A study of $CP$ violation in $B^{\pm} \rightarrow DK^{\pm}$ and $B^{\pm} \rightarrow D\pi^{\pm}$ decays with $D \rightarrow K_{S}^{0}K^{\pm}\pi^{\mp}$ final states

LHCb Collaboration

Abstract

A first study of $CP$ violation in the decay modes $B^{\pm} \rightarrow [K_{S}^{0}K^{\pm}\pi^{\mp}]_{0}h^{\pm}$ and $B^{\pm} \rightarrow [K_{S}^{0}K^{\mp}\pi^{\pm}]_{0}h^{\pm}$, where $h$ labels a $K$ or $\pi$ meson and $D$ labels a $D^0$ or $\bar{D}^0$ meson, is performed. The analysis uses the LHCb data set collected in $pp$ collisions, corresponding to an integrated luminosity of 3 fb$^{-1}$. The analysis is sensitive to the $CP$-violating CKM phase $\gamma$ through seven observables: one charge asymmetry in each of the four modes and three ratios of the charge-integrated yields. The results are consistent with measurements of $\gamma$ using other decay modes.

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1. Introduction

A precise measurement of the unitarity triangle angle $\gamma = \arg(-V_{ud}V_{ub}^{\ast})$ is one of the most important tests of the Cabibbo Kobayashi Maskawa (CKM) mechanism. This parameter can be accessed through measurements of observables in decays of charged $B$ mesons to a neutral $D$ meson and a kaon or pion, where $D$ labels a $D^0$ or $\bar{D}^0$ meson decaying to a particular final state accessible to $D^0$ and $\bar{D}^0$. Such decays are sensitive to $\gamma$ through the interference between $b \rightarrow cDs$ and $b \rightarrow uDs$ amplitudes. They offer an attractive means to measure $\gamma$ because the effect of physics beyond the Standard Model is expected to be negligible, thus allowing interesting comparisons with other measurements where such effects could be larger.

The determination of $\gamma$ using $B^{\pm} \rightarrow DK^{\pm}$ decays was first proposed for $D$ decays to the $CP$-eigenstates $K^{+}K^{-}$ and $\pi^{+}\pi^{-}$ (so-called “GLW” analysis) [1,2]. Subsequently, the analysis of the $K^{\pm}\pi^{\mp}$ final state was proposed (named “ADS”) [3,4], where the suppression between the colour favoured $B^{-} \rightarrow D^{0}\bar{K}^{-}$ and suppressed $B^{-} \rightarrow D^{0}K^{-}$ decays is compensated by the CKM suppression of the $D^{0} \rightarrow K^{+}\pi^{-}$ decay relative to $\bar{D}^{0} \rightarrow K^{\mp}\pi^{\pm}$, resulting in large interference. The LHCb Collaboration has published the two-body ADS and GLW analyses [5], the Dalitz analysis of the decay $B^{\pm} \rightarrow [K_{S}^{0}h^{\pm}\pi^{\mp}]_{0}K^{\pm}$ ($h = \pi, K$) [6] and the ADS-like analysis of the decay mode $B^{\pm} \rightarrow [K_{S}^{0}\pi^{+}\pi^{\mp}]_{0}K^{\pm}$ [7], where $[X]_{0}$ indicates a given final state $X$ produced by the decay of the $D$ meson. These measurements have recently been combined to yield the result $\gamma = (72.0^{+14.7}_{-15.6})^\circ$ [8], which is in agreement with the results obtained by the BaBar and Belle Collaborations of $\gamma = (69^{+17}_{-10})^\circ$ [9] and $\gamma = (68^{+15}_{-14})^\circ$ [10], respectively. In analogy to studies in charged $B$ meson decays, sensitivity to $\gamma$ can also be gained from decays of neutral $B$ mesons, as has been demonstrated in the LHCb analysis of $B^{\pm} \rightarrow [K_{S}^{0}K^{\mp}]_{0}K^{\pm}0$ decays [11].

