Optimal Storage Investment and Management under Uncertainty

It is costly to avoid outages!

Joachim Geske and Richard Green
Imperial College Business School
London, United Kingdom
j.geske@imperial.ac.uk, r.green@imperial.ac.uk

***Abstract*—Subject of this analysis is to show how storage is operated optimally under renewable and load uncertainty in the electricity system context. We estimate a homogeneous Markov Chain representation of the residual load in Germany in 2014 on an hourly basis and design a very simple dynamic stochastic electricity system model with non-intermittent generation technologies and storage. We compare these results to Perfect foresight findings and identify a significant over estimation of the storage potential under perfect foresight.**

Keywords—Storage, Electricity system, Uncertainty

I. Introduction

Storage has the technical potential to increase efficiency of electrical systems significantly - especially in the context of integrating intermittent renewable technologies. This is achieved by shifting energy from periods of low demand to periods of high demand. Thus, the utilization of medium load power plants is increased and the utilization of peak load power plants is reduced. The full extent of efficiency gain is achieved if generation capacity is adapted to the "equilibrated" load situation - with a higher base load and lower peak load share. In this case, the installed fossil generation capacity falls below peak load level. Since the amount of energy stored is generally limited, there is a risk of outages in cases of prolonged demand peaks. This problem does not occur in perfect foresight based analyses that are still the paradigm of electrical system analysis. The subject of this analysis is to show how storage is operated optimally under renewable and load uncertainty in the system context.

We estimate a homogeneous Markov Chain representation of the residual load in Germany in 2014 on an hourly basis (section 2) and design (section 3) a very simple dynamic stochastic electricity system model with fossil generation technologies and storage (a Markov Decision Process). This model is solved in section 4 for a stationary state using numerical methods (linear optimization) and the optimal storage strategy is presented. In section 5 this optimal strategy is compared to the optimal solution derived under perfect foresight of explicit drawings of the stochastic load process. Thus, the perfect foresight "error" can be quantified.

It is shown that under uncertainty at high demand an increasing share of the storage is "frozen" in its charged state to avoid lost load (outages). Therefore, the "buffer share" of the storage is not used for equilibration of load any more. Furthermore, this buffer state of charge is established, if necessary, even in periods of high demand, so that the storage operation stresses the system.

To implement the full efficiency potential of storage, generating capacities must be reduced. Peak load can then exceed installed capacity. If this is the case, in the optimum under load uncertainty a storage buffer is created and maintained. This is not required in optimal storage operation under perfect foresight assumption. Therefore, efficiency gains caused by the usage of storage are overestimated in analyses based on perfect foresight. We will quantify the extent of this overestimation.

When using storage to prevent outages it has to be decided whether it is operated "inefficiently" with respect to “full” capacity adjustments, or “efficiently” and peak load capacity is not decommissioned “one for one”. This analysis refines the assessment of the economic potential of electricity storage, thus contributing to more effective planning of energy systems of the future, where outages are avoided.

II. MODELING RESIDUAL LOAD IN GERMANY AS MARKOV CHAIN

In this section, it is examined to what extent the modelling of the residual load of Germany in 2014 in hourly resolution a) as a homogeneous Markov Chain and b) in 10 GW steps is empirically valid[1]. The residual load is defined as load reduced by renewable generation - in the case of Germany, mainly wind and solar power.

		Load	Wind	Solar	Residual Load
Total	GWh	504166	51443	32816	419906
Day Mean	GWh	1381	141	90	1150
Max	GW	79	29	24	78
Min	GW	35	0	0	14

Table 1: Benchmarks of residual load in Germany

The annual load in Germany amounts to 500 TWh. 10% of this load are generated by wind and 7% by solar. Herewith the residual load is reduced to 83%. The peak load is 79 GW. At maximum 29 GW of wind and 24 GW solar respectively are produced. Nevertheless, the peak of the residual load remains at 78 GW, which reflects the intermittent character of renewable generation. The load will never fall below 35 GW; the minimum of the residual load, however, is only half of the lower bound (14 GW). It is obvious that renewables a) cover a substantial share of the load (17%), b) halve the minimum load but c) do not decrease peak load (the residual load duration curve is shown in figure 1, red).

