This work is a numerical investigation on the influence of viscoelastic effects on the aerodynamics of integrally actuated membrane wings. For that purpose, a high-fidelity electro-aeromechanical computational model of wings made of dielectric elastomers has been developed. The structural model is based on a geometrically non-linear description and a non-linear electro-viscoelastic constitutive material law. It is implicitly coupled with a fluid solver based on a finite-volume discretisation of the unsteady Navier–Stokes equations. The resulting framework is used for the evaluation of the dynamics of passive and integrally actuated membrane wings at low Reynolds numbers under hyperelastic and viscoelastic assumptions on the constitutive model. Numerical simulations show that the damping introduced by viscoelastic stresses can significantly reduce the amplitude of membrane oscillations and modify key features in the coupled system dynamics. The estimated wing performance metrics are in good agreement with previous experimental observations and demonstrate the need of including rate-dependent effects to correctly capture the coupled system dynamics, in particular, for highly compliant membranes.

© 2016 The Authors. Published by Elsevier Ltd. This is an open access article under the CC BY license (http://creativecommons.org/licenses/by/4.0/).
High-fidelity computational modelling is an essential ingredient to understand the physical phenomena behind the complex global behaviour of actuated membrane wings. Structural and fluid details have to be accurately represented to capture the strong coupled-system dynamics. A big effort has been done towards the high-fidelity modelling of low Reynolds number flows around membranes. Viscous laminar solvers (Smith and Shyy, 1995; Tiomkin et al., 2011), turbulent models coupled with transition models (Lian and Shyy, 2007) and LES models (Gordnier, 2009; Visbal, 2009) have been extensively used in order to reproduce the unsteady flow phenomena typical of membrane wings. From the structural point of view, a similarly high level of fidelity is often missing in these models, since typically a nonlinear geometric solver is coupled with a linear elastic constitutive law. Indeed, despite membrane strains easily reaching values above 100%, relatively few papers (Lian and Shyy, 2007; Stanford et al., 2008; Buoso and Palacios, 2015, among others) have considered hyperelastic material behaviour to characterise the large deformations of highly compliant wings.

Experimental testing of DEs has identified strong rate-dependent material behaviour, even for small deformation rates (Ask et al., 2012; Mokarram et al., 2012). To date, only Wissler and Mazza (2007), Fox and Goulbourne (2009) and Mokarram et al. (2012) have published fully relaxed experimental data for the characterisation of the purely elastic constitutive behaviour of acrylic DEs. However, most previously proposed constitutive models for DEs (Fox and Goulbourne, 2009; Carpi et al., 2011) have neglected the viscous stresses typical of these polymers. Several phenomenological models have been proposed to model rate dependent effects. Wissler and Mazza (2007) have suggested a viscoelastic model based on Prony series, while Ask et al. (2012) and Mokarram et al. (2012) have used the multiplicative decomposition of the deformation tensor to define a nonlinear viscoelastic model for uni-axial tensile cases. More recently, Li et al. (2013) have shown that material constants obtained from uni-axial or equi-biaxial loading conditions predicted a significantly different behaviour, highlighting the importance of the boundary conditions for the material identification. This was found to be linked to the hyperelastic behaviour of thin membranes, which significantly differs from more conventional (bulk) material behaviour.

It is therefore expected that the coupled electro-aeromechanical response of DE membrane wings will be significantly affected by the viscoelastic characteristics of the material. Recently, the authors have presented a high-fidelity model for actuated wings that assumes a purely hyperelastic material behaviour (Buoso and Palacios, 2015). Therefore, this paper proposes a nonlinear viscoelastic model suitable for DE membrane wings that will be coupled with that electro-aeromechanical model and used to investigate the impact of viscoelastic effects in the wing dynamics.

Section 2 introduces the mathematical model for the constitutive material behaviour, which is developed in the finite deformation framework. Section 3 describes the analytical dynamic model for the identification of the viscoelastic material properties from experimental data, and Section 4 outlines the fully coupled electro-aeromechanical model for membrane wings. This is finally used in Section 5 to evaluate the behaviour of passive and actuated membrane wings for the hyperelastic and viscoelastic material models, providing the basis for a comparison of the impact of the structural level of fidelity in the characterisation of the aeroelastic system.
2. Constitutive model for dielectric elastomers

In the isotropic constitutive model of this paper, the total instantaneous mechanical stresses will be assumed to be the sum of elastic and viscoelastic components. It is also assumed the independence of the electrostatic stresses from the viscoelastic behaviour of the material, so that they only depend on the purely elastic deformation (Suo, 2010). The evolution of the electrostatic forces is further assumed to be instantaneous, since its time scale is several orders of magnitude faster than the mechanical ones. Under these assumptions, the material constitutive model is described through the definition of an energy function \( W \) of the form

\[
W = W_\infty + W_\nu + W_e,
\]

where \( W_\infty, W_\nu, \) and \( W_e \) are the energy functions defining the hyperelastic equilibrium, viscoelastic and electrostatic stress states, respectively.

The large displacement framework, the hyperelastic equilibrium function \( W_\infty \), and the electrostatic model are briefly described in Section 2.1 before introducing the rate-dependent part of the model in Section 2.2.

2.1. Hyperelastic material model

The hyperelastic equilibrium constitutive model \( W_\infty \) of Eq. (1) is developed in the framework of large deformations. Its formulation is based on the deformation gradient, \( F \), which is defined in the usual way as the gradient of the mapping function from the reference configuration to the deformed configuration (Bonet, 2001). Commonly, \( F \) is split into its volumetric and deviatoric components (Flory, 1961) by means of \( J = \det F \), and \( F = \hat{F} \, F \), respectively. The deviatoric stresses are expressed in terms of the invariants of the right Cauchy–Green deformation tensor, \( \mathbf{C} = \frac{1}{J} \mathbf{F}^\top \mathbf{F} \), while the volumetric stress component is defined as a function of the variation in volume, \( J \) (Bonet and Wood, 2008). The energy function \( W_\infty \) is defined as

\[
W_\infty = U_\infty(J) + D_\infty(T_1, I_2),
\]

where \( U_\infty(j) \) is the volumetric energy function and \( D_\infty(T_1, I_2) \) is the deviatoric free-energy function, which is expressed in terms of the first and second invariants of \( \mathbf{C} \). The mechanical stresses for the deformed configuration, \( \sigma_m \), are obtained as

\[
\sigma_m = \frac{1}{J} \mathbf{F}^\top \mathbf{S} \mathbf{F},
\]

