Multichannel algorithms for seismic reflectivity inversion

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Abstract

Seismic reflectivity inversion is a deconvolution process for quantitatively extracting the reflectivity series and depicting the layered subsurface structure. The conventional method is a single channel inversion and cannot clearly characterise stratified structures, especially from seismic data with low signal-to-noise ratio. Because it is implemented on a trace-by-trace basis, the continuity along reflections in the original seismic data is deteriorated in the inversion results. We propose here multichannel inversion algorithms that apply the information of adjacent traces during seismic reflectivity inversion. Explicitly, we incorporate a spatial prediction filter into the conventional Cauchy-constrained inversion method. We verify the validity and feasibility of the method using field data experiments and find an improved lateral continuity and clearer structures achieved by the multichannel algorithms. Finally, we compare the performance of three multichannel algorithms and merit the effectiveness based on the lateral coherency and structure characterisation of the inverted reflectivity profiles, and the residual energy of the seismic data at the same time.

Keywords: reflectivity inversion, Cauchy-constrained inversion, multichannel algorithm, spatial prediction filter

(Some figures may appear in colour only in the online journal)

Introduction

Reflectivity inversion is a pivotal step in seismic signal processing and quantitative inversion, because a reflectivity series holds the information of elastic parameters of the subsurface media, such as the impedance etc (Wang 2016). The inverted reflection series can be used to recover subsurface distribution of stratified layers and their lithology properties. Theoretically, the calculation of the reflectivity series is simply to remove the effect of wavelet from an observed seismic trace (Robinson and Treitel 1980). This inversion procedure is commonly referred to as deconvolution in the geophysics field. However, seismic reflectivity inversion is ill-conditioned by nature, and the most fundamental method is based on the least-squares criterion, with the aim of minimising the data misfit between the field observation and a synthetic trace formed by the inverted reflectivity series. Because the least-squares method assumes the noise within the data having a zero mean and Gaussian distribution, and the priori model for the reflectivity being Gaussian as well, the inversion result is not spiky and sparse enough to be treated as a seismic reflectivity series.

The $L_1$-norm constraint and the Cauchy constraint can be applied to the least-squares inversion, in order to generate a seismic reflectivity series with a longer-tailed distribution than the Gaussian distribution, and thus to increase the effective bandwidth of the retrieved reflectivity series (Levy and Fulagar 1981, Sacchi\textit{ et al} 1998). The $L_1$-norm constrained inversion can be further generalised as $L_p$-norm constrained inversion by adjusting the parameters according to different input seismic traces (Debye and van Riel 1990). A spectral sparse Bayesian learning reflectivity inversion implements sequentially adding, deleting or re-estimating hyper-parameters, attempting to avoid the pre-setting of the number of non-zero reflectivity series (Yuan and Wang 2013). The basis
pursuit (BP) algorithm which builds a wedge reflector matrix to represent seismic reflectivity inversion can also generate a very sparse solution in the $L_1$-norm sense.

However, these inversion methods with either the Cauchy constraint or the $L_1$-norm constraint are implemented in a trace-by-trace fashion. This implementation may lead to laterally dispersed reflections in seismic reflectivity profiles, especially when seismic data sets have a low signal-to-noise ratio (SNR). In order to improve the lateral coherency of the inverted reflectivity events, a multichannel method incorporates the information of adjacent traces into the inversion process. Idier and Goussard (1993) modelled the reflectivity field as a Markov–Bernoulli random field since seismic reflectivity profile can be characterised by local continuity. Then the multichannel inversion was carried out by a suboptimal maximum a posterior (MAP) estimator. Assuming that the reflectivity possesses the Bernoulli–Gaussian distribution which corresponds to a layered Earth model, Kaaresen and Taxt (1998) implemented the so-called Iterated Window Maximization method (IWM) in the blind multichannel inversion to achieve lateral continuity of the reflectors. Heimer and Cohen (2009) implemented Viterbi algorithm and Ram et al. (2010) implemented Markov Chain Monte Carlo (MCMC) method, both of which are also based on the model of Markov–Bernoulli random field, for seismic reflectivity inversion.

