A technology selection and operation (TSO) optimisation model for distributed energy systems: Mathematical formulation and case study

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Abstract

This paper presents a model which simultaneously optimises the selection and operation of technologies for distributed energy systems in buildings. The Technology Selection and Operation (TSO) model enables a new approach for the optimal selection and operation of energy system technologies that encompasses whole life costing, carbon emissions as well as real-time energy prices and demands; thus, providing a more comprehensive result than current methods. Utilizing historic metered energy demands, projected energy prices and a portfolio of available technologies, the mathematical model simultaneously solves for an optimal technology selection and operational strategy for a determined building based on a preferred objective: minimizing cost and/or minimizing carbon emissions. The TSO is a comprehensive and novel techno-economic model, capable of providing decision makers an optimal selection from a portfolio of available energy technologies. The current portfolio of available technologies is composed of various combined heat and power (CHP) and organic Rankine cycle (ORC) units. The TSO model framework is data-driven and therefore presents a high level of flexibility with respect to time granularity, period of analysis and the technology portfolio. A case study depicts the capabilities of the model; optimisation results under different temporal arrangements and technology options are showcased. Overall, the TSO model provides meaningful insights that allow stakeholders to make technology investment decisions with greater assurance.

1 Introduction

The implementation of distributed energy systems (DES) has gained popularity in recent years. Energy efficiency, rising energy costs, transmission and distribution infrastructure constraints as well as sustainability issues have increased the attractiveness of the implementation of DES in urban settings; particularly commercial, institutional and other large buildings.

DES can avoid electricity transmission and distribution losses as the electricity generated is used locally, possibly resulting in a more energy efficient system [1]. In addition to this, the use of co-generation technologies in distributed systems enables point-of-use heat generation. Co-generation technologies typically have high net first-law efficiencies but are more difficult to benefit from in centralised form given that heat is not as easily distributed over longer distances as electricity. DES can more easily benefit from co-generation technologies and the high energy efficiency they provide [2].

As a consequence of this energy efficiency, there is a potential for lower energy resource consumption which could also translate into lower purchase expenditure and reduced carbon emissions. Virtuously, reduced carbon emissions lower the cost incurred from carbon emissions (e.g. carbon tax or emission allowances) and electricity transmission and distribution costs are also avoided.

This potential reduction in energy costs and carbon emissions as well as increased energy security provided by distributed systems make them appealing to investors [3]. Decision makers face a challenge when trying to select which on-site generation technology to implement. There is a broad range of options from which to select and the outcome of a selection comes with great uncertainty. Different variables such as size, price and performance come into play that can make the search for the best option a challenging task.

Simulations and mixed-integer programming (MIP) optimization models have assisted decision-makers in such tasks [1,2,4–10]. Diverse features and levels of complexity have been explored; the simplest model approaches consider constant energy demands and fixed energy costs while complex approaches consider fluctuating energy prices, variable energy demands, part-load efficiency integration, a carbon price and the possibility of selling electricity exported to the grid.

This paper introduces the Technology Selection and Operation (TSO) model, a tool to provide decision makers the optimal energy technology selection and operational strategy for implementation of distributed energy systems. The TSO model considers all the elements of complexity aforementioned and introduces the novel features of fuel flexibility and projections for energy costs, carbon price and grid carbon intensity. It aims to address the financial and technical uncertainty that DES investments bring by using these projections and analysing the best option over a multi-year period.

The TSO model works jointly with a database which includes a portfolio of available energy technologies. The model can be implemented for single or multi-building studies as long as energy demands are known. Because the database is independent from the model, it can also be easily adapted to include additional or alternative technologies.

Although DER optimisation has been an area widely explored as will be evidenced in the literature review section, this model presents strengths against similar models by:

- Providing a portfolio of existing technologies with both technical and economic parameters which can be easily expanded
- Incorporating real-time pricing data for both electricity import and export which integrate energy as well as ancillary service charges
- Considering projections of grid carbon intensity, carbon cost, and energy resource costs
- Performing a multi-year period optimisation
- Flexibility of time granularity

The data-driven structure of the model allows for these parameters to be updated easily to provide real-world technology selection and not simply an optimal theoretical capacity. This model covers the areas of complexity that similar models do and adds by introducing detail and long-term perspective through the use of the projections for carbon intensity, carbon cost and energy resource costs, facilitating greater assurance in DER investments. These parameters are inherently subject to temporal variability and need to be considered in DER investment planning. Additionally, the model provides flexibility with regards to the time granularity of this model as will be evidenced in the case study; it can accept simplifications of time domains depending the complexity the user desires. Finally, the model presents a high level of detail in the carbon accounting of the energy resources used.

The TSO model has been utilised as a tool for guiding decision-making for DER investments, specifically in the case of bio-methane fuelled CHP deployment in supermarket buildings for a large food retailer in the UK. A case study of the use of the TSO model will be presented to demonstrate the capabilities of its implementation and showcase its results. The case study involves a commercial building in the UK. The inputs used for the case study are described in detail and the techno-economic results obtained from the optimisation are presented.

This paper is structured as follows: Section 2 highlights previous literature on the selection and operation of technologies for DES. Section 3 defines the problem statement while Section 4 describes the model structure. The mathematical formulation of the model is provided and explained in Section 5 and the case study is presented in Section 6. Finally, Section 7 concludes with some remarks.

2 Literature Review

There is an extensive amount of literature describing different approaches for designing DES, ranging from standard recommendations to simulations [1,4–6] and mathematical programming models [2,7–9,11]. A broad range of factors integrate and influence DES design: diverse energy technologies, types of application (i.e. industrial, commercial or domestic), energy resource pricing (i.e. fixed or variable) and time periods of study among others. A great part of the literature available on DES incorporates fuelled co-generation units; these include combined heat and power (CHP), mainly gas-engine [1,8,11], or a variation of this such as combined cooling, heating and power (CCHP) [4,5,12]. Although some of the studies included the integration of other types of elements such as renewable energy technologies (e.g. solar PV) [2,7], the main element of the system is usually the CHP unit because the other elements cannot guarantee dispatchability [13]. It was also identified from the literature that most formulations applied an energy technology and resource conversion method similar to the Resource Technology Network (RTN) [14].

Besides traditional and standard sizing methods for CHPs such as [15], individual sizing or strategy optimization have been studied in [4,7,16]. [4] optimized system design by simulating a constant-load system, focusing on the size of the prime mover. [7] optimized the operational strategy of a district heating system with three existing CHP units. However, [1] argued that system sizing and operational strategy should be an integrated approach because of the potential impact it may have; it is mentioned that an integrated strategy can achieve greater optimality in terms of the desired objective.

Cho et al. [5] and Hueffed and Mago [6] provided simulations of different operational strategies under different unit sizes to evaluate impacts in annual cost, primary energy consumption and carbon dioxide emissions. These work under fixed energy prices. [11] created a mixed-integer non-linear programming model (MINLP) for a single objective optimization (cost) that considered time-of-use (TOU) tariffs, carbon tax, variable energy demand, part load efficiencies according to interpolation based on specific load points and linear scalability for capital costs. It focused on micro-CHP for domestic purposes. [17] introduced a MIP model for multi-objective optimization (annual cost and greenhouse emissions) for commercial buildings and considered various generation and conversion technologies with different upper and lower capacity constraints, performing a multi-period optimization; it contemplated fixed energy resource prices, power demands and technology efficiencies.

Ghadimi et al. [1] provided a simulation of an integrated approach and improve the linear scalability costs and fixed efficiency approaches used in the aforementioned studies by empirically modelling equations that allow to more accurately represent these. Their simulation focused on large systems for manufacturing plants, with a unit capacity ranging from 800 to 2200 kW that can evaluate energy cost, net present value (NPV) and carbon emissions of the system. However, they did not enable possible revenue from selling surplus electricity to the grid. [8] proposed a MINLP multi-objective model for integrated optimization for a residential complex and enabled the possibility of selling surplus electricity and heat to the grid; like [1], it considered TOU tariffs, variable energy demands and it made use of similar empirical models for scaling costs and determining part-load efficiencies. The previous two studies allowed for the use a single specific technology (CHP) that can vary in size, while [2,9,17] enabled the possibility to select from various technologies (e.g. gas-engine, fuel cells, biomass CHP) by incorporating in their MILP models a binary variable which represented if a technology was selected or not. These three lacked the improved method of empirically modelling seen in Refs. [1,8].

Akbari [3] claimed that many studies ignore uncertainties that could alter the outcome of the optimisations, possibly misleading the decision making process. For example, commodity prices can vary and affect the projected financial outcomes of these investments. Some of the previous studies considered simply fixed energy prices for their inputs, others considered TOU electricity tariffs but even those are provided for a single year even though they may vary from year to year. Other factors such as carbon taxes and grid carbon intensity can also vary through the lifetime of a project. [1] increased gas and electricity costs by a flat percentage each year, trying to predict or account for these future variations. However, historic data has shown that this is not necessarily the case. The DER-CAM tool [18] has been in development since 2000 and has been able to integrate a broad range of technologies while achieving complexity. It has been the most comprehensive tool to date and even provides a graphical user interface. However, it has not yet integrated long-term time-variable parameters which influence DER investments. These variations and uncertainties can affect the projected outcomes of the investments and have to be considered when guiding decision making for investments.

From the literature reviewed, it was identified that an area of opportunity exists for a practical tool that can serve as a guide for decision-makers and which is capable of more accurately representing typical real-life situations. Although the mentioned simulations and models provide an optimal solution, they do not provide an accurate real-life solution because unit capacities selected have thus far been theoretical and thus can be different from what is available in the market. Moreover, prices for units and efficiencies can only be approximated and will inherently vary between unit models. Energy prices and demands should be represented as accurately as possible to what occurs in real-life, most importantly by considering their variability. Additionally, a forward looking assessment is important as dealing with the effect of uncertainties is critical; projections of costs and time-variable parameters are required for greater investment confidence and accuracy. The TSO model was developed with the aim to incorporate all these considerations and is described in the following sections.

3 Problem Formulation

The TSO model serves as a tool for decision makers to determine the optimal on-site energy system technology and operational strategy for implementation in a building. Therefore, it simultaneously has to solve for the optimal selection of a technology (from a portfolio of available options) and determine the operational strategy of such technology considering fluctuating energy prices and demands. In this work, the preferred technology is to satisfy a building's heat and electricity demands over a given period of time consisting of specific time intervals. This is performed with the objective of minimising: a) cost, b) greenhouse gas emissions or c) a multi-objective combination of the two.

