Maximum Performance of Piezoelectric Energy Harvesters When Coupled to Interface Circuits

Lindsay M. Miller, Alwyn D. T. Elliott, Student Member, IEEE, Paul D. Mitcheson, Senior Member, IEEE,
Einar Halvorsen, Member, IEEE, Igor Paprotny, Member, IEEE, and Paul K. Wright

Abstract—This paper presents a complete optimization of a piezoelectric vibration energy harvesting system, including a piezoelectric transducer, a power conditioning circuit with full semiconductor device models, a battery and passive components. The optimization is done within a framework, which models the combined mechanical and electrical elements of a complete piezoelectric vibration energy harvesting system. To realize the optimization, an optimal electrical damping is achieved using a single-supply pre-biasing circuit with a buck converter to charge the battery. The model is implemented in MATLAB and verified in SPICE. The results of the full system model are used to find the mechanical and electrical system parameters required to maximize the power output. The model, therefore, yields the upper bound of the output power and the system effectiveness of complete piezoelectric energy harvesting systems and, hence, provides both a benchmark for assessing the effectiveness of existing harvesters and a framework to design the optimized harvesters. It is also shown that the increased acceleration does not always result in increased power generation as a larger damping force is required, forcing a geometry change of the harvester to avoid exceeding the piezoelectric breakdown voltage. Similarly, increasing available volume may not result in increased power generation because of the difficulty of resonating the beam at certain frequencies whilst utilizing the entire volume. A maximum system effectiveness of 48% is shown to be achievable at 100 Hz for a 3.38-cm³ generator.

Index Terms—Energy harvesting, piezoelectric transducers, optimization, vibration-to-electric energy conversion.

I. INTRODUCTION

ENERGY can be harvested from environmental vibrations using piezoelectric [1], [2], electromagnetic [3], [4], or electrostatic [5] transduction to couple the mechanical and electrical domains. Because the power requirements of electronics are decreasing [6], [7] and the number of wireless electronic devices used in everyday life is increasing, there is a rapidly growing interest in energy harvesters as a continuously available power supply to recharge a secondary battery. One of the main application areas of interest is in powering wireless sensor network nodes.

The harvested energy can be stored in a battery or capacitor, which serves as a buffer between the energy harvester’s supply and the load’s demand. The energy harvester’s a.c. output voltage must be converted to d.c. and conditioned to charge the energy storage element. A number of informative studies have been previously presented that focus on the design and behaviour of the energy harvesting transducer with an idealized impedance-matched resistive load [8]–[10], or that focus on power electronics while assuming a fixed set of idealized transducer parameters [11]–[13]. However, energy harvesting systems inherently couple the electrical and mechanical domains and so the design and behaviour of each system affects that of the other.

In order to design an energy harvesting system that effectively converts the maximum possible mechanical energy into electrical energy, it is necessary to have a model that incorporates the electromechanical transducer, the power conditioning electronics and the energy storage element. Additionally, it is not possible to determine which type of energy harvesting transducer is best to use (electrostatic, electromagnetic, or piezoelectric), given a specific input vibration source and available volume, without performing a full system analysis that allows a fair comparison of the three technologies.

One of the first examples of a full system model in the literature presents a piezoelectric harvester coupled to a pulsed resonant converter circuit [14]. However, this paper does not conduct an optimization of the harvester or the circuit parameters. A detailed study of electrostatic energy harvesting systems was presented in [15], in which the maximum system effectiveness (usable power output / maximum possible power generation within volume) was determined for a given input excitation and available volume.
In this paper we present a model that captures the coupled behaviour of a piezoelectric energy harvester and the power conditioning circuitry with full semiconductor device models, allowing optimal system design in piezoelectric energy harvesting for the first time. Given an allowed device size and the expected input excitation, the model enables a user to choose harvester device dimensions and circuit component values for optimal power output. In conjunction with [15] the model presented here provides a way to determine whether a piezoelectric or electrostatic energy harvester system performs better in a given operating environment.

The rest of this paper is organized as follows. Section II presents background information on the methodology, the mechanical, piezoelectric, and circuit definitions, and the model parametrisation. Sections III through V present the key equations describing the piezoelectric transducer, power conditioning circuit, and energy storage circuit, while section VI discusses the system effectiveness. Details on those derivations are in Appendices A and B. Finally, simulation and results are discussed in sections VII and VIII, followed by conclusions. A list of terms can be found in Appendix C.

II. BACKGROUND

A. Scope of Analysis

The goal of this study is to determine a piezoelectric energy harvesting system’s maximum power output, and corresponding effectiveness and system parameters as a function of input acceleration, frequency and system volume. The optimisation maximises output power over a range of mechanical configurations and transistor areas. The SSPB circuit is used as the power conditioning circuit since it was determined in [16] to be the most efficient implementation to date which performed the needed operations to achieve optimally controlled Coulomb-damping for electrical power extraction. It was shown to achieve twice the theoretical power output compared with the next best scheme and a practical extraction. It was shown to achieve twice the theoretical power output compared with the next best scheme and a practical extraction. It was shown to achieve twice the theoretical power output compared with the next best scheme and a practical extraction. It was shown to achieve twice the theoretical power output compared with the next best scheme and a practical extraction. It was shown to achieve twice the theoretical power output compared with the next best scheme and a practical extraction. It was shown to achieve twice the theoretical power output compared with the next best scheme and a practical extraction. It was shown to achieve twice the theoretical power output compared with the next best scheme and a practical extraction. It was shown to achieve twice the theoretical power output compared with the next best scheme and a practical extraction. It was shown to achieve twice the theoretical power output compared with the next best scheme and a practical extraction. It was shown to achieve twice the theoretical power output compared with the next best scheme and a practical extraction. It was shown to achieve twice the theoretical power output compared with the next best scheme and a practical extraction. It was shown to achieve twice the theoretical power output compared with the next best scheme and a practical extraction. It was shown to achieve twice the theoretical power output compared with the next best scheme and a practical extraction. It was shown to achieve twice the theoretical power output compared with the next best scheme and a practical extraction. It was shown to achieve twice the theoretical power output compared with the next best scheme and a practical extraction. It was shown to achieve twice the theoretical power output compared with the next best scheme and a practical extraction. It was shown to achieve twice the theoretical power output compared with the next best scheme and a practical extraction. It was shown to achieve twice the theoretical power output compared with the next best scheme and a practical extraction. It was shown to achieve twice the theoretical power output compared with the next best scheme and a practical extraction.

B. Definitions of Terms

The following terms have been defined before in the literature and are used throughout this paper.