To improve the spatial continuity of a seismic reflectivity profile, the FX prediction filter can be applied to reduce the spatial differences of adjacent seismic reflectivity traces. The reflectivity is first obtained by the Cauchy-constraint inversion method, and then the FX prediction filter is applied on the result. Those filtered traces are then used as the initial model for the new iteration process of the inversion. In this paper, we develop a multichannel reflectivity inversion algorithm in which the prediction operator is not only applied to enhance the initial model of the iteration process but also integrated into the whole inversion procedure. Inverted reflectivity profiles are expected to have an improved lateral continuity and a better structure characterisation.

This paper is organised as follows. First, we overview the conventional single channel reflectivity inversion with the Cauchy constraint, and the concept of FX prediction filtering. Then, we describe basic implementation details of the proposed multichannel inversion algorithm and validate the method using a field data set. Furthermore, we propose two modified multichannel algorithms by using different implementation procedures. We present a comparative demonstration for these three methods, and provide a conclusion in the last part.

### Single channel reflectivity inversion algorithm with the Cauchy constraint

The Earth convolution model can be represented in a matrix-vector form as

$$ \mathbf{d} = \mathbf{W} \mathbf{r} + \mathbf{n}, \quad (1) $$

where $\mathbf{d}$ denotes the seismic trace, $\mathbf{W}$ is a cyclic matrix representing the discrete convolution operation of seismic wavelet, $\mathbf{r}$ represents the reflectivity, and $\mathbf{n}$ refers to the additive noise. In this paper, the wavelet is assumed to be known, which has been estimated based on high-order statistics (Lu and Wang 2007).

By minimising $||\mathbf{W} \mathbf{r} - \mathbf{d}||^2$, the result of the least-squares method is obtained as

$$ \mathbf{r} = (\mathbf{W}^T \mathbf{W} + \mu \mathbf{L}^{-1})^{-1} \mathbf{W}^T \mathbf{d}, \quad (2) $$

where $\mu$ is a pre-whitening parameter used to stabilise the matrix inverse calculation (Wang 2016).

If the reflectivity coefficients are assumed to be Cauchy distributed and have a zero mean, by combining the Cauchy prior and Gaussian data noise likelihood together, the posterior distribution of reflectivity $\mathbf{r}$ for the seismic signal $\mathbf{d}$ can be expressed as

$$ p(\mathbf{r} | \mathbf{d}) \propto \exp \left[ -\frac{1}{2} (\mathbf{d} - \mathbf{W} \mathbf{r})^T \mathbf{C}_d^{-1} (\mathbf{d} - \mathbf{W} \mathbf{r}) + \sum_{i=0}^{N-1} \ln \left( \frac{1}{\pi \lambda^2 + r_i^2} \right) \right], \quad (3) $$

where $\mathbf{C}_d$ is the data covariance matrix, $N$ denotes the number of seismic reflectivity time samples, and $\lambda$ is the Cauchy distribution parameter, controlling the sparseness of $\mathbf{r}$.

By maximising the posterior probability $p(\mathbf{r} | \mathbf{d})$, the solution of the least-squares inversion with the Cauchy constraint can be represented as

$$ \mathbf{r} = (\mathbf{W}^T \mathbf{C}_d^{-1} \mathbf{W} + \frac{2}{\lambda^2} \mathbf{Q})^{-1} \mathbf{W}^T \mathbf{C}_d^{-1} \mathbf{d}, \quad (4) $$

where $\mathbf{Q}$ is a diagonal matrix derived by Wang (2003):

$$ \mathbf{Q} = \text{diag} \left\{ \left( 1 + \frac{r_i^2}{\lambda^2} \right)^{-1} \right\}, \quad i = 0, 1, \cdots, N - 1, \quad (5) $$

and $N$ is the number of reflectivity samples. Equation (4) can be solved iteratively using the conjugate gradient (CG) method. However, since this inversion process is carried out in a trace-by-trace manner, the lateral continuity of the retrieved reflectivity might be deteriorated if the input seismic signals are contaminated by the background noise.

