Optimization of vehicle and pedestrian signals for isolated intersections

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Abstract

In most traffic signal optimization problems, pedestrian traffic at an intersection receives minor consideration compared to vehicular traffic, and is usually considered in the form of simplistic and exogenous constraints (e.g., minimum green time). This, however, could render the resulting signal timings sub-optimal, especially in dense urban areas with significant pedestrian traffic or when two-stage pedestrian crosswalks are present. This paper proposes a convex (quadratic) programming approach to optimize traffic signal timings for an isolated intersection with one- and two-stage crosswalks with undersaturated vehicular traffic condition. Both vehicle and pedestrian traffic are integrated into a unified framework. The total weighted delay of pedestrians and vehicles is adopted as the objective function. Both vehicle delay and pedestrian delay at different types of crosswalks (i.e. one- or two-stage) are modeled and converted into quadratic forms. Capacity constraints of the refuge island at a two-stage crosswalk is formulated. A case study verifies the effectiveness of the consideration of pedestrian delay as well as geometric and spatial constraints (e.g., available space on the refuge island) in the signal optimization. A further analysis shows that a two-stage crosswalk may outperform a one-stage crosswalk in terms of both vehicle and pedestrian delays in some circumstances.

Keywords
Traffic signal optimization, vehicle and pedestrian delay, spatial constraint, quadratic programing, isolated intersection
1 Introduction

With rapid urbanization and motorization especially in developing countries, daily commuters in dense urban areas suffer from heavy traffic congestion that leads to delays, environmental deterioration, and economic loss (Koonce et al., 2008). As reported in National Transportation Operations Coalition (NTOC) (2012), delays at traffic signals on major roadways are estimated to be 295 million vehicle-hours. Improving traffic signal operation has a significant impact on the efficiency of transportation systems in dense urban areas, potentially more effective than any other operational measure in the traffic engineering toolkit (NTOC, 2012).

Abundant studies on signal optimization have been conducted at isolated intersections, along corridors, or at network levels. A comprehensive literature review of off-line traffic signal optimization methods is provided in Wong et al. (2005). Conventionally, signal optimization are stage-based (Allsop, 1981; Webster, 1958) or group-based (phase-based) (Heydecker, 1992; Silcock, 1997). Compatible traffic movements are grouped and move together in the same time window, called a stage, in the stage-based approach. Green time is then allocated to each stage. In contrast, there is no need to specify a stage structure in the group-based approach. Green time is directly allocated to each movement. Obviously, the group-based approach is more flexible and potentially more efficient. More recently, it has been reported that signals could be optimized together with lane markings to further improve traffic condition (Wong and Heydecker, 2011; Wong and Wong, 2003a, b). These various approaches adopt similar optimization objectives; that is, the minimization of vehicle delay and maximization of intersection capacity with undersaturated and oversaturated vehicular traffic, respectively.

Based on the signal optimization for isolated intersections, signal coordination for multiple intersections along corridors is investigated. MAXBAND (Little et al., 1981) and MULTIBAND (Stamatiadis and Gartner, 1996) are proposed to signalize intersections along arterial roads. The stage-, group-, and lane-based methods are extended to the optimization of area traffic signals (Li and Gan, 1999; Wong and Wong, 2002; Wong and Yang, 1999). In addition, relatively new methods (e.g., fuzzy approaches and reinforcement learning algorithms) are applied to cope with complicated models and address computational burdens in signal optimization problems (Murat and Kikuchi, 2008; Ozan et al., 2015).

Moreover, additional concerns such as traffic flow uncertainty and vehicle-derived emissions are taken into consideration in signal optimization. The real-world traffic flows are time-varying and stochastic in nature (Cascetta et al., 2006; Szeto and Lo, 2005), even for the same time period across days of the same category (Yin, 2008). In general, robust signal optimization and on-line (adaptive) signal control are available to address flow uncertainty and influence traffic in an adaptive manner. Using robust optimization, signal parameters (e.g., cycle lengths and green splits) are predetermined, for example, by a scenario-based model using historical data in order to maintain satisfactory performance under flow fluctuations (Yin, 2008; Zhang et al., 2010). Robust optimization has also been applied to handle uncertainties in emission estimation arising from a macroscopic traffic modeling approach (Han et al. 2016). On the side of on-line optimization, real-time traffic measurements and prediction of future demand are utilized to dynamically adjust signal plans (Liu et al., 2015; Tong et al., 2015). With growing attention to the negative impacts of traffic on air quality and public health, sustainable traffic control measures have gained much attention. Minimization of vehicle emissions is regarded as one of the objectives in the signal
optimization, mainly by means of analytical models (Khalighi and Christofa, 2015; Ma and Nakamura, 2010) and traffic microsimulations (Mascia et al., 2016; “Brian” Park et al., 2009; Zhang et al., 2013). In addition, signal prioritization for emergency vehicles and public transportation (Head et al., 2006) is also investigated.

The numerous studies reviewed above focus on vehicles and their delays, with little or no attention given to crossing pedestrian traffic. In fact, most of these studies only implicitly consider pedestrians by imposing constraints such as minimum green time and pedestrian clearance time. Pedestrian delay is not included in the objectives of the signal optimization problems. This seems reasonable in situations where vehicular traffic dominates pedestrian traffic, and the former receives far greater priorities. However, this does not apply to dense metropolitan areas with significant pedestrian traffic. Indeed, very limited studies have provided insights into this issue. Li et al. (2010b) propose a traffic signal optimization strategy that considers both vehicular and pedestrian traffic. Vehicle and pedestrian delays are calculated by a deterministic queuing model. Wang and Tian (2010) develop a model in an analytical form to estimate pedestrian delay at a two-stage crosswalk. Ma et al. (2014) model pedestrian delay in the optimization of signal timing for an isolated intersection with exclusive pedestrian phases. In these studies, however, vehicle and pedestrian delay models are nonconvex, and global optima cannot be guaranteed. Moreover, only one-stage crosswalks have been investigated so far. Notably, two-stage crossings become increasingly popular and preferred by traffic engineers when a crosswalk at an intersection is long and a safe median refuge island is available. For example, in MOHURD (2011) a refuge island is requested when a crosswalk is longer than 16 m. However, the limited space on the refuge island is usually ignored. Insufficient space on the island may impose restrictive constraints to the signal optimization problems, while raising safety concerns. Although the extended cell transmission model in Zhang and Chang (2014) may provide a potential way to address these issues, their optimization problem is solved with heuristic methods, which does not guarantee global optimality and may be computationally expensive. Therefore, a rigorous and effective signal design framework for intersections with two-stage crosswalks is needed to minimize vehicle delay as well as provide a high level of service for pedestrians.