- Given:
  - The parameters of a building or area in which the energy system will be implemented consisting of its:
    - Heat and electricity demands (i.e. historically metered, assigned per time period)
    - Location, in order to determine its transmission and distribution charges
  - A technology portfolio comprising all the different energy technologies which the model can select from featuring:
    - Capacities, used in conjunction with thermal and electrical efficiencies to calculate heat and power outputs at different operational levels
    - Costs (i.e. capital and operational, per annum considering project life and discount rate)
    - Efficiencies (i.e. electrical and thermal; inclusive of part load efficiencies)
  - Energy and carbon parameters:
    - Price of purchased electricity from the grid (i.e. per time interval, including projections for future years)
    - Price of gas (i.e. fixed per year, including projections for future years)
    - Price of diesel (i.e. fixed per year, including projections for future years)
    - Price of electricity sold to the grid per time interval (i.e. per time interval, including projections for future years)
    - Carbon factors of electricity and fuel (i.e. fixed for fuels, including projections for future years in the case of electricity)

        - Carbon tax or cost of carbon allowance as well as any other regulatory surcharges (including projections for future years)
- Determine:
    - The technology selection (if any, of one of the available energy technologies for the system)
    - The technology operation (i.e. operational schedule for each interval of the defined time period)
- Subject to:
    - Satisfying electricity and heat demands of the building for each time interval considered
    - Technical and financial constraints
- In order to achieve (any of the below):
    - Minimise cost
    - Minimise GHG emissions
    - Minimise a combined function of the two

4 Model Structure

The TSO model makes use of the Resource Technology Networks (RTN) methodology [14] to represent energy resources, conversion technologies and energy transfer networks. A diagram representing the RTN employed for this model is illustrated in Figure 1. Energy resources are the inputs and output of energy conversion technologies; those employed by this model are: natural gas, diesel, bio-methane, electricity and heat. Energy conversion technologies are those that can convert an energy resource into a different one (e.g. a CHP unit converting a fuel into electricity and heat); those adapted to this model are: gas boilers, CHP units, combined CHP and ORC arrangements and diesel generators. Electricity demand can be satisfied by on-site generation, the electricity grid (i.e. import) or a mix of the two. The model can also decide to sell electricity to the grid (i.e. export). Excess heat is simply disposed by releasing it to the environment. Heat demand can be satisfied through on-site generation (e.g. gas boiler or CHP); temperature level of demand is assumed as 50 ˚C, typical of commercial buildings. The model considers heat and electricity demands as net loads that can come from a single or multi-set of buildings. However, any special considerations of power flow or thermal constraints within these loads are out of scope in this version of the model.

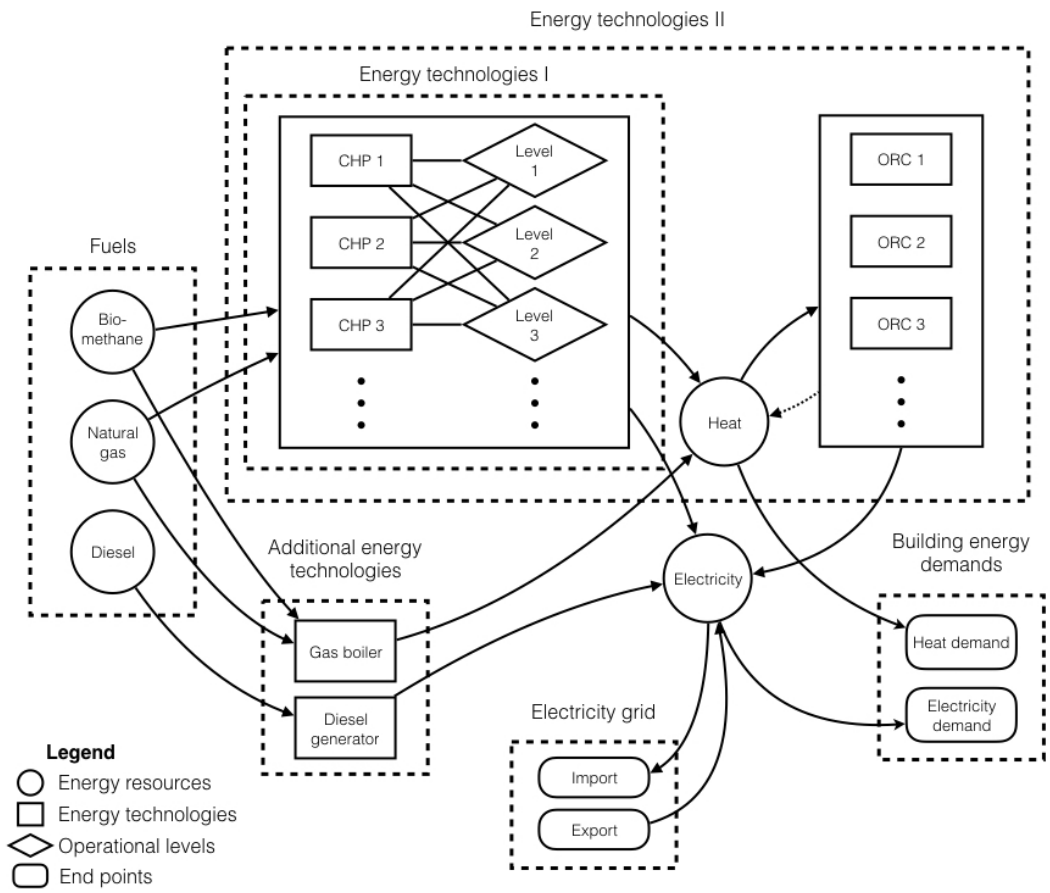

Figure 1. Simplified representation of the RTN used for the TSO model.

4.1 Technologies and levels

The current version of the model database includes 23 CHP technologies (gas-engine) in its portfolio with capacities ranging from 90 kW to 2 MW (*Energy technologies I* in Figure 1). In addition to these, the possibility of having CHP-ORC (organic Rankine cycle) arrangements also exists for each of the CHP units just mentioned, together with a compatible ORC unit size and are included as additional technologies to the previous (*Energy technologies II*). The model can also consider existing diesel generators and gas boilers (*Additional energy technologies*) that may already be installed; these can be explicitly defined with their actual capacity and efficiency. Utilisation of existing diesel generators for peak-price avoidance of electricity charges has become common practice in the UK and hence their use has been enabled for analysis in the model. However, heat rejected from the diesel generator or the gas boiler is not assumed to be redirected to an ORC; existing assets would require retrofit of a heat recovery element to enable directing heat to the ORC for which cost and feasibility would vary from case to case.

Therefore, only CHP-ORC arrangements can exist in the current version of the TSO model as these are assumed to be installed together as a package.

Energy technologies I and II are given as the set of technologies j that can operate at different levels l. The operational levels considered in this version of the model are: 50%, 60%, 70%, 80%, 90% and 100%. Operational levels can be expanded as desired to provide results with a higher level of accuracy but will incur an increase in computational time as they increase the number of possible combinations that may exist for the solution. Discrete levels were explicitly defined instead of allowing the model to freely choose any value for this reason and others that will be explained next. CHP technologies are generally limited to operate between 50-100% of their rated capacity [1] and confirmed from unit manufacturer recommendations, therefore levels below 50% are not considered. Additionally, explicit levels facilitate the accurate relationship of part-load efficiency; a precise part-load efficiency as indicated by the unit manufacturer is assigned for each level and for each of the units available in the portfolio. Furthermore, discrete levels also enable the possibility of creating a simplified operational schedule. CHP-ORC arrangements account for the additional electrical output and the reduction in heat output, consequence of the efficiency of the Rankine cycle, in their operational levels. The model is also enabled to choose working in CHP-only mode. The additional energy technologies such as the boiler and the generator can vary their output and are only constrained by their capacity; for these efficiency is a fixed parameter unlike Energy technologies I and II.

Table 1 demonstrates the various capacity CHP technologies incorporated in the technology portfolio. The table also shows relevant parameters of the technologies such as electrical and thermal efficiencies at different load levels (part-load efficiencies for other operating levels were interpolated from these values). These parameters were sourced from manufacturer data sheets [19–41]. Additionally, an indication of the ORC capacity (for CHP-ORC arrangements) is provided. These were sized and paired to each CHP according to the heat output of the latter at full output; for the purpose of this model the CHP-ORC arrangements can be considered as a package. ORC capacity is calculated upon the theoretical assumption that 13.7%[1] of CHP thermal output is effectively converted into electrical output, although the possibility exists to update the technology portfolio parameters with specific efficiencies provided by the manufacturer. The heat output from the CHP units specified in this table is effectively recovered at 120˚C (according to the manufacturer) and is assumed to be directed into the ORC.

Figure 2 provides indicative installed and maintenance cost curves for the technology portfolio; indicative data was collected from an industrial partner and Ref. [42].

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[1] According to the endoreversible, also Chambadal-Novikov, efficiency considering 20 ˚C as the low temperature and 120 ˚C (output of the CHP) as the high temperature of the cycle.

