Geometric and Hydraulic Void Constrictions in Granular Media

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**ABSTRACT:** Constrictions in the void space between soil particles govern hydraulic conductivity, internal stability and filtration performance of sands and gravels. Various analytical, numerical and image-based methods have been proposed to measure void constrictions based solely on analysis of particle and void geometry. These geometric constrictions are increasingly being used in models to predict hydraulic conductivity or filtration performance. However, both of these phenomena depend not only on the void geometry, but also on the directions and magnitudes of fluid velocities within the void space.

This paper presents Computational Fluid Dynamics (CFD) simulations performed on micro-Computed Tomography (microCT) images of voids in real sands, as well as idealised materials generated by Discrete Element Modelling (DEM). Laminar flow conditions are considered and an alternative definition of a void constriction is presented, the hydraulic constriction, which is based on fluid velocities rather than void geometry. The data show that for laminar flow, where Darcy’s law is applicable, the position, size and orientation of hydraulic and geometric constrictions share many similarities, but there are measurable differences which should be considered in hydraulic conductivity and filtration analyses.

**Key words:** void constriction, micro-Computed Tomography, Computational Fluid Dynamics, filters, permeability

**Introduction**

Void constrictions (sometimes called “pore throats”) are typically defined as “the narrowest segments of the pores” (Khilar and Fogler, 1998); here these are taken to be “geometric constrictions” as they are identified solely on the basis of void geometry. In the study of porous rocks, hydraulic conductivity is commonly predicted based on the size of void constrictions, either by empirical relations using mercury intrusion porosimetry (Rezaee et al., 2006) or by analysis of 3D images (Andrew et al., 2015; Valvatne and Blunt, 2004).
Recently geotechnical engineers have begun using geometric constrictions to examine hydraulic conductivity in soils (Jaafar and Likos, 2011; Jang et al., 2011; Kress et al., 2012; Likos and Jaafar, 2013) and geometric constriction sizes have been related to hydraulic conductivity empirically (Indraratna et al., 2012).

Void constriction sizes are a key factor in the design of granular filters and the assessment of internal erosion by suffusion. Geometric constriction sizes have been used to examine particle transport probabilistically (Humes, 1996; Indraratna et al., 2015; Silveira, 1993; To et al., 2015) or using simple network models (Indraratna et al., 2007; Kenney et al., 1985; Kim and Whittle, 2009; Locke and Indraratna, 2000).

The transport of fluid or fine particles across constrictions will depend not only on the void geometry, but also on the local magnitudes and directions of fluid velocities within the void space. Constrictions in sandstones form tube-like throats (Doyen, 1988), shown schematically in Figure 1(a) and it is reasonable to assume that the fluid velocity (black arrows) will align roughly orthogonal to the geometric constriction. Sands, having a much higher porosity, will have more open constrictions with less clearly defined throats (Lindquist et al. 2000), as shown schematically in Figure 1(b). Here the velocities could align orthogonal to the geometric constriction (black arrows) or, alternatively, the fluid could retain its original direction and cross the geometric constriction at some angle (grey arrow), but there is no simple way to determine which of these cases will occur, or whether the flow direction is somewhere between these two extreme cases. Figure 1(c) shows a possible intermediate flow direction (grey arrows) and the most constrained point along the fluid’s path can be called the “hydraulic constriction” (grey line).

This contribution aims to examine the extent of the difference between geometric and hydraulic constrictions. The paper firstly outlines how Computational Fluid Dynamics (CFD)
was used to estimate fluid velocities within the void space and identify the hydraulic
constrictions. The approach was applied to void topographies obtained by micro-Computed
Tomography (microCT) imaging of real sands and also from Discrete Element Modelling
(DEM) with idealised spherical particles. The orientations, positions and sizes of the
geometric and hydraulic constrictions are compared for the samples considered.

