The structural performance and frequency filtering effects of perforated metal to composite joints

A thesis submitted for the degree of Doctor of Philosophy (PhD)

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Abstract

Glass fibre reinforced polymer (GFRP) composites are becoming increasingly common place in structural load bearing applications. Many of these applications require that loads from the GFRP are transmitted to steel elements. The aim of this thesis was to investigate the performance of joints between steel and vinyl-ester GFRP, comparing the tension strength at quasi-static loading rates with that at impact loading rates. The effect of loading rate on the mode II fracture toughness of the GFRP material was also investigated in order to establish a link between the increase in joint strength under dynamic loads and the increase in mode II fracture toughness under dynamic loads. The effect that typical manufacturing flaws have on the performance of the joints was also assessed. Specimens were manufactured using the vacuum assisted resin transfer moulding process, steel adherends were co-bonded during manufacture.

The investigation was undertaken using both physical tests and numerical analyses, physical tests on joints were conducted at both quasi-static and impact loading rates. The effect of loading rate on the mode II fracture toughness of vinyl-ester GFRP was investigated using four point bending end notch flexure tests at a range of loading rates. Digital image correlation techniques were used to measure crack propagation during the end notch flexure testing.

The phononic crystal behaviour of perforated steel adherends was investigated in order to demonstrate that attenuation of in-plane stress waves could be achieved at frequencies relevant to the likely joint applications. Finite element analyses and plots of the in-plane wave dispersion relations were employed for the frequency filtering studies.

It was shown that the tension strength of the joints was significantly higher under impact loads than quasi-static loads and that this increase in strength could be attributed to an increase in the mode II fracture toughness of the GFRP at the higher loading rate. The perforations in the steel plates enhanced both the static and impact tension strength of the joints, the joints were found to be relatively insensitive to the types of manufacturing flaw that were assessed. It was demonstrated that finite element surface based cohesive behaviour was able to effectively simulate the debond failures in the physical specimens. The investigation into frequency filtering effects has shown that finite steel plates with resin filled perforations are capable of attenuating in-plane waves at frequencies likely to be generated during a blast event.
Declaration

I confirm that the work in this thesis is my own and I give appropriate references and citations whenever I refer to or describe or quote from the work of others, whether published or unpublished.

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Reuben Brambleby 2016
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Chapter 1

Introduction

1.1 Background to the research

1.1.1 Applications for metal to composite joints

This thesis considers load bearing joints between a glass-fibre reinforced polymer composite material and steel. Fibre reinforced polymer (FRP) composites are comprised of a polymer matrix material, which surrounds a fibrous reinforcing material. The fibres are the primary load carrying component in an FRP composite, providing high tensile strength and stiffness. The polymer matrix material binds the fibres together, distributes load into the fibres and protects the fibres from abrasion and moisture. FRP composites are designed and manufactured in such a way that they have good mechanical properties which are properties not provided by the constituent materials in isolation.

Figure 1.1: The Vestas V47 series wind turbine uses glass fibre reinforced blades [1, 2].
FRP composites are becoming increasingly commonplace in a wide range of applications and in many manufacturing sectors, including the automotive, energy and defence industries. One typical example of the use of FRP is the Vestas V47 series wind turbine, shown in Figure 1.1. This model of wind generator uses Glass-fibre Reinforced Polymer (GFRP) composite material for its 47 m diameter turbines blades. Another more recent example is the deckhouse for the USS Zumwalt, Figure 1.2b. This four storey deckhouse is constructed from carbon fibre composite (CFRP) sandwich materials and measures 155 ft (47.2 m) in length and is over 50 ft (15.2 m) tall [3].

![USS Zumwalt on sea trials](image1.png) ![CFRP deckhouse for USS Zumwalt](image2.png)

Figure 1.2: The US Navy use FRP for major structural elements on their ships.

The applications outlined above make use of different FRP composite materials. There are an increasingly wide variety of different FRPs available, each with distinct attributes. However, when compared with more traditional materials such as steel, the benefits typically provided by most FRP composites include:

- A high strength to weight ratio.
- Good corrosion resistance.
- The ability to take on complex shapes.
- A low radar and electromagnetic signature (this aspect is primarily relevant to defence applications).

There are two important disadvantages that are common to many FRP composites. Firstly they tend to have a low absolute strength when compared with steel. Secondly they have low damage tolerance, which in turn means that they tend to behave poorly when subjected to high stress concentrations. Due to these two shortcomings, it is often necessary for engineers to integrate both FRP composites and metallic materials into their designs. Typically, FRP would be used in locations where lower weight has greatest benefit and metal would be used in locations where the loads are higher. Naturally, when metals and FRP composites are structurally integrated in this manner, then a metal to FRP joint is required. The joint must transfer the load from one material to the other. The Vestas V47 and the USS Zumwalt are examples
of applications in which metal to composite joints are required. The GFRP rotor blades of the V47 must be joined to the steel main-shaft of the generator. The CFRP deckhouse of the Zumwalt must be joined to the steel hull of the ship.

In a review of the applications for FRP materials in naval ships and submarines, Mouritz et al [6] determined that these platforms afford a broad range of opportunities in which composite materials can be employed to beneficial effect. The applications that they cite include bulkheads, decking, funnels and enclosures. However Mouritz et al found that higher costs are tending to inhibit the adoption of composites, despite the benefits that they would deliver. The bulk of these additional costs are derived from the joints between the composite component and the main steel structure [6]. Part of the reason for high cost joints can be ascribed to the lack of design rules and empirical design data for steel to composite joints. Another contributing factor is the absence of significant historical design experience relating to the use of adhesives in structural joints.

1.1.2 Methods for joining metal to FRP composites

The standard methods for forming joints in FRP materials fall into three categories; bonded joints [7, 8], bolted joints [9, 8] and bonded-bolted joints [10, 8]. All three of these methods are equally applicable to joints between metal and FRP materials.

Over recent decades a significant amount of time has been devoted to investigating, understanding and designing bolted joints in FRP materials [11, 12]. There are many instances in which bolted FRP joints have been used to good effect, for example the USS Zumwalt deckhouse in Figure 1.2b is secured using bolted joints [13]. However, a number of authors observe that bolted joints are sub-optimal as a joining method for FRP materials due to the non-ductile nature of FRP [14] and the fact that bolted joints tend to impose higher structural weight and greater fabrication costs [7].

A novel design of bonded metal-to-FRP joint has been proposed by Unden [15]. The concept is to perforate the metal adherend and to partially embed this perforated plate within the fibre reinforcement, prior to infusing the fibres with the polymer resin. During the resin infusion process, the liquid resin flows into the perforations in the plate. The resin curing process then takes place thus forming a bonded joint between the perforated metal adherend and the FRP adherend. A modified version of this perforated joint has been successfully tested under static tension loads [16].

The author considers this form of perforated joint to be a good candidate for providing economical and robust joints between FRP and metal components. However, the joint must be shown to perform adequately under impact loading rates and to perform well when the joint contains manufacturing flaws. It will also be beneficial
to show that relatively simple numerical modelling techniques can be used to predict the behaviour of this type of joint.

The perforations in the metal adherend can be placed on a repetitive grid, this opens up an interesting additional area for investigation. The perforations can be considered as forming a phononic crystal lattice [17] which can be used to attenuate the propagation of in plane stress waves passing through the joint.

1.2 Primary objectives of the research

The objectives of the research covered by this thesis are as follows:

1. To study the strength of hybrid joints between vinyl-ester GFRP and perforated stainless steel plates. The goal here is to demonstrate that perforated joints perform well under dynamic loads and that the perforations deliver an increase in dynamic strength compared to non-perforated joints.

2. To introduce intentional manufacturing flaws into the joints and to study the effect that these have on joint performance at both static and impact loading rates. The goal here is to demonstrate that the perforated joints are robust and do not suffer from excessive sensitivity to manufacturing faults.

3. To investigate frequency filtering effects on stress waves that pass through the joints. The main goal here is to determine whether frequency filtering in the joints is active under impact loading and to find a perforation geometry that will deliver attenuation of stress waves at frequencies that are relevant to the likely joint applications.

4. To investigate the efficacy of using finite element surface based cohesive behaviour to model failure in the joints.

5. To study the loading rate effect on mode II fracture toughness of the vinyl-ester GFRP. The main goals here are to establish if there is a loading rate effect on toughness and to determine whether this influences the dynamic strength of the joints.

1.3 Outline approach for the research

There are three interconnected strands to the approach that has been undertaken in order to meet the research objectives. An individual chapter is devoted to each strand:
1. Strength testing of Steel-GFRP joints under static and dynamic loads.
   - Both non-perforated and perforated joints will be tested in order to establish the enhancement provided by the perforations.
   - Both static and dynamic load tests will be conducted. The static results will be used as a baseline against which to assess the performance under dynamic loads.
   - Finite element modelling of the Steel-GFRP joints will be carried out and the results calibrated against the physical test results.

2. Mode II fracture toughness testing of the GFRP material at a range of load displacement rates.

3. Analytical, numerical and physical analysis of elastic wave propagation in perforated steel plates.

1.4 Layout of the thesis

Literature review
Following this introductory chapter, chapter 2 provides a review of the literature relating to the structural performance and frequency filtering effects of bonded joints between steel and vinyl-ester GFRP. The review covers the types of joint that are commonly used for joining fibre reinforced composite materials and the main factors that influence the performance of these joints. It then focuses on joints between steel and GFRP discussing various joint geometries that can be seen in the literature. The final type of bonded joints discussed are steel-GFRP joints with perforated steel adherends.

Due to the fact that the thesis considers dynamically loaded joints, the literature review covers loading rate effects on GFRP and stainless steel under axial loads. Loading rate effects on the fracture toughness of vinyl-ester GFRP are also reviewed, fracture toughness was found to be the dominant material property governing the strength of the tested joints.

The next two sections of the review cover fracture toughness testing of FRP materials and the use of the digital image correlation technique for measuring crack growth in FRP materials. These reviews provide important background information in relation to the toughness testing results that are reported in the thesis as well as validation of the testing techniques employed. The penultimate section of the review discusses frequency filtering effects in perforated plates.
The chapter closes with some concluding remarks for the literature review. These remarks identify current limitations in the literature and thus identify the novel contribution of the work covered by this thesis.

**Joint strength testing**

Chapter 3 covers the joint strength testing that was carried out. The chapter starts with an introduction and provides an outline of the joint testing programme that was conducted. Sections 3 - 6 relate to the first set of joint specimens that were tested; specimen design, specimen manufacture, testing and results are each covered in turn. Sections 7 - 10 relate to the second set of joint specimens and follow the same format as sections 3-6. Section 11 provides a review of the physical test results while section 12 covers finite element modelling of the joint. The chapter closes with an overview discussion of the joint testing.

**Toughness testing**

Chapter 4 covers the 4ENF fracture toughness testing that was carried out in order to identify any loading rate effect on toughness of the vinyl-ester GFRP material. The chapter provides an introduction then sections 2 - 4 cover test fixture design, specimen design and specimen manufacture. Section 5 covers the 4ENF test procedure and section 6 discusses the non-linear load versus displacement behaviour of the 4ENF test, this non-linear behaviour is critical to the interpretation and reduction of the test data. The fracture toughness test results are given in section 7.

Section 8 presents finite element analyses of the 4ENF test, these were used to:

- Assess the significance of inertial effects on the 4ENF test at higher loading rates.
- Assess the importance of friction between the crack faces of the 4ENF specimen.
- Provide data which was used to linearise the test data.

The chapter closes with concluding remarks which discuss significant aspects of the test method and test results followed by a discussion of the overall conclusions that can be drawn from the 4ENF testing.

**Frequency filtering effects**

Chapter 5 covers frequency filtering effects in perforated plates. The introduction to the chapter provides an outline of the factors that provided motivation for the study and an outline of the investigation strategy that was adopted. Sections 2 - 4 cover the semi-analytical, numerical and physical testing methods that were employed for the investigation. The chapter closes with the conclusions that can be taken from the frequency filtering study.
Conclusions and recommendations

The final chapter provides a summary of the main findings from the joint strength testing, toughness testing and frequency filtering aspects of the thesis and goes on to provide recommendations for future work in relation to each of the three aspects.
Chapter 2

Literature Review

2.1 Introduction

This chapter provides a brief literature review covering the primary strands of the investigation that is reported in this thesis. The review aims to provide an overview of the critical aspects in each particular subject area and to outline the context for the work that has been undertaken.

2.2 Joints in fibre reinforced polymer materials

Structural joints in Fibre Reinforced Polymer (FRP) materials can be formed using mechanical fasteners, adhesives or a combination of these. A thorough review of the important parameters relevant to mechanically fastened joints in FRPs is provided by Godwin & Matthews [9]. Equivalent reviews for adhesively bonded joints are given by Matthews et al [18] and Banea [7]. A thorough treatment of the design of adhesively bonded joints is given by Hart-Smith [19], this work continues to be cited on a regular basis in connection with the design of adhesive joints.

Mechanically fastened FRP joints have been used successfully in a wide variety of applications, but they have distinct disadvantages compared with adhesively bonded joints:

- The fasteners, typically bolts, generate stress concentrations in the FRP. FRP materials tend to be brittle and are therefore particularly susceptible to stress concentrations.
- Forming holes in the FRP material to accommodate the fixings generally leads to reinforcing fibres being cut, thus weakening the material.
- Mechanically fastened joints tend to be heavier and to occupy more space than adhesively bonded joints.

Bonded joints are more continuous, leading to better stress distribution. They also have the potential to provide a lighter weight solution and provide greater design
flexibility [7]. The result of these benefits is that the adoption of bonded joints in structural FRP components has increased in recent years, thus providing opportunity for further research into their performance.

2.2.1 Adhesive joints in FRP materials

There are a number of joint configurations that are commonly used for adhesively bonded joints in FRP materials, a useful review of the common geometries is provided by Banea and da Silva [7]. The most commonly used configurations for FRP adhesive joints are shown in Figure 2.1. It can be seen that single and double lap joints have the simplest geometry while the scarf and stepped lap joints have the benefit of providing a planar joint, i.e. the surface steps that are present in the single and double lap joints are avoided. For applications in which the stepped geometry of the single and double lapped joints is acceptable these joint types are often preferred due to the reduced complexity in terms of joint manufacture [7].

Both single lap and double lap joints suffer from significant moments being induced in the joint as a result of eccentricity in the load paths through the joint. In the case of a single lap joint under axial load this eccentricity leads to global rotation of the joint. In the case of a double lap joint under axial load the symmetry of the double lap joint overcomes this problem of global joint rotation. However, the load path eccentricity in the double lap joint nonetheless generates internal moments which must be balanced by through thickness peeling stresses within the joint. These through thickness stresses are discussed further in Section 2.2.1.1. Global load path eccentricity is avoided by both the scarf and stepped lap joints.

Hart-Smith [19] studied the behaviour of adhesive joints and provided a thorough analysis of the adhesive stresses that are induced in the various common joint geometries. Figure 2.2, which reproduces two figures originally by Hart-Smith, compares the non-linear distribution of adhesive shear stress in typical double lap and stepped lap joint configurations. It can be seen that a concentration in shear stress occurs at each end of the double lap joint and that this stress concentration profile tends to be repeated for each step in the stepped lap joint. The peel stresses that are discussed
A literature review on the subject of double-lap joints revealed a number of factors that can be expected to have some influence on the strength of the joints:

- Peel stresses.
- Surface preparation.
- A mismatch in the stiffness of the adherends.
- A mismatch in the thermal expansion coefficients of the adherends.

### 2.2.1.1 Peel stresses

It appears that Hart-Smith [19] was one of the first authors to study peel stresses in FRP joints [18]. He noted that bending rotation at the tip of an overlapping adherend tends to cause the tip to peel away from the substrate adherend. A diagram showing how these peeling stresses are generated in a double lap joint is provided in Figure 2.3.
and can be described as follows:

- There is an inherent eccentricity between the axial stress in the overlapping adherend and the shear stress at the interface between this adherend and the adhesive layer.
- This inherent eccentricity in the load path generates an induced moment in the overlapping adherend.
- In order to achieve static equilibrium in the joint, through thickness stresses occur. These stresses are a maximum at the tip of the overlapping adherend.
- When the joint is under tension the induced through thickness stress is a peeling stress.

As discussed by Banea and da Silva, and shown in Figure 2.4, these peel stresses can be particularly important for laminated FRP materials. The through thickness stress can induce delaminations due to the low through thickness strength that is an inherent property of many FRP materials. However, in the case of the joint specimens tested for this thesis, peel stresses were not found to dominate the behaviour of the joint. There are two main contributing factors here:

- The overlapping adherends are only approximately 1 mm thick which delivers a small load eccentricity and therefore small peel stresses.
- The substrate adherend is steel, therefore through thickness stresses of the magnitude generated by the adhesive are not significant relative to the strength of the substrate.

### 2.2.1.2 Surface preparation

For adhesively bonded joints it is critically important that the bond surfaces are prepared appropriately, failure in this regard can reduce both the strength and durability
Melograna and Grenestedt [21] carried out a study in which twelve different steel surface pre-treatment strategies were tested. The treated surface was an AL-6XN stainless steel adherend, the adhesive was Dow Derakane 510A-40 vinyl-ester. Each treatment strategy was ranked in accordance with the pull-off tension strength that was generated by the specimen. Their favoured pre-treatment process used a priming system from Poly Fiber and had the following steps:

- Grit blasting.
- Cleaning with a solvent, C-2200.
- Application of an epoxy catalyst, EP430.
- Specimen manufacture.

Although the design of the author’s joint specimens was based on a design proposed by Melograna and Grenestedt, the surface preparation was downgraded from the Poly Fiber preparation that they used. This is discussed in more detail in Chapter 3.

### 2.2.1.3 Stiffness mismatch of adherends

Hart-Smith demonstrated analytically that when the joint is under load, there is a non-linear variation of shear stress in the adhesive of a bonded double lap joint [22]. As shown in Figure 2.5, the shear stress in the adhesive rises towards the tip of each adherend and is at a minimum part way along the bond-line. The asymmetry of the stress distribution in Figure 2.5 is due to a stiffness mismatch of the adherends, the inner adherend has larger strains than the outer adherends. Shear strain in the

![Figure 2.4: Banea - Peel stresses in a double lap joint [7].](image)
adhesive is at a peak towards the tip of each outer adherend due to the fact that, at this point, axial strain is at a maximum for the inner adherend and at a minimum for the outer adherend. The same reasoning can be used to explain the rise in shear stress at the tip of the inner adherend.

It is evident from Figure 2.5 that the bond strength of the joint would be increased if the thickness of the outer adherends was reduced. This reduction would increase the strain in the outer adherends and therefore deliver a more balance stress distribution in the adhesive. Hart-Smith also considers stepped lap and scarf joints [19], the bond strength for both of these alternatives is significantly higher than for a double lap joint due to the matched adherend strain over the length of the joint.

The philosophy for the joint specimens that were tested for this thesis was to keep the fabrication simple. A joint design with tapered adherends would tend to diverge from this philosophy, particularly in terms of the fibre reinforcement. The reinforcement would also have to be tapered and laying up the glass fibres to generate this taper would be a time consuming manual operation. As discussed in Section 2.2.2.3, stiffness mismatch in the author’s joint specimens is mitigated by using perforated steel adherends.

2.2.1.4 Thermal mismatch of adherends

Hart-Smith demonstrated that a mismatch in the thermal expansion coefficients of the two adherends in a double-lap joint will lead to residual stresses in the joint, see Figure 2.6. For Figure 2.6 Hart-Smith had in mind high performance composite material. During manufacture this material would be cured at high temperature in
an auto-clave and would be subjected to significantly lower temperatures in service. For joints formed using graphite / epoxy material Hart-Smith quotes a temperature range of $+350^\circ F$ (cure temperature) down to $-67^\circ F$ (service temperature), Hart-Smith refers to this as the "temperature differential - $\Delta T$".

The test specimens used here were cured at room temperature as prescribed by the resin manufacturer [23] and subsequently tested at room temperature, although the exothermic reaction of the resin curing process did generate a significant rise in temperature during manufacture. The author did not request for the test specimen cure temperature to be recorded, but the temperature differential for the test specimens was certainly lower than that quoted by Hart-Smith. The residual stresses generated due to this temperature differential are an unfortunate inherent but unavoidable aspect of co-bonded steel-GFRP joints.

### 2.2.2 Adhesive joints between steel and GFRP

The need for creating load bearing joints between steel and GFRP material has already been discussed. These joints can be necessary for example when joining components for which weight is critical to components for which strength and ductility are critical. The weight critical nature of the GFRP component often means that this component is configured as a sandwich panel, i.e. GFRP outer sheets enclosing a lightweight core material, see Figure 2.7.
2.2.2.1 Steel-GFRP joints under axial loading

Wright et al [24] studied four different configurations of steel-GFRP joint under axial tension and compression loads. Their joints are depicted in Figures 2.7a and 2.7b and can be described as follows:

- **Type A:**
  - GFRP / balsa sandwich bonded into an asymmetrical steel “tuning fork” component.
  - GFRP / balsa sandwich bonded into a symmetrical steel “tuning fork” component.

- **Type B:**
  - Steel plate asymmetrically bonded into the GFRP tail of a GFRP / balsa sandwich.
  - Steel plate symmetrically bonded into the GFRP tail of a GFRP / balsa sandwich.

Joint type A was fabricated using the secondary bonding technique, i.e. the steel component and sandwich panel were manufactured separately and subsequently joined using adhesive, although Wright et al point out that the co-bonding technique could also be employed. Joint type B was manufactured using a co-bonding process. In the co-bonding process the steel plate is placed between the appropriate layers of glass fibre reinforcement during manufacture of the sandwich panel, therefore the resin that becomes the matrix of the sandwich panel also acts as the adhesive which bonds the sandwich panel to the steel plate.

The constituent materials for the GFRP used by Wright et al [24] were E-glass reinforcement and vinyl-ester resin. An epoxy adhesive was used to form the secondary
bond for the type A joints. Wright et al considered joint type A to be the most promising in terms of strength, although it can be seen that type B will be less susceptible to manufacturing tolerances and employs fewer manufacturing steps.

Wright et al studied the behaviour and performance of joint type B using finite element analysis. For joint type A the behaviour was studied using both finite element analysis and physical testing. The physical testing carried out on joint type A incorporated quasi-static tests under both tension and compression as well as fatigue tests under both tension and compression. In all cases the tested joints failed as a result of de-bonding at the bond line between the GFRP and steel plate. The de-bonding initiated at the tip of the steel plates when the joint was in tension and at the embedded tip of the sandwich panel when the joint was in compression. Wright et al mention the importance of peel stresses for the compression case, presumably these peel stresses are developed under compression due to the diverging geometry of the steel portion of the type A joint. Wright et al concluded that:

- Bonded connections between steel and GFRP sandwich panels can be designed to achieve good mechanical performance.
- Symmetric joint geometry was found to be preferable in terms of mechanical performance, although applications may require the adoption of asymmetrical geometry.
- Fatigue testing of the joints demonstrated good performance under fatigue load conditions.

### 2.2.2.2 Steel-GFRP joints under flexural loading

This section covers work on the performance of steel-GFRP joints under flexural loading. Although the joint designs and test methods are not directly related to those covered in this thesis, the review helps to put the joints studied here in context and also to show recent developments in joint design.

Clifford et al [25] considered a joint design closely based on the asymmetrical Type B joint of Wright et al [24], see Figure 2.7. The joints were manufactured via a proprietary resin infusion process using the following materials:

- GFRP Resin - Derakane 411-C50 vinyl-ester.
- GFRP Reinforcement - Chonorat twill weave E-glass fabric.
- Sandwich core - End grain balsa wood and Divinycell® PVC foam.
- Steel plate - Mild steel.

Clifford et al found that under flexural loading their initial joint design, Type A in Figure 2.8, failed in a brittle manner. As indicated in Figure 2.8b this joint failed due to delamination of the sandwich panel at the tip of the embedded steel plate. This failure mode was attributed to the presence of stress concentrations caused by the
low flexural stiffness of the joint at the embedded tip of the steel plate. Two methods for mitigating against these stress concentrations were tested:

1. The embedded length of the steel plate was extended, Type B in Figure 2.8b.
2. The tapered portion of the balsa core was replaced with Divinycell® PVC foam, Type C in Figure 2.8b.

Both of these methods were found to both increase the strength of the joint and to increase the displacement at failure, i.e. the joints failed in a less brittle manner.

Boyd et al [26] extended the work of Clifford et al by carrying out tests under compression, fatigue and flexural loading using joints with a similar geometry to that used by Clifford et al. The geometry of the joints tested by Boyd et al was based on the helicopter hanger to deck joint used in the La Fayette class frigate from the French navy [26], the geometry is shown in Figure 2.9. The materials used to fabricate the joints were as follows:

- GFRP Resin - Derakane 411-C50 vinyl-ester.
- GFRP Reinforcement - Chonorat E-glass woven roving.
- Sandwich core - Baltek AL600-10 balsa wood.
- Steel plate - 6mm thick mild steel.
There were three phases to the testing programme employed by Boyd et al:

1. Determine the ultimate compression strength of the joint.
2. Characterize the compression fatigue life of the joint by applying cyclic compression loads at various load amplitudes and numbers of cycles.
3. Residual strength testing of the fatigued joints:
   - Residual compression strength.
   - Residual flexural strength.

Under quasi-static compression loads failure in the joint initiated via delamination of the GFRP skin from the balsa core adjacent to the tip of the embedded steel plate, this delamination extended as the load was increased. Under compression loads the joint failed in a brittle manner. For joints that had been subjected to prior fatigue loading, in which the fatigue loading is comparable to that expected for the in service condition, Boyd et al found that:

- The ultimate residual compression strength of the joint is not significantly reduced.
- There is little reduction in axial stiffness of the joint.
- The ultimate residual flexural strength of the joint is not significantly reduced.

### 2.2.2.3 Perforated Steel-GFRP joints

Unden and Rider [15] proposed a laminated steel to FRP joint. The steel adherends in their design were provided with an array of circular perforations, the spacing of which gradually decreases along the length of each steel adherend. The purpose of the perforations in the steel plates was twofold:

1. To permit the resin of the FRP to penetrate the holes and therefore provide some mechanical connection between the steel and the FRP adherends.
2. To mitigate against the stiffness mismatch between the steel and FRP adherends.

Melograna and Grenestedt [16] subsequently adopted some aspects of the Unden and Rider joint design. They carried out quasi-static tension tests on a variety of steel-GFRP double lap joints, some of the details of their specimens are as follows:
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- Steel adherend - 2.6 mm thick x 20 mm wide, grade AL-6XN stainless steel.
- GFRP adherend - 5 mm thick x 20 mm wide, 510A-40 vinyl-ester reinforced with 4 layers of TH-4000 tri-axial stitched fabric.
- The VARTM process was used to manufacture the joint specimens.
- A range of lap lengths between 25 mm and 50 mm were tested.
- Some of the specimens were not perforated, some had circular perforations the remainder had triangular perforations.
- Some of the specimens had enhanced (Poly Fiber) surface preparation, the remainder had a basic degreasing and grit blasting surface preparation.
- For some of the perforated joints the perforations were manually filled with fibre reinforced resin prior to infusing the joint.
- Tension tests were carried out at a cross head displacement rate of 1.0 mm/minute.
- Joint strengths ranged from 9.6 kN to 22.5 kN

The principal findings from the Melograna and Grenestedt joint tests were as follows:

- The perforated joints were approximately 25% stronger than the non-perforated joints. This increase was similar for both the enhanced surface treatment and the standard surface treatment.
- The joints with enhanced surface treatment were approximately 13% stronger than those with standard surface treatment.
- Filling the perforations with fibre reinforced resin did not increase the strength of the joint.
- The spread of their tension test results was greater for the joints with standard surface treatment.

Melograna and Grenestedt concluded that two mechanisms contributed to the higher strength of the perforated joints. The first mechanism was reduced stiffness mismatch between the GFRP and steel adherends. The graduated spacing of the perforations in the steel adherend provided a lower effective stiffness towards the tip of the adherend and gradually greater effective stiffness along the length of the joint. The second mechanism was mechanical interlock between the GFRP and steel adherends, the resin within the perforations can be thought of as being monolithic with the GFRP adherends.

In the tests outlined above Melograna and Grenestedt have shown that perforated Steel-GFRP joints can deliver increased strength under quasi-static loading rates. A literature search was undertaken to determine whether test results for dynamically loaded perforated joints had been published, no data was found. Therefore the Melograna and Grenestedt joint design was considered to be a good basis for the development of a perforated joint specimen that would be tested under impact loading rates.
2.3 Strain rate effects on stainless-steel and GFRP under tension loads

One of the significant aspects of the testing that was carried out in support of this thesis is the comparison of the quasi-static and dynamic tension strengths of the steel-GFRP joints. It is therefore interesting to consider what effect the change in loading rate will have on the steel and GFRP materials to be used for the specimens tested here.

The quasi-static joint testing that was carried out by Melograna and Grenestedt [16] can be used to make an initial estimate of the stresses, strains and strain rates that can be expected for the two adherend materials to be tested here. These initial estimates are as follows:

- **Steel adherend:**
  - Maximum stress $\approx 450 \text{ N/mm}^2$
  - Quasi-static strain rate $\approx 1 \times 10^{-3} \text{s}^{-1}$
  - Dynamic strain rate $\approx 20 \text{s}^{-1}$

- **GFRP adherend:**
  - Maximum stress $\approx 220 \text{ N/mm}^2$
  - Quasi-static strain rate $\approx 0.1 \times 10^{-3} \text{s}^{-1}$
  - Dynamic strain rate $\approx 10 \text{s}^{-1}$

2.3.1 Strain rate effects on vinyl-ester GFRP

As outlined in Section 2.3 the GFRP strain rates that are applicable here range from approximately $0.1 \times 10^{-3} \text{s}^{-1}$ to approximately $10 \text{s}^{-1}$. Unfortunately there is limited data in the literature regarding the effects of changes in strain rate over this range for glass fibre reinforced vinyl-ester composite material.

Plaseied and Fatemi [27] provide data for vinyl-ester polymer over this range of strain rates and demonstrate that there are significant rate effects on both modulus and strength. However the tension strength and stiffness of GFRP material are dominated by the performance of the reinforcement material rather than the matrix material [28]. Therefore the data from Plaseied and Fatemi is only of background interest here.

As can be seen in Figure 2.10a, South et al [29] show that strain rate has little influence on tension modulus for vinyl-ester GFRP up to strain rates of $1 \text{s}^{-1}$. However, South et al also provide the data shown in Figure 2.10b which demonstrates that over a much wider range of strain rates there is a significant effect on tension modulus for
CHAPTER 2. LITERATURE REVIEW

(a) Modulus versus strain rate for a glass/vinyl-ester composite. (b) Modulus versus strain rate for glass fibers and a glass/epoxy composite.

Figure 2.10: Rate dependence of tension modulus for GFRP [29].

Figure 2.11: Effect of strain rate on the tensile modulus of GFRP [30].
Note: 1. Horizontal dashed black line added by the author. 2. Original paper has incorrect label for vertical axis, label added by author.

epoxy GFRP. Unfortunately the data from South et al is not sufficient to determine the transition point at which strain rate starts to be significant in terms of the mechanical properties of GFRP.

Shokrieh and Omidi [30] used a servo-hydraulic test setup to study the mechanical properties of epoxy GFRP material over a range of strain rates from $1 \times 10^{-3}$ s$^{-1}$ to 100 s$^{-1}$. Their plot of tensile modulus versus strain rate is shown in Figure 2.11. It can be seen that, over a strain rate range of $1 \times 10^{-3}$ s$^{-1}$ to 10 s$^{-1}$, the tension modulus increases by approximately 2 GPa or 5%.

From Figure 2.10a [29] and Figure 2.11 [30] we can see that the strain rate effect on both vinyl-ester GFRP and epoxy GFRP is small for strain rates up to 1 s$^{-1}$. For strain rates between 1 s$^{-1}$ and 10 s$^{-1}$ we must assume that the behaviour for both vinyl-ester GFRP and epoxy GFRP is dominated by the glass fibre reinforcement and that we can therefore assume that rate effect shown in Figure 2.11 is also valid for vinyl-ester GFRP. Using this argument we can deduce that the rate effect on the mechanical properties of vinyl-ester GFRP under tension loads is likely to be small for the range of strain rates expected in the tests carried out here.
2.3.2 Strain rate effects on grade 304 stainless steel

The steel material used for the hybrid steel-GFRP joints tested here was grade 304 stainless steel. Joshua et al. [31] have reported the effect of tension strain rate on grade 304 stainless steel. Their specimens were tested at a range of tension loading rates which delivered strain rates in the range $0.125 \times 10^{-3}$ s$^{-1}$ to 400 s$^{-1}$, see Figure 2.12a and Table 2.1.

![Figure 2.12a](image1)

![Figure 2.12b](image2)

Figure 2.12: True stress versus true strain for alloy 304L in tension at 24 °C [31].

<table>
<thead>
<tr>
<th>Strain rate (s$^{-1}$)</th>
<th>0.2% Yield stress (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000125</td>
<td>300</td>
</tr>
<tr>
<td>0.00125</td>
<td>307</td>
</tr>
<tr>
<td>0.0125</td>
<td>328</td>
</tr>
<tr>
<td>0.125</td>
<td>351</td>
</tr>
<tr>
<td>1.25</td>
<td>361</td>
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<tr>
<td>10</td>
<td>382</td>
</tr>
<tr>
<td>100</td>
<td>438</td>
</tr>
<tr>
<td>400</td>
<td>480</td>
</tr>
</tbody>
</table>

Table 2.1: Extract from Table IV “Tensile properties of 304L (Average of three tests)” [31].

Figure 2.12b shows an annotated version of the Joshua et al. [31] results, the curves for strain rates of $1.25 \times 10^{-3}$ s$^{-1}$ and 10 s$^{-1}$ have been highlighted and the corresponding strain at 450 N/mm² has been identified. It is clear that a significant drop in steel strain should be expected when increasing the strain rate from $1 \times 10^{-3}$ s$^{-1}$ to 20 s$^{-1}$. By taking measurements from a larger plot of Figure 2.12b, and conservatively using the curve for 10 s$^{-1}$ strain rate, it can be determined that a reduction in strain of
approximately 35% can be expected for the higher strain rate.

If all other parameters remain the same then a reduction in steel strain at a given load pertains to an increase in the stiffness mismatch between the steel and GFRP adherends. An increase in the stiffness mismatch of the two adherends will tend to increase the stress in the adhesive and therefore, if all other parameters are unchanged, will lead to a reduction in joint strength [22]. It will be seen in Chapter 3 that reduced strain in the steel adherend at higher loading rates did not turn out to be a dominant factor governing the strength of the joint specimens.

2.4 Loading rate effects on the toughness of vinyl-ester GFRP composites

A review of the effect that loading rate has on the toughness properties of polymer composites was carried out by Jacob et al [32]. Much of the literature covered in the review relates to carbon epoxy composites but some of the literature discussed other materials including glass reinforced vinyl-ester. In general, across all the materials and for both mode I and mode II toughness, there is very little consensus regarding whether increasing the loading rate tends to increase, decrease or have negligible effect on the toughness properties.

2.4.1 Loading rate effect on mode I toughness

Although mode I fracture toughness (\(G_{IC}\)) is not directly related to the tests carried out for this thesis, it is interesting to review the literature regarding the effect that loading rate has on \(G_{IC}\). Unfortunately there is very little in the literature regarding this effect for vinyl-ester GFRP materials and none of the papers reviewed by Jacob et al [32] consider mode I toughness for vinyl-ester.

2.4.1.1 Carbon / epoxy composite

The review paper by Jacob et al [32] covers a diverse range of matrix and reinforcement materials. In terms of this thesis the most relevant material that is covered by Jacob et al for \(G_{IC}\) is carbon fibre reinforced epoxy composite (epoxy CFRP). Six of the papers reviewed by Jacob et al consider \(G_{IC}\) for epoxy CFRP, three of these suggest that \(G_{IC}\) is rate insensitive [33, 34, 35], two suggest that \(G_{IC}\) increases as loading rate increases [36, 37] and the final paper suggests that \(G_{IC}\) decreases as loading rate increases [38].

2.4.1.2 Glass / vinyl-ester composite

A short paper by Compston et al [39] presents a method that can be used to directly relate \(G_{IC}\) for un-reinforced resin with \(G_{IC}\) for the corresponding composite material.
They present test data for un-reinforced Derakane 8084 vinyl-ester resin and Derakane 8084 vinyl-ester GFRP material in order to compare $G_{IC}$ of the un-reinforced vinyl-ester with that of the corresponding GFRP.

Compston et al carried out mode I toughness tests on both the vinyl-ester and the composite over a range of load displacement rates, from 1 mm/min up to 100 mm/min. The crack opening displacement rate was measured during each test. For both the vinyl-ester and the composite, they found that crack opening displacement rate was proportional to loading rate and that $G_{IC}$ reduced as loading rate increased.

The Derakane 8084 resin used by Compston et al is an elastomer modified vinyl-ester which delivers superior elongation and toughness properties compared with the Derakane 510A-40 resin used here [40]. While the Owens Corning R25 Advantex® uni-directional glass fibre reinforcement they used for the GFRP specimens has a different form, i.e. it is uni-directional rather than a stitched fabric, it uses very similar E-glass fibres to the TTX1330 fabric used here.

From the available literature it is difficult to conclusively state what effect loading rate has on $G_{IC}$ for vinyl-ester GFRP, but it would appear that there may be a drop on $G_{IC}$ as loading rate increases.

### 2.4.2 Loading rate effects on mode II toughness

Mode II fracture toughness ($G_{IIC}$) of the GFRP material was found to be the cohesive parameter that had most influence over the strength of the joints tested here. Again there is only limited coverage in the literature regarding the effect of loading rate on $G_{IIC}$ for vinyl-ester GFRP, two relevant papers are outlined below.

#### 2.4.2.1 Compston, Jar and Davies

Compston et al [41] used Double Cantilever Beam (DCB), three point bending End Notch Flexure (ENF) and Central Notch Flexure (CNF) tests to study the matrix effect on the quasi-static and dynamic fracture toughness of glass-fibre composites. The three test types were used to generate toughness data as follows:

- DCB test - quasi-static Mode I toughness.
- ENF test - quasi-static Mode II toughness.
- CNF test - dynamic Mode II toughness.

The primary focus of the study was to establish the extent to which matrix toughness influenced the interlaminar fracture toughness of the corresponding composite material. However, as part of the study they provided a comparison between $G_{IIC}$ at quasi-static loading rates and at drop weight loading rates (up to an impactor speed of 3 m/s). Various matrix materials were tested, the matrix material of interest here
was Atlac 580 vinyl-ester supplied by DSM. All specimens were reinforced using E-glass fibres supplied by Verotex.

Compston et al [41] found that $G_{IIc}$ of the vinyl-ester composite dropped from approximately 3 kJ/m$^2$ under quasi-static loading down to approximately 1.8 kJ/m$^2$ under dynamic loading.

### 2.4.2.2 Compston, Jar, Burchill and Takahashi

Compston et al [42] carried out a follow up study which again considered the effect of matrix toughness and loading rate on $G_{IIc}$ for vinyl-ester GFRP composites. The focus of this second study was to investigate the extent to which the improved toughness of rubber-modified vinyl-esters was capable of delivering an increase in the mode II toughness of the corresponding GFRP material. The reinforcing material used was Owens Corning R25H E-glass fibre. Only one of the matrix materials that was used is of interest here, this was Dow Derakane 411-45 which is an epoxy-based vinyl-ester resin similar to the resin used for the specimens covered in this thesis.

Compston et al [42] conclude that loading rate has a negligible effect on the fracture toughness of their specimens. However, as shown in Figure 2.13, their raw data suggests that increasing the loading rate tends to increase $G_{IIc}$. It is interesting that the rate effect has been dismissed as negligible, reasons for this may include:

- Compston et al note that they changed test method for tests that were carried out above 1000 mm/min, they therefore doubt the validity of the results from these higher rates.
- The data in Figure 2.13 relates to mode II toughness for the load at which crack propagation is initiated. The authors also calculated the maximum mode II toughness. These values of maximum toughness show a less significant rate effect.

As with mode I fracture toughness it is difficult to conclusively state what effect loading has on $G_{IIc}$. However, this second paper from Compston et al [42] gives a hint that the mode II fracture toughness of some vinyl-ester composites may increase as loading rate is increased.

### 2.5 Toughness testing of FRP materials

Having completed the tests on the first set of double-lap joint specimens it was clear that the joints were stronger under dynamic loading than under quasi-static loading. Using finite element (FE) models of the joint specimens it was established that mode II fracture toughness was the dominant property controlling bond strength of the joints. As identified in the literature review, see Section 2.4.2, it is apparent that:
Figure 2.13: Extract of Figure 5 from Compston et al [42] showing mode II fracture toughness \( (G_{IIc}, \text{closed symbols}) \) and flexural modulus \( (E_f, \text{open symbols}) \) versus loading rate.

(a) There is no general consensus regarding whether loading rate does influence mode II toughness.

(b) The extent of published data on the subject, particularly in relation to vinyl-ester, is limited.

The author therefore chose to carry out mode II fracture toughness testing of the 510A-40 vinyl-ester resin that was used for the joint specimens. The testing was carried out at a range of load displacement rates in order to expose loading rate effects on mode II toughness. The following provides an overview of the critical aspects related to toughness testing.

### 2.5.1 Fracture toughness

The concept of fracture toughness comes from the field of Linear Elastic Fracture Mechanics (LEFM). A rigorous introduction to LEFM is beyond the scope of this literature review, but LEFM relies on a concept developed by Griffiths [43] which balances the strain energy in a material with the energy required to generate new surface in the material, i.e. the energy required for crack growth. Griffiths demonstrated that there is a limiting crack length related to stable crack growth in a material under stress. That is to say that a crack may exhibit stable growth initially, i.e. the crack will only grow if the stress is increased, but once the limiting crack length has
been reached then crack growth becomes unstable. This is due to the fact that when crack length exceeds the critical crack length there is sufficient strain energy in the material to cause rupture without any increase in applied stress.

This energy balance concept can be extended in order to define critical energy release rate, $G_\text{C}$. This is the work (energy) needed to generate a unit increase in crack area. Critical energy release rate, $G_\text{C}$, is the parameter that is generally used to quantify the toughness of FRP composites. A thorough explanation of the concepts behind fracture toughness can be found in one of the numerous texts that deal with LEFM, for example Broek’s Elementary Engineering Fracture Mechanics [43].

Due to the fact that FRP composite materials are neither homogeneous nor isotropic, their toughness varies with loading direction. The standard designations for the three orthogonal loading directions are illustrated in Figure 2.14, mode I relates to the opening, prying, direction, mode II relates to the longitudinal shear direction and mode III relates to the transverse shear direction. A specific test method and test fixture is required for measuring each toughness mode.

### 2.5.2 Testing methods for mode II toughness

Due to the fact that there was so little data regarding the loading rate effect on mode II toughness of vinyl-ester composites and that there is such poor consensus on the subject in the literature, the author was encouraged to investigate the effect by carrying out some tests using the same GFRP material as were used for the joint specimens.

