CODED EXCITATION FOR LOW-SNR SYSTEMS AND EMATS

by

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Declaration of originality

The content of this thesis is my own work, with supervision from Dr. Frederic Cegla and Professor Peter Cawley. Where I have made use of the work of others, I have made this clear and provided appropriate references.

Julio Isla 09/02/2017
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Abstract

This thesis addresses two main subjects: the development of efficient electromagnetic-acoustic transducers (EMATs) and the synthesis of coded sequences for pulse-echo mode. EMATs are desirable because no mechanical contact with the sample is required; however, EMATs are inherently inefficient. To improve their performance, a ferromagnetic core surrounded by permanent magnets whose like poles face the core is proposed as the bias magnetic field source; this configuration can outperform a single magnet by an order of magnitude. Coil configurations that result in linear and radial polarisations of the ultrasonic waves are also compared; the linear polarisation was found to yield higher mode purity and penetration depths. Furthermore, the optimal impedance of the EMAT coil is discussed.

Although improvements in EMAT sensitivity of up to 20 dB were achieved, this is not enough to obtain adequate signal-to-noise ratios (SNR) (> 30 dB) without averaging over a long period of time. Therefore, efficient encoding techniques to achieve a greater SNR over shorter periods of time were investigated. The maximum length of conventional coded sequences and hence the maximum SNR increase is limited by the location of the closest reflectors. In this thesis, coded sequences that have receive intervals are introduced so that their overall length and therefore the SNR increase is not affected by the location of the reflectors; the proposed sequences can produce a given SNR increase an order of magnitude faster than averaging. These sequences are then used to drive EMATs using less than 0.5 W and a repetition rate of 10 Hz; this rate can be perceived by a human inspector as quasi-real-time. Moreover, a set of pseudo-orthogonal sequences that have common receive intervals can simultaneously be transmitted through several transducers. This makes it possible to increase the number of
active elements in an array, which combined with synthetic focusing, can increase the resolution and contrast of the resulting image without affecting the frame rate.

Binary quantisation is also investigated in this thesis. It can be used with low-SNR systems resulting in minimal loss of information while reducing the data throughput and the complexity of the electronics, especially in arrays with many elements. The theory behind binary quantisation is reviewed and the maximum input SNR range that does not cause unacceptable distortions is investigated.

Finally, the first low-power, pulse-echo EMAT phased array system is proposed. This array can perform similarly to conventional piezoelectric arrays mounted on a wedge to excite angled shear wave in the sample. Racetrack coils are used as the array elements laid out in an overlapping pattern that minimises inter-element crosstalk and results in an array element width and pitch equivalent to 1 and 2/3 of the wavelength respectively. Coded sequences that have receive intervals are used to reduce the drive power to just a few watts (24 Vpp). The performance of an 8-channel, 1-MHz prototype in the detection of surface cracks which have a length of less than 1 mm is reported.
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Contents

1 Introduction 18
   1.1 Motivation ........................................ 18
      1.1.1 EMATs for intrinsically-safe monitoring and arrays ... 18
      1.1.2 Low-SNR regimes and coded sequences with receive intervals 22
   1.2 Outline of thesis ..................................... 24

2 Optimisation of the Bias Magnetic Field of Shear Wave EMATs 27
   2.1 Introduction ........................................ 27
   2.2 Background ........................................ 30
      2.2.1 EMATs based on the Lorentz force ............... 30
      2.2.2 Bias magnetic field formulation ................ 31
   2.3 Concentric arrangement of permanent magnets surrounding a ferromagnetic core .............. 32
      2.3.1 Ideal configuration and practical approximations ... 32
      2.3.2 Finite element simulations of the ideal core-magnet configuration ................ 34
      2.3.3 Magnetic flux density distribution of the ideal core-magnet configuration ............. 35
      2.3.4 Comparison with other configurations ............. 37
   2.4 Influence of the ultrasound aperture area and polarisation on the signal strength and mode purity .............. 39
      2.4.1 Finite element models of ultrasonic apertures which have radial and linear polarisations .............. 39
4 Binary Quantisation

4.1 Introduction ........................................ 85
4.2 Binary Quantisation and Averaging ................. 88
  4.2.1 Transfer function of the binary quantiser after averaging . 88
  4.2.2 Quantisation errors and SNR ........................ 91
  4.2.3 Limits of binary quantisation ...................... 92
4.3 Numerical Simulations ................................ 94
  4.3.1 Expected value at the output of the quantiser ....... 94
  4.3.2 Output SNR .................................. 96
4.4 Experimental Results ................................. 100
4.5 Summary ............................................ 105

5 Pulse-echo coded excitation: single channel 107

5.1 Introduction ......................................... 107
5.2 Overview of proposed coded excitation ............... 109
5.3 Background on Coded Excitation ..................... 113
  5.3.1 Previous work ................................ 113
  5.3.2 Merit Factor .................................. 114
  5.3.3 SNR increase .................................. 115
5.4 Properties and synthesis of sequences with receive intervals ...... 118
  5.4.1 Synthesis of sequences with receive intervals ....... 119
  5.4.2 Random distribution of receive and transmit intervals for even sampling of the medium ..................... 121
  5.4.3 SNR and optimal ratio of transmit and receive intervals . 122
  5.4.4 Comparison between sequences with receive intervals and averaging ...................................... 125
  5.4.5 Periodic sequences with receive intervals: continuous transmission ........................................ 127
  5.4.6 Burst modulation and multiple reflectors .............. 128
5.5 Application example: fast low-power EMAT .......... 130
  5.5.1 Experimental Setup ................................ 131
5.5.2 Results .............................................. 132
5.5.3 Discussion of results ............................ 137
5.6 Summary .............................................. 138

6 Pulse-echo coded excitation: arrays .......... 140
   6.1 Introduction ....................................... 140
   6.2 Background: Synthetic focusing using every transmit-receive path 145
   6.3 Random sequence set with receive gaps ............. 146
      6.3.1 System overview ................................ 146
      6.3.2 SNR increase .................................. 149
      6.3.3 Comparison with systems that do not use coded excitation 150
      6.3.4 SNR adjusted for modulation and uneven wave paths . . 152
   6.4 Numerical simulations ............................. 153
   6.5 Discussion ........................................ 158
   6.6 Summary ........................................... 160

7 Low-Power Phased EMAT Array .................. 162
   7.1 Introduction ....................................... 162
   7.2 Array design ...................................... 165
      7.2.1 Bias magnetic field ............................ 165
      7.2.2 Radiation pattern of single coils ............... 171
      7.2.3 Crosstalk between array elements ................ 173
   7.3 Results .......................................... 175
      7.3.1 Simulations ................................... 175
      7.3.2 Experiments .................................. 178
   7.4 Summary .......................................... 180

8 Conclusions .......................................... 182
   8.1 Review of the thesis findings ..................... 182
   8.2 Main contributions ................................ 185
   8.3 Areas for future work ............................. 187

References ............................................. 189
# List of Figures

1.1 ET210 EMAT-based monitoring system (Permasense Ltd., Horsham, UK). .......................................................... 22

2.1 Conventional EMAT comprising a single magnet and a pancake coil. 30

2.2 Concentric arrangement of ferromagnetic core and permanent magnets with a butterfly coil. ................................................. 33

2.3 Simulations of absolute magnetic flux density distribution of different EMAT configurations. .................................................... 36

2.4 Normal $B_n$ (black) and tangential $B_t$ (grey) flux density distributions simulations at 0.1 mm under the specimen surface for different EMAT configurations. .................................................... 36

2.5 Signals from finite element simulations. a-c) Radially polarised apertures (grey traces). d-f) Linearly polarised apertures (black traces). .......................................................... 41

2.6 Longitudinal mode generation in radially polarised shear aperture (axisymmetric view). .......................................................... 43

2.7 Maximum amplitude of the first echoes of the simulated signals for different apertures diameters, polarisation and specimen thicknesses. .......................................................... 45

2.8 Simulations of the signal strength according to equation (2.10) vs. EMAT overall diameter for the core-magnet arrangement. ........ 47

2.9 Simulation of the maxima of $B_n$ (continuous) and $B_t$ (dashed) at 0.1 mm beneath the specimen surface vs. specimen thickness. 50

2.10 Core-magnet arrangement used in the experiments. ................. 51
LIST OF FIGURES

2.11 Magnetic flux density of a core-magnet arrangement and a cylindrical magnet. ........................................ 53
2.12 D-shape coil with 22 turns of enamelled wire used in the experiments. 54
2.13 Signals from pulse-echo measurements on a 50 mm thick mild steel specimen. ........................................ 55
2.14 Relative signal amplitude for the single magnets of 10 and 20 mm diameters and the core-magnet arrangement. 57

3.1 Transformer circuit for modelling the interaction between the EMAT coil and the eddy currents on transmission. 63
3.2 Admittance curves of the pancake-coil EMAT on steel with a series capacitor of 1 nF and a lift-off of 0 mm. ........ 68
3.3 Effect of series resonance on transmission. ....................... 69
3.4 Multilayer coil. .................................................. 70
3.5 EMAT with switching board (a) used to select the optimal number of layers of the EMAT coil for the measurement set-up shown (b). 72
3.6 Amplitude of the received signal (squared) vs. number of coil layers. 73
3.7 Transformer circuit for modelling the interaction of the eddy currents and the EMAT coil on reception. ............... 75
3.8 Operational amplifier in non-inverting configuration, coil and eddy currents with their respective noise sources. .......... 78
3.9 Effect of parallel resonance on reception. .......................... 82
3.10 Impact of the coil inductance on the signal strength on reception. 82

4.1 N-channel binary acquisition system. ........................... 86
4.2 Stages of binary quantisation. .................................... 89
4.3 a) Cumulative distribution function (CDF) of the standard normal distribution, \( F \).  b) Repetitions of \( X(t_0) = s(t_0) + Y(t_0) \) with \( s(t_0) = 1 \). .................................................. 90
4.4 Mean value before and after quantisation of a normal distribution with \( \sigma = 1 \) for \( 10^4 \) sets of a) \( 10^2 \) and b) \( 10^4 \) samples. ............... 95
4.5 SNR before and after quantisation. 10^4 sets of a) 10^2 and b) 10^4 realisations. ........................................ 97
4.6 Output of equation (4.10) for SNR inputs between −5 and 15 dB using 10^4 sets of 10^2, 10^3 and 10^4 realisations. ......................... 98
4.7 Input SNR that yields SNR_{max} (--), 10 \log_{10} N − SNR_{max} (\cdots), and input SNR where saturation occurs 10% of the time with a probability of 0.9 (labelled Sat.>10%) vs. the number of added realisations N. .................................................. 99
4.8 Experimental set-up using ultrasonic transducers. .................... 101
4.9 Comparator input (black) and output (grey). .......................... 101
4.10 Receive signal (before comparator) after 500 averages. .............. 102
4.11 First echo under different excitations and post-processing conditions. 103
5.1 Pulse-echo system with close and far reflectors. ....................... 108
5.2 Different types of excitations for pulse-echo transducers. ............ 110
5.3 Fundamental steps of the proposed coded excitation. .................. 112
5.4 Random distribution of receive intervals in a sequence. ............... 120
5.5 SNR of the sequence with receive intervals, SNR_{\text{gaps}}, vs. the probability of having a transmit interval, p_1, for different input SNRs, SNR_{\text{in}}. ....................................................................... 124
5.6 Ratio of transmit and total number of intervals when averaging, t = \frac{N}{L}, and input SNR, SNR_{\text{in}}, for which the SNR obtained after using the sequences with receive intervals is approximately the same to that of averaging, i.e., \alpha \approx 1. .................................................. 127
5.7 Experimental setup using low-power custom-made electronics and sequences with receive intervals to drive a commercial EMAT employing 4.5 Vpp only. ................................. 132
5.8 Echoes from 20 mm-thick steel block. ..................................... 134
5.9 Signals using linearly polarised shear wave EMAT ..................... 136
5.10 Experimentally observed SNR vs. number of averages N or equivalent sequence length L/4. .............................................. 137
<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.1</td>
<td>Proposed system architecture to reduce data throughput.</td>
<td>144</td>
</tr>
<tr>
<td>6.2</td>
<td>Full matrix capture.</td>
<td>145</td>
</tr>
<tr>
<td>6.3</td>
<td>Synthetic focusing using a random set of sequences and receive intervals.</td>
<td>147</td>
</tr>
<tr>
<td>6.4</td>
<td>Ratio of transmit and total number of intervals, ( t = \frac{N \cdot M}{L} ), and input SNR, SNR_in, for which the SNR obtained when using the proposed sequences set is the same as that obtained with averaging, i.e. ( \alpha = 1 ).</td>
<td>152</td>
</tr>
<tr>
<td>6.5</td>
<td>Finite element model.</td>
<td>154</td>
</tr>
<tr>
<td>6.6</td>
<td>Focused images.</td>
<td>156</td>
</tr>
<tr>
<td>6.7</td>
<td>Predicted upper bound using equation (6.10) (curves) and simulated results (circle markers) for the image SNR using ( M = 16 ) elements.</td>
<td>157</td>
</tr>
<tr>
<td>7.1</td>
<td>Alternative configurations of angled shear wave phased arrays that can be used to detect surface cracks.</td>
<td>164</td>
</tr>
<tr>
<td>7.2</td>
<td>Sketch of proposed EMAT array showing the overlapping pattern of the coils and the ferromagnetic core abutted by magnets with like poles facing the core.</td>
<td>166</td>
</tr>
<tr>
<td>7.3</td>
<td>Photo of the EMAT array prototype that was built.</td>
<td>166</td>
</tr>
<tr>
<td>7.4</td>
<td>2D simulations of the distribution of the magnetic flux density in the ferromagnetic core, magnet (Neodymium N42, ( B_r = 1.32 ) T), and sample.</td>
<td>168</td>
</tr>
<tr>
<td>7.5</td>
<td>2D simulations of the normal components of the flux density over aluminium and steel samples.</td>
<td>170</td>
</tr>
<tr>
<td>7.6</td>
<td>Distribution of the normal components of the magnetic flux density over the surface of an aluminium sample (simulations and experiments).</td>
<td>171</td>
</tr>
<tr>
<td>7.7</td>
<td>2D simulation of two shear line sources of opposite phase in steel 10 ( \mu )s after the burst excitation.</td>
<td>172</td>
</tr>
<tr>
<td>7.8</td>
<td>Measured crosstalk between two pairs of coils.</td>
<td>174</td>
</tr>
</tbody>
</table>
7.9  2D model of an aluminium block that has a slot. .............. 175

7.10  Focused image that corresponds to a 30-mm aluminium block which
       has a slot (defect). .................................. 177

7.11  Experimental setup: an 8-channel EMAT array is placed over an
       aluminium block that has a slot on the back wall; the slot cross-
       section is 0.2-mm wide and 0.8-mm high. ..................... 179
List of Tables

3.1 Relative amplitude of the signals corresponding to a 4-layer coil at 1 MHz ........................................ 73
3.2 Capacitance values (5% tolerance) ........................................ 74
Chapter 1

Introduction

1.1 Motivation

1.1.1 EMATs for intrinsically-safe monitoring and arrays

The initial motivation of this thesis was to develop the first intrinsically-safe monitoring system for thickness gauging of steel pipes in the oil and gas industry based on electromagnetic-acoustic transducers (EMATs). Many critical areas of pipelines around the world require regular inspection/monitoring to determine their operational status, to see whether corrosion or erosion has developed or to look for cracks. This monitoring process helps to minimise the occurrence of large-scale disasters but also offers data that allows to develop more accurate maintenance schedules, which can help to effectively avoid the cost of oil and gas plant shutdown. Moreover, permanently installed transducers are particularly useful where frequent measurements are required and the access is costly or dangerous.

EMATs are highly desirable for these tasks because they do not require mechanical contact with the specimen and can be used to inspect through coating layers such as anti-corrosion paints [1-4]. They are relatively simple transducers, which comprise a coil that carries alternating current and a magnet. EMAT testing requires minimal or no surface preparation, as opposed to current technology, which is based on piezoelectric transducers which require mechanical coupling to
1. Introduction

the area that is to be inspected. Mechanical coupling requires surface preparation such as the removal of the surface coating, welding of studs or adhesive bonding of supporting parts [5]. It is therefore clear that EMATs offer a significant advantage speeding up the installation process and avoiding any damage to the corrosion protection of the area which is to be inspected.

In the non-destructive evaluation (NDE) field, EMATs have been used to generate guided waves for the inspection of plates and pipes [6, 7] and also Rayleigh waves on the surface of the specimens [8, 9]. They have been employed for the detection of defects using bulk waves [10–13]. EMAT designs for high-temperature (250° and 800°C) inspection has also been reported [14, 15]. Furthermore, EMATs are particularly useful to inspect moving samples [16].

Literature on EMATs is heavily focused on describing their complex but versatile operating mechanisms [6–9, 17–21]; however, hardly any work has focused on optimising their design to increase their efficiency. Part of this thesis tries to fill this gap by investigating how to increase the transduction efficiency of an EMAT which is to be used for thickness gauging of mild steel pipes. EMATs have two main generation mechanisms, namely the Lorentz force and magnetostriction. Under the Lorentz force, the alternating current that circulates through the EMAT coil induces eddy currents in the specimen, the eddy currents interact with the bias magnetic field of the EMAT and this generates Lorentz force components, which subsequently excite ultrasonic waves [1, 4]. On the other hand, magnetostriction is a property of ferromagnetic materials that causes them to change their shape or dimensions when subject to magnetisation. Recent work suggests that the Lorentz force is the most efficient mechanism in mild steel [1, 22–25], but this is still a source of debate. Magnetostriction requires a specific magnetic field biasing for optimal operation, and this optimal biasing varies with the type of material and the separation between the EMAT and the specimen [26]. Therefore, the EMAT design needs to be carefully optimised for each individual application. Conversely, the greater the bias magnetic field, the greater the Lorentz force components, and therefore the more intense the ultrasonic waves that are generated [P1]. For this reason, the work presented here focuses on the Lorentz force as the
main transduction mechanism and maximisation of the bias magnetic field.

In the initial stages of this project, both pulse-echo and pitch-catch configurations were considered. The pitch-catch configuration consists of two different EMATs (or at least two different coils) each one acting simultaneously either as a transmitter or as a receiver. This allows to transmit long coded sequences from the transmit to the receive EMAT so that the transmitted power can be reduced by cross-correlating the received signal with the transmitted one, a technique known as pulse-compression and which can reduce the electronics noise faster in comparison to averaging [3, 27–31].

It was concluded that the pitch-catch configuration is inherently more inefficient when both transducers have to be placed on the same side of a test sample which has parallel walls because the radiation pattern of the transducers prevents most of the radiated energy from travelling to the receive transducer. This is contrary to the pulse-echo configuration, wherein a single transducer operates as a receiver and a transmitter alternatively, and hence the transducer can be designed in such a way that a significant portion of the radiated energy is gathered after the waves reflect on the back-wall [P1].

EMATs can be designed to generate shear or longitudinal waves. High-purity, single-mode excitation is important because it facilitates the detection of the reflected echoes. Shear waves were initially selected due to their shorter wavelength, which allows for a higher resolution at any given frequency, and later it was concluded that shear-waves EMATs are more compact and produce greater magnetic flux densities, i.e., greater signal intensities [P1].

Another interesting issue that was investigated was the polarisation of the ultrasonic waves. Most commercial EMATs used for thickness gauging are based on the radial polarisation; this can be achieved by placing a single magnet on top of a pancake-like coil. The linear polarisation is achieved by placing a single magnet on top of a straight section of a coil. The linear polarisation was found to produce higher single-mode purity and greater penetration depth [P1].

With all this information at hand, a configuration for the source of the bias magnetic field of shear-waves EMATs which consists of a core surrounded by
permanent magnets with like poles facing the core was explored \[32-34\]. One of
the main contributions of this thesis is the study of the optimal dimensions of
such a configuration for a given overall volume and active area \([P1]\), i.e., the area
where the ultrasound waves of interest are generated. It was found that, under
certain conditions, this configuration can outperform a single magnet by an order
of magnitude.

The optimal impedance of EMATs is also addressed here and light is shed
on the importance of maximising the power transfer from the driver to the eddy
currents induced by the EMAT coil rather than to the EMAT coil itself; although
EMATs have been around for many decades such a methodology is not clear in
the literature. It is also shown how the coil impedance can be tuned, similarly to
a variable impedance transformer, to maximise the ultrasonic waves \([P2]\), thereby
simplifying the components of the driving circuitry. Additionally, some discussion
on the optimal coil impedance on reception and the receiver configuration is
provided.

The results obtained in these different areas here investigated became the core
technology of the first commercial, intrinsically-safe, EMAT monitoring system:
the ET210, commercialised by Permasense Ltd., Horsham, UK. A photo of the
sensor is shown in Fig. 1.1. This system can operate continuously on batteries
for several years providing information about the thickness of the pipes in the oil
and gas industry.

Finally, the attention is turned to EMAT arrays. There are very few EMAT
phased arrays reported in the literature for bulk waves, and probably the only
one commercially available is from Innerspec Technologies, Inc., US \([35]\). The
design of EMAT arrays is challenging because the strong electromagnetic crosstalk
between their elements deteriorates the array performance and also small array
elements (coils) are needed to obtain adequate radiation patterns; smaller coils
require greater excitation powers, which they cannot withstand.

A solution to those central problems is proposed, which consists in laying
out the array elements in a way that minimises crosstalk and element size and
that maximises the coupling to the specimen \([P7]\). Additionally, the amount of
1. Introduction

**Figure 1.1:** ET210 EMAT-based monitoring system (Permasense Ltd., Horsham, UK, an Emerson company, US). Image from http://www.permasense.com/technology.php (Oct. 2016).

...power required by smaller elements is reduced to just a few watts and 24 Vpp by using a new type of coded sequences [P5] and [P6]. The proposed array generates shear waves, similarly to many conventional piezoelectric arrays that use a wedge to convert longitudinal waves into shear waves [36, 37], but with the intrinsic advantages of EMATs. Such arrays are desirable for the detection of small surface cracks that can be produced by, for example, material fatigue [38–40].

### 1.1.2 Low-SNR regimes and coded sequences with receive intervals

Low-power EMATs produce low signal-to-noise ratios SNRs, where the signal is comparable to or lower than the electronic noise at the input of the receiver. It was reported in [41, 42] that once the signal is below the noise level, a comparator can be used to binary quantise the signal. The SNR is later increased by means of averaging or pulse-compression resulting in a negligible loss of SNR compared to a high-resolution analog-to-digital converter (ADC). The use of a comparator in place of a conventional ADC significantly reduces the complexity of the electronics and the data throughput, especially in array systems where the channel count is high. Due to the impact of this methodology in the development of highly-dense and miniaturised electronics, the performance of the comparator in the presence...
of higher-SNR inputs was studied in order to understand how the dynamic range of the binary quantiser can be extended using post-processing techniques [P3].

This thesis also focuses on pulse-compression techniques as a means to quickly increase the SNR of the received signals. Pulse-compression can be accomplished by using coded sequences. For example, a binary coded sequence can be used to control the polarity of bursts subsequently transmitted, which results in a chain of bursts with different polarities. The received signal is then cross-correlated with the transmitted chain of bursts, which yields a signal similar to that obtained when transmitting a single burst but with a much greater SNR.

I found that under a low-SNR regime, the well-known high-performance coded sequences, e.g., Golay codes [43], Barker codes [44, 45], Legendre sequences [45], etc., do not offer any substantial advantage with respect to an arbitrarily random sequence [P4] and [P9]. This is an important observation because it simplifies the selection of any optimal code.

Despite the improvement in EMAT transduction presented in this thesis, low-power EMATs operating in pulse-echo could not be used in real-time with conventional pulse-compression techniques. The problem with conventional pulse-compression techniques is that the length of the excitation, which determines the SNR increase, is limited by the location of the closest reflectors. Moreover, when there are far reflectors (or strong reverberations inside a specimen) averaging takes long due to the wait time imposed between transmissions, which is required for reverberations to die out before a new acquisition can be started.

To solve this problem, I proposed to introduce random receive intervals within long coded sequences, such that on average, the same number of reflections from each reflector can be received irrespective of the reflector location. This technique permits commercial EMATs (e.g., Part No. 274A0272, Innerspec, USA) to be used in quasi-real-time with only 5 Vpp, otherwise needing 600 Vpp or more [P4] and [P5].

It is important to highlight that the use of these new coded sequences extends beyond any particular EMAT, transduction mechanism or field (i.e., radar, sonar, industrial or medical ultrasound). Therefore, many applications previ-
1. Introduction

Previously thought impractical can now be reconsidered, for example, the use of more inefficient but maybe cheaper or miniaturised transducers [46–55].

Another key advantage of the use of long coded sequences that have receive intervals in the low-SNR regime is that non-correlated or pseudo-orthogonal sequences can be transmitted in parallel through each array element operating in pulse-echo mode [P6] and [P9]. Such sequences find immediate application in low-power EMAT phased arrays [P7]. The design of EMAT phased arrays is challenging because smaller elements (coils) are needed to obtain good resolution and steering capabilities, but smaller elements require higher power, which they cannot withstand in practice.

Likewise, these sequences could solve the problem encountered in dense arrays, such as 2D matrix arrays used for 3D ultrasound [56–60], where the number of focal points that can be attained on transmission is limited by the frame rate and the time the ultrasonic wave takes to propagate to and back from the reflectors. For example, the X6-1 xMATRIX array probe (Philips Medical Systems, Andover, MA, USA) has 9212 elements but the information that can be recovered from all of its elements in parallel is limited without suitable coded sequences and acquisition systems. By firing and receiving on all of the channels in parallel, any focal point can be synthetically reconstructed, which increases resolution while maintaining an adequate frame rate [P6] and [P9]. This is something not possible with conventional sequences, mainly because they do not have receive intervals.

1.2 Outline of thesis

In this thesis, two main topics are addressed: EMATs and coded sequences. The development of low-power EMATs was the initial motivation, so Chapters 2 and 3 are devoted to the optimisation of the bias magnetic field of shear-wave EMATs and to the optimal EMAT impedance respectively.

Given that low-power EMATs produce low-SNR outputs, the use of binary quantisation is investigated in Chapter 4 as an alternative to standard ADCs. Having found that by introducing receive intervals in long coded sequences, low-
SNR transducers (e.g., low-power EMATs) can be used in quasi-real-time, the main focus of the thesis is then turned to this subject in Chapter 6, while Chapter 6 elaborates on the use of parallel sequences that have receive intervals. With this idea in mind, the first pulse-echo, low-power EMAT phased array is proposed in Chapter 7. Finally, conclusions are drawn.

In more detail, the chapters are structured as follows:

**Chapter 2** discusses the design of a Lorentz-force shear-wave EMAT for thickness gauging. Special emphasis is placed on the optimisation of the design elements that correspond to the bias magnetic field of the EMAT. A configuration that consists of several magnets axi-symmetrically arranged around a ferromagnetic core with like poles facing the core is investigated in detail. The linear and radial polarisations of shear waves are also compared.

**Chapter 3** investigates the optimal impedance of EMATs on transmission and reception. The first part of the chapter addresses the optimal impedance on transmission. A transformer circuit is used to model the interaction between the EMAT coil and the eddy currents that are generated beneath the coil in the conducting specimen. Expressions for the coil impedance that produces the maximum efficiency and the maximum power transfer on transmission are presented. A tunable coil that consists of several identical thin layers vertically stacked and independently accessed is used to tune the coil inductance without affecting the radiation pattern of the EMAT. The second part of the chapter discusses the optimal impedance on reception and how the noise sources of the receive amplifier affect it.

**Chapter 4** deals with the use of binary quantisation. The theory behind binary quantisation is reviewed, mainly from previous work on wireless sensor networks, and the maximum input SNR range where binary quantisation is viable is investigated.

**Chapter 6** presents a new approach to coded excitation, wherein receive intervals are introduced within long coded sequences. As a result, the code length is
no longer limited by the distance to the closest reflector and greater SNRs can be realised. The optimal distribution of the receive intervals and the resulting SNRs are discussed. An example of an EMAT being driven with 4.5 V in quasi-real-time is shown.

Chapter 6 introduces a pseudo-orthogonal set of randomly coded sequences that have receive intervals, which can be transmitted in parallel in pulse-echo arrays. This leads to a novel system architecture for low-SNR regimes in which drivers and receives are controlled by digital lines thereby reducing data throughput by roughly an order of magnitude; this architecture is valuable with dense, high-frequency arrays. The synthesis of the pseudo-orthogonal set and the resulting SNRs are discussed.

Chapter 7 introduces an EMAT phased array that consists of narrow racetrack coils as the array elements, which are laid out in an overlapping pattern that minimises inter-element crosstalk and results in an array pitch equivalent to 2/3 of the wavelength. Coded sequences with receive intervals are used to reduce the peak power of the driver to just 4.8 W (24 Vpp and 200 mA). A ferromagnetic core surrounded by magnets is employed to increase the bias magnetic field of the array and to reduce the generation of ultrasonic waves inside the magnets. An 8-channel prototype is shown to be capable of imaging surface cracks which have cross-section area of less than 1 mm$^2$; this prototype operates at a central frequency of 1 MHz based on the Lorentz force.
Chapter 2

Optimisation of the Bias Magnetic Field of Shear Wave EMATs

2.1 Introduction

Most of the material in this chapter was extracted with minor modifications from [P1] (with permission from IEEE) and also from [P8].

Electromagnetic-acoustic transducers (EMATs) do not require direct contact with the specimen under test, they outperform piezoelectric transducers in applications where couplant cannot be used, where there are unfavourable coating layers, or simply when non-contact transduction is required [1–4]. Moreover, EMATs are very versatile and various configurations exist that can be used to generate a whole range of ultrasonic waves, for example, longitudinal, shear and different types of guided waves [6–9, 17–21].

However, EMATs commonly operate at hundreds of volts, which tends to make the electronics bulky. Moreover, any devices operating at such voltage levels are unsuitable in environments where intrinsic safety certification is required, for example, in the oil and gas industry [61]. Lower voltages can be used but averaging is then required to increase the signal-to-noise ratio (SNR). In many applications the properties of the specimen under test change while the test is being run, and this corrupts the results produced by averaging. Therefore, it is necessary to study how to increase the sensitivity of EMATs so that they can be
driven by compact, low voltage and intrinsically safe electronics. The outcome of this study can be applied to extending the battery life of battery-powered EMATs, which may lead to their use in long-term monitoring applications.

EMATs comprise a source for the bias magnetic field, such as that generated by permanent magnets, and a coil carrying alternating current [1, 2]. EMATs may rely on various transduction mechanisms, but this material focuses on the Lorentz force as the predominant transduction mechanism in mild steel and ignores the influence of other mechanisms such as magnetostriction; this assumption is sound based on [1, 22–25]. When considering the Lorentz force as the predominant transduction mechanism, the bias magnetic field is of paramount importance, since the strength of the signal/wave increases in proportion to its magnitude on both transmission and reception.

According to recent publications, the strength of the bias magnetic field of EMATs can be increased by introducing other configurations more complex than a single magnet, which can be grouped under magnetic flux concentrators and repelling magnet configurations. In [62], a soft magnetic ribbon was placed between a permanent magnet and the coil. This acted as a flux concentrator for the bias magnetic field, which increased the flux density and the signal strength. In [63] a capped cone of ferromagnetic material was placed between a magnet and a ferromagnetic specimen. The capped cone concentrated the flux from the wider area of the magnet into a smaller area over the specimen, thereby increasing the flux density.

Two magnets arranged in a repelling configuration were employed in [64, 65]. An increase in the magnetic flux density by a factor of almost two compared to a single magnet was achieved in the area between the repelling magnets. However, this improvement was constrained to small areas between the repelling magnets and hence its application was limited to small ultrasound apertures.

Configurations that combine flux concentration and repulsion mechanisms have already been proposed [32–34]. These basically consisted of a core of ferromagnetic (magnetically permeable) material surrounded by magnets with like poles facing the core. A strong magnetic field resulted inside the core, due to the
repulsion between the magnets, which then escaped through the two remaining faces that are normal to the axis of symmetry. However, the dimensions of the core and magnets that maximised the magnetic flux density within a given ultrasound aperture and/or produce ultrasonic waves with high mode purity are still unknown.

Despite all these studies on EMATs, there remains a need for more quantitative analyses of their optimal configurations and dimensions, such as that conducted in [66]. In [66] the effect of the ratio between the width of the magnet and coil area on the strength of the signal was investigated; the authors concluded that beyond a certain ratio no major improvement in the signal strength is possible.