where \( \mathbf{S} \) is the second Piola–Kirchhoff stress tensor. The final step is the specification of a form for the volumetric and deviatoric components of the free-energy functions in (2). The material model proposed by Gent (1996) will be used for \( D_\infty \) as

\[
D_\infty = -\frac{\mu J_m}{2} \log \left( 1 - \frac{T_1}{I_2} \right),
\]

where \( \mu \) and \( J_m \) are the elastic shear modulus and the limiting stretch value, respectively, and \( I_1 = \text{tr} \mathbf{C} \). The volumetric energy function is defined as

\[
U_\infty(j) = K(j - 1)^2,
\]

where \( K \) is the compressibility modulus of the material. Finally, the electromechanical stresses, \( \sigma_e \), associated to the electromechanical energy function \( W_e \) in (1), are finally added to the mechanical stresses in (3) as in (Suo, 2010)

\[
\sigma_e = \epsilon \mathbf{E} \otimes \mathbf{E} - \frac{1}{2} \epsilon (\mathbf{E} \cdot \mathbf{E}) \mathbf{I},
\]

where \( \epsilon \) is the material dielectric constant, \( \mathbf{E} \) is the electric field vector in the deformed coordinate system and \( \mathbf{I} \) is the identity second order tensor. The coefficients of (4) and (5), as well as the dielectric constant in (6), have to be determined from experimental data on the specific material of interest. Buoso and Palacios (2015) have obtained a good fitting of the experimental results for an inflated VHB4905 membrane with \( \mu = 20 \text{ kPa}, J_m = 100, K = 3.5 \times 10^6 \text{ Pa} \) and \( \epsilon = 2.7 \epsilon_0 \), where \( \epsilon_0 \) is the vacuum dielectric constant.

2.2. Viscoelastic material model

The formulation of the viscoelastic constitutive model \( W_e \) of Eq. (1) will be based on \( N \) time dependent internal tensors \( \mathbf{F}_t \) representing an equivalent number of viscous mechanisms included in the model. This will extend the model of Mokarram et al. (2012) for loading conditions typical of membrane wings. For each value of \( \mathbf{F}_{t_\alpha} \) with \( \alpha = 1, \ldots, N \), through the multiplicative decomposition of the deviatoric deformation gradient, \( \mathbf{F} \), the viscoelastic tensor \( \mathbf{F}_e \) is obtained

\[
\mathbf{F}_{e,\alpha} = \mathbf{F} \mathbf{F}_{t_\alpha}^{-1}, \quad \text{for } \alpha = 1, \ldots, N,
\]
which maps the intermediate configuration into the fully relaxed with the same $F$. These new viscoelastic tensors are used in the definition of the non-linear viscoelastic constitutive model.

It is assumed, as in Buoso and Palacios (2015), isotropic behaviour, which is a reasonable approximation of the material behaviour in the equi-biaxial stresses considered for the wing in this work. The viscoelastic free-energy function is defined in terms of the invariants of the right Cauchy–Green deformation tensors of the viscoelastic tensors, $C_{e,a} = F^T_{e,a} F_{e,a}$. This results in a mechanical free-energy function of the form

$$W_v = \sum_{a=1}^{N} W_{v,a}(I_{1,e}, T_{2,e}),$$

where $W_{v,a}(I_{1,e}, T_{2,e})$ is the general form of constitutive model related to the $\alpha$-th mechanism and $I_{1,e}$ and $T_{2,e}$ are the first and second invariants of $C_{e,a}$. The non-equilibrium evolution law, in this work, is in the form

$$\dot{C}_{i,a} = \frac{1}{\tau_a} [\dot{C} - C_{i,a}],$$

which is a linearisation of the law proposed by Bonet and Wood (2008), where $C_{i,a} = F^T_{i,a} F_{i,a}$ is the right Cauchy deformation tensor of $F_{i,a}$ and $\tau_a$ is the time constant relative to the $\alpha$-th relaxation mechanism considered. The linearised evolution law has shown to be a good representation of the material behaviour for the relatively small range of stretches and strain-rates considered for membrane wing applications. As in the hyperelastic constitutive model, the form of the viscoelastic energy function has to be specified. In this paper it is described with the Neo-Hookean model

$$W_{v,a} = \frac{\mu_{v,a}}{2}(I_{1,e} - 3),$$

where $\mu_{v,a}$ is the viscoelastic shear modulus of the $\alpha$-th relaxation mechanism.

3. Identification of material constants

The characterisation of the viscoelastic material model for the selected material requires the identification of $2N$ coefficients, $N$ shear moduli $\mu_{v,a}$ from (11) and $N$ relaxation constants $\tau_a$ from (10). In this section an analytical, linear dynamic model for axi-symmetrical inflated membranes is derived for their identification. The boundary conditions in this case are similar to those of membrane wings, and hence the material characterisation in this case allows a realistic representation of the performance of membrane wings. The model is then used to reproduce the experimental results of Fox and Goulbourne (2008, 2009) for the characterisation of the rate-dependent behaviour of the VHB4905 acrylic elastomer. In their experiments, a circular prestretched, inflated membrane is actuated with both mechanical and electrical loadings. Buoso and Palacios (2015) have presented a static non-linear analytical model for the identification of the material constant in $W_\infty$ in Eq. (1), which, in this work, is assumed to be known. The total stresses are obtained through the superimposition of the viscoelastic and hyperelastic ones. This simplification holds in the linear model used for the characterisation, and has shown to be a good approximation of the linear response regime. The dynamic model is obtained through the linearisation of the model around the reference configuration of the experiments. In the analytical model, the circular membrane has diameter $2a$ and radial prestretch $\lambda_p$. The initial thickness is $H$ and, assuming incompressibility, the thickness of the stretched membrane is $h = H\lambda_p^{-2}$. The deformed shape of the membrane, when inflated, is assumed to by spherical.

From Newton’s second law for a membrane element of infinitesimal area subjected to a uniform pressure difference $\Delta P$ and uniform tension $T$, we obtain

$$\rho h \frac{\partial^2 z}{\partial t^2} = -\frac{2T}{R} + \Delta P,$$

where $\rho$ is the membrane density, $z$ is the vertical displacement and $R$ is the local curvature radius. In the case of actuated DEs, the tension $T$ is the sum of the mechanical hyperelastic and viscoelastic in-plane stresses and the in-plane Maxwell stress for incompressible materials, $-\sigma^2$ (Suo, 2010).

