Fundamental diagrams of airport surface traffic: models and applications

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Abstract

This paper reveals and explores the flow characteristics of airport surface network on both mesoscopic and macroscopic levels. We propose an efficient modeling approach based on the cell transmission model for simulating the spatio-temporal evolution of flow and congestion on taxiway and apron networks. The existence of link-based fundamental diagram that expresses the functional relationship between link density and flow is demonstrated using empirical data collected in Guangzhou Baiyun airport. The proposed CTM-based network model is shown to be an efficient and accurate method capable of supporting air traffic prediction and decision support. In addition, using both CTM-based computation and empirical data, we further reveal the existence of an aggregate relationship between traffic density and runway throughput, which is referred to as macroscopic fundamental diagram (MFD) in the literature of road traffic. The MFD on the airport surface is analyzed in depth, and utilized to devise robust off-block control strategies under uncertainties, which is shown to significantly outperform existing off-block control methods.

Keywords: airport surface operation, cell transmission model, fundamental diagram, macroscopic fundamental diagram, off-block optimization

1. Introduction

Air traffic flow management seeks to balance air traffic demand and supply in such a way that improves air traffic efficiency and reduces flight delays. As the most stringent bottleneck in the air traffic network, airport surface is often subject to severe traffic congestion, such as stop-and-go movement on airport surface, excessive runway queuing, and even gridlock, which leads to the increase of taxiing time, fuel burn and emission. Severe congestion facing departure traffic at airport surface has now become a major challenge in air traffic management (Balakrishnan and Jung, 2007).

Earlier work on the congestion mitigation for departure traffic on airport surface has primarily focused on the optimization of taxiway operation, which is typically formulated as mixed integer linear programs (Marín and Codina, 2008), or solved with metaheuristic methods (García et al., 2005). Taxiing optimization problems seek to assign to each aircraft taxiing route and required time of arrival (RTA) at the key nodes along the route to avoid conflicts and improve taxiing efficiency (Yang, 2012). In 2009, EUROCONTROL proposed the operational concept of DMAN (Departure Manager), and that “maintaining the optimal number of aircraft on the taxiway” is a key undertaking of DMAN (Burgain, 2010).

The key to the understanding and mitigation of airport surface congestion is to capture, in an effective way, traffic flow characteristics and mechanisms under which congestion forms, propagates, and dissipates. Traditional methods in this regard include statistical methods (Shumsky, 1995; Idris et al., 2001), stochastic queuing models (Pujet et al., 1999; Simaiakis and Balakrishnan, 2009), and microsimulation methods (Couluris et al., 2008; Nakamura et al., 2010; Ryota, 2013). However, statistical methods and queuing network models do not capture the flow characteristics of
traffic in the surface network (such as acceleration, deceleration, merging and diverging behavior) or congestion caused by factors other than runway queuing (such as delay at network nodes due to conflict). Although microsimulation models, if well calibrated, could accurately describe the network traffic dynamics, they typically require substantial computational time and do not provide timely and robust outcome for real-time application.

Given the similarities between airport surface traffic and network car traffic, and inspired by the mesoscopic traffic modeling (Daganzo, 1994, 1995), this paper proposes an adaptation of the cell transmission model (CTM) to describe and predict the dynamics of flow and congestion on airport surface networks. Mesoscopic traffic models ignore certain granularities at the individual level, and capture the aggregate dynamics in a robust and computationally efficient way. In particular, we describe the flow propagation within links using the fundamental diagram (FD), which expresses the relationship between link flow and density. Junction movements such as merge, diverge, and crossover are modeled using the notions of demand and supply that interact under specific protocols. Moreover, modeling scenarios unique to airport surface, such as runway queuing and apron traffic, are integrated into the CTM framework. Overall, the proposed mesoscopic modeling framework is flexible in accommodating different network topology and flow characteristics, and computationally tractable to support strategic planning and real-time operation.

To the best of the authors’ knowledge, this study is the first to explore the link-based FD on airport surface for the modeling and simulation of surface traffic. The FD is calibrated and validated using empirical data from Guangzhou Baiyun Airport (ZGGG) in China. As an immediate application, we use the CTM to derive a network-level macroscopic fundamental diagram (MFD), which is again validated using empirical data. The MFD (Daganzo, 2007) characterizes the aggregate behavior of the network traffic in terms of average density and trip completion rate, in a parsimonious way capable of capturing the key demand-supply interactions. The MFD aims at reducing the modeling complexity while capturing the main dynamic features of congestion. The main concept is to utilize the MFD models to manage large-scale networks in hierarchical control approach, having the capability of integrating the collective behavior of the network in the local control decisions. The MFD is the basis for a variety of flow control algorithms aiming at maximizing the network performance of a particular transportation system, see Section 2.2.

The same concept of the MFD can be applied for airport surface traffic. In a recent study (Simaaiakis et al., 2014), an aggregate curve, which relates the jet take-off rate as a function of the number of departing aircraft on the ground, is utilized to devise a simple yet effective off-block control. However, that method embraces a simplistic bang-bang control strategy, with idealized curve parameterized by the number of arrivals and absence of uncertainties. In this paper, we first demonstrate the existence of an MFD curve for airport surface, and then we propose a robust off-block feedback control that incorporates a range of uncertainties in the MFD, arrival rate, and taxi-out traffic. Different variants of the robust control are compared with the one from Simaiakis et al. (2014) using the CTM-based simulation platform for the case study of ZGGG, and are shown to yield superior performance in terms of reducing total delay and runway queuing.

The main contribution of this paper may be summarized as follows:

- We propose a mesoscopic dynamic model for airport surface networks based on the CTM. The existence and validity of the FD is shown with empirical data, so is the accuracy of the proposed model in capturing network-wide propagation of flow and congestion.

- Using the CTM-based simulation platform, we derive a network-level MFD for the Guangzhou Baiyun Airport, which is verified against an empirical MFD for the same network. This further proves the consistency of the propose mesoscopic model with aggregated dynamics on a network level.

- Several new robust off-block control strategies are proposed based on the MFD, which outperform existing ones (Simaaiakis et al., 2014) in terms of reducing departure delays and runway queuing in a variety of uncertain environments. This is demonstrated on the CTM-based simulation platform.

The CTM-based mesoscopic network model is sufficiently general to support a wide range of tasks involved in the design, planning, and operation of airport surface traffic. It captures realistic taxiing traffic dynamics such as free-flow, trailing, turning maneuver, queuing, and holding, with intuitive parameters and rules. Its computational efficiency over microsimulation, as adequately shown for vehicular traffic, makes it an attractive modeling platform for airport traffic research. These have been shown in this paper through the discovery of a MFD for the airport surface traffic and the evaluation of a number of off-block control strategies.
The rest of this paper is organized as follows. Section 2 provides a review of relevant literature on departure traffic modeling and macroscopic fundamental diagram. In Section 3 we present the cell transmission model and its adaptation to airport surface network. Section 4 reveals the MFD on airport surface based on empirical data, and devise several MFD-based robust off-block control methods to mitigating surface congestion on airports. We show the details of the CTM on airport surface and validate its modeling accuracy using empirical data in Section 5. Section 6 presents a case study of the off-block controls on a real-world airport operational environment. Finally, Section 7 provides some concluding remarks.

2. Literature review

2.1. Departure traffic management

There are two aspects of departure traffic management at airports. The first aspect focuses on the prediction of taxiing time, which is a crucial task for Airport Collaborative Decision Making (A-CDM) to compute Calculated Off-block Time (COBT) based on allocated takeoff time slot. Shumsky (1995) considers a range of explanatory factors, such as airline, departure runway, and departure demand, in order to predict aircraft taxiing time. Idris et al. (2001) identify main factors that affect the taxiing time and establish forecast model based on regression. However, they do not explicitly account for the runway service processes, and therefore do not link taxiing delay with runway capacity constraints. A stochastic queuing model is proposed by Pujet et al. (1999), who define the taxiing time as the sum of a deterministic travel time on taxiways and a stochastic queuing time at the end of the runway captured by a stochastic queuing process. Khadilkar and Balakrishnan (2014) simulate the aircraft taxi-out process by representing the airport surface as a network. The total taxi time, as the sum of travel times on different links of the same path, can be calculated by introducing a set of suitable random processes to model the distribution of link travel times. The development of database systems brings large amount of empirical taxiing data that supports surface taxi flow modeling and optimization. Zhang et al. (2010) propose an ordered response model by combining a series of variables and an iterative algorithm for predicting taxi-out time. Considering the uncertainty in taxiing time, fuzzy rule (Chen et al., 2011) and machine learning method (Lee et al., 2015) have been adopted to enhance the robustness of prediction during daily operations. Zhang and Wang (2017) develop a new method relying on and econometrics regression models to estimate taxiing performance.

