SEISMIC REFLECTIVITY AND IMPEDANCE INVERSION IN MULTICHANNEL FASHION

by

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Declaration of Originality

I herewith certify that the research reported in this thesis is of the author's own, conducted in the Centre for Reservoir Geophysics (CRG), Department of Earth Science and Engineering, Imperial College London from October 2010 to September 2014. Any other work mentioned in thesis has been properly referenced or acknowledged.
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Abstract

Seismic reflectivity inversion is an important step in both signal processing and quantitative interpretation since reflectivity contains the information of impedance and other elastic parameters. Conventional methods assume stratified media and perform deconvolution on seismic data trace by trace. However, when using these single-channel methods, the lateral coherency of the result may be affected when the input seismic traces have low signal-to-noise ratios (SNRs) or complex structures. In this thesis, the development of multichannel inversion algorithms will be investigated to improve the continuity of the reflectivity profiles and suppress the noise.

An example of a widely-used seismic reflectivity inversion method that is applied to single-channel seismic trace is the basis pursuit method. In this thesis, an FX prediction filter is incorporated with the conventional BP method, to investigate the potential benefit of multichannel implementation. Since the dictionary used in BP is huge in size, the matrix operations are very time consuming, yielding a long overall computational time. To improve the efficiency, a GPU-accelerated basis pursuit method is further implemented under CUDA architecture. Numerical results with the same accuracy are obtained and a speedup factor up to 145 for the whole process has been achieved.

To implement a multichannel reflectivity inversion, the curvelet transform is employed. A comparative study on the performance of the curvelet deconvolution with two other widely used methods, the least-squares method and $L_p$-norm deconvolution, is further conducted. Since the deconvolution based on the curvelet transform offers a good trade-off between the lateral continuity and sparseness, the curvelet deconvolution result is used as the initial model to enhance the $L_p$-norm deconvolution. Numerical results show that the lateral continuity of the spiky reflectivity profile can be further improved.

Moreover, I develop a proper multichannel deconvolution method based on the Cauchy constraint. In this algorithm, a multichannel prediction operator is integrated
into the iteration process. In this way, the information of the adjacent traces is exploited during the inversion procedure. This method can provide results with improved lateral coherency, better structure characterisation ability and lower residual energy ratio. I also develop two modified processes based on the original Cauchy constrained multichannel method. These two modifications can give better quality for the reflectivity inversion results, when compared with the original algorithm.

Finally, using a similar concept as the multichannel deconvolution method with the Cauchy constraint, I apply the multichannel inversion algorithm to seismic impedance inversion, with the input reflectivity series obtained from the previous inversion steps. Numerical results show impedance profiles with better structure identification and lateral continuity.
I would like to express my gratitude to my supervisor, Professor Yanghua Wang for his guidance in the field of geophysics during the period of my Ph. D. study. With his help I have a deep understanding in seismic reflectivity inversion and seismic impedance inversion.

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Abbreviations

AI Acoustic impedance
API Application programming interface
BP Basis pursuit
CEI Converted wave elastic impedance
CIP Common image point
CG Conjugate gradient
CPU Central processing unit
CRI Converted wave ray impedance
CRP Constant ray parameter
CUBLAS CUDA basic linear algebra subroutines
CUDA Compute unified device architecture
EI Elastic impedance
GLI Generalized linear inversion
GPU Graphics processing unit
LP Linear programming
MAP Maximum a posteriori
MBRF Markov-Bernoulli random field
MMs Matrix multiplications
MP Matching pursuit
MPI Multiple prediction through inversion
MVMs Matrix-vector multiplications
OSs Operating systems
PDF Probability density function
RI Ray impedance
SNR Signal-to-noise ratio
SU Seismic Unix
VA Viterbi algorithm
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Chapter 1  Introduction

In seismic reflectivity and ray impedance inversions, the goal is to obtain a high-fidelity image of subsurface layers for geological interpretation. When we generate seismic data we use sources that have a finite length and frequency content which, along with the addition of noise, affect our ability to image layers. Thus, different seismic inversion algorithms are developed to reduce the effects of using a non-ideal source and noise. The main steps of seismic reflectivity and ray impedance inversions undertaken in this thesis are shown in Figure 1.1.

As seen in Figure 1.1, the procedure involves four steps:

Step 1: Data preparation and construct constant ray parameter profiles (CRP). In the initialisation and preparation stage, offset-domain common image point (CIP) gathers are provided by SINOPEC Shengli Oilfield Company. As the ray impedance is defined in the ray parameter domain, the input field data sets used for seismic reflectivity inversion and ray impedance inversion in this thesis should be constant ray parameter profiles. To construct
CRP profiles, first bending ray-tracing algorithm (Wang, 2003; Lu, 2010; Zhang, 2010) is adopted to transform offset-domain CIP gathers into ray parameter domain CIP gathers. After that, CRP profiles are obtained by stacking the CIP gathers within the selected range of ray parameters.

Step 2: Mixed-phase wavelet estimation. For seismic reflectivity inversion, the wavelet estimation process cannot be ignored. Only by removing the seismic wavelet can one obtain the feature of the reflectivity series. Therefore, extracting the wavelet precisely is the precondition of acquiring accurate and high resolution reflectivity series. In this step, the concept of seismic wavelet does not only refer to the source wavelet. In fact, it is corresponding to the observed response of the seismic source energy propagating through the complex subsurface ray path (Lu, 2010; Zhang, 2010). As a result, the wavelet estimation in this step refers to the mixed-phase wavelet estimation.

Step 3: Seismic reflectivity inversion. After obtaining the wavelet, reflectivity inversion can be undertaken. Since seismic reflectivity inversion is ill-conditioned by nature, different constraints have to be applied to tackle this problem. In the field of seismic reflectivity inversion, reflectivity is often assumed to be white, and this assumption has been widely used in various methods such as deconvolution based on the Cauchy constraint (Amundsen, 1991; Sacchi and Ulrych, 1995; 1996; Sacchi, 1997), $L_p$-norm deconvolution (Debye and van Riel, 1990) and linear programming scheme via Fourier transform (Oldenburg et al., 1983). However, it was pointed out that this assumption is invalid at precritical incidence under the critical reflection theorem (Fokkema and Ziolkowski, 1987). In this thesis, the conventional white assumption is used, since a reflectivity profile that is sparse and spiky can lead to a ray impedance profile with high resolution. As a result, the focus is on the development and improvement of the sparse algorithms, which can provide reflectivity results with broad bandwidths.

Step 4: Seismic ray impedance inversion. After the reflectivity inversion, the obtained result can be used for the inversion of seismic impedance, which is very important for discriminating lithology. In this thesis, I will study the topic of seismic ray impedance (RI) inversion, due to the reason that when comparing with acoustic impedance (AI) and elastic
impedance (EI), RI can classify and identify different lithological properties better. The features of the ray impedance will be discussed in this chapter later.

Due to the inherent relationship of mixed-phase wavelet estimation, seismic reflectivity inversion and ray impedance inversion, in this chapter, these three procedures will be discussed with the goal of having a full understanding of the reflectivity inversion procedure and obtaining a better calculation result.

This introductory chapter is organised as follows: first of all, a brief introduction is given for mixed-phase wavelet estimation. The basic concept and research background of seismic reflectivity inversion will then be presented and discussed, followed by the concept of seismic ray impedance inversion. Finally, an outline of this thesis is provided.

1.1 Mixed-phase wavelet estimation

Seismic wavelet extraction is one of the long-standing important topics in the field of seismic data processing. To seismic exploration techniques, common ways for improving the resolution is to utilise the deconvolution to minimise the effect of the wavelet. Since seismic wavelet connects the seismic traces with reflectivity series, it is essential also for geological interpretation. Generally, there are two ways for seismic wavelet estimation. One is by using well logs directly (Buland and Omre, 2003; Ma et al., 2010), while the other one is based on statistics (Matson, 2000; van der Baan, 2008; Rietsch, 1997a and b). In this thesis, I adopt statistical method for estimating the wavelet.

The wavelet estimation starts from zero-phase and minimum phase wavelet estimation, but they both are rough approximations (Robinson, 1967; Peacock and Treitel, 1969; Ziolkowski, 1991). During the propagation of the wavelet, the phase and amplitude will alter because of the consequence of the earth’s absorption, near-surface effects and some other reasons (Lu, 2010). Therefore, mixed-phase wavelet estimation methods have been widely studied in the field of geophysics. For example, a fourth-order cumulant matching technique (Lazear, 1993) attempts to obtain a proper result through the iterations by making the fourth-order cumulant of the estimated wavelet match the cumulant of the seismic data. To
solve the matching problem of the windowed cumulant, Velis and Ulrych (1996) used the simulated annealing strategy. Misra and Sacchi (2006) suggested parameterisation of a mixed-phase wavelet as the convolution result of a minimum phase wavelet and an all-pass operator.

The procedure of the mixed-phase wavelet estimation based on high order statistics (Mendel, 1991; Lu and Wang, 2007; Lu, 2010) adopted in my thesis can be summarised by the following three steps.

Step 1: Estimate the zero-phase wavelet. If the reflectivity is assumed to be white, the power spectrum of the wavelet can be calculated using the autocorrelation of seismic trace.

Step 2: Estimate the constant phase based on the kurtosis, so as to generate a constant-phase wavelet. For the phase of the wavelet, the simplest case is to assume the phase as frequency independent. This is the so-called constant phase. A straightforward implementation is to rotate the seismic trace with different angles, and find the angle that corresponds to the largest kurtosis value. Then this angle is treated as the most likely wavelet phase (Longbottom et al., 1988), the details of this process are shown below.

A phase-rotated trace $c(t, \theta)$ can be computed from the original trace $d(t)$ by (Levy and Oldenburg, 1987; van der Baan, 2008; van der Baan and Fomel, 2009)

$$c(t, \theta) = d(t) \cos \theta - H[d(t)] \sin \theta,$$

where $H[d(t)]$ is the Hilbert transform of the original trace $d(t)$. The kurtosis of this rotated trace is defined as

$$k(\theta) = \frac{\sum c^4(t, \theta) / N}{\left(\sum c^2(t, \theta) / N\right)^2},$$

where $N$ is the number of time samples. When the phase-rotated trace exhibits the highest kurtosis value, it means that the trace has been corrected with a constant phase.

An example is shown in Figure 1.2. For this seismic trace, a series of kurtosis values corresponding to different phase angles are calculated. As illustrated in Figure 1.2 (b), the value of kurtosis reaches its peak at 28 degrees. Therefore, we can obtain the wavelet result with a corrected phase, as shown in Figure 1.2 (c).
Figure 1.2 (a) A seismic trace. (b) Calculated kurtosis of the phase-rotated trace. (c) The constant-phase wavelet estimated. (Lu, 2010)

Step 3: Estimate the mixed-phase wavelet iteratively, where the constant-phase wavelet obtained from Step 2 is used as the initial model. Mixed-phase wavelet estimation by high order statistics method (Lu, 2010) is adopted in this procedure.

The $k$-th order moment function of a signal $x(t)$ can be described as (Mendel, 1991)

$$m_k(\tau_1, \tau_2, \cdots \tau_{k-1}) = E\{x(t) \cdot x(t + \tau_1) \cdots x(t + \tau_{k-1})\},$$

where $E$ is the statistical expectation.
The $k$-th order cumulant function can be described as (Velis and Ulrych, 1996)

$$
c_k^r(t_1, t_2, \cdots, t_{k-1}) = E\{x(t) x(t + t_1) \cdots x(t + t_{k-1})\} - E\{g(t) g(t + t_1) \cdots g(t + t_{k-1})\},
$$

(1.4)

where $g(t)$ is the equivalent Gaussian process which possesses the same second-order statistics as $x(t)$. It can be seen that cumulant is able to be used to identify the deviation of a process from Gaussian process.

Based on the convolution model, the seismic model in the form of high-order statistics called Bartlett-Brillinger-Rosenblatt equation can be obtained (Mendel, 1991):

$$
c_k^d(t_1, \cdots, t_{k-1}) = c_k^r(t_1, \cdots, t_{k-1}) * m_k^w(t_1, \cdots, t_{k-1}) + c_k^w(t_1, \cdots, t_{k-1}),
$$

(1.5)

where $c_k^d(t_1, \cdots, t_{k-1})$, $c_k^r(t_1, \cdots, t_{k-1})$ and $c_k^w(t_1, \cdots, t_{k-1})$ are the $k$th order cumulant of seismic trace, reflectivity series and noise, respectively. $m_k^w(t_1, \cdots, t_{k-1})$ is the $k$th order moment of the seismic wavelet.

Here the noise is assumed to be of Gaussian distribution, as a result, its third and higher order cumulants are equal to zero. Assuming $r(t)$ is independent identically distributed (IID) and non-Gaussian, the cumulant of $r(t)$ can be expressed as

$$
c_k^r(t_1, \cdots, t_{k-1}) = \begin{cases} 
g_k^r, & \text{for } t_1 = \cdots = t_{k-1} = 0 \\
0, & \text{otherwise} 
\end{cases}
$$

(1.6)

where $g_k^r = c_k^r(0, \cdots, 0)$ is the origin value of the cumulant of the reflectivity. As a result, Equation (1.5) can be simplified into the form below:

$$
c_k^d(t_1, \cdots, t_{k-1}) = g_k^r m_k^w(t_1, \cdots, t_{k-1}).
$$

(1.7)

Equation (1.7) shows that the only difference between the $k$th ($k>2$) order cumulant of the seismic trace and the $k$th order moment of the wavelet is a scalar.

In practical cases, after applying a 3D smoothing-tapering window to Equation (1.7), the moment of the wavelet can be approximated as (Velis and Ulrych, 1996)

$$
\hat{m}_4^w(t_1, t_2, t_3) = \frac{1}{g_4^r} \tilde{a}(t_1, t_2, t_3) \hat{c}_4^d(t_1, t_2, t_3),
$$

(1.8)
where \( a(\tau_1, \tau_2, \tau_3) \) is the 3D window function:

\[
a(\tau_1, \tau_2, \tau_3) = d(\tau_1)d(\tau_2)d(\tau_3)d(\tau_2 - \tau_1)d(\tau_3 - \tau_2)d(\tau_3 - \tau_1).
\]  

(1.9)

In Velis and Ulrych (1996), \( d(\tau) \) is the Parzen window, which can be expressed as:

\[
d(\tau) = \begin{cases} 
1 - 6|\tau|/L^2 + 6|\tau|/L^3, & |\tau| \leq L/2 \\
2(1 - |\tau|/L)^3, & L/2 \leq |\tau| \leq L \\
0, & |\tau| > L
\end{cases}
\]

(1.10)

where \( L \) is the computation scale.

The mixed-phase wavelet then can be solved iteratively from the constant-phase wavelet by minimizing the difference between the fourth-order moment of the estimated wavelet and the windowed fourth-order cumulant of the seismic trace:

\[
J(\omega) = \sum_{\tau_1=-q}^{q} \sum_{\tau_2=-q}^{q} \sum_{\tau_3=-q}^{q} [\hat{m}_w^w(\tau_1, \tau_2, \tau_3) - \hat{m}_w^w(\tau_1, \tau_2, \tau_3)]^2,
\]

(1.11)

where \( q \) is the sample number of the wavelet. Equation (1.11) can be solved by updating the obtained constant-phase wavelet with an iterative linear inversion method.

Using the constant-phase wavelet shown in Figure 1.2 (c) as the initial model, the resultant wavelet obtained by mixed-phase wavelet estimation is shown in Figure 1.3.

![Figure 1.3 The mixed-phase wavelet estimated. (Lu, 2010)](image)

Theoretically, a series of different wavelets can be obtained from a seismic profile. However, wavelet variation may introduce more uncertainty in further inversions. As a result, an average wavelet is estimated for the whole seismic profile in practical cases (Zhang, 2010;
Lu, 2010). Figure 1.4 shows the input seismic profile and estimated average wavelet. A constant ray parameter \( p=80\text{ms/km} \) profile is selected for general purposes. The wavelet is extracted using the high order statistics method discussed above.

![Figure 1.4](image)

Figure 1.4 (a) A field data set. (b) Wavelet extracted from the field data set.

### 1.2 Seismic reflectivity inversion

Seismic reflectivity inversion is based on the well-known convolution model. Assuming the earth is composed of a series of flat layers with certain velocity and density, the impedance contrasts between the adjacent layers cause the reflections (Yilmaz, 2001; Mousa, 2011). Seismic trace is defined as the convolution result of the reflectivity and wavelet with additive noise:

\[
d(t) = w(t)^* r(t) + n(t),
\]

where \( d(t) \) is the observed seismic trace, \( w(t) \) is the seismic wavelet, while \( r(t) \) is the reflectivity series, and \( n(t) \) is the additive noise.

The matrix-vector form of the Earth convolution model can be represented as

\[
d = Wr + n,
\]
where \( \mathbf{d} \) is the vector which represents seismic trace, \( \mathbf{r} \) is the vector of seismic reflectivity, \( \mathbf{n} \) is the vector of noise, and \( \mathbf{W} \) is the convolution matrix which is constituted by the elements of the wavelet. It is properly padded with zeros in order to express discrete convolution:

\[
\mathbf{W} = \begin{bmatrix}
w_0 & 0 & \cdots & 0 \\
w_1 & w_0 & \cdots & 0 \\
\vdots & \ddots & \ddots & \vdots \\
w_{k-1} & \cdots & w_1 & 0 \\
0 & w_{k-1} & \cdots & w_1 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & w_{k-1}
\end{bmatrix}.
\] (1.14)

Equation (1.13) is the basic equation used for the reflectivity inversion, where the wavelet is assumed to be known, and has been estimated using the method based on high-order statistics introduced in Section 1.1 (Lu and Wang, 2007; Lu, 2010).

When carrying out the deconvolution procedure based on the convolution model, since the wavelet is band-limited, the seismic trace obtained is finite and thus cannot provide a clear image of the reflectors. On the other hand, the expected reflectivity is broadband. Since we want to extract a broadband output from a band-limited input, the reflectivity which fits the seismic trace is non-unique. Proper constraints have to be applied in order to obtain suitable results.

The most fundamental method for seismic reflectivity inversion is based on the least-squares criterion, with the aim of minimising the misfit between the field observation and synthetic traces formed by the inverted reflectivity series (Robinson and Treitel, 1980). When employing the least-squares method, the noise which contaminates the data is assumed to be Gaussian white noise, and the \textit{a priori} model for the reflectivity is assumed to be Gaussian as well. The reflectivity series inverted by the least-squares method is, however, not spiky and sparse enough, thus not able to provide high resolution reflectivity images.

To improve the resolution of the least-squares solution, the \( L_1 \)-norm (Levy and Fullagar, 1981) and Cauchy norm (Sacchi and Ulrych, 1995; 1996; Sacchi, 1997 and Sacchi et al., 1998) can be applied. Both of them can provide distributions which are longer-tailed than the Gaussian distribution for seismic reflectivity sequences, and thus are able to increase the effective bandwidth of the retrieved results. \( L_1 \)-norm deconvolution can be further generalised.
as $L_p$-norm deconvolution (Debye and van Riel, 1990; Zhang, 2010) in order to obtain better results as the values of parameters can be adjusted according to different input seismic traces.