Consider next a reference static configuration identified by the parameters $\Delta P_0$, $T_0$, $z_0$ and $R_0$. Expanding (12) from the reference configuration and linearising it is obtained

$$\rho h \frac{\partial^2 z}{\partial t^2} = -\frac{2\delta T}{R_0} - \frac{2T_0}{R_0^2} \delta R + \delta P,$$

where $\delta T$ is the variation of in-plane tension considering the hyperelastic, viscous and Maxwell contributions and $\delta R$ is the variation of $R$ due to $z$. The variation of the hyperelastic stresses, in the range of validity of this linearised model, is
negligible, thus \( \delta T \) is written as the sum of the contribution of viscoelastic and Maxwell stresses. Linearising the expression of \( \delta R \), and moving into the Laplace domain, we obtained
\[
\mathcal{L}(\delta R) = \frac{z^2_0 - a^2}{2z_0^2} \mathcal{L}(z),
\]
(14)
where \( \mathcal{L}(\bullet) \) identifies the Laplace transform. Considering a linearised form of the viscoelastic stresses, \( \mathcal{L}(\delta T) \) can be written as
\[
\mathcal{L}(\delta T) = \mathcal{L}(\delta \sigma_v) + \sum_{a=1}^{N} \Omega(x_a, s) \mathcal{L}(z),
\]
(15)
where \( N \) is the number of viscoelastic mechanisms considered, \( \Gamma = \Gamma(a, h_0, R_0) \) is a geometrical function depending on the initial conditions that linearises the variation of the length of the spherical cap, \( \Omega(x_a, s) \) is the transfer function from the membrane stretch \( (\Gamma \mathcal{L}(z)) \) to the viscous stresses, and \( \delta \sigma_v \) refers to the variation of the Maxwell stresses due to the actuation with the voltage \( \Phi \)
\[
\sigma_v = -\epsilon \left( \frac{\Phi}{H_p^2} \right)^2 H_p^2 a^2.
\]
(16)
The viscoelastic model proposed in Section 2.2 is linearised and coupled with the evolution law giving the following expression for \( \Omega(x_a, s) \) in (15):
\[
\Omega(x_a, s) = \frac{\mu_0 s}{T_a + s}.
\]
(17)
Substituting (15) and (14) into the Laplace transform of in (13) the linear dynamics of the system is
\[
\mathcal{L}(z) = \frac{2 \mathcal{L}(\sigma_{\text{max}}) R_0^{-1} + \mathcal{L}(\delta P)}{\rho \text{s}^2 + \sum_{a=1}^{N} \Omega(x_a, s) \mathcal{L}(z) - \frac{2T_0 z_0^2 - a^2}{R_0^2}}
\]
(18)
Eq. (18) represents the linear response of the inflated membrane when a variation in pressure and voltage is applied over the reference configuration considered. A voltage or a pressure input is characterised by the same transfer function, except for a constant gain for the electrical input, \(-2\epsilon \Phi^2 / HR_0\).

The fluid used in the experiment is air and it will be modelled as an ideal gas. Since the membrane is deforming, the enclosed volume of fluid in the camber will also vary, defining a variation of the internal pressure. Considering a linearised expression for the variation of the fluid volume with the displacement of the membrane \( z \) we can write
\[
\mathcal{L}(\delta P_i) = -\frac{\pi}{2} (a^2 + z_0^2) \frac{P_0}{V_0} \mathcal{L}(z),
\]
(19)
where \( \delta P \) denotes the variation of the pressure in the fluid due to the displacement of the membrane and \( P_0 \) and \( V_0 \) are the initial air pressure and volume. Coupling (18) with (19) we have
\[
\mathcal{L}(z) = \frac{2 \mathcal{L}(\sigma_{\text{max}}) R_0^{-1} + \mathcal{L}(\delta P)}{\rho \text{s}^2 + \sum_{a=1}^{N} \Omega(x_a, s) \mathcal{L}(z) - \frac{2T_0 z_0^2 - a^2}{R_0^2} - \frac{\pi}{2} (a^2 + z_0^2) \frac{P_0}{V_0}}
\]
(20)
This analytical result can be now used to reproduce those in the experiments of Fox and Goulbourne, 2009. In the experimental set-up considered, a circular prestretched membrane is inflated with a bias pressure \( \Delta P_0 = 80 \text{ Pa} \) and then actuated with a harmonic voltage. The VHB4905 membrane has a radius \( a = 88.9 \text{ mm} \) with a radial prestretch \( \lambda_p = 3.5 \). The applied voltage amplitude goes from 1 to 5 kV. The influence of the initial air volume will be calculated with a parametric study. For a detailed description of the experiments the reader is referred to the work of Fox and Goulbourne (2008, 2009). Using (20), the transfer function of the membrane is defined. Since the variation of the elastic stresses is negligible in the linear model, the dynamics of the system will depend only on the variation of the viscoelastic and Maxwell stresses. From this reference condition, the membrane is then actuated with a sinusoidal voltage with amplitude \( \Phi_0 = 1.5 \text{ kV} \) and various frequencies, \( f_v \). The Maxwell stress is
\[
\sigma_v = -\epsilon \left( \frac{\Phi_0}{H_p^2} \sin (2\pi f_v t) \right)^2 = -\epsilon \left( \frac{\Phi_0}{H_p^2} \right)^2 \frac{1 - \cos (4\pi f_v t)}{2}.
\]
(21)
The resulting Maxwell stress has a constant mean value \(-2\epsilon \Phi_0^2 / H_p^2 \), which is added directly to the initial tension of the reference configuration. The final input in the system is then only a time-dependent signal that is evolving at twice the voltage actuation frequency \( f_v \).

From the experimental data by Fox and Goulbourne (2008, 2009), amplitudes and phase delays of the membrane response are obtained and used in a least-square fitting problem to identify the viscoelastic coefficients in the transfer function (20). Only one viscoelastic mechanism was found to be necessary for the applications in this work. The coefficients
are \( \mu_0 = 8.14 \text{ MPa} \) and \( \tau_0 = 5.40 \times 10^{-4} \text{ s} \), values which are in the range of the parameters proposed by Mokarram et al. (2012), who have obtained a similar model for uni-axial loading conditions. Since the constitutive model will be used for membrane wings, which are dominated by out of plane loading conditions, the model from Mokarram et al. (2012) will predict a different membrane response as experimentally observed by Li et al. (2013). The results of the constitutive model just derived are plotted against the experimental data from Fox and Goulbourne (2008, 2009) in Fig. 1 for a purely mechanical and a voltage actuated case. The comparison shows a good agreement for both low and high frequencies. The first natural frequency of the membrane from the dynamic linearised model is found to be 72 Hz, which is close to the 70 Hz reported in the experiments.