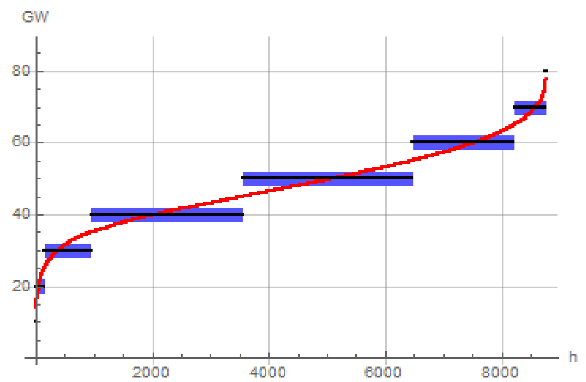

Figure 1: Load Duration: German data 2014 (red curve), rounded German data 2014 (black step function), stationary distribution of the Markov process (blue bar step function)

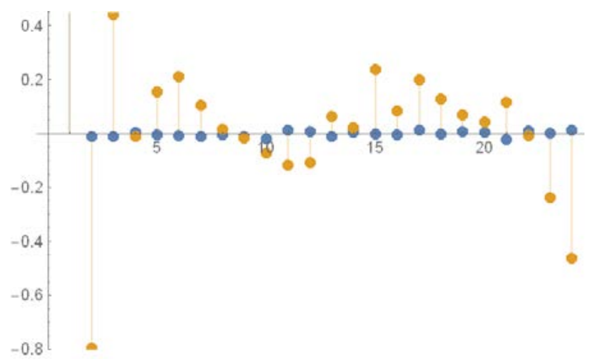

Figure 2: Partial autocorrelation plot of the German residual load 2014 (orange) and of a simulated path of the Markov Chain

[1] See [13] and [14] for a discussion of Markov Load modelling.

The (consistent) maximum likelihood estimators of the transition matrix of a homogeneous Markov chain are the numbers of state transitions normalized per line. The estimator of the transition matrix for the residual load in Germany is:

$$P = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.02 & 0.79 & 0.19 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.03 & 0.8 & 0.17 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.05 & 0.81 & 0.14 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.12 & 0.76 & 0.12 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.2 & 0.73 & 0.07 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.22 & 0.77 & 0.01 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.45 & 0.55 \end{pmatrix}$$

The estimated Markov chain is a) "irreducible", thus all states are mutually accessible, b) "aperiodic", i.e. the greatest common divisor of the return times to the initial states is one and c) all states are "positive recurrent", i.e. with probability 1 there is a return to the initial state and the anticipated return time is finite. In this case, there is a steady state distribution q with $\lim_{t\to\infty} p(t) = \lim_{t\to\infty} p(0)P^t = q$, independent of the initial distribution. q is the solution of the linear system of equations $q = P\,q$, $q1 = 1$. The solution of this system for the estimated Markov Chain P results in the stationary distribution:

$$q\ [\%] = \{<1, 1, 9, 29, 34, 20, 6, <1\}.$$

If these probabilities are interpreted as interval length and the states are ordered, q can be interpreted as load duration. Compared with the original data, the approximation of the discretized load (figure 1, black step function) by the Markov Chain (blue step function) is accurate.

Intuitively the estimated Markov Chain is a sound representation of the frequency of the state transitions of the residual load. It is therefore not surprising that also the long-term distribution of the residual load is well approximated. However, periodic structures, such as daily, weekly and annual rhythms cannot be approximated by aperiodic first order Markov chains. Thus, the partial autocorrelation coefficients of the residual load (figure 2) are significant (5% ± 0.02) for almost all lags (orange points). In contrast, the coefficients of the Markov process are only significant for a lag of one hour.

As an illustration of the two processes, respectively the difference between them, in figure 3: a) a 500 hours segment of the residual load in Germany, b) a 500 hours segment of the rounded (to 10 GW) residual load in Germany and c) a 500 hours simulation of the estimated Markov Chain is shown. It is not obvious that there is a fundamental difference between time series b) and c).