The inclusion of additional $B^{\pm} \rightarrow DK^{\pm}$ modes can provide further constraints on $\gamma$. In this Letter, an analysis of the $D \rightarrow K_{S}^{0}K^{\pm}\pi^{\mp}$ final states is performed, the first ADS-like analysis to use singly Cabibbo-suppressed (SCS) decays. The two decays, $B^{\pm} \rightarrow [K_{S}^{0}K^{\pm}\pi^{\mp}]_{0}h^{\pm}$ and $B^{\pm} \rightarrow [K_{S}^{0}K^{\mp}\pi^{\pm}]_{0}h^{\pm}$, are distinguished by the charge of the $K^{\pm}$ from the decay of the $D$ meson relative to the charge of the $B$ meson, so that the former is labelled “same sign” (SS) and the latter “opposite sign” (OS).

In order to interpret $CP$-violating effects using these three-body decays it is necessary to account for the variation of the $D$ decay strong phase over its Dalitz plot due to the presence of resonances between the particles in the final state. Instead of employing an amplitude model to describe this phase variation, direct measurements of the phase made by the CLEO Collaboration are used, which are averaged over large regions of the Dalitz plot [12]. The same CLEO study indicates that this averaging can be employed without a large loss of sensitivity. The use of the CLEO results avoids the need to introduce a systematic uncertainty resulting from an amplitude model description.

The analysis uses the full 2011 and 2012 LHCb $pp$ collision data sets, corresponding to integrated luminosities of 1 and 2 fb$^{-1}$ and centre-of-mass energies of $\sqrt{s} = 7$ TeV and 8 TeV, respectively. The results are measurements of $CP$-violating observables that can be interpreted in terms of $\gamma$ and other hadronic parameters of the $B^{\pm}$ meson decay.
2. Formalism

The $B^+ \rightarrow [K_S^0 K^+\pi^-]_D K^+$ can proceed via a $D^0$ or $\bar{D}^0$ meson, so that the decay amplitude is the sum of two amplitudes that interfere,

$$A(m^2_{K_S^0 K^+ K^-}, m^2_{K_S^0 \pi^+ \pi^-}) = A_{D^0}(m^2_{K_S^0 K^+}, m^2_{K_S^0 \pi^+}) + r_B e^{i\beta_B} A_{D^0}(m^2_{K_S^0 K^-}, m^2_{K_S^0 \pi^-}),$$

where $A_{D^0}(m^2_{K_S^0 K^+}, m^2_{K_S^0 \pi^+})$ are the $D^0$ and $\bar{D}^0$ decay amplitudes to a specific point in the $K_S^0 K^+ \pi^-$ Dalitz plot. The amplitude ratio $r_B$ is $[\frac{|A(B^+\rightarrow D^0 K^+)|}{|A(B^+\rightarrow \bar{D}^0 K^+)|}] = 0.089 \pm 0.009$ [8] and $\beta_B$ is the strong phase difference between the $B^+ \rightarrow D^0 K^+$ and $B^+ \rightarrow \bar{D}^0 K^+$ decays. The calculation of the decay rate in a region of the Dalitz plot requires the evaluation of the integral of the interference term between the two $D$ decay amplitudes over that region. Measurements have been made by the CLEO Collaboration [12], where quantum-correlated $D$ decays are used to determine the integral of the interference term directly in the form of a "coherence factor", $\kappa_{K_S^0 K^+}$ and an average strong phase difference, $\delta_{K_S^0 K^+}$, as first proposed in Ref. [13]. The coherence factor can take a value between 0 and 1 and is defined through the expression

$$\kappa_{K_S^0 K^+} = \frac{\int A^{*}_{\text{int}}(m^2_{K_S^0 K^+ \pi^-}, m^2_{K_S^0 \pi^+}) dK_{S^0} d\pi^+}{\int A_{\text{int}}(m^2_{K_S^0 K^+ \pi^-}, m^2_{K_S^0 \pi^+}) dK_{S^0} d\pi^+},$$