In this chapter, several EMAT configurations that can be used for thickness gauging applications are investigated and their performance is compared. Special emphasis is given to the configuration that consists of a ferromagnetic core surrounded by repelling magnets. Additionally, I study the combined effect of the magnet configuration and the size of the ultrasound aperture on the signal quality and intensity; the difference between linear and radial polarisations of shear-wave EMATs is also addressed.

The organisation of this chapter is as follows: firstly the theory behind the Lorentz force, as the predominant transduction mechanism of EMATs in mild steel, and the bias magnetic field is introduced. Secondly, the configuration consisting of a ferromagnetic core surrounded by permanent magnets is described. The distribution of the magnetic flux density within this configuration and the specimen is studied by using finite element (FE) simulations. The results are then compared with other configurations. Later, the effect of the size of the ultrasound aperture on the signal strength and the mode purity of the ultrasonic waves in pulse-echo mode are investigated using FE analysis. Following that, the optimal dimensions of the core-magnet arrangement that maximise the signals in pulse-echo mode are investigated and compared to other configurations. Finally, experimental results that show the superior performance of the optimised core-magnet arrangement are presented and conclusions are drawn.
2. Optimisation of the Bias Magnetic Field of Shear Wave EMATs

2.2 Background

2.2.1 EMATs based on the Lorentz force

EMATs comprise a bias magnetic field and a coil that carries alternating current. One of the simplest and commonly used configurations for EMATs is shown in Fig. 2.1. It consists of a single cylindrical permanent magnet placed on top of a pancake-like coil such that radially polarised waves are generated [17].

![Figure 2.1: Conventional EMAT comprising a single magnet and a pancake coil.](image)

EMATs can rely on various transduction mechanisms (i.e. the Lorentz force, magnetostriction, etc.) depending on their configuration and the specimen properties [1, 2]. In mild steel, the main focus of this thesis, the Lorentz force is assumed to be the predominant transduction mechanism for bulk waves; this assumption is sound based on [1, 22–25]. It can be understood as follows: firstly the coil of the EMAT induces eddy currents on the surface of a conductive specimen whose path tends to mimic that of the coil. Then these eddy currents with density $J_e$ interact with the bias magnetic field, whose flux density is $B$, and the resulting Lorentz force density on the charged particles (electrons) is given by

$$f = J_e \times B.$$  \hspace{1cm} (2.1)

The charged particles interact with the atomic structure of the material which results in deformations that generate ultrasonic waves. In this chapter $B$ refers to
the bias magnetic field due to permanent magnets only, any other contributions to the bias magnetic field, e.g. due to the eddy currents, is neglected.

The inverse effect also applies whereby an ultrasonic wave forces the charged particles to move which, combined with a bias magnetic field, produce eddy currents under the surface of a conductive specimen with density

$$J'_e = \sigma (v \times B),$$

(2.2)

where $\sigma$ is the conductivity of the material and $v$ is the velocity of the charged particles. These eddy currents are then inductively picked up by the coil of the EMAT. It should be noted that in the case of an EMAT operating in pulse-echo, the resulting signal is proportional to the square of the magnetic flux density $B$ since it contributes twice, on transmission and reception.

### 2.2.2 Bias magnetic field formulation

The magnetic field $H$ and the magnetic flux density $B$ are related as follows

$$B = \mu_0 (H + M)$$

(2.3)

where $M$ is the magnetisation vector. The magnetisation vector can be expressed as

$$M = \chi_m H + M_0$$

(2.4)

where $M_0$ is the remanent magnetisation and $\chi_m$ the susceptibility of the material. A useful figure of merit for permanent magnets is their remanent flux density
\[ B_r = \mu_0 M_0. \] For static fields

\[ \nabla \times H = 0 \]  \hspace{1cm} (2.5)

so a scalar magnetic potential \( V_m \) can be defined as

\[ H = -\nabla V_m. \]  \hspace{1cm} (2.6)

Substituting this back into (2.3) and using the fact that

\[ \nabla \cdot B = 0, \]  \hspace{1cm} (2.7)

it is obtained

\[ \nabla \cdot (-\mu \nabla V_m + B_r) = 0 \]  \hspace{1cm} (2.8)

where \( \mu = \mu_0 (\chi_m + 1) \). As discussed later in this chapter, equation (2.8) can be solved using FE methods in order to obtain the distribution of \( H \) within a given volume where a permanent magnet is present.

### 2.3 Concentric arrangement of permanent magnets surrounding a ferromagnetic core

#### 2.3.1 Ideal configuration and practical approximations

First, an ideal configuration is introduced to facilitate the FE simulations of the concentric arrangement of permanent magnets surrounding a ferromagnetic core. This ideal core-magnet configuration consists of a cylindrical magnet with its remanent flux density \( B_r \) oriented towards its centre, where there is a permeable
core (Fig. 2.2a-b). This ideal approach is useful to simplify simulations due to its cylindrical symmetry. However, in practice it is difficult to achieve such an orientation of $B_r$ with only one magnet. By ideal configuration I mean that it consists of a magnet whose remnant flux density is oriented towards the centre, which is difficult to make in practice; the word ideal does not imply that the configuration itself is necessarily the optimal global solution to the problems or scenarios discussed in this chapter. A good approximation can be obtained by several magnets, each having $B_r$ oriented towards the ferromagnetic core. Figure 2.2c shows an example using four magnets.

![Diagram of core-magnet configuration](image)

**Figure 2.2:** Concentric arrangement of ferromagnetic core and permanent magnets with a butterfly coil. a) Front view. b) Top view of ideal configuration with cylindrical symmetry. c) Top view of practical configuration with four magnets. d) Direction of currents in the butterfly coil.

The rationale behind this core-magnet configuration can be understood as follows: magnets with like poles facing each other produce an increase in the magnetic flux density in the space between them. An extra increase in the flux density is obtained by filling the gap between the magnets with a permeable (ferromagnetic) material. It is interesting to mention that permanent magnet arrangements with different magnet orientations in combination with permeable materials have been used in the past to significantly increase the magnetic flux density that can be generated by the magnet arrangement compared to a single magnet. These magnet arrangements can achieve 3 T [67,68], 4 T [69] and 5 T [70]; however, they were developed for other purposes not related to EMATs.
Moreover, the use of the permeable core provides other advantages. For example, a flat distribution of the normal component of the flux density $B_n$ can be obtained under the core for certain core and magnet dimensions. All of the cases discussed herein approximate this flat distribution of $B_n$. The core can also be made of non-conductive material such as ferrite or laminated iron so that eddy currents, and the subsequent ultrasonic waves generated in the core, are almost eliminated.

2.3.2 Finite element simulations of the ideal core-magnet configuration

In order to study and quantify the bias magnetic field generated by a cylindrical single magnet on the surface of a specimen, a finite-element model was simulated in COMSOL Multiphysics 4.3b using the Magnetic Field, No Current interface of the AC/DC module model library. This interface solves equation (2.8) by means of a Lagrange element formulation [71]. An axisymmetric (two-dimensional) model was employed as a result of the magnet cylindrical symmetry.

The maximum length of the quadrilateral mesh elements (distance between furthest nodes) was set to less than 0.1 mm under the core. Then, the length of the elements was increased progressively so that the elements furthest from the core and the magnets reached a maximum length of 2 mm. A convergence test was conducted to confirm that results did not change by more than 1% when using a denser mesh.

The space surrounding the structure was modelled as air. The height of the area simulated as air was three times the height of the structure whereas the width of the area simulated as air was twice the width of the structure. Magnetic insulation boundaries were employed to enclose the region modelled as air; the axis of symmetry did not require a magnetic insulation boundary. The magnet had a height of 20 mm, a width (inner to outer radii difference) of 6.2 mm and $B_r = 1.42$ T [72]. $B_r$ was oriented towards a soft iron core in the centre, which had a radius of 5 mm. The magnet-core arrangement was positioned above a soft
2. Optimisation of the Bias Magnetic Field of Shear Wave EMATs

iron specimen, whose thickness was 10 mm. A 0.5 mm-gap was left between the magnet-core arrangement and the specimen.

Due to the fact that the permeability of soft iron changes with the intensity of the field \( H \), a curve relating \( B \) vs. \( H \) has to be used in the simulations to obtain accurate results. This curve was obtained from the COMSOL library [71]. However, no significant changes were found in the distribution of \( B_n \) beneath the core when using other curves (e.g. for mild steel) because the strength of the field was very high in that region and therefore the different materials saturated easily.

2.3.3 Magnetic flux density distribution of the ideal core-magnet configuration

Figure 2.3a shows the results from the simulation. The absolute value of the flux density distribution is plotted using a grey scale in Tesla while the black lines represent the field lines. The field lines pass through the magnet towards the core, where their density increases as a result of repulsion. The lines then escape through the bottom and top sides of the core. Since the ferromagnetic specimen creates a higher permeability path compared to air, the flux density in the surface of the specimen beneath the core experiences a greater increase.

At frequencies above a few tens of kilohertz, the eddy currents induced by the EMAT coil concentrate within a few micrometres under the surface of mild steel specimens due to the electromagnetic skin effect. That is where the transduction mechanism takes place and hence it is sufficient to study the normal and tangential components of the flux density (\( B_n \) and \( B_t \) respectively) within that region in order to fully characterise the transduction mechanism. The flux density was found to vary little between 0 and 0.1 mm below the surface of the specimen; therefore, simulations that employ resolutions higher than 0.1 mm will not add any extra information to the results but will increase the computational burden.
2. Optimisation of the Bias Magnetic Field of Shear Wave EMATs

Figure 2.3: Simulations of absolute magnetic flux density distribution of different EMAT configurations represented by a grey scale in Tesla. Field lines are plotted with black lines.

Figure 2.4: Normal $B_n$ (black) and tangential $B_t$ (grey) flux density distributions simulations at 0.1 mm under the specimen surface for different EMAT configurations. In a) and b) the vertical dashed lines on the left indicate the radius of the core/capped cone face closest to the specimen whereas the ones on the right indicate the outer diameter of the EMAT. The vertical dashed line in c) indicates the outer diameter of the EMAT.
The distribution of $B_n$ and $B_t$ at 0.1 mm below the surface of the specimen that correspond to Fig. 2.3a are plotted in Fig. 2.4a. It can be observed that $B_n$ is fairly constant beneath the core, where it reaches values above 2.5 T and then drops abruptly. When a butterfly coil is used, as in Fig. 2.2a-b, with this distribution of $B_n$, shear waves with single polarity and high mode purity are generated under the core.

The highest values of $B_t$ appear under the magnet and its mean value there is roughly half of that achieved by $B_n$ under the core. $B_t$ is responsible for the generation of longitudinal waves, which are of much lower intensity than the shear waves in this configuration.

2.3.4 Comparison with other configurations

The core-magnet arrangement is now compared to other EMAT configurations. A cylindrical magnet is depicted in Fig. 2.3c, only half of the cross-section of the magnet is shown due to its symmetry. To give a fair comparison, the volume of this single magnet is made equal to that of the core and magnet combined of Fig. 2.3a. Its $B_r = 1.42$ T is also the same but oriented towards the specimen.

Figure 2.4c shows $B_n$ and $B_t$ distributions that correspond to the single magnet case at 0.1 mm below the surface of the specimen. The highest values of $B_n$ are reached beneath the magnet and these do not increase far above 1.2 T regardless of the size of the magnet. The maximum values of $B_t$ occur at the edge of the magnet, and they are of comparable magnitude to those of $B_n$.

It is important to highlight that the use of a single magnet to produce single-polarisation shear waves is impractical because the coil tracks must have a single orientation under the magnet and a return path elsewhere. This return path adds extra volume to the EMAT. A pancake-like coil, as in Fig. 2.1, is a volume efficient solution for a single magnet but radially polarised waves are generated instead. The disadvantages of the radially polarised waves are addressed later on.

Figure 2.3b depicts the same single magnet but on top of a capped cone of ferromagnetic material as reported in [63]. The radius of the bottom face of the capped cone is 5 mm, equal to the radius of the core-magnet arrangements.
of Fig. 2.3a to give a fair comparison, whereas the height of the capped cone is 5 mm. Different values were explored through simulations revealing that when the cone height was either increased or decreased by a few millimetres, no significant differences can be found in the results.

In Fig. 2.3b, the field lines from the magnet go through the capped cone where the flux density increases as the transversal area of the cone decreases. Most of the lines do not abandon the cone before entering the specimen because a high permeability path exists between the cone and the specimen. However, when the gap between the bottom of the cone and the specimen increases, a high reluctance region is introduced and a significant part of the field leaves the cone without reaching the specimen. This makes this particular configuration very sensitive to the EMAT lift-off (i.e. the gap between the bottom of the cone and the specimen).

Figure 2.4b shows the corresponding $B_n$ and $B_t$ distributions at 0.1 mm below the surface of the specimen. It can be observed that the behaviour of the field within the footprint of the cone is similar to that of Fig. 2.4a but with smaller values.

Overall, the use of any conical core has a detrimental effect on the intensity of the field in the specimen, especially when the lift-off increases. This is one problem found in [32, 33], where a core-magnet arrangement was proposed similar to that of Fig. 2.3a, but whose core bottom section had a conical shape. As a result, the magnets surrounding the core had to be separated from the specimen. I observed that in the case of a core-magnet arrangement, the greatest flux densities within the specimen are achieved when the bottom face of the core is aligned with that of the surrounding magnets as in Fig. 2.2a-b.
2. Optimisation of the Bias Magnetic Field of Shear Wave EMATs

2.4 Influence of the ultrasound aperture area and polarisation on the signal strength and mode purity

When extracting the time of flights of signals for thickness gauging applications, the signals received should have very low distortion and the greatest amplitudes possible in order to avoid errors. Therefore, it is important to understand how the aperture area and polarisation affect the received signals. Finite element simulations using the COMSOL Multiphysics 4.3b Solid Mechanics module were carried out to investigate that.

2.4.1 Finite element models of ultrasonic apertures which have radial and linear polarisations

In a first model a circular aperture with radial polarisation, as in Fig. 2.1, is investigated. An axisymmetric (two-dimensional) model sufficed because cylindrical symmetry can be exploited. In a second model the same aperture area is studied but using linear polarisation, as in Fig. 2.2, and a three-dimensional model due to the lack of cylindrical symmetry. In both models the general procedure is a) to apply a boundary load to the surface of a mild steel specimen within the aperture area, b) to simulate the propagation of the ultrasonic waves and c) to record the velocity of the reflected ultrasonic waves on the same surface where the boundary load is applied.

The axisymmetric model comprised a rectangular region simulated as mild steel (density 7850 kg/m$^3$, Young modulus 205 GPa and Poisson ratio 0.28). The thickness of this region was set to 5, 10 and 15 mm in a parametric study. The width of this region matched the area of the aperture and was varied from 2.5 to 10 mm in 2.5 mm intervals. One side of the rectangle was set as the axis of symmetry whereas the other side was coupled to multiple absorbing regions as described in [73] – this is to reduce wave reflections from this side. The outer boundary of the absorbing region parallel to the axis of symmetry was fixed to
prevent the displacement of the whole structure when the load was applied; the remaining boundaries were free to move.

The mesh employed quadrilateral elements with maximum length (distance from the furthest nodes) of 0.27 mm, this is roughly 1/6 th the shear wavelength in mild steel at 2 MHz (1.6 mm). A time domain simulation was conducted using time steps with a period equivalent to a sampling frequency of 32 MHz. The excitation signal was a 3-cycle Hanning tone-burst centred at 2 MHz. The excitation was applied as a force (boundary load) tangential to the surface. The force density on the surface was in the order of a few nN/m² – exact values are not relevant since only relative values in this range are of interest for the analysis of the results. Once every parameter was specified, a convergence test was conducted to confirm that neither a denser mesh nor a higher sampling frequency improved the results by more than 2%.

Results from simulations are shown in Fig. 2.5a-c, these are the sum of the radial component of the velocity of the reflected signals at the excitation nodes. The signals are normalised to the maximum value of the first echo to establish a qualitative comparison between the signals. Note that some of the excitation bursts are clipped because of this. The aperture diameter and the specimen thickness are specified in each case.

The three-dimensional model used to investigate the linearly polarised aperture was obtained by revolving the two-dimensional model a quarter of a circumference around its axis of symmetry. Then, symmetric and antisymmetric boundaries were used appropriately with respect to the orientation of the load, which was applied in a direction parallel to the symmetric boundary. The remaining settings were the same used in the two-dimensional simulation, except for the use of tetrahedral elements in the mesh. To establish a fair comparison, the same aperture diameters and specimen thicknesses of the two-dimensional model were simulated. The tangential component to the aperture area of the velocity of the reflected signals (also parallel to the symmetric boundary) is shown for each case in Fig. 2.5d-f.
Figure 2.5: Signals from finite element simulations. a-c) Radially polarised apertures (grey traces). d-f) Linearly polarised apertures (black traces). All of the signals are normalised to the maximum value of the first echo.
2.4.2 Signal Distortion in Pulse-Echo Mode

By visual inspection of the signals in Fig. 2.5, three main observations can be drawn: a) radially polarised apertures produce higher distortion, b) distortion decreases when the aperture area increases, and c) distortion decreases when the thickness increases. I analysed the wave fields in each particular case and determined that the distortion of the signals was caused by the direct generation of the longitudinal mode and by mode conversion upon reflections. In Fig. 2.5c echoes due to the longitudinal and shear modes are labelled as L and S respectively whereas the mode converted is labelled L+S.

In the case of the radially polarised shear apertures, the longitudinal mode is generated in its centre and on the edge. This is because the discontinuity of the applied forces in those regions causes compression and rarefaction creating a point source in the centre of the aperture and a curved line source on the edge. Figure 2.6a shows an axisymmetric view of a radially polarised aperture where the location of the longitudinal apertures can be appreciated. The reflection paths of the pure modes, shear and longitudinal, are also shown. Figure 2.6b shows an example of mode conversion from longitudinal to shear (L+S); other combinations are omitted for simplicity.

On the other hand, when the aperture is linearly polarised, the longitudinal mode is mainly produced on that section of the edge of the aperture perpendicular to the applied force. This explains why the linearly polarised aperture produces less distortion (or a weaker longitudinal mode) than the radial one.

Distortion decreases when the aperture area increases because the proportion of the area that generates the shear mode increases with respect to the length of the line (and point) sources that generate the longitudinal mode. And finally, distortion decreases when the thickness increases because the line sources are more affected by beam spreading than the larger areas that generate the shear mode.
Figure 2.6: Longitudinal mode generation in radially polarised shear aperture (axisymmetric view). a) Traction force discontinuities generates compression and rarefaction which triggers longitudinal point (centre) and line (edge) sources. b) Example of mode conversion L+S; other possible combinations are omitted for simplicity.
2.4.3 Signal Amplitude in Pulse-Echo Mode

The following inequation sets an upper bound for the intensity of the reflected signals $S$ given a transducer area $A$

$$S \leq \iint G_t G_r dA,$$  \hspace{1cm} (2.9)

where $dA$ is the infinitesimal area of the aperture while $G_t$ and $G_r$ are the transduction gains on transmission and reception respectively. This inequation is necessary to account for the beam spreading effect.

Assuming that the Lorentz force is the main transduction mechanism and that eddy currents follow a uniform distribution, inequation (2.9) can be rewritten such that the transduction gains $G_t$ and $G_r$ are expressed as a function of the magnetic flux density $B$. This is because in pulse-echo the magnetic flux density distribution and aperture area are the same on reception and transmission. Provided the coil shape is also the same on reception and transmission, it can be written

$$S \leq \alpha' \iint B^2 dA$$ \hspace{1cm} (2.10)

where $\alpha'$ is a constant.

Figure 2.7 shows the amplitude of the first echo of the simulated signals for different aperture diameters and specimen thicknesses. The black markers correspond to the linearly-polarised apertures whereas the grey markers represent the radially polarised apertures. For simplicity, and without loss of generality, $G_t$ and $G_r$ can be assumed constant and combined into a single constant $\alpha$. The black continuous curve in Fig. 2.7 was obtained by evaluating the upper bound of (2.9) and choosing $\alpha$ so that the curve matches the marker with the greatest value in the figure.
2. Optimisation of the Bias Magnetic Field of Shear Wave EMATs

Figure 2.7: Maximum amplitude of the first echoes of the simulated signals for different apertures diameters, polarisation and specimen thicknesses. The black continuous curve was obtained by evaluating the equality in (2.9) and choosing $\alpha$ so that the curve matches the marker with the greatest value.

It can be observed in the figure that the markers that correspond to aperture diameters larger than 10 mm and specimen thicknesses smaller than 15 mm lie very close to the solid black curve. In other words, the amplitude $S$ of each of the reflected echoes that correspond to these markers are very close to their maximum value, which is the right-hand side of inequation (2.9). This is because in these cases the beam spreading is negligible.

In general, the linearly polarised aperture (black markers) achieves greater echo amplitudes than the radially polarised aperture (grey markers) for each aperture diameter and specimen thickness. The difference in echo amplitudes between linear and radial apertures increases as the aperture diameter decreases and the specimen thickness increases. Note that for 10 mm apertures and thicknesses greater than 15 mm this difference is greater than 3 dB.
2.5 Optimal dimensions of the core-magnet arrangement

In this section, the optimal dimensions of the core-magnet arrangement are explored; the objective being to maximise equation (2.10). Results from the core-magnet arrangement are compared with those from other configurations, especially the single magnet EMAT. At the end of this section, the effect of the specimen thickness on the magnetic flux density distribution is investigated.

2.5.1 Effect of EMAT height and core and overall diameter

In order to investigate the optimal design parameters of the ideal cylindrical core-magnet arrangement of Fig. 2.2a, a finite-element model was simulated in COMSOL Multiphysics 4.3b employing cylindrical symmetry. A parametric study was conducted where the height of the arrangement was set to 10, 20 and 40 mm; the diameter of the core was set to 10 and 20 mm; and the width of the magnet (inner to outer radii difference) was changed to obtain EMAT overall diameters between 15 and 50 mm. The remanent flux density was set to \( B_r = 1.42 \text{T} \) (N52 [72]) and oriented towards a soft iron core in the centre. Both magnet and core had a lift-off of 0.5 mm from a soft iron specimen that had a thickness of 10 mm. As in Sec. 2.3.2, a curve relating \( B \) vs. \( H \) was employed for soft iron, which was obtained from the COMSOL library [71].

The resulting distribution of \( B_n \) under the surface of the specimen was squared and integrated over the area of the core to compute the expected signal strength based on equation (2.10) – results are shown in Fig. 2.8a. Continuous grey and black curves represent 5 and 10 mm diameter cores respectively whereas the dashed grey curves correspond to 20 mm diameter cores. Different EMAT heights were employed (10, 20 and 40 mm) for each core diameter represented by each family of curves. The position of the curves in the family is positively correlated with the height of the EMAT such that the smallest heights correspond to the curves with the smaller values. Curves were arbitrarily normalised since only the relative values are of interest.
Figure 2.8: Simulations of the signal strength according to equation (2.10) vs. EMAT overall diameter for the core-magnet arrangement. a) Continuous grey and black curves represent 5 and 10 mm diameter cores respectively whereas the dashed grey curves correspond to 20 mm diameter cores; for each diameter there are different EMAT heights (10, 20 and 40 mm) that correspond to each family of curves from bottom to top. b) Black and grey curves represent lift-offs of 0.5 and 2 mm; the overall height in each case is 20 mm; continuous curves represent the envelope of the resulting family of curves for different core diameters of the core-magnet arrangement (fourth order interpolation for core diameters between 5 and 25 mm in 5 mm intervals) whereas the dashed curves correspond to the single magnet. Curves were arbitrarily normalised. The circle markers correspond to a three-dimensional model of an EMAT with a height of 20 mm, a core with a square base of 10 × 10 mm and magnets with a rectangular base of 5 × 10 mm as in Fig. 2.2c.

Overall the improvement when increasing the height of the core-magnet arrangement from 10 to 20 mm is nearly 3 dB; from 20 mm onwards, the signal
increase is not greater than 2 dB. Also, an increase in the width of the magnet (inner to outer radii difference) produces a steep increase in the signal when the width is less than the radius of the core. When the magnet width is greater than the radius of the core, the signal still increases but at a lower rate. Then it can be inferred that for each overall EMAT area, there is a core diameter that maximises the signal strength.

To investigate the effect of lift-off on the signal intensity, the calculations were repeated for core diameters between 5 and 25 mm in intervals of 5 mm using an overall height of 20 mm and lift-offs of 0.5 and 2 mm. Figure 2.8b shows the envelope of the resulting family of curves using a fourth order interpolation – continuous black and grey curves correspond to 0.5 and 2 mm lift-offs respectively. For example, it can be observed that when the lift-off is 0.5 mm, the optimal overall diameter for a 10 mm diameter core is roughly 17 mm whereas for a 20 mm diameter core is roughly 30 mm (see corresponding curves in Fig. 2.8a). Then the optimal ratio between the core and the overall EMAT diameter is roughly 2 : 3 for these cases.

If the lift-off increases to 2 mm, the signal drops in general. The difference between the curves of 0.5 and 2 mm lift-offs is 5 and 10 dB for lower and higher overall diameters respectively, so the larger the core the smaller the signal drop when increasing the lift-off. Moreover, this drop also implies that the optimal ratio between the core and the overall diameter is roughly 1 : 2 for a lift-off of 2 mm.

Finally, a core-magnet arrangement with a core that has a square base of 10 × 10 mm and magnets with a rectangular base of 5 × 10 mm, as sketched in Fig. 2.2c, was simulated using a three-dimensional model. This type of core-magnet arrangement is easier to build in practice than the cylindrical version. The height of the core-magnet arrangement was set to 20 mm and the rest of the parameters of the simulation were kept the same. The relative amplitude of the signal and outer diameter of the EMAT for this configuration are shown in Fig. 2.8 using a circle marker. The results show that the signal is 2.5 dB smaller when using this configuration compared to the ideal cylindrical case irrespective
of the lift-off. This can be explained by the loss of magnetic material in the overall volume of the EMAT. The performance of the cuboid core and magnets using other dimensions were not further explored because it is computationally expensive.

### 2.5.2 Comparison with other configurations

Using equation (2.10), the performance of the core-magnet arrangement was compared with those of a single magnet, as in the EMAT of Fig. 2.3c, and a single magnet with a capped cone beneath it (Fig. 2.3b). The distribution of $B_n$, required to calculate the expected signal strength in equation (2.10), was obtained through simulations as in Sec. 2.3.3 and 2.5.1.

To give an unbiased comparison, the diameter of the bottom base of the capped cone was set to 10 and 20 mm to match the diameter of the core of the core-magnet arrangement of Fig. 2.8b; the height of the capped cone was set to 5 mm. For each configuration, the magnet height was set to 20 mm and the width/diameter was changed so that the resulting diameter of the EMATs was between 5 and 50 mm. All the remaining parameters of the simulations were the same as those used to obtain the results of Fig. 2.3 and 2.8.

The strength of the signal produced by the single magnet according to equation (2.10) was plotted in Fig. 2.8b using dashed lines (black and grey for 0.5 and 2 mm lift-offs respectively). It can be observed that when the lift-off is 0.5 mm, the optimal core-magnet arrangement outperforms the single magnet by more than 3 dB when the overall diameter is greater than 20 mm.

In practice, the difference between the optimal core-magnet arrangement and the single magnet should be greater because the single magnet EMAT is assumed to have a linearly polarised aperture. Such an aperture requires extra space for the return path of the coil, which increases the EMAT overall diameter. Moreover, if a pancake-like coil were to be used, the resulting wave would be radially-polarised, which also produces weaker (and more distorted) signals as discussed in Sec. 2.4.

The grey traces in Fig. 2.8b show how the signal intensity is affected when the lift-off of the EMAT increases to 2 mm. Compared to the core-magnet arrange-
ment, the single magnet EMAT is less affected by a lift-off increase, especially those that have a smaller ultrasonic aperture diameter.

The single magnet with a capped cone beneath it was found to produce the weakest signals among all of the configurations. Different heights for the capped cone were explored without significant changes in the results. Results are not shown for the sake of brevity.

2.5.3 Influence of the specimen thickness

In the following simulations a single magnet (40 mm height and 20 mm diameter) and a core-magnet arrangement (10 mm diameter core, 50 mm outer diameter and 40 mm height) both with a lift-off of 1 mm from a ferromagnetic specimen were employed. The thickness of the ferromagnetic specimen was varied between 2 and 25 mm. The maximum of the normal flux density $B_n$ and tangential flux density $B_t$ was computed for every case at 0.1 mm below the surface; results are shown in Fig. 2.9.

Regardless of the EMAT configuration and its dimensions, there is a minimum thickness, whose exact value depends upon the EMAT configuration and dimensions, in Fig. 2.9 this is roughly 7 mm, where $B_n$ starts decreasing and $B_t$ increasing. The reason is that for thin specimens the field is trapped inside
the saturated specimen, due to its permeability being higher than air, and consequently the field is forced to bend immediately when entering the specimen in order to reach the opposite pole.

2.6 Experiments

The goal of the experiments was to assess the accuracy of equation (2.10) and the simulations so that results presented in Fig. 2.8 could be validated. To do so, the magnetic field produced by a single magnet and the core-magnet arrangement were measured and compared to simulations in Sec. 2.6.1. Also, the signals produced by different magnet configurations on the same coil were investigated and then compared to simulation results in Sec. 2.6.2.

2.6.1 Magnetic field measurements

A photograph of the core-magnet arrangement that was constructed is shown in Fig. 2.10. It consists of four cuboid magnets with dimensions $40 \times 10 \times 20 \text{ mm}$, which were made out of Neodymium N42 with remanent flux density $B_r = 1.32 \text{ T}$ (Part No. F401020-1, Magnet Expert Ltd., UK). Like poles of each magnet were oriented towards a ferromagnetic mild steel core, which had dimensions $40 \times 10 \times 10 \text{ mm}$. The magnets and the core were encased in a cylindrical container made out of polymer.

![Core-magnet arrangement used in the experiments. A black polymer casing holds the magnets and the core. The height of the magnets and the core is 40 mm. The core base is $10 \times 10 \text{ mm}$ and the magnet bases are $10 \times 20 \text{ mm}$.](image-url)
Two single magnets were built by stacking four 10-mm-thick magnets to mimic 40-mm-height magnets. Two different sets using diameters of 10 and 20 mm were built (Part No. F674-4 and F646-1 respectively, Magnet Expert Ltd., UK).

The magnetic flux density from the stacked magnets and the core-magnet arrangement were measured using a Gaussmeter (GM08, Hirst Magnetic, UK) and a transverse Hall probe (TP002, Hirst Magnetic, UK). Both a single magnet and the core-magnet arrangement were placed on top of a mild steel block (20 × 20 × 5 mm) using a plastic layer that had a thickness of 1 mm. The plastic layer had a slot along which the Hall probe was slid in 0.5 mm steps from the centre of the magnets or core. In the case of the core-magnet arrangement, the slot was aligned with one of the magnets and the centre of the core.

Figures 2.11a-b show the measured magnetic flux density of the core-magnet arrangement (a) and single magnet (b) using circle markers. Two sets of measurements are shown for each case to provide an overview of the variability of the results. The solid lines in the figures correspond to simulations; the procedures to conduct these simulations were the same as those described in Sec. 2.3 for cylindrical and three-dimensional models. Overall, there is good agreement between simulations and measurements.
2. Optimisation of the Bias Magnetic Field of Shear Wave EMATs

![Graph](image)

**Figure 2.11:** Magnetic flux density of a core-magnet arrangement and a cylindrical magnet. a) Core-magnet arrangement consisting of four cuboid magnets with dimensions $40 \times 10 \times 20$ mm and a mild steel core with dimensions $40 \times 10 \times 10$ mm; the field is measured from the centre of the core and beneath one of the magnets. b) Cylindrical magnet with diameter of $10$ mm and a height of $40$ mm. All magnets are made out of Neodymium N42 with remanent flux density $B_r = 1.32$ T. Circle markers correspond to the measured fields and solid black lines to simulations. The dashed curve in (a) corresponds to a simulation of the core-magnet arrangement using cylindrical symmetry.

