Quantum discontinuity between zero and infinitesimal graviton mass with a Λ term

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Abstract

We show that the recently demonstrated absence of the usual discontinuity for massive spin 2 with a Λ term is an artifact of the tree approximation, and that the discontinuity reappears at one loop.

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An old question is whether the graviton has exactly zero mass or perhaps a small but non-zero mass. This issue seemed to have been resolved by van Dam and Veltman [1] and, independently, Zakharov [2] when they noted that there is a discrete difference between the propagator of a strictly massless graviton and that of a graviton with mass M in the $M \to 0$ limit. This difference gives rise to a discontinuity between the corresponding amplitudes involving graviton exchange. In particular, the bending of light by the sun in the massive case is only $3/4$ of the experimentally confirmed massless case, thus ruling out a massive graviton.

Recently, however, the masslessness of the graviton has been called into question by two papers [3,4] pointing out that the van Dam-Veltman-Zakharov discontinuity disappears if, instead of being Minkowski, the background spacetime is anti-de Sitter (AdS). The same result in de Sitter space had earlier been obtained in [5,6]. In fact, as shown below, this can be extended to any Einstein space satisfying

$$R_{\mu\nu} = \Lambda g_{\mu\nu} \tag{1}$$

with a non-zero cosmological constant $\Lambda \neq 0$ provided $M^2/\Lambda \to 0$.

These results remain surprising, however, since the massive graviton retains five degrees of freedom, while the massless one only has two. Although these extra states decouple from a covariantly conserved stress tensor for $M^2/\Lambda \to 0$, yielding a smooth classical limit, they are nevertheless still present in the theory, suggesting that a discontinuity may remain at the quantum level. In this letter, we demonstrate that this is indeed the case by calculating the one loop graviton vacuum amplitude for a massive graviton and showing that it does not reproduce the result for the massless case in the limit. Thus the apparent absence of the discontinuity is only an artifact of the tree approximation and the discontinuity reappears at one loop.

In order to handle a massive graviton and resulting loss of general covariance, we employ the alternative "Stückelberg" formalism [7,6]. This formalism reproduces a tree-level amplitude for conserved sources which agrees with Ref. [4] for general Λ and M, but confirms the naively expected discontinuity in the determinant describing the one-loop effective action for the background configuration.

We work in four dimensions with Euclidean signature $(+ + + +)$. As in Ref. [4], our starting point is the action

$$S[h_{\mu\nu}, T_{\mu\nu}] = S_L[h_{\mu\nu}] + S_M[h_{\mu\nu}] + S_T[h \cdot T], \tag{2}$$

where S_L is the Einstein-Hilbert action with cosmological constant $S_E = -\frac{1}{16\pi G} \int \mathrm{d}^4 x \sqrt{\hat{g}} (\hat{R} - 2\Lambda)$, linearized about a background metric $g_{\mu\nu}$ satisfying Eq. (1) according to $\hat{g}_{\mu\nu} = g_{\mu\nu} + \kappa h_{\mu\nu}$ where $\kappa^2 = 32\pi G$. For such a background, this linearized action for $h_{\mu\nu}$ is

$$S_L = \int \mathrm{d}^4 x \sqrt{g} \Big[\tfrac{1}{2} \tilde{h}^{\mu\nu} \left(-g_{\mu\rho} g_{\nu\sigma} \Box - 2 R_{\mu\rho\nu\sigma} \right) h^{\rho\sigma}$$
$$- \nabla^\rho \tilde{h}_{\rho\mu} \nabla^\sigma \tilde{h}_\sigma{}^\mu \Big], \tag{3}$$

where $\tilde{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2} g_{\mu\nu} h^\sigma{}_\sigma$. The linearized Lagrangian S_L has a gauge symmetry described by a vector $\xi_\mu(x)$

$$h_{\mu\nu} \rightarrow h_{\mu\nu} + 2\nabla_{(\mu}\xi_{\nu)}\,. \tag{4}$$

All indices are raised and lowered with respect to the metric $g_{\mu\nu}$ and ∇_μ is taken to be covariant with $\nabla_\mu g_{\lambda\sigma} = 0$. The Pauli-Fierz spin-2 mass term

$$S_M = \tfrac{M^2}{2} \int \mathrm{d}^4x \sqrt{g} \left[h^{\mu\nu}h_{\mu\nu} - (h^\mu{}_\mu)^2 \right] \tag{5}$$

breaks the symmetry (4). Finally, the source term is given by

$$S_T = \int d^4x \sqrt{g}\, h_{\mu\nu} T^{\mu\nu}\,. \tag{6}$$

As in Ref. [4], we apply the simplifying assumption that $T_{\mu\nu}$ is conserved with respect to the background metric, $\nabla_\mu T^{\mu\nu} = 0$, although it will become clear how this assumption may be relaxed, if needed.

