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Droplet Preferential Concentration in Homogeneous and Isotropic Turbulence

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submitted in partial fulfilment of the requirements for the degree of Doctor of Philosophy of the Imperial College London
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Declaration of Originality

The work included in this thesis, unless where the reference is mentioned, is the result of the study carried out by Huan Lian.
Abstract

In particle-laden turbulent flow, it has been found both experimentally and numerically that when the particle response time is similar to a turbulent characteristic timescale, particles tend to preferentially concentrate and form clusters. This phenomenon of non-uniform particle dispersion has been referred as preferential concentration.

The thesis studies experimentally the preferential concentration of poly-dispersed droplets in homogeneous and isotropic turbulence generated in the facility referred to as the ‘box of turbulence’ and includes comparisons with Direct Numerical Simulations (DNS). It discusses the effect of poly-dispersion on droplet preferential concentration, temporal evolution of droplet clustering and the turbulent mechanisms (i.e. topological turbulent flow patterns) that may be responsible for the droplet clustering dynamics.

The thesis is structured into six chapters. Motivations, theoretical background and related literature of this work are discussed in Chapter 1. Chapter 2 describes the experimental setup and the applied laser diagnostic techniques. Chapter 3 focuses on the effect of poly-dispersion on droplet preferential concentration. The techniques used in quantifying the preferential concentration are the Radial Distribution Function (RDF) and Voronoï analysis. An image processing method for locating droplets from droplet Mie-scattering images has been proposed and evaluated. Chapter 4 reports the time-resolved dispersion measurements of poly-dispersed droplets. The fine scale topological turbulent patterns (i.e. zero velocity/acceleration) are extracted from the fluid flow velocity measurements, considering the effect of experimental noise, and are observed to follow a non-uniform spatial distribution and form clusters. The clustering of zero velocity/acceleration points are quantified
by RDF and Voronoï analysis and compared with the dispersed droplet clusters. A cluster identification method based on the mean shift pattern space analysis and the Voronoï tessellation has been proposed and applied to all the temporally resolved images to obtain cluster time scale and length scale statistics. Chapter 5 compares the results from experiments and corresponding DNS calculations using the same data processing methods. The clustering of experimentally and numerically acquired zero velocity/acceleration points and dispersed droplets are quantified and compared. Chapter 6 is the conclusion of the thesis and possible directions of future work.
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To my families
Contents

List of Figures 5

List of tables 15

1 Introduction 17
   1.1 Motivation ........................................ 17
   1.2 Theoretical Background .......................... 22
       1.2.1 Turbulence .................................. 22
       1.2.2 Dispersed Phase Properties ................. 28
       1.2.3 Phase Coupling ............................... 32
   1.3 Experimental Turbulence Generation Facilities 36
   1.4 Aim of the Present Work ........................... 40
   1.5 Outline of the Thesis ............................... 46

2 Experimental Setup 49
   2.1 ‘Box of Turbulence’ Facility ....................... 49
   2.2 Particle Image Velocimetry (PIV) .................. 53
   2.3 Laser Doppler Anemometry (LDA) ................... 55
   2.4 Phase Doppler Anemometry (PDA) ................... 58
   2.5 Gaussian Beam Optics .............................. 61
   2.6 Optical Arrangement for the Experiment .......... 63
CONTENTS

2.7 Summary ....................................................... 64

3 Clustering of Poly-dispersed Droplets .................. 65

3.1 Experimental Arrangement ............................... 66

3.1.1 Turbulence Generation ................................. 67

3.1.2 Turbulence Characterisation .......................... 68

3.1.3 Droplet Generation and Characterisation ........... 80

3.2 Mie-Scattering Intensity Image Processing: Bandpass Filter ........ 86

3.2.1 Bandpass Filter ......................................... 87

3.2.2 Selection of Bandpass Filtering Cut-off Value ....... 89

3.3 Quantifying Droplet Clustering ........................... 95

3.3.1 Radial Distribution Function (RDF) .................... 95

3.3.2 Voronoï Analysis ......................................... 97

3.3.3 Stokes Number Scaling for Poly-dispersed Droplets .... 99

3.3.4 Effect of Turbulent Reynolds number ................. 104

3.4 Discussion on Effect of Droplet Poly-dispersity ....... 105

3.4.1 Stokes Number Distribution ............................ 106

3.4.2 Radial Distribution Function (RDF) .................... 108

3.4.3 Voronoï Analysis ......................................... 108

3.5 Summary ....................................................... 111

4 Time Resolved Statistics .................................. 114

4.1 Evaluation of Turbulent Topological Characteristics .... 115

4.1.1 Evaluation of Turbulent Topological Characteristics .... 116

4.1.2 Effect of Experimental Noise on the Topological Structure and Digital Filtering ......................... 119

4.1.3 Discussions on the number of zero velocity points .... 139

4.2 Cluster Identification and Time-dependant Tracking .... 141
CONTENTS

4.2.1 Mean Shift Feature Space Analysis .................................. 143
4.2.2 Droplet Cluster Identification ........................................ 149
4.2.3 Time-dependent Droplet Cluster Tracking ............................. 154
4.2.4 Optimised Bandwidth Selection ...................................... 157
4.3 Time-resolved Investigation of Clustering of Droplets ............... 159
4.3.1 Experimental Setup ..................................................... 159
4.3.2 Turbulent Flow Characteristics ...................................... 160
4.3.3 Droplet Size Distribution ............................................. 168
4.3.4 Radial Distribution Function (RDF) .................................. 168
4.3.5 Vorotoï Analysis ....................................................... 172
4.3.6 Droplet Cluster Identification and Temporal Tracking ............. 181
4.4 Time-resolved Investigation of Clustering of Turbulent Zero Veloc- 
ity/Acceleration Points .................................................. 185
4.4.1 Radial Distribution Function (RDF) .................................. 185
4.4.2 Vorotoï Analysis ....................................................... 187
4.4.3 Turbulent Zero Velocity/Zero Acceleration Points Cluster 
Identification and Temporal Tracking ................................... 190
4.5 Droplet Clustering Self-similarity and Sweep-Stick Mechanism ...... 192
4.5.1 Clustering Self-similarity ............................................... 195
4.5.2 Sweep-Stick Mechanism ............................................... 197
4.6 Summary ............................................................................ 199

5 Comparison between Experiments and DNS ............................. 206
5.1 Direct Numerical Simulation (DNS) ....................................... 208
5.1.1 DNS of Mallouppas et al. (2013a) (Imperial) ..................... 208
5.1.2 Johns Hopkins University Turbulence Database (JHTDB) .... 210
5.1.3 Topological Structure of Turbulence ................................. 211
5.2 Quantitative Comparison with Experiment ............................. 212
List of Figures

1.1 Preferential concentration observed experimentally and numerically. (a) Vortical flow structures in multi-phase flows can influence the spatial distribution of the dispersed phase, causing large concentration fluctuations (Hardalupas and Horender, 2003a); (b) Mono-size particles clustering and centrifuging observed in DNS (Fevrier et al., 2005) . . 18
1.2 Four group combustion modes of a droplet cloud (Chiu et al., 1982) . 19
1.3 Schematic of energy cascade . . . . . . . . . . . . . . . . . . . . . . . . 24
1.4 Length scales at very high Reynolds number . . . . . . . . . . . . . 26
1.5 Map of flow regimes in particle-laden flows (Elghobashi, 1991) . . . 33
1.6 Schematic of the coupling effect . . . . . . . . . . . . . . . . . . . . . 33
1.7 Schematic of the box counting methods . . . . . . . . . . . . . . . . . .35
1.8 Schematic of the Radial Distribution Function (RDF) . . . . . . . . 36
1.9 Schematic of a wind tunnel (Monchaux et al., 2010) . . . . . . . . 37
1.10 Schematic of counter-rotating disk device (a) pumping mode; (b) shear mode (La Porta et al., 2000) . . . . . . . . . . . . . . . . . . . . 38
LIST OF FIGURES

1.11 Schematic of the existing ‘Box of Turbulence’ (a) Birouk’s closed box with eight fans in the cubic corner (Birouk et al., 1996); (b) Hwang’s closed box with eight speakers in the cubic corner (Hwang and Eaton, 2004); (c) Goepfert’s open chamber box with six woofers (Goepfert et al., 2009); (d) Charalampous’s open chamber box with eight woofers (Charalampous and Hardalupas, 2010) ........................................... 39

1.12 Zone classification in a typical plane. light grey with dots: convergence zones; dark grey with dots: eddy zones; light grey: stream zones; dark grey: rotational zones (Squires and Eaton, 1991) ........................................... 42

1.13 Particle density contours in an x-y plane for St = 0.52 (Squires and Eaton, 1991) ........................................... 43

1.14 Zone classification and particle number density contours, zone colour same as Fig. 1.12 (Squires and Eaton, 1991) ........................................... 43

1.15 DNS spatial dispersion of inertial particles and turbulent stagnations points (Goto and Vassilicos, 2006) (a),(c),(e) Spatial distribution of inertial particles St = 1.9; (b) (d) (f) Spatial distribution of zero-acceleration points ........................................... 44

2.1 Schematic of perforated PVC plate with 55 holes of diameter of 6mm arranged in triangular mesh pattern ........................................... 50

2.2 Experimental setup of ‘box of turbulence’ (a) schematic of the box of turbulence; (b) experimental setup of the box of turbulence ........................................... 51

2.3 Woofer control strategy ........................................... 52

2.4 Setup of 2D PIV system (Adrian and Westerweel, 2011) ........................................... 54

2.5 Schematic of Doppler shift of moving particles ........................................... 55

2.6 Fringe pattern in the intersection area of LDA measurement ........................................... 57

2.7 Block diagram of a Laser Doppler Anemometry using monomode laser diodes for frequency shift generation (Tropea, 1995) ........................................... 57
LIST OF FIGURES

2.8 Phase Doppler Anemometry Alignment (a) geometry and co-ordinate system of the PDA. (b) arrangement of the collection apertures of the receiving optics (c) droplet in the probe volume with Gaussian intensity profile (Hardalupas and Liu, 1997) .......................... 60

2.9 Diameter of a Gaussian beam (Griot, 2007) ........................................ 61

2.10 Gaussian beam propagation in changing wavefront radius (Griot, 2007) 61

2.11 Gaussian beam focusing and formation of the beam waist (Griot, 2007) 62

2.12 Light sheet optics using three cylindrical lenses (Raffel et al., 2007). 63

3.1 Schematic illustrating the definition of temporal mean, spatial mean and ensemble mean of velocity obtained by 2D PIV ........................................ 69

3.2 Two-point velocity correlation coefficient ............................................ 72

3.3 Mean velocity contours (a) $Re_{\lambda} = 107$; (b) $Re_{\lambda} = 145$; (c) $Re_{\lambda} = 185$; (d) $Re_{\lambda} = 213$ .......................................................... 75

3.4 R.m.s velocity ratio (a) $Re_{\lambda} = 107$; (b) $Re_{\lambda} = 145$; (c) $Re_{\lambda} = 185$; (d) $Re_{\lambda} = 213$ .......................................................... 76

3.5 Probability distribution of normalised fluctuating velocity (a) $Re_{\lambda} = 107$; (b) $Re_{\lambda} = 145$; (c) $Re_{\lambda} = 185$; (d) $Re_{\lambda} = 213$ .......................................................... 78

3.6 Two dimensional energy spectrum (a) $Re_{\lambda} = 107$; (b) $Re_{\lambda} = 145$; (c) $Re_{\lambda} = 185$; (d) $Re_{\lambda} = 213$ .......................................................... 79

3.7 Number weighted droplet size distribution (a) $Re_{\lambda} = 107$; (b) $Re_{\lambda} = 145$; (c) $Re_{\lambda} = 185$; (d) $Re_{\lambda} = 213$ .......................................................... 82

3.8 Number weighted droplet size cumulative distribution (a) $Re_{\lambda} = 107$; (b) $Re_{\lambda} = 145$; (c) $Re_{\lambda} = 185$; (d) $Re_{\lambda} = 213$ .......................................................... 83

3.9 Number-weighted droplet size distribution for the case with Sauter mean diameter of 45 $\mu$m with vertical lines indicating the various characteristic droplet diameters (Lian et al., 2013) ................................ 84
LIST OF FIGURES

3.10 Experimental droplet images (a) A typical recorded Mie-scattering image of droplets in the ‘box of turbulence’ (b) Band-pass filtered image based on the Gaussian and boxcar convolution ................................................................. 90
3.11 Bandpass filtered droplet images (a) Bandpass filtered image based on the Gaussian and boxcar convolution; (b) Locations of local intensity maxima indicating individual droplet, used for the quantification of droplet clustering ........................................................................................................ 90
3.12 Non-dimensional universal image quality index $Q$ as a function of convolution kernel spread ................................................................................................................................. 92
3.13 Average number of droplets detected with the convolution kernel spread as $\pm 5\sigma$, $\pm 6\sigma$ and $\pm 7\sigma$ ........................................................................................................................................... 93
3.14 Averaged RDF with the convolution kernel spread as $\pm 5\sigma$, $\pm 6\sigma$ and $\pm 7\sigma$. The radial distance is normalised by the Kolmogorov length scale $\eta$ 94
3.15 RDFs for different droplet size distributions and flow with the turbulent Reynolds number of 185 ................................................................................................................................. 96
3.16 Flow with turbulent Reynolds number of 185 (a) Normalized PDFs of Voronoï areas for different droplet size distributions. The PDF for RPP is superimposed as dashed line; (b) Standard deviation of normalized Voronoï area as a function of droplet Sauter mean diameter. The standard deviation for RPP is 0.53. ................................................................. 98
3.17 Standard deviation of the Voronoï area as a function of droplet Stokes number, defined according to various representative diameters of the droplet size distribution, for flows with turbulent Reynolds numbers of 107, 145, 185 and 213 ................................................................................................................................. 102
LIST OF FIGURES

3.18 Variation of standard deviation of Voronoi areas as a function of droplet Stokes number, based on $DN_{60\%}$, for flows with turbulent Reynolds number of 107, 145, 185 and 213. Error bars indicate the statistical uncertainties for a confidence level of 95% ............................. 104

3.19 Probability of droplet Stokes number for experiment with $Re_\lambda = 107$ ................................. 107

3.20 RDFs of droplets generated by ultrasonic atomizer and air-assist atomizer ............................... 109

3.21 Voronoi analysis (a) P.d.f.s of normalised Voronoï area; (b) P.d.f.s of the standard deviation of normalised Voronoï area .................................. 109

4.1 The topological categories of the critical points in two dimensional velocity field (Perry and Chong, 1987; Cardesa et al., 2013) ......................... 118

4.2 The effect of adding Gaussian noise of different variance $\sigma$ to the dataset on velocity invariants joint p.d.f.s (Buxton et al. (2011) Fig.11(c)) .... 121

4.3 The joint p.d.f.s of velocity invariants of homogeneous, isotropic turbulence from JHU DNS simulation with added Gaussian noise and Salt & Pepper noise ...................................................... 125

4.4 Power spectral density estimation with optimal Wiener Filter .................. 128

4.5 The joint p.d.f.s of velocity invariants of homogeneous, isotropic turbulence from experiment, processed by Median Filter, Wiener Filter and linear coupling of Median Filter and Wiener Filter ......................... 131

4.6 Instantaneous zero velocity points identified with different Median Filter window size. (a) raw PIV data; (b) velocity field filtered by $2 \times 2$ window sized Median Filter; (c) velocity field filtered by $3 \times 3$ window sized Median Filter; (d) velocity field filtered by $5 \times 5$ window sized Median Filter; (e) noise free DNS data .............................. 133

4.7 Quantification of clustering of turbulent zero mean velocity with Radial Distribution Function (RDF) ....................................................... 135
4.8 Quantification of clustering of turbulent zero mean velocity with Voronoï Analysis ................................................. 137
4.9 The number of zero velocity points per unit volume versus $L/\eta$ in DNS (solid circle) and KS for $p = 5/3$ and different values $V/u'$; Eq. 4.16 is shown as dashed lines (Davila and Vassilicos, 2003) .................. 139
4.10 p.d.f.s of the zero velocity/acceleration points measured by 2D PIV ........................................... 140
4.11 Human visualisation of temporal evolved droplets with corresponding Voronoï cells. spatial resolution is 45.5 $\mu m$, 900 pixels corresponds to approximately 4.1 $mm$, about 250 times of the Kolmogorov length scale of the presented turbulent flow .................... 142
4.12 Example of a feature space. (a) A $400 \times 276$ color image. (b) Corresponding l*u*v color space with 110,400 data points (Comaniciu and Meer, 2002) ........................................... 144
4.13 Illustration of Mean Shift feature space analysis (a) Spatial representation of 110,400 sample points; (b) Decomposition based on the mean shift algorithm; (c) Trajectories of the mean shift procedure, the red dots of centre of mass define the decomposition (Comaniciu and Meer, 2002) ........................................... 148
4.14 Mean shift approach combined with Voronoï analysis for droplet cluster identification for 2 consecutive droplet distribution images with time difference of $3 \times 10^{-4}$ s ............................................. 150
4.15 Examine mean shift cluster identification scheme (a) Experimental droplets cluster identification (c) Normalised Voronoï area p.d.f. for clustering droplets (e) Cluster length scale p.d.f. for clustering droplets; (b) Randomly distributed particles and cluster identification (d) Normalised Voronoï area p.d.f. for random particle distribution (f) Cluster length scale p.d.f. for random particle distribution ............. 153
LIST OF FIGURES

4.16 Histogram of cluster area in 9 consecutive images with time difference 
\[ dt = 0.0003s \] .................................................. 155

4.17 Bhattacharyya coefficient of probability distribution of cluster areas in 
9 consecutive frames with time difference \[ dt = 0.0003s \] .............. 156

4.18 Bhattacharyya coefficient for experiment with droplets dispersed in 
flow with \( Re_\lambda = 235 \) and droplet \( D_{32} = 55 \mu m \) .......... 157

4.19 Time-resolved experimental setup of the ‘Box of turbulence’ ........ 160

4.20 Mean velocity contours (a) \( Re_\lambda = 97 \) (b) \( Re_\lambda = 127 \) (c) \( Re_\lambda = 147 \) (d) 
\[ Re_\lambda = 235 \] .................................................. 162

4.21 R.m.s velocity ratio (a) \( Re_\lambda = 97 \) (b) \( Re_\lambda = 127 \) (c) \( Re_\lambda = 147 \) (d) 
\[ Re_\lambda = 235 \] .................................................. 163

4.22 Probability distribution of normalised fluctuating velocity (a) \( Re_\lambda = 
97 \) (b) \( Re_\lambda = 127 \) (c) \( Re_\lambda = 147 \) (d) \( Re_\lambda = 235 \) ............. 164

4.23 Two dimensional energy spectrum (a) \( Re_\lambda = 97 \) (b) \( Re_\lambda = 127 \) (c) \( Re_\lambda = 147 \) (d) \( Re_\lambda = 235 \) ............. 165

4.24 Number based droplet size probability (a) \( Re_\lambda = 97 \) (b) \( Re_\lambda = 127 \) (c) 
\[ Re_\lambda = 147 \) (d) \[ Re_\lambda = 235 \] .................................................. 169

4.25 Number based droplet size cumulative distribution (a) \( Re_\lambda = 97 \) (b) 
\[ Re_\lambda = 127 \) (c) \[ Re_\lambda = 147 \) (d) \[ Re_\lambda = 235 \] ............. 170

4.26 Radial Distribution Function representing droplet clustering for the 
four turbulent conditions (a) \( Re_\lambda = 97 \) (b) \( Re_\lambda = 127 \) (c) \( Re_\lambda = 147 
(d) \[ Re_\lambda = 235 \] .................................................. 173

4.27 Probability distribution function of the normalised Voronoi area (a) 
\[ Re_\lambda = 97 \) (b) \[ Re_\lambda = 127 \) (c) \[ Re_\lambda = 147 \) (d) \[ Re_\lambda = 235 \] ............. 174
LIST OF FIGURES

4.28 Temporal evolution of the standard deviations of the normalised Voronoï areas for the experiment $Re_\lambda = 147$, droplet $St = 1.18$, time step $= 3.3 \times 10^{-4}$ s, represented as a function of the time normalised by the Kolmogorov time scale $\tau_k = 0.003$s ........................................... 175

4.29 Probability of standard deviation of normalised Voronoï area (a) $Re_\lambda = 97$ (b) $Re_\lambda = 127$ (c) $Re_\lambda = 147$ (d) $Re_\lambda = 235$ ......................... 176

4.30 Autocorrelation coefficient functions of the fluctuation of the standard deviation of the normalised Voronoï area for the four turbulent flows and different droplet size distributions. Droplet Stokes number calculated based on the arithmetic mean droplet diameter $D_{10}$. (a) $Re_\lambda = 97$ (b) $Re_\lambda = 127$ (c) $Re_\lambda = 147$ (d) $Re_\lambda = 235$ ......................... 178

4.31 Fourier spectrum of the standard deviation of the normalised Voronoï area for the four turbulent flows and different droplet size distributions. Droplet Stokes number calculated based on the arithmetic mean droplet diameter $D_{10}$. (a) $Re_\lambda = 97$ (b) $Re_\lambda = 127$ (c) $Re_\lambda = 147$ (d) $Re_\lambda = 235$ ................................. 180

4.32 Standard Deviation of normalised Voronoï area for turbulent Reynolds number $Re_\lambda = 97, 127, 147$ and $235$ ............................................. 181

4.33 Probability distribution of number of droplet clusters per image ($45mm^2$) identified by the mean shift combined with Voronoï cluster identification technique. (a) $Re_\lambda = 97$; (b) $Re_\lambda = 127$; (c) $Re_\lambda = 147$; (d) $Re_\lambda = 235$ ................................. 183

4.34 Probability distribution of droplet cluster length scale derived by the mean shift combined Voronoï cluster identification technique. (a) $Re_\lambda = 97$; (b) $Re_\lambda = 127$; (a) $Re_\lambda = 147$; (a) $Re_\lambda = 235$ ......................... 184

4.35 Radial Distribution Function (RDF) of zero velocity points, zero acceleration points ................................................. 186
LIST OF FIGURES

4.36 P.d.f.s of normalised Voronoï areas ........................................ 187
4.37 Auto-correlation coefficient of the fluctuations of standard deviation of
normalised Voronoï areas of zero velocity/zero acceleration points of
the turbulent flow ................................................................. 188
4.38 Fourier spectra of fluctuations of standard deviation of normalised
Voronoï areas of zero velocity/zero acceleration points of the turbulent
flow .................................................................................. 189
4.39 Standard deviation of normalised Voronoï areas of zero velocity/zero
acceleration points of the turbulent flow .................................... 190
4.40 P.d.f.s of cluster length scales of zero velocity/zero acceleration points
of the fluid flow turbulence based on mean shift analysis ............ 191
4.41 DNS pair correlation function of (a) inertial particles $St = 0.4$,
(b) zero-acceleration points (solid circle) and zero-velocity points (circle),
(c) inertial particle ($St = 1.9$, solid circle) and zero-acceleration points
(circle) (Chen et al., 2006) .................................................... 193
4.42 P.d.f.s of identified void area of dispersed particles (a) result of (Boffetta
et al., 2004) based on box counting method (b) result of Monchaux et al.
(2010) based on Voronoï analysis .............................................. 195
4.43 Voronoï analysis in identifying void areas (Monchaux et al., 2010) .. 196
4.44 Normalized void area p.d.f.s of droplet clusters in ‘box of turbulence’
defined by Voronoï analysis (a) $Re_\lambda = 97$; (b) $Re_\lambda = 127$; (c) $Re_\lambda =
147$; (d) $Re_\lambda = 235$ ......................................................... 197
4.45 Radial Distribution Function of dispersed droplets and turbulent
stagnation points measured in the ‘box of turbulence’ (a) $Re_\lambda = 97$; (b)
$Re_\lambda = 127$; (a) $Re_\lambda = 147$; (a) $Re_\lambda = 235$ ......................... 198
LIST OF FIGURES

4.46 Probability distribution function of the normalised Voronoi area of dispersed droplet and turbulent stagnation points measured in the ‘box of turbulence’ (a) $Re_\lambda = 97$ (b) $Re_\lambda = 127$ (c) $Re_\lambda = 147$ (d) $Re_\lambda = 235$

5.1 Joint p.d.f.s of velocity invariants $p$ and $q$. (a) JHU forced isotropic turbulence of $Re_\lambda = 433$; (b) Malloupas et al. (2013a) forced isotropic turbulence $Re_\lambda = 108$

5.2 One example spatial distribution of zero velocity points (a) JHU forced isotropic turbulence $Re_\lambda = 433$; (b) Imperial forced isotropic turbulence $Re_\lambda = 100$

5.3 Radial Disstribution Function (RDF) of DNS and experiment

5.4 Probability of normalised Voronoï areas of zero velocity/acceleration points, dispersed droplets from DNS and experiment

5.5 Autocorrelation coefficient of time-resolved fluctuating standard deviation of normalised Voronoï area

5.6 P.d.f.s of typical cluster length scale identified by mean shift algorithm
List of Tables

1.1 Definitions of mean droplet diameters ........................................ 30

3.1 Turbulent velocity statistics measured by 2DPIV with repetition rate  
of 10 Hz .................................................................................. 74

3.2 Turbulent quantities measured by 2DPIV with repetition rate of 10 Hz 78

3.3 Characteristics of droplets present in the turbulent flow with $Re_\lambda =$  
107, 145, 185 and 213, diameters unit in $\mu m$ ................................. 85

3.4 Settling velocity of dispersed droplets ........................................... 86

4.1 Turbulent velocity statistics measured by 2DPIV with repetition rate  
of 1500 Hz .............................................................................. 161

4.2 Turbulent quantities measured by 2DPIV with repetition rate of 1500  
Hz .......................................................................................... 166

4.3 Statistics of the invariants of the reduced velocity gradient tensor  
measured by 2DPIV with repetition rate of 1500 Hz ......................... 167

4.4 Statistics of the principal strain rate measured by 2DPIV with  
repetition rate of 1500 Hz ...................................................... 168

4.5 Characteristics of droplets present in the turbulent flow with $Re_\lambda =$  
97, 127, 147 and 235, diameters unit in $\mu m$ ................................. 171

5.1 Multi-phase DNS simulation parameters (Mallouppas et al., 2013a) . 209

5.2 Turbulent and dispersed droplet quantities of Imperial DNS dataset . 209
5.3 JHTDB forced isotropic turbulence DNS parameters .......... 210
5.4 Turbulent quantities of JHTDB DNS dataset ................. 211
Chapter 1

Introduction

1.1 Motivation

Liquid-gaseous two phase flows are generally involved in various industrial and environmental applications including spray combustion, rocket propulsion, spray drying, pharmaceutical process, and warm rain initiation in convective cloud. The mechanism of each application remains complicated for various physical processes involved. For example, the complexity of studying \textit{spray combustion} mechanism is raised by several interacting physical processes apart from the two-phase flow interactions between droplets and turbulent flow, including spray formation, droplet breakup, droplet evaporation, heat transfer and flame propagation. To fully understand the spray combustion mechanism, each process needs to be studied separately to provide quantitative information before considering coupling effects between them. This thesis is particularly focused on studying the mechanism of the two-phase flow interactions between droplets and turbulent flow. Many interesting and fundamental questions regarding droplets dispersion due to turbulence are investigated, addressed and discussed.

Droplet dispersion is affected by the surrounding turbulence which is known
to be characterised by a series of *length and time scales* (Tennekes and Lumley, 1982). The droplet dynamics vary with different droplet properties (i.e. droplet size, droplet density, etc.), turbulent eddy length scales and the relative droplet-fluid velocities. When the droplet response time is much smaller than the turbulent characteristic time, the droplets can follow the surrounding flow well and result in spatially uniform dispersion. Similarly, when the droplet response time is much larger than the turbulent characteristic time, the droplets are very little affected by turbulence and tend to disperse uniformly within flow field. However, despite the traditional theory which suggests that turbulence applies a random force on the dispersed phase (Tchen, 1947), resulting in a statistical random dispersion, it has been found both experimentally and numerically (Maxey, 1987; Squires and Eaton, 1991; Eaton and Fessler, 1994) that, when droplet response time is similar to turbulence characteristic time, droplets tend to preferentially concentrate and form clusters at regions of low vorticity and high strain rate. The Stokes number defined in Eq. 1.21 is used to characterise the dispersion behavior. This phenomena of non-uniform particle dispersion has been widely studied in the past two decades and referred as *preferential concentration* Fig. 1.1.

*Figure 1.1:* Preferential concentration observed experimentally and numerically. (a) Vortical flow structures in multi-phase flows can influence the spatial distribution of the dispersed phase, causing large concentration fluctuations (Hardalupas and Horender, 2003a); (b) Mono-size particles clustering and centrifuging observed in DNS (Fevrier et al., 2005)
Preferential concentration and the formation of clusters has profound influence on spray combustor performance (Zimmer et al., 2003), particle settling (Aliseda et al., 2002; Wang and Maxey, 1993), and the development of convective cloud in warm rain initiation process (Sundaram and Collins, 1997; Shaw et al., 1998; Siebert et al., 2010).

For spray combustion, the formation of clusters leads to higher local concentration and local void region with variations to the local mean number density, which can influence the liquid-fueled combustor performance through fuel oxidizer process, which has a direct impact on combustion efficiency and exhaust emission. In a fuel dense region, the oxidizer can not mix and interact with each fuel droplet and its ambient fuel vapor, resulting a cloud of droplets burn as a group. This phenomenon was described and defined by Chiu et al. (1982) as group combustion.

![Figure 1.2: Four group combustion modes of a droplet cloud](Chiu et al., 1982)

Four different group combustion modes are determined by a single group
combustion number $G$, defined as

$$G = \frac{t_c}{t_g} = \frac{4\pi a_c^2 an}{3} = \left(\frac{a_c}{a_g}\right)^2$$

(1.1)

where $t_c = \frac{a_c^2}{D_T}$ is the diffusive time of heat into the droplet cloud, $t_g = (4\pi D_T an / 3)^{-1}$ is the characteristic time scale of mass and thermal energy variation in the ambient vaporized liquid fuel gas, $a_c$ is the radius of quasi-stationary monodispersed droplet cloud, with $n$ droplets and each droplet radius of $a$ and density $\rho_d$. $D_T$ is the oxidizing gas thermal diffusivity with a density of $\rho$. $a_g = (4\pi na / 3)^{-1/2}$ is the characteristic thermal penetration length in the droplets cloud.

According to the definition Eq.1.1, large values of $G$ indicate restrained combustion and vaporization with long heat diffusive time, small values of $G$ imply enhanced combustion and evaporation with a short diffusive time. The four combustion modes illustrated in Fig. 1.2 are

- $G < 10^{-2}$: Single droplet combustion, with a complete combustion.
- $10^{-2} < G < 1$: Internal group combustion, with inner vaporization preheating zone and outer combustion zone.
- $10^{-1} < G < 1$: External group combustion, with limited vaporization and combustion compared with the internal group combustion
- $G > 10^2$: External sheath combustion, combustion occurs in a thin outer layer of fuel vapor

The preferential concentration, which of the focus of this thesis, modifies the distribution of liquid fuel droplets in the ambient turbulent gaseous flow, which could shift the combustion mode from one to another. Thus the understanding of
the preferential concentration is essential in studying the spray combustion at the fundamental level.

For the particle settling velocity, Aliseda et al. (2002) set up experiments in a horizontal wind tunnel with isotropic decaying turbulence with conveying mean velocity and studied the effect of preferential concentration on the settling velocity of heavy particles. It was found that clusters greatly enhance the particle settling velocity. Local concentration map was generated for illustration and a ‘box counting’ analysis method was applied to quantify preferential concentration.

For the development of convective cloud in the warm rain initiation process, Sundaram and Collins (1997) studied the inter-particle collisions as a function of turbulence parameters and particle properties numerically with DNS simulation. Turbulence was set to be isotropic with small mean velocity, and mono-dispersed particles were used for the collision study. Statistical description of how particle preferential concentration and particle de-correlation affects collision frequency was given for the whole Stokes number (defined in Eq.1.21) range. It was suggested that the collision frequency depends on both particle size and preferential concentration, which was quantified by using the Radial Distribution Function (RDF). Shaw et al. (1998) suggested that turbulent-induced preferential concentration could affect supersaturation of clouds which can greatly enhance droplet growth in warm rain initiation process. This study was based on DNS simulation of isotropic turbulence with constant vertical velocity in cumulus cloud and poly-dispersed cloud droplets. Siebert et al. (2010) reviewed recent field and laboratory studies towards understanding the role of turbulence on droplets in clouds and suggested that droplet acceleration in cloud due to interactions with flow turbulence exceed the acceleration due to gravity. Also, intermittency at small turbulent scale was confirmed to exist in stratocumulus cloud.

Thus, the cluster formation and interactions with turbulence is important for
the understanding of spray combustion, droplet settling and the development of convective cloud in the warm rain initiation process. To fully understand the two-phase flows physical process described above, knowledge of the characteristics of each phase is required thus being presented followed by information on coupling effects and interactions between them.

In this Chapter, theoretical background of turbulence is briefly summarized first with emphasis on different turbulent lengths and time scales. Descriptions of dispersed phase properties and settling velocity parameters, including particle size distribution, particle response time, particle Stokes number are stated afterward. With separate information of each phase, coupling effects are then defined and statistical quantification techniques on the preferential concentration effect are provided. A review of literature with emphasis on experimental turbulence generation methods and recent findings on preferential concentration is summarized followed by the aim of the present work and the outline of the thesis.

1.2 Theoretical Background

1.2.1 Turbulence

Turbulence Definition Turbulent flows are not a feature of fluids but of fluid flows (Tennekes and Lumley, 1982). Turbulence is created from instabilities of laminar flows and generally characterised as random, continuous, diffusive, three-dimensional and dissipative. A dimensionless number (Eq.1.2) introduced by Osborne Reynolds (Reynolds, 1985) has been used to differentiate laminar and turbulent flows, which is considered as the ratio of inertia to viscous terms. Turbulence occurs at larger Reynolds numbers.

\[ Re = \frac{UL}{\nu} \]  

(1.2)
In the above equation, $U$ is a characteristic velocity of the flow, $L$ is a characteristic macro-scale length scale and $\nu$ is the kinematic viscosity.

For appropriately large values of the turbulent Reynolds number, laminar flows can transit to turbulent flows. The transition can be prevented by energy dissipation, a mechanism that consumes turbulent kinetic energy. Turbulence is not able to maintain itself and would decay back to laminar flows without sufficient energy input.

The random nature of turbulence is directly revealed by random flow velocity fluctuations. The velocity is decomposed into mean and fluctuating components (Eq.3.3) for all subsequent statistical study.

$$u(t) = \bar{u} + u'$$  \hspace{1cm} (1.3)

where $\bar{u}$ represent the time averaged mean velocity and $u'$ donates the fluctuation component. Detailed definition of averaging process is included in Chapter 3 Fig. 3.1.

The mathematical approach used to describe random distributed quantities is referred as Reynolds decomposition. By applying Reynolds decompositions, Navier-Stokes equations can be simplified and the non-linear term Reynolds stress $\tau_{ij}$, that gives rise to the turbulence, can be extracted.

**Turbulent Scales** Richardson introduced the concept of energy cascade based on observations of different size eddies spread across the turbulent flows, as shown in Fig. 1.3.

Energy cascade outlines the turbulence structures that consist of eddies with different size. The dimension of the largest eddy is comparable with the characteristic length scale of turbulent flow, while most of the eddies are much smaller and the smallest eddy size depends on the Reynolds number of turbulence. The largest eddies break up into smaller ones because of inertial instability. Smaller eddies break up into even smaller structures while the energy is passed from the larger turbulent eddies to smaller ones without viscosity dissipation. When the eddies are too small for
specific turbulent Reynolds number, energy dissipates due to viscosity. Richardson summarised this matter in 1922 as *Big whorls have litter whorls, which feed on their velocity; and little whorls have lessor whorls, and so onto viscosity.* The life span for the energy containing eddies with specific dimension is called *turn-over-time* defined in Eq.1.4

\[ \tau = \frac{l}{u'} \]  

(1.4)

where \( l \) is the length scale of the energy containing eddies and \( u' \) is the characteristic velocity of turbulence.

Kolmogorov (1941) studied *energy cascade* theory and proposed three hypotheses to determine the smallest eddies size, providing a more vivid picture of turbulent energy flux.

*Kolmogorov’s hypothesis of local isotropy:* At sufficiently high Reynolds number, the small scale \((l \ll l_{0})\) turbulent motions are statistically isotropic. Local isotropy hypothesis means small eddies are isotropic and not affected by boundary conditions. A length scale \( l_{EI} \) is introduced to marginate anisotropic and isotropic scale range. The scale \( l < l_{EI} \) is generally referred as *universal equilibrium range.*
Kolmogorov's first similarity hypothesis: In every turbulent flow at sufficiently high Reynolds number, the statistics of the small-scale motions have a universal form that is uniquely determined by $\nu$ and $\varepsilon$. With the first similarity hypothesis, Kolmogorov microscale $\eta$ representing the smallest dissipative eddy size can be determined by the following definitions along with relevant velocity scale $u_\eta$ and time scale $\tau_\eta$.

\begin{equation}
\eta = \left(\frac{\nu^3}{\varepsilon}\right)^{\frac{1}{4}} \tag{1.5}
\end{equation}

\begin{equation}
u_\eta = (\varepsilon \nu)^{\frac{1}{4}} \tag{1.6}
\end{equation}

\begin{equation}
\tau_\eta = \left(\frac{\nu}{\varepsilon}\right)^{\frac{1}{2}} \tag{1.7}
\end{equation}

Kolmogorov's second similarity hypothesis: In every turbulent flow at sufficiently high Reynolds number, the statistics of the motions at scale $l$ in the range $\eta \ll l_{EI}$ have a universal form that is uniquely determined by $\varepsilon$, independent of $\nu$.