As a comparison point, a simulation of the core-magnet arrangement using cylindrical symmetry is shown in Fig. 2.11a with a dashed curve. This cylindrical configuration corresponds to a 10-mm-diameter core with an overall diameter of 50 mm. It produces a greater flux density but is also more difficult to build in practice.
2.6.2 Signals from single magnet and core-magnet arrangement configurations

A D-shape coil was built as sketched in Fig. 2.12 by winding 22 turns of enamelled wire with a diameter of 0.4 mm so that the resulting width of the coil in its straight section is 10 mm; the D-shape coil is easy to build and produces linearly polarised waves under its straight section. One core-magnet arrangement and two single magnets with diameters 10 and 20 mm were placed over the straight section of the D-coil one at a time, as shown in Fig. 2.12. The same coil was used in each case to avoid different coil shapes/impedances affecting the results.

![Figure 2.12: D-shape coil with 22 turns of enamelled wire used in the experiments. In different experiments, the circular magnets of diameter 10 mm and 20 mm and the 10 x 10 mm square section core were centred in the straight region of the D-coil as shown.](image)

The coil was connected to a pulse-echo system (WaveMaker-Duet – custom made for the NDE group of Imperial College London), which comprises a driver with an output trigger signal and a receive pre-amplifier. The driver of the system was configured to excite the coil with a 3 cycle Hann tone-burst at 2 MHz with a maximum peak-to-peak current of 200 mA. The shape and intensity of the tone-burst were monitored throughout the experiments with a non-invasive current sensor (Bergoz CT-B0.1-B) connected to an oscilloscope (LeCroy WaveRunner 44Xi).

The receive pre-amplifier of the system was set to amplify the signal by 60 dB and then the output of the receiver was in the millivolt range. The output of the receive pre-amplifier and the output trigger signal from the driver were
also connected to the oscilloscope. For each EMAT the acquired signals were synchronised with the trigger signal from the driver and then averaged 4000 times in the oscilloscope, so that electrical random noise was attenuated far below any coherent noise present. Any remaining coherent noise could be due to the generation and reverberation of ultrasonic waves inside the magnets and/or the electronics.

Figure 2.13 shows the signals from the measurements on a 50-mm-thick specimen (with parallel flat surfaces and free from defects) corresponding to the three cases tested. Since the purpose of the experiments was to compare different configurations and the absolute voltage measured is dependent on the instrumentation used, the results were arbitrarily normalised – note that the amplitude scale of Fig. 2.13a-c is different in each case.

![Figure 2.13](image)

**Figure 2.13:** Signals from pulse-echo measurements on a 50 mm thick mild steel specimen. a) Core-magnet arrangement. b) 20 mm diameter single magnet. c) 10 mm diameter single magnet. Signals are normalized to the maximum value and plotted with different scales in each case.

The reflected echoes from the back-wall of the specimen can be observed at the
right end of the plots at approximately 33 $\mu$s. As expected, the core-magnet arrangement produced the strongest signals followed by the 20-mm-diameter single magnet. The coherent noise floor is the same in each case and can be seen in Fig. 2.13c to be about $\pm 0.05$; the noise amplitude corresponds to roughly 5% of the signal produced by the core-magnet arrangement.

In Fig. 2.13b two echoes were identified at approximately 21 and 28$\mu$s ($6.4\mu s$ difference), which corresponds to the thickness of the 10-mm-height magnets that were stacked to mimic the single magnet EMAT. The echoes are likely to be reverberations inside the magnets, which have also been observed previously [74]. Similar echoes are likely to be present in Fig. 2.13c but they could not be identified due to the high level of the noise.

Finally, the relative signal amplitudes of the three EMATs were estimated using equation (2.10), where the magnetic flux density distribution beneath the surface of the specimen was obtained from simulations; simulation details are discussed in Sec. 2.3 for the cylindrical and three-dimensional models. The estimated lift-off for each configuration was 0.5 mm. Simulated and experimental results are given in Fig. 2.14. The results are normalised to the values corresponding to that of the 20-mm-diameter magnet to simplify the analysis of the results.

The asterisk markers show the measured (relative) signal amplitude for each configuration. The cross markers correspond to three-dimensional simulations taking into account the contribution of the field beyond the core/magnet, i.e. considering the nearby coil sections, while the triangle markers only consider the field beneath the intersection of the coil and the core or cylindrical magnets, i.e. the active aperture. The circle markers correspond to the maximum flux density from simulations using cylindrical symmetry over the core active aperture area. In the case of the core-magnet arrangement this corresponds to a core diameter of 10 mm and an overall diameter of 50 mm.

In general, there is good agreement between measured and simulated results which confirms the validity of equation (2.10). In every case, the contribution of the field outside the active aperture is relatively small, less than 1.5 dB, with
2. Optimisation of the Bias Magnetic Field of Shear Wave EMATs

Figure 2.14: Relative signal amplitude for the single magnets of 10 and 20 mm diameters and the core-magnet arrangement. Results are normalised with respect to the 20 mm diameter magnet.

the greater difference corresponding to the core-magnet arrangement. It can be concluded that when the diameter of the ultrasound aperture is 10 mm, the core-magnet arrangement produces signals roughly 20 dB greater compared to a single magnet.

In the core-magnet arrangement case, the experimentally measured values are roughly 3 dB higher than the simulated results (using the 20-mm-diameter magnet as the reference point). A combination of several factors could have caused this difference, for example, dimensional differences, the signals from the 10- and 20-mm-diameter magnets being overestimated due to the coherent noise of the amplifier or the influence of other transduction mechanisms, such as magnetostriction.

2.7 Summary

This chapter presented an in-depth analysis of the bias magnetic field strength and resulting signal amplitude of different magnet configurations for shear wave EMATs on mild steel. The Lorentz force was assumed to be the predominant transduction mechanism neglecting any other mechanisms such as magnetostriction; experimental results showed no contradiction with this assumption. A par-
2. Optimisation of the Bias Magnetic Field of Shear Wave EMATs

ticular core-magnet arrangement was found to produce pulse-echo signals, from a flat back-wall, greater than 3 – 6 dB compared to a single magnet when EMATs of the same overall volume were compared – the exact difference depends on the EMAT geometry in each particular case. The configuration that consists of a single magnet with a ferromagnetic capped cone beneath it was found to perform worse than a single magnet of equivalent overall volume. The core-magnet arrangement also produces signals 20 dB greater than a single magnet when the aperture diameter is 10 mm. This is due to the high magnetic flux density generated beneath the core in the centre of the EMAT, which can exceed 3 T.

The enhanced performance of the presented core-magnet arrangement relies on increasing the magnetic flux density within a given area of interest by exploiting two mechanisms: a) repulsion between magnets and b) the flux guide formed by both the ferromagnetic core between the repelling magnets and the ferromagnetic specimen itself. The effective use of these mechanisms could be in principle generalised to others EMAT configurations.

It was also found that for each overall diameter of the core-magnet arrangement there is a core diameter that maximises the signal. When the height of the core-magnet arrangement is 20 mm and the lift-off is 0.5 mm, the optimal ratio between the core and overall diameter is roughly 2 : 3 for core diameters between 10 and 25 mm. This optimal ratio decreases to roughly 1 : 2 when the lift-off increases to 2 mm. It should be highlighted that these ratios only apply to their respective configurations when placed on mild steel specimens.

Moreover, another finding was that linearly polarised apertures, as in the core-magnet arrangement with a butterfly or D-shape coil, produce pulse-echo signals with greater amplitude and less distortion than the radially polarised apertures, as in the case of a single magnet with a pancake-like coil. Distortion is caused by the generation of the longitudinal mode, whose effect decreases when either the area of the aperture or the thickness of the specimen increases.

In general, the core-magnet arrangement produces high purity shear waves but this is limited by the thickness of the ferromagnetic sample. The tangential component of the magnetic flux density is responsible for the intensity of longit-
udinal mode, which causes distortion, and it was found to increase steeply for mild steel samples with thickness below 7 mm.
Chapter 3

EMATs Optimal Impedance

3.1 Introduction

This chapter is divided into two main sections that address the optimal impedance of EMATs on transmission and reception. Most of the first part, related to the study of the optimal impedance on transmission, has been extracted from [P2] (with permission from AIP Publishing LLC).

Regardless of the transduction mechanism (Lorentz force, magnetostriction, etc. [2]) the impedance of the EMAT coil plays an important role in increasing the strength of the signal. When reviewing the scarce literature on this subject [65, 75–77], the tendency is to build coils by trial and error and then design a matching network so that the driving source transfers the maximum power to the coil.

This approach, however, has no control over the amount of power that goes to the coil resistance and that is therefore wasted. A better approach is to design an EMAT coil where the induced eddy currents are maximised for a given source power. It is also important to understand how a certain configuration is affected by frequency shifts or lift-off changes from the specimen. These are paramount because the intensity of the ultrasonic waves increases with the intensity of the eddy currents when using Lorentz-force EMATs, which is the main transduction mechanism of interest to this thesis. Moreover, matching networks could introduce losses and parasitic capacitances that degrade the performance of the
system, overwrite complicate the design, affect the bandwidth and/or require extra components.

On reception, the problem of finding the optimal impedance is more complex due to the presence of noise sources from the electronics and the EMAT itself. It is unclear, to the best of the author's knowledge, which coil impedance and receive amplifier configuration maximise the SNR. Interestingly, these questions have already been investigated for piezoelectric transducers, for example, in [78] a model of the noise sources of a receive amplifier and a piezoelectric sensor was presented and the characteristics of the amplifier discussed in order to maximise the SNR; and in [79] the fundamental noise limit of accelerometers (a type of piezoelectric transducer) was investigated and helped understand the maximum achievable SNR of these transducers.

The organisation of this chapter is as follows: first, an electrical transformer circuit is proposed to model the interaction between the EMAT coil and the eddy currents on transmission. Following this, the optimal coil impedance and electrical configuration for maximum efficiency and power transfer to the eddy currents are investigated. Using a tunable coil, experiments are conducted to show how the optimal conditions change with frequency, resonance, and coil lift-off from the specimen. Then, a similar model is proposed for reception and later extended to include the main noise sources present in order to find a configuration that maximises the SNR. Experiments are conducted using a tunable coil to investigate the effect of the coil impedance on the received signal. Conclusions are drawn at the end.

3.2 Optimal coil impedance on transmission

Intuitively, coils with a higher inductance will produce a better coupling with the eddy currents. However, the higher the inductance of the coil, the lower the current that flows through it and therefore the lower the power that can be transferred into it. Conversely, a low inductance allows for higher currents and higher power transfers, but because of the poorer coupling, most of the power
is dissipated in the coil and driver resistance rather than in the eddy currents themselves. Because of these conflicting effects, there is a clear need to study how the coil impedance affects the power transfer to the eddy currents.

This problem is very similar to that encountered in wireless power transfer [80–82] and radio frequency identification (RFID) [83, 84] systems, where one transmit coil transfers energy to a receive coil. In those cases most of the effort has been placed in maximising the efficiency of the energy transfer; however, with EMATs maximum power transfer is often preferred over maximum efficiency, for a higher SNR can be realised in a shorter period of time.

Among the configurations studied in this thesis, those at resonance were found to produce the best results. The benefits of electrical resonance on EMATs have also been reported in [65].

In this chapter, a transformer circuit is employed to model the interaction between the EMAT coil and the eddy currents in the specimen. Similar models have already been employed to study the impedance of coils for eddy current testing techniques [85, 86]. Starting from this model, the optimal electrical configurations and coil impedances that produce the highest power transfers to the eddy currents and the maximum efficiencies on transmission are studied. To support this analysis, a tunable coil that consists of stacked identical thin coil layers independently accessed is introduced so that the overall coil inductance can be modified while maintaining the radiation pattern of the EMAT unaffected.

### 3.2.1 Transformer model

The interaction of the EMAT coil and the eddy currents on transmission is modelled using the transformer circuit of Fig. 3.1. Variations of this model have been widely employed for eddy current testing in the past [85, 86].

A series of simplifications have been introduced. Firstly, the effect of the eddy currents in the magnet on top of the coil is disregarded. For these currents could be attenuated by, for example, laminating the magnet to increase the resistance of the eddy currents or increasing the distance between the magnet and the coil. Secondly, the parasitic capacitance of the coil, cable and driving source
are not considered. This is a reasonable assumption within the frequency range of interest in this thesis (less than 3 MHz). Radiation and hysteresis losses are not considered either because the coupling and losses of the eddy currents are assumed to be predominant. Finally, the coil and the driver resistance are both modelled as a single resistor $R_c$ for mathematical convenience – this is possible because they are connected in series.

![Transformer circuit](image)

**Figure 3.1:** Transformer circuit for modelling the interaction between the EMAT coil and the eddy currents on transmission.

In the circuit of Fig. 3.1, $R_e$ is the resistance of the eddy currents. The voltage at both sides of the transformer, i.e., at the inductors representing the EMAT coil and eddy currents with inductances $L_e$ and $L_c$ respectively can be found as

$$
\begin{bmatrix}
U_c \\
U_e
\end{bmatrix} = j
\begin{bmatrix}
X_c & X_M \\
X_M & X_e
\end{bmatrix}
\begin{bmatrix}
I_c \\
I_e
\end{bmatrix},
$$

where $X_c = \omega L_c$, $X_e = \omega L_e$, $X_M = k \sqrt{X_c X_e}$, $\omega$ is the angular frequency, $j$ the imaginary unit, and $k \in [0, 1]$ is the coupling factor between the coil and the eddy currents. An ideal voltage source $V$ and a capacitor $C$ are connected in series with the coil; the capacitor is used to compensate the reactive component seen by the source.

### 3.2.2 Maximum power transfer

Maximum efficiency and power transfer are the two main optimal conditions on transmission. Maximum efficiency is obtained by maximising the ratio between
the power dissipated by the eddy currents and all of the loads (including the eddy currents) combined whereas maximum power transfer is achieved when the power dissipated by the eddy currents is maximised irrespective of the other loads. In practice the latter is preferred for the SNR can be increased faster.

The power dissipated by the eddy currents is

$$P_e = \frac{|I_e|^2 R_e}{2},$$

(3.2)

where $I_e$ is the peak current. After some manipulation this yields (Appendix A.1)

$$P_e = \frac{V^2}{R_e + jX_c - jX_{Cap} + \frac{X_M^2}{Z_e}} \left( \frac{X_M}{Z_e} \right)^2 \frac{R_e}{2},$$

(3.3)

where $X_{Cap} = \frac{1}{\omega C}$ is the capacitor reactance and $Z_e = R_e + j\omega L_e$ the eddy currents impedance.

If $X_{Cap}$ is chosen so that the complex components in the denominator cancel out at a resonant frequency $\omega_0$, $P_e$ increases and equation (3.3) can be simplified to

$$P_e = \frac{V^2}{2 \left( \frac{R_e|Z_e|}{X_M\sqrt{R_e}} + \frac{X_M\sqrt{R_e}}{|Z_e|} \right)^2}.$$  

(3.4)

To further increase $P_e$, the denominator of equation (3.4) can be minimised. First let us assume that the shape of the eddy currents does not change, i.e., $R_e$ and $L_e$ remain fixed as well as $k$ for a given $\omega_0$. Then, since $R_e > 0$ (because of the driver resistance) and $R_e$ is a monotonically increasing function of $L_e$ for a given $\omega_0$, the denominator has to be a convex function. So there is an optimal value of $L_e$ for which $P_e$ reaches its maximum – note that $X_M = \omega_0 k \sqrt{L_e L_c}$.

To simplify the analysis, let $R_e$ be constant when $L_e$ changes with the number of turns in the coil. This is a reasonable approximation since the output imped-
EMATs Optimal Impedance

The impedance of an amplifier is in many cases a few tens of ohms whereas the impedance of a coil (this is in air without the effect of the eddy currents taken into account) is just a few ohms. Hence, equation (3.4) reaches its maximum when

\[ L_c = \frac{R_c (R_c^2 + \omega_0^2 L_e^2)}{\omega_0^2 k^2 R_e L_e}. \]  

(3.5)

It may happen that the resistance of the coil is comparable to that of the driver and therefore \( R_c \) cannot be considered constant when changing \( L_c \) which leads to a different expression for the optimal \( L_c \). Moreover, regardless of the restrictions on \( R_c \), it may also occur that the optimal \( L_c \) is either too small or too big in order to be achieved in practice while maintaining \( R_e, L_e \) and \( k \) constant for a given \( \omega_0 \). Notwithstanding all these, the presented methodology provides a useful insight into the problem.

When equation (3.5) is substituted into equation (3.4), it gives

\[ P_{\text{max}} = \frac{V^2}{8R_c}, \]  

(3.6)

which is the maximum power dissipated by both the eddy currents and the coil and driver resistances under the maximum power transfer condition.

Maximising equation (3.2) is equivalent to simply maximising the eddy current \( I_e \), for a given eddy current resistance \( R_e \). However, the use of (3.2) is sounder from the electrical point of view and also simplifies the analysis. The final step in maximising \( I_e \) is to combine equations (3.2) and (3.6)

\[ |I_e| = \frac{V}{2\sqrt{R_e R_c}}. \]  

(3.7)

This formally confirms the intuitive result that under maximum power transfer condition \( I_e \) increases by decreasing \( R_e \) and \( R_c \) for a given \( V \).

This suggests that in the cases where only a certain region of the coil/eddy
current path is used to generate the active ultrasonic aperture based on the Lorentz force, once an optimal coupling has been achieved, the remaining section of the coil/eddy current path should be designed such that they have the lowest resistances. This basically means that excessively large, non-active sections of a coil should be avoided.

### 3.2.3 Maximum efficiency

The efficiency of an EMAT coil is the ratio between the power dissipated by the eddy currents $P_e$ and all of the loads (including $P_e$)

$$\eta = \frac{P_e}{P_e + P_c}, \quad (3.8)$$

where $P_c$ is the power dissipated by the coil-driver resistance $R_c$. It can be shown (Appendix A.2) that

$$\eta = \frac{1}{1 + \frac{R_c}{kX_c} \left( \frac{R_c}{X_e} + \frac{X_e}{R_e} \right)}. \quad (3.9)$$

From this expression it can be concluded that to maximise efficiency, $k$ should be maximised as well as the ratio $\frac{X_e}{R_c}$, which is basically the quality factor of the coil and driver combined.

The maximum efficiency condition gives useful insights into the design of the coil and electronics in general. Nonetheless, there are other significant losses in the electronics, mainly in the receive electronics and control unit, which are not considered in equation (3.8). In practice, these losses are minimised by keeping the system running for the shortest period of time possible under maximum power transfer.
3.2.4 Results

To support the analysis derived from the model of Fig. 3.1, the effect of the series capacitor on the signal amplitude is investigated, a tunable coil is introduced to study its optimal impedance under maximum power transfer and the effect of lift-off and frequency changes on the optimal impedance are addressed.

3.2.4.1 Impact of series resonance on the signal strength

The use of a resonant series capacitor increases the power delivered to the load by cancelling out the reactive components seen by the driving source as shown in equations (3.3) and (3.4). This tuning procedure is a necessary step in the process of finding the impedance of the coil that yields the maximum power transfer condition. In the following experiment, the effect of resonance on the signal strength is quantified for a given EMAT design.

Two identical EMATs with pancake-like coils were arranged at opposite sides of a steel specimen that has a thickness of 40 mm. Each EMAT comprises a two-layer-coil with 26 turns per layer and an overall coil thickness of 0.5 mm with inner and outer diameters of 10 and 30 mm. A cylindrical Neodymium-N42 magnet [72] with diameter and height of 30 and 20 mm respectively was placed on top of the coil with a 2-mm-gap in between. These EMATs generate shear waves with radial polarisation similar to that used in [6].

The transmit EMAT was connected in series with a 1 nF capacitor as shown in the model of Fig. 3.1. The value of the capacitor was chosen so that, when placed on top of the steel sample with no lift-off, a purely resistive impedance of 40 Ohm appeared at the coil terminals at roughly 1 MHz. This 40 Ohm value, though arbitrary, is known to be easily handled by standard driving sources. The impedance value was confirmed by using an impedance analyser (SinePhase Z-Check 16777k, SinePhase Instruments GmbH, Hinterbruehl, Austria). The resulting admittance curves are shown in Fig. 3.2.

The behaviour of a series resonant system can be better understood by using admittance rather than impedance curves. It can be observed that the system resonates at roughly 950 kHz with a real admittance of approximately 25 mS.
(40 Ohm). At this point, the current through the circuit reaches a maximum (assuming a purely resistive source). The profile of the real part of the impedance and the impedance magnitude (not shown) describe a bell-like shape whose bandwidth is \( \sim 0.4 \text{ MHz} \) and therefore does not affect tone-bursts excitations that have more than 3 cycles, which have a bandwidth smaller than 0.4 MHz at 1 MHz.

The transmit EMAT was connected to the driver output of a WaveMaker-Duet system (custom made for the NDE group of Imperial College London). The impedance of the driver was assumed to be predominantly real. The driver of the system was set to generate a 4-cycle-tone-burst apodized with a Hann-like window using a maximum amplitude of 40 Vpp at 1 MHz. The series capacitor was not connected during this operation. A derivation of the driver output was connected to an oscilloscope (LeCroy WaveRunner 44Xi) through a high impedance buffer in the WaveMaker-Duet system so that this reference could be used to read the driver output voltage without affecting the load.

The receive EMAT was connected to a 60 dB gain pre-amplifier in the WaveMaker-Duet system and its output to one channel of the oscilloscope. The driver trigger output of the WaveMaker-Duet system was also connected to another channel of the oscilloscope. Then the central frequency of the tone-burst was varied from 0.4 to 1.4 MHz in steps of 50 KHz. By inspecting the signal it was confirmed...
that no saturation of the driver occurred. Received signals corresponding to each central frequency were synchronised with the driver trigger output and averaged 4000 times to increase the SNR above 40 dB. The maximum voltage recorded was in the range of tens of millivolts. The results were arbitrarily normalised since the purpose of the experiments was to compare different configurations and the absolute voltage measured is dependent on the instrumentation used.

Then the envelope of the signal was extracted by means of the Hilbert transform. The results with and without the series capacitor connected to the driver are plotted in Fig. 3.3. The vertical axis shows the peak amplitude of the envelope for each frequency tested. The main observation is that the amplitude of the signal increases from 0.6 to 1.4 MHz when the series capacitor is connected. However, for this particular example, the improvement is generally no bigger than 3 dB. Note that the highest amplitudes occur near 0.8 MHz, which is not necessarily the resonant frequency of the driver-coil; other factors such as the change of the EMAT radiation pattern with frequency may have a stronger influence in the amplitude of the signals within this frequency range.

3.2.4.2 Optimal number of turns for maximum power transfer

A coil was constructed by stacking identical thin coil layers as shown in Fig. 3.4a. By using this coil the shape and therefore the impedance of the eddy current path does not change, but the impedance of the coil can be modified by choosing
a different number of layers. This allows the maximum power transfer condition to be investigated.

(a) Cross-section of a butterfly coil with a magnet on top.

(b) Equivalent electric circuit.

Figure 3.4: Multilayer coil. a) Cross-section of a butterfly coil. b) Equivalent electric circuit showing the connexion of the terminals to the layers.

The equivalent electrical circuit of the multilayer coil is simply a set of coils connected in series as sketched in Fig. 3.4b, where each layer can be accessed independently. If, for example, two layers are found to perform best, only terminal 2 is connected while the rest are left open. The unconnected layers will not affect the results because no current flows through them. Note that the inductance of the coil is not necessarily the sum of the inductance of layers due to the mutual coupling between them.

The thickness of this multilayer coil must be kept to a minimum, otherwise, the farthest layers from the specimen will be poorly coupled to the sample, and the condition requiring the coupling factor $k$ to be constant will not be satisfied. For this reason, the coil was built using multilayer printed circuit board technology.

A 5-layer butterfly coil was built using a layer thickness of 80 $\mu$m and 16 turns per layer. A magnet that has a diameter of 10 mm and a height of 20 mm was
placed in the middle of the coil leaving a gap of 1 mm in between them. This configuration radiates shear waves with linear polarisation from the middle of the coil.

The EMAT was placed on one side of 40-mm-thick steel specimen that had a shear-wave piezoelectric transducer (Panametrics-V151) on the opposite side. Both transducer polarisations and locations were aligned so as to maximise the signal amplitude.

A switching board was connected to the coil terminals to control the active number of layers (Fig. 3.5-a) in series with a capacitance decade box (Tenma-72-7265) and the driver output of a WaveMaker-Duet system (Fig. 3.5-b). The capacitance box was used to select the capacitance value that maximises the signal for a given number of layers in the coil; the capacitance was varied from 0.2 to 7 nF in 0.1-nF steps. The piezoelectric transducer was connected to the pre-amplifier of the WaveMaker-Duet system that has a 40-dB gain and its output to an oscilloscope (LeCroy WaveRunner 44Xi, Teledyne LeCroy, New York, US). The driver trigger output of the WaveMaker-Duet system was also connected to the oscilloscope.

The driver of the system was set to generate an 8-cycle tone-burst with a Hann-window apodization and a maximum amplitude of 40 Vpp at 1 and 2 MHz. The driver output was split and connected to the oscilloscope through a high impedance buffer in order to keep a reading of the driver output and confirm there was no distortion in the signal due to saturation. Received signals were synchronised for each measurement using the driver trigger output and averaged 20 times in the oscilloscope to increase the SNR. The maximum voltage recorded was a few hundred millivolts; results were arbitrarily normalised as only relative comparisons of the experimental data were conducted.

First, the frequency was set to 1 MHz and the EMAT lift-off changed to 0, 1 and 2 mm using calibrated non-conductive polymer sheets. Results are shown in Fig. 3.6a with the curves being normalised with respect to their maximum value; Table 3.1 gives the relative amplitude of the curves for a 4-layer coil. The capacitance values that maximised the signal are given in Table 3.2. The
Figure 3.5: EMAT with switching board (a) used to select the optimal number of layers of the EMAT coil for the measurement set-up shown (b).
Table 3.1: Relative amplitude of the signals corresponding to a 4-layer coil at 1 MHz.

<table>
<thead>
<tr>
<th>Lift-off (mm)</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative amplitude (squared)</td>
<td>1</td>
<td>0.65</td>
<td>0.28</td>
</tr>
</tbody>
</table>

Amplitude decrease should be attributed, not only to the coil lift-off but also to the magnet lift-off from the specimen, which reduces the bias magnetic field and hence the Lorentz force components.

Figure 3.6: Amplitude of the received signal (squared) vs. number of coil layers. a) 0-, 1-, and 2-mm lift-off at 1 MHz. b) 1 and 2 MHz with 0-mm lift-off. The amplitude of each curve is normalised with respect to its maximum.

The curves in Fig. 3.6a suggest that the optimal coil inductance corresponds to 4 layers – this is when the maximum power transfer condition should have been
satisfied. Once the curves are normalised, there is a small difference between them. This means that a) changes in the value of the coupling factor $k$ with lift-off are small and cannot be observed due to the coarse change in the inductance when switching between layers, and/or b) the inductance of the coil increases with the distance from the ferromagnetic specimen. The latter could be, for example, due to magnetic saturation in the sample caused by the bias magnetic field of the EMAT magnet – recall that when a ferromagnetic material is saturated its permeability decreases [87, 88], and as the bias magnetic field weakens (because the magnet moves away from the specimen) the permeability increases which can increase the inductance of the coil. Another explanation is that the path of the eddy current and hence its inductance and resistance change in a way that the results remain roughly the same.

A further observation is that there is a small difference between the capacitance values when the lift-off changes (Table 3.2). Overall, for this particular design and frequency, once the optimal number of turns and capacitance have been set, the coil and driver will operate close to the maximum power transfer condition regardless of the lift-off within that range. The same results were found for different ferromagnetic mild steel samples to rule out any dependencies on the particular specimen used.

The experiments were repeated at 2 MHz using no lift-off. Results are plotted in Fig. 3.6b; capacitance values are also given in Table 3.2. As the frequency increases, fewer layers are required to reach the maximum power transfer condition, the optimal point correspond to 3 layers in this case compared to the 4 layers needed at 1 MHz. It is important to highlight that at lower frequencies, for example at a few kilohertz, many turns should be required to achieved the

### Table 3.2: Capacitance values (5% tolerance).

<table>
<thead>
<tr>
<th>Freq.</th>
<th>Lift-off</th>
<th>1 layer</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 MHz</td>
<td>2 mm</td>
<td>4 nF</td>
<td>2.2 nF</td>
<td>1.2 nF</td>
<td>.6 nF</td>
<td>.4 nF</td>
</tr>
<tr>
<td>1 MHz</td>
<td>1 mm</td>
<td>5 nF</td>
<td>3 nF</td>
<td>1.4 nF</td>
<td>7 nF</td>
<td>.4 nF</td>
</tr>
<tr>
<td>1 MHz</td>
<td>0 mm</td>
<td>4 nF</td>
<td>3 nF</td>
<td>1.6 nF</td>
<td>.9 nF</td>
<td>.6 nF</td>
</tr>
<tr>
<td>2 MHz</td>
<td>0 mm</td>
<td>2 nF</td>
<td>.7 nF</td>
<td>.4 nF</td>
<td>.2 nF</td>
<td>&lt;.2 nF</td>
</tr>
</tbody>
</table>
maximum power transfer condition and this may not be physically possible for certain coil geometries.

3.3 Optimal coil impedance for maximum SNR on reception

On reception, the optimal impedance of the EMAT coil is that which produces the maximum SNR for a given receive electronics. First, this section explores which impedance and configuration maximise the signal strength. Then, the noise sources are discussed.

3.3.1 Signal strength

On reception, the electromotive force (EMF) resulting from the ultrasonic wave interacting with the bias magnetic field of the EMAT – see equation (2.2) – is modelled as a voltage source $V$ in series with the eddy current path resistance $R_e$ and inductance $L_e$. The electrical model on reception is shown in Fig. 3.7.

![Figure 3.7: Transformer circuit for modelling the interaction of the eddy currents and the EMAT coil on reception.](image)

Firstly, the impedance of the coil that produces the highest voltages at node 1 is investigated. This is because on reception the goal is to sense the highest voltage possible from the EMF, which is done by connecting an amplifier with a high input impedance at node 1. Secondly, it is shown that when a capacitor is connected across the coil to form a resonant configuration, the voltage at the coil terminals increases. This is caused by a resonant effect similar to that observed in tank-circuits found in many radio amplifiers.
In the case where there is no capacitor, the terminals of the coil are open and there is no current flow through $R_c$, hence the absolute voltage at node 1 is

$$|U_s| = \frac{V \cdot k \cdot \omega \sqrt{L_c L_e}}{|Z_e|}. \quad (3.10)$$

In this case $U_s$ unconditionally increases with $L_c$ and $k$, so in principle the greater the number of turns or the closer the coil to the specimen, the better. If $Z_e$ also decreases, $U_s$ should increase as well. These suggest that excessively large non-active sections of a coil should be avoided.

When there is a capacitor connected parallel to the coil (Fig. 3.7), the absolute voltage at node 1 is

$$|U_{Cap}| = |I_c X_{Cap}|, \quad (3.11)$$

where $X_{Cap} = \frac{1}{\omega C}$ is the reactance of the capacitor. Any losses in the capacitor are neglected. After some algebra, this yields

$$|U_C| = \frac{V \cdot X_{Cap} X_M}{|Z_e (Z'_c - jX_{Cap}) + X_M^2|}, \quad (3.12)$$

where $Z'_c = R'_c + j\omega L_c$ is the impedance of the coil.

Now the conditions under which the voltage at node 1 is higher with a capacitor than without it, i.e., when $|U_C| > |U_s|$, are shown. The first step is to simplify the denominator of (3.12) by making the reactance of the capacitor equal to that of the coil $X_{Cap} = X_c$. Since this is not necessarily the optimum value for $X_{Cap}$, because the complex components of the denominator are not completely
cancelled, it is written after some manipulations

\[
|U_C| \geq \frac{V \cdot X_M}{\left| \frac{Z_e}{Q'_c} + k^2 X_e \right|}, \tag{3.13}
\]

where \( Q'_c = \frac{\omega L_c}{R'_c} \) is the quality factor of the coil over a non-conducting material with equivalent permeability to that of the specimen. Because the numerator of equation (3.10) and inequation (3.13) are the same, the inequation \(|U_C| > |U_s|\) can be simplified to

\[
\left| \frac{Z_e}{Q'_c} + k^2 X_e \right| < |Z_e|. \tag{3.14}
\]

After some manipulation, the above inequation is found to hold if (Appendix A.3)

\[
Q'_c > \frac{1 + Q_e}{1 + Q_e (1 - k^2)}, \tag{3.15}
\]

where \( Q_e = \frac{\omega L_e}{R_e} \). Given that \( \max \{k\} = 1 \),

\[
Q'_c > 1 + Q_e. \tag{3.16}
\]

This condition should be satisfied at high frequencies (e.g., greater than 1 MHz) when using multi-turn coils. The higher the value of \( Q'_c \), the higher the voltage increase when using a parallel capacitor. Another advantage of using the capacitor is that the resonant tank-circuit formed with the coil acts as a band-pass filter and reduces the out-of-band noise.

