On the validity of the quasi-steady-turbulence hypothesis in representing the effects of large scales on small scales in boundary layers

Lionel Agostini and Michael Leschziner

Citation: Physics of Fluids 28, 045102 (2016); doi: 10.1063/1.4944735

View online: http://dx.doi.org/10.1063/1.4944735

View Table of Contents: http://scitation.aip.org/content/aip/journal/pof2/28/4?ver=pdfcov

Published by the AIP Publishing

Articles you may be interested in

Skewness-induced asymmetric modulation of small-scale turbulence by large-scale structures
Phys. Fluids 28, 015110 (2016); 10.1063/1.4939718

Towed body measurements of flow noise from a turbulent boundary layer under sea conditions

Numerical study of the primary instability in a separated boundary layer transition under elevated free-stream turbulence

Anisotropy in pair dispersion of inertial particles in turbulent channel flow
Phys. Fluids 24, 073305 (2012); 10.1063/1.4737655

A wall-layer model for large-eddy simulations of turbulent flows with/out pressure gradient
On the validity of the quasi-steady-turbulence hypothesis in representing the effects of large scales on small scales in boundary layers

Lionel Agostini and Michael Leschziner

1Mechanical and Aerospace Engineering, The Ohio State University, Columbus, Ohio 43210, USA
2Department of Aeronautics, Imperial College London, South Kensington, London SW7 2AZ, United Kingdom

(Received 12 October 2015; accepted 11 March 2016; published online 1 April 2016)

The “quasi-steady hypothesis,” as understood in the context of large-scale/small-scale interactions in near-wall turbulence, rests on the assumption that the small scales near the wall react within very short time scales to changes imposed on them by energetic large scales whose length scales differ by at least one order of magnitude and whose energy reaches a maximum in the middle to the outer portion of the log-law layer. A key statistical manifestation of this assumption is that scaling the small-scale motions with the large-scale wall-friction-velocity footprints renders the small-scale statistics universal. This hypothesis is examined here by reference to direct numerical simulation (DNS) data for channel flow at \( Re_{\tau} \approx 4200 \), subjected to a large-scale/small-scale separation by the empirical mode decomposition method. Flow properties examined include the mean velocity, second moments, joint probability density functions, and skewness. It is shown that the validity of the hypothesis depends on the particular property being considered and on the range of length scales of structures included within the large-scale spectrum. The quasi-steady hypothesis is found to be well justified for the mean velocity and streamwise energy of the small scales up to \( y^+ \sim O(80) \), but only up to \( y^+ \sim O(30) \) for other properties.

I. INTRODUCTION

Major experimental and computational efforts have been made over the past decade to increase our insight into statistical and structural properties of turbulent boundary layers. A recent review by Jiménez, focusing on DNS-derived information, provides an excellent illustration of this fresh impetus, building upon knowledge gained in the 1970 and 1980.

One particular focus of attention has been on the interaction between large-scale, coherent structures within the log-law layer and small-scale turbulence, the latter both within that layer and in the semi-viscous near-wall layer. Extensive experimental campaigns by groups around Marusic, Mathis, Hutchins Smits, Ganapathisubramani, and McKeon have identified a variety of new features and interactions, the most important among which are “footprinting” and “modulation” effects by outer energetic structures on the small-scale motions throughout the shear layer. These studies culminated with an empirical predictive model, proposed by Mathis et al., which expresses the statistical properties of the near-wall layer in terms of a “universal” (generic) small-scale field and convective shifts and amplitude modulation provoked by large-scale motions that are associated with structures in the outer portion of the log-law region. Alongside these experimental investigations, DNS studies by Lozano-Durán and Jiménez, Cosсу and Hwang, Chung and McKeon, Schlatter and Örlü, Agostini and Leschziner, and Lee and Moser have examined similar large-scale/small-scale interactions and/or the wall-normal hierarchical eddy scales relating to Townsend’s attached-eddy hypothesis. While these computational studies have inevitably been constrained to lower Reynolds numbers than those possible in experimental measurements, they have nevertheless confirmed, reinforced, and augmented experimental observations and findings for
example, those relating to wall-normal variations in the correlation between large-scale motions and
the energy of small-scale turbulence.