6 Background and Definitions

7 Geometric Void Constrictions

8 In developing his classic filter criterion, Terzaghi estimated that “the pore size [constriction
size] of a broadly-graded filter comprises at maximum 1/5th of the diameter of the biggest
grain of the finest fraction of the filter materials” (Fannin, 2008), but this value was not
measurable and had to be estimated from laboratory tests. Since then many attempts have
been made to estimate constriction sizes analytically based on particle sizes, typically by
assuming spherical particles with idealised arrangements (e.g. Humes, 1996; Kenney et al.,
1985; Locke and Indraratna, 2000; Silveira et al., 1975).

9 More recently attempts have been made to measure constriction sizes directly. Referring to
Figure 2(a), DEM simulations can provide the 3D positions and radii of model soils
comprising spherical particles and Delaunay Triangulation based-methods (Reboul et al.,
2010; Shire and O’Sullivan, 2012) define geometric constrictions along the triangular
surfaces formed between three neighbouring particles. The 3D void geometry of real soils
can be obtained by microCT imaging; where reconstruction of x-ray scans from multiple
directions provides a 3D image of density within a sample. Each voxel (volume pixel) in the
image can be labelled as either particle or void based on density and atomic number. The
constrictions can then be located by either filling the void space with spheres and locating the
smallest spheres (the maximal ball method (Dong & Blunt 2009, Figure 2(b)), by thinning the
void space to a line and finding the closest point on this line to the particles (the medial axis
method (Binner et al. 2010; Homberg et al. 2014, Figure 2(c)) or subdividing the void space
into separate void volumes and defining the constrictions as the boundaries between voids
(the watershed segmentation method (Taylor et al. 2015, Figure 2(d)). Taylor et al. (2015)
demonstrated that all four methods give similar constriction size distributions.

Pore-scale Velocity Modelling

Computational Fluid Dynamics (CFD) involves solving the equations of mass and
momentum conservation locally over discretised geometry and time. The current study used
the open source CFD solver OpenFOAM (OpenFOAM Foundation, 2015) as described in
Raeini et al. (2012) and also used in a number of previous studies on porous rocks
(Mostaghimi et al., 2013; Pereira Nunes et al., 2015; Piller et al., 2014). OpenFOAM uses a
finite volume method, discretising the geometry into a mesh of polyhedral cells, applying
initial pressures, velocities and boundary conditions, then iteratively adjusting the boundary
conditions and re-solving the conservation equations until the solution converges to the
desired boundary conditions. Further details of the solution process are beyond the scope of
this paper, but more information can be found in Raeini et al. (2012) and at
www.openfoam.org (OpenFOAM Foundation, 2015).Typical outputs from an OpenFOAM
simulation are a 3D map of pressures and velocities at cell centres, as shown in Figure 3 for
an example with spherical particles.

Hydraulic Void Constrictions

In Figure 3(b), the lightest shades of grey represent the highest velocities and these clearly
occur in the narrow regions between particles, i.e. near the geometric constrictions. This
follows from consideration of continuity; for steady state conditions the product of velocity
and area is constant and so logically the highest velocities will occur where the cross
sectional area is smallest. The hydraulic constriction is defined to be the planar surface that
(i) passes through the point where the velocity attains a local maximum, (ii) has a normal vector orientated in the direction of the local maximum velocity vector, and (iii) extends across the void space to terminate at the particle surfaces, as shown in Figure 1(c). It is interesting to note that, in their contribution to an American Association of Petroleum Geologists handbook, Hartmann and Beaumont (1999) provide the following definition: “the absolute size of a pore throat [void constriction] is the radius of a circle drawn perpendicular to fluid flow and fitting within its narrowest point”; the emphasis on an orientation perpendicular to fluid flow is consistent with the new definition of hydraulic constriction, however the location at the “narrowest point” implies a geometric constriction.

Two validations were performed to compare geometric and hydraulic constrictions in a relatively simple, symmetrical void geometry. For the voxelised image of a periodically constricted tube, as shown in Figure 4(a), the calculated velocities were all within ±2% of the analytical solution given in Sisavath et al. (2001), which was deemed acceptable and the small error is believed to be caused by the voxelised tube surface. Geometric constrictions are overlain as light grey circles in Figure 4(b) and a 2D section showing central velocities as a grey-scale is shown in Figure 4(c). A simple cubic packing (Figure 4(d, e & f)) was analysed to provide a simple void geometry more similar to the voids in a granular soil. In both of these validation cases the geometric and hydraulic constrictions are coincident, as expected for these simple, symmetrical geometries.