The second research direction focuses on collaborated operation on airport surface, aiming at an integrated solution for surface congestion mitigation. Kim et al. (2009) study gate assignment and off-block sequence optimization as means to effectively alleviate ramp congestion. Simaiakis and Balakrishnan (2009) simulate surface traffic from the gates to the runway using queuing models and illustrate that a system-wide optimization of queuing patterns has the potential to achieve further reduction in taxiing time and emissions. To enhance the robustness of the control strategy under congestion, Carr et al. (2005) employ the modeling and optimization of dynamic queues with random factors included in the surface operation. Khadilkar and Balakrishnan (2014) design a control algorithm to minimize the delay of aircraft while limiting the impact on throughput for a complete airport surface network. Simaiakis et al. (2014) demonstrate the feasibility of dynamically adjusting the off-block rate to maintain the so-called critical value of taxiing-out traffic, in order to maximize the departure throughput. In order to refine aircraft pushback management during rush hour, aircraft pushback slot allocation based on congestion pricing (aka “external cost of surface congestion”) was explored while considering monetary compensation based on the quality of the surface operations (Liu et al., 2017). To support real-time operation, a mixed-integer linear programming (MILP) approach, which allows a 15-second time resolution of decisions, is proposed to optimize the time-based taxing routes Evertse and Visser (2017). However, taxiing time uncertainty remains a crucial factor that can compromise the effectiveness of solutions. A robust optimization approach for metering aircraft departures under uncertainty in the taxi-out process is proposed by Murca (2017) to dynamically determine an optimal and robust sequence and schedule of aircraft releases from the gate.

2.2. Macroscopic fundamental diagram

Modeling of transportation networks based on MFDs has created scientific attention to many disciplines within and outside the transportation field. The MFDs were found to well describe the traffic flow dynamics of a congested urban network, e.g. Daganzo (2007); Geroliminis and Daganzo (2008); Buisson and Ladier (2009); Mazloumian et
al. (2010); Mahmassani et al. (1987); Olszewski et al. (1995); Ortigosa et al. (2014); Leclercq et al. (2014); Gayah et al. (2011) and many others. The MFDs can respectively link between network vehicle density (or accumulation) and network space-mean flow (or trip completion flow). The MFD has the following link-form: network flow increases as accumulation increases up to a critical accumulation where the flow is maximized, and if the accumulation increases beyond the critical one, then the flow decreases until it reaches zero at gridlock. Different MFD-based state space dynamic models for control design, describing the traffic flow dynamics of separate or multi-region systems, have been founded (in one form or another) relying on vehicle conservation equations Keyvan-Ekbatani et al. (2012); Geroliminis et al. (2013); Haddad et al. (2013).

MFDs have been showed to enable the design of elegant management strategies to improve mobility and decrease delays in large road networks. Among the different proposed management architectures based on MFD, the perimeter feedback control, starting with the work Daganzo (2007), is one of the most promising new and rapidly developing. In perimeter control, the idea is to manipulate the transfer flows along the perimeter of an urban region. MFD-based perimeter control has been proposed for single- and multi-region cities in e.g. Daganzo (2007); Keyvan-Ekbatani et al. (2012); Geroliminis et al. (2013); Aboudolas and Geroliminis (2013); Zhang et al. (2013). Different control approaches have been used to solve the perimeter control problems, e.g., the Model Predictive Control approach has been used in Geroliminis et al. (2013); Haddad et al. (2013), and the classical feedback control approach has been implemented in Keyvan-Ekbatani et al. (2012); Aboudolas and Geroliminis (2013), where a Proportional-Integral (PI) perimeter controller has been designed for an urban region in Keyvan-Ekbatani et al. (2012), while in Aboudolas and Geroliminis (2013) a multivariable feedback regulator for multiple regions has been designed.

Recent research efforts have been devoted to enhance the traffic aggregate flow modeling and to develop applicable perimeter control schemes (Leclercq et al., 2015; Laval and Castrillon, 2015; Haddad, 2015; Ramezani et al., 2015; Mahmassani et al., 2013; Keyvan-Ekbatani et al., 2015; Haddad et al., 2016; Mariotte et al., 2017; Yildirimoglu et al., 2014; Kouvelas et al., 2017; Arnott, 2013; Fosgerau, 2015). Recent works aim at improving the MFD models in various aspects: stochastic approximations for uncertain MFDs are introduced in Laval and Castrillon (2015), while partially uncertain (interval) MFDs with upper and lower bounds are integrated in the flow dynamics in Haddad and Shraiber (2014) and Haddad (2015). Based on the improved models, enhanced perimeter control schemes have been developed, e.g. robust perimeter controllers have been designed to systematically take into account uncertainties in MFD-based dynamics in Haddad (2015) and Haddad and Shraiber (2014).

Previous works of perimeter control with MFD dynamics for urban networks inspired us to develop off-block control with MFD dynamics for airport surface networks, see Section 4. The perimeter control aims at metering the traffic flow entering the urban region, similarly to the off-block control which aims at metering (or holding at gates) the airplane flow rate entering the airport surface.

3. CTM-based modeling of airport surface

An airport surface network consists of directed links and junctions. Figure 1 shows the surface network of Guangzhou Baiyun airport in 2014, which consists of runway, taxiway, and apron networks. These sub-networks have diverse configurations and flow characteristics. In particular, the taxiway network connects the runway network with the apron network, and is where most merging, diverging, and crossover behavior take place. It is also the main location that generates congestion and delay.

3.1. Fundamental diagrams of aircraft flow at airport surface

Similar to road traffic, the aircraft movement at airport surface is characterized by the balance between demand and supply, which are typically captured by a density-flow relationship known as the fundamental diagram for road traffic (Daganzo, 1994). In order to empirically study the flow characteristics of airport surface, we choose aircraft movement data collected form Guangzhou Baiyun airport on July 22, 2014, which are distinguished by apron and taxiway. Figure 2 shows the density-speed-flow relationships. The FDs in Figure 2 were established for all the links of apron and taxiway sub-networks. This is justified by the general taxing regulations, which imply that air traffic (both taxi-in and taxi-out) along each link in the same sub-network are homogeneous. Moreover, the density values are derived from all the aircraft traveling along the links, including both taxi-in and taxi-out.
Figure 1: The surface network of Baiyun International Airport (ZGGG), consisting of the runways, the taxiway network, and the apron network.

Figure 2 shows two distinct phases for both apron and taxiway movements; that is, a free-flow phase and a congested phase. Overall, the relationships are consistent with the fundamental diagrams in road traffic, with the following distinctive features.

- In the free-phase, due to different taxiing speed limitations of different airlines and/or aircraft types, the variation in speed is relatively high in the low-density region. This is apparent from the density-velocity diagram.
- There exist qualitative differences between the fundamental diagrams of apron and taxiway. Firstly, the con-
gested region of taxiway traffic is much more scattered because the speed is much higher in taxiways and prone to high variations. Another reason for the low variation in the congested phase of apron traffic is the gate assignment strategies applied by the airport operation center that proactively avoids potential conflicts in the apron area.

- Extremely high density is hardly observed in apron area. The reason is that the controllers tend to hold aircraft at the gates when a foreseeable conflict comes up. In contrast, severe queuing is much more common on taxiways especially at the runway queuing area.

3.2. The CTM with application to airport surface

As suggested by the empirical data (Figure 2), we construct the fundamental diagram (FD) for the movement of aircraft on the airport surface. The FD describes the relationship between aircraft density and flow along a homogeneous segment. The cell transmission model (Daganzo, 1994, 1995) is a widely used mesoscopic traffic flow model, which is based on the Godunov discretization of the well-known Lighthill-Whitham-Richards kinematic wave model (Lighthill and Whitham, 1955; Richards, 1956). The CTM is based on a trapezoidal or triangular fundamental diagram (see Figure 3), and propagates flow and congestion through straightforward bookkeeping, which admits efficient computation for large and complex road or airport surface networks.

![Figure 3: Density-flow relationship on traffic flow at apron or taxiway links. Left: Trapezoidal; right: Triangular.](image)

It is important to note that the surface network links are uni-directional for all the taxi-in and taxi-out aircraft for the following reasons. To avoid head-on conflict and to reduce the workload of ground controllers, all the taxiing routes, which link the runway and the gates, are pre-determined and traverse uni-directional links. This means, once the runway direction and mode is determined, only one traveling direction is permitted for each link, for both taxi-in and taxi-out aircraft.

Each link in the network is partitioned into a number of cells of appropriate size, along with a time discretization following the Courant-Friedrichs-Lewy (CFL) condition (Courant et al., 1928). The fundamental diagrams corresponding to different types of sub-networks are used to derive cell and transmission parameters. In Section 3.3, the flow propagation through ordinary, merge, diverge, and crossover cells are described by integrating the modeling procedure from Daganzo (1994) and Daganzo (1995) with the unique characteristics of surface traffic. In Section 3.4 we discuss some unique modeling scenarios specific to airport surface, and introduce novel techniques to capture their essential operational features. To keep the paper concise, in the following presentation we omit widely distributed materials on CTM, and focus only on parts of the modeling that are unique to taxiing traffic on airport surface.