The specimen and fixture arrangements of the four standard mode II tests are shown in Figure 2.15. The stabilised ENF and four point ENF (4ENF) configurations are both relatively recent developments and both aim to deliver stable crack growth during the test. Stable crack growth is inherent in the 4ENF configuration [45] while the stabilised ENF utilises an electronic control system to measure crack length and
adjust the loading rate to maintain stable crack growth [46]. Due to the relative simplicity of the 4ENF fixture and test setup, this was the test method used for the toughness testing in this thesis.

As can be seen in Figure 2.15 all of the configurations are bending tests and each requires a specimen that is provided with a pre-cracked region at one end. The pre-cracked region behaves as two independent flexural arms while the remainder of the specimen is monolithic. Crack propagation is caused by the stress concentration at the tip of the cracked region.

2.5.3 The 4ENF test method

The 4ENF test was first described by Martin and Davidson [45], they suggested that it had the following advantages over conventional tests (presumably meaning three point bend ENF tests):

- The method has stable crack growth under displacement controlled loading.
- Compliance calibration can be carried out directly from the test data rather than requiring a second pass as is the case for conventional tests.
- The effects of friction are reduced due to the absence of shear between the loading rollers.

They provide an expression for energy release rate (ERR) which is derived from classical beam theory (CBT), although they do not provide the full derivation. The author has derived the CBT equations of bending for the 4ENF setup, the derivation is given in Appendix D and is shown to agree with Martin and Davidson expression for ERR.

Martin and Davidson state that "... the connection between the loading fixture and the load frame may be fixed or pinned." In fact, as can be seen in Figure 2.15, the
deflection of the 4ENF specimen must be asymmetrical. The cracked portion is more flexible than the monolithic portion so the deflection of the left hand loading roller must be greater than that of the right. This is not compatible with a load frame that is fixed against rotation due to the fact that typical displacements for a 4ENF test are not negligible. For example Martin and Davidson report test displacements of approximately 2.7 mm.

2.5.3.1 Effects of friction in the 4ENF test

Schuecker and Davidson [47] have studied the effects of friction on both the 3ENF and 4ENF tests. The work was in response to reports that the 4ENF test produces high values for mode II critical energy release rate, \( G_{\text{IIc}} \), compared with the 3ENF test. They convey reports of 4ENF values of \( G_{\text{IIc}} \) that were approximately 9\% higher for non-precracked specimens and 21\% higher for precracked specimens.

A note to clarify terminology. In this context the term precrack refers to the process of advancing the delamination crack from the de-bond insert prior to carrying out the toughness test. It is generally accepted [44] that a resin rich region is likely to exist at the tip of the debond insert and for this reason the ENF specimen should be precracked prior to carrying out the toughness test. The precrack moves the crack past the spurious resin rich region.

Schuecker and Davidson [47] show that the effect that friction has on the 4ENF test is in the range 5\% to 11\%, the higher value relates to a larger load roller distance, both values relate to a coefficient of friction \( \mu = 0.5 \). The effect on the 3ENF test is just under half as big. They note that the effect is greater for 4ENF tests because friction only has a significant influence at the roller locations, of which there are a greater number in the 4ENF test.

The author has used FE analysis to examine the effect of friction on the test geometry that had been used for the 4ENF tests in this thesis. The results of this examination are given in Section 4.8.1.4 and Section 4.9.2.

2.5.3.2 Effects of geometric nonlinearities

Sun and Davidson [48] presented work which investigated the effects of friction and geometric non-linearities on 4ENF tests. They used FE models which incorporated nominally rigid support and loading rollers to simulate the geometrical non-linearity of a 4ENF test. They used the following procedure to calculate the non-linear energy release rate:
• Set up the FE model with crack length $a$.
• Apply a predetermined displacement to the model and record the strain energy $U_a$.
• Release a set of nodal constraints in order that the crack advances to $a + \delta a$.
  Record the strain energy $U_{a+\delta a}$.
• Calculate the simulated non-linear energy release rate $G_{NL} = \frac{1}{B} \left( \frac{U_a - U_{a+\delta a}}{\delta a} \right)$, where $B$ = specimen width.

This method permitted direct control of crack growth in the FE model and consequently provided the opportunity to determine the effect that various fixture and specimen parameters have on the linearity of measured energy release rate. However, the method is not well suited to correcting for non-linearities in crack propagation FE results or physical 4ENF tests.

2.6 The measurement of crack propagation in FRP materials using DIC

The Digital Image Correlation (DIC) technique [49, 50] was used to measure crack propagation during the 4ENF testing that was carried out here. DIC is an optical measurement technique that provides full field strain measurement, it relies on the digital comparison of sequential images which have been captured during a specimen test. A random speckle pattern is applied to the surface of the specimen and the sequential deformation of this speckle pattern, as strain is generated in the specimen, is measured and recorded using DIC in order to provide the measured strain field across the surface of the specimen. The DIC analysis can provide the measured strain field for each recorded image.

The technique has become widely used in materials testing and was used by Crammond et al [50] in order to measure strains in a GFRP single lap joint specimen that was tested in a drop weight test rig. Crammond et al were particularly interested in using DIC to study the development of cracks in the adhesive as load in the joint increased. The development of the critical crack at the root of the adhesive in the single lap joint can be seen in Figure 2.16, micro cracks forming early in the test are just visible in the left hand image while a larger crack that has formed at a later stage of the test is visible in the right hand image.

Figure 2.17 shows results from a DIC analysis of the root of the adhesive, the same region as is seen in Figure 2.16. The DIC shear strain field for the root region in the adhesive is shown for three points on the loading curve, the development of shear strain can be seen in the images. At load increment 3 the large apparent strain across the crack is clearly visible and thus the rate of propagation of the crack during the
test can be ascertained.

The work by Crammond et al demonstrates that DIC is a viable technique for measuring crack propagation in fibre reinforced composite materials.

![Figure 2.16: Initiation and growth of a crack at the root of the adhesive in a single lap joint [50].](image)

2.7 Frequency filtering effects

Chapter 5 considers the frequency filtering of in plane stress waves travelling through a perforated joint. The interest is in considering the perforated steel adherend of the joint to be a phononic crystal material and to investigate whether attenuation of significant frequency components of a dynamic stress wave will occur as they pass through the perforated plate.

Phononic crystal materials were first introduced by Kushwaha et al [17], their interest was related to both the theoretical aspects and the practical applications of creating a material that would inhibit the transmission of vibrations in certain bands of frequencies. They mention, for example, creating a vibration free environment for highly sensitive mechanical systems. Since this introduction by Kushwaha there has
Chapter 2. Literature Review

been significant interest in phononic crystal materials.

Jensen [51] presented a study of 1 dimensional and 2 dimensional (1D and 2D) lattice systems that act as phononic crystals and are able to attenuate waves in certain bands of frequencies. Jensen’s paper is particularly interesting for two main reasons:

1. The paper studies both theoretical phenomena in infinite space and physical phenomena in finite space.

2. The paper presents a method for using systems of point masses and massless springs to model 1D and 2D infinite media and hence to investigate frequency filtering (band gap) behaviour.

Jensen also considered the effect that viscous damping has on the presence of band gaps in mass-spring systems. For mass-spring systems that provided a significant band gap, Jensen demonstrated that there was no significant change in either the width of the band gap or the attenuation in the band gap for a range of critical damping ratios up to 1% critical damping.

There is a large body of work on the subject of phononic materials and many aspects have been studied. Wang and Wang [52] have presented a work of particular interest which discusses circular perforations in infinite plates and shows that complete band gaps can be achieved. Considering this in combination with Jensen’s demonstration that finite systems are able to deliver attenuation over certain bands of frequencies it is reasonable to assume that finite plates with circular holes may be able to deliver attenuation of propagated waves. On this basis, spring-mass models and FE models are used in Chapter 5 to assess the potential for perforated steel plates to attenuate in-plane wave propagation.

2.8 Concluding remarks for the literature review

This chapter has provided a review of the available literature in relation to the two primary strands of the thesis and also in relation to supplementary topics that support these primary strands. The following comments aim to provide a context for the work in the following chapters.

2.8.1 Structural performance of steel-GFRP bonded joints

The behaviour and performance of bonded joints between steel and FRP materials is quite well understood. However the behaviour of bonded joints incorporating perforated steel plates, particularly under higher rates of loading, is not well documented. It is expected that perforated joints of this type can provide an effective and economical method for increasing the structural performance of steel-GFRP joints. In
most applications for this type of joint it is necessary to demonstrate that the joint performs well under both static and dynamic loading conditions.

2.8.2 The effect of loading rate on mode II fracture toughness of vinyl-ester GFRP

Having carried out physical tests and FE modelling of both perforated and non-perforated steel-GFRP joints the author was able to establish that:

(a) The tested joints fail at a higher load when the load is applied at a higher rate.
(b) Mode II toughness is the dominant cohesive parameter in terms of joint strength.
(c) There is a lack of consensus in the literature regarding the effect that loading rate has on mode II toughness of vinyl-ester resin.

Thus it was important to carry out a set of tests to establish whether the vinyl-ester GFRP material that was used for the joints did exhibit a loading rate effect on mode II fracture toughness. This would in turn establish whether the observed increase in joint strength could be attributed to an increase in fracture toughness.

2.8.3 Frequency filtering effects in perforated steel-GFRP joints

It is well known that periodic structures can exhibit frequency filtering effects, i.e. periodic structures are able to attenuate the passage of dynamic stress waves for certain bands of wave frequency. It has been shown that if perforated plates are provided with perforations in the form of a periodic lattice, then these plates can exhibit frequency filtering effects in relation to the propagation of stress waves in the plane of the plate. It is therefore anticipated that perforated steel-GFRP joints can deliver frequency filtering effects.

Thus it interesting to study the filtering effect of perforated steel plates with the aim of identifying perforation geometries that will deliver filtering effects at frequencies that are relevant to the expected applications of the steel-GFRP joints.
Chapter 3

Strength Testing Of Steel-GFRP Joints

3.1 Introduction

Before providing details of the joint strength tests that were carried out as part of this study, it will be useful to review the inception of the work as this informed both the chosen programme of testing and the design of the joint specimens.

As discussed in section 2.2.2.3, Melograna et al [16] have shown that Steel-GFRP joints incorporating perforated steel plates are capable of sustaining significantly greater static tension loads than comparable joints with non-perforated steel plates. Selected representative results from the Melograna tests are given in Table 3.1.

The motivation for the joint testing reported here is twofold. The first aim is to extend the work undertaken by Melograna et al by testing joints under dynamic tension loads. The purpose of this testing is to determine whether the strength enhancements that have been reported for static loading can be replicated for dynamic loading. The second aim is to generate joint strength data against which we can calibrate our FE models of the joints and thus show that the FE models are capable of predicting the performance of such joints.

<table>
<thead>
<tr>
<th>Perforation type</th>
<th>Without surface primer</th>
<th>With surface primer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-perforated</td>
<td>736</td>
<td>916</td>
</tr>
<tr>
<td>7-9 Rows / circular</td>
<td>977</td>
<td>1051</td>
</tr>
</tbody>
</table>

Table 3.1: Average joint strength (kN/mm width) reported by Melograna et al [16] for non-perforated joints and perforated joints with between 7 and 9 rows of circular holes.
The strongest of the perforated joints that were tested by Melograna et al [16] had between 7 and 9 rows of perforations. Their results show that both circular and triangular perforations lead to similar levels of performance in terms of enhancing the strength of the joints. Their perforated joints exhibit higher strengths than their non-perforated joints. However, as shown by Wang et al [52], plates incorporating a lattice of circular perforations can attenuate elastic waves travelling in the plane of the perforated plate. This attenuation of elastic wave propagation is of interest in the current project, therefore the circular perforation geometry that was used by Melograna et al [16] and has been used as the starting point for the joint design tested in this study.

The geometry of Melograna’s [16] joint with 9 rows of circular perforations is shown in Figure 3.1. The circular perforations range from 1 mm in radius up to 2 mm in radius with a 0.125 mm radius increment for each row. The perforations are arranged on a triangular grid which has 5 mm side lengths. The materials used for the joint are given in Table 3.2.

In summary then, the purpose of the strength testing carried out here is to determine whether the enhancement in static load bond strength, that was demonstrated by Melograna for perforated joints [16], is also available under dynamic loads.

### 3.2 Initial programme of joint tests

In order to provide a baseline against which to assess the results of the dynamic load tests, a matching set of quasi-static load tests was also conducted. For brevity, these quasi-static tests will subsequently be referred to as static tests.

<table>
<thead>
<tr>
<th>Steel plate</th>
<th>GFRP reinforcement</th>
<th>GFRP resin</th>
</tr>
</thead>
<tbody>
<tr>
<td>12 gauge Stainless</td>
<td>BTI</td>
<td>Dow Derakane</td>
</tr>
<tr>
<td>grade AL-6XN</td>
<td>TH-4000</td>
<td>510A-40</td>
</tr>
</tbody>
</table>

Table 3.2: The materials used for the joints tested by Melograna et al [16].
CHAPTER 3. STRENGTH TESTING OF STEEL-GFRP JOINTS

### Static tension tests
- Non-perforated Joints Without flaw \(\text{STNP-1,2,3}\)
- Non-perforated Joints With flaw \(\text{STNP-F-1,2,3}\)
- Perforated Joints Without flaw \(\text{STPE-1,2,3}\)
- Perforated Joints With flaw \(\text{STPE-F-1,2,3}\)

### Dynamic tension tests
- Non-perforated Joints Without flaw \(\text{DYNP-1,2,3}\)
- Non-perforated Joints With flaw \(\text{DYNP-F-1,2,3}\)
- Perforated Joints Without flaw \(\text{DYPE-1,2,3}\)
- Perforated Joints With flaw \(\text{DYPE-F-1,2,3}\)

Table 3.3: Initial programme of joint tests, showing the specimen labelling system used for the first set of joint specimens.

In order to provide a baseline against which to assess the effect of steel plate perforations, for both static and dynamic loads, a matching set of tests was conducted on joints without perforations in the steel plate.

Additionally, in order to ascertain the susceptibility of the joint design to defects that may occur during joint manufacture, an intentional flaw was added to half of the specimens. Thus, to incorporate the various combinations of joint type and loading type, the testing programme for the first set of joint specimens comprised eight specimen types. These are tabulated in Table 3.3.

Once the tests on the first set of joint specimens had been successfully completed, then USNA offered to manufacture a second set of specimens and a decision was needed to determine the purpose and design of this second set. Two alternative options for the second set of specimens were considered. The first option was to manufacture a nominally identical set of specimens, the aim of this option would have been to give us a larger set of repeated data and thus a more statistically reliable set of results. The second option was to manufacture specimens with nominally identical materials but with different joint geometry, the aim of this option would have been to generate a second set of data points. We could then test the FE model with this second set of points and thus improve the calibration of the model. On the basis that the second option would provide further insight into the behaviour and failure mechanisms of the joint, a second perforated joint geometry was proposed. The design of this second set of specimens is covered in Section 3.7.
CHAPTER 3. STRENGTH TESTING OF STEEL-GFRP JOINTS

3.3 First set of joint specimens - specimen design

This section provides some background information regarding the design of the first set of joint specimens. The reasoning that led to some of the more significant design decisions, which were necessary in order to finalise the specimen design, are outlined. A sketch of the joint design is given in Figure 3.2, detailed drawings defining the materials and geometry of the first set of joint specimens, drawings DWTS-01 and DWTS-02, are given in Appendix A. Tests were carried out on both perforated and non-perforated joints, the arrangement shown in Figure 3.2 is common to both types of joint.

3.3.1 Selection of the steel plate material

The initial intention for the first set of joint specimens was to use identical materials and geometry to those used by Melograna et al [16] for their 9 row joints, see Figure 3.1 and Table 3.2. Unfortunately this proved to be impossible, firstly due to the fact that our funder dictated that we use a less expensive grade of steel and secondly due to the fact that the reinforcement used by Melograna was no longer in production.

The USNA were partners on the project and had been assigned the task of manufacturing the test specimens. It was established that USNA had 12 gauge (approximately 2.8mm thick) grade 304 stainless steel plate in stock. In order to save costs this grade 304 plate was chosen to be used for the test specimens. Grade 304 is a lower steel grade than the AL-6XN steel used by Melograna, a comparison of the principal mechanical properties of the two steel types is given in Table 3.4.
Table 3.4: Comparison of the principal mechanical properties for grade AL-6XN and grade 304 stainless steel [53, 54].

<table>
<thead>
<tr>
<th>Grade</th>
<th>Young’s Modulus (GPa)</th>
<th>Yield Strength (MPa)</th>
<th>Tensile Strength (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AL-6XN</td>
<td>N/A</td>
<td>345</td>
<td>760</td>
</tr>
<tr>
<td>304</td>
<td>193</td>
<td>290</td>
<td>621</td>
</tr>
</tbody>
</table>

In order for the tests to deliver the required information, the specimens must fail within the joint region of the specimen and the failure must be related to the strength of the bond between the two adherends. Thus the joint must be weaker than both of the adherends, including within the perforated portion of the steel plate. It can be seen from Table 3.4 that the grade 304 steel used here has approximately 80% of the tensile strength of the steel that was used by Melograna. Consequently, during the specimen design process, there was some concern that the strength of the joint geometry used by Melograna may be too strong for the grade 304 steel. Hence failure may occur within the steel adherend prior to failure of the bond. As a result of this concern it was necessary to weaken the joint by modifying its geometry, thus ensuring that the joint would fail at the bond-line prior to failure of the steel adherend.

From the analyses that have been carried out by Hart-Smith [19], we can see that the most effective method for reducing the strength of a double-lap joint is to increase the stiffness imbalance of the two adherends. More stiffness imbalance leads to a higher stress concentration at the tips of the adherends and therefore lower ultimate strength.

As discussed in Section 2.2.2.3 the graduated perforations used by Melograna et al [16] provide a mechanism for mitigation of the stiffness imbalance that exists between the GFRP and steel adherends. Thus the target of reducing the joint strength by increasing the stiffness imbalance between the two adherends can be achieved by removing one of the rows of perforations, dropping from the 9 rows used by Melograna to 8 rows. As shown in Figure 3.3 the joint design for the first set of specimens had a 40mm lap length with eight rows of circular perforations. The perforations ranged from 1.125 mm radius up to 2 mm radius in 0.125 mm radius increments. This can be compared with the Melograna joint shown in Figure 3.1, this incorporates 9 rows of perforations ranging from 1 mm up to 2 mm, also in 0.125mm increments.
CHAPTER 3. STRENGTH TESTING OF STEEL-GFRP JOINTS

3.3.2 Selection of the GFRP material components

3.3.2.1 Selection of the GFRP reinforcement

The reinforcement used by Melograna et al was BTI TH-4000 [16], this was a stitched glass-fibre fabric. Unfortunately this was no longer being manufactured so it was necessary to find a suitable alternative. After some discussions with suppliers the USNA determined that Owens Corning were able to supply TTX1330 glass fibre reinforcement which provided the same stitched tri-axial construction and very similar areal weights when compared with BTI TH-4000. The areal weights of the two fabrics are given in Table 3.5.

The TTX1330 fabric uses Owens Corning Advantex® E-CR glass fibres [55], these provide improved corrosion resistance when compared with standard E-glass [56] and, as shown in Table 3.6, very similar mechanical properties compared with E-glass.

---

**Table 3.5: Comparison of TH-4000 [16] and TTX1330 [55] glass fibre reinforcement fabrics.**

<table>
<thead>
<tr>
<th>Fibre orientation</th>
<th>Areal weights (g/m²)</th>
<th>TH-4000</th>
<th>TTX1330</th>
</tr>
</thead>
<tbody>
<tr>
<td>90°</td>
<td>540</td>
<td>520</td>
<td></td>
</tr>
<tr>
<td>+45°</td>
<td>405</td>
<td>401</td>
<td></td>
</tr>
<tr>
<td>-45°</td>
<td>405</td>
<td>401</td>
<td></td>
</tr>
</tbody>
</table>

---

**Figure 3.3: Geometry of the perforations for the first set of joint specimens.**
<table>
<thead>
<tr>
<th>Property</th>
<th>Units</th>
<th>E-glass</th>
<th>ECR-glass</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tensile strength</td>
<td>N/mm²</td>
<td>3400</td>
<td>3300</td>
</tr>
<tr>
<td>Tensile modulus</td>
<td>kN/mm²</td>
<td>72</td>
<td>72</td>
</tr>
<tr>
<td>Elongation at failure</td>
<td>%</td>
<td>4.8</td>
<td>4.8</td>
</tr>
</tbody>
</table>

Table 3.6: Comparison of mechanical properties for E-glass and ECR-glass [57].

<table>
<thead>
<tr>
<th>Property</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tensile strength</td>
<td>N/mm²</td>
<td>86</td>
</tr>
<tr>
<td>Tensile modulus</td>
<td>N/mm²</td>
<td>3400</td>
</tr>
<tr>
<td>Elongation at yield</td>
<td>%</td>
<td>4-5</td>
</tr>
</tbody>
</table>

Table 3.7: Principal mechanical properties for un-reinforced Derakane 510A-40 vinyl-ester resin [23].

### 3.3.2.2 Selection of the GFRP resin

In contrast to the steel and reinforcement materials, it was possible to use the identical product for the GFRP resin as was used by Melograna et al, namely Derakane 510A-40 resin which is produced by Ashland. This is an epoxy vinyl-ester resin designed to offer a high degree of fire retardance, good resistance to chemical attack and room temperature curing [23]. Some of the principal mechanical properties are given in Table 3.7.

### 3.3.3 Influence of the test set-up on specimen design

So far the discussion regarding specimen design has centred on the joint itself. The joint represents only part of the overall specimen, which had to be designed to suit the test method and test fixtures to be employed during the tests.

The static specimens will be tested using an Instron screw jack universal test machine, while the dynamic specimens will be tested in an Instron Dynatup drop weight test machine, see Figure 3.4. Each type of test machine employs a different mechanism for gripping the specimen and hence applying the load to the specimen being tested. These differences in grip have an influence on the design of the test specimens.

### 3.3.3.1 Static specimens

Standard wedge grips are available for use on the screw jack test machine, see Figure 3.5. These standard grips are designed to accommodate a wide range of specimen materials and thicknesses, as a result no particular specimen design features are required for the steel plate adherend of the static specimens. However, there was some concern that the crushing action of the ridged grips may cause surface damage to the
GFRP. As a result of this concern the thickness of the GFRP adherend was increased by two layers of stitched fabric in the gripped region.

3.3.3.2 Dynamic specimens

The dynamic specimens were tested in the Imperial Tension Adapter (ITA) which is a bespoke fixture designed at Imperial College London. The ITA transforms the dynamic drop weight impact load generated by the Dynatip test rig into a dynamic tension load within the specimen which is mounted in the ITA.

The principal components of the ITA are identified in Figure 3.6. The operation of the ITA is shown in Figure 3.8 and can be described as follows. The impactor, or tup, of the Dynatip drop-weight rig impacts the upper yoke of the ITA. The top yoke transfers the tup impact load laterally into the two compression struts, which in turn transmit the compression load down to the bottom yoke. The test specimen is held between this bottom yoke and the cross-beams of the ITA. The cross-beams are fixed rigidly to the baseplate of the test rig and are thus held in a fixed position throughout the test. So the top of the specimen is held in a fixed position, by the cross-beams, and the bottom of the specimen is pulled downward, by the bottom yoke, thus the desired tension impact load is generated in the test specimen.

The standard method for attaching the top of the specimen to the cross-beams of
CHAPTER 3. STRENGTH TESTING OF STEEL-GFRP JOINTS

Figure 3.5: Mechanical wedge grips are used for gripping the static specimens.

the ITA is via an M12 threaded socket which is mounted on a spherical bearing, see Figures 3.6 and 3.7. The spherical bearing accommodates specimen misalignment, for example due to minor misalignment of the specimen within the ITA or due to specimen manufacturing tolerances. It was useful to take advantage of this inherent adaptability, therefore the dynamic specimens were provided with grade 304 M12 threaded rods which were welded to the steel plate adherend.

The standard method for attaching the bottom of the specimen to the bottom yoke of the ITA also utilises a threaded rod. Two alternatives for attaching the bottom of the dynamic specimens to the bottom yoke of the ITA were considered. Option 1 was to use the standard bottom yoke by incorporating a threaded rod at the bottom of the specimen in addition to the threaded rod at the top. Option 2 was to modify the bottom yoke in order to interface directly with the GFRP adherend. After considering a number of design alternatives, option 2 was chosen.

The final design for the bottom yoke is shown in Figure 3.7, it can be seen that this new bottom yoke acts as a simple bolted grip device. The force of the clamp bolts generates a through thickness squashing load in the lower end of the GFRP portion of the specimen. This squashing load provides sufficient friction between the faces of the clamping yoke and the GFRP portion of the specimen such that the specimen does not slip during the test. Outline calculations for this yoke design are given in Appendix B.
CHAPTER 3. STRENGTH TESTING OF STEEL-GFRP JOINTS

3.3.4 Design of intentional flaws

The author chose to incorporate intentional flaws into the design of the joint specimens. The aim of these flaws was to replicate the type of flaw that may occur in practice during the manufacture of a hybrid joint. The flaws would thus permit an investigation into the susceptibility of the joint design to these typical manufacturing flaws. It was presumed that one likely type of manufacturing flaw would be a localised reduction in the strength of the bond between the GFRP and steel plate. During manufacture of the joint such a flaw could be brought about, for example, by poor de-greasing of the steel plate or local contamination of the steel surface after degreasing. Either of these examples would be likely to lead to poor localised adhesion of the GFRP resin onto the steel plate.
Due to the failure modes for double lap joints that are reported in the literature and were discussed in Section 2.2.1.1, it was anticipated that failure of the joints was likely to initiate at the tip of the GFRP laps. On this basis the designed flaws for the first set of specimens were placed beneath the tip of the GFRP overlap. The intention was that the weakened bond at the tip of the GFRP overlap would lead to early initiation of joint failure.

The chosen design of intentional flaw is shown in Figure 3.9. The flaw comprised two 8 mm diameter semi-circular pieces of PTFE tape which were adhered to each side of the steel plate prior to completion of the specimen manufacture. The PTFE tape provides a very poor surface for the resin to bond to, this creates a local area with significantly impaired bond and thus recreates the poor adhesion of a manufacturing flaw. As previously identified in Table 3.3 half of the first set of specimens were provided with these designed flaws.

### 3.3.5 Steel surface preparation

Section 2.2.1.2 reviewed various strategies that can be employed for improving the strength of the adhesive bond between polymer resins and steel. From this review it
The tup impacts on the top of the ITA
When the tup impacts on the top yoke the struts are compressed
Compression in the struts causes tension in the specimen
The specimen has failed in tension

1. Before The Tup Impacts The ITA
2. As The Tup Impacts The Top Of The ITA
3. After The Impact Test

Figure 3.8: Operation of the Imperial Tension Adapter.

Semi-circular piece of self adhesive PTFE tape beneath the tips of the GRP overlaps. Both faces of steel plate.

8mm diameter

Steel adherend
GRP adherend

View on face of steel adherend
View on edge of joint

Figure 3.9: Design of the intentional flaw for the first set of specimens.

can be ascertained that the minimum preparation which would be considered credible in practice is some form of surface roughening, such as grit blasting, and subsequent degreasing. A study by Melograna et al [21] has shown that significant additional adhesion strength can be achieved by applying proprietary surface primers to the steel prior to forming the adhesive bond. Melograna et al used one such priming system, Poly Fiber®, to enhance the strength of their perforated joint specimens [16].

It can be seen that if the use of a steel priming system is avoided and the steel surface preparation for the joints is limited to grit blasting and degreasing, then the strength of the joints will be reduced. This expected reduction in strength was considered to be beneficial in the light of the enforced change of steel grade that was discussed in Section 3.3.1. The lower strength would help to ensure that the specimens tested here would fail due to bond strength rather than due to steel adherend strength.
Therefore grit blasting and de-greasing were the only surface treatments that were specified for the steel bond surfaces of the test specimens. The intention was to provide a consistent and repeatable strength of bond between the two adherends, which would be economical to reproduce in practice.

### 3.3.6 Instrumentation

As shown in Figure 3.2 each of the joint specimens was instrumented with two pairs of strain gauges, one pair on the steel adherend and one pair on the GFRP adherend. The two gauges in each pair were placed directly opposite each other on either face of the corresponding adherend. Each pair of gauges was connected in a Wheatstone bridge configuration, the Wheatstone bridge configuration is discussed in Section 3.5.3.4.

Strain gauges are not necessary for the static specimens in their own right. The static specimens will be tested in a universal test machine which incorporates a load cell. This load cell provides the required applied load data. However, the strain gauges and test machine load cell that are used for the static specimens permit the generation of strain versus load data for the GFRP and steel adherend material. This strain versus load data, in combination with the strains measured on the dynamic specimens, will be used to calculate the load that is developed in the dynamic specimens.

The load that is applied to the dynamic specimens is not measured directly in the dynamic test setup. This is because the load cell in the Dynatup rig does measure the tup impact force, but this cannot be used to provide data for the specimen load. This is because the various components of mass and stiffness that are present in the dynamic test setup, in particular those within the ITA fixture, will modify the shape and magnitude of the load pulse. The maximum tup load is not necessarily equal to the maximum specimen load.

A piezo-resistive accelerometer (type 7270A by Endevco) is attached to the bottom yoke of the ITA fixture. Double integration of the acceleration data provided by this sensor provides displacement data for the bottom of the dynamic specimens.

### 3.4 First set of joint specimens - specimen manufacture

The Vacuum Assisted Resin Transfer Moulding (VARTM) process was used to manufacture the test specimens for this project. VARTM is suitable for the manufacture of composites which use a resin that can be room temperature cured rather than a resin which requires the use of an autoclave (a pressurised oven). Figure 3.10 shows
the principal components required for the VARTM process.

Figure 3.10: The principal components forming the Vacuum Assisted Resin Transfer Moulding setup. (Note, the vertical axis is exaggerated to aid clarity)

Referring to Figure 3.10, the VARTM setup used for manufacturing the joint test specimens comprised the following principal components:

- A flat and rigid mould base. In this case a steel topped workbench.
- A layer of release fabric. This prevents adhesion of the GFRP to the mould base.
- The layers of reinforcement that are to be infused, this layer also includes the steel adherends.
- A layer of porous peel ply.
- A layer of breather / bleed fabric. This permits evacuation of the air within the mould and subsequently permits the infused resin to spread evenly.
- The vacuum bag film. An impermeable plastic sheet which seals the mould and compresses the reinforcement to a uniform thickness once the vacuum is applied.
- A perforated tube to deliver the resin.
- A perforated tube to allow the air within the mould be be removed, thus generating the required vacuum.
a) Laying up the glass reinforcement onto a layer of release film. Seen here are two of the four layers of fabric that will form the GRP adherends.

b) Placing the steel plate within the mould. A water-jet cutter has been used to form the perforations and to partially cut the individual adherends.

c) Taping down the VARTM vacuum bag to form a seal around the perimeter of the mould.

d) Infusing the specimens by evacuating the VARTM mould, from the right, and allowing de-gased resin to enter the mould, from the left.

e) The completed plate of incorporating 24 specimens, following infusion and room temperature curing of the resin.

f) Completed specimens after water-jet cutting the individual specimens from the completed plate.

Figure 3.11: Selected steps from the manufacturing process for the first set of joint test specimens.
CHAPTER 3. STRENGTH TESTING OF STEEL-GFRP JOINTS

The VARTM process relies on a vacuum being generated within the mould prior to delivery of the resin material. The vacuum compresses the reinforcement to a uniform thickness and subsequently promotes the flow and even distribution of the resin during the resin infusion phase. Figure 3.11c shows the final stage of mould construction, prior to the vacuum being applied. Figure 3.11d shows the mould with the vacuum applied and the resin infusion phase well under way, the transition between the light and dark area within the mould is the resin infusion front. The resin infusion front progresses from the resin delivery pipe, on the left hand side of the mould, across the mould towards the vacuum pipe, on the right hand side of the mould.

Once the resin infusion process was complete then the delivery and vacuum pipes were locked off to retain the vacuum within the mould. The infused mould was then left for the resin curing process to take place, a total duration of approximately 5 days.

3.5 First set of joint specimens - test setup

3.5.1 Static tests

The displacement controlled static tests were carried out in an Instron 5980 series double column screw jack universal test machine. Instron Bluehill 3 test control software, which was running on a PC dedicated to the test machine, was used to control the tests. The Bluehill software and dedicated PC were also used as a backup data logger for the load and displacement data derived directly from the test machine. The backup data which was logged by Bluehill was useful to confirm that the correct calibration parameters were being used to calculate load and displacement from the primary data. The Bluehill data logger records data in load and displacement units while the primary data logger records voltage data which must be converted to the relevant units of measurement. The overall setup for the static joint tests can be seen in Figure 3.12a, Figure 3.12b provides a zoomed in view of a test specimen in the grips of the static test machine.

Primary data capture was carried out using a PC workstation which was fitted with a high speed data acquisition card. A Fylde FE-H379-TA high speed transducer amplifier was used to condition, amplify and balance the signals from the strain gauge Wheatstone bridges as well as providing the Wheatstone bridge excitation voltage. Five of the analogue voltage input channels on the data acquisition card were used during the static tests and one digital voltage output channel was used, see Table 3.8.

The load, displacement and strain channels probably require no additional explanation. The acoustic emission input and video trigger output were used to control the high speed video camera that was used to capture the propagation of failure in the joints. For the first few static joint tests a system of manually triggering the video
(a) Overall arrangement.  
(b) A test specimen in the static test machine.

Figure 3.12: Test setup for the static joint tests.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Signal type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load</td>
<td>Analogue input</td>
<td>From test machine</td>
</tr>
<tr>
<td>Displacement</td>
<td>Analogue input</td>
<td>From test machine</td>
</tr>
<tr>
<td>Steel strain</td>
<td>Analogue input</td>
<td>From Fylde amplifier</td>
</tr>
<tr>
<td>GFRP strain</td>
<td>Analogue input</td>
<td>From Fylde amplifier</td>
</tr>
<tr>
<td>Acoustic emission</td>
<td>Analogue input</td>
<td>From piezo transducer</td>
</tr>
<tr>
<td>Video trigger</td>
<td>Digital output</td>
<td>To video camera trigger</td>
</tr>
</tbody>
</table>

Table 3.8: Utilisation of the data logger channels for static joint tests.

camera was used, this proved to be rather unsatisfactory due to the slow and variable human reaction time inherent in this method. The only reliable approach using this method was to configure the camera so that it would capture video over a few seconds. Due to memory limitations on the camera, this capture duration required a relatively slow video frame rate of 1000 frames per second. The slow frame rates were not appropriate for capturing the joint failure event which typically lasted only 600 micro-seconds under static loading.

The solution was to fix a small electro-mechanical speaker [60] to one of the GFRP overlaps on the joint being tested and to use the high speed data logger to measure the electrical output of this speaker. Small inexpensive mylar cone speakers are readily available from suppliers such as Maplin Electronics.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load displacement rate</td>
<td>1 mm / minute</td>
</tr>
<tr>
<td>Test end trigger</td>
<td>Load drops to 40% of max.</td>
</tr>
<tr>
<td>Data acquisition rate</td>
<td>1000 samples / second</td>
</tr>
<tr>
<td>Video frame rate</td>
<td>10,000 frames / second</td>
</tr>
</tbody>
</table>

Table 3.9: Static joint test parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drop weight energy</td>
<td>≈ 45 J</td>
</tr>
<tr>
<td>Drop weight mass</td>
<td>≈ 5.25 kg</td>
</tr>
<tr>
<td>Data acquisition rate</td>
<td>10 MHz</td>
</tr>
<tr>
<td>Video frame rate</td>
<td>100,000 frames / second</td>
</tr>
</tbody>
</table>

Table 3.10: Dynamic joint test parameters.

In this application the speaker is actually being used as a transducer to detect the cracking sounds emitted from the test specimen as it fails. The data logging card was configured to detect a trigger event when the output from the transducer exceeded a threshold voltage. Once this trigger event occurred, then the card was configured to provide a digital output signal. This output signal was connected to the trigger input of the video camera and thus caused the camera to start recording. After some experimentation this method proved to be reliable and allowed joint failure events to be captured at a frame rate of 10,000 frames per second. Both the Bluehill test control software and the data logger were triggered manually at the start of the test.

3.5.2 Dynamic tests

The dynamic tests were carried out using a Dynatup drop weight system and the Imperial Tension Adapter (ITA) which transforms the drop weight load into a tension load within the specimen. The overall test setup for the dynamic tests can be see in Figure 3.13, the test parameters are shown in Table 3.10. The arrangements for specimen instrumentation and data capture were similar to those described for the static testing, as indicated in Table 3.11 the main differences relate to triggering the video / data capture and measuring the ITA acceleration. The drop weight test parameters given in Table 3.10 generate a tup impact velocity of approximately 4.1 ms\(^{-1}\).
3.5.2.1 Triggering the video and data capture

The Dynatup system is designed to facilitate the capture of short duration events and therefore has a built in optical trigger system. This consists of a plate, which is attached to the sliding drop-weight, and an optical switch which is triggered when the sliding plate passes through. The height of the optical switch can be adjusted manually so that the trigger signal is sent at an appropriate time for the instrumentation equipment. The output of this optical trigger was used to trigger both the high speed video camera and the high speed data logger.

3.5.2.2 Acceleration measurement

A bespoke Imperial College accelerometer amplifier is used to condition and amplify the signal from the piezo accelerometer that is attached to the bottom yoke of the ITA. The output of this amplifier is fed to one of the data logger channels in order that ITA displacements can be calculated.
CHAPTER 3. STRENGTH TESTING OF STEEL-GFRP JOINTS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tup Load</td>
<td>From test machine</td>
</tr>
<tr>
<td>Acceleration</td>
<td>Bespoke accelerometer amplifier</td>
</tr>
<tr>
<td>Steel strain</td>
<td>From Fylde amplifier</td>
</tr>
<tr>
<td>GFRP strain</td>
<td>From Fylde amplifier</td>
</tr>
<tr>
<td>Optical trigger</td>
<td>From test machine</td>
</tr>
</tbody>
</table>

Table 3.11: Utilisation of the data logger channels for dynamic joint tests.

3.5.3 Processing the raw results data

Both the primary and Instron data loggers provide the results data in the form of Comma Separated Value (CSV) files. These are text files in which numerical data is provided in a tabular format, each column within a given row of the table is separated by a prescribed character. The standard prescribed character is the comma, but in this case the primary data logger uses a space character. Each row in the table gives the time increment for that set of values and the voltage values recorded for each channel at the corresponding time increment.

Typically, the files from the primary data logger had over 130,000 rows of data for the dynamic specimens and over 500,000 rows of data for the static specimens. The large data sets from the static specimens were due to the fact that a) the minimum logging rate available on the primary data logger was 1000 samples/second and b) the logging software for the primary data logger required that a pre-determined logging duration was defined prior to running the test. A conservative (long) logging duration was chosen in order to ensure that the captured data covered the full duration of the test right through to failure of the specimen.

These large data sets were not convenient to work with so the Python [61] programming language was used to write a script which processed the raw data. Listings of the Python scripts that were developed for completion of this thesis are provided in Appendix C. The data processing steps that were carried out using the script were as follows:

1. Cropping. A short period of the data that was measured prior to the test starting was retained in order that the zero offset value could be calculated. Apart from this the redundant data points at the start and end of the original data set were removed.

2. Filtering. A low pass filter is applied to the data in order to limit the effects of anti-aliasing during the decimation process [62].

3. Decimation. A term from the field of digital signal processing [62] which refers
to reducing the size of a set of discretised data. The reduced set of data is generated by re-sampling the original data at a reduced sample frequency.

4. Apply a zero offset. In general the logged signals were not at zero volts prior to the start of the test, this was due to electrical noise and an inherent voltage offset in the instrumentation. Data that was logged prior to the start of each test was manually selected, these values were averaged and this average was used as the zero offset value to be subtracted from each data point.

5. Apply the appropriate conversion factor. The raw data represented the voltage value recorded for each channel, it was therefore necessary to convert these voltage values into the correct engineering units for the type of data being represented.

Thus a specific conversion factor is required for each channel depending on the type of data being measured. The evaluation of the conversion factors required for load, displacement, acceleration and strain data are outlined in the following sections.

### 3.5.3.1 Load

The load conversion factor is defined by the Instron test software. It defines the multiplication factor that is required in order to convert the Instron load channel output voltage into a load value in Newtons. For the static testing, using the Bluehill 3 software, this is a user defined conversion factor which depends on the load range and voltage range that have been selected by the user:

\[
\text{Full scale output signal} = \pm 10\text{ Volts} \\
\text{Full scale load} = \pm 50\text{ kN} \\
\text{Load Conversion Factor} = \frac{50,000\text{ N}}{10\text{ Volts}} \\
\text{LCF} = 5000
\]

For the dynamic testing the load conversion factor is a predefined load cell calibration factor provided by Instron with the specific load cell. In this case the load conversion factor is 30.315 kN/Volt.

### 3.5.3.2 Displacement

The displacement conversion factor is only valid for the static tests, it is a user defined conversion factor which depends on the displacement range and voltage range that
have been selected by the user:

\[
\begin{align*}
\text{Full scale output signal} &= \pm 10 \text{ Volts} \\
\text{Full scale displacement} &= \pm 50 \text{ mm} \\
\text{Displacement Conversion Factor} &= \frac{50 \text{ mm}}{10 \text{ Volts}} \\
\text{DCF} &= 5
\end{align*}
\]

3.5.3.3 Acceleration

The acceleration conversion factor is only valid for the dynamic tests, it is a predefined accelerometer calibration factor provided by Endevco the transducer manufacturer. In this case the conversion factor is 7.884 ms\(^{-2}\)/Volt. As previously mentioned, the acceleration data is used in the dynamic test setup to calculate the displacement of the bottom yoke of the ITA.

3.5.3.4 Strain

The strain conversion factor defines the multiplication factor that is required in order to convert the strain gauge transducer amplifier output voltage into a value of measured strain. The conversion factor depends on a number of parameters.

The gauge factor is the primary strain gauge parameter and is determined by the strain gauge manufacturer, it defines the strain sensitivity of the gauge:

\[
GF = \frac{\Delta \text{resistance}}{\text{resistance}} \frac{\text{resistance}}{\text{strain}}
\]

where:

\[
\Delta \text{resistance} = \text{The change in electrical resistance that results from the applied strain.}
\]
\[
\text{resistance} = \text{The nominal electrical resistance of the gauge with no applied strain.}
\]
\[
\text{strain} = \text{The mechanical strain applied to (measured by) the gauge.}
\]

The strain gauges are connected in a Wheatstone bridge configuration. Referring to Figure 3.14, resistors \(R_{G1}\) and \(R_{G2}\) represent a pair of strain gauges on either side of the specimen, while resistors \(R_{D1}\) and \(R_{D2}\) are fixed resistors which have nominally the same resistance as the zero strain resistance of the strain gauges. The resistance of gauge G1 increases as the tensile strain in gauge G1 increases, this change in resistance tends to reduce the voltage at point A in Figure 3.14. Conversely the resistance of gauge G2 decreases as the tensile strain in gauge G2 decreases, this change in resistance tends to reduce the voltage at point B thus negating the effect of the change.
in strain of gauge G1. From this behaviour it can be determined that the Wheatstone bridge configuration will inhibit an output signal when the specimen undergoes bending strain, but will promote an output signal when the specimen undergoes axial strain [63]. This behaviour is beneficial as small manufacturing tolerances or small misalignments in the test set up can induce bending strain in the specimen, but it is only axial strain that is of interest here.