Identifying Constrictions in 3D Images

The steps involved in locating geometric and hydraulic constrictions in both real and ideal sand samples are summarised in Figure 5. The following section provides details of each step.
Void Geometry from 3D Images

Two samples of sub-angular Leighton Buzzard Sand were considered (mean particle sphericity 0.9, mean aspect ratio 0.75, mean convexity 0.95, measured by laser scanning as described in Altuhafi et al. (2013)). Noting that constriction sizes are dependent on coefficient of uniformity, $C_u$ (Kenney et al., 1985), the sample “Sand-Cu3” had particle sizes between 0.3-2.0 mm and a $C_u$ of 3, while “Sand-Cu1.5” had particle sizes between 0.425-1.0 mm and a $C_u$ of 1.5. Triaxial specimens 38 mm in diameter were prepared using dry deposition (Ishihara 1993) at approximately 70% relative density. The dry samples were then consolidated isotropically to 30 kPa by applying a cell pressure using air. As described in Fonseca et al. (2013), samples were impregnated with epoxy resin, by connecting an elevated reservoir of resin to the base of the sample and applying a small air suction ($\approx 1$ kPa) to the top of the sample. Once the resin had set, 9 mm diameter sub-samples were cored from the centre of the triaxial samples to achieve a high ratio of grain size to voxel size, as explained by Fonseca et al. (2012). The cores were scanned using a Nikon XT-H-225 microCT scanner, producing images with a voxel size of approximately $10 \times 10 \times 10 \mu m$. Image processing included median filtering and thresholding, using the method of Otsu (1979) to produce binary images where each voxel is either particle or void. Sub-volumes from the resulting 3D images are shown in Figure 6(a&b), where the grey areas are particles and the voids have been left transparent.

A model soil with $C_u$ of 3 (DEM Cu-3) was produced by using the centre coordinates and radii of spherical particles from a DEM simulation to generate a voxelised 3D image (Figure 6(c)). Geometric constrictions were located using the watershed segmentation method and Figure 7 shows examples of geometric constriction centres calculated for the DEM-Cu3 image using three different methods (Watershed, Medial Axis and Delaunay triangulation),
which supports the assertion by Taylor et al. (2015) that the geometric constriction locations do not vary significantly amongst these methods.

**Computational Fluid Dynamics Simulations**

An open source graphical user interface program, HELYX-OS (Engys, 2015), was used to run OpenFOAM; HELYX-OS includes a simple but effective mesh generation algorithm. For simulations to converge within 24 hours on a desktop computer with 144GB RAM, the full microCT images could not be analysed and so sub-volumes comprising $400 \times 400 \times 400$ voxels were used, as shown by the white outlines in Figure 6, where 400 voxels represents approximately 5 to $7 \times D_{50}$ (median particle diameter).

The voxelised void geometry was imported into the CFD solver as a 3D surface in StereoLithography (.stl) format. Voxelised images of 3D objects overestimate surface area by approximately 50% (Rajon et al., 2002), due to the stepped nature of the voxelised surface. This increase in surface area could have a major impact on fluid flows, so the .stl surface was smoothed using the ‘Smooth’ function in the 3D design software Rhino 5.0 (Robert McNeel & Associates, 2015). An example of a sand particle before and after smoothing is shown in Figures 8(a&b). After smoothing, a finite volume mesh was generated with mesh cells approximately equal to the microCT voxel size.