where $A^{*}_{\text{int}}(m^2_{K_S^0 K^+ \pi^-}, m^2_{K_S^0 \pi^+}) = \frac{1}{2} \int A_{\text{int}}(m^2_{K_S^0 K^+ \pi^-}, m^2_{K_S^0 \pi^+}) dK_{S^0} d\pi^+$. This avoids the significant modelling uncertainty incurred by the determination of the strong phase difference between the $D^0$ and $\bar{D}^0$ amplitudes at each point in the Dalitz region from an amplitude model. The decay rates, $\Gamma$, to the four final states can therefore be expressed as

$$\Gamma_{\text{SS,DK}}^{K_S^0 K^+} = z \left[ 1 + r_B^2 + 2r_B e^{i\beta_B} \cos(\delta_B - \gamma - \delta_{K_S^0 K^+}) \right],$$

$$\Gamma_{\text{OS,DK}} = z \left[ 1 + r_B^2 + 2r_B e^{i\beta_B} \cos(\delta_B - \gamma + \delta_{K_S^0 K^+}) \right]$$

where $r_D$ is the amplitude ratio for $D^0 \rightarrow K_S^0 K^+ \pi^-$ with respect to $D^0 \rightarrow K_{S^0}^0 K^- \pi^-$ and $z$ is the normalisation factor. Analogous equations can be written for the $B^+ \rightarrow D^0 \pi^+$ system, with $r_B^2$ and $\delta_B^2$ replacing $r_B$ and $\delta_B$, respectively. Less interference is expected in the $B^+ \rightarrow D^0 \pi^+$ system, with the value of $r_B^2$ much higher, approximately 0.0015 [8]. These expressions receive small corrections from mixing in the charm system, which, though accounted for in Section 7, are not explicitly written here. At the current level of precision these corrections have a negligible effect on the final results.

Observables constructed using the decay rates in Eq. (3) have a sensitivity to $\gamma$ that depends upon the value of the coherence factor, with a higher coherence corresponding to greater sensitivity. The CLEO Collaboration measured the coherence factor and average strong phase difference in two regions of the Dalitz plot: firstly across the whole Dalitz plot, and secondly within a region $\pm 100 \text{MeV}/c^2$ around the $K^*(892)^\pm$ resonance, which decays to $K_{S^0}^0 \pi^\pm$, where, though the sample size is diminished, the coherence is higher. The measured values are $\kappa_{K^0_{S^0} K^\pm}$ = 0.73 $\pm$ 0.08 and $\delta_{K^0_{S^0} K^\pm}$ = 8.3 $\pm$ 15.2$^\circ$ for the whole Dalitz plot, and $\kappa_{K^0_{S^0} K^\pm}$ = 1.00 $\pm$ 0.16 and $\delta_{K^0_{S^0} K^\pm}$ = 26.5 $\pm$ 15.8$^\circ$ in the restricted region. The branching fraction ratio of $D^0 \rightarrow K_{S^0}^0 K^+ \pi^-$ to $D^0 \rightarrow K_{S^0}^0 K^- \pi^+$ decays was found to be 0.592 $\pm$ 0.044 in the whole Dalitz plot and 0.356 $\pm$ 0.034 in the restricted region [12]. Eight yields are measured in this analysis, where seven observables are constructed. The charge asymmetry is measured in each of the four decay modes, defined as $A_{SS,DK} = \frac{N_{K^0_{S^0} K^+ \pi^-} - N_{K^0_{S^0} K^- \pi^+}}{N_{K^0_{S^0} K^+ \pi^-} + N_{K^0_{S^0} K^- \pi^+}}$ for the $B^+ \rightarrow [K_{S^0}^0 K^+ \pi^+]_D K^+$ mode and analogously for the other modes. The ratios of $B^+ \rightarrow D^0 K^+$ and $B^+ \rightarrow D^+ \pi^+$ yields are determined for the SS and OS decays, $R_{DK/D^0 SS}$ and $R_{D^0/D^+/OS}$ respectively, and the ratio of SS to OS $B^+ \rightarrow D^+ \pi^+$ yields, $R_{SS/OS}$, is measured. The measurements are performed both for the whole Dalitz plot and in the restricted region around the $K^*(892)^\pm$ resonance.