### Multichannel reflectivity inversion algorithm with the Cauchy constraint

If we have a multichannel prediction operator for seismic traces (see the appendix), we assume the prediction filter, $\mathbf{P}$, being also applicable for seismic reflectivity traces,

$$ R_k(f) = \sum_{j=1}^{L} P(j) \tilde{R}_{k-j}(f), \quad k = L + 1, \cdots, N_u, \quad (6) $$

$$ R_k(f) = \sum_{j=1}^{L} P(j) \tilde{R}_{k+j}(f), \quad k = 1, \cdots, N_u - L, \quad (7) $$

where $\tilde{R}_k$ represents the reflectivity before spatial prediction, $R_k$ is the reflectivity after prediction and $N_u$ refers to the number of traces.

With multichannel FX prediction filter, the information of adjacent traces can be applied during the inversion process.
To incorporate the prediction filter into the inversion procedure, equation (1) can be rewritten as

\[ \mathbf{d} = \mathbf{W} \mathbf{F}^{-1} \mathbf{P} \mathbf{F} \mathbf{r} + \mathbf{n}, \]  

where \( \mathbf{r} \) is the time domain representation of the reflectivity before spatial smoothing. \( \mathbf{F} \) denotes the Fourier transform and \( \mathbf{F}^{-1} \) represents the inverse Fourier transform. Re-writing \( \mathbf{W} \mathbf{F}^{-1} \mathbf{P} \mathbf{F} \) as a new operator \( \mathbf{L} \), equation (8) becomes

\[ \mathbf{d} = \mathbf{L} \mathbf{r} + \mathbf{n}, \]  

Therefore, the reflectivity series before filtering, which is calculated using the proposed Cauchy-constrained, multichannel inversion method, can be expressed as

\[ \mathbf{r} = \left( \mathbf{L}^T \mathbf{C}_d^{-1} \mathbf{L} + \frac{2}{\lambda^2} \mathbf{Q} \right)^{-1} \mathbf{L}^T \mathbf{C}_d^{-1} \mathbf{d}, \]  

where \( \mathbf{C}_d \) is the data covariance matrix calculated from the input traces.

The procedure of the Cauchy-constrained multichannel inversion is designed as what follows.

1. Initialising \( \mathbf{r} \) using equation (2), set the residual \( \mathbf{e} \) as the difference between \( \mathbf{L}^T \mathbf{C}_d^{-1} \mathbf{d} \) and \( \mathbf{L}^T \mathbf{C}_d^{-1} \mathbf{L} + 2 \lambda^{-2} \mathbf{Q} \mathbf{r} \), and the search direction \( \mathbf{p} \) is equal to \( \mathbf{e} \).

2. Starting the CG process using equation (11), where \( k \) is the number of CG iterations:

\[ \mathbf{u}^{(k)} = \left( \mathbf{L}^T \mathbf{C}_d^{-1} \mathbf{L} + \frac{2}{\lambda^2} \mathbf{Q} \right)^{-1} \mathbf{L}^T \mathbf{C}_d^{-1} \mathbf{d}, \]

\[ \mathbf{a}_k = \frac{\langle \mathbf{e}^{(k)}, \mathbf{u}^{(k)} \rangle}{\langle \mathbf{p}^{(k)}, \mathbf{u}^{(k)} \rangle}, \]

\[ \mathbf{r}^{(k+1)} = \mathbf{r}^{(k)} + \mathbf{a}_k \mathbf{p}^{(k)} \]