The fluid boundary conditions for a CFD simulation are shown in Figure 9. A no slip condition was applied to all particle surfaces and symmetry conditions were applied to external surfaces parallel to the flow direction, to prevent flow out from the sides of the model. A small pressure difference (0.001 kPa) was applied between the inlet and outlet boundaries, representing a hydraulic gradient of approximately 0.025, which is quite low for a seepage flow through soil. This value was selected to ensure low Reynolds numbers and hence laminar flow. The resultant discharge velocities ($v_d$, flow per unit area) were
approximately 0.055 to 0.065 mm/s; Harr (1990) proposes the use of the average particle diameter $\bar{D}$ in the equation for Reynolds number, $Re = \frac{\rho_w \bar{D} v_d}{\mu}$ where $\rho_w=$ density of water and $\mu=$ viscosity of water, giving $0.4 < Re < 0.6$ (i.e. $<1$, the lower limit of $Re$ values for laminar flow stipulated by Harr (1990)). Thus the findings from the simulations are relevant where Darcy’s law is applicable, as assumed in most geotechnical analyses. Velocities perpendicular to the flow direction were set to zero at the inlet and outlet boundaries to prevent circulating flows across the inlet or outlet boundary, which prevent convergence of the CFD solution.

Referring to Table 1, one CFD simulation was performed for each of the three images, with flow in the positive Z direction ($Z^+$), representing upward vertical flow in the physical specimens. To examine the effect of flow direction, three additional simulations were performed on the DEM-Cu3 image, for flow in the negative Z direction ($Z^-$), the positive X direction and the positive Y direction. To check if hydraulic constrictions are sensitive to the magnitude of hydraulic gradient, the DEM-Cu3 $Z^+$ simulation was repeated with the input boundary pressure doubled, however this resulted in no change to the hydraulic constrictions and hence the results from this simulation are not presented. Results from all other simulations (comprising mesh centre coordinates, pressure values and velocity components) were loaded into MATLAB (The MathWorks Inc., 2013) and the data were interpolated to obtain pressures and velocity components in the X, Y and Z directions and the velocity magnitude at each voxel centre.

**Locating Hydraulic Constrictions**

Local maxima were sought by considering voxels with velocity magnitudes greater than those in all of the 26 neighbouring voxels (making up a $3 \times 3 \times 3$ cube voxel neighborhood). To reduce the search time and allow direct comparison between hydraulic and geometric
constriction pairs, the hydraulic constrictions were identified locally in the vicinity of each geometric constriction, rather than simply by scanning the whole image for local maxima. As shown schematically in Figure 10, searching over too large a region around a geometric constriction may generate multiple velocity maxima, associated with other geometric constrictions. The region size was selected to extend in each direction by approximately half of the mean particle radius (15-25 voxels) and where multiple velocity maxima were located the maximum closest to the geometric constriction was recorded as the corresponding hydraulic constriction.

Figure 11 outlines a procedure to visualise a plane perpendicular to the velocity vector at the hydraulic constriction centre. First a large circular disk is generated perpendicular to the maximal velocity vector and any particle voxels on this plane are removed, as shown in Figure 11(a). Due to the relatively open nature of voids in soils, the plane often extends into neighbouring voids and throats and the surface needs to be subdivided. In this case, portions of the plane passing into neighbouring voids will be associated with separate velocity maxima, as shown in Figure 11(b). Watershed segmentation (here performed using the ‘watershed’ function in MATLAB) was used to sub-divide the plane based on the separate velocity maxima, resulting in the four regions shown in Figure 11(c). The hydraulic constriction plane is defined as the region containing the hydraulic constriction centre, which is compared graphically with the corresponding geometric constriction in Figure 11(d).

This procedure is applied for each geometric constriction within the CFD sub-volume of the image. Typical examples of constrictions pairs from the Sand-Cu3 image are shown in Figure 12, with flow in the upward (Z+) direction. In Figure 12(a) the geometric and hydraulic constrictions are almost coincident, the only difference being that geometric constriction surface from the watershed segmentation method is not required to be planar. Figure 12(b) shows a constriction pair which appear similar in shape and size, but which have a significant
offset. In this case the geometric constriction is made up of two separate planes with a sharp angle between the two (a similar situation is evident in Figure 11(d)). Rather than forming two separate constrictions, the hydraulic constriction criteria define a single constriction. Based on visual inspection of a large number of constrictions, the majority of hydraulic constrictions are represented by these two cases.