Table 1. Technical parameters of technology portfolio [19–41]. *(Part-load efficiencies for operating levels other than those in this table were interpolated from these values.)*

		Electrical efficiency			Thermal Efficiency			
CHP Capacity (kW_e)	**Thermal output (kW_{th})**	Full-load	75% load	50% load	Full-load	75% load	50% load	**ORC capacity (kW_e)**
90	163	32.1%	30.0%	30.0%	58.2%	59.8%	61.1%	22
100	175	32.8%	30.8%	30.8%	57.4%	59.3%	60.9%	24
110	186	33.4%	31.5%	31.5%	56.5%	58.8%	60.6%	25
122	198	34.2%	32.4%	32.4%	55.4%	58.0%	60.2%	27
135	219	34.1%	31.8%	31.8%	55.2%	55.8%	55.1%	30
150	233	35.1%	32.8%	32.8%	54.6%	55.7%	55.4%	32
165	285	32.7%	30.3%	30.3%	56.4%	56.4%	55.3%	39
185	309	33.6%	31.2%	31.2%	56.1%	56.5%	55.8%	42
205	242	39.6%	38.4%	38.4%	46.8%	47.6%	51.3%	33
210	337	34.6%	32.3%	32.3%	55.6%	56.4%	56.1%	46
229	356	35.4%	33.1%	33.1%	55.1%	56.3%	56.3%	49
254	321	37.4%	36.5%	36.5%	47.2%	47.7%	49.4%	44
310	357	37.8%	36.7%	36.7%	43.5%	46.5%	51.4%	49
376	398	38.7%	37.4%	37.4%	41.0%	44.4%	49.0%	55
405	513	38.7%	38.0%	38.0%	49.0%	51.0%	53.0%	70
426	464	38.5%	37.4%	37.4%	41.9%	45.8%	50.8%	64
502	538	38.0%	38.0%	38.0%	40.7%	44.4%	49.0%	74
530	648	39.5%	38.8%	38.8%	48.3%	48.8%	49.5%	89
775	823	42.3%	40.8%	40.8%	44.9%	45.8%	48.2%	113
1,284	1323	43.2%	41.8%	41.8%	44.5%	45.7%	47.3%	181
1,519	1402	44.2%	43.3%	43.3%	40.8%	42.0%	43.3%	192
2,024	1901	44.3%	43.4%	43.4%	41.6%	42.6%	43.8%	260

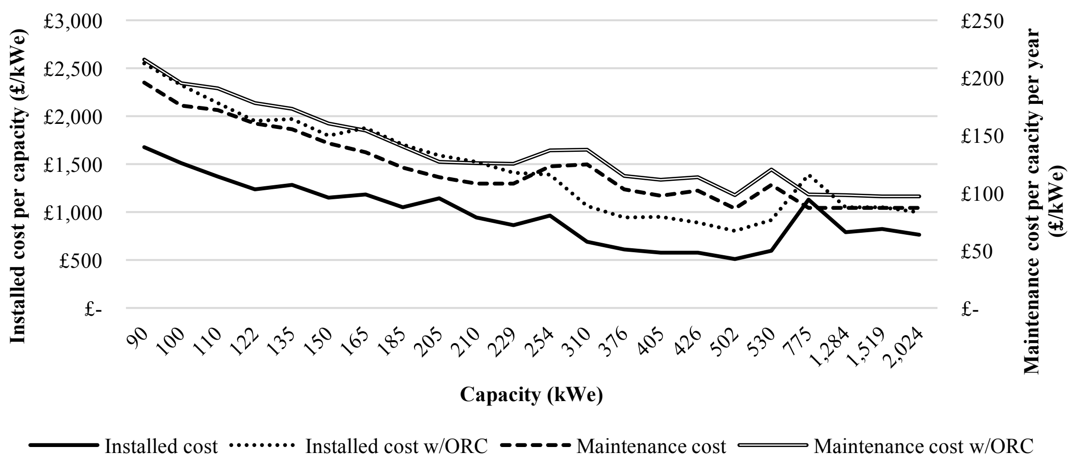

Figure 2. Indicative installed and maintenance cost curves for the technology portfolio.

4.2 Time

Time is represented in three levels: half-hour intervals, days and years. The TSO model is currently configured for the UK and therefore days consist of half-hourly intervals (i.e. 48, following the UK electricity market structure) and years consist of days. Groups of days are characterized into 'types' of days in order to simplify the solution. The optimisation can also consider periods consisting of one or several years,_the current configuration is set to optimise for the initial five years of operation. In summary, the time level structure allows energy prices and demands to be specific for each time interval of each day type of each year.

4.3 Resources

Energy resources are the inputs and outputs of the energy conversion technologies. Resource prices can be applied for each specific time interval if known or generalized into a higher time level. Electricity and fuel prices applied for this model were obtained from Imperial College's Real-time Energy Pricing Research [43]. Electricity has a specific price per half-hour interval for both import and export (following UK electricity market settlement periods), and is provided characterized into four types of days: summer (March to October) weekday, summer weekend, winter (November to February) weekday and winter weekend. Electricity prices are specific per region according to each Distribution Network Operator (DNO); import and export prices integrate electricity wholesale prices, distribution & transmission charges and other ancillary fees. Although for the purpose of this study prices are input in a half-hour resolution to represent wholesale electricity market prices for a large commercial consumer, they can be simplified into time-of-use (TOU), seasonal or fixed tariffs. Figure 3 shows an example of the electricity tariffs applied for the model from Imperial College's Real-time Energy Pricing Research. Fuels are considered to have a fixed price per year (

Table 3). Projections on the future prices of these energy resources according to Refs. [43] and [44] are also implemented in the model and shown in Table 2. Bio-methane is assumed to be sourced from the natural gas grid; it can be purchased at a premium and offset when injected into the gas grid at a different location. Specific fuel use may be constrained if desired. The case study will present an example of an optimisation in which gas use is constrained to natural gas, restricting the use of bio-methane.

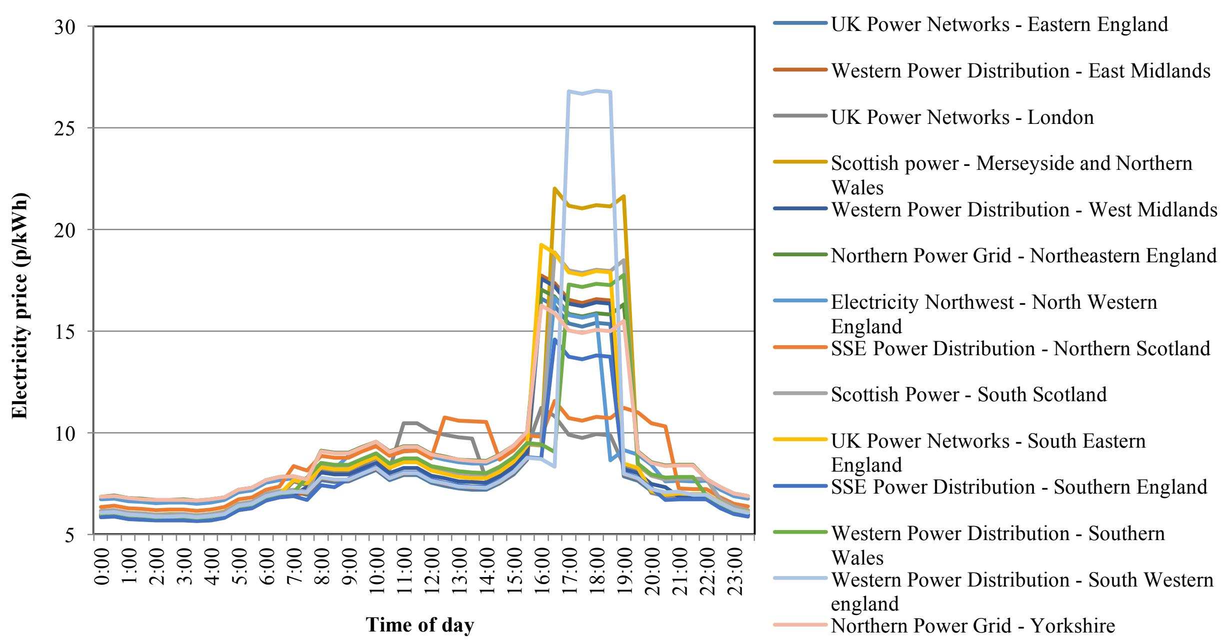

Figure 3. Example electricity tariffs (commercial) according to time of day and Distribution Network Operator (DNO); indicative for a weekday between March and October 2016 [43].

Table 2. Energy resource price projections adopted from Imperial College's Real-time Energy Pricing Research [43] *and* [44].

Energy resource	2016	2017	2018	2019	2020
Natural gas (p/kWh)	2.44	2.45	2.50	2.60	2.70
Bio-methane (p/kWh)	2.69	2.70	2.75	2.86	2.98
Diesel (p/kWh)	4.12	4.30	4.43	4.56	4.70
Electricity (% change)	-	-2.13%	-1.65%	+3.43%	+0.39%

Table 3. Pricing time-levels and carbon factors for the model's input resources.

Input resource	Pricing time-level	Carbon factor (gCO_2/kWh)
Natural Gas	Annually fixed	184.45
Bio-methane	Annually fixed	0.41

Electricity import	Half-hour interval specific	462.19[2]
Diesel	Annually fixed	244.35

4.4 Carbon

Fuel and electricity imports are also assigned a carbon factor which allows for the accounting of the resultant carbon emissions of burning these fuels as well as embodied emissions from electricity imported from the grid. These can be observed in

Table 3. Carbon factors configured for this model are sourced from DEFRA's Greenhouse Gas Conversion Factor Repository [45].

4.5 Objectives seeked

Decision makers for DES investments, such as facility or energy managers, building services engineers and even financial directors, usually seek to minimise cost and/or environmental impact when they consider implementing a distributed energy system. Energy resources and conversion processes influence these variables; both cost and emissions are function of decision variables in the problem. Therefore, the TSO model incorporates three separate objective functions. The first objective seeks solely to minimise the cost of satisfying energy demands for the period, which is the result of the sum of capital, carbon, operating and maintenance costs. The second objective function seeks solely to minimise the emissions that result from satisfying these demands for the period, which are the sum of emissions from burning fuels and embodied emissions from electricity imported from the grid. A third function seeks to minimise the weighted sum of the total cost and total emissions of the period with the purpose of understanding the trade-offs when combining a set of objectives. The values of the weights are user defined and can be manipulated to perform a sequence of optimisations to produce a Pareto-optimal set, or could reflect a view on carbon prices. This Pareto-optimal set can help evaluate in visual form the economic and environmental trade-offs of shifting weight from cost to environmental impact and vice versa.

5 Mathematical Formulation

A MILP modelling strategy was implemented to represent the problem, thus allowing the use of continuous, discrete and binary variables for the representation of the system. Binary variables, like in [17], were employed in the model to represent the selection (1 value), or not (0 value), of technologies

[2] For year 2016. The parameter for grid carbon intensity is set per year and is allowed to be defined for future years.

and operational levels. The mathematical formulation of the problem takes root in the energy balance of the system, which is presented below.