The basis pursuit (BP) algorithm, which is based on the $L_1$-norm criterion, was recently introduced for seismic reflectivity inversion (Zhang, 2010; Zhang and Castagna, 2011; Zhang et al., 2013). When this method is applied to an underdetermined system of linear equations, the solution can be exact, and in the meantime, be very sparse in the $L_1$-norm sense (Donoho and Johnstone, 1994; Chen et al., 2001). When using this method, the reflector pair from the top and bottom of a bed is treated as a superposition of even and odd dipoles. Then, a wedge reflector matrix, which is defined as a collection of these dipoles, can be used to represent the reflectivity. After that, the wedge seismic response matrix can be obtained by convolving the wavelet with the wedge reflector matrix. Seismic traces can be represented as the multiplication result of the wedge seismic response matrix and corresponding coefficients and the sparse constraint is proposed on the coefficients. After obtaining the results of the coefficients, seismic reflectivity can be calculated by multiplying the coefficients with wedge reflector matrix. This process will be discussed in detail in Chapter 2.

Mallat and Zhang first proposed the theory of signal decomposition on an over-complete dictionary and introduced the matching pursuit (MP) algorithm in 1993. Matching pursuit expands the signal into a linear combination of wavelets (or atoms) in a redundant dictionary. Gabor wavelet, Morlet wavelet and Ricker wavelet can be used to build the dictionary (Liu, 2005). It was found that Morlet wavelet can denote the energy attenuation and velocity dispersion of acoustic waves propagating through porous media (Liu and Marfurt, 2005; Wang, 2007; 2010). The MP algorithm was also employed to obtain the reflectivity series by searching for the best-fit wavelet and subtracting it from the seismic traces iteratively until the residual energy of the residual trace is below a predefined threshold (Nguyen, 2008; Nguyen and Castagna, 2010). The matching pursuit algorithm shares the same principle as the basis pursuit algorithm to decompose the input signal into a series of atoms. The difference is that matching pursuit is a path-dependent process, while basis pursuit can provide a single global solution which minimises the $L_1$-norm of the coefficient vector. As a result, basis pursuit is an optimal method, by which the inversion process is stable even when the
elements in the dictionary are not orthogonal to each other and, at the same time, the interference problem between the elements can be relieved.

Furthermore, much research has been undertaken on the methods of probability and statistics based on the pre-assumed models. All these methods first assume a pre-defined model for seismic reflectivity. Then, the probabilities of the reflectivity with different patterns are calculated and the one with the largest probability is considered as the final result. Mendel et al. (1981) modelled the reflectivity series as a Bernoulli-Gaussian process and then implemented the maximum likelihood estimation using the single most likely replacement algorithm for inverting reflectivity series (Mendel et al., 1981; Kormylo and Mendel, 1982). Methods such as stochastic expectation maximization algorithm (Celeux and Diebolt, 1985; Celeux et al., 1996) and maximum posterior mode algorithm (Chalmond, 1989; Cheng et al., 1996) based on the adaptive Gaussian mixtures model (Lavielle, 1995; Santamaria et al., 1999 and Rosec et al., 2003) have also been developed and implemented.

Apart from the numerous single-channel deconvolution methods mentioned above, several multichannel deconvolution methods have also been investigated. For example, Idier and Goussard (1993) modelled the reflectivity field as a Markov-Bernoulli random field (MBRF) since seismic reflectivity profile can be characterised by local continuity. After the modelling, the multichannel deconvolution was carried out by a suboptimal maximum a posteriori (MAP) estimator. Assuming that the reflectivity possesses a Bernoulli-Gaussian distribution which corresponds to a layered earth model, iterated window maximization method could be adopted to implement the blind multichannel deconvolution and to achieve the lateral continuity of reflectors (Kaaresen, 1997; Kaaresen and Taxt, 1998). In a multichannel blind deconvolution using dynamic programming (Heimer et al., 2007; Heimer and Cohen, 2008), although the result was more continuous, the lateral discontinuity of the layers was not considered during the deconvolution process. The Viterbi algorithm (Forney, 1973; Heimer, 2008; Heimer and Cohen, 2009) and Markov Chain Monte Carlo (MCMC) method (Ram, 2009; Ram and Cohen, 2010), which were also based on MBRF, were implemented for seismic reflectivity inversion considering the discontinuity of the structures.
1.3 Assessing the performance of seismic reflectivity inversion algorithms

To evaluate the quality of the retrieved seismic reflectivity profiles, two main aspects need to be considered:

(1) The recovery of the frequency bandwidth:
In theory, reflectivity sequences are spikes with infinite bandwidth. However, due to the restriction of the Nyquist frequency, reflectivity can be only assumed as sparse impulses with a certain probability distribution. It is known that the sharper the reflectivity is, the higher the resolution of the seismic impedance profile will be. The recovery of the reflectivity bandwidth is an important aspect to evaluate the performance of seismic reflectivity inversion algorithms.

(2) The resemblance to the original data:
The resemblance to the original data can be measured by calculating the energy difference (residual energy) between the original traces and synthetic traces. The residual energy ratio can be expressed as

$$residual\ energy\ ratio = \left| \frac{\text{input\ data\ energy} - \text{synthetic\ data\ energy}}{\text{input\ data\ energy}} \right|.$$ (1.15)

Ideally, residual energy should be only noise-related. The least-squares method can provide results with the least residual energy as it minimizes the difference between the synthetic and original traces. However, the retrieved layers of the least-squares solution cannot be identified clearly, and in turn, affecting the resolution of the reflectivity and ray impedance result. If one increases the frequency bandwidth of the result by making it sparser, some small structure will be eliminated and the residual energy will be increased, while the resolution of the result can be improved. Therefore, a good balance between these two aspects has to be made during the inversion process.

Figure 1.5 shows the synthetic seismic traces, wavelet and reflectivity of the Marmousi model. From Figure 1.5 (c) it can be seen that ideally, the expected reflectivity is broadband,
while at the same time, the structures of the layers are preserved. However, in practice, real input seismic data is contaminated with noise. As a result, in the inversion process, noise may sometimes be considered as effective signal, and reflectors at wrong positions may be obtained when using the least-squares method. To reduce the effect of the noise, sparse constraint can be adopted, but sometimes the effective signal may be removed if the constraint is too strong. Until now, there is no method which clearly describes the optimal balance between the broadband and residual energy ratio quantitatively, in the field of seismic reflectivity inversion. As Wang (2011) wrote in his book, during seismic reflectivity inversion, there is a parameter which is used to control the strength of the constraint, and we can only adjust the value of this parameter to make a compromise between these two aspects.
As an example, Figure 1.6 shows the variation of the residual energy ratio by adjusting the constraint parameter as a function of the iteration step (Wang, 2011). In general, for all the evaluated constraint parameters, the first iteration gives a least-squares solution with
minimum residual energy ratio, and the second iteration yields an increased value. This is because sparse algorithms suppress small values in order to suppress the noise and increase the bandwidth of the result. After several more iterations, the residual energy ratio becomes stable and the algorithm reaches the convergence. Furthermore, it can be observed that the larger the constraint parameter is, the stronger the sparse constraint is. This results in a wider frequency bandwidth and a higher residual energy ratio, as more structures are eliminated. By choosing a small constraint parameter, the residual energy ratio can be reduced, but the effective bandwidth and noise suppression capability will be affected. Therefore, an intermediate parameter is normally selected in order to have a compromise. In this case, the constraint parameter is chosen to be 8. In this thesis, based on the input data set, the parameter that can make a compromise between the bandwidth and residual energy ratio is chosen. In order to have a fair comparison, for a certain data set, same constraint parameter used by the single-channel method is used to evaluate the performance of the developed multichannel algorithm. As will be seen in the next several chapters, multichannel algorithms can provide results with less residual energy at a relatively broad bandwidth, when comparing with the results obtained by single-channel sparse algorithms.

![Figure 1.6 Variation of the residual energy ratio by adjusting the constraint parameter as a function of the iteration step (Wang, 2011).](image-url)
1.4 Seismic impedance inversion

After obtaining the reflectivity series, ray impedance inversion can be carried out in order to interpret the lithology.

Although there are some other methods, e.g. band-limited impedance inversion, for calculating ray impedance directly from the input seismic data, we tend to extract the reflectivity series first, and then derive seismic ray impedance from the retrieved reflectivity series. The reason for using these two steps is that we can quantitatively control the quality of the inversion results in each step. Especially, the sparseness and continuity of the reflectivity series can be modified adaptively. By improving the lateral continuity of the inverted reflectivity profile, the lateral continuity of the ray impedance around the target layers will be enhanced, the distribution of the layers can be clearly presented and different lithology will be shown by the value of ray impedance. Furthermore, by obtaining high-fidelity images of reflectivity, more detailed structural information can be identified. In this section, the concept of ray impedance and its relationship with seismic reflectivity series is introduced.

Seismic acoustic impedance (AI) is a powerful tool for discriminating between reservoirs that contain hydrocarbons and water. However, when using AI in inversion, some important parameters such as S-wave velocity, shear modulus, and bulk modulus have been neglected. Seismic elastic impedance (EI) is introduced as a generalisation of AI (Connolly, 1999). However, there are still two assumptions which may not be able to simulate real-life problems. The first assumption is that the incident angle and transmitted angle are assumed to be identical, which violates the Snell’s law. The other one is that in EI there is a parameter $K=\alpha^2/\beta^2$, where $\alpha$ is P-wave velocity and $\beta$ is S-wave velocity, is assumed to be a constant. To obtain a more precise impedance function, ray impedance is introduced (Wang, 1999; 2003) to eliminate these disadvantages.

Ray impedance can be treated as the EI generalisation which honours the Snell’s law strictly. On both sides of the interface, the ray parameters ($\rho$) are set as a constant and angles are set as variables to deal with. Seismic ray impedance is first introduced by Wang (1999) by deviating from the truncated quadratic approximation of the PP-wave reflection.
coefficients expression. Assuming there are two adjacent elastic media separated by an interface, and the P-wave velocities of these two layers are defined as $\alpha_1$ and $\alpha_2$, S-wave velocities are defined as $\beta_1$ and $\beta_2$, densities are represented by $\rho_1$ and $\rho_2$, respectively, seismic reflection coefficients can be expressed as (Wang, 1999)

$$R(p) \approx R_f(p) - \frac{2\Delta \mu}{\rho} p^2,$$  \hspace{1cm} (1.16)

where $R(p)$ represents the reflectivity coefficients, and $\mu$ is the shear moduli, $\rho$ is the average density, and

$$R_f(p) = \frac{\rho_2 q_{\alpha_1} - \rho_1 q_{\alpha_2}}{\rho_2 q_{\alpha_1} + \rho_1 q_{\alpha_2}},$$  \hspace{1cm} (1.17)

in which $q_{\alpha_1}$ and $q_{\alpha_2}$ are vertical slowness, and the difference of the shear moduli can be expressed as $\Delta \mu = \mu_2 - \mu_1 = \rho_2 \beta_2^2 - \rho_1 \beta_1^2$.

If RI is assumed to have the similar property as EI, the relationship between the seismic reflectivity and ray impedance for the $i^{th}$ layer can be written as

$$R(p) = \frac{Z_{i+1}(p) - Z_i(p)}{Z_{i+1}(p) + Z_i(p)} \approx \frac{1}{2} \ln \frac{Z_{i+1}(p)}{Z_i(p)},$$  \hspace{1cm} (1.18)

where $Z_i(p)$ and $Z_{i+1}(p)$ represent the upper and lower layers’ seismic ray impedances across an interface $i$. With equation (1.16) and (1.18), an explicit definition of ray impedance can be obtained (Wang, 2003):

$$Z_i(p) = \frac{\rho_i \alpha_i}{\sqrt{1 - \alpha_i^2 p^2}} (1 - \beta_i^2 p^2)^{2 \eta^{i+2}},$$  \hspace{1cm} (1.19)

where $\eta = (\Delta \rho/\rho)/(\Delta \beta/\beta)$ is assumed to be a constant. Compared with the EI, RI can produce inversion results that allow more confident interpretations (Lu, 2010; Zhang, 2010).

In order to implement the joint inversion of PP and PS waves, Duffaut et al. (2000) proposed the converted wave elastic impedance (CEI) based on the linearized approximation of PS-wave reflectivity coefficients. Similar to the derivation of RI, Zhang (2010) derived the form of the converted PS-wave ray impedance (CRI) based on the expression of the CEI. The procedure of the PS-wave inversion is similar to that of the PP-wave inversion, with an
additional step which carries out the PS-wave event registration (Zhang and Wang, 2010). The CRI result can be compared with the RI value and the correlation between them can be studied. Once the coefficient for modification has been calculated, the inversion of RI constrained CRI can be completed. In this thesis, I will mainly focus on the inversion of RI, as our main objective is to investigate the improvement of the multichannel algorithms. In the future, multichannel CRI inversion can be implemented in order to have a better joint inversion result.

1.5 Outline of the thesis

This thesis consists of seven chapters including the introduction and conclusions. The conventional single-channel methods are summarised and discussed in Chapter 2. Several multichannel seismic inversion methods are proposed and tested from Chapter 3 to Chapter 6, followed by the conclusions in the last chapter. These algorithms are implemented in C and the field seismic data sets used during the process are all in SU (Seismic Unix) format. A brief outline of the thesis is given below.

Chapter 2, Single-channel reflectivity inversion methods. The chapter provides a detailed introduction and discussion on different methods, including the least-squares method with the Cauchy constraint, $L_p$-norm deconvolution and basis pursuit algorithm, for single-channel reflectivity inversion. As the size of the wedge dictionary constructed in the BP algorithm is much larger than the size of the input data, matrix operations are very time consuming. The BP method is accelerated by parallelisation using a Graphics Processing Unit (GPU). Due to the highly parallel structure of the GPU and the proposed full parallelisation scheme, the time consumption of the method is significantly decreased, with an overall speedup factor of 145 in the best-case scenario. This value is expected to be further increased for larger input data sets due to better load balancing. Numerical tests show the potential of using basis pursuit in practical large-scale problems.

Chapter 3, FX prediction filtering combined basis pursuit method for seismic reflectivity inversion. To obtain desired inversion results with enhanced lateral continuity,
FX prediction filtering is incorporated into the process of basis pursuit. To be specific, first the reflectivity is obtained using the conventional single-channel basis pursuit method, and then FX prediction filtering process is applied to the calculated reflectivity series. This filtered result is adopted as the new initial model to re-start the reflectivity inversion. This loop is repeated until a satisfactory result is obtained. As a result, the reflectivity profiles are cleaner and more laterally continuous.

**Chapter 4, Curvelet transform enhanced $L_p$-norm deconvolution.** The curvelet deconvolution refers to seismic reflectivity inversion based on the curvelet transform. Since the curvelet transform is a multi-scale and multi-directional transform, when using it to model the reflectivity, the signal is represented effectively by large coefficients and the coefficients corresponding to random noise decay rapidly. I conduct a comparative study to investigate the performance of the curvelet deconvolution, least-squares inversion method and $L_p$-norm deconvolution. It is shown that by using the curvelet deconvolution, the inverted reflectivity profiles have better noise suppression performance and higher resolution than those obtained by the least-squares method. Its results also excel those obtained by the $L_p$-norm deconvolution in terms of the lateral continuity because the curvelet deconvolution is a multichannel method. Since the curvelet deconvolution offers a good trade-off between the lateral continuity and sparseness, I use the result obtained by the curvelet deconvolution, instead of the least-squares solution, as the initial model for enhancing the $L_p$-norm deconvolution.

**Chapter 5, Multichannel reflectivity inversion with the Cauchy constraint.** Due to complicated structures and low signal-to-noise ratios (SNRs) of some input seismic traces, the conventional Cauchy constrained deconvolution method, which is performed in a trace-by-trace manner, may produce results that the continuity along the reflectors is deteriorated. In this chapter, I develop a multichannel algorithm with the Cauchy constraint to perform the inversion process. The validity and feasibility of this method are demonstrated by the field data set, showing that the proposed method can effectively improve the lateral continuity, provide a clearer description of the structure of the reflectivity profiles, and reduce the residual energy ratio. Two enhanced methods are then developed based on the originally proposed multichannel method and can achieve better results.
Chapter 6. Seismic impedance inversion in a multichannel manner. Seismic ray impedance inversion, using the generalised linear inversion (GLI) algorithm, is also implemented conventionally in a trace-by-trace fashion. In this thesis, I apply a multichannel algorithm for seismic ray impedance inversion, during which the multichannel prediction filter is incorporated into the conventional GLI method similar to the mechanism of the multichannel reflectivity inversion. Several tests are carried out on field data sets to prove the performance of the multichannel method.

Chapter 7 Conclusions and future work. In this chapter, conclusions corresponding to my research will be provided and some prospects for future research on multichannel seismic inversion methods and high performance computing algorithms for geophysics will be addressed.
Chapter 2  Single-channel reflectivity inversion methods

As the starting point and foundation of the multichannel reflectivity inversion methods, the theories and implementation details of several popular single-channel seismic reflectivity inversion methods are discussed in this chapter. These methods include the least-squares inversion with the Cauchy constraint, $L_p$-norm deconvolution, and basis pursuit method.

2.1 Least-squares inversion with the Cauchy constraint

In the least-squares inversion method, the error between the actual seismic data and desired output is minimised in the least-squares sense. Based on the convolution model, a seismic trace can be expressed as the convolution of seismic wavelet and reflectivity:

$$d_k = \sum_j w_j r_{k-j} + n_k.$$  \hspace{1cm} (2.1)

To minimise the difference between the field and synthetic traces, an objective function can be defined as

$$J = \sum_k \left( d_k - \sum_j w_j r_{k-j} \right)^2.$$  \hspace{1cm} (2.2)

By setting $\partial J / \partial r_j = 0$, an equation system in the matrix-vector form can be obtained:

$$W^T W r = W^T d.$$  \hspace{1cm} (2.3)

If matrix $W$ is singular or nearly singular, the square matrix $W^T W$ is not invertible. A small perturbation factor $\mu$ has to be added to the diagonal of $W^T W$ so that the inverse of matrix $(W^T W + \mu I)$ exists and the inversion is stabilised. In the context of signal processing, this process is called as pre-whitening (Robinson and Treitel, 1980). The final form of the least-squares solution can be written as

$$r = (W^T W + \mu I)^{-1} W^T d,$$  \hspace{1cm} (2.4)

where $\mu$ is the pre-whitening parameter.
Figure 2.1 shows the Marmousi model without any noise and the reflectivity obtained by the least-squares method. Seismic wavelet of this data set is shown in Figure 1.5 (b). The residual energy ratio corresponding to this result is only 0.85%. It can be observed that the resolution of the result is not very high since the sparse constraint is not applied.

Figure 2.1 (a) Seismic traces of the Marmousi model. (b) Seismic reflectivity obtained by the least-squares inversion.
Figure 2.2 (a) and (b) illustrate a field data set and the corresponding reflectivity profile inverted by the least-squares inversion. This data set is a zoomed-in version of the data shown in Figure 1.4 (a), with the time from 0.2 to 0.6 second and trace number from 400 to 500. It is also a CRP profile with a ray parameter of 80ms/km. Figure 2.2 (c) is the synthetic profile built by convolving the estimated mixed-phase wavelet shown in Figure 1.4 (b), with the least-squares solution, while Figure 2.2 (d) is the difference between the synthetic and original seismic data sets. The value of the residual energy ratio is only 1.36%. However, the obtained reflectivity result does not agree with the sparse and spiky assumptions of the reflectivity series and thus cannot be used for blocky (e.g., each layer can be clearly identified) impedance inversion. To solve this problem, different kinds of improvements have been proposed.
Figure 2.2 (a) A field data set. (b) Seismic reflectivity obtained by the least-squares inversion. (c) Synthetic seismic profile. (d) Difference between the original and synthetic seismic profiles.

