European Union money market will lend to another participant bank in the EU money market.
Because banks involved with EURIBOR are the largest participants in the EU money market, it has become the benchmark for short-term interest rates.

3. Longstaff et al. (2005) and their previous papers introduced the Refcorp curve as a benchmark curve, alongside with Treasury curves and Swap curves for a comparison. They show that Refcorp bonds have the same default risk as Treasury bonds, yet provide a better liquidity with less specialness to the market than the Treasury bonds. Thus, they believe that the Refcorp curve may provide a more accurate measure of the riskless curve than the Treasury Curve. In this thesis, we only take the bond spread over government bonds and swap rates, hence Refcorp is not considered.

Hull et al. (2004) argue that credit default swaps can be used to imply the risk-free rate assumed by traders, the risk-free rate used appears to be approximately equal to the LIBOR/swap rate minus 10 basis points on average, they argue that this estimate is plausible due to the counterparty risk of the CDS seller. The credit risk in a swap rate is the credit risk from making a series of 6-month loans to AA-rated counterparties and 10 basis points is a reasonable default risk premium for an AA-rated 6-month instrument. This is the first paper to incorporate the counterparty risk of swap issuer.

5.3.2 Data Collection and Process

For the Euro Benchmark Curve, we collect data for the constant maturity 3-month, 6-month, each year between 1-year to 10-year, and 15-year. Firstly we convert the annual par rates into semi annual basis, we then interpolate the data into semi-annual intervals using cubic spline method, these par rates are then bootstrapped to provide a discount curve at semi-annual intervals. We then use the relationship below to derive zero rates from the discount curve. Zero rates at other maturities are then derived by linearly interpolate the zero rates at semi-annual tenor points, discount factor can then be calculated using equation (5.2).
We collect Euro Swap Annual rates (Bloomberg ID: I53) for 1-month, 3-month, 6-month, 9-month, each year between 1-year to 10-year, and 15-year from the Bloomberg system. We follow the same algorithm as described above for the Euro Benchmark Curve to obtain swap discount functions. In both Euro benchmark curve and swap curve case, data up to 15 years or less are collected since all of the corporate bonds in our sample have a maturity of 15 years and less.

5.3.3 CDS Data

We select the underlying reference entities given the following standard:

We first select companies that frequently show up in CDS index iTraxx Europe series 1-11 for investment grade, and iTraxx Crossover series 1-11 for the lower boundary of investment grade. The iTraxx Europe index is the most widely traded of all the indices, it is composed of the most liquid 125 CDS referencing European investment grade credits, subject to certain sector rules as determined by the IIC and also as determined by the SEC. There is also significant volume, in nominal values, of trading in the HiVol and Crossover indices.

To guarantee the popularity in terms of trading, which indicates high liquidity in terms of CDS and bond prices, we pick up reference entities that have appeared more than 5 series out of 11 for further investigation.

Companies are chosen to be diversified to represent different industries such as autos & industrials, consumers, energy, financials, and telecommunications. Recovery rate is chosen to be 40%, which is the average recovery rate from iTraxx Europe Series 11. Although various research has been done to show that

\[ DF = \frac{1}{(1 +ZR)^{d/365}} \]  

2HiVol is a subset of the main index consisting of what are seen as the most risky 30 constituents at the time the index is constructed. Crossover is constructed in a similar way but is composed of 50 sub-investment grade credits.
a constant recovery rate does not underperform the dynamically structured recovery rate such as using beta distribution. Furthermore, it does not play a big part on the pricing of CDS price or default component by changing the level of recovery rate.

The contributing banks within Markit iTraxx Europe\(^3\) index are leading investment banks, ABN AMRO (defaulted), Dresdner Kleinwort, Bank of America, BNP Paribas, Barclays Capital, Citibank, Credit Suisse, Deutsche Bank, Goldman Sachs, HSBC, JPMorgan, Morgan Stanley, RBS, UBS, and Wells Fargo, from where we can see a great overlap with the panel of EURIBOR banks\(^4\).

Our credit default swap data consists of a set of CDS spread quotes provided by Datastream, which is a composite price from the contributors listed above. The data contains 55 individual CDS quotes and covers 3 years of CDS quotes from July 1, 2004 to June 30, 2007. Each quote contains the date of the quotes, the name of the reference entity, and the spread is quoted in basis points. The reference entity may be a financial institution such as BNP Paribas, an automobile company such as Volkswagen, or a telecommunication company such as Vodafone. During the period covered by the data CDS quotes are provided on 55 named entities. The CDS rate quoted for any particular CDS depends on the term of the CDS and the credit quality of the underlying asset. Majority of the quotes lie within the range of 0 and 100 basis points for the period between July 2004 and June 2007.

\(^3\)The Markit iTraxx index family provides the market standards for investing, trading and hedging and thus helps improve market liquidity. Index trades have increased rapidly in recent years and represent more than 40% of overall credit derivatives volume. The rules-based Markit iTraxx indices are comprised of the most liquid names in the European and Asian markets. The selection methodology ensures that the indices are replicable and represent the most liquid, traded part of the market.

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5.3.4 Corporate Spread

5.3.4.1 Bond Data Selection

One obvious component of spread is the expected loss on corporate bond due to default.

We adapt the same bracketing technique as mentioned in Longstaff’s paper for companies with more than one bond available, although we do not eliminate companies that have only one bond in the market\(^5\). The advantage of using bracketing bonds for the same entity is that it provides a better approximation of the credit spread at 5 year horizon, by reducing the idiosyncratic risk that is embedded in individual bonds.

We use the same underlyings as in the selection of reference entities in the CDS spread, certain criteria are performed in order to filter out bond with poor maintenance history or quality:

- Only SEC-registered Euro bonds are included;
- We pick the period between July 1, 2004, and June 30, 2007 as observation period, which means we need complete quotes from both the CDS and bond markets between the observation period, therefore candidate bonds should exist before July 1, 2004, and mature after June 30, 2007.
- where possible, larger issuers are chosen. Issuers with total notional amount of less than 10 million euros are excluded.
- Only bonds with straight fixed-coupon are chosen, floating coupon bonds and zero coupon bonds are excluded.
- Bonds with convertible features such as callable or puttable bonds are excluded.

\(^5\)we find that companies with only one bond available in the market, their bond tends to have a strong indication to the CDS spread, whereas for companies with two many bonds available, the indicative effect tends to be diluted, and companies with many bonds tend to have much poorer liquidity than the single bond company.
In order to select reference entities with bracketing bonds, at least two bonds are included, as the 5-year maturity date for the period between the beginning and end of observation date (July 1, 2004 to June 30, 2007), is within the period between July 1, 2009 to June 30, 2012. We first attempt to find a bond with a maturity shorter than 5 years as the first observation date for the company, which means we need to find a bond that has maturity between July 1, 2007 to June 30, 2009. Also, we need to find bonds with a maturity longer than 5 years of the last observation date for each company, which means we need to find bonds with maturity later than June 30, 2009. Once the bonds defining the lower and upper limit of the bracketing interval are selected, we then select bonds with intermediate maturity dates to provide a roughly equal spaced coverage of the bracketing interval. Some filtering of the bond yield data is necessary, which concerns yields that change dramatically over a short period of time or yields that have more than a month of missing data for any period between the observation dates, which in fact eliminate a large portion of bond data available from Datastream, a few underlying entities are excluded due to the poor quality of their bond data.

5.3.4.2 Credit Spread Derivation from Bond Data

To compute the corporate spread the following procedure is used:

1. The bond yields from Datastream are all default prices, hence clean pricing, therefore, for each observation, we need to add accrued interest onto the clean price to get the dirty price of the bond.

2. For each corporate bond in the bracketing set, we calculate the theoretical bond price by discounting all the outstanding coupons plus principle back to each observation date using the discount curve of either benchmark curve or swap curve, we then solve for the yield-to-maturity on a riskless bond with the same maturity date and coupon rate.

3. Subtracting either the riskless benchmark yield or swap yield from the
yield on the corporate bond gives the yield spread for that particular corporate bond over benchmark curve and swap curve.

4. Regress the yield spreads for each individual bonds in the bracketing set on their maturities in order to obtain a five-year-horizon yield spread for the firm. We then use the fitted value of the regression at a five-year horizon as the estimate of the corporate spread for the firm.

5. Check the results of the spread over the benchmark and swap curve with the statics from Datastream and find out that the results are robust.

Fig. 5.1 plots the yield spread over benchmark and yield spread over swap curve, together with credit default swap premium from the market during the sample period. In the model independent approach, the credit default swap premium is used as the estimate of the default component of the corporate spread. It shows a strong trend of corporate bond spread with CDS premium, however, the CDS spread shows a much closer range with bond spread over swap rather than bond spread over benchmark curve. This is why previous studies such as Longstaff et al. (2005)\cite{58} conclude that swap spread provides a much better proxy for default swap, and there is no clear indication of counterparty risk once swap rates are picked as risk-less curve. However, they ignore counterparty risk in the CDS premium, we show later in fig. 5.2 that by taking the difference between the bond spread over benchmark curve and bond spread over swap curve as a counterparty risk premium, and add it onto CDS premium from the market, the newly adjusted CDS premium will not contain counterparty risk and provide a much closer relationship to the bond spread over benchmark curve, which is theoretically the risk-free curve.

5.3.5 CDS Spread with Counterparty Spread Proxy

As introduced before, this chapter assumes that the interest rate curve generated from the government bond market has no default risk, whereas swap rates issued by the participating EURIBOR banks tend to have counterparty credit risk,
The CDS price has a much closer trend with spread over swap curve in terms of trend and range than the bond spread over benchmark curve. Reference Entity: Deutsche Telekom, Michelin; Sample period: July 1, 2004 - June 30, 2007.

Figure 5.1: Plot of Spread over benchmark curve (SP), spread over swap curve (SWSP), and CDS premium (with counterparty risk)
Figure 5.2: The difference between corporate bond spread over euro benchmark curve and spread over euro swap curve
This shows a strong trend across bonds referencing different entities. We use this difference between the two spreads as a proxy to represent the counterparty risk, and add it on top of credit spread quotes from the CDS market.

Therefore we use the spread difference between bond spread over benchmark curve and bond spread over swap curve to represent the spread of counterparty risk, as shown in figure 5.2.

By assuming the independence between the risk-free curves and the CDS premium, we use the difference between the Euro benchmark curve and the Euro swap curve as a proxy to represent the counterparty risk, and add it on to CDS spread from the market. Note that the interest rate swaps are also issued by counterparties from the participating banks in iTraxx index or with similar credit quality. This will guarantee that the adjusted CDS data is counterparty-risk-free, and we assume there is no liquidity risk as we have picked up the most liquid 5-year CDS spread. Detailed explanation can be found in Longstaff et al. (2005)\[58\].

Fig. 5.3 plots the yield spread over benchmark and yield spread over swap curve, credit default swap premium from the market, and adjusted CDS pre-

\[58\] In case there is any liquidity risk left, we take the difference between the credit spread from the bond market and the adjusted credit spread from the CDS market, and assume it to be the liquidity risk difference between the CDS and bond market.
mium with an added counterparty default proxy, during the sample period. In the model independent approach, the credit-default swap premium is used as the estimate of the default component of the corporate spread. It shows a strong trend of corporate bond spread with the CDS premium, by adding the counterparty default proxy.