5.1 Energy balance

Equation (1) states that electricity demand, e^d, at each point in time (t, d, y) has to be equal to import, e^i, minus export, e^e, plus production from a technology (if selected), e^t, and the diesel generator (if available), e^g. Heat demand, h^d, also has to be satisfied by the production of heat through the selected technology, h^t, and the production of heat from the boiler, h^b, as shown in Equation (2). Heat production is allowed to exceed the demand.

$$e^d{}_{tdy} = e^i{}_{tdy} - e^e{}_{tdy} + e^t{}_{tdy} + e^g{}_{tdy} \tag{1}$$

$$h^d{}_{tdy} \leq h^t{}_{tdy} + h^b{}_{tdy} \tag{2}$$

Electricity and heat production from the technologies are the result of the multiplication of variable β and the parameters power output, O^p, or heat output, O^h, respectively, shown in Equations (3) and (4). Variable β represents the state at which both a technology has been selected and a level for that technology is active; it will only have a value of 1 for a single combination of technology j and level l in each point in time (t, d, y). Equation (5) restricts the output of the generator, e^g, to its capacity, CAP^{gen}.

$$e^t{}_{tdy} = \sum_j \sum_l \beta_{jltdy} \, O^p{}_{jl} \tag{3}$$

$$h^t{}_{tdy} = \sum_j \sum_l \beta_{jltdy} \, O^h{}_{jl} \tag{4}$$

$$e^g{}_{tdy} \leq CAP^{gen} \tag{5}$$

5.2 Technology and active level constraints

A technology constraint was used to restrict the selection technologies, τ, to a maximum of 1 for this study as shown in Equation (6). This value can be modified if there is a need to allow for the selection of more than one technology. For evaluation of the baseline case, in which no distributed energy system is yet implemented, this value is constrained to zero. It is infeasible to have more than one active level of operation at a specific point in time (e.g. the technology can not run at 50% and 100% operating levels at the same time). Therefore, a constraint, α, is applied for this restriction as shown in Equation (7).

$$\sum_{j} \tau_j \leq 1 \tag{6}$$

$$\sum_{l} \alpha_{ltdy} \leq 1 \tag{7}$$

5.3 Nonlinearity circumvention

The status represented by variable β at which one technology is selected and an operational level is active at a certain point in time could possibly be obtained by the product of the two binary variables τ and α. However, this results in non-linearity, which would require mixed-integer non-linear programming (MINLP) as opposed to MILP. MINLP however comes with drawbacks as it is difficult to solve and global optimality is not guaranteed [46]. A common MILP technique that allows maintaining linearity when using binary variables while providing the intended output for variable β is achieved through the set of Equations (8 – 10):

$$\beta_{jltdy} \geq \tau_j + \alpha_{ltdy} - 1 \tag{8}$$

$$\beta_{jltdy} \leq \tau_j \tag{9}$$

$$\beta_{jltdy} \leq \alpha_{ltdy} \tag{10}$$

5.4 Resource use

The rate of gas use, G, at a certain point in time (*t, d, y*) is the sum of gas use from the selected technology and the gas boiler (Equation 11). The gas used by the technology is calculated through the division of electricity production at that point in time by the efficiency, $\eta^t_{(e)}$, of that technology $\boldsymbol{j}$ and operational level $\boldsymbol{l}$. The gas used by the boiler is simply the division of the heat production from the boiler by the efficiency of the same, η^b. The gas use can consist of a mix of bio-methane (BM) and natural gas (NG) (Equation 12). The rate of diesel use, D, is calculated by dividing the electricity generation at a specific interval by the efficiency of the generator, η^g, as shown in Equation (13).

$$G_{tdy} = \sum_{j} \sum_{l} \left(\frac{\beta_{jltdy}\, O^p{}_{jl}}{\eta^t_{(e)}{}_{jl}} \right) + \left(\frac{h^b{}_{tdy}}{\eta^b} \right) \tag{11}$$

$$G_{tdy} = BM_{tdy} + NG_{tdy} \tag{12}$$

$$D_{tdy} = \frac{e^{g}{}_{tdy}}{\eta^{g}} \tag{13}$$

5.5 Emissions

The carbon factors of natural gas, bio-methane, diesel and electricity are used in order to calculate the carbon emissions for each time interval. Fuel and electricity consumption rates (kW) are multiplied by the duration of the time interval, σ, in order to represent the total quantity of resource used (kWh). Carbon emissions, GHG (tCO_2e), can then be obtained by the multiplication of each energy resource use with its corresponding carbon factor, F(tCO_2e/kWh), and are represented in terms of tonnes of carbon dioxide equivalent.

$$GHG_{tdy} = \left(NG_{tdy}\, F^{NG} + BM_{tdy}\, F^{BM} + D_{tdy}\, F^{D} + e^{i}{}_{tdy}\, F^{E}{}_{y}\right) \sigma \tag{14}$$

5.6 Costs

The TSO model considers capital, carbon, operating and maintenance costs when performing the optimisation. Costs are individually brought by summation to the highest time level (years) from the level at which they initially exist. Then, using a present value multiplier, PVM, they are again individually summed to provide the cost for the entire period of analysis. A multiplier, dt, is implemented as shown in Equations (17-18, 21) to account for the number of days each day-type occurs in a year. A duration multiplier, σ, is used to transform rates of use into total resource used each time interval.

The capital cost of the acquisition and installation of the energy technology is spread throughout the lifetime of the project (assumed to be 15 years) into annual costs, providing the possibility of analysing shorter time periods without compromising the functionality of the optimization. The resultant capital cost for the period, C^{c}, consists of the sum of the annualised capital cost of the technologies selected, AC^{c}, for each year of the period brought to present value (Equation 15).

$$C^{c} = \sum_{y} \sum_{j} AC^{c}{}_{j}\, \tau_j\, PVM_y \tag{15}$$

The resultant maintenance cost for the period, C^{m}, consists of the sum of the annual maintenance cost for the technologies selected, AC^{m}, for each year of the period brought to present value.

$$C^{m} = \sum_{y} \sum_{j} AC^{m}{}_{j}\, \tau_j\, PVM_y \tag{16}$$

The resultant operational cost, C^o, is obtained from the fuel and electricity expense and a deduction from the revenue from exporting electricity to the grid. Each resource use is multiplied by its specific resource price, p, at each point in time before being brought to the highest time level. These are summed for each time interval, day and year of the period.

$$C^o = \sum_y \sum_d \sum_t \Big(NG_{tdy}\, {p^{NG}}_y + BM_{tdy}\, {p^{BM}}_y + D_{tdy} {p^D}_y + {e^i}_{tdy}\, {p^{e,import}}_{tdy} - {e^e}_{tdy}\, {p^{e,export}}_{tdy} \Big)\, dt_d\, PVM_y\, \sigma \tag{17}$$

The resultant cost incurred from carbon emissions for the period, C^{GHG}, is calculated from summation of the amount of emissions for each interval while again using the multipliers, dt and PVM, to account for numbers of days for day-types and to bring to present value. The carbon price, p^c, is the multiplier used to convert emissions into monetary value.

$$C^{GHG} = \sum_y \sum_d \sum_t GHG_{tdy}\, dt_d\, PVM_y\, {p^c}_y \tag{18}$$

The total cost incurred for the period, C^t, is finally the sum of all other costs:

$$C^t = C^c + C^m + C^o + C^{GHG} \tag{19}$$

5.7 Objective Functions

Three objective functions can be solved by the model. For all three, the objective is to minimise the value of the function: f_1 is equivalent to the total cost for the period (Equation 20), f_2 is equivalent to the total carbon emissions of the period (Equation 21) and f_3 is a weighed sum of total cost and total emissions (Equation 22). In the third equation ω_1 and ω_2 are user defined parameters that give weight to total cost and total emissions respectively.

$$f_1 = C^t = C^c + C^m + C^o + C^{GHG} \tag{20}$$

$$f_2 = GHG^{total} = \sum_y \sum_d \sum_t GHG_{tdy}\, dt_d \tag{21}$$

$$f_3 = \omega_1\, C^t + \omega_2\, GHG^{total} \tag{22}$$

6 Case Study

This section demonstrates the capabilities of the TSO model through the case study of a commercial building from a large food retailer in the UK. Table 4 provides some of the characteristics of the building used for this case study.

Table 4. Characteristics of building used for the case study.

Avg. heat demand (kW)	**181**
Avg. electricity demand (kW)	419
HPR	0.43
DNO region	Western Power Distribution
Floor space (m^2)	4,550

6.1 Inputs and assumptions

The TSO model was populated with relevant parameters for the analysis in addition to the technology database. This set of data which drives the model consisted of the energy demands of the building and energy resource costs. The TSO model required heat and electricity demands for each time interval of the period of study. Historic metered demands for this building were available and provided the consumption of heat and electricity of the building for each half-hour of the year 2014. These values were converted from total energy use for each interval (kWh) into rates of energy use (kW).

As mentioned in Section 4.2, days may be grouped into types in order to simplify the computation of the solution. This avoids solving for each individual day by instead generalizing into day-types. For this case study, the model was solved in separate instances using different grouping arrangements. The first one (Grouping A) consisted of the same four day types used by the electricity pricing structure (summer weekday, summer weekend, winter weekday and winter weekend). The second (Grouping B) consisted of 39 day-types, grouping into 13 four-week periods in a year and distinguishing between weekdays, Saturdays and Sundays. Grouping into 13 four-week periods provides a more accurate structure than division into the 12 months of the year as months have variable number of days and begin at different points of the week; organisational budgeting is also often split into four-week periods. For orientation, Week 1 begins with the first Monday of the year.

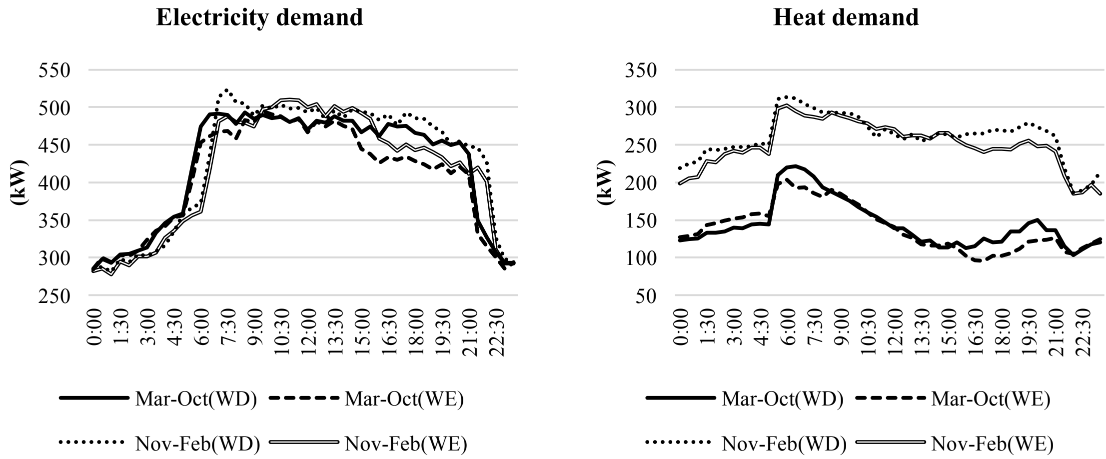

Figure 4. Example load profiles of the commercial building analysed in this case study, represented under Grouping A. (Left) electricity demand and (right) heat demand. Weekdays (WD), weekends (WE).

