FATIGUE CRACK GROWTH AND FRACTURE IN
STEELS USED FOR HIGH PRESSURE TUBING

by

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ABSTRACT

The growth of fatigue cracks and the resulting safe or catastrophic failure in thick walled tubing used in high pressure applications has been of considerable concern for some time. This thesis attempts to describe how such failure can be predicted using linear elastic fracture mechanics. This approach requires fatigue crack growth and fracture toughness data for the tubing material as well as a value for the stress intensity factor of a semi-elliptical crack propagating in the radial direction through the tubing.

These and other properties were found in both the radial (transverse) and axial (longitudinal) directions for AISI 4333 low alloy steel obtained by four different manufacturing routes to assess the influence of vacuum refinement, hot reduction by rolling or Assel elongation, and additional cold reduction. The effect of mean stress, residual stress, temperature, material inhomogeneity and specimen geometry was obtained for one sample of the material. Stress intensity factor calibrations for established and unusual specimens were found using experimental and numerical methods.

A comprehensive instrumentation system was developed which included an AC electrical potential system to monitor crack growth and a microcomputer to carefully conduct complex fatigue tests. Operator time was reduced and the accuracy of test results was greatly improved.

Finally, a model which can accommodate radial as well as axial fatigue crack propagation data has been proposed to predict the growth of a semi-elliptical crack in thick walled tubing using the material properties obtained previously. The influence on fatigue crack growth of the shape and size of the initial flaw as well as material anisotropy was investigated using this model. The dependence of the crack shape on the applied pressure was also shown.
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CONTENTS

Title Page 1
Abstract 2
Acknowledgements 3
Contents 4
Nomenclature 9

CHAPTER 1: INTRODUCTION 11

CHAPTER 2: REVIEW OF THE LITERATURE 15

2.1 General 15
2.2 Linear Elastic Fracture Mechanics (LEFM) 16
2.3 Initiation 19
2.4 Propagation 20
2.4.1 Propagation - effect of inclusions 21
2.4.2 Effect of grain size and hardness on fatigue crack propagation 22
2.4.3 The influence of stress ratio 22
2.4.4 Temperature 23
2.5 Fracture Toughness 24
2.6 Stress Intensity Factor Calibrations for Surface Cracks in Thick Walled Tubing 25
2.7 Tubing 30
2.8 Residual Stress Produced by Autofrettage 38
2.8.1 The relaxation of residual stress 39
2.8.2 Growth of fatigue cracks through a residual stress field 40
5.4.1 The boundary integral equation (BIE) method 97
5.4.2 Experimental methods 100
5.4.3 Application to the compact tension specimen (CKS) 101
5.4.4 Application to a DCB specimen 102
5.4.5 Application to the C-shaped specimen 103
5.4.6 Application to SR specimens 105
5.4.7 Application to RING specimens 106

CHAPTER 6: EXPERIMENTS 108
6.1 Variables - Fatigue Crack Growth Tests 108
   6.1.1 Stress intensity factor range 108
   6.1.2 Radial and axial tubing properties 109
   6.1.3 Residual stress and work hardening as a result of autofrettage 109
   6.1.4 Temperature 110
   6.1.5 Mean stress 110
   6.1.6 Frequency of loading 110
   6.1.7 Specimen thickness 110
   6.1.8 Specimen geometry 111
   6.1.9 Variation with tube radius 111
6.2 Procedure - Fatigue Crack Growth 111
   6.2.1 Mechanical test set-up 112
   6.2.2 Setting up the computer 112
   6.2.3 Measurements and real-time analysis 114
   6.2.4 Reanalysis of results 114
6.3 Precautions - Fatigue Crack Growth 115
6.4 Variables - Fracture Toughness Tests 117
6.4.1 Radial and axial tubing properties 117
6.4.2 Residual stress and work hardening as a result of autofrettage 117
6.4.3 Temperature 117
6.5 Procedure - Fracture Toughness Tests 117
6.6 Precautions - Fracture Toughness Tests 118

CHAPTER 7: RESULTS 119
7.1 General 119
7.2 Repeatability 122
7.3 Mean Stress 122
7.4 Autofrettaged Material 123
7.5 Specimen Breadth 126
7.6 Fatigue Frequency 126
7.7 Temperature 126
7.8 DCB Specimens 127
7.9 Split Ring (SR) Specimens 128
7.10 RING Specimens 130
7.11 Comparison of Radial and Axial Fatigue Crack Growth Rates 131
7.12 Comparison of Fatigue Crack Growth Between Materials 133
7.13 Constant ΔK Tests 133
7.14 Fracture Toughness Tests 135
7.15 Photographs of Fatigue and Fracture Surfaces 138

CHAPTER 8: DISCUSSION 141
8.1 Experimental Techniques 141
8.2 Specimens 144
8.3 Fatigue Crack Propagation Results
   8.3.1 Mean stress
   8.3.2 Temperature
   8.3.3 Anisotropy
   8.3.4 Residual stress

8.4 Fracture Toughness Tests

CHAPTER 9: SIMULATION OF FATIGUE CRACK GROWTH IN THICK WALLED TUBING

9.1 Introduction
9.2 The Model
   9.2.1 Limitations of the model
9.3 Results
   9.3.1 Influence of initial flaw size and shape
   9.3.2 Changes in constants $C$ and $m$ of the Paris equation
   9.3.3 Anisotropic material properties
   9.3.4 Effect of the magnitude of pressure on the crack shape
   9.3.5 Tubing geometry
9.4 Prediction of Fatigue Crack Growth in the Tubing Used in This Investigation
9.5 Discussion

CHAPTER 10: CONCLUSIONS AND RECOMMENDATIONS

References
Figures
Plates
NOMENCLATURE

Only the more frequently used symbols are listed.

\( a \) : crack length
\( a, b \) : major and minor half axes of an elliptical crack. In the case of a semi-elliptical crack in thick walled tubing, \( b \) is in the radial direction
\( B \) : specimen breadth
\( c \) : crack length (growing inwards)
\( C \) : (i) compliance (ii) constant in the Paris equation, \( \frac{da}{dN} = c \Delta K^m \)
\( E \) : Young's modulus
\( i \) : \( i \)th increment
\( k \) : diameter ratio of a cylinder
\( K \) : stress intensity factor
\( \Delta K \) : range of stress intensity factor, \( \Delta K = K_{max} - K_{min} \)
\( m \) : constant in the Paris equation
\( N \) : number of fatigue cycles
\( p \) : pressure
\( P \) : load
\( \Delta P \) : range of load
\( R \) : (i) stress ratio, \( R = K_{min}/K_{max} \) (ii) tubing radius
\( V \) : displacement
\( W \) : wall thickness or specimen width
\( x \) : coordinate

Greek Symbols
\( \theta \) : parametric angle
\( \nu \) : Poisson's ratio
\[ \sigma : \text{normal stress} \]
\[ \tau : \text{shear stress} \]
\[ \phi : \text{complete elliptic integral of the second kind} \]

**Subscripts**
- \( a \): axial
- \( f \): final
- \( i \): (i) initial
  (ii) inside
- \( I \): mode I
- \( \max \): maximum
- \( \text{mean} \): mean
- \( \min \): minimum
- \( n \): notch
- \( o \): outside
- \( r \): radial
- \( \text{th} \): threshold
- \( y_s \): yield strength
- \( \theta \): hoop direction
The past decade has witnessed a general improvement in material manufacturing processes and increased sophistication in engineering design. The consequential improved confidence in materials and designs has allowed smaller margins of safety to be used with a resultant reduction in weight and cost of components. High strength materials may see sufficiently high service stresses to induce and propagate cracks, particularly if flaws or stress concentrations are present. Since these materials usually have a low crack resistance (fracture toughness), the structure may fail at stresses below the designed service stress. The mode of failure is often catastrophic, accompanied by very little yielding and may cause significant damage to plant or loss of life.

Thick wall tubing used in the chemical, nuclear and armaments industries are usually manufactured from high quality and high strength low alloy steels. In order to improve the efficiency of the plant or structure, the trend has been increasingly towards large bore, thicker walled tubing. For instance, the trend amongst low density polyethylene (LDPE) manufacturers has been away from small bore, multi-line reactor facilities towards larger bore tubular reactors with perhaps only one or two lines per plant. Operating pressures and temperatures have also increased.

The failure of high strength low alloy steel tubing used in LDPE plants by fatigue crack growth and fracture is the subject of this investigation. Tubular reactors in these plants are subjected to gas at 250 °C and mean pressures of 250 MPa with superimposed fatigue loads ranging from a small high frequency pressure ripple of between 20 MPa and 50 MPa caused by reciprocating compressors to the infrequent large unloading
cycle associated with plant shutdown. This latter cycle may occur about once a week and most often follows the chemical decomposition of polyethylene within the reactor. Some plants, depending on the reactor and chemical process used, are further subjected to a bump cycle where pressure is released by about 40 MPa to 80 MPa at thirty second to two minute intervals. The bump cycle also removes encrusted polyethylene from the reactor walls. A typical reactor with a life expectancy of over ten years may therefore experience up to ten million bump cycles, considerably more cycles of compressor pressure ripple, and some five hundred repeated pressure cycles.

Three modes of failure in thick wall tubing have been observed in practice. A fatigue crack grows from the bore of the tubing in a stable manner until it reaches the outside, whereupon gas leakage occurs. This is the preferred mode of failure since the leak can be detected by instrumentation before any harm is done. In the second mode of failure, the crack reaches a critical size before leakage occurs, with consequential fast fracture resulting in catastrophic failure and potentially disastrous consequences. The final mode of failure is a combination of the other two, where fast fracture occurs a finite number of cycles after the commencement of otherwise stable gas leakage. The outcome of this mode of failure is clearly dependent on the speed of response of the emergency plant shutdown mechanisms.

Conventional techniques based on S-N curve analysis, although providing important design data, have proved inadequate in the proper characterisation of the failure mechanism. A fracture mechanics approach provides an attractive alternative for the description of fatigue crack growth and fast fracture in the tubing. This discipline has been generally developed during the past three decades to describe the failure of structures containing flaws, such as inclusions or cracks, which fail at lower loads
than calculated by conventional design criteria based on tensile strength, yield strength and buckling stress.

Linear elastic fracture mechanics utilises continuum mechanics principles to describe the stress field in the vicinity of a crack tip or front. It can be shown that the stresses are singular in that region, being inversely proportional to the square root of the distance from the crack front. The constant or proportionality is called the stress intensity factor, \( K \), which as a consequence of describing the stresses in the vicinity of a crack, provides a measure of the rate of crack growth in fatigue and determines the conditions for the occurrence of fast fracture.

This investigation attempts to use fracture mechanics principles to describe fatigue crack growth and fracture in tubing. Tests have been performed on simply shaped specimens cut from the tubing in order to measure the required material properties. Hereafter, a mathematical model can relate the simple material properties to the more complex behaviour of the tubing. The use of simple specimens has allowed material variables, such as chemical composition, structural refinement and manufacturing route, to be investigated under different conditions of temperature, residual stress as a result of autofrettage, fatigue frequency and mean stress. The effect of material anisotropy could also readily be investigated.

Clearly, a sizeable number of tests would have to be performed in order to measure all the variables described above. A typical fatigue test may take 4 to 5 man-days to perform; a considerable length of time if 100 tests are required. This period can be reduced to less than half if a computer system can be used to control a test, take measurements and subsequently (or in real time) provide an analysis. Significant additional advantages of increased accuracy and resolution can be realised, with the added ability to perform complex tests, such as constant stress
intensity factor and fatigue threshold tests. These advantages were too obvious to be easily ignored, resulting in the design and construction of a micro-computer based control, measurement and analysis system to be used with a servo-hydraulic testing machine. An alternating current (AC) electrical potential system (EPS), which is used to measure crack length and to give an indication of initiation in fracture toughness tests, was also developed.

Finally, the material parameters from the tests performed on the simple specimens are used, together with relationships describing the stress intensity factor of an elliptical surface crack in the thick walled tubing, to indicate how failures may be predicted. Some suggestions are also included for further work which may lead to an improvement in the performance of thick walled tubing.
CHAPTER 2
REVIEW OF THE LITERATURE

2.1 GENERAL

The phenomenon of fatigue and the consequential material failure of a component has been the topic of intense investigation during the past century. The traditional method used to predict the life of a component has been through the use of $S-N$ curves where the life of a component is plotted against the applied stress amplitude.

It is now well known that failure as a result of fatigue loading occurs in three phases. The crack initiates at the surface or blunt notch and propagates until a critical size is reached, whereafter fracture of the remaining section occurs. The total time occupied by the initiation and propagation stages determines the life of the component. The relative proportion of times occupied by these two phases of crack growth may depend on surface finish, residual stresses, material microstructure such as inclusions, or the presence of small cracks introduced during manufacture. Various environments, manufacturing processes and loading conditions can affect crack growth and any test should be conducted under similar conditions. Alternatively, the effect of these variables should be clearly understood in order to apply the data obtained under different conditions. Finally, the combination of load, crack shape and crack size to cause fracture should be known. In many applications, a knowledge of whether failure will occur under a safe or catastrophic mode is also of primary importance.

Fracture mechanics is particularly suited to describing fatigue crack propagation and fracture and its use is therefore becoming increasingly popular. This review of the literature has been conducted with particular emphasis to the application of fracture mechanics to the prediction of the
failure of low alloy steel, thick wall tubing.

The following sections, therefore, review initiation, propagation and fracture in materials similar to those used in this investigation, followed by a review of stress intensity factor calibrations used previously to describe cracks in thick walled tubing. A section on past work in predicting the life of thick walled tubing subjected to pulsating internal pressure is also included. Finally, this chapter is concluded with a review of investigations of the propagation of cracks through residual stress fields.

2.2 LINEAR ELASTIC FRACTURE MECHANICS (LEFM)

A description of elementary fracture mechanics can be found in a variety of texts (see, for example, Knott (1973) and Broek (1974)). This section is therefore limited to the explanation of the expressions used in the later text.

For an infinite plate subjected to a remote tensile stress, $\sigma$, the stress field around the tip of a sharp crack with length $a$ can be expressed in the general form:

$$\sigma_{i,j} = \sigma \sqrt{\frac{a}{2\pi r}} f_{i,j}(\theta)$$  \hspace{1cm} (2.1)

where $\sigma_{i,j}$ are the direct and shear stresses at the polar coordinate $(r, \theta)$. The vector $r$ is measured from the crack tip and is inclined at an angle $\theta$ from the crack plane. $f_{i,j}(\theta)$ is a known function initially proposed by Westergaard (1939).

The stress intensity factor, $K_I$, where the subscript $I$ stands for mode I, can in the above case of an infinite plate be written as:

$$K_I = \sigma \sqrt{\pi a}$$
In the case of finite bodies, a more general expression is required, hence:

\[ K_I = \sigma \sqrt{\pi a} f(a, x_c) \]  

(2.2)

where \( f(a, x_c) \) is a function peculiar to a given geometry with characteristic dimensions \( x_c \).

The above expression is often written as \( K_I = Y \sigma \sqrt{a} \) or as the non-dimensional expression:

\[ \frac{K_I B \sqrt{W}}{P} = f\left(\frac{a}{W}\right) \]  

(2.3)

In the case of a compact tension specimen (see BS5447:1977 or ASTM E399-78a) with width \( W \), breadth \( B \) and load \( P \), the expression:

\[ f\left(\frac{a}{W}\right) = \sqrt{\frac{a}{W}} \left[ 29.6 - 185.5 \left(\frac{a}{W}\right) + 655.7 \left(\frac{a}{W}\right)^2 - 1017 \left(\frac{a}{W}\right)^3 + 630.9 \left(\frac{a}{W}\right)^4 \right] \]  

(2.4)

is used.

The form of equation (2.3) has been used throughout the text when describing the stress intensity factor for common and unusual specimens (Chapter 5) and thick walled tubing (Chapter 9). The subscript \( I \) will, however, be omitted from the symbol \( K \) since only the opening mode is assumed throughout this work.

An expression for the function \( f(a/W) \) is often found using the Griffith energy criterion (see, for example, Broek (1974)) which relates the energy required to increase the crack length by an infinitesimal increment, \( da \), to the elastic energy released as a result of that crack growth. For the case of plane strain, the equation:

\[ \frac{K B \sqrt{W}}{P} = f\left(\frac{a}{W}\right) = \sqrt{\frac{1}{2(1-v^2)}} \frac{\partial (BEC)}{\partial (a/W)} \]  

(2.5)
can be readily derived. The compliance $C = \partial V / \partial P$ can be found by measuring the displacement $V$ at the loading line due to an applied load, $P$. In the case of a linear response, the non-dimensionalised compliance can be simplified to $B E V / P$. $\nu$ is Poisson's ratio and $E$ is Young's modulus.

A cyclic load leads to a range in the stress intensity factor, $\Delta K$, which equals the difference between the maximum and minimum values or $\Delta K = K_{\text{max}} - K_{\text{min}}$. The ratio $K_{\text{min}} / K_{\text{max}}$ is called the stress ratio, $R$. The fatigue crack growth rate, $da/dN$, where $N$ is the number of fatigue cycles, can in general be expressed as a function of $\Delta K$, $R$, temperature, environment, residual stress, loading history, etc. Hence:

$$ \frac{da}{dN} = f(\Delta K, R, \text{temperature, environment, ...}) $$  \hspace{1cm} (2.6)

Paris & Erdogan (1963) found that fatigue crack growth, under a given set of constant conditions, satisfies:

$$ \frac{da}{dN} = C (\Delta K)^m $$  \hspace{1cm} (2.7)

where $C$ and $m$ are constants. Deviations from this form occur at very low crack growth rates close to the threshold $\Delta K$ value, and at rates approaching fracture.

The Paris relationship (equation (2.7)) describes fatigue crack propagation and holds true in the range $10^{-9}$ m/cycle < $da/dN$ < $10^{-6}$ m/cycle. At lower growth rates, there appears to exist a threshold $\Delta K_{\text{th}}$ below which crack growth does not occur. Values for $\Delta K_{\text{th}}$ of 4 to 5 MPa$\sqrt{m}$ for low alloy steel have been obtained by various investigators, including Pook (1972).
2.3 INITIATION

The fatigue life of smooth specimens in which a crack has not yet nucleated also involves crack initiation. Although this investigation does not include tests with the object of characterising the initiation process, some understanding of initiation is required in order to specify the initial flaw size used in later predictions of component life.

Inclusions play a dominant role in initiation and hence literature in this field should be carefully reviewed. The classic work of Stulen, Cummings & Schutte (1956) showed that cracks initiate at inclusions in AISI 4340 steel and that large inclusions provide nucleation sites for high-cycle fatigue. Thornton (1971) critically reviewed a number of studies attempting to relate inclusion content and size to the fatigue strength in low alloy, high strength steels. He concluded that no general trend exists, although in some cases increasing inclusion content and size apparently decreased fatigue strength.

In a more recent study on the initiation and early growth of fatigue cracks in high strength steel, Lankford (1977) used replication techniques to evaluate the results in terms of microstructure and fracture mechanics. Two hot rolled AISI 4340 steels with yield strengths of 890 MPa and 1190 MPa were used. The average prior austenite grain size was 8 μm with manganese sulphide and calcium aluminate inclusions. Fatigue cracks nucleated from the latter which ranged in diameter from 5 μm to 20 μm. It was found that inclusions in both steels are able to debond on the very first load cycle and they do so at stresses below the fatigue limit. In low strength steels, initiation was related to slip band formation whilst initiation was slipless for higher strength levels. Initial cracks can only grow to a size corresponding to that required for stable fatigue crack propagation if that crack is associated with a debonded inclusion. Only about one hundredth of the life of the specimen was required to initiate a
microcrack.

Lankford also found that the early stages of growth are punctuated by retardation periods in which the separation correlates well with the prior austenitic grain size. Early growth was consistently transgranular and it appears that these grain boundaries serve as barriers to crack growth. The initiation process is therefore affected by grain size. Finally, the author shows that the microcrack growth data on low alloy steels correlates with the fracture mechanics approach over the entire growth regime.

Braglia, Hertzberg & Roberts (1979) and Forman (1972) have investigated initiation from various radius notches and flaws. Although their work is not of direct consequence in this investigation, they both show that the total fatigue process is propagation controlled.

It would, therefore, appear that as long as an initial flaw of sufficient size can be found, fatigue crack propagation fracture mechanics can be used to predict the total life of the thick walled tubing.

2.4 PROPAGATION

The sigmoidal nature of the fatigue crack propagation curve which can be divided into three stages from I to III is shown in Figure 2.1. A recent general description of fatigue crack propagation has been provided by Pook (1979).

In addition to the Paris relationship given in Section 2.2, numerous other laws of fatigue crack propagation have been proposed in the literature and a summary has been compiled by Hoeppner & Krupp (1974). Although a particular law is suited to similar conditions to those used to collect the data, it is often found that the same law provides an unsatisfactory description for other data. The significant scatter in the results of inadequately instrumented tests may be partly to blame.
2.4.1 Propagation - Effect of Inclusions

It is important to review the role of inclusions on fatigue crack propagation to determine what benefits can be derived from vacuum refining and the influence of inclusions in causing anisotropic fatigue crack propagation properties in wrought materials.

Thornton (1972) investigated the influence on stage II fatigue crack growth of three types of inclusions in cast AISI 4340 steel. In specimens which contained well dispersed spherical inclusions, the slowest growth rates were associated with the highest volume fracture of inclusions. On the other hand, specimens with irregularly shaped interdendritic inclusions, such as type II sulphides, showed the highest crack growth rate despite the lower volume fraction of inclusions. Thornton therefore concluded that fatigue crack propagation rates are more sensitive to morphology, composition and distribution variations than the total volume fraction of inclusions.

The anisotropy of fatigue crack propagation in hot rolled Mn-C steel plate was investigated by Heiser & Hertzberg (1972). It was found that the dependence of fatigue crack growth rate was a function of microstructure and orientation of microstructural constituents relative to the fracture plane. The fatigue crack growth rate was the highest in the longitudinal direction in martensite-ferrite microstructures. Local fracture at inclusion/matrix interfaces was considered to be responsible for the anisotropic crack growth behaviour. The ease with which such localised fracture processes occur is related to the orientation of inclusions. The measured anisotropy also increased with increasing stress intensity factor range.

More recently, Priddle (1977) investigated the effect of orientation in a high strength steel, En24. No mention of manufacturing route could be found but specimens were produced to determine the
longitudinal and transverse fatigue crack growth properties. Fatigue crack growth rates in the longitudinal direction were generally four times faster than in the transverse direction. The author believed that this effect can be expected as the crack front transverses a greater area of slag and non-metallic inclusions along the grain than across the grain.

2.4.2 Effect of Grain Size and Hardness on Fatigue Crack Propagation

In tests on vacuum-arc remelted, silicon-modified 4340 (300-M) low alloy steel, Ritchie (1977) found that decreasing the strength of the steel leads to a marked increase in the threshold stress intensity factor, $\Delta K_{th}$, and a reduction in near-threshold crack propagation rates. Coarsening the grain size also leads to lower near-threshold crack growth rates, although the threshold itself remains unchanged. No clear reasons were given why the threshold, $\Delta K_{th}$, and near-threshold crack propagation rates show a large dependence on strength and grain size.

2.4.3 The Influence of Stress Ratio

The stress ratio, $R$, has been found to influence the fatigue crack growth rate and the threshold level of the stress intensity factor. In a review on the threshold dependence on $R$, Vosikovsky (1979) proposed the empirical relationship:

$$\Delta K_{th} = \Delta K_{th}(R=0) - B R$$

(2.8)

where $\Delta K_{th}$ is the threshold stress intensity factor, and $B$ is a constant. Values for $\Delta K_{th}(R=0)$ and $B$ are 5.55 MPa\(\sqrt{m}\) and 3.91 MPa\(\sqrt{m}\), respectively, for En24 (Cooke, Irving, Booth & Beevers (1975)). This linear relationship fits the measured data very well over the whole range of $R = 0$ to 1.
The influence of $R$ in the linear portion (stage II) of the fatigue crack growth curve is less pronounced.

Cooke et al. (1975) show that the effect of $R$ becomes insignificant in vacuum for En24 low alloy steel, and fatigue crack growth rates are reduced by a factor of 30 when compared with a laboratory air environment. This result appears to indicate that the dependence of fatigue crack growth rate on mean load should perhaps rather be attributed to the influence of the environment. Clearly, more work in this field is required.

2.4.4 Temperature

Since the operating temperature of the LDPE reactor tubing approaches 300°C, fatigue crack propagation and fracture data at elevated temperatures should also be reviewed.

Pook & Beveridge (1973) have obtained threshold fatigue crack growth data on two ferritic steels at 300°C which show similar results to those obtained at room temperature. One of the steels investigated was a low alloy 0.16% C steel with a yield strength of 360 MPa. The authors argue that since fatigue crack growth is a strain-controlled process, both $\Delta K_{th}$ and $\Delta K$ for a given crack growth can be expected to be proportional to Young's modulus, which at 300°C is only slightly different than at room temperature. It is interesting to note, however, that their results at 300°C generally predicted slightly higher $\Delta K_{th}$ values than at room temperature.

In contrast, tests performed by McHenry & Pense (1973) on a low alloy quench and tempered 0.16% C steel with a yield strength of 700 MPa, obtained fatigue crack propagation rate results that at 260°C and 430°C were, respectively, 1.6 and 2.9 times higher than at room temperature. Increases of this magnitude may be too high to be attributed to the
temperature dependence of Young's modulus alone.

2.5 FRACTURE TOUGHNESS

The fracture toughness of low alloy AISI 4340 or En24 type steels can primarily be affected by heat treatment and microstructural cleanliness.

Generally speaking, the hardenability of these steels is sufficient to provide full martensite upon quenching, for the section thicknesses in which they are used, but the choice of tempering temperature has a significant effect on the fracture toughness. Pellisier (1968) investigated the effect of tempering temperature of air and vacuum melted 4340 steels. As expected, the tensile strengths of both steels declined almost linearly with tempering temperature, whereas the yield strengths increased slightly to a maximum at 260°C and then decreased at about the same rate as the tensile strength. Fracture toughness remained low and nearly constant before rising abruptly at tempering temperatures of about 320°C and 420°C for the vacuum and air melted steels, respectively. In the constant region, the fracture toughness of the vacuum melted steel was about 100% higher than the air melted steel.

A similar variation in tempering temperature was observed in En24 by Robinson & Tuck (1972), who obtained fracture toughness values of 45 MPa√m up to a tempering temperature of 350°C. This value increased dramatically to 80 MPa√m when tempered at 450°C. No correlation could be found between microstructural features and the sudden rise in fracture toughness. Robinson & Tuck also investigated the effect of grain size and found that a reduction in prior austenite grain size from ASTM 5-6 to ASTM 12-13 resulted in an increase in fracture toughness from 44 MPa√m to 82 MPa√m.

When plotting fracture toughness against yield strength, a fourfold drop is observed for a moderate increase in yield strength from 1100 MPa to 1600 MPa (see Wanhill (1978)). Considerably higher fracture toughness
values can be found at high tensile strengths if unconventional heat
treatments are followed when austenitising from 1200 C as opposed to 870 C
(Parker & Zackay (1975), Wood (1975)). However, any advantage that may
have been gained from this procedure appears to be lost if tempering
temperatures above 350 C are used.

Despite the importance of control of alloy phases, the fracture
toughness of high strength steels can most readily be improved by micro-
structural cleanliness (Hahn & Rosenfield (1973)), since inclusions provide
easy nucleation sites for large voids; hence, the current trend to vacuum
induction melting. A reduction in sulphur content, for instance, from
0.049% to 0.008% increases fracture toughness by 50% (Pellisier (1968)).
Toughness also decreases with increasing volume fraction of inclusions,
although the trend is less strong at higher strength levels (Hahn &
Rosenfield (1973)).

Not all second phase particles are equally detrimental and manganese
sulphide appears to have a dominating influence in 4340 steel. Finally,
Priddle (1972) investigated the effect of anisotropy in En24 steel and
found fracture toughness values of 47 MPa√m and 95 MPa√m in the longitudinal
and axial directions, respectively.

2.6 STRESS INTENSITY FACTOR CALIBRATIONS FOR SURFACE CRACKS IN THICK
WALLED TUBING

Values for the stress intensity factor as the crack propagates
through the wall of thick walled tubing is essential for a reasonable
estimate of the life and mode of failure of the structure. Since there
exists a stress gradient across the wall of the tube and the shape of the
crack is usually that of an ellipse, a three-dimensional calculation is
essential and computers with large core memory and fast execution times
are therefore required. A highly efficient three-dimensional analysis
(Tan (1979)) costs £400 sterling per run for a single crack size and tube geometry when run at a commercial computer centre. It is therefore not surprising that three-dimensional analyses have only become prevalent in recent years with the introduction of more efficient mathematical techniques and cheaper larger computers.

It may be argued, however, that since most of the life of pressurised thick walled tubing is spent at low stress intensity factors and hence at short cracks, estimated values derived from semi-elliptical cracks in a finite thickness plate subjected to uniform tension and pressure acting in the crack will provide adequate results.

The solution of Irwin (1962) provides a useful example of such an approach. For a semi-elliptical flaw with minor and major axes $b$ and $a$, respectively, and $b$ in the wall thickness direction, the value for the stress intensity factor at an angle $\theta$ to the major axis is:

$$K = 1.12 \frac{a\sqrt{\pi b}}{\phi} \left(\sin^2 \theta + \frac{b^2}{a^2} \cos^2 \theta\right)^{\frac{1}{2}} \tag{2.9}$$

where $\phi$ is an elliptical integral of the second kind and may be approximated by:

$$\phi = \frac{3\pi}{\delta} + \frac{\pi b^2}{a^2} \tag{2.10}$$

The gross hoop stress, $\sigma_\theta$, which includes the effect of pressure, $p$, acting in the crack, can be expressed as:

$$\sigma_\theta = p + p \frac{k^2+1}{k^2-1} \tag{2.11}$$

where $k$ is the ratio of outside to inside tube diameters. Combining the above equations and considering a tube with $k = 2$ and a crack with an aspect ratio $b/a = 0.8$, results in:
\[
\frac{K}{p\sqrt{\pi b}} = \begin{cases} 
2.09 \text{ at } \theta = 0^\circ \\
1.87 \text{ at } \theta = 90^\circ
\end{cases}
\]

No plastic zone correction need be included, since the stress intensity factor at low crack lengths is usually small.

The other alternative for simplifying a three-dimensional problem to two dimensions is to consider a straight-fronted crack in an arbitrary stress environment.

The simple form of the stress intensity factor \( K = \alpha \sqrt{\pi b} \) was modified using an expression by Folias (1965) to take the difference in the stress distribution between plate and cylinder into account. The Dugdale correction for plasticity at the crack tip was also used, resulting in an equation of the form:

\[
K = \sigma_\theta \sqrt{\pi b_\theta} \left\{ 1 + \left[ C_o + C_1 \ln \left( \frac{b_\theta^2}{R W} \right) \right] \frac{b_\theta^2}{R W} \right\} \quad (2.12)
\]

where:

\[
b_\theta = \frac{b}{(\cos \frac{\pi}{2} \frac{\sigma_\theta}{\sigma_{ys}})}
\]

and \( b \) is the crack length, \( \sigma_\theta \) the hoop stress, \( \sigma_{ys} \) the yield stress, \( R \) the tube radius, and \( W \) the wall thickness. \( C_o \) and \( C_1 \) are constants.

Bowie (1956) solved the problem of an infinite plate containing a hole with a crack in the radial direction and subjected to biaxial tension. This configuration is equivalent to a long straight-fronted axial crack in an internally pressurised cylinder of infinitely large diameter ratio.

Underwood, Lasselle, Scanlon & Hussain (1970) obtained a \( K \) calibration for an internally pressurised thick wall cylinder with a straight radial notch from a compliance test. This calibration nearly coincides with a semi-infinite plate solution simulating both the hoop stress, \( \sigma_\theta \), due to pressure \( p \), and the direct effect of pressure in the notch of depth \( b \).
that is:

\[
K = 1.12 \sigma_0 \sqrt{a} + 1.13 \frac{p}{\pi b}
\]  \hspace{1cm} (2.13)

Note the hoop stress at the bore of the tubing should be used in the above expression.

This unexpected agreement, particularly for values of \( b/W \) up to 0.6, was explained by the combination of bending constraint and drop of hoop stress in the cylinder wall.

The above results agree with a modified mapping collocation technique used by Bowie & Freese (1972). A radial crack emanating from the internal boundary of a circular ring, loaded by uniform external tension on the outer boundary, was considered. This loading distribution corresponds to an internally pressurised cylinder.

Similar solutions were obtained by Shannon (1974) using a finite element technique which showed good agreement with the work of the previously mentioned authors.

To arrive at an improved description of a semi-elliptical crack in thick walled tubing, Underwood (1972) combined the solution of Bowie & Freese with that of Rice & Levy (1970). Since this result is based on a solution for a long straight-fronted crack, it can be expected that inaccuracy may occur for large \( b/a \) and \( b/W \) ratios (see equation (2.9) for notation).

A curvature correction factor for an elliptical surface crack in a flat plate was incorporated by Kobayashi, Polvanich, Emery & Love (1979). Again, only an approximate solution can be expected.

Numerous three-dimensional finite element analyses have been attempted in order to obtain the stress intensity factor for specific values of diameter ratio, \( k \), and crack shape and depth. Marcal (1972) and Pereira (1977) modelled only one eighth of the cylinder and diametrically opposite
crack growth is therefore assumed. Other solutions which model one quarter of the cylinder have been obtained by Blackburn & Hellen (1977), McGowan & Raymund (1979) and Atluri & Kathiresan (1979). These sections, considered in the above models, are shown diagrammatically in Figure 2.2.

A recent alternative to the finite element method is the more efficient boundary integral equation (BIE) method. Heliot, Labbens & Pellisier-Tanon (1979) have used this method to solve the semi-elliptical surface problem in relatively thin walled cylinders with a diameter ratio of 1.1. Tan & Fenner (1980), however, have obtained the stress intensity factor for a semi-elliptical crack in tubing for the diameter ratios of 2 and 3, and crack depths ranging from 20% to 80% of the wall thickness. It was assumed that the crack shape remains constant throughout this range and an ellipse aspect ratio of 0.8 was used. One quarter of the cylinder was modelled. Their results show that the use of a two-dimensional analysis of a straight-fronted axial crack gives values of $K$ which are generally about twice as high as the three-dimensional solution. Also, it appears that, moving around the crack from the point of deepest penetration (X in Figure 2.2), the value of $K$ decreases gradually to a minimum (point Y) and then increases more rapidly as the internal free surface (point Z) is approached.

Three-dimensional numerical solutions are very expensive to use and are limited to particular values of ellipse aspect ratio, $b/a$, diameter ratio, $k$, and crack depth, $b$. An approximate method has recently been obtained by Williams (1980) to solve the problem of a semi-elliptical crack in internally pressurised thick walled tubing. He suggests the approximation of treating an ellipse as a circle of the same area. The relationships obtained are:

$$
\frac{K}{\rho \sqrt{\pi B}} = \frac{1}{\pi} \frac{8k^2}{k^2 - 1} \left( \sin^2 \theta + \frac{b^2}{a^2} \cos^2 \theta \right)^{\frac{1}{3}} f(a)
$$

(2.14)
where:

\[
f(a) = \frac{1}{2} \left( \frac{2-a^2}{1-a^2} - \frac{a}{(1-a^2)^{3/2}} \tan^{-1} \left( \frac{1-a}{1+a} \right)^{1/2} \right), \quad a < 1 \tag{2.15}
\]

and:

\[
f(a) = \frac{1}{2} \left( \frac{2-a^2}{1-a^2} + \frac{a}{2(a^2-1)^{3/2}} \ln \left\{ \frac{1+((a-1)/(a+1))^{1/2}}{1-((a-1)/(a+1))^{1/2}} \right\} \right), \quad a > 1 \tag{2.16}
\]

and:

\[
f(1) = \frac{11}{16}
\]

Also:

\[a = \sin \theta \left( \frac{a}{b} \right)^{1/2} \frac{b}{W} (k-1) \tag{2.17}\]

\[a, b, \theta \text{ and } \phi \] have the same meaning as in equation (2.9). Comparison with Tan & Fenner (1980) for \( k = 2 \) and \( b/a = 0.8 \) results in errors less than 6% if \( b/W \) is limited to under 0.5. Good agreement with Irwin is also obtained if the front surface correction factor is taken into account (Williams deliberately does not include it). The method is therefore of considerable value for fatigue life prediction.

2.7 TUBING

Failure of thick walled tubing has been of concern as long ago as the eighteenth century when brittle fracture of cast guns was commonplace. In one of the first reported series of experiments, Rodman (1861) found that the compressive residual stresses induced by cooling the bore when casting guns significantly improved their endurance.

More recently, fatigue failures in high pressure polyethylene plants prompted Morrison, Crossland & Parry (1956) to study failure in thick walled tubing in greater detail. Hereafter followed various papers from the same and other authors (Morrison, Crossland & Parry (1959,1960), Parry
(1956,1965), Austin & Crossland (1965), Burns & Parry (1967)), giving fatigue results for a wide range of tubing materials. Their findings can be summarised as follows.

(i) The fatigue limits of repeated pressure tests correlated very well when compared on the basis of the maximum shear stress at the bore.

(ii) There appeared to be an unexpected relationship where the limiting range of shear stress required to cause failure in a reversed torsion fatigue test equalled the fatigue limit of the tubing, expressed as the range of shear stress at the bore of the cylinder.

(iii) Compressive residual stresses induced by pressurising the tube in order to cause extensive yielding, a process known as autofrettage, has a significantly beneficial effect.

(iv) Increasing the mean stress had a detrimental effect.

(v) High pressure fluid must have some damaging effect on the metal. (Frost & Burns (1968) subsequently showed that this fluid effect is a physical and not a chemical effect.)

Haslam (1969) attempted to correlate the fatigue limit of thick walled cylinders, subjected to repeated internal pressures, with the fatigue limits of the cylinder material in uniaxial tension and pure torsion. For correlation of the fatigue strength using Gough's elliptical criterion, he found it was necessary to describe the effect of high pressure fluid on the bore surface.

An empirical expression for the effective bore hoop stress, \( \sigma_e \), as a function of the outside/inside diameter ratio, \( k \), and pressure, \( p \), was found to be of the form:
which gave good agreement with experimental results. Haslam deliberately did not attempt to define the physical action of the oil on the bore surface.

It is interesting to note that the effect of pressure on the bore as Haslam expresses it is as high as \( \delta p (k + 1) \). From the fracture mechanics viewpoint, the pressure acting in the crack would be simply added to the bore hoop stress and hence these two approaches of equivalent stress would only be equivalent if \( k = 4 \). Reference to Haslam, however, shows agreement with \( k \) values as low as 1.22 which results in an additional stress due to pressure in the crack of about 2.5\( \delta p \).

In a later paper, Haslam (1972) used the same expression of the effective bore hoop stress to predict the life of the thick walled tubing subjected to internal pulsating pressure using fracture mechanics principles. The stress intensity factor for a surface flaw of depth \( b \) and shape parameter \( Q \) in a semi-finite plate was used:

\[
K = \sigma_e \left( \pi b \frac{1.21}{Q} \right)^{\frac{1}{k}}
\]  (2.19)

Since values for \( Q \) were found to lie between 1.1 and 1.4, the above expression could conveniently be reduced to:

\[
K = \sigma_e \left( \pi b \right)^{\frac{1}{k}}
\]  (2.20)

A value for the life, \( N \), of the tubing, assuming that the fatigue crack propagation satisfies the Paris equation, can be expressed as:
\[ N = \frac{2}{m-2} \sum_{i=0}^{m-2} \frac{1}{C \Delta \sigma e^{m/2}} \left[ b_i^{1-m/2} - b_i^{1-m/2} \right] \]  

(2.21)

where \( b_i \) and \( b_f \) are the initial and final crack lengths, respectively. The value for \( b_i \) is more critical than \( b_f \), and the condition for crack propagation:

\[ b_i (\frac{\Delta \sigma}{2})^3 > D \]  

(2.22)

attributed to Frost (1965), where \( \Delta \sigma \) is the stress range and \( D \) a material property, was used. A value for \( b_i \) of 2 \( \mu m \) was obtained in this way. Haslam obtained good agreement with experimental results when using this approach.

A similar approach to Haslam was used by Wessel & Mager (1971), except that the effect of pressure on the bore (or in the crack) was not taken into account.

Quoting results found in other investigations, Tomkins (1973) believes that the endurance limit of thick walled tubing is plasticity controlled and that the von Mises yield criterion should be used. Therefore, the fatigue limit of a thick walled cylinder can be predicted from uniaxial fatigue data, provided that the basis of equivalent stress is used to relate the two different stress systems.