5.4 Liquidity risk

We assume there is no liquidity risk in the CDS data, as we pick up the 5-year CDS data which is recognised to be the most liquid out of all the CDS spreads for the same underlyings, and we only pick up the 'frequent shower' of the major underlyings within the iTraxx series 1-11. Another important reason is that the contractual nature of credit default swaps, rather than being a security, makes them far less sensitive to liquidity or convenience yield effects. This means that the types of supply and demand pressures that may affect corporate bonds are much less likely to influence credit default swaps, where as for securities supply is fixed. Since new credit default swaps can always be created rather than being “squeezed” like the underlying corporate bonds, which means even if an investor wants to liquidate a credit default swap position, it may be less costly to enter into a new swap in the opposite direction rather than trying to sell the current position. Thus, the liquidity of the current position is less relevant given one’s ability to replicate swap cash flows through other contracts. Finally to support our assumption, Blance, Brennan, and Marsh (2003) find that credit derivative market is more liquid than corporate bond markets in the sense that new information is impounded into CDS premia more rapidly than into corporate bond prices. A detailed explanation can be found in Longstaff (2005).

7 Only reference entities that showed up in more than 5 series within the past 11 series are selected to reduce the possible effect of liquidity.
Figure 5.3: Plot of Spread over benchmark curve (SP), spread over swap curve (SWSP), CDS premium, and adjusted counterparty-risk-free CDS premium. By adding the counterparty default proxy brings the CDS price to a similar level to the corporate bond spread.
Chapter 6

The CIR Default Component

Calibration

Notation of Variables

\[ \lambda_t^A \]  default intensity of protection buyer

\[ \lambda_t^P \]  default intensity of protection seller

\[ D(T) \]  the value of a riskless zero-coupon bond with maturity \( T \)

\[ \lambda_t \]  the intensity process \( \lambda_t \), we assume that

\[ \alpha, \beta, \sigma \]  are all positive constants, \( \alpha/\beta \) is the mean-reverting average

\[ \beta \]  mean-reverting speed

\[ \sigma \]  mean-reverting volatility

\[ Z_{\lambda} \]  a standard Brownian motion

\[ \gamma_t \]  the liquidity process

\[ \eta \]  a positive constant
Another standard Brownian motion that is independent of $Z_\lambda$

c  the continuous coupon rate of a corporate bond

$CB(c, w, T)$  corporate bond price

$r_t$  short interest rate

$s$  the premium of a CDS paid by the protection buyer, which is paid continuously

$P(s, T)$  the present value of the premium leg of a CDS

$PR(w, T)$  the value of the protection leg of a CDS

$w$  weight in the expected average value of $\lambda_t w$.  

$\lambda^{CDS}$  average of calibrated default intensities
In this section we first introduce the model Longstaff et al. (2005) [58], it is based on the framework introduced by Duffie and Singleton (1997) [25], where the default intensity follows a CIR process. Based on this, Longstaff then calibrates the parameters of the intensity process and liquidity component from bond and CDS data, and calculate the default component.

In our approach we use the same intensity model as in Duffie and Singleton, however, we use the same intensity model to calculate the total intensity using both CDS (counterparty-risk adjusted) and bond data. We then take the difference between the intensity of CDS and bond data and put it as liquidity risk. In doing this we simplify the model by reducing a separate step for bond data. Also we find the parameter approximation performs better than Longstaff’s approach as by using Longstaff’s parameter approximation method, the liquidity risk has very dramatic fluctuation which does not have explanatory power in the real world. As we introduced before, using the counterparty-risk adjusted CDS price reduce, if not eliminate, the impact counterparty play in CDS quotes, it provides a less unbiased to the corporate bond spread over benchmark curve, rather than close to the swap curve as previous studies conclude.

6.1 The Longstaff Approach

In the paper Longstaff (2005) [58], they assume that the CDS premium equals the default component for the company’s bonds. Comparing the CDS premium for a 5-year contract directly with the corporate spread for a 5-year bond provides a simple model-independent measure of the percentage size of the default component. It is important to stress that this approach generally produces a biased measure of the default component. Duffie (1999) [22] states that the CDS premium should equal to the spread between corporate and riskless floating-rate notes. However, Duffie and Liu (2001) [23] show that the corporate bond spread over riskless bonds is generally not equal to the spread between corporate and riskless floating-rate notes. In general, the effect of the bias is to underestimate
the size of the default component in investment grade bonds, and vice versa for below investment grade bonds. This can be avoided by using an explicit credit model to make the adjustment from floating-rate to fixed-rate spreads. Accordingly, in Longstaff’s approach, they develop a simple closed-form model for valuing credit-sensitive contracts and securities within the well-known reduced-form framework of Duffie (1998)\textsuperscript{[21]}, Lando (1998)\textsuperscript{[53]}, Duffie and Singleton (1997, 1999)\textsuperscript{[25, 26]} and others. Once fitted to the data, their model can be used to provide direct estimates of the default component of the spread implied by credit-default premia.

Following Duffie and Singleton (1997)\textsuperscript{[25]}, let $r_t$ denote the riskless rate, $\lambda_t$ the intensity of the Poisson process governing default, and $\gamma_t$ the liquidity process that is used to capture the extra return investors may require above and beyond compensation for credit risk, from holding corporate bond rather than riskless securities. All of $r_t$, $\lambda_t$, and $\gamma_t$ are stochastic and evolve independently of each other. According to Longstaff, this assumption simplifies the model but has little effect on the empirical results. As in Lando (1998)\textsuperscript{[53]}, we assume that a bondholder recovers a fraction of $1 - w$ of the par value of the bond in the event of default, which means $w$ is the loss rate, if $w = 0.6$, then the recovery rate is 0.4.

Therefore, the value of a riskless zero-coupon bond $D(T)$ with maturity $T$ is expressed as\footnote{Given the independence assumption, we do not need to specify the risk-neutral dynamics of the riskless rate to solve for credit default swap premia and corporate bond prices.}

$$D(T) = \mathbb{E} \left[ \exp \left( - \int_0^T r_t dt \right) \right].$$

(6.1)

To specify the risk-neutral dynamics of the intensity process $\lambda_t$, we assume that

$$d\lambda = (\alpha - \beta \lambda) dt + \sigma \sqrt{\lambda} dZ,$$

(6.2)

where $\alpha$, $\beta$, and $\sigma$ are all positive constants, and $Z$ is a standard Brownian
motion. This is the same approach Cox, Ingersoll and Ross (CIR) develop in 1985 for short-term interest rate model. It allows both mean reverting and conditional heteroskedasticity in corporate spreads and guarantees that the intensity process is always non-negative. The risk-neutral dynamics of the liquidity process $\gamma_t$ is

$$d\gamma = \eta dZ_\gamma,$$  \hfill (6.3)

where $\eta$ is a positive constant and $Z_\gamma$ is another standard Brownian motion that is independent of $Z_\lambda$. This allows the liquidity process to take on both positive and negative values.

Following Duffie (1998) [21], Lando (1998) [53], Duffie and Singleton (1999) [22] and others, it is straightforward to represent the values of corporate bonds and the premium and protection legs of a CDS contract as simple expectations under the risk-neutral measure. Let $c$ denote the coupon rate of a corporate bond, which is a continuous term. Therefore the price of this corporate bond $CB(c, w, T)$ can be expressed as

$$CB(c, w, T) = E\left[c \int_0^T \exp\left(-\int_0^t r_s + \lambda_s + \gamma_s ds\right) dt\right] + E\left[\exp\left(-\int_0^T r_t + \lambda_t + \gamma_t dt\right)\right] + E\left[(1 - w) \int_0^T \lambda_t \exp\left(-\int_0^t r_s + \lambda_s + \gamma_s ds\right) dt\right].$$ \hfill (6.4)

In which the first term in this expression is the present value of the promised coupon payments by the bond contract, the second term is the present value of the promised principal payment, and the third term is the present value of recovery payment once the underlying defaults. In each term, cashflows are discounted at the credit and liquidity adjusted rate $r_t + \lambda_t + \gamma_t$.

For the valuation of a CDS contract, recall that swaps are contracts, not
securities, hence the contractual nature of CDS makes them far less sensitive to liquidity effect. Longstaff et al. (2005) explain in details the difference in response to liquidity effect between contracts and securities. We select underlyings from the 125 names in iTraxx Europe, out of all their CDS contracts we pick up 5-year maturity which is believed to be the most liquid contracts given the same underlyings. Therefore, given the discussion above, we assume liquidity process $\gamma_t$ only apply to cashflows from corporate bonds, but not to cashflows from CDS contracts. Alternatively, $\gamma_t$ can also be considered as the difference between the liquidity of corporate securities and the liquidity of CDS contracts.

Let $s$ denotes the premium paid by the protection buyer, and assume that the premium is paid continuously, the present value of the premium leg of a CDS can be expressed as

$$P(s, T) = E\left[ s \int_0^T \exp \left( - \int_0^t r_s + \lambda_s ds \right) dt \right]. \quad (6.5)$$

Similarly, the value of the protection leg is

$$PR(w, T) = E\left[ w \int_0^T \lambda_t \exp \left( - \int_0^t r_s + \lambda_s ds \right) dt \right]. \quad (6.6)$$

Setting the premium leg equal to the protection leg and solve for the premium gives

$$s = \frac{E\left[ w \int_0^T \lambda_t \exp \left( - \int_0^t r_s + \lambda_s ds \right) dt \right]}{E\left[ \int_0^T \exp \left( - \int_0^t r_s + \lambda_s ds \right) dt \right]}.$$

(6.7)

From which we can see if $\lambda_t$ is not stochastic, the premium is simply $\lambda w$. Even if $\lambda_t$ is stochastic, the premium can be expressed as a weighted average present value of $\lambda_t w$.

2In general, given the negative correlation between $\lambda_t$ and $\exp \left( - \int_0^t \lambda_s ds \right)$, the premium should be less than the expected average value of $\lambda_t w$. Therefore, given the square-root dynamic for the intensity process $\lambda_t$ and
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the liquidity process $\gamma_t$, Longstaff et al. (2005)\textsuperscript{58} derive a closed-form solution for the expectation terms in equations (6.4) and (6.10). Detailed derivation can be found in Appendix A. Therefore the value of a corporate bond is

$$CB(c, w, T) = c \int_0^T A(t) \exp(B(t) \lambda) C(t) D(t) e^{-\gamma t} dt + A(t) \exp(B(T) \lambda) C(T) D(T) e^{-\gamma T} + (1 - w) \int_0^T \exp(B(t) \lambda) C(t) D(t) (G(t) + H(t) \lambda) e^{-\gamma t} dt,$$

where

$$A(t) = \exp\left(\frac{\alpha (\beta + \phi)}{\sigma^2} t \left(1 - \frac{1 - \kappa}{1 - \kappa e^{\phi t}}\right)^{\frac{2\beta}{\sigma^2}}\right),$$

$$B(t) = \frac{\beta - \phi}{\sigma^2} + \frac{2\phi}{\sigma^2 (1 - \kappa e^{\phi t})},$$

$$C(t) = \exp\left(\frac{\eta^2 t^3}{6}\right),$$

$$G(t) = \frac{\alpha}{\phi} (e^{\phi t} - 1) \exp\left(\frac{\alpha (\beta + \phi)}{\sigma^2} t \left(1 - \frac{1 - \kappa}{1 - \kappa e^{\phi t}}\right)^{\frac{2\beta}{\sigma^2} + 1}\right),$$

$$H(t) = \exp\left(\frac{\alpha (\beta + \phi) + \phi \sigma^2}{\sigma^2} t \left(1 - \frac{1 - \kappa}{1 - \kappa e^{\phi t}}\right)^{\frac{2\beta + 2}{\sigma^2}}\right),$$

$$\phi = \sqrt{2\sigma^2 + \beta^2},$$

and

$$\kappa = \frac{\beta + \phi}{\beta - \phi}.$$

And the closed-form solution for CDS premium is
With these closed-form solutions, Longstaff et al. (2005) fits the model to match simultaneously the CDS premia and prices for a set of bracketing bonds with maturities span around the 5-year horizon of the CDS contract. They first choose a set of values for parameters $\alpha$, $\beta$, $\sigma$, and $\eta$, and then calibrate the exact default intensity from the CDS premium. They then put the parameters and default intensity into the bond prices, and derive the process of the liquidity for each observation date. Different sets of parameters are then chosen and the one with the least Root-Mean-Squared-Error (RMSE) is picked up to represent the parameters for default intensity and liquidity process.