Demands were analysed to identify how representative each simplification is. An analysis of the standard deviation of the demands under different grouping arrangements was made. Table 5 presents the average of the standard deviation of the historical demand figures. Taking Grouping A as an example, the standard deviation was calculated for each half hour interval of the historical demand data corresponding to the days that fit within that grouping. This provided the standard deviation of demand for each half hour for each of the four day types under that grouping. The values of the standard deviation for each half hour and day were then averaged to provide an average for the grouping arrangement representative of the whole year. A low average for the standard deviation indicates that there is little variation between actual demands being generalised into a certain grouping. A high value would identify a high variation between the demands, in which case a different grouping may better characterize the demand data. Table 5 demonstrates that there is a slight benefit obtained from Grouping A as opposed to generalising demand into a single day type. However, a greater benefit can be perceived with Grouping B. As evidenced through an energy audit from a case study of a similar commercial building [47] and through the iteration of this analysis, it was confirmed that energy demands were more similar between a Saturday and a weekday than a Sunday. Hence, the reason to differentiate Saturdays and Sundays in Grouping B.

Table 5. Average standard deviation of demands under different grouping arrangements.

	1 day (annual average)	**4 days (Grouping A)**	**39 days (Grouping B)**

Electricity	15.1	13.5	8.2
Heat	30.3	27.0	16.5

Section 4.3 spoke about the energy pricing included in the model database and used for this case study. The building lies within the region operated by Western Power Distribution (South Wales) and electricity prices were allocated accordingly. The carbon price applied for this study was the cost of the Carbon Reduction Commitment (CRC) allowance[3], £16.10/tCO_2e in 2015 [48] and was assumed to increase 2.5% each year. The carbon intensity of the grid was assumed to decrease 1.5% per annum. Discount rate for the project was assumed to be 8%.

A baseline case representing the business-as-usual scenario without an energy system, simply importing electricity from the grid and generating its heat through the already-present gas boiler, is used as comparison against the optimisation results. The model was solved for five instances in total, represented below as scenarios, providing results for the minimum cost and minimum GHG emissions optimisations:

- **Scenario 1**: Grouping A and entire portfolio of technologies;
- **Scenario 2**: Grouping A and restricted portfolio of technologies (CHP only);
- **Scenario 3**: Grouping B and entire portfolio of technologies;
- **Scenario 4**: Grouping B and restricted portfolio of technologies (CHP only);
- **Scenario 5**: Grouping B and entire portfolio of technologies, restricted to natural gas use.

6.2 Results

This case study's optimisation results for each of the scenarios are summarised in Table 6, which shows the key economic and sustainability indicators. Because the period of optimisation consisted of five years, cash flows after the fifth year were assumed to be the average of the first years and a cash flow analysis was carried out in order to calculate the net present value (NPV) of the investment benefit, return of investment (ROI), internal rate of return (IRR) and payback period. The lifetime of the projects was assumed to be 15 years.

Two different baselines are obtained for the two different day grouping arrangements. These are calculated by constraining electricity provision through grid import and heat provision through gas import and boiler use for each of the day-types, scaled to a full year. Scenarios 1 and 3 which involve different time granularities, or day groupings, under the full portfolio of technologies enabled selected the same

[3] The CRC is a scheme set by the UK government to improve energy efficiency and reduce emissions of large energy users not covered by the Emissions Trading Scheme (ETS) or Climate Change Agreements (CCAs) [48].

technology capacities for both the cost and emissions optimisations and provide similar financial indicators. The same happens for Scenarios 2 and 4 under a CHP-only constrained portfolio. However, for this case a different capacity is selected for the emission reduction optimisation in Scenario 4. For Scenarios 1 through 4, the optimisation tends to select higher capacity units to satisfy all the building demands by on-site generation. This happens because it utilises bio-methane as a fuel which makes on-site generation less carbon-intensive than importing electricity from the grid. Scenario 5, which is similar to Scenario 4 but with fuel use restricted to natural gas, also selects a 400 kW_e capacity unit coupled to an ORC for the cost optimisation. For the emissions optimisation it does not decide to size up as in other scenarios as it does not predict as much carbon benefit through on-site generation with natural gas as other scenarios do with bio-methane. All scenarios and optimisations result in lower costs than the baseline in this case, otherwise the model would not decide to select a technology for the cost optimisations. The natural gas to bio-methane ratio is provided on the right-most column of Table 6. On all scenarios except 5 (where fuel use is constrained to natural gas) and the baseline, the optimisation results select bio-methane entirely as the fuel. The results demonstrate that the premium in bio-methane price compared to natural gas for this case study is not high enough as for the model to select natural gas as the fuel in the cost optimisation. This is due to the fact that this premium cost is offset by the carbon cost avoided through the use of bio-methane. Possibly, a different result would be obtained if the premium of bio-methane over natural gas was higher or the cost of the carbon tax/allowance was lower.

Table 6. Summary of the key results of each scenario and optimisation.

	Optimisation Objective	**CHP capacity**	**Cost**	**Emissions**	**Capital Investment**	**NPV benefit**	**Abatement cost (£/tCO2e)**	**ROI**	**IRR**	**Payback (yr)**	**NG/BM ratio**
Scenarios 1 and 2	Baseline	-	£1,874,280	9859	-	-	-	-		-	1
Scenario 1	Cost	400 kW_e (w/ORC)	£1,394,796 (-26%)	382 (-96.12%)	-£386,327	£957,239	-48.11	248%	40%	4	0
	Emissions	530 kW_e (w/ORC)	£1,626,167 (-13%)	21 (-99.79%)	-£485,141	£470,310	-22.83	97%	21%	6	0
Scenario 2	Cost	500 kW_e	£1,514,754 (-19%)	818 (-91.70%)	-£258,000	£720,467	-37.16	279%	43%	3	0
	Emissions	530 kW_e	£1,614,107 (-14%)	22 (-99.77%)	-£316,940	£523,304	-24.06	165%	30%	5	0
Scenarios 3,4 and 5	Baseline	-	£1,875,649	9882	-	-	-	-		-	1
Scenario 3	Cost	400 kW_e (w/ORC)	£1,398,100 (-25%)	279 (-97.17%)	-£386,327	£954,249	-47.29	247%	39%	4	0
	Emissions	530 kW_e	£1,510,956	18	-£485,141	£730,668	-34.59	151%	28%	5	0

		(w/ORC)	(-19%)	(-99.82%)							
Scenario 4	Cost	500 kW_e	£1,497,550 (-20.15%)	581 (-94.12%)	-£258,000	£757,248	-38.13	294%	44%	3	0
	Emissions	770 kW_e	£1,867,226 (-0.4%)	21 (-99.79%)	-£875,000	£23,749	1.52	3%	8%	15	0
Scenario 5	Cost	400 kW_e (w/ORC)	£1,504,419 (-20%)	7611 (-22.98%)	-£386,327	£789,425	-95.58	204%	34%	4	1
	Emissions	400 kW_e (w/ORC)	£1,523,328 (-19%)	7579 (-23.31%)	-£386,327	£751,789	-89.96	195%	33%	4	1

Figure 5 presents a graphical visualisation of the cost and emission reductions specific to each of the scenarios and optimisations. Baseline points can be observed on the upper left corner, while emissions optimisation points are located on the right edge for most of the scenarios. Scenario 5 differs from other scenarios as it uses natural gas as a fuel for the CHP unit and does not achieve emission reductions beyond 30%.

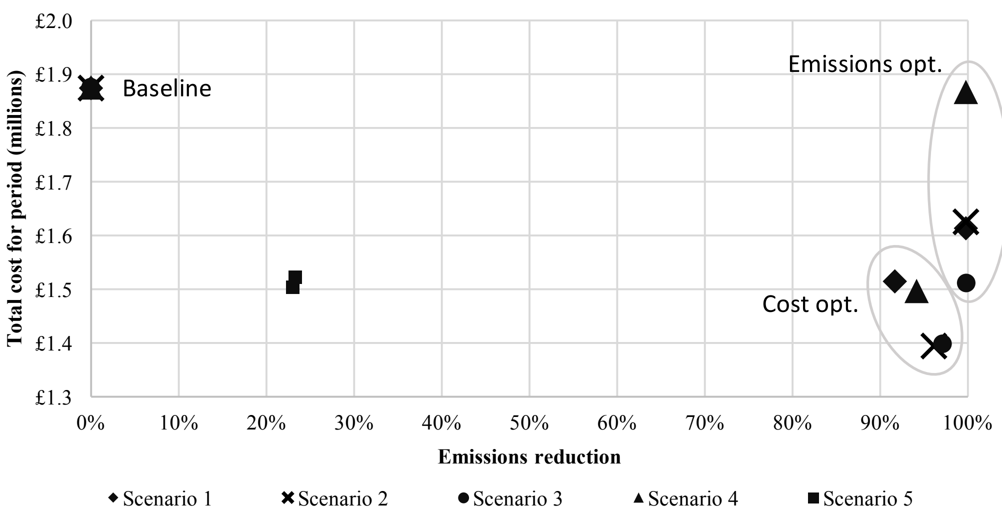

Figure 5. Costs for period and emissions reductions for each of the scenarios.

It can be observed that for all the scenarios both cost and emission optimisations result in a lower cost for the period whilst achieving emission reductions. Also for all of the scenarios, there appears to be a trade-off between optimising for cost and optimising for emissions reduction. Minimum cost optimisations can achieve 90+% emission reduction in almost all scenarios except Scenario 5. However, in order to achieve higher emission reductions as in the emission optimisation case, the cost for the period increases. This occurs in part because the model may decide to size-up the technology in comparison to the cost optimisation, as well as because on-site generation takes place irrespective of whether it is more economically convenient than importing electricity from the grid and generating heat through the boiler at a certain interval. Figure 6 demonstrates this trade-off between cost and emissions through the results of a sequence of optimisations shifting weight between objectives in the multi-objective function (Equation 22). The example shown represents Scenario 2.

