Get Real

Paul Whelan

Alternative title: "Bond Markets"

Submitted in partial fulfillment of the requirements for the degree of
Doctor of Philosophy of Imperial College London
Abstract

I study questions related to risk premia in real bond markets. First, I document novel evidence that factors explaining excess returns for nominal Treasuries are also common to the real term structure. This suggests that sources of bond predictability should be interpreted in the context of the real consumption risks as opposed to the dynamics of inflation. Next, I investigate the role of monetary policy as a source of time-varying priced risk. I use both high-frequency and low-frequency approaches to show that monetary policy is non-neutral in the sense of affecting bond risk premia. I conclude by studying a general equilibrium term structure model with multiple agents who disagree about the unobservable model for the economy. These agents are induced to engage in speculative trading because of their beliefs, which in turn generates endogenously time-varying risk premia. My results show that speculation can help explain low short term interest rates, time-varying expected returns, and path-dependence in the cross-section of yields.
## Contents

1 Real and Nominal Yields
   I Empirical Results .................................................. 12
      A Yield Data ........................................ 12
      B Slopes ........................................ 13
      C Sharpe Ratios ....................................... 13
      D Real Campbell and Shiller (1991a) ....................... 14
      E Real Cochrane and Piazzesi (2005a) ....................... 14
      F The Cross-section of Yields ............................. 16
      G The Inflation Risk Premium ................................. 18
      H Liquidity Explanations ................................. 19
   II Equilibrium Implications ........................................ 23
      A Long Run Risk ...................................... 23
      B Habit ........................................... 24
      C Heterogeneous Agents .................................. 26
   III Risk Factors .................................................. 30
      A Economic Uncertainty .................................. 30
      B Consumption Surplus .................................. 31
      C Differences in Belief ................................... 32
   IV Understanding Equilibrium Risk Compensation ............... 33
   V Conclusion ....................................................... 34
   VI Appendix: Figures ........................................ 35
   VII Appendix: Tables ........................................ 44

2 Monetary Policy and Bond Risk Premia ........................ 54
   I High Frequency Identification .................................. 58
      A The federal funds rate ................................ 58
      B The federal funds futures market ....................... 59
      C The impact on Treasury bond yields ..................... 61
      D The Timing of Expected Fed Interventions ............... 61
      E Target versus Path shocks ................................ 62
      F Risk Premia ........................................ 64
   II Data : Low Frequency Identification ....................... 65
      A Survey data ........................................ 65
      B Quality of survey data ................................ 67
      C Macroeconomic activity data ......................... 68
      D Bond data ......................................... 68
   III Overview of identification schemes ....................... 68
### 3 Term Structure Models and Differences in Belief

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. Theoretical Framework</td>
<td>98</td>
</tr>
<tr>
<td>A. The Homogeneous Benchmark Economy</td>
<td>99</td>
</tr>
<tr>
<td>B. Disagreement</td>
<td>100</td>
</tr>
<tr>
<td>C. Disagreement and Optimal Learning</td>
<td>100</td>
</tr>
<tr>
<td>D. Disagreement and the Radon-Nikodym Derivative</td>
<td>102</td>
</tr>
<tr>
<td>E. Individual Agent Problem</td>
<td>102</td>
</tr>
<tr>
<td>F. Representative Agent Problem</td>
<td>103</td>
</tr>
<tr>
<td>G. Equilibrium Risk Allocation</td>
<td>106</td>
</tr>
<tr>
<td>H. The Term Structure of Bond Prices</td>
<td>106</td>
</tr>
<tr>
<td>I. The Yield Curve</td>
<td>108</td>
</tr>
<tr>
<td>J. Instantaneous Risk Premia</td>
<td>111</td>
</tr>
<tr>
<td>II. Testable Implications</td>
<td>113</td>
</tr>
<tr>
<td>III. Data</td>
<td>114</td>
</tr>
<tr>
<td>IV. Empirical Results</td>
<td>116</td>
</tr>
<tr>
<td>V. Conclusion</td>
<td>126</td>
</tr>
<tr>
<td>VI. Appendix A: Proofs</td>
<td>127</td>
</tr>
<tr>
<td>A. Single Agent Term Structure of Interest Rates</td>
<td>127</td>
</tr>
<tr>
<td>B. Fundamental System</td>
<td>127</td>
</tr>
<tr>
<td>C. Learning</td>
<td>128</td>
</tr>
<tr>
<td>D. Term Structure of Interest Rates</td>
<td>128</td>
</tr>
<tr>
<td>E. Belief System</td>
<td>129</td>
</tr>
<tr>
<td>F. Joint Distribution</td>
<td>130</td>
</tr>
<tr>
<td>VII. Appendix B: Data</td>
<td>133</td>
</tr>
<tr>
<td>A. Realised Returns</td>
<td>133</td>
</tr>
<tr>
<td>B. Macro Data</td>
<td>133</td>
</tr>
<tr>
<td>VIII. Appendix C: Figures</td>
<td>135</td>
</tr>
</tbody>
</table>
List of Figures

1.1 Real and Nominal 3 month Yields ........................................ 35
1.2 Real and Nominal Slopes .................................................. 35
1.3 Real and Nominal Excess Returns ....................................... 36
1.4 Real and Nominal Forecasting Factor Loadings ....................... 37
1.5 Real and Nominal Forward Rate Implied Risk Premia .................. 38
1.6 Principle Component Loadings of Residual Yield Variation ......... 38
1.7 Unspanned Principle Components: US .................................. 39
1.8 Unspanned Principle Components: UK .................................. 39
1.9 10-year Inflation Risk Premium ......................................... 40
1.10 Liquidity Variables ....................................................... 40
1.11 Liquidity Factor and Liquidity Adjusted Inflation Risk Premia .. 41
1.12 Long Run Risk Factors ................................................... 41
1.13 Consumption Surplus Ratios ............................................ 42
1.14 Belief Dispersion: US .................................................... 42
1.15 Belief Dispersion: UK ..................................................... 43

2.1 Treasury Yields ......................................................... 80
2.2 Expected vs Unexpected Changes ........................................ 80
2.3 Cumulative Shocks ........................................................ 81
2.4 Futures Predictions ........................................................ 81
2.5 Futures Predictions ........................................................ 82
2.6 Target vs Timing Shock .................................................. 83
2.7 All Shocks ................................................................. 84
2.8 Target vs Path Shocks .................................................... 84
2.9 Daily Cochrane-Piazzesi .................................................. 85
2.10 Federal Funds Rate Forecasts ........................................... 85
2.11 Consensus Macro Forecasts ............................................. 86
2.12 The Performance of BCFFS Forecasts ................................ 87
2.13 PathShock ................................................................. 88
2.14 Comparing Shocks ........................................................ 88
2.15 Counter-cyclicality of PathShock ....................................... 89
2.16 Monetary policy shocks and yield curve information ............... 89

3.1 Short Rate Sensitivity to Disagreement ................................ 135
3.2 Term Structure Example 1: Sentiment Economy ....................... 135
3.3 Term Structure Example 2: Symmetric Economy ....................... 136
3.4 Term Structure Example 2: Symmetric Economy ....................... 136
3.5 Term Structure Example 3: Pessimistic Economy ....................... 136
3.6 Term Structure Example 4: Optimistic Economy ......................................... 137
3.7 Risk Premia Example 1: Symmetric Economy ............................................ 137
3.8 Risk Premia Example 2: Pessimistic Economy ........................................... 138
3.9 Risk Premia Example 2: Optimistic Economy ............................................. 139
3.10 Survey Extract: 1Q ahead Interest Rate Forecasts ...................................... 140
3.11 BlueChip Financial Forecasts Contributors ................................................. 140
3.12 Differences in Belief Proxies ........................................................................ 141
3.13 Real and Nominal Short Term Interest Rates ................................................. 142
3.14 Path Dependence Example ........................................................................... 143
3.15 Predicting Disagreement about Real Growth ................................................. 144
# List of Tables

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>Summary Statistics: US Treasury Curves</td>
</tr>
<tr>
<td>1.2</td>
<td>Summary Statistics: UK Treasury Curves</td>
</tr>
<tr>
<td>1.3</td>
<td>U.S Campbell-Schiller Projections</td>
</tr>
<tr>
<td>1.4</td>
<td>U.K Campbell-Schiller Projections</td>
</tr>
<tr>
<td>1.5</td>
<td>Real and Nominal Cochrane-Piazzesi Regressions: US</td>
</tr>
<tr>
<td>1.6</td>
<td>Real and Nominal Cochrane-Piazzesi Regressions: UK</td>
</tr>
<tr>
<td>1.7</td>
<td>Principle Component Decomposition</td>
</tr>
<tr>
<td>1.8</td>
<td>Real - Nominal Yields and Macroeconomic Expectations: US</td>
</tr>
<tr>
<td>1.9</td>
<td>Real - Nominal Yields and Macroeconomic Expectations: UK</td>
</tr>
<tr>
<td>1.10</td>
<td>Summary Statistics: US Consumer Price Inflation</td>
</tr>
<tr>
<td>1.11</td>
<td>Summary Statistics: UK Consumer Price Inflation</td>
</tr>
<tr>
<td>1.12</td>
<td>10-year Inflation Risk Premium</td>
</tr>
<tr>
<td>1.13</td>
<td>Heterogenous Agent Models</td>
</tr>
<tr>
<td>1.15</td>
<td>Risk Premium Factors on Equilibrium Proxies: UK</td>
</tr>
<tr>
<td>2.1</td>
<td>Summary Statistics</td>
</tr>
<tr>
<td>2.2</td>
<td>FOMC Announcements: Expected vs Unexpected Changes</td>
</tr>
<tr>
<td>2.3</td>
<td>FOMC Announcements: Timing vs Level</td>
</tr>
<tr>
<td>2.4</td>
<td>Factor Decomposition</td>
</tr>
<tr>
<td>2.5</td>
<td>FOMC Announcements: Target vs Path</td>
</tr>
<tr>
<td>2.6</td>
<td>Daily Risk Premium Regressions</td>
</tr>
<tr>
<td>2.7</td>
<td>Taylor rule specifications</td>
</tr>
<tr>
<td>2.8</td>
<td>Path Shocks vs Target Shocks</td>
</tr>
<tr>
<td>2.9</td>
<td>Risk Premium Proxies</td>
</tr>
<tr>
<td>2.10</td>
<td>Bond Return Predictability: (PathShock)</td>
</tr>
<tr>
<td>2.11</td>
<td>Bond Return Predictability: (PathShock) and Real Activity</td>
</tr>
<tr>
<td>2.12</td>
<td>Bond Return Predictability: (PathShock) and Inflation</td>
</tr>
<tr>
<td>3.1</td>
<td>Calibrated Parameters</td>
</tr>
<tr>
<td>3.2</td>
<td>Short Rate Regressions</td>
</tr>
<tr>
<td>3.3</td>
<td>Short Rate Regressions</td>
</tr>
<tr>
<td>3.4</td>
<td>Short Rate Regressions</td>
</tr>
<tr>
<td>3.5</td>
<td>Path Dependent Forecasting Regression</td>
</tr>
<tr>
<td>3.6</td>
<td>Return Predictability Regressions</td>
</tr>
<tr>
<td>3.7</td>
<td>Return Predictability Robustness</td>
</tr>
</tbody>
</table>
Declaration of Originality

‘I herewith certify that this thesis constitutes my own work and that all material, which is not my own work, has been properly acknowledged’

Paul Whelan
Copyright

‘The copyright of this thesis rests with the author and is made available under a Creative Commons Attribution Non-Commercial No Derivatives licence. Researchers are free to copy, distribute or transmit the thesis on the condition that they attribute it, that they do not use it for commercial purposes and that they do not alter, transform or build upon it. For any reuse or redistribution, researchers must make clear to others the licence terms of this work.’

Paul Whelan
Acknowledgments

Firstly, I thank Professor Andrea Buraschi for his transfer of knowledge. This thesis is the result of countably infinite discussions with him through which I discovered ‘the questions’ and found some answers. There are other beautiful minds to thank but they shall be dealt with another day.

Secondly, I thank Andrea Carnelli, for confusion, late nights, and for wine. Not necessarily in that order.

Thirdly, I thank my family, my mother, my father, my sisters, for unbounded love.

Finally, I thank those that loved me, in the past, in the present, and in the future . . . and one more thank you, above all else, I thank me.
Chapter 1

Real and Nominal Yields

The consensus among fixed income researchers is that expected returns on long term nominal bonds in excess of short dated securities are predictable ahead of time. This is consistent with this view that short term interest rates are pro-cyclical, driven by the properties of expected fundamentals, while long term interest rates are counter-cyclical, driven by the properties of risk premia. Well studied forecasting factors include forward spreads (Fama and Bliss (1987)), the slope of the yield curve (Campbell and Shiller (1991a)), and an affine function of the forward curve (Cochrane and Piazzesi (2005a)). However, there exists little consensus about the economic source of this predictability.

In this chapter I document that since the advent of inflation protected securities, for both U.S. and U.K. bond markets, there appears to be very little difference between real and nominal risk premia. This suggests the dominant source of predictable variation in expected excess returns must be a risk due to the real stochastic discount factor.

Understanding the properties of real bonds is an important question that has so far received relatively little attention. In this respect, I contribute along a number of dimensions. First, I document that Sharpe ratios on real bonds are at least as large as nominal bonds over the sample period 2000.1 – 2011.12, at close to 0.43 across maturity. Excluding the 2008 crisis Sharpe ratios on TIPS (0.55) are 50% higher than Sharpe ratios of nominal Treasuries (0.35). This difference is large enough that if one were to construct a tangency portfolio based on sample averages, volatilities, and correlations, this portfolio would be almost dominated by TIPS. This empirical puzzle is even more striking in terms of magnitude than the Mehra and Prescott (1985) puzzle for equities. Second, I show that the real term structure is upward sloping and the slopes in both markets are strongly correlated. For the sample period 2000.1-2011.12 the correlation of 10-5 yields spreads is 0.74 and excluding observations around the collapse of Lehman Brothers the correlation rises to 0.90. This is intriguing since the slope of the yield curve is known to capture information about the joint dynamics of investor preferences and risk factors.

Moving to joint dynamics of risk premia I compare the ability of today’s term structures to forecast future term structures. Re-visiting the classic expectations hypothesis test of Campbell and Shiller (1991a) I show that real and nominal projection coefficients are negative, and quantitatively similar in both economic and statistical terms. Next, I build real ($CP_r^t$) and nominal ($CP^n_r^t$) return forecasting factors by

1. The exact portfolio weights are 90% TIPS, 10% equity, and 0% nominal bonds
adapting the approach of Cochrane and Piazzesi (2005a). Real $CP_r$ inherits many of the properties of nominal $CP^g$. Firstly, both factors load on the forward curve with a ‘tent’ shaped pattern, which is increasing in maturity across maturity implying a single factor structure for real-nominal risk premia. Secondly, both factors contain substantial information about expected excess returns: in the U.S a single factor explains 9% of the variance of 3-month returns on 10-year real bonds, and 15% of the variance of 3-month returns on 10-year nominal bonds. Thirdly, both $CP_r$ and $CP^g$ are counter-cyclical peaking in 2002 and again in 2008/2009. Excluding the crisis period the correlation between the series is 0.56, while the full sample correlation is 0.46, meaning that expected excess returns, and hence risk premia, display large co-movement across markets.

Next, I examine the cross-sectional properties of yields. Using principle components extracted from real and nominal curves separately, I find the covariance matrix of nominal yields is well explained by the covariance matrix of real yields. I find 76% of the variance of a nominal U.S level is explained by the real level, and 60% of the nominal U.S slope is explained by the real slope. Taken together this implies 75% of the variance of the nominal term structure is explained by the first two principle components of the real term structure. This finding is consistent with evidence from the UK. Since variations in nominal yields can be decomposed as variations in the real yields, expected inflation and inflation risk premia, this implies that the role of inflation is relatively small.

Considering projections of yields on fundamentals I study residual components orthogonal to growth rates. Remarkably, the information contained in these residuals is almost identical across markets. Considering principle components of the residuals across maturity I obtain a ‘level’ and ‘slope’ factors that are moving in lockstep. This result is important since it suggests that factors common to both term structures are unrelated to GDP growth or inflation. I examine the robustness of these findings across a number of dimensions. Collecting inflation expectations from a number of distinct survey sources I study the difference between long term nominal and real yields after adjusting for inflation. Between 2000.1 – 2011.12 the inflation risk premium in the U.S has averaged just -14 basis points, while in the U.K the average inflation risk premium is similarly small. Since liquidity considerations complicate the measurement of real risk premia, I examine the ability of liquidity proxies to explain a small inflation risk premium. I reject this as an alternative. Finally, I examine the link between factors extracted from real bonds and well studied nominal risk premium proxies: encouraging for a real risk premium explanation, information available from the real bonds explains 57% of the variance of a Cochrane and Piazzesi (2005a) factor, and 33% of a volatility premium factor from Le and Singleton (2013a).

Finally, I conclude by reviewing specifications for the stochastic discount factor that introduce either time-varying quantities of risk, or time-varying prices of risk, or both. For both the U.S and U.K I proxy for the conditional variance of fundamentals from survey data following Bansal and Shaliastovich (2013), estimate a consumption surplus proxy following Wachter (2006), and consider dispersion in beliefs measures along the lines of Buraschi and Whelan (2012a). Through projections on risk factors I find evidence
that the dominant source of risk is the conditional volatility of gdp growth, lending support for a real quantity of risks explanation.

The rest of this chapter is organised as follows. Section I reviews stylised facts across real and nominal markets. Section II reviews what three alternative asset pricing models say about real and nominal bonds. Section III proposes observable proxies for the risk factors associated with these economies. Section IV concludes by discussing underlying economic sources for real risk premia.

I. Empirical Results

A. Yield Data

U.S inflation protected Treasuries were first issued in 1997 which adjust to the all urban consumer price index with a 3-month lag. In the early years of issue this market suffered significant liquidity problems (see, for example Roll (2004)) and our sample focuses on the period 2000.01 - 2012.12 from which I collect nominal and TIPS zero coupon bonds estimated by Gürkaynak, Sack, and Wright (2006, 2010) (GSW). GSW construct zero coupon yields by fitting the Nelson-Siegel-Svensson functional form to market quoted coupon bonds and is publicly available from the Federal Reserve Board site. I also use short-term nominal interest rates with 3 and 6-month maturities from the Fama-Bliss T-bill files available from CRSP.

Nominal zero coupon yields are available in the U.K since 1971 while inflation linked gilts have been on issue since 1985. Both the coupon payments and the principal are adjusted to the General Index of Retail Prices with a variable lag depending on the sample period.\(^2\) I obtain both sets of prices from the Bank of England site which are estimated with a penalty based spline (Anderson and Sleath (2001)). At the short end of the nominal curve I obtain 3 and 6 month LIBOR rates from Bloomberg.

Studying real versus nominal risk premia requires observations on short term real rates. Unfortunately, short term inflation linked bills are not issued by either the U.S or the U.K. I proxy for these following Campbell and Shiller (1996) who estimate a VAR model including the ex post real return on a 3-month nominal bills, the nominal bill yield, and lagged annual inflation rate.

Solving the VAR forward I build ex-ante forecasts for date \(t\) 3-month real rates, which are displayed with their nominal counterparts in figure 1.1. The left panel displays estimates for the U.S while the right panel displays estimates for the U.K. I find short term real and nominal rates are positively correlated over this sample, and that U.S real rates were negative in the years 2002 - 2004, in 2008, and 2010 onwards. For the U.K real rates dropped into the negative region in the aftermaths of the financial crisis.

B. Slopes

The U.S nominal term structure slopes upward, from 3.5% at 5-years to 4.5% at 10-years. This is well known. Less well known is that the real curve is also upward sloping, from 1.7% at 5-years to 2.2% at 10-years. Yield curves in the U.K are flatter but both slopes are again upward sloping. Figure 1.2 plots the time series dynamics for U.S real (left panel) and nominal (right panel) slopes between 10-years and \{5, 6, 7, 8, 9\}-years. The series indicate that both term structures were flat between the years 2000 − 2001 / 2006 − 2007, steep in years 2003 − 2004 / 2008, and remained so until the of the sample. The dynamics of the two slopes track each other closely, the full sample correlation between the 10 − 5 year slopes is 0.74, and excluding the massive negative shock to long term real yields in the crisis is 0.90. This is intriguing because the slope of the term structure is revealing about the joint dynamics of investor preferences and risk factors.

C. Sharpe Ratios

I construct 3-month excess returns on both real and nominal bonds maturing between 5 and 10-years from

$$r_{x_t}^{(n)} = p_t^{(n-0.25)} - p_t^{(n)} - y_t^{(0.25)}$$

where $p_t^{(n)}$ is a log price of a real / nominal n-year bond, and $y_t^{r/S,(0.25)}$ is the real / nominal 3-month rate. Figure 1.3 displays time series for (annualised) realised excess returns on 5-year bonds. The co-movement is large. In the U.S, excluding (including) the crisis the correlation is 0.83 (0.64) and for the U.K is 0.75 (0.46). Both real bond return series display massive negative returns in the aftermath of Lehman brothers which are not present on the nominal curves. The term structure of excess returns is upward sloping in the U.S from 1.10% to 1.75% for nominal bonds, and 1.20% to 1.65% for real bonds, between 5 and 10-years. Real and nominal excess returns in the U.K are around 1% lower but again upward sloping. U.S nominal Sharpe ratios are relatively flat from 0.44 to 0.38 and close to 0.43 across maturity on the real curve. Excluding the crisis, for the sample period 2000.1 - 2008.1, the Sharpe ratio on nominal bonds was much lower at $\sim$ 0.35 across maturity, while the Sharpe ratio on TIPS was higher at $\sim$ 0.55 across maturity. This is an important statistic since positive Sharpe ratios on real bonds provide an important moment from which to benchmark competing models. For example, in the long run risk model of Bansal and Yaron (2004) the Sharpe ratio on real bonds is negative when agents have a preference for early resolution of uncertainty. Moreover, in the habit model of Campbell and Cochrane (1999) the

---

3 Studying real versus nominal (return) risk premia requires observations on short term real rates. Unfortunately, short term inflation linked bills are not issued by either the U.S or the U.K. I proxy for the short term real rates following Campbell and Shiller (1996) as discussed above.

4 Fleckenstein, Longstaff, and Lustig (2010) also point this out

5 Using a no-arbitrage model Duffee (2010) estimates Sharpe ratios on nominal bonds to be (statistically) downward sloping. In general, statistical comparisons of Sharpe ratios rely on assumptions about the data generating process. For a generalised method of moments approach to this problem see Christie (2005).
Sharpe ratio on real bonds is negative when the risk free real rate is pro-cyclical.

[ insert figure 1.3 about here]

D. Real Campbell and Shiller (1991a)

Campbell and Shiller (1991a) study the information content in long-maturity yields spreads for future interest rate changes on long return bonds; thus examine the information content contained in date $t$ yields for expected returns. The Campbell-Schiller projection coefficients are the loadings in the following regression

$$y_{t+m}^{(n-m)} - y_t^{(n)} = \text{const} + \beta_{nm} \left( \frac{m}{n-m} \right) (y_t^{(n)} - y_t^{(m)}) + \text{error}_{t+m}$$  \hspace{1cm} (1.1)

Absent of a time-varying risk premium all variation in the real yield spread is due to variation in expected future real short rates, which implies $\beta_{nm} = 1$ for all $n,m$. For the nominal Treasury curve Campbell and Shiller (1991a) find that, not only is $\beta_{nm} \neq 1$, but is in fact negative. They conclude that there exists an element of predictability in bond returns and that this is due to current yield spreads embedding information about term premia, thus compensation for risk.

[ insert table 1.3 and 1.4 about here]

Table 1.3 reports the estimates for equation 1.6 for the U.S, while table 1.4 reports estimates for the U.K. The regression uses 3-month overlapping observations, which I correct for using a GMM-correction with 3 Newey-West lags, for $n = \{60, 72, 84, 96, 108, 120\}$-months and $m = 3$-months. Mirroring the results of Campbell-Schiller I find negative projection coefficients for all maturities. For the U.S both real and nominal loadings are statically significant at close to the 1% level. The magnitude of the nominal projection coefficients are larger than the real coefficients and increasing in maturity while the real projection coefficients are flat across maturity. For example, the nominal (real) coefficient is $-0.20$ ($-0.19$) on a 6-year bond and $-0.29$ ($-0.19$) on a 10-year bond. In terms of predictable variation, the $R^2$'s ranging between $7\%$ - $12\%$ for nominal bonds and $9\%$ - $10\%$ on real bonds. Considering the U.K projection coefficients I find a very similar pattern, which is important since some authors have argued rejections of the expectation hypothesis is a U.S based phenomenon.

E. Real Cochrane and Piazzesi (2005a)

Figure 1.3 showed the dynamics of realised excess returns on real and nominal bonds have large co-movements. This section examines whether the dynamics of expected excess returns are also co-moving. Given a date $t$ cross-section of bond yields all information regarding future interest rates (and thus expected returns) is summarised in the shape of the term structure today. Linear combinations of yields suffice to characterise risk factors through yield curve inversion.$^6$ Building on this notion Cochrane and Piazzesi

$^6$Specifically, assume $N$ bond yields are measured without error. Then, stacking these yields into the vector $y^N = AN + BX_t$, I can solve for the risk factors through inversion as $X_t = (B^N)^{-1} (y^N - AN)$ so long as the matrix $B^N$ is non-singular.
(2005a) show that an affine function of forward rates embeds substantial information for the dynamics of excess returns. The Cochrane-Piazzesi return forecasting factor, $CP_t$, is a tent-shaped linear combination of forward rates that predicts excess returns on bonds with $R^2$ statistics as high as 43% (in their sample period) and has been shown to capture $\sim 99\%$ of the time-variation in expected returns.

I adapt the bond risk premium estimates proposed by Cochrane and Piazzesi (2005a, 2008) for inflation protected securities. Specifically, I project 3-month excess returns on 3-month forward rate spreads. Common factors are then formed from real and nominal forward rates by factorizing the first stage regression as

$$r_{xt+1} = \alpha + \gamma'(f_t^{[5,10]} - y_t^{(0.25)}) + \epsilon_{t+0.25}$$

$$CP_t = \gamma'(f_t^{[5,10]} - y_t^{(0.25)})$$

Real $CP_t^r$ inherits many of the properties of nominal $CP_t^\delta$. Firstly, figure 1.4 shows a tent shaped factor structure is clearly visible on both real and nominal curves, for both the U.S and the U.K. Interpreting the ‘shape’ of the factor loadings is debatable since, as argued by Dai, Singleton, and Yang (2004), including yields of close maturity on the right hand side introduces collinearity problems. More importantly, the magnitude of the tent shape is increasing in maturity implying that a single factor predicts increasing risk premia on long term bonds. Secondly, while embedding substantial information about future returns $CP_t^r$ explains less than 1% of the variance of contemporaneous yield changes.\(^7\)

Figure 1.5 plots time-series for real and nominal risk premium proxies, which display countercyclical behaviour peaking in 2002 and again in 2008/2009. Excluding the crisis period the correlation between the series is 0.56 while the full sample correlation is 0.46. This means that both realised and expected returns co-moving across markets. I evaluate the statistical significance of this finding by projecting returns onto $CP_t^r$ and $CP_t^\delta$ in second stage regression

$$r_{x_t+0.25}^{\delta,(5:10)} = \alpha_n + b_n CP_t^\delta + \epsilon_t^{(n)}$$

$$r_{x_t+0.25}^{r,(5:10)} = \alpha_n + b_n CP_t^r + \epsilon_t^{(n)}$$

Tables 1.5 (US) and 1.6 (UK) report point estimates, t-statistics, and $R^2$’s computed via a 2-stage GMM approach that corrects for generated regressors. For the both countries real and nominal loadings are statistically significant at the 1% level. The magnitude of the nominal loadings are close to the real loadings. In the U.S ranging from 0.71 (5-years) to 1.22 (10-years) for nominal expected returns, and from 0.81 (5-years) to 1.15 (10-years) for real expected returns. In terms of relative predictability, nominal U.S

\(^7\) I do not report this result to save space but refer the reader to table 8 of the NBER version of Cochrane and Piazzesi (2005a) for details.
bonds appear marginally more predictable than real bonds, with $R^2$’s of $\sim 16\%$ compared to $\sim 9\%$ for real bonds. In the U.K this result flips, where real bonds appears more than twice as predictable, with nominal $R^2$’s from 4% to 10% compared to real $R^2$’s from 19% to 21%.

The results of this section suggest a large proportion of the variation in nominal bond risk premia is common to the real term structure, and that the dominant source of bond predictability should be interpreted in the context of the real stochastic discount factor, as opposed to the dynamics of inflation.

[ insert tables 1.5 and 1.6 about here]

F. The Cross-section of Yields

Variations in nominal yields can be decomposed as variations in the real term structure, expected inflation plus inflation risk premia. The previous section argued the existence of a common risk premium component across real and nominal markets. In this section I try to learn about the joint dynamics of real and nominal markets by decomposing term structures into a component explained by conditional first moments of economic growth and an orthogonal unexplained component.

Litterman and Scheinkman (1991) decompose term structures into level, slope and curvature factors via a decomposition of the covariance matrix of yields. For the sample period 2000.1 – 2011.12 I perform this decomposition for real and nominal yields including maturities $n = 3, \ldots, 10$. Table 1.7 reports the results for U.K and U.S yield curves. As expected, one finds a level factor explains the vast majority of variation: $\sim 97\%$ for U.S real and nominal curves, and $\sim 99\%$ for U.K real and nominal curves. Generally, the variance explained by the level factor would be lower, between 85% – 90%, but here I discard maturities less than 3-years which contribute orthogonal variation. A slope factor is also present in both term structures, loading negatively on maturities 3, 4, 5, 6, positively on maturities 7, 8, 9, 10; explains around 2% of variation in the U.S and 1/2 % of variation in the U.K. The key question I want to address, however, is whether nominal term structure factors are explained by real term structure factors.

I regress nominal PCs on corresponding real PCs:

\[
\begin{align*}
\text{Level}^{\%}_t &= \alpha_l + \beta_l \text{Level}^r_t + \varepsilon^l_t \\
\text{Slope}^8_t &= \alpha_s + \beta_s \text{Slope}^r_t + \varepsilon^s_t \\
\text{Curvature}^3_t &= \alpha_c + \beta_c \text{Curvature}^r_t + \varepsilon^c_t
\end{align*}
\]

The final row reports the $R^2$ from these regressions. Considering level projections I find 76% of the variance of U.S nominal level factor is explained by the real level factor, and 60% of the nominal slope factor is explained by the real slope factor. Taken together this implies 75% of the variance of the nominal term structure is explained by the first two principle components of the real term structure. This pattern

---

8Stacking yields of different maturities, $y_t^{(n)}$, one first decomposes the covariance matrix of yields as $Q\Lambda Q^\top$. The diagonal elements of $\Lambda$ contain eigenvalues and columns of $Q$ contain eigenvectors. Ordering the eigenvalues from largest to smallest as $\lambda_1, \lambda_2, \lambda_3$ with associated eigenvectors $q_1, q_2, q_3$, the first three PCs are given by $x_{it} = q_i y_t^{(n)}$. 

16
is repeated for the U.K. The explanatory power of nominal factors by real factors can be understood as a latent interpretation for the co-movement between real and nominal forward spreads (figure 1.2).

Macro-finance term structure models attempt to link latent factors to economic fundamentals (see, for example, Diebold, Rudebusch, et al. (2006) or Dewachter, Lyrio, and Maes (2006)). This approach is related to a vast literature that studies the relationship between inflation expectations and the level of the term structure, or the ability of the slope of the term structure to predict future real economic activity. These models have been successful in explaining large persistence across-maturities in terms of highly persistent fundamentals but from an equilibrium perspective cannot explain time-variation in risk compensation. For example, consider a regression of yield levels on survey expectations about growth rates

\[ y_r^{T(3:10)} = \text{const} + \beta_y E_t[g] + \beta_{\pi} E_t[\pi] + \varepsilon_{r, T(3:10)} \]

Tables 1.8 and 1.9 reports point estimates and statistics. For the U.S, expected inflation explains 48% and 35% of nominal yield variation at 5 and 10-years maturities and is statistically significant at the 1% level, while gdp growth rate is insignificant. Obviously, this implies that more than half of the variation in nominal yields is orthogonal to conditional first moments of fundamentals. Projections of real yields show much smaller explanatory power by conditional first moments. Put simply, between 85% and 90% of the variation in real yields is orthogonal to the expected path of the economy. In the U.K the proportion of explained variance is larger, at > 60% for both real and nominal yields, but again a significant component remains unexplained.

The residual in this regression contains substantial information that one can use to learn about the stochastic discount factor. Bikbov and Chernov (2008) extract a similar set of residuals from a no-arbitrage model and link them to monetary and fiscal shocks based on their correlation with a measures of liquidity and measures of government debt. Duffee (2013) finds that nominal yield residuals of this type account for in excess of 60% of all yield variation but is unlikely to be explained by Treasury specific supply and demand effects. I repeat the principle component decomposition on \( \varepsilon_{r, T(3:10)} \) and \( \varepsilon_{\pi, T(3:10)} \) after removing expectations of fundamentals. Associated eigenvector loadings are displayed in figure 1.6. In both countries yield sensitivities to factors are remarkably similar in shape and magnitude, displaying familiar level, slope, and curvature patterns; I denote the first two principle components \( UN_L \) and \( UN_S \).

Figures 1.7 and 1.8 shows the time-series dynamics these factors are highly correlated. Since expected fundamentals are orthogonal to \( UN \) this suggest the presence of a significant common risk premia. In fact, both real and nominal \( UN_L \) and \( UN_S \) are moving in near lockstep and must be common factors to real and nominal term structures. In the U.S the level factor is rising between 1990 – 2000 and declining

G. The Inflation Risk Premium

How large is the inflation risk premium? The preceding sections argued that the dominant source of predictable variation in nominal excess returns belonged to the real stochastic discount factor. This finding is interesting since extant literature largely explains time-variation in nominal excess returns through compensation for inflation risks and not compensation for real consumption risks. However, this also implies that the inflation risk premium should be small. I examine the robustness of our findings by studying the difference between long term yields on nominal versus real bonds after adjusting for expected inflation.

Before the availability of inflation protected securities estimation relied upon one of two methods: (i) combining macroeconomic fundamentals such as price indices or the money supply, with structural models for the real and nominal pricing kernels; or (ii) building hypothetical indexed bond yields from ex-post observations of nominal yields and inflation. For example, using a real business cycle model, Buraschi and Jiltsov (2005) derive a closed form solution linking the inflation risk premium to the money supply and productivity shocks to estimate the inflation risk premium while Campbell and Shiller (1996) use a VAR approach. More recent evidence has utilised the availability of inflation indexed securities. Hörndahl, Tristani, and Vestin (2008) solve and calibrate a general equilibrium model with habit persistence and nominal rigidities while Grishchenko and Huang (2012) use a regression based approach to measure the inflation risk premium. Both authors report positive estimates a few basis points in magnitude.

Computing compensation for inflation shocks requires a proxy for the market’s assessment of average future inflation: $E_t \left[ \frac{1}{T} \sum_{t=1}^{T} \pi_{t+i} \right]$. These tables 1.10 and 1.11 display summary statistics for year-on-year log growth rates recorded at monthly frequencies for a cross-section of consumer price indices for both the U.S and U.K. These table suggest something important. Excluding energy prices, the annual growth rate of inflation over a broad set of price indices has averaged 2.0% in the U.S and 2.5% in the U.K in the last 25 years. Under the assumption that inflation linked Treasuries are good proxies for real rates a back of envelope calculation says that the unconditional inflation risk premium should be small, $\sim 0.24\%$ for 5-year U.S bonds, and $\sim 0.26\%$ for 5-year U.K bonds.

In order to provide a conditional estimate of the inflation risk premium I collect long horizon inflation forecasts from a set of sources. For the U.S I obtain 10-year forecasts for the average rate of consumer price inflation from (i) Survey of Professional Forecasters, (ii) BlueChip Economic Indicators, (iii) Consensus Economics, (iv) the Office for Management and Budget Responsibility; and (v) The Congressional Budget Office. For the U.K I use long horizon forecasts for average growth rates of the retail price index from
Consensus Economics. For each survey source, the dataset represents semi-annually reported inflation for each of the next 5-years plus an average for the following 5-year period.

Figure 1.9 computes the 10-year inflation risk premium implied by the inflation adjusted difference between nominal and real yields. The left panel displays estimates for the U.S across each of the 5 sources for long term inflation expectations, while the right panel displays estimates for the U.K backed out from Consensus Economics. The U.S full-sample inflation risk premium is -14 basis points and statistically different from zero. This estimate is much smaller but arguably consistent with the structural estimate provided by Buraschi and Jiltsov (2005) who estimate a massive 150 basis point inflation risk premium on the 10-year bond during at the end of monetary policy experiment (1982) but a steadily declining premium that reached 40 basis points in 2000, the start of our sample. The time series properties display two notable episodes, around 2002 and late 2008 / 2009 where the inflation risk premium reached −100 basis points. These periods coincide with known deflationary scares. On November 6th 2002 the Fed cut its overnight rate an aggressive 50 basis points to a 40-year low of 1.25% in a bid to stimulate slowing consumer spending. In a speech to the Economic Club of New York City one month later, Alan Greenspan, the Chairman of the Federal Reserve Board, noted that ‘a major objective of the recent heightened level of scrutiny is to ensure that any latent deflationary pressures are appropriately addressed well before they became a problem’. In the aftermath of Lehman brother’s collapse, a second deflation scare was widely reported in the press leading Nouriel Roubini to argue that the current economic state was at risk of ‘stag-deflation’, a recession coinciding with a period of deflation.⁹

In the U.K I find the 10-year inflation risk premium was close to zero between 2001 - 2008. In subsequent years the retail price index fell dramatically while long term inflation expectations remained relatively stable. As a result, breakeven rates dropped below expected inflation leading to an estimated −150 basis point inflation adjusted spread. This did not escape the attention of Bank of England, whose February 2009 Inflation report carried a section discussing likely impact of deflation and warned the retail price index would continue to fall in coming months.¹⁰

H. Liquidity Explanations

Several authors have documented liquidity issues in the U.S TIPS market. D’Amico, Kim, and Wei (2008) argue that the TIPS liquidity premium was as high as 120 basis points in 1999 and trended down to 10 basis points in 2004. The liquidity of the TIPS market complicates measurement of the inflation risk premium since illiquid securities command lower prices hence higher yields masking the true spread between real and nominal yields. One might worry that the observed co-movement between real and nominal

---


¹⁰See page 33 here: [www.bankofengland.co.uk/publications/Documents/inflationreport/ir09feb.pdf](http://www.bankofengland.co.uk/publications/Documents/inflationreport/ir09feb.pdf)
risk premia were in fact due to a co-movement between liquidity risk premia and nominal risk premia that is revealed through inflation protected markets. In this section I examine this alternative and reject it.

I propose a liquidity adjustment to the inflation indexed bonds that uses a regression model along the lines of Grishchenko and Huang (2012) and Pflueger and Viceira (2013). Absent an indexation mis-match, hypothetical real yields and inflation indexed bonds are linked by a premium \( L_y(t,T) \):

\[
y^\text{TIPS}_t = y(t,T) + L_y(t,T).
\]  

(1.2)

This implies that observed break-even rates are complicated by a liquidity distortion

\[
be^L(t,T) = y^S(t,T) - y_t - L^y(t,T) = E_t \left[ \frac{1}{\tau} \sum_{t} \pi_{t+t} \right] + IRP^d_y(t,T) \] 

(1.3)

where \( IRP^d_y(t,T) \) denotes the distorted inflation risk premium on an \( T \) maturity bond. Given an observable proxy for liquidity I can in principle recovering the true inflation risk premium by adding back in the liquidity premium: \( IRP_y(t,T) = IRP^d_y(t,T) + L_y(t,T) \). I employ three independent variables to form this adjustment:

1. **Noise** \(_t\) : the yield curve fitting error, defined as a root mean squared error between market quoted real par yields and those implied by a Nelson-Siegel-Svensson model. The measure is the TIPS counterpart to the noise measure studied in Hu, Pan, and Wang (2010) who argue that hidden liquidity states for the overall financial market can be extracted by the amount of arbitrage opportunities present in U.S Treasuries. Since arbitrage forces tend to smooth out yield curves times of abundant arbitrage capital should be times of low average fitting errors. As such, ‘noise’ should be informative about liquidity conditions in the TIPS market. For the U.S I compute the difference between implied par-yields from the GSW dataset and market traded quotes published on the Federal Reserve board site. For the U.K I obtain an analogous measure estimated by the Macro Analysis division of the Bank of England.

2. **Turn** \(_t\) : I construct a measure of relative transaction volume for inflation linked market for both the U.S and the U.K. The measure is taken to be the ratio of inflation index turnover relative to the total Treasury market turnover and is potentially informative about TIPS market liquidity conditions when agents face search frictions in the sense of Duffie, Gárleanu, and Pedersen (2005). For the U.S this data is available from the Federal Reserve board and in the U.K form the Office of National Statistics.

3. **BidAsk** \(_t\) : The spread between the prices to buy and sell simultaneously market quoted bonds. The spread compensates dealers for inventory risk, order costs, and the cost of trading against informed investors, and is a general measure of the overall liquidity of the market. Bid-Ask spreads are available from Bloomberg.