3. The LHCb detector and data set

The LHCb detector [14] is a single-arm forward spectrometer covering the pseudorapidity range $2 < \eta < 5$, designed for the study of particles containing $b$ or $c$ quarks. The detector includes a high-precision tracking system consisting of a silicon-strip vertex detector surrounding the interaction point, also called a primary vertex (PV). This stage, which applies a full event reconstruction. The software trigger consists of a hardware stage, based on information from the calorimeter and muon systems, followed by a software stage, which applies a full event reconstruction. The software trigger searches for a track with large $p_T$ and large $IP$ with respect to any $pp$ interaction point, also called a primary vertex (PV). This track is then required to be part of a two-, three- or four-track secondary vertex with a high $p_T$ sum, significantly displaced from any PV. A multivariate algorithm [16] is used for the identification of secondary vertices consistent with the decay of a $b$ hadron.

Samples of around two million $B^\pm \rightarrow [K_{S^0}^0 K^+ \pi^\pm]_D K^+$ and two million $B^\pm \rightarrow [K_{S^0}^0 K^+ \pi^\pm]_D K^+$ decays are simulated to be used in the analysis, along with similarly-sized samples of $B^\pm \rightarrow [K_{S^0}^0 K^+ \pi^-]_D \pi^\pm$, $B^\pm \rightarrow [K_{S^0}^0 K^- \pi^+]_D \pi^\pm$ and $B^\pm \rightarrow [K_{S^0}^0 K^+ \pi^- \pi^+]_D \pi^\pm$ decays that are used to study potential backgrounds. In the simulation, $pp$ collisions are generated using PyTHIA [17] with a specific LHCb configuration [18]. Decays of hadronic particles are described by EVTGEN [19], in which final state radiation is generated using PHOTOS [20]. The interaction of the generated particles with the detector and its response are implemented using the GEANT4 toolkit [21] as described in Ref. [22].

4. Candidate selection

Candidate $B \rightarrow [K_{S^0}^0 K^+ \pi^\pm]_D K$ and $B \rightarrow [K_{S^0}^0 K^+ \pi^\pm]_D \pi$ decays are reconstructed in events selected by the trigger and then the candidate momenta are refit, constraining the masses of the neutral $D$ and $K_{S^0}^0$ mesons to their known values [23] and the $B^\pm$ meson to originate from the primary vertex [24]. Candidates where the $K_{S^0}^0$ decay is reconstructed using "long" pion tracks, which
leave hits in the VELO and downstream tracking stations, are ana-
ysed separately from those reconstructed using “downstream” pion
tracks, which only leave hits in tracking stations beyond the VELO.
The signal candidates in the former category are reconstructed with a
better invariant mass resolution.