### 6.1.1 Extension and Simplification of the Longstaff Model

In this thesis, we use similar terms for default intensity, however, we treat the counterparty-adjusted CDS premium and bond spread over benchmark curve equally, and calculate their individual default intensity using equation (6.10), which is the same method to solve default swap premium for a CDS contract. However, since we have derived 5 year spread over benchmark curve from the bond market data already, we just use that spread as a total spread and derive the total intensity of credit and liquidity, which mean the calibrated intensity will equal to the sum of $\lambda_s + \gamma_s$ for bond market, and $\lambda_s$ only for CDS market data. By assuming no correlation between the intensity of the reference entity and the liquidity factor of corporate bond, we take the difference between the intensities derived from the bond market (spread over benchmark curve) and CDS market (counterparty-risk adjusted CDS premium) and leave it as the liquidity difference between the two markets.

$$s = \frac{w \int_0^T \exp (B (t) \lambda) D (t) (G (t) + H (t) \lambda) \, dt}{\int_0^T A (t) \exp (B (t) \lambda) D (t) \, dt}.$$  \hspace{1cm} (6.10)
equation of corporate bond price as in eq. (6.8), we simply derive it from the
difference between the intensities. It is more efficient in a way if there are too
many parameters in a equation to calibrate/approximate, the ultimate result
may not be as representable and will be lack of practical sense.

6.2 The Volkswagen Case Study

In order to demonstrate the methodology introduced above, we choose certain
firms to illustrate the effect of counterparty risk and liquidity risk. Volkswagen
is picked from the 55 underlying firms to represent the typical, not necessary the
best, results of the companies we study in this thesis. As introduced before, two
major types of VW data are used in this case study: CDS premia and corporate
bond yields.

As usual, CDS data is the mid rate on VW from Datastream during the
period from July 1 2004 to June 30 2007, which is a composite price of major
contributing banks. This means quotations should be representative of counter-
parties across the entire CDS market. In order to match each CDS spread rate
across the 3-year observation horizon with a 5 year maturity, we need to select
representative bonds issued on VW and regress out a credit default spread with
a 5 year maturity.

Since we have picked CDS data with all 5-year maturities, it would make
sense to have a matching 5-year bond available for each observation date of the
3 years horizon that selected, however, this is not possible to find in reality. To
address this problem, we select a set of bonds with maturities that bracket the
5-year maturity of the CDS from 2009 to 2012, 5 bonds on VW are selected
with maturity listed on the right, trying to provide an equally spaced interval,
these 5 bonds are all fixed-rate straight bond Euro-dominated debt obligations
of VW and do not have any embedded options. To minimise the effects of
illiquidity, only bonds that are registered with SEC are included in the set.
The coupon rates for these bonds range from 4.125% to 5.375%, the maturities
range from March 10, 2008 to May 22, 2013. We refer this set of bonds as the bracketing set for VW. The bond yields are then obtained by manipulating each bond with a coupon bond with same maturity and coupon rate using the Euro benchmark curve and Euro swap curve as their riskfree alternative. Differences are then taken between the VW bonds and its risk-free alternative to get the bond spread over benchmark curve and swap curve. We conduct a number of robustness comparisons using data for the bonds from Datastream to verify that our data are reliable.

The spreads are as shown in figure 6.1, the first graph is the calculated spread over benchmark curve and the middle graph is the calculated spread over swap curve, the bottom graph is calculated by regressing on the 5 representative bonds for each observation date to get the bond spread over benchmark/swap with a 5 year maturity in order to match the same format as the CDS data.

As one can see, there is a significant difference between the bond spread over benchmark curve and bond spread over swap curve, which we can use as the spread for counterparty risk and add it onto CDS spread to make it counterparty risk free. Hence the first plot of figure 6.2 shows the adjusted CDS spread after the counterparty effect component is added on. Together with the bond spread over benchmark curve and swap curve. One can see the CDS spread is brought to a similar level to the spread over benchmark curve, rather than showing a misleading “close” relationship with the bond spread over swap curve.

To estimate the discount function $D(T)$ for each observation date, we firstly identify the riskless curve as the generic Euro benchmark curve which is a composite of French and German government bonds. This is done by collecting Euro Benchmark curve rates for the constant maturity 3-month, 6-month, each year from 1-year to 10-year and 15-year. We then apply a cubic-spline method to interpolate the par rates at semi-annual tenor points, which are then bootstrapped to provide zero rates at semi-annual intervals. In order to obtain zero rates and then discount factor at other maturities, linear interpolation is applied directly between adjacent semi-annual tenor points.
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Figure 6.1: Calculated Bond spread over benchmark curve and bond spread over swap curve
Obtained from regressing 5 representative bonds issued by Volkswagen over a 5-year horizon.
To estimate the parameters for the intensity process $\lambda^C_t$ for the CDS spread, we first pick up trial values for parameters $\alpha^C$, $\beta^C$, and $\sigma^C$ for the CIR credit spread process on the counterparty risk adjusted CDS premium. For each set of parameters, we solve for the default intensity $\lambda^C_t$ value for each observation date. We then try on different set values of the three parameters to get the global minimum of the RMSE (root-mean-square-error) value, with the condition that $\alpha$, $\beta$, and $\sigma$ are all positive. We then adjust the outcome value of $\alpha/\beta$ to make sure it is in sync with the long term mean of the intensity, and also the value $\sigma\sqrt{\lambda}$ to make sure it is representable to the volatility of the credit spread. We then do the same for the 5-year bond spread to calibrate parameters $\alpha^B$, $\beta^B$, and $\sigma^B$ which provide the best fit for $\lambda^C_t$ which provide in-sync value of long-term mean and volatility of the spread. We notice later as in Table 6.3 that the mean reverting speed factor $\beta$ for all the CDS and bond spreads proves relatively stable across all the companies. By changing $\beta$ it only causes a parallel shift to the solved intensity values, therefore it makes sense to keep the mean-reverting speed stable to provide all the intensity values on the same scale. Throughout this procedure, we hold the loss rate $w$ constant at 60%, which means the recovery rate is 40%, we find that the estimation results are identical when other values of $w$ are used.

This estimation procedure has several advantages: firstly, by fitting intensity to a cross section of bonds with maturities that bracket the maturity of the credit default swap (counterparty-risk adjusted), the effect of any measurement error in individual bond prices on the results are minimised. We average out the effect of idiosyncratic pricing errors in individual bonds. However, companies that have only one bond in the fixed income market are still included, we find that the bond spread of these companies has a strong correlation with the CDS premium as it is the only available bond in the market, therefore CDS seller and investors take the bond price as an important indicator when it comes to price the default swap of the same underlying.

Therefore, the estimated dynamics of the intensity process for CDS spread
CHAPTER 6. THE CIR DEFAULT COMPONENT CALIBRATION

for VW is

\[ d\lambda^C = (0.001658 - 0.28137\lambda^C)dt + 0.01950\sqrt{\lambda^C}dZ^C. \]  (6.11)

Similarly, the estimated dynamics of the intensity process for bond spread over benchmark for VW is

\[ d\lambda^B = (0.001349 - 0.28231\lambda^B)dt + 0.01837\sqrt{\lambda^B}dZ^B. \]  (6.12)

Figure 6.2 shows the calibrated default intensity of Volkswagen in the second graph, it illustrates a strong relationship between the CDS derived intensity \( \lambda^C \) and bond derived intensity \( \lambda^B \). The liquidity effect, which is the difference between the CDS intensity and Bond intensity, contributes less than 20% on average of the CDS intensity. This shows relatively small liquidity effect from the bond market, majority of the intensity is explained by the default risk of the underlying reference entity.

It also makes sense that the CDS intensity is higher than the bond intensity as the CDS market is generally more liquid than the bond market, which mean investors would like to pay higher price for higher liquid instruments, hence, the flight to liquidity.

6.3 Alternative Explanation

6.3.1 Flight to Liquidity

Longstaff (2004)[57] examines the existence of a flight to liquidity premium in Treasury bond prices by comparing them with prices of bonds issued by Refcorp, a U.S. Government agency. These bonds are guaranteed by the Treasury, therefore shares the same credit risk as Treasury bonds. A large liquidity premium in Treasury bonds, which can be more than 15% of the value of some Treasury bonds, is found. This liquidity premium is believed to be related to changes in consumer confidence, the amount of Treasury debt available to in-
Figure 6.2: Volkswagen’s credit spreads and decomposition
The first graph plots Volkswagen’s bond spread over benchmark curve and swap curve individually, and credit default swap rates before and after the adjustment of counterparty risk proxy; The second graph shows the calibrated default intensity from VW’s adjusted CDS spread and intensity from VW’s bond spread. The liquidity curve shows the difference between the two intensity processes.
vestors, and flows into equity and money market mutual funds. This suggests that the popularity of Treasury bonds directly affects their value.

He finds that during the past decade, there are often large liquidity premia in Treasury bond prices. In some cases, these premia can represent as much as 10%-15% of the value of the Treasury bond. An explanatory analysis reveals that these flight to liquidity premia are related to a variety of market sentiment measures, such as changes in consumer confidence and in the amount of funds flowing into equity and money market mutual funds. Furthermore, the flight to liquidity premia are directly related to changes in the supply of Treasury securities available to investors resulting from the recent treasury buyback program.

Same concerns can arise in our study, however, it involves the liquidity difference between Euro Government bonds and swaps rates, which are both believed to be liquidly exchanged. Swaps, however, are believed to be more liquid than Euro Government Bonds. This could affect the explanatory power of using the Euro Government bond rates instead of the Euro Swap rates as riskless benchmark. This means the usual counterparty spread premium of an average 15-20 basis points can partially be explained by the liquidity difference between the Euro Government Bond rates and the Euro Swap rates.

6.4 Concluding Remarks

Looking at Table 6.3 below we can see the parameters for each underlying within our study describing the property of their default intensity process, one may notice that financial institutions show a much lower long-term mean ($\alpha$) for the change of default intensity, and also a lower mean of the default intensity itself, comparing to other industries. This could be due to the fact that the financial institutions are relatively higher rated than the rest of the sample population; also the banks below are already contributors of the counterparties, hence when subtracting the counterparty risk from the credit spread, (by actually adding
the difference between the government bond spread and the swap spread on top of credit spread), their own default risk is partially subtracted/eliminated.