The impact of the rate-dependent effects on the global (linear) dynamic behaviour is shown in Fig. 2a. The plot compares the transfer functions of the amplitude \( z \) for a voltage input \( \Phi \) in the case of the elastic (blue dashed line) and viscoelastic (red solid line) material models defined from Eq. (20). The main effect of the rate-dependent stresses is observed at the larger frequencies, while for slower dynamics viscoelastic mechanisms can fully relax. Importantly, viscoelastic effects increase the natural frequency of the membrane, and, for higher frequencies, make the structure effectively stiffer and damp the amplitude of the oscillations. It is assumed that the values obtained through the material identification present a certain amount of uncertainty. In Fig. 2a it is shown that the effect of an uncertainty of 20% in both coefficients of the viscoelastic model on the dynamic behaviour of the membrane. It can be seen that the main differences appear at the resonance frequency of the system, but are still a small deviation from the reference values as compared with the difference with the purely hyperelastic case.

The membrane model derived in this section is obtained from the linearisation of both the geometrical formulation (for the calculation of the membrane length) and of the material formulation (linearisation of the viscoelastic and hyperelastic stresses). However, the constitutive model identified via the analytical model will be used in a non-linear description when evaluating the performance of the actuated wing. Fig. 2b compares the membrane response in the linear (analytical) and non-linear (finite-element) dynamic models. The comparison shows a very good agreement between the analytical and finite-element solutions, with an increasing discrepancy when moving towards the system resonance (due to the larger...
oscillations, which amplify non-linear effects). It has to be noted that, for the FE model, the Bode plot has been obtained running a simulation for each frequency considered with a voltage amplitude of \( \Phi_{FE} = 2 \text{kV} \) and calculating the ratio of the amplitude of the steady state response over \( \Phi_{FE}^2 \). In this sense, the finite-element solution depends on the amplitude of the applied voltage, but has also showed a good agreement with the analytical result. This is justifying the use of a linear model for the material identification, allowing a very low computational cost of the least-square fitting procedure. For frequencies close to the resonance, which is the limit of validity of the analytical solution, there is a discrepancy of around 20%.

4. Coupled electro-aeromechanical model

The main characteristics of the electro-aeromechanical model used in this section have been presented in Buoso and Palacios (2015), where its implementation has been verified against the relevant literature and the effect of actuation on the aerodynamic performance has been investigated. However, those results assumed a purely hyperelastic material, and that will be modified here to include the viscoelastic constitutive behaviour characteristic of DEs.

The fluid solver is based on the finite volume discretisation of the unsteady Navier–Stokes equations implemented in STAR-CCM+ (CD-Adapco, 2013). The Reynolds number is limited to 2500, allowing the direct implicit integration of the unsteady equations without turbulence models or subgrid schemes. Second-order accuracy in time and space is considered. The fluid domain extends 100 chords in every direction and it is schematically represented in Fig. 3, together with a zoom on the wing. The leading and trailing edges are modelled as rigid supports, whose detailed geometry is not considered in this work. The membrane walls are modelled with a no-slip boundary condition. The outer boundaries of the domain extend for 100 chords length in every direction. At the domain inlet the velocity magnitude and direction are set as boundary conditions, while at the outlet a constant pressure is specified. The other two boundaries are modelled as free-stream boundaries, where the direction of the velocity, the Mach number and the pressure are specified. The domain is discretised with a structured mesh with 400 elements in every direction from the wing to the boundaries, and 400 elements on the membrane walls. The resulting mesh consists of one million elements, whose smaller dimension, on the membrane walls and wake, is 50 \( \mu \text{m} \) end exponentially grows towards the boundaries. The domain size and mesh density have been selected after a convergence exercise which considered mean value, amplitude and frequency of the oscillations of the aerodynamic coefficients of the wing.

The implicit dynamic structural model includes a geometrical nonlinear membrane solver based on 3D solid hybrid elements from Abaqus (Dassault System, 2013). The model accounts for the thickness degree of freedom of the structure to allow the definition of the full Maxwell stress tensor (Buoso and Palacios, 2015). Also when the aerodynamic loading determines small membrane deformations, which could be captured with a linear constitutive model, when considering actuated cases the large deformations that can be obtained requires the use of a non-linear material description. This is demonstrated in Fig. 4 which compares the deformation of a circular inflated membrane considering non-linear and linearised material descriptions coupled with a non-linear geometric formulation. The prestretch is \( \lambda_0 = 1.02 \), which provides the linearisation condition of the material model. From the comparison it is clear that the linearised material description suffices for the analysis of the passive response, but when considering actuated cases the assumptions lead to significant discrepancies in the final membrane deformation. Since the prestretch level, aerodynamic loading and voltage-to-thickness ratio of the actuation case are similar to those of the wing considered in this work, a non-linear material description is then used throughout the analysis.

Leading and trailing edges are pinned at the lower edge and periodic boundary conditions are imposed at the lateral edges to preserve the two-dimensionality of the problem. The constitutive model includes the rate-dependent material
behaviour introduced in Section 2. A total of 800 elements are used in the chordwise direction, which gave convergence of the dynamic properties of the structure, and only one in the thickness direction.

A common interface to exchange field data is defined for both models. The pressure and viscous stresses on the membrane surfaces are passed from the fluid to the structural model in the form of nodal forces interpolated using the shape functions of the elements of the finite-element solver. The displacements and velocities of the membrane, after the calculation of the equilibrium configuration, are passed in the other way. An implicit time-marching coupling is considered to account for the strong fluid–structure coupling. The coupling time step is $5 \times 10^{-4}$ s with a maximum of 10 subiterations for step convergence.