To improve the result, the Cauchy constraint can be employed for the inversion problem (Amundsen, 1991). The generalised Cauchy probability density function (PDF) is defined as

\[
p(x) = \frac{1}{\pi \lambda^2 + (x-x_0)^2},
\]

(2.5)

where \( x_0 \) is the parameter which specifies the location of the peak value of the distribution, and \( \lambda \) is used to specify the half width at half maximum value of the probability density. Figure 2.3 shows the comparison between the probability density functions of the Cauchy and Gaussian distributions schematically; they share the same central probability density. Since the Cauchy distribution is longer-tailed than the Gaussian distribution, it is closer to the real seismic reflectivity distribution (Walden and Hosken, 1986). Thus, the Cauchy distribution is more suitable than the Gaussian distribution for seismic reflectivity inversion.
If the components of an $N_t$-dimensional event $\mathbf{x} = \{x_0, x_1, ..., x_{N_t-1}\}$ are independent with each other, the joint probability of the event will be

$$p(\mathbf{x}) = p(x_0)p(x_1)\cdots p(x_{N_t-1}).$$

(2.6)

If the reflectivity coefficients are assumed to be Cauchy distributed and have a zero mean, the joint possibility of the $N_t$-dimensional reflectivity sequences $\mathbf{r}$ can be expressed as

$$p(\mathbf{r}) = \prod_{i=0}^{N_t-1} \frac{1}{\pi \lambda^2 + r_i^2}.$$  

(2.7)

By combining the Cauchy prior and Gaussian data noise likelihood using Bayes’ theorem, the posterior distribution of the reflectivity $\mathbf{r}$ for seismic signal $\mathbf{d}$ can be expressed as (Tarantola and Valette, 1982; Tarantola, 1987; Scales andTenorio, 2001; Ulrych et al., 2001)

$$p(\mathbf{r} | \mathbf{d}) \propto \exp\left[-\frac{1}{2}(\mathbf{d} - \mathbf{W}\mathbf{r})^T \mathbf{C}_d^{-1}(\mathbf{d} - \mathbf{W}\mathbf{r}) + \sum_{i=0}^{N_t-1} \ln\left(\frac{1}{\pi \lambda^2 + r_i^2}\right)\right],$$

(2.8)

where $\mathbf{C}_d$ is the data covariance matrix. With equation (2.8), it can be observed that the objective function that needs to be minimised is

$$J = \|\mathbf{d} - \mathbf{W}\mathbf{r}\|^2_2 + \mu_{\text{cauchy}} \left[ N_t \ln(\pi\lambda) + \sum_{i=0}^{N_t-1} \ln(1 + \frac{r_i^2}{\lambda^2}) \right].$$

(2.9)

By minimising the objective function, the least-squares solution with the Cauchy constraint can be represented by

$$\mathbf{r} = (\mathbf{W}^T\mathbf{C}_d^{-1}\mathbf{W} + \mu_{\text{cauchy}} \frac{2}{\lambda^2} \mathbf{D})^{-1} \mathbf{W}^T \mathbf{C}_d^{-1} \mathbf{d},$$

(2.10)
where \( \lambda \) is called the Cauchy distribution parameter, which can be used to control the sparseness of \( r \), \( \mu_{\text{cauchy}} \) is the weighting factor of the Cauchy constraint, and \( D \) is a diagonal matrix derived by Wang (2003):

\[
D = \text{diag}\{1 + (r_i^2 / \lambda^2)\}^{-1}, \quad i = 0, \ldots, N_t - 1.
\] (2.11)

This problem (2.10) can be solved iteratively using the conjugate gradient (CG) method. The least-squares solution is used as the initial model, and then the reflectivity calculated each time is substituted into equation (2.11) for the next iteration.

Figure 2.4 shows the inverted result for the Marmousi model using the Cauchy constrained algorithm. It can be seen that the resolution of the reflectivity profile is significantly improved, while the residual energy ratio of this result is 0.87%, which is only slightly higher than that obtained by the least-squares method. By comparing the result of Figure 2.1 (b) and Figure 2.4, it can be concluded that when the input data is clean, the residual energy ratios associated with the least-squares and Cauchy constrained methods are both very small, verifying that the structures have been well preserved. On the other hand, the Cauchy constrained method can improve the frequency bandwidth of the reflectivity profile effectively, which will benefit the following layered ray impedance inversion.

![Figure 2.4 Seismic reflectivity obtained by the least-squares inversion with the Cauchy constraint.](image-url)
Figure 2.5 shows the seismic reflectivity profile obtained from the field data set shown in Figure 2.2 (a) using the least-squares method with the Cauchy constraint. The residual energy ratio associated with this result is 9.25%. When compared with the least-squares inversion, although the residual energy ratio has increased, the result is more sparse and spiky so that the layers can be clearly identified. It means that the frequency bandwidth of the inverted reflectivity is increased. Having reflectivity profiles with wider bandwidth, the image resolution of the following inverted ray impedance can be improved, and, as a result, more detailed structural information can be identified.

![Seismic reflectivity obtained by the least-squares inversion with the Cauchy constraint.](image)

**Figure 2.5** Seismic reflectivity obtained by the least-squares inversion with the Cauchy constraint.

### 2.2 $L_p$-norm deconvolution

The $L_p$-norm deconvolution is also an alternative to enhance the standard least-squares method. Different norms correspond to different distributions: the $L_\infty$-norm corresponds to a uniform distribution, the $L_2$-norm corresponds to a Gaussian distribution, the $L_1$-norm corresponds to an exponential distribution and the $L_0$-norm is associated with a variable that
is sparsely distributed (Debeye and van Riel, 1990). When using the $L_p$-norm as the a priori knowledge of the reflectivity distribution, the density function of an event $x$ can be expressed as

$$
\rho(x) = \exp \left\{ -\frac{1}{2} \langle |x|^{p/2} \rangle^T W_x \langle |x|^{p/2} \rangle \right\},
$$

(2.12)

where $W_x$ represents a weighting matrix. The formula of a general $L_p$-norm deconvolution problem can be written as

$$
\mu L_p(r) + L_q(n),
$$

(2.13)

where $L_q(n)$ represents the $L_q$-norm of the noise, $L_p(r)$ represents the $L_p$-norm of the reflectivity, and $\mu$ is a trade-off parameter which reflects the compromise between the accuracy and sparsity of the reflectivity. With equation (2.13), in the time domain, the objective function of the $L_p$-norm deconvolution can be expressed as

$$
J = L_q(d - Wr) + \mu L_p(r).
$$

(2.14)

When the reflectivity is sparse and spiky and the noise is zero mean and possesses a Gaussian distribution, it is better to solve the problem by minimising the $L_0$-norm of the reflectivity and the $L_2$-norm of the noise. However, the $L_0$-norm minimization process is difficult to implement in an algorithm. It is feasible to use the $L_1$-norm minimization as an approximate optimal implementation (Donoho, 2004; Ramirez et al., 2013). For the $L_1$-norm deconvolution, the parameter $q$ and $p$ in equation (2.14) are set as 2 and 1 separately:

$$
J = L_2(d - Wr) + \mu L_1(r).
$$

(2.15)

Although it can provide a favourable result of reflectivity function by overcoming the adverse effects of band-limitation, sometimes $L_1$-norm may not be ideal for all kinds of input seismic traces.

For a general $L_p$-norm deconvolution, $p$ is set as a variable, instead of using $L_1$-norm directly. The objective function of the practical $L_p$-norm deconvolution can be written as

$$
J = \|d - Wr\|_p^2 + \mu_p \|r\|_p,
$$

(2.16)

where $\mu_p$ is the weighting factor of the $L_p$-norm constraint. The solution can be represented as

$$
r = (W^T C_d^{-1} W + \mu_p \|r\|^{p-2})^{-1} W^T C_d^{-1} d.
$$

(2.17)
The result of the \( L_p \)-norm deconvolution is calculated in an iterative way similar to the least-squares reflectivity inversion with the Cauchy constraint. In Zhang’s thesis (Zhang, 2010), cases of \( p \) between 0.1 and 0.01 were tested and conclusions can be obtained that the sparseness and frequency spectrum are not improved obviously when \( p \) equals to 0.01, while the residual energy is increased significantly. As a result, the case with \( p=0.1 \) is ideal to obtain an optimised solution.

The weighting factor \( \mu \) for the sparseness term is another criterion. When it is small, a better performance in high frequency and a worse result in low frequency can be obtained and vice versa when it is large. Numerical experiments by Zhang show that it is suitable to choose a weighting factor between 0.2 and 0.1. In real case situations, experiments should be carried out to find the optimal values of these parameters.

Figure 2.6 is the reflectivity result of the Marmousi model inverted by \( L_p \)-norm deconvolution. The residual energy ratio is 0.88%. It can also be concluded that \( L_p \)-norm deconvolution can provide inverted result with high resolution and low residual energy ratio when the input seismic data set is not contaminated by noise.

![Figure 2.6](image)

**Figure 2.6** Seismic reflectivity obtained by the \( L_p \)-norm deconvolution.
Figure 2.7 is the reflectivity retrieved by the $L_p$-norm deconvolution from the same field data we used in Section 2.1, the corresponding residual energy ratio is 9.33%. It can be observed that the results obtained by the $L_p$-norm deconvolution and least-squares method with the Cauchy constraint are very similar to each other; both of them can provide good results with wide bandwidth at a relatively low residual energy ratio.

Figure 2.7 Seismic reflectivity obtained by the $L_p$-norm deconvolution.

2.3 Basis pursuit method

Basis pursuit is a method for decomposing a signal into an optimal superposition of dictionary elements. It should be noted that “optimal” means having the smallest $L_1$-norm of the coefficients among all such decompositions. This method was originally applied to underdetermined systems of linear equations which should be satisfied and, in the meantime, the solution should be very sparse (Chen, 1995).

To implement the BP method for the reflectivity inversion, first, the seismic profile can be seen as consisting of a number of beds (Zhang, 2010; Zhang and Castagna, 2011). Since
the reflections from the top and bottom of a bed can be considered as reflector pairs, dipole decomposition can be applied. All reflector pairs will be divided into the superposition of even \((r_e)\) and odd \((r_o)\) pairs, as shown in Figure 2.8.

![Figure 2.8](image)

**Figure 2.8** The real reflector pair \(r_1\) and \(r_2\) can be represented as the weighted sum of even and odd pairs.

Assuming the sample rate is \(\Delta t\) and the time thickness between the two reflectors in Figure 2.8 is \(n\Delta t\), even and odd dipoles can be expressed as

\[
\begin{align*}
  r_e(t) &= \delta(t) + \delta(t + n\Delta t) \\
  r_o(t) &= \delta(t) - \delta(t + n\Delta t) \\
\end{align*}
\]  

(2.18)

Taking the time scale of the whole trace into consideration, the reflector dipoles in equation (2.18) can be rewritten as

\[
\begin{align*}
  r_e(t, l, n) &= \delta(t - l\Delta t) + \delta(t - l\Delta t + n\Delta t) \\
  r_o(t, l, n) &= \delta(t - l\Delta t) - \delta(t - l\Delta t + n\Delta t),
\end{align*}
\]  

(2.19)

where \(l\) represents the time scale, ranging from the first sample point to the last one.

By applying dipole decomposition, the interference of the seismic response between the layers can be avoided, providing an increased resolution for the retrieved result. The reflectivity series then can be represented as

\[
r(t) = \sum_{n=0}^{n_{\text{max}}} \sum_{l=1}^{l_{\text{max}}} \left( a_{n,l} r_e(t, l, n) + b_{n,l} r_o(t, l, n) \right),
\]

(2.20)

where \(a_{n,l}\) and \(b_{n,l}\) are the coefficients corresponding to even and odd reflectivity pairs, respectively. To include all possible layer thickness, \(n\) ranges from zero to a constant \(n_{\text{max}}\).
which represents the maximum layer thickness of the pairs. A single impulse is also included in order to properly represent the layer thickness which is larger than the maximum value.

Equation (2.20) can be rewritten into the matrix-vector equation form, which can be expressed as

\[
\mathbf{r} = a_{11} + a_{12} + \ldots + a_{2n} 1 + \ldots + b_{11} 0 + b_{12} + \ldots + b_{2n} -1 + \ldots
\]

\[
\begin{bmatrix}
1 & 0 & \ldots & 1 & 0 & \ldots \\
1 & 1 & \ldots & -1 & 1 & \ldots \\
0 & 1 & \ldots & 0 & -1 & \ldots \\
\vdots & \vdots & \ddots & \vdots & \vdots & \ddots \\
\vdots & \vdots & \ddots & \vdots & \vdots & \ddots \\
\vdots & \vdots & \ddots & \vdots & \vdots & \ddots \\
\end{bmatrix}
\begin{bmatrix}
a_{11} \\
a_{12} \\
\vdots \\
b_{11} \\
b_{12} \\
\vdots \\
\end{bmatrix}
\]

(2.21)

As a result, we can have the equation:

\[
\mathbf{r} = \mathbf{G}' \mathbf{m},
\]

(2.22)

where \( \mathbf{r} \) represents the vector of the reflectivity, \( \mathbf{m} \) is the coefficient vector, constituted by \( a_{n,l} \) and \( b_{n,l} \), and \( \mathbf{G}' \) is called as the wedge reflector matrix, which is formed by even and odd pairs. The wedge reflector matrix is a collection of dipole reflectors with the interval increasing from zero to a specific interval. Since the wedge model is an over-complete dictionary for decomposing the reflectivity, the coefficient vector \( \mathbf{m} \) is sparser than \( \mathbf{r} \), which means that \( \mathbf{m} \) is suitable to be solved under the \( L_1 \)-norm constraint.

By convolving both sides of equation (2.22) with the known wavelet, a matrix-vector equation of a seismic trace can be obtained:

\[
\mathbf{d} = \mathbf{G} \mathbf{m},
\]

(2.23)
where \( \mathbf{d} \) represents the vector of seismic trace, and \( \mathbf{G} \) is the wedge seismic response matrix calculated by convolving the known wavelet with \( \mathbf{G}' \).

In the inversion process, the initial value of the coefficient vector \( \mathbf{m} \) is calculated as the solution of the least-squares method:

\[
\mathbf{m} = (\mathbf{G}^T \mathbf{G})^{-1} \mathbf{G}^T \mathbf{d}.
\]

Given this initial model, \( \mathbf{m} \) is solved by minimising the objective function below iteratively:

\[
J = \| \mathbf{d} - \mathbf{Wr} \|_2 + \lambda \| \mathbf{m} \|_1
= \| \mathbf{d} - \mathbf{WG}' \mathbf{m} \|_2 + \lambda \| \mathbf{m} \|_1,
\]

where \( \lambda \) is the trade-off parameter for controlling the balance between the data misfit and \( L_1 \)-norm model constraint. Note that the value of \( \lambda \) has to be chosen carefully according to the input traces.

In the BP algorithm, equation (2.25) is first transformed into a linear programming (LP) problem, and then a method called primal-dual log-barrier LP algorithm (Chen et al., 2001; Kim et al., 2007) is applied to solve it. Once the model vector \( \mathbf{m} \) is obtained, the reflectivity series can be generated through equation (2.22).

Unlike \( L_2 \)-norm problems, no direct solution is available for \( L_1 \)-norm problems, following translations can be made in order to change the form of the objective function:

\[
\mathbf{m} \equiv \mathbf{u} - \mathbf{v}, \quad \mathbf{x} = \begin{pmatrix} \mathbf{u} \\ \mathbf{v} \end{pmatrix}, \quad x_i \geq 0. \tag{2.26}
\]

Then

\[
\lambda \| \mathbf{n} \|_1 = \lambda \| \mathbf{u} - \mathbf{v} \|_1 \leq \lambda (\| \mathbf{u} \|_1 + \| \mathbf{v} \|_1) = \lambda (\mathbf{u} + \mathbf{v}) = \mathbf{c}^T \mathbf{x},
\]

where \( \mathbf{c}^T = \lambda [1 \ 1 \ \cdots] \) is the vector which has the same number of elements of \( \mathbf{x} \). As a result, the objective function can be rewritten as

\[
J = \| \mathbf{d} - \mathbf{Ax} \|_2 + \mathbf{c}^T \mathbf{x}, \tag{2.28}
\]

where \( \mathbf{A} = [\mathbf{G}, -\mathbf{G}] \).

With equation (2.28), the original inversion problem can be reformulated as the standard form of a linear program:
This equation can be called the *primal* linear program, which is equivalent to a *dual* linear program:

\[
\begin{align*}
\max d^T y \ s.t. \ A^T y + z &= c, \quad z_i \geq 0, \\
\end{align*}
\]

with \(y\) a vector that has the same number of elements as seismic trace vector, \(z\) is a vector having the same number of elements as \(x\). In this primal and dual equation system, \(x\) is a primal variable while \(y\) and \(z\) are called dual variables.

For this *primal-dual* problem, we have three parameters:

\[
\begin{align*}
\text{Primal infeasibility} & \quad \|d - Ax\|_2, \\
\text{Dual infeasibility} & \quad \|c - z - A^T y\|_2, \\
\text{Duality gap} & \quad c^T x - d^T y,
\end{align*}
\]

where *Duality gap* represents the difference between the primal objective and the dual objective.

Theoretically, \((x^*, y^*, z^*)\) can be the solution for this linear optimization problem if and only if *primal infeasibility*, *dual infeasibility* and *duality gap* are all zero. However, such solution is normally very difficult to achieve if this problem is solved by numerical techniques. Therefore, in practice, suitable thresholds of *primal infeasibility*, *dual infeasibility* and *duality gap* should be set. The values of \((x, y, z)\) that can make the values of these three parameters all smaller than the predefined thresholds are treated as the optimal results of the problem.

Using the primal-dual log-barrier LP algorithm, two small perturbation parameters \(\delta\) and \(\gamma\) are introduced and the first-order necessary conditions of this problem can be written as

\[
F(x, y, z) = \begin{cases}
A^T y + z - \gamma^2 x - c \\
A x + \delta^2 y - d \\
Z x - \mu e
\end{cases} = 0,
\]
where $Z$ is a diagonal matrix composed of the elements of $z$. With the Newton-Gaussian step method, this linear system can be solved iteratively by updating the solution with Newton search directions ($\Delta x$, $\Delta y$, $\Delta z$) satisfying:

\[
F(x, y, z) + (\Delta x, \Delta y, \Delta z) \begin{bmatrix}
\frac{\partial F}{\partial x}(x, y, z) \\
\frac{\partial F}{\partial y}(x, y, z) \\
\frac{\partial F}{\partial z}(x, y, z)
\end{bmatrix} = 0.
\] (2.33)

With equation (2.33), we obtain:

\[
\begin{bmatrix}
-\gamma^2\Delta x + A^T\Delta y + \Delta z \\
A\Delta x + \delta^2\Delta y \\
Z\Delta x + X\Delta z
\end{bmatrix} = \begin{bmatrix}
t \\
r \\
v
\end{bmatrix},
\] (2.34)

with

\[
t = c + \gamma^2 x - z - A^T y \\
r = d - Ax - \delta^2 y \\
v = \mu e - Zx
\] (2.35)

Initialising using the current solution estimate equation (2.24), the updating steps can be solved from the following equation:

\[
(ADA^T + \delta^2 I)\Delta y = r - AD(X^{-1}v - t)
\]

\[
\Delta x = DA^T \Delta y + D(X^{-1}v - t)
\] (2.36)

\[
\Delta z = X^{-1}(v - Z\Delta x)
\]

\[
D = (X^{-1}Z + \gamma^2 I)^{-1},
\]

where $X$ is a diagonal matrix composed of the elements of $x$. ($x$, $y$, $z$) are updated with ($\Delta x$, $\Delta y$, $\Delta z$) Newton-Gaussian step and will approach to optimal results during this process; the values of primal infeasibility, dual infeasibility and duality gap will decrease. When they are all under the predefined thresholds, the iteration process is terminated. With the obtained $x$, $m$ can be calculated by subtracting the second half part of $x$ ($v$) from the first half part of $x$ ($u$). After that, basis pursuit for the reflectivity inversion has been successfully implemented.
In order to validate the performance of the conventional BP method, a field data set, as shown in Figure 2.9 (a), is used as an example, and the results after different iterations is given in Figure 2.9 (c)-(f). To evaluate the performance of this method, the variation of the residual energy against iteration steps is provided, as seen in Figure 2.10. It is observed that the result after the first iteration step produces the minimum residual energy ratio since it is the least-squares solution, after that there is a jump in the amount of residual energy in the next several steps. As discussed in Section 1.3, this is because in order to make the result sparse and eliminate the noise, the BP method suppresses small structures. By changing the value of $\lambda$ in equation (2.25), a figure similar to Figure 1.6 can be obtained, and the value that can make a compromise between the residual energy ratio and frequency bandwidth has been chosen. After several more iterations, the residual energy becomes stable, which means the procedure has converged and the iteration can be stopped, and we can see the obtained result is continuous and has a high resolution. Since this input field data set contains some noise, the residual energy ratio is relatively high.