In order to address this gap, this paper presents a convex (quadratic) programming approach to optimize fixed signals for vehicles and pedestrians at an isolated intersection with one- and two-stage crosswalks with undersaturated vehicular traffic.¹ A unified signal timing optimization framework is developed, in which the delays of both vehicles and pedestrians are explicitly accounted for. The impacts of spatial and temporal constraints, including space limitation of the refuge island and the occupancy of the island, are explicitly captured and modeled through linear formulae. Next, a series of quadratic formulae are derived to approximate vehicle and pedestrian delays at the signalized intersection with one- and two-stage crosswalks. The feasibility of the proposed delay model is validated. The signal optimization problem is decomposed into a series of quadratic programming problems, which guarantees global optimality and maintains computational tractability, allowing computationally intensive tasks such as real-time signal control and signal coordination to be subsequently developed. The proposed model can be readily

¹The model is built assuming undersaturated vehicular traffic conditions as vehicle delay is part of the objective to be minimized. In case of oversaturation, maximization of intersection capacity tends to be used as the objective. In this latter case, all of the proposed linear constraints remain effective, and the signal optimization problem can be formulated as a linear program when the reserve capacity is adopted (Wong and Heydecker, 2011; Wong and Wong, 2003a, b).
solved by many existing commercial solvers efficiently, which serves well the purpose for practical applications. The effectiveness of the proposed model is validated by numerical examples.

The remainder of this paper is structured as follows. Section 2 presents the signal optimization problem and some key notations. In Section 3, the quadratic programming models are developed. In order to illustrate the model, a numerical study is conducted in Section 4. Finally, conclusions and recommendations are provided in Section 5.

## 2 Problem description and notations

### 2.1 Signal optimization for intersections with crosswalks

A typical intersection with four approaches is illustrated in Fig. 1. Generally, a one- or two-stage pedestrian crosswalk is applied in each approach. As a common example, it is assumed that Approach 1 and Approach 3 are on the main road; Approach 2 and Approach 4 are on a minor road. There are four approach lanes and three exit lanes in Approach 1 and Approach 3, two approach lanes and two exit lanes in Approach 2 and Approach 4. Two-stage crosswalks are used in Approach 1 and Approach 3, and one-stage crosswalks are used in Approach 2 and Approach 4. A refuge island should be installed, for example, between the 1st segment and the 2nd segment of the two-stage crosswalk in Approach 1, for pedestrian safety (Li et al., 2010a). A two-stage crosswalk has two separate pedestrian signal phases as a general case. That is, the signals for the 1st and 2nd segments are independent.

![Fig. 1 A typical intersection with four approaches.](image-url)

In practice, a typical North American dual-ring, eight-phase controller is usually used. The controller in Fig. 2 is implemented in this study. Phase 1 and Phase 8 are assumed to be adjacent to reduce pedestrian clearance time due to high pedestrian traffic. The proposed model can also be
applied to other phase structures. Pedestrian movements on the 1\textsuperscript{st} and 2\textsuperscript{nd} segments in Approach 1 are illustrated as an example. Obviously, vehicle and pedestrian signals are related to each other. For example, pedestrians can cross over the 1\textsuperscript{st} segment of the two-stage crosswalk in Approach 1 during Phase 1 and Phase 8. Undersaturated vehicular traffic is assumed in which case vehicle delay is supposed to be minimized. Therefore, the remaining problem is to optimize vehicle and pedestrian signals for the intersection for minimal vehicle and pedestrian delays. Fixed signal timings are applied.

![Dual-ring, eight-phase controller.](image)

**2.2 Notations**

To facilitate the presentation of the model, key notations to be applied hereafter are summarized in Table 1.

<table>
<thead>
<tr>
<th><strong>Table 1 Parameters and Variables.</strong></th>
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<tbody>
<tr>
<td><strong>General parameters:</strong></td>
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<tr>
<td>( M ): A sufficiently large positive constant</td>
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<tr>
<td><strong>Traffic parameters:</strong></td>
</tr>
<tr>
<td>( \beta ): Weighting parameter, ( 0 \leq \beta \leq 1 )</td>
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<tr>
<td>( r_1, r_2 ): Arrival rates of pedestrians on Side I and Side II of Approach 1 (ped/s)</td>
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<tr>
<td>( r_p^p ): Arrival rate of vehicles in Phase ( p ) (pcu/s), ( p = 1, \ldots, 8 )</td>
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<tr>
<td>( s_p ): Saturation flow rate of vehicles in Phase ( p ) (pcu/s), ( p = 1, \ldots, 8 )</td>
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<tr>
<td>( p_{p,\max} ): Maximum acceptable degree of saturation of Phase ( p ), ( p = 1, \ldots, 8 )</td>
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<tr>
<td>( C_1, C_2 ): Minimum and maximum signal cycle lengths (s)</td>
</tr>
<tr>
<td>( g_{d,\min} ): Minimum green time for drivers to avoid stop-and-go motions (s)</td>
</tr>
<tr>
<td>( g_{1,\min} ): Minimum Walk interval for crossing pedestrians on Side I of Approach 1 (s)</td>
</tr>
<tr>
<td>( g_{2,\min} ): Minimum Walk interval for crossing pedestrians on Side II of Approach 1 (s)</td>
</tr>
<tr>
<td>( \tau_1, \tau_2 ): Time to cross over the 1\textsuperscript{st} and 2\textsuperscript{nd} segments of the crosswalk in Approach 1 if a two-stage crosswalk is applied (s)</td>
</tr>
<tr>
<td>( \tau ): Time to cross over the crosswalk in Approach 1 if a one-stage crosswalk is applied (s)</td>
</tr>
<tr>
<td>( N_{\max} ): Maximum number of waiting pedestrians that can be accommodated on the refuge island at the two-stage crosswalk in Approach 1</td>
</tr>
<tr>
<td><strong>Decision variables:</strong></td>
</tr>
<tr>
<td>( C ): Signal cycle length (s)</td>
</tr>
</tbody>
</table>
\( \alpha_i \): 1, if Case I is optimal for a two-stage crosswalk in Approach 1; 0, otherwise. \( i = 1, ..., 10 \)

\( t_p \): Ending time of Phase \( p \) (s), \( p = 1, ..., 8 \)

\( g_p \): Green time of Phase \( p \) (s), \( p = 1, ..., 8 \)

\( t_w \): Starting time of the Walk interval for the crosswalk in Approach 1 if a one-stage crosswalk is applied (s)

\( t_f \): Starting time of the flashing Don’t Walk (FDW) clearance interval for the crosswalk in Approach 1 if a one-stage crosswalk is applied (s)

\( t_r \): Starting time of the Don’t Walk or Stop interval for the crosswalk in Approach 1 if a one-stage crosswalk is applied (s)

\( t_{g1}^{1}, t_{g1}^{2} \): Starting time of the Walk intervals for the 1st and 2nd segments of the crosswalk in Approach 1 if a two-stage crosswalk is applied (s)

\( t_{f1}^{1}, t_{f1}^{2} \): Starting time of the FDW clearance intervals for the 1st and 2nd segments of the crosswalk in Approach 1 if a two-stage crosswalk is applied (s)

\( t_{r1}^{1}, t_{r1}^{2} \): Starting times of the Don’t Walk or Stop intervals for the 1st and 2nd segments of the crosswalk in Approach 1 if a two-stage crosswalk is applied (s)

**Intermediate variables:**

\( g_{p, \text{min}} \): Minimum green time of Phase \( p \) to clear queues (s), \( p = 1, ..., 8 \)

