Flotation Circuit Optimisation and Design

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Declaration

Originality

I declare that no portion of the work referred to in this thesis has been submitted in support of an application for another degree or qualification at this or any other university or institute of learning. All work presented in this thesis is my own except that which is appropriately referenced.

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Weimeng Hu
Abstract

Froth flotation is a widely used and versatile mineral processing method for concentrating metal ores. A finely ground ore feed is processed through a flotation circuit consisting of interlinked cells where mineral particles are separated from waste material by using their differences in surface properties. The layout of the circuit has been found to greatly affect the overall flotation performance. While industrial flotation circuit design has in the past relied on experience, due to the complex nature of the process, only small scale circuit optimisations using simple flotation models were reported in literature.

This work proposes a new system capable of automatic generation of optimal circuit designs for any given feed. The system combines a circuit simulator containing detailed froth-phase flotation models with a robust genetic algorithm to search through possible layouts and to produce the global optimal result.

An empirical model to predict the pulp phase flotation rate constant was developed, which was used together with physics-based models describing the froth recovery and entrainment factor to simulate the flotation process. Three feed models with different complexities were also developed. Comparable flotation performance was observed between a modelled 10-cell rougher circuit and the experimental results from Northparkes copper concentration plant.

Through the genetic algorithm, optimal layouts were obtained for circuits consisted of 3 to 10 cells. Layouts containing only rougher cells were able to recover a maximum amount of mineral and were found optimal for smaller circuits, whilst inclusion of cleaner cells in the optimal layouts was found more beneficial for larger circuits. This circuit modelling and optimisation system was used to study the sensitivity of flotation performance and optimal layouts to variations in feed particle size and grade. The results showed strong correlations between the variables and froth phase behaviours which determined the concentrate recovery and grade, the optimal layouts were, however, robust and relatively resilient to the changes in feed conditions.
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Chapter 1

Introduction

1.1 Motivation

Demand for metals such as copper has increased significantly in recent years to support rapid infrastructure developments and expansions in countries like China and India. Rich mineral ore reserves, however, are depleting, and more regulations for a cleaner production of metals have been put in place to reduce environmental impact of the extraction processes. As a result, the mining industry has a great incentive to improve process efficiency, such that more metal can be produced from the same amount of ore even the grade is low, which in turn can also reduce the amount of pollution and increase sustainability.

Froth flotation, as a highly versatile mineral processing method to recover minerals from ores, has been used for over 100 years, and today the process treats billions of tonnes of ore annually. It uses the difference in surface properties to physically separate minerals from waste materials (gangue). Finely ground ores are mixed with chemical reagents to induce hydrophobicity to mineral particles, leaving gangue particles hydrophilic. The mixture is transferred to an aerated water tank (a flotation cell), where hydrophobic particles adhere to air bubbles and rise to the surface forming a froth layer and are subsequently collected as a mineral-rich concentrate.

As the separation efficiency of a single flotation cell is limited, multiple cells are used to improve the recovery of valuable minerals. These cells are connected
and form a flotation circuit, the layout of which has been found to greatly affect the  
overall performance of the circuit. Flotation circuit design has primarily relied on  
experience; to develop a system that automatically generates optimal layout for any  
given ore feed, there are two aspects needed to be addressed: circuit simulation and  
optimisation.

Development of mathematical models that simulate the flotation process started  
shortly after its invention. General models that summarise the process have been  
reported in literature as early as the 1940s. In recent years, the focus has shifted  
to studying the behaviours of particles in the froth phase, which plays an impor-
tant role in determining separation performance but was previously overlooked and  
poorly understood. Due to the complexity of the three-phase flotation process, em-
pirical relationships are commonly used to describe the material transfer in the froth  
phase, which can be process specific and difficult to use predictively. In addition,  
during circuit optimisations, a vast number of simulations needs to be carried out  
which requires a level of simplicity in the flotation models. As a result, the choice and  
construction of the mathematical models for simulating flotation circuits is critical  
for the following optimisation stage.

The number of possible layouts of a circuit is an exponential function of the  
size of the circuit (number of cells). There can be as many as 800,000 possible  
arrangements for a circuit of 5 cells (see later Section 3.3), though many of them  
may prove to be duplicative, incorrect and ineffective. Conventional optimisation  
methods have been used to solve circuit optimisation problems, but are often found  
to be inefficient and prone to finding local optimal results; other methods require  
linearisation of modelling equations and objective functions which can be difficult  
to achieve in a complex system such as flotation. Advances in optimisation algo-
rints and computational power presents possibilities for new robust methods to be  
developed, improved and combined with flotation models including the froth phase  
to optimise circuit layout and analyse performance.

Previous studies have used empirical flotation models with various optimisa-
tion methods to solve up to 6-cell circuit optimisation problems (Mehrotra, 1988;  
Yingling, 1993; Méndez et al., 2009). The aim of this research is to construct a  
physics-based pulp and froth-phase flotation model for individual cells and combine  
them into a circuit simulator that is capable of predicting the effects of variations  
in feed and operating conditions on flotation performance, and to integrate this
simulator with a global optimisation methods so that it can not only automatically
generate optimal circuit designs but also work with larger circuits while using less
computational power. After successfully developing the simulation and optimisation
system, comparison will be made between the results and industrial experimental
data to ensure its reliability. Studies of the effect of key flotation parameters such
as particle size and feed grade on single-cell performance and optimal layouts can
then be carried out. This system can also act as an analytical tool for investigating
the sensitivity of flotation performance to changes in many design variables.

1.2 Organisation of the thesis

The thesis is organised in line with this introduction. Chapter 2 present the
background of the mineral processing industry and the important role of froth flota-
tion, followed by a description of the structure and mechanisms inside flotation
froths. A review of the published work on modelling flotation processes is given,
which is followed by a detailed summary of the optimisation methods used in the
past to address the circuit design problems.

Chapter 3 describes the details of models used in this work to simulate flotation
processes and a genetic algorithm used to optimise circuit layouts. Validation of
the models is provided at the end of this chapter by comparing the results from a
modelled 10-cell rougher circuit to experimental data.

The developed circuit simulation and optimisation system is used in Chapter 4
to test the effect of variation in circuit size on flotation performance and optimal
layouts. The impact of changes in financial parameters on the choice of optimal
circuits is also examined.

In Chapter 5, 6-cell circuits are optimised for a range of particle sizes and feed
grades to study the sensitivity of the optimal layouts to variations in feed conditions.
The trends of froth phase key variable such as froth recovery and entrainment factor
in both circuits and a single cell are predicted.

The final chapter, Chapter 6 summarises the results from all previous chapters
and draws conclusions of this work. Further work that could be carried out to
improve the simulation and optimisation system is discussed.
One refereed publication produced from this work is given in Appendix.
Chapter 2

Literature review

This chapter begins with an introduction to mineral processing; a series of processes that enriches the ores and produces concentrates. It is followed by a detailed description of froth flotation, one of the most widely used and versatile methods of ore concentration. The mechanisms of material recovery in the flotation process are described, which include true flotation, entrainment and the role of froth stability (air recovery). Various approaches of modelling the process are discussed.

Employing multiple flotation cells in a circuit to improve the separation of valuable mineral from waste requires optimisation of the layout of these cells. A review of optimisation techniques is presented, which covers some conventional methods that have been applied to optimising circuit layout, and also potential application of new approaches. A genetic algorithm as a robust and global search method has been recently introduced to circuit optimisation problems, this is described in detail.

2.1 Mineral processing

Production of metals is one of the oldest applied sciences, and it continues to be a fundamental industry that builds our civilisation. Metals are found naturally in native form or more often as minerals contained in ore bodies in the crust of the earth. Most ores are mixtures of extractable minerals and waste gangue materials (Wills, 2005). Extracting metals from their ores can be done by three methods; hydrometallurgy, electrowinning, and the most common pyrometallurgy. However,
Chapter 2 - Literature review

The pyrometallurgical method, i.e. smelting, requires a large amount of energy. It is therefore of interest to reduce the amount of gangue that is sent to the smelter with the minerals.

Mineral processing is a series of physical processes during which valuable minerals are firstly liberated from the gangue, then concentrated to produce an enriched product containing the majority of the valuable materials (the concentrate). Mineral processing reduces both smelter energy cost and metal losses (Wills, 2005).

In the liberation stage, valuable minerals are freed from surrounding gangue by crushing and grinding the mined ore to fine particles. This process of size reduction is called comminution (see Fig. 2.1). The size of the particles is important as particles need to be small enough so that the minerals can be liberated but not too small to avoid wasting energy in grinding. The particle size also affects the concentration process.

![Figure 2.1: Breaking of ore to smaller fragments results in particles with varying degrees of liberation. The shaded area represent the valuable minerals](image)

The concentration of minerals is carried out using the differences in physical properties between minerals and gangue, such as appearance, density, electrical conductivity, magnetic and surface properties. One of the most important methods of concentration is froth flotation, which exploits differences in surface properties of mineral and gangue species.

The route of pyrometallurgically producing metal, including the stages of mineral processing, is shown in a flowsheet in Fig. 2.2.

### 2.2 Froth flotation

Froth flotation is a process for separating valuable solid raw materials from waste that utilises the difference in the surface properties of the raw materials.
Froth flotation has been used for over 100 years, its development has had a profound effect on the mineral industry (Fuerstenau et al., 2007). In addition, flotation has been applied in other areas such as de-inking of recycling paper and waste water treatment.

A process describing the mixing of a crushed metal sulphide ore with oil which floated after being stirred in water was patented by William Haynes in 1860, this is considered to be the first record of potential use of the surface properties for separating minerals (Ives, 1984). The flotation process was not commercialised until 1877 by Bessel brothers in Germany to clean graphite ore. Early developments of the process were made in Australia between 1900 and 1910, and froth flotation was first introduced to the United States to recover copper from its sulphide ore in 1911. Improvements were constantly made in the process, and the production of copper by flotation increased rapidly from 1950. Soon afterwards, the application of froth flotation was extended for treating phosphate and iron ores (Fuerstenau et al., 2007). By the end of the twentieth century, nearly 2 billion tonnes of ore are treated worldwide using flotation techniques (Fuerstenau, 1999). In the world’s copper production alone, flotation is responsible for about 80% (Davenport et al., 2002) of the copper produced, which totalled 15 million tonnes in 2014 (U.S. Geological Survey, 2015).

Today, froth flotation is most widely used for concentrating oxide, phosphate and especially sulphide minerals (Lynch et al., 1981). The mineral ore is ground to fine particles of typically 10 to 100µm in diameter before introducing to the flotation process. The sulphides can be rendered hydrophobic by coating the particles with
appropriate chemical compounds and leaving unwanted gangue particles hydrophilic. Separation processes are designed to exploit this difference in surface properties. As a result, the particle mixture is transferred to an aerated water tank where the air bubbles collide with the particles. The hydrophobic sulphides adhere to the bubbles and rise to the surface to form a froth layer, which is collected as the concentrate. The remaining pulp exits the tank as the tailings. A diagram illustrating the flotation process in a cell is shown in Fig. 2.3

![Diagram of a flotation cell showing the material flows](image)

**Figure 2.3:** Diagram of a flotation cell showing the material flows

Flotation separation performance is measured by the quality and quantity of the concentrate, i.e. the grade and recovery. The materials in the concentrate not only include the hydrophobic minerals that are collected by being attached to the bubbles, but also some of the hydrophilic gangue that are carried in the upward flow, and are subsequently entrained in water channels between bubbles and trapped in the froth. Increasing the amount of minerals reporting to the concentrate can improve the recovery, and decreasing the amount of gangue can improve the grade.

### 2.2.1 Froth structure

Froth formation and removal are the last steps of the flotation process. The froth structure, and the behaviour of the materials in the froth layer play a major role in determining the flotation performance.
Relatively small and spherical bubbles with a diameter of a few millimetres enter the bottom of the froth. The bubbles coalesce as they rise to the surface, where large polyhedral bubbles up to 200\( \text{mm} \) can be found. Fig. 2.4 shows the progression of the froth structure from the pulp-froth interface to the surface.

![Diagram of froth structure](image)

**Figure 2.4:** Comparison of the froth structure at different froth height (Ross, 1998)

At the surface, large bubbles are prone to coalescence and bursting. During these events, all or part of the bubble structure ruptures. Particles that were attached to the bubble fall through the froth, and can re-attach to other bubbles or drop back to the pulp.

A detailed schematic of the froth structure near the surface is shown in Fig. 2.5. The thin films dividing the bubbles are called lamellae, three of which meet along an edge called a Plateau border. Plateau borders hold the majority of the liquid in the froth and form a continuous network of channels throughout the froth, meeting at vertices (Weaire and Hutzler, 1999). The bubbles in the froth rise upwards carrying the attached particles, while the water flows along the Plateau borders downwards with the entrained particles. It is worth noting that the net motion of the entrained particles is not necessarily downwards for all particles, in fact, the majority of the gangue particles recovered in the concentrate are dragged upwards by the rising bubbles and carried in the overflowing froth.
The structure of the froth varies with height. Water from bubble lamellae drains into the Plateau borders and thins the lamellae which leaves the froth dryer at the top. Horizontal movement of the froth stretches the lamellae as the flow diverges for overflowing the cell lip. These behaviours are affected by the velocity that air travels through the cell, and they encourage coalescence and bursting of bubbles (Neethling and Cilliers, 2008). Overall, if bubble coalescence and bursting can be controlled and minimised, the froth will become more stable and provide larger bubble surface area available for attached particles, which can improve the recovery.

2.3 Flotation models

Modelling the flotation process has been the subject of decades of research, in order to understand and simulate the complex three phase process (solid, liquid and gas). There has been various approaches to construct mathematical models of flotation in the literature, from fundamental physics of the separation process to empirical relationships in experimental observations. The choice of the approach depends on the point of view of the authors and the expected usage of the model, and the focus will be different for plant design, process analysis and performance optimisation. Due to the complex nature of the flotation process, it is important to realise the weakness and limitation of any model.
2.3.1 Overall kinetic approach

Instead of constructing models that incorporate detailed mass transfers of particle species within pulp and froth phase, earlier models took a macroscopic approach towards modelling flotation processes. These models tend to consider the recovery of particles in a flotation cell as a whole to simplify the simulation.

Probability models

A probability model was developed by Schuhmann (1942) which considered the rate of particles being successfully recovered to the concentrate $k_x$ (flotation rate constant) as the combination of the probability of particle-bubble collision $P_c$ and subsequent adhesion $P_a$, together with a froth stability factor $F$ (Eq. (2.1)):

$$k_x = P_c P_a F$$  \hspace{1cm} (2.1)

The froth stability factor accounts for the fraction of attached particles that drops back to the pulp after bubble coalescence and bursting (it should be noted that $F$ accounts for froth phase mass transfers, and is different from the probability of detachment $P_d$ in later sections which only involves pulp phase). The probabilities and the stability factor are likely to be affected by the feed and operating conditions in the flotation cell, and these parameters can be difficult to measure. As a result, this probability model can be difficult to implement.

Instead of using the rate constant, Kelsall (1961) simplified the concept of the probabilities such as collision and adhesion to a single variable $P_s$, the probability of successfully recovery, and used it for modelling continuous multiple flotation cells (a flotation bank). Kelsall suggested the mass flowrate of particles in the tailings $M_{tail}$ at steady state is directly related to the feed $M_{feed}$ in a single cell as in Eq. (2.2):

$$M_{tail} = M_{feed} (1 - P_s)$$  \hspace{1cm} (2.2)

If $P_s$ is assumed to be constant in every cell and there are $n$ cells in the bank, the final tailings can be expressed using Eq. (2.3):

$$M_{circuit,tail} = M_{circuit,feed} (1 - P_s)^n$$  \hspace{1cm} (2.3)
Although the measurements of the feed and tailings are simple, similar to the model proposed by Schuhmann, this model does not have input options for the feed nor operating conditions, which limits its application. However the use of the probabilities and rate constant to describe the flotation mechanics are widely accepted and are also seen in a more common first-order kinetic modelling approach.