\[ \mathbf{e}^{(k+1)} = \mathbf{e}^{(k)} - \mathbf{a}_k \mathbf{u}^{(k)} \]

\[ \beta_k = \frac{\langle \mathbf{e}^{(k+1)}, \mathbf{e}^{(k+1)} \rangle}{\langle \mathbf{e}^{(k)}, \mathbf{e}^{(k)} \rangle}, \]

\[ \mathbf{p}^{(k+1)} = \mathbf{e}^{(k+1)} + \beta_k \mathbf{p}^{(k)}, \]

where \( (\mathbf{a}, \mathbf{b}) \) represents the inner product of vectors \( \mathbf{a} \) and \( \mathbf{b} \), and \( \alpha_k \) and \( \beta_k \) are vectors storing values of the step length corresponding to each trace.

3. Re-initialising \( \mathbf{r} \) using the values calculated by CG method, which is described as equation (11), and starting a new iteration process from Step 1. Repeating the iteration until a satisfactory result of \( \mathbf{r} \) is obtained.

4. Calculating the final reflectivity \( \mathbf{r} = \mathbf{F}^{-1} \mathbf{P} \mathbf{F} \mathbf{r} \).

The CG method is originally employed for single trace inversion, and the iteration process of one trace is repeated until the residue \( \mathbf{e} \) is below a predefined threshold \( \mathbf{e}_0 \). In our implementation, although the CG process shown in equation (11) seems to be the same as the conventional CG method, it has been extended to a multichannel version. Since the operator \( \mathbf{L} \) is a multichannel operator, the iterations of all traces are performed simultaneously. Once the residual of one certain trace is under the threshold, the values \( \mathbf{r}, \mathbf{e}, \mathbf{d}, \) and \( \mathbf{p} \) of this trace are stored, while the rest traces will keep updating in the iteration process. The CG process will not stop until the residual of all traces are below the threshold.

To implement the new operator \( \mathbf{L} = \mathbf{W} \mathbf{F}^{-1} \mathbf{P} \mathbf{F} \) in the CG method, first the multichannel matrices (which store the information of \( \mathbf{r}, \mathbf{d}, \) or \( \mathbf{p} \) correspondingly) is transformed into the frequency domain, and then the prediction filter \( \mathbf{P} \) is applied to the obtained frequency domain matrices. Once the filtering process has been performed, the matrix is transformed back into the time domain. The convolution matrix \( \mathbf{W} \) is then applied to obtain the final result of the operator \( \mathbf{L} \). The adjoint operator \( \mathbf{L}^T = \mathbf{F}^{-1} \mathbf{P}^T \mathbf{F} \mathbf{W}^T \) can be accomplished in a similar way. Note that here \( \mathbf{W} \) represents a crosscorrelation, as it is the adjoint process of convolution (Claerbout 1992). The FX prediction process \( (\mathbf{F}^{-1} \mathbf{P} \mathbf{F}) \) is performed by simply replacing the prediction filter \( \mathbf{P} \) by its adjoint \( \mathbf{P}^T \).

Let us demonstrate the method using a field seismic profile (figure 1(a)). The retrieved reflectivity profile (figure 1(b)) using the proposed multichannel inversion method shows the effective preservation of lateral continuity, whereas the structure of the reflectivity profile is clearly presented.

To gain an insight into the retrieved reflectivity profile, data with trace number from 400 to 500 and time between 0.4 and 0.6s is selected (figure 2(a)). The reflectivity profile obtained by single channel inversion (figure 2(b)) is filtered using the spatial prediction filter (figure 2(c)). Although the prediction filter can smooth the reflectivity, some detailed structure is also eliminated while the noise is spread around the layers. For the multichannel inversion, we compare the reflectivity profiles retrieved after Step 3, namely \( \mathbf{r} \) (figure 2(d)), and after Step 4, which is the final reflectivity result \( \mathbf{r} \) (figure 2(e)), a filtered version of \( \mathbf{r} \). The reflectivity profile \( \mathbf{r} \) (figure 2(d)) has a better lateral coherency than that obtained by the single channel method (figure 2(b)). Comparing figures 2(c) and (e) indicates that the stratification of the reflectivity profile is clearer under the multichannel mechanism.