The Wheatstone bridge equation [63] gives the relationship between the bridge output voltage, $V_O$, and the strain applied to the gauges:

$$\text{Strain} = \epsilon = \frac{2 \times V_O}{GF \times V_{Ex}}$$

where:

$V_O$ = Wheatstone bridge output voltage.

$V_{Ex}$ = The strain gauge excitation voltage provided by the transducer amplifier.

$GF$ = Gauge factor.

The Wheatstone bridge output is conditioned and amplified by the Fylde transducer amplifier, therefore the strain conversion factor must relate amplifier output voltage to applied strain:

$$\text{Strain Conversion Factor} = \frac{\epsilon}{V_{amp}} = \frac{2}{Gain \times GF \times V_{Ex}}$$

where:

$V_{amp}$ = The amplifier output voltage $= Gain \times V_0$

$Gain$ = The voltage gain provided by the transducer amplifier.
3.6 First set of joint specimens - test results

3.6.1 Failure load

This section presents the results from the strength testing of the first set of joint specimens. The results from the first set of static tests are shown Figure 3.15, results from the first set of dynamic tests are shown in Figures 3.16 and 3.17. In all three figures the line type and line colour combinations have been chosen to give two main groups of data, each with two sub-groups, the groups are shown in Table 3.12 and Table 3.13. Numerical results for mean failure load are given in Table 3.14. Two plots are given for the dynamic tests because it is useful to record both the extension to failure and time to failure.

<table>
<thead>
<tr>
<th>Main group</th>
<th>Sub group</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Perforated Joints</td>
<td>STPE-F</td>
<td>With intentional flaw</td>
</tr>
<tr>
<td></td>
<td>STPE</td>
<td>Without intentional flaw</td>
</tr>
<tr>
<td>Non-perforated Joints</td>
<td>STNP-F</td>
<td>With intentional flaw</td>
</tr>
<tr>
<td></td>
<td>STNP</td>
<td>Without intentional flaw</td>
</tr>
</tbody>
</table>

Table 3.12: Data groupings for presentation of the static joint test results.

<table>
<thead>
<tr>
<th>Main group</th>
<th>Sub group</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Perforated Joints</td>
<td>DYPE-F</td>
<td>With intentional flaw</td>
</tr>
<tr>
<td></td>
<td>DYPE</td>
<td>Without intentional flaw</td>
</tr>
<tr>
<td>Non-perforated Joints</td>
<td>DYNP-F</td>
<td>With intentional flaw</td>
</tr>
<tr>
<td></td>
<td>DYNP</td>
<td>Without intentional flaw</td>
</tr>
</tbody>
</table>

Table 3.13: Data groupings for presentation of the dynamic joint test results.
Figure 3.15: Load versus extension plots for the first set of static joint specimens.

Figure 3.16: Load versus extension plots for the first set of dynamic joint specimens.

Figure 3.17: Load versus time plots for the first set of dynamic joint specimens.
Table 3.14: Mean failure loads from the first set of joint tests. (Note, bracketed values indicate the spread in results as a percentage of the mean value)

<table>
<thead>
<tr>
<th></th>
<th>Static</th>
<th>Dynamic</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Perforated joints</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>without flaws</td>
<td>10.8 kN (3%)</td>
<td>15.3 kN (5%)</td>
</tr>
<tr>
<td><strong>Perforated joints</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>with flaws</td>
<td>11.0 kN (5%)</td>
<td>16.0 kN (2%)</td>
</tr>
<tr>
<td><strong>Non-perforated joints</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>without flaws</td>
<td>4.4 kN (56%)</td>
<td>6.0 kN (15%)</td>
</tr>
<tr>
<td><strong>Non-perforated joints</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>with flaws</td>
<td>4.5 kN (32%)</td>
<td>6.2 kN (26%)</td>
</tr>
</tbody>
</table>

3.6.2 Propagation of failure within the joint

When using finite element analysis to model the behaviour of the joints it is informative to compare the physical tests with the analysis models in terms of the manner in which failure propagates along the joint.

In Figure 3.18 four frames from a high speed video are shown, the video was captured during one of the dynamic test on specimen DYPE-3 from the first set of specimens. The frame images, which are taken from frames 150, 206, 230 and 236 of the video, are grouped into four pairs. The left hand image of each pair is the raw image extracted directly from the Vision Research Phantom Camera Control (PCC) software. The right hand image in each pair has had a high pass filter applied using the PCC Software. The high pass filter has the effect of enhancing the edges within the original image, the distinction between the debonded areas and the intact areas in the raw image become enhanced and more obvious via the high pass filtering. The graph on the right hand side of Figure 3.18 shows the GFRP strain versus time plot taken from the DYPE-3 results.

Figure 3.19 shows images of a typical perforated specimen from the first set after testing. All of the specimens from the first set failed in a very well defined manner, for example:

- There was a clean de-bond between the two adherends.
- There was a clean fracture of the GFRP adherend from the resin within the perforations of the perforated specimens.
- Both sides of the GFRP adherend became de-bonded from the steel adherend.
CHAPTER 3. STRENGTH TESTING OF STEEL-GFRP JOINTS

3.7 Second set of joint specimens - specimen design

As discussed in Section 3.2 a second set of joints was designed and tested, this set had a modified joint geometry in order to generate a second set of data points which could be used to check the calibration of the FE model. Unfortunately the manufacture of this second set of joints was not successful, the joints suffered from poor resin impregnation. The poor impregnation caused the GRP adherends to fail prematurely.
For completeness, despite the fact that the second set of joints did not provide useful test data, the work carried out in connection with the second set of joints is covered in the following section of the thesis.

3.7.1 Selection of materials

One criteria for the new specimen design was to change the joint geometry in such a way that the change would influence only one significant joint strength parameter. If the geometrical changes influence more than one significant strength parameter then it becomes difficult to conclusively attribute any change in joint performance to one particular aspect of the joint design. For this reason the materials used for the second set of specimens were the same as those used for the first set of specimens. Choosing the same materials was also beneficial in terms of costs due to the fact that surplus material from the first set was still available at USNA. The same specification for steel plate surface preparation will also be used.

3.7.2 Perforation geometry

As previously discussed, and as shown in Figure 3.3, the perforated joints from the first set of specimens incorporated 8 rows of perforations and a 40mm total joint length. The test results from the first set of specimens demonstrated that, for the same overall joint length, the perforated joints were significantly stronger than the non-perforated joints.

The test data from the first set of specimens was used to calibrate the FE joint model as follows. The test results from the non-perforated joints were used to calibrate a non-perforated FE model, this calibrated model provided data defining the characteristics of the bond between the vinyl-ester GFRP and the steel plate. The test results for the perforated joints were then used to calibrate a perforated joint FE model. The previously determined characteristics for the vinyl-ester to steel bond were used in this perforated joint FE model and the calibrated model was used to determine data defining the characteristics of the bond between the vinyl-ester GFRP and the un-reinforced vinyl-ester that fills the perforations in the perforated joints.

A cohesive traction separation model was utilised for defining the bond characteristics in the FE models. One difficulty with the FE model calibration methodology outlined above, when using a cohesive traction separation definition, is the fact that the traction separation definition requires multiple parameters. In order to gain more confidence regarding the accuracy of the calibration of these parameters, it is beneficial to have a second set of test data. This test data would need to be derived from physical tests on a different joint geometry. A second perforated joint FE model, with new perforation geometry, would then be used to check the accuracy of the pre-
viously calibrated traction separation parameters. The calibration of the parameters can then be improved if necessary.

Based on this discussion regarding a new perforation geometry, combined with the requirement to change only one significant joint strength parameter, the design criteria for the new geometry can be defined.

### 3.7.2.1 Overall lap length

It was shown by Hart-Smith [19] that the strength of a double-lap bonded joint can be influenced by the joint lap length. Therefore the 40mm overall lap length that was used in the first set of specimens should be retained for the second set, in order to prevent changes in strength being the result of changes in lap length.

### 3.7.2.2 Stiffness mismatch

It was shown by Hart-Smith [19] that the strength of a double-lap bonded joint will be reduced if the mismatch of adherend stiffness is significant. In the first set of perforated specimens, adherend stiffness mismatch was mitigated using graduated perforation diameters. Therefore the maximum and minimum perforation diameters that were used in the first set should also be used in the second set. A maximum diameter of 4mm and minimum diameter of 2.25mm. This will provide the same degree of stiffness mitigation for the two sets of specimens.

### 3.7.2.3 Length of overlap beyond the last row of perforations

The perforation geometry used for the first set of specimens provides approximately 6mm of additional GFRP overlap beyond the last row of perforations. This additional overlap was considered important in order to accommodate specimen fabrication tolerances. The fabric reinforcement was cut by hand and then placed in the mould by hand, therefore it is likely that some of the fibres will not extend to the full 40mm joint length. Despite these tolerances it is important that the reinforcing fibres are embedded in resin beyond the last perforation. If this reinforcement embedment is not available then the full effect of the last row of perforations may not be mobilised and the joint strength reduced. Therefore the length of lap beyond the last row of perforations should not be reduced significantly below 6mm.

### 3.7.2.4 Chosen perforation geometry

The perforation geometry that was chosen for the second set of joint specimens is shown in Figure 3.20. The design incorporates 7 rows of circular holes ranging from 2.25 mm diameter to 4 mm diameter.
3.7.3 Intentional flaw design

The test results from the first set of specimens revealed no appreciable difference in strength between the joints that incorporated intentional flaws and those that did not. As a result it was necessary to use an alternative flaw design for the second set of specimens.

As discussed in Section 3.6.2, joint failure did not initiate at the tip of the GFRP laps as was anticipated. In the high speed videos that were captured during the joint testing it can be seen that failure was initiated by de-bonding from the steel plate at the tip of the steel plate. Therefore, in order to induce early initiation of failure, the location of the new flaw needed to be at the tip of the steel plate.

It is useful to restate that the intention of the flaw is to replicate the effect of poor surface preparation of the steel plate. It can be seen from Figure 3.21 that simply moving the flaw from one end of the joint to the other would largely obscure one of the perforations. It is likely that this would result in poor penetration of resin into that particular perforation. This type of intentional flaw would not replicate poor preparation of the steel plate and was therefore not considered to be a viable flaw design for the perforated joints.

An alternative approach was required for the new flaw design. Rather than using PTFE tape to cause localised poor adhesion, the new design of flaw specified a local area of steel plate that would not be roughened by grit blasting. The original, as rolled, surface of the plate is smooth and provides a much poorer surface for the resin to bond to, see Figure 3.22. This approach was equally suitable for both perforated and non-perforated joints.
PTFE flaw at tip of steel plate, possible flaw location for second set of specimens.

PTFE flaw at tip of GRP lap as used for first set of specimens.

Figure 3.21: The problem with moving the semi-circular flaw to the tip of the steel plate.

5mm wide flaw on both faces of the steel plate. Steel plate is masked off during grit blasting to leave original non-roughened surface.

Figure 3.22: Design of the intentional flaw for the second set of specimens.

Using this new flaw design for the non-perforated joints would lead to test results being available for non-perforated joints without flaws, non-perforated joints with flaw type 1 and non-perforated joints with flaw type 2. These results could be compared to investigate the effect that the two types of flaw have on joint performance.

Unfortunately the situation was less straightforward for the perforated joints. If the new flaw design was used in conjunction with the new perforation design then there would be no opportunity to compare the results for the original perforation geometry with results for the new flaw design on the original perforation geometry. There was also some concern that joints with the new perforation geometry may behave differently and that failure may initiate at the tip of the GFRP laps for these joints. If this occurred then, as before, the perforation would have little influence on strength.

Table 3.15 helps to explain the decision making process that was used to determine which combination of flaw design and perforation design to use for the second set of specimens. In table 3.15, original flaw refers to the semi-circular PTFE flaw below
CHAPTER 3. STRENGTH TESTING OF STEEL-GFRP JOINTS

Table 3.15: Tabulated decision making process for the joint design to be used with the second set of specimens.

the tip of the GFRP lap, new flaw refers to the non-roughened strip at the tip of the steel plate. The option of using both of these flaws on the same joint was also considered. The options for perforation design were the 8 row design used for the first set of specimens and the 7 row design shown in Figure 3.20. Five of the options can be discounted for the reasons given in Table 3.15. Therefore the option of using the original perforation design for the second set of flawed specimens was the chosen option, which may initially seem counter intuitive. Drawings defining the materials and geometry of the second set of joint specimens, drawings DWTS-03, DWTS-04, DWTS-05 and DWTS-06, are given in Appendix A.

The proposed test programme for the second set of joint specimens follows the same format as the programme for the first set of specimens and is given in Table 3.16.

Table 3.16: Test programme for the second set of joint specimens, showing the specimen labelling system. (Note, D1 refers to design version 1, D2 refers to design version 2)
3.7.4 Steel surface preparation and instrumentation

Except for in the local area of the intentional flaws, the steel surface preparation for the second set of specimens was nominally the same as the first set. Namely grit blasting and degreasing. Even though grit blasting was not required at the flaw locations, degreasing was still specified. Instrumentation for the second set of specimens was identical to the first set.

3.8 Second set of joint specimens - specimen manufacture

In common with the first set of joint specimens, the second set were manufactured at USNA using the VARTM process. Unfortunately the technicians at USNA had some difficulties with achieving a sufficiently long gel time for the second set. The term gel time refers to the time interval for which a resin retains a low viscosity after the curing catalyst has been added. A low viscosity resin is required for the VARTM process in order to draw the resin through the reinforcement and to eliminate any voids in the finished GFRP. If the gel time is too short then the resin viscosity increases too rapidly, i.e. the resin turns from a liquid to a gel, and full impregnation of the VARTM mould is not possible. Incomplete resin impregnation can in fact be seen in Figure 3.11e, the light area towards the bottom right of the completed plate is non-impregnated reinforcement. In this case though, the non-impregnated region does not affect the portion of the GFRP that incorporates the specimens.

The first attempt at manufacturing the second set of specimens was rejected. The short gel time was evident during the infusion phase of manufacture. Once the GFRP had been removed from the mould it was visually apparent that insufficient resin impregnation had occurred for many of the specimens within the plate. Some time later a second attempt was made. An improvement in gel time was achieved, but it was still shorter than when the first set of specimens were produced. The specimens were completed and delivered to ICL for testing, but it was visually apparent that the specimens were of a poorer quality than the first set. The images in Figure 3.23 show specimens from the second set in which poor resin impregnation is apparent.

3.9 Second set of joint specimens - test methodology

The test setup and test methodology that were used for the second set of specimens were predominantly the same as those used for the first set of specimens. The only change was due to the fact that some of the second set of perforated joint specimens only partially failed on the first pass of the test.
CHAPTER 3. STRENGTH TESTING OF STEEL-GFRP JOINTS

3.9.1 Static tests

With the static specimens the partial failure would lead to a drop in applied load, which would cause the Bluehill testing software to halt the test. The initial partial failure was a delamination and / or splitting failure in the GFRP adherend, the actual joint appeared to have remained intact.

For the specimens in which this partial failure occurred, the test was restarted to check that the peak strength had been found during the first pass. As discussed in Section 3.10 only one specimen attained a greater strength on the second pass.

3.9.2 Dynamic tests

As was the case with the static specimens, a delamination and / or splitting failure of the GFRP adherend occurred with some of the dynamic perforated specimens. During this partial failure a significant portion of the drop weight impact energy was dissipated via the generation of new failure surfaces. As a result of this energy dissipation there was insufficient energy to cause complete failure in the joint itself, for example one side of the double lap joint remained intact.

In order to overcome this problem of not achieving failure in the joint, the drop weight impact energy was increased from 45 Joules to 60 Joules. This ensured that the joint failed in the first test and thus that the ultimate strength of the specimen
was measured.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drop weight energy</td>
<td>$\approx 60.1$ J</td>
</tr>
<tr>
<td>Drop weight mass</td>
<td>$\approx 5.39$ kg</td>
</tr>
<tr>
<td>Data acquisition rate</td>
<td>10 MHz</td>
</tr>
<tr>
<td>Video frame rate</td>
<td>100,000 frames / second</td>
</tr>
</tbody>
</table>

Table 3.17: Dynamic joint test parameters for the second set of specimens.

### 3.10 Second set of joint specimens - test results

This section presents the results from the strength testing of the second set of joint specimens. Results from the second set of static tests are shown in Figure 3.24 and results from the second set of dynamic tests are shown in Figure 3.25. As before the line type and line colour combinations have been chosen to help identify perforated joints versus non-perforated joints as well as joints with intentional flaws versus joints without intentional flaws. Numerical results for mean failure loads of the second set of specimens are given in Table 3.18.

![Figure 3.24: Load versus displacement plots for the second set of static joint specimens.](image)
While it is useful to see all of the dynamic results on the same graph, Figure 3.25 presents a rather confusing set of result traces, therefore the data is subdivided further.

Figure 3.26 shows the data for the non-perforated dynamically loaded joints from the second set of specimens. Figure 3.27 shows the results for a selection of dynamically loaded specimens from the second set, these specimens all performed in a fairly consistent manner. Figure 3.28 shows the data for specimens DYPE2-1 and DYPE-F-3, the load versus time plots for these two specimens do not follow the same profile as the plots in Figure 3.27.

Specimen DYPE2-1 remained partially intact after the test, hence the trace remains at over 10kN. Specimen DYPE2-1 began to fail at less than 2kN applied load, hence the load climbs at a much slower rate than the other specimens. The failure load result for DYPE2-F-3 is considered valid, but the failure load for DYPE2-1 is not considered to be valid and is therefore not included in the numerical results given in Table 3.18.
CHAPTER 3. STRENGTH TESTING OF STEEL-GFRP JOINTS

Figure 3.26: Load versus displacement plots for the second set of dynamically loaded non-perforated static joint specimens.

Figure 3.27: Load versus time plots for four specimens from the second set of dynamic joint specimens.

Figure 3.28: Unusual behaviour of two specimens from the second set of dynamic joint specimens.
### Table 3.18: Mean failure loads from the second set of joint tests. (Note, bracketed values indicate the spread in results as a % of the average value. Data for DYPE2-1 has not been included)

<table>
<thead>
<tr>
<th></th>
<th>Static</th>
<th>Dynamic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perforated joints</td>
<td></td>
<td></td>
</tr>
<tr>
<td>without flaws</td>
<td>14.3 kN (5%)</td>
<td>11.9 kN (2%)</td>
</tr>
<tr>
<td>Perforated joints</td>
<td></td>
<td></td>
</tr>
<tr>
<td>with flaws</td>
<td>13.0 kN (5%)</td>
<td>12.2 kN (5%)</td>
</tr>
<tr>
<td>Non-perforated joints</td>
<td></td>
<td></td>
</tr>
<tr>
<td>without flaws</td>
<td>6.9 kN (8%)</td>
<td>8.5 kN (6%)</td>
</tr>
<tr>
<td>Non-perforated joints</td>
<td></td>
<td></td>
</tr>
<tr>
<td>with flaws</td>
<td>8.1 kN (1%)</td>
<td>8.6 kN (5%)</td>
</tr>
</tbody>
</table>

3.11 **Review of the results**

Having provided the results for both the first and second sets of joint specimens in Sections 3.6 and 3.10, it will be useful to review some interesting aspects and to make comparisons between the two sets of results.

3.11.1 **Review of the results from the first set**

3.11.1.1 **Intentional flaw**

It is clear from the results given in Table 3.14 that the semi-circular intentional flaw has a small influence on the strength of the joint. The effect appears to be smaller than the size of the scatter in the failure load results.

When designing the intentional flaw it was expected that the joint would begin to fail, primarily due to mode I peeling stresses, at the tip of the GFRP adherend overlaps. Thus it was anticipated that failure would initiate earlier in joints with the semi-circular flaw.

As was shown in Figure 3.18, for perforated joints, and as can be seen in Figure 3.29, for non-perforated joints, failure in the joints actually began at the tip of the steel plate. Based on these findings regarding the propagation of failure in the joints, it is not surprising that the semi-circular flaws did not cause early initiation of joint failure. It therefore seems reasonable to combine the data for specimens with and without intentional flaws, so that the results from the first set of specimens can be considered as one group containing the statically loaded specimens and another group...
3.11.1.2 Strength enhancement due to the perforations

The results show that the joints with perforated steel plates are significantly stronger, by a factor of approximately 2.5, than the non-perforated joints. This is the case for both statically loaded and dynamically loaded joints. This perforation enhancement factor is significantly greater than that reported by Melograna which was a factor of approximately 1.3 [16].

The greater enhancement factor found here, compared with that of Melograna, is...
probably due to the fact that the steel surface preparation used for the specimens in this study was downgraded from that used by Melograna, the reasons for this were discussed in Section 3.3.5. The downgraded surface preparation will have a much greater impact on the non-perforated joints than on the perforated joints.

### 3.11.1.3 Strength increase due to loading rate

It is evident from the results that the dynamically loaded joints are stronger than the statically loaded joints, this is the case for both the perforated and non-perforated joints. This was an interesting result which is discussed further in Chapter 4.

### 3.11.1.4 Scatter in failure load results

The test results for the non-perforated joints exhibit significantly more scatter than the results for the perforated joints. This is likely to be due to a number of factors:

- Most of the strength of the perforated joints is derived from the mechanical interlock of the resin in the perforations:
  - The interlock strength is mobilised by the chemical bonds between the resin in the perforations and the resin that forms the GFRP matrix material.
  - The resin formulation is designed to give reliable consistent mechanical properties as long as the mixing and curing is performed correctly.

- All of the strength of the non-perforated joints is derived from the adhesive bond between the GFRP and steel adherends:
  - The steel plate surface preparation is critical for achieving a good bond strength between the GFRP and steel adherends.
  - Preparation of the steel surfaces of the specimens tested here was a one off manual operation.
  - During the specimen design process it was intended that the surface preparation procedure would be discussed and agreed with the fabrication team. This step in the process did not occur.
  - The specification for the steel surface preparation was rather loose and open to interpretation.

If a more detailed specification for the steel surface preparation had been provided, it is likely that the scatter in the results for the non-perforated specimens would be reduced.

### 3.11.1.5 Drop in stiffness of the non-perforated static specimens

The load versus extension plots for the statically loaded non-perforated specimens, Figure 3.15, exhibit a distinct drop in stiffness as the joint approaches failure. The
same behaviour is not apparent in the statically loaded perforated specimens. This behaviour can also be explained by considering the propagation of failure along the joint.

The images in Figure 3.29 show a gradual extension of the de-bonded region between the two adherends. In the de-bonded region the thin GFRP laps are carrying the whole joint load. Therefore, in the non-perforated joints, the thickness of load carrying GFRP drops from approximately 5 mm beyond the joint to approximately $2 \times 1.3 \, \text{mm} = 2.6 \, \text{mm}$ within the de-bonded region.

If we now consider the perforated joint images shown in Figure 3.18, in particular the image for Frame 230. It can be seen that the bond between the GFRP adherend and the resin that has filled the steel perforations is still largely intact at peak load. Therefore the length over which the thin GFRP laps are carrying the peak joint load is significantly smaller for the perforated joints.

The thin GFRP laps become part of the load string at high loads in the non-perforated tests, this is the cause of the change in specimen stiffness for the non-perforated tests. It is assumed that the same effect is occurring in the dynamic non-perforated tests, but the dynamics (oscillation) of the specimen - ITA system tends to mask the effect.

3.11.2 Review of the results from the second set

Unfortunately the performance of the second set of specimens was rather disappointing. Delamination within the main body of the GFRP adherend occurred over a significant length of the perforated specimens prior to failure of the joint itself. This is significant because as the GFRP becomes delaminated, then some of the laminates become less effective. This causes local through thickness variations in the axial stiffness of the GFRP adherend, which in turn causes eccentric axial loads within the specimen. The eccentric loading generates stress concentrations in the joint and these are likely to lead to premature failure of the specimen. Examples of this premature delamination failure are shown in Figures 3.30, 3.31 and 3.32. Due to the premature failure of the dynamically loaded perforated joints, the results for these specimens will be discounted from the discussion.

An additional complication is apparent when comparing the strength of the non-perforated joints from the second set with those of the first set. The second set are significantly stronger than the first, for example the second set of non-perforated joints without flaws are over 50% stronger than those from the first set. This one group of specimens have nominally the same joint design for both the first set and second set. The most likely explanation for this difference in strength is a difference in the steel surface preparation. For example the grit-blasting may have led to greater
Before the test
Body of GRP delaminates
Specimen Fails
Delamination within the main body of the GRP adherend. Some of the $0^\circ$ fibres pull away from the joint

Figure 3.30: High speed video images of a perforated dynamic joint test showing delamination within the GFRP adherend leading to failure of the specimen with only partial failure of the joint itself.

Figure 3.31: A perforated dynamic joint from the second set. The core of the GFRP adherend has delaminated from the GFRP laps during the test. This has lead to premature failure of the specimen.

surface roughness in the second set and therefore an improved adhesive bond between the GFRP and steel.

Table 3.20 provides a comparison between the first and second set of specimens, the effect that the three main specimen parameters have on joint strength are identified. These main parameters are discussed in more detail below.
Figure 3.32: A perforated dynamic joint from the second set. The GFRP adherend has delaminated during the test, leaving part of the GFRP laminate still adhered to the steel plate.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Effect on strength</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>First set</strong></td>
<td><strong>Second set</strong></td>
</tr>
<tr>
<td>Perforations</td>
<td>Positive</td>
</tr>
<tr>
<td>Flaw</td>
<td></td>
</tr>
<tr>
<td>Perforated joints</td>
<td>No effect</td>
</tr>
<tr>
<td>Non-perforated joints</td>
<td>No effect</td>
</tr>
<tr>
<td>Dynamic loading</td>
<td></td>
</tr>
<tr>
<td>Perforated joints</td>
<td>Positive</td>
</tr>
<tr>
<td>Non-perforated joints</td>
<td>Positive</td>
</tr>
</tbody>
</table>

Table 3.20: Comparison of the first and second set of specimens in terms of the effect that the three main specimen parameters have on strength.

3.11.2.1 Intentional flaw

It is difficult to make any conclusive statements regarding the 5 mm wide intentional flaw used for the second set of specimens. It would be expected that the flaw would have greatest effect on the non-perforated joints and would tend to lower the strength of the joints. The effect should be greater because the surface area of the flaw is much greater for the non-perforated joints than it is for the perforated joints. The strength should be lower because the adhesion of the resin to the smooth flaw should be weaker than the adhesion to the roughened steel surfaces beyond the flaw.

In fact the results do not match the expected behaviour. The statically loaded non-perforated joints with the flaw are significantly stronger than the corresponding joints without the flaw. The flaw has a negligible effect on the dynamically loaded non-perforated joints. The flaw does appear to weaken the statically loaded perforated joints, but, in the light of the anomalous flaw results for the rest of the specimens, it is difficult to give this result much credence.
3.11.2.2 Strength enhancement due to the perforations

Despite the problems with the premature failure of the perforated specimens, it is clear that the perforations have enhanced the strength of the second set under both static and dynamic loads.

3.11.2.3 Strength enhancement due to loading rate

Although the results are not entirely conclusive in terms of the perforated joints, the weight of the evidence that has been gathered during the two rounds of joint testing suggests that dynamic loading does increase the strength of the joints. It seems reasonable to assume that if the GFRP had been manufactured successfully for the second set of specimens then the dynamically loaded perforated joints would exhibit greater strength than the statically loaded joints.

3.11.2.4 Apparent stiffness of the non-perforated static specimens

The load versus displacement plots for the non-perforated static specimens, Figure 3.24, show some unusual behaviour. This was due to the GFRP adherend slipping in the grips of the test machine. The same grips and tightening procedure were used throughout the testing, yet this slippage still occurred in some of the specimens. It may have been due to the presence of non-impregnated glass fibres on the surface of the GFRP.

3.12 Finite element modelling of the joints

Finite element (FE) models of both the perforated and non-perforated joint specimens have been developed. The primary aims for these models were:

- To confirm that surface based traction separation cohesive damage model was appropriate for modelling both the propagation of failure in the joints and the tension strength of the joints.

- To determine which cohesive damage parameters were dominant in terms of the strength of the tested joints.

3.13 Details of the models

Figure 3.33 shows the Abaqus model for the perforated joints. Eight node linear 3D elements with reduced integration and hourglass control (C3D8R) were used for the model, the explicit solver was used due to the inclusion of cohesive surfaces. The through thickness symmetry of the joint was utilised, this permitted modelling only half of the joint. The material properties for the steel and GFRP adherends are given in Table 3.21. As indicated in Figure 3.33, axial load was generated in the
### Table 3.21: Properties of the materials used in the joint specimen FE models.

<table>
<thead>
<tr>
<th>Material</th>
<th>Flexural modulus N/mm²</th>
<th>Poisson’s ratio</th>
<th>Density kg/m³</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>190,000</td>
<td>0.3</td>
<td>7,850</td>
</tr>
<tr>
<td>GFRP</td>
<td>20,000</td>
<td>0.44</td>
<td>2,200</td>
</tr>
<tr>
<td>Resin</td>
<td>3,400</td>
<td>0.3</td>
<td>1,120</td>
</tr>
</tbody>
</table>

FE model by applying vertical displacement to the bottom edge of the GFRP plate while holding the top edge of the steel plate in a fixed position. Through thickness symmetry of the joint is modelled by applying z-axis restraint at the mid-thickness plane of the joint.

![Perforated joint FE model](image)

#### 3.14 Determining the appropriate cohesive parameters

At the outset of this FE modelling exercise the cohesive parameters for the Steel-GFRP and Resin-GFRP interfaces were not known. As shown in Table 3.22, there are six parameters that define the traction separation cohesive behaviour, strictly speaking there are nine parameters but the values for axial shear (mode II) were also used for the transverse shear direction (mode III).

The first stage in the process of finding appropriate values for the FE cohesive parameters was to understand whether one of the fracture directions dominated the behaviour of the joint. Figure 3.34a shows the load versus displacement plots for five non-perforated joint FE analyses. Each analysis used a different set of cohesive
parameters, the mode II parameters were kept constant while the mode I parameters were modified ranging ±50% either side of a baseline. Figure 3.34b shows a complementary set of load versus displacement plots, for these analyses the mode I parameters were kept constant while the mode II parameters were modified ranging ±50% either side of a baseline.

It is clear from Figure 3.34 that mode II cohesive zone parameters dominate the strength of the non-perforated joint model. Having completed this first stage, one set of unknowns has been eliminated from the problem. It is now only mode II cohesive parameters for Steel-GFRP and Resin-GFRP interfaces that need to be found. The second stage of the process also deals only with the non-perforated joint model. This model does not have any Resin-GFRP interfaces so it is only necessary to work with the Steel-GFRP interface during the second stage.

The process for finding an appropriate set of cohesive parameters for the Steel-GFRP interface, the second stage, is as follows:

1. Carry out a literature search to find initial estimates for the cohesive parameters for the two interfaces. This was relatively easy for the Resin-GFRP interface but proved much more difficult for the Steel-GFRP interface.

2. Carry out an FE analysis for the non-perforated joint and compare the ultimate strength of the model with the strength that was developed in the physical tests.

3. Adjust the fracture energy parameter of the cohesive material in the appropriate direction, i.e. to make the model stronger or weaker, and repeat the exercise until the strength of the FE model converges with the test strength.

The approach that was adopted for adjusting the cohesive parameters is indicated in Figure 3.35 which gives a graphical representation of three notional cohesive traction separation models.
Figure 3.35: A similar triangles approach is taken when adjusting the cohesive parameters.

The baseline model in Figure 3.35 (black line) has:

- initial stiffness $k_a = 10,000 \text{ N/mm}$
- initiation criteria $\sigma_a = 10 \text{ N/mm}^2$
- fracture energy $G_a = 300 \text{ J/m}^2$

The model indicated by the blue line has:

- initial stiffness $k_b = 10,000 \text{ N/mm}$
- initiation criteria $\sigma_b = 1.5 \times 10 \text{ N/mm}^2$
- fracture energy $G_b = 1.5^2 \times 300 \text{ J/m}^2$

The initiation criteria and fracture energy are scaled in order to retain the overall "shape" of the cohesive definition while fracture toughness is increased.

At the end of the second stage, parameters had been determined for the Steel-GFRP cohesive bond. In the third stage these Steel-GFRP parameters were used in the perforated joint FE model for the Steel-GFRP surfaces. Cohesive parameters for the Resin-GFRP interface were then found by repeating the same iterative search method that was used in the second stage. The cohesive parameters for the Steel-GFRP interface and the Resin-GFRP interface that were finally adopted are shown in Table 3.22.
3.15 Results from the joint specimen FE model

3.15.1 Load versus displacement plots

Plots of load versus displacement for the two joint types are given in Figure 3.36. When comparing these with the results from the physical joint tests it must be remembered that the FE model only represents half of the joint, so the FE joint strength appears to be half that of the physical joint.

3.15.2 Comparing the behaviour of the FE and physical joints

The images in Figure 3.37 show individual frames taken from the high speed video camera. They show that failure in the non-perforated joints begins at the tip of the steel adherend and progresses towards the tip of the GFRP laps. The green dotted lines that have been added in an attempt to highlight the delamination front. This failure mode was prevalent in all of the non-perforated joint specimens. The only caveat is that a small area of delamination at the tip of the GFRP overlap developed prior to failure in two of the joints. Despite this, the dominant failure mode for these two atypical joints was still from the tip of the steel adherend.

Figure 3.38 shows images taken from the non-perforated joint FE model. The colours in the images represent the cohesive surface damage parameter which defines stiffness degradation of the cohesive surface. Blue indicates either zero stiffness degradation or that the surface is not a cohesive surface. Red indicates 100% stiffness degradation, i.e. the cohesive has failed. The plots in Figure 3.38 demonstrate that failure in the non-perforated joint FE model progresses in a similar manner to the physical tests.

The images, shown in Figure 3.39, of one of the perforated joint tests were taken with the high speed video camera. The progression of damage in the joint can be seen as lighter grey regions. Damage again initiates at the tip of the steel adherend and progresses towards the tip of the GFRP laps. It can be seen that GFRP has de-bonded

<table>
<thead>
<tr>
<th></th>
<th>GFRP-steel</th>
<th>GFRP-resin</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mode I</td>
<td>Mode II</td>
</tr>
<tr>
<td><strong>Initial Stiffness</strong></td>
<td>10,000</td>
<td>4,000</td>
</tr>
<tr>
<td>(N/mm)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Initiation Criteria</strong></td>
<td>2.70</td>
<td>9.42</td>
</tr>
<tr>
<td>(N/mm$^2$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Fracture Energy</strong></td>
<td>0.07</td>
<td>0.14</td>
</tr>
<tr>
<td>($10^3$ J/m$^2$)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3.22: Parameters for the cohesive element traction-separation laws used in the joint specimen models.
from the steel plate for almost the full length of the perforated region before the joint finally fails. The peak strength in the joint results primarily from the bond between the GFRP laps and the resin in the perforations.

Figure 3.40 shows images taken from the perforated joint FE model, the colour coding again represents the cohesive surface damage parameter. It can be seen that progression of failure in the model is very similar to the progression seen in the physical tests.

3.16 Observations relating to the FE models of the joints

The results from the FE modelling of the joints show that surface based cohesive behaviour is an effective method of modelling the macro effects of bond-line failure in double lap Steel-GFRP joints. The similarity between the observed progression of damage in the physical tests and the progression of damage exhibited by the FE
models is encouraging and gives additional confirmation that the physical behaviour is being captured by the FE models.

A systematic method for calibrating the parameters of the surface based cohesive model was used and proved to be effective. It is unfortunate that data for a second joint geometry was not available as this would provide a further check on the validity of the parameter calibration. The surface based cohesive model is beneficial as it avoids the mesh refinement problems that can be encountered when using cohesive elements. The FE models of the joints demonstrate that mode II fracture toughness is the most dominant cohesive parameter in relation to tensions strength of the double lap joint geometry tested here.

It is important to remember that the joints were designed to fail at the bondline, this dramatically simplified the modelling problem. The second set of joint specimens suffered from poor resin infusion, which is another problem entirely, but failure behaviour similar to that shown by the second set could be exhibited by well manu-
factured specimens with a stronger joint design, i.e. the specimens could fail in the adherend rather than at the bondline. The FE models used in this thesis would not be able to capture that type of adherend failure.

3.17 Discussion concerning the joint testing

The joint testing programme reported here had two main objectives:

1. To determine whether the strength enhancements that perforations provide to statically loaded joints [16] can be replicated for dynamically loaded joints.

2. To generate joint strength data against which FE models of the joints can be calibrated and thus show that the FE models are capable of predicting the performance of the joints.

The tests results show that perforations do provide significant strength enhancement for dynamically loaded joints, the perforation enhancement under dynamic loading is similar to that under static loading. The results also indicate that there is a loading rate effect on the strength of both perforated and non-perforated joints. The dynamically loaded joints exhibit greater strength than the corresponding statically loaded joints.

From the FE modelling of the joints it was apparent that mode II toughness of the resin is the property that has greatest impact on the tension strength of the joints. It was therefore postulated that mode II fracture toughness of the resin increases as loading rate increases, this lead to a new avenue of investigation which will be discussed further in Chapter 4.
Chapter 4

Mode II Toughness Testing

4.1 Introduction

4.1.1 Background

This chapter covers the loading rate specific, mode II fracture toughness testing that was carried out by the author. The interest in mode II fracture toughness, and in particular its sensitivity to loading rate, arose from the first set of double-lap joint tests that were reported in Chapter 3. A review of the results from these double-lap joint tests demonstrated that, when tested at impact loading rates, the joints were stronger than when they were tested at quasi-static loading rates.

It was shown in Chapter 3 that finite element analyses of the double lap joints show that mode II fracture toughness, expressed as the mode II critical strain energy release rate \( G_{IIc} \), is the cohesive parameter with most influence on the strength of the joints. Therefore the author speculates that the mode II critical fracture toughness of the vinyl-ester polymer resin used for the joint specimens is greater at impact loading rates than at quasi-static loading rates. The author has carried out four point bending End Notch Flexure (4ENF) tests at various rates of loading in order to confirm whether any sensitivity of \( G_{IIc} \) to loading rate could be detected.

The testing discussed in Chapter 3 was concerned with the strength of hybrid double-lap joints which incorporated steel and GFRP adherends. The purpose of the joint specimens was to test different aspects of the joint design, the specimens were therefore designed to fail at the bond-line between the two materials rather than within the bodies of the adherends themselves. Half of the joints were plain non-perforated joints, for these specimens the bond was simply between the GFRP and the steel plate. The other half of the joints were perforated joints in which the steel plate was provided with circular holes, during the manufacture of the specimens these holes became filled with the vinyl-ester resin that was used to form the GFRP adherend. As a result, for the perforated joints, the bond-line was more complex including interfaces...
CHAPTER 4. MODE II TOUGHNESS TESTING

For the non-perforated joints there is a single type of bond-line interface.

For the perforated joints there are two types of bond-line interface.

Figure 4.1: Steel plates from double-lap joints that have previously been tested to failure.

between GFRP and steel as well as between GFRP and vinyl-ester resin. This point is clarified by the images of the steel plates from tested joints shown in Figure 4.1.

The fracture toughness of interest here then, is the toughness of the bond:

1. Between the GFRP and the steel.
2. Between the GFRP and the resin in the perforations.

4.1.2 Fracture toughness

Fracture toughness is a material property, it quantifies the ability of the material to resist crack growth under the application of load. The concept of fracture toughness comes from the field of fracture mechanics. Fracture mechanics relies on the energy balance approach which was established by Griffith [43], this approach equates a) the strain energy released by a material as a crack in the material grows and b) the energy absorbed by the material in order to generate new surface area in the material when the crack grows. Energy is absorbed by the material due to adhesive bonds in the material being broken as the crack grows and new surface area is created.

The fundamental equation for experimental evaluation of the critical Energy Release Rate (ERR) of a material is [64]:

\[
G_C = \frac{P_C^2 \frac{dC}{da}}{2B} \tag{4.1}
\]

where: 
\(P_C\) = load required to cause the crack to propagate
\(\frac{dC}{da}\) = rate of change of specimen compliance with crack length
\(B\) = width of the specimen

In general, material toughness is a function of load direction. That is to say, it is dependent on the relationship between the direction of the applied load and the direction of crack growth, see Figure 4.2. For many materials the toughness is lowest in mode I [44] and crack growth would naturally occur in a direction consistent with mode I. However, in fibre reinforced composite materials crack growth is constrained...
by the presence of the reinforcing fibres which tend to arrest the propagation of cracks. In this case the mode I crack growth can be inhibited leading to mode II and mode III crack growth. In the joint specimens that were discussed in Chapter 3, crack growth was not only constrained by the presence of reinforcement within the GFRP but also as a result of the change in material at the bond-line.

Figure 4.2: Schematic diagrams showing the three modes of loading that can lead to crack growth [44].

4.1.3 Test configurations used to evaluate mode II toughness

Within the field of composite materials there are four principal test configurations that are used to determine Mode II fracture toughness [46], see Figure 4.3:

- Three point bending End Notch Flexure test (3ENF):
  - The configuration uses three point bending.
  - A standard test fixture can be used.
  - The configuration does not deliver stable crack growth so only initiation values for $G_{IIc}$ are yielded by the 3ENF test.

Figure 4.3: Test configurations for Mode II fracture toughness testing [46].
• Four point bending End Notch Flexure test (4ENF):
  – The configuration uses four point bending.
  – A standard test fixture can be used.
  – The configuration delivers stable crack growth.

• Stabilised End Notch Flexure test:
  – The shear displacement rate of the crack must be measured.
  – Sophisticated test control software is required to ensure that the shear
displacement rate of the crack remains constant throughout the test. Thus
the configuration delivers stable crack growth.
  – The configuration uses three point bending.

• End Loaded Split test:
  – The configuration uses cantilever bending.
  – A specialised test fixture is required. One end of the specimen must be
clamped in a fixture which is rotationally rigid, but which can slide later-
ally.
  – The configuration delivers stable crack growth.

In each case the specimen has a monolithic region and a pre-cracked region, the initial
pre-cracked region is formed when the specimens are manufactured. In the case of
the specimens tested here, a debond insert was placed between the second and third
laminates of the four laminate lay-up, i.e. at mid-thickness of the specimen, prior
to resin infusion, see Figure 4.7. Each of the test configurations places the specimen
in bending, the resulting bending stress concentration at the tip of the pre-cracked
region is the driver for crack propagation during the test.

Stable crack growth is desirable for the fracture toughness tests carried out here be-
cause the rate of crack growth tends to be more closely related to loading rate when
crack growth is stable. The crack propagation rate during a 3ENF test is largely
independent of loading rate [44]. A test that provides stable crack growth is also ben-
eficial due to the fact that multiple data points are delivered from each test specimen
rather than only the single crack initiation data point.