**First-order kinetic**

Many batch flotation tests that have been conducted in the laboratory have indicated that a first order kinetic model can be used to describe the essential nature of the flotation process (King, 2001). This model considers the flotation process in a flotation cell as a reaction between bubbles and particles in a chemical reactor.

The earliest example of this approach was described by Sutherland (1948) based on hydrodynamics of a single particle. This first-order rate flotation model is widely used in industry. It suggests that the rate at which particles are recovered to the concentrate is proportional to the concentration of the particles in the pulp phase $C_{\text{pulp}}$ (Eq. (2.4)):

$$\frac{dC_{\text{pulp}}}{dt} = -k \ C_{\text{pulp}} \quad (2.4)$$

If the initial concentration of the pulp is $C_{\text{feed}}$, Eq. (2.4) can be integrated to:

$$C_{\text{pulp}} = C_{\text{feed}} \exp(-k \ t) \quad (2.5)$$

Eq. (2.5) can be easily applied to a flotation system with more than one particle species. Particle species are often classified into fast floating and slow floating based on the value of rate constant (Kelsall, 1961), or even more sub-classes (Imaizuni and Inoue, 1965). The variation of the rate constant between species is likely caused by the degree of liberation and the size of particles (Jameson, 2012).

Industrially, flotation plants typically employ continuous flotation cells. The chemical reactor analogy is again applicable, such that flotation cells are considered to be equivalent to CSTRs (continuously-stirred tank reactors). Jowett and Safvi (1960) reported observing first-order kinetics also in continuous flotation processes.

The recovery $R$ is defined as the fraction of particle species in the feed that are collected in the concentrate. This recovery is commonly calculated as a function of
the rate constant and the mean residence time $\tau$ as in Eq. (2.6) (Arbiter and Harris, 1962):

$$R = \frac{k \tau}{1 + k \tau}$$  \hspace{1cm} (2.6)

The residence time can vary from species to species, its distribution is discussed in detail by Woodburn et al. (1969).

Eq. (2.6) was adopted by Mehrotra and Kapur (1974) and Dey et al. (1989) to calculate the concentrate and tailings flowrate (see Eqs. (2.7) and (2.8)):

$$M_{conc} = M_{feed} R = M_{feed} \frac{k \tau}{1 + k \tau}$$  \hspace{1cm} (2.7)

$$M_{tail} = M_{feed} (1 - R) = M_{feed} \frac{1}{1 + k \tau}$$  \hspace{1cm} (2.8)

The mass flowrate of every component in the concentrate and tailings can be found using these equations, which can then be summed up to determine the overall flowrate of the streams and to evaluate the flotation performance of the cell.

The overall flotation kinetic approach is easily accessible and widely used in industry, however due to the lack of detailed modelling, it provides limited insight into froth phase behaviours which has been found to play an important role in determining flotation performance.

### 2.3.2 Froth phase modelling approach

Recent work in modelling the flotation process (Franzidis and Manlapig, 1999; Gorain et al., 1998a, 1999; Runge et al., 1998; Savassi et al., 1998; Ghobadi et al., 2011) uses the first order kinetic approach as basis and expands its dimension by categorising the means of mass transfer between pulp, froth phase and concentrate individually.

Flotation can be divided into four sub processes considering both pulp phase and froth behaviours which include (Laplante et al., 1989) (see Fig. 2.6):

1. Selective material transfer from the pulp to froth phase by attachment to bubbles;
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Figure 2.6: Methods of mass transfer in flotation processes

2. Non-selective material transfer from the pulp to froth phase by entrainment;
3. Loss of material from the froth to pulp phase due to bubble coalescence and bursting, and subsequent liquid drainage;
4. Material transfer from the froth phase to the concentrate.

Another point of view of the flotation kinetics, which is often applied in more recent literature, is to regard:

- the selective material transfer from the pulp to froth (item 1 in Laplante’s list) then to concentrate (part of item 3 and 4 that accounts for the selective transferred particles) as recovery by true flotation (the fraction of detached material that is suspended in froth and carried over to the concentrate is still considered to be a part of the true flotation (Neethling, 2008)),
- and non-selective material transfer from the pulp to froth (item 2) then to concentrate (part of item 3 and 4 that accounts for the non-selective transferred particles) as recovery by entrainment.

It should be noted that the loss of material through drainage is included in the consideration of the two ways of recovery.

Recovery by true flotation

The material recovery by true flotation is essentially a two stage process including pulp phase and froth phase. In the pulp phase, particles attach to air bubbles and rise to the pulp-froth interface. The attachment and transfer are often considered to be a first order process. In the froth phase, the attached particles can be carried in the overflowing froth and report the concentrate. However a fraction
of particles will become detached from air bubbles when the lamellae rupture during coalescence and bursting. Within this fraction some particles will re-attach to new bubbles, others will return to the pulp. As the result, the recovery of by true flotation $R_{TF}$ can be expressed as a function of the pulp recovery $R_{pulp}$ and froth recovery $R_f$ as in Eq. (2.9) (Franzidis and Manlapig, 1999):

$$R_{TF} = \frac{R_f}{R_f R_{pulp} + 1 - R_f}$$  (2.9)

Pulp-phase kinetics

A pulp-phase only first order rate constant is developed by Gorain et al. (1997) and Gorain et al. (1998b) to describe the attachment and transfer processes. It redefines the rate constant as the product of the bubble surface area flux $S_b$ and particle floatability. The work of Yoon (2000) develops the formerly mentioned concept of the probabilities by Schuhmann (1942) and suggests the floatability is a combined term of probability of particle-bubble collision, adhesion and detachment in the pulp phase. As the result, the pulp-phase first order rate constant can be calculated using Eq. (2.10):

$$k = S_b P_c P_a (1 - P_d)$$  (2.10)

Selective transfer in the pulp is greatly affected by particle-bubble attachment, which is the result of the induced hydrophobicity of the mineral particles after mixing with chemicals called collectors. Collectors are reagents added the ore feed before the flotation process (during a conditioning period) which render the mineral particles hydrophobic. Collectors adsorb molecules or ions on the mineral surface, and reduce the stability of the hydrated layer separating the mineral from the air bubble when mixed in the flotation cell, which in turn facilitates particle-bubble attachment (Wills, 2005).

Fig. 2.7 shows a simple illustration of collectors adsorbing on mineral surface with its polar group leaving the a non-polar ‘tail’ on the outer surface. The probability of particle-bubble adhesion term in the floatability is a reflection of the particle’s hydrophobicity. The more hydrophobic the particles become, the greater the contact angle between the particle surface and the bubble, the greater the force that is required to break the particle-bubble agglomerate, hence the greater probability of
adhesion after collision.

Froth-phase recovery

In the froth phase, the attached particles can become detached during bubble coalescence and bursting, such that not all attached particles that enter the froth are recovered to the concentrate. Yianatos et al. (1988) found that $5 - 20\%$ of particles return from the froth to the collection zone (pulp) by disengagement from the bubbles in an industrial flotation column (though this fraction of the particles is originally transferred to the froth by both true flotation and entrainment).

A parameter called froth recovery $R_f$ is used to quantify the successful recovery of the attached particles. It is defined as the fraction of particles attached to air bubbles entering the froth phase, that reports to the concentrate (Finch and Dobby, 1990). The work of Franzidis and Manlapig (1999) suggested that $R_f$ is an exponential function of the froth residence time $\tau_{froth}$ (the ratio of froth height and superficial air velocity) as in Eq. (2.11):

$$R_f = \exp (-\beta \tau_{froth})$$

(2.11)

where $\beta$ is a parameter describing the mobility and stability of the froth (Gorain et al., 1998a).

Neethling (2008) treated the froth recovery as a combination of the recoveries of the particles that remain attached to bubble films and those that become detached but still report to the concentrate as a freely moving particle within the water in the
froth. The author introduced $f$ as the fraction of particles attached to the bubble film that become detached when it ruptures. So that the recovery of the particles that remain attached in the froth can be represented by Eq. (2.12).

$$R_{attach} = \left( \frac{d_{b, in}}{d_{b, out}} \right)^f \alpha$$  \hspace{1cm} (2.12)

where $d_{b, in}$ is the bubble size at the pulp-froth interface, $d_{b, out}$ is the bubble size overflowing the cell lip and $\alpha$ is the air recovery (the fraction of air entering the cell that overflows as unbroken bubbles, indicating the froth stability).

Detached particles settle downwards in the froth. The settling velocity $v_{set}$ at a particular height in the froth can be equal to the upward fluid velocity. The bubble size at this height $d_{b, crit}$ is estimated by Eq. (2.13):

$$d_{b, crit} = \begin{cases} \sqrt{\frac{\alpha}{\alpha v_g}} \frac{d_{b, out}}{2} & \text{if } \alpha < \frac{1}{2} \\ \sqrt{\frac{2v_{set}}{v_g}} d_{b, out} & \text{if } \alpha \geq \frac{1}{2} \end{cases}$$  \hspace{1cm} (2.13)

where $v_g$ is the upwards speed of the air travelling through the froth (superficial air velocity).

Only those particles that fall off in lower froth regions where bubbles are smaller than $d_{b, crit}$ will contribute to the material lost from the froth, so that the froth recovery can be expressed as in Eq. (2.14):

$$R_{froth} \approx \left( \frac{d_{b, in}}{d_{b, crit}} \right)^f$$  \hspace{1cm} (2.14)

Substituting Eq. (2.13) into Eq. (2.14):

$$R_{froth} \approx \begin{cases} \left( \frac{\alpha v_g}{v_{set}} \right)^{\frac{1}{2}} \left( \frac{d_{b, in}}{d_{b, out}} \right)^f & \text{if } \alpha < \frac{1}{2} \\ \left( \frac{v_g}{2} \frac{v_{set}}{v_g} \right)^{\frac{1}{2}} \left( \frac{d_{b, in}}{d_{b, out}} \right)^f & \text{if } \alpha \geq \frac{1}{2} \end{cases}$$  \hspace{1cm} (2.15)

Both methods of predicting $R_f$ have their merits, the experimental function is easy to use to simulate an existing flotation process while the physics-based approximation can provide more details in process design and analysis.
Recovery by entrainment

Entrainment is the process by which particles enter the base of a flotation froth and are transferred up and out of the flotation cell suspended in the water between bubbles (Smith and Warren, 1989). Unattached particles are carried upwards in the water into the froth by non-selective entrainment, and then suspended in the water in the Plateau borders and vertices. As a result, the entrained recovery is proportional to the amount of water recovered in the concentrate (Engelbrecht and Woodburn, 1975; Trahar, 1981).

Entrainment factor

The degree of entrainment or the entrainment factor, $Ent$, is defined as the ratio of the entrainment recovery of solids $R_{ent}$ to the recovery of water $R_w$ as in Eq. (2.16):

$$R_{ent} = Ent \ R_w$$

The entrainment factor is particle species specific. It is a strong function of particle size (Johnson, 1972) and becomes less significant for particles larger than 50µm (Smith and Warren, 1989).

The value of $Ent$ is often determined experimentally from plant data, and Savassi et al. (1998) developed an empirical model which links the entrainment to particle size $d_p$ (Eq. (2.17)):

$$Ent = \frac{1}{2} \exp(2.292(d_p/\xi)^{1+\frac{ln\delta}{\xi}}} + \exp(-2.292d_p/\xi)$$

where $\xi$ is an entrainment parameter and $\delta$ is a drainage parameter.

The work of Savassi et al. (1998), Zheng and Knopjes (2005) and Zheng et al. (2006) also considered the effect of solids suspension in the pulp phase on entrainment, which is represented by a classification function and added to Eq. (2.16).

On the other hand, a theoretical model for the entrainment factor was developed by Neethling and Cilliers (2009) using physics-based models for the liquid and...
solids behaviour in the froth (see Eq. (2.18)):

\[
Ent_x \approx \begin{cases} 
\exp \left( -\frac{v_{sett,x} 1.5 h_{froth}}{D_{axial} \sqrt{v_g (1-\alpha)}} \right) & \text{if } \alpha < \frac{1}{2} \\
\exp \left( -\frac{2v_{sett,x} 1.5 h_{froth}}{D_{axial} \sqrt{v_g}} \right) & \text{if } \alpha \geq \frac{1}{2}
\end{cases}
\]  

(2.18)

where \( h_{froth} \) is the froth height from the pulp-froth interface to the bursting surface. Solids motion in the froth includes particle dispersion \( D_{axial} \) and settling. The contribution of air to the entrainment process is taken into account by the inclusion of the superficial air velocity and air recovery.

### Water recovery

Water recovery is used together with the entrainment factor in Eq. (2.16) to calculate the entrained recovery. It is defined as the fraction of the water entering the flotation cell that is recovered in the concentrate.

Savassi et al. (1998) used an empirical model which relates water recovery \( R_w \) to the froth residence time \( \tau_{froth} \):

\[
R_w = 63.905 \; \tau_{froth}^{-0.926}
\]  

(2.19)

Harris (2000) modelled the water recovery in two sequential processes, a first-order kinetic equation accounts for the water recovered from the pulp to froth phase, and an exponential function of the mean froth residence time accounts for water recovered from the froth to concentrate.

Neethling et al. (2003b) studied the recovery of water in a steady-state stable flowing column of foam. The majority of water is held and moves in the Plateau borders in the foam. This concept is transferred and applied to determine the the recovery of water in froth flotation by the authors in Neethling et al. (2003a). The amount of water in the concentrate is suggested to be a function of the length of Plateau borders per volume of froth \( \lambda \) and the air recovery as in Eq. (2.20):

\[
Q_{water} = \begin{cases} 
\frac{A_{sett} v_g^2 \lambda}{k_1} \alpha (1 - \alpha) & \text{if } \alpha < \frac{1}{2} \\
\frac{A_{sett} v_g^2 \lambda}{4k_1} & \text{if } \alpha \geq \frac{1}{2}
\end{cases}
\]  

(2.20)
where \( A_{\text{cell}} \) is the cross-section area of the flotation cell and \( k_1 \) is a constant representing the balance between gravity and viscosity.

### 2.3.3 Froth stability

**Air recovery**

Froth stability is commonly reported in literature as a key factor affecting the flotation performance (Subrahmanyam and Forssberg, 1988; Barbian et al., 2005; Hatfield, 1975; Morar et al., 2012). Froth stability is essentially reflected by the inverse of the bubble film failure caused by bubble coalescence in the froth and bursting at the froth surface. However there is no specific criterion to quantify froth stability (Farrokhpay, 2011). A number of parameters are used as indicators of froth stability, such as froth height, bubble growth, bubble loading, froth velocity, surface burst rate and air recovery.

In recent work (Morar et al., 2012; Ventura-Medina and Cilliers, 2002), the bubble burst rate and air recovery are chosen to reflect the froth stability by quantifying the amount of air loss in the froth phase. These parameters are measured by applying image analysis to video footage of overflowing froth, which is then related to the flotation performance. The air recovery and burst rate can be easily calculated from each other if the overflowing bubble size is known.

Air recovery is defined as the volume fraction of air that overflows the weir in the form of unbroken bubbles. It is first introduced by Woodburn et al. (1994) to express the flow of bubble surface area overflowing the cell lip \( \Psi_b \) as in Eq. (2.21):

\[
\Psi_b = \alpha \frac{Q_a S_b}{Q_{\text{air}}} \quad (2.21)
\]

where \( S_b \) is the bubble surface area flux.