In the limited life region, Tomkins states that the vast proportion of any fatigue endurance is taken up in fatigue propagation. Hence, for 99% of the life, the crack grows at a steady rate which is dependent primarily on the applied stress range, \( \Delta \sigma \), and the current crack length, \( b \):

\[ \frac{dB}{dN} = B f(\Delta \sigma) g(b) \]  

(2.23)

In the initial shear controlled growth stage, crack growth occurs under a
positive shear increment only so the driving stress and strain are $\tau^+$ and $\gamma^+$ (the superscript + indicates the positive shear increment). Then:

$$\frac{db}{dN} = B_2 \gamma^+ (\tau^+)^2 b$$

(2.24)

and $B_2$ is a material constant. Pressure in the crack has no effect on the shear stress.

In the later crack propagation stage perpendicular to the hoop stress, the growth law becomes:

$$\frac{db}{dN} = B_3 \gamma^+ (\sigma^+)^2 b$$

(2.25)

and $B_3$ is a material property.

But since fluid can enter in the crack, the driving stress becomes $\sigma^+ p$, or by Lame:

$$\sigma^+ p = p \frac{k^2 - 1}{k^2 + 1} + p$$

$$= 2p \frac{k^2}{k^2 + 1}$$

$$= 2\tau$$

(2.26)

hence:

$$\frac{db}{dN} = B_3 \gamma^+ (2\tau^+)^2 b$$

(2.27)

which explains the anomaly found by the researchers at Bristol University; that the fatigue life of tubing can be found by comparing the shear stress at the bore with twice the shear stress amplitude obtained from torsion tests. It is also important to note that the effect of the stress gradient in the cylinder has not been taken into account.

Frost & Sharples (1978) presented a method of predicting both the
fatigue limit and life of a repeatedly pressurised thick walled cylinder made from a wrought material, based on the assumption that both the fatigue limit and life are a consequence of mode I fatigue crack growth from an inherent defect.

Using the expression for an edge defect, with a crack shape parameter, \( \lambda \), the threshold intensity factor can be obtained from a repeated tension plain fatigue limit, \( \sigma_t \), of a transverse specimen, or:

\[
\Delta K_{th} = \lambda \sigma_t (\pi b_o)^{\frac{1}{2}}
\]  (2.28)

where \( b_o \) is the defect depth. Note also that the shape parameters used by Haslam and Frost are related by \( Q = 1.21/\lambda^2 \).

If internal pressure can be assumed to act in the crack, the value of \( \sigma \) used to calculate \( K \) will be the hoop stress plus the internal pressure, as was previously postulated by Tomkins (1973). Hence, in a cylinder:

\[
\Delta K_{th} = \lambda \left( P_L + \frac{P_L}{k^2 - 1} \right) (\pi b_o)^{\frac{1}{2}}
\]  (2.29)

where \( P_L \) is the internal pressure corresponding to the fatigue limit.

By direct comparison of equations (2.27) and (2.28):

\[
P_L = \frac{\sigma_t (k^2 - 1)}{2k^2}
\]  (2.30)

Calculations of the fatigue life were obtained using a similar relationship as found by Haslam (1972). Again, the effect of the stress gradient in the cylinder was not taken into account.

Underwood & Throop (1979) compared \( K \) solutions for internal surface cracks in pressurised cylinders. The expression by Underwood (1972) was found to provide adequate representation of results up to \( b/W = 0.5 \) when
compared to experimental results by Smith, Jolles & Hu (1975) and Hussain (1976).

The authors note that in cannon, the crack starts in the approximate shape of a semi-ellipse with a small aspect ratio \( b/a \) (the same notation defined earlier in this section is used). This ratio increases as the crack deepens. The shape of the crack is important since the authors' results show that crack shape effects can produce a \( K \) value of less than one half that for straight-fronted cracks.

A series of tests were conducted in tubing with outside and inside diameters of 362 mm and 180 mm, respectively, and the progress of the crack was monitored using ultrasonics. Results show that the life of tubing with an initial aspect ratio \( b/a = 0.024 \) was seven times less than \( b/a = 1.0 \). The initial depth, \( b_0 \), obtained by electric-discharge-machining into the inner surface of each specimen was constant at 6.35 mm, whilst \( a_0 \) was varied to provide different aspect ratios.

Life predictions were made using the Paris equation and expressing the stress intensity factor as:

\[
K = f_k p \left( \pi b \right)^{1/2}
\]  

(2.31)

where \( f_k \) was treated as a constant and found from the Underwood (1972) expression at a suitable \( b/W \). An expression for the cycle life, \( N \), was obtained for gun steel:

\[
N = \frac{2(1/\sqrt{b}) - 1/\sqrt{a})}{6.52 \times 10^{-12} \left(f_k p \sqrt{\pi} \right)^3}
\]  

(2.32)

where:

\[
\frac{db}{dN} = 6.52 \times 10^{-12} \Delta K^3 \text{ m/cycle}
\]

(2.33)

and \( \frac{db}{dN} \) is expressed in the units m/cycle and \( \Delta K \) in MPa\(\sqrt{m}\).
Comparison of predicted with experimental results was good when a constant value for $f_K$ at $b/W = 0.25$ was used.

Finally, Underwood & Throop note that because cracks start with a low value of $b/a$ and grow with a steady increase in $b/a$, a method for including this changing crack shape in the description of crack growth will have the very significant advantages of ease of application and verification.

None of the references reviewed up to now have considered the effect of anisotropy and local changes in material properties in heavily worked thick walled tube. All the predictions cited here have only considered the stress intensity factor at the deepest point in the tube and transverse fatigue propagation properties.

Hodulak, Kordisch, Kunzelmann & Sommer (1979) have investigated the growth of part-through cracks from surface notches using artificially marked fracture surfaces. Experimental results for all specimen and crack geometries show that erroneous estimates of the growth of part-through cracks in thick walled metal specimens are obtained when calculated on the basis of the stress intensity factor distribution along the crack front and on an assumed uniform response of the material throughout the specimen volume. Also, the development of the crack shape has been found to depend on the load applied to the component.

In their model of a semi-ellipse in a plate, the authors take the influence of the front and rear surfaces into account, resulting in a changing crack shape which has an unusually small $b/a$ ratio at low $b/W$ values and an unusually large $b/a$ ratio at high $b/W$ values. Recall that the variable $b$ is measured in the direction of crack growth.

Since this behaviour is not necessarily seen in practice, a locally variable crack resistance model is also included. It is postulated that the material on the surfaces has a higher crack resistance than that of the specimen interior. A retardation of a portion of the crack front in
the front surface layer decreases the slope of the curve $a = f(b)$ in the initial stages of crack growth. In contrast, when the crack approaches the rear surface, its growth in the thickness direction is retarded in the outer surface layer and the slope of the curve $a = f(b)$ increases.

The authors believe that this qualitative treatment may contribute to a better understanding of the growth of a part-through crack in thick walled metal specimens.

2.8 RESIDUAL STRESS PRODUCED BY AUTOFRETTAGE

Autofrettage (see, for example, Manning (1950), Franklin & Morrison (1960), Gerlach (1979)) provides an effective method of developing compressive residual stress at the bore of the tubing which therefore reduces the operating bore stress. This overstraining process produces the desired residual stress when a thick walled cylinder is subjected to sufficient pressure to cause plastic deformation to some extent through the wall thickness of the cylinder. When the pressure is released, those regions which have undergone plastic deformation are only able to recover elastically, resulting in the formation of a residual stress distribution. Although increasing the autofrettage pressure leads to a larger residual compressive stress at the bore, the correspondingly higher tensile stress at the tube outside may lead to the growth of cracks at structural discontinuities on the tube exterior (Kapp & Eisenstadt (1979)). Therefore, depending on a given application, an optimum autofrettage pressure usually exists.

Various methods exist in order to calculate the residual stresses in tubes. Elastic-perfectly-plastic analyses with the Tresca and von Mises yield criteria can be found in numerous text-books on strength of materials. Manning (1945), on the other hand, provided a numerical method which takes work hardening into account. Further details are given in Chapter 4.
2.8.1 The Relaxation of Residual Stress

The gradient of residual hoop stress is very steep at the bore of the tube and some relaxation due to fatigue loading and time can occur which may also depend on temperature.

Dawson & Jackson (1969) conducted tests on AISI 4340 material to determine what the effect of time and temperature would have on the residual stress. Small-size autofrettaged cylinders were subjected to temperature varying from 200°C to 450°C for times from 1 to 72 hours. The residual stress was determined using a modification of the Sachs (1927) boring method. The authors observed that the residual bore hoop stress decreased to 46% of the original stress after 24 hours at 340°C and to 44% after the same period at 450°C. An additional 48 hours at 450°C only reduced the residual bore hoop stress by a further 3% to 41% of the original stress. The authors believe that this relaxation can be predicted on the basis of the creep relaxation properties of the material.

The only reference that could be found in the literature on the effect of fatigue loading on the relaxation of residual stress was a paper by Radhakrishnan & Prasad (1976), who conducted tests on low strength 0.23% C steel subjected to repeated fatigue cycles. The magnitude of the fatigue cycle was about 80% of the tensile stress initially used to prestrain the specimens. Tests on four values of residual stress show that the residual stresses are reduced to zero after about $10^5$ cycles. These results are not directly applicable to this investigation since a different material was used and the fatigue stresses were exceptionally high. Nevertheless, their results do indicate that the effect of fatigue loading is not negligible and should perhaps be taken into account when analysing fatigue crack growth data in autofrettaged tubes.
2.8.2 Growth of Fatigue Cracks Through a Residual Stress Field

Even if the residual stress in a component could be computed, its effect on fatigue crack propagation would be difficult to assess; the residual stress distribution is often very complex and its response to the presence of a growing crack is unclear. Underwood, Pook & Sharples (1977) investigated the fatigue crack propagation through a measured residual stress field in a low alloy forged steel with a yield strength of 1210 MPa, fracture toughness of 145 MPa√m and a chemical composition similar to the material used in this investigation. Residual stress in three-point bend specimens was produced by using a localised plastic deformation process. Although the probable residual stress distribution must satisfy the conditions of zero resultant force and moment, a stress distribution that remained linear across the full width of the specimen was assumed. This allowed the stress distribution to be presented by the superposition of a pure bending and a uniform tension stress distribution. Fatigue crack growth data were obtained for both the as-received and residual stressed specimens.

A value for ΔK which attempted to include the effect of residual stress was expressed as:

\[ ΔK_R = \left( K_{\text{max}} + K_R \right) - \left( K_{\text{min}} + K_{\text{op}} \right) \]  

(2.34)

where ΔK\text{R} is the effective ΔK including effects of residual stress, K\text{max} and K\text{min} are the extreme values of applied K, and K\text{R} and K\text{op} are the K values which simulate the effect of residual stress at the point of maximum and minimum load during the cycle. In the case of a compressive radial stress, crack closure can occur, resulting in the delayed opening of the crack at some point in the cycle above the minimum load.

The authors consider both notched and cracked specimens with
their idealised residual stress distributions. In the case of the notched specimen, the residual stress distribution is shifted to the end of the notch and is independent of the applied load. The cracked specimen, however, can close completely under the influence of a compressive residual stress and will transmit load across its faces. When the crack specimen is loaded, the residual stress distribution is similar to that of the notched case. The residual stress distribution will therefore shift, depending whether the crack is open or closed. Using this argument, the authors obtain the equations:

\[
\Delta K_R = \Delta K_\alpha + K_R \quad \text{for cracked specimens} \tag{2.35}
\]

and:

\[
\Delta K_R = \Delta K_\alpha + 2K_R \quad \text{for notched specimens} \tag{2.36}
\]

where \( \Delta K_\alpha \) is the applied \( \Delta K \).

Two methods to obtain \( K_R \) are suggested. The first method uses the simple expression:

\[
K_R = \sigma_{ave}^R (\pi a)^{\frac{1}{2}} \tag{2.37}
\]

where \( \sigma_{ave}^R \) is the average value of the redistributed residual stress near the tip of a crack with length \( a \). A superposition of a pure bending and a uniform tension stress intensity factor is used in the other approach. This latter approach does not take redistribution of residual stress into account.

An analysis of results showed that the effect of residual stress remained significant, even at \( a/W = 0.3 \). When the redistribution of residual stress was taken into account, then reasonable agreement with stress-free results was obtained. These results also seemed to indicate
that the effect of residual stress is independent of crack length. The authors believe that this may have been due to the local closing effect on the crack tip which occurs over an area which is small relative to crack length.

In a subsequent paper, Underwood & Throop (1979) considered the effect of residual stress on the stress intensity factor in a completely different manner. In a paper on fatigue life prediction in high pressure thick walled tubing, they argue that an estimate of the residual stress distribution, which is known over the distance of the notch depth, can be obtained by the superposition of two other $K$ solutions; an arbitrary stress applied to the notch faces to balance the effect of residual stress resulting in $K = 0$, and an equal and opposite distribution to the arbitrary stress distribution. These stress distributions can be described by combinations of the $K$ solutions by Brueckner (1960), Bentham & Koiter (1972) and Tada, Paris & Irwin (1973). Hence:

$$K = \left\{ 1.12 \sigma_0 - 0.88 (\sigma_0 - \sigma_a) \right\} \sqrt{\pi a}$$  \hspace{1cm} (2.38)

where $\sigma_0$ and $\sigma_a$ are the residual stresses at the bore and crack front, respectively. This value for $K$ should be summed with the external applied $K$ to obtain the driving force for crack growth. Only short crack lengths with a linear residual stress distribution can be considered. Underwood does not compare this method with that argued previously (Underwood, Pook & Sharples (1977)).

Although not of direct bearing, a recent paper by Kapp & Eisenstadt (1979) on the fatigue crack growth in externally flawed, autofrettaged thick walled cylinders and rings nevertheless provides useful information. Ring specimens which were autofrettaged to different degrees of overstrain were repeatedly loaded in compression allowing a
crack to grow inwards from an external notch. Crack growth rates were presented as a function of crack length, \( \alpha \), measured from the tube outside. From this curve, it could readily be observed that the crack growth rate generally increases with increasing amounts of overstrain. At a crack depth of 0.4 \( \alpha/w \), where \( w \) is the wall width, cracks propagated twice as fast in the 50\% overstrained specimens and nearly an order of magnitude faster in the 100\% overstrained specimens when compared to the as-received specimens. The authors believe this may be attributed to the increase in the \( R = R_{\text{in}}/R_{\text{max}} \) ratio during fatigue loading due to the tensile residual stresses from autofrettage. \( R \) may change from about 0.1 in the as-received case to as high as 0.8 in the 100\% overstrained tube. They draw attention to recent investigations (see Section 2.4.3) which describe the dependence of crack growth rate on \( R \). It was found that growth rate increases with an increase in \( R \), the amount depending on the value of \( R \) and \( \Delta K \). The maximum increase occurs at a \( \Delta K \) value close to the threshold; tests on the autofrettaged tubing were conducted at low \( \Delta K \) values from 11 MPa\( \sqrt{m} \) to 17 MPa\( \sqrt{m} \). Due to the limited data available, no further characterisation of the effect of autofrettage was made.

Since residual stresses are often present in weldments, it is not surprising that some work on the fatigue crack growth through residual stresses has been attempted in that field. In common with the other work reviewed in this section, Glinka (1979) used a superposition method assuming that the effective stress intensity factor, \( K_\varepsilon \), can be computed as a sum of the residual stress intensity factor, \( K_R = f(\sigma_R) \), and the applied stress intensity factor, \( K = f(\sigma) \). It was assumed that residual stresses do not diminish under fatigue loading. The author believes this assumption overestimates the residual stress effect, but is reasonable from the practical point of view. For calculating \( K_R \) in a centre cracked plate, Kanazawa's (1961) formula:
\[ K_R = \int_{-\alpha}^{\alpha} \sigma_R(x) \left( \frac{2 \sin \left( \frac{(\pi(a+x))/\hat{W}}{W} \right)}{\hat{W} \sin \left( \frac{2 \pi a}{\hat{W}} \right) \sin \left( \frac{(\pi(a-x))/\hat{W}}{W} \right)} \right)^{1/2} dx \]  

was used, where \( \alpha \) is the half crack length, \( \hat{W} \) is the plate width, and the \( x \) axis lies in the crack plane with \( x = 0 \) when \( \alpha = 0 \). The true residual stress distribution was approximated by simplified rectangular distributions. Hereafter, Forman's (1967) equation, which includes the \( R \) ratio \( (R = K_{min}/K_{max}) \) was used to obtain life predictions. Good agreement with experimental results was obtained. Glinka also notes that the effect of residual stress on fatigue crack growth rate depends on the magnitude of the applied load. The lower the applied load, the higher the residual stress effect. Also, higher applied loads cause greater residual stress relaxation.
CHAPTER 3
EQUIPMENT

Common with many other technologies, state of the art material testing facilities have seen a shift towards closed loop testing machines which can be controlled by simple electrical signals. Closed loop machines are particularly important in crack growth studies where large changes in compliance can occur which would result in load changes if simple screw or resonant machines are used. The servo-hydraulic testing machine is an example of a closed loop machine able to perform a variety of materials tests. An additional advantage of using electrical signals to drive the testing machine is the ability to run complex tests. These may include constant $\Delta K$ tests, fatigue threshold tests or simulated service loading. It is common in these tests to derive the driving signal from a digital computer which could also be used to log data and present the test results (see, for instance, Mindlin & Landgraf (1976), Sherrat (1978), Ewing (1979) and Nowack et al (1979)).

The arguments in favour of a system that includes a servo-hydraulic testing machine controlled by a digital computer are presented in this chapter. It is shown that such a facility can perform complex tests, obtain and analyse data to significantly greater accuracy than previously possible, save operator times and improve the machine's turnover.

Unfortunately, the cost of a digital computer and its ancillaries, when obtained from the manufacturers of testing machines, doubles the already high costs of the testing machine. Some of the cost is due to the use of mini- and not micro-computers as well as the cost of the software, usually written in the low level language, ASSEMBLER.

The cost of a micro-computer based system was investigated, together with any time limitations that may occur if programming was done in BASIC.
The results of this study were encouraging and one year of the author's PhD studies was spent in the design and construction of a system based on a Commodore 2000/3000 series micro-computer. The system, which has undergone its first phase of development, is described in this chapter.

It has been understood in this discussion that the testing machine and other transducers require or provide analogue inputs or outputs. This is fortunately the case with most modern instruments. The measurement of crack length is more complex and hence an electrical potential system was developed to fulfil this requirement.

Plates 3.1 to 3.3 show the 100 kN and 250 kN Mayes servo-hydraulic testing machines, micro-computers, electrical potential systems and other equipment used in this investigation. These photographs are described in greater detail later in the text.

3.1 MEASUREMENT PHILOSOPHY

To obtain the fatigue relationship (see Section 2.2)

\[
\frac{da}{dn} = \Delta K (a/W, \Delta P, \text{specimen geometry})
\]

(3.1)

under certain constant conditions such as fatigue frequency, temperature and \(R\) ratio, values for \(a\), \(N\) and \(P\) are required. This assumes that a relationship already exists for the stress intensity factor, \(K\). As the crack propagates under constant amplitude loading, \(\Delta K\) will increase steadily, resulting in a corresponding increase of fatigue crack growth rate as determined by equation (3.1).

If a function for \(K\) does not exist for the specimen under test, then a relationship for \(K\) can be obtained during the same test in which fatigue crack growth data are being acquired. The equation for plane strain (see Section 2.2)
\[
\frac{K_B \sqrt{H}}{P} = \sqrt{\frac{1}{2(1-v^2)} \frac{\delta(BEC)}{\delta(a/W)}}
\]  

(3.2)
is used, where \( C = \partial V/\partial P \) and \( V \) = displacement along the loading line.

It is well known that finding the differential of experimental data is prone to unusually large errors. Also, some functions show large changes for small changes in their independent variables. Hence, in equations (3.1) and (3.2), considerable care must be taken in the calculation of \( da/dN \), \( \Delta K \) at high \( a/H \), \( \partial V/\partial P \) and \( \partial C/\partial a \). Crack length, \( a \), appears in all these relationships and it is unfortunate that this parameter is the most difficult to measure. Hence, considerable care has to be taken to construct an electrical potential system (EPS) able to obtain consistently accurate values of crack length.

The value of \( V \) also causes difficulty since the change in \( V \) is very small, particularly for short crack lengths. Furthermore, the measurement of \( V \) along the loading line creates problems in the fixing of the transducer to the specimen. Therefore, reasonable attention has also been given to the design of LVDT and strain gauge type transducers.

Before a test is commenced, it must be decided at what interval of \( a \) or \( N \) readings must be taken. It is instructive to consider two extreme cases:

1. Readings are taken at each single increment of \( N \) and \( da/dN \) and \( \Delta K \) calculated. Considerable scatter may be seen due to the possibly quantised nature of crack growth (Kfouri (1979)). This curve will, however, show localised cracking rates and could stimulate or confirm theories on fatigue crack growth. When these results are filtered or integrated, the usual fatigue crack growth curve should be obtained.
2. Researchers very often measure crack length by microscope only. Not only is this method susceptible to large errors due to the limitation in its accuracy (typically 100 μm), but different researchers are known to interpret the location of the tip of the crack differently. Furthermore, due to the tedium of this method, very few readings are generally taken. Various authors have used sometimes involved techniques in an attempt to analyse such results. These methods range from finite difference methods (Mukherjee (1972)), high order polynomial fits (Davies & Feddersen (1973)), simple curve fitting methods to portions of data (Smith (1973)), and more complex methods (McCartney & Cooper (1972)). Erroneous and erratic curves frequently result.

In practice, the number of readings taken during a test should be sufficiently high to show any local effects and correctly define a curve, whilst not so high so as to create difficulties in analysis and data storage. An optimum probably lies between 70 and 200 points on a $\frac{da}{dN}$ versus $\Delta K$ curve.

Automation of the fatigue crack growth test will result in a greater number of more accurate measurements as well as reduced operator time.

The simplest form of automation would require a $y-t$ chart recorder continuously recording crack length using a measuring system such as the EPS. Hereafter, a calibration curve must be used to obtain actual values of crack length. If it is assumed that $N$ varies linearly with time and that $\Delta P$ remains constant, then values of $\frac{da}{dN}$ and $\Delta K$ can be calculated. The time required for analysis may be long, since values have to be read from the chart recorder and typed into a computer.

A computer connected directly to the testing machine would allow a higher level of automation to be attained. Control of the machine is
possible, accuracy is further improved and operator time reduced to a minimum.

The advantages of automation can be summarised in Table 3.1.

### Table 3.1

**Comparison of Test Efficiency for a Typical 24 Hour Test**

<table>
<thead>
<tr>
<th>Test Method</th>
<th>Degree of Sophistication</th>
<th>Man Days</th>
<th>Accuracy in $\Delta a$ (%)</th>
<th>Number of Readings Taken</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manual</td>
<td></td>
<td>$\frac{1}{2}$</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>Simple automation</td>
<td></td>
<td>$\frac{1}{2}$</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Computer automation</td>
<td></td>
<td>$\frac{1}{2}$</td>
<td>1</td>
<td>$\frac{1}{2}$</td>
</tr>
</tbody>
</table>

Computer control of the testing machine is essential if complex control functions such as constant $\Delta K$ and threshold tests are to be conducted. Constant $\Delta K$ tests are used to determine the relationship between fatigue crack growth and $a/W$. This technique allows the effect of inhomogeneity of castings, large heat treated sections or heavily work hardened materials to be investigated. Contoured double cantilever beam (DCB) specimens have previously been used to maintain constant $\Delta K$ conditions for constant $\Delta P$ loading. These specimens are machined at substantial cost and hence a technique which reduces $\Delta P$ in order to maintain $\Delta K$ constant has significant advantages.

Threshold tests require a slowly decreasing $\Delta P$ to be applied to the specimen as the crack propagates. If $\Delta P$ decreases too rapidly, then the crack may arrest prematurely due to crack retardation. Too slow a decreasing $\Delta P$, on the other hand, may miss the threshold completely, since $\Delta K$ can increase faster due to crack growth than decrease due to the reduction in load. The best technique could be to decrease $\Delta P$ in large
steps at first and in successively smaller steps as the threshold is reached.

These two tasks are typical applications where the computer can readily control a servo-hydraulic testing machine to perform complex operations without further capital outlay. The further advantages of increased accuracy and reduction of operator time, as well as the real time calculation of such parameters as $K$ for unusual specimens, makes a computer controlled servo-hydraulic testing machine a very attractive proposition.

3.2 MEASUREMENT OF CRACK LENGTH

Various methods can be used to determine crack length. These include optical methods, acoustic emission, eddy currents, X-ray diffraction, laser interferometry, displacement gauges and the electrical potential technique. Klintworth & Webster (1979) and Ritchie & Bathe (1979) provide a short survey of these methods.

The electrical potential method, also called the potential drop method, has been used most widely because it is relatively cheap, can be used for cyclic loading and can be adapted for high temperature and environmental testing.

In this method, an electric field is formed in the specimen by passing a current through it. The field changes as the crack lengthens, resulting in a change in the potential difference across two sensing leads suitably placed symmetrically across the crack. This potential difference can be related to crack length by a calibration curve, assuming no change in the current passed through the specimen for a constant temperature.

The hardware required for a simple electrical potential method is shown in Figure 3.1. A stabilised current is passed through the specimen whilst the potential difference across the crack is measured by a suitable
amplifier. In a direct current (DC) system, the current density in the thickness direction is uniform, resulting in an average measurement of crack length. Despite large currents in the order of 50 A being used, the input to the amplifier is only a few microvolts.

An alternating current (AC) system can be used to provide significantly higher specimen output voltages for the same current by utilising a magnetic material's inherent skin effect. Here, the current prefers to flow on the outside surface of the specimen, resulting in a higher impedance and hence higher output voltage. The calibration curve no longer provides an indication of average crack length but some value between the crack length at the specimen's surface and an average value.

Differences between the AC and DC systems are compared in Table 3.2.

It is undesirable to over-amplify the potential difference across the sensing leads since significant noise and amplifier temperature drift may result. The design of the hardware is therefore important. Equally important are the positions of the current and sensing leads since their positions alter the calibration curve for the system. A method of optimising the positions of current and sensing leads using finite element and boundary integral equation techniques has been described by Klintworth & Webster (1979).

The dominant criteria in this optimisation are best combinations of sensitivity, repeatability and, to a lesser extend, linearity.

An AC electrical potential system was initially designed and reported by Klintworth (1977). This system consisted of a 50 A power supply which was modulated in the form of a square wave by switching a regulator, buffered by several power transistors in an on/off fashion at the required AC frequency of 3.3 kHz. A large resistor in series with the specimen was used to maintain constant current conditions. A narrow band amplifier consisting of 3 Wein bridge filters was used with simple bridge
TABLE 3.2
Differences Between AC and DC Electrical Potential Systems

<table>
<thead>
<tr>
<th>Measurements</th>
<th>AC</th>
<th>DC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Due to the skin effect, ( \alpha ) lies between average and surface values</td>
<td>1. Reads average crack length</td>
<td></td>
</tr>
<tr>
<td>2. No thermocouple effects</td>
<td>2. Thermocouple effects due to dissimilar metals in specimen and amplifier</td>
<td></td>
</tr>
<tr>
<td>3. Conductance of the specimen material changes with temperature</td>
<td>3. Ditto</td>
<td></td>
</tr>
<tr>
<td>4. Sensitive to spatial position of current and sensing leads</td>
<td>4. Less sensitive</td>
<td></td>
</tr>
<tr>
<td>5. Wires should be twisted. This is difficult at high temperature</td>
<td>5. Wires should be shielded</td>
<td></td>
</tr>
<tr>
<td>6. A minimum, associated with crack initiation, is displayed</td>
<td>6. Monotonically increasing function with load</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Hardware</th>
<th>AC</th>
<th>DC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Due to high specimen output per unit input current, a low current power supply is only required</td>
<td>1. High currents are necessary</td>
<td></td>
</tr>
<tr>
<td>2. The current must be modulated</td>
<td>2. No modulation necessary</td>
<td></td>
</tr>
<tr>
<td>3. Narrow band amplification can be used, resulting in excellent noise rejection</td>
<td>3. Chopper stabilised DC amplifiers are usually used</td>
<td></td>
</tr>
<tr>
<td>4. Cost of hardware usually lower due to the smaller power supply</td>
<td>4. Expensive power supply due to higher currents used</td>
<td></td>
</tr>
</tbody>
</table>
rectifier demodulation. This system, although proving the feasibility of the system, was plagued by temperature dependent errors. This was not surprising since low cost components were used throughout.

The oscillator, which drives the regulator, was particularly susceptible to temperature. This caused a frequency mismatch between the source frequency and the natural frequency of the narrow band amplifier, resulting in abnormally high errors due to the Gaussian nature of the narrow band curve. The oscillator was therefore replaced with a crystal controlled design which provided excellent frequency stability.

Where previously the oscillator frequency was adjusted to match the natural frequency of the narrow band amplifier, this was no longer possible with the crystal controlled version. This necessitated a change in the natural frequency of the narrow band amplifier.

Although it is possible to calculate new values of the passive components used in the Wein bridge design, the values required in practice invariably differ due to second order effects. Development of this design was discontinued in favour of a redesign using higher quality components.

The square wave previously used was abandoned in favour of a sinusoidal waveform since it was thought that the harmonics may have been the cause of second order effects in the narrow band amplifier. A precision oscillator BB4423 with a frequency stability of 50 ppm/C and output stability of 0.05%/C was used to drive the same regulator and power transistors previously used. Some redesign of the regulator circuit was also necessary since it was now driven with a reference voltage and no longer switched in an on/off manner. A large series resistor was still used to obtain constant current conditions. An RMS current of 10.8 A is obtained from this unit, which is shown in the centre of Plate 3.1.

A completely new design of the narrow band amplifier was made. Two BBUAF41 filters were used and these were configured as a 4 pole Butterworth
filter with \( Q = 8 \) (\( Q \)-factor = natural frequency/3dB bandwidth). A schematic arrangement of the circuit is shown in Figure 3.2. The input to the narrow band amplifier is buffered by a unity gain difference input operational amplifier. This is followed by a pair of cascaded 2 pole active filters with an overall gain of about 200. A potentiometer located before the next stage of amplification adjusts the attenuation of the amplifier. The filtered output of the amplifier is demodulated by a RMS to DC converter with an accuracy of 0.2%. An operational amplifier with a gain of 2 provides the EPS output. The natural frequency of the amplifier is 3230 Hz and the gain as a function of frequency is shown in Figure 3.3. It can be seen that a small change in frequency causes negligible change in the amplifier's gain however other frequencies are strongly attenuated.

This combination of AC power source and narrow band amplifier was used for many of the experiments reported later in this thesis.

During one stage of the testing programme, two servo-hydraulic testing machines were used which necessitated the construction of another electrical potential system. The opportunity was taken to redesign the AC power source using VMOS technology. VMOS power field effect transistors can be driven by very low currents, yet can deliver high currents combined with low ON resistance. A schematic diagram of the circuit is shown in Figure 3.4.

A precision oscillator provides a reference signal at the natural frequency of the narrow band amplifier. This signal is attenuated by a potentiometer before a positive mean value is applied by an operational amplifier. A constant voltage across a stable precision resistor is maintained by a feedback loop formed by an operational amplifier driving a VMOS power FET. This circuit results in a constant alternating current of 4.75 A obtained using a 10 A single voltage power supply; a
considerable improvement in the efficiency of the system. In Plate 3.2, the improved AC power supply is shown on the right of the load frame and the narrow band amplifier can be seen on top of the testing machine's control cabinet.

The AC electrical potential system provides the user with an additional tool. A characteristic decrease in potential output is seen as the crack tip opening displacement or stress intensity factor increases (see Okumura et al (1980) and Bachmann & Munz (1976)). Figure 3.5 shows this dependence which can be used in fracture toughness tests to give an indication when slow crack growth commences.

Stability of the system is excellent if the specimen temperature can be kept constant. A curve relating the potential across the sensing leads with temperature is shown in Figure 3.6. This effect may be eliminated if the specimen temperature can be monitored throughout the test and the EPS output modified using a previously obtained function relating EPS output to temperature. This has not been done in this study but is recommended for future work.

Plate 3.3 shows a specimen with attached current and sensing leads. Current leads were attached to the specimen using OBA brass bolts screwed into previously tapped holes. A brass washer was placed between the specimen and current lead to ensure an even and repeatable contact for CKS and DCB specimens (see Section 5.1 for specimen details). The detail is a little different in RING, SR and C specimens. Here, the current lead is held between the bolt head and a nut and the bolt screwed into the specimen until contact is made with the bottom of the tapped hole. The sensing leads were made of 650 μm diameter soft iron wire spot welded to the specimen using 60 J energy and 50 N force. The leads could easily be positioned to ±500 μm. Figure 3.7 shows details of current and sensing lead positions for various specimens.
3.3 OTHER MEASUREMENTS

Load and ram position were obtained directly from the servo-hydraulic testing machine's instrumentation. Output terminals on the front panel allow these values to be recorded remotely. Temperature measurement for the hot tests was obtained from Chromel/Alumel thermocouples attached to the specimen by inserting the thermocouple junction into a small drilled hole and peening the hole closed using a centre punch. Two thermocouples were placed symmetrically across the crack.

A travelling microscope was used to record optical crack length.

Accurate values of displacement of the specimen along the loading line is required when compliance measurements are made. As the C-shaped specimen was not yet a standard specimen (ASTM E399-78a) when this work was commenced, careful attention was paid to the development of a suitable transducer system for that specimen.

Three different types of transducers were tried on C-shaped specimens. These are shown in Figure 3.8. In transducer I, an integral LVDT was fixed to the side of a shackle in order to measure the displacement between shackles. Errors were caused by bending of the pins as well as axial misalignment of the shackles. Transducer II shows a clip gauge made from spring steel fixed in position in the specimen's loading line. Two strain gauges were attached on the inside and outside of the spring clip. Shallow holes were drilled into the specimen to be used as sockets for the location of the rounded pins bolted to the clip gauge. This type of location would not, of course, restrict the clip gauge to one plane; instead, it could rotate on an axis perpendicular to the plane of the crack. Although the results were encouraging, errors did seem to result from this rotation. Transducer III, also shown in detail in Figure 3.9 and in Plate 3.3, proved to be the most successful. Needles from needle bearings were glued to the specimen in the loading line. The displacement
gauge, containing an LVDT, is located on these needles. The transducer contains a linear bearing which allows vertical motion of the upper and lower locating beams. These beams cannot rotate with respect to one another due to their positive location on the needles. Note the upper and lower beams are made from glass reinforced epoxy in order to insulate the transducer from the specimen and so not change the EPS calibration curve. A small spring pushes the beams apart. The calibration curve for this transducer is shown in Figure 3.10.

3.4 MICRO-COMPUTER CONTROL, DATA ACQUISITION AND REAL TIME ANALYSIS OF FRACTURE MECHANICS TESTS

Mini-computers have been associated with servo-hydraulic testing machines for quite some time but have unfortunately been very expensive, often more costly than the testing machine itself. The high cost is understandable since digital computers have to be interfaced to an analogue servo-hydraulic testing machine by analogue to digital converters (ADC) and digital to analogue converters (DAC). Other miscellaneous items such as cycle counters and machine trips must also be interfaced to the computer. Furthermore, programs have to be written to perform a variety of tests and the cost of software, particularly purpose-written software, is considerable.

Micro-computers have steadily decreased in cost and often cost less than the annual maintenance contract of one mini-computer. The cost of the interfacing to the testing machine is unchanged but the cost of software can also be reduced if it is written in a high level language such as BASIC, with only short routines written in ASSEMBLER. This alternative thus offers significant cost advantages to institutions where specialist knowledge of materials testing as well as software is available.

This section aims at describing the use of a micro-computer in the
field of material testing. For a more general description of how micro-
computers work and how they are applied in a laboratory environment, the
reader is referred to Klintworth (1979) and Klintworth & Webster (1980).

3.4.1 Control

The ability of a micro-computer to perform control functions on a servo-hydraulic testing machine allows many complex testing operations to be performed. Some of these operations are described in the sections below.

Control functions can be performed as a result of a computer being able to measure parameters, perform calculations as well as provide a new controlling signal to the testing machine. In this manner, the computer forms part of a closed control loop with the testing machine.

The interaction of the computer control and the servo-hydraulic closed loops is shown in Figure 3.11 and explained below. For simplicity, the case of load control is assumed. The computer generates an output which represents the load which must be applied to the specimen. The load cell continually measures the load and provides the feedback to the servo-amplifier which adjusts the servo-valve opening until the actual load equals the required input load. The servo-hydraulic closed loop responds rapidly in attempting to maintain control. The micro-computer closed loop, however, has deliberately been made to respond more slowly by introducing damping coefficients and the command input signal to the testing machine will only change if the servo-hydraulic loop has been unable to maintain control. This condition can occur, for instance, when the gain setting on the servo-amplifier is too low during dynamic loading.

I. Compensation for compliance increase

The above paragraph describes a typical simple interaction
between computer and testing machine during a conventional constant load amplitude fatigue test. As the crack grows, the compliance of the specimen increases with a resulting reduction in the mechanical component gain of the closed loop. This loss of gain should be compensated for by an increase in the gain setting of the servo-amplifier. In the absence of computer control or the intervention of an operator, it will be found that the load steadily decreases. The micro-computer closed loop artificially compensates for the low gain of the servo-amplifier by increasing the command input, until the correct load is once again achieved.

II. Constant $\Delta K$ tests

Constant $\Delta K$ tests can readily be achieved under computer control. Assume a relationship (see Section 2.2)

\[
\frac{K_B \sqrt{\pi}}{P} = f(a/W)
\]  

(3.3)

exists for a specimen which must be loaded at a constant value, $\Delta K_0$. Then, equation (3.3) can be written as

\[
\Delta P = \frac{\Delta K_0 B \sqrt{\pi}}{f(a/W)}
\]  

(3.4)

which defines the command input to the testing machine in order to maintain constant $\Delta K$ conditions. A flow chart depicting this operation is shown in Figure 3.12. It can be seen from the diagram that the computer only updates the value of $\Delta P$ once a given increment in EPS, $\Delta V$, is exceeded. This will typically occur between 70 and 300 times during the fatigue crack growth of one specimen.
III. Threshold or near threshold $\Delta K$ tests

When commencing a test, it is necessary to subject the specimen to a reasonably high value of $\Delta K$ to initiate cracking from a blunt notch. The value of $\Delta K_{th}$ is substantially lower and a decreasing load is required soon after crack growth commences to achieve $\Delta K_{th}$. The effect of crack retardation is well known (see, for instance, McCartney (1978), Gemma et al (1977), Matsuoka & Tanaka (1978) and Obianyor & Miller (1978)) and care must be taken not to decrease $\Delta P$ too quickly, particularly close to $\Delta K_{th}$ and so prematurely arrest the crack.

Two approaches are described which can be used to obtain $\Delta K_{th}$.

A. Estimating $\Delta K_{th}$

This method requires a good estimation of $\Delta K_{th}$. In this technique, load is decreased, initially in coarse steps, but in gradually finer steps as the estimated value of $\Delta K_{th}$ is reached. A quadratic function for $\Delta P$ is chosen which decreases with each finite increment, $\Delta V$, of EPS output for $M$ such increments. The nature of this function, shown in Figure 3.13, has been chosen such that its slope is zero when threshold conditions of fatigue crack growth are reached. Start of test conditions are shown with the subscript $s$.

The quadratic relationship for the $i$th decrement in $\Delta P$ is

$$\Delta P = \Delta P_s - 2 (\Delta P_s - \Delta P_{th})(i/M) + (\Delta P_s - \Delta P_{th})(i/M)^2$$  \hspace{1cm} (3.5)

B. Specifying the required threshold $da/dN$

This approach is probably superior since a threshold crack growth rate, $(da/dN)_{th}$, can readily be defined. In this case, the instantaneous crack growth rate is compared with the required value at
threshold and reducing $\Delta K$ in stages until threshold is achieved.

Equation (3.4) defined the conditions under which a constant $\Delta K$ test ($\Delta K = \Delta K_0$) can be conducted. A similar approach can be used for reducing $\Delta K$ tests but with $\Delta K_0$ replaced by $\mathcal{f}\Delta K$, where $\mathcal{f}$ is a predetermined fraction by which $\Delta K$ must be reduced from its previous value. Hence, the new load range can be expressed as

$$\Delta P = \frac{\mathcal{f}\Delta K B \sqrt{N}}{\mathcal{f}(a/W)}$$  \hspace{1cm} (3.5)$$

The value of $da/dN$ is compared with the required threshold value. Stepping down will continue until this value is reached.