Considering the dual benefits of generating stable crack growth and using a standard
test fixture, the 4ENF test configuration was selected for the mode II toughness
testing that was carried out here.
4.2 Test fixture design

4.2.1 Adapting the standard Instron fixture

A standard Instron four point bending fixture was available in the ICL structural engineering laboratory. As can be seen in Figure 4.4 this fixture consisted of adjustable lower rollers, which can be fixed to the bed of the test machine, and adjustable upper rollers which can be attached to the moveable cross head of the test machine. The fixture is designed for typical metallic specimens which have constant stiffness over their length. The deflected shape of these typical specimens would be symmetrical, therefore a rotationally rigid fixture is acceptable.

The bending stiffness of 4ENF specimens is not constant over the whole length of the specimen, the cracked portion of the specimen is less stiff than the monolithic portion. As a result it was necessary to modify the standard Instron fixture to permit rotation of the upper rollers. Figure 4.4 shows the initial pivot that was fabricated in order to permit rotation of the upper rollers. The pivot incorporates a loading roller, which is part of an existing three point bending fixture, and a new roller bearing plate which is screwed to the top of the lower rollers of the four point bending fixture. The drawing for this pivot, RBP-1, is given in Appendix A.

It soon became apparent that this fixture modification was not appropriate. Because of the high pivot point the two loading rollers did not remain centered between the two support rollers. As can be seen in Figure 4.4b, the right hand side of the specimen is more flexible than the left hand side, this causes the upper rollers to rotate. This rotation causes the upper rollers to move to the left, thus they are no longer centered between the support rollers.
4.2.2 The new low-pivot fixture

Cross beam to be placed in the test machine grips
Fixture pivot point placed close to the specimen
Rollers from the standard Instron fixture are re-used

Figure 4.5: Diagram showing the design of a new low-pivot upper roller fixture.

Once it had been established that the simple adaptation to the Instron fixture was not adequate, a new low-pivot fixture was designed and fabricated. The intention for the design of the low-pivot fixture was to place the pivot point at the same level as the centre of the two upper rollers. With the pivot point at this height, the nominal contact points for the two upper rollers would remain equidistant from the centre-line of the fixture as the fixture rotates, i.e., load symmetry would be maintained. However, the four ENF test procedure requires that the position of the crack tip is measured during the test, in order to make this measurement the side of the specimen must be visible. Taking this measurement requirement into account it was necessary to raise the level of the pivot point slightly, therefore forcing a small compromise in terms of the design of the fixture. Figure 4.6 shows the low-pivot fixture with a load roller height offset of 5 mm, it can be seen that the asymmetry is less than 0.5% of the 50 mm load roller span. This was considered to be an acceptable compromise and small compared with the likely margin of error in the test results as a whole.

The compromise in pivot height has a small effect on load symmetry when the fixture rotates.

Figure 4.6: Fixture symmetry is slightly compromised due to raised pivot position.
It is worth making two additional points regarding the error due to fixture rotation:

- The above assessment of load eccentricity due to fixture rotation is based on the nominal contact points. The term "nominal contact points" refers to the position of the contact between the upper rollers and a perfectly horizontal specimen. In fact the specimen will not be horizontal, it will deflect due to the applied load, so the above assessment is an approximation.

- A linearisation procedure is carried out during 4ENF data reduction, this is discussed in Section 4.7.2. The actual fixture geometry is used as part of this procedure, therefore the approximation due to the "nominal contact points" is accounted for during linearisation.

The fabrication drawing for the new low-pivot fixture is given in Appendix A.

4.3 4ENF specimen design

Two types of 4ENF specimen are required in order to examine $G_{IIc}$ for the two bondline interfaces that occur in the hybrid joint specimens, one to test $G_{IIc}$ for the GFRP-to-resin interface and another to test $G_{IIc}$ for the GFRP-to-steel interface.

However, it is worth noting that the results of the joint testing that was discussed in Chapter 3 demonstrate that the perforated joints are significantly stronger than the non-perforated joints. For the perforated joints it was also seen that:

1. The GFRP laps had become de-bonded from the steel plate prior to failure of the joint.

2. The perforated joints exhibited a shear failure at the interface between the GFRP laps and the resin in the perforations of the steel plate, i.e. the resin in the perforations remained largely intact in the tested specimens

Thus it can be seen that mode II failure of the GFRP-to-resin interface dominates the strength of the strongest joints that were tested.

4.3.1 GFRP-GFRP 4ENF specimens

The geometry used for the GFRP-GFRP 4ENF specimens is given in Figure 4.7. The materials used are the same as those used for the GFRP adherends of the joint specimens, namely Owens Corning TTX1330 stitched glass fabric and Dow Derekane 510A-40 vinyl-ester resin. The length and width of the specimens are as suggested by Hodgkinson [44] for 3ENF specimens. Hodgkinson suggests a GFRP specimen thickness of approximately 5 mm while the DoD Composite Materials Handbook [65] suggests that the thickness should be in the range 3 - 5 mm. Four layers of TTX1330
tri-axial fabric give a laminate thickness of approximately 4.5 mm while six layers would give approximately 6.75 mm. Four layers of fabric are used for the specimens tested here giving a specimen thickness of approximately 4.5 mm. It should be noted that an even number of fabric layers are required to permit the debond insert being placed at mid-thickness.

The debond insert material used was 0.0005 inch (0.013 mm) thick Teflon™ Fluorinated Ethylene Propylene (FEP) Film supplied by CS Hyde Company in Illinois, USA. FEP is a Polytetrafluoroethylene (PTFE) based film material that provides the low friction and anti-stick properties that are required for the debond insert in 4ENF specimens [44].

![Figure 4.7: The geometry of the GFRP-GFRP 4ENF test specimen.](image)

### 4.3.2 Steel-GFRP 4ENF specimens

The geometry used for the Steel-GFRP 4ENF specimens is given in Figure 4.8. The materials used are the same as those for the GFRP-GFRP specimens with the addition of 32 gauge (2.38 mm thick) grade 304 steel for the steel portion. The grade and thickness of the steel plate were chosen for convenience, this material was used for the joint specimens discussed in Chapter 3, and was readily available at the United States Naval Academy. The same is true for the GFRP resin and reinforcement materials. The thickness of the GFRP portion was chosen so that, within the cracked region of the specimen, the flexural stiffness of GFRP and steel plates were similar. Table 4.1 provides a comparison between the flexural stiffness of the steel plate and two thicknesses of GFRP plate. A GFRP thickness corresponding to four layers of TTX1330 fabric provides a closer match for flexural stiffness than five layers, therefore the Steel-GFRP specimens use 4 layers of fabric giving a GFRP thickness of approximately 4.5 mm.

The specification for steel surface preparation was identical to that specified for the joint specimens, i.e. grit blasting and de-greasing. This preparation is only required for the steel surface that will be bonded to the GFRP.
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<table>
<thead>
<tr>
<th>Material &amp; thickness</th>
<th>Young’s modulus $\times 10^3$N/mm$^2$</th>
<th>$2^{\text{nd}}$ moment of area mm$^4$</th>
<th>Flexural stiffness $\times 10^6$Nmm$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel - 2.38 mm</td>
<td>193</td>
<td>23</td>
<td>4.3</td>
</tr>
<tr>
<td>GFRP - 4 layers</td>
<td>20</td>
<td>152</td>
<td>3.0</td>
</tr>
<tr>
<td>GFRP - 5 layers</td>
<td>20</td>
<td>297</td>
<td>5.9</td>
</tr>
</tbody>
</table>

Table 4.1: Comparison of flexural stiffness between the steel plate and two thicknesses of GFRP plate.

![Diagram](https://example.com/diagram)

**Figure 4.8:** The geometry of the Steel-GFRP 4ENF test specimen.

#### 4.4 Specimen manufacture

In common with the joint specimens the 4ENF specimens were manufactured using the Vacuum Assisted Resin Transfer Moulding process. For each type of specimen, i.e. GFRP-GFRP and Steel-GFRP, a continuous plate was manufactured, from which the individual specimens were cut using a water-jet cutter.

The GFRP-GFRP specimens were successful, unfortunately the design of the Steel-GFRP specimens was not successful. Figure 4.9 shows one of the Steel-GFRP 4ENF specimens, the specimen has not been tested but there is a clear separation of the GFRP and steel plates. To varying degrees this type of separation occurred with all of the Steel-GFRP specimens and was often apparent at both ends of the specimen, not only at the end that had the debond insert.

![Image](https://example.com/image)

**Figure 4.9:** A close up view of an un-tested Steel-GFRP 4ENF specimen showing the separation of the GFRP and steel plates.
It is likely that the primary cause for the separation of the two materials in the Steel-GFRP specimens is a combination of curing shrinkage of the GFRP plate and mis-matched thermal expansion properties of the two materials. Ashland [23] state that the volumetric curing shrinkage of un-reinforced 510A-40 resin is approximately 8% given by and mis-matched thermal expansion properties of the GFRP and steel materials.

Testing of the Steel-GFRP specimens was not successful, therefore data for mode II fracture toughness of the vinyl-ester to steel bond was not determined. This was unfortunate, but it has been seen from both the physical joint testing and the FE models of the joint that the critical bond interface in the perforated joints was the interface between the GFRP laps and the resin in the steel perforations. This is a vinyl-ester to vinyl-ester bond interface so the GFRP-GFRP 4ENF specimens provide the most significant data in relation to this critical interface. Therefore data from the GFRP-GFRP 4ENF specimens will provide data that relates to the highest performing joints tested here.

4.5 4ENF testing

4.5.1 Test setup and instrumentation

An overall schematic diagram of the 4ENF test setup is shown in Figure 4.10, a photograph of a 4ENF specimen in the hydraulic universal test machine is shown in Figure 4.11. The high speed video camera (HSV) and associated high intensity direct current lighting can be seen in front of the test machine in Figure 4.11.

The 4ENF test is fundamentally a displacement controlled four point bending test. Applied load, displacement and crack length must be recorded during the test. As was the case for the joint strength tests, the load and displacement data for the 4ENF tests were recorded using both the Instron Bluehill software and the PC workstation data logger. The workstation data logger, the primary data logger, was required in order to acquire data at a suitable rate for the faster 4ENF loading rates. The Bluehill software was a useful backup to ensure that the correct calibration parameters were being used to convert the primary load and displacement data. Bluehill was used to set the load displacement rate as well as the displacement range to be used for the test.

Table 4.2 shows the load displacement rates that were used for the 4ENF tests. An Instron 5980 Series screw jack Universal Test Machine (UTM), equipped with a 2580 series load cell, was used for rates up to and including 750 mm/min. An Instron 8802 servo-hydraulic UTM, equipped with a Dynacell dynamic load cell, was used for the faster loading rates. The data sampling frequency and video frame rates that were used for each of the test rates are also given in Table 4.2, it can be seen that there is a
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Table 4.2: 4ENF tests were carried out at a range of load displacement rates using either a screw jack or hydraulic test machine.

<table>
<thead>
<tr>
<th>Load rate mm/min</th>
<th>Test machine</th>
<th>Data sampling frequency - Hz</th>
<th>Video frame rate - Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Screw jack</td>
<td>40,000</td>
<td>60</td>
</tr>
<tr>
<td>50</td>
<td>Screw jack</td>
<td>1,200</td>
<td>1,200</td>
</tr>
<tr>
<td>350</td>
<td>Screw jack</td>
<td>8,000</td>
<td>8,000</td>
</tr>
<tr>
<td>750</td>
<td>Screw jack</td>
<td>17,000</td>
<td>17,000</td>
</tr>
<tr>
<td>1500</td>
<td>Hydraulic</td>
<td>41,000</td>
<td>41,000</td>
</tr>
<tr>
<td>3000</td>
<td>Hydraulic</td>
<td>41,000</td>
<td>41,000</td>
</tr>
<tr>
<td>6000</td>
<td>Hydraulic</td>
<td>50,000</td>
<td>50,000</td>
</tr>
</tbody>
</table>

discrepancy between the 1 mm/min tests and the faster tests. This is due to the fact that once the 1 mm/min data had been analysed, it was determined that the data sampling frequency could be reduced while the video frame rate should be increased. Also, the test setup was modified slightly after the 1 mm/min tests, the clock signal output from the HSV was used as the external clock for the high speed analogue data acquisition card. Thus the data points and video frames were synchronised for tests with loading rates of 50 mm/min and above.

4.5.2 Aspects specific to the hydraulic UTM test setup

The tests that were carried out in the hydraulic UTM were too fast for manual triggering of the data logger and HSV. To overcome this, an electronic trigger was set up in such a way that once the moving cross-head of the hydraulic UTM moved down to a pre-set position then the contacts became closed and the camera / logger were triggered. This electronic triggering is indicated in Figure 4.10 and is labelled as trigger contacts.

From the images and diagrams of the four point bending setup that have already been seen in this chapter, e.g. Figure 4.3, it can be deduced that the usual fixture orientation is to place the support rollers (those with the wider spacing) below the specimen and the load rollers (those with the closer spacing) above the specimen. However, as shown in Figure 4.13b, the opposite orientation was used for the tests in the hydraulic UTM. This upside down orientation was used because the Dynacell load transducer is placed below the specimen in the hydraulic UTM. The standard Instron fixture that was used as the support rollers for the 4ENF fixture has a significantly larger mass than the bespoke low-pivot fixture that was used for the load rollers. Placing the smaller mass on the load measurement side of the specimen reduces the effect of inertia on the load measurements when dynamic changes in applied load occur. Dynamic changes in load occur during a 4ENF test because the crack advances in a
Figure 4.10: Schematic diagram of the 4ENF test setup and instrumentation.

In a stepwise fashion, the applied load drops at each stepwise crack increment.

In spite of reducing the effect of inertia as described above, the stepwise changes in applied load still generated some resonant ringing in the load data. The inherent damping in the load-cell / fixture system attenuated this ringing. However, the attenuation rate was not very high, consequently at loading rates above 750 mm/min the ringing amplitude was still significant by the time the next crack propagation event occurred. A low pass filter was applied to the results data in order to combat this problem for the faster loading rates.

Figure 4.12 shows the load data for a 4ENF test that was carried out at 6000 mm/min. The grey load trace shows the resonant ringing in the load data, while the blue load trace shows the stepwise load data that is exposed by applying a low pass filter.
4.5.3 Crack propagation measurement

Crack propagation measurements for the 4ENF tests were carried out using the two dimensional Digital Image Correlation technique (DIC). DIC is an optical method which uses sequential images of a specimen under test to determine the frame by frame full field deformation vector across the surface of the specimen [66]. For the 4ENF tests carried out here, the raw images for DIC analysis were taken from the HSV video that was captured for each specimen. The StrainMaster software [66], from La Vision GmbH, was then used to carry out the DIC analysis. The method for using the HSV video and StrainMaster software to measure 4ENF crack propagation is outlined below:

1. The HSV camera software was used to convert the HSV video file into a .AVI file which StrainMaster can deal with. As part of this conversion:
   - The start and end of each video were truncated to reduce file size and analysis time.
   - The time stamp was added to the video if it was not already present.
   - The following .AVI conversion settings were used: Frame Rate = 5 fps, Video Compressor = Microsoft Video 1, Compression Quality = 65%
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Figure 4.13: Comparing the screw jack and hydraulic test setups.

2. The .AVI file was imported into the StrainMaster DIC analysis software:
   - The number of frames to be analysed was set while importing the .AVI file.
   - Typically 200 frames were used for the DIC analysis of each specimen.

3. The calibration scale of the images was set:
   - StrainMaster needs a scaling factor to size the images correctly, i.e. pixels per mm.
   - A graph paper calibration image was used to determine the image scaling factor.

4. The StrainMaster software was used to carry out the DIC analysis of the imported video frames.

5. The StrainMaster software was used to generate shear-strain plots from the DIC analysis.

6. Individual shear-strain images corresponding to critical points in the load versus displacement data were selected:
   - The critical points in the load versus displacement data are identified, see Section 4.7.1.
   - Each critical point in the data is correlated with an individual shear-strain image.
   - Correlation is assisted by use of the time stamp which is shown on the shear-strain image and is available in the load data.

7. The StrainMaster software was used to measure the crack length:
   - The crack length measurement is required for each of the critical points in the results data.
• The StrainMaster graphical cursor was used to determine the coordinates of three points along the crack.
• The length of the crack was approximated by fitting a circular arc to these three points.

Figure 4.14 provides four images which are shear-strain plots of a 4ENF test, they were generated using the StrainMaster DIC software as described above. The progression of the 4ENF crack is clearly visible and the measured crack lengths are given in the sub-caption for each image.

![DIC images](image-url)

(a) a = 2.5mm  
(b) a = 11.6mm  
(c) a = 17.5mm  
(d) a = 30.6mm

Figure 4.14: Typical DIC images from a 4ENF test showing crack growth during the test (a = crack length).

As identified in step 3 above, a calibration image is required in order to define the image scaling factor that StrainMaster uses. The setup for creating one of these calibration images is shown in Figure 4.15.

The fixture and HSV camera are positioned ready for a 4ENF test to be carried out. An 80 mm x 50 mm piece of graph paper is taped to the side of a 4ENF specimen and this specimen is placed in the fixture. The image size, resolution and focus settings on the camera are all adjusted to suit the 4ENF test that is to follow. A single frame is captured on the camera, this frame incorporates the required camera setup and shows the graph paper. The 10 mm x 10 mm grid on this calibration image provides reference points, which are a known distance apart, to give StrainMaster the required image calibration data. A new calibration image is required any time that the camera position or camera setup is changed.

### 4.5.4 Specimen preparation

There are two steps that are required in order to prepare the manufactured specimens so that they are ready for testing: 1) pre-crack the specimen, 2) apply a speckle pattern that is suitable for DIC analysis.
4.5.4.1 Pre-cracking the specimen

As described above, the specimens are manufactured with a debond insert which provides the initial pre-crack at the mid-thickness of the specimen. However, Hodgkinson [44] suggests that the pre-crack in a 4ENF specimen should be extended from the tip of the debond insert prior to carrying out the test. This extension of the pre-crack is carried out in order to avoid a false initial value of measured $G_{IIc}$, which can occur due to the resin rich region at the tip of the insert.

Hodgkinson [44] suggests that the pre-crack should be extended by applying three point bending to the specimen. This will cause the pre-crack to extend but to become arrested at the point of load application. Thus the amount by which the pre-crack is extended will be determined by the position of the specimen in the three point bending fixture. Figure 4.16 shows a 4ENF specimen that has been pre-cracked by applying three-point-bending. The rotation of the low pivot point 4ENF fixture has been restricted by placing a thick steel plate on one side of the fixture. This causes the fixture to act as an asymmetrical three point bending fixture.

Figure 4.16: Pre-cracking of a 4ENF specimen. The 4ENF fixture is converted into a form of three-point-bending fixture by restricting rotation of the fixture.
4.5.4.2 DIC speckle pattern

In the DIC technique, patterns on the surface of the specimen are sub-divided into subsets which can be identified in each of the images being analysed. Any displacement of these subsets, from one image to the next, can be identified and used to build up a vector of surface displacements which vary with time. From this displacement vector field, the variation of strain with time can be calculated [66].

The DIC technique is most effective when the images being analysed contain a random pattern which contains a wide range of light intensity levels. Such a pattern is readily achieved by spraying the surface of the specimen with white paint and then applying a random speckle pattern onto this white surface. For the 4ENF specimens discussed here, the author used a can of matt white enamel spray paint and an art blow-pen to generate the DIC speckle pattern. Figure 4.17 shows a typical 4ENF specimen with the DIC speckle applied to one edge. The art blow-pen is a felt tip pen mounted inside a two part plastic tube which acts as a form of airbrush. Blowing into the top of the plastic tube causes air to flow through the tube, this air flow draws ink from the pen tip which is then spattered from a hole in the bottom of the tube. By adjusting the rate of air flow and the distance of the blow-pen from the surface of the specimen, the size and density of the speckles can be varied.

Figure 4.17: A 4ENF specimen showing the edge speckle pattern to be used for DIC analysis.

4.6 Load displacement behaviour of the 4ENF test

Before considering the results and data reduction for the 4ENF testing, it will be useful to discuss the non-linear load displacement behaviour that is exhibited by a 4ENF specimen and the various sources of this non-linearity.

4.6.1 4ENF behaviour according to classical beam theory

Classical Beam Theory (CBT) can be used to predict the load versus displacement behaviour of a 4ENF specimen under small deflections. A number of authors have used CBT to derive an expression for the rate of change of specimen compliance with crack length \( \frac{dC}{da} \). This is used in conjunction with the equation for energy release rate (ERR), Equation 4.1, to give an equation for ERR that is based on CBT [45, 48]. The author was not able to find a CBT derivation for 4ENF specimen compliance in
Figure 4.18: The geometry of the 4ENF test and fixture.

the literature, therefore a derivation is provided in Appendix D.

The derivation in Appendix D shows that the CBT equation for 4ENF specimen compliance is:

\[
C = \frac{1}{EI_m} \left( \frac{11S^3}{12} + \frac{3S^2a}{4} \right)
\]  \hspace{1cm} (4.2)

where: 
- \( C \) = specimen compliance = \frac{vertical \ displacement}{vertical \ load}
- \( E \) = flexural elastic modulus
- \( I_m \) = second moment of area of the un-cracked portion of the specimen
- \( S \) = roller spacing (see Figure 4.18)
- \( a \) = crack length \((0 \leq a \leq 2S)\)

Validation of Equation 4.2 is also provided in Appendix D.3. The validation combines the fundamental equation for ERR, Equation 4.1, with Equation 4.2 to derive Equation 4.3 which is the CBT equation for ERR that is given by Martin [45]:

\[
G = \frac{3P^2S^2}{8B^2D}
\]  \hspace{1cm} (4.3)

where: 
- \( P \) = applied load
- \( S \) = outer roller spacing
- \( B \) = breadth of the specimen
- \( D \) = flexural rigidity of the specimen
- \( I_m \) = \frac{EI_m}{B}

\( I_m \) is the second moment of area of the monolithic (un-cracked) portion of the 4ENF specimen.
As would be expected, Equation 4.2 shows that the CBT load versus displacement behaviour of a 4ENF specimen is linear. That is to say that the CBT compliance does not vary for a given specimen, fixture geometry and crack length. Equation 4.2 also shows that the CBT specimen compliance varies linearly with crack length.

**4.6.2 4ENF behaviour in a physical test**

While the CBT equations give a useful baseline insight into the bending behaviour of a 4ENF specimen, the linear behaviour that is predicted by CBT is not apparent during a 4ENF physical test. The non-linearity that occurs during a 4ENF physical test is primarily due to the large specimen displacements and rotations that occur during the test[48].

The non-linearity exhibited by the 4ENF test can be illustrated using fixed crack length Finite Element (FE) models, Figure 4.19 shows plots from two such FE models. The linearised load versus displacement response (dashed lines) is plotted alongside the raw load versus displacement response. The plots are for crack lengths of \( a = 0 \) mm and \( a = 35 \) mm. The FE models have the same specimen and fixture geometry as the 4ENF test setup used here, but the crack length in the FE models does not change as load point displacement is applied. It can be seen that the FE models exhibit a stiffening phase, up to a vertical displacement of between 5 mm and 7 mm for this geometry, and a subsequent softening phase. The FE model that was used to generate the load versus displacement data in Figure 4.19 is discussed in detail in Section 4.8.2.

**4.6.2.1 Stiffening phase**

During the stiffening phase of the 4ENF test, the apparent stiffness of the specimen increases as displacement increases. This increase in stiffness is primarily due to
flexural rotation in the specimen and the resulting shift in specimen-roller contact points. Figure 4.20 shows two video frames captured during a 4ENF test, one frame from the start of the test and another from the end of the test. It can be seen that large displacements and rotations have been imposed on the specimen during the test. White markers have been added to the images, the markers indicate both the rotation of the specimen and the shift in specimen-roller contact point.

![Figure 4.20: Large displacements and rotations occur during a 4ENF test.](a) Start of a 4ENF test. (b) End of a 4ENF test.

Figure 4.20 shows the nominal (un-deflected) geometry of the 4ENF test. Figure 4.21 shows the specimen deflection that would occur later in the test. It can be seen that the rotation of the specimen-roller contact points leads to a reduction in the horizontal outer span distance ($S_E$). The reduction in outer span distance is the sum of $\Delta S_D$ and $\Delta S_E$, as shown in Figure 4.21.

For small to medium displacements of the specimen, the additional specimen bending moment that is generated by the horizontal component of the outer roller reactions ($R_A$ & $R_E$) is small. This is because the magnitude of this horizontal component is smaller than the vertical component and the lever arm for the horizontal component is also small.

During the stiffening phase of the test, the bending moment in the specimen is dominated by the horizontal outer span distances ($S_A$ & $S_E$), i.e. Maximum moment in the specimen = $M_{max} \approx \frac{Load}{2} \times S_E$. Therefore a reduction in $S_E$ results in a reduction in bending moment and a resulting apparent increase in specimen stiffness.

![Figure 4.21: 4ENF span shortening under large deflections.](horizontal displacement of roller contact point)
4.6.2.2 Softening phase

During the softening phase, the apparent stiffness of the specimen reduces as deflection increases. This reduction in stiffness is primarily due to the fact that the horizontal component of the outer roller reactions ($R_A$ & $R_E$) becomes significant.

It can be seen in Figure 4.21 that the rotation of the specimen about the outer roller ($\theta_E$) leads to a reaction at E which has a horizontal component. Horizontal equilibrium is maintained primarily by an equal and opposite horizontal component at A. These two horizontal components, allied with the vertical displacement, generate additional bending moment in the specimen. As the vertical displacement increases into the softening phase then the outer span continues to shorten, as it did in the stiffening phase, but the effect of the horizontal component of the reaction at the outer supports begins to dominate. The increasing influence of the horizontal components of the reactions causes the apparent stiffness to fall.

4.6.3 Linearisation of the load data

The 4ENF data reduction method that is proposed by Hodgkinson [44] incorporates the experimental compliance calibration technique. This technique is based on the assumption that specimen compliance varies linearly with crack length. It can be seen from Equation 4.2 that this assumption is in accordance with CBT.

However, both crack length and specimen displacement vary during a 4ENF test. Under these conditions, it can be seen from the discussion above that the 4ENF test will not deliver a linear response for compliance versus crack length. Therefore the load data measured from the 4ENF tests must be linearised before applying the compliance calibration technique to calculate fracture toughness. The geometric non-linearity of the 4ENF test is also evident in the energy release rate plots provided by Sun and Davidson [48].

4.6.3.1 Generating load correction factor curves

The load linearisation is achieved by applying a load correction factor to the measured loads. The load correction factor is a function of both displacement and crack length, load correction curves for fixed crack lengths of $a = 0$ mm and $a = 35$ mm are shown in Figure 4.22.
As expected, the load correction factor is unity at 0 mm displacement. The curves also reflect the stiffening and softening phases that were discussed in Section 4.6.2, i.e. correction factor reduces to a minimum between 5 mm and 7 mm (depending on the crack length) then rises again during the softening phase.

The curve of load correction factor versus displacement, for a given fixed crack length, was found using fixed crack length FE models. The fixed crack length FE models are described in Section 4.8.2. The method for generating the load correction curve for a given crack length, $CF_a$, is described below:

$$CF_a = \frac{P_{lin}}{P_{FE}}$$

where: $P_{FE}$ = load measured from the fixed crack FE model  
$P_{lin}$ = linearised load  
$= k_a \Delta_{FE}$  
$k_a$ = small displacement stiffness of the fixed crack FE model  
$\Delta_{FE}$ = displacement measured from the fixed crack FE model

$$\therefore CF_a = \frac{k_a \Delta_{FE}}{P_{FE}}$$

$k_a$ has a single value for each fixed crack length. $k_a$ can be found using a linear regression over the initial (small deflection) load versus displacement data from the fixed crack model.
Polynomial regression is now used to find a quadratic curve to fit the discrete $CF_a$ versus $\Delta_{FE}$ data points. This gives a pair of polynomial coefficients for each fixed crack length:

$$CF_a = A_a \Delta_{FE} + B_a \Delta_{FE}^2$$

where: $A_a$ and $B_a$ are the polynomial regression coefficients for crack length $a$.

This method was used to generate six pairs of polynomial coefficients corresponding to the load correction factors for six fixed crack length FE models, with crack lengths of $a = \{0, 7, 14, 21, 28, 35\}$.

4.6.3.2 Using the load correction factor curves

The six quadratic load correction factor curves, which are defined by the six pairs of polynomial coefficients, can be used to form a correction factor surface as illustrated in Figure 4.23. The height of the surface (Y axis) represents the load correction factor and the X and Z axes represent displacement and crack length respectively.

![Figure 4.23: 4ENF load correction factor surface.](image)

Each of the six bold black lines in Figure 4.23 represents one of the fixed crack length load correction curves. The surface is not defined for displacements of less than 4 mm, this portion of the surface is not required due to the fact that none of the tests exhibited crack propagation at displacements of less than 4 mm.
The red lines in Figure 4.23 are plots of load correction factor for a typical 4ENF physical test. The solid red line is plotted in 3D space while the dashed red line is plotted in 2D space using the correction factor versus displacement plane. The red triangle markers represent critical data points that have been extracted from the 4ENF test data. The method for determining the linearised load for a 4ENF test data point is as follows:

1. Extract the values of displacement, \( \Delta_t \), crack length, \( a_t \), and load, \( P_t \), from the test data.

2. Use the six pairs of polynomial coefficients, \( A_a \& B_a \) for \( a = \{0, 7, 14, 21, 28, 35\} \), to calculate six values of load correction factor, \( CF_a = A_a \Delta_t + B_a \Delta_t^2 \).

3. Use polynomial regression to fit a third order polynomial to the six \( CF_a \) versus \( a \) data points, giving \( CF = Da + Ea^2 + Fa^3 \).

4. Use the three polynomial coefficients to calculate the required load correction factor, \( CF_t = Da_t + Ea_t^2 + Fa_t^3 \).

5. Calculate the linearised load, \( P_{lin} = CF_t \times P_t \).

Linearisation of the measured 4ENF load data is incorporated into the test data reduction process which is covered in detail in Section 4.7.2.

4.7 4ENF test results

This section describes a) how the results data was processed and b) the data reduction method used to extract critical mode II fracture toughness results. It concludes with the presentation of the mode II toughness results.

4.7.1 Processing the 4ENF test data

For each of the 4ENF tests that were carried out, the results data comprised the following elements:

- Measured load. Discrete data points with a sampled frequency as shown in Table 4.2.
- Measured displacement. Discrete data points with a sampled frequency as shown in Table 4.2.
- Measured crack lengths. Discrete data points, one for each crack propagation event.
- A time stamp with each type of data to allow the three types to be synchronised.

The tests generated large sets of load and displacement data which were rather cumbersome to manipulate. Also the first 30-40\% of each data set was redundant due to
Figure 4.24: An example of the graphical output from the Python script that was used for 4ENF data processing.
the fact that no crack propagation occurred until displacement exceeded 4 mm. The Python programming language [61] was used to write scripts which facilitated data processing of the 4ENF test results, this processing included:

- Removal of redundant data from the start and end of the data set.
- Application of zeroing offsets for the load and displacement data.
- Application of low pass filters:
  - Filtering was required in order to limit the effects of anti-aliasing when data is decimated [62].
  - Filtering was also required for the specimens tested at rates above 750 mm/min due to resonance in the load measurement string (see Section 4.5.2).
- Decimation of the data. The number of data points in the results data set is reduced by re-sampling the original data at a reduced sample frequency [62].
  - The rate of data acquisition during each test was intentionally high in order to ensure that all of the interesting data was captured. On one hand each test is an unrepeatable event so it is useful to err on the side of caution in terms of data acquisition. On the other hand reducing the number of data points once the data has been captured is a relatively simple task.
  - The original and decimated data can be examined to ensure that all significant information is captured in the decimated data set.
- Conversion of the raw data, measured in volts, to the appropriate engineering units.
- Identification of critical points in the data set and extraction of load, displacement and time data for these points. Critical points are those points just prior to a crack propagation event (see Figure 4.24).

The final output from the Python script is discrete pairs of load - displacement data, one pair corresponding to each critical point in the test. An Excel spreadsheet was used to combine this load - displacement data with the crack length data in order to carry out the final data reduction and calculation of $G_{IIc}$ for each 4ENF specimen.

Listings of the Python scripts that were developed for completion of this thesis are provided in Appendix C.
4.7.2 Data reduction

The steps that are required in order to carry out the data reduction and hence to calculate the mode II fracture toughness, $G_{IIc}$, are as follows:

1. Measure the width of the specimen.

2. For each critical data point:
   - Calculate the load correction factor (see Section 4.6.3) and determine the corrected load, $P_C$.
   - Calculate the specimen compliance, $C = \frac{\Delta}{P_C}$.

3. Carry out a linear regression analysis for the crack length versus compliance data points:
   - This gives $C = \frac{dC}{da}a + C_{a=0}$.
   - The rate of change of compliance with crack length ($\frac{dC}{da}$) is required for the calculation of fracture toughness.

4. Use the ERR equation, 4.1, to calculate the mode II fracture toughness for each critical data point.

This procedure was carried out using an Excel spreadsheet. Data for one of the 4ENF specimens is given in Table 4.3 and Figure 4.25.

<table>
<thead>
<tr>
<th>Load</th>
<th>Displacement</th>
<th>Crack length</th>
<th>Corrected load</th>
<th>Compliance</th>
<th>Toughness</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>mm/mm</td>
<td>mm/mm</td>
<td>N</td>
<td>mm/N</td>
<td>Nmm/mm²</td>
</tr>
<tr>
<td>829</td>
<td>5.7</td>
<td>5</td>
<td>744</td>
<td>0.0077</td>
<td>2.89</td>
</tr>
<tr>
<td>771</td>
<td>6.2</td>
<td>10</td>
<td>689</td>
<td>0.0089</td>
<td>2.48</td>
</tr>
<tr>
<td>736</td>
<td>6.6</td>
<td>15</td>
<td>657</td>
<td>0.0100</td>
<td>2.26</td>
</tr>
<tr>
<td>755</td>
<td>7.5</td>
<td>21</td>
<td>679</td>
<td>0.0111</td>
<td>2.41</td>
</tr>
<tr>
<td>744</td>
<td>8.1</td>
<td>25</td>
<td>674</td>
<td>0.0120</td>
<td>2.37</td>
</tr>
<tr>
<td>794</td>
<td>9.4</td>
<td>30</td>
<td>737</td>
<td>0.0128</td>
<td>2.83</td>
</tr>
<tr>
<td>768</td>
<td>10.3</td>
<td>35</td>
<td>721</td>
<td>0.0142</td>
<td>2.72</td>
</tr>
</tbody>
</table>

Notes:
   i) $1 \text{ Nmm/mm}^2 = 10^3 \text{J/m}^2$
   ii) $\frac{dC}{da} = 0.209 \times 10^{-3} \text{N}^{-1}$

Table 4.3: Test results for 4ENF specimen GG-8 tested at 1mm/minute.
Figure 4.25: Plot of compliance and toughness data for 4ENF specimen GG-8. This specimen was tested at 1 mm/minute.

4.7.3 Results for the GFRP-GFRP specimens

This section provides results from the 4ENF testing in graphical and tabular format. Figures 4.26, 4.27 and 4.28 show the compliance and toughness plots for selected specimens that were tested at 50, 750 and 6000 mm/minute. Figure 4.25 shows the compliance and toughness plot for a specimen tested at 1 mm/minute.

The toughness plots for all four of these selected specimens are shown on the same axes in Figure 4.29. The figure suggests that, for a given specimen, the fracture toughness that relates to the first crack propagation event tends to be higher than subsequent values of toughness. Two toughness values have been chosen to characterise the toughness for each specimen:

1. The initial toughness. The toughness relating to the first propagation event for a given specimen, see Figure 4.25.

2. The average toughness. The average of the toughness values for all crack propagation events for a given specimen.

The primary purpose for carrying out the 4ENF tests was to determine if the mode II fracture toughness of the specimens is influenced by the rate at which load is applied. Figures 4.30 and 4.31 provide plots of fracture toughness against loading rate. For both figures a linear regression analysis is used to determine the best straight line fit through the data, the equation of this line is also shown in each figure. Figure 4.30 shows initial toughness values, Figure 4.31 shows average toughness values.
Figure 4.26: Plot of compliance and toughness for 4ENF specimen GG-10. This specimen was tested at 50 mm/minute.

Figure 4.27: Plot of compliance and toughness for 4ENF specimen GG-18. This specimen was tested at 750 mm/minute.

Figure 4.28: Plot of compliance and toughness for 4ENF specimen GG-33. This specimen was tested at 6000 mm/minute.
CHAPTER 4. MODE II TOUGHNESS TESTING

Figure 4.29: Plot of toughness for 4ENF specimens GG-8, GG-10, GG-18 and GG-33.

<table>
<thead>
<tr>
<th>Loading rate (mm/min)</th>
<th>Toughness (J/m²)</th>
<th>Loading rate (mm/min)</th>
<th>Toughness (J/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.81</td>
<td>1</td>
<td>2.89</td>
</tr>
<tr>
<td>1</td>
<td>3.48</td>
<td>1</td>
<td>3.60</td>
</tr>
<tr>
<td>50</td>
<td>3.33</td>
<td>749</td>
<td>3.50</td>
</tr>
<tr>
<td>50</td>
<td>3.63</td>
<td>747</td>
<td>3.26</td>
</tr>
<tr>
<td>50</td>
<td>3.54</td>
<td>749</td>
<td>4.23</td>
</tr>
<tr>
<td>349</td>
<td>3.03</td>
<td>3020</td>
<td>3.33</td>
</tr>
<tr>
<td>350</td>
<td>3.24</td>
<td>6033</td>
<td>3.26</td>
</tr>
<tr>
<td>352</td>
<td>3.59</td>
<td>6059</td>
<td>3.03</td>
</tr>
</tbody>
</table>

Table 4.4: Fracture toughness versus loading rate results.

Figure 4.30: Plot of initial mode II toughness versus loading rate.
Figure 4.31: Plot of average mode II toughness versus loading rate.

<table>
<thead>
<tr>
<th>Regression results</th>
<th>Increase in $G_{IIC}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slope</td>
<td>Intercept</td>
</tr>
<tr>
<td>-----------------</td>
<td>------------</td>
</tr>
<tr>
<td>Initial toughness</td>
<td>7.94E-5</td>
</tr>
<tr>
<td>Average toughness</td>
<td>3.01E-5</td>
</tr>
</tbody>
</table>

Note: % increase in toughness relates to changing the loading rate from 1 mm/minute to 6000 mm/minute.

Table 4.5: Results from the regression analysis of the initial and average toughness versus loading rate data.

Table 4.4 provides the loading rate, initial toughness and average toughness for each of the specimens, the specimens are shown in ascending order of loading rate. Table 4.5 shows the results of the linear regression analysis for the initial and average toughness data, it also provides the percentage increase in toughness that relates to a change in loading rate from 1 mm/minute to 6000 mm/minute for each regression analysis.

4.8 FE analysis in support of ENF testing

Two distinct types of FE model have been used to support the 4ENF testing that is reported here. The first type was used to simulate the 4ENF physical tests that were discussed in Section 4.5. This 4ENF FE model was primarily used to investigate the importance of inertia and friction with regard to the 4ENF physical testing. The second type of FE model, fixed crack length models, were used to generate the load correction curves that were discussed in Section 4.6.3.
4.8.1 FE model of the 4ENF physical tests

The FE model for the 4ENF physical tests is a 2D plane stress model with a plane stress thickness of 20 mm, it was analysed using the explicit the dynamic explicit procedure within Abaqus 6.12 [67].

The specimen is modelled as two 2 mm thick layers of homogeneous isotropic material, the GFRP material, which are bonded together by a 0.1 mm thick layer of cohesive material, see Figure 4.32. The pre-cracked portion of the specimen is modelled by placing a 0.1 mm thick layer of homogeneous isotropic material between the upper and lower layers of the specimen. The pre-crack material is held in position using a single tie node which is indicated in Figure 4.32. The two interfaces between the pre-crack material and the upper and lower GFRP layers is defined using frictionless sliding contact behaviour. The GFRP and pre-crack layers use CPS4R elements, these are 4-node bilinear reduced integration elements with hour glass control [67]. The cohesive layer uses COH2D4, 4-node cohesive elements.

The upper and lower arms in the FE model of the 4ENF test specimens were modelled as 2D homogeneous regions. This approach does not mimic the laminated nature of the GFRP material, but the FE model was able to adequately replicate both the stiffness and crack propagation behaviour of the physical tests as indicated in Figure 4.35. The approach of using homogeneous 2D FE models of ENF specimens has successfully been employed by other authors such as Sun and Davidson [48].

The two lower rollers are modelled as analytical rigid parts using circular arcs of 5 mm radius. The pivoting portion of the 4ENF fixture is modelled as a single analytical rigid part which incorporates the two 5 mm radius loading rollers and the single central pivot point. Contact between the rollers and the GFRP surfaces are defined using frictionless sliding contact behaviour. The overall geometry of the model matches the test setup, i.e. overall specimen length = 150 mm and S = 25 mm.

The two lower rollers are held rigidly in position, displacement controlled load is applied by moving the central pivot point of the upper rollers downward. This pivot point is free to rotate about axis-3 but is held in position on axis-1. The bottom layer of GFRP is held in position on axis-1 using the centre node on the bottom edge, as indicated in Figure 4.32.

For the 4ENF FE model, it was found that the governing factor for determining the maximum acceptable mesh size was related to contact discretisation noise rather than bending stiffness or bending stress. When the horizontal dimension of the element size is significantly larger than 0.25 mm then the load versus displacement response tends to become stepped. This is due to the discretised surface to surface contacts
where frictionless contact is defined. The stepped load response can be better understood by taking the contact stress between one of the lower rollers and the adjacent GFRP as an example. In this case the load does not transfer smoothly from one node to the next as the GFRP slides across the roller. Instead a stick-slip type of behaviour is apparent as contact is transferred from one node to the next. This load transfer causes a step change in the angle of the reaction at the roller. This step change of angle is the source of the steps in the load versus displacement behaviour of the specimen. Reducing the element size tends to reduce the size of the load steps.

Having established that a mesh size of 0.25 mm x 0.25 mm provided an acceptably low level of discretisation noise, a mesh sensitivity analysis was carried out. This analysis showed that a square mesh with 0.25 mm side length, using bi-linear elements, provides good accuracy in terms of the stiffness of the model. The upper and lower layers of the 4ENF FE model are each 2 mm thick, therefore the 0.25 mm mesh gives 8 elements through the thickness of each layer.

4.8.1.1 FE model material properties

An elastic isotropic material definition is used for both the GFRP and pre-crack filler material in the 4ENF FE model. The material properties for the FE model were determined from physical tension tests on GFRP coupons and are given in Table 4.6. A traction separation definition [67] is used to specify the properties of the cohesive material, see Table 4.7. The traction separation parameters are shown graphically in Figure 4.33 and are defined as follows:
### Chapter 4. Mode II Toughness Testing

<table>
<thead>
<tr>
<th>Flexural modulus</th>
<th>Poisson’s ratio</th>
<th>Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>N/mm²</td>
<td></td>
<td>kg/m³²</td>
</tr>
<tr>
<td>20,000</td>
<td>0.44</td>
<td>2,200</td>
</tr>
</tbody>
</table>

Table 4.6: Properties for the GFRP material used in the 4ENF FE models.