Figure 1.10 displays time series for U.S liquidity proxies.
Monthly expectations of long term inflation expectations are unavailable from surveys. I employ a Phillips curve VAR with the state vector $x_t = [\text{CPIU}, \text{IP}, \text{EMPL}, \text{UNEM}]$ where I include year-on-year log growth rates in the the industrial production index (IP), the Index of Help Wanted Advertising in the Newspapers (HELP), the civil U.S. employment (EMPL), and unemployment rate (UNEM). For the U.K the state vector includes the Retail Price Index, to which inflation linked Gilts are adjusted, the unemployment rate, and the growth rate in retail sales. For both sets of forecasts I estimate the VAR recursively using a 10-year history of monthly observations preceding date $t$. Given long term inflation rates for 5, 7, and 10-year maturities I run the regression  

$$-1 \times \left( y^T(t,T) - y^{TIPS}(t,T) - E_t \left[ \frac{1}{T} \sum_{i=1}^{\tau} \pi_{t+i} \right] \right) = \alpha + \beta_1 Noise_t + \beta_2 \log \text{Turn}_t + \beta_3 \text{BidAsk}_t + \varepsilon_t.$$  

Assuming the liquidity variables are not explaining variation in the IRP one can think of the constant in this regression having two components, one due to inflation and a one due to liquidity such that $\alpha = \alpha_{LIQ} + \alpha_{IRP}$. Our approach to separate these components is to normalise the fitting liquidity factor such that it’s in sample minimum reaches zero. The resulting time series is the fitted value $L(t,T) = \alpha_{liq} + \hat{\beta}_1 Noise_t + \hat{\beta}_2 \log \text{Turn}_t + \hat{\beta}_3 \text{BidAsk}_t$  

The left panel of figure 1.11 plots the estimated liquidity adjustment for 5, 7, and 10 year maturities. The time-series confirms the findings of Grishchenko and Huang (2012) and Pflueger and Viceira (2013) who both document the existence of a liquidity premium in the early years of our sample (2000 - 2003), followed by a massive liquidity shock in excess of 100 basis points during the 2008/2009 financial crisis. Given this liquidity adjustment for each inflation forecast $i$ I recover an estimate of the ‘true’ inflation risk premium as follows  

$$\text{IRP}^i(t,T) = \text{IRP}^{d,i}(t,T) + L^i(t,T) = b e^L(t,T) - E_t \left[ \frac{1}{T} \sum_{i=1}^{\tau} \pi_{t+i} \right] + L^i(t,T)$$  

The right panel of figure 1.11 plots the liquidity adjusted inflation risk premia. For the U.K I obtain results that are quantitively the same as the U.S counterpart but omit these in order to save space. The adjusted estimates suggest that the findings presented above are unlikely to be driven by liquidity differentials. For the 10-year maturity, the sample average across each of the three inflation forecasts is $-6.5$ basis points and statistically different from zero at the 1% level, consistent with the unadjusted estimates. More importantly, the dynamics of adjusted versus unadjusted inflation risk premia are also consistent with the 2002/2008/2009 deflation scare interpretation clearly visible. The overall size of the inflation risk premium remains small which I take as evidence in favour of a real risk premium story being the dominant driving source for nominal return predictability.\footnote{These results are available on request.}
Alternative Risk Premium Proxies

The previous section showed that $U_L$ and $U_S$ are priced risk factors and that $U_S$ is a factor responsible for time-variation in risk premia. Here, I examine the link between these factors and risk premium proxies that have been suggested by the extant literature. I consider two proxies, the first is the 3-month real and nominal Cochrane and Piazzesi (2005a) factors discussed above. The second is a volatility state filtered by Le and Singleton (2013a) from a no-arbitrage term structure model that has the interpretation of a time-varying quantity of risk factor.

Table 1.12 reports point estimates and t-statistics from a regression of risk premium proxies on $U_L$ and $U_S$. The first and second rows reports regressions $V_t$ with on the left hand side. The sample period using nominal term structure factors is 1990.1–2007.1 and for real term structure factors is 2000.1–2007.1. The remaining rows report projections of adapted Cochrane and Piazzesi (2005a) factors over the common sample period 2000.1–2007.1.

Two results emerge. The first is that $U_S$ is positively related to these risk premium proxies. The second, more surprising observation, is that the nominal risk premium factors are explained equally well by the real $U$’s as they are by the nominal $U$’s. Both regressions show that $V_t$ is positively related to the slope factors with t-statistics that are significant the 1% level, while the level factor loading is close to zero and insignificant. In both sample periods the unspanned slope explains a large fraction of the variance of $V_t$ with an $R^2$ of 60% and 33%, respectively. In an unreported regression using the nominal factors over the period 2000.1–2007.1 I obtain a near identical result. While this regression uses prices on the left and right, importantly the risk premium factor is extracted from the cross-section of nominal yields yet is explained by a factor living in the space of real yields.

The remaining rows report projection adapted Cochrane and Piazzesi (2005a) factors over the common sample period 2000.1–2007.1. A large proportion of $CP_t^r$ is explained by $U_L^{r,s}$, $U_S^{r,s}$, where all t-statistics are significant at 1%. More importantly, the predictable variation due to nominal factors is almost the same that due to real factors, with $R^2$’s are 67% and 57%, respectively. Second, and symmetrically, both real and nominal slope factors are explaining $CP_t^r$ in a similar fashion, with $R^2$’s are 32% and 37%, respectively.

In terms of equilibrium models, these findings suggest that specifications for the nominal term structure must account for the fact that a dominant component of the dynamics of excess returns must come from a source of risk that belongs to the real term structure. In the next section I discuss possible explanations for what this source of risk might be.
II. Equilibrium Implications

Many asset pricing models are capable of generating positive nominal Sharpe ratios through compensation for inflation risks. It is not so easy to generate upward sloping real curves. Backus, Gregory, and Zin (1989) make this point clear within the context of a monetary Mehra and Prescott (1985) economy with CRRA preferences. These authors argue to generate real Sharpe ratios as large as those observed in the U.S Treasury market risk aversion should be around 10. More troubling, however, the real term structure is upward sloping only if consumption growth is negatively autocorrelated. Since quarterly consumption growth has small positive autocorrelation the average risk premium for inflation protected markets should be negative. I review briefly three alternative economies that introduce time-varying quantities of risk or time-varying prices of risk, and study which has better potential to explain the joint behaviour of real and nominal interest rates.

A. Long Run Risk

Long run risk economies introduce time-varying quantities of risk in combination with non-separable preferences. In power utility models risk aversion ($\gamma$) and elasticity of intertemporal substitution ($1/\psi$) are tightly restricted: they are the reciprocal of each other. This implies that investors who are averse to consumption volatility across states must also be averse to consumption volatility over time. It is not clear why this should be the case. Epstein and Zin (1989) and Weil (1989) develop generalised expected utility building on the Kreps and Porteus (1978) preferences, allowing for separation of these concepts. Epstein-Zin (EZ) preferences define utility recursively by

$$U_t = \left\{ (1 - \delta) c_t^{1-\gamma} \theta + \delta (E_t[U_{t+1}^{1-\gamma}])^{1/\theta} \right\}^{\theta/(\theta-1)}$$

where $0 < \delta < 1$ determines the rate of time preference, and $\theta \equiv \frac{1-\gamma}{1-1/\psi}$. The special case $\gamma = 1/\psi$ corresponds to time-separable power utility.

The representative maximises lifetime utility subject to the standard budget constraint $W_{t+1} = R_{w,t+1} (W_t - C_t)$, where $W_t$ is the representative agent wealth that includes human capital, and $R_{w,t+1}$ is the return on the wealth portfolio. The real stochastic discount factor implied by this formulation is:

$$m_{t+1} = \log(M_{t+1}) = \theta \log(\delta) - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1) r_{w,t+1},$$

where $r_{w,t+1}$ is the real log return on the wealth portfolio, and $\Delta c_{t+1}$ is log consumption growth.

One important pricing implication of EZ preferences is that state variables that affect consumption dynamics will be priced through the the return on the wealth portfolio $r_{w,t+1}$. Bansal and Yaron (2004)
consider an economy with forecastable consumption growth subject to time-varying conditional volatility:

\[ \Delta c_{t+1} = \mu_c + g_t + \sigma_c \epsilon_{c,t+1}, \]
\[ g_{t+1} = \rho g_t + \sigma_g \epsilon_{g,t+1} \]
\[ \sigma^2_{t+1} = \sigma^2 - \nu (\sigma^2 - \sigma^2_t) + \sigma_w \epsilon_{w,t+1} \]

where all shocks are i.i.d orthogonal. Ignoring a convexity term the risk premium on real bonds is given by

\[ E_t[hprx(n)_{t,t+1}] = \beta_{n,c} \lambda_c \sigma^2_c + \beta_{n,g} \lambda_g \sigma^2_g + \beta_{n,w} \lambda_w \sigma^2_w \]

where the betas are determined endogenously from the joint dynamics of the SDF and returns. The market price of consumption shocks is \( \gamma \) as in power utility. If the covariance between consumption shocks and expected consumption shocks is positive (negative) then real bonds are a hedge (risky). The market price of consumption growth risk is positive when agents have a preference for the early resolution of uncertainty, which is the case when \( \gamma > 1/\psi \). The market price of volatility risk is negative with preference for early resolution.

Through time-varying quantities, long run risk economies also generate implications for return predictability. Recently, Bansal and Shaliastovich (2013) extend the benchmark economy to study nominal yields in the presence of time-varying inflation uncertainty. In equilibrium, both real and nominal yields are increasing in economic uncertainty. The implications for consumption risk is identical to Bansal and Yaron (2004). However, due to a non-neutrality assumption inflation risk is priced on the real curve, and since it signals lower real uncertainty in the future its sign is the opposite to consumption risk.

**Implications:** Long run risk and economic uncertainty have the effect of decreasing (increasing) average real bond risk premia when agents have an early (late) preference for the resolution of uncertainty. This is because the beta on consumption growth is negative and the beta on volatility shocks is positive. Therefore, the shape of the real yield curve is revealing about the preference the resolution of uncertainty, one of the key ideas motivating recursive utility. Moreover, when agents have a preference for early (late) resolution of uncertainty, real bond risk premia will be decreasing (increasing) in the conditional variance of expected consumption growth. If inflation is non-neutral as in Bansal and Shaliastovich (2013), the effect of inflation volatility is opposite to real volatility.

**B. Habit**

Habit economies introduce variation in risk premia through an alternative channel: time-varying prices of risk. When investors form habits, current utility depends not only on current consumption, but also on a reference point that depends on past consumption. The key insight to habit formation is that marginal utility is volatile even when consumption is smooth, because utility is derived from consumption relative to some ‘standard of living’, and prices of risk vary counter-cyclically.
Habit models come in ratio or difference form. In addition, habit formation can be *internal*, in which consumption choices factors effects on future habit, or *external* where the habit has no affect on consumption policy and in this sense is a true ‘externality’. For ease of exposition and since ratio models imply constant risk premia I focus on external difference models as in Campbell and Cochrane (1999) and Wachter (2006).

The representative agent maximises the following objective

$$E_t \sum_{i=0}^{\infty} \delta^{i} \frac{(C_{t+i} - H_{t+i})^{1-\gamma} - 1}{1-\gamma}$$

where $\gamma$ denotes the utility curvature parameter and $H_t$ is the habit level. Consumption growth is assumed to follow a random walk

$$\delta c_{t+1} = g + \varepsilon_{c,t+1}, \quad \varepsilon_{c,t+1} \sim N(0, \sigma_c^2)$$

Current utility depends on past consumption through an autoregressive process for the surplus ratio $S_t = \frac{C_t}{C_t}$. To ensure utility is well defined most models follow Campbell-Cochrane by specifying an AR(1) process for the log surplus ratio ($s_t = \log S_t$)

$$s_{t+1} = (1-\phi)\bar{s} + \phi s_t + \lambda(s_t)\varepsilon_{c,t+1}$$

where $\phi$ is the persistence of habit, and $\lambda(s_t)$ is the sensitivity function that controls the response of habit to consumption innovations. The local coefficient of relative risk aversion is time-varying given by $\gamma^*(t) = -C_t UCC/U = \gamma/S_t$. Risk aversion rises as $S_t$ declines, that is, bad states occur after negative consumption shocks as consumption is pushed towards the standard of living. In habit models countercyclical risk aversion is generated endogenously. The stochastic discount factor is given by

$$M_{t+1} = \delta \left( \frac{S_{t+1}C_{t+1}}{S_t C_t} \right)$$

Taking logs and using the dynamics of consumption and surplus the riskfree real rate is given by

$$r_t = -\log(\delta) + \gamma g - \gamma(1-\phi)(s_t - \bar{s}) - \frac{\gamma^2 \sigma_c^2}{2} [\lambda(s_t) + 1]$$

A drop in consumption surplus has two effects on the real risk free rate: (i) marginal utility is high in bad times, when surplus is low, but since surplus is mean-reverting so is marginal utility. Consumers anticipate marginal utility will fall in the future and would like to smooth intertemporally, borrowing against the future driving up the risk free interest rate; and (ii) Precautionary savings lowers borrowing costs since surplus and its conditional volatility are negatively correlated. As uncertainty rises, investors are more willing to save and this drives down the risk free rate.

Habit models introduce additional flexibility to the risk free rate which is useful in matching the ob-
served stability of the risk free rate. For example, Campbell and Cochrane (1999) choose the sensitivity function so that intertemporal substitution and precautionary savings effects offset exactly so the riskless rate is constant. Indeed, Backus, Chernov, and Zin (2011) argue that a key ingredient in matching discount factor entropy bounds is the ability to offset variation entering short rates. When short rates are constant so are bond risk premia. Building on the Campbell-Cochrane economy, Wachter (2006) generalises the offsetting effect of intertemporal smoothing versus precautionary savings so that the risk free rate can be written as

\[ r_t = \text{const} - b(s_t - \bar{s}) \]

The free parameter \( b \) has an economic interpretation. When \( b < 0 \), intertemporal smoothing dominates and interest rates are low in high marginal utility states, and when \( b > 0 \) precautionary savings dominates and interest rates are high in high marginal utility states. Bond prices in Wachter (2006) are unavailable in closed form but up to a second order Taylor expansion bond risk premia can be written as

\[ E_t[hprx^{(n)}_{t,t+1}] \sim \delta(1 + r_t) \frac{\gamma^*(t)}{C_t} \text{cov}_t[(C_{t+1} - X_{t+1}), r^{(n)}_{t+1}] \]

**Implications:** The real bond risk premium depends on the covariance between bond returns and surplus consumption levels. If the covariance are positive (negative) they pay off in good (bad) times and are risky (insurance). The average slope of the term structure depends crucially on whether real rates are pro-cyclical or not. In particular, if real rates are low in bad times then bond returns will be high and covary negatively with surplus; thus, the real term structure should be downward sloping. Furthermore, a decrease (increase) in consumption, given habits, will decrease (increase) the surplus ratio and increase (decrease) expected excess returns.

### C. Heterogeneous Agents

Mehra and Prescott (1985) assume the existence of a representative agent who consumes aggregate consumption. Since aggregate consumption is smooth, so is marginal utility. Habit and long run risk models tackle this problem by introducing time-non-separable preferences. An alternative possibility is that aggregate consumption is a poor proxy for individual consumption. Indeed, using panel data Zeldes (1989) and Parker and Preston (2002) show that the consumption of stockholders is more volatile, and more more correlated with market returns, than non-stockholders. Attanasio and Low (2004) report that per capita volatility of consumption is up to twelve times larger than aggregate consumption.

Models with heterogeneous agents address the smoothness in aggregate consumption by allowing investors to form individual consumption policies based on their beliefs. The key insight of this literature is that, if agents can trade, the equilibrium SDF is affected by disagreement. Consider two agents, \( a \) and \( b \), each representing its own class with separate (absolutely continuous) subjective probability measures on the data generating process, denoted as \( dP^a_t \) and \( dP^b_t \). Given a filtered probability space the difference in beliefs between the two agents can be conveniently summarized by the Radon-Nikodym derivative.
\[ \eta_t = \frac{dP_t^b}{dP_t^a}, \] so that for any random variable \( X_t \) that is \( F_t \)-measurable,

\[ E^b(X_T|F_t) = E^a(\eta_T X_T|F_t), \quad \text{with } \eta_t = 1. \]  

(1.4)

In the literature, the Radon-Nikodym derivative \( \eta_t \) is either assumed as an exogenous process or obtained as the outcome of an optimal learning problem in which agents have different prior beliefs.\(^{12}\) Independent of its microfoundations, disagreement among agents affects the distribution of consumption in equilibrium, for \( T < \infty \). Agents trade to equate ex-ante expected marginal utility of consumption, \( E^a u'(c_t^a|F_t) = E^b u'(c_t^b|F_t) \). Thus, using (3.16), any frictionless equilibrium requires that \( E^a u'(c_t^a|F_t) = E^a(\eta_T u'(c_T^b|F_t)) \) so that innovations in \( \eta_t \) necessarily imply a different allocation of state-contingent consumption \( c_t^a \) and \( c_t^b \) between the two agents. Optimists will trade to shift consumption to states of the world in which their subjective probabilities are the highest, in exchange for a lower consumption in those states they deem less likely.

Indeed, in equilibrium, agents must have different individual stochastic discount factors, which need to satisfy \( \mathcal{M}_t^a = \eta_t \mathcal{M}_t^b \). Thus, ex-post marginal utilities will not be equal since ex-ante beliefs drive a wedge between agent-specific consumption. In some sense, subjective beliefs generate endogenous risk since they make individual consumption stochastic even when aggregate consumption is smooth and can thus generate long run risk. On the other hand, the pricing measure inherits a time-varying price of risk, so provide provide micro foundations for time-varying risk aversion as in a habit economy.

**Equilibrium Consumption**

The link between equilibrium asset prices and disagreement depends on the aggregation properties of the model. In complete markets, Cuoco and He (1994) show how the competitive equilibrium solution can be obtained from the problem of a central planner.\(^{13}\) A representative agent \( U^* \) can be constructed as a stochastic weighted average \([1, \lambda_t]\) of the marginal utilities of the two agents:

\[ U^*(c_t, \lambda) := \max \left\{ q_t u_a(c_a(t)) + \lambda_t q_t u_b(c_b(t)) \right\} \]

\[ s.t \ (i) \sum_i c_i^t = c_t \forall t \]

Normalising the weight on agent \( a \), a necessary condition for a social optimum is \( u'_a(c_a(t)) = \lambda_t u'_b(c_b) \). From the first order condition of the individual agent problems this implies \( \lambda_t \frac{u_a'(c_a(t))}{u_b'(c_b(t))} = \frac{\alpha_a \mathcal{M}^a(t)}{\alpha_b \mathcal{M}^b(t)} \).

Imposing the aggregate resource constraint \((c_t = c_t^a + c_t^b)\) one obtains individual consumption policies and

\(^{12}\) Scheinkman and Xiong (2003), Buraschi and Jiltsov (2006), Dumas, Kurshesv, and Uppal (2009), and Buraschi and Whelan (2012a), all study economies in which the process \( \eta_t \) arise from investors’ different prior knowledge about the informativeness of signals and the dynamics of unobservable economic variables. See Kurz (1994) for a discussion on the microfoundations of disagreement. Dumas, Kurshesv, and Uppal (2009) label the process \( \eta_t \) ‘sentiment’.

\(^{13}\) Constantinides (1982) extends Negishi (1960)’s results and proves the existence of a representative agent with heterogeneous preferences and endowments but with homogeneous beliefs. In an incomplete market setting with homogeneous agents Cuoco and He (1994) show a representative agent can be constructed from a social welfare function with stochastic weights. Basak (2000) discuss the aggregation properties in economies with heterogeneous beliefs but complete markets. He shows that a representative can be constructed from a stochastic weighted average of individuals marginal utilities.
the stochastic discount factors for the representative

\[ c_t^a = \frac{c_t}{1 + \eta_t^{1/\gamma}} \quad \text{and} \quad c_t^b = c_t \eta_t^{1/\gamma} \left( 1 + \frac{1}{\gamma} \right) \]

\[ M_t^r = \varrho_t c_t^{-\gamma} \mathcal{H} (\eta_t) \]

where \( \mathcal{H} (\eta) = \left( 1 + \eta_t^{1/\gamma} \right)^\gamma \).

**Effective Risk Aversion**

In equilibrium, agents must have different individual stochastic discount factors. This implies an ex-post wedge between marginal utilities which is generated by beliefs about fundamentals, rather than fundamentals themselves, resulting in a ‘distortion’ \( \mathcal{H}(\eta_t) \) to the pricing measure with respect to the standard CRRA kernel. As in a habit economy, the local curvature of the representative investor’s utility function is now time-varying and given by

\[ \gamma^*(t) = -c_t \frac{U_{cc}}{U_c} = \gamma \left( 1 + \frac{c_t^a + c_t^b}{c_t^a} H(\eta)^{-2} \right) \]

which is strictly larger than \( \gamma \). For equal consumption shares this reduces to

\[ \gamma^*(t) = \gamma (1 + 2^{-1+\gamma}) \].

This shows that, contrary to the Mehra and Prescott (1985) puzzle, it is possible for small risk aversion to generate large local curvature at the representative level. The intuition is the following. When agents have low risk aversion, they are very willing to engage in speculative trades based on their beliefs, which generates large endogenous risk to the counterparty whose forecasting model performs poorly ex-post. For very large levels of risk aversion, agents are very unwilling to trade and the economy collapses to the degenerate single agent case. Motivated by this observation, table 1.13 collects discount factors and risk free rates from popular models in the literature, and classifies them as (i) myopic \( (\gamma = 1) \); (ii) risk sharing \( (\gamma > 1) \); (iii) speculative \( (\gamma < 1) \); or (iv) frictions based models.

**Subjective Discount Factors**

The real state price densities for agent \( n = a, b \) will be given by

\[ \frac{dM_t^n}{M_t^n} = -r(t) dt - \kappa_{c,n}(t) d\hat{W}_c^x - \kappa_{x,n}(t) d\hat{W}_x^x \]

where \( \hat{W}_c^x \) are subjective consumption shocks, and \( \hat{W}_x^x \) are shocks from any signals that affect the Radon-Nikodym derivative of agents beliefs. For example, in Buraschi and Jiltsov (2006) agents learn about the growth rate of consumption from publicly available signals. Since the dynamics of disagreement affect equilibrium allocations disagreement on the informativeness of signals can have real economic effects. In
a nominal context, Illeditsch, Gallmeyer, Heyerdahl-Larsen, and Ehling (2009) and Buraschi and Whelan (2012a) derive optimal filters in which shocks to the inflation get priced on the real discount factor because inflation contains information about future real growth. In these economies, disagreement about inflation can have real economic consequences; thus, represents a form of inflation non-neutrality.

The second column of table 1.13 presents a list of discount factors from the literature. The shared property of these models is that belief dynamics break the tight link between risk compensation and second moments of fundamentals. When agents have subjective beliefs, individual consumption is stochastic even if aggregate consumption is i.i.d. Investors consume more (less) in states of where they perceive high aggregate cash flows, at a lower (higher) marginal utility, because they believe these states as more (less) likely. Real prices of risk are time-varying and special cases of:

\[
\kappa_{c,a}(t) = \gamma \sigma_c + \omega_a^b \psi_t^b \\
\kappa_{c,b}(t) = \gamma \sigma_c - \omega_b^a \psi_t^a \\
\kappa_{x,a}(t) = \omega_a^b \psi_t^x \\
\kappa_{x,b}(t) = \omega_b^a \psi_t^x
\]

where \(\psi_t^x\) is disagreement on publicly available information useful for learning. An important insight is that the belief adjustment to risk prices contain two components, one which is backward looking dependent on past trades \((\omega_t^\eta)\) and one which is forward looking \((\psi_t)\) that depending on the future (perceived) investment opportunity set.

Real and nominal short rates are obtained as an application of Ito’s lemma to either stochastic discount factor

\[
r_t = \delta + \gamma \left( \omega_a \hat{g}_t^a + \omega_b \hat{g}_t^b \right) - \frac{1}{2} \gamma (1 + \gamma) \sigma_c^2 + \frac{1}{2} \frac{(\gamma - 1)}{\gamma} \omega_a \omega_b \Psi_t^a \Psi_t^b \\
r_t^* = r_t + \left( \omega_a \hat{\pi}_t^a + \omega_b \hat{\pi}_t^b \right) - \sigma_q^2
\]

where \(\omega_i(\eta_t) = c_i^t/c_t\) is investor’s \(i\) total consumption share, and \(\Psi_t\) is a vector of disagreements that includes consumption growth and any signals used to learn the state of the economy. In the absence of heterogeneous beliefs short term interest rates are given by the Lucas solution. This includes terms reflecting time preference, precautionary savings demand, and a wealth/substitution effects.\(^{14}\) In the heterogeneous case, short rates include two new components. The first is a belief distortion due to speculative trading that took place in the past. This term biases observed short rates towards the belief of the agent who has greater purchasing power since they are wealthier, or because the other agent is constrained. The second component is due to a hedging (speculative) demand. Holding the wealth weighted average belief constant its impact is symmetric and quadratic in disagreement. The magnitude of the effect is largest when the two agents have equal consumption shares \(\omega_a = \omega_b\). The sign of the effect, however, depends on whether \(\gamma\) is greater or smaller than 1. Investors disagree about the correct model for the economy, but choose to trade since they expect to gain a larger consumption share tomorrow. Given that today’s

\(^{14}\) The intuition is the standard one. The larger expected consumption growth, the higher the demand for current consumption, the lower the demand for savings, and the higher the interest rate required to clear the bond market.
consumption is fixed, interest rates must fall when $\gamma < 1$ due to the substitution effect, and must rise when $\gamma > 1$ due to the wealth effect.

**Implications:** A necessary condition for trade is that agents agree on the price of date $t$ securities, which in the absence of arbitrage, must be consistent with a set of positive state prices. Since state prices are probabilities multiplied by marginal utilities, subjective beliefs imply agents choose different levels of consumption. Furthermore, when beliefs are stochastic, so are their consumption streams. In a heterogeneous agent equilibrium, differences in beliefs represent a ‘non-fundamental’ source of priced risk, and thus should be correlated with the cross-sectional and time-series properties of bond markets.

III. Risk Factors

A. Economic Uncertainty

In long run risk economies recursive preferences time-variation in risk compensation arises from the dynamics of the quantity of risk, i.e. heteroskedasticity in the conditional mean of consumption growth and/or inflation. In equilibrium, bond risk premia depends on the beta of yields on uncertainty, and prices of risk attached to consumption and inflation risk. To construct proxies of consumption and inflation uncertainty, I follow a similar approach to Bansal and Shaliastovich (2013) and Le and Singleton (2013a), who propose using survey forecasts as proxies for macroeconomic expectations.

For the U.S I use BlueChip Financial ForecastsForecasts (BCFF) to construct expected growth rates. BCFF is a monthly publication providing extensive panel data on survey data from professional economists working at leading financial institutions and service companies.\textsuperscript{15} Forecasted variables include Treasury yields and economic fundamentals. While the exact timing of the surveys are not published, the survey is usually conducted between the 25\textsuperscript{th} and 27\textsuperscript{th} of the month and mailed to subscribers within the first 5 days of the subsequent month. The resulting dataset represents an extensive and unique dataset to investigate the role of formation of expectations about the macroeconomy.

The horizon of BCFFS forecasts ranges from the end of the current quarter to 5 quarters ahead (6 from January 1997). I obtain a set of constant maturity forecasts (from 1 to 4 quarters ahead) for individual respondents by interpolating linearly between adjacent horizons. Macroeconomic forecasts are expressed as annualized percentage changes between subsequent quarters: I obtain compound growth forecasts by chaining subsequent quarterly forecasts. For instance, suppose that as of April 2000, the 1Q- and 2Q-ahead GDP forecasts of agent $n$ are 5.00 and 6.00, respectively. This means that the agent expects GDP to increase by $(1 + \frac{5.00}{400})$ between April 2000 (the month of the forecast) and June 2000 (the

\textsuperscript{15} The BCFF paper archive was obtained from Wolters Kluwer and entered manually. The digitization process required inputting around 750,000 entries of named forecasts plus quality control checking and was completed in a joint venture with the Federal Reserve Board.
end of current quarter), and by \(1 + \frac{6.00}{360}\) between end of June 2000 (the end of current quarter) and the end of September 2000 (the end of the next quarter). The (annualized) compound growth rate between April 2000 and September 2000 is obtained as \(2 \cdot (1 + \frac{5.00}{360}) \cdot (1 + \frac{6.00}{360})\).

Consensus forecasts for horizons \(h = 1, 2, 3, 4\) are then defined as the cross-sectional median of all respondents at time \(t\). Next, I use an ARMA(1, 1) model to demean 1-year consensus expectations, and fit a GARCH(1, 1) to the residuals. The conditional volatilities implied by the GARCH model then serve as proxies for economic uncertainty.

In the U.K I use Consensus Economics, a comparable survey to BCFF, but which provides forecasts for gdp growth and the retail price index, to which inflation protected GILTS are indexed. The survey provides two forecast horizons, an average for the remaining months of the current year, and an average for the following year. I match short and long term forecasts for each contributor and interpolate linearly as with BCFF. Given an implied constant maturity forecast I compute uncertainty proxies as above. Figure 1.12 plots the resulting time-series estimates for real and inflation volatility for the U.S (left) panel and the U.K (right panel).

\[\text{Figure 1.12 plots the resulting time-series estimates for real and inflation volatility for the U.S (left) panel and the U.K (right panel).}\]

\[\text{[insert figure 1.12 about here]}\]

\[\text{B. Consumption Surplus}\]

In Lucas economies with habit preferences as in Campbell and Cochrane (1999), predictability arises in equilibrium because of an endogenously time-varying price of risk. Shocks to the current endowment affect the wedge between consumption and habit, thus inducing time-variation in the price of risk. The impact of consumption shocks on interest rates depends on the relative importance of the intertemporal consumption smoothing and precautionary savings effects. A negative shock to consumption tends to increase the short term interest rate through the consumption smoothing channel: agents expect surplus consumption to recover, so they borrow more against future consumption to smooth their consumption path. At the same time, the shock tends to reduce interest rates through a precautionary savings channel: the conditional volatility of surplus rises when its level drops, inducing agents to save more. The endogenous process for log surplus follows a complicated non-linear autoregressive process, however, up to an approximation follows an exponentially weighted average of consumption as one would expect from an external habit model.

Following Wachter (2006) I construct a proxy of consumption surplus \(s_t\) as a weighted average of 10 years of monthly consumption growth rates:

\[s_t = \sum_{j=1}^{120} \phi^j \Delta c_{t-j},\]

where for U.S surplus I set \(\phi = 0.95\) to match the the monthly autocorrelation of S&P price/dividend ratio,
and in the U.K \( \phi = 0.83 \) to match the the quarterly autocorrelation of \( \text{FTSE100} \) price/dividend ratio. Consumption data for the U.S is seasonally adjusted, real per-capita consumption of nondurables and services (from the Bureau of Economic Analysis), and for the U.K quarterly real per-capita consumption expenditure (from Global Insights). For the U.K I linearly interpolate to obtain a monthly time-series.

\[ \text{[insert figure 1.13 about here]} \]

C. Differences in Belief

I obtain proxies for heterogeneity about real growth and inflation from BCFF and Consensus Economics surveys. From matched individual forecasts I can proxy the for belief dispersion in a number of ways. For example, the left panel of figure 1.14 plots time-series for the cross-sectional standard deviation, inter-quartile range, and entropy for real GDP forecasts.\(^{16}\) The large co-movement in dispersion measures suggests the underlying distribution is relatively stable and free some outliers. The right panel compares dispersion in gdp (\( \psi_g \)) and inflation (\( \psi_\pi \)) forecasts, computed from cross-sectional mean-absolute-deviations, to an political uncertainty (\( UnC_t \)) factor studied in Baker, Bloom, and Davis (2013).\(^{17}\)

\[ \text{[insert figure 1.14 about here]} \]

Both real and inflation dispersion has a business cycle component, peaking in each of the previous three NBER recessions (1990/1991, 2001, 2007/2009). This is interesting as large disagreement is often reported at this stage of the cycle. Comparing \( UnC_t \) to our measures for belief dispersion I find positive co-movement with both \( \psi_\pi \) (correlation = 0.42) and \( \psi_g \) (correlation = 0.48), which is somewhat surprising given that the weight assigned to forecaster disagreement about inflation in this index is just 1/6. The remaining components of the index are 1/2 a broad-based news index, 1/6 a tax expiration index and 1/6 a government purchases disagreement measure. Taken together these measures suggest the existence of a common component in the formation of expectations, which is important since systematic variation is required for priced return predictability. Moreover, macroeconomic disagreement is correlated with a broad measure of political uncertainty which could be important from a policy perspective, depending on the real economic effects of investor heterogeneity.

\[ \text{[insert figure 1.15 about here]} \]

Figure 1.15 plots disagreement series for U.K gdp and retail price inflation computed from the mean-absolute deviation in the cross-section. The U.K has witnessed two recessions in the last 20 years, in 1990/1991 and 2008/2009. Repeating the pattern found in the U.S, disagreement peaks during these periods lending additional support to a link between heterogeneity and bad times. A final comment is worth mentioning. Figure 1.15 plots the dispersion in inflation forecasts standardised by the consensus (median) estimate, which shows that between late 2008 and 2010 disagreement about the future path of

\(^{16}\)Entropy is estimated as \( p \times \log(p) \) from a histogram fitted to the cross-sectional belief distribution.

\(^{17}\)The economic uncertainty proxy plotted here is available for down from \url{www.policyuncertainty.com}
inflation was nearly 10 times its historical average. This implies that even though the level of inflation was at a 20-year low, there were extreme views in the survey, some people expected rapid inflation while others expected rapid deflation. If investors truly risk share based on their beliefs then this is a time when risk premia is exchanged through insurance contracts.

IV. Understanding Equilibrium Risk Compensation

Section III proposed proxies for the state variables associated time-varying prices of risk, quantities of risk, and subjective beliefs. Using the common factors extracted from real yields I try to learn about risk premium dynamics via running regression of unexplained yield portfolios on risk factors. For the residual slope factor I run

\[ UN^S = const + \beta_1 vol_t[g] + \beta_2 vol_t[\pi] + \gamma surplus_t + \phi_1 DiB_t(g) + \phi_2 DiB_t(\pi) + \epsilon_t \]

where \( vol_t[g] \) and \( vol_t[\pi] \) are expected gdp and inflation conditional volatilities, \( surplus_t \) is consumption surplus ratio, and \( DiB_t(g) \) and \( DiB_t(\pi) \) are the cross-sectional dispersion in gdp and inflation forecasts.

Tables 1.14 and 1.15 report results grouped by asset pricing model. For each model, I consider the slope factor studied above since this account for observable variation in bond risk premia. Motivated by the large correlation between real and nominal factors in the sample period 2001.1 – 2007.1 but mindful of the statistical implications over short sample periods I run regression using \( UN^S \) on the left for the sample 1990.1 – 2007.1

First, considering long run risk explanations I find the volatility of growth is positively related to the slope factor, while the volatility of inflation is negatively related. The left and right hand sides in this regression are standardised so the point estimates imply a 1 standard deviation shock to \( vol_t[g] \) raises the left hand side 61 basis points. The point estimate on \( vol_t[g] \) is 0.61 (\( t = 4.25 \)) and \( vol_t[\pi] \) is −0.19 (\( t = −2.71 \)) so that the economic significance The \( R^2 \) is 32%, explaining a large fraction of variation in the slope factor which meaning that the volatility of growth drives up returns on long term bonds. I note that the signs on our volatility proxies are opposite to the signs obtained by Bansal and Shaliastovich (2013). However, these are the signs one should expect if the real term structure is risky (upward sloping) and compensation for uncertainty in long run risks is primarily due to the real discount factor. On the other hand, this presents a challenge to the benchmark long run risk model because, as discussed above, if real bonds command a positive risk premium this implies agents with recursive preferences have a late preference for the resolution of uncertainty. Unfortunately, with late preference for the resolution of uncertainty long run risk models cannot explain at the same time the equity risk premium.

Second, considering habit models, \( surplus_t \) loads negatively and is statistically significant, with a t-stat equal to −4.24. A one standard deviation negative shock to surplus lowers the slope factor yields
46 basis points, consistent with the equilibrium interpretation that low surplus is a bad state of the world, in bad states real interest rates are low; thus, hedge marginally utility. The $R^2$ in this regression is 21%. It is encouraging to find a clear link between a consumption based risk factor and time-variation in bond risk premia. However, given the obviously pro-cyclicality of the short term real risk free rate (see figure 1.1) and estimated sign on consumption surplus, this implies benchmark external habit models are unable to explain an upward sloping real term structure.

Third, considering heterogeneous belief models, belief proxies explain 9% of the slope factor variation, which is driven entirely by real disagreement which loads positively with t-stats equal to 3.13. The signs on real disagreement are consistent with an economy where $\gamma < 1$, in which case wealth effects dominates causing agents speculate on their beliefs, increasing endogenous risk and driving up the volatility of long term bonds. In the case of equity markets this channel is studied in detail by David (2008) and in the case of bond markets by Buraschi and Whelan (2012a).

The bottom two rows presents a horse race of risk factors. I note that point estimates on $vol_t[\sigma] / vol_t[\pi]$ and $\text{surplus}_t$ have the same sign are individually significance at the 1% level. However, the explanatory power of the belief variables is subsumed by the alternative risk factor proxies. Interestingly, combining models explain 41% of slope variation, suggesting a potential role for both habit and long run risk economies.

[insert tables 1.14 and 1.15 about here ]

V. Conclusion

This chapter studied the co-movement of risk premia across real and nominal Treasury bond markets. I documented that factors explaining time variation in expected excess returns of nominal bonds are also common to the real term structure, and that this finding is robust to different methods for measuring the dynamics of risk premia. This suggests that the dominant source of bond predictability arises from the real term structure. I build on this finding and study the implications for long-run risk, habit, and heterogeneous agent models, and argue the likely explanation is due to time-varying quantities of risk.
VI. Appendix: Figures

Figure 1.1. Real and Nominal 3 month Yields.
Figure displays time series of real and nominal 3-month risk free rates. We proxy for ex-ante real rates following Campbell and Shiller (1996) using a VAR that includes the ex-post real return on a 3-month nominal bills, the nominal bill yield, and lagged annual inflation rate. Solving the VAR forward we build ex-ante forecasts for date $t$ measurable 3-month real rates. The left panel displays estimates for the U.S while the right panel displays estimates for the U.K.

Figure 1.2. Real and Nominal Slopes.
Figure shows yields curve slopes between 10-years and {5, 6, 7, 8, 9}-years for TIPS (left panel) and nominal Treasuries (right panel).
Figure 1.3. Real and Nominal Excess Returns:
Figure displays realised excess returns computed from 3-month holding periods returns in excess of the 3-month risk free interest rate. The time-series records returns at the date they are realised. Black lines represent 5-year real bonds while dark red lines represent 5-year nominal bonds.
Figure 1.4. Real and Nominal Forecasting Factor Loadings:

Figure displays the $\gamma$'s in the following regression

$$\frac{1}{3} \sum_{[5\;7\;10]} r_{x,t,t+0.25}^{(n)} = \bar{\alpha} + \gamma' \left( f_{t,7}^{[5\;7\;10]} - y_{t}^{(0.25)} \right) + \epsilon_{t,t+0.25}$$

where returns and forward rates are either real or nominal. Each panel displays the factor loadings used to build 3-month real and nominal Cochrane and Piazzesi (2005a) forecasting factors. The left panel displays loadings for the U.K and the right panel loadings for the U.S. The top panels display nominal loadings and the bottom panels display real loadings.
Figure 1.5. Real and Nominal Forward Rate Implied Risk Premia:
In a first stage the average 3-month excess returns across for maturities 5 – 10 are projected on date \( t \) forward rates. The fitted value from this regression is a linear combination forward rates that predicts returns of all maturities. Red lines show the implied nominal bond risk premium \((CP^N_t)\) and black lines the real bond risk premium \((CP^R_t)\).

Figure 1.6. Principle Component Loadings of Residual Yield Variation:
Figure displays loadings from a principle component decomposition of the residuals in a regression of yields on macroeconomic expectations. The left panel reports U.S loadings and the right panel reports U.K loadings. Solid lines report loadings from nominal yields and dashed lines loadings from real yields.
Figure 1.7. Unspanned Principle Components: US
Figure displays the first two principle components of the component of yields which is orthogonal to macro economic expectations. In a first stage we projection yields onto survey expectations of gdp growth and inflation. In a second stage we take principle components of the residuals. The left panel displays level factors while the right panel displays slope factors.

Figure 1.8. Unspanned Principle Components: UK
Figure displays the first two principle components of the component of yields which is orthogonal to macro economic expectations. In a first stage we projection yields onto survey expectations of gdp growth and inflation. In a second stage we take principle components of the residuals. The left panel displays level factors while the right panel displays slope factors.
Figure 1.9. 10-Year Inflation Risk Premium:
Figure displays the 10-year inflation risk premium computed as $IRP(t,T) = \hat{y}(t,T) - \hat{y}(t,T) - E_t \left[ \frac{1}{\tau} \sum_{i} \pi_{t+i} \right]$. Average expected inflation rates are obtained from long horizon surveys from various sources. Left panel shows U.S estimates and the right panel U.K estimates.

Figure 1.10. Liquidity Variables:
Figure displays time series of U.S liquidity variables used to build liquidity adjusted inflation risk premium estimates. $Noise_t$ is the yield curve fitting error, defined as a root mean squared error between market quoted par yields and those implied by a Nelson-Siegel-Svensson model. $Turn_t$ is log ratio of inflation linked transaction volume to nominal transaction volumes. $BidAsk_t$ is the spread between the prices to buy and sell simultaneously market quoted TIPS.
Figure 1.11. Liquidity Factor and Liquidity Adjusted Inflation Risk Premia
The left panel plots the estimated an liquidity adjustment factor [5, 7, 10] year maturity bonds. The right panel displays liquidity adjusted inflation risk premia for [5, 7, 10] year maturity bonds.

Figure 1.12. Long Run Risk Factors
Figure displays the conditional volatility of expected gdp growth and inflation. First we fit a time-series model to survey implied 1-year growth rates. Next we fit a GARCH(1,1) model to the residuals. Blue lines plot the volatility of expected gdp and green lies the volatility of expected inflation. Left panel displays estimates for the U.S and the right panel estimates for the U.K.
Figure 1.13. Consumption Surplus Ratios
Figure plots consumption surplus factors for the U.S (left panel) and the U.K (right panel). Factors are constructed using a weighted average of 10 years past monthly log consumption growth rates.

Figure 1.14. Belief Dispersion: US
Left panel plots cross-sectional standard deviation, inter-quartile range, and entropy in real growth forecasts from BCFF. The right panel plots the mean-absolute deviation in real growth and inflation forecasts, and a political uncertainty factor.
Figure 1.15. Belief Dispersion: UK
Left panel plots the Mean-absolute deviation in real growth and inflation forecasts from Consensus Economics. The right panel plots mean-absolute deviation in the cross-section standardised by the consensus estimate.
VII. Appendix: Tables

Table 1.1. Summary Statistics: US Treasury Curves
The top panel reports statistics for U.S nominal zero coupon bonds, and the bottom panel B reports statistics for U.S TIPS curves. Both term structures contain 144 observations for each maturity. Interest rates are obtained from Gürkaynak, Sack, and Wright (2006, 2010).