By measuring the overflow velocity of the froth \( v_f \), its height above the cell lip \( h \) and the length of the lip \( w \) experimentally, the air recovery can be determined using Eq. (2.22) (Ventura-Medina and Cilliers, 2002):

\[
\alpha = \frac{v_f h w \zeta}{Q_{\text{air}}} \quad (2.22)
\]
where $Q_{air}$ is the volumetric flowrate of air introduced to the cell, and $\zeta$ is the fraction of air in the overflowing froth and can be assumed to be 1.

The air recovery is a good indicator of the froth stability. Its measurement is uncomplicated and non-intrusive, such that it can be readily applied in the flotation models (Eqs. (2.15), (2.18) and (2.20)) to simulate flotation processes and predict the performance.

### 2.4 Flotation performance - grade and recovery

The performance of mineral concentration processes can be reflected by measurement of the product concentrate stream, such as (Wills, 2005):

- **Yield** - mass flowrate of the concentrate stream,
- **Recovery** - the percentage of the total valuable contained in the feed stream that is recovered to the concentrate,
- **Grade** - the percentage of the valuable contained in the concentrate,
- **Ratio of concentration** - the mass ratio of the feed to the concentrate,
- **Enrichment ratio** - the ratio of the grade of the concentrate to the grade of the feed.

In froth flotation, the performance is usually quantified using recovery and grade, which are the quantity and quality of the concentrate product.

The recovery is found using Eq. (2.23), and the grade is found using Eq. (2.24):

$$Recovery = \frac{M_{conc,metal}}{M_{feed,metal}} \quad (2.23)$$

$$Grade = \frac{M_{conc,metal}}{M_{conc,solids}} \quad (2.24)$$

It should be noted that the grade of the concentrate is limited by the mineralogical nature of the ore feed. For a flotation plant treating a chalcopyrite ore, as the metal content of the pure chalcopyrite mineral (CuFeS$_2$) is only 34.62%, the
concentrate grade from the plant will not be able to exceed this percentage due the physical nature of the separation process.

The recovery and grade are competing qualities of the concentrate, meaning an improvement in recovery is likely to cause a deterioration in the grade, and vice versa. Fig. 2.8 is a typical grade-recovery curve demonstrating the inverse relationship between the two qualities.

![Grade-Recovery Curve](image.png)

**Figure 2.8:** Typical grade-recovery curve

### 2.5 Flotation circuits

Complete separation of valuables from the gangue in a single pass is seldom possible (Mehrotra and Kapur, 1974). Hence, flotation cells are connected in series as a bank of cells to improve the overall separation efficiency, i.e. the final recovery and grade. Each bank (which can be a single cell) is distinguished by its general purpose, and is often referred as,

- **Rougher** - the first bank of cells that treats the circuit feed, where the majority of the valuable material will be recovered,

- **Scavenger** - the bank that treats the barren tails from the roughers, where the recovery is maximised before discarding the tailings,

- **Cleaner** - the bank that cleans the concentrates from the roughers to reduce the amount of gangue in the product, the tailings of the cleaners contain a significant amount of valuable and are often recycled back to the roughers.
A froth flotation system, a circuit, consists of interconnected rougher, cleaner and scavenger banks. The design and arrangement of individual flotation cells within banks and arrangement of linkages between banks determines the overall performance of the circuit. For the purposes of this study, all flotation cells in a circuit are identical, irrespective of the rougher, cleaner and scavenger banks they are situated in.

Industrial flotation circuit designs have been heavily based on empirical relationships whilst plant configuration and optimisation rely more upon past-experience (Mehrotra, 1988). In searching for an optimal circuit design, building a mathematical model to simulate flotation circuits as accurately as possible will be the initial step.

As a flotation circuit consists of more than one cell, the possible ways of arranging the cells increases with circuit size. Fig. 2.9 shows all the possible layouts for a 4-cell circuit. With an appropriate searching mechanism, layouts that satisfy both technical and economic constraints can be found.

### 2.6 Optimisation techniques

A optimum searching mechanism is required to explore possible circuit configurations in order to find the layout that yields the “best” performance while meeting all the constraints. Unlike optimising usual non-linear mathematical systems which are often continuous and consist of differentiable equations, flotation circuit optimisations require complex simulation to be carried out first (if detailed models are used), as the performance is unique to each circuit layout and the number of possible layout increases exponentially with circuit size, therefore the optimisation search space is not only discrete but also vast. Therefore it is essential to employ a robust optimisation technique that is capable of finding the global optimal layout within a reasonable time.

#### 2.6.1 Conventional circuit optimisation

Flotation circuit optimisation has been studied using various approaches with the majority adopting conventional optimisation methods (Yingling (1993) and
Figure 2.9: All possible layouts for a 4-cell flotation circuit (Guria et al., 2005a)
Méndez et al. (2009)). These methods can be classified into three major groups; calculus-based, enumerative, and random.

Mehrotra and Kapur (1974) used calculus-based methods for circuit optimisation. The methods include Indirect or Direct search. Indirect search seeks local optima by solving a non-linear set of equations, which involves assigning zero to the gradient of the objective function. An optimal position can be found by limiting the search to the positions where the gradients become zero in all directions. Direct search methods (hill climbing), on the other hand, pursue local optima by following the function in the direction related to the local gradient. However, both methods only hunt for the optimal point in the vicinity of the present position, which depends on the initial value when addressing a multi-optimum problem. The result is not guaranteed to be the global optimum, but local optima can serve as design guides (Mehrotra and Kapur, 1974).

Enumerative search evaluates every possible location in the search space (every possible layout in the case circuit optimisations) and is therefore inefficient (Goldberg, 1989). Yingling (1990) applied the method to a sequence of small scale searches, and Schena et al. (1996) used it to secure the global optimal circuit design with a financial objective function. This method is limited to solving relatively small problems.

Random search algorithms evaluate the performance of a circuit at a random location in the search space, and the result is retained until it is succeeded by a better circuit at another random location. The method was chosen by Kapur et al. (1991) to optimise feed parameters in flotation circuits. However, random search also has low efficiency.

2.6.2 Mathematical programming

By using superstructures to represent possible arrangements of a flotation circuit, such as a systematic description of the circuit shown previously in Fig. 2.9, it is possible to formulate the optimisation problem in a mathematical programming manner, which can be solved using linear programming (LP), mixed integer non-linear programming (MINLP) and mixed integer linear programming (MILP). However, detailed flotation models, such as inclusion of froth-phase modelling, are difficult to incorporate into the programming problem.
Yingling (1993) classified early work in flotation circuit layout optimisation into two groups. The first group used the first-order flotation rate modelling approach and conventional optimisation methods. The second group used structure parameters and enhancement factors instead of a direct flotation model to describe the separation process in a flotation circuit. The flotation model or industrial data was implemented through bounds on the enhancement factors which affect the material outputs from a flotation bank. These bounds, mass balances and constraints together with an objective function, described the flotation process in terms of linear equations and inequalities. Optimal layouts were produced by using LP to solve the linearised optimisation problem.

The use of a financial objective function by Schena et al. (1996) and Schena et al. (1997) resulted in bringing various non-linearities into the circuit layout optimisation problem. The non-linear constrained optimisation problem was decomposed into sequential sub-problems which were solved using MINLP (Schena et al., 1997). Cisternas et al. (2004) and Cisternas et al. (2006) developed two hierarchical super-structures to represent a flotation circuit. This breaking down of the optimisation problem facilitated the linearisation of the financial objective function using a Taylor series, such that MILP could be used to optimise the circuit layout. Through the use of MINLP and MILP, a diversity of arrangements and equipment such as banks with different numbers of cells, regrinding units and flotation columns was allowed to be considered in flotation circuit designs.

### 2.6.3 Logic programming

In addition to traditional searching method, a human-thinking-resembling, problem solving algorithm of Abductive and Inductive Logic Programming (Abductive ILP) (Tamaddoni-Nezhad et al., 2006) was studied to investigate the feasibility of applying such method to the optimal flotation circuit searching problem.

Logic programming is a method that adopts human logical reasoning processes into computer program to solve a problem. It is declarative programming through which problems are expressed by definitions and statements and without instructions for how the problem is solved. An execution mechanism is then used to manipulate the information contained in the program so as to derive the solution to a given problem efficiently and systematically.
The logical reasoning used in the programming is the process which uses arguments, statements, premises and axioms to define whether a statement is true or false, resulting in a logical or illogical reasoning. Deduction, induction and abduction are the three kinds of logical reasoning. Given a precondition, a conclusion, and a rule that the precondition implies the conclusion:

- **Deduction** is the process determining the conclusion. It utilises the rule and its precondition to create a conclusion. Deductive reasoning does not provide new information, it only rearranges information that is already known into a new statement or conclusion;

- **Induction** is the process determining the rule. It is learning the rule after numerous examples of the conclusion following the precondition;

- **Abduction** is the process determining the precondition. It uses the conclusion and the rule to support that the precondition could explain the conclusion assuming that the most plausible conclusion is also the correct one.

However, for the problem of optimising circuit layouts faced in this research, due to the lack of vast amount of industrial data of plant configuration and effects on the separation efficiency, it is very difficult to employ the Abductive ILP, also the results and conclusions generated by such method will be doubtful and unreliable. Therefore logic programming is not appropriate for the flotation flowsheet design problem.

### 2.6.4 Genetic algorithms

An adaptive heuristic search technique, genetic algorithm (GA), has been developed (Holland, 1975; Goldberg, 1989) as a powerful search tool capable of finding global solutions. The technique is based on the mechanics of natural selection and natural genetics. It uses the concept of survival of the fittest, in combination with randomised structure information exchange to mimic the process of natural evolution (Goldberg, 1989). In contrast to conventional searching algorithms, GA starts with a pool of randomly generated initial guesses of the decision variables (cell linkage) which spreads over the search space. In addition, values of the objective functions are used in GA instead of their gradients, and as a result the complexity of modelling
is reduced (Guria et al., 2005b). The efficiency of the algorithm is dramatically improved (fitness guided search) compared to random search. Although GA has been used widely in a number of disciplines, it is a relatively new optimisation search algorithm for flotation circuit optimisation.

Applications of GAs to flotation optimisation were initially proposed by Guria et al. (2005a), Guria et al. (2005b) and Guria et al. (2006). These studies used the overall first order flotation rate approach developed by Mehrotra and Kapur (1974) (reviewed in Subsection 2.3.1) to model the flotation process. The GA was applied to the problem to evolve from an initial population of circuit designs. The authors were able to solve for multi-objectives with a jumping gene adaptation to improve the algorithm efficiency. However, the flotation model did not account for entrainment nor the effects of operating conditions, such as air flowrate. The model also lacked a solid demonstration that the optimal result was the global solution. More recently, GA was employed with additional process-based constraints by Ghobadi et al. (2011). Entrainment was considered in addition to first order kinetics.

The scale of the process in the studies from both Guria et al. (2006) and Ghobadi et al. (2011) was relatively small (2 to 4 cell/bank circuits). These studies were focused mainly on solving optimisation problems rather than investigating the effects of the underpinning operating variables. However the authors demonstrated successful applications of genetic algorithms in optimising circuit layouts. The algorithms have the potential of finding global optimal solutions in discrete search space which is important for addressing the circuit optimisation problem.

2.7 Summary

This chapter has introduced the fundamental physics of froth flotation with a detailed description of the froth structure. A survey of literature has shown various approaches towards modelling the three-phase process. The material recovery to the concentrate has been considered to be the result of either true flotation or entrainment in recent work, while traditionally it has been summarised by an overall first order kinetics model. Empirical flotation models have been developed and used to predict the flotation performance, however these models require fitting parameters from experimental data and their lack of inputs of feed and operating conditions
limits the applicability in circuit design and optimisation. Physics based models have been used to describe the particle and liquid behaviours in the froth phase, which are more suited for purpose of this research.

Flotation circuits contain multiple cells and can be simulated by applying the flotation models to each unit. The overall performance is quantified using recovery and grade of the final concentrate. The arrangement of flotation cells determines the overall performance of the circuit, such that it is of interest to optimise the circuit layout of a flotation plant.

Several searching methods have been introduced to the circuit layout optimisation problems. The conventional methods that have been used to optimise circuits are prone to finding local optima and can be inefficient. Using linear programming addresses these problems but can be difficult to apply to complex flotation models. Genetic algorithms, on the other hand, have been shown capable of finding global optimal circuit layouts and need no linearisation of the flotation models and objective functions.

In the following chapter, a methodology for circuit optimisation will be introduced. The methodology will employ physics-based flotation models to simulate circuits, combined with a robust genetic algorithm to optimise circuit layouts.
Chapter 3

Flotation models and optimisation methods

In a flotation cell, there are two routes for materials to be recovered from the feed to the concentrate, through true flotation and entrainment. True flotation is a selective recovery process which comprising pulp-phase kinetics and froth-phase mechanics, while entrainment is a non-selective process which depends on froth-phase fluid dynamics and particle motion. Through these two routes, mineral-rich concentrates are produced from flotation cells, which are combined and reprocessed in the flotation circuit and form the final concentrate. Flotation cell and circuit performance are determined by quantity of the mineral recovered, the recovery, and the quality of the concentrate (amount of the gangue contained), the grade. The flotation circuit performance is dependent on the ways which cells are arranged within, such that by changing the circuit layout, the recovery and grade of the final concentrate can vary significantly. As a result, improvements in terms of financial benefits can be made by performing a circuit layout optimisation.

This chapter will detail the models that are used to simulate the flotation process in a single cell and in a circuit. Firstly, the material mass balance for a single cell will be performed, which introduces the two routes of recovery. As the pulp-phase governing variable in the true flotation, the flotation rate constant will be modelled based on the equations and data from prior work. Following this an air recovery model reflecting froth stability will be described in detail. This air recovery is a key variable in determining the froth recovery, froth-phase governing variable in the true flotation, and later the entrainment factor. Together with a water recovery
model, the material recovery by entrainment will be developed through the use of an entrainment factor. Using these models, the performance of cells and circuit (through iteration to achieve steady state in a circuit) can be determined, which prepares the grounds for circuit layout optimisation.

Finally, to improve the performance of flotation circuits, the layout of the circuits requires to be systematically designed instead of relying on past experiences, and to be optimised to maximise the financial benefits gained from concentrating the mineral feed. A global optimisation method, a genetic algorithm, will be adopted to design and optimise the flotation circuit layout for a given number of cells. The optimisation begins with a simple coding to represent circuit layouts. Represented circuits undergo modelling and by using a smelter return formula the product values are calculated which provide balance between recovery and grade. The genetic algorithm will be used to manipulate the layout representation and generate an optimal circuit design.

3.1 Flotation models for a single cell

A diagram of a flotation cell is shown in Fig. 3.1, it shows the mechanisms of material recovery to the concentrate, true flotation and entrainment (note that the froth is uniform and continuous at the surface of the cell, the gaps in the froth are for the purpose of clear illustrations of the particle motions).

True flotation recovery refers to the materials being attached to the bubbles (due to their hydrophobicity), transferred to the froth, and finally reported to the concentrate. It is illustrated by the orange-coloured arrows at the right hand side of the cell. True flotation is governed by the pulp-phase flotation rate constant and the froth recovery.

The flotation rate constant is the rate of material that leaves the pulp-phase as attached particles on the air bubbles. In this research, relationships and data from the literature are used to determine the flotation rate constant, which is found to be a function of the particle size and the degree of the mineral liberation. The selective mass transfer of the hydrophobic material from the pulp phase to pulp-froth interface rate constant is illustrated in Fig. 3.1 as a solid orange arrow. The froth recovery is the fraction of attached particles entering the froth that reports to
the concentrate, this includes the particles that become detached after entering the froth but still report to the concentrate as freely moving particles within the water in the froth. The two dashed orange arrows in Fig. 3.1 illustrate the recovery of particles that are attached to and detached from air bubbles.

The entrainment recovery refers to the materials being carried in the up flowing current that are subsequently trapped in the Plateau borders (channels between bubbles) of the froth and reported to the concentrate. It is a non-selective recovery where the surface property difference between mineral and gangue particles has little effect, this is illustrated in Fig. 3.1 as two dashed blue arrows.