![Trace number](image)

(a) Trace number

(b) Time (ms)

(c) Trace number

(d) Time (s)
Figure 2.9 (a) A field data set. (b) Wavelet of the field data set. (c) Seismic reflectivity obtained by the conventional BP method after 3th iteration. (d) Seismic reflectivity obtained by the conventional BP method after 5th iteration. (e) Seismic reflectivity obtained by the conventional BP method after 7th iteration. (f) Seismic reflectivity obtained by the conventional BP method after 9th iteration.
What is more, Figure 2.11 illustrates the amplitude spectra of the inverted reflectivity series after different numbers of iterations. The black curve represents the spectrum of the input seismic traces and all the other coloured curves are associated with the BP method solutions after different iterations (from bottom to top denoting iteration step from 1 to 7). From the figure it can be seen that the high frequency components of the inverted reflectivity are compensated gradually as the iteration number increases. By analysing these figures it can be concluded that when the input data has a relatively high signal-to-noise ratio (SNR), the BP method is able to provide a sparse and continuous result, and also improve the effective bandwidth of the result.

Figure 2.11 The amplitude spectra of seismic reflectivity sequences against number of iterations (black curve is associated with the original trace, the other curves from bottom to top denotes an iteration number from 1 to 7).
Figure 2.12 provides the inverted result of the Marmousi model by the BP method. Similar to the results by the Cauchy constrained and $L_p$-norm deconvolution, the inverted reflectivity has a broad bandwidth and a low residual energy ratio, which is only 0.93%. All these results of the Marmousi model show that the inversion methods with the sparse constraint are able to achieve inverted reflectivity with high resolution. At the same time, the structures are well preserved when the input data has a high SNR.

![Seismic reflectivity obtained by the conventional BP method.](image)

**Figure 2.12** Seismic reflectivity obtained by the conventional BP method.

### 2.4 GPU-accelerated basis pursuit method

As mentioned above, in the basis pursuit algorithm, even and odd dipoles are composed to form a dictionary for decomposing reflector pairs. However, since the size of the dictionary is huge, a large-scale wedge reflector matrix has to be constructed and used for later calculation. Therefore, the computational time is considerable for solving the reflectivity inversion problems using the BP method.

In order to reduce the calculation time and to improve the efficiency of the algorithm, a straightforward and efficient way is to develop a parallel code for the original method. Among various possible implementations for high performance computing, the Graphics
Processing Unit (GPU) is a good candidate due to its highly parallel structure. It is much more effective to process large blocks of data than the general-purpose Central Processing Unit (CPU). GPU has a high computational density and memory bandwidth, and was first popularized by NVIDIA as GeForce 256 in 1999. It was originally used for real-time rendering to relieve the stress of the CPU. In 2006, NVIDIA introduced the first GPU based on Tesla architecture (G80), and it was no longer confined to manipulating graphics.

In 2007, NVIDIA released Compute Unified Device Architecture (CUDA), a parallel computing architecture for NVIDIA GPUs implementation. It is a kind of Application Programming Interface (API) which supports standard languages such as C and FORTRAN to be used on the GPU and it is supported by most operating systems (OSs) such as Windows, Mac OS and Linux. As a parallel computing platform and programming model, CUDA is often adopted for GPU-based code implementation. It enables high-level languages to be used for the GPU related programming and it allows C programs capable of taking advantage of the GPU’s ability to operate on large matrices in parallel, while still making use of the CPU when it is necessary (Sanders and Kandrot, 2011).

CUDA Basic Linear Algebra Subroutines (CUBLAS) is an implementation of BLAS (Basic Linear Algebra Subprograms) library on top of CUDA runtime. It allows users to get access to the computational resources of the GPU in a more convenient way. It should be noted that in order to call CUBLAS functions, the input matrix must be first transformed into vector with column major in the GPU memory space. The obtained result is also a column-major stored vector, and must be transformed back to matrix form with row-major storage before being used by other non-CUDA functions (CUBLAS Library User Guide, 2013).

Nowadays, the GPU-based parallel technique has gained broad attention in computational physics, for its suitability for floating point operation and parallel operation. In the area of geophysics and geoscience, applications of GPUs are also of great interest. For example, a GPU-based algorithm has been developed for solving 3D anisotropic elastic wave equations (Weiss and Shragge, 2013). By using a GPU, the modelling of seismic data for 3D anisotropic elastic models can have less computation time and lower hardware requirement. Moreover, 3D Fourier migration has been employed using the GPU to get a satisfactory result
with much lower time consumption (Zhang et al., 2009). Multiple prediction through inversion (MPI) based on the GPU has also been implemented by Zhang and Wang (2012).

The difference between the CPUs and GPUs is as illustrated in Figure 2.13. For the former, only a few cores are employed in the structures, making them better to be adopted for serial processing. GPUs contain more cores when compared with the CPUs, enabling them to handle multiple tasks simultaneously and efficiently. Due to their different features, it is powerful to combine these two types of processing units together in many algorithms, where the sequential part is processed by the CPUs and the compute intensive functions can be allocated to the GPUs.

![Figure 2.13 The comparison of the number of cores of the CPU and GPU (Adapted from NVIDIA’s website).](image)

In this section, we implement a parallel BP algorithm based on the GPU technique. The flowchart of the GPU accelerated seismic reflectivity inversion using the basis pursuit method is shown in Figure 2.14 (Wang and Wang, 2014a).
Figure 2.14 Workflow of the GPU accelerated seismic reflectivity inversion using the basis pursuit method.

As discussed in Section 2.3, the core problems to be solved during the process of the BP method are the calculation of the initial reflectivity model by the least-squares method:

\[ m = (G^T G)^{-1} G^T d, \]  

(2.37)

and the calculation of Newton search directions \((\Delta x, \Delta y, \Delta z)\):

\[ (ADA^T + \delta^2 I)\Delta y = r - AD(X^{-1}v - t) \]

\[ \Delta x = DA^T \Delta y + D(X^{-1}v - t) \]  

(2.38)

\[ \Delta z = X^{-1}(v - Z\Delta x) \]

\[ D = (X^{-1}Z + \gamma^2 I)^{-1}, \]

where

\[ t = c + \gamma^2 x - z - A^T y \]

\[ r = d - Ax - \delta^2 y \]

\[ v = \mu e - Zx \]

(2.39)

In equation (2.37), if we assume that the maximum interval of the dipoles is \(N\), and the number of sample points of the input data is \(nt\), the size of matrix \(G\) will be \(nt \times [(2N + 1) \cdot nt - (N + 1) \cdot N]\), which is much larger than the size of the input data.
Moreover, in equation (2.38) and (2.39), since matrix $A$ is constructed by two $G$s, the size of it is $nt \times [2 \cdot (2N + 1) \cdot nt - 2 \cdot (N + 1) \cdot N]$, which is double in the column size when compared with matrix $G$. Therefore, the calculation time for the matrix multiplications (MMs) and matrix-vector multiplications (MVMs) in the iteration process is considerably long, especially for solving equation (2.38).

These operations can be fully parallelized using CUBLAS. First of all, the MMs can be parallelized by calling the function `cublasSgemm` in CUBLAS and the MVMs can be parallelized by using `cublasSgemv`.

Besides, the conjugate gradient (CG) method, which is applied to solve equation (2.37) and (2.38) iteratively, should also be parallelized. In addition to the two functions mentioned above, `cublasSaxpy` is needed in the CG method to multiply a vector with a scalar and then add it to another vector; `cublasScal` is applied to scale the vector by a defined scalar and `cublasSdot` is used to compute the dot product of two vectors. The pseudo codes of the non-parallelized CG method and parallelized CG method for solving problem $Ax=b$ are given below:

Non-parallelized CG method

$p^{(0)} = r^{(0)} = b - Ax^{(0)}$ for some initial guess $x^{(0)}$, then for $k=1, 2, 3…$

$u^{(k)} = Ap^{(k)}$

$\alpha_k = \frac{(r^{(k)}, r^{(k)})}{(p^{(k)}, u^{(k)})}$

$x^{(k+1)} = x^{(k)} + \alpha_k p^{(k)}$

$r^{(k+1)} = r^{(k)} - \alpha_k u^{(k)}$

$\beta_k = \frac{(r^{(k+1)}, r^{(k+1)})}{(r^{(k)}, r^{(k)})}$

$p^{(k+1)} = r^{(k+1)} + \beta_k p^{(k)}$

convergence check

Parallelized CG method

Initialise the method, then for $k=1, 2, 3…$

`cublasSgemv(A, p^{(k)}, u^{(k)})`;
\[\alpha_k = \text{cublasSdot}(r^{(k)}, r^{(k)}) / \text{cublasSdot}(p^{(k)}, u^{(k)});\]
\[x^{(k+1)} = \text{cublasSaxpy}(\alpha_k, p^{(k)}, x^{(k)});\]
\[r^{(k+1)} = \text{cublasSaxpy}(-\alpha_k, u^{(k)}, r^{(k)});\]
\[\beta_k = \text{cublasSdot}(r^{(k+1)}, r^{(k+1)}) / \text{cublasSdot}(r^{(k)}, r^{(k)});\]
\[\text{cublasScale}(\beta_k, p^{(k)});\]
\[p^{(k+1)} = \text{cublasSaxpy}(1, r^{(k)}, p^{(k)});\]

As the GPU version basis pursuit algorithm has been implemented, data with different sizes are adopted to test the performance of the parallel code, comparing with the original CPU version.

Two computing environments are used to test the serial and parallel basis pursuit code, respectively. The serial code is performed on the platform with an Intel Xeon Quad-Core X5647 (2.93 GHz) CPU and 3 GB dedicated memory (RAM); while the GPU-accelerated BP algorithm is tested using a 448 cores NVIDIA Tesla C2050 GPU.

The first numerical experiment employs matrices with different sizes, from 200×200 to 1600×1600, to evaluate the performance of the CG method. The maximum CG iteration number is set to 200000 in order to have a suitable calculation time for both the CPU- and GPU-based CG methods. The computational times of these two methods using matrices of different sizes are as shown in Figure 2.15 (a), (b) and (c), respectively. By comparing these figures we can observe that there is a significant decrease in the calculation time when using the GPU-parallelized CG method. Furthermore, with an increase in the matrix size, the time cost by the CPU-based CG method is growing more rapidly, exhibiting a time complexity of \(O(N)\), when compared with those obtained by the GPU-based CG method. Figure 2.15 (d) demonstrates the speedup factor as a function of the matrix size. Speedup factor is defined as the calculation time of the CPU-based code over that of the GPU-based algorithm. It can be seen that the speedup factor is improved as the matrix size increases due to a better load balancing. The maximum speedup factor for the tested data is 92. It can be concluded that the larger the input matrix is, the more efficient the GPU-based code operates when compared with the CPU-based CG code.
Figure 2.16 (a) Calculation time for the GPU-based CG method. (b) Calculation time for the CPU-based CG method. (c) Calculation time for the CPU- and GPU-based CG methods. (d) Speedup factor for the CPU- and GPU-based CG methods. Matrix sizes vary from 200×200 to 1600×1600.

Moreover, the computational consumption for an individual iteration step of the BP process is tested. The test data sets have 20 traces with the number of sample points ranging from 200 to 1600. Calculation times of the serial and parallel codes are shown from Figure 2.16 (a) to (c), while the speedup factor is further demonstrated in Figure 2.16 (d). Similar conclusion can be obtained that the GPU accelerated BP process is much more efficient than the original CPU version. And the speedup factor increases as the number of sample points of the input seismic traces increases.
Figure 2.16 (a) Calculation time for an individual iteration step of the GPU-based BP method. (b) Calculation time for an individual iteration step of the CPU-based BP method. (c) Calculation time for an individual iteration step of the CPU- and GPU-based BP methods. (d) Speedup factor for an individual iteration step of the CPU- and GPU-based BP methods. Seismic data sample points vary from 200 to 1600.

After the performance evaluation, a small real data set, which contains 20 traces and 251 sample points, is used to further investigate the performance of the GPU-accelerated BP algorithm. Figure 2.17 shows the original seismic profile, seismic wavelet obtained by high order statistics method introduced in Section 1.1, and the results obtained by the serial and parallel BP method, respectively. The calculation time of the serial and parallel codes, as well as the speedup factor, are also provided, as shown in Table 2.1. It can be seen that the result obtained by the GPU-parallelized code has the same precision as its counterpart, while the calculation time is reduced significantly, especially in the BP process which processes matrices with larger size and involves more MMs and MVMs.
Figure 2.17 (a) Input seismic traces. (b) Wavelet extracted from the small data set. (c) Seismic reflectivity obtained by the CPU-based basis pursuit algorithm. (d) Seismic reflectivity obtained by the GPU-based basis pursuit algorithm.

Table 2.1 Calculation time of the CPU, GPU-based BP algorithms and speedup factor for a small data set.

<table>
<thead>
<tr>
<th>Process</th>
<th>GPU (s)</th>
<th>CPU (s)</th>
<th>Speedup Factor (CPU/GPU)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Least-squares method (LS) for solving equation (2.37)</td>
<td>1</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Basis Pursuit (BP) for solving equation (2.38)</td>
<td>23</td>
<td>1419</td>
<td>62</td>
</tr>
<tr>
<td>Total</td>
<td>24</td>
<td>1423</td>
<td>59</td>
</tr>
</tbody>
</table>

Numerical test is also performed using a field data set with 701 traces and 226 sample points. Figure 2.18 shows the original data, the corresponding wavelet and results obtained by
different versions of codes. Table 2.2 provides the calculation time and corresponding speedup factor of these two methods. Since the sizes of matrix $G$ and $A$ are controlled by the number of sample points of the input data, and the sample points number of this real data is close to that of the small data we used above, their speedup factors are similar to each other.
Figure 2.18 (a) Input seismic traces. (b) Wavelet extracted from the field data set. (c) Seismic reflectivity obtained by the CPU-based basis pursuit algorithm. (d) Seismic reflectivity obtained by the GPU-based basis pursuit algorithm.

Table 2.2 Calculation time of the CPU, GPU-based BP algorithms and speedup factor for a field data set.

<table>
<thead>
<tr>
<th>Process</th>
<th>GPU (s)</th>
<th>CPU (s)</th>
<th>Speedup Factor (CPU/GPU)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Least-squares method (LS) for solving equation (2.37)</td>
<td>8</td>
<td>23</td>
<td>3</td>
</tr>
<tr>
<td>Basis Pursuit (BP) for solving equation (2.38)</td>
<td>1059</td>
<td>57795</td>
<td>55</td>
</tr>
<tr>
<td>Total</td>
<td>1067</td>
<td>57818</td>
<td>54</td>
</tr>
</tbody>
</table>

The performance of the developed parallel code is further tested using a large field data set with 1461 traces and 501 points, as shown in Figure 2.19 (a). The wavelet associated with this data set is shown in Figure 1.4 (b). The results obtained by the CPU- and GPU-based BP algorithm are illustrated in Figure 2.19 (b) and (c), respectively. We can hardly identify any difference between these two results, confirming that the same precision is kept for the developed BP method. The computational time of the serial and parallel codes are compared in Table 2.3. It can be concluded that with a data containing more sample points, the speedup factor of the BP process can be further improved; up to 147 can be achieved in this case because of an improved load balancing.
Figure 2.19 (a) Input seismic traces. (b) Seismic reflectivity obtained by the CPU-based basis pursuit algorithm. (c) Seismic reflectivity obtained by the GPU-based basis pursuit algorithm.
Table 2.3 Calculation time of the CPU, GPU-based BP algorithms and speedup factor for a large field data set.

<table>
<thead>
<tr>
<th>Process</th>
<th>GPU (s)</th>
<th>CPU (s)</th>
<th>Speedup Factor (CPU/GPU)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Least-squares method (LS) for solving equation (2.37)</td>
<td>62</td>
<td>398</td>
<td>6</td>
</tr>
<tr>
<td>Basis Pursuit (BP) for solving equation (2.38)</td>
<td>6394</td>
<td>937563</td>
<td>147</td>
</tr>
<tr>
<td>Total</td>
<td>6456</td>
<td>937961</td>
<td>145</td>
</tr>
</tbody>
</table>

2.5 Conclusions

This chapter has reviewed the details of several popular conventional reflectivity inversion methods, including the Cauchy constrained least-squares method, $L_p$-norm deconvolution and basis pursuit method. These methods basically are implemented in a single-channel fashion. The numerical experiments show that when the input seismic profile has a relatively high signal-to-noise ratio (SNR), all these three methods are able to provide sparse and spiky results, which can thus depict the underlying structure clearly.

Furthermore, a GPU-based parallel algorithm has been implemented for the basis pursuit method, and the efficiency of the method can be improved significantly. The GPU is a highly parallel structure which can manipulate computer image efficiently. It is much faster than the CPU because it has thousands of cores which are optimized for executing multiple tasks at the same time while the CPU only has a few cores, thus is more suitable for sequential serial processing.

In the basis pursuit method, since the dictionary used to represent the seismic reflectivity is composed of large amounts of even and odd dipoles, matrices used in this algorithm are very large and the matrix operations are tedious. As a result, the inversion process is very time-consuming. In order to decrease the computational time and therefore improve the efficiency, we have developed a GPU-accelerated BP algorithm based on CUDA. The implementation details are presented and the performance of the developed code is evaluated
by matrices and data with different sizes. What is more, several field data sets are utilized to investigate the performance of the accelerated algorithm. It can be seen that by fully parallelizing the matrix multiplications, matrix-vector multiplications and employing the parallelized conjugate gradient method, the calculation time has been significantly decreased with an overall speedup factor up to 145. This improvement shows the potential of applying the basis pursuit method for practical large-scale reflectivity inversion problems.
Chapter 3  FX prediction filtering combined basis pursuit method for seismic reflectivity inversion

When using the basis pursuit (BP) method for seismic reflectivity inversion, we use even and odd dipoles to build a dictionary for decomposing the reflector pairs (Zhang, 2010; 2011). Once the corresponding coefficients have been obtained, the reflectivity series can be calculated by multiplying the coefficients with the dictionary. Since this method is normally implemented on a trace-by-trace basis, there may be a lack of lateral coherence for the retrieved seismic reflectivity profiles. The reflectivity profiles are likely to be noisy and lateral discontinuous, especially for input data with high noise backgrounds.