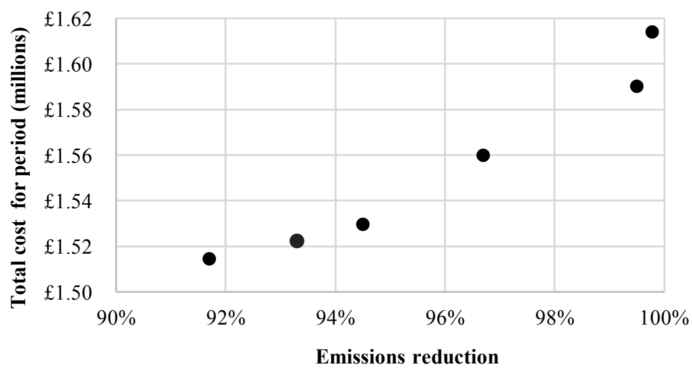

Figure 6. Results from sequence of optimisations shifting weight between cost objective and emissions objective (Scenario 2).

Figure 7 demonstrates the optimal operational schedule for the selected unit and each time period (the example shown represents the cost optimisation of Scenario 4). The numbers represent the percentage load of the units (e.g. 0 = OFF and 100 = full load). It can be observed that the model optimisation indicates that the unit selected should run at lower levels or be OFF during Saturdays and Sundays. This possibly occurs because electricity demands as well as prices are lower during the weekends than during the weekdays. The cost benefit from on-site generation during those intervals is not as attractive and competitively matched through electricity import.

	WD													SAT													SUN												
	Weeks 1-4 WD	Weeks 5-8 WD	Weeks 9-12 WD	Weeks 13-16 WD	Weeks 17-20 WD	Weeks 21-24 WD	Weeks 25-28 WD	Weeks 29-32 WD	Weeks 33-36 WD	Weeks 37-40 WD	Weeks 41-44 WD	Weeks 45-48 WD	Weeks 49-52 WD	Weeks 1-4 SAT	Weeks 5-8 SAT	Weeks 9-12 SAT	Weeks 13-16 SAT	Weeks 17-20 SAT	Weeks 21-24 SAT	Weeks 25-28 SAT	Weeks 29-32 SAT	Weeks 33-36 SAT	Weeks 37-40 SAT	Weeks 41-44 SAT	Weeks 45-48 SAT	Weeks 49-52 SAT	Weeks 1-4 SUN	Weeks 5-8 SUN	Weeks 9-12 SUN	Weeks 13-16 SUN	Weeks 17-20 SUN	Weeks 21-24 SUN	Weeks 25-28 SUN	Weeks 29-32 SUN	Weeks 33-36 SUN	Weeks 37-40 SUN	Weeks 41-44 SUN	Weeks 45-48 SUN	Weeks 49-52 SUN
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19:30	100	100	100	90	90	90	90	100	90	90	90	90	90	90	90	90	90	80	80	80	90	80	80	90	90	90	80	80	75	75	75	75	75	75	80	80	75	80	80
20:00	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	80	80	80	90	80	80	90	90	80	75	75	75	75	75	75	75	75	80	80	70	75	80
20:30	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	80	90	90	80	90	90	90	75	75	75	75	75	75	75	75	80	80	75	75	75
21:00	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	80	90	80	90	80	80	80	90	90	75	75	70	75	75	75	75	75	75	75	70	75	75
21:30	90	90	90	90	90	90	90	90	90	90	90	90	90	90	90	80	90	80	90	90	90	90	90	90	90	80	75	75	75	75	75	75	75	75	80	80	70	75	80
22:00	80	80	80	80	80	80	80	90	80	90	80	80	80	80	80	80	80	80	80	80	90	80	80	80	80	80	75	75	70	75	75	75	75	75	75	75	70	75	75
22:30	70	70	60	60	60	70	60	70	70	75	70	70	60	70	70	60	60	60	60	60	0	60	70	70	70	60	70	60	50	60	60	60	60	0	60	70	60	60	60
23:00	60	60	60	60	60	60	60	0	60	70	60	60	60	60	60	50	60	60	60	0	0	60	60	60	60	60	60	60	50	60	60	60	0	0	60	0	50	50	60
23:30	60	60	50	60	60	60	60	0	60	60	60	60	60	60	60	50	50	50	50	0	0	0	0	0	60	60	60	60	50	50	50	50	0	0	0	50	50	60	60

Figure 7. Example operational schedule for minimum cost optimisation (Scenario 4, first year).

Figure 8 shows an example of the difference in operating costs through the selected operation versus the baseline, in this case also for Scenario 4 as in Figure 7. The numbers represent the savings amount (£) for each specific time interval. It can be observed that the model makes a decision to run the selected technology at full load in periods in which it can obtain a greater benefit in terms of cost and decides to turn it off when importing electricity from the grid and generating heat on-site is more convenient.