The reconstructed masses of the \(D\) and \(K^0_S\) mesons are re-
quired to be within 25 MeV/c\(^2\) and 15 MeV/c\(^2\), respectively, of
their known values. Candidate \(B^+ \to D^+ K^\pm\) decays are separated
from \(B^0 \to D^0 \pi^\pm\) decays by using PID information from the RICH
detectors. A boosted decision tree (BDT) [25,26] that has been de-
developed for the analysis of the topologically similar decay mode
\(B^\pm \to [K^2_S h^3_1 h^0_1] h^\pm\) is applied to the reconstructed candidates.
The BDT was trained using simulated signal decays, generated uni-
formly over the \(D^0\) Dalitz plot, and background candidates taken
from the \(B^+\) invariant mass region in data between 5700 and
7000 MeV/c\(^2\). It exploits the displacement of tracks from the de-
cays of long-lived particles with respect to the PV through the use
of \(γ^2\) variables, where \(γ^2\) is defined as the difference in \(χ^2\) of a
given PV fit with and without the considered track. The BDT also
employs the \(B^0\) and \(D^0\) candidate momenta, an isolation variable
sensitive to the separation of the tracks used to construct the \(B^\pm\)
candidate from other tracks in the event, and the \(χ^2\) per degree of
freedom of the decay refit. In addition to the requirement placed
on the BDT response variable, each composite candidate is required to
have a vector displacement of production and decay vertices
that aligns closely to its reconstructed momenta. The cosine of
the angle between the displacement and momentum vectors is re-
quired to be less than 0.142 rad for the \(K^0_S\) and \(D^0\) candidates,
and less than 0.0141 rad (0.0100 rad) for long (downstream) \(B^\pm\)
candidates.

Additional requirements are used to suppress backgrounds from
specific processes. Contamination from \(B\) decays that do not con-
tain an intermediate \(D\) meson is minimised by placing a minimum
threshold of 0.2 MeV on the decay time of the \(D\) candidate. A po-
tential background could arise from processes where a pion is
misidentified as a kaon or vice versa. One example is the relatively
abundant mode \(B^0 \to [K^0_S π^+ π^-] h^0_1\), which has a branching frac-
tion around ten times larger than the signal. These are suppressed by
placing requirements on both the \(D^0\) daughter pion and kaon,
making use of PID information. For \(K^0_S\) candidates formed from
long tracks, the flight distance \(χ^2\) of the candidate is used to sup-
press background from \(B^0 \to [K^0_S π^+ π^-] h^0_1\) decays. Where
multiple candidates are found belonging to the same event, the can-
didate with the lowest value of the refit \(χ^2\) per degree of
freedom is retained and any others are discarded, leading to a re-
duction in the sample size of approximately 0.3%.

The \(B^\pm\) invariant mass spectra are shown in Fig. 1 for can-
didates selected in the whole \(D\) Dalitz plot, overlaid with a para-
metric fit described in Section 5. The \(D\) Dalitz plots are shown in
Fig. 2 for the \(B^+ \to DK^\pm\) and \(B^0 \to D\pi^\pm\) candidates that fall
within a nominal \(B^+\) signal region in \(B^\pm\) invariant mass
(5247–5317 MeV/c\(^2\)). The dominant \(K^*(892)\) resonance is clearly
visible within a horizontal band, and the window around this res-
onance used in the analysis is indicated.

5. Invariant mass fit

In order to determine the signal yields in each decay mode, si-
multaneous fits are performed to the \(B^\pm\) invariant mass spectra in the
range 5110 MeV/c\(^2\) to 5800 MeV/c\(^2\) in the different modes,
both for candidates in the whole \(D\) Dalitz plot, and for only those
inside the restricted region around the \(K^*(892)\) resonance. The
data samples are split according to the year in which the data were
taken, the decay mode, the \(K^0_S\) type and the charge of the \(B\)
candidate. The fit is parameterised in terms of the observables described
in Section 2, rather than varying each signal yield in each category
independently.

The probability density function (PDF) used to model the signal
component is a modified Gaussian function with asymmetric tails,
where the unnormalised form is given by

\[
f(m; m_0, α_L, α_R, σ) = \left\{ \begin{array}{ll}
    \exp\left[-(m - m_0)^2 / (2α^2 + α_L(m - m_0)^2)\right] & \text{for } m < m_0, \\
    \exp\left[-(m - m_0)^2 / (2α^2 + α_R(m - m_0)^2)\right] & \text{for } m > m_0,
\end{array} \right.
\]