The notion of “two kinds of structures [that] are different enough for the elementary structures of
the viscous layer to spend most of their ‘lives’ in an environment defined by a single large-scale
modulations” — a quote of Jiménez[1] — has recently led Chernyshenko et al.[20] to propose a
Quasi-Steady (QS) theoretical description, which is based on the supposition that the statistics of
the near-wall small-scale turbulence is “universal,” if scaled by the spatially and temporally local,
footprint-modified wall-friction velocity. As the strength of the footprints rises with the Reynolds
number, universality in the present sense is also equivalent to Reynolds-number independence. The
QS theory starts from expansions of the velocity and turbulence intensity in terms of the large-scale
modified (i.e., unsteady) friction velocity, truncated after the linear term, and it ends with theoret-
ically derived wall-normal correlations, on the one hand, between large-scale motions at any $y^+$
location and an outer location at which the large-scale structures are most intense (representing
“footprinting”), and on the other hand, between the amplitude-modulated small-scale intensity at
any $y^+$ location and fluctuations in the large-scale friction velocity, or in the large-scale velocity
itself (representing “modulation”). At the time of writing, Chernyshenko et al. use DNS data of
Agostini et al.,[18,21,22] at $Re_e = 1020$, to validate the theory, subject to a particular filtering method
they use for separating the large-scale and small-scale motions. Contemporaneously, the present
authors, Agostini and Leschziner,[18,23] analysed the same data set in an effort to shed light on the
validity of the QS description, offering cautious support for the theory, subject to a significant level
of uncertainty posed by severe limitations in the availability of wall-normal data.

The present study makes use of DNS data for channel flow $Re_e \approx 4200$, obtained by Lozano-
Durán and Jiménez,[12,13] to examine the extent to which the QS concept underpinning theoretical
framework of Chernyshenko et al. is valid. The data are first subjected to a small-scale/large-scale
separation by the use of the empirical mode decomposition (EMD), previously employed by Agos-
tini and Leschziner,[18,23] to analyse near-wall interactions at $Re_e = 1020$. The large-scale field so
extracted then allows footprint-induced skin-friction variations to be examined, and conditional
statistics of small-scale motions to be obtained, demonstrating the extent to which quasi-steadiness
is satisfied.

II. THE DNS DATA AND THEIR ANALYSIS

The DNS data forming the basis of the present study were downloaded from a database
created by Lozano-Durán and Jiménez.[12,13] The DNS was conducted with a spectral code, accord-

ing to well-established quality criteria, over a domain $L_x = 2\pi h$, $L_z = \pi h$, with a grid containing
$3072 \times 3072 \times 1081$ nodes and cell dimensions $\Delta x^+ = 12.8$, $\Delta z^+ = 6.4$, and $\Delta y_{\text{max}}^+ = 10.7$. Down-
loaded data consisted of full-volume snapshot at 13 time levels separated by 3000 wall-scaled time
units.

As noted in Section I, the EMD has been used[24] to separate the large from the small(er)

scales. In essence, the EMD, extended to 2d spatial fields in Agostini and Leschziner,[18,23] and
Agostini et al.,[21] is an algorithm that produces physically meaningful modal representations of
data derived from arbitrary non-stationary or spatially varying processes, including amplitude-
and frequency-modulated 1-d and 2-d signals. The EMD splits any signal into a set of Intrinsic
Mode Functions (IMFs) based purely on the local characteristic time/space scales of the signal.

The method requires no pre-determined functional elements, such as Fourier or wavelet functions.
Rather, the IMFs are the EMD-generated basis functions, which arise purely from the given signal
itself. Unlike Fourier or wavelet-based methods, the EMD does not require filters to separate the
scales and does involve filter-induced loss of energy, and the resulting modes are mutually ortho-
gonal. On the other hand, the number of EMD modes selected to separate scales and algorithmic
details associated with the representation of signal envelopes and iterative stopping criterion influ-
ce the separation of the scales. As will emerge, this separation is, in turn, influential in respect of
identifying the validity of the QS concept: the larger the size with which the large scales are associ-
cated, the lower is their energy and the more faithfully the QS concept is satisfied. It is important,
therefore, to include results here that expose the properties of the separation process, as done previously by the authors for their own DNS data at $Re_{\tau} = 1020$.\(^{18}\)