6.4.1 Bond Spread Difference as Counterparty Default Risk Proxy

By taking the difference between the government bond yield and swap yield as the proxy for global counterparty risk, and adding it onto the CDS spread, we bring the default component to the same level as the spread derived from corporate bonds. In which the counterparty-risk adjusted CDS spread represents only the pure credit spread over the underlying reference entity’s default risks. Whereas the original CDS spread provided by the market still has the impurity of counterparty default risk. For a firm who does not have sufficient information in the bond market, one can easily extend the same approach and obtain its counterparty-risk-free credit spread, by adding the counterparty spread on top of its credit spread from the CDS market.
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### Table 6.3: Table of parameters under CIR framework for credit spreads

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The table shows long term mean, mean reversion speed, and volatility for each of the sample 55 underlying entities. Default intensity is calculated given the processed credit spread without the effect of counterparty risk.
Chapter 7

Crisis Time Study

7.1 Introduction

In the previous chapter, we investigated the existence and impact of counterparty risk and quantify the counterparty risk by using the difference between the spread over benchmark and spread over swap as a proxy, using data spanning July 2004 to June 2007. In this chapter, we use a different set of data from July 2007 to June 2009, in order to see how the counterparty risk changes during the credit crunch period.

We further validate our approach by doing a comparison with Mercurio (2009)[61], whose study on the interest swap rates discrepancy on counterparty risk matches our findings. By finding great similarity we can conclude that our proxy for counterparty risk can be extended from credit swap market to interest rate swap market.

7.2 Alternative Counterparty Proxy

Mercurio (2009)[61] describes the major changes that occurred in the quotes of market rates after 2007 sub-prime mortgage crisis. He comments on the
lost analogies and consistencies of rates once closely interconnected and hints on a possible, simple way to formally reconcile them, part of which mentioned the discrepancy of “last” one-month EONIA rates and one-month deposit rates, from November 14th, 2005 to November 12, 2008, it can be observed that the basis was well below 10 bps until August 2007. However, since then it has started to move erratically around different levels, which is believed to be due to counterparty risk.

He notices that once compatible rates, such as 3x6 EONIA forward rates and 3x6 FRA rates, three-month LIBOR rate and six-month LIBOR rate, began to diverge substantially since the credit crunch of the sub-prime mortgage market, producing a clear segmentation of market rates.

Now the FRA cannot be priced as a trivial forward on a LIBOR rate, as it used to be, at least approximately. One possible explanation is the increased perception of bank-vs-bank counterparty/liquidity risk after the burst of sub-prime crisis. The presence of counterparty/liquidity risk in LIBOR market quotes is often estimated based on the difference between LIBOR rates and rates theoretically free of such counterparty/liquidity risk, such as EONIA. Thus now the risk-free rate is taken to be EONIA, whereas LIBOR would be a different default risky rate.

7.2.1 Figure Comparison

All those rates, which were very closely interconnected, suddenly became different objects, each one incorporating their own liquidity or credit premium. Take 1m EONIA and 1m deposit rate for example. EONIA is the weighted average of overnight Euro Inter-bank Offer Rates for inter-bank loans, it is the standard interest rate for Euro currency deposits. The European Central Bank is responsible for calculating the EONIA every day. Fig. 7.1 shows the spread between 1m EONIA rates, which is believed to be sensitive to counterparty risk, and 1m

\[\text{such as } 3x6 \text{ EONIA forward rates vs. } 3x6 \text{ FRA rates, and three-month LIBOR rate and six-month LIBOR rate, those pair of rates were once compatible, however, diverge substantially once the sub-prime mortgage crisis has started.}\]
deposit rate, which is believed to be counterparty-risk free.

We notice that the jump in counterparty spread is strongly linked to the major events during the sub-prime crisis, to be more specific, the default of banks, the major events first kick off at Bear Stearns’ $3.2bn bail-out of its struggling hedge funds in June 2007, it is followed by the write-down in bad loans and mortgage-backed securities, and reported losses by big banks and their subsidiary hedge funds, such as UBS, Merrill Lynch, Citigroup, Goldman Sachs, and Northern Rock, Bank of America, Deutsche Bank, HSBC, BNP Paribas near the end of 2007 at different levels. In March 2008, Bear Stearns was acquired by JP Morgan Chase for just $240 million under FED pressure in an emergency rescue takeover.

The credit crisis reached its peak when in September 2008, on September 7th, Federal takes over mortgage finance agencies Fannie Mae and Freddie Mac and effectively nationalised them; On Sept. 14th, Merrill Lynch was taken over by bank of America Corp; On 15th September, Lehman Brothers, after days of searching frantically for a buyer, Lehman Brothers filed for bankruptcy protection, becoming the first major bank to collapse since the start of the credit crisis; The next day, on 16th September, the US Federal Reserve announced an $85 rescue package for AIG, the country’s biggest insurance company, to save it from bankruptcy. And the day after that, Lloyds TSB announced it is to take over Britain’s biggest mortgage lender HBOS in a £12bn deal creating a banking giant holding close to one-third of the UK’s savings and mortgage market. Then on 25th September, in the largest bank failure yet in the United States, Washington Mutual, a giant mortgage lender, who had assets valued at $307bn, is closed down by regulators and sold to JPMorgan Chase, not to mention the partial nationalisation of European banking and insurance giant Fortis, nationalisation of mortgage lender Bradford & Bingley in Britain, the Icelandic government took control and nationalised the country’s three major banks, Glitnir, after the company faces short-term funding problems; the bail-
Figure 7.1: Euro 1m EONIA rates vs. 1m deposit rates from 15 Jan. 2004 to 20 Aug. 2009
Source: Bloomberg.

Figure 7.2: Plot of 3M, 1Y and 5Y counterparty spread
The difference between Euro Swap rate and Euro Benchmark rate is defined as counterparty-risk proxy, and compared to Mercurio (2009), who uses the difference between 1M EONIA vs. Deposit rates.
out of Dexia, which all happened in a very short period of time at the end of September 2008.

All the major events mentioned above damage the credibility among banks as counterparty and major player of vast majority of the derivatives in the credit market, and hence the big jump in terms of counterparty risk can all be observed in figure 7.1 and 7.2.

Fig. 7.2 shows the strong similarity between Mercurio (2009)’s finding and the findings within this chapter, where we obtain counterparty spread by taking the difference between EURIBOR\(^2\) and Euro Benchmark rates. We compare the 1M counterparty risk in Mercurio (2009)\(^61\) to a range of term structure of 3M, 1Y, and 5Y, we can see the counterparty spread does exist before crisis period, where as the difference between EONIA and deposit rate in Mercurio’s paper is close to zero. Also the 3M counterparty risk is the most extreme in terms of volatility and magnitude, it is close to the 1M counterparty spread in Mercurio’s paper and shares the same shape and match at peaks. Also as one can see from the counterparty spread of 1Y and 5Y, the spread tend to scale down and become less volatile due to the longer time horizon into the future, rather than very spot sensitive at current date like 1M and 3M. We believe that using the difference between Euro swap rate and Euro Benchmark rate provides a more consistent proxy before and after the credit crunch, rather than the abrupt jumps of the 1m proxy for counterparty risk. In doing this, we also monitor the term structure of counterparty risk and observe its shape and behaviour at different time such as normal and excited state of the market, which we will discuss in detail later.

The above investigation validate our counterparty spread as a sound proxy for approximating counterparty risk, therefore, we carry on using the same framework as in Chapter 5 to calibrate spread dynamic parameters using the

\(^2\)EURIBOR: The rate of interest at which panel banks borrow funds from other panel banks, in marketable size, in the EU interbank market. In other words, this is the rate at which participant banks within the European Union money market will lend to another participant bank in the EU money market. Because banks involved with EURIBOR are the largest participants in the EU money market, this rate has become the benchmark for short-term interest rates.
Cox–Ingersoll–Ross model, during the sub-prime mortgage crisis period.

### 7.3 Carry-On study on Crisis Time

Carrying on the same theory and methodology from previous chapter on 5 year CDS counterparty risk by taking the difference between bond spread over the Euro Benchmark curve and bond spread over the swap curve. We replace the date using July 2nd 2007 till June 30th 2009, taking the difference between spread over Euro benchmark and spread over swap as a proxy for counterparty risk, and add it to the CDS spread and Bond spread to derive counterparty-risk free spread curve. We then follow the same CIR model for credit default intensity and calibrate daily intensity rates based on the counterparty-risk adjusted credit spread. Table 7.3 below shows calibrated parameters for CDS spread derived intensity process, with long term mean of \( \lambda \), volatility \( \sigma \), and average of CDS spread intensity all much higher than “peaceful” time between July 2004 to June 2007, mean reversion spread become more diversified and lower than “peaceful” time, which makes default intensity more diverse.

From figure 7.3 we could see the CDS market and bond market still have a very good connection in terms of representing the default risk of the underlying entity. However, the CDS market provides a much more liquid in terms of response to market shocks, where as bond market does similar trend as CDS spread given market condition with much less volatile.

Blue line shows daily intensity rates calibrated from the CDS spread plus counterparty spread; red line shows intensity calibrated from the bond spread plus counterparty spread; green line shows the difference between intensities derived from both markets and represents the liquidity different between these two markets. As one could observe, the CDS market provides much more volatile intensity rates, and responds to market information faster than bond market. On the other hand, the bond market remains a good candidate to represent the credit quality of an underlying firm for keeping up the same magnitude of
## CHAPTER 7. CRISIS TIME STUDY

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<th>β</th>
<th>σ</th>
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CHAPTER 7. CRISIS TIME STUDY

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<th>$\beta$</th>
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Table 7.3: Credit spread parameters under CIR model
During credit crunch period (July 2007 - June 2009).

For the financial markets, bond spreads tend to be higher than CDS spreads, which shows a higher anxiety from the fixed income market rather than credit derivatives market. Energy and Telecommunication markets show a close relationship between intensities from CDS market and bond market, given they are usually big corporations and issue a well diversified number of bonds, and are liquidly traded in the bond market. Whereas for automobile market, which is indirectly influenced by the credit crunch, shows a rather slow response from the bond market. Intensity from the bond market is relatively smoother and smaller in magnitude than the default intensity from the CDS market, investors in the bond market for automobile companies only start “panicking” after the September 2008 events. One can see the intensity only begin to increase dramatically and chase up with the CDS spread intensity, which also shows an illiquid bond market in that industry.

One of the promising prospects in a liquid CDS market is that it provides better information on the term structure of credit risk for specific issuers. First of all, issuing a CDS on a particular firm does not change the capital structure
Figure 7.3: Sample plots of default intensity rate $\lambda$ in Financial and Energy sector
Blue line shows default intensity from CDS market, red line shows default intensity derived from bond market. All the CDS spreads are counterparty-risk adjusted.
Figure 7.4: Sample plot of default intensity rate $\lambda$ in TMT and Autos sector
Blue line shows default intensity from the CDS market, red line shows default
intensity derived from the bond market. All the CDS spreads are counterparty-
risk adjusted.
of the firm, which makes it reasonable to assume that the price behaves linearly in the underlying amount insured over wider intervals. Secondly, the maturities of CDS swaps can be chosen independently from the maturity structure of debt chosen by the firm. This means it is possible to have a full term structure of credit spreads derived from the CDS prices even for names with few bonds outstanding.