The second similarity hypothesis separates universal equilibrium range into inertial subrange and dissipation range. The marginal length scale is defined as $l_{DI}$. To summarize, Fig. 1.4 shows different eddy sizes with various length scales.

Homogeneous and Isotropic Turbulence Homogeneous and isotropic turbulence (HIT) is the ideal case in the study of turbulence. HIT is especially desirable for the study of multi-phase flows because of simplifications that appear in turbulent models. The definition of HIT is provided below together with practical verification methods based on instantaneous flow field velocity information.

Homogeneity describes the uniformity of turbulence in each direction and isotropy refers to the uniformity in each orientation. Precise definitions of homogeneity and isotropy are defined below (Pope, 2009).
In a simple domain, let $x^0, x^1, ..., x^N$ represent a series of points within the domain. Distance and velocity difference can be defined as

$$y = x - x^0$$  \hspace{1cm} (1.8)

$$v(y) = U(x, t) - U(x^0, t)$$  \hspace{1cm} (1.9)

**Homogeneity:** The turbulence is locally homogeneous in the domain $\varsigma$, if for every fixed $N$ and $y^n$, the $N$-point velocity PDF $f_N$ is independent of $x^0$ and $U(x^0, t)$.

**Isotropy:** The turbulence is locally isotropic in the domain $\varsigma$, if it is locally homogeneous and if in addition the velocity PDF $f_N$ is independent of $x^0$ and $U(x^0, t)$, which donate the position and velocity in any coordinate system obtained by rotation and reflections of the coordinate axes.

According to the definition stated above, the homogeneity and isotropy of turbulence can be verified by comparing velocity PDF with Gaussian distribution profile. Close following of the Gaussian profile suggests homogeneity. In order to statistically quantify the homogeneity of the turbulent flow generated in the centre
area of a ‘box of turbulence’, spatial correlation coefficient of the velocity fluctuations between two neighbouring points are calculated from the instantaneous flow field velocity information. The two-point velocity correlation can be briefly expressed in tensor form as

$$R_{ij}(r, x, t) = \langle u_i(x, t)u_j(x + r, t) \rangle$$

(1.10)

The corresponding two-dimensional longitudinal velocity correlation coefficient in $x$ and $y$ directions are defined as

$$F_{11}(r) = \frac{\langle u_1(x_1, x_2)u_1(x_1 + r, x_2) \rangle}{u_{1,rms}^2}$$

(1.11)

$$F_{22}(r) = \frac{\langle u_2(x_1, x_2)u_2(x_1, x_2 + r) \rangle}{u_{2,rms}^2}$$

(1.12)

where $u_1$ is the velocity component in $x$ direction and $u_2$ is the velocity in $y$ direction. $\langle - \rangle$ suggests spatial and ensemble average described in details in Chapter 3, and subscript $rms$ refers the root mean square of the corresponding velocity fluctuations.

The lateral velocity correlation coefficient in $x$ and $y$ directions are defined as

$$G_{11}(r) = \frac{\langle u_1(x_1, x_2)u_1(x_1, x_2 + r) \rangle}{u_{1,rms}^2}$$

(1.13)

$$G_{22}(r) = \frac{\langle u_2(x_1, x_2)u_2(x_1 + r, x_2) \rangle}{u_{2,rms}^2}$$

(1.14)

A good agreement between the $x$ and $y$ direction of both the longitudinal and the lateral spatial velocity correlation coefficients indicates the flow field to be homogeneous.
1.2.2 Dispersed Phase Properties

**Particle Size Distributions** Dispersed particles are classified into mono-dispersed and poly-dispersed. The mono-dispersed particles tend to have uniform size and the poly-dispersed particles have size distribution with wide range. The size of particles dispersed in carrier phase plays an important role in multi-phase flows.

For studies of atomized liquids in turbulent flows, the droplet size can reflect both the atomising nozzle conditions and specific carrier phase turbulence characterizations. Statistical methods are developed to quantify droplet dimensions. Spherical droplets are characterized by diameters and non-spherical droplets are quantified by a series of equivalent diameters. The discussions below are limited to spherical droplet diameters.

Some statistical parameters used to quantify distribution functions are summarized below, including mode, mean, Sauter mean diameter, median, variance as well as some special applications in liquid atomization.

For droplet diameter distributions, mode suggests the point with maximum frequency.

Mean measures the average of particle diameters. It can also be calculated from the frequency distribution defined below.

\[
\mu = \int_0^{D_{\text{max}}} D f(D) dD
\]  

where, \(D\) is the droplet diameter and \(f(D)\) is the frequency corresponds to certain diameter.

Mean droplet diameter not only provides dimensional information but also indicates droplet surface area, droplet volume characterizations. There are several methods to quantify mean droplet diameter summarized in Table 1.1 (Bayvel and Orzechowski, 1993) in which all the descriptive parameters follow the definitions given
CHAPTER 1. INTRODUCTION

in Eq.1.16.

\[ D_{pq} = \sqrt[p-q]{\frac{\sum_{i=1}^{m} D_i^p \triangle n_i}{\sum_{i=1}^{m} D_i^q \triangle n_i}} \] (1.16)

_Sauter mean diameter_ \( D_{32} \) is widely used in combustion spray studies and can be viewed as the ratio of droplet volume to surface area (Crowe et al., 1997).

_Variance_ describes the spread of droplet sizes. With small variance, droplets tend to be monodispersed and with large variance, droplets are polydispersed and droplet sizes vary over a larger range. The variance can be calculated as below. (Eq.1.17)

\[ \sigma^2 = \int_0^{D_{max}} (D - \mu)^2 f(D) dD \] (1.17)

where \( \mu \) is the _mean_ diameter defined in Eq.1.15.

Generally, droplets can be classified as monodisperse when \( \sigma^2 < 0.1 \).

_Median_ is the diameter when frequency of a cumulative size distribution is 0.5.

A frequently used size distribution function to correlate droplet size measurement is _log-normal distribution_. It is derived from Gaussian distribution defined as

\[ f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{1}{2}\left(\frac{x - \mu}{\sigma}\right)^2\right] \] (1.18)

where \( \sigma \) is the standard deviation and \( \mu \) is the mean.

The log-normal distribution replace the variable \( x \) by log of the droplet diameter and is defined as

\[ f(x) = \frac{1}{\sqrt{2\pi}\sigma_0} \exp\left[-\frac{1}{2}\left(\frac{\ln D - \mu_0}{\sigma_0}\right)^2\right] dD \] \( D \) (1.19)
<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>Symbol</th>
<th>Name</th>
<th>Application</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>$D_{10}$</td>
<td>Arithmetic</td>
<td>Comparison of disperse systems</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>$D_{20}$</td>
<td>Surface</td>
<td>Surface area control, surface phenomena, i.e. absorption, vaporization</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>$D_{30}$</td>
<td>Volume</td>
<td>Volume control, volumetric phenomena</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>$D_{21}$</td>
<td>Relative surface</td>
<td>Drop disintegration, absorption</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>$D_{31}$</td>
<td>Relative volume, Probert’s</td>
<td>Evaporation, molecular diffusion, combustion</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>$D_{32}$</td>
<td>Volume-surface, Sauter’s</td>
<td>Drop range, mass transfer, heat transfer</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>$D_{43}$</td>
<td>Mass, de Brouckere’s or Herdan’s</td>
<td>Drop fractionation, combustion</td>
</tr>
</tbody>
</table>
**Droplet Response Time and Stokes Number** The response time of a droplet to turbulent flow field is an important parameter in the study of multi-phase flows. The momentum response time $\tau_p$ of a droplet refers to the time that a droplet needs to follow a change of velocity in the flow field (Crowe *et al.*, 1998), which is defined as

$$\tau_p = \frac{\rho_p D^2}{18\mu}$$

(1.20)

where $\rho_p$ is the density of dispersed droplets, $D$ is the droplet characteristic diameter and $\mu$ is the fluid dynamic viscosity.

The droplet Stokes number $St$, defined in Eq.1.21, represents the ratio of droplet response time to the turbulent characteristic time scale, where $\tau_p$ is the droplet response time, $\tau_k$ is the turbulent characteristic time scale. The Kolmogorov time scale is widely used as the turbulent characteristic time scale in defining the droplet Stokes number in the community of multi-phase flows.

$$St = \frac{\tau_p}{\tau_k}$$

(1.21)

**Droplet Settling Parameter** The ratio of terminal velocity of droplets $W$ and turbulent flow r.m.s. velocity $u_{i,\text{rms}}$ is defined as the parameter, which describes the effect of gravity on droplets. If the ratio of the droplet terminal velocity and turbulent velocity fluctuations is less than 1, the influence of gravity is not considered significant.

$$S_v = \frac{W}{u_{i,\text{rms}}}$$

(1.22)

The terminal velocity of the droplets $W$ was defined by Maxey (1987) as

$$W = \frac{m_p g}{6\pi \eta D_p} = \frac{\rho_p g d_p^2}{18\nu}$$

(1.23)
where $\nu$ is the dynamic viscosity of turbulent flow, $\rho_p$ is the density of dispersed droplets, $g$ is the gravitational acceleration 9.8 $m/s^2$, and $d_p$ is the droplet diameter.

### 1.2.3 Phase Coupling

The interaction between different phases in multi-phase flows can be classified into *one way*, *two way* or *four way couplings*. If only one phase affects the other without any reversing effect, the phenomenon is referred as *one way* coupling. If there are mutual influences between different phases, the interaction is noted as *two way* coupling. For very dense suspension, when collisions occur in the dispersed phase, the term *four way coupling* is used. Most studies focus on the dilute dispersions, avoiding the complexities raised by reversing effects and collisions. The type of interactions are generally classified in terms of droplet diameter, volume fraction $\Phi_v$ and mass fraction $\Phi_m$. A dimensionless figure (Fig. 1.5) summarizing the monodispersed particel laden flow classification was contributed by Elghobashi (1991).

In Fig. 1.5, the volume fraction $\Phi_v$ is defined as

$$\Phi_v = \frac{MV_p}{V}$$

where $M$ is the number of droplets, $V_p$ is the volume of the droplets, $V$ is the volume occupied by the droplets and the fluid. The distance between the centres of the two neighbouring droplets is represented using $S$. $d$ is the droplet arithmetic mean diameter and $\tau_p$ is the droplet response time, $\tau_k$ is the Kolmogorov time scale and $\tau_e$ is the energy containing eddies turn over time.

The influence of turbulence on the dispersed phase exists in all the three classifications, while the dispersed droplet effect is ignored in one-way coupling. Fig. 1.6 illustrates the coupling effect between the carrier phase and the dispersed phase.
CHAPTER 1. INTRODUCTION

Figure 1.5: Map of flow regimes in particle-laden flows (Elghobashi, 1991)

Figure 1.6: Schematic of the coupling effect
**Preferential concentration** refers to the phenomenon that when the droplet Stokes number around unity, droplets tend to respond to turbulent flow and accumulate preferentially and form clusters in the region of high strain rate and low vorticity. The influence of turbulence is not significant when the droplet Stokes number is much larger than unity, with droplets inertia dominates and thus being little affected by turbulence, or much smaller than unity, with droplets following turbulent flow. In order to quantify the magnitude of preferential concentration, Monchaux *et al.* (2012) provides a good review on the existing techniques. Three of the most common approaches are briefly discussed in the following Section:

- **Box counting.** (Fessler *et al.*, 1994)
- **Radial distribution function (RDF).** (Sundaram and Collins, 1997)
- **Voronoi analysis.** (Monchaux *et al.*, 2010)

**Box counting** The box counting method proposed by Fessler *et al.* (1994) divides an acquired image into a number of uniform size boxes with certain grid length, as shown in Fig. 1.7. The number of droplets in each box is counted for comparison with randomly distributed droplets following the random Poisson distribution.

A statistic descriptive quantity $D$ defined in Eq.1.25 provides information of how strong the preferential concentration or clustering would be.

$$D(l) = \frac{(\sigma_e - \sigma_p)}{\lambda}$$ (1.25)

where $\sigma_e$ is the standard deviation of the experimental distribution and $\sigma_p$ corresponds to the standard deviation of the Poisson distribution. $\lambda$ is the average number of droplets per box. $l$ is the selected box grid length. A large difference away from the random Poisson process represented with large $D$, suggests the presence of strong
preferential concentration.

**Radial distribution function (RDF)** The radial distribution function (RDF) introduced by Sundaram and Collins (1997) is a statistical descriptive quantity in terms of local droplet relative densities. It shows the relative droplet density within a ring of a certain radius and is defined as

$$RDF = \frac{N(r_i)A}{A(r_i)N}$$

(1.26)

where $N(r_i)$ is the number of droplets in a ring with a width of $2dr$ and of the radius $r_i$ from the center chosen randomly as the location of one droplet. $A(r_i)$ is the area between $r - dr$ and $r + dr$. $A$ and $N$ is the total area and number of droplets in the investigated region.

**Voronoi analysis** The Voronoï analysis is classically used to identify galaxy clusters and study granular systems. Monchaux et al. (2010) applied this technique to quantify the droplet preferential concentration and overcomes several disadvantages of the above mentioned methods. Both the box counting method and the RDF require the selection of a particular length scale and the computation efficiency is relatively
low. While the Voronoï analysis does not require any particular length scale. The Voronoï diagram is a method to decompose two dimensional space into a set of independent cells, which are the ensemble of points closer than any others to a certain droplet. The area of each Voronoï cell is the inverse of the local droplet concentration. Therefore, the probability density function (p.d.f.) of the measured Voronoï cell areas can be used as a descriptive quantity of the preferential concentration. Compared with the p.d.f. of the random Poisson process, the experimental p.d.f.s provide quantitative description of the clustering.

1.3 Experimental Turbulence Generation Facilities

Homogeneous and isotropic turbulence, which is defined in the previous Section, has been studied by many researchers due to the simplifications of turbulent models. Although the homogeneous and isotropic turbulence remains a complex non-linear system, it is relatively easier to model as a single phase and also as the carrier phase in the study of multi-phase flows. For this reason, homogeneous and isotropic turbulence has been widely studied numerically mainly with Direct Numerical Simulation (DNS), and Large Eddy Simulation (LES). Despite its popularity in numerical studies, it has long been difficult to generate homogeneous and isotropic
turbulence experimentally. There are two general approaches to generate turbulence. One is to generate spatial decaying turbulence with mean shear stress in wind or water tunnels with active or passive grids (Comte-Bellot and Stanley, 1966; Uberoi and Wallis, 1967). Another approach is to generate statistically stationary turbulence to exclude the mean shear stress with counter-rotating disks in a tank (Douady et al., 1991; La Porta et al., 2000), or with fans or woofers in a cubic box (Birouk et al., 1996; Hwang and Eaton, 2004; Goepfert et al., 2009; Charalampous and Hardalupas, 2010).

For the first category, wind/water tunnel is the main facility with active/passive grid generated turbulence with conveying mean velocity. Large scale turbulence is induced due to mean shear stress. Wind/water tunnels are popular research facilities to study fluid mechanics and the multi-phase flows. Active/passive grids are used to generate turbulence with different characteristics. A schematic showing a typical wind tunnel facility is in Fig. 1.9. The flow field is visualized by laser diagnostic techniques. Descriptions of the optical instrumentation used in this thesis are included in the next Chapter.

![Figure 1.9: Schematic of a wind tunnel (Monchaux et al., 2010)](image)

For the second category, La Porta et al. (2000) described a device with two cylindrical counter-rotating disks. There are two flow generation modes of this
device. Shear flow can be generated with the *shear mode*. Statistically stationary homogeneous and isotropic turbulence can be generated with the *pumping mode*. This device can generate bulky turbulence with large Reynolds number up to a few thousand. A schematic of this ‘disk’ device is shown in **Fig. 1.10**

![Figure 1.10: Schematic of counter-rotating disk device (a) pumping mode; (b) shear mode (La Porta et al., 2000)](image1)

Apart from the counter-rotating device, Birouk *et al.* (1996) proposed the idea of generating the homogeneous and isotropic turbulence with small mean flow to exclude large scale motion of the mean shear in a constrained box with eight fans. Eight fans were placed at each corner of the cubic box as the energy inputs to the system. Hwang and Eaton (2004) developed the idea and replaced the fans with eight woofers. The improvement increased the system stability and enabled generation of intensified turbulence with relative high Reynolds number. Goepfert *et al.* (2009) generated stationary homogeneous and isotropic turbulence in an open chamber with six woofers. The open chamber configuration enabled a better optical access for flow characterization. Charalampous and Hardalupas (2010) developed the design of the box with eight woofers and open chamber. This configuration makes it easier to mount the particle seedings and atomizer vertically while simultaneously maintaining the optical access. Schematic of the four cubic boxes is shown in **Fig. 1.11**.
CHAPTER 1. INTRODUCTION

Figure 1.11: Schematic of the existing ‘Box of Turbulence’ (a) Birouk’s closed box with eight fans in the cubic corner (Birouk et al., 1996); (b) Hwang’s closed box with eight speakers in the cubic corner (Hwang and Eaton, 2004); (c) Goepfert’s open chamber box with six woofers (Goepfert et al., 2009); (d) Charalampous’s open chamber box with eight woofers (Charalampous and Hardalupas, 2010)
CHAPTER 1. INTRODUCTION

The ‘box of turbulence’ is a cubic-shaped device that generates homogeneous and isotropic turbulence with small mean flow. Compared to the turbulence generated by the active/passive grid in the wind/water tunnel, the box of turbulence has the advantage of excluding mean shear stress, which causes large scale motion. By excluding the mean shear further, the configuration simplifies the associated turbulence models which are of particular interest to the study of multi-phase flow interactions.

1.4 Aim of the Present Work

Quantifications of preferential concentration have been widely studied numerically during the past two decades based on Direct Numerical Simulation (DNS). However, experimental study remains scarce (Wood et al., 2005; Salazar et al., 2008; Saw et al., 2008; Monchaux et al., 2010). Wood et al. (2005) compared box counting method (Eaton and Fessler, 1994) and two-dimensional Radial Distribution Function (RDF) (Sundaram and Collins, 1997) in quantifying mono-dispersed solid particle preferential concentration within homogeneous and isotropic turbulence with very small mean flow. Strongest clustering was found to occur on the length scale of 10 times Kolmogorov time scale with particle Stokes number near unity. Salazar et al. (2008) presented three-dimensional RDF quantification of mono-dispersed solid particle preferential concentration within homogeneous and isotropic turbulence with small mean flow by acquiring information with holographic Particle Image Velocimetry (PIV) and compared the experimental results with DNS simulations. Saw et al. (2008) quantified droplet pair correlations to study clustering of dense inertial water droplet generated by four spray nozzles in a wind tunnel with high Reynolds number turbulence. Monchaux et al. (2010) proposed Voronoï analysis to quantify preferential concentration of poly-dispersed droplets generated by air-assist atomizer in isotropic
turbulence with conveying mean flow in wind tunnel. None of those experimental studies considered the influence of particle/droplet size distribution on preferential concentration. However, Zimmer et al. (2003) proposed that the instantaneous spatial distribution of droplet diameters may affect spray combustion behaviour. Warhaft (2009) reviewed the recent advances in the study of droplet motion and suggested that the droplet size influence should be considered and studied systematically. Bordas et al. (2011) presented the study of turbulent influence on droplet diameters generated by air-assist nozzle in the spatial decaying turbulence with conveying mean velocity.

None of these experiments considered the influence of particle/droplet size distribution on preferential concentration. To further contribute to the existing literature, this thesis studies the effect of poly-dispersity of liquid droplets on preferential concentration. Study of droplet size distribution with and without turbulence will be presented and the role of droplet size in the formation of clusters will be discussed.

The aim of the thesis is to understand the droplet preferential concentration within homogeneous and isotropic turbulence, so the underlying mechanism that governs the preferential concentration is of great interest. Monchaux et al. (2012) produced a good review on the preferential concentration including a summary of the physical mechanisms that were studied in the past two decades.

Simultaneous visualisation of the turbulent vorticity and particle concentration fields has been obtained by DNS (Squires and Eaton, 1991; Eaton and Fessler, 1994; Wang and Maxey, 1993). There is a strong evidence from this research that the particles tend to preferentially concentrate in the regions of high strain and low vorticity due to particles inertia. In the work of Squires and Eaton (1991), four different flow zone were classified and defined as convergence zone, eddies, streams and rotational zones, as shown in Fig. 1.12. The particle density contours with particle Stokes number 0.52 is shown in an x-y plane in Fig. 1.13. The two figures
were overlapped for an instantaneous visualisation of the turbulent structure to the concentrating field as shown in Fig. 1.14.

As shown in Fig. 1.14, in the regions of eddy zones and rotational zones, the particle number density is significantly lower than that in the convergence zones. This mechanism has been widely studied numerically (Tanaka et al., 2002; Bec, 2005; Tagawa et al., 2012). Although the particle preferential concentration has been observed occurring in regions of low vorticity and high strain, the statistics of vorticity and strain rate tensor do not fully correlate with the presence of clusters. Also, the concept of ‘vortex’ or ‘eddy’ remains vaguely defined. To fully resolve the governing mechanism of particle preferential concentration, there is a need to either define a ‘vortex’ objectively or alternatively, and study the correlations between some well-defined turbulent structures with presence of particle clusters. Due to the complex morphology of instantaneous ‘vortices’, attempts for an alternative description are of particular interest.
Fig. 1.13: Particle density contours in an x-y plane for $St = 0.52$ (Squires and Eaton, 1991)

Fig. 1.14: Zone classification and particle number density contours, zone colour same as Fig. 1.12 (Squires and Eaton, 1991)
Goto and Vassilicos (2006) and Chen et al. (2006) introduced a new physical governing mechanism of preferential concentration by numerically investigating the possible role of certain topological turbulent structures with DNS. The locations of the zero velocity points and the zero acceleration points were compared with the dense region of particle clusters Fig. 1.15.

Figure 1.15: DNS spatial dispersion of inertial particles and turbulent stagnations points (Goto and Vassilicos, 2006) (a),(c),(e) Spatial distribution of inertial particles $St = 1.9$; (b) (d) (f) Spatial distribution of zero-acceleration points

Goto and Vassilicos (2008) extended their research into 3D turbulence and defined the sweep-stick mechanism, which refers that the fluid acceleration field is swept by the
local fluid velocity $u$ and particles tend to stick to and move with zero acceleration points $a = 0$. The sweep-stick mechanism attracted much attention but not yet fully examined especially through experiments. Thus, the experimental evaluation of sweep-stick mechanism with homogeneous and isotropic turbulence experimentally is one goal of this thesis.

This thesis will study the poly-dispersed droplet preferential concentration and experimentally investigate the mechanism for cluster formation (Eaton and Fessler, 1994; Goto and Vassilicos, 2008). In terms of choosing the method for experimentally generate turbulence, the ‘box of turbulence’ is the best choice because of the following advantages in the work of this thesis. Firstly, the ‘box of turbulence’ can generate homogeneous and isotropic turbulence with extremely small mean shear so that individual study of large and small scale motions of turbulence without considering the large motion caused by the mean shear is possible. Secondly, the homogeneous and isotropic turbulence is an ideal turbulence environment which simplifies the complex mechanisms raised by the presence of different phases especially in the study of multi-phase flows. Also, it is a lot easier to compare experiments with numerical simulations, when the carrier phase is stationary homogeneous and isotropic turbulence. Thus, the study of this thesis, aiming to understand the physics of droplet cluster formation and phase interactions, is based on the ‘box of turbulence’.

In summary, this thesis consists of two parts, the experimental part and the comparison with the DNS simulations of particle dispersion (Mallouppas et al., 2013b,c). This thesis develops post processing algorithms and provides comparisons between experimental results and the DNS simulations performed by Mallouppas et al. (2013b,c). The experimental part of this thesis begins with studies of poly-dispersed droplet dispersion in the homogeneous and isotropic turbulence generated in the ‘box of turbulence’. The mechanism for droplet clustering formation (Eaton and Fessler, 1994; Goto and Vassilicos, 2008) will be discussed and evaluated experimentally. The
DNS data will be processed with the same post-processing algorithm applied to the experimental data and comparisons will be given. The following questions are of particular interest to this thesis, and some ideas towards answering each question will be provided.

1. What is the effect of poly-dispersity on droplet preferential concentration?

2. How is a droplet cluster formed temporally? Is there any characteristic time/length scale of one particular droplet cluster?

3. Do droplet clusters respond to the topological structure of zero velocity points/zero acceleration points of turbulent flow, as observed by Goto and Vassilicos (2006); Chen et al. (2006)? Can the proposed sweep-stick mechanism be observed experimentally?

4. Does the DNS simulation agree with the experiments?

1.5 Outline of the Thesis

This Chapter described with the motivation of this thesis for the study of poly-dispersed droplet preferential concentration, which is essential for the understanding of spray combustion, particle settling and warm rain initiation. Basic background theories on the carrier phase turbulence, dispersed phase poly-dispersed droplets and two phase interactions are provided, followed by a brief review of the experimental approach in generating turbulence. The research of the present work is detailed and summarised by five Chapters.

Chapter 2 provides information on the experimental setup of the ‘box of turbulence’ and a brief walkthrough on the laser diagnostic techniques applied to
the experiments, including Particle Image Velocimetry (PIV) and Laser Doppler Anemometry (LDA) for the turbulence quantification, Phase Doppler Anemometry (PDA) for dispersed droplet sizing. Descriptions of the Gaussian beam optics are included, emphasising on the laser sheet optical arrangement.

Chapter 3 focuses on studying the influence of the poly-dispersed droplets on preferential concentration. Droplets with different size distributions and Sauter mean size ranging from $25 \ \mu m$ to $95 \ \mu m$ were generated and introduced in the ‘box of turbulence’ with four different turbulent intensities associated with turbulent Reynolds numbers $Re_\lambda = 107, 145, 185, 213$. An image processing method has been proposed to locate each droplet in the acquired experimental images, based on the Gaussian kernel and boxcar kernel convolution bandpass filter. Preferential concentration is quantified with the Radial Distribution Function (RDF) and the Voronoï analysis. Discussions of the first question in the previous Section are detailed in this Chapter (Lian et al., 2013).

Chapter 4 records a series of temporally-resolved experiments. Droplets with different size distribution and Sauter mean diameter ranging from $25 \ \mu m$ to $95 \ \mu m$ were generated and introduced in the ‘box of turbulence’ with four different turbulent intensities $Re_\lambda = 98, 127, 147, 235$. Turbulence topological structures of zero velocity points and zero acceleration points are calculated from 2DPIV result as an estimate of the three dimensional structures (Goto and Vassilicos, 2006; Chen et al., 2006; Cardesa et al., 2013). A droplet cluster identification and tracking algorithm has been developed, based on the non-parametric mean shift algorithm (Comaniciu and Ramesh, 2000). Discussions of the second and the third questions in the previous Section are detailed in this Chapter.

Chapter 5 compares the temporal resolved DNS simulations of $Re_\lambda = 108$ (Mallouppas et al., 2013b,c) and the Johns Hopkins University Turbulence Database of forced isotropic turbulence of $Re_\lambda = 433$ (Perlman et al., 2007; Li et al., 2008)
with the high speed experimental measurements $Re_\lambda = 98, 127, 147, 235$ presented in Chapter 4. The consistency of DNS and experimental results is discussed.

Chapter 6 lists the conclusions of this thesis and the scope for future work.
Chapter 2

Experimental Setup

Chapter 1. indicated that the study of this thesis is limited to statistically stationary homogeneous and isotropic turbulence generated in the ‘box of turbulence’. This chapter begins with descriptions of the ‘box of turbulence’ constructed facility of Charalampous and Hardalupas (2010). Possible improvements are discussed in order to characterize turbulence and dispersed droplets, appropriate optical instrumentation is needed. For turbulence characterization, PIV and LDA are introduced into the system. PDA is applied to droplet size characterization in the present study. The principles of each technique and the reason for choosing these techniques are discussed in the second part of this Chapter.

2.1 ‘Box of Turbulence’ Facility

The term ‘box of turbulence’ is defined in previous Chapter with a summary of up to date existing facilities sharing similar approach. A facility following the open chamber configuration proposed by Goepfert et al. (2009) has been developed in our lab (Charalampous and Hardalupas, 2010). This open chamber configuration provides good optical access. PIV and LDA are integrated in this facility. Descriptions of optical instrumentation are discussed in details in the following sections. In this
section, emphasis is drawn only on the ‘box of turbulence’ facility.

**Facility** The facility was built within a cubic frame box, which is constructed by one meter length aluminium profiles. Eight loudspeakers from Davis acoustic (Model No. 20MC8A) were mounted on special designed aluminium plates, installed at eight corners of the cubic box, all pointing at the center of the cubic box. The vibrations of speaker membrane during operation induce ambient air motions and uniform arrays of jets are produced by mounting perforated PVC plates with holes of diameter of 6 mm arranged in triangular mesh pattern on top of each loudspeaker. There is a total number of 55 holes on each plate as shown in Fig. 2.1. Four pairs of opposing jet arrays are set up with a distance of 590 mm between each opposing pair. The experimental arrangement of ‘box of turbulence’ with eight loudspeakers can be viewed in Fig. 2.2. Homogeneity and isotropy are achieved with a volume of 40 mm × 40 mm × 40 mm at the centre of the box. The ‘box of turbulence’ is located in a closed lab room equipped with air purifier to avoid contaminant interference.

![Image of a perforated PVC plate]

**Figure 2.1:** Schematic of perforated PVC plate with 55 holes of diameter of 6 mm arranged in triangular mesh pattern

Supplemental frames are assembled on top of the ‘box of turbulence’ to support particle screw feeders and spray atomiser. Particles or droplets are fed with gravity towards the centre of the box for further investigation.

**Control strategy** The intensity of synthetic jets is directly relevant to how the speaker membrane vibrates which is controlled by sine wave voltage signal
characterized by its frequency, amplitude and phase. When the frequency of the sine wave increases, the membranes of the loudspeakers tend to vibrate faster. The vibrations are enhanced when the amplitude of the sine wave increases and the phase of sine wave determines axial movement direction of speaker membrane. In our case, by controlling the amplitude of sine wave that drives each loudspeaker, the intensity of synthetic jet scan be controlled precisely. The sine waves are generated by 16 bit National Instruments analogue output card with 8 channels (PCI6733), controlled by program developed in Labview environment. To provide sufficient voltage to drive the speakers, the power outputs of these sine waves are then amplified by Behringer Europower EP2500 audio stereo amplifiers. The amplification of each channel is set as constant, because no precise control can be achieved with the chosen approach. As a consequence, only the analogue card with high resolution provides precise control of the amplitude of sine wave at each channel. Refer to Fig. 2.3 for sine wave controlled woofer wiring.

Limitations Ideally, each loudspeaker is identical and all are positioned pointing
towards the cubic box center. Homogeneity and isotropy should be easily reached by inputting same amplitude of each sine wave that drives corresponding speaker. However, in practice, the balancing of the box requires careful adjustments to each of the amplitude of the sine wave. The generated turbulence is highly sensitive to actual loudspeaker configurations. The difficulty in reaching local homogeneity and isotropy by finding corresponding amplitudes of each sine wave could be caused by several unavoidable reasons: (a) Manufacturing imperfections result in differences between each loudspeaker. (b) Each speaker can not be ideally positioned to point exactly at the box centre. (c) Nominally identical models of loudspeakers may have small differences at the response to the input signals. For these reasons, finding the corresponding amplitudes, referred as balancing the box, remains a time consuming process and need to be repeated on a daily basis.

The open chamber configuration provides good optical access. To quantify the flow velocity in the ‘box of turbulence’ and to characterise dispersed phase properties, the following optical techniques have been applied to the ‘box of turbulence’. The following sections provide detailed descriptions on these techniques, including basic arrangement and principle, experimental setup and processed quantities obtained from the measurements.
CHAPTER 2. EXPERIMENTAL SETUP

- 2D Particle Image Velocimetry (PIV): to characterise flow velocity in x and y directions.
- Laser Doppler Velocimetry (LDV): to characterise flow velocity in z direction.
- Phase Doppler Anemometer (PDA): to characterise the droplet size.

2.2 Particle Image Velocimetry (PIV)

The instantaneous velocity of the flow field is very important for the characterisation of turbulent flows. In the past decades, single point intrusive measurement, i.e. hot wire anemometry, is widely applied to velocity measurements in laboratory and industry. However, this single point sensor cannot be used in the high turbulence environment without mean flow of the ‘box of turbulence’. Particle Image Velocimetry (PIV) is an optical, non-intrusive technique, developed over the past two decades to measure instantaneous velocity distribution across a plane of the flow field (LaVision GmbH, 2007; Raffel et al., 2007; Adrian and Westerweel, 2011), as determined by the location of a laser sheet.

Before obtaining measurements, the flow field is seeded with tracer particles, which are assumed well follow the flow motions. In our case, we use a fog generator to produce micron-sized particles. A fog generator is located at one end of the room so that the ‘box of turbulence’ can be seeded with tracer droplets homogeneously. Circular laser beams are generated by New Wave Nd:YAG laser at 532 nm with the maximum pulse energy 120 mJ per pulse and then converted to laser sheets by a series of cylindrical lenses. The measuring area and tracer droplets are illuminated by a pulsed laser sheet twice with a small time difference, which is determined mainly by the flow field velocity. The displacements of tracer droplets are recorded by capturing the scattered light intensity with a PCO Sencicam inter-frame CCD camera, which is placed perpendicular to the illuminated plane and pointing to the measuring area.
in the ‘box of turbulence’ center, with a high resolution of 1376 $\times$ 1040 pixels. The recording speed of this camera can be up to 10 frames per second and inter-frame time for PIV measurements can be as short as 500 ns. An interference filter, which only allow light with particular wavelength through, was installed in front of the camera lens to reduce background noise. The particle displacement information can be recorded by one image two exposure or two images two exposures, followed by different post processing techniques (auto correlation or cross correlation) to obtain velocity information. The approach based on two images and two exposures was used in the current experiment. The obtained two images are both divided into small cells referred as *interrogation window*, and each window is evaluated with cross correlation to calculate the velocity vectors. A typical 2D PIV alignment schematic is shown in Fig. 2.4.

Figure 2.4: Setup of 2D PIV system (Adrian and Westerweel, 2011)
By characterizing the flow field with PIV technique, two-dimensional instantaneous spatial velocity information is extracted from the generated flow field. The velocity PDFs, mean and standard deviation of velocity fluctuations, two-point velocity correlations as well as the energy spectrum can be calculated from the measured velocity data. The calculation is required in order to verify the homogeneity and isotropy within the imaging region, while statistically stationarity is confirmed with vector calculation algorithm provided by Lavision 7.2 software. The third direction velocity is measured by Laser Doppler Anemometry (LDA), which is discussed below.

### 2.3 Laser Doppler Anemometry (LDA)

Apart from Particle Image Velocimetry (PIV), Laser Doppler Anemometry (LDA) has been used to measure particle velocities on the basis of Doppler shift that the scattered light frequency of a moving object can be shifted from incident light frequency. Doppler shift $f_D$ is defined as the shift of frequency between incident light $f$ and scattering light $f + f_D$ (see Fig. 2.5). In practice, a photomultiplier tube (PMT), which transfers photon energy into corresponding current, is used to detect the frequency of the resulting signal.

![Figure 2.5: Schematic of Doppler shift of moving particles](image)

Doppler shift can be defined as follows.
\[ f_D = \frac{2V}{\lambda} \cos \beta \sin \frac{\alpha}{2} \]  

(2.1)

where, \( \beta \) is the angle between bisector of angle ABC and moving particle velocity, \( \alpha \) is the angle between photo detector and incident light and \( \lambda \) is the incident light wavelength. Theoretically, particle velocity can be calculated with alignment detecting Doppler shift shown in Fig. 2.5. However, the Doppler shift \( f_D \) is relatively small comparing to incident light frequency. As a consequence, a technique with two incident laser beams of same intensity and frequency is used to measure the Doppler shift in order to minimize the estimation uncertainty. The measuring area at the intersection point and the measured particle velocity has to be perpendicular to the bisector of two beams. With two incident beams, there are two scattered intensities from particles crossing the probe volume. The scattered intensities with equal amplitude and slightly different frequency are modulated resulting in periodic variation referred as a beat. The resulting beat frequency of the Doppler signal is defined as

\[ f_D = \frac{|f_{D1} - f_{D2}|}{2} = \frac{2V}{\lambda} \sin \frac{\theta}{2} \]  

(2.2)

where \( \theta \) is the angle between the two incident laser beams.

The interference fringes generated by two coherent incident laser beams, shown in Fig. 2.6, are another method to interpret detected signals. According to light interference theories, the fringe spacing of the interference pattern formed inside the probe volume is defined as

\[ d_f = \frac{\lambda}{2 \sin(\theta/2)} \]  

(2.3)

Particle movement with a velocity \( V \) perpendicular to the fringes (bisector of two incident beams) is measured. The intensity of the scattered light corresponds to the intensity of fringes. As a result, signal bursts with different amplitude are shown in
Fig. 2.6 and the frequency can be calculated as

\[ f_D = \frac{V}{d_f} = \frac{2V}{\lambda} \sin(\theta/2) \]  

(2.4)

Figure 2.6: Fringe pattern in the intersection area of LDA measurement

The method described above can not provide particle velocity direction information. To avoid directional ambiguity, one of the incident beams is shifted with a frequency \( f_s \) by an acoustic-optical device named Bragg cell and move the fringe pattern at a speed of \( V_s = f_s d_f \) toward the unshifted beam. So the directional ambiguity is removed by shifting the detected frequency \( f_D \) up and down from \( f_s \) Eq.2.5.

\[ f_D = |f_s + \frac{2V}{\lambda} \sin(\theta/2)| \]  

(2.5)

Figure 2.7: Block diagram of a Laser Doppler Anemometry using monomode laser diodes for frequency shift generation (Tropea, 1995)
A practical alignment of LDA system is shown in Fig. 2.7. Back scattered alignment is more convenient because it allows both transmitting and receiving optics integrated in the same tube.