FX prediction filtering has been incorporated into the conventional sparse deconvolution process. The reflectivity is obtained first by the inversion method based on the Cauchy constraint, and then FX prediction filtering process is applied on it. The filtered traces are used as the new initial model for the new iteration process (Wang et al., 2006; 2007; Wang and Sacchi, 2008). This method, called as a “multichannel” deconvolution, can provide results with better coherent information. With a similar mechanism, I attempt to improve the results obtained by the conventional BP algorithm.

This chapter first illustrates the numerical results retrieved by the conventional single-channel BP method. The performance of the BP algorithm is verified using clean synthetic data sets. In the meantime, the disadvantages of this method under the high noise background can be observed. After that, FX prediction filtering is integrated into the process of the conventional BP algorithm. A modified version of the BP method which incorporates FX prediction filtering is proposed and tested by both synthetic and real data sets. The validity and feasibility of this method will be proved, showing the inverted seismic reflectivity profiles with better lateral continuity.
3.1 Motivation

The conventional basis pursuit method for seismic reflectivity inversion has been successfully implemented in Chapter 2. Figure 3.1 (a) shows a synthetic model containing a horizontal and a dip structure, which is used as the input to test this method. Figure 3.1 (b) shows the wavelet corresponding to this synthetic data set. Figure 3.1 (c) illustrates the reflectivity profile obtained by the conventional BP method, which shows a very clear and continuous structure.

![Figure 3.1](image)

**Figure 3.1** (a) Synthetic seismogram. (b) Wavelet of the synthetic data set. (c) Seismic reflectivity obtained by the standard BP method.

However, as mentioned above, if the signal-to-noise ratio (SNR) of the input traces is low, the lateral coherence of the inverted reflectivity series may be deteriorated. As an example, external noise with an SNR of 2.0 is added to the same synthetic data, and shown in Figure
3.2 (a). The noise was added using the suaddnoise function of the Seismic Unix package (Stockwell, 1997; Cohen and Stockwell, 2008). The output noisy profile is computed by adding a scaled noise with a Gaussian probability distribution to the original signal:

\[ \text{output} = \text{signal} + \text{scale} \times \text{noise}, \]

where \( \text{scale} \) is defined as

\[ \text{scale} = \frac{\text{abs max (signal)} / \sqrt{2}}{\text{SNR} \sqrt{\text{energy per sample}}}. \]

Figure 3.2 (b), the result obtained by the standard single-channel BP method, demonstrates that the noise is largely removed while some coherent signal is also eliminated. The residual energy ratio between the synthetic profile calculated from the inverted reflectivity and the clean input data is 54.73%, which is really large. As a result, the lateral coherency, structure characterisation and noise suppression performance of the reflectivity profile are severely affected.

![Figure 3.2](image)

**Figure 3.2** (a) Synthetic seismogram with random noise (SNR=2.0). (b) Seismic reflectivity obtained by the standard BP method.
Figure 3.3 (a) gives the seismic profile obtained by adding random noise on the Marmousi model. This model has multiple layers, which is more realistic than the synthetic data set shown in Figure 3.2. The reflectivity obtained by the conventional BP algorithm is shown in Figure 3.3 (b). It can be seen that even if the SNR is not very low (SNR=10.0), the quality of the inverted result is affected, providing a noisy reflectivity profile. Furthermore, some effective structure is removed when compared with the clean Marmousi model, with a residual energy ratio of 9.34%.

**Figure 3.3** (a) Marmousi model with random noise (SNR=10.0). (b) Seismic reflectivity obtained by the standard BP method.
To improve the lateral coherency, a FX prediction filter is applied to improve the performance of the BP method. The process of the improved method follows a similar process as the structure preserving multichannel deconvolution method (Wang et al., 2006; 2007; Wang and Sacchi, 2008).

### 3.2 FX prediction filtering

A FX prediction filter is based on the assumption that a seismic image can be modelled as a composition of linearly-coherent reflections (Harrison, 1990). Under the linear assumption, the function of seismic event is predictable in the frequency domain. Thus a filter can be built to predict the lateral structure of seismic events.

If seismic events are assumed to be the sum of the convolution of each wavelet and its corresponding impulses, we have

\[
d(t, x) = \sum_{i=1}^{N_t} w_i(t) * \delta(t - g_i(x)),
\]

where \( w_i(t) \) is the temporal wavelet associated with the reflector with the index \( i \) and \( g_i(x) \) is the delay function that defines the shape of the events.

Taking the Fourier transform, we obtain

\[
d(\omega, x) = \sum_{i=1}^{N_t} W_i(\omega) e^{-j Jo_{i}(x)} ,
\]

where \( W_i(\omega) \) is the Fourier transform of the wavelet \( w_i(t) \). If the events are assumed to be linear, our model equation becomes

\[
d(\omega, x) = \sum_{i=1}^{N_t} W_i(\omega) e^{-j s_{i}x} ,
\]

in which \( s_i \) is a measure of the slope of event \( i \).

From equation (3.5) it can be observed that when assuming the seismic events are linear, the function \( d(\omega, x) \) is strictly sinusoidal and predictable in the \( x \) direction. Consequently,
the trace can be predicted by the preceding or following traces with a forward-backward prediction filter (Canales, 1984; Spitz, 1991; Wang, 1999 and Wang, 2002).

Letting the vector \( \mathbf{P} \) be the prediction operator of length \( L \) and the vector \( \mathbf{d} \) be the predicted value of \( \mathbf{d}' \), then for each frequency, we have

\[
\mathbf{d} = \mathbf{D}' \mathbf{P},
\]

where \( \mathbf{D}' \) is the convolution matrix built based on \( \mathbf{d}' \). Letting \( \mathbf{D}'' \) represent the transposed complex conjugate of \( \mathbf{D}' \), equation (3.6) can be rewritten as

\[
\mathbf{D}'' \mathbf{d} = \mathbf{D}'' \mathbf{D}' \mathbf{P}.
\]

By defining the complex autocorrelation matrix \( \mathbf{R} \) by

\[
\mathbf{R} = \mathbf{D}'' \mathbf{D}',
\]

and the complex cross-correlation matrix \( \mathbf{g} \) as

\[
\mathbf{g} = \mathbf{D}'' \mathbf{d}.
\]

Finally we can have

\[
\mathbf{g} = \mathbf{R} \mathbf{P}.
\]

Equation (3.10) can be rewritten into the matrix form of its real and imaginary parts, which is called as the Wiener problem (Treitel, 1974):

\[
\begin{pmatrix}
\mathbf{g}_\text{Re} \\
\mathbf{g}_\text{Im}
\end{pmatrix} =
\begin{pmatrix}
\mathbf{R}_\text{Re} - \mathbf{R}_\text{Im} \\
\mathbf{R}_\text{Im} \quad \mathbf{R}_\text{Re}
\end{pmatrix}
\begin{pmatrix}
\mathbf{P}_\text{Re} \\
\mathbf{P}_\text{Im}
\end{pmatrix}.
\]

\( \mathbf{P} \) can then be solved by recursive algorithms with great efficiency. LU decomposition and forward and backward substitution can be applied to solve equation (3.11).

Solving equation (3.11) will enable us to obtain the prediction filter in the \( +x \) direction. To calculate the prediction filter in the \( -x \) direction, the matrix problem turns out to be

\[
\mathbf{g}^* = \mathbf{R}' \mathbf{P}_-,\]

where \( * \) denotes the complex conjugate of a matrix, and \( \mathbf{P}_- \) represents the prediction filter in the \( -x \) direction.

Taking the complex conjugate on both sides of equation (3.12), we have

\[
\mathbf{g} = \mathbf{R} \mathbf{P}_-^*.
\]
Equation (3.13) shows that the prediction filter in the \( -x \) direction can be seen as the complex conjugate of the filter in the \( +x \) direction. As a result, the two-sided prediction filter can be built simply by expanding the one-sided operator with its complex conjugate.

### 3.3 FX prediction filtering combined BP method

With the FX prediction filter defined in the previous section, the prediction equation for seismic traces can be expressed as

\[
D_k(f) = \sum_{j=1}^{L} P_j(f) D'_{k-j}(f) \quad k = L + 1, \ldots, Ntr, \tag{3.14}
\]

\[
D_k^*(f) = \sum_{j=1}^{L} P_j(f) D'^*_{k-j}(f) \quad k = 1, \ldots, Ntr - L, \tag{3.15}
\]

where \( Ntr \) is the number of seismic traces, \( P_i \) refers to the prediction filter of length \( L \), \( D_i \) represents the frequency domain seismic trace after spatial smoothing and \( D'_i \) represents the trace before spatial smoothing. The prediction filter \( P \) can be calculated from the original seismic data using the algorithm introduced in the last section.

Assuming the input seismic trace and the inverted reflectivity portray similar spatial continuity, the reflectivity can also be predicted with the same filter:

\[
R_k(f) = \sum_{j=1}^{L} P_j(f) R'_{k-j}(f) \quad k = L + 1, \ldots, Ntr, \tag{3.16}
\]

\[
R^*_k(f) = \sum_{j=1}^{L} P_j(f) R'^*_{k-j}(f) \quad k = 1, \ldots, Ntr - L, \tag{3.17}
\]

where \( R_i \) represents the frequency domain reflectivity after spatial smoothing and \( R'_i \) represents the reflectivity before spatial smoothing. Here the filter \( P \) is the multichannel operator calculated from the original seismic data.

The procedure for combining the FX prediction filter with the conventional BP method can be described as (Wang and Wang, 2013a)

**Step 1:** Compute the reflectivity series \( r' \) using the conventional BP method.

**Step 2:** Apply the FX prediction filter \( P \) to \( r' \), in order to obtain a filtered reflectivity \( r \), where the random noise can be removed from the reflectivity profile and the lateral continuity is enhanced. Note that since the prediction filter is defined in the
frequency domain and the BP method is implemented in the time domain, we must first transform the reflectivity traces into the frequency domain, and then transform them back into the time domain after the prediction filtering process.

Step 3: Convolve the filtered reflectivity $r$ with the known wavelet to generate synthetic traces $d$.

Step 4: Re-calculate the initial value of the coefficient vector $m$ using the synthetic traces calculated from Step 3.

Step 5: Repeat the BP process by minimising the value of $J$ iteratively with the initial value $m$ which is updated in Step 4 until a satisfactory result is obtained.

The workflow of the FX prediction filtering combined BP method is shown in Figure 3.4:

![Figure 3.4 Workflow of the FX prediction filtering combined BP method.](image-url)
Due to the predictable nature of seismic traces, the lateral coherency of the final reflectivity profile can be improved by the proposed process which incorporates the FX prediction filter into the BP algorithm.

### 3.4 Numerical results

To verify the validity of the improved BP method, the same noisy synthetic data with an SNR of 2.0, as shown in Figure 3.2 (a), is used. Figures 3.5 (a) is the result obtained by the standard BP method, which is noisy and lateral discontinuous. If applying the FX prediction filter directly onto the result of the conventional BP method, we have a result shown in Figure 3.5 (b). The reflectivity profile is still noisy and the structure is not clear enough, although the residual energy ratio has been reduced a bit to 48.11%. The reflectivity profile retrieved using the FX prediction filtering combined BP method, illustrated in Figure 3.5 (c), is clearer and more continuous, while the residual energy ratio is reduced to only 21.09%.
Figure 3.5 (a) Seismic reflectivity obtained by the standard BP method. (b) Seismic reflectivity after FX prediction filtering. (c) Seismic reflectivity obtained by the FX prediction filtering combined BP method.

Figure 3.6 (a) to (c) show the reflectivity profiles associated with the noisy Marmousi model obtained by the standard and enhanced methods, respectively. By applying the FX prediction filter directly onto the result from the single-channel method shown in Figure 3.6 (a), the lateral coherency of the result can be improved, and the residual energy ratio is reduced from 9.34% to 8.06%. However, the details are not clear, as shown in Figure 3.6 (b). After using the FX prediction filtering combined BP algorithm, the continuity of the reflectivity profile is improved and the details are better demonstrated with a residual energy ratio of only 5.37%, as shown in Figure 3.6 (c).
Figure 3.6 (a) Seismic reflectivity obtained by the standard BP method. (b) Seismic reflectivity after FX prediction filtering. (c) Seismic reflectivity obtained by the FX prediction filtering combined BP method.

Figure 3.7 (a) is a field seismic profile of 701 traces. Figures 3.7 (b) and (c) illustrate the retrieved results calculated by the standard BP method and by applying FX prediction filtering directly on the single-channel method’s result. Figure 3.7 (d) illustrates the retrieved result calculated by the proposed FX prediction filtering combined BP algorithm.
Figure 3.7 (b) shows a relatively discontinuous result. After applying the FX prediction filter directly, some detailed structure has been blurred, as observed in Figure 3.7 (c), while the reflectivity profile has been smoothed. Using our proposed method combining FX prediction with the BP inversion, the detailed structure of the reflectivity profile is better presented and the lateral coherency is improved.

Figure 3.7 (a) A field data set. (b) Seismic reflectivity obtained by the standard BP method. (c) Seismic reflectivity after FX prediction filtering. (d) Seismic reflectivity obtained by the FX prediction filtering combined BP method.
Figure 3.8 compares the amplitude spectra of the reflectivity profiles obtained using these three processes. The black solid line denotes the amplitude spectrum of the original seismic trace and the coloured lines represent the spectra of the reflectivity profiles inverted by different methods described above. When using the conventional single-channel BP method, it can be seen from Figure 3.8 (a) that the inverted result has a broad bandwidth. Figure 3.8 (b) shows that by using the conventional BP method with FX prediction filtering, the spectrum has been affected. On the other hand, as illustrated in Figure 3.8 (c), although the spectrum of the result obtained by the proposed method is affected when comparing with the conventional BP result, it is still able to provide marginal improvement in frequency bandwidth when compared with that shown in Figure 3.8 (b). As a result, the FX prediction filtering combined BP method can provide clearer and more continuous reflectivity profiles, without significantly affecting the effective frequency bandwidth of the reflectivity series.

![Amplitude Spectra](image)

**Figure 3.8** (a) The amplitude spectrum of seismic reflectivity sequences inverted by the standard BP method. (b) The amplitude spectrum of FX prediction filtered seismic reflectivity sequences. (c) The amplitude spectrum of seismic reflectivity sequences inverted by the FX prediction filtering combined BP method. (The black solid line denotes the amplitude spectrum of the original seismic trace and the coloured lines represent the spectra of the reflectivity profiles inverted by different methods)
Moreover, a larger field data set is used. Figure 3.9 shows the original traces and the inverted reflectivity sequences using the conventional BP, conventional BP with FX prediction filtering and FX prediction filtering combined BP algorithm, respectively. The estimated wavelet for this data set is shown in Figure 1.4 (b). Figure 3.10 shows the zoomed-in version of these profiles, with trace number from 400 to 500 and time from 0.4 to 0.6 s. It can also be observed that when applying the FX prediction filter directly on the single-channel result, the detailed structure of the result can hardly be identified. On the other hand, when using the proposed FX prediction filtering combined BP method, the resolution and noise suppression performance of the inverted result is relatively better.
Figure 3.9 (a) A field data set. (b) Seismic reflectivity obtained by the standard BP method. (c) Seismic reflectivity after FX prediction filtering. (d) Seismic reflectivity obtained by the FX prediction filtering combined BP method.
Figure 3.10 (a) Zoomed-in field data set. (b) Zoomed-in seismic reflectivity obtained by the standard BP method. (c) Zoomed-in seismic reflectivity after FX prediction filtering. (d) Zoomed-in seismic reflectivity obtained by the FX prediction filtering combined BP method.

Figure 3.11 provides the amplitude spectra for the results obtained by the standard single-channel BP method, applying FX prediction filtering directly, and FX prediction filtering combined BP method respectively. While FX prediction filtered result and FX prediction filtering combined BP method’s result have similar frequency spectra with a relatively wide frequency band, the latter can improve the spatial continuity of the inverted reflectivity sequences with a relatively better resolution.
Figure 3.11 (a) The amplitude spectrum of seismic reflectivity sequences inverted by the standard BP method. (b) The amplitude spectrum of FX prediction filtered seismic reflectivity sequences. (c) The amplitude spectrum of seismic reflectivity sequences inverted by the FX prediction filtering combined BP method. (The black solid line denotes the amplitude spectrum of the original seismic trace and the coloured lines represent the spectra of the reflectivity profiles inverted by different methods)

3.5 Conclusions

The basis pursuit method can be used for seismic reflectivity inversion. It can provide a sparse and clean seismic reflectivity profile when the input seismic traces profile has a relatively high signal-to-noise ratio. However, the lateral coherency of the retrieved reflectivity may be deteriorated under high noise background when it is solved by the conventional single-channel BP method.

To tackle this problem, FX prediction filtering has been incorporated into the procedure of the standard BP algorithm. The reflectivity is extracted with the sparseness controlled by the BP method and the lateral continuity is improved by FX prediction filtering. Both
synthetic and field data sets have shown that the proposed method can reduce the noise on the reflectivity profiles whilst preserving the coherent information.

However, the process described in this chapter is not essentially the “multichannel”, as the multichannel FX prediction operator $\mathbf{P}$ is only adopted to improve the initial model of the seismic reflectivity and is applied outside the whole iteration steps of the BP algorithm.
Chapter 4  Curvelet transform enhanced $L_p$-norm deconvolution

Seismic reflectivity inversion conventionally poses a sparse constraint on the inversion problem directly. However, in some situations, the inverted seismic reflectivity is so sparse that the underlying structure is destroyed. To tackle this problem, different models have been proposed to represent the reflectivity sequences. For example, when using the basis pursuit method (Zhang, 2010; Zhang and Castagna, 2011), a collection of even and odd dipoles is adopted to model the reflectivity. Therefore, the sparse constraint is exerted on the corresponding coefficients, reducing the loss of the geological structure. However, these methods are normally implemented in a trace-by-trace manner and thus there may be a lack of lateral coherency for the retrieved results.

The curvelet transform was first implemented as a multi-scale and multi-directional transform. It has been proved that the curvelet transform can provide a sparse representation for smooth objects with edges such as seismic events (Candes and Donoho, 1999; 2002 and Candes et al., 2005). After transforming the data into the curvelet domain, the coefficients, which represent the noise and effective signal, can be separated clearly. Due to such property, the curvelet transform has been widely adopted in seismic signal processing. Non-parametric seismic data recovery has been implemented with the curvelet transform (Hennenfent and Herrmann, 2008) and can recover data with up to 80% traces missing. Moreover, it plays an important role in random, coherent and incoherent noise attenuation (Neelamani et al., 2008; Kumar, 2009), showing a better performance when comparing with the conventional methods such as median filter and FX deconvolution.

Furthermore, since the structure of seismic reflectivity along the layers can be seen as curves, the curvelet transform has been applied to model the reflectivity. The non-spiky seismic reflectivity inversion was developed with the curvelet transform (Hennenfent and Herrmann, 2005; Kumar and Herrmann, 2008). It is shown that seismic deconvolution with
the multichannel curvelet operator can exploit the continuity along the reflectors by promoting the curvelet-domain sparsity.

In this chapter, we first implement curvelet deconvolution by applying the sparsity constraint in the curvelet domain. A comparative study is then conducted to investigate the performance of three deconvolution methods, including curvelet deconvolution, least-squares inversion and $L_p$-norm deconvolution. Both synthetic and field data sets will be used in this comparative study. Based on the study, we further propose an enhanced $L_p$-norm deconvolution method by using the result obtained by the curvelet deconvolution as the initial model during the $L_p$-norm inversion process. Numerical results show that the lateral continuity of the spiky reflectivity sequences inverted by the proposed method can be further improved.