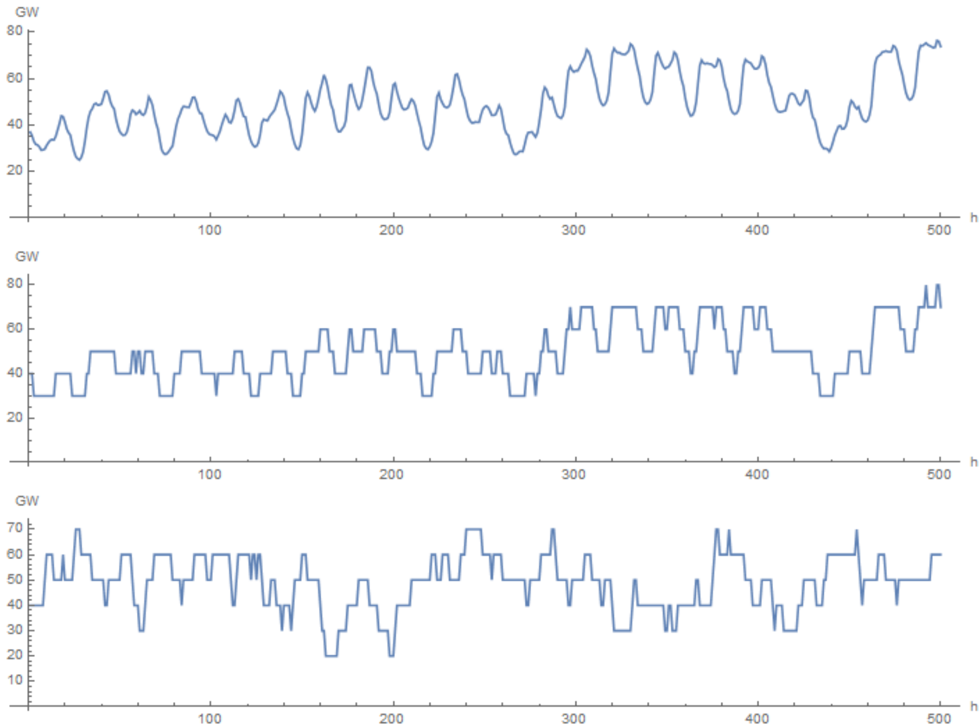

Figure 3: a) 500 hours segment of the residual load in Germany 2014, .
b) 500 hours segment of the rounded residual load in Germany 2014,
c) 500 hours simulation of the estimated Markov Chain

In summary, the modelling of the residual load as a first order homogeneous Markov Chain represents the load transitions and the load duration very well. With this interpretation, a stochastic energy system model is formulated in section 3. Nevertheless, the order of load changes is random and does not reflect the natural daily, weekly and seasonal rhythms. Therefore, there is substantially more uncertainty about the future residual load development represented in the model than in reality. Thus the Markov approach to residual load modelling would be more appropriate for a scenario with a higher share of intermittent renewables. The order of the load transitions is relevant for the operation of the storage.

III. Electricity System Model

In the following welfare-maximizing capacity-, dispatch- and storage-decisions are determined to derive the social value of electricity storage. Welfare is interpreted as system cost. In the following case study the availability of a free storage capacity $\hat{S}$[2] = 300 GWh (equivalent to 6 hours average load in Germany; for an overview of storage potential in Europe see [15]) is assumed. It is then possible to compare the system cost in a scenario with storage to a scenario without storage and thus to determine the value of storage in terms of system cost avoidance. This comparison is on the one hand conducted under perfect foresight (Det) of the residual load D_t [GW] and on the other hand under stochastic residual load (Sto), as described in the previous chapter, to identify the impact of the two assumptions on the valuation of a storage option.

Electricity can be generated by a portfolio of non-intermittent technologies - modelled as stack in contrast to unit commitment. The vector $x_t \geq 0$ [GW] describes the related production per hour. To apply these generation technologies initial investments with capacities k [GW] have to be made (green field approach). The capacity limits the non-intermittent generation $k \geq x_t$. s_t [GW] corresponds to the charging or discharging of the storage per hour. The stored energy is S_t [GWh] ($\hat{S} \geq S_t \geq 0$). There are neither politically motivated interventions (CO_2 prices) nor

[2] The Symbol S-hat is used with the same meaning as S-bar.

technical restrictions that impact the use of specific technologies - neither distribution effects are taken into account (copper plate assumption). The alternatives are evaluated in a system cost approach that includes fixed costs $c^{var}\ k$ and variable costs $c^{var}\ x_t$ over 40 years. In the deterministic as well as in the stochastic model the time resolution is one hour. In the stochastic model only a single hour is modelled and then extrapolated to 40 years, while in the deterministic model the result of a year is extrapolated. Costs are not discounted but modelled as long-term variable average costs in €/hour[3].

