User-Centric Interference Nulling in Downlink Multi-Antenna Heterogeneous Networks

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Abstract—In heterogeneous networks (HetNets), strong interference due to spectrum reuse affects each user’s signal-to-interference ratio (SIR), and hence is one limiting factor of network performance. In this paper, we propose a user-centric interference nulling (IN) scheme in a downlink large-scale HetNet to improve coverage/outage probability by improving each user’s SIR. This IN scheme utilizes at most maximum IN degree of freedom (DoF) at each macro-BS to avoid interference to uniformly selected macro (pico) users with signal-to-individual-interference ratio (SIIR) below a macro (pico) IN threshold, where the maximum IN DoF and the two IN thresholds are three design parameters. Using tools from stochastic geometry, we first obtain a tractable expression of the coverage (equivalently outage) probability. Then, we obtain the asymptotic expressions of the coverage/outage probability in the low and high SIR threshold regimes. The analytical results indicate that the maximum IN DoF can affect the order gain of the outage probability in the low SIR threshold regime, but cannot affect the order gain of the coverage probability in the high SIR threshold regime. Moreover, we characterize the optimal maximum IN DoF which optimizes the asymptotic coverage/outage probability. Finally, numerical results show that the proposed scheme can achieve good gains in coverage/outage probability over some baseline schemes.

Index Terms—Heterogeneous networks, multiple antennas, inter-tier interference coordination, stochastic geometry, optimization.

I. INTRODUCTION

Heterogeneous wireless networks (HetNets), i.e., the deployment of low power small cell base stations (BSs) overlaid with conventional large power macro-BSs, provide a powerful approach to meet the massive growth in traffic demands by aggressively reusing existing spectrum assets [1], [2]. However, spectrum reuse in HetNets causes strong interference. This affects the signal-to-interference ratio (SIR) of each user, and hence is one of the limiting factors of network performance. Interference management techniques are thus desirable in HetNets [3]. One such technique is interference cooperation. For example, in [4]–[6], different interference cooperation strategies are considered and their performances are analyzed for large-scale HetNets under random models in which the locations of BSs and users are spatially distributed as independent homogeneous Poisson point processes (PPPs) [7], [8]. However, in [4]–[6], the cooperation clusters are formed to favor a typical user located at the origin of the network (referred to as the typical user) only, and hence, the analytical performance of the typical user is better than the actual network performance (of all users). In addition, [4]–[6] only consider single-antenna BSs. Orthogonalizing the time or frequency resource allocated to macro cells and small cells can also mitigate interference in HetNets. One such technique is almost blank subframes (ABS) in 3GPP LTE [9]. In ABS, the time or frequency resource is partitioned, whereby offloaded users and the other users are served using different portions of the resource in HetNets with offloading. The performance of ABS in large-scale HetNets with offloading is analyzed in [9] using tools from stochastic geometry. Note that ABS focuses on mitigating the interference of offloaded users, and [9] only considers single-antenna BSs.

Deploying multiple antennas at each BS in HetNets can further improve network performance. With multiple antennas, besides boosting signals to desired users, more effective interference management techniques can be implemented [10]–[15]. For example, in [10]–[13], the authors consider HetNets with a single multi-antenna macro-BS and multiple multi-antenna small-BSs, where the multiple antennas at the macro-BS are used for serving its scheduled users as well as mitigating the interference to some small cell users using different interference coordination schemes. These schemes are analyzed and shown to improve the network performance. Note that since only one macro-BS is considered in [10]–[13], the analytical results obtained in [10]–[13] cannot reflect the macro-tier interference, and thus may not offer accurate insights for practical HetNets. In [14], [15], large-scale multi-antenna HetNets are considered. Specifically, [14] considers offloading, and proposes an interference nulling (IN) scheme where some degree of freedom at each macro-BS can be used for avoiding its interference to some of its offloaded users. The rate coverage probability is analyzed and optimized by optimizing the amount of degree of freedom (DoF) for interference nulling. However, the IN scheme proposed in [14] only improves the performance of scheduled offloaded users, and scheduled offloaded users are selected by the corresponding macro-BS for interference nulling with equal probability. In [15], a fixed number of BSs which provide the strongest average re-
ceived power for the typical user form a cluster, and adopt an interference coordination scheme where the BSs in each cluster mitigate interference to users in this cluster. The coverage probability is analyzed based on the assumption that the BSs in each cluster are the strongest BSs of all the users in this cluster.

The investigation of interference management techniques in large-scale single-tier multi-antenna cellular networks is less involved than that in large-scale multi-antenna HetNets, and hence has been more extensively conducted. In [16]–[18], all the BSs are grouped into disjoint clusters. Coordination [16], [17] and cooperation [18] are performed among the BSs within each cluster to mitigate intra-cluster interference. Specifically, [16] and [17] design disjoint BS clustering from a transmitter’s point of view and fail to consider each user’s interference situation. The dynamic clustering proposed in [18] considers all the users’ signal and interference situations to optimize the network performance. However, it requires centralized control and may not be suitable for large networks. Recently, a novel distributed user-centric IN scheme, which takes account of each user’s desired signal strength and interference level, is proposed and analyzed for (single-tier) multi-antenna small cell networks in [19]. However, in [19], the maximum DoF for IN (i.e., maximum IN DoF) at each BS is not adjustable, and thus cannot properly utilize resource in small cell networks. Moreover, directly applying the scheme in [19] to HetNets cannot fully exploit different properties of macro and pico users in HetNets.

In this paper, we consider a downlink large-scale two-tier multi-antenna HetNet and propose a user-centric IN scheme to improve the coverage probability by improving each user’s SIR. This scheme has three design parameters: the maximum IN DoF $U$, and the IN thresholds for macro and pico users, respectively. In this scheme, each scheduled macro (pico) user first sends an IN request to a macro-BS$^1$ if the power ratio of its desired signal and the interference from the macro-BS, referred to as the signal-to-individual-interference ratio (SIIR), is below the IN threshold for macro (pico) users. Then, each macro-BS utilizes zero-forcing beamforming (ZFBF) precoder to avoid interference to at most $U$ scheduled users which send IN requests to it as well as boost the desired signal to its scheduled user. In general, the performance analysis and optimization of interference management techniques in large-scale multi-antenna HetNets are very challenging, mainly due to i) the statistical dependence among macro-BSs and pico-BSs [10], ii) the complex distribution of a desired signal using multi-antenna communication schemes, and iii) the complicated interference distribution caused by interference management techniques (e.g., beamforming). Our main contributions are summarized below.

- We obtain a tractable expression of the coverage (equivalently outage) probability, by adopting appropriate approximations and utilizing tools from stochastic geometry.
- We obtain the asymptotic expressions of the coverage/outage probability in the low and high SIR threshold regimes, using series expansions of special functions. The analytical results indicate that the maximum IN DoF can affect the order gain of the outage probability in the low SIR threshold regime, but cannot affect the order gain of the coverage probability in the high SIR threshold regime; the IN thresholds only affect the coefficients of the coverage/outage probability in the low and high SIR threshold regimes.
- We consider the optimizations of the maximum IN DoF for given IN thresholds in the two asymptotic regimes, which are challenging integer programming problems with very complicated objective functions. By exploiting the structure of each objective function, we characterize the optimal maximum IN DoF. The optimization results reveal that the IN scheme can linearly improve the outage probability in the low SIR threshold regime, but cannot improve the coverage probability in the high SIR threshold regime.
- We show that the IN scheme can achieve good gains in coverage/outage probability over a maximum ratio beamforming scheme and a user-centric ABS scheme, using numerical results.

II. NETWORK MODEL

We consider a two-tier HetNet where a macro-cell tier is overlaid with a pico-cell tier, as shown in Fig. 1. The locations of macro-BSs and pico-BSs are spatially distributed as two independent homogeneous Poisson point processes (PPPs) $\Phi_1$ and $\Phi_2$ with densities $\lambda_1$ and $\lambda_2$, respectively. The locations of users are also distributed as an independent homogeneous PPP $\Phi_u$ with density $\lambda_u$. Without loss of generality, denote the macro-cell tier as the 1st tier and the pico-cell tier as the 2nd tier. We focus on the downlink scenario. The macro-BSs and the pico-BSs share the same spectrum concurrently. Each macro-BS has $N_1$ antennas with total transmission power $P_1$, each pico-BS has $N_2$ antennas with total transmission power $P_2$, and each user has a single antenna. Assume $N_1 > N_2$. We consider both path loss and small-scale fading. Specifically, due to path loss, transmitted signals from the $j$th tier with distance $r$ are attenuated by a factor $r^{-\alpha_j}$, where $\alpha_j > 2$ is the path loss exponent of the $j$th tier and $j = 1, 2$. For small-scale fading, we assume Rayleigh fading channels.

A. User Association

We assume open access [4]. User $i$ (denoted as $u_i$) is associated with the BS which provides the maximum long-term average (over small-scale fading) received power

$^1$Note that, compared to a pico-BS, a macro-BS usually causes stronger interference due to larger transmit power, and has a better capability of performing spatial cancellation due to a larger number of transmit antennas. Thus, it is more advisable to perform IN at macro-BSs.
among all the macro-BSs and pico-BSs. This associated BS is called the serving BS of user \(i\). Note that within each tier, the nearest BS to user \(i\) provides the strongest long-term average received power in this tier. User \(i\) is thus associated with (the nearest BS in) the \(j^\text{th}\) tier, if\(^2\)

\[
j_i^j = \arg \max_{j \in \{1,2\}} P_j Z_{i,j}^{-\alpha_i},
\]

where \(Z_{i,j}\) is the distance between user \(i\) and its nearest BS in the \(j^\text{th}\) tier. We refer to the users associated with the macro-cell tier as the macro-users, denoted as \(U_1 = \{u_i | P_1 Z_{i,1}^{-\alpha_i} \geq P_2 Z_{i,2}^{-\alpha_i}\}\), and the users associated with the pico-cell tier as the pico-users, denoted as \(U_2 = \{u_i | P_2 Z_{i,2}^{-\alpha_i} > P_1 Z_{i,1}^{-\alpha_i}\}\). All the users can be partitioned into two disjoint user sets: \(U_1\) and \(U_2\).

After the user association, each BS schedules its associated users according to TDMA, i.e., scheduling one user in each time slot. Hence, there is no intra-cell interference.