The reason to introduce this technique is to extract instantaneous velocity information in the third direction. Together with the two-dimensional spatial velocity data measured by PIV. In this way, a complete three-dimensional characterisation of the flow field is achieved by the combination of 2D PIV and LDA.

### 2.4 Phase Doppler Anemometry (PDA)

Phase Doppler Anemometry (PDA) is a non-intrusive laser technique to measure spherical droplet diameter simultaneously with droplet velocities for various flow conditions. A fiberPDA system (57X80) from Dantec is integrated for droplet characterization in the open chamber of the ‘box of turbulence’. It is a single point measurement based on the concept of Doppler frequency shift corresponding to droplet diameter and velocity as an extension of Laser Doppler Anemometry (LDA). For LDA, the amplitude modulated signal frequency can be acquired in terms of fringe pattern to calculate droplet velocity. Same principle applies to PDA for velocity acquisition. However, PDA is also capable of measuring spherical droplet size simultaneously by introducing a second photo-detector to receive the scattered light intensity. There is a difference in optical path length between the scattered light recorded at the two photo-detectors arranged at different angular positions. A Doppler burst is recorded by each photo-detector with equal frequency, but different phase which is dependant on the size of the droplets. This is the overall idea of PDA technique. The equation govern the relationship between phase difference and spherical droplet size is shown below in Eq.2.6
\[ \Phi_{ij} = \Phi_j - \Phi_i = \frac{\pi}{\lambda} D \times (\beta_j - \beta_i) \]  

(2.6)

from which a linear relationship between droplet size and phase difference can be detected, where geometry factor \( \beta \) depends on scattering mode and the three angles \( \theta, \varphi_i \) and \( \psi_i \) which refer to intersection angle between two incident beams, scattering angle measured from transmitting optics and azimuthal angle. The scattering mode can be divided into three different types: reflection, 1st order refraction and 2nd order refraction. For relationship between \( \beta_i \) and each scattering mode including methods to change the three angles, refer to Dantec (2006); Hardalupas and Liu (1997).

Theoretically, a typical PDA system consists of two photo-detectors. However, to solve the \( 2\pi \) ambiguity problem, which could confuse the droplet size when phase difference exceeds \( 2\pi \), Dantec uses three detectors to achieve high resolution and large measurement range by comparing phase differences of two detector pairs. The comparison of phase difference between two detector pairs can also provide spherical validation information. Curvatures of spherical surface can be measured at two different locations and identical curvature is expected for spherical droplets.

The conventional PDA system in our lab consists of the following components: Ar:Ion laser, laser controller, laser probe and receiving optics (including front lens, aperture plate, composite lens, alignment eyepiece and small filter selector). The receiving optics used in our lab is 57X80 : 112mm PDA receiving probe from Dantec. A typical PDA setup is shown in Fig. 2.8.

The dispersion of poly-dispersed droplets is of primary interest in the study of this thesis, so that PDA is used to characterise dispersed droplet size distributions.
Figure 2.8: Phase Doppler Anemometry Alignment (a) geometry and coordinate system of the PDA. (b) arrangement of the collection apertures of the receiving optics (c) droplet in the probe volume with Gaussian intensity profile (Hardalupas and Liu, 1997)
2.5 Gaussian Beam Optics

In most laser applications, including in this thesis, the laser beam propagation can be assumed as a laser beam with an ideal Gaussian intensity distribution that corresponds to the theoretical $TEM_{00}$ laser mode (Griot, 2007). The diameter of a Gaussian beam can be defined as shown in Fig. 2.9

The $1/e^2$ diameter is where the beam intensity falls to the 13.5% of the peak intensity and the FWHM refers to the full width at half maximum diameter where the intensity has fallen to 50% of the peak.

Without applying any optics, the Gaussian beam propagates with changes in wavefront radius as shown in Fig. 2.10

If the Gaussian beam passes through focusing optics, the ‘expanding’ Gaussian
CHAPTER 2. EXPERIMENTAL SETUP

beam can be converged/focused in a certain direction and forms the beam waist 

\[ \omega_0 = \left( \frac{z\lambda}{\pi} \right)^{1/2} \]  

(2.7)

where \( \lambda \) is the laser wavelength.

The Reyleigh range \( z_R \) over which the radius is \( \sqrt{\omega_0} \) is defined as

\[ z_R = \frac{\pi \omega_0^2}{\lambda} \]  

(2.8)

In practice, the recording area should be within the Reyleigh range from the laser beam waist to ensure there is no significant laser intensity difference in the whole experimental image.

With the knowledge of the Gaussian beam propagation and beam focusing, a desired laser sheet to illuminate the area of interest can be generated by a series of cylindrical lenses. One possible arrangement of the cylindrical lens is shown in 

**Fig. 2.11.** Two concave lenses are used to expand the laser beam to a laser sheet in one direction with a constant height and one convex lens is used to focus the laser sheet in the perpendicular direction to form the beam waist and corresponding Reyleigh range.
2.6 Optical Arrangement for the Experiment

The optical arrangement for the experiments reported in this thesis is described below.

For the experiment recorded in Chapter 3, a New Wave Nd:YAG laser at 532 nm with maximum energy of 120 mJ per pulse was used at the repetition frequency of 10 Hz. A PCO Sensicam inter-frame CCD camera with a resolution of 1,024 × 1,024 pixels with integration window size of 59 mm and spatial resolution of 58 µm/pixel. The laser sheet optics consists of two cylindrical lens, a -50 mm concave lens and a 500 mm convex lens resulting in the laser sheet thickness around 0.1 mm.

For the temporal resolved experiment recorded in Chapter 4, an Edgewave, IS-series, Nd:YAG laser operating at 532 nm was pulsed at high speed (between 25 Hz and 3 kHz) in order to illuminate a plane through the centre of the 'box'. A planar laser sheet was shaped by a -50 mm concave lens and a 500 mm convex lens with approximate thickness of 0.1 mm and was aligned at the centre of an illuminated Area of Interest (AOI) of around 45x45 mm² at the box centre. The laser sheet illuminated the droplets and the intensity of the scattered light was recorded by a Photron APX CMOS camera (1024 × 1024 pixel) used with a 105 mm lens f/2.8, leading to a linear magnification of 0.3 and resulting in spatial resolution of 45.5 µm/pixel.
2.7 Summary

Chapter 2. provides a description of the facility developed in our lab referred as ‘box of turbulence’, followed by a brief walkthrough on the laser diagnostic techniques applied to the experiments, including Particle Image Velocimetry (PIV) and Laser Doppler Anemometry (LDA) for the turbulence visualisation, Phase Doppler Anemometry (PDA) for the dispersed droplet sizing. Descriptions of the Gaussian beam optics are also included emphasising the laser sheet optics arrangement.

The following Chapter focuses on studying the influence of the poly-dispersed droplets on the preferential concentration.
Chapter 3

Clustering of Poly-dispersed Droplets

Particle laden turbulence covers a wide range of environmental and industrial applications, yet remaining one of the least understood topics for the complexity raised by the presence of turbulence itself and the interactions between different phases. The facility of ‘box of turbulence’ detailed in the previous Chapter is capable to generate homogeneous and isotropic turbulence without mean shear which simplifies the turbulent flow field and makes it ideal for the experimental study of multiphase flows with droplets or particles. This Chapter reports an experiment conducted in the ‘box of turbulence’ observing the preferential concentration of poly-dispersed water droplets. The preferential concentration of poly-dispersed droplets with a range of Sauter mean diameters between 25 $\mu m$ and 95 $\mu m$ has been studied experimentally in stationary homogeneous isotropic turbulence with four different turbulent intensities associated with $Re_\lambda = 107, 145, 185, 213$. The techniques used in quantifying the preferential concentration are the Radial Distribution Function (RDF) and the Voronoï analysis. An image processing method for locating droplets on recorded Mie-scattering intensity images has been proposed and evaluated.
This Chapter begins with a description of the experimental setup for turbulence generation and droplet injection. The turbulent flow velocity was measured by Particle Image Velocimetry (PIV) and the injected droplet size distributions were measured by Phase Doppler Anemometry (PDA). The spatial distribution of droplets was recorded as Mie-scattering intensity images. Turbulent velocity and droplet size characteristics are described, including definitions of statistics. The following Section records a novel image processing method extracting droplet locations from the experimental Mie-scattering intensity images based on a bandpass filtering method. Detailed examination of the bandpass filtering parameters and their influence on droplet clustering quantification is presented. Quantification and discussion of droplet preferential concentration are presented in the third Section. The Section described the methodologies of the Radial Distribution Function (RDF) and Voronoï Analysis and explains first the ways that quantify droplet clustering. Finally, the effect of droplet poly-dispersity is discussed by comparing the clustering behaviour of poly-dispersed droplets with similar Sauter mean diameter, but different size spreads for turbulent Reynolds number $Re_\lambda = 107$.

3.1 Experimental Arrangement

This Section describes the experimental arrangement for the characterisation of the stationary homogeneous and isotropic turbulence, the quantification of the two-dimensional turbulent velocity vector fields, the verification of the turbulent flow homogeneity and isotropy and the arrangement for droplet injection and measurements of droplet size distribution.
3.1.1 Turbulence Generation

The stationary homogeneous and isotropic turbulence has been generated in a facility ‘Box of turbulence’ described in detail in the previous Chapter. The instantaneous velocity of the air flow turbulence was measured by PIV. The laser beam was generated by a New Wave Nd:YAG laser at 532 nm with maximum energy of 120 mJ per pulse. Glycol droplets with size less than 3 µm were introduced in the flow as tracer particles. The measurement area was illuminated twice with a small time difference by a laser sheet generated by a series of cylindrical lenses based on the principle of Gaussian beam optics. The parameter of the laser sheet optics and the laser sheet is detailed in Chapter 2. The displacement of tracer particles were recorded as double-frame/double exposure images with a PCO Sensicam inter-frame CCD camera with a resolution of 1,376 × 1,040 pixels with spatial resolution of 58 µm/pixel. Two identical recording cameras were arranged to obtain PIV measurements on two normal planes. The first camera records particle images on the XY plane, while the second camera records particle images in the XZ plane of Fig. 2.2. By introducing a mirror placed 45° to the plane XZ, both cameras can be placed perpendicular to the laser sheet and the second camera placed parallel to the first camera. This arrangement makes the optical alignment easier. Illumination of the two crossing XY and XZ planes can be achieved by rotating the laser sheet optics by 90°. Combining the recorded information from the two cameras, the carrier phase velocity is measured in all three directions. This two-camera arrangement enabled a more efficient approach to evaluate the characteristics of the flow turbulence. After establishing that the turbulence was stationary on both plane XY and XZ, the turbulent isotropy was evaluated on plane XY only. All the Mie-scattering intensity images of droplets were recorded also on XY plane.
3.1.2 Turbulence Characterisation

A revisit of the turbulent statistics is provided first, followed by the description of turbulent characterization in the experimental facility.

Most of the statistical analysis of turbulence is based on averaging over space, time or both. These averaging processes are usually referred as the temporal mean, the spatial mean and the ensemble mean. Definitions of the averaging processes are included in this Section. The averaging processes can be applied on many turbulent quantities i.e. velocity, vorticity, strain rate etc. The application of these averaging processes over the experimental two-dimensional PIV velocity vector field are presented.

The Temporal Mean, Spatial Mean and Ensemble Mean

The *temporal mean* for \( N \) realizations of a discrete sampling signal \( u_{i}^{(n)}(x, t) \) is defined as

\[
\overline{u_i(x)} = \frac{1}{N} \sum_{n=1}^{N} u_i^{(n)}(x) \ n = 1, 2, \ldots N \tag{3.1}
\]

where \( u_i \) is the fluctuating velocity in the \( x_i \) direction. For a typical two-dimensional PIV measurement in one particular direction, \( N \) realizations of a \( p \times q \) sized instantaneous vector field can be obtained with a separation time \( \Delta t \) which is directly determined by the PIV repetition rate. Applying the temporal averaging on the PIV measured turbulent velocity results in a \( p \times q \) sized vector field with the velocity elements as the local average over the \( N \) realizations.

The *spatial mean* is the averaging of the spatially distributed signal as one single value \( \langle u_i^n \rangle \). The spatial average of a \( p \times q \) sized instantaneous PIV velocity vector field in the \( x_i \) direction returns a single scalar value, which is the magnitude of spatially averaged velocity in \( x_i \) direction.

The *ensemble average* for \( N \) realizations of a discrete sampling signal \( u_i^{(n)}(x, t) \) is
denoted as $\langle u_i \rangle$ and defined as

$$\langle u_i \rangle = \frac{1}{N} \sum_{n=1}^{N} \langle u_i^n \rangle \quad n = 1, 2, ... N \quad (3.2)$$

The ensemble average of $N$ realizations of a $p \times q$ sized instantaneous PIV vector field in the $x_i$ direction can be viewed as the temporal average of the spatial averaged velocity vector, hence its dimension is the same as a spatial mean which is a single scalar value, as the magnitude of spatial-temporal averaged velocity in $x_i$ direction.

The averaging procedure discussed above is generally applicable to two dimensional vector fields. The application on PIV measured velocity vector field is defined as an example with the averaging process illustrated in Fig. 3.1.

**Fluctuating Velocity and R.M.S. Velocity** Random nature of turbulence is directly revealed by random velocity components. Most of the turbulent statistical studies are based on the concept of *averaging*. The directly derived quantities *fluctuating velocity* and the *root mean square velocity* are the start points of most of subsequent turbulent quantitative studies.

The *fluctuating velocity* is defined as (Eq.3.3), where $\overline{u_i}$ represents the temporal
mean velocity and $u'_i$ donates the fluctuation velocity. For the 2DPIV measurement, temporal mean of the $N$ realizations is calculated and then used as the deduction of each instantaneous realization to for the fluctuating components.

$$u_i(t) = \overline{u_i} + u'_i$$  (3.3)

The mathematical technique used to describe random distributed quantities is referred as Reynolds decomposition. By applying Reynolds decompositions, Navier-Stokes equations can be simplified and non-linear term Reynolds stress $\tau_{ij}$, that gives rise to the turbulence, can be extracted.

The root mean square velocity is a statistical measure of the varying quantities magnitude. It is defined locally as

$$u_{i,\text{rms}}(x) = \sqrt{\frac{1}{N} \sum_{n=1}^{N} \left( u_{i}^{(n)}(x) - \overline{u_{i}}(x) \right)^2}$$  (3.4)

For the 2DPIV measurement, the r.m.s velocity is calculated in the similar approach as the calculation of the fluctuating velocity.

**Energy Spectrum** Both the one-dimensional and the two-dimensional turbulent kinetic energy were calculated from the obtained PIV vector maps. The one-dimensional energy spectrum was defined in Eq.3.5 based on the corresponding two-point correlation functions (George, 2006).

$$F_{i,j}^{(1)}(k) = \frac{1}{2\pi} \int_{-\frac{T}{2}}^{+\frac{T}{2}} e^{-ikr} B_{i,j}(r,0,0) \, dr$$  (3.5)

where the two-point correlation function is defined in Eq.3.6.

$$B_{i,j}^{(1)}(\vec{r},t) = \int_{-\frac{T}{2}}^{+\frac{T}{2}} \left\langle \hat{u}_i(\vec{k}', t) \hat{u}_j^*(\vec{k}, t) \right\rangle e^{i(k'_m-k_p)} \, d\vec{k} \int_{-\frac{T}{2}}^{+\frac{T}{2}} \, d\vec{k}'$$  (3.6)

The calculation procedure started by subtracting the temporal mean flow velocity
and then performing fast Fourier transform (FFT) of the fluctuating velocities at each row of the velocity vector map and averaging over all the images. The universal Kolmogorov spectrum (Kolmogorov, 1941) is defined in Eq.3.7

\[ E_{\alpha\alpha}(k_{\alpha}) = \frac{18}{55} C_k(\varepsilon)^{\frac{2}{3}} (k_{\alpha})^{-\frac{5}{3}} \] (3.7)

where, \( C_k \) is the constant used as 1.5 by Pope (2009), \( k_{\alpha} \) is the wavenumber defined by Eq.3.8

\[ \vec{k}_{\alpha} = \frac{2\pi}{r_{\alpha}} \] (3.8)

\( \varepsilon \) is the turbulent dissipation rate which is calculated from the velocity gradient field. The complexity raised by the discrete velocity gradient calculation leads to a separate Section discussing the turbulent dissipation rate and relevant quantities.

The two dimensional energy spectrum was calculated from the squared one-dimensional spectrum in the longitudinal and lateral directions.

**Normalized two-point velocity correlation coefficients** defined in Eq.1.11 Eq.1.14 are calculated for the turbulent condition \( Re_\lambda = 185 \) and shown in Fig. 3.2.

No parabolic behaviour can be observed at the origin due to lack of spatial resolution. The values of the correlations did not reach to zero because the measurement domain was too small. Therefore, we estimated the integral length scale by integrating the correlation function after extrapolating it at large radial distances using a polynomial function. However, it should be noted that the estimated integral length scale has some uncertainty due to the extrapolation process. For the presented flow with \( Re_\lambda = 185 \), the estimated integral length scale was calculated from u vector and v vector to be about 42.2 and 43.5 m, respectively.

**Turbulent Dissipation Rate** To sustain the turbulent flow, the system should receive energy supply continuously because the turbulent kinetic energy is converted
into thermal internal energy with the *turbulent dissipation rate* $\varepsilon$. The turbulent dissipation rate is an important parameter in the study of turbulence because it is essential in defining turbulent scaling parameters. It is, for example, defined in the textbook Pope (2009) as

$$
\varepsilon = 2\nu \langle s_{ij} s_{ij} \rangle = \nu \left\langle \frac{\partial u_i}{\partial x_j} \frac{\partial u_i}{\partial x_j} + \frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_i} \right\rangle
$$

(3.9)

where $s_{ij}$ is the fluctuating rate of strain tensor, $u_i$ is the fluctuating velocity in the $x_i$ direction and $\nu$ is the kinematic viscosity of the turbulent flow.

For the numerical simulations, the turbulent dissipation rate is calculated directly from the spatial velocity gradients. For the experimental studies, the ‘traditional’ approach is to measure the turbulent velocity with a hot wire probe in a wind tunnel while invoking the Taylor’s frozen hypothesis (Taylor, 1938) so that the spatial velocity gradient can be converted from the temporal derivatives. However, the hot-wire measurement provides one point velocity information only or a small region with a multiple hot-wires setup.

Two dimensional multi-point velocity information is available by measuring with
CHAPTER 3. CLUSTERING OF POLY-DISPERSED DROPLETS

Particle Image Velocimetry (PIV). Direct estimation of the dissipation rate can be made in a relative larger region from the spatial velocity derivatives (Tanaka and Eaton, 2007), and the indirect methods rely on the inertial subrange scaling. Direct method has been used in the current study.

Under the assumption of homogeneity and isotropy, the turbulent energy dissipation rate defined in Eq.3.9 can be simplified as (de Jong et al., 2008)

\[ \varepsilon = 15 \nu \left\langle \left( \frac{\partial u_1}{\partial x_1} \right)^2 \right\rangle \]  

(3.10)

and in two dimensions,

\[ \varepsilon = 4 \nu \left[ \left\langle \left( \frac{\partial u_1}{\partial x_1} \right)^2 \right\rangle + \left\langle \left( \frac{\partial u_2}{\partial x_2} \right)^2 \right\rangle + \left\langle \frac{\partial u_1}{\partial x_1} \frac{\partial u_2}{\partial x_2} \right\rangle + \frac{3}{4} \left\langle \left( \frac{\partial u_2}{\partial x_1} + \frac{\partial u_1}{\partial x_2} \right)^2 \right\rangle \right] \]  

(3.11)

The calculation of the turbulent dissipation rate depends on the turbulent velocity gradients measured from the PIV velocity vector field.

**Turbulent Scaling Quantities** The other quantities were calculated based on the dissipation rate with the following definitions.

The ensemble averaged turbulent kinetic energy \( q^2 \) is defined in Eq.3.12.

\[ q^2 = \left\langle q^2(x,y) \right\rangle = \left\langle 3 \times \frac{u_{1,rms}^2(x,y) + u_{2,rms}^2(x,y)}{2} \right\rangle \]  

(3.12)

The Reynolds number \( Re \) associated with the Taylor micro-scale is defined in Eq.3.13

\[ Re_\lambda \approx \frac{\lambda (q^2/3)^{1/2}}{\nu} \]  

(3.13)

where \( \nu \) is the kinematic viscosity of the ambient gaseous flow field, which is \( 1.57 \times 10^{-5} \text{m}^2/\text{s} \) for the air at the atmospheric pressure and 20°C.
CHAPTER 3. CLUSTERING OF POLY-DISPERSED DROPLETS

The Taylor micro-scale $\lambda$ is defined in Eq. 3.14

$$\lambda \equiv \left( \frac{5\nu q^2}{\varepsilon} \right)^{1/2}$$  \hspace{1cm} (3.14)

The Kolmogorov time scale $\tau_k$ and length scale $\eta$ are defined in Eq. 3.15 and Eq. 3.16 respectively.

$$\tau_k \equiv \left( \frac{\nu}{\varepsilon} \right)^{1/2}$$  \hspace{1cm} (3.15)

$$\eta \equiv \left( \frac{\nu^3}{\varepsilon} \right)^{1/4}$$  \hspace{1cm} (3.16)

The spatial contour of the temporal averaged mean velocity magnitude $\sqrt{u_1^2 + u_2^2}$ were plotted by averaging 1024 vector map obtained from PIV images with a recording rate of 25 Hz for all the turbulent conditions as shown in Fig. 3.3. The temporal mean velocities are less than the magnitude of 0.1 m/s, approximating zero mean flows and the stationary turbulent statistics.

The spatial contours of the ratio of the velocity fluctuations in the two orthogonal flow directions associated with the plane of the laser sheet, $u_{1,rms}/u_{2,rms}$, are presented in Fig. 3.4. The results show that the range of values are between 0.9 and 1.1 centred around 1. This confirms the spatial isotropy of turbulence within the area of interest. The absolute values of the ensemble mean velocity fluctuations and rms velocity are summarised in Table 3.1

| Table 3.1: Turbulent velocity statistics measured by 2DPIV with repetition rate of 10 Hz |
|---|---|---|---|---|
| $\bar{u}$ (m/s) | Exp.1 | -0.012 | Exp.2 | -0.002 | Exp.3 | -0.004 | Exp.4 | 0.011 |
| $\bar{v}$ (m/s) | 0.031 | -0.004 | -0.033 | 0.008 |
| $u_{rms}$ (m/s) | 0.350 | 0.417 | 0.526 | 0.608 |
| $v_{rms}$ (m/s) | 0.348 | 0.421 | 0.543 | 0.621 |
Figure 3.3: Mean velocity contours (a) $Re_{\lambda} = 107$; (b) $Re_{\lambda} = 145$; (c) $Re_{\lambda} = 185$; (d) $Re_{\lambda} = 213$
Fluctuating velocity in both $x$ and $y$ directions are normalised by the r.m.s. velocity and the normalised fluctuating velocity p.d.f.s are plotted in Fig. 3.5. The experimental normalised fluctuating velocity p.d.f.s follow Gaussian profiles in most regions. However, deviance is observed around zero mean in all the turbulent conditions. For measurements obtained in ‘box of turbulence’ constructed in other research groups, as briefed in Chapter 1, the normalised fluctuating velocity p.d.f.s measured by Hwang and Eaton (2004) follow well the Gaussian profile, while Goepfert
et al. (2009) showed similar deviance in the normalised fluctuating velocity p.d.f.s. Goepfert et al. (2009) obtained the velocity measurements with both PIV and LDV and the deviance remained, and argued that the deviance is possibly due to the flow behaviour at the intersecting area of air jets and can hardly be avoided when generating turbulence in ‘box of turbulence’ with woofers. It is also possible that the fluctuating velocity around zero mean is more easily contaminated by the experimental noise. Thus, to assess the homogeneity and isotropy of turbulence, more quantifications are needed. The two-point velocity correlation coefficients defined in Eq.1.10–1.14 can be used to assess the homogeneity and isotropy. The turbulent energy spectrum defined in Eq.3.5 is calculated based on the Fourier transform of the two-point velocity correlation function. Thus, the turbulent energy spectrum is calculated and shown below in order to assess the homogeneity and isotropy.

Both the one-dimensional and the two-dimensional turbulent kinetic energy were calculated from the obtained PIV velocity vector maps. The one-dimensional energy spectrum was defined in Eq.3.5 based on the corresponding two-point correlation functions (George, 2006).

The two dimensional energy spectrum was calculated from the squared one-dimensional spectrum in the longitudinal and lateral directions and shown in Fig. 3.6. The two dimensional energy spectrum in x and y directions agree reasonably well in the low wave number regions, suggesting the turbulence in the centre of the ‘box of turbulence’ is homogeneous and isotropic. However, the two dimensional energy spectrum deviate from the -5/3 slope in high wave number regions, which is also observed by Hwang and Eaton (2004) and Goepfert et al. (2009). This is possibly due to that the effect of experimental noise is more significant in the high wave number regions of fine scale turbulence.

The turbulent descriptive quantities for the four different turbulent conditions are summarised in Table 3.2.
Figure 3.5: Probability distribution of normalised fluctuating velocity
(a) $Re_\lambda = 107$; (b) $Re_\lambda = 145$; (c) $Re_\lambda = 185$; (d) $Re_\lambda = 213$

Table 3.2: Turbulent quantities measured by 2DPIV with repetition rate of 10 Hz

<table>
<thead>
<tr>
<th></th>
<th>Exp.1</th>
<th>Exp.2</th>
<th>Exp.3</th>
<th>Exp.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q^2$ (m$^2$/s$^2$)</td>
<td>0.36</td>
<td>0.52</td>
<td>0.85</td>
<td>1.13</td>
</tr>
<tr>
<td>$\varepsilon$ (m$^2$/s$^3$)</td>
<td>1.28</td>
<td>1.48</td>
<td>2.38</td>
<td>3.15</td>
</tr>
<tr>
<td>$Re_\lambda$</td>
<td>107</td>
<td>145</td>
<td>185</td>
<td>213</td>
</tr>
<tr>
<td>$\tau_k$ (ms)</td>
<td>3.4</td>
<td>3.2</td>
<td>2.5</td>
<td>2.2</td>
</tr>
<tr>
<td>$\eta$ (mm)</td>
<td>0.23</td>
<td>0.22</td>
<td>0.19</td>
<td>0.18</td>
</tr>
<tr>
<td>$\tau_{varpsilon}$ (ms)</td>
<td>0.14</td>
<td>0.18</td>
<td>0.18</td>
<td>0.18</td>
</tr>
</tbody>
</table>
Figure 3.6: Two-dimensional energy spectrum (a) $Re_{\lambda} = 107$; (b) $Re_{\lambda} = 145$; (c) $Re_{\lambda} = 185$; (d) $Re_{\lambda} = 213$
3.1.3 Droplet Generation and Characterisation

Poly-dispersed droplets are generated by an air-assist atomizer (Spraying systems, Model 1/4J, full cone spray). The droplet sizes are controlled by adjusting the inlet water flow rate and air pressure. Larger droplet mean diameters can be achieved by increasing the water flow rate, while keeping the air pressure constant. The atomizer was installed at the top of the ‘box of turbulence’ with an adjustable positioning device, in order to optimize the operation for various experimental conditions. The atomizer nozzle was placed 0.8 m above the central region of the ‘box of turbulence’. This ensured that the droplet momentum was negligible when entering the ‘box’. The droplet size distributions present in the ‘box of turbulence’ were measured by a fibre Phase Doppler Anemometry (PDA) system (57×80) from Dantec forming an optical focusing probe volume at the centre of the ‘box of turbulence’. The optical set-up of the PDA allowed a maximum droplet size of 160 µm to be measured.

To acquire sufficient data to describe the droplet size distribution, 1,500 to 9,000 validated droplets were measured for each experiment. The typical statistical uncertainty for the measured droplet size distributions and representative mean diameters was around 3 % based on the number of samples that were considered, following suggestions of Tate (1982). The main statistical parameters applied to quantify the droplet size are the droplet arithmetic mean diameter, $D_{10}$, and the Sauter mean diameter, the ratio of the droplet volume to droplet surface area, $D_{32}$. These were calculated according to the general definition of Eq.3.17 (Bayvel and Orzechowski, 1993).

$$D_{pq} = \sqrt[p]{\frac{\sum_{i=1}^{m} D_i^p \Delta n_i}{\sum_{i=1}^{m} D_i^q \Delta n_i}}$$ (3.17)
CHAPTER 3. CLUSTERING OF POLY-DISPERSED DROPLETS

Number-weighted droplet size distributions for different operating conditions of the atomizer, measured within the flow with turbulent Reynolds number of 107, 145, 185, 213 are shown in Fig. 3.7.

The corresponding droplet cumulative size distributions are shown in Fig. 3.8. The droplet number-based diameters $DN_{x\%}$ representing diameters carrying different fractions of the cumulative number of droplets in a size distribution and droplet volume-based diameters $DV_{x\%}$ representing diameters carrying different fractions of the cumulative volume of the droplet size distribution were calculated. $DN_{40\%}$ and $DN_{60\%}$ are derived from the cumulative droplet number-based size distributions and indicate respectively the diameters below which 40 and 60 % of the total number of droplets in the spray are present. The relevant definitions of the representative diameters for a specific size distribution is shown in Fig. 3.9. Representative droplet diameters based on a number-weighted size distribution may be more appropriate for scaling the behaviour of droplet preferential concentration, which is identified by droplet clusters with large number density and voids with low number density of droplets relative to the average across the imaging region. Volume-weighted size distributions and the corresponding cumulative distributions are also commonly used and are converted from number-weighted distributions after considering the volume of each droplet size. Representative diameters, $DV_{x\%}$, associated with volume-based size distributions, are those below which the respective fraction x of the total liquid volume in the spray is present. The droplet size spread is also characterized based on the volume-based size distribution with relative diameter span factor $\Delta_{DRSF}$ defined in Eq. 3.18 (Bayvel and Orzechowski, 1993)

$$\Delta_{DRSF} = \frac{DV_{90\%} - DV_{10\%}}{DV_{50\%}}$$

Although the droplet size measurements were obtained at the centre of the
Figure 3.7: Number weighted droplet size distribution (a) $Re_\lambda = 107$; (b) $Re_\lambda = 145$; (c) $Re_\lambda = 185$; (d) $Re_\lambda = 213$
Figure 3.8: Number weighted droplet size cumulative distribution (a) $Re_\lambda = 107$; (b) $Re_\lambda = 145$; (c) $Re_\lambda = 185$; (d) $Re_\lambda = 213$
‘box of turbulence’, the droplet size distributions remained the same within the statistical uncertainty of the measurements elsewhere. This was verified by obtaining measurements at different locations. This is expected due to the lack of droplet momentum when entering the box. Each measurement represents one set of experiments and is characterized by the droplet Sauter mean diameter $D_{32}$ (Eq.3.17) ranging from 25 to 95 $\mu m$. Different size spreads have been achieved and characterized accordingly for each experiment. Experiments on quantification of droplet clustering were limited to droplet size distributions with relatively narrow size spreads, and for this reason, the range of the considered Sauter mean diameters was limited between 25 and 75 $\mu m$. However, the proposed image processing method has been applied and tested for experiments with droplet Sauter mean diameter in the full range of 25 to 95 $\mu m$. A summary of the various representative droplet diameters and droplet size spread of the size distributions for four different turbulent Reynolds numbers is shown.

Figure 3.9: Number-weighted droplet size distribution for the case with Sauter mean diameter of 45 $\mu m$ with vertical lines indicating the various characteristic droplet diameters (Lian et al., 2013)
in Table 3.3 The considered diameters are $D_{10}$ and $D_{32}$ according to the definition of (Eq.3.17); for example, $DN_{40\%}$ of 4.3 microns means that 60 % of the droplets are larger than 4.3 microns, and $DV_{5\%}$ of 10.6 microns means that droplet with size larger than 10.6 microns represents 95 % of the total liquid volume. Finally, $\Delta_{DRSF}$ characterizing the droplet size spread of the distribution is according to Eq.3.18.

Table 3.3: Characteristics of droplets present in the turbulent flow with $Re_{\lambda}$
= 107, 145, 185 and 213, diameters unit in $\mu m$

<table>
<thead>
<tr>
<th>Experiment No.</th>
<th>$D_{10}$</th>
<th>$D_{32}$</th>
<th>$DN_{40%}$</th>
<th>$DN_{60%}$</th>
<th>$DV_{5%}$</th>
<th>$DV_{50%}$</th>
<th>$\Delta_{DRSF}$</th>
</tr>
</thead>
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<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
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<tr>
<td>No.1</td>
<td>8.5</td>
<td>25±1.2</td>
<td>4.3</td>
<td>6.9</td>
<td>10.6</td>
<td>30.6</td>
<td>2.4</td>
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<tr>
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<td>11.2</td>
<td>35±2.8</td>
<td>5.2</td>
<td>8.9</td>
<td>15.0</td>
<td>45.8</td>
<td>2.1</td>
</tr>
<tr>
<td>No.3</td>
<td>14.1</td>
<td>45±0.9</td>
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<td>11.3</td>
<td>19.3</td>
<td>56.9</td>
<td>1.6</td>
</tr>
<tr>
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<td>55±1.4</td>
<td>7.5</td>
<td>13.3</td>
<td>22.9</td>
<td>66.7</td>
<td>1.4</td>
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<tr>
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<td>21.0</td>
<td>65±1.4</td>
<td>9.4</td>
<td>17.0</td>
<td>29.1</td>
<td>77.3</td>
<td>1.2</td>
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<tr>
<td>No.6</td>
<td>25.3</td>
<td>75±2.7</td>
<td>11.8</td>
<td>21.0</td>
<td>33.5</td>
<td>84.9</td>
<td>1.0</td>
</tr>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
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<td>5.7</td>
<td>8.9</td>
<td>27.7</td>
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<td>19.0</td>
<td>57.1</td>
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<td>24.7</td>
<td>72.8</td>
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<td>25.6</td>
<td>62.1</td>
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<td>32.2</td>
<td>31.3</td>
<td>75.3</td>
<td>1.2</td>
</tr>
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</table>

The gravitational settling velocity of the droplets was defined by Maxey (1987) as
CHAPTER 3. CLUSTERING OF POLY-DISPERSED DROPLETS

\[ W = \frac{m_p g}{6\pi r_p \mu} = \frac{\rho g d_p^2}{18\mu} \tag{3.19} \]

where \( \mu \) is the dynamic viscosity of air at room temperature with the value of \( 1.9 \times 10^{-5} \text{ kg/m/s} \), \( \rho \) is the density of water valued \( 1,000 \text{ kg/m}^3 \), \( g \) is the gravitational acceleration \( 9.8 \text{ m/s}^2 \); and \( d_p \) is the droplet diameter in \( m \). A summary of droplet gravitational settling velocity and its ratio with turbulent velocity \( u_{i,rms} \) is given in Table 3.4. The ratio of the droplet settling velocity and turbulent velocity fluctuations is less than 1 for all the experimental conditions, so that the influence of gravity is small compared to turbulent strain in the presented study.

<table>
<thead>
<tr>
<th>( d_p (\mu m) )</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>80</th>
<th>90</th>
<th>100</th>
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<td>( W (m/s) )</td>
<td>0.01</td>
<td>0.03</td>
<td>0.05</td>
<td>0.07</td>
<td>0.10</td>
<td>0.14</td>
<td>0.18</td>
<td>0.23</td>
<td>0.29</td>
</tr>
<tr>
<td>( \sqrt{u_{1,rms}^2+u_{2,rms}^2} )</td>
<td>( Re_\lambda = 107 )</td>
<td>0.03</td>
<td>0.07</td>
<td>0.13</td>
<td>0.20</td>
<td>0.29</td>
<td>0.40</td>
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<td>0.66</td>
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<td>( Re_\lambda = 145 )</td>
<td>0.03</td>
<td>0.06</td>
<td>0.11</td>
<td>0.17</td>
<td>0.25</td>
<td>0.33</td>
<td>0.44</td>
<td>0.55</td>
<td>0.68</td>
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<tr>
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<td>0.05</td>
<td>0.08</td>
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<td>0.19</td>
<td>0.26</td>
<td>0.34</td>
<td>0.43</td>
<td>0.54</td>
</tr>
<tr>
<td>( Re_\lambda = 213 )</td>
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<td>0.04</td>
<td>0.07</td>
<td>0.12</td>
<td>0.17</td>
<td>0.22</td>
<td>0.30</td>
<td>0.38</td>
<td>0.47</td>
</tr>
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</table>

### 3.2 Mie-Scattering Intensity Image Processing:

**Bandpass Filter**

The Mie-scattering from the laser illuminated water droplets has been recorded with single frame/single exposure image by a PCO Sensicam inter-frame CCD camera with a resolution of \( 1,024 \times 1,024 \)\ pixels. The experimentally recorded images were stored as grey-scale TIFF formatted files. For each experiment with certain turbulent Reynolds number, eight measurements with different droplet size distributions were obtained with 1,000 instantaneous Mie-scattering intensity droplet images each.
3.2.1 Bandpass Filter

The Mie-scattering intensity images are usually contaminated with experimental noise. The locations of droplets are essential in the quantification of preferential concentration, thus there is a need to develop an image processing method to eliminate the experimental noise and locate the droplets. A literature review of the image processing method applied to the study of preferential concentration is briefed below.

Fallon and Rogers (2002) applied image intensity threshold for image filtering and reported measurements based on the assumption that intensity threshold does not affect measured statistics. This assumption may not be valid when poly-dispersed droplets are considered, since scattering intensity depends on the droplet sizes. Salazar et al. (2008) used high-pass particle size threshold $d_c$ for particle identification and assumed a $d_c$ level independent of the experimental conditions. However, these studies did not evaluate the performance of the image processing methods on the droplet identification and the quantification of preferential concentration. A detailed image processing method in locating droplets is proposed and evaluated against droplet identification and the quantification of preferential concentration.

