FLOW PAST A BLUFF BODY WITH A WAVY STAGNATION FACE

R. M. DAREKAR and S. J. SHERWIN†
Department of Aeronautics, Imperial College of Science, Technology and Medicine, London, SW7 2BY, UK.

(Received 7 September 2000, and in final form 7 September 2000)

Numerical investigations have been performed to study the flow past square section cylinders with a spanwise geometric deformation leading to a stagnation face with a sinusoidal waviness. The computations were performed using a spectral/hp element solver over a range of Reynolds numbers from 10 to 500.

Starting from fully developed shedding past a straight cylinder at a Reynolds number of 100, a sufficiently high waviness is impulsively introduced resulting in the stabilisation of the near-wake to a time-independent state. The steady nature of the near-wake is associated with a reduction in total drag of about 16% at a Reynolds number of 100 as compared with the straight, non-wavy cylinder. Further increases in the amplitude of the waviness lead to the emergence of hairpin vortices from the near-wake region, similar to the wake of a sphere at low Reynolds numbers.

At higher Reynolds numbers the drag reduction increases substantially, e.g. at a Reynolds number of 500 it is 34%, principally due to the increase in drag of the non-wavy cylinder. Alternative methods based on three-dimensional forms of bleed are investigated to suppress the von-Kármán vortex street of a straight, non-wavy cylinder.

© 1999 Academic Press

1. INTRODUCTION

To reduce the drag and weaken the vortex shedding of two-dimensional bluff bodies, three-dimensional disturbances can be introduced into the base geometry. Tanner (1972) introduced a broken separation line along the trailing edge of a blunt aerofoil. He applied a stepwise deformation to the trailing edge and observed that by increasing the depth of the steps, larger base drag reductions are obtained. The maximum drag reduction obtained using this method was 64%. The study of this drag reduction technique (segmented trailing edge) was then continued by Rodriguez (1991) and Petrusma & Gai (1994).

Bearman & Tombazis (1993) and Tombazis & Bearman (1997) investigated the three dimensional features of the wake behind a blunt-based model with a wavy trailing edge at a Reynolds number of 40,000. They observed that the introduction of a spanwise waviness at the trailing edge fixed the positions of vortex dislocations along the span of the body. Increasing the wave steepness, defined as the ratio of peak to peak wave height divided

†author for correspondence
by the wavelength, increased the base pressure which resulted in a drag reduction. The maximum drag reduction of 34% at a Reynolds number of 40,000 occurred for a wave steepness of 0.14. Based on these facts, they concluded that encouraging the formation of dislocations in the wake reduces the drag.

More recently Bearman & Owen (1998a,b) continued the above work by applying the waviness at the leading edge of a rectangular cross-section body. They observed that a mild disturbance (wave steepness of only 0.06 to 0.09) resulted in the complete suppression of vortex shedding and substantial drag reduction of at least 30% at a Reynolds number of 40,000.

The aim of the current work is to carry out well resolved numerical simulations of the flow past three-dimensional bluff bodies with a sinusoidal stagnation surface. The geometries used in this work have both sinusoidal front and rear faces. In the work of Bearman & Owen (1998a,b), only the front face was wavy for the rectangular cross-section bodies. However, in their experiments, similar results were also obtained with a flat plate which had both wavy front and rear faces. This latter result would suggest that the wavy trailing edge does not influence the qualitative observations made during the experiments of Bearman & Owen (1998a,b).

This paper is outlined as follows. Section 2 details the problem definition and introduces the non-dimensional parameters as well as the numerical method. In section 3, we present the main results of the effect of varying the Reynolds number for a cylinder with a constant amplitude of waviness. In addition, the effects of various forms of three-dimensional bleed on the wake topology of a straight, non-wavy cylinder are investigated.