5. Numerical results

The first set of results that will be presented in Section 5.1 will investigate the effect of viscoelastic stresses on the aerodynamics of non-actuated elastomeric membranes for different levels of prestretch. Section 5.2 presents the effects of integral actuation for a very compliant case, which is a typical example of membrane wings’ applications. All results are non-dimensionalised with the chord length, $c$, free stream velocity, $V_\infty$, flow density, $\rho$, and angle of attack, $\alpha$. In particular, frequencies are expressed in terms of the chord-based Strouhal number, $St = f_c / V_\infty$, and time histories are expressed as function of the non-dimensional time $t^* = tV_\infty / c$. This approach, which is often adopted in the fluids community, allows to relate the frequencies of the flow features to the characteristic frequency of the Kármán vortex street, which shows a strong correlation with Reynolds number and angle of attack. For a flexible membrane, the dynamics of the system, including vortex shedding, are determined by the interaction of the fluid and structural fields. As it will be seen from the numerical results proposed in this section, for the same Reynolds number and angle of attack, the shedding frequency strongly depends on the membrane characteristics. This has been experimentally observed by Arbos-Torrent et al. (2013) and Gursul et al. (2014), among others.

5.1. Passive wing

A 2D wing in a 2500 Reynolds number air-flow is considered. The free-stream velocity is $V_\infty = 1.45$ m/s, the chord length is $c = 0.03$ m and the initial thickness is $h/c = 1.6 \times 10^{-3}$. The material considered is the acrylic DE of Section 3. The values of membrane prestretch, $\lambda_0$, will be chosen between 1.02 and 1.50 to investigate the influence of rate-dependent effects in relation to the wing compliance. For the hyperelastic material model, only the two limiting values are investigated, $\lambda_0 = 1.02$ and 1.50, which correspond to a very compliant and stiff cases, respectively. Two angles of attack are considered, $\alpha = 4^\circ$ and $8^\circ$. For the smallest angle, a steady state equilibrium configuration is observed for all prestretch levels. Both the hyperelastic and viscoelastic constitutive models, when the same prestretch is considered, give the same deformed shape because in a static case rate-dependent effects are not active. For $\lambda_0 = 1.02$, the maximum mean camber amplitude is $\gamma/c = 1.7\%$ at $x/c = 0.42$. For the stiffer wing, because of the reduced compliance, the camber amplitude is reduced to $\gamma/c = 0.17\%$, with a maximum amplitude point at $x/c = 0.41$. The corresponding mean lift coefficients are $C_l = 0.47$ for $\lambda_0 = 1.02$ and $C_l = 0.39$ for $\lambda_0 = 1.50$. Both values of lift are quite close, indicating that for such small cambers, the incidence angle is the main contributor to the lift of the wing. This is in agreement with the experimental findings from Attar et al. (2012), who found a reduced influence of the level of prestrain in the $C_l$ values for low angles of attack and Reynolds numbers.

In the second flow condition investigated, for $\alpha = 8^\circ$, self-excited oscillations of the membrane are observed due to the shedding of leading edge vortices (LEV), which are characteristic of thin profiles and flat plates in low Reynolds number flows (Yarusevych et al., 2009). The proximity of the vortex to the wing, together with the reduced pressure gradient from the membrane shape adaptation, allows the flow to reattach at about 50% of the chord. In this case, the presence of higher modes has been experimentally observed (Huang and Lin, 1995; Huang et al., 2001). Fig. 5 shows the computed mean membrane shapes and in Fig. 6 the non-dimensional amplitude of oscillations over the mean value is plotted against the non-dimensional chord wise position $x/c$ and time $t^*$. Comparing the two hyperelastic cases, red circles and stars in Fig. 5

![Fig. 4. Static membrane displacement for the linearised and non-linear hyperelastic constitutive models coupled with a non-linear geometric description.](image)
and Fig. 6a and j, reveals that increasing the prestretch leads to a smaller mean camber, in agreement with the experimental results of Song et al. (2008). A reduction of the amplitude of the structural oscillations of an order of magnitude is also observed. The maximum amplitude drops from $\gamma = 3.16$ for to $\gamma = 0.47$ with a chord-wise position of the maximum amplitude of $x/c = 0.43$. Both displacement histories show the dominance of the first membrane mode, which couples with the shedding of the LEV. The dynamics is similar to the results from the experimental work by Song et al. (2008) and Rojratsirikul et al. (2009), who observed a predominant first membrane mode for low Reynolds numbers and angles of attack.

Critically, in this dynamic case the contribution of rate-dependent effects can lead to very different dynamics between the two constitutive models. The influence of the viscous components in the material is investigated for different levels of prestretch in the range of values considered for the hyperelastic cases. For the lower value, $\lambda_0 = 1.02$, the two constitutive models show a markedly different behaviour. The maximum camber of the membrane mean shape is around 12% smaller because of viscoelastic effects. Also the amplitude of the oscillation is reduced by an order of magnitude (Fig. 6a and b), and the dominant modes switches from the first to the second one. Since viscous stresses depend on rate-dependent effects, the large strain rates of the deformations in the viscoelastic case define large contribution of viscoelastic stresses with a resulting high damping of the oscillatory behaviour. This explains the lower amplitude, but to have an insight of the mechanism determining the mode switch it is necessary to consider also the flow-field characteristics, which will be done shortly.

Increasing the level of prestretch determines, on the one side, a stiffening of the structure visible from the reduction in the mean deformed shape in Fig. 5, and on the other a reduction of the effects of viscous stresses, because of the smaller strain-rates. This results in an increasing contribution of the first membrane mode in the global structural evolution, and a reduction of the influence of the second one. A local peak in the amplitude of the oscillations is identified for $\lambda_0 = 1.05$, indicating a strong coupling with the flow dynamics. The membrane behaviour gradually converges to the one of the stiffer wing with the increase of the prestretch, and for $\lambda_0 = 1.50$ the hyperelastic and viscoelastic constitutive models behave in the same manner (Fig. 6j and i).

In order to investigate the effects of viscoelastic stresses into the aerodynamic performance of the wing, the instantaneous lift coefficient, $C_l$, is here considered as a global metric of flow behaviour. As expected from experimental observations (Attar et al., 2012), the mean $C_l$ for the hyperelastic cases is very similar regardless of the prestretch levels, around 0.85. The dominant frequency of the evolution increases with prestrain, from $St = 0.4$ to 0.5, but in can always be associated to the coupling of the first membrane mode with the evolution of the LEV. For the two hyperelastic cases in the figure, a relatively small second peak can also be identified in their spectral content, which corresponds to the coupling of the second structural mode with the second shedding mode. $C_l$ histories and the spectral content for the other viscoelastic cases, for $\lambda_0$ from 1.02 to 1.10, are shown in Fig. 7. After these values of prestretch, the lift coefficient does not show major differences in frequency content or amplitude, and gradually converged to the stiffer case. For this reason, and for clarity, those evolution histories have not been included in the plot.