The energy harvesting system effectiveness is given by

$$\eta_{\text{sys}} = \eta_{\text{coup}} \times \eta_{\text{extraction}} \times \eta_{\text{conv}} = \frac{E_{\text{out}}}{E_{\text{max}}}, \quad (1)$$

where the coupling effectiveness, $\eta_{\text{coup}}$, extraction efficiency, $\eta_{\text{extraction}}$- and conversion efficiency, $\eta_{\text{conv}}$ are

$$\eta_{\text{coup}} = \frac{E_{\text{coup}}}{E_{\text{max}}}, \quad \eta_{\text{extraction}} = \frac{E_{\text{harv}}}{E_{\text{coup}}}, \quad \eta_{\text{conv}} = \frac{E_{\text{out}}}{E_{\text{harv}}}. \quad (2)$$

$E_{\text{coup}}$ is the energy generated by the piezoelectric transducer as a result of coupling to the mechanical input vibrations, $E_{\text{harv}}$ is the energy harvested after the losses of the interface circuit are accounted for, $E_{\text{out}}$ is the final output energy from the system in to the energy storage element, and $E_{\text{max}}$ is the opportunity energy available from the acceleration and frequency of the vibration source for a given available volume and proof mass density. Most studies on energy harvesting focus on maximizing only one of the ratios in (2), but it is necessary to maximize the product of these in order to obtain the highest system effectiveness.

It should be noted that effectiveness and efficiency are different concepts [18]. Efficiency is defined as the output energy divided by the actual input energy, whereas effectiveness is the output energy divided by the opportunity energy, $E_{\text{max}}$. Note that maximising the power output for piezoelectric energy harvesting systems will coincide with maximising the effectiveness in the case that the volume and acceleration are given.

C. Mechanical Structure of the Harvester

Piezoelectric energy harvesters are typically modelled as velocity damped resonant generators (VDRGs) [19] when they are connected to resistive or bridge rectifier loads, but when they are connected to certain types of power conditioning circuits, including the SSPB circuit considered here, the piezoelectric harvester behaves like a Coulomb damped resonant generator (CDRG) [20]. Operating the piezoelectric harvester with an SSPB circuit rather than a rectifier is advantageous because the optimal electrical damping at resonance cannot be achieved in many cases with a bridge rectifier due to the low electromechanical coupling factor [16]. The CDRG modeling framework is well-known [19], [21], enabling coupled analysis of the transducer and power electronics.

The generic mass-spring-damper system used to model a CDRG is shown in Fig. 1. The motion of the mass is constrained within a cubic box with side length $S$, as shown in Fig. 2. The mass is assumed to be symmetric about the beam. Gold is assumed to be the proof mass material because of its high density and MEMS compatibility. The width of the beam and mass are assumed to be equal to the width of the available box side length, $S$. The proof mass thickness, $H_m$, is set at $S/2$ such that the proof mass occupies close to half of the volume available in the cube, which is the optimal fraction for resonant
operation in the displacement constrained case [19]. There is a trade-off between beam length, $L_b$, and mass length, $L_m$, since they must sum to a length of $S$. The trade-off also affects the beam spring constant, $K_0 = \frac{m_0 \omega^2}{2}$. The model used in this study accounts for rotation of the the proof-mass through an angle, $\theta$, to a maximum possible deflection of $Z_0$, which is defined as the maximum vertical displacement of the centre of mass.

The piezoelectric laminar composite beam serves as the spring in this system. The beam layers, from bottom to top, consist of a silicon layer of variable thickness, $t_s$, an oxide layer 1 $\mu$m thick, $t_o$, and a piezoelectric layer 1.5 $\mu$m thick, $t_p$. This study assumes an aluminium nitride thin film piezoelectric layer that covers the entire beam except for the mass area. The mechanical contribution of the electrodes, sandwiching the piezoelectric layer, is neglected as the electrodes are several orders of magnitude thinner than the other layers. In this study it is assumed that the mass spring damper system operates at resonance and is vacuum packaged such that it is reasonable to neglect viscous air damping [6].

D. Single-Supply Pre-Biasing Circuit

The SSPB circuit allows a piezoelectric harvester to operate with a controlled Coulomb-damping force. There is, however, an overhead in terms of circuit losses and control power and, therefore, the SSPB circuit is likely to produce an efficient harvester system when the required electrical damping is greater than what can be achieved with other interface circuits (e.g. a simple rectifier circuit). The schematic for the SSPB circuit is shown in Fig. 3. The operation of the circuit is described in detail in [16]. The current source, $I_0$, (charge displacement due to strain in the piezoelectric material) and capacitor, $C_0$, (capacitance of the piezoelectric material) in parallel represent the electrical port from the piezoelectric transducer. The mechanical part of the harvester as an equivalent circuit is not shown here but can be found in [2].

The circuit components to the left of the dashed line are the SSPB circuit, and those to the right of the dotted line are the buck converter circuit. The DC bus capacitor $C_{\text{int}}$ is assumed to be at least an order of magnitude larger than $C_0$ so that the voltage on $C_{\text{int}}$ can be considered constant over one cycle of the beam’s motion. The voltage across $C_{\text{int}}$ is referred to as $V_{\text{cc}}$, the supply voltage, throughout this study. The voltage across $C_0$ varies as the piezoelectric transducer moves through its cycle.

A minimum of six MOSFETs are required to achieve the necessary conduction and blocking states, but only three MOSFETs are blocking at any given time. For example, MOSFETs $M1$, $M2$, and $M6$ will block at the same time if $V_{\text{PB}} > V_{\text{cc}}$ and $V_{\text{PB}} > 0$, where $V_{\text{PB}}$ is the pre-bias voltage given in (5), while $M1$, $M3$, and $M6$ will be blocking if $0 < V_{\text{PB}} < V_{\text{cc}}$. In this study it is assumed that all of the MOSFETs have the same semiconductor cross sectional area, $A_{\text{semi}}$, because they all conduct the same peak and average currents, however they are designed for different blocking voltages.

It is assumed that the same volume, $S^3$, is allocated to the circuit as was allocated to the transducer. This volume is mostly taken up by inductors and is divided evenly among $L$ and $L_{\text{back}}$. The battery itself, $V_{\text{batt}}$, for energy storage is not included in system volume since it doesn’t affect harvester power density and is dependent on application.

In order to make the model as realistic as possible, three types of losses are taken into account in the SSPB circuit. First, losses occur as charge redistributes onto MOSFET parasitic capacitances when switches flip state, thus altering the charge on $C_0$. Second, there is a leakage current associated with any MOSFET that is blocking voltage. Third, resistive conduction losses in the inductors and MOSFETs must be accounted for. As switching always occurs at zero current due to resonant operation of the circuit, switching losses in the MOSFETs are neglected.

E. Energy Storage

As mentioned in section II-D, the portion of the circuit in Fig. 3 to the right of the dotted line is a buck converter whose task is to convert the voltage output from the SSPB circuit to the proper level to charge the battery, $V_{\text{batt}}$, which is assumed to be a 1.5 V battery in this study.

The switches that allow current to flow from $C_{\text{int}}$ to $V_{\text{batt}}$ are assumed in this study to operate once per cycle, which is a conservative assumption. In reality, they may be operated less frequently if the voltage on $C_{\text{int}}$ changes little per vibration period.