The proposed image processing method, based on the bandpass filtering (Croker and Grier, 1996), can generate binary images of droplet locations from the recorded Mie-scattering gray scale intensity images. The gray sale Mie-scattering images are stored as TIFF format and can be imported into the MATLAB environment as UINT8 image arrays, which assign integers between 0 and 255 as the intensity of each pixel, with the value 0 correspond to black and 255 to white. The first step in locating droplets is to filter the acquired instantaneous raw image using a band-pass filter established with the two-dimensional Gaussian convolution as the high pass filter and the two-dimensional boxcar convolution as the low pass filter. The filtered image is the difference between the Gaussian convoluted high-pass filtered image and the boxcar convoluted low-pass image. The final binary images with located droplets
are achieved by calculating the local intensity maxima of the filtered image. The quantification on the preferential concentration is based on these binary images with droplet locations.

The two-dimensional Gaussian kernel is defined as

\[
G(x, y) = \frac{1}{2\pi\sigma} e^{-\frac{x^2+y^2}{2\sigma^2}} \tag{3.20}
\]

where \(\sigma\) is the standard deviation of the Gaussian kernel. The spread of the Gaussian convolution kernel should be larger than \(\pm 3\sigma\), according to the statistical three-sigma rule.

The two-dimensional boxcar kernel is defined as

\[
F(x, y) = \begin{cases} 
1 & \text{if } -a \leq x \leq +a \text{ and } -a \leq y \leq +a \\
0 & \text{else}
\end{cases} \tag{3.21}
\]

where \(a\) sets the boundary for the boxcar kernel, \(x\) and \(y\) are the local coordinate of the filtered image. The value of \(a\) in Eq.3.21 should be set the same as the spread of the Gaussian convolution kernel. Details in choosing the spread for the band pass filter kernels are discussed in this Chapter. The local intensity maxima of the band pass filtered image were calculated to identify each droplet location.

The procedure to identify the droplet locations is summarised as the following steps.

- Import the recorded TIFF formatted images into MATLAB as UINT8/UNIT16 array;
- Convert the content in the UINT8/UNIT16 arrays into double precision data format;
- Perform the bandpass filter on the double precision intensity matrix with the
Gaussian convolution as the high pass filter and the boxcar convolution as the low pass filter;

- Locate individual droplet by extracting the local maxima from the bandpass filtered image data matrix;

- Assign the pixel values as 1 for the droplet centres and the rest as 0 to form the binary image arrays.

A typical recorded raw image and the corresponding band-pass filtered images are shown in Fig. 3.10. By comparing the two frames, a good correlation is observed between the bandpass filtered image and the raw image with consistency in capturing the observable droplet structure and at the same time, the background noises are well eliminated. The steps to extract the local maxima and form the binary image array are shown in Fig. 3.11. It is obvious that a good correlation between the two images is achieved. Judging by human visualisation, the proposed bandpass filter in identifying droplets from the Mie scattering intensity images is efficient in maintaining the droplet structure and pruning out the noise. However, a quantitative evaluation of this method should be performed and the optimum bandpass filter parameters for all the experimental conditions need to be decided.

3.2.2 Selection of Bandpass Filtering Cut-off Value

Several instantaneous droplet Mie scattering images were processed with a series of image processing parameters. The combination of the parameters that result in the best visual inspection was further test through the $Q$ factor criterion (Zhou and Bovik, 2002) and the calculated Radial Distribution Function (RDF), used to quantify the droplet preferential concentration, is the ultimate goal in developing this image processing method.
Figure 3.10: Experimental droplet images (a) A typical recorded Mie-scattering image of droplets in the ‘box of turbulence’ (b) Band-pass filtered image based on the Gaussian and boxcar convolution

Figure 3.11: Bandpass filtered droplet images (a) Bandpass filtered image based on the Gaussian and boxcar convolution; (b) Locations of local intensity maxima indicating individual droplet, used for the quantification of droplet clustering
The $Q$ index proposed by Zhou and Bovik (2002) is a non-dimensional universal image quality evaluation parameter that has been widely used in the signal processing community. The $Q$ index is capable to indicate loss of correlation, luminance distortion and contrast distortion between the benchmark image and the test image. It has been introduced in the current study to examine the behaviour of image processing parameters by calculating the $Q$ index between the experimental raw images and the bandpass filtered images. The $Q$ index is defined as

$$Q = \frac{\sigma_{xy}}{\sigma_x \sigma_y} \cdot \frac{2\overline{x} \cdot \overline{y}}{(\overline{x})^2 + (\overline{y})^2} \cdot \frac{2\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2}$$  \hspace{1cm} (3.22)$$

where $x = x_i \mid i = 1, 2, \ldots n$ and $y = y_i \mid i = 1, 2, \ldots n$ are the digitised intensity signals of the experimental raw image and the bandpass filtered image, $\overline{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$; $\overline{y} = \frac{1}{n} \sum_{i=1}^{n} y_i$; $\sigma_x^2 = \frac{1}{n-1} \sum_{i=1}^{n} n(x_i - \overline{x})^2$; $\sigma_y^2 = \frac{1}{n-1} \sum_{i=1}^{n} n(y_i - \overline{y})^2$; $\sigma_{xy} = \frac{1}{n-1} \sum_{i=1}^{n} n(x_i - \overline{x})(y_i - \overline{y})$, and $n$ is the image resolution which is 1024 in the current study.

As the standard deviation of the image intensity can be viewed as an estimation of the image contrast, the first part of Eq.3.22 represents the correlation between two images, the second component represents the luminance distortion and the last part shows the contrast distortion. The overall $Q$ index is computed by averaging over each local $Q$ index with detailed description in Zhou and Bovik (2002).

To determine the appropriate convolution kernel spread, the experimental images were processed with different spread ranging from $\pm \sigma$ to $\pm 20 \sigma$. The $Q$ index is calculated for each convolution kernel spread for both Gaussian convolution and the boxcar convolution and plotted as a function of the kernel spread to evaluate its behaviour. As shown in Fig. 3.12, the $Q$ index peaks around $\pm 5 \sigma$ and $\pm 7 \sigma$, indicating that the least loss of correlation, luminance distortion and contrast distortion occur in this range comparing to other kernel spreads investigated.
In determining one unique kernel spread to process the Mie scattering intensity images for all the experimental conditions, there is a need to process and test the behaviour of ±5σ, ±6σ and ±7σ on the images that cover all the experimental conditions. The bandpass filter has been applied to identify droplet locations for 8 measurement conditions with the turbulent Reynolds number of 185. Each measurement condition has droplets with different Sauter mean diameter and size distribution spread. The average number of droplets detected from 1,000 raw images for each measurement is plotted in Fig. 3.13.

It appears that the number of droplets detected does not vary greatly to the convolution kernel spread, especially for the measurements with droplet Sauter mean diameter of 85 \( \mu m \) and 95 \( \mu m \). For the measurements with smaller droplet Sauter mean diameter, ±5σ convolution kernel spread performs slightly better with 2% more droplets detected. Dips are observed for the number of detected droplets of the measurements with Sauter mean diameter of 25 \( \mu m \) and 55\( \mu m \). This is mainly due to the nonlinear dependance of the number of droplets introduced from the atomisation
process controlled by the operating air and liquid flow rates of the air blast atomiser, not as a result of the measurement uncertainty.

The Radial Distribution Function (RDF), defined in Eq.3.23, is calculated from the droplet locations identified by the image processing method proposed. Fig. 3.14 shows the averaged RDF from 1,000 images for each of the 8 measurement conditions with turbulent Reynolds number of 185. A logarithmic axis has been used in Fig. 3.14 to reduce the overlapping of the curves. There is not a significant difference shown in the RDF resulting from the convolution kernel spread applied, ±5σ, ±6σ and ±7σ.

To summarise the steps in determining the convolution kernel spread, the first step is to limit the kernel spread to ±5σ, ±6σ and ±7σ with the evaluation of the corresponding universal image evaluation index Q; the second step is to examine the behaviour of ±5σ, ±6σ and ±7σ on the quantities of detected droplet number and the RDF for the images that cover all the experimental conditions. According
Figure 3.14: Averaged RDF with the convolution kernel spread as ±5σ, ±6σ and ±7σ. The radial distance is normalised by the Kolmogorov length scale η.
to Fig. 3.13, $\pm 5\sigma$ spread is slightly more efficient in detecting droplets with Sauter diameters smaller than 75$\mu m$ and return about 2% more droplets compared to the applications with $\pm 6\sigma$ and $\pm 7\sigma$. However, there is a negligible influence on the RDF from the image processing procedure with the convolution kernel spread of $\pm 5\sigma$, $\pm 6\sigma$ and $\pm 7\sigma$. Finally, the decision of the unique convolution kernel spread used in all the image processing scheme relies also on human visual inspection that $\pm 5\sigma$ seems to produce the best representation of the droplets identified from the raw experimental images. Thus, $\pm 5\sigma$ convolution kernel spread has been applied to the images in this thesis to locate droplets with the proposed image processing method from the Mie scattering intensity images acquired from all the experimental conditions.

3.3 Quantifying Droplet Clustering

3.3.1 Radial Distribution Function (RDF)

The RDF proposed by Sundaram and Collins (1997) has been mentioned in Chapter one, a brief revisit is included in this Section. The RDF is defined as

$$RDF = \frac{N(r_i)A}{A(r_i)N}$$

where $N(r_i)$ is the number of particles in a ring with a width of $2dr$ and of the radius $r_i$ from the center chosen randomly as the location of one particle. $A(r_i)$ is the area between $r - dr$ and $r + dr$. $A$ and $N$ is the total area and number of particles in the investigated region. The RDF indicates the droplet number density within a ring of a specific radius relative to the spatial average droplet number density in the area of interest. Increased droplet number density in the ring of one radius results in a larger RDF value.

The RDF is valued unity for randomly distributed droplets. In the presence of
droplet preferential concentration, the RDF values are larger than unity and for the void regions RDF values are smaller than unity. The minimum radius is around 3 to 4 times of Kolmogorov length scale. This is determined by the spatial resolution and the number density of the droplets in the flow. The selection of ring width \( dr \) is a compromise between having enough droplets within each of the rings and maintaining appropriate resolution. However, provided that the RDF converges after averaging over all the images, the resulting function is not dependant to the choice of \( dr \). Examples of the RDFs for 5 measurements with the turbulent Reynolds number of 185 are shown in Fig. 3.15.

![Figure 3.15: RDFs for different droplet size distributions and flow with the turbulent Reynolds number of 185](image)

The RDF values for the droplet Sauter mean diameter \( D_{32} \) of 55\( \mu m \) are larger than those for other measurements. This suggests that the strongest preferential concentration occurs for the measurement with droplet Sauter mean diameter of 55\( \mu m \). The RDF for the measurement with \( D_{32} \) of 65\( \mu m \) has the second largest value, as shown in Fig. 3.15, while the other measurements appear to have similar degree of preferential concentration with overlapping RDF curves.
The radial distance where the RDF value becomes unity can be viewed as a statistical description of the characteristic cluster length scale. Therefore, for the turbulence with \( Re_\lambda = 185 \), the droplet cluster length scale varies between 20 \( \eta \) and 30 \( \eta \), where \( \eta \) is the Kolmogorov length scale. The measurement with droplet Sauter mean diameter of 55\( \mu \text{m} \), which is with the highest magnitude of preferential concentration, is shown in Fig. 3.15 with the largest cluster length scale. This suggests a positive relation might exist between magnitude of preferential concentration and the characteristic cluster length scale. It should be noted that the characteristic cluster length scales measured for the mono-sized particles Eaton and Fessler (1994) are of the order of 10\( \eta \), which is of a similar order of magnitude as presented in the present work.

### 3.3.2 Voronoï Analysis

Monchaux et al. (2010) introduce the Voronoï tessellation in quantifying the preferential concentration, which is a method to decompose two dimensional space into a set of independent cells that the ensemble of points closer than any others to a certain particle. The area of each Voronoï cell is the inverse of the local particle concentration. Thus, the probability density function p.d.f. of the measured Voronoï cell areas can be used as a descriptive quantity of the preferential concentration. The distribution function of the normalised Voronoï cell areas of randomly distributed droplets is represented by the Random Poisson Process(RPP) defined by Ferenc and Neda (2007) as

\[
f_{2D}(y) = \frac{343}{15} \sqrt{\frac{7}{2\pi}} \cdot y^{5/2} \cdot e^{-7y/2}
\]  

(3.24)

The standard deviation of 2D RPP Voronoï areas is known to be \( \sigma_{RPP} = 0.53 \). If the droplets preferentially concentrate, the standard deviation of the
Voronoï area should be larger than 0.53 and a large deviant suggests a large magnitude of preferential concentration. Therefore, the standard deviation of the normalized Voronoï areas can be regarded as the descriptive scalar of the preferential concentration intensities.

The normalized probability density functions (PDFs) of the Voronoï areas, for different size distributions and for flows with turbulent Reynolds number of 185, are shown in Fig. 3.16(a).

![Figure 3.16](image)

**Figure 3.16:** Flow with turbulent Reynolds number of 185 (a) Normalized PDFs of Voronoï areas for different droplet size distributions. The PDF for RPP is superimposed as dashed line; (b) Standard deviation of normalized Voronoï area as a function of droplet Sauter mean diameter. The standard deviation for RPP is 0.53.

The dashed line represents the RPP distribution function in Fig. 3.16(a). The results show that the measured p.d.f.s deviate from RPP, which indicates the presence of droplet preferential concentration. The standard deviation of the normalized Voronoï areas, averaged over 1,000 images for each experimental condition, is presented in Fig. 3.16(b) for flow with turbulent Reynolds number of 185 as a function of droplet Sauter mean diameter. All the standard deviations are larger than 0.53, the corresponding RPP value, confirming the presence of preferential concentration. The results also show that the maximum value of standard deviations and, therefore, the strongest preferential concentration occurs for droplet size...
distribution with $D_{32}$ of 55 µm. The second largest is for $D_{32}$ of 65 µm, while the other conditions appear to have similar degree of preferential concentration since the standard deviation of the normalized Voronoï area are similar, in agreement with indication from the RDF analysis as shown in Fig. 3.15.

3.3.3 Stokes Number Scaling for Poly-dispersed Droplets

The Stokes number, defined in Eq.3.25, is the primary parameter used to scale preferential concentration.

$$St = \frac{\tau_p}{\tau_k} = \left( \frac{d}{\eta} \right)^2 \frac{1 + 2\Gamma}{36}$$  (3.25)

where $\tau_p$ represents the droplet response time, $\tau_k$ is the characteristic turbulent timescale, $d$ and $\eta$ are the droplet diameter and characteristic turbulent length scale, respectively. The characteristic flow timescale was selected to be the Kolmogorov timescale as many previous workers (Wang and Maxey, 1993; Fallon and Rogers, 2002; Wood et al., 2005). $\Gamma$ denotes the density ratio between the dispersed and the carrier phase. For poly-dispersed droplet-laden flows, the presence of various droplet size distributions with different mean diameters and size spreads results in large variation of the value of the Stokes number. In order to identify a representative droplet Stokes number for each size distribution and link it to resulting preferential droplet concentration, it is desirable to establish an appropriate diameter of the measured droplet size distribution for which the Stokes number will describe appropriately the clustering behaviour of the poly-dispersed droplets. For mono-sized particle-laden flows, it has been found both numerically and experimentally that the strongest preferential concentration occurs when the Stokes number is around unity (Crowe et al., 1985; Maxey, 1987; Hardalupas et al., 1990, 1992; Squires and Eaton, 1991; Longmire and Eaton, 1992; Eaton and Fessler, 1994; Wood et al., 2005; Salazar
et al., 2008; Saw et al., 2008; Monchaux et al., 2010; Bateson and Aliseda, 2012). Studies with poly-dispersed droplets tend to conclude ambiguously that the strongest preferential concentration occurs when the Stokes number is significantly different than unity. For example, Monchaux et al. (2010) considered poly-dispersed droplet-laden flows and reported that the strongest preferential concentration occurs for St around 2 to 3. The fact that the Stokes number deviates from unity, when strongest preferential concentration occurs, is due to the selection of the representative diameter of the size distribution that was used for the calculation of the droplet Stokes number of Eq.3.25. Therefore, for poly-dispersed droplets, an evaluation of the resulting Stokes number values, based on different droplet representative diameters of the size distribution, is required. Thus, we compare the values of the Stokes number based on droplet mean diameters $D_{10}$, $D_{32}$ defined in Eq.3.25, number-weighted droplet diameter $DN_{40\%}$, $DN_{60\%}$, and volume-weighted droplet diameter $DV_{5\%}$ and $DV_{50\%}$ defined in the Section of droplet characteristics. The relative relation of those characteristic droplet diameters is shown in Fig.3.9. The standard deviation of the variation of the normalized Voronoï areas as a function of droplet Stokes number for flow with $Re_\lambda = 185$ is presented in Fig.3.17(c). Each line corresponds to the estimation based on a different characteristic droplet diameter. Fig.3.17(c) shows that the standard deviation of the Voronoï areas, which indicates the degree of preferential concentration, becomes maximum for corresponding droplet Stokes numbers ranging from 0.5 to 10 for experiments with $Re_\lambda = 185$. This wide range of values demonstrates why previous studies have observed variability at the value of the Stokes number for which the strongest preferential concentration occurs. This also demonstrates the need to identify the representative diameter for a poly-dispersed droplet size distribution that is appropriate for the definition of the Stokes number, so that it can be around unity when maximum preferential concentration is observed. It should be noted that the standard deviation of the normalized Voronoï area is
an unambiguous parameter indicating the degree of droplet clustering. The current Section focuses only on the Voronoï analysis for its advance in providing single scalar quantifying the preferential concentration intensities. Quantifications based on RDF are included in order to provide comparisons with the results from the Voronoï data and estimation of the cluster length scale. The agreement between the two approaches supports better our findings. **Fig. 3.17(c)** shows that three droplet representative diameters, namely $D_{10}$ (arithmetic mean), $D_{N60\%}$ (the diameter below which 60 \% of the total number of droplets in the spray is present) and $D_{V5\%}$ (the diameter which carries 5\% of the liquid volume in the spray), lead to the least bias from the unity value of Stokes number when the standard deviation of Voronoï areas is maximized. The results for the three different turbulent conditions with $Re_\lambda = 107$, 145 and 213 are shown in **Fig. 3.17(a),(b),(d)** respectively. We can also observe ascending curves of standard deviation of Voronoï areas with Stokes number, except of one stand-alone point for $Re_\lambda = 107$, which is more likely due to experimental uncertainty. **Fig. 3.17(a), (b), (d)** also show that the rising curves of standard deviations become maximum for a droplet Stokes number around unity, when the Stokes number is based on $D_{10}$, $D_{N60\%}$ and $D_{V5\%}$. Attempts to scale droplet preferential concentration with a Stokes number based on other droplet diameters lead to ascending curves of standard deviation of Voronoï areas for droplet Stokes numbers much larger than unity, which is not expected from theoretical and experimental studies with mono-sized particles. It should be noted that **Fig. 3.17** shows that the droplet distribution always has a non-random component and the standard deviation of the Voronoï areas never approaches 0.53 (the value for random distributions) for small Stokes numbers. This is because the droplet Stokes number never becomes small enough to eliminate the preferential concentration. However, when the same analysis was applied to the concentration of seeding particles, introduced in the flow for PIV measurements of the flow turbulence characteristics, their distribution was always random.
Figure 3.17: Standard deviation of the Voronoi area as a function of droplet Stokes number, defined according to various representative diameters of the droplet size distribution, for flows with turbulent Reynolds numbers of 107, 145, 185 and 213.
It may be suggested that the consideration of around 60% of the total number of droplets in a poly-dispersed spray is appropriate to scale the preferential concentration. However, when considering a volume-based description of the size distribution, the diameter carrying 5% of the total liquid volume is appropriate for the identification of the preferential droplet concentration. This suggests that preferential concentration occurs mainly for droplet sizes that carry around 5% of the liquid volume of the spray, which raises some questions about the practical significance of preferential concentration. The most probable droplet diameter has been used in the poly-dispersed clustering study of Monchaux et al. (2010). The most probable droplet diameter does not necessarily have a response timescale similar to the Kolmogorov timescale for which the strongest preferential concentration appears. The Stokes numbers based on $D_{10}$, $DN_{60\%}$ and $DV_{5\%}$ are similar, and it appears that the strongest preferential concentration occurs when the Stokes number derived from the representative droplet diameters is around unity. Thus, we propose the use of $D_{10}$, $DN_{60\%}$ and $DV_{5\%}$ in studying clustering of the poly-dispersed droplets with certain size spread shape. It is possible that the dynamics of the droplets with such diameters governs the overall clustering behaviour of the poly-dispersed phase.

Despite the wide range of conditions of the current study, further evaluation is required for the verification of the generality of the proposed representative diameters for scaling of preferential concentration at different flows and sprays and the associated practical significance. It should be noted that the size of most droplets studied is less than 150 microns (shown in Fig. 3.7), while the Kolmogorov length scale for all flow conditions is larger than 180 microns (refer to Table 3.2). We also consider very dilute suspension with the volume fraction (Elghobashi, 1994) in the range of $10^{-6}$. Thus, the finite size effect (Qureshi et al., 2007) and the collective effect (Aliseda et al., 2002) do not affect the droplet clustering behaviour in the presented work. The current conclusions are related to the observed shape of the droplet size distribution, which
is typical for a wide range of sprays. However, if the shape of the size distribution deviates completely from the observed shapes in the current work, the appropriate representative diameters may change.

### 3.3.4 Effect of Turbulent Reynolds number

The previous Section proposed a droplet Stokes number based on $D_{10}$, $D_{N60}$ and $D_{V5}$ to scale preferential concentration. In the current Section, we will use this droplet Stokes number to evaluate the influence of the turbulent Reynolds number on preferential concentration. The standard deviation of the Voronoi areas, representing the degree of droplet clustering, as a function of the Stokes number, based on $D_{N60}$, for different turbulent Reynolds number is presented in Fig. 3.18. Error bars indicate the corresponding statistical uncertainties for a confidence level of 95%.

![Figure 3.18](image)

**Figure 3.18:** Variation of standard deviation of Voronoi areas as a function of droplet Stokes number, based on $D_{N60}$, for flows with turbulent Reynolds number of 107, 145, 185 and 213. Error bars indicate the statistical uncertainties for a confidence level of 95%. It is obvious
that the statistical errors are relatively large, so that it appears to be little dependence of preferential concentration on flow turbulent intensities. Droplets appear to cluster more, as the Stokes number is approaching unity for different flow turbulent intensity. Although some scatter is present in the results, the degree of preferential concentration appears to have little dependence on flow turbulent intensities. This finding is in contrast to a recent DNS simulation by Tagawa et al. (2012), which proposed that both light and heavy particles tend to have stronger preferential concentration for more intense flow turbulence. A recent experimental work by Obligado et al. (2011) covered a wider range of Stokes number and suggested that there might be a positive dependence of preferential concentration on the turbulent Reynolds number. However, in our case, due to the presence of uncertainties and lack of data points at higher Stokes number range, we could not see such a trend. Based on the presented data, there seems to be little dependence of the standard deviation of Voronoï areas on the turbulent Stokes number. Thus, more experiments covering a wider range of droplet Stokes numbers and turbulent Reynolds numbers is required to evaluate the effect of turbulent intensity on preferential concentration.

3.4 Discussion on Effect of Droplet Poly-dispersity

Discussions in the previous Sections are based on droplets with relative wide size spread, characterised by the relative diameter span factors $\Delta_{DRSF}$ defined in Eq.3.18, smaller than 3 larger than unity. The conclusions drawn above are suggested valid only for droplets with such droplet size distributions. To discuss the effect of droplet poly-disersity, droplets with Sauter mean diameter of 75$\mu$m but narrower size spread characterised by $\Delta_{DRSF} = 0.4$ were generated by an ultra-sonic atomiser produced by Sono-Tek to compare with the droplets generated by conventional air-assist atomiser with Sauter mean diameter of 75 $\mu$m and $\Delta_{DRSF} = 1.0$. It should be noted that the
relative diameter span factor is defined based on volumetric cumulative droplet size
distribution, it does not necessarily correlate with the number weighted droplet size
distribution.

The experimental setup was identical to that in the study of narrow distributed
droplets. This Section begins with description of measured size spread followed
by discussions on the clustering behaviour of the two sets of droplets evaluated by
Voronoï analysis.

3.4.1 Stokes Number Distribution

The droplets generated by ultrasonic atomiser have negligible momentum as the main
characteristic of ultrasonic atomiser. Gravitational settling velocity properties remain
similar for the two sets generated by ultrasonic atomiser and air-assist atomiser.
Thus, a control study can be achieved by ensuring the turbulent condition that the
measurement of dispersed droplets generated by ultrasonic atomiser obtained is with
the same turbulent condition of the droplets generated by air-assist atomiser with
turbulent Reynolds number $Re_\lambda = 107$. The droplet size distribution was measured
by PDA at the centre of the ‘box of turbulence’.

According to the definition of particle/droplet Stokes number Eq.3.25, the
measured droplet size distributions result in distributions of droplet Stokes number
as shown in Fig. 3.19.

The most probable Stokes number of droplets generated by the ultrasonic
atomiser is around unity while the Stokes number of droplets generated by air-assist
atomiser is around 0.1. There is a large variance of the Stokes number distribution for
both cases but the Stokes number distribution for droplets generated by ultrasonic
atomiser clearly represents a wider spread compared to that of the droplets generated
by air-assist atomiser.
Figure 3.19: Probability of droplet Stokes number for experiment with $Re_\lambda = 107$.
CHAPTER 3. CLUSTERING OF POLY-DISPERSED DROPLETS

3.4.2 Radial Distribution Function (RDF)

The obtained Mie-scattering intensity images are processed with the bandpass filtering method, proposed in this Chapter, to extract droplet coordinates. The Radial Distribution Function (RDF), defined in Eq. 3.23 is calculated. It is shown in Fig. 3.20 that the magnitude of RDF of droplets generated by ultrasonic atomizer is larger than the RDF of droplets generated by air-assist atomizer in the all scale, indicating the magnitude of preferential concentration of droplets generated by ultrasonic atomizer is larger than the droplets generated by air-assist atomizer. The RDF also indicates that the average lengthscale of the droplet clusters is around 15 $\eta$ for the wider size distribution of the air-assist atomizer and increases to around 40 $\eta$ for the narrower size distribution of the ultrasonic atomizer. This is an indication that the cluster lengthscale is larger when most droplets have Stokes number around 1 when partial droplet response leads to stronger clustering.

3.4.3 Voronoï Analysis

Voronoï analysis was performed on the droplet coordinate obtained to evaluate the non-uniform spatial distribution of preferential concentration. The p.d.f.s of the normalised Voronoï area is shown in Fig. 3.21(a). Both cases clearly deviate from the Random Poisson Process, indicating droplets are preferentially concentrated and form clusters. The p.d.f. of the droplets generated by ultra-sonic atomiser shows a larger deviance, suggesting stronger clustering, in agreement with the quantification of RDF. The standard deviation of normalised Voronoï area is a scalar indicator of the clustering magnitude.

To demonstrate the variance of standard deviation of normalised Voronoï areas, the p.d.f.s of the standard deviation of normalised Voronoï area are plotted for 1024 recorded Mie-scattering intensity images each in Fig. 3.21(b). For both set of data, the most probable standard deviation of normalised Voronoï area is around 1, while
Figure 3.20: RDFs of droplets generated by ultrasonic atomizer and air-assist atomizer

Figure 3.21: Voronoï analysis (a) P.d.f.s of normalised Voronoï area; (b) P.d.f.s of the standard deviation of normalised Voronoï area
the most probable Stokes number for droplets generated by air-assist atomiser is around 0.1 and for droplets generated by ultra-sonic atomiser is around 1. However, it is clearly indicated that there is a larger variance of the standard deviation of normalised Voronoï area for the droplets with a wider size distribution, which could be explained as droplets with different size respond to governing turbulent structures differently.

To summarize, by comparing the clustering behavior of droplets with different spread of the size distribution, larger variance of clustering magnitude is observed for narrower droplet size distribution. The most probable magnitude of clustering represented by the most probable standard deviation of Voronoï area does not correlate with the most probable number based droplet size/droplet Stokes number. Possible explanation to these findings is as follows. The overall droplet response to turbulent structures and the corresponding non-uniform spatial distribution are more likely to be due to the different dispersion of individual droplet sizes rather than a group behavior. As a result, the more poly-dispersed droplets with wider size distribution have a wider range of Stokes numbers and, therefore, the observed droplet clustering is not so pronounced leading to lower variance of magnitude of clustering. The narrower size distribution of the ultrasonic atomizer includes droplets with most probable Stokes number around 1, which is the value of the Stokes number leading to higher clustering. Therefore, the narrower size distribution leads to higher amplitude of droplet clustering. These findings should raise the awareness of researchers studying particle/droplet preferential concentration that the poly-dispersity should be considered when the size distribution spreads wide other than relying solely on the most probable statistics.
3.5 Summary

The preferential concentration of poly-dispersed droplets with Sauter mean diameter between 25 and 95 $\mu$m with four different flow turbulent intensities ($Re_\lambda = 107, 145, 185$ and 213) has been studied. A band-pass filter-based image processing method is proposed for the identification of droplet locations on images of scattered light intensity from droplets. The effectiveness of the method was assessed for various image qualities, and the influence of the image processing parameters was evaluated on the quantification of droplet clustering. RDF and Voronoï analysis were applied to quantify droplet preferential concentration. An appropriate Stokes number for poly-dispersed droplets is proposed for scaling of droplet clustering and the role of turbulence on clustering behaviour is discussed. Finally, the effect of poly-dispersity is examined by comparing the clustering behaviour of droplets with different size spreads. A summary of the findings is listed below.

1. The influence of image processing parameters of a bandpass filtering method to detect droplet locations and quantify droplet clustering was presented and verified by considering the universal image quality $Q$ index. This allowed the quantification of the degree of droplet preferential concentration for poly-dispersed droplets with wide range of size distributions, as summarised in Table 3.3.

2. The preferential concentration was quantified by the RDF method and Voronoï analysis and the results are in agreement. It was concluded that the strongest preferential concentration occurred when a proposed Stokes number, based on $D_{10}$ (the arithmetic mean droplet diameter), $DN_{60\%}$ (the diameter associated with 60% of the total number of droplets in the spray) or $DV_{5\%}$ (the diameter that carries 5% of the liquid volume in the spray), is around unity. This suggests that the preferential concentration of poly-dispersed droplets is
governed by sizes carrying a large number of small droplets and a small fraction of liquid volume in a spray.

3. The RDF method indicated that the length scale of droplet clusters varies between 20 and 30 times the Kolmogorov length scale of the flow turbulence for all size distributions and flow conditions. This is of the same order of 10 times the Kolmogorov scale for mono-sized particles of previous studies (Fessler et al., 1994; Aliseda et al., 2002; Wood et al., 2005).

4. Droplet preferential concentration appears to have little dependence on turbulent Reynolds number when the proposed Stokes number remains the same. This finding is in contrast to recent DNS results (Tagawa et al., 2012), which suggests that further investigation of the effect of turbulence is required.

5. The effect of poly-dispersity is found to be significant for droplet preferential concentration. Larger variance of clustering magnitude is observed for narrower droplet size distribution. The most probable magnitude of clustering represented by the most probable standard deviation of Voronoï area does not correlate with the most probable number based droplet size/droplet Stokes number. Possible explanation to these findings is as follows. The overall droplet response to turbulent structures and the corresponding non-uniform spatial distribution are more likely to be due to the different dispersion of individual droplet sizes rather than a group behavior. As a result, the more poly-dispersed droplets with wider size distribution have a wider range of Stokes numbers and, therefore, the observed droplet clustering is not so pronounced leading to lower variance of magnitude of clustering. The narrower size distribution of the ultrasonic atomizer includes droplets with most probable Stokes number around 1, which is the value of the Stokes number leading to higher clustering. Therefore, the narrower size distribution leads to higher amplitude of droplet
clustering. Also, the finding that lack of correlation between most probable magnitude of clustering and most probable number based droplet Stokes number could be an indirect support for the theory that the clustering behaviour of droplets with similar responding time scale of the Kolmogorov time scale is of the most pronounced.
Chapter 4

Time Resolved Statistics

The present Chapter reports a temporal resolved experimental study of clustering of poly-dispersed droplets in the ‘box of turbulence’ and aims to answer the following questions:

1. How does the droplet clustering develop as a function of time?

2. What is the typical cluster length scale?

3. What is the typical cluster time scale?

4. Do the droplet clusters correspond to the clustering of turbulent stagnation points, zero acceleration points?

5. What are the possible turbulent structures that are responsible for the droplet preferential concentration?

Following the approach in the previous chapter in quantifying droplet clustering, namely, 2D Radial Distribution Function (RDF) and 2D Voronoï analysis, the first question is answered by evaluating the overall degree of droplet clustering in the area of interest and resulting in temporal evolution of the descriptive statistical quantities. To answer the second and the third question, individual droplet clusters need to be
defined and tracked in time. A novel morphology quantification approach in defining and tracking droplet clusters in time is described in this Chapter, so that the temporal evolution of droplet cluster can be tracked and characteristic cluster length and time scale can be extracted.

This Chapter evaluate time dependent turbulent flow and droplet dispersion. It begins with theory of topological turbulent flow patterns based on critical theory including discussions of the effect of experimental noise. The morphology quantification technique of clustering based on mean shift feature space analysis and Voronoi space tessellation is detailed as follows. The time-resolved experiments are reported with quantifications of clustering of both dispersed droplets and turbulent topological flow patterns zero velocity/acceleration points. The sweep-stick mechanism proposed by Chen et al. (2006); Goto and Vassilicos (2006, 2008); Coleman and Vassilicos (2009) suggesting the correlation between dispersed droplets and turbulent zero velocity/acceleration points is discussed and examined with the time-resolved experimental results.

4.1 Evaluation of Turbulent Topological Characteristics

In order to conceptually visualise coherent flow structures in turbulent flows, Perry and Chong (1987) described and classified the turbulent topological flow patterns using the critical points concept, where the slopes of the flow streamlines are indeterminate. These topological flow patterns are defined by invariants of velocity gradient that can be measured experimentally. This Section details the approach to derive topological flow patterns from conventional two dimensional PIV velocity measurement and discusses the effect of experimental noise and de-noise methods applied.
4.1.1 Evaluation of Turbulent Topological Characteristics

Evaluation of topological turbulent flow patterns is proposed by Perry and Chong (1987); Chong et al. (1990) and based on the critical points concept (Otino, 1989). To describe this concept, the velocity invariants derived from velocity gradient tensor need to be defined first.

The complete three dimensional Cartesian tensor of the velocity gradient is given by $A$

$$
A = \begin{bmatrix}
\frac{\partial u_1}{\partial x_1} & \frac{\partial u_1}{\partial x_2} & \frac{\partial u_1}{\partial x_3} \\
\frac{\partial u_2}{\partial x_1} & \frac{\partial u_2}{\partial x_2} & \frac{\partial u_2}{\partial x_3} \\
\frac{\partial u_3}{\partial x_1} & \frac{\partial u_3}{\partial x_2} & \frac{\partial u_3}{\partial x_3}
\end{bmatrix}
$$

The three similarity invariants of $A$ are determined by the following characteristic polynomial (Eq.4.1).

$$\alpha^3 + P\alpha^2 + Q\alpha + R = 0 \quad (4.1)$$

where

$$P = -A_{ii} = -tr(A) \quad (4.2)$$

$$Q = \frac{1}{2}[P^2 - A_{ij}A_{ji}] = \frac{1}{2}[P^2 - tr(AA)] \quad (4.3)$$

$$R = \frac{1}{3}P^3 - PQ - \frac{1}{3}A_{ij}A_{jk}A_{ki} = -det(A) \quad (4.4)$$

where $P$ and $Q$ is defined by the trace of matrix $A$, $R$ is defined by the determinant of matrix $A$.

The reduced two dimensional velocity gradient tensor $\tilde{A}$ is a first-order Cartesian tensor, as the upper left block of the three dimensional velocity gradient tensor.
The characteristic polynomial of $\tilde{A}$ is defined as follows. (Eq.4.5).

$$\alpha^2 + p\alpha + q = 0 \tag{4.5}$$

where

$$p = -\tilde{A}_{ii} = -tr(\tilde{A}) = -\left(\frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2}\right) \tag{4.6}$$

$$q = -\det(\tilde{A}) = \frac{\partial u_1}{\partial x_1} \frac{\partial u_2}{\partial x_2} - \frac{\partial u_1}{\partial x_2} \frac{\partial u_2}{\partial x_1} \tag{4.7}$$

The eigenvalues of $\tilde{A}$ can be calculated by solving the characteristic polynomial equation (Eq.4.5), so that are uniquely determined by the characteristic polynomial coefficient $p$ and $q$. The classification of the critical points located at $x$ are determined by the local eigenvalue of $\tilde{A}$ (Chong et al., 1990). As a consequence, the categories of the critical points depend entirely on the values of the characteristic polynomial coefficient $p$ and $q$.

The topological categories of the critical points in two dimensional velocity field was shown in a form of a phase portrait of the characteristic polynomial coefficient $p$ and $q$ in the work of Cardesa et al. (2013), who studied the invariants of the reduced 2×2 velocity gradient tensor for various flow conditions. A reproduction of the Perry and Chong (1987); Cardesa et al. (2013) phase portrait is shown in Fig. 4.1.

Five different topological categories of the critical points are defined in the phase portrait.

1. *unstable node* if $p < 0$ and $q > 0$ and $q < p^2/4$;

2. *unstable focus* if $p < 0$ and $q > 0$ and $q > p^2/4$;
3. **stable node** if $p > 0$ and $q > 0$ and $q < p^2/4$;

4. **stable focus** if $p > 0$ and $q > 0$ and $q > p^2/4$;

5. **saddle points** if $p < 0$ and $q < 0$.