Therefore, we can conclude that the proposed method has a better performance than a single channel inversion method with the multichannel FX prediction filter. This is because in the proposed algorithm, the FX prediction filter is adopted during the iteration steps in order to stabilise the process, resulting in improved lateral continuity and structure characterisation.

Two modified multichannel algorithms with the Cauchy constraint

In the previous section, the proposed multichannel algorithm was validated and the improvements have been observed in terms of lateral coherency and structure characterisation. As the prediction filter \( \mathbf{P} \) calculated from seismic traces was applied to the reflectivity data, the result might be not accurate. Thus, we suggest two different implementation procedures.

Modification 1. Seismic reflectivity is assumed to be predictable in the frequency domain, it is more suitable to calculate the prediction
filter directly from the reflectivity. In the originally proposed multichannel method, \( P \) is calculated from the input traces since at the beginning of the inversion process, the value of the reflectivity is unknown. In **Modification 1**, the problem is overcome by modifying the iteration process. At the beginning, \( P \) is initialised as an identity matrix. Then in Step 3, the prediction filter \( P \) is calculated from \( \tilde{r} \) which has been obtained in Step 2, and \( P \) thus can be used in the next iteration which re-starts from Step 1. Therefore, the value of \( P \) is updated at each iteration step.

**Modification 2.** The reflectivity is not assumed to be predictable. Only the predictable nature of seismic trace is applied here. In this case, the prediction filter can only be calculated from the input data.

If the reflectivity is not predictable with \( P \), the predictable nature of the input traces is adopted, yielding

\[
D_k(f) = \sum_{j=1}^{L} P_j(f) \tilde{D}_{k-j}(f), \quad k = L + 1, \ldots, N_t, \quad (12)
\]

\[
D_k^*(f) = \sum_{j=1}^{L} P_j^*(f) \tilde{D}^*_{k+j}(f), \quad k = 1, \ldots, N_t - L, \quad (13)
\]

where \( \tilde{D}_k \) represents the seismic trace before spatial prediction, and \( D_k \) represents the seismic trace after prediction.

Seismic trace equation (1) can be rewritten as

\[
d = F^{-1}P\tilde{d} + n = F^{-1}PF\tilde{r} + n, \quad (14)
\]

where \( \tilde{d} \) is the time domain representation of the seismic trace before spatial smoothing. The operator \( L \) can be expressed as \( L = F^{-1}PF \) and used in the iteration process as shown in the original multichannel method. The reflectivity series then can be obtained with this new operator.

**Comparative study of the three multichannel algorithms**

A comparative study is carried out to evaluate the performance of these three algorithms. First of all, a synthetic data (SNR = 2.0) containing a flat and dip structure is used and shown in figure 3(b). Originally, the clean synthetic data can be obtained by generating spikes to form a flat and a dip layer. Then a zero-phase wavelet is convolved with these spikes to form the clean synthetic data (figure 3(a)). The angle between the flat and the dip layer is 60°. There are 60 traces and the sampling rate is 0.2. The noisy signal is computed by adding a scaled noise with a Gaussian probability distribution to the original signal:

\[
\text{noisy single} = \text{signal} + \text{scale} \times \text{noise}, \quad (15)
\]

where ‘scale’ is defined as

\[
\text{scale} = \frac{\max(\text{abs(signal)})/\sqrt{2}}{\text{SNR/energy per sample}}. \quad (16)
\]

where SNR is the signal-to-noise ratio. As demonstrated in figure 3(c), the result obtained by single channel inversion...
is noisy and laterally discontinuous. Although the originally proposed multichannel inversion method can provide clearer reflectivity profile, some information of the structure is also eliminated, as shown in figure 3(d). The results obtained by the two modified methods are demonstrated in figures 3(e) and (f), respectively. Improved retrieved reflectivity profiles are presented with better noise reduction and structure characterisation.