![Graph](image)

Figure 4.33: Representation of the traction separation response of the cohesive material.

- **Initial stiffness.** This parameter defines the elastic response of the cohesive material while the applied stress remains below the initiation criteria.

- **Initiation criteria.** This defines the maximum stress that can be generated in the cohesive material, the stiffness of the cohesive begins to degrade once the initiation criteria has been met.

- **Fracture energy.** As indicated in Figure 4.33, a linear softening behaviour is used for the cohesive material. Therefore the fracture energy dictates the rate at which the stiffness of the cohesive material is degraded if separation continues to grow after the initiation criteria has been met. The total area under the traction separation response curve equates to the fracture energy.

<table>
<thead>
<tr>
<th></th>
<th>Mode I</th>
<th>Mode II</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Initial Stiffness</strong></td>
<td>10,000</td>
<td>4,000</td>
</tr>
<tr>
<td>(N/mm)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Initiation Criteria</strong></td>
<td>10</td>
<td>30</td>
</tr>
<tr>
<td>(N/mm²)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Fracture Energy</strong></td>
<td>0.75</td>
<td>2.80</td>
</tr>
<tr>
<td>($10^3$ J/m²)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.7: Parameters for the cohesive element traction-separation law used in the 4ENF FE model.
4.8.1.2 Results from the 4ENF FE model

Figure 4.34 provides graphical output from the 4ENF FE model at a load point displacement of approximately 10 mm. At the right hand end of the specimen it can be seen that the upper and lower layers are not acting monolithically. The layers are acting independently in terms of flexure due to the mid-thickness crack. The distribution of flexural stress indicates the transition from independent to monolithic flexure that occurs at the crack tip. The location of the crack tip is also apparent from the distribution of cohesive stiffness degradation. For the cohesive stiffness degradation plot, red indicates full degradation (zero stiffness), blue indicates zero degradation.

![Stress distribution plot from the 4ENF FE model showing flexural stress distribution and cohesive stiffness degradation close to the crack tip.](image)

Figure 4.34: Stress distribution plot from the 4ENF FE model showing flexural stress distribution and cohesive stiffness degradation close to the crack tip.

Figure 4.35 shows the load versus displacement results for both the 4ENF FE model and physical test specimen GG-10, which was tested at a loading rate of 50 mm/minute. It can be seen that there is generally good agreement between the FE and physical results, although the FE model does not predict the high initial toughness of the physical test nor the stepwise crack propagation.

4.8.1.3 Significance of inertial effects in the 4ENF tests

Having established that the 4ENF FE model provides a reasonably representation of the behaviour of the physical tests, the FE model was used to investigate the significance of specimen inertia. The fastest loading rate that was used for the 4ENF tests was 6000 mm/minute, this is significantly faster than the loading rates that are typically used for materials testing in the ICL structural engineering laboratory. Testing rates in the region of 1 mm/minute to 10 mm/minute are more usual in a structural engineering context. The higher loading rates generate higher accelerations in the
specimen, there was a concern that the combination of mass, stiffness and acceleration within the specimens may influence their flexural behaviour.

In order to confirm whether specimen inertia was a significant factor, the 4ENF FE model was set up to generate a load displacement rate of 6000 mm/minute and to provide the following energy magnitudes in the results data:

- External work done. This is the work done by the load that is applied to the 4ENF model, it is obtained by using the Abaqus output variable ALLWK [67].
- Damage dissipation energy. This is the energy that is dissipated by damage in the cohesive elements, it is obtained by using the Abaqus variable ALLDMD [67].
- Strain energy. This is obtained by using the Abaqus output variable ALLSE [67].
- Kinetic energy. This is obtained by using the Abaqus output variable ALLKE [67].

The energy plots in Figure 4.36 come from the 4ENF FE model that has a loading rate of 6000 mm/min. They show the relative magnitudes of the specimen energies listed above. It can be seen from Figure 4.36 that the kinetic energy in the specimen is negligibly small when compared with either the total work done by the external load or the energy that is dissipated by cohesive damage. The maximum kinetic energy is approximately 0.008 Joules while the total external work done prior to the start of crack propagation is approximately 1.1 Joules and the total energy that is dissipated in crack growth by the end of the test is approximately 2.6 Joules.
It can be concluded that kinetic energy in the specimen, and therefore inertial effects in the specimen, are not important at the rates of loading used for the 4ENF.

4.8.1.4 Significance of friction effects in the 4ENF tests

It has been shown that friction within a 4ENF specimen can have a significant effect on the perceived mode II fracture toughness for some specimen geometries [48]. The effects of friction on the 4ENF specimens were investigated by modifying the 4ENF FE model in order to generate friction in the pre-cracked region. It was only necessary to incorporate the friction model within the pre-crack region because, once a crack has developed, the surfaces of the cracked region do not come into contact with one another. If the test was continued to give very large load point displacements then the crack would close, however crack closure does not occur for the displacements used in the 4ENF test.
The FE model was modified so that friction is present in two locations i) between the upper edge of the pre-crack filler material and the upper GFRP layer, ii) between the lower edge of the pre-crack filler material and the lower GFRP layer. The Abaqus penalty friction formulation [67] was used as a replacement for the frictionless surface to surface interaction, see Section 4.8.1. A classical Coulomb friction model was used to define the friction behaviour [68], the fixed coefficient of friction was set at $\mu = 0.35$. This value for $\mu$ was established by Davidson et al [69] using a hinged-plate sliding block test on carbon/epoxy plates with Teflon inserts.

The energy plots in Figure 4.37 come from the 4ENF FE model that incorporates friction in the pre-cracked region. It can be seen that the energy that is dissipated by the friction is much smaller than the other energies, but not negligibly small. The energy that has been dissipated in friction by the end of the test equates to approximately 11% of the damage energy. This demonstrates that friction is not negligible for this test geometry. The significance of this, in terms of the primary findings from the 4ENF testing, will be discussed in Section 4.9.2.
4.8.2 Fixed crack length FE model

As discussed in Section 4.6.3.1, six fixed crack length models were used to generate the load correction data. These models had crack lengths of \( a = 0, 7, 14, 21, 28, 35 \) mm. The fixed crack length FE models are very similar to the friction free 4ENF FE models described in Section 4.8.1, but have two differences. The first difference is that the cohesive material for the fixed crack models has an artificially high value for the cohesive initiation criteria, this is to prevent cohesive failure and thus prevent crack propagation. The mode I and mode II initiation stresses are set to 10,000 N/mm\(^2\) and 30,000 N/mm\(^2\) respectively.

Figure 4.38 provides graphical output from the 4ENF fixed crack length model, crack length \( a = 7 \) mm, at a load point displacement of approximately 10 mm. As with the crack propagation FE model shown in Figure 4.34, the transition between the cracked portion and the monolithic portion of the specimen can be seen in the flexural stress distribution.

The second difference for the fixed crack models is the length of the pre-crack region. It can be seen in Figure 4.32 that the length of the pre-crack region in the 4ENF FE model is \( 2 \times S = 50 \) mm. So initially the crack tip is located directly below the right hand loading roller. This is also the case for the fixed crack length model with \( a = 0 \) mm, but for the other five models the pre-crack region is extended and the cohesive region shortened to suit the required crack length.

4.9 Concluding remarks regarding the ENF testing

4.9.1 The drop in rate effect for large crack lengths

The apparent loading rate effect on the mode II toughness of the specimens tested here is over twice as large for the initial crack propagation event compared with the toughness averaged over all crack propagation events for a given specimen. It may
be more meaningful to express this in terms of crack length. The loading rate effect is much more apparent for short crack lengths, \(a < 15 \text{ mm}\), but is less apparent for long crack lengths, \(a > 25 \text{ mm}\). The cause of the apparent drop in rate effect on mode II toughness as crack length increases is not known. However, the high speed videos of the physical 4ENF tests expose two factors which may be important: i) the crack tends to migrate away from the mid-thickness of the specimen for large crack lengths and ii) the crack tends to open under large displacements.

4.9.1.1 Crack migration

The primary cause of the observed crack migration that occurred in the specimens, is probably the fact that the laminates either side of the crack are 45° laminates rather than 0° laminates. By considering the second moment of area of the cracked portion of the 4ENF specimen, we can see how crack migration influences the apparent toughness:

For a crack at mid-thickness:

\[
I_C = I_T + I_B
\]

\[
= \frac{B}{12} \left[ \left( \frac{h}{2} \right)^3 + \left( \frac{h}{2} \right)^3 \right]
\]

where: 
- \(I_C\) = second moment of area of the cracked portion
- \(I_T\) = second moment of area of the top arm
- \(I_B\) = second moment of area of the bottom arm
- \(B\) = breadth of the specimen (\(B = 20 \text{ mm}\))
- \(h\) = height of the monolithic portion of the specimen (\(h = 4 \text{ mm}\))

But for a crack that is not at mid-thickness:

\[
I_C = I_T + I_B
\]

\[
= \frac{B}{12} \left[ \left( 1 + e \right) \left( \frac{h}{2} \right)^3 + \left( 1 - e \right) \left( \frac{h}{2} \right)^3 \right]
\]

where: \(e\) = crack eccentricity, \(-1 \leq e \leq 1\)
- when \(e = -1\) then the crack is at the bottom
- when \(e = 0\) then the crack is at mid thickness
- when \(e = +1\) then the crack is at the top

A plot of \(I_T\), \(I_B\) and \(I_C\) versus crack eccentricity (\(e\)) is shown in Figure 4.39. It can be seen that the minimum flexural stiffness of the cracked portion of the specimen occurs when the crack is at mid-thickness, \(e = 0\). Minimum flexural stiffness equates to
maximum compliance. Therefore, for a given crack length, the specimen compliance reduces as the crack migrates away from mid-thickness.

Figure 4.39: Plot of crack eccentricity versus cracked portion second moment of area.

Referring to the fundamental equation for energy release rate, Equation 4.1 which is restated below, we can see that a reduction in compliance will lead to a reduction in apparent energy release rate.

\[ G_{IIc} = \frac{P_C^2 \frac{dC}{da}}{2B} \]

where: \( P_C \) = load required to cause the crack to propagate

\( \frac{dC}{da} \) = rate of change of specimen compliance with crack length

\( B \) = width of the specimen

It is clear that the observed crack migration will tend to cause a reduction in the apparent toughness, it seems likely that this is a significant factor in the observed drop in toughness as crack length increases. Unfortunately this does not fully explain why a loading rate effect is much less significant at large crack lengths.

### 4.9.1.2 Crack opening

The tendency for the crack to open under large crack lengths may be due to a combination of fracture debris within the crack and interference between the rough crack surfaces above and below the crack itself. The opening of the crack will tend to induce mode I fracture in addition to the intended mode II fracture. This mixed mode fracture will modify the measured apparent toughness. Mode I toughness is lower than that for mode II so crack opening may also be a contributing factor in the observed drop in toughness as crack length increases. Again, unfortunately this does not fully explain why a loading rate effect is much less significant at large crack lengths.

### 4.9.2 Friction effects

It was found that crack face friction is likely to have an important effect on the apparent mode II toughness of the specimens tested here. However, according to the
classical laws of friction [68] the static coefficient of friction, $\mu_s$, is greater than the kinetic coefficient of friction, $\mu_k$, also $\mu_k$ is independent of sliding velocity.

If we consider these two laws in conjunction with the fact that crack face friction increases the apparent mode II toughness, then we can see that crack face friction is not causing the rate effect on mode II toughness.

4.9.3 Broad conclusions

There is significant scatter in the measured fracture toughness throughout the range of loading rates. However, the results suggest that there is a loading rate effect on the mode II fracture toughness of the Derakane 510A-40 vinyl-ester GFRP. This conclusion is supported by observations made during the joint testing that is covered in Chapter 3 of this thesis. The joint testing demonstrated that the steel-GFRP joints failed at a significantly higher load under drop weight loading rates than they did under quasi-static loading rates. The FE models of the steel-GFRP joints suggested that mode II fracture toughness is the dominant parameter in terms of joint strength. The mode II toughness testing covered in this chapter suggests that mode II toughness increases as loading rate increases, therefore it seems reasonable to assume that the enhanced joint strength can largely be attributed to an increase in the mode II toughness of the vinyl-ester resin used to manufacture the joints.

There is a lack of consensus regarding the effects of loading rate on mode II fracture toughness for FRP materials [32] and limited published data concerning rate effects on mode II toughness for vinyl-ester composites in particular.

Compston et al [42] have published results concerning the rate effect on mode II toughness of various vinyl-ester GFRP materials. All but one of the resins that they studied were toughened resins, the un-toughened resin was Derakane 411-45 vinyl-ester. Figure 4.40 reproduces their figure 5a which gives results for $G_{IIc-init}$ of the 411-45 resin. The trend line and the triangular reference point at 6000 mm/minute have been added by the author. With the exception of the data points at 1000 mm/minute their results suggest that increasing the loading rate does increase the mode II toughness. The red triangle has been added at 6000 mm/minute, this point represents a 17 % increase in $G_{IIc-init}$ over the rate range of 1 mm/minute to 6000 mm/minute. The corresponding increase in mode II toughness on Figure 4.30 is 14 % so there appears to be reasonable agreement between the two sets of results.
It is interesting to note that Compston et al report that there was no significant rate effect on mode II toughness, part of the reasoning for this is a change in the experimental method once the loading rate exceeded 1000 mm/minute. They expressed concern that the change in measurement method may have influenced the results.

Figure 4.40: Reproduction of Figure 5a from Compston [42].
Chapter 5

Frequency Filtering Effects in Perforated Steel-GFRP Joints

5.1 Introduction

5.1.1 Motivation

In recent decades there has been significant interest in periodic composite materials which are capable of suppressing and / or directing elastic waves [70]. Periodic composite materials, which are often referred to as phononic crystal (PC) materials, are inhomogeneous elastic materials in which the elastic properties vary in a periodic manner.

The first PC materials were proposed by Kushwaha et al [17], they considered nickel and aluminium and combined these two materials to form a regular lattice of cylinders (material A) within a background (material B) as shown in Figure 5.1a. They considered two different cases, i) nickel cylinders in an aluminium matrix and ii) aluminium cylinders in a nickel matrix. When these two materials are combined to form this regular lattice material and the material is subjected to elastic in plane waves, then the different elastic properties of the two materials interact modifying the wave as it is propagated. The speed of the elastic wave is different in each of the two materials, this leads to refraction, reflection and interference which influence wave propagation.

Following the introduction of PC materials by Kushwaha et al [17], many authors have studied different forms of PC material. These studies have covered a range materials, lattice arrangements and analysis techniques. A paper by Wang and Wang [52] shows that perforated aluminium plates can be used to create a PC material that suppresses wave propagation within certain bands of frequencies. The work on PC materials in general, and in particular the demonstration that perforated metal plates can be used to form PC materials, is what has led to the interest in investigating the frequency
filtering effects that may occur in the perforated joints that are being studied in this thesis.

5.1.2 An opportunity for utilising frequency filtering in joints

There may be an opportunity to take advantage of frequency filtering within perforated metal-composite joints similar to those that were discussed in Chapter 3. For example, this type of joint could be used to mount a fibre reinforced blast resistant bulkhead within the steel hull of a ship. This bulkhead could be designed to primarily resist the blast load via membrane action. In this situation the bulkhead would impart significant axial tension loads to the joint around its perimeter.

It may be possible to employ frequency filtering within such a bulkhead mounting joint. The joint and supporting structure must resist the dynamic blast load, some of the frequencies which constitute this dynamic load could be attenuated by the joint. This attenuation would reduce the load that is placed on the supporting structure. With the filter joint in place, a larger proportion of the blast energy would be dissipated in the bulkhead, thus reducing the energy transferred to the supporting structure. When considering wave propagation and frequency filtering in the joint, it will be useful to consider the bandwidth of frequencies that are likely to be encountered in this blast resisting bulkhead scenario.

The work here is limited to axial joint loads. Therefore it is the frequency content of the axial stress pulse within the bulkhead that is of interest. A crude assessment of the frequency content of this stress pulse can be made by assuming that the shape of the stress pulse, i.e. the profile in terms of amplitude versus time, will be similar to the shape of the blast wave pressure versus time profile. The frequency content of a
blast wave profile can be determined using Fourier decomposition.

In the literature, the shape of a blast wave pressure-time profile is often idealised as an exponential decay [51] [52] [71]. Smith and Hetherington [71] provide charts for calculating the duration of a blast pressure pulse in relation to charge stand off distance and size of the charge. The shortest pressure pulse duration given by Smith and Hetherington for a charge equivalent to 1 kg of TNT is approximately 0.2 milliseconds, this relates to a stand off distance of approximately 0.3 m. Smith and Hetherington show that, in general, a larger stand off distance delivers a longer pressure pulse duration.

In addition to the theoretical pulse durations given by Smith and Hetherington, it is pertinent to consider blast pressures that have been measured in full scale physical tests. Borvik et al [72] provide physical test data from blast tests that were carried out on shipping containers. The tests that were carried out by Borvik et al represent a blast load of 4000 kg of TNT at a stand off distance of 120 m, their measured pressure-time plots show a pressure pulse duration of approximately 80 milliseconds.

Figures 5.2 and 5.3 both show stress pulses, that have an exponential decay, alongside the Fourier decomposition for the pulse. It can be seen from Figure 5.2 that the frequency content of a 0.2 millisecond pulse is small, i.e. $<1\%$ of maximum amplitude, above approximately 400 kHz. It can be seen from Figure 5.3 that the frequency content of an 80 millisecond pulse is small above approximately 1000 Hz. We can therefore infer that, in order for the frequency filtering of the joint to have any mean-

![Exponential stress pulse.](image1)

![Fourier decomposition of the pulse.](image2)

Figure 5.2: Frequency content of a 0.2 ms exponential pulse.
meaningful effect in the proposed bulkhead scenario, the joint needs to deliver significant attenuation of frequencies below say 400 kHz. Any attenuation of frequencies above this will have little effect on stress pulses that are generated by blast pressures.

5.1.3 Wave propagation in periodic structures

An insight into how PC materials are capable of suppressing the propagation of waves that have certain frequencies, can be gained by considering an infinitely thick planar laminated material. The laminated material comprises alternate layers of two different materials. There is a plane wave passing through the laminated material, this wave is propagating in a direction perpendicular to the plane of the laminates. At every interface between two laminates, some of the incoming wave energy is transferred to a transmitted wave while the remaining energy is transferred to a reflected wave. The waves that are reflected from each laminate interface will interfere with one another.

In the particular cases where the interference of these reflected waves is constructive, then all of the incident wave energy becomes reflected and propagation though the laminate is not possible. In the cases where the interference of the reflected waves is destructive, then none of the wave energy is reflected, it is all transmitted. From this example of a laminated material it is clear that there needs to be a close relationship between the physical spacing of the laminates and the wave length of the incident wave in order for interference to occur.
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5.1.4 Strategy for investigating frequency filtering effects

The approach that will be used for studying the filtering effects in perforated metal to composite joints has three strands:

1. Band gap analysis.
2. Finite element analysis.
3. Physical testing.

5.2 Band gap analysis

5.2.1 Introduction

Band gap analysis is an analytical method that uses the dispersion relation of a material to expose any frequency bands for which wave propagation is suppressed. Our interest here is restricted to the propagation of elastic waves, phonons, within a two-dimensional (2D) planar periodic material of the type shown in Figure 5.1a. Figure 5.1a represents an infinite plate of 2D phononic crystal which is periodic in the $x$ and $y$ directions, it is taken as homogeneous in the third direction. In order to aid the discussion regarding how band gap analysis has been used here, it is useful to introduce some important terminology:\footnote{It should be noted that this introduction to terminology has been intentionally limited to suit the narrow scope of interest that is relevant here. Standard texts should be consulted for a broader and more rigorous introduction [73, 74].}

- **Phase velocity ($v_\Phi$).** The velocity at which any fixed phase, for example the peak, of a sinusoidal wave travels through a material. Loosely this can be considered as synonymous with wave speed for a sinusoid.

- **Wavenumber ($k$).** The wavenumber of a given wave can be considered as the spatial frequency of that wave. It can be expressed either as $k = \frac{2\pi}{\lambda}$ or as $k = \frac{1}{\lambda}$, where $\lambda$ is wavelength. The former is used in this thesis.

- **Wave vector ($k$).** A vector defining the magnitude of the $x$ and $y$ components of the wave number of a given wave, $k = (k_x, k_y)$.

- **Dispersion relation.** The dispersion relation of a medium expresses the relationship between phase velocity, frequency and wave vector for that medium. The dispersion relation for a material can be shown graphically by plotting frequency versus wavenumber for the material.

- **Unit cell.** The most fundamental / smallest repeating entity within a periodic material. Wave propagation within a periodic material can be fully described by studying a single unit cell [73]. The periodic materials studied here are limited to those with square unit cells.
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Figure 5.4: Example plot of a dispersion relation showing a band gap.

- Lattice constant. The physical dimension of the unit cell, denoted here as $a$. Only square unit cells are considered here, therefore the lattice constants for the vertical and horizontal directions are equal.

- Irreducible Brillouin zone. Strictly speaking this is defined in the reciprocal lattice space \[74\], but the significance here is that it defines the smallest set of wave vectors that are required in order to fully define wave propagation in a unit cell and therefore to define wave propagation in the corresponding periodic material, see Figure 5.1b.

Phononic band gap analysis of a periodic material involves plotting the dispersion relation for elastic waves in that material and checking for horizontal gaps in the plot. As shown in Figure 5.4, a band gap is seen as an empty horizontal band on the dispersion relation plot. The presence of a band gap indicates that the material does not support wave propagation for frequencies in the band gap. For the initial assessments carried out here, only complete band gaps are considered, i.e. band gaps that cover all wave directions.

5.2.2 Band gap analysis using mass-spring models

The use of mass-spring models for demonstrating the dynamic behaviour of periodic materials has been promoted by Jensen \[51\]. As indicated in Figure 5.5 a mass-spring model comprises point masses and massless springs to represent the periodic material being modelled. The springs, which only have axial stiffness, provide nearest neighbour interaction between the masses. The mass of each point and stiffness of each spring is determined in accordance with the material being represented and the spacing of the masses.
The dynamic behaviour of the mass spring system can be determined by solving the set of multi-degree of freedom equations of motion for the whole system.

### 5.2.3 Dispersion relation for a mass-spring model

The dispersion relation for a mass-spring model can be found using the following steps:

1. Determine the equations of motion (EOM) for a general mass in the model. One equation for horizontal motion and one for vertical motion (the springs only have axial stiffness, therefore the masses only have translational degrees of freedom).

2. Express the displacement and acceleration of the general mass in terms of a harmonic plane wave travelling through the model.

3. Substitute these general plane wave expressions for displacement and acceleration into the EOM.

4. Generate a set of simultaneous plane wave EOM covering all of the masses within the unit cell. Periodic boundary conditions should be applied at the perimeter of the unit cell.

5. Solve the eigenvalue problem for these simultaneous EOM. This is an iterative process, there is a unique eigenvalue problem for each wave vector within the Brillouin zone. Therefore each face of the Brillouin zone must be discretised and the eigenvalues must be found for each of these discrete wave vectors.

6. The dispersion relation is generated by plotting eigen-frequencies versus wave vector for each of the discrete wave vectors.

#### 5.2.3.1 EOM for horizontal displacement of mass \( n \)

In order to define the equation of motion for a general mass in the mass-spring lattice, it is necessary to define a method for calculating the mass number for each of the masses which are nearest neighbours to the current mass. Figure 5.5 helps to explain the method for calculating these neighbour numbers. The mass at the bottom left is denoted as mass 0, the masses are numbered consecutively from left to right and bottom to top, so the top right mass has the largest mass number. As shown in Figure 5.5 the mass numbers for the masses to the left and right of mass \( n \) are simply \( n-1 \) and \( n+1 \) respectively. Masses on adjacent rows can be calculated if we know the total number of columns in the mass-spring lattice, the number of columns is denoted as \( C \).
It is also necessary to identify the springs that connect mass \( n \) to each of its neighbours, springs are identified using two subscripts separated by a comma, \( k_{m,s} \). Where \( m \) is a mass number, \( 0 \leq m \leq N \), and \( s \) is a spring number, \( 0 \leq s \leq 3 \).

Figure 5.6 helps to explain the spring numbering strategy by giving the spring identification for the eight springs that are attached to mass 4. Four springs \( (k_{4,0}, k_{4,1}, k_{4,2} \& k_{4,3}) \) are considered to "belong" to mass 4, while the remaining springs "belong" to the neighbouring masses. The vertical springs are given \( s = 0 \), the remaining springs are numbered consecutively in an anti-clockwise direction.

This mass and spring numbering strategy leads to the following EOM for the hori-
zontal degree of freedom of general mass $n$:

$$m_n \ddot{u}_n = \frac{1}{2} k_{n-C-1,3} (u_{n-C-1} - u_n + v_{n-C-1} - v_n)$$

$$+ \frac{1}{2} k_{n,1} (u_{n-C+1} - u_n - v_{n-C+1} + v_n)$$

$$+ k_{n-1,2} (u_{n-1} - u_n) + k_{n,2} (u_{n+1} - u_n)$$

$$+ \frac{1}{2} k_{n+C-1,1} (u_{n+C-1} - u_n - v_{n+C-1} + v_n)$$

$$+ \frac{1}{2} k_{n,3} (u_{n+C+1} - u_n + v_{n+C+1} - v_n)$$

(5.1)

where: $m_n = \text{mass of mass } n$

$u_n = \text{horizontal displacement of mass } n$

$\ddot{u}_n = \text{horizontal acceleration of mass } n$

$v_n = \text{vertical displacement of mass } n$

$k = \text{spring stiffness}$

A plane wave solution for the EOM is required in order to generate a dispersion plot. For horizontal motion, this can be achieved by expressing motion of mass $n$ as follows:

$$u_n = A_n e^{i(\gamma_x js + \gamma_y ks - \omega t - \theta)}$$

$$\ddot{u}_n = -\omega^2 A_n e^{i(\gamma_x js + \gamma_y ks - \omega t - \theta)}$$

where: $A_n = \text{wave amplitude for mass } n$

$j = \text{column number of mass } n \ (j = 0 \text{ at left hand column})$

$k = \text{row number of mass } n \ (k = 0 \text{ at bottom row})$

$s = \text{physical spacing of the masses}$

$\gamma_x = \text{horizontal component of the wave vector}$

$\gamma_y = \text{vertical component of the wave vector}$

$\omega = \text{wave frequency}$

$t = \text{time}$

$\theta = \text{phase angle}$

It is convenient to decompose these wave solutions into fixed and varying terms. The fixed term can then be combined with the wave amplitude to give a complex wave amplitude:

$$u_n = A_n e^{-i\theta} e^{i(\gamma_x js + \gamma_y ks - \omega t)}$$

$$= S_n e^{i(\gamma_x js + \gamma_y ks - \omega t)}$$

$$\ddot{u}_n = -\omega^2 A_n e^{-i\theta} e^{i(\gamma_x js + \gamma_y ks - \omega t)}$$

$$= -\omega^2 S_n e^{i(\gamma_x js + \gamma_y ks - \omega t)}$$
Similarly for vertical motion:

\[ v_n = T_n e^{i(\gamma x s + \gamma y k s - \omega t)} \]
\[ \ddot{v}_n = -\omega^2 T_n e^{i(\gamma x s + \gamma y k s - \omega t)} \]

where: \( S_n \) is the horizontal component of complex wave amplitude for mass \( n \)
\( T_n \) is the vertical component of complex wave amplitude for mass \( n \)

Substituting these plane wave expressions into equation 5.1 and grouping the common displacement terms together, gives the following wave solution for horizontal motion of mass \( n \) in terms of the motion of its nearest neighbours:

\[
0_x = -S_{n-C-1}e^{-ia(\gamma x + \gamma y)\frac{1}{2}k_{n-C-1}} - T_{n-C-1}e^{-ia(\gamma x + \gamma y)\frac{1}{2}k_{n-C-1,3}} - S_{n-C+1}e^{ia(\gamma x - \gamma y)\frac{1}{2}k_{n-1,1}} + T_{n-C+1}e^{ia(\gamma x - \gamma y)\frac{1}{2}k_{n-1,1}} - S_{n-1}e^{-ia\gamma y_k_{n-1,2}} - S_n(-\frac{1}{2}k_{n-1} - k_{n,2} - \frac{1}{2}k_{n,3} - \frac{1}{2}k_{n-C-1,3} - k_{n-1,2} - \frac{1}{2}k_{n+C-1,1}) - T_n(\frac{1}{2}k_{n-1} - \frac{1}{2}k_{n,3} - \frac{1}{2}k_{n-C-1,3} + \frac{1}{2}k_{n+C-1,1}) - S_{n+1}e^{ia\gamma y}k_{n,2} - S_{n+C-1}e^{ia(-\gamma x + \gamma y)\frac{1}{2}k_{n+C-1,1}} + T_{n+C-1}e^{ia(-\gamma x + \gamma y)\frac{1}{2}k_{n+C-1,1}} - S_{n+C+1}e^{ia(\gamma x + \gamma y)\frac{1}{2}k_{n,3}} - T_{n+C+1}e^{ia(\gamma x + \gamma y)\frac{1}{2}k_{n,3}} - S_n\omega^2 m_n \]

(5.2)

5.2.3.2 EOM for vertical displacement of mass \( n \)

Following a similar process for vertical displacement of mass \( n \), leads to the following wave solution for vertical motion of mass \( n \) in terms of its nearest neighbours:

\[
0_y = -T_{n-C-1}e^{-ia(\gamma x + \gamma y)\frac{1}{2}k_{n-C-1,3}} - S_{n-C-1}e^{-ia(\gamma x + \gamma y)\frac{1}{2}k_{n-C-1,3}} - T_{n-C}e^{-ia\gamma y}k_{n,0} - T_{n-C+1}e^{ia(\gamma x - \gamma y)\frac{1}{2}k_{n,1}} + S_{n-C+1}e^{ia(\gamma x - \gamma y)\frac{1}{2}k_{n,1}} - T_n(-k_{n,0} - \frac{1}{2}k_{n,1} - \frac{1}{2}k_{n,3} - k_{n+C,0} - \frac{1}{2}k_{n+C-1,1} - \frac{1}{2}k_{n-C-1,3}) - S_n(\frac{1}{2}k_{n,1} - \frac{1}{2}k_{n,3} - \frac{1}{2}k_{n-C-1,3} + \frac{1}{2}k_{n+C-1,1}) - T_{n+C-1}e^{ia(-\gamma x + \gamma y)\frac{1}{2}k_{n+C-1,1}} + S_{n+C-1}e^{ia(-\gamma x + \gamma y)\frac{1}{2}k_{n+C-1,1}} - T_{n+C}e^{ia\gamma y}k_{n+C,0} - T_{n+C+1}e^{ia(\gamma x + \gamma y)\frac{1}{2}k_{n,3}} - S_{n+C+1}e^{ia(\gamma x + \gamma y)\frac{1}{2}k_{n,3}} - T_n\omega^2 m_n \]

(5.3)
5.2.3.3 Eigenvalue problem

Equations 5.2 and 5.3 can be combined to give the following eigenvalue problem:

\[ 0 = [K_e - \omega^2 M] U \quad (5.4) \]

where:  
- \( K_e \) = modified stiffness matrix  
- \( M \) = mass matrix  
- \( U \) = matrix of wave amplitudes, incorporating \( S_n \) and \( T_n \) terms

It can be seen from Equations 5.2 and 5.3 that the modified stiffness matrix, \( K_e \), incorporates exponential and wave vector terms as well as spring stiffnesses. Hence the comment in Section 5.2.3 regarding discretisation of the Brillouin zone. A unique modified stiffness matrix must be determined for each of the discrete wave vectors, hence a unique set of eigen frequencies is obtained for each wave vector.

5.2.3.4 Defining the periodic boundary condition

When building the modified stiffness matrix, \( K_e \), the periodic boundary conditions must be respected. Figure 5.6 helps to explain the periodic boundary condition, the masses in Figure 5.6 with a dashed line are dummy masses which assist with visualisation of the periodic boundary condition. The periodic boundary condition defines a cyclic or wrap around boundary which can be exemplified by mass 3 in Figure 5.6. Physically, the nearest neighbour masses for mass 3 are masses 0, 1, 4, 7 and 6. However, when building \( K_e \), masses 2, 5 and 8 must also be incorporated into the equations of motion for mass 3.

5.2.4 Python script for band gap analysis

The Python programming language [61] was used to write a script which would generate the dispersion relation for a PC material. The script is limited to working with two different materials, which are defined by their density and elastic modulus, and is limited to rectangular mass-spring lattice models. It is also limited to PC materials which have a rectangular lattice arrangement. The Python listing for the script is given in Appendix C, the algorithm that is employed for the script is as follows:

- **Initialisation**
  - Define the Irreducible Brillouin Zone (IBZ) relevant to the system being modelled.
  - Determine values for mass and spring stiffness as defined by the material properties and mass spacing.
  - Setup the diagonal mass matrix, \( M \).
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- **Main body**
  
  - Outer loop, this defines each side of the IBZ in turn.
    - Inner loop, this defines each discrete point on the current side of the IBZ.
      - Define the wave vector \((\gamma_x, \gamma_y)\) for the current point on the IBZ.
      - Build the modified stiffness matrix \(K_e\) for the current wave number vector.
      - Calculate the eigen solutions of \(\text{det}|K_e - \omega^2 M| = 0\).
      - Sort the eigen values, \(\omega^2\), into ascending order.
      - Store the values of \((\gamma_x, \gamma_y)\) & \(\omega\) to be plotted later. \((\gamma_x, \gamma_y)\) represents the abscissa and the corresponding values for \(\omega\) represent the ordinates.
  - Create the dispersion plot using the stored values of \((\gamma_x, \gamma_y)\) & \(\omega\).

5.2.4.1 Validation of the band gap script

The band gap script was validated by modelling the type 1 PC material which was defined and analysed by Jensen [51]. Jensen represented the material as a 5 x 5 mass-spring unit cell which incorporates a 3 x 3 inclusion of a stiffer material as shown in the bottom right corner of Figure 5.7. Jensen provided the band structure, dispersion relation, for this 5 x 5 mass-spring unit cell. The author used the Python script, described above, to generate the dispersion relation for the same unit cell. Figure 5.7 shows an overlay of the dispersion relation given by Jensen, black solid lines, and the dispersion relation generated by the author using the Python script, circular markers only. The dispersion relation that was generated by the Python script is a good match with that provided by Jensen. This is taken as validation that the script is functioning correctly.

5.2.5 Plates with circular holes

The band gap script has been used to investigate PC materials formed from steel plates with circular perforations. In each case the perforations are placed on a square grid at 5 mm centre-to-centre on both the x and y axes, this gives a lattice constant \(a = 5mm\). The mass-spring models each use a grid of 21 x 21 masses to represent the 5 mm square unit cell.

The perforation geometries that have been investigated, and the height of any band gap that the geometry provides, are given in Table 5.1. The dispersion plots and mass-spring lattices for three of the investigated geometries are shown in Figures 5.8, 5.9 and 5.10. Neither of the remaining two geometries, \(\frac{d}{a} = 0.24\) and \(\frac{d}{a} = 0.52\), provided a band gap.
In Table 5.1, LU refers to lattice units. A lattice unit is the horizontal or vertical distance that can be attributed to a single mass in the mass-spring model. For the models referred to in Table 5.1, \( LU = \frac{5mm}{20} = 0.25mm \). The ratio of perforation diameter:lattice constant, \( d/a \), is used to characterise each model. It can be seen from these results that for circular perforations on a square grid, \( d/a \) must approach unity in order for a significant band gap to be present in the dispersion relation.

### 5.3 Finite element analysis

#### 5.3.1 Introduction

Band gap analysis provides information regarding theoretical steady state wave propagation in a PC material with infinite in-plane dimensions. Finite element analysis is
Figure 5.8: Dispersion relation for a steel plate with circular holes, $\frac{d}{a} = 0.71$.

Figure 5.9: Dispersion relation for a steel plate with circular holes, $\frac{d}{a} = 0.81$.

Figure 5.10: Dispersion relation for a steel plate with circular holes, $\frac{d}{a} = 0.90$. 
used here to study the wave propagation behaviour of finite plates of a 2D periodic material. The periodic material is in the form of perforated steel plates in which the perforations are placed on a regular grid, either a square grid or a grid formed of equilateral triangles. Analysis is also carried out on non-perforated plates in order to define a baseline response against which the perforated plates can be compared. There are a number of objectives for the FE analyses:

- To confirm that a finite plate, which has perforation geometry corresponding to that shown in Figure 5.10, can attenuate in plane waves in the frequency range 325 kHz to 415 kHz as predicted by the band gap analysis.

- To compare a finite plate that has perforations on a rectangular grid, as in Figure 5.10, with a finite plate that has perforations on a triangular grid. A triangular grid was used for the physical perforated joints tested in Chapter 3.

- To briefly investigate the effect of reducing the size of the finite plate, first in the transverse direction and then in the axial direction

- To determine the effect of resin in the perforations rather than air.

### 5.3.2 The FE model

All of the wave propagation FE models are plane stress models, the 8x8 rectangular grid model is depicted in Figure 5.11. The left hand edge of the plate is held in position using Abaqus Multi-Point Constraints (MPCs), these constrain the three degrees of freedom (DoF) of each node along the left hand edge to match the displacement / rotation of the corresponding DoF of a reference node, labelled as “React” in Figure 5.11. The “React” node is fixed in position for all three DoF.

The right hand edge of the plate also has MPCs which constrain the three DoF of each node along the edge so that they match the displacement / rotation of the reference node labelled ”Load” in Figure 5.11. The ”Load” node is fixed in terms of both rotation and displacement in the vertical direction, a concentrated load is applied to ”Load” in the horizontal direction. The top and bottom edges of the plate are both restrained against movement in the vertical direction.
The perforations have a diameter of 4.5 mm and are set out on a 5 mm grid. For the rectangular grid, as shown in Figure 5.11, this perforation geometry corresponds to that of the dispersion relation shown in Figure 5.10.

5.3.2.1 Transfer function

Frequency filtering in the FE model is checked by plotting the frequency transfer function, \( H(f) \), for the model in question. The frequency transfer function is defined as:

\[
H(f) = \frac{Y(f)}{X(f)}
\]

where:
- \( H(f) \) = transfer function of the plate
- \( X(f) \) = input amplitude at frequency \( f \)
- \( Y(f) \) = output amplitude at frequency \( f \)

Rather than applying load to the model at individual discrete frequencies, the model is subjected to a short duration pulse load which comprises a wide range of frequencies. The applied load and the corresponding reaction force are recorded and subsequently decomposed to the frequency domain using a Discrete Fourier Transform (DFT) [75, 76]. The transfer function, \( H(f) \), is then calculated at each discrete frequency that is generated by the DFT.

5.3.2.2 Applied load

Figure 5.12a shows the short Gaussian pulse shape that is used to define the time domain amplitude of the load that is applied to the FE models. As can be seen in Figure 5.12b, this pulse generates the broad bandwidth of input frequencies, \( X(f) \), that are necessary in order to determine \( H(f) \) for the plate being analysed.

The Gaussian pulse, defined by Equation 5.5, is used here because it delivers a Gaus-
Figure 5.12: Gaussian pulse shape that defines the input load for the FE models.

\[ X(t) = Ne^{-\frac{(t-t_0)^2}{2c^2}} \]  

where:  
- \( X(t) \) = the load pulse  
- \( N \) = Magnitude of the applied load  
- \( t \) = time  
- \( t_0 \) = time at which the pulse reaches a maximum  
  \( = \frac{P_w}{2} \)  
- \( c \) = width parameter  
  \( = \frac{P_w}{10} \)  
- \( P_w \) = required pulse width in seconds

The load pulse is applied to a single MPC reference node which is restrained to move only on the horizontal axis. The MPCs along the right hand edge of the plate constrain each node on the edge to follow the movement of the reference node. Therefore the load pulse is applied to the plate as a plane wave which travels in a direction perpendicular to the right hand edge of the plate.
5.3.3 Results from the wave propagation FE models

Figures 5.13 to 5.17 show the transfer functions for each of the FE models alongside images from the models to show the plate geometry. Each plot also shows the transfer function for a non-perforated plate of the same overall size, these non-perforated transfer functions provide a baseline against which the perforated transfer function can be compared.

Figure 5.13: FEM Transfer function for a rectangular 8x8 grid.

Figure 5.14: FEM Transfer function for a triangular 8x8 grid.
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Figure 5.15: FEM Transfer function for a rectangular 8x8 grid with resin filled holes.

Figure 5.16: FEM Transfer function for a rectangular 8x3 grid.

Figure 5.17: FEM Transfer function for a rectangular 3x8 grid.
5.3.3.1 FE analysis of a low frequency PC material

It is interesting to consider perforated plates that are able to attenuate waves at audio frequencies, i.e. less the 20 kHz. As shown in Figure 5.18 and Figure 5.19 steel plates with 140 mm diameter holes on a 160 mm diameter grid would provide attenuation at 10 kHz.

5.3.4 Observations relating to the wave propagation FE results

It can be seen from Figure 5.13 that the finite perforated plate with holes on a rectangular grid does attenuate wave propagation over a similar bandwidth to that predicted by the dispersion relation in Figure 5.10. The maximum attenuation factor is $\alpha \approx 0.012$. There are additional frequency bands over which the wave is attenuated,
but at lower frequencies these are very narrow and at higher frequencies any attenuation is not likely to be meaningful in terms of improving blast load performance of a joint.

Figure 5.14 shows that changing the perforation geometry to a triangular grid has a significant impact on the propagation behaviour. The band of attenuation in the 300 kHz to 400 kHz band is no longer apparent, the only wide bands of attenuation are now at much higher frequencies.

As shown in Figure 5.15, incorporating resin in the 8x8 perforated plate has a significant effect on the transfer function. The main attenuation band has become narrower, but also deeper with a maximum attenuation factor \( \alpha \approx 0.005 \). The behaviour at lower frequencies has also been modified revealing a broader attenuation band with maximum attenuation, at approximately 200 kHz, of \( \alpha \approx 0.1 \).