III. Model Parametrisation: Electrical Damping

As previously described, the mechanical behaviour of a piezoelectric harvester connected to an SSPB circuit is that of a CDRG. The damping force for the CDRG is set by the
pre-bias voltage [20]. Hence the goal of this section is to determine the pre-bias voltage that must be applied to the piezoelectric beam to secure the greatest possible deflection of the proof mass within the confines of the volume. The magnitude of that ideal deflection is found by numerically solving equation (3), which is derived from the geometric constraints shown in Fig. 2, for \( Z_1 \).

\[
S = \frac{Z_1}{2} + \sqrt{\frac{L_m^2}{4} + \frac{H_m^2}{4} \sin \left( \theta_{11} Z_1 + \tan^{-1} \left( \frac{H_m}{L_m} \right) \right)}
\]  

(3)

where \( S \) is length of box side, \( Z_1 \) is the maximum possible vertical displacement of the centre of the mass within the confines of the package, \( L_m \) is length of the mass, \( H_m \) is thickness of the mass, and \( \theta_{11} \) is the rotation angle per unit vertical displacement from the neutral position, as shown in Fig. 2. The maximum angle of rotation from the neutral position is \( \theta \), which is equal to \( Z_1 \theta_{11} \).

From [20], the optimal Coulomb damping force on a displacement constrained CDRG harvester operating at resonance is

\[
F_{optCZ} = \frac{\pi}{4} m \omega_{\text{input}} Z_1 Y_0 = \frac{\pi}{4} m A_{\text{input}},
\]

(4)

where \( m \) is mass, \( \omega_{\text{input}} \) is input frequency, \( Y_0 \) is input vibration amplitude, and \( A_{\text{input}} \) is input acceleration. The pre-bias voltage, \( V_{PB} \), needed to obtain this damping force is

\[
V_{PB} = \left( \frac{\pi}{4} m A_{\text{input}} - \frac{\Gamma^2 Z_1}{C_0} \right) \frac{1}{\Gamma}
\]

(5)

where \( \Gamma \) is the transduction factor [20]. This expression allows the optimal damping to be set based on the harvester’s design.

It is possible for the optimal value of pre-bias voltage, \( V_{PB} \), to be negative if acceleration is small, but \( V_{PB} \) may not be less than \(-|\Gamma| Z_1 / C_0\) or the system becomes a forced oscillator rather than an energy harvester. Fig. 4 illustrates the difference in the piezoelectric voltage waveform for positive and negative pre-bias voltages. In the case of positive pre-bias voltage, the voltage on the piezoelectric beam does not change sign during a half-cycle, while in the case of negative pre-bias voltage it always changes sign.

![Fig. 4. The difference between waveforms when a positive versus negative pre-bias voltage is required.](image)

IV. CALCULATION OF \( E_{\text{coup}} \) AND \( E_{\text{harv}} \)

In this section, detailed circuit equations are presented which allow the coupling effectiveness and extraction efficiency to be calculated as a function of the system mechanical parameters and semiconductor device area.

In order to calculate the coupling effectiveness and extraction efficiency for the SSPB circuit, equations describing the piezoelectric voltage waveform must be derived, accounting for circuit non-idealities, such as transistor parasitic capacitance and leakage currents.

The steps of circuit operation (Fig. 5) are as follows: 1) the beam approaches its maximum displacement under open circuit conditions; 2) one set of switches close briefly to discharge the piezoelectric’s capacitor, \( C_0 \), when the beam reaches its maximum; 3) all switches open briefly; 4) the other set of switches close briefly to pre-bias \( C_0 \); 5) \( C_0 \) is in open circuit conditions as the beam moves through the half-cycle to its minimum displacement; and 6) same set of switches as in step 4 close briefly to discharge \( C_0 \) into \( C_{\text{int}} \) therefore extracting electrical energy. Thus the piezoelectric beam is in open circuit conditions most of the time, with switches flipping only when the beam reaches its extremes of travel. A detailed description of the circuit operation can be found in [16].

There is an additional step in this cycle that is advantageous only if the voltage remaining on \( C_0 \) after step 2, called \( V_{\text{rem}} \), is of the opposite sign as \( V_{PB} \), as depicted in Fig. 5. This extra step is a very brief short circuit that occurs between steps 3 and 4 to bring the voltage on \( C_0 \) to zero and is called a “forced return to zero” (FRTZ). It is beneficial in this case because it reduces the energy required to pre-bias the piezoelectric capacitor during step 4. It can be seen in Fig. 5 that if \( V_{\text{rem}} \) is negative, shorting the piezoelectric capacitor to zero before pre-biasing it requires more energy and is therefore not beneficial. The FRTZ operation is described in detail in [22].

Fig. 5 also illustrates two non-idealities that result in energy losses. The first is a difference between the ideal and actual pre-bias voltage: the pre-bias voltage that was determined in equation (5) will be called \( V_{PBend} \), which is the voltage...
on the piezoelectric beam as it begins its half cycle motion. In contrast, \( V_{PBstart} \) is the pre-bias voltage required in order that \( V_{PBend} \) remains on \( C_0 \) after charge redistribution when the switches flip state.

The second is a difference between ideal and actual generated piezoelectric voltage. The ideal voltage generated by the piezoelectric beam during its motion over a half cycle is \( V_{po} = \frac{\Delta Z}{C_0} \). However leakage currents and charge sharing with device capacitances reduces this voltage. Note that \( V_{end,ideal} \) in the figure is defined as \( V_{PBend} + 2V_{po} \).

We will now proceed to calculate \( V_{PBstart}, V_{PBend}, V_{end,actual} \) and \( V_{rem} \) as a function of circuit parameters. If \( V_{PBstart} \) and \( V_{PBend} \) were identical then the required \( V_{cc} \) needed to supply the appropriate pre-bias is

\[
V_{cc} = 2\gamma^2V_{po} + V_{PBend}(1 + \gamma^2)
\]

(6)

where \( \gamma = \exp(-\pi/2Q) \) and \( Q \) is the Q-factor of the RLC resonant charging path. The derivation of (6) is given in Appendix A. In reality \( V_{PBstart} \) and \( V_{PBend} \) are not the same due to charge sharing and in addition \( V_{po} \) is also non-ideal due to charge sharing and charge leakage. Consequently (6) is used as a starting point to iterate a time domain simulation which calculates true values for \( V_{PBstart}, V_{PBend}, V_{end,actual}, V_{rem} \) and \( V_{cc} \).

Note that by increasing the circuit Q-factor by increasing the semiconductor area to reduce resistance does not indefinitely improve circuit performance because it increases parasitic capacitances, which can reduce system output power. If the iteration results in a negative \( V_{cc} \), the solutions are discarded because the system is then a forced oscillator.

The voltages that the MOSFETs must be able to block are given in [17] as

\[
\begin{align*}
V_{B,HiP} &= 2V_{po} + V_{PBend} \\
V_{B,LoP} &= 2V_{po} + V_{PBend} - V_{cc} \\
V_{B,LoN} &= V_{cc}
\end{align*}
\]

(7)

where HiP, LoP, and LoN refer to the high and low side \( p^- \) and \( n^- \) type MOSFETs respectively.