<table>
<thead>
<tr>
<th>Maturity</th>
<th>5 Year</th>
<th>6 Year</th>
<th>7 Year</th>
<th>8 Year</th>
<th>9 Year</th>
<th>10 Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.0357</td>
<td>0.0379</td>
<td>0.0399</td>
<td>0.0417</td>
<td>0.0433</td>
<td>0.0447</td>
</tr>
<tr>
<td>Std Dev</td>
<td>0.0133</td>
<td>0.0121</td>
<td>0.0111</td>
<td>0.0103</td>
<td>0.0096</td>
<td>0.0090</td>
</tr>
<tr>
<td>Min</td>
<td>0.0095</td>
<td>0.0120</td>
<td>0.0144</td>
<td>0.0167</td>
<td>0.0187</td>
<td>0.0206</td>
</tr>
<tr>
<td>Max</td>
<td>0.0663</td>
<td>0.0664</td>
<td>0.0665</td>
<td>0.0667</td>
<td>0.0668</td>
<td>0.0670</td>
</tr>
<tr>
<td>Skew</td>
<td>0.1424</td>
<td>0.1098</td>
<td>0.0801</td>
<td>0.0494</td>
<td>0.0163</td>
<td>−0.0191</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>2.4908</td>
<td>2.6886</td>
<td>2.8810</td>
<td>3.0539</td>
<td>3.1982</td>
<td>3.3097</td>
</tr>
<tr>
<td>1st Lag Auto</td>
<td>0.9459</td>
<td>0.9373</td>
<td>0.9284</td>
<td>0.9193</td>
<td>0.9105</td>
<td>0.9022</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Maturity</th>
<th>5 Year</th>
<th>6 Year</th>
<th>7 Year</th>
<th>8 Year</th>
<th>9 Year</th>
<th>10 Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.0172</td>
<td>0.0186</td>
<td>0.0197</td>
<td>0.0206</td>
<td>0.0214</td>
<td>0.0221</td>
</tr>
<tr>
<td>Std Dev</td>
<td>0.0122</td>
<td>0.0115</td>
<td>0.0109</td>
<td>0.0103</td>
<td>0.0098</td>
<td>0.0093</td>
</tr>
<tr>
<td>Min</td>
<td>−0.0084</td>
<td>−0.0065</td>
<td>−0.0045</td>
<td>−0.0027</td>
<td>−0.0010</td>
<td>0.0006</td>
</tr>
<tr>
<td>Max</td>
<td>0.0428</td>
<td>0.0430</td>
<td>0.0431</td>
<td>0.0430</td>
<td>0.0430</td>
<td>0.0429</td>
</tr>
<tr>
<td>Skew</td>
<td>0.1466</td>
<td>0.1561</td>
<td>0.1789</td>
<td>0.2079</td>
<td>0.2398</td>
<td>0.2729</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>2.5091</td>
<td>2.5714</td>
<td>2.6320</td>
<td>2.6883</td>
<td>2.7402</td>
<td>2.7877</td>
</tr>
<tr>
<td>1st Lag Auto</td>
<td>0.9376</td>
<td>0.9372</td>
<td>0.9362</td>
<td>0.9349</td>
<td>0.9335</td>
<td>0.9320</td>
</tr>
</tbody>
</table>
Table 1.2. Summary Statistics: UK Treasury Curves
The top panel reports statistics for U.K nominal zero coupon bonds, and the bottom panel B reports statistics for U.K inflation protected curves. Both term structures contain 285 observations for each maturity. Interest rates are obtained from the Bank of England website.

<table>
<thead>
<tr>
<th>Maturity</th>
<th>5 Year</th>
<th>6 Year</th>
<th>7 Year</th>
<th>8 Year</th>
<th>9 Year</th>
<th>10 Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.0545</td>
<td>0.0553</td>
<td>0.0559</td>
<td>0.0564</td>
<td>0.0568</td>
<td>0.0572</td>
</tr>
<tr>
<td>Std Dev</td>
<td>0.0269</td>
<td>0.0262</td>
<td>0.0255</td>
<td>0.0248</td>
<td>0.0242</td>
<td>0.0237</td>
</tr>
<tr>
<td>Min</td>
<td>0.0045</td>
<td>0.0068</td>
<td>0.0091</td>
<td>0.0112</td>
<td>0.0133</td>
<td>0.0152</td>
</tr>
<tr>
<td>Max</td>
<td>0.1293</td>
<td>0.1283</td>
<td>0.1273</td>
<td>0.1263</td>
<td>0.1251</td>
<td>0.1237</td>
</tr>
<tr>
<td>Skew</td>
<td>0.3171</td>
<td>0.3858</td>
<td>0.4533</td>
<td>0.5142</td>
<td>0.5656</td>
<td>0.6067</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>2.8269</td>
<td>2.8098</td>
<td>2.7937</td>
<td>2.7718</td>
<td>2.7402</td>
<td>2.6973</td>
</tr>
<tr>
<td>1st Lag Auto</td>
<td>0.9840</td>
<td>0.9844</td>
<td>0.9846</td>
<td>0.9848</td>
<td>0.9850</td>
<td>0.9853</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Maturity</th>
<th>5 Year</th>
<th>6 Year</th>
<th>7 Year</th>
<th>8 Year</th>
<th>9 Year</th>
<th>10 Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.0209</td>
<td>0.0212</td>
<td>0.0216</td>
<td>0.0218</td>
<td>0.0221</td>
<td>0.0223</td>
</tr>
<tr>
<td>Std Dev</td>
<td>0.0158</td>
<td>0.0152</td>
<td>0.0147</td>
<td>0.0144</td>
<td>0.0141</td>
<td>0.0139</td>
</tr>
<tr>
<td>Min</td>
<td>-0.0237</td>
<td>-0.0214</td>
<td>-0.0193</td>
<td>-0.0173</td>
<td>-0.0155</td>
<td>-0.0139</td>
</tr>
<tr>
<td>Max</td>
<td>0.0480</td>
<td>0.0487</td>
<td>0.0491</td>
<td>0.0494</td>
<td>0.0495</td>
<td>0.0495</td>
</tr>
<tr>
<td>Skew</td>
<td>-0.9632</td>
<td>-0.8545</td>
<td>-0.7386</td>
<td>-0.6248</td>
<td>-0.5175</td>
<td>-0.4192</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>3.2350</td>
<td>3.1433</td>
<td>3.0226</td>
<td>2.8930</td>
<td>2.7651</td>
<td>2.6442</td>
</tr>
<tr>
<td>1st Lag Auto</td>
<td>0.9761</td>
<td>0.9776</td>
<td>0.9789</td>
<td>0.9802</td>
<td>0.9813</td>
<td>0.9823</td>
</tr>
</tbody>
</table>
Table 1.3. U.S Campbell-Schiller Projections
Table replicates the Campbell and Shiller (1991a) projection coefficients from the following regression
\[ y_{t+n-m}^{(n)} - y_t^{(n)} = \text{const} + \beta_{nm} \left( \frac{m}{n-m} \right) (y_t^{(n)} - y_t^{(m)}) + \text{error}_{t+m} \]  
(1.5)
for \( n = \{60, 72, 84, 96, 108, 120\} \)-months and \( m = 3 \)-months.

<table>
<thead>
<tr>
<th>Maturity</th>
<th>Nominal</th>
<th>Real</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \beta_{nm} )</td>
<td>( \beta_{nm} )</td>
</tr>
<tr>
<td>60</td>
<td>-0.17</td>
<td>-0.20</td>
</tr>
<tr>
<td>72</td>
<td>-0.20</td>
<td>-0.19</td>
</tr>
<tr>
<td>84</td>
<td>-0.23</td>
<td>-0.19</td>
</tr>
<tr>
<td>96</td>
<td>-0.26</td>
<td>-0.18</td>
</tr>
<tr>
<td>108</td>
<td>-0.28</td>
<td>-0.18</td>
</tr>
<tr>
<td>120</td>
<td>-0.29</td>
<td>-0.18</td>
</tr>
<tr>
<td></td>
<td>t-stat</td>
<td>t-stat</td>
</tr>
<tr>
<td>60</td>
<td>(-2.41)</td>
<td>(-2.49)</td>
</tr>
<tr>
<td>72</td>
<td>(-2.50)</td>
<td>(-2.44)</td>
</tr>
<tr>
<td>84</td>
<td>(-2.64)</td>
<td>(-2.43)</td>
</tr>
<tr>
<td>96</td>
<td>(-2.81)</td>
<td>(-2.45)</td>
</tr>
<tr>
<td>108</td>
<td>(-2.99)</td>
<td>(-2.49)</td>
</tr>
<tr>
<td>120</td>
<td>(-3.16)</td>
<td>(-2.55)</td>
</tr>
<tr>
<td></td>
<td>( R^2 )</td>
<td>( R^2 )</td>
</tr>
<tr>
<td>60</td>
<td>0.07</td>
<td>0.10</td>
</tr>
<tr>
<td>72</td>
<td>0.08</td>
<td>0.09</td>
</tr>
<tr>
<td>84</td>
<td>0.10</td>
<td>0.09</td>
</tr>
<tr>
<td>96</td>
<td>0.11</td>
<td>0.09</td>
</tr>
<tr>
<td>108</td>
<td>0.12</td>
<td>0.09</td>
</tr>
<tr>
<td>120</td>
<td>0.12</td>
<td>0.09</td>
</tr>
</tbody>
</table>

Table 1.4. U.K Campbell-Schiller Projections
Table replicates the Campbell and Shiller (1991a) projection coefficients from the following regression
\[ y_{t+n-m}^{(n)} - y_t^{(n)} = \text{const} + \beta_{nm} \left( \frac{m}{n-m} \right) (y_t^{(n)} - y_t^{(m)}) + \text{error}_{t+m} \]  
(1.6)
for \( n = \{60, 72, 84, 96, 108, 120\} \)-months and \( m = 3 \)-months.

<table>
<thead>
<tr>
<th>Maturity</th>
<th>Nominal</th>
<th>Real</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \beta_{nm} )</td>
<td>( \beta_{nm} )</td>
</tr>
<tr>
<td>60</td>
<td>-0.38</td>
<td>-0.23</td>
</tr>
<tr>
<td>72</td>
<td>-0.39</td>
<td>-0.22</td>
</tr>
<tr>
<td>84</td>
<td>-0.40</td>
<td>-0.20</td>
</tr>
<tr>
<td>96</td>
<td>-0.40</td>
<td>-0.19</td>
</tr>
<tr>
<td>108</td>
<td>-0.41</td>
<td>-0.18</td>
</tr>
<tr>
<td>120</td>
<td>-0.42</td>
<td>-0.16</td>
</tr>
<tr>
<td></td>
<td>t-stat</td>
<td>t-stat</td>
</tr>
<tr>
<td>60</td>
<td>(-3.70)</td>
<td>(-2.55)</td>
</tr>
<tr>
<td>72</td>
<td>(-3.78)</td>
<td>(-2.41)</td>
</tr>
<tr>
<td>84</td>
<td>(-3.84)</td>
<td>(-2.30)</td>
</tr>
<tr>
<td>96</td>
<td>(-3.89)</td>
<td>(-2.19)</td>
</tr>
<tr>
<td>108</td>
<td>(-3.93)</td>
<td>(-2.10)</td>
</tr>
<tr>
<td>120</td>
<td>(-3.96)</td>
<td>(-2.01)</td>
</tr>
<tr>
<td></td>
<td>( R^2 )</td>
<td>( R^2 )</td>
</tr>
<tr>
<td>60</td>
<td>0.20</td>
<td>0.11</td>
</tr>
<tr>
<td>72</td>
<td>0.22</td>
<td>0.10</td>
</tr>
<tr>
<td>84</td>
<td>0.21</td>
<td>0.09</td>
</tr>
<tr>
<td>96</td>
<td>0.22</td>
<td>0.09</td>
</tr>
<tr>
<td>108</td>
<td>0.21</td>
<td>0.08</td>
</tr>
<tr>
<td>120</td>
<td>0.22</td>
<td>0.07</td>
</tr>
</tbody>
</table>
Table 1.5. Real and Nominal Cochrane-Piazzesi Regressions: US

Table reports regression of 3-month excess returns on n-year real and nominal bonds on a forecasting factor constructed from an affine combination of date t forward rates ($CP_t$):

$$hprx_t^{(n)} = E_t[p_t^{(n-3)}] - r_t^m = const + \beta CP_t + \epsilon_t^{(n)}.$$ 

t-statistics, reported in ( )’s, are corrected for autocorrelation and heteroskedasticity using the Hansen and Hodrick (1983) GMM correction with 3 Newey-West lags. Sample Period: 2000.1 - 2011.12. The left panel reports point estimates from nominal bond returns on nominal $CP_t$ and the right panel reports real bond returns on real $CP_t$.

<table>
<thead>
<tr>
<th></th>
<th>$const$</th>
<th>$CP_t^8$ $\bar{R}^2$</th>
<th>$const$</th>
<th>$CP_t^9$ $\bar{R}^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$hprx_t^{(5)}$</td>
<td>0.06</td>
<td>0.71</td>
<td>0.16</td>
<td>0.02</td>
</tr>
<tr>
<td>(0.16)</td>
<td>(4.47)</td>
<td>(0.03)</td>
<td>(2.44)</td>
<td></td>
</tr>
<tr>
<td>$hprx_t^{(6)}$</td>
<td>0.03</td>
<td>0.85</td>
<td>0.17</td>
<td>0.01</td>
</tr>
<tr>
<td>(0.07)</td>
<td>(4.53)</td>
<td>(0.01)</td>
<td>(2.55)</td>
<td></td>
</tr>
<tr>
<td>$hprx_t^{(7)}$</td>
<td>0.00</td>
<td>0.98</td>
<td>0.17</td>
<td>-0.01</td>
</tr>
<tr>
<td>(0.00)</td>
<td>(4.47)</td>
<td>(-0.01)</td>
<td>(2.63)</td>
<td></td>
</tr>
<tr>
<td>$hprx_t^{(8)}$</td>
<td>-0.02</td>
<td>1.08</td>
<td>0.17</td>
<td>-0.01</td>
</tr>
<tr>
<td>(-0.04)</td>
<td>(4.34)</td>
<td>(-0.02)</td>
<td>(2.69)</td>
<td></td>
</tr>
<tr>
<td>$hprx_t^{(9)}$</td>
<td>-0.03</td>
<td>1.16</td>
<td>0.16</td>
<td>-0.01</td>
</tr>
<tr>
<td>(-0.05)</td>
<td>(4.17)</td>
<td>(-0.01)</td>
<td>(2.74)</td>
<td></td>
</tr>
<tr>
<td>$hprx_t^{(10)}$</td>
<td>-0.03</td>
<td>1.22</td>
<td>0.15</td>
<td>0.00</td>
</tr>
<tr>
<td>(-0.05)</td>
<td>(3.98)</td>
<td>(0.00)</td>
<td>(2.76)</td>
<td></td>
</tr>
</tbody>
</table>
Table 1.6. Real and Nominal Cochrane-Piazzesi Regressions: UK
Table reports regression of 3-month excess returns on n-year real and nominal bonds on a forecasting factor constructed from an affine combination of date $t$ forward rates ($CP_t$):

$$hprx_t^{(n)} = E_t[p_{t+3}^{(n-3)}] - p_t^{(n)} - r^m_t = const + \beta CP_t + \varepsilon_t^{(n)},$$

t-statistics, reported in ( )’s, are corrected for autocorrelation and heteroskedasticity using the Hansen and Hodrick (1983) GMM correction with 3 Newey-West lags. Sample Period: 2000.1 - 2011.12. The left panel reports point estimates from nominal bond returns on nominal $CP^n_t$ and the right panel reports real bond returns on real $CP^r_t$.

<table>
<thead>
<tr>
<th></th>
<th>const</th>
<th>$CP^n_t$</th>
<th>$\overline{R}^2$</th>
<th>const</th>
<th>$CP^r_t$</th>
<th>$\overline{R}^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$hprx_t^{(5)}$</td>
<td>0.18</td>
<td>0.53</td>
<td>0.04</td>
<td>0.03</td>
<td>0.81</td>
<td>0.19</td>
</tr>
<tr>
<td></td>
<td>(0.52)</td>
<td>(2.50)</td>
<td>(0.11)</td>
<td>(2.81)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$hprx_t^{(6)}$</td>
<td>0.12</td>
<td>0.74</td>
<td>0.06</td>
<td>0.00</td>
<td>0.93</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td>(0.31)</td>
<td>(2.92)</td>
<td>(-0.01)</td>
<td>(3.00)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$hprx_t^{(7)}$</td>
<td>0.04</td>
<td>0.94</td>
<td>0.08</td>
<td>-0.02</td>
<td>1.01</td>
<td>0.21</td>
</tr>
<tr>
<td></td>
<td>(0.10)</td>
<td>(3.25)</td>
<td>(-0.06)</td>
<td>(3.14)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$hprx_t^{(8)}$</td>
<td>-0.04</td>
<td>1.12</td>
<td>0.09</td>
<td>-0.02</td>
<td>1.06</td>
<td>0.21</td>
</tr>
<tr>
<td></td>
<td>(-0.09)</td>
<td>(3.46)</td>
<td>(-0.06)</td>
<td>(3.25)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$hprx_t^{(9)}$</td>
<td>-0.12</td>
<td>1.28</td>
<td>0.10</td>
<td>-0.01</td>
<td>1.09</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td>(-0.25)</td>
<td>(3.54)</td>
<td>(-0.02)</td>
<td>(3.34)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$hprx_t^{(10)}$</td>
<td>-0.18</td>
<td>1.39</td>
<td>0.10</td>
<td>0.02</td>
<td>1.10</td>
<td>0.19</td>
</tr>
<tr>
<td></td>
<td>(-0.37)</td>
<td>(3.49)</td>
<td>(0.05)</td>
<td>(3.42)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 1.7. Principle Component Decomposition
Table reports eigenvalue decomposition of the covariance matrix of of yields. The first two rows report the percentages explained by each orthogonal factor for nominal and real yields, respectively. The final row report the percentage explained ($R^2$) from a regression of nominal factors on real factors. Sample period 2000.1-2011.1

<table>
<thead>
<tr>
<th></th>
<th>Level</th>
<th>Slope</th>
<th>Curvature</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>U.S Treasuries</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>% of $cov(y_t^S)$ explained</td>
<td>97.00</td>
<td>1.94</td>
<td>0.06</td>
</tr>
<tr>
<td>% of $cov(y_t^R)$ explained</td>
<td>97.61</td>
<td>2.37</td>
<td>0.02</td>
</tr>
<tr>
<td>% Nom factor explained by real factor</td>
<td>0.76</td>
<td>0.60</td>
<td>0.22</td>
</tr>
<tr>
<td><strong>U.K Treasuries</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>% of $cov(y_t^S)$ explained</td>
<td>99.41</td>
<td>0.58</td>
<td>0.01</td>
</tr>
<tr>
<td>% of $cov(y_t^R)$ explained</td>
<td>99.51</td>
<td>0.47</td>
<td>0.01</td>
</tr>
<tr>
<td>% Nom factor explained by real factor</td>
<td>0.76</td>
<td>0.19</td>
<td>0.01</td>
</tr>
</tbody>
</table>
### Table 1.8. Real - Nominal Yields and Macroeconomic Expectations: US
Table reports regression of 5 and 10-year yields on 1-year expected GDP growth and inflation. The top panel reports regressions using nominal yields while the bottom panel reports real yields. t-statistics reported in ( ) are corrected for generalised heteroskedasticity and serial correlation. Sample period 2000.1 – 2011.12.

<table>
<thead>
<tr>
<th></th>
<th>const</th>
<th>$E_t(g)$</th>
<th>$E(\pi)$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_t^{(5)}$</td>
<td>2.63</td>
<td>0.07</td>
<td>1.51</td>
<td>0.35</td>
</tr>
<tr>
<td></td>
<td>(0.62)</td>
<td>(0.49)</td>
<td>(2.90)</td>
<td></td>
</tr>
<tr>
<td>$y_t^{(10)}$</td>
<td>3.79</td>
<td>0.57</td>
<td>1.21</td>
<td>0.28</td>
</tr>
<tr>
<td></td>
<td>(0.89)</td>
<td>(0.68)</td>
<td>(1.80)</td>
<td></td>
</tr>
</tbody>
</table>

### Table 1.9. Real - Nominal Yields and Macroeconomic Expectations: UK
Table reports regression of 5 and 10-year yields on 1-year expected GDP growth and inflation. The top panel reports regressions using nominal yields while the bottom panel reports real yields. t-statistics reported in ( ) are corrected for generalised heteroskedasticity and serial correlation. Sample period 2000.1 – 2011.12.

<table>
<thead>
<tr>
<th></th>
<th>const</th>
<th>$E_t(g)$</th>
<th>$E(\pi)$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_t^{(5)}$</td>
<td>4.92</td>
<td>1.17</td>
<td>1.34</td>
<td>0.63</td>
</tr>
<tr>
<td></td>
<td>(6.86)</td>
<td>(5.52)</td>
<td>(2.74)</td>
<td></td>
</tr>
<tr>
<td>$y_t^{(10)}$</td>
<td>5.29</td>
<td>0.82</td>
<td>1.10</td>
<td>0.64</td>
</tr>
<tr>
<td></td>
<td>(10.17)</td>
<td>(5.60)</td>
<td>(3.15)</td>
<td></td>
</tr>
<tr>
<td>$y_t^{r(5)}$</td>
<td>4.07</td>
<td>0.49</td>
<td>1.36</td>
<td>0.61</td>
</tr>
<tr>
<td></td>
<td>(9.66)</td>
<td>(3.01)</td>
<td>(5.00)</td>
<td></td>
</tr>
<tr>
<td>$y_t^{r(10)}$</td>
<td>3.37</td>
<td>0.40</td>
<td>0.99</td>
<td>0.63</td>
</tr>
<tr>
<td></td>
<td>(10.92)</td>
<td>(3.56)</td>
<td>(5.30)</td>
<td></td>
</tr>
</tbody>
</table>
Table 1.10. Summary Statistics: US Consumer Price Inflation

<table>
<thead>
<tr>
<th>Maturity</th>
<th>AUCNS</th>
<th>APPNS</th>
<th>ENGNS</th>
<th>MEDNS</th>
<th>TRNNS</th>
<th>UFDSL</th>
<th>SAS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.0268</td>
<td>0.0027</td>
<td>0.0417</td>
<td>0.0445</td>
<td>0.0280</td>
<td>0.0272</td>
<td>0.0319</td>
</tr>
<tr>
<td>Std Dev</td>
<td>0.0123</td>
<td>0.0207</td>
<td>0.1064</td>
<td>0.0160</td>
<td>0.0489</td>
<td>0.0133</td>
<td>0.0101</td>
</tr>
<tr>
<td>skew</td>
<td>-0.3477</td>
<td>0.6365</td>
<td>-0.6901</td>
<td>1.5539</td>
<td>-1.0225</td>
<td>0.4928</td>
<td>0.3688</td>
</tr>
<tr>
<td>kurt</td>
<td>-0.3477</td>
<td>0.6365</td>
<td>-0.6901</td>
<td>1.5539</td>
<td>-1.0225</td>
<td>0.4928</td>
<td>0.3688</td>
</tr>
<tr>
<td>Min</td>
<td>-0.0212</td>
<td>-0.0407</td>
<td>-0.3298</td>
<td>0.0245</td>
<td>-0.1538</td>
<td>-0.0067</td>
<td>0.0071</td>
</tr>
<tr>
<td>Max</td>
<td>0.0610</td>
<td>0.0607</td>
<td>0.2986</td>
<td>0.0924</td>
<td>0.1353</td>
<td>0.0656</td>
<td>0.0628</td>
</tr>
<tr>
<td>1st Lag Auto</td>
<td>0.9390</td>
<td>0.9430</td>
<td>0.9198</td>
<td>0.9842</td>
<td>0.9145</td>
<td>0.9496</td>
<td>0.9837</td>
</tr>
</tbody>
</table>

Table 1.11. Summary Statistics: UK Consumer Price Inflation

<table>
<thead>
<tr>
<th>Maturity</th>
<th>D7BT</th>
<th>D7BU</th>
<th>D7BW</th>
<th>D7BZ</th>
<th>D7C2</th>
<th>D7C7</th>
<th>D7CH</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.0275</td>
<td>0.0298</td>
<td>-0.0282</td>
<td>0.0350</td>
<td>0.0363</td>
<td>0.0310</td>
<td>0.0528</td>
</tr>
<tr>
<td>Std Dev</td>
<td>0.0172</td>
<td>0.0274</td>
<td>0.0364</td>
<td>0.0246</td>
<td>0.0256</td>
<td>0.0152</td>
<td>0.0910</td>
</tr>
<tr>
<td>skew</td>
<td>1.4089</td>
<td>0.6491</td>
<td>0.2678</td>
<td>0.6308</td>
<td>0.3619</td>
<td>0.8934</td>
<td>1.1422</td>
</tr>
<tr>
<td>kurt</td>
<td>1.4089</td>
<td>0.6491</td>
<td>0.2678</td>
<td>0.6308</td>
<td>0.3619</td>
<td>0.8934</td>
<td>1.1422</td>
</tr>
<tr>
<td>Min</td>
<td>0.0054</td>
<td>-0.0238</td>
<td>-0.1087</td>
<td>-0.0340</td>
<td>-0.0259</td>
<td>0.0047</td>
<td>-0.0761</td>
</tr>
<tr>
<td>Max</td>
<td>0.0816</td>
<td>0.1231</td>
<td>0.0535</td>
<td>0.1079</td>
<td>0.1069</td>
<td>0.0750</td>
<td>0.3327</td>
</tr>
<tr>
<td>1st Lag Auto</td>
<td>0.9807</td>
<td>0.9587</td>
<td>0.9652</td>
<td>0.8720</td>
<td>0.9308</td>
<td>0.9463</td>
<td>0.9100</td>
</tr>
</tbody>
</table>

Table 1.12. Risk Premium Proxies on Term Structure Factors
Tables reports regressions of a volatility state from Le and Singleton (2013a) and 3-month Cochrane-Piazzesi factors extracted from real ($CP_t^R$) and nominal ($CP_t^N$) yields. t-statistics reported in brackets are corrected for generalised heteroskedasticity and autocorrelation.

<table>
<thead>
<tr>
<th>$U_t^S$</th>
<th>$U_t^N$</th>
<th>$\bar{R}^2$</th>
<th>$U_t^S$</th>
<th>$U_t^N$</th>
<th>$\bar{R}^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_t$</td>
<td>0.00</td>
<td>1.80</td>
<td>0.60</td>
<td>0.26</td>
<td>1.94</td>
</tr>
<tr>
<td>(0.05)</td>
<td>(8.44)</td>
<td>(1.51)</td>
<td>(4.52)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$CP_t^S$</td>
<td>0.32</td>
<td>0.75</td>
<td>0.67</td>
<td>0.30</td>
<td>0.73</td>
</tr>
<tr>
<td>(6.80)</td>
<td>(4.85)</td>
<td>(5.00)</td>
<td>(3.78)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$CP_t^N$</td>
<td>0.05</td>
<td>1.02</td>
<td>0.32</td>
<td>0.09</td>
<td>1.27</td>
</tr>
<tr>
<td>(0.90)</td>
<td>(5.03)</td>
<td>(1.31)</td>
<td>(5.53)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 1.13. Heterogenous Agent Models: In Xiong and Yan (2010), agents disagree on the long-term properties of the inflation process assumed to be controlled by the central bank. Since $\pi_t$ is observable, $r_t$ is affected neither by $\eta_t$ nor $\psi_t$. The parameter $\hat{\mu}_t$ is the belief of the representative agent, where $\hat{\mu}_t \equiv \frac{1}{1+\gamma} \mu_t + \frac{\gamma}{1+\gamma} \hat{\mu}_t^{\ast}$ and $\hat{\mu}_t^{\ast} \equiv E[\beta^{\frac{1}{\gamma}}]$. Let the consumption shares be $\omega_{ai}(\eta_t) = c_i(t)/D_t$, with $\omega_{ai}(\eta_t) = 1 - \omega_{bi}(\eta_t)$; when $u(c_t) = \frac{1}{2} \sum_i c_i(t)^2$, in equilibrium $\omega_{ai}(\eta_t) = \frac{1}{1+\gamma}$. The variable $X'$ defines the wealth of agent $i = a, b$. In Ehling et. al. (2012) the function $V(h_i)$ depends on the habit surplus $h_i$ and $f(t)$ is the sharing rule. The stochastic process $\lambda_t$ in Chen, Joslin, and Tran (2012) is a jump intensity.

<table>
<thead>
<tr>
<th>Frictions</th>
<th>$M_t = \nu(C'_t)H(\eta)_t$</th>
<th>$r_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Myopic Models ($\gamma = 1$)</td>
<td>$M_t^{\ast}(1 + \zeta \eta_t) \frac{\pi_t + (X^a/X)\eta_t^a - \sigma_a^2}{\pi_t + \delta + \mu_d - \sigma_d^2}$</td>
<td>as above</td>
</tr>
<tr>
<td>Xiong and Yan (2010)</td>
<td>$M_t^{\ast}(1 + \zeta \eta_t) \frac{\pi_t + (X^a/X)\eta_t^a - \sigma_a^2}{\pi_t + \delta + \mu_d - \sigma_d^2}$</td>
<td>as above</td>
</tr>
<tr>
<td>Risk-sharing Models ($\gamma &gt; 1$)</td>
<td>$M_t^{\ast}(1 + \zeta \eta_t^{1/\gamma}) \frac{\pi_t + (X^a/X)\eta_t^a - \sigma_a^2}{\pi_t + \delta + \mu_d - \sigma_d^2}$</td>
<td>as above</td>
</tr>
<tr>
<td>Buraschi and Jiltsov (2006)</td>
<td>$M_t^{\ast}(1 + \zeta \eta_t^{1/\gamma}) \frac{\pi_t + (X^a/X)\eta_t^a - \sigma_a^2}{\pi_t + \delta + \mu_d - \sigma_d^2}$</td>
<td>as above</td>
</tr>
<tr>
<td>Dumas, Kurshev, and Uppal (2009)</td>
<td>as above</td>
<td>as above</td>
</tr>
<tr>
<td>Buraschi, Trojani, and Vedolin (2009)</td>
<td>as above</td>
<td>as above</td>
</tr>
<tr>
<td>Ehling, Gallmeyer,</td>
<td>as above</td>
<td>as above</td>
</tr>
<tr>
<td>Heyerdahl-Larsen, Illeditsch (2012)</td>
<td>as above</td>
<td>as above</td>
</tr>
<tr>
<td>Speculative Models: ($\gamma &lt; 1$)</td>
<td>$M_t^{\ast}(1 + \zeta \eta_t^{1/\gamma}) \frac{\pi_t + (X^a/X)\eta_t^a - \sigma_a^2}{\pi_t + \delta + \mu_d - \sigma_d^2}$</td>
<td>as above</td>
</tr>
<tr>
<td>David (2008)</td>
<td>as above</td>
<td>as above</td>
</tr>
<tr>
<td>Buraschi and Whelan (2012)</td>
<td>as above</td>
<td>as above</td>
</tr>
<tr>
<td>Frictions</td>
<td>$M_t^{\ast} \rightarrow \text{Optimist}$</td>
<td>$r^*(\eta_t) \rightarrow \text{Optimist}$</td>
</tr>
<tr>
<td>Scheinkman and Xiong (2003)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hong, Srer, Yu (2013)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 1.14. Risk Premium Factors on Equilibrium Proxies: US
Table reports regressions $\mathcal{U}_t$ factor on equilibrium risk factor proxies. First, we run the regression:
\[ y_t^{(3,10)} = \text{const} + \beta_{g} E_t[g] + \beta_{\pi} E_t[\pi] + \text{error}_t^{(3,10)} \]
The residuals in this regression are the variation in yields unexplained by macro expectations. In a second stage we perform a principle component decomposition of error. t-statistics, reported in ( )'s, are corrected for auto-correlation. $R^2$ reports the adjusted $R^2$. All right hand variables are standardized. A constant is included by not reported. Sample Period: 1990.1 - 2007.1

<table>
<thead>
<tr>
<th></th>
<th>vol$_t$(g)</th>
<th>vol$_t$((\pi))</th>
<th>surplus$_t$</th>
<th>DiB$_t$(g)</th>
<th>DiB$_t$((\pi))</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathcal{U}_S$</td>
<td>0.61</td>
<td>-0.19</td>
<td></td>
<td></td>
<td></td>
<td>0.32</td>
</tr>
<tr>
<td></td>
<td>(4.25)</td>
<td>(-2.71)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mathcal{U}_S$</td>
<td></td>
<td></td>
<td>-0.46</td>
<td></td>
<td></td>
<td>0.21</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(-4.24)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mathcal{U}_S$</td>
<td></td>
<td></td>
<td>0.32</td>
<td>-0.03</td>
<td></td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(3.13)</td>
<td>(-0.34)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mathcal{U}_S$</td>
<td>0.51</td>
<td>-0.25</td>
<td>-0.36</td>
<td>-0.05</td>
<td>-0.08</td>
<td>0.41</td>
</tr>
<tr>
<td></td>
<td>(3.03)</td>
<td>(-3.40)</td>
<td>(-2.76)</td>
<td>(-0.42)</td>
<td>(-0.90)</td>
<td></td>
</tr>
</tbody>
</table>

Table 1.15. Risk Premium Factors on Equilibrium Proxies: UK
Table reports regressions $\mathcal{U}_t$ factor on equilibrium risk factor proxies. First, we run the regression:
\[ y_t^{(3,10)} = \text{const} + \beta_{g} E_t[g] + \beta_{\pi} E_t[\pi] + \text{error}_t^{(3,10)} \]
The residuals in this regression are the variation in yields unexplained by macro expectations. In a second stage we perform a principle component decomposition of error. t-statistics, reported in ( )'s, are corrected for auto-correlation. $R^2$ reports the adjusted $R^2$. All right hand variables are standardized. A constant is included by not reported. Sample Period: 1990.1 - 2007.1

<table>
<thead>
<tr>
<th></th>
<th>vol$_t$(g)</th>
<th>vol$_t$((\pi))</th>
<th>surplus$_t$</th>
<th>DiB$_t$(g)</th>
<th>DiB$_t$((\pi))</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathcal{U}_S$</td>
<td>0.08</td>
<td>-0.22</td>
<td></td>
<td></td>
<td></td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>(0.75)</td>
<td>(-3.12)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mathcal{U}_S$</td>
<td></td>
<td></td>
<td>-0.21</td>
<td></td>
<td></td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(-1.59)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mathcal{U}_S$</td>
<td></td>
<td></td>
<td>-0.03</td>
<td>0.52</td>
<td></td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(-0.35)</td>
<td>(4.32)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mathcal{U}_S$</td>
<td>-0.03</td>
<td>-0.33</td>
<td>-0.05</td>
<td>0.01</td>
<td>0.57</td>
<td>0.35</td>
</tr>
<tr>
<td></td>
<td>(-0.30)</td>
<td>(-5.65)</td>
<td>(-0.42)</td>
<td>(0.12)</td>
<td>(4.97)</td>
<td></td>
</tr>
</tbody>
</table>

53
Chapter 2

Monetary Policy and Bond Risk Premia

Monetary policy and bond prices are connected via two channels. Firstly, central banks use the nominal short-term rate as a primary policy instrument. Secondly, the absence of arbitrage opportunities implies that bond yields reflect risk neutral expectations about future short rates. Hence, both institutional features (short rate as a policy instrument) and economic restrictions (no arbitrage) enforce a fundamental link between monetary policy and entire term structure of interest rates. Indeed, a large literature documents strong responses in yields to news about monetary policy (see, for instance, Kuttner (2001a), Fleming and Piazzesi (2005), and Gurkaynak, Sack, and Swanson (2005)). However, little is known about whether there is a link between monetary policy and bond risk premia.

Empirically, measuring the actions of monetary policy is a challenging task. A first difficulty is related to the fact that a significant component of policy actions reflects the systematic response of the policy instrument to the macro-economic environment, rather than exogenous policy shocks. In practice, researchers make identifying assumptions to be able to disentangle the systematic component from the monetary policy shock. Needless to say, the dynamic properties of the resulting decomposition are highly dependent on these assumptions (Christiano, Eichenbaum, and Evans (1999)). A second difficulty is that data on short-term target changes is unlikely to capture the richness of policy decisions. For instance, market participants may fully foresee target rate changes, but be considerably surprised about the path of future policy as inferred from the statements of the members of the policy committee: in these circumstances, a measure of monetary policy shocks based on the policy instrument alone may significantly underestimate the extent of exogenous variation in monetary policy. This concern is particularly important for our setting, since monetary policy is known to influence long term yields more via path than target surprises (Gurkaynak, Sack, and Swanson (2005)). This possibility is widely understood by policymakers:

*The current funds rate imperfectly measures policy stimulus because the most important economic decisions, such as a family’s decision to buy a new home or a firm’s decision to acquire new capital goods, depend much more on longer-term interest rates, such as mortgage rates and corporate bond rates, than on the federal funds rate. Long-term rates, in turn, depend primarily not on the current funds rate but on how financial market participants expect the funds rate and other short-term rates to evolve over time.*’ (Bernanke (2004)).

The first half of this chapter takes a high-frequency (event study) approach to question the link be-
tween monetary policy and bond risk premia. Using a sample of 164 FOMC meetings (January 1990 - June 2008) I document that Treasury yields react strongly to the expected path of policy and that this channel is linked to changes in a daily risk premium proxy. In the second half I address the same question from a low-frequency (time-series) perspective. I develop a novel measure of market participants’ expectations about the path of monetary policy, available at monthly frequency, and show this measure contains significant information about bond return predictability.

I summarise my high-frequency findings as follows:

The Federal funds futures market is remarkably accurate at predicting changes to the Federal funds target rate. The futures markets was correct in predicting no change in the effective rate in 67 out of 72 occasions, while it was correct at predicting a change in 66 out of 92 occasions. The market does, however, make mistakes. On 26 occasions the market forecasted a change while the FOMC left the rate unchanged. On 5 occasions the futures market predicted no change while in fact there was a change, and on 3 occasions the market forecasted a change with the wrong sign (20-Dec-1991, 18-Apr-2001, 17-Sep-2001). It is interesting to note that on each of these occasions the forecasting error committed was the same: futures contracts were forecasting an increase in the rate while the FOMC reduced the rate, and at the same time these events occurred during recessions.

Updating the findings of Kuttner (2001b) I find that raw target changes have limited impact on Treasury yields beyond 1-year maturity. However, decomposing target changes into expected versus unexpected components I find that unexpected changes are strongly significant in explaining changes in Treasury yields at maturities between 1 and 5 years, while there is no statistical effect due to expected changes for these maturities. These results confirm the conclusion of Kuttner (2001b) that the reason raw target changes appear unimportant is due to the polluting effect of the expected component.

A number of authors have argued that market beliefs about the timing and path of policy decisions are an important channel for explaining changes in long term bond yields (see, for example, Bernanke and Kuttner (2005), Gurkaynak, Sack, and Swanson (2005) or Gurkaynak (2005)). I study the evidence on Fed timing by decomposing the change in the 1-quarter Federal funds futures rate around FOMC meetings into two components. The first component is the ‘target shock’ defined as the change in the front month rate, and the second component is the ‘timing’ shock defined as the difference between the change in the 3-month rate and the front month rate. Regressing the 1-day change in bond yields around FOMC announcements on target and timing shocks I find timing is highly statistically significant and increasingly important for long-term bonds. Next, I review the evidence presented by Gurkaynak, Sack, and Swanson (2005) that expectations about the entire path of monetary policy matter on announcement days. I combine two datasets that include information about policy expectations over the coming year, and estimate two orthogonal factors associated with movements to the front-end of the futures curve, and a second linked to future policy decisions: I label these factors ‘surprise’ and ‘path’,
respectively. Regressing the 1-day change in bond yields around FOMC announcements on surprise and
path shocks I find that while 3 month bills are more sensitive to target shocks, long-dated bonds are more
sensitive to path shocks. For the 5 year bond, the slope coefficient on the path shock is 2.5 times larger
than the slope coefficient on the target shock. For the 10 year bond, only path shocks are statistically
significant, with a t-statistic of 3.31. These results confirm that target rate changes are not the most
important component driving long-term yield changes.

What is the channel via which bond yields react to monetary policy? I test the conjecture that mon-
etary policy affects compensation for risk by constructing a daily risk premium proxy along the lines of
Cochrane and Piazzesi (2005b). Adapting their methodology I obtain forward rate loadings on 1-year
excess returns at monthly frequency and apply these loadings to the daily forward curve. The resulting
time series \( CP_t \) represents a risk premium proxy available before and after FOMC meetings. I then run
regressions of changes in \( CP_t \) on target and path factors and find a strong correlation between changes
in risk premia and policy shocks. The t-statistics are \(-2.46\) and \(2.8\), on target and path, respectively.
Moreover the signs are economically intuitive. The loading on the target factor is negative: unexpected
drops to the target rate are correlated with an increases in risk premia, i.e., the risk premium is larger in
bad states. The coefficient on the path factor is positive: an unexpected increase in the expected future
target rate increases risk premia, consistent with the result of a positive link between the path factor and
long term yields.