As in the case at the lower Reynolds number, two-dimensional snapshots of instantaneous velocity fluctuations were decomposed into $N$ intrinsic modes (plus a residual), of which $N - 1$ are held to represent the small scales, while the $N$-th and the residual represent the large scales. The dependence of the scale-separation process on $N$, up to $N_{\text{max}} = 5$, will be discussed below. The velocity field is thus decomposed as $U_i = u_{i,SS} + u_{i,LS} + \langle U_i \rangle_{x,z,t}$, where $u_{i,SS}$ are the small-scale-fluctuation components, $u_{i,LS}$ are the large-scale-fluctuation components, and $\langle U_i \rangle_{x,z,t}$ are the space/time-averaged velocity components.

The application of the EMD (with $N = 4$) to 2D snapshots leads to the typical representation shown in Figure 1(b) for the small-scale streamwise-velocity fluctuations and in 1(c) for the large-scale field, in the latter of which the islands surrounded by the line contours are areas within which the large-scale motions fall within the 10% tails (by area) of the large-scale probability density function (PDF) (note that only one half of the streamwise computational domain is shown in Figure 1). The choice of 10% is arbitrary, and this level is motivated purely by the wish to

FIG. 1. Snapshot of streamwise velocity at $y^+ \approx 12.5$: (a) complete signal, (b) small-scale motions, and (c) large-scale velocity fluctuations: islands with red/blue boundaries identify positive/negative large-scale fluctuations within the extreme 10% tails (by area) of the PDF of the large-scale fluctuations.
explore the response of the conditionally averaged small-scale to the extremes of the large scale footprints. The sensitivity of the physical statements derived from the conditional averaging to the portions of the large-scale PDF tails adopted within the range 5%–40% is discussed in Agostini and Leschziner.\textsuperscript{23}

Statistical properties of the decomposition process are illustrated in Figures 2 and 3. Figure 2(a) shows pre-multiplied spanwise ($\lambda_z^+$) spectra for the streamwise-velocity fluctuations contained in

![Graph showing energy spectra for modes $i=3, 4, 5$ at $y^+=12.5$.]

![Graph showing profiles of small-scale and large-scale energy components for $N=3$, $N=4$, and $N=5$.]

**FIG. 2.** Dependence of the streamwise-fluctuations energy on the choice of EMD modes: (a) pre-multiplied energy spectra of modes $i=3, 4, 5$ ($N=5$ being the maximum number of modes considered) at $y^+=12.5$; (b) profiles of small-scale and large-scale energy components for the choices $N=3$ (chained red lines), $N=4$ (solid black lines), and $N=5$ (dashed blue lines).

![Two-point correlation maps in $x-y$ plane: (a) and (b) large-scale/large-scale correlation relative to $y^+=250$; (a) $N=4$; (b) $N=5$; (c) small-scale/small-scale correlation relative to $y^+=250$, $N=4$; (d) small-scale/small-scale correlation relative to $y^+=12.5$, $N=4$.]

**FIG. 3.** Two-point correlation maps in $x-y$ plane: (a) and (b) large-scale/large-scale correlation relative to $y^+=250$; (a) $N=4$; (b) $N=5$; (c) small-scale/small-scale correlation relative to $y^+=250$, $N=4$; (d) small-scale/small-scale correlation relative to $y^+=12.5$, $N=4$. 