The MOSFET on-state resistance is

\[
R_{on} = \frac{k_{epi}V_B^2}{A_{semi}},
\]

(8)

where the resistance of the high or low voltage \( p^- \) or \( n^- \) type MOSFET is found by using the appropriate \( k_{epi} \) constant and \( V_B \) term [23].

The sum of the on-state resistance of the all MOSFETs in the RLC resonant path is denoted \( R_{mos} \). The inductor’s resistance, \( R_L \), is specified through the ratio \( c = R_L/R_{mos} \) and its value is modelled by the Brooks coil form [24]

\[
L = K_LR_LV_L^{\frac{3}{2}}
\]

(9)

where \( K_L \) is a constant and \( V_L \) is the inductor voltage. The Q-factor of a series RLC circuit is \( Q = \sqrt{L/C_0/(R_{mos} + R_L)} \) which, by use of the definitions above, can be solved with respect the semiconductor area to give

\[
A_{semi} = \left( \frac{(1 + c)^2Q^2C_0}{cK_LV_L^{2/3}} \right) \sum (k_{epi}V_B^2)
\]

(10)

where the summation accounts for the different resistances of each MOSFET due to differing required blocking voltages and differences in hole and electron carrier mobilities. The constants used are \( k_{epiN} = 2 \times 10^{-11} \Omega m^2V^{-2}, k_{epiP} \approx 3k_{epiN}, K_L = 0.23 H \Omega^{-1}m^{-2} \) and \( c = 1 \) [15], [23]. Both the SSPB inductor and the buck inductor are assumed to occupy half of the volume, \( V_L = 0.55^3 \) each.

Using the above expressions, the losses that will be incurred during operation of the system can now be calculated, which are associated with three effects: leakage current through the MOSFETs in the off-state; charge sharing due to parasitic capacitance when the MOSFETs are switched; and on-state conduction loss. The leakage current is

\[
l_I = k_{lj}A_{semi}((V_0 - V_{operation})V_B)^{\frac{1}{2}},
\]

(11)

where \( k_{lj} = 3.9 \times 10^{-4} \Omega m^2V^{-1} \), \( V_0 \) is the diode threshold voltage of 0.7 V, and \( V_{operation} \) is the reverse bias voltage across the diode. The parasitic capacitance is given by

\[
C_j = k_{cj}A_{semi}((V_0 - V_{operation})V_B)^{\frac{1}{2}},
\]

(12)

where \( k_{cj} = 1.1 \times 10^{-3} \text{Cm}^{-2} \) [15], [23]. Equations (11) and (12) can also be found in [23].

The general form of the equations used to calculate energy losses associated with leakage, charge sharing and conduction are respectively

\[
\begin{align*}
E_{Iloss} &= l_1V_{cc}\Delta t \\
E_{Closs} &= Q_JV_{cc} \\
E_{Rloss} &= I_2^2(R_{mos} + R_L)\Delta t
\end{align*}
\]

(13)-(15)

where \( Q_J \) is charge on the MOSFET, \( I_1 \) is current in the inductor and \( \Delta t \) is the on-state conduction time.

Now that the leakage current and the junction capacitances have been parametrised as in (11) and (12), the imperfections to the idealised SSPB waveform (both illustrated in Fig. 5) can be calculated for any chosen value of semiconductor area. In other words, \( V_{PBstart}, V_{PBend}, V_{end,actual}, V_{rem} \) and \( V_{cc} \) can now be computed. This calculation is explained in detail in Appendix B.

With all of the voltages of the non-ideal system determined, the next step is to calculate the leakage current that occurs continuously through either MOSFET \( M3 \) or \( M6 \) (see Fig. 3) and the resulting energy loss over 1 mechanical harvester cycle are given by

\[
\begin{align*}
E_{I31} &= k_{lj}A_{semi}((V_0 + V_{cc})V_{B,LoN})^{\frac{1}{2}} \\
E_{I31,loss} &= E_{I31}V_{cc} \\
E_{I31,loss} &= \frac{f_1V_{cc}^2}{2f_1}
\end{align*}
\]

(16)-(17)

where \( f_1 \) is the resonant frequency of the energy harvester. Reverse recovery losses in the anti parallel diodes are zero due to the fact the diodes never conduct.

The energy generated into \( C_{int} \) (Fig. 3) due to mechanical excitation of the harvester can now be calculated. The energy generated on \( C_0 \) as the beam moved is:

\[
E_{coup} = \frac{1}{2}C_0 \left( V_{PBend} + 2V_{po} - V_{PBend}^2 \right).
\]

(18)
The energy used to pre-bias the piezoelectric capacitor is
\[ E_{PB} = C_0 V_{cc} (V_{P_{	ext{Start}}} + V_{\text{rem}}). \] (19)
However, if the FRTZ case is implemented,
\[ E_{PB} = C_0 V_{cc} V_{P_{	ext{Start}}}. \] (20)
The energy generated in discharging the piezoelectric capacitor from \( V_{\text{end}} \) to \( V_{\text{rem}} \) is
\[ E_{\text{extract}} = C_0 V_{cc} (V_{P_{\text{end}}} + 2V_{po} - V_{\text{rem}}) \] (21)
such that the energy harvested, as in energy put back into the power supply per half cycle is calculated as
\[ E_{\text{harv}} = E_{\text{extract}} - E_{PB} - E_{\text{iq, loss}}. \] (22)

V. CALCULATION OF \( E_{\text{out}} \) OF ENERGY STORAGE CIRCUIT
Expressions for the energy harvested have been obtained, but the losses incurred during the charging of the energy storage component through the buck converter need to be accounted for.

There are two phases of operation of the buck converter: the first is with \( M8 \) closed and \( M7 \) open so that the harvested energy is transferred through the inductor to the battery, and the second is with switch \( M7 \) closed and \( M8 \) open so that energy freewheels through the inductor and is disconnected. For steady state operation, the goal is to remove only as much charge from the intermediate capacitor, \( C_{\text{int}} \), into the battery as was put onto the intermediate capacitor during that energy harvesting cycle, a quantity given by
\[ Q_{eq} = \frac{E_{\text{harv}}}{V_{cc}}. \] (23)
The required peak buck inductor current required for steady state operation is
\[ I_{\text{req}} = \left( \frac{2E_{\text{harv}}}{L_{\text{buck}}} \right)^{\frac{1}{2}}, \] (24)
where \( L_{\text{buck}} \) is the value of the inductor in the buck converter circuits, and is found using the following expressions:
\[ R_{L_{\text{buck}}} = \frac{k_{\text{epi,N}}V_{B, Loo}N^2}{A_{\text{semi}}} \] (25)
\[ L_{\text{buck}} = K_1 R_{L_{\text{buck}}} V_I^2, \] (26)
where \( R_{L_{\text{buck}}} \) is the resistance in the inductor of the buck converter circuit.