I summarise my low-frequency findings as follows:

I construct an empirical proxy for path shocks from the residuals of a Taylor rule estimated on sur-
vey expectations. Central to this analysis is a new data set that includes joint expectations about the
target rate (fed fund rate) and economic fundamentals (GDP and inflation). This data set is compiled at
monthly frequency and is available in a panel data form so that, for each individual, I have observations
on the expected counterparts to both the left and right-hand side of the Taylor rule. Importantly, this
dataset allows us to empirically identify a measure of path shocks without making assumptions about
the data generating process in the mind of agents. While real-time and truly forward-looking, the use
of survey forecasts may introduce additional concerns. In order to ensure that the survey biases are
quantitatively negligible I conduct a host of quality checks. When I compare the properties of subjective
macro expectations to those obtained from traditional econometric benchmarks, I find that the errors of
consensus forecasts are in absolute value lower than their econometric counterparts. This is especially
ture for the Fed fund rates forecasts. This result is interesting and highlights the potential importance of
structural breaks in the conduct of policy decisions. Another advantage of the panel structure of the data
is that it circumvents the need to use pre-aggregated consensus data, which may in itself bias the results.
Indeed, I compare models based on pre-aggregated consensus data, or panel data, using (i) pooled OLS,
(ii) fixed effects, (iii) random effects, and find that a panel data approach is preferable to procedures
based on pre-aggregated consensus data.
To understand the relative importance of target versus path shocks I consider three alternative shocks: (i) the residuals from an orthogonalised monthly VAR (Christiano, Eichenbaum, and Evans (1996)); (ii) the daily change to the 1-month Federal funds futures rates around FOMC announcements (Bernanke and Kuttner (2005)); and (iii) the daily change in the 6-month euro-dollar rate around FOMC announcements (Cochrane and Piazzesi (2002)). An intriguing yet robust finding is that target shocks are negatively correlated to path shocks. This is interesting since one learns that short-term policy actions are linked to the formation of expectations about future policy actions. Moreover, path shocks are on average countercyclical, different than target shocks, which are procyclical. This observation is consistent with a term structure of interest rates in which the short end is pro-cyclical driven primarily by target shocks and the long end is counter-cyclical driven primarily driven by risk compensation.

Finally, I ask whether path shocks represent an empirically important source of variation in bond risk premia. Summarising, I find that path shocks feature strong co-movement with risk premium factors based on yield curve information such as the slope of the yield curve, or those proposed by Cochrane and Piazzesi (2005b) and Le and Singleton (2013b). In classical return predictability regressions I find that a one standard deviation change to path shocks predicts an \( \sim 0.40 \) standard deviation change to expected excess returns on 5-year bonds. The statistical significance is also large: the null of no predictability is always rejected at the 1\% level and path shocks alone account for 15\% of the variance of excess returns. I show that this evidence is: (i) robust across a variety of Taylor rule specifications; (ii) present both in full sample and in subsamples that exclude the financial crisis; and (iii) survives after controlling for the contemporaneous levels of macroeconomic activity.

The rest of the paper is organised as follows. After a brief overview the existing literature, Section I discusses the mechanics of the Federal funds market and studies the reaction of the nominal Treasury curve to FOMC decisions. Section II describes the data used in the subsequent low frequency analysis. Section III reviews the details behind the construction of monetary policy shocks, while Section IV presents the key evidence on bond return predictability and monetary policy in the time-series.

**Literature Overview**

This paper surveys two streams of related literature. The first studies the high frequency identification of monetary policy shocks and their impact on returns and volatilities of Treasury securities. This literature uses FOMC announcements to identify shocks to market participant’s expectations about the current and future monetary policy stance. The canonical study is Kuttner (2001a) who is the first to separate expected versus unexpected components of monetary policy from federal funds futures and shows that unexpected shocks explain a large fraction of yield changes around FOMC announcements. Fleming and Piazzesi (2005) find that the impact of surprise components on yield changes depends on the slope of the term structure. Gurkaynak, Sack, and Swanson (2005), on the other hand, show that target rate shocks represent only a part of the information set used by market participants around an-
nouncements. The authors that proxies for news about the future path of policy that is orthogonal to
front month shocks but explains a large fraction of the variance of daily changes in long dated Treasuries. The effects of monetary policy are not limited to U.S bond markets. In the context of equities, Bernanke and Kuttner (2005) find that unanticipated cuts to target rate result in large positive shocks to the stock market; Basistha and Kurov (2013) document a large intra-day response of energy futures prices to the surprise component of announcements; and Andersson (2007) shows a feedback between U.S and euro area policy announcements.

The second stream studies the low-frequency impact of monetary policy on the term structure. This literature has two approaches. The first links reduced-form term structure models with the conjecture that the monetary authorities controls the path of the short rate. Under no arbitrage, yields are risk neutral expectations of future short rates; thus, introducing a map between the time-series properties of monetary policy and long term interest rates. Some notable applications of this approach include Piazzesi (2005), Ang, Dong, and Piazzesi (2007) , or Mönch (2008), and Chun (2011). A second, more challenging, approach studies monetary policy from a general equilibrium perspective. Real business cycle models that include reactions functions to the real economy and/or endogenous inflation dynamics have greatly advanced our understanding of the economic drivers of nominal yields. For recent examples see Kung (2014) and Gallmeyer, Hollifield, Palomino, and Zin (2007b) who study Taylor rules with representative agents having Epstein-Zin preferences, or Buraschi and Jiltsov (2007a) and Campbell, Pfueger, and Viceira (2013) who combine habit formation and monetary policy.

I. High Frequency Identification

A. The federal funds rate

The federal funds rate is the interest rate at which depository institutions actively trade balances held at the Federal Reserve, called federal funds, with each other, usually overnight, on an uncollateralized basis. Institutions with surplus balances in their accounts lend those balances to institutions with deficits. The effective federal funds rate is a weighted average of all federal funds transactions for a group of federal funds brokers who report to the Federal Reserve Bank of New York each day. The nominal rate is a target set by the governors of the Federal Reserve, which they enforce primarily by open market operations. The effective Fed funds rate generally lies within a range of that target rate, as the Federal Reserve cannot set an exact value through open market operations.

Figure 2.1 shows the evolution over time of the target rate, the Fed funds rate, and Treasury yields at 6 month, 2 year and 5 year maturities from 1982. Three results emerge. First, the target is persistent which

[^1]Piazzesi (2005) studies bond prices when target rates follow a jump process; Ang, Dong, and Piazzesi (2007) explore no-arbitrage restrictions implied by a variety of Taylor rule specifications; and Chun (2011) studies the link between inflation, GDP forecasts and bond yields in a forward looking Taylor rule specification that incorporates surveys.
the literature refers to as interest rate smoothing. Moreover, target changes are often followed by additional changes in the same direction, which the literature refers as policy inertia. For a detailed discussion of Federal funds-rate targeting procedures related to smoothing and inertia see Goodfriend (1991). Second, cycles of monetary policy easing are associated with large spreads between long term and short term bonds. As the target rate is lowered during recessions the short term treasury rate tracks its path (the short end is pro-cyclical) while the spread between long and short maturity bond yields often moves in the opposite direction (the term premium is counter-cyclical). This has motivated an extensive literature that investigates the behaviour of bond risk premia using time series methods and low frequency data. A third piece of evidence relates to the high frequency component of changes in yields around FOMC meetings when the Fed changes the target rate. An important literature emerged to study these affects. Notable contributions to the FOMC announcements literature at daily frequencies include Cochrane and Piazzesi (2002) and Bernanke and Kuttner (2005) who study the response of Treasury yields and equities, respectively, while Fleming and Piazzesi (2005) and Gurkaynak, Sack, and Swanson (2005) study asset pricing implications using intra-day data.

B. The federal funds futures market

The Chicago Board of Trade (CBOT) began offering federal funds futures contracts in October 1988. Unlike T-bill futures contracts, where the contract is for the T-bill rate on a specific day, the federal funds futures contract is the simple average of the daily effective federal funds rate during the month of the contract. The CBOT offers futures contracts ranging from the current month to 24 months out. Contracts have a nominal value of $5 million, and their settlement price is equal to 100 minus the average of the effective federal funds rate for the month of the contract. Hence, a market price of 96.5 for a one-month contract on September 15 means that the current futures rate for October is 3.5 percent (100 – 96.5). Hamilton (2009) studies the properties of daily changes in the prices fed funds futures contracts and concludes that they provide an excellent measure of markets expectation of near-term changes in Fed policy. In the following I utilise changes in daily contracts to review the evidence on the ‘high frequency’ impact of policy decisions on the yield curve.

To illustrate the nature of my empirical experiment I spell out some federal funds rate algebra. Suppose that at time \( t \) the Fed has a planned FOMC meeting. The day before, at time \( t-1 \), the prevailing effective Fed fund rate is \( r^{0}_{t-1} \) and the price of the front-month Federal funds rate future contract is \( f^{0}_{t-1} \). Then, the equilibrium Fed fund future price is equal to the weighted average of the current effective rate and the expected Fed fund rate over the remaining life of the contract plus a risk premium \( \eta^{0}_{t-1} \)

\[
f^{0}_{t-1} = d \frac{r^{0}_{t}}{m} + \frac{m-d}{m} \bar{r}_{t-1} + \eta^{0}_{t-1},
\]

where \( r^{0}_{t} \) is the target rate announced after the FOMC meeting, \( d \) is the day of the month, and \( m \) is the number of days in the month. Since immediately after the FOMC announcement the price of the front
month contract is

\[ f_t^0 = \frac{d}{m} r_{t-1}^0 + \frac{m-d}{m} r_t^0 + \eta_t^0, \]

I can compute the surprise component \( e_t^0 \equiv r_t^0 - E_{t-1} r_t^0 \) as

\[ e_t^0 = \frac{m}{m-d} \left[ f_t^0 - f_{t-1}^0 \right] - \frac{m}{m-d} \left[ \eta_t^0 - \eta_{t-1}^0 \right]. \]

Assuming that over an interval of one day the risk premium variation \( \frac{m}{m-d} \left[ \eta_t^0 - \eta_{t-1}^0 \right] \) is negligible, I can use the dynamics of Fed fund futures to identify the expected and unexpected components of target rate changes: \( e_t^0 \approx \frac{m}{m-d} \left[ f_t^0 - f_{t-1}^0 \right] \).

\[ r_t^0 - r_{t-1}^0 = E_{t-1} (r_t^0 - r_{t-1}^0) + e_t^0. \]

The expected component is obtained as a residual, i.e. \( [r_t^0 - r_{t-1}^0] - e_t^0 \). Table 2.1 and figure 2.2 summarize the results. The first three columns of Table 2.1 report summary statistics for the change in the total Fed fund rate, the expected, and unexpected components, respectively. I find that a large component of changes in Fed fund is anticipated by the market. Indeed, the variance of the expected component is 0.0457 while the variance of the unexpected component is 0.0353. I also find that during this period the mean unexpected change in Fed fund rate has been negative implying that, on average, the Fed has surprised markets by reducing rates more aggressively than the market expected. This feature can be visualized by plotting the cumulative unexpected shocks over time. Indeed, since 1990 the overall cumulative unexpected shock in Fed fund rates has been \(-5\% \) (figure 2.3)

Our sample includes 164 FOMC meetings. The futures market predicted no change in the effective rate (or less than 10bp) in 72 occasions. In 67 out of these 72 occasions, the Fed indeed did not change the target rate. The left panel of figure 2.4, plots these events; the red bars indicate those occasions in which future markets predicted no-change while a change occurred. Moreover, future markets predicted a change in the effective rate (for at least 10bp) in 92 occasions. In 66 out of these 92 cases, indeed the target rate was changed. However, in 26 of these meetings the FOMC decided to leave the rate unchanged. The right panel of figure 2.4 shows these events; the red bars indicate those occasions in which future markets predicted a change while a change did not occur. Moreover, in 3 of these 66 occasions the future market forecasted the change with the wrong sign. The dates of these meetings are: (i) 20-Dec-1991, (ii) 18-Apr-2001, (iii) 17-Sep-2001. Interestingly, in all these events the type of forecasting error committed by the market was the same: future contracts were forecasting an increase in the rate while the FOMC decided to reduce the rate. All these events occurred during recessions. The evidence from the second event is quite revealing, due to the magnitude of the forecasting error and its timing. March 2001 marks the end of an economic expansion period that, according to NBER, started in March 1991 and lasted
exactly 10 years, the longest in the NBER’s chronology. Nonetheless, on 18 April 2001 future markets were forecasting an increase of at least 50 basis points. Instead, the FOMC decided to drop the rate by 50 basis points to address the start of an economic recession. The forecast error committed around the 17 September 2001 FOMC event is more idiosyncratic in nature since it is related to an emergency meeting held by the Fed following the September 11 events when markets were closed.

In summary, the implied expected change in the target rate by the futures market is remarkably accurate. However, I find that the average absolute magnitude of the unexpected component is significant and similar in size to the expected component.

C. The impact on Treasury bond yields

The distinction between expected and unexpected components in the dynamics of Fed funds rate becomes immediately apparent when I investigate the link between Fed decisions and changes in Treasury bond yields. I use data on (implied) zero coupon bond yields from January 1990 to June 2008 and regress the 1-day change in bond yields around FOMC announcements on the target changes. I use nominal and TIPS zero coupon bonds estimated by Gürkaynak, Sack, and Wright (2006, 2010) (GSW) and short-term nominal interest rates with 3 and 6-month maturities from the Fama-Bliss T-bill files available from CRSP.\(^2\) The results are summarized in table 2.2. The top panel shows that most of the impact of the total change in Fed fund rates on Treasury yields is limited to maturities up to 1 year. After this maturity, the effect is somewhat muted and the slope coefficient is not statistically significant. However, when I decompose the total change in two components (bottom panel), one finds that unexpected changes are strongly significant in explaining changes in Treasury bond yields at maturities between 1 and 5 years. On the other hand, for these maturities the effect of the expected component is indistinguishable from noise. Thus, the reason raw target changes are not significant for longer maturities is due to the polluting effect of the expected component. This is confirmed by the substantially larger \(R^2\) obtained when I run the regression on the two terms separately. In addition to statistical significance, the results show that at short maturities most of the magnitude of yield changes is due to unexpected variations in the target Fed fund rate. The marginal impact of the unexpected component on the 3 month T-bill rate is twice as large as the expected component.

D. The Timing of Expected Fed Interventions

The previous results show that on five occasions futures market predicted no change in the target rate while in fact there was a change. Moreover, in three cases futures markets predicted changes in target

\(^2\)GSW construct zero coupon yields by fitting the Nelson-Siegel-Svensson functional form to market quoted coupon bonds and is publicly available from the Federal Reserve Board site
rates with the wrong sign. One possible interpretation is that markets may have forecasted incorrectly the timing of decisions that were relatively inevitable. Both Gurkaynak (2005) and Bernanke and Kuttner (2005) discuss this channel as a particularly important component to explain changes in medium and long term bond yields as these are sensitive not only to the target rate decided at the next FOMC meeting but also to future policy decisions. To investigate this conjecture figure 2.5 plots unexpected changes in the front month futures contract (y-axis) with respect to changes in the 1-quarter (3rd month) contract (x-axis). The red line in the plot is the 45 degrees lines; the blue line is the least squares fit. I find that the slope coefficient is far from being one. Moreover, for large positive front month changes the least square fit is below the 45 degrees line suggesting that the three month contract responds less than the front month contract. Similarly, for large negative front month changes, changes in the three month contract plots above the 45 degree line. The \( R^2 \) of a linear regression is 70\%, suggesting that unexpected changes in the target rate obtained from the dynamics of the front-month future contract are not perfectly correlated with changes at longer horizons. A possible reason is that the market interprets the information content of FOMC decisions in the context of the timing of future policy decisions. While these decisions are useful signals, their information content to infer future target rate decisions likely depends on the state of the business cycle and the market surely looks at other signals, such as statements by policy makers.

Thus, in what follows I distinguish between two components of the change in the 1-quarter contract. The first component is the change in the front month contract (the ‘target’ shock); the second component (the ‘timing’ shock) is defined as the difference between the shock to the 4 month shock and to the front-month contract (the vertical distance of the data points from the 45 degree line). Figure 2.6 summarizes some stylized features of ‘target’ and ‘timing’ shocks. I find that the economic magnitude of ‘timing’ shocks are comparable to ‘target’ shocks. Moreover, in several occasions target and timing shocks are negative correlated, moving in opposite directions.

This is a potentially important channel to explain the dynamics of long-term yields. To investigate this channel, I run a regression using the same data and sample period of Table 2.2. Indeed, I find that the timing factor is as important as the target factor explaining variations in the Treasury bond market (see Table 2.3). Moreover, when I compare the \( R^2 \) of Table 2.2, Panel B, to those in Table 2.3 I find that the timing factor substantially increases the explanatory power for long-term bond, with the \( R^2 \) for the 5 year note increasing from 10\% to 17\%.

\[ \text{[ insert figure 2.5 about here ]} \]

\[ \text{[ insert figure 2.6 about here ]} \]

\[ \text{[ insert table 2.2 and 2.3 about here ]} \]

\textbf{E. Target versus Path shocks}

Gurkaynak, Sack, and Swanson (2005) conjecture that the surprise component of target rate changes conveys only part of the news about monetary policy on announcement days: market participants update
their beliefs by interpreting statements about monetary policy which give information about the path of future policy. Following this intuition the authors construct a ‘path factor’ from high frequency data that is by construction orthogonal to surprise shocks to the front months. Indeed, the important role played by the timing shocks suggests that monetary policy shocks may be characterized by different frequency components. To review this evidence more formally, I combine two datasets that include data containing information about policy expectations over the coming year. The first two time series provide information on futures on the front-month and three-month-ahead Federal funds contracts. The remaining three time series includes data on longer maturity Eurodollar future contracts with maturities of two-, three-, and four-quarter-ahead. At these maturities Eurodollar futures are significantly more liquid than Federal fund futures contracts. Over a period of 12 months there are an average of 8 FOMC meetings. Thus, this data includes price information that is sensitive to the decisions in the following eight FOMC meetings. Figure 2.7 shows the magnitude of the five original shocks over time. It is apparent that in several occasions small shocks to the front month future were contemporaneously associated to large shocks to longer dated futures.

I stack the shocks to these five contracts around each FOMC announcements in a $T \times 5$ matrix and perform a principle component decomposition. The decomposition of the variance explained by the first three principal components (PCs) $F_t$ is summarized in table 2.4. Indeed I find that a single factor can only explain 62% of the total variance of the shocks to the term structure of future rates around FOMC meeting. A second factor is needed to explain about 33% of the remaining variation across maturity. This is strongly suggestive of a multi-dimensional factor structure underlying the evolution of future monetary policy.

Since the identification of the principal components is only linked to their relative explanatory power for the overall variance, one does not have an interpretation of what the factors might be. To help economic understanding, following Gurkaynak, Sack, and Swanson (2005), I perform a rotation of the first two PCs. Let the original PCs $F_t = [PC_1, PC_2]$. I define the new rotated factors $Z_t = F_t U$, where the matrix $U$ is such that $Z^2_t$ is orthogonal to the shocks to the front month future, $E(Z^2_t e^0_t) = 0$. Because this rotation generates a component that is orthogonal to $e^0_t$, I label the second factor $Z^2_t$ as the path factor. By construction, this factor includes information that affects the expected path for monetary policy but is orthogonal to decisions about the current target rate. This also implies that the dynamics of $e^0_t$ can only be explained by $Z^1_t$. Thus, I label the first rotated principal component $Z^1_t$ the surprise factor. Moreover, to identify the factors uniquely, I impose that $Z^1_t$ have unit variance and $E(Z^1_t Z^3_t) = 0$.

Figure 2.8 summarizes the difference between the target shock $Z^1_t$ and the path shock $Z^2_t$. I find that target and path shocks move in the same direction in 94 of the 164 FOMC meetings. On the other hand, in 70 occasions, they moved in opposite directions.
The result confirms my conjecture about the existence of two distinct components in monetary policy shocks. The first associated to movements in the front-end of the future curve and a second one linked to future policy decisions. Similar to the importance of distinguishing between expected and unexpected components in Fed funds decisions, it is likely that long dated Treasury bonds are more sensitive to path shocks than to target shocks.

To investigate I run a contemporaneous regression of changes in Treasury yields on both types of shocks. Table 2.5 summarizes the results. Indeed, I find that while the yield on the 3 month T-Bill is more sensitive to target shocks, both economically and statistically, the opposite is true for long-dated Treasury bonds. For the 5 year bond, the slope coefficient on the path shock is 2.5 times larger than the slope coefficient on the target shock. For the 10 year bond, only path shocks are statistically significant, with a t-statistics of 3.31. This confirms that target rate changes in themselves are not the most important component driving long-term bond changes.

F. Risk Premia

Cochrane and Piazzesi (2005b) argue that risk premia are revealed by the cross-sectional shape of the yield curve. In what follows I adapt their methodology to investigate the extent to which FOMC announcements affect risk premia. Let \( r_{x_{t+12}}^{(n)} \) be the 12-month excess return on a \( n \)-year bond and let \( f_t = [f_t^{(1)}, f_t^{(3)}, f_t^{(5)}, f_t^{(7)}, f_t^{(9)}] \) be a vector of five forward rates. The spanned component of expected risk premia by running the following regression

\[
\begin{align*}
r_{x_{t+12}}^{(n)} &= \alpha_n + \beta_1 f_t^{(1)} + \beta_2 f_t^{(3)} + \beta_3 f_t^{(5)} + \beta_4 f_t^{(7)} + \beta_5 f_t^{(9)} + \epsilon_t^{(n)} \\
&= b_n (\gamma_0 + \gamma_1 f_t^{(1)} + \gamma_2 f_t^{(3)} + \gamma_3 f_t^{(5)} + \gamma_4 f_t^{(7)} + \gamma_5 f_t^{(9)}) + \epsilon_t^{(n)}
\end{align*}
\]

from which the CP-factor is defined as \( CP_t = \gamma_1 f_t \). I estimate the previous regression sampling forward rates at monthly frequency then apply the factor loadings to the daily forward curve to obtain a 1-year risk premium proxy at daily frequency. Figure 2.9 compares the behavior of the CP-factor at monthly and daily frequency. It can be noticed that in the sample period up to 1990, the daily series shows substantially higher volatility than then monthly one, ans especially so in the 80s.

In table 2.6 I report results of a regression that studies the contemporaneous link between the changes in the \( CP_t \) factor and the Target and Path factors:

\[
\Delta CP_t = const + \beta_{\text{surprise}} t + \gamma_{\text{path}} t + \epsilon_t
\]
Indeed I find a strong correlation between changes in risk premia and the two monetary policy factors. The t-statistics are $-2.46$ and $2.8$, respectively. Moreover the signs is economically intuitive: unexpected drops in the target rate is correlated with an increase in risk premia. The magnitude of this effect is larger in bad states of the world. The slope coefficient on the path factor is positive: an unexpected increase in the expected future fund rates increases risk premia. This is consistent with the earlier result of a positive link between the path factor and long term yields and supports the null hypothesis of a significant risk premium channel.

II. Data : Low Frequency Identification

This section describes the data sets I use to identify the ‘low frequency’ component of monetary policy shocks. The sample I study is available at monthly frequency and runs from January 1990 to August 2012.

A. Survey data

I use survey forecasts from BlueChip Financial Forecasts Indicators (BCFF) to construct a new measure of monetary policy shocks. BCFF is a monthly publication providing extensive panel data on the expectations of professional economists working at leading financial institutions and service companies. Forecasted variables include Treasury yields and economic fundamentals. While the exact timing of the surveys is not published, the survey is usually conducted between the 25th and 27th of the month and mailed to subscribers within the first 5 days of the subsequent month. The resulting dataset represents an extensive and unique dataset to investigate the role of formation of expectations about monetary policy shocks.

The horizon of BCFFS forecasts ranges from the end of the current quarter to 5 quarters ahead (6 from January 1997). I obtain a set of constant maturity forecasts (from 1 to 4 quarters ahead) by interpolating linearly between adjacent horizons. Macroeconomic forecasts are expressed as annualized percentage changes between subsequent quarters: I obtain compound growth forecasts by chaining subsequent quarterly forecasts.\footnote{For instance, suppose that as of April 2000, the 1Q- and 2Q-ahead GDP forecasts of agent $n$ are 5.00 and 6.00, respectively. This means that the agent expects GDP to increase by $(1 + \frac{5.00}{100})$ between April 2000 (the month of the forecast) and June 2000 (the end of current quarter), and by $(1 + \frac{6.00}{100})$ between end of June 2000 (the end of current quarter) and the end of September 2000 (the end of the next quarter). The (annualized) compound growth rate between April 2000 and September 2000 is obtained as \(\left((1 + \frac{5.00}{100}) \cdot (1 + \frac{6.00}{100})\right)^\frac{3}{2} - 1\).}

The resulting dataset of forecasts can be described as follows. Let $Z_t$ denote the time-$t$ realization of the economic or financial variable of interest, and let $E^n_{\cdot|\Omega_{n,t}}$ denote the expectation operator under the subjective measure of agent $n$ and conditional on her time-$t$ information set $\Omega_{n,t}$. The data manipulations
described above allows us to obtain \( Z_{n,t,h}^e \), the forecast of \( Z_{t+h} \) made by agent \( n \) at time \( t \):

\[
Z_{n,t,h}^e \triangleq E^n [Z_{t+h} | \Omega_{n,t}],
\]  

(2.1)

for quarterly horizons out to 1 year, \( h = 3, 6, 9, 12 \) months. Notice that this representation allows for incomplete information \( (\Omega_{n,t}) \), and difference in priors about the data-generating process (the expectation is taken under the subjective measure); the only assumption is that forecasts be rational in the sense of Muth (1961). I also construct consensus forecasts \( Z_{C,t,h}^e \), defined as the cross-sectional mean of the forecasts by all respondents at time \( t \):

\[
Z_{C,t,h}^e \triangleq \frac{1}{N} \sum_{n=1}^{N} Z_{n,t,h}^e,
\]  

(2.2)

where \( N \) denotes the size of the cross-section of forecasters.

The forecasts used here are real GDP (Real GNP until February 1992), Consumer Price Inflation, and the Federal Funds rate. Since real GDP, CPI, and federal funds rates are available at different frequencies (quarterly, monthly and daily, respectively), the quarterly values that the survey participants are asked to forecast are defined in different fashions. Let \( GDP_q(t) \), \( CPI_m(t,j) \), and \( FF_{d(t,j)} \) denote, respectively: (i) the seasonally adjusted value of real GDP at the end of the quarter that includes month \( t \); (ii) the seasonally adjusted value of CPI at the end of the \( j \)-th month of the quarter that includes month \( t \); and (iii) the value of the federal funds rate at the end of the \( j \)-th day of the quarter that includes month \( t \) (assumed to be 90, for simplicity). For each horizon \( h \), survey participants are asked to forecast are \( g_{q(t+h)} \), the quarter-over-prior-quarter percent change of seasonally-adjusted real GDP, expressed as an annualized rate:

\[
g_{q(t+h)} \triangleq \left( \frac{GDP_{q(t+h)}}{GDP_{q(t+h-1)}} \right)^4 - 1; \tag{2.3}
\]

\( \pi_{q(t+h)} \), the quarter-over-prior-quarter percent change of the intra-quarter average of seasonally-adjusted CPI, expressed as an annualized rate:

\[
\pi_{q(t+h)} \triangleq \left( \frac{1/3 \sum_{m=1}^{3} CPI_{m(t+h,j)}}{1/3 \sum_{j=1}^{3} CPI_{m(t+h-1,j)}} \right)^4 - 1; \tag{2.4}
\]

and, \( f_{q(t+h)} \), the average of intra-quarter daily federal funds rates:

\[
f_{q(t+h)} \triangleq 1/90 \sum_{j=1}^{90} FF_{d(t+h,j)}. \tag{2.5}
\]

I denote the time-\( t \) forecasts of agent \( n \) for \( g_{q(t+h)} \), \( \pi_{q(t+h)} \), and \( f_{q(t+h)} \) by \( g_{n,t,h}^e \), \( \pi_{n,t,h}^e \), and \( f_{n,t,h}^e \), respectively. Figure 3.10 and 2.11 show the dynamics of the 1-year ahead consensus forecasts for the federal funds rate, inflation, and GDP growth.
I construct expected output gaps as follows. From the Bureau of Economic Analysis (BEA) I obtain real GDP from and interpolate linearly to obtain monthly values. Then, I fit a Hodrick-Prescott filter (with a smoothing parameter of 14,400) to log output \( y_t = \log(Y_t) \) and estimate the mean growth rate of the economy \( g_t^* \) as the average log difference of output. I construct potential output \( Y_t^* \) by taking the (exponential of the) trend component of the filtered series, and construct conditional estimates of future potential output (common across agents) as \( E[Y_{t+h}^*|\Omega_t] = Y_t^* \exp(g_t^* \cdot h \cdot 3) \). Next, I obtain estimates of actual output using individual GDP growth forecasts, \( E^n[Y_{t+h}|\Omega_{n,t}] = Y_t \cdot \left(1 + \frac{g_{e,n,t}}{400}\right) \cdot \left(1 + \frac{g_{e,n,t+2}}{400}\right) \ldots \left(1 + \frac{g_{e,n,t+h}}{400}\right) \). Finally, I construct the percentage projected output gap for horizon \( h \) as \( x_{n,t,h}^e = E^n[x_{t+h}|\Omega_{n,t}] = \left(\frac{E^n[Y_{t+h}|\Omega_{n,t}]}{E[Y_{t+h}|\Omega_t]} - 1\right) \cdot 100 \). Since this definition of gap may suffer from look-ahead biases, I also construct real time output gaps by fitting the Hodrick-Prescott filter and estimating mean growth rates recursively over a 10-years look-backwards rolling window. The results are only marginally affected.

B. Quality of survey data

Survey data forecasts feature a number of advantages over forecasts implied by econometric approaches such as VARs (vector autoregressions). First, the specification of the VAR may not coincide with the data generating process in the mind of the agents. Second, agents may, contrary to the econometrician, observe a structural break in the sample of interest. Third, even if both the econometrician and the agents observe the data generating process, VAR forecasts still suffer from estimation error. Survey data allows obtaining direct measures of agents’ expectations, dispensing with the need to posit and estimate a data generating process for the variable of interest.

I summarize the cross-sectional and time-series properties of BCFFS expectations by comparing their performance to an econometric benchmark. In particular, I first construct the sample equivalents of the forecasts for federal funds, real GDP growth rate, and CPI growth rate. Next, I assume that the quantities of interest \( \Delta f_t, \ g_t, \ \pi_t \) follow a first-order VAR. I fit the VAR recursively using a 25 years rolling window (100 quarterly observations), and use the estimated parameters to construct the benchmark forecasts. Since macro-economic data are released with a month lag, I always drop the last observations when estimating the VAR to ensure that forecasts are based on the actual real-time information set of agents. Finally, I compare the forecasting errors of BCFFS versus VAR(1) expectations. Figure 2.12 summarizes the magnitude of BCFFS forecast errors (1 quarter horizon) relative to VAR forecast errors. The plots on the left represent the errors from a cross-sectional perspective: they show the time series of the number of agents in the cross-section whose forecast error is, in absolute value, less than the absolute value of the VAR forecast error. The plots on the right, on the other hand, summarize the forecasting ability of consensus (mean) forecasts: they show the difference between the absolute value of the VAR

---

4The construction of this expectations implicitly assumes that output is lognormally distributed and ignores a Jensen’s inequality term, which is quantitatively negligible.
forecast error and the absolute value of the average BCFFS forecast error, so that a value above zero means that, on a specific quarter, the consensus forecasts performs better than the VAR forecast. The figure suggests that there is a strong time-series component in the ability of BCFFS surveys to beat VAR forecasts. Overall, the errors of consensus forecasts are, in absolute value, less than the forecast error of the VAR 85% (FF), 43% (GDP), and 65% (CPI) of the times.

[Insert Figure 2.12 about here.]

C. Macroeconomic activity data

I construct a proxy for the level of macroeconomic activity by following Ludvigson and Ng (2009b) and Buraschi and Whelan (2012b). Ludvigson and Ng (2009b) find strong evidence linking bond returns to variations in the level of economic growth rate factors by running return predictability regressions on the principle components from a large panel of real, nominal, and price-based variables. The identity and sources of the dataset are described in Ludvigson and Ng (2009b); following Buraschi and Whelan (2012b), I drop all price based information in order to interpret the resulting panel as a pure growth rate factor. Examples of price variables removed include: S&P dividend yield, the Federal Funds (FF) rate; 10 year T-bond; 10 year - FF term spread; Baa - FF default spread; and the dollar-Yen exchange rate. A small number of discontinued macro series are replaced with appropriate alternatives or dropped.\footnote{Further details of the construction and macro series included are given in the appendix of Buraschi and Whelan (2012b).} I take the first principle component of the resulting dataset of 99 macro series as a proxy for the conditional mean of consumption growth, $g_t$.\footnote{$g_t$ explains around 90% of the unconditional variance of the panel of macroeconomic activity series.}

D. Bond data

I use Fama-Bliss data from CRSP of zero coupon bond prices (available at monthly frequency) with maturities between 1 and 5 years. The following notation is adopted. Define the date $t$ log price of a $n$-year discount bond as $p_t^{(n)}$. The yield of a bond is defined as $y_t^{(n)} = -\frac{1}{n}p_t^{(n)}$. The date-$t$ 1-year forward rate for the year from $t+n-1$ and $t+n$ is $f_t^{(n)} = p_t^{(n)} - p_{t+1}^{(n+1)}$. The log holding period return is the realised return on an $n$-year maturity bond bought at date $t$ and sold as an $(n-1)$-year maturity bond at date $t+12$:

$$ r_t^{(n)} = p_{t+12}^{(n-1)} - p_t^{(n)} . $$

(2.6)

Excess holding period returns are denoted by:

$$ rx_t^{(n)} = r_t^{(n)} - y_t^{(1)} . $$

(2.7)

III. Overview of identification schemes

This section describes the methodology I use to construct monetary policy shocks. First, I provide a brief overview of the alternative approaches employed by the literature. Second, I introduce my identification
scheme based on a Taylor rule and a panel of forecast data. Third, I report the statistical properties of
the constructed series.

Since much of the decisions taken by the monetary authorities reflect non-monetary developments in
the economy, the literature that studies the effect of monetary policy is typically concerned with policy
shocks, rather than policy actions per se (Christiano, Eichenbaum, and Evans (1999)). A classic inter-
pretation of monetary policy shocks is that they reflect changes in the preferences of the central bank,
which may arise from shift in the weights of the views or political influence of the members of the policy
committee (Christiano, Eichenbaum, and Evans (1999)).

In general, the strategies that seek to identify monetary policy shocks can be classified as being based
on a policy rule, or not. The idea behind strategies based on policy rules is to impose as much structure on
the feedback rule as needed to decompose policy actions into systematic and non-systematic components
(policy shocks). In particular, researchers following this approach must make assumptions about the pol-
icy instrument, the functional form of the feedback rule, the arguments of the rule, and, importantly, the
interaction of the arguments of the rule and monetary policy shocks. A common assumption, which under-
lies the entire literature on Taylor rules,\textsuperscript{7} is that monetary policy shocks are orthogonal to the arguments
of the rule, so that they can be estimated from regression residuals. Christiano, Eichenbaum, and Evans
(1999) note that the economic content of this assumption is that the macroeconomic variables that the
monetary policy authority looks at in setting policy are predetermined relative to the policy shocks: these
variables do not respond to contemporaneous realizations of monetary policy shocks.

The second family of identification strategies, on the other hand, infers monetary policy shocks from
data that suggests exogenous monetary policy actions. An example of this identification philosophy is
Romer and Romer (1989), who use historical records to identify large monetary disturbances not caused
by macroeconomic developments. Similarly, Romer and Romer (2004) use quantitative and narrative
records to infer intended funds rates changes around FOMC meetings, and take into account the forecasts
of the Federal Reserve to remove anticipatory components.

A. Constructing a measure of path shocks

As discussed in the high-frequency identification sections above there are two types of monetary policy
shocks: target shocks and path shocks. Target shocks represent exogenous variation in the conduct of
monetary policy as reflected by the current behavior of the policy instrument (the target rate). Path
shocks, on the other hand, capture variation in the projected path of monetary policy. Intuitively, path
shocks reflect the surprises about future policy that can be inferred, for instance, from FOMC statements
or interviews of members of the policy committee. Path shocks, unlike target shocks, cannot be easily
identified via Taylor rule regressions because the FOMC does not reveal the projected evolution for the

\textsuperscript{7}The literature on Taylor rules is extensive: see, for instance, Taylor (1993), Clarida, Gali, and Gertler (2000),
Ang, Dong, and Piazzesi (2007).
This is unfortunate, since path shocks represent the quantitatively most important source of variation in bond yields, especially for long maturities (Gurkaynak, Sack, and Swanson (2005)). My data on federal funds forecasts, however, allows me to measure the projected evolution for the path of monetary policy from the perspective of market participants; since I also observe market participants’ expectations about macroeconomic developments I am able to identify path shocks by taking the residuals from a policy feedback rule estimated on federal funds and macro forecasts. This section describes the Taylor specification I adopt and discusses the restrictions it implies for forecast data.

Let \( \pi_t \) denote the change in the price level from quarter \( t - 1 \) to quarter \( t \), annualized and in percentage points. Similarly, let \( x_t \) denote the output gap in quarter \( t \), in percentage points. As standard in the literature, the output gap is defined as \( x_t = \left( \frac{Y_t}{Y^*_t} - 1 \right) \cdot 100 \), where \( Y_t \) and \( Y^*_t \) denote actual and potential output at time \( t \), respectively. Finally, let \( f_t \) denote the time \( t \) Federal Funds rate, with long run mean \( f \). The level of the federal funds rate can be decomposed into a systematic component \( f_t^* \) (the Taylor rule), and an orthogonal shock \( u_t \) (the target shock):

\[
\begin{align*}
  f_t &= f_t^* + u_t \\
  &= f + \beta (\pi_t - \pi) + \gamma x_t + u_t
\end{align*}
\]

where \( u_t \perp f_t^* \). This characterization has a simple interpretation. The feedback rule \( f_t^* \) captures the systematic component of monetary policy. In the absence of disturbances to the economy and monetary policy shocks, the federal funds rate is constant and equal to \( f \). If output deviates from its potential level, or inflation from its target, the central bank intervenes to stabilize the economy: the parameters \( \beta \) and \( \gamma \) capture the sensitivity to inflation and output stabilization, respectively. The target shock \( u_t \), on the other hand, captures the non-systematic component of monetary policy: the orthogonality between \( u_t \) and the arguments of the Taylor rule means that it can be estimated as the residual of a simple time series regression of federal funds onto inflation and gap.

In practice, it has been observed that the central bank behaves less responsively to the state of the economy than implied by the benchmark Taylor rule, consistent with the idea that the central bank may have preferences over the degree of variability of federal funds rates. Also, the central bank may wish to respond to macro aggregates that are not realizing at time \( t \). In order to accommodate policy inertia and backward-/forward-looking policies, the benchmark rule can be extended to include lagged federal funds

---

8 Only recently has there has been a change in the disclosure about future policy actions by the Fed, so-called, forward guidance.
and lags/leads in its arguments, so that realized federal funds rates are described by:

\[
f_t = \rho(L)f_{t-1} + (1 - \rho)f_t^* + u_t \\
= \rho(L)f_{t-1} + (1 - \rho)\left( f + \beta (\pi_{t+j} - \pi^*) + \gamma x_{t+k} \right) + u_t, \tag{2.10}
\]

where \(\rho(L) = \rho_1 + \rho_2 L + \ldots + \rho_m L^{m-1}\) and \(\rho = \rho(1)\) capture the degree of interest rate smoothing. Assuming that agents know the functional form and parameters of the policy rule, while they can disagree about the future evolution of macroeconomic variables, the time-\(t\) expectation of \(f_{t+h}\) of agent \(n\) is given by:

\[
E^n [f_{t+h} | \Omega_{n,t}] = E^n [\rho(L)f_{t+h-1} | \Omega_{n,t}] \\
+ (1 - \rho) \left( f + \beta (E^n [\pi_{t+h+j} | \Omega_{n,t}] - \pi^*) + \gamma E^n [x_{t+h+k} | \Omega_{n,t}] \right) \\
+ E^n [u_{t+h} | \Omega_{n,t}], \tag{2.11}
\]

which, using the notation introduced in the data section, can be re-written as:

\[
f_{n,t,h}^e = \rho_1 f_{n,t,h-1}^e + \ldots + \rho_m f_{n,t,h-m}^e \\
+ (1 - \rho) \left( f + \beta (\pi_{n,t,h+j}^e - \pi^*) + \gamma x_{n,t,h+k}^e \right) + u_{n,t,h}^e, \tag{2.12}
\]

or, in its consensus form, as:

\[
f_{C,t,h}^e = \rho_1 f_{C,t,h-1}^e + \ldots + \rho_m f_{C,t,h-m}^e \\
+ (1 - \rho) \left( f + \beta (\pi_{C,t,h+j}^e - \pi^*) + \gamma x_{C,t,h+k}^e \right) + u_{C,t,h}^e. \tag{2.13}
\]

This expression shows that the assumption that agents believe in a Taylor rule has two implications. First, it implies a restriction on the comovement of forecasts of federal funds, inflation, output gap, and monetary policy shocks. Second, it suggests that subjective expectations about future monetary policy shocks (path shocks) can be recovered from a panel of macroeconomic and financial forecasts.

### B. Empirical features of the monetary policy shocks measure

All the specifications examined amount to choices for \(h\), the horizon of the federal funds forecasts, and \(i\), the horizon of the output gap and inflation forecasts, in the general model:

\[
f_{n,t,h}^e = \rho_1 f_{n,t,h-1}^e + \rho_2 f_{n,t,h-2}^e + (1 - \rho) \left( f + \beta (\pi_{n,t,i}^e - \pi^*) + \gamma x_{n,t,i}^e \right) + u_{n,t,h}^e. \tag{2.14}
\]
all specifications include 2 smoothing terms, which are necessary to remove the persistent component of federal funds forecasts as in Clarida, Gali, and Gertler (2000). I consider three families of specifications based on the choice of the horizons of the forecasts for the federal funds and the arguments of the Taylor rule: 2 are contemporaneous, and 1 is forward looking. For each family, output gap is either constructed using full sample information (GAP 1), or recursively (GAP 2). This gives a total of 6 specifications, that are summarized in Table 2.7.

[Insert Table 2.7 about here.]

In general, these models could be estimated (i) using consensus data, via OLS; or, using panel data, via: (ii) pooled OLS (POLS); (iii) fixed effects (FE); (iv) random effects (RE). Table 2.7 reports the test-statistics and p-values for two tests: (i) an F-test for the joint significance of individual effects, and (ii) the Hausman test for the null hypothesis that the difference between random and fixed coefficients is not systematic. In all cases, the F-statistic for the joint significance of agent dummies rejects the null of no significance; furthermore, the Hausman test always rejects the null of random effects. Taken together, these results indicate that fixed effects is preferable over estimation procedures based on consensus, pooled OLS, and random effects.