\( D_p \): Vehicle delay in Phase \( p \) (s), \( p = 1, ..., 8 \)

\( D_{\text{per}}^m \): Total pedestrian delay at the crosswalk in Approach \( m \) (s), \( m = 1, 2, 3, 4 \)

\( D_1^1, D_2^1 \): 1st stage delays of waiting pedestrians on Side I and Side II of Approach 1 (s)

\( D_2^2, D_3^2 \): 2nd stage delays of waiting pedestrians on the refuge island from Side I and Side II of Approach 1 (s)

\( n_1, n_2 \): Accumulated pedestrians on Side I and II of Approach 1 when the Walk intervals start

\( n_{t1}, n_{t2} \): Introduced variables for the convenience of modeling

### 3 Model formulation

A one- or two-stage crosswalk is usually applied in each approach of an intersection, as shown in Fig. 1. The signals of one- and two-stage crosswalks in Approach 1 are described in detail as an example to illustrate the modeling of pedestrian constraints and delay. As shown in Fig. 3, a precedence graph is employed to represent the dual-ring eight phases in Fig. 2 to capture the relations between vehicle and pedestrian signals.
Pedestrians wait on Side I and Side II until the starts of the Walk intervals (i.e., \( t_g, t_g^1 \) and \( t_g^2 \)). If a one-stage crosswalk is used in Approach 1, pedestrians can cross only during Phase 8. The pedestrian Walk interval starts when Phase 8 starts and the Stop interval starts when Phase 8 ends to reduce pedestrian delay (i.e., \( t_g = t_7, t_r = t_8 \)). If a two-stage crosswalk is used, two separate pedestrian signal phases on the 1st and 2nd segments are available. Crossing pedestrians can wait on the refuge island. Pedestrians can cross over the 1st segment during Phase 1 and Phase 8, and the 2nd segment during Phase 3 and Phase 4 (i.e., Phase 7 and Phase 8). To make the best use of vehicle phases, the pedestrian Walk interval on the 1st segment and Phase 8 start at the same time (\( t_g^1 = t_7 \)); the Stop interval on the 2nd segment starts when Phase 4 ends (\( t_r^2 = t_4 \)). Note that Phases 3, 4 and Phases 1, 8 may not be fully utilized for crossing pedestrians owing to the limited space on the refuge island (i.e., \( t_g^2 \geq t_2, t_g^1 \leq C + t_1 \)).

### 3.1 Vehicle-related constraints

In this section we present constraints pertaining to vehicular traffic at the intersection, including cycle time, phase sequences, and minimum green time. As the signal timings are fixed and cyclic, the following constraints are expressed for only one cycle.

#### 3.1.1 Cycle length

It is assumed that the minimum and maximum signal cycles, denoted as \( C_1 \) and \( C_2 \), respectively, are given in advance.

\[
C_1 \leq C \leq C_2
\]  

#### 3.1.2 Precedence relationships of signal phases

As shown in Fig. 3, the start time \( t_0 \) of the cycle is set at zero as the reference time:

\[
t_0 = 0
\]  

Phase 2 and Phase 6 terminate at the same time:

\[
t_2 = t_6
\]  

Both Phase 4 and Phase 8 terminate at the time of \( C \):

\[
t_4 = t_8 = C
\]

#### 3.1.3 Green time of each phase

For simplicity, green time of each phase is regarded as effective green duration. The green duration of Phase \( p \) is calculated as

\[
t_p - t_{p-1} = g_p \quad p = 1, \ldots, 4, 6, \ldots, 8
\]

\[
t_5 - t_0 = g_5
\]

#### 3.1.4 Minimum green time

On the basis of the arrival rate and the saturation flow rate in Phase \( p \), as shown in the following Fig. 4 (a), the minimum green time of each phase to clear vehicle accumulation during the red time is calculated as

\[
g_{p, \text{min}} = \frac{r^p(C - g_p)}{s_p - r^p} \quad p = 1, 2, \ldots, 8
\]
The green time $g_p$ should be no less than the minimum value $g_{p,\text{min}}$:

$$g_{p,\text{min}} \leq g_p \quad p = 1,2,\ldots,8$$  \hfill (8)

Substitute Eq. (7) for $g_{p,\text{min}}$, and Eq. (8) can be modified as

$$r_v^p C \leq s_p g_p \quad p = 1,2,\ldots,8$$  \hfill (9)

Furthermore, another minimum green time $g_{d,\text{min}}$ is set so that drivers can have sufficient time to prepare the changes of signals to avoid sudden stop-and-go motions:

$$g_p \geq g_{d,\text{min}} \quad p = 1,2,\ldots,8$$  \hfill (10)

![Fig. 4 Delay models: (a) vehicle delay model; (b) pedestrian delay model.](image)

3.1.5 Maximum acceptable degree of saturation

The saturation degree of Phase $p$ is calculated as

$$\rho_p = \frac{r_v^p}{s_p g_p C} \quad p = 1,2,\ldots,8$$  \hfill (11)

To ensure the service level, a maximum acceptable degree of saturation $\rho_{p,\text{max}}$ is set:

$$\rho_p \leq \rho_{p,\text{max}}$$  \hfill (12)

Substitute the expression (11) for $\rho_p$, and Eq. (12) can be modified as

$$r_v^p C \leq \rho_{p,\text{max}} s_p g_p \quad p = 1,2,\ldots,8$$  \hfill (13)

Note that these constraints should be set carefully since they may render the problem infeasible under high vehicle demand.

3.2 Pedestrian-related constraints of one-stage crosswalks

3.2.1 Starting time of the pedestrian Walk interval

As shown in Fig. 3, pedestrians can cross over the intersection at a one-stage crosswalk only during Phase 8 for safety concerns. To make the best use of Phase 8, the pedestrian Walk interval starts when Phase 7 ends:

$$t_g = t_7$$  \hfill (14)

3.2.2 Starting time of the Do-not Walk or Stop interval

For similar purposes, the Don’t Walk or Stop interval starts when Phase 8 ends to provide
pedestrians more crossing time:
\[ t_r = t_g \]  

### 3.2.3 Starting time of the flashing Do-not Walk (FDW) clearance interval

Given the layout of the intersection, the FDW clearance interval \( \tau \) is determined. Therefore, the starting time of the clearance interval is calculated as
\[ t_f = t_r - \tau \]  

### 3.2.4 Minimum pedestrian Walk intervals

To satisfy the crossing demand on Side I and Side II, minimum pedestrian Walk intervals on both sides should be guaranteed:
\[ t_f - t_g \geq g_{1,min} \]  
\[ t_f - t_g \geq g_{2,min} \]  

The minimum values can be calibrated according to pedestrian demand as discussed in Lao et al. (2009).