3.3. Ordinary, merge, diverge and crossover cells

We consider the cell size $\Delta x$ and time increment $\Delta t$ such that the CFL condition $\frac{\Delta x}{\Delta t} \geq v$ holds, where $v$ denotes the free-flow speed. The following notations are used in this paper.
\( v_i \): Forward wave speed (free-flow speed) of cell \( i \); see Figure 3
\( w_i \): Backward wave speed of cell \( i \); see Figure 3
\( Q_i \): Maximum number of aircraft that can be transmitted through cell \( i \) within one time step
\( m_i(t) \): The number of aircraft in cell \( i \) during time step \( t \)
\( y_i(t) \): The number of aircraft entering cell \( i \) from its upstream cell \( i - 1 \) during time step \( t \)
\( M_i \): The holding capacity of cell \( i \) (i.e. the product of the cell length and jam density)

Unlike car traffic, a minimum spatial headway between two taxiing aircraft must be strictly maintained according to International Civil Aviation Organization (ICAO) regulation (not less than 50 meters) due to aircraft vortex. It is vital to investigate the specific separation standard when modeling individual airports. The spatial headway matrix adopted in the Baiyun airport, which is selected as the case study in this paper, is shown in Table 1 (Jiang et al., 2013).

<table>
<thead>
<tr>
<th>Type of leading aircraft</th>
<th>Type of trailing aircraft</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light (L)</td>
<td>Light (L)</td>
</tr>
<tr>
<td></td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>100</td>
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<tr>
<td></td>
<td>100</td>
</tr>
<tr>
<td>Medium (M)</td>
<td>200</td>
</tr>
<tr>
<td></td>
<td>200</td>
</tr>
<tr>
<td></td>
<td>200</td>
</tr>
<tr>
<td>Heavy (H)</td>
<td>300</td>
</tr>
<tr>
<td></td>
<td>300</td>
</tr>
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<td></td>
<td>300</td>
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</tbody>
</table>

We let \( p_{ab} \) be the probability that the leading aircraft is of type \( a \) and the trailing aircraft is of type \( b \); let \( D_{ab} \) be the minimum separation as shown in Table 1 \((a, b \in \{L, M, H\})\). The probability \( p_{ab} \) can be estimated using field data. Then, the average minimum separation \( \bar{D} \) can be estimated as \( \bar{D} = \sum_{a,b} p_{ab} D_{ab} \). Therefore, the maximum number of aircraft a cell of size \( \Delta x \) can hold can be approximated as

\[
M_i = \frac{\Delta x}{\bar{L} + \bar{D}} \tag{3.1}
\]

where \( \bar{L} \) denotes the average aircraft length.

3.3.1. Ordinary cells

The fundamental recursion for the dynamics in ordinary cells is given as (Daganzo, 1994, 1995)

\[
\begin{align*}
\dot{m}_i(t + 1) &= m_i(t) + y_i(t) - y_{i+1}(t) \\
y_i(t) &= \min \left\{ m_{i-1}(t), Q_i, \frac{w_i}{v_i}(M_i - m_i(t)) \right\}
\end{align*} \tag{3.2}
\]

In addition, we define the demand and supply of a cell as:

\[
\begin{align*}
D_i(t) &= \min \{ Q_i, m_i(t) \} \\
S_i(t) &= \min \{ Q_i, \frac{w_i}{v_i}(M_i - m_i(t)) \}
\end{align*} \tag{3.3}
\]

The demand \( D_i(t) \) represents the maximum number of aircraft that can be sent from cell \( i \) during time step \( t \), and the supply \( S_i(t) \) represents the maximum number of aircraft that can be received by cell \( i \) during time step \( t \).

Before discussing the junction models, we note that an important distinction between car traffic and airport surface traffic is the existence of a point-capacity constraint (indicated as \( M \) in Figure 4) at the junctions (merge/diverge/crossover) of the airport surface. This is mainly due to wingspan collision avoidance as illustrated in Figure 4. In practice, a circular protection zone of radius \( R \) poses the separation constraints in addition to the vortex-induced minimum separation \( \bar{D} \). However, given the current aircraft sizes, such a circular zone is usually no bigger than 50m in radius, which is much smaller than the vortex-induced separation (see Table 1). Thus, when both constraints apply, the latter always prevail. In addition, the vortex-induced separation constraint only applies when the leading and trailing aircraft are traveling in the same direction. These facts will be crucial for our discussion of the junction models below.
3.3.2. Merge cells

We first consider the merge junction shown in Figure 4. As can be seen from the figure, any two aircraft exiting from the upstream cells will face the same direction of travel (i.e. in the downstream cell), therefore the vortex-induced separation constraint applies here, while the wingspan constraint can be ignored.

We let $y_{1 \rightarrow 3}(t)$ be the number of aircraft transmitted from cell 1 to cell 3 during time step $t$; similar notations are used below with obvious meanings. The following demand and supply constraints must be satisfied:

$$y_{1 \rightarrow 3}(t) \leq D_1(t), \quad y_{2 \rightarrow 3}(t) \leq D_2(t), \quad y_{1 \rightarrow 3}(t) + y_{2 \rightarrow 3}(t) \leq S_3(t)$$

(3.4)

In addition, we assume that flow coming from cells 1 and 2 receive certain priorities, $p_1 > 0$ and $p_2 > 0$ such that $p_1 + p_2 = 1$. The air traffic controller should ensure the equity of aircraft operations besides safety and efficiency. In merge scenarios, the upstream taxiway segment with more aircraft is normally assigned higher priority to avoid accumulation of congestion on that segment. The demand-based priority rule (Jin and Zhang, 2003) is selected in this paper to best interpret such an equity principle. This rule stipulates that the discharge flows $y_{1 \rightarrow 3}(t)$ and $y_{2 \rightarrow 3}(t)$ follow the same ratio as the demand-based priorities $p_1(t)$ and $p_2(t)$. The solution is given explicitly as:

$$
\begin{align*}
    y_{1 \rightarrow 3}(t) &= \min \left\{ D_1(t), \ p_1(t) \cdot \min \left\{ S_3(t), \ D_1(t) + D_2(t) \right\} \right\}, & p_1(t) &= \frac{D_1(t)}{D_1(t) + D_2(t)} \\
    y_{2 \rightarrow 3}(t) &= \min \left\{ D_2(t), \ p_2(t) \cdot \min \left\{ S_3(t), \ D_1(t) + D_2(t) \right\} \right\}, & p_2(t) &= \frac{D_2(t)}{D_1(t) + D_2(t)}
\end{align*}
$$

(3.5)

3.3.3. Diverge cells

We focus on the diverge junction shown in Figure 4. Since aircraft discharged from the upstream cell travel in the same direction before turning to the downstream cell, only the vortex-induced separation constraint applies.
It is assumed that flow leaving cell 4 advances to cells 5 and 6 according to some turning ratios \( \alpha_{4 \rightarrow 5}(t) \geq 0 \) and \( \alpha_{4 \rightarrow 6}(t) \geq 0 \), which sum up to be one. In view of (3.4) and the flow maximization principle, it is easy to derive the following solution for the diverge junction:

\[
y_{4 \rightarrow 5}(t) = \alpha_{4 \rightarrow 5}(t) \cdot \min \left\{ D_4(t), \frac{S_5(t)}{\alpha_{4 \rightarrow 5}(t)}, \frac{S_6(t)}{\alpha_{4 \rightarrow 6}(t)} \right\} \tag{3.6}
\]

\[
y_{4 \rightarrow 6}(t) = \alpha_{4 \rightarrow 6}(t) \cdot \min \left\{ D_4(t), \frac{S_5(t)}{\alpha_{4 \rightarrow 5}(t)}, \frac{S_6(t)}{\alpha_{4 \rightarrow 6}(t)} \right\}
\]

Note that the turning ratios \( \alpha_{4 \rightarrow 5}(t) \) and \( \alpha_{4 \rightarrow 6}(t) \) are not exogenously given; rather, they are determined by the pre-defined aircraft routing information, which is the input of the simulation. In this regard, the CTM-based network simulation is similar to the dynamic network loading problem extensively studied in the dynamic traffic assignment literature (Friesz et al., 2013); and further details are omitted here.

### 3.3.4. Crossover cells

We now focus on the crossover junction shown in Figure 4. Given that no turning is allowed at the junction \(^1\), we make the following observation:

1. only the vortex-induced separation constraint is applied when two or more consecutive aircraft from the same approach advance through the junction without interruption from the other approach;
2. only the wingspan constraint is applied when the two upstream approaches take turn to discharge aircraft.

Here, we define the demand-based priorities \( p_7, p_8 \) for the upstream cells 7 and 8, respectively, following the formula (3.5). Without loss of generality, we assume that \( D_7 \leq D_8 \), so that \( p_7 \leq p_8 \). Then, the probability of case (2) is \( p_8 - p_7 \), and the probability of case (1) is \( 2p_7 \). Therefore, the Average Spatial Headway (ASH) is computed as:

\[
\text{ASH} = (2p_7) \cdot 2R \text{ (meter/aircraft)} + (p_8 - p_7) \cdot \bar{D} \text{ (meter/aircraft)}
\]

where \( R \) is the radius of the wingspan-induced protection zone, and \( \bar{D} \) is the vortex-induced minimum separation. Assuming that aircraft move at the free-flow speed when crossing the junction, the point capacity at the crossover junction is estimated as:

\[
C_M = \frac{v}{\text{ASH}} = \frac{v(D_7 + D_8)}{4D_7R + (D_8 - D_7)\bar{D}} \tag{3.7}
\]

where \( v \) denotes the free-flow speed. Note that (3.7) is valid only when \( \max(D_7, D_8) > 0 \); otherwise, if both demands are zero no junction model is needed.