Given one of the 6 Taylor rule specifications, I construct a measure of policy path shocks, $\text{PathShock}_t$, by taking the cross-sectional average of the estimated residuals:

$$\text{PathShock}_t = \frac{1}{N} \sum_{n=1}^{N} u_{n,t,h}.$$  

Figure 2.13 plots the time series dynamics for each specification. The message here is that, regardless of the specification, the statistical properties are very similar and the co-movement across $\text{PathShock}_t$ is always very high. In what follows I study the economics of path shocks focusing on specification 2, in both Federal funds and macro forecasts are for the 1-year horizon, and the output gap is constructed recursively using only information available at date $t$. However, the quantitative message of the paper is robust to different choices for $\text{PathShock}_i$. An online appendix presents an expanded set of results for $i = 1, \ldots, 6$ for the empirical tests that follow.

[Insert Figure 2.13 about here.]

C. Low-Frequency versus High-Frequency Shocks

To help understand the information content in $\text{PathShock}_t$ I compare its dynamics with three alternative measures for monetary policy shocks studied by the literature: (i) the residuals from an orthogonalised monthly VAR (Christiano, Eichenbaum, and Evans (1996)); (ii) the daily change to the 1-month Federal funds futures rates around FOMC announcements (Bernanke and Kuttner (2005)); and (iii) the daily change in the 6-month euro-dollar rate around FOMC announcements (Cochrane and Piazzesi (2002)).
Firstly, I follow Christiano, Eichenbaum, and Evans (1996) and construct a monthly VAR: \( BZ_t = A(L)Z_{t-1} + \Sigma \eta_t \). The data vector \( Z_t \) is given by \( Z_t = [EMP_t, CPI_t, PCOM_t, FF_t] \) where \( EMP_t \) is the logarithm of Nonfarm payroll employment, \( CPI_t \) is the logarithm of the consumer price index, and \( PCOM_t \) is growth rate in commodity price index. I identify the system by orthogonalizing the shocks as in CEE, using the order given by \( Z_t \). This implies that shocks to the Fed fund rate has no contemporaneous effect on the other economic variables. I specify the VAR with \( L = 6 \) monthly lags. The estimated policy shocks are given by

\[
\eta^{\text{cee}}(t) = i_4 \Sigma^{-1} \left[ \hat{B}Z_t - \hat{A}(L)Z_{t-1} \right]
\]  

(2.17)

where \( i_4 = [0, 0, 0, 1] \), from which I recover the policy shocks.

The second and third shock measures are based on daily data. As in Bernanke and Kuttner (2005) I measure the surprise component in target rate changes from the change in the 30-Day Federal Funds Futures contract price relative to the day prior to the FOMC meeting. The contract’s settlement price is based on the monthly average federal funds rate so the surprise change must be scaled up by a factor related to remaining duration of the contract. Specifically, given target change on day \( d \) of month \( m \), I compute the unexpected target rate change as

\[
\eta^{\text{bk}}(t) = \frac{D - d}{D} \left( f_{m,d}^0 - f_{m,d-1}^0 \right)
\]  

(2.18)

where \( f_{m,d}^0 \) is the current-month futures rate and \( D \) is the number of days in the month. Since the effective federal funds rate tracks the target rate closely \( \eta_t \) provides a timely measure of the surprise component to target rate changes. For the third measure, I follow Cochrane and Piazzesi (2002) and define the target shock \( \eta^{\text{cp}} \) as the daily change in the 3-month euro-dollar rate around target changes

\[
\eta^{\text{cp}}(t) = \left( e_{1m,d}^1 - e_{1m,d-1}^1 \right)
\]  

(2.19)

Figure 2.14 shows that while the three series are generally positively correlated their dynamics shows several periods in which the information content of the three series is rather different. Indeed, Cochrane and Piazzesi (2002) argue that ‘If in one year the Fed worries about inflation, but in another year it places more weight on unemployment, market forecasts will adapt, but vector auto-regressions (VARs) may not adapt and thus may incorrectly interpret anticipated actions to be shocks.’ The \( \eta^{\text{cee}}(t) \) target shocks assume a time invariant VAR structure. While ‘pure’ unanticipated changes, captured by \( \eta^{\text{cp}}(t) \) and \( \eta^{\text{bk}}(t) \), proxy temporary variation in the preferences of the Fed, they may not convey long term information about the expected path of monetary policy. Constructing \( \text{PathShock}_t \) from surveys allows a real-time assessment of how agents expected the stance of the monetary policy to evolve over time.

To study the link between realised policy shocks and beliefs about future shocks I run regression of \( \text{PathShock}_t \) on the three target shocks described above. Panel A of table 2.8 reports results for path

\[\text{These are traded on the Chicago Board of Trade where the implied futures rate is 100 minus the contract price.}\]
shocks on target shocks, while panel B reports path shocks on a 6-month lagged summation of past target shocks. Considering first the statistical link I find slope coefficients that are consistently negative across specification for $\eta_i^t$. The estimated loads are convincingly significant, 5/6 of the loadings are significant at the 1% level. While the $R^2$'s of (noisy) contemporaneous shocks on path shocks are low as expected, the lagged $\eta^p(t)$ and $\eta^b(t)$, explain 17% and 13% of the predictable variation in $PathShock_t$, respectively. More interesting than statistical significance I find that target shocks are negatively correlated to expectations about the future stance of policy. This is interesting since I learn a non-trivial link between observed short-term policy actions (target shocks) and the formation of expectations about future policy actions (path shocks).

[ Insert table 2.8 about here ]

In general the dependence of path shocks on target shocks could take zero, positive, negative values. If agents form expectations about future policy ignoring current actions I should expect a zero loading in table 2.8. Alternatively, if agents believe policy shocks are subject to regime shifts and learn that they will revert in the future, the loadings should be negative. This observation is consistent with Sims and Zha (2006) who finds that variance of structural disturbances in Taylor rule regressions are subject to regime shifting components that are short lived. Alternatively, positive loadings could be rationalised through a number of behavioural biases such ‘representativeness’ (Tversky and Kahneman (1974)). In this case agents place put too much weight on recent experiences, such as a series of negative rate cuts, and extrapolate this as the likely path of policy going forward.

A revealing episode to learn about this link is given by the joint dynamics of target rate changes and $PathShock_t$ during the dotcom bubble bust. This was a period that witnessed a large shift in the non-systematic component of U.S monetary policy. In 1996 Alan Greenspan, then the Chairman of the Federal Reserve, tried unsuccessfully to talk down the market in the face of an over heating technology sector. However, the housing market and dotcom sector rally continued. In response between June 1996 and May 2000 the Fed raised the target rate from 4.75% to 6.5%. This represented a shift from standard Taylor rule logic since CPI inflation average just $\sim$ 2.20% over this period. However, the path of monetary policy was quickly reversed on September 11th 2001. In response to the Trade Center attacks the Fed acted quickly with a series of rate cuts to shore up market confidence. To support the stock market, between January 2001 and January 2002 The Fed lower the target rate from 6.5% to 1.75%. The market forecast was significantly surprised by 4 of these rate changes as evidenced by the large negative realizations for both $\eta^p(t)$ and $\eta^b(t)$ shocks (see figure 2.14). However, over this period $PathShock_t$ actually rose. This implies that, as market participants observed large swings in contemporaneous Taylor rule residuals, they inferred the stance of the Fed would revert in the future.

---

This narrative and the negative loadings in table 2.15 are consistent with the hypothesis that extreme policy response will be reversed in the future. This implies path shocks are, on average countercyclical, different than target shocks, which are procyclical. Figure 2.13 makes the counter-cyclicality of PathShock clear by comparing the time-series dynamics of path shocks to a proxy of macroeconomic activity ($g_t$) discussed in the data section above, and NBER recession dates (shaded grey areas). In all three recessions, PathShock rises when macroeconomic activity drops. Since bond risk premia are known to be countercyclical (Duffee (2002), Cochrane and Piazzesi (2005b)), this motivates the natural question of whether its co-movement with PathShock is also quantitatively important, which constitutes the topic of the next section.

D. Path Shocks and Risk Premium Proxies

To investigate whether PathShock, extracted from macroeconomic surveys, is a potential source of priced risk, I first investigate its co-movement with proxies of bond risk premia proposed by other well-known independent studies. I consider three proxies for bond risk premia, $Z_t$: (i) the slope of the yield curve as studied by Campbell and Shiller (1991b) ($\text{Slope}_t = y_5(t) - y_1(t)$); (ii) the forward rate factor of Cochrane and Piazzesi (2005b) ($\text{CP}_t$), and (iii) the two volatility factors constructed by Le and Singleton (2013b) ($\text{LS}_{1t}$ and $\text{LS}_{2t}$). I assess the link between path shocks via linear regressions:

$$\text{PathShock}_t = \text{const.} + Z'_t \beta + \epsilon_t. \quad (2.20)$$

Table 2.9 presents the results of the regressions. There are two conclusions to be drawn. Regardless of the bond risk premium proxy there is a strong positive statistically significant link to PathShock. Firstly, consistent with the notation that path shocks are counter-cyclical the loading on the slope of the yield curve is positive, with a t-stat equal to 2.19, and an $R^2$ equal to 8%: when agents expect monetary policy to revert from a loosening cycle long term bonds command risk positive risk premium. Second, PathShock co-move positively with $\text{CP}_t$ and $\text{LS}_{2t}$ (t-stats above 2.2 and 2.8, and $R^2$ equal to 13% and 24%, respectively), suggesting that beliefs about changes to the future stance of policy are linked to contemporaneous bond risk premia.$^{11}$ Figure 2.16 makes this point clear graphically by plotting PathShock versus the three risk premium proxies.

Table 2.9 presents the results of the regressions. There are two conclusions to be drawn. Regardless of the bond risk premium proxy there is a strong positive statistically significant link to PathShock. Firstly, consistent with the notation that path shocks are counter-cyclical the loading on the slope of the yield curve is positive, with a t-stat equal to 2.19, and an $R^2$ equal to 8%: when agents expect monetary policy to revert from a loosening cycle long term bonds command risk positive risk premium. Second, PathShock co-move positively with $\text{CP}_t$ and $\text{LS}_{2t}$ (t-stats above 2.2 and 2.8, and $R^2$ equal to 13% and 24%, respectively), suggesting that beliefs about changes to the future stance of policy are linked to contemporaneous bond risk premia.$^{11}$ Figure 2.16 makes this point clear graphically by plotting PathShock versus the three risk premium proxies.

11 The factor $LS_{2t}$ is the dominating risk premium factor in Le and Singleton (2013b)
expectations may differ from the expectations of the marginal investor. The ability of yields to span path shocks largely depends on the relative difference in dimensions of the state vector under the physical versus agents subjective measure. Duffee (2011) discusses this point in the context of the invertibility of the current yield curve to reveal information relevant for bond risk premia. Moreover, the fact that a proxy for path shocks is correlated with a set of (spanned) risk premia proxies suggests that expectations of future monetary policy affect the shape of the yield today curve not only through physical expectations of the short rate but also through a change of measure.

IV. Bond return predictability

This section establishes the key empirical result of this chapter: path shocks drive the time variation of bond risk premia. I obtain these results via classic return predictability projections of one-year holding period bond excess returns on lagged \( \text{PathShock} \), and test for the statistical and economic significance of the slope coefficient.

I verify the robustness of the results from a variety of perspectives. Firstly, an online appendix presents an expanded set of results for all specifications of \( \text{PathShock} \), bond maturities, and subsamples. Second, I assess the statistical and economic significance of the slope coefficients in predictive regressions for bonds with maturities ranging between 2 and 5 years controlling for alternative risk factor proxies. Finally, I analyze the stability of the coefficients in a subsample that excludes the crisis.

A. Predictability Regressions

Table 2.10 reports the estimation output of regressions:

\[
x^{(n)}_{t+12} = \text{const.} + \beta^{(n)}_{PS} \text{PathShock}_{t} + \epsilon^{(n)}_{t+12},
\]

where bond maturities \( n \) range from 2 to 5 years. The estimates in the left panel are for the entire sample (the last excess return is defined between 2011:7 and 2012:7), while those on the right exclude the financial crisis (the last excess return is defined between 2007:6 and 2008:6). Reported \( \hat{R}^2 \) are adjusted, and all t-statistics employ Newey-West standard errors (18 lags); all left and right hand side variables are standardized, so that the coefficients can be interpreted as standard deviation changes of the regressand for a unit standard deviation change in the regressor.

[Insert Table 2.10 about here.]

In all projections, \( \text{PathShock} \) explains the time variation in bond excess returns in an economically and statistically significant fashion. Estimated slope coefficients are between 0.35 and 0.40: a one standard deviation increase in path shocks predicts, on average, an increase of fitted excess returns by \( \sim 40\% \) of its unconditional standard deviation. Adjusted \( R^2 \) range between 12.00% and 15.53%, so that monetary policy path shocks also explain a large fraction of the overall variation of realized excess returns. Also, the statistical significance is large: all t-statistics reject the null hypothesis of no predictability at the 1% level.
Besides a slight decline of the economic magnitude of the slope coefficients for longer maturity bonds, there are no noticeable patterns across specifications of the right hand side variable and sample period. Overall, the results suggest that path shocks are not only a statistically and economically significant, but also robust predictor of future realized excess returns.

B. Real Growth and Inflation

Previous literature on return predictability in bond markets has highlighted the importance of real activity, which is one of the systematic ingredients of the Taylor rule. To evaluate the marginal contribution of the policy shocks versus the systematic component of the rule, I follow Ludvigson and Ng (2009b) and construct a real activity factor ($g_t$) from the first principle component from a large panel of macroeconomic indicators that includes 104 individual macro time series available at monthly frequency. This is different than the real argument of the Taylor used to estimate path shocks but is consistent with the Fed responding to deviations in real activity from a target. In addition, this provides a tougher test for the marginal ability of path shocks to forecast returns since the forecasting power of $g_t$ is well documented.

Table 2.11 repeats the predictability regressions above controlling for $g_t$. The loadings on $g_t$ are negative and significant for all maturities; both the statistical and economic significance are strongest for $n = 2$ and decrease in maturity. The inclusion of $g_t$ in the set of regressors leads to a sizable increase in the adjusted $R^2$ relative to the univariate case, especially for short maturities and in the 1990-2007 sample. In the case of bonds with 2 years maturities, the average increase in $R^2$ is between 10% and 15% in the 1990-2007 period, and between 5% and 10% for the full sample; the effect decreases in maturity, becoming negligible for 5 years bonds. Nevertheless, the economic and statistical significance of expected monetary policy path shocks is hardly affected. The conclusion to this section is that the information content in PathShock relevant for the bond risk premium is not subsumed by the contemporaneous level of macroeconomic activity.

A second potential issue is that expected inflation is endogenous to the monetary policy shocks. For instance, Gallmeyer, Hollifield, Palomino, and Zin (2007a) discuss an economy with recursive preferences and monetary policy. In this economy inflation is endogenous to a Taylor rule which helps match the historical level of long term yields by introducing negative autocorrelation to the pricing kernel. Moreover, to the extent that monetary policy affects inflation and inflation is priced in nominal bond returns, PathShock may affect bond returns through an inflation channel. For these reasons, I run a second test controlling for consensus expectations on inflation from BlueChip surveys. Table 2.12 summarizes the results. I find that in the sample period excluding the financial crisis, expected inflation does indeed contain marginal information on bond risk premia. The slope coefficients are positive with comparable economic magnitude of PathShock. However, PathShock retains statistical significance even after controlling for inflation. Moreover, in the sample period including the crisis, the predictable content of expected inflation disappears while PathShock maintains its statistical significance.

[Insert Table 2.11 and 2.12 about here.]
C. Discussion: PathShock and the financial crisis

Tables 2.10 and 2.11 document that the economic and statistical significance of predictability is largely unaffected by the inclusion of the last financial crisis in the sample. This may sound, at first sight, a little surprising. Our predictor, \textit{PathShock}, is based on the notion that the federal fund rate is the instrument of monetary policy, a tool that has lost its flexibility and effectiveness in the context of the ZLB (zero lower bound) characterizing the US monetary landscape since the end of 2008.\footnote{During the first turmoil and Lehman's collapse, the Fed engaged in a particularly intense series of target rate cuts: between 18 September 2007 and 16 December 2008, the target fed funds rate was decreased on each of the 10 FOMC meetings, going from 4.75\% down to a 0\%-0.25\% range. Between 16 December 2008 and December 2012, the Fed maintained the target rate in the 0\%-0.25\% range uninterruptedly.} Unable to cut federal funds targets any further, US monetary authorities have started considering forward guidance\footnote{Forward guidance represents the result of a decade-long process of changes in the strategy underpinning policy communication. The structure of FOMC statements has been modified to include: (i) an economic outlook, in January 2000; (ii) qualitative statements about future policy inclinations, in August 2003; (iii) calendar-based guidance, in August 2011; (iv) outcome-based guidance, in December 2012.} and QE (Quantitative Easing)\footnote{Quantitative Easing policies consist of purchases, by the central bank, of specified quantities of long term financial assets. Our sample includes two instances of QE policies: (i) QE1, between late 2008 and 2009; and (ii) QE2, between the second quarters of 2010 and 2011. While QE1 consisted of purchases of MBS, Treasuries, and Agency securities, QE2 focused only on the purchase on long term Treasury securities. See Gagnon, Raskin, Remache, and Sack (2011) and Krishnamurthy and Vissing-Jorgensen (2010) for further details about QE policies and their quantitative impact on financial securities.} as alternative policy instruments (see Woodford (2012) for an extensive discussion).

Given such profound changes in the way that monetary policy is implemented, are measures of monetary policy shocks based on Taylor rule residuals appropriate and, more specifically, is \textit{PathShock} suitable to measure exogenous variation in monetary policy? \textit{Target} shocks, measured as residuals from Taylor rules estimated on \textit{current} federal funds, are indeed meaningless for two reasons. First, Taylor rules imply negative nominal federal funds rates in negative GDP growth and low inflation scenarios, thus ceasing to be adequate representations of the systematic and exogenous components of monetary policy. Second, surprises about \textit{current} federal funds targets are little informative about how monetary policy is actually conducted in practice, since the Federal Reserve Bank has, de facto, switched its policy instrument from \textit{current} federal funds targets to forward guidance (at least temporarily). These criticisms, however, do not apply to residuals from Taylor rules estimated over expected \textit{future} federal funds, and therefore to \textit{PathShock}. First, despite the 0\%-0.25\% range imposed by the Fed onto \textit{current} federal funds rates since December 2008, \textit{expected} federal funds rates have featured noticeable volatility over the same period (see Figure ??). Second, \textit{expected} future short rates are precisely the instrument of policies based on forward guidance: being a residual from a Taylor rule estimated on expectations of \textit{future} short rates, \textit{PathShock} is, by construction, a measure of the exogenous variation in forward guidance. As a consequence, \textit{PathShock} is particularly suitable to measure exogenous monetary policy shocks in the recent monetary environment.
V. Conclusion

Much of the term structure literature uses latent factors to model the dynamics of yields without attempting any economic interpretation. Motivated by the link between monetary policy and the short term interest rate more recent literature has moved towards an explicit role for policy shocks within no-arbitrage term structure models. This paper asks a more fundamental question: does monetary policy represent a source of priced risk in bond markets and is it important for understanding time-variation in the bond risk premium?

To answer this question I examine the reaction of nominal Treasury yields to monetary policy shocks around FOMC meetings. I construct two types of shock using Federal funds futures and long dated Eurodollar futures: (i) target shocks that reflect current behavior of the policy instrument; and (ii) path shocks that capture variation in the projected path of monetary policy. I document that while short dated nominal Treasuries react strongly to target shocks, changes to long dated yields are mainly driven by path shocks. Exploring the economic source of this variation I construct a bond risk premium proxy available at daily frequency and show that path shocks command a positive adjusted to risk compensation.

Next, I construct a time-series equivalent for FOMC paths shocks: \(PathShock\). This measure is constructed in real-time from the residuals of Taylor rules estimated on survey forecasts of federal funds rates, GDP growth, and inflation. Through classical return predictability regressions I establish that \(PathShock\)'s account for 10%-15% of the variance of one-year excess returns for bond maturities between 2 and 5 years and are significant at the 1% level.

These results have implications for both asset pricing and monetary policy. First, the evidence suggests that macro-finance models of the term structure should include a role for monetary policy not only through physical expectations of future short rates but also through risk-adjusted ones. Second, the results indicate that monetary authorities can learn about market expectations for the path of policy from real-time survey forecasts, and therefore represent a valuable tool in implementing policy today.
VI. Appendix: Figures

**Figure 2.1. Treasury Yields:**
Figure plots time series of the Federal funds target rate, the Fed funds effect rate, and Treasury yields at 6 month, 2 year and 5 year maturities from 1982 - 2014.

**Figure 2.2. Expected vs Unexpected Changes:**
Figure plots expected versus unexpected changes in the front month Federal funds futures around all FOMC meetings. Our sample includes 164 FOMC meetings between 1990.1 and 2008.6
**Figure 2.3. Cumulative Shocks:**
Figure plots the cumulative sum of shocks to front and 3rd month contract Federal funds futures ($e_0,e_3$), and 6, 9 and 12 month eurodollar rates($E_6,E_9,E_{12}$).

**Figure 2.4. Futures Predictions:**
Figure plots predicted changes by the future market. The left panel plots those occasions when the future markets predicted no change in the effective rate, and the red bars indicate those occasions when a change did occurred. The right panel shows the occasions when future markets predicted a change, and the red bars indicate those occasions when a change did actually not occur.
Figure 2.5. Futures Predictions:
Figure plots unexpected changes in the front month futures contract (y-axis) with respect to changes in the 1-quarter (3rd month) contract (x-axis). The red line in the plot is the 45 degrees lines; the blue line is the least squares fit.

\[ y = -0.01 + 0.39 \times \]

\[ R^2 = 0.69 \]
Figure 2.6. Target vs Timing Shock:
Figure plots the time-series behaviour of two components of the change in the 1-quarter contract: (i) the change in the front month contract (the ‘target’ shock); (ii) the difference between the shock to the 4 month shock and to the front-month contract (the ‘timing’ shock)
Figure 2.7. All Shocks:
Figures plots shocks to the front and third month Federal funds futures and shocks to 6, 9 and 12 month eurodollar rates around FOMC announcements.

Figure 2.8. Target vs Path Shocks:
Figures plots target versus path shocks. The left panel plots those occasion when these moved in the same direction while the right panel plots those occasions when the shocks moved in opposite directions.
Figure 2.9. Daily Cochrane-Piazzesi:
Figure compares the behavior of the Cochrane and Piazzesi (2005b) return forecasting factor constructed at monthly and daily frequency.

Figure 2.10. Federal Funds Rate Forecasts:
Figure plots 1-quarter to 4-quarter consensus forecasts for the level of the Federal funds rate in percentage points.
Figure 2.11. Consensus Macro Forecasts:
This figure plots the time series of consensus forecasts for 1-year inflation ($\pi_{C,t,12}$), GDP growth ($g_{C,t,12}$) and proxy for the level of macroeconomic activity ($g_t$) described in the data section. Inflation and GDP forecasts are plotted against the right y-axis in percentage points. Macroeconomic activity is plotted on the left axis in standardised units.
Figure 2.12. The Performance of BCFFS Forecasts
The figure plots the time series of the number of agents in the cross-section whose forecast error is, in absolute value, less than the absolute value of the VAR forecast error (left panels), and the difference between the absolute value of the VAR forecast error and the absolute value of the average BCFFS forecast error (right panels). Forecast horizon: one quarter.
This figure plots monetary policy path shocks $PathShock$, constructed as cross-sectional averages of the residuals from Taylor rules estimated over a panel of forecast data. Each series corresponds to one of the 6 specifications described in Table 2.7. Sample period: 1990:1 - 2011:7.

Figure 2.14. Comparing Shocks:
Figure plots $PathShock$ against three proxies for target shocks proposed by the literature: (i) the residuals in a monthly orthogonalised VAR ($\eta_{\text{cee}}$); (ii) the 1-day change in the 3 month euro-dollar rate around FOMC announcements ($\eta_{\text{cp}}$); and (iii) the 1-day change in the 1-month Federal funds futures rate around FOMC announcements ($\eta_{\text{bk}}$).
Figure 2.15. Counter-cyclicality of PathShock
This figure plots monetary policy path shocks PathShock (specification 1), and macroeconomic activity, $g$. Areas shaded in gray indicate NBER recessions. Sample period: 1990:1 - 2011:7. Time series are standardised for easy comparison.

Figure 2.16. Monetary policy shocks and yield curve information
This figure plots PathShock against three risk premium proxies: (i) the slope of the yield curve $\text{Slope}_t = y_t^{(5)} - y_t^{(1)}$; (ii) the forward rate factor of Cochrane and Piazzesi (2005b) ($CP_t$); and (iii) a volatility factor from Le and Singleton (2013b) ($LS^2_t$).
VII. Appendix: Table

Table 2.1. Summary Statistics:
Table reports summary statistics for expected versus unexpected changes in the front month Federal funds futures rate, the change in the 3rd month contract, and the change in 6, 9 and 12 month eurodollar rates. Our sample includes 164 FOMC meetings between 1990.1 and 2008.6.

<table>
<thead>
<tr>
<th></th>
<th>Δr</th>
<th>E[Δr]</th>
<th>Δr − E[Δr]</th>
<th>ΔFF3</th>
<th>ΔE6</th>
<th>ΔE9</th>
<th>ΔE12</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>−0.0259</td>
<td>0.0060</td>
<td>−0.0319</td>
<td>−0.0226</td>
<td>−0.0138</td>
<td>−0.0155</td>
<td>−0.0140</td>
</tr>
<tr>
<td>var</td>
<td>0.0503</td>
<td>0.0457</td>
<td>0.0353</td>
<td>0.0079</td>
<td>0.0255</td>
<td>0.0202</td>
<td>0.0178</td>
</tr>
</tbody>
</table>

Table 2.2. FOMC Announcements: Expected vs Unexpected Changes
Table reports regressions of the daily change in {3, 6}-month bills rates, and {1, 3, 5, 10}-year note rates on target changes (top panel), and expected versus unexpected components of the target change (bottom panel). OLS t-stats are reported in parenthesis.

<table>
<thead>
<tr>
<th>Target Rate Changes</th>
<th>Bill 3m</th>
<th>Bill 6m</th>
<th>1yr</th>
<th>3yr</th>
<th>5yr</th>
<th>10yr</th>
</tr>
</thead>
<tbody>
<tr>
<td>target</td>
<td>0.20</td>
<td>0.16</td>
<td>0.11</td>
<td>0.05</td>
<td>0.02</td>
<td>−0.01</td>
</tr>
<tr>
<td>(8.33)</td>
<td>(5.97)</td>
<td>(2.96)</td>
<td>(1.34)</td>
<td>(0.62)</td>
<td>(−0.38)</td>
<td></td>
</tr>
<tr>
<td>R²</td>
<td>0.37</td>
<td>0.28</td>
<td>0.11</td>
<td>0.02</td>
<td>−0.00</td>
<td>−0.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Expected vs Unexpected Changes</th>
<th>Bill 3m</th>
<th>Bill 6m</th>
<th>1yr</th>
<th>3yr</th>
<th>5yr</th>
<th>10yr</th>
</tr>
</thead>
<tbody>
<tr>
<td>expected</td>
<td>0.15</td>
<td>0.10</td>
<td>0.05</td>
<td>0.00</td>
<td>−0.02</td>
<td>−0.04</td>
</tr>
<tr>
<td>(5.84)</td>
<td>(5.75)</td>
<td>(1.56)</td>
<td>(0.02)</td>
<td>(−0.73)</td>
<td>(−1.44)</td>
<td></td>
</tr>
<tr>
<td>unexpected</td>
<td>0.30</td>
<td>0.29</td>
<td>0.22</td>
<td>0.15</td>
<td>0.11</td>
<td>0.04</td>
</tr>
<tr>
<td>(6.15)</td>
<td>(5.50)</td>
<td>(3.70)</td>
<td>(2.52)</td>
<td>(2.23)</td>
<td>(1.12)</td>
<td></td>
</tr>
<tr>
<td>R²</td>
<td>0.50</td>
<td>0.50</td>
<td>0.30</td>
<td>0.13</td>
<td>0.10</td>
<td>0.04</td>
</tr>
</tbody>
</table>

Table 2.3. FOMC Announcements: Timing vs Level
Table reports regressions of the daily change in {3, 6}-month bills rates, and {1, 3, 5, 10}-year note rates on level and timing shocks. OLS t-stats are reported in parenthesis.

<table>
<thead>
<tr>
<th>Timing vs Level</th>
<th>Bill 3m</th>
<th>Bill 6m</th>
<th>1yr</th>
<th>3yr</th>
<th>5yr</th>
<th>10yr</th>
</tr>
</thead>
<tbody>
<tr>
<td>level</td>
<td>0.55</td>
<td>0.60</td>
<td>0.50</td>
<td>0.44</td>
<td>0.36</td>
<td>0.22</td>
</tr>
<tr>
<td>(5.70)</td>
<td>(6.34)</td>
<td>(4.38)</td>
<td>(3.62)</td>
<td>(3.16)</td>
<td>(2.44)</td>
<td></td>
</tr>
<tr>
<td>timing</td>
<td>0.54</td>
<td>0.60</td>
<td>0.49</td>
<td>0.48</td>
<td>0.39</td>
<td>0.27</td>
</tr>
<tr>
<td>(5.52)</td>
<td>(6.41)</td>
<td>(4.13)</td>
<td>(3.38)</td>
<td>(2.70)</td>
<td>(2.12)</td>
<td></td>
</tr>
<tr>
<td>R²</td>
<td>0.46</td>
<td>0.59</td>
<td>0.40</td>
<td>0.23</td>
<td>0.17</td>
<td>0.07</td>
</tr>
</tbody>
</table>
We stack the shocks to five contracts (2 Fed funds and 3 Eurodollar) around each FOMC announcement in a $T \times 5$ matrix and perform a principle component decomposition. This table reports a decomposition of the variance explained by the first three principal components of the covariance matrix of the shocks. OLS t-stats are reported in parenthesis.

<table>
<thead>
<tr>
<th>PC1</th>
<th>PC2</th>
<th>PC3</th>
</tr>
</thead>
<tbody>
<tr>
<td>%</td>
<td>61.98</td>
<td>32.97</td>
</tr>
</tbody>
</table>

Table 2.5. FOMC Announcements: Target vs Path
Table reports regressions of the daily change in $\{3, 6\}$-month bills rates, and $\{1, 3, 5, 10\}$-year note rates on surprise and path shocks. OLS t-stats are reported in parenthesis.

<table>
<thead>
<tr>
<th></th>
<th>Bill 3m</th>
<th>Bill 6m</th>
<th>1yr</th>
<th>3yr</th>
<th>5yr</th>
<th>10yr</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surprise</td>
<td>0.68</td>
<td>0.67</td>
<td>0.47</td>
<td>0.28</td>
<td>0.22</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>(4.00)</td>
<td>(4.27)</td>
<td>(3.28)</td>
<td>(2.50)</td>
<td>(2.41)</td>
<td>(1.15)</td>
</tr>
<tr>
<td>Path</td>
<td>0.17</td>
<td>0.32</td>
<td>0.49</td>
<td>0.57</td>
<td>0.57</td>
<td>0.56</td>
</tr>
<tr>
<td></td>
<td>(1.87)</td>
<td>(2.86)</td>
<td>(3.11)</td>
<td>(3.16)</td>
<td>(3.20)</td>
<td>(3.31)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.52</td>
<td>0.63</td>
<td>0.55</td>
<td>0.45</td>
<td>0.41</td>
<td>0.33</td>
</tr>
</tbody>
</table>

Table 2.6. Daily Risk Premium Regressions:
Table reports regressions of the daily change in a Cochrane and Piazzesi (2005b) factor constructed at daily frequency on surprise and path shocks:

$$\Delta CP_t = \text{const} + \beta_{\text{surprise}} + \gamma_{\text{path}} + \varepsilon_t$$

OLS t-stats are reported in parenthesis.

<table>
<thead>
<tr>
<th></th>
<th>$\Delta CP_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>surprise</td>
<td>$-0.20$</td>
</tr>
<tr>
<td></td>
<td>($-2.46$)</td>
</tr>
<tr>
<td>Path</td>
<td>0.31</td>
</tr>
<tr>
<td></td>
<td>(2.89)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.10</td>
</tr>
</tbody>
</table>
Table 2.7. Taylor rule specifications
This table describes different specifications of the general Taylor rule model
\[ f_{n,t,h} = \rho_1 f_{n,t,h-1} + \rho_2 f_{n,t,h-2} + (1 - \rho)(f + \beta (\pi_{n,t,i}^e - \pi^e) + \gamma x_{n,t,i}^e) + u_{n,t,h}. \]
The first row contains the horizon, in months, of the federal funds rate forecast. The second row contains the horizon, in months, of inflation and output gap forecasts. The third row describes the type of output gap employed; output gap 1 is constructed using full sample information, while output gap 2 is constructed recursively. The fourth and fifth rows report the test-statistic and p-value of the F-test for the joint significance of the individual effects. The final two rows report the test-statistic and p-value of the Hausman test for systematic differences between random and fixed effects coefficients (null hypothesis: random effects is appropriate).

<table>
<thead>
<tr>
<th>Specification</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>h</td>
<td>12</td>
<td>12</td>
<td>9</td>
<td>9</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>i</td>
<td>12</td>
<td>12</td>
<td>9</td>
<td>9</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>GAP</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>F(FE)</td>
<td>7.69</td>
<td>7.56</td>
<td>8.86</td>
<td>8.85</td>
<td>8.19</td>
<td>8.31</td>
</tr>
<tr>
<td></td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Hausman</td>
<td>52.91</td>
<td>50.61</td>
<td>32.71</td>
<td>37.17</td>
<td>31.28</td>
<td>36.23</td>
</tr>
<tr>
<td></td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Table 2.8. Path Shocks vs Target Shocks
Table reports results of a regression of \( PathShock_t \) on test three proxies for target shocks that have been studied in the literature: (i) the residuals in a monthly orthogonalised VAR \( (\eta_{cex}^e) \); (ii) the 1-day change in the 3 month euro-dollar rate around FOMC announcements \( (\eta_{cp}^e) \); and (iii) the 1-day change in the 1-month Federal funds futures rate around FOMC announcements \( (\eta_{bk}^e) \). Panel A reports loadings, t-statistics (White standard errors) and \( R^2 \) from
\[ PathShock_t = \alpha + \beta \eta_{cex}^e + \varepsilon_t \]
while Panel B reports regressions of \( PackShock_t \) on a 6-month moving sum of past \( \eta_{cex}^e \) shocks
\[ PathShock_t = \alpha + \beta \sum_{k=1}^{6} \eta_{cex}^{e,k} + \varepsilon_t \]

<table>
<thead>
<tr>
<th>( \eta_{cex}^e )</th>
<th>( \eta_{cp}^e )</th>
<th>( \eta_{bk}^e )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>-0.04</td>
<td>-0.03</td>
</tr>
<tr>
<td>t-stat</td>
<td>-2.03</td>
<td>-0.99</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.02</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Panel A

| \( \beta \)      | -0.02            | -0.06            |
| t-stat           | -2.77            | -4.97            | -4.43           |
| \( R^2 \)        | 0.06             | 0.15             | 0.10             |

Panel B
Table 2.9. Risk Premium Proxies

The table reports the results from regressions of $PathShock$ on bond risk premia proxies extracted from date $t$ yield curve information:

$$PathShock_t = \text{const.} + \beta Z_t + \epsilon_t$$

The proxies for yield based risk premium proxies $Z_t$ are the slope of the yield curve as in Campbell and Shiller (1991b) ($\text{Slope}_t = y_5^{(5)} - y_1^{(1)}$), the forward rate factor of Cochrane and Piazzesi (2005b) ($CP_t$), and the two volatility factors estimated by Le and Singleton (2013b) ($LS_{1t}$ and $LS_{2t}$). T-statistics, reported below in parenthesis are corrected for auto-correlation and heteroskedasticity using Newey-West errors (18 lags). Both left and right hand variables are standardized. A constant is included but not reported. Sample period: 1990:1 - 2007:12.

<table>
<thead>
<tr>
<th>$Slope_t$</th>
<th>$CP_t$</th>
<th>$LS_{1t}$</th>
<th>$LS_{2t}$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.29</td>
<td></td>
<td></td>
<td>0.08</td>
</tr>
<tr>
<td>t-stat</td>
<td>(2.19)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.36</td>
<td></td>
<td></td>
<td>0.12</td>
</tr>
<tr>
<td>t-stat</td>
<td>(3.03)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.06</td>
<td>0.52</td>
<td>0.23</td>
<td></td>
</tr>
<tr>
<td>t-stat</td>
<td>(0.32)</td>
<td>(2.94)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2.10. Bond Return Predictability: $PathShock$

The table reports the output from regressions of annual bond excess returns on a constant and expected monetary policy shocks:

$$r_x^{(n)}_{t+12} = \text{const.} + \beta^{(n)}_{PathShock} + \epsilon^{(n)}_{t+12}.$$  

Bond maturities ($n$) range from 2 to 5 years. Each panel reports the results for one of the 6 proxies of expected monetary policy described by Table 2.7. The left panels report the results for the full sample (the last observation is the excess return that realized between 2011:7 and 2012:7), while the right panels report the results for the sample excluding the crisis (the last observation is the excess return that realized between 2007:6 and 2008:6). T-statistics, reported below in parenthesis are corrected for auto-correlation and heteroskedasticity using Newey-West errors (18 lags). $\bar{R}^2$ is the adjusted $R^2$. Both left and right hand variables are standardized. A constant is included but not reported.

<table>
<thead>
<tr>
<th>$n$</th>
<th>$PathShock$</th>
<th>$\bar{R}^2$</th>
<th>$PathShock$</th>
<th>$\bar{R}^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.38</td>
<td>14.00%</td>
<td>0.40</td>
<td>15.53%</td>
</tr>
<tr>
<td></td>
<td>(3.13)</td>
<td>(3.07)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.37</td>
<td>13.73%</td>
<td>0.39</td>
<td>14.62%</td>
</tr>
<tr>
<td></td>
<td>(3.19)</td>
<td>(3.00)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.38</td>
<td>14.05%</td>
<td>0.38</td>
<td>13.70%</td>
</tr>
<tr>
<td></td>
<td>(3.35)</td>
<td>(2.89)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.35</td>
<td>12.05%</td>
<td>0.35</td>
<td>12.00%</td>
</tr>
<tr>
<td></td>
<td>(3.19)</td>
<td>(2.77)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 2.11. Bond Return Predictability: PathShock and $g_t$

The table reports the output from regressions of annual bond excess returns on a constant, expected monetary policy shocks, and levels of macroeconomic activity:

$$rx_t^{(n)} = const + \beta_{PathShock}^{(n)} \text{PathShock} + \beta_g^{(n)} g_t + \epsilon_t^{(n)}$$

Bond maturities ($n$) range from 2 to 5 years. The left panels report the results for the full sample (the last observation is the excess return that realized between 2011:7 and 2012:7), while the right panels report the results for the sample excluding the crisis (the last observation is the excess return that realized between 2007:6 and 2008:6). T-statistics, reported below in parenthesis are corrected for auto-correlation and heteroskedasticity using Newey-West errors (18 lags). $\bar{R}^2$ is the adjusted $R^2$. Both left and right hand variables are standardized. A constant is included but not reported.

<table>
<thead>
<tr>
<th>$n$</th>
<th>PathShock</th>
<th>$g$</th>
<th>$\bar{R}^2$</th>
<th>PathShock</th>
<th>$g$</th>
<th>$\bar{R}^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.35</td>
<td>-0.31</td>
<td>22.97%</td>
<td>0.23</td>
<td>-0.40</td>
<td>28.04%</td>
</tr>
<tr>
<td></td>
<td>(3.32)</td>
<td>(-2.70)</td>
<td>(1.85)</td>
<td>(-4.24)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.35</td>
<td>-0.28</td>
<td>21.16%</td>
<td>0.25</td>
<td>-0.32</td>
<td>22.54%</td>
</tr>
<tr>
<td></td>
<td>(3.30)</td>
<td>(-2.89)</td>
<td>(1.95)</td>
<td>(-3.57)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.36</td>
<td>-0.23</td>
<td>18.89%</td>
<td>0.26</td>
<td>-0.26</td>
<td>18.78%</td>
</tr>
<tr>
<td></td>
<td>(3.27)</td>
<td>(-2.41)</td>
<td>(2.00)</td>
<td>(-3.00)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.33</td>
<td>-0.20</td>
<td>15.90%</td>
<td>0.27</td>
<td>-0.19</td>
<td>14.45%</td>
</tr>
<tr>
<td></td>
<td>(3.10)</td>
<td>(-2.50)</td>
<td>(2.08)</td>
<td>(-2.32)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2.12. Bond Return Predictability: PathShock and $E[\pi_t]$

The table reports the output from regressions of annual bond excess returns on a constant, expected monetary policy shocks, the consensus forecast for the 4Q-ahead ahead rate of inflation activity:

$$rx_t^{(n)} = const + \beta_{PathShock}^{(n)} \text{PathShock} + \beta_{E[\pi_t]}^{(n)} E[\pi_t] + \epsilon_t^{(n)}$$

Bond maturities ($n$) range from 2 to 5 years. The left panels report the results for the full sample (the last observation is the excess return that realized between 2011:7 and 2012:7), while the right panels report the results for the sample excluding the crisis (the last observation is the excess return that realized between 2007:6 and 2008:6). T-statistics, reported below in parenthesis are corrected for auto-correlation and heteroskedasticity using Newey-West errors (18 lags). $\bar{R}^2$ is the adjusted $R^2$. Both left and right hand variables are standardized. A constant is included but not reported.