B. Performance Metric

In this paper, we study the performance of the typical user denoted as \(u_0\), which is a scheduled user located at the origin [20]. Since HetNets are interference-limited, we ignore the thermal noise in the analysis of this paper. Note that the analytical results with thermal noise can be obtained in a similar way [21]. The coverage probability of \(u_0\) is defined as the probability that the SIR of \(u_0\) is larger than a threshold [4], i.e.,

\[
S(\beta) \triangleq \Pr (\text{SIR}_0 > \beta)
\]

where \(\beta\) is the SIR threshold. The outage probability of \(u_0\) is defined as the probability that the SIR of \(u_0\) is smaller than or equal to a threshold, i.e., \(1 - S(\beta)\). The coverage/outage probability provides the cumulative distribution function (c.d.f.) of the random SIR over the entire network [7]. In Sections IV, V and VI, we shall analyze the coverage/outage probability in the general, low and high SIR threshold regimes, separately.

III. USER-CENTRIC INTERFERENCE NULLING SCHEME

In this section, we first elaborate on a user-centric IN scheme. Then, we obtain some distributions related to this scheme.

A. Scheme Description

First, we refer to an interfering macro-BS which causes the SIIR at scheduled user \(i\) in the \(j^\text{th}\) tier (\(u_i \in U_j\)) below threshold \(T_j \geq 1\) as a potential IN macro-BS of \(u_i\), where \(j = 1, 2\). We refer to \(T_j\) as the IN threshold for the \(j^\text{th}\) tier. Mathematically, interfering macro-BS \(\ell\) is a potential IN macro-BS of scheduled user \(u_i \in U_j\) if

\[
P_\ell \frac{Z_{\ell,ji}^{-\alpha}}{P_1 Z_{i,1}^{-\alpha}} < T_j,
\]

where \(D_{\ell,wi}\) is the distance from macro-BS \(\ell\) to \(u_i\). Note that \(T_1\) and \(T_2\) are two design parameters of the IN scheme. In each time slot, each scheduled user sends IN requests to all of its potential IN macro-BSs. We refer to the scheduled users which send IN requests

\[2\]In the user association procedure, the first antenna is normally used to transmit signal (using the total transmission power of each BS) for received power distribution according to LTE standards.

to interfering macro-BS \(\ell\) as the potential IN users of interfering macro-BS \(\ell\) (in this time slot). We introduce another design parameter \(U \in \{0, 1, \cdots, N_1 - 1\}\) of this IN scheme, referred to as the maximum IN DoF. Consider a particular time slot. Let \(K_\ell\) denote the number of the potential IN users of interfering macro-BS \(\ell\). Note that \(T_1 = T_2 = 1\) implies \(K_i = 0\). Consider two cases in the following. i) If \(K_\ell > 0\) and \(U > 0\), macro-BS \(\ell\) makes use of at most \(U\) DoF to perform IN to some of its potential IN users. In particular, if \(0 < K_\ell \leq U\), macro-BS \(\ell\) can perform IN to all of its \(K_\ell\) potential IN users using \(K_\ell\) DoF; if \(K_\ell > U\), macro-BS \(\ell\) randomly selects \(U\) out of its \(K_\ell\) potential IN users according to the uniform distribution, and perform IN to the selected \(U\) users using \(U\) DoF. Hence, in this case, macro-BS \(\ell\) performs IN to \(u_{IN,\ell} \triangleq \min (U, K_\ell)\) potential IN users (referred to as the IN users of macro-BS \(\ell\)) using \(u_{IN,\ell}\) DoF (referred to as the IN DoF of macro-BS \(\ell\)). ii) If \(K_\ell = 0\) or \(U = 0\), macro-BS \(\ell\) does not perform IN. In this case, we let \(u_{IN,\ell} = 0\). In both cases, \(N_1 - u_{IN,\ell}\) DoF at macro-BS \(\ell\) is used for boosting the desired signal to its scheduled user.

Now, we introduce the precoding vectors at macro-BSs in the IN scheme. Consider two cases in the following. i) If \(K_\ell > 0\) and \(U > 0\), macro-BS \(\ell\) utilizes the low-complexity ZFBF precoder to serve its scheduled user and simultaneously perform IN to its \(u_{IN,\ell}\) IN users. Specifically, denote \(H_{1,\ell} = [h_{1,\ell} \ b_{1,\ell} \ \cdots \ b_{1,\ell} u_{IN,\ell}]^T\), where \(h_{1,\ell} \sim \mathcal{CN}_{N_1,1}(0_{N_1 \times 1}, I_{N_1})\) denotes the channel vector between macro-BS \(\ell\) and its scheduled user, and \(b_{1,\ell} \sim \mathcal{CN}_{N_1,1}(0_{N_1 \times 1}, I_{N_1})\) denotes the channel vector between macro-BS \(\ell\) and its \(i\)th IN user \((i = 1, \cdots, u_{IN,\ell})\). The ZFBF precoding matrix at macro-BS \(\ell\) is designed to be the pseudo-inverse of \(H_{1,\ell}\), i.e., \(W_{1,\ell} = H_{1,\ell}^H (H_{1,\ell} H_{1,\ell}^H)^{-1}\) and the ZFBF vector at macro-BS \(\ell\) is designed to be \(f_{1,\ell} = \frac{w_{1,\ell}}{\|w_{1,\ell}\|}\), where \(w_{1,\ell}\) is the first column of \(W_{1,\ell}\) [22]. ii) If \(K_\ell = 0\) or \(U = 0\), macro-BS \(\ell\) uses the maximal ratio transmission (MRT) precoder to serve its scheduled user, which is a special case of the ZFBF precoder introduced for \(K_\ell > 0\) and
$U > 0$, and can be readily obtained from it by letting $u_{IN, \ell} = 0$, i.e., $\mathbf{h}_{\ell} = \mathbf{h}_{0,\ell}^0$. Next, we introduce the precoding vectors at pico-BSs. Each pico-BS utilizes the MRT precoder to serve its scheduled user. Specifically, the beamforming vector at pico-BS $\ell$ is $\mathbf{f}_{\ell,0} = \frac{\mathbf{h}_{\ell,0}^0}{\|\mathbf{h}_{\ell,0}^0\|}$, where $\mathbf{h}_{\ell,0} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_N)$ denotes the channel vector between pico-BS $\ell$ and its scheduled user. Note that the simple beamforming scheme (without interference management) can be included in the IN scheme as a special case by letting $T_1 = T_2 = 1$ and/or $U = 0$. Note that all the analytical results in this paper hold for the beamforming vector at pico-BS $\ell$ and its scheduled user. Specifically, the SIR of the typical user $u_0 \in U_j$ is given by

$$\text{SIR}_{j,0} = \frac{P_j Y_j^\alpha_0 |\mathbf{h}_{0,0}^0 f_{j,0}|^2}{P_1 I_{1,0} + P_2 I_{2,0} + P_3 I_{3,0} + P_4 I_{4,0}}$$

(2)

where $I_{j,1C} = \sum_{\ell \in \Phi_{j,1C}} D_{\ell,0}^2 |\mathbf{h}_{0,0}^0 f_{\ell,0}|^2$, $I_{j,1O} = \sum_{\ell \in \Phi_{j,1O}} D_{\ell,0}^2 |\mathbf{h}_{0,0}^0 f_{\ell,0}|^2$, and $I_{j,2} = \sum_{\ell \in \Phi_{j,2}} D_{\ell,0}^2 |\mathbf{h}_{2,0}^0 f_{\ell,2}|^2$.

**B. Preliminary Results**

In this part, we evaluate some distributions related to the IN scheme, which will be used to calculate the coverage probability in (1). These distributions are based on approximations, the accuracy of which will be verified in Section IV. We first calculate the probability mass function (p.m.f.) of the number of the potential IN users of an arbitrary (chosen uniformly at random) macro-BS, denoted as $K_0$. The p.m.f. of $K_0$ depends on the point processes formed by the scheduled macro and pico users, which are related to but not PPPs [24]. For analytical tractability, we approximate the scheduled macro and pico users as two independent PPPs with densities $\lambda_1$ and $\lambda_2$, respectively. Note that approximating the scheduled users as a homogeneous PPP has been considered in existing papers (see e.g., [24]).

**Lemma 1**: The p.m.f. of $K_0$ is given by

$$\Pr(K_0 = k) = \frac{\tilde{L}(T_1, T_2)^k}{k!} \exp(-\tilde{L}(T_1, T_2))$$

(3)

where $k = 0, 1, \cdots$ and $\tilde{L}(T_1, T_2) = \tilde{L}_1(T_1) + \tilde{L}_2(T_2)$ with

$$\tilde{L}_j(T_j) = 2\pi \lambda_j \int_0^\infty \frac{r \cdot \alpha_j}{\gamma_j} f_j(y) dy dr, \quad j = 1, 2.$$

(4)

Here, the p.d.f.s of $Y_j$ ($j = 1, 2$) are given as follows [25, Lemma 4]:

$$f_j(y) = \frac{2 \pi \lambda_1}{A_1} y \exp\left(-\pi \left(\lambda_1 y^2 + \lambda_2 \left(\frac{P_2}{P_1}\right) \frac{z_{\pi^2}}{y^2} \right)\right)$$

(5)

and

$$f_j(y) = \frac{2 \pi \lambda_2}{A_2} y \exp\left(-\pi \left(\lambda_1 \left(\frac{P_1}{P_2}\right) \frac{z_{\pi^2}}{y^2} + \lambda_2 y^2\right)\right)$$

(6)

where $A_j \triangleq \Pr(u_0 \in U_j)$ ($j = 1, 2$) are given by

$$A_1 = 2 \pi \lambda_1 \int_0^\infty z \exp\left(-\pi \lambda_2 \left(\frac{P_2}{P_1}\right) \frac{z_{\pi^2}}{y^2} \right) \times \exp\left(-\pi \lambda_1 z^2\right) dz$$

(7)

and

$$A_2 = 2 \pi \lambda_2 \int_0^\infty z \exp\left(-\pi \lambda_1 \left(\frac{P_1}{P_2}\right) \frac{z_{\pi^2}}{y^2} \right) \times \exp\left(-\pi \lambda_2 z^2\right) dz.$$

(8)
Proof: See Appendix A.