The form $\mathcal{f}\Delta K$ has the advantage of reducing the magnitude of each step as $\Delta K$ is reduced, resulting in a smooth transition to $\Delta K_{th}$.

IV. Dynamic/static transitions

Despite the use of an automated crack monitoring system such as the EPS, it is prudent to confirm the crack length reading using an optical microscope. Optical measurements are best made when the specimen is subjected to only a static load. Computer control ensures that the transitions between dynamic and static load and back to dynamic loading are executed smoothly.

V. Loading functions

The computer system is able to generate complex loading functions such as ramp, pace and hold, and random loading functions. Usually, very little software is required; a ramp function used in $K_{Ic}$ tests can be generated using only four BASIC statements.
VI. Fault conditions

Fault conditions such as the failure of a pull rod or a drop in furnace temperature can be monitored by the computer which could then shut down the test.

3.4.2 Measurements

Analogue to digital converters (ADC) associated with computers show significant advantages over conventional means of data logging such as chart recorders and digital paper tape punches. They typically have an accuracy of 0.025%, can read numerous channels and are able to measure both slow and fast transients. Some ADC contain a programmable gain amplifier which allows a different gain to be selected for any given channel under software control. For instance, a slow steady thermocouple output of only 10 mV can be read with equal ease and accuracy as a fast changing load with an output varying between -10 V and 10 V.

In the testing machine application, the ADC would measure the following parameters.

(i) Load: Static values. Maximum and minimum values found by sampling a dynamic load and comparing the instantaneous value to the previously found maximum and minimum values.

(ii) Displacement: As in Load above.

(iii) Hysteresis loops: Load and displacement can be measured at cycle frequencies up to 100 Hz, allowing 100 coordinates to be sampled per cycle (micro-computer performance).

(iv) Electrical potential system.

(v) Strains.

(vi) Temperature.
The number of fatigue cycles to which a specimen is subjected must also be measured. This can be achieved by comparing the input or output waveforms to set upper and lower values using a comparator. The digital output from the comparator can be counted by a bank of digital counters which are interfaced directly to the micro-computer. A 3 byte counter bank provides cycle counts up to 16.7 million with a discrimination of 1 cycle.

3.4.3 Analysis

Analysis is required in control operations such as constant $\Delta K$ tests as well as processing measurements taken during the test. Parameters required for control must be computed in real time, whilst test results may be calculated after the test. Real time analysis, however, does provide the researcher with an instantaneous indication of the test's performance.

An example of the analysis required in a crack growth test can be found in Section 3.4.6.

3.4.4 Micro-Computer Hardware and Interface to a Servo-Hydraulic Testing Machine

Most servo-hydraulic testing machines provide analogue output terminals for recording of load, displacement and strain. An analogue command input is also usually included to allow external command of the machine. Figure 3.14 shows a micro-computer interfaced to a servo-hydraulic testing machine. This system corresponds to the first prototype commissioned in 1978. An 8 kbyte Commodore PET micro-computer with digital tape recorder and printer was interfaced to an 8 channel 8 bit DAC and to an 8 channel 12 bit ADC. These units provide the analogue input and outputs to the computer. The 12 bit accuracy ADC allowed
voltages in the range -10 V to 9.995 V to be measured with a discrimination better than 5 mV, whilst the DAC provided potential outputs in the range 0 V to 9.96 V in steps of 40 mV. These voltage ranges agree with those used by the testing machine.

Measurements of maximum and minimum load, instantaneous load, displacement and EPS were measured by the ADC. A filter was required since the outputs from the servo-hydraulic testing machine proved to contain substantial high frequency noise. The input impedance of the ADC is very high, hence simple passive RC filters could be used without attenuating the signal.

To provide a sine wave where the amplitude and the mean value could be changed by the computer required the use of multiplying and adding unit. This unit multiplied a 10 V amplitude sine wave or other signal with one of the DAC outputs, divided it by 10 and added the result to another DAC output. Changing the first DAC output altered the amplitude, whilst the second varied the mean. Four other outputs from the DAC were used to plot graphs on an analogue plotter. The seventh DAC output was used to reset the memory module which provides maximum and minimum load values. Resetting this module cancels the previously stored values and allows new extremes to be read.

An IEEE-488/RS232C interface was used to interface the serial printer to the micro-computer.

The prototype system is shown in the foreground of Plate 3.1 and in Plate 3.3.

An improved version of a micro-computer control and data acquisition system to that described in the preceding paragraphs is shown in Figure 3.15. A larger 32 kbyte Commodore PET allows greater programming freedom. The tape recorder and printer remain unchanged. A photograph of the system is shown in Plate 3.2.
The accuracy of the ADC is unchanged but the improved version includes a programmable gain instrumentation amplifier which allows gains between 1 and 1024 to be selected by software. This allows a wide range of transducers to be measured by the system. Individual passive filters are connected in line with the cable from each transducer.

A new concept of measuring maximum and minimum values has been adopted in the improved version, resulting in improved accuracy and a significant decrease in the number of cables required to connect the computer to the testing machine. A fast machine code program was written to sample waveforms up to 100 Hz and determine their maximum and minimum values. This routine is not restricted to load only but can sample strains or displacements too.

A high resolution signal conditioner replaces two DAC channels and the multiplying and adding unit. This is achieved by summing the output of one 12 bit DAC with another 12 bit multiplying DAC. The reference input of the multiplying DAC is derived from the signal conditioner. This increased accuracy for control allows loads, strains or displacements to be controlled to 0.025% of their full range. This is a considerable improvement over the prototype system which could control with an accuracy of only 0.4% of the half range.

A cycle counting unit has been added to the improved version. One cycle is counted with each transition of an input signal going from +3 V to -3 V. In this manner, spurious counts due to noise are eliminated. The counter derives its input from the signal generator output.

A four channel 8 bit DAC has been provided for general purpose work such as graph plotting.

These units assembled in a 483 mm standard racking frame are shown in Plate 3.4. The ribbon cable emerging from the left of the frame...
forms the only required connection to the micro-computer.

3.4.5 Description of the Interface Units

The functional description and operation of the units required to interface the micro-computer to the testing machine are described here. Photographs of these units are shown in Plates 3.5 to 3.7.

Analogue to digital converter (ADC)

Figure 3.16 shows the functional layout of the ADC. When the micro-computer addresses the ADC, the decoding logic holds the selected values of channel and gain in the latches provided. A differential input analogue multiplexer is used to give good common mode noise rejection performance. The programmable gain instrumentation amplifier (PGIA) requires between 120 μs and 250 μs to settle following a sudden change in its input value before the A/D converter can be triggered to commence conversion. During the conversion time of 25 μs, the sample and hold amplifier ensures that the analogue signal remains constant. The 12 bit digital output from the A/D converter must be multiplexed to the 8 bit data bus as two separate bytes using the two digital multiplexers shown.

A machine code program provides the correct sequence and timing of the triggers to the various components. A flow diagram is shown in Figure 3.17 describing the sequence of operations.

The BASIC commands to access the ADC are

SYS(ADC) : X = USR(G(I)+CH)

The SYS and USR commands are general purpose BASIC commands to interact with machine code routines. The letters ADC represent a constant which is determined by the location of the machine code routine in memory.
Constants such as ADC may have to be changed as the machine code routines are updated. To ensure correct use of these routines, the new updated values of these constants used as arguments to the SYS command are displayed on the computer's visual display unit (VDU) when the machine code routine is initialised. CH is a value in the range 1 to 8 and determines the input channel number to be read by the ADC with a gain of $2^I$, $I = 0$ to 10. Due to the complex operation of the PGIA, an array $G$ has to be defined to return a coded gain value to the machine code routine. This array, which can be found in Table 3.3, is also displayed by the machine code routine.

**TABLE 3.3**

Array $G(I)$ for ADC Gains of $2^I$

<table>
<thead>
<tr>
<th>I</th>
<th>Gain</th>
<th>$G(I)$ for 8 CH</th>
<th>$G(I)$ for 32 CH</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>8</td>
<td>256</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>16</td>
<td>512</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>40</td>
<td>1280</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
<td>48</td>
<td>1536</td>
</tr>
<tr>
<td>5</td>
<td>32</td>
<td>64</td>
<td>2048</td>
</tr>
<tr>
<td>6</td>
<td>64</td>
<td>72</td>
<td>2304</td>
</tr>
<tr>
<td>7</td>
<td>128</td>
<td>80</td>
<td>4096</td>
</tr>
<tr>
<td>8</td>
<td>256</td>
<td>96</td>
<td>3072</td>
</tr>
<tr>
<td>9</td>
<td>512</td>
<td>104</td>
<td>3328</td>
</tr>
<tr>
<td>10</td>
<td>1024</td>
<td>112</td>
<td>3584</td>
</tr>
</tbody>
</table>

The output of the ADC is equated to the variable $X$ in the above example. The relationship between $X$ and the potential measured at its inputs is
when \( X = 32768 = 2^{15} \)
then the ADC input = 10 V for a gain of 1

or

when \( X = 32768 = 2^{15} \)
then the ADC input = \( 10/2^I \) V for a gain of \( 2^I \)

Hence, if the value of potential is required in volts, then the expression

\[
SYS(ADC) : V = \frac{USR(G(I) + CH)}{32768 \times 2^I}
\]

can be used.

The versatility of the software approach of controlling the ADC allows a fast routine to be written in machine code which can find the maximum and minimum values of an input waveform applied at any channel or gain. The simplified flow diagram of this routine is shown in Figure 3.18. A sample is taken every 120 \( \mu s \) and compared with the previous maximum and minimum value stored in memory. Favourable comparison results in a new stored value of maximum and/or minimum. This operation is repeated 256 times to provide a 30 ms sampling period. Up to 256 of these sampling periods can be selected by the user before the routine returns to BASIC, resulting in a maximum 7 s sampling time. This routine can be repeatably called by BASIC if longer sampling periods are required.

The BASIC instructions for this routine are

\[
SYS(MAX) : X = \text{USR}(G(I) + CH)
\]

\[
SYS(MIN) : Y = \text{USR}(\text{DUMMY VALUE})
\]
MAX and MIN are constant values pointing to the address where the maximum and minimum routines are stored. G(I) and CH are required in the same manner as the ADC. The maximum value is placed in the variable X, whilst the minimum is placed in Y.

The number of 30 ms loops can be changed by the BASIC command

```
POKE MAXFLAG, N
```

where N = the number of 30 ms loops and MAXFLAG is a constant determined and displayed by the machine code routine.

Signal conditioner (SC)

The functional layout of the SC is shown in Figure 3.19. The SC consists of two 12 bit D/A converters which have been configured to condition an input signal by providing an output of the same waveform but with different amplitude and mean values. The multiplying D/A converter attenuates an up to 10 V amplitude input signal to provide a new amplitude determined by the digital input to the D/A converter. A mean value is obtained by another conventional D/A converter according to its digital input. The amplitude and mean are then summed to provide the SC output.

A system of latches is required by each 12 bit D/A converter to ensure that the input remains constant until a new 12 bit data word is available. The operation of these latches is as follows. The decoding logic allows data to be latched into latch B. This forms the least significant nibble (half a byte or 4 bits) of the 12 bits required by the D/A converter. The data stored in latches A and C remain unchanged and so does the output of the D/A converter. The most significant byte of data is now latched into latch A and, simultaneously, the previously latched data in B is transferred to latch C, in this way providing a new
set of 12 bits to the D/A converter in one operation.

The following BASIC commands are used to define the mean and amplitude

\[ \text{SYS(MEAN)} : \ DU = \ \text{USR(MN)} \]
\[ \text{SYS(AMPL)} : \ DU = \ \text{USR(AM)} \]

where MEAN and AMPL are constants determined by the location of the relevant machine code routines and DU is a dummy variable. MN and AM are the mean and amplitude values to be placed into the D/A converters and must be scaled by the relationship

10 V output requires \( 32768 = 2^{15} \) D/A converter units

Hence, if the mean and amplitude are to be expressed in volts, VM and VA, respectively, then the following expressions can be used

\[ \text{SYS(MEAN)} : \ DU = \ \text{USR(INT(VM \times 3276.8))} \]
\[ \text{SYS(AMPL)} : \ DU = \ \text{USR(INT(VA \times 3276.8))} \]

Note: The argument of the USR command must lie between 32767 and -32767 to prevent errors occurring in the transition between BASIC and machine code routines.

Digital to analogue converter (DAC)

The four channel 8 bit DAC can provide voltages in the range 0 V to 9.96 V. These outputs can be used for plotting curves on x-y recorders, the input to variable voltage power regulators, relay
operations, etc.

No special machine code routines are required since BASIC statements can be used directly for 8 bit interfaces. The command is

\[
\text{POKE DAC + CH, X}
\]

where DAC = 40935, CH is a channel number in the range 1 to 4, and X is the DAC output and must not exceed 255. The variable X and the potential in volts is defined by the relationship

\[
10 \text{ V output requires } 256 = 2^8 \text{ DAC units}
\]

**Counter**

The functional layout of the counter is shown in Figure 3.20. The analogue input is compared in a pair of comparators to upper and lower limits which can be adjusted at will. These units are usually set at +3 V and -3 V, which ensures noise-free response. The digital output from the comparator is counted by a bank of 3 cascaded 8 bit counters which can be read by the micro-computer in turn when the decoding logic activates each buffer. The reset line sets all the counters to zero.

The BASIC statement to read the number of counted cycles N is

\[
N = \text{PEEK(CT)} + 256 \times (\text{PEEK(CT + 1)} + 256 \times \text{PEEK(CT + 2)})
\]

where \(\text{CT} = 40944\).

The counters can be reset by the statement

\[
\text{DU = PEEK(CT + 3)}
\]
where DU = dummy variable.

**Bus board**

Tristate Schmidt trigger buffers have been provided on the bus board to ensure reliable communication between the micro-computer and interface unit. This approach also prevents electrical faults in one unit affecting the other.

**Power supply**

The power supply contains an unusual safety feature to prevent the interface unit from again becoming operational after a power failure. This allows the operator to first initialise the control output voltages to a known state before the test is continued.

### 3.4.6 Software

Once suitable hardware exists to interface the micro-computer to the testing machine, software is required to access these interfaces in order to take measurements, provide an analysis and finally output new command signals. Various programming languages exist such as the high level languages FORTRAN and BASIC and low level languages such as ASSEMBLER. Programming in BASIC is relatively easy but provides only slow computational speeds, whilst ASSEMBLER routines run at the micro-computer's optimum speed but are difficult to program. A further difference exists in high level programs which can either use interpreters or compilers to reduce high level commands to long sequences of machine code. Programs which use compilers must first be compiled before they can be run and hence they are more time-consuming to debug than interpretive programs which can be run immediately. Compiled programs do, however, run faster than interpreted programs.
Software required for fatigue work tends to be involved but high computing speeds for analysis are generally not required. As a result, a BASIC program interspersed with machine code routines previously written in ASSEMBLER, has been used to control, take measurements and analyse fatigue crack growth tests.

Before the long fatigue crack propagation program is described, it may be purposeful to describe how the interface commands are used within a simple program. An interesting example is a program used to generate a loading ramp and take measurements during a fracture test.

The program listed in Figure 3.21 was written to generate a loading ramp when it was found that the usual ramp and hold module in the testing machine no longer functioned. A short routine was also included to take measurements of load, ram displacement (if the specimen displacement transducer malfunctioned during the test), specimen displacement and EPS. The program, which only took one hour to conceive and enter into the computer, can be divided into four main parts.

Lines 1000 to 1160 define arrays and constants, set the signal conditioner output or (testing machine input) to zero, and invite values for the title of the test and the load range used.

Lines 1200 to 1350 generate the loading ramp and take the required measurements. The load is ramped in 2047 increments until full range is reached. Typically, this gives increments of 2.5 N in a 5 kN load range test. Measurements are taken before each load increment but only stored every fourth load increment to reduce the amount of data accumulated. The FOR/NEXT loop in lines 1240 and 1250 is used to provide a delay between successive load increments in order to achieve the loading rate required for valid $K_{Ic}$ tests.

The routine used to dump data on tape is listed in lines 1400 to 1450. The data tape can be used at a later stage to plot different
combinations of curves such as load and EPS as a function of specimen displacement or ram displacement. Alternatively, since data were required at constant time intervals, curves plotted as a function of time can also be obtained.

The final routine, lines 1500 to 1570, is used to print every fourth measurement on a printer. This printout is not particularly useful but does serve as a back-up copy should the data tape be destroyed through misuse.

3.4.7 Fatigue Crack Growth Software

The software used for the fatigue crack growth tests is shown in flow chart form in Figures 3.22 to 3.24 and as a remarked listing in Figure 3.25. The program can be broadly divided into three principal routines with numerous subroutines.

The first routine, INITIALISATION, is shown in Figure 3.22 and corresponds to lines 0 to 197 in the program listing. The primary purpose of this routine is to initialise constants and variables and to define functions.

Test details are contained in DATA statements as opposed to the usual method of interactively keying information from the keyboard in a question/answer fashion. This approach is preferred since it is frequently found that the information required by the program does not differ very much from test to test. Hence, only those values contained in the DATA statements which need to be changed have to be updated, and unchanged values can be quickly skipped over using the micro-computer's editor.

Functions used to evaluate the values of stress intensity factor for various specimens used in this investigation are defined in this routine. A cubic polynomial representing the EPS calibration curve is also included.
The INITIALISATION routine, which is executed only once, ends in the STANDBY mode where a small static load is placed on the specimen. In the STANDBY mode, the user is invited to input a new value of load, or the word EXIT can be entered whereupon the program enters the CONTROL routine.

The CONTROL routine is shown in Figure 3.23 and in lines 198 to 290 of the listed program. The program circulates in this loop for a large proportion of the running time of the fatigue test. The primary function of the CONTROL routine is to ascertain if a significant amount of crack growth has taken place by monitoring the EPS output, and to ensure that the load applied to the specimen follows the required function.

At the start of the loop, values of the EPS output and cycle count are monitored and displayed on the computer's VDU. If the EPS output exceeds a maximum value, or if the character S is typed on the keyboard, then the CONTROL routine exits to the STANDBY mode. This maximum EPS value may correspond to a maximum value of crack length or to a value which is nearly out of range of the EPS's narrow band amplifier.

Measurements must be taken whenever the crack has grown by a given increment. For this purpose, the EPS output is monitored and the MEASURE routine is entered if the present EPS output exceeds the previous output by a given EPS increment. Typing the character M on the keyboard will force an exit into the MEASURE routine.

A by-pass loop exists which prevents an exit from the CONTROL routine if a required minimum number of cycles since the previous exit to the MEASURE routine has not been exceeded. This ensures that the computer is always able to control the test between successive measurements and is particularly important at high $\Delta K$ values when large gain changes occur.

The measurement of the maximum and minimum of the loading cycle
allows mean and amplitude values to be calculated. A control error term can be computed which is the difference between the measured and required mean and amplitude values. In an attempt to maintain control, a new command (or driving) signal is calculated which is the sum of the previous command signal and a given fraction of the error term. The value of this fraction determines the damping factor of the computer's response in an attempt to maintain control. A check is made to ensure that the difference between measured and command mean and amplitude values does not exceed 50%; otherwise the test is aborted and the STANDBY mode is entered.

The primary function of the MEASURE routine, shown in Figure 3.24 and lines 500 to 1710 of the program listing, is to take measurements and perform an analysis using these measurements. Results from the analysis are not only required to present fatigue crack growth data, but also to calculate various control parameters during the test.

Two options exist in the standard fatigue crack growth program. The RAMP option slowly halts the fatigue cycle and ramps down the load whilst taking readings of load, displacement and EPS output. These measurements allow compliance $C$ and hence an experimental stress intensity factor $K$ to be calculated for any specimen geometry. A suitable transducer to measure displacement along the specimen's loading line must also be available. Since the EPS output is dependent on the value of $K$, it is preferable to measure EPS output at the same $K$ throughout the test. This option provides EPS output values at various loads and hence an accurate value of crack length can be calculated. The second or CONTINUOUS fatigue option continues fatigue cycling during the MEASURE routine and compliance, and hence $K$ values, cannot be calculated.

It is important to define the term computation domain (CD) before continuing the discussion on the MEASURE routine. The analysis of the data in this program is performed using simple techniques on closely
spaced, accurate data. Typically, 100 sets of readings may be taken during a test. Where differentials to a curve are required, a quadratic curve is fitted to an odd number of adjacent data. This group of data, on which a calculation is to be performed, is termed the computation domain. Whenever measurements on a particular computation domain have been completed, the computation domain shifts by one measurement to the right to include a new measurement and disregard the least recent measurement.

The routine MEASURE commences by specifying the number of measurements included in the computation domain. Hereafter, a transition from DYNAMIC to STATIC load will take place if an optical measurement of crack length is required, a new data tape is to be inserted into the recorder or if the RAMP option has been chosen. This transition in loading takes place smoothly to avoid bumping of the specimen which may cause adverse effects. Fatigue loading would otherwise continue. At this stage, the counter, which contains the number of cycles since the last measurement, is read and added to the previous total. The cycle counter is then reset. A value of EPS output is also read and will be used in the analysis if a better static value is not obtained.

An opportunity to measure the crack length optically can be requested at specified measurements. Both near and far sides should be measured and typed into the computer. An average value is calculated.

If the RAMP option has been selected, the load is reduced in steps while readings of load, displacement and EPS output are taken. The computer first reads the present values of load before ramping down to the next value of load.

An approximate value of stress intensity factor is calculated and compared with the required value at which the EPS output is valid. This EPS output value should, wherever possible, be used for crack length
calculations. If the calculated value of stress intensity factor is out of range, then the EPS value at the highest load is used. The dynamic EPS value is only used in the case of the CONTINUOUS fatigue option.

Compliance and stress intensity factors can be found experimentally if the RAMP option is selected. A least squares curve fitting subroutine is used to fit a linear curve to the displacement and load data obtained whilst the load was ramped down. A value for $\partial C/\partial a$ can be obtained by differentiating the quadratic curve fitted to compliance versus crack length data in the computation domain. The gradient is found at both the centre and right hand sides of the computation domain. A comparison of these results gives an indication of the accuracy of the method used. Experimental $K$ and $\Delta K$ values can then be found using equation (3.2). The corresponding analytical values of the stress intensity factor are computed using a standard function.

Crack growth rate, $da/dN$, is found by the same method of differentiating a quadratic curve fitted to data in the computation domain. Curves of $a/W$ versus $N$, EPS output versus $a/W$ and $da/dN$ versus $\Delta K$ (or versus $a/W$ in the case of constant $\Delta K$ tests) can be plotted on an analogue plotter using the subroutine PLOT. The structure of this routine would not differ greatly if a digital plotter were used.

Observations and results are sent to the printer and dumped on a data tape. This tape can be transferred to disc and used in further reanalysis programs run on another similar micro-computer in more comfortable surroundings.

The computation domain is shifted one measurement to the right to allow a new measurement to be included at the next pass of the MEASURE routine.

Parameters can now be calculated for the CONTROL routine.
Section 3.4.1 details the techniques used in control operations. The concept of a multiplication factor is used to calculate new load values for quadratic decreasing load functions and constant $\Delta K$ tests. The new load is equal to the product of the initial specified load and the multiplication factor. In the case of a quadratic decreasing load function used to initiate a crack from a blunt notch, the multiplication factor, $MF$, reduces from a constant, $FI$, down to unity over ID passes of the MEASURE routine (see line 1691). If a constant $\Delta K$ test is also required, then the multiplication factor will continue to decrease below unity in order to maintain $\Delta K$ constant.

A new value of EPS output is calculated which equals the previous value plus a given increment. This new value must be exceeded before the MEASURE routine is again entered.

If necessary, the specimen undergoes a smooth transition from static to dynamic load by first sampling the static load and then entering the STATIC to DYNAMIC subroutine.

The eleven subroutines used by the three principal routines are sufficiently straightforward not to require a description in the text.
CHAPTER 4
MATERIALS

Thick wall high pressure tubing is manufactured by a few specialist companies using different grades of low alloy steel and manufacturing processes. Rogan (1979), who conducted conventional fatigue tests on large varieties of tubing, found that the fatigue life and mode of failure can be affected by these variables.

In order to investigate fatigue crack growth and fracture from a fracture mechanics viewpoint, four tubes of similar material but different conditions were selected to provide a reasonable cross-section of grades of steel and manufacturing route. All four tube materials tested were manufactured by the Timken Company*.

4.1 STEEL GRADES

Timken use two high strength low alloy steels in the manufacture of seamless high pressure tubing: 4333 M4 which contains 0.33% to 0.35% C, 0.3% Mo, 1.7% Ni and 0.8% Cr, whilst 4333 M6 contains an additional 0.5% Ni and 0.3% Mo to improve hardenability. These steels are available in aircraft quality air melt vacuum degassed and consumable arc remelt grades. Both grades contain only minute traces of sulphur and phosphorous, but the latter grade contains particularly low levels of impurity elements, resulting in generally isotropic material properties. Details of the chemical specification can be found in Table 4.2.

4.2 MANUFACTURING ROUTE

The tubing is manufactured using one of two methods, as shown in

* The Timken Company, Steel Division, Canton, Ohio, USA.
Figure 4.7. The first mill in the manufacturing route is the Mannesmann type piercing mill which is used in both methods. A hot solid round billet at forging temperature is inserted between two revolving rolls set at an angle which force the revolving billet over a plug, thereby forming a tube.

In the conventional method, the tube, while still hot, is then processed by the rolling mill where the tube passes between grooved rotating rolls and over a plug two or three times to reduce the wall to the desired thickness. Hereafter, grooves which may remain in the tube are ironed out by the reeler where the still hot tube is forced over a smooth plug in the same type of motion used by the piercing mill.

The reducing mill is used in both methods where the tube is rolled through sets of grooved rolls set at right angles to each other to reduce the diameter of the tube. Finally, a rotary sizer produces the final required outside diameter and rounds up the oval tubing coming from the reducing mill. The operation of this mill is similar to the reeler.

In the second method, an Assel elongator is used instead of the rolling mill and reeler. After leaving the piercing mill, a bar is inserted into the still hot tube. The tube and bar are then fed through three revolving rolls set at angles to each other. These rolls are designed and installed in such a manner as to elongate the tube and reduce its wall thickness. As a result of this process, the tube is twisted considerably more than would be obtained by the rolling mill/reeler route, resulting in the possibility of anisotropic material properties.

The hot rolled or hot Asselled tube may be further conditioned by cold reduction. Rotorolled tubing is produced on a machine designed to make large degree reductions in one pass of the tube. The machine operates by cold swaging the tube between two semi-circular dies containing tapered grooves, which rock back and forth while the tube is alternately
advanced and rotated over a long mandrel. Cold working imparts higher strength and better surface finish to the tubing.

The tubing is finished and heat treated before being sold as hot worked or cold reduced tubing.

4.3 SELECTION OF MATERIAL FOR TESTS

Material properties may be directly affected by the amount of reduction of the outside diameter (sink) and reduction in the wall thickness (draft), since the resulting mechanical work controls the degree to which the longitudinal structure, which originates from the billet, is broken up. Further cold reduction may also have a beneficial effect. Finally, the chemical composition, level of impurity elements and heat treatment may also affect fatigue crack propagation and fracture properties.

Four different materials were selected from some 15 available types of tubing. At the time this selection was made, it was hoped that a large range of fatigue crack propagation and fracture properties would be found. Table 4.1 compares the different materials used in these tests. Each material is identified by a code letter, such as DM and V. Manufacturing routes, tube dimensions and the $k$ ratio ($k = \text{outside/inside diameter}$) are shown. The final column indicates the principal reason why the tube was chosen for this investigation. Table 4.2 details the chemical specification of the tubing.

It is apparent (Crofton (1980)) that inclusions can initiate and contribute to the propagation of fatigue cracks. Also, the fracture toughness of high strength steels can be most readily improved by greater microstructure cleanliness (see Wanhill (1978)) because inclusions provide easy nucleation sites for large voids. Vacuum refinement is therefore one of the most important methods to improve the crack resistance of high strength steel.
Hot or cold deformation may produce an anisotropy due to the alignment of crystallographic axes of the grains into a preferred orientation or texture. It also causes mechanical fibering: elongated grains and bands of elongated inclusions and second phase particles. Finally, deformation may introduce residual stresses.

Before tests were commenced, it was therefore anticipated that the V tubing would show best fatigue crack growth and fracture properties due to the vacuum remelt process and cold reduction. The relationship between fracture toughness and yield strength is well known and hence fracture properties may be reduced as a result of its higher strength (see Table 4.1). U tubing, on the other hand, may have poor corresponding properties as no additional refinement was undertaken, the tube was not cold reduced and tube dimensions were considerably larger than the other tubing considered. The hot Asselled H tubing had shown a peculiar spiral fatigue failure in laboratory high pressure tests on the tubing (Rogan (1979)). This tendency may be the result of the exceptional twisting of the tubing in the Assel elongator. Finally, DM tubing was known (Rogan (1979)) to have poor transverse impact strength; it was believed that this was due to inadequate heat treatment.

4.4 MATERIAL PROPERTIES

Some properties which can be used to characterise the four selected materials are shown in Tables 4.1 to 4.4.

Tensile yield strength and ultimate strength values determined by Rogan (1979) are shown in Table 4.1. The small difference between these two strength level measurements indicates the nearly elastic-perfectly-plastic nature common to many low alloy steels. Similar tests conducted by Qattan (1980) show no variation in ultimate tensile strength between the radial, hoop and axial directions, indicating isotropic behaviour.
Considerable care was taken in the measurement of both impact and hardness values for the materials used in this investigation. Charpy V-notch impact values are presented in Table 4.3. Four specimen types were cut from each slice of tubing to measure axial (longitudinal) and radial (transverse) impact strengths. Specimens were cut close to the inside bore as well as outside surface of the tubing. Each combination of orientation and position was repeated three times to provide an indication of repeatability and scatter in material properties and testing technique. Average values are shown in the four columns labelled "axial/inside" to "radial/outside" and the tolerance associated with each value shows the variation in the three repeated tests on a particular combination of orientation and position. The "overall" column gives average impact values for the whole tubing and is computed by taking the aggregate of the previous four columns. The associated tolerance indicates the variation of impact values as a function of orientation and position and therefore a measure of anisotropy. It is notable that the hot Asselled tubing H gives the greatest variation as may be expected. U tubing shows the highest impact strength which may be a consequence of its slightly lower tensile strength.

Hardness measurements were subsequently made on the broken Charpy specimens. The results are shown in Table 4.4 and the same rules concerning tolerances apply. These variations in hardness are small and can be attributed to scatter and experimental error. As far as hardness and tensile strength are concerned, the tube material appears to be isotropic.

A brief comment on the relationship between impact, hardness and yield strength results is in order. These properties for the four materials investigated can be expressed as:
Impact : DM < V < H < U
Hardness : V = DM > H > U
Yield Strength : V > DM > H > U

The relationship between these properties is as would be expected, with the exception of DM material which shows low impact properties.

4.5 AUTOFRETTAGE

Tests were performed on DM tubing which was autofrettaged by pressurising the tube to 660 MPa. Specimens used in these tests are denoted by a suffix A. Figure 4.1 shows the loading and unloading curve found by measuring the hoop strain at the outside diameter.

Measurements of the imparted residual stress were made using a similar technique to the Sachs (1927) boring method, where thin layers of material are machined from the inside bore of the tubing whilst the change in strain is measured using strain gauges on the outside diameter. Precautions were taken to reduce additional residual and temperature stresses from being produced by the machining process.

An expression recently described by Williams, Gray & Hodgkinson (1980) was used to relate the measured strains and amount of material removed to the initial residual stress. The value for the residual hoop stress can be expressed as:

\[
\sigma_\theta = \frac{x}{E} \left( 1 - x^2 \right) \left( \Delta \varepsilon_\theta - \frac{\Delta \varepsilon_\theta}{2} (1 + x^2) \right)
\]

where \(x\) is the ratio of the present (machined) bore diameter to outside diameter, \(E\) is Young's modulus, and \(\Delta \varepsilon_\theta\) is the change in strain from the initial unbored strain, measured on the outside diameter of the tubing.

A typical curve of change in strain as a function of tube radius as
expressed by the ratio \( \alpha/W \), where \( W \) is the wall thickness, can be seen in Figure 4.2. The residual hoop stress calculated using equation (4.2) is also shown. The differential of strain required for this equation was found by fitting a quadratic polynomial to each combination of fifteen adjacent points and analytically finding the gradient to the fitted curve. Note that the gradient of stress close to the bore is very steep which could result in large errors in predicting the residual hoop stress at the bore.

Readings were taken from four strain gauges placed evenly around the outside diameter of the tubing. Relatively large differences between these gauges were measured, as shown in Figure 4.3 and the corresponding hoop stress curves in Figure 4.4.

Measurements were made of the residual stresses in four specimens. Two specimens had been recently autofrettaged (less than two months before the measurement), whilst two other specimens had experienced fatigue loading in excess of one million cycles below the fatigue limit in laboratory fatigue testing machines. The temperature in these tests seldomly rises above 60 C. The residual stresses measured are shown in Figure 4.5, where it can clearly be seen that the residual stress close to the bore is reduced as a consequence of fatigue loading and/or time. The results from the specimen RSI DM are poor due to early experimental difficulties, but the general trend is similar to the other tests.

The residual stress distribution can be predicted using simple elastic-perfectly-plastic (EPP) behaviour. Two expressions can be derived, depending on whether the Tresca or von Mises yield criteria are used to obtain the residual hoop stress at the bore of the tubing:

Tresca:

\[
\sigma_\theta = \sigma_y \alpha - \frac{p_\alpha}{k^2 - 1} \quad (4.2)
\]
von Mises: \[ \sigma_\theta = \frac{2}{\sqrt{3}} \sigma_y \sigma_p - \frac{p_\alpha (\frac{2k^2}{k^2 - 1})}{k^2 - 1} \tag{4.3} \]

where \( \sigma_\theta \) is the residual hoop stress at the bore, \( \sigma_y \) is the yield stress, \( p_\alpha \) is the autofrettage pressure, and \( k \) is the ratio of outside to inside bore diameters. Substituting values for DM tubing and using a yield stress of 1100 MPa (average of actual yield and ultimate tensile stresses) resulted in values of -620 MPa and -450 MPa, respectively.

Comparison with Figure 4.5 shows that assuming von Mises and EPP behaviour provides reasonably accurate values of maximum residual stress. The difference can readily be attributed to relaxation and experimental inaccuracies. Franklin & Morrison (1960) used the maximum shear stress criterion and found their prediction inadequate; concluding that the assumption of elastic release of pressure was to blame. The simple calculation above shows that their conclusion may not be altogether correct.

4.6 FATIGUE DATA

Figure 4.6 shows fatigue data in the form of S-N curves obtained by Rogan (1979) for all four materials in both the as-received and autofrettaged conditions.

The high cyclic internal pressure fatigue tests were carried out on special purpose machines where the pressure cycle is generated by a ram reciprocating through an all metal lip seal. The specimens were short lengths of tubing threaded at both ends with no other machining of the tube inside and outside diameters. A pressure cycle with a high mean of 200 MPa was used.

The curves in Figure 4.6 should be examined in terms of the bore shear stress, \( \tau \), or the sum of hoop stress, \( \sigma_\theta \), and internal pressure, \( p \),
in the crack (see Frost & Sharples (1978)):

\[ 2\tau = \sigma_0 + p = p \frac{2k^2}{k^2 - 1} \]  

(4.4)

where \( k \) is the outside/inside diameter ratio. Both H and DM tubing yield values of 280 MPa, whilst U and V tubing give 250 MPa and 220 MPa, respectively. Rogan (1979) attributes the significantly poorer performance of V tubing to an inferior bore surface finish.

The effect of autofrettage on V tubing is particularly striking and reflects industry's common belief that autofrettage is most useful in reducing the detrimental effect of defects and poor bore finish. When comparing autofrettaged to as-received tubing, it can be seen that autofrettage has only improved the fatigue limit of DM tubing by 80 MPa.
<table>
<thead>
<tr>
<th>Material</th>
<th>Manufacturing Route</th>
<th>Outside Diameter (mm)</th>
<th>Bore Diameter (mm)</th>
<th>k Ratio</th>
<th>$\sigma_{ys}$ (GPa)</th>
<th>$\sigma_u$ (GPa)</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>DM 4333 M4</td>
<td>Vacuum degassed Hot rolled Cold reduced</td>
<td>124</td>
<td>59</td>
<td>2.1</td>
<td>1.069</td>
<td>1.151</td>
<td>Batch had low transverse impact strength</td>
</tr>
<tr>
<td>H 4333 M6</td>
<td>Vacuum degassed Hot Asselled</td>
<td>118</td>
<td>51</td>
<td>2.32</td>
<td>1.055</td>
<td>1.161</td>
<td>Hot Asselled</td>
</tr>
<tr>
<td>U 4333 M6</td>
<td>Vacuum degassed Hot rolled</td>
<td>140</td>
<td>59</td>
<td>2.55</td>
<td>1.048</td>
<td>1.131</td>
<td>Large bore and high k ratio</td>
</tr>
<tr>
<td>V 4333 M6</td>
<td>Vacuum remelt Hot rolled Cold reduced</td>
<td>114</td>
<td>51</td>
<td>2.25</td>
<td>1.214</td>
<td>1.317</td>
<td>Vacuum remelt</td>
</tr>
</tbody>
</table>
TABLE 4.2
Chemical Composition for the Four Low Alloy High Strength Steels Investigated

<table>
<thead>
<tr>
<th>Material</th>
<th>Material Type</th>
<th>C</th>
<th>Mn</th>
<th>P</th>
<th>S</th>
<th>Si</th>
<th>Cr</th>
<th>Ni</th>
<th>Mo</th>
<th>Cu</th>
<th>Sn</th>
<th>Al</th>
</tr>
</thead>
<tbody>
<tr>
<td>DM</td>
<td>4333 M4</td>
<td>0.34*</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.8*</td>
<td>1.7*</td>
<td>0.3*</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>H</td>
<td>4333 M6</td>
<td>0.34*</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.8*</td>
<td>2.2*</td>
<td>0.6*</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>U</td>
<td>4333 M6</td>
<td>0.34</td>
<td>0.94</td>
<td>0.009</td>
<td>0.009</td>
<td>0.27</td>
<td>1.02</td>
<td>2.30</td>
<td>0.57</td>
<td>0.14</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>V</td>
<td>4333 M6</td>
<td>0.36</td>
<td>0.83</td>
<td>0.009</td>
<td>0.004</td>
<td>0.29</td>
<td>0.87</td>
<td>2.13</td>
<td>0.51</td>
<td>0.12</td>
<td>0.009</td>
<td>0.055</td>
</tr>
</tbody>
</table>

* Specified values only
TABLE 4.3
Values for Charpy V-Notch Tests for Axial and Radial Orientations and Inside and Outside Positions for Various Materials

Tolerance values in first four columns indicate bounds in the scatter of three different specimens tested whereas the last column shows variation in the first four columns

<table>
<thead>
<tr>
<th>Material</th>
<th>Orientation and Position (joules)</th>
<th>Overall</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Axial Inside</td>
<td>Axial Outside</td>
</tr>
<tr>
<td>DM</td>
<td>31 ±0 -1</td>
<td>32 ±2 -1</td>
</tr>
<tr>
<td>H</td>
<td>33 ±1 -0</td>
<td>43 ±2 -2</td>
</tr>
<tr>
<td>U</td>
<td>49 ±2 -3</td>
<td>50 ±0 -0</td>
</tr>
<tr>
<td>V</td>
<td>34 ±1 -0</td>
<td>37 ±1 -0</td>
</tr>
</tbody>
</table>

TABLE 4.4
Values for Hardness Taken from Specimens Used in the Charpy V-Notch Impact Tests

For significance of tolerance values, see Table 4.3

<table>
<thead>
<tr>
<th>Material</th>
<th>Orientation and Position (Vickers pyramid numeral)</th>
<th>Overall</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Axial Inside</td>
<td>Axial Outside</td>
</tr>
<tr>
<td>DM</td>
<td>381 ±4 -5</td>
<td>379 ±11 -9</td>
</tr>
<tr>
<td>H</td>
<td>368 ±10 -10</td>
<td>369 ±10 -10</td>
</tr>
<tr>
<td>U</td>
<td>351 ±6 -10</td>
<td>354 ±8 -9</td>
</tr>
<tr>
<td>V</td>
<td>381 ±6 -6</td>
<td>382 ±6 -9</td>
</tr>
</tbody>
</table>
CHAPTER 5
SPECIMENS

During manufacture, the tubing undergoes considerable mechanical working in order to improve its mechanical properties as well as to form the tube into the required dimensions. This leads to material property differences in the axial (longitudinal) and radial (transverse) orientations of the tubing. Fatigue crack growth and fracture properties may therefore also be dependent on these orientations.