Overall, \( Q'_c \) should be as high as possible in order to increase the strength of the signal at node 1. However, the maximum \( Q'_c \) is limited by the minimum bandwidth required for the system and the parasitic capacitance. Hence, the coil inductance (or the number of turns) should be increased up to a point where the
Figure 3.8: Operational amplifier in non-inverting configuration, coil and eddy currents with their respective noise sources.

coil becomes self-resonant due to the parasitic capacitance of the coil and cables. Though this may be regarded as optimal on reception, if a self-resonant coil is used on transmission, it becomes more difficult to drive (inject current into the coil) because its impedance increases. As a result, the optimal impedance of the coil is not necessarily the same on transmission and reception.

This section has shed light on how to select the coil impedance to increase the signal strength, but for this analysis to be complete the effect of the noise sources has to be included. That is addressed in the next section.

3.3.2 Noise sources

In order to sense the voltage at node 1 in Fig. 3.7 without disturbing the circuit, a high-input-impedance amplifier is needed. Any other amplifier with input impedance comparable to that of the circuit at node 1 will cause a potential drop at node 1 and possibly deteriorate the SNR. An operational amplifier in a non-inverting configuration is an elemental high-input-impedance amplifier. The noise sources present when using this type of amplifier will now be investigated considering it is connected as shown in the circuit of Fig. 3.8, where the capacitor noise has been neglected.

Each resistor in the circuit consists of an ideal (noiseless) resistor in series with a random voltage noise source equivalent to its Johnson noise which has
uniform voltage density \[e = \sqrt{4k_BTR},\] (3.17)

where \(k_B\) is the Boltzmann constant, \(T\) is the resistor temperature and \(R\) the value of the resistor.

The operational amplifier itself has three different noise sources. A voltage noise source \(e_n\) associated with the amplifier differential inputs and two current noise sources \(i_{np}\) and \(i_{nn}\) that correspond to each input. These are constant values defined by the designer of the amplifier. Common values for a low-noise amplifier could be roughly 1.2 nV/√Hz and 3 pA/√Hz respectively (see ADA4930, Analog Devices, Norwood, US), but some could also have very low current noise at the expense of a greater voltage noise depending on the transistor technology.

Since the sources are treated as uncorrelated, the total squared noise density at the input of the amplifier is the sum of the square values of the voltage density due to each individual source at that point

\[e_T^2 = e_e^2 + e_c^2 + e_n^2 + e_{2f}^2 + (i_{np} |Z_s|| - jX_C|)^2 + (i_{nn} R_g |R_f|^2)^2\] (3.18)

where \{||\} indicates a parallel combination and \(\bar{e}_e\) and \(\bar{e}_c\) are the noise voltage densities at the input of the amplifier due to \(e_e\) and \(e_c\) respectively

\[\bar{e}_e = e_e \left| \frac{X_{Cap}X_M}{Z_e (Z_e' - jX_{Cap}) + X_M^2} \right|\] (3.19)

and

\[\bar{e}_c = e_c \left| \frac{X_C}{Z_s - jX_{Cap}} \right|,\] (3.20)
where

\[ Z_s = Z'_c + \frac{X^2}{Z_e} \]  

(3.21)

is the impedance of the coil over a conductive specimen seen from the amplifier as shown in Fig. 3.8. Finally \( e^2_{gf} \) is the voltage density of \( R_g \parallel R_f \). A high gain is desired in this first stage of the amplifier so that the signal can be rapidly amplified from the noise threshold overcoming the noise from further stages. This implies that \( R_g \ll R_f \) and hence \( R_g \approx R_g \parallel R_f \).

The total voltage noise referred to the output (RTO) of the amplifier within the frequency band of interest is

\[ E_{RTO} = G \sqrt{\int e^2 \, df}, \]  

(3.22)

where \( G = 1 + R_f/R_g \) is the gain of the amplifier.

It should be highlighted that both the signal source due to the EMF \( V \) and the noise source of the eddy currents \( e_e \) have identical transfer functions to the input of the amplifier, (3.12) and (3.19) respectively. Because of this, \( e_e \) determines the maximum SNR that can be achieved. Then to maximise the SNR, the inductance of the coil should be increased up to the point where \( \bar{e}_e \) becomes the predominant source of noise in equation (3.18). Once this condition is met, the maximum SNR has been reached, and any increase in the coil inductance or attenuation of the amplifier noise will not further increase the SNR.

The models presented are based on high-input-impedance amplifiers such as non-inverting operational amplifiers. This configuration is believed to simplify the analysis and to offer a better performance, but as classical electronics literature points out the difference between inverting (low-input-impedance) and non-inverting configurations could be as simple as a matter of ground reference. Therefore, the use of optimal inverting configurations cannot completely be ruled
Moreover, the models presented are single ended, which simplify the analysis, yet EMATs are suitable for differential configurations. This is simply achieved by using a balanced coil with a central tap. Differential configurations have two main advantages: high common mode noise rejection and averaging of their differential inputs. Additionally, the coil central tap makes the electronics easier to bias, which may be convenient from a design point of view.

3.3.3 Results

Firstly, the effect of resonance is quantified for a particular coil on reception. Secondly, a tunable coil with fixed shape is introduced to study its optimal impedance for maximum signal strength on reception. Finally, some noise measurements are attempted.

3.3.3.1 Impact of parallel resonance on the signal

Two identical EMATs that have a pancake-like coil were arranged at opposite sides of a steel specimen as described in Sec. 3.2.4.1 except for a 1 nF capacitor connected in parallel to the receive coil. The receive amplifier of the WaveMaker-Duet system is assumed to have high input impedance. The results with and without the parallel capacitor are plotted in Fig. 3.9. The highest amplitudes are observed near the resonant frequency at 0.8 MHz. The signal increases when the capacitor is connected by roughly 6 dB at 0.8 MHz. This suggests that inequation (3.13) holds for these frequencies, as predicted. In this example, the effect of parallel resonance on reception is two times greater compared to series resonance on transmission (see Fig. 3.3).

3.3.3.2 Impact of coil inductance on the signal

The multilayer EMAT used in Sec. 3.2.4.2 was connected in parallel to the capacitance decade box (Tenma-72-7265) to investigated how the inductance of the EMAT affects the receive signal amplitude. The capacitance box was used to
select the capacitance value that maximises the signal for each selected number of layers. The capacitance was varied from 0.2 to 7 nF in 0.1 nF steps. A piezoelectric transducer was connected to the output of the WaveMaker-Duet system and the receive EMAT to its receive amplifier input. This experiment is similar to that described in Sec. 3.2.4.2.

Results are shown in Fig. 3.10 at 1 and 2 MHz, where the lift-off was 0 mm. The curves are normalised with respect to their maximum value. In both cases the amplitude increases with the number of layers as expected in this regime according to equation (3.10).
3.3.3.3 Noise measurements attempt

The same tunable coil of previous experiments was connected to an ultra-low-noise amplifier made in house to observe changes in the noise level when the inductance of the coil and the lift-off change. The whole experimental setup was enclosed in a Faraday cage and batteries were used to supply power to the amplifier such that interference from the environment was greatly attenuated.

However, no changes in the noise were observed when the coil inductance or lift-off was changed. This suggests that for this coil-amplifier setup $\bar{e}_c$ is not the dominant source of noise. Then, a) the maximum theoretical SNR has not been reached and therefore improvements in the electronics can further increase the SNR, and b) the highest coil inductance should be employed in this particular case. Note that the maximum coil inductance is limited by the self-resonant condition due to the parasitic capacitance of the coil and cables.

Because no changes were observed in the noise level and no further electronics development was pursued, no numerical investigation of the equations related to the noise sources could be conducted. Nonetheless, the model of Fig. 3.8 is still believed to be useful for other types of coils or less noisy electronics.

3.4 Summary

An electrical model has been presented to investigate the optimal impedance of the coil of an EMAT based on the Lorentz force as the main transduction mechanism. It was discussed how the number of turns in the coil can be controlled to balance the load between the coil, the driver and the eddy currents such that maximum power can be delivered to the eddy currents. As a result, an external matching network may be unnecessary in most cases. This is important because the standard practice is to use an external matching network, which has a lack of control over the load balance thereof. Additionally, matching networks require more elements, introduce losses and parasitic capacitances that overcomplicate the design and affect performance.

A resonant series circuit is formed when a capacitor is connected in series with
an EMAT and a resistive driving source, which maximises the current through the coil at the resonant frequency. Series resonance on transmission was found to increase the signal strength by roughly 2 dB in the example investigated whereas parallel resonance on reception increased the signal by approximately 6 dB.

A tunable coil was proposed whose inductance could be changed without affecting the radiation pattern of the EMAT. It was found that the number of turns that satisfy the maximum power transfer condition is almost invariant to the coil lift-off in the range of 0 to 2 mm over mild steel specimens. Moreover, if the frequency increases, fewer turns in the coil are required to satisfy the maximum power transfer condition; this was observed for 1 and 2 MHz.

On reception, the signal increased with the coil inductance in the example investigated, but no changes were observed in the noise at the output of the receive amplifier. This implies that a) further improvement in the electronics could be beneficial and that b) the highest attainable coil inductance should produce the best SNR in this case. As a result, the optimal coil impedance is not necessarily the same on transmission and reception.

Another observation is that the resistance of the source and coil should be the smallest possible to maximise the eddy currents. This implies that when only a section of the coil is used in the ultrasonic aperture, once the optimum coupling has been achieved, the return path should be as short as possible.
Chapter 4

Binary Quantisation

4.1 Introduction

Most of the material in this chapter was extracted from [P3] (with permission from IEEE) with minor modifications.

Many ultrasound applications produce signals which are weak and potentially fall below the noise level at the receiver. However, after quantisation, the signal-to-noise ratio (SNR) is increased by ensemble averaging and filtering or pulse-compression techniques. This is possible because the excitation signals are recurrent. Some examples of applications where the received signals are below the noise threshold can be found in [3] and [P1] for electromagnetic acoustic transducers, [90] for piezoelectric paints, [27] for photo-acoustic imaging, [28,31,91,92] for air-coupled ultrasound, and [30,93,94] for guided ultrasonic waves. Several other applications exist for the inspection of highly attenuating materials such as Inconel [95] and distance and displacement measurements using ultrasound [96,97].

In those cases the information has been shown to be recovered using quantisation levels which are not much bigger than the signal itself [27,91,93,98–100] – the explanation of how this is possible was attributed to the effect of dithering [101–103]. Of particular interest is the work in [91], where it was reported that the information can be recovered using binary (one-bit) quantisation only. The same result was reported in [41] (a decade before) where binary quantisa-
tion was employed with time-reversal techniques and pulse-compression without degrading the spatial or temporal resolution of an array of sensors.

These findings have an important implication in the acquisition of signals embedded in noise since no analog-to-digital converters (ADCs) are then required for they could be replaced by a comparator and a binary latch. An example of such a system is shown in Fig. 4.1, where the output of each transducer or analog channel is connected to an amplifier. After the amplifier there is an antialiasing filter to remove the high frequency components followed by a comparator that acts as a one-bit ADC. The comparator may require a latch control signal to synchronise its output with the digital system. However, the dashed rectangle in the figure highlights that, depending on the digital interface, neither the comparator nor the latch control signal may be required, and hence the analog channel could be directly connected to the digital input.

By using comparators the size of the digital bus is greatly simplified to one digital line per channel and therefore data throughput is reduced. In general, without an ADC the acquisition system becomes faster, more compact and energy efficient. All this is especially attractive for applications that require arrays with many channels and high sampling rates, where the sampling rate can be as high as the system clock. The maximum sampling rate of standard ADCs is usually less than the system clock.

The binary quantisation of noisy signals has been investigated extensively

Figure 4.1: $N$-channel binary acquisition system. The sampling frequency can be as high as the system clock frequency. External comparators and/or latches may not be necessary in some cases.
in the past years, mainly in the field of wireless sensor networks (WSN) [42, 104, 105], where the motivations were also the limited power and bandwidth of the acquisition and data transmission systems. It is necessary to emphasise that binary quantisation is actually employed to later estimate a parameter of interest, in this case the signal embedded in noise, and not to necessarily reconstruct the exact sampled signal – see [106] for a discussion on this.

One of the main findings has been that when the signals are below the noise threshold, the difference between binary quantisation and ideal quantisation, i.e., using ADCs with infinite bits, is roughly only 2 dB [41, 42], and that this difference increases as the SNR increases [42, 104]. Further work has also been conducted to select the optimum threshold for the binary comparator [104, 105, 107].

For signals with greater SNR, i.e., above the noise threshold, the work has been focused on incorporating some control input before quantisation or adding extra quantisation levels [108]. However, this approach introduces extra complexity in the acquisition system. The main goal here is to investigate the conditions under which a simple system, as described in Fig. 4.1, can be employed for ultrasonic applications.

Information about the input SNR range where binary quantisation is of practical interest is not readily available. In this chapter, I review the theory of binary quantisation from previous work (mainly that related to WSNs) and then investigate the input SNR range of practical interest for ultrasound applications.

This chapter is organised as follows: first, the theory related to binary quantisation from previous work is presented, then the maximum input SNR that can be employed is investigated theoretically. Following this, some numerical simulations are carried out to corroborate the theoretical results. Experiments with binary-quantised ultrasound signals are presented and finally conclusions are drawn.
4.2 Binary Quantisation and Averaging

The theory behind binary quantisation has been reported in [42, 104]. However, in this section it is reviewed again in a way that highlights how the different sources of error affect the results. The main sources of error are: a) the error introduced by the binary quantisation itself and b) the error caused when only a limited number of quantised samples or realisations are added (averaged).

4.2.1 Transfer function of the binary quantiser after averaging

Consider a stationary random or stochastic process \( Y(t) \), where \( t \in \mathbb{Z} \) hereinafter, that has \( N \) independent copies \( Y_1(t), \ldots, Y_N(t) \), which are just time functions. Say this stochastic process represents the electrical noise introduced by an amplifier. At any instant \( t_i \), \( Y(t_i) \) is a random variable, whereas \( Y_n(t_i) \) is just a number.

Let \( s(t) \) be a deterministic signal invariant to each copy of \( Y(t) \), in this case it can be said that \( s(t) \) is recurrent, and let

\[
X(t) = s(t) + Y(t).
\]  

(4.1)

Figure 4.2 shows the addition of \( s(t) \) to each of the \( N \) values taken by the copies of \( Y(t) \), which are the input to binary quantisers with corresponding outputs \( Q_1(t), \ldots, Q_N(t) \), where \( Q(t) \) is the stochastic process that represents the output of the \( N \) quantisers.

The output of the \( n \) binary quantiser can take the following values at \( t_i \)

\[
Q_n(t_i) = \begin{cases} 
1 & X_n(t_i) > 0 \\
-1 & X_n(t_i) \leq 0 
\end{cases}
\]  

(4.2)
4. Binary Quantisation

![Diagram of binary quantisation stages](image)

**Figure 4.2:** Stages of binary quantisation. $N$ realisations are added after the comparators, which produces an integer number $c_N$. The output of the quantisers has to be “expanded” to compensate for the non-linear “compressing” behaviour of $E[Q]$.

To simplify the notation we write that the expected value of $Q$ for any $t$, is

$$E[Q] = \overline{F}_X(0) - F_X(0),$$

where $F_X(x)$ is the cumulative distribution function (CDF) of $X$ (at any $t$) and $\overline{F}_X = 1 - F_X$. $F_X$ is equal to the CDF of $Y$ offset by $s$. Hereinafter $t$ can be dropped at any time to simplify the notation.

If $Y$ is assumed to be normally distributed, the following equation can be derived based on the fact that $X$ is then normally distributed with mean $s(t)$ and standard deviation $\sigma_y$.

$$E[Q(t)] = 1 - 2F\left[-\frac{s(t)}{\sigma_y}\right],$$

where $F$ is the CDF of the standard normal distribution (mean $\mu = 0$ and standard deviation $\sigma = 1$). In this case $Y$ acts as a noisy carrier for the signal $s$, which is the foundation of dithering. It is interesting to highlight that when $\sigma_y \to 0$, $E[Q(t)] \to -1$ if $s(t) < 0$, otherwise $E[Q(t)] \to 1$.

Equation (4.4) can be understood intuitively based on Fig. 4.3a-b, where $F$ is plotted (Fig. 4.3a) together with several repetitions or realisations of $X$ at $t_0$ with $s(t_0) = 1$ (Fig. 4.3b). If the number of realisations at either side of a certain threshold and their distribution are known, then the mean value of the
4. Binary Quantisation

Figure 4.3: a) Cumulative distribution function (CDF) of the standard normal distribution, \( F \). b) Repetitions of \( X(t_0) = s(t_0) + Y(t_0) \) with \( s(t_0) = 1 \).

distribution can be estimated relative to its standard deviation, \( \sigma_y \).

Let the result after adding \( N \) copies of \( Q \) be

\[
c_N(t) = \sum_{n=1}^{N} Q_n(t) .
\] (4.5)

Note \( c_N(t) \in \mathbb{Z} \) with \( c_N(t) \in [-N, N] \), which introduces a round-off error.

Due to \( F \) being a non-linear function, equation (4.4) describes a type of non-linear quantisation similar to that of \( \mu \) and A-law companders [89], where a “compression function” – equation (4.4) – is uniformly quantised by \( 2N + 1 \) levels after adding \( N \) copies of it. To compensate for the non-linearity introduced by the “compression function”, an “expansion function” is required, which is basically
the inverse of equation (4.4). Hence, after compensation

\[ s_{N}(t) = -F^{-1}\left[\frac{N - c_{N}(t)}{2N}\right], \tag{4.6} \]

where \( F^{-1} \) is the inverse of \( F \).

Since equation (4.6) is subject to random variations any time \( N \) copies of \( Q \) are added, for completeness we say that \( s_{N}(t) \) is a copy of a random process \( S_{N}(t) \) and then

\[ s(t) \propto E[S_{N}(t)] + e(t), \quad -N < c_{N}(t) < N. \tag{4.7} \]

where \( e(t) \) is the round-off error that appears due to the fact \( c_{N}(t) \in \mathbb{Z} \). It will be shown that \( e(t) \) is negligible compared to the standard deviation of \( S_{N}(t) \), \( \sigma_{S_{N}} \), when \( N \) is large and \( -N < c_{N}(t) < N \).

### 4.2.2 Quantisation errors and SNR

The variance after adding \( N \) copies of \( Q \) is (see Appendix A.4)

\[ \sigma_{Q,N}^{2} = 4N \cdot F\left(\frac{s}{\sigma_{y}}\right) \cdot \overline{F}\left(\frac{s}{\sigma_{y}}\right), \tag{4.8} \]

where \( \overline{F} = 1 - F \); note that \( t \) has been dropped to simplify the notation.

Now, suppose \( N \) is large and \( -N < c_{N} < N \), then the standard deviation of \( S_{N} \) can be approximated as (see Appendix A.5 for the rationale behind this approximation)

\[ \sigma_{S_{N}} \approx \begin{cases} 
  s_{N} + F^{-1}\left(\frac{N - c_{N} - \sigma_{Q,N}}{2N}\right) & N > c_{N} \geq 0 \\
  s_{N} - F^{-1}\left(\frac{N - c_{N} + \sigma_{Q,N}}{2N}\right) & -N < c_{N} < 0 
\end{cases} \tag{4.9} \]

In [104, 105] close-form Cramer-Rao and Chernoff bounds are used to estimate
this variance; however, I found that the approximation in (4.9) produced accurate results for all the values that were simulated.

Additionally, the signal-to-noise ratio (SNR) at the output of the binary quantiser can be approximated as

\[
\text{SNR} \approx \frac{E[S_N]}{\sigma_{SN}} - N < c_N < N. \tag{4.10}
\]

It is interesting to investigate the SNR when \( \frac{s}{\sigma_y} \ll 1 \). In this case \( F \) can be regarded as a linear function of \( s \) (see Fig. 4.3a). Therefore, the SNR is the same before and after the expansion operation. Then, if the round-off error \( e \) in equation (4.7) is negligible, the following approximation for the SNR is obtained, see \[41, 42]\,

\[
\text{SNR}\bigg|_{\frac{s}{\sigma_y} \ll 1} \approx \frac{N \cdot E[Q]}{\sigma_{Q,N}} \approx \frac{s}{\sigma_y} \sqrt{\frac{2}{\pi} N}. \tag{4.11}
\]

When \( \frac{s}{\sigma_y} \ll 1 \), the resulting SNR after binary quantisation and \( N \) realisations added is just roughly 0.8 times (2 dB) smaller than without any quantisation at all, i.e., an ADC that uses infinite quantisation levels and produces an SNR = \( \frac{s}{\sigma_y} \sqrt{N} \). Note that \( F'(0) = \sqrt{\frac{2}{\pi}} \), where \( F' \) is the derivative of \( F \).

### 4.2.3 Limits of binary quantisation

If \( c_N \in \{-N, N\} \), then \( s_N \in \{-\infty, \infty\} \) even when \( -\infty < s < \infty \). By substituting \( c_N \in \{-N + 0.5, N - 0.5\} \) in equation (4.6) the upper and lower bounds that define the quantiser range where the round-off error \( e \) takes finite values is obtained

\[
-F^{-1}\left(1 - \frac{1}{4N}\right) > s_N > -F^{-1}\left(\frac{1}{4N}\right). \tag{4.12}
\]
To prevent $s_N$ from being infinite in the event $c_N \in \{-N, N\}$, $s_N$ can simply be truncated to the closer of these bounds; this of course introduces a significant round-off error.

The impact of $e$ on the results can be inferred by the number of times that $c_N \in \{-N, N\}$ occurs in $N$ realisations. Then, it is useful to find the probability of reaching the condition $c_N = N$ for a given $\frac{s}{\sigma_y}$. This is the probability of obtaining $Q_n = 1$, i.e., $\bar{F}(\frac{s}{\sigma_y})$, for each of the $N$ realisations of $X$

$$p_N = \left[ \bar{F}\left(\frac{s}{\sigma_y}\right) \right]^N. \quad (4.13)$$

Moreover, to numerically investigate the standard deviation at the output of the quantiser for $N$ added realisations ($\sigma_{s_N}$), $M$ sets with $N$ realisations each have to be assessed. The probability of having $c_N = N$ $L$ times in $M$ realisations follows the binomial distribution, then

$$p_L = \binom{M}{L} p_N^L (1 - p_N)^{M-L} \quad (4.14)$$

while the probability of having $c_N = N$ $L$ or more times in $M$ realisations is the cumulative probability of having $c_N = N$ from $L$ to $M$ times

$$p_{L,cum} = \sum_{k=L}^{M} \binom{M}{k} p_N^k (1 - p_N)^{M-k}. \quad (4.15)$$

Equation (4.15) can be used to predict, for example, the value of $s$ for which $c_N = N$ occurs more than 10% of the time, i.e., $L = 0.1M$, with a probability of say 0.9. This may be used to indicate when $e$ has a significant impact on the results.

It is equally useful to know the probability of $c_N = N$ occurring at least once in $M$ realisations, which is equivalent to the complement of the probability of
\[ c_N = N \text{ not occurring, i.e., } 1 - p_N, \text{ in } M \text{ realisations} \]

\[ p_{1,cum} = 1 - (1 - p_N)^M. \]  \hfill (4.16)

### 4.3 Numerical Simulations

A set of \(10^2\) and \(10^4\) samples were normally distributed with \(\sigma = 1\) to obtain the random variable \(Y(t_i)\). The mean of the distribution was varied from \(-5\) to \(15\) dB in intervals of \(1\) dB to simulate \(s\). Each sample was binary quantised, then added (averaged) and expanded using equation (4.6); this process is summarised in Fig. 4.2. Hereinafter, for the sake of brevity, these three operations will be referred to as quantisation. The maximum/minimum value of each realisation after the expansion operation was limited to the upper/lower bound in equation (4.12) so that infinite results were avoided. Each step was repeated \(10^4\) times to investigate the expected values and SNRs at the output of the quantiser.

#### 4.3.1 Expected value at the output of the quantiser

Figure 4.4 shows the expected value at the output of the quantiser for different inputs defined as the ratio between the mean and the standard deviation of the set of samples, which is basically the distribution mean \(s\) since \(\sigma = 1\). The circle markers correspond to the simulated sets of (a) \(10^2\) and (b) \(10^4\) added samples. The dot markers represent the theoretical expected values according to equation (4.6). The continuous line represents the ideal acquisition process, where there is no saturation or round-off error \(e\). The vertical dotted line (labelled Sat.> 1) indicates the occurrence of saturation at least once with probability of \(10^{-4}\); this is basically the value of \(s\) for which equation (4.16) yields \(10^{-4}\). The other vertical dotted line (labelled Sat.> 10\%\)) indicates the occurrence of saturation 10\% of the time with a probability of 0.9; this is the value of \(s\) for which equation (4.15) gives 0.9 with \(L = 0.1M\).

In general there is good agreement between the theory presented above and
Figure 4.4: Mean value before and after quantisation of a normal distribution with \( \sigma = 1 \) for \( 10^4 \) sets of a) \( 10^2 \) and b) \( 10^4 \) samples. The continuous line represents the expected signal without saturation error. The vertical dotted line (Sat.>1) indicates the occurrence of saturation at least once with a probability of \( 10^{-4} \). The vertical dotted line (Sat.>10\%) indicates the occurrence of saturation 10\% of the time with a probability of 0.9.
the simulations. Equation (4.15) can be used to predict the value of the input mean where the linearity of the system changes. Moreover, when the maximum/minimum value of each realisation is truncated using equation (4.12) so that the result is not infinite, the input range that produces a linear output is extended from the first occurrence of saturation (marked by Sat. $> 1$) to roughly where saturation occurs 10% of the time. This increase is approximately 5 and 1 dB for the sets of $10^2$ and $10^4$ samples respectively; note that truncation has a greater impact on the set with fewer samples. Overall, the greater the number of samples (equivalent in practice to the number of averages) in a set, the greater the bounds in equation (4.12) and therefore the greater the input range of the quantiser.

4.3.2 Output SNR

Figure 4.5 shows the SNR before and after quantisation. The SNR of each simulation is computed as the ratio of the mean and standard deviation of the set (circle markers). The dot markers correspond to the outputs of equation (4.10). The continuous line is the theoretical result assuming there is no saturation or round-off error; this is calculated by replacing $c_N$ by $N \cdot E[Q]$ in equations (4.6) and (4.9). The dashed line represents the resulting SNR without quantisation, i.e., the standard deviation of the sum of all of the samples in a set. The vertical dotted lines labelled Sat. $> 1$ and Sat. $> 10\%$ are the same as in the previous figure.

Again, the theory presented above and the simulations match well before saturation takes place (below the input SNR marked by the vertical dotted line labelled Sat. $> 1$). This confirms that the round-off error $e$ is negligible in this interval. Note that for an input SNR below $-5$ dB the difference between binary quantisation and no quantisation at all (dashed line) is roughly 2 dB as predicted by equation (4.11). In general, the resulting SNR after binary quantisation is always smaller than the SNR without any quantisation at all. The resulting SNR produced by any other type of quantisation, e.g., 2- or 12-bit quantisation, should lie between these two cases.

For input SNR values between the dotted lines Sat. $> 1$ and Sat. $> 10\%$ satura-
Figure 4.5: SNR before and after quantisation. $10^4$ sets of a) $10^2$ and b) $10^4$ realisations. The continuous line represents the expected SNR without saturation or round-off error. The vertical dotted line (Sat. $> 1$) indicates the occurrence of saturation at least once with a probability of $10^{-4}$. The vertical dotted line (Sat. $> 10\%$) indicates the occurrence of saturation $10\%$ of the time with a probability of 0.9. The dashed line is the resulting SNR without quantisation.
4. Binary Quantisation

Figure 4.6: Output of equation (4.10) for SNR inputs between $-5$ and $15$ dB using $10^4$ sets of $10^2$, $10^3$ and $10^4$ realisations.

...tion causes the output SNR to be overestimated by no more than 2 dB. Note that the distance between the dotted lines shortens as the number of added samples in the set increases. Results that correspond to an input SNR beyond the line Sat.$>10\%$ should be ignored as errors due to saturation are significant and the information is lost.

Figure 4.6 shows the outputs of equation (4.10) for an input SNR between $-5$ and $15$ dB using $10^4$ sets of $10^2$, $10^3$ and $10^4$ realisations. The curves are vertically offset and the output SNR increases as a function of $N$ when the input SNR is below $8-12$ dB. The maximum output SNR ($\text{SNR}_{\text{max}}$) occurs when the input SNR is roughly 4 dB. $\text{SNR}_{\text{max}}$ is slightly smaller than the number of realisations in a decibel scale ($10 \log_{10} N$).

In Fig. 4.7 the difference $10 \log_{10} N - \text{SNR}_{\text{max}}$ is plotted with a dashed-dotted curve for $10^2$ to $10^6$ realisations. From the curve the following expression can be used as a good estimate of $\text{SNR}_{\text{max}}$ when $N > 10^3$

\[
\text{SNR}_{\text{max}} \approx 10 \log_{10} N - 2 \quad N > 10^3,
\]  

(4.17)

The input SNR that yields $\text{SNR}_{\text{max}}$ is also shown in Fig. 4.7 (dashed-dotted curve). This confirms that $\text{SNR}_{\text{max}}$ occurs when the input SNR is roughly 4 dB.
Figure 4.7: Input SNR that yields $\text{SNR}_{\text{max}}$ (---), $10\log_{10} N - \text{SNR}_{\text{max}}$ (-----), and input SNR where saturation occurs 10% of the time with a probability of 0.9 (labelled Sat. $>10\%$) vs. the number of added realisations $N$. The dotted and continuous lines labelled Sat. $>10\%$ correspond to sets of 10 and $10^4$ samples.

as previously observed.

Finally, the dotted and continuous curves in Fig. 4.7 indicate the input SNR where saturation occurs 10% of the time with a probability of 0.9 for sets of 10 and $10^4$ samples respectively. Note that the size of the set has a minor effect on the results. These curves are good approximations to the maximum input SNR that produces a non-distorted output. For example, when $N = 10^3$, the maximum input SNR that produces a non-distorted output is approximately 9 dB, and this only increases by roughly 4 dB when $N = 10^6$. In the interval $N \in [10^3, 10^6]$ the maximum input SNR ($\text{SNR}_{\text{max,in}}$) can be approximated as

$$\text{SNR}_{\text{max,in}} \approx \frac{4}{3} \log_{10} N + 4 \quad N \in [10^3, 10^6]. \quad (4.18)$$

Overall, a minimum bound for the input SNR range (difference between maximum and minimum input SNR in decibels), which can also be understood as the input signal dynamic range, can be approximated as

$$D > 10 \log_{10} N - \text{SNR}_{\text{min}} + 2 \quad N \in [10^3, 10^6], \quad (4.19)$$
where $\text{SNR}_{\text{min}}$ is the minimum tolerable SNR after quantisation and averaging (defined for each application beforehand). As an example, if $\text{SNR}_{\text{min}} = 20 \text{ dB}$ and it is desired $D > 8 \text{ dB}$, then $N > 100$. The dynamic range $D$ is therefore tunable by adjusting the number of averages $N$. This means that the dynamic range can be increased at the cost of decreased measurement speed in order to suit the requirements of different applications. In general, binary quantisation offers a lower input SNR range compared to standard ADCs. This is because ADCs can be thought of as a superposition of offset binary quantisers. However, once the signals are embedded in noise, the advantage of using a standard ADC is only a $2 \text{ dB}$ increase in SNR. Filtering may also increase the SNR further by removing the noise components outside the frequency band of interest.