Figure 12(c) presents a case where the geometric constriction is oriented at approximately 50° to the $Z^+$ direction and the fluid crosses the geometric constriction at an angle, rather than aligning orthogonally to it, resulting in a hydraulic constriction with very different position, size and orientation to the geometric constriction. Figure 12(d) shows an extreme case, where the geometric constriction being analysed is roughly perpendicular to the $Z^+$ direction. Here the closest hydraulic constriction clearly corresponds to the neighbouring geometric constriction and there is no velocity maximum closer to the geometric constriction being analysed, i.e. there is no concentration of flow across this geometric constriction. In these cases the geometric constriction is said to have “no valid hydraulic constriction”. For the case in Figure 12(d) the hydraulic constriction shown will be identified twice (once for this geometric constriction and a second time for the neighbouring constriction). These duplicated hydraulic constrictions are removed at the end of the analysis, retaining only the closest pair of geometric and hydraulic constrictions.

**Results**

Geometric and hydraulic constriction pairs were located for all six CFD simulations and the numbers of geometric and hydraulic constrictions for each simulation are presented in Table 1. In this section, pairs of geometric and hydraulic constrictions are compared in terms of orientation, position and size.
**Constriction Orientations**

The 3D orientations of hydraulic constrictions, defined by the velocity vector at each constriction centre, are presented in Figure 13 in terms of 2D angles on the ZX and ZY planes. The rose diagram bins shown in Figure 13(a) indicate the frequency of hydraulic constrictions for DEM-Cu3 (Z+ flow), for 20° bins and presented as percentage of the total number of hydraulic constrictions. For clarity, only outlines of the frequency distributions (dashed black line in Figure 13(a)) are shown for the other CFD simulations. For Z+ flow more than 50% of hydraulic constrictions are oriented within 30° either side of the Z+ axis and approximately 85% are within 60° either side of the Z+ axis. The distribution is not perfectly symmetrical about the Z axis due to a small degree of heterogeneity caused by the small sample size. When the flow direction is reversed (Z- flow, shown as a dashed grey line in Figure 13(a)) the constriction orientations are very similar to the Z+ results rotated by 180° (shown as a solid grey line); the small differences are discussed below.

Figures 13 (b&c) show the hydraulic constriction orientations for all the CFD simulations. In Figure 13(b) the DEM-Cu3 Z+, Z- and X simulations produce similar distributions, oriented towards their respective flow direction. In Figure 13(b) the Y direction is out of the page and the relative frequencies for Y flow (dashed line with triangles) are approximately equal over the full 360° range, suggesting no significant anisotropy between the vertical (Z) and horizontal (X) directions. Similar results are obtained in the ZY plane, as shown in Figure 13(c). The light and dark grey lines in Figures 13(b&c) represent Sand-Cu3 and Sand-Cu1.5 respectively and both show a clear trend towards the Z+ direction but with wider spread than the DEM-Cu3 results, with approximately 45% within 30° and 80% within 60° either side of the Z+ axis.

The orientation of geometric constrictions is more difficult to assess, as they form irregular surfaces, rather than flat planes. The voxels making up each geometric constriction can be
analysed using Principal Component Analysis (PCA) to find major, minor and intermediate
directions of the voxel point cloud. The minor PCA direction indicates the orientation in
which the point cloud has the smallest dimension, in this case perpendicular to the geometric
constriction surface. The MATLAB function ‘princomp’ was used to determine minor PCA
orientations for all geometric constrictions and the results for Sand-Cu3 (Z+ flow direction)
are shown in Figure 14, with frequency as a percentage of the total number of geometric
constriction and using a 20° bin size. The PCA directions calculated here have no meaningful
‘forward’ and ‘backward’ directions (unlike the velocity vectors used for hydraulic
constrictions) and hence the orientation angles only range from 0 to 180° in Figure 14. It
should also be noted that, due to their stepped shape, voxelised surfaces produce PCA vectors
aligned preferentially towards the coordinate axes, as is evident for the data in Figure 14.