4.1 Curvelet transform

The curvelet transform is a multi-scale and multi-directional transform that can decompose the image (data) into harmonic scales. Curvelets can be obtained by rotations and translations of a “mother” curvelet $\varphi_j$, which can be expressed as (Candes et al., 2005)

$$\varphi_{j,l,k}(x) = \varphi_j(R_{\theta_l}(x - x_{j,l}^{(l)}))$$

(4.1)

As seen from equation (4.1), curvelets can be characterised by three indices: $j$ which represents the scale, $l$ which is associated with different angles, and $k$ which corresponds to different positions.

These three parameters play important roles in the definition of the curvelet transform (Candes et al., 2005):

1. Scale parameter $j$: different scales correspond to different frequency bands.
2. $l$, which is used to define an equispaced sequence of rotation angles:
   $$\theta_l = 2\pi \cdot 2^{-\lceil j/2 \rceil} \cdot l \quad \text{with} \quad l = 0,1,... \quad \text{and} \quad 0 \leq \theta_l < 2\pi .$$
3. $k$, which is used to define the position of the curvelets:
\[ x^{(j,k)}_x = R^{-1}_\theta(k_1 \cdot 2^{-j}, k_2 \cdot 2^{-j/2}), \]
where \( x \) and \( k \) represent the points in a two dimensional space, and \( R_\theta \) represents the rotation by \( \theta \) radians:

\[
R_\theta = \begin{pmatrix}
\cos\theta & \sin\theta \\
-\sin\theta & \cos\theta
\end{pmatrix}.
\]  (4.2)

\( R^{-1}_\theta \) is the invert of \( R_\theta \):

\[
R^{-1}_\theta = R_\theta^T = R_{-\theta}.
\]  (4.3)

With the above definitions, the curvelet transform has several properties:

1. **Tight frame:**
   An arbitrary function \( f \) can be expanded as a series of curvelets:
   \[
f = \sum_{j,l,k} \langle f, \varphi_{j,l,k} \rangle \varphi_{j,l,k},
   \]
   and it fulfills the Parseval relation:
   \[
   \sum_{j,l,k} |\langle f, \varphi_{j,l,k} \rangle|^2 = \|f\|^2.
   \]

2. **Parabolic scaling:**
   The effective length and width of curvelets obey the anisotropy scaling relation:
   \[
   \text{width} \approx \text{length}^2.
   \]

3. **Oscillatory behaviour.**
   Curvelets are strictly localized in the frequency domain, they can be thought as little pyramids with many directions and positions at each length scale. Figure 4.1 shows the tiling of the digital curvelet transform in the frequency domain. While on the other hand, in the time-space domain, they are needle like with both ends tapering off while smooth along and oscillatory across the ridge, as can be observed from Figure 4.2 (Wu and Hung, 2013).
After transforming the data into the curvelet domain, only a few curvelet coefficients will remain large while the others will decay rapidly. As a result, the curvelet transform is a suitable tool for providing a sparse representation of the seismic signal profile. Wavelets have also been used for signal decomposition, however, in the frequency domain, they lack directionality. Only when the orientation of wavelets is perpendicular to the interfaces, the coefficients of the data will decay rapidly, so the wavelet coefficients corresponding to the seismic signal are not as sparse as that of the curvelets, which means that the curvelet transform is more suitable to be used in sparse inversion problem than wavelets.
The curvelet coefficients \( c \) can be obtained by calculating the inner products of data \( r \) and \( \varphi_{j,l,k} \):

\[
c_{j,l,k} = \langle r, \varphi_{j,l,k} \rangle .
\]  

(4.4)

To simplify the form of the equation above, we can adopt a matrix-vector form:

\[
e = Cr ,
\]

(4.5)

where \( C \) represents the curvelet transform operator and \( e \) is the vector of coefficients corresponding to data \( r \).

Since curvelets are tight frames, the adjoint operator \( C^T \) is equal to the pseudo-inverse of the curvelet operator \( C \) (Candes and Donoho, 2002). Thus we can have the following reconstruction equation:

\[
r = C^T e .
\]

(4.6)

The curvelet transform can map the signal into distinct sets of curvelet coefficients. The values of the coefficients corresponding to the effective signal are very large. On the other hand, the energy of random noise is spread out over all frequencies and dips. Therefore, the curvelet transform maps the random noise into a large number of weak amplitude curvelet coefficients. As a result, signal and noise have minimal overlap in the transform domain, and sparse deconvolution method can be utilised to suppress the noise while at the same time invert the reflection series. The algorithm is developed based on the fast discrete curvelet transform via wrapping (Candes et al., 2005).

### 4.2 Curvelet deconvolution

As introduced in Chapter 1, seismic reflectivity inversion is based on the well-known convolution model. Conventionally, in order to obtain a sparse reflectivity result, \( L_1 \)-norm of the “model reflectivity” can be used to regularize the reflectivity inversion process by using the objective function:

\[
J = \|d - Wr\|_2^2 + \lambda \|r\|_1 .
\]

(4.7)
As the structure of the reflectors along the layers can be treated as curves, curvelet coefficients are adopted to represent seismic reflectivity. The effective signal will concentrate on large curvelet coefficients and random noise will be mapped on weak amplitude coefficients. The reflectivity can then be represented by the reconstruction equation: \( r = C^T c \), where \( c \) is the curvelet coefficient vector corresponding to seismic reflectivity.

Since the curvelet coefficients are sparse, the objective function can be rewritten as

\[
J = \| d - WC^T c \|^2_L + \lambda \| c \|_1.
\]

(4.8)

Solving this objective function, we can obtain the result of \( c \). Reflectivity is then calculated by \( r = C^T c \). This method can be called curvelet deconvolution.

The curvelet operator is defined on a data set of size \( Nt \times Nt \), where \( Nt \) is the number of traces and \( Nt \) is the number of sample points of each trace. Therefore, the curvelet operator is a multichannel operator working on the whole seismic data set rather than a single trace. During the deconvolution process, all traces are put in together and processed simultaneously, and the curvelet coefficients approach the reflection coefficients of the real earth layers as the iteration proceeds. Consequently, the inherent continuity of the layers will be preserved.

### 4.3 Numerical results for a comparative study

In order to evaluate the capability of the curvelet transform based reflectivity inversion method, a comparative study is conducted and its performance is compared with the least-squares inversion and \( L_p \)-norm deconvolution. The synthetic data set with a horizontal and a dip structure is first used for the comparison. Figure 4.3 illustrates the reflectivity profiles inverted by these three methods. By observing these results, we can come to the conclusion that when the input data is clean, these three methods can all portray the structure of seismic events clearly. The only difference between them is that the frequency bandwidth of the reflectivity series obtained by the deconvolution with the curvelet transform is narrower than that of the \( L_p \)-norm deconvolution, but wider than that of the least-squares method.
Figure 4.3 (a) Synthetic seismogram. (b) Seismic reflectivity obtained by the least-squares inversion. (c) Seismic reflectivity obtained by the $L_p$-norm deconvolution. (d) Seismic reflectivity obtained by the curvelet deconvolution.
To test the performance of these methods under low signal-to-noise ratio (SNR) circumstance, external random noise with an SNR of 2.0 is added to the original clean synthetic data. By applying these three different deconvolution methods on the obtained noisy synthetic data, it can be seen that when using the least-squares method, the inverted result is so noisy that we can hardly recognise the structure of the seismic event. The residual energy ratio associated with the least-squares solution is calculated to be 165.45%, which is even larger than the energy of the effective signal. On the other hand, although the $L_p$-norm deconvolution can largely suppress noise, some structures are eliminated. Figure 4.4 (d) shows the result of the inversion with the curvelet frames. The residual energy ratios corresponding to the results shown in Figure 4.4 (c) and (d) are 42.81% and 35.67%, respectively. It can be concluded that the curvelet deconvolution is able to suppress the noise on the seismic profile effectively when compared with the least-squares solution, and at the same time conserve the character of the seismic events when compared with the $L_p$-norm deconvolution.
Figure 4.4 (a) Synthetic seismogram with random noise (SNR=2.0). (b) Seismic reflectivity obtained by the least-squares inversion. (c) Seismic reflectivity obtained by the $L_p$-norm deconvolution. (d) Seismic reflectivity obtained by the curvelet deconvolution.

Furthermore, the noisy Marmousi model is used to test these methods, and the results are as shown in Figure 4.5. When the input seismic traces are clean, the least-squares method can provide the result with the lowest residual since this method is based on the principle that the difference between the original and synthetic traces is minimized. However, when the input data is noisy, the noise existed on the seismic profile may be treated as effective signal and used to retrieve the reflectors at wrong positions. Therefore, the inverted result will deviate far from the optimal solution and the residual associated with the least-squares method will be large. With this example, the residual energy ratio associated with the least-squares solution, as shown in Figure 4.5 (b), is 12.76%. When using the $L_p$-norm deconvolution and curvelet deconvolution, the ratios are only 7.82% and 5.91%, respectively. It means that more noise has been suppressed. On the other hand, the lateral continuity of the reflectivity profile obtained by the curvelet deconvolution is better than that of the $L_p$-norm deconvolution.
Figure 4.5 (a) Marmousi model with random noise (SNR=10.0). (b) Seismic reflectivity obtained by the least-squares inversion. (c) Seismic reflectivity obtained by the $L_p$-norm deconvolution. (d) Seismic reflectivity obtained by the curvelet deconvolution.

A small stack is further adopted to evaluate the performance of these three deconvolution methods. Figure 4.6 (a) shows the small stack used for the test, Figure 4.6 (b) shows the result obtained by the conventional least-squares method, while Figure 4.6 (c) and Figure 4.6
are the results obtained by the $L_p$-norm deconvolution and deconvolution based on the curvelet transform, respectively. From these figures it can be seen that the stratification of Figure 4.6 ($d$) is better than that of the least-squares method in Figure 4.6 ($b$), as denoted by the black circle. On the other hand, although the result of the deconvolution based on the curvelet transform is not as spiky and sparse as the result of the $L_p$-norm deconvolution, the resulting lateral continuity is superior. Therefore, we can conclude that the curvelet deconvolution can be regarded as an intermediate method between the least-squares inversion and the $L_p$-norm deconvolution, as it can offer a trade-off between the lateral continuity and sparseness.

(a)  (b)
A field data set has also been employed to test the difference between these three methods. Figure 4.7 shows the profile of the input seismic traces, the result of the least-squares method, reflectivity obtained by the $L_p$-norm deconvolution and deconvolution based on the curvelet transform, respectively. Figure 4.8 provides a detailed view of the seismic data with trace number from 400 to 500 and time from 0.4 to 0.6 s and the corresponding inverted reflectivity profiles. It can be concluded that the reflectivity inversion based on the curvelet transform can suppress the noise in an effective manner, and has a higher resolution than that of the least-squares inversion. This is because in the curvelet domain, the curvelet coefficients of the noise are much weaker than that of the effective signal, and by eliminating these small curvelet coefficients we can suppress the noise effectively. On the other hand, the lateral coherent information of the structure is better preserved when comparing with the reflectivity profile obtained by the $L_p$-norm deconvolution due to the multichannel mechanism.
Trace number

Time (s)

(a)
Figure 4.7 (a) A field data set. (b) Seismic reflectivity obtained by the least-squares inversion. (c) Seismic reflectivity obtained by the $L_p$-norm deconvolution. (d) Seismic reflectivity obtained by the curvelet deconvolution.
Figure 4.8 (a) Zoomed-in field data set. (b) Zoomed-in seismic reflectivity obtained by the least-squares inversion. (c) Zoomed-in seismic reflectivity obtained by the $L_p$-norm deconvolution. (d) Zoomed-in seismic reflectivity obtained by the curvelet deconvolution.
4.4 Curvelet transform enhanced $L_p$-norm deconvolution

Conventionally, as discussed in Chapter 2, result obtained by the least-squares inversion is used as the initial model for the $L_p$-norm deconvolution, the flowchart of this process is shown in Figure 4.9 (a). As observed in the previous section, since the curvelet deconvolution has its own advantages when comparing with the $L_p$-norm deconvolution and least-squares inversion, we can use the result of the deconvolution based on the curvelet transform to enhance the process of the traditional $L_p$-norm deconvolution (Wang and Wang, 2014b). The flowchart of the curvelet transform enhanced $L_p$-norm deconvolution is shown in Figure 4.9 (b). Originally, with the initial model obtained by the least-squares method, the $L_p$-norm deconvolution is started iteratively, and after several iterations, the final result of seismic reflectivity can be outputted. With the newly proposed process, the initialization process is undertaken by the curvelet deconvolution.

Figure 4.9 (a) Flowchart of the conventional $L_p$-norm deconvolution. (b) Flowchart of the curvelet transform enhanced $L_p$-norm deconvolution.
After using the result from the curvelet deconvolution as the initial model instead of that from the least-squares method during the inversion process, the retrieved seismic reflectivity series profile of the noisy synthetic data is cleaner and some missing structures are corrected when comparing with the result by the conventional $L_p$-norm deconvolution, as can be seen from Figure 4.10. The residual energy ratio is reduced from 42.81% to 28.51%.

![Image](image.png)

**Figure 4.10** (a) Seismic reflectivity obtained by the conventional $L_p$-norm deconvolution. (b) Seismic reflectivity obtained by the curvelet transform enhanced $L_p$-norm deconvolution.

The conventional and enhanced processes can be further applied to invert the noisy Marmousi model shown in Figure 4.5 (a). After using the curvelet deconvolution enhanced $L_p$-norm deconvolution, the noise existed on the reflectivity profile can be suppressed, and the residual energy ratio is reduced from 7.82% to 4.71%, while the lateral coherency is improved, as shown in Figure 4.11 (b).
Figure 4.11 (a) Seismic reflectivity obtained by the conventional $L_p$-norm deconvolution. (b) Seismic reflectivity obtained by the curvelet transform enhanced $L_p$-norm deconvolution.

The same small stack shown in Figure 4.6 (a) is also used to test the performance of the enhanced $L_p$-norm deconvolution. As shown in Figure 4.12 (a) and (b), it can be observed that the missing structures are recovered and the lateral continuity of the inverted reflectivity is improved when using the proposed method, especially for the structure denoted by the black circle.
Figure 4.12 (a) Seismic reflectivity obtained by the conventional $L_p$-norm deconvolution. (b) Seismic reflectivity obtained by the curvelet transform enhanced $L_p$-norm deconvolution.

Moreover, the test has been carried out using the same field data set shown in Figure 4.7 (a). Figure 4.13 gives the retrieved reflectivity result obtained by the conventional $L_p$-norm deconvolution and $L_p$-norm deconvolution enhanced by the curvelet transform, respectively. It can be seen that when using the conventional method, the detailed layers are mixed up together and difficult to be identified. When using the enhanced process, the stratification between the layers is clearer.
Figure 4.13 (a) Seismic reflectivity obtained by the conventional $L_p$-norm deconvolution. (b) Seismic reflectivity obtained by the curvelet transform enhanced $L_p$-norm deconvolution.

Since the image of the whole traces may not be clear enough for comparing these two processes, we further demonstrate the detailed reflectivity profiles with trace number from 400 to 500 and time from 0.4 to 0.6 s of Figure 4.13, as shown in Figure 4.14. It can be seen that by applying the result of the deconvolution based on the curvelet transform as the new initial model, the structure of the reflectivity profile is more coherent, which means that our proposed method is superior in terms of the spatial continuity.
Figure 4.14 (a) Zoomed-in seismic reflectivity obtained by the conventional $L_p$-norm deconvolution. (b) Zoomed-in seismic reflectivity obtained by the curvelet transform enhanced $L_p$-norm deconvolution.

Figure 4.15 (a) and (b) shows the amplitude spectrum of the reflectivity inverted by the conventional and enhanced $L_p$-norm deconvolution, respectively. Both methods can provide results with broad bandwidths, showing that the curvelet transform enhanced $L_p$-norm deconvolution can improve the structure coherency of the result, while conserving the resolution at the same time.

Figure 4.15 (a) The amplitude spectrum of seismic reflectivity sequences inverted by the conventional $L_p$-norm deconvolution. (b) The amplitude spectrum of seismic reflectivity sequences inverted by the curvelet transform enhanced $L_p$-norm deconvolution. (The black solid line denotes the amplitude spectrum of the original seismic trace and the coloured lines represent the spectra of the reflectivity profiles inverted by different methods)
4.5 Conclusions

The curvelet transform is able to provide a sparse representation for seismic reflectivity. In this chapter, first the multichannel curvelet operator was applied during the inversion process to obtain a better lateral continuity. A comparative study was then conducted to evaluate the performance of the curvelet deconvolution, least-squares inversion and $L_p$-norm deconvolution. It can be seen that the deconvolution based on the curvelet transform can provide the reflectivity profiles which are cleaner and with higher resolution, when compared with those obtained by the least-squares method. On the other hand, although its results are not that sparse and spiky, it can produce profiles which are more continuous than those obtained by the $L_p$-norm deconvolution as it is a multichannel method.

In order to improve the lateral continuity of the spiky seismic reflectivity, an enhanced $L_p$-norm deconvolution procedure was proposed by applying the result obtained by the curvelet deconvolution as the initial model instead of the least-squares solution. Numerical results using a noisy synthetic data set, a small stack and a large field data set have validated the proposed method, showing that the obtained spiky reflectivity profile has better lateral coherency and lower residual energy ratio, while the frequency bandwidth is still very broad. The curvelet transform enhanced $L_p$-norm deconvolution can be treated as a semi-multichannel method, since the multichannel method, curvelet deconvolution, is employed only to improve the initial model of seismic reflectivity. While the basic method utilized in this inversion process is the $L_p$-norm deconvolution, which is still a single-channel method.
Chapter 5  Multichannel reflectivity inversion with the Cauchy constraint

As introduced in previous chapters, the reflectivity series inverted by the least-squares method is not spiky and sparse enough. Alternatively, the Cauchy distribution (Sacchi and Ulrych, 1995), which is longer-tailed than the Gaussian distribution, can be used to increase the effective bandwidth of the retrieved reflectivity. However, deconvolution with the Cauchy constraint is also implemented in a trace-by-trace style. This may lead to noisy and lateral discontinuous reflectivity profiles, especially when the input seismic data has a complicated structure and low signal-to-noise ratio (SNR).

Multichannel deconvolution, which introduces the information of the adjacent traces during the inversion process, is a possible way to improve the lateral continuity as well as to provide better structure characterization. A multichannel deconvolution method has been developed for noise reduction (Wang and Sacchi, 2009), where the FX prediction filter was applied as the smoothing operator to reduce the spatial differences of the adjacent seismic traces. FX and FXY prediction filters have also been employed in the spectral decomposition process (Bonar and Sacchi, 2011 and 2013). The time frequency representation of the data is de-noised as it is generated and finally a spatially smooth result which maintains the resolution can be obtained. In this chapter, I develop a multichannel deconvolution algorithm based on the Cauchy criterion for solving the reflectivity inversion problems. Since the information of the adjacent traces is introduced by the FX prediction filter during the inversion process, the inverted reflectivity profiles are expected to have an improved lateral continuity and a better structure characterisation.

In this chapter, the motivation for developing multichannel reflectivity inversion with the Cauchy constraint is introduced at first. Implementation details of the proposed multichannel deconvolution algorithm are then described. This algorithm is validated by the result of a field data set. Furthermore, we propose two modified multichannel algorithms using different implementation procedures. A comparative study for these three methods is finally presented
and numerical results are demonstrated and analysed to compare the performance of these methods.

### 5.1 Motivation

When using the conventional single-channel reflectivity inversion method with the Cauchy constraint, seismic reflectivity is initialized as

\[ r = (W^TW + \mu d)^{-1} W^Td. \]  

(5.1)

The result of the Cauchy constrained deconvolution can be written as

\[ r = (W^T C_d^{-1} W + \mu_{cauchy} \frac{2}{\lambda^2} D^{-1} W^T C_d^{-1} d). \]  

(5.2)

where \( C_d \) is the data covariance matrix calculated from the input traces.