The monetary value of unsatisfied demand (*VoLL*, value of lost load) has been quantified in comprehensive empirical studies. Following [11], the social planner can decide upon capacity and production such that - assuming the unsatisfied demand is valued with this *VoLL* = 100 max $\{c^{var}\}$[4] – a satisfaction of the complete demand is discarded in favour of lower system costs. The satisfaction of the residual load is therefore not modelled as a restriction, but incorporated as part of the objective function[5]. This has the particular advantage that an endogenous determination of reserve capacity is possible within the model. The latter is described as desirable by [18] for future electricity system modelling approaches that are able to consider and to evaluate the effects of the inclusion of additional intermittent renewables quantitatively.

Variable and fixed costs of non-intermittent generation technologies based on [20] are presented in table 2:

Technology		**Coal**	**IGCC**	**Combust turbine**	**Combined cycle**	**Nuclear**
Variable Cost (Fuel+OM)	€/MWh	27	25	55	40	22
Fix cost	€/KW	2000	2500	650	800	3250

Table 2: technology specific cost data; constant lifetime of the plants of 40x365x24[h/plant], Source: [20]

The welfare-maximizing capacity-, dispatch- and storage-decisions under the perfect foresight assumption are the solution of the following problem with T = 365×24 and η_{Det} = 40×T:

$$\min_{x_t,k,s_t} c^{fix}k + \frac{\mu_{Det}}{T}\sum_{t=1}^{T} c^{var}x_t + VOLL\ y_t \tag{1}$$

$$y_t \geq D_t + s_t - x_t \tag{2}$$

$$\text{s.t.}: \overline{S} \geq S_t = S_{t-1} + s_t \geq 0 \tag{3}$$

$$\text{s.t.}: k \geq x_t \geq 0 \tag{4}$$

$$\text{s.t.}: y_t \geq 0 \tag{5}$$

Unlike in the perfect foresight model in the case of uncertainty the information available about the residual load will be considered as well as the impact of dispatch and storage decisions on the future revenues. These requirements are met in the stochastic dynamic programming

---

[3] To give up discounting avoids implausible unloading in storage models. Similarly Tijms [6] explains: „For many applications of Markov decision theory this criterion is the most appropriate optimality criterion. The average cost criterion is particularly appropriate when many state transitions occur in a relatively short time."

[4] For a literature review see [7], [8], [9], [10].

[5] Modeling the *VoLL* reminds of the „penalty"-approach to the numerical solution of constrained nonlinear optimization problems. This analogy can be used to apply an (intuitive) theorem that sheds light on the modelling approach: The penalty solution converges to the solution of the constrained problem, as *VoLL* tends to infinity. Proof: penalty methods e.g. Luenberger (1984).

approach by the determination of an optimal strategy (policy: for an overview of stochastic analyses in the energy sector see e.g. [12], [16], [17]). This strategy consists of decision rules for each time. Every decision rule allows to select an action on the basis of the occurring state from a distribution of alternatives (actions). In our case, the state of the system consists of the residual load D_t, the energy stored S_t and the initially fixed non-intermittent generation capacities k and the storage capacity $\hat{S}$. The decisions (actions) in each period are the level of non-intermittent generation x_t and the change of the state of charge s_t [6]. A decision rule f_t assigns a distribution over the actions $(s_{t,},x_t)$ at each state $(S_t,D_t,k,\hat{S})$: $(s_{t,},x_t) = f\,(S_t,D_t,k,\hat{S})$. A strategy π consists of decision rules for each time $\pi = (f_1, f_2,\ldots)$, esp. a stationary decision rule $\pi = (f, f,\ldots)$.