Reuse of AIP Publishing content is subject to the terms at: https://publishing.aip.org/authors/rights-and-permissions. Downloaded to IP: 155.198.175.139 On: Wed, 06 Apr 2016 10:33:21
EMD modes $i = 3 - N$, $N = 5$, at $y^+ = 12.5$. This figure serves to illustrate the progressive shift of the large scales towards higher wavelengths and lower energy with increasing mode count $i$. If $N = 4$ is chosen as the maximum number of modes, the large-scale fluctuations are contained within wavelengths $\lambda^+_z > 500$, with the maximum at $\lambda^+_z \approx 800$, while $N = 5$ implies that the large-scale wavelengths are in the range $\lambda^+_z > 1000$, this value being close to the Fourier cut-off adopted by Mathis, Marusic, and Hutchins.\textsuperscript{11,25,26} Attention is drawn to the fact that the large-scale energy for $N = 5$, say, is not merely that contained within the spectrum for $i = N = 5$, but is the sum of this and any residual beyond that mode.
Figure 2(b) shows the dependence of proportions of small-scale and large-scale streamwise-fluctuation energies as a function of $N$, for $N = 3$–5. As expected, increasing the number of modes $N − 1$ to represent the small-scale motions increases their energy and decreases that of the large scales. For $N = 5$, the small-scale energy profile (dashed blue line) has characteristics that comply with those reported by Marusic$^{25}$ for a range of Reynolds number. This applies, in particular, to the magnitude of the peak and the near-logarithmic variation in the outer region of the profile. It is noted, however, that the shape of the profile in this outer region begins to bulge upwards, indicating the inclusion of a proportion of the scales reasonably deemed to be “large.” Hence, it could be argued that $N = 4$, rather than 5, would be the most appropriate choice. As regards the large-scale energy, one point to highlight is that this energy, whilst elevated across the entire log-layer, reaches a maximum at $y^+ = 12.5$, the location proposed by Marusic et al.$^{11,25}$

The fact that the large-scale energy is high right down to $y^+ ≈ 10$ is indicative of a high level of wall-normal coherence of the large-scale motions, a characteristic that is brought to light in quantitative terms in the two-point-correlation map presented in Figure 3. Figures 3(a), 3(c), and 3(d) arise for $N = 4$ and pertain, respectively, to the $y^+$-wise variations of the correlation coefficients for the large scales at any location $y^+$ relative to $y^+ = 250$, for the small scales relative to the location $y^+ = 250$, and for the small scales relative to $y^+ = 12.5$. The choice of $y^+ = 250$ complies with the location $y^+ = 3.9\sqrt{Re}_τ$ claimed by Marusic et al.$^{11,25}$ to correspond to that at which the most energetic outer structures occur — although, as noted earlier, Figure 2(b) suggests that this position is closer to $y^+ = 500$. Figure 3(b) corresponds to Figure 3(a), but arises from the choice $N = 5$. The abscissa, $Δx^+$, is the $x$-wise distance between the locations for which the correlation coefficient is computed. Figures 3(a) and 3(b) show the correlation between the large scales at $y^+ ≈ 1$ and 250 to be around 0.7 and 0.8, respectively, with the lag in the locus of maximum correlation being around $Δx^+ ≈ 1000$. The higher near-wall value of the correlation coefficient for $N = 5$ indicates that footprinting is especially pronounced and highly coherent in the case of the largest outer scales. In contrast, the correlation between small-scale motions is highly localized around the reference $y^+$ location, while the $x$-wise elongation of the contours suggests that the small-scales are correlated over a distance of $Δx^+ ≈ 700$. Finally, Figure 3(d) demonstrates that the correlation between small-scale motions in the buffer layer, $y^+ = 12.5$, and those below it remains high throughout the viscous sublayer, but diminishes quickly above the buffer layer.

Figure 4 shows pre-multiplied cross-flow spectra in terms of the spanwise wavelength $λ_z^+$. Figure 4(a) relates to the total-fluctuation field, while Figures 4(b)–4(e) arise from the EMD decomposition the first two obtained with $N = 4$ and the last two with $N = 5$. In each of the two rows, the lowest $N − 1$ modes are included in the left-hand-side plot, while the sum of the $N$th mode

![Figure 5](image-url)
and the residual are included in the right-hand-side plot. Both sets of spectra, for \( N = 4 \) and 5, convey a fairly clean separation of the large scales from the small scales, the latter showing a peak in the buffer layer at the generally accepted streak-separation length \( \lambda_z^+ \approx 100 \), while the former is concentrated in the outer region of the flow, within the range 0.2-0.4 times the channel half-height, which is broadly in accord with observations by Agostini and Leschziner\(^{18,23} \) for \( Re_\tau = 1020 \).