2. PROBLEM DEFINITION & SIMULATION METHOD

We are interested in the flow past a square section cylinder with a waviness in both the front and rear faces as shown in Figure 1. The wavy cylinder is defined by the peak to peak wave height $W$, the wavelength $\lambda$ and the base height $D$. The free-stream velocity $U_\infty$ is aligned with the $x$-axis (streamwise direction), the span of the cylinder is aligned to the $z$-axis (spanwise or cross-flow direction) and finally the $y$-axis will be denoted as the vertical direction. We define the Reynolds number based on the base height $D$, as $Re = U_\infty D/\nu$ where $\nu$ is the kinematic viscosity of the fluid. The maximum denotes

![Figure 1. Schematic of the cylinder with the waviness at both the leading and trailing edge surfaces.](image-url)
the most upstream cross-section (peak), while the minimum denotes the most downstream cross-section (valley). Furthermore, the waviness of the centre line (defined as a line going through the centroid of the body along the span) can be expressed mathematically in the following form:

\[ \xi(z) = -\frac{W}{2} \cos\left(\frac{2\pi}{\lambda}z\right) \]  

(1)

2.1. Dimensionless Length Scales

In contrast to the standard flow past a non-wavy cylinder, we have now introduced two extra length scales \( W \) and \( \lambda \). Whilst the non-wavy case can be completely characterised by the Reynolds number based upon the base height \( D \), we now have a further two independent length parameters that we choose to define as \( W/\lambda \) and \( \lambda/D \). Further, by differentiating equation (1) with respect to \( z \) gives an expression for the slope of the waviness \( d\xi/dz \):

\[ \frac{d\xi}{dz} = \frac{\pi W}{\lambda} \sin\left(\frac{2\pi}{\lambda}z\right) = A_s \sin\left(\frac{2\pi}{\lambda}z\right) \]  

(2)

Where \( A_s = \pi W/\lambda \) is the maximum magnitude of the slope and depends on \( W/\lambda \) which is defined as the wave steepness by (Bearman & Owen, 1998a; Bearman & Owen, 1998b). The slope of the waviness which is proportional to the wave steepness reaches a maximum value of \( A_s \) at the inflection points \( z = \lambda/4, 3\lambda/4 \).

2.2. Simulation Method

A parallel spectral element code \( \text{NuEka} \) (Sherwin & Kamiadakis 1995) was employed to solve the three-dimensional incompressible Navier-Stokes equations. Spectral element methods have been widely used in the past for the prediction of bluff body flows due to their high spatial accuracy. Solution refinement can be obtained either by refining the mesh (h-refinement) or increasing the polynomial order \( P \) (P-refinement). To incorporate the wavy geometries, a geometric mapping is introduced as previously adopted by Newman (1996) and Evangelinos (1999). Application of the mapping to the Navier-Stokes equations leads to the modified set of equations:

\[ \frac{\partial u}{\partial t} + (u \cdot \nabla)u = -\nabla p + \frac{1}{Re} \nabla^2 u + A(u, p, \xi) \]

\[ \nabla \cdot u = 0 \]  

(3)

where \( A(u, p, \xi) = [A_x, A_y, A_z]^T \) is the d’Alembert forcing term which as reported in Dakekar & Sherwin (2000) has non-zero pressure contributions of the form:

\[ A_z = \frac{\pi W}{\lambda} \sin\left(\frac{2\pi}{\lambda}z\right) \frac{\partial p}{\partial x} \]  

(4)

\[ A_x = \frac{\pi W}{\lambda} \sin\left(\frac{2\pi}{\lambda}z\right) \frac{\partial p}{\partial z} - \pi^2 \left(\frac{W}{\lambda}\right)^2 \sin^2\left(\frac{2\pi}{\lambda}z\right) \frac{\partial p}{\partial x} 
- 2W^2 \pi^2 \frac{W}{\lambda^2} \cos\left(\frac{2\pi}{\lambda}z\right) \]  

(5)
3. RESULTS & DISCUSSION

3.1. PARAMETER SPACE STUDY

In Darekar & Sherwin (2000), the wavelength $\lambda$ and amplitude $W$ of the waviness were varied at $Re = 100$ introducing different degrees of geometric three-dimensionality in the wake. From consideration of the wake topology in terms of the Jeong & Hussain (1995) identification criteria and force characteristics, the effect of introducing a wavy stagnation face was classified into five distinct regimes. A summary of the parameter space study is shown in Figure 2.

For a mild geometric waviness we obtain regime I, where the formation of the wake at the base of the cylinder is mildly three-dimensional and the force history is similar to the straight cylinder. However as the wake evolves further downstream there is a discernible deformation of the spanwise Kármán vortices (see Figure 3(a)).

Figure 2. Summary of the parameter space study, $\lambda/D$ versus $W/\lambda$ at $Re = 100$. (o) regime I; (o) regime II(type A); (c) regime II(type B); (●) regime III(type A); (■) regime III(type B), From Darekar & Sherwin (2000).

For a slightly higher level of waviness we obtain a transitional regime denoted as regime II. In this regime the effects of the geometric waviness in the formation region are more dominant. A large spanwise curvature appears in the top and bottom shear layers and an associated drop occurs in mean drag and the root mean square of the lift. Further downstream we observe the formation of streamwise vortices associated with the sinusoidal spanwise deformation of the distorted Kármán vortices. Within this transitional regime II, we make a further distinction between normalised wavelengths above and below $\lambda/D \approx 5.6$. For $\lambda/D \leq 5.6$, denoted as regime II (type A), we observe a time periodic state with a single frequency where streamwise vortices occur in the braids and connect adjacent Kármán vortices (see Figure 3(b)). These streamwise vortices have some analogy with the streamwise structures of the mode A instability (see Williamson 1996). However for $\lambda/D > 5.6$, denoted as regime II (type B), we observe a beating phenomenon where the
wake topology alternates between a mildly three-dimensional state similar to regime I and a highly three-dimensional state similar to regime II (type A).

Finally, when the amplitude of the waviness is sufficiently large, we obtain a near-base region which is completely steady leading to a significant reduction in mean drag and lift tending to zero. This region of maximal drag reduction is denoted by regime III and within this regime, there is no evidence of a Kármán vortex wake. Once again we can identify two sub-regimes. In the first case, regime III (type A), the flow is completely steady and has only been observed for $\lambda/D \leq 5.6$. It is in regime III (type A) that the most significant drag reduction occurs. The wake topology is similar to that shown in Figure 5(a). However, when $\lambda/D > 5.6$, hairpin vortices are shed periodically from the almost steady near-base region. In this regime, defined as regime III (type B), a small unsteadiness appears in the near-wake due to the shedding of these hairpin vortices. We note that the wake topology of the hairpin vortices in regime III (type B) resembles that of a sphere at low Reynolds numbers (see Figure 5(b)). It was shown that the smallest wave steepness $W/\lambda$ to force the flow into regime III (type A) occurs at around a value of $\lambda/D \approx 5.6$. This wavelength has a similar length scale as the mode A transition of the wake of a straight, non-wavy square section cylinder and the primary spacing of similar sign vortices in a Kármán vortex street.

### 3.2. Reynolds Number Effect

The effect of the Reynolds number on the forces and wake topology of the wavy cylinder is investigated for the particular case of $(\lambda/D, W/\lambda) = (5.6, 0.1667)$. The numerical simulations were performed over a range of Reynolds numbers from 10 to 500 and are compared with experimental data for the straight and wavy bodies up to a Reynolds number of 40,000. The results for the total drag coefficient against the Reynolds number are summarised in Figure 4.

For the straight cylinder, at low Reynolds numbers the flow is steady and a pair of symmetric vortices forms in the near-wake. In this regime, the drag is relatively high and has a strong component due to skin friction from the boundary layer. For example, at $Re = 10$, the total drag is 3.20, 32% of which is due to wall shear stresses. As the Reynolds number is increased the shear stress component of the total drag decreases and the length of the closed recirculating region increases, e.g. at $Re = 40$, the total drag is reduced to 1.71
and only 16% is due to skin friction.

At a critical Reynolds number $Re_1$, the steady flow becomes unstable and bifurcates to a two-dimensional time-periodic flow, resulting in the well-known Kármán vortex street in the wake. This first transition, known as the primary instability, is the result of a Hopf bifurcation. For the square section cylinder, the experimental value for $Re_1$ reported in Sobanskar et al. (1997) is $Re_1 = 47 \pm 2$, whereas previous numerical computations place the critical Reynolds number for the onset of vortex shedding near $Re_1 = 51.2$ (Sobanskar et al. 1998) and $Re_1 = 53$ (Kelkar & Patankar 1992), respectively. Beyond this critical Reynolds number, the drag deviates from the steady and symmetric curve as shown in Figure 4.

An interesting flow pattern then develops on the top and bottom surfaces with increasing Reynolds number as shown in the simulations of Robichaux et al. (1999). In this work, they show that at low Reynolds numbers, the shear-layers remain attached to the top and bottom surfaces. The shear-layers then separate from the trailing edge, forming the von Kármán vortex street in the wake. As the Reynolds number is increased to around $Re = 120$, the shear-layers separate from the leading edge but reattach a short distance downstream, thus forming small unsteady recirculating cells on the top and bottom surfaces. Finally, at around a Reynolds number of 150, the shear-layers separate from the leading edge without reattachment. They also report that the two-dimensional wake becomes unstable to three-dimensional perturbations at around $Re = 161$ but the associated spanwise wavelength is
Figure 5. Perspective view showing the three-dimensional wake of the wavy cylinder ($\lambda/D, W/\lambda$) = (5,6,0.1667) at (a) $Re = 80$; (b) $Re = 100$; (c) $Re = 200$ and (d) $Re = 500$.

not given. The onset of Mode A for the square section cylinder is given as $Re = 162 \pm 12$. The lower bound of the onset of three-dimensionality seems to be very close to the Reynolds number at which the flow separates from the leading edge without reattachment. From the experimental data of Okajima (1995), the total drag increases sharply from $Re \approx 200$. It should be noted that this trend does not occur for the circular cylinder and is probably due to the fixed leading edge separation points. It is not yet clear why there is a large drop in the drag coefficient after $Re = 1000$ but it would seem to be due to the shear-layer transition (Okajima 2000, private communication). Further increasing the Reynolds number leads to a monotonic increase in the drag coefficient.

For the wavy body, the transition process is distinctively different. In the low Reynolds number regime, the drag is similar to that of the straight cylinder (see Figure 4). For example, at $Re = 10$, the total drag is 3.19, 32% of which is due to viscous shear stresses and at $Re = 40$, the total drag is 1.67 where 18% is due to skin friction. The wake topology is steady and symmetric which results in a net zero lift force. The drag coefficient follows the steady and symmetric curve shown in Figure 4. Beyond the critical Reynolds number for the onset of vortex shedding of the straight cylinder ($Re_c \approx 47$), the three-dimensional wake remains steady and symmetric. The topology of the wake at $Re = 80$ is shown in Figure 5(a). The drag continues to follow the steady and symmetric curve until $Re \approx 100$. 
At around this critical Reynolds number, transition to unsteadiness occurs in the wake where the drag coefficient deviates from the steady and symmetric curve (see Figure 4). A staggered array of hairpin vortices emerge from the near-wake region as can be seen from Figure 5(b). The distinct structure of these hairpin vortices persists until \( Re = 200 \), after which smaller scale instabilities are induced as shown by Figure 5(d) at \( Re = 500 \). As can be seen, the wavy stagnation face has delayed the onset of unsteadiness to \( Re = 100 \) and the antisymmetric mode has changed from a Kármán vortex street to a street composed of hairpin vortices, similar to the wake of a sphere at low Reynolds numbers.

Defining drag reduction with respect to the drag level of the straight cylinder, at \( Re = 80 \) the drag reduction is about 13%, whereas at \( Re = 100, 200 \) and 500, it is 16%, 20% and 34%, respectively. It can be appreciated that the drag reduction at \( Re = 500 \) has substantially increased from that at \( Re = 100 \) and is comparable to the experiments of Owen (1997) at \( Re = 40,000 \). The increase in the drag reduction with increasing Reynolds number is principally due to the fact that for a straight, non-wavy cylinder, the shear-layers separate from the leading edge from \( Re \approx 170 \) whereas at \( Re = 100 \) the shear-layers remain attached until they separate from the trailing edge. Therefore, the leading edge separation leads to a higher drag for the straight, non-wavy cylinder. However, from the numerical simulation at higher Reynolds numbers, the asymptotic level of the drag coefficient of the wavy cylinder does not change significantly from that at \( Re = 100 \) (see Figure 4). Therefore the higher drag reduction, at higher Reynolds numbers, is mainly due to the increase in the drag coefficient of the straight, non-wavy cylinder.

3.3. Three-dimensional Bleed

In Section 2.2, it was shown that the waviness can be interpreted as a forcing function to the flow past a straight, non-wavy cylinder. The spanwise forcing term \( A_z \), given by equation (4), depends on the streamwise pressure gradient and the \( z \)-derivative of the shape of the waviness. The streamwise pressure gradient \( \partial p / \partial x \) will be large near the stagnation face and will always be positive. The forcing term \( A_z \) will be positive between \( 0 < z < \lambda / 2 \) and negative between \( \lambda / 2 < z < \lambda \). This forcing therefore sets up a spanwise or cross-flow velocity component, \( w \), along the leading edge surface going from a position corresponding to the maximum towards a position corresponding to the minimum of the wavy cylinder. In Darekar & Sherwin (2000), it was shown that the streamwise forcing term \( A_x \), given by equation (5), is mainly dominated by the term \( -\pi^2 (W/\lambda)^2 \sin^2((2\pi/\lambda)z)(1/\rho)\partial p / \partial x \) which is negative along the leading edge and reaches a maximum in the region of the inflection points. The streamwise forcing term, \( A_x \), will thus slow down the two-dimensional \( u \) component of the velocity more significantly close to the inflection points.

These observations motivated the interpretation of the wavy stagnation face as a combination of promoting a cross-flow at the stagnation face and slowing down the flow near to the inflection points. To simulate this effect, the influence on the unsteady wake of a straight cylinder due to various forms of surface bleed has been investigated at a spanlength of \( \lambda / D = 5.6 \) at \( Re = 100 \). Two types of bleed were imposed as boundary conditions on the cylinder’s surface: (i) a cross-flow bleed on the stagnation face and (ii) a vertical bleed on the top and bottom surfaces. The following mathematical formulations were used for each of the three-dimensional forms of bleed:

- Cross-flow bleed on the stagnation face, \( w_{cr} = 0.14 \sin(\frac{2\pi z}{\lambda}) \left( -\frac{1}{\rho \partial x} \right) (y - 0.5)(y + 0.5) \)
Figure 6. Perspective view showing the three-dimensional wake for the two forms of bleed (a) on the stagnation face and (b) on the top & bottom surfaces.

- bleed on the top & bottom surfaces, \( v_{bl} = \pm 0.01 e^{(-1000z^2)}(1 + 200e^{-(z-2.5)^2}) \)

The cross-flow bleed was specified on the whole stagnation face of the straight, non-wavy cylinder. The bleed has a sinusoidal variation in the spanwise \( z \) direction and a parabolic variation in the \( y \) direction. It reaches a local maximum of 0.14 i.e 14% of \( U_{\infty} \) at \( z = 1.4, 4.2 \). The results showed that the near-wake was stabilised to a time-independent state. The wake topology is shown in Figure 6(a) and has strong qualitative similarities with that of the wavy cylinder in the steady regime (see Figure 5(a)). The stabilisation of the near-wake resulted in a drag reduction of about 15% and in a zero lift force.

The bleed on the top and bottom surface was specified by modifying the velocity boundary conditions on the top and bottom surfaces to \((u, v, w) = (0, \pm v_{bl}, 0)\). The distribution is exponential in both the streamwise \( x \) and spanwise \( z \) directions. A maximum velocity of about 2 occurs at the midspan of the cylinder. The topology of the wake is shown in Figure 6(b) and has strong similarities to the unsteady flow past the wavy cylinder at \( Re = 100 \) (see Figure 5(b)). The lift force dropped to nearly zero and the drag reduction was equal to about 11%.

Current work is focusing on the minimum amount of bleed necessary to stabilise the near-wake. A comparison of this minimum amount of bleed will be done for the stagnation face cross-flow, the top and bottom surface bleed as well as the base bleed of the straight cylinder. It is anticipated that the stagnation face cross-flow will be the most effective means of stabilising the near-wake as perturbations are introduced very early on in the shedding process and hence have a longer convective time to disrupt the shedding mechanism.

4. CONCLUSION

It was shown that the wavy stagnation face delayed the onset of unsteadiness to \( Re \approx 100 \) for the particular case of \((\lambda/D, W/\lambda) = (5.6, 0.1667)\). In contrast to the standard von-Kármán vortex street of a straight cylinder, the unsteady wake of the wavy cylinder is composed of a staggered array of hairpin vortices, which is similar to the wake of sphere at low Reynolds numbers. The drag was found to be substantially lower than that of a straight cylinder. This drag reduction increased with increasing Reynolds number. Alternative methods, based on three-dimensional forms of bleed, have been shown to suppress the von-Kármán
BLUFF BODY WITH A WAVY STAGNATION FACE

vortex street of a straight, non-wavy cylinder, leading to similar wake structures to that found for the wavy cylinder.

ACKNOWLEDGEMENTS

The authors would like to acknowledge several inspiring discussions and critical feedback from J.C. Owen, Prof. P.W. Bearman and Dr. D. Barkley. The first author is supported by the Edmund Davis Trust fund awarded by the University of London. Computational resources were provided by the Computer Services for Academic Research (CSAR) under the EPSRC grant number GR/M08424 and the Imperial College Parallel Computing Centre.

REFERENCES