It would clearly be difficult to conduct a test on a section of tubing such that both axial and radial fatigue crack growth or fracture properties could be investigated. For this reason, specimens were cut from the tubing in such a manner that only one orientation is investigated at any one time.

5.1 SPECIMEN TYPES

Five specimen types have been used in this investigation which are shown in Figure 5.1.

Axial fatigue crack growth properties were investigated using compact tension specimens (CKS) and double cantilever beam (DCB) specimens. Dimensional constraints of the tubing under investigation restricted the width of the specimens to about 21 mm. These specimens were loaded using pins placed through the specimen pin holes resulting in crack growth in the direction shown. Side grooves were machined in the DCB specimens to guide the crack down the centre axis of the specimen as well as provide greater constraint on the crack front. It is possible to cut four CKS or DCB specimens from a given length of tubing at angular positions 0°, 90°, 180° and 270°, although sometimes, for convenience of specimen manufacture, only 90° and 270° specimens were cut. In order to investigate changes in
axial properties as a result of radial position, DCB specimens were cut from the inside and outside positions of the tube. The width of these specimens was limited to 10 mm. Only CKS were used to obtain fracture properties. Figures 5.2 and 5.3 show the dimensions of the CKS and DCB specimens.

Three specimen types, C-shaped, split-ring (SR) and RING specimens, were used in an attempt to determine fatigue crack growth and fracture properties in the radial or transverse direction. C-shaped specimens were used most frequently in this investigation as they are relatively straightforward to test and two specimens can be cut from a slice of tubing. Properties close to the bore of the tubing are particularly important and specimens without precut slots are therefore required. The length of the arms of the C-shaped specimen are too short to provide a sufficiently large stress intensity factor close to the bore of the tubing when reasonable loads are applied. The split-ring (SR) specimen, on the other hand, requires loads less than half that of the C-shaped specimen. As a result, the SR specimen was used where properties close to the bore were required. Both C-shaped and SR specimens were loaded by pins placed through the specimen pin holes. The RING specimen has the geometry of a complete slice of tubing and is loaded by two pins on the inside of the tube. This specimen is probably the easiest to manufacture but very high loads are needed to provide the required stress intensity factor. It is therefore not well suited to obtain properties close to the bore of the tubing. Hoop residual stresses in the tubing are least affected in this type of specimen when parted from the tubing, which makes the RING specimen particularly useful in the study of crack growth through residual stress fields formed by autofrettage.

Figures 5.4 and 5.5 show the dimensions of the C-shaped and SR specimens. Dimensions for the SR specimen can also be used for the RING
specimen.

The bore and outside of the tubing were usually not machined in the radial specimens described in the preceding paragraphs. In practice, however, the bore spirals down the length of the tubing resulting in the bore and tubing outside not lying concentric to one another. Eccentricities larger than 1 mm were removed by machining the outside concentric with the bore of the tubing. The bore of the tubing is never machined.

The CKS is covered by BS5447:1977 and ASTM E399. The current edition of the latter standard (E399-78a) now also includes the C-shaped specimen.

5.2 THICKNESS REQUIREMENTS

Valid fracture toughness tests require that the minimum specimen thickness, \( B \), exceeds \( 2.5 \left( \frac{K_{Ic}}{\sigma_y} \right)^2 \), where \( K_{Ic} \) is the critical mode I fracture toughness and \( \sigma_y \) is the 0.2% offset yield strength. Data supplied by a manufacturer at the commencement of the investigation indicated that a critical fracture toughness value of around 70 MPa√m could be expected, which with a yield strength of 1 GPa would result in a minimum required thickness of 12 mm.

The specimen thickness for a CKS is limited by the inside and outside radii \( R_i \) and \( R_o \), respectively, and the half specimen height, \( H \). A theoretical maximum value of \( B \) can be easily determined and expressed as

\[
B = \sqrt{R_o^2 - H^2 - R_i^2}.
\]

Inserting values for tubing of DM material, for instance, would result in a maximum thickness \( B \) of 24 mm. Practical constraints reduce this value to about 21 mm, which was used universally throughout the tests. No maximum thickness limit exists for radial type specimens (although ASTM E399-78a specifies the range \( B = 0.25 \nu \) to 0.5 \( \nu \)) and a \( B \) value of 25 mm was taken to be sufficient.
5.3 ELECTRICAL POTENTIAL CALIBRATION CURVES

The calibration curve for an EPS relates crack length to the potential difference across the sensing leads or specimen output. The calibration curve can be optimised by assessing various combinations of current and sensing lead positions. The following EPS behaviour is required.

A steep curve yields good sensitivity and a high absolute value reduces the effects of noise and improves stability. The deviation from a given calibration curve as a result of mispositioning the sensing leads should be small to give good repeatability. A linear function provides constant sensitivity for all crack lengths and simplifies calibration procedure.

A convenient technique to optimise the positions of current and sensing leads using boundary integral equation and finite element methods has been described by Klintworth & Webster (1979). These are two-dimensional methods which ignore the effect of specimen thickness; consequently, AC and DC techniques are treated alike. The AC EPS calibration curve may deviate from the optimum curve obtained as a result of the presence of magnetic materials close to the specimen. The influence of specimen geometry and its surroundings on the EPS is considered below.

Figure 5.6 compares two calibration curves for identical CKS specimens with a curve obtained using the BIE method. The positions of current and sensing leads are shown in Figure 3.7 with specimen dimensions detailed in Figure 5.2. Curve A has been obtained from a specimen loaded using ferritic steel shackles with a 3.3 kHz AC EPS, whilst curve B corresponds to the same specimen but loaded using austenitic (non-magnetic) stainless steel shackles. The slopes of these two curves are similar which indicates the existence of a constant residual potential, perhaps due to a change in the magnetic earth loop. The predicted curve does not
follow the same slope, however, which may be due to the assumption of a two-dimensional model in the calculations, whereas in the real specimen of finite thickness a skin effect will exist.

The effect of specimen thickness is clearly demonstrated in Figure 5.7 which shows two C-shaped specimens of different thickness. Identical specimen geometries were otherwise used as shown in Figure 5.4 with current and sensing lead positions shown in Figure 3.7. Similar shackles were used. The difference in these curves can be readily explained if a specimen is considered having both front/back and side surfaces which conduct the alternating current in the characteristic skin-layer associated with magnetic materials. The relative change in the areas of these surfaces would result in a new calibration curve.

Comparison between Figures 5.6 and 5.7 shows the effect of specimen type on the EPS calibration curves. The curves of Figure 5.6 lie close to curve B of Figure 5.7. Curve A of Figure 5.7 has the best sensitivity and absolute output of all the curves shown in Figures 5.6 and 5.7. The results of between 3 and 6 specimens have been plotted on each curve. Repeatability is generally good with the exception of C-shaped specimens at low crack lengths. The scatter in this region is due to the sensing leads being placed too close to the crack tip. This error is not particularly significant since the stress intensity factor shows little change due to an error in $a$ at low crack lengths.

5.4 STRESS INTENSITY FACTOR CALIBRATION

It is essential in fracture and fatigue crack growth studies that a reliable function for the stress intensity factor exists. Whereas the stress intensity factor need only be known in the range $a/W = 0.45$ to 0.55 for fracture work, a considerably larger range, normally $a/W = 0.25$ to 0.75, is required in fatigue crack growth investigations.
Stress intensity factor functions for standard specimens, such as the CKS, are well documented in the literature (for example, BS5447:1977, ASTM E399-78a, Srawley (1976) and Saxena & Hudak (1978)). Variations from these standard specimens, a change in the usual $a/W$ range or new types of specimens necessitate the use of analytical, numerical or experimental methods to determine the stress intensity factor.

5.4.1 The Boundary Integral Equation (BIE) Method

The BIE numerical method was used to compute the stress intensity factor for CKS, DCB, C, SR and RING specimens. In this method, integral equations are used to relate the boundary tractions and displacements, which when solved yield the displacements, tractions and stress tensor on the whole boundary. From these data, displacements and stresses at any specified interior point may be obtained.

In practice, numerical solutions to the BIE are found by discretising the boundary of a body into elements where the unknowns are related by simple algebraic functions. A two-dimensional problem is thus reduced to specifying the coordinates and boundary conditions on the body's (single dimension) boundary. Redundant interior points need not be specified. This leads to a substantial reduction in the cost of data preparation.

A typical mesh is drawn in Figure 5.8 which shows the elements generated for a CKS. Only half the specimen has been modelled for reasons of symmetry. Elements containing three nodes per element, and hence a quadratic variation of variables across the element, start at the back of the specimen and move around the half specimen in an anti-clockwise direction. A number represents a node, hence element 1, for instance, is defined by nodes 1, 2 and 3. For simplicity, only the units digit of a number larger than 9 is shown. Element 32 completes the outer
boundary and a new boundary is generated in a clockwise direction, starting with element 33 so defining the pin hole. Element 40 completes the pin hole. Note that the line defining the boundary has not been completed in elements 32 and 40 in order to clearly show the start and end of each boundary.

Some attempt has been made to increase the density of elements in those areas where the largest changes in stress can be expected. This technique can be extended further by concentrating elements close to the crack tip. This has not been done here since it involves a partial regeneration of the mesh for each crack length considered.

Boundary conditions specified consist of a fixed vertical displacement on elements to the right of the crack tip and a fixed horizontal displacement on element 1 to prevent the specimen moving in the horizontal direction. A vertical, uniformly distributed load was placed on elements 34 and 35 to simulate pin loading.

The two-dimensional BIE program used in this investigation was developed by Remzi (1980) and yielded results for tractions, displacements, stresses and strains, from which values of the stress intensity factor could be calculated.

Various techniques to determine $K_I$ (see, for example, Chan et al (1970) and Gallagher (1978)) from BIE stress/strain analysis exist which can be classified as direct and indirect methods. Direct methods can be sub-divided into displacement methods and stress methods which are based on the displacement and stress field solutions, respectively, in the vicinity of the crack front. The strain energy release rate provides the basis of the indirect method.

The displacement method utilises the relationship for the vertical displacement, $u_2$, at a distance $r$ from the crack front


\[ \sigma_2 = \frac{4K_I}{H' \sqrt{2\pi r}} \]  

(5.1)

\( H' \) is the material stiffness which is equal to Young's modulus, \( E \), for plane stress and \( E/(1-v^2) \) for plane strain. Equation (5.1) can be transposed to yield a value for \( K_I \), called \( K_I^* \), at a given \( r \) and displacement \( V \). Hence

\[ K_I^* = \frac{H' \sqrt{2\pi}}{4} \frac{V}{\sqrt{r}} \]  

(5.2)

As equation (5.2) is only true as \( r \) approaches zero and \( V \) is not well defined by the BIE method close to the point of singularity, \( K_I \) must be extrapolated from values of \( K_I^* \) found at relatively high \( r \) values.

A similar technique is used with the stress method, but the relationships

\[ \sigma_2 = \frac{K_I}{\sqrt{2\pi r}} \]  

(5.3)

and

\[ K_I^* = \sigma \sqrt{2\pi r} \]  

(5.4)

are used, where \( \sigma \) is the stress at distance \( r \) from the crack front.

The indirect method utilises the change in energy of the stress field as a result of an increment in crack length which, when the applied load \( P \), acting in the loading line is considered, can be expressed as (see Section 2.2)

\[ \frac{K_I B \sqrt{W}}{P} = \sqrt{\frac{1}{2(1-v^2)}} \frac{\partial(BE/V}{\partial(a/W)} \]  

(5.5)

for conditions of plane strain. Since the energy of the crack tip only contributes a small amount to the overall energy of the body, it can be argued that an accurate representation of the crack front stress is
unnecessary. This provides a considerable advantage over the direct methods where elements should be closely spaced around the crack front.

Stress intensity factor results for the specimens presented in the following sections were obtained using the indirect method. The specimen compliance was computed by considering the displacement along the loading line for a given applied load. The stress intensity factor was generally found by differentiating a quadratic curve fitted to three adjacent compliance versus crack length data points, although other methods, described later, were also used.

5.4.2 Experimental Methods

Stress intensity factors were computed using the indirect energy method for C-shaped specimens from data obtained during fatigue crack growth tests usually performed under constant $\Delta K$ conditions. Running the test under these conditions ensures that excessive displacements are not obtained at high crack length since the mean load, $P_{\text{mean}}$, and load amplitude, $\Delta P$, slowly decrease during the test. Values for compliance were obtained at about 100 increments in crack length during the test by fitting a linear curve to displacement versus load data obtained whilst unloading the specimen from a load of $P_{\text{mean}} + \Delta P/2$ in six steps down to $P_{\text{mean}}$. Fatigue cracking and not machine slotting the specimen allows compliance measurements to be read under sharp crack conditions (see, for example, Bonesteel et al (1978)).

The differential of compliance with respect to crack length $\frac{\alpha(BE'/P)}{\alpha(a/W)}$ required in equation (5.5) was found by first fitting a quadratic curve to seven adjacent compliance versus crack length data points and then analytically finding the gradient at the centre of the fitted curve (see Software Section 3.4.6).
5.4.3 **Application to the Compact Tension Specimen (CKS)**

Standards for compact tension specimens specify major dimensions, including the pin hole diameter, as ratios of the net specimen width. Fatigue crack growth tests generally require larger diameter pins, particularly where insulating spacers are used to separate the specimen electrically from the shackles, thereby considerably increasing the bending stresses imposed on the pins.

The BIE method was applied to the CKS in order to ascertain the effect of pin hole diameter as well as confirm that the program correctly predicts the same stress intensity factor as defined by the standards.

Figure 5.8 shows the BIE mesh used for a CKS with a net width \( W \) of 41.7 mm (see Figure 5.2). Figure 5.9 shows the non-dimensionalised compliance for three different pin hole diameters. 10 mm pins were initially used since hardened and ground pins were available in that dimension but considerable breakages were experienced. Pins of 12.7 mm diameter were subsequently used, whilst 10.42 mm corresponds to the pin hole diameter specified by the standards. The effect of changing the pin hole diameter is to shift the compliance curves, which were obtained by measuring the displacement at the top of the pin hole, vertically by a constant amount. Since the differential of a constant zero, this shift does not show in the stress intensity factor. A compliance curve derived by Saxena & Hudak (1978) is shown for comparison.

Figure 5.10 shows the non-dimensional stress intensity factor found by the BIE method compared to functions by BS5447:1977 and Srawley (1976). The latter is said to provide greater accuracy at high and low values of crack length. Comparison is favourable considering the relatively crude method with which the derivative of compliance is calculated.
5.4.4 Application to a DCB Specimen

The compliance curve for a DCB specimen is shown in Figure 5.11 with dimensions given in Figure 5.3. The corresponding stress intensity factor curve is shown in Figure 5.12. DCB specimens usually contain side grooves where \( B \) is the net width of the crack plane. The BIE method used in this investigation, being a two-dimensional model, is clearly unable to describe the influence of side grooves. Comparison with the relationships by Mostovoy et al (1967)

\[
\frac{BEV}{P} = 8 \frac{W}{H} \left\{ \left( \frac{a}{W} \right)^3 + \left( \frac{H}{W} \right)^2 + \frac{a}{W} \right\}
\]

and

\[
\frac{KB \sqrt{W}}{P} = \sqrt{\frac{B}{Bn} \frac{4}{1 - \nu^2}} \frac{W}{H} \left\{ 3 \left( \frac{a}{W} \right)^2 + \left( \frac{H}{W} \right)^2 + 1 \right\}
\]

is excellent when \( a_o \), a rotation correction factor expressed in the form of an increase in crack length, equals \( 0.4 H \). Mostovoy et al (1967) and Klintworth (1977) found experimentally, however, that a value of \( 0.6 H \) and \( 0.7 H \), respectively, gave good agreement with side grooved specimens. The additional \( 0.2 H \) and \( 0.3 H \) probably depends on the depth of side grooves and may be attributed to their influence in increasing the specimen compliance.

Bonesteel (1978) considers compliance and stress intensity factor curves derived from specimens without side grooves with fatigue cracks and machined slits. Both compliance and stress intensity factor curves from the fatigue cracked specimens lie below those of the machine slitted specimens.

The latter specimen curves have been plotted in Figures 5.11 and 5.12. There is good agreement between the BIE method and Bonesteel between \( a/W = 0.45 \) to 0.65 when the stress intensity factor is considered.

The curve attributed to Srawley (1976) and which takes side
grooves into account understandably overestimates the value of stress intensity factor for non-side grooved specimens.

5.4.5 Application to the C-shaped Specimen

Figure 5.13 shows the 40 element mesh first used to model a C-shaped specimen with dimensions given in Figure 5.4. Similar boundary conditions to those used in the CKS were used. Some doubt was expressed in the ability of the mesh to accurately describe body displacements at large values of crack length, where the density of the lines of stress is particularly high through the unbroken ligament of the specimen. A refined mesh with 54 elements, called the fine mesh, was generated and is shown in Figures 5.14 and 5.15. The latter figure shows an enlarged view of the crack line or plane. It can be seen that the additional 14 elements have been distributed in this region.

Compliance curves for both the coarse and fine meshes applied to a C-shaped specimen of DM geometry have been plotted in Figure 5.16. Agreement is good for $a/W$ values up to 0.6, whereafter the coarse mesh results display lower values, probably due to the reasons expressed above. The same figure also shows quadratic curves which have been fitted to successive groups of three adjacent compliance values obtained from the fine mesh. The differential of the resultant compliance quadratic curves is used in the calculation of the stress intensity factor. Due to the strongly increasing nature of the specimen compliance, this method appears to overestimate the gradient and hence stress intensity factor. Alternative methods to determine the differential of compliance with respect to crack length, such as running the program twice with original and perturbed crack lengths, may provide more confident results.

Figure 5.17 shows the corresponding stress intensity factor curves for the fine and coarse meshes. Fifth degree polynomial fits have
been fitted to these curves. An expression by Underwood & Kendall (1977), later incorporated in ASTM E399-78a, is also shown, together with values found during a constant \( \Delta K \) fatigue crack growth experiment. There is generally good agreement between curves, particularly when errors discussed in the preceding paragraphs are taken into account. The most significant discrepancy may occur in the region \( \rho/W = 0.3 \) to 0.6 where the Underwood & Kendall curve lies lower than the others. This error may result from the different radial positions of the pin hole in the DM specimen used in the experiment and BIE program (see Figure 5.4) compared to that specified by Underwood & Kendall.

Curves from the BIE method using a coarse mesh, experiment and Underwood & Kendall are compared for C-shaped specimens of \( u \) and \( v \) geometries in Figures 5.18 and 5.19. The coarse BIE mesh appears to be less sensitive with these dimensions than the DM geometry, resulting in better comparison. Agreement between experimental data and Underwood & Kendall is excellent, except in the region \( \rho/W = 0.3 \) to 0.6 where the Underwood & Kendall curve again lies marginally lower than the other curves.

The two hiccups seen in the experimental results of Figure 5.19 can probably be attributed to experimental errors such as a movement of the displacement transducer or an incorrect EPS reading due to an electrical short occurring between the cracked surfaces. Of some interest is curve B in the same figure which shows the large errors that can be experienced when a high order polynomial curve, fitted to the complete range of compliance data, is differentiated in order to obtain the stress intensity factor.

Due to the generally good agreement with both BIE and experimental results, the relationship by Underwood & Kendall was used for the calculation of stress intensity factor for all C-shaped specimens.
5.4.6 Application to SR Specimens

Although the slit ring specimen is an extreme case of the C-shaped specimen with a large \( \frac{X}{W} \) ratio, it would be reckless to assume the relationship by Underwood & Kendall also applies in this type of specimen. Since the BIE method's ability to satisfactorily calculate the stress intensity factor had been confirmed in the C-shaped specimens, it can confidently be applied to the SR specimen.

Figure 5.20 shows the mesh used for the SR specimen, with dimensions given in Figure 5.5, with a detailed view of the elements in the crack line region shown in Figure 5.15. Note that the straight lines connecting adjacent nodes actually follow a quadratic function, and are not straight lines as shown here. Figure 5.21 shows the stress intensity factor calculated by a BIE method with coarse and fine meshes, a finite element method adapted by Ang (1980) and a curve for C-shaped specimens by Underwood & Kendall. The coarse BIE mesh yielded poor results as already expounded. Two methods to find the gradient of compliance were used with the fine mesh BIE results. Whereas the method where a quadratic curve is fitted to three adjacent points and so determining the gradient produced results which were too high, a simple central difference method gave lower more reasonable values. The results of this latter method interestingly correspond very closely with the relationship for C-shaped specimens by Underwood & Kendall, demonstrating the wide range of application that relationship enjoys.

An important application of the SR specimen is to obtain crack growth rates at low \( \frac{a}{W} \) values. The curve obtained from the BIE method extends down to \( \frac{a}{W} = 0.05 \). Lower values can be obtained using a model of an edge crack in a finite width sheet. A similar stress system to the SR specimen for a \( \frac{a}{W} \ll 1 \) can be obtained by superposition of a bending moment, \( M \), with uniaxial tensile load, \( P \), which respectively result in
stress intensity factors (Rooke & Cartwright (1976))

\[
\frac{KB\sqrt{W}}{M} = 6\sqrt{\pi} \left( \frac{a}{W} \right) \{1.12 - 1.39 \left( \frac{a}{W} \right) \} \tag{5.8}
\]

and

\[
\frac{KB\sqrt{W}}{P} = \sqrt{\pi} \left( \frac{a}{W} \right) \{1.12 - 0.23 \left( \frac{a}{W} \right) \} \tag{5.9}
\]

Only the first two terms are shown since small cracks are considered. Superimposing the above stress intensity factors and replacing \( M \) with \( P(X + W/2) \) gives

\[
\frac{KB\sqrt{W}}{P} = \sqrt{\pi} \left\{ 11.91 \left( \frac{X}{W} \right) + 7.94 - \left( \frac{a}{W} \right) (14.78 \left( \frac{X}{W} \right) + 7.80) \right\} \tag{5.10}
\]

The single curve shown in Figure 5.22, which contains the term \( \sqrt{a/W} \), fits both the BIE data and equation (5.10) and hence this curve was used as the \( K \) calibration for the SR specimen.

5.4.7 Application to RING Specimens

The RING specimen has previously been used by Jones (1974) in toughness tests and by Crymble, Goldthorpe & Austin (1977), who experimentally verified analytical and semi-empirical \( K \) calibrations for thick wall cylinders with single and multiple straight fronted cracks. The RING specimen was initially used in this investigation to investigate the growth of a fatigue crack through the residual stress field induced by autofrettage. Since fatigue crack growth values close to the bore of the tube are important, stress intensity factor values for low \( a/W \) ratios were of considerable interest.

The mesh used for the RING specimen is shown in Figure 5.23 and with the same dimensions as the SR specimen. A uniformly distributed load is placed on elements 27 and 28, whilst fixed boundary displacement
conditions in the vertical direction were applied to elements 18 to 21 and on all elements in the crack plane to the right of the crack. No pin holes needed to be modelled.

The compliance curve for the RING specimen subjected to a central tensile load at the bore is shown in Figure 5.24. Local quadratic curves have been fitted to each combination of three adjacent data points. The goodness of fit is satisfactory due to the gentle change in the slope of the curve.

The resulting $K$ calibration is shown in Figure 5.25 with a fifth degree polynomial fit to the data given as curve A. This curve provides an inadequate description of the stress intensity factor. Plotting the data on a new set of axes and fitting a fourth degree polynomial as shown in Figure 5.26 provides substantially better results. This new curve is also shown in Figure 5.25 as curve B.
A variety of parameters such as degree of autofrettage, temperature and manufacturing route may affect the resistance of the tubing to failure. A set of experiments was therefore devised to investigate how the performance of various tubing materials from different manufacturing routes was affected when subjected to various conditions.

6.1 VARIABLES - FATIGUE CRACK GROWTH TESTS

Fatigue crack growth tests were performed under a limited number of different conditions on all four materials, whilst tests to determine the effects of other parameters such as temperature and residual strain were performed on DM material alone. The parameters investigated are described below. The list is clearly not complete since other effects, such as pressure and environment, have not been investigated.

6.1.1 Stress Intensity Factor Range

A changing stress intensity factor range, $\Delta K$, accompanies a growing crack in thick wall tubing subjected to pulsating internal pressure. The value of $\Delta K$ may begin at approximately $10 \text{ MPa}\sqrt{m}$, which corresponds to an elliptical crack 100 $\mu$m deep in tubing of diameter ratio 2 under a pressure fluctuation of 250 MPa, and increase until fracture or leak occurs. Since a flaw size of 100 $\mu$m can readily be found in thick wall tubing, it would appear that threshold values are not necessarily important.

A similar $\Delta K$ range should be investigated in experiments on simple specimens cut from tubing. These tests are conducted at constant load range, $\Delta P$, which allows $\Delta K$ to increase as the crack length or $a/W$
grows. If the tests are correctly planned and no experimental difficulties are encountered, a full range of $\Delta K$ from 10 MPa\(\sqrt{m}\) to fracture can be obtained from one specimen.

6.1.2 Radial and Axial Tubing Properties

Using C-shaped and RING specimens to determine radial fatigue crack growth rates, and CKS and DCB specimens for the corresponding properties in the axial direction, allows the effect of tube anisotropy to be investigated. Both radial and axial properties were found for all four tubing materials.

6.1.3 Residual Stress and Work Hardening as a Result of Autofrettage

The process of autofrettage, where residual compressive stresses are induced at the bore of the tubing, can alter the fatigue crack properties by work hardening the material as well as providing residual compressive stresses which must be overcome before a crack can open and propagate. Autofrettage is therefore a commonly used method to improve the tubing’s resistance to crack growth. Tests were conducted on DM material only, to determine the effect of autofrettage in both the axial and radial directions at room as well as elevated temperatures.

Two types of specimen were used to determine properties in the radial direction since this direction is most affected by autofrettage. RING specimens probably still retain most of the residual stresses originally present in the autofrettaged tubing, whereas C-shaped specimens are subjected to a redistributed system of stresses. The combined use of both specimen types may therefore indicate how residual stresses affect crack growth and what material changes have taken place as a result of autofrettage.
6.1.4 Temperature

Most tests were conducted at room temperature. Plant temperatures normally run substantially higher at about 250°C which may influence fatigue crack growth. Temperature may also have a further effect on the residual stresses in autofrettaged tubing.

Additional hot tests at 300°C were performed on DM material only in the as-received and autofrettaged conditions and in both radial and axial directions.

6.1.5 Mean Stress

It is well known that the loading cycle can affect fatigue crack growth properties. To simulate repeated pressure and high mean pressure loading cycles seen in the plant, two extremes of $R$ ratio or $K_{min}/K_{max}$ were chosen. Tests were performed on DM material only at $R$ ratios of 0.05 and 0.5. Tests on other material and tests on autofrettaged tubing were performed at $R = 0.5$.

6.1.6 Frequency of Loading

The frequency of the loading cycle could affect the fatigue crack growth properties. Frequencies of 5 Hz and 30 Hz were compared in the radial direction of DM tubing. Other tests were performed at between 10 Hz and 30 Hz, depending on the length of crack in the specimen. Long cracks lead to very large displacements and hence the power required to fatigue cycle the specimen may exceed the capacity of the machine unless the frequency is reduced.

6.1.7 Specimen Thickness

Due to dimensional constraints, the thickness of the axial specimens could not be increased and hence changes in specimen thickness
could not be easily investigated in this direction. Radial specimens, however, are not limited in thickness and specimens 40 mm thick were compared with the usual 25 mm thick specimens in DM materials.

6.1.8 Specimen Geometry

In fracture mechanics, the stress intensity factor is used to characterise fatigue crack growth rate. Specimens of different geometries cut from the same tubing should therefore yield identical results if stress intensity factor calibrations are known for those specimens. In order to ensure that the fatigue crack growth process is properly quantified, specimens of different geometry were cut from the DM tube in both radial and axial directions.

6.1.9 Variation with Tube Radius

In the conventional test where \( \Delta P \) is kept constant and \( \Delta K \) rises as \( \frac{a}{W} \) increases, the effect of tube radius cannot be readily investigated. A constant \( \Delta K \) test, on the other hand, allows tubing properties to be investigated as a function of radius for a given constant \( \Delta K \). Constant \( \Delta K \) tests were therefore used on C-shaped specimens for all tubing materials investigated.

The effect of tube radius for properties in the axial direction was investigated in DM material only using DCB specimens cut from the inside and outside of the tubing wall.

6.2 PROCEDURE - FATIGUE CRACK GROWTH

The procedure followed during a fatigue crack growth test is greatly simplified when using computer control. Section 3.4.6 describes the software used to control the test.
6.2.1 Mechanical Test Set-Up

With the servo-hydraulic testing machine in displacement control and the computer disconnected for safety reasons, a specimen was placed in the shackles. The pins were greased with an anti-fretting compound. Care was taken to see that the positive current lead was closest to the insulated shackle (the load cell is insulated from the testing frame) and that insulating washers were placed between the specimen and shackles. The twisted current and sensing leads were secured so that they remained in the same position throughout the test. A small residual load equivalent to, say, 10% of the total load was then applied to the specimen.

In order to go from displacement to load control, it is best to engage the standby module. This was achieved by adjusting the load setting on the standby module to equal that displayed on the digital load readout before switching in the standby load. The control mode switch was then changed from displacement to load control. Finally, a check was made that the static load input was set to zero, the signal generator was switched out from the testing machine and that the signal generator output potentiometer was set to 10.0.

In high temperature tests, a temperature controlled furnace was placed around the specimen. A constant temperature was maintained before the test was commenced. Temperature measurements were taken at two places on the specimen using Chromel/Alumel thermocouples.

6.2.2 Setting Up the Computer

Program variables were set in subroutine 2000 of the program listed in Figure 3.25. The micro-computer’s editor was quickly used to type in new test details or leave other details unchanged from the previous test. Briefly, values for the following variables are required.
(i) Test description.
(ii) Constant $\Delta P$ or $\Delta K$ test.
(iii) Ramp or continuous test options. The ramp option is chosen when experimental $K$ values are to be found.
(iv) Number of measurements on one side of the data tape.
(v) Plot option required?
(vi) Specimen type and dimensions.
(vii) Material properties: Young's modulus and Poisson's ratio.
(viii) Transducer ranges.
(ix) Frequency of test.
(x) Mean load.
(xi) Load amplitude.
(xii) Standby load.
(xiii) Overload required to initiate crack, decreasing quadratically for a specified number of cycles.
(xiv) Maximum EPS output allowed.
(xv) EPS increment before next measurement cycle.
(xvi) Value of $K$ for proper EPS reading.
(xvii) EPS calibration curve to find $\alpha/W$.
(xviii) Axis definitions for plotted graphs.

Once these parameters were typed in and a fresh data tape placed in the tape recorder, computer control was instituted by typing RUN. The computer was programmed to automatically go into its standby mode in order to match the computer generated load to the present load applied to the specimen. This was achieved by, firstly, reconnecting the computer to the testing machine's input (this was disconnected for safety reasons) and typing the same load on the computer keyboard that was displayed on the load readout. The testing machine's standby load was then disengaged and
computer control was possible.

6.2.3 Measurements and Real-Time Analysis

Measurements were taken during the test and a real-time analysis performed (see Section 3.4.6). Observations and results were printed on the printer and recorded for further reanalysis on a data tape.

6.2.4 Reanalysis of Results

Following the conclusion of the test, reanalysis may be required or results presented in a more professional manner. This is possible since all data obtained during the experiment are stored on a data tape.

It has been the approach in the Mechanical Engineering Department at Imperial College to provide a reanalysis station which can be used to service computer systems, actively used to control and obtain measurements in material tests. The reanalysis station is provided with a floppy disc unit, a high quality digital plotter and a direct link with the College's Computer Centre. Data from tape are transferred to disc to allow faster reading of data for further, sometimes repeated, reanalysis. Reanalysis of results may be required if, for instance, a correction to the EPS calibration curves must be made to take bowing into account. The AC EPS measures crack length as a result of the parting of the two side surfaces by the newly formed crack as well as the creation of two new cracked surfaces. Hence, the AC EPS does not fully take crack bowing into account. Crack bowing was therefore measured at the end of the test and all crack length readings measured by the AC EPS were multiplied by a so-called bowing factor. The bowing factor was found by dividing the area of the crack plane calculated, assuming the crack was not bowed, into the actual area of the crack plane.
The output from the digital plotter can be used in publications directly. Most curves in this thesis were obtained in this manner. The link to the Computer Centre can be used to transfer files if further, more complex calculations are required.

6.3 PRECAUTIONS - FATIGUE CRACK GROWTH

A variety of precautions were adhered to during fatigue crack growth tests.

Overloading the specimen during the fatigue crack cycle or when an optical crack length is required can cause retardation of crack growth. Furthermore, since the plastic zone size caused by a dynamic load is smaller than that of a static load, the load applied to the specimen when reading the crack length optically should be about half the maximum of the load cycle. Crack retardation difficulties are experienced when decreasing the load to obtain threshold values. Considerable care should also be taken not to decrease the load too rapidly during these tests (see Section 3.4.1).

Some difficulty was experienced when propagating a crack from a blunt notch. Cracks often propagate from one corner instead of a straight crack front perpendicular to the direction of crack growth. A chevron shaped crack is sometimes used to encourage cracking at the centre but this approach cannot be used when properties close to the bore of the tube are to be investigated. A reasonable solution was found in scratching the blunt machined notch in the centre (as seen from a plan view) which encouraged crack growth in a stable, straight fronted manner.

Cracks were also initiated using only a short scriber scratch at the bore of the unnotched SR specimens. Care had to be taken to round off the sharp corners at the bore of the tubing close to the crack plane to prevent cracks initiating there. This technique proved less successful.
with autofrettaged tubing where the centrally placed scratch grew as an elliptical crack for a considerable way before crack growth on the specimen sides commenced. This type of growth is difficult to quantify when applied to blunt cracks using an AC EPS. High initial $\Delta K$ values appeared to encourage stable crack growth.

Optical crack length should be measured on both sides of the specimen in order to take into account any skew growth. This includes the DCB specimen where a duplicate reading is important since measurement of crack length of DCB specimens is very difficult.

The accuracy and repeatability of the AC EPS can be affected in a number of ways. The current and sensing leads must not be moved during a test and temperature changes of the specimen must be avoided or taken into account. A conducting clip gauge can affect an existing calibration curve.

When compliance measurements are made, care should be taken not to take load and displacement measurements that are so low that crack closure occurs. On the other hand, too high values of load will result in the specimen being overloaded.

Side grooves need to be machined into DCB specimens to guide the crack along the centre of the specimen. An arm of the specimen may break off if the side grooves are insufficiently deep. A region of instability appears to exist when the side grooves are just deep enough to prevent breaking of an arm but not sufficiently deep to totally contain the crack. This instability results in crack propagation in a wave-like motion frequently crossing the crack plane, and this may lead to incorrect results.

Pin breakages can go unnoticed during the test and only discovered when the specimen is removed. Sufficiently large pins should be selected for the specimen under test and the breadth of shackle used.
6.4 VARIABLES - FRACTURE TOUGHNESS TESTS

Fracture toughness tests were performed on the same materials that were subjected to fatigue crack growth tests. Additional tests were performed on DM material to investigate the effect of temperature and residual stress.

The following material parameters were investigated.

6.4.1 Radial and Axial Tubing Properties

Radial fracture toughness values were obtained using C-shaped specimens, whilst axial values were found using CKS. Both radial and axial properties were found for all materials.

6.4.2 Residual Stress and Work Hardening as a Result of Autofrettage

Tests were conducted on DM material only to determine the effect of autofrettage in both axial and radial directions under room as well as elevated temperatures.

6.4.3 Temperature

Most tests were conducted at room temperature, whereas additional tests were performed at 300 C on DM material only in both the as-received and autofrettaged conditions.

6.5 PROCEDURE - FRACTURE TOUGHNESS TESTS

Fracture toughness tests were performed according to BS5447:1977. Each specimen was fatigue precracked immediately before commencing with the fracture toughness test.

A clip gauge containing four strain gauges in a full Wheatstone bridge was used to measure displacement on CKS, whilst an LVDT based displacement gauge, described in Section 3.3, was used on C-shaped specimens. The EPS
was used to monitor electro-magnetic changes in the specimen as it was loaded. The load cell in the testing machine was used to record load.

The micro-computer was used to generate a loading ramp and take up to 500 readings per test of the above parameters as well as ram position of the servo-hydraulic testing machine. This latter reading was taken as a precaution and could be used instead of the specimen displacement in extreme cases. These measurements were printed on the printer and recorded on data tape. The computer program is described in Section 3.4.6.

Values of load and EPS output were plotted against specimen displacement on a dual channel x-y recorder. The 10 V maximum EPS output signal was offset with a 9 V battery and the difference plotted using a 25 mV/cm scale. 500 mV/cm scales were used for load and specimen displacement readings. Tests at 300 C were conducted in a temperature controlled furnace. The temperature of the specimen was measured using two Chromel-Alumel thermocouples.

6.6 PRECAUTIONS - FRACTURE TOUGHNESS TESTS

The precautions detailed in BS5447:1977 should be adhered to.
CHAPTER 7

RESULTS

Results of experiments to investigate those variables discussed in Chapter 6 are presented in this chapter. Graphs are discussed first, followed by photographs of the fatigue and fracture surfaces.

7.1 GENERAL

Before the effect of variables is considered and comparisons made between different materials, it may be worthwhile to discuss the two basic curve types that are used to present these results.

Figure 7.1 shows the growth of a fatigue crack, non-dimensionalised with respect to the specimen width, $W$, as the number of cycles, $N$, is accumulated in a constant $\Delta P$ test. The shape of the curve displays an even increase in the crack growth rate, $da/dN$, until fracture occurs. Since the load range $\Delta P$ has remained constant but $a/W$ has increased, the value of stress intensity range, $\Delta K$, has also increased. The numerical computation of the gradient to this curve, using a method described in Chapter 3, results in Figure 7.3 which shows $da/dN$ plotted against $\Delta K$. The good resolution has been made possible through the combination of an AC electrical potential system (EPS), which has a discrimination of 20 $\mu$m for a specimen width, $W$, of say 30 mm, and an automated computer system.

Two observations are important. The test commenced from a blunt notch at a higher value of load than used for the remainder of the test. The load was decreased linearly (a quadratic decreasing function was used in later tests) during the first 10 measurements, resulting in a leftward and downward going curve which can be seen in Figure 7.3 and more clearly in Figure 7.5. This portion of the curve is not shown in later figures. It will suffice to note that once the crack is sharp, some curves such as
Figure 7.3 follow the same line going both up and down the $\frac{da}{dN}$ versus $\Delta K$ curve, whilst some curves such as Figure 7.5 do not. Consequently, Figure 7.5 shows signs of a false threshold; a real threshold may be as low as 4 MPa$\sqrt{m}$ (Pook (1972)). This may be due to a too rapid load reduction or skew crack propagation. The other important observation concerns the accuracy and scatter of the results which is related to experimental technique and the method in which $\frac{da}{dN}$ is obtained from the $a$ versus $N$ data. A quadratic curve is generally fitted to seven adjacent $a$ versus $N$ points which allows the gradient to be calculated at the centre point. The use of seven points allows any minor scatter to be reduced, resulting in a smooth final crack growth rate curve. The slight waviness of Figure 7.3 can probably be attributed to temperature variations affecting the EPS, small variations in the load or inherent material inhomogeneity. The jump at A is due to a fatigue frequency change from 30 Hz to 20 Hz.

The other curve type used to present results is shown in Figure 7.4 which has been obtained from a constant $\Delta K$ test. The abscissa is now $a/W$, whilst the ordinate remains unchanged. Only every alternate point has been plotted. This type of test is primarily used to determine the variation in crack growth rate as a function of tube radius. The $a$ versus $N$ curve from which Figure 7.4 was derived is shown in Figure 7.2. The linear nature of the curve indicates sound constant $\Delta K$ testing technique and an unchanging crack growth rate with the tube radius between $a/W = 0.2$ to 0.8.

The slightly lower crack growth rate seen at low $a/W$ values (Figure 7.4) is often due to the initially blunt crack. The same technique of slowly reducing the applied load is also used in this type of test. Constant $\Delta K$ are very difficult to conduct since an accurate EPS calibration must exist and the test must be perfectly repeatable. This is particularly
difficult with the AC EPS which is sensitive to the position of current and sensing leads in space. Small deviations in \( \Delta K \) may therefore occur, resulting in correspondingly larger deviations due to the greater than unity power to which \( \Delta K \) is raised in the Paris equation. As a result, constant \( \Delta K \) tests are almost impossible to conduct successfully at \( \Delta K \) values close to the threshold value.