Table 2 shows fabric tensor components for the geometric constriction orientations, where
the tensor is calculated using equation 1 (Satake, 1982):

\[ \Phi_{ij} = \frac{1}{N} \sum_{N} n_i n_j \]  

[1]

where \( N \) is the number of constrictions and \( n_i \) is the constriction normal unit vector. In a
perfectly isotropic material the \( \Phi_{xx}, \Phi_{yy} \) and \( \Phi_{zz} \) components should all equal 1/3 and a
\( \Phi_{zz} \) value above 1/3 suggests that constrictions tend to face more towards the Z direction
than the X or Y directions. The solid black lines in Figure 14 are approximately symmetrical
about the Z axis but show larger frequencies in the Z direction than the X or Y directions,
which corresponds with the small degree of anisotropy evident in Table 2. The dashed lines
in Figure 14 represent only those geometric constrictions which have valid hydraulic
constrictions and there is a significant reduction in the number of constrictions orientated
more than about 45° from the Z axis, suggesting that geometric constrictions oriented >45°
from the flow direction are less likely to form hydraulic constrictions. Nonetheless, the
relative frequencies close to 0 and 180° are still higher in Figure 14 than for hydraulic constrictions in Figure 13 (b&c), proving that while some geometric constrictions at angles >45° do form hydraulic constrictions, the hydraulic constrictions must be oriented at a significant angle to the geometric constriction.

For all simulations, the values in Table 1 suggest that approximately 60% of geometric constrictions form valid hydraulic constrictions. The slight increase in this proportion for Sand-Cu1.5 may be due to a larger degree of anisotropy, favouring the Z direction, as shown in Table 2.

**Distance between Geometric and Hydraulic Constrictions**

To assess whether the hydraulic and geometric constrictions lie within the same inter-particle throat, Figure 15 shows relative frequencies of the distance between corresponding pairs of geometric and hydraulic constrictions for all CFD simulations, normalised by the diameter of the smallest particle in each image (D₀). The peak in Figure 15 is located at approximately 0.05 to 0.07 × D₀ (10-14% of the smallest particle radius), which represents only 2 voxels difference between constriction pairs in the 3D image. Approximately 80% of constriction pairs are within 0.15 × D₀ apart (30% of the smallest particle radius) and graphical inspection indicates that these pairs are within the same narrow throat in the void space. Graphical inspection of pairs with distances of more than 0.5 × D₀ relate to one of three cases:

- Constriction pairs within the same throat, but between particles much larger than D₀,
- Large differences between the orientations of the two constrictions
- Hydraulic constrictions that span two or more geometric constrictions, as in Figures 11(d) & 12(b).

Figure 16(a) shows a typical 3D example of hydraulic constrictions for Z+ and Z- flow through the same geometric constriction, seen from two different orientations. The Z+ and Z-
planes indicate hydraulic constrictions which are similar in shape and size to the geometric constriction and which are offset by only a few voxels, however this offset is not random as both hydraulic constrictions form slightly before the geometric constriction in terms of the flow direction. Figure 16(b) illustrates Z+, X and Y hydraulic constrictions for a geometric constriction surface roughly parallel to the Y axis, hence the Y hydraulic constriction is very different to the geometric constriction, while the Z+ and X hydraulic constrictions are roughly similar, but not identical.

**Constriction Size Comparison**

The distance measured from a geometric constriction centre to the nearest particle surface defines the size of the constriction (equalling the radius of a sphere which can just fit at this point in the void space). The distance from each void voxel to the nearest particle can be calculated for a 3D image using a Distance Map algorithm, such as the ‘bwdist’ function in MATLAB. Distance map values across a hydraulic constriction plane are shown in 3D as shaded voxels in Figure 17(a) and as 2D contours in Figure 17(b), while velocity magnitudes perpendicular to the hydraulic constriction plane are shown in Figure 17(c). In Figures 17(b&c) black crosses represent maximum distance map values while black circles represent the maximum velocity (hydraulic constriction centre). Figures 1(c) and 12(c) show that, where fluid crosses the geometric constriction at an angle, the hydraulic constriction plane will have a greater total area than the geometric constriction plane. However, as shown in Figures 1(b), 11(b) and 17(c), the majority of the fluid flow will be concentrated though a channel smaller than the geometric constriction and offset from its centre, hence the hydraulic constriction can be considered smaller than the geometric constriction. Clearly the definition of hydraulic constriction size is subjective, but here it is defined as the distance map value at the hydraulic constriction centre (black circle in Figure 17(b)), as this represents the radius of
a sphere (or a cylinder) passing the hydraulic constriction centre and this allows direct comparison against geometric constriction sizes.