	WD													SAT													SUN												
	Weeks 1-4 WD	Weeks 5-8 WD	Weeks 9-12 WD	Weeks 13-16 WD	Weeks 17-20 WD	Weeks 21-24 WD	Weeks 25-28 WD	Weeks 29-32 WD	Weeks 33-36 WD	Weeks 37-40 WD	Weeks 41-44 WD	Weeks 45-48 WD	Weeks 49-52 WD	Weeks 1-4 SAT	Weeks 5-8 SAT	Weeks 9-12 SAT	Weeks 13-16 SAT	Weeks 17-20 SAT	Weeks 21-24 SAT	Weeks 25-28 SAT	Weeks 29-32 SAT	Weeks 33-36 SAT	Weeks 37-40 SAT	Weeks 41-44 SAT	Weeks 45-48 SAT	Weeks 49-52 SAT	Weeks 1-4 SUN	Weeks 5-8 SUN	Weeks 9-12 SUN	Weeks 13-16 SUN	Weeks 17-20 SUN	Weeks 21-24 SUN	Weeks 25-28 SUN	Weeks 29-32 SUN	Weeks 33-36 SUN	Weeks 37-40 SUN	Weeks 41-44 SUN	Weeks 45-48 SUN	Weeks 49-52 SUN
0:00	3.6	4.2	2.4	2.2	2.1	1.7	0.5	0.0	0.7	1.2	1.4	3.0	3.7	2.8	3.8	3.1	2.3	2.0	1.9	0.4	0.0	0.8	0.7	1.0	2.9	3.9	4.3	4.2	2.6	2.5	2.0	1.6	0.3	0.0	0.8	0.9	1.1	3.0	4.4
0:30	3.6	4.3	2.6	2.3	2.2	1.9	0.6	0.0	0.8	1.3	1.5	3.3	3.8	3.0	4.2	2.9	2.0	1.7	1.5	0.0	0.0	0.5	0.4	0.8	3.2	4.2	4.5	4.5	2.3	2.1	1.7	1.3	0.2	0.0	0.5	0.8	0.6	3.5	4.5
1:00	3.4	3.9	2.5	2.1	2.0	1.5	0.3	0.0	0.5	1.1	1.2	3.1	3.6	3.2	4.3	3.1	2.2	1.9	1.6	0.0	0.0	0.6	0.5	1.0	3.4	4.4	4.8	4.4	2.6	2.2	1.8	1.4	0.4	0.0	0.6	0.7	0.6	3.6	4.6
1:30	3.7	4.2	2.7	2.4	2.2	1.7	0.4	0.0	0.6	1.0	1.3	3.3	3.7	3.0	4.4	3.5	2.6	2.1	1.7	0.3	0.0	0.7	0.5	1.2	3.1	4.1	4.7	4.4	3.0	2.5	2.1	1.5	0.5	0.0	0.7	0.8	0.8	3.4	4.6
2:00	3.4	4.1	2.6	2.2	2.0	1.5	0.3	0.0	0.5	1.0	1.3	3.2	3.6	2.9	4.4	3.4	2.4	2.0	1.7	0.4	0.0	0.6	0.4	1.2	3.3	3.8	4.4	4.3	2.9	2.4	2.0	1.4	0.6	0.0	0.6	0.9	1.5	3.4	4.5
2:30	3.4	4.1	2.8	2.3	2.0	1.5	0.4	0.0	0.5	1.0	1.3	3.1	3.5	4.1	4.7	3.2	2.2	1.8	1.4	0.5	0.0	0.4	0.2	1.0	3.1	4.0	4.5	4.4	2.9	2.3	1.8	1.2	0.4	0.0	0.5	0.8	1.4	3.4	4.5
3:00	2.8	3.5	2.9	2.5	2.1	1.5	0.6	0.0	0.6	1.1	1.5	2.6	2.9	4.1	4.3	3.2	2.2	1.9	1.4	0.6	0.0	0.5	0.2	1.0	3.0	4.0	4.3	4.1	3.0	2.3	1.9	1.1	0.3	0.0	0.6	0.7	1.7	3.2	4.2
3:30	2.6	3.3	2.4	2.3	2.0	1.4	0.6	0.0	0.6	1.1	1.5	2.4	2.7	3.2	4.1	2.9	2.1	1.8	0.6	0.1	0.0	0.4	0.5	0.1	2.7	4.2	3.9	3.9	2.8	2.2	1.8	1.3	0.7	0.0	0.6	0.1	2.4	2.9	3.3
4:00	2.9	3.4	3.0	2.6	2.1	1.6	0.8	0.0	0.7	1.2	1.5	2.6	2.9	3.9	4.1	3.3	2.1	1.8	1.0	0.5	0.0	0.4	1.0	1.1	2.7	3.5	3.7	3.7	2.8	2.2	1.7	1.0	0.5	0.0	0.7	0.6	1.3	2.7	3.8
4:30	2.7	3.3	3.2	2.9	2.4	1.9	1.0	0.2	1.0	1.5	1.9	2.3	2.6	2.9	3.5	3.3	2.2	1.8	1.1	0.5	0.0	0.6	0.8	0.9	1.9	2.9	3.1	3.0	2.8	2.1	1.7	0.9	0.4	0.0	0.6	0.5	1.3	2.1	3.2
5:00	4.4	5.2	3.6	3.7	3.3	2.7	2.1	1.0	2.0	2.8	2.9	4.2	4.6	2.6	3.2	3.1	2.1	1.7	0.6	0.2	0.0	0.8	0.7	0.1	1.8	3.3	3.3	3.1	2.7	2.2	1.9	1.5	0.7	0.0	0.4	0.1	1.9	2.1	3.0
5:30	5.6	5.6	5.0	5.1	5.0	3.3	3.7	2.2	2.9	4.4	5.0	5.5	5.2	4.3	4.5	3.8	2.2	1.9	1.7	1.7	0.2	1.4	1.7	2.6	4.2	3.9	3.8	3.6	2.9	3.0	3.0	1.5	1.5	0.2	1.4	2.0	2.2	3.9	4.4
6:00	7.8	7.6	5.9	5.9	6.0	4.3	4.5	3.2	4.2	5.8	6.3	7.5	7.5	4.8	4.9	3.6	2.3	1.7	2.0	1.7	0.1	1.5	1.7	2.7	4.8	4.8	4.0	4.1	2.8	2.7	2.9	1.3	1.3	0.4	1.3	1.6	2.4	4.3	4.7
6:30	8.6	8.6	6.6	6.5	6.8	5.3	5.3	4.6	5.3	6.1	6.8	8.2	8.1	5.2	5.5	3.8	2.1	1.6	2.2	1.8	1.0	2.0	1.9	2.0	5.1	5.3	4.7	4.7	2.5	2.8	3.3	1.5	1.5	0.3	0.9	0.8	1.7	5.1	5.5
7:00	9.4	9.6	7.7	7.3	7.1	5.8	5.4	4.8	5.9	6.5	6.8	9.1	9.1	5.0	5.2	4.3	2.7	2.3	2.4	2.0	1.3	2.7	2.4	3.0	4.8	5.1	4.3	4.2	3.2	3.7	3.7	2.0	1.4	0.5	1.6	1.8	2.7	4.4	5.2
7:30	13.9	14.1	9.5	9.1	9.0	7.7	7.0	6.4	7.7	8.3	8.5	13.6	13.7	5.9	6.1	4.2	2.6	2.2	2.2	1.7	1.0	2.4	2.2	2.9	5.5	6.0	5.2	5.1	3.1	3.6	3.5	1.9	1.4	0.5	1.6	1.7	2.7	5.0	5.9
8:00	13.0	13.2	11.6	11.1	11.1	9.6	8.9	8.1	9.3	10.4	10.7	12.6	12.8	6.1	6.1	3.8	2.8	2.3	1.4	0.9	0.6	1.8	1.8	1.7	5.6	6.2	6.1	6.1	3.8	2.9	3.2	1.7	1.0	0.2	1.1	1.0	1.8	5.2	6.2
8:30	13.0	13.1	11.1	10.8	10.6	9.2	8.2	7.5	8.9	9.5	10.3	12.6	12.7	6.1	6.1	4.1	3.4	3.5	1.9	1.3	0.2	1.8	1.9	2.4	5.7	6.1	6.3	6.3	4.2	3.5	2.8	1.9	1.3	0.4	1.3	2.0	2.9	4.9	6.1
9:00	12.8	12.6	10.9	10.4	10.2	8.9	7.9	7.2	8.4	9.1	9.7	12.1	12.1	7.9	7.7	4.4	3.5	3.4	2.2	1.5	0.5	1.9	2.2	2.7	7.4	7.6	7.9	7.8	4.2	3.7	3.0	2.0	1.5	0.5	1.7	2.6	3.3	6.4	7.7
9:30	12.6	12.7	12.1	11.7	11.4	10.1	9.0	8.5	9.6	10.5	11.0	12.1	12.4	7.4	7.7	4.2	3.2	2.4	1.4	1.1	0.1	1.6	2.2	2.2	6.8	7.6	7.6	7.6	4.1	3.2	3.4	2.3	1.4	0.4	1.5	1.9	3.9	6.3	7.6
10:00	13.2	13.3	12.8	12.5	11.8	10.4	9.6	9.3	9.9	10.8	11.7	12.4	12.6	8.2	8.3	4.3	3.4	3.2	1.9	1.2	0.4	1.5	1.7	2.0	7.7	8.2	8.4	7.9	4.1	3.3	2.9	2.0	1.3	0.7	1.8	2.4	2.8	7.0	8.0
10:30	12.5	12.9	11.4	11.3	10.1	8.8	8.1	7.7	8.1	9.1	10.2	12.2	12.4	7.9	8.0	4.0	3.0	2.9	1.4	0.8	0.2	1.1	1.5	1.6	7.3	7.8	8.1	8.0	3.8	3.0	2.4	1.7	0.9	0.3	1.5	2.1	3.0	6.6	7.9
11:00	10.2	11.2	12.0	12.1	10.8	9.4	8.5	8.4	8.9	9.5	10.3	10.4	9.8	7.9	8.0	3.9	3.0	1.9	0.7	0.8	0.0	0.8	1.7	1.8	7.1	7.8	8.2	8.1	3.8	2.9	3.0	2.0	0.6	0.1	1.0	1.6	3.3	6.7	8.0
11:30	10.3	10.9	12.0	11.8	10.5	9.2	8.4	8.4	8.8	9.3	10.3	10.0	9.9	8.3	8.3	3.6	2.7	2.4	0.8	0.2	0.0	0.6	1.0	1.1	7.6	8.2	8.4	8.4	3.6	3.0	1.9	1.2	0.3	0.0	0.9	1.3	2.2	6.9	8.4
12:00	10.0	10.7	10.5	10.5	8.9	7.7	6.9	6.9	7.3	7.9	8.9	9.6	9.5	8.9	9.0	3.4	2.5	2.1	0.8	0.0	0.0	0.2	0.9	0.9	7.9	8.6	9.0	8.6	3.3	2.7	1.7	0.9	0.0	0.0	0.5	0.9	1.9	7.3	8.6
12:30	9.5	10.4	10.3	10.3	8.5	7.6	6.5	6.7	7.1	7.2	8.9	9.1	9.2	9.0	9.3	3.3	2.4	1.0	0.0	0.0	0.0	0.1	0.9	1.0	8.1	8.8	9.3	9.0	3.2	2.4	2.2	1.2	0.0	0.0	0.3	0.0	2.3	7.4	9.0
13:00	9.2	9.5	9.7	9.2	8.0	6.6	5.7	5.8	6.6	6.6	8.1	8.4	8.8	9.1	8.8	3.2	2.4	1.7	0.7	0.0	0.0	0.0	0.1	0.5	8.4	8.9	9.0	8.6	3.1	2.4	1.2	0.2	0.0	0.0	0.1	0.3	1.7	7.1	8.7
13:30	9.0	9.2	9.6	9.0	7.6	6.3	5.5	5.8	6.4	6.2	8.0	8.5	8.6	8.4	8.6	3.2	2.3	1.5	0.5	0.0	0.0	0.0	0.0	0.4	7.4	8.2	8.5	8.2	3.0	2.4	0.9	0.0	0.0	0.0	0.0	0.0	1.5	6.8	8.1
14:00	8.9	9.7	9.6	9.3	7.7	6.7	5.5	5.8	6.3	5.8	7.8	8.2	8.5	8.2	8.3	3.2	2.4	0.4	0.0	0.0	0.0	0.0	0.5	0.6	7.0	8.1	8.2	7.9	3.2	2.3	1.5	0.6	0.0	0.0	0.0	0.0	2.1	6.6	7.6
14:30	9.2	9.6	10.4	10.0	8.4	7.2	6.6	6.8	7.2	7.1	8.9	8.4	8.7	8.3	8.1	3.2	2.3	1.3	0.3	0.0	0.0	0.0	0.0	0.4	7.0	7.9	8.4	7.9	3.3	2.4	0.9	0.0	0.0	0.0	0.0	0.0	1.6	6.7	7.7
15:00	10.4	10.9	11.6	11.3	9.8	8.5	7.9	8.2	8.6	8.4	10.2	9.7	9.5	10.1	10.0	3.3	2.5	1.6	0.3	0.0	0.0	0.0	0.0	0.8	9.0	9.8	10.4	9.7	3.5	2.4	1.0	0.0	0.0	0.0	0.0	0.3	1.9	8.3	9.3
15:30	10.6	10.9	14.1	13.7	12.2	11.3	10.1	10.4	10.9	10.2	12.3	9.9	9.7	9.4	9.3	3.6	2.4	0.6	0.2	0.0	0.0	0.0	0.3	1.0	8.1	9.1	9.3	8.9	3.1	2.5	1.8	0.5	0.0	0.0	0.1	0.0	2.2	7.4	8.3
16:00	10.8	10.9	13.4	13.0	11.4	10.2	9.5	9.9	10.1	10.0	11.5	9.7	10.0	10.5	10.0	4.7	3.2	2.1	1.9	0.7	0.5	1.1	1.0	1.7	9.4	10.0	8.7	8.1	3.3	3.6	2.4	0.9	0.8	0.5	1.0	1.0	1.9	8.0	9.3
16:30	11.2	11.5	12.5	11.9	10.4	9.2	8.7	9.0	9.2	9.1	10.7	10.5	10.4	12.9	12.8	4.9	3.2	2.0	1.5	0.7	0.7	1.3	1.2	1.7	11.8	12.4	9.8	9.5	2.6	3.7	2.4	1.0	0.8	0.5	0.9	1.2	1.9	9.8	10.8
17:00	126.9	127.5	34.1	33.3	31.7	31.1	29.9	30.2	30.3	29.9	32.3	126.4	126.4	14.2	13.9	4.8	3.2	1.1	0.9	1.0	1.1	0.9	1.5	2.3	12.7	12.8	10.7	10.7	3.4	3.5	3.1	2.1	0.4	0.4	0.7	0.4	0.0	11.4	11.9
17:30	128.8	129.1	33.9	33.0	31.6	30.3	29.8	30.2	30.3	30.4	32.3	128.1	127.5	14.2	14.0	4.9	3.2	2.1	1.5	0.9	0.8	1.4	1.5	1.9	13.0	13.1	10.5	10.5	2.6	3.9	2.5	1.3	0.7	0.6	0.8	1.2	2.0	10.8	12.0
18:00	128.9	129.9	34.6	33.6	32.0	30.7	30.3	30.7	30.7	30.8	32.9	127.7	127.2	13.4	13.5	4.6	3.0	1.8	1.3	0.5	0.4	1.0	1.2	1.8	11.9	12.0	10.0	9.9	2.3	3.8	2.3	0.9	0.5	0.3	0.7	0.9	1.7	10.2	11.2
18:30	37.3	38.6	34.8	34.0	32.3	31.4	30.1	30.4	30.6	30.3	32.9	37.0	36.7	12.1	12.1	5.0	3.0	1.3	0.9	1.1	0.8	0.9	1.6	2.8	10.7	10.7	8.9	9.0	3.3	3.7	3.2	2.0	0.2	0.3	1.2	0.4	0.0	9.5	10.0
19:00	36.6	37.0	36.0	35.1	33.5	32.1	31.2	31.5	31.8	32.5	34.2	35.6	35.5	8.6	8.3	5.2	3.4	2.4	1.8	0.9	1.0	1.5	1.8	2.6	7.3	7.8	6.3	6.2	2.7	4.0	2.9	1.6	0.8	0.7	1.5	1.8	2.0	6.5	7.6
19:30	11.8	12.3	11.2	10.5	9.4	7.9	7.0	6.9	7.7	8.1	9.6	10.9	10.9	8.5	8.4	5.3	3.5	2.5	2.3	1.1	1.3	1.8	2.2	2.6	7.3	7.4	5.9	6.1	2.7	4.1	3.6	1.9	0.9	0.8	1.7	2.1	2.1	6.3	7.3
20:00	10.6	11.5	9.6	9.1	7.8	6.8	5.5	5.2	6.0	6.4	8.2	10.5	10.2	7.9	7.9	6.2	4.6	3.4	2.7	2.5	2.1	2.2	3.5	4.4	6.6	6.8	5.7	5.7	4.5	5.2	5.3	3.4	1.8	1.5	3.0	2.7	1.0	6.4	6.9
20:30	10.4	10.8	9.1	8.3	7.2	5.8	4.8	4.6	5.6	6.1	7.4	9.8	10.0	7.8	7.6	6.1	4.5	3.9	3.3	1.9	2.1	2.7	3.5	3.4	6.8	7.3	5.7	5.6	3.5	4.9	4.4	2.8	1.9	1.6	2.9	3.2	3.2	5.9	7.1
21:00	9.7	10.2	8.8	8.4	7.4	6.0	4.7	4.7	5.7	6.3	7.3	9.1	8.9	7.8	7.7	5.9	4.5	3.9	3.3	2.1	2.1	2.9	3.4	3.1	6.5	6.8	5.6	5.5	3.3	4.9	4.3	2.9	1.9	1.8	2.8	3.2	2.8	5.9	6.6
21:30	8.5	9.4	8.3	7.8	6.9	6.2	4.7	4.5	5.2	6.0	6.6	8.1	8.1	6.7	6.6	4.9	4.1	3.4	2.1	1.8	1.3	2.1	2.9	3.0	5.1	5.8	5.3	5.2	4.0	4.1	3.8	3.3	1.0	1.1	2.1	1.8	0.6	4.7	5.5
22:00	5.6	5.9	5.1	4.9	4.5	3.8	2.4	2.2	2.9	3.8	3.9	4.6	5.2	5.7	5.3	3.7	3.5	3.2	2.2	1.2	0.7	1.7	1.9	2.7	4.3	5.2	5.4	5.1	2.7	3.4	3.0	2.5	0.8	0.8	1.6	2.0	1.6	4.2	5.1
22:30	4.7	4.5	3.2	3.1	3.0	2.3	1.0	0.5	1.2	2.5	2.5	3.2	3.8	4.6	4.3	2.9	2.8	2.5	1.7	0.7	0.0	1.0	1.6	2.0	3.5	4.1	4.7	4.6	2.3	2.7	2.5	2.0	0.2	0.0	0.9	1.3	1.2	3.2	3.8
23:00	3.3	3.8	2.7	2.7	2.5	2.0	0.7	0.0	0.7	1.5	1.6	2.7	3.6	4.3	4.1	1.9	2.1	1.6	0.7	0.0	0.0	0.3	0.7	0.7	2.9	3.9	4.8	5.1	2.4	1.7	1.6	1.4	0.0	0.0	0.2	0.0	0.0	3.1	3.4
23:30	3.6	3.9	2.4	2.3	2.3	1.8	0.5	0.0	0.7	1.2	1.4	2.7	3.4	2.8	3.7	2.1	1.6	1.3	1.0	0.0	0.0	0.0	0.0	0.0	3.1	3.8	4.1	3.7	1.8	1.6	1.3	0.8	0.0	0.0	0.0	0.0	0.4	2.7	3.7