where \(m\) is the reconstructed mass, \(m_0\) is the mean \(B\) mass and \(σ\)
determines the width of the function. The \(α_L, α_R\) parameters govern
the shape of the tail. The mean \(B\) mass is shared among all catego-
ries but is allowed to differ according to the year in which the data were
collected. The \(α\) parameters are fixed to the values de-
termined in the earlier analysis of \(B^\pm \to [K^0_S π^+ π^-] h^0_1\) [6].
The \(α_L, α_R\) parameters are common to the \(B^+ \to D^\pi^+\) and \(B^0 \to D^K^\pm\),
SS and OS categories, and are allowed to vary in the fit. Only the
width parameters \(σ(B^\pm \to DK^\pm)\) are allowed to vary in the fit.
The ratios \(σ(B^\pm \to Dπ^+) / σ(B^\pm \to DK^\pm)\) are fixed according to
studies of the similar mode \(B^0 \to D^π^+\) and \(B^0 \to D^K^\pm\),
SS and OS categories, and are allowed to vary in the fit. The yields in the vari-
ous \(D^\pi^\pm\) decay modes and different charges, and all the \(B^\pm \to DK^\pm\)
yields, are determined using the observables described in Sec-
ton 2, rather than being fitted directly.

In addition to the signal PDF, two background PDFs are re-
quired. The first background PDF models candidates formed from
random combinations of tracks and is represented by a linear func-
tion. In the fit within the restricted Dalitz region, where the sam-
ple size is significantly smaller, the slope of the linear function
fitting the \(B^\pm \to Dπ^\pm\) data is fixed to the value determined in
the fit to the whole Dalitz plot. The second background PDF ac-
counts for contamination from partially reconstructed processes.

Given that the contamination is dominated by those processes that
involve a real \(D^0 \to K^0_S K^± π^\pm\) decay, the PDF is fixed to the shape
determined from the more abundant mode \(B^0 \to [K^± π^\pm] h^0_1\).

The yields of both these background components are free to vary in
each data category.

A further significant background is present in the \(B^\pm \to DK^\pm\)
samples due to \(π \to K\) misidentification of the much more abun-
dant \(B^0 \to Dπ^\pm\) mode. This background is modelled in the \(B^\pm \to
DK^\pm\) spectrum using a Crystal Ball function [27], where the pa-
rameters of the function are common to all data categories in
the fit and are allowed to vary. The yield of the background in
the \(B^\pm \to DK^\pm\) samples is fixed with respect to the fitted \(B^\pm \to
Dπ^\pm\) signal yield using knowledge of the RICH particle identifi-
cation efficiencies that is obtained from data using samples of
\(D^{±} \to [K^± π^\pm] h^0_1\) decays. The efficiency for kaons to be selected is
found to be around 84% and the misidentification rate for pions is
around 4%.

Production and detection asymmetries are accounted for, fol-
lowing the same procedure as in Refs. [5,7]. Values for the \(B^\pm\)
production and \(K\) detection asymmetries are assigned such that the
combination of production and detection asymmetries corresponds to
the raw asymmetry observed in \(B^\pm \to J/ψ K^\pm\) decays [28].
The detection asymmetry assigned is \(-0.5 ± 0.7%\) for each unit
of strangeness in the final state to account for the differing inter-
actions of \(K^\pm\) and \(K^-\) mesons with the detector material. An anal-
ogous asymmetry is present for pions, though it is expected to be
much smaller, and the detection asymmetry assigned is \(0.0 ± 0.7%\).
Any potential asymmetry arising from a difference between the re-
sponses of the left and right sides of the detector is minimised by
Fig. 1. Distributions of $B^\pm$ invariant mass of the SS and OS samples for the (a, c, e, g) $B^+ \to D K^+$ and (b, d, f, h) $B^\pm \to D \pi^\pm$ candidates in the full data sample. The fits are shown for (a, b, e, f) $B^+$ and (c, d, g, h) $B^-$ candidates. Fit PDFs are superimposed.