There are four sources of losses in the buck converter to account for. First, because there is always one MOSFET off, there is loss due to a constant leakage current that is given by twice the value of the expression in equation (17)
\[ E_{\text{iq,loss}} = \frac{2i_{\text{dq}} V_{cc}}{2 f_1}. \] (27)
Second, there is always one MOSFET on, so there is loss due to charge sharing on blocking junctions:
\[ E_{\text{C_{loss}}} = 4k_{c_j} A_{\text{semi}} V_{cc}, \] (28)
which comes from [23]. Third and fourth, there are conduction losses in the inductor and devices during charging and freewheeling to account for. To calculate these losses, it must first be determined at what time the buck converter switches from discharging \( C_{\text{int}} \) into the battery to freewheeling, \( t_{\text{switch}} \), and at what time the current falls to zero during freewheeling, \( t_{\text{f}} \). This is done by first solving the differential equation for current through the inductor during \( C_{\text{int}} \) discharging phase, given by
\[ \frac{di_{\text{ind, ch}}}{dt} = \frac{V_{cc} - V_{\text{batt}}}{L_{\text{buck}}} - \frac{i_{\text{ind, ch}} R_{\text{loss, total}}}{L_{\text{buck}}}, \] (29)
\[ R_{\text{loss, total}} = (1 + \varepsilon) \frac{k_{\text{epi,N}} V_{B, Loo} N^2}{A_{\text{semi}}} \] (30)
where \( R_{\text{loss, total}} \) is the total resistance in the buck converter circuit.

This gives the values for current into the inductor over time, \( i_{\text{ind, ch}} \), which are then compared with the current required for steady state operation, \( I_{\text{req}} \), and the time at which these currents are equal is the time when the switches flip to begin the freewheeling phase. A differential equation for current through the inductor during the freewheeling phase is then solved
\[ \frac{di_{\text{ind, fw}}}{dt} = -\frac{V_{\text{batt}} - i_{\text{ind, fw}} R_{\text{loss, total}}}{L_{\text{buck}}}, \] (31)
and the time at which the current falls to zero in the inductor is determined by observing when the current \( i_{\text{ind, fw}} = 0 \). Then, the total resistive losses are calculated by integrating as follows:
\[ E_{\text{R_{loss, total}}} = \int_0^{t_{\text{switch}}} i_{\text{ind, ch}}^2 R_{\text{loss, total}} dt \]
\[ + \int_{t_{\text{switch}}}^{t_{\text{f}}} i_{\text{ind, fw}}^2 R_{\text{loss, total}} dt. \] (32)

During this calculation, checks are done to ensure that \( V_{cc} > V_{\text{batt}} \), that \( E_{\text{harv}} > 0 \), and that the intermediate capacitor was discharged enough to maintain steady state operation. Subtracting those four energy losses in the buck converter circuit from \( E_{\text{harv}} \) gives the net energy and power generated according to
\[ E_{\text{net}} = E_{\text{harv}} - E_{\text{buck, loss}} - E_{\text{C_{loss}}} - E_{\text{R_{loss, total}}} \]
\[ P_{\text{out}} = 2 f E_{\text{out}}. \] (34)

VI. MODEL PARAMETERIZATION: SYSTEM EFFECTIVENESS AND OUTPUT POWER
Having obtained expressions for energy generated by the piezoelectric transducer, energy required for pre-biasing, and energy losses through all parts of the system, it is now possible to calculate the coupling effectiveness, extraction efficiency, conversion efficiency, and system effectiveness. First, as discussed in [19], the maximum possible theoretical power available to be harvested is given by
\[ P_{\text{max}} = \frac{1}{2} Y_w \omega^2 m Z_l \] (35)
where the following substitutions were used:
\[ A_{\text{input}} = Y_w \omega^2, \quad m = \frac{\rho_{\text{mass}} S^2}{2}, \quad Z_l = \frac{S}{4} \] (36)
The system effectiveness is then found by dividing $P_{\text{out}}$ by this maximum theoretically available power

$$\eta_{\text{sys}} = \frac{P_{\text{out}}}{P_{\text{max}}} = \frac{E_{\text{out}}}{E_{\text{max}}} = \eta_{\text{coup}} \times \eta_{\text{extraction}} \times \eta_{\text{conv}}. \tag{37}$$

Now, combining equation (35) with (18) and (22) (converted from energy to power), and (34), terms are substituted into (2) and (37) to find the coupling effectiveness, conversion efficiency, and system effectiveness. With the system equations now defined, the following sections will discuss simulation methods and results.

### VII. SIMULATION METHOD

The equations were solved using MATLAB and verified in SPICE. The optimisation (maximising $E_{\text{out}}$ and hence maximising effectiveness at each chosen geometry) was performed by sweeping all parameters of interest over all their ranges ensuring the entire state space was searched and consequently guaranteeing that a global optimum was found.

$S$ was varied from 1 to 15 mm since devices smaller than 1 mm do not produce enough power to be of interest and devices greater than 15 mm approach the limits of what makes sense to fabricate with MEMS technology. Piezoelectric harvesters in general can certainly perform very well at size scales larger than 15 mm, but this study is restricted to MEMS fabricated technologies. The reason for scoping the study in such a manner is that the properties and manufacturing processes of bulk piezoelectric ceramics differ greatly than those for piezoelectric thin films, so they should be compared separately.

$A_{\text{input}}$ was varied from 0.01 to 100 ms$^{-2}$, since that range encompasses the regime of many ambient vibration sources [25] and allows comparison with the results of [15]. Circuit inversion efficiency, $\gamma$, was varied from 0.5 to 0.99 because values under 0.5 result in a very poor system performance and above 0.99 are not practical. Additionally, $\omega_{\text{input}}$ was a variable input to the model. Results are shown for 1 Hz, 100 Hz, and 1000 Hz cases, which represent the full limits of the range that occur for ambient vibration sources [8], [25]. The constant parameter values used as inputs to the model are given in Table I.

The algorithm followed the steps detailed in the previous several sections, but are summarized in Table II for convenience. A limit was placed on the maximum voltage allowed on the piezoelectric capacitance based on a value for the dielectric breakdown voltage of 500 MV m$^{-1}$ for an aluminium nitride thin film [26]. If this voltage was exceeded, due to a very high optimal pre-bias voltage required, the mass displacement was reduced, which in turn decreased the induced voltage and required pre-bias voltage.

### VIII. RESULTS AND DISCUSSION

The point of maximum power on a contour plot of $L_b$ and $\gamma$ was found and saved for a single value of box size, $S$ and $A_{\text{input}}$ (not shown). A $15 \times 15$ matrix of $S$ and $A_{\text{input}}$ values was run such that the points of maximum power for all 225 cases could be plotted as a function of $S$ and $A_{\text{input}}$. The points of max power and max $\eta_{\text{sys}}$ plotted as a function of $S$ and $A_{\text{input}}$ were not similar, as shown in the plots. This is because an increase in acceleration or size would usually increase power, however may not necessarily increase system effectiveness (also discussed [27]). This process was conducted for frequencies of 1, 100, and 1000 Hz. The results were verified using SPICE.