A final general observation can be made when viewing Figure 7.5 which shows the fatigue crack growth for a C-shaped specimen of DM material at a temperature of 300°C. Scatter is more significant than Figure 7.3 which shows a room temperature test on the same specimen geometry and material. It would appear that the scatter is either due to temperature fluctuations in the EPS system or due to larger material scatter at high temperatures.

The effect of statically overloading the specimen is vividly demonstrated in Figure 7.6 where a CKS was deliberately over-loaded whilst optical measurements of crack length were made. A straight line has been drawn through subsequent points so that the order in which the points were plotted can be easily followed. Only three points have been used to fit a quadratic curve to \( a \) versus \( N \) data in an attempt to obtain an instantaneous fatigue crack growth rate. Nevertheless, this technique still has some small integrating or smoothing effect and this must be kept in mind when examining the effect of over-loads in Figure 7.6. From comparisons with other curves, it will be shown later that the material has no knowledge of previous over-loading once the crack has grown out of the over-loaded region.

Another peculiar feature shown in Figure 7.6 is the crack growth obtained from a skew crack. The crack commenced at A from a blunt notch and high \( \Delta K \) which was reduced in steps until point B, as was explained previously. Examination of the specimen after the test showed that a skew crack had developed at one corner only. A short period of apparently
rapid increasing crack growth and steady crack growth followed, seen as BC in the figure. This may not be a true reflection of the propagation of a crack but merely the interpretation of the straightening out of the crack by the AC EPS which was previously calibrated using straight cracks.

Up to now, experimental curves have been shown using hollow circle-like points. This technique cannot be used, however, when similar curves are to be compared, since the points are too closely spaced to distinguish one point from another. A system has therefore been adopted where straight lines joining adjacent points is used. This method has the advantage of still showing scatter in the curve. Numbers have been drawn on the curve to distinguish one curve from the other.

7.2 REPEATABILITY

Most of the tests conducted were repeated to ensure accurate results. Repeatability was generally excellent, as can be seen in Figure 7.7, which also shows results from a test where a specimen loading pin has broken. Also, the effect of an over-load does not change the general trend of the curve but only affects fatigue crack growth results immediately after the over-load. Note that the method used to determine $da/dN$ fits a quadratic curve to seven adjacent points and it therefore has the effect of reducing the changes resulting from the over-load.

7.3 MEAN STRESS

The effect of mean stress is demonstrated by tests conducted using two different $R$ ratios of 0.05 and 0.5. Fatigue crack growth rates in the radial direction are shown in Figure 7.8 and in the axial direction in Figure 7.9. The slope or Paris constant, $m$, of these curves does not appear to change significantly with $R$ ratio. Note curve 1 of Figure 7.9 represents three curves previously shown in Figure 7.7. Comparisons
between axial and radial fatigue crack growth rates are made in Section 7.11.

7.4 AUTOFRETTAGED MATERIAL

Tests were performed on C-shaped specimens cut from autofrettaged tubing to compare fatigue crack growth and fracture properties with the original as-received tubing. The results from these fatigue crack growth tests are shown in Figure 7.10. Considerable difficulty was experienced in initiating cracks from the blunt machined notch, leading to often very skew propagating cracks. The angle of the skew crack front, measured with respect to a normally propagating crack front, and the depth, $a$, to which it had grown when the growth rate was $10^{-7}$ m/cycle, is also shown in the figure. The appearance of a threshold is immediately obvious from the figure and appears to be strongest for short crack lengths. (Compare the relative position of each fatigue crack growth curve as a function of its crack length, $a$, at $10^{-7}$ m/cycle.) Artificially high crack growth rates appear to result from skew crack growth as shown by curves 4 and 5 which have skew crack fronts of 20° and 17°, respectively.

The difficulty experienced in initiating a crack in these autofrettaged material tests, and the appearance of an artificially high threshold, indicates the existence of residual stresses. An analysis of residual stresses confirms this belief. Figure 7.11 shows how these stresses change from the original distribution in the tubing (curve 1) to that of a cracked C-shaped or SR specimen (curve 3). Curve 1 has been obtained from Figure 4.5 where the residual stress distribution has been found by the Sachs boring method on a RING specimen. When the RING is parted, the resultant moment acting on any one place is released elastically. This moment can either be calculated by integrating curve 1 or by measuring the deflection of the two new faces when the ring is parted.
to form a split ring. This latter method was adopted and the expression (Williams et al (1980)):

$$\frac{\sigma_\theta}{E} = \frac{1}{\pi} \frac{\delta}{w} \left(\frac{k-1}{k+2}\right)^2$$  \hspace{1cm} (7.1)

provides a useful first approximation. $\sigma_\theta$ is the residual elastic bore hoop stress, $\delta$ is the deflection of the parted faces measured at their centres, $w$ is the wall width, $k$ is the ratio of the outside to inside tube diameters, and $E$ is Young's modulus. Substituting values for DM tubing and $\delta = 0.43$ mm yields a residual elastic bore hoop stress of 110 MPa which is represented by curve 2. Subtracting curve 2 from curve 1 leads to curve 3, the residual stress pattern for the C-shaped and SR specimens.

Clearly, the distribution will have to change as the crack extends into the specimen and must reach zero for the maximum crack length. As a first approximation, a linearly reducing distribution could be assumed and curve 4 shows a typical residual stress distribution when the crack is half way through the wall thickness. (Residual stress will probably relax faster than the linear relationship proposed since the high crack tip stresses have been ignored.)

Having examined the residual stress distribution, it may be possible to explain why curve 1 of Figure 7.10 has the highest threshold. Also, note that the abscissa $\Delta K$ is the applied stress intensity factor, whereas an effective $\Delta K$, to be discussed later, would take residual stresses into account, possibly resulting in similar curves as previously shown for as-received material. Changes in material fatigue crack growth properties can only be examined if the fatigue crack growth curve already takes residual stresses into account.

Figure 7.12 compares as-received and autofrettage tests in the radial direction, plotted against the applied $\Delta K$. Only those autofrettage
material tests which showed non-angled crack fronts have been plotted. From the curves, it would appear that once the crack had extended sufficiently deeply into the material \((a/W = 0.4)\), the threshold type behaviour ceases and the curve of the as-received material is followed. Alternatively, residual stresses become insignificant when fatigue crack growth rates exceed \(6 \times 10^{-8} \text{ m/cycle}\). These and later (see Figure 7.23) observations are important when considering fatigue crack growth and tube fracture and will be discussed later.

No threshold behaviour is observed in tests performed on autofrettaged material in the axial direction, as can be seen in Figure 7.13 which compares as-received and autofrettaged material results. The residual stress distribution across the breadth, \(B\), of a CKS may be similar to that shown in Figure 7.11 but of lesser magnitude due to some stressed material being cut off during the making of the specimen. Nevertheless, since no external forces are present, the tensile and compressive forces must balance, and if no material changes have occurred as a result of autofrettage, similar fatigue crack growth curves could be expected. The residual stress distribution with its tensile stresses at the specimen centre may cause greater bowing in the autofrettaged material rests. Examination of as-received and autofrettaged CKS specimens in Plates 7.4 and 7.5, respectively, appear to show some signs of this tendency.

The autofrettaged material curves 1 and 2 of Figure 7.13 lie below the as-received curves. As argued above, the lower observed crack propagation rates probably are not a result of residual stresses. Work hardening of the material may have resulted, since approximately one half of the specimen breadth exceeded the yield stress during autofrettage. Finally, the effect of material scatter should not be ignored and when examining the curves presented in this chapter as a whole, there appears
to be a tendency to observe greater scatter in material properties in the axial direction.

7.5 **SPECIMEN BREADTH**

Tests were conducted on different specimen breadths in the radial direction and the results are plotted in Figure 7.14. There appears to be little difference between the two breadths, although marginally lower fatigue crack growth rates are observed for the thicker specimens.

7.6 **FATIGUE FREQUENCY**

To examine the effect of fatigue frequency, a test was conducted at a frequency of 5 Hz. Figure 7.15 compares this result with another test using a higher frequency, but no variation due to fatigue frequency was observed.

7.7 **TEMPERATURE**

The effect of temperature can be seen from Figures 7.16 to 7.19 where fatigue crack growth obtained at 300°C is compared with room temperature results.

Comparison of radial fatigue crack growth for as-received tubing is shown in Figure 7.16. Virtually no difference can be observed between the two different temperature tests. Greater scatter, however, accompanies the high temperature results (see Section 7.1).

Results from tests in the axial direction are given in Figure 7.17. A greater distribution in fatigue crack growth results is observed in the axial direction and results from CKS5DM appear to be particularly high. Taking this into account, no effect of temperature can be observed.

Although a higher crack growth rate is observed from the high temperature test on autofretted material in the radial direction, as
shown in curve 4 of Figure 7.18, the crack growth of this test was considerably more advanced than the others ($a = 17.5$ mm at $da/dN = 10^{-7}$ m/cycle, compared to $a = 11$ mm to 14 mm for the other curves at the same fatigue crack growth rate). This may account for the lower threshold behaviour, as discussed in Section 7.4.

The final measurement on the effect of temperature is shown in Figure 7.19, where fatigue crack growth in the axial direction of autofrettaged material is considered. Again, in the range considered, no temperature dependence can be distinguished.

Of interest in this figure and the other high temperature results is the greater scatter observed in high temperature tests. The hump in curve 4 of Figure 7.19 at a $\Delta K$ of about 15 MPa$\sqrt{m}$ has no particular cause and must therefore be attributed to either material properties or EPS malfunction, such as a metal particle shorting the crack faces.

7.8 DCB SPECIMENS

Fatigue crack propagation results from six side-grooved specimens are shown in Figure 7.20 and compared with results from CKS tests. In order that axial fatigue crack propagation results can be obtained as a function of tube radius, a pair of narrow DCB specimens were cut from both close to the bore (curves 6 and 7) and close to the outside diameter (curves 4 and 5) of the tubing. The notch breadth of these specimens ranged from 3.9 mm to 4.7 mm. Two broader DCB specimens were also tested to obtain average values and these specimens had notch breadths of 9.2 mm and 14.4 mm, respectively (c.f. CKS breadth of 20.8 mm).

Generally, the results compare reasonably well with each other and with the two curves derived from tests on CKS. Curve 5 lies considerably lower, however, despite considerable care taken to check the experimental procedure and analysis. Like curve 5, curve 4 describes fatigue crack
growth close to the tube outside diameter. If some scatter can be tolerated, it is possible to draw a single line through both curves 4 and 5. This line would have a gradient, \( m \), of 4.9 and may be indicative of the considerable mechanical work that the tube outside has undergone. Results obtained close to the bore show a similar trend to those of the CKS and broad DCB specimens, where the gradient is about 3.0.

In conclusion to this section, it must be stressed that the test results on DCB specimens should be treated with some scepticism since the notch breadth was as low as 3.7 mm in some cases. Not only are the conditions of stress at the crack front in some doubt, the crack plane is so narrow that local changes in material properties may become significant.

7.9 SPLIT RING (SR) SPECIMENS

SR specimens were intended to be used to obtain fatigue crack growth data close to the bore of the tube. This proved difficult as the crack preferred to grow from a corner or grow in a bowed thumbnail fashion. Some examples of these difficulties have been recorded in Plate 7.3. The AC EPS is not suited to follow these shapes of cracks, even if an appropriate calibration curve did exist, since the relative changes in skin and crack face surfaces becomes too complex. Some peculiar trends will therefore be noticed in the SR data. Tests on SR specimens, performed both at 20 °C and 300 °C for as-received and autofrettaged material, are shown in the next three figures. In all figures, test results from C-shaped specimens are also shown (curve 1 in all figures) for comparison purposes. The expression given in Figure 5.22 was used to calculate \( \Delta K \).

Curves 2, 3 and 4 in Figure 7.21 have been obtained from a single SR specimen for as-received material at room temperature. The crack was initiated from a short scratch scribed on the centre of the tube bore, using a 50% over-load which was reduced rapidly as the crack began to
propagate. Curve 2 shows the fatigue crack growth rate once the load was constant, but before the bowed thumbnail crack had become straight. Probable crack profiles at the beginning and end of curve 2, shown in the same figure, may be the cause of the high measured fatigue crack growth rates. It must also be kept in mind that the AC EPS was calibrated using straight-fronted and not strongly bowed cracks. It is unlikely that a poor $K$ calibration of the specimen is to blame since an error in the order of 100% would be necessary.

The load was decreased and the subsequent crack growth is presented in curve 3. When the crack growth rate became too high, the load was again reduced, leading to curve 4. The agreement between curves 3 and 4 and collectively with curve 1, the fatigue crack growth rate under identical conditions obtained using a C-shaped specimen is excellent. Plate 7.3 shows a photograph of the specimen's fatigue surface.

Test results for as-received material at a temperature of 300 C using an SR specimen are shown in Figure 7.22. A similar trend to that shown in curve 2 in Figure 7.21 is also observed for short cracks. This effect may also have been caused by a non-straight crack front as shown in the sketch in Figure 7.22. The reason why the curve deviates from that of the C-shaped specimen at high $\Delta K$ is not clear. The maximum crack length is only 19 mm and hence large errors due to a poor $K$ calibration curve are not likely. Also, Figure 7.21 shows good and repeatable crack growth data at similar $a/W$ values.

Results from an autofrettaged material test are given in Figure 7.23. Generally, straight crack growth was observed for short crack lengths, resulting in accurate fatigue crack growth rates given in curve 2 of Figure 7.23. Hereafter, the load was reduced, leading to curve 3 which shows the same influence of residual stresses as discussed previously, despite its relatively long crack length. Possible reasons for this
peculiar behaviour are discussed later.

7.10 RING SPECIMENS

Tests on RING specimens were attempted prior to the use of the SR specimen to determine fatigue crack growth properties of the material close to the bore of the tube. Control of these tests was particularly difficult as exceptionally large loads were required to initiate a crack. Thereafter, fatigue crack growth accelerated to very high values if the load was not immediately decreased. This behaviour can readily be explained by examining Figure 5.25 which relates the stress intensity factor, \( K \), to crack length. The stress intensity factor rises rapidly for the first 10\% of crack growth. Due to the steep slope of this curve, large errors in \( K \) can occur.

Between \( a/W \) values of 0.35 and 0.75, an almost constant value of \( K \) is observed. Crack growth in this interval under constant \( \Delta P \) conditions will therefore not yield a wide range of fatigue crack growth results.

It is also interesting to note that the SR specimens require approximately fifteen times less load for the same stress intensity factor at an \( a/W \) value of 0.5 than required by the RING specimen.

Results from two reasonably successful tests on RING specimens are shown in Figures 7.24 and 7.25 for as-received and autofrettaged material, respectively, at room temperature. Both curves show the same trend of rapidly increasing fatigue crack growth rates followed by constant crack growth rate. Altogether, 50 points have been plotted at the same crack growth rate of about \( 10^{-7} \) m/cycle in Figure 7.24 and some 30 points between \( 8 \times 10^{-8} \) and \( 10^{-7} \) m/cycle in Figure 7.25. The results are not sufficiently accurate to make any further reliable conclusions.
7.11 COMPARISON OF RADIAL AND AXIAL FATIGUE CRACK GROWTH RATES

Fatigue crack growth rates were obtained in both the radial and axial directions for all four material types investigated. An $R$ ratio of 0.5 was chosen with the exception of DM material where tests were also performed at $R = 0.05$. The tests were conducted at room temperature. Constants $C$ and $m$ used in the Paris equation, $da/dN = C (\Delta K)^m$, are given in Table 7.1.

For DM material and an $R$ ratio of 0.05, Figure 7.26 shows that the fatigue crack growth rates in the axial direction are almost twice as high as in the radial direction for the same value of $\Delta K$.

The difference in fatigue crack growth rates in the two directions appears to decrease with increasing $R$ ratio, as can be seen in Figure 7.27. Scatter about an average Paris line is greater but curve 1 has consistently shown abnormally high rates when compared to tests conducted under identical conditions.

Tests on the hot Asselled tube H have provided interesting data, different from those of other materials tested. The most noticeable difference is that the axial fatigue growth rate curve is lower than that in the radial direction and lies at a different slope to results from all other tests conducted in this investigation. The results seem to be repeatable, as shown in Figure 7.28. Considerably larger scatter from a straight continuous line is observed when compared to results from other materials. There is also no obvious experimental explanation for the two humps at A and B, possibly indicating some material inhomogeneity.

Examination under a scanning electron microscope may provide interesting results.

Results from tubes U and V shown in Figures 7.29 and 7.30, respectively, show little difference between axial and radial fatigue crack growth rates.
TABLE 7.1

*C and m Values for the Paris Equation* \( \frac{da}{dN} = C \Delta K^m \), Where \( \Delta K \) and \( \frac{da}{dN} \) Have the Units MPa\( \sqrt{m} \) and pm/cycle, Respectively

All tests conducted at room temperature at \( R = 0.5 \), with the exception of DM* where \( R = 0.05 \)

<table>
<thead>
<tr>
<th>Material</th>
<th>Radial</th>
<th></th>
<th></th>
<th>Axial</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( C )</td>
<td>( m )</td>
<td>( C )</td>
<td>( m )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DM*</td>
<td>4.11</td>
<td>2.87</td>
<td>5.39</td>
<td>2.97</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DM</td>
<td>16.2</td>
<td>2.64</td>
<td>13.4</td>
<td>2.91</td>
<td></td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>19.6</td>
<td>2.66</td>
<td>0.921</td>
<td>3.62</td>
<td></td>
<td></td>
</tr>
<tr>
<td>U</td>
<td>24.8</td>
<td>2.57</td>
<td>24.8</td>
<td>2.57</td>
<td></td>
<td></td>
</tr>
<tr>
<td>V</td>
<td>39.2</td>
<td>2.40</td>
<td>39.2</td>
<td>2.40</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
7.12 COMPARISON OF FATIGUE CRACK GROWTH BETWEEN MATERIALS

Comparison of radial (transverse) fatigue crack growth for the four materials investigated is shown in Figure 7.31. Differences between materials in this direction appear to be small and therefore relatively insensitive to the degree of vacuum refinement and manufacturing route.

Longitudinal fatigue crack growth is more sensitive to these parameters as can be seen from Figure 7.32, which compares crack growth in the axial direction. The most obvious difference occurs with the hot Asselled tubing which has been twisted during manufacture, imparting improved resistance to axial fatigue crack growth. DM tubing shows the worst axial crack growth, particularly at high values of $\Delta K$. Similar fatigue crack growth rates are obtained for U and V tubing.

7.13 CONSTANT $\Delta K$ TESTS

Constant $\Delta K$ tests were performed for two principal reasons; to determine if radial fatigue crack growth rates varied as a function of radius and to obtain a calibration for the stress intensity factor as described in Chapter 5.

It is essential during constant $\Delta K$ tests to have a reliable method of measuring crack length since small differences in its measurement cause large differences in $\Delta K$ resulting in even larger changes in $\frac{da}{dN}$. Difficulties were experienced with the AC EPS which was sensitive to current and sensing lead placement and therefore repeatability between EPS calibration curves was not always satisfactory in these sensitive tests. In conventional constant $\Delta P$ tests, sufficient optical crack length readings were taken to allow individual EPS calibration curves for each test to be obtained.

Figure 7.33 shows one of the worst constant $\Delta K$ tests where $\Delta K$ varied from 40 MPa$\sqrt{m}$ to 55 MPa$\sqrt{m}$. The first five points should be ignored since
the crack is still blunt in this region. The curve corrected by using the Paris $C$ and $m$ constants in Table 7.1 is shown in Figure 7.34 and displays almost flat fatigue crack growth at a $\Delta K$ value of 46 MPa$\sqrt{m}$. No explanation is offered for the humps in the curve at the beginning and centre of the curve.

Results from various constant $\Delta K$ tests on DM material are shown in Figure 7.35. Curves 2 and 4 have been obtained from two different tests using the same $\Delta K$ value of 58 MPa$\sqrt{m}$. Repeatability for $a/W$ values less than 0.6 is good but considerable deviation occurs hereafter, indicating the sensitivity of this type of test. Fatigue crack growth rate as a function of $\Delta K$ becomes less linear at these high values of $\Delta K$ which probably contributes to the errors observed in these two curves. Curves 3 and 1 correspond to lower $\Delta K$ values of 46 MPa$\sqrt{m}$ and 35 MPa$\sqrt{m}$, respectively. All four curves have a relatively flat appearance, indicating no change in radial fatigue crack growth rate as a function of radius for the $a/W$ values considered.

Curves 5 and 6 in the same figure correspond to tests performed on autofrettaged material. Comparison of curves 1 and 5 which were obtained from tests performed at the same $\Delta K$ value of 35 MPa$\sqrt{m}$ demonstrates the considerable influence of autofretage residual stresses. Of particular interest is the nearly constant difference between curves, implying that the residual stress distribution does not decrease in magnitude and the distribution moves ahead of the crack as it advances. These tests were not repeated and hence positive conclusions should not yet be made.

Curve 6 shows the result of a test on autofrettaged material performed at a low $\Delta K$ value of 15 MPa$\sqrt{m}$. The curve of $da/dN$ against $\Delta K$ (see Figure 7.10) is nearly vertical at this point and hence curve 6 is prone to exceptionally large errors. Nonetheless, the behaviour depicted by this curve is very similar to what one would expect. The effect of residual
stress is greatest at low values of $a/W$ and progressively decreases for increasing $a/W$ as described by the model in Figure 7.11. The trend shown in this curve contradicts that of curve 5 and a greater volume of work in this field is clearly necessary.

Figure 7.36 shows results of constant $\Delta K$ tests performed on various tube materials. The good agreement between curves obtained under similar $\Delta K$ values supports the general technique used to conduct these tests. The results from most of these tests were also used to obtain $K$ calibration curves for the C-shaped specimens described in Chapter 5.

7.14 FRACTURE TOUGHNESS TESTS

Fracture toughness tests were performed on all four materials with additional tests on DM material to investigate the effect of temperature and autofrettage. All the tests were conducted in accordance with BS5447:1977 and many of the tests were additionally instrumented with the AC EPS which shows a characteristic minimum in potential as the specimen is loaded.

Figure 7.37 shows a typical load displacement curve for a C-shaped specimen at room temperature. The relationship of the AC EPS with displacement is also shown. The minimum in this curve corresponds exactly with the construction to obtain the provisional fracture load, $P_n$. This test, in common with other room temperature tests on C-shaped specimens, showed excessive non-linearity and hence failed to fulfil the conditions for a valid $K_{IC}$ test.

A typical test result for a CKS at room temperature is shown in Figure 7.38. The load displacement curve is more linear than obtained from C-shaped specimens and these tests only failed the condition of $B > 2.5 (K_p / \sigma_y)^2$. The minimum in the EPS curve does not correspond with $P_Q$ but the load corresponding to this minimum is very similar in magnitude.
No explanation can be offered for the presence of the hump A in the 
EPS curves shown in Figures 7.37 and 7.38. This hump was observed in 90% 
of all the EPS curves obtained during fracture toughness tests. 

Tests at 300°C are shown for C-shaped and CKS in Figures 7.39 and 
7.40. Improved linearity is observed in the C-shaped specimen at high 
temperature. The minimum in the EPS curve again provides a useful 
measurement of the magnitude of $P_Q$. It is also interesting to note that 
the same pronounced ripple in the load and EPS trace is observed prior to 
failure in high temperature tests. 

Values for the provisional fracture toughness, $K_{Q, \text{EPS}}$ (the value 
of $K$ at the minimum in the EPS curve), and $K_{\text{max}}$ are tabulated in Table 7.2 
for all the mode I fracture toughness tests conducted. 

The following conclusions can be made when considering $K_{\text{max}}$ values. 

(i) There is a 16% reduction in fracture toughness when 
increasing the temperature from 20°C to 300°C. 
(ii) Similar results are obtained for as-received and 
autofrettaged materials. 
(iii) When comparing radial with axial values for the following 
materials:

<table>
<thead>
<tr>
<th>Material</th>
<th>Radial</th>
<th>Axial</th>
</tr>
</thead>
<tbody>
<tr>
<td>DM</td>
<td>116%</td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>107%</td>
<td></td>
</tr>
<tr>
<td>U</td>
<td>102%</td>
<td></td>
</tr>
<tr>
<td>V</td>
<td>100%</td>
<td></td>
</tr>
</tbody>
</table>

(iv) Comparison between tube material:

Radial : $U > H > V > DM$
Axial : $U > V > H > DM$
<table>
<thead>
<tr>
<th>Material</th>
<th>Radial (MPa√m)</th>
<th>Axial (MPa√m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$K_Q$</td>
<td>$K_{EPS}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DM</td>
<td>20 C</td>
<td>114.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>121.9</td>
</tr>
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<td></td>
<td>300 C</td>
<td>107.3</td>
</tr>
<tr>
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<td>127.8</td>
</tr>
<tr>
<td></td>
<td>300 C</td>
<td>102.8</td>
</tr>
<tr>
<td></td>
<td></td>
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</tr>
<tr>
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<td>126.0</td>
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<tr>
<td>V</td>
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<td>122.9</td>
</tr>
<tr>
<td></td>
<td>136.7</td>
<td>131.6</td>
</tr>
</tbody>
</table>

* Estimates only due to poor clip gauge reading
7.15 PHOTOGRAPIHS OF FATIGUE AND FRACTURE SURFACES

Plate 7.1 shows two groups of CKS, C-shaped and SR specimens from autofrettaged (top group) and as-received DM material. These tests were conducted at high temperature which has left an oxide on the fracture surfaces. The oxide changes colour as the stress intensity factor or crack opening increases and may be attributed to the mass of oxygen that can diffuse into the crack to oxidise the freshly exposed crack surfaces. The colour spectrum is: brown, ochre, pale green, pale blue, ending with dark blue on the fast fracture surfaces.

The different colours, which in Plate 7.1 are distinguished by different shades of grey, allow the progress of the crack to be followed. A strongly bowed crack at short crack lengths can be seen clearly in the autofrettaged specimen CKS21DMA (see Section 7.4) as opposed to the as-received specimen CKS20DM which shows an almost straight crack front. Specimen C20DMA shows signs of skew crack growth which appears to be a tendency in tests on materials containing residual compressive stresses.

Note that the SR specimens have no machined notch; crack growth commences from the bore of the tubing.

Fatigue surfaces from tests where very low values of fatigue crack growth rate were measured are shown in Plate 7.2. Areas where crack growth has been low have a copper-brown appearance which can be seen as the dark grey areas in Plate 7.2. Note the skew and uneven crack front where low crack growth rates have been obtained, indicating a high degree of sensitivity to small changes in $\Delta K$ at low stress intensity factors.

A RING specimen, R2DM, is also shown in Plate 7.2 where crack growth commenced from a 1 mm deep machined notch. Observe the fine texture close to the bore and the constant, more coarse texture across the remainder of the surface.

Plate 7.3 shows some of the difficulties experienced when initiating
crack growth from a short scriber scratch at the centre of the specimen on the bore of the tubing. Specimen SR6DM cracked on a plane 10 mm below the required plane and hence outside the two sensing leads of the EPS. Consequently, no crack growth readings could be taken. The bowed crack front prior to fast fracture can be seen. Specimen SR1DM commenced with a bowed crack front which soon developed into straight fronted crack growth. Distinct bands of different $\Delta K$ ranges can be seen in SR5DMA. Initially, a high value of $\Delta K$ was used which, once initiation had occurred, resulted in very rapid crack growth. The load was decreased twice before the test was stopped. Crack growth occurred from a corner in SR1DMA. The crack showed no tendency to straighten itself out.

Fatigue crack growth and fracture toughness specimens from DM tubing are shown in Plate 7.4. The top two fracture toughness (denoted FT) specimens were tested at 300 C. The crack fronts of all the specimens show some shallow bowing. The fatigue crack growth surfaces are smooth without obvious overload striations. Some signs of directionality can be observed on the fracture surfaces. With the exception of larger shear lips, there do not appear to be any obvious differences in appearance between the hot and room temperature fracture toughness tests.

Plate 7.5 shows similar specimens for the autofrettaged DM tubing, except the hot tests are shown at the bottom of the photograph. Note the more pronounced bowing in the autofrettaged CKS specimens compared to the autofrettaged C-shaped and as-received CKS and C-shaped specimens, which may be attributed to the residual stress distribution which, in the CKS specimen, is tensile in the centre and compressive at the specimen sides (see Section 7.4). No other differences in the fracture appearance between as-received and autofrettaged tubing are discernible.

Plates 7.6, 7.7 and 7.8 show fatigue crack growth and fracture toughness specimens for H, U and V tubing, respectively. There are very
little signs of directionality in the fracture surfaces of these specimens and the shear lips in all fracture toughness specimens are relatively large.

Finally, Plate 7.9 compares the fatigue crack growth surfaces of constant $\Delta K$ tests performed on the four different materials. Compared to V material, which was tested at the same $\Delta K$, a coarser grain structure can be observed in the DM material.
CHAPTER 8
DISCUSSION

The primary task of this investigation was to obtain accurate fatigue crack propagation and fracture toughness properties which would be used, in conjunction with a suitable model, to predict the growth of a crack through thick walled tubing. The description of such a model has been reserved for Chapter 9 and this chapter is therefore limited to a discussion of the results presented in the previous chapter. A short discussion on the techniques used in an attempt to obtain these results is also included.

8.1 EXPERIMENTAL TECHNIQUES

It unfortunately occurs all too often that experiments are conducted without proper regard to instrumentation, precautions and analysis of the results. Fatigue crack growth tests appear to be particularly susceptible to this attitude since instrumentation can be complex, the tests are drawn out and often monotonous and analysis of the observations requires frequent numerical or graphical differentiation. It is for these reasons that special emphasis was paid to the development of an accurate automated testing method.

Arguments in favour of the micro-computer system were presented in Chapter 3, together with a brief description of the system. An AC electrical potential system (EPS) to monitor crack growth was also described. Now that the results have been presented, it may be worthwhile to review the applicability of these two systems in obtaining fatigue crack growth data.

Probably the best indication of the accuracy achieved by the equipment is to obtain a measure of the discrimination, scatter and repeatability observed in the test results. Figure 7.3 shows the results
of a test where the discrimination is excellent and the scatter is negligible. Repeatability can be gauged from Figure 7.7, which again is excellent. The system also appears to be sufficiently sensitive since the effect on the fatigue crack growth rate of an overload (Figure 7.6) and material inhomogeneity (Figure 7.28) can be readily gauged.

The 12 bit analogue to digital (ADC) and digital to analogue (DAC) converters, each with an accuracy of 0.024%, provided more than adequate performance in the tests conducted in this investigation. It can be argued that improved resolution may be required when very small changes in short cracks are to be measured such as in initiation tests. Additional instrumentation such as high accuracy integrating voltmeters should then be used, although the converging times of these devices are too slow to be of general use.

The precautions incorporated in the micro-computer software, to ensure that the test piece was not overloaded and that all transitions between dynamic and static loading modes occurred smoothly, assisted in realising reliable results. In short, the opportunity for operator error has been considerably reduced.

The technique used to determine the crack growth rate, $\frac{da}{dn}$, by first fitting a quadratic curve to sets of adjacent points and then finding the derivative to that curve, appeared to provide satisfactory results. It is interesting to note that this technique is virtually identical to that proposed in the ASTM standard E647-78T (1979), which is a tentative test method for determining constant load amplitude fatigue crack growth data.

Accurate measurements of crack length were obtained using the AC electrical potential system (EPS). An indication of the suitability of this method to measure crack growth could be gauged by observing the EPS output monitored on the micro-computer's visual display unit during the
test. The system was so sensitive that a crack could sometimes be clearly observed to grow in quantised steps after short periods of no growth. The minimum crack growth that could be measured was approximately 10 \( \mu m \).

The AC power supply and narrow band amplifier were carefully designed to provide excellent temperature stability. However, the conductivity of the specimen material is also a function of temperature which can be a source of long term instability. This is a common problem with AC as well as DC electrical potential systems. A solution can be found in either a measurement of the conductance of the material by placing sensing leads in a region where the electric field is independent of crack length, such as a dummy specimen placed in series with the first, or by simply measuring the temperature of the specimen and computing the new conductance. The latter method has the advantage that only low cost temperature sensing instrumentation is required as opposed to the acquisition of an additional narrow band amplifier. This facility for temperature compensation was not included in this study but is strongly recommended to simplify experimentation in any future work.

Some difficulty in obtaining repeatable results was encountered with the AC EPS used in this investigation. This appears to arise from what may be described as an earth loop problem. It is well known that electrical equipment should be earthed at one point only and this philosophy was also adopted with the AC EPS. However, since the large currents flowing through the current leads cause these leads to act as a powerful aerial, a second earth point may be set up resulting in the formation of an earth loop. A different position of the current leads provides a new earth loop and hence a different EPS output. Problems with repeatability can therefore be experienced since it is difficult to ensure that the same positions of the current leads are maintained from specimen to specimen. The above disadvantage, however, can be overcome
if individual EPS calibration curves are obtained for each test. This approach was adopted in this investigation.

8.2 SPECIMENS

At the outset of this investigation, it was thought imperative to obtain axial as well as radial fatigue crack propagation and fracture toughness properties of the tubing. Fatigue crack propagation through a residual stress field was also of interest. These requirements necessitated the use of altogether five different specimen types which were described in Chapter 5. Furthermore, to ensure accurate results, experimental and numerical calculations of the stress intensity calibrations for some of these specimens were obtained. It is the intention of this chapter to review the suitability of these specimens.

The CKS and DCB specimens were used to obtain axial fatigue crack growth properties. The DCB specimen was largely only used to confirm that the fatigue crack growth results obtained in the axial direction were independent of the specimen geometry. An examination of Figure 7.20 appears to support this presumption. The reason for not using DCB specimens more often can be realised when comparing the range of stress intensity factor as a function of crack length for these two specimens. Referring to Figures 5.10 and 5.12, it can be seen that for a useful range of crack growth from $a/W = 0.3$ to $0.75$, the stress intensity factor increases by 600% for the CKS, whilst it only increases by 200% in the case of the DCB specimen. Furthermore, the cost of manufacture must be higher in the DCB specimen and the side grooves which are required to contain the crack may affect the fatigue crack growth rate.

The diameter of the loading pins are specified by the standards BS5447:1977, ASTM E399-78a and the new tentative standard for constant amplitude fatigue crack growth tests, ASTM E647-78T. It was, however,
found if pins of the specified diameter were used, frequent pin failures would result despite a careful selection of the pin material and heat treatment. A possible reason for this difficulty may have been due to the excessive gap between the specimen and the shackle which is necessary for the proper electrical insulation required by the electrical potential system. Since only a 20% increase in the pin diameter results in 80% higher strength, an obvious solution may be to increase the pin diameter as long as the stress intensity factor calibration of the specimen is not affected. It is therefore encouraging to note from Figure 5.9 that an increase of 22% in the pin diameter produces only a very small and constant change in the compliance. Now, since equation (2.5) relates the compliance to the stress intensity factor and the derivative of a constant is zero, no change in the stress intensity calibration due to a change in the pin diameter is expected.

Radial fatigue crack propagation properties were determined using the C-shaped and split-ring (SR) specimens. It can be seen from Figures 7.21 and 7.22 that, despite difficulties encountered at short crack lengths in the SR specimen, the agreement between results from the C-shaped and SR specimens are generally favourable.

The geometries of the C-shaped and SR specimens are similar except that the dimension X (see Figures 5.4 and 5.5) has been increased considerably in the SR specimen. This results in a stress intensity factor for the SR specimen which is about twice as high for a given load and crack length as in the C-shaped specimen (see Figures 5.17 and 5.21), and hence the SR specimen was used to obtain fatigue crack propagation properties close to the bore of the tubing without the use of a notch. Considerable difficulty was encountered in propagating a straight crack starting at a short scriber scratch on the bore of the tubing, as was reported in Section 7.9. The reason for this behaviour is clear when
examining Figure 5.21 which shows a high rate of increase in $\Delta$ at short crack lengths. This results in marginally deeper areas along the crack front growing considerably faster until they are restrained by the remaining lagging crack front. A possible solution may be to very accurately machine a shallow sharp notch and thereafter rounding off the sharp edges close to the notch with the hope that this will encourage straight-fronted crack growth. Alternatively, a new geometry specimen should be developed which displays a slowly increasing or constant stress intensity factor with increase in crack length close to the bore of the tubing.

The experimentally derived values of the stress intensity factor calibration for the C-shaped specimen generally agreed very well with the expression by Underwood & Kendall (1977) which is now incorporated in ASTM E399-78a. In two of the three experimental curves obtained, which are shown in Figures 5.17 to 5.19, the Underwood curve underestimated the stress intensity factor by up to 7% in the range $a/W = 0.3$ to 0.6. A possible, although unlikely, reason for this discrepancy may be due to the different values of the $X/W$ ratio which were used in this investigation, as well as a different radial dimension between the centre of the pin hole and the outside diameter. For the sake of convenience, call this latter dimension $RH$. The values of these dimensions specified by the standard are $X/W = 0$ or 0.5 and $RH = 0.25 W$, whereas the specimens used in this investigation have $X/W$ values between 0 and 0.5 and $RH = 0.58 W$ for DM tubing and 0.4 $W$ for the other tubing. $RH = 0.4 W$ was used by Underwood & Kendall (1974). Furthermore, the pin hole diameter is specified as 0.25 $W$ in the standard, whereas 12.7 mm diameter pins (approximately 0.4 $W$) were used in this investigation.

It is also preferable to compare the Underwood curve with the boundary integral equation (BIE) result. The BIE method appeared to provide
accurate values of displacement which allowed the compliance to be calculated at various values of crack length. This curve was subsequently differentiated to provide the stress intensity factor. Clearly, this process can only be accurate in the limit, and hence values of compliance at a pair of closely spaced crack lengths are required. Unfortunately, the program output did not contain sufficient significant figures to make this computation possible, and a source listing of the program was not made available to the author. Hence, it would not be prudent to compare the BIE results critically with the Underwood expression at this stage.

No experimental $K$ calibration was obtained for the SR specimen. However, it is interesting to note that the BIE solution corresponds very closely with the Underwood expression.

The RING specimen was primarily chosen to investigate the propagation of a fatigue crack through a residual stress field. The stress intensity calibration is shown in Figure 5.25 which shows a rapidly increasing function until $a/W = 0.35$, whereafter the stress intensity factor remains almost constant. The rapidly rising portion will cause difficulty when attempting to obtain straight-fronted crack growth, whilst the constant region of the stress intensity factor produces a limited range of fatigue crack growth data from a constant load test. This latter influence can clearly be seen from Figures 7.24 and 7.25, where 50 and 30 points, respectively, have been plotted at the same value of $\Delta K$. Also from the as-received material results shown in Figure 7.24, it can be seen that the C-shaped and RING specimens do not agree very well. This may be due to a combination of an inaccurate $K$ calibration derived from the BIE method and the effect of residual stress which may even be present in an as-received RING specimen.

Despite the poor results from the RING specimen shown in Figure 7.24, an indication of the influence of the residual stress field on the fatigue
crack growth rate in an autofrettaged RING specimen can be gauged in Figure 7.25. The point where the crack length exceeds 9 mm has been marked in Figure 7.24 and it can be seen clearly that the $\Delta K$ should be increased from about 20 $\text{MPa}\sqrt{\text{m}}$ to 30 $\text{MPa}\sqrt{\text{m}}$ or the curve in that region should be moved about 8 mm to the right. The data corresponding to a crack length of 9 mm in the autofrettaged RING specimen has also been marked in Figure 7.25 and since it can be argued that the error in the $K$ calibration at a given crack length should be the same, the data in excess of 9 mm in this figure should also be moved to the right by a similar amount. The influence of the additional residual stress in the RING specimen compared to those found in the C-shaped specimen can then be clearly seen in the form of a higher apparent threshold $\Delta K$ value.

8.3 FATIGUE CRACK PROPAGATION RESULTS

At the start of the investigation, a careful selection of variables was made which were thought to affect the life of the thick walled tubing. Since different degrees of chemical refinement are used and the tubing is heavily worked during manufacture, axial and radial fatigue crack growth properties were measured in four different materials, each having followed a different manufacturing route. From these results, it may be possible to choose the most beneficial manufacturing route or to simply predict the life of the tubing made from a particular material.

In practice, the tubing is subjected to various conditions of temperature and fatigue load. The tubing is also often autofrettaged in an attempt to improve its life. Hence, in one sample of material, the influence of fatigue frequency, mean stress, temperature, and residual stress was also investigated.