### 4.4 Experimental Results

Ultrasound signals were recorded before and after a comparator as shown in Fig. 4.8. Two transducers (Panametrics V106, Olympus, MA 02453, USA) were placed on one side of a 300-mm-thick aluminium block that has a cylindrical shape. One of the transducers acted as a transmitter and the other as a receiver. The edge of the block on the transducers side was rounded to avoid reflections of surface waves. The transmit transducer was connected to a driver, which also triggered an oscilloscope (LeCroy WaveRunner 44Xi). The signal from the receive transducer was amplified and filtered, after which the signal split into two, one cable connecting directly to the scope and the other entering the scope via a comparator (ADCMP600, Analog Devices, Norwood, US). The scope has an 8-bit resolution ADC and $2 \text{ mV}$ minimum sensitivity; the sampling rate was set to $100 \text{ MHz}$. The driver, amplifier and filter are independent units of the WaveMaker-Duet system (custom made for the NDE group of Imperial College London).

The driver was set to transmit a 5-cycle tone-burst with a Hann apodization at a central frequency of $2 \text{ MHz}$. The amplifier gain was set to $60 \text{ dB}$. The response of the band-pass filter in the WaveMaker-Duet system is assumed to encompass the
Figure 4.8: Experimental set-up using ultrasonic transducers. Signals are recorded before and after the comparator and later averaged.

Figure 4.9: Comparator input (black) and output (grey). The output has been normalised to fit the figure.

tone-burst frequency band and known to have a cut-off frequency below 10 MHz. The comparator reference level was calibrated with a potentiometer such that the mean value of the resulting signal at the output was in the middle of the comparator output range; this was to maximise the input range of the comparator.

In Fig. 4.9 signals before and after the comparator (black and grey curve respectively) are shown. The output of the comparator indicates when the noise is above or below 0 mV in the figure. The shortest time interval between the comparator transitions, i.e., the shortest pulse, is determined by the comparator and noise bandwidth. The shortest pulse is equivalent to the effective sampling rate of the binary signal. The duration of the shortest pulse is below 10 ns, so the effective sampling frequency is greater than 100 MHz, which is fifty times greater than the tone-burst central frequency.

Initially, the driver excitation intensity was set such that the received echoes were just below the noise threshold. The received signals were averaged 500 times. Figure 4.10 shows the averaged signals before the comparator stage. The section of the signal marked as receiver noise, just before the first echo, is known
to contain noise from the receive amplifier only – there is no mode conversion or any other coherent noise source present. This was confirmed by comparing different averaged signals, where no correlation was found. The noise computed in this section is used as a reference throughout the experiments.

Figure 4.11a-c shows the first echo after 500 averages under different excitations and post-processing conditions. The thick grey trace (8-bit) corresponds to the signal before the comparator, the dashed line (1-bit) to the signal after the comparator and the continuous black line (Expanded) to the signal after the comparator expanded using equation (4.6). Before expanding the signal using equation (4.6), the signal after the comparator was normalised to its maximum value 611.8 mV. Then, the output of the expansion operation was limited as indicated in equation (4.12) to avoid infinite results.

Figure 4.11a shows the first echo which corresponds to an input SNR of \(-5.8\) dB. This value was estimated by computing the SNR of the signal before the comparator and after 500 averages, which yields roughly \(21.2\) dB since 500 averages increase the SNR by approximately \(27\) dB. The SNR is estimated as the ratio of the maximum value of the signal and the variance of the noise in the interval highlighted in Fig. 4.10. All of the curves in Fig. 4.11a were normalised to their maximum value and as expected a good match was found because when the input SNR is below zero, the comparator shows a linear response (after averaging). The SNR difference between the signals before and after the comparator (8-bit and 1-bit respectively) is 2.1 dB as closely predicted by equation (4.11) for this low input SNR regime.
4. Binary Quantisation

Figure 4.11: First echo under different excitations and post-processing conditions. Thick grey trace (8-bit) corresponds to the signal before the comparator, dashed line (1-bit) is the signal after the comparator and the continuous black line (1-bit expanded) is the signal after the comparator expanded according to equation (4.6); all these traces consist of 500 averages. a) Input SNR ≈ −5.8 dB. b) Input SNR ≈ 7 dB. c) Input SNR ≈ 7 dB with comparator output scaled by 1.05 and then offset by 0.11. d) Signal with SNR ≈ 7 dB (8-bit no ave.).
Then, the excitation was increased by approximately 13 dB and the results shown in Fig. 4.11b. The SNR of the 8-bit signal is estimated at 33.9 dB after averaging. The input SNR is now roughly 7 dB, which produces a non-linear response of the comparator as shown by the dashed line, and the output of the expansion operation (continuous lines) gives a highly distorted echo. This is due to a imperfect calibration of the comparator. To correct this, the normalised signal after the comparator was scaled by 1.05 and then offset by 0.11. As a result of this fine-tuning, both the signal before the comparator and after the expansion operation matched very well as can be appreciated in Fig. 4.11c.

However, the input SNR in this case is close to the maximum input SNR ($\text{SNR}_{\text{max,in}}$), where saturation starts occurring. Therefore, the SNR after binary quantisation deviates from its maximum value, see Fig 4.6. The small distortions in the inset of Fig. 4.11c are a consequence of this. Nonetheless, these can be alleviated by filtering out the signal. It is interesting to highlight that the input SNR for this example is very close to the corresponding $\text{SNR}_{\text{max,in}}$ for 500 averages, see equation (4.18) or Fig. 4.7. Hence, if the input SNR is increased any further, saturation will take place and the expansion operation will not prevent severe distortion from occurring.

To conclude the discussion, a visual example of a simulated single copy of the signal before the comparator is shown in Fig. 4.11d (8-bit no ave.); the estimated SNR of this signal is 7 dB. This signal was simulated by adding noise normally distributed with a variance of 27 dB to the signal before the comparator after 500 averages. By visual inspection some sections of the signal appear to lie solely on one side of the comparator threshold at zero amplitude, yet binary quantisation can recover this signal with low distortion after averaging several repetitions.

Finally, it is important to highlight that in this chapter averaging is used as a means to increasing the SNR after binary acquisition due to its simplicity; however, in many applications averaging over a large number of realisations may be lengthy and hence impractical. In such cases the use of pulse-compression may be preferred as discussed in the next chapter. Pulse-compression based on coded sequences can be readily understood as a weighted averaging process,
where the aim of using the weights is to accomplish some further post-processing, such as reducing interference between adjacent bursts. Therefore, all of the results reported for averaging herein apply to coded pulse-compression. Any advantage of pulse-compression over averaging lies solely on the post-processing stage and does not directly affect the binary acquisition mechanism here discussed. The analysis of the effect of other pulse-compression techniques on binary quantisation, e.g., chirp signals, is out of the scope of this thesis.

4.5 Summary

In this chapter the theory of binary quantisation of recurrent signals embedded in noise was reviewed in detail. Binary quantisation and averaging can be understood as a non-linear acquisition process similar to standard companding techniques where an expansion function is required to compensate for non-linearities introduced in the process.

The input SNR where binary quantisation is of practical value for ultrasound applications was investigated, and it was found that in most cases binary quantisation can only be employed when the input SNR is below 8 dB. Hence, the input SNR of the binary quantiser is significantly smaller compared to standard ADCs, which can be understood as a set of offset binary quantisers. Moreover, the maximum SNR after binary quantisation and averaging can be estimated as $10 \log_{10} N - 2$; therefore, at least a few hundred of averages are required to produce an SNR at the output greater than 20 dB.

However, the fact that there is only a 2 dB difference between binary quantisation and ideal quantisation at all when the signals are below the noise threshold has an important implication in the quantisation of signals embedded in noise because standard ADCs can be replaced by comparators and binary latches, and in some cases even the analog channel may be directly connected to the digital input. All this is especially attractive for applications that require arrays with many channels and high sampling rates, where the sampling rate could be as high as the system clock rate, which in general permits the electronics to be more
compact and faster and to consume less energy.
Chapter 5

Pulse-echo coded excitation: single channel

5.1 Introduction

Most of the material in this chapter was extracted from [P4] (with permission from IEEE) with minor modifications.

Pulse-compression has been in use for decades to increase the signal-to-noise ratio (SNR) and resolution in radar [109,110], sonar [111,112], medical [93,113–118] and industrial ultrasound [3,27,28,30,31,91]. It consists of transmitting a modulated and/or coded excitation, which is then correlated with the received signal such that received echoes become shorter in duration and higher in intensity, thereby increasing the system resolution and SNR. Pulse-compression is a faster alternative to averaging because, when averaging, a wait time is required between consecutive excitations, where the energy in the medium that is being inspected dies out so that there is not interference between the excitations.

The two main approaches to pulse-compression are chirp signals and coded excitation (or sequences). Chirp signals are obtained by frequency-modulating the excitation; the increase in SNR and resolution depends on the chirp length and bandwidth [118]. Coded sequences operate in a slightly different way, most often by coding the polarity of concatenated bursts according to a binary sequence, i.e., a sequence composed of 1s and 0s or +1s and −1s [91]. In any case a good
5. Pulse-echo coded excitation: single channel

Figure 5.1: Pulse-echo system with close and far reflectors.

approximation to the single initial burst is obtained when correlating the received signal with the transmitted sequence, hence the term compression.

In the applications that initially motivated this work, namely low-power excitation of electromagnetic-acoustic transducers [3] and [P1], piezoelectric paints [119], photo-acoustic imaging [27], air-coupled ultrasound [28, 31, 91, 92], and guided ultrasonic waves [30, 93], the received signals lie below the noise threshold. Therefore an increase in the SNR of more than 30 dB is required to accurately extract the information from the signals. The goal in those scenarios is to transmit the longest sequence or chirp signal possible to achieve the highest SNR increase, but in a pulse-echo system the distance between the closest reflector and the transmit/receive source limits their length. This problem is critical when reflectors are simultaneously located very close to and very far from the transmit/receive source, see Fig. 5.1. In this scenario long sequences cannot be transmitted and averaging takes a longer time due to the need for long receive intervals between transmissions so that the echoes from the farthest reflector do not overlap.

Figure 5.2a shows a common scenario of a low-SNR pulse-echo system in industrial ultrasound affected by the problem of close and far reflectors. It consists of a metal block with a transmit-receive transducer on the front-wall of the block. The objective is to find the thickness of the block, i.e., the location of the opposite parallel back-wall. The back-wall itself acts as the closest reflector, whereas the wave reverberation between the walls behaves as reflections from far reflectors. The received signals when using averaging are shown in Fig. 5.2b. After each transmission there are several echoes that decay progressively and hence some wait time between transmissions is necessary to avoid interference; this makes
averaging a lengthy process. Figure 5.2c shows the received signals when transmitting a sequence or a chirp signal. The location of the back-wall limits the length of the excitation and therefore the SNR increase. If the excitation overlaps the reflection from the back-wall, the information is lost because it is not possible to receive while transmitting.

In this chapter, I propose a solution to these problems by introducing receive intervals within a coded sequence in which reception can take place while the sequence is being transmitted, see Fig. 5.2d. Hence, the overall sequence length and SNR increase is independent of the location of the reflectors. The aim of this idea is to increase the SNR without compromising the overall duration of the measurement so that the pulse-echo system can still respond to fast changes in the medium that is being inspected.

The organisation of this chapter is as follows: first, the proposed methodology is briefly introduced. Then, the autocorrelation properties of standard sequences and the corresponding SNR increase are discussed in Sec. 5.3. The properties and construction of coded sequences with receive intervals are introduced in Sec. 5.4. In Sec. 5.5 experimental results are presented. After discussing the results, conclusions are drawn.

5.2 Overview of proposed coded excitation

Figure 5.3 shows the fundamental steps of the proposed methodology. There are three main stages: 1) sequence synthesis, 2) propagation through the medium and reception, and 3) post-processing. First, two sequences are generated, sequence $X$ controls the polarity of the burst ($+1$ or $-1$) whereas $G$ controls the transmit/receive intervals ($1$ corresponds to transmission and $0$ to reception).

In practice the transmitted sequences are not a train of delta functions but concatenated band-limited bursts, $B$. These band-limited bursts are necessary due to the limited bandwidth of the electronics and transducers. The bursts are assumed to be optimal; this means that their length can be the maximum possible without overlapping the closest reflector and that they can utilise all the available
Figure 5.2: Different types of excitations for pulse-echo transducers. a) Transducer operating in pulse-echo mode. b) Received signal when using averages. c) Received signals when using a sequence. d) Proposed sequence with receive intervals.
bandwidth. Moreover, there is no restriction on the type of burst; they can be square pulses, chirp signals or multiple cycles with a certain apodization.

Before the sequences can be modulated by the burst $B$, they have to be up-sampled by $Q$ samples to match the burst length. By doing so $X'$ and $G'$ are obtained. Then $X'$ is convolved with $B$, which yields $X''$. On the other hand $G'$ is convolved with a rectangular pulse, which gives $G''$. The transmitted sequence $Z''$ is simply the multiplication of the elements of $X''$ and $G''$.

The reflected signals can be modelled as the convolution of the medium impulse response and the excited sequence $Z''$; an example is shown in Fig. 5.3. At the receiver, the reflected signals are combined with noise from the electronics $Y$. The complement $\tilde{G}'' = 1 - G''$ is used to zero the received signal when transmission is on. This is basically the role of the transmit/receive (T/R) switch in the electronics. In practice the T/R switching occurs before the receiver noise is added to the signal; however, in Fig. 5.3 the order of the operation is flipped to indicate that reception does not take place during transmission and the signals are zeroed. Recall that transmission and reception cannot occur simultaneously due to the limited dynamic range of the receiver since a single transducer acts as transmitter and receiver in pulse-echo.

The modulated medium response can be partially recovered by cross-correlating the received signals with $X' \cdot G'$; this is the compression stage. Note that cross-correlation is equivalent to convolution with one of the terms being time-reversed. The recovered medium response will have a noise component (not shown in the figure) that corresponds to the electronics. Interference between bursts in the sequence will also affect the recovered medium response. Further noise reduction is possible by cross-correlating the result with the burst $B$, which is the matched-filtering stage. However, matched filtering distorts the originally transmitted burst, as can be appreciated in Fig. 5.3.

It has to be highlighted that the steps in Fig. 5.3 have been arranged in such a way that they correspond to fundamental steps of the methodology proposed so that it is easier to understand; however, this is not necessarily an efficient way of implementing it. For example, $Z''$ can be obtained by simply convolving $B$ with
**Figure 5.3:** Fundamental steps of the proposed coded excitation. The operators \{\cdot\}, \{\ast\} and \{\star\} indicate multiplication, convolution and cross-correlation respectively.
the up-sampled result of $X \cdot G$ and the last compression and matched-filtering operations combined, which is equivalent to cross-correlating the received signal with the transmitted signal $Z'' = (X' \cdot G') \ast B$, where $\ast$ indicates convolution.

In the next sections, I investigate the optimal synthesis of these random sequences with receive intervals and their expected SNR.

## 5.3 Background on Coded Excitation

In this section the attention is focused on coded sequences, especially binary coded sequences, wherein the polarity of the bursts or symbols in a sequence is changed. These are simpler to implement than non-binary ones. There are no restrictions on the bursts other than their bandwidth not exceeding that of the system and their length not overlapping the closest reflector; note a burst can be a chirp signal. In this section, I review the previous work on coded sequences and then discuss their merit factor; this is central to proving the optimality of the sequences with receive intervals proposed herein.

### 5.3.1 Previous work

Overall, the performance of a sequence relies on its autocorrelation properties. Ideally, its autocorrelation should be a delta function but this cannot be achieved with a single sequence. The quest for "good" sequences started around the middle of the last century [120–123] and still continues today [124–126]; see [127–129] for a comprehensive review of the different sequences. Among the key binary sequences known so far are those named after Barker [44, 130], and Legendre [45], as well as maximum length register sequences [131]. This list is not exhaustive and other sequences can be found in the literature [128], though some may be considered either as special cases or family members of those previously mentioned.

One of the most elegant solutions to the imperfection of the autocorrelation properties of a single sequence can be found in [120], whereby paired complementary sequences produce a perfect delta function when their corresponding
autocorrelations are added together; this was later extended to orthogonal complementary sets of sequences in [121]. Another solution is to use sequences that achieve zero or very low autocorrelation values only in certain intervals of interest [132–136]. In general, there has been a tremendous interest in improving the autocorrelation properties of sequences, mainly by means of optimisation strategies, for example, [125,135–140], and also in efficient ways of processing and obtaining them [134,141,142].

The fact that good or perfect autocorrelation can be (partially) achieved is highly relevant; however, there are certain scenarios where the SNR at the input of the amplifier is low [3,27,28,30,91–93,119], [P1] and [P3], and in these cases good autocorrelation properties are not essential. Indeed, in this section, I show that when the SNR is low (i.e., the signal magnitude is comparable to or below the noise level) the choice of the sequence is relatively unimportant and a simple random sequence that has a uniform distribution of +1s and −1s will suffice in most cases.

### 5.3.2 Merit Factor

Let \( X \) be a sequence of \( N \) elements, where each element \( x \) takes on values +1 or −1. The aperiodic autocorrelation of this sequence at shift \( k \) is

\[
c_k = \sum_{j=0}^{N-k-1} x_j x_{j+k}, \quad k = 0, \cdots, N-1.
\]

(5.1)

Golay introduced the *merit factor*, \( F \), of a sequence [43] to compare and measure its performance

\[
F = \frac{N^2}{2 \sum_{k=1}^{N-1} c_k^2}.
\]

(5.2)

The merit factor is basically the ratio between the energy at shift zero and the combined energy of the rest of the shifts or autocorrelation sidelobes. The factor 2

\[
\]
is included to compensate for the tapering effect of the aperiodic autocorrelation. The merit factor can be understood as measure of how similar the autocorrelation result is to a delta function; for the sake of simplicity it should be assumed that the elements \( c_k, k \in [1, N - 1] \), have a zero mean. A random binary sequence with \(+1s\) and \(-1s\) has \( F \approx 1 \) on average for large \( N \) \([43]\); a Barker sequence of 13 elements, which is the longest known, has \( F \approx 14.08 \) \([44, 45]\); Golay sequences have \( F \approx 3 \), the added autocorrelations of the Golay complementary sequences have of course \( F = \infty \), i.e., a delta; while Legendre sequences can achieve \( F \approx 6 \) \([45]\).

Finding sequences with an optimal merit factor for a given length by extensive search is computationally demanding; the best known cases from 60 to 200 elements are limited to \( F \approx 10 \) \([128, 129]\). Longer sequences are expected to have \( \max \{ F \} < 6 \) since no sequence with a higher merit factor has been found, though this remains a conjecture \([129]\).

### 5.3.3 SNR increase

When adding (averaging) \( N \) received signals from identical excitations, the resulting SNR is

\[
\text{SNR}_{\text{avg}} = N \cdot \text{SNR}_{\text{in}},
\]

(5.3)

where the input SNR, \( \text{SNR}_{\text{in}} \), is defined as

\[
\text{SNR}_{\text{in}} = \frac{s^2}{\sigma^2_{\text{in}}},
\]

(5.4)

where \( s \) is the magnitude of the received signal, assuming that the excitation is an impulse and that there is only one point-like reflector, and \( \sigma^2_{\text{in}} \) is the variance of the received noise, which has zero mean. In most ultrasound systems, the received noise is mainly due to electrical noise of the receive amplifier – for simplicity this noise can be assumed to be additive white Gaussian noise. Clearly the actual
received signal can take on other values after noise is added, but here $s$ refers to
the ideal received signal prior to additive noise.

In order to simplify the analysis and to focus the attention on the sequences
themselves two assumptions have been made: a) the excitation is an impulse and
b) there is only one point-like reflector. This means that the receive sequence
only takes on $+s$ and $-s$ values. I then discuss the effect of modulation and
multiple reflectors in Sec. 5.4.6.

When using coded excitation, the cross-correlation of the received signal and
the transmitted sequence introduces noise as a result of the interference between
the coded bursts in the sequence. This interference is of (pseudo) random nature
and behaves similarly to the electrical noise of the receiver. Therefore, it is
referred to simply as noise and measured in the same way since its effect is not
different from that of electrical noise.

Let the transmitted sequence be of length $N$ with unity magnitude and let the
received sequence take on values $+s$ and $-s$. Then, the energy at shift $k = 0$ after
cross-correlation is $(N \cdot s)^2$, while the sample variance of the noise introduced by
the cross-correlation is $\sigma_s^2$, which can be defined as

$$\sigma_s^2 = \frac{2s^2}{N - 1} \sum_{k=1}^{N-1} c_k^2 \approx \frac{N \cdot s^2}{F}, \quad (5.5)$$

when $N$ is large. The factor 2 in equation (5.5) has been added to compensate
for the tapering effect the aperiodic cross-correlation has on $c_k$.

Now let $Y$ be a sequence of independent and identically (normally) distributed
(i.i.d.) elements $y_j$ with zero mean and variance $\sigma^2$; say this sequence represents
the noise added at the receiver. The sample variance of the result of cross-
correlating $Y$ with the transmitted sequence can be approximated, if $N$ is large,

$$\sigma_Y^2 \approx E \left[ \frac{2}{N - 1} \sum_{k=1}^{N-1} d_k^2 \right], \quad (5.6)$$
where \( E[\cdot] \) denotes the expected value and \( d_k \) are the coefficients of the cross-correlation for each shift \( k \). Since each \( d_k \) is i.i.d with zero mean

\[
\sigma_Y^2 \approx \frac{2}{N-1} \sum_{k=1}^{N-1} E[d_k^2], 
\]  
(5.7)

\[
d_k = \sum_{j=0}^{N-k-1} y_j x_{j+k}, \quad k \in [1, N-1]. 
\]  
(5.8)

Due to each \( y_j \) and \( x_{j+k} \) being also i.i.d. with zero mean,

\[
E[d_k^2] = \sum_{j=0}^{N-k-1} E[y_j^2] E[x_{j+k}^2] 
= \sigma^2 (N-k), \quad k \in [1, N-1] 
\]  
(5.9)

Hence,

\[
\sigma_Y^2 \approx N\sigma^2. 
\]  
(5.10)

Finally, given that the noise introduced by the sequence is independent of the noise introduced by \( Y \), the SNR of the aperiodic cross-correlation can be approximated, when \( N \) is large, as

\[
\text{SNR}_s = \frac{(N \cdot s)^2}{\sigma_s^2 + \sigma_Y^2} \approx \frac{N}{\frac{1}{F} + \frac{1}{\text{SNR}_{\text{in}}}}. 
\]  
(5.11)

There are two special cases of interest in equation (5.11)

\[
\text{SNR}_s \approx \begin{cases} 
N \cdot \text{SNR}_{\text{in}} & \text{if } F \gg \text{SNR}_{\text{in}} \\
N \cdot F & \text{if } F \ll \text{SNR}_{\text{in}} 
\end{cases}. 
\]  
(5.12)
If $F \gg SNR_{in}$, the SNR increase due to coded excitation is that of averaging—see equation (5.3). Moreover, there is no benefit in using sequences with $F > 1$ (i.e., other than random sequences, which achieve $F \approx 1$ when $N$ is large) to increase the SNR when $SNR_{in} \ll 1$. Note that even the complementary Golay sequences, which can perfectly cancel the sequence noise [120, 121], yield no advantage in this case.

Interestingly, many scenarios exist where either $SNR_{in} \sim 1$ or $SNR_{in} \ll 1$ and hence a significant number of averages or long sequences are required (commonly $N > 1000$) to produce a satisfactory SNR, which often needs to be in the order of $30 - 50$ dB. These scenarios are usually found in systems that rely on inefficient/poor transducers or constraints on the excitation power [3, 27, 28, 30, 91-93, 119], [P1] and [P3].

Conversely, if $F \ll SNR_{in}$, the $SNR_s$ is independent of $SNR_{in}$, and if $SNR_{in}$ is high, it may happen that $SNR_{in} > SNR_s$ for a given $N$ due to the noise introduced by the sequence during the cross-correlation operation. In these cases special attention should be paid to increasing the merit factor $F$ and hence to the use of complementary Golay sequences and zero autocorrelation zone sequences [132-136].

### 5.4 Properties and synthesis of sequences with receive intervals

In a pulse-echo system the length of the excitation, be that a coded sequence or a chirp signal, is limited by the distance between the closest reflector and the transmit/receive source, see Fig. 5.1. In order to transmit longer sequences, receive intervals are introduced in the sequences. In this section the rationale behind this approach is explained and the optimal interval distribution within a given sequence is discussed.
5. Pulse-echo coded excitation: single channel

5.4.1 Synthesis of sequences with receive intervals

It is desirable to create a ternary random sequence $Z$ that takes on values $+1$, $-1$, and 0, where the values $+1$ and $-1$ codify the excitation and the zeroes allows for reception to take place, hence the name receive interval. Such a ternary sequence can be synthesised as follows. Let $X$ be a binary sequence of length $L$ that takes on values $+1$ and $-1$ and let $G$ be another binary sequence also of length $L$ that takes on values $+1$ and 0. Sequence $Z$ can be obtained as

$$Z = X \cdot G = (x_0g_0, x_1g_1, \ldots, x_{L-1}g_{L-1}),$$

(5.13)

where each $x_j$ and $g_j$ are i.i.d.. The process described by equation (5.13) was shown in Fig. 5.3 but including the burst modulation. The sequence $G$ controls the location of the receive intervals ($g_j = 0$) and transmit intervals ($g_j = 1$), whereas $X$ controls the polarity ($\pm 1$) of the bursts during the transmit interval. For simplicity, and without loss of generality, each transmit/receive intervals is considered to be of the same length. The expected SNR when using $Z$ is quantified in Sec. 5.4.3, where each term of equation (5.13) is instrumental in the mathematical formulation.

Now, I introduce the concept of transmit-receive pairs within the sequence $G$. The transmit-receive pairs are responsible for the amount of energy received from reflectors, which is discussed in Sec. 5.4.2. Let $\overline{g}_j$ be the complement of $g_j$ defined as

$$\overline{g}_j = 1 - g_j,$$

(5.14)

then $\{g_j, g_{j+m}\}$ is said to be a transmit-receive pair of length $m$ if $g_j\overline{g}_{j+m} = 1$. As an example, a pair of length $m = 5$ is shown in Fig. 5.4.

The pair length is defined as a range of time differences between any two transmit and receive states. Therefore, a given pair length corresponds to the
Figure 5.4: Random distribution of receive intervals in a sequence. A burst is sent in each transmit interval (sequence $G_{\text{high level}}$) and the reflected echo can be received only if its arrival time matches the occurrence of a receive interval (sequence $G_{\text{low level}}$).

The total travel time range of a wave to and back from a potential reflector. The larger the pair length, the longer the travel time, and hence the deeper the potential reflector. The amount of energy received from a given reflector is proportional to the total number of transmit and receive pairs of a pair length that corresponds to the depth of the reflector. For this reason, a uniform probability distribution of pair lengths is desired in order to have a uniform sensitivity with respect to the reflector depth. In the next section it is discussed how to ensure this condition, whereas the expected amount of energy received from a given reflector is investigated in Sec. 5.4.3.

Figure 5.4 shows an example of a sequence $G$ together with the time-of-flight and distance travelled by waves generated in transmit intervals of $G$. For example, the wave generated in $g_0$ reflects back from reflectors 1 and 2 and the first reflection arrives at $g_3$, so this reflector location is said to require a pair of length $m = 3$. Equally, the second reflection arrives at $g_6$ and hence this reflector is said to require a pair $m = 6$.

In a pulse-echo system, where transmission and reception cannot occur simultaneously due to the limited dynamic range of the receiver, a transmit-receive pair may not exist for a given transmit interval and reflector location. For example, in Fig. 5.4 $\{g_0, g_6\}$ is not a valid pair because $g_0g_6 = 0$ and hence the
reflection cannot be received.

5.4.2 Random distribution of receive and transmit intervals for even sampling of the medium

In a pulse echo system it is important to ensure equal sensitivity to every reflector regardless of its location in the interrogated space. When using the sequences with receive intervals and ignoring beam spread and directivity effects, this is equivalent to obtaining the same number of reflections, i.e., the same amount of energy, from any point-like reflector irrespective of its location. More formally, this is to obtain the same number of reflections \( r \) for any transmit-receive pair of length \( m \) up to a length \( M \).

This condition can approximately be satisfied by any random binary sequence \( G \) when \( L \) is large and \( L \gg M \). To prove this, let the expected number of reflections that correspond to a reflector, whose distance from the transducer corresponds to a transmit-receive pair of length \( m \), be

\[
 r_m = E \left[ \sum_{j=0}^{L-m-1} g_j \overline{g}_{j+m} \right] \quad m \in [1, M]. \tag{5.15}
\]

Note that \( r_m \) is basically the expected total number of valid transmit-receive pairs, i.e., those that yield \( g_j \overline{g}_{j+m} = 1 \), for each shift \( m \).

Let \( p_1 \) be the probability of having a transmit interval defined as

\[
 p_1 = E [g_j] = 1 - E [\overline{g}_j]. \tag{5.16}
\]

As every \( g_j \) in equation (5.15) is i.i.d.,

\[
 r_m = p_1 (1 - p_1) (L - m) \quad m \in [1, M]. \tag{5.17}
\]
Then if $L \gg M$,

$$r = r_m|_{L \gg M} \approx p_1 (1 - p_1) L. \quad (5.18)$$

In practice the location of the furthest reflector is limited to an equivalent transmit-receive pair of length $M$. By using sequences of length $L \gg M$, the number of reflections from any possible reflector location within such a finite distance can be homogenised, as shown by equation (5.18). This greatly simplifies the formulae for the quantification of the sequence SNR in the next section.

5.4.3 SNR and optimal ratio of transmit and receive intervals

Having discussed that a random distribution of transmit-receive intervals guarantees that the same number of reflections $r$ be received irrespective of the reflector location within a finite distance from the source, the next step is to investigate the optimal number or proportion of transmit-receive intervals in a sequence, i.e., find the optimal $p_1$. The optimal number of transmit intervals $p_1 L$ is that which yields the maximum SNR for a given sequence $G$ of length $L$. To obtain the SNR of a sequence with receive intervals, the total received energy, the noise from the sequence and the added noise at the receiver need to be found as follows.

Let $Y$ be the noise added at the receiver. To estimate the sample variance after the cross-correlation of $Y$ with the transmitted sequence $Z$, $\sigma^2_{YG}$, the steps from equations (5.6) to (5.10) can be repeated. When $M$ is large, $\sigma^2_{YG}$ can be approximated as

$$\sigma^2_{YG} \approx \frac{1}{M} \sum_{k=1}^{M} e_k^2 \approx E[e_k^2] \quad M \ll L. \quad (5.19)$$

where $e_k$ are the coefficients of the cross-correlation for each shift $k$. Note that the factor 2 has been dropped with respect to equation (5.6) because $M \ll L$ shifts.
are used to obtain $\sigma_{YG}^2$ and therefore the tapering effect of the cross-correlation can be neglected.

$$E \left[ e_k^2 \right] = \sum_{j=0}^{L-k-1} E \left[ z_j^2 \overline{g}_{j+k} y_{j+k}^2 \right], \quad k \in [1, M], \quad (5.20)$$

where $z_j = x_j g_j$ are the elements of $Z$ and $\overline{g}_j y_j$ are the elements of the received sequence; note that $\overline{g}_j^2 = \overline{g}_j$. Finally

$$\sigma_{YG}^2 \approx r \sigma^2 \quad M \ll L. \quad (5.21)$$

Now the sample variance of the noise introduced by the sequence itself is investigated following the same steps. Say $M$ is large, then

$$\sigma_{SG}^2 \approx \frac{1}{M} \sum_{k=1}^{M} f_k^2 \approx E \left[ f_k^2 \right] \quad M \ll L, \quad (5.22)$$

where $f_k$ are the coefficients of the cross-correlation for each shift $k$ and

$$E \left[ f_k^2 \right] = s^2 \sum_{j=m}^{L-k-1} E \left[ z_j^2 \overline{g}_{j+k} z_{j-m+k}^2 \right], \quad m \in [1, M], \quad (5.23)$$

where $z_{j-m} = g_{j-m} x_{j-m} s$ are the elements of the reflected sequence (i.e., the transmitted sequence $z_j$ scaled by $s$ and shifted by $m$) while the actual received sequence is $\overline{g}_j z_{j-m} s$. Note that

$$\sigma_{SG}^2 \approx p_1 r s^2 \quad M \ll L. \quad (5.24)$$

According to equation (5.18), $r$ reflections are received and since each reflection has magnitude $s$, the total received energy at shift $k = m$ is approximately
(r \cdot s)^2$ when $L \gg M$. Hence, when $M$ is large and $L \gg M$,

$$\text{SNR}_{\text{gaps}} \approx \frac{(r \cdot s)^2}{\sigma_{SG}^2 + \sigma_{YG}^2} \approx \frac{r}{p_1 + \frac{1}{\text{SNR}_{\text{in}}}}. \quad (5.25)$$

Figure 5.5 shows $\text{SNR}_{\text{gaps}}$ vs. $p_1$ for different $\text{SNR}_{\text{in}}$; $L$ has been set to $10^4$ to provide a numerical example. There are two extreme cases of interest

$$\text{SNR}_{\text{gaps}} \approx \begin{cases} r \cdot \text{SNR}_{\text{in}} & \text{if } \text{SNR}_{\text{in}} \ll \frac{1}{p_1} \\ (1 - p_1) L & \text{if } \text{SNR}_{\text{in}} \gg \frac{1}{p_1} \end{cases} \quad (5.26)$$

Given that $\max \{r\} = 0.25L$, which occurs for $p_1 = 0.5$, then $\max \{\text{SNR}_{\text{gaps}}\}\big|_{\text{SNR}_{\text{in}} \leq 2} = 0.25L \cdot \text{SNR}_{\text{in}}$. Conversely, if $\text{SNR}_{\text{in}} \gg \frac{1}{p_1}$, $\text{SNR}_{\text{gaps}}$ is independent of $\text{SNR}_{\text{in}}$, and if $\text{SNR}_{\text{in}}$ is high, it may happen that $\text{SNR}_{\text{in}} > \text{SNR}_{\text{gaps}}$, in which case the use of the sequences is detrimental.