The transfer functions shown in Figure 5.16 and Figure 5.17 confirm what would intuitively have been expected. The attenuation factor is greater when the propagated wave must travel through a larger number of perforations. The transfer function for the 8x3 plate is very similar to that for the 8x8 plate, in both cases there are 8 perforations in the direction of wave propagation. However the transfer function for the 3x8 plate, with 3 perforations in the direction of wave propagation, shows a significantly lower attenuation factor \( \alpha \approx 0.04 \).
5.4 Physical tests

5.4.1 Introduction

Physical wave propagation tests have been carried out on joint specimens in order to investigate frequency filtering in the joints. Three perforated and three non-perforated joint specimens, from the set of specimens discussed in Chapter 3, were tested prior to carrying out the main strength test. The test setup and instrumentation was the same as that described for the drop weight tests in Chapter 3, but the impact energy was reduced from 45 Joules down to approximately 8 Joules in order to ensure that the specimens were not damaged during the wave propagation tests. The principal parameters for the test setup are given in Table 5.2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drop weight energy</td>
<td>$\approx$ 8 J</td>
</tr>
<tr>
<td>Drop weight mass</td>
<td>$\approx$ 5.05 kg</td>
</tr>
<tr>
<td>Data acquisition rate</td>
<td>1 MHz</td>
</tr>
</tbody>
</table>

Table 5.2: Principal test parameters for the wave propagation tests.

5.4.2 Working with the results data

As was the case with the joint strength testing, signals from the strain gauges attached to the GFRP and steel portions of the specimens were logged using the high speed data acquisition system. The results files were again analysed using a Python script which makes use of SciPy [76] which is the scientific computing package for Python. SciPy is used to carry out Fourier decomposition of the strain gauge signal by calculating the discrete Fourier transform of the logged data. Figure 5.20 shows GFRP strain versus time along with the corresponding frequency spectrum for one of the wave propagation tests. Two frequency plots of the same data are given in order to show the frequency content over a 30 kHz bandwidth as well as an 8 kHz bandwidth.

It is interesting to compare the pulse width generated in the wave propagation physical test, Figure 5.20, with the pulse width generated by a blast event, Figures 5.2 and 5.3. It can be seen that the 4 ms pulse width from the wave propagation physical test is reasonably representative as it lies between the two blast event pulse widths of 0.2 ms and 80 ms.

The identical process is used for the data from the strain gauge attached to the steel portion of the specimen. The two sets of frequency domain data are each normalised and then compared. By comparing these two frequency spectra it is possible to
determine whether the frequency content of the propagated wave has been modified, filtered, while travelling from the GFRP portion of the specimen, through the joint and into the steel portion of the specimen. The results for perforated specimens are compared with those for non-perforated specimens, again the non-perforated results provide a baseline for comparison.

5.4.3 Test results

It can be seen in Figure 5.20 that the amplitudes of the component frequencies that constitute the load pulse become very small, <0.1 % of maximum amplitude, above 15 kHz. Unfortunately, based on the findings in Section 5.2 and Section 5.3, it seems unlikely that the load pulse being generated in the Dynatup rig is short enough to expose any frequency filtering effects in the joint.

An overlay of the GFRP and steel strain gauge data for the first perforated specimen is given in Figure 5.21. The relative stiffness of the GFRP and steel materials is reflected in the relative amplitudes of the strain pulse in each of the two materials. Overlays of the two frequency spectra are also provided, again using two different bandwidths.

Differences between the two spectra can be seen, most clearly in the narrow bandwidth plot on the right. However, if we compare the narrow bandwidth plots for all three non-perforated specimens, see Figure 5.22, it can be seen that the behaviour of the specimens appears to vary significantly considering that the specimens are nominally the same. A similar comparison can be made for the three perforated wave propagation specimens, the narrow bandwidth spectra for these specimens are given in Figure 5.23.
Figure 5.21: Perforated specimen 1

Figure 5.22: Frequency spectra for the non-perforated wave propagation specimens.

Figure 5.23: Frequency spectra for the perforated wave propagation specimens.
5.4.4 Observations relating to the physical test results

Unfortunately it is difficult to make any firm conclusions based on this physical test data. The results do not appear to contradict any of the results from the band gap analysis or the finite element analysis, but the tests are not capable of generating the high frequency elastic waves that are required to expose any frequency filtering that may be occurring with the joint and perforation geometry being tested.

It seems reasonable to assume that the apparent variation in specimen behaviour, which is observed in Figures 5.22 and 5.23, is the result of random noise in the measurements rather than significant variations in the physical behaviour of each specimen.

5.5 Concluding remarks regarding frequency filtering

It has been shown that finite dimensioned steel plates incorporating a regular grid of circular perforations, can deliver frequency filtering effects for in plane wave propagation. Circular perforations on a rectangular grid have been compared with circular perforations on a triangular grid, the rectangular grid provides filtering at lower frequencies and with a greater degree of attenuation. It has also been shown that perforated steel plates in which the circular perforations are filled with resin will also provide frequency filtering albeit with a modified transfer function compared with empty perforations.
Chapter 6

Conclusions and
Recommendations

6.1 Introduction

This thesis aims to provide an improved understanding of the performance and behaviour of bonded perforated Steel-GFRP joints under dynamic loads. The work is part of a research programme which aims to encourage the increased adoption of FRP composite materials for designs in which their physical properties can deliver improved performance.

It has been seen from the literature that the adoption of FRP composites is hindered by a lack of empirical data, this is particularly true in a naval environment where the FRP components must interface with a steel primary structure. One step towards increased adoption of FRP composites is to find robust and economical techniques for joining metals and FRP. This must then be demonstrated by providing sufficient empirical data to demonstrate the performance and behaviour of joints employing the technique.

This thesis proposes perforated Steel-GFRP joints as an effective joining technique and uses four interconnected approaches to investigate their performance:

1. Testing of physical joint specimens at static and impact loading rates.
2. Finite element modelling of the bonded joint.
3. Mode II fracture toughness testing using a four point bending fixture and a range of loading rates.
4. An analytical, numerical and physical study of the perforated plate in relation to its behaviour as a phononic crystal material.
CHAPTER 6. CONCLUSIONS AND RECOMMENDATIONS

6.2 Summary of the main findings

Taken as a whole the testing programme has successfully demonstrated that perforated Steel-GFRP joints can be fabricated using standard sheet-metal and FRP manufacturing techniques. The test results confirm that the joints perform well at both static and impact loading rates under tensile loading conditions.

The main findings from each strand of the research are summarised in the following sections.

6.2.1 Strength testing of Steel-GFRP joints

The Steel-GFRP joint testing has shown that the perforated joints perform well in a number of areas namely:

- The graduated perforations mitigate the stiffness mismatch of the two adherends as well as providing a degree of mechanical interlock.
- The perforations provide a significant increase in joint strength under both static and dynamic loads.
- The test results suggest that the perforated joints are not susceptible to manufacturing defects related to surface preparation of the steel bond surface.

Finite element analyses of the perforated joints have shown that a surface based cohesive model can successfully be employed to capture the de-bond failure that is observed during the physical joint tests. The cohesive model has been calibrated so that the failure load predicted by the finite element model matches that of the physical tests. The surface based cohesive model was used to demonstrate that mode II fracture toughness is the dominant cohesive parameter for the double lap joints tested here.

In light of the mode II fracture toughness results and the finite element analysis of the joints, it is reasonable to assume that the increase in joint strength at higher rates of loading is largely due to a corresponding increase in mode II fracture toughness of the vinyl-ester resin.

6.2.2 Mode II toughness testing

The 4ENF test programme has revealed a loading rate effect for the mode II toughness of Derakane 510A-40 vinyl-ester GFRP. As the 4ENF loading rate is increased over the range 1 to 6000 mm/minute, the critical energy release rate, \( G_{\text{IIc}} \), is seen to increase by approximately 14%. This result compares favourably with results from
Compston et al [42], they report that $G_{\text{IC,init}}$ of 411-45 vinyl-ester resin increases by approximately 17% over the same 1 to 6000 mm/minute range of loading rates.

Finite element analysis, undertaken in support of the 4ENF testing, has demonstrated that the effects of specimen inertia are negligible for the 4ENF tests carried out here, including those at the highest loading rate of 6000 mm/minute.

### 6.2.3 Frequency filtering effects in perforated Steel-GFRP joints

Wave propagation FE analysis has been used to show that phononic crystal (PC) material can be formed using finite perforated plates with circular holes placed on a triangular grid. In terms of in-plane stress wave propagation, the triangular grid was found to deliver less attenuation and attenuation at a higher frequency than circular holes on a similar sized rectangular grid.

It has been demonstrated that a PC material formed using a 5 mm grid, matching the grid used for the perforated joint specimens, will not deliver frequency filtering effects (attenuation of stress waves) at frequencies that are useful in terms of the stress wave generated by a typical blast load. For a 5 mm rectangular grid, an attenuation band occurs between approximately 200 kHz and 400 kHz. The stress pulse from a typical blast load will only contain significant amplitudes below approximately 10 kHz.

### 6.3 Recommendations for future work

#### 6.3.1 Strength testing of Steel-GFRP joints

If further Steel-GFRP joints are to be designed and fabricated, then it would be beneficial to find an appropriate standardised formal specification for steel surface treatment. Reducing the variability of this aspect of the joint manufacture would be beneficial in terms of repeatability of test results.

The joints tested here have been shown to perform well under tension loads. If the joints are to be developed further then they should be tested under a more realistic loading regime. It is recommended that some example applications for the joints are found, the joint loading regime for this application should then be examined. A test specimen which will subject the joints to a realistic combination of loads can then be designed.
6.3.2 Finite element modelling of the joint

6.3.2.1 Further calibration of the cohesive model

In conjunction with further physical joint tests it would be beneficial to carry out further modelling of the joint. These additional test results and models would be used to check and improve the calibration of the cohesive surface model.

6.3.2.2 Parametric studies

The finite element model of the joint could be used to carry out a parametric study of the joint geometry. For example, the following parameters could be adjusted in the model in order to ascertain the impact that the change has on the behaviour and strength of the joint:

- Steel perforations:
  - Number of perforations.
  - Size range of the perforations.
  - Layout of the perforations.

- Overall joint geometry:
  - GFRP lap length.
  - GFRP lap thickness.
  - Steel plate thickness.

- Manufacturing flaw type and geometry.

The results of these parametric studies could be used to enhance the strength of the joint as well as to generate design guidance for future joint designs.

6.3.3 Mode II toughness testing

With respect to the mode II toughness testing, the recommendations for future work are as follows:

- If further mode II testing is carried out then specimens fabricated with unidirectional reinforcement would be recommended. This should control the problematic crack migration that was observed here.

- Specimens with a debond insert length of approximately 40 mm would be recommended, this would allow the specimens to be placed centrally in the 4ENF fixture.

- It would be beneficial to use smooth platens for both the top and bottom of the VARTM mould:
– This would provide a smooth surface on both the top and bottom of the specimens.
– The smooth surfaces should reduce the effects of friction between the specimen and the fixture rollers.
– Reduced friction may reduce scatter in the results.

• The use of high speed video equipment, and the subsequent crack length measurement using DIC analysis, worked well and would be recommended for future end notched flexure testing.

• The use of the video camera and DIC analysis could be extended to include the measurement of displacement.
  – DIC displacement measurement could be considered as providing a more direct measurement of specimen displacement.
  – The approach may be particularly useful if loading rates are to be increased further.

6.3.4 Frequency filtering effects in perforated Steel-GFRP joints

With respect to frequency filtering effects in the perforated joints, it would seem that the main aim is to deliver attenuation at lower frequencies than are currently being achieved. This requires an approach that does not make use of very large perforations which would not be practical. Mass-in-mass or meta-material arrangements may be an avenue that could deliver lower attenuation frequencies.
References


REFERENCES


REFERENCES


REFERENCES


Appendices
Appendix A

Specimen and test fixture drawings

Specimen drawings

The drawings of the test specimens are given on the following pages, the drawings have been scaled down to suit the current A4 page size. These drawings were sent to the United States Naval Academy where they were used to manufacture the specimens which were then sent to Imperial College London to carry out the testing.

<table>
<thead>
<tr>
<th>First joint specimens</th>
<th>Number</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>DWTS-01</td>
<td>Drop weight test specimens - General arrangement &amp; steel plate bond details</td>
<td></td>
</tr>
<tr>
<td>DWTS-02</td>
<td>Drop weight test specimens - Component details</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Second joint specimens</th>
<th>Number</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>DWTS-03</td>
<td>Second set of joint specimens - General arrangement of specimens without flaws</td>
<td></td>
</tr>
<tr>
<td>DWTS-04</td>
<td>Second set of joint specimens - Component details for specimens without flaws</td>
<td></td>
</tr>
<tr>
<td>DWTS-05</td>
<td>Second set of joint specimens - General arrangement of specimens with flaws</td>
<td></td>
</tr>
<tr>
<td>DWTS-06</td>
<td>Second set of joint specimens - Component details for specimens with flaws</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ENF specimens</th>
<th>Number</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>ENFS-01</td>
<td>End notched flexure specimens</td>
<td></td>
</tr>
</tbody>
</table>

Table A.1: Test specimen drawing numbers and titles.
NOTES:
1. This drawing covers 12 specimens:
   3 No. with perforated steel plates
   3 No. with perforated steel plates and de-bond flaws
   3 No. with solid steel plates
   3 No. with solid steel plates and de-bond flaws
2. The specimens should be labelled as follows:
   With perforated steel plates
   With perforated steel plates and de-bond flaws
   With solid steel plates
   With solid steel plates and de-bond flaws
3. In addition two re-usable clamping plates are required.
4. For the overall arrangement see drawing DWTS-01
5. For perforation and flaw geometry see drawing DWTS-01
6. Items to be confirmed by USNA and agreed with Imperial College:
   a. Surface treatment to steel plates that will be bonded to GRP
   b. Roughening treatment that will be applied to the clamp plates

REV A

SIDE VIEW OF THREADED STEEL ROD

GRADE 304 M12 threaded rod

SIDE VIEW OF GRP PLATE

TOP VIEW OF GRP PLATE

GRP plates are to be molded with steel plates in place during resin infusion

TOP VIEW OF STEEL JOINT PLATE

Note that a variety of plate types are required. The dimensions given here are common to all types but each
has a different GRP bond condition, see drawing DWTS-01.

GENERAL NOTES:

GRP material:
- T3X/1330 reinforcement with surface weight of 1330 grm²
- Dow Derakane 510A40 resin

GRP layup to be:
0° ± 40%
+45° ± 30%

Tapes for de-bonding flaps to be discussed, for example TFE single sided self adhesive tape 5 mils thick (0.127mm)

Steel surface treatment to be agreed prior to manufacture, typically:
- Grip blasting
- Solvent cleaning
- Application of epoxy primer
- Light sanding

ACCELEROMETER

FRINGES

2 No. threaded holes for 10mm long M3 bolts

CLAMPING PLATE
TWO NO. REQUIRED

ROUGHED AREA TO INCREASE CLAMP MOUNTING

DRAWING TITLE

PROJECT DESCRIPTIVE DETAILS

PERFORMANCE & FREQUENCY FILTERING OF HYBRID METAL TO COMPOSITE JOINTS

DRAWING NUMBER

DROP WEIGHT TEST SPECIMENS - Component details

SCALE: NTS

REV.: A

DATE: Nov 2012

Imperial College
London
APPENDIX A SPECIMEN AND TEST FIXTURE DRAWINGS

PRODUCED BY AN AUTODESK EDUCATIONAL PRODUCT

OVERALL EDGE VIEW OF DYNAMIC SPECIMEN
to be used in the Imperial Tension Adapter

A perforated joint is indicated here. Half of the joints are to be non-perforated.

OVERALL FACE VIEW OF DYNAMIC SPECIMEN
to be used in the Imperial Tension Adapter

NOTES:
1. This drawing covers 12 specimens:
   - 3 for Dynamic Loading with perforated steel plates
   - 3 for Static Loading with perforated steel plates
   - 3 for Dynamic Loading with non-perforated steel plates
   - 3 for Static Loading with non-perforated steel plates

2. For component dimensions see drawing DWTS-04.

OVERALL EDGE VIEW OF PSEUDO STATIC SPECIMEN
to be used in a universal test machine

A perforated joint is indicated here. Half of the joints are to be non-perforated.

OVERALL FACE VIEW OF PSEUDO STATIC SPECIMEN
to be used in a universal test machine

NOTES: ONLY SEVEN COLUMNS OF HOLES IN THE SPECIMENS WITHOUT FLAWS

General Notes:

General Notes:

GNE Project Reference:
N00014-12-1-0583
IC Project Number:
P33718_CIST

Dimensions in millimeters unless noted otherwise.

Do not scale from this drawing, only written dimensions are to be used.

Project: PERFORMANCE & FREQUENCY FILTERING OF HYBRID METAL TO COMPOSITE JOINTS
Drawing Title: SECOND SET OF JOINT SPECIMENS - General Arrangement of specimens without flaws
Number: DWTS-03

Scale: NTS
Rev.: -
Date: Dec 2013

Imperial College London

PRODUCED BY AN AUTODESK EDUCATIONAL PRODUCT
APPENDIX A. SPECIMEN AND TEST FIXTURE DRAWINGS

SIDE VIEW OF THREADED STEEL ROD

Slot in end of bolt to be snug fit over plate

Dashed line indicates the extent of the bonded GRP overlap.

MT12 grade 8.8 partially threaded bolt with head removed.
Grade 8.8 has a 0.2% proof stress of >= 540N/mm² (60ksi)

SIDE VIEW OF GRP PLATE

Dashed line indicates the extent of the steel plate embedment.

TOP VIEW OF GRP PLATE

GRP plates are to be molded with steel plates in place during resin infusion

TOP VIEW OF GRP PLATE

GRP plates are to be molded with steel plates in place during resin infusion

TOP VIEW OF STEEL JOINT PLATE

Note that both perforated and non-perforated steel plates are required. The dimensions given here are common to both types.

TOP VIEW OF STEEL JOINT PLATE

Note that both perforated and non-perforated steel plates are required. The dimensions given here are common to both types.

STATIC LOAD

SPECIMEN GEOMETRY

3 specimens with perforated steel plate
3 specimens with solid steel plate

DYNAMIC LOAD

SPECIMEN GEOMETRY

3 specimens with perforated steel plate
3 specimens with solid steel plate

General Notes:

QRN Project Reference: N00014-12-1-4483

IC Project Number: P33718_CIST

Dimensions in millimeters unless noted otherwise. Do not scale from this drawing, only written dimensions are to be used.

Drawing to be read in conjunction with drawing DWT-09

Steel plates to be Grade 304 stainless steel

Vacuum Infused GRP:
  - TTX1330 reinforcement with surface weight of 1330 g/m²
  - Continuous filament glass mat placed at mid thickness of GRP to make up difference in thickness between steel plate and GRP layup
  - Dow Duraskare 510A-80 resin

Steel surface treatment to be identical to that used for the previous drop weight specimens, e.g. grit blasting and degreasing.

Longitudinal tolerance of specimens to be better than 0.5mm in both width and thickness axes.

Proposed GRP Layup:

- 45° TTX1330 - 1
- 45° TTX1330 - 3
- 45° TTX1330 - 4
- 45° TTX1330 - 4
- 90° TTX1330 - 2
General Notes:

- CNR Project Reference: N00014-12-1-0683
- IC Project Number: P33718_CIST
- Dimensions in millimeters unless noted otherwise.
- Do not scale this drawing, only written dimensions are to be used.

**INTENTIONAL FLAW DETAILS**

- Intentional flaws are to be created on both sides of the steel plate for all 12 of the specimens with intentional flaws.
- A 3mm wide strip of each side of each steel plate is to be masked off during the grit blasting to prevent grit ablation.
- In all cases the whole joint is to be degreased after grit blasting.

**NOTES:**

1. This drawing covers 12 specimens, each including intentional flaws:
   - 3 for Dynamic Loading with perforated steel plates
   - 3 for Static Loading with perforated steel plates
   - 3 for Dynamic Loading with non-perforated steel plates
   - 3 for Static Loading with non-perforated steel plates
2. For component dimensions see drawing DWTS-06.

**NOTE:** EIGHT COLUMNS OF HOLES IN THE SPECIMENS 

- Circular holes in the perforated plate are set out using an equilateral triangular grid with 5mm side lengths.
- The radius of each hole in a given column is constant.
- Specimens with flaws have the same perforation geometry as provided for the first set of specimens manufactured in 2013.

**Drawing Details:**

- **Title:** SECOND SET OF JOINT SPECIMENS - General Arrangement of specimens with flaws
- **Number:** DWTS-05
- **Scale:** NTS
- **Date:** Dec 2013

**Construction Details:**

- Perforated plate with flaw
- Non-perforated plate with flaw
- Joint being tested
- GRP plate
- Steel plate
- M12 bolt with head removed 35mm long threaded portion (1.75mm pitch)
- Lock nut
- Strain gauges will be applied to the non-threaded portion
- Filled weld between M12 bolt and steel plate
- A perforated joint is indicated here. Half of the joints are to be non-perforated.
APPENDIX A. SPECIMEN AND TEST FIXTURE DRAWINGS

GRP - Steel Mode II Specimen

15 number specimens required

GRP - GRP Mode II Specimen

15 number specimens required

General Notes:
- General Notes: Dimensions in millimeters unless noted otherwise. Do not scale from this drawing, only written dimensions are to be used.
- Steel plates to be Grade 304 stainless steel. Steel surface treatment to be identical to that used for the previous drop weight specimens, e.g. grit blasting and degreasing.
- Vacuum infused GRP:
  - TTX1330 reinforcement with surface weight of 1330 g/m²
  - Dow Derakane 510A-40 resin

Proposed GRP Layout:

Debond material to be 0.5 mm thick (0.015in) PTFE film (Teflon), NOT single sided Teflon tape.

Test fixture drawings

In addition to the modified ITA bottom yoke, which is shown on drawings DWTS-01 and DWTS-02, it was also necessary to design fixtures for the end notched flexure testing. Drawings for these fixtures are given on the following pages.
New roller bearing plate (side view)

4No. M4 threaded holes each side of plate

4No. M4 threaded holes through plate

New roller bearing plate (top view)

Existing loading roller (new threaded holes required).

New roller bearing plate (to be fabricated).

Existing Instron fixture (shown here for illustration only)

5No. M4 threaded holes each side of plate

Existing loading roller. Main body to be drilled & tapped.
Appendix B

Imperial tension adaptor bolted grip

This appendix provides the design calculations and finite element analysis results that were used to redesign the bottom yolk of the Imperial College Tension Adaptor. The new bottom yolk has two parts which are used to clamp the GRP portion of the test specimens, bolts are used to generate the clamp gripping force.
Design of a reusable clamping plate for the perforated joint drop weight tests

Beam elements representing the clamp bolts. The clamping load is generated by applying a fixed y-axis displacement of upper beam node, thus the relaxation of bolt tension load under specimen tension is modelled. Multi-point constraints connect the bolt beam elements to the clamp plate.

Clamp plate, T6061-T6 Aluminium

Solid GRP clamp region of specimen

Specimen tension load

Figure 1 - Clamp FEM analysis model

Design approach

- The GRP plate will be gripped between two aluminium clamp plates. The clamping force will be generated by tightening four M10 bolts. See drawings DWTS-01 and DWTS-02.
- The clamp plates will subsequently be bolted to the arms of the Imperial College tension adapter.
- A coefficient of friction of 0.3 between the clamp plates and the GRP Plate will be assumed (this value has been used historically at IC).
- In reality, during the physical testing, the clamp force will be estimated by applying a measured tightening torque to the clamp bolts. Advisory note AD184 from the Steel Construction Institute (SCI) suggests the following for calculating bolt tension based on bolt torque:

\[ F = \frac{5T}{d} \pm 30\% \]

where

- \( F = \) the bolt tension force
- \( T = \) the bolt torque
- \( d = \) nominal bolt diameter

- Data for the bearing strength of the GRP is not available but the quoted value for the tension strength of the Derakane 510A-40 resin is 86N/mm². It is assumed that the bearing strength of the GRP manufactured with this resin will be greater than the tension strength therefore the design bearing stress will be limited to 86N/mm²
- Principle material properties adopted:

<table>
<thead>
<tr>
<th>Material</th>
<th>E(_{zz}) (GPa)</th>
<th>E(_{yy}) (GPa)</th>
<th>(\nu_{xz})</th>
<th>(\nu_{xy})</th>
<th>Tension Yield Strength (Mpa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GRP</td>
<td>24</td>
<td>6.73</td>
<td>0.26</td>
<td>0.30</td>
<td></td>
</tr>
<tr>
<td>Aluminium 6061-T6</td>
<td>69</td>
<td>0.33</td>
<td></td>
<td></td>
<td>255</td>
</tr>
</tbody>
</table>

APPENDIX B. IMPERIAL TENSION ADAPTOR BOLTED GRIP
Based on SCI AD184 it is assumed that the clamping force generated by tightening the bolts can be 30% greater than intended. As a result the bolt clamping force used in the FEM model is chosen to give a maximum GRP clamping stress of $0.7 \times 86\text{N/mm}^2 = 60\text{N/mm}^2$.

As a result the initial bolt clamping force used in the model is 14.9kN. As can be seen overleaf the maximum GRP clamping stress occurs just prior to the onset of specimen slip and is approximately 60N/mm$^2$.

Two analysis steps are used in the model:
1. Application of clamp bolt load.
2. Application of displacement controlled specimen tension.

The maximum tension load generated in the specimen is 14.9kN, a plot of Extension versus Tension from the FEM model is shown in Figure 3. The model represents only half of the specimen and one side of the clamp so this represents a specimen tension of 29.8kN.

The maximum failure load quoted in the Melograna et. al. (2002) paper is 1125N/mm width of joint for a 20mm wide joint, i.e. 22.5kN. Due to changes in steel plate grade and thickness it is expected that the failure load of the joints proposed here will be less than 60% of this load.

$$\text{Factor of Safety (FOS) against clamp failure} = \frac{29.8\text{kN}}{(0.6 \times 22.5\text{kN})} = 2.21$$

A FOS of >2 is considered acceptable.
Design of a reusable clamping plate for the perforated joint drop weight tests

**APPENDIX B. IMPERIAL TENSION ADAPTOR BOLTED GRIP**

**DATE:** 30 Nov 2012

**PROJ:** N00014-12-1-0563

**Design of a reusable clamping plate for the perforated joint drop weight tests**
Appendix C

Python source code

Reduce Joint Tension Test Data

A Python script to reduce .CSV results data from a joint tension test and plot the results.
# For drop weight experimental test data
# Read data from a csv file, preferably one which has already been decimated
# This version uses material data files which define a polynomial expression
# for the stress v strain plot of the material stiffness curve.

import csv
import os
import numpy as np
import scipy.integrate
import tkFileDialog, tkMessageBox
import Tkinter as tk
import matplotlib.pyplot as plt
from matplotlib.widgets import Slider, Button, RadioButtons, CheckButtons
from matplotlib.patches import Rectangle
import itertools
from bisect import bisect_left

def stonum(numstr):
    try: return int(numstr)
    except ValueError: return float(numstr)

def getnames(def_file, def_dir):
    root = tk.Tk()
    root.withdraw()
    fileandpath = tkFileDialog.askopenfilename(defaultextension='.txt', initialdir=def_dir, initialfile=def_file, title='SELECT THE RAW DATA CSV FILE')
    with open(fileandpath, 'rb') as f:
        # Read the header row from the CSV and use this row in conjunction with the user defined
        # chanDict{} dictionary to create keys[] an ordered list of the keys in data{}. The order
        # is important as this is the column order used by the CSV file.
        line = f.readline()
        f.seek(0)
        if ',' in line: delim = ','
        elif '\t' in line: delim = '\t'
        else:
            tkMessageBox.showerror("Error", "Invalid file format")
            return 0
        fname = os.path.basename(fileandpath)               # Get filename but with the extension
        specname = os.path.splitext(fname)[0]               # Get the filename without extension
        # Remove any portion of the name after '_' if '_' exists in the name
        try:
            specname = specname[:specname.index('_')]
        except:
            pass
        return specname, fileandpath

def readconfig(specname, fname):
    # Create a specimen configuration dictionary for a given
    # specimen from the specified configuration file.
    # ['Specimen': 'specimen name', 'Ch-0': {'Type': '???', 'Material': '???', etc . . .},
    #                              'Ch-1': {'Type': '???', 'Material': '???', etc . . .},
    #                              'Ch-2': etc . . .
    config_dict = {'Specimen': specname}                # Initialise the configuration dict
    with open(fname, 'rb') as f:
        reader = csv.DictReader(f)
        for line in reader:
            if line['Ident'].upper() == specname.upper(): break
        if line['Ident'].upper() <> specname.upper():
            sys.exit('Specimen name not found in configuration file')
            # config_dict = {'Specimen': specname} # Initialise the configuration dict
            for config in reader:
                # config is a dictionary where the value of ['Ident'] is the current channel name
                # we want to turn this name into a key within dict_config, remove the ['Ident']
                # key from then make the current channel dictionary the value of the key
                if config['Ident'] == 'NewSpecimen':
                    break
                Ident = config.pop('Ident')
                if Ident[0] == 'Ch-':
                    config_dict[Ident] = config
                    for k in config:
                        try:
                            config_dict[Ident][k] = stonum(config_dict[Ident][k])
                            config_dict[Ident][k] = stonum(config_dict[Ident][k])
                        except:
                            pass
    return config_dict

# ...
config[k] = float(config[k])
except Exception:
    pass
else:
    # Therefore it must be specimen dimension data
    # There's a good chance that the specimen dimension data won't be available so . . .
    try:
        # This is a fudge. The dimensional information will actually be in
        # the 'Type' key of the DictReader
        config_dict[Ident] = float(config["Type"])
    except:
        config_dict[Ident] = float('nan')
return config_dict

def makeresdict(specname):
    global config_dict

    # Set up the data common to all results for this specimen
    # First initialise the resdict['common'] dictionary within the resdict dictionary
    resdict = {'Name': specname,
               'Common': {k: None for k in ['MatCurves', 'ZeroStt', 'ZeroEnd']})

    resdict['Common']['TimeRaw'] = [] # Special case as this needs to be a list
    resdict['Common']['TimeDecim'] = [] # Special case as this needs to be a list
    resdict['Common']['Types'] = [] # Special case as this needs to be a list
    resdict['Common']['Flag'] = {'Flag': False, 'Chant': []}

    # Add the specimen geometry data from the config dictionary
    for k in ['SSteel304_Width', 'SSteel304_Thick', 'GRP_Width', 'GRP_Thick', 'M12_Diam']:
        resdict['Common'][k] = config_dict[k]

    # Set up each of the channel dictionaries within {resdict}
    for ck in config_dict.keys():
        if ck[3:] == 'Ch-
        Type = config_dict[ck]['Type']
        if Type == 'Trigger':
            resdict[ck] = {'Type': 'Trigger', 'Raw': []}
        elif Type == 'Strain':
            # Set up the keys that will be single values
            resdict[ck] = {k: None for k in ['Material', 'MatCurve', 'Excitation', 'Gain', 'CF', 'Units', 'Max', 'Min']}
            resdict[ck]['Type'] = 'Strain'
            resdict[ck]['Zero'] = 0
            # Set up the keys that will be lists
            resdict[ck].update({k: [] for k in ['Raw', 'Strain', 'Stress', 'Load']})
        elif Type == 'Accel':
            # Set up the keys that will be single values
            resdict[ck] = {k: None for k in ['CF', 'Units', 'Zero', 'Max', 'Min']}
            resdict[ck]['Type'] = 'Accel'
            # Set up the keys that will be lists
            resdict[ck].update({k: [] for k in ['Raw', 'Accel', 'Veloc', 'Displ']})
        elif Type == 'Load':
            # Set up the keys that will be single values
            resdict[ck] = {k: None for k in ['Material', 'CF', 'Units', 'Max', 'Min']}
            resdict[ck]['Type'] = 'Load'
            resdict[ck]['Zero'] = 0
            # Set up the keys that will be lists
            resdict[ck].update({k: [] for k in ['Raw', 'Load']})
        elif Type == 'Displ':
            # Set up the keys that will be single values
            resdict[ck] = {k: None for k in ['CF', 'Units', 'Max', 'Min']}
            resdict[ck]['Type'] = 'Displ'
            resdict[ck]['Zero'] = 0
            # Set up the keys that will be lists
            resdict[ck].update({k: [] for k in ['Raw', 'Displ']})
        else:
            resdict[ck] = {'Type': 'N/A', 'Raw': []}
    return resdict

# Read the given material files and store them in res_dict['Common']
# POLYNOMIAL REGRESSION material data files are required
def mats2resdict(matfile):
    mat_dict = {'File': '',
                'SSteel304_Static': {'NumTerms': 0, 'Terms': [], 'LwrStrain': 0, 'UprStrain': 0},
                'GRP_Static': {'NumTerms': 0, 'Terms': [], 'LwrStrain': 0, 'UprStrain': 0},
                'M12_Elastic': {'NumTerms': 0, 'Terms': [], 'LwrStrain': 0, 'UprStrain': 0}}
    with open(matfile, 'rb') as f:
        reader = csv.DictReader(f, delimiter='	')
        for line in reader:
            material = line['Ident']
if mat_dict.has_key(material):
    keys = ['NumTerms', 'UprStrain', 'LwrStrain']
    for k in keys:
        mat_dict[material][k] = stonum(line[k])
        for indx in range(1, int(line['NumTerms']) + 1):
            mat_dict[material]['Terms'].append(float(line['Term' + str(indx)]))
else:
    return 'Material mismatch'

res_dict['Common']['MatCurves'] = mat_dict
return 1

# Copy data given in the specimen configuration dictionary into the results dictionary

def config2resdict():
    global res_dict, config_dict
    readmaterials(matfiles)
    for outkey in config_dict.keys():
        if res_dict['Common'].has_key(outkey):
            res_dict['Common'][outkey] = config_dict[outkey]
        for inkey in config_dict[outkey]:
            if res_dict[outkey].has_key(inkey):
                res_dict[outkey][inkey] = config_dict[outkey][inkey]
    res_dict['Common']['MatCurves'] = readmaterials(matfiles)

# Read the raw data from the specified file into res_dict

def readrawdata(data_file):
    global res_dict
    with open(data_file, 'rb') as f:
        reader = csv.DictReader(f, delimiter='\t')
        for line in reader:
            res_dict['Common']['TimeRaw'].append(float(line['time(sec)']))
            for k in ['Ch-0', 'Ch-1', 'Ch-2', 'Ch-3', 'Ch-4', 'Ch-5']:
                res_dict[k]['Raw'].append(float(line[k]))

# If any two channels have the material then the data represents strain gauges on opposite faces
# of a plate of that material. We want to calculate and store the average data for these two
# strain gauges too therefore in this case add another key to res_dict to store the average values.
# Build a list of the materials used by the current configuration
chans = ['Ch-0', 'Ch-1', 'Ch-2', 'Ch-3', 'Ch-4', 'Ch-5']

# Make a list of (channel, material) tuple pairs of channels using the same material as another channel
# First make a list of channels with Strain data
mats = [(x, res_dict[x]['Material']) for x in chans if res_dict[x]['Type'] == 'Strain']

# Break mats into a tuple of channels and corresponding tuple of materials
mats = zip(*mats)

# Make a list of the channels involved in the multi use of the given material
chans = [mats[0][x] for x in range(len(mats[1])) if mats[1].count(mats[1][x]) > 1]

# Then we want to store average values
res_dict['Common']['Avg']['Flag'] = True
res_dict['Common']['Avg']['Chans'] = chans
res_dict['Avg']['Type'] = 'Strain'
res_dict['Avg']['Material'] = 'Average'

# Overseer the calculation and storage of the results

def calcres():
    global res_dict
    for k in ['Ch-0', 'Ch-1', 'Ch-2', 'Ch-3', 'Ch-4', 'Ch-5']:
        chandict = res_dict[k]
        Type = chandict['Type']
        if Type == 'Strain':
            straincalcs(chandict, res_dict['Common'])
            print 'Strain done'
        elif Type == 'Accel':
            accelcalcs(chandict, res_dict['Common'])
            print 'Accel done'
        elif Type == 'Load':
            loadcalcs(chandict, res_dict['Common'])
            print 'Load done'
        elif Type == 'Displ':
            dispcalcs(chandict, res_dict['Common'])
            print 'Displ done'

    # If averaged values are required then calculate them
    # NOTE only deals with the averaging of pairs of channels
    if res_dict['Common']['Avg']['Flag']:
        chanA = res_dict['Common']['Avg']['Chans'][0]
        chanB = res_dict['Common']['Avg']['Chans'][1]
        calcaverage(res_dict[chanA], res_dict[chanB], res_dict['Avg'])
Calculate the strain, stress and load for a given channel
Store the results in res-dict

```python
def straincalcs(chan, common):
    # Calculate the zero offset for this channel
    zstt = common['ZeroStt']
    zend = common['ZeroEnd']
    zero = np.average(chan['Raw'][zstt:zend])
    chan['Zero'] = zero

    # Calculate the strain results for this channel
    GF = chan['GF']
    CF = chan['CF']
    Excite = chan['Excitation']
    Gain = chan['Gain']
    Material = chan['Material']
    factor = CF / GF / Gain / Excite
    chan['Strain'] = [factor*(x-zero) for x in chan['Raw']]

    # Calculate the stress results for this channel
    chan['Stress'] = [straintostress(x, Material) for x in chan['Strain']]

    # Calculate the load results for this channel
    Material = Material[:Material.find('_')]  # Truncate the material name to suit how data is stored in res_dict['Common']
    if Material == 'M12':
        area = (np.pi * common['M12_Diam']**2) / 4
    else:
        width = common[Material + '_Width']
        thick = common[Material + '_Thick']
        area = width * thick
    chan['Load'] = [area * x for x in chan['Stress']]
```

Function that uses a polynomial regression type of material file
Calculate a single stress value given a strain and material curve

```python
def straintostress(strain, Material):
    MatCurve = res_dict['Common']['MatCurves'][Material]
    if strain > MatCurve['UprStrain'] or strain < MatCurve['LwrStrain']:
        sys.exit('Strain out of range for ' + Material)
    stress = 0
    for trm in range(MatCurve['NumTerms']):
        stress += MatCurve['Terms'][trm] * strain**(trm+1)
    return stress
```

Calculate the load data for the load cell channel
Store the results in res-dict

```python
def loadcalcs(chan, common):
    # Calculate the zero offset for this channel
    zstt = common['ZeroStt']
    zend = common['ZeroEnd']
    zero = np.average(chan['Raw'][zstt:zend])
    chan['Zero'] = zero
    fact = chan['CF']
    chan['Load'] = [fact * (x-zero) for x in chan['Raw']]
```

Calculate average

```python
def calcaverage(chanA, chanB, chanAvg):
    valtypes = ['Strain', 'Stress', 'Load']
    for vtype in valtypes:
        vals1 = chanA[vtype]
        vals2 = chanB[vtype]
        chanAvg[vtype] = [(a+b)/2.0 for a,b in zip(vals1, vals2)]
```

Calculate acceleration

```python
def accelcalcs(chan, common):
    # Calculate the zero offset for this channel
    zstt = common['ZeroStt']
    zend = common['ZeroEnd']
    zero = np.average(chan['Raw'][zstt:zend])
    chan['Zero'] = zero
    fact = 1.0 / (chan['CF'] * 1E-7)
    chan['Accel'] = [fact * (x-zero) for x in chan['Raw']]
```

Calculate the velocity and displacement

```python
chan['Accel_decim'] = [chan['Accel'][0]]
chan['Veloc'] = [0.0]
chan['Displ'] = [0.0]
chan['Time'] = [common['TimeRaw'][0]]
decim = len(common['TimeRaw'][0]) / 1000
for indx in range(1, len(common['TimeRaw'][0])):
    xVals = common['TimeRaw'][:indx+decim:decim]
    yVals = chan['Accel'][0:indx*decim:decim]
```
CHAN['Accel_decim'].append(yVals[-1])
CHAN['Veloc'].append(simp(yVals, xVals))

yVals = CHAN['Veloc'][0:indx]
CHAN['Displ'].append(1000*simp(yVals, xVals))
CHAN['Time'].append(xVals[-1])

def dscalcs(chan, common):
    zstt = common['ZeroStt']
    zend = common['ZeroEnd']
    zero = np.average(chan['Raw'][zstt:zend])
    chan['Zero'] = zero
    fact = chan['CF']
    chan['Displ'] = [fact * (x-zero) for x in chan['Raw']]

# Set up the overall figure with subplots and widgets
def setupFig(specname):
    global subs, subs1ax2, interact
    fig, subs = plt.subplots(1, 2, figsize=(15,10))
    mngr = plt.get_current_fig_manager()
    mngr.window.Maximize()
    plt.tight_layout()
    fig.subplots_adjust(left=0.04, right=0.94, bottom=0.2, top=0.94, wspace=0.25, hspace=0.35)
    subs1ax2 = subs[1].twinx()
    subs1ax2.axis('off')
    title = specname + ' - Dynamic Test'
    fig.suptitle(title, fontsize=18, fontweight='bold')

    # Set up the widgets on the figure
    # First find the data required to define the limits etc for the sliders
    zstt = res_dict['Common']['TimeRaw'][0]
    zend = res_dict['Common']['TimeRaw'][-1]

    # Set up the sliders
    axPEnd = plt.axes([0.1, 0.1, 0.5, 0.02])
    axZStt = plt.axes([0.1, 0.05, 0.5, 0.02])
    axZEnd = plt.axes([0.1, 0.01, 0.5, 0.02])
    slPEnd = Slider(axPEnd, 'Horiz Lim', zstt, zend, valinit=zend)
    slPEnd.valtext.set_visible(False)
    slPEnd.on_changed(horizlim)
    slZStt = Slider(axZStt, 'Zero Start', zstt, zend, valinit=zstt)
    slZStt.valtext.set_visible(False)
    slZStt.on_changed(zeroref)
    slZEnd = Slider(axZEnd, 'Zero End', zstt, zend, valinit=zstt + (zend-zstt)/10)
    slZEnd.valtext.set_visible(False)
    slZEnd.on_changed(zeroref)
    print 'sliders set up'