The dependence of fatigue crack growth on these variables is discussed below.
8.3.1 Mean Stress

It is generally believed (see Pook (1979)) that stage II fatigue crack propagation is largely independent of the mean stress or stress ratio $R$. A greater dependence in the threshold or stage I region has been observed by various authors and their results are reviewed in Section 2.4.3.

In this investigation, measurements were made on DM tubing in both the axial and radial directions at two stress ratios of $R = 0.05$ and $R = 0.5$. These fatigue crack propagation results correspond to the stage II regime. A distinct dependence on $R$ ratio was observed with the fatigue crack propagation results at $R = 0.5$, exceeding those at $R = 0.05$ by a factor of between 1.6 and 3 depending on the value of the stress intensity factor, $\Delta K$. The influence is strongest at low $\Delta K$ values. Similar relationships were observed in both the axial and radial directions, although larger scatter in the axial direction makes a reliable interpretation more difficult.

The dependence of the fatigue crack growth rate on the $R$ ratio is important since, in practice, the thick walled tubing may be subjected to various load cycles of different $R$ ratio. Cooke et al (1975) have found, however, that tests performed in vacuum show no $R$ ratio dependence (see Section 2.4.3). The relationship between fatigue crack growth rate and the $R$ ratio observed in air may therefore not be the same in a polyethylene gas environment.

8.3.2 Temperature

The effect of temperature was investigated in both the axial and radial directions, for as-received as well as autofrettaged material, by conducting tests at 300 C. This temperature corresponds to that used in low density polyethylene reactor tubing. No temperature dependence was
observed when comparing results at 300 C with those obtained at room temperature. This result is in agreement with that found by Pook & Beveridge (1973) and contradicts the observations by McHenry & Pense (1973), who obtained fatigue crack propagation rates at 260 C which were 1.6 times higher than at room temperature.

Of further interest is the greater scatter which appeared to accompany the high temperature results. This may be due to small temperature fluctuations affecting the conductance of the specimen or the bridging of the crack faces by oxide or other particles, both of which would affect the EPS output, or alternatively there may be larger material scatter at high temperatures.

8.3.3 Anisotropy

A comparison of axial and radial fatigue crack growth properties for the four materials investigated provides a variety of contrasting results. The U and V tubing display isotropic material behaviour since by looking at Figures 7.29 and 7.30, it is not possible to distinguish between their radial and axial fatigue crack growth rates at a given $\Delta K$.

Different types of anisotropy are displayed by DM and H tubing. In the DM tubing shown in Figures 7.26 and 7.27, the axial fatigue growth rates are consistently higher than the radial direction for both $R$ ratios of 0.05 and 0.5. The fatigue crack growth curves in the axial and radial direction are approximately parallel to one another with fatigue crack growth rates in the axial direction being almost twice as high as in the radial direction for an $R$ ratio of 0.05, and this difference falls to a factor of about 1.5 for $R = 0.5$.

Completely different behaviour is observed in the H tubing, as can be seen in Figure 7.28. This tubing has been hot Asselled and hence probably worked more than the corresponding rolled tubing. Whereas
in the DM tubing the axial direction displayed faster fatigue crack growth rates, slower axial crack growth rates are found in the H tubing. Since the slopes of the two curves (see Figure 7.28) are so different, four times slower growth is observed in the axial direction at a $\Delta K$ value of 7 MPa$\sqrt{m}$, whilst little difference in the crack growth rates can be distinguished at $\Delta K$ values higher than 20 MPa$\sqrt{m}$.

Narrow DCB specimens cut from DM tubing close to the outside of the tubing also showed lower axial fatigue crack propagation results (see Figure 7.20) to those found in the radial direction. This behaviour is similar to that found in the H tubing. Referring to Figure 4.7, it may be argued that the same twisting action on the tubing outside is obtained in the reeler as is experienced by almost the whole wall thickness when the tubing is reduced in the Assel elongator. The improved fatigue crack propagation properties in the axial direction may therefore perhaps be attributed to the hot reduction of the tubing which is also associated with a twisting action.

Without undertaking a thorough metallurgical study of the fatigue surfaces, it is difficult to account for the results seen in DM and H tubing described above. It is clear, however, from the literature, which has been reviewed in Section 2.4.1, that inclusions can influence the fatigue crack propagation rates in the axial and radial directions. Priddle (1977), for instance, found that the axial fatigue crack growth rate in the axial direction exceeded that in the radial direction by a factor of four.

The DM material is known (see Chapter 4) to have low transverse (radial) impact properties, and since fracture is also dependent on the population and nature of inclusions as described in Section 2.5, it could perhaps be argued that the DM tubing has a larger population of inclusions than found in the U and V tubing, both of which have good, almost isotropic
fatigue crack propagation and fracture properties. Hence, a higher fatigue crack growth rate in the axial direction can be expected compared to that found in the radial direction.

Continuing the argument that inclusions can also influence the fatigue crack growth rate of the H tubing, it may be supposed that, whilst hot rolling only elongates the inclusions, the hot Assel or twisting action breaks up the long inclusions into shorter sections. These "chopped up" inclusions may provide improved fatigue crack resistance, which gradually becomes less effective as the stress intensity factor increases; a result also observed by Heiser & Hertzberg (1972). This process also appears to cause greater material inhomogeneity as can be seen from the scatter in Figure 7.28.

Comparing the U with the V tubing, which have, respectively, been manufactured using the air melt vacuum degassed and vacuum arc remelt processes, it would appear that isotropic stage II fatigue crack propagation properties can be expected from both these grades.

Furthermore, no difference in the radial crack propagation results of the four materials can be observed in Figure 7.31. Hence, no obvious advantage appears to be gained by vacuum arc remelting as opposed to the less expensive vacuum degassing as far as stage II fatigue crack propagation is concerned. Also, the radial fatigue crack propagation does not appear to be a function of radius as shown in Figures 7.35 and 7.36, although successful tests were not conducted close to the bore of the tubing.

Finally, it should be stressed that the comments made above only apply to stage II fatigue crack propagation. Results from initiation and stage I fatigue crack propagation may show different trends to those found in this investigation.
8.3.4 Residual Stress

Tests were performed on C-shaped and SR specimens which were cut from autofrettaged tubing. An approximate calculation of the residual stress distribution which is shown in Figure 7.11 indicates that the maximum compressive residual stress at the inside radius of the specimen may be in the region of -290 MPa and hence a considerable influence on the fatigue crack propagation results may be expected. Some comments on the effect of residual stress were made in Sections 7.4 and 7.9 and these are summarised below.

(i) A threshold type appearance is observed in material containing compressive residual stress at the bore of the tubing when the fatigue crack growth rate is plotted against the applied $\Delta \kappa$. The apparent threshold value appears to be dependent on the crack length or, alternatively, the depth that the crack has penetrated into the residual stress field.

(ii) The effect of the residual stress disappears either once the crack has extended sufficiently far into the material or, alternatively, once a crack growth rate of approximately $6 \times 10^{-8}$ m/cycle or an applied $\Delta \kappa$ of 22 MPa√m has been exceeded.

(iii) Fatigue crack growth at a $\Delta \kappa$ value greater than 22 MPa√m showed no retardation as a result of residual stress whatsoever, irrespective of the length of the crack. Subsequently decreasing the applied $\Delta \kappa$ again showed the influence of residual stress by decreasing the fatigue crack growth rate, despite the crack length exceeding $a/W = 0.5$. Furthermore, by examining the two curves shown in Figure 7.23, the higher value of apparent threshold $\Delta \kappa$ is associated with the higher value of crack length, whereas the trend seen in (i) above
showed decreasing apparent $\Delta K$ threshold values with increasing initial crack length. It would appear from the above statements that the apparent threshold $\Delta K$ decreases to a minimum and then begins to increase as the crack length increases.

A brief explanation of how the residual stress distribution retained its original form but decreased in magnitude as it moved ahead of the crack was proposed in Section 7.4. This simple model allowed the influence of residual stress on the apparent threshold behaviour to be understood.

A more fundamental approach has been adopted in the qualitative model to describe the influence of residual stress explained below.

The residual stress distribution in a C-shaped or SR specimen shown in curve 3 of Figure 7.11 is assumed to be independent of the crack length $a/W$ at a plane remote from the crack plane. This must hold true since halving an SR specimen to form a C-shaped specimen does not alter the residual stress distribution. The second basic assumption is that a negative stress intensity factor does not contribute to fatigue crack growth.

The model is described in conjunction with Figure 8.1. The residual stress distribution described in Figure 7.11 is repeated in frame A, together with an assumed stress intensity factor calibration in frame B, which must be overcome before the crack can open. Although the concept of a negative residual stress intensity factor is not entirely correct, it does provide the means to obtain the final effective stress intensity factor. Note also that the actual distribution of the residual stress intensity factor is unknown and, in drawing the distribution shown here, an attempt was made to take the effect of both load and crack length into
account. It can be seen from the $K$ calibration for a C-shaped specimen in Figure 5.17 that at low crack lengths the load is the dominant variable, whereas at high crack lengths a small change in crack length produces a large change in the stress intensity factor. Finally, at $a = W$, the residual load must reduce to zero and hence $K = 0$.

Two applied alternating loads of different magnitude and with an $R$ ratio of about 0.5 have been considered. The resulting stress distributions are shown in frame C and the corresponding stress intensity factor calibrations are shown in frame D. The range of stress and stress intensity factor are represented by curves showing the maximum and minimum values of these parameters.

Finally, the effective remote stress distribution and stress intensity factor have been calculated as the sum of their residual and applied values, and are shown in frames E and F, respectively. Looking at the effective stress intensity factor (the positive component has been shaded), it can clearly be seen that until a crack length, $a_o$, has been reached, a significant portion of the applied stress intensity factor is negative (seen as the unshaded area) and hence does not contribute to the growth of the crack. An apparent threshold is therefore observed and its dependence on crack length can be readily realised.

When a high load is applied to the specimen, the effective $\Delta K$ equals the applied $\Delta K$ because the minimum applied load is able to completely overcome the residual stress in the specimen. Only an $R$ ratio dependence will be observed in these tests. Now, if the load is reduced, part of the effective $\Delta K$ will again become negative and an apparent threshold will once again be observed. This behaviour was found in Figure 7.23.

A final comment should be made concerning the dependence of the apparent threshold on crack length. It can be seen in Figure 7.10 that a short crack leads to a high apparent threshold, whereas Figure 7.23
shows the converse to be true. Here, a long crack also shows a high apparent threshold value. This apparent contradiction can be explained by the model. Looking at the residual stress intensity factor calibration in Figure 8.2, which is identical to that drawn in frame B of Figure 8.1, it can be readily argued that if a constant $\Delta K$ at $R = 0.5$ is added to that curve, a new effective $\Delta K$ value will be obtained. Only the positive component of the effective $\Delta K$, which has been shaded, can assist in the growth of the crack. Now, whenever part of the effective $\Delta K$ falls below zero, an apparent threshold will be seen when the fatigue crack propagation rate is plotted against the applied $\Delta K$. For the constant $\Delta K$ drawn in Figure 8.2, it is obvious that apparent threshold behaviour will be observed on both sides of $a_m$ and the apparent threshold $\Delta K$ will increase for crack lengths less than $a_1$ as well as increase for crack lengths greater than $a_2$.

8.4 FRACTURE TOUGHNESS TESTS

It is evident from the work reviewed in Section 2.5 that fracture toughness results are influenced by heat treatment as well as microstructural cleanliness. A two-fold variation in the fracture toughness due to these variables is not uncommon. Hence, whereas little difference in the stage II fatigue crack growth properties was observed for the four materials investigated, a considerably larger distribution could be expected in their fracture toughness properties.

Both fracture toughness and Charpy V-notch (CVN) properties were found. Due to their smaller size, CVN specimens can more easily measure local material properties and hence both axial and radial directions were investigated close to the bore as well as close to the outside of the tubing. Only average values of axial and radial fracture toughness were obtained.
Comparing the axial and radial fracture toughness ($K_{\text{max}}$) results presented in Section 7.14, almost isotropic behaviour is observed in both the U and V tubing. Anisotropic fracture toughness values are found in the DM and H tubing, where the axial fracture toughness values are 15% and 7% less than found in the radial direction. A similar relationship was found in the fatigue crack propagation results.

Various authors, including Parry & Lazzer (1969), Barsom & Rolfe (1970) and Iwadate et al (1977), have found a close correlation between fracture toughness and CVN tests. It is therefore justified to compare the CVN results given in Table 4.3 in the same manner as the fracture toughness results were examined previously. The greatest difference between the axial and radial CVN values occurs in the DM material, where a 6% difference is found. This result is consistent with that found in the fracture toughness tests, although a smaller percentage difference was obtained in the CVN result. It is also interesting to note that no difference in the average axial and radial CVN results were observed in the H tubing.

Of greater interest in the CVN results is the considerable difference found between inside and outside tubing impact properties in the H tubing. In the axial direction, this difference exceeds 23%, although no difference in the hardness (see Table 4.4) can be found, and hence this difference cannot be attributed to differential heat treatment. Since the other general cause of reduced fracture toughness (see Section 2.5) is an increase in the volume fracture of inclusions or the presence of a detrimental type of inclusion, a microscopic examination of the fracture surfaces may provide a likely reason for this peculiar behaviour. In this investigation, however, it is sufficient to attribute poor fracture toughness properties at the bore of the tubing to the manufacturing process which includes the hot reduction of the tubing using the Assel elongator.
The differences between the tubing inside and outside impact values is not as significant for the other materials investigated. It is somewhat surprising, however, that very little difference is observed for the U tubing which has only been degassed, whereas an 8% difference occurs in the vacuum arc remelt V tubing. The U tubing which shows the least difference in the inside and outside impact properties is physically larger than the other tubing (see Table 4.1) and if the initial billet size is the same for all tubing, it may be argued that the amount of hot work in the U tubing must be lower than that experienced in the other tubing. Hence, the microstructural difference between the inside and outside of the U tubing may be very low and since the fracture toughness is sensitive to the microstructure, this may account for the negligible difference observed between the inside and outside impact properties.

When comparing the fracture toughness and impact results for the four different materials amongst one another, it is immediately obvious that the U tubing shows the highest values, whilst poor properties are found in the DM tubing. The results from the latter tubing are consistent with the belief that this tubing contains a higher volume fraction of detrimental inclusions such as manganese sulphides. The U tubing has slightly lower values of yield strength and hardness than the other tubing, which may account for its disproportionately higher fracture toughness. The vacuum arc remelt tubing V may be expected to have good fracture toughness properties due to its considerable refinement but its high yield strength has the effect of reducing this value. Unfortunately, due to the different yield strengths of the materials used and the sensitivity of fracture toughness on this property, a direct comparison of the fracture toughness of the vacuum arc remelt and vacuum degassed process cannot be made.

A 16% decrease in the fracture toughness at 300 C compared to values
obtained at room temperature can be attributed to the temperature dependence of Young's modulus.

Finally, the relationships between fracture toughness and Charpy V-notch impact strength in the upper shelf region found by Barsom & Rolfe (1970) should be examined more closely. A large number of useful impact tests have been performed on similar tubing material in the past and CVN specimens are smaller and hence local material properties can be obtained.

The original expression by Barsom & Rolfe, rewritten in SI units, is:

\[
\frac{K_Ic}{\sigma_{ys}}^2 = \frac{0.66}{\sigma_{ys}} (CVN - \frac{\sigma_{ys}}{100})
\]

where the stress intensity factor, \( K_Ic \), is expressed in MPa√m, the yield strength, \( \sigma_{ys} \), in MPa, and the CVN in joules.

Average radial fracture toughness values obtained in this way from the impact data given in Table 4.3 yield values for DM, H, U and V tubing of 126 MPa√m, 137 MPa√m, 160 MPa√m and 140 MPa√m, respectively.

Comparison with the \( K_q \) or \( K_{EPS} \) values in Table 7.2 shows that the Barsom & Rolfe relationship probably overestimates the fracture toughness of the four tubing materials by 9%, 5%, 27% and 17%, respectively, whereas if this comparison is made with the \( K_{max} \) values in the same table, these differences are reduced to 0%, 0%, 4% and 7%, respectively.
CHAPTER 9
SIMULATION OF FATIGUE CRACK GROWTH
IN THICK WALLED TUBING

9.1 INTRODUCTION

Numerous attempts have been made during the past decade to predict the life of thick walled tubing which is subjected to internal pulsating pressure. The complexity of these solutions has ranged from simple approximations of a straight-fronted crack propagating through a constant stress distribution, to involved numerical techniques to obtain accurate stress intensity factor values at considerable expense. Both extremes suffer from inflexibility; the over-simplified model inadequately describes the problem, whilst numerical techniques are time-consuming and costly. Until such time that micro-computers become as powerful as today's mainframe computers, a more favourable solution must be sought in a combination of numerical and approximate analytical techniques. This chapter attempts to describe such a method.

It is apparent from the review of previous work in predicting the life of tubing (Section 2.7) that a model is required which can accommodate a crack shape which is allowed to vary from that of an initial flaw, such as an inclusion (≈ 20 μm), to its final shape at safe failure or fracture. This requirement is expressed explicitly by Underwood & Throop (1979) who observed an elliptical crack at tube failure with an aspect ratio \((b/a)\) of 0.39 which had started from a machined notch with \(b/a = 0.062\). Hodulak et al (1979) have attempted to qualitatively describe the reasons for this behaviour by including the influence of both free surfaces as well as material changes.

The distribution of the stress intensity factor along the perimeter of the crack front and fatigue crack propagation data in both the axial
and radial directions are required to predict the shape of the fatigue crack. Results presented in Chapter 7 have shown very different fatigue crack growth rates in these directions for some materials which may influence the life of the tubing, yet all the tube life predictions reviewed in Chapter 2 have assumed isotropic material properties. This does not come as a surprise since there is little point in obtaining both axial and radial fatigue crack growth properties if the crack shape cannot adapt to suit such anisotropic material behaviour.

The initial shape of a scratch, inclusion or multiple adjacent inclusions may also influence the life of the tubing. The model should allow such cases to be simulated.

Finally, a recent preliminary investigation (Lees (1980)) using sophisticated strain gauge and data logging instrumentation to follow the growth of a crack through thick walled tubing has provided evidence of the influence of the magnitude of pressure on the crack shape in commercial quality low alloy steel. No such influence could, however, be detected in higher quality steel. It would be satisfying if the model could also predict similar behaviour.

9.2 THE MODEL

The model for fatigue crack growth in thick walled tubing which allows the shape of the crack to vary is based on the expression derived by Williams (1980) for a semi-elliptical crack in a Lamé stress field which also includes pressure in the crack. The author, whose work was briefly reviewed in Section 2.6, obtained an analytical expression for a penny-shaped crack subjected to an axisymmetric pressure distribution. An approximate solution for an ellipse was then obtained by treating the ellipse as a circle of the same area. Hence, the expression for a semi-elliptical crack in thick walled tubing:
\[
\frac{K}{p \sqrt{\pi b}} = \frac{1}{\phi} \frac{2k^2}{k^2 - 1} \left( \sin^2 \theta + \frac{b^2}{a^2} \cos^2 \theta \right)^{\frac{1}{2}} f(\alpha) \tag{9.1}
\]

where \( p \) is the internal pressure, and \( k \) is the outside to inside diameter ratio. \( a \) and \( b \) are the major and minor half axes of the ellipse with \( b \) in the radial direction. \( \theta \) is the parametric angle such that 
\[
x = a \cos \theta, \\
y = b \sin \theta,
\]
with \( x \) and \( y \) falling on the major and minor axes, respectively. \( \phi \) is a complete elliptical integral of the second kind which can be approximated by a series expansion shown in equation (2.10). Finally, \( f(\alpha) \) is a function containing the variables \( k, W \) (the wall thickness), \( a \), \( b \) and \( \theta \), and can be found in equations (2.15) to (2.17).

For a given tube geometry and internal pulsating pressure, \( \Delta p \), the Paris equation (see Section 2.2) was used to obtain the new crack shape and size after the \((i+1)\)th increment of \( \Delta N \) cycles crack growth, namely:

\[
a_{i+1} = a_i + C_a \Delta K(a_i, b_i, \theta=0) \Delta N \tag{9.2}
\]

\[
b_{i+1} = b_i + C_r \Delta K(a_i, b_i, \theta=\pi/2) \Delta N \tag{9.3}
\]

and:

\[
N_{i+1} = N_i + \Delta N \tag{9.4}
\]

where subscripts \( a \) and \( r \) denote axial and radial fatigue crack growth material properties, respectively.

The use of the Paris equation in the above expressions implies a linear relationship between \( \log (da/\Delta N) \) and \( \log (\Delta K) \) from a threshold stress intensity factor, \( \Delta K_{th} \), to fracture.

This simple numerical formation was run on a micro-computer and the crack growth at a given pressure, tube geometry and initial flaw size and shape, was simulated by allowing the crack to grow in 100 to 300 increments.
A single computer run which included plotting of the crack front and printing of selected information every fifth increment took about five minutes to execute. The BASIC program used to obtain the results presented in the following section is shown in Figure 9.22. The equations (9.2) to (9.4) have been programmed in lines 1630 to 1760, whilst the Williams calculation of the stress intensity factor is computed in the subroutine, lines 1810 to 1990. Note that the command !PRINT was used to communicate with an HP7225A digital plotter using HP-GL instructions, whilst PRINT#1 transmitted data to the printer.

9.2.1 Limitations of the Model

Before the results are discussed, it may be worthwhile to point out possible limitations of the model.

Williams has compared his approximate solution with a boundary integral equation solution by Tan & Fenner (1980). For $k = 2$ and $b/a = 0.8$, good agreement is obtained in the range $b/W = 0.2$ to 0.5 (Tan & Fenner obtained data in the range $b/W = 0.2$ to 0.8). Hereafter, the solution by Williams appears to underestimate the radial stress intensity factor ($\theta = \pi/2$) and at $b/W = 0.8$, the Williams solution is 26% too low. Errors in the axial stress intensity factor ($\theta = 0$) are not as high and are less than 6% in the whole $b/W$ range.

Similar errors in the stress intensity factor for $k = 3$ are found in the radial direction, but in the axial direction errors are generally higher across the whole range and the stress intensity factor is overestimated by about 14%.

The Williams solution is derived from a semi-circular shaped crack in a stress field. The solution for an ellipse with a low aspect ratio may therefore be less accurate.

Finally, it is assumed that initiation occupies a negligible
number of cycles (see Section 2.3) and the Paris equation was assumed to
be valid from threshold to fracture.

9.3 RESULTS

Computer runs were made to investigate the influence of the following
variables.

(i) Initial $b/a$ ratio ranging from 0.05 to 1.0 for small ($\approx 20 \mu m$)
    and large ($\approx 1 mm$) cracks.
(ii) Changes in $C$ or $m$ of the Paris equation, but retaining
    isotropic material properties.
(iii) Anisotropic material properties simulated by changes in $C$ or
    $m$ of the Paris equation.
(iv) The dependence of crack shape on pressure in both isotropic
    and anisotropic materials.
(v) Geometry: Changes in outside/inside diameter ratio, $\kappa$, and
tube thickness, $\omega$.

The results are presented in the form of a graph and a table, and
collectively they are referred to as a figure. The graph shows a series
of contours which signifies a crack front as it propagates through the
tubing wall. The spacing of the grid drawn on the graph is in units of
0.1 $\omega$; the abscissa lies at the bore of the tubing, whilst the top end of
the ordinate lies at the outside of the tubing. Information of the tube
geometry, material properties, initial crack size and shape, and pressure
is included with each graph. The total number of cycles to failure is
also shown.

The table gives information of the number of cycles taken to reach a
particular contour, the values of $a$, $b$ and $b/a$, the radial stress intensity
factor at the apex of the crack front, and the axial stress intensity factor at the tube bore.

Since there is little point in plotting crack fronts for very short crack lengths, contours have only been drawn when the radial crack length exceeds 500 \( \mu m \). Each line of the table which has also been plotted is denoted with an asterisk.

In the subsequent computer runs used to investigate the influence of various parameters, the following tube geometry, crack shape and size, and material properties were used for reference.

Geometry:
\[
\begin{align*}
\kappa &= 2 \\
W &= 30 \text{ mm}
\end{align*}
\]
Initial crack size and shape:
\[
\begin{align*}
b &= 20 \mu m \\
b/a &= 0.5
\end{align*}
\]
Pressure range:
\[
\begin{align*}
p &= 220 \text{ MPa}
\end{align*}
\]
Material properties (isotropic):
\[
\begin{align*}
C &= 40 \times 10^{-12} \\
m &= 2.4
\end{align*}
\]

Note that the units of \( C \) are such that \( \Delta K \) and \( da/dN \) are expressed in MPa\(\sqrt{\text{m}} \) and \( \text{m/cycle} \), respectively. Also, the material properties are similar to those found in the V tubing. The other dimensions are fictitious and have been chosen as round numbers to facilitate comparison.

The effect of a particular variable was investigated by altering only that variable and keeping the others unchanged.

9.3.1 Influence of Initial Flaw Size and Shape

Figures 9.1 to 9.3 demonstrate the effect of three different initial \( b/a \) ratios of 0.05, 0.5 and 1.0 for a small initial crack with \( b = 20 \mu m \). No difference in the crack profile can be observed from the
figures, but a closer examination of $b/a$ values in the tables indicates that the crack shapes become virtually identical with $b/a$ exceeding 0.9 after a crack size of about $b = 0.5$ mm. It is also interesting to note that this occurs at approximately the half life of the tubing.

The number of cycles to failure is clearly dependent on the initial flaw shape; a flaw with $b/a = 0.05$ compared to a flaw of the same depth ($b = 20$ μm), but $b/a = 1$ will fail $47 \times 10^3$ cycles or 30% earlier. Elongated inclusions or a collection of round inclusions lying side by side may therefore be particularly harmful to the fatigue strength of the tubing.

Initial $b/a$ ratios of 0.05 and 1.0 but with a large ($b = 1$ mm) initial crack were also investigated. Figure 9.4 clearly shows that the crack only commences to grow significantly in the axial direction when $b/a$ exceeds 0.2. This is probably due to the large difference between the radial and axial stress intensity factors at low $b/a$ ratios. The eventual $b/a$ ratio at failure is 0.62, which is surprisingly close to 0.67 which was found in the small initial crack examples discussed previously.

The growth from a large initial crack ($b = 1$ mm) and $b/a = 1.0$ is shown in Figure 9.5 and is almost identical in shape to that where the crack grew from a small initial crack (see Figures 9.1 to 9.3). This result can be expected since the $b/a$ ratio of all the small initial crack problems discussed up to now are close to unity at a crack size of $b = 1$ mm.

Finally, it is interesting to note that tubing with an initial flaw of $b/a = 0.05$ and $b = 1$ mm also fails about 30% earlier than tubing with the same sized crack but with $b/a = 1$. A similar dependence on the $b/a$ ratio was observed for crack growth from small flaws ($= 20$ μm). This agreement cannot be explained at this stage and may be coincidental.
9.3.2 Changes in Constants $c$ and $m$ of the Paris Equation

On examination of equations (9.2) and (9.3), it can be seen that the Paris constant, $C$, and number of cycles, $\Delta N$, are separable. Hence, an increase in $C$ will lead to a corresponding decrease in the total number of cycles to failure. The crack shape will also clearly be independent of $C$. The above argument cannot apply to changes in the exponent $m$ and hence a change in crack shape may be expected. These two cases are considered below.

The value of $C$ was increased by an order of magnitude above the reference in both the axial and radial directions with the value of $m$ unchanged. The crack shape and number of cycles to failure are shown in Figure 9.6. The crack shape appears to be identical to that obtained for the previous value of $C$ (Figure 9.2), but the life has been reduced by a factor of ten. This result, therefore, confirms the argument above.

It is also important to investigate the influence of a different exponent $m$ in the Paris equation. A value of 3.6 was selected as a reasonable value which has been measured in some tubing (see, for example, tubing H). In order to maintain reasonable crack growth rates, the Paris constant $C$ was reduced such that the Paris curve with a high value intersects the usual or reference curve at $\Delta K = 10$ MPa$\sqrt{m}$.

Figure 9.7 shows the results from the computer run with a high Paris exponent. The life of the tubing under these material conditions is similar to that obtained with the usual $m$ value (see Figure 9.2) and supports the viewpoint that the standard and high $m$ Paris curves should intersect at 10 MPa$\sqrt{m}$ in order to maintain reasonable crack growth rates. A comparison of crack shape with Figure 9.2 shows that similar $b/a$ ratios are obtained up to $b = 6$ mm ($b/W = 0.2$), whereafter the high $m$ results yield lower $b/a$ values, and at failure the aspect ratio is 0.61 compared to 0.67 found in Figure 9.2. It may therefore be concluded that, even in
isotropic materials, the crack shape is dependent on material properties (Paris exponent $m$).

9.3.3 Anisotropic Material Properties

The influence of anisotropic material properties was investigated in a similar way to that described in Section 9.3.2 above, except that changes in the Paris constants were only made in the axial direction. Individual increases in the Paris constants $C$ (by an order of magnitude) and $m$ (from 2.4 to 3.6) were considered. The results from these two simulations of material anisotropy are shown in Figures 9.8 and 9.9.

In contrast to cases where isotropic material behaviour was modelled and where the $b/a$ ratio increased from its initial value of 0.5 to almost unity before falling to 0.67 at fracture, the simulation of anisotropic behaviour using a high $C$ value in the axial direction decreases immediately and ends at $b/a = 0.21$. This behaviour is clearly demonstrated in Figure 9.8.

Two further observations can be made from this simulation. Despite the crack growth rate in the axial direction being an order of magnitude higher than in the radial direction, the $b/a$ ratio is only about three times smaller than obtained in the isotropic case (Figure 9.2). Also, a long crack with a small $b/a$ ratio leads to a larger radial stress intensity factor and a smaller axial stress intensity factor when compared to a semi-circular crack shape.

The second method where anisotropic material properties are simulated using a high $m$ value in the axial direction provides an interesting contrast. The large variation in $b/a$ can be seen in Figure 9.9. The initial $b/a$ ratio increases rapidly and reaches a maximum of 1.26 at $b = 100 \, \mu m$, before reducing to a final value of 0.22. This
behaviour can be expected since the axial crack growth below $\Delta K = 10 \text{ MPa}^{\sqrt{m}}$ is retarded, whilst significantly higher crack growth rates in the axial direction are obtained thereafter.

Of further interest is the number of cycles to failure which is similar to that obtained in the isotropic case (Figure 9.2). An immediate response may be that the life of the tubing is controlled by radial fatigue propagation alone. Closer examination, however, of Figures 9.9 and 9.2 show that after half the life of the tubing, the crack in the anisotropic case has only propagated half as far in the radial direction as in the isotropic case.

9.3.4 Effect of the Magnitude of Pressure on the Crack Shape

The influence of the magnitude of pressure on the shape of the crack has been observed by Sommer et al (1979) and Lees (1980). Before using the model to predict the shape of the crack for isotropic and anisotropic cases, it may be worthwhile to first examine equations (9.2) and (9.3). Using the notation $\Delta a_i = a_{i+1} - a_i$ and $\Delta b_i = b_{i+1} - b_i$, it is possible to combine these equations into:

$$\frac{\Delta b_i}{\Delta a_i} = \frac{C_a \Delta K(a_{i+1}, b_{i+1}, \theta = \pi/2)}{C_a \Delta K(a_i, b_i, \theta = 0)}$$  \hspace{1cm} (9.5)

Also, since $\Delta K = \Delta p \sqrt{2} g(a, b, \theta, k, m)$, where the function $g$ is expressed as the right hand side of equation (9.1), the above equation can be written:

$$\frac{\Delta b_i}{\Delta a_i} = \frac{C_a \Delta K(a_{i+1}, b_{i+1}, \theta)}{C_a \Delta K(a_i, b_i, \theta)}$$  \hspace{1cm} (9.6)

where the subscripts to the function $g$ have the same meaning as before.

It is obvious from equation (9.6) that the pressure term is eliminated if the Paris exponents in the radial and axial directions are
the same. An influence of the magnitude of pressure could therefore only be expected in anisotropic materials where a difference exists in the Paris exponents in the two directions. Various examples were run to confirm these presumptions.

Two contrasting simulations at a higher pressure range of 280 MPa were performed to determine if any change in crack shape would occur in isotropic material as a result of using a different range of pressure. The two computer runs used the initial conditions of $b/a = 0.5$ with $b = 20 \, \mu m$, and $b/a = 0.05$ with $b = 1 \, mm$. The results are shown in Figures 9.10 and 9.11, respectively. These figures should be compared with Figures 9.2 and 9.4 which were obtained using a low pressure of 220 MPa. No change in crack shape can be observed.

Anisotropic material behaviour where the Paris $c$ has been increased by an order of magnitude in the axial direction also shows no pressure dependence on the crack shape. Figure 9.12 gives the results for a small flaw with $b/a = 0.5$ and a pressure of 280 MPa. The corresponding results obtained at 220 MPa are shown in Figure 9.8.

Finally, anisotropic material behaviour can also be modelled by a change in the Paris exponent $m$ which was described in Figure 9.9. A similar simulation run at a pressure of 280 MPa yields different $b/a$ ratios as can be seen in Figure 9.13. The $b/a$ ratios at high pressure generally appear to be lower than at low pressure, indicating a faster axial crack growth rate. The final $b/a$ values for the 220 MPa and 280 MPa pressure ranges are 0.22 and 0.19, respectively.

Although this result is probably not of any great significance, it does, however, confirm the observations by Lees (1980) where higher pressures appeared to decrease the $b/a$ ratio in poor (and hence probably anisotropic) materials only.
9.3.5 Tubing Geometry

It can readily be seen from the expressions derived by Williams (1980) to obtain the stress intensity factor and given in equations (9.1) and (2.15) to (2.17) that semi-elliptical cracks in geometrically similar tubing with different values of wall thickness, \( W \), but the same value of \( b/W \), will lead to identical values of \( K/(p \sqrt{\pi b}) \).

Now, if equations (9.2) and (9.3) are non-dimensionalised and a new function, \( g \), is included, where \( g = K/(p \sqrt{\pi b}) \), and \( g \) is a function of \( b/W \) for geometrically similar tubing, then:

\[
\frac{\Delta a}{W} = C_\alpha \left[ g(b/W) \right]^{m_a} \left[ \frac{m}{W} \right]^{m_a/2-1} \sigma N, \tag{9.7}
\]

and similarly in the radial direction.

Hence, the life of the tubing would appear to be a function of \( W^{1-m/2} \) if \( m = m_a = m_b \). For example, if \( m = 2.4 \), a tubing with dimensions half as large as another will last 15% longer if the initial flaw size is also half as large.

The above assumption of geometrical similarity which includes the crack size is not necessarily true since the initial flaw size may be a function of manufacturing process which may be similar for a wide range of differently sized tubing. To simulate this case, a solution was obtained using the model with \( W \) increased to 60 mm, whilst keeping the \( k \) ratio and the initial flaw size constant at 2 and 20 \( \mu \)m, respectively. The result is shown in Figure 9.14 which should be compared to the solution described in Figure 9.2. A similar \( b/a \) ratio was obtained and the difference in the number of cycles to failure between these two solutions with different \( W \) values is only 2% when the same flaw size was used.

A comment on the stress intensity factor values at failure
should also be made. Since the function $g$ or $K/(p\sqrt{b})$ is not a function of the wall thickness, $w$, the value of $K$ at failure should be a function of $\sqrt{b}$ only for geometrically similar tubing subjected to the same internal pressure. This conclusion is confirmed upon examination of Figures 9.2 and 9.14.

The influence of the diameter ratio, $k$, on crack growth through thick walled tubing was also investigated. The pressure for a diameter ratio $k$ of 3 was increased to give the same bore hoop stress, using Lamé's equations, as was used in previous runs with $k = 2$. The results are shown in Figure 9.15. Comparing this figure with Figure 9.2, it can be seen that the life of the tubing is considerably reduced if the equivalent pressure is calculated in this way. Calculating the equivalent pressure on the basis of the sum of bore hoop stress and pressure in the crack (or the maximum bore shear stress) results in a life of $154 \times 10^3$ cycles which is only slightly higher than found in the $k = 2$ case. A comparison of these two results is interesting since a similar result was reported by workers at Bristol University two decades ago (see Section 2.7). The value of $b/a$ at failure for the $k = 3$ tubing is 0.58 and hence it would appear that the crack shape may be dependent on the $k$ ratio.

9.4 PREDICTION OF FATIGUE CRACK GROWTH IN THE TUBING USED IN THIS INVESTIGATION

Similar figures to those presented in Section 9.3 were also obtained for the four tubing materials investigated in this work using the fatigue crack growth results presented in Chapter 7. These figures, together with life predictions at different pressures, are discussed in this section.

The pressure used in the computer runs shown in Figures 9.16 to 9.20 was chosen to give the same bore hoop stress in each tubing. Also, $R$ ratios of 0.5 were chosen, with the exception of DM tubing, where an
additional R ratio of 0.05 was considered, since the tests used to obtain the material properties were restricted to this R ratio.

Figure 9.16 shows the crack growth in DM tubing for an R ratio of 0.05, whilst Figure 9.17 gives details of crack growth in the same tubing but R = 0.5. Due to the difference in the Paris exponent m, different crack shapes in these two cases can be expected and final values of \( b/a = 0.45 \) and 0.39, respectively, were found. Also, the tubing subjected to a low mean pressure has a useful life more than twice as long as the tubing subjected to a high mean pressure.

It can be seen from Figure 9.18 that a very high value of \( b/a \) is observed (\( b/a = 1.58 \) at \( b = 130 \mu m \)) in H tubing before reaching a final value of 0.34. This is due to the considerable anisotropy in this tubing's fatigue crack growth properties.

Curves for U and V tubing are shown in Figures 9.19 and 9.20, respectively. Final values of \( b/a \) for these two cases are 0.6 and 0.64.

The predicted final \( b/a \) values should be compared with values from tests on short cylindrical specimens obtained by Rogan (1979). It was generally found that the experimental values were considerably higher than those obtained in the model and values for DM, H, U and V tubing were found to be 0.77, 0.56, 0.91 and 0.91, respectively. It is encouraging to note, however, that the correct order is predicted for the four materials.

There appear to be two reasons for this discrepancy. The Williams solution underestimates the radial stress intensity factor for crack lengths greater than \( b/W = 0.5 \) and hence predicts crack shapes with \( b/a \) values that are too low. Also, the length of the specimens used by Rogan may have been too short to allow the crack to develop properly.

If measurements of the crack shape are made at \( b/W = 0.5 \), then the model should provide accurate results. The \( b/a \) values at this crack
length for DM, H, U and V tubing are 0.48, 0.46, 0.71 and 0.75, respectively. These values are still too small, hence it would appear that the specimens used by Rogan may have been too short or some other unknown reason may be the cause of this result.

A series of computer runs were executed at different pressures for the four types of tubing investigated in this work, in order to predict an S-N type curve. The fatigue crack growth material properties and tubing geometry can be found in Figures 9.17 to 9.20. An $R$ ratio of 0.5 and a $b/a$ ratio for the initial flaw of 0.5 were used.

An accurate estimate of the initial flaw size is important to predict the life of the tubing, whilst the initial flaw size, together with the threshold fatigue crack growth rate, are used to obtain the fatigue limit. It is interesting to review the work of other researchers in this regard. Lankford (1977) found that cracks initiated from calcium aluminate inclusions which ranged in diameter from 5 $\mu$m to 20 $\mu$m. Haslam (1972), on the other hand, estimated an initial flaw size of 2 $\mu$m using a condition attributed to Frost (1965). Finally, Smith (1977) has argued that the minimum fatigue crack size for which continuum considerations should hold is at least 50 times that of a minimum fatigue substructural element, i.e. a subgrain slip band. If this is 0.5 $\mu$m, then an initial crack size of 25 $\mu$m can be expected.

Looking at the approximate fatigue limits obtained by Rogan (1979), and shown in Figure 4.6, result in initial flaw sizes $b$ for DM, H, U and V tubing of 26 $\mu$m, 24 $\mu$m, 31 $\mu$m and 31 $\mu$m, respectively, if $\Delta K_{th} = 3.5$ MPa$\sqrt{m}$ and $b/a = 0.5$. Decreasing $\Delta K_{th}$ to 3.9 MPa$\sqrt{m}$ results in a corresponding reduction in initial flaw size to 20 $\mu$m, 18 $\mu$m, 24 $\mu$m and 24 $\mu$m, respectively. According to Cooke et al (1975), a threshold stress intensity factor in this region can be expected, although a large error in this estimate may be expected since threshold fatigue crack propagation
tests are difficult to conduct.