Note that \( L(T_1, T_2) \) represents the average number of IN requests of the scheduled users received by an arbitrary macro-BS, and \( L_j(T_j) \) represents the average number of IN requests of the scheduled users in the \( j \)-th tier received by an arbitrary macro-BS. From (4), we can easily see that \( L_j(T_j) \) and \( L(T_1, T_2) \) increase with \( T_1 \) and \( T_2 \).

Next, we calculate the p.m.f. of the number of the IN users of an arbitrary (chosen uniformly at random) macro-BS \( u_{IN,0} \) and the preliminary results obtained in Section II-B, we have the following theorem.

\[
\Pr (u_{IN,0} = u) = \sum_{k=0}^{\infty} \Pr (K_0 = k) , \quad \text{for } 0 \leq u < U
\]
\[
\Pr (K_0 = k) , \quad \text{for } u = U
\]

Now, we calculate the probability that an arbitrary (chosen uniformly at random) potential IN macro-BS of \( u_0 \) selects \( u_0 \) for IN, referred to as the IN probability and denoted as \( p_u(U, T_1, T_2) \), based on Lemma 1.

**Lemma 2:** The p.m.f. of \( u_{IN,0} \), is given by

\[
p_c(U, T_1, T_2) \approx \left( \sum_{k=0}^{u-1} \frac{L(T_1, T_2)^k}{k!} + U \sum_{k=U}^{\infty} \frac{L(T_1, T_2)^k}{(k+1)!} \right) \times \exp\left(-L(T_1, T_2)\right)
\]

**Proof:** See Appendix B.

Note that different potential IN macro-BSs of \( u_0 \) select \( u_0 \) for IN independently (as the numbers of the potential IN users of these macro-BSs are dependent). For analytical tractability, we assume that different potential IN macro-BSs of \( u_0 \) select \( u_0 \) for IN independently. Using independent thinning, \( u_0 \)'s potential IN macro-BSs which do not select \( u_0 \) for IN can be approximated by a homogeneous PPP with density \( p_c(U, T_1, T_2) \lambda_1 \), where \( p_c(U, T_1, T_2) \lambda_1 \)

1. **IV. COVERAGE PROBABILITY—GENERAL SIR THRESHOLD REGIME**

In this section, we investigate the coverage probability in the general SIR threshold regime. By total probability theorem and the preliminary results obtained in Section III-B, we have the following theorem.

**Theorem 1 (Coverage Probability):** Under design parameters \( U, T_1 \) and \( T_2 \), we have: 1) coverage probability of a macro-user: \( S_1(\beta, U, T_1, T_2) \triangleq \Pr (SIR_0 > \beta | u_0 \in U_1) \), given in (9); 2) coverage probability of a pico-user: \( S_2(\beta, U, T_1, T_2) \triangleq \Pr (SIR_0 > \beta | u_0 \in U_2) \), given in (10); 3) overall coverage probability \( S(\beta, U, T_1, T_2) = A_1 S_1(\beta, U, T_1, T_2) + A_2 S_2(\beta, U, T_1, T_2) \), where \( A_1 \triangleq \Pr (u_0 \in U_2) \) and \( A_2 = \Pr (u_0 \in U_1) \).

IV. COVERAGE PROBABILITY—GENERAL SIR THRESHOLD REGIME

\[
S_1(\beta, U, T_1, T_2) = \frac{N \cdot \exp(-\beta)}{\Delta L(T_1, T_2)}
\]

where \( N \) denotes the complementary incomplete beta function, \( \Delta L(T_1, T_2) = L(T_1, T_2) - L(T_1) - L(T_2) \) and \( L(T_1) \) and \( L(T_2) \) are the average number of IN requests of the scheduled users in the macro-BSs and \( \Delta L(T_1, T_2) \) is the total number of IN requests of the scheduled users received by an arbitrary macro-BS. Note that different potential IN macro-BSs of \( u_0 \) select \( u_0 \) for IN independently. Using independent thinning, \( u_0 \)'s potential IN macro-BSs which do not select \( u_0 \) for IN can be approximated by a homogeneous PPP with density \( p_c(U, T_1, T_2) \lambda_1 \), where \( p_c(U, T_1, T_2) \lambda_1 \).

Fig. 2. Coverage probability versus maximum IN DoF and SIR threshold. \( N_1 = 10, N_2 = 8, \alpha_1 = 4.5, \alpha_2 = 4.7, \Delta L = 15 \) dB, \( \lambda_1 = 0.0005 \) nodes/m\(^2\), and \( \lambda_2 = 0.001 \) nodes/m\(^2\). For the Monte Carlo results, we use a two-dimensional square area 240^2 m^2 to simulate the large-scale HetNet and 10^6 random realizations to obtain the average coverage probability. The computation time using Monte-Carlo simulations is about 250 times of that using the analytical results, demonstrating that the analytical results are more tractable than Monte-Carlo simulations.

**Proof:** See Appendix C.

V. ASYMPTOTIC OUTAGE PROBABILITY ANALYSIS—LOW SIR THRESHOLD REGIME

In this section, we analyze and optimize the complement of the coverage probability, i.e., the outage probability of the IN scheme in the low SIR threshold regime, i.e., \( \beta \to 0 \).

A. Asymptotic Outage Probability Analysis

In this part, we analyze the asymptotic outage probability \( \Pr (SIR_0 < \beta) \) of the IN scheme when \( \beta \to 0 \). First, as
\[ S_1(\beta, U, T_1, T_2) = \sum_{u=0}^{U} \text{Pr}(\text{uIN}, 0 = u) \int_{0}^{\infty} \sum_{n = 0}^{N_s - u - 1} \sum_{(n_1, n_2, n_3) \in N_n} \left( \begin{array}{c} n \\ n_1, n_2, n_3 \end{array} \right) \tilde{p}^{(n_1)}_{1,1,c} \left( U, \beta y, T \right) \]

\[ \times \tilde{L}^{(n_2)}_{I_{1,1,O}} \left( \beta y, T \right) \tilde{L}^{(n_3)}_{I_{1,2}} \left( \frac{P_T y}{P_1}, \left( \frac{P_1}{P_2} \right) \alpha, \frac{\alpha_1}{\alpha_2} \right) f_Y(y) dy \]

\[ S_2(\beta, U, T_1, T_2) = \int_{0}^{\infty} \sum_{n = 0}^{N_s - u - 1} \sum_{(n_1, n_2, n_3) \in N_n} n \left( \begin{array}{c} n \\ n_1, n_2, n_3 \end{array} \right) \tilde{L}^{(n_1)}_{I_{2,1,c}} \left( U, \beta P_1 P_2 y, \left( \frac{P_1}{P_2} \right) \alpha, \frac{\alpha_1}{\alpha_2} \right) \]

\[ \times \tilde{L}^{(n_2)}_{I_{2,1,O}} \left( \beta P_2 y, \left( \frac{P_1 T}{P_2} \right) \alpha, \frac{\alpha_1}{\alpha_2} \right) \tilde{L}^{(n_3)}_{I_{2,2}} \left( \beta y \alpha_2, \left( \frac{\alpha_1}{\alpha_2} \right) f_Y(y) dy \right) \]

\[ \tilde{p}^{(n)}_{I_{j,k}}(U, s, r_{j,1C}, r_{j,1G}) = \exp \left( - \left( B' \left( \frac{2}{\alpha_1} 1 - \frac{2}{\alpha_1} \frac{1}{1 + s r_{j,1C}} \right) - B' \left( \frac{2}{\alpha_1} 1 - \frac{2}{\alpha_1} \frac{1}{1 + s r_{j,1O}} \right) \right) \right) \]

\[ \times \tilde{p}(U, T_1, T_2) \lambda_1 S \]

\[ \times \left( B' \left( \frac{1}{\alpha_1} 1 + \frac{2}{\alpha_1} \frac{1}{1 + s r_{j,1G}} \right) - B' \left( \frac{1}{\alpha_1} 1 + \frac{2}{\alpha_1} \frac{1}{1 + s r_{j,1O}} \right) \right) \]

\[ \times \prod_{a=1}^{n} \left( \frac{2 \pi \lambda J(k)}{\alpha_J(k)} \right)^{2 \pi \lambda_J(k)} s r_{j,k} \]

Then, we define the coefficient of the asymptotic outage probability: \( \lim_{\beta \to 0} \frac{\log \text{Pr}(\text{SIR} < \beta)}{\log \beta} \). Leveraging the order gain and the coefficient of the outage probability, we shall characterize the key behavior of the complex outage probability in the low SIR threshold regime.

Recently, a tractable approach has been proposed in [27] to characterize the order gain for a class of communication schemes in wireless networks which satisfy certain conditions. However, this approach does not provide tractable analytical expressions for the coefficient of the asymptotic outage probability for most of the schemes using multiple antennas in this class. By utilizing series expansion of some special functions and dominated convergence theorem, we characterize both the order gain and the coefficient of the asymptotic outage probability of the IN scheme in multi-antenna HetNets.

**Theorem 2 (Asymptotic Outage Probability):** Under design parameters \( U, T_1 \) and \( T_2 \), when \( \beta \to 0 \), we have:\(^3\) 1) outage probability of a macro-user:

\[ b(U, T_1, T_2) = \begin{cases} A_2 b_1(U, T_1, T_2), & U < N_1 - N_2 \\ A_1 b_1(U, T_1, T_2) + A_2 b_2(U, T_1, T_2), & U = N_1 - N_2 \\ A_1 b_1(U, T_1, T_2), & U > N_1 - N_2 \end{cases} \]

Here, \( b_j(U, T_1, T_2) \) is given in (14) with \( U_j = U \) and \( P_j = \text{Pr}(\text{uIN}, 0 = U) \) if \( j = 1 \) and \( T_1, T_2 > 1 \); \( U_j = 0 \) and \( P_j = 1 \), otherwise. Moreover, \( b_2(U, T_1, T_2) \) decreases with \( U \).

