1. Abstract

- Low power Continuous Wavelet Transform (CWT) circuits are essential for use in wearable, long term physiological monitoring systems.
- Present analogue CWT circuits implement the CWT as a bandpass filter and rely on taking a mathematical approximation of the wanted mother wavelet function.
- We present an alternative approximation technique based upon the use of standard filter approximations: Butterworth, Chebyshev and Bessel bandpass filters.

2. Low power CWT circuits

- At a single analysis scale the CWT is a bandpass filtering operation\(^1\).
- The required bandpass filter must have an impulse response function:
  \[
  h(t) = \frac{1}{\sqrt{a}} \psi\left(\frac{t}{a}\right)
  \]
  
  where \(a\) is the analysis scale and \(\psi(t)\) is the mother wavelet function.
- The bandpass filter can be either the analogue or digital domain:
  - Analogue approaches are popular as they have very low power consumption, typically in the order of nano-Watts\(^1,\,2\).
  - However, analogue filters can only implement an approximation of \(h(t)\).

3. Current mathematical approximation

- Using the Mexican hat mother wavelet as an example, the ideal \(h(t)\) impulse response is:
  \[
  h(t) = -\frac{a^{1/4}}{\sqrt{\sqrt{2}\pi}} \cdot \sqrt{\frac{a}{3}} s(t)
  \]
  
  where a delay \(T\) has been introduced and the denominator has been expanded as a truncated Maclaurin series.
- The resulting impulse response function is:
  \[
  H(s) = \frac{1}{\sqrt{a}} \sqrt{\frac{a}{3}} s(T) + \frac{1}{\sqrt{a}} \sqrt{\frac{a}{3}} s(T) + \cdots
  \]

5. Demonstration of operation

- To demonstrate the utility of the proposed approximations we use them to perform a CWT analysis of ECG data.
- The ECG record is taken from the MIT-BIT noise stress database\(^3\):
  - It is measured ambulatory ECG corrupted by baseline wander, muscle artefact and electrode motion artefact.
- For each approximation method we use 98 bandpass filters in parallel:
  - For correct wavelet operation the gain of each filter changes with the centre frequency \(\omega_c\), as:
  \[
  H(s) \rightarrow \frac{3.65}{\omega_c} \times H(s)
  \]
  - Centre frequencies are logarithmically spaced.
- Below, colour represents the absolute output signal from each filter versus time and the filter centre frequency for the ECG signal above:

Compared to the ideal Mexican hat CWT all the approximations have a delay present at low frequencies.
- Nevertheless, similar ECG information is extracted in all cases.
- The Mexican hat approximations proposed here thus work, and provide alternatives to the current mathematical approach.
- Furthermore, there is a large body of work on the design of circuits implementing Butterworth, Chebyshev and Bessel transfer functions, easing the circuit level design required.

For comparison, the more traditional frequency domain view:

- As the filter order increases, better approximations to the Mexican hat impulse response are achieved.
- Thus Butterworth, Chebyshev and Bessel bandpass filters can approximate the wanted \(h(t)\) function.
- The parameters used to achieve this:

\begin{itemize}
  
  \item **Filter**
  - Butterworth
  - Centre freq. = 15 rads\(^-1\), \(Q = 1.5\)
  
  \item **Chebyshev type I**
  - Centre freq. = 15 rads\(^-1\), \(Q = 1.35\)
  - Ripple (\(dB\)) = 0.8
  
  \item **Chebyshev type II**
  - Centre freq. = 7.5 rads\(^-1\), \(Q = 0.55\)
  - Ripple (\(dB\)) = 20
  
  \item **Bessel**
  - Centre freq. = 15 rads\(^-1\), \(Q = 1.35\)
\end{itemize}

- By varying these parameters other mother wavedlets can also be approximated.

\(^2\) S. A. P. Haddad and W. A. Serdijn, Ultra low-power biomedical signal processing: An analog wavelet approach for pacemakers. Springer. 2009.