In addition to reproducing the classical correlators for the sources $T_{\mu\nu}$ given in Ref. [4], we would also like to determine whether the one-loop effective action for the background field configuration $g_{\mu\nu}$ is continuous in the $M^2/\Lambda \rightarrow 0$ limit. Since many of the relevant calculations have already been given for the propagation of massless $M^2 = 0$ gravitons in Ref. [8], we now proceed directly to the case $M^2 \neq 0$.

We employ the path-integral (PI) formalism and define the generating functional

$$Z[g, T] = \int \mathcal{D}h\, e^{-(S_L[h]+S_M[h]+S_T[h\cdot T])}\,. \tag{7}$$

Since this theory no longer contains a gauge symmetry of the form (4), one could, in principle, proceed directly by calculating the propagators and determinants associated with the quadratic form implied by $S_L + S_M$ without the appearance of any ghost sector. Indeed the propagators were computed directly in Ref. [4] using a suitable component decomposition. Instead, we introduce the gauge symmetry (4) using a Stückelberg [7] formulation. This will help to make contact with the operators appearing in Ref. [8] for the pure massless case.

In the Stückelberg formalism, the gauge symmetry of the massless theory is restored by introducing an auxiliary vector field V_μ. We first multiply $Z[g, T]$ by an integration $\int \mathcal{D}V$ over all configurations of this decoupled field, and then perform the shift $h_{\mu\nu} \rightarrow h_{\mu\nu} - 2M^{-1}\nabla_{(\mu}V_{\nu)}$. Since S_L and S_T are gauge invariant in themselves, the only effect of this shift is to make the replacement

$$S_M[h_{\mu\nu}] \rightarrow S_M[h_{\mu\nu} - 2M^{-1}\nabla_{(\mu}V_{\nu)}] \tag{8}$$

in (7). Thus S_M becomes a "Stückelberg mass", and gauge invariance is restored under the simultaneous shift

$$V_\mu \rightarrow V_\mu + M\xi_\mu\,, \tag{9}$$

along with the original transformation (4).

We now gauge fix by identifying V with the longitudinal part of $\tilde{h}$, namely $MV_\mu = \nabla^\rho \tilde{h}_{\rho\mu}$. This choice is made in order to simplify the relevant operators appearing in the action, and is accomplished by adding to the action the gauge-fixing term

$$S_{\rm gf} = \int {\rm d}^4x \sqrt{g} \left(\nabla^\rho \tilde{h}_{\rho\mu} - MV_\mu \right) \left(\nabla^\rho \tilde{h}_\rho^\mu - MV^\mu \right) . \tag{10}$$

This has the effect of canceling the last term in equation (3) in addition to the cross-term in S_M between V and (the traceless component of) $h_{\mu\nu}$.

To properly account for interactions with the background metric, we must also include a Faddeev-Popov determinant connected with the variation of this gauge condition under (4) and (9). It is straightforward to show that the appropriate determinant is

$$\text{Det} \left[(-\Box + M^2)\delta_\mu^\lambda - R_\mu^\lambda \right] = \text{Det} \left[\Delta(\tfrac{1}{2},\tfrac{1}{2}) - 2\Lambda + M^2 \right] , \tag{11}$$

where the second-order vector spin operator is defined by $\Delta(\tfrac{1}{2},\tfrac{1}{2})\xi_\mu \equiv -\Box\xi_\mu + R_{\mu\nu}\xi^\nu$ [8] and we have exploited the Einstein condition (1) for the background metric.

After gauge fixing, there remains a coupling proportional to $h^\sigma{}_\sigma \nabla \cdot V$ which can be eliminated by making the change of variables $V_\mu \to V_\mu + \alpha\nabla_\mu h^\sigma{}_\sigma$. It turns out that the remaining component $h_{\mu\nu}$ becomes completely decoupled from V_μ if we choose $\alpha = M/(-4\Lambda + 2M^2)$. This shift is a convenient but not crucial step in our calculation; our results will also hold in the apparently singular case when $2\Lambda = M^2 \neq 0$.