### 3.3 Pedestrian-related constraints of two-stage crosswalks

#### 3.3.1 Potentially optimal cases

Pedestrian signal phases on the 1st and 2nd segments are separate. As a result, it is difficult to model pedestrian delay and the waiting pedestrian number on the refuge island. The arrival time of the pedestrians from Side II at the refuge island is between \([t_g^2 + \tau_2, t_f^2]\) while the Walk interval on the 1st segment is \([t_g^1, t_f^1]\). Similarly, the arrival time of the pedestrians from Side I is between \([t_g^3 + \tau_1, t_f^1]\) while the Walk interval on the 2nd segment is \([t_g^2, t_f^2]\). According to the relative positions of these intervals, ten potentially optimal cases are identified for each two-stage crosswalk. The optimal pedestrian signals at the crosswalk are produced in one of the ten cases. This is specified by
\[ \sum_{i=1}^{10} \alpha_i = 1 \]  

where \( \alpha_i = 1 \) if Case \( i \) is optimal; otherwise, \( \alpha_i = 0 \) \((i = 1, ..., 10)\).

Case 1 is shown in Fig. 5 as an example (we refer the reader to Appendix I for Cases 2–10), which only shows the traffic pattern in one cycle. Fig. 5 (a) shows the precedence graph when the following relationships of the above intervals hold:
\[ t_g^2 + \tau_2 \leq t_1^2 < t_f^1 \leq t_f^2 \]  
\[ t_f^1 > t_1^3 + \tau_1 \geq t_f^3 > t_g^2 \]  

In Fig. 5 (b), the dark line represents the number of pedestrians who arrive at the refuge island from Side II in one cycle length; the blue line shows the pedestrian Walk interval on the 1st segment. Fig. 5 (c) shows the number of waiting pedestrians on the island who come from Side II, which is derived from Fig. 5 (b). The shaded area represents the delay of these pedestrians. Similarly, Fig. 5 (d) and (e) show the information of pedestrians from Side I. Fig. 5 (f) shows the total number of pedestrians on the island from both sides which is derived by combining Fig. 5 (c) and (e).
3.3.2 Starting time of the pedestrian Walk interval on the 2nd segment

In all cases, the starting time of the Walk interval on the 2nd segment should be no earlier than the ending time of Phase 2 ($t_2$):

$$t_2^2 \geq t_2$$  \hspace{1cm} (22)

The earliest arrival time at the refuge island from Side II is $t_2^2 + \tau_2$. The Walk interval on the 1st segment is $[t_g^2, t_f^2]$. In Cases 1–4 (refer to Fig. 5 and Appendix 1), $t_2^2 + \tau_2$ and $[t_g^1, t_f^1]$ satisfy the following constraint (23). Considering the constraint (22), the following constraint (24) should hold as well. Eq. (23) and Eq. (24) are effective only when $\alpha_i = 1 \ (i = 1, 2, 3, 4)$.

$$t_g^2 + \tau_2 \leq t_f^1 + (1 - \sum_{i=1}^{4} \alpha_i)M$$ \hspace{1cm} (23)

$$t_2 + \tau_2 < t_f^1 + (1 - \sum_{i=1}^{4} \alpha_i)M$$ \hspace{1cm} (24)

In Cases 5–8, $t_g^2$ is taken as $t_2$, which can be specified by Eq. (22) and Eq. (25). At the same time, $t_2 + \tau_2$ (or $t_g^2 + \tau_2$) and $[t_g^1, t_f^1]$ satisfy the following constraint (26). Eq. (25) and Eq. (26) are effective only when $\alpha_i = 1 \ (i = 5, 6, 7, 8)$.

$$t_g^2 \leq t_2 + \left(1 - \sum_{i=5}^{8} \alpha_i\right)M$$ \hspace{1cm} (25)

$$t_g^1 - \left(1 - \sum_{i=5}^{8} \alpha_i\right)M \leq t_2 + \tau_2 < t_f^1 + \left(1 - \sum_{i=5}^{8} \alpha_i\right)M$$ \hspace{1cm} (26)

In Cases 9–10, $t_g^2 + \tau_2 \geq t_f^1$, which can be realized by Eq. (27) and Eq. (22). Eq. (27) is effective only when $\alpha_i = 1 \ (i = 9, 10)$. 

**Fig. 5** Case 1: (a) precedence graph representation, (b) arrivals at the refuge island from Side II, (c) accumulated waiting pedestrians on the refuge island from Side II, (d) arrivals at the refuge island from Side I, (e) accumulated waiting pedestrians on the refuge island from Side I, and (f) total waiting pedestrian number on the refuge island. Possible maximum numbers are marked by a red square or triangle.
\[ t_2 + \tau_2 \geq t_f^1 - \left( 1 - \sum_{i=9}^{10} \alpha_i \right) M \] (27)

### 3.3.3 Starting time of the pedestrian Walk interval on the 1st segment

In all cases, the pedestrian Walk interval on the 1st segment and Phase 8 start at the same time to reduce pedestrian delay:

\[ t_g^1 = t_f \] (28)

The earliest arrival time at the refuge island from Side I is \( t_g^1 + \tau_1 \). The Walk interval on the 2nd segment is \([t_g^2, t_f^2]\). In Cases 2\(n + 1\) (\(n = 0,1,2,3,4\)) (refer to Fig. 5 and Appendix I), \( t_g^1 + \tau_1 \) and \([t_g^2, t_f^2]\) satisfy the following constraint (29). All pedestrians from Side I have to wait on the refuge island.

\[ t_g^1 + \tau_1 \geq t_f^2 - \left( 1 - \sum_{n=0}^{4} \alpha_{2n+1} \right) M \] (29)

In Cases 2\(n + 2\) (\(n = 0,1,2,3,4\), \(t_g^1 + \tau_1 \) and \([t_g^2, t_f^2]\) satisfy the following constraint (30). Pedestrians arriving at the refuge island from Side I during \([t_g^1 + \tau_1, t_f^2]\) can proceed without delay while the others have to wait on the island.

\[ t_f^2 + \left( 1 - \sum_{n=0}^{4} \alpha_{2n+2} \right) M > t_g^1 + \tau_1 \geq t_g^2 - \left( 1 - \sum_{n=0}^{4} \alpha_{2n+2} \right) M \] (30)

### 3.3.4 Starting time of Don’t Walk or Stop intervals

Pedestrians can cross over the 1st segment only during Phase 8 and Phase 1. Therefore, the starting time of the Don’t Walk or Stop interval on the 1st segment \( (t_r^1) \) should be no later than the ending time of Phase 1 for safety concerns:

\[ t_r^1 \leq t_1 + C \] (31)

The starting time of the Don’t Walk or Stop interval on the 2nd segment is set as the ending time of Phase 8 to maximize available pedestrian crossing time:

\[ t_r^2 = t_8 \] (32)

### 3.3.5 Starting time of FDW clearance intervals

Given the layout of the intersection, the FDW clearance intervals on the 1st and 2nd segments \((\tau_1 \text{ and } \tau_2)\) are determined. Therefore, the starting time of the clearance intervals on both segments \((t_f^1 \text{ and } t_f^2)\) are calculated as

\[ t_f^1 = t_r^1 - \tau_1 \] (33)

\[ t_f^2 = t_r^2 - \tau_2 \] (34)

In all cases, \( t_r^1 \) (i.e., \( t_f^1 + \tau_1 \)) is larger than \( t_f^2 \) to make use of available crossing time on the 2nd segment for pedestrians from Side I:

\[ t_r^1 = t_f^1 + \tau_1 \geq t_f^2 \] (35)