Using these definitions, the optimization of the energy system, with residual load modelled by the Markov Chain $P(D_{t+1}|D_t)$ can be described as a Markov Decision Process (For an introduction to the topic see e.g. [1], [2], [3], [4] and [5]). Therefore, the simplifying approximation of a long but finite horizon by an infinite one is applied. The optimal strategy is a solution of the problem

$$\min_{k,\pi} c^{fix}k + \mu_{Sto} \lim_{T\to\infty} \frac{1}{T+1} \mathrm{E}\Big[\sum\nolimits_{t=0}^{T} c^{var}x_t + VoLL(D_t + s_t - x_t)^+\Big] \quad (6)$$

considering η_{Sto} = 365×40×24, the capacity restriction $k \geq x_t \geq 0$, the state transitions a) deterministic: $\hat{S} \geq S_t \geq 0$, and b) stochastic: $P(D_{t+1}|D_t)$ and the initial conditions $(S_0, D_0, \hat{S})$. Theorems of existence and structure of an optimal strategy are proved e.g. in [19].

The problem can be decomposed to a dispatch&storage management problem and a capacity optimization. The dispatch problem can be solved separately. This reduces complexity considerably. The solution of the dispatch problem x_t at given capacity is simple[7]: The technology with lowest variable cost is used as long as its capacity is exhausted and the next more "expensive" technology has to be used and so forth. This optimum dispatch solution makes it possible to define a cost function in which the load D that exceeds the non-intermittent capacity $k = (k_1,\ldots,k_I)$ is valued with the *VoLL*:

$$C(D,k) = \begin{cases} c_1^{var}D & 0 \leq D < k_1 \\ \sum\nolimits_{j=1}^{i} k_j c_j^{var} + c_{i+1}^{var}\left(D - \sum\nolimits_{j=1}^{i} k_j\right) & k_i \leq D < k_{i+1} \\ \sum\nolimits_{j=1}^{I} k_j c_j^{var} + VoLL\left(D - \sum\nolimits_{j=1}^{i} k_j\right) & k_I < D \end{cases} \quad (7)$$

Thus the decision rules simplify to $s_t = f\,(S_t,D_t \mid k,\hat{S})$ respectively the strategy to $\pi(k,\hat{S}) = (f,f,\ldots)$. The decomposition of the problem is then: Determine

$$V\big(k|S_0, D_0, \overline{S}\big) = \min_{\pi(k,\overline{S})} \lim_{T\to\infty} \frac{1}{T+1} \mathrm{E}\Big[\sum\nolimits_{t=0}^{T} C(D_t + s_t|k)\Big] \quad (8)$$

with respect to the deterministic state transition $\hat{S} \geq S_{t+1} = S_t + s_t \geq 0$ and the stochastic $P(D_{t+1}|D_t)$. Determine the capacity k by solving

[6] This optimization problem is therefore decisively more complex, than e.g. the original stochastic optimization problem of the Real Business Cycle Theory. In the latter only storage management has to be optimized without considering the optimization of initial conditions (capacities).
[7] It can be shown that the following rule satisfies the Kuhn-Tucker conditions of the corresponding linear program.

$$\min_{k} c^{fix} k + \mu_{Sto} V(k|S_0, D_0, \overline{S}) \tag{9}$$

Both problems can be solved sequentially. With the discrete states {1,..., $|\hat{S}|$} in 10 GWh and {1,…,|D|}in 10 GW steps the admissible actions in a state are

$$A\left((S,D)|k,\overline{S}\right) = \{s \in \mathbb{N} |\ 1 \le s + S \le \overline{S}, 1 \le s + D \le \mathbb{1}k\} \tag{10}$$

Applied to the storage problem [19] proves that from the solution of the linear program:

$$\max_{d((S,D),s)} \sum_{(S,D),s \in A(S,D)} d\left((S,D),s\right) C(D+s|k) \tag{11}$$

$$\sum_{(S,D),s \in A(S,D)} d\left((S,D),s\right) = 1 \tag{12}$$

$$d\left((S,D),s\right) \ge 0 \tag{13}$$

$$\sum_{s \in A(S,D)} d\left((S,D),s\right) = \sum_{S',D',s \in A(S',D')} d\left((S',D'),s\right) Prob\left((S',D'),(S,D),s\right) \tag{14}$$

$$Prob\left((S',D'),(S,D),s\right) = \begin{cases} P(D'|D) & S' = s + S \\ 0 & otherwise \end{cases} \tag{15}$$

the decision rule f [8]can be derived as follows:

$$Prob\left(s|(S,D)\right) = \frac{d\left((S,D),s\right)}{\sum_{s' \in A(i)} d\left((S,D),s'\right)}. \tag{16}$$

Thereby $V(k \mid S_0, D_0, \hat{S})$ can be determined. This allows the solution of the capacity problem with a hill-climbing approach. It proves efficient to include a search direction that is capacity preserving with substitutions of "adjacent"[9] technologies. The solution of a case with a storage capacity of 300 GWh is described in detail in the following section.