For the most compliant case of this work, $\lambda_0 = 1.02$, the mean lift coefficient with a viscoelastic material model is $C_l = 0.74$. The dynamic response shows low amplitude oscillations with a dominant frequency at the second system resonance, $St = 1.1$, as seen in Fig. 7b. The signal shows only a small contribution of the first aeroelastic frequency which becomes more and more dominant as the prestretch increases, as seen in the structural deformations in Fig. 6. The excitation of the first resonance determines higher oscillations of the $C_l$ and a larger mean value, which reaches its peak for $\lambda_0 = 1.05$ with $C_l = 0.91$. This coincides with the peak in membrane oscillations discussed above. A further increase in the prestretch leads to a converging behaviour of the lift evolution to the rigid case and a shifting of the resonant peaks towards higher frequencies.

To provide a better understanding of the different system behaviour when considering the two constitutive modelling assumptions, Fig. 8 compares the mean values, $C_l$, and reduced frequencies, $St$, of the lift evolution for the hyperelastic and viscoelastic constitutive assumptions for $\alpha = 8^\circ$. A logarithmic scale has been used for the prestretch to avoid the clustering of the results for the more compliant values. As previously observed, the mean values converge to the same behaviour for increasing values of prestretch, while showing a noticeable reduction of the $C_l$ for the viscoelastic case and large membrane

![Fig. 5. Mean wing shape comparison for the hyperelastic and viscoelastic wings, Re = 2500, $\alpha = 8^\circ$.](image-url)
The analysis of the main reduced frequencies of the response shows, for both constitutive models the net division of the frequency values associated to the first, second and third aeroelastic modes, respectively. For the more compliant cases rate-dependent effects determine a stiffening of the aeroelastic system, in accordance with the predictions of the analytical model in Section 3. The system response frequencies for the two constitutive models converge for stiffer wings and approach the values of the rigid wing case for large values of prestretch.

5.1.1. Analysis of coupled system dynamics

It is interesting to understand the physical mechanisms behind the consistent change in the global aeroelastic performance when introducing viscoelastic effects. As observed by Song et al. (2008) membrane wings are very sensible to slight variations in the flow conditions, and in particular to the vortex shedding. They argued that the shedding frequencies, in a rigid case, are well-defined parameters of the problem but, in an aeroelastic case, the system will respond as a resonator tuning its behaviour to the closest mode in frequency, as it has been identified in the numerical results of this work.

Thus, the flow characteristics have been identified with a simulation of a rigid flat plate of the same geometry of the undeformed wing for \( \alpha = 8^\circ \). The lift coefficient evolution history and its spectral content are shown in Fig. 9. Fig. 10 shows the snapshots of the vorticity field around the wing for the minimum (Fig. 10a) and maximum (Fig. 10b) values of the lift coefficient history.

The flow structures around the wing, shown in Fig. 10, are similar to the ones observed by Gursul et al. (2005), defined as “dual vortex” structure, which is typical of cases with low incidence angles. The vortex shed from the leading edge, because of the proximity with the wing surface, interacts with the boundary layer of the suction surface giving rise to boundary layer separation and the formation of a vortex of opposite sign vorticity. The presence of reattached flow has been seen to be one of the causes leading to the presence of higher shedding modes in fluid (Huang and Lin, 1995; Huang et al., 2001).

The main contribution of the first shedding frequency at \( \text{St} = 0.62 \), which is determining the large amplitude oscillation of the lift coefficient, is clearly visible. Higher modes in the shedding, which have a smaller influence of the global aerodynamic performance, are at a \( \text{St} = 1.26 \) and \( \text{St} = 1.8 \). This is in agreement with the experimental results from Rojratsirikul et al. (2009), who found that for low Reynolds and prestretched membranes, the shedding frequency is around 0.5–0.6 when coupled with the first membrane mode. A second mode, with a St number close to unity was also found for many angles of attack and it is usually related to the shedding due to the shear layer instability. Commonly (Rojratsirikul et al., 2009; Gordnier, 2009), a different formulation of the St number is also proposed considering the vertical distance of the leading- and trailing-edge in the normal direction to the flow, \( \text{St}_v = \text{St} \sin(\alpha) \). With this metric, the shear layer shedding for flat plates and rigid aerofoils, usually falls in the range of 0.16–0.22 (Rojratsirikul et al., 2009). In this work it was found that...
the second mode resonates around $St_d = 0.17$, which is the same value as that obtained experimentally by Rojratsirikul et al. (2009).

When considering a fluid–structure interaction problem, a locking of the system response should be expected around those frequencies. It has been experimentally found that at low Reynolds number and low angles of attack the membrane response is dominated by the first natural mode, which couples with the evolution of the LEV generated from the sharp leading edge of the wing (Song et al., 2008; Rojratsirikul et al., 2010; Arbos-Torrent et al., 2013). The coupling with the first natural mode determines, as seen from the contour plots of Fig. 6, large amplitude oscillations. This has been found to be the main contributor to the lift evolution and has been observed also in the numerical results of this work. In particular, for the $\lambda_0 = 1.05$ case, the enhanced aerodynamic performance are due to the strong coupling of the structural oscillations with the flow dynamics. For the lowest prestretch values, the dominance of the second mode can be linked to the contribution of the viscoelastic stresses. The large damping they introduce in the system prevents the coupling with the first natural mode of the membrane from developing and shifts the dynamics to a characteristic mode which involves

---

![Graph showing lift coefficient for the hyperelastic material model and prestretches from $\lambda_0 = 1.02$ to 1.10, $Re = 2500$ and $\alpha = 8°$.](image-url)

Fig. 7. Lift coefficient for the hyperelastic material model and prestretches from $\lambda_0 = 1.02$ to 1.10, $Re = 2500$ and $\alpha = 8°$. The graphs illustrate the behavior of the lift coefficient ($C_l$) over time ($t^*$) and its spectral content over mean value.
lower displacements. This case shows a similar behaviour of aeroelastic systems with a low Weber number, which measures the ratio between the aerodynamic loading and the membrane stiffness. Song et al. (2008) found that when reducing the Weber number the second natural mode assumes a dominant role in the system evolution. It is opinion of the authors that the switch of the system dynamics to the second mode can be mainly explained by the damping characteristics of viscoelastic stresses. From the purely structural point of view, the first mode for this very compliant case shows large viscous damping, due to relaxation of viscous mechanisms. When moving towards higher frequencies the system exhibits a stiffer behaviour with a reduced effect of damping. For this reason the system can lock the main evolution frequency with the second fluid shedding frequency.