    # Set up the buttons
    axRefr = plt.axes([0.8, 0.01, 0.1, 0.04])
    btRefr = Button(axRefr, 'Get New File')
    btRefr.on_clicked(newfile)
    axSave = plt.axes([0.8, 0.06, 0.1, 0.04])
    btSave = Button(axSave, 'Save CSV Data')
    btSave.on_clicked(savefile)
    axPlot = plt.axes([0.8, 0.11, 0.1, 0.04])
    btPlot = Button(axPlot, 'Single Plot')
    btPlot.on_clicked(singleplot)
    axRestype = plt.axes([0.62, 0.02, 0.06, .08])
    axRestype.set_title('Select Result Type', fontsize=10, fontweight='bold')
    btRestype = RadioButtons(axRestype, ('Strain', 'Stress', 'Load', 'Displ'), active=0)
    btRestype.on_clicked(chngtype)
    interact = {'resType': 'Strain', 'horizlim': zend, 'slZStt': slZStt, 'slZEnd': slZEnd, 'slPEnd': slPEnd, 'btRefr': btRefr, 'btType': btRestype}

    print 'buttons setup'

drawRaw()                                    # Send the raw data to Matplotlib
print 'Raw data plotted'
zeroRef(None)                               # Add the initial "zero box" to MatplotlibLib
calcRes()                                   # Calculate the results and store them in res_dict
print 'Results have been calculated'
APPENDIX C. PYTHON SOURCE CODE

```python
line_dict = drwResult()
interact['btInvrt'] = checkbutt(line_dict)
plt.show()
return fig, subs, interact

# Set up the change sign check boxes
def checkbutt(line_dict):
    if line_dict.has_key('Avg'): line_dict.pop('Avg')
axInvrt = plt.axes([0.72, 0.02, 0.06, .08])
axInvrt.set_title('Select Results To Invert', fontsize=10, fontweight='bold')
btInvrt = CheckButtons(axInvrt, line_dict.keys(), [False]*(len(line_dict)+1))
for label in btInvrt.labels:
    col = line_dict[label.get_text()]
    label.set_color(col)
bttInvrt.on_clicked(chnginvrt)
return btInvrt

# When the upper limit for the x axis is adjusted
def horizlim(label):
    subs[1].cla()
    subs1ax2.cla()
    subs1ax2.axis('off')
    hlim = interact['slPEnd'].val
    interact['horizlim'] = hlim
drwResult()

# When the zero range sliders are adjusted:
def zeroref(label):
    # Determine the current slider values
    zstt = interact['slZStt'].val
    zend = interact['slZEnd'].val
    # Save the slider values into res-dict
    res_dict['Common']['ZeroStt'] = bisect_left(res_dict['Common']['TimeRaw'], zstt)
    res_dict['Common']['ZeroEnd'] = bisect_left(res_dict['Common']['TimeRaw'], zend)
    # Draw a patch to show the zero range
    yscale = subs[0].axis()[2:]
    if interact.has_key('zpatch'):
        interact['zpatch'].remove()
    interact['zpatch'] = subs[0].add_patch(Rectangle((zstt, yscale[0]/2.0), zend-zstt,
                                                     (yscale[1]-yscale[0])/2.0, color='0.8'))
    if label <> None: # I.E. if this call results from the sliders being changed
        subs[1].cla() # Tell Matplotlib to delete the current load plots
        subs1ax2.cla()
        subs1ax2.axis('off')
        plt.draw() # Update the screen plot
calcres() # Re-calculate the results, e.g. using a new zero range definition
drwResult()

# Update the calculations and load plot when the Update button is pressed
def newfile(label):
    global interact
    interact = 'newfile'
    plt.close()

def savefile(label):
    chans = ['Ch-0', 'Ch-1', 'Ch-2', 'Ch-3', 'Ch-4', 'Ch-5', 'Avg']
types = ['Strain', 'Stress', 'Load']
head = [['Common'], ['TimeRaw']]
data = [res_dict['Common']['TimeRaw']]
for typ in types:
    for chan in chans:
        if res_dict.has_key(chan) and res_dict[chan].has_key(typ):
            head[0].append(typ)
            head[1].append(res_dict[chan]['Material'])
            data.append(res_dict[chan][typ])
forchan in chans:
    if res_dict.has_key(chan) and res_dict[chan].has_key('Displ'): # FUDGE
        head[0] += 3*[chan]
        for t in ['Accel', 'Veloc', 'Displ']:
            data.append(res_dict[chan][t])
            head[1] += [t, t, t]
```

break
transdata = zip(*data)
fname = def_dir + '/' + specname + '.res'
print 'Saving  -  ' + fname
with open(fname, 'wb') as f:
    wr = csv.writer(f, quoting=csv.QUOTE_NONE)
    wr.writerow(head[0])
    wr.writerow(head[1])
    for line in transdata:
        wr.writerow(line)

#The type of plot that is requested has changed
def chngtype(label):
    interact['resType'] = label
    subs[1].cla()  # Tell Matplotlib to delete the current load plots
    subs1ax2.axis('off')
    interact['btInvert'].ax.cla()
    plt.draw()
    line_dict = drwResult()
    interact['btInvert'] = checkbutt(line_dict)
    plt.draw()

#The channels to invert have changed
def chnginvrt(label):
    res_dict[label]['CF'] *= -1
    subs[1].cla()
    subs1ax2.cla()
    subs1ax2.axis('off')
calcres()
drwResult()

#Send Raw data to Matplotlib

def drwRaw():
    global res_dict
    markers = itertools.cycle(['D', 's', 'o', 'x', '+', '*', '^', ''])
    subs[0].set_title('Raw Data', fontsize=16)
    xVals = res_dict['Common']['TimeRaw']
    plotdecim = int(len(xVals) / 1000)
    markspace = int(len(xVals) / 15)
    rawcurves = [x for x in res_dict.keys() if type(res_dict[x]) == dict and res_dict[x].get('Raw', None)]
    for curve in rawcurves:
        yVals = res_dict[curve]['Raw']
        label = res_dict[curve]['Type']
        if label == 'Strain':
            label = res_dict[curve]['Material'] + ' Strain'
        subs[0].plot(xVals[::plotdecim], yVals[::plotdecim], label = label, marker=markers.next(), markevery=markspace)
        subs[0].legend(loc='best')
        subs[0].set_xlabel('Time - s', fontsize=14)
        subs[0].set_ylabel('Volts', fontsize=14)
        subs[0].xaxis.grid(True, ls='--', lw=1)

def drwResult():
    global res_dict
    xVals = res_dict['Common']['TimeRaw']
    yVals = res_dict['Ch-3']['Raw']
    fig2 = plt.figure()
    plt.plot(xVals, yVals)
    plt.show()
subs1ax2.axis('on')
xVals = res_dict[curves[0]]['Time']
yAccel = res_dict[curves[0]]['Accel_decim']
yDispl = res_dict[curves[0]]['Displ']
subs[1].plot(xVals[:plotdecim], yAccel[:plotdecim], label = 'Accel', marker=markers.next(), markevery=markspace)
subs1ax2.plot(xVals[:plotdecim], yDispl[:plotdecim], color='r', label = 'Displ', marker=markers.next(), markevery=markspace)

subs[1].legend(loc=1)
subs1ax2.legend(loc=2)
subs[1].set_xlabel('Time - s', fontsize=14, fontweight='bold')
subs[1].set_ylabel('Acceleration - m/s$^2$', fontsize=14, fontweight='bold')
subs1ax2.set_ylabel('Displacement - mm', fontsize=14, fontweight='bold')
subs[1].xaxis.grid(True, ls='--', lw=1)
else:
    for curve in curves:
        yVals = res_dict[curve][restype]
        label = res_dict[curve]['Material']
        line = subs[1].plot(xVals[:plotdecim], yVals[:plotdecim], label = label, marker=markers.next(), markevery=markspace)
        line_dict[curve] = line[0].get_color()
subs[1].legend(loc='best')
subs[1].set_xlabel('Time - s', fontsize=14, fontweight='bold')
subs[1].set_ylabel(restype, fontsize=14, fontweight='bold')
#subs1.ax[0].xaxis.grid(True, ls='--', lw=1)
subs[1].grid(True, ls='--', lw=1)

    #Don't show negative values on the plot
    #Also x axis HACK to solve non auto scale problem
    axlims = list(subs[1].axis())
    axlims[0] = min(xVals)
    axlims[1] = interact['horizlim']
    axlims[2] = 0
    subs[1].axis(axlims)

plt.draw()
return line_dict

def singleplot(label):
    chans = ['Ch-0', 'Ch-1', 'Ch-2', 'Ch-3', 'Ch-4', 'Ch-5']
    if res_dict['Common']['Avg']['Flag']:
        chans.append('Avg')
    restype = interact['resType']
    if restype == 'Load':
        wanted = ['Strain', 'Load']
    else:
        wanted = ['Strain']
    plt.figure(2)
    mngr = plt.get_current_fig_manager()
    mngr.window.Maximize()
    maxtime = interact['horizlim']
    maxindx = next(i for i, v in enumerate(res_dict['Common']['TimeRaw']) if v>=maxtime)
    xVals = res_dict['Common']['TimeRaw'][:maxindx]
    markers = itertools.cycle(['D', 's', 'o', 'x', 'h', '+', '*', '^'])
    markspace = int(len(xVals) / 10)
    curves = [x for x in chans if res_dict[x]['Type'] in wanted]
    line_dict = {}
    for curve in curves:
        yVals = res_dict[curve][restype][:maxindx]
        label = '{:<17}{}{:>9.1f}'.format(res_dict[curve]['Material'], '- Max Val =', max(yVals))
        line = plt.plot(xVals, yVals, label = label, marker=markers.next(), markevery=markspace)
        line_dict[curve] = line[0].get_color()
    plt.legend(prop={'family': 'monospace'}, loc='best')
    ax = plt.gca()
    ax.tick_params(axis='both', which='major', labelsize=14)
    plt.xlabel('Time - s', fontsize=18, fontweight='bold')
    if restype == 'Load': restype = 'Load - N'
    plt.ylabel(restype, fontsize=18, fontweight='bold')
    plt.grid(True, ls='--', lw=1)
    #Don't show negative values on the plot
    axlims = list(plt.axis())
    axlims[1] = interact['horizlim']
    axlims[2] = 0
    plt.axis(axlims)
APPENDIX C. PYTHON SOURCE CODE

```python
title = specname + ' - ' + restype[:restype.find(' ')] + 's'
plt.title(title, fontsize=20, fontweight='bold')
plt.show()

def test(res_dict):
    print
    print '--- TEST ---
    
    for k in res_dict.keys():
        if k == 'Common':
            print 'START = = = = COMMON'
            print
            for ki in res_dict[k].keys():
                print ki
                print res_dict[k][ki]
                print
            print 'END = = = = COMMON'
            print
        else:
            print k
            print res_dict[k]
            print
    

def test2():
    Material = 'M12_Elastic'
    for strain in range(-5, 5):
        print straintostress(strain / 100.0, Material)

def addfiggrids():
    for a in range(1, 10):
        vert = fig.add_axes([a/10.0, 0, 0, 1])
        vert.set_yticks([])
        for s in ['left', 'right']:
            vert.spines[s].set_linestyle('dashed')
            vert.spines[s].set_color('g')
        horiz = fig.add_axes([0, a/10.0, 1, 0])
        horiz.set_xticks([])
        for s in ['top', 'bottom']:
            horiz.spines[s].set_linestyle('dashed')
            horiz.spines[s].set_color('g')

def printdict(d):
    if not isinstance(d, dict):
        print 'That is not a dictionary'
        return
    for outrk in d.keys():
        print outrk
        if isinstance(d[outrk], dict):
            for innrk in d[outrk].keys():
                print '   '+innrk
        else: print '   This key is not a dict'

# Define the configuration and material curve files to be used
confpath = 'C:/Users/RB3810/Documents/Hybrid_Joints/Joints Perforated/Testing/' + '131205-Second Batch from USNA/Python/
config_file = confpath+'DYXX2 Logged Measurement Configuration Data.csv'
config_file = confpath+'STXX2 Logged Measurement Configuration Data.csv'
matfile = confpath+'MaterialRegressionData.csv'
def_file = 'DYPE2-1_decim.txt'
def_dir = 'C:/Users/RB3810/Documents/Hybrid Joints/Joints Perforated/Testing/' + '131205-Second Batch from USNA/DropWeight'
while 1==1:
    specname, data_file = getnames(def_file, def_dir)  # Get the specimen name and data file name
    print 'Specimen name -> ' + specname[specname.rfind('/')+1:]
    print 'Configuration file -> ' + config_file[config_file.rfind('/')+1:]
    print 'Material file -> ' + matfile[matfile.rfind('/')+1:]
    print
    config_dict = readconfig(specname, config_file)  # Read the specimen configuration file into
    # the specimen configuration dictionary
    print 'config dict is done'
    res_dict = makeresdict(specname)  # Make the empty res_dict
    print 'res dict is done'
    mats = mats2resdict(matfile)  
    print 'mats res dict is done'
```

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if mats <> 1:                       #Read the material data files into res_dict
    print 'Error - ' + mats
    sys.exit('Error with Material data')

config2resdict()                    #Transfer data from the specimen configuration dict into res_dict
print 'config transferred to resdict'
readrawdata(data_file)              #Read the raw results data into res_dict
filterrawdata()                     #raw data has been read'

fig, subs, interact = setupFig(specname)    #Set up the figure

if type(interact) == type(''):      #Interact has changed to a string so the user asked for a new file
    res_dict.clear()
    config_dict.clear()
    #fig.clear()
    plt.close('all')
else:                               #Interact is still a dict so the user has closed the window
    break

plt.close('all')
APPENDIX C. PYTHON SOURCE CODE

Generate 4ENF Critical Points Results

A Python script to work with .CSV results that have been created during a 4ENF test.

```python
import matplotlib
matplotlib.use('TkAgg')

from matplotlib.backends.backend_tkagg import FigureCanvasTkAgg, NavigationToolbar2TkAgg
import matplotlib.pyplot as plt
import Tkinter as Tk
import tkFileDialog, tkMessageBox
import sys
import os
import csv
from scipy.signal import butter, filtfilt
import itertools
from bisect import bisect_left
import numpy as np

def destroy():
    root.destroy()

def findvals(targLoad, targDisp):
    sttindx = data_dict['trigindx']
    width = 5  # We're going to treat values within +/-3 x Resolution of the targLoad
    # as interesting, then search those values for the closest to the target displacement
    if data_dict['Filtered']['Flag']:
        data = data_dict['Filtered']
    else:
        data = data_dict['Data']

    Loads = data['Load'][sttindx:]
    Disps = data['Disp'][sttindx:]
    LRange = max(Loads) - min(Loads)
    DRange = max(Disps) - min(Disps)
    small = DRange / 10000

    indxMax = Disps.index(max(Disps))  # The data is likely to include the unloading of the specimen but
    # we want to ignore that portion of the data

    diffs = list(set([abs(Disps[indx] - Disps[indx-1]) for indx in range(1,50)]))
    diffs.sort()
    if diffs[0] < small: Resolution = diffs[1]
    else: Resolution = diffs[0]

    indxLo = bisect_left(Disps, targDisp - width*Resolution, 0, indxMax)
    indxHi = bisect_left(Disps, targDisp + width*Resolution, 0, indxMax)
    dist = max(DRange, LRange)
    for indx in range(indxLo, indxHi):
        if ((Disps[indx] - targDisp)/DRange)**2 + ((Loads[indx] - targLoad)/LRange)**2 < dist:
            minindx = indx

    return sttindx + minindx

def getnames(def_file, def_dir):
    global root
    fileandpath = tkFileDialog.askopenfilename(defaultextension='.txt', initialdir=def_dir,
                                              initialfile=def_file, title='SELECT THE RAW DATA CSV FILE')

    try:
        with open(fileandpath, 'rb') as f:
            line = f.readline()  # Read the header row from the CSV and use this row in conjunction with the user defined
            if not(' ' in line or ',' in line or '\' in line):
                tkMessageBox.showerror("Error", "Invalid file format")
                root.destroy()
                sys.exit('Invalid file format')
            else:
                keys = line.split()  # Create a list of keys from the header row
                data.keys()  # Create a dictionary with keys as keys and values as empty lists

    except Exception:
        tkMessageBox.showerror("Error", "No file was selected!")
        root.destroy()
        sys.exit('No file was selected')

    fname = os.path.basename(fileandpath)  # Get filename but with the extension
    specname = os.path.splitext(fname)[0]  # Get the filename without extension

    if '_' in specname:
        specname = specname[:specname.index('_')]

    return specname.upper(), fileandpath
```
# Read the raw data from the specified file into res_dict

```python
def readconfigdata(config_file):
    with open(config_file, 'rb') as f:
        try:
            # Is it a CSV file?
            dialect = csv.Sniffer().sniff(f.read(10))
            line = f.readline()
            if ',' in line:
                print config_file + ' is a Comma delimited file'
                delim = ','
            elif '\t' in line:
                print config_file + ' is a Tab delimited file'
                delim = '\t'
            elif ' ' in line:
                print config_file + ' is a Space delimited file'
                delim = ' ' 
            else:
                tkMessageBox.showerror("Error", 'The file is not delimited by either commas, tabs or spaces')
                sys.exit('File delimiter error')
            except csv.Error:
                tkMessageBox.showerror("Error", 'That is not a valid .CSV file')
                sys.exit('File not CSV error')
            f.seek(0)
            config_dict = {}
            reader = csv.DictReader(f, delimiter=delim)
            for line in reader:
                curName = line['FileName']
                config_dict[curName] = {} 
                if '' in line.keys(): line.pop('')
                for k in line:
                    if k == 'FileName': continue
                    try:
                        config_dict[curName][k] = float(line[k])
                    except ValueError:
                        config_dict[curName][k] = line[k]
                config_dict[curName]['LoadFact'] = config_dict[curName]['LoadFS_Newtons'] / config_dict[curName]['LoadFS_Volts']
                config_dict[curName]['DisplFact'] = config_dict[curName]['DisplFS_mm'] / config_dict[curName]['DisplFS_Volts']
            return config_dict
```

def displayconfig(config_dict, specname):
    data = [['DisplChan', 'DisplFS_Volts', 'DisplFS_mm', 'DisplFact'], ['LoadChan', 'LoadFS_Volts', 'LoadFS_Newtons', 'LoadFact'], ['TrigChan', 'TrigFS_Volts', 'ClockPeriod'], ['L - mm', 'a - mm', 'SL - mm']]
    config_info = config_dict[specname]
    print 'Configuration for specimen -> ' + specname
    for chan in data:
        print
        for info in chan:
            print '   {:16s}'.format(info) + str(config_info[info])
        print

# Read the raw data from the specified file into res_dict

```python
def readrawdata(data_file):
    global data_dict, specname
    with open(data_file, 'rb') as f:
        try:
            # Is it a CSV file?
            dialect = csv.Sniffer().sniff(f.read(10))
            line = f.readline()
            if ',' in line:
                print 'Comma delimited data file'
                delim = ','
            elif '\t' in line:
                print 'Tab delimited data file'
                delim = '\t'
            elif ' ' in line:
                print 'Space delimited data file'
                delim = ' ' 
            else:
                tkMessageBox.showerror("Error", 'The file is not delimited by either commas, tabs or spaces')
                sys.exit('File delimiter error')
            except csv.Error:
                tkMessageBox.showerror("Error", 'That is not a valid .CSV file')
                sys.exit('File not CSV error')
            f.seek(0)
```
reader = csv.DictReader(f, delimiter=delim)
keys = reader.fieldnames
data_dict['Data'] = {k:[] for k in keys if k!='' for line in reader: if '' in line.keys(): line.pop('')
        for k in line.keys(): data_dict['Data'][k].append(float(line[k]))
data_dict['DispFact'] = config_dict[specname]['DispFact']
data_dict['LoadFact'] = config_dict[specname]['LoadFact']
data_dict['CutOff'] = config_dict[specname]['CutOff']
data_dict['Data']['Disp'] = [data_dict['DispFact']*val for val in data_dict['Data']['Ch-1']]
data_dict['Data']['Load'] = [data_dict['LoadFact']*val for val in data_dict['Data']['Ch-4']]
Nyquist = 1.0 / (2*(data_dict['Data']['time(sec)'][1] - data_dict['Data']['time(sec)'][0]))
data_dict['Decimated'] = {k:[] for k in data_dict['Data'].keys() if not ('Ch' in k)}
data_dict['Filtered']['Flag'] = False:
data_dict['Filtered']['Load'] = {}
data_dict['Filtered']['Disp'] = {}
data_dict['CurveTypes'] = () for key in ['Trig', 'Disp', 'Load', 'Load2']:
configkey = key + 'Chan'
data_dict['CurveTypes'][config_dict[specname][configkey]] = key
for key in data_dict['ToPlot']:
        if 'Ch-2' in data_dict['ToPlot']:
                data_dict['ToPlot'].remove('Ch-2')
colours = itertools.cycle(['k', 'r', 'm', 'g', 'c', 'y'])
data_dict['LineCols'] = {k:colours.next() for k in data_dict['ToPlot']}
def findtrigger():
        trigvals = np.array(data_dict['Data']['Ch-0'])
        threshold = (trigvals.max() + trigvals.min()) / 2.0
        indx = np.argmax(trigvals<threshold)
        print 'Length of trigger data = {0:d}, trigger threshold value = {1:.2f}'.format(len(trigvals), threshold)
def syncclock(specname):
        zeroindx = data_dict['Data']['Index'][data_dict['trigindx']]
        data_dict['Data']['time(sec)'] = [(period * (i-zeroindx)) for i in data_dict['Data']['Index']]
global axes, markers

sttindx = data_dict['trigindx']
ax = event.inaxes
tbar = axes['fig'].canvas.toolbar
if event.button==1 and ax == axes['axLvD'] and tbar.mode == '':
    if markers:
        nxt = max(markers.keys()) + 1
    else:
        nxt = 1
    txtoset = (0.005*(ax.axis()[1] - ax.axis()[0]), 0.005*(ax.axis()[3] - ax.axis()[2]))
    posit = [event.xdata, event.ydata]
    indx = findvals(posit[1], posit[0])
    print '-' * 100
    print 'cursor location: Displ={:6.3f}, Load={:5.1f}'.format(posit[0], posit[1])
    print 'Found values: Displ={:6.3f}, Load={:6.1f}'.format(data_dict['Data']['Displ'][indx], data_dict['Data']['Load'][indx])
    if data_dict['Filtered']['Flag']:
        data = data_dict['Filtered']
    else:
        data = data_dict['Data']
    posit[0] = data['Displ'][indx]
    posit[1] = data['Load'][indx]
    ax.plot(posit[0], posit[1], 'o', markersize=8)  # Plot the marker
    ax.annotate(str(nxt), xy=posit, fontsize=16, xytext=(posit[0]+txtoset[0], posit[1]+txtoset[1]))
    markers[nxt] = dict.fromkeys(['Load', 'Displ', 'Time'])
    markers[nxt]['Load'] = data['Load'][indx]
    markers[nxt]['Displ'] = data['Displ'][indx]
    markers[nxt]['Time'] = data_dict['Data']['time(sec)'][indx]
    axes['fig'].canvas.draw()
    print 'Point number {:3d} > Time = {:5.3f}  Load = {:6.1f}  Displ = {:6.4f}'.format(nxt, data_dict['Data']['time(sec)'][indx], data['Load'][indx], data['Displ'][indx])
# Send Raw data to Matplotlib

def drwRaw():
    global data_dict, axes
    axRaw = axes['axRaw']
    markers = itertools.cycle(['D', 's', 'o', 'x', 'h', '+', '*', '^'])
    xVals = data_dict['Data']['time(sec)']
    markspace = int(len(xVals) / 15)
    # curvetype = {'Ch-0': '_Trig', 'Ch-1': '_Displ', 'Ch-4': '_Load', 'Ch-2': '_Load2'}
    for curve in data_dict['ToPlot']:
        if len(data_dict['Data'][curve]) == 0: continue
        col = data_dict['LineCols'][curve]
        label = curve + '_' + data_dict['CurveTypes'][curve]
        yVals = data_dict['Data'][curve]
        axRaw.plot(xVals, yVals, label=label, color=col, marker=markers.next(), markevery=markspace)
    axRaw.legend(loc='best')

def applyfilter():
    if data_dict['CutOff'] == 'N/A':
        return
    # data_dict['CutOff'] provides an appropriate predetermined normalised 3dB cut off frequency
    b, a = butter(9, data_dict['CutOff'])
    # The decimation value, number of data points to step forward when decimating
    # for key in ['Load', 'Displ']:
    data_dict['Filtered'][key] = list(filtfilt(b, a, data_dict['Data'][key]))
    def drwFiltered():
        dispchan = [k for k, v in data_dict['CurveTypes'].items() if v=='Displ'][0]
        dispcol = data_dict['LineCols'][dispchan]
        loadchan = [k for k, v in data_dict['CurveTypes'].items() if v=='Load'][0]
        loadcol = data_dict['LineCols'][loadchan]
        xVals = data_dict['Data']['time(sec)']
        if data_dict['Filtered']['Flag']:
            print 'has been filtered'
            axes['axDisp'].plot(xVals, data_dict['Data']['Displ'], color='0.75')
            axes['axDisp'].plot(xVals, data_dict['Filtered']['Displ'], color=dispcol)
            axes['axLoad'].plot(xVals, data_dict['Data']['Load'], color='0.75')
            axes['axLoad'].plot(xVals, data_dict['Filtered']['Load'], color=loadcol)
        else:
            print 'has not been filtered'
            axes['axDisp'].plot(xVals, data_dict['Data']['Displ'], color='0.75')
            axes['axDisp'].plot(xVals, data_dict['Filtered']['Displ'], color=dispcol)
            axes['axLoad'].plot(xVals, data_dict['Data']['Load'], color='0.75')
            axes['axLoad'].plot(xVals, data_dict['Filtered']['Load'], color=loadcol)
def drwLvDispl():
    if data_dict['Filtered']['Flag']:
        load = data_dict['Data']['Load']
        displ = data_dict['Data']['Displ']
        axes['axLvD'].plot(displ, load, color='0.75')
        load = data_dict['Filtered']['Load']
        displ = data_dict['Filtered']['Displ']
        axes['axLvD'].plot(displ, load, color='b')
    else:
        load = data_dict['Data']['Load']
        displ = data_dict['Data']['Displ']
        axes['axLvD'].plot(displ, load, color='b')

#Send Raw data to Matplotlib
def drwResult():
    global data_dict, interact
    #sttindx = data_dict['trigindx']
    #endtime = data_dict['endtime']
    xVals = data_dict['Filtered']['Displ'][#[sttindx:endtime]
    yVals = data_dict['Filtered']['Load'][#[sttindx:endtime]
    subs[1].plot(xVals, yVals, color='b')
    #subs[1].plot(6, 510, 'o')
    interact['canvas'].draw()

def horizlim(val):
    global data_dict
    print 'horizlim = '+str(val)
    indx = bisect_left(data_dict['Filtered']['time(sec)'], int(val))
    print 'indx = '+str(indx)
    if indx >= len(data_dict['Filtered']['time(sec)']): indx = -1
    print 'indx = '+str(indx)
    data_dict['endtime'] = indx
    subs[1].lines = []
    drwResult()

def designfilt(ws):
    #wp, ws = normalised pass and stop frequencies
    #gpass = maximum loss in the pass band (flatness of pass band)
    #gstop = minimum attenuation in the stop band
    b, a = butter(9, ws)
    return b, a

def filtdecim():
    b, a = butter(9, data_dict['CutOff'])
    decim = int(1 / (8*data_dict['CutOff']))    #The decimation value, number of data points to step
    #forward when decimating
    for key in data_dict['Filtered'].keys():
        if key in ['Index', 'time(sec)']: continue
        print key
        data_dict['Filtered'][key] = filtfilt(b, a, data_dict['Data'][key])[::decim]

axes = {} 
interact = {} 
markers = {} 

#APPENDIX C. PYTHON SOURCE CODE

APPENDIX C. PYTHON SOURCE CODE

def drwLvDispl():
    if data_dict['Filtered']['Flag']:
        load = data_dict['Data']['Load']
        displ = data_dict['Data']['Displ']
        axes['axLvD'].plot(displ, load, color='0.75')
        load = data_dict['Filtered']['Load']
        displ = data_dict['Filtered']['Displ']
        axes['axLvD'].plot(displ, load, color='b')
    else:
        load = data_dict['Data']['Load']
        displ = data_dict['Data']['Displ']
        axes['axLvD'].plot(displ, load, color='b')

#Send Raw data to Matplotlib
def drwResult():
    global data_dict, interact
    #sttindx = data_dict['trigindx']
    #endtime = data_dict['endtime']
    xVals = data_dict['Filtered']['Displ'][#[sttindx:endtime]
    yVals = data_dict['Filtered']['Load'][#[sttindx:endtime]
    subs[1].plot(xVals, yVals, color='b')
    #subs[1].plot(6, 510, 'o')
    interact['canvas'].draw()

def horizlim(val):
    global data_dict
    print 'horizlim = '+str(val)
    indx = bisect_left(data_dict['Filtered']['time(sec)'], int(val))
    print 'indx = '+str(indx)
    if indx >= len(data_dict['Filtered']['time(sec)']): indx = -1
    print 'indx = '+str(indx)
    data_dict['endtime'] = indx
    subs[1].lines = []
    drwResult()

def designfilt(ws):
    #wp, ws = normalised pass and stop frequencies
    #gpass = maximum loss in the pass band (flatness of pass band)
    #gstop = minimum attenuation in the stop band
    b, a = butter(9, ws)
    return b, a

def filtdecim():
    b, a = butter(9, data_dict['CutOff'])
    decim = int(1 / (8*data_dict['CutOff']))    #The decimation value, number of data points to step
    #forward when decimating
    for key in data_dict['Filtered'].keys():
        if key in ['Index', 'time(sec)']: continue
        print key
        data_dict['Filtered'][key] = filtfilt(b, a, data_dict['Data'][key])[::decim]

axes = {} 
interact = {} 
markers = {} 

#APPENDIX C. PYTHON SOURCE CODE

APPENDIX C. PYTHON SOURCE CODE

def drwLvDispl():
    if data_dict['Filtered']['Flag']:
        load = data_dict['Data']['Load']
        displ = data_dict['Data']['Displ']
        axes['axLvD'].plot(displ, load, color='0.75')
        load = data_dict['Filtered']['Load']
        displ = data_dict['Filtered']['Displ']
        axes['axLvD'].plot(displ, load, color='b')
    else:
        load = data_dict['Data']['Load']
        displ = data_dict['Data']['Displ']
        axes['axLvD'].plot(displ, load, color='b')

#Send Raw data to Matplotlib
def drwResult():
    global data_dict, interact
    #sttindx = data_dict['trigindx']
    #endtime = data_dict['endtime']
    xVals = data_dict['Filtered']['Displ'][#[sttindx:endtime]
    yVals = data_dict['Filtered']['Load'][#[sttindx:endtime]
    subs[1].plot(xVals, yVals, color='b')
    #subs[1].plot(6, 510, 'o')
    interact['canvas'].draw()

def horizlim(val):
    global data_dict
    print 'horizlim = '+str(val)
    indx = bisect_left(data_dict['Filtered']['time(sec)'], int(val))
    print 'indx = '+str(indx)
    if indx >= len(data_dict['Filtered']['time(sec)']): indx = -1
    print 'indx = '+str(indx)
    data_dict['endtime'] = indx
    subs[1].lines = []
    drwResult()

def designfilt(ws):
    #wp, ws = normalised pass and stop frequencies
    #gpass = maximum loss in the pass band (flatness of pass band)
    #gstop = minimum attenuation in the stop band
    b, a = butter(9, ws)
    return b, a

def filtdecim():
    b, a = butter(9, data_dict['CutOff'])
    decim = int(1 / (8*data_dict['CutOff']))    #The decimation value, number of data points to step
    #forward when decimating
    for key in data_dict['Filtered'].keys():
        if key in ['Index', 'time(sec)']: continue
        print key
        data_dict['Filtered'][key] = filtfilt(b, a, data_dict['Data'][key])[::decim]

axes = {} 
interact = {} 
markers = {} 

#APPENDIX C. PYTHON SOURCE CODE

APPENDIX C. PYTHON SOURCE CODE

def drwLvDispl():
    if data_dict['Filtered']['Flag']:
        load = data_dict['Data']['Load']
        displ = data_dict['Data']['Displ']
        axes['axLvD'].plot(displ, load, color='0.75')
        load = data_dict['Filtered']['Load']
        displ = data_dict['Filtered']['Displ']
        axes['axLvD'].plot(displ, load, color='b')
    else:
        load = data_dict['Data']['Load']
        displ = data_dict['Data']['Displ']
        axes['axLvD'].plot(displ, load, color='b')

#Send Raw data to Matplotlib
def drwResult():
    global data_dict, interact
    #sttindx = data_dict['trigindx']
    #endtime = data_dict['endtime']
    xVals = data_dict['Filtered']['Displ'][#[sttindx:endtime]
    yVals = data_dict['Filtered']['Load'][#[sttindx:endtime]
    subs[1].plot(xVals, yVals, color='b')
    #subs[1].plot(6, 510, 'o')
    interact['canvas'].draw()
print "Trigger index = {:d}, trigger time = {:.4f} seconds".format(trigindx, data_dict['Data']['time(sec)'][trigindx])

setupFig(specname)
drwRaw()
applyfilter()
drwFiltered()
drwLvDispl()
plt.show()

print
print
print 'End time = {:.4f}'.format(data_dict['Data']['time(sec)'][-1])
print '-' * 40
print specname
print '{:<12}|{:>13.4f}|{:>13.4f}|{:>13.4f}|'.format('Point number', 'Time', 'Load', 'Displ')
for key in markers:
    print '{:<12d}|{:>13.4f}|{:>13.1f}|{:>13.4f}|'.format(key, markers[key]['Time'], markers[key]['Load'], markers[key]['Displ'])
Generate a Dispersion Plot

A Python script to generate a dispersion plot using a 2D mass-spring lattice.

```python
import numpy as np
import matplotlib.pyplot as plt
import matplotlib.lines as mlines
from matplotlib.patches import Rectangle
from scipy import linalg
import time
import win32ui
import sys
from traceback import print_exc as PExc

# Set up data defining the model and simulation
Plate = 0.005                # Overall size of the plate in both directions in m
Thick = 0.0024           # Material thickness in m
GamStep = 15            # Number of subdivisions along the Wavenumber axis
Curves = 8             # Number of dispersion curves to be plotted
NumRows = 21
NumCols = 21
NumMass = NumRows * NumCols
A = Plate / NumCols
GapFreq = 45000                 # Frequency known to be somewhere in the band gap. This is used when making the mode shape field plot. See DrwSField function.
MatMat = (200, 0.3, 7850) # Matrix material properties - Epoxy
IncMat = (2E-6, 0.3, 7E-6)  # Inclusion material properties - Steel
MatMat = (200, 0.3, 7850) # Matrix material properties - Epoxy
IncMat = (2E-6, 0.3, 7E-6)  # Inclusion material properties - Steel
SFields = [[0, int(GamStep/4), int(2*GamStep/4), int(3*GamStep/4)]]
#IncDef = [['RP', (1,1), (3,3)]] # 5x5 Jensen Type 1
#IncDef = [['RP', (2,2), (7,7)]] # 10x10 Jensen Type 1
#IncDef = [['RP', (4,4), (15,15)]] # 20x20 Jensen Type 1
#IncDef = [['RP', (2,2), (15,15)]] # 25x25 Jensen Type 1
#IncDef = [['RP', (3,3), (23,23)]] # 30x30 Jensen Type 1
#IncDef = [['RP', (4,4), (23,23)]] # 40x40 Jensen Type 1

21x21 Fat Cross
Name = str(NumCols)+'x'+str(NumRows)+'_Cross
IncDef = [['HL', (9, 2), 17], ['HL', (10, 2), 17], ['VL', (2, 9), 17], ['VL', (2,10), 17]]

GapFreq = 370                 # Frequency known to be somewhere in the band gap. This is used when making the mode shape field plot. See DrwSField function.
#IncDef = [['RP', (1,1), (3,3)]] # 5x5 Jensen Type 1
#IncDef = [['RP', (2,2), (7,7)]] # 10x10 Jensen Type 1
#IncDef = [['RP', (4,4), (15,15)]] # 20x20 Jensen Type 1
#IncDef = [['RP', (2,2), (15,15)]] # 25x25 Jensen Type 1
#IncDef = [['RP', (3,3), (23,23)]] # 30x30 Jensen Type 1
#IncDef = [['RP', (4,4), (23,23)]] # 40x40 Jensen Type 1
```
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# IncDef = [['CI', 2]]  # Terms to generate Ke matrix for Horizontal Acceleration
# IncDef = [['RP', (9, 1), (11, 19)]]  # Terms for Horiz acceleration of current mass due to Horiz degrees of freedom of other masses
# IncDef = [['RP', (1, 4), (9, 6)]]  # These are the terms other than those for the current mass

# Set up empty arrays for the mass and spring definition, these arrays just define the model and are used later to generate the mass and stiffness matrices to be used in the calculation
Masses = np.zeros((NumMass), dtype=int)  # 1D Array defining material type of each mass
Springs = np.zeros((NumMass, 4), dtype=float)  # 2D Array defining the spring stiffnesses
Springs[:, 0] = 1.0  # Rows zero define material type of each mass
Springs[:, 1] = 0.0  # Row number corresponds to the mass associated with these springs
Springs[:, 2] = 0.0  # The four values on that row correspond to the stiffness of each of the four springs
Springs[:, 3] = 0.0  # The zeroth spring is the spring vertically below the respective mass

# Set up empty Mass matrix, this is the real mass matrix used in the dispersion calculation
M = np.zeros((2*NumMass, 2*NumMass), dtype=float)  # Double the number of elements are needed to accommodate two degrees of freedom
# Set up an empty matrix to hold the data to be plotted (the dispersion results)
Disp = np.empty((NumMass+1, 3*GamStep+1), dtype=complex)

# Create a list of empty tuples. This will hold data so that the dispersion plot can be populated
# with points showing where the mode shape field plots relate to. Each tuple will be:
FieldPnts = []  # [(0,0,0)] * len(SFields)
Check = []

# Ancilliary functions - drawing etc
def BuildIncl():
    # Build a list of masses that are part of the inclusion
    # Definition consists of a list with the following syntax options:
    # Rectangular perimeter filled with inclusion masses
    # 'RP', (BRow,LCol), (TRow,RCol)
    # BRow, LCol = Row and column of bottom left corner
    # TRow, RClo = Row and column of top right corner
    # Horizontal line of masses - 'HL', (LRow, LCol), (RCol)
    # LRow, LCol = Row and column of the mass at left hand end of the line
    # Vertical line of masses - 'VL', (BRow, BCol), (TRow)
    # BRow, BCol = Row and column of the mass at the bottom of the line
    # TRow = Row of the mass at the top of the line
    Inc = []
    for Indx in range(0, len(IncDef)):
        Item = IncDef[Indx]
        print Item
        if Item[0] == 'RP':
            for Row in range(Item[1][0], Item[2][0]+1):
                for Col in range(Item[1][1], Item[2][1]+1):
                    Inc.append(Col + Row*NumCols)
        elif Item[0] == 'HL':
            Row = Item[1][0]
            for Col in range(Item[1][1], Item[2][1]+1):
                Inc.append(Col + Row*NumCols)
        elif Item[0] == 'VL':
            Col = Item[1][1]
            for Row in range(Item[1][0], Item[2][0]+1):
                Inc.append(Col + Row*NumCols)
        elif Item[0] == 'CI':
            print 'Circle'
            Cent = (NumRows-1)/2.0
            Rad = float(Item[1])
            Bot = int((Cent - Rad) + 0.5)
            Top = int((Cent + Rad) + 0.5)
            for Row in range(Bot, Top+1):
                Vrt = Row - Cent
                Lft = int((Cent - Rad * np.cos(np.arcsin(Vrt/Rad))) + 0.5)
                Rgt = NumCols - Lft
                print Row, Lft, Rgt, Vrt, Rad
                for Col in range(Lft, Rgt):
                    Inc.append(Col + Row*NumCols)
        else:
            MBox('Your inclusion definition type is not recognised. Type = '+Item)
            print Item
            sys.exit(0)
    return Inc

def DrwSpring(Mass, Spring, Stt, End, Thk, fig):
    # Draw springs between the masses
    # Stt = coords of 1st mass, End = coords of 2nd mass, fig = figure to be drawn on
    Divis = ((0.1, 0.3, 1), (0.3, 0.7, Thk), (0.7, 0.9, 1))
    # Each spring is drawn with 3 segments
    # Divis = ((Start this far along, End this far along, Thickness), ...)
    Vect = (End[0]-Stt[0], End[1]-Stt[1])
    # Vector from 1st mass to 2nd mass
    for Indx in range(0, len(Divis)):
        # Draw each segment of spring
        SegStt = (Stt[0]+Divis[Indx][0]*Vect[0], Stt[1]+Divis[Indx][0]*Vect[1])
        SegEnd = (Stt[0]+Divis[Indx][1]*Vect[0], Stt[1]+Divis[Indx][1]*Vect[1])
        Line = mlines.Line2D((SegStt[0], SegEnd[0]), (SegStt[1], SegEnd[1]),
                              color='k', linewidth=Divis[Indx][2])
        fig.gca().add_artist(Line)
        Ann = '$' + str(Mass) + '_' + str(Spring) + '$'
        XOff = 0.05
        YOff = 0.3
        if Spring == 2: YOff = 0.3
        elif Spring == 3: XOff = -0.75
        Pos = (XOff + Stt[0] + Vect[0]/3.5, YOff + Stt[1] + Vect[1]/4.5)
        plt.annotate(Ann, xy=Pos, size=12, color='b', family='Arial', fontweight='bold')
return DrwSpring

def DrwLattice(Rad = 0.5):
    # First setup a dictionary which defines the line thickness to use when drawing each spring
    # It assumes there are a maximum of 6 different spring stiffnesses and defines a unique
    # thickness for each, this thickness is not proportional to stiffness.