IV. OPTIMAL STRATEGY OF THE STOCHASTIC MODEL

The optimal stationary strategy π for the case of the residual load determined in section 2 and a storage capacity of 300 GWh is shown in figure 4. Eleven grid lines of the state of charge are plotted in 30-GWh steps on the vertical axis[10]; eight grid lines in steps of 10 GW mark the residual load on the horizontal axis[11]. At each intersection of the grid lines changes in the state of charge are indicated by directed arrows. The length of these arrows corresponds to the quantity of change in 10 GWh steps. A black ring denotes an unchanged state of charge. The

[8] which assigns to each alternative $s \in A(S,D)$ for a given state (S,D) a probability,

[9] - with respect to the fixed costs -

[10] The index 1 corresponds to 0 GWh.

[11] The index 1 corresponds to 10 GW.

stationary probabilities of the Markov decision process with the optimal strategy are visualized at the grid points by the size of the areas of blue circles. To assess the state of the energy system, non-intermittent production and marginal costs of production are shown at each grid point.

The optimal capacity k = (50, 0, 0, 20, 0) sums up to 70 GW with expected system costs of 556726 million euros. Therefore, the residual load of 80 GW cannot be covered without unloading the storage. If the state of charge is zero, it cannot be discharged any further and load is lost (red rectangle). This state is valued by the social planner with the extreme marginal cost of 5190 euros.

Storage is used to equilibrate the residual load longer at higher levels by charging and discharging. This increases utilization of non-intermittent generation capacity and therefore efficiency. For this purpose the storage is charged below a residual load of 50 GW and discharged above – the stronger, the greater the deviation from 50 GW – except the storage is fully charged or has reached a lower limit of 50 GWh. Thus a load of 50 GW is achieved over a wide range of the grid, making an expansion of base load capacity profitable. Equilibration is limited by three forms of „stabilization actions“ to reduce the risk of losing load:

1. It is apparent that the storage is discharged below 50 GWh only at peak load. At lower residual load it is charged below this bound - even at the high load of 60 GW. Thereby capacity is fully utilized. This "reserve" reflects in very low probabilities of staying in a state of charge below 50 GWh.
2. At peak load, the storage is discharged system-equilibrating above a state of charge of 150 GWh in 30 GW steps. Below this level discharging is reduced to 10 GW steps thus contributing to keep the system at full capacity and thus delaying lost load.
3. Also below the peak load level - at 60 GW - system-equilibrating discharge is stopped below a state of charge of 100 GWh. Full load is thereby accepted to keep a higher „distance“ to lost load.