<table>
<thead>
<tr>
<th>$n$</th>
<th>PathShock</th>
<th>$E[\pi_t]$</th>
<th>$\bar{R}^2$</th>
<th>PathShock</th>
<th>$E[\pi_t]$</th>
<th>$\bar{R}^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>rx(2)</td>
<td>0.35</td>
<td>0.29</td>
<td>0.22</td>
<td>0.39</td>
<td>0.39</td>
<td>0.30</td>
</tr>
<tr>
<td></td>
<td>(2.94)</td>
<td>(2.36)</td>
<td>(3.20)</td>
<td>(3.05)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>rx(3)</td>
<td>0.36</td>
<td>0.21</td>
<td>0.18</td>
<td>0.38</td>
<td>0.32</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>(2.95)</td>
<td>(1.64)</td>
<td>(2.88)</td>
<td>(2.45)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>rx(4)</td>
<td>0.36</td>
<td>0.18</td>
<td>0.17</td>
<td>0.37</td>
<td>0.31</td>
<td>0.23</td>
</tr>
<tr>
<td></td>
<td>(3.12)</td>
<td>(1.43)</td>
<td>(2.74)</td>
<td>(2.35)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>rx(5)</td>
<td>0.34</td>
<td>0.12</td>
<td>0.13</td>
<td>0.35</td>
<td>0.28</td>
<td>0.19</td>
</tr>
<tr>
<td></td>
<td>(3.01)</td>
<td>(0.93)</td>
<td>(2.59)</td>
<td>(2.10)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Chapter 3

Term Structure Models and Differences in Belief

This chapter investigates the implications of heterogeneity in macroeconomic beliefs for the term structure of interest rates. When moving from single to multiple agent models several properties of asset prices change. Differences in beliefs affect the equilibrium pricing kernel impacting, at the same time, the dynamics of short term interest rates, the shape of the yield curve, and expected bond risk premia. I highlight three sets of empirical implications for the term structure of interest rates and empirically investigate these properties using historical data on traders beliefs about the state of the economy.

When agents have different models to interpret public information, they are rationally induced to speculate in asset markets. In this case, the equilibrium stochastic discount factor is distorted by a component that depends on the characteristics of beliefs. This implies that bond prices generally deviate from those emerging in a homogeneous economy in which the representative agent is endowed with the consensus belief. Indeed, in a heterogeneous economy the beliefs of the representative agent include an aggregation bias that depends on the relative wealth distribution of the agents.\(^1\) I consider an economy with stochastic disagreement, learning and non-myopic agents and derive closed-form solutions for bond prices that allow us to investigate the marginal effect of different characteristics of the economy on the shape of the yield curve and bond risk premia.\(^2\).

A first set of implications relates to the behavior of the short term interest rate. When agents are non-myopic, the short term interest rate is affected by disagreement in two ways. The first is due to the effect of the wealth weighted aggregation bias of the beliefs beliefs. This effect is history-dependent: as past disagreement affects agents’ speculative positions and events unfold, wealth is redistributed endogenously over time toward the agent whose model happened to align more closely with the data generating process. The second effect is due to the optimal hedging demand and depends on whether EIS is greater or smaller than 1. As agents rely on different models, shocks to fundamentals endogenously change the perceived investment opportunity set, affecting agents future speculative trading opportunities. This occurs even if

---

\(^1\)See Jouini, Marin, and Napp (2010) for a discussion on aggregation biases; see, moreover, Xiong and Yan (2010) for an insightful discussion of the implications of disagreement on bond prices when agents are myopic (\(\gamma = 1\)).

\(^2\)Important contributions that discuss the asset pricing implications of heterogeneity in beliefs include Basak (2005), Buraschi and Jiltsov (2006), David (2008), (Buraschi, Trojani, and Vedolin, 2011, 2010), Bhamra and Uppal (2011), and Gallmeyer and Hollifield (2008).
fundamentals are homoskedastic as long as agents agree to disagree. For $EIS > 1$, larger differences in beliefs reduce the short term interest rate (negative hedging demand). This is because the substitution effect dominates the wealth effect and agents reduce current consumption when the investment (speculative) opportunity set is potentially increased by beliefs shocks. The opposite holds for $EIS < 1$ (positive hedging demand). This knife edge case for short term interest rates provides an interesting testable restriction that makes bond markets a special laboratory to distinguish across alternative speculative models.

The second set of implications relates to the shape of the term structure. When the endowment process follows an affine dynamics, bond yields are given by the sum of an exponentially affine function in the expected economic growth plus a quadratic function in disagreement. I calibrate the economy and show how wealth distribution, risk aversion and disagreement interact to affect the shape of the yield curve. A robust feature is that when $\gamma \neq 1$, the yield curve can be upward sloping even when the equivalent economy with homogeneous investors would give rise to a decreasing yield curve. This is interesting at the light of the well known difficulty for simple single agent economies to produce upward sloping term structures. This feature crucially depends on the size of the term premium and it does not hold when agents are myopic. Moreover, bond prices depend on beliefs in a history-dependent way. Since equilibrium prices depend on agents wealth distribution, which is the result of the way agents traded and shared risks in the past, distant lags of disagreement should be statistically significant to explain the dynamics of both short term interest rates and today's cross-section of yields.

The third set of implications that I study are about the properties of bond risk premia. In these economies, agents marginal utility do not equate in equilibrium state-by-state, even when markets are dynamically complete. Each agent has his own stochastic discount factor and shocks to their individual beliefs cannot be fully insured, independently of the number of contingent claims. This is similar to the effect of labor income shocks in incomplete markets economies, with the difference that the asset pricing effects do not require frictions for these individual shocks to affect prices. Agents' specific prices of risk are proportional to the level of differences in beliefs as this induces larger volatility in individual consumption shares.

When I study long-run expected bond risk premia under the measure of an unbiased econometrician, I find that their properties differ from those arising in homogeneous economy in several respects. They are given by three components. The first is standard Lucas compensation for consumption that is proportional to the product of risk aversion $\gamma$ and the volatility of aggregate consumption. It is well known that this term requires a large risk aversion to generate a bond risk premia similar to those that are observable in the data. The second component, on the other hand, is inversely proportional to $\gamma$. Indeed, for lower levels of risk aversion agents are willing to speculate more, so that the endogenous quantity of risk that each agent faces is larger. The net effect of this non linearity is that for $\gamma$ below a given threshold, the risk premium can be high even if $\gamma$ is low. Moreover, an implication is that bond risk premia depend on lagged disagreement. This is due to the fact that sample periods with larger belief dispersions imply larger
subsequent redistribution of wealth and more volatile consumption/wealth ratios that, in turn, implies larger ex-ante conditional bond risk premia. The third component arises from agents’ demand to hedge future changes in their expected marginal utility due to future shocks to the distribution of beliefs. This term is increasing in risk aversion $\gamma$ and it is due to the fact that future changes in beliefs (either their own or of the other agent) will necessarily affect future wealth redistribution. It is worth noting that the last term can be significant even if today disagreement is zero. Indeed, even in this case, agents know that they will disagree tomorrow, almost surely. Speculation makes higher order beliefs play a first order effect on long-run risk premia. These properties suggest a clear set of testable implications: disagreement give rise to endogenous predictability in bond excess returns even if fundamentals are homoskedastic. Moreover, the lower the risk aversion, the higher the amount of speculation, and the more distant lags of disagreement may affect the time variation in bond risk premia.

These questions relate to a vast literature that documents the difficulty to reconcile the properties of the yield curve with homogeneous agents macro models (Duffee (2012)). A stream of the literature shows strong evidence of bond return predictability. However, the properties of risk compensation in Treasury markets appear orthogonal to the properties of bond conditional second moments (see Duffee (2002) or Dai and Singleton (2002)). This has motivated a search for flexible specifications for the price of risk in reduced-form models, that often do not have simple counterparts in known structural models.\footnote{To address this issue, structural models have been proposed that are capable of generating counter-cyclical risk premia. Example include habit models (Wachter (2006) Buraschi and Jiltsov (2007b)), long run risk models (Bansal and Shaliastovich (2012)), and ambiguity aversion models (Gagliardini, Porchia, and Trojani (2009), Ulrich (2011)). While habit models have proven successful on some aspects, they imply a tight link between past consumption and bond expected excess returns that can generate excess short term interest rate volatility. Long run risk models generate time-varying risk premia via a stochastic quantity of risk. Ambiguity models, on the other hand, can generate rich specifications for the price of risk, with the caveat that risk factors are inherently unobservable.}

I build what is arguably the richest data set on the distribution of expectations of professional forecasters for a broad set of macroeconomic variables. This data set merges all the historical paper archives of BlueChip surveys and is unique in that it is available at a monthly frequency, covers a long history, and it is based on a large and stable cross-section of forecasters.\footnote{The Survey of Professional Forecasters is available only at quarterly frequency and, especially in some periods, it has a more restricted cross-section of forecasters. Previously, the commercially available BlueChip economic digital files started only in 2007.}

I obtain a number of empirical results. First, I find that both real and nominal short term interest rates are negatively related to current disagreement about fundamentals. In both cases real disagreement is statistically significant with t-statistics of -3.30 and -4.01, respectively. These results supports heterogeneous beliefs models in which $EIS > 1$. The results are confirmed when I restate the test in terms of changes in interest rates controlling for the Fama-Bliss one-year forward spot spread and/or the Cochrane-Piazzesi factor.
Second, I find a strong effect of contemporaneous real disagreement on the slope of the term structure. A one standard deviation shock to disagreement raises the slope of the yield curve by 0.25 standard deviations with a t-statistic of 4.11. When I test whether past beliefs contain useful information about today’s cross-section of yields, I find that adding a 12 month and then a 24 months lag to the regression for the level of interest rates raises the $R^2$ to 7% then 27%, respectively. Lagged disagreement is significant at the 1% level. I find that lagged disagreement is strongly significant also for the slope of the yield curve. When I add a 12 month lag and a 24 month lag I obtain an $R^2$ of 32% then 49%, respectively. This is an intriguing result that suggests that past speculative activity, as proxied by lagged disagreement, has a large effect of the shape of the yield curve today, consistent with the hypothesis that agents are trading on their beliefs. These results are consistent with models with $\gamma < 1$. I run a series of robustness check to investigate whether the result is due to any spurious features of the time series.

Third, when I run one-year holding period excess returns regression I find that a substantial amount of return predictability is coming from both contemporaneous and lagged dispersion terms. In the case of 5-year bonds, the $R^2$ of the regression is 16% and the t-statistics on 3 month lagged disagreement is 3.52. A similar result holds also in the case of longer maturity bonds. I examine the robustness of these findings to a number of alternative candidate risk factors. I check whether information in lagged disagreement is already subsumed by the level of economic activity (as in Ludvigson and Ng (2009a)). While the results confirm the statistical significance of such a business cycle factor, disagreement seems to capture different information and its significance persists. I also control for the potential effect of the dynamics of second moments of fundamentals. This channel is discussed in Bansal and Yaron (2004) and Bansal and Shaliastovich (2013) and it is potentially important since one may conjecture that beliefs dispersion and economic uncertainty are correlated. I also control for the political uncertainty factor studied in Baker, Bloom, and Davis (2012). In both cases, I find that the factor loadings and significance on disagreement are almost unchanged. This result is interesting since it suggests that individual consumption volatility matters more for explaining time variation in risk premia than aggregate consumption volatility, as predicted by the model.

I. Theoretical Framework

This section studies the properties of bond markets in a multiple agent economy with heterogeneous beliefs. We begin from a simple structural homoskedastic economy with a representative agent. Then extending this setting to multiple agents we derive implications for both the cross-section of yields and the time-series properties of risk premia. In the following section we take the model to the data.
A. The Homogeneous Benchmark Economy

Consider a simple endowment economy in which a representative agent has CRRA preferences $u'(c_t) = e^{-\delta t}c_t^{-\gamma}$. The growth rate of endowment is a linear combination of a vector of factors $g_t$, with

$$dD_t/D_t = g_t dt + \sigma_D dW^D_t$$  
$$dg_t = -\kappa_g (g_t - \theta) dt + \sigma_g dW^g_t, \quad \text{with} \quad \rho_{Dg} = E\left(dW^D_t dW^g_t\right)$$  

When short run shocks ($\epsilon_t$) are constant, i.e. $\beta g_t = g_0$, so are interest rates and the term structure is flat. When $g_t$ is stochastic, bond prices $P(t,T)$ can be computed by solving for the Euler condition $P(t,T) = E_t [\mathcal{M}_T/\mathcal{M}_t]$. In this economy, the term structure is exponentially affine in $g_t$ and given by

$$P(t,T) = g_r \exp[A(\tau) + B(\tau) g_t]$$

where $g_r = e^{-\delta \tau}$. Bond prices belong to the exponentially affine class as characterised by Duffie and Kan (1996) and $A(\tau)$ and $B(\tau)$ are functions of time until maturity ($\tau$) and the structural parameters of the economy (see appendix):

$$A(\tau) = \frac{1}{2} \gamma (\gamma + 1) \sigma^2 g \tau + \left(\frac{-\gamma \theta}{\kappa_g} + \frac{\gamma^2}{\kappa_g^2} \sigma_D \sigma_g g \right) (\kappa_g \tau + e^{-\kappa_g \tau} - 1)$$

$$B(\tau) = \frac{-\gamma}{\kappa_g} (1 - e^{-\kappa_g \tau})$$

In this setting, instantaneous bond excess returns are equal to

$$E_t[r_{rt,t+dt}^0(T)] = -E_t\left[\frac{dP_t}{P_t} \frac{dM_t}{M_t}\right] = \frac{1}{P_t} \frac{\partial P_t}{\partial g} \sigma_g \gamma \sigma_D \{dW^g_t, dW^D_t\}$$

$$= \gamma \sigma_D \rho_{Dg} \times \frac{\sigma_g B(\tau)}{\text{Real Price of Risk}} \times \frac{\text{Quantity of Risk}}{\text{Real Price of Risk}}$$

When short run shocks ($W^D_t$) are correlated with long run shocks ($W^g_t$) the simple benchmark model restricts expected excess returns to be a scaled multiple of the volatility of macroeconomic fundamentals.\footnote{This is the case when $D_t$ is assumed to be a deterministic affine transformation of $g_t$, i.e. $D_t = \exp(\beta' g_t)$.} As $E(dW^D_t dW^g_t) \to 0$ the risk premium on bonds converges to zero. The tight connection between
first and second conditional moments of bond returns has been discussed in the literature as a problematic feature of early models of the term structure (see Duffee (2002) and Dai and Singleton (2000)). Moreover, while yield dynamics are predictable because the conditional growth rate of the economy is time-varying, expected excess returns are constant hence completely unpredictable. To break the tight link between conditional moments and introduce time-variation in risk compensation the literature has focused on two directions: (i) models with time-varying quantities of risk (as in Bansal and Yaron (2004) and Bansal and Shaliastovich (2013)) or (ii) models with time-varying prices of risk (as in the habit models of Buraschi and Jiltsov (2007b) and Wachter (2006)). In the following we study an alternative channel in which both prices and quantities of risk are time-varying.

B. Disagreement

The drift of the consumption process is unobservable which means the objective measure is not defined on either agents’ filtration. In such situations it is easy to imagine the emergence of disagreement about the correct model for the economy (for discussion along these lines see Hansen, Heaton, and Li (2008) and Pastor and Stambaugh (2000)). The literature has generally focused on two channels for belief dispersion: (i) subjective priors as in Basak (2000) or Buraschi and Jiltsov (2006); and (ii) subjective models as in David (2008) or Dumas, Kurshev, and Uppal (2009). In both cases agents have common information sets and ‘agree to disagree’ about how to process information which, mathematically, is represented by different filtered probability spaces \( \{\Omega, F_t^i, \mathcal{P}^i\} \). Regardless of how heterogeneity arises, since \( D_t \) is observable, consistent perceptions of dividend innovations require that

\[
dW_t^{D,i} = \sigma_D^{-1} (dD_t/D_t - g_t^i) dt = dW_t^{D} + \text{error}_{i}^{t} dt
\]

(3.10)

where we have defined standardised forecast error of agent \( i \) as \( \text{error}_{i}^{t} \). Since the above holds for both agents subjective innovations are related by

\[
dW_t^{D,b} = dW_t^{D,a} + \sigma_D^{-1} (g_t^a - g_t^b) dt = dW_t^{D,a} + \psi_t dt
\]

(3.11)

where the scaled disagreement process is defined as \( \psi_t \). The follow section derives a disagreement process by requiring agents form rational posterior forecasts, in the sense that, they each solve an optimal filtering problem.

C. Disagreement and Optimal Learning

Agents learn in a Bayesian fashion by filtering the state dynamics from observations of the dividend process and publicly available information represented by a filtration \( \mathcal{F}_t \). Denote agent \( i \)’s conditional

\[^6\text{Hansen, Heaton, and Li (2008) argue about the existence of significant measurement challenges in quantifying the long-run risk-return trade-off and that ‘the same statistical challenges that plague econometricians presumably also plague market participants’. Pastor and Stambaugh (2000) discuss the statistical properties of predictive systems when the predictors are autocorrelated but } \kappa \text{ is not known.} \]
forecast $$\hat{g}_t = E_t^i [g_t | F_t]$$ and posterior variance $$\nu_t^i = E_t^i [(\hat{g}_t^i - g_t)^2 | F_t]$$. Since state dynamics are conditionally Gaussian, standard linear filtering results allow a closed form solution for a posteriori means and variances. Consider a rotation of our state space by writing 3.1 and 3.2 in terms of independent Brownian motions $$(W_t^1, W_t^2)$$:

$$W_t^D = W_t^1$$ \hspace{1cm}(3.12)

$$W_t^g = \rho_i W_t^1 + \sqrt{1 - \rho_i^2} W_t^2$$ \hspace{1cm}(3.13)

where $$\rho_i$$ is an agent specific parameter that determines the perceived correlation between shocks to the level of consumption versus shocks to the growth rate of consumption. Investors have identical information sets that include realisations of the dividend process and a signal that is correlated with the stochastic growth rate of the economy $$\{D_t, s_t\}_{t=0}^T$$. The filtering problem contains three independent Brownian motions and two measurement equations

$$dD_t/D_t = g_t dt + \sigma_D dW_t^1,$$

$$ds_t = \phi_t dW_t^2 + \sqrt{1 - \phi_t^2} dW_t^3,$$

where as in Scheinkman and Xiong (2003) we introduce a parameter $$\phi_t$$ that determines the reaction of subjective expectations to the arrival of signal shocks. Applying the results of Lipster and Shiryayev (1974) (theorem 12.7. Page 36) the optimal linear in terms of our original Brownians is given by

$$d\hat{g}_t^i = -\kappa g (\hat{g}_t^i - \theta) dt + \left( \frac{\gamma^*_i}{\sigma_D} + \sigma_g \rho_i \right)d\hat{W}_t^{D,i} + \left( \sigma_g \sqrt{1 - \rho_i^2 \phi_t} \right) ds_t$$

$$\gamma^*_i = \left( -\kappa_g + \rho_i \frac{\sigma_g}{\sigma_D} \right) + \sqrt{\left( \kappa_g + \rho_i \frac{\sigma_g}{\sigma_D} \right)^2 + \left( \frac{\sigma_g}{\sigma_D} \right)^2 \left[ 1 - \phi_t^2 (1 - \rho_i^2) - \rho_i^2 \right]}$$ \hspace{1cm}(3.14)

The diffusion for standardised disagreement can then be written as

$$d\psi_t = \sigma^{-1}_D d(\hat{g}_t^a - \hat{g}_t^b)$$

$$= \left[ -\kappa_g (\hat{g}_t^a - \hat{g}_t^b) - \frac{\sigma_g}{\sigma_D} (\hat{g}_t^a - \hat{g}_t^b) \right] dt + \sigma_D^{-1} (\sigma_g^a D - \sigma_g^b D) d\hat{W}_t^{D,a} + \sigma_D^{-1} (\sigma_g^a s - \sigma_g^b s) ds_t$$

$$= -\kappa_{\psi} \psi_t dt + \sigma_{\psi,D} d\hat{W}_t^{D,a} + \sigma_{\psi,s} ds_t$$

Note that the disagreement process has a zero long run mean implying that in the long run agents agree on the state of the economy, but conditionally disagreement can take both positive and negative values, i.e., growth rate optimists can become growth rate pessimists and vice-versa.
D. Disagreement and the Radon-Nikodym Derivative

Heterogeneous beliefs models depart from the traditional setting by assuming that agents disagree on some features of the conditional distribution of the fundamentals. They have different models that lead them to different empirical likelihoods. Special cases include disagreement on some parameters values or different priors. The key insight of this literature is that, if agents can trade, the equilibrium SDF is affected by disagreement. Consider two agents, \( a \) and \( b \), each representing its own class with separate subjective probability measures on the data generating process, denoted as \( dP_a^t \) and \( dP_b^t \).

Let \((\Omega, \mathcal{F}, \{\mathcal{F}_t^D\}, P_i^t)\) be the filtered probability space of agent \( i = a, b \) with \( \mathcal{F}_t^D \) being the filtration generated by observations on the dividend process \( D_t \). The difference in beliefs between the two agents can be conveniently summarized by the Radon-Nikodym derivative \( \eta_t = \frac{dP_b^t}{dP_a^t} \), so that for any random variable \( X_t \) that is \( \mathcal{F}_t \)-measurable,

\[
E^b(X_T|\mathcal{F}_t^D) = E^a(\eta_T X_T|\mathcal{F}_t^D) \quad \text{with} \quad \eta_0 = 1.
\] (3.16)

The restriction that agents’ measures are absolutely continuous, equation 3.11, and an application of Girsanov implies that the Radon-Nikodym derivative of agent \( b \)'s measure with respect to agent \( a \)'s measure is given by:

\[
\eta_t = \frac{dP_b^t}{dP_a^t} = \eta_0 \exp \left( -\frac{1}{2} \int_0^t \psi'_s \psi_s ds - \int_0^t \psi_s dW_s^a \right)
\]

Moreover, its diffusion satisfies

\[
d\eta_t / \eta_t = -\psi_t d\hat{W}_{D,a}^t
\] (3.17)

which is a martingale on \([0, t]\) with respect to \( P^a \) so long as Novikov’s condition holds:

\[
E^a_t \exp \left( 1/2 \int_0^t \psi'_s \psi_s ds \right) < \infty.
\] (3.18)

which can be viewed as a stationarity restriction on agents’ learning.

E. Individual Agent Problem

We examine agents with time separable utility functions

\[ u'_t = c_t^{-\gamma}, \text{ time preferences } \varrho_t = \exp[-\int_0^t \rho(s)ds], \]

and an infinite sequence of endowments \( e_i^t \). When markets are dynamically complete, an equilibrium is defined by a unique stochastic discount factor \( M_i^t \) for each agent and a consumption plan \( c_i^t \) that solves the following intertemporal problem

\[
\max_{\{c_i^t, M_i^t\}} E^a_0 \int_0^\infty \varrho_t u(c_t^t)dt \text{ subject to } E^a_0 \int_0^\infty M_i^t [c_i^t - e_i^t] dt \leq 0
\]

such that markets clear, i.e. \( \sum_i c_i^t = D_t \) for \( \forall t \). The first order conditions imply that the optimal consumption policies are of the form

\[ c_i^t = \left( \varrho_t / (\alpha_i M_i^t) \right)^{1/\gamma}, \]

where \( \alpha_i \) is the Lagrange multiplier associated with the static budget constraint of agent \( i \).

\[ \eta_t = \frac{dP_b^t}{dP_a^t} \text{ is a martingale with respect to agent as’ measure. In order to find a valid change of measure from the perspective of agents b one simply re-define the Radon-Nikodym derivative as } \eta_t = \frac{dP_a^t}{dP_a^t}. \]

102
Given a subjective discount factor a necessary conditions for consumption markets to clear is that agents trade until ex-ante expected marginal utilities equate. The equilibrium price of a random payoff $X_T$ then satisfies $E^b \left( \frac{u'_a(c(T))}{u'_b(c(T))} X_T \right) = E^a \left( \frac{u'_a(c(T))}{u'_b(c(T))} X_T \right)$. It is easy to show that this implies the Radon-Nikodym derivative must be equal to the ratio of the stochastic discount factors of the two agents

$$\eta_t = \frac{\alpha_b}{\alpha_a} \frac{u'_a(c_t)}{u'_b(c_t)} = \frac{\mathcal{M}^a_t}{\mathcal{M}^b_t}.$$  
(3.19)

Regardless of the micro foundations for heterogeneity, disagreement among agents affects the equilibrium distribution of consumption for all $T < \infty$, implying a different allocation of state-contingent consumption $c^a_t$ and $c^b_t$ for the two agents.

Since markets are complete the stochastic discount factor for each agent is unique. Furthermore, the absence of arbitrage implies the existence of $d\mathcal{M}^a_t = - \left( r^a_t dt + \kappa^a_t d\hat{W}^{D,a}_t \right)$, where $\kappa^a_t$ contains the market prices of risk for consumption shocks. From equation 3.19 it follows that

$$d\eta_t/\eta_t = - \left( \kappa^a_t - \kappa^b_t \right) d\hat{W}^{D,a}_t.$$  
(3.20)

One notices that the disagreement process coincides with the wedge between subjective prices of risk. It should then be clear that the date $t$ cross-sectional distribution of consumption depends on date $t - dt$ distribution of beliefs.

### F. Representative Agent Problem

The dynamic properties of bond prices depend on the characteristics of the stochastic discount factor of the representative agent. In complete markets, Basak (2000) extends Cuoco and He (1994) approach to show how the competitive equilibrium solution can be obtained from the solution of a central planner problem. A representative investor utility function can be constructed is a weighted average of each individual utilities:

$$U^*(D(t), \lambda) := \max_{c_a(t)+c_b(t)=D(t)} \left\{ \lambda^a_t \theta_t u_a(c_a(t)) + \lambda^b_t \theta_t u_b(c_b(t)) \right\}.$$  
(3.21)

Normalising the weight on agent $a$, a necessary condition for a social optimum is $u'_a(c_a(t)) = \lambda_t u'_b(c_b)$. From the first order condition of the individual agent problems we see that is achieved if the stochastic weight is set equal to $\lambda_t = \frac{\alpha_t u'_a(c_a(t))}{\alpha_t u'_b(c_b(t))} = \frac{\alpha_t M^a_t(t)}{\alpha_t M^b_t(t)}$. This implies that the relative weight of the second set of agents must be proportional to the Radon-Nikodym process $\eta_t$: i.e. $\lambda_t = \frac{\alpha_t}{\alpha_b} \eta_t$. Moreover, since the Lagrange multipliers are constant, the diffusion of the Radon-Nykdomy process coincides with the dynamics of the relative weight: $d\eta_t/\eta_t = d\lambda_t/\lambda_t$. This confirms that $\psi_t$ directly affects the relative

---

8 Constantinides (1982) extends Negishi (1960)’s results and proves the existence of a representative agent with heterogeneous preferences and endowments but with homogeneous beliefs. In an incomplete market setting with homogeneous agents Cuoco and He (1994) show a representative agent can be constructed from a social welfare function with stochastic weights. Basak (2000) discuss the aggregation properties in economies with heterogeneous beliefs but complete markets. He shows that a representative can be constructed from a stochastic weighted average of individuals marginal utilities.
marginal utility of the two set of agents in equilibrium. The stochastic discount factor of the representative agent is obtained from

\[ M_t^* = u'_a(c_a(t)) \frac{\partial c_a}{\partial D} + \lambda(t)u'_b(c_b(t)) \frac{\partial c_b}{\partial D} = u'_a \left( \frac{\partial c_a}{\partial D} + \frac{\partial c_b}{\partial D} \right) = \alpha_a M_a(t) = \lambda(t) \alpha_b M_b(t) \] (3.22)

which from the second fundamental theorem of welfare economics is also competitive equilibrium. Finally, imposing the aggregate resource constraint \((D_t = e^a_t + e^b_t)\) one obtains individual consumption policies and the stochastic discount factors for the pricing measure

\[ c_a(t) = \frac{D_t}{1 + \eta_t^{1/\gamma}} \] (3.23)

\[ c_b(t) = D_t \frac{\eta^{1/\gamma}}{1 + \eta_t^{1/\gamma}} \] (3.24)

\[ M_t^* = \frac{\rho t D^{-\gamma}}{\gamma} \left( 1 + \eta_t^{1/\gamma} \right)^{-\gamma} \] (3.25)

Notice, the heterogeneous agent stochastic discount factor is the product of the SDF of a homogeneous CRRA economy and a function \(H(\eta_t)\) of disagreement. In fact, most models in the literature give rise to equilibrium discount factors that can be cast in this form. Moreover, the volatility of this distortion depends on difference in beliefs which emerges as an additional state variable.

**The Real Short Rate**

Market completeness and the absence of arbitrage guarantee the existence of a unique (possibly subjective) stochastic discount factor for each measure that is absolutely continuous with respect to the risk neutral measure (Harrison and Kreps (1979)). Applying ito’s lemma to equation 3.25 and equating drift and diffusion coefficients to \(dM_t^n = - \left( r_t dt + \kappa^n(t) d\hat{W}_t^{D,n} \right)\) the equilibrium short rate is given by

\[ r_f = \rho - \frac{1}{2} \gamma(\gamma + 1) \sigma^2 + \gamma \beta (\omega_a(\eta_t) \hat{g}^a_t + \omega_b(\eta_t) \hat{g}^b_t) + \frac{\gamma - 1}{2\gamma} \omega_a(\eta_t) \omega_b(\eta_t) \psi_t^2, \] (3.26)

where \(\omega_i(\eta_t) = c_i^t/D_t\) is investor’s \(i\) total consumption share.

When the wealth distribution is symmetric \((\eta_t = 1)\) and disagreement is zero \((\psi_t = 0)\) the short term interest rate is given by the Lucas solution. In the heterogeneous case, the short term interest rates includes two new terms. The first is \(\left[ \omega_a(\eta_t) \hat{g}^a_t + \omega_b(\eta_t) \hat{g}^b_t \right]\) and is due to the standard wealth effect. The larger the expected growth opportunity, the higher the demand for current consumption, the lower the demand for savings, thus the higher the interest rate. However, when \(\eta_t \neq 1\), this term differs from the consensus belief \(\frac{1}{2} \hat{g}^a_t + \frac{1}{2} \hat{g}^b_t\). Speculative activity undertaken in the past affects agents’ relative wealth today and this term biases the short rate towards the belief of the agent who has been relatively more
successful.\(^9\) The implications of this term for the term structure are rich. For example, an immediate implication is that the short rate, and hence the entire yield curve, is path dependent even though state dynamics are Markovian.\(^{10}\) The second term is due to speculative demand given by the product of \(\omega_a(\eta_t)\omega_b(\eta_t)\) and \(\psi_t^2\).

To understand the intuition for this effect consider the diffusion for the relative wealth of agent \(a\)\(^{11}\)

\[
d\omega_a = \frac{\gamma - 1}{2\gamma} \omega_a(\eta_t)\omega_b(\eta_t)\psi_t^2 \left[ (\gamma - 1) + 2\gamma \omega_b(\eta_t) \right] - \frac{1}{\gamma} \omega_a(\eta_t)\omega_b(\eta_t)\psi_t dW_t^D
\]

from which we identify the speculative demand in the drift of individual consumption. The reason is because an increase in \(\psi_t\) changes the investment opportunity set, as it increases speculative opportunities between agents. The sign of the effect depends on whether \(\gamma\) is greater or smaller than 1. For \(\gamma > 1\) the wealth effect dominates: speculation raises the drift of planned consumption, which is fixed today; thus, interest rates must rise to clear the market. When \(\gamma < 1\) the substitution effect dominates: speculation increases expected returns raising the price of current consumption relative to future consumption, lowering the drift of planned consumption; thus, interest rates must fall. To visualise the interaction of risk aversion and disagreement on the short rate consider its sensitivity with respect to \(\psi_t\)\(^{12}\)

\[
\frac{\partial r}{\partial \psi} = -\gamma \sigma_D \left( \frac{\eta_t^{1/\gamma}}{1 + \eta_t^{1/\gamma}} \right) + \left( \frac{\gamma - 1}{\gamma} \right) \frac{\eta_t^{1/\gamma}}{(1 + \eta_t^{1/\gamma})^2} \psi_t
\]

Figure 3.1 summarizes the results. The left panel shows that when \(\gamma < 1\) the substitution effect dominates and short term interest rates are negatively related to disagreement, \(\partial r/\partial \psi < 0\). Moreover, in this region \(\partial r/\partial \psi\) is decreasing in the level of disagreement. For \(\gamma > 1\) there exists a threshold for \(\psi\) above which the wealth effect dominates so that \(\partial r/\partial \psi > 0\). The right panel shows the behavior of \(\partial r/\partial \psi\) as a continuous function of \(\gamma\) for different levels of disagreement. In economies with larger disagreement and relative risk aversions, short term interest rates are increasing in disagreement; on the other hand, in economies with low risk aversion and low relative risk aversion, interest rates are decreasing in disagreement.

\footnote{\(\gamma\) Jouini and Napp (2006) also construct a consensus investor whose SDF prices the term structure and contains an aggregation bias.}

\footnote{\(\eta_t\) is not Markovian while the couple \((\eta_t, \psi_t)\) is Markovian.}

\footnote{An analogous diffusion is obtained for agent \(b\) under his measure.}

\footnote{The gradient with respect to the state \((\eta_t^a, \eta_t, \psi_t)\) is obtained by re-writing the short rate as}

\[
r_t = \delta - \frac{1}{2} \eta_t (\gamma + 1) \sigma_D^2 + \gamma g^a - \psi_t \left( \frac{\eta_t^{1/\gamma}}{1 + \eta_t^{1/\gamma}} \right) \left( \gamma \sigma_D - \frac{\gamma - 1}{2\gamma} \frac{1}{1 + \eta_t^{1/\gamma}} \psi_t \right)
\]
G. Equilibrium Risk Allocation

Market completeness and the absence of arbitrage ensures agent specific market prices of risk are given by

\[ \kappa^i_g(t) = \kappa^i_g(t) = \gamma \sigma_D^i + \xi^i(\eta_t, \psi_t) \]

where

\[ \xi^i(\eta_t, \psi_t) = \begin{cases} \omega_b(\eta_t)\psi_t & \text{for } i = a \\ -\omega_a(\eta_t)\psi_t & \text{for } i = b \end{cases} \]

An important insight is that the belief adjustment contains two components, one which is backward looking depending on past trades (\(\omega^n_t\)) and one which is forward looking (\(\psi_t\)) depending on the future (perceived) investment opportunity set. The price of risk for agent \(a\) is increasing in the relative consumption share of agent \(b\) since this increases the risk exposure of agent \(a\) to agent \(b\)'s beliefs. Similarly for agent \(b\). The sign of the belief adjustment depends on who is the relative optimist / pessimist. Optimists trade to shift consumption to states of the world in which their subjective probabilities are the highest, in exchange for a lower consumption in those states they deem less likely. Effectively they sell insurance to pessimists. If a bad state is revealed tomorrow not only will the optimist be poor but he will also have to pay out to the pessimist. Optimists expect this so their risk prices are higher. Finally, since markets are complete agents equilibrium allocations must imply perfect risk sharing. To see this, note that wealth weighted average prices of risk are the same as in the benchmark homogeneous economy

\[ \omega^a_t \kappa^a_t + \omega^b_t \kappa^b_t = \omega^a_t [\gamma \sigma_D + \omega^b_t \psi_t] + \omega^b_t [\gamma \sigma_D - \omega^a_t \psi_t] = \gamma \sigma_D \]

H. The Term Structure of Bond Prices

The date \(t\) price of an \(T\)-period default free (real) zero-coupon bond is:

\[ P^T_t = \frac{1}{M^T_t} E^i_t [\varrho_{t,T} M^T_t] \]

\[ P^T_t = E^i_t \left[ \varrho_{t,T} \left( \frac{D_T}{D_t} \right)^{-\gamma} \left( 1 + \frac{\eta_t^{1/\gamma}}{1 + \eta_t^{1/\gamma}} \right)^\gamma \right] \]

where the expectation is taken under agent \(i\)'s measure. Defining \(x_T = \ln D_T\) and \(y_T = \ln \eta_T\) solving for bond prices requires computing the joint moment generating

\[ \phi_{x,y}(T; u_1, u_2) = E^i_t \left( e^{u_1 x_T + u_2 y_T} \right) \]

which is a function of the ‘fundamental’ system \(\phi_x\) derived above and the ‘belief’ system

\[ \phi_y = \phi_{x,y}(T; 0, u_2) = E^i_t \left( e^{u_2 y_T} \right) \]

In equilibrium the solution can be equivalently computed with respect to any probability measure since \(P(t,T) = E^a(M^T_t) = E^b(M^T_t) = E^*(M^T_t)\).
Given a solution for characteristic function of \((D_T, \eta_T)\) we can recover the joint density via inversion which allows us to compute the price of any contingent claim.\textsuperscript{14} Setting \(u_1 = -\gamma\) I evaluate the (inverse) bilateral Laplace transform at \(iu_2\) using a Fractional Fast Fourier Transform (FrFFT) using the algorithm suggested by Chourdakis (2004).\textsuperscript{15}

**Theorem 1 (Bond Prices).** The term structure of bond prices is equal to the product of two deterministic functions. The first is exponentially affine in the posterior growth rate of the endowment; the second is quadratic function of differences in beliefs

\[
P(t, T) = \varrho \phi_x(\tau; -\gamma) \int_0^\infty g(y, T) \left[ \frac{1}{\pi} \int_0^\infty e^{-iu_2yT} e^{iu_2y} \phi_y(\tau; -\gamma, u_2) du_2 \right] dy(T) \tag{3.35}
\]

where

\[
g(y, s) = (1 + (e^{\psi_y s})^\gamma)^\gamma \tag{3.36}
\]
\[
\phi_x(\tau; -\gamma) = e^{A(\tau) + B(\tau) g^2} \tag{3.37}
\]
\[
\phi_y(\tau; u_1, u_2) = e^{L(\tau) + M(\tau) \psi + N(\tau) \psi^2} \tag{3.38}
\]

where \(A(\tau), B(\tau), L(\tau), M(\tau), N(\tau)\) are functions of time and the structural parameters of the economy, known in closed form.

The dependence of bond prices on \(g^2\) is exponentially affine because the dividend process is conditionally Gaussian. Under incomplete information and learning the term structure also explicitly depends on the difference in beliefs \(\psi^2\). The dependence on these factors is exponentially quadratic. For the case of integer \(\gamma\) I can exploit the binomial expansion to obtain an exact analytical result in terms of the state vector \((g^2, \eta_t, \psi^2)\):

\[
P(t, T) = \varrho \phi_x(\tau; -\gamma)(1 + \eta_1^{-\gamma})^{-\gamma} \sum_{j=0}^{\gamma} \binom{\gamma}{j} (\eta_1)^{j/\gamma} \phi_y(\tau; -\gamma, j/\gamma)
\]

from which we see that bond prices in the heterogeneous agent economy are a wealth weighted average of quadratic term structures. The myopic (log utility) for yields is obtained as a special case:

**Corollary 1 (Myopic Term Structures).** When agents are myopic heterogeneous equilibrium bond prices are given as wealth weighted averages of fictitious homogeneous equilibrium bond prices. For the gamma = 1 case we see from equation 3.26 that squared disagreement does not enter the term structure solution. This is because agents are myopic hence there is no speculative demand to their optimal portfolios. One

\textsuperscript{14}The solution given by Xiong and Yan (2010) only applies to the case of log utility investors. In a portfolio selection context with irrational investors Dumas, Kurshev, and Uppal (2009) show that the joint density of \(D_t\) and \(\eta_t\) can be computed in semi-closed form by Fourier inversion. The spirit of this approach follows methods developed in the option pricing literature by (for example) Heston (1993), Carr and Madan (1999), and Duffie, Pan, and Singleton (2000) for equity, and Chacko and Das (2002) for interest rates.

\textsuperscript{15}For an overview of numerical inversion recipes in finance see Kahl and Lord (2010)
obtains

\[ P(t, T) = \omega^a(t, T) + \omega^b(t, T). \]

I. The Yield Curve

Given the solution for bond prices given in equation 3.35 a solution for (log) yields for any value of risk aversion \( \gamma \in \mathbb{R}^+ \) can be computed from

\[ y(t, T) = \delta + a(\tau) + b(\tau)g_t^\alpha + D(\tau, \eta_t, \psi_t^g) \]

(3.39)

The term structure that falls outside the exponentially affine class (Duffie and Kan (1996)). Notice that the homogeneous benchmark solution appears in the solution, the heterogeneous agent term structure includes an adjustment \( D(\tau, \eta_t, \psi_t^g) \) that depends explicitly on both relative wealths and dispersion in beliefs. Specifically the term structure is a wealth weighted average of quadratic functions of disagreement.

I calibrate our model with parameters quantitively similar those used in Brennan and Xia (2001). To calibrate the correlation \( \rho_{c,g}^i \) between consumption shocks and growth rate shocks I compute a VAR(1) between the date \( t \) realised quarterly growth rate of real GDP, and the date \( t - 4 \) 1-quarter forecast for GDP computed from surveys.\(^{16}\) During the sample period January 1999 and December 2011 we find that the correlation between short run shocks \( (dW^D) \) and long run shocks \( (dW^g) \) is 0.23. In our calibration I choose subjective \( \rho_{c,g}^e = 0.20 \) and \( \rho_{c,g}^b = 0.90 \) to emphasise that fact that short term real rates in a standard Lucas economy should be pro-cyclical and the bond risk premium should be negative, i.e., bonds hedge consumption shocks and the term structure is downward sloping. I set the time rate of preference \( \delta = 0 \) since this only affects the average level of the term structure. I set the long run mean of consumption growth equal to 3% and its persistence 0.40, which implies a half life for growth rate shocks of 1.73 years, and the volatility of consumption is set equal to 5%. Table 3.1 reports the parameter set.

<table>
<thead>
<tr>
<th>Table 3.1. Calibrated Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta )</td>
</tr>
<tr>
<td>0</td>
</tr>
</tbody>
</table>

To gain insight on the implications of heterogeneity for bond markets I consider the following stylised examples. Note, for simplicity I assume (a) the unobservable objective \( g_t \) as its long run mean, i.e., \( g_t = \theta = 3\% \); (b) and the agents beliefs are symmetric with respect to \( g_t \), i.e., they have a mean preserving spread around \( \theta \).