The result from equation (5.1) is substituted into the diagonal matrix \( D \) in equation (5.2) and the reflectivity can be solved using the conjugate gradient (CG) method. The initial residual \( e \) is set as the difference between \( W^T C_d^{-1} d \) and \( (W^T C_d^{-1} W + \mu_{cauchy} \frac{2}{\lambda^2} D)r \), and the initial search direction \( p \) is set as equal to \( e \):

\[
\begin{align*}
  u^{(k)} &= (W^T C_d^{-1} W + \mu_{cauchy} \frac{2}{\lambda^2} D)p^{(k)} \\
  \alpha_k &= \frac{(e^{(k)}, e^{(k)})}{(p^{(k)}, u^{(k)})} \\
  r^{(k+1)} &= r^{(k)} + \alpha_k p^{(k)} \\
  e^{(k+1)} &= e^{(k)} - \alpha_k u^{(k)} \\
  \beta_k &= \frac{(e^{(k+1)}, e^{(k+1)})}{(e^{(k)}, e^{(k)})} \\
  p^{(k+1)} &= e^{(k+1)} + \beta_k p^{(k)}
\end{align*}
\]

(5.3)

This CG calculation process is carried out in a trace-by-trace manner, which means only one seismic trace is applied at each iteration step and the reflectivity of a single trace will be retrieved as the output. Figure 5.1 shows the result obtained by the single-channel inversion based on the Cauchy constraint. It can be seen that when the input data is clean, this method can provide a satisfactory result.
Figure 5.1 (a) Synthetic seismogram. (b) Seismic reflectivity obtained by the single-channel reflectivity inversion with the Cauchy constraint.

However, as discussed above, because this method is a single-channel method, the lateral continuity and the noise suppression performance of the inverted reflectivity may deteriorate if the input seismic signal is contaminated by background noise, as is shown in Figure 5.2. With this noisy input, the noise suppression and structure characterization performances of this method are both affected.
Figure 5.2 (a) Synthetic seismogram with random noise (SNR=2.0). (b) Seismic reflectivity obtained by the single-channel reflectivity inversion with the Cauchy constraint.

The inverted reflectivity for the noisy Marmousi model using the single-channel Cauchy constrained deconvolution is shown in Figure 5.3. Similarly, it can be observed that the lateral continuity of the reflectivity profile has been affected and there are still some noise existed. The value of the residual energy ratio is 8.58%. Therefore, multichannel algorithm is developed to tackle these problems.
Figure 5.3 (a) Marmousi model with random noise (SNR=10.0). (b) Seismic reflectivity obtained by the single-channel reflectivity inversion with the Cauchy constraint.

5.2 Multichannel reflectivity inversion with the Cauchy constraint

As discussed in the previous section, since the traces are processed separately when using the single-channel methods, the reflectivity profiles may be laterally discontinuous and noisy. To tackle this problem, a multichannel algorithm for seismic reflectivity inversion is proposed in this section.

Assuming seismic events are linear, seismic traces have a predictable nature in the FX domain. Consequently, the trace can be predicted by the preceding or following traces with a prediction filter (Spitz, 1991; Wang, 1999). If the reflectivity series are assumed to share a similar spatial continuity with the seismic traces, the prediction filter $P$, which is a multichannel operator for seismic traces, is also applicable for reflectivity (Wang, 2006; 2007; 2008). Under this assumption, we have

$$R_k(f) = \sum_{j=1}^{L} P_j(f) R_{k-j}(f) \quad k = L + 1, \ldots, N_t,$$  \hspace{1cm} (5.4)

$$R_k^*(f) = \sum_{j=1}^{L} P_j^*(f) R_{k+j}^*(f) \quad k = 1, \ldots, N_t - L,$$  \hspace{1cm} (5.5)
where $R'_k$ represents the reflectivity before spatial prediction, $R_k$ represents the reflectivity after prediction, $Ntr$ is the number of traces and $L$ is the length of prediction filter.

With the multichannel FX prediction filter, the information of the adjacent traces can be applied during the inversion process. To incorporate the prediction filter into the deconvolution procedure, the convolution model in equation (1.13) can be rewritten as

$$d = \mathbf{W} \mathbf{F}^{-1} \mathbf{P} \mathbf{F} \mathbf{r}' + \mathbf{n},$$  \hspace{1cm} (5.6)

in which $\mathbf{r}'$ is the time domain representation of the reflectivity sequences before spatial smoothing, $\mathbf{F}$ denotes the Fourier transform and $\mathbf{F}^{-1}$ represents the inverse Fourier transform.

Re-writing $\mathbf{W} \mathbf{F}^{-1} \mathbf{P} \mathbf{F}$ as a new operator $\mathbf{L}$, equation (5.6) can be expressed as

$$d = \mathbf{L} \mathbf{r}'+ \mathbf{n}.$$  \hspace{1cm} (5.7)

Therefore, the reflectivity series before filtering, which is calculated using the proposed multichannel Cauchy constrained deconvolution method, can be expressed as

$$\mathbf{r}' = (\mathbf{L}^T \mathbf{C}_d^{-1} \mathbf{L} + \mu_{\text{cauchy}} \frac{2}{\lambda^2} \mathbf{D})^{-1} \mathbf{L}^T \mathbf{C}_d^{-1} \mathbf{d},$$  \hspace{1cm} (5.8)

The form of this result is very similar to that of the single-channel method shown in equation (5.2), however, they are quite different. Since $\mathbf{L}$ is a multichannel operator, equation (5.8) represents the result corresponding to the whole seismic section.

With equation (5.8), the procedure of the multichannel deconvolution with the Cauchy constraint is designed as (Wang and Wang, 2013b)

Step 1: Initialize the value of $\mathbf{r}'$ using equation (5.1), then set the residual $\mathbf{e}$ as the difference between $\mathbf{L}^T \mathbf{C}_d^{-1} \mathbf{d}$ and $(\mathbf{L}^T \mathbf{C}_d^{-1} \mathbf{L} + \frac{2}{\lambda^2} \mathbf{D}) \mathbf{r}'$, and the search direction $\mathbf{p}$ is set to the value of $\mathbf{e}$.

Step 2: Start the multichannel CG iteration process, where $k$ is the number of iterations, and $\alpha_k$ and $\beta_k$ are vectors storing the values of the step length corresponding to each trace:
\[ u^{(k)} = (L^T C_d^{-1} L + \mu_{\text{cauchy}} \frac{2}{\lambda^2} D)p^{(k)} \]
\[ \alpha_k = \frac{(e^{(k)}, e^{(k)})}{(p^{(k)}, u^{(k)})} \]
\[ r^{(k+1)} = r^{(k)} + \alpha_k p^{(k)} \]
\[ e^{(k+1)} = e^{(k)} - \alpha_k u^{(k)} \]
\[ \beta_k = \frac{(e^{(k+1)}, e^{(k+1)})}{(e^{(k)}, e^{(k)})} \]
\[ p^{(k+1)} = e^{(k+1)} + \beta_k p^{(k)} \]

(5.9)

Step 3: Re-initialize \( r' \) using the values calculated by the CG method, and start a new iteration process from Step 1. Repeat this iteration process until a satisfactory result of \( r' \) is obtained.

Step 4: Calculate the reflectivity \( r \) using the equation \( r = F^{-1} P F r' \).

In the conventional single-channel Cauchy constrained deconvolution method, \( r' \), \( e \), \( d \) and \( p \) are vectors that only store the values corresponding to a single trace. However, in multichannel deconvolution, since the prediction filter \( P \) is a multichannel operator, the calculated data for all seismic traces are collected to form the multichannel matrices, and appropriate information needed for further calculation are stored. Correspondingly, the process of the CG algorithm needs to be modified in the multichannel version. The CG method is originally employed for single trace deconvolution, and the iteration process of one trace is repeated until the residue \( e \) is below a predefined threshold \( e_0 \). In our implementation of the proposed multichannel deconvolution method, the iterations of all traces are performed simultaneously. Once the residual of one certain trace is under the threshold, the result of that trace is stored while other traces are still being updated. The CG process continues until the residuals of all traces are below the threshold.

To implement the new operator \( L = W F^{-1} P F \) in the CG method, first the multichannel matrix (e.g. \( r' \), \( d \) or \( p \)) is transformed into the frequency domain, and then the prediction filter \( P \) is applied to the obtained frequency domain matrix. Once the filtering process has been performed, the matrix is transformed back into the time domain. The convolution matrix \( W \) is then applied to obtain the final result of the operator \( L \). The adjoint operator \( L^T = F^{-1} P^T F W^T \) can be accomplished in a similar way. Note that here \( W^T \) represents cross-correlation, as it is
the adjoint process of convolution (Claerbout, 1992). The FX prediction process \((F^\dagger P^T F)\) is performed by simply replacing the prediction filter \(P\) by its adjoint \(P^T\).

### 5.3 Numerical results

The proposed method is first tested using a post-stack field data set, as seen in Figure 5.4 (a). Figure 5.4 (b) and (c) show the reflectivity profiles inverted by the single-channel method with the Cauchy constraint and by applying the prediction filter directly on the single-channel result, respectively. Figure 5.4 (d) and (e) show the retrieved reflectivity profile using the proposed multichannel deconvolution method after Step 3 and Step 4, respectively. It can be seen that by applying the information of the adjacent traces during the inversion process, the background noise has been effectively suppressed while the detailed structure of the reflectivity profile is clearly presented.
Trace number

Time (s)

200  400  600  800  1000  1200  1400
Figure 5.4 (a) A field data set. (b) Seismic reflectivity obtained by single-channel deconvolution. (c) Seismic reflectivity obtained by single-channel deconvolution with FX prediction filtering. (d) Seismic reflectivity obtained by multichannel deconvolution after Step 3. (e) Seismic reflectivity obtained by multichannel deconvolution after Step 4.
To gain an insight into the property of the retrieved reflectivity profile, the same data with trace number from 400 to 500 and time between 0.4 and 0.6 s is further selected, as shown in Figure 5.5 (a). The reflectivity illustrated in Figure 5.5 (b) is obtained by the single-channel deconvolution, having a corresponding residual energy ratio of 19.51%. Figure 5.5 (c) is the result calculated by applying the prediction filter directly to Figure 5.5 (b), and the residual energy ratio is reduced to 17.86%. It can be seen that although the prediction filter can smooth the reflectivity, some detailed structures are also eliminated. Figure 5.5 (d) gives the reflectivity profile retrieved using the proposed multichannel method after Step 3, namely \( r' \), with a residual energy ratio of 14.55%. By applying the prediction filter on \( r' \), the final reflectivity result of our multichannel method, say \( r \), is obtained, as shown in Figure 5.5 (e). The residual energy ratio of the result in Figure 5.5 (e) is further reduced to 10.34%.

It is observed that the reflectivity profile \( r' \) as shown in Figure 5.5 (d) has a better lateral coherency than that obtained by the single-channel method in Figure 5.5 (b). The lateral continuity can be further improved by applying the prediction filter on \( r' \), as shown in Figure 5.5 (e). Moreover, when comparing Figure 5.5 (c) and 5.5 (e), it is observed that the stratification of the reflectivity profile is clearer under the multichannel mechanism. We can conclude that our proposed method has a better performance than single-channel deconvolution method with a prediction filter. This is because in our algorithm, the multichannel FX prediction filter is adopted during the iteration steps in order to stabilize the process, resulting in improved lateral continuity and structure characterization.
Figure 5.5 (a) Zoomed-in field data set. (b) Zoomed-in seismic reflectivity obtained by single-channel deconvolution. (c) Zoomed-in seismic reflectivity obtained by single-channel deconvolution with FX prediction filtering. (d) Zoomed-in seismic reflectivity obtained by multichannel deconvolution after Step 3. (e) Zoomed-in seismic reflectivity obtained by multichannel deconvolution after Step 4.
5.4 Two modified multichannel algorithms

In the previous section, the proposed multichannel algorithm was validated and the inverted reflectivity profiles have shown improvements in terms of the lateral coherency and structure characterization. However, it is not accurate to calculate the prediction filter $P$ from seismic traces if the reflectivity is assumed to be predictable. In this section, the originally proposed multichannel deconvolution process is further modified using two different implementation procedures.

**Modification 1**: Seismic reflectivity is assumed to be predictable and the prediction filter is calculated directly from the reflectivity.

If the reflectivity is assumed to be predictable in the frequency domain, it is more suitable to calculate the prediction filter directly from the reflectivity sequences. This assumption is not contradictory to equation (2.6). This is because in equation (2.6), the reflectivity samples in one trace are assumed to be independent, while in this chapter, it is assumed that the predictable nature of the reflectivity exists between trace and trace. In the originally proposed multichannel method, $P$ is calculated from the input traces since at the beginning of the inversion process, the value of the reflectivity is unknown. In Modification 1, this problem is overcome by modifying the iteration process. At the beginning of the Modification 1 algorithm, $P$ is initialized as an identity matrix. Then in Step 3, the prediction filter $P$ is calculated from $r'$ which has been obtained in Step 2, and $P$ thus can be used in the next iteration which re-starts from Step 1. Therefore, the value of $P$ is updated corresponding to the reflectivity result obtained at each iteration.

**Modification 2**: The reflectivity is not assumed to be predictable. Only the predictable nature of seismic trace is applied here. In this case, the prediction filter can only be calculated from the input data.

If the reflectivity is not predictable by $P$, the predictable nature of the input traces is adopted, yielding

$$D_k(f) = \sum_{j=1}^L P_j(f) D'_{k-j}(f) \quad k = L + 1, \ldots, Ntr,$$  \hspace{1cm} (5.10)
where $D_k'$ represents the seismic trace before spatial prediction, and $D_k$ represents the seismic trace after prediction.

Then equation (1.13) can be rewritten as

$$
d = F^{-1}PFd^{'} + n
= F^{-1}PFWr^{'} + n,
$$

where $d'$ is the time domain representation of the seismic trace before spatial smoothing. The operator $L$ can be expressed as $L = F^{-1}PFW$ and adopted in the iteration process as shown in the originally proposed multichannel method in Section 5.2. The reflectivity series then can be obtained with this new operator.

### 5.5 Comparative study of the three proposed multichannel algorithms

In this section, a comparative study is carried out to evaluate the performance of these three algorithms. First of all, a synthetic data containing a horizontal and dip structure is used, as seen in Figure 5.6 (a). The SNR of the input data is set to 2.0. As demonstrated in Figure 5.6 (b), the result obtained by single-channel deconvolution is noisy and laterally discontinuous. After applying the prediction filter onto the result by single-channel method, Figure 5.6 (c) can be obtained, which is not spiky and sparse enough, and at the same time, the structure is not clearly presented.

Although the originally proposed multichannel method can provide clearer reflectivity profile, some information of the structure is also eliminated, as shown in Figure 5.6 (d). The results obtained by the two modified methods are demonstrated in Figure 5.6 (e) and 5.6 (f), respectively. The improved retrieved reflectivity profiles are presented with better noise reduction and structure characterization ability.

Furthermore, we compare the residual energy ratio of these three methods to evaluate the energy difference between the retrieved and the clean synthetic data. When using the conventional single-channel method, the residual energy ratio is calculated to be 45.2592 %.
After applying the FX prediction filter onto the result by single-channel method, the residual energy ratio is reduced to 34.947%, while the residual energy ratios of the originally proposed multichannel method, Modification 1, and 2 are 20.4751 %, 9.49378 % and 13.268 %, respectively. Therefore, it can be concluded that Modification 1 and 2 can provide even lower residual energy ratio.
Figure 5.6 (a) Synthetic seismogram with random noise (SNR=2.0). (b) Seismic reflectivity obtained by single-channel deconvolution. (c) Seismic reflectivity obtained by single-channel deconvolution with FX prediction filtering. (d) Seismic reflectivity obtained by the originally proposed multichannel algorithm. (e) Seismic reflectivity obtained by Modification 1. (f) Seismic reflectivity obtained by Modification 2.

Different algorithms have also been applied to invert the noisy Marmousi model. When using the conventional single-channel algorithm, the residual energy ratio is 8.58%. After applying the FX prediction filter directly on Figure 5.7 (a), this value is reduced to 7.72%. However, the resolution of the inverted profile is affected. If the originally proposed multichannel algorithm is adopted, the residual energy ratio is only 4.55%. After using
Modification 1 and 2, this value can be further reduced to 2.02% and 3.72%, respectively, and the lateral continuity of the inverted reflectivity is significantly improved.

(a)

(b)
Figure 5.7 (a) Seismic reflectivity obtained by single-channel deconvolution. (b) Seismic reflectivity obtained by single-channel deconvolution with FX prediction filtering. (c) Seismic reflectivity obtained by the originally proposed multichannel algorithm. (d) Seismic reflectivity obtained by Modification 1. (e) Seismic reflectivity obtained by Modification 2.

Another comparison is performed using the same field data set as shown in Figure 5.4 (a). The results obtained from these different processes are presented from Figure 5.8 (a) to 5.8 (f) and the zoomed-in view of these results are illustrated from Figure 5.9 (a) to 5.9 (f). It comes to the same conclusion that Modification 1 and 2 are preferred as they can provide reflectivity profiles with clearer and more refined structures, as denoted by the black circle, where the stratification between the detailed layers can be observed.
(a)

(b)

(c)
Figure 5.8 (a) Seismic reflectivity obtained by the originally proposed multichannel algorithm after Step 3. (b) Seismic reflectivity obtained by the originally proposed multichannel algorithm after Step 4. (c) Seismic reflectivity obtained by Modification 1 after Step 3. (d) Seismic reflectivity obtained by Modification 1 after
Step 4. (e) Seismic reflectivity obtained by Modification 2 after Step 3. (f) Seismic reflectivity obtained by Modification 2 after Step 4.
Figure 5.9 (a) Zoomed-in seismic reflectivity obtained by the originally proposed multichannel algorithm after Step 3. (b) Zoomed-in seismic reflectivity obtained by the originally proposed multichannel algorithm after Step 4. (c) Zoomed-in seismic reflectivity obtained by Modification 1 after Step 3. (d) Zoomed-in seismic reflectivity obtained by Modification 1 after Step 4. (e) Zoomed-in seismic reflectivity obtained by Modification 2 after Step 3. (f) Zoomed-in seismic reflectivity obtained by Modification 2 after Step 4.

The amplitude spectra of the reflectivity series obtained by the originally proposed multichannel algorithm, Modification 1 and Modification 2 are shown in Figure 5.10. It is observed that these three methods can still provide results with relatively broad bandwidth.
Figure 5.10  (a) The amplitude spectrum of seismic reflectivity sequences inverted by the single-channel deconvolution. (b) The amplitude spectrum of seismic reflectivity sequences inverted by the originally proposed multichannel algorithm after Step 4. (c) The amplitude spectrum of seismic reflectivity sequences inverted by Modification 1 after Step 4. (d) The amplitude spectrum of seismic reflectivity sequences inverted by Modification 2 after Step 4. (The black solid line denotes the amplitude spectrum of the original seismic trace and the coloured lines represent the spectra of the reflectivity profiles inverted by different methods)

5.6 Conclusions

Most of the methods for solving seismic reflectivity inversion problems are based on a trace-by-trace principle and the retrieved reflectivity profiles may lack lateral coherence, especially when the input data are contaminated by background noise and/or have a complicated structure. In this chapter, a multichannel deconvolution method with the Cauchy constraint has been proposed to tackle this problem. Since the information of the adjacent traces was applied during the inversion process, the lateral continuity was increased while the noise could be suppressed, as confirmed by the numerical results.