The defined saddle points of velocity vector field are the stationary points, thus also referred as ‘zero velocity points’. Applying the same concept of saddle points on the acceleration vector field, the saddle points are referred as ‘zero acceleration points’.

In the study of particle preferential concentration, Goto and Vassilicos (2006, 2008); Coleman and Vassilicos (2009) proposed sweep stick mechanism explaining the self-similarity observed by Boffetta et al. (2004), which refers that the fluid acceleration field is *swept* by the local fluid velocity $\mathbf{u}$ and particles tend to *stick* to and move with zero acceleration points $\mathbf{a} = \mathbf{0}$. The term *zero velocity point* and *zero
acceleration point are defined according to the turbulent topological characteristics described above with the saddle points defined in the velocity vector field as the zero velocity points, and the zero acceleration points correspond to that in the acceleration field.

4.1.2 Effect of Experimental Noise on the Topological Structure and Digital Filtering

The topological flow patterns defined in the previous Section by invariants of velocity gradients can be measured experimentally. Due to the discrete differentiation approach to calculate velocity derivatives, both the two-dimensional and three-dimensional experimental measurements of the velocity gradient are largely affected by the measurement spatial resolution, the velocity vector spacing, and the existing noise. Previous work discussing the effect of resolution on fine scale turbulence indicate that the coarsely resolved velocity vector fields result in underestimating the magnitude of velocity gradient and errors in turbulent quantities (Antonia et al., 1994; Worth et al., 2010). In addition, the three main sources of noise on the Particle Image Velocimetry (PIV) velocity field has been described by Christensen (2004) as due to the electrical noise of the camera, the bias error due to pixel peak locking and the gradient noise due to the local random velocity gradient within the field. Westerweel (2000) pointed out that the PIV measurement precision also depends on the sub-pixel displacement error in the processing algorithm. The effects of resolution and noise on the velocity derivatives determine the accuracy of the fine scale turbulent quantities and turbulent kinematic features estimation, and may therefore, affect the understandings of the physical processes evolved. Thus, to provide reasonable flow visualizations and interpretations of the evolving physical process, there is a need to discuss the resolution and noise effect on topological flow patterns defined upon the phase portrait of the the velocity invariants joint probability distribution functions.
Buxton et al. (2011) studied the effect of resolution and noise on the three-dimensional fine scale turbulent kinematic features suggesting a minimum resolution of $2.5\eta$ for the vector spacing in the study of fine scale turbulent structures. The p.d.f.s of the eigenvalues of the rate of strain rate tensor remain the same peak location whereas the tails appear narrower when the vector spacing resolution is decreased. The divergence errors pronounce with coarser vector spacing resolution and higher Gaussian noise. The shape of the velocity invariants $Q$ and $R$ joint p.d.f.s is sensitive to the vector spacing resolution and noise that the ‘Vieillefosse tail’ diminishes with decreased resolutions or increased Gaussian noise level as shown in Fig. 4.2. The effect of adding Gaussian noise of different variance $\sigma$ to the DNS dataset on velocity invariants joint p.d.f.s is plotted in Fig. 4.2. The significance of the experimental noise is greater than that of the vector spacing resolution on the fine scale turbulent kinematic features discussed.

Cardesa et al. (2013) reported the joint p.d.f.s of the two-dimensional velocity invariants. They appear similar in shape for various experimental flow conditions measured by standard two dimensional PIV, and a good agreement can be observed between the experiments and Direct Numerical Simulations (DNS). Therefore joint p.d.f.s and the fine scale turbulent kinematic features measured by the planar PIV is suggested to be less susceptible to noise and aliasing errors in comparison to that observed with 3D lawer diagnostics. However, a thorough evaluation of the effect of image noise can be performed by adding artificial noise on the calculated flow velocity from Direct Numerical Simulations (DNS).

In the current work, the effect of image noise is evaluated by adding random noise on the DNS data from Johns Hopkins University (JHU) open resource turbulence database (Perlman et al., 2007; Li et al., 2008) of forced homogeneous and isotropic turbulence in a $1024^3$ periodic cube. Three filtering approaches to eliminate the
Figure 4.2: The effect of adding Gaussian noise of different variance $\sigma$ to the dataset on velocity invariants joint p.d.f.s (Buxton et al. (2011) Fig.11(c))
added image noise and spurious vectors are considered, namely (a) Median Filter (Westerweel, 1994), (b) Wiener Filter (Press et al., 1988) and (c) Linear coupling of Median Filter and Wiener Filter. The above methods are also applied to measured 2DPIV images of the flow velocity of homogenous and isotropic turbulence, obtained from the ‘box of turbulence’ facility described in detail in Chapter 2. The goal is to quantify the effect of image noise and establish appropriate filtering approaches for the processing of experimental 2DPIV images of the flow in the ‘box of turbulence’ to eliminate the experimental noise at a satisfactory level in order to identify the twodimensional topological turbulent flow patterns.

**Noise Model** The concept and theory of digital noise transfer function (Point Spread Function (PSF)), defined and developed in the area of image processing, is briefed below to provide foundations of ‘noise’ elimination in the PIV measured velocity vector fields.

The noise present in a digital image is usually modeled as either impulse noise or Gaussian noise. The impulse noise is also generally referred to as ‘Salt & Pepper noise’ and assumed to be Poisson distributed with the form:

\[ p(k) = e^{-\lambda} \frac{\lambda^k}{k!} \]  

Given a specific window size, the probability of having \( k \) pixels affected by noise yields \( p(k) \) and \( \lambda \) is the average number of affected pixels and also represents the variance of the Poisson distribution. The ‘Salt & Pepper noise’ tends to have a local effect on individual pixels.

The Gaussian noise is associated with the random noise value at a given pixel, which is drawn from a Gaussian distribution with given expectation and variance, and affects the intensity value of each pixel in the whole image area. The random noise field from these two noise sources is added to the ‘true’ pixel intensity value,
thus referred to as additive noise.

Apart from the noise presented in a digital image, the blurring of an optical system is measured by the Point Spread Function (PSF), which expresses how much the input intensity value affects the output intensity value over locations of pixels when imaging a point source (Petrou and Bosdogianni, 1999). The Line Spread Function (LSF) is the PSF along a given direction and could be measured by the scanning edge-knife technique (Soulopoulos et al., 2014) as the 1st order derivative of the Edge Spread Function (ESF) or estimated as with a Gaussian profile when the image intensifier is not employed in the optical system. The Fourier transform of the Line Spread Function is an important parameter in the process of digital filtering and is generally referred to as the Modulation Transfer Function (MTF).

A measured digital image is blurred by the PSF and superimposed by the additive noise, which is modeled as Salt & Pepper noise or Gaussian noise or a combination of these two noise sources. In the context of fluid mechanics, the velocity vector fields measured by PIV are of interest for noise elimination. The biases occurred, reflecting the complicated effects of the digital image noise and the PIV processing algorithms, have been summarized by Christensen (2004); Westerweel (1994). However, a noise model can be developed for the evaluation of topological characteristics of turbulent flows to guide the development of the noise elimination techniques.

To establish the noise model and observe the influence of noise on the topological flow patterns, the two defined noise sources in the community of image processing, Gaussian noise and Salt & Pepper noise, have been added linearly to the ‘un-blurred’ ‘noise-free’ two-dimensional velocity vector fields acquired from the DNS data of Johns Hopkins University (JHU) open resource turbulence database (Perlman et al., 2007; Li et al., 2008) with the linear expression of:

$$\mathbf{u}_{i,p}(x, y) = \mathbf{u}_{i,DNS}(x, y) + n_g(x, y) + n_{sp}(x, y)$$  \hspace{1cm} (4.9)
where $u_{i,p}(x,y)$ is the pseudo velocity component with artificial additive noise at location $(x,y)$; $u_{i,DNS}(x,y)$ is the local velocity component from the DNS database of JHU; $n_g(x,y)$ is the local intensity of noise with Gaussian profile; $n_{sp}(x,y)$ is the local intensity of Salt & Pepper noise with Poisson distribution.

Zero mean Gaussian noise with variance of 0.001 and 0.005 and Salt & Pepper noise with density of 0.001 and 0.005 have been separately added with the other term set as zero, corresponding to value of $10^3$ times the turbulent flow r.m.s. (root mean square) velocity $u_{r.m.s}=3.5$ m/s, the same levels as those used by Buxton et al. (2011) for the study of the effect of noise on three dimensional conditions. Linear combination of the Gaussian noise with variance of 0.005 and the Salt & Pepper noise density of 0.005 has also been tested and the resulting joint p.d.f.s of velocity invariants are shown in Fig. 4.3.

Fig. 4.3 shows that the addition of different levels of zero mean Gaussian noise modifies significantly the shape of the joint p.d.f.s. The tails of the p.d.f.s are more pronounced for higher magnitude of Gaussian noise and the shape expands along the horizontal axis. The observed skewness of the joint p.d.f.s from the DNS is reduced when Gaussian noise is artificially added. The addition of Salt & Pepper noise modifies the shape of the joint p.d.f.s greatly, when the magnitude of density is 0.005. The skewness could no longer be observed and the pointy edges are rounded as shown by the thick dash line.

To effect on the joint p.d.f.s in the presence of both Gaussian and Salt & Pepper noise is tested by linearly adding up the two noise sources with zero mean Gaussian variance of 0.005 and Salt & Pepper noise density of 0.005. It is shown by the green solid line in Fig. 4.3 that the corruption of Salt & Pepper noise is improved after introducing the Gaussian noise. The shape of the joint p.d.f.s of the velocity invariants calculated from the raw experimental data is shown by dotted thick green line. It is indicated that the joint p.d.f.s of the raw experimental data is in transition...
Figure 4.3: The joint p.d.f.s of velocity invariants of homogeneous, isotropic turbulence from JHU DNS simulation with added Gaussian noise and Salt & Pepper noise.
between the joint p.d.f.s of the combined Gaussian and Salt & Pepper noise and the Salt & Pepper with density level of 0.005, suggesting that the Salt & Pepper noise is dominant in the velocity vector fields measured by PIV. In summary, an effective noise model is established for PIV measured velocity data with the ability to mimic the effect of experimental noise on the topological characteristics of turbulent flow, i.e. the joint p.d.f.s of velocity invariants. Since the topological turbulent flow patterns are dened according to the joint p.d.f.s phase portrait, the topological turbulent flow patterns are sensitive to the introduced Gaussian noise and Salt & Pepper noise with the later being more pronounced. This noise model can then be used to provide guidance for the development of digital filtering techniques for noise elimination from PIV velocity data in order to evaluate the topological turbulent flow patterns.

Digital Filtering Techniques This Section presents the theory of digital filtering behind the Wiener Filter and the Median Filter approaches in order to enhance and restore the noise-free images with the application of these digital filtering techniques on the two-dimensional velocity vector field. The Wiener Filter is a technique to restore the image blurred by PSF and mainly the Gaussian additive noise. The Median Filter is an effective method to remove impulsive Salt & Pepper noise.

The theory of the Wiener filter is briefed first. In the following discussion, it should be noted that lower case expressions donate image intensity values in physical space and upper case expressions donate image intensity values in Fourier space. An image blurred by PSF and with additive noise can be expressed as

\[ f_M(r) = \sum_{-\infty}^{\infty} \sum_{-\infty}^{\infty} h(r - r')f(r')dr' + n(r) \]  \hspace{1cm} (4.10)

where \( f_M(r) \) is the measured image, \( f(r) \) is the un-blurred noise-free image, \( h(r) \) is the PSF of the optical system and \( n(r) \) is the corrupting additive noise. The Fourier
form of this expression is:

\[ F_M(k) = H(k)F(k) + N(k) \]  \hspace{1cm} (4.11)

Applying a digital filter, the filtered image can be expressed as:

\[ f_F(r) = \sum_{-\infty}^{\infty} g(r - r')f_M(r')dr' \]  \hspace{1cm} (4.12)

with its Fourier form as:

\[ F_F(k) = F_M(k)G(k) \]  \hspace{1cm} (4.13)

The least square error between the un-blurred noise-free image and the filtered image is defined as:

\[ e^2 = E[f(r) - f_F(r)]^2 \]  \hspace{1cm} (4.14)

where \( E \) donates the expectation.

The Wiener Filter is an optimal linear solution to the least squares error estimation between the ideal image and the filtered image. Its Fourier form has been obtained (Petrou and Bosdogianni, 1999) as:

\[ G(k) = \frac{H^*(k)}{|H(k)|^2 + \frac{S_n(k)}{S(k)}} \]  \hspace{1cm} (4.15)

where \( G(k) \) is the Fourier form of optimal Wiener Filter, \( H(k) \) is the Fourier form of PSF of the degradation process, \( H^*(k) \) is the conjugate of \( H(k) \). The spectral densities of the additive noise and un-blurred noise-free image are \( S_n(k) \) and \( S(k) \).

The construction of Wiener Filter requires estimation or measurement of the PSF. In the current work, the PSF is represented by a Gaussian profile, which is expected for most of the optical systems without the presence of an image intensifier that alters
the PSF with a longer tail.

In the context of fluid mechanics, where the Wiener Filter is applied to the twodimensional velocity vector field, the estimation of spectral density of noise and unblurred noise-free ‘image’ is achieved via kinetic energy power spectrum following the approach of Soulopoulos et al. (2014). The kinetic energy power spectrum of the turbulent flow condition of Reynolds number $Re_\lambda = 147$ is shown in Fig. 4.4.

Characterisation of the turbulent flow is discussed in the following Section.

![Figure 4.4: Power spectral density estimation with optimal Wiener Filter](image)

According to the procedure described in Soulopoulos et al. (2014), power spectral density estimation is shown in Fig. 4.4. $F_M(k)$ represents the measured kinetic energy power spectrum of turbulent flow with Reynolds number of 147. $N(k)$ is the estimated noise spectrum using cubic spline extrapolation from the measured power spectrum. The filtered energy spectrum is the deduction between the two shown as $F_M(k) - N(k)$. 

$E_{\text{mn}}(k)$: Measured kinetic energy power spectrum of turbulent flow with Reynolds number of 147.

$E_{\text{np}}(k)$: Estimated noise spectrum using cubic spline extrapolation from the measured power spectrum.

$F_M(k)$: Measured kinetic energy power spectrum of turbulent flow with Reynolds number of 147.

$N(k)$: Estimated noise spectrum using cubic spline extrapolation from the measured power spectrum.

$F_M(k) - N(k)$: Filtered energy spectrum.
The term of Eq. 4.15 can be simplified as a constant \( \Gamma \) following suggestions by Petrou and Bosdogianni (1999). In the current work, it is approximated as 1/10.

The procedure applying Wiener Filter on the two-dimensional velocity vector field is summarized as follows:

1. Evaluate the Modulation Transfer Function (MTF), which is the Fourier transform of the Point Spread Function (PSF). The PSF can be approximated as a Gaussian profile or measured with scanning knife-edge technique.

2. Estimate the constant \( \Gamma \) from the kinetic energy power spectrum.

3. Calculate the two-dimensional Fourier Transform of the instantaneous velocity vector field measured by PIV, \( F(k) \).

4. Multiply \( F(k) \) point by point with the constructed Wiener Filter \( G(k) \).

5. Calculate the inverse Fourier transform to obtain the filtered instantaneous velocity vector field, \( f_f(r) \).

6. Calculate the fluctuating velocity vector field from the filtered instantaneous velocity vector field for velocity gradient and invariants estimation.

The two-dimensional raw experimental velocity vector fields are processed with the constructed Wiener Filter, and the resulting joint p.d.f.s of velocity invariants are plotted in Fig. 4.5 with thick black dashed line. The shape of the joint p.d.f.s is similar to that of the DNS data with Salt & Pepper noise of density level 0.005, suggesting that the Wiener Filter removes certain levels of Gaussian noise leaving the velocity vector fields dominated by Salt & Pepper noise.

The Median Filter is a non-linear digital de-noising method widely used by the image processing community to remove the impulsive Salt & Pepper noise, which, according to Westerweel (1994), represents similar properties as that of spurious velocity vectors occurring in PIV measurements. It replaces the current pixel entry...
with the median value of the neighbouring pixel entries within the square filter window size of $n \times n$, where $n$ is usually set as an integer less than 10. The median filter with different window sizes ($2 \times 2$, $3 \times 3$ and $5 \times 5$) has been applied to the experimental PIV data, as shown in Fig. 4.5. The $p$ and $q$ joint p.d.f.s shrink and follow the 'ideal shape', identified by the DNS data without any noise, better with a larger window size. It is observed that the median filter effectively removes the spurious velocity vectors.

As indicated by the noise model established in the previous Section, the velocity vector fields are contaminated by both the Gaussian and the Salt & Pepper noise with the latter dominating. The combined Median Filter and the Wiener Filter seem to be the solution to filter the experimental velocity vector fields thus tested and shown in Fig. 4.5. No significant improvement but only a bit smoothing is observed comparing to that filtered by a $5 \times 5$ window sized Median Filter. It should be noted that the sequence of applying the Wiener Filter and the Median Filter on the images does not modify the result. This confirms the suggestion that the Salt & Pepper noise, effectively the spurious vector in velocity vector fields, is the dominant factor in the noise model influencing the evaluation of the topological characteristics of the turbulent flow.

There are limitations of the current proposed filtering technique that the shape of the joint p.d.f.s could not be fully ‘recovered’ to coincide with the joint p.d.f.s of un-blurred, noise-free velocity vector invariants from DNS data. This is possibly due to the fact that the coupling of Gaussian noise and Salt & Pepper noise does not comply a linear relationship as modelled in Eq.4.9, thus the digital filtering techniques needed are not linear operators. To fully resolve the noise elimination in evaluating topological characteristics of turbulent flows requires constructing a detailed non-linear noise model and modifying further the digital filtering techniques. However, this is beyond the content of the current work.
CHAPTER 4. TIME RESOLVED STATISTICS

Figure 4.5: The joint p.d.f.s of velocity invariants of homogeneous, isotropic turbulence from experiment, processed by Median Filter, Wiener Filter and linear coupling of Median Filter and Wiener Filter
To summarize, this part described the application of the Wiener Filter and the Median Filter to eliminate the blurring of PSF, Gaussian noise and Salt & Pepper noise from two-dimensional turbulent velocity vector fields. It confirms the finding in the noise model Section that the Salt & Pepper noise, which is assumed equivalent to spurious vector, is dominating in the evaluation of topological characteristics of turbulent flows. A Median Filter with window size of $5 \times 5$ is proved to be an effective and efficient approach to eliminate noise in the current application with very limited improvement provided by the Wiener Filter. Thus, a Median Filter with window size of $5 \times 5$ is used in the subsequent study extracting the turbulent topological patterns, namely saddle points, which are also referred to as zero velocity points for the velocity vector fields.

To demonstrate the influence of median filtering of the PIV measured velocity data on the identified topological turbulent zero velocity points, an instantaneous distribution of zero velocity points, identified with different median filter window sizes, is shown in Figure 4.6.

Detailed flow structures can be observed after filtering the PIV velocity with a $2 \times 2$ Median Filter. The morphology of the zero velocity points remains and the main structures are captured after filtering with $3 \times 3$ and $5 \times 5$ Median Filters. It is also observed that the representation of zero velocity points with $5 \times 5$ Median Filter is similar to that of the noise free DNS data, forming clear-edged clusters. The statements rely on visual inspection of Figure 4.6. Quantitative description on the clustering of the zero velocity points is included in the following part.

The saddle points, one of the turbulent topological flow patterns, are of interest in the study of droplet-laden flows, since they are possibly responsible for the preferential concentration of dispersed particles (Chen et al., 2006; Goto and Vassilicos, 2006, 2008; Coleman and Vassilicos, 2009). Thus, the influence of the noise elimination digital filtering technique is examined on the clustering behavior of the saddle points.
Figure 4.6: Instantaneous zero velocity points identified with different Median Filter window size. (a) raw PIV data; (b) velocity field filtered by $2 \times 2$ window sized Median Filter; (c) velocity field filtered by $3 \times 3$ window sized Median Filter; (d) velocity field filtered by $5 \times 5$ window sized Median Filter; (e) noise free DNS data
of velocity vector fields, also referred to as the zero velocity points. In the following
discussion, zero velocity points are used to represent the saddle points of velocity
vector fields. The clustering behavior of the zero velocity points is quantified by two
methods, namely the Radial Distribution Function (RDF) and Voronoï analysis.

The Radial Distribution Function (RDF) was proposed by Sundaram and Collins
(1997) and defined in Eq.3.23.

Fig. 4.7 shows the RDF of zero velocity points derived from raw, median filtered
experimental data and noise-free DNS data. It is expected that the data filtered by
5 × 5 Median Filter window size are measured with the highest clustering intensity
and the ones from the raw experimental data return the least degree of clustering,
agreeing with the visualization of Fig. 4.6. This is illustrated in Fig. 4.7, where the
dotted RDF curve marked with triangular symbols linked to the data filtered by 5 × 5
Median Filter, is slightly above the other three curves of the data filtered by 3 × 3
Median Filter, the data filtered by 2 × 2 Median Filter and the raw experimental
data for lengthscales smaller than 10. Also, it is interesting that there is a fairly good
spatial correlation for lengthscales larger than 10 between all the RDFs regardless of
the data noise level. It seems that, although the application of the Median Filter
significantly alters the shape of the joint p.d.f.s of the velocity invariants, the RDF
measurements of the clustering of the turbulent zero velocity points are not affected
much by the applied Median Filter. The magnitude of the RDF of the noise free
DNS data is higher than the experimental result, indicating stronger clustering of
zero velocity points occurring in the numerical simulation. This could possibly be
due to the higher turbulent intensity represented by turbulent Reynolds number of
$Re_\lambda = 433$ compared to the considered experimental condition of $Re_\lambda = 147$. However,
the physical mechanisms leading to the clustering of turbulent zero velocity points is
not the focus of the current part, but is discussed in the following Section.

The Voronoï analysis decomposes the two-dimensional space into individual
Figure 4.7: Quantification of clustering of turbulent zero mean velocity with Radial Distribution Function (RDF)
Voronoi cells that correspond to each zero velocity points with ensemble points closer to it than any other points. The Voronoï cell area $A$ is the inverse of the local concentration. Thus, the distribution function of Voronoï cell areas can provide information of the local clustering level. The probability distribution function of the normalized Voronoï cell areas $A/A_{\text{avg}}$ has been applied to the quantification of preferential concentration of particles (Monchaux et al., 2010).

Voronoï analysis has been applied to evaluate the clustering of the zero velocity points. The normalised Voronoï area p.d.f.s of zero velocity points derived from raw and median filtered data are shown in Fig. 4.8. It is consistent with both the visual inspection of Fig. 4.6 and the results for the RDF of Fig. 4.7 that the noise-free DNS data and the experimental data filtered by $5 \times 5$ median filter window size show a stronger degree of clustering. The remaining differences between the noise-free DNS results indicate that the filtering approach is successful in eliminating most of the noise. This is supported by the p.d.f.s of Voronoï areas showing the largest deviation from the Random Poisson Process (R.P.P.) of Voronoï areas. It should be noted that the zero velocity points of the raw experimental data are more likely to be distributed randomly. However, the normalized Voronoï area p.d.f.s are more sensitive to the denoising process in comparison to the robustness observed in the RDF analysis. This could possibly be due to the fact that Voronoï analysis is better capable in capturing the clustering statistics contributed by each individual zero velocity point, since the averaging occurs once per measurement frame. Also, it should be noted that the normalized Voronoï area p.d.f.s of zero velocity points are calculated based on the PIV vector field with vector spacing of 8 pixels, effectively $364 \, \mu m$ and improvement of the spatial resolution of the measurements was not possible with the available cameras.

The clustering of turbulent zero velocity points derived from raw and median filtered experimental data is quantified by RDF and Voronoï analysis. It is consistent
Figure 4.8: Quantification of clustering of turbulent zero mean velocity with Voronoi Analysis
that zero velocity points of the data processed by Median Filter window size of $5 \times 5$ returns the strongest degree of clustering. However, there is no significant difference in terms of clustering magnitude compared to other data sets to distinguish the appropriate Median Filter window size. Thus, due to the fact that the joint p.d.f.s of velocity invariants processed by Median Filter with $5 \times 5$ window size shows the best fit to the un-blurred, noise-free joint p.d.f.s, the visualization of the zero velocity points is appropriate for noise elimination in evaluating turbulent topological flow patterns. Therefore, the current analysis of the clustering behaviour of zero velocity points can be used to correlate with the clustering behaviour of dispersed droplets by the flow turbulence. This study is described in the following Section.

To summarize, the experimental evaluation of the topological characteristics of the turbulent flow generated in the ‘box of turbulence’ requires study of noise effect on velocity invariants. The joint p.d.f.s of noise contaminated velocity invariants are found with different shape than the noise-free joint p.d.f.s of velocity of DNS data from the Johns Hopkins University (JHU) open resource turbulence database. A noise model based on Gaussian noise and impulsive Salt & Pepper noise is established by adding artificial noise to the velocity vector field of DNS data from JHU database. Digital filtering methods, based on Median and Wiener Filters, are chosen to eliminate the modelled noise source and examined their capacity to restore the joint p.d.f.s of velocity invariants to that of DNS data. The clustering of turbulent zero velocity points is quantified using Radial Distribution Function (RDF) and Voronoï analysis combined with median filtered velocity of different median filter window size. It has been found that a Median Filter with window size $5 \times 5$ is the effective and efficient approach in identifying the two-dimensional topological turbulent flow patterns, which eliminate the experimental noise to a satisfactory level.
CHAPTER 4. TIME RESOLVED STATISTICS

4.1.3 Discussions on the number of zero velocity points

Zero velocity points calculated by Newton-Raphson method from Kinematic Simulation (KS) and Direct Numerical Simulation (DNS) were reported in Davila and Vassilicos (2003). The number of zero velocity points per unit volume were given by the following expression and shown in Fig. 4.9 as function of $L/\eta$.

$$n_s \approx C_s L^{-3} \left( \frac{L}{\eta} \right)^{D_s}$$

(4.16)

where $D_s = 2$ and $C_s$ is a dimensionless number representing fractal dimension, $L$ is integral length scale and $\eta$ is the Kolmogorov length scale.

![Figure 4.9: The number of zero velocity points per unit volume versus $L/\eta$ in DNS (solid circle) and KS for $p = 5/3$ and different values $V/u'$; Eq.4.16 is shown as dashed lines (Davila and Vassilicos, 2003)](image)

For the current experimental conditions, the numbers of zero velocity points per unit volume are estimated in the range of $10^3$ to $10^4$ as shown in Fig. 4.9, with the ratio between $L/\eta$ approximated around 200.

Following the different method detailed in the previous section, the zero velocity points are calculated from 2D PIV measurements in a domain of $40mm \times 40mm$, and from 2D DNS slices (Malloupas et al., 2013a; Perlman et al., 2007; Li et al., 2008). The p.d.f.s of the number of zero velocity points is shown in Fig. 4.10.
Figure 4.10: p.d.f.s of the zero velocity/acceleration points measured by 2D PIV
Fig. 4.10 shows that the absolute number of zero velocity point in 2D PIV measured velocity fields is in the range of $10^3$ to $10^4$, and with a larger variance comparing to the p.d.f.s of DNS results. The number of zero velocity points of both 2D experiment and DNS are in the same range, indicating that the stagnation points generated by opposing jets in the ‘box of turbulence’ are more likely to be negligible.

One possible reason for the difference in the number of zero velocity points derived from the method detailed above and the Newton-Raphson method (Davila and Vassilicos, 2003), is 2D projection distortion of 3D turbulent topological critical points. Comparing the calculation of zero velocity points from DNS data with both 3D velocity invariants and 2D velocity invariant, could provide quantitative information of such distortion, which is one of the future work of this thesis.

### 4.2 Cluster Identification and Time-dependant Tracking

This Chapter will present a time-dependent analysis of the droplet distribution patters, as measured in the ‘box of turbulence’. However, this Section presents a method used to analyse the measured time-dependent images in order to obtain information on droplet clustering.

Existing quantification techniques for droplet clustering rely on statistics lacking connections to human visual inspection of time-dependant droplet dispersion images. A summary of these quantification methods was presented by Monchaux et al. (2012). A specific droplet cluster can be observed by eye in temporal evolution of individual droplet clusters analysed by Voronoï analysis. This Section starts by presenting an example of a temporal sequence of droplets identified by experimental images using the image processing of Chapter 3, in Fig. 4.11. In addition, the Voronoï analysis areas are superimposed.
Figure 4.11: Human visualisation of temporal evolved droplets with corresponding Voronoï cells. Spatial resolution is 45.5 µm, 900 pixels corresponds to approximately 4.1 mm, about 250 times of the Kolmogorov length scale of the presented turbulent flow.
The sequence of images of Fig. 4.11 shows a droplet cluster, circled on the images, with its initial lengthscale of $3\eta$, that is gradually reduced in size over a timescale of $1.5\tau_k$, when the droplet cluster disappeared. The velocity of the cluster within this period was found to be similar to the turbulent fluctuations of the fluid flow. To evaluate the behaviour of individual droplet clusters, there is a need to develop a technique to identify individual droplet clusters, as detected by human visual inspection, and track them as a function of time. In this Section, a novel algorithm based on mean shift feature space analysis (Comaniciu and Ramesh, 2000; Comaniciu and Meer, 2002) and Voronoï tessellation was proposed to fulfil this goal and provide an estimation of the temporal evolution of the droplet cluster length scale. This Section begins with the theory of mean shift feature space analysis followed by the combination of Voronoï space tessellation in identifying and tracking each individual droplet cluster. Its application on temporal resolved experiments in studying droplet preferential concentration is briefly discussed with details presented in the following Sections.

### 4.2.1 Mean Shift Feature Space Analysis

The first step in order to track a cluster is to identify and define a cloud of droplets as ‘a cluster’. In the community of pattern recognition, the non-parametric mean-shift algorithm has been a widely applied approach in the feature space analysis and real-time tracking (Comaniciu and Meer, 2002). A feature space can be regarded as the empirical probability function (p.d.f) of its represented parameter, for example, the three-dimensional $l^*u^*v^*$ color space for color images. An example of the feature space is shown in Fig. 4.12.

The $l^*u^*v^*$ color space shown in Fig. 4.12 is a color space concept adopted by the International Commission on Illumination (CIE) in 1976, as a simple-to-compute transformation of color images, also referred as CIELUV. The hue, saturation and
lightness of a color image can be transformed and reversed according to the defined and adopted Chromaticity Diagram. Detailed definition regarding the transformation into \( l^*u^*v^* \) color space can be found in Fairchild (1997). For typical images, \( u^* \) and \( v^* \) range \( \pm 100 \). By definition, \( 0 \leq l^* \leq 100 \). Fig. 4.12 is shown for the purpose of introducing the mean shift feature space analysis illustrated below in Fig. 4.13.

The one-dimensional intensity space for grayscale images or the one-dimensional binary space for binary images is also a representation of digital images. In the present study of droplet preferential concentration, binary images with droplets, identified as the component of a droplet cluster, have been produced with Voronoï analysis. As a consequence, the p.d.f.s of the one-dimensional binary space are of particular interest in terms of cluster identification.

By applying the mean shift algorithm, one ‘cluster’ can be defined by the identified centre of mass. Multivariate kernel density estimation is a popular density estimation method (Comaniciu and Meer, 2002). Given \( n \) sample points \( x_i \) for \( i = 1, \ldots, n \) in the \( d \)-dimensional space \( \mathbb{R}^d \), the density at the point \( \mathbf{x} \) can be estimated by the definition of Eq.4.17.
\[ \hat{f}(x) = \frac{1}{n} \sum_{i=1}^{n} K_H(x - x_i) \quad (4.17) \]

where \( K(x) \) is the kernel function and \( H \) is a symmetric positive definite \( d \times d \) bandwidth matrix defined as

\[ K_H = H^{-1/2} K(H^{-1/2} x) \quad (4.18) \]

The bandwidth matrix \( H \) can be defined as fully parameterized. However, in practice, the bandwidth matrix \( H \) is usually chosen proportional to the identity matrix and expressed as \( H = h^2 I \) (Comaniciu and Meer, 2002). Details in choosing the bandwidth matrix \( H \) is discussed in the following Section. The kernel density estimation Eq.4.17 becomes

\[ \hat{f}(x) = \frac{1}{nh^d} \sum_{i=1}^{n} K \left( \frac{x - x_i}{h} \right) \quad (4.19) \]

where \( h \) is the bandwidth parameter defined. \( d \) represents the \( d \) dimensional space \( \mathbb{R}^d \). For the one dimensional binary space in the study of droplet preferential concentration, \( d \) yields one.

The Gaussian multivariate kernel \( K_N(x) \) is one of the most commonly used kernel for \( K(x) \) and is defined as

\[ K_N(x) = (2\pi)^{-d/2} e^{-\frac{1}{2} \|x\|^2} \quad (4.20) \]

Employing the Gaussian kernel profile of Eq.4.20 in Eq.4.19, the kernel density estimation can be rewritten as

\[ \hat{f}_K(x) = \frac{1}{nh^d} \sum_{i=1}^{n} k \left( \left\| \frac{x - x_i}{h} \right\|^2 \right) \quad (4.21) \]
The density at each point $x$ is effectively the average weighted density by each of the sample points $x_i$ for $i = 1, ..., n$.

The zero points of the density gradient $\nabla f(x) = 0$ represent the centre of mass for each point $x$. The mean shift procedure is to locate those zero density gradient points.

The density gradient can be obtained by the gradient of the density estimator defined in Eq.4.22.

$$\nabla \hat{f}_K(x) = \frac{2}{nh^{d+2}} \sum_{i=1}^{n} (x - x_i)k' \left( \left\| \frac{x - x_i}{h} \right\|^2 \right)$$  \hspace{1cm} (4.22)

where $k'$ represents the derivative of multivariate kernel $k$. The following expression Eq.4.23 is defined.

$$g(x) = -k'(x)$$  \hspace{1cm} (4.23)

Introducing Eq.4.23 into Eq.4.22 yields,

$$\nabla \hat{f}_K(x) = \frac{2}{nh^{d+2}} \left[ \sum_{i=1}^{n} g \left( \left\| \frac{x - x_i}{h} \right\|^2 \right) \right] \left[ \frac{\sum_{i=1}^{n} x_i g \left( \left\| \frac{x - x_i}{h} \right\|^2 \right)}{\sum_{i=1}^{n} g \left( \left\| \frac{x - x_i}{h} \right\|^2 \right)} - x \right]$$  \hspace{1cm} (4.24)

The first term of the product of Eq.4.24 is proportional to the density estimate at location $x$ as defined in Eq.4.21. Thus, the second term of the product in Eq.4.24 is effectively the vector which always point towards the direction of maximum increase
in the density, and is widely referred as the *mean shift vector*, defined in Eq.4.25.

\[
m_{h,G}(x) = \frac{\sum_{i=1}^{n} x_i g \left( \| \frac{x-x_i}{h} \|^2 \right)}{\sum_{i=1}^{n} g \left( \| \frac{x-x_i}{h} \|^2 \right)} - x
\] (4.25)

The mean shift vector can shift a point of interest \( x \) to its corresponding *centre of mass* and define a path leading to this zero density gradient point. This procedure is an adaptive gradient ascent approach. In the regions with low density, the mean shift steps are significantly larger than those in the dense regions. Near the *centre of mass*, the mean shift analysis is more refined with small shift steps. The behaviour of the mean shift feature space analysis is expressed in Fig. 4.13. (Comaniciu and Meer, 2002).

**Fig. 4.13** (a) is the spatial representation of 110,400 sample points of the two components \( l^* \) and \( u^* \) of the \( l^*u^*v^* \) colour space shown in **Fig. 4.12**(b), marked with black dots, **Fig. 4.13**(b) is the decomposition of these sample points based on the mean shift analysis on the point of interest to the same *centre of mass* (i.e. ‘peaks’) and define the sample points bounded by he point of interests as the component of one sample points ‘cluster’. **Fig. 4.13** (c) is the spatial density contour with trajectories of the mean shift procedure shown converging to the *centre of mass* marked with large red dots.

The mean shift procedure in order to identify the ‘peak of density’, also referred as ‘center of mass’, can be summarised as

1. compute the mean shift vector \( m_{h,G}(x) \) of Eq.4.25;
2. shift the point of interest \( x \) with the mean shift vector;
3. repeat the above two steps until approaching the points where \( \nabla f(x) \) of Eq.4.22 equals zero, which also yields, \( m_{h,G}(x) = 0 \).
Figure 4.13: Illustration of Mean Shift feature space analysis (a) Spatial representation of 110,400 sample points; (b) Decomposition based on the mean shift algorithm; (c) Trajectories of the mean shift procedure, the red dots of centre of mass define the decomposition (Comaniciu and Meer, 2002)
4.2.2 Droplet Cluster Identification

The mean shift algorithm requires dividing the two-dimensional space with fixed length grids as the points of interest for the density gradient estimation. In the application of identifying real world targets in three dimensional l*u*v* color space, for example human figure, still life object or moving vehicles etc, the fixed length grids serve well the purpose due to the fact that the representations of targets in l*u*v* color space are usually clearly distinguished against background features. However, the choice of a fixed grid size causes ambiguity in identifying droplet clusters as shown in previous work of Fessler et al. (1994). Thus, to avoid this ambiguity, the vertices of the Voronoï cells are used in the mean shift procedures instead of the grid vertices. The cluster identification procedure based on the mean shift feature space analysis and the Voronoï tessellations is explained as follows.

1. The two-dimensional space is divided into Voronoï cells corresponding to each individual droplet. The vertices of polygon Voronoï cells are defined as points of interest for density gradient estimation in the mean shift analysis. Note that defining vertices of polygon Voronoï cell as points of interest is the major development to adopt mean shift feature space analysis in order to identify droplet clusters.