The plots in Figs. 6, 7, 8, 9, and 10 show the power output, system effectiveness, coupling effectiveness, extraction efficiency, and conversion efficiency, respectively, at 100 Hz input frequency with a gold proof mass. Table III lists the output values for beam geometry, quality factor, inductor value, and MOSFET semiconductor area that correspond to the optimal configuration at 100 Hz.

Figs. 11, 12, 13, and 14 show power and system effectiveness of the same system operating at 1 Hz and 1000 Hz.
It is clear that the range of box size and accelerations at which the system is functional is greatly diminished at very low or very high frequencies. There are two reasons for the nearly non-existent operating regime at 1 Hz. First, for small values of $S$, no combination of beam length and thickness exists that can satisfy the requirement that the transducer resonant
frequency match the low driving frequency. This prevents any
functional systems until $S$ is greater than approximately 1 cm.
Second, at accelerations above a certain threshold, the system
is unable to provide enough electrical damping to the harvester
to prevent it from hitting the end-stops, which is not allowed
in this model due to the damage this would cause. This is due
in part to the large mass required at low frequencies requiring
a very large damping force.

The 1000 Hz case also has limited functionality for two
main reasons. At low accelerations, a net loss in power occurs
due to the losses in the SSPB and buck circuit caused by
device leakage, capacitive sharing and conduction. As the
level of acceleration increases, the extracted energy increases,
overcoming these losses, and so power can be extracted. A maximum length of $S$ also exists due to no combination
of beam length and thickness existing to resonate at the high
driving frequency. However unlike in the 1 Hz case, the beam
can be assumed shorter than $S$ and therefore will resonate,
however the system effectiveness is severely reduced due to
the underutilisation of the volume.

The limitations discussed in the preceding two paragraphs
apply to the limits in operating regime for the 100 Hz case
as well, but to a far lesser extent. One other factor relating to
the reason for the drop off in system effectiveness and power
generation at large accelerations at 100 Hz is that the limit
for dielectric breakdown voltage of the piezoelectric material
is surpassed, meaning that the displacement of the mass must
be reduced to decrease the piezoelectric induced voltage and
the required pre-bias voltage.

The optimization was run using gold for the mass material
because of its high density (19320 kg m$^{-3}$) and compatibility
with MEMS processing. However, the results with a silicon
proof mass are shown in Table III along side those with
gold for comparison. If cost is a concern, and silicon is
not dense enough to get the desired performance, tungsten
(19300 kg m$^{-3}$) or nickel (8900 kg m$^{-3}$) may be used instead
of gold. As expected, the power output from a transducer with
a gold proof mass is higher than that with a silicon proof mass.
However, system effectiveness from the silicon mass is higher
than the gold, because a greater pre-bias voltage is required to
damp the motion of the heavier proof mass within the confined
volume, and that requires a larger supply voltage and more
energy for the pre-biasing, undermining system effectiveness.

The results for piezoelectric energy harvesting systems
presented here can now be compared with electrostatic systems
from [15], where an analogous study was done. It can be
determined that piezoelectric systems generate higher power
output from a 100 Hz excitation frequency when acceleration
and device size are relatively large ($A_{\text{input}}$ greater than about
0.1 ms$^{-2}$ and $S$ greater than about 5 mm). However, electro-
static systems generate higher power output at $\omega_{\text{input}}$ of 100 Hz
if $A_{\text{input}}$ is low or $S$ is constrained to be small.

IX. Conclusion

A framework was developed in order to investigate the full
system effectiveness of a piezoelectric harvester coupled to a
SSPB circuit and a buck converter, including the semiconductor
device models, and a battery, to maximise power generation
within a specific volume.

A parameter sweep over system geometric dimensions and
circuit inversion coefficient was conducted to find the optimal
value of system parameters for a given input size, $S$, input
frequency, $\omega_{\text{input}}$, and input acceleration, $A_{\text{input}}$. Subsequently,
the size and acceleration were swept while holding frequency
fixed at 1, 100, or 1000 Hz to find the power output and
effectiveness of the energy harvesting system over a range of
operating conditions.

The operating envelope of the system has limits related
to box size, $S$, input acceleration, $A_{\text{input}}$, and input
frequency, $\omega_{\text{input}}$. Generally, at low values of $A_{\text{input}}$, the energy
losses in the system result in a negative net energy gain. Whilst
at high values of $A_{\text{input}}$ the system becomes non-functional
when it is not able to provide a large enough pre-bias voltage
to prevent the mass from crashing into the box limits, which
would violate the requirements of the model. Alternatively, the
system can become less effective when the mass displacement
is constrained to reduce the induced voltage and pre-bias
voltage, to ensure the total voltage across the piezoelectric
capacitance does not exceed the dielectric breakdown voltage.
Thus, larger $A_{\text{input}}$ is not always better for system performance
as might have been expected.

Finally, since it is a requirement of the model that the
beam resonant frequency matches the input frequency, the
system operating envelope is limited by the fact that some
combinations of $\omega_{\text{input}}$ and $S$ have no geometric solutions.
This is due to the fact that the resonant frequency is inversely

---

**TABLE III**

<table>
<thead>
<tr>
<th></th>
<th>With Silicon Mass</th>
<th>With Gold Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{\text{max}}$</td>
<td>$A_{\text{input}}$</td>
<td>$A_{\text{input}}$</td>
</tr>
<tr>
<td>$\omega_{\text{input}}$</td>
<td>$1.2e-9$</td>
<td>$1.2e-9$</td>
</tr>
<tr>
<td>$S$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\eta_{\text{sys}}$</td>
<td>0.27</td>
<td>0.27</td>
</tr>
<tr>
<td>$P$</td>
<td>0.0003</td>
<td>0.0005</td>
</tr>
<tr>
<td>$I_{\text{r}}$</td>
<td>0.25</td>
<td>0.13</td>
</tr>
<tr>
<td>$t_{\text{r}}$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$Q$</td>
<td>23.58</td>
<td>23.58</td>
</tr>
<tr>
<td>$L_{\text{back}}$</td>
<td>15</td>
<td>19</td>
</tr>
<tr>
<td>$A_{\text{semi}}$</td>
<td>2.2</td>
<td>3.0</td>
</tr>
<tr>
<td>$A_{\text{input}}$</td>
<td>0.27</td>
<td>0.27</td>
</tr>
</tbody>
</table>

**Fig. 14.** System effectiveness at 1000 Hz.
proportional to harvester length and proof mass, which are defined with respect to $S$.

These limitations of piezoelectric systems, when compared with an electrostatic system model [15], lead to the finding that electrostatic harvesting systems produce more power from a 100 Hz driving frequency when acceleration and device box size are less than 0.04 ms^{-2} and 1.73 mm^3, while piezoelectric systems generate more power when acceleration or device size is larger. It is interesting to note that, unlike electrostatic harvester systems, the resonator and transducer elements are the same structure for piezoelectric systems, thus limiting geometric design choices.