Because pedestrians arriving at the refuge island from Side I earlier than \( t_f^2 \) can proceed without delay.
In Cases 1, 2, 5, and 6, \( t_f^1 \) and \( t_r^2 \) are related in the following Eq. (36). However, Eq. (37) holds in Cases 3, 4, 7, and 8.

\[
\begin{align*}
    t_f^1 & \leq t_r^2 + (1 - \alpha_1 - \alpha_2 - \alpha_5 - \alpha_6)M \\
    t_f^1 & > t_r^2 - (1 - \alpha_3 - \alpha_4 - \alpha_7 - \alpha_8)M
\end{align*}
\] (36) (37)

### 3.3.6 Minimum pedestrian Walk intervals

To satisfy the crossing demand on Side I and Side II, minimum pedestrian Walk intervals at both sides should be guaranteed:

\[
\begin{align*}
    t_f^1 - t_g^1 & \geq g_{1,min} \\
    t_f^2 - t_g^2 & \geq g_{2,min}
\end{align*}
\] (38) (39)

As the pedestrian demand on Side I and Side II may differ, \( g_{1,min} \) and \( g_{2,min} \) can take different values.

### 3.3.7 Number of waiting pedestrians on the refuge island

The total number of waiting pedestrians on the refuge island can be easily identified in each case, for example, as the points marked by a red square and a red triangle in Fig. 5 (f). It is shown that there are two possible maximum values in Case 1. These values can be easily expressed with linear formulae derived from Fig. 5 (c) and (e), and are then constrained by the capacity of the refuge island.

The following Eq. (40) and Eq. (41) show the capacity constraints in a unified form for all cases. These equations mean that both of the two maximum values should not exceed the island capacity \( N_{max} \). In each case with determined \( \alpha_i (i = 1, \ldots, 10) \), the following formulae can be simplified. The calculation of the pedestrian number on the island is broken into several parts for the convenience of illustration.

\[
\begin{align*}
    n_{t_2} & = \sum_{i=1}^{2} \alpha_i + n_2 \sum_{i=1}^{4} \alpha_i + r_1 \left( t_g^1 - (t_f^2 + \tau) \right) \sum_{i=1}^{4} \alpha_i \leq N_{max} \\
    n_{t_1} + n_{t_2} & \leq N_{max}
\end{align*}
\] (40) (41)

where \( n_{t_1} \), \( n_{t_2} \), and \( n_2 \) are calculated in Eq. (42), Eq. (43), and Eq. (44), respectively:

\[
\begin{align*}
    n_{t_1} & = r_1 \left( t_f^1 - t_f^2 \right) \sum_{n=0}^{2} \alpha_{2n+2} + r_1 C \sum_{n=0}^{2} \alpha_{2n+1} \\
    n_{t_2} & = r_2 \left( t_f^2 - t_f^1 \right) \left( \sum_{i=1}^{2} \alpha_i + \sum_{i=5}^{6} \alpha_i \right) + r_2 C \sum_{i=9}^{10} \alpha_i \\
    n_2 & = r_2 \left( C - (t_f^2 - t_g^2) \right)
\end{align*}
\] (42) (43) (44)

### 3.4 Delay models

#### 3.4.1 Vehicle delay

The deterministic queuing model in Li et al. (2010b) is applied. Vehicle delay \( D_p \) in each
Phase $p$ is calculated as the shaded area in Fig. 4 (a):

$$D_p = \frac{r_p^p s_p (C - g_p)^2}{2(s_p - v_p^p)} \quad p = 1, 2, ..., 8$$  \hspace{1cm} (45)

Although this delay model is simple, it will produce similar delay compared with other models such as the shock wave delay model and the steady-stage stochastic delay model under undersaturated vehicular traffic (Dion et al., 2004).

### 3.4.2 Pedestrian delay at one-stage crosswalks

Take the one-stage crosswalk in Approach 1 in Fig. 3 as an example, the delays on Side I and Side II are calculated as the shaded area in Fig. 4 (b) by use of the commonly used pedestrian delay model in the Highway Capacity Manual (HCM) (TRB, 2010):

$$D_j^f = \frac{r_j^f (C - (t_f - t_g))^2}{2} \quad j = 1, 2$$  \hspace{1cm} (46)

Therefore, the total pedestrian delay in Approach 1 is finally determined in the following Eq. (47):

$$D_{per}^1 = \sum_{j=1}^{2} D_j^f$$  \hspace{1cm} (47)

### 3.4.3 Pedestrian delay at two-stage crosswalks

The calculation of pedestrian delay at a two-stage crosswalk is complicated. The delay of pedestrians from either Side I or Side II is the sum of the experienced delays at the first and second stages (i.e., $D_j^f$ and $D_j^s$, respectively):

$$D_{per}^2 = \sum_{j=1}^{2} \sum_{k=1}^{2} D_k^j$$  \hspace{1cm} (48)

The delay at the first stage $D_j^f$ is calculated in a similar way to Eq. (46):

$$D_j^f = \frac{r_j^f (C - (t_f - t_g))^2}{2} \quad j = 1, 2$$  \hspace{1cm} (49)

The delay of pedestrians from Side I at the second stage $D_j^s$ is calculated as the shaded area, for example, in Fig. 5 (e) for Case 1. Similarly, $D_j^s$ in other cases can also be expressed by quadratic formulae. Eqs. (50)–(53) are the formulae to calculate $D_j^s$ for all cases in a unified form. The calculation of $D_j^s$ is broken into several parts for the convenience of illustration.

$$D_j^s = d_{11} + d_{12}$$  \hspace{1cm} (50)

$$d_{11} = \frac{r_1^s}{2} (t_1^f - t_2^f)^2 \sum_{n=0}^{4} \alpha_{2n+1} + \frac{r_1^s}{2} (t_1^s - t_2^s)^2 \sum_{n=0}^{4} \alpha_{2n+2}$$  \hspace{1cm} (51)

$$d_{12} = n_1 (t_1^f - t_2^f) \sum_{n=0}^{4} \alpha_{2n+1} + n_1 (C + t_2^f - t_1^f) \left( \sum_{i=1}^{4} \alpha_i + \sum_{i=9}^{10} \alpha_i \right) + n_1 (C + t_2 - t_1^f) \sum_{i=5}^{8} \alpha_i$$  \hspace{1cm} (52)
where $n_{t1}$ is calculated in Eq. (42), and $n_1$ is calculated as

$$n_1 = r_1 (C - (t_f^1 - t_g^1))$$

Note that only certain terms in the above equations remain in each case with determined $\alpha_i$ ($i = 1, \ldots, 10$).