Results for Sand-Cu3 are presented in Figure 18(a) and show a clear correlation between geometric and hydraulic constriction sizes, but with hydraulic constrictions typically smaller than their corresponding geometric constriction, as explained in Figure 17(b). Results for all simulations are presented in Figure 18(b), where the difference in size is expressed as a percentage of the geometric constriction size and plotted in terms of relative frequency. All simulations show a clear peak at approximately +10%, suggesting that geometric constrictions are on average 10% larger than the corresponding hydraulic constrictions. The standard deviation is quite large (≈40%), suggesting that hydraulic constrictions will regularly range from 50% smaller to 30% larger than geometric constrictions. Relatively smaller hydraulic constrictions occur where the fluid crosses the constriction at a significant angle (as in Figures 1(b) & 17(b)), while relatively larger hydraulic constrictions occur where flow merges across several geometric constrictions, producing a hydraulic constriction further out in the void throat (as in Figures 11(d) & 12(b)).

Figure 19(a) shows cumulative Constriction Size Distributions (CSDs) for Sand-Cu3 (black) and Sand-Cu1.5 (grey), ignoring any constrictions smaller than 0.155 × D₀, as these may relate to imaging defects (Taylor et al., 2015). The solid lines include all constrictions in the image while the dotted lines only include geometric constrictions which produced valid hydraulic constrictions. The solid and dotted lines are similar for these materials, suggesting that existing methods to measure geometric constrictions are adequate to determine the CSDs. This result suggests that when there is a small degree of anisotropy in the orientation of geometric constrictions (Table 2), the range of constriction sizes is approximately similar at all orientations. CSDs based on hydraulic constriction sizes are shown in Figure 19(a) as dashed lines with crosses and typically plot to the left of the geometric constriction sizes,
corresponding to the earlier observation of a 10% difference in size on average. Figure 19(b) shows similar results for DEM-Cu3 (black lines) and also shows CSD results for geometric constrictions from the same image using the Medial Axis and Delaunay methods, indicating that the CSD results are not very sensitive to the method used to find geometric constrictions.

Conclusions

Constrictions in the void space in sandy soils play a key role in determining hydraulic conductivity and filtration behaviour; this contribution has considered how to quantify constriction topology. In this paper, Computational Fluid Dynamics (CFD) simulations that considered laminar fluid flow in the pore space were used to examine fluid velocities in the vicinity of geometric constrictions. The pore geometries used in the CFD simulations were extracted from microCT images and DEM simulations. The “hydraulic constriction” is an alternative definition of a void constriction, based on local maxima of fluid velocities, and a methodology for locating these hydraulic constrictions was presented.

Geometric constrictions have several advantages, notably that they provide a conceptually simple way to interpret filtration phenomena and there are various analytical, numerical and image-based methods to measure geometric constrictions in the literature. However, the results presented here indicate that knowing the size, position and orientation of geometric constrictions is not sufficient to fully understand the transport of fluid through the void space in sands. Identifying hydraulic constrictions is computationally expensive and hence is not recommended in all studies. Engineers wishing to use geometric constrictions to predict fluid behaviour should be aware of the key similarities and differences found between the two types of constriction which, for the laminar flow situations considered here, are summarised as follows:
• For geometric constrictions oriented within roughly 30° of the flow direction (hydraulic gradient direction), hydraulic and geometric constrictions are very similar in terms of orientation, position and size.