In the absence of accurate measured values of initial crack size for the materials used in this investigation, a value of \( b = 20 \ \mu m \) was chosen as a likely flaw size. Using this and other previously mentioned values for the parameters required in the model, predicted values of tubing life as a function of pressure were obtained. These results are shown in Figure 9.21 and in Table 9.1.

Although these predictions correspond reasonably well with the limited number of experimental results available, it was thought prudent to also examine the effect of using an initial flaw size of only 5 \( \mu m \). It can be seen from Figure 9.21 that this curve obtained for DM material overestimates the useful life of the tubing and this estimate of initial crack size is therefore probably too small. Increasing the \( b/a \) ratio would similarly move the predicted curves to the right (compare Figures 9.2 and 9.3).

The fatigue limit is clearly more difficult to predict since accurate values of the flaw shape and size and the threshold stress intensity factor are required. Both these parameters are difficult to measure and therefore large errors may result. As a consequence, it was thought that it would be reckless to draw fatigue limits for the various curves in Figure 9.21. Nevertheless, using \( \Delta \kappa_{th} = 3.5 \ \text{MPa}\sqrt{\text{m}} \), \( b/a = 0.5 \) and \( b = 20 \ \mu m \), fatigue limits of 210 MPa, 222 MPa, 230 MPa and 220 MPa were obtained for DM, H, U and V tubing, respectively.

A final reassurance of the validity of the model proposed in this chapter is obtained by comparing the results with a similar prediction, using the data provided by Tan & Fenner (1980) for an elliptical crack with \( b/a = 0.8 \) in tubing with \( k = 2 \). A quadratic curve was first fitted to the radial stress intensity data which included an additional point at the tube bore obtained from the expression by Irwin (1962) (see Section 2.6):
TABLE 9.1

Number of Cycles to Failure for Various Tubing and Pressure with $R = 0.5$ and an Initial Flaw Shape and Size of $b = 20 \mu m$ and $b/a = 0.5$

<table>
<thead>
<tr>
<th>Pressure (MPa)</th>
<th>Life of Tubing ($10^3$ cycles)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DM</td>
</tr>
<tr>
<td>200</td>
<td>234</td>
</tr>
<tr>
<td>220</td>
<td>182</td>
</tr>
<tr>
<td>240</td>
<td>149</td>
</tr>
<tr>
<td>260</td>
<td>118</td>
</tr>
<tr>
<td>280</td>
<td>97</td>
</tr>
<tr>
<td>300</td>
<td>81</td>
</tr>
<tr>
<td>320</td>
<td>69</td>
</tr>
<tr>
<td>340</td>
<td>60</td>
</tr>
</tbody>
</table>
Predictions of the number of cycles to failure using this expression were generally 10% lower than those obtained from the model described in this chapter. Isotropic material properties of $C = 16.2 \times 10^{-12}$ and $m = 2.64$ were assumed in both cases ($da/dN$ in m/cycle and $\Delta K$ in MPa$m^{1/2}$).

9.5 DISCUSSION

The model described in this chapter appears to provide reasonably accurate predictions of the life of the tubing. It is not exceptional in this respect since similar claims have been made in the other papers reviewed in Section 2.7.

However, due to its ability to accommodate both the radial and axial fatigue crack propagation properties, the model is able to provide additional useful information which is summarised below.

Initial elliptical flaws with a low aspect ratio are liable to cause earlier tube failure since they appear to be able to develop to a stable shape more quickly. From this point onwards, the number of cycles to failure is the same, no matter what the shape of the initial flaw was.

As a consequence of the above observation, it is not possible to determine from the appearance of the fracture surface and final crack shape at failure whether a crack initiated from a round or elongated flaw. The life of the tubing will provide more information, since an elongated initial flaw will result in a shorter tubing life. It would not be possible, however, from a macroscopic examination of the crack shape and life of the tubing, to distinguish between the tube failure as a result of a small elongated flaw and a large round flaw.

The Paris constant, $C$, appears to have no effect on the crack shape
in isotropic materials, although the crack shape appears to be dependent on the Paris exponent, $m$.

Anisotropic materials can be easily simulated by a change in the Paris constants $C$ and $m$. A change in $m$ appears to exaggerate the variation in $b/a$ as the crack propagates through the wall of the tube.

The influence of the magnitude of pressure on the crack shape has been examined using the model. It has been argued analytically as well as by way of examples that the magnitude of pressure will only influence the crack shape if there is a difference in the Paris exponent, $m$, between the axial and radial directions.

Some comment should be made concerning the prediction of the life and fatigue limit of tubing. In addition to obtaining stage II fatigue crack propagation properties in both the axial and radial directions, estimates of the initial crack size and shape as well as threshold stress intensity factor are important. Values for these last three parameters were not obtained in this investigation.

Values of the initial flaw shape and size and $\Delta K_{th}$ may best be found from simple tests on polished tensile fatigue specimens used to obtain the transverse fatigue limit of the tubing material. Subsequent microscopic examination of the fracture surface may reveal the shape and size of the flaw which caused the initiation of the fatigue crack. Hereafter, the value of the threshold stress intensity factor can be obtained from a simple fracture mechanics analysis of the specimen. Also note that this method of obtaining $\Delta K_{th}$ may be faster than using fracture mechanics type specimens because tensile specimens can be loaded at considerably higher frequencies due to their lower compliance.

The effect of the initial flaw shape and size on the life of the tubing has already been discussed. Another factor which may influence the number of cycles to failure is the degree to which the pressure acts in the crack.
Also, the error resulting from the approximation that there is an abrupt transition between stage II crack growth and $\Delta K_{th}$, as opposed to the real situation where a stage I region exists, can affect the prediction.

The model used in this investigation can clearly be improved. The first changes may be to include a correction to the Williams solution for $b/W$ values greater than 0.5 as well as a correction for the plane stress conditions at the bore of the tubing. In this latter respect, it may be worthwhile to use the stress intensity factor a little way into the tube wall, at say $\theta = 15^\circ$, to calculate the propagation in the axial direction. Clearly, more runs of the program by Tan & Fenner (1980) are required to allow these corrections to be made.

Comparisons between the model and experimental results were made on the basis of $S-N$ type curves. The only proper comparison must be against data obtained by monitoring the growth of the crack through the tube wall. Further work in this field is required.

Finally, a brief comment should be made with regard to fast fracture in thick walled tubing. The Williams model provides a complex relationship between the stress intensity factor and the aspect ratio, $b/a$. The following qualitative relationship can be described which can be seen in the figures presented in this chapter.

(i) A rounded crack with a high aspect ratio has an axial $K$ value which is higher than that in the radial direction.

(ii) The converse is true for an elongated crack where the radial $K$ value is higher than the axial value.

Hence, it may be argued that fast fracture from an initially rounded crack will alternatively occur in the axial and radial directions in a see-saw or oscillating fashion. Similar behaviour could be expected from an
Initially elongated crack, except the first cycle of fast growth would occur in the radial direction. This type of behaviour is demonstrated in Figure 9.1, except that it shows the fatigue crack growth from an initially elongated flaw.
10.1 CONCLUSIONS

The object of this investigation, as set out in the introduction, was to predict the fatigue failure and fracture of thick walled tubing using linear elastic fracture mechanics. The approach assumed here, in common with previous similar work by other authors, has been to relate the fatigue crack growth and fracture properties, found by conducting tests on simple specimens, to the actual behaviour of the tubing using a suitable model.

This investigation differs from its predecessors in a number of ways. Both axial and radial fatigue crack growth rates and fracture toughness values were obtained from specimens cut from the tubing. Also, the model is not limited to fatigue crack propagation in the radial direction alone; both axial and radial crack growth rates can be accommodated, allowing the crack to change shape as it propagates. Considerable attention was given to the development of an automated materials testing facility which allowed accurate values of fatigue crack growth to be obtained with a significant reduction of operator intervention. This facility included a micro-computer suitably interfaced to the servo-hydraulic testing machine, as well as an AC electrical potential system to measure crack growth. The effect of temperature, mean and residual stress, fatigue frequency and manufacturing route on the fatigue crack growth rate of the tubing material were investigated using this equipment. Fracture toughness and a selected number of other material properties were also obtained.

Although the separate fields of fracture mechanics and failure in thick walled tubing (seen from the general S-N curve point of view) have both enjoyed active research at Imperial College for quite some time, the
merging of these fields in an attempt to describe the behaviour of thick walled tubing more accurately is still relatively young. This investigation is one of the first to make use of this new attitude and could be viewed as a forerunner in this field. As a result, the list of recommendations for future work is relatively long. Nevertheless, when compared with previous work in the prediction of the failure in thick walled tubing, this investigation has provided considerable data on the fatigue crack propagation and fracture toughness of steels used in the manufacture of thick walled tubing. Also, a new model to predict the life of the tubing has been proposed which has a greater amount of freedom in its application than previously known models.

More detailed conclusions of the results presented in the various sections of this work are drawn below.

A micro-computer based system used to control fatigue crack propagation tests as well as to obtain and analyse the data has been criticised in the past as being too slow in execution time and the memory insufficiently large. It was claimed that a mini-computer is required to adequately perform these tasks. From the results presented in this investigation, however, it is evident that the micro-computer based system has performed satisfactorily in both constant $\Delta P$ and $\Delta K$ tests. No difficulty was encountered in the measurement of waveforms up to 100 Hz, which is probably the limiting frequency in material testing, and the 12 bit or 0.024% resolution of the analogue to digital converter (ADC) and signal conditioner (SC) proved to be adequate. The software described in Chapter 3 was found to be sufficiently general purpose to conduct constant $\Delta P$, $\Delta K$ and threshold $\Delta K$ tests. The various techniques written in BASIC and used in the program, such as to obtain the derivative of the crack length as the number of fatigue cycles increased and the execution of a smooth transition from dynamic to static load and vice versa, performed
adequately. One of the greatest advantages of the micro-computer system, however, was the increased turnover of tests and the considerable reduction in the tedium of conducting these tests.

An AC electrical potential system (EPS) was developed which provided a discrimination in crack length of between 10 μm and 20 μm. The system was also used in fracture toughness tests to find the point where the initiation of slow crack growth occurred. The AC EPS performed very well with the exception of two difficulties: the change in the specimen conductance with temperature, and the sensitivity of the current lead position in space caused problems with repeatability. The former problem is also found in the DC EPS and can be readily solved by measuring the conductance or temperature change of the specimen and consequently modifying the calibration curve.

Five specimen types were used to obtain fatigue crack growth results. To ensure accurate $K$ calibrations for common (CKS, DCB and C-shaped) specimens and to obtain new $K$ calibrations for unusual specimens (split-ring (SR) and RING), experimental and numerical solutions were found. The boundary integral equation (BIE) method was used in the latter technique which proved to be a simple and fast way of finding the $K$ calibration for virtually any specimen which can be modelled in two dimensions.

Numerous interesting conclusions can be drawn from these solutions. In the CKS, an increase in the pin diameter (solutions were obtained for an increase of about 20%) caused no change in the stress intensity factor. Hence, pin failures in fatigue tests can be virtually eliminated through the use of larger diameter pins. For the C-shaped specimen, the agreement between the expression by Underwood (now ASTM E399-78a), the BIE method and the experimental data was generally good, although the Underwood curve may give slightly lower results in the range $a/W = 0.3$ to 0.6. It would appear that the SR specimen can be used to obtain fatigue crack
growth properties close to the bore of the tubing if a reliable method can be found to obtain even, straight-fronted crack growth. This difficulty is caused by the rapidly increasing $K$ value close to the bore of the tubing. Alternatively, a new specimen should be developed which shows a more gradual increase in $K$. The RING specimen can be used to evaluate the growth of a crack through a residual stress field, although very high loads are required. This specimen also shows a strongly increasing $K$ value up to $a/W = 0.35$, whereafter it remains relatively constant.

The results of tests performed on one sample of AISI 4333 vacuum degassed, cold reduced material to determine the effects of mean stress, temperature, residual stress, fatigue frequency and specimen breadth, $B$, can be summarised as follows. Note that only stage II fatigue crack growth data were obtained.

In the stage II fatigue crack growth region, a distinct dependence on $R$ ratio was observed. Increasing the $R$ ratio to 0.5 led to fatigue crack growth rates which were between 1.6 and 3 times higher than those obtained at 0.05. A greater influence at low $\Delta K$ values was observed and similar trends were found in both the axial and radial directions.

No influence of temperature was observed in the fatigue crack growth rates in both directions for an increase in temperature from 20°C to 300°C. A reduction in the fracture toughness of 16% in both directions was found for the above temperature range.

The fatigue frequency did not influence the fatigue crack propagation results in the range 5 Hz to 30 Hz. An increase in the specimen breadth from 25 mm to 40 mm in a C-shaped specimen gave the same fatigue crack propagation results.

Tests conducted on radial specimens cut from autofrettaged tubing displayed an apparent threshold of above 10 MPa m$^{-1}$ when plotting $da/dN$. 

against applied $\Delta K$. This apparent threshold value is a function of crack length and the apparent threshold $\Delta K$ decreased with increasing crack length for short crack lengths, whilst the converse was found for long crack lengths. Also, the effect of residual stress vanished for applied $\Delta K$ values exceeding about 22 MPa$\sqrt{m}$. The above trends have been explained qualitatively using a simple model. No effect of residual stress could be observed in the axial specimen, although the crack appeared more bowed due to the tensile residual stress in the centre of the specimen in the autofrettaged material.

Anisotropy was measured in the tubing using specimens cut such that fatigue crack growth and fracture properties could be investigated in the radial and axial specimens. The anisotropic fatigue crack growth and fracture toughness results seen in DM tubing appeared to be caused by a greater or more detrimental inclusion population than found in the other tubing. No metallurgical examination was conducted, however, and hence this conclusion should be viewed tentatively at this stage. The twisting nature of the hot reduction in the manufacture of the hot Asselled tubing H and the outside of the other tubing appeared to result in axial fatigue crack growth properties that were significantly lower than found in the radial direction. Fatigue crack growth rates in the radial direction were unaffected by the manufacturing route. Both the axial as well as radial fracture toughness were affected, however, and Charpy V-notch impact values close to the bore of the tubing were significantly lower than those found at the outside for the hot Asselled tubing.

No significant difference in the stage II fatigue crack growth and fracture toughness could be found between the air melt vacuum degassed and the vacuum remelt processes, although there are indications that the latter process may give rise to better fracture properties. Similarly, no difference could be distinguished between the hot reduced and the
additionally cold reduced materials. It must be stressed, however, that too many variables were investigated using too few material types. A more controlled environment where only one variable is changed at a time may prove to be more beneficial in finding the influence of these variables on the fatigue crack growth rate and fracture toughness properties.

These material properties were used in a model in an attempt to predict the life of the tubing as well as determine the influence of various material and geometric variables on the growth of a crack through thick walled tubing.

The model is based on a solution by Williams (1980) which describes the stress intensity factor for a semi-elliptical crack in thick walled tubing. The model allows the crack to grow independently in both the axial and radial directions, depending on the respective values of the stress intensity factor and fatigue crack growth rate in those directions at a given time. As a result, it was found that the aspect ratio of the semi-ellipse can, for instance, increase from 0.05 to 0.95 and then decrease to 0.67 at failure.

Using assumed values of the initial flaw size and threshold \( \Delta K \), predictions of the life of the tubing were made, and these results are similar to those found in \( S-N \) type tests using thick walled tubing. Various conditions of initial flaw shape and size, changes in isotropic and anisotropic material properties, pressure magnitude and geometric variables were considered using the model. The results from these simulations can be summarised as follows.

(i) Initial elliptical flaws with a low aspect ratio will cause earlier failure since they appear to develop to a stable crack shape more quickly.
(ii) The Paris constant, $c$, has no effect on the crack shape in isotropic materials, but the shape is dependent on the exponent, $m$.

(iii) Anisotropic tubing can be predicted using different Paris constants, $C$ and $m$, in the two directions.

(iv) The magnitude of pressure influences the crack shape in anisotropic tubing only when different Paris exponents, $m$, are found in the axial and radial directions.

(v) Fast fracture from a flaw may occur in a see-saw fashion since a low aspect ratio ellipse has the highest $K$ in the radial direction, whilst a high aspect ratio leads to a high $K$ value in the axial direction.

10.2 RECOMMENDATIONS FOR FUTURE WORK

This investigation formed a small part of a larger project to investigate the failure of thick walled tubing subjected to a pulsating internal pressure. As a result, a long list of recommendations for further work is presented here.

The micro-computer system performed very well and hence requires no modification. Further attention should be given to the AC electrical potential system (EPS) in an effort to improve its repeatability. To eliminate the temperature dependence of the specimen conductance, a thermocouple should be attached to the specimen which can be measured using a free ADC channel. A calibration curve of conductance as a function of temperature can then be used to correct the initial EPS calibration curve. An attempt should also be made to reduce the earth loop effect peculiar to the AC EPS which has been described earlier. The complete electrical
insulation of the specimen or the more elaborate twisting and screening of
the current leads may provide improved repeatability.

The $K$ calibration of the RING specimen described in Chapter 5 may be
incorrect and it may be purposeful to generate a new mesh for the BIE
program to allow more elements to be included in the unbroken ligament of
the RING. The format of the BIE output should be improved to give more
accurate values of displacement and hence stress intensity factor.

A metallurgical examination of the fatigue and fracture surfaces
should be made in an attempt to describe the results of tests on various
tubing. A new specimen should be developed to find fatigue crack growth
properties close to the bore of the tubing which displays a moderate
increase in the stress intensity factor for a given change in crack length.
Valid fracture toughness values may be sought using broader specimens and
using non-linear elastic fracture mechanics if necessary.

Interesting results were obtained from the autofrettaged tests and
considerably more work is required in this field. Using either or both
experimental compliance techniques and the BIE program, it should be
possible to calculate the fictitious stress intensity factor caused by the
residual stress distribution. In the experimental method for measuring
the compliance, a change in slope should accompany the opening of the
crack. Inserting the value of load found at the knee of the compliance
curve into the existing $K$ calibration of the C-shaped specimen should give
the required fictitious value of $K$. The model described in Chapter 8,
and shown in Figure 8.1, can then be checked against a constant $\Delta K$ test
performed as given in Figure 8.2. Finally, similar tests should be
conducted on the RING specimen where the residual stress distribution is
similar to that found in the thick walled tubing.

It was shown in Chapter 4 that the residual stress decreases with
time or the application of an alternating load without the growth of a
crack. If a compressive residual stress is used to prolong the life of the tubing, it is important to quantify this decrease. Tests should therefore be conducted to investigate the influence of time, the load magnitude and frequency, and the temperature on the residual stress. The use of simple specimens where the residual stress can be readily determined, should perhaps be used in preference to tests on RING specimens.

The model to predict the life of the tubing as described in Chapter 9 should be modified to also include large $b/W$ ratios. The accuracy of the model when using small aspect ratios should be determined. Actual values of the initial flaw size and threshold $\Delta K$ should be found and, finally, a test should be conducted to find the stage I fatigue crack growth and hence determine the error in assuming stage II crack growth right down to $\Delta K_{th}$. 
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FIG 2.1 THE SIGMOIDAL NATURE OF THE FATIGUE CRACK PROPAGATION CURVE
1/8 SECTION OF TUBING

ABC'D' EFG'H'

CRACK IN PLANES IJEF & KLGH

1/4 SECTION OF TUBING

ABCD EFGH

CRACK IN PLANE IJEF ONLY

FIG 2.2 SECTIONS OF TUBING USED IN MODELS FOR NUMERICAL ANALYSIS
FIG 9.1 SIMPLE ELECTRICAL POTENTIAL SYSTEM (EPS)
FIG 3.2 NARROW BAND AMPLIFIER
FIG 3.3 GAIN AS A FUNCTION OF FREQUENCY FOR THE NARROW BAND AMPLIFIER. \( f_0 = 3230 \text{Hz} \)
FIG 3.4 VMOS AC CONSTANT CURRENT POWER SOURCE
FIG 3.5 THE DEPENDENCE OF THE AC ELECTRICAL POTENTIAL ON THE CRACK OPENING DISPLACEMENT
FIG 3.6 TEMPERATURE DEPENDENCE OF EPS AT CONSTANT $\alpha$. 

$V = 2.691 + 1.857 \times 10^{-4} T + 1.45 \times 10^{-6} T^2$
FIG 3.7 POSITIONS OF CURRENT AND SENSING LEADS FOR VARIOUS SPECIMENS
FIG 3.8 DISPLACEMENT TRANSDUCERS FOR C-SHAPED SPECIMENS
FIG 3.9 TRANSDUCER TO MEASURE DISPLACEMENT IN THE LOADING LINE OF A C-SHAPED SPECIMEN (TYPE III)
FIG 3.10 CALIBRATION CURVE OF THE TYPE III TRANSDUCER
FIG 3.11 MICRO-COMPUTER AND SERVO-HYDRAULIC CONTROL LOOPS
FIG 3.12 CONSTANT ΔK TEST

1. MONITOR EPS OUTPUT V
2. If $V_i > V_{i-1} + \Delta V$, then:
   - CALCULATE $\alpha$ FROM CALIBRATION CURVE
   - FIND $\Delta P$ FROM $\Delta P = \Delta KB f W / f (\alpha/W)$
   - DRIVE TESTING MACHINE WITH NEW $\Delta P$
   - NO
FIG 3.13 QUADRATIC DECREASING FUNCTION FOR $\Delta P$
IN ORDER TO OBTAIN $\Delta K_{th}$
FIG 3.14  PROTOTYPE MICRO-COMPUTER SYSTEM AND INTERFACE TO TESTING MACHINE.
FIG 3.15 IMPROVED MICRO-COMPUTER SYSTEM AND INTERFACE TO TESTING MACHINE
FIG 3.16 FUNCTIONAL LAYOUT OF THE ADC
FIG 3.17 ADC MACHINE CODE SOFTWARE
FIG 3.18 MACHINE CODE SOFTWARE TO FIND MAXIMUM AND MINIMUM VALUES OF ANY ADC CHANNEL.
FIG 3.19 FUNCTIONAL LAYOUT OF THE SIGNAL CONDITIONER
FIG 3.20 FUNCTIONAL LAYOUT OF THE CYCLE COUNTER
FIG 3.21 SIMPLE PROGRAM TO DEMONSTRATE SOFTWARE USED WITH A TESTING MACHINE.
FIG 3.22 FATIGUE CRACK GROWTH SOFTWARE - START

START OF TEST

SET CONSTANTS

READ TEST DETAILS

INITIALIZE VARIABLES

DEFINE FUNCTIONS

PRINT TEST DETAILS TO TAPE & PRINTER

RESET CYCLE COUNTER

STANDBY
FIG 3.23 FATIGUE CRACK GROWTH SOFTWARE - CONTROL
MEASURE

DEFINE COMPUTATION DOMAIN (CD)

RAMP OPTICAL ø END OF TAPE

READ COUNTER AND RESET IT. V=DYNAMIC EPS

CHANGE DATA TAPE IF FULL

OPTICAL ø REQUIRED

 YES INPUT NEAR AND FARSIDE OPTICAL ø VALS

 NO RAMPDOWN SELECTED

 MEASURE PRESENT STATIC LOAD

 RAMP DOWN BY ONE INCREMENT FROM THIS LOAD

 MEASURE EPS DISPLACEMENT AND LOAD

 CALCULATE APPROXIMATE K FROM EXPRESSION

 K (APPROX) = K (EPS)

 YES V=MEASURED EPS

 NO END OF RAMPDOWN

 ð=f(V) FROM EPS CAL CURVE

 FIG 3.24 FATIGUE CRACK GROWTH SOFTWARE – MEASUREMENTS
FIG 3.24 CONTINUED
FIG 3.24 CONTINUED
REM GENERAL PURPOSE FATIGUE CRACK GROWTH PROGRAM  G C KLINTWORTH

PRINT"EPS=ADC CH2" :REM SELECT CORRECT CHANNELS
PRINT"DISP=ADC CH4"
PRINT"EPS=ADC CH2"
PRINT"SC OUTPUT=MAYES INPUT"
PRINT"SIG GEN OUTPUT=SC INPUT"
PRINT"X AXIS=DAC CH1"
PRINT"Y AXIS=DAC CH2"
PRINT"L/R=DAC CH3"
PRINT"PEN=DAC CH4"

O=107*256 :REM DEFINE ADDRESSES
ADC=Q+8
MEAN=Q+29
AMPL=Q+38
MAX=Q+56
MIN=Q+65
POKE Q+633,4 :REM SET MAX/MIN SAMPLE TIME AT 4*30 MS

SYS(MEAN):DU=USR(0)
SYS(AMPL):DU=USR(0)

OPEN 2,1,2
OPEN 1,9
OPEN 3,3

B$=" 
D$="999999999"
Z$="000000"
N$=" 123456789"
R$=CHR$(13)
T0=40944
T1=T0+1
T2=T0+2
T3=T0+3
Z=0
TM=590
CB=3276.8
CD=25.6
M8=256
C=255
CF=60
CH=20
CP=80
DA=4935
H%=5
SD=.75
SE=.33
NI=2
RD=10

REM

FIG 3.25 FATIGUE CRACK GROWTH PROGRAM (13 PAGES)
GOSUB2000 : REM FETCH TEST DETAILS
71 CC=EP/CR : REM LENGTH OF DYNAMIC/STATIC TRANSITION
72 CL=PC/CB : REM GIVES EPS FROM ADC READING
73 CM=DC/CB : REM LOAD
74 CA=YM/((1-NU*NU)*2) : REM MATERIAL CONSTANT
75 MF=FI : REM MAGNIF. FACTOR FOR FANCY FUNCTIONS
77 VS=PS/PC : REM MEASURED MEAN LOAD (VOLTS)
78 VM=PA/PC : REM AMPLITUDE
79 VK=VS*MF : REM DRIVING MEAN LOAD (VOLTS)
80 VL=VA*MF : REM AMPLITUDE
85 REM
90 DIMG(2,3),U(4),O(2),P(4),DT(5),VT(5),PT(5),KT(5)
95 REM
90 DEFNA(X)=A0+A1*X+A2*X*X+A3*X*X
100 ONSPGOTO110,140,160,180 : REM C/CKS/DC3/SPECIAL
110 DEFFNU(X)=((18.23/X/X-J06.2/X+379.7-582*X+369*X*X)*X+2.5
120 DEFFNK(X)=FNU(X)*(1+1.54*C/X+S.5*X)*(1+0.22*(1-SQR(X)))*W/RO)/B/SGR(W)
130 GOTO190
140 DEFFNK(X)=(29.6/X/X-185.5/X+655.7-1017*X+638.9*X*X)*X*X*SQR(X/U)*B
150 GOTO190
160 DEFFNK(X)=SQR(4*(3*(X*U+.7*HH2+H*H)/(BN*(1-NU*NU)*B*H)+3.43+X*5))/B/SQR(W)
190 DEFFND(X)=(X+ABS(X)>/2+SGN(INK(X+ABS(X))/5J0))«<C-X>
192 REM
195 GOSUB3000 : REM OUTPUT TEST DETAILS
196 DU=PEEK(T3) : REM RESET COUNTER
197 GOSUB7000 : REM GOTO STANDBY
199 REM
200 REM CONTROL ROUTINE
201 X=2 : REM READ ADC CHANNEL 2 IE. EPS
202 GOSUB5000
203 VE=X*CC : REM SPECIMEN OUTPUT
210 PRINT"hsssssssssEPS SPEC OUTPUT ";
212 PRINTVE"w V"B$
214 REM
216 T0=PEEK(T0) : REM READ COUNTER REGISTERS
217 T1=PEEK(T1)
218 T2=PEEK(T2)
219 T=INT(T0%+H8*(T1%+H8*T2%)) : REM COMPUTE NO. OF CYCLES SO FAR
220 PRINT"sCYCLES SINCE PREV. READING";T;"w (hrs) AGO
221 REM
222 GETA$: : REM GET A CHARACTER FROM KEYBOARD
223 IF A$="S"ORVE>YMTHENGOSUB7000 : REM GOTO STANDBY IF 'S' OR EPS > MAX
224 IF T<THENTHEN240 : REM FATIGUE & CONTROL FOR > TM CYCLES
225 IF VE<VOOR$="H"THEN600 : REM MEASUREMENT LOOP IF 'H' OR EPS OK
230 REM
REM FIND MAX & MIN OF ADC CHANNEL 3
REM IE. LOAD
REM MEASURED MEAN LOAD
REM AMPLITUDE
REM PREVENT CONTROL IN 1ST CYCLE
REM DRIVING MEAN LOAD (VOLTS)
REM AMPLITUDE

PRINT "MAX="Y" MIN="X"

PS=(Y+X)*CL/2
PA=(Y-X)*CL/2

IFFL=0 THEN FL=1:GOTO200

VK=VK+(VS*MF-PS/PC)/NI
VL=VL+(VA*MF-PA/PC)/NI
PRINT VK
PRINT VL

REM STANDY IF OUT OF CONTROL
IF ABS(VS-VK/MF)>.5*VS0RABS(VA-VL/MF)>.5*VATHEN GOSUB7000

REM CALCULATE NEW MEAN
REM CHECK IF INTEGER
REM OUTPUT NEW AMPLITUDE
REM BACK TO START OF CONTROL LOOP

REM MEASUREMENT & ANALYSIS ROUTINE
REM J SETS RHS OF COMPUTATION DOMAIN
REM NO. OF DATA IN DOMAIN (STARTS AT 3)
REM DESTINATION STATIC LOAD VALUE
REM CHECK IF RAMP, OPT. A OR TAPE END
REM YES! GO STATIC
REM READ COUNTER REGISTERS &
REM COMEER TOTAL COUNT
REM TOTAL CYCLES = OLD TOTAL + INCR.
REM RESET COUNTERS
REM
REM V=VE
REM OPTICAL A REQD?
REM OPTICAL A READ?

PRINT "HSSSSSSSSSSSSS%/F OPT CRACK a (m), NEXT TEST NO."
INPUT AO, AR, NS
AO=(A0+AR)*.5
PRINT "H", AO, AR"M"
REM COMPUTE AVERAGE OPTICAL A

REM MEAN LOAD
REM CHECK IF INTEGER
REM OUTPUT NEW AMPLITUDE
REM BACK TO START OF CONTROL LOOP
IF T=\text{"C"} THEN Z_0

\text{FORM} = \text{"O"} \text{TM}\% 

X = 5 

\text{GOSUB5000}

LV = X / CB

\text{SYS(HEAN)}

\text{FORM} = \text{"O"} \text{TQRD}

DU = (LV + ((VS + VA) * (1 - SE * M / MZ) * MF * SD - LV) / RD * MT) * CB

\text{GOSUB5000}

DU = \text{USR(DU)}

\text{NEXT}

\text{GOSUB5000}

X = 2

\text{GOSUB5000}

X = 4

\text{GOSUB5000}

\text{DT(M)} = X \times \text{CM}

X = 5

\text{GOSUB5000}

PT(M) = X \times \text{CL}

\text{KT(M)} = F(N)(FNA(VT(M)) / W) \times \text{PT(M)}

\text{IF} K = Z \text{THEN} \text{VT} = \text{VT}(0)

\text{IF} \text{K} < \text{K} \text{AND} K = Z \text{THEN} \text{VT} = \text{VT}(0)

\text{NEXT}

\text{REM}

\text{A(J)} = F(N)(V)

\text{EM} = \text{INT}(J / 2)

\text{REM}

\text{IF} FZ < 2 \text{AND} \text{T} = \text{"C"} \text{THEN} Z_0

\text{REM}

\text{NO COMPL. IF NO DATA OR CONT. TEST}

\text{REM}

\text{FORM} = \text{"O"} \text{TM}\%

\text{X(M)} = \text{PT}(M)

\text{Y(M)} = \text{DT}(M)

\text{N} = \text{MZ}

\text{D} = 1

\text{GOSUB6000}

\text{CO(J)} = 0(1)

\text{REM}

\text{NO. OF DATA POINTS}

\text{REM}

\text{POLYNOMIAL DEGREE}

\text{REM}

\text{LEAST SQUARES SUBROUTINE}

\text{REM}

\text{COMPLIANCE = GRADIENT}
1005 IF J<2 THEN 1360
1040 FORM=0 TO J
1041 X(M)=A(M)
1042 Y(M)=CO(M)
1043 NEXT
1044 N=J
1045 D=2
1046 GOSUB 6000
1100 RF=0(1)+2*0(2)*A(J)
1101 IF RF<Z THEN RF=Z
1110 RC=0(1)+2*0(2)*A(J-EM)
1111 IF RC<Z THEN RC=Z
1210 UF=SQ(R(CA+RF/BN)
1211 UC=SQ(R(CA+RC/BN)
1250 HF=UF*PR
1260 REM
1310 FORM=0 TO J
1311 X(M)=T(M)
1312 Y(M)=A(M)
1313 NEXT
1314 N=J
1315 D=2
1316 GOSUB 6000
1350 SF=0(1)+2*0(2)*T(J)
1351 SC=0(1)+2*0(2)*T(J-EM)
1355 REM
1356 UE=FMK(A(J)/U)
1357 PR=PA+PA
1358 HE=UE*PR
1359 REM
1360 IFPLS="M" THEN 1500
1400 IFLS="K" THEN 1420
1410 IFSF=EM THEN 1415
1411 X=LOG(HE/03)/LOG(R3)*C
1412 Y=LOG(SF/04)/LOG(R4)*C
1413 N=1
1414 GOSUB 8000
1415 GOTO 1430
1420 IFSC=EM THEN 1430
1421 X=(A(J-EM)/U-01)/R1*C
1422 Y=LOG(SC/04)/LOG(R4)+C
1423 N=1
1424 GOSUB 8000
1430 X=C-(A(J)/U-01)/R1*C
1431 Y=(T(J)-05)/R5+C
1432 N=Z
1433 GOSUB 8000
1440 IF JX>NTTHEN 1500
1441 X=(AO/U-01)/R1*C
1442 Y=(V-02)/R2+C
1443 N=Z
1444 GOSUB 8000
1445 REM
1446 REM SKIP IF INSUFFIENT DATA
1447 REM SET UP LEAST SQUARES DATA FOR JC/DA
1448 REM QUADRATIC
1449 REM GRADIENT AT MAX VALUE OF DOMAIN
1450 REM CHECK FOR NEGATIVE VALUE
1451 REM GRADIENT AT CENTRE OF DOMAIN
1452 REM CALCULATE CORRESPONDING K/P
1453 REM CALCULATE CORRESPONDING DEL K
1454 REM DEFINE LEAST SQUARES DATA FOR DA/DN
1455 REM QUADRATIC
1456 REM DA/DN AT RHS OF DOMAIN
1457 REM CENTRE
1458 REM K/P BY FORMULA
1459 REM FATIGUE LOAD RANGE
1460 REM DEL K BY FORMULA
1461 REM PLOT GRAPHS?
1462 REM PLOT DA/DN (RHS) VS DEL K
1463 REM GRAPH 1
1464 REM PLOTTING SUBROUTINE
1465 REM OR PLOT DA/DN (CENTRE) VS A/W
1466 REM GRAPH 1
1467 REM PLOT A VS N
1468 REM GRAPH 0
1469 REM IF OPTICAL A AVAILABLE THEN PLOT IT
1470 REM ALSO ON GRAPH 0
1500 IF X<> THEN 1510 :REM END OF TAPE?
1501 PRINT "TURN OVER TAPE GIVE GOOD LEADER" :REM END OF TAPE?
1502 REM
1510 S=1:K=4:X=I:GOSUB 4000 :REM PRINT RESULTS.
1512 K=10:X=T(J):GOSUB 4000 :REM SUBROUTINE 4000 FORMATS VARIABLE X
1520 S=4:X=C(J):GOSUB 4000 :REM IN FIELD LENGTH K, WITH S SPACES
1521 S=1:X=U:GOSUB 4000 :REM BETWEEN FIELDS.
1522 X=S:F:GOSUB 4000 :REM ALSO PRINTS DATA TO TAPE.
1530 S=4:X=H:GOSUB 4000
1531 PRINT1
1535 S=1:K=4:X=F:GOSUB 4000
1551 K=10:X=V:GOSUB 4000
1552 X=A(J):GOSUB 4000
1560 PRINT1, ";
1561 X=U:GOSUB 4000
1562 X=S:GOSUB 4000
1570 X=H:GOSUB 4000
1571 PRINT1, ";
1572 PRINT1
1575 PRINT1, " ";
1576 X=S:GOSUB 4000
1577 X=P:GOSUB 4000
1579 X=M:X=U:GOSUB 4000
1580 X=U:GOSUB 4000
1581 X=U:GOSUB 4000
1582 X=H:E:S:GOSUB 4000
1583 PRINT1:PRINT1
1584 S=1
1586 FORM=0:TM
1587 X=P:GOSUB 4000
1588 NEXT
1589 PRINT1
1591 FORM=0:TM
1592 X=O:TM:GOSUB 4000
1593 NEXT
1594 PRINT1
1600 FORM=0:TM
1601 X=V:TM:GOSUB 4000
1602 NEXT
1603 PRINT1
1604 PRINT1
1605 PRINT1
1607 REM
1610 IF J<ETHEN1680
1620 FORM=1TOJ
1621 N=M-1
1622 A(N)=A(M)
1623 CO(N)=CO(M)
1624 T(N)=T(M)
1625 NEXT
1670 REM
1680 NT=NS
1681 IZ=IZ+1
1683 GETA$
1684$ IF A$="S"$THENSTOP
1685 REM
1686 IF IZ>1THEN1695
1690 REM DROP LOAD USING QUADRATIC FUNCTION FOR FIRST ID MEASUREMENT CYCLES
1691 MF=(FI-1)/ID*ID*IZ*IZ+2*(1-FI)/ID*IZ+FI
1692 KC=FNK(A(J)/U)
1693 GOTO1700
1695 IF A$="K"$THEN MF=KC/FNK(A(J)/U)
1700 FL=Z
1701 V0=VE+VI
1702 IF KF=0THEN200
1703 SYS(ADC)
1704 X=USR(S)/CB
1705 GOSUB950$0
1706 DU=PEEK(T3)
1710 GOTO200
1900 REM----------------------
1910 REM TEST DETAILS SUBROUTINE
1920 REM
1930 REM TEST DETAILS, CONST P OR K, (R)AMP OR (C)ONTINUOUS, NO. OF MEASURE-
1940 REM MENT CYCLES ON TAPE, PLOT REQD Y OR N, SPECIMEN TYPE TYPE NO.(1 TO 4)
1950 DATA SPECIMEN C20DM, 300C 12/6/80, P, C, 300, N, 1
1960 READSP$, TT$, TN$, TE, PL$, SP
1970 REM
1980 REM LOAD RANGE/V, DISP RANGE/V, MEAN LOAD N, AMPL LOAD N, FRED,
1990 REM DEFAULT STANDBY LOAD AS FRACTION OF MAX LOAD, EPS SPECIMEN OUTPUT/V,
2000 DATA 5000, 1E-3, 9E3, 3E3, 20, 2, 20, 0, 2, 20
2010 READPC, DC, PS, PA, F, SB, EP, FI, ID
2020 REM
2030 REM MAX EPS VAL, K FOR EPS VAL, NO. OF DATA IN COMPUTATION DOMAIN (STARTS
2040 REM AT ZERO), EPS INCREMENT FOR NEXT MEASUREMENT CYCLE
2050 DATA 1.9E-3, 20E6, 6, .5E-5
2060 READVM, KV, E, VI
2070 REM
2080 REM B, BN, H, W, C, X (C SPEC), RO (C SPEC), YOUNGS MODULUS, POISSON'S
2090 REM RATIO
2100 DATA .025, .025, .025, .0313, .0521, .0166, .0616, 207E9, .3
2110 READB, BN, H, W, CK, CX, RO, YM, NU
2120 REM
2130 REM ORIGIN AND RANGES OF 5 POSSIBLE AXES OF GRAPHS - SEE LINES 1400+
2140 DATA 0, 1, 0, 5E-3, 3E6, 20, 1E-13, 1E5, 9.2E3
2150 READOI, R1, R2, R3, R4, R5, R6
2160 REM
2170 REM COEFFICIENTS OF EPS CALIBRATION POLYNOMIAL, A=F(EPS)
2180 DATA -9.29E-3, 54.267, -38191.7, 12639163
2190 READA0, A1, A2, A3
2200 RETURN
2210 REM-----------------------
2220 REM OUTPUT TEST DETAILS SUBROUTINE
2230 REM
2240 FORM=1 TO 2: REM PRINT TO PRINTER & TAPE
2250 PRINTHB, DO$
2260 PRINTHB, "CONST "; TT$; " RAMP/CONT "; TN$
2270 PRINTHB, "LD & DSP CAL"; PC; "N/V"; DC; "m/V
2280 PRINTHB, "MN LD"; PS; " AMP"; PA; "N
2290 PRINTHB, "YNGS MOD"; YM; "Pa"; PNS RAT; "N
2300 PRINTHB, "U"; "W"; "CX"; CX; " C"; CK; " H"; H; " RO"; RO; " B"; B; " BN"; BN; " (m)
2310 PRINTHB, "PTS TO CURVE FIT"; E
2320 PRINTHB, "EPS CAL"; EP; "U/V"; " del K": PRINTHB, " LOAD&DISP\EPS"
2330 PRINTHB, " da/dN\ao"
2340 RETURN
3970 REM -----------------------------------------------
3980 REM  FORMATTING SUBROUTINE FOR PRINTOUT
3990 REM
4000 SS=STR$(X)
4010 IF X=0 THEN 4100
4011 L=LEN(SS)
4012 I=ABS(X)
4013 IF I<10 OR I>UL THEN 4080
4040 IF I<10 THEN UL=VAL(LEFT$(SS,K-1)+RIGHT$(SS,10-K))
4060 FORM=KTOL
4070 IF MID$(SS,1)<>"." THEN 4065
4080 M=M-1
4090 NEXT
4100 M=L
4101 S$=LEFT$(SS,2)+"."+MID$(SS,3,K-7)+"E+0"+MID$(NS,M,1)
4102 GOTO 4100
4130 IF K<10 OR K>UL THEN 4090
4160 S$=LEFT$(SS,K-4)+RIGHT$(SS,4)
4180 GOTO 4100
4190 S$=LEFT$(SS,K)+LEFT$(SS,L-4)+RIGHT$(SS,4)
4210 GOTO 4100
4230 RETURN