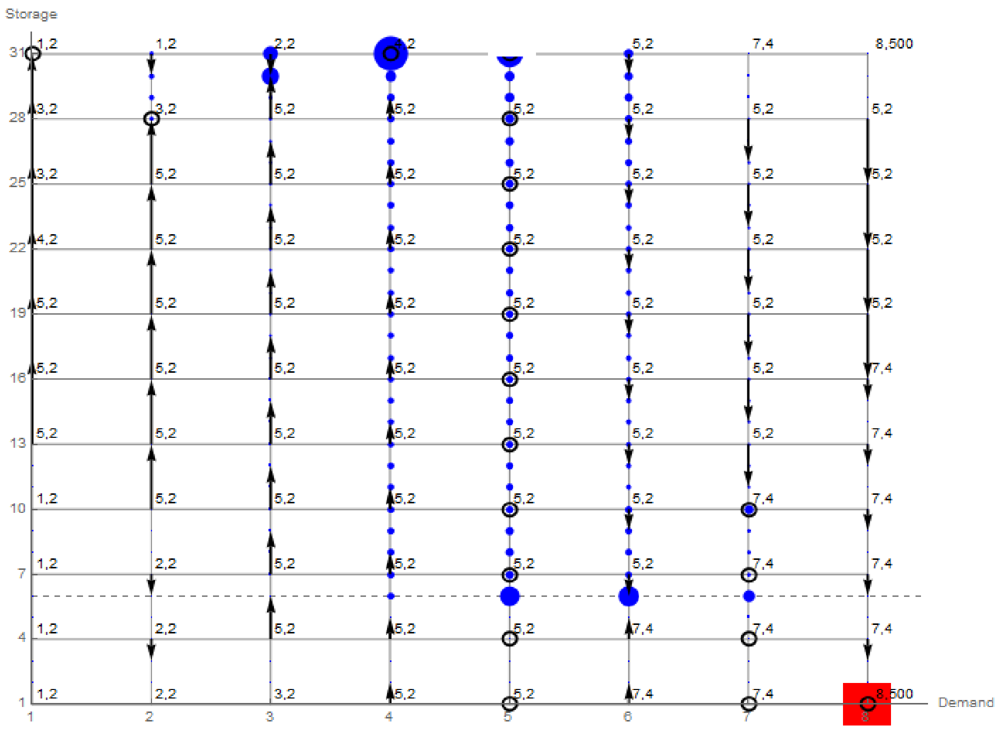

Figure 4: Optimal charging strategy and stationary probabilities of the Markov Decision Process; numbers: non-intermittent generation, marginal cost [Eurocents/kWh]

By increasing the utilization of non-intermittent generation, the load duration curve proceeds flatter from a higher pedestal than without storage (without figure). Compared with the perfect foresight model the amount of "Peak Shaving" is not achieved. Therefore, system cost reduction by including a 300 GWh storage is lower under stochastic residual load than under perfect foresight of the same residual load. If the perfect foresight solution is categorized by the frequency of being in a specific state, the corresponding probabilities can be entered in a state diagram (Figure 5). It becomes apparent that the state of charge is distributed more evenly than in the stochastic model (Figure 6).

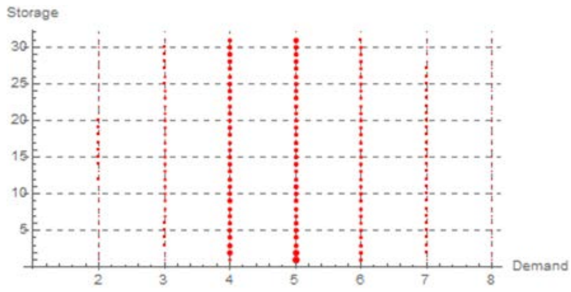

Figure 5: Probabilities perfect foresight storage model

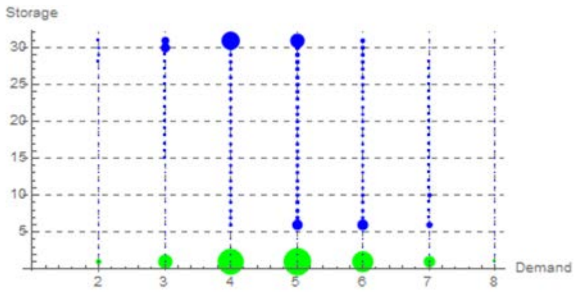

Figure 6: Probabilities perfect foresight without storage model (green) and stochastic storage model

V. CONCLUSION

Table 3 shows the system cost and capacity of non-intermittent generation distinct by information scenarios and available storage capacity. Under the perfect foresight hypothesis with the residual load for Germany in 2014, system costs can be reduced by 3.6% using 300 GWh storage capacity. This is achieved by a 10% expansion of the base load capacity and a halving average- and peak load-capacity. If - instead of the residual load data of Germany - 20 simulated time series from the residual load Markov modelling are used, then the storage option decreases system cost by 4.4% in the perfect foresight case. Again, the base load capacity is increased by 10%, peak load by about 1 GW and mean load is reduced to less than half.

Under stochastic residual load without storage, the installed capacity rises by 10 GW and thus system costs rise by 2.4%. Storage enables an extension of the base load capacity of 10 GW, a reduction in the mean load capacity and an abandonment of peak load capacity. Therefore system costs fall by 2.9%. Compared to perfect foresight modelling the relative reduction of system costs reduces by 50%. The consideration of unpredictable changes in residual load and thus holding reserves to avoid lost load seemingly reduced efficiency gains by energy storage. In fact, the gain in efficiency by storage of a perfect foresight modelling is overestimated.