The flotation models introduced in this work describe the particle behaviour in froth phase which simulate the froth and entrained recovery at steady state. With the aim of constructing a model to simulate the flotation process in a single cell, various principles and relationships are used, described in five subsections: Mass balancing, pulp-phase flotation rate constant, air recovery model, true flotation recovery and entrained recovery.
3.1.1 Material mass balance

Assuming the flotation process in a cell is a physical separation process and there is no change in the chemical and physical nature of the materials throughout, the sum of the mass flowrate (product of the volumetric flowrate \(Q\) and concentration \(C\)) of a material species \(x\) in the concentrate (conc) and tailings (tail) always matches the feed (feed) at steady-state due to the conservation of mass.

\[
Q_{\text{feed}} = Q_{\text{conc}} + Q_{\text{tail}} \tag{3.1}
\]

\[
Q_{\text{feed}}C_{\text{feed},x} = Q_{\text{conc}}C_{\text{conc},x} + Q_{\text{tail}}C_{\text{tail},x} \tag{3.2}
\]

The concentrate flow can be represented by the combination of the two routes of material recovery, true flotation and entrainment.

The true flotation term is governed by the apparent flotation rate, which is described in terms of specific rate constant \(k_x\) and froth recovery factor \(R_{f,x}\). The recovery of material through true flotation in a flotation cell at steady-state is often predicted in terms of a CSTR (continuous stirred tank reactor) model (Guria et al., 2005b). Defining \(M\) as the mass flow of material and \(V_{\text{cell}}\) as the volume of the pulp phase in the flotation cell, the product of \(k_x\) and \(R_{f,x}\) is used in place of the rate of reaction:

\[
M_{\text{conc},x,\text{true flotation}} = V_{\text{cell}}k_xR_{f,x}C_{\text{tail},x} \tag{3.3}
\]

The recovery of entrained material is proportional to the water recovered in the concentrate \(Q_{\text{water}}\) and depends on the material concentration in the pulp \(C_{\text{tail},x}\); it is estimated through the use of an entrainment factor \(Ent_x\), which is defined as the recovery of the entrained species divided by the recovery of water. The water recovered is approximated to be the same as the volumetric concentrate flowrate \(Q_{\text{conc}}\) as the water constitutes the greatest fraction in the concentrate (Neethling and Cilliers, 2009).

\[
M_{\text{conc},x,\text{entrainment}} = Q_{\text{water}}Ent_xC_{\text{tail},x} \approx Q_{\text{conc}}Ent_xC_{\text{tail},x} \tag{3.4}
\]

Substituting \(Q_{\text{conc}}C_{\text{conc},x}\) in Eq. (3.2) with \(M_{\text{conc}}\) from Eq. (3.3) and Eq. (3.4)
leads to:

\[ Q_{\text{feed}}C_{\text{feed},x} = (V_{\text{cell}}k_x R_{f,x} C_{\text{tail},x} + Q_{\text{conc}} E_{\text{nt},x} C_{\text{tail},x}) + Q_{\text{tail}} C_{\text{tail},x} \quad (3.5) \]

To easily calculate the concentrate and tailings stream from a given feed, Eq. (3.5) is rearranged so that the concentration of species \( x \) in the tailings and concentrate, \( C_{\text{tail},x} \) and \( C_{\text{conc},x} \), can be expressed as:

\[ C_{\text{tail},x} = \frac{Q_{\text{feed}}C_{\text{feed},x}}{V_{\text{cell}}k_x R_{f,x} + Q_{\text{conc}} E_{\text{nt},x} + Q_{\text{tail}}} \quad (3.6) \]

\[ C_{\text{conc},x} = \frac{V_{\text{cell}}k_x R_{f,x} C_{\text{tail},x} + Q_{\text{conc}} E_{\text{nt},x} C_{\text{tail},x}}{Q_{\text{conc}}} \quad (3.7) \]

### 3.1.2 True flotation - flotation kinetics

The flotation kinetics of particles in the flotation system, the flotation rate constants, are dependent on the chemical and the physical nature of the particles and the froth. While chemistry aspects such as the effects of the collector (surfactants that are used to render selected minerals hydrophobic) and the frother (chemicals that are added to stabilise bubble formation in the pulp phase and in turn to create a reasonably stable froth) are important, however, the focus is on physical aspects in this research, such as particle size, liberations, and the effect of these parameters on flotation rates constant. These physical properties are closely related to the mineralogy of the ore and the mineral liberation processes, which play a fundamental and direct role in determining the flotation kinetics.

The flotation rate constant used in this research describes the rate of the attached particles entering the froth phase. This rate constant is particle species specific and for pulp phase only, and it should be distinguished from the apparent flotation rate (the product of the specific rate constant and froth recovery) and the overall rate constant (accounting both true flotation and entrainment recovery) usually seen in simple flotation models in literature (Méndez et al., 2009).

There have been researchers who have studied the relationship between the specific flotation rate constant, liberation and particle size (Welsby et al., 2010; Muganda et al., 2011; Jameson, 2012). In this research, the kinetics are determined
using the methods and data from this prior work.

**Effect of liberation**

Liberation is the release of the valuable minerals from the gangue, which is accomplished by comminution. In flotation kinetics, the extent of liberation directly affects the surface property of mineral particles, the more the valuable mineral surface is exposed, the more hydrophobic the particles can become.

The work of Jameson (2012) demonstrated using data from another study (Welsby et al., 2010) that the ratio of the rate constant of partially liberated, $k_x$, to fully liberated particles, $k_{max}$, is dependent only on the degree of liberation $L_x$, regardless of the size of the particle:

$$\frac{k_x}{k_{max}} = C_1 \ L_x \ \ e^{(C_2\ L_x\ C_3)}$$  \hspace{1cm} (3.8)

where $C_1$, $C_2$ and $C_3$ are constants.

In the work of Welsby et al. (2010) which the above liberation equation is based on, the liberation is defined as the mass fraction of the mineral in the particles instead of free surface area to simplify the calculations. The author found that defining mineral liberation by particle composition was comparable to defining liberation by free surface area.

**Effect of particle size**

The liberation Eq. (3.8) is used in conjunction with a model, developed using experimental data provided in Muganda et al. (2011), to predict the rate constant for any particle species of given size and degree of liberation. The experiments were conducted using chalcopyrite and pyrite conditioned with sodium dicseryl dithiophosphate (DTP) and potassium amyl xanthate (KAX) at high and low advancing contact angle regimes. Five sets of applicable experimental data are used, the $k_{max}$ (rate constant of fully liberated mineral particles) is plotted in Fig. 3.2. The $k_{max}$ used in this literature is a modified flotation rate constant and is for all floatability components in the size fraction. It takes into account the use of froth recovery to predict true-flotation material recovery (not the previously mentioned apparent
flotation rate nor the overall rate constant), which therefore makes the data suitable to use in the models in this thesis.

Averages of the data of the five experiments are calculated, which are used to fit a model (Eq. (3.9)) of the effect of particle size on the rate constant. The fitted model uses the formula structure of a gamma distribution which gives a close approximation to the graph of the experimental data.

\[
k_{\text{max},x} = \frac{C_4 C_5 \left( \frac{d_{\text{p},x}}{C_6} \right)^{(C_4 - 1)} e^{-\left( \frac{d_{\text{p},x}}{C_6} \right)^{C_4}}}{C_6} + C_7
\]  

(3.9)

where \( C_4, C_5, C_6 \) and \( C_7 \) are constants. This model is plotted in Fig. 3.3 together with experimental averages from the literature for comparison.

The general shape of the modelled \( k_{\text{max}} \) in Fig. 3.3 (sharp increase followed by decrease as particle size increases) is also reported in Welsby et al. (2010). However, there is limited data presented in the work of Welsby and other literature that can be used to model the effect of particle size on \( k_{\text{max}} \). For the purpose of this research, the \( k_{\text{max}} \) model in Eq. (3.9) can be used to approximate a continuous spectrum of flotation rates by a small number of discrete components, though experiments should be carried out to determine the value of the constant to ensure the reliability of the results.

Combining Eq. (3.8) and Eq. (3.9), it is now possible to predict the rate con-
constant of any particle species of given degree of liberation and size:

\[ k_x = C_1 C_4 C_5 \left( \frac{d_{p,x}}{C_6} \right)^{(C_4 - 1)} L_x e \left[ C_2 L_x C_3 \left( \frac{d_{p,x}}{C_6} \right)^{C_4} \right] + C_7 \]  

(3.10)

It is important to note that the constants in the combined rate constant formula define the shape of the graph, therefore the constants are unique to a specific ore feed. In this work, the constants in Eq. (3.8) are given the values suggested by Jameson (2012); the constants in Eq. (3.9) take the values that fit the experimental averages closely. The values of the constants are tabulated below (Table 3.1):

<table>
<thead>
<tr>
<th>( C_1 )</th>
<th>( C_2 )</th>
<th>( C_3 )</th>
<th>( C_4 )</th>
<th>( C_5 )</th>
<th>( C_6 )</th>
<th>( C_7 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.27</td>
<td>1.3</td>
<td>10.8</td>
<td>2.1</td>
<td>( 3.5 \times 10^{-6} )</td>
<td>0.00012</td>
<td>0.0083</td>
</tr>
</tbody>
</table>

**Table 3.1: Constants in the rate constant model**

**Floatability**

In addition to the effect of particle size, the work of Gorain et al. (1997) and Gorain et al. (1998b) found that the flotation rate constant relates to bubble surface area flux \( S_b \) (\( S_b \) combines the effect of bubble size, gas hold-up and superficial air velocity) very well. As a result, a linear relationship between the rate constant \( k_x \)
and the bubble surface area flux $S_b$ suggested by the literature is adopted:

$$k_x = P_{\text{float,x}}S_b$$  \hspace{1cm} (3.11)

where $P_{\text{float,x}}$ is the floatability of the particle species $x$. The bubble surface area flux is a function of the superficial air velocity $v_g$ (a measurement of the air motion in the flotation cell, calculated by dividing the volumetric air flowrate by the cell cross-section area) and the bubble size at the pulp-froth interface $d_{b,\text{in}}$:

$$S_b = \frac{6v_g}{d_{b,\text{in}}}$$  \hspace{1cm} (3.12)

$d_{b,\text{in}}$ is found empirically to be also a function of $v_g$ (Nesset et al., 2007):

$$d_{b,\text{in}} = d_0 + d_A v_g^{d_B}$$  \hspace{1cm} (3.13)

where $d_0$ is the bubble size at $v_g = 0$, which is of a value of 0.0005m. $d_A$ and $d_B$ are constants that are dependent on the system chemistry and bubble generation mechanism, they are of 0.001 and 0.5 respectively in the model in (Nesset et al., 2007).

**The combined kinetics model**

With the aim of building an model for the flotation kinetics which takes into account the effects of both air flow, mineral liberation and particle size, the rate constant can be expressed as the product of floatability and bubble surface flux as in Eq. (3.11). While $S_b$ accounts for the effect of air flow, $P_{\text{float,x}}$ is governed by the liberation and size. Assuming that the experimental data used to determine the rate kinetics in the work of Muganda et al. (2011) was obtained from tests done at $v_g$ of 0.01m s$^{-1}$ (which gives $S_b$ a value of 100s$^{-1}$), the rate constant model in Eq. (3.10) can be rearranged to express $P_{\text{float,x}}$ as:

$$P_{\text{float,x}} = \frac{C_1 C_4 C_5 \left( \frac{d_{p,x}}{C_6} \right)^{C_4 - 1} L_x e^{\left[ C_7 L_x^{C_3} \left( \frac{d_{p,x}}{C_6} \right)^{C_4} \right]}}{100 C_6}$$  \hspace{1cm} (3.14)

Substituting Eq. (3.14) back into Eq. (3.11), the rate constant can now be fully expressed in terms of the air flow $v_g$, mineral liberation $L_x$, particle size $d_{p,x}$ and
Chapter 3 - Flotation models and optimisation methods

constants:

\[
k_x = \frac{v_g C_1 C_4 C_5 \left( \frac{d_{p,x}}{C_6} \right)^{(C_4-1)} L_x e^{C_2 L_x C_3 - \left( \frac{d_{p,x}}{C_6} \right)^C_4} + v_g C_6 C_7}{16.67 (d_0 + d_A v_g d_A) C_6}
\]  

(3.15)

3.1.3 Air recovery

The air added in the flotation cell is one of the key elements that controls the process in both the pulp and froth phase. The air travels through the cell in the form of bubbles and exits as being trapped in the froth or being released to the atmosphere when bubbles burst. The air recovery, which is the fraction of air entering the cell that overflows as unbroken bubbles, indicates the froth stability and is linked closely to the flotation performance (it is sometimes measured as bubble bursting rate in other approaches in the literature (Murphy et al., 1996)).

The flotation performance is measured by the recovery and grade, which are essentially the amount of the mineral and gangue in the concentrate. The top layer of the froth overflows the cell lip and carries these materials from the froth phase to the concentrate. This top froth layer is made of unbroken bubbles. Using the air recovery together with the inlet air velocity and other parameters, it is possible to calculate the surface area of the overflowing bubbles where the majority of mineral particles are attached, and calculate the total length of the Plateau border where the majority of gangue particles are trapped.

Experimentally, readings of the overflowing froth height (the distance between the froth surface and the cell lip at the edge of a cell) are taken, and the overflow velocity (the speed at which the froth surface flows across the lip) is measured by performing image analysis on the froth surface at the edge of the cell. The amount of air overflows as unbroken bubbles is approximated by the volume of the overflowing froth, which is the product of the overflowing froth height and overflow velocity. The air recovery can now be calculated as the volume ratio of the overflowing froth to the air entering the cell.

In this thesis, an air recovery model is developed using data from experiments carried out at a copper operation in Australia (Hadler et al., 2010). A potential maximum value of the air recovery \( \alpha_{max} \) for a flotation cell is dependent on the superficial air velocity \( v_g \) base on the plant data, it is assumed to follow a quadratic
trend which introduces a peak of 45% at 0.01 \text{m s}^{-1} \) (see Eq. (3.16)):

\[
\alpha_{\text{max}} = a_A v_g^2 + a_B v_g + a_C
\]  

(3.16)

where \( a_A, a_B \) and \( a_C \) are constants, and they are given the value of -35750, 715 and -3.125 to allow the air recovery to follow the trend of the plant data (see Fig. 3.4).