Lando (2004)\cite{Lando2004} also compared the factors that influence the credit spread between the CDS market and bond market, and explains the reason why CDS spreads could be larger bond spreads. An important difference between default swaps and bonds is the definition of a credit event. The ISDA (International Swap and Derivatives Association) standard documentation for default swaps includes restructuring as a credit event - even if no loss is caused to the bond holders by the restructuring. The tendency for the CDS to have a wider definition of default than that of rating agencies would tend to increase the CDS premium compared with the bond spread. A further argument could be that the CDS buyer has no influence through covenants on the decisions made by the issuer, in contrast with the owner of a reference bond. Additionally, the cost of shorting a bond through a reverse repo also contributes to a larger CDS spread. However, on the other hand, the desire of insurance companies and hedge funds to sell CDS contracts as a means to obtain “unfunded” exposure to credit risk could lower CDS spread, although little empirical evidence is available to date.

7.4 Conclusion

In this chapter we studied the term structure of counterparty spread into details. We first extend our model into credit crunch period and examine the default risk of the same 55 reference entities used in previous chapter. We collect their 5 year credit spreads from July 2007 - June 2009, combined with 5 year Euro benchmark curve as risk-free rates, and 5 year EURIBOR Swap rates as interest rate which considered to carry counterparty risk on top of riskless
rate. Counterparty risk spread is approximated by taking the difference between the two set of rates according to the same maturities. We then calibrate the coefficients following the CIR process of intensity model and found a much higher long-term mean of the intensity for those companies, with a higher volatility, and relatively smaller mean-reversion speed. This indicates the intensity processes are more dispersed with a high level of average comparing to the same set of coefficients during July 2004 - June 2007.

We obtain a good proxy of counterparty risk, and by comparing our findings with Mercurio (2009)\cite{61}, and validate our belief of the existence of counterparty risk, and even further, show that our counterparty spread provides a more consistent observation than that of Mercurio (2009)\cite{61}. About 25 basis points counterparty spread is found during peaceful market, which can soar up to more than 100 basis points during stress period of the economy.
Chapter 8

The Term Structure of the Counterparty Spread

Notation of Variables

\( f_i \) log-normal distributed LIBOR forwards rates

\( \mu_i \) drift component of the log-normal process

\( \sigma_i \) variance of the log-normal process

\( a, b, c, d \) parameters to determine the shape of the instantaneous volatility

\( \sigma_i (t, T_i) \) the instantaneous volatility of the \( i \)th forward rate at time \( t \).

\( T_i \) the expiry of the \( i \)th forward rate

\( t \) calendar time

\( \tau \) time-to-maturity
In this Chapter, we extend the counterparty credit spread from 5-year horizon to a term structure from 1 year to 10 years. We first look into the term structure of interest rates and credit default spread. We then look into the term structure of counterparty credit risk and its dynamics during the credit crisis period and found great similarity between the term structure of counterparty spread, and the term structure of instantaneous volatility from Libor-Market-Model (LMM) Rebonato first introduced in 2001. We then adapt the way Rebonato calibrates parameters to describe the shape of instantaneous volatility under LMM model, and calibrate the parameters to describe the shape of the counterparty spread, using Markov Regima Switching.

8.1 The Term Structure of the Interest Rates

The term structure of interest rates refers to the relationship between bonds of different terms, a yield curve is observed when interest rates of bonds are plotted against their terms. The shape of the yield curve reflects the market’s expectation of future interest rates and the conditions for monetary policy.

Usually longer term interest rates are higher than shorter term ones. This is called a “normal yield curve” and is thought to reflect the higher inflation-risk premium investor’s demand for longer term bonds. This is the yield curve shape that forms during normal market conditions, wherein investors generally believe that there will be no significant changes in the economy. During such conditions, investors expect higher yields for fixed income instruments with long-term maturities that occur further into the future.

The yield curve can become “steep” when the difference between long and short term interest rates is large. This reflects a “loose” monetary policy which means credit and money is readily available in an economy. This situation usually develops at an early stage of the economic cycle when a country’s monetary authorities are trying to stimulate the economy after a recession or slowdown in economic growth. The low short term interest rates reflect the easy availability
of money and low or declining inflation. Higher longer term interest rates reflect investor’s fear of future inflation.

When interest rates increase due to higher inflation expectations and tighter monetary policy, a small or negligible difference between short and long term interest rates occurs later in the economic cycle. This is called a “flat” yield curve and higher short term rates reflect less available money, as monetary policy is tightened and higher inflation later in the economic cycle.

This curve shape indicates that the market is sending mixed signals to investors, who are interpreting interest rate movements in various ways. During such condition, it is difficult for the market to determine whether interest rates will move significantly in either direction in the future. A flat yield curve usually occurs when the market is making a transition that emits different but simultaneous indications of what interest rates will do. In rare cases wherein longer-term interest rates decline, a flat curve can sometimes lead to an inverted curve.

Conditions wherein the expectations of investors are completely the inverse of a normal yield curve. In such abnormal market condition, bonds with longer maturity are expected to offer lower yields than bonds with shorter maturity. This indicates that the market currently expects interest rates to decline in the future.

Tight monetary policy results in short term interest rates being higher than longer term rates, which occurs as a shortage of money and credit drives up the cost of short term capital. Longer term rates stay lower as investors expect an eventual loosening of monetary policy and declining inflation. This increase the demand for long term bonds which lock in the higher long term rates.

Financial academics and economists have developed theories to explain the shape of yield curve. Mathematically, the yield curve can be used to predict interest rates at future dates.
8.2 The Term Structure of Credit Default Spread

Below is a figure sourced from Lando (1997) [52], which simulates term structure of credit default spread for different ratings. It can be seen that for lower rating firms, their credit spreads tend to increase over the years, as they are expected to downgrade to lower class and contain higher default risk. Whereas for lower rating class, the credit spread tend to decrease as in the next few year will be crucial period for those firms. Once they have past that stage, they are expected to have improved structure of assets and credibility.

The same concepts apply to the bond market, for investment grade bonds, the probability of default in a year tends to be an increasing function of time, this is because the bond issuer is initially considered to be creditworthy, the more time that elapses, the greater the possibility that its financial health will decline; on the other hand, for bonds with a poor credit rating, the probability of default is often a decreasing function of time. This is because for bonds with a poor credit rating, the next year or two can be crucial period for the underlying firm, the longer it survives, the greater the chance that its financial health will improve.

The schedule of credit spread of an issuer as a function of maturity is attracting a lot of attention today with the development of the bond to credit paradigm. Insurance instruments such as Credit Default Swaps are becoming liquid for maturities up to five or ten years. In reduced form model, the term structure of credit spreads is often captured by a default intensity parameter which is assumed to be a function of time and spot.

As Henrotte (2003) [38] pointed out, tweaking the default intensity does the job and yields simple numerical procedures, however, this is achieved at the cost of hiding the stochastic structure of the default process. The term structure of credit spread contains key information which is revealed in a time-homogeneous framework with a few constant parameters. Henrotte calibrates the parameters of a time-homogeneous regime-switching model, which reveals far more informa-
CHAPTER 8. THE TERM STRUCTURE OF THE COUNTERPARTY SPREAD

Figure 8.1: The simulated term structure of spreads in basis points for credit classes AAA, AA, A, and BBB as a function of maturity. These spreads assume risk neutrality and zero recovery rate. They are calculated based on the generator matrix from Lando (1997).

Figure 8.2: The simulated term structure of spreads in basis points for credit classes BB, B, CCC as a function of maturity. These spreads assume risk neutrality and a zero recovery rate. They are based on the generator matrix from Lando (1997).
tion on the underlying stochastic nature of the problem than using a seemingly simpler model with fewer parameters which must be tweaked every period.

Take Volkswagen for example, we take snapshots of its credit spread to represent the dynamic of its term structure, fig. 8.3 shows the term structure patterns of Volkswagen from July 2007 until August 2009, which obtains the major time period of credit crisis up-to-date. As one can observe, since July 2007, the credit crunch start to cause serious concern in the financial market, with all the write-down of credit debt and failure of pricing for structured finance products, financial institutions start to announce big losses in their loan and structured finance sector. The credit spread for the underlying entities start to shift parallel upwards to represent a higher probability of default. The shape of their term structure does not change and stays upward sloping. This means that the chances for the underlying to default in the long term are still higher than that of the short term.

However, since September 2008, default events, nationalisation and acquisition of a few major banks took place around the globe, the market started to crash even further. Investors start to panic and consider current situation could not be worse. Therefore, as can be seen in the second graph of fig. 8.3, the short end spread starts to pick up dramatically and out-win the longer spread. Spreads from both the short end and long end of the term structure maintain at peak level of their credit spread history. This phenomenon continues until the end of 2008 and into 2009, where market starts to normalise and calm down from what has happened in Sept 2008. The pattern of term structure starts to shift downwards, however, maintain the decreasing shape. This indicates that the confidence from the credit market remains low given what has happened.

This change of patterns for term structure of CDS spread with/without counterparty risk applies to most of the 55 sample underlying names.

Notice the spreads in fig. 8.3 come directly from the CDS market, which means counterparty risk was ignored. In order to account counterparty risk into the picture, we take the term structure of spread difference between the Euro
Figure 8.3: Terms structure patterns of Volkswagen spreads from July 2007 - Aug 2009
Spreads are obtained directly from Datastream (without counterparty-risk adjustment), source: Datastream.
Figure 8.4: Terms structure patterns of Volkswagen spreads from July 2007 - Aug 2009
Spreads are obtained directly from Datastream, and adjusted to be counterparty-risk free (by adding the counterparty spread on top of original CDS spread), source: Datastream.
swap rates and the Euro Benchmark rates as the representative for counterparty risk spread, same as we did in previous chapter for 5 year maturity but extend the horizon from 1 year to 10 years. We later add onto the term structure of credit spread for individual underlying entities, to represent the “true” credit risk of the underlying reference entities, as shown in fig. 8.4. As we could observe, the changes in term structure shape maintains similar progress, however, shows smoother progress in terms of the difference of credit spread between the short term and the long term. The spread level maintains at an even higher level. This is because during the credit crisis, the market’s confidence on financial institutions as counterparty issuers has reached its lowest point in history. Therefore the counterparty add-on is much greater than that in pre-crisis time (as shown in fig. 7.2). The chance of default for underlying entities are higher than that of those spreads observed directly from the CDS market. We will study the shape of term structure for counterparty spread in more details in latter section of this chapter.

8.2.1 The Swap Spread versus the Corporate-Bond Spread

Investment banks, which are the major dealers in the derivatives market and key players as counterparties for all the deals, shares a special unique role in the financial market. Majority of them are rates AA, or even AA, however, their credit spread should not alter too much into the future like other AA firms.

Lando (2004)[54] also compared the difference between the government yield curve, swap curve and AA rating spread curves. Although majority of the banks, who are the issuers of interest rate swap, and credit default swap, have ratings of AA or above, they are not expected to default in neither the short end nor longer end of the spread curve. Therefore, the swap yield curve shows very narrow spread with the AA rating spread at the short end; however, diverse from the AA curve and shows a more parallel shape on top of government bond at the long end, .

Lando (2004)[54] presents a simple model in which he obtains explicit solu-
tions using affine models. He takes into account both the stochastic variation in the six-month LIBOR rate and the possible deterioration of credit quality through rating downgrades of a particular issuer. His model can be easily extended to include more factors to better capture slope effects of the term structure.