• For geometric constrictions oriented between roughly 30° and 60° from the flow direction:
  o The orientation of the hydraulic constrictions may be very different from the geometric constrictions, i.e. the fluid crosses the constriction at a significant angle.
  o Hydraulic constrictions typically occur close to geometric constrictions (within a distance of about 10% of the smallest particle diameter),
  o The maximum velocity (i.e. the maximum concentration of flow) does not occur at the geometric centre of the constriction and hence the apparent size of the hydraulic constriction may be roughly 10% to 50% smaller than the geometric constriction.

• For geometric constrictions oriented between roughly 60° and 90° from the flow direction:
  o Differences in terms of orientation, position and size may be similar to those noted above for 30° to 60°, only more pronounced,
  o Geometric constrictions may not form valid hydraulic constrictions, suggesting negligible flow across the geometric constriction. In all the simulations presented here, this occurred in approximately 40% of all geometric constrictions.

• For the materials analysed, consideration of the fluid flow resulted in very little change to Constriction Size Distribution determined from geometric constrictions.
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Notation List

$\Phi_{ij}$ = 3D Fabric tensor component in the i,j direction;

$D_0$, $D_{50}$ = Particle diameters corresponding to 0% passing and 50% passing by mass;

$N$ = Number of constrictions in the image;

$n_i$, $n_j$ = Constriction normal unit vector in the i and j directions;

$X$, $Y$, $Z^+$, $Z^-$ = Orthogonal flow directions.

References


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OpenFOAM Foundation, 2015. OpenFOAM.


FIG 1. Schematic fluid flow through geometric constrictions in a) sandstone and b) sand. c) Intermediate flow direction and hydraulic constriction.


FIG 3. 2D section through CFD simulation showing: a) Pressures, b) Velocities
FIG 4. Periodically Constricted Tube and Simple Cubic Packing. (a & b) 3D images showing solid geometry, (c & d) 3D cut-aways showing void constrictions, (e & f) Central 2D sections showing fluid velocities.

FIG 5. Flow chart outlining the algorithm to locate hydraulic constrictions

FIG 6. 3D images of materials analysed: a) Sand-Cu3, b) Sand-Cu1.5, c) DEM-Cu3
FIG 7. Examples of geometric constriction centres using the Watershed, Medial Axis and Delaunay triangulation methods.

FIG 8. 3D image of sand particle a) before and b) after voxel smoothing in Rhino

FIG 9. 3D image of DEM-Cu3 sub-volume, showing CFD boundary conditions for Z+ flow
FIG 10. Typical section showing local velocity maxima in search region around geometric constriction

FIG 11. Identifying Hydraulic constriction plane: a) Remove particle voxels, b) Segment based on velocities, c) Result of velocity segmentation, d) Hydraulic vs geometric constrictions in 3D
FIG 12. 3D examples of geometric (black) and nearest hydraulic (grey) constrictions in Sand-Cu3: a) Good match, b) Offset, c) Different orientation, d) no valid hydraulic constriction

FIG 13. Distribution of hydraulic constriction velocity vectors: a) Example of data divided into 20° bins and resulting outline, b) Results for all simulations, on ZX plane, c) Results for all simulations, on ZY plane

FIG 15. Distance between corresponding geometric and hydraulic constriction centres, as a proportion of smallest particle diameter ($D_0$), bin size is 1 voxel (0.025 to 0.035 $\times$ $D_0$).

FIG 17. Typical hydraulic constriction in Sand-Cu3: a) 3D view shaded by distance map values, b) Distance map values on constriction plane, c) Velocities on constriction plane.

FIG 18. Distance map values at constriction centres: a) Hydraulic vs geometric constrictions in Sand-Cu3, b) Relative difference between geometric and hydraulic distance map values, with 10% bin size.

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*Sub-volume refers to 400 x 400 x 400 voxel sub-volume used in CFD simulation.

Table 1 – Number of geometric and hydraulic constrictions for all CFD simulations.

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Table 2 – Fabric Tensor components of geometric constriction orientations.