4970 REM -----------------------------------------------
4980 REM  ADC DIGITAL FILTER SUBROUTINE
4990 REM
5000 SS=0
5010 SYS(ADC)
5020 FOR I=1 TO CH
5030 SS=SS+USR(X) :REM FIND AVERAGE OF CH SAMPLES FROM
5040 NEXT :REM CHANNEL X AND PLACE RESULT IN X
5050 X=SS/CH
5060 RETURN

5470 REM -----------------------------------------------
5480 REM  CHECK SIGNAL CONDITIONER OUTPUT SUBROUTINE
5490 REM
5500 IF ABS(DU)>32767 THEN GOSUB 7000 :REM MAX INTEGER ALLOWED = 32767
5510 RETURN
5970 REM........................................................................
5980 REM LEAST SQUARES POLYNOMIAL SUBROUTINE
5990 REM
6000 D1=D+1
6001 FORI=0 TO 2*D
6002 U(I)=Z
6003 NEXT
6070 FORI=0 TO D
6071 O(I)=Z
6072 NEXT
6073 FORK=0 TO N
6074 S=1
6090 FORI=0 TO 2*B
6091 U(I)=U(I)+S
6092 IFI<=D THEN O(I)=O(I)+S*Y(K)
6110 S=S*X(K)
6111 NEXT
6112 NEXT
6120 FORI=0 TO D
6121 FORJL=0 TO D
6122 G(I,JL)=U(I+JL)
6123 NEXT
6124 6(1,D1)=O(I)
6125 NEXT
6150 FOR I=0 TO D-1
6151 FORJL=I+1 TO D
6152 IFG(JL,I)=Z THEN 6210
6170 FORK=I+1 TO D
6172 NEXT
6200 G(JL,D1)=G(JL,D1)-G(I,D1)*G(JL,I)/G(I,I)
6210 NEXT
6211 NEXT
6215 IFG(D,D)=Z THEN PRINT 1, "ZERODIV6220": GOTO 6230
6220 O(D)=G(D,D1)/G(D,D)
6230 FORI=0 TO D-1
6231 K=O-I-1
6240 FORJL=K+1 TO D
6241 G(K,D1)=G(K,D1)-O(JL)*G(K,JL)
6250 NEXT
6251 IFG(K,K)=Z THEN PRINT 1, "ZERODIV6250": GOTO 6270
6260 O(K)=G(K,D1)/G(K,K)
6270 NEXT
6380 RETURN
6970 REM----------------------------------------------
6980 REM STANDBY SUBROUTINE
6990 REM
7000 PRINT"cssrSTANDBY"
7002 SX=SB
7004 GOSUB9000
7010 JS=J
7012 IF J>E-1 THEN JS=E-1
7020 REM
7030 T0%=PEEK(T0)
7031 T1%=PEEK(T1)
7032 T2%=PEEK(T2)
7033 T=INT(T0%+M8*(M8*T2%))
7040 T(JS)=T(JS)+T
7042 PRINT"sLOAD (N)=";
7044 X=5
7046 GOSUB5000
7049 PRINTX+CL;8$
7050 PRINT"sDISP («)=";
7051 X=4
7052 GOSUB5000
7053 PRINTX+CM;B$
7054 PRINT"sEPS (V)=";
7055 X=2
7056 GOSUB5000
7057 PRINTX+CC;B$
7059 INPUT"ssNEW ST LD(N),OR'EX'":$;
7060 IF LEFT$(S$)="EX" THEN 7200
7062 PRINT"hss"
7064 IF LEFT$(S$,2)="EX" THEN 7200
7066 SYS(MEAN)
7068 DU=CB*VAL(S$)/PC
7070 SYS(USR)
7072 DU=USR(DU)
7074 GOSUB5000
7075 X=2
7076 GOSUB5000
7077 PRINTX+CC;B$
7079 INPUT"ssNEW ST LD(N),OR'EX'":$;
7080 IF LEFT$(S$)="EX" THEN 7200
7082 PRINT"hss"
7084 IF LEFT$(S$,2)="EX" THEN 7200
7086 SYS(MEAN)
7088 DU=CB*VAL(S$)/PC
7090 SYS(USR)
7092 DU=USR(DU)
7094 GOSUB5000
7095 X=2
7096 GOSUB5000
7097 PRINTX+CC;B$
7100 REM
7100 PRINT"cssrFATIGUE"
7102 PRINT" rSoSTANDBY, rMoEASURE"
7104 DU=PEEK(T3)
7106 FL=0
7108 SYS(ADC)
7110 X=USR(5)/CB
7112 REM STAY IN STANDBY
7114 REM
7115 REM STANDBY AS FRACTION OF MAX LOAD
7116 REM DYNAMIC TO STATIC
7117 REM DEFINE ELEMENT OF ARRAY T
7118 REM READ COUNTER REGISTERS
7119 REM COMPUTE THE NO. OF CYCLES
7120 REM ADD TO CURRENT TOTAL
7121 REM READ ADC CHANNEL 5 IE. LOAD
7122 REM READ ADC CHANNEL 4 IE. DISPLACEMENT
7123 REM READ ADC CHANNEL 2 IE. EPS
7124 REM INVITE NEW LOAD OR EXIT TO FATIGUE
7125 REM EXIT?
7126 REM OUTPUT NEW LOAD
7127 REM STAY IN STANDBY
7128 REM EXIT
7129 REM RESET COUNTERS
7131 REM TO DYNAMIC
7220 RETURN
7970 REM ---------------------------------------------
7980 REM ANALOGUE PLOTTER PLOTTING SUBROUTINE
7990 REM
8000 IFS1THEN8200 :REM PLOT FRAME ONCE ONLY
8040 DATA0,0,255,0,255,255,0,255,0,0 :REM READ FRAME DETAILS
8042 FORM=0TO4
8044 READ0%()%,P%()
8046 NEXT
8060 FORK=0TO1 :REM GRAPHS Ø 3 1
8062 POKED+4,Z :REM PEN UP
8064 POKED+3,K+CP
8070 POKED+1,0%() :REM GOTO LOWER LEFT CORNER
8072 POKED+2,P%() :REM TIME DELAY
8074 GOSUB9800 :REM DOWN
8076 POKED+4,CP :REM PLOT FRAME
8080 FORM=1TO4
8100 FORK=1TO4
8102 POKED+1,0%() :REM GOTO LOWER LEFT CORNER
8104 POKED+2,P%() :REM PEN UP
8106 GOSUB9800
8110 NEXT
8120 POKED+4,Z :REM PEN UP
8140 NEXT
8150 DATA-2,-1,2,-1,2,1,-2,1,-2,-1 :REM READ DATA TO PLOT SMALL SQUARE
8152 FORM=0TO4
8154 READ0%(),P%():REM READ DATA TO PLOT SMALL SQUARE
8155 NEXT
8156 S=1
8200 POKED+3,N+CP :REM GRAPH Ø OR 1
8202 POKED+1,FND(X+O%()) :REM LOWER LEFT CORNER OF DATA POINT
8204 POKED+2,FND(Y+P%()) :REM PEN DOWN
8206 GOSUB9800
8220 POKED+4,CP :REM PLOT DATA SQUARE
8222 GOSUB9800
8224 FORM=1TO4
8240 POKED+1,FND(X+O%())
8242 POKED+2,FND(Y+P%())
8244 GOSUB9800
8260 NEXT
8262 GOSUB9800
8264 POKED+4,Z :REM PEN UP
8268 GOSUB9800
8300 RETURN
8970 REM-----------------------------------------------
8980 REM DYNAMIC TO STATIC TRANSITION SUBROUTINE
8990 REM
9000 FORM=0 TO CG
9001 SYS(AMPL)
9002 DU=VL*(1-H/CG)*CB : REM AMPLITUDE GOES TO 0
9003 GOSUB5500
9004 DU=USR(DU)
9010 SYS(MEAN)
9011 DU=(VK+(VL+VK)*SX-VK)/CG*H)*CB : REM MEAN LOAD GOES TO SX*(MAX LOAD)
9012 GOSUB5500
9013 DU=USR(DU)
9014 NEXT
9020 RETURN
9470 REM-----------------------------------------------
9480 REM STATIC TO DYNAMIC TRANSITION SUBROUTINE
9490 REM
9500 FORM=Z TO CG
9501 SYS(MEAN)
9502 DU=(X+(VK-X)*H/CG)*CB : REM MEAN GOES FROM STATIC VALUE TO
9503 GOSUB5500 : REM PREVIOUS MEAN VALUE OF FAT. CYCLE
9504 DU=USR(DU)
9510 SYS(AMPL)
9511 DU=VL+H/CG+CB : REM AMPLITUDE RETURNS TO PREVIOUS VALUE
9512 GOSUB5500
9513 DU=USR(DU)
9514 NEXT
9520 RETURN
9770 REM-----------------------------------------------
9780 REM TIME DELAY SUBROUTINE
9790 REM
9800 TT=TI
9810 IF TT+30 THEN 9810 : REM WAIT FOR .5 SECOND
9830 RETURN
9990 REM

END OF FIGURE 3.25
FIG 4.1 HOOP STRAIN DURING AUTOFRETTAGE
ROGAN (1980)
FIG 4.2 RESIDUAL STRESS MEASUREMENT FOR RS2DM
FIG 4.3 COMPARISON OF 4 STRAIN GAUGES USED FOR RS4DM
FIG 4.4 COMPARISON OF RESIDUAL HOOP STRESSES FOR RS4DM FOUND USING 4 STRAIN GAUGES.
FIG 4.5 COMPARISON OF RESIDUAL STRESSES MEASURED IN FOUR DIFFERENT SPECIMENS OF THE SAME GEOMETRY
Fig 4.6 Fatigue Results for Tubes DM, H, U & V
Rogan (1979)
FIG 4.7 MANUFACTURING ROUTE OF THICK-WALLED TUBING
A COMPARISON OF THE TWO METHODS USED
FIG 5.1 A RANGE OF SPECIMENS CUT FROM THICK-WALLED TUBING
FIG 5.2 DIMENSIONS OF THE COMPACT TENSION SPECIMEN (CKS)

PREVIOUSLY 10mm
FIG 5.3 DIMENSIONS OF THE DCB SPECIMEN

SEE FIG 7.20
FOR $B_n$ VALUES
$B_n = 0.48$
FIG 5.4 DIMENSIONS FOR THE C-SHAPED SPECIMEN
FIG 5.5 SPLIT-RING (SR) SPECIMEN
DM TUBING ONLY

THICKNESS = 25
FIG 5.6  EPS CALIBRATION CURVES FOR SOME CKS SPECIMENS
FIG 5.7 EPS CALIBRATION CURVES FOR C-SHAPED SPECIMENS
FIG 5.8  BIE MESH FOR A CKS SPECIMEN
FIG 5.9 COMPLIANCE CURVES FOR STANDARD CKS GEOMETRIES USING A BIE METHOD. DIFFERENT PIN HOLE DIAMETERS.
FIG 5.10 STRESS INTENSITY FACTOR FOR A CKS OF STANDARD GEOMETRY FOUND BY A BIE METHOD.
FIG 5.11 COMPLIANCE FOR DCB SPECIMEN USING A BIE METHOD
FIG 5.12 STRESS INTENSITY FACTOR FOR A DCB SPECIMEN USING A BIE METHOD.
FIG 5.13 COARSE BIE MESH FOR A C-SHAPED SPECIMEN
FIG 5.14 FINE BIE MESH FOR A C-SHAPED SPECIMEN
FIG 5.15 ENLARGED VIEW SHOWING ELEMENTS ALONG THE CRACK PLANE OF A FINE DIE MESH FOR A C-SHAPED SPECIMEN.
FIG 5.16 COMPLIANCE FOR A C-SHAPED SPECIMEN OF DM GEOMETRY USING A BIE METHOD.
FIG 5.17 STRESS INTENSITY FACTOR FOR A C-SHAPED SPECIMEN OF DM GEOMETRY USING A BIE METHOD.
FIG 5.18 STRESS INTENSITY FACTOR FOR A C-SHAPED SPECIMEN OF U GEOMETRY USING A BIE METHOD.
FIG 5.19 STRESS INTENSITY FACTOR FOR A C SHAPED SPECIMEN OF V GEOMETRY USING A BIE METHOD.
FIG 5.20  FINE BIE MESH FOR A SR SPECIMEN
FIG 5.21 STRESS INTENSITY FACTOR FOR A SR SPECIMEN USING A BIE METHOD.

* SEE FIG 5.22 FOR AN EXPRESSION WHICH INCLUDES ALL a/W.
FIG 5.22 STRESS INTENSITY FACTOR FOR A SR SPECIMEN USING A BIE METHOD.
Fig 5.24 Compliance for a ring specimen of DM geometry using a BIE method
FIG 5.25 STRESS INTENSITY FACTOR FOR A RING SPECIMEN OF DM GEOMETRY USING A FINE BIE MESH
FIG 5.26 STRESS INTENSITY FACTOR OF A RING SPECIMEN OF DM GEOMETRY. DIFFERENT ORDINATE USED.
FIG 7.1 CRACK GROWTH WITH NUMBER OF CYCLES FOR C90M

$R = 0.05$ $T = 20^\circ C$ $F = 30/20Hz$
FIG 7.2 CRACK GROWTH WITH NUMBER OF CYCLES FOR C100DM (CONSTANT ΔK)

$R = 0.5 \quad T = 20°C \quad F = 20Hz$
FIG 7.3 FATIGUE CRACK PROPAGATION CURVE FOR C90M

$R = 0.05 \ T = 20^\circ C \ F = 30/20\ Hz$
FIG 7.4 FATIGUE CRACK PROPAGATION CURVE FOR C100DM
FIG 7.5 FATIGUE CRACK PROPAGATION CURVE FOR C22DM
FIG 7.6 FATIGUE CRACK PROPAGATION CURVE FOR CKS10M
DEMONSTRATION OF THE EFFECT OF OVERLOAD AND SKEW
CRACK GROWTH. ONLY 3 DATA POINTS WERE USED TO
FIND da/dN.

\[ R = 0.05 \quad T = 20{\text{C}} \quad F = 30/15\text{Hz} \]

- OVERLOAD

START FROM A BLUNT NOTCH

STEPPING DOWN

SKEW CRACK GROWTH

A

B
FIG 7.7 FATIGUE CRACK PROPAGATION CURVE FOR CKSXDM
DEMONSTRATION OF TEST REPEATABILITY
FIG 7.8 FATIGUE CRACK PROPAGATION CURVE FOR CXDM
THE EFFECT OF MEAN STRESS IN THE RADIAL DIRECTION
FIG 7.9 FATIGUE CRACK PROPAGATION CURVE FOR CKSXDM
THE EFFECT OF MEAN STRESS IN THE AXIAL DIRECTION
FIG 7.10 FATIGUE CRACK PROPAGATION CURVE FOR CXDMA AUTOFRETTAGED TUBING IN THE RADIAL DIRECTION
FIG 7.11 RESIDUAL STRESSES IN C-SHAPED AND SR SPECIMENS
FIG 7.12 FATIGUE CRACK PROPAGATION CURVE FOR CXDM/DMA
THE EFFECT OF AUTOFRETTAGE IN THE RADIAL DIRECTION
FIG. 13  FATIGUE CRACK PROPAGATION CURVE FOR CKSXD/DM/A
THE EFFECT OF AUTOFRETTAGE IN THE AXIAL DIRECTION
FIG 7.14 FATIGUE CRACK PROPAGATION CURVE FOR CXDM
THE EFFECT OF SPECIMEN BREADTH

- 294 -
FIG7.15 FATIGUE CRACK PROPAGATION CURVE FOR CXDM
THE EFFECT OF FREQUENCY

1 C70M F=5Hz
2 C20M F=30/20/15Hz

R=0.05 T=20C
FIG. 7.16  FATIGUE CRACK PROPAGATION CURVE FOR CXDM
TEMPERATURE DEPENDENCE IN THE RADIAL DIRECTION
FIG 7.17 FATIGUE CRACK PROPAGATION CURVE FOR CKSXDM TEMPERATURE DEPENDENCE IN THE AXIAL DIRECTION
FIG 7.18 FATIGUE CRACK PROPAGATION CURVE FOR CXDMA TEMPERATURE DEPENDENCE OF AUTOFRETTAGED TUBING IN THE RADIAL DIRECTION
FIG 7.19 FATIGUE CRACK PROPAGATION CURVE FOR CKSXDMA TEMPERATURE DEPENDENCE OF AUTOFRETTAGED TUBING IN THE AXIAL DIRECTION
FIG 7.20 FATIGUE CRACK PROPAGATION CURVE FOR DCBXDM
FIG 7.21 FATIGUE CRACK PROPAGATION CURVE FOR SRIDM
FIG 7.22 FATIGUE CRACK PROPAGATION CURVE FOR SR20DM
FIG 7.23 FATIGUE CRACK PROPAGATION CURVE FOR SR5DMA

**Graph Details**

- **Conditions:**
  - $R = 0.5$
  - $T = 20°C$
  - $F = 10/20/30Hz$

- **Crack Lengths:**
  - $a = 14\, \text{mm}$
  - $a = 13.6\, \text{mm (C8DMA)}$
  - $a = 6\, \text{mm}$
  - $a = 7.5\, \text{mm}$
  - $a = 20\, \text{mm}$
  - $a = 15\, \text{mm}$

- **Y-axis:**
  - $da/dN$ (m/cycle)
  - $10^{-9}$, $10^{-8}$, $10^{-7}$, $10^{-6}$

- **X-axis:**
  - $\delta K$ (MPa $\sqrt{m}$)
  - $10$, $100$
FIG. 24  FATIGUE CRACK PROPAGATION CURVE FOR R2DM
$\frac{da}{dN}$ (m/cycle)

**Figure 7.25 Fatigue Crack Propagation Curve for R2DMA**

- $R = 0.5$
- $T = 20^\circ C$
- $F = 20 Hz$

Delta $K$ (MPa m$^{1/2}$) vs. $1/N$

Points at $a = 18$ mm and $a = 9$ mm
FIG 7.26 FATIGUE CRACK PROPAGATION CURVE FOR C/CKSXDM
THE EFFECT OF ANISOTROPY IN DM TUBING
FIG 7.27 FATIGUE CRACK PROPAGATION CURVE FOR C/CKSXDM
THE EFFECT OF ANISOTROPY IN DM TUBING
308

FIG 7.28 FATIGUE CRACK PROPAGATION CURVE FOR C/CKSXH
THE EFFECT OF ANISOTROPY IN H TUBING
FIG7.29 FATIGUE CRACK PROPAGATION CURVE FOR C/CKSXU
THE EFFECT OF ANISOTROPY IN U TUBING
FIG 7.30 FATIGUE CRACK PROPAGATION CURVE FOR C/CKSXV
THE EFFECT OF ANISOTROPY IN V TUBING
FIG. 7.31 FATIGUE CRACK PROPAGATION CURVE FOR CxVARIOUS MATERIAL COMPARISON IN THE RADIAL DIRECTION
FIG 7.32 FATIGUE CRACK PROPAGATION CURVE FOR CKS XVARIOUS MATERIAL COMPARISON IN THE AXIAL DIRECTION
FIG 7.33 FATIGUE CRACK PROPAGATION CURVE FOR C11DM CONSTANT DELTA K TEST BEFORE CORRECTION
$\delta K = 48 \text{ MPa}\sqrt{\text{m}}$

FIG 7.34 FATIGUE CRACK PROPAGATION CURVE FOR CORRECTED CONSTANT DELTA K RESULTS
FIG 7.35 FATIGUE CRACK PROPAGATION CURVE FOR CXDM CONSTANT DELTA K RESULTS
FIG 7.36 FATIGUE CRACK PROPAGATION CURVE FOR CXVARIABLES CONSTANT DELTA K RESULTS
FIG 7.37  FRACTURE TOUGHNESS TEST EPS & LOAD BEHAVIOUR FOR C10M AT 20°C
FIG 7.38  FRACTURE TOUGHNESS TEST EPS & LOAD BEHAVIOUR FOR CKS3H AT 20°C
FIG 7.39 FRACTURE TOUGHNESS TEST LOAD & EPS BEHAVIOUR OF C21DM AT 300°C
FIG. 7.40 FRACTURE TOUGHNESS TEST LOAD & EPS BEHAVIOUR OF CKS20DMA AT 300°C
FIG 8.1 A MODEL DESCRIBING THE EFFECT OF RESIDUAL STRESS ON THE GROWTH OF A FATIGUE CRACK
FIG 8.2 THE APPARENT THRESHOLD K AT SHORT AND LONG CRACK LENGTHS. DEMONSTRATED BY A CONSTANT $\Delta K$ TEST.
FIG 9.1 SMALL CRACK & $b/a=0.05$

INITIAL $b/a$, $W$ (mm), $k$ $0.05$ $30$ $2$

$C$ & $m$ (RADIAL) $4E-11$ $2.4$

$C$ & $m$ (AXIAL) $4E-11$ $2.4$

PRESSURE RANGE (MPa) $220$

INITIAL CRACK LENGTH $b_i = 20 \, \mu m$
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<th>N (CYCLES)</th>
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FIG 9.1 CONTINUED.
FIG 9.2 SMALL CRACK & b/a=0.5

INITIAL b/a, W (mm), k = 0.5 30 2
C & m (RADIAL) 4E-11 2.4
C & m (AXIAL) 4E-11 2.4
PRESSURE RANGE (MPa) 220
INITIAL CRACK LENGTH b_i = 20 μm
### INITIAL b/a, W (mm), k

| C & m (RADIAL) | 4E-11 | 2.4 |
| C & m (AXIAL) | 4E-11 | 2.4 |

### PRESSURE RANGE (MPa) 220

### INITIAL CRACK LENGTH b1 = 20 μm

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**FIG 9.2 CONTINUED.**
N = 157500 CYCLES

INITIAL $b/a$, $W$ (mm), $k$ 1 30 2
C & m (RADIAL) 4E-11 2.4
C & m (AXIAL) 4E-11 2.4
PRESSURE RANGE (MPa) 220
INITIAL CRACK LENGTH $b_i = 20 \mu m$

FIG 9.3 SMALL CRACK & $b/a$=1
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FIG 9.3 CONTINUED.
$N = 37000$ CYCLES

INITIAL $b/a$, $W$ (mm), $k = 0.05$ 30  2

$C & m$ (RADIAL) 4E-11  2.4

$C & m$ (AXIAL) 4E-11  2.4

PRESSURE RANGE (MPa)  220

INITIAL CRACK LENGTH $b_i = 10000$ $\mu m$

FIG 9.4 LARGE CRACK & $b/a=0.05$
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<th>K(AXIAL) (MPa/(\sqrt{\text{m}}))</th>
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**FIG 9.4 CONTINUED.**
FIG 9.5 LARGE CRACK & b/a=1

INITIAL b/a, W (mm), k 1 30 2
C & m (RADIAL) 4E-11 2.4
C & m (AXIAL) 4E-11 2.4
PRESSURE RANGE (MPa) 220
INITIAL CRACK LENGTH bi = 1000 µm
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<th>$a$ (nm)</th>
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**FIG 9.5 CONTINUED.**
HEWLETT PACKARD

\[ N = 14850 \text{ CYCLES} \]

Initial \( b/a, W \) (mm), \( k \) .5 30 2

C & \( m \) (RADIAL) 4E-10 2.4
C & \( m \) (AXIAL) 4E-10 2.4

Pressure Range (MPa) 220

Initial Crack Length \( b_i = 20 \mu m \)

**FIG 9.6** ISOTROPIC & LARGE PARIS C
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FIG 9.6 CONTINUED.
INITIAL $b/a$, $W$ (mm), $k$. $5 \ 30 \ 2$

C & $m$ (RADIAL) $2.52\times10^{-12}$ $3.6$

C & $m$ (AXIAL) $2.52\times10^{-12}$ $3.6$

PRESSURE RANGE (MPa) $220$

INITIAL CRACK LENGTH $b_i = 20 \ \mu m$

FIG 9.7 ISOTROPIC & HIGH PARIS $m$
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**FIG 9.7 CONTINUED.**
Fig. 9.8 Anisotropic, Large Paris C in Axial Direction

N = 89000 Cycles

Initial $b/a, W$ (mm), $k = 0.5$

- $C \& m$ (Radial) $4E-11$ 2.4
- $C \& m$ (Axial) $4E-10$ 2.4

Pressure Range (MPa) 220

Initial Crack Length $b_i = 20 \mu m$
INITIAL b/a, U (nm), K  

C & m (RADIAL)  4E-11  2.4  30  2
C & m (AXIAL)  4E-19  2.4

PRESSURE RANGE (MPa)  220

INITIAL CRACK LENGTH b_i = 20 \mu m

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FIG 9.8 CONTINUED.
Fig 9.9 Anisotropic. High Paris m in axial direction.
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FIG 9.9 CONTINUED.
FIG 9.10  ISOTROPIC. HIGH PRESSURE.

N = 82250 CYCLES

INITIAL b/a, W (mm) , k .5 30 2
C & m (RADIAL) 4E-11 2.4
C & m (AXIAL) 4E-11 2.4
PRESSURE RANGE (MPa) 280
INITIAL CRACK LENGTH bi = 20 μm
### Table

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<th>a (mm)</th>
<th>b/a</th>
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FIG 9.10 CONTINUED.
FIG 9.11 ISOTROPIC. HIGH PRESSURE & $b/a=0.05$ AT $b=1\text{mm}$
INITIAL \( b/a, W (\text{mm}), k \) \( 0.03 \) \( 30 \) \( 2 \)

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<th>( a ) (mm)</th>
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<th>( K(\text{AXIAL}) ) (MPa/( \mu ))</th>
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FIG 9.11 CONTINUED.
ANISOTROPIC (LARGE PARIS C). HIGH PRESSURE

INITIAL \( b/a, W \) (mm), \( k \) 0.5 30 2
\( C & m \) (RADIAL) \( 4 \times 10^{-11} \) 2.4
\( C & m \) (AXIAL) \( 4 \times 10^{-10} \) 2.4
PRESSURE RANGE (MPa) 280
INITIAL CRACK LENGTH \( b_i = 20 \) \( \mu \text{m} \)
INITIAL $b/a$, $W$ (mm), $k$  

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<th>$a$ (mm)</th>
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<th>$K$(AXIAL) (MPa)</th>
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**FIG 9.13 ANISOTROPIC (HIGH PARIS $m$). HIGH PRESSURE**

INITIAL $b/a$, $W$ (mm), $k$ 0.5 30 2
C & $m$ (RADIAL) 4E-11 2.4
C & $m$ (AXIAL) 2.52E-12 3.6
PRESSURE RANGE (MPa) 280
INITIAL CRACK LENGTH $b_i$ = 20 $\mu$m
### INITIAL \( b/a, U \text{ (mm)} \), \( k \)

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FIG 9.13 CONTINUED.
FIG 9.14 LARGE WALL THICKNESS

INITIAL $b/a, w$ (mm), $k = 0.5$ 60 2
$C_0$ & $m$ (RADIAL) $4 \times 10^{-11}$ 2.4
$C_0$ & $m$ (AXIAL) $4 \times 10^{-11}$ 2.4
PRESSURE RANGE (MPa) 220
INITIAL CRACK LENGTH $b_i = 20 \ \mu$m
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FIG 9.14 CONTINUED.
N = 117000 CYCLES

INITIAL $b/a$, $W$ (mm), $k = 0.5$

$C & m$ (RADIAL) $4E-11$ 2.4

$C & m$ (AXIAL) $4E-11$ 2.4

PRESSURE RANGE (MPa) 293.3

INITIAL CRACK LENGTH $b_i = 20 \mu m$

FIG 9.15 LARGE DIAMETER RATIO
### Table

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<th>b/a</th>
<th>K RADIAL (MPa/ft)</th>
<th>K AXIAL (MPa/ft)</th>
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**FIG 9.15 CONTINUED.**
FIG 9.16 DM TUBING $R=0.05$

INITIAL $b/a$, $W$ (mm), $k$ 32.5 2.1
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$C & m$ (AXIAL) 5.39000001E-12 2.97
PRESSURE RANGE (MPa) 291
INITIAL CRACK LENGTH $b_i = 20 \mu m$
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FIG 9.16 CONTINUED.
INITIAL $b/a, W_{(mm)}$, $k$.5 32.5 2.1
$C & m$ (RADIAL) 1.62E-11 2.64
$C & m$ (AXIAL) 1.34E-11 2.91
PRESSURE RANGE (MPa) 291
INITIAL CRACK LENGTH $b_i = 20 \mu m$

FIG 9.17 DM TUBING $R=0.5$
### Table 1

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**Notes:**
- *C & n (RADIAL)*
- *C & n (AXIAL)*
- PRESSURE RANGE (MPa) 231
- INITIAL CRACK LENGTH b1 = 20 µm

*FIG 9.17 CONTINUED.*
FIG 9.18  H TUBING  R=0.5

INITIAL $b/a, W$ (mm), $k$ .5 33.5 2.32
$C & m$ (RADIAL) 1.96E-11 2.66
$C & m$ (AXIAL) 9.21E-13 3.62
PRESSURE RANGE (MPa) 252
INITIAL CRACK LENGTH $b_i = 20 \, \mu$m

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FIG 9.18 CONTINUED.
FIG 9.19 U TUBING R=0.5

INITIAL $b/a$, $W$ (mm), $k$ 0.5 42.5 2.55
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$C$ & $m$ (AXIAL) 2.48E-11 2.57
PRESSURE RANGE (MPa) 2 68
INITIAL CRACK LENGTH $b_i = 20$ μm
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FIG 9.19 CONTINUED.
FIG 9.20 V TUBING R=0.5

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PRESSURE RANGE (MPa) 246
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<td>8.055E+03</td>
<td>60.412E+03</td>
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</tr>
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<td>55.0844</td>
<td>6.548E+03</td>
<td>165.800E+03</td>
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</tr>
<tr>
<td>140000</td>
<td>39.3580</td>
<td>65.0834</td>
<td>6.046E+03</td>
<td>211.200E+03</td>
<td></td>
</tr>
<tr>
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<td>49.2328</td>
<td>75.0834</td>
<td>5.544E+03</td>
<td>262.700E+03</td>
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</tr>
</tbody>
</table>

**FIG 9.20 CONTINUED.**
FIG 9.21 PREDICTED RESULTS FOR TUBES DM, H, U & V
(EXPERIMENTAL RESULTS FROM ROGAN (1979))
1000 REM....................................................................................................................
1010 REM CRACK GROWTH IN THICK WALLED TUBING
1020 REM....................................................................................................................
1030 REM
1040 REM INPUT ROUTINE
1050 REM
1060 INPUT"READ DATA FROM DATA STATEMENT";Q$:IFQ$="*THEN1110#
1070 READA,B,A,W,K,CA,HA,CR,HR,DE,PR,TR
1080 DATA3E-6,0.3,.0313,2.23,39.2E-12,2.40,39.2E-12,2.40,1000,244E4,4E4
1090 GOSUB2150#
1100 GOTO1110#
1110 INPUT"START B (M), B/A, W & K";B,BA,W,K
1120 INPUT"PARIS C (M/CYCLE) & M (AXIAL)";CA,MA
1130 INPUT"PARIS C (M/CYCLE) & M (RADIAL)";CR,MR
1140 INPUT"THRESHOLD DELTA K (MPA/M);TR;TR=TR=1E6
1150 INPUT"DELTA N, PRESSURE (PA)";DE,PR
1160 REM
1170 REM SET VARIABLES
1180 REM
1190 Z=4tN1=1;N2=2:N5=5
1200 M1=1-tM2=.25tM3=3/64tM4=11/16tM5=.5tH6=1.5
1210 E6=1E-6
1220 LI=.5E-3;BA=.83W
1230 PI=3.14159265tP2=PI/2
1240 A=B/BA*K2=K*K
1250 FL=1:O=Q
1260 Q$=":S$=":
1270 REM
1280 REM CALCULATE CRACK SIZE FOR THRESHOLD AT GIVEN PRESSURE & TUBING
1290 BA=B/AtTH=p/2:GOSUB1810#:BS=(TR/PR/K1)+2/p
1300 PRINT"B AT DELTA K (THRESHOLD)";BS*1E6"MICROMETRES"
1310 INPUT"PRESS TO USE THIS VALUE";Q$
1320 IFQ$="*THENBS=BS
1330 A=B/BA
1340 REM
1350 REM PRINT DETAILS
1360 REM
1370 REM
1380 REM
1390 OPEN1,10
1400 PRINTN1,0$"INITIAL b/a, W (mm) , k",BA,W=1E3,K
1410 PRINTN1,0$"C & M (RADIAL)";CR,MR
1420 PRINTN1,0$"C & M (AXIAL)";CA,MA
1430 PRINTN1,0$"PRESSURE RANGE (MPa)";PR/1E6
1440 PRINTN1,0$"INITIAL CRACK LENGTH bi = ";B=1E6" um";PRINTN1
1450 PRINTN1:PRINTN1,0$" N b a k "
1460 PRINTN1," k(RADIAL) k(AXIAL)"
1470 PRINTN1,0$"(CYCLES) (mm) (mm) (MPa/m)(MPa/n)"
1480 PRINTN1," (MPa/m) (MPa/n)"
1490 REM PRINT DETAILS ON PLOT
1500 IPRINT"P;PA5000,-500;SI.18, .25;SL.2;LD";
1510 IPRINT"INITIAL b/a, W (mm) , k ";BA,W=1E3,K
1520 IPRINT"C & M (RADIAL)";CR,MR
1530 IPRINT"C & M (AXIAL)";CA,MA
1540 IPRINT"PRESSURE RANGE (MPa)";PR/1E6
1550 IPRINT"INITIAL CRACK LENGTH bi = ";B=1E6" um"
1560 IPRINTCHR$(3);"IN1100,3000,10199,6000;";
1370 REM
1380 REM PRINT & PLOT DETAILS OF INITIAL CRACK
1390 REM
1400 BA=B/A:TH=Z:.05SUB1810;KA=KI*PR*SQR(PI*B)*E6
1410 TH=P2:.05SUB1810;KR=KI*PR*SQR(PI*B)*E6
1420 G0SUB2939;G0SUB2339
1430 REM
1440 REM OBTAIN K FOR CRACK AND ALLOW IT TO GROW FOR DE CYCLES
1450 REM
1460 BA=B/A;TH=Z:.05SUB1810;KA=KI*PR*SQR(PI*B)*E6
1470 NA=4CA*(KA+MA)*DE
1480 TH=P2:.05SUB1810;KR=KI*PR*SQR(PI*B)*E6
1490 BT=B
1500 D=B+CR*(KR+HR)*DE
1510 A=NA
1520 H=H+DE
1530 D=Q+M1:IFD=M5ANDB)G0SUB2939
1540 IFD=M5THEN.05SUB1810;D=Z
1550 IFD=BT>68THENDE=DE/N2
1560 IFD<WHEN1660
1570 IFD=2WHEN1779
1580 G0SUB2939;G0SUB2339
1590 !PRINT"IN;PA45@;31@;LB=""N" CYCLES"CHR(3)";";
1600 STOP
1610 REM
1620 REM FIND K USING WILLIAMS (1990) APPROXIMATION
1630 REM
1640 E2=P2*(M1-M2*(M1-BA*BA)-M3*((N1-BA*BA)+M2))
1650 RA=SQR(3A*BA*COS(TH)+M2+2*SIN(TH)+M2)
1660 AL=SIN(TH)*(D/4*(K-M1)/SQR(3A))
1670 IFAL=N1THENH4:G0TO1989
1680 AA=N1-AL+AL
1690 F1=M5*(1+AA)/AA
1700 SQ=(M1-AL)/(M1+AL)
1710 IFAL>M1THEN1989
1720 SQ=SQR(SQ)
1730 F=F1-AL/(AA+M6)*ATN(SQ)
1740 G0TO1989
1750 SQ=SQR(-SQ)
1760 F2=LO6((M1+SA)/(M1-SA))
1770 F=F1+AL/(M2*(-AA)*M6)*F2
1780 K1=M2*K2/(K2-M1)/E2*SQR(RA)*F
1790 RETURN
REM PRINT ROUTINE ON HP7225A
REM PLOTTING ROUTINE ON HP7223A
REM PRINT\$0$""CHR$(13);         2030
PL=PI:FL=FL$M1:IF FL=M1THEN PL=2 2040
FORD=0 TO 40               2050
TH=PL-FL*PI/40*TD         2060
CO=COS(TH):SI=SIN(TH)   2070
X=INT(A*CO+SC+4500)     2080
Y=INT(B*SI+SC)             2090
PRINT"X","Y";  2100
IF TD=0 THEN PRINT"PD";  2110
NEXT                         2120
PRINT"PU";  2130
RETURN                         2140
UP=3E3:SC=WP/W              2150
PRINT"IN";  2160
PRINT"IP;1100,3000,10100,6000";  2170
PRINT"SC;9999,8,3999";  2180
PRINT"IP;1100,3000,10100,6000";  2190
PRINT"PA4500,0;YT";  2200
FOR I=0 TO 10               2210
PRINT"PD;PA4500,"(WP/10*I)";YT";  2220
NEXT                         2230
PRINT"PU;TL1.5,0;PA0,0";  2240
FOR I=0 TO 30               2250
PRINT"PD;PA"INT(WP/10*I)";XT";  2260
NEXT                         2270
PRINT"PU";  2280
RETURN                         2290
REM PRINT ROUTINE           2300
REM                         2310
M$=STR$(M)+S$:PRINT\$1,O$LEFT$(M$,10);         2320
B$=STR$(B*1000)+S$:PRINT\$1,LEFT$(B$,8)" ";       2330
A$=STR$(A*1000)+S$:PRINT\$1,LEFT$(A$,8)" ";       2340
K$=STR$(KR)+S$:PRINT\$1,LEFT$(KR$,8)" ";       2350
KA$=STR$(KA)+S$:PRINT\$1,LEFT$(KA$,8)" ";       2360
RETURN                         2370

FIG 9.22 BASIC PROGRAM USED TO SIMULATE CRACK GROWTH THROUGH THICK WALLED TUBING.
Plate 3.1: General view of the testing facility used in this investigation
Plate 3.2: The advanced micro-computer and AC electrical potential systems
Plate 3.3: A C-shaped specimen undergoing a fatigue crack growth test
Plate 3.4: The interface unit between the testing machine and micro-computer
Plate 3.5: The analogue to digital converter (ADC) and signal conditioner (SC) modules
Plate 3.6: The cycle counter and bus board modules
Plate 3.7: The power supply and digital to analogue (DAC) modules
Plate 7.1: View of fatigue surfaces (high temperature)
Plate 7.2: View of fracture surfaces (low fatigue crack growth rate)
Plate 7.3: View of fatigue surfaces (difficulties encountered with SR specimens)
Plate 7.4: View of fatigue and fracture surfaces (DM tubing)
Plate 7.5: View of fatigue and fracture surfaces (DMA tubing)
Plate 7.6: View of fatigue and fracture surfaces (H tubing)
Plate 7.7: View of fatigue and fracture surfaces (U tubing)
Plate 7.8: View of fatigue and fracture surfaces (V tubing)
Plate 7.9: View of fatigue surfaces for various tubing (constant $\Delta K$ tests)