To understand the effects of the switching of the coupling with the first and second modes, the case with \( \lambda_0 = 1.02 \) is investigated more in detail, comparing the hyperelastic and viscoelastic behaviours. The selection of this case has multiple reasons. First of all, this case has shown large differences in the dynamic responses, and supports the need of accounting for
viscoelastic effects in the constitutive model for DE membrane wings. In addition, a higher wing compliance brings significantly positive aerodynamic benefits in the near stall region and under flow disturbances (Rojratsirikul et al., 2009) and therefore it is of particular interest. Results are shown in Fig. 11, which compares the $C_l$ time histories and frequency content of the two cases with $\lambda_0 = 1.02$. In the hyperelastic case, the dominant resonant frequency is at $St = 0.39$, which is linked to the first natural mode. It also shows secondary effects at $St = 0.78$ and $St = 1.3$, related to the excitation of the second and third membrane modes, respectively (Fig. 11b).

For the viscoelastic material model the coupled system evolution is dominated by a frequency at $St = 1.1$ (Fig. 11b), which corresponds to the second membrane mode and has shifted upwards as compared to the hyperelastic case. This is because rate-dependent effects, as observed in Section 2.2, increase system frequencies. In this case, secondary effects are caused by the excitation of the third and first membrane modal shapes at $St = 1.3$ and 0.5, respectively.

Snapshots of the instantaneous vorticity field around the membrane, for the same conditions ($\lambda_0 = 1.02$ and $\alpha = 8^\circ$), are shown in Figs. 12 and 13 for the hyperelastic and viscoelastic material models, respectively. The times shown can be related to local peaks in Fig. 11a. In Fig. 12a the membrane is at its lowest position and generates a leading-edge vortex. It is clearly visible in Fig. 12b and it dominates the system evolution coupling with the first membrane mode and giving rise to large values of structural and $C_l$ oscillations. The LEV travels downstream the wing in successive snapshots, followed by the point of maximum amplitude of the membrane deformation. The amplitude increases and gets to its maximum in Fig. 12b when the vortex detaches. The maximum amplitude point moves then upstream, Fig. 12c, and the membrane quickly returns to its lowest configuration, Fig. 12d, when the vortex is travelling downstream in the wake.

For the viscoelastic material model the vorticity field evolution is shown in Fig. 13, which shows the instantaneous vorticity for local peaks of the lift coefficients in Fig. 11a in a similar time window as in the hyperelastic case. In this case the main contribution is the shedding of vortices from the trailing edge at $St = 1.1$, which couples with the second membrane mode. The snapshots in Fig. 13 show smaller vortices as compared with the hyperelastic case, which develop at twice the frequency (the snapshots cover in fact two oscillation cycles of the dominant frequency). The low pressure regions

---

**Fig. 11.** Lift coefficient for the hyperelastic and viscoelastic material models for $\lambda_0 = 1.02$, $Re = 2500$ and $\alpha = 8^\circ$.

**Fig. 12.** Positive (red) and negative (blue) vorticity contour plot around the wing for the hyperelastic material model, $Re = 2500$, $\alpha = 8^\circ$ and $\lambda_0 = 1.02$. Link to animated version: Re2500_alpha8_prestretch_102_elastic.gif. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)
generated by the vortices in Fig. 13a and c are smaller than the one from the strong LEV in Fig. 12, and in Fig. 13b and d it can be seen that the coherence of the vortical structures is much weaker in Fig. 12b. The smaller oscillations generated by this second modal shape are predominant as compared to the first natural one, and the separation vortices they generate reduce the effect of the first mode shedding on the wing.

In fact, these smaller vortices break the large structural coherence of the leading-edge vortex, reducing its strength and weakening its positive contribution on the aerodynamic performance. As a result, lift time histories show a lower mean value, the absence of the large amplitude oscillations due to the leading-edge vortex and an increased amplitude of the oscillations due to the shedding of the small vortices associated with the second shedding mode. Similar observations have been experimentally made by Song et al. (2008), who have highlighted that the presence of separated flow regions on the suction side weakens the leading-edge vortex and its effect on the membrane.

5.2. Actuated wing

It has been demonstrated that viscoelastic effects can significantly modify the system dynamics for non-actuated wings, in particular for compliant cases. It was also stated that, a compliant membrane is beneficial for the global aerodynamic performance, so it is finally of interest to investigate the effect of viscoelastic stresses for the actuated wing. The lowest prestretch considered in Section 5.1, $\lambda_0 = 1.02$, will be investigated. The angle of attack selected is $\alpha = 4^\circ$, which defines a static equilibrium in the reference condition. A static case has been selected as reference to remove from the reference condition the differences due to rate-dependent effects. The actuation signals are harmonic functions with a constant amplitude $\phi_0 = 500$ V and a frequency ranging from $St_v = 0$ to $St_v = 0.3$. For the static actuation case, $St_v = 0$, the amplitude of the voltage considered is $\Phi_{static} = \phi_0 / \sqrt{2}$, which corresponds to the mean voltage amplitude in the dynamic cases. The variation of the mean lift coefficient and the amplitude of the oscillations of the instantaneous $C_l$ is shown in Fig. 14 as function of the actuation voltage $St_v$, under hyperelastic and viscoelastic material assumptions.

As observed for the passive case, for a static actuation both constitutive models lead to the same performance, because of the absence of rate-dependent effects. The increased wing compliance due to the actuation determines a higher lift coefficient, with an increase of 23%. When the wing is dynamically actuated, rate-dependent effects have a major contribution in the system dynamics, markedly modifying the wing aerodynamic performance. In the hyperelastic case, the initial drop in the mean lift coefficient is progressively compensated when the actuation frequency moves towards the resonance peak of the system, at $St = 0.3$, which gives an increase of about 30% of $C_l$. For frequencies above the first system resonance, the aerodynamic performance rapidly degrade. This is likely associated to the local peak in the amplitude of the oscillations of the $C_l$, suggesting that large amplitudes introduce large average drag. These observations are in agreement with the experimental results from Attar et al. (2012). The lift coefficient has another peak around the second resonant frequency, with $\Delta C_l / C_l \approx 15\%$. Although this is a local maximum, the value is substantially lower than the static case, and hence suggests that for this low Reynolds number flow, a fast actuation will reduce its effectiveness for membrane wings showing hyperelastic material behaviour. When rate-dependent effects are included, as seen in the passive case, the system dynamics is significantly modified. The mean $C_l$ value shows to be almost independent on the actuation frequency in the range of $St$ numbers considered, with a value of the average increment that sets around 21% above the reference condition. Moreover, $C_l$ oscillations are much reduced as compared to the hyperelastic case, as it is seen in Fig. 14b. The maximum instantaneous $C_l$ obtained through the actuation is about 50%, lower than the hyperelastic case, which was found to be up to 200%. The global
behaviour of the wing for the viscoelastic constitutive model shows then a low sensibility to the actuation frequency. These observations are in good agreement with the experimental results from Curet et al. (2014), who have also found that, for low angles of attack, the DE wings exhibited a very low dependence on the actuation frequency.