The delay of pedestrians from Side II at the second stage $D_{2}^2$ is calculated as the shaded area, for example, in Fig. 5 (c) for Case 1. Similar to $D_{1}^1$, $D_{2}^2$ in all cases is calculated by Eqs. (54)–(56) in a unified form:

$$D_{2}^2 = d_{21} + d_{22}$$

$$d_{21} = \frac{r_2}{2} (t_r^2 - t_f^1)^2 \left( \sum_{i=1}^{2} \alpha_i + \sum_{i=5}^{6} \alpha_i \right) + \frac{r_2}{2} (t_g^1 - (t_g^2 + \tau_2))^2 \sum_{i=1}^{4} \alpha_i + \frac{r_2}{2} (t_f^2 - t_g^2)^2 \sum_{i=9}^{10} \alpha_i$$

$$d_{22} = n_2 (t_f^2 - t_g^2) \left( \alpha_i \sum_{i=9}^{10} \alpha_i + n_2 (t_g^1 - (t_g^2 + \tau_2)) \sum_{i=1}^{4} \alpha_i \right) + n_{t2} (C + t_f^1 - t_r^1) \left( \sum_{i=1}^{2} \alpha_i + \sum_{i=5}^{6} \alpha_i + \sum_{i=9}^{10} \alpha_i \right)$$

where $n_{t2}$, $n_2$ are calculated in the above Eq. (43) and Eq. (44), respectively. In this way, the total pedestrian delay $D_{1\text{per}}^1$ in Approach 1 can be expressed by a quadratic formula in each case with fixed $\alpha_i$ ($i = 1, \ldots, 10$).

The formulae of vehicle delay (Eq. (45)) and pedestrian delay at a one-stage crosswalk (Eq. (46)) are convex, quadratic while the formula of pedestrian delay at a two-stage crosswalk is nonconvex due to the terms (52) and (56). As a result, no global optima is guaranteed although there are many existing algorithms to solve a nonconvex, quadratic model efficiently for local optima. In this study, the following convex, quadratic formulae Eqs. (57) and (58) are proposed to replace Eqs. (52) and (56), respectively, to calculate approximate pedestrian delay. Note that Eqs. (57) and (58) are the transformations of Eqs. (52) and (56) in form but have no specific meanings.

$$d_{12} = \frac{y_1 n_1}{2} + \frac{y_2 (t_f^1 - t_g^1)^2}{2} \sum_{n=0}^{4} \alpha_{2n+1} + \frac{y_3 n_{t1}^2}{2}$$

$$+ \frac{y_2 (C + t_g^1 - t_f^1)^2}{2} \left( \sum_{i=1}^{2} \alpha_i + \sum_{i=9}^{10} \alpha_i \right) + \frac{y_2 (C + t_2 - t_f^2)^2}{2} \sum_{i=5}^{8} \alpha_i$$
$$d'_{22} = \frac{y_3 n_2}{2} \left( \sum_{i=1}^{4} \alpha_i + \sum_{i=9}^{10} \alpha_i \right) + \frac{y_4 (t_2^2 - t_2^2)}{2} \left( \sum_{i=1}^{10} \alpha_i + \sum_{i=9}^{10} \alpha_i \right)$$

$$+ \frac{y_3 n_{t2}^2 + y_4 (C + t_2^1 - t_2^2)}{2} \left( \sum_{i=1}^{2} \alpha_i + \sum_{i=5}^{6} \alpha_i + \sum_{i=9}^{10} \alpha_i \right)$$

where $y_1 = 1.948$, $y_2 = 0.149$, $y_3 = 1.626$, and $y_4 = 0.011$ are factors, which are calibrated using signal samples randomly generated satisfying the constraints in Eqs. (1)–(13) and (19)–(39). Therefore, the delays $D^1_i$ and $D^2_i$ are modified in the following Eqs. (59) and (60), whose feasibility is verified in the numerical tests.

$$D^1_i = d_{11} + d'_{12} \quad (59)$$

$$D^2_i = d_{21} + d'_{22} \quad (60)$$

### 3.5 Optimization models

The objective function of the optimization model is to minimize vehicle delay and pedestrian delay in each approach with a crosswalk:

$$\min \beta \sum_{p=1}^{8} D_p + (1 - \beta) \sum_{m=1}^{4} D_{m\text{per}}$$

where $\beta \ (0 \leq \beta \leq 1)$ is a weighting parameter, and it can be calibrated by the economic losses of vehicle and pedestrian delays. $D_p$ is calculated in the above Eq. (45). $D_{m\text{per}}$ is set at zero if there is no crosswalk in Approach $m$; otherwise, it can be calculated in a similar way to $D_{1\text{per}}^1$ in the above Eq. (47) or Eq. (48).

The constraints of vehicle signals are shown in Eqs. (1)–(13). If a one-stage crosswalk is applied in Approach 1, the constraints of pedestrian signals are specified by Eqs. (14)–(18). If a two-stage crosswalk is applied instead, Eqs. (19)–(44) are effective. In addition, similar constraints should be added when a one- or two-stage crosswalk is used in other approaches.

As discussed above, the signal optimization problem is decomposed into several cases. In each case with determined $\alpha_i$ $(i = 1, ..., 10)$, all constraints are linear and the objective function is convex, quadratic, which is a convex, quadratic programming model. Many algorithms such as interior-point methods are available to solve such kind of models efficiently for global optima. Therefore, the optimal signals are produced after solving a series of convex, quadratic programming models. There are $10^n$ potentially optimal cases if $n$ approaches use two-stage crosswalks at the intersection. However, the solving process is quite efficient, because the number of intersection approaches is usually small in the real world.

It is noted that pedestrians may also cross over the 2nd segment in Approach 1 when Phase 2 and Phase 5 are both active as shown in Fig. 2. This requires that the overlap time is long enough to satisfy the constraints of the minimum pedestrian Walk interval and the FDW clearance interval on the 2nd segment; all waiting pedestrians on Side II can be accommodated on the refuge island. However, these conditions are not likely to be satisfied, especially with large pedestrian traffic that is assumed in this study.
4 Numerical examples

4.1 Experimental data

In order to demonstrate the applicability of the proposed model and the effectiveness of the resulting signal timing, we consider the intersection shown in Fig. 1 for the numerical experiment. The dual-ring, eight-phase controller is shown in Fig. 2. Relevant parameters are summarized in Table 2. The saturation flow rates \( s_p \) are calculated as the product of a base saturation flow rate and adjustment factors as suggested in HCM. Vehicular traffic demand is set according to the saturation flow rates. Owing to the lack of pedestrian field data, pedestrian parameters for the four approaches are assumed to be identical. The typical minimum pedestrian Walk interval and pedestrian speed can be found in Wang and Tian (2010). Pedestrian-related parameters are set accordingly. The maximum acceptable pedestrian number on the refuge island \( N_{\text{max}} \) should be specified in practice, for example, at a certain level of service according to HCM. The weighting parameter \( \beta \) in Eq. (61) is 0.6 in this study, as the basic unit costs of $6 per vehicle per hour and $4 per pedestrian per hour are suggested for vehicle and pedestrian delays, respectively, in Ma et al. (2014).

The computer program for the optimization model is written in C# and solved using Gurobi 6.0.4 (Gurobi Optimization, Inc., 2016). All the computational tests are performed on a PC equipped with an Intel 2.8 GHz CPU with 4 GB memory. To produce the optimal solution, the model is solved for one hundred (10\(^2\)) cases within 1 s, which serves well the purpose for engineering applications.