**Proof:** See Appendix D.

From Theorem 2, we clearly see that the maximum IN DoF \( U \) and the IN thresholds \( (T_1, T_2) \) affect the asymptotic behavior of the outage probability in dramatically different ways. Specifically, \( U \) can affect the order gain, while \( (T_1, T_2) \) can only affect the coefficient. In addition, we see that \( U \) affects the order gain of the asymptotic outage probability through affecting the order gain of the asymptotic macro-user outage probability. On the other hand, in this paper, IN is only performed at macro-BSs, and \( U \) is the upper bound on the actual DoF for IN in the ZFBF precoder (which is random due to the randomness

\(^3\) \( f(\beta) \to 0 \) \( g(\beta) \) means \( \lim_{\beta \to 0} \frac{f(\beta)}{g(\beta)} = 1 \).
Theorem 2
In this part, we characterize the optimal maximum IN DoF
thresholds in the asymptotic outage probability than the IN thresholds.

B. Asymptotic Outage Probability Optimization

Therefore, Fig. 3 verifies Theorem 2 and shows that the asymptotic outage probability in the low SIR threshold regime provides a reasonable approximation for the outage probability when the SIR threshold is below -5 dB.

B. Asymptotic Outage Probability Optimization

From Theorem 2, we know that $U$ has a larger impact on the asymptotic outage probability than the IN thresholds. In this part, we characterize the optimal maximum IN DoF $U^*(\beta, T_1, T_2)$ which minimizes the asymptotic outage probability given in Theorem 2 (maximizes the asymptotic coverage probability) for given thresholds $T_1$ and $T_2$, i.e.,

$$U^*(\beta, T_1, T_2) \triangleq \arg \min_{U \in \{0, 1, \ldots, N_1-1\}} b(U, T_1, T_2) \beta^{\min\{N_1-U, N_2\}},$$

(15)

Lemma 4 (Optimality Property of $U^*(\beta, T_1, T_2)$):

$\exists \beta > 0$ such that for all $\beta < \beta$, we have$^4$

$$U^*(\beta, T_1, T_2) \begin{cases} N_1 - N_2 - 1, & \text{if } A_2b_2(N_1 - N_2 - 1, T_1, T_2) \\ < A_2b_1(N_1 - N_2, T_1, T_2), & \text{otherwise} \end{cases},$$

Proof: See Appendix E.

Lemma 4 indicates that in the low threshold regime, the IN scheme achieves the optimal asymptotic outage probability when reserving $N_3$ or $N_2 + 1$ DoF at each macro-BS to boost the desired signal to its scheduled user, which is comparable to the $N_2$ DoF used at each pico-BS to boost the desired signal to its scheduled user. The reason is that in the low threshold regime, the network performance is mainly limited by the worst users. Balancing the DoF for boosting signals to all the users effectively improves the performance of the worst users.

Fig. 4 plots the outage probability versus the maximum IN DoF in the small SIR threshold regime. From Fig. 4, we can see that $U^*(\beta, T_1, T_2) = N_1 - N_2 - 1$ or $N_1 - N_2$ at small $\beta$. This verifies Lemma 4. Fig. 5 shows that the asymptotically optimal solution in Lemma 4 for the low SIR threshold regime is optimal when the SIR threshold is below -4 dB, as it is the same as the optimal solution optimizing the coverage probability in Theorem 1 for the general SIR threshold regime. Therefore, the asymptotically optimal solution in Lemma 4 provides good guidance on choosing effective maximum IN DoF when the SIR threshold is relatively small.

$^4$Lemma 4 is similar to Theorem 3 of our previous work [14]. The reason is that the two interference management schemes in this paper and [14] are both based on IN. One difference is that the proposed scheme in this paper aims to improve the performance of all users with low SIR, while the scheme in [14] only improves the performance of offloaded users.

Fig. 3. Outage probability versus SIR threshold in the low SIR threshold regime. $N_1 = 10$, $N_2 = 8$, $U = 6$, $\alpha_1 = 4.5$, $\alpha_2 = 4.7$, $\lambda_1 = 15$ dB, $\lambda_2 = 0.005$ nodes/m$^2$, and $\lambda_2 = 0.001$ nodes/m$^2$. of the network topology). Therefore, the result of the order gain in Theorem 2 extends the existing order gain result in single-tier cellular networks where the DoF for IN in the ZFBF precoder is deterministic [17].

Fig. 3 plots the outage probability versus the SIR threshold in the low SIR threshold regime. We see from Fig. 3 that when the SIR threshold is small, the “Analytical” curves, which are plotted using Theorem 1, are reasonably close to the “Asymptotic” curves, which are plotted using Theorem 2. In addition, from Fig. 3, we clearly see that the outage probability curves with the same $U$ have the same slope (indicating the same order gain), and there is a shift between two outage probability curves with the same $U$ but different $(T_1, T_2)$ (indicating different coefficients). Therefore, Fig. 3 verifies Theorem 2, and shows that the asymptotic outage probability in the low SIR threshold regime provides a reasonable approximation for the outage probability when the SIR threshold is below -5 dB.
Then, we define the coefficient of the coverage probability, i.e., the exponent of coverage probability in two scenarios, i.e., order gain and the coefficient of the coverage probability, as

\[ \lim_{N \to \infty} \frac{\Pr (\text{SIR} > \beta)}{\log \beta}. \tag{16} \]

Then, we define the coefficient of the asymptotic coverage probability: \( \lim_{N \to \infty} \frac{\Pr (\text{SIR} > \beta)}{\log \beta} \). Similarly, leveraging the order gain and the coefficient of the coverage probability, we shall characterize the key behavior of the complex coverage probability in the high SIR threshold regime. In the following, we analyze the asymptotic coverage probability in two scenarios, i.e., \( \alpha_1 \neq \alpha_2 \) and \( \alpha_1 = \alpha_2 \).

When \( \alpha_1 \neq \alpha_2 \), it turns out to be difficult to obtain the expression of the asymptotic coverage probability. Thus, we derive lower and upper bounds on the asymptotic coverage probability.

**VI. ASYMPTOTIC COVERAGE PROBABILITY ANALYSIS—HIGH SIR THRESHOLD REGIME**

In this section, we analyze and optimize the coverage probability of the IN scheme in the high SIR threshold regime, i.e., \( \beta \to \infty \).

**A. Asymptotic Coverage Probability Analysis**

In this part, we analyze the asymptotic coverage probability of the IN scheme when \( \beta \to \infty \). First, we define the order gain of the coverage probability (in interference-limited systems), i.e., the exponent of coverage probability as the SIR threshold increases to infinity:

\[ d_c = \lim_{\beta \to \infty} \frac{\Pr (\text{SIR} > \beta)}{\log \beta}. \]  

Then, we define the coefficient of the asymptotic coverage probability: \( \lim_{N \to \infty} \frac{\Pr (\text{SIR} > \beta)}{\log \beta} \). Similarly, leveraging the order gain and the coefficient of the coverage probability, we shall characterize the key behavior of the complex coverage probability in the high SIR threshold regime. In the following, we analyze the asymptotic coverage probability in two scenarios, i.e., \( \alpha_1 \neq \alpha_2 \) and \( \alpha_1 = \alpha_2 \).

When \( \alpha_1 \neq \alpha_2 \), it turns out to be difficult to obtain the expression of the asymptotic coverage probability. Thus, we derive lower and upper bounds on the asymptotic coverage probability.

\[ f(\beta) \beta^{-\frac{1}{\alpha_1}} g(\beta) \text{ means } \lim_{\beta \to \infty} \frac{f(\beta)}{g(\beta)} = 1. \]
\[ \eta_1(U, T_1, T_2) = \frac{\pi \lambda_1}{A_1} \sum_{u=0}^{U} \Pr(u_{IN,0} = u) \sum_{n=0}^{N_{T} - u - 1} \sum_{n_2=0}^{n} \sum_{(p_0)_{a,z_1} \in M_{n_2}} \sum_{(q_a)_{a,z_1} \in M_{n_2}} \frac{n_2!}{\prod_{a=1}^{n_2} p_a!} \]

\[ \times \prod_{a=1}^{n_2} \left( \frac{2 \pi \lambda_1 B \left( 1 + \frac{2}{\alpha}, a - \frac{2}{\alpha} \right)}{\prod_{a=1}^{n_2} q_a!} \right) \prod_{a=1}^{n_2} \left( \frac{2 \pi \lambda_2}{\alpha} \frac{P_1}{P_2} \right) \frac{2}{\alpha} B \left( 1 + \frac{2}{\alpha}, a - \frac{2}{\alpha} \right) \]

\[ \times \left( \frac{2 \pi \lambda_1}{\alpha} B \left( \frac{2}{\alpha}, 1 - \frac{2}{\alpha} \right) \right)^{-\frac{n_2}{1}} \sum_{a=1}^{n_2} \sum_{q_a=1}^{q_a - 1} \Gamma \left( \frac{n_2}{\alpha} \sum_{a=1}^{n_2} p_a + \sum_{a=1}^{n_2} q_a + 1 \right) \]

\[ \eta_2 = \frac{\pi \lambda_2}{A_2} \sum_{n=0}^{N_{T} - n_2 - 1} \frac{n}{n_2!} \sum_{n_2=0}^{n} \sum_{(p_0)_{a,z_1} \in M_{n_2}} \sum_{(q_a)_{a,z_1} \in M_{n_2}} \frac{n_2!}{\prod_{a=1}^{n_2} p_a!} \]

\[ \times \prod_{a=1}^{n_2} \left( \frac{2 \pi \lambda_1}{\alpha} \frac{P_1}{P_2} \right) \frac{2}{\alpha} B \left( 1 + \frac{2}{\alpha}, a - \frac{2}{\alpha} \right) \left( \frac{2 \pi \lambda_2}{\alpha} \frac{P_1}{P_2} \right) \frac{2}{\alpha} B \left( 1 + \frac{2}{\alpha}, a - \frac{2}{\alpha} \right) \]

\[ \times \left( \frac{2 \pi \lambda_1}{\alpha} B \left( \frac{2}{\alpha}, 1 - \frac{2}{\alpha} \right) \right)^{-\frac{n_2}{1}} \sum_{a=1}^{n_2} \sum_{q_a=1}^{q_a - 1} \Gamma \left( \frac{n_2}{\alpha} \sum_{a=1}^{n_2} p_a + \sum_{a=1}^{n_2} q_a + 1 \right) \]