To highlight the tensor structure of the gauge-fixed action, we decompose the metric fluctuation $h_{\mu\nu}$ into its traceless and scalar parts: $\phi_{\mu\nu} \equiv h_{\mu\nu} - \tfrac{1}{4}g_{\mu\nu}h^\sigma{}_\sigma$, and $\phi \equiv h^\sigma{}_\sigma$. The source may similarly be split into its irreducible components $j_{\mu\nu}$ and j, so that $T_{\mu\nu} = j_{\mu\nu} + \tfrac{1}{4}g_{\mu\nu}j$. The gauge-fixed action then becomes

$$\begin{aligned}
\tilde{S} = \int {\rm d}^4x \sqrt{g} \Big[&\tfrac{1}{2}\phi^{\mu\nu} \left(\Delta(1,1) - 2\Lambda + M^2 \right) \phi_{\mu\nu} \\
&- \tfrac{1}{8}\left(\tfrac{-2\Lambda+3M^2}{-2\Lambda+M^2} \right) \phi \left(\Delta(0,0) - 2\Lambda + M^2 \right) \phi \\
&+ V^\mu(\Delta(\tfrac{1}{2},\tfrac{1}{2}) - 2\Lambda + M^2)V_\mu - (\nabla \cdot V)^2 \\
&+ \phi_{\mu\nu}j^{\mu\nu} + \tfrac{1}{4}\phi j \Big].
\end{aligned} \tag{12}$$

The second-order spin operators are the scalar Laplacian $\Delta(0,0) \equiv -\Box$ and the Lichnerowicz operator for symmetric rank-2 tensors $\Delta(1,1)\phi_{\mu\nu} = -\Box\phi + R_{\mu\tau}\phi_\nu^\tau + R_{\nu\tau}\phi_\mu^\tau - 2R_{\mu\rho\nu\tau}\phi^{\rho\tau}$ [8].

At this point, the tree-level amplitude for the current $T_{\mu\nu}$ can be read from the action (12) directly and is given by

$$\begin{aligned}
A[T] = \tfrac{1}{4}\Big[&2\,j^{\mu\nu} \left(\Delta(1,1) - 2\Lambda + M^2 \right)^{-1} j_{\mu\nu} \\
&- \tfrac{1}{2}\left(\tfrac{-2\Lambda+M^2}{-2\Lambda+3M^2} \right) j \left(\Delta(0,0) - 2\Lambda + M^2 \right)^{-1} j \Big],
\end{aligned} \tag{13}$$

since there are no sources for V_μ. Writing $j_{\mu\nu}$ and j in terms of $T_{\mu\nu}$, we find that

$$\begin{aligned}
A[T] = \tfrac{1}{4}\Big[&2\,T^{\mu\nu} \left(\Delta(1,1) - 2\Lambda + M^2 \right)^{-1} T_{\mu\nu} \\
&- \left(\tfrac{-2\Lambda+2M^2}{-2\Lambda+3M^2} \right) T_\mu^\mu \left(\Delta(0,0) - 2\Lambda + M^2 \right)^{-1} T_\mu^\mu \Big],
\end{aligned} \tag{14}$$

in agreement with the result of Ref. [4], modulo an overall convention-dependent factor of 1/4; (the removable singularity at $\Box = -4\Lambda/3$ reported in Ref. [4] is absent in this

formulation). We note here that there would be sources for the Stückelberg fields if one were to relax the assumption of a conserved stress tensor. In this case, one needs only to account for the shifts in $h_{\mu\nu}$ and V_μ to see how $T_{\mu\nu}$ contributes to sources for V_μ (and χ below).

Together, (12) and (11) provide a representation of Z with no manifest vector gauge symmetry. The terms involving V_μ in (12) correspond to a massive Maxwell action in the Einstein background with an effective mass $m^2 = M^2 - 2\Lambda$ which breaks the would-be $U(1)$ invariance whenever $m^2 \neq 0$. As before, we impose gauge symmetry in the Stückelberg formalism by introducing a scalar field χ and making the change of variables $V_\mu \rightarrow V_\mu - M^{-1}\nabla_\mu\chi$. Only the penultimate line of (12) changes as a result of this shift.