The concentration of hydrophobic mineral particles in the feed, the feed grade, affects the bubble solids loading which in turn provides stabilisation of the bubbles (Barbian et al., 2007). The froth stability is also found to be strongly dependent on the size of particle (Aktas et al., 2008). Since the air recovery is used to reflect the froth stability, the effects of changes in the feed metal grade and particle size on the air recovery are taken into consideration, such that the overall air recovery becomes a combination of \( \alpha_{\text{max}}, f_{FG} \) and \( f_{PS} \):

\[
\alpha = f_{FG} f_{PS} \alpha_{\text{max}}
\]  

(3.17)

\( f_{FG} \) is the the effects of the feed grade, which is defined as the ratio of the feed metal grade of the cell \( G_{\text{feed,cell}} \) to the circuit \( G_{\text{feed,circuit}} \). If the feed grade of a cell is lower than the feed grade of the circuit (e.g. cells down a flotation bank), the air
recovery decreases (see Eq. (3.18)).

\[
f_{FG} = \begin{cases} 
G_{feed,cell}/G_{feed,circuit} & \text{if } G_{feed,cell} \leq G_{feed,circuit} \\
1 & \text{otherwise}
\end{cases}
\]  

(3.18)

\(f_{PS}\) is the the effects of the particle size, which allow air recovery to decrease as the average particle size of the cell’s feed \(d_{p,avg,cell}\) increases above 40\(\mu m\) (see Eq. (3.19)).

\[
f_{PS} = \begin{cases} 
 f_{PS,min} & \text{if } d_{p,avg,cell} < 40\mu m \\
1 + (f_{PS,min} - 1) \left(\frac{d_{p,avg,cell} - 40}{200 - 40}\right) & \text{if } 40\mu m \leq d_{p,avg,cell} \leq 200\mu m \\
1 & \text{if } d_{p,avg,cell} > 200\mu m
\end{cases}
\]  

(3.19)

3.1.4 True flotation - froth recovery

The froth recovery is defined as the fraction of particles entering the froth phase attached to bubbles which are later recovered to the concentrate either attached or unattached. It has been modelled using Eq. (3.20) (Neethling, 2008). These equations include the effects of superficial air velocity (air rate) \(v_g\), particle settling velocity \(v_{set,x}\) and air recovery \(\alpha\):

\[
R_{f,x} \approx \begin{cases} 
\left[\frac{\alpha(1-\alpha)v_g}{v_{set,x}}\right]^\frac{f}{2} \left(\frac{d_{b,in}}{d_{b,out}}\right)^f & \text{if } \alpha < \frac{1}{2} \\
\left(\frac{v_g}{4v_{set,x}}\right)^\frac{f}{2} \left(\frac{d_{b,in}}{d_{b,out}}\right)^f & \text{if } \alpha \geq \frac{1}{2}
\end{cases}
\]  

(3.20)

Here \(f\) is the fraction of material that becomes detached from the vanishing interface during a coalescence event, \(d_{b,in}\) is the bubble size at the pulp-froth interface and \(d_{b,out}\) is the bubble size overflowing the cell lip.

For air recoveries lower than 50\%, the froth recovery is highly dependent on \(\alpha\). In this study, \(f\) was assumed to be unity, that is, when bubbles coalesce, all the material is detached from the coalescing surface. The particle size \(d_{p,x}\) is also important in the froth recovery; it contributes to the particle terminal velocity \(v_{term,x}\) and determines whether a particle will be carried upwards in the froth and collected.
to the concentrate (Neethling and Cilliers, 2002).

\[ v_{\text{term},x} = \frac{g (\rho_x - \rho_{\text{water}}) d_{p,x}^2}{18\mu_{\text{slurry}}} \]  

(3.21)

\[ v_{\text{set},x} = \frac{v_{\text{term},x} (1 - \phi) 4.65}{3} \]  

(3.22)

Here \( \rho_x \) is the solid density of species \( x \) and \( \phi \) is the volumetric fraction of solids in the Plateau borders of the froth. \( \mu_{\text{slurry}} \) is the viscosity of the slurry in the Plateau borders, and is estimated from \( \phi \) (Coulson and Richardson, 1993):

\[ \mu_{\text{slurry}} = \mu_{\text{water}} \exp \left( 2.5 \frac{\phi}{1 - 0.609\phi} \right) \]  

(3.23)

The volumetric fraction of solids and the viscosity of the slurry, in both the froth and the concentrate are assumed to be the same as \( \phi \) and \( \mu_{\text{slurry}} \).

Ideally, these properties would take account of the particles and water which are attached to and in the bubble film as well as those in the Plateau borders, and they are likely to vary depends on the horizontal and vertical positions in the froth. However, without performing detailed modelling of the froth phase, it is not possible to predict the precise values of these properties of the froth and concentrate. Thus, an assumption is made that the values of the volumetric fraction of solids and the viscosity of the slurry can be used interchangeably in the Plateau borders, across the froth and in the concentrate.

### 3.1.5 Entrained flotation recovery

In contrast to true flotation, entrainment is the process by which unattached particles enter the froth (not governed by the rate constant) at the pulp-froth interface in the channels between the bubbles (Plateau borders) and are recovered to the concentrate. It has been found that the entrained material recovery is proportional to the water recovered to the concentrate (Engelbrecht and Woodburn, 1975; Trahar, 1981).

The water recovery depends on the liquid behaviour in Plateau borders and is governed by gravity, viscous drag and capillary suction in the froth as proposed by
Neethling et al. (2003a):

\[ Q_{\text{conc}} \approx Q_{\text{water}} = \begin{cases} \frac{A_{cell} v_s^2 \lambda}{k_1} \alpha (1 - \alpha) & \text{if } \alpha < \frac{1}{2} \\ \frac{A_{cell} v_s^2 \lambda}{4k_1} & \text{if } \alpha \geq \frac{1}{2} \end{cases} \]

(3.24)

where \( A_{cell} \) is the cross-section area of the flotation cell. \( \lambda \) is the length of Plateau borders per volume of froth (Neethling et al., 2003a) where the bubbles are assumed to be Kelvin cells (a polyhedron structure which was thought to form the most efficient bubble foam until the recent discovery of the Weaire-Phelan structure):

\[ \lambda = \frac{6.815}{d_{b,\text{out}}^2} \]

(3.25)

The constant \( k_1 \) in the water recovery model represents the balance between gravity and viscosity (Neethling et al., 2003a):

\[ k_1 = \frac{\rho_{\text{slurry}} g}{3C_{pb} \mu_{\text{slurry}}} \]

(3.26)

where \( C_{pb} \) is the viscous drag coefficient in the Plateau borders. Its value is taken to be 49 due to the immobile interface in flotation froths (Neethling et al., 2003a). \( \rho_{\text{slurry}} \) is the density of the concentrate:

\[ \rho_{\text{slurry}} = \phi \rho_{\text{solids}} + (1 - \phi) \rho_{\text{water}} \]

(3.27)

To quantify the recovery of entrained material using the water recovery, the entrainment factor is introduced to incorporate the effects of liquid motion, particle settling and particle dispersion (Neethling and Cilliers, 2009).

\[ Ent_x \approx \begin{cases} \exp \left( -\frac{v_{set,x}^{1.5} h_{\text{froth}}}{D_{\text{axial}} \sqrt{g \alpha (1-\alpha)}} \right) & \text{if } \alpha < \frac{1}{2} \\ \exp \left( -\frac{2v_{set,x}^{1.5} h_{\text{froth}}}{D_{\text{axial}} \sqrt{g \alpha (1-\alpha)}} \right) & \text{if } \alpha \geq \frac{1}{2} \end{cases} \]

(3.28)

where \( h_{\text{froth}} \) is the froth height from the pulp-froth interface to the bursting surface, and \( D_{\text{axial}} \) is the axial dispersion coefficient. \( D_{\text{axial}} \) has been found to be a function of relative liquid velocity and the Peclet number \( Pe \), and this relationship is simplified
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to (Neethling and Cilliers, 2009):

\[ D_{axial} \approx \frac{v_g^{1.5}}{\sqrt{k_1 \left( \sqrt{3} - \frac{\pi}{2} \right) Pe}} \]  \hspace{1cm} (3.29)

where \( Pe \) is taken to be 0.15 (Lee et al., 2005).

### 3.1.6 Integrated cell and circuit simulator

In order to simulate the flotation process in a cell and determine its concentrate and tailings flowrate and composition, the models described previously are combined and solved for a given feed.

Firstly, the properties of the feed are given (flowrate, density, grade and particle size), which are first used to calculate the rate constants of each particle species. An iterative approach is then taken to solving the equations, and this is shown schematically in Fig. 3.5. The steps of the iteration is described here:

**Figure 3.5:** The iteration cycle for determining flotation performance in the simulator
1. An initial value for the volumetric solids fraction of the concentrate is taken as an input. The density of the solids in the concentrate is initially assumed to be the same as that for the feed. This allows the density and viscosity of the concentrate to be evaluated using Eqs. (3.27) and (3.23) respectively.

2. Knowing the individual density of each solid species and the particle size from the given feed, the particle settling velocity is predicted using the Stokes’ Law in Eqs. (3.21) and (3.22). The gravity and viscosity balance is calculated using Eq. (3.26).

3. Subsequently the volumetric concentrate flowrate, froth recovery and axial dispersion coefficient are found using Eqs. (3.24) and (3.20) and (3.29) according to the air recovery value.

4. Next, by performing a mass balance for the flotation cell and using Eq. (3.28), the volumetric tailings flowrate and the entrainment factor are obtained.

5. The concentrations of solid species in the tailings and concentrate streams are determined by performing the mass balances in Eqs. (3.6) and (3.7).

6. Finally, knowing the concentrations in the concentrate stream allows the initially guessed volumetric solids fraction and assumed solids density to be recalculated for the next iteration cycle.

    The iteration is repeated until the volumetric solids fraction naturally converges, following which the mass flow of each particle species in streams can be calculated as the product the concentration \((C_{\text{conc},x}, C_{\text{tail},x})\) and volumetric flowrate \((Q_{\text{conc}}, Q_{\text{tail}})\).

    After the first cell in a flotation circuit is modelled using the method above, the resulting concentrate and tailings are either new feeds to downstream or a part of the circuit output. The iterative flotation modelling is repeated for the remaining cells in the circuit. The circuit concentrate and tailings are the sum of each relative outputting streams from the cells.

    Recycling streams are often present in a circuit, which are needed to be combined with the feed streams connected to cells. However, there is no material in recycling streams until the original cells have been modelled. To solve this issue, a
second iteration of circuit modelling is used. The circuit is modelled cell by cell as described above initially with empty recycling streams.

In the next iteration, the circuit is modelled again with the recycling streams taking the values (flowrate of all particle species and water) from the previous iteration result. This iteration of circuit modelling is thought to be similar to starting the flotation process in an circuit with empty cells in real situation. The iteration ends when there is little difference in the stream flowrate of every cell between three successive iteration results, in which case it is considered that the circuit is at steady-state. The performance, grade and recovery, of the circuit can be calculated using the final modelling result.

The volumetric solids fraction iteration and the steady-state iteration are fixed-point systems, and the solutions are attractive fixed points. This means the iterations always converge (the iterations are stopped if the difference in solutions between two consecutive cycles is less than 0.0001%), and usually within 20 cycles.

This modelling approach incorporates detailed froth models allowing the effects of design and operating variables such as cell dimensions and air inlet to be taken in to account in the flotation performance. However, in contrast to the first order rate modelling approach where the mass balances of the cells are usually solved instantaneously, this double iterative modelling requires relatively intensive computation. In addition, the detailed froth models require more inputs such as bubble sizes and froth height, which can be difficult to measure or estimate. Moreover, this approach allows the prediction of circuit performance for any feed conditions and circuit layout, once rate constants and other parameters have been established.

### 3.2 Feed modelling and operating variables

Feed from the flotation circuit at Northparkes Mine, New South Wales Australia is used as the base for all optimisation problems in this work. The feed is assumed to comprise of chalcopyrite and uniform gangue, which has a copper metal grade of 0.5% (chalcopyrite mineral grade of 1.44%).

Three models are considered to approximate the feed stream with increasing complexity, and they are referred to by the number of particle species included in the model.
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The first model consists of two particle species (2PS), a complete liberation of the chalcopyrite from the gangue is assumed. As the result, the particles in the feed are classified into either pure mineral or pure gangue of a single size. The second model consists of four particle species (4PS), in which another size class is introduced. The previous pure mineral and pure gangue particles remain in a fine size class, while pure gangue and middlings (half mineral and half gangue by mass) particles constitute the coarser (double the size of the fine) class. In the last model, fifteen particle species (15PS), a distribution of particles is taken into account with five size classes to produce a closer approximation to reality. The size classes ranges from 1 to 500 micron and each size class contains mineral, middlings and gangue particles based on the liberation.

These feed models are used to show the sensitivity of the flotation performance and the optimal circuits to the models.

3.2.1 Two particle species (2PS) - Perfect liberation

The feed from Northparkes is taken as the base for modelling the particles that enter the flotation process. The mass flowrate of the solids feed is 400 t h\(^{-1}\), it is mixed with 600 t h\(^{-1}\) of water which results in a solid content of 40%. The stream is considered to be containing fully liberated chalcopyrite particles as the only mineral, the remaining solid material is treated as uniform gangue. Both the mineral and the gangue are of 50 \(\mu m\) in diameter. Details of the modelled feed are given in Table 3.2:

<table>
<thead>
<tr>
<th></th>
<th>Mineral</th>
<th>Gangue</th>
<th>Water</th>
<th>Sum/Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feed flowrate</td>
<td>t h(^{-1})</td>
<td>5.78</td>
<td>394.22</td>
<td>600</td>
</tr>
<tr>
<td></td>
<td>kg s(^{-1})</td>
<td>1.6</td>
<td>110.56</td>
<td>166.67</td>
</tr>
<tr>
<td>Density</td>
<td>kg m(^{-3})</td>
<td>4500</td>
<td>3000</td>
<td>1000</td>
</tr>
<tr>
<td>Volumetric flowrate</td>
<td>m(^3) hr(^{-1})</td>
<td>1.28</td>
<td>131.41</td>
<td>600</td>
</tr>
<tr>
<td></td>
<td>m(^3) s(^{-1})</td>
<td>3.57 \times 10(^{-4})</td>
<td>3.65 \times 10(^{-2})</td>
<td>0.167</td>
</tr>
<tr>
<td>Solid fraction</td>
<td>%wt</td>
<td>1.44 (metal: 0.5)</td>
<td>98.56</td>
<td>N/A</td>
</tr>
<tr>
<td>Metal content</td>
<td>%wt</td>
<td>34.62</td>
<td>17.31</td>
<td>0</td>
</tr>
<tr>
<td>Floatability</td>
<td>m(^{-1}) s(^{-1})</td>
<td>2.80 \times 10(^{-4})</td>
<td>7.63 \times 10(^{-9})</td>
<td>N/A</td>
</tr>
<tr>
<td>Particle size</td>
<td>m</td>
<td>50 \times 10(^{-6})</td>
<td>50 \times 10(^{-6})</td>
<td>N/A</td>
</tr>
</tbody>
</table>
It is worth noting that the floatability of the particle species is determined by its size and degree of liberation using the flotation kinetics model derived in Subsection 3.1.2. At a superficial air velocity of 0.01 m s\(^{-1}\), the floatability rate constant for mineral particles at 50\(\mu\)m is calculated to be 2.80 \times 10^{-2} m^{-1} s^{-1} and 7.63 \times 10^{-7} m^{-1} s^{-1} for gangue (pure gangue particles are treated as particles containing 0.01% mineral in order to be marginally floatable). The values of these modelled rate constants are comparable to the ones from a plant survey conducted (Smith and Cilliers, 2010), in which 40\(\mu\)m mineral particles are floated at a rate of 8.00 \times 10^{-3} m^{-1} s^{-1} and gangue particles are at 3.00 \times 10^{-7} m^{-1} s^{-1}.

This feed model is relatively simple, but it represents an ideal state of the ore feed as the result of perfect grinding from the mills (perfect liberation of the mineral grains from gangue). The model is first used to test the flotation circuit optimisation system.