Furthermore, we compare these three methods quantitatively, by evaluating the relative residual energy difference between the retrieved and original clean data. The residual profiles associated with different processes are shown in figure 4. When using the conventional single channel method, the residual energy ratio is calculated to be 43.6447%, while the residual energy ratios of the originally proposed multichannel method, Modification 1, and 2 are 20.4751%, 9.49378% and 13.268%,
respectively. Therefore, Modification 1 and 2 can provide lower residual energy ratio. From figure 4, it can be seen that when using the multichannel methods, the noise existed on the reflectivity profile has been eliminated, when comparing with the single channel method’s result. Furthermore, we can see that when using Modification 1 and Modification 2, the amplitude of the deviation of the traces obtained by the results from the clean input traces is smaller, which means the structure loss is less.
Figure 5. (a) Reflectivity profile obtained by the originally proposed multichannel algorithm after Step 3. (b) Reflectivity profile obtained by the originally proposed multichannel algorithm after Step 4. (c) Reflectivity profile obtained by Modification 1 after Step 3. (d) Reflectivity profile obtained by Modification 1 after Step 4. (e) Reflectivity profile obtained by Modification 2 after Step 3. (f) Reflectivity profile obtained by Modification 2 after Step 4.
Another comparison is performed using the same field data as shown in figure 2(a). The same conclusion is obtained that Modification 1 and Modification 2 are preferred as they can provide reflectivity profiles with clearer and more refined structures, as illustrated from figures 5(a)–(f). The residuals associated with the single channel and multichannel methods are shown in figure 6. By comparing the residual profiles, it can be seen that the amplitude of residuals associated with the multichannel methods are lower than that of the single channel method. Furthermore, the continuous structures shown in the
residuals of the proposed methods are less, which means the lateral continuity of the seismic events are better preserved, especially when using Modification 1 and 2.

**Conclusions**

If seismic reflectivity inversion methods are based on a trace-by-trace principle, the retrieved reflectivity profiles might lack of lateral coherence, especially when the input data are contaminated by background noise or have complex structures. The multichannel inversion method can increase the lateral continuity and largely suppress the noise, since it exploits the information of adjacent traces during the inversion process. Moreover, a comparative study using both synthetic and field data indicates that the two modified algorithms can further improve the reflectivity profiles, including increased lateral continuity, better structure characterisation and stratification, and lower residual energy ratio.

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**Appendix: FX prediction filtering**

Seismic coherent events are assumed to be the sum of the convolutions of wavelets and their corresponding impulses:

$$d(t, x) = \sum_{i=1}^{N} w_i(t) * \delta(t - g(x)),$$  \hspace{1cm} (A.1)

where $w_i(t)$ is the temporal wavelet associated with the reflection with the index $i$ and $g(x)$ is the delay function that defines the shape of the coherent events in a seismic profile. Taking the Fourier transform of equation (A.1), we obtain

$$d(\omega, x) = \sum_{i=1}^{N} W_i(\omega) e^{-j\omega g(x)},$$  \hspace{1cm} (A.2)

where $W_i(\omega)$ represents the Fourier transform of wavelet $w_i(t)$. If the coherent events are assumed to be linear, $g(x) = s_i x$, the model equation (A.2) becomes

$$d(\omega, x) = \sum_{i=1}^{N} W_i(\omega) e^{-j\omega s_i x},$$  \hspace{1cm} (A.3)