By construction, the resulting action is now invariant under the gauge transformation

$$
\begin{aligned}
V_\mu &\rightarrow V_\mu + \nabla_\mu\zeta\,, \\
\chi &\rightarrow \chi + M\zeta\,.
\end{aligned}
\tag{15}
$$

One can then choose a gauge-condition to simplify the shifted action. It is useful to associate the longitudinal component of V with χ according to $M\nabla \cdot V = (-2\Lambda + M^2)\chi$. This is done by the addition of a gauge-fixing term

$$
S'_{\rm gf} = \int \mathrm{d}^4x \sqrt{g} \left(\nabla \cdot V - \tfrac{-2\Lambda+M^2}{M}\chi \right)^2\,,
\tag{16}
$$

along with a corresponding scalar Faddeev-Popov determinant

$$
\mathrm{Det}\left[\Delta(0,0) - 2\Lambda + M^2\right]\,.
\tag{17}
$$

The $(\nabla \cdot V)^2$ and $\chi\nabla \cdot V$ terms in (16) are designed to cancel against corresponding terms in the shifted version of (12). If any $\phi\nabla \cdot V$ coupling had remained in Eq. (12), (for instance, if we had not performed the shift $V_\mu \rightarrow V_\mu + \alpha\nabla_\mu\phi$), then this too could have been eliminated by incorporating an appropriate ϕ dependence in the gauge-fixing (16). The additional mixing between ϕ and χ would present no difficulty. Hence, the results (if not the method) presented here also hold if $M^2 = 2\Lambda$. The case $M^2 = 2\Lambda/3$ must be treated separately [9], however, since from (12) the trace mode completely decouples, leaving only four degrees of freedom instead of five [10].

The final completely gauged-fixed action is now given by (12) with $-(\nabla \cdot V)^2$ replaced by the quadratic scalar term

$$
\tfrac{-2\Lambda+M^2}{M^2}\chi\left(\Delta(0,0) - 2\Lambda + M^2\right)\chi\,.
\tag{18}
$$

Along with the addition to the two Faddeev-Popov determinants (11) and (17), this provides a complete description of Z, including couplings to the background metric. We can integrate over all species to find the first quantum correction

$$
\begin{aligned}
Z[g,T] \propto\ & e^{-A[T]}\mathrm{Det}\left[\Delta(\tfrac{1}{2},\tfrac{1}{2}) - 2\Lambda + M^2\right]\mathrm{Det}\left[\Delta(0,0) - 2\Lambda + M^2\right]\mathrm{Det}\left[\Delta(1,1) - 2\Lambda + M^2\right]^{-1/2} \\
& \times\mathrm{Det}\left[\Delta(\tfrac{1}{2},\tfrac{1}{2}) - 2\Lambda + M^2\right]^{-1/2}\mathrm{Det}\left[\Delta(0,0) - 2\Lambda + M^2\right]^{-1/2}\mathrm{Det}\left[\Delta(0,0) - 2\Lambda + M^2\right]^{-1/2},
\end{aligned}
\tag{19}
$$

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where the operator $\Delta(1,1) - 2\Lambda + M^2$ arises in the traceless sector $\phi^{\mu\nu}$ sector so its determinant refers to traceless modes only.

In addition to the propagator (14), this allows us to compute the one-loop contribution

$$\Gamma^{(1)}[g] = -\ln Z[g,0] = -\tfrac{1}{2}\ln\mathrm{Det}\Big[\Delta(\tfrac{1}{2},\tfrac{1}{2}) - 2\Lambda + M^2\Big] + \tfrac{1}{2}\ln\mathrm{Det}\Big[\Delta(1,1) - 2\Lambda + M^2\Big] \quad (20)$$

to the effective action for the Einstein background $g_{\mu\nu}$. This is now to be compared with the one loop contribution in the strictly massless case [8]

$$\Gamma^{(1)}[g] = -\ln Z[g,0] = -\ln\mathrm{Det}\Big[\Delta(\tfrac{1}{2},\tfrac{1}{2}) - 2\Lambda\Big] + \tfrac{1}{2}\ln\mathrm{Det}\Big[\Delta(1,1) - 2\Lambda\Big] + \tfrac{1}{2}\ln\mathrm{Det}\Big[\Delta(0,0) - 2\Lambda\Big]$$

$$(21)$$

The difference in these two expressions reflects the fact that 5 degrees of freedom are being propagated around the loop in the massive case and only 2 in the massless case. Denoting the dimension of the spin (A, B) representation by $D(A,B) = (2A + 1)(2B + 1)$, we count $D(1,1) - D(1/2,1/2) = 5$ for the massive case, while $D(1,1) - 2D(1/2,1/2) + D(0,0) = 2$ for the massless one.