As shown in figure 8.5, Lando’s model produces exactly the kind of pattern that has been observed empirically with swaps rates close to AA rates in the short end and increasing spread between AA rates and the swap rate as maturity grows.
8.3 The Term Structure of the Counterparty Spread

Using the same approach as in Chapter 5 and 6 to approximate the counterparty risk, we extend the counterparty proxy from 5-year spread only to a term structure of 1 to 10 years. We take the difference between the spread of swap curve and the European Benchmark curve as the counterparty spread, given each tenor year within the term structure. Fig. 8.6 shows the historic term structure of counterparty spread, by taking the difference between swap rates from EURIBOR and risk-free rates from Euro Benchmark Curve, the figure shows clearly how term structure of counterparty spread/risk over the years including the credit crunch period.

Similar to instantaneous volatility from the LIBOR Market Model we will introduce later, the counterparty spread shows a humped shape during “normal” time with the peak of term structure mostly at maturity of 3 years. From January 2005 until March 2006, the counterparty spread level remains low between around 10 basis points to 25 basis points. The term structure flatten out during pre-credit-crunch period from April 2006 to June 2007, which shows the market is giving mixed signals of the future counterparty risk. However, the level of counterparty risk remains the same as the “peaceful” period Investors are holding their bonds/credit default spread of reference entities and waiting for the direction market is going.

From July 2007- January 2009, credit crunch hit the market hard with two major events period at July 2007 till March 2008, when many investment banks announce huge losses and Bear Stearn’s default. This increases the counterparty spread as investors is getting anxious about financial institutions as counterparty of interest rate derivatives and credit risk derivatives. By September 2008, with the failure to save Lehman Brothers, one of the top four investment banks, and acquisition of Merill Lynch, together with many other bankruptcy, nationalisation and acquisition events happening then, the market starts to collapse. Investors lose faith of the financial institutions as counterparty and write-down
CHAPTER 8. THE TERM STRUCTURE OF THE COUNTERPARTY SPREAD

Figure 8.6: Historic Term Structure of Counterparty Credit Spread by taking the difference between EURIBOR Swap Rates and Euro Benchmark Curve from January 2005 to October 2009.
of mortgaged back security market continues, the counterparty spread sour to as high as 100 basis points with the short end of the term structure reaching almost 200 basis points. The term structure of counterparty spread shows a monotonically decreasing shape.

Things start to pick up in 2009, with the counterparty spread decrease within 100 basis points level, by keeping the same decreasing shape of “excited” period. This shows although the conditions have improved, financial institutions are still cautious about lending money to each other, or issuing interest rates swaps and credit default swaps to each other as counterparties. By September 2009, the counterparty spread is getting closer to its level before the credit crunch, spanning 20 to 60 basis points, after the market has had a major shuffle in 2008, with all the “damage” has been done.

This state of affairs would therefore give rise during “normal” period to a maximum in the market uncertainty in the intermediate-maturity region. However, during “excited” periods of market turmoil, there is a lack of consensus about the short-term actions of the monetary authorities, the spread at the short end can become very high. In this case, the counterparty credit spread should sharply increase at the short end, and the hump would disappear.

A financial justification for the empirically-observed existence of a hump can be explained as follows: at the short end of the maturity of counterparty spread are influenced by the actions of the monetary authorities, which tend to signal their intentions well ahead of their rate decisions. Surprises at the very short end are therefore rare and in “normal” period, the uncertainty in the front Credit Default Spread contracts tends to decline as they approach expiry. At the other end of the maturity spectrum the variation of market expectations about very distant spread rates is mainly driven by changing expectations about long-term inflation. Again, the presumed future actions of central banks, which often operate with an inflation target, would act to reduce the long-term volatility of nominal rates (at least if one assumes that real rates are little volatile). The greatest uncertainty resides in the intermediate region as tightening or loosening
regime can easily be reversed or continued beyond what is originally anticipated.

8.4 Instantaneous Volatility Functional Form for LMM

As mentioned in earlier sections, we found great similarity between the shape for the term structure of counterparty spread and that of the instantaneous volatilities of Libor Market Model. This section gives an introduction of the Libor Market Model and the function that is used to describe the shape of the term structure, under the framework of Markov Regime Switching.

It is well known that in the Black model for caplet, a closed form formula for the price of caplet was derived assuming that the forward rates are log-normal distributed and have constant volatility. The volatility implied by Black model of a caplet is the volatility with which the Black formula returns the market quoted price of the caplet. The LMM is constructed in a way that the LIBOR forwards rates are log-normally distributed under the associated measure where the Black formula applies. Once these functions are assigned, not only will the stochastic innovation be fully specified, but also the deterministic (no-arbitrage) drift component is defined.

$$\frac{df_i}{f_i} = \mu_i (\{\sigma_j\}, \{f_j\}) dt + \sigma_i (t, T_i) dz_i.$$  

Prescribing desirable instantaneous volatility functions is therefore the pricing problem in all the HJM-inspired pricing models (LIBOR Market Model, or the swap-rate Market Model). A functional form is used to represent the instantaneous volatility which should lead to a reduction of number of parameters and ultimately to a more stable calibration procedure. The parametrisation of the volatility term structure proposed in Rebonato (1999) is tested most extensively in the literature:
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Figure 8.7: Example of each parameter’s impact on the term structure shape with initial values of $a=-0.08$, $b=0.06$, $c=0.40$ and $d=0.15$.

$$\sigma_i(t, T_i) = \sigma_i(\tau) = [a + b\tau_i] \exp(-c\tau_i) + d,$$  \hspace{1cm} (8.1)

$$\tau_i = T_i - t,$$ \hspace{1cm} (8.2)

where $T_i$ is the expiry of the $i$th forward rate, $t$ is calendar time and $\sigma_i(t, T_i)$ is the instantaneous volatility of the $i$th forward rate at time $t$.

The functional form in eq. (8.1) has become the starting point for a popular stochastic-volatility extension of the LIBOR market model, some or all of the parameters (or their logarithms) are given a mean-reverting (Ornstein-Uhlenbeck) process.

As we can see in fig. 8.7, the change of shape due to the change of different parameters within equation (8.1). The value of $a$ determines the spread at very short end, whereas it has no impact in the longer maturity. It determines the shape of spread term structure from humped shape to monotonically decreasing; value of $b$ determines the volume of the peak at the hump; $c$ determines the...
time when peak happens, whereas \( d \) shifts the spread term structure parallel onto different levels.

The functional form of eq. (8.1) has the advantage of being simple yet affording a transparent interpretation for the parameters and their combinations. As time to maturity \( \tau \) goes to zero, instantaneous and average volatility coincide:

\[
\sigma_t^T = a + d = \lim_{T \to 0} \hat{\sigma}(t),
\]

(8.3)

With \( c \) positive, the volatility tends to \( d \) for large time-to-maturity \( \tau \), and is equal to \( a + d \) at \( \tau = 0 \) which corresponds to option expiry. When \( b, c > 0 \) there is a maximum value at

\[
\tilde{\tau} = 1/c - a/b.
\]

(8.4)

### 8.4.1 Markov Regime Switching

Markov Regime Switching can be applied in different sectors and aspects in Finance and Economics to model a variety of different phenomena, including asset prices and market crashes. The first financial model to use a Markov chain was the regime-switching model of James D. Hamilton in 1989, where a Markov chain is employed to model switches between periods of high volatility and low volatility of asset returns. A more recent example is the Markov Switching Multifactor asset pricing model, which builds upon the convenience of earlier regime-switching models. It uses an arbitrarily large Markov chain to drive the level of volatility of asset returns. Markov chains are heavily used in dynamic macroeconomics, an example is to use Markov chains to exogenous model prices of equity in a general equilibrium setting.

Bollen, Gray, and Whaley (2000) examine the ability of regime-switching models to capture the dynamics of foreign exchange rates. They test the ability of the models to fit foreign exchange rate data in-sample and forecast variance out-of-sample. They find that a regime-switching model with independent shifts
in mean and variance exhibits a closer fit and more accurate variance forecasts than a range of other models. They then use exchange-traded currency options to determine whether market prices reflect regime-switching information and find that observed option prices are significantly different from their theoretical levels determined by a regime-switching option valuation model and that a simulated trading strategy based on regime-switching option valuation generates higher profits than standard single-regime alternatives. Overall, their results indicate that regime-switching models may have practical implications for investors and captures the dynamics of exchange rates better than alternative time series models.

In credit risk, Das et al. (2006) [16] provide a comprehensive empirical investigation of how default probabilities co-vary using a database of issuer level default probabilities for the period 1987-2000. The database they select provides a unique opportunity to understand how default risk behaves both in the cross-section of firms and in the time series of all public non-financial firms in the U.S.. In order to account for their observation that default probabilities and correlation vary with economic events, they allow the economy-wide default risk to be regime-dependent. Strong support are found for a two-regime model, with a high default regime and a low default regime, the high default regime has a mean default level more than twice as high as the low default regime. They also demonstrate that each regime shows a different correlation structure, wherein default correlations are higher in the high default regime compared with those in the low default regime. Therefore, systematic variation in joint default risk may be modelled within a simple reduced-form framework, which allows default risk to be quantified at a portfolio level.

Jarrow et al. (1997) [44] presents a simple Markov model for valuing risky debt that explicitly incorporates a firm’s credit rating as an indicator of the likelihood of default. They provide an arbitrage-free model for the term structure of credit risk spreads and their evolution through time. The model is based on Jarrow and Turnbull (1995) [47], with the bankruptcy process following a
discrete state space, continuous time, time-homogeneous Markov chain in credit ratings. This model is useful for pricing and hedging corporate debt with embedded options, for pricing and hedging: OTC derivatives with counterparty risk; or foreign government bonds subject to default risk; or credit derivatives; and for risk management in general.

Joshi & Rebonato (2001) present an extension of the LIBOR market model, which allows the stochastic instantaneous volatility of the forward rates in a displaced diffusion setting. They successfully extend the deterministic volatility case to the stochastic volatility case while keeping all the powerful and important approximations. They also show that the market caplet surface across strikes and maturities can be well recovered even after reducing the number of the structure of volatility.

Rebonato & Kainth (2003) introduce a two-regime, Markov chain extension to the LMM model, where the unobservable instantaneous-volatility process migrates between two states, one of which is associated with the parameters that give a monotonically-decreasing term structure of the instantaneous volatility, and the other with the parameters associate with a humped shape. Rebonato (2006) suggests that a two-regime Markov chain approach may be more successful and better financially motivated, more generally, his study highlights the shortcomings of purely time-dependent or time-homogeneous approaches. He concludes that neither time-homogeneity nor time dependence constitute a desirable modelling approach, and that the possibility of the instantaneous volatility migrating between a normal to an excited state is likely to be a necessary ingredient for a convincing description of the dynamics of the swaption surface.