The flows discharged from cells 7 and 8 are constrained by not only the supplies of their respective downstream cells, but also the point capacity (3.7). The latter \( (C_M) \) is shared by both incoming approaches, and we employ a similar, demand-based priority assignment as in (3.5)

\[
\begin{align*}
y_{7 \rightarrow 9}(t) &= \left\{ D_7(t), \frac{D_7(t)}{D_7(t) + D_8(t)} \cdot C_M \cdot \Delta t, S_9(t) \right\} \\
y_{8 \rightarrow 10}(t) &= \left\{ D_8(t), \frac{D_8(t)}{D_7(t) + D_8(t)} \cdot C_M \cdot \Delta t, S_{10}(t) \right\}
\end{align*}
\tag{3.8}
\]

### 3.4. Modeling scenarios specific to airport surface

#### 3.4.1. Runway queuing

Departing aircraft wait at the end of the runway before taking off. Therefore, the cells towards the end of the runway are the main queuing area, whose average density will influence, not just one, but all of their upstream cells simultaneously. This is due to the fact that ground control tends to meter aircraft from entering the parallel artery runway from the by-pass taxiway when the runway queuing area reaches certain level of saturation; see Figure 5.

\(^1\)If turning is allowed, then we have a 2-by-2 junction model, which can be treated in similar ways as the merge/diverge junctions. Due to space limitation, in this paper we do not perform an exhaustive discussion of different junction geometries, and instead refer the reader to Garavello et al. (2016) for a more extensive coverage.
There are different ways to mathematically represent such a control mechanism. In this paper, we let $N_q$ be the holding capacity (in number of aircraft) of the queuing area at the end of the runway, and $M_q(t)$ be the total traffic volume at time $t$ in the parallel artery taxiway. We also denote by $Q_{j+z\rightarrow i+n}(t)$ the transmission capacity (in number of aircraft) from cell $j + z$ to cell $i + n$ (see Figure 5) during time step $t$. When ground control is applied in view of the congestion in the queuing area, the discharge flow from the by-pass taxiway cell $j + z$ is reduced according to:

$$Q_{j+z\rightarrow i+n}(t) = \min\{Q_{i+n}, \Psi_{j+z\rightarrow i+n}(t)\}, \quad (3.9)$$

where $Q_{i+n}$ is the transmission capacity of cell $i + n$, and

$$\Psi_{j+z\rightarrow i+n}(t) = \max\{N_q - M_q(t), 0\} \times \gamma \quad (3.10)$$

Here, $\gamma \in (0, 1)$ is some priority parameter that can be derived either exogenously according to expert experience or endogenously, e.g. proportional to the cell $j + z$'s demand, as is done in this paper. The treatment of the flow control at other by-pass taxiway cells is completely similar.

### 3.4.2. Runway network

In many airport surface networks, the runway serves as both the source (sink) of arriving (departing) air traffic. As shown in the left picture of Figure 6, the runway itself may be modeled as a single cell (since it can be occupied by no more than one aircraft at the same time for safety reasons) located at the center of a merge-diverge intersection.

In contrast to how $2\times2$ intersections are typically modeled for vehicular traffic (Garavello et al., 2016), the runway network has some unique characteristics. First, arrival traffic through cell $i - 1$ has significant priority over departure
traffic through cell $i-2$. Typically, the capacity trade-off for departure and arrival traffic is captured through the fundamental diagram of runway, also known as the envelope model (Gilbo, 1993). An example is shown in the right picture of Figure 6, whose derivation is detailed later in Section 5.1. Due to the absolute priority of landing flow over departure flow at final approach, the cell $i-1$ always remains in the free-flow state. Then the left-over capacity of the runway, which is given by the fundamental diagram, is assigned to departure traffic from cell $i-2$. Such a system may be expressed using the following set of equations:

$$Q_i(t) = F(y_{i-1\to j}(t))$$ (3.11)

$$y_{i-2\to i}(t) = \min\left\{m_{i-2}(t), Q_i, Q_{i-2}, Q_j(t)\right\}$$ (3.12)

$$y_{i-i+2}(t) = y_{i-1\to j}(t)$$ (3.13)

Here, $y_{i-1\to j}(t)$ denotes the arrival rate and is exogenously given, $Q_i(t)$ denotes the left-over capacity available to departure traffic. Eqn (3.11) expresses the runway fundamental diagram; (3.12) determines the departure traffic flow; (3.13) simply states that, due to the high priority and free-flow condition for arrival traffic, the flow from the runway into the taxiway network is the same as the arrival flow (assuming that the time on the runway is negligible).

### 3.4.3. Hybrid CTM of apron traffic flow

Apron cells can be categorized into apron taxiing cells and stand cells. The latter are the destination of inbound flights and the origin of outbound flights; therefore, they are treated as both sink and source cells. Due to the highly complicated configuration of stands and taxiing routes within the apron area, we employ a mesoscopic modelling approach by ignoring some fine granularities as follows. We first aggregate different stands into sink/source cells $j+1, \ldots, j+n$ according to their spatial proximity and routing information. We then partition the apron taxiways into cells $i+1, \ldots, i+n$, which are then connected to the aggregated sink/source cells; see Figure 7. Using the notion of aggregate sink/source cells, we may define their sending and receiving capacities as usual, based on the physical characteristics of taxi links, flight schedules, and the occupancy of stands. These modeling details are omitted here.

![Figure 7: Cell representation of the apron area.](image-url)

### 4. The macroscopic fundamental diagram and off-block control

An empirical aggregate relation may exist for the airport surface network (Simaiakis and Balakrishnan, 2010; Simaiakis et al., 2014; Sandberg et al., 2013). We investigate the existence of an MFD for the airport surface network. The MFD characterizes the aggregate behavior of the airport surface network in terms of occupancy and throughput, in a parsimonious way yet capable of capturing the key demand-supply relationship. We define the following key quantities:

- **Arrival rate** $a(t)$: The number of arriving aircraft in a given time interval $t$;
- **Departure rate** $d(t)$: The number of departing aircraft in a given time interval $t$;
- **Taxi-out traffic** $n(t)$: The number of outbound aircraft taxiing on the surface network at the end of the time interval $t$.  

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Remark 4.1. When formulating the airport surface MFDs, two choices exist regarding the "occupancy" and the "throughput":

1. set "occupancy" as the total taxiing density (taxi-in and taxi-out combined), and set "throughput" as runway departure;
2. set "occupancy" as the taxi-out traffic density only, and set "throughput" as runway departure.

The first case is aligned with conventional, road network MFDs, while the second case is chosen in this paper for surface MFDs. This is because taxi-in and taxi-out traffic compete for surface capacity, and in the first case a fixed "occupancy" could correspond to numerous combinations of taxi-in and taxi-out traffic, causing the runway departure throughput to vary wildly. To quantitatively justify this observation, we compute the correlation between "occupancy" and "throughput" as 0.422 (first case) and 0.876 (second case), based on the empirical data from Guangzhou Baiyun airport (ZGGG). Moreover, for each "occupancy" value, the standard deviations of "throughput" in the first and second cases are 5.6 (aircraft) and 2.2 (aircraft), respectively. These results suggest that a stronger functional relationship exists in the proposed form of MFD, and promises better control performance for off-block optimization.

Based on the empirical data collected from ZGGG, we are able to demonstrate the existence of a macroscopic fundamental diagram as shown in Figure 8. Details of the data are presented later in Section 5.1. The MFDs are constructed as follows. We consider a sequence of time windows of 15 minutes, and calculate the corresponding arrival/departure rates according to the given flight schedule. The taxi-out traffic within this 15-min window is averaged over the number of taxiing-out aircraft recorded at higher time resolution based on the 4-D trajectory data collected via Surface Surveillance Radar. As Figure 8(a) shows, the surface MFD is not only a function of the taxi-out traffic \( n(t) \), but also depends on the arrival rate \( a(t) \); this marks a clear distinction with the MFD studied in conventional road traffic networks. Such a multi-variate MFD is dissected by different values of the arrival rate in Figure 8(b).

By analyzing the empirical distribution of the arrival rate \( a(t) \), one could aggregate the MFD by \( a(t) \) and obtain a single-variate function, which is shown in Figure 8(c) with a 3rd-order polynomial fitting. Finally in Figure 8(d), we take the upper and lower bounds on the family of curves shown in Figure 8(c), and fit them respectively with 2nd- and 3rd-order polynomials.

4.1. Off-block control

Off-block control problem aims at finding the rate at which aircraft are pushed away from the gates, such that certain objectives (e.g. departure rate, delay) are optimized. The existence of macroscopic fundamental diagrams provides unique opportunity to address the network control problem as extensively reviewed in Section 2.2. In the rest of this section, we consider several off-block control strategies, based on a developed MFD model for airport surface network. A comparison between the different strategies is carried out later for a case study in Section 6.