Future research should focus on increasing the useful operating envelope of the full piezoelectric harvester system. It may be of interest to investigate meso- rather than micro-scale systems, where bulk piezoelectric materials with much higher coupling coefficients may be used in place of thin film piezoelectrics, thus improving operation at low frequencies and high accelerations. It may also be fruitful to modify the model to allow the mass to strike the limits of the box in order to potentially increase the operation regime.

**APPENDIX A**

**DERIVATION OF $V_{cc}$**

To derive the supply voltage, $V_{cc}$, required to achieve the desired pre-bias voltage, we look at the voltage in two phases of the circuit operation: first, from $V_{end}$ to $V_{rem}$, and then from $V_{rem}$ to $V_{PBstart}$. Equations for these two phases are found by considering a simple circuit with the supply voltage as a battery, an open switch, and an inductor, and a capacitor initially with either $V_{end}$ or $-V_{rem}$ on it. When the switch is closed, a resonant path from the capacitor to the battery via the inductor and switch is formed.

In the first case, the resonant discharge path causes the capacitor voltage, $V_{end}$, to decrease by the voltage difference across the capacitor, $V_{cc} - V_{end}$, multiplied by $1 + \gamma$, where $\gamma$ is the fractional capacitor voltage conserved by an RLC circuit with a quality factor $Q$. Note in a circuit with an infinite $Q$-factor, $\gamma$ will tend to 1. The first equation (for part of circuit operation from $V_{end}$ to $V_{rem}$) the voltage remaining is:

$$V_{rem} = -V_{end}\gamma + V_{cc}(1 + \gamma)$$  \hspace{1cm} (38)

For the second equation (for part of circuit operation from $-V_{rem}$ to $V_{PBstart}$) the polarity of $V_{PBstart}$ must be taken into account:

If $V_{PBstart} \geq 0$

$$V_{PBstart} = V_{rem}\gamma + (V_{cc} - V_{rem})(1 + \gamma)$$  \hspace{1cm} (39)

If $V_{PBstart} < 0$

$$V_{PBstart} = -V_{rem}\gamma + (V_{cc} - V_{rem})(1 + \gamma)$$  \hspace{1cm} (40)

Substituting (38) into the relevant $V_{PBstart}$ gives the required $V_{cc}$:

If $V_{PBstart} \geq 0$

$$V_{cc} = \frac{\gamma^2 2V_{po} + V_{PBstart}(\gamma^2 - 1)}{(1 + \gamma)^2}$$  \hspace{1cm} (41)

If $V_{PBstart} < 0$

$$V_{cc} = \frac{\gamma^2 2V_{po} + V_{PBstart}(\gamma^2 - 1)}{(1 + \gamma)^2}$$  \hspace{1cm} (42)

**APPENDIX B**

**DERIVATION OF $V_{PBstart}$ AND $V_{PBend}$**

The objective is to derive the relationship between the voltages on the piezoelectric capacitor before and after the switches are flipped, which results in charge redistribution. The original pre-bias voltage is $V_{PBstart}$, and the voltage remaining after charges redistribute is $V_{PBend}$. Starting with the fact that charge is conserved so $Q_{start} = Q_{end}$, we can write

$$V_{PBstart}C_0 = V_{PBend}C_0 + \sum Q_{diodes}.$$  \hspace{1cm} (43)

Then, using equation (12) and $Q = CV$,

$$Q_j = \frac{k_{cj}A_{semi}}{\sqrt{V_B}} \int_{V_{op2}}^{V_{op1}} \frac{1}{\sqrt{V_0 - V_{operation}}} dV_{operation}.$$  \hspace{1cm} (44)

Computing the integral to find the charge on each diode capacitance active in the circuit and solving for $V_{PBstart}$ yields the necessary starting pre-bias voltage.

The expression for the starting pre-bias voltage depends on the ending pre-bias voltage value relative to the supply voltage value and zero. There are four different cases, as follows.

If $V_{PBend} \geq V_{cc}$ and $V_{PBend} > 0$, then

$$V_{PBstart} = V_{PBend} + \frac{2k_{cj}A_{semi} ((V_o + V_{PBend})\frac{1}{2} - (V_o)\frac{1}{2})}{C_0 (V_{B,HiP})\frac{1}{2}}$$  \hspace{1cm} (45)

If $V_{PBend} < V_{cc}$ and $V_{PBend} > 0$, then

$$V_{PBstart} = V_{PBend} + \frac{2k_{cj}A_{semi} ((V_o + V_{PBend})\frac{1}{2} - (V_o)\frac{1}{2})}{C_0 (V_{B,HiP})\frac{1}{2}}$$  \hspace{1cm} (46)

If $V_{PBend} \leq V_{cc}$ and $V_{PBend} < 0$, then

$$V_{PBstart} = V_{PBend} - \frac{2k_{cj}A_{semi} ((V_o - V_{PBend})\frac{1}{2} - (V_o)\frac{1}{2})}{C_0 (V_{B,HiP})\frac{1}{2}}$$  \hspace{1cm} (47)

If $V_{PBend} \geq V_{cc}$ and $V_{PBend} < 0$, then

$$V_{PBstart} = V_{PBend} - \frac{2k_{cj}A_{semi} ((V_o - V_{PBend})\frac{1}{2} - (V_o)\frac{1}{2})}{C_0 (V_{B,HiP})\frac{1}{2}}$$  \hspace{1cm} (48)
And finally, if $V_{PBend} > -V_{cc}$ and $V_{PBend} < 0$, then

$$V_{PBstart} = V_{PBend} - \frac{2k_{cj}A_{semi}}{C_0} \left( (V_o - V_{PBend})^2 - (V_o)^2 \right) - \frac{2k_{cj}A_{semi}}{C_0} \left( (V_o + V_{cc})^2 - (V_o + V_{cc} + V_{PBend})^2 \right).$$

At this point the first iteration of calculations that would be conducted in the computer algorithm is complete, but the losses that were illustrated in Fig. 5 have not yet been accounted for in calculation of $V_{po}$. There are both losses due to charge redistribution and leakage current losses in the reverse biased MOSFETs. To find an expression for $V_{po}$ that takes these losses into account, we derive an equation for the voltage on the piezoelectric capacitor, $V_{piezo}$, over time using Kirchhoff’s current law at the node between $M1$ and $M2$ in Fig. 3. Again, these expressions depend on the value of $V_{piezo}$ relative to $V_{cc}$ and zero. If $V_{piezo} \geq V_{cc}$ and $V_{piezo} > 0$, then