\[ \xi_1 = \frac{\pi \lambda_1 \alpha_{\text{max}}}{A_1 \alpha_1} \left( \frac{2 \pi \lambda_1}{\alpha_1} B \left( \frac{2}{\alpha_1}, 1 - \frac{2}{\alpha_1} \right) + \frac{2 \pi \lambda_2}{\alpha_2} \left( \frac{P_1}{P_2} \right) \frac{2}{\alpha_2} B \left( \frac{2}{\alpha_2}, 1 - \frac{2}{\alpha_2} \right) \right) \]

\[ \xi_2 = \frac{\pi \lambda_1 \alpha_{\text{max}}}{A_1 \alpha_2} \left( \frac{2 \pi \lambda_1}{\alpha_1} \frac{P_1}{P_2} \frac{2}{\alpha_1} B \left( \frac{2}{\alpha_1}, 1 - \frac{2}{\alpha_1} \right) + \frac{2 \pi \lambda_2}{\alpha_2} \left( \frac{P_1}{P_2} \right) \frac{2}{\alpha_2} B \left( \frac{2}{\alpha_2}, 1 - \frac{2}{\alpha_2} \right) \right) \]

\[ c_1 (U, T_1, T_2) = \frac{\pi \lambda_1}{A_1} \sum_{u=0}^{U} \Pr(u_{IN,0} = u) \sum_{n=0}^{N_{T} - u - 1} \frac{1}{n!} \sum_{n_2=0}^{n} \sum_{(p_0)_{a,z_1} \in M_{n_2}} \sum_{(q_a)_{a,z_1} \in M_{n_2}} \frac{n_2!}{\prod_{a=1}^{n_2} p_a!} \]

\[ \times \prod_{a=1}^{n_2} \left( \frac{2 \pi \lambda_1 B \left( 1 + \frac{2}{\alpha}, a - \frac{2}{\alpha} \right)}{\prod_{a=1}^{n_2} q_a!} \right) \prod_{a=1}^{n_2} \left( \frac{2 \pi \lambda_2}{\alpha} \frac{P_1}{P_2} \right) \frac{2}{\alpha} B \left( 1 + \frac{2}{\alpha}, a - \frac{2}{\alpha} \right) \]

\[ \times \left( \frac{2 \pi \lambda_1}{\alpha} B \left( \frac{2}{\alpha}, 1 - \frac{2}{\alpha} \right) \right)^{-\frac{n_2}{1}} \sum_{a=1}^{n_2} \sum_{q_a=1}^{q_a - 1} \Gamma \left( \frac{n_2}{\alpha} \sum_{a=1}^{n_2} p_a + \sum_{a=1}^{n_2} q_a + 1 \right) \]

\[ c_2 (T_1, T_2) = \frac{\pi \lambda_2}{A_2} \sum_{n=0}^{N_{T} - n_2 - 1} \frac{1}{n!} \sum_{n_2=0}^{n} \sum_{(p_0)_{a,z_1} \in M_{n_2}} \sum_{(q_a)_{a,z_1} \in M_{n_2}} \frac{n_2!}{\prod_{a=1}^{n_2} p_a!} \]

\[ \times \prod_{a=1}^{n_2} \left( \frac{2 \pi \lambda_1}{\alpha} \frac{P_1}{P_2} \right) \frac{2}{\alpha} B \left( 1 + \frac{2}{\alpha}, a - \frac{2}{\alpha} \right) \prod_{a=1}^{n_2} \left( \frac{2 \pi \lambda_2}{\alpha} \frac{P_1}{P_2} \right) \frac{2}{\alpha} B \left( 1 + \frac{2}{\alpha}, a - \frac{2}{\alpha} \right) \]

\[ \times \left( \frac{2 \pi \lambda_1}{\alpha} B \left( \frac{2}{\alpha}, 1 - \frac{2}{\alpha} \right) \right)^{-\frac{n_2}{1}} \sum_{a=1}^{n_2} \sum_{q_a=1}^{q_a - 1} \Gamma \left( \frac{n_2}{\alpha} \sum_{a=1}^{n_2} p_a + \sum_{a=1}^{n_2} q_a + 1 \right) \]
Analytical (T\textsubscript{1}=T\textsubscript{2}=6, U=3)
Analytical (T\textsubscript{1}=T\textsubscript{2}=6, U=3)
Analytical (T\textsubscript{1}=T\textsubscript{2}=6, U=3)
Asympt. UB (T\textsubscript{1}=T\textsubscript{2}=2, U=3)
Asympt. UB (T\textsubscript{1}=T\textsubscript{2}=2, U=3)
Asympt. UB (T\textsubscript{1}=T\textsubscript{2}=6, U=3)
Asympt. UB (T\textsubscript{1}=T\textsubscript{2}=6, U=3)
Asympt. UB (T\textsubscript{1}=T\textsubscript{2}=6, U=3)
Asympt. UB (T\textsubscript{1}=T\textsubscript{2}=6, U=3)

Fig. 6. Coverage probability versus SIR threshold in the high SIR threshold regime. N\textsubscript{1} = 10, N\textsubscript{2} = 8, λ\textsubscript{T} = 15 dB, λ\textsubscript{1} = 0.0005 nodes/m\textsuperscript{2}, and λ\textsubscript{2} = 0.001 nodes/m\textsuperscript{2}.

forming scheme without interference management when β \to ∞. In addition, T\textsubscript{1} and T\textsubscript{2} affect the coefficient of the upper bound on the asymptotic coverage probability when α\textsubscript{1} ≠ α\textsubscript{2} and the coefficient of the asymptotic coverage probability when α\textsubscript{1} = α\textsubscript{2}. U affects the coefficient of the upper bound on the asymptotic coverage probability when α\textsubscript{1} ≠ α\textsubscript{2} and the coefficient of the asymptotic coverage probability when α\textsubscript{1} = α\textsubscript{2}, through affecting the upper bound on the asymptotic coverage probability of a macro-user when α\textsubscript{1} ≠ α\textsubscript{2} and the asymptotic coverage probability of a macro-user when α\textsubscript{1} = α\textsubscript{2}, respectively.

Fig. 6 plots the coverage probability versus the SIR threshold in the high SIR threshold regime for α\textsubscript{1} ≠ α\textsubscript{2} and α\textsubscript{1} = α\textsubscript{2}, respectively. We see from Fig. 6(a) that when α\textsubscript{1} ≠ α\textsubscript{2}, the “Analytical” curves, which are plotted using Theorem 1, are bounded by the corresponding “Asymptotic” upper bound curves and lower bound curve, which are plotted using Theorem 3. Note that there is only one “Asymptotic” lower bound curve, as the asymptotic lower bound is independent of U and (T\textsubscript{1}, T\textsubscript{2}). In addition, from Fig. 6(a), we clearly see that the coverage probability curves with different U or (T\textsubscript{1}, T\textsubscript{2}) have slightly different slopes (indicating different order gains), and there is a small shift between any two coverage probability curves with different U or (T\textsubscript{1}, T\textsubscript{2}) (indicating different coefficients). On the other hand, we see from Fig. 6(b) that when α\textsubscript{1} = α\textsubscript{2}, the “Analytical” curves, which are plotted using \textit{S}(β, U, T\textsubscript{1}, T\textsubscript{2}) in Theorem 1, are reasonably close to the “Asymptotic” curves, which are plotted using Theorem 4. In addition, from Fig. 6(b), we clearly see that the coverage probability curves with different U or (T\textsubscript{1}, T\textsubscript{2}) have the same slope (indicating the same order gain), and there is a shift between any two coverage probability curves with different U or (T\textsubscript{1}, T\textsubscript{2}) (indicating different coefficients). Therefore, Fig. 6 verifies Theorem 3 and Theorem 4, and shows that the asymptotic coverage probability in the high SIR threshold regime provides a reasonable approximation for the coverage probability when the SIR threshold is above 13 dB.

B. Asymptotic Coverage Probability Optimization

In this part, we characterize the optimal maximum IN DoF U\textsuperscript{*}(β, T\textsubscript{1}, T\textsubscript{2}) which maximizes the upper bound on the asymptotic coverage probability given in Theorem 3 when α\textsubscript{1} ≠ α\textsubscript{2} and the asymptotic coverage probability given in Theorem 4 when α\textsubscript{1} = α\textsubscript{2}, for given thresholds T\textsubscript{1} and T\textsubscript{2}, i.e.,

\[
U\textsuperscript{*}(β, T\textsubscript{1}, T\textsubscript{2}) = \arg \max_{U \in \{0,1,\ldots,N\textsubscript{1}−1\}} c^{ab}(U, T\textsubscript{1}, T\textsubscript{2}) β^{−\frac{2}{\max α}} \text{, } \alpha\textsubscript{1} ≠ α\textsubscript{2} = \arg \max_{U \in \{0,1,\ldots,N\textsubscript{1}−1\}} \left\{ (A\textsubscript{1}c\textsubscript{1}(U, T\textsubscript{1}, T\textsubscript{2}) + A\textsubscript{2}c\textsubscript{2}(T\textsubscript{1}, T\textsubscript{2})) β^{−\frac{1}{\alpha}} \right\} \text{, } \alpha\textsubscript{1} = α\textsubscript{2} = \arg \max_{U \in \{0,1,\ldots,N\textsubscript{1}−1\}} c^{ab}(U, T\textsubscript{1}, T\textsubscript{2}) \text{, } \alpha\textsubscript{1} ≠ α\textsubscript{2} = \arg \max_{U \in \{0,1,\ldots,N\textsubscript{1}−1\}} c\textsubscript{1}(U, T\textsubscript{1}, T\textsubscript{2}) \text{, } \alpha\textsubscript{1} = α\textsubscript{2}.
\]

(24)

Note that U does not affect the lower bound on the asymptotic coverage probability given in Theorem 3.

Lemma 5 (Optimality Property of U\textsuperscript{*}(β, T\textsubscript{1}, T\textsubscript{2})): There exists β < ∞ such that for all β > β, we have U\textsuperscript{*}(β, T\textsubscript{1}, T\textsubscript{2}) = 0 for arbitrary α\textsubscript{1} and α\textsubscript{2}.

\textbf{Proof:} See Appendix H.

Lemma 5 indicates that performing IN will not improve the asymptotic coverage probability in the high SIR threshold regime. The reason is that in the high SIR threshold regime, the overall coverage probability is mainly contributed by cell center users, which have much better performance than cell edge users. Using all N\textsubscript{1} DoF at each macro-BS to boost the desired signal to its scheduled user can effectively improve the coverage probability of a cell center macro-user, and hence improve the overall coverage probability.