2. Perform the mean shift converging on the points of interest: vertices of polygon Voronoï cells.

3. The vertices of Voronoï cells that converge to the same ‘peak’/‘center of mass’ are identified as one vertices group. So far, the two-dimensional space is divided into vertices groups. Each vertices group is made up of a number of completer Voronoï cells and incomplete Voronoï cells.

4. The droplets contained in complete Voronoï cells in one vertices group are identified as one droplet cluster.
5. The droplets contained in incomplete Voronoï cells between neighbouring identified droplet clusters can be defined as voids.

Example of applying the described cluster identification procedure is shown in Fig. 4.14 on two consecutive droplet images with a time difference of $3 \times 10^{-4} \text{s}$. The mean shift algorithm with the chosen bandwidth is capable to define a cluster corresponding to human visual inspection. Parameterising the bandwidth for the mean shift algorithm affects the respondence to human visual inspection. Optimal bandwidth selection is discussed in the following Section.

Two sample frames of the full experimental images with clusters identified are shown in Fig. 4.14, giving a brief idea that the proposed cluster identification scheme capable to track clusters as a function of time. It can be seen in Fig. 4.14 that clusters could move, rotate, transform in shapes, merge and disappear. Detailed study of the cluster tracking is included in the following Sections.

The advantage of introducing the Voronoï tessellation in the mean shift feature space decomposition is to avoid the effect of fixed grid size. Also, due to the capability
of Voronoï tessellation to track each droplet individually, the non-rigid shape of the clusters can be identified quantitatively. The statistics of a typical cluster length scale can be deduced by characterizing the cluster length scale \( l_c \) (Eq.4.26) as the diameter \( d_c \) of a circle that occupies the same area as that of a non-rigid shaped droplet cluster \( A_{nc} \).

\[
l_c = d_c \tag{4.26}
\]

with

\[
\pi \left( \frac{d_c}{2} \right)^2 = A_{nc} \tag{4.27}
\]

where \( A_{nc} \) is directly derived as the total area of the Voronoï cells that are defined as one group based on the Voronoï mean shift analysis.

The mean shift converging analysis, combined with Voronoï space tessellation, has been used to identify instantaneous droplet clusters and define corresponding cluster length scales from the experimental Mie-scattering intensity images. The procedure to identify instantaneous droplet clusters and corresponding cluster length scales from experimental Mie-scattering intensity images is listed as the following steps.

1. Perform the band-pass filter described in Chapter 3 to locate droplets on instantaneous Mie-scattering intensity images.

2. Perform the cluster identification procedure described above.

3. The diameter \( d_c \) of a circle that occupies the same area as that of a non-rigid shaped droplet cluster \( A_{nc} \) is defined as the cluster length scale.

In order to examine the proposed cluster identification technique based on mean shift feature space analysis combined with Voronoï space tessellation, randomly dispersed particles are generated numerically. Those together with preferentially
distributed particles observed in experiments are processed for cluster identification and cluster length scale estimation. Results are shown in Fig. 4.15

Fig. 4.15(a)(c)(e) correspond to experimental images with preferentially concentrated droplets. Fig. 4.15(a) shows the identified droplet clusters with the defining Voronoi cell vertices marked with different colour. The droplet clusters with different length scales observed by visual inspection can be distinguished with the proposed cluster identification approach with their centers identified and shown in Fig. 4.15(a). Fig. 4.15(c) shows the normalised Voronoi area p.d.f. for clustering droplets, which clearly deviates from the Random Poisson Process (RPP) representing preferential concentration. Fig. 4.15(e) shows an example calculation of the cluster length scale p.d.f. averaged over 2048 experimental images with $1024 \times 1024$ pixel and a spatial resolution of $45.8\mu m$ per pixel. The p.d.f. of the sample cluster length scale has maximum at around $10 mm$, suggesting that the most probable cluster length scale observed in the example images is around $10 mm$.

Randomly dispersed particles are generated numerically with MATLAB function ‘rand’, with the same image resolution and domain as achieved in the experiment. The proposed cluster identification method is examined by processing those randomly distributed particles. It is expected that no particular clusters/cluster length scales would be identified. Fig. 4.15(b) shows the identified pseudo-clusters with the defining Voronoi cell vertices marked with different colour. It appears that these pseudo-clusters have similar length scales. Fig. 4.15(d) shows the normalised Voronoi area p.d.f. for randomly distributed droplets that follows the random Poisson process well. Fig. 4.15(f) is the cluster length scale p.d.f. of randomly distributed particles averaged over 1500 sample realisations. There is not a spread of cluster length scale which is expected for randomly dispersed particles.

Thus, the proposed cluster identification method based on mean shift feature space analysis and Voronoi tessellation with chosen bandwidth parameter is capable in
Figure 4.15: Examine mean shift cluster identification scheme (a) Experimental droplets cluster identification (c) Normalised Voronoï area p.d.f. for clustering droplets (e) Cluster length scale p.d.f. for clustering droplets; (b) Randomly distributed particles and cluster identification (d) Normalised Voronoï area p.d.f. for random particle distribution (f) Cluster length scale p.d.f. for random particle distribution
quantifying cluster length scale in a way that comparable to human visual inspection. It also satisfies expectations for randomly distributed droplets.

4.2.3 Time-dependent Droplet Cluster Tracking

Morphological droplet cluster identification achieved by the proposed algorithm is the first step to track individual cluster in time. However, as shown in Fig. 4.14, the complexity raised by cluster movement and transformation makes morphological tracking difficult. Alternatively, measurement of the overlap between two discrete probability distributions of the cluster area can be used to indicate the similarity between two frames of dispersed droplets. The probability distribution of 9 consecutive distribution images of dispersed droplets with the time difference \( dt = 0.0003\text{s} \) is shown in Fig. 4.16.

In order to define the similarity of two discrete distributions indicating the closeness of the two images of dispersed droplets, Bhattacharyya coefficient (Bhattacharyya, 1943) is used given by Eq. 4.28.

\[
\rho(p, p') = \sum_{i=1}^{N} \sqrt{p(i)p'(i)}
\]  

(4.28)

where \( p(i) \) and \( p'(i) \) represents the two probability distributions in comparison, yields \( \sum_{i=1}^{N} p(i) = \sum_{i=1}^{N} p'(i) = 1 \). The Bhattacharyya coefficient represent geometric interpretation as the cosine of the angle between two vectors. Thus, during the measurement for two identical distribution \( p(i) = p'(i) \), the Bahattacharyya coefficient is

\[
\rho(p, p') = \sum_{i=1}^{N} \sqrt{p(i)p'(i)} = \sum_{i=1}^{N} \sqrt{p(i)p(i)} = \sum_{i=1}^{N} p(i) = 1
\]  

(4.29)

In the work of Comaniciu and Meer (2002), the Bhattacharyya coefficient
Figure 4.16: Histogram of cluster area in 9 consecutive images with time difference $dt = 0.0003s$
definition is modified to represent the metric distance between distribution as

\[ d(p, p') = \sqrt{1 - \rho(p, p')} \] (4.30)

In the current thesis, the calculation of Bhattacharyya coefficient follow the approach of Kailath (1967) and Comaniciu and Meer (2002). Sample calculation corresponds to the probability distribution shown in Fig. 4.16. It is shown in Fig. 4.17 that the Bhattacharyya coefficient between image 1 and itself yields zero and it is consistent that largest deviation is observed between image 1 and image 6 with the Bhattacharyya coefficient peaks at frame 6.

![Figure 4.17: Bhattacharyya coefficient of probability distribution of cluster areas in 9 consecutive frames with time difference \( dt = 0.0003 \) s](image)

The Bhattacharyya coefficient has been calculated from 2048 realisations of experiment of images of droplets dispersed in flow with \( Re_\lambda = 235 \) and droplet \( D_{32} = 55 \mu m \), aiming to find out the typical time scale the droplet clusters maintain
themselves. Fig. 4.18 shows an overall increasing trend of the Bhattacharyya coefficient with multiple values around zero, suggesting occurrence of similar probability distribution of cluster area possibly because of the limited sample number of droplet clusters identified. Also, the variation of Bhattacharyya coefficient becomes significant in the range of $10^0$ to $10^1$ Kolmogorov time scale, indicating the droplet clusters do not maintain themselves for more than 10 times the Kolmogorov time scale.

![Bhattacharyya coefficient for experiment with droplets dispersed in flow with $Re_\lambda = 235$ and droplet $D_{32} = 55 \mu m$](image)

**Figure 4.18:** Bhattacharyya coefficient for experiment with droplets dispersed in flow with $Re_\lambda = 235$ and droplet $D_{32} = 55 \mu m$

### 4.2.4 Optimised Bandwidth Selection

In the process of mean shift feature space analysis, the bandwidth matrix $H$ is the single parameter affecting the outcome of the droplet cluster identification. In the community of pattern recognition, the selection of bandwidth has been commonly assessed empirically (Comaniciu and Meer, 2002) for the tracking object and is usually
characterised by specific colour representation histogram. However, in the application of identifying clusters with a non-rigid shape, a more rigorous approach is required since there is no benchmark histogram to assist the identification process.

In the work of this thesis, the optimal bandwidth is defined as the one that minimises the mean integrated square error (MISE) that achieved the best compromise between the bias and variance in the kernel density estimation. Although kernel density estimation is not needed in the mean shift feature space analysis, it has been performed for the purpose of determining the optimal bandwidth. The mean integrated square error (MISE) is defined as

\[
MISE(x) = E \int (\hat{f}(x) - f(x))^2 \, dx
\]  

(4.31)

where \(\hat{f}(x)\) is the kernel density estimation defined in Eq. 4.19. The time resolved experiments reported in this Chapter is characterised by different turbulent Reynolds number, droplet size distribution and number density. Thus, the optimal bandwidth that minimise MISE is slightly different according to the sample image processed. In order to maintain the consistency in the cluster identification scheme based on the mean shift feature space analysis, the same bandwidth matrix has been used for the processing of all the experimental data following the procedures summarised in the last Section. The selection of this ‘universal’ fixed bandwidth matrix is based on the kernel density estimation for the measurements characterised by droplet Sauter mean diameter \(D_{32} = 55\mu m\) and turbulent Reynolds number \(Re_\lambda = 235\).
CHAPTER 4. TIME RESOLVED STATISTICS

4.3 Time-resolved Investigation of Clustering of Droplets

4.3.1 Experimental Setup

The generation of a volume of homogeneous and isotropic turbulence without mean flow, known as "Box of Turbulence", was detailed in the previous Chapters. Homogeneous and isotropic turbulence with different intensities where generated in the centre of the 'box'. Four different turbulent intensities were achieved in the time-resolved investigations. Characteristics of the time resolved turbulent flow field can be found in the following Section.

Poly-disperse droplets were introduced from an air-assist atomizer mounted around 1m above the ‘box’, in order to ensure that droplet momentum dissipates before they reach the box. The droplet size distributions were measured by means of Phase Doppler Annemometry (PDA) and could be varied widely by controlling the air and liquid flow rates as described in Chapter 3. An Edgewave, IS-series, Nd:YAG laser operating at 532 nm was pulsed at high speed (between 25 Hz and 3 kHz) in order to illuminate a plane through the centre of the ‘box’. A planar laser sheet was shaped by a series of cylindrical optics with approximate thickness of 0.1 mm and was aligned at the centre of an illuminated Area of Interest (AOI) of around 45x45 mm² at the ‘box’ centre. The laser sheet illuminated the droplets and the intensity of the Mie-scattered light intensity was recorded. The actual experimental setting is shown in Fig. 4.19

The air flow characteristics of the ‘box of turbulence’ were measured by double frames Particle Image Velocimetry(PIV) with a time lag of 100 µs between two frames after introducing fine glycol droplets with size less than 3 µm, generated by a VIVID stage fog generator. The instantaneous double frame, double exposure PIV measurements and the single frame spatial droplet distribution images were captured
by a Photron APX CMOS camera (1024 pixel × 1024 pixel) used with a 105mm lens f/2.8, leading to a linear magnification of 0.3 and resulting in spatial resolution of 45.5 µm/pixel. The recording rates of the camera for the PIV measurements and the instantaneous light scattering intensity images of the droplets were set from 25 Hz up to 1500 Hz and 3000 Hz respectively.

4.3.2 Turbulent Flow Characteristics

Four different turbulent intensities were generated with the experimental setup described above. The homogeneity and isotropy are examined with aforementioned statistics from velocity gradient tensor and strain rate tensor. The *velocity stagnation points* and *zero acceleration points* are derived from the time-dependent flow velocity so as to test the *sweep-stick mechanism*. Descriptions of flow fields quantities are detailed in this Section, followed by quantifications of the droplet preferential concentration with the same techniques used in the previous Chapter.

Double-frame, double-exposure Image pairs were processed with the PIV software Davis 7.2 from Lavision GmbH to acquire velocity vector fields. The 46.6 mm × 46.6 mm image pairs were processed twice with the initial interrogation window size of
64 × 64 with 50% overlap and the final window size of 16 × 16 with 50% overlap, resulting in a 128 × 128 vector field. The resulting vector spacing is 364 µm, 8 times of the spatial resolution 45.5 µm/pixel.

The spatial contour of the temporal averaged mean velocity magnitude \( \sqrt{u_1^2 + u_2^2} \) were plotted by averaging 1024 images with the recording rate of 25 Hz for all the turbulent conditions Fig. 4.20. The temporal mean velocities are less than the magnitude of 0.15 m/s, approximating zero mean velocity flows and the stationary turbulent statistics. The turbulent flow velocity conditions are summarise in Table 4.1. The uncertainties are estimated based on the confidential interval of 99%.

<table>
<thead>
<tr>
<th></th>
<th>Exp.1</th>
<th>Exp.2</th>
<th>Exp.3</th>
<th>Exp.4</th>
</tr>
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<tr>
<td>(&lt;u&gt;) (m/s)</td>
<td>-0.016 ± 0.000</td>
<td>-0.022 ± 0.000</td>
<td>0.049 ± 0.000</td>
<td>0.028 ± 0.001</td>
</tr>
<tr>
<td>(&lt;v&gt;) (m/s)</td>
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<td>0.072 ± 0.000</td>
<td>-0.001 ± 0.000</td>
<td>0.0637 ± 0.001</td>
</tr>
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<td>(&lt;u_{rms}&gt;) (m/s)</td>
<td>0.384 ± 0.000</td>
<td>0.505 ± 0.001</td>
<td>0.746 ± 0.003</td>
<td>0.866 ± 0.001</td>
</tr>
<tr>
<td>(&lt;v_{rms}&gt;) (m/s)</td>
<td>0.380 ± 0.000</td>
<td>0.512 ± 0.001</td>
<td>0.734 ± 0.003</td>
<td>0.849 ± 0.001</td>
</tr>
</tbody>
</table>

The absolute spatial-temporal mean velocity magnitude for the four conditions are: \( \langle u_a \rangle = 0.02 m/s, \langle u_b \rangle = 0.07 m/s, \langle u_c \rangle = 0.05 m/s, \langle u_d \rangle = 0.07 m/s \), which confirms that the mean velocity is negligible in all the experimental turbulent conditions.

The spatial contours of the ratio of the velocity fluctuations in the two orthogonal flow directions associated with the plane of the laser sheet, \( u_{1,rms}/u_{2,rms} \), are presented in Fig. 4.21. The results show that the range of values are between 0.9 and 1.1 centred around 1. This confirms the spatial isotropy of turbulence within the area of interest. The absolute values of the spatial-temporal mean velocity fluctuations are summarised in Table 4.1

Both the one-dimensional and the two-dimensional turbulent kinetic energy were calculated from the obtained PIV vector maps. The one-dimensional energy spectrum
Figure 4.20: Mean velocity contours (a) $Re_\lambda = 97$ (b) $Re_\lambda = 127$ (c) $Re_\lambda = 147$ (d) $Re_\lambda = 235$
Figure 4.21: R.m.s velocity ratio (a) $Re_\lambda = 97$ (b) $Re_\lambda = 127$ (c) $Re_\lambda = 147$ (d) $Re_\lambda = 235$
Figure 4.22: Probability distribution of normalised fluctuating velocity (a) 
$Re_\lambda = 97$ (b) $Re_\lambda = 127$ (c) $Re_\lambda = 147$ (d) $Re_\lambda = 235$
was defined in Eq.3.5 based on the corresponding two-point correlation functions (George, 2006).

The two dimensional energy spectrum was calculated from the squared one-dimensional spectrum in the longitudinal and lateral directions and shown in Fig. 4.23. The two dimensional energy spectrum in $x$ and $y$ direction agree well, suggesting that the turbulence in the centre of the ‘box of turbulence’ is homogeneous and isotropic.

The other quantities were calculated based on the dissipation rate with the definitions defined in the Chapter 3. The turbulent descriptive quantities for the four different turbulent conditions are summarised in Table 4.2.
Table 4.2: Turbulent quantities measured by 2DPIV with repetition rate of 1500 Hz

<table>
<thead>
<tr>
<th></th>
<th>Exp.1</th>
<th>Exp.2</th>
<th>Exp.3</th>
<th>Exp.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q^2$ ($m^2/s^2$)</td>
<td>0.441 ± 0.000</td>
<td>0.789 ± 0.002</td>
<td>2.148 ± 0.025</td>
<td>2.262 ± 0.006</td>
</tr>
<tr>
<td>$\varepsilon$ ($m^2/s^3$)</td>
<td>4.966 ± 0.085</td>
<td>10.499 ± 0.116</td>
<td>19.692 ± 0.4581</td>
<td>16.151 ± 0.251</td>
</tr>
<tr>
<td>$Re_\lambda$</td>
<td>97.854 ± 0.357</td>
<td>126.749 ± 0.440</td>
<td>147.407 ± 0.773</td>
<td>234.888 ± 0.590</td>
</tr>
<tr>
<td>$\lambda$ (m)</td>
<td>0.004 ± 0.000</td>
<td>0.004 ± 0.000</td>
<td>0.003 ± 0.000</td>
<td>0.004 ± 0.000</td>
</tr>
<tr>
<td>$\tau_k$ (s)</td>
<td>0.003 ± 0.000</td>
<td>0.002 ± 0.000</td>
<td>0.001 ± 0.000</td>
<td>0.001 ± 0.000</td>
</tr>
<tr>
<td>$\eta$ (mm)</td>
<td>0.193 ± 0.000</td>
<td>0.167 ± 0.000</td>
<td>0.134 ± 0.000</td>
<td>0.136 ± 0.000</td>
</tr>
<tr>
<td>$\tau_{\varphi\epsilon}$ (m)</td>
<td>0.123 ± 0.001</td>
<td>0.122 ± 0.001</td>
<td>0.069 ± 0.000</td>
<td>0.129 ± 0.000</td>
</tr>
<tr>
<td>$\Lambda$ (m)</td>
<td>0.031 ± 0.000</td>
<td>0.039 ± 0.000</td>
<td>0.032 ± 0.000</td>
<td>0.072 ± 0.002</td>
</tr>
</tbody>
</table>

The turbulence homogeneity and isotropy have been examined previously with $u_{r.m.s}$ ratio contour, two-point velocity correlation coefficient and both one-dimensional and two-dimensional energy spectrums. Apart from defining the saddle points: velocity stagnation points and zero acceleration points, the properties of characteristic polynomial coefficient $p$ and $q$ statistics can also be considered in examining the turbulent homogeneity and isotropy (Cardesa et al., 2013).

The average of $p$ defined in Eq.4.32 should approach zero under the assumption of stationary homogeneous and isotropic turbulence for which $\langle u_1 \rangle = \langle u_2 \rangle = 0$.

$$\langle p \rangle = \left\langle -\left(\frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2}\right) \right\rangle = - \left\langle \frac{\partial u_1}{\partial x_1} \right\rangle - \left\langle \frac{\partial u_2}{\partial x_2} \right\rangle = - \frac{\partial}{\partial x_1} \langle u_1 \rangle - \frac{\partial}{\partial x_2} \langle u_2 \rangle = 0 \quad (4.32)$$

The average of $q$ defined in Eq.4.33 should also approach zero with that an equal expression of $\left\langle \frac{\partial u_1}{\partial x_2} \frac{\partial u_2}{\partial x_1} \right\rangle = \left\langle \frac{\partial u_1}{\partial x_1} \frac{\partial u_2}{\partial x_2} \right\rangle$ (George and Hussein, 1991).

$$\langle q \rangle = \left\langle \left(\frac{\partial u_1}{\partial x_2} \frac{\partial u_2}{\partial x_1}\right) - \left(\frac{\partial u_1}{\partial x_1} \frac{\partial u_2}{\partial x_2}\right) \right\rangle = \left\langle \frac{\partial u_1}{\partial x_2} \frac{\partial u_2}{\partial x_1} \right\rangle - \left\langle \frac{\partial u_1}{\partial x_1} \frac{\partial u_2}{\partial x_2} \right\rangle = 0 \quad (4.33)$$

The behaviour of the eigenvalues of the strain-rate tensor is another indication of
the turbulent homogeneity and isotropy. The three-dimensional strain-rate tensor $S$ is defined as

$$S = \begin{bmatrix}
a_{11} & d_{12} & d_{13} \\
d_{12} & a_{22} & d_{23} \\
d_{13} & d_{23} & -(a_{11} + a_{22})
\end{bmatrix}$$

The reduced two dimensional strain-rate tensor $\tilde{S}$ is the upper left part of the above $3 \times 3$ matrix.

$$\tilde{S} = \begin{bmatrix}
a_{11} & d_{12} \\
d_{12} & a_{22}
\end{bmatrix}$$

where $a_{11} = \partial u_1 / \partial x_1$, $a_{22} = \partial u_2 / \partial x_2$, $d_{12} = ((\partial u_1 / \partial x_2) + (\partial u_2 / \partial x_1))/2$.

The eigenvalues of the two dimensional strain rate matrix $\tilde{\Lambda}_{1,2}$ are defined in Eq.4.34

$$\tilde{\Lambda}_{1,2} = \frac{a_{11} + a_{22}}{2} \pm \frac{1}{2} \sqrt{(a_{11} + a_{22})^2 - 4(a_{11}a_{22} - d_{12}^2)} \quad (4.34)$$

If the turbulence is homogeneous and isotropic, immediately the following expression is true.

$$\langle a_{11} + a_{22} \rangle = \langle p \rangle = 0 \quad (4.35)$$

Properties of the $p$ and $q$ statistics calculated from Eq.4.32 and Eq.4.33 are summarised in Table 4.3. The averages are calculated with 99% of the confidential interval.

The ratio of the two principal strain rate of the four experimental conditions are listed in Table 4.4, which confirms the homogeneity and isotropy of turbulence.

### Table 4.3: Statistics of the invariants of the reduced velocity gradient tensor measured by 2DPIV with repetition rate of 1500 Hz

<table>
<thead>
<tr>
<th></th>
<th>Exp.1</th>
<th>Exp.2</th>
<th>Exp.3</th>
<th>Exp.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$&lt;p&gt;$</td>
<td>-0.0000 ± 0.0003</td>
<td>-0.0000 ± 0.0002</td>
<td>-0.0000 ± 0.0003</td>
<td>-0.0000 ± 0.0003</td>
</tr>
<tr>
<td>$&lt;q&gt;$</td>
<td>-0.0002 ± 0.0004</td>
<td>-0.0007 ± 0.0005</td>
<td>-0.0043 ± 0.0008</td>
<td>-0.0011 ± 0.0005</td>
</tr>
<tr>
<td>$&lt;pq&gt;$</td>
<td>-0.0143 ± 0.0033</td>
<td>-0.0142 ± 0.0175</td>
<td>-0.0063 ± 0.0060</td>
<td>-0.0182 ± 0.0044</td>
</tr>
</tbody>
</table>
Table 4.4: Statistics of the principal strain rate measured by 2DPIV with repetition rate of 1500 Hz

<table>
<thead>
<tr>
<th>Exp.1</th>
<th>Exp.2</th>
<th>Exp.3</th>
<th>Exp.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\langle \Lambda_1/\Lambda_2 \rangle)</td>
<td>-1.0000</td>
<td>-1.0000</td>
<td>-1.0000</td>
</tr>
</tbody>
</table>

4.3.3 Droplet Size Distribution

The droplets are generated by an air-assist atomiser mounted at the top of the ‘box of turbulence’. The droplet size distributions used for the four experimental conditions are presented in Fig. 4.24

Table 4.5 provides a summary of droplet mean size and size spread. Droplet Stokes number defined in Eq.3.25 is calculated based on the arithmetic mean diameter \(D_{10}\) according to the conclusion in Chapter 3 that \(D_{10}\) (the arithmetic mean diameter) and \(DN_{60}\%\) (the diameter below which 60 % of the total number of droplets in the spray is present) are found to be more appropriate, comparing to other droplet mean diameter definitions in calculating droplet Stokes number in the study of droplet preferential concentration, that the strongest droplet clustering occurs when the scaled droplet Stokes number is around unity. Descriptions on the droplet Stokes number scaling based on differently defined droplet mean diameter can be found in Chapter 3. In the studies reported in the following Sections of this Chapter, droplet Stokes number is always calculated based on the arithmetic mean droplet diameter as shown in Table 4.5.

4.3.4 Radial Distribution Function (RDF)

The Radial distribution function(RDF) defined in Eq.3.23 has been calculated for the four experimental turbulent conditions \(Re_\lambda = 97, 127, 147 \) and 235 and shown
Figure 4.24: Number based droplet size probability (a) $Re_\lambda = 97$ (b) $Re_\lambda = 127$ (c) $Re_\lambda = 147$ (d) $Re_\lambda = 235$
Figure 4.25: Number based droplet size cumulative distribution (a) $Re_\lambda = 97$ (b) $Re_\lambda = 127$ (c) $Re_\lambda = 147$ (d) $Re_\lambda = 235$
Table 4.5: Characteristics of droplets present in the turbulent flow with $Re_\lambda$ = 97, 127, 147 and 235, diameters unit in $\mu m$

<table>
<thead>
<tr>
<th>Exp. No.</th>
<th>$D_{10}$</th>
<th>$D_{32}$</th>
<th>$DN_{40%}$</th>
<th>$DN_{60%}$</th>
<th>$Dv_{5%}$</th>
<th>$Dv_{50%}$</th>
<th>$\Delta_{DRSF}$</th>
<th>$St_{D_{10}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Re_\lambda = 97$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No.1</td>
<td>15.0</td>
<td>25.1</td>
<td>10.8</td>
<td>14.3</td>
<td>11.9</td>
<td>28.3</td>
<td>1.7</td>
<td>0.28</td>
</tr>
<tr>
<td>No.2</td>
<td>17.5</td>
<td>25.1</td>
<td>11.9</td>
<td>16.3</td>
<td>14.8</td>
<td>39.4</td>
<td>1.9</td>
<td>0.38</td>
</tr>
<tr>
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<td>25.1</td>
<td>13.7</td>
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<td>56.6</td>
<td>1.2</td>
<td>0.62</td>
</tr>
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<td>63.6</td>
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<td>32.4</td>
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<td>86.6</td>
<td>0.8</td>
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</tr>
<tr>
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<td>25.1</td>
<td>20.3</td>
<td>34.2</td>
<td>43.4</td>
<td>105.0</td>
<td>0.8</td>
<td>1.84</td>
</tr>
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<td>25.1</td>
<td>22.0</td>
<td>35.3</td>
<td>46.7</td>
<td>116.5</td>
<td>0.7</td>
<td>2.09</td>
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<tr>
<td>$Re_\lambda = 127$</td>
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<td></td>
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</tr>
<tr>
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<td>25.1</td>
<td>12.6</td>
<td>16.8</td>
<td>13.4</td>
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<td>1.4</td>
<td>0.49</td>
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<td>25.1</td>
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<td>110.1</td>
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<td>37.2</td>
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</tr>
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<td>25.7</td>
<td>52.4</td>
<td>1.4</td>
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<td>28.0</td>
<td>39.8</td>
<td>27.0</td>
<td>60.2</td>
<td>1.2</td>
<td>3.28</td>
</tr>
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<td>29.5</td>
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<td>1.0</td>
<td>4.09</td>
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<td>30.2</td>
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<td>42.7</td>
<td>93.6</td>
<td>0.8</td>
<td>5.23</td>
</tr>
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<td>31.2</td>
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<td>55.3</td>
<td>112.1</td>
<td>0.7</td>
<td>7.04</td>
</tr>
</tbody>
</table>
in Fig. 4.26. The droplet Stokes number is calculated based on the arithmetic mean diameter as shown in the last column of Table 4.5. The RDF analysis presents statistically the clustering behavior with different length scales \( r \) in comparison to the turbulent Kolmogorov length scale \( \eta \). The intersection point with unity has been used to indicate the typical cluster length scale. It is shown in Fig. 4.26 that the typical droplet cluster length scale is in the range of 20 to 30 times the Kolmogorov length scale for the four experimental conditions.

4.3.5 Voronoï Analysis

To quantify the droplet preferential concentration, the Voronoï analysis proposed by Monchaux et al. (2010) has also been used. Descriptions of this technique have been discussed in the previous Chapters. The probability distribution function of the normalised Voronoï areas for the four different experimental conditions are shown in Fig. 4.27. It is clear that for the four experimental conditions, the experimental p.d.f.s deviate from the Random Poisson Process, representing droplet preferential concentration.

The standard deviation \( \sigma_\nu \) of the normalised Voronoï area is the parameter that represents the magnitude of droplet preferential concentration. The droplets shows a stronger tendency to concentrate preferentially if \( \sigma_\nu \) is larger than that of the Random Poisson Process (RPP), which is \( \sigma_{\nu,RPP} = 0.53 \). Thus, the standard deviation \( \sigma_\nu \) can be defined as a function of time \( t \) in order to describe the temporal evolution of droplet preferential concentration. The temporal evolution of clustering for turbulent \( Re_\lambda = 147 \) and droplet Stokes number \( St = 1.18 \) \((D_{32} = 55 \mu m)\) is shown in Fig. 4.28 as an example.

The instantaneous evolution of the standard deviation \( \sigma_\nu \) seems a stochastic process so that statistical analysis is included as follows. The probability distribution of the standard deviation of the normalised Voronoï area is shown in Fig. 4.29.
Figure 4.26: Radial Distribution Function representing droplet clustering for the four turbulent conditions (a) $Re_{\lambda} = 97$ (b) $Re_{\lambda} = 127$ (c) $Re_{\lambda} = 147$ (d) $Re_{\lambda} = 235$
CHAPTER 4. TIME RESOLVED STATISTICS

Figure 4.27: Probability distribution function of the normalised Voronoï area
(a) $Re_\lambda = 97$ (b) $Re_\lambda = 127$ (c) $Re_\lambda = 147$ (d) $Re_\lambda = 235$
It is indicated that clustering occurs for each measurement at the four turbulent conditions, since all the standard deviations are larger than the value of 0.53 for the Random Poisson Process (RPP). Also, the probability distributions show significant variation of the magnitude of droplet clustering around its most probable value. The results also indicate differences for different droplet Stokes numbers and also for different turbulent flow conditions.

In order to analyse the similarity between clustering observations represented by the standard deviation of the normalised Voronoi area as a function of the time lag, the auto-correlation coefficient of the standard deviation of normalised Voronoi area is defined in Eq.4.37, and is calculated for all the measurements. It should be noted that the mean value of the standard deviation of normalised Voronoi area has been extracted, so the ‘fluctuating’ standard deviations $\sigma'_v(t)$ are used for the auto-correlation calculation. The fluctuating standard deviation of normalised
Figure 4.29: Probability of standard deviation of normalised Voronoi area
(a) $Re_\lambda = 97$ (b) $Re_\lambda = 127$ (c) $Re_\lambda = 147$ (d) $Re_\lambda = 235$
Voronoï area is defined in Eq. 4.36.

\[ \sigma'_\nu(t) = \sigma_\nu(t) - \bar{\sigma}_\nu \]  

(4.36)

\[ \rho(\tau) = \frac{c(\tau)}{c_0} = \frac{\langle \sigma'_\nu(t)\sigma'_\nu(t+\tau) \rangle}{\langle \sigma'^2_\nu \rangle} \]  

(4.37)

The auto-correlation coefficient for the time dependent fluctuation of the standard deviation of the normalised Voronoï area are represented in Fig. 4.30 of four experimental conditions \( Re_\lambda = 97, 127, 147, 235 \). The time scale when the auto-correlation coefficient becomes zero for the first time is in the range of \( 10^{-1} \) and \( 10^1 \) times of Kolmogorov time scale with most of the measurements in the range of \( 10^1 \).

In order to obtain a compact representation of the temporal evolution of droplet clustering, the Fourier form of \( \sigma_\nu(t) \) is defined in this Section as

\[ \hat{\sigma}_\nu(f) = \int_{-\infty}^{+\infty} dt e^{-i2\pi ft} \sigma'_\nu(t) \]  

(4.38)

with the finite discrete Fourier term defined as

\[ \hat{\sigma}_\nu(f) = \sum_{n=0}^{N-1} e^{-i2\pi fn\Delta t} \sigma_{vn}\Delta t \]  

(4.39)

and the discrete frequency defined as

\[ f = \frac{m}{N\Delta t}; m = 0, 1, 2, ..., N - 1 \]  

(4.40)

For each experimental condition with one specific turbulent intensity, 8 sets of measurements with different droplet size distribution and Sauter mean diameter ranging from \( 25 \, \mu m \) to \( 95 \, \mu m \) have been recorded, resulting in 32 measurements in total. The corresponding droplet Stokes number were calculated based on the
Figure 4.30: Autocorrelation coefficient functions of the fluctuation of the standard deviation of the normalised Voronoï area for the four turbulent flows and different droplet size distributions. Droplet Stokes number calculated based on the arithmetic mean droplet diameter $D_{10}$. (a) $Re_\lambda = 97$ (b) $Re_\lambda = 127$ (c) $Re_\lambda = 147$ (d) $Re_\lambda = 235$
CHAPTER 4. TIME RESOLVED STATISTICS

The arithmetic mean droplet diameter $D_{10}$ and shown in Table 4.5. For each measurement, 2048 images have been recorded at the rate $3000 \text{Hz}$, covering a period of 0.68s. The Fourier spectrums $\hat{\sigma}_\nu(f)$ of the fluctuations of the standard deviation of the normalised Voronoï area $\sigma'_\nu$ are calculated for the 32 measurements with four experimental conditions and shown in Fig. 4.31.

The statistical convergence of the Fourier spectra for each measurement is poor due to the fact that only 2048 data points are used for each calculation. However, such temporal Fourier spectra are represented for the first time and indicate that a power law form following the $-5/3$ slope may exist.

The arithmetic averaged standard deviation of normalised Voronoï area is the descriptive scalar representing clustering degree. The standard deviation for the Random Poisson Process (R.P.P) is 0.53, shown as the bold black dash line in Fig. 4.32. Larger values than that of R.P.P suggests stronger degree of droplet preferential concentration. The error bar in Fig. 4.32 represents the confidence interval based on confidence level of 95%. The droplet Stokes numbers are calculated based on arithmetic droplet mean size shown in Table 4.5 according to Eq. 3.25. For the four turbulent conditions $Re_\lambda = 97, 127, 147, 235$, the strongest clustering occurs for Stokes number around unity. The standard deviations of normalised Voronoï area for $Re_\lambda = 235$ are significantly larger than that of the other three turbulent conditions, which is suggestive that stronger preferential concentration would be expected for higher turbulent Reynolds number. However, comparisons between $Re_\lambda = 97, 127$ and 147 show no clear evidence for this trend. It has been observed by DNS (Tagawa et al., 2012) and experiments (Obligado et al., 2011) that stronger preferential concentration corresponds to intense turbulence. The experiment presented here seems to support this trend, since there is a large difference between turbulent Reynolds number $Re_\lambda = 235$ and the rest. However, it is difficult to draw such a conclusion when one intends to compare clustering behaviour with relatively similar turbulent Reynolds number,
Figure 4.31: Fourier spectrum of the standard deviation of the normalised Voronoi area for the four turbulent flows and different droplet size distributions. Droplet Stokes number calculated based on the arithmetic mean droplet diameter $D_{10}$. (a) $Re_\lambda = 97$ (b) $Re_\lambda = 127$ (c) $Re_\lambda = 147$ (d) $Re_\lambda = 235$
possibly due to the large variance of standard deviation of normalised Voronoï shown in Fig. 4.29.

In summary, it is shown in Fig. 4.32 that droplet clustering is stronger with turbulent $Re_\lambda = 235$ than for the other three turbulent conditions. However, by comparing all four turbulent conditions, there is no clear dependence of the clustering degree on the turbulent Reynolds number. The clustering degree is only weakly dependant to turbulent Reynolds number when one is significantly higher than other conditions.

### 4.3.6 Droplet Cluster Identification and Temporal Tracking

The probability distributions of the number of identified clusters for the 32 measurement conditions for four turbulent Reynolds number $Re_\lambda = 97$, 107, 127 and 235 are shown in Fig. 4.33. There are about 10 clusters identified per image for all the experimental conditions. The physical dimensions of each image is $45 \text{ mm} \times$
45 mm. This suggests that the probability of the turbulent flow structures that are responsible for the formation of droplet clusters is around 10 per 45 mm$^2$.

The instantaneous cluster length scale can be calculated following this procedure for the 32 measurement conditions for four turbulent conditions. The p.d.f.s of the droplet cluster length scale for each measurement based on 2048 images for the four turbulent conditions plotted in Fig. 4.34.

The most probable cluster length scale (circle diameter) for all four turbulent conditions is shown in Fig. 4.34. The p.d.f.s of Fig. 4.34 show that the lengthscale of the droplet clusters can vary between values below 10 times the Kolmogorov lengthscale up to values around 300 times the Kolmogorov lengthscale. This information is presented experimentally for the first time. This shows that the droplet cluster size can be affected by flow structures with wide range of length scales, since the droplet clusters are expected to occur due to the turbulent flow. Therefore, it is interesting to consider the available lengthscale of relevant structures in the turbulent flow.