where $s_i$ is a measure of the slopes of the coherent events. It is observed that the function $d(\omega, x)$ is strictly sinusoidal and predictable in the $x$ direction. Consequently, the trace can be predicted by the preceding or following traces with a spatial prediction filter (Spitz 1991, Wang 1999, 2002).
Let vector $\mathbf{P}$ be the prediction operator of length $L$, $\mathbf{d}$ be the seismic data before spatial smoothing and the vector $\hat{\mathbf{d}}$ be the predicted value of $\mathbf{d}$ for each frequency, we have

$$
\hat{\mathbf{d}} = \mathbf{D}\mathbf{P},
$$

(A.4)

where $\mathbf{D}$ is the convolution matrix of $\mathbf{d}$. If $\mathbf{D}^H$ represents the transposed complex conjugate of $\mathbf{D}$, equation (A.4) can be rewritten as

$$
\mathbf{D}^H \mathbf{d} = \mathbf{D}^H \mathbf{P}. \;
$$

(A.5)

By defining the complex autocorrelation matrix $\mathbf{R}$ as

$$
\mathbf{R} = \mathbf{D}^H \mathbf{D},
$$

(A.6)

and the complex cross-correlation matrix $\mathbf{g}$ as

$$
\mathbf{g} = \mathbf{D}^H \mathbf{d},
$$

(A.7)

we finally have

$$
\mathbf{g} = \mathbf{R}\mathbf{P}.
$$

(A.8)

Following Treitel (1974), equation (A.8) can be rewritten into the matrix form of its real and imaginary parts, which is the Wiener problem:

$$
\begin{pmatrix}
\mathbf{g}_{\text{Re}} \\
\mathbf{g}_{\text{Im}}
\end{pmatrix} =
\begin{pmatrix}
\mathbf{R}_{\text{Re}} & -\mathbf{R}_{\text{Im}} \\
\mathbf{R}_{\text{Im}} & \mathbf{R}_{\text{Re}}
\end{pmatrix}
\begin{pmatrix}
\mathbf{P}_{\text{Re}} \\
\mathbf{P}_{\text{Im}}
\end{pmatrix},
$$

(A.9)

where $\mathbf{P}$ can then be solved by recursive algorithms with great efficiency. Here LU decomposition and forward and backward substitution are applied to solve equation (A.9), enabling us to obtain the prediction filter in the $+x$ direction. To calculate the prediction filter in the $-x$ direction, the matrix problem becomes

$$
\mathbf{g}^* = \mathbf{R}^* \mathbf{P}^*,
$$

(A.10)

where $^*$ denotes the complex conjugate of a matrix and $\mathbf{P}^*$ represents the prediction operator in the $-x$ direction. Taking the complex conjugate on both sides of equation (A.10), we have

$$
\mathbf{g} = \mathbf{R}^* \mathbf{P}^*.
$$

(A.11)

Equation (A.11) shows that the prediction filter in the $-x$ direction is the complex conjugate of the filter in the $+x$ direction. Therefore, we can build the two-sided prediction filter by simply expanding the one-sided operator with its complex conjugate.

References


Kaaresen K F and Taxt T 1998 Multichannel blind deconvolution of seismic signals *Geophysics* 63 2093–107

Levy S and Fulagar P K 1981 Reconstruction of a sparse spike train from a portion of its spectrum and application to high-resolution deconvolution *Geophysics* 46 1235–43


Spitz S 1991 Seismic trace interpolation in the $F$-$X$ domain *Geophysics* 56 785–94

Treitel S 1974 The complex Wiener prediction filter *Geophysics* 39 169–73

Wang Y 1999 Random noise attenuation using forward-backward linear prediction *J. Seismic Explor.* 8 133–42

Wang Y 2002 Seismic trace interpolation in the $F$-$X$-$Y$ domain *Geophysics* 67 1232–9

Wang Y 2003 Multiple attenuation: coping with the spatial truncation effect in the Radon transform domain *Geophys. Prospect.* 51 75–87


Yuan S Y and Wang S X 2013 Spectral sparse Bayesian learning reflectivity inversion *Geophys. Prospect.* 61 735–46