Fig. 7 plots the coverage probability versus the maximum IN DoF in the high SIR threshold regime. From Fig. 7, we can see that U\textsuperscript{*}(β, T\textsubscript{1}, T\textsubscript{2}) = 0. This verifies Lemma 5. In addition, we can observe that the coverage probability decreases with the maximum IN DoF. Fig. 8 shows that the asymptotically optimal solution in Lemma 5 for the high SIR threshold regime is optimal when the SIR threshold is above 16 dB, as it is the same as the optimal
The user-centric ABS scheme has three design parameters, i.e., \( \alpha \) and \( \lambda \). The user-centric ABS scheme is a modified version of the existing ABS scheme in 3GPP. Theorem 1 provides good guidance on choosing effective maximum IN DoF when the SIR threshold is relatively high.

### VII. Numerical Experiments

In this section, we compare the proposed user-centric IN scheme with two baseline schemes. One is a simple beamforming scheme (without interference management), which can be treated as a special case of our IN scheme by setting \( U = 0 \) and/or \( T_1 = T_2 = 1 \). The other is a modified version of the existing ABS scheme in 3GPP-LTE, referred to as the user-centric ABS scheme. The user-centric ABS scheme has three design parameters, i.e., a resource partition parameter \( \eta \) and two thresholds \( T_j \) and \( T_{j+1} \), where \( T_j \) (\( j = 1, 2 \)) is the threshold for the \( j \)-th tier. We define a potential ABS macro-BS of a scheduled user in a similar way to a potential IN macro-BS of a scheduled user in the user-centric IN scheme. In each slot, each scheduled user sends ABS requests to all of its potential ABS macro-BSs. We define the potential ABS users of a macro-BS in a similar way to the potential IN users of a macro-BS in the user-centric IN scheme. The fraction of resource is allocated to the remaining BSs to serve their own scheduled users. Then, for given \( T_1 \) and \( T_2 \), we choose the optimal \( \eta \) to maximize the coverage probability of the user-centric IN scheme. Under this user-centric ABS scheme, each scheduled potential ABS pico-user or macro-user whose serving macro-BS is not a potential ABS macro-BS can avoid the interference from all its potential ABS macro-BSs via resource partition in ABS.

Note that the benefit of the proposed user-centric IN scheme compared to the simple beamforming scheme is that it can optimally allocate DoF in boosting desired signals and managing interference. Thus, the performance of the proposed user-centric IN scheme is always better than that of the simple beamforming scheme. One benefit of the proposed user-centric IN scheme compared to the user-centric ABS is that it does not have (time or frequency) resource sacrifice. On the other hand, one loss of the proposed user-centric IN scheme compared to the user-centric ABS is due to the DoF reduction for boosting desired signals to macro-users.

Fig. 9 illustrates the coverage probability versus the number of antennas at each macro-BS \( N_1 \). From Fig. 9, we can observe that the proposed user-centric IN scheme and the user-centric ABS outperform the simple beamforming scheme, demonstrating the importance of interference management in the parameter region considered in this figure. In addition, the proposed user-centric IN scheme outperforms the user-centric ABS when \( N_1 \) is relatively large. The reason is as follows. When \( N_1 \) is relatively large, for serving macro-users, the loss of the user-centric ABS caused by (time or frequency) resource sacrifice (due to resource partition) is large, while the loss of the proposed user-centric IN scheme caused by DoF reduction (due to performing IN) is small. Fig. 10 illustrates the coverage probability versus the path loss exponent in the macro-cell tier \( \alpha_1 \). From Fig. 10, we can observe that the proposed user-centric IN scheme outperforms the user-centric ABS when \( \alpha_1 \) is relatively large. The reason is as follows. When \( \alpha_1 \) is large, the loss of the proposed user-
centric IN scheme due to the DoF reduction for boosting desired signals to macro-users is small. Fig. 11 illustrates the user-centric IN scheme outperforms the user-centric ABS when $P_1/P_2$ is relatively large. This is because when $P_1/P_2$ is relatively large, for serving macro-users, the loss of the user-centric ABS caused by (time or frequency) resource sacrifice (due to resource partition) is large.

VIII. CONCLUSIONS

In this paper, we proposed a user-centric IN scheme in downlink two-tier multi-antenna HetNets. Using tools from stochastic geometry, we first obtained a tractable expression of the coverage probability. Then, we analyzed the asymptotic coverage/outage probabilities in the low and high SIR threshold regimes. The analytical results indicate that the maximum IN DoF and the IN thresholds affect the asymptotic coverage/outage probability in dramatically different ways. Moreover, we characterized the optimal maximum IN DoF which optimizes the asymptotic coverage/outage probability in each asymptotic regime. Finally, numerical results showed that the user-centric IN scheme can achieve good gains in coverage/outage probability over existing schemes.

APPENDIX

A. Proof of Lemma 1

According to Slivnyak’s theorem [28], we focus on a macro-BS located at the origin, referred to as macro-BS 0. Note that both scheduled macro and pico users may send IN requests to macro-BS 0. We first characterize the probability that a scheduled macro-user sends an IN request to macro-BS 0. Denote $R_{1i}$ as the distance between macro-BS 0 and a randomly selected (according to the uniform distribution) scheduled macro-user, referred to as scheduled macro-user $i$. Assume that the scheduled macro-users form a homogeneous PPP with density $\lambda_1$. Conditioned on $R_{1i} = r$, scheduled macro-user $i$ sends an IN request to macro-BS 0 with probability

$$ p_{1i,R_{1i}}(r, T_1) = \text{Pr}(T_1 \frac{1}{r} r < Y_1 < r) = \int_{T_1}^{r} \frac{1}{r} f_{Y_1}(y)dy $$

(25)

where $f_{Y_1}(y)$ is the p.d.f. of $Y_1$ given by (5) [25, Lemma 4]. Then, the scheduled macro-user density at distance $r$ away from macro-BS 0 is $p_{1i,R_{1i}}(r, T_1)$ $\lambda_1$. This indicates that the scheduled macro-users at distance $r$ away from macro-BS 0 which send IN requests to macro-BS 0 form an inhomogeneous PPP with density $p_{1i,R_{1i}}(r, T_1) \lambda_1$. Next, we characterize the probability that a scheduled pico-user sends an IN request to macro-BS 0. Denote $R_{2i}$ as the distance between macro-BS 0 and a randomly selected (according to the uniform distribution) scheduled pico-user, referred to as scheduled pico-user $i$. Similarly, we assume that the scheduled pico-users form a homogeneous PPP with density $\lambda_2$, and it is independent of the PPP formed by the scheduled macro-users. Then, we can show that the scheduled pico-users at distance $r$ away from macro-BS 0 which send IN requests to macro-BS 0 form an inhomogeneous PPP with density $p_{1i,R_{2i}}(r, T_1) \lambda_2$. Finally, numerical results showed that the user-centric
we have on its own distances to each of its potential IN macro-BS, 

\[ \frac{p_{2i,R_2}(r,T_2)}{p_{2i,R_2}(r,r)} < Y_2 \leq \left( \frac{p_{2i,R_2}(r)}{P_{T}} \right) \]

\[ = \int \left( \frac{f_{Y}(y)}{f_{Y}(y)} \right)^{2} f_{Y}(y) dy. \]  

(26)

Note that \( f_{Y}(y) \) is the p.d.f. of \( Y_2 \) given by (6) [25, Lemma 4].

By the superposition property of PPPs [28], the scheduled macro-users and the scheduled pico-users at distance \( r \) away from macro-BS 0 which send IN requests to macro-BS 0, i.e., the potential IN users of macro-BS 0, still form an inhomogeneous PPP with density \( p_{1i,R_1}(r,T_1)\lambda_1 + p_{2i,R_2}(r,T_2)\lambda_2 \). Therefore, the number of the potential IN users of macro-BS 0 is Poisson distributed with parameter (mean) \( L(T_1,T_2) = 2\pi \int_{0}^{\infty} (p_{1i,R_1}(r,T_1)\lambda_1 + p_{2i,R_2}(r,T_2)\lambda_2) \) dr = \( L(T_1) + L(T_2) \).

\( \text{B. Proof of Lemma 3} \)

Let \( p_c(U,T_1,T_2) \) denote the probability that an arbitrary potential IN macro-BS of \( u_0 \) selects \( u_0 \) for IN when it has \( K \) potential IN users besides \( u_0 \). If \( K + 1 \leq U \), \( p_c(U,T_1,T_2,K) = 1 \); if \( K + 1 > U \), \( p_c(U,T_1,T_2,K) = \frac{U}{K+1} \), as the selection is according to the uniform distribution. Thus, for \( K \), we have \( p_c(U,T_1,T_2,K) = \min \left\{ \frac{U}{K+1}, 1 \right\} \). Averaging over \( K \), we have \( p_c(U,T_1,T_2) = \min \left\{ \frac{U}{K+1}, 1 \right\} \). As shown in [19], each scheduled user will send the IN request based on its own distances to each of its potential IN macro-BSs and its serving BS, which are independent of the other scheduled users. Thus, given that \( u_0 \) has sent the request to the potential IN macro-BS, \( K \) follows the same distribution as \( K_0 \). Therefore, we have

\[ p_c(U,T_1,T_2) = E \left[ \min \left\{ \frac{U}{K+1}, 1 \right\} \right] \]

\[ = \sum_{k=0}^{U-1} \Pr(K = k) + \sum_{k=U}^{\infty} \frac{U}{k+1} \Pr(K = k). \]  

(27)

Substituting (3) into (27), we have the final result.

\( \text{C. Proof of Theorem 1} \)

Let \( R_{j,1c} \) and \( R_{j,1o} \) denote the minimum and maximum possible distances between \( u_0 \in U_j \) and its nearest and furthest macro-interferers (among \( u_0 \)'s potential IN macro-BSs which do not select \( u_0 \) for IN), respectively. Let \( R_{j,2} \) denote the minimum possible distance between \( u_0 \in U_j \) and its nearest pico-interferer. The relationships between \( R_{j,1c} \), \( R_{j,1o} \), \( R_{j,2} \), and \( Y_j \), respectively, are shown in Table I. Based on (2) and conditioned on \( Y_j = y \), we have (28) where (a) is due to \( |h_{j,0,0}-1|^{2} \backsim \text{Gamma}(M_j,1) \), (b) is due to Multinomial Theorem, and \( L^{(n)}(s,r) \triangleq E[(-I)^{n} \exp(-sI)] \) denotes the \( n \)th-order derivative of the Laplace transform of random variable \( I \), i.e., \( L_I(s) \triangleq E[I^{n}] \).