\(^{16}\) Real GDP is obtained from http://research.stlouisfed.org/fred2/. Survey forecasts for GDP growth are from BlueChip Financial indicators and discussed in the data section that follows.
1. \( g_t^a - g_t^b = 0 \) and \( \omega_t^a \in [0, 1] \) (Belief Risk), \( \gamma = [0.5, 1.0, 2.0] \)

2. \( \omega_a(t) = 0.50 \) (Symmetric Economy): \( g_t^a - g_t^b \in [0\% : 3\%] \), \( \gamma = [0.5, 1.0, 2.0, 3.0] \)

3. \( \omega_a(t) = 0.25 \) (Pessimistic Economy): \( g_t^a - g_t^b \in [0\% : 3\%] \), \( \gamma = [0.5, 1.0, 2.0, 3.0] \)

4. \( \omega_a(t) = 0.75 \) (Optimistic Economy): \( g_t^a - g_t^b \in [0\% : 3\%] \), \( \gamma = [0.5, 1.0, 2.0, 3.0] \)

**Example 1: Belief Risk**

Figure 3.2 (left panel) shows that when agents have log utility (\( \gamma = 1 \)), agents are myopic and term structures are bounded above and below by affine homogeneous term structures (Xiong and Yan (2010)). Since \( \rho_{c,g} > 0 \) for \( i = a,b \) the term structures in both homogeneous and heterogeneous economies is downward sloping. On the other hand, figure 3.2 (right panel) shows that when \( \gamma \neq 1 \) and both investors are present in the economy (\( \omega_a \approx \omega_b \approx 0.5 \)) yields for all maturities are higher than in the homogeneous case. The reason is: agents know even though they agree today, almost surely, they will disagree tomorrow.

Equilibrium clearing of consumption markets requires

\[
E^b (D_T | F_t) = E^a (\eta_T D_T | F_t).
\]

However, future consumption sharing depends not only on today’s disagreement but also on the path of future disagreement. The implications of path dependence in consumption sharing is best appreciated by considering impulse response functions for the dividend process versus the ratio of relative wealths. Denote \( D_D T \) and \( D_\omega T \) row vectors that measure the response of the dividend process and relative wealth of agent \( a \) (under his measure) at time \( T \) to a 1 unit shock to \((d\tilde{W}^T_{D,a}, ds_t)_t\).

\[
DD_T = D_T \left[ \sigma_D + \frac{1 - e^{-\kappa_g(T-t)}}{\kappa_g} \left[ \sigma_{\eta D} , \sigma_{\eta s} \right] \right]
\]

\[
D_\omega T = \frac{1}{\gamma} \psi^a_T + \int_t^T e^{-\kappa_\phi(u-t)} \psi^a_T du + \int_t^T e^{-\kappa_\phi(u-t)} d\tilde{W}^T_{\eta a} \left[ \sigma_{\psi,D} , \sigma_{\psi,s} \right]
\]

The important point to note is that, given a shock today and holding all future shocks at zero, the path of the dividend process is deterministic, while the path of the consumption sharing rule remains stochastic because of the path dependence due to future disagreement. In the context of bond markets path dependence plays a particular important role since empirical work on time-variation in bond risk premia focuses on long run returns (Fama and Bliss (1987), Campbell and Shiller (1991a), Cochrane and Piazzesi (2005a)), which depend on the long run prices of risk. To see how long term bonds depend on long run prices consider the impulse response function for \( P(\tau) = E^a_T \left[ \frac{\Delta^*(T)}{\Delta^*(t)} \right] \) (computed using equation 3.25):

\[
DP(\tau) = E^a_T \left[ \frac{\Delta M^*(T)}{\Delta M^*(t)} \right] = E^a_T \left[ \frac{\Delta M^*(T)}{\Delta M^*(t)} \left( \gamma \frac{DD_T}{D_T} + \omega_T \frac{D_\eta T}{\eta T} \right) \right]
\]

\(^{17}\) Computing continuous time impulse response functions is straightforward using Malliavin calculus, which extends standard calculus of variations to stochastic process defined on a Wiener space. Specifically, given a Wiener functional \( F = f(W_t, \ldots, W_m) \), the Malliavin derivative \( DF \) computes the change in \( F \) due to a change in the path of \( W \). For an extensive application of Malliavin calculus in finance see Detemple, Garcia, and Rindisbacher (2003).
The long run price of risk is identified as $\kappa^a(T) = \frac{\mathcal{D}M^*(T)}{\mathcal{M}^*(T)} = \gamma \frac{\mathcal{D}T}{\mathcal{D}T} + \omega^T \frac{\mathcal{D}T}{\mathcal{D}T}$. Expanding the derivative I obtain

$$\kappa^a(T) = \kappa^a(t) + L(T, \eta_t, \psi_t)$$

where $\kappa^a(t)$ is the instantaneous price of risk and

$$L(T, \eta_t, \psi_t) = -B(\tau)[\sigma_gD, \sigma_g] + (\omega^T - \omega^T)\psi_t - \omega^T \int_t^T e^{-\kappa(\psi-u)} \psi^g u du + \int_t^T e^{-\kappa(\psi-u)} d\hat{W}^{D,a} \cdot [\sigma_{\psi,D}, \sigma_{\psi,s}] \cdot [\sigma_{\psi,D}, \sigma_{\psi,s}]$$

In multiple agent economies long-run returns depend not only on the local properties of $\eta_t$ but also on the stream of future values of $\eta_{t+1}$ and hence on the path of future beliefs. This means that long run prices of risk differ from instantaneous prices of risk via a (path) dependence on disagreement. Moreover, some shocks which are not priced instantaneously, such as shocks to the signal $d\eta_t$, can command long run risk compensation. Figure 3.2 shows that this leads to an increase in bond yields, especially for medium and long maturity bonds. In unreported results we obtain similar results for $\gamma < 1$. The increase is the highest for $\omega^a = 0.5$ when agents have equal market power. As a consequence, although yield curves in the homogeneous economies are downward-sloping, yield curves in the heterogeneous economies can be upward sloping and have a humped shape at medium maturities. This is interesting since a humped-shape yield curve is common in the data and empirical evidence suggests that concavity of the yield curve (the level of medium maturities relative to the average of short- and long-term maturities) is a good proxy for the level of bond risk premia (Cochrane and Piazzesi (2005a) or Campbell, Sunderam, and Viceira (2009)).

EXAMPLE 2: SYMMETRIC ECONOMY

Figure 3.3 shows the shape of the term structure for a symmetric economy. The central panel shows that for the log case ($\gamma = 1$) the term structure is always downward for the same reason as the $\gamma = 1$ case discussed directly above. For the $\gamma \neq 1$ case the front end of the curve crucially depends on the level of risk aversion (see also David (2008) who also discusses this point). For $\gamma < 1$ short term interest rates are very low and bond yields are generally increasing in their duration. On the other hand, when $\gamma > 1$ short term interest rates are high and the term structure can become negatively sloped for high disagreement.

Figure 3.4 shows how the average level (defined as $\frac{1}{n} \sum_{i=1}^n y^n$) and slope of the term structure respond to an increase of disagreement. The response of the shape of the yield curve to disagreement crucially depends on the trade off between the wealth and substitution effects. For $\gamma < 1$ as disagreement increases:

---

18 John Maynard Keynes famously wrote ‘It is not a case of choosing those that, to the best of one’s judgment, are really the prettiest, nor even those that average opinion genuinely thinks the prettiest. We have reached the third degree where we devote our intelligences to anticipating what average opinion expects the average opinion to be. And there are some, I believe, who practice the fourth, fifth and higher degrees.’ Keynes (1937).
(a) the level of the yield curve decreases (left panel), and (b) the slope increases (right panel). On the other hand, for $\gamma > 1$ as disagreement increases: (a) the level of the yield curve increases, (ii) the slope decreases.

[ Insert figures 3.3 and 3.4 and about here ]

EXAMPLE 3 & 4: PESSIMISTIC AND OPTIMISTIC ECONOMIES

Figure 3.5 shows the shape of the term structure for an economy populated mainly by pessimists, so that $\omega_a(t) = 0.25$. The central panel shows that in the $\gamma = 1$ case, bond yields are decreasing in disagreement. In this economy an increase in the mean preserving spread between $\tilde{g}_a^t$ and $\tilde{g}_b^t$ increases the difference between the arithmetic consensus growth rate and the wealth-weighted average of the expected growth rates (i.e. $(\omega_a(\eta_t)\tilde{g}_a^t + \omega_b(\eta_t)\tilde{g}_b^t)$). Thus, an increase in disagreement increases also the aggregation bias toward the pessimists. As a result the level of the term structure falls (since beliefs have no intertemporal effect). In an economy populated mainly by optimists, as in figure 3.5 with $\omega_a(t) = 0.75$, the opposite holds: an increase in disagreement, increases the aggregation bias toward the optimist so short rates and the level rise.

When $\gamma \neq 1$, one needs to account also for the trade-off between the wealth and substitution effects. Consider first an economy dominated by pessimists (see figure 3.5, right panel). For $\gamma > 1$ short term rates increase with disagreement so that for large disagreements $\psi^g_t$ the slope becomes negative, while for $\gamma < 1$ short term interest rates fall and the slope becomes positive. Interest rates at the long end do not respond so aggressively since disagreement is expected to mean-revert: in our example the half life for $d\psi^g_t$ is 0.88 years compared with the half life for $dg_t$ of 1.73 years. Studying the $\gamma > 1$ case in more detail one can see the non-linearities in the short rate due to the trade-off between the aggregation bias and the intertemporal substitution effects. For small values of disagreement, this results in a level shift; for large values of disagreement this results in significant change of the slope. Figure 3.5 plots the equivalent results for the optimistic economy which follows similar logic.

J. Instantaneous Risk Premia

The absence of arbitrage and complete markets ensure equilibrium expected excess returns for each agent are given by $E_t[hprx_{i,t+\delta}^{(P)}] = \kappa^i(t)\sigma_{P,D}^T(t)$, where $\sigma_{P,D}^T(t)$ measures risk exposure to $d\tilde{W}_{i,D}^t$, and prices of risk for agent $i = a, b$ are given by equation 3.29. Notice, subjective expected returns can be opposite in sign. For example, consider the case of an economy populated mainly by pessimists: since a smaller mass of optimists must absorb the total risk transfer their risk prices are larger (in magnitude) than the pessimists. Moreover, optimists prices of risk will be positive while pessimists will be negative, and therefore expected excess returns on long term bonds will be opposite for both agents.

\footnote{In the context of equity markets Jouini and Napp (2006) argue that aggregate pessimism can lead to an increase of the market price of risk and to a decrease of the risk free rate, and therefore help to resolve the equity premium puzzle.}
To understand the link between subjective physical measures, subjective risk neutral measures, and the true physical measure first note that

\[
P(t, T) = E_t^a \left[ \frac{\mathcal{M}_T}{\mathcal{M}_t} \right] = E_t^a \left[ \frac{\lambda_T}{\lambda_t} \frac{\mathcal{M}_T}{\mathcal{M}_t} \right] = E_t^a \left[ \frac{\eta_T}{\eta_t} \frac{\mathcal{M}_T}{\mathcal{M}_t} \right] = E_t^b \left[ \frac{\mathcal{M}_T}{\mathcal{M}_t} \right]
\]

bonds can be priced equivalently under the measure of either agent. The discount factor for each agent can be written as

\[
\frac{\mathcal{M}_T}{\mathcal{M}_t} = e^{-\int_t^T r_u du H_T/H_t}
\]

where \(H_T/H_t\) is an exponential martingale satisfying \(dH_t = -\kappa(t) dW_t^{D,a}\). From Girsanov it follows that the drift of a maturity \(T\) bond under the risk neutral measure, and thus the risk premium, is given by

\[
\mu^Q_i(t, T) = \mu^P_i(t, T) - \kappa^i \sigma_{P,D}(t, T).
\]

(3.40)

Moreover, subjective physical drifts are related to the drift observed by unbiased econometrician (who knows the true \(g_t\)) by

\[
\mu^P_e(t, T) = \mu^P_i(t, T) - \text{error}^e_i(t) \sigma_P(t, T)
\]

(3.41)

where \(\text{error}^e_i(t)\) is defined in equation 3.10 above. Combining 3.40, 3.41, and 3.30 it follows that the wealth weighted average risk premium from the perspective of the econometrician is

\[
\mu^P_e(t, T) = \left( \omega^a_t \mu^Q_a(t, T) + \omega^b_t \mu^Q_b(t, T) \right) - \left( \omega^\phi_t \sigma_{P,D}^a(t, T) + \omega^\psi_t \sigma_{P,s}^a(t, T) \right) \sigma_P(t, T) + \sum_{i=a,b} \omega^\phi_t \text{error}^e_i(t) \sigma_P(t, T)
\]

(3.42)

In what follows I re-visit example economies 2 (symmetric), 3 (pessimistic), and 4 (optimistic) by computing expected returns from the perspective of the unbiased econometrician. This requires computing bond sensitivities to \(dW_t^{D}\) and \(dW_t^s\) shocks

\[
[\sigma_{P,D}(t, T), \sigma_{P,s}(t, T)] = \frac{1}{P(t, T)} \left[ \frac{\partial P(t, T)}{\partial g^a}, \frac{\partial P(t, T)}{\partial \eta}, \frac{\partial P(t, T)}{\partial \psi} \right] \left[ \begin{array}{cc} \sigma_{g,D} & \sigma_{g,s} \\ \sigma_{\psi,D} & \sigma_{\psi,s} \end{array} \right] \left( \psi_t \right)
\]

(3.42)

which from equation 3.35 are known in semi-closed and given in the appendix. Notice, in heterogeneous agent models volatility depends explicitly on \(\eta_t\) and \(\psi_t\) and since the term structure is quadratic in \(\psi_t\) bond sensitivities to \(dW_t^{D}\) shocks become stochastic.

**Example 1: Symmetric Economy**

Figure 3.7 summarizes the link between instantaneous risk premia and disagreement for different levels of relative risk aversion. When the economy is equally populated by pessimists and optimists, bond risk premia are decreasing in disagreement when \(\gamma \leq 1\) while they are relatively flat when \(\gamma > 1\). Even for \(\gamma = 3\), however, it can be noticed that the sensitivity of equilibrium risk premia to disagreement is not...
very large in the symmetric economy. The reason is that in the calibrated symmetric economy agents forecast errors (almost) cancel out. Thus, the large positive distortion in the price of risk of one agent is broadly compensated by the negative distortion a negative distortion in the price of risk of the other.

The negative average risk premium is due to the assumption in the calibration that $\rho_{c,g} > 0$. In this case bonds are hedges and always command a negative risk premium in the symmetric economy. The larger effect on risk premia for $\gamma < 1$ is because in this case agents are very willing to substitute intertemporally driving up bond volatilities as agents engage in trading activity. Notice that belief risk has no bearing on instantaneous risk premia.

Example 2 & 3: Pessimistic and Optimistic Economies
Consider first excess returns for the pessimistic economy (figure 3.8). At zero disagreement the economy is populated by homogeneous investors in which bonds are hedges and risk premia slope downward across maturity. For all values of $\gamma$ the risk premium is increasing (in magnitude) in disagreement: large instantaneous differences in belief predict negative returns on long term bonds. This effect comes from two channels: (i) the objective price of risk is positively skewed, i.e., biased towards the forecast error of the pessimists; (ii) bond volatility is increasing (in magnitude) in disagreement, driven by an increase in trade.

Considering now excess returns for the optimistic economy. I see for small values of disagreement bonds remain hedges, however, for intermediate and large values of disagreement bond risk premia change sign. This is an intriguing feature of the model since empirically expected returns on bonds take both positive and negative values (see, for example, Dai and Singleton (2002) or Cochrane and Piazzesi (2005a)). Moreover, for large values of disagreement long term bonds command larger risk compensation regardless of the magnitude of $\gamma$. The flipping of the risk premium follows the same logic as the pessimist case: (i) the objective price of risk is negatively skewed and can become negative for large enough disagreements; (ii) bond sensitivities to $dW_t^D$ shocks are again negative and increasing in disagreement.

II. Testable Implications

The implications of heterogeneous agent models for term structure studied the previous section suggests a number of empirical tests. In the empirical sections below we focus on the following hypothesis:

- $H_{01}$: Short Term Interest Rate.
  - disagreement is a state variable driving the short term interest rate due to an intertemporal hedging demand. The sign of the effect depends on the wealth vs substitution effects dominate, i.e., whether $\gamma$ is bigger than or smaller than one.
• **H02**: Yield Curves, Intertemporal demand, and Wealth Weighted Beliefs.
  
  – since disagreement drives short term interest rates it necessarily affects the level and slope of the term structure. Moreover, the level and slope of the term structure inherit a wealth weighted aggregation bias from the short rate. To the extent that past differences in belief proxy for contemporaneous fluctuations in wealth, distance lags of disagreement should affect today’s cross-section of yields.

• **H03**: Expected Returns, Relative Weights, and Future Beliefs.
  
  – from the perspective of an unbiased econometrician, average optimism (pessimism) drives positive (negative) variation in the bond risk premium due to a bias in the wealth weighted belief. Since the history of speculate bets determines today’s relative wealths, the history of disagreement should predict risk premia after controlling for contemporaneous disagreement. Moreover, while disagreement does not drive bond risk premia instantaneous it drives long run returns.

### III. Data

I use an extensive dataset on the distribution of beliefs to learn about the relative importance of the channels through which heterogeneity can affect asset prices. This section discusses the data sources and construction of variables designed to proxy for the state vector \((g_t, \eta_t, \psi_t^\theta)\). The appendix contains descriptions of the remaining variables used in the empirical sections.

BlueChip Financial Forecasts Indicators (BCFF) is a monthly publication providing extensive panel data on expectations by agents who are working at institutions active in financial markets. Unfortunately, digital copies of BCFF are only currently available since 2001. I obtained, however, the complete BCFF paper archive directly from Wolters Kluwer and entered the data manually. The digitization process required inputting around 750,000 entries of named forecasts plus quality control checking and was completed in a joint venture with the Federal Reserve Board. The resulting dataset represents an extensive and unique dataset to investigate the role of formation of expectations about the compensation for bearing interest rate risk. Each month, BlueChip carry out surveys of professional economists from leading financial institutions and service companies regarding all maturities of the yield curve and economic fundamentals and are asked to give point forecasts at quarterly horizons out to 5-quarters ahead (6 from January 1997). While exact timings of the surveys are not published, the survey is usually conducted between the 25th and 27th of the month and mailed to subscribers within the first 5 days of the subsequent month, thus our empirical analysis is unaffected by biases induced by staleness or overlapping observations between returns and responses.\(^{20}\)

An extract from the paper archives is shown in figure 3.10.

\[^{20}\text{An exception to the general rule was the survey for the January 1996 issue when non-essential offices of the U.S. government were shut down due to a budgetary impasse and at the same time a massive snow storm covered Washington, DC: \url{www.nytimes.com/1996/01/04/us/battle-over-budget-effects-paralysis-brought-shutdown-begins-seep-private-sector.html}. As a result, the survey was delayed a week.}\]
BCFF is attractive along a number of dimensions with respect to alternative commonly studied surveys. First, the number of participants in the survey is stable over time. On average 45 respondents are surveyed with standard deviations of 2.8. The left panel of figure 3.11 plots a time-series of the number of respondents and right panel plots a histogram of respondent numbers. Only on rare occasions are survey numbers less than 40 and no business cycle patterns are visible. In the Survey of Professional Forecasters, on the other hand, the distribution of respondents displays significant variability. While the mean number of respondents is around 40, the standard deviation is 13, and in some years the number of contributors is as low as 9. While in the early 70’s the number of SPF forecasters was around 60, it decreased in two major steps in the mid 1970’s and mid 1980’s to as low as 14 forecasters in 1990 and if one restricts the attention to forecasters who participated to at least 8 surveys, this limits the number of data point considerably. Second, while our dataset is available at a monthly frequency, SPF is available only at quarterly frequency. Third, the SPF survey has been administered by different agencies over the years which have changed the form of questions. For a detailed discussion on the issues related to SPF see D’Amico and Orphanides (2008) and Giordani and Soderlind (2003). Other well known surveys, such as the ‘University of Michigan Survey of Consumers’ do not provide point estimates from individual survey respondents. Finally, BCFF survey forecasts for both macro variables and interest rates are highly competitive with respect to the out-of-sample performance of sophisticated econometric models. Recent studies documenting the quality of BCFF forecasts include Chun (2012), Faust and Wright (2012), and Buraschi, Carnelli, and Whelan (2013).

To proxy for macro economic disagreement I use 1-quarter ahead point forecasts on real GDP and the GDP deflator for consumption growth and inflation disagreement, respectively. Individual forecasts allow us to proxy for belief dispersion in a number of ways. In what follows I choose to proxy for belief heterogeneity from the cross-sectional inter-quartile range and check the robustness of our findings with alternative proxies. In our empirical work I focus on implications for real disagreement ($\psi_g^t$) but also investigate disagreement about inflation ($\psi_\pi^t$) as an alternative determinant for bond markets as argued recently by Wright (2011), Ehling, Gallmeyer, Heyerdahl-Larsen, and Illeditsch (2013), and Hong, Sraer, and Yu (2013).

Figure 3.12 plots the time series for our macroeconomic disagreement proxy along with an economic policy uncertainty factor ($U\pi C_t$) studied by Baker, Bloom, and Davis (2012). Disagreement on the real economy has a significant business cycle component: in all previous three NBER economic recessions since 1990, $\psi_g$ is low before the recessions and it increases to peak at the end of the recessions (this occurs in 1991, 2002, and 2009). This is interesting as large disagreement is often reported at this stage of the cycle. Comparing $U\pi C_t$ to our measures for belief dispersion I find large co-movement with $\psi_g$ (correlation = 0.58), which is somewhat surprising given that the weight assigned to forecaster
disagreement about inflation in this index is just 1/6. The remaining components of the index are 1/2 a broad-based news index, 1/6 a tax expiration index and 1/6 a government purchases disagreement measure. Taken together these measures suggest the existence of a common component in the formation of expectations which is important since systematic variation is required for time-variation in priced risk compensation.

[Insert figure 3.12 here.]

IV. Empirical Results

\( H_{01} : \text{Short Term Interest Rate} \)

The model predicts that \( \psi_t^g \) should drive short term interest rates even after controlling for fundamentals \((g_t)\). This is a result of an optimal hedging demand as opposed to some behavioural bias. More interestingly, the link between \( \psi_t \) and \( r_t \) depends crucially on whether agents are risk tolerant \((\gamma < 1)\) or risk averse \((\gamma > 1)\). When \( \gamma < 1 \) (or \( EIS > 1 \)) positive shocks to disagreement map induce negative shock to \( r_t \) since the substitution effect dominates, while \( \gamma > 1 \) the should occur since the wealth effect dominates.

I compute time-series for the 3-month real interest rate by subtracting the 3-month expected inflation from BCFF from the 3-month nominal interest rate:

\[
y_t^{(3m)} = y_t^{(3m)} - \beta E_t[\pi(t + 3m)]
\]

Figure 3.13 plots the resulting 3-month real and nominal rate along with consensus GDP growth \((E_t[g(t + 3m)])\) for the sample period January 1990 - December 2011.

[Insert figure 3.13 about here]

Next I estimate OLS regressions of real and nominal rates on contemporaneous disagreement about real GDP growth after controlling for consensus real GDP and inflation. Table 3.2 reports the results. I standardise all right hand variables to be zero mean with unit variance which implies the constant each regression represents the sample mean of 3-month real and nominal interest rates. Considering the factor loadings I find both real and nominal short term interest rates are negatively to \( \psi_t^g \) after controlling for expected fundamentals. In both cases real disagreement is statistically significant with t-statistics of \(-3.30\) and \(-4.01\), respectively. The empirical evidence supports heterogeneous beliefs models with \( \gamma < 1 \). Indeed, in this case an increase in disagreement reduces the demand for current consumption so short rates must fall.

---

of the United Kingdom - labeled the initial sign of the recovery from the S&L recession as ‘green shoots’. Many years later, Ben Bernanke used the same words in a well-known ‘CBS 60 Minutes’ interview in 2009, which was counterpointed by Nouriel Roubini who argued his disagreement and labeled those signs as ‘yellow weeds’. The data indeed confirm that macro-disagreement is usually pervasive in this phase of the cycle.
Table 3.2. Short Rate Regressions

OLS projections of real \((y_r^{(3m)})\) and nominal \((y_{(3m)})\) interest rates on disagreement about 1-quarter GDP growth \((\psi^g)\) and consensus expectations \((E[\pi^{(3m)}], E[g^{(3m)}])\). t-statistics are corrected for autocorrelation and heteroskedasticity. Sample Period: 1990.1 - 2011.12

| \(y_r^{(3m)}\) & \(E[\pi^{(3m)}]\) & \(E[g^{(3m)}]\) & \(\psi^g\) & \(R^2\) |
|---|---|---|---|---|
| \(y_t\) | 1.06 | -0.34 | 0.04 |
| (5.42) | (2.28) |
| \(y^{(3m)}\) & 1.06 | -0.02 | -0.00 |
| (5.29) | (-0.13) |
| \(y_t\) | 1.06 | -0.40 | -0.59 | 0.07 |
| (5.58) | (-1.79) | (-3.30) |
| \$^{(3m)}\) & 3.69 | 1.40 | -0.48 | -0.68 | 0.47 |
| (20.85) | (8.83) | (-2.61) | (-4.01) |

A concern with the above short rate regression is that the very short end of the yield curve is controlled by Federal reserve policy. If the Fed were reacting to market uncertainty (as proxied by dispersion in beliefs) the negative loadings should not be interpreted in the context of equilibrium risk sharing, but rather the result of monetary policy. Indeed, as discussed in the data section above, our proxy for belief dispersion shares a common component with the economic policy uncertainty factor from Baker, Bloom, and Davis (2012) (figure 3.12). In the context of equity markets Buraschi, Trojani, and Vedolin (2011) find a strong link between both firm specific and market wide uncertainty (volatility risk premia) and dispersion in beliefs. However, while the Fed does control the Federal funds rate (FF) by adjusting the target rate (TR), the extent to which it chooses to do this is unclear. Moreover, the empirical link between the FF and rates of longer maturities is weak. For example, for the period 1982 - 2012 Fama (2013) finds that when the Fed chooses to adjust the TR they move strongly towards \textit{existing} short rates. Moreover, changes in maturities greater than 6-months have almost no correlation with the TR changes. The conclusion from Fama (2013) is that most of the variation in open market rates is orthogonal to actions of the Fed. However, if the Fed cannot influence interest rates then what does? I ask the extent to which disagreement affects 1-year interest rates by considering an augmented Fama and Bliss (1987) complementarity regression,

\[
y^{(1)}_{t+1} - y^{(1)}_t = \alpha + \beta_1 (I^{(1)}_t - y^{(1)}_t) + \beta_2 CP_t + \beta_3 \psi_t^g + \epsilon_{t+1},
\]

for 1-year yield changes in the 1-year interest rate on the 1-year forward-spot spread, the return forecasting factor \((CP_t)\) from Cochrane and Piazzesi (2005a), and differences in belief \((\psi_t^g)\).\textsuperscript{24} Table 3.3 replicates these results in our overlapping sample period. First, row (i) shows that the forward spot spread \textit{does} contain some information for expected spot changes. The beta on the forward spot spread is 0.32 (the expectation hypothesis predicts 1.00) and the t-stat is significant at the 10% level. However, after con-

\textsuperscript{24} This approach closely follows Cochrane and Piazzesi (2005a) who find that, not only is there a factor in the cross-section of yields that drives risk prices, but also that this factor predicts lower short rates in the future. I construct \(CP_t\) in sample using 5-forward rates as in Cochrane and Piazzesi (2005a)
trolling for $CP_t$ I find that, conditional of a steep forward curve, the Cochrane-Piazzesi factor predicts an economically large drop of short term rates over the following year with a standardized point estimate of $-0.46$ and a t-stat of $-3.38$. Row (iii) repeats this exercise replacing $CP_t$ with disagreement about the real economy. Interestingly, I find an almost identical result to row (ii), with loadings (t-stats) of $-0.39$ ($-4.05$), so that not only is disagreement about real growth contemporaneously negatively correlated with short rates (as discussed above) but also forecasts declining short rates in the future. Finally, row (iv) includes the forward spot spread, the Cochrane-Piazzesi factor, and disagreement. The results show that while the effect of including $CP_t$ and disagreement in the Fama-Bliss complementarity regression have quantitatively similar predictions, they contain largely orthogonal information: the $R^2$ rises to 35% including all right hand variables, the economic magnitude of the loadings is unaltered, and both factors remain significant at the 1% level.

<table>
<thead>
<tr>
<th>Table 3.3. Short Rate Regressions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Forecasting regression of 1-year changes in the 1-year yield on the $(1,2)$ forward spot spread, $CP_t$, and $\psi_t$. t-statistics are corrected for autocorrelation and heteroskedasticity. Both left and right hand variables are standardized. Sample Period: 1990.1 - 2011.12</td>
</tr>
<tr>
<td>$f_{t,1}^{1,2}$</td>
</tr>
<tr>
<td>(i)</td>
</tr>
<tr>
<td>(1.87)</td>
</tr>
<tr>
<td>(ii)</td>
</tr>
<tr>
<td>(2.90)</td>
</tr>
<tr>
<td>(iii)</td>
</tr>
<tr>
<td>(2.45)</td>
</tr>
<tr>
<td>(iv)</td>
</tr>
<tr>
<td>(3.41)</td>
</tr>
</tbody>
</table>

$H_{02}$: Yield Curves, Intertemporal demand, and Wealth Weighted Beliefs

A vast literature studies the empirical properties of bond markets from the perspective of reduced form latent factor models. For instance, in affine models date-$t$ risk factors are completely summarised by linear combinations of date-$t$ yields through yield curve inversion (see, for example, Duffee (2002) or Dai and Singleton (2002)). The resulting latent factors are usually labelled level, slope and curvature due to the shape of their factor loadings on yields. While this literature has made great advances in understanding the statistical properties of yields there is little consensus for what these latent factors represent. In the context of our equilibrium treatment I have a joint testable implication that both the level and slope of the term structure are a function of $\psi(t)$ and $\omega_i(t)$. This is an important but challenging test for the model since I do not observe the cross-sectional distribution of wealth.

I propose a simple test based on the model implication that variations in $\omega_i$ require that agents hold subjective (optimal) portfolio allocations due to past differences in belief. Thus, I propose to test the
effect of wealth fluctuations using lags of difference in belief. To understand of why lagged disagreement should matter empirically consider the stylised example, depicted in figure 3.14:

Assume a 3-period sample where in the first node agents have equal wealth ($\omega_a^0 = \omega_b^0$) but that agent $a$ is the relative optimist ($g_a^0 > g_b^0$). If in the next period a good (up) state is revealed the wealth of the optimist will be larger than the pessimist ($\omega_a^u > \omega_b^u$) since his prediction for fundamentals was ex-post more accurate than the pessimist. For simplicity assume that the realised endowment and signals are such that the optimist does not update his beliefs much, since he already predicted a good state, but that the pessimist has a large forecast revision ($g_b^u > g_b^o$). If in the next period a bad state occurs there will be a redistribution of wealth towards the pessimist ($\omega_b^u < \omega_b^{ud}$) that will depend not only on beliefs from the previous (up) node but also on beliefs from the initial period. Consider now an alternative scenario in which a bad (down) state occurs following by a good (up) state. Initially the wealth distribution is shifted towards the pessimist ($\omega_a^d < \omega_b^d$) but then subsequently redistributed towards the optimist ($\omega_a^d < \omega_a^{du}$). Comparing the short rates in the final nodes since both agents have higher (lower) growth rate forecasts estimates in the up-up (down-down) states I have that $r_{uu} > r_{dd}$. However, while in general $r_{ud} \neq r_{du}$ without knowing the path of disagreement (portfolio choices) I cannot rank the short rate in these nodes $r_{ud} \not\preceq r_{du}$. In other words, the interest rate tree is non-recombing since it depend not only the realisation of the economy but also on agent’s beliefs along each path. In the context of our empirical tests past disagreement ($\psi_{t-\tau}$) contains important information about today’s cross-sectional of yields, conditional on contemporaneous disagreement ($\psi_t$), since the distribution of wealth depends jointly on the path of beliefs and actual realisations of the economy.

To test this hypothesis I measure the level of the term structure as $Level_t = \frac{1}{5} \sum_{n=1}^{5} y_t^{(n)}$ and the slope as $Slope_t = y_t^{(5)} - y_t^{(1)}$. Next, I run contemporaneous regressions of the level and slope of the term structure on disagreement and lagged disagreement:

$$Level(t) = const + \beta_1 \psi^g(t) + \beta_2 \psi^g(t-12) + \beta_3 \psi^g(t-24) + \varepsilon_{level}^t$$

$$Slope(t) = const + \gamma_1 \psi^g(t) + \gamma_2 \psi^g(t-12) + \gamma_3 \psi^g(t-24) + \varepsilon_{slope}^t$$

Table 3.4 reports the results. Considering the effect of contemporaneous disagreement on the level (i) I find a negative relationship but the factor loading is not significant and the $R^2$ is zero. Next, considering the slope (ii) I find a strong effect in terms of both statistical and economic significance. Since both left and right hand variables are standardised the estimated loadings are the response to a one standard deviation shock to disagreement: a 1-standard deviation shock to $\psi^g(t)$ raises the slope of the yield curve by 0.25 standard deviations with a t-statistic is 4.11.

Specifications (iii) – (vi) tests whether past beliefs contain useful information about today’s cross-section of yields. Adding $\times 1$ lag then $\times 2$ lags of $\psi$ to the $Level_t$ regression raises the $R^2$ to 7% then 27%,
Table 3.4. Short Rate Regressions
OLS regression of the $\text{Level}_t = \frac{1}{5} \sum_{n=1}^{5} y_t(n)$ and $\text{Slope}_t = y_t(5) - y_t(1)$ of the term structure on disagreement and lagged disagreement factors. $t$-statistics are corrected for autocorrelation and heteroskedasticity. Both left and right hand variables are standardized. A constant is included by not reported. Sample Period: 1990.1 - 2011.12

<table>
<thead>
<tr>
<th></th>
<th>$\psi^g(t)$</th>
<th>$\psi^g(t-12)$</th>
<th>$\psi^g(t-24)$</th>
<th>$\bar{R}^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) Level$_t$</td>
<td>-0.04</td>
<td></td>
<td></td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>(-0.48)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(ii) Slope$_t$</td>
<td>0.25</td>
<td></td>
<td></td>
<td>0.06</td>
</tr>
<tr>
<td></td>
<td>(4.11)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(iii) Level$_t$</td>
<td>-0.07</td>
<td>-0.24</td>
<td></td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>(-0.82)</td>
<td>(-2.87)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(iv) Slope$_t$</td>
<td>0.17</td>
<td>0.51</td>
<td></td>
<td>0.32</td>
</tr>
<tr>
<td></td>
<td>(2.76)</td>
<td>(7.17)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(v) Level$_t$</td>
<td>-0.31</td>
<td>-0.20</td>
<td>-0.25</td>
<td>0.27</td>
</tr>
<tr>
<td></td>
<td>(-3.95)</td>
<td>(-2.73)</td>
<td>(-4.09)</td>
<td></td>
</tr>
<tr>
<td>(vi) Slope$_t$</td>
<td>0.26</td>
<td>0.41</td>
<td>0.41</td>
<td>0.49</td>
</tr>
<tr>
<td></td>
<td>(4.92)</td>
<td>(6.06)</td>
<td>(9.23)</td>
<td></td>
</tr>
</tbody>
</table>

respectively. In specification (v) the estimates on disagreement are significant at the 1% level and the factor loadings are consistently negative. A similar pattern is repeated for the Slope$_t$ regressions where I find adding $\times 1$ then $\times 2$ lags of $\psi$ raises the $R^2$ to 32% then 49%, respectively. Again, in specification (vi) I find all the estimates are significant at the 1% but this time the loadings are consistently positive. This is an intriguing result that suggests past speculative activity has a large effect of the shape of the yield curve today, consistent with our hypothesis that agents are trading on their beliefs. One concern with the above regressions is that since $\psi_t^g$ is measured with error the significance of the lagged terms could arise if lagged disagreement is helping to forecasting the true contemporaneous state of disagreement. However, if the dependence on past disagreement were arising spuriously then lagged disagreement should have no significance on its own. Table 3.5 tests this conjecture which confirms that past disagreement is individually statistically significant in these tests.

Table 3.5. Path Dependent Forecasting Regression

<table>
<thead>
<tr>
<th></th>
<th>$\psi^g(t-12)$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level$_t$</td>
<td>-0.37</td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td>(-4.26)</td>
<td></td>
</tr>
<tr>
<td>Slope$_t$</td>
<td>0.56</td>
<td>0.31</td>
</tr>
<tr>
<td></td>
<td>(7.13)</td>
<td></td>
</tr>
</tbody>
</table>

As a final test of the link between bond yields and disagreement I consider the ability of the Level$_t$ and Slope$_t$ to forecast future belief heterogeneity. Indeed, if date $t$ yields are mappings from date $t$ risk factors they should contain information about the predictable component of the state vector. Thus, an alternative
test for the determinants of bond markets is to ask whether Treasury yields contain information about the future dynamics of the conjectured state vector. The macro-forecasting literature has used this logic to answer the question of whether GDP growth is important for bonds by testing if there is information content in the slope of the term structure about future economic activity (see, for example, Harvey (1988) and Estrella and Hardouvelis (1991)). I follow this approach by running a forecasting regression for changes in $\psi(g)$ on the date $t$ level and slopes.

$$\psi_{t+h}(g) - \psi_t(g) = \text{const} + \beta_1 \text{Level}_t + \beta_2 \text{Slope}_t + \varepsilon_{t+h}.$$

Figure 3.15 summarises the results graphically, the left panel plots point estimates and 1-standard error bounds, while the right panel plots the $R^2$ for horizon $t + \tau$. I find a striking result that the slope of the yield curve has large predictable content for changes in $\psi(g)$: conditional on observing a steep yield curve today I expect disagreement to be lower in the future suggesting the slope contains information about the mean reversion characteristics for dispersion. This result is economically and statistically meaningful: the factor loadings are significant at the 1% for horizons greater than 9 months, and the degree of variance explained is large, peaking at $\sim 25\%$ at the 18-month horizon.

$H_{03}$: Expected Returns, Relative Weights, and Future Beliefs

The previous section studied the role of intertemporal demand versus the effect of relative wealth fluctuation in the space of yields. In this section I study risk premia: the third testable implication of the model is that time-variation in bond risk premia is driven by (i) variations in relative wealths generate (instantaneous returns); and (ii) beliefs about future beliefs (long run returns). This is an important additional test of the model since, while the space of yields provides information on the drift of the stochastic discount factor, the space of excess returns gives us information on its diffusion component (prices of risk). As above, I propose to test relative wealth fluctuations from lags of disagreement. The underlying assumption is that the ex-post distribution of wealth is shifted towards the agent who holds relatively more accurate forecasts. Thus, conditional on a sample period with a series of large belief dispersions I should expect a large subsequent redistribution of wealth.

I repeat briefly how long run return variation arises with multiple agent economies. Optimists insure pessimists against bad state in exchange for premium in good states. This implies optimists’ prices of risk are higher than pessimists but because of perfect risk sharing wealth weighted prices of risk are equal to those in a standard Lucas economy. However, long run prices of risk (the impulse response of the stochastic discount factor) depend on today’s disagreement $\psi(t)$ and the path of future disagreements $\psi(t + h)$. Therefore, while under the objective measure $\psi(t)$ does not matter instantaneously, over long run horizons it drives risk compensation.
I study these hypotheses by asking whether lags of disagreement, \( \psi^g(t-h) \), contain information about future excess returns on \( n = 2, 5 \) and 10-year zeros

\[
hrx_{t,t+12}^{(n)} = const + \beta_1 \psi^g(t) + \beta_2 \psi^g(t-h) + \varepsilon_{t,t+12}^{(n)}
\]

in addition to contemporaneous disagreement, \( \psi^g(t) \). Table 3.6 reports point estimates and statistics for \( h = 1, 3 \) and 6 month lagged disagreement. Considering expected excess returns on 2-year bonds I find that \( \psi^g(t-h) \) is statistically significant at the 1% level for all lags. Since left and right hand variables are standardised, the point estimates represent the response of 1-year excess returns to a 1-standard deviation shock to disagreement. Therefore, the economic significance of the lagged terms is of equal importance to the contemporaneous terms: 0.22 versus 0.34 for \( h = 3 \), and 0.31 versus 0.24 for \( h = 6 \). In terms of predictable variation the \( \mathcal{R}^2 \)'s range from 12% to 24%. For comparison, repeating this exercise as a univariate forecasting regression with 6-month lagged \( \psi^g(t-6) \) on the right hand side I obtain a t-statistic of 3.77 and an \( R^2 \) of 15%. Considering next expected excess returns on 5-year bonds I a similar pattern in terms of point estimates, t-statistics and \( \mathcal{R}^2 \), suggesting that a substantial amount of return predictability is coming from both contemporaneous dispersion and lagged dispersion terms. Finally, considering the estimated loadings on 10-year bonds, I find an intriguing result that while the lagged terms are highly significant (t-stat = 3.58, 2.61, 2.33) for each \( h \), the contemporaneous terms are statistically indistinguishable from zero. Overall, these results suggest there is substantial information about expected returns embedded in distant lags of disagreement consistent with the model implication that \( \omega^i(t) \) is a function of past \( \psi^g(t-h) \), and contemporaneous disagreement \( \psi^g(t) \) due to time-variation in long run returns.

I examine the robustness of these findings to a number of alternative candidate risk factors. To save space I report regressions for 5-year maturity bonds on 6-month lagged disagreement \( \psi(t-6) \) and contemporaneous disagreement \( \psi^g(t) \).