It is indicated by the Radial Distribution Function (RDF) of Fig. 4.26 in the previous Section that the typical average cluster radius is in the range of 20 to 30 times Kolmogorov length scale, effectively 40 to 50 times Kolmogorov length scale in viewed as the diameter of the rings. The results on the cluster length scale based on two entirely different approaches agree fairly well. It should be noted that the RDF provides the average value and the p.d.f.s of droplet cluster length scale shown in Fig. 4.34 are not Gaussian.
Figure 4.33: Probability distribution of number of droplet clusters per image (45 mm²) identified by the mean shift combined with Voronoi cluster identification technique. (a) $Re_\lambda = 97$; (b) $Re_\lambda = 127$; (c) $Re_\lambda = 147$; (d) $Re_\lambda = 235$. 

Number of identified clusters vs. Probability for different values of $St$. The plots show the distribution of clusters for different Reynolds numbers.
Figure 4.34: Probability distribution of droplet cluster length scale derived by the mean shift combined Voronoi cluster identification technique. (a) $Re_\lambda = 97$; (b) $Re_\lambda = 127$; (a) $Re_\lambda = 147$; (a) $Re_\lambda = 235$
4.4 Time-resolved Investigation of Clustering of Turbulent Zero Velocity/Acceleration Points

The temporal characteristics of the turbulent flow are evaluated here in order to try to understand the way that the turbulent flow interacts with the droplets and affect dispersion.

Turbulent topological characteristics of zero velocity points and zero acceleration points were defined in the first Section of this Chapter. As shown in Fig. 4.6, non-uniform distribution of those saddle points has been observed. In order to quantify this non-uniformity, Radial Distribution Function (RDF) and the Voronoï analysis have been applied on the turbulent velocity/acceleration field obtained experimentally. Turbulent velocity vector fields have been measured with temporal resolution of 1,500 Hz using high speed PIV. The acceleration fields were derived directly from the velocity fields by extracting the two consecutive velocity vector map and divided by the time difference $dt = 0.0006s$. The effect of experimental noise on defining the turbulent topological characteristics has been identified and quantified in the first Section of this Chapter and $5 \times 5$ median filter has been applied to the experimentally obtained velocity vector field in order to eliminate the experimental noise effect in the measured velocity vector field.

4.4.1 Radial Distribution Function (RDF)

The Radial distribution function (RDF) defined in Eq.3.23 has been calculated for the zero velocity/acceleration points of the four experimental turbulent conditions $Re_{\lambda} = 97,127,147,235$ and each averaged over 1024 vector maps.

It is shown in Fig. 4.35 that all the zero velocity points demonstrate stronger clustering behaviour than the zero acceleration points. However, there is no clear dependance of the clustering degree on the turbulent Reynolds number. Fig. 4.35
Figure 4.35: Radial Distribution Function (RDF) of zero velocity points, zero acceleration points
also shows that the average length scale of the clusters of the zero velocity/zero acceleration points of the turbulent flow is around 10 times Kolmogorov lengthscale, which is in qualitative agreement with the lengthscale of the droplet clusters. Also, it is clear that the spatial distribution of the zero velocity/zero acceleration points of the flow is not random, but some clustering behaviour is present. These characteristics will be considered later in the following Sections.

4.4.2 Voronoï Analysis

Voronoï analysis have been performed on the zero velocity and zero acceleration field. The p.d.f.s of the normalised Voronoï area are shown in Fig. 4.36. It is clear that the p.d.f.s of the zero velocity and zero acceleration field deviate from the Random Poisson Process (RPP) and zero velocity points demonstrate stronger preferential concentration in agreement with observations for the RDF.

![Figure 4.36: P.d.f.s of normalised Voronoï areas](image-url)
correlation coefficient function of the temporal fluctuations of the standard deviation of normalised Voronoï areas of the zero velocity/zero acceleration points of the flow were calculated and shown in Fig. 4.37. For turbulent $Re_\lambda = 147$ and 235, the normalised time scale is between $10^0$ to $10^1$ times the Kolmogorov timescale for zero velocity/acceleration points to become uncorrelated. The normalised time scales for zero velocity/acceleration points are less than $10^{-1} t/\tau_k$ for $Re_\lambda = 97$ and 107. It seems that with higher turbulent Reynolds number, the saddle points maintain themselves remain correlate for relatively longer period of time. These timescales tend to be shorter than those of the lifetime of the droplet clusters.

![Figure 4.37: Auto-correlation coefficient of the fluctuations of standard deviation of normalised Voronoï areas of zero velocity/zero acceleration points of the turbulent flow](image)

The Fourier spectra of the fluctuations of the standard deviations of normalised
Voronoi area of turbulent zero velocity/acceleration points are shown in **Fig. 4.38**. The spectra of saddle points do not follow the power law form as observed in the spectra of droplet cluster fluctuations shown in **Fig. 4.31**.

![Fourier spectra of fluctuations of standard deviation of normalised Voronoi areas of zero velocity/zero acceleration points of the turbulent flow](image)

**Figure 4.38:** Fourier spectra of fluctuations of standard deviation of normalised Voronoi areas of zero velocity/zero acceleration points of the turbulent flow

The averaged standard deviation of normalised Voronoi areas over 1024 vector maps are plotted for different turbulent Reynolds numbers in **Fig. 4.39**. The error bars represent uncertainties with confidence interval of 95%. The behaviour is consistent with observations for RDFs would show that the turbulent zero velocity points demonstrate stronger clustering than the zero acceleration points and there is no clear dependance of the clustering degree on the turbulent Reynolds number.
4.4.3 Turbulent Zero Velocity/Zero Acceleration Points

Cluster Identification and Temporal Tracking

Individual clusters of zero velocity/acceleration points were identified by the mean shift feature space analysis proposed in the second Section of this Chapter. The p.d.f.s of cluster length scale defined in Eq.4.27 and Eq.4.26 were presented in Fig. 4.40. The most probable cluster length scale of zero velocity points is larger than the cluster length scale of zero acceleration points, and the most probable cluster length scales for both zero velocity points and zero acceleration points are all around 50 times the Kolmogorov length scale, consistent to the most probable cluster length scale of dispersed droplets.
Figure 4.40: P.d.f.s of cluster length scales of zero velocity/zero acceleration points of the fluid flow turbulence based on mean shift analysis.
4.5 Droplet Clustering Self-similarity and Sweep-Stick Mechanism

Droplet preferential concentration is a turbulent induced phenomenon that its governing mechanism is of interest to many researchers. One physical mechanism considering the turbulent vortical regions was reported by Eaton and Fessler (1994) that when droplet response time is similar to turbulent characteristic time, droplets tend to preferentially concentrate and form clusters with regions of low vorticity and high strain rate of the flow fluid. This was demonstrated through simultaneous visualisation of the turbulent vorticity and particle concentration fields. Rouson and Eaton (2000) tried to correlate the particle concentration non-randomness with the coherent topological turbulent flow structures in a channel flow using DNS by comparing the joint p.d.f.s of velocity invariants sampled at fluid grid points and particle cluster locations. It was suggested that the near wall vortical topological structures, referred as ‘focii’, correlate with particle clustering, while particle clustering does not correlate with turbulent topological structures away from the wall. As most of the enstrophy accumulates at the small dissipative scale for turbulence with Kolmogorov power law spectrum, the approach focusing on vorticity observes predominantly the effect of smallest eddies. Due to the fact that the PIV measurements and Mie-scattering droplet imaging within the ‘box of turbulence’ were not taken simultaneously, direct comparisons between two phases, aiming to examine the correlation between regions of low vorticity and dispersed droplets, are not possible.

Boffetta et al. (2004) observed multi-scale structure of particle clustering with self-similar nature in fully developed two-dimensional homogeneous and isotropic turbulence by presenting power law form of the p.d.f.s of the void area. Chen et al. (2006) calculated the pair correlation function defined as Eq.4.41, which is directly
linked with the Radial Distribution Function defined in Eq.3.23

\[ m(r) = \frac{\langle (\delta N)^2 \rangle_r}{\langle N \rangle_r^2} - \frac{1}{\langle N \rangle_r} \]  \hspace{1cm} (4.41)

where \( N \) is the number of particles in each box, \( \langle N \rangle_r \) is the averaged number of particles over all boxes, \( \langle (\delta N)^2 \rangle_r = \langle (N - \langle N \rangle)^2 \rangle_r \) is the average number of particles over all box with different sizes.

A good correlation was presented by Chen et al. (2006) and reproduced in Fig. 4.41 between the pair correlation function of particles and turbulent zero-acceleration points, suggesting that particle clustering mimic the clustering of zero-acceleration points in two-dimensional isotropic turbulence.

Goto and Vassilicos (2006, 2008) proposed a *sweep-stick mechanism* aiming to explain the multi-scale structure of clustering that the fluid acceleration field is *swept* by the local fluid velocity \( \mathbf{u} \) and particles tend to *stick* to and move with zero acceleration points \( \mathbf{a} = 0 \). The sweep-stick mechanism is different from the centrifugal mechanism associated with the turbulent flow vorticity, since it considers the effect of eddies with multiple length scales, thus providing explanations to the multi-scale structures of particle clustering. Coleman and Vassilicos (2009) provided a
CHAPTER 4. TIME RESOLVED STATISTICS

quantitative measure of the correlation by introducing the correlation function defined as Eq. 4.42 between the two data sets of particle clustering and zero-acceleration points.

\[ C^a_b(r) = \frac{\sum_{i=1}^{N(r)} [(n^a_i - \langle n_a \rangle)(n^b_i - \langle n_b \rangle)]}{\left( \sum_{i=1}^{N(r)} [(n^a_i - \langle n_a \rangle)^2(n^b_i - \langle n_b \rangle)^2] \right)^{1/2}} \]  

(4.42)

where \( n^a_i \) is the number of particles of type \( a \) in box \( i \), \( \langle n^a \rangle \) is the mean particle number of particles of type \( a \) in box with size of \( r \). It should be noted that \( C^a_b(r) \) is bounded between -1 and 1, with \( C^a_a(r) = 1 \) suggesting existence of perfect positive correlation and \( C^a_b(r) = 0 \) for the uncorrelated data set \( a \) and \( b \). A positive correlation has been observed between the data sets of particle clusters and zero-acceleration points. The \textit{stick} behavior was further tested by introducing velocity to the acceleration field, and the p.d.f.s of the velocity of the acceleration field and the particle velocity coincided, suggesting statistically that particles tend to ‘stick’ to zero-acceleration points and ‘move’ with them.

To summarize, the particle/droplet preferential concentration has been evaluated by DNS and showed that multi-scale structures that dispersed particles ‘stick’ and ‘move’ with the turbulent zero acceleration points, which are swept by the local velocity field of turbulence. The centrifugal effect is predominate, when one considers the effect of turbulent dissipative scales only, where most of the enstrophy accumulates, hence the strongest preferential concentration occurs for Stokes number around unity, when the Kolmogorov time scale is used for normalization. To examine the sweep stick mechanism and the multi-scale structure of particle clustering, comparison between the experiments is needed and presented in the current Section.

In this Section, the temporal resolved experimental data acquired within the ‘box of turbulence’ described in the previous Section has been used to examine the sweep stick mechanism. The zero-velocity points and zero-acceleration points have been extracted for the four turbulent conditions \( Re_\lambda = 97, 128, 147, 235 \) after
filtering out the experimental noise. The droplet locations have been derived from the Mie-scattering intensity images based on the band-pass filtering image processing method proposed in Chapter 3. It should be noted that the measurements of the gaseous turbulence and the liquid phase have been obtained separately. Although instantaneous measurement of both phases are most desirable in comparison with DNS, however, due to the limit of current optical diagnostic techniques, it is difficult to obtain instantaneous measurements.

### 4.5.1 Clustering Self-similarity

Boffetta et al. (2004) suggested the multi-scale structuring of particle clustering by observing the power law form of p.d.f.s of void areas, based on box counting method Fig. 4.42(a). Geometry statistics of the voids directly derived from the Voronoï analysis (Monchaux et al., 2010) without the ambiguity raised by choosing box length is shown in Fig. 4.42(b). It is shown that the power law form of void area p.d.f.s remains consistent. Fig. 4.42 is reproduced from Boffetta et al. (2004) and Monchaux et al. (2010).

![Figure 4.42: P.d.f.s of identified void area of dispersed particles (a) result of (Boffetta et al., 2004) based on box counting method (b) result of Monchaux et al. (2010) based on Voronoï analysis](image)

Voronoï analysis has been used in the current study since it avoids the ambiguity
raised by different box lengths. The approach proposed by Monchaux et al. (2010) is briefed below before moving to the experimental results. In order to present the void area p.d.f.s, the normalized Voronoï area p.d.f.s intersect twice with the Random Poisson Process (RPP), as shown in Fig. 4.43 (a). The intersection points $V_c$ and $V_v$ define the cluster/void that the Voronoï cells whose area is smaller than that of $V_c$ are considered as components of clusters and the Voronoï cells with larger area than that of $V_v$ are expected to represent voids. Fig. 4.43 (b) shows the ratio of the two p.d.f.s presented in Fig. 4.43(a). Visualization of clusters (dark gray) and voids (light gray) is shown in Fig. 4.43 (c). Fig. 4.43 is reproduced from Monchaux et al. (2010).

![Figure 4.43: Voronoï analysis in identifying void areas (Monchaux et al., 2010)](image)

The void area p.d.f.s of dispersed droplets for the four experimental conditions in the ‘box of turbulence’ are shown in Fig. 4.44. The tails of all the experimental p.d.f.s appear to follow power law form with -2 slope, as observed by Monchaux et al. (2010), suggesting appearance of the multi-scale structure of droplet clustering. Another supporting evidence for the multi-scale structure is that the typical cluster length scale is found to be independent to droplet Stokes number when calculated,
both from the Radial Distribution Function (RDF) and the mean shift feature space analysis combined with Voronoï tessellation.

### 4.5.2 Sweep-Stick Mechanism

The Radial Distribution Functions (RDF) of the zero-velocity points, zero-acceleration points and droplets are calculated and averaged over 1024 PIV velocity vector fields, 1024 processed Mie-scattering intensity images of droplets and shown

Figure 4.44: Normalized void area p.d.f.s of droplet clusters in ‘box of turbulence’ defined by Voronoï analysis (a) $Re_\lambda = 97$; (b) $Re_\lambda = 127$; (c) $Re_\lambda = 147$; (d) $Re_\lambda = 235$
Radial Distribution Function

(a)

(b)

(c)

(d)

Figure 4.45: Radial Distribution Function of dispersed droplets and turbulent stagnation points measured in the ‘box of turbulence’ (a) $Re_\lambda = 97$; (b) $Re_\lambda = 127$; (c) $Re_\lambda = 147$; (d) $Re_\lambda = 235$

in Fig. 4.45. It shows a good correlation between the three RDFs agreeing with the findings of Chen et al. (2006).

The p.d.f.s of the normalised Voronoï area for both the dispersed droplets and the zero velocity/acceleration points are shown in Fig. 4.46. After the intersecting points with RPP that the corresponding Voronoï cells identified as the void by Monchaux et al. (2010), the p.d.f.s of the dispersed droplets coincide with that of turbulent zero velocity points. The intersecting points that define ‘clusters’ by Monchaux et al.
(2010) are similar for the zero velocity/acceleration points and dispersed droplets. In the ‘intermediate’ range between the two defining intersecting points, the p.d.f.s of the dispersed droplets coincide with that of turbulent zero acceleration points. If one try to visualise what is suggested by this figure, it appears that the zero velocity points distribute non-uniformly and form void areas, the zero velocity points ‘sweep’ the zero acceleration points forming ‘smaller’ clusters and the droplets ‘stick’ with those zero acceleration points resulting in the coinciding of the p.d.f.s between the intersecting points. The droplets ‘stick’ with the zero acceleration points and, at the same time, are constrained by the zero velocity points. Thus, the droplet void areas are limited by the zero velocity void areas with the p.d.f.s after the intersecting points defining ‘voids’ appearing similar. It should be noted that as the zero velocity points/zero acceleration points are derived from velocity vector field with $5 \times 5$ median filter, the effect of experimental noise and de-noising methods applied introduce the uncertainties of the findings.

4.6 Summary

This Chapter reports a time resolved experiment studying the droplet preferential concentration in homogeneous and isotropic turbulence. The time resolved PIV flow velocity measurements of carrier phase turbulence and the Mie-scattering intensity images of dispersed phase droplets were acquired separately in the ‘box of turbulence’. The turbulent topological characteristics were evaluated based on the two dimensional PIV measurements with the effect of experimental noise effect identified and discussed. A novel droplet cluster identification technique was proposed and evaluated with the advantage of identifying individual instantaneous droplet clusters corresponding to visual inspection of droplet dispersions, thus enabling the morphological instantaneous droplet cluster length scale calculation.
Figure 4.46: Probability distribution function of the normalised Voronoï area of dispersed droplet and turbulent stagnation points measured in the ‘box of turbulence’ (a) $Re_\lambda = 97$ (b) $Re_\lambda = 127$ (c) $Re_\lambda = 147$ (d) $Re_\lambda = 235$
The time resolved investigations of the preferential concentration of dispersed droplets and turbulent topological zero velocity/acceleration points were quantified by Radial Distribution Function (RDF), Voronoï analysis and the mean shift cluster identification scheme proposed. Evaluations of the theory of clustering self-similarity and responsible sweep-stick mechanism were included to complete this Chapter. The main findings of this Chapter are summarised as follows.

1. The topological turbulent flow patterns are defined and found sensitive to the Gaussian noise and Salt & Pepper noise due to the fact that the shape of the invariant $p$ and $q$ joint p.d.f.s is observed to be altered by adding zero mean Gaussian noise to the DNS data (Fig. 4.3). A noise model is established by adding Gaussian noise and Salt & Pepper noise to the velocity vector fields of the DNS velocity data from JHU database. The Salt & Pepper noise is found to be the main noise source in the evaluation of topological characteristics of turbulent flows. A digital Filtering technique has been developed to eliminate the noise. Median and Wiener Filters have been applied and their effectiveness in removing noise examined. It has been observed that the $5 \times 5$ Median Filter returns the best fit of the joint p.d.f.s to the John Hopkins DNS benchmark data comparing with $3 \times 3$ and $2 \times 2$ Median Filter and Wiener Filter as shown Fig. 4.5.

The clustering of the turbulent zero velocity points is quantified by Radial Distribution Function (Sundaram and Collins, 1997) and Voronoï analysis (Monchaux et al., 2010). The RDF measurements of the clustering of the turbulent velocity saddle points are not affected much by the median filtering applied, while the median filter de-noising process results in noticeable difference in the normalized Voronoï area p.d.f.s. The statistics of Voronoï analysis depend on two-dimensional space tessellation of non-rigid Voronoï cells, while the statistics of RDF are based on pseudo rings and circles that divide the space
and result in lower sensitivity to the applied filtering process.

Thus, Median Filter with $5 \times 5$ window size is found to be the efficient and effective filtering approach to eliminate the experimental noise at a satisfactory level, so that the joint p.d.f.s of filtered velocity invariants fit closely enough to the noise free p.d.f.s. of DNS data in order to evaluate the topological characteristics of the flow.

2. Techniques quantifying preferential concentration have been developed by researchers focusing on the clustering statistics, while morphological descriptions based on instantaneous droplet cluster visual appearance have not been proposed up to date. A novel cluster identification scheme based on mean shift feature space analysis widely applied by the community of pattern recognition has been proposed and evaluated in this Chapter. The novelty and advance of this technique is two folds. Firstly, by combining the Voronoï space tessellation, the routine mean shift feature space analysis has been further developed particularly in identifying non-rigid individual clusters that move rotate and transform in shapes. Secondly, once individual clusters have been identified, morphological cluster length scale can be estimated as the diameter of circular area with the same area as the non-rigid cluster area. Also, the mean shift feature space analysis enabled cluster temporal tracking by correlating the cluster area histograms. The Bhattacharyya coefficient has been calculated examining the histogram similarity and the similarity vanishes within a few recording frames for time scales of $10^{-1}$ to $10^0$ times the Kolmogorov time scale.

3. Analysis, based on the Radial Distribution Function (RDF), quantified the temporally and spatially averaged droplet cluster length scale for all the considered droplet size distributions and turbulent conditions $Re_\lambda = 97, 107,$
127, 235 and found that the droplet cluster length scale is in the range of 20\eta to 30\eta, zero velocity point cluster length scale around 20\eta, zero acceleration point cluster length scale around 10\eta. The morphological cluster length scale was defined based on the mean shift feature space analysis and found to be around 10\text{mm} (around 50\eta) for droplets and zero velocity points, and less than 10\text{mm} for zero acceleration points for all the four experimental conditions. The cluster length scale quantified by RDF is effectively the radius of a circle, which after doubling to convert to diameter is consistent to the length scale calculated from morphological cluster length scale approach. Thus, the cluster length scale for zero velocity points and droplets is similar, while the zero acceleration points tend to form ‘smaller’ clusters.

4. Voronoï analysis has been performed to evaluate the temporal evolution of preferential concentration of droplets. The standard deviation of the normalised Voronoï area can be viewed as a single scalar descriptor of the magnitude of spatially averaged droplet cluster. For dispersed droplets, according to the p.d.f. of the time dependant standard deviation of normalised Voronoï area, the temporal variation of the magnitude of droplet clustering is up to 50% around the mean value. The time scale of the lifetime of droplet cluster is in the range of 10^1 to 10^2 times the Kolmogorov timescale, corresponding to the time when the auto correlation coefficient of the fluctuations of the standard deviation of normalised Voronoï area reach zero for the first time. Also, the instantaneous droplet clusters tend to survive longer when the turbulent Reynolds number is higher. The temporal Fourier spectra of the fluctuations of standard deviation of normalised Voronoï area follow power law with -5/3 slope. For zero velocity/acceleration points, the time scale over which they remain clustered is in the range of 10^0 to 10^1 times the Kolmogorov timescale and clusters survive longer with higher turbulent Reynolds number. Thus, the
cluster time scales for zero velocity/acceleration points are smaller than those of the dispersed droplets.

5. The effect of turbulent Reynolds number on the magnitude of droplet clustering has been studied by both RDF and Voronoï analysis. It is consistent that there is no clear dependence of the clustering degree on the turbulent Reynolds number. The clustering degree is only weakly dependent on turbulent Reynolds number when one is significantly higher than other conditions. This could possibly be due to the temporal evolution detailed in finding 3 that cluster survive longer for higher turbulent Reynolds number.

6. Clustering self-similarity has been observed with temporal Fourier spectra of the fluctuations of standard deviation of normalised Voronoï area follow power law with -5/3 slope and the void area p.d.f.s follow power law with -2 slope. The current measurements were the first attempt trying to provide experimental support for the sweep-stick mechanism proposed by Chen et al. (2006); Goto and Vassilicos (2006, 2008); Coleman and Vassilicos (2009) responsible for the self-similarity of droplet clustering. The local velocity ‘sweep’ the zero acceleration points and the droplets ‘stick’ with those zero acceleration points. Such ‘picture’ is suggested by the following terms.

   (a) Qualitative good correlation between the RDF of zero velocity/acceleration points and droplet distribution suggesting the clustering of zero velocity points, zero acceleration points and droplets is related.

   (b) Cluster length scales, quantified both by RDF and mean shift feature space analysis, of zero velocity points and droplets are similar, while the zero acceleration points tend to form ‘smaller’ clusters, suggesting that the zero acceleration points are swept by the velocity field.

   (c) The time scales of clusters, derived from the autocorrelation coefficient
of fluctuations of standard deviation of normalised Voronoi area of zero velocity/acceleration points are similar but smaller than those of the dispersed droplets, suggesting the preferential concentration of zero velocity/acceleration points is possibly responsible for the clustering of dispersed droplets.

(d) The p.d.f.s of normalised Voronoi area show that after the intersecting points with RPP, the corresponding Voronoi cells are identified as voids, and the p.d.f.s of the dispersed droplet voids coincide with those of turbulent zero velocity points. The intersecting points that define ‘clusters’ are similar for the zero velocity/acceleration points and dispersed droplet. In the ‘intermediate’ range between the ‘cluster’/‘void’ defining intersecting points, the p.d.f.s of the dispersed droplets coincide with that of turbulent zero acceleration points. It appears that the zero velocity points distribute non-uniformly and form void areas, the local velocity ‘sweep’ the zero acceleration points forming ‘smaller’ clusters and the droplets ‘stick’ with those zero acceleration points resulting in the coincide part of the p.d.f.s between the intersecting points. The droplets ‘stick’ with the zero acceleration points, and at the same time, are constrained by the zero velocity points so that the droplets void areas are limited by the zero velocity points void areas. In this way, the p.d.f.s of the intersecting points defining ‘voids’ appear similar.
Chapter 5

Comparison between Experiments and DNS

Preferential concentration of dispersed particles/droplets has been widely studied using Direct Numerical Simulation (DNS), especially in the dilute suspension with one way coupling or two way coupling (Maxey, 1987; Squires and Eaton, 1991; Longmire and Eaton, 1992; Eaton and Fessler, 1994; Wang and Maxey, 1993; Fessler et al., 1994; Sundaram and Collins, 1997; Shaw et al., 1998; Tanaka et al., 2002; Aliseda et al., 2002; Bec, 2005; Goto and Vassilicos, 2006; Chen et al., 2006; Goto and Vassilicos, 2008; Salazar et al., 2008; Coleman and Vassilicos, 2009; Tagawa et al., 2012). It has been suggested by Squires and Eaton (1991); Longmire and Eaton (1992); Eaton and Fessler (1994) that the governing mechanism of preferential concentration of dispersed particles/droplets is the turbulent vortex structure affecting droplets accumulation in regions of high strain and low vorticity. Goto and Vassilicos (2006); Chen et al. (2006); Goto and Vassilicos (2008); Coleman and Vassilicos (2009) proposed that the turbulent topological patterns, zero velocity points and zero acceleration points are responsible for the preferential concentration of dispersed droplets that droplets tend to ‘stick’ with zero acceleration points which are swept by local velocity and
form spatial non-uniform distributions. The preferential concentration of dispersed droplets has been found influential in the applications of gravitational settling velocity (Maxey, 1987; Aliseda et al., 2002), warm rain initiation (Shaw et al., 1998). A more thorough review of turbulent dispersed multiphase flows covering the topic of preferential concentration in gaseous turbulent flow is included in Balachandar and Eaton (2010).

A few direct comparison with experimental data of these numerical studies have been made (Salazar et al., 2008). Therefore, there is a need to provide such a comparison and it is the goal of this Chapter. A direct comparison with experiment is provided in this Chapter discussing the ‘sweep and stick’ mechanism proposed by Goto and Vassilicos (2006); Chen et al. (2006); Goto and Vassilicos (2008); Coleman and Vassilicos (2009)

Two sets of Direct Numerical Simulation (DNS) data are used for comparison between the experimental results and the Johns Hopkins University (JHU) open source turbulent database (Perlman et al., 2007; Li et al., 2008) and the DNS of Mallouppas et al. (2013a). The JHU data set simulates turbulence only, while those of Mallouppas et al. (2013a) simulate both carrier turbulent phase and dispersed droplet phase. The Chapter begins with brief descriptions of the DNS simulations, followed by the evaluation of velocity gradient invariants and topological characteristics of turbulent flow, in order to define zero velocity and zero acceleration points. The clustering behaviour of the zero velocity and zero acceleration points are quantified by Radial Distribution Function (RDF) and Voronoï analysis and mean shift feature space analysis and compared with the experimental results. It should be noted that this thesis used the DNS datasets only without contributions to the DNS simulation.
5.1 Direct Numerical Simulation (DNS)

5.1.1 DNS of Mallouppas et al. (2013a) (Imperial)

Simulation of turbulence This Section describes the Direct Numerical Simulation (DNS) of Mallouppas et al. (2013a). The simulation is to investigate the clustering of dispersed droplets in forced homogeneous and isotropic turbulence at Taylor Reynolds number of $Re_\lambda = 108$.

Without kinetic energy input, turbulence would decay and dissipate due to the presence of viscosity. Hence, in order to simulate homogeneous, isotropic, non-decaying turbulence with DNS, a type of forcing scheme must be imposed on the Navier-Stokes equations to prevent turbulence decaying. The novelty of the simulation is the new forcing scheme imposed (Mallouppas et al., 2013a).

Three-dimensional DNS of homogeneous and isotropic turbulence on $180^3$ periodic cubic box of length 0.069 m was performed.

The momentum equations solved are shown in Eq.5.1

$$\frac{\partial(\alpha_f v_{f,j})}{\partial t} + \frac{\partial(\alpha_f v_{f,j} v_{f,i})}{\partial x_i} = -\frac{1}{\rho_f} \frac{\partial \alpha_f p}{\partial x_j} + \frac{1}{\rho_f} \frac{\partial(\alpha_f \tau_{ij})}{\partial x_i} + T_j + \Pi_j$$

(5.1)

where $p$ is the pressure of the fluid, $\tau_{ij}$ as the stress tensor, $T_j$ represents the source term used in the forcing scheme proposed by Mallouppas et al. (2013a) to sustain the turbulence kinetic energy, $\Pi_j$ is the inter-phase momentum exchange, which is zero for one-way coupling conditions, and can be formulated to study two-way couplings. One-way coupling is assumed in the current study.

Simulation of dispersed Droplets

The particles are simulated following the Newton’s 2nd law as

$$m_p \frac{dv_{p,i}}{dt} = \beta \frac{V_p}{\alpha_p} (v_{f,p,i} - v_{p,i}) + F_{pp,i}$$

(5.2)
where $F_{pp,i}$ is the force caused by inter-particle collisions and ignored in the current simulation. $m_p$ is the mass of the particle, $v_{f\theta,p,i}$ is the undistributed turbulent velocity, $v_{p,i}$ is the particle velocity and $\beta$ is the drag term.

The simulation parameters for the turbulence and the dispersed phase are summarised in Table 5.1.

### Table 5.1: Multi-phase DNS simulation parameters (Mallouppas et al., 2013a)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domain size $L(m^3)$</td>
<td>0.069$^4$</td>
</tr>
<tr>
<td>Grid</td>
<td>180$^3$</td>
</tr>
<tr>
<td>Kinematic viscosity $\nu_f(m^2s^{-1})$</td>
<td>$1.47 \times 10^{-5}$</td>
</tr>
<tr>
<td>Fluid density $\rho(kgm^{-3})$</td>
<td>1.17</td>
</tr>
<tr>
<td>Simulation time-step $\Delta t(s)$</td>
<td>0.0003</td>
</tr>
<tr>
<td>Frames</td>
<td>350</td>
</tr>
</tbody>
</table>

### Table 5.2: Turbulent and dispersed droplet quantities of Imperial DNS dataset

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Experiment</th>
<th>Imperial DNS</th>
</tr>
</thead>
<tbody>
<tr>
<td>r.m.s. velocity x direction $u_{r.m.s.}$</td>
<td>0.505</td>
<td>0.431</td>
</tr>
<tr>
<td>r.m.s. velocity y direction $v_{r.m.s.}$</td>
<td>0.502</td>
<td>0.425</td>
</tr>
<tr>
<td>Kinetic energy $q^2(m^2s^2)$</td>
<td>0.789</td>
<td>0.558</td>
</tr>
<tr>
<td>Dissipation rate $\varepsilon(m^2s^3)$</td>
<td>10.499</td>
<td>3.001</td>
</tr>
<tr>
<td>Taylor-scaled Reynolds number $Re_\lambda$</td>
<td>126.7</td>
<td>107.9</td>
</tr>
<tr>
<td>Taylor micro-scale $\lambda(m)$</td>
<td>0.004</td>
<td>0.0037</td>
</tr>
<tr>
<td>Kolmogorov time scale $\tau_k(s)$</td>
<td>0.002</td>
<td>0.002</td>
</tr>
<tr>
<td>Kolmogorov length scale $\eta(mm)$</td>
<td>0.167</td>
<td>0.184</td>
</tr>
<tr>
<td>Eddy turnover time $\tau_{\varepsilon}(s)$</td>
<td>0.122</td>
<td>0.094</td>
</tr>
<tr>
<td>Integral length scale $\Lambda(m)$</td>
<td>0.039</td>
<td>0.027</td>
</tr>
<tr>
<td>Vector spacing $\Delta x(mm)$</td>
<td>0.364($\approx 2.17\eta$)</td>
<td>0.383($\approx 2.08\eta$)</td>
</tr>
<tr>
<td>Temporal step $\Delta t(s)$</td>
<td>0.0006 turbulence</td>
<td>0.0003 turbulence</td>
</tr>
<tr>
<td>No. of time frames $N$</td>
<td>1024 turbulence</td>
<td>350 turbulence</td>
</tr>
<tr>
<td>Time coverage $T(s)$</td>
<td>0.614($\approx 307\tau_k$)</td>
<td>0.105($\approx 53\tau_k$)</td>
</tr>
<tr>
<td>Spherical droplet diameter $d_p(\mu m)$</td>
<td>$8.0 \times 10^{-6}$</td>
<td>$8.0 \times 10^{-6}$</td>
</tr>
<tr>
<td>Droplet Stokes number $St$ based on $D_{10}$</td>
<td>1.18</td>
<td>0.86</td>
</tr>
<tr>
<td>Droplet density $\rho_p(kgm^{-3})$</td>
<td>1.000</td>
<td>1.000</td>
</tr>
</tbody>
</table>

*Experiment and DNS with same PSD*
The statistical characteristics of turbulence and dispersed droplets are shown in Table 5.2. The DNS simulation performed by Mallouppas et al. (2013a) is comparable to the experimental data with similar turbulent intensities and similar spatial resolution normalised by the corresponding Kolmogorov length scale.

### 5.1.2 Johns Hopkins University Turbulence Database (JHTDB)

The DNS data of forced isotropic turbulence by Perlman et al. (2007); Li et al. (2008) is on a $1024^3$ periodic grid in a domain of $2\pi \times 2\pi \times 2\pi$ with energy injected by keeping the total energy constant that their wave-number magnitude is less or equal to 2. 1024 frames full three dimensional components of velocity and pressure vectors are generated and stored after the simulation reaches statistical stationary state. The simulation parameters are summarised in Table 5.3. 350 frames of two dimensional velocity data with time step of $dt = 0.004s$ were queried from the JHTDB database and processed in comparison with the DNS of Mallouppas et al. (2013a) and experimental results.

<table>
<thead>
<tr>
<th>Fluid-phase</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Domain $L^3$</td>
<td>$2\pi^3$</td>
</tr>
<tr>
<td>Grid</td>
<td>$1024^3$</td>
</tr>
<tr>
<td>Kinematic viscosity $\nu_f (m^2 s^{-1})$</td>
<td>0.000185</td>
</tr>
<tr>
<td>Simulation time-step $\Delta t (s)$</td>
<td>0.0002</td>
</tr>
<tr>
<td>Storage time-step $\Delta t (s)$</td>
<td>0.004</td>
</tr>
<tr>
<td>Frames</td>
<td>350</td>
</tr>
</tbody>
</table>

Statistical characteristics of turbulence are summarised in Table 5.4. The listed statistics for JHTDB DNS are provided by Perlman et al. (2007); Li et al. (2008).
### Table 5.4: Turbulent quantities of JHTDB DNS dataset

<table>
<thead>
<tr>
<th></th>
<th>Experiment</th>
<th>JHTDB DNS</th>
</tr>
</thead>
<tbody>
<tr>
<td>average r.m.s. Velocity $u_{i,r.m.s.}$ (m/s)</td>
<td>0.503</td>
<td>0.681</td>
</tr>
<tr>
<td>Kinetic energy $q^2$ (m$^2$/s$^2$)</td>
<td>0.789</td>
<td>0.695</td>
</tr>
<tr>
<td>Dissipation rate $\varepsilon$ (m$^2$/s$^3$)</td>
<td>10.499</td>
<td>0.093</td>
</tr>
<tr>
<td>Taylor-scaled Reynolds number $Re_\lambda$</td>
<td>126.7</td>
<td>433</td>
</tr>
<tr>
<td>Taylor micro-scale $\lambda$ (m)</td>
<td>0.004</td>
<td>0.118</td>
</tr>
<tr>
<td>Kolmogorov time scale $\tau_k$ (s)</td>
<td>0.002</td>
<td>0.045</td>
</tr>
<tr>
<td>Kolmogorov length scale $\eta$ (mm)</td>
<td>0.167</td>
<td>2.870</td>
</tr>
<tr>
<td>Eddy turnover time $\tau_{\varepsilon}$ (s)</td>
<td>0.122</td>
<td>2.020</td>
</tr>
<tr>
<td>Integral length scale $\Lambda$ (m)</td>
<td>0.039</td>
<td>1.376</td>
</tr>
<tr>
<td>Vector spacing $\Delta x$ (mm)</td>
<td>0.364 ($\approx 2.17\eta$)</td>
<td>6.136 ($\approx 2.14\eta$)</td>
</tr>
<tr>
<td>Temporal step $\Delta t$ (s)</td>
<td>0.0006 turbulence</td>
<td>0.004 turbulence</td>
</tr>
<tr>
<td>No. of time frames $N$</td>
<td>1024 turbulence</td>
<td>350 turbulence</td>
</tr>
<tr>
<td>Time coverage $T$ (s)</td>
<td>0.614 ($\approx 307\tau_k$)</td>
<td>2.0 ($\approx 44\tau_k$)</td>
</tr>
</tbody>
</table>

The JHTDB DNS simulation is with similar spatial resolution normalised by the Kolmogorov length scale of the comparatives experimental data, but the turbulent intensity is higher with turbulent Reynolds number of 433.

#### 5.1.3 Topological Structure of Turbulence

This Section describes the process of evaluating turbulent topological characteristics and defining the zero velocity and zero acceleration points in the two DNS datasets. Three-dimensional turbulent velocity vector fields have been simulated in both sets of data. To compare with the two-dimensional experimental measurements, two-dimensional snapshots are obtained at one particular $z$ plane of DNS data.