8.5 Counterparty Credit Spread under the Markov Regime Switching

The framework for the two-regime, Markov chain approach is as follows:
\[ CS_i (t, T_i) = y_i CS^n_i (t, T_i) + (1 - y_i) CS^x_i (t, T_i), \]
\[ CS^n_i (t, T_i) = [a^n_i + b^n_i (T - t)] \exp (-c^n_i (T - t)) + d^n_i, \]
\[ CS^x_i (t, T_i) = [a^x_i + b^x_i (T - t)] \exp (-c^x_i (T - t)) + d^x_i. \]  

The two different sets of coefficients \( \{a^n, b^n, c^n, d^n\} \) and \( \{a^x, b^x, c^x, d^x\} \) are associated with the normal (superscript \( n \)) and excited state (superscript \( x \)) and the latent variable, \( y_i \), takes value 0 or 1 following a two-state Markov-chain process between the normal state and excited state, with transition probabilities:

\[ P = \begin{bmatrix} P_{nn} & P_{nx} \\ P_{xn} & P_{xx} \end{bmatrix} = \begin{bmatrix} P_{nn} & 1 - P_{nn} \\ 1 - P_{xx} & P_{xx} \end{bmatrix}. \]  

where \( P_{xn} \) is the probability of going from the excited to the normal state. The unconditional probabilities of being in the normal and excited state are:

\[ q_n = \frac{1 - P_{xx}}{2 - P_{nn} - P_{xx}} \quad \text{and} \quad q_x = \frac{1 - P_{nn}}{2 - P_{nn} - P_{xx}}. \]  

The interpretation of the two states as “normal” and “excited” has an intuitively motivation to the procedure. It fits well with the empirical observation of sharp transitions in the swaption implied-volatility matrix between well-defined states. Our intuition is that the system should spend most of its time in a normal state, and experience short transitions to the excited state.

### 8.5.1 Parameter Constraints

The parameters that need to fit are:

- \( a_n, b_n, c_n \), and \( d_n \) describing the normal volatility curve;
- \( a_x, b_x, c_x \), and \( d_x \) describing the excited volatility curve;
- \( P_{nn} \) and \( P_{xx} \) describing the Markov transition matrix.

Here we make the counterparty spread non-deterministic via the following
stochastic mean-reverting behaviour for the coefficients \( \{a^n, b^n, c^n, d^n\} \) or their logarithm under “normal” state, and \( \{a^x, b^x, c^x, d^x\} \) or their logarithm under “excited” state, respectively as appropriate:

\[
\begin{align*}
    da_t &= \beta_a (\theta_a - a_t) \, dt + \sigma_a(t) \, dz^a_t \\
    db_t &= \beta_b (\theta_b - b_t) \, dt + \sigma_b(t) \, dz^b_t \\
    d\ln c_t &= \beta_c (\theta_c - \ln c_t) \, dt + \sigma_c(t) \, dz^c_t \\
    d\ln d_t &= \beta_d (\theta_d - \ln d_t) \, dt + \sigma_d(t) \, dz^d_t
\end{align*}
\] (8.8-11)

All the Brownian increments within the formula above are uncorrelated with each other and \( \theta \) denotes the long-term mean of those coefficients individually under either “normal” or “excited” routine, and \( \beta \) denotes the mean reversion speed for those coefficient under each state.

\[
P = \begin{bmatrix} P_{nn} & P_{nx} \\ P_{xn} & P_{xx} \end{bmatrix} = \begin{bmatrix} P_{nn} & 1 - P_{nn} \\ 1 - P_{xx} & P_{xx} \end{bmatrix}.
\] (8.12)

8.6 Results

Given the framework of credit spread determined by a different set of coefficient: \( \{a^n, b^n, c^n, d^n\} \) under “normal” state, and \( \{a^x, b^x, c^x, d^x\} \) under “excited” state, we calibrate the coefficient by shape and level of term structure of counterparty spread for each observation date. However, what we do different from Rebonato & Kainth (2003) is to determine the “normal” or “excited” state purely upon the counterparty spread level, rather than the shape of the term structure. It is clear to us, although some rare dates during the credit crisis the term structure of counterparty spread shows a hockey-stick humped shape, that does not indicate that the market is back to a normal state, the belief in the counterparty credibility for those dates is still weak, hence we have to define them as “excited” state. We define the 1 year counterparty as the most sensitive point to
CHAPTER 8. THE TERM STRUCTURE OF THE COUNTERPARTY SPREAD

<table>
<thead>
<tr>
<th></th>
<th>Normal Regime</th>
<th>Excited Regime</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>-1.237</td>
<td>-3.533</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.109</td>
<td>0.204</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>3.523</td>
<td>8.808</td>
</tr>
<tr>
<td>$\ln a^n$</td>
<td>1.441</td>
<td>-1.763</td>
</tr>
<tr>
<td>$\ln b^n$</td>
<td>-1.005</td>
<td>1.756</td>
</tr>
<tr>
<td>$\ln c^n$</td>
<td>-0.261</td>
<td>-0.889</td>
</tr>
<tr>
<td>$\ln d^n$</td>
<td>1.141</td>
<td>-1.763</td>
</tr>
<tr>
<td>$a^x$</td>
<td>0.146</td>
<td>0.217</td>
</tr>
<tr>
<td>$b^x$</td>
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<td>0.892</td>
</tr>
<tr>
<td>$c^x$</td>
<td>1.615</td>
<td>4.269</td>
</tr>
<tr>
<td>$d^x$</td>
<td>0.892</td>
<td>4.998</td>
</tr>
</tbody>
</table>

Table 8.1: Calibrated parameters to describe counterparty spread term structure under “normal” and “excited” regime, where for each regime, $\theta$ represents long-term mean, $\beta$ represents mean-reversion speed, and $\sigma$ represents volatility.

respond to current market situation and this is a major coefficient to determine the shape of the term structure of counterparty spread. If it has risen above 50 basis point, then the counterparties are in an “excited” state, the rest are all defined as “normal” state.

Once the dates are categorised as “normal” or “excited” by our criteria, and individual daily coefficients of $\{a, b, c, d\}$ are calibrated, we separate the coefficients individually by the state it occurs, and further calibrate their long-term mean and mean-reversion speed as well as volatility. Take coefficient $a$ for example, which determines the level of counterparty spread in the long end of the term structure. We first gather all the values of $a^n$ under normal state, and calibrate its long term mean and mean-reversion speed as well as volatility, we do the same for the rest of the parameters for $b^n, c^n$ and $d^n$ and repeat our calibration for $\{a^x, b^x, c^x, d^x\}$. Notice that for coefficients $c$ and $d$, logarithmic values are taken to guarantee strictly positive value for those coefficients.

By separating our observations from 2004 - 2009 into “normal” or “excited” states, we obtain two sets of values for the mean-reverting process $\{a^n, b^n, c^n, d^n\}$ and $\{a^x, b^x, c^x, d^x\}$, or their logarithm under each state, bearing their impact on the shape of counterparty term-structure.

8.6.1 Transition Matrix

When it comes to transition matrix calibration, $P = \begin{bmatrix} P_{nn} & P_{nx} \\ P_{xn} & P_{xx} \end{bmatrix}$ can choose different values to converge to a same set of eigenvalues to represent the unconditional probability of the two regimes. Although we have chosen a time span
of 5 years of data, it is still worth to expand the time horizon further to look at historical probability for excited events to happen.

In here, we assume the transition matrix takes the value of

\[
P = \begin{bmatrix}
P_{nn} & P_{nx} \\
P_{xn} & P_{xx}
\end{bmatrix} = \begin{bmatrix}
0.894 & 0.106 \\
0.599 & 0.400
\end{bmatrix}
\]

. Then we obtain the unconditional of “normal” regime’s probability to be 0.85, and the unconditional of “excited” regime to be \((1 - 0.85) = 0.15\). Different possible values of matrix \(P\) can be assigned to represent one’s belief of market behaviour.

8.7 Conclusion

This chapter looks at the shape of term structure of credit spread with/without counterparty risk. We mainly focus on the term structure of counterparty spread and compare its similarity towards the shape of term structure of instantaneous volatility of the LIBOR Market Model Rebonato first introduced in 2001, it is later improved by making a two-regime Markov model, and even making the coefficients of each state to be stochastic and calibrate parameters for each coefficients. We find the two-regime switching concept is necessary for counterparty spread calibration, as the term structure of counterparty spread and its level does change during the credit crunch and it is very distinct from that of a peaceful time.
Chapter 9

Conclusion

In this Chapter we describe a few suggestions for further research and conclude by summarising the contributions made in this thesis.

9.1 Suggestion for Future Research

There are several factors this thesis does not take into account:

9.1.1 Central Clearing House

A central clearing house for the CDS market is called for by the regulators to eliminate the counterparty risk, it is demanded both in the academic world and among practitioners. However, the example of mortgage agencies such as Fannie Mae and Freddie Mac, to provide a non-default guarantee as a mediator, is a good example that a central clearing house is not a solution for all \footnote{Fannie Mae and Freddie Mac were being placed into conservator-ship of the FHFA.} cautions are still required when booking the CDS contract correctly and monitor the underlying entity's default risk sensibly.
9.1.2 Netting and Collateralisation

This thesis ignores the effect from netting agreement and collateralisation. Netting and collateralisation with frequent margin adjustments are essential for counterparties that are involved in various derivatives and security financing trades (SFT). This is because even in the absence of reference entity default, a failure of a major counterparty could lead to losses across the financial system. Upon the default of the counterparty, OTC derivatives would be immediately and significantly re-priced, with credit spreads likely widening dramatically. It is standard practice for financial institutions to enter derivative contracts documented on Master Agreements as recommended by the ISDA (international Swaps and Derivative Association). All ISDA contract holders are ranked pari passu to senior debt, in terms of claims on a defaulting counterparty. Margin agreements require banks to post different levels of collateral on their outstanding contracts depending on the current mark-to-market of the contract. Another advantage of trading within the ISDA framework is the provision of netting. Netting agreements come into action in the case of actual counterparty default. Without such agreements, a surviving counterparty would legally have to fully meet its obligations to the defaulting counterparty, while only being left with a claim on its dues from the same.

The counterparty risk after collateralisation and establishment of netting agreement with other derivatives traded with the same counterparty can be further investigated. However, the counterparty effect after collateralisation, and netting after counterparty default is difficult to quantify, that is why this paper ignores this effect and leave space for further study.

9.1.3 Other Non-default Components

In Chapter 5 and 6, when comparing the default components derived from both the bond market and the CDS market, we assume the difference between the intensities from the two markets purely comes from the liquidity risk, a non-
default component. Further investigation can be done to find other determinants of the non-default components on this difference from both a cross-sectional and time-series perspective using a number of explanatory variables such as tax and liquidity-related bid-ask spread.

9.1.4 Stochastic Intensity

For the contagion model in Chapter 4, instead of making the default intensity time-variant, a possible alternative could be to make the default intensity a stochastic process. However, the mathematical solution might be difficult to find, where a Monte-Carlo simulation might need to be employed.

9.2 Concluding Remark

In this thesis we focus on the counterparty risk aspect of credit default swap pricing. In Chapter 3 and 4, we firstly carry out a theoretical analysis based on the contagion model introduced by Jarrow & Yu (2001) and extend the framework from discrete premium rate into continuous premium rate, by applying the change of numeraire technique. We further extend it into three-firm model, where protection buyer is defaultable. Chapter 5 carries out an empirical work to find counterparty risk for credit default swaps. We introduce a counterparty risk proxy by taking the difference between 5-year credit spread over swap rate, and 5-year credit spread over benchmark rate, we then obtain the counterparty-risk-free spread after this treatment and apply the Cox-Ingersoll-Ross framework to model the credit spread dynamic of 55 sample underlying entities. Chapter 6 investigates the counterparty risk during credit crunch period using the same CIR model. Chapter 7 studies the shape of the term structure of the counterparty credit spread by adopting the function Rebonato (2006) uses to approximate the shape of the term structure of instantaneous volatility under Libor-Market-Model, where additional Markov Regime Switching is applied in order to represent the “normal” and “excited” condition before, during
and after the sub-prime crisis..