A further correction is included to account for non-uniformities in the acceptance over the Dalitz plot. This efficiency correction primarily affects the $R_{SS}$ observable, given the difference in the Dalitz distributions for the two $D$ meson decay modes. A correction factor, $\zeta$, is found by combining the LHCb acceptance, extracted from the simulated signal sample, and amplitude models, $A_{SS,OS}(m_{K^0_S K}, m_{K^0_S \pi})$, for the Dalitz distributions of the SS or OS decays,

$$\zeta \equiv \frac{\int_D dm_{K^0_S} dm_{K^0_S} \frac{e(m^2_{K^0_S K}, m^2_{K^0_S \pi})}{|A_{OS}(m^2_{K^0_S K}, m^2_{K^0_S \pi})|^2} \times |A_{SS}(m^2_{K^0_S K}, m^2_{K^0_S \pi})|^2}{\int_D dm_{K^0_S} dm_{K^0_S} \frac{e(m^2_{K^0_S K}, m^2_{K^0_S \pi})}{|A_{OS}(m^2_{K^0_S K}, m^2_{K^0_S \pi})|^2} \times |A_{SS}(m^2_{K^0_S K}, m^2_{K^0_S \pi})|^2},$$

where $e(m^2_{K^0_S K}, m^2_{K^0_S \pi})$ is the efficiency at a point in the Dalitz plot. The typical deviation of $\zeta$ from unity is found to be around 5%. The acceptance is illustrated in Fig. 3, where bins of variable size are used to ensure that statistical fluctuations due to the finite size of the simulated sample are negligible. The Dalitz distributions are
determined using the fact that little interference is expected in $B^\pm \to D\pi^\pm$ decays and, therefore, the flavour of the $D$ meson is effectively tagged by the charge of the pion. In this case, the Dalitz distributions are given by considering the relevant $D^0$ decay ($D^{0} \to K^0_S K^- \pi^+$ for SS and $D^{0} \to K^0_S K^+ \pi^-$ for OS). These $D^0$ decay Dalitz distributions are known and amplitude models from CLEO are available [12] from which the Dalitz distributions can be extracted.

Due to the restricted sample size under study, small biases exist in the determination of the observables. The biases are determined by generating and fitting a large number of simulated samples using input values obtained from the fit to data, and are typically found to be around 2%. The fit results are corrected accordingly.

The fit projections, with long and downstream $K_S^0$-type categories merged and 2011 and 2012 data combined, are given for the fit to the whole Dalitz plot in Fig. 1. The signal purity in a nominal mass range from 5247 MeV/c$^2$ to 5317 MeV/c$^2$ is around 85% for the $B^\pm \to DK\pm$ samples and 96% for the $B^\pm \to D\pi^\pm$ samples. The signal yields derived from the fits to both the whole and restricted region of the Dalitz plot are given in Table 1. The fitted values of the observables are given in Table 2, including their systematic uncertainties as discussed in Section 6. The only significant difference between the observables fitted in the two regions is for the value of $K_{SS,OS}$. This ratio is expected to differ significantly, given that the fraction of $D^0 \to K^0_SK^+\pi^-$ decays that are expected to lie inside the restricted portion of the Dalitz plot is around 75%, whereas for $D^0 \to K^0_SK^-\pi^+$ the fraction is around 44% [12]. This accounts for the higher value of $K_{SS,OS}$ in the restricted region. The ratios between the $B^\pm \to DK\pm$ and $B^\pm \to D\pi^\pm$ yields are consistent with that measured in the LHCb analysis of $B^\pm \to [K\pi]|_{dh^\pm}$, $0.0774 \pm 0.0012 \pm 0.0018$ [5]. The $CP$ asymmetries are consistent with zero in the $B^\pm \to D\pi^\pm$ system, where the effect of interference is expected to be small. The asymmetries in the $B^\pm \to DK^\pm$ system, $A_{SS,DK}$ and $A_{OS,DK}$, which have the highest sensitivity to $\gamma$ are all compatible with zero at the 2$\sigma$ level. The correlations between $R_{SS,OS}$ ratio and the ratios $R_{DK/DD,SS}$ and $R_{DK/DD,OS}$ are $-16\% (-13\%)$ and $+16\% (+16\%)$, respectively, for the fit to the whole Dalitz plot ($K^*(892)^\pm$ resonance). The correlation between the $R_{DK/DD,SS}$ and $R_{DK/DD,OS}$ ratios is $+11\% (+15\%)$. Correlations between the asymmetry observables are all less than 1% and are neglected.