Moreover, the originally proposed multichannel method was improved by applying two other different implementation procedures. A comparative study using both synthetic and field data sets was performed to evaluate the effectiveness of the two modified methods. Calculation results from both synthetic and field data sets showed that the two modified algorithms could further improve the reflectivity profiles, including increased lateral continuity, better structure characterization and stratification, and lower residual energy ratio. This indicates the potential of applying our algorithms to practical seismic data sets and
applications. The multichannel reflectivity inversion method based on the Cauchy constraint described in this chapter can be considered as the real multichannel method because the multichannel prediction operator is integrated into the iteration process of the inversion. With this method, the information of the adjacent traces is veritably applied during each iteration step of the deconvolution procedure, thus can provide better lateral continuity and structure identification of the inverted reflectivity profiles.
Chapter 6  Seismic impedance inversion in a multichannel manner

Since the reflectivity has been obtained, seismic ray impedance (RI) inversion can be started. RI inversion can provide the impedance profiles which reflect the lithology. However, if the input seismic reflectivity sequences have low signal-to-noise ratios (SNRs) or with complex structures, the lateral continuity and structure identification ability of the inverted ray impedance profile will be deteriorated. In this chapter, after introducing the conventional single-channel seismic impedance inversion, I introduce a multichannel impedance inversion algorithm, in order to overcome the drawbacks of the original single-channel method.

6.1 Multichannel impedance inversion

For a constant ray parameter \( p \), there is a relationship between the reflectivity and seismic ray impedances of the upper and lower layers, which can be expressed as (Wang, 2003)

\[
R(p) = \frac{Z_{i+1}(p) - Z_i(p)}{Z_{i+1}(p) + Z_i(p)},
\]

(6.1)

where \( R(p) \) represents the reflectivity, and \( Z_i(p) \) and \( Z_{i+1}(p) \) represent the seismic ray impedances of the layers above and below an interface \( i \), respectively.

To implement the impedance inversion, the objective function is

\[
J(Z) = \|R_{cal}(Z) - R_{obs}(Z)\|_2^2,
\]

(6.2)

where \( R_{obs}(Z) \) is the observed reflectivity series, while \( R_{cal}(Z) \) is the theoretical reflectivity series. Both of them are vectors having the length equal to the input seismic trace.

For a Generalized Linear Inversion (GLI) scheme (Cook and Schneider, 1983), equation (6.1) can be expanded using Taylor series and written into the matrix-vector form, as shown in equation (6.3):
\[ R(Z) = R(Z_0) + G(Z - Z_0), \] (6.3)

where \( Z \) is the ray impedance profile to be solved, \( Z_0 \) is the initial model of the ray impedance profile and \( G \) is the partial derivative matrix of the reflectivity sequences regarding ray impedance values. Lu (2010) and Zhang (2010) used this GLI scheme to solve the single-channel ray impedance inversion problem.

In the previous chapter, the lateral continuity of the reflectivity profile is assumed to be predictable by a FX prediction filter. This FX prediction operator can also be incorporated into the impedance inversion process, with the aim of improving the quality of the inverted impedance profiles:

\[ J(Z) = \| R_{cal}(F^{-1}PFZ) - R_{abs}(Z) \|_2^2, \] (6.4)

where \( P \) is the FX prediction operator, and \( F \) is the Fourier transform operator with respect to the time. For this multichannel inversion, since it involves spatial prediction, the conventional GLI scheme will not be suitable.

The multichannel inversion procedure consists of the following steps:

Step 1: Start with a linear inversion, the result is used as the initial model \( Z_0 \).

Step 2: Apply the prediction filter on \( Z_0 \) to obtain \( \tilde{Z}_0 \). Then calculate \( \tilde{R}_{cal} \) and matrix \( \tilde{G} \) using \( \tilde{Z}_0 \).

Step 3: Calculate \( \delta R \). Then the objective function in equation (6.4) can be linearized as

\[ J(Z) = \| \tilde{G} \delta Z - \delta R \|_2^2, \] (6.5)

where \( \delta R = R_{obs} - \tilde{R}_{cal} \), \( \delta Z = Z - \tilde{Z}_0 \), and \( \delta Z \) is the value that needs to be solved for updating the value of the ray impedance. The solution can be obtained as

\[ (\tilde{G}^T \tilde{G} + \lambda I)\delta Z = \tilde{G}^T \delta \tilde{R}(F^{-1}PFZ). \] (6.6)

where \( \lambda \) is a stabilization factor. Equation (6.6) can be solved iteratively by the conjugate gradient (CG) method.
6.2 Numerical results

In this section, the performance of the multichannel seismic impedance inversion method is evaluated using two field data sets. To fairly compare this multichannel RI inversion method with the previously proposed single-channel RI inversion method, both seismic reflectivity sequences inverted by the conventional single-channel BP method and single-channel deconvolution with the Cauchy constraint are used as the input for each RI method.

Figure 6.1 shows the original seismic profile, initial ray impedance model, impedance inverted by the single-channel and multichannel seismic impedance inversion methods, respectively. The initial impedance model is provided by SINOPEC Shengli Oilfield Company. The input reflectivity is retrieved by the conventional single-channel basis pursuit method introduced in Chapter 2. By comparing the impedance profiles obtained by these two impedance inversion methods, it can be seen that the lateral continuity of the multichannel impedance inversion result is relatively better, indicating a more stable inversion process. The spatial variation of the impedance values by the proposed method are finely characterised, as shown in Figure 6.1 (d).
Figure 6.1 (a) The original seismic profile. (b) The initial ray impedance model. (c) The impedance result obtained by the single-channel impedance inversion. (d) The impedance result obtained by the multichannel impedance inversion.
The original seismic profile, initial model and impedance results by different RI methods for another field data set are shown from Figure 6.2 (a) to (d). The input seismic reflectivity series here are obtained using the single-channel reflectivity inversion method with the Cauchy constraint introduced in Chapter 5. Similar conclusion can be obtained that by expanding the conventional single-channel generalized linear inversion method into the multichannel version, we can get clearer and more continuous seismic impedance profile.
Figure 6.2 (a) The original seismic profile. (b) The initial ray impedance model. (c) The impedance result obtained by the single-channel impedance inversion. (d) The impedance result obtained by the multichannel impedance inversion.

6.3 Comparative study of the multichannel ray impedance inversion method with input reflectivity obtained by different mechanisms

In this thesis, seismic reflectivity inversion and seismic impedance inversion in multichannel fashion have been proposed in Chapter 5 and 6, respectively. From the
numerical results, it has been observed that the multichannel methods can effectively improve the quality of seismic reflectivity and impedance profiles. In this section, a comparative study of two processes is conducted. The first process uses the reflectivity profile from the single-channel method as the input, while the second employs the seismic reflectivity inverted by the multichannel method as the input, and the impedance results from both processes are calculated by the proposed multichannel ray impedance method.

Figure 6.3 (a) to (c) show the results of the multichannel ray impedance inversion, whose inputs are the reflectivity sequences inverted by the multichannel reflectivity inversion with the Cauchy constraint (originally proposed, Modification 1 and Modification 2, respectively). These results are compared with that obtained by the proposed multichannel impedance inversion, whose input is calculated from the single-channel reflectivity inversion based on the Cauchy constraint, as shown in Figure 6.3 (d).

It is observed that the inverted impedance profiles are very similar to each other, as these methods share a similar multichannel mechanism. However, we can still find that the impedance profiles shown from Figure 6.3 (a) to (c), which are associated with multichannel reflectivity inversion process, demonstrate structures with better continuity when compared with the result calculated from the reflectivity inverted by the single-channel method in Figure 6.3 (d). This is due to the fact that when using the multichannel reflectivity inversion, the input for impedance inversion can be quantitatively controlled and enhanced delicately, which can improve the performance of impedance inversion from the very beginning. However, we still need more information, such as well logs, to have a complete evaluation of these processes.
Figure 6.3 Multichannel impedance inversion with its input reflectivity inverted by (a) multichannel reflectivity inversion with the Cauchy constraint (originally proposed), (b) multichannel reflectivity inversion with the Cauchy constraint (Modification 1), (c) multichannel reflectivity inversion with the Cauchy constraint (Modification 2), and (d) single-channel reflectivity inversion with the Cauchy constraint.

6.4 Conclusions

In this chapter, the multichannel ray impedance inversion method based on the FX prediction operator was introduced. Two field data sets were used to compare the performance of the single-channel and multichannel ray impedance inversions. It was shown that the multichannel ray impedance inversion scheme can provide smoother results, indicating a more stable inversion process. The obtained impedance profiles had a better lateral coherency, while the variation of the values of the impedance can be clearly observed, when compared with the conventional single-channel impedance inversion method.

Furthermore, I compared the results of multichannel RI inversion method, having the input reflectivity series from different reflectivity inversion methods. These inputs were obtained by a), the multichannel reflectivity inversion method with the Cauchy constraint (originally proposed method), b), Modification 1, c), Modification 2 and d), single-channel Cauchy constrained reflectivity inversion, respectively. It was shown that the results from these methods were very similar to each other, all of which can provide improved ray
impedance profiles with better structure characterization ability and lateral continuity. In the future, the comparison between the impedance results corresponding to single-channel and multichannel reflectivity inversion methods can be studied with more information such as well logs, in order to further evaluate the advantages and disadvantages of the proposed multichannel methods.

In fact, this implementation is not truly multichannel, as the FX prediction filter is not applied to the inversion as a constraint, but only applied to the result of the each iteration. Further research direction is to apply the FX prediction as a constraint on the “model”, which will be a highly non-linear problem.
Chapter 7  Conclusions and future work

This thesis focuses on the development of the multichannel inversion methods for seismic reflectivity series and ray impedance. My study is based on the well-known convolution model, where the wavelet is assumed to be known and has already been estimated by the method based on high order statistics. The innovative point of my research is the concept of “multichannel” during the process of seismic reflectivity and ray impedance inversions.

7.1 General conclusions

In this thesis, I first implemented the algorithms for single-channel reflectivity inversion, including the least-squares inversion based on the Cauchy constraint, conventional \( L_p \)-norm deconvolution and single-channel basis pursuit method. Since the wedge reflector and seismic response matrices built during the BP process are much larger, when compared with the size of the input seismic data, the computational consumption of this method is huge. To improve the efficiency of the BP algorithm, the GPU-based BP method was implemented. With this parallel algorithm, the precision of the inverted results was preserved, while the calculation time of the method was significantly decreased.

After that, the performance of the basis pursuit algorithm was tested under both high and low noise background. It was seen that there were deteriorations in both noise attenuation ability and lateral coherency of the inverted reflectivity sequences. FX prediction filtering was combined with the basis pursuit method for the data with low SNRs, to provide the reflectivity profiles with better continuity. In this method, FX prediction filtering process was applied outside the whole basis pursuit iteration process, thus this method can be seen as a pseudo multichannel method.

A multichannel deconvolution method based on the curvelet transform was then introduced in Chapter 4. A comparative study of several inversion methods, including the least-squares inversion, \( L_p \)-norm deconvolution and curvelet deconvolution, was presented.
Furthermore, due to the advantages of the curvelet deconvolution, the result obtained by this method was used as the new initial model, to enhance the lateral continuity of the spiky result retrieved by the $L_p$-norm deconvolution. Since the multichannel mechanism was only utilized to enhance the initial model of the inversion process, this combination can be seen as a semi-multichannel inversion method.

In Chapter 5, I developed a multichannel reflectivity inversion method based on the Cauchy constraint, by introducing the information of the adjacent traces into the conventional single-channel Cauchy deconvolution process. The obtained reflectivity profiles were more continuous and had better structure stratification than those inverted by the conventional single-channel deconvolution based on the Cauchy constraint. Two enhanced processes were then developed based on the originally proposed multichannel method and both of them can achieve even better results. When using the proposed methods, the connection between the adjacent traces was introduced by the multichannel prediction filter, which was used as an operator inside the iteration process. Therefore, this multichannel deconvolution with the Cauchy constraint is a real multichannel method.

Finally, following the concept of multichannel reflectivity inversion, the conventional generalized linear inversion algorithm for seismic ray impedance inversion can be expanded to a multichannel version, and the lateral continuity and structure identification ability of the retrieved impedance profiles can be improved.

The performance of the methods introduced in Chapters 3, 4 and 5 were validated by numerical examples, confirming that the multichannel deconvolution with the Cauchy constraint can give results with the best spatial continuity and lowest residual energy, when compared with other proposed methods. At the same time, the frequency bandwidth of this method was relatively broadband. The detailed stratification between the layers could also be preserved. This is because the multichannel algorithm with the Cauchy constraint is a real multichannel process, during which the prediction operator is incorporated into every single step.

With all these studies, it can be found that seismic inversion in a multichannel version is feasible. After introducing the relationship between the forward and backward traces into the inversion process, the detailed structures of the inverted reflectivity and impedance profiles
will be better illustrated and the spatial coherency will be improved. Thus, the multichannel methods are considered to have the potential to be applied to practical applications.

7.2 Applying other operators for multichannel inversion

By using the FX prediction operator in both the FX prediction filtering combined BP method and multichannel deconvolution based on the Cauchy constraint, seismic reflectivity series profiles with improved quality have been obtained. This mechanism can be further expanded to 3D data sets, while the FXY prediction filter (Chase, 1992; Gulunay et al., 1993; Gulunay, 2000; Wang, 2002) can be employed.

However, it should be noted that the FX prediction filter is based on the assumption that the seismic events are linear. In cases where the input seismic profile is very complicated, the assumption is not valid, and the retrieved results will definitely be deteriorated.

As described in Chapter 6, the multichannel seismic ray impedance inversion also used the FX prediction operator. Thus the result obtained by this method may also face the same problem, if the structure of the input seismic event does not obey the linear assumption. As a result, when carrying out the multichannel methods with the FX prediction filter, it is necessary to check whether the input traces meet this criterion. Furthermore, we can develop new multichannel algorithms by integrating other multichannel operators for different types of seismic data.

7.3 Multichannel reflectivity inversion methods based on statistics and probability

Besides the methods I implemented and developed in this thesis, there are other approaches to implement the multichannel reflectivity inversion. For example, methods based on statistics and probability, such as the iterated window maximization method developed by Kaaresen and Taxt (1998), the maximum \emph{a posteriori} estimation based on Markov-Bernoulli random field (MBRF) (Idier and Goussard, 1993) and Viterbi algorithm, which is also based
on MBRF (Forney, 1973; Heimer, 2008; Heimer and Cohen, 2009), have been studied during
the past decades.

When using the MBRF, the reflectivity $r$ is defined as an $Nt \times Ntr$ matrix, where $Ntr$ is the
trace number and $Nt$ is the number of time sample points. As seen in Figure 7.1, $q \in \{0, 1\}^{Ntr \times Nt}$ is a binary matrix representing whether there exists a reflector in the reflectivity
matrix $r$. If a reflector exists in row $n$ and column $m$ of $r$, then $q(n, m) = 1$; otherwise $q(n, m) = 0$. Three binary matrices $t^+, t^-, t^0 \in \{0, 1\}^{Ntr \times Nt}$, which are called transition variables, are defined to show the ascending, horizontal and descending transitions between reflectors, respectively. To be more detailed, $t^+(n, m) = 1$ if $q(n, m) = q(n - 1, m + 1) = 1$, and these two reflectors belong to the same layer boundary, otherwise $t^+(n, m) = 0$. $t^-(n, m)$ and $t^0(n, m)$ share the similar property.

Using $p$ to represent the probability, the characteristic parameters of the Bernoulli
distributions are defined as $\mu^+ = p\{t^+(n, m) = 1\}$, $\mu^- = p\{t^-(n, m) = 1\}$, $\mu^0 = p\{t^0(n, m) = 1\}$, $\lambda = p\{q(n, m) = 1\}$ and the probability of the discontinuities along layer boundaries is defined
as $p\{q(n, m) = 1\} t^+(n + 1, m - 1) = 0, t^-(n, m - 1) = 0, t^0(n - 1, m - 1) = 0\} = \varepsilon$.

---

![Figure 7.1 Location and transition variables.](image-url)
These definitions describe a Markov-Bernoulli random field of reflectors with Gauss amplitudes, which is homogeneous and symmetric. Based on the above definitions, we can simulate various reflectivity fields by changing the values of different parameters. Here, time sample number $N_t$ is set as 500, trace number $N_{tr}$ is set as 500. Using the values listed in Table 7.1, we can obtain four different models respectively, as seen in Figure 7.2. From the four models we can see that $\lambda$ is used to control the appearance of reflectors, the larger $\lambda$ is, and the more non-zero points in reflectivity profile there are. $\varepsilon$ controls the appearance of new boundaries while $\mu', \mu$ and $\mu$ manage the direction of layers in the profile.

<table>
<thead>
<tr>
<th>Model 1</th>
<th>0.05</th>
<th>0.0001</th>
<th>0.0084</th>
<th>0.0335</th>
<th>0.0084</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 2</td>
<td>0.053</td>
<td>0.0003</td>
<td>0.0084</td>
<td>0.0335</td>
<td>0.0084</td>
</tr>
<tr>
<td>Model 3</td>
<td>0.099</td>
<td>0.0001</td>
<td>0.0168</td>
<td>0.067</td>
<td>0.0168</td>
</tr>
<tr>
<td>Model 4</td>
<td>0.05</td>
<td>0.0001</td>
<td>0.0044</td>
<td>0.0335</td>
<td>0.0124</td>
</tr>
</tbody>
</table>

Table 7.1 Parameters used to create the reflectivity models.
Figure 7.2 Simulated models (a) Model 1. (b) Model 2. (c) Model 3. (d) Model 4.

Similar to the MBRF, methods based on statistics and probability often first simulate a model to represent seismic reflectivity. After that, based on this pre-defined model, the probability of different patterns of the reflectivity sequences will be calculated and compared. Finally, the pattern with the largest probability will be chosen as the result.

When using such methods, the number of parameters to be considered are often much larger than that of the methods that were proposed in the previous chapters. Consequently, the statistics and probability based methods are very computationally intensive. The GPU acceleration technique is considered to be a possible way of improving the efficiency. It is a very interesting and challenging topic and I will extend my research in this field in order to have a deeper understanding on seismic inversion.

7.4 High performance computing for geophysics

The GPU-based basis pursuit algorithm has been developed and presented in Chapter 2. However, only matrix multiplications, matrix-vector multiplications and conjugate gradient
method were parallelized in the parallel code. In the near future, other steps in this method, for example the Fourier transform and inverse Fourier transform will also be parallelized by the GPU, and will result in a further decrease in the computational time for the parallel BP algorithm.

Furthermore, basis pursuit was recently used for analysing pre-stack data sets (Zhang et al., 2013). However, since this algorithm is computational intensive, the non-parallel version of the BP method can only be applied to very small-scale seismic data. With the improved parallelization schemes, the BP method can be further applied for processing larger-scale pre-stack data sets.

The calculation time of other inversion methods mentioned in the previous chapters, including the multichannel deconvolution based on the Cauchy constraint, curvelet transform enhanced $L_p$-norm deconvolution and multichannel seismic ray impedance inversion, can also be decreased by the GPU in the future.

Finally, in practice, we often need to process seismic data of huge dimensions. As a result, it is very time-consuming, not only for seismic reflectivity inversion and ray impedance inversion, but also for other processes such as migration, multiple attenuation and full waveform inversion. Therefore, it is highly required to develop the GPU-based codes for various topics of geophysics. In addition, other ways for acceleration such as OpenMP and MPI can be investigated and combined with the existing GPU algorithms, to further improve the efficiency of the parallel codes.


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