In order to facilitate the discussion below, we first describe the continuous-time dynamic of the surface network, where the network’s demand-supply relationship has been approximated with the macroscopic fundamental diagram(s). We denote by \( a(t) \) and \( q(t) \) (in, e.g. aircraft/15min) the arrival flow (in the final approach) and demand flow (at the gates), respectively. Both \( a(t) \) and \( q(t) \) are obtained from flight schedules and are assumed known in this formulation. An off-block rate control is introduced at the gates, where \( u(t) \in [0,1] \) meters the demand flow; i.e. \( u(t) \cdot q(t) \) is the off-block rate at time \( t \). The dimensionless control input \( u(t) \) controls the ratio of the demand flow to be pushed back; e.g. \( u(t) = 0 \) means that no aircraft is pushed back, and \( u(t) = 1 \) means the aircraft are pushed as scheduled with no delay. Driven by more practical operational constraints, we consider the following more general constraints

\[
\begin{align*}
    u_{\text{min}} &\leq u(t) \leq u_{\text{max}}
\end{align*}
\]

where \( u_{\text{max}} \leq 1 \) and \( u_{\text{min}} \geq 0 \).

The system dynamic is given as below based on the conservation of aircraft:

\[
\frac{dn(t)}{dt} = u(t) \cdot q(t) - G(n(t); a(t))
\]
where \( n(t) \) is the number of taxi-out aircraft. Here, the family of fundamental diagrams as seen in Figure 8(b) are parameterized by the arrival rate \( a(t) \). For any given \( a(t) \), we denote by \( n^*(t) \) the critical value that corresponds to the apex of the MFD \( G(\cdot, a(t)) \).

4.1.1. Control algorithm from Simaiakis et al. (2014)

The first control strategy is to maintain the taxi-out traffic at the critical value corresponding to the apex of the MFD associated with the current arrival rate. By employing this strategy the departure rate (i.e. the throughput of the surface network) is kept maximum. The following algorithm, adapted from Simaiakis et al. (2014), summarizes such a control strategy.

Figure 8: The macroscopic fundamental diagram of the ZGGG surface network traffic. (a): the Macroscopic Fundamental Surface; (b): the family of MFDs corresponding to different arrival rates (AR); (c): the average of the family of MFDs based on the frequency of the corresponding arrival rates; (d): Upper and lower bounds of the family of MFDs based on the 95% interval. The MFDs in (c) and (d) are fitted with second- or third-order polynomials.
Algorithm 1 (Simaiakis et al., 2014)

**Step 1** At the end of the $k$-th time period, forecast the arrival rate $a(k + 1)$. Find the critical taxi-out traffic value $n^*(k + 1)$ corresponding to $a(k + 1)$ for the next time period.

**Step 2** Obtain the taxi-out traffic $n(k)$, and approximate the predicted departure rate using the MFD:

$$d(k + 1) = G(n(k); a(k + 1))$$

**Step 3** Calculate the off-block rate during the time period $k + 1$ as:

$$u(k + 1) = \max \left[ u_{\text{max}}(k + 1), n^*(k + 1) - n(k) + d(k + 1), 0 \right]$$

where $u_{\text{max}}(k + 1)$ is the maximum off-block rate during time interval $k$.

**Step 4** Apply the off-block rate $u(k + 1)$ for the period $k + 1$. Set $k = k + 1$ and go to **Step 1**.

4.1.2. Discussion

Algorithm 1, proposed in Simaiakis et al. (2014), is actually a variant of the optimal bang-bang solution of the control problem. The control decision is switched between the upper and lower bounds of the control input, i.e., in Step 3, the off-block rate is switched between the maximum (available) off-block rate, i.e. $\max \left[ u_{\text{max}}(k + 1), n^*(k + 1) - n(k) + d(k + 1) \right]$, and the minimum value of “0”. Note that the shape of the aggregate curve proposed in Simaiakis et al. (2014) is different than the MFD shape proposed in this paper. Figure 8 shows that the departure rate increases as taxi-out traffic increases up to a critical accumulation where the departure rate is maximized, and if the taxi-out increases beyond the critical one, then the departure decreases, while in Figure 1 in Simaiakis et al. (2014), the departure rate maintains constant if taxi-out increases beyond the critical one.

For urban networks, the bang-bang policy that aims at maintaining the critical value of accumulation, which is the number of cars corresponding to the apex of the MFD, has been derived and investigated in Daganzo (2007) and Haddad (2017). While the MFD shape for urban networks was assumed to be invariant, the MFD for airport surface networks is parameterized according to the arrival rate $a(t)$, as shown in Figure 8(b). In the following, we derive the optimal bang-bang solution of the off-block control problem for airport surface networks.

The optimal control problem is defined as follows: manipulate the off-block control rate $u(t)$ to maximize the throughput of the surface network, defined as

$$J = \int_0^T G(n(t), a(t))dt$$

subject to the dynamic equation (4.15), and the control constraint (4.14). Note that $a(t)$ is assumed to be known, and here is treated as a priori known parameter. An optimal solution is derived for the off-block control problem with free end state, i.e. (4.14)–(4.16) and $n(t_f)$ is free. The Pontryagin maximum principle (PMP) is used to solve the problem, see Pontryagin et al. (1962). The Hamiltonian, denoted by $H$, is formed as

$$H = p(t) \cdot (a(t) \cdot q(t) - G(n(t), a(t))) + G(n(t), a(t))$$

where $p(t)$ is the costate variable that satisfies

$$\frac{dp}{dn} = -\frac{\partial H}{\partial n} = (p(t) - 1) \cdot \frac{dG}{dn}$$

Recall that the end state $n(t_f)$ is free, hence according to PMP it follows that $p(t_f) = 0$. The Hamiltonian must be maximized over the control variable $u(t)$ subject to the control constraint (4.14). The optimal control solution obtained
by \( \max_q H \) in (4.17) is

\[
u(t) = \begin{cases} 
  u_{\max} & \text{if } p(t) > 0 \quad (\text{i.e. } dG/dn > 0), \\
  u_{\min} & \text{if } p(t) < 0 \quad (\text{i.e. } dG/dn < 0).
\end{cases}
\]  

(4.19)

A switching point \( t_s \) [sec] is the time instant that satisfies \( p(t_s) = 0 \). If there exists a switching point, then the optimal policy will be determined by that point. The initial condition of the costate should be chosen such that the resulting solution satisfies \( p(t_f) \) and optimality condition (4.19). One should consider two cases: (i) uncongested regime i.e. \( dp/dt > 0 \) and (ii) congested regime, i.e. \( dG/dn < 0 \). Let us first consider case (i). Note that if \( p(0) > 1 \) then \( dp/dt > 0 \), see (4.18), but this is a contradiction since \( p(t_f) = 0 \), hence lets assume that \( p(t) < 1 \). Since \( p(t) = 0 \), \( p(t) \) must be decreasing for \( 0 \leq t < t_f \), i.e. \( dp/dt < 0 \), which implies \( p(t) > 0 \) for \( 0 \leq t < t_f \) according to (4.18), therefore the optimal control is \( u(t) = u_{\max} \), see (4.19). In this case, there will be no switching point \( t_s \) otherwise \( dp/dt > 0 \) for \( t_s \leq t \leq t_f \), however \( p(t_f) = 0 \) is not satisfied, therefore \( t_s = t_f \). Case (ii) holds in a similar way.

In other words, the optimal bang-bang policy (4.19) states that if the airport surface network is uncongested, then aircraft are pushed as scheduled with minimum delays at gates, corresponding to \( u_{\max} \). When the airport surface is congested, then the aircraft is pushed back after maximum delays at gates, corresponding to \( u_{\min} \). Indeed, the derived bang-bang control law is aligned with the intuitive solution. Note that in real time operations, this policy with the practical operational constraints \( u_{\max} \) and \( u_{\min} \) can be activated after reaching some threshold of taxi-out traffic.

4.2. Robust off-block feedback control

In the previous subsection, the optimal bang-bang policy has been derived for a well-defined MFD with a priori known \( a(t) \). This corresponds to the situation where the predicted arrival rate \( a(t) \) is available, and without any prediction error. In this subsection, the MFD function for airport surface network, which approximates the aggregate relationship between taxi-out traffic and departure rate variables, is considered as an interval function that might change within specified bounds.

In the following, we design a robust off-block controller for an airport surface network with the uncertain MFD representation. The MFD-based model in (4.15) is used for the off-block feedback control design, taking into account that the MFD can vary between upper and lower bounds, related to the parameter \( a(t) \). Figure 8(b) provides a shape of such partially uncertain (interval) MFD, with upper and lower bound curves.

The robust off-block controller is designed to handle uncertainty in the MFD. The controller should perform well for different values of \( a(t) \), and not necessary for the exact predicted values. Therefore, first the formulated nonlinear model is transformed to a linear model with MFD and parameter uncertainties, where the parameter uncertainty includes not only the nonlinearity and that the predicted arrival is not known in advance, but also uncertainty in the demand \( q(t) \). Then, we develop a robust controller for the developed uncertainty linear model, i.e. for a given fixed PI-controller with proportional \( K_p \) and integrator \( K_I \) gains, the controller stabilizes the linear system against all uncertainties.