<table>
<thead>
<tr>
<th>Parameters-related parameters</th>
<th>Traffic demand</th>
<th>Pedestrian-related parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s_p )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3537 pcu/h ((p = 1, 5))</td>
<td>( r_1^1 ) 300 pcu/h</td>
<td>( r_1 ) 1620 ped/h</td>
</tr>
<tr>
<td>3789 pcu/h ((p = 2, 6))</td>
<td>( r_1^2 ) 550 pcu/h</td>
<td>( r_2 ) 1080 ped/h</td>
</tr>
<tr>
<td>1965 pcu/h ((p = 3, 7))</td>
<td>( r_1^3 ) 100 pcu/h</td>
<td>( \tau_1 ) 9 s (for Approaches 1 and 3)</td>
</tr>
<tr>
<td>2105 pcu/h ((p = 4, 8))</td>
<td>( r_1^4 ) 300 pcu/h</td>
<td>( \tau_2 ) 12 s (for Approaches 1 and 3)</td>
</tr>
<tr>
<td>( \rho_{p,\text{max}} )</td>
<td>( r_0^5 ) 320 pcu/h</td>
<td>( \tau ) 12 s (for Approaches 2 and 4)</td>
</tr>
<tr>
<td>( C_1 )</td>
<td>( r_0^6 ) 500 pcu/h</td>
<td>( g_{1,\text{min}} ) 5 s</td>
</tr>
<tr>
<td>( C_2 )</td>
<td>( r_0^7 ) 200 pcu/h</td>
<td>( g_{2,\text{min}} ) 5 s</td>
</tr>
<tr>
<td>( g_{d,\text{min}} )</td>
<td>( r_0^8 ) 100 pcu/h</td>
<td>( N_{\text{max}} ) 20</td>
</tr>
<tr>
<td>( \beta )</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.6</td>
<td></td>
</tr>
</tbody>
</table>

4.2 Feasibility of approximate delay functions

In the modelling process, the convex, quadratic formulae of approximate pedestrian delays, Eqs. (57) and (58), are proposed to replace the nonconvex, quadratic formulae Eqs. (52) and (56), respectively, so that the original nonconvex, quadratic programming model turns into a convex, quadratic programming model. A simplified intersection with only one two-stage crosswalk in Approach 1 derived from Fig. 1 is taken to verify the feasibility of the proposed approximate formulae. Three hundred feasible signal plans are randomly generated satisfying the basic
constraints (i.e., Eqs. (2)-(13) and Eqs. (19)-(39)) with a fixed cycle length of 120 s. Then the weighted pedestrian and vehicle delay can be calculated by Eq. (61). If the original nonconvex formulae Eqs. (52) and (56) are used in Eq. (61), the calculated delay is denoted as accurate delay. If the approximating convex formulae Eqs. (57) and (58) are used instead, the calculated delay is denoted as approximate delay. The relationship between these two kinds of delays is depicted in Fig. 6.

![Fig. 6 Relationship between accurate delay and approximate delay: (a) \( \beta = 0.1 \), (b) \( \beta = 0.9 \).](image)

Fig. 6 shows that the accurate and approximate delays are not equal due to the different analytical expressions (i.e., Eqs. (52), (56), (57), and (58)). However, they have a highly positive correlation no matter a larger weight is given to pedestrian delay (\( \beta = 0.1 \)) or vehicle delay (\( \beta = 0.9 \)). This means that a signal plan with a lower approximate delay is more likely to result in a lower accurate delay. Therefore, it is feasible to use the approximate delay in the objective function in the signal optimization model instead of the accurate delay. In addition, more tests show that the factors \( \gamma_1, \gamma_2, \gamma_3, \) and \( \gamma_4 \) in Eq. (57) and Eq. (58) are robust, and maintain stable performance when vehicle and pedestrian demand vary.

### 4.3 Optimization results and discussion

The optimal vehicle and pedestrian signal plan is shown in Fig. 7. The ending times of the eight vehicle phases are illustrated as well as the starting times of the pedestrian Walk interval, the FDW interval, and the Don’t Walk or Stop interval at a one- or two-stage crosswalk in each approach. Phases 2, 4, 6, and 8 are noticeably longer than the other phases due to the relative large through vehicular traffic and pedestrian traffic. The optimal cycle length is the minimum value 60 s, which is consistent with the intuition that a shorter cycle length is usually preferred when delay minimization is used as the optimization objective. Although a cycle length of 60 s may seem short, all constraints are satisfied with current vehicle and pedestrian demand. The two-stage crosswalks in Approach 1 and Approach 3 make it possible to accommodate crossing pedestrians in a short cycle. A larger cycle length will be produced if pedestrian demand increases.
To validate the effectiveness of the proposed model in this paper, the pedestrian delay in the objective function and the capacity constraints of the refuge island at a two-stage crosswalk are removed for comparison. Three levels of pedestrian demand are tested, which is the product of the basic pedestrian demand in Table 2 and a factor $\mu_{ped}$. The optimization results are shown in Table 3. Under low pedestrian demand ($\mu_{ped} = 0.5$), vehicle delay decreases by ~8% after pedestrian delay is removed from the objective function while pedestrian delay increases by ~58%. Therefore, the inclusion of pedestrian delay in the objective makes a significant difference, especially from the perspective of pedestrian traffic. Actually, in that case, the refuge islands at the two-stage crosswalks in Approach 1 and Approach 3 suffice to accommodate crossing pedestrians. The capacity constraints of the refuge islands are loose. Therefore, no differences are observed when the capacity constraints are further relaxed. As pedestrian demand increases ($\mu_{ped} = 1$), the capacity constraints become tight. Feasible solutions are reduced greatly. As a result, the increase of pedestrian delay is only 5.56% while no decrease of vehicle delay occurs after the removal of pedestrian delay from the objective. As expected, however, pedestrian delay increases and vehicle delay decreases in the same way as the case with $\mu_{ped} = 0.5$ after the relaxation of the island capacity constraints. With further increasing pedestrian demand ($\mu_{ped} = 2$), the optimization results remain the same as the case with $\mu_{ped} = 1$ due to the capacity constraints. Note that the proposed model may be infeasible with extremely high pedestrian demand due to the limited space on the refuge island in Approach 1 and Approach 3.

It is also observed that same results are produced with different levels of pedestrian demand when neither the pedestrian delay nor the capacity constraints of the refuge islands is considered. That is, the total vehicle delay, the relative decrease of vehicle delay, and the relative increase of pedestrian delay are the same although the total pedestrian delay increases with increasing pedestrian demand. This is intuitive because pedestrian demand is supposed to have no impacts on the optimization results of the model that does not consider the pedestrian delay in the objective and the limited space on the refuge islands.
### Table 3
Optimization results.