It remains to check that there is no conspiracy among the eigenvalues of these operators that would make these two expressions coincide. To show this, it suffices to calculate the coefficients in the heat-kernel expansion for the graviton propagator associated with $S_L + S_M$, and compare it with the massless case given in Ref. [8]. The coefficient functions $b_k^{(\Lambda)}$ in the expansion

$$\mathrm{Tr}\, e^{-\Delta^{(\Lambda)}t} = \sum_{k=0}^{\infty} t^{(k-4)/2}\int \mathrm{d}^4x\sqrt{g}\, b_k^{(\Lambda)} \quad (22)$$

were calculated in Ref. [8] for general "spin operators" $\Delta^{(\Lambda)}(A,B) \equiv \Delta(A,B) - 2\Lambda$ with the result

$$180(4\pi)^2 b_4^{(\Lambda)}(1,1) = 189 R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 756\Lambda^2\,,$$
$$180(4\pi)^2 b_4^{(\Lambda)}(\tfrac{1}{2},\tfrac{1}{2}) = -11 R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + 984\Lambda^2\,,$$
$$180(4\pi)^2 b_4^{(\Lambda)}(0,0) = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + 636\Lambda^2\,. \quad (23)$$

It is straightforward to extend those results to relevant massive operators $\Delta^{(\Lambda,M)}(A,B) \equiv \Delta(A,B) - 2\Lambda + M^2$. The coefficients $b_k^{(\Lambda,M)}(A,B)$ for these operators are perfectly smooth functions of M^2. Thus, as $M^2 \to 0$, we obtain

$$180(4\pi)^2 b_4^{(\Lambda,M)}(\text{total})$$
$$= 180(4\pi)^2\Big[b_4^{(\Lambda,M)}(1,1) - b_4^{(\Lambda,M)}(\tfrac{1}{2},\tfrac{1}{2})\Big]$$
$$\to 200 R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 1740\Lambda^2\,, \quad (24)$$

which clearly differs from the $M^2 = 0$ result

$$180(4\pi)^2 b_4^{(\Lambda)}(\text{total})$$
$$= 180(4\pi)^2\Big[b_4^{(\Lambda)}(1,1) - 2b_4^{(\Lambda)}(1/2,1/2) + b_4^{(\Lambda)}(0,0)\Big]$$
$$= 212 R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 2088\Lambda^2\,. \quad (25)$$

(These one-loop differences between massive and massless spin 2 in the $\Lambda = 0$ case are well-known [11]). Even in the case of backgrounds with constant curvature

$$R_{\mu\nu\rho\sigma} = \tfrac{1}{3}\Lambda(g_{\mu\nu}g_{\rho\sigma} - g_{\mu\rho}g_{\nu\sigma}),$$
$$R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} = \tfrac{8}{3}\Lambda^2, \tag{26}$$

there is no cancellation. Thus we conclude that the absence of the discontinuity between the $M^2 \to 0$ and $M^2 = 0$ results for massive spin 2, demonstrated in Ref. [4,3], is an artifact of the tree approximation and that the discontinuity itself persists at one loop.

That the full quantum theory is discontinuous is not surprising considering the different degrees of freedom for the two cases. However, as seen in (12), the three additional longitudinal degrees of freedom of the massive graviton do not couple to a conserved stress tensor. Thus, in the absence of any self-couplings (or at tree-level), the additional longitudinal modes would decouple from matter, yielding a smooth $M^2 \to 0$ limit. Nevertheless, due to these self-couplings (seen here as couplings to the background metric in the linearized approach), these additional modes do not decouple, thus yielding the resulting discontinuity in the massless limit. (This result also suggests that the situation would be similar for the spin-$\tfrac{3}{2}$ case considered in Ref. [12,13].) Of course, these one loop effects are very small and so experiments such as the bending of light would still not be able to distinguish a massless graviton from a very light graviton in the presence of a non-zero cosmologocal constant.

We finish with the important caveat that the $M \to 0$ discontinuity for fixed Λ of the massless limit of massive spin-2 we have demonstrated applies to fields described by the action appearing in (7) discussed in Ref. [4,3]. One may question whether this is a suitable action to describe the interaction of massive gravitons. We are not necessarily ruling out a smooth limit for other actions that might appear in Kaluza-Klein or brane-world models, for example. Indeed one would expect a smooth limit if the mass is acquired spontaneously [14] rather than through an explicit Pauli-Fierz term. In conventional Kaluza-Klein models, however, this limit, though smooth, would also be the decompactification limit and would result in massless gravitons in the higher dimension rather than four dimensions. A closer examination would be necessary to discern the form of the effective action describing the trapped graviton of the brane-world scenario of Refs. [15,16].

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