	Without storage	**300 GWh storage**	**Change in system cost**
Perfect foresight Residual load 2014	570768 {41,13,9,0,13}	550030 {45.6,8.1,4.4,0,5.5}	-3.6%
Perfect foresight Markov Load	559010 {40,0,10,20,0}	534196 {44.5,0,5.9,5.6,0.8}	-4.4%
Stochastic model Markov Load	573082 {40,0,10,20,10}	556726 {50,0,0,20,0}	-2.9%

Table 3: System cost and capacities of non-intermittent generation

References

[1] Baeuerle, N., Rieder, U. (2011): Markov Decision Processes with Applications to Finance. Springer.

[2] Bertsekas, D.P. (2001, 2005): Dynamic programming and optimal control. Vol. I, II. Athena Scientific, Belmont, MA.

[3] Feinberg, E.A. and Shwartz, A. (2002): Handbook of Markov decision processes. Kluwer Academic Publishers, Boston, MA.

[4] Hernandez-Lerma, O. and Lasserre, J.B. (1996): Discrete-time Markov control processes. Springer-Verlag, New York.

[5] Puterman, M.L. (1994): Markov decision processes: discrete stochastic dynamic programming, John Wiley & Sons, New York.

[6] H.C. Tijms (2003): *A First Course in Stochastic Models,* John Wiley & Sons, Ltd. ISBNs: 0-471-49880-7 (HB)

[7] *London Economics International LLC,* 2013: "Estimating the Value of Lost Load".

[8] C. Growitsch, R. Malischek, S. Nick and H. Wetzel, 2013: "The Costs of Power Interruptions in Germany- an assessment in the light of the Energiewende", EWI Working Paper No 13/07.

[9] London Economics International LLC, 2013: „The Value of Lost Load (VoLL) for Electricity in Great Britain", Final Report for OFGEM and DECC.

[10] London Economics International LLC, 2013: "Estimating the Value of Lost Load".

[11] Qiu and Girgis (1993): „Optimization of power-system reliability level by stochastic-programming", Electric Power System Research 26(2), 87-95.

[12] A. Ruszcynski and A. Shapiro, Eds., Handbooks in OR & MS, Vol. 10, 2003, Chapter 10. Stochastic Programming Models in Energy, Stein W. Wallace, Stein-Erik Fleten. (MEMO/Literatur/stochastic models/ letztes)

[13] K. Brokish, J. Kirtley, 2009: "Pitfalls of modeling wind power using Markov Chains", Power Systems Conference and Exposition, 2009. PSCE'09. IEEE/PES. 2009. 1-6.

[14] A. Shamshad, M.A. Bawadi, W.M.A. Wan Hussin, T.A. Majid, S.A.M. Sanusi, 2003: "First and second order Markov chain models for synthetic generation of wind speed time series", Energy 30 (2005) 693–708.

[15] F. Ess, L. Haefke, J. Hobohm, F. Peter, M. Wünsch, 2012: "Bedeutung der internationalen Wasserkraft-Speicherung für die Energiewende", Prognos AG, Studie für den Weltenergierat – Deutschland e.V.

[16] S. Rebennack, J. Kallrath, "Stochastic Optimization in energy systems – With Practical Implementations in GAMS" ISBN 978-3-8348-2135-5

[17] J. Kallrath, P.M. Pardalos, M. Scheidt (Eds.), 2009: "Optimiziation in the energy Industry", 2009 ISBN 978-3-540-88965-6

[18] Andreas ZUCKER, Timothée HINCHLIFFE, Amanda SPISTO (2013): "Assessing Storage Value in Electricity Markets - A literature review", European Comission, doi: 10.2790/89242

[19] Sheldon, M. Ross, "Introduction to stochastic dynamic programming", 1983.

[20] R. Tidball, J. Bluestein, N. Rodriguez, S. Knoke, 2010: "Cost and Performance Assumptions for Modeling Electricity Generation Technologies", NREL is a national laboratory of the U.S. Department of Energy, Office of Energy Efficiency & Renewable Energy, operated by the Alliance for Sustainable Energy, LLC.