The reduced two-dimensional velocity invariants defined in the previous Chapter are evaluated for the two sets of DNS. The joint p.d.f.s of the velocity invariants $p$ and $q$ are shwon in Fig. 5.1.

The shape of the joint p.d.f.s, shown in Fig. 5.1, are almost identical between two independent DNS dataset and all converge to the curve $p = q^2/4$, agreeing with the
work of Cardesa et al. (2013), who suggested that the joint p.d.f.s for the reduced velocity invariants $p$ and $q$ would appear similar for various turbulent flow condition.

The zero velocity points and the zero acceleration points are extracted following the approach of Chong et al. (1990) based on the phase portrait of Fig. 5.1. One example spatial distribution of instantaneous zero velocity points for JHU dataset and Imperial dataset are shown in Fig. 5.2. The zero velocity points from both JHU and Imperial show clear tendency to distribute non-uniformly and form clusters, qualitatively agree with the spatial distribution of instantaneous experimental zero velocity points with appropriate noise elimination algorithm applied as shown in Fig. 4.6. Quantitative comparison with experiment of preferential concentration of zero velocity/acceleration points is discussed below.

5.2 Quantitative Comparison with Experiment

Turbulent topological structure: zero velocity/acceleration points are extracted from both sets of DNS data based on the critical points theory described briefly in the
last Section of this Chapter and in details in the previous Chapter. The clustering behaviour of zero velocity/acceleration points are evaluated by Radial Distribution Function (RDF), Voronoï analysis and mean shift feature space analysis in terms of typical time scale, cluster length scale and validation of sweep-stick mechanism.

Poly-dispersed droplets with $D_{32} = 55 \mu m$, $St = 0.86$ based on arithmetic mean diameter $D_{10} = 25 \mu m$ are simulated by Malloupas et al. (2013a). The preferential concentration of numerically acquired dispersed droplets is quantified by Radial Distribution Function (RDF), Voronoï analysis and mean shift feature space analysis. Comparisons between numerical simulation and experiment are made based on these methods.

### 5.2.1 Radial Distribution Function

Radial Distribution Functions defined in Eq.3.23 quantifying spatial dispersion of zero velocity/acceleration points of two-dimensional slices of 3D forced isotropic turbulence simulation from Imperial DNS of $Re_\lambda = 108$ and Johns Hopkins Turbulence Database.
of $Re_\lambda = 433$ are plotted in Fig. 5.3. Solid lines represent the RDF of zero velocity points and dash lines represent the RDF of zero acceleration points. The Imperial simulation with $Re_\lambda = 108$ is marked in black and the JHU simulation with $Re_\lambda = 433$ is marked in red and the time resolved experiment with $Re_\lambda = 126$ is marked in green.

It is consistent that the zero velocity points show stronger magnitude of clustering for both DNSs and experiment comparing to zero acceleration points. There is a relatively good agreement between the DNS with $Re_\lambda = 108$ and experiment with $Re_\lambda = 126$ on the RDF of zero velocity/acceleration points. The DNS simulated poly-dispersed droplets with turbulent Reynolds number of $Re_\lambda = 108$ preferentially concentrate with RDF deviates from unity but much weaker comparing to the clustering of turbulent zero velocity/acceleration points with absolute value of RDF smaller than others. Although the droplet Stokes numbers are in similar range, the DNS simulated poly-dispersed droplets is with smaller magnitude of clustering comparing to the experimental result. This is possibly due to the significant larger number of droplets in one instantaneous DNS snapshot of around 20,000 comparing to the experimental image of 5,000, that RDF tends to capture more clusters with smaller size of DNS simulated droplets. The reason for the difference in droplet numbers in one 2D slice with DNS set with same volume fraction of experiment, might be that the image processing method used to locate droplets from experimentally acquired Mie-scattering intensity image exclude those droplets out of focus and those partially outside laser sheet thickness of $0.1 \text{mm}$, while the DNS snapshot is capable of tracking and capturing each droplet.

The typical cluster length scales, defined as the intersecting point with unity, are in the range of $10^1$ to $10^2$ times the Kolmogorov length scale for all the turbulent zero velocity/acceleration points. The JHTDB forced isotropic turbulent zero velocity/acceleration points with $Re_\lambda = 433$ form larger ‘clusters’ comparing
to the zero velocity/acceleration points of Imperial simulation and experiment. The poly-dispersed droplets simulated by Malloupas et al. (2013a) form ‘clusters’ with typical length scale smaller than $10^1$. For the Imperial DNS of $Re_\lambda = 108$, the tails of the RDFs for zero velocity/acceleration and poly-dispersed droplet appear correlated, suggesting that it is possible that the zero velocity/acceleration points are responsible for the clustering of dispersed droplets, as proposed by Chen et al. (2006); Goto and Vassilicos (2006, 2008); Coleman and Vassilicos (2009).

![Figure 5.3: Radial Distribution Function (RDF) of DNS and experiment](image)

**Figure 5.3:** Radial Distribution Function (RDF) of DNS and experiment
5.2.2 Voronoï Analysis

Voronoï analysis has been performed on the zero velocity points and zero acceleration points of the aforementioned DNS datasets. The corresponding p.d.f.s of the normalised Voronoï area are shown in Fig. 5.4.

![Figure 5.4: Probability of normalised Voronoï areas of zero velocity/acceleration points, dispersed droplets from DNS and experiment](image)

The p.d.f.s of the dispersed phase clustering in Imperial simulation with $Re_\lambda = 108$ tend to follow the R.P.P. and show small magnitude of clustering. However, the
p.d.f.s of experimental behaviour of the dispersed phase with similar Stokes number as the simulated droplets has larger deviance and magnitude of clustering. Simulation with different volume fraction and effective number of droplets per snapshot should be performed to compare with the statistics of zero velocity/acceleration points. The zero velocity/acceleration points of Imperial simulation and JHU simulation are observed to concentrate preferentially with the p.d.f.s of the normalised Voronoi area deviating from the Random Poisson Process (R.P.P.). Also, the p.d.f.s of normalised Voronoi area of zero velocity/acceleration points acquired from Imperial simulation and JHU simulation correlate well between the intersecting points defining ‘cluster’ and ‘void’ but lack correlation in the region defining ‘voids’ with larger value of normalised Voronoi area. The p.d.f.s of experimental zero velocity/acceleration points do not correlate with the simulation, this could possibly be due to the noise effect on the fine scale turbulent structures. However, there is a possibility that the DNS calculation suffer from the limited range of the computational space, thus can not consider the interaction with the rest of the flow present around the central region of the ‘box of turbulence’.

Thus, the clustering of zero velocity/acceleration points is consist between the Imperial simulation and JHU simulation based on Voronoi analysis, despite the different turbulent Reynolds number were achieved. However, clustering of experimentally and numerically acquired zero velocity/acceleration points does not correlated based on Voronoi analysis.

The timescale over which the clusters of zero velocity/acceleration points of DNS survive is discussed. Following the approach detailed in Chapter 4, the autocorrelation coefficient (defined in Eq.4.37) of time resolved fluctuation of standard deviation of normalised Voronoi area is plotted in Fig. 5.5. Experimental autocorrelation coefficient of time resolved standard deviation of normalised Voronoi area is also shown in Fig. 5.5 for comparison. The autocorrelation coefficient approaches to zero
in the range of $10^{-1}$ and $10^1$ times Kolmogorov time scale for the standard deviation of normalised Voronoï area of all experimental acquired zero velocity/acceleration points. This is in agreement with the observations of the DNS results. It is also noted that higher levels of oscillation remain in the computational results of JHU, which may be due to the reduced number of samples.

Figure 5.5: Autocorrelation coefficient of time-resolved fluctuating standard deviation of normalised Voronoï area
5.2.3 Mean Shift Cluster Identification and Temporal Tracking

Mean shift feature space analysis combined with Voronoï space tessellation has been done with the simulations in the aim of finding the typical cluster length scale. Detailed description about the mean shift feature space analysis can be found in Chapter 4. The p.d.f.s of typical cluster length scale derived with 350 frames of data in both simulations are shown in Fig. 5.6. The zero velocity/acceleration points obtained from JHU simulation are with the largest cluster length scale in the range of $10^2$ Kolmogorov length scale, and the dispersed phase of Imperial simulation of $Re_\lambda = 108$ represents smallest cluster length scale in the range of $10^1$ times Kolmogorov length scale. The zero velocity/acceleration points of Imperial simulation of $Re_\lambda = 108$ are characterised by the cluster length scale in the same range of $10^1$ times Kolmogorov length scale, but larger than that of the dispersed phase. The relative relationship of the derived cluster length scale from the proposed mean shift feature space analysis is consistent with the RDF analysis. The Imperial simulation is also consistent with the experiment indicating that the typical cluster length scales for zero velocity/acceleration points and dispersed droplets are all in the range of $10^1$ times the Kolmogorov length scale.

The convergence of p.d.f.s of typical cluster length scale of numerically acquired zero velocity/acceleration and dispersed droplets does not match well with the experimental results (as shown in Fig. 4.34 and Fig. 4.40). This is due to the lack of time resolved frames used in calculation. The number of frames used in experimental study is 2048, while the number of frames in simulation is only 350. Thus, lack of convergence is expected and can be improved by acquiring more numerical frames of velocity vector field and dispersed droplets.

The findings of the quantitative comparison between DNS and experiment are summarised as follows.
CHAPTER 5. COMPARISON BETWEEN EXPERIMENTS AND DNS

Figure 5.6: P.d.f.s of typical cluster length scale identified by mean shift algorithm
• Good agreement was found between Imperial simulation and experiment for
the typical cluster length scales, defined statistically by Radial Distribution
Function (RDF) and morphologically by mean shift feature space analysis.
These are in the range of $10^1$ times the Kolmogorov length scale for turbulent
zero velocity/acceleration points and dispersed droplets. The cluster length
scale of the zero velocity/acceleration points acquired from JHU forced
isotropic turbulence is slightly larger than the cluster length scale of zero
velocity/acceleration points and dispersed phase in Imperial simulation and
experiment, but within reasonable range.

• Good agreement was found between Imperial simulation and experiment that
the typical time scales of the lifetime of clusters defined by the point where the
auto-correlation coefficient of fluctuations of standard deviation of normalised
Voronoï area, are in the range of $10^{-1}$ to $10^1$ times the Kolmogorov timescale.

• Correlations between the RDFs of zero velocity/acceleration points and
dispersed droplets acquired from simulation and experiment are observed.
However, there is no close correlation between the p.d.f.s of normalised
Voronoï area of zero velocity/acceleration points and the dispersed droplets
acquired from DNSs and experiment.

5.3 Further Discussions

The consistency of data acquired from Imperial DNS, Johns Hopkins University open
source turbulence database and experiments conducted in the ‘box of turbulence’
detailed in the previous Chapter is summarised and discussed in this Section.
5.3.1 Consistency of Data between Imperial and JHU Forced Isotropic Turbulence DNS Simulations

Turbulent zero velocity/acceleration points are extracted from Imperial 3D forced homogeneous and isotropic turbulence DNS and JHTDB 3D forced isotropic turbulence DNS. The shape of the joint p.d.f. of the reduced two dimensional velocity invariants $p$ and $q$ appear similar, agreeing with the work of Cardesa et al. (2013) who have proved the joint p.d.f.s for the reduced velocity invariants $p$ and $q$ would appear similar in various turbulent flow conditions. The zero velocity/acceleration points from both simulations are observed to concentrate preferentially. Radial Distribution Function (RDF), Voronoï analysis and mean shift feature space analysis are applied to quantify the clustering behaviour of the zero velocity/acceleration points.

There is a good correlation between the RDFs of zero velocity/acceleration points of Imperial DNS and JHTDB simulation. The typical cluster length scale, defined statistically by the intersecting points with unity, of the zero velocity/acceleration points of JHTDB simulation is found slightly larger than that of the Imperial simulation but within the same range of magnitude. For Voronoï analysis, the p.d.f.s of normalised Voronoï area of the zero velocity/acceleration points of JHTDB simulation and Imperial simulation agree well between the points defining ‘clusters’ and ‘voids’ but the probability of voids are slightly higher in the Imperial simulation than that of the JHTDB simulation. The time scale that the auto-correlation coefficient of the fluctuations of standard deviation of normalised Voronoï area approaches zero is in the range of $10^{-1}$ to $10^4$ times the Kolmogorov time scale for the zero velocity/acceleration points of Imperial simulation and zero acceleration points of JHTDB simulation, but the zero velocity points of JHTDB simulation are observed with a longer time scale in the range of $10^1$ times Kolmogorov time scale.

The cluster length scales defined via morphological descriptive mean shift feature space analysis of the zero velocity/acceleration points of JHTDB simulation are in
the range of $10^2$ times the Kolmogorov length scale, while the cluster length scales of zero velocity/acceleration points are in the range of $10^1$ times the Kolmogorov length scale. The fact that JHTDB simulation returns ‘larger clusters’ is consistent with the findings from RDF analysis. However, the length scale of clusters of zero velocity/acceleration points of JHTDB simulation is in the range of $10^1$ quantified by RDF and $10^2$ times of the Kolmogorov length scale in the mean shift feature space analysis.

The reason explaining the disagreement between the Imperial simulation and JHTDB simulation could be that the JHTDB forced isotropic turbulence has larger domain/spatial resolution and temporal resolution. However, the normalised spatial resolution by the Kolmogorov length scale is similar in both simulations.

### 5.3.2 Consistency of Data between Imperial DNS and Experiment

The two-phase DNS of Imperial includes information of carrier phase turbulence and dispersed droplet phase. The clustering behaviour of zero velocity/acceleration points and dispersed droplets are quantified by Radial Distribution Function (RDF), Voronoï analysis and mean shift feature space analysis in comparison to the result of experimental data detailed in Chapter 4.

The RDFs of turbulent zero velocity/acceleration points of Imperial simulations agree well with experiments and represent good correlation between dispersed droplets acquired both numerically and experimentally, aiming to examine the sweep stick mechanism proposed and discussed by Chen et al. (2006); Goto and Vassilicos (2006, 2008); Coleman and Vassilicos (2009). Although the RDF of dispersed droplet of Imperial simulation represents weak preferential concentration, the trend of the RDF correlates with the simulated turbulent zero velocity/acceleration points. The possible reason for this disagreement is that the average number of droplets per image of the
DNS is larger than that observed in experimental condition. If the DNS could cover a wider range of droplet Stokes numbers and with similar average number of droplets, a thorough comparison with experiment is possible, which could be the future scope of this thesis.

The Voronoï analysis, however, does not represent consistency in the p.d.f.s of normalised Voronoï area of zero velocity/acceleration points between simulation and experiment. This could be due to the de-noising median filter applied to the experimental velocity vector field which alters the shape of the p.d.f.s of normalised Voronoï area. The effect of median filter on the Voronoï analysis of experimentally acquired zero velocity/acceleration points are discussed in detail in Chapter 4. The time scales of clusters of zero velocity/acceleration points in the simulation and experiment are in the range of $10^{-1}$ times the Kolmogorov time scale and the time scales of clusters of dispersed droplets are in the range of $10^0$ to $10^1$ times the Kolmogorov time scale. The fact that ‘clusters’ of dispersed droplets maintain themselves longer than the ‘clusters’ of zero velocity/acceleration points is consistent in experiment and simulation.

The cluster length scales, defined by the mean shift feature space analysis detailed in Chapter 4, of the zero velocity/acceleration points are in the range of $10^4$ times the Kolmogorov length scale for both DNS and experiment. The cluster length scale of dispersed droplets of simulation is in the range of $10^0$ times the Kolmogorov length scale while the cluster length scale of dispersed droplet of experiment is in the range of $10^1$ times the Kolmogorov length scale. In experimental results, the cluster length scales of zero velocity points and dispersed droplets are similar, while zero acceleration points tend to form ‘smaller’ clusters. The zero velocity/acceleration points are with similar cluster length scale in the DNS, while dispersed droplet tend to form ‘smaller’ clusters possibly due to larger number of droplets.
5.4 Summary

Three dimensional two-phase DNS from Imperial and three dimensional forced isotropic turbulence DNS from JHTDB are processed and compared in this Chapter. The turbulent zero velocity/acceleration points are extracted from both simulations. The clustering behaviours are quantified by Radial Distribution Function (RDF), Voronoï analysis and mean shift feature space analysis. The purpose of this Chapter is to evaluate the numerically acquired zero velocity/acceleration points and droplet dispersion compare to the findings from the time-resolved experiment reported in Chapter 4.

The findings of this Chapter is summarised as follows.

- In the two-phase DNS of Imperial, the clustering of zero velocity/acceleration points and dispersed droplets are quantified by RDF, Voronoï analysis and mean shift feature space analysis. Correlation between the RDFs of dispersed droplets and turbulent zero velocity/acceleration points is observed suggesting that the zero velocity/acceleration points could be responsible for the droplet preferential concentration. The DNS simulated dispersed droplets are with Stokes number of 0.86 but represent weak preferential concentration. Although the droplet Stokes numbers are in similar range, the DNS simulated poly-dispersed droplets is with smaller magnitude of clustering comparing to the experimental result. This could possibly due to the significant larger number of droplets in one instantaneous DNS snapshot of around 20,000 comparing to the experimental image of 5,000, that RDF tend to capture more clusters with smaller size of DNS simulated droplets. The reason for the difference in droplet numbers in one 2D slice might be that the image processing method to locate droplets from experimentally acquired Mie-scattering intensity image exclude those droplets out of focus and those partially outside laser sheet thickness of 0.1mm, while the
DNS snapshot is capable of tracking and capturing each droplet. As a result, to make a thorough comparison with experiment, simulation with different volume fraction and effectively number of droplets per snapshot should be performed to provide a thorough comparison.

- For the turbulent phase simulation by Imperial and JHTDB. The shape of the joint p.d.f. of the reduced two dimensional velocity invariants $p$ and $q$ appear similar, agreeing with the work of Cardesa et al. (2013) who have proved the joint p.d.f.s for the reduced velocity invariants $p$ and $q$ would appear similar for various turbulent flow conditions. According to Voronoï analysis and mean shift feature space analysis, the zero velocity/acceleration points of JHTDB simulation tend to form slightly ‘larger clusters’ and maintain themselves a bit longer in comparison to the zero velocity/acceleration points of Imperial simulation. This is possibly due to the large spatial/temporal resolution of JHTDB simulation. The RDFs of zero velocity/acceleration points correlate relatively well.

- For the cluster morphology of zero velocity/acceleration points from simulation of Imperial and JHTDB and experimental data, good agreement was found between Imperial simulation and experiment for the typical cluster length scales, defined statistically by Radial Distribution Function (RDF) and morphologically by mean shift feature space analysis. The cluster length scales are in the range of $10^1$ times of the Kolmogorov length scale for turbulent zero velocity/acceleration points and dispersed droplets. The cluster length scale of the zero velocity/acceleration points acquired from JHU forced isotropic turbulence is slightly larger than the cluster length scale of zero velocity/acceleration points and dispersed phase in Imperial simulation and experiment, but within reasonable range. Good agreement was found between
simulation and experiment that the typical cluster lifetimes, defined by the point where the auto-correlation coefficient of the fluctuations of the standard deviation of normalised Voronoi area approaches zero, are in the range of $10^{-1}$ to $10^1$ times the Kolmogorov timescale.
Chapter 6

Conclusions

The thesis studies the preferential concentration of poly-dispersed droplet experimentally in homogeneous and isotropic turbulence generated in a facility referred to as the ‘box of turbulence’ and includes comparisons with Direct Numerical Simulation (DNS), providing discussions on the topics of the effect of droplet poly-dispersion on droplet preferential concentration, temporal evolution of droplet clustering and the turbulent mechanisms (i.e. topological turbulent flow patterns) that are responsible for droplet clustering dynamics.

The thesis is structured into six Chapters. Motivation, theoretical background and related literatures of this work are discussed in Chapter 1. Chapter 2 consists of experimental setup and descriptions of the applied laser diagnostic techniques. Chapter 3 focuses on the effect of droplet poly-dispersion on droplet preferential concentration. The techniques used in quantifying the preferential concentration are the Radial Distribution Function (RDF) and the Voronoï analysis. An image processing method for locating droplets from experimental Mie-scattering intensity images has been proposed and evaluated. Chapter 4 reports temporally resolved experiments on poly-dispersed droplets and quantifies the temporal evolution of droplet clustering. A droplet cluster identification method based on the mean shift
pattern space analysis and the Voronoi tessellation has been proposed and applied to all the temporal resolved data to obtain cluster time scale and length scale statistics. The fine scale topological turbulent flow patterns (i.e. zero velocity points and zero acceleration points) extracted from the experiment are observed to disperse non-uniformly and form clusters. The clustering of these turbulent stagnation points and the dispersed droplets are evaluated by RDF, Voronoi analysis and the proposed meanshift feature space analysis. The theory of clustering self-similarity is examined and the ‘sweep-stick mechanism’ responsible for the self similarity of clustering is discussed. Chapter 5 presents a comparison between DNS computations with the experiments of Chapter 4. This Chapter summarises the findings of the thesis and possible directions for future work.

6.1 Summary of Findings

6.1.1 Poly-dispersed Droplet Clustering

The preferential concentration of poly-dispersed droplets with Sauter mean diameter between 25 and 95 µm with four different flow turbulent intensities ($Re_\lambda = 107, 145, 185$ and 213) has been studied for homogeneous, isotropic turbulence. A band-pass filter-based image processing method is proposed for the identification of droplet locations on images of scattered light intensity from droplets. The effectiveness of the method was assessed for various image qualities, and the influence of the image processing parameters was evaluated on the quantification of droplet clustering. RDF and Voronoi analysis were applied to quantify droplet preferential concentration. An appropriate Stokes number for poly-dispersed droplets is proposed for scaling of droplet clustering and the role of turbulence on clustering behaviour is discussed. Finally, the effect of droplet poly-dispersity is examined by comparing the clustering behaviour of droplets with different size spreads. A summary of the findings is listed
• The influence of image processing parameters in the bandpass filtering method on the detection of droplet locations and quantify droplet clustering was presented and verified by considering the universal image quality $Q$ index. This allowed the quantification of the degree of droplet preferential concentration for poly-dispersed droplets with wide range of size distributions, as summarised in Table 3.3.

• The preferential concentration was quantified by the RDF method and Voronoi analysis and the results are in agreement. It was concluded that the strongest preferential concentration occurred when the proposed Stokes number, based on $D_{10}$ (the arithmetic mean droplet diameter), $DN_{60\%}$ (the diameter associated with 60% of the total number of droplets in the spray) or $DV_{5\%}$ (the diameter that carries 5% of the liquid volume in the spray), is around unity. This suggests that the preferential concentration of poly-dispersed droplets is governed by the sizes carrying a large number of fraction of droplets and a small fraction of liquid volume in a spray.

• The RDF method indicated that the length scale of droplet clusters varies between 20 and 30 times the Kolmogorov length scale of the flow turbulence for all size distributions and flow conditions. This is of the same order of magnitude as 10 times the Kolmogorov scale found for mono-sized particles of previous studies (Fessler et al., 1994; Aliseda et al., 2002; Wood et al., 2005).

• Droplet preferential concentration appears to have little dependence on turbulent Reynolds number when the proposed Stokes number remains the same. This finding is in contrast to recent DNS results (Tagawa et al., 2012), which suggest that further investigation of the effect of turbulence is required.
• The effect of droplet poly-dispersity is found to be significant on the droplet preferential concentration as larger variance of clustering magnitude is observed for wider droplet size distribution. The most probable magnitude of clustering represented by the most probable standard deviation of Voronoï area does not correlate with the most probable number based droplet size/droplet Stokes number. Possible explanations of these findings are two folds. Firstly, the overall droplet response to turbulent structures and the resulting non-uniform spatial droplet dispersion are more likely to be due to the combination of dispersion of individual droplet rather than a group behaviour. As a result, poly-dispersed droplets with wider size distribution are expected to have larger magnitude of clustering. Secondly, the lack of correlation between most probable magnitude of clustering and most probable number based droplet Stokes number could be an indirect support for the theory that the clustering behaviour of droplets with similar response time scale as the Kolmogorov time scale is more pronounced.

6.1.2 Temporal Evolution of Droplet Clustering

A time resolved experiment studying the droplet preferential concentration in homogeneous and isotropic turbulence is reported in this thesis. The time resolved PIV velocity measurements of carrier phase turbulence and the Mie-scattering intensity images of dispersed phase droplets were acquired separately in the ‘box of turbulence’. The turbulent topological characteristics were evaluated based on the two dimensional velocity measurements with the effect of experimental noise identified and discussed. A novel cluster identification technique was proposed and evaluated with the advantage of identifying instantaneous single droplet cluster corresponding to those observed by visual inspection thus enabling the morphological cluster length scale definition. The time resolved investigations of the preferential concentration of dispersed droplets and turbulent topological zero velocity/acceleration points were
quantified by Radial Distribution Function (RDF), Voronoï analysis and the proposed mean shift cluster identification scheme. Evaluation of the theory of clustering self-similarity and responsible sweep-stick mechanism were included. The main findings of this time resolved experiment are summarised as follows.

• The topological turbulent flow patterns are defined and found sensitive to experimental noise since the shape of the invariant $p$ and $q$ joint p.d.f.s is altered by adding zero mean Gaussian noise or Salt & Pepper noise to the DNS data. A noise model based on Gaussian noise and impulsive Salt & Pepper noise is established by adding artificial noise to the velocity vector field of DNS data from JHU database. Digital filtering methods, based on Median and Wiener Filters, are chosen to eliminate the modelled noise source and examined their capacity to restore the joint p.d.f.s of velocity invariants to that of DNS data. The performance of the Median Filter with different filtering windows and Wiener Filter has been examined in terms of alternations to the shape of the joint p.d.f.s and quantitative descriptions of clustering of the zero velocity points, defined in the theory of topological turbulent pattern. It has been observed that the $5 \times 5$ Median Filter returns the best fit of the joint p.d.f.s to the John Hopkins DNS benchmark data comparing with $3 \times 3$ and $2 \times 2$ Median Filter and Wiener Filter, as shown in Fig. 4.5.

The RDF measurements of the clustering of the turbulent velocity saddle points are not affected much by the Median Filtering applied, while the Median Filter de-noising process results in noticeable difference in the normalised Voronoï area p.d.f.s. The statistics of Voronoï analysis depend on two dimensional space tessellation in non-rigid Voronoï cells, while the statistics of RDF based on pseudo rings and circles that divide the space is approximate and result in less sensitivity to the applied filtering process applied. It has been found that a Median Filter with window size of $5 \times 5$ is the effective and efficient approach
in identifying the two-dimensional topological turbulent flow patterns, which eliminates the experimental noise to a satisfactory level.

• Techniques quantifying preferential concentration have been developed by researchers focusing on the clustering statistics, while the morphological descriptions based on cluster visual appearance have not been presented up to date. A novel cluster identification scheme based on mean shift feature space analysis widely applied by the community of pattern recognition has been proposed and evaluated. The novelty and advantage of this technique is two folds. Firstly, by combining the Voronoï space tessellation, the routine mean shift feature space analysis has been further developed particularly in identifying non-rigid individual clusters that move, rotate and transform in shapes. Secondly, once individual cluster has been identified, morphological cluster length scale can be estimated as the diameter of circular area with the same area as non-rigid cluster. Also, the mean shift feature space analysis enabled cluster temporal tracking by correlating the cluster area histograms.

• Analysis based on the Radial Distribution Function (RDF), quantified the temporally and spatially averaged cluster length scale for all the considered droplet size distributions and turbulent flow conditions $Re_{\lambda} = 98, 107, 127, 235$. The droplet cluster length scale was in the range of $20\eta$ to $30\eta$, the zero velocity point cluster length scale around $20\eta$, and the zero acceleration point cluster length scale around $10\eta$. The morphological cluster length scale estimated from the mean shift feature space analysis was found to be around $10\mu m$ (around $50\eta$) for droplets and zero velocity points, and less than $50\eta$ for zero acceleration points for all the four experimental conditions. The cluster lengthscale is the diameter of a circular area with the same area as the identified non-rigid cluster area. The cluster length scale, quantified by RDF, is effectively the radius of
a circle, converting to diameter for comparison with the morphological cluster
length scale, which shows that the statistical and morphological defined cluster
length scales are consistent. Thus, the cluster length scales for zero velocity
points and droplets are similar, while the zero acceleration points tend to form
‘smaller’ clusters.

- Voronoï analysis has been performed to evaluate the temporal evolution of
droplet preferential concentration for the standard deviation of the normalised
Voronoï area, which can be viewed as a single scalar descriptor of spatially
averaged magnitude of clustering. For dispersed droplets, according to the
p.d.f. of the time dependant standard deviation of normalised Voronoï area, the
temporal variation of the magnitude of droplet clustering is up to 50% around
the mean value. The time scale of cluster is in the range of $10^1$ to $10^2$ times the
Kolmogorov time scale, identified when the auto correlation coefficient function
of the fluctuations of the standard deviation of normalised Voronoï area reach
zero. Also, the clusters tend to survive longer when the turbulent Reynolds
number is higher. The temporal Fourier spectra of the fluctuations of standard
deviation of normalised Voronoï area follow power law with $-5/3$ slope. For zero
velocity/acceleration points, the time scale for the magnitudes of clustering
remain in the range of $10^0$ to $10^1$ times the Kolmogorov time scale and clusters
survive longer with higher turbulent Reynolds number. Thus, the cluster time
scales for zero velocity/acceleration points are smaller than that of the dispersed
droplets.

- The effect of turbulent Reynolds number on the magnitude of clustering has
been studied by both RDF and Voronoï analysis. It is consistent that there
is no clear dependence of the degree of clustering on the turbulent Reynolds
number. The degree of clustering is only weakly dependent on turbulent
Reynolds number, when one is significantly higher than other conditions.

### 6.1.3 Sweep-Stick Mechanism

Clustering self-similarity has been observed at the Fourier spectra of the temporal fluctuations of the standard deviation of normalised Voronoï area, which follows a power law with -5/3 slope, and the void area p.d.f.s which follow a power law with -2 slope. The current measurements were the first attempt trying to provide experimental support for the sweep-stick mechanism proposed by Chen et al. (2006); Goto and Vassilicos (2006, 2008); Coleman and Vassilicos (2009) responsible for the self-similarity of droplet clustering with the concept that the zero velocity points distribute non-uniformly and form clusters and void areas. The zero velocity points ‘sweep’ the zero acceleration points and the droplets ‘stick’ with those zero acceleration points. Such ‘picture’ is suggested by the following observations.

- Good qualitative correlation between the RDF of zero velocity/acceleration points and droplets suggesting that the clustering of zero velocity points, zero acceleration points and droplets is related.

- Cluster length scale, quantified both by RDF and mean shift feature space analysis, of zero velocity points and droplets are similar while the zero acceleration points tend to form ‘smaller’ clusters, suggesting that the zero acceleration points are swept by the local velocity.

- The time scales of clusters, derived from the autocorrelation coefficient of the fluctuation of the standard deviation of normalised Voronoï area, of zero velocity/acceleration points are similar and smaller than that of the dispersed droplets, suggesting that the preferential concentration of zero velocity/acceleration points is possibly responsible for the clustering of dispersed droplets.
The p.d.f.s of normalised Voronoï area show that, after the intersecting points with R.P.P., where the corresponding Voronoï cells are identified as voids, the p.d.f.s of normalised Voronoï area of the dispersed droplets coincide with that of turbulent zero velocity points. The intersecting points that define ‘clusters’ are similar for the zero velocity/acceleration points and dispersed droplets. In the ‘intermediate’ range between the ‘cluster’/‘void’ defining intersecting points, the p.d.f.s of the dispersed droplets coincide with that of turbulent zero acceleration points. It appears that the zero velocity points distribute non-uniformly and form void areas, the zero velocity points ‘sweep’ the zero acceleration points forming ‘smaller’ clusters and the droplets ‘stick’ with those zero acceleration points resulting in the coincidence of the p.d.f.s between the intersecting points. The droplets ‘stick’ with the zero acceleration points and, at the same time, constrained by the zero velocity points so that the droplet void areas are limited by the zero velocity void areas and the p.d.f.s after the intersecting points with RPP defining ‘void’ appear similar.

### 6.1.4 Comparison between Experiments and DNS

Three dimensional two-phase DNS simulation from Imperial Mallouppas et al. (2013a) and three dimensional forced isotropic turbulence DNS simulation from Johns Hopkins University Turbulence Database (JHTDB) are processed and compared. The turbulent topological structure of zero velocity/acceleration points are extracted from both simulations and the clustering behaviour is quantified by Radial Distribution Function (RDF), Voronoï analysis and mean shift feature space analysis. The purpose is to evaluate the dispersion of numerically acquired zero velocity/acceleration points and droplets to compare with the findings from the time-resolved experiment and assist the discussion of governing mechanism of dispersed droplet preferential concentration.
The findings of comparison between simulation and experiments are summarised below.

- In the two-phase DNS simulation of Imperial, the clustering of zero velocity/acceleration points and dispersed droplets are quantified by RDF, Voronoï analysis and mean shift feature space analysis. Correlation between the RDFs of dispersed droplets and turbulent zero velocity/acceleration points is observed suggesting that the zero velocity/acceleration points could be responsible for the droplet preferential concentration and examine the sweep-stick mechanism proposed by Chen et al. (2006); Goto and Vassilicos (2006, 2008); Coleman and Vassilicos (2009). The DNS simulated dispersed droplets are with Stokes number of 0.86 but represent weak preferential concentration. Although the droplet Stokes numbers are in similar range, the DNS simulated poly-dispersed droplets is with smaller magnitude of clustering comparing to the experimental result. This could possibly be due to the significant larger number of droplets in one instantaneous DNS snapshot of around 20,000 comparing to the experimental image of 5,000, that RDF tend to capture more clusters with smaller size of DNS simulated droplets. The reason for the difference in droplet numbers in one 2D slice might be that the image processing method to locate droplets from experimentally acquired Mie-scattering intensity image exclude those droplets out of focus and those partially outside laser sheet thickness of 0.1\(mm\), while the DNS snapshot is capable of tracking and capturing each droplet. As a result, to make a thorough comparison with experiment, simulation with different volume fraction and effectively number of droplets per snapshot should be performed to provide a thorough comparison.

- For the turbulent flow simulation by Imperial and JHTDB. The shape of the joint p.d.f. of the reduced two dimensional velocity invariants \(p\) and \(q\) appear similar, agreeing with the work of Cardesa et al. (2013) who have proved
that the joint p.d.f.s for the reduced velocity invariants \( p \) and \( q \) would appear similar for various turbulent flow condition. According to Voronoï analysis and mean shift feature space analysis, the zero velocity/acceleration points of JHTDB simulation tend to form slightly ‘larger clusters’ and survive a bit longer compared to the zero velocity/acceleration points of Imperial simulation possibly due to the larger spatial/temporal resolution of JHTDB simulation. The RDFs of zero velocity/acceleration points correlate relatively well.

- For the cluster morphology of zero velocity/acceleration points from simulation of Imperial and JHTDB and experimental data, good agreement was found between Imperial simulation and experiments showing that the typical cluster length scales, defined statistically by Radial Distribution Function (RDF) and morphologically by mean shift feature space analysis, are in the range of \( 10^1 \) times the Kolmogorov length scale for turbulent zero velocity/acceleration points and dispersed droplets. The cluster length scale of the zero velocity/acceleration points acquired from JHU forced isotropic turbulence is slightly larger than the cluster length scale of zero velocity/acceleration points and dispersed phase in Imperial simulation and experiment, but within reasonable range. Good agreement was found between simulation and experiment showing that the typical time scales of clusters lifetimes, defined by the point where the auto-correlation coefficient of fluctuations of standard deviation of normalised Voronoï area crosses zero, are in the range of \( 10^{-1} \) to \( 10^1 \) times the Kolmogorov timescale.
6.2 Scope for Future work

6.2.1 Experimental work

Since self-similarity of droplet clustering has been observed, the ‘sweep and stick’ mechanism is promising to explain the observed phenomenon. It is desirable to obtain simultaneous measurement of two phases so that the velocity of acceleration points and velocity of droplets can be compared in time, as achieved numerically by Coleman and Vassilicos (2009). This can be achieved by combining ILIDS and PIV techniques (Hardalupas et al., 2010), but significant effort is required to achieve such measurement in the ‘box of turbulence’.

The measurement of the three dimensional structure of droplet clusters is another direction for future experimental works. Such experimental technique is not widely available. Some attempts have been made based on holographic methods (Katz and Sheng, 2010), but the application of such techniques in the ‘box of turbulence’ is also a significant development.

6.2.2 Direct Numeric Simulations

In current two-phase DNS simulations, there is a significant larger number of droplets in one instantaneous DNS snapshot of around 20,000 comparing to the experimental image of 5,000. As a result, it is expected to observe underestimated cluster length scale by both RDF and mean shift feature space analysis and lack of correlation in the p.d.f.s of normalised Voronoï area in the DNS simulated dispersed droplet. In order to make a thorough comparison with experiment, simulation with different volume fraction and effectively number of droplets per snapshot is required to provide a thorough comparison.
6.2.3 Mean Shift Tracking

The mean shift combined Voronoï feature space analysis is a very promising technique providing morphological descriptions of clusters. In the current work, fixed optimal bandwidth has been used. However, applications of variable bandwidth could make the mean shift combined Voronoï feature space analysis more powerful to morphologically represent the self-similarity property of droplet clustering. This approach can also be used to calculate the shape of the observed clusters and quantify its temporal evolution.
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Appendix

The publications produced from this thesis are listed below.


7. H. Lian, G. Charalampous, Y. Hardalupas. Time resolved investigation of
clustering of droplets in a box of turbulence. 8th International Conference on Multiphase Flows ICMF, Jeju, Korea, May 26 - 31 (2013)