The large differences in the system dynamics for the two constitutive models can be explained using the analytical structural models of Section 3. In fact, the investigation of the structural dynamics alone provides very good insight into the main dynamic performance of the fully coupled actuated system. The model derived in Section 3 is therefore modified for an actuated 2D membrane wing. The reference pressure distribution used is \( \Delta P = \frac{1}{2} \rho V^2 Cl \). For the passive case and \( \alpha = 4^\circ \) this is \( \Delta P \approx 1 \) Pa. The actuation of the wing determines a relaxation of the membrane tension, because of the non-zero mean value. Its effect is included in the initial configuration, assuming that the reference pressure remains the same, although this is strictly valid only under linear assumptions. Fig. 15 shows the structural linear frequency response function between applied voltage and the actuated wing for both the elastic and viscoelastic material behaviours. For low enough frequencies, the effect of viscoelastic stresses is negligible, as for the inflated circular membrane. When the forcing frequency gets close to the resonance, the viscoelastic wing dynamics is significantly damped. The effect of the mean value of the actuation, \( \Phi_0/\sqrt{2} \), is essentially to increase the membrane compliance, which increases the amplitude of the displacements for both static and dynamic actuation and reduces the natural frequencies of the system. For large values of \( \Phi_0 \), the structural system behaves like a first-order low band pass filter.

To verify the trends predicted by the structural analytical model, the aeroelastic wing is actuated with three signals of amplitudes \( \Phi_0 = 150, 250 \) and \( 500 \) V, respectively, and a linearly varying frequency \( St_v \) between 0 and 0.8 in a simulation time window of \( \Delta t^* = 90 \). The frequency content of the resulting signal spans up to \( St_v = 3.3 \). Results are shown in Fig. 16 were the lift coefficient histories are compared. In the hyperelastic case and for the lower voltage (Fig. 16a) the maximum increase of the \( Cl \), close to the first resonance point, is about 11%. This is reduced to 3% for the viscoelastic model. Note also that both signals have the same mean value, which is only 1% higher than the reference value with no actuation. This confirms that for low amplitude oscillations the time-averaged response is directly linked to the mean value of the actuation signal. The prediction of the linearised hyperelastic model is obviously overestimating the amplitude of oscillations at the resonance point, but in the viscoelastic case a good correlation can be identified between the high-fidelity fully coupled model and the trend from the analytical linear model. At higher frequencies viscoelastic effects play a dominant role in the dynamic response of the aeroelastic system, reducing the system oscillations and increasing the value of resonant frequencies.

If the amplitude of the actuation is increased to \( 250 \) V, Fig. 16b, the mean \( Cl \) increases by around 3% as with respect to the passive case. This is consistent with the predicted behaviour from the linearised model. In addition, the amplitude of the oscillations is increased and the resonance frequencies are reduced for both the elastic and viscoelastic wings. The ratio
between the mean values is roughly the same as the second power as the ratio of the mean amplitudes, strong linearity in the problem. A similar behaviour is shown by the viscoelastic wing.

When increasing further the amplitude to 500 V, Fig. 16c, the system behaviour considerably diverges from the linear assumption in the elastic case and shows a broader frequency spectra related to the excitation of the higher membrane modes and a stronger interaction with the fluid. For the viscoelastic material model, instead, the oscillation amplitude remains bounded to a much lower value, around 30% of the initial $C_l$, in agreement with the prediction of the linearised model.

6. Conclusions

Viscoelastic effects, which are negligible in steady state case and at low frequencies, can play a dominant role in the dynamic response of actuated DE wings, which grows as the compliance increases. In the dynamic cases, the increased in-plane tension due to the resultant viscoelastic contribution reduces the mean camber and increases the coupling frequency with the flow. Coupling phenomena and flow characteristics obtained from the numerical model were found to be in agreement with the experimental results in the literature and highlight the need for an accurate material model in the numerical prediction of the complex fluid–structure interaction around compliant membranes. In particular, it has been shown how rate-dependent stresses can switch the dominant coupling mode of the membrane with the vortex shedding. In the configuration that was analysed, the switch was from the first to the second system modes. In the actuated case, the overall aerodynamic behaviour of the DE wing benefits from viscoelastic stresses that reduce the oscillations and avoid the significant loss in performance that is observed in the elastic case for high actuation frequencies.

In the context of membrane control, dynamic actuation can still produce an increment of 30% in the instantaneous lift coefficient. This is lower than what would be obtained under elastic material assumptions, which indicates a reduced authority of the control under dynamic integral actuation. However, this degradation of performance is balanced by the reduced sensibility of the viscoelastic wing to flow disturbances, as seen from the linearised model proposed in this work. As a result, viscoelastic damping determines a more stable case with a higher overall efficiency. As it has been seen, the strong couplings and complex dynamical response of the resulting system make high-fidelity computational models a key ingredient in the development of DE-based actuation for MAV wings. This paper highlights and quantifies the large impact that the assumptions on the wing material can have on the prediction of the overall system response.

Fig. 16. Lift coefficient evolution of the actuated wing for the hyperelastic and viscoelastic constitutive models. $\Phi = \Phi_0 \sin(2\pi St_v t^*)$, with $St_v$ linearly varying from 0 to 0.8.
Acknowledgements

The authors acknowledge the financial support of the European Office of Aerospace Research & Development Grant FA8655-12-1-204 of the US Air Force Office of Scientific Research and the UK Engineering and Physical Sciences Research Council Grant EP/J002070/1, “Towards Biologically-Inspired Active-Compliant-Wing Micro-Air-Vehicles”. All results in this paper are available as open data. Information on access can be found at http://www.imperial.ac.uk/aeroelastics/software.

References


Huang, R., Shyy, W., 2007. Laminar-turbulent transition of a low Reynolds number rigid or flexible airfoil. AIAA Journal 45, 1501–1513.