Moreover, $\text{SNR}_{\text{gaps}}$ is a concave function of $p_1$ for any $\text{SNR}_{\text{in}} < \infty$. Then,
there exists a value of $p_1$ that maximises $SNR_{\text{gaps}}$ for each $SNR_{\text{in}}$

$$p_{1,\text{max}} = \arg \max_{p_1} \{SNR_{\text{gaps}}\} \leq 0.5 \quad (5.27)$$

and hence the maximum $SNR_{\text{gaps}}$ is

$$SNR_{\text{gaps,max}} \approx \frac{(1 - p_{1,\text{max}}) L}{1 + \frac{1}{p_{1,\text{max}}SNR_{\text{in}}}}. \quad (5.28)$$

An important fact is that $p_1 = 0.5$ performs nearly optimal once $SNR_{\text{in}} < 0 \text{dB}$.

The figure of merit of the sequence $F$ was not included in equation (5.25) because this equation is intended to be used with random sequences that do not have any predefined structure and for which $F \approx 1$ when $L$ is large. This is because I conjecture that it should be difficult to obtain a sequence that produces $F > 1$ when random receive intervals are used due to the structure of the transmitted sequence being affected by these intervals.

Finally, it is worth mentioning that sequences whose elements take on values $+1, -1$ and $0$ have been reported in the literature and are known as ternary sequences [143–145]. However, the insertion of the zeroes aims to improve the sequence autocorrelation properties and is not initially intended for reception to take place. Furthermore, these sequences do not necessarily satisfy $p_1 = 0.5$, which is required when $SNR_{\text{in}} \ll 2$.

5.4.4 Comparison between sequences with receive intervals and averaging

Once we assume that the sequence burst is optimal (i.e., their length can be the maximum possible without overlapping the closest reflector and they can utilise all the available bandwidth) and that the sequence burst can also be a chirp signal, averaging using the same sequence burst is the only relevant comparison point against the sequences with receive intervals. In this section I investigate
which of these techniques yields the highest SNR increase in a given amount of
time subject to initial conditions. To do so, I introduce the following ratio

\[ \alpha = \frac{\text{SNR}_{\text{gaps,max}}}{\text{SNR}_{\text{avg}}} \approx \frac{1 - p_{1,\text{max}}}{t \left( \text{SNR}_{\text{in}} + \frac{1}{p_{1,\text{max}}} \right)}, \quad (5.29) \]

\[ t = \frac{N}{L}, \quad (5.30) \]

where \( N \) is simply the number of averages – or to establish a comparison point
against the sequences, the number of transmit intervals when averaging – and \( L \)
is the total number of transmit and receive intervals for both the sequences and
averaging. Note that the number of transmit intervals in a sequence of length \( L \)
is \( p_1 L \), or \( p_{1,\text{max}} L \) in the case of equation (5.29). As before, transmit and receive
intervals are considered of equal length without loss of generality.

Figure 5.6 shows the values of \( t \) and \( \text{SNR}_{\text{in}} \) for which \( \alpha \approx 1 \). For any combination of \( t \) and \( \text{SNR}_{\text{in}} \) values below the curve, \( \alpha > 1 \), and hence the sequences with
receive intervals produce a greater SNR; otherwise, averaging produces a higher
SNR. A desired \( t \) for a given \( \text{SNR}_{\text{in}} \) may not be achieved when averaging due to
many receive intervals being required to avoid interference between transmissions,
e.g., when wave reverberations inside the specimen are significant.

Consider the extreme case in equation (5.29) where \( \text{SNR}_{\text{in}} \ll 2 \) and for which
\( p_1 = 0.5 \) is known to be optimal, then

\[ \alpha \bigg|_{\text{SNR}_{\text{in}} \ll 2, p_1 = 0.5} \approx \frac{1}{4t}. \quad (5.31) \]

This means that when \( \text{SNR}_{\text{in}} \ll 2 \) and \( t < \frac{1}{4} \), i.e., when the wait time between
averages necessary to avoid interference is greater than 4 intervals (1 transmit and
3 receive intervals), the sequence with receive intervals produces a greater SNR.
Finding scenarios where there is no interference using 3 or less receive intervals
when averaging is rare in practice. A common scenario in pulse-echo ultrasound systems is to use more than 40 receive intervals when averaging – the receiver is on for 40 times the transmit length to avoid interference. In such a case the SNR achieved by the sequence can be at least 20 dB greater provided $\text{SNR}_{\text{in}} \ll 2$ and $p_1 = 0.5$.

### 5.4.5 Periodic sequences with receive intervals: continuous transmission

Let the sequence $\hat{Z}$ be infinite with period $L$ and elements

\[
\hat{z}_{j+qL} = z_j \quad j \in [0, L - 1],
\]  

(5.32)

where $q$ is an integer and the elements $z_j$ are defined in equation (5.13). In the same way $\hat{g}_j$ can be defined from $g_j$.

Say $\hat{Z}$ is transmitted and the received signal is cross-correlated with $\hat{Z}$ shifted
by \( n \). The expected value of the periodic cross-correlation of a finite number of samples \( L \) is then

\[
E \left[ \hat{f}_{k,n} \right] = s \sum_{j=0}^{L-1} E \left[ \hat{z}_{j-n} \hat{g}_{j-n+k} \hat{z}_{j-m+k} \right]
\]

\[
= \begin{cases} 
  r \cdot s & k = m - n + qL, \quad m, k \in [1, L-1] \\
  0 & \text{otherwise}
\end{cases}
\]

(5.33)

Since \( \hat{Z} \) and \( \hat{G} \) have period \( L \), for every \( n \) there exists a value of \( k \) in the interval \([1, L-1]\) for which \( E \left[ \hat{f}_{k,n} \right] = r \cdot s \). This means that the sequence \( \hat{Z} \) can be transmitted continuously and at any instant \( n \), reflections within a time-of-flight of \( m < L - 1 \) can be recovered after cross-correlating \( L \) received elements. Note the same \( \text{SNR}_{\text{gaps}} \) is obtained when replacing \( Z \) by \( \hat{Z} \).

By transmitting a sequence with finite period \( L \), a significant amount of memory and computing power is saved but the time-of-flight of the furthest reflection has to be less than \( L - 1 \) to prevent these reflections from being seen as coherent interference. This is equivalent to waiting for the energy in a specimen to die out between transmissions when using averaging. The importance of being able to transmit/receive continuously is that it significantly reduces any delays in the system when processing the sequences, which then reduces the time the system takes to respond to changes in the medium.

### 5.4.6 Burst modulation and multiple reflectors

Let \( s_i \) be the magnitude of the received echo from reflector \( i \), then equation (5.25) can be rewritten for a reflector \( i' \) as

\[
\text{SNR}_{\text{gaps}}' \approx \frac{L (1 - p_1)}{\sum_{i=1}^{R} s_i^2 + \frac{1}{p_i \text{SNR}_{\text{in}}}}
\]

(5.34)

where \( R \) is the total number of reflectors, the reflectors are assumed to be well resolved, and \( \text{SNR}_{\text{in}} \) is defined with respect to \( s_{i'} \). The existence of multiple
reflectors increases the sequence noise; however, this increase is not significant if there are just a few dominant reflectors, as happens to be the case in most practical scenarios.

To include the effect of the burst modulation, let us assume that: the sequences are up-sampled to match the burst length, as shown in Fig. 5.3; that the normalised modulated burst $B$ has variance $\sigma_B^2$ and mean zero; and that the received sequences are correlated with the up-sampled but unmodulated sequences $X' \cdot G'$ in the compression stage as shown in Fig. 5.3. This process can be understood as a shifted combination of the transmitted sequence weighted by the burst samples. Since only the peak of the burst is of interest in the numerator, this process has no effect on it. Note that only the transmitted sequences are modulated, which has no effect on the right hand term of the denominator either. Hence, the result of the modulation on transmission is simply the left hand term of the denominator multiplied by the variance $\sigma_B^2$

$$\text{SNR}_{\text{gaps}}'' \approx \frac{L (1 - p_1)}{\sigma_B^2 \sum_{i=1}^{R} s_i^2 + \frac{1}{\rho_1 \text{SNR}_m}}. \quad (5.35)$$

Typically $\sigma_B^2 \sim 0.2$ for normalised bursts, so modulation reduces the sequence noise at the expense of longer excitations.

It should be highlighted that discontinuities may occur between adjacent intervals. The magnitude of the discontinuities is given by a) the difference in the number of reflections of two contiguous transmit-receive pair lengths – for very long sequences this difference is negligible and can also be compensated on the post-processing stage because the number of reflections from every transmit-receive pair length is known a priori – and b) the phase change of the electronics noise between non-adjacent receive intervals due to the noise being band-limited. Overall, the effect of these discontinuities is negligible and can be further attenuated by filtering.

An additional SNR increase is possible by using matched filtering, see Fig. 5.3. The effect of matched filtering on the SNR can be found elsewhere, e.g. [89].
Basically, if the signal is assumed to have a white noise component, mainly due to the electronics noise, the increase in the SNR corresponds to the energy of the burst. However, this result cannot be immediately extrapolated to the noise introduced by the sequences because it mostly shares the same frequency band of the burst. I empirically found that an acceptable approximation of the resulting sequence noise (left hand term of the denominator) is as follows

\[
\text{SNR}''_{\text{gaps}} \approx \frac{L (1 - p_1) Q \sigma_B^2}{2 Q \sigma_B^2 \sigma_{BB}^2 \sum_{i=1}^{R} s_i^2 + \frac{1}{p_1 \text{SNR}_\text{in}}},
\]

where \( Q \) is the number of samples of the burst and \( \sigma_{BB}^2 \) is the variance of the normalised auto-correlation of the burst. It is interesting to note that when the left hand term of the denominator is negligible, which corresponds to a regime where the sequences perform optimally, the SNR increase due to matched filtering is \( Q \sigma_B^2 \), i.e., the energy of the burst.

### 5.5 Application example: fast low-power EMAT

In this section a sequence with receive intervals was applied to an industrial ultrasound example, which consists of an electromagnetic-acoustic transducer (EMAT) driven with only 4.5 Vpp (peak-to-peak) and a maximum current of 150 mA (less than 0.5 W). The main advantage of using EMATs is that, unlike standard piezoelectric transducers, they do not require direct contact with the specimen. However, EMATs are notorious for requiring very high excitation voltages, commonly in excess of a few hundred volts and powers greater than 1 kW [6,19,66,146,147]. In certain scenarios high powers are not permissible, e.g. in explosive environments, such as refineries, or where compact/miniaturised electronics is wanted. Moreover, high-power electronics requires bigger components and more space to dissipate the heat. The use of sequences with receive intervals presented in this chapter can be used to reduce the excitation power while keeping the overall duration of the measurement short in these scenarios.
5.5.1 Experimental Setup

The experimental setup is shown in Fig. 5.7a-b. An EMAT (Part No. 274A0272, Innerspec, USA) was placed on top of a mild steel block, which had a thickness of 20 mm. This is a pancake coil EMAT that generates radially polarised shear waves within a circular aperture, which has an outer diameter of roughly 20 mm.

The main objective of this setup is to obtain a signal that can be used to estimate the thickness of the steel block. In this particular case it is convenient to use the coded sequences with receive intervals because a) the back-wall can be close and therefore long chirp signals or standard sequences cannot be used and b) the steel block offers low attenuation to the wave, which reverberates inside the specimen for a long time, making averaging lengthy due to the wait time needed between transmissions.

A custom-made transmit-receive electronic circuit was developed for the experiment. This circuit was solely powered by the USB port of a standard personal computer (PC), which can deliver a maximum of 5 V and 5 W. The electronics consists of a balanced transmitter with a maximum output voltage of 4.5 Vpp and maximum output current of 150 mA, hence the maximum peak power is less than 0.34 W. The receiver provided a gain of roughly 60 dB and both transmitter and receiver have a bandwidth greater than 5 MHz.

A device (Handyscope-HS5, TiePie, Netherlands) that consists of a signal generator and an analog-to-digital converter (ADC) was employed to drive the custom-made transmitter (driver) and to digitise the output of the custom-made receive amplifier. The Handyscope-HS5 communicates with a PC via the USB port. Both the signal generator and the ADC of the Handyscope-HS5 were sampled at 100 MHz.

In a second setup, the EMAT was connected to the transmit-receive system PowerBox H (Innerspec, USA) – provided by the manufacturer of the EMAT – without changing the EMAT position on the steel block. This setup is not shown for the sake of brevity. The PowerBox H was set to drive the EMAT at 1200 Vpp, which according to the manufacturer can produce a peak power of 8000 W. A 3-cycle pulsed burst at a central frequency of 2.5 MHz was transmitted. The
number of averages in the system was set to zero and the repetition rate to 30 bursts per second to avoid any interference from subsequent excitations. The receive amplifier gain was set to 60 dB.

### 5.5.2 Results

The signals obtained using the PowerBox H (Innerspec, USA) were matched-filtered with a 3-cycle Hanning window centred at 2.5 MHz to produce a fairer comparison with the cross-correlation output of the sequences; the output of the filter is shown in Fig. 5.8a. Multiple echoes that correspond to the back- and front-walls of the steel block can be observed to decay progressively. Smaller echoes produced by mode conversion (from shear to longitudinal waves and vice versa)
can also be observed between the main echoes. These mode-coverted echoes act as coherent noise, which is dominant over the electrical random noise of the electronics. Therefore, there is not much gain in increasing any further the transmitted power, the length of the sequence or the number of averages in order to increase the SNR because the coherent noise will increase proportionally.

To drive the custom-made electronics shown in Fig. 5.7b) a sequence of length $2^{14} = 16384$ was generated with equal number of receive and transmit intervals randomly distributed as described in Fig. 5.3. The sequence burst consisted of a 3-cycle Hanning window centred at 2.5 MHz similarly to the excitation used for the PowerBox H. The length of the sequence intervals was set to 6 times the burst duration totalling $7.2 \mu s$; this produces a blind zone of roughly 10 mm within a steel specimen when using a shear-wave transducer. This was necessary to permit the energy in the transducer to die out, so that the receive electronics does not saturate. Overall, the total duration of the sequence was 118 ms; a system that processes the data at this rate can be considered quasi-real-time for inspections that use hand-held transducers.

The received signals were zeroed at the transmission intervals to eliminate any noise introduced during this stage and then correlated with the transmitted sequence, see Fig. 5.3; the results are shown in Fig. 5.8b. The first echoes can be clearly identified from the noise threshold. In general, the noise level of Fig. 5.8b appears to be, by visual inspection, just slightly greater than that of Fig. 5.8a. This noise could either be noise introduced by the sequence or random electrical noise – the latter mainly due to the receive amplifier. Nonetheless, it is clear that a similar performance can be achieved even with a drastic reduction in the excitation power.

To investigate the performance of the sequence in more detail, an EMAT that produces shear waves linearly polarised [P1] was used instead. This EMAT achieves a higher mode purity and a more collimate radiation pattern, so that a thicker sample can be used where the echoes are well separated and mode conversion can be considered negligible. The new specimen is an aluminium block with dimension $80 \times 80 \times 150 \text{mm}^3$, and the transducer was placed in the
5. Pulse-echo coded excitation: single channel

Figure 5.8: Echoes from 20 mm-thick steel block. a) Signal from the Innerspec system using 1200 Vpp excitation. b) Signal from custom-made electronics using a sequence with receive intervals. Signals are normalised to the maximum value of the first echo.
middle of one of the 80 × 150 mm² faces.

All the parameters remained the same except for the sampling frequency, which was reduced to 20 MHz, so that the acquisition system can handle longer sequences. First, \(2^{17} = 131072\) signals were averaged; the result is shown in Fig. 5.9a, where the inset shows the transmitted burst. The objective was to find a region in the signal where the dominant source of noise corresponds to the receive electronics. Ring down from the coil, noise from the T/R switches and mode conversion can be observed before the first echo, but after the first echo the noise from the receive electronics dominates. Hence, a good approximation can be obtained by computing the variance of the noise between the first two echoes, as indicated by the horizontal brace.

Figure 5.9b shows the signals after \(N = 2^{14}\) averages and the result being cross-correlated with the excitation burst. In this input SNR regime a signal with the same output SNR can be obtained by using a sequence that has a length \(L = 2^{16} = 65536\) and intervals 6 times longer than the transmitted burst, as shown in Fig. 5.9c.

The SNR after averaging \(N = \{2^{10}, 2^{12}, 2^{14}\}\) signals and convolving the result with the excited burst is shown in Fig. 5.10 with circle markers. The SNR produced by sequences of length \(L = \{2^{12}, 2^{13}, \ldots, 2^{17}\}\) with intervals 6 times longer than the transmitted burst is shown with asterisk markers.

To evaluate equation (5.36), so that experimental and theoretical results can be compared, the input SNR, \(\text{SNR}_{\text{in}}\), was estimated from the SNR of the resulting signal after \(N = 2^{14}\) averages by dividing it by the number of averages and the variance of the burst \(\sigma_B^2 = 0.26\) and the number of samples of the burst \(Q = 25\); this yielded \(\text{SNR}_{\text{in}} = -19.5\) dB. Also, the variance of the normalised auto-correlation of the burst was \(\sigma_{BB}^2 = 0.16\) and the sum of the peak value of the first four echoes normalised to the first one was \(\sum_{i=1}^{4} s_i^2 \approx 1.3\). With all this information at hand, the output of equation (5.36) was plotted in Fig. 5.10 for different \(L\) values as shown by the continuous line. Overall there is good agreement between the SNRs obtained using averaging, the sequences (with intervals 6 times longer than the burst duration) and equation (5.36).
Figure 5.9: Signals using linearly polarised shear wave EMAT [P1]. a) $2^{17}$ averages; the inset shows the transmitted burst. b) $2^{14}$ averages. c) Sequence of length $2^{16}$. 

(a) $2^{17}$ averages.

(b) $2^{14}$ averages.

(c) Sequence of length $2^{16}$. 
5. Pulse-echo coded excitation: single channel

Figure 5.10: Experimentally observed SNR vs. number of averages $N$ or equivalent sequence length $L/4$. The input SNR is $\text{SNR}_{\text{in}} = -19.5$ dB.

It should be noted that under this low input SNR regime ($\text{SNR}_{\text{in}} = -19.5$ dB), the right hand term of denominator of equation (5.36) is predominant over the left hand term, therefore the noise introduced by the sequence itself is insignificant. Moreover, in this case the increase in the sequence noise due to the multiple reflectors is less than 30%.

### 5.5.3 Discussion of results

The main conclusion from the experiments is that a significant power reduction in the excitation can be obtained by using coded sequences with receive intervals while still being able to obtain a quasi-real-time response. Note that had averaging been used with the custom-made electronics, the wait time between transmissions would have needed to be around 1 ms and the number of averages needed $2^{12} = 4096$, which corresponds to a total duration of around 4 s (40 times longer than the length of the coded sequences).

The power delivered by the PowerBox H was expected to be in the order of 8000 W. A similar signal was obtained by the custom-made electronics driven with the proposed sequences using a mere 0.34 W, which corresponds to a difference of more than 40 dB. The exact power reduction achieved by the custom-made electronics when using the sequences (compared to the PowerBox H) should be
interpreted with care because the noise performance of the receive electronics of both systems has a direct impact on the SNR of the received signal; note the noise performance of the PowerBox H and the custom-made electronics were not compared.

The importance of fast and quiet switching electronics and also of active damping in procuring a short transmission interval should be highlighted. A drawback of the custom-made electronics employed was that the transmission interval was excessively large (6 times the duration of the excitation burst). This was necessary to attenuate any remaining energy in the EMAT coil after the excitation and to prevent the receive amplifier from entering into saturation during reception. Here, both transmit and receive intervals were set to the same length to simplify the analysis. However, when large transmission intervals are necessary to wait for any ring down or switching noise to die out, it should be considered to only increase the length of those transmit intervals that are followed by a receive interval. The use of active damping, whereby the transducer terminals are clamped or grounded for some time after the excitation to quickly attenuate the excess ringing in the transducer should be investigated.

It is also worth mentioning that when the received signals lie close to or below the noise threshold, as in [3, 27, 28, 30, 91–93, 119], [P1] and [P3], analog-to-digital converters (ADCs) can be replaced by comparators with negligible (2 dB) loss of information [41, 42, 104] and [P3]. This may result in a faster, more compact and efficient electronics.

5.6 Summary

Pulse-compression has been used for decades to increase the SNR without significantly increasing the overall duration of the measurement but current pulse-compression techniques cannot be used in pulse-echo when a significant SNR increase is needed and there are close reflectors. Here, I present a solution to that problem, which consists in inserting receive intervals in a coded sequence.

When the input SNR is low (< 10 dB) or there are far reflectors present, the
sequences with receive intervals are much faster than averaging or can produce an extra SNR increase for the same overall measurement duration. In general, the sequences can outperform averaging by more than 20 dB in many cases.

I also show that under low input SNR a simple random codification of the sequence using equal number of receive and transmit intervals of equal length randomly distributed performs optimally. Moreover, a sequence of any given length can be continuously transmitted without pauses, which increases the refresh rate of the system.

An application of these sequences in industrial ultrasound was presented. It was shown that an EMAT can be driven with 4.5 Vpp obtaining a clear signal in quasi-real-time; commercially available systems require 1200 Vpp for similar performance.

Future work should be related to the development of fast-switching electronics and the use of the proposed sequences with receive intervals in parallel channels as in medical/industrial ultrasound arrays, where their pseudo-orthogonality can be exploited.
Chapter 6

Pulse-echo coded excitation: arrays

6.1 Introduction

The use of phased arrays is ubiquitous in the fields of radar, sonar, medical and industrial ultrasound. There has always been a quest for arrays with a greater number of elements so that more information can be gathered about the medium, which usually translates into higher contrast and lateral resolution [37, 148]. Currently, in medical ultrasound the number of array elements has soared from a few hundred to thousands, e.g., as in the X6-1 xMATRIX array transducer (Philips Medical Systems, Andover, MA, USA) and the system reported by Gennison et al. [149], the main aim being to use dense 2D probes to increase the quality of 3D ultrasound imaging [56-60].

This trend gains more attention as more sensitive and denser probes are developed [46-55], mainly due to recent advances in micro-machined ultrasonic transducer (CMUT) technology. Meanwhile, progress is also being made on the instrumentation towards the full control of every array element [150] and higher probe-electronics integration [58, 151-157]. For example, in [158] an array that has a density of 1061 elements per mm² and operates at a central frequency of 18.6 MHz was fabricated using piezoelectric micro-machined ultrasonic transducers (PMUT), in [159] a 64-element array was integrated into a biopsy needle, and in [154] a 48-receive element system was integrated into an intravascular ultrasound (IVUS) probe.
Despite these achievements, the full capability of dense probes is yet to be exploited because a) they have smaller elements, which produce less intense waves/signals, and b) the number of independent transmit-receive element pairs that can be processed is limited by the overall duration of the measurement or the desired frame rate. Increasing the number of independent transmit-receive pairs that can be processed is important because it helps increase the overall resolution [37, 148]. Currently, the number of independent transmit-receive pairs processed is limited by the waiting time required between excitations to avoid interference [151].

Although all of the transmit-receive pairs in an array are rarely or never used simultaneously due to instrumentation and processing limitations, some alternatives that combined certain groups of elements have been proposed at a cost of less resolution and contrast. For example, row-column combination of transmit and receive elements [49, 156], where, say, the rows of the array act as transmitters whereas the columns act as receivers.

Plane-wave excitation is also widely used to increase resolution [149, 160–164], whereby plane waves are excited using a different inclination angle; the greater the inclination angle and the smaller the angle steps, the greater the resolution. Plane-wave excitation yields higher intensity signals compared with firing elements individually. Plane-wave excitation with very fast acquisition systems enable two recent key achievement in ultrasound technology, namely fast sub-wavelength resolution [165], though this requires the use of invasive contrast agents (see also [166]), and the study of viscoelastic properties of tissue through shear imaging [167].

In general, the number of plane waves with different angles that can be transmitted is limited by the waiting time required between transmissions and, therefore, this technique becomes less time-efficient for 3D applications, where the number of plane waves with different inclination angles increases due to the extra dimension [149, 168]. Moreover, plane waves generated by finite apertures are just approximations constrained to a limited region under the aperture and to a maximum number of inclination angles.
The use of sparse arrays is another technique that allows to achieve high resolution with moderate grating lobes using less array elements, whereby fewer elements are sparsely distributed with a mean distance between elements greater than that required to avoid grating lobes [169–171]. The array lateral resolution is given by the length of the array in the corresponding dimension, whereas the inter-element spacing is responsible for the undesired grating lobes. Although sparse arrays reduce high peak values in the grating lobes, the overall radiation outside the main beam increases [172], which results in higher interference on average from reflectors outside the focal zone.

In general, dense array probes have smaller elements, which radiate less power, and hence the SNR of the received signals has to be increased by, for example, using pulse-compression techniques, such as coded sequences with good autocorrelation properties; good autocorrelation properties are required to minimise the noise introduced by the sequences. Orthogonal sequences can speed up the number of transmit-receive pairs to be processed because there is no waiting time between independent element excitations. A tremendous effort has been placed in developing sequences with good autocorrelation and orthogonality properties in the last decades [121, 133, 173–176] with most of the effort being vested in the communication field, such as in code-division multiple-access (CDMA).

One central problem in the use of coded sequences is that a single set of coded sequences cannot achieve good autocorrelation and orthogonality properties simultaneously. However, it is essential to highlight the work of [121], wherein complementary sets of sequences were introduced, which achieve perfect autocorrelation and orthogonality. That work extended to orthogonal channels the complementary principle introduced in [120], which permits perfect autocorrelation to be achieved. An alternative to complementary sequences is the use of zero-autocorrelation-zone sequences [132–136]. These sequences only achieve perfect autocorrelation within a given interval, which suffices in many practical applications, and therefore many constraints in the synthesis of the sequences can be relaxed. These sequences are mainly reported for single channel applications, but in principle they could be used in orthogonal multi-channel systems.
Further discussions on the use of coded sequences in ultrasound can be found in [118,177-179].

Regardless of all the progress made in the synthesis of coded sequences, there is still a main limitation in their use: the sequences have to be long in order to achieve a sufficient degree of orthogonality and SNR increase, but their length is limited by the location of the closest reflectors [178,179], as discussed in the previous chapter.

In this chapter, I extend my previous work on single-channel coded sequences discussed in Chapter 5, [P4] and [P5] by proposing a set of random sequences for multi-channel applications which have receive intervals. As a result, the information of every transmit-receive pair can be acquired simultaneously while the overall length of the sequences is no longer limited by the location of the closest reflector. This is an important result because there exist many applications where the receive SNR is intrinsically low and there are reflectors close to the transmit-receive elements [3,27,28,30,91-93,119]. Moreover, these sequences permit dense array transducers to be designed such that their elements produce low-SNR signals, thereby relaxing many design and construction constraints while enabling higher low-power electronics integration.

An advantageous situation when using low-SNR arrays is that a significant simplification of the electronics and data throughput reduction are possible. This is because when the received signals lie close to or below the noise threshold, analog-to-digital converters (ADCs) can be replaced by comparators with a negligible (2 dB) decrease in SNR [41,42,104]. In Chapter 4 and [P3], I investigated the maximum possible dynamic range of binary quantisation under different conditions and found that binary quantisation can be used when the input SNR is lower than 8 dB. Comparators also operate at higher sampling rates, which is particularly useful for high frequency arrays [48,152]. Furthermore, binary excitation combined with pulse-width modulation techniques have been shown to control the shape of the transmitted waves with similar performance to that of digital-to-analog converters (DACs) [180]. Hence, a simplified pulse-echo array acquisition system, whereby each array element is controlled by a one-bit digital...
Figure 6.1: Proposed system architecture to reduce data throughput. Each array element is fully controlled by a digital line. ADCs can be replaced by comparators because the received signal is close to or below the noise threshold. LNA stands for low-noise amplifier.

line, becomes possible. All this results in a faster, more compact, and higher spatial resolution system but with an inherently lower dynamic range. A diagram of such a system is shown in Fig. 6.1.

The organisation of this chapter is as follows. First, there is a background section to recall synthetic image focusing. Then, a multi-channel random sequence set with receive gaps is introduced, along with the formulae for the resulting focused image SNR. After that, simulations are carried out and the effect of the sequence set size on the image SNR is discussed for different scenarios. Finally, the conclusions are given.
Figure 6.2: Full matrix capture. Each array element acts as a transmitter at a time and the wave path between each transmit and receive elements is recorded.

6.2 Background: Synthetic focusing using every transmit-receive path

The ability to focus the ultrasonic waves on transmission and reception significantly increases resolution at a cost of extra post-processing. Full synthetic transmit-receive focusing is usually known to the non-destructive evaluation (NDE) community as the total focusing method (TFM) [181]. To implement TFM, each transmit-receive path is acquired; this is also known in NDE as full matrix capture (FMC). This process is shown in Fig. 6.2, where, for example, \( h_{21} \) is the path from transmit element 1 to receive element 2.

In the two-dimensional case, the goal is to form an image \( I_{xy} \), where \( x \) and \( y \) are the coordinates of the image. To accomplish this, each path is delayed following a focal law so that they add up coherently at the coordinate of interest. Hence,

\[
I_{xy} = \sum_a \sum_b h'_{ab} (t) \cdot d_{abxy} (t) ,
\]

(6.1)

\[
d_{abxy} (t) = \gamma_{ab} \delta \left[ t - \sqrt{\frac{(x_a - x)^2 + y^2}{c} + \frac{(x_b - x)^2 + y^2}{c}} \right]
\]

(6.2)

where \( h'_{ab} \) is the recorded path, \( a \) and \( b \) are the indexes of the receive and transmit elements, \( \gamma_{ab} \) is a constant, which can be used, for example, to create apodiza-
tion effects, \( c \) is the speed of the wave, and \( \delta \) is a delta function. Equation (6.1) can readily be extended to the three-dimensional case where a volume is obtained instead of an image. Also, the Fourier transform can be employed to significantly speed up computations [181–183]. Equation (6.1) is the simplest version of TFM; further processing techniques have also been investigated with the aim of increasing the quality of the focused image [184, 185].

6.3 Random sequence set with receive gaps

In this section I introduce a set of random sequences that can be excited simultaneously through each of the array elements. The received sequences are cross-correlated with the transmitted ones and then the path that corresponds to each transmit-receive pair is recovered. This results in a faster way of producing full synthetic focusing.

6.3.1 System overview

Figure 6.3 shows an overview of the proposed system. On the left hand side of the figure \( M \) transmitters are excited simultaneously, each with a different random binary sequence \( (s_1, s_2, \ldots s_M) \), where each sequence has the same amplitude and number of receive intervals and is uncorrelated with the rest. These sequences take on values 1, \(-1\) or 0, and can be obtained by multiplying a random sequence common to every channel, which controls the occurrence of transmit and receive intervals and take on values 1 (transmit) and 0 (receive), by another sequence that taken on values 1 and \(-1\), which controls the polarity of the bursts of each channel independently. The grey bands in Fig. 6.3 correspond to the transmit intervals. To simplify the analysis, the bursts are initially considered to be delta functions, later on this is extended to any type of burst.