### 3.2.2 Four particle species (4PS) - Fine and coarse classes

The idealised 2PS model is unrealistic, and feed particles are likely to contain both mineral and gangue. In this feed model, particles are distributed in two size classes, fine and coarse. The coarse particles have the double the diameter of the fine, and the mass ratio of the fine to coarse is 1 : 2 in the feed. To provide comparability to the previous feed model (perfect liberation), the volume based average size of the fine and coarse mixture are kept the same of 50\(\mu\)m. As a result, the fine particles are 30\(\mu\)m in diameter and the coarse particles are 60\(\mu\)m in diameter. It is assumed that in the fine particle class, perfect liberation remain true, but middlings species of half mineral and half gangue is present instead of pure mineral particles in the coarse class. The detail of the Fine and Coarse feed model is tabulated in Table 3.3:

The floatability of the middlings is dependent on the degree of liberation of the mineral. This degree is often measured in terms of surface liberation that is the fraction of the surface area of the particle covered by the mineral only. However for the ease of interpretation and calculation, the mass fraction of the mineral in the particle is used as a substitute in determining the floatability of the middlings.
### Table 3.3: Four particle species (4PS) model

<table>
<thead>
<tr>
<th>Feed flowrate</th>
<th>Mineral</th>
<th>Middlings</th>
<th>Gangue F</th>
<th>Gangue C</th>
<th>Water</th>
<th>Sum/Ave.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t h^{-1}$</td>
<td>2.89</td>
<td>5.78</td>
<td>130.44</td>
<td>260.89</td>
<td>600</td>
<td>1000</td>
</tr>
<tr>
<td>$kg s^{-1}$</td>
<td>0.8</td>
<td>1.6</td>
<td>36.23</td>
<td>72.47</td>
<td>166.7</td>
<td>277.78</td>
</tr>
<tr>
<td>Density</td>
<td>$kg m^{-3}$</td>
<td>4500</td>
<td>3600</td>
<td>3000</td>
<td>3000</td>
<td>1000</td>
</tr>
<tr>
<td>Volumetric flowrate</td>
<td>$m^3 hr^{-1}$</td>
<td>0.64</td>
<td>1.6</td>
<td>43.48</td>
<td>86.96</td>
<td>600</td>
</tr>
<tr>
<td></td>
<td>$m^3 s^{-1}$</td>
<td>$1.78 \times 10^{-4}$</td>
<td>$4.46 \times 10^{-4}$</td>
<td>$1.21 \times 10^{-2}$</td>
<td>$2.42 \times 10^{-2}$</td>
<td>0.167</td>
</tr>
<tr>
<td>Solid fraction</td>
<td>%wt</td>
<td>1.44 (metal: 0.25)</td>
<td>7.22 (metal: 0.50)</td>
<td>65.22</td>
<td>32.61</td>
<td>N/A</td>
</tr>
<tr>
<td>Metal content</td>
<td>%wt</td>
<td>34.62</td>
<td>17.31</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Floatability</td>
<td>$m^{-1} s^{-1}$</td>
<td>$2.07 \times 10^{-4}$</td>
<td>$4.18 \times 10^{-5}$</td>
<td>$5.65 \times 10^{-9}$</td>
<td>$9.35 \times 10^{-9}$</td>
<td>N/A</td>
</tr>
<tr>
<td>Particle size</td>
<td>$m$</td>
<td>$30 \times 10^{-6}$</td>
<td>$60 \times 10^{-6}$</td>
<td>$30 \times 10^{-6}$</td>
<td>$60 \times 10^{-6}$</td>
<td>N/A</td>
</tr>
</tbody>
</table>

#### 3.2.3 Fifteen particle species (15PS) - Liberation and size distribution

In order to create a detailed feed model that is accurate and close to reality, five particle size classes are used for the feed: $0 – 10\mu m$, $10 – 50\mu m$, $50 – 100\mu m$, $100 – 200\mu m$ and $200 – 500\mu m$. The mass fraction of the particles is each size class is assumed to follow Rosin-Rammler distribution, and the spread is determined by the value of $P80$ and $m$ (parameter describing the spread of the distribution) (see Fig. 3.6).

![Particle size distribution for $P80$ of 25 to 125$\mu m$](image)

Figure 3.6: Particle size distribution for $P80$ of 25 to 125$\mu m$

To achieve the same volume based average size of $50\mu m$, $P80$ and $m$ are set to $78.9\mu m$ and 1.5 respectively (it is worth noting that $P80$, the diameter at which 80% of the particles are finer, is a measure of size distribution but not an average size).
Within each size class, the copper metal grade is kept constant at 0.5% (1.44% chalcopyrite), and particles are sorted into three liberation types: pure mineral, pure gangue and middling (50% mineral and 50% gangue). It is assumed that the mineral and gangue are fully liberated in the smallest size class (no middlings), and the liberation decreases as the particle size increases. The mass fraction of the mineral, middlings and gangue particles within each size class are adjusted to maintain the 0.5% metal grade. In total, this feed comprise 15 particle species. The detail of 15PS model is tabulated in Table 3.4.

Although the mass fractions of middlings species in the small particle size classes are small, the middlings content increases significantly as particle size increases assuming liberation is an inverse function of size. An arbitrarily relationship is drawn which links the division of metal content within the size classes and the particle size of the class. As the result, the mineral and middlings species have a copper mass ratio of 5 : 0 in the 0 – 10µm size class (middlings is absent), and this ratio decreases to 4 : 1 in the 10 – 50µm class, 3 : 2 in 50 – 100µm class, 2 : 3 in 100 – 200µm class and finally 1 : 4 in 200 – 500µm class. It is important to maintain these ratios to provide comparability in the results, especially in later chapters when the average particle size P80 is varied (varying P80 will results in significant changes in mass fractions of middlings across size classes but not within each size class).
Table 3.4: Fifteen particle species (15PS) model

<table>
<thead>
<tr>
<th></th>
<th>Unit</th>
<th>Particle species</th>
<th>Solids</th>
<th>Water</th>
<th>Sum/Ave</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size class</td>
<td>µm</td>
<td>0 10 (5)</td>
<td>10 50 (22)</td>
<td>50 100 (71)</td>
<td>100 200 (141)</td>
</tr>
<tr>
<td>Size fraction</td>
<td>%wt</td>
<td>7.0028</td>
<td>48.587</td>
<td>34.342</td>
<td>9.9163</td>
</tr>
<tr>
<td>Mineral grade</td>
<td>%</td>
<td>1.44 (0.5)</td>
<td>1.44 (0.5)</td>
<td>1.44 (0.5)</td>
<td>1.44 (0.5)</td>
</tr>
<tr>
<td>Liberation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lab. fraction</td>
<td>%wt</td>
<td>1.44</td>
<td>98.56</td>
<td>1.16</td>
<td>0.58</td>
</tr>
<tr>
<td>Mass fraction</td>
<td>%wt</td>
<td>0.1011</td>
<td>6.9017</td>
<td>0.5614</td>
<td>0.2807</td>
</tr>
<tr>
<td>Metal content</td>
<td>%wt</td>
<td>34.62</td>
<td>17.31</td>
<td>0</td>
<td>34.62</td>
</tr>
<tr>
<td>Feed rate</td>
<td>t h⁻¹</td>
<td>0.4046</td>
<td>27.607</td>
<td>2.2455</td>
<td>1.1228</td>
</tr>
<tr>
<td></td>
<td>kg s⁻¹</td>
<td>0.1124</td>
<td>7.6685</td>
<td>0.6238</td>
<td>0.3119</td>
</tr>
<tr>
<td>Density</td>
<td>kg m⁻³</td>
<td>4500</td>
<td>3600</td>
<td>3000</td>
<td>4500</td>
</tr>
<tr>
<td>Volume</td>
<td>m³ hr⁻¹</td>
<td>0.0809</td>
<td>9.2023</td>
<td>4.99</td>
<td>3.12</td>
</tr>
<tr>
<td></td>
<td>m³ s⁻¹</td>
<td>2.50E-5</td>
<td>0</td>
<td>0.0026</td>
<td>1.38E-4</td>
</tr>
<tr>
<td>Floatability</td>
<td>m⁻¹ s⁻¹</td>
<td>0.0001</td>
<td>1.37E-5</td>
<td>2.74E-9</td>
<td>0.0002</td>
</tr>
</tbody>
</table>
3.2.4 Cell design and operating conditions

Industrially, cells are designed based on their specific function, different handling requirements such as feed tonnage and residence time will be determining factors in the cell dimensions. However, in order to give a higher comparability and produce a relatively general result, the flotation cells used in the simulated circuits here are identical. The cells are cylindrical, due to the simplicity of flotation models, detailed cell modification such as launder and impeller designs are not considered. In addition to the feed conditions and the cell dimension, the flotation models also require input of operating conditions, the detail of which is tabulated below (Table 3.5):

<table>
<thead>
<tr>
<th>Table 3.5:</th>
<th>Cell dimensions and operating conditions based on plant data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cell cross-sectional area</td>
<td>$m^2$</td>
</tr>
<tr>
<td>Cell diameter</td>
<td>$m$</td>
</tr>
<tr>
<td>Pulp volume</td>
<td>$m^3$</td>
</tr>
<tr>
<td>Froth depth</td>
<td>$m$</td>
</tr>
<tr>
<td>Bubble size at zero air rate</td>
<td>$m$</td>
</tr>
<tr>
<td>Air volumetric flowrate</td>
<td>$m^3 \text{ s}^{-1}$</td>
</tr>
<tr>
<td>Superficial air velocity (air rate)</td>
<td>$m \text{ s}^{-1}$</td>
</tr>
</tbody>
</table>

3.3 Circuit layout optimisation

A typical froth flotation circuit consists of four to eight flotation banks, with five to twenty cells within each bank (Bourke, 2002). The possible number of different circuit configurations is extremely large.

Every cell outputs two product streams; concentrate and tailings. In a flotation system consisting of $n$ cells, each cell’s output stream can be connected to one of $n$ destinations, every other cell ($n-1$) and the circuit output (1) which excludes self-recycles. It is fruitless to direct the tailings to the same destination as the concentrate. As a result, there are $n \times (n-1)$ ways to connect a cell in the system. Taking all cells into account, the number of different configurations rises exponentially to $(n \times (n-1))^n$. However, a circuit must have at least two product streams, which means that at least one concentrate and one tailings from any two
cells are taken to be the products, so the number of configurations is corrected to 
\((n \times (n - 1))^{n-1}\). The feed stream can be introduced to any cell in the circuit, which 
increases the number of configurations by a factor of \(n\) to \(n^n \times (n - 1)^{n-1}\). This sug-
gests that even for a 5-cell circuit, there are 800,000 possible circuit arrangements 
(see Fig. 3.7), although not all are valid and many are equivalent to one another.

![Figure 3.7: Number of possible configurations for circuits of different size](image)

Most of the possible designs are suboptimal or invalid as a result of technical faults or illogical configuration (e.g. recycling the tailings of the scavenger to the cleaner). Consequently, adopting an optimisation method is critical to first screen out the flawed designs, and to generate the optimum design which satisfies both technical and economic constrains.

### 3.4 Genetic algorithm methodology

In order for the genetic algorithm methodology to be applied to flotation cir-
cuits, cell linkages (i.e. the destination of output streams) must be coded as a finite-
length string. A population of strings representing flotation circuit layouts are first 
generated (parent population); for each of these a single fitness value of performance 
is determined (based on efficiency or profit, for example). Daughter strings of the 
same population size are randomly selected from the original strings. It is a fitness proportionate selection (roulette wheel selection), as the greater the fitness of the original string, the more likely it is to be selected as a daughter. This means that the
best strings are expected to be selected more than once in the process. To encourage the survival of the circuit layouts which yield better performance, the current best string (highest fitness) is always included as a daughter. After obtaining the daughter population, every two strings are mated, so that the genetic information between two randomly chosen string positions is swapped. This process is termed ‘Crossover’; hence the daughter strings now contain features from both of their parents. For every parameter, there is a slight possibility that it undergoes mutation. The value of the mutating bit is randomly reassigned during this process. Although this minor alteration may seem insignificant, the operation is an efficient means of preventing unification of the whole population and avoiding being trapped at only local optima. The fitness value is re-evaluated for every daughter string to enable the selection process to take place again. One selection-crossover-evaluation cycle is referred to as one generation, and the cycle is repeated until a designated number of generations is reached or the fitness satisfies minimum criteria (e.g., minimum concentrate grade). The final daughter string is the solution to the optimisation problem. For in-depth discussions on GAs see Goldberg (1989) and Mitchell (1998).

The GA operations described above is shown in Fig. 3.8:

![Figure 3.8: The GA cycle for circuit performance optimisation](image)

### 3.4.1 The string representation of a circuit

A flotation circuit configuration is coded in a systematic way to represent the linkages of every cell. Instead of a binary form, a decimal digit string is used to
symbolise the connections between cells for a straightforward and concise representation. Cells in an \( n \)-cell circuit are numbered from 1 to \( n \). In the decimal string, each digit represents a stream in the circuit. The position of the digit indicates the origin of the stream and whether this stream is a concentrate or tailings; the value of the digit denotes the destination cell of the stream.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1,2,...,( n )</td>
<td>0,2,3,...,( n )</td>
<td>0,2,3,...,( n )</td>
<td>0,1,3,...,( n )</td>
<td>0,1,3,...,( n )</td>
<td>...</td>
<td>0,1,...,( n-1 )</td>
</tr>
</tbody>
</table>

Figure 3.9: Possible destinations of streams

As seen from Fig. 3.9, the first digit block in the string representation denotes the destination of the circuit feed (the cell which the external feed is connected to), and every two blocks followed are the destinations of the concentrate and tailings stream from Cell 1 to Cell \( n \). The streams which exit the circuit are assigned with ‘0’. An example circuit is shown below in Fig. 3.10, it is coded as ‘1 23 01 24 20’. The exiting horizontal arrows from the sides of the cells represent tailings outflows, while vertical arrows from the bottom of the cells represent concentrate outflows. The circuit feed is connected to Cell 1 which begins the circuit string representation with ‘1’; it is followed by Cell 1’s concentrate destination ‘2’ and tailings destination ‘3’; the next pair of the digit represents Cell 2’s concentrate and tailings destination of ‘0’ and ‘1’ respectively. In the same manner, the rest of cells are joined into the string to give ‘1 23 01 24 20’.

![Figure 3.10: A simple circuit configuration coded as ‘1 23 01 24 20’](image)

3.4.2 GA parameters

When implementing a GA, various algorithm settings, such as population size, GA cycle length, crossover rate and mutation possibility must be decided. The reliability of the result, the production of true optimum, is heavily dependent on
the values set for these parameters, which are interrelated. Although the parameter settings are specific to the optimisation problem being considered, many researchers use these settings that have been successful in previous studies (Mitchell, 1998).

In the algorithm used in this study, the crossover rate was set to 1, which means every pair of circuits undergoes crossover attempts. Post-crossover circuits were checked against the circuit validation criteria. The crossover was reversed if the process had been attempted five times and the circuits remained flawed. This ‘uncrossed’ portion of the population decreased the apparent crossover rate to approximately 0.7. Having a crossover rate lower than 1 prevents a premature convergence of the result due to allele loss (loss of circuit variety when unique pieces of string representation is lost in crossover), which in turn is caused by discarding high performance circuits faster than crossover can produce improvements.

The mutation rate is used to prevent unification of the population at local optima and maintain genetic diversity. In later GA cycles, however, it is desirable for convergence to arise as the optimal solution can be delayed or partially prevented if a large number of mutations occur. According to previous work (Fogarty, 1989; Bäck, 1993), the effect of mutation rate on the efficiency of the algorithm is dependent on the nature of the optimisation problem (e.g. finite or infinite search space; whether the starting parent population is a matrix of all zeros or random digits) and fitness function, such that it is arguable whether the implementation of a time-dependent variation of the mutation rate can accelerate the optimisation. A typical mutation rate of 0.005 is suggested in the work of Goldberg (1989) and used in this study.

De Jong (1975) and later Schaffer et al. (1989) performed a systematic investigation on the effect of population size and GA cycle length on the algorithm performance. It was found that the most efficient population size was 20 to 100 individuals, the mutation rate was 0.001 to 0.01.

Although these parameters were proven reliable and efficient in these studies, the complexity of the optimisation problem differs from one problem to the next. Therefore a range of population sizes and GA cycle lengths were tested (see Subsection 3.4.5).