In line with the spectra in Figure 2(a), an increase in \( N \) leads to a larger proportion of the scales, in terms of their wavelength and also energy, being included in the small-scale spectra, whilst being removed from the large-scale spectra. As noted already, by reference to Figure 2(b), the spectra at \( N = 5 \) suggest that an excessive proportion of intermediate-to-large-scale motions is interpreted as belonging to the small-scale field, despite the fact that a cut-off wavelength at \( \lambda_z \) in excess of 1000 is the level advocated by Mathis \textit{et al.}\(^{26} \) as being appropriate. In previous studies by the present authors,\(^{18,23} \) \( N = 4 \) was chosen for \( Re_\tau = 1020 \), but for that relatively low Reynolds number, \( N = 5 \) would emphatically have implied an excessively high cut-off value.

### III. QUASI-STEADY-TURBULENCE CHARACTERISTICS

With the results in Section II claimed to justify the validity of the large-scale/small-scale separation process, attention is turned next to an investigation of the QS concept. As explained in Section I, the approach taken here is to sample small scales across two-dimensional (\( x-z \) plane) snapshots, conditional on areas in which the large-scale velocity fluctuations are highly positive or

![Graphs](https://example.com/graphs.png)

**FIG. 6.** Profiles of streamwise second moment in patches of extreme positive (red) and extreme negative (blue) regions of large-scale velocity fluctuations: (a) and (b) moment scaled with mean friction velocity; (a) \( N = 4 \); (b) \( N = 5 \); (c) and (d) conditionally averaged moment scaled with space/time-varying friction velocity associated with large-scale footprints; (c) \( N = 4 \); (d) \( N = 5 \).
highly negative, the cut off being at ±10% (by area) within the tails of the large-scale PDF. The QS proposal is thus deemed to apply if the statistics, normalized by the local friction velocity (within the ±10% patches), collapse. Results are given in Figures 5-11, for both EMD decomposition choices $N = 4$ and 5, so as to convey the sensitivity of the validity of the QS concept to constraints applied to the size and energy of the large-scale modes.

Figures 5–7 show, respectively, mean-velocity, streamwise normal stress, and Reynolds-stress profiles. The profiles in blue and red always relate to low-speed and high-speed large-scale fluctuations, respectively. A focus on the mean velocity, Figure 5, suggests that the QS concept is valid up to around $y^+ \approx 200–300$—i.e., well into the log-law region. However, the Reynolds-stress profiles suggest a considerably more restricted applicability, the degree of correspondence with the QS concept depending only modestly on $N$. For each of the two $N$ values, any stress is shown twice: one is scaled with the mean friction velocity and the other scaled conditionally with the local/instantaneous large-scale shear velocity.

Except for the streamwise normal stress, the collapse of the locally scaled stress profiles extends only to $y^+ \approx 20–30$, the latter value pertaining to $N = 5$. In contrast, the streamwise normal stress displays a good level of collapse across the buffer layer and into the log-layer, up to $y^+ \approx 80$ and 100, for $N = 4$ and 5, respectively. For $N = 5$, it is noticeable that the profile of the streamwise moment relating to negative large-scale fluctuations displays a marked elevation around $y^+ \approx 200–300$, and this justifies the views, expressed earlier by reference to the spectra, Figure 2, as well profiles of streamwise small-scale energy, Figure 4(d), that an excessive proportion of the large-scale structures contributes to the small-scale field.
FIG. 8. Conditionally averaged PDFs of small-scale fluctuations scaled with space/time-varying friction velocity in patches of extreme positive (red) and extreme negative (blue) regions of large-scale footprints, $y_{LS}^+ = 8$: (a) $u_{SS}$ PDFs, $N = 4$; (b) $u_{SS}$ PDFs, $N = 5$; (c) joint $u_{SS} - v_{SS}$ PDFs, $N = 4$; (d) joint $u_{SS} - v_{SS}$ PDFs, $N = 5$; (e) joint $u_{SS} - w_{SS}$ PDFs, $N = 4$; (f) joint $u_{SS} - w_{SS}$ PDFs, $N = 5$. PDF contours correspond to 0.1–0.9 of the PDF height at constant increment 0.1, subject to total PDF volume normalized to 1.