A robust off-block feedback controller is designed in the following steps, similarly to the steps described in Haddad and Shraiber (2014) for designing perimeter control of urban networks.

Deriving necessary condition for steady-state period. The feedback controller should be designed such that the control constraint \( 0 \leq u(t) \leq 1 \) is satisfied during steady-state period. In the steady-state period it holds that the derivative of the state variable (4.15) is equal to zero, i.e. \( dn/dt = 0 \). Hence, one can isolate the control input in steady-state \( u_{ss} \) from (4.15) with \( dn/dt = 0 \), and then derive a necessary condition by satisfying the control constraint \( 0 \leq u_{ss} \leq 1 \). Let \( n_{ss} \) (aircraft), \( q_{ss} \), and \( a_{ss} \) (aircraft/5min) be respectively the state variable, demand, and arrival flows at steady-state. One gets at steady-state \( u_{ss} = G(n_{ss}, a_{ss})/q_{ss} \). Since \( u_{ss} \) should satisfy the constraints \( 0 \leq u_{ss} \leq 1 \), one respectively gets the following necessary condition \( 0 \leq G(n_{ss}, a_{ss}) \leq q_{ss} \).

Deriving a control design model. We simplify the nonlinear system equation (4.15) by deriving a linear model with parametric uncertainty, whereas the uncertainty should capture the nonlinearity dynamics. Hence, we first define \( q(t) = b(t) \cdot q \) with a parameter \( b(t) \) as \( b_{\min} \leq b(t) \leq b_{\max} \) and \( q \) is a known constant demand. Note that (4.15) is a nonlinear equation that depends only on one state variable \( n(t) \), but has two parameters \( b_{\min} \leq b(t) \leq b_{\max} \) and
$a_{\text{min}} \leq a(t) \leq a_{\text{max}}$. Let us now linearize the nonlinear system model (4.15) around a set-point $(\hat{n}, \hat{u})$, one gets the following linear model with two uncertain parameters $\tilde{a}$ and $\tilde{b}$

$$\frac{d\Delta n(t)}{dt} = A(\tilde{a}) \cdot \Delta n(t) + B(\tilde{b}) \cdot \Delta u(t),$$

(4.20)

where $\Delta n(t) = n(t) - \hat{n}$ and $\Delta u(t) = u(t) - \hat{u}$, and

$$A(\tilde{a}) = -\frac{dG(\hat{n}, a)}{dn},$$

(4.21)

$$B(\tilde{b}) = q \cdot b(t).$$

(4.22)

The uncertain equation (4.20), (4.21), and (4.22) with uncertain parameters $\tilde{a}$ and $\tilde{b}$ approximates the original nonlinear system (4.15) near the set-point $(\hat{n}, \hat{u})$. In the following, we design a robust controller that stabilizes the system against all uncertainty in $a(t)$ and $b(t)$.

**Developing a closed-loop block diagram and its transfer function.** We develop the closed-loop block diagram and transfer function as follows. First, we apply Laplace transformation to the control design model (4.20), which is transformed to the frequency domain as follows

$$\Delta N(s) = \frac{B(\tilde{b})}{s - A(\tilde{a})} \cdot \Delta U(s).$$

(4.23)

We design a robust PI-controller with proportional $K_P$ gain and integral $K_I$ gain, which is $K_P + \frac{K_I}{s}$. Finally, we close the loop with the reference $\Delta N_{ss}$ as shown in Fig. 9. Note that $\Delta n_{ss}(t) = n_{ss}(t) - \hat{n}$ and $\Delta n(t) = n(t) - \hat{n}$. Corresponding to the block diagram (without input and output disturbances) in Fig. 9, the closed-loop transfer function is calculated as follows

$$CL_{\Delta n_{ss}, \Delta N} = \frac{K_P B s + K_I B}{s^2 + (K_P B - A)s + K_I B}.$$  

(4.24)

**Designing PI control parameters.** Stability of the closed-loop function $CL_{\Delta n_{ss}, \Delta N}$ is determined by its poles $\lambda_i$, $i = 1, 2$, or simply the roots of the denominator equation in (4.24), i.e.

$$\lambda_{1,2} = \frac{-(K_P B - A) \pm \sqrt{(K_P B - A)^2 - 4K_I B}}{2}.$$  

(4.25)

For stability, the real part of every pole must be negative, i.e. $\Re(\lambda_{1,2}) < 0$. Therefore, the closed-loop transfer function $CL_{\Delta n_{ss}, \Delta N}$ is stable if

$$K_P < \frac{A}{B},$$

(4.26)

$$K_I > \frac{(K_P B - A)^2}{4B}.$$  

(4.27)

For a robust PI controller, taking into account the uncertainty bounds $A_{\text{min}}, A_{\text{max}}, B_{\text{min}}, B_{\text{max}}$ and its combination, $K_P$ and $K_I$ are determined to tackle the worst case scenario.
**Robust off-block PI controller in discrete formulation.** The robust PI controller in discrete time formulation can be implemented as follows

\[ \Delta u(k) = \Delta u(k - 1) + K_P[e(k) - e(k - 1)] + K_I e(k), \]  

(4.28)

where \( e(k) = \Delta n(k) - \Delta n_{ss}(k) = n(k) - n_{ss}(k) \), \( \Delta u(k) = u(k) - \hat{u} \). Algorithm 2 summarizes the robust off-block control strategy.

**Algorithm 2**

**Step 1** Choose an operating point, \((\hat{n}, \hat{u})\).

**Step 2** Define the upper and lower bound parameters, \(a_{\text{max}}, a_{\text{min}}, b_{\text{max}}, b_{\text{min}}\).

**Step 3** Determine the uncertainty bounds \(A_{\text{min}}, A_{\text{max}}, B_{\text{min}}, \text{ and } B_{\text{max}}\) according to (4.21) and (4.22).

**Step 4** Determine the controller parameters \(K_P\) and \(K_I\) such that (4.26) and (4.27) hold for all combinations of uncertainty bounds.

**Step 5** At the end of each time period \(k\), estimate the taxi-out traffic \(n(k)\), and calculate \(e(k)\).

**Step 6** Calculate the (deviated) off-block rate \(\Delta u(k)\) during the time period \(k\) according to (4.28).

**Step 7** Apply the off-block rate \(u(k)\) for the next time period. Set \(k = k + 1\) and go to **Step 1**.

5. Case study of the cell transmission model for airport surface networks

**5.1. Description of the test site and data**

We consider data collected from Guangzhou Baiyun International Airport (ICAO code: ZGGG) ground operation. This airport has a surface network consisting of more than 250 nodes and links (see Figure 1), and is constantly operating at high-density level (saturation or over-saturation). However, limited by the airspace capacity and a lack of effective decision support system, the airport suffers from long taxiing-out time and runway queuing time.

Operational data were collected from ZGGG and consist of (1) Updated Flight Plan Data (UFPD, May to October 2014) and (2) 4-Dimensional Trajectory Data (4-DTD, collected for 24 hours on 22 July 2014). The UFPD include critical attributes of each flight such as flight number, Estimated/Actual Time of Departure (ETD/ATD), Estimated/Actual Time of Arrival (ETA/ATA), Calculated/Actual Off-Block Time (COBT/AOBT), runway in use (RWY) and gate (Gate) information. Table 2 illustrates the UFPD for some sample flights. The UFPD is used to create simulated traffic scenarios in accordance with actual operation.

<table>
<thead>
<tr>
<th>Flight No.</th>
<th>ETD</th>
<th>COBT</th>
<th>AOBT</th>
<th>ATD</th>
<th>ATA</th>
<th>RWY</th>
<th>Gate</th>
</tr>
</thead>
<tbody>
<tr>
<td>CCA4308</td>
<td>13:30</td>
<td>13:03</td>
<td>13:06</td>
<td>13:51</td>
<td>02R</td>
<td>21</td>
<td></td>
</tr>
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<td>CSN6342</td>
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<td>13:05</td>
<td>13:10</td>
<td>13:53</td>
<td>02L</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>KAL866</td>
<td>13:35</td>
<td>13:10</td>
<td>13:10</td>
<td>13:55</td>
<td>02L</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>FDX6024</td>
<td>13:35</td>
<td>13:11</td>
<td></td>
<td></td>
<td>02R</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>CSS6871</td>
<td>13:04</td>
<td></td>
<td></td>
<td></td>
<td>02L</td>
<td>76</td>
<td></td>
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<td>14:00</td>
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<td></td>
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<tr>
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<td>13:35</td>
<td>13:14</td>
<td>13:16</td>
<td>14:03</td>
<td>02L</td>
<td>33</td>
<td></td>
</tr>
</tbody>
</table>

For each flight, the 4-DTD is an ordered sequence of 4-tuples representing the position (longitude, latitude, height) and time, and with a time step size of 15 seconds. The data were collected via Surface Surveillance Radar (SSR). The
taxiing speed and heading profile of each aircraft can be constructed. For example, Figure 10 illustrates the speed and heading profiles of a particular aircraft, Figure 11 shows all the departing and arriving aircraft trajectories for part of ZGGG on 22 July 2014. The 4-DTD are utilized to establish the fundamental diagram \(^2\) (see Figure 2), and define the taxi route of each aircraft. They also serve as the base scenarios to be compared with different off-block control strategies proposed in this paper. Below is a summary of the data:

- FDs are generated using 4-D trajectory data collected in a 24-hr time span on July 22, 2014.
- Runway capacity envelop and MFDs are established using UFPD from May to October 2014
- In our case study, different off-block control strategies are applied to the operations during 13:00-17:00 on July 22, 2014. The actual off-block control strategies during that period were utilized to generate the base control scenario.