Now, we calculate \( L_I(s) \) and \( L^{(n)}_I(s) \), respectively. First, \( L_{f,j,1c}(s) \) can be calculated as follows:

\[ L_{f,j,1c}(s) = E_{\Phi,j,1c} \left[ \exp \left( -s \sum_{\ell \in \Phi_{j,1c}} D_{j,1c}^{-\alpha_1} g_{j,1,c} \right) \right] \]

\[ = \prod \left[ E_{\Phi_j,1c} \left[ \exp \left( -s D_{i,0}^{-\alpha_1} g_{j,1,c} \right) \right] \right] \]

\[ = \exp \left( -2\pi p_c(U,T_1,T_2) \lambda_1 \int_{r_{j,1o}}^{r_{j,1c}} \left( 1 - \frac{1}{1 + \frac{s}{\lambda_1}} \right) dr \right) \]

\[ \triangleq L_{f,j,1c}(U,s,r_{j,1c},r_{j,1o}) \]  

(29)

where \( g_{j,1,c} \triangleq \left| h_{j,1,0} f_{j,1,c} \right|^{2} \), (c) is obtained by noting that \( g_{j,1,c} (\ell \in \Phi_{j,1c}) \) are mutually independent, (d) is due to \( \left| h_{j,1,0} f_{j,1,c} \right|^{2} \sim \text{Gamma}(1,1) \) (i.e., \( \text{Exp}(1) \)), and (e) is obtained by using the probability generating function of a PPP [28]. Further, by first letting \( s \alpha \rightarrow t \) (i.e., \( \frac{\alpha}{s} = t^{-\alpha} \)) and then \( \frac{\alpha}{s} \rightarrow w \), we have \( \int_{1/\alpha}^{1} \left( 1 - \frac{1}{1 + \frac{s}{\lambda_1}} \right) dr = \frac{\alpha}{s} \int_{1}^{1/\alpha} \left( 1 + \frac{s}{\lambda_1} \right)^{1/\alpha - 1} (1 - w)^{1/\alpha - 1} dw \). By the definition of \( B'(a,b,z) \), we can obtain

\[ L_{f,j,1c}(U,s,r_{j,1c},r_{j,1o}) \]

\[ = \exp \left( \frac{2\pi}{\alpha_1} \lambda_1 s \frac{\alpha}{\alpha} \right) \left( B' \left( \frac{2}{\alpha_1}, 1 - \frac{2}{\alpha_1}, \frac{1}{1 + \alpha \lambda_1} \right) - B' \left( \frac{2}{\alpha_1}, 1 - \frac{2}{\alpha_1}, \frac{1}{1 + \alpha \lambda_1} \right) \right) p_c(U,T_1,T_2). \]  

(30)

Next, based on (29) and utilizing Faà di Bruno’s formula [29], \( L^{(n)}_{f,j,1c}(s) \) can be calculated as follows:

\[ L^{(n)}_{f,j,1c}(s) = \sum_{\ell \in \Phi_{j,1c}} \left( \frac{n_1}{\alpha_1} \right) m_a \]

\[ \times \prod_{a=1}^{n_1} \left( \frac{2\pi p_c(U,T_1,T_2) \lambda_1}{a!} \int_{r_{j,1c}}^{r_{j,1o}} \frac{d^a}{da^a} \left( 1 + \frac{s}{\lambda_1} \right) dr \right) m_a \]

TABLE I

<table>
<thead>
<tr>
<th>Parameter Values</th>
<th>( R_{j,1c} )</th>
<th>( R_{j,1o} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( y_1 )</td>
<td>( T_1 )</td>
</tr>
<tr>
<td>2</td>
<td>( y_2 )</td>
<td>( T_2 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( R_{j,2} )</th>
<th>( Y_1 )</th>
<th>( (\bar{L})_{y_2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( y_1 )</td>
<td>( T_1 )</td>
</tr>
<tr>
<td>2</td>
<td>( y_2 )</td>
<td>( T_2 )</td>
</tr>
</tbody>
</table>
\[ \Pr (\text{SIR}_j > \beta | u_0 \in U_j, Y_j = y) = \Pr \left( \left\| h^*_{Y_0, 0} f_{0, 0} \right\|^2 > \beta g^{y_2} \left( \frac{P_j}{P_j} I_{1,1C} + \frac{P_j}{P_j} I_{1,1O} + \frac{P_j}{P_j} I_{1,2} \right) \right) \]

\[ \begin{align*}
\text{E}_{I_{1,1C}, I_{1,1O}, I_{1,2}} & \left[ \exp \left( -\beta g^{y_2} \left( \frac{P_j}{P_j} I_{1,1C} + \frac{P_j}{P_j} I_{1,1O} + \frac{P_j}{P_j} I_{1,2} \right) \right) \right] \\
& = \sum_{n=0}^{M_j-1} \frac{(\beta g^{y_2})^n}{n!} \sum_{(n_a)_{a=1}^{3} \in \mathbb{N}_n} \left( \prod_{a=1}^{3} \left( \frac{P_j}{P_j} \right)^{n_a} \right) \times \exp \left( -\beta g^{y_2} \left( \frac{P_j}{P_j} I_{1,1C} + \frac{P_j}{P_j} I_{1,1O} + \frac{P_j}{P_j} I_{1,2} \right) \right)
\end{align*} \]

\[ \text{E}_{I_{1,1C}, I_{1,1O}, I_{1,2}} \left[ \exp \left( -\beta g^{y_2} \left( \frac{P_j}{P_j} I_{1,1C} + \frac{P_j}{P_j} I_{1,1O} + \frac{P_j}{P_j} I_{1,2} \right) \right) \right] \\
= \sum_{n=0}^{M_j-1} \frac{(\beta g^{y_2})^n}{n!} \sum_{(n_a)_{a=1}^{3} \in \mathbb{N}_n} \left( \prod_{a=1}^{3} \left( \frac{P_j}{P_j} \right)^{n_a} \right) \times \exp \left( -\beta g^{y_2} \left( \frac{P_j}{P_j} I_{1,1C} + \frac{P_j}{P_j} I_{1,1O} + \frac{P_j}{P_j} I_{1,2} \right) \right)
\]

Now, we calculate \[ \lim_{\beta \to 0} \int_{0}^{\infty} \sum_{n=M_j}^{\infty} \mathcal{T}_j | \mathcal{Y}_j (n, y, U, T_1, T_2, \beta) \]\
\[ = \lim_{\beta \to 0} \int_{0}^{\infty} \sum_{n=M_j}^{\infty} P_j \mathcal{T}_j | \mathcal{Y}_j (n, y, U, T_1, T_2, \beta) \]\

Finally, removing the conditions on \( Y_j = y \) and after some algebraic manipulations, we can obtain the final result.

\[ \text{D. Proof of Theorem 2} \]

Conditioned on \( Y_j = y \), we have (31) where (a) is due to \( \left\| h_{0,0}^* f_{0,0} \right\|^2 \sim \text{Gamma}(M_j, 1) \), (b) is due to Multinomial Theorem, and (c) is due to similar calculations in Appendix C. Removing the condition on \( Y_j = y \), we have

\[ 1 - S_j (U, T_1, T_2, \beta) = \int_{0}^{\infty} \sum_{n=M_j}^{\infty} \mathcal{T}_j | \mathcal{Y}_j (n, y, U, T_1, T_2, \beta) \]\

Moreover, utilizing dominated convergence theorem, we can show that

\[ \lim_{\beta \to 0} \int_{0}^{\infty} \sum_{n=M_j}^{\infty} \mathcal{T}_j | \mathcal{Y}_j (n, y, U, T_1, T_2, \beta) \]\

Finally, \( f(x) = o(g(x)) \) means \( \lim_{x \to 0} \frac{f(x)}{g(x)} = 0 \).
1 − Pr(SIR_{j,0} > \beta | u_0 \in U_j, Y_j = y) = Pr \left( \|f_{j,00,0} \|^2 \leq \beta y^{n_y} \left( \frac{P_1}{P_j} I_{j,1C} + \frac{P_2}{P_j} I_{j,1O} + \frac{P_2}{P_j} I_{j,2} \right) \right)

= \exp \left( -\beta y^{n_y} \left( \frac{P_1}{P_j} I_{j,1C} + \frac{P_2}{P_j} I_{j,1O} + \frac{P_2}{P_j} I_{j,2} \right) \right) \sum_{n = M_j}^{\infty} \frac{(\beta y^{n_y})^n}{n!} \left( \frac{P_1}{P_j} I_{j,1C} + \frac{P_2}{P_j} I_{j,1O} + \frac{P_2}{P_j} I_{j,2} \right)^n

\leq \sum_{n = M_j}^{\infty} \frac{(\beta y^{n_y})^n}{n!} \sum_{(n_1, n_2, n_3) \in \mathbb{N}_n} \left( \frac{P_1}{P_j} \right)^{n_1} \left( \frac{P_2}{P_j} \right)^{n_2} \left( \frac{P_2}{P_j} \right)^{n_3} I_{j,1C}^{(n_1)}(s) I_{j,1O}^{(n_2)}(s) I_{j,2}^{(n_3)}(s) |_{s = \beta y^{n_y} \frac{P_j}{P_1}}

\text{Hence, substituting (33), (34) and (35) into (32), and after some algebraic manipulations, we obtain Results 1), 2) and 3) in Theorem 2. To complete the proof, we now show that b_2(U, T_1, T_2) decreases with U. This can be proved by noting that i) b_2(U, T_1, T_2) is an increasing function of }\ p_c(U, T_1, T_2), \text{ and ii) }\ p_c(U, T_1, T_2) \text{ decreases with } U \text{ (which can be easily shown by (27)).}