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Stiff = list(set(Springs.ravel())) #Create a list of the stiffnesses that are used
Stiff.sort #Put the stiffness list in order

ThkDict = {}
for Indx in range(0, len(Stiff)):
    ThkDict[Stiff[Indx]] = 3 + 4*Indx

#Draw a rectangular lattice
#A = mass spacing
#Oset = x & y offset from 0,0 origin
#Rad = radius used when drawing the masses
Oset = 5
XSze = 2*Oset + 1000*A*(NumCols-1)
YSze = 2*Oset + 1000*A*(NumRows-1)

fig = plt.figure(figsize=(1.5*NumCols, 1.5*NumRows))
ax = plt.axes([0,0,1,1])
plt.box(on=False)
ax.set_xticks([])
ax.set_yticks([])
plt.xlim([0, XSze])
plt.ylim([0, YSze])

Lattice = []
for R in range(0, NumRows):
    for C in range(0, NumCols):
        Xpos = Oset + 1000*A*C
        Ypos = Oset + 1000*A*R
        if R == 0: #First row
            if C < NumCols - 1: #Not last column
                Spring = [0, 0, 1, 1]
            else:
                Spring = [0, 0, 0, 0]
        elif R == NumRows - 1: #Last column
            Spring = [1, 0, 0, 0]
        elif C == NumCols - 1:
            Spring = [1, 1, 1, 0]
        else:
            Spring = [1, 1, 1, 1]
        Lattice += [[Xpos, Ypos, Spring]]

#Now draw the lattice using Matplotlib

fig = plt.figure(figsize=(8,8))
plt.xlim([-1, XSze+1])
plt.ylim([-1, YSze+1])

#Build the lattice definition [[Xpos, Ypos, [Spr1, Spr2, Spr3, Spr4]] . . . . ]
#Xpos, Ypos = location of the Nth mass
#Spr1 is to (j,k-1), Spr2 is to (j+1,k-1), Spr3 is to (j+1,k), Spr4 is to (j+1,k+1)
Lattice = []
for R in range(0, NumRows):
    for C in range(0, NumCols):
        Xpos = Oset + 1000*A*C
        Ypos = Oset + 1000*A*R
        if R == 0: #First row
            if C < NumCols - 1: #Not last column
                Spring = [0, 0, 1, 1]
            else:
                Spring = [0, 0, 0, 0]
        elif R == NumRows - 1: #Last column
            Spring = [1, 0, 0, 0]
        elif C == NumCols - 1:
            Spring = [1, 1, 1, 0]
        else:
            Spring = [1, 1, 1, 1]
        Lattice += [[Xpos, Ypos, Spring]]

#Now draw the lattice using Matplotlib

fig = plt.figure(figsize=(1.5*NumCols, 1.5*NumRows))
ax = plt.axes([0,0,1,1])
plt.box(on=False)
ax.set_xticks([])
ax.set_yticks([])
plt.xlim([0, XSze])
plt.ylim([0, YSze])

fig = plt.figure(figsize=(1.5*NumCols, 1.5*NumRows))
#Spr1 is to (j,k-1), Spr2 is to (j+1,k-1), Spr3 is to (j+1,k), Spr4 is to (j+1,k+1)

Lattice = []
for R in range(0, NumRows):
    for C in range(0, NumCols):
        if R == 0:                      #First row
            if C < NumCols - 1:           #Not last column
                Spring = [0, 0, 1, 1]
            else:
                Spring = [0, 0, 0, 0]
        else:
            if C == NumCols - 1:          #Last column
                Spring = [1, 0, 0, 0]
            elif R == NumRows - 1:        #Last row
                Spring = [1, 1, 1, 0]
            else:                       #Inside main body of lattice
                Spring = [1, 1, 1, 1]
        Lattice += [[C, R, Spring]]

SThresh = min(max(KInc), max(KMat))     #Find greatest stiffness of a soft spring, this is used
#to draw thicker springs where stiffer springs occur
for N in range(0, NumMass):
    #Get the masses that are adjacent to the current mass
    Adjac = Adjacent(N)
    #Get the masses at the other ends of the springs
    Far = [Adjac[1], Adjac[2], Adjac[5], Adjac[8]]
    Xpos, Ypos = Lattice[N][1:2]  #Position of current mass
    #Draw the springs if they exist
    for S in range(0, 4):
        if Lattice[N][2][S]:            #Is there a spring in this position?
            End = (Lattice[Far[S]][0], Lattice[Far[S]][1])  #Get position of mass at far end
            Lw = 1 + 1.25*(Springs[N][S] > SThresh)
            Line = mlines.Line2D((Xpos, End[0]), (Ypos, End[1]), color='0.6', zorder = 1, linewidth=Lw)
            fig.gca().add_artist(Line)

#Now draw the masses
if M[2*N][2*N] == Mass[1]:
    Circle = plt.Circle((Xpos, Ypos), Rad, edgecolor='0.7', facecolor='0.7', zorder = 2)
else:
    Circle = plt.Circle((Xpos, Ypos), Rad, edgecolor='k', facecolor='k', zorder = 2)
    fig.gca().add_artist(Circle)
fig.savefig('Lattice'+Name+'.pdf')

def DrwSField(w, S, GammaX, GammaY):
    #Make a 2D field plot of a mode shape
    #w = array of eigen values found for the current wave numbers
    #S = array of eigen values found for the current wave numbers
    #GammaX, GammaY are the current wave numbers
    #GapFreq = Global object giving an approximate value for the middle of the band gap
    #First find frequencies of the first mode above GapFreq and the first mode below GapFreq
    #then find the index of these values in the original array of eigen values, these 2 indices
    #point to the mode shapes of interest within S. Below & Above are both 2 element lists.
    #Below = [Lowest frequency that is below band gap, Index within w]
    #Above = [Highest frequency that is above band gap, Index within w]
    Below = [max(x for x in w if x<GapFreq)]  #Highest frequency that is below band gap
    Above = [min(x for x in w if x>GapFreq)]  #Lowest frequency that is above band gap
    Below.append(np.where(w==Below[0])[0][0]) #Index of Below[0]
    Above.append(np.where(w==Above[0])[0][0]) #Index of Above[0]
    #Convert the omega^2 values to kHz values to aid user interpretation
    Below[0] = np.sqrt(Below[0].real)/2/pi/1000
    Above[0] = np.sqrt(Above[0].real)/2/pi/1000
    #Assemble the data used to make the field plots.
    #The data is a subset of S, i.e. it is one of the eigenvectors within S broken down into
    #Horizontal.real, Horizontal.imag, Vertical.real, Vertical.imag
    #We need to create two field plots, one for the mode shape above the gap and one for shape
    #below the gap.
    for Indx in [Below[1], Above[1]]:  
        #Make a list containing four empty arrays to hold the field data
        Fields = []
        for cnt in range(4):
            Fields.append(np.empty((NumRows, NumCols), dtype=float))
        for row in range(NumRows):
            for col in range(NumCols):
                Fields[0][row, col] = S[2*(NumCols*row + col)][Indx].real
                Fields[1][row, col] = S[2*(NumCols*row + col)+1][Indx].real
                Fields[2][row, col] = S[2*(NumCols*row + col)+1][Indx].imag
                Fields[3][row, col] = S[2*(NumCols*row + col)+1][Indx].imag
        sub = []
        f = np.sqrt(w[Indx].real)/2/pi/1000
        MainTitle = '\text{Wave amplitudes in' 'x'$gamma_X$ = {:.2f}, '$gamma_Y$ = {:.2f}, '$omega$ = {:.2E}' \n'.format(GammaX, GammaY, f)
        SubTitles = ['Horiz.real', 'Horiz.imag', 'Vert.real', 'Vert.imag']
        plt.figure()
        plt.title(MainTitle)
        plt.suptitle(SubTitles)
        plt.show()
fig = plt.figure()
plt.figtext(0.5, 0.94, MainTitle, ha='center', color='black', weight='bold')
for cnt in range(4):
    sub.append(fig.add_subplot(221+cnt))
    plt.imshow(Fields[cnt])
    plt.colorbar()
    sub[cnt].invert_yaxis()
    sub[cnt].set_title(SubTitles[cnt], fontsize=12)
    #AmpPDF.savefig(fig)

return (Below[0], Above[0])

#Functions used in the Dispersion calculation

def Material(Type):
    KMat = [0, 0]
    KInc = [0, 0]
    Mass = [0, 0]
    if Type == "RJB":
        G = MatMat[0] * 1E9 / 2 / (1+MatMat[1]) #Shear modulus of the matrix material (GPa)
        KMat[1] = G * Thick #Stiffness for diagonal matrix spring (kg/s²)
        KMat[0] = MatMat[0] * 1E9 * Thick - KMat[1] #Stiffness for axial matrix spring (kg/s²)
        G = IncMat[0] * 1E9 / 2 / (1+IncMat[1]) #Shear modulus of the inclusion material (GPa)
        KInc[1] = G * Thick #Stiffness for diagonal inclusion spring (kg/s²)
        KInc[0] = IncMat[0] * 1E9 * Thick - KInc[1] #Stiffness for axial inclusion spring (kg/s²)
    elif Type == "Jensen":
        Mass = (1.82E-2, 4.53E-2) #Mass of masses, mass in kg
            #Matrix material, Inclusion material)
        KIncAx = 70.9E9 #Stiffness of the inclusion on axis springs (kg/s²)
        KIncDi = KIncAx/2 #Stiffness of the inclusion diagonal springs (kg/s²)
        KmatAx = 4.10E9 #Stiffness of the body material on axis springs (kg/s²)
        KmatDi = KmatAx/2 #Stiffness of the body material diagonal springs (kg/s²)
        KInc = [KIncAx, KIncDi]
        KMat = [KmatAx, KmatDi]
    else:
        MBox('Calculation type not recognised!')
sy.exit(0)
return KMat, KInc, Mass #Material

def Adjacent(N):
    Adj = [0]*9 #Empty 9 element list
    if N in Inc:
        Masses[N] = 1
    #Calculate the row that N is in
    j = N % NumCols
    #Calculate the column that N is in
    k = N / NumCols
    #Iterate over the 9 neighbour masses row by row
    for r in range(-1, 2):
        for c in range(-1, 2):
            Neighbj = (j+c)%NumCols #j is column of current mass c is colm of neighbour
            Neighbk = (k+r)%NumRows #k is row of current mass r is row of neighbour
            #Modulo division (%) takes care of wrap around
            N = Neighbj + NumCols*Neighbk
            #Calculate element number given by Modj, Modk
            Adj[3*(1+r)+c+1] = N #Build the list of adjacent element numbers
    return Adj #Adjacent

def BuildM():
    #First build Masses, a matrix with an integer for each mass which defines the material type
    #of that mass, 0 = Matrix material, 1 = Inclusion material
    for N in range(0, NumMass):
        if N in Inc:
            Masses[N] = 1
    #Build M, the "true" mass matrix which will be used in the eigenvalue calculation
    for N in range(0, NumMass):
        M[2*N, 2*N] = Masses[N]
    print 'BuildM = DONE'
    return #BuildM

def BuildSprings():
    #Springs is a [NumRows][NumCols] list with each element being a list of four floats
    #Each element gives the stiffness of the four springs associated with that mass
    #Spring material is the matrix material except for springs that are within the inclusion,
    #thus if the current node is of matrix material then we know to also make the spring of
    #matrix material. If the current node is of inclusion material then we need to check the
    #material of each of the four nodes (j,k+1), (j+1,k+1), (j+1,k), (j+1,k-1), the material of
    #these nodes will determine the material to be used for the associated spring.
    FarNode = [1, 2, 5, 8] #We need to know the mass number of masses at
for N in range(NumMass):
    if Masses[N]:
        Adj = Adjacent(N)
        for Far in range(0, 4):
            FarMass = Adj[FarNode[Far]]
            if Masses[FarMass]:
                Springs[N][Far] = KInc[Far%2]
            else:
                Springs[N][Far] = KMat[Far%2]
        else:
            for Far in range(0, 4):
                Springs[N][Far] = KMat[Far%2]
    else:
        for Far in range(0, 4):
            Springs[N][Far] = KMat[Far%2]
print 'BuildSprings = DONE'
return #BuildSprings

def BuildKe(GammaX, GammaY):
    # Ke is "stiffness" matrix. Double the number of elements are needed in order to accommodate
    # the two degrees of freedom
    Ke = np.zeros((2*NumMass, 2*NumMass), dtype=complex)
    for N in range(0, NumMass):
        Adjac = Adjacent(N)
            #Point to each mass in turn
            #Build the Ke elements which relate to masses other than the current mass
            #for Term in range(0, 6):  #Point to each equation term
                #Horizontal acceleration
                #Horizontal freedom
                Trms = HAccHFree[Term]  #Get the current list of equation terms
                if Trms[0]:
                    AdjMass = Adjac[Trms[1]]  #Find the actual number of the adjacent mass
                    SprMass = Adjac[Trms[5]]
                    SprNumb = Trms[6]
                #Vertical freedom
                Trms = HAccVFree[Term]  #Get the current list of equation terms
                if Trms[0]:
                    AdjMass = Adjac[Trms[1]]  #Find the actual number of the adjacent mass
                    SprMass = Adjac[Trms[5]]
                    SprNumb = Trms[6]
            #Vertical acceleration
            #Horizontal freedom
            Trms = VAccHFree[Term]  #Get the current list of equation terms
            if Trms[0]:
                AdjMass = Adjac[Trms[1]]  #Find the actual number of the adjacent mass
                SprMass = Adjac[Trms[5]]
                SprNumb = Trms[6]
            #Vertical freedom
            Trms = VAccVFree[Term]  #Get the current list of equation
            if Trms[0]:
                AdjMass = Adjac[Trms[1]]  #Find the actual number of the adjacent mass
                SprMass = Adjac[Trms[5]]
                SprNumb = Trms[6]
        #Build the Ke elements which relate to the current mass
        for Term in range(0, 6):
            #Horizontal acceleration of current mass, horizontal degrees of freedom
            Trms = HAccHFreeCur[Term]  #Get the current list of equation terms
            if Trms[0]:
                AdjMass = Adjac[Trms[1]]  #Find the actual number of the adjacent mass
                SprMass = Adjac[Trms[5]]
                SprNumb = Trms[6]
                Stiff = Springs[AdjMass][Trms[2]]
                Ke[2*N][2*N] = Trms[0] * Stiff
            #Vertical acceleration of current mass, vertical degrees of freedom
            Trms = VAccVFreeCur[Term]
            if Trms[0]:
                AdjMass = Adjac[Trms[1]]  #Find the actual number of the adjacent mass
                SprMass = Adjac[Trms[5]]
                SprNumb = Trms[6]
                Stiff = Springs[AdjMass][Trms[2]]
                Ke[2*N][2*N] = Trms[0] * Stiff

APPENDIX C. PYTHON SOURCE CODE
def Dispersion():

    #Define the three sides of the 2D irreducible Brillouin zone
    #M to GAMMA, GAMMA to X, X to M
    #[(XOffSet, Rate of X Change, YOffSet, Rate of Y Change), (Ditto crnr 2), (Ditto crnr 3)]
    GamDef = [(pi, -1, pi, -1),  #M to GAMMA
               (0, 1, 0, 0),    #GAMMA to X
               (pi, 0, 0, 1)]   #X to M

    for BrillSide in range(0, 3):  #Point to each side of the Brillouin zone in GamDef
        BZoneX = GamDef[BrillSide][:2]  #OFFSET, MULTIPLY gives X axis of this side of B Zone
        BZoneY = GamDef[BrillSide][2:]  #OFFSET, MULTIPLY gives Y axis of this side of B Zone

        CurTime = time.asctime(time.localtime(time.time()))[4:19]
        print 'Starting Brillouin zone GamX = {:.3f} / GamY = {:.3f} - time = {}'
        .format(BZoneX[0], BZoneY[0], CurTime)

        for Cnt in range(GamStep):  #Define the Wavenumber increments
            XVal = BrillSide * GamStep + Cnt
            Gamma = pi * Cnt / float(GamStep)
            GammaX = BZoneX[0] + BZoneX[1] * Gamma
            GammaY = BZoneY[0] + BZoneY[1] * Gamma
            Ke = BuildKe(GammaX/NumCols, GammaY/NumRows)
            w, S = linalg.eig(Ke, M)   #Solve the eigen problem
            w.sort()  #Order the w^2 results

            #Store the data so it can be plotted later
            Disp[0, XVal] = XVal  #First store the X value
            for row in range(NumMass):  #Then points for each of the curves
                Disp[row+1, XVal] = np.sqrt(w[row]) / (2*pi) / 1000

            #Repeat the first data set at the end of the data so that the dispersion plot "wraps around"
            XVal += 1
            Disp[0, XVal] = XVal
            for row in range(NumMass):  #Then points for each of the curves
                Disp[row+1, XVal] = Disp[row+1, 0]

    def CheckRes(w, S, Gam, Ke, M):
        #NOTE: w needs to be sorted

        #Check the omega results to find:
        #the size of the largest imaginary value in relation to the range of real values,
        #repeated omega values,
        #Largest imaginary omega value
        ResInfo = {'Gam': Gam, 'RMax': w.real.max(), 'RMin': w.real.min(), 
                   'IMax': max(abs(w.imag.max()),abs(w.imag.min())), 'Reps': [], 'Cholesky': ''}

        #Find "equal" omega.real values, use 1/1E9 x the maximum omega.real value as the threshold
        #for equality comparison
        Repat = [-1]
        Thresh = ResInfo['RMax'] / 1.0E9
        for Indx in range(1, len(w)):
            if w[Indx].real - w[Indx-1].real < Thresh and w[Indx].real - Repat[-1] > Thresh:
                Repat.append(w[Indx].real)

        #Actual Omega won't be less than zero
        for Indx in range(len(w)): 
            if w[Indx].real <= Thresh:
                w[Indx] = 0

        try:
            np.linalg.cholesky(Ke)
        except:
            Cholesky = 'Ke is posdef'

        try:
            np.linalg.cholesky(M)
        except:
            Cholesky += ' - M is posdef'

        return ResInfo, Repat, Cholesky
ResInfo['Cholesky'] = Cholesky
ResInfo['Reps'] = Repeat[1:]

return ResInfo

def CheckOP():
    MinOmegaR = np.sqrt(min([x['RMin'] for x in Check]))
    MaxOmegaR = np.sqrt(max([x['RMax'] for x in Check]))
    MaxOmegaI = np.sqrt(max([x['IMax'] for x in Check]))
    MaxRatio = max([x['IMax']/x['RMax'] for x in Check])
    MinRatio = min([x['IMax']/x['RMax'] for x in Check])

    print('Min Omega.real = {:.4E}    Max Omega.real = {:.4E}    Max Omega.imag = {:.4E} (Hertz)'.format(MinOmegaR/2/pi, MaxOmegaR/2/pi, MaxOmegaI/2/pi)
    print 'Max Ratio {:.4E}    Min Ratio {:.4E}'.format(MaxRatio, MinRatio)
# # # # # # # # # # # # # # # # # # # # # # # # # # # # # # # # # # # # # # # # # 

Main Body
try:
    KMat, KInc, Mass = Material("RJB")
    Masses = np.zeros((NumMass), dtype=int)  # 1D Array defining material type of each mass
    Springs = np.zeros((NumMass, 4), dtype=float)
    M = np.zeros((2*NumMass, 2*NumMass), dtype=float)
    Inc = BuildInc()
    BuildM()
    BuildSprings()
    DrwBigLattice()
    Dispersion()
    curBelow = Below = 0
    curAbove = Above = 10*GapFreq
    # Find the height of the band gap at GapFreq
    for indx in range(1, Disp.shape[1]):
        if min(Disp[indx]) < GapFreq:
            curBelow = max(x for x in Disp[indx] if x<GapFreq).real  # Highest frequency that is below band gap
        if max(Disp[indx]) > GapFreq:
            curAbove = min(x for x in Disp[indx] if x>GapFreq).real  # Lowest frequency that is above band gap
        if curBelow > Below: Below = curBelow
        if curAbove < Above: Above = curAbove

    print 'Bottom of gap = {:.1f} kHz, Top of gap = {:.1f} kHz, Height of gap = {:.1f} kHz'.format(Below, Above, Above-Below)
    # Set up the plot to show the results
    fig = plt.figure()
    plt.xlabel(r'Wavevector, $\bf{\gamma N}$', position=(0.5,0), fontsize=16)
    plt.ylabel(r'Frequency, $\frac{\omega}{2\pi}$ (kHz)', position=(0,0.5), fontsize=16)
    title = 'Dispersion Relation - '+Name
    title += 'MassInc = {:.3E} kg, MassMat = {:.3E} kg
    title += r'KIncAx = {:.3E} kg/s^2, KMatAx = {:.3E} kg/s^2/bf ^2'.format(KInc[0], KMat[0])
    plt.title(title, fontsize=12, fontweight='bold')
    MaxYval = np.max(Disp[1:Curves].real)
    plt.ylim(ymin=0, ymax=800)
    plt.xticks([0, GamStep, 2*GamStep, 3*GamStep], ['M', r'$\gamma M$', 'X', 'M'])
    plt.vlines(GamStep, 0, 1.1*MaxYval, linestyle='--', color='0.5')
    plt.vlines(2*GamStep, 0, 1.1*MaxYval, linestyle='--', color='0.5')
    plt.tight_layout()

    # Plot the dispersion curves for Indx in range(1, NumMass):
    # The plot style for all curves of a given model are the same so we need to label
    # each line per model so that the legend is not over populated
    plt.plot(Disp[0].real, Disp[Indx].real, # X & Y values
    linewidth=2)
    ax = plt.gca()
    ax.add_patch(Rectangle((0, Below), Disp.shape[1]-1, (Above-Below), color="0.7")
    plt.text(0.7, 0.06, 'Gap = {:.1f} kHz'.format(GapFreq), transform=ax.transAxes,
    fontsize=13, color = 'g')
    fig.savefig('Dispersion'+Name+'.pdf')
except:
    print 'Exception captured and file closed.'
PExc()
Appendix D

4ENF Classical beam theory equations

D.1 Derivation of the 4ENF deflection equation

This appendix provides a derivation of the classical beam theory (small deflection) bending equations relating to the four point bending end notch flexure test:

- The equations for deflection within the four critical segments of the beam are derived.
- The equation for fixture load point deflection is then developed.
- Specimen compliance in terms of crack length is determined for the fixture geometry used in the physical testing (S=25mm).
- The deflection equations are checked by ensuring that the equations give deflection continuity from one segment to the next.

![Figure D.1: Schematic of the four point bending setup.](image)

\[ E = \text{Young's Modulus, constant for whole specimen.} \]

\[ I_m = \text{Second moment of area of the monolithic portion of the specimen.} \]

\[ I_m = \frac{Bh^3}{12} \]

\[ I_c = \text{Second moment of area of one arm of the cracked portion of the specimen.} \]

\[ I_c = \frac{I_m}{R}, \text{ for a mid thickness crack } R = 8 \]

\[ P = \frac{\text{Total applied load}}{2} \]
This assumes a geometrically symmetrical and pinned four point bending fixture, thus both point loads and both reactions are all equal.

Small deflection theory bending equation

\[ EI \frac{d^2 y}{dx^2} = M \]

**Left span (A to B)**

\[ M = Px \]

\[ EI_m \frac{d^2 y_{AB}}{dx^2} = x \]

\[ EI_m \frac{dy_{AB}}{dx} = \int xdx \]

\[ = \frac{x^2}{2} + C_1 \] (D.1)

\[ EI_m \frac{y_{AB}}{P} = \int \frac{x^2}{2} + C_1 dx \]

\[ = \frac{x^3}{6} + C_1 x + C_2 \] (D.2)

**Middle span up to the crack tip (B to C)**

\[ M = Px - P(x - S) \]

\[ = PS \]

\[ EI_m \frac{d^2 y_{BC}}{dx^2} = S \]

\[ EI_m \frac{dy_{BC}}{dx} = \int Sdx \]

\[ = Sx + C_3 \] (D.3)

\[ EI_m \frac{y_{BC}}{P} = \int Sx + C_3 dx \]

\[ = \frac{Sx^2}{2} + C_3 x + C_4 \] (D.4)
APPENDIX D. 4ENF CLASSICAL BEAM THEORY EQUATIONS

Middle span past the crack tip (C to D)

\[ M = PS \]
\[ \frac{2EI_m}{PR} \frac{d^2y_{CD}}{dx^2} = S \]
\[ \frac{2EI_m}{PR} \frac{dy_{CD}}{dx} = \int Sdx \]
\[ \frac{EI_m}{P} \frac{dy_{CD}}{dx} = \frac{R}{2} \int Sdx \]
\[ = 4Sx + C_5 \] (D.5)

\[ \frac{EI_m}{P} y_{CD} = \int 4Sx + C_5 dx \]
\[ = 2Sx^2 + C_5x + C_6 \] (D.6)

Right span (D to E)

\[ M = Px - P(x - S) - P(x - 3S) \]
\[ = -Px + 4PS \]
\[ \frac{2EI_m}{R} \frac{d^2y_{DE}}{dx^2} = -Px + 4PS \]
\[ \frac{EI_m}{P} \frac{d^2y_{DE}}{dx^2} = -4x + 16S \]
\[ \frac{EI_m}{P} \frac{dy_{DE}}{dx} = \int -4x + 16Sdx \]
\[ = -2x^2 + 16Sx + C_7 \] (D.7)

\[ \frac{EI_m}{P} y_{DE} = \int -2x^2 + 16Sx + C_7 dx \]
\[ = -\frac{2x^3}{3} + 8Sx^2 + C_7x + C_8 \] (D.8)

Boundary conditions

Condition 1
\[ y = 0 \text{ @ } x = 0 \]

\[ \frac{EI_m}{P} y_{AB} = \frac{x^3}{6} + C_1x + C_2 \ldots \text{ Eqn D.2} \]
\[ 0 = C_2 \]
Condition 2

\( y = 0 \) @ \( x = 4S \)

\[ \frac{EIm}{P} y_{DE} = \frac{-2x^3}{3} + 8Sx^2 + C_7x + C_8 \ldots \quad \text{Eqn D.8} \]

\[ 0 = \frac{-128S^3}{3} + 128S^3 + 4SC_7 + C_8 \]

\[ C_7 = \frac{-64S^2}{3} - \frac{C_8}{4S} \]

Condition 3

\[ @ \quad x = S \quad \text{Eqn D.1} = \text{Eqn D.3} \]

\[ \frac{S^2}{2} + C_1 = S^2 + C_3 \]

\[ C_1 = \frac{S^2}{2} + C_3 \]

Condition 4

\[ @ \quad x = S \quad \text{Eqn D.2} = \text{Eqn D.4} \]

\[ \frac{S^3}{6} + C_1S = \frac{S^3}{2} + C_3 + C_4 \]

\[ \frac{S^3}{6} + \frac{S^3}{2} + C_2S = \frac{S^3}{2} + C_3S + C_4 \]

\[ C_4 = \frac{S^3}{6} \]

Condition 5

\[ @ \quad x = 3S - a \quad \text{Eqn D.3} = \text{Eqn D.5} \]

\[ S(3S - a) + C_3 = 4S(3S - a) + C_5 \]

\[ C_3 = 9S^2 - 3Sa + C_5 \]

Condition 6

\[ @ \quad x = 3S - a \quad \text{Eqn D.4} = \text{Eqn D.6} \quad (a^2 = 9S^2 + a^2 - 6Sa) \]

\[ \frac{Sx^2}{2} + C_3x + C_4 = 2Sx^2 + C_5x + C_6 \]

\[ \frac{9S^3 + Sa^2 - 6S^2a}{2} + 3C_3S - C_3a + \frac{S^3}{6} = \ldots \]

\[ = \frac{9RS^3 + RSA^2 - 6RS^2a}{4} + 3SC_5 - C_3a + C_6 \]
\[
\frac{9S^3 + 6Sa^2 - 6S^2a}{2} + 3C_3S - C_3a + \frac{S^3}{6} = \ldots = 18S^3 + 2Sa^2 - 12S^2a + 3SC_5 - C_5a + C_6
\]

\[
\frac{27S^3 + 6S^3 - 108S^3}{6} + \frac{6S^2a + 24S^2a}{2} + 3C_3S - C_3a - 3SC_5 + C_5a = C_6
\]

\[
C_6 = \frac{-40S^3}{3} - \frac{3Sa^2}{2} + 9S^2a + C_5(3S - a) + C_5(a - 3S)
\]

**Condition 7**

\[\begin{align*}
@ x = 3S & \quad \text{Eqn D.5 = Eqn D.7} \\
4Sx + C_5 &= -2x^2 + 16Sx + C_7 \\
\frac{24S^2}{2} + C_5 &= -\frac{72S^2}{4} + 48S^2 + C_7 \\
C_5 &= C_7 + 18S^2
\end{align*}\]

**Condition 8**

\[\begin{align*}
@ x = 3S & \quad \text{Eqn D.6 = Eqn D.8} \\
2Sx^2 + C_5x + C_6 &= -\frac{2x^3}{3} + 8Sx^2 + C_7x + C_8 \\
\frac{216S^3 + 216S^3 - 864S^3}{12} + 3SC_5 + C_6 &= 3SC_7 + C_8 \\
C_8 &= -36S^3 + 3SC_5 + C_6 - 3SC_7
\end{align*}\]

**Find constants**

**Use conditions 5 & 6 to find \( C_6 \)**

\[
\begin{align*}
C_3 &= 9S^2 - 3Sa + C_5 \\
C_6 &= \frac{-40S^3}{3} - \frac{3Sa^2}{2} + 9S^2a + C_5(3S - a) + C_5(a - 3S)
\end{align*}
\]

expanding \( C_3 \) term of condition 6

\[
\begin{align*}
C_3(3S - a) &= 27S^3 - 9S^2a + 3SC_5 - 9S^2a + 3Sa^2 - C_5a \\
&= 27S^3 - 18S^2a + 3SC_5 + 3Sa^2 - C_5a
\end{align*}
\]

\[
\begin{align*}
C_6 &= \frac{-40S^3}{3} - \frac{3Sa^2}{2} + 9S^2a \ldots \\
&\quad + 27S^3 + 3Sa^2 - 18S^2a + 3SC_5 - C_5a \ldots + C_5a - 3SC_5
\end{align*}
\]
Use conditions 7 & 8 and $C_6$ to find $C_8$

\[
C_7 = C_7 + 18S^2 \\
C_8 = -36S^3 + 3SC_5 + C_6 - 3SC_7 \\
\quad = -36S^3 + 3SC_7 + 54S^3 + C_6 - 3SC_7 \\
\quad = 18S^3 + C_6 \\
\quad = 18S^3 + \frac{41S^3}{3} + \frac{3S a^2}{2} - 9S^2a \\
C_8 = \frac{95S^3}{3} + \frac{3S a^2}{2} - 9S^2a
\]

Use condition 2 to find $C_7$

\[
C_7 = -\frac{64S^2}{3} - \frac{C_8}{4S} \\
\quad = -\frac{64S^2}{3} - \frac{95S^2}{12} + \frac{3a^2}{8} + \frac{9Sa}{4} \\
C_7 = -\frac{351S^2}{12} - \frac{3a^2}{8} + \frac{9Sa}{4}
\]

Use condition 7 to find $C_5$

\[
C_5 = C_7 + 18S^2 \\
\quad = -\frac{351S^2}{12} - \frac{3a^2}{8} + \frac{9Sa}{4} + 18S^2 \\
C_5 = -\frac{135S^2}{12} - \frac{3a^2}{8} + \frac{9Sa}{4}
\]

Use condition 5 to find $C_3$

\[
C_3 = 9S^2 - 3Sa + C_5 \\
\quad = 9S^2 - 3Sa + \frac{-135S^2}{12} - \frac{3a^2}{8} + \frac{9Sa}{4} \\
C_3 = -\frac{27S^2}{12} - \frac{3a^2}{8} - \frac{3Sa}{4}
\]

Use condition 3 to find $C_1$

\[
C_1 = \frac{S^2}{2} + C_3 \\
\quad = \frac{S^2}{2} + \frac{-27S^2}{12} - \frac{3a^2}{8} - \frac{3Sa}{4} \\
C_1 = -\frac{21S^2}{12} - \frac{3a^2}{8} - \frac{3Sa}{4}
\]
APPENDIX D. 4ENF CLASSICAL BEAM THEORY EQUATIONS

Deflection equations

\begin{align*}
\text{Deflection equations} \\
y_{AB} &= \frac{P}{EI_m} \left( \frac{x^3}{6} + x \left( -\frac{21S^2}{12} - \frac{3a^2}{8} - \frac{3Sa}{4} \right) \right) \\
y_{BC} &= \frac{P}{EI_m} \left( \frac{Sx^2}{2} + x \left( -\frac{27S^2}{12} - \frac{3a^2}{8} - \frac{3Sa}{4} \right) + \frac{S^3}{6} \right) \\
y_{CD} &= \frac{P}{EI_m} \left( 2Sx^2 + x \left( -\frac{135S^2}{12} - \frac{3a^2}{8} + \frac{9Sa}{4} \right) + \frac{41S^3}{3} + \frac{3Sa^2}{2} - 9S^2a \right) \\
y_{DE} &= \frac{P}{EI_m} \left( -\frac{2x^3}{3} + 8Sx^2 + x \left( -\frac{351S^2}{12} - \frac{3a^2}{8} + \frac{9Sa}{4} \right) + \frac{95S^3}{3} + \frac{3Sa^2}{2} - 9S^2a \right)
\end{align*}

Deflection at load point \((y_{LP})\)

The deflection at the load point is the average of \(y_B\) and \(y_D\).

\begin{align*}
y_B &= y_{AB} \quad @ \quad x = S \\
y_{AB} &= \frac{P}{EI_m} \left( \frac{x^3}{6} + x \left( -\frac{21S^2}{12} - \frac{3a^2}{8} - \frac{3Sa}{4} \right) \right) \\
y_B &= \frac{P}{EI_m} \left( -\frac{19S^3}{12} - \frac{3Sa^2}{8} - \frac{3S^2a}{4} \right) \\
y_D &= y_{DE} \quad @ \quad x = 3S \\
y_{DE} &= \frac{P}{EI_m} \left( -\frac{2x^3}{3} + 8Sx^2 + x \left( -\frac{351S^2}{12} - \frac{3a^2}{8} + \frac{9Sa}{4} \right) + \frac{95S^3}{3} + \frac{3Sa^2}{2} - 9S^2a \right) \\
y_D &= \frac{P}{EI_m} \left( -\frac{54S^3}{3} + 72S^3 - \frac{1053S^2}{12} - \frac{9Sa^2}{8} + \frac{27S^2a}{4} + \frac{95S^3}{3} + \frac{3Sa^2}{2} - 9S^2a \right)
\end{align*}

\begin{align*}
y_{LP} &= \frac{y_B + y_D}{2} \quad \text{and} \quad P = \frac{P_{Tot}}{2} \\
&= \frac{P_{Tot}}{4EI_m} \left( -\frac{19S^3}{12} - \frac{3Sa^2}{8} - \frac{3S^2a}{4} - \frac{25S^3}{12} + \frac{3Sa^2}{8} - \frac{9S^2a}{4} \right) \\
&= \frac{P_{Tot}}{EI_m} \left( -\frac{11S^3}{12} - \frac{3S^2a}{4} \right)
\end{align*}

\text{(D.9)}

Check the 4ENF deflection equations

Check that the deflection at the supports is zero and that deflection continuity is maintained from one equation to the next.

\text{Check } y_{AB} \quad @ \quad x = 0

\begin{align*}
y_{AB} &= \frac{P}{EI_m} \left( \frac{x^3}{6} + x \left( -\frac{21S^2}{12} - \frac{3a^2}{8} - \frac{3Sa}{4} \right) \right) \\
y_{AB} &= 0 \quad @ \quad x = 0 \quad \checkmark
\end{align*}
Check \( y_{DE} @ x = 4S \)

\[
y_{DE} = \frac{P}{EI_m} \left( -\frac{2x^3}{3} + 8Sx^2 + x \left( -\frac{351S^2}{12} - \frac{3a^2}{8} + \frac{9Sa}{4} \right) + \frac{95S^3}{3} + \frac{3Sa^2}{2} - 9S^2a \right)
\]

\[
= \frac{P}{EI_m} \left( -\frac{128S^3}{3} + 128S^3 - \frac{1404S^3}{12} - \frac{12Sa^2}{8} + \frac{36S^2a}{4} + \frac{95S^3}{3} + \frac{3Sa^2}{2} - 9S^2a \right)
\]

\[
= \frac{P}{EI_m} \left( -\frac{512S^3 + 1536S^3 - 1404S^3 + 380S^3}{12} \right)
\]

\[
y_{DE} = 0 @ x = 4S \quad \checkmark
\]

Check \( y_{AB} = y_{BC} @ x = S \)

\[
y_{AB} = \frac{P}{EI_m} \left( \frac{x^3}{6} + x \left( -\frac{21S^2}{12} - \frac{3a^2}{8} - \frac{3Sa}{4} \right) \right)
\]

\[
= \frac{P}{EI_m} \left( \frac{S^3}{6} - \frac{21S^3}{12} - \frac{3a^2S}{8} - \frac{3S^2a}{4} \right)
\]

\[
= \frac{P}{EI_m} \left( -\frac{19S^3}{12} - \frac{3Sa^2}{8} - \frac{3S^2a}{4} \right)
\]

\[
y_{BC} = \frac{P}{EI_m} \left( \frac{Sx^2}{2} + x \left( -\frac{27S^2}{12} - \frac{3a^2}{8} - \frac{3Sa}{4} \right) + \frac{S^3}{6} \right)
\]

\[
= \frac{P}{EI_m} \left( \frac{Sx^3}{2} - \frac{27S^3}{12} - \frac{3Sa^2}{8} - \frac{3S^2a}{4} + \frac{S^3}{6} \right)
\]

\[
= \frac{P}{EI_m} \left( -\frac{19S^3}{12} - \frac{3Sa^2}{8} - \frac{3S^2a}{4} \right)
\]

\[
y_{AB} = y_{BC} @ x = S \quad \checkmark
\]
Check \( y_{BC} = y_{CD} \) @ \( x = 3S - a \) \( (x^2 = 9S^2 - 6Sa + a^2) \)

\[
y_{BC} = \frac{P}{EI_m} \left( \frac{Sx^2}{2} + x \left( \frac{-27S^2}{12} - \frac{3a^2}{8} - \frac{3Sa}{4} \right) + \frac{S^3}{6} \right) = \frac{P}{EI_m} \left( \frac{9S^3 - 6S^2a + Sa^2}{2} + \frac{-81S^2 + 27S^2a}{12} + \frac{-9S^2a + 3S^3}{8} + \frac{-9S^2a + 3S^2a}{4} + \frac{S^3}{6} \right) = \frac{P}{EI_m} \left( \frac{-25S^3}{12} - 3S^2a + \frac{Sa^2}{8} + \frac{3a^3}{8} \right)
\]

\[
y_{CD} = \frac{P}{EI_m} \left( 2Sx^2 + x \left( \frac{-135S^2}{12} - \frac{3a^2}{8} + \frac{9Sa}{4} \right) + \frac{41S^3}{3} + \frac{3S^2a}{2} - 9S^2a \right) = \frac{P}{EI_m} \left( 18S^3 - 12S^2a + 2Sa^2 + \frac{-405S^3 + 135S^2a}{12} + \frac{-9S^2a + 3S^3}{8} + \frac{27S^2a - 9Sa^2}{4} \right) + \ldots = \frac{P}{EI_m} \left( \frac{-25S^3}{12} - 3S^2a + \frac{Sa^2}{8} + \frac{3a^3}{8} \right)
\]

\[
y_{BC} = y_{CD} \quad \@ \quad x = 3S
\]

Check \( y_{CD} = y_{DE} \) @ \( x = 3S \)

\[
y_{CD} = \frac{P}{EI_m} \left( 2Sx^2 + x \left( \frac{-135S^2}{12} - \frac{3a^2}{8} + \frac{9Sa}{4} \right) + \frac{41S^3}{3} + \frac{3S^2a}{2} - 9S^2a \right) = \frac{P}{EI_m} \left( 18S^3 - 12S^2a + 2Sa^2 + \frac{-405S^3 + 135S^2a}{12} + \frac{-9S^2a + 3S^3}{8} + \frac{27S^2a - 9Sa^2}{4} \right) = \frac{P}{EI_m} \left( \frac{-25S^3}{12} + 3S^2a + \frac{Sa^2}{8} + \frac{3a^3}{8} \right)
\]

\[
y_{DE} = \frac{P}{EI_m} \left( \frac{2x^3}{3} + 8Sx^2 + x \left( \frac{-351S^2}{12} - \frac{3a^2}{8} + \frac{9Sa}{4} \right) + \frac{95S^3}{3} + \frac{3S^2a}{2} - 9S^2a \right) = \frac{P}{EI_m} \left( -54S^3 + 72S^3 - \frac{1053S^2a}{12} + \frac{-9S^2a + 27S^2a}{4} + \frac{95S^3}{3} + \frac{3S^2a}{2} - 9S^2a \right) = \frac{P}{EI_m} \left( \frac{-25S^3}{12} + 3S^2a + \frac{Sa^2}{8} + \frac{9S^2a}{4} \right) = \frac{P}{EI_m} \left( \frac{-25S^3}{12} + 3S^2a + \frac{Sa^2}{8} + \frac{9S^2a}{4} \right)
\]

\[
y_{CD} = y_{DE} \quad \@ \quad x = 3S
\]
D.2 Derivation of the 4ENF compliance equation

The equation for load point deflection, Equation D.9, is used to derive the 4ENF compliance equation.

\[
\text{Compliance} = C = \frac{-y_{LP}}{P_{Tot}} = \frac{1}{EI_m} \left(\frac{11S^3}{12} + \frac{3S^2a}{4}\right) \tag{D.10}
\]

Compliance when \( S = 25 \text{ mm} \)

The physical test setup uses \( S = 25 \text{ mm} \).

\[
C = \frac{-y_{LP}}{P_{Tot}} = \frac{1}{EI_m} \left(\frac{171875}{12} + \frac{1875a}{4}\right)
\]

D.3 Validation of the 4ENF compliance equation

The 4ENF compliance equation, Equation D.10, can be checked by comparing it against the 4ENF energy release rate (ERR) equation given by Martin [45]. The ERR equation given by Martin is:

\[
G = \frac{3P^2S^2}{8B^2D} \tag{D.11}
\]

where:
- \( P \) = applied load
- \( S \) = outer roller spacing
- \( B \) = breadth of the specimen
- \( D \) = flexural rigidity of the specimen
  
  \( D = \frac{EI_m}{B} \)

\( I_m \) is the second moment of area of the monolithic (un-cracked) portion of the 4ENF specimen. Substituting for \( D \) in Equation D.11 gives:

\[
G = \frac{3P^2S^2}{8BEI_m} \tag{D.12}
\]

The fundamental equation for ERR given by Broek [43], Equation 4.1, shows that ERR is proportional to the rate of change of compliance with crack length (\( G \propto \frac{dC}{da} \)). Equation D.10 can be used to determine \( \frac{dC}{da} \):

\[
C = \frac{1}{EI_m} \left(\frac{11S^3}{12} + \frac{3S^2a}{4}\right) \tag{Equation D.10}
\]

\[\therefore \quad \frac{dC}{da} = \frac{3S^2}{4EI_m} \tag{D.13}\]
Substituting for $\frac{dC}{da}$ into Equation 4.1:

$$G = \frac{P^2 dC}{2Br_a} \quad \text{(Equation 4.1 (Broek [43]))}$$

$$= \frac{3P^2 S^2}{8BEI_m} \quad \text{(D.14)}$$

Equation D.14 and Equation D.12 are equal, this demonstrates that the equation for 4ENF compliance, Equation D.10, agrees with the equation for 4ENF ERR that is given by Marti [45], Equation D.11.
Appendix E

Plots of the 4ENF test results

4ENF-GG-7 tested at 1 mm/minute

The results for specimen GG-7 were not captured by the high speed logger due to a problem with the trigger signal. The results from the Instron logger were used to generate toughness results for this specimen.

4ENF-GG-8 tested at 1 mm/minute
4ENF-GG-13 tested at 1 mm/minute

4ENF-GG-9 tested at 50 mm/minute
APPENDIX E. PLOTS OF THE 4ENF TEST RESULTS

4ENF-GG-10 tested at 50 mm/minute

4ENF-GG-11 tested at 50 mm/minute
APPENDIX E. PLOTS OF THE 4ENF TEST RESULTS

4ENF-GG-12 tested at 50 mm/minute

4ENF-GG-14 tested at 350 mm/minute
4ENF-GG-15 tested at 350 mm/minute

APPENDIX E. PLOTS OF THE 4ENF TEST RESULTS

4ENF-GG-17 tested at 350 mm/minute
4ENF-GG-18 tested at 750 mm/minute

4ENF-GG-19 tested at 750 mm/minute
4ENF-GG-20 tested at 750 mm/minute

4ENF-GG-21 tested at 1500 mm/minute
APPENDIX E. PLOTS OF THE 4ENF TEST RESULTS

4ENF-GG-22 tested at 1500 mm/minute

4ENF-GG-23 tested at 1500 mm/minute
APPENDIX E. PLOTS OF THE 4ENF TEST RESULTS

4ENF-GG-28 tested at 3000 mm/minute

4ENF-GG-29 tested at 3000 mm/minute
4ENF-GG-31 tested at 6000 mm/minute

4ENF-GG-32 tested at 6000 mm/minute
**APPENDIX E. PLOTS OF THE 4ENF TEST RESULTS**

4ENF-GG-33 tested at 6000 mm/minute

4ENF-GG-34 tested at 6000 mm/minute
4ENF-GG-35 tested at 6000 mm/minute