<table>
<thead>
<tr>
<th>Pedestrian demand level</th>
<th>Pedestrian delays in objective</th>
<th>Limited space on refuge island</th>
<th>Vehicle delay (s)</th>
<th>Pedestrian delay (s)</th>
<th>Weighted delay (s)</th>
<th>Equivalent delay for different vehicle occupancy (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>√</td>
<td>√</td>
<td>684.88</td>
<td>1781.40</td>
<td>1123.49</td>
<td>2466.28 2808.73 3151.17 3493.61 3836.05</td>
</tr>
<tr>
<td>$\mu_{ped}^* = 0.5$</td>
<td>×</td>
<td>√</td>
<td>630.28</td>
<td>2809.39</td>
<td>1501.92</td>
<td>3439.66 3754.80 4069.94 4385.08 4700.22</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Increase (%) -7.97 57.71 33.68 39.47 33.68 29.16 25.52 22.53</td>
</tr>
<tr>
<td></td>
<td>×</td>
<td>×</td>
<td>630.28</td>
<td>2809.39</td>
<td>1501.92</td>
<td>3439.66 3754.80 4069.94 4385.08 4700.22</td>
</tr>
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<td></td>
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<td></td>
<td></td>
<td>Increase (%) -7.97 57.71 33.68 39.47 33.68 29.16 25.52 22.53</td>
</tr>
<tr>
<td>$\mu_{ped} = 1$</td>
<td>√</td>
<td>√</td>
<td>684.88</td>
<td>3760.80</td>
<td>1915.25</td>
<td>4445.68 4788.13 5130.57 5473.01 5815.45</td>
</tr>
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<td></td>
<td>×</td>
<td>√</td>
<td>684.88</td>
<td>3760.80</td>
<td>1915.25</td>
<td>4445.68 4788.13 5130.57 5473.01 5815.45</td>
</tr>
<tr>
<td></td>
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<td></td>
<td></td>
<td>Increase (%) 0.00 5.56 4.31 4.66 4.31 4.01 3.75 3.52</td>
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<tr>
<td></td>
<td>×</td>
<td>×</td>
<td>630.28</td>
<td>5618.78</td>
<td>2625.68</td>
<td>6249.05 6564.19 6879.33 7194.47 7509.61</td>
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<td></td>
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<td></td>
<td></td>
<td></td>
<td>Increase (%) -7.97 57.71 33.68 39.47 33.68 29.16 25.52 22.53</td>
</tr>
<tr>
<td>$\mu_{ped} = 2$</td>
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<td>√</td>
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<td></td>
<td></td>
<td></td>
<td>Increase (%) -7.97 57.71 33.68 39.47 33.68 29.16 25.52 22.53</td>
</tr>
</tbody>
</table>

*Three levels of pedestrian demand are set by $\mu_{ped1}$ and $\mu_{ped2}$. 
4.4 Comparison between two- and one-stage crosswalks

It is widely believed that the refuge island of a two-stage crosswalk benefits pedestrian traffic at the expense of traffic operational efficiency. Because one extra lane might be available for vehicular traffic otherwise. Nevertheless, it should be recognized that more green time can be allocated to vehicle flows as a result of the reduced FDW clearance interval when a one-stage crosswalk is replaced by a two-stage one. Therefore, it is a trade-off between spatial and temporal resources. A further analysis is conducted to investigate the impact of a two-stage crosswalk on vehicular traffic compared with a one-stage crosswalk.

The two-stage crosswalks in Approach 1 and Approach 3 in Fig. 1 are replaced by one-stage crosswalks so that one extra lane is added for through traffic in both approaches. Because, in the real world, the main traffic flows at an intersection of a main road and a secondary road are usually through flows. The FDW clearance intervals in Arm 1 and Arm 3 are prolonged to 24 s accordingly. For better illustration, the pedestrian volumes in Table 2 are halved and the cycle length is taken as 100 s. Flow factors $\mu_{13}$ and $\mu_{24}$ are introduced to represent the varying through flows in Approaches 1, 3 and Approaches 2, 4, respectively. The vehicle arrival rates in Phases 2, 4, 6, and 8 in Table 2 are modified as $r_{v}^2 = 550\mu_{13}$, $r_{v}^4 = 300\mu_{24}$, $r_{v}^6 = 500\mu_{13}$, and $r_{v}^8 = 100\mu_{24}$, respectively.

The relative increase of pedestrian and vehicle delays after one-stage crosswalks are installed in Approach 1 and Approach 3 are portrayed in Fig. 8 (a) and (b), respectively. Obviously, a two-stage crosswalk benefits pedestrian traffic (i.e., positive impacts). The increase of pedestrian delay exceeds 7.5% with varying traffic flows. In contrast, the impact of a two-stage crosswalk can be either positive or negative in terms of vehicle delay. In most cases, the impact is negative (i.e., increasing vehicle delay), which means that the benefits got by setting a central refuge island is relatively smaller than the capacity drop resulted by reducing a lane. However, the impact is positive (i.e., decreasing vehicle delay) with relatively low flows of through traffic in the four Approaches (e.g., $\mu_{13} = 0.6$, $\mu_{24} = 0.6$). This means that the reduced vehicle delay, owing to the shortened FDW clearance time by the installed refuge island at a two-stage crosswalk, dominates. Therefore, a two-stage crosswalk at an intersection may benefit both vehicular traffic and pedestrian traffic in some circumstances.

![Fig. 8](image_url)
replaced by one-stage crosswalks: (a) total pedestrian delay, and (b) total vehicle delay.

5 Conclusions and recommendations

This paper presents a convex (quadratic) programming approach to optimize signals for both vehicular and pedestrian traffic at an isolated intersection with one- and two-stage crosswalks. The optimization model assumes undersaturated condition for vehicular traffic. The objective is to minimize vehicle and pedestrian delays. Quadratic forms of approximate delays are proposed in the objective function. The limited space on the refuge island at a two-stage crosswalk is explicitly taken into consideration, and is captured in linear forms. The proposed model can be solved efficiently for global optima by existing solvers, which serves well the purpose for further engineering applications.

Several numerical studies have shown the effectiveness of the proposed signal optimization approach. A correlation analysis is conducted to verify the quadratic approximation of the delays. In particular, a highly positive correlation is found between the exact delay (calculated by the original nonconvex formulae) and approximate delay (the proposed quadratic formulae). The optimization results account for the limited space on the refuge island at a two-stage crosswalk, and the pedestrian delay to be minimized. Further analyses show that a two-stage crosswalk at an intersection can benefit both vehicular and pedestrian traffic in some circumstances.

The model is built assuming undersaturated vehicular traffic condition as primary objective is to minimize delays. Under oversaturation conditions, the maximization of intersection capacity may be considered as the objective instead. In this case, all of the proposed linear constraints remain effective, and the signal optimization problem can be formulated as a linear program when reserve capacity is adopted (Wong and Heydecker, 2011; Wong and Wong, 2003a, b). The proposed framework will be further validated using field data in a future study. In order to cope with significant traffic flow variations, adaptive signal plans may be devised and optimized for different time periods (e.g., peak and off-peak) or day categories (e.g. weekdays and holidays). The extension of the proposed model to actuated control and signal coordination has been conceived. Other concerns such as the interactions among pedestrian platoons and the integration of long and short cycle lengths will also be explored.

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Appendix I

Fig. 9 Case 2.

Fig. 10 Case 3.

Fig. 11 Case 4.
Fig. 12 Case 5.

Fig. 13 Case 6.

Fig. 14 Case 7.
Fig. 15 Case 8.

Fig. 16 Case 9.

Fig. 17 Case 10.
Reference


Kunming, China, pp. 275-284.