3.4.3 Circuit validation rules

The random circuit generated in the parent population at the beginning of the GA and the daughter circuits evolved after crossover and mutation are validated against several criteria. This is done to eliminate the designs with technical faults or unreasoned linkages, so that a design is only valid if there are:

- no cell self-recycles (as no accumulation is allowed, recycling the concentrate or the tailings stream of a cell back to itself is essentially bypassing this cell);
- at least one feed stream with materials flowing is connected to every cell (this makes sure that every cell is utilise in the circuit);
- at least one concentrate and one tailings stream from the cells that exit the circuit as products (this avoids endless recycle and bypassing the whole circuit);
- the concentrate and tailings from a single cell do not flow to the same destination (no separation is achieved by a cell if both its product streams are fed to the same destination);

The above criteria do not guarantee a valid circuit and thus the final check is to see if the iteratively solved mass balance converges, which it must for a valid circuit.

3.4.4 Circuit optimisation fitness function

After every valid circuit design is simulated, a finance-based fitness function is used to evaluate the circuit performance by providing a single quantitative value, therefore enabling the comparison of different circuit configurations. The fitness function and revenue calculation used in this study are given in Eqs. (3.30) and (3.31).

\[ \text{Circuit Fitness} = \frac{Revenue_{\text{circuit}} - Revenue_{\text{min}}}{Revenue_{\text{max}} - Revenue_{\text{min}}} \]  

(3.30)

The circuit revenues are calculated using the financial function (a net smelter return formula) suggested by Schena et al. (1996):

\[ Revenue_{\text{circuit}} = M_{\text{conc}} p(G_{\text{conc}} - u)(q - Rf c) - M_{\text{conc}} Tr c \]  

(3.31)
where $M_{\text{conc}}$ is the circuit concentrate flowrate,

$p$ is the fraction paid by smelter,

$G_{\text{conc}}$ is the concentrate metal grade,

$u$ is the grade penalty,

$q$ is the metal price,

$Rfc$ is the refining charge,

$Trc$ is the treatment charge.

$Revenue_{\text{min}}$ is the minimum possible profit observed when 50% of minerals and gangue in the circuit feed are recovered in the final concentrate (if less mineral or more gangue is recovered, the tailings becomes more profitable), whereas $Revenue_{\text{max}}$ is the maximum profit observed when all the minerals in the circuit feed are recovered in the final concentrate without any of the gangue.

The values of the financial parameters in the net smelter return formula are taken from internet sources (Lazenby, 2012) and listed below (Table 3.6):

<table>
<thead>
<tr>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Metal (copper) price</td>
<td>$ (t\text{ metal})^{-1}$ 8000</td>
</tr>
<tr>
<td>Refining charge</td>
<td>$Rfc$ $ (t\text{ metal})^{-1}$ 60</td>
</tr>
<tr>
<td>Treatment charge</td>
<td>$Trc$ $ (t\text{ concentrate})^{-1}$ 130</td>
</tr>
<tr>
<td>Grade penalty</td>
<td>$u$ % 1.5</td>
</tr>
<tr>
<td>Fraction paid by smelter</td>
<td>$p$ % 97.5</td>
</tr>
</tbody>
</table>

The circuit with the highest fitness value among the randomly generated strings is recorded as the current optimal circuit design, and it is overwritten by new optimal circuit designs as the GA cycle progresses. At the end of the cycle, the circuit yielding the highest fitness function will be, therefore, the optimal circuit for a given set of conditions.

The improvement of the average fitness of the population through GA cycles follows a logarithmic shape, the tail of which levels off at the fitness value of the optimal circuit. Fig. 3.11 shows a typical progress of the average fitness of a 6-cell circuit optimisation problem. The population size set for the example in Fig. 3.11 was 1296, 576 GA cycles were carried out, and the test was repeated 20 times.
3.4.5 Sensitivity of GA Parameters

Due to the probabilistic nature of GA, the final result may be suboptimal if the population size or the evolution cycles used are insufficient. It is therefore necessary to investigate the reliability of the final result, and to determine the optimal settings of the GA parameters.

Comparison of GA with Brute Force Optimisation

There is a finite number of ways of arranging the flotation cells in a circuit. For flotation circuits with a relatively small number of cells (i.e. fewer than 7 cells), every possible circuit configuration can be tested in order to determine the optimal circuit design. This brute-force enumerative method (BF) enables the optimal circuit generated by the GA to be validated. It was found that the circuit with the highest fitness value (the optimal circuit with best performance) generated by the GA was the same as that found using the BF technique. However, in a 6-cell circuit layout optimisation, the GA was able to find the optimum within only 1% of the computational time required by the BF method.
Selecting the optimal GA parameters

Both population size (number of circuits in each cycle) and number of cycles (selection, crossover, mutation and evaluation) can be varied in the GA, with higher numbers of both indicating higher confidence in finding the global optimum whilst increasing computational time. A trade-off, therefore, is required to consider the requirements of both the flotation simulator and the GA.

A 6-cell circuit (using the simple feed model, 2PS, which is described in Section 3.2) was optimised using five different GA cycle sizes and six population sizes. The reproducibility of the GA program was tested for 30 cases. For each case the test was repeated 25 times. The percentage of the results which matched the optimum found by BF is shown in Fig. 3.12. The maximum number of circuits attempted by the GA program was the product of the population size and the number of the GA cycles performed; the ratio of this value to the number of all possible configurations is used to show the maximum search space covered by the GA. Increasing this coverage brings the reproducibility closer to 100% (see Fig. 3.12).

![Figure 3.12](image)

**Figure 3.12:** Effect of population size and number of cycles on the number of times the optimal circuit is obtained (optimum reproducibility)

It was found that the maximum number of attempts must be at least 0.1% of the total number of possible configurations (coverage of 0.1%) to obtain a reproducible optimal design (reproducibility ≥ 96%). This observation was valid for 4 and 5-cell circuits in addition to the 6-cell circuit shown in Fig. 3.12. Although this requirement of coverage decreased as the number of cells in the circuit increased, in
the optimisations carried out in this study, the population size and number of cycles were set to the smallest values that gave 0.1% coverage to ensure the obtained result was the global solution.

The requirement of the computational time for the various GA population size and cycle settings is shown in Fig. 3.13. A typical optimisation of a 6-cell problem takes 3600s to complete for 2PS feed model, 4000s for 4PS model and 9400s for 15PS model (the complexity of the feed model increase from 2PS to 15PS, which is describe in Section 3.2).

![Figure 3.13: Effect of population size and number of cycles on the computational time required](image)

### 3.4.6 GA parallelisation

Throughout the GA flotation circuit optimisation program, the fitness evaluation, during which circuits are simulated, is the most computational heavy task. As mentioned before, a typical optimisation of a 6-cell 15PS problem takes about 3 hours to complete, the computational time increases to 1.5 days and 2 weeks for 8-cell and 10-cell problems. Therefore, to enable the GA program to handle optimisation problem of large circuits and complex models, reductions in computational time are critical. There are two ways to improve the time required to solve optimisation problem, one is to reduce the calculations and iteration loops in the program, the other is to increase the computational power.

Room for improvements is limited to reduce calculations and iteration loops,
the calculations in the circuit simulation are essential and iteration loops are designed so that they are terminated as soon as convergence is confirmed. As a result, the focus is on providing additional computational power to shorten the program running time.

**Multithreading**

Conventional programs are designed to be executed by a single processor unit (single threading, where a thread is an independently managed sequence of instructions), however today’s computer usually consists more than one processors with a common shared memory. The shared-memory parallel, or multiprocessor, machines have the capability to run programs in parallel (multithreading), in other words, multiple processors can work together to execute a single program by sharing the calculation tasks. The GA circuit optimisation program can be parallelised and its implementation are relatively easy and promise substantial gains in performance (Chapman et al., 2007).

In this study, the GA program is parallelised using OpenMP. OpenMP is a shared-memory application programming interface, it enables the creation of shared-memory parallel programs. OpenMP supports the fork-join programming model (see Fig. 3.14), which is particularly suitable for GA parallelisation, and the details of the parallelisation is described in the subsection below.

![Figure 3.14: OpenMP fork-join programming model. The program starts as a single thread of execution, it is divided into a team of threads at the beginning of a parallel region and joined at the end.](image)
Parallel GA

A GA parallelisation method called master-slave parallelisation, also known as distributed fitness evaluation, is used. This method performs the evaluation of the individuals (flotation circuit simulation and fitness evaluation) and the application of genetic operators (parent generation, selection, crossover and mutation) in parallel.

A master thread initially makes preparation for starting the GA circuit optimisation program by initialising variables and setting up programming structures to store feed and circuit information. Once the preparation is done, the master thread begins the first GA operation, generating the parent population, here the parallel programming is first introduced. The master thread assigns a fraction of the population to each of the processors available. As there is no need to communicate between processors, these slave processors operate independently to generate random circuits, then simulate the circuits and evaluate the fitness of circuits. After the parent generation, the population now contains valid circuits with evaluated fitness values and is handed back to the master thread to initialise the next GA operation, selecting the daughter population. In a similar manner, the selection, crossover and mutation are all done in parallel by the slave threads, and between each operation, the population is managed by the master thread.

The GA cycles become a repetition of the fork-join programming model, and by parallelising the intensive calculations using OpenMP, the time required to complete the GA cycles is significantly reduced. The circuit optimisations performed in this study were done using a workstation which had 12 processor cores, by running the GA program in parallel the same flotation circuit optimisation problem ran up to 11 times faster.

3.5 Down the bank performance study and experimental data

Having obtained the circuit simulator by combining the flotation models, a simple circuit consisting of 10 cells is used to test the flotation performance and the feed models, results are compared with the plant data. The cells are connected in
series where the tailings of each cell are the feed stream to the next. The flowsheet of this circuit layout, is shown in Fig. 3.15.

![Figure 3.15: Rougher circuit layout containing 8 cells in series](image)

In Fig. 3.15, the exiting horizontal arrows represent tailings outflows, and the vertical arrows represent concentrate. Although this circuit has a single bank of cells, complex circuits often have more than one bank. Each bank (which can be just a single tank) is characterised by its general purpose, often referred to as ‘rougher’, ‘cleaner’, ‘re-cleaner’ or ‘scavenger’. The circuit in Fig. 3.15 is a rougher circuit, it is denoted as ‘$Rs$’.

The experimental data used in this study was gathered during a plant survey carried out at Northparkes copper concentrator in Australia in 2007. The flotation circuit consisted of a 4-cell rougher bank (Cell 1 to 4), a 4-cell scavenger bank (Cell 5 to 8) and a cleaner sub-circuit (Hadler et al., 2010). The ore from Northparkes contained mainly chalcopyrite and bornite as valuable minerals. Given that the mineralogy detail (ratio of chalcopyrite to bornite) is unavailable, it is assumed that all the copper bearing minerals are chalcopyrite. The feed had a metal grade of 0.55% with a total flowrate of 1023 t h$^{-1}$, 40.39%wt of which was solid material (the values of the modelled feed are 0.50%, 1000 t h$^{-1}$ and 40.00%wt respectively). The averages of experimental data from five tests were calculated and used to compare with the simulated results. It should be noted that the presence of the bornite in the experimental data has an effect on later calculations of material recovery, solid volume fraction and cumulative grade due to its greater copper content 63.3% compared to 34.6% in chalcopyrite. Differences between the experimental and simulated results are to be expected.

Here, the study of flotation performance down the bank is discussed in six subsections; mineral recovery, air recovery, froth recovery, entrainment factor, water recovery and flotation performance (grade and recovery). The mineral recovery dominated by the true flotation process is analysed first. The next two subsections describe the air recovery and froth recovery variation down the bank, which are the governing parameters of the true flotation recovery. The entrainment process
is responsible for the majority of the gangue recovered and it is explained through the analysis of the water recovery and entrainment factor down the bank. Last, the cumulative grade and recovery curves are plotted for the circuit and compared with experimental result.

3.5.1 Mineral recovery down the bank

The 10-cell Rs circuit is simulated using the three feed models (defined in Section 3.2). The results showing the cumulative amount of mineral recovered in the circuit are plotted in Fig. 3.16 together with experimental data.

![Graph showing mineral recovery down the bank](image)

**Figure 3.16:** Comparison of the cumulative mineral recovery between the 10-cell rougher circuits and the 8-cell rougher/scavenger bank from experimental data. 2PS, 4PS and 15PS are results from the modelled feed with different numbers of particle species considered, Exp is from experimental data.

The feed contains a decreasing amount of valuable mineral as it passes through the rougher bank, while the amount of gangue remaining in the streams has little change due to its large quantity. As a result, whilst the cumulative recovery increases down the bank, the rate at which it does so decreases. This is shown as the levelling of the increasing recovery of mineral down the bank in Fig. 3.16.

The perfect liberation of the mineral in the 2PS feed model increases its recovery early in the bank, which is shown as the elevated solid circular data compared to other results in Fig. 3.16. The inclusion of the middlings species reduces the recovery for the 4PS and 15PS models.
The experimental data shows the greatest amount of total mineral being recovered even with eight cells in the circuit compared to the ten-cell simulated results. However, the experimental data has the least amount of mineral recovered in the first cell, it is compensated for by the higher recovery down the bank. The high but slow (less recovery in the first cell) mineral recovery can be a result of assuming all mineral species in Northparkes experimental data are chalcopyrite, while in reality bornite is also present in the system of an unknown amount. The experimental result is measured in terms of copper recovered, so that the amount of mineral recovered is deduced from the all-chalcopyrite assumption. Having the same copper content in the concentrate, the absence of bornite can result in an overestimate of the mineral recovery. In addition, the bornite in the real feed can result in the slower recovery as seen in Fig. 3.16 if the rate constants of bornite particles are lower than chalcopyrite.

### 3.5.2 Air recovery

The flotation performance is closely linked to the air recovery of the cell. The air recovery is a function of the superficial air velocity \( v_g \), which considers the effects of the cell’s feed grade and average particle size. As \( v_g \) is kept the same in all cells, and the average particle size of the feed in each cell is dominated by the gangue species which shows little change in the circuit, the variation of the air recovery is therefore mainly caused by the deceasing feed grade down the bank. The recovery of mineral species lowers the mineral content remaining in the tailings streams, i.e. the feed grade down the bank. The lower the feed grade, the lower the froth stability, the smaller the air recovery. The air recovery is plotted against the cell number in Fig. 3.17, which also includes the averaged data from the plant survey.

The experimental data shows a relatively mild decrease of \( \alpha \) in the front 4 cells (the rougher bank) and low values in the rear 4 cells (the scavenger bank). Differences can be seen between the experimental data and the values from the three feed models, but the general magnitude and decreasing trend are comparable.
Chapter 3 - Flotation models and optimisation methods

3.5.3 Rate constant and froth recovery

More than 99.5% of the mineral recovered is through true flotation (see Fig. 3.18). It is important to understand this true flotation process which is governed by the flotation rate constant and froth recovery.

The rate constant is dependent on the degree of liberation, the diameter of the particle species and the bubble surface area flux $S_b$ (Eq. (3.11) in Subsection 3.1.2). $S_b$ is determined by the superficial air velocity $v_g$. Because there is no change in the
The froth recovery is the fraction of attached particles entering the froth phase that is recovered through true flotation. As mentioned in Subsection 2.3.2, a portion of particles becomes detached from the bubbles in the froth, but is carried in the up-flowing of the froth, and nevertheless reports to the concentrate, this portion is accounted in the froth recovery and true flotation. The froth recovery is a species specific variable, which means that every particle species in every feed model has its own froth recovery. To study the variation in froth recovery and later the entrainment factor down the bank, the mineral, middlings and gangue species of the most abundant particle size class of 10 – 50µm in the 15PS feed model. This size class accounts for 48.6% of the material in the feed.

The variation of the froth recoveries for the three species down the bank are shown in Fig. 3.19.