Based on the 4-DTD, the key parameters of the calibrated fundamental diagrams have been shown in Figure 3. A time step of 30 seconds is chosen based on a trade-off analysis of the modeling accuracy and the computational efficiency. The resulting cell network, shown in Figure 12, has 14 apron cells (220 meters in length), 47 taxiway cells (400 meters in length). The runway queuing capacity \(N_q\) (see Eqn (3.10)) is 8 (aircraft).

The fundamental diagram of the runway cell has been shown in Figure 6, which is calibrated using runway data from May to October 2014. The capacity envelop, which is shown as the blue curve, is obtained using the quantile regression method with frequency constraint (Yang et al., 2015).

5.2. Validation of the network cell transmission model

The CTM-based airport surface network simulation is validated on three different levels. On the individual cell level, the proposed CTM simulation captures the phase transition of surface traffic, namely from the free-flow to the congested states. Figure 13 shows the density-flow and density-speed plots on a cell level, for both the empirical and simulated surface traffic. The mean and standard deviation of the absolute and relative errors are summarized in Table 3. First, the results show that the proposed CTM-based simulation captures the cell dynamics with reasonable accuracy. The results also suggest that the errors of apron modeling are lower than those of taxiway modeling. The reason is similar to those mentioned in Section 3.1: The deviation of the traffic from the fundamental diagram is larger on the taxiways than the apron area, due to the small speed variation and active gate assignment strategies occurring on the latter.
Figure 11: Departure and arrival trajectories on part of ZGGG network on 22 July 2014.

Figure 12: The cell representation of the ZGGG airport surface network.

Table 3: Cell-based simulation error. Unit for speed: Knot (=0.51m/s); unit for flow: aircraft/5 min.

<table>
<thead>
<tr>
<th>Cell-based Speed</th>
<th>Cell-based Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>Apron Cells</td>
<td>Taxiway Cells</td>
</tr>
<tr>
<td>Abs. Error</td>
<td>Rel. Error</td>
</tr>
<tr>
<td>Mean</td>
<td>Std.</td>
</tr>
<tr>
<td>2.17</td>
<td>1.89</td>
</tr>
<tr>
<td>5.52</td>
<td>2.33</td>
</tr>
</tbody>
</table>

For the purpose of calibrating the FD on a link level, it suffices to collect data within a time span of 24 hours on a single day, which is a common practice in mesoscopic modeling.

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On the taxiway level, we consider the arterial taxiway T41-T47 (see Figure 12), which includes the runway queuing area. In order to evaluate the accuracy of the simulation in terms of predicting the temporo-spatial evolution of traffic congestion on the arterial taxiway, we compare the empirical data with the simulated data in terms of time-dependent cell occupancies. For each cell $i \in \{T41, \ldots, T47\}$ and each time step $t$, the cell occupancy from the CTM-based simulation is given by $m_i(t)$. On the other hand, the empirical 4-D trajectory data need to be converted to the cell occupancies since the number of aircraft in a single cell during the 30-second time step is not a constant. To do this, we define the effective occupancy of a cell as

$$m_E = \sum_{j=1}^{N} e \times \frac{l_j}{\Delta x}$$

where $N$ is the number of aircraft present in the cell, and $l_j$ is the distance from the aircraft $j$ to the end of the cell, both recorded at the end of the 30-second time step. $\Delta x$ denotes the length of the cell, and $e$ is the unit aircraft so that the unit of $m_E$ is number of aircraft. The effective occupancy treats the individual aircraft as a continuum in space.

Finally, on the network level, one of the most important variables for surface operation is departure rate at the runway, which is a direct indicator of the efficiency of the surface network. Figure 15 compares the empirical and simulated departure rates for every 15-min time window from 13:00 to 16:45, under the same off-block strategy and arrival traffic profile.

Based on the CTM-based simulation, we are able to construct the network MFD, which is expressed as a function of both taxi-out traffic and arrival rate. In Figure 16, the simulated MFD is compared with the empirical MFD. It can be seen that the CTM-based network simulation captures the aggregate demand-supply relationship and the nonlinear congestion effect on the surface network. Specifically, we partition the domain $[0, 13] \times [0, 17]$ into $14 \times 18 = 252$ grids, and compute the absolute and relative differences between the two functions at these grid points. The absolute errors have a mean of 0.76 aircraft/15min and standard deviation of 0.71 aircraft/15min; the relative errors have a mean of 7.9% and standard deviation of 10.2%.

6. Case study of different off-block control strategies

In order to compare several control strategies on a fair ground, we simulate the operational period 13:00-17:00 on 22 July 2014 based on the UFPD and 4-DTD with different off-block controls. The actual off-block operation on that
day is considered the base scenario. The arrival rate $a(t)$ during this period ranges from 6 to 10 (aircraft/15min).

By applying different assumptions and levels of conservatism to Algorithms 1-2, we create the following four control variants.

- **Control Strategy I.** We apply Algorithm 1 with the critical taxi-out traffic given as:

  $$n^*(t) = \begin{cases} 
  10 & \text{if } a(t) = 6 \\
  10 & \text{if } a(t) = 7 \\
  9 & \text{if } a(t) = 8 \\
  9 & \text{if } a(t) = 9 \\
  8 & \text{if } a(t) = 10 
  \end{cases} \quad (6.29)$$

- **Control Strategy II.** We apply Algorithm 2 with $a(t)$ (in aircraft/15min) known, and $a_{\min} = a_{\max} = a(t)$. This corresponds to the situation where the predicted arrival rate $a(t)$ is available, and no prediction error is assumed in the algorithm.
Control Strategy III. This is similar to Control Strategy II but assuming a random prediction error for $a(t)$ by up to $\pm 1$ (aircraft/15min); i.e. $a_{\text{min}} = a(t) - 1$, $a_{\text{max}} = a(t) + 1$. Note that for a 15-min period, a prediction error by 1 aircraft is considered rather significant in the current ATM system, and this strategy is more conservative than Control Strategy II.

Control Strategy IV. We follow Algorithm 2 and assume that no predicted arrival rate $a(t)$ is available. The upper and lower bounds $[a_{\text{min}}, a_{\text{max}}] \subset [6, 10]$ (aircraft/15min) are selected based on historical information. This strategy is the most conservative as it ignores knowledge of the predicted arrival rate.

Furthermore, the base scenario is considered, which corresponds to the actual operation on 22 July 2014. To compare all the control scenarios on the same ground and to eliminate, from the base scenario, other actual causes of delay apart from inefficient off-block control, we consider the following:

Base Control Strategy. We use the actual off-block rate on 22 July 2014 as input of the CTM-based simulation.

We consider two simulation scenarios for testing the above control strategies:

Test Scenario A. The arrival rate is known exactly with no prediction error.

Test Scenario B. The arrival rate prediction is known, but with an up to $\pm 1$ (aircraft/15min) random prediction error.

Control Strategies I-IV are evaluated in Test Scenarios A & B (in Scenario B, random experiments are independently repeated 10 times). Five Key Performance Indicators (KPIs) are considered to quantify the effectiveness of the proposed controls.

1. Reduction of total delay compared to the Base Control Strategy;
2. Increase of apron delay compared to the Base Control Strategy;
3. Maximum size of runway queuing throughout the operational horizon;
4. Increase of average taxiing speed compared to the Base Control Strategy; and
5. Utilization of departure slot.

The testing results are summarized in Table 4, and lead to the following observations. First of all, all control strategies achieve a more efficient surface operation over the base control scenario, by reducing network-wide delay and increasing taxiing speed. Strategy I, which is based on Algorithm 1 and Simaiakis et al. (2014), tends to maximize departure slots by operating around the critical (sometimes over-critical) taxi-out traffic. The effect is that most traffic
and queuing are shifted from the apron area to the end of runway, resulting in a higher utilization of departure slot at the expenses of increased runway queuing and higher network-wide delay. In comparison, Strategies II-III suppress runway queuing by allowing congestion to be built up in the apron area instead of the taxiway network, thereby reducing the network-wide total delay and increasing the taxiing speed compared to Strategy I. Finally, Strategy IV has the most total delay among the four, partially due to its conservative control policy that tends to hold the aircraft at the gates to make room for the uncertain arrival rate at the runway.

Table 4: Comparison of KPIs for four control strategies (I-IV) under two test scenarios (A-B). For Scenario B, the results are averaged based on 10 independent runs.