Figure 8. Example cost difference between operation under scenario optimisation strategy and baseline (Scenario 4, first year).

7 Conclusion

This paper presented the TSO model for the optimal selection and operation of energy system technologies for buildings. The paper focused on describing the mathematical formulation behind the model, as well as the problem statement and model structure. The model consisted of a mixed-integer linear programming (MILP) mathematical model; each of its equations, parameters and variables have been described above. The paper also demonstrated the capabilities of the model through a case study of a commercial building.

It was demonstrated that this model avoids shortcomings of similar models and more accurately provides guidance when making DES investments. The model incorporates real-time energy pricing and demands, fuel flexibility, projections for various parameters and a diverse portfolio of technology options. The nature of the model's framework provides great flexibility for extending technology options or time granularity if desired.

It was proved that the MILP optimisation model can generate results which guide decision makers in the implementation of distributed energy systems. The case study of a commercial building in the UK showcased the functionality of the model in separate instances using different time granularities and technology options, represented as five different scenarios. Results were obtained and summarised as key indicators such as cost, emission reductions, investment returns among others.

In this case, it was demonstrated that the optimal selection and operation of energy technologies resulted in a lower cost of satisfying a building's energy demands while providing emission reductions in comparison to the business-as-usual case. The TSO model indicated for this case study that implementing a CHP system resulted in emission reductions against the baseline of 90+% for the scenarios using bio-methane and 20+% for the scenario with natural gas. The results of the scenarios with CHP + ORC arrangements also proved lower costs and higher emission reductions than CHP-only scenarios, mainly because of the heat to power ratio of the commercial building analysed in the study.

The model is clearly data-driven. Hence, its solution performance relies heavily on the quality of the data (technology parameters, energy demands and energy resource prices) provided to it. Therefore, an effort has to be made to maintain the data in order to guarantee its quality. Additionally, because the model relies on historic metered energy demands for a building, these can only be approximated for an initial solution and then updated to optimise the operational strategy once the DES is installed if this data is not available.

Further work can consist of expanding the database to enable configurations for other countries besides the UK; this would consist of energy resource pricing and energy technology research in the desired country of study. The technology portfolio could also be expanded to include other energy technologies (e.g. fuel cells) and the model could be reconfigured to enable the use of energy storage technologies. Finally, the author identifies that a graphical user interface (GUI) would serve useful to simplify the use of the model for end-users.

The TSO model proves to be a suitable and comprehensive tool for guiding decision makers. It can select the optimal technology from a portfolio of available options and define an operational schedule favouring the economic performance of the investment. The TSO model has been already utilised as a tool for DER investment optimisation, specifically for the case of bio-methane fuelled CHP/CHP-ORC deployment in supermarket buildings of a large food retailer in the UK. The organisation has been able to invest in projects resulting in 70+% emission reductions while providing attractive returns on investment, getting them closer to achieve their sustainability targets and providing reliability of energy supply in their operations.

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Nomenclature

Indices and sets

j	Energy technologies
l	Operational levels
t	Intervals
d	Days
y	Years

Parameters

${e^d}_{tdy}$	Electricity demand at time interval t of day d and year y (kW)
${h^d}_{tdy}$	Heat demand at time interval t of day d and year y (kW)
${O^p}_{jl}$	Power output of technology j at operating level l (kW)
${O^h}_{jl}$	Heat output of technology j at operating level l (kW)
${\eta^t_{(e)}}_{jl}$	Electrical efficiency of technology j at operating level l
η^b	Thermal efficiency of the gas boiler
η^g	Electrical efficiency of the generator
σ	Time duration of intervals (hr)
CAP^{gen}	Capacity of diesel generator
dt_d	Number of days of type d in a year
F^{NG}	Carbon factor of natural gas (tCO_2e/kWh)
F^{BM}	Carbon factor of bio-methane (tCO_2e/kWh)
F^D	Carbon factor of diesel (tCO_2e/kWh)
${F^E}_y$	Carbon factor of electricity (tCO_2e/kWh) at year y
${AC^c}_j$	Annualised capital cost of technology j
${AC^m}_j$	Annual maintenance cost of technology j
${p^{NG}}_y$	Price of natural gas at year y
${p^{BM}}_y$	Price of natural gas at year y
${p^{e,import}}_{tdy}$	Price of electricity import at time interval t of day d and year y
${p^{e,export}}_{tdy}$	Price of electricity export at time interval t of day d and year y
${p^c}_y$	Price of carbon emissions at year y
PVM_y	Present value multiplier for

	year y
ω_1	Weight coefficient for cost in objective function Z_3
ω_2	Weight coefficient for carbon emissions in objective function Z_3

Binary Variables

τ_j	Defines the selection of technology j
α_{ltdy}	Defines the active status of operational level l at time interval t of day d and year y
β_{jltdy}	Defines the status at which both a technology j has been selected and operational level l is active at time interval t of day d and year y

Positive Variables

${e^i}_{tdy}$	Electricity import at time interval t of day d and year y (kW)
${e^e}_{tdy}$	Electricity export at time interval t of day d and year y (kW)
${e^t}_{tdy}$	Electricity generated by the selected technology(ies) at time interval t of day d and year y (kW)
${e^g}_{tdy}$	Electricity generated by the diesel generator at time interval t of day d and year y (kW)
${h^t}_{tdy}$	Heat generated by the selected technology(ies) at time interval t of day d and year y (kW)
${h^b}_{tdy}$	Heat generated by the gas boiler at time interval t of day d and year y (kW)
G_{tdy}	Rate of gas use at time interval t of day d and year y (kW)
BM_{tdy}	Rate of bio-methane use at time interval t of day d and year y (kW)
NG_{tdy}	Rate of natural gas use at time interval t of day d and year y (kW)
D_{tdy}	Rate of diesel use at time interval t of day d and year y (kW)

Free variables

GHG_{tdy}	Greenhouse gas emissions for time interval t of day d and year y (tCO_2e)
C^c	Total capital cost for the period (£)
C^m	Total maintenance cost for the period (£)
C^o	Total operating cost for the period (£)
C^{GHG}	Total cost incurred from carbon emissions for the period (£)
C^t	Grand total cost for the period (£)
GHG^{total}	Total GHG emissions for period (tCO_2e)
f_1	Objective function 1, cost
f_2	Objective function 2, GHG emissions
f_3	Objective function 3, multi-objective

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