The transmitted sequences generate \( M^2 \) wave paths \( (h_{11}, h_{12}, \ldots , h_{MM}) \). These paths carry information about the medium, and are simply the sequences delayed and scaled by certain amounts (assuming the medium behaves as a linear system). There is a noise source associated to each receiver input \( (n_1, n_2, \ldots n_M) \) due to
Figure 6.3: Synthesizing focusing using a random set of sequences and receive intervals.
6. Pulse-echo coded excitation: arrays

**Algorithm 6.1** Sequence synthesis and image focusing.

1. Input sequence length, $L$, number of elements, $M$, and burst, $b$, to be transmitted of length $Q$.

2. Synthesise the sequences
   
   (a) Create binary random matrix of size $L \times M$ that takes on values 1 and −1.
   
   (b) Create binary random vector, $v$, of length $L$ that takes on values 0 and 1.
   
   (c) Multiply each element of the $M$ rows of the random matrix by the corresponding elements of $v$.
   
   (d) Up-sample the $M$ rows of the resulting random matrix by $Q$ samples.
   
   (e) Convolve each of the $M$ rows with burst $b$.

3. Transmit each of the $M$ rows of the matrix through a different element.

4. Record the received matrix using the complement of vector $v$ (up-sampled by $Q$) to activate the receiver when zeroes are being transmitted.

5. Cross-correlate each of the $M$ rows of the received matrix with each of the rows of the transmitted matrix to recover the $M^2$ paths.

6. Compute the focused image by summing every path, delayed according to equation (6.2), for each pixel in the image.

the receive electronics and/or the transducer. All of these sources are considered to be uncorrelated to one another.

The $M^2$ paths can be recovered by cross-correlating the received signals with the sequences initially transmitted; the operator {∗} stands for cross-correlation. The recovered wave paths ($h'_{11}, h'_{12}, \ldots, h'_{MM}$) are contaminated with noise due to the self-interference of the sequences and the receive electronics. Finally, once the sequence paths are recovered, they are delayed according to equation (6.1) so that the focused image can be obtained. The overall process is summarised in Algorithm 6.1.
6.3.2 SNR increase

Based on equation (5.28), the expected SNR of each recovered path is

$$\text{SNR}_h \approx \frac{(1 - p_1) L}{M + \frac{1}{p_1 \text{SNR}_{in}}}$$

(6.3)

where \(\text{SNR}_{in}\) is the ratio of the power between one isolated received sequence and the receiver noise source; for simplicity it is assumed that all of the received sequences have the same amplitude. The sequence noise, given by the left hand term of the denominator, increases from 1 to \(M\) with respect to equation (5.28) due to the interference of the extra \(M - 1\) sequences that are combined at the receiver input.

Once each wave path is recovered, they are delayed according to equation (6.1) to obtain the focused image. We now assume that no wave path is the same and hence they are uncorrelated, except at the focal point; for this step note that cross-correlation is not commutative when the inputs have no symmetry, i.e. \(s_a \ast s_{a'} \neq s_{a'} \ast s_a\), as in our case. Finally, the SNR of the image for a single point-like reflector can be defined as the ratio of the energy at the focal point, where each received sequence adds up coherently, and the variance of a region in the image that does not include the focal point or any focusing artefacts. Since \(M^2\) paths are combined, the resulting SNR is equivalent to the numerator of equation (6.3) times \(M^4\) and the denominator times \(M^2\), which yields

$$\text{SNR}_I \leq \frac{(1 - p_1) L \cdot M}{1 + \frac{1}{p_1 \text{SNR}_{in} M^2}}.$$ \hspace{1cm} (6.4)

The inequality symbol indicates that this is an upper bound for \(\text{SNR}_I\). In practice this upper bound is difficult to reach due to coherent interference caused by the focusing algorithm and unequal intensity of the wave paths. In more detail, a) some adjacent elements may have the same sequence path, mainly because time and spatial dimensions are discretized, therefore violating the independence
hypothesis; b) elements further from the reflector receive less intense signals; c) elements and reflectors are non-isotropic and therefore the received/transmitted wave intensity varies with the orientation of the reflectors relative to the element; d) paths from different reflectors may interfere with each other; and e) direct wave incidence from transmitting elements that propagates through the array probe or along the surface of the specimen can be seen as coherent interference (noise).

Equation (6.4) leads to an interesting case

\[
\text{SNR}_1 \leq 0.25L \cdot M^2 \text{SNR}_{\text{in}}, \quad \text{SNR}_{\text{in}} \ll \frac{2}{M}. \quad (6.5)
\]

In this regime (\(\text{SNR}_{\text{in}} \ll 2/M\)) the noise introduced by the electronics is greater than that of the sequences. Hence, the random sequences are said to perform optimally, with \(p_1 = 0.5\).

Another interesting observation is that, provided that \(L\) is sufficiently large, it is possible to have \(L < M\) without critically increasing the interference between the sequences. It should be noted that a sequence set that achieves perfect orthogonality will require \(L \geq M\).

### 6.3.3 Comparison with systems that do not use coded excitation

When using the proposed sequence set to obtain every transmit-receive path, the overall duration of the process is given by \(L\). Now, we repeat the analysis to compare the expected SNR when averaging single excitations for the same overall duration \(L\), which gives

\[
\text{SNR}_{\text{ave}} = N \cdot M^2 \text{SNR}_{\text{in}}, \quad (6.6)
\]

where \(N\) is the number of excitations (transmit intervals) per channel that are to be averaged.
Let the ratio between the number of transmit intervals and the total number of intervals be

\[ t = \frac{N \cdot M}{L}. \]  

(6.7)

Note that when averaging, at least \( M \) transmit intervals and \( M \) receive intervals are required. This implies \( L \geq 2M \) and hence \( t \leq 0.5 \). Finally, the ratio between the SNR using coded excitation and averaging is

\[ \alpha = \frac{\text{SNR}_{I,\text{max}}}{\text{SNR}_{\text{ave}}} = \frac{(1 - p_{1,\text{max}})}{t \left( \text{SNR}_{\text{in}} + \frac{1}{p_{1,\text{max}}M} \right)}, \]

(6.8)

where \( p_{1,\text{max}} \) is the value of \( p_1 \) that maximises \( \text{SNR}_I \). Of particular interest is the case

\[ \alpha \big|_{\text{SNR}_{\text{in}} \ll \frac{2}{M} p_{1,\text{max}} = 0.5} \approx \frac{M}{4t} \geq 1, \]

(6.9)

where, since \( t \leq 0.5 \) and \( M \geq 2 \), the proposed sequence set performs optimally irrespective of \( t \) or \( M \).

Figure 6.4 shows \( t \) vs. \( \text{SNR}_{\text{in}} \) subject to \( \alpha = 1 \) for \( M \in \{1, 10, 10^4\} \) using continuous curves. The dashed line corresponds to a typical maximum value of \( t = 0.025 \), equivalent to 39 receive intervals per transmit interval. For any combination of \( t \) and \( \text{SNR}_{\text{in}} \) that lies below the curves, \( \alpha \geq 1 \) and hence the proposed sequence set yields a higher SNR. Note that once \( M \geq 10 \) the behaviour of the curves does not differ much. As a rule of thumb, once \( \text{SNR}_{\text{in}} < 15 \text{ dB} \) the proposed sequence set can be said to perform optimally. To give an example, say \( M = 10^3 \), \( \text{SNR}_{\text{in}} = -10 \text{ dB} \) and \( t = 0.025 \), then \( \alpha \approx 196 \) and therefore the proposed sequence set yields an extra 23-dB-SNR increase for the same excitation duration in comparison to averaging.
6. Pulse-echo coded excitation: arrays

Figure 6.4: Ratio of transmit and total number of intervals, \( t = \frac{N \cdot M}{L} \), and input SNR, \( \text{SNR}_{\text{in}} \), for which the SNR obtained when using the proposed sequences set is the same as that obtained with averaging, i.e. \( \alpha = 1 \). For any combination of \( t \) and \( \text{SNR}_{\text{in}} \) values below the curve \( \alpha > 1 \). The dashed grey curve shows a typical maximum value of \( t \) for pulse-echo ultrasound system.

6.3.4 SNR adjusted for modulation and uneven wave paths

Equation (6.4) gives a quick estimation of the performance of the sequences. However, the effect of the sequence modulation and uneven wave paths has to be taken into account to obtain more accurate results. We should highlight that modulation can also be used to increase the SNR while increasing resolution by using chirp signals [113–115, 118].

The random sequence set, as shown in Fig. 6.3 with its expected SNR given in equation (6.4), is limited to delta bursts. Modulation can be readily incorporated by up-sampling the sequences by the relevant amount and convolving the result with a more complex burst. After some algebra (see Appendix A.6), it can be shown that

\[
\text{SNR}_{\text{mod}} \leq \frac{L (1 - p_1) \left( \sum_{i=1}^{M} \sum_{j=1}^{M} w_{ij} \right)^2}{\sigma_b^2 M \sum_{i=1}^{M} \sum_{j=1}^{M} w_{ij}^2 + \frac{M^2}{p_1 \text{SNR}_{\text{in}}}},
\]

(6.10)

where \( w_{ij} \) is the amplitude of the received echo from transmitter \( i \) and receiver \( j \), and \( \sigma_b^2 \) is the variance of the modulated burst, which is assumed to have mean
6. Pulse-echo coded excitation: arrays

zero. SNR$_{in}$ is the input SNR with respect to the strongest reflection. Equation (6.10) is an upper bound for SNR$_{Imod}$ due to the effect of coherent noise. Note that if $w_{ij} = 1$, $\forall i, j$, and $\sigma_{symb}^2 = 1$, equation (6.10) reduces to (6.4).

In the case of a point-like reflector, the coefficients $w_{ij}$ can be obtained by simple algebra taking into account the effect of beam spread and element directivity [181]. For example, in the two-dimensional case, the losses due to beam spread with respect to the $i^{th}$ array element are

$$\beta_i = \frac{y'}{\sqrt{(x_i - x')^2 + (y')^2}}$$

(6.11)

where $x'$ and $y'$ are the locations of the reflector with respect to the linear array. The role of $y'$ in the numerator is to normalise $\beta_i$ to the distance between the reflector and the linear array. In this particular case, it can be shown based on [186] that $\beta_i^2$ is an acceptable approximation to the directivity of point-like array elements, assuming that the main acceleration component of the array elements is normal to the array/specimen surface. Note that then, $w_{ij} = (\beta_i \beta_j)^3$.

It should be noted that due to the received signal becoming weaker as the array is extended in at least one direction for any given reflector location while the electric noise is the same for every channel, there exists a point beyond which adding more elements to the array will decrease the SNR of the image, though spatial resolution and contrast may keep increasing further.

6.4 Numerical simulations

In this section, the performance of the sequences is investigated by simulating a 16-element array using the Pressure Acoustic and Solid Mechanics (transient) modules of COMSOL MultiPhysics 5.2 (COMSOL Inc., Massachusetts, USA). Figure 6.5 shows the two-dimensional finite element model used. Each element consists of a line, which has a length equal to a quarter of the wavelength at 1 MHz, spaced by half a wavelength (from the centre of the element). The me-
Figure 6.5: Finite element model.

dium is simulated as water and a circle with a diameter equal to a quarter of a wavelength is placed 7.3 mm beneath the 10\textsuperscript{th} and 11\textsuperscript{th} elements. The material within the circle is simulated as cooper to create a significant acoustic impedance change with respect to water.

Absorbing layers were placed at the boundaries to attenuate any reflections by more than 40 dB. The absorbing layers were simulated in a similar way as described in [73]. 16 absorbing layers were employed, the most inner one had a width of a quarter of a wavelength and this was increased following a quadratic law so that the outer layer was half a wavelength wide. In each absorbing layer the dynamic and bulk viscosity were increased following a cubic law so that the value of the outer region was 200 Pa·s. Free-triangular elements were used for the mesh. By setting the maximum size of the elements to one sixth of the wavelength, the focused imaged was not affected by more than 1\% with respect to its maximum value. This value was selected by the authors as an acceptable compromise between accuracy and simulation time.

In an initial simulation each array element was excited individually by a 3-cycle burst that has a Hann apodization and a central frequency of 1 MHz using a sampling frequency of 16 MHz. The excitation was specified as the normal component of the acceleration along the line that defines the element. For each excited element the received signals corresponding to all of the elements were recorded, which resulted in 16\textsuperscript{2} signals. The received signals were obtained as the
added normal acceleration component (with respect to the line that defines the element) of the end points of the line which defines the elements.

Equation (6.1) was employed to produce a focused image using a regular spatial grid with a resolution of one sixteenth of the wavelength. Results are shown in Fig. 6.6(a). The circle that simulates the cooper wire can be easily identified. No reflections from the absorbing region or focusing artifacts are observed within this 40 dB dynamic range, except at the corresponding location of the array elements.

The simulations were repeated using the proposed sequence set. Three random sets of $M = 16$ sequences with length $L \in \{250, 10^3\}$ and $p_t = 0.5$ were generated. Each sequence in the set was up-sampled so that the transmitted burst has a $4 \mu s$ length. Then the sequences were convolved with a 3-cycle burst that has a Hann apodization and a central frequency of 1 MHz. This left a $1 \mu s$ guard gap to allow any ringing from the excitation to die out before the receive intervals start.

During the simulation, the acceleration of the 16 excitation regions was recorded. The resulting 16 signals were correlated with the original non-modulated (but up-sampled) ones; the modulation was omitted on reception to avoid the effect of matched filtering. By doing so, the $16^2$ possible paths were recovered. Figure 6.6(b-c) shows the focused image when using the sequences with lengths 250 and $10^3$ respectively. Note that the peak background noise due to the sequences is roughly $-20$ dB in (b) and $-30$ dB in (c).

To simulate the effect of electrical noise, each of the 16 received sequences in the set of length $10^3$ were combined with different normally distributed sequences. The variance of the normally distributed sequences was selected so that the input SNR is $\text{SNR}_{\text{in}} = 0$ dB with respect to the peak value of the reflection from the simulated defect to the closest transducer.

Results from the simulation are shown in Fig. 6.6(d). It can be appreciated that the addition of noise which has a variance equivalent to the signal peak value squared, i.e. $\text{SNR}_{\text{in}} = 0$ dB, has almost no effect on the quality of the image compared to Fig. 6.6(c). This is due to the combined averaging mechanisms of the sequences and the focusing algorithm.

To compare the results of the simulations with those predicted by equation
Figure 6.6: Focused images. a) Elements excited one at a time; no sequences used, SNR$_{in} = \infty$ dB. b) Sequence length 250, SNR$_{in} = \infty$ dB. c) Sequence length $10^3$, SNR$_{in} = \infty$ dB. d) Sequence length $10^3$, SNR$_{in} = 0$ dB. Colour scale is in dB.
(6.10), the SNR of the focused image was computed for the cases $\text{SNR}_{in} \in \{\infty, 0, -10, -20\} \text{ dB}$ and $L \in \{250, 500, 10^3\}$, where $p_1 = 0.5$. The image SNR, $\text{SNR}_{I, \text{mod}}$, was calculated as the inverse of the variance after normalising the image by the peak amplitude of the region corresponding to the defect. The variance was computed on the bottom half of the image where no reflections or focusing artefacts are expected.

When using equation (6.10), the variance of the burst was computed for the strongest received echo yielding $\sigma_b^2 = 0.2$. The sums of the weights $w_{ij}$ yield 0.55 for the numerator and 0.57 for the right hand term of the denominator of equation (6.10).

In Fig. 6.7, the circle markers correspond to the simulated results and the curves to the expected upper bound for $\text{SNR}_{I, \text{mod}}$, as given by equation (6.10), for $\text{SNR}_{in} \in \{\infty, 0, -10, -20\} \text{ dB}$ and $L \in [250, 10^3]$. In general, there is just a slight offset (less than 3 dB) between simulated and theoretical results, which is consistent in most cases. This slight difference was expected since equation (6.10) is just an upper bound for $\text{SNR}_{I, \text{mod}}$. Note that when $\text{SNR}_{in} = \infty \text{ dB}$ and $L = 250$ the difference is greater than 3 dB with respect to the theoretical case; this can safely be ignored because it is a result of the insufficient length of the sequence, which consequently does not behave completely randomly.
6.5 Discussion

In this chapter I have introduced a set of random sequences with receive intervals that behave optimally when the input SNR is low (typically below 15 dB). The examples presented only show a marginal benefit when using the proposed set because it is limited to just a few elements. However, the validity of the methodology is clear, which can readily be extrapolated to those cases where a significantly greater number of elements can be used as well as longer sequences.

For example, say an array with $M = 10^3$ elements is to be used at a centre frequency of 5 MHz. Such dense arrays have already been reported, e.g. the X6-1 xMATRIX (Philips Medical Systems, Andover, MA, USA) and the one reported in [149], which consist of 9212 and 1024 elements respectively. Let us assume that each burst is composed of three cycles and one guard gap that has a length equivalent to one cycle. Also, say that the overall acquisition duration should be 50 frames per second. In that case, it is possible to use a sequence as long as $L = 25000$ intervals, which yields an SNR of more than 70 dB according to equation (6.4). The resulting SNR could be further increased by using chirp modulation [113–115,118] at the expense of greater bandwidth and burst length.

Although higher SNRs may have been claimed elsewhere, the proposed sequences uniquely provide all of the wave paths within the imposed frame rate. This translates into higher lateral resolution and contrast throughout the inspected volume [37, 148]. Another advantage of the proposed sequences is that they can be transmitted continuously [P4], i.e. there is no sequence start or end. This is useful when implementing pipeline-like architectures to reduce post-processing latency.

If single excitation were to be used to retrieve all of the wave paths, the frame rate in the above example would be less than one frame per second without resorting to any time averages and assuming that more than 39 intervals per transmit interval are required. Using orthogonal sequences, such as in [121], would not significantly speed up the acquisition of all of the wave paths because the length of the sequences that can be excited are limited by the location of the
closest reflector. In general, this highlights the trade-offs between sequence length (frame rate), SNR and spatial resolution (number of wave paths processed).

It should also be noted that dense arrays produce signals with lower SNR because they consist of smaller elements. In those scenarios, the proposed sequences perform optimally if $\text{SNR}_{\text{in}} < 15 \, \text{dB}$ (see Fig. 6.4). Besides, low-SNR images can also be found in other cases, for example, in materials that have grain noise [187-190] or strong speckle in tissue [183, 191-194]. Given that this spatial noise is uncorrelated with the sequences and the electronics noise, the proposed sequences will perform optimally in these scenarios, typically when $\text{SNR}_{\text{i}} < 50 \, \text{dB}$. However, there are scenarios where the sequences may not be applicable due to their limited dynamic range. For example, when pressure-wave arrays are used on elastic materials, substantial surface (Rayleigh) waves are generated, which reach the receiver elements with greater intensity than the reflected echoes and therefore deteriorate the dynamic range of the resulting image.

A central drawback in the use of dense arrays is the very large data throughput and post-processing required. For example, sampling $10^3$ elements at 70 MHz requires a data throughput of roughly 70 Gbit/s, if binary quantisation is used, see Chapter 4. Therefore, a dedicated state-of-the-art architecture is required. In [150] a system that comprises 128 channels sampled at 70 MHz using 12-bit ADCs was reported; this system produces a data throughput of 108 Gbit/s. In [149] an array of 1024 elements was reported, whereby signals from 512 elements are acquired simultaneously. Since the reported centre frequency was 3 MHz, the data throughput should be greater than $50 - 100 \, \text{Gbit/s}$. In [195] a data throughput greater than $60 - 100 \, \text{Gbit/s}$ seems to have been achieved, as 128 elements are sampled at 60 MHz.

In general, it can be concluded that the use of binary quantisation combined with the sequences permits more transmit-receive pairs to be acquired, which translates into higher lateral resolution and contrast, at higher equivalent frame rates while using similar or less data throughput compared to reported technology. This is possible at the expense of increased post-processing and lower dynamic range (see Sec. 6.3.3 for a discussion on measurement duration, number of array
elements and SNR). When the medium is intrinsically noisy, due to, e.g., grain noise [187–190] or speckle in tissue [183,192,193], using more transmit-receive elements may be beneficial over more time averages or longer sequences used to attenuate the electronics noise due to the extra spatial averaging power and resolution.

The extensive post-processing required for synthetic beam-forming, especially when processing three-dimensional images, is a central limitation of dense arrays. Currently, only a few tens or fewer focal laws or plane waves with different inclinations are used on transmission, and this is already on the edge of the processing capabilities when attempting effective rates of $10^4$ frames per second on a few hundreds of array elements [149,195]. Conversely, when using the proposed random sequences, the number of wave paths to be synthetically processed soars by at least one order of magnitude. This implies a significant increase in resolution but also lower effective frame rates. Although the post-processing of the proposed sequences is highly parallelizable, it is so far the main expected bottleneck. Nevertheless, in [195] the authors discussed an expected increase in parallel processing capabilities in the near future using graphics processing units (GPUs); a recent example of the use of GPUs for beam-forming can be found in [196].

### 6.6 Summary

In this chapter I have introduced a set of random sequences which have receive gaps. The main advantages of the proposed sequences are that the overall length is no longer limited by the location of the closest reflectors and that every transmit-receive element pair can be processed simultaneously, therefore solving a central problem in coded excitation for dense arrays.

The proposed set is shown to be an optimal solution, at least one order of magnitude faster than the sequential excitation of individual elements when the received signals have low SNR (typically below 15 dB) or the resulting focused image does not require high signal-to-coherent-noise ratio (typically not much greater than 50 dB). The method is therefore particularly valuable with dense
arrays, where smaller elements produce weaker signals, and when the medium is intrinsically noisy due to, for example, grain noise in elastic materials or strong speckle in tissue.

Moreover, when binary quantisation is used along with the proposed sequences in low-SNR scenarios, the volume of data that must be transmitted is reduced by roughly one order of magnitude without affecting the SNR. Consequently, an acquisition architecture is proposed whereby each array element is fully controlled by a one-bit digital line. This permits more transmit-receive pairs to be acquired, which translates into higher spatial resolutions and image contrast, at higher frame rates while using similar data throughputs comparable to that of current technology but at the expense of additional post-processing.

The main applications areas of this work are related to dense high-frequency arrays for fast, high-resolution 3D ultrasound due to the possibility of handling simultaneously many low-SNR channels using less data throughput.
Chapter 7

Low-Power Phased EMAT Array: a Feasibility Study of Surface Crack Detection

7.1 Introduction

Electromagnetic-acoustic transducers (EMATs) basically consists of a magnet and coil and do not require mechanical contact with the specimen under test [1,4,75]. EMATs are widely used in non-destructive evaluation (NDE) to speed up the inspection process because ultrasonic couplant is not required and coatings thinner than a couple of millimetres do not need to be removed [17,74,197,198]. However, EMATs produce less intense signals in comparison to piezoelectric elements and therefore more attention to their design optimization is required [199–201] as well as the use of signal processing techniques, such as pulse-compression, to increase the signal-to-noise ratio (SNR) [3].

EMAT phased arrays would be advantageous in general over standard monolithic transducers because, as any ultrasonic phased array, they can generate different electrically-controlled ultrasonic fields and rapidly visualise the internal structure of the specimen through focused images [37]. But because EMATs have to commonly be excited using powers in excess of 1 kW [75], this has hindered the development of bulk-wave EMAT phased arrays, mainly because the array
elements have to be smaller than a wavelength at the central frequency in order for the array to perform satisfactorily [37, 148]. In metals, this is typically less than 5 mm in the frequency range from 0.5 to 2 MHz. These smaller elements are weaker radiators and cannot withstand driving powers in excess of 1kW.

Because of that, there is virtually no literature on bulk-wave EMAT phased arrays, albeit for an 8-element system reported in [202, 203], whose performance detecting defects is unclear. The majority of EMAT designs that have been reported so far where the ultrasonic beam can be electrically steered or deflected consist of a magnet on top of meander coil [10–13]. These designs are very convenient for their simplicity since only one active element or coil is required. However, the central beam of the ultrasonic radiation pattern is steered by changing the central frequency; this limits the resolution (i.e. the capacity of the system to resolve defects) and the steering capabilities.

In this Chapter, I make use of the recent development in EMAT technology reported previous chapters, namely a methodology for increasing the bias magnetic field of shear wave EMATs [P1] and a new type of coded sequence for pulse-echo mode operation [P4] and [P5], in order to reduce the excitation power to less than 4.8 W (24 Vpp and 200mA) so that racetrack coils with dimensions \(3.2 \times 18 \text{ mm}^2\) can be employed. The dimensions of these coils can be considered narrow for conventional, reported EMATs.

My main contributions is the study of the radiation pattern of racetrack coils as the array elements and the coil layout. Coils are laid out overlapping 1/3 of their area in their shortest dimension, which helps reduce the crosstalk between the coils, i.e., the array elements, to less than -15 dB and creates a periodic pattern where the separation between the array elements, i.e., the array pitch is 2.1 mm.

As a result, I present the first pulse-echo EMAT phased array which generates shear waves similarly to many conventional piezoelectric arrays which are mounted on a wedge to convert the wave mode from longitudinal to shear [36, 37], see Fig. 7.1. Such configurations can be used to detect small surface cracks that can be produced by material fatigue [38–40, 197] and to inspect welds [204, 205].
Figure 7.1: Alternative configurations of angled shear wave phased arrays that can be used to detect surface cracks. a) A piezoelectric array mounted on a wedge. b) A shear-wave EMAT array. The piezoelectric version has the disadvantages of requiring couplant and retaining longitudinal and shear wave reverberations in the wedge, which increases the background noise.
The outline of this chapter is as follows. First, the design elements of the proposed EMAT array are discussed in detail, namely the ferromagnetic core-magnet arrangement and the layout of the coils. In this section, the source of the bias magnetic field is simulated using finite elements (FE) and aluminium and steel specimens, the radiation patterns of the racetrack coils is simulated using FE, and the crosstalk between adjacent coils is measured. Then, the performance of an 8-channel EMAT array which operates at a central frequency of 1 MHz in detecting a surface crack is simulated using FE, followed by an experiment to corroborate the results. Finally, the conclusions are drawn.

7.2 Array design

As in Chapter 2 and [P1], I assume that the Lorentz force is the dominant transduction mechanism. A sketch of the proposed array is shown in Fig. 7.2. The coils of the array are laid out in an overlapping pattern under a ferromagnetic core. Each coil constitutes an element of the array so that the array equivalent pitch, i.e., the separation between adjacent coils, is 2/3 of the coil width. The real prototype is shown in Fig. 7.3.

7.2.1 Bias magnetic field

The bias magnetic field of the EMAT array is produced by a ferromagnetic core abutted by two magnets with like poles facing the core. This configuration exploits two basic mechanisms to increase the bias magnetic field under the core: repulsion between magnets and low reluctance paths [P1].

The width of the core defines the active regions of the coils, and also the focusing capabilities in the passive dimension, i.e., the dimension where there is no electronic beam steering; the width of the passive dimension is commonly referred to as elevation. A 15-mm core width was selected to match the common elevations encountered in commercial arrays, which are normally between 10 and 20 mm for 1 MHz probes (e.g., see array probe models 2L8-DGS1 or 1.5L16-A4 from Olympus Scientific Solutions Americas Inc., MA, USA). The height of the
Figure 7.2: Sketch of proposed EMAT array showing the overlapping pattern of the coils and the ferromagnetic core abutted by magnets with like poles facing the core. The circles and crosses in the top view indicate that the currents in the coils leave and enter the plane of the figure respectively.

Figure 7.3: Photo of the EMAT array prototype that was built.
core (21 mm) and the dimensions of the magnets (cross section 20x10 mm$^2$) were chosen by me as a good compromise between the intensity of the bias magnetic field and the overall volume of the array. A study of such a compromise was presented in [P1] for axisymmetric configurations.

In order to study and quantify the bias magnetic field generated by the core-magnet arrangement on the surface of the specimen, a finite-element model was simulated in COMSOL Multiphysics 5.2 (COMSOL Inc., Massachusetts, USA) using the Magnetic Field (No Current) interface of the AC/DC module model library. A symmetric (two-dimensional) model was employed as shown in Fig. 7.4; the axis of symmetry is on the left-hand side of the figure. In our case, a two-dimensional model is a good approximation because the core-magnet arrangement is larger in the active direction of the array, i.e., the direction on which the coils are laid out, with respect to the other direction.

The dimensions of the cross-section of the core in the model of Fig. 7.4 are 7.5 x 21 mm$^2$ (15 x 21 mm$^2$ in the real core). The dimensions of the magnet cross-section are 10 x 20 mm$^2$. Aluminium (a) and mild (b) steel samples which have dimensions 20 x 60 mm$^2$ are simulated beneath the magnet and the core. The lift-off between the magnets and the sample is 3 mm, whereas the lift-off between the core and the sample is 2 mm. These lift-offs correspond to the thickness of the housing of the array, the coils and a non-conductive coating layer over the sample surface. Finally, the model is enclosed in a domain modelled as air which has dimensions 100 x 120 mm$^2$ and is surrounded by magnetic insulation boundaries.

The mesh elements of the model had a quadrilateral shape and a maximum length (distance between the furthest nodes) of less than 0.1 mm under the core. The length of the elements was increased progressively until the elements furthest from the core and the magnets reached a maximum length of 2 mm. A convergence test was conducted to confirm that the results did not change by more than 1% when using a denser mesh; this value was selected by me as a good compromise between accuracy and run length of the simulation. The remanence of the magnet was set to $B_r = 1.32$ T, equivalent to Neodymium N42 (Hitachi Metals America, Ltd., "Permanent Magnets", 2015) and oriented towards the core in the centre.
7. Low-Power Phased EMAT Array

Figure 7.4: 2D simulations of the distribution of the magnetic flux density in the ferromagnetic core, magnet (Neodymium N42, $B_r = 1.32 \, \text{T}$), and sample. a) Aluminium sample. b) Mild steel sample. The colour scale corresponds to the absolute value of the flux density in Tesla (T), whereas black curves represent flux lines.
Given that the permeability of steel changes with the intensity of the magnetic field $H$, a curve that relates $B$ and $H$ was used in the simulations; this curve was obtained from the COMSOL library. Other $B$ vs. $H$ curves, e.g., for mild steel, were also used finding no significant differences in the results under the core; this is because the strength of the field in that region is such that the different materials saturated easily [P1].

The simulation results are shown in Fig. 7.4a and b for aluminium and steel respectively. The colour scale corresponds to the absolute value of the flux density in Tesla (T), whereas black curves represent flux lines. The distribution of the field is similar to that observed in [P1] for the axisymmetric case. The field is concentrated beneath the core, where the field lines are perpendicular to the surface of the sample. The main difference between Fig. 7.4a and b is the increase in the flux density in the presence of the steel sample due to the existence of a lower reluctance path between the core and the ferromagnetic sample.

A detailed distribution of the normal components of the field at the surface of the samples is given in Fig. 7.5. The dashed and continuous black curves correspond to the core-magnet arrangement over the aluminium and steel samples respectively. The distribution of the flux components is similar in both cases with the maximum values appearing under the core and then decaying rapidly. The flux density is twice as large in the steel sample under the core compared to the aluminium sample; in pulse-echo mode, this corresponds to a 12-dB signal increase because the signals double on both transmission and reception.

To offer a quick, fair comparison point, a single magnet which has a cross-section area equal to that of the core and magnet combined ($35 \times 20 \text{mm}^2$) was simulated. The lift-off of the magnet relative to the sample was 2 mm, $B_r = 1.32\, \text{T}$ and the magnetic field was oriented towards the sample. The dashed and continuous red curves in Fig. 7.5 correspond to the single magnet over aluminium and steel samples, respectively. There is no significant difference in the maximum flux density between the core-magnet arrangement and the single magnet when placed over the aluminium sample. However, over the steel sample, the core-
magnet arrangement produces roughly double the flux density compared to the single magnet.