<table>
<thead>
<tr>
<th>Control Strategy</th>
<th>Reduction of total delay</th>
<th>Increase of apron delay</th>
<th>Max. runway queuing</th>
<th>Increase of average taxiing speed</th>
<th>Utilization of Departure slot</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>428 min</td>
<td>39 min</td>
<td>5 aircraft</td>
<td>19.9%</td>
<td>98.6%</td>
</tr>
<tr>
<td>II</td>
<td>509 min</td>
<td>62 min</td>
<td>4 aircraft</td>
<td>23.9%</td>
<td>96.0%</td>
</tr>
<tr>
<td>III</td>
<td>498 min</td>
<td>69 min</td>
<td>4 aircraft</td>
<td>22.7%</td>
<td>94.6%</td>
</tr>
<tr>
<td>IV</td>
<td>341 min</td>
<td>78 min</td>
<td>3 aircraft</td>
<td>27.7%</td>
<td>92.0%</td>
</tr>
</tbody>
</table>

Table 5: The differences between the KPIs in Test Scenario B and the KPIs in Test Scenario A

<table>
<thead>
<tr>
<th>Control Strategy</th>
<th>Reduction of total delay</th>
<th>Increase of apron delay</th>
<th>Max. runway queuing</th>
<th>Increase of average taxiing speed</th>
<th>Utilization of Departure slot</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>-37 min</td>
<td>+4 min</td>
<td>0 aircraft</td>
<td>-2.0%</td>
<td>-2.6%</td>
</tr>
<tr>
<td>II</td>
<td>-26 min</td>
<td>-1 min</td>
<td>0 aircraft</td>
<td>-1.4%</td>
<td>-1.4%</td>
</tr>
<tr>
<td>III</td>
<td>-20 min</td>
<td>-6 min</td>
<td>0 aircraft</td>
<td>+0.9%</td>
<td>0.0%</td>
</tr>
<tr>
<td>IV</td>
<td>+40 min</td>
<td>-3 min</td>
<td>0 aircraft</td>
<td>+0.7%</td>
<td>0.0%</td>
</tr>
</tbody>
</table>

A comparison across Test Scenarios A and B reveals the advantage of the robust approach. For ease of comparison, Table 5 shows the differences in the four strategies’ performances under the deterministic (Test Scenario A) and stochastic (Test Scenario B) environment. In terms of the following positive indicators (i.e the larger the better): reduction of total delay, increase of average taxiing speed, and utilization of departure slot, Control Strategy I suffers with uncertainty (scenario B). In contrast, the levels of performance degradation are less severe for Strategies II-III; and Strategy IV even has a positive gain under Test Scenario B. These results provide sufficient evidence that Control Strategies II-IV are far more robust than Strategy I when the arrival rate is subject to random prediction error.

We visualize the control strategies in the contour plots in Figure 17, which clearly show the different levels of conservatism adopted by Strategies I-IV. From Strategy I to IV, the controlled taxi-out traffic corresponding to each arrival rate has a tendency to shift south (to smaller values). In particular, in Strategy I, many controlled points are distributed in the over-critical branches of the curves, while such points are diminishing for strategies II, III and IV. This indicates that the more conservative a control strategy is, the lower the off-block rate it generates given the same information and level of uncertainty.

Finally, we test the performance of the control strategies with increased congestion for departure by simulating the following hypothetical scenario. We keep the flight plan for departures, and scale up uniformly the arrival demand by 30%. Such an increase will impact the departing traffic in two ways. First, the runway capacity allocated to departing aircraft will be further reduced as arrivals have high priority; second, more taxi-in traffic will occupy more network space, thereby aggravating network congestion for departures. The factor of 30% is chosen following the
Figure 17: The arrival rate and controlled taxi-out traffic at each 15-min period. The black dashed lines indicate the critical taxi-out traffic on the MFD corresponding to different arrival rates.

consideration that the arrival rate cannot exceed the upper bound for arrival capacity. Such a scenario corresponds to the over-saturation of both arrival and departure traffic, possibly following massive flight delays caused by, say severe weather conditions. The test results are summarized in Table 6. Note that due to increased arrival demand, we no longer compare these results with the base case.

Table 6: Increased arrival demand: Comparison of KPIs for four control strategies (I-IV) under two test scenarios (A-B). For Scenario B, the results are averaged based on 10 independent runs.

<table>
<thead>
<tr>
<th>Total delay</th>
<th>Apron delay</th>
<th>Max. runway queuing</th>
<th>Average taxiing speed</th>
<th>Utilization of Departure slot</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test Scenario A (deterministic)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Control Strategy I</td>
<td>745 min</td>
<td>451 min</td>
<td>5 aircraft</td>
<td>8.4 m/s</td>
</tr>
<tr>
<td>Control Strategy II</td>
<td>682 min</td>
<td>594 min</td>
<td>3 aircraft</td>
<td>9.9 m/s</td>
</tr>
<tr>
<td>Control Strategy III</td>
<td>718 min</td>
<td>611 min</td>
<td>4 aircraft</td>
<td>9.5 m/s</td>
</tr>
<tr>
<td>Control Strategy IV</td>
<td>805 min</td>
<td>721 min</td>
<td>2 aircraft</td>
<td>10.2 m/s</td>
</tr>
<tr>
<td>Test Scenario B (stochastic)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Control Strategy I</td>
<td>792 min</td>
<td>471 min</td>
<td>5 aircraft</td>
<td>7.9 m/s</td>
</tr>
<tr>
<td>Control Strategy II</td>
<td>694 min</td>
<td>599 min</td>
<td>3 aircraft</td>
<td>9.6 m/s</td>
</tr>
<tr>
<td>Control Strategy III</td>
<td>727 min</td>
<td>606 min</td>
<td>3 aircraft</td>
<td>9.3 m/s</td>
</tr>
<tr>
<td>Control Strategy IV</td>
<td>837 min</td>
<td>765 min</td>
<td>2 aircraft</td>
<td>10.3 m/s</td>
</tr>
</tbody>
</table>

Table 6 shows similar results across the four control strategies as before. The robust Strategies II and III outperform Strategy I in terms of reducing total delay and increasing taxiing speed. Strategy I tends to shift delay in the apron area to the taxiway and runway queuing area, which is a typical feature of the bang-bang type control. Strategy IV is overly conservative and suppresses heavily runway queuing by holding aircraft from being pushed off. It is also noted that Strategies II and III are more robust against uncertainty, as the deterioration of KPIs in Scenario B compared to Scenario A is minimal for these controls.
7. Conclusion and future research

This paper explores the flow characteristics of airport surface network on both mesoscopic and macroscopic levels. We propose an efficient modeling approach based on the cell transmission model for simulating the spatio-temporal propagation of flow and congestion on airport surface. The proposed model stipulates the existence of link-based fundamental diagram that expresses the functional relationship between link density and flow, which is validated using empirical data collected in Guangzhou Baiyun airport. The CTM-based network model is shown to be an efficient and accurate simulation method capable of supporting a wide range of applications such as off-block optimization, gate assignment, route planning, and scheduling.

Using both CTM-based simulation and empirical data, we further reveal the existence of an aggregate relationship between traffic density and runway throughput, which is referred to as macroscopic fundamental diagram in the literature of road traffic. The MFD is utilized in this paper to devise robust off-block control strategies with uncertainties, which is shown to outperform existing off-block control methods, which corresponds to a simple bang-bang control.

This paper is the first to systematically study the fundamental relationship between “occupancy” and “throughput” on mesoscopic and macroscopic levels, which manifest themselves as the link-based FD and MFD, respectively. Both analytical computation and empirical evidence are employed to verify these notions and develop efficient network simulation and optimization techniques that hold promise in supporting further air traffic management initiatives such as integrated Arrival Manager-Departure Manager-Surface Manager.

The proposed simulation and control frameworks are meant to treat general airport surface networks with varying type and configuration. In 2015, a third runway was built in ZGGG, which may impact the MFDs in a fundamental way, although the exact influence is yet to be assessed with quantification (by empirical data or simulation). The proposed analyses of the surface MFD, as well as the CTM-based simulation platform, are transferrable to treat general surface networks of varying sizes and types. The comparison of the MFDs before and after the addition of the third runway in ZGGG is a topic worthy of investigation as future research.

Dynamic spatial clustering methods have been investigated for MFDs on large-scale road traffic networks with heterogeneous density distribution of vehicles, and provides a new perspective on MFD-related modeling (Ramezani et al., 2015; Saeedmanesh and Geroliminis, 2017). For airport surface networks, as studied in this paper, clustering was not employed because, unlike large-scale road networks, the airport surface network is relatively sparse in terms of both topology and traffic flow. Moreover, the distribution of congestion and airport surface is highly related to the physical layout, utilization of aprons and runways, and the time-varying (arrival) patterns. Therefore, the airport surface network is highly susceptible to frequent and localized congestion (Khadilkar and Balakrishnan, 2013). Partitioning of the airport surface into regions with homogeneous density profiles is a difficult task and will likely encounter greater complexity and uncertainties in the individual regional MFDs. Nevertheless, the clustering approach could be potentially useful for studying airports with certain configuration, size, and dynamics. This will be pursued as future research.

References