$$i_p = i_{C_0} + i_{2C} + i_{21} + i_{1C} + i_{11}. \quad (49)$$

If $V_{piezo} < V_{cc}$ and $V_{piezo} > 0$, then

$$i_p = i_{C_0} - i_{3C} - i_{31} + i_{1C} + i_{11}. \quad (50)$$

If $V_{piezo} \leq -V_{cc}$ and $V_{piezo} < 0$, then

$$i_p = i_{C_0} + i_{4C} - i_{41} - i_{5C} - i_{51}. \quad (51)$$

If $V_{piezo} > -V_{cc}$ and $V_{piezo} < 0$, then

$$i_p = i_{C_0} + i_{6C} + i_{61} - i_{4C} - i_{41}. \quad (52)$$

The subscript $p$ refers to the piezoelectric capacitor, $C$ refers to a parasitic capacitive current, $l$ refers to leakage current, and the numerical subscripts indicate the corresponding MOSFET switch under consideration (the subscript $C_0$ indicates the piezoelectric capacitor). The expressions for each of the currents can be found using equations (8), (11), and (12) with the appropriate voltages inserted as follows:

$$i_p = \Gamma \frac{dV_{piezo}}{dt} \quad i_{C_0} = C_0 \frac{dV_{piezo}}{dt} \quad i_{1C} = \frac{dV_{piezo}}{dt} \left( \frac{k_{cj}A_{semi}}{C_0} \right) \frac{1}{2} \left( (V_o + V_{piezo} + V_{cc})V_{B,LoN} \right)^{1/2} \quad i_{2C} = \frac{dV_{piezo}}{dt} \left( \frac{k_{cj}A_{semi}}{C_0} \right) \frac{1}{2} \left( (V_o + V_{piezo} - V_{cc})V_{B,HiP}\right)^{1/2} \quad i_{3C} = \frac{dV_{piezo}}{dt} \left( \frac{k_{cj}A_{semi}}{C_0} \right) \frac{1}{2} \left( (V_o - V_{piezo} + V_{cc})V_{B,LoP} \right)^{1/2} \quad i_{4C} = \frac{dV_{piezo}}{dt} \left( \frac{k_{cj}A_{semi}}{C_0} \right) \frac{1}{2} \left( (V_o - V_{piezo} - V_{cc})V_{B,HiP} \right)^{1/2} \quad i_{5C} = \frac{dV_{piezo}}{dt} \left( \frac{k_{cj}A_{semi}}{C_0} \right) \frac{1}{2} \left( (V_o - V_{piezo} - V_{cc})V_{B,HiP} \right)^{1/2} \quad i_{6C} = \frac{dV_{piezo}}{dt} \left( \frac{k_{cj}A_{semi}}{C_0} \right) \frac{1}{2} \left( (V_o + V_{piezo} + V_{cc})V_{B,LoN} \right)^{1/2}.$$

Substituting these currents into the appropriate Kirchhoff current law equation, a differential equation can be obtained that gives the value of the voltage on the piezoelectric capacitor as a function of time over a half cycle. Taking the value of the piezoelectric voltage, $V_{piezo}$, at the end of its half cycle gives its maximum magnitude, corresponding to the maximum magnitude of displacement, and the improved value of $V_{po}$ can then be calculated to be

$$V_{po, improved} = \frac{1}{2} (V_{piezo, lastvalue} - V_{PBend}). \quad (54)$$

In a computer algorithm, it would now be necessary to recalculate all of the expressions from equation (6) to (54) since all of the subsequent terms depend on $V_{po}$ and all of the improved voltage values with losses taken into account are used. These calculations are conducted iteratively until
the change in the values of $V_{po}$, $V_{cc}$, and $V_{PB\text{start}}$ is less than 1% between subsequent iterations. The last voltage to be calculated is the voltage that remains after discharging the piezoelectric capacitor, $V_{\text{rem}}$. It can be zero under certain conditions but can also be positive or negative. It is given by

$$V_{\text{rem}} = \frac{V_{cc}(1 + \gamma)}{\gamma} - V_{\text{end}}$$

(55)

As mentioned in section IV, the FRTZ circuit step is advantageous and is implemented if $V_{\text{rem}}$ has the opposite sign as $V_{\text{end}}$, as it does in Fig. 5.

APPENDIX C

LIST OF TERMS

Tables IV, V, and VI in this appendix describe all of the terms used in this paper that are not already listed as fixed input parameters in Table I or as optimized output parameters in Table III.

REFERENCES


Lindsay M. Miller received the Ph.D. degree in 2012. She conducts research on energy harvesting from ambient vibration or thermal sources. Her Ph.D. dissertation focused on passively self-tuning resonant systems, low-frequency piezoelectric MEMS devices, and energy harvester system integration. She is currently the Director of Materials Integration at Alphabet Energy, a Hayward, California-based thermoelectric waste-heat-to-power company founded in 2009.

Alwyn D. T. Elliott (S’11) received the M.Eng. degree in electrical and electronic engineering, winning the Nujira Prize for the best final year analogue electronics project 2011, and the Ph.D. degree from Imperial College London, London, U.K., in 2015. He is currently with the Control and Power Research Group, Imperial College London, researching low-power electronics. He was the Local Organizing Committee Chair for PowerMEMS 2013 at the Royal Society, London.

Paul D. Mitcheson (SM’12) received the M.Eng. degree in electrical and electronic engineering, and the Ph.D. degree from Imperial College London, London, U.K., in 2001 and 2005, respectively. He is currently a Reader in Electrical Energy Conversion with the Control and Power Research Group, Imperial College London. His research interests are energy harvesting, power electronics, and wireless power transfer to provide power to applications in circumstances where batteries and cables are not suitable. He is a Fellow of the Higher Education Academy and sits on the Executive Committee of the U.K. Power Electronics Centre.

Einar Halvorsen (M’03) received the Siv.Ing. degree in physical electronics from the Norwegian Institute of Technology, Trondheim, Norway, in 1991, and the Dr.Ing. degree in physics from the Norwegian University of Science and Technology, Trondheim, in 1996. He has worked both in academia and the microelectronics industry. He is with the Department of Micro and Nanosystem Technology, University College of Southeast Norway, Horten, Norway. His current main research interest is in theory, design, and modeling of microelectromechanical devices, in particular energy harvesting devices.

Igor Paprotny (M’10) received the Engineering Diploma degree in mechatronics from the NKI College of Engineering, Oslo, Norway, in 1994, the B.S. and M.S. degrees in industrial engineering from Arizona State University, Tempe, in 1999 and 2001, respectively, and the Ph.D. degree in computer science from Dartmouth College in 2008. He is an Assistant Professor with the University of Illinois at Chicago. His research interests include applications of microsystem technologies to energy harvesting, smart grid sensing, air-microfluidics, and microrobotics.

Paul K. Wright is the A. Martin Berlin Professor of Mechanical Engineering and the Director of the Berkeley Energy and Climate Institute. His research takes place in the Advanced Manufacturing for Energy Laboratory. Funds from industry, foundations, the federal government, and the California Energy Commission, support an integrated research program on the resilience and analytics of energy systems. Topics cover a broad spectrum, including MEMS-sensors for electrical and gas distribution systems, energy harvesting, 3-D printing of storage systems, and demand response and condition monitoring. He is involved in the start-ups Imprint Energy and Wireless Industrial Technologies.