Firstly, column \( (i) \) reports estimates from a univariate predictability regression including a single lag of disagreement on the right hand side. This regression confirms the forecasting power of lagged disagreement from table 3.6: the point estimate (t-stat) is 0.35 (3.56) and the \( R^2 \) is 12%. Column \( (ii) \) repeats this univariate regression with \( \psi(t) \) which shows that contemporaneous disagreement is also a strong predictor of expected returns with a point estimate (t-stat) equal to 0.33 (3.61) and the \( R^2 \) is 11%

Secondly, I check whether the information in dispersion factors is subsumed by fundamentals. To test this I construct a set of macro-activity factors following Ludvigson and Ng (2009a). The data appendix describes the construction of three macro return forecasting factors \( \{M1_t, M2_t, M3_t\} \). Constructing these factors I confirm the predictable content for expected returns available from a large panel of aggregates. For instance, in an unreported regression including only the macro factors the \( R^2 \) is 14% and each factor is significant. Columns \( (iii) \) and \( (iv) \) test the marginal contribution of \( \psi^g(t-h) \) and then \( \psi^g(t) \) after controlling for the macro factors. The estimated loadings are encouraging for the model since the information content in dispersion factors is largely orthogonal to the macro-economy: the point estimates
are close to columns (i) and (ii) and the significance is unaffected. Adding both dispersion factors to the regression the adjusted $R^2$ rises to 23% and, importantly, the disagreement factors are significant at the 1% level. This result is important since it suggests that (a) variations in aggregate macro-activity important for risk compensation are unrelated to relative wealth fluctuations; (b) higher order beliefs are an important determine of long run bond risk premia.

Thirdly, in long run risk economies (Bansal and Yaron (2004)) with recursive preferences predictability can arise from the dynamics of second moments (economic uncertainty) of fundamentals. One might suspect that the reported results could arise because beliefs are reacting to macro volatility. To address this concern I construct conditional variances of gdp growth and inflation following Bansal and Shaliastovich (2013), denoted $\sigma_t^2(g)$ and $\sigma_t^2(\pi)$, and consider the uncertainty factor studied by Baker, Bloom, and Davis (2012), denoted $UnC_t$. Column (vi) and (vii) show that these factors have no marginal contribution in explaining risk premia. This is interesting since it suggests that individual consumption volatility matters more for explaining time variation in risk premia than aggregate consumption volatility, as predicted by the model.
Table 3.6. Return Predictability Regressions

This table reports estimates from OLS regressions of annual \((t \rightarrow t + 12)\) excess returns of 2, 5 and 10-year zero-coupon bonds on disagreement \(\psi_t^g\) and lagged disagreement \(\psi(t-h)\) for horizons \(h = 1, \ldots, 12:\)

\[
hprx_{t,t+12}^{(n)} = \text{const} + \beta_1 \psi(t) + \beta_2 \psi(t-h) + \epsilon_{t,t+12}^{(n)}
\]

t-statistics reported in parentheses are corrected for autocorrelation and heteroskedasticity using the Hansen and Hodrick (1983) GMM correction computed with Newey-West lags. Both left and right hand variable are standardised. A constant is included but not reported. \(R^2\) reports the adjusted \(R^2\). Sample Period: 1990.1 - 2011.12

<table>
<thead>
<tr>
<th>(h = )</th>
<th>1m</th>
<th>3m</th>
<th>6m</th>
</tr>
</thead>
<tbody>
<tr>
<td>(n = 2)</td>
<td>(\psi^g(t))</td>
<td>0.20</td>
<td>0.22</td>
</tr>
<tr>
<td></td>
<td>(3.51)</td>
<td>(3.01)</td>
<td>(3.86)</td>
</tr>
<tr>
<td></td>
<td>(\psi^g(t-h))</td>
<td>0.30</td>
<td>0.34</td>
</tr>
<tr>
<td></td>
<td>(4.38)</td>
<td>(4.27)</td>
<td>(2.93)</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.21</td>
<td>0.24</td>
<td>0.23</td>
</tr>
<tr>
<td>(n = 5)</td>
<td>(\psi^g(t))</td>
<td>0.14</td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td>(2.58)</td>
<td>(2.06)</td>
<td>(2.79)</td>
</tr>
<tr>
<td></td>
<td>(\psi^g(t-h))</td>
<td>0.25</td>
<td>0.30</td>
</tr>
<tr>
<td></td>
<td>(3.69)</td>
<td>(3.52)</td>
<td>(2.81)</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.13</td>
<td>0.16</td>
<td>0.15</td>
</tr>
<tr>
<td>(n = 10)</td>
<td>(\psi^g(t))</td>
<td>0.04</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>(0.76)</td>
<td>(1.08)</td>
<td>(1.34)</td>
</tr>
<tr>
<td></td>
<td>(\psi^g(t-h))</td>
<td>0.21</td>
<td>0.21</td>
</tr>
<tr>
<td></td>
<td>(3.58)</td>
<td>(2.61)</td>
<td>(2.33)</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.05</td>
<td>0.06</td>
<td>0.07</td>
</tr>
</tbody>
</table>
Table 3.7. Return Predictability Robustness
This table reports estimates from OLS regressions of annual \((t \rightarrow t + 12)\) excess returns of 5-year zero-coupon bonds 6-month lagged disagreement \(\psi(t - 6)\) controlling for alternative risk factor proxies.

\[
hprx_{t,t+12}^{(5)} = \text{const} + \beta_{\psi} \psi(t - 6) + \gamma \text{Controls}_t + \varepsilon_{t,t+12}^{(5)}
\]

_t-statistics_ reported in parentheses are corrected for autocorrelation and heteroskedasticity using the Hansen and Hodrick (1983) GMM correction computed with Newey-West lags. Both left and right hand variable are standardised. A constant is included but not reported. _\bar{R}^2_ reports the adjusted _R^2_. Sample Period: 1990.1 - 2011.12

<table>
<thead>
<tr>
<th></th>
<th>(i)</th>
<th>(ii)</th>
<th>(iii)</th>
<th>(iv)</th>
<th>(v)</th>
<th>(vi)</th>
<th>(vii)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\psi(t - 6))</td>
<td>0.35</td>
<td>0.34</td>
<td>0.23</td>
<td>0.24</td>
<td>0.21</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(3.56)</td>
<td>(3.35)</td>
<td>(2.46)</td>
<td>(2.54)</td>
<td>(2.24)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\psi(t))</td>
<td>0.33</td>
<td>0.34</td>
<td>0.25</td>
<td>0.18</td>
<td>0.19</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(3.61)</td>
<td>(3.52)</td>
<td>(2.96)</td>
<td>(2.12)</td>
<td>(2.53)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(M_{1t})</td>
<td>0.11</td>
<td>0.10</td>
<td>0.10</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.96)</td>
<td>(3.45)</td>
<td>(2.60)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(M_{2t})</td>
<td>0.15</td>
<td>0.19</td>
<td>0.14</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.19)</td>
<td>(2.60)</td>
<td>(1.90)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(M_{3t})</td>
<td>0.17</td>
<td>0.18</td>
<td>0.21</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.56)</td>
<td>(1.95)</td>
<td>(2.38)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\sigma^2(g))</td>
<td>-0.07</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-0.47)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\sigma^2(\pi))</td>
<td>0.20</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.27)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(UnC_t)</td>
<td>0.12</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.34)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\bar{R}^2)</td>
<td>0.12</td>
<td>0.11</td>
<td>0.19</td>
<td>0.19</td>
<td>0.23</td>
<td>0.16</td>
<td>0.16</td>
</tr>
</tbody>
</table>
V. Conclusion

A vast empirical literature documents the existence of a predictable component in bond excess returns. At the same time, it shows that it is difficult to reconcile the dynamics of excess bond returns with simple homogeneous agents models. I investigate two questions. First, what is the extent to which heterogeneity in macroeconomic beliefs matter in Treasury bond markets? Is this specific dimension of heterogeneity empirically important or can it be abstracted away within the simpler single agent paradigm? Second, what is the exact channel through which disagreement affects asset prices, and which models are consistent with the data?

To answer these questions I derive a general equilibrium economy with multiple agents who hold subjective beliefs regarding unobservable growth rates. In equilibrium, agents engage in speculative trade because of disagreement, which in turn generates endogenously time-varying wealth fluctuations. I study a set of testable restrictions about short term interest rates and the cross-sectional distribution of yields that help distinguish myopic, speculative, and risk sharing models. Moreover, we show that the properties of long run returns depend crucially on forward looking beliefs.

My empirical results suggest that: (i) heterogeneity is an important channel that cannot be abstracted way; (ii) agents have a coefficient of relative risk aversion $\gamma < 1$ ($EIS > 1$); (iii) both the cross-section of yields and the bond risk premium are path dependent due to past speculation; and (iv) that long run returns differ from instantaneous returns because of the risk of changing future beliefs.
VI. Appendix A: Proofs

A. Single Agent Term Structure of Interest Rates

The date \( t \) price of an \( T \)-period default free zero-coupon bond is:

\[
P_t^T = \frac{1}{M_t} E_t \left[ g_t, T, M_T \right] = E_t \left[ g_{t,T} \left( \frac{D_T}{D_t} \right)^{-\gamma} \right]
\]

Defining \( x_T = \ln D_T \) solving for bond prices requires computing the following joint moment generating:

\[
\phi_x(T; u_1) = E_t \left( e^{x_1 x_T} \right).
\]

We denote this the ‘fundamental system’.

B. Fundamental System

From Feynman-Kac this function satisfies the following partial differential equation

\[
0 \equiv \mathcal{D} \phi_x + \frac{\partial \phi_x}{\partial t} (D_t, g_t, t, T; u_1)
\]

with initial condition \( \phi_x(t; u_1) = D(t)^{\gamma} \) and where \( \mathcal{D} \) is the Dynkin operator for the multivariate process \((D_t, g_t)\). Applying the operator we have

\[
\begin{align*}
\frac{\partial \phi_x}{\partial D} D_t g_t - \frac{\partial \phi_x}{\partial g} \kappa_g (g_t - \theta) + \frac{1}{2} \frac{\partial^2 \phi_x}{\partial D^2} D_t^2 \sigma_D^2 + \frac{1}{2} \frac{\partial^2 \phi_x}{\partial g^2} \left[ \sigma_D^2 + \sigma_g^2 \right] + \frac{\partial^2 \phi_x}{\partial D \partial g} D_t \sigma_D \sigma_g + \frac{\partial \phi_x}{\partial t} &= 0
\end{align*}
\]

Defining \( \tau = T - t \) the solution has the following form

\[
\phi_x = D_t^{u_1} E_t \left[ \left( \frac{D_T}{D_t} \right)^{u_1} \right] = D_t^{u_1} e^{A(\tau,u_1)+B(\tau,u_1)g_t}
\]

(3.44)

Taking partials with respect to \( D_t \) we obtain

\[
u_1 \tilde{g}_t - B(\tau)[\kappa_g (g_t - \theta)] + \frac{1}{2} u_1 (u_1 - 1) \sigma_D^2 + \frac{1}{2} B^2(\tau) \sigma_g^2 + u_1 B(\tau) \rho \sigma_D \sigma_g + \left[ \frac{\partial A}{\partial t} + \frac{\partial B}{\partial t} g_t \right] = 0
\]

which are separable ODE’s

\[
\begin{align*}
\frac{\partial A}{\partial \tau} &= \frac{1}{2} u_1 (u_1 - 1) \sigma_D^2 + [\kappa_g \theta + u_1 \rho \sigma_D \sigma_g] B(\tau) + \frac{1}{2} B(\tau)^2 \sigma_g^2 \\frac{\partial B}{\partial \tau} + \kappa_g B(\tau) &= u_1
\end{align*}
\]

with solutions given by

\[
A(\tau) = \frac{1}{2} u_1 (u_1 - 1) \sigma_D^2 \tau + \left( \frac{u_1 \theta}{\kappa_g} + \frac{u_1^2}{\kappa_g^2} \sigma_D \sigma_g, D \right) \left( \kappa_g \tau + e^{-\kappa_g \tau} - 1 \right)
\]

\[
+ \frac{1}{4} \frac{u_1^2}{\kappa_g^2} \left( \sigma_D^2 + \sigma_g^2 \right) \left( 2 \kappa_g \tau + 4 e^{-\kappa_g \tau} - e^{-2 \kappa_g \tau} - 3 \right)
\]

\[
B(\tau) = \frac{u_1}{\kappa_g} (1 - e^{-\kappa_g \tau})
\]

Identifying \( u_1 = -\gamma \) we obtain the functions reported in the body of the paper.
C. Learning

Denote agent \( i \)'s conditional forecast \( \hat{g}_i = E_i[ g_i | \mathcal{F}_t ] \) and posterior variance \( \nu_i = E_i[ (\hat{g}_i - g_i)^2 | \mathcal{F}_t ] \). Write the correlated Brownians in terms of our rotated Brownians we have for \( 3.1 \) and \( 3.2 \) in terms of the independent Brownian motions \( \{ W_1^D, W_2^D \} \):

\[
W_i^D = W_i^1 \quad (3.45)
\]
\[
W_i^g = \rho_i W_i^1 + \sqrt{1 - \rho_i^2} W_i^2 \quad (3.46)
\]

where \( \rho_i \) is an agent specific parameter. The filtering problem contains three independent Brownian motions and two measurement equations.

\[
dD/dt = g_i dt + \sigma_d dW_i^1,
ds_i = \phi_i dW_i^2 + \sqrt{1 - \phi_i^2} dW_i^3,
\]

Writing the unobservable state in terms of our original Brownians follows as reported in the body of the paper.

Using notation consistent with Lipster and Shiryayev (1974) we can re-write the measurement equation in vector form

\[
dY_t = \left[ \begin{array}{c} dD/dt \quad ds_i \end{array} \right] \quad \text{so that}
\]

\[
dY_t = (A_0 + A_1 \hat{g}_i) dt + B_1 dW_i^1 + B_2 dW_i^2 + B_3 dW_i^3
\]

At the same time, consistent with this notation we can write the state dynamics as:

\[
d\hat{g}_i = \left( \begin{array}{c} \frac{a_0}{\kappa_g} + \frac{a_1}{-\kappa_g} \hat{g}_i \\ \sigma_g \rho_i \end{array} \right) dt + \left( \begin{array}{c} b_1 \\ \sigma_g \rho_i \end{array} \right) dW_i^1 + \left( \begin{array}{c} b_2 \\ \sigma_g \sqrt{1 - \rho_i^2} \end{array} \right) dW_i^2 + \left( \begin{array}{c} b_3 \\ 0 \end{array} \right) dW_i^3
\]

Applying the results of Lipster and Shiryayev (1974) (theorem 12.7. Page 36) the optimal linear in terms of our original Brownians follows as reported in the body of the paper.

D. Term Structure of Interest Rates

The date \( t \) price of an \( T \)-period default free zero-coupon bond is:

\[
P^T_t = \frac{1}{M_t} E_i^t \left[ g_t \gamma T \right]
\]
\[
P^T_t = E_i^t \left[ g_t \left( D_T \right)^{-\gamma} \left( \frac{1 + \eta T^{1/\gamma}}{1 + \eta t^{1/\gamma}} \right)^\gamma \right]
\]

where the expectation is taken under agent \( i \)'s measure. Defining \( x_T = \ln D_T \) and \( y_T = \ln \eta_T \) we need to compute the following joint moment generating:

\[
\phi_{x,y}(T; u_1, u_2) = E_i^t \left( e^{x_T + u_1 y_T} \right).
\]  

(3.47)
E. Belief System

Setting \( u_1 = 0 \) in equation F we obtain the following moment generating function for \( \eta_T \)

\[ \phi_y = \phi_{\eta,y}(T;0,u_2) = E_t(\exp^{u_2\eta_T}). \]  

(3.48)

Feynman-Kac implies the following partial differential equation

\[ 0 \equiv D\phi_y + \frac{\partial \phi_y}{\partial t}(\eta,\psi,t;u_2) \]

with initial condition \( \phi_y(t;u_2) = \eta(t)^{u_2} \). Applying the operator we have

\[ -\frac{\partial \phi_y}{\partial \eta} \psi_1 + \frac{1}{2}\frac{\partial^2 \phi_y}{\partial \eta^2} \eta^2_1 \psi_1^2 + \frac{1}{2} \frac{\partial^2 \phi_y}{\partial \psi^2}(\sigma_{\psi,D}^2 + \sigma_{\psi,s}^2) - \frac{\partial^2 \phi_y}{\partial \psi \partial \eta} \eta_p \psi_1 \sigma_{\psi,D} + \frac{\partial \phi_y}{\partial t} = 0 \]

whose solution is affine in the extended state vector \( (\psi_1, \psi_1') \)

\[ \phi_y = \eta_t^{u_2} E_t \left[ \left( \frac{\eta_T}{\eta} \right)^{u_2} \right] = \eta_t^{u_2} \exp^{L(\tau)+M(\tau)\psi+N(\tau)\psi_1^2} \]  

(3.49)

Taking partials with respect to \( \eta \) we obtain

\[ -[M(\tau) + 2N(\tau)\psi_1] \kappa \psi_1 + \frac{1}{2}u_2(u_2 - 1) \psi_1^2 + \frac{1}{2}(\sigma_{\psi,D}^2 + \sigma_{\psi,s}^2)[2N(\tau) + M(\tau)^2 + 4M(\tau)N(\tau)\psi_1 + 4N(\tau)^2\psi_1^2] \]

\[ -[M(\tau) + 2N(\tau)\psi_1]u_2 \sigma_{\psi,D} \psi_1 + \left[ \frac{L(\tau)}{\partial \tau} + \frac{M(\tau)}{\partial \tau} \psi_1 + \frac{N(\tau)}{\partial \tau} \psi_1^2 \right] = 0 \]

Defining \( c_1 = -(\kappa_0 + u_2 \sigma_{\psi,D}) \), \( c_2 = \frac{1}{2}u_2(u_2 - 1) \), \( c_3 = \frac{1}{2}(\sigma_{\psi,D}^2 + \sigma_{\psi,s}^2) \) we collect the following ODEs

\[ \frac{\partial L}{\partial \tau} = c_3[2N(\tau) + M(\tau)^2] \]

(3.50)

\[ \frac{\partial M}{\partial \tau} = [4c_3N(\tau) + c_1]M(\tau) \]

(3.51)

\[ \frac{\partial N}{\partial \tau} = 4c_3N(\tau)^2 + 2c_1N(\tau) + c_2 \]

(3.52)

The last ODE is a constant coefficient Riccati equation whose solution is given by

\[ N(\tau) = \frac{q}{2c_3} \frac{1}{\tilde{c}^2 - 4c_3 c_2} - \left( \frac{q + c_1}{4c_3} \right) \]

where \( q = \sqrt{\tilde{c}^2 - 4c_3 c_2} \) and \( \tilde{c} = \frac{q - c_1}{q + c_1} \)

Solutions for \( L(\tau) \) and \( M(\tau) \) can be computed in closed form but we omit these to save space.

**Solving the Constant Coefficient (scalar) Riccati Equation**

We derive a general solution so the constant coefficient Riccati equations \( y(x) \):

\[ \frac{dy(x)}{dx} = ay(x)^2 + by(x) + c \]

which we can reduce to a second order linear equation via the substitution \( y(x) = -\frac{1}{a} \frac{w''(x)}{w'(x)} \) yielding

\[ w'' - bw' + c = 0. \]

The general solution to this ODE is \( w(x) = C_1 e^{r_1x} + C_2 e^{r_2x} \) where \( r_1 \) and \( r_2 \) are the positive and negative roots

\[ r_{1,2} = \frac{b \pm \sqrt{b^2 - 4ac}}{2} = \frac{b \pm q}{2} \]

129
and \( C_{1,2} \) are constants determined by the boundary condition. A general solution is then
\[
y(x) = -\frac{1}{a} \frac{C_1 e^{x^a} + C_2 e^{x^b}}{C_1 e^{x^a} + C_2 e^{x^b}} = -\frac{1}{a} \frac{\hat{c} e^{x^a} + r_2}{\hat{c} e^{x^a} + 1} = -\frac{q}{a e^{x^a} + 1} - \left( \frac{b + q}{2a} \right)
\]
where the last step follows by some partial fractions algebra. This result is applied above with an appropriate boundary condition.

F. Joint Distribution

We now derive the joint moment generating function:
\[
\phi_{x,y}(T; u_1, u_2) = E^t \left( e^{x^a T + u_2 y} \right)
\]
which from Feynman-Kac satisfies the following partial differential equation
\[
0 \equiv D \phi_{x,y} + \frac{\partial \phi_{x,y}}{\partial t} (D, \eta, g^s, \psi, t, T; u_1, u_2)
\]
with initial condition \( \phi_{x,y}(t; \epsilon, \chi) = D(t)^{i\eta(t)\psi(t)} \) and where \( D \) is the Dynkin operator for the multivariate process \((D, \eta, g^s, \psi)\).

We solve for prices under the measure of agent \( a \) and so all parameters and expectations in the following should be understood from his perspective. Applying the operator (and dropping subscripts on \( \phi_{x,y} \)) we have
\[
\frac{\partial \phi}{\partial D} D g^s - \frac{\partial \phi}{\partial g^s} \kappa_s (g^s - \theta) - \frac{\partial \phi}{\partial \psi} \kappa_\psi \psi_t + \frac{1}{2} \frac{\partial^2 \phi}{\partial D^2} D^2 g^s + \frac{1}{2} \frac{\partial^2 \phi}{\partial \psi^2} \eta^2 \psi_t^2 + \frac{1}{2} \frac{\partial^2 \phi}{\partial \psi \eta^2} \eta \psi_t \psi_s + \frac{1}{2} \frac{\partial^2 \phi}{\partial \kappa_\psi \eta^2} \eta \psi_t \psi_s + \frac{1}{2} \frac{\partial^2 \phi}{\partial \psi \kappa_\psi} \kappa_\psi \psi_t \psi_s + \frac{1}{2} \frac{\partial^2 \phi}{\partial \kappa_\psi \kappa_\psi} \kappa_\psi \psi_t \psi_s
\]
\[
one - \frac{\partial^2 \phi}{\partial \psi \psi_s} \psi_t \psi_s + \frac{\partial^2 \phi}{\partial g^s \psi_s} [\sigma_\psi, \sigma_s, \psi_s] + \frac{\partial^2 \phi}{\partial \psi \psi_s} \psi_t \psi_s + \frac{\partial \phi}{\partial t} = 0
\]
The solution takes the following form
\[
\phi(T) = D_T^{i\eta_1} \eta_2 E_t \left( \left( \frac{D_T}{D_t} \right)^{i\eta_1} \right) E_t \left[ \left( \frac{\eta_2}{\eta_1} \right)^{u_2} \right] = D_T^{i\eta_1} \eta_2 \phi_s(T; u_1) \phi_y(T; u_2)
\]
where \( \phi_s(T; u_1) = e^{A(T, u_1) + B(T, u_1) \psi} \) and \( \phi_y(T; u_1, u_2) = e^{L(T, \psi) + M(T, \psi) \psi + N(T, \psi) \psi^2} \)

Taking partials with respect to \( D_t \) and \( \eta_1 \) allows us to factor out terms independent of \( \psi \). The solutions for \( A(T) \) and \( B(T) \) are then as above. Next, terms in \( \psi \) solve
\[
\frac{\partial \phi}{\partial \psi} \left[ -\kappa_\psi \psi_t + u_1 \sigma_{\psi, \psi} \psi_t - u_2 \sigma_{\psi, \psi} \psi_t + \frac{u_1}{\kappa_\psi} \left( 1 - e^{-\kappa_\psi T} \right) \psi_t \right] + \frac{1}{2} \frac{\partial^2 \phi}{\partial \psi^2} \left[ \sigma_{\psi, \psi}^2 + \sigma_{\psi, \psi}^2 \right] + \phi_y u_1 \left( -u_2 \sigma_{\psi, \psi} - \frac{1}{\kappa_\psi} \left( 1 - e^{-\kappa_\psi T} \right) u_2 \sigma_{\psi, \psi} \right) \psi_t + \phi_y \frac{1}{2} u_2 \left( u_2 - 1 \right) \psi_t^2 + \frac{\partial \phi}{\partial t} = 0
\]
which we write compactly as
\[
\frac{\partial \phi}{\partial \psi} \left[ c_1 \psi_t + u_1 b_1(\tau) \right] + \frac{\partial^2 \phi}{\partial \psi^2} c_3 + \phi_y u_1 b_2(\tau) \psi_t + \phi_y c_2 \psi_t^2 + \frac{\partial \phi}{\partial t} = 0
\]
where \( c_1, c_2, c_3 \) are as above and \( b_1(\tau) \) and \( b_2(\tau) \) defined appropriately. Taking partials we find
\[
\left[ M(T) + 2N(T) \psi_t \right] c_1 \psi_t + u_1 b_1(\tau) + c_2 \left[ 2N(T) + M(T) \right] + c_3 \left[ 2N(T) + M(T) \right] + N(T) \psi_t + 4N(T) \psi_t^2
\]
\[
\left[ b_2(\tau) \psi_t + c_2 \psi_t^2 + \frac{L(T)}{\partial t} \right] + \frac{M(T)}{\partial t} \psi_t + \frac{N(T)}{\partial t} \psi_t^2 = 0
\]
from which we obtain the following system of ODE’s

\[
\frac{\partial L}{\partial \tau} = u_1 M(\tau) b_1(\tau) + c_3 [2N(\tau) + M(\tau)^2] \\
\frac{\partial M}{\partial \tau} = u_2 b_2(\tau) + [4c_3 N(\tau) + c_1] M(\tau) \\
\frac{\partial N}{\partial \tau} = 4c_3 N(\tau)^2 + 2c_2 N(\tau) + c_2.
\]

(3.53) 
(3.54) 
(3.55)

Firstly, notice that setting \( u_1 = 0 \) we recover equations 3.50 - 3.52 which characterise the density for \( \eta(T) \). Equation 3.55 is a again a constant coefficient Ricatti equation whose solution is given in the solution for the ‘belief system’. The solution for \( M(\tau) \) is given by

\[
M(\tau) = \frac{u_1 u_2}{\kappa_0 q(\kappa_0 - q)(\kappa_0 + q)} e^{-\kappa_0 \tau} \left( -\tilde{c} (\kappa_0 - q)(\kappa_0 + q) e^{\tau(\kappa_0 + 2q)} \right) \times \left( \kappa_0 \sigma_D + \sigma_D \kappa_0 + \kappa_0 e^{\tau(\kappa_0 + q)} (\tilde{c} - 1) \sigma_D (\kappa_0 - q)(\kappa_0 + q) + \sigma_D (\tilde{c} - 1) \sigma_D (\kappa_0 + q) \right) \\
- \tilde{c} \sigma_D \kappa_0 e^{\tau q} + (\kappa_0 - q)(\kappa_0 + q) e^{\tau q} (\kappa_0 \sigma_D + \sigma_D \kappa_0 + \sigma_D (\kappa_0 - q)^2 )
\]

\[\text{(3.56)}\]

The solution for \( L(\tau) \) follows by direct integration.

**Solving \( \gamma \in \mathbb{R}^+ \)**

Given a solution for characteristic function of \((D_T, \eta_T)\) we can recover the joint density via inversion which allows us to compute the price of any contingent claim. Setting \( u_1 = -\gamma \) we recover the joint transition density via inversion by evaluating the (inverse) bilateral Laplace transform at \( u_2 \), which equivalent the computing the continuous time Fourier transform

\[
P(t, T) = \phi_T(\tau; -\gamma) g(y, t) t^{-1} \int_0^\infty g(y, T) \frac{1}{\pi} \int_0^\infty e^{-iu_2 \tau} \phi_2(\tau; -\gamma, u_2) du_2 \] dB

(3.57)

where

\[
g(y, s) = \left(1 + e^{s \gamma} \right)^\gamma
\]

(3.58)

\[
\phi_2(\tau; u_1) = e^{A(\tau) + B(\tau) \eta_1}
\]

(3.59)

\[
\phi_2(\tau; u_1, u_2) = e^{L(\tau) + M(\tau) \eta_1 + N(\tau) \eta_2^2}
\]

(3.60)

with \( \{A(\tau), B(\tau), L(\tau), M(\tau), N(\tau)\} \) reported in the main body of the paper.

**Solving \( \gamma \in \mathbb{N} \)**

For integer \( \gamma \) a closed form solution for bond prices can be found using the binomial expansion:

\[
P_T = \phi_T e^{\frac{D_T}{D_T}} \left[ \left( \frac{D_T}{D_T} \right)^{-\gamma} \left( \frac{1}{1 + \eta_T \gamma} \right) \right]^{-\gamma} \]

\[
= \phi_T (\omega_T^\gamma \sum_{j=0}^\gamma \left( \frac{\omega_T^j}{\omega_T^{j/\gamma}} \right)^{j/\gamma} \left( \frac{D_T}{D_T} \right)^{-\gamma} \left( \frac{\eta_T}{\eta_T} \right)^{j/\gamma} \]

\[
= \phi_T F_g(\omega_T^\gamma, T, -\gamma)(\omega_T^\gamma \sum_{j=0}^\gamma \left( \frac{\omega_T^j}{\omega_T^{j/\gamma}} \right)^{j/\gamma} \left( \frac{\omega_T^j}{\omega_T^{j/\gamma}} \right)^{j/\gamma} F_\phi(\psi, T; -\gamma, j/\gamma)
\]

**Myopic Term Structures**

When agents are myopic heterogeneous equilibrium bond prices are given as wealth weighted averages of fictitious
homogeneous equilibrium bond prices:

\[ P(t,T) = g_r \phi^a(\tau; -1) \omega^a \left( 1 + \frac{\omega^a}{\omega^b} e^{\gamma} \right), \]
\[ = g_r (\omega^a \phi^a(\tau; -1) + \omega^b e^{\gamma} + L(\tau) + M(\tau) s^2 \gamma / \sigma_D), \]
\[ = g_r (\omega^a \phi^a(\tau; -1) + \omega^b e^{\gamma} + L(\tau) + B(\tau) s^2), \]
\[ = \omega^a P^a(t,T) + \omega^b P_b(t,T). \]

since \( M(\tau, u_1 = -\gamma) = -\sigma_D B(\tau, u_1 = -\gamma).^{25} \)

**Bond Sensitivities**

\[
\frac{\partial \ln P(t,T)}{\partial g^a} = B(\tau) \\
\frac{\partial \ln P(t,T)}{\partial \eta} = \int_0^\infty \left[ g(y,T) \int_0^\infty e^{-iy} e^{(u_2 - 1)\gamma} \phi(y; \gamma, u_2) du_2 \right] dy(T) - \frac{\eta^{(1-\gamma)/\gamma}}{1 + \eta^{1-\gamma}} \\
\frac{\partial \ln P(t,T)}{\partial \psi} = \int_0^\infty \left[ g(y,T) \int_0^\infty e^{-iy} e^{(u_2 - 1)\gamma} \phi(y; \gamma, u_2) [M(\tau) + 2N(\gamma) \psi] du_2 \right] dy(T) \\
\int_0^\infty g(y,T) \int_0^\infty e^{-iy} e^{(u_2 - 1)\gamma} \phi(y; \gamma, u_2) du_2 \right] dy(T)
\]

---

^{25} An alternative proof is as follows. Myopic agents consume wealth at a rate equal to their time rate of preference: \( c_t^r = \rho W_t^a \). Since agents must agree on tradable prices, \( E^a_t(\frac{1}{2} P(t + 1, T)) = E^a_t(\frac{1}{2} P(t + 1, T)) \) this implies \( \eta_T = W_T^a / W_T^b \) serves as a valid change of measure. Imposing market clearing, \( c_t^r + c_t^b = D_t \), we obtain \( c_t^r = \frac{1}{\gamma} D_t \) and \( c_t^b = \frac{\gamma}{\rho} D_t \).

Substituting \( c_t^a \) into the price of a bond from the perspective of agent \( a \), and after a change of measure we obtain the desired result.
VII. Appendix B: Data

A. Realised Returns

For Treasury bonds data, I use (unsmoothed) discount bonds dataset for maturities 1 to 10 years kindly provided by Robert Bliss. I introduce notation along the lines of Cochrane and Piazzesi (2005a) by defining the date \( t \) log price of a \( n \)-year discount bond as:

\[
p_t^{(n)} = \log \text{price of } n \text{-year zero coupon bond.}
\]

The yield of a bond is defined as

\[
y_t^{(n)} = -\frac{1}{n} p_t^{(n)}.
\]

The date-\( t \)-1-year forward rate for the year from \( t + n - 1 \) and \( t + n \) is

\[
f_t^{(n)} = p_t^{(n)} - p_t^{(n+1)}.
\]

The log holding period return is the realised return on an \( n \)-year maturity bond bought at date \( t \) and sold as an \((n-1)\)-year maturity bond at date \( t+12 \):

\[
r_t^{(n)} = p_{t+12}^{(n-1)} - p_t^{(n)}.
\]

Excess holding period returns are denoted by:

\[
r_{x,t+12}^{(n)} = r_t^{(n)} - y_t^{(1)}.
\]

B. Macro Data

In dynamic single agent equilibrium models of the term structure, factors linked to the marginal productivity of capital account for the dynamics of interest rates. In such models, the marginal rate of substitution is tightly connected to the marginal rate of transformation which drives the term structure. Unfortunately, linking variation in yields to observable macro-economic quantities has met limited success. There are two fundamental problems: Firstly, shocks to real growth and/or inflation explain just a small fraction of shocks to nominal yields. Secondly, there appears to be very little covariance between bond returns and macro-factors that demand compensation in standard consumption based models.\(^{26}\) An notable exception is Ludvigson and Ng (2009a) who find strong evidence linking variations in the level of macro-economic activity, obtained from principal components of a very large set of macroeconomic variables, to the time variations of expected excess bond returns. I study the marginal contribution of disagreement after controlling for the macro-activity factors of Ludvigson and Ng (2009a). Different than in their approach, however, I drop any price based information from their initial information set.\(^{27}\) This allows us to interpret the factors as pure ‘macro’ that capture the current state of economic activity. After removing price based information from the panel I end up with 99 macro series, from which I compute the first eight principal components.\(^{28}\) From the resulting dataset I compute the first eight principal components \( F_1^{[1:8]} \) then construct a single ‘pure’ macro return forecasting factor from a regression of average holding period returns on 1 to 10-year bonds:\(^{29}\)

\[
\frac{1}{n} \sum_{n=1}^{10} r_{x,t+12}^{(n)} = \pi + \gamma' F_t^{[1:8]} + \varepsilon_{t+12}
\]

\[
LN_t = \gamma' F_t^{[1:8]}
\]

In long run risk economies (Bansal and Yaron (2004)) with recursive preferences predictability arises from the dynamics of second moments (economic uncertainty) of the conditional growth rate of fundamentals. Recently, Bansal and Shaliastovich (2013) discuss the implications of this class of models for bond markets. In their model, equilibrium bond prices depend on 4-factors while risk compensation is time-varying with conditional second moments depending on both inflation non-neutrally and whether agents have a preference for early or late resolution of uncertainty.

\(^{26}\) Duffee (2012) provides a critical review of macro-finance models and their ability explain bond markets and concludes and concludes that the link between bond markets and the macro-economy is at best weak.

\(^{27}\) Examples of price variables removed include: S&P dividend yield, the Federal Funds (FF) rate; 10 year T-bond; 10 year - FF term spread; Baa - FF default spread; and the dollar-Yen exchange rate. A small number of discontinued macro series were replaced with appropriate alternatives or dropped.

\(^{28}\) A description of each series along with its data source is available on the authors websites.

\(^{29}\) I refer the reading to Ludvigson and Ng (2009a) for additional details.
I proxy for economic uncertainty following Bansal and Shaliastovich (2013) who seeks to exploit the information about future volatility contained in date $t$ yields using survey forecasts of gdp growth and inflation. First, I fit a bivariate VAR(1) to inflation and GDP expectations:

$$g_{t+1}^e = 0.63 + 0.86 g_t^e - 0.08 \pi_t^e + \epsilon_{g,t+1}$$

$$\pi_{t+1}^e = 0.93 - 0.20 g_t^e + 0.87 \pi_t^e + \epsilon_{\pi,t+1},$$

where $\pi_t^e = E_t^c[\pi_{t+12}]$ and $g_t^e = E_t^c[g_{t+12}]$ for ease of notation. Similar to Bansal and Shaliastovich (2013), I find that both processes feature high persistence (their AR(1) coefficient for inflation expectations is 0.99, however), and that inflation has a non-neutral effect on growth: the loading of consumption growth on lagged inflation is strongly significant. Next, I regress squared residuals between $t$ and $t+12$ on time-$t$ yields:

$$\sum_{k=1}^{12} \epsilon_{g,t+k}^2 = 0.19 + 0.21 y_t^{(1)} - 0.55 y_t^{(3)} + 0.30 y_t^{(5)} - 0.04 y_t^{(10)} + \eta_{g,t+1} R^2 = 0.07$$

$$\sum_{k=1}^{12} \epsilon_{\pi,t+k}^2 = 0.21 + 0.04 y_t^{(1)} + 0.04 y_t^{(3)} - 0.24 y_t^{(5)} + 0.16 y_t^{(10)} + \eta_{\pi,t+1} R^2 = 0.08,$$

and take fitted values as an estimate of conditional variances $\sigma^2(g)$ and $\sigma^2(\pi)$. 

134
VIII. Appendix C: Figures

Figure 3.1. Short Rate Sensitivity to Disagreement:
The left panel plots the sensitivity of the short rate with respect to disagreement as a function of the level of disagreement for risk aversion levels above and below one, while the right panel plots the short rate sensitivity with respect to disagreement as a function of risk aversion for different levels of disagreement.

Figure 3.2. Term Structure Example 1: Sentiment Economy
Figure plots the term structure of interest rates for an economy with zero disagreement but varies the relative wealths between the optimistic and pessimist agents.
Figure 3.3. Term Structure Example 2: Symmetric Economy
Figure plots the term structure of interest rates for an economy with a symmetric wealth distribution $\omega^a_t = \omega^b_t = 0.50$ with $g^a_t - g^b_t \in [0\% : 3\%]$ for various levels of risk aversion above and below 1.

Figure 3.4. Term Structure Example 2: Symmetric Economy
Left Panel: Level = $\frac{1}{n} \sum^n_i y^n_i$. Right Panel: Slope = $y^i_t - r_t$

Figure 3.5. Term Structure Example 3: Pessimistic Economy
Figure plots the term structure of interest rates for an economy which is on average pessimistic $\omega^a_t < \omega^b_t$ with $g^a_t - g^b_t \in [0\% : 3\%]$ for various levels of risk aversion above and below 1.
Figure 3.6. Term Structure Example 4: Optimistic Economy
Figure plots the term structure of interest rates for an economy which is on average optimistic $\omega_a > \omega_b$ with $g_a - g_b \in [0\% : 3\%]$ for various levels of risk aversion above and below 1.

Figure 3.7. Risk Premia Example 1: Symmetric Economy
Figure plots the term structure of risk premia (instantaneous expected excess returns) for an economy with a symmetric wealth distribution $\omega_a = \omega_b$ with $g_a - g_b \in [0\% : 3\%]$ for risk aversions 0.5, 1.0, 2.0.
Figure 3.8. Risk Premia Example 2: Pessimistic Economy

Figure plots the term structure of risk premia (instantaneous expected excess returns) for an economy which is on average pessimist $\omega_a < \omega_b$ with $g_a - g_b \in [0\% : 3\%]$ for risk aversions 0.5, 1.0, 2.0.
Figure 3.9. Risk Premia Example 2: Optimistic Economy

Figure plots the term structure of risk premia (instantaneous expected excess returns) for an economy which is on average optimistic \( \omega_a > \omega_b \) with \( g^a_t - g^b_t \in [0\% : 3\%] \) for risk aversions 0.5, 1.0, 2.0.
Figure 3.10. Survey Extract: 1Q ahead Interest Rate Forecasts:
Each month BlueChip Financial Forecasts (BCFF) carries out surveys of professional economists who are asked to forecast a large cross-section of interest rates from 1-quarter to 5-quarter ahead.

Figure 3.11. BlueChip Financial Forecasts Contributors
The left panel plots a time-series of the average number of respondents to the BlueChip Financial Forecasts survey. The right panel plots the distribution of respondents. Sample Period: January 1990 - December 2011.
Figure 3.12. Differences in Belief Proxies
Figure plots time-series for differences in belief about the 1-quarter GDP growth ($\psi(g)$) along with an uncertainty proxy studied by Baker, Bloom, and Davis (2012). Sample period: January 1990 - December 2011.
Figure 3.13. Real and Nominal Short Term Interest Rates

Figure plots the time-series of the 3-month nominal interest rate (blue line) taken from the Fama-Bliss short rate files on CRSP. The real short rate is constructed by subtracting 3-month consensus inflation expected from BCFF:

\[ y_{t}^{3m\text{ real}} = y_{t}^{3m\text{ Nom}} - \beta E_{t}[\pi(t + 3m)] \]

Also plotted is 3-month expected GDP growth \( E_{t}[g(t + 3m)] \). Sample period: January 1990 - December 2011
Figure 3.14. Path Dependence Example:
Figure plots a stylised binomial tree for the evolution of beliefs and relative wealths. The tree begins at the zeroth node where agents have equal wealths but agent $a$ is the optimist. In subsequent periods agents revise their beliefs and there is a redistribution of wealth based on the path of beliefs from previous periods.
Figure 3.15. Predicting Disagreement about Real Growth

The left panel plots the factor loadings ($\beta_1, \beta_2$) for horizon $h$ in the forecasting regression

$$\psi_{t+h}(g) - \psi_t(g) = \text{const} + \beta_1 \text{Level}_t + \beta_2 \text{Slope}_t + \epsilon_{t+h}.$$ 

Vertical lines plot standard error bounds. The right panel plots associated $R^2$’s for each horizon. January 1990 - December 2011.
Bibliography


———, 2008, Decomposing the yield curve, Graduate School of Business, University of Chicago, Working Paper.


D’Amico, Stefania, Don Kim, and Min Wei, 2008, Tips from tips: the informational content of treasury inflation-protected security prices, .


Duffee, G., 2010, Sharpe ratios in term structure models, Johns Hopkins University, working paper.


Faust, Jon, and Jonathan Wright, 2012, Forecasting inflation, *manuscript, Johns Hopkins University.*


Fleming, Michael, and Monika Piazzesi, 2005, Monetary policy tick by tick, Stanford University working paper.


Grishchenko, Olesya, and Jing-zhi Jay Huang, 2012, The inflation risk premium: Evidence from the tips market, Available at SSRN 2147016.


Gurkaynak, Refet S, 2005, Using federal funds futures contracts for monetary policy analysis . vol. 5 (Divisions of Research & Statistics and Monetary Affairs, Federal Reserve Board).


Woodford, Michael, 2012, Methods of policy accommodation at the interest-rate lower bound, working paper Columbia University.