![Figure 3.19](image)

**Figure 3.19:** Comparison of the froth recovery down the bank for 10-cell rougher circuit between the 10 – 50µm species in 15PS feed model only

The froth recovery decreases down the bank which corresponds to experimental observations by Savassi (1998). The difference in $R_f$ between the species is minimal and is only caused by the density difference, the mineral particles are of 4500$kg m^{-3}$, the middlings are 3600$kg m^{-3}$ and the gangue is 3000$kg m^{-3}$. In addition to the air recovery, the froth recovery is also affected by the particles’ settling behaviour in the Plateau borders; the faster the settling velocity $v_{set}$, the smaller the $R_f$. The settling velocities of the species are shown in Fig. 3.20.
The settling rate increases with increasing particle size and density and decreases with increasing particle concentration. In this case, the sizes of the particles are unchanged within the 10 – 50µm size class, but as less mineral remains in the streams down the bank, the particle concentration in the Plateau borders decreases sequentially after each cell which results in the increase in $v_{set}$.

### 3.5.4 Solids volume fraction and water recovery

The particle concentration in the Plateau borders is represented as the volume fraction of all solid materials $\phi$, which is a fundamental property of the froth phase. $\phi$ is assumed to be the same in the Plateau borders, froth and concentrate (explained in Subsection 3.1.4). The variation of $\phi$ down the bank for 15PS is shown in Fig. 3.21 together with the other models and experimental data from Northparkes.

In Fig. 3.21, the values of the solids volume fraction decreases exponentially for all three simulated cases, which indicate that the rate of water recovery does not drop as fast as the solids recovery down the bank. The average results from the experiments showed a decrease in concentrate solids volume fraction down the rougher and scavenger banks. However the reduction in the experimental $\phi$ from the first rougher (42.9%) to the eighth scavenger cell (30.1%) was found to be only 12.8% compared to an average drop of 31.0% from the modelled 10-cell circuits.

The high solids volume fraction in the experimental data can be a result of
assuming all-chalcopyrite in the feed. As stated in Subsection 3.5.1, the assumption of the lack of bornite likely leads to an overestimate of the mineral recovery, which in turn leads to an overestimate of the solids volume fraction. However this difference in the solids volume fraction between the simulated results and experimental data does not affect the modelled recoveries, which in turn does not affect the reliability of the simulation.

The water recovery in the concentrate is used to calculate the solids volume fraction, and it directly affects the material recovery by entrainment. The amount of water recovered down the bank is shown in Fig. 3.22.
As less floatable material remains in the streams after each cell, the bubble loading down the bank is expected to decrease (Hadler et al., 2006). Decreasing bubble loading results in reduced bubble stability, and in turn encourages coalescence and increases the overflowing bubble size from cell to cell. Larger bubbles reduce the length of Plateau borders per volume of froth $\lambda$, which restricts the water recovery. Therefore, the amount of water recovered decreases down the bank as seen in Fig. 3.22 for all simulated results and experimental data. The fast mineral recovery in the 15PS feed model results in its low water recovery. Despite the difference in the solids volume fraction between the simulated results and experimental data, the water recoveries are of similar values and the same decreasing trend down the bank.

### 3.5.5 Gangue recovery down the bank

Having addressed the mineral recovery down the bank, it is important to analyse the amount of gangue recovered in the cells, which is used to determine the quality of the product, i.e. the concentrate grade. The cumulative amount of gangue recovered in the circuit is plotted in Fig. 3.23.

![Figure 3.23](image)

*Figure 3.23:* Comparison of the cumulative gangue recovery between the 10-cell rougher circuits and the 8-cell rougher/scavenger bank from experimental data

It should be noted that as with mineral and middlings particles, pure gangue species can be recovered by true flotation and entrainment. The rate constant of the pure gangue species is assumed to have the value of a species containing 0.01%
mineral. The amount of the pure gangue species recovered by true flotation is, therefore, insignificant compared to by entrainment.

The 2PS feed model has the lowest gangue recovery among the simulated circuits. Most of the mineral particles are recovered in the first cell, leaving a barren feed to the rest of the circuit. The depletion of the floatable material in the feed streams down the bank significantly reduces the froth stability (seen as a drop in air recovery in the second cell in Fig. 3.17). The depletion also reduces the solids volume fraction in the Plateau borders and increases the settling velocity. This high settling velocity, low air and water recovery after the first cell greatly restrict the entrainment process. At the same time, the lack of middlings results in 96% of the gangue particles in the concentrate being recovered by entrainment. Consequently, these combined effects lead to low and horizontal trend of gangue recovery of the 2PS model in Fig. 3.23.

Middlings species are present in the 4PS and 15PS models, which contain 50%wt of gangue material. The rate constants of the middlings species are high enough to enable a portion of the species to be recovered by true flotation. Although the gangue recovered as part of the middlings only contributes to 35% and 22% of the total gangue in the circuit concentrate for 4PS and 15PS models respectively, these floatable materials help to generate a more stable froth and relatively encourage the recovery by entrainment down the bank. As a result, 4PS (with highest middlings content) has the largest amount of gangue recovered followed by 15PS. The gangue recovery decreases down the bank for both models, it is also true for 2PS but less visible in Fig. 3.23.

In contrast to the simulated circuits, the experimental result of the gangue recovery is significantly lower. As stated previously, the lack of bornite leads to an overestimate of the mineral recovery, this also causes an underestimate of the gangue recovery. For example, a concentrate stream of 10t h\(^{-1}\) has a copper grade of 20%, if all copper bearing minerals are chalcopyrite, this concentrate will contain 5.8t h\(^{-1}\) chalcopyrite and 4.2t h\(^{-1}\) gangue; on the other hand, if all copper bearing minerals are bornite, this concentrate will contain only 3.2t h\(^{-1}\) bornite and 6.8t h\(^{-1}\) gangue. Therefore, it requires ore mineralogy details in order to have a better comparison for the gangue recovery in this case study.
3.5.6 Entrainment factor

Entrainment accounts for the majority of the pure gangue species recovered in the concentrate due to their low values of rate constant and the dominant gangue concentration in the pulp phase. The entrainment factors down the bank in shown in Fig. 3.24.

![Figure 3.24: Comparison of the entrainment factor down the bank for 10-cell rougher circuit between the 10 – 50µm species in 15PS feed model](image)

The entrainment factor depends heavily on the particle size in terms of the settling velocity as large particles are more difficult to be carried in the water channels of the Plateau borders. However in this case study, the 10 – 50µm size class in 15PS feed model is taken as an example to show the entrainment factors for mineral, middlings and gangue species down the bank, the species are of the same representative size of 22µm and do not vary from cell to cell. Therefore the variation of entrainment factor along the circuit is affected by parameters other than the particle size, such as solids volume fraction and air recovery.

In Fig. 3.24, for each particle species, the entrainment factor decreases down the bank as the air recovery and solids volume fraction decreases and settling velocity increases. The difference between the species in each cell is solely caused by the variation in their densities. The densest mineral species settle fastest and therefore have the lowest entrainment factor.

Despite having the same air flow to identical cells in the 10-cell rougher circuit, the depletion of the minerals down the bank plays a significant role in determining
and varying the key flotation variables. The froth recovery and entrainment factor of the same species decrease down stream as the conditions in the cells ‘deteriorates’ (air recovery and solids volume fraction decreases, particles settle faster). This shows the importance of including froth models that incorporate froth stability in the flotation models.

### 3.5.7 Grade and recovery down the bank

Finally, the flotation performance results (in terms of cumulative grade and recovery) of the three simulated circuits are shown in Fig. 3.25 together with experimental data.

![Comparison of the flotation performance (cumulative grade and recovery) between the 10-cell rougher circuits for the three feed models and the 8-cell rougher/scavenger bank from experimental data](image)

**Figure 3.25:** Comparison of the flotation performance (cumulative grade and recovery) between the 10-cell rougher circuits for the three feed models and the 8-cell rougher/scavenger bank from experimental data

In general, the rate of cumulative recovery increase decreases down the bank, as the amount of valuable mineral that remained in the feed decreases, while the abundance of gangue materials results in more gangue being recovered thus the decreasing the cumulative grade along the circuit. In Fig. 3.25, this observation is true in the case of the 4PS, 15PS feed models and the Northparkes data. However, it is unusual to see that the cumulative concentrate grade for 2PS increases down the bank. This is caused by the lack of middlings species in the feed model and less gangue entrainment as discussed in previous subsections.
The all-chalcopyrite assumption for the experimental data results in an overestimate of the mineral recovery and an underestimate of the gangue recovery, which in turn leads to the high grades seen in Fig. 3.25.

3.5.8 Summary

Due to the complex nature of the flotation process, it is difficult to construct simple models that capture the behaviour of the system, which require less computational power in the GA circuit optimisation while maintaining a high accuracy in simulating real flotation circuit.

The results of this basic circuit simulation have shown that the flotation performance is sensitive to the feed models used. It is shown that the presence of middlings species in 4PS 15PS feed models is essential in mitigating the differences between pure mineral and gangue species and providing a uniform flotation behaviour, it helps to produce a closer approximation to real flotation processes.

Down the 10-cell bank, the cumulative recovery increases, while the grade decreases as the feed grade to each subsequent cell decreases with an exception of the circuit using 2PS feed model. In the froth phase, both froth recovery and entrainment factor decrease down the bank, which are caused by the decreasing air recovery and increasing settling velocity. The differences in the flotation performance and froth behaviours between the simulated circuits are mainly the result of the variation in the amount of middlings in the feed models.

All copper bearing minerals are assumed to be chalcopyrite in the experimental data, whereas the real feed contained bornite. Due to lack of the ore mineralogy details, this assumption results in creating differences in material recovery, solid volume fraction and concentrate grades between the experimental data and simulated results.

The down the bank results have demonstrated the variation of key froth phase parameters from cell to cell. As the result, the flotation and feed models are considered adequate for the work on circuit design.

The proposed cell and circuit modelling methods can be used to simulate existing flotation circuits, which provide opportunities to optimise the operating conditions and improve flotation performance. The methods can also be used to
predict the separation and financial potential for a ore feed, which facilitate the

designing of new plants. However, in order for the cell and circuit modelling methods
to be transferred into industrial practice, extensive experimental work is required to
determine the parameters in the feed and flotation models, such as mineralogy of
the ore feed and the constants in flotation rates.
Chapter 4

Effect of circuit size and financial function on optimal layout and performance

Having established the circuit simulator and GA optimisation program, it is now possible to test the effect of variations in the design variables used in the models, such as circuit size and financial parameters, on circuit performance and optimal layouts.

The mineral separation achieved in a single cell is limited, the flotation performance can be generally improved by expanding the size of the circuit. This chapter starts with a study on optimising circuits with 3 to 10 cells. The sensitivity of the optimal layouts to feed models will be investigated.

During layout optimisation, circuits are optimised for producing concentrates of high financial value. It is therefore important to analyse the effect of the financial parameters on determining the optimal circuit layout. The flotation performance of the optimal and suboptimal circuit will be compared, and the balance between recovery and grade provided by the financial function will be studied.
4.1 Optimal circuits with increasing circuit size

Limited separation of mineral from gangue is achieved in a single cell, expanding a circuit by employing more cells generally improves the flotation performance, particularly with regards to recovery. On the other hand, installing more cells raises the capital expenditure, operating and maintenance costs. It is of interest to connect the additional units to the ‘best’ place, where they will provide the highest financial benefit together with the rest of the circuit, in order to establish whether the benefit is able to outweigh the costs.

Circuits consisting of 3 to 10 flotation cells have been optimised using the genetic algorithm for the three feed models. All feed models are based on the same industrial feed, which is based on that from Northparkes copper concentrator. As stated previously, the flowrates, density and the grade of the feed in each of the models remain identical, the only differences are the number of particle species modelled (based on the size and degree of liberation), and how the particles are distributed in various size class. The flotation performance shown as grade and recovery curves, of 3 to 10 cells for the 2PS, 4PS and 15PS models is given in Fig. 4.1.

![Figure 4.1: Comparison of the flotation performance for optimal 3 to 10-cell circuits between the three feed models](image)

As more flotation cells are introduced into the circuit, the grade and recovery for the optimal circuit layouts both increase regardless of the feed model used compared to the decreasing grade observed in the straight roughers circuits previously. For the 2PS model, the optimal circuits produce concentrates with considerably
higher grade and recovery compared to the other models. Additionally, the graph of $2PS$ presents a unique stepped trend where the recovery and grade rise alternately. For $4PS$ and $15PS$ circuits, the recoveries are relatively lower than $2PS$ due to the inclusion of the middlings in the feed. The flotation rate of the middlings species is lower compared to the pure mineral, such that a fraction of the middlings cannot be recovered to the concentrate, which limits the recovery of the circuits. Half of the material in the recovered middlings is gangue, and this also reduces the final grade for these circuits.

The step changes in the grade and recovery curves in Fig. 4.1 are indications of variations in the optimal circuit layout. The finance-based fitness function described in Subsection 3.4.4 is the mechanism that directly governs the optimal layout of a circuit, in which the grade and recovery are translated to the product value of the circuits through the use of the smelter contract. The step changes in the grade and recovery are levelled in the calculation of the product value, the graphs of which are plotted in Fig. 4.2 for the optimal circuits.

![Figure 4.2](image_url)  
**Figure 4.2:** Comparison of the product value for optimal 3 to 10-cell circuits between the three feed models

The circuit product value increases as the circuit size expands, and the trend gradually levels off for all three feed models. The smooth graphs in Fig. 4.2 illustrates that despite the differences in the grade and recovery between adjacent optimal circuits (the layouts of which can be different), the potential in the product value is limited and predictable by increasing the number of cells in a circuit.

However, it should be noted that the product is valued as dollar per tonne of Run of Mine, a marginal improvement in the product value often equals to significant
long-term financial benefit especially for circuits processing large quantities of ores, which in turn demonstrates the benefit circuit expansion and optimisation.

4.1.1 Optimal circuits found using $2PS$ model

From the flotation performance graphs (Fig. 4.1), 4 step changes are observed in the grade and recovery curve for the simple $2PS$ feed model. The steps corresponds to 4 different optimal layouts when the number of cells in the circuit increased from 3 to 10.

The initial optimal circuits of 3 and 4 cells are found to consist only of rougher cells that are connected in series (Fig. 4.3).

![Figure 4.3: Optimal 3 and 4-cell circuit layout, $Rs$, for $2PS$ model](image)

This straight bank of roughers is the simplest type of circuit layout, it is referred as roughers layout or ‘$Rs$’. $Rs$ designates the concentrate outflows from every cell to report to the final concentrate stream, which provides the maximum possible recovery for the circuit. This is shown previously in Fig. 3.25 and Fig. 4.1 in the graphs of $2PS$ model (note that Fig. 3.25 shows the cumulative grade and recovery for the 10-cell $Rs$ circuit, whereas Fig. 4.1 shows the ‘circuit-wise’ grade and recovery of difference optimal circuits consisting of 3 to 10 cells). The cells down the bank process the tailings of the previous unit to recover the mineral remained in the stream. The recovery of the first cell, Cell 1, is 83.1%, such that less than 17% of the mineral in the original feed is fed to Cell 2. After Cell 4, the fraction of copper in the circuit tailings contains is reduced to 4.75%, which is equivalent to a grade of 0.024%.

The decreasing feed grade in the stream discourages the roughers layout as the circuits expands. A penalty is paid for the amount of gangue material in the final concentrate product in the form of a refining charge $Rfc$. As a result, circuit designs are driven to generate layouts that are capable of producing concentrate of a higher grade with no or little reduction in mineral recovery. For optimal circuits of 5 and 6
cells, a cleaner cell is added instead in the circuits referred as rougher-cleaner layout or ‘RC’ (Fig. 4.4).

![Diagram of optimal 5 and 6-cell circuit layout, RC, for 2PS model](image)

**Figure 4.4:** Optimal 5 and 6-cell circuit layout, RC, for 2PS model

The concentrate outflows from the roughers are mixed and passed through the cleaner to remove the gangue that has been recovered through entrainment, and the tailings from the cleaner is recