Control of centrifugal compressors via model predictive control for enhanced oil recovery applications

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Abstract: This paper proposes a control system for integrated pressure and surge control of centrifugal compressors for enhanced oil recovery application. The proposed control system is based on linear model predictive control. A fully validated non-linear dynamic model was developed in order to simulate the operation of the compressor at full and partial load. The model of the compression system includes a main process line with the compressor and a recycle line with the antisurge recycle valve. Different disturbance and control tuning scenarios were tested and the response of the model predictive controller was analysed, evaluated and also compared with a traditional control system. Temperature effects have been taken into account in the model of the process and in the constraint formulation of the MPC optimization problem. The results show that the proposed control technique is able to meet the process demand while preventing surge and also minimizing the amount of gas recycle.

Keywords: MPC, compressor, surge, control, driver torque, recycle, carbon dioxide, supercritical.

1. INTRODUCTION

Enhanced Oil Recovery (EOR) methods are commonly used in industry to recover oil from onshore and offshore reservoirs after primary and secondary extraction (Sobers et al., 2013). Among the non-thermal gas injection methods, carbon dioxide floods have been used for EOR (Thomas, 2008). CO2 has already been used in the past for oil recovery however this method has been recently integrated with carbon storage for the reduction of atmospheric emissions (Ravagnani et al., 2009).

For the purposes of enhanced oil recovery and carbon dioxide storage, CO2 must be compressed to supercritical conditions. For this type of application, the phase transition takes place inside a multistage centrifugal compressor. The operation of this type of machine is limited by surge. Surge is a dynamic instability of the gas that causes flow reversal inside the machine. When the compressor is surging, the oscillatory behaviour of the gas flow causes vibrations that can damage blades, casing and bearings (Boyce, 2012). In industrial practice, surge control still relies on avoidance control. Although many solutions based on active control have been proposed (Arnulfi et al., 2006), they were not implemented on industrial-size compressors due mainly to the cost and reliability of the additional devices they require (Uddin and Gravdahl, 2012).

Avoidance control for centrifugal compressors relies on the recycle of part of the compressed gas in order to increase the inlet flow rate of the compressors. When the recycle valve opens a compressor becomes a multiple-input multiple-output (MIMO) system. Model predictive control (MPC) is considered the most appropriate control for this type of system (Seborg et al., 2004). In the literature it has already been demonstrated that model predictive control was applicable for the control of complex compression systems (Smeulers et al., 1999, Øvervåg, 2013) and for surge prevention via closed coupled valve (Johansen, 2002) and drive torque actuation (Cortinovis et al., 2012). However the minimization of the recycle flow rate and the temperature effects have not previously been taken into account.

This paper proposes the use of MPC for the integrated control of pressure and surge in centrifugal compressor applications. The amount of gas recycled for surge prevention is minimized by control tuning and the temperature constraints have been included in the MPC formulation.

The structure of the paper is the following. In Section 2 the model of the compressor is presented. In Section 3 an overview on traditional compressor control is given. It is then followed by the description of the implemented model predictive controller and its design. In Section 4 the paper includes the MPC tuning, the scenarios for the validation of the control system and the results of the dynamic simulations. Finally, Section 5 presents the conclusions of the work.

2. MODEL OF THE COMPRESSOR

2.1 Mathematical model of the compressor

The model of the compression system is a non-linear one-dimensional dynamic model that includes a main process line and a recycle line. It is represented in Figure 1. The main process line includes inlet valve, outlet valve, compressor, duct and plenum. The recycle line includes the antisurge recycle valve that is used to prevent surge occurrence. Hot gas recycle should be limited over time because it can...
overheat the machine. On the other hand it reduces the time delay of the system as a smaller amount of gas is stored along the recycle line (Botros, 2011).

The system includes also two nodes. The first node represents the physical point where the freshly fed gas mixes with the recycled gas, while the second node represents the physical point where the compressed gas splits between delivered gas and recycled gas. Variables $m_{in}$, $m_{out}$ and $m_r$ are the gas flow rate respectively through inlet, outlet and antisurge valve. $m$ is the gas flow rate that enters the compressor and it is monitored for surge control, while $m_{pout}$ is the gas flow rate that leaves the plenum. $p_{in}$ and $p_{out}$ are the inlet and outlet pressures of the system. $p_{o1}$, $p_{o2}$ and $p$ are respectively the compressor inlet pressure, compressor outlet pressure and plenum pressure. $p$ is monitored for pressure control.

The model of the compressor is based on a well-established model present in the literature that includes a compressor, a plenum and an outlet throttle valve (Greitzer, 1976). This model was further developed by Fink et al. (1992) in order to include the dynamic of the rotating shaft connecting driver and compressor. Gravdahl and Egeland (1999) proposed a further modification by expressing the torque of the compressor $\tau_c$ as a function of shaft rotational velocity $\omega$ and mass flow rate $m$ while Gravdahl et al. (2002) proposed to use the torque of the driver $\tau_d$ as input variable of the model instead of the rotational shaft speed $N$. This last model was the reference for this work and was modified according to Morini et al. (2007) in order to include also the recycle loop.

The equations of the model include the mass and the momentum balance of the compressor, the moment of momentum balance of the rotating shaft, the compressor torque and characteristic (Gravdahl et al., 2002). They also include the equations of the flow through inlet, outlet and recycle valve (Morini et al., 2007). The equations are the following:

1. $\frac{dp}{dt} = \frac{a_{o1}^2}{V}(m - m_{pout})$
2. $\frac{dm}{dt} = \frac{A_1}{L}(\Psi_c p_{o1} - p)$
3. $\frac{d\omega}{dt} = \int \left(\tau_d - \tau_c\right) dt$
4. $\tau_c = \mu r_2^2 \omega m$
5. $\Psi_c = \frac{p_{o2}}{p_{o1}} = \Psi_c(\omega, m)$
6. $m_{in} = k_{in} \sqrt{\rho_{in}(p_{in} - p_{o1})}$
7. $m_{out} = k_{out} \sqrt{\rho (p - p_{out})}$
8. $m_r = k_r \sqrt{\rho_r (p_r - p_{o1})}$

where $a_{o1}^2$ is the sonic velocity at ambient condition, $V$ is the volume of the plenum, $A_1$ is the duct throughflow area, $L$ is the duct length, $\Psi_c$ is the compressor characteristic, $J$ is the total inertia of the system, $\mu$ is the slip factor, $r_2$ is the impeller radius, $k_{in}$, $k_{out}$, $k_r$ are the constants for respectively inlet, outlet and antisurge valve, $\rho_{in}$, $\rho$, $\rho_r$ are the density of respectively $m_{in}$, $m$ and $m_r$.

In this paper corrected compressor maps have been used in order to define the surge line as a function of pressure ratio, rotational shaft speed, inlet pressure and inlet temperature of the gas. The temperature of the gas entering the machine ($T_{in}$) has been estimated as a function of the temperature of the freshly fed gas ($T_{in}$), the temperature of the recycled gas ($T_{o2}$) and the mass flow rates of these two flows (respectively $m_{in}$ and $m_r$), according to the following equation:

$$m_{in} \int_{T_{ref}}^{T_{in}} c_p(T)dT + m_r \int_{T_{ref}}^{T_{o2}} c_p(T)dT = m \int_{T_{ref}}^{T_{o1}} c_p(T)dT$$

where $T_{ref}$ is the reference temperature and the heat capacity of the gas mixture $c_p$ is evaluated at the temperature $T$ according to:

$$c_p(T) = \sum_{i=1}^{2} x_i c_{p,i}(T)$$

where $i$ is the number of components of the gas and $x_i$ is their mass fraction. The outlet temperature of the compressor $T_{o2}$ is estimated according to the performance maps provided by the supplier of the compressor.

2.2 Case study and model validation

The case study presented in this paper refers to a multistage centrifugal compressor arranged in a single shaft.
Table 1. Typical parameter values

<table>
<thead>
<tr>
<th>Parameter name</th>
<th>Parameter value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_{eq}$</td>
<td>0.001-0.005 s$^{-1}$ m$^{-1}$</td>
</tr>
<tr>
<td>$\bar{L}$</td>
<td>0.001-0.005 m</td>
</tr>
<tr>
<td>$\frac{1}{f}$</td>
<td>0.5-2 kg$^{-1}$ m$^{-2}$</td>
</tr>
<tr>
<td>$\mu_{v}^{2}$</td>
<td>0.01-0.05 m$^{-2}$</td>
</tr>
<tr>
<td>$k_{in}$</td>
<td>1-2.5 kg$^{-1}$ m$^{-2}$</td>
</tr>
<tr>
<td>$k_{out}$</td>
<td>1-2.5 kg$^{1/2}$ m$^{1/2}$</td>
</tr>
<tr>
<td>$k_{r}$</td>
<td>1-2.5 kg$^{1/2}$ m$^{1/2}$</td>
</tr>
</tbody>
</table>

configuration. The fourth and last stage of compression was selected for the present analysis and its target pressure ratio is 2.85. After the calculations were complete all the other process variables reported in the paper were scaled to be 1 at their design point due to non-disclosure agreement with the industrial partners of the project. The driver is an asynchronous electric motor that allows variable speed operation. The process fluid is a mixture of carbon dioxide and water with small percentages of light hydrocarbons. The Span and Wagner equation of state (Span and Wagner, 1996) was selected in order to estimate the thermodynamic properties of the gas.

The model of the compressor was validated against data coming from the industrial case study. Process data sheets and compressor performance maps were used to validate the model during steady state simulations while an industrial simulator, provided by the project partner ESD Simulation Training, was used to validate the dynamic behaviour during transients between steady states. The agreement between the available transient behaviours and the model presented in the paper was satisfactory. The model was then implemented in MATLAB Simulink and the ordinary differential equations were solved numerically using the MATLAB function ode45. Although the values pertaining to the model may not be disclosed, some typical values are presented in Table 1.

### 3. MODEL PREDICTIVE CONTROLLER

#### 3.1 Traditional PID control

The task of the control system of a compressor is to deliver the fluid to the downstream part of the process at the desired pressure, while avoiding surge. In the industrial practice two separate PID controllers are usually employed: the pressure controller and the antisurge controller.

The pressure controller has a cascade control structure. The slave loop is a speed controller. Its set point is the output of the master loop and the manipulated variable is the torque of the driver. The master loop of the cascade controller is a pressure controller. Its controlled variable is the outlet pressure of the compressor, while its output is the remote set point of the slave loop. The pressure controller has been tuned using initially the lambda tuning technique and then trial and error testing.

The antisurge controller continuously monitors the inlet flow rate of the compressor, which is its controlled variable. If the flow goes below its lower limit the controller opens the recycle valve that allows part of the gas to be recycled back to the inlet of the machine. This lower limit is called antisurge control line. The antisurge controller has been tuned in order to be able to open the antisurge valve within 2 seconds.

The interaction between pressure controller and surge controller is strong and they can end up pushing the compression system in opposite directions, as will be demonstrated in the Section 4.

#### 3.2 Representation of the surge margin

Usually both surge and control lines are plotted on the compressor map and therefore their distance from the operating point is easily identifiable. However this type of visualisation, even if very common in both academia and industry, can be misleading. The reason is that the surge line depends on both inlet pressure and temperature and therefore is affected by process disturbances and also by the opening of the recycle valve. The corrected compressor maps can be useful when the inlet conditions are different from the reference conditions however not when they continuously change over time as it happens during a process disturbance. Therefore a different way to visualise the proximity to surge is suggested in this paper. The proximity of the machine to surge is represented as the distance between the inlet mass flow rate of the compressor $m$ and the surge control mass flow rate $m_{crtu}$.

#### 3.3 MPC controller

In order to avoid the interaction between different controllers, an MPC controller was designed, implemented and tuned in order to control both pressure and surge.

Figure 2 is the schematic representation of the system controlled by the MPC controller. The plant is defined by its states $x_m$. The process inputs are the disturbances $d_x$ and the manipulated variables $u_x$. The process outputs are $y_x$. These outputs are the measured variable of the MPC controller. These variables are compared with their reference or set points and the MPC solves a constrained optimization.
problem. The MPC is based on the linearized version of the plant model. The constraints are the lower and upper boundaries for both controlled variables \( y_i \) and manipulated variable \( u_j \). The optimisation function contains weights for both manipulated variables and process output variables. For the control problem presented in this paper, states \( x_m \) are \( p \), \( m \) and \( \omega \), disturbances \( d_k \) are \( p_{in} \) and \( p_{out} \), manipulated variables \( u_j \) are \( t_d \) and the position of the antisurge valve \( ASV \), outputs \( y_i \) are pressure \( p \), mass flow rate \( m \), rotational speed \( N \) and compressor outlet temperature \( T_{out} \). \( p \) is the controlled variable as it has to be at its set point, while \( m \), \( N \) and \( T_{out} \) have to be within their operating range, according to the following equations:

\[
m \geq m_{ctrl} = m_{surge} + \text{margin} \tag{11}
\]

\[
N_{min} \leq N \leq N_{max} \tag{12}
\]

\[
T_{out} \leq T_{out,max} \tag{13}
\]

Minimum and maximum rotational shaft speeds depends on the driver while the constraint on the maximum temperature guarantees the integrity of the machine during hot gas recycle.

4. SIMULATION RESULTS

4.1 Tuning of the model predictive controller

The MPC controller has been tuned in order to guarantee good pressure control while avoiding as much as possible the opening of the antisurge valve. The reason for doing that is that gas recycle increases the operating cost of the system as more gas must be compressed by the machine without being delivered. Three different sets of control tuning parameters have been defined and they have been summarised in Table 2. The first tuning set was called ‘set 1’ and it is better performing with regards to pressure control. The second tuning set was called ‘set 2’ and it is more robust towards boundary disturbance. Both these two tuning sets aim at the minimisation of the opening of the recycle valve. A third set of tuning parameters, called ‘set 3’, was defined. It performs well in terms of pressure control however it does not minimise the gas recycle. The control tuning parameters are called weights in the MPC formulation. The simulation scenarios tested in this paper come from the literature and also from industrial practice (Dukle and Narayanan, 2003, Patel et al., 2007, Wu et al., 2007). They have been proposed in the past for the validation of antisurge controller.

4.2 Simulation scenarios and performance parameters

The first validation scenario includes process disturbances that can affect the operation of the plant. Inlet and outlet pressures of the system were selected as disturbance variables. The second validation scenario is the load pattern. The pressure set point was changed and the response of the system was recorded. The third scenario includes the step closure of inlet and outlet valves of the system. Various simulations were run within these three scenarios and some representative results have been reported in the paper.

The response of the control system was evaluated using graphical comparison and also via two different performance parameters. The first parameter is called \( M_{dimles} \) and it represents the dimensionless total amount of gas recycled during a certain disturbance:

\[
M_{dimles} = \frac{\int_{t=0}^{t_{final}} m_{d} \, dt}{\int_{t=0}^{t_{final}} m_{d} \, dt} \tag{14}
\]

where \( t_{final} \) represents the time interval considered for the analysis. The second parameter is the Integral of Squared Error (ISE), where the error is the difference between the controlled variable \( p \) and its set point \( p_{SP} \) over time:

\[
ISE = \int_{t=0}^{t_{final}} (p(t) - p_{SP}(t))^2 \, dt \tag{15}
\]

4.3 Results

In figures 3 and 4 the inputs and the outputs of the plant are represented for a process boundary disturbance and different control configurations. The disturbance is a positive pulse change of the outlet pressure of the system \( p_{out} \). The positive step change takes place at time \( t=100 \) seconds while the negative step change takes place at \( t=800 \) seconds. In Figure 3 the response of the system under the control of a traditional

![Fig. 3. PI control of process disturbance - inputs (a) and outputs (b)](image)

Table 2. Control tuning parameters

<table>
<thead>
<tr>
<th>Tuning set</th>
<th>( t_d )</th>
<th>( ASV )</th>
<th>( p )</th>
<th>( m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Set 1</td>
<td>0</td>
<td>10</td>
<td>1</td>
<td>0.08</td>
</tr>
<tr>
<td>Set 2</td>
<td>0</td>
<td>10</td>
<td>1</td>
<td>0.72</td>
</tr>
<tr>
<td>Set 3</td>
<td>0</td>
<td>0.1</td>
<td>1</td>
<td>0.08</td>
</tr>
</tbody>
</table>
PI controller is represented. In the first graph (Figure 3-a) the driver torque (left) and the opening of the antisurge valve (right) are represented. These two variables are the manipulated variables of the compression system. In the second graph (Figure 3-b) the compressor outlet pressure and its set point (left) and the mass flow rate and its lower limit (right) are represented. These variables are the main controlled variables of the compression system. When \( p_{\text{out}} \) increases the pressure controller reduces \( \tau_d \) in order to reduce \( p \). This action reduces the flow rate through the machine \( m \) as well. When this variable becomes equal to the surge control value \( m_{\text{ctrl}} \) the antisurge controller opens the antisurge valve. However this action causes the reduction of the pressure \( p \). Therefore the pressure controller decreases \( \tau_d \) and \( m \) increases. When \( m \) becomes bigger than \( m_{\text{ctrl}} \) the antisurge controller closes the antisurge valve. The consequence is the increase of \( p \) above its set point, that brings the pressure controller to reduce \( \tau_d \) and therefore reproduces the same behaviour. The result is the oscillation of the system that is interrupted only by the end of the pulse disturbance. When the outlet pressure of the system goes back to its design value, the system stabilise to the previous steady state point.

Figure 4 represents the response of the system under the same disturbance but controlled via MPC. The tuning set 1 was employed for the model predictive controller. Following the process disturbance, the MPC controller reduces \( \tau_d \) while barely moves \( ASV \). The pressure \( p \) is kept within its constraints but not tightly closer to its set point as this would force the control system to open the antisurge valve. These results demonstrate that the MPC controller is able to control the outlet pressure without causing the oscillation of the system.

Other simulations were run in order to compare the first and second tuning sets. A summary of the results is collected in Tables 3 and 4. For disturbances such as step change of outlet pressure \( p_{\text{out}} \), pressure set point \( p_{\text{sp}} \), inlet valve \( V_{\text{in}} \) and outlet valve \( V_{\text{out}} \), \( M_{\text{dimles}} \) and \( ISE \) have been estimated.

In all the tested cases the controller under tuning set 1 has allowed to recycle a smaller amount of gas (Table 3) while better controlling the pressure (Table 4). The only occurrence in which the controller under tuning set 1 has a higher \( ISE \) than the controller under tuning set 2 was due to saturation of the torque. In fact in this case the speed of the driver arrived to its maximum value.

Disturbance rejection of the pulse disturbance of the outlet pressure was also performed using the third tuning set. This allowed a much tighter control of the pressure however it involved a bigger amount of gas recycled over the duration of the transient (Table 5).

### 5. Conclusion

Different disturbance scenarios and controller tuning were tested and the results demonstrate that the proposed controller is effective for both disturbance rejection and set point tracking. The results demonstrate that the MPC controller is able to control the outlet pressure of the compressor while avoiding surge. They also demonstrate that the MPC controller is more suitable than PI controller for this multiple-input multiple-output process system. Under certain disturbances it is not possible to keep the pressure at its set point while avoiding surge without recycling. Therefore the tuning of the controller was performed in order to give priority to respectively the minimisation of the gas recycle (tuning set 1), the stability and protection of the system under aggressive disturbances (tuning set 2) and the control of the outlet pressure (tuning set 3). In all these cases the MPC controller performed as requested. The decision regarding the

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**Table 3. Dimensionless amount of gas recycled \( M_{\text{dimles}} \)**

<table>
<thead>
<tr>
<th>Disturbance</th>
<th>Set 2</th>
<th>Set 1</th>
<th>Relative difference %</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_{\text{out}} )</td>
<td>0.148</td>
<td>0.090</td>
<td>-39.6</td>
</tr>
<tr>
<td>( p_{\text{sp}} )</td>
<td>0.130</td>
<td>0.078</td>
<td>-39.8</td>
</tr>
<tr>
<td>( V_{\text{in}} )</td>
<td>0.179</td>
<td>0.004</td>
<td>-97.8</td>
</tr>
<tr>
<td>( V_{\text{out}} )</td>
<td>0.222</td>
<td>0.090</td>
<td>-59.5</td>
</tr>
</tbody>
</table>

**Table 4. Integral of the square error for pressure control \( ISE \)**

<table>
<thead>
<tr>
<th>Disturbance</th>
<th>Set 2</th>
<th>Set 1</th>
<th>Relative difference %</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_{\text{out}} )</td>
<td>3.54·10^3</td>
<td>1.38·10^4</td>
<td>-60.9</td>
</tr>
<tr>
<td>( p_{\text{sp}} )</td>
<td>2.64·10^3</td>
<td>1.01·10^4</td>
<td>-61.8</td>
</tr>
<tr>
<td>( V_{\text{in}} )</td>
<td>4.12·10^3</td>
<td>4.22·10^3</td>
<td>2.3</td>
</tr>
<tr>
<td>( V_{\text{out}} )</td>
<td>8.42·10^4</td>
<td>1.42·10^4</td>
<td>-83.2</td>
</tr>
</tbody>
</table>

**Table 5. Comparison between tight and loose recycle minimisation**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Set 1</th>
<th>Set 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_{\text{dimles}} )</td>
<td>0.229</td>
<td>1.769</td>
</tr>
<tr>
<td>( ISE )</td>
<td>1.68·10^3</td>
<td>5.67·10^3</td>
</tr>
</tbody>
</table>

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Fig. 4. MPC controller of process disturbance – input (a) and outputs (b)
type of tuning to adopt depends on many factors and cannot be generalised. Possible saturation of the manipulated variable must be taken into account as it can reduce the performance of the control system.

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