Finally, to validate the simulations, a Gaussmeter (GM08, Hirst Magnetic, UK) which has a transverse Hall probe (TP002, Hirst Magnetic, UK) was used to measure the flux density under the core-magnet arrangement in air; this is equivalent to the case where the core-magnet arrangement is placed over an aluminium sample. The ferromagnetic core of the real prototype shown in Fig. 7.3, was made out of mild steel and has dimensions $15 \times 23 \times 86 \text{mm}^3$. The core is abutted by 4 magnets that have dimensions $10 \times 20 \times 20 \text{mm}^3$ such that like poles face the core. The core height is slightly larger than in the 2D model for ease of construction. The core-magnet arrangement was simulated in a 3D model to take into account the changes in the design with respect to the 2D model. For example, there is a 2-mm gap between the magnets, a 0.5-mm gap between the magnets and the core that was filled with glue, and $1 \times 1 \text{mm}^2$ grooves on the surface of the core.

The results are given in Fig. 7.6, where the circle and square markers correspond to measurements taken on the edge and centre of the core, respectively. The continuous red curve corresponds to 3D simulations and the dashed black curve
to 2D simulations. There is almost no difference between the measurements taken at the end and centre of the array, which implies that fringe effects are negligible. In general, the 3D simulation fits the experimental measurements better than the 2D.

Experiments were not conducted over a steel sample because the probe housing was fabricated using a fragile polymer that could easily be broken if exposed to the magnetic forces between the ferromagnetic sample and the probe; however, these simulations are expected to be accurate according to [P1].

### 7.2.2 Radiation pattern of single coils

The radiation pattern of an array is determined by its elements, in this case the racetrack coils, or more precisely the path of the eddy currents induced by them. The radiation pattern of the individual coils was simulated in COMSOL Multiphysics 5.2 (COMSOL Inc., Massachusetts, USA) using the Solid Mechanics module. The eddy currents induced by the coils were approximated as two line sources of opposite phase; a two-dimensional simulation suffices because the racetrack shape of the coil is long and narrow.

The two-dimensional model consists of a semi-circular region, see Fig. 7.7,
simulated as steel, which has a density of 7850 kg/m³, a Young’s modulus of 205 GPa and a Poisson’s ratio of 0.28. The radius of the semi-circular region is 60 mm. The mesh consisted of quadrilateral elements that had a maximum length (distance from the furthest nodes) of 0.53 mm; this is roughly 1/6 of the shear wavelength in steel at 1 MHz, where the shear velocity is 3.2 mm/s.

The excitation signal consisted of a 3-cycle Hann tone-burst centred at 1 MHz. The sampling frequency was 16 MHz. This excitation was applied as prescribed displacements of opposite phase tangential to the surface at two points symmetrically located around the centre of the semicircle (marked by the red circles in

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure_7.7.png}
\caption{2D simulation of two shear line sources of opposite phase in steel 10 \( \mu s \) after the burst excitation. The distance between the sources (red circles) is a) \( \frac{1}{4} \), b) \( \frac{1}{2} \) and c) 1 shear wavelength. The absolute total displacement is normalised with respect of the maximum of the plots.}
\end{figure}
Fig. 7.7) to simulate the shear line sources. The distance between the two sources was set to 1/4, 1/2 and 1 shear wavelength (3.2 mm). A convergence test was conducted to confirm that neither a denser mesh nor a higher sampling frequency improved the results by more than 2%; I consider that this value is an acceptable compromise between the overall run length of the simulation and its accuracy.

Figure 7.7 shows the absolute value of the displacements 10 μs after the burst excitation. The distance between the line sources in Fig. 7.7a-c is 1/4, 1/2 and 1 shear wavelength, respectively. In general, four types of waves can be observed, Rayleigh waves on the surface, longitudinal waves travelling at an angle of roughly 30° with respect to the plane defined by the surface, shear waves at roughly 60°, and head waves for angles greater than the critical angle (roughly 33°), see [206].

When the distance between the sources exceeds half wavelength, the wave packets split in two; having more than one wave packet is not desirable because it deteriorates the resolution of the system and can lead to artifacts in the resulting focused image. In general, the distance between the line sources, i.e., the width of the coil, poses a trade-off between ease of construction and the distortion of the wave packet. In practice, an adequate separation of the sources is between 1/2 and 1 wavelength; at 1 MHz this corresponds to 1.6 and 3.2 mm respectively.

All these results also apply to aluminium samples, because aluminium and steel have similar wave velocities.

### 7.2.3 Crosstalk between array elements

To evaluate the crosstalk between the array elements, two pairs of racetrack coils were built such that the wire bundles were as compact as possible; these have mean dimensions 3.2 × 18 mm². The coils were built by winding 12 and 15 turns of magnet (enamelled) 52-AWG wire, which has a diameter of approximately 20 μm. One of the coils was connected to a custom made driver that is able to produce a maximum output of 24 Vpp and 250 mA. The other coil was connected to the input of an oscilloscope (LeCroy WaveRunner 44Xi). The coils were initially laid out on top of each other and then carefully slid in the direction perpendicular to their straight section while recording the amplitude of the signal. The recorded
Figure 7.8: Measured crosstalk between two pairs of coils.

values are given in Fig. 7.8, where the vertical axis shows the relative amplitude of the signals (attenuation) and the horizontal axis indicates the distance between the centres of the coils normalised by the width of the coil (3.2 mm). The measurements were recorded using two different pairs of coils in order to assess the impact of the design tolerances on the results.

It can be observed in Fig. 7.8 that the difference between the two pairs of coils can be as high as 5 dB. This can be due to errors in the alignment of the coils or a different coil mean width. When the distance between the coils is roughly 2/3 of the coil width, the crosstalk reaches a local minimum of approximately −27 dB. This is because the coils are balanced, i.e., the magnetic flux within the area shared by the coils has opposite direction but equal magnitude to the flux outside this area.

The existence of a local minimum at roughly 2/3 of the coil width is convenient to create a regular overlapping pattern. Following such a pattern, the third coil would be located at 4/3 of the coil width, where the crosstalk is roughly −15 dB. From the fourth coil onwards, the crosstalk is below −25 dB as can be extrapolated from Fig. 7.8. Although piezoelectric probes may achieve smaller crosstalk values, this level of isolation is enough to produce a good degree of independence between the elements – a necessary condition to obtain focused images of adequate quality.
7. Low-Power Phased EMAT Array

![Image: Diagram of a 2D model of an aluminium block with a slot]

**Figure 7.9**: 2D model of an aluminium block that has a slot.

7.3 Results

7.3.1 Simulations

In this section an 8-element EMAT array is simulated using the Solid Mechanics (transient) module of COMSOL Multiphysics 5.2 (COMSOL Inc., Massachusetts, USA). Figure 7.9 shows a two-dimensional finite element model of an aluminium block, which have a density of 2700 kg/m³, a Young’s modulus of 70 GPa and a Poisson’s ratio of 0.33. The block has a 0.2 × 0.8 mm² slot as shown in Fig. 7.9.

Absorbing layers are placed at the side boundaries of the model to attenuate any outgoing wave by more than 40 dB. The absorbing layers were simulated as in [73], 16 absorbing layers are employed. The innermost layer has a width equivalent to a quarter of a wavelength and this is increased following a quadratic law so that the width of the outer layer is half a wavelength. In each absorbing layer the mass damping parameter is increased following a cubic law so that the value of the outer region is 2 · 10⁷ s⁻¹.

Free-triangular elements are used in the mesh of the FE model. The maximum size of the elements corresponds to 1/6 of the wavelength. When using a finer mesh, the results only changed by less than 2%; this was deemed to be a good compromise between overall run length and accuracy.

Each element (coil) of the EMAT array is simulated as two point loads separated by 3.2 mm (roughly the wavelength of the shear waves at 1 MHz in aluminium) which exert tangential forces of opposite phase on the surface of the
aluminium block as in Fig. 7.7. Each element overlaps adjacent ones such that the array pitch is 2/3 of the element width.

Each element is excited individually by a 3-cycle burst centred at 1 MHz and apodized using a Hanning window; the sampling frequency was 16 MHz. For each excited element, the signals received on all of the array elements are recorded, which results in a total of 64 signals. The received signals are obtained by subtracting the tangential displacements at the loading points of each element.

Equation (6.1) is employed to produce a focused image using a regular spatial grid equivalent to 1/16 of the wavelength at 1 MHz and a wave velocity of 3.1 mm/s. Results are shown in Fig. 7.10a, where the back wall of the aluminium block is marked using a white horizontal line. Both the back wall and the slot in the back wall can be clearly identified. Focusing artifacts also appear at the right-hand side of Fig. 7.10a. It was confirmed that the focusing artifacts disappear when the distance between the two point loads of the elements or the frequency is reduced; this can be attributed to the effect of the side/grating lobes.

Reflections from the back wall are only recovered beneath the array by those elements close to the edge. This is a result of the radiation pattern of the array elements, which have their main lobes oriented at roughly 60° from the surface of the array, see Fig. 7.7c.

Moreover, the centre of the focal region that corresponds to the defect in Fig. 7.10a lies slightly below the original location of the defect. This is because in equation (6.1) the array elements are assumed to radiate cylindrical shear waves; however, head waves (see Fig. 7.7c) predominate in this region with respect to the shear waves due the orientation of the array elements with respect to the defect. These head waves travel faster, which causes the downward shift effect of the focal point. This aberration could be fixed using a more sophisticated focusing algorithm since the radiation pattern of the individual elements is known, but that is out of the scope of this paper.
Figure 7.10: Focused image that corresponds to a 30-mm aluminium block which has a slot (defect). The array consists of 8 elements, which have a width of 3.2 mm and a pitch of 2.1 mm. The central frequency of the excitation is 1 MHz. The white and red horizontal lines correspond to the back wall of the aluminium block and the location of the array respectively. a) 2D simulation. b) Experimental results.
7. Low-Power Phased EMAT Array

7.3.2 Experiments

The EMAT array prototype that was built is shown in Fig. 7.3; details of the core and magnet dimensions are given in Sec. 7.2.1. As in previous simulations, only 8 channels are used to simplify the instrumentation. The coils are wound such that the mean distance between the wire bundles is 3.2 mm, which roughly corresponds to a wavelength (3.1 mm) at 1 MHz in aluminium. The distance between the coils, i.e., the array pitch is 2/3 of the coil width. The effective elevation of this array is 15 mm (Fig. 7.3); this is determined by the core width. Each coil comprises 12 turns of magnet (enamelled) 52-AWG wire, which has a diameter of approximately \(20 \mu m\).

Each coil is connected to a custom made transmit-receive electronics board, comprising a driver which can deliver 24 Vpp and 200 mA, a transmit-receive switch to protect the receive amplifier and an ultra-low-noise, 60-dB-gain receive amplifier. The input of the driver and the output of the receive amplifier are connected to a digital-to-analog converter (DAC) and to an analog-to-digital converter (ADC) respectively. The DAC and the ADC are embedded in a programmable acquisition system (Handyscope-HS5, TiePie, Netherlands) and both use a sampling frequency of 20 MHz. This acquisition system is set to transmit a binary sequence as described in Chapter 5, \([P4]\) and \([P5]\). The sequence consists of \(L = 2^{14}\) transmit and receive intervals randomly distributed, where each interval has \(Q = 480\) samples. Each transmit interval consists of a 3-cycle, Hann-tapered bursts centred at 1 MHz, whose polarity is randomly chosen. This produces an SNR increase of more than 36 dB. Each interval has a length of \(24 \mu s\), which yields a sequence with an overall length of roughly 100 ms. The intervals of the sequence are 8 times longer than the bursts to allow the electronics to recover from excitation before receiving starts.

The EMAT array is placed over a 30-mm-thick aluminium block that has a slot on the back wall as shown in Fig. 7.11; the slot cross-section is 0.2-mm wide and 0.8-mm high. The horizontal distance from the closest coil of the array to the defect is 22.5 mm. The signals from all of the 64 transmit-receive coil pairs are recorded after transmitting the coded sequence one element at the time.
Figure 7.11: Experimental setup: an 8-channel EMAT array is placed over an aluminium block that has a slot on the back wall; the slot cross-section is 0.2-mm wide and 0.8-mm high. Only 4 connectors can be seen from the front; the other 4 are at the back.

The amplitude of the received echoes is in the order of 0.5 mV and the root-mean-square value of the noise is 2 mV (both after a 60-dB amplification); this corresponds to an SNR of roughly -12 and 24 dB at the receiver input and after cross-correlating the digitised signals with the transmitted sequence respectively. The superposition of the different channels when using equation (6.1) to produce the focused image also has an averaging effect equivalent to a 9-dB-SNR increase for 8 channels. As a result, the overall image SNR is greater than 33 dB.

All of the recorded signals were visually inspected and 12 out of 64 were found to be highly distorted. Most of the distorted signals correspond to the cases where the same transmit and receive element are used; this is due to poor performance of the instrumentation, mainly caused by a slow recovery from saturation, which subsequently affects the performance of the sequences. The distorted signals were discarded and the rest used in equation (6.1) to obtain the focused image.

The focused image is shown in Fig. 7.10b. Overall, the focal regions of interest, namely the defect and the back wall, match well with the simulations in Fig. 7.10a despite the fact that signals from some transmit-receive pairs were discarded; this also indicates that crosstalk is sufficiently low, as expected from the results of Fig. 7.8. The focal regions are slightly larger in Fig. 7.10b, which could be due to a narrower bandwidth or ringing of the transducer.

Several artifacts can be seen in Fig. 7.10b, for example, the semicircle-like
artifact that is closer to the array, which is suspected to have been produced by the generation of longitudinal modes inside the ferromagnetic core of the array. According to [P1], shear transducers, as the racetrack coils of the EMAT array, can generate/receive longitudinal modes due to the discontinuity of the traction forces on the surface of the specimen, and since the displacements that correspond to the longitudinal mode are aligned with the bias magnetic field, the longitudinal mode can propagate unaffected. The use of a ferrite core could reduce the eddy currents and attenuated the waves in the core further. The rest of the artifacts in Fig. 7.10b are believed to be associated with the transmit-receive switching functionality of the electronics and its low recovery from saturation, which affect the performance of the sequences.

7.4 Summary

This chapter reports the first fully-steerable, pulse-echo EMAT array, which can be used to image defects that have a cross-section area of $0.2 \times 0.8 \text{mm}^2$ using excitations signals of only 24 Vpp and 200mA per channel. By using coded sequences with receive intervals, the excitation power can be reduced so that small coils with dimensions comparable to the ultrasonic wavelength are not damaged. For example, the racetrack coils of the proposed arrays, which have dimensions $3.2 \times 18 \text{mm}^2$, cannot be driven at voltages greater than a few hundred volts because insulator breakdown can occur.

The array elements consist of racetrack coils that mainly generate broad-beam shear waves at roughly $60^\circ$ from the surface of the array. The orientation of this shear-wave beam is similar to that of conventional piezoelectric arrays mounted on a wedge. The layout of the coils yields a compact design which has a sub-wavelength pitch of approximately 2 mm and reduces the crosstalk between the array elements to less than $-15 \text{dB}$. A comparison of experimental and simulated data showed that these crosstalk levels are acceptable. A core-magnet arrangement is used to increase the bias magnetic field over the eddy currents; compared with a single magnet, the core-magnet arrangement can produce signals whose
amplitudes are 10-dB greater.
Chapter 8

Conclusions

8.1 Review of the thesis findings

This thesis focuses on two main subjects: the development of efficient electromagnetic-acoustic transducers (EMATs) and the synthesis of coded sequence for pulse-echo mode, low-signal-to-noise ratio (low-SNR) systems.

Chapter 2 presented a study of the optimisation of the bias magnetic field of shear-wave EMATs, where different magnet configurations were investigated and design curves to help with the selection of the best operating point for maximum signal strength were provided. The Lorentz force was assumed to be the predominant transduction mechanism neglecting any other mechanisms such as magnetostriction; experimental results showed no contradiction with this assumption. A ferromagnetic core surrounded by magnets which have like poles facing the core was found to produce the best results. The corresponding signals can be greater than $3 - 6\, \text{dB}$ and $20\, \text{dB}$ compared to a single magnet for the same overall volume and aperture area respectively. The enhanced performance of the presented core-magnet arrangement relies on two mechanisms: a) repulsion between magnets and b) the low reluctance path formed by both the ferromagnetic core between the repelling magnets and the ferromagnetic specimen itself. For each overall diameter of the core-magnet arrangement, there is a core diameter that maximises the signal. When the height of the core-magnet arrangement is 20 mm and the lift-off is 0.5 mm, the optimal ratio between the core and overall diameter
is roughly 2 : 3 for core diameters between 10 and 25 mm. This optimal ratio decreases to roughly 1 : 2 when the lift-off increases to 2 mm. It should be highlighted that these ratios only apply to their respective configurations when placed on mild steel specimens. Another finding was that linearly polarised apertures produce pulse-echo signals with greater amplitude and less distortion than the radially polarised apertures. It was also found that the tangential component of the magnetic flux density, responsible for the generation of the longitudinal mode, which causes distortion, increase steeply for mild steel samples with thickness below 7 mm.

Chapter 3 investigated the optimal impedance of an EMAT coil. It was discussed how the number of turns in the coil can be controlled to balance the load between the coil, the driver and the eddy currents such that maximum power can be delivered to the eddy currents. Series resonance on transmission was found to increase the signal strength by roughly 2 dB whereas parallel resonance on reception increased the signal by approximately 6 dB. A tunable coil was proposed whose inductance could be changed without affecting the radiation pattern of the EMAT. The number of turns that satisfy the maximum power transfer condition was found to be almost invariant to the coil lift-off in the range of 0 to 2 mm over mild steel specimens. The theoretical model developed indicates that the resistance of the source and coil should be the smallest possible to maximise the eddy currents. This implies that when only a section of the coil is used in the ultrasonic aperture, once the optimum coupling has been achieved, the return path should be as short as possible. The effect of the noise sources when selecting the coil impedance on reception was also discussed.

Chapter 4 reviewed the principle behind binary quantisation and studied the input SNR where binary quantisation is of practical value for ultrasound applications. In most cases, binary quantisation can only be employed when the input SNR is below 8 dB. The maximum SNR after binary quantisation and averaging can be estimated as $10 \log_{10} N - 2$, therefore, at least a few hundred of averages are required to produce an SNR at the output greater than 20 dB. By using binary quantisation, standard ADCs can be replaced by comparators and binary
latches, and in some cases, even the analog channel may be directly connected to the digital input. All this is especially attractive for applications that require arrays with many channels and high sampling rates, where the sampling rate could be as high as the system clock rate. In general, this permits the electronics to be more compact and faster and to consume less energy.

Chapter 5 introduced a new methodology to use pulse-compression in pulse-echo systems. The novel aspect is the insertion of receive intervals. When the input SNR is low (< 10 dB), the sequences with receive intervals are much faster than averaging or can produce an extra SNR increase for the same overall measurement duration. In general, the sequences can outperform averaging by more than 20 dB in many cases. Under low input SNR a simple random codification of the sequence using an equal number of receive and transmit intervals of equal length randomly distributed in time performs optimally. Moreover, a sequence of any given length can be continuously transmitted without pauses, which increases the refresh rate of the system. It was shown that an EMAT can be driven with 4.5 Vpp (< 0.5 W) obtaining a clear signal in quasi-real-time; commercially available systems require 1200 Vpp (8 kW) for similar performance.

Chapter 6 extends the methodology of coded sequences which have receive gaps to a set of pseudo-orthogonal sequences to be used with arrays. As a result, every transmit-receive element pair can be processed simultaneously. The proposed set is shown to be an optimal solution, at least one order of magnitude faster than the sequential excitation of individual elements when the received signals have low SNR (typically below 15 dB) or the resulting focused image does not require high signal-to-coherent-noise ratio (typically not much greater than 50 dB). The method is therefore particularly valuable with dense arrays, where smaller elements produce weaker signals, and when the medium is intrinsically noisy due to, for example, grain noise in elastic materials or strong speckle in tissue. Moreover, when binary quantisation is used along with the proposed sequences in low-SNR scenarios, the volume of data that must be transmitted is reduced by roughly one order of magnitude without affecting the SNR. This permits more transmit-receive pairs to be acquired, which translates into higher
8. Conclusions

spatial resolutions and image contrast, at higher frame rates while using similar data throughputs comparable to that of current technology but at the expense of additional post-processing.

Chapter 7 reports the first low-power, pulse-echo EMAT phased array, which can be used to image defects that have a cross-section area of $0.2 \times 0.8 \text{mm}^2$ using excitations signals of only 24 Vpp and 200 mA per channel. By using coded sequences with receive intervals, the excitation power can be reduced so that small coils with dimensions comparable to the ultrasonic wavelength are not damaged. For example, the racetrack coils of the proposed arrays, which have dimensions $3.2 \times 18 \text{mm}^2$, cannot be driven at voltages greater than a few hundred volts because insulator breakdown can occur. The array elements consist of racetrack coils that mainly generate broad-beam shear waves at roughly $60^\circ$ from the surface of the array. The orientation of this shear-wave beam is similar to that of conventional piezoelectric arrays mounted on a wedge. The layout of the coils yields a compact design which has a sub-wavelength pitch of approximately 2 mm and reduces the crosstalk between the array elements to less than $-15 \text{dB}$; a comparison of experimental and simulated data showed that these crosstalk levels are acceptable. A core-magnet arrangement is used to increase the bias magnetic field over the eddy currents; compared with a single magnet, the core-magnet arrangement can produce signals whose amplitudes are 10-dB greater.

8.2 Main contributions

This thesis reports several novel findings across different areas. All the improvements achieved in the design of shear-wave EMATs (Chapter 2 and 3) along with a better understanding of binary acquisition system for low-SNR signals (Chapter 4) led to the development of the first, low-power, intrinsically-safe EMAT monitoring technology (ET210, Permasense [Emerson], Horsham, UK). This was a significant step in the EMAT field because EMATs were historically believed to be too inefficient for such a task.

The introduction of the coded sequences with receive intervals (Chapter 5)
allowed, for the first time, that pulse-echo EMATs be driven using low powers (4.5 Vpp and 100 mA) in quasi-real-time (10 Hz repetition rate). These coded sequences are applicable to any transduction mechanism, e.g., piezoelectricity, antennas, guided wave, etc., and can be used to reduce the driving power or to increase the range of the area which is been inspected. It also permits more inefficient and usually cheaper transducers to be employed.

Coded sequences with receive intervals can be transmitted in parallel in a pseudo-orthogonal set (Chapter 6). As a result, the duration of the measurement is not limited by the number of channels, and, given that these sequences are optimal when the received signals have low SNR (typically below 15 dB), binary quantisation can be used (Chapter 4) significantly reducing the data throughput. This could solve a central problem in medical ultrasound technology, where high-frequency (> 5 MHz) dense arrays which have thousand of elements have been reported. When using very dense probes, all of the transmit-receive pairs of the array probe cannot be recorded simultaneously without coded excitation due to the duration of the time-of-flight of the ultrasonic waves. Furthermore, as the driving frequency increases and the size of the elements shrinks, the elements produce signals of lesser intensity; the use of coded excitation can increase the SNR of these signals.

Finally, by combining all the knowledge of the design of efficient shear-wave EMATs and the use of coded sequences (Chapters 2-5), the first, low-power (24 Vpp and 200 mA per channel), pulse-echo, EMAT phased arrays is introduced in Chapter 7. This array can be used to image defects that have a cross-section area of $0.2 \times 0.8 \text{mm}^2$. The fact that EMATs have to commonly be excited using powers in excess of 1 kW hindered the development of EMAT phased arrays, mainly because the array coils have to be smaller than a wavelength at the central frequency and these smaller coils are weaker radiators and cannot withstand driving powers in excess of 1kW.
8.3 Areas for future work

Several areas regarding EMATs and coded sequences should be explored further. EMATs do not require direct contact with the specimen under test; this makes them suitable for high-temperature applications. There are two main areas to be investigated regarding high-temperature EMATs. First, the material permeability, resistivity, and the attenuation of the different types of ultrasonic waves are known to change when the temperature raises, but the combined effect of all these changes on the transduction mechanism remains unexplored. Secondly, in principle, EMATs could be designed to withstand temperatures greater than 300°C without coolant for years; however, the challenges of such designs, any compromises in performance and their long-term stability remain to be addressed. Such designs would be desirable for permanent monitoring of high-temperature components.

Another interesting area is the use of EMATs for low-frequency (< 200 KHz) guided ultrasonic waves applications. The methodology developed in this thesis to increase the bias magnetic field of EMATs and to choose the optimal impedance can be applied there. It is shown in Chapter 3 that at lower frequencies more turns are required for a coil impedance to be optimal (Chapter 3). In the KHz range, reaching such a condition may be challenging and alternative ways of designing or interconnecting coils need to be explored. Moreover, the coded sequences with receive intervals presented in this thesis can be used to increase the inspection range.

Future development on EMAT phased arrays should also be considered. Thanks to the use of coded sequences, the size of the EMAT coils can be reduced. This thesis shows that the coils can be laid out in a balanced configuration to reduce crosstalk. Other configurations that achieve the same balanced can be investigated, for example, to obtain a 2D matrix probe. Designing coils that operate at frequencies greater than 3 MHz is also challenging because of their small size and the negative effects of the parasitic capacitance. The development of EMAT phased arrays using these design concepts can additionally be extended to low-
frequency (< 200 KHz) guided ultrasonic waves.

A very promising area is the use of pseudo-orthogonal sequences that have receive intervals to drive high-frequency (> 5 MHz), dense arrays, as those used for 3D medical ultrasound. However, this requires a dedicated acquisition system where each channel is driven and sampled using only one digital line. The use of PCI-express input-output cards could be a good starting point. Data throughputs in the order of 50 – 100 Gbit/s have been reported, and this is enough to support more than 1000 parallel channels when using the proposed sequences and binary quantisation. Moreover, processing the sequences and the synthetic focusing algorithms require extensive computation, but this can be achieved with commercial GPUs. To ensure that the sequences perform efficiently, foremost attention should also be given to the design of ultra-fast and -quiet active switches and damping in the acquisition systems.
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Publications and patent applications list


Appendix A

Derivation of Equations

A.1 Equation 3.3

In order to obtain equation (3.3), $I_e$ in equation (3.2) should be written as a function of the source voltage $V$. This is achieved by noting that in Fig. 3.1

\[ U_e = -I_e R_e \]  \hspace{1cm} (A.1)

and

\[ V = I_c R_e - j I_c X_{Cap} + U_c \]  \hspace{1cm} (A.2)

and that according to equation (3.1)

\[ U_e = I_c \cdot j X_M + I_e \cdot j X_e \]  \hspace{1cm} (A.3)

and

\[ U_c = I_c \cdot j X_e + I_e \cdot j X_M. \]  \hspace{1cm} (A.4)
Combining equations (A.1) and (A.2) and using $Z_e = R_e + jX_e$

$$\frac{I_c}{I_e} = -\frac{Z_e}{jX_M}. \quad (A.5)$$

Combining equations (A.2) and (A.4)

$$V = I_c (R_e + jX_e) - I_e X_{Cap} + I_e \cdot jX_M. \quad (A.6)$$

Finally, combining equations (A.5) and (A.6)

$$|I_e|^2 = \frac{V^2}{X_M + X_e X_{M}} \frac{Z_e}{X_M (R_e + jX_e - jX_{Cap})}^2. \quad (A.7)$$

### A.2 Equation 3.9

Equation (3.8) can be written as

$$\eta = \frac{|I_e|^2 R_e}{|I_e|^2 R_e + |I_c|^2 R_e}. \quad (A.8)$$

The squared module of equation (A.5) is

$$\frac{|I_c|^2}{|I_e|^2} = \left(\frac{R_e}{X_M}\right)^2 + \left(\frac{L_e}{X_M}\right)^2. \quad (A.9)$$

This is substituted back in equation (A.8) using $X_M = k \sqrt{X_e X_c}$ to obtain equation (3.9) after a few manipulations.
A.3 Equation 3.15

Equation (3.14) can be written as

\[ Q'_e > \frac{|Z_e|}{X_e} - k^2, \]  

(A.10)

where

\[ \frac{|Z_e|}{X_e} \leq \frac{1}{Q_e} + 1 \]  

(A.11)

and \( Q_e = \frac{X_e}{R_e} \). Since \( \frac{|Z_e|}{X_e} \) is a positive number, inequation (3.15) holds unconditionally when substituting (A.11) in (A.10).

A.4 Variance after adding \( N \) realisations of \( Q \)

To obtain equation (4.8), we first find the variance of \( Q \)

\[ \sigma_Q^2 = E [Q^2] - E [Q]^2. \]  

(A.12)

Note that \( E [Q^2] = 1 \) and \( E [Q] \) is given in equation (4.4), hence

\[ \sigma_Q^2 = 4F \left( -\frac{s}{\sigma_y} \right) + 4F \left( -\frac{s}{\sigma_y} \right)^2. \]

Since \( F = 1 - F \) and the sign of \( s \) does not affect the result, we can write

\[ \sigma_Q^2 = 4F \left( \frac{s}{\sigma_y} \right) \overline{F} \left( \frac{s}{\sigma_y} \right). \]  

(A.13)
Given that the copies of $Q$ are identical and independently distributed (i.i.d.), the variance after adding $N$ copies is $\sigma_{Q,N}^2 = N\sigma_Q^2$.

### A.5 Estimation of the standard deviation

Equation (4.9) is empirically presented without a proof and later its accuracy is corroborated in Sec. 4.3.2. Nonetheless, the rationale behind this equation for the case $N > c_N > 0$ is discussed here, which can be readily extended to the remaining case. The need for two cases arises in order to avoid exceeding the domain of equation (4.9) during numerical computations. First note that when $N$ is large and $-N < c_N < N$, the argument of $F^{-1}$ in equation (4.9) can be approximated to

$$\frac{N - c_N - \sigma_{Q,N}}{2N} \approx 0.5 - \frac{E[Q(t)]}{2} - \frac{\sigma_Q}{2\sqrt{N}},$$

where $\sigma_Q^2$ is defined in Appendix A.4. In the case where $F^{-1}$ behaves linearly, for example when $|E[Q(t)]|$ and $\frac{\sigma_Q}{\sqrt{N}}$ are smaller than 0.5, the distribution of the realisations of the ensemble of $Q$ lie mostly within the linear regime and hence equation (4.9) can be shown to hold by using equation (2.5); however, this is not the case in the non-linear region of $F^{-1}$. Nonetheless, in the non-linear region, where the ratio $\frac{|s|}{\sigma_y}$ increases, $|E[Q(t)]|$ increases while $\sigma_Q$ decreases (see Appendix A.4), hence a non-linear operation over the distribution of the ensemble of $Q$ may still have a quasi-linear impact over its standard deviation within a local interval, as predicted by equation (4.9) and later corroborated in Sec. 4.3.2 for the interval of interest.
A. Derivation of Equations

A.6 Derivation of the sequence SNR when using modulation and uneven paths

To obtain equation (6.10), let $w_{ij}$ be the coefficients that correspond to the amplitude of the received echo from transmitter $i$ and receiver $j$. With uneven wave paths, equation (6.3) can be rewritten for transmitter $i'$ and receiver $j'$ as

$$\text{SNR}' \approx \frac{L(1 - p_1) w_{i'j'}^2}{\sum_{i=1}^{M} w_{ij}^2 + \frac{1}{p_1\text{SNR}_{in}}}$$  \hspace{1cm} (A.14)

where $\text{SNR}_{in}$ is the input SNR with respect to the strongest reflection. Combining all the transmitters for receiver $j'$

$$\text{SNR}'' \approx \frac{L(1 - p_1) \left( \sum_{i=1}^{M} w_{ij} \right)^2}{M \sum_{i=1}^{M} w_{ij}^2 + \frac{M}{p_1\text{SNR}_{in}}}$$  \hspace{1cm} (A.15)

Note that the right hand term of the denominator, which simulates the electrical noise, is not affected in this process.

To include the effect of the modulation, let us assume that the sequences are up-sampled to match the burst length, that the modulated burst has variance $\sigma_{\text{symb}}^2$ and mean zero, and that the received sequences are correlated with the up-sampled but unmodulated sequences. This process can be understood as a shifted combination of the transmitted sequence weighted by the burst samples. Since only the peak of the burst is of interest in the numerator, the weighting has no effect on it. Note that only the transmitted sequences are modulated, which has no effect on the right hand term of the denominator either. Hence, the result of the modulation on transmission is simply the variance $\sigma_{\text{symb}}^2$ multiplying the left hand term of the denominator. Finally, by combining all of the receiver elements $j$, equation (6.10) is obtained.