E. Proof of Lemma 4

First, we characterize the maximum order gain. When }\ U \in \{0, 1, \ldots, N_1 - N_2\}, \text{ we have }\ N_1 - U \geq N_2, \text{ implying }\ min\{N_1 - U, N_2\} = N_2. \text{ When }\ U \in \{N_1 - N_2 + 1, \ldots, N_1 - 1\}, \text{ we have }\ N_1 - U < N_2, \text{ implying }\ min\{N_1 - U, N_2\} = N_1 - U < N_2. \text{ Thus, we can show that the maximum order gain is }\ max_{U \in \{0, 1, \ldots, N_1 - 1\}} |\ min\{N_1 - U, N_2\} = N_2, \text{ achieved at any } U \in \{0, 1, \ldots, N_1 - N_2\}. \text{ Next, we compare the coefficients of }\ \beta^{N_2} \text{ achieved at different } U \in \{0, 1, \ldots, N_1 - N_2\}. \text{ We consider two cases. i) When }\ U < N_1 - N_2, \text{ as } b_2(U, T_1, T_2) \text{ decreases with } U, \text{ the coefficients satisfy } A_2b_2(N_1 - N_2 - 1, T_1, T_2) < A_2b_2(N_1 - N_2 - 2, T_1, T_2) < \ldots < A_2b_2(0, T_1, T_2). \text{ ii) When } U = N_1 - N_2, \text{ the coefficient of }\ \beta^{N_2} \text{ is }\ A_2b_1(N_1 - N_2, T_1, T_2) + A_2b_2(N_1 - N_2, T_1, T_2). \text{ Therefore, we can complete the proof.}

F. Proof of Theorem 3

1) Upper Bound: Let }\ S_j(Y_j, y, \beta, U, T_1, T_2) = \Pr(\text{SIR}_{j,0} > \beta | u_0 \in U_j, Y_j = y) \text{ denote the conditional SIR coverage probability. Then, from Theorem 1 (Appendix C), } S_j(\beta, U, T_1, T_2) \text{ can be written as}

\begin{equation}
S_j(\beta, U, T_1, T_2) = \int_0^\infty S_j(Y_j, y, \beta, U, T_1, T_2) f_Y(y) dy
\end{equation}
Let \( \tilde{f}_Y(y) = \frac{2\pi \lambda_j}{\alpha_1} y \exp \left( -\pi \lambda_j y^2 \right) \).

Let \( \tilde{S}_{jY} (y, \beta, U, T_1, T_2) \) with \( \tilde{g}_j (y, \beta, U, T_1, T_2) = \exp \left( -c_j(\beta) \beta^{\frac{a_j}{\alpha_j}} y^2 \right) \). Then, we have

\[
\int_0^\infty \tilde{S}_{jY} (y, \beta, U, T_1, T_2) f_Y(y) dy 
\]

\[
\leq \int_0^\infty \tilde{S}_{j,Y} (y, \beta, U, T_1, T_2) \tilde{f}_Y(y) dy
\]

where (a) is due to \( g_j (y, \beta, U, T_1, T_2) < \tilde{g}_j (y, \beta, U, T_1, T_2) \) and \( f_Y(y) < \tilde{f}_Y(y) \), and (b) is due to \( \tilde{g}_j (y, \beta, U, T_1, T_2) \) and \( f_Y(y) < \tilde{f}_Y(y) \).

To calculate the order of \( \beta \) for (38) as \( \beta \to \infty \), we first calculate the orders of \( \beta \) for (c) and \( c_j(\beta) \) as \( \beta \to \infty \).

We note that \( B'(a, b, z) = B(a, b) - \frac{a}{z} + o(z^a) \), as \( z \to 0 \). Thus, as \( \beta \to \infty \), we have

\[
c_j(\beta) = \frac{2\pi \lambda_j}{\alpha_1} \left( \frac{P_2}{P_j} \right)^{\frac{1}{\alpha_j}} \left( \frac{\alpha_1}{2} p_c(U, T_1, T_2) \left( T_{j,Y}^{\alpha_1} - 1 \right) \beta^{\frac{a_j}{\alpha_j}} + B \left( \frac{2}{\alpha_1}, 1 - \frac{2}{\alpha_1} \right) - \frac{\alpha_1}{2} \left( \frac{T_j}{\beta} \right)^{\frac{1}{\alpha_1}} + o \left( \beta^{\frac{a_j}{\alpha_j}} \right) \right)
\]

(40)

where \( \tilde{S}_{j,Y} (y, \beta, U, T_1, T_2) = \mu_j(\beta, U, T_1, T_2) \tilde{g}_j (y, \beta, U, T_1, T_2) \) with \( \tilde{g}_j (y, \beta, U, T_1, T_2) = \exp \left( -c_j(\beta) \beta^{\frac{a_j}{\alpha_j}} y^2 \right) \).

(41)

We can obtain the order of \( \beta \) for each term corresponding to a choice for \( n_i \) \( i = 1, \ldots, n \) and \( m_i \) \( i = 1, \ldots, n \), in (38) as \( \beta \to \infty \): \( \beta^{-\frac{a_j}{\alpha_j}} \sum_{i=1}^n m_i - \frac{a_j}{\alpha_j} \), which can be maximized when \( n_i = 0 \). Hence, we order the terms the upper bound: \( \beta^{-\frac{a_j}{\alpha_j}} \). Moreover, based on (38), (39), (40) and after some algebraic manipulation, we obtain the expressions of \( \eta(U, T_1, T_2) \) and \( \eta_2 \).

2) Lower Bound: First, we note that \( S_j (\beta, U, T_1, T_2) \) can be rewritten as

\[
S_j (\beta, U, T_1, T_2) = \int_0^\infty \tilde{S}_{j,Y} (y, \beta, U, T_1, T_2) f_Y(y) dy
\]

\[
+ \int_1^\infty \tilde{S}_{j,Y} (y, \beta, U, T_1, T_2) f_Y(y) dy
\]

\[
\geq \int_0^1 \tilde{S}_{j,Y} (y, \beta, U, T_1, T_2) f_Y(y) dy
\]

\[
\geq \int_0^1 \tilde{S}_{j,Y} (y, \beta, U, T_1, T_2) f_Y(y) dy
\]

where \( \tilde{S}_{j,Y} (y, \beta, U, T_1, T_2) = \mu_j(\beta, U, T_1, T_2) \tilde{g}_j (y, \beta, U, T_1, T_2) \) with \( \tilde{g}_j (y, \beta, U, T_1, T_2) = \exp \left( -c_j(\beta) \beta^{\frac{a_j}{\alpha_j}} y^2 \right) \).

(42)
\[ \frac{\sum_{i=1}^{n} m_i + \sum_{i=1}^{n} q_i}{\alpha n} - \frac{\sum_{i=1}^{n} m_i + \sum_{i=1}^{n} q_i}{\min{\alpha n}} \times \beta^{-\sum_{i=1}^{n} m_i + \sum_{i=1}^{n} q_i} = 0, \]

which can be maximized when \( n_1 = n_2 = n_3 = 0 \), i.e., \( n = 0 \). Hence, we obtain the order of the lower bound as \( \frac{1}{\alpha n} \min{\alpha n} \).

Moreover, based on \( B(a,b,z) = \frac{1}{a+b+1} + o(1-z) \) as \( z \to 1 \) and after some algebraic manipulation, we obtain the expression of \( \xi \).

**G. Proof of Theorem 4**

When \( \alpha_1 = \alpha_2 = \alpha \), from (37), we have

\[
\Pr (\text{SIR} \geq \beta) = \sum_{n=0}^{M_1-1} \sum_{a_1}^{n_1} \sum_{a_2}^{n_2} \sum_{a_3}^{n_3} \pi_{\lambda_j} \frac{A_j}{A_3} e^{\beta} \int_0^\infty \left( c_1 \beta x + c_2 \beta \beta x + a_1 + a_2 \right) y^2 dy \\
\times \exp \left( - \frac{c_1 \beta x + c_2 \beta \beta x + a_1 + a_2}{y} \right) dy \\
\times \frac{\pi_{\lambda_j} A_j}{A_3} e^{\beta} \Gamma \left( \frac{a_1}{n_1}, m_1 + \frac{n_2}{a_1} p_1 + \frac{n_3}{a_1} q_1 + 1 \right) \left( a_1 + a_2 \right) \\
+ \left( c_1 (\beta + c_2 (\beta)) \beta \beta \right) - \frac{n_1}{a_1} a_{1m} - \frac{n_2}{a_1} p_{1m} - \frac{n_3}{a_1} q_{1m} - 1
\]

(43)

When \( \beta \to \infty \), based on (39)-(41), we can obtain the order of \( \beta \) for each term corresponding to a choice for \( n_1, (n_{a_1})_{a_1=1}^{n_1} \in N_1 \) and \( (m_{a_1})_{a_1=1}^{n_2} \in M_1 \) in (43) as:

\[ \beta^{-1} \frac{1}{a_1} \sum_{a_1}^{n_1} m_1 - \frac{1}{a_1} \sum_{a_1}^{n_2} p_2 \]

which can be maximized when \( n_1 = 0 \). Hence, the order is \( \frac{1}{\alpha n} \min{\alpha n} \).

Moreover, after some algebraic manipulation, we can obtain the expressions of \( c_1 (U, T_1, T_2) \) and \( c_2 (T_1, T_2) \).

**H. Proof of Lemma 4**

We solve the optimization problem for \( \alpha_1 = \alpha_2 \). When \( \alpha_1 \neq \alpha_2 \), the optimization problem can be solved in a similar way and is omitted due to page limit. First, we rewrite \( c_1 (U, T_1, T_2) \) in (22) as

\[ c_1 (U, T_1, T_2) = \frac{1}{U} \sum_{u=0}^{U} \Pr (u_{T_1,U} = u) f(u), \]

where \( f(u) \) denotes the expression after \( \Pr (u_{T_0,N} = u) \) in (22). It can be easily verified that \( f(u) \) is a decreasing function of \( u \). By Lemma 2, we have

\[ c_1 (U, T_1, T_2) = \sum_{u=0}^{U} \Pr (K_0 = u) f(u) + \sum_{k=U+1}^{\infty} \Pr (K_0 = k) f(U) \]

(24). Thus, we have

\[ c_1 (U, T_1, T_2) = \frac{1}{U} (f(u) + f(U)) \sum_{k=U+1}^{\infty} \Pr (K_0 = k) < 0. \]

Therefore, we can show \( \frac{U^* (\beta, T_1, T_2) = 0}{} \)

**REFERENCES**


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