### 6. Systematic uncertainties

The largest single source of systematic uncertainty is the knowledge of the efficiency correction factor that multiplies the $R_{SS,OS}$ observable. This uncertainty has three sources: the uncertainties on the CLEO amplitude models, the granularity of the Dalitz divisions in which the acceptance is determined, and the limited size of the simulated sample available to determine the LHCb acceptance. Of these, it is the modelling uncertainty that is dominant. In addition, an uncertainty is assigned to account for the fact that interference is neglected in the computation of the efficiency correction factor, which is shared between the $D\pi^\pm$ and $DK^-\pi^+$ systems.

Uncertainties on the parameters that are fixed in the PDF are propagated to the observables by repeating the fit to data whilst...
the restricted region are given in Table 4. These result in the smallest uncertainties on the experimental ob-
ces.

In Table 3, the sources of systematic uncertainty are given for each observable in the fit to the whole Dalitz plot. Similarly those for the fit in the restricted region are given in Table 4.

7. Interpretation and conclusions

The sensitivity of this result to the CKM angle \( \gamma \) is investigated by employing a frequentist method to scan the \( \gamma - r_B \) parameter space and calculate the \( \chi^2 \) probability at each point, varying each fixed parameter according to its uncertainty. An additional systematic uncertainty is calculated for the fit to the restricted \( K^*(892)^\pm \) region, where the \( D\pi^\pm \) combinatorial background slopes are fixed to the values determined in the fit to the whole Dalitz plot.

Uncertainties are assigned to account for the errors on the \( B^\pm \) production asymmetry and the \( K^\pm \) and \( \pi^\pm \) detection asymmetries. The effect of the detection asymmetry depends on the pion and kaon content of the final state, and the resulting systematic uncertainty is largest for the \( A_{SS,DK} \) and \( A_{SS,DT} \) observables.

The absolute uncertainties on the particle identification efficiencies are small, typically around 0.3% for kaon efficiencies and 0.03% for pion efficiencies. Of the four main sources of systematic error, these result in the smallest uncertainties on the experimental observables.

In Table 3, the sources of systematic uncertainty are given for each observable in the fit to the whole Dalitz plot. Similarly those for the fit in the restricted region are given in Table 4.

![Fig. 4. Scans of the \( \chi^2 \) probabilities over the \( \gamma - r_B \) parameter space for (a) the whole Dalitz fit and (b) the fit inside the \( K^* \) region (b). The contours are the usual \( n\sigma \) profile likelihood contours, where \( \Delta \chi^2 = n^2 \) with \( n = 1 \) (dark blue), 2 (medium blue), and 3 (light blue). The 2\( \sigma \) contour encloses almost all of the parameter space shown, so a central value of \( \gamma \) and relevant bounds are not extracted. The result is seen to be compatible with the current LHCb measurement of \( \gamma \), indicated by the point at \( (\gamma = 72.0^\circ \text{ and } r_B = 0.089) \), at a level between 1 and 2\( \sigma \).](image-url)

References

[1] M. Gronau, D. London, How to determine all the angles of the unitarity triangle from $B^0 \to K^{(*)}\pi^0$ and $B^\pm \to D^\pm \phi$, Phys. Lett. B 253 (1991) 483.


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