An extended view of the QS characteristics is offered by the PDFs shown in Figures 8–10 and obtained for $N = 4$ and 5. Each figure relates to a particular value of $y^+$ and includes two sets of PDFs, one set for $N = 4$ and the other for $N = 5$. Any one set comprises one-dimensional PDFs for $u_{SS}^+$ and joint $(u_{SS}^+ - v_{SS}^+)$ and $(u_{SS}^+ - w_{SS}^+)$ PDFs, all fluctuations being conditionally scaled with the local large-scale friction velocity. It is important to point out here that any pair of PDFs, for positive and negative large-scale fluctuations, was derived at one and the same value of the wall...
距离与大尺度修正摩擦速度 $u_{r,LS}$ 相比，它们不具有相同的物理 $y$ 值（大尺度摩擦速度的波动主要在大约 $\pm 20\%$ 范围内）。另外，使用 skewness PDF 作为其特性的一个指标，它提供了壁面法向的分布图，图 11 中给出了 $N = 4$ 和 $5$ 时的两个分布图。

总体而言，PDFs 在近壁区域 ($y^+ < 30$) 的合并情况良好，尤其是 $N = 5$。同样，skewness 分布图的合并情况更好，$N = 5$。观测到的与 $N$ 的依赖性是预期的，因为大尺度对于 $N = 5$ 是不仅大，而且具有更长的时间尺度，使得小尺度有效地调整到大尺度的变化。

图 9。图 8 的图注，但 $y^+ = 12.5$。
For $N = 4$, significant differences in the PDFs for positive and negative large-scale fluctuations arise beyond the lower portion of the viscous sublayer. In particular, the skewness of the $u'_{SS}$ PDF associated with positive large-scale motions is substantially higher. The physical implications of this bias, and the interactions at play are discussed extensively in Agostini and Leschziner\textsuperscript{27} and are not repeated here in detail. In essence, the high negative skewness reflects an increased level of “splatting” of small-scale motions at the wall due to large-scale sweeping motions. This splatting is implied by more pronounced pear-shaped distortions in the $(u'_{SS} - w'_{SS})$ PDF, indicating higher spanwise fluctuations associated with positive $u_{SS}$ fluctuations, i.e., sweeping motions. Differences are smaller in the case of $N = 5$ mainly because the large-scale motions are weaker and hence also

FIG. 10. See caption of Figure 8, but for $y^+ = 30$. 
FIG. 11. Conditionally averaged wall-normal skewness profiles of PDFs of small-scale streamwise fluctuations in extreme positive (red) and extreme negative (blue) regions of large-scale velocity fluctuations: (a) scaled with mean friction velocity, $N = 4$; (b) scaled with mean friction velocity, $N = 5$; (c) scaled with space/time-varying large-scale friction velocity, $N = 4$; (d) scaled with space/time-varying large-scale friction velocity, $N = 5$.

the associated sweeps and ejections. In other words, a wider range of larger scales increasingly contribute to the small-scale PDFs, and the dependence of the small-scale PDFs on the large-scale motions is less pronounced.

IV. CONCLUSIONS

Overall, the study has demonstrated that the QS hypothesis has a fair level of validity in the near-wall layer. The extent of this validity depends, however, on the statistical property in question and the length-scale cut-off—enforced or implied—separating the small-scale from the large-scale structures.

The QS hypothesis fares especially well in respect of the streamwise velocity and streamwise energy, extending to $y^+ \sim O(80)$, but its applicability is much more restricted when the focus is on other second moments and PDFs, in which case the hypothesis applies to the viscous and the buffer layers only.

As expected, the QS representation improves with a progressive restriction of the range of large scales towards larger wavelengths — in which case, these scales also change more slowly, allowing the smaller scales to adjust more readily to large-scale fluctuations. The problem with progressively restricting the large-scale motions to larger length-scale values is that an excessive proportion of medium-to-large-scale structures is then included in the small-scale spectrum, and this tends to blur the distinction between large and small. Moreover, as the range of large-scales is reduced and
ACKNOWLEDGMENTS

The authors are very grateful to Professor J. Jiménez, Dr. A. Lozano-Durán, and Dr. J. A. Sillero for sharing their DNS database.

The authors would also like to thank Professor S. Chernyshenko and Mr. Chi Zhang for their useful input during discussions of this work.

8 I. Jacobi and B. McKeon, “Phase relationships between large and small scales in the turbulent boundary layer,” Exp. Fluids 54, 113 (2013).