### 9.2.1 Contagion Default Model

Theoretical work is firstly done to demonstrate the impact of joint default events between reference entity, protection seller and protection buyer on the CDS spread. A contagion model is applied, it assumes a discrete constant default intensity under the reduced-form framework, which is introduced by Jarrow & Yu (2001)[48]. In Chapter 3, we extend the previous study by making the premium payment a continuous process, a closed form solution is then provided based on the change of measure technique introduced in Collins-Dufresne et al. (2002)[11] in the valuation procedure of the swap rate. Using the change of measure solves the problem of violation of jump conditions under Cox processes for defaultable credit derivatives, instead of assuming a primary-secondary framework to overcome the integrations. More indicative and simple results of the swap premium calculation is found by extending to a continuous-time premium rate instead of its discrete counterpart. The results validate the assumption that higher the default intensity of protection seller, lower the swap premium, and vice versa. On the other hand, by fixing the intensity rate constant, the longer the settlement period, the more likely the counterparty will default within the period, and the less a protection buyer would pay for protection against the default of the reference entity.

Carrying on with the contagion framework, Chapter 4 extends the counterparty risk model to a three-firm model. This makes the protection buyer defaultable with the rest of the parties. However, the results show that protection seller’s default probability does not play an as important role as protection seller and reference entity, similar results is derived as the two-firm model. Settlement period and counterparty B’s total intensity rate still play a crucial role in the swap premium calculation, which is in sync with the two-firm model.

In order to make our findings more generic, we relax some of the assumptions and make the intensity rates time-variant by simulating an economy upturn and
downturn every decade. In a two-firm defaultable environment, the premium rate does not vary much from the constant intensity rate case, in fact if settlement period is relatively small, the time-variant intensity case generates close results to the constant intensity example.

### 9.2.2 CIR Model for CDS spread

In Chapter 5 and 6, a counterparty spread proxy is done by taking the difference between spread over government bond and spread over swap yield. Counterparty spread is then added onto the CDS spread from the CDS or bond market to represent the real default spread of the corporate bond issuer. In doing so one can retain true credit spread of a firm who does not have sufficient information from the bond market by adding the counterparty spread on top of credit spread from the CDS market. 5-year credit spread is extracted from the bond market by using the bracketing technique, which is to calculate the credit spread of all the bonds of a company of different maturities and regress the credit spread of 5-year maturity.

Once the counterparty-risk-free credit spread is obtained, we calibrate the dynamic process of default intensity using the CIR model used for short-term interest rates. We found that the mean-reversion speed is similar across all the 55 sample companies, we also found that using the government bond provides a good estimate for risk-free rate. Comparing the default intensity between CDS market and the default intensity from bond market, we found that the spread from CDS market provides better liquidity and responds more sensitively to market signals.

In Chapter 6 we study the term structure of counterparty spread into details. We extend our model into credit crunch period and examine the default risk of the same 55 reference entities used in Chapter 5. Counterparty risk spread is approximated by taking the difference between the swap rates and government bond rates according to the same maturities. The default intensity process is then calibrated to follow the CIR process. A much higher long-term mean of
the intensity is found for the sample companies, with a higher volatility, and relatively smaller mean-reversion speed. This indicates the intensity process are more dispersed with a high level of average comparing to the same set of coefficients during July 2004 - June 2007.

We compare our proxy for counterparty risk with the one proposed in Mercurio (2009)[61], we validate our belief of the existence of counterparty risk, and show that our counterparty spread provides a more consistent observation than that of Mercurio (2009)[61]. About 25 basis points counterparty spread is found in our study during peaceful “normal” market, however, it can soar up to more than 100 basis points during stress period of the economy.

Chapter 7 looks at the shape of the term structure of credit spread with/without counterparty risk. We focus on the term structure of counterparty spread and compare its similarity towards the shape of term structure of instantaneous volatility of the LIBOR Market Model Rebonato first introduced in 2001. We improve it by making a two-regime Markov model, and even more, we make the coefficients of each state to be stochastic when calibrating parameters for each coefficients. The two-regime switching concept is found to be necessary for counterparty spread calibration, as the term structure of counterparty spread and its level changes dramatically during credit crunch period, which is very distinct from that of a peaceful time.
Appendix A

Markov Chains

In Mathematics, a Markov Chain, named after Andrey Markov, is a random process where all information about the future is contained in the present state (i.e. one does not need to examine the past to determine the future). If a process has the Markov property, its future states only depend on the present state, and are independent of its past states. In other words, the observation of the present state fully captures all the information that could influence the future evolution of the process. Making this a stochastic process means that all the states transitions are probabilistic.

At each step the system may change its state from the current state to another state, if not remain in the same state, according to a probability distribution. The change of state is called a transition, and the probabilities linked with transitions are called transition probabilities.

A Markov chain is a sequence of random variables $X_1, X_2, X_3, \ldots$ with the Markov property, which means given the present state, the future and past states are independent:

$$\Pr(X_{n+1} = x \mid X_1 = x_1, X_2 = x_2, \ldots, X_n = x_n) = \Pr(X_{n+1} = x \mid X_n = x_n).$$ (A.1)
The possible values of $X_i$ form the state space of the chain, namely, set $S$. Continuous-time Markov processes have a continuous index, and time-homogeneous Markov chains are processes such as

$$\Pr(X_{n+1} = x \mid X_n = y) = \Pr(X_n = x \mid X_{n-1} = y). \quad (A.2)$$

for all $n$. Which means the probability of transition is independent of $n$.

The probability of going from state $i$ to state $j$ in $n$ time steps is

$$p_{ij}^{(n)} = \Pr(X_n = j \mid X_0 = i), \quad (A.3)$$

and the single-step transition is

$$p_{ij} = \Pr(X_1 = j \mid X_0 = i), \quad (A.4)$$

For a time-homogeneous Markov chain:

$$p_{ij}^{(n)} = \Pr(X_{n+k} = j \mid X_k = i), \quad (A.5)$$

and

$$p_{ij} = \Pr(X_{k+1} = j \mid X_k = i). \quad (A.6)$$

Therefore the $n$-step transition satisfies the Chapman-Kolmogorov equation, that for any $k$ such that $0 < k < n$,

$$p_{ij}^{(n)} = \sum_{r \in S} p_{ri}^{(k)} p_{rj}^{(n-k)},$$

where $S$ is the state space of the Markov chain. The marginal distribution $\Pr(X_n = x)$ is the distribution over states at time $n$, the initial distribution is $\Pr(X_0 = x)$. The evolution of the process through one time step is described by
\begin{align*}
\Pr(X_n = j) &= \sum_{r \in S} p_{rj} \Pr(X_{n-1} = r) = \sum_{r \in S} p_{rj}^{(n)} \Pr(X_0 = r) .
\end{align*}

Where the superscript \((n)\) is an index instead of an exponent.

### A.1 Example of Markov Regime Switching

Here, two regimes are chosen, and they are called “normal” and “excited” states. Given the economic state on the preceding day, the transition probabilities between states can be represented by a transition matrix:

\[
P = \begin{bmatrix}
P_{nn} & P_{nx} \\
P_{xn} & P_{xx}
\end{bmatrix} = \begin{bmatrix}
P_{nn} & 1 - P_{nn} \\
1 - P_{xx} & P_{xx}
\end{bmatrix} = \begin{bmatrix}
0.9 & 0.1 \\
0.6 & 0.4
\end{bmatrix}
\]

The matrix \(P\) represents the weather model in which a normal market condition is 90% likely to be followed by another normal day, whereas a excited day is 40% likely to be followed by another excited day. \(P_{ij}\) is the probability that given a day is of type \(i\), it will be followed by a day of type \(j\). Notice that the rows of \(P\) sum to 1, which is because \(P\) is a stochastic matrix.

Predicting the market condition: the market condition at day 0 is known to be normal, this is represented by a vector in which the “normal” entry is 1, and the “excited” entry is 0:

\[
x^{(0)} = \begin{bmatrix} 1 & 0 \end{bmatrix} , 
\]

(A.7)

the weather on day 1 can be predicted as:

\[
x^{(1)} = x^{(0)} P = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 0.9 & 0.1 \\
0.6 & 0.4
\end{bmatrix} = \begin{bmatrix} 0.9 & 0.1 
\end{bmatrix} .
\]

(A.8)

Thus, there is a 90% chance that the next day, day 1, will also be “normal”.

\footnote{The market condition can also be set as “normal” entry is 0, and the “excited” entry is 1, however, this is only a initial setup and does not affect long term probability of “normal” or “excited” once we have reached the steady state distribution.}
APPENDIX A. MARKOV CHAINS

The market condition on day 2 can be predicted in the same way:

\[ x^{(2)} = x^{(1)}P = x^{(0)}P^2 = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 0.9 & 0.1 \\ 0.6 & 0.4 \end{bmatrix}^2 = \begin{bmatrix} 0.87 & 0.13 \end{bmatrix}. \quad (A.9) \]

To make it more general: \( x^{(n)} = x^{(n-1)}P = x^{(0)}P^n \).

Steady state of the market condition

In this example, predictions for the market condition on more distant days are becoming inaccurate increasingly and tend towards a steady state vector. This vector represents the probabilities of “normal” and “excited” stage on all days, and is independent of the initial market condition.

The steady state vector is defined as:

\[ q = \lim_{n \to \infty} x^{(n)} \quad (A.10) \]

But only converges to a strictly positive vector if \( P \) is a regular transition matrix, which means there is at least one \( P^n \) with all non-zero entries. Since \( q \) is independent from initial conditions, it must be unchanged when transformed by \( P \). This makes it an eigenvector (with eigenvalue 1), which means it can be derived from \( P \). For the market condition example:

\[
\begin{align*}
P &= \begin{bmatrix} 0.9 & 0.1 \\ 0.6 & 0.4 \end{bmatrix} \\
qP &= q = qI \\
q(P - I) &= 0 \\
q \begin{bmatrix} 0.9 & 0.1 \\ 0.6 & 0.4 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} &= q \begin{bmatrix} -0.1 & 0.1 \\ 0.6 & -0.6 \end{bmatrix} \\
\begin{bmatrix} q_1 & q_2 \end{bmatrix} \begin{bmatrix} -0.1 & 0.1 \\ 0.6 & -0.6 \end{bmatrix} &= \begin{bmatrix} 0 & 0 \end{bmatrix}
\end{align*}
\]
Which means $-0.1q_1 + 0.6q_2 = 0$ and since they are probability vector we know that $q_1 + q_2 = 1$.

Solving this pair of simultaneous equations gives us the steady state distribution:

\[
\begin{bmatrix}
q_1 \\
q_2
\end{bmatrix} = \begin{bmatrix}
0.857 & 0.143
\end{bmatrix}
\]

In conclusion, in the long term, 86% of the days are “normal” market status, whereas the rest 14% are “excited” market.

### A.2 Real-World probability vs. Risk-Neutral Probabilities

The default probabilities implied from bond yields are risk-neutral probabilities of default. This can be shown from the calculations which assume expected default losses can be discounted at the risk-free rate. This risk-neutral valuation principle shows that this is a valid procedure providing the expected loss to be calculated in a risk-neutral world. On the other hand, the default probabilities implied from historical data are real-world default probabilities.
Bibliography


[38] Henrotte, P., 2003, The case for time homogeneity, HEC School of Management, working paper.


[40] Huang, J., and M. Huang, 2002, How Much of the Corporate-Treasury Yield Spread is Due to Credit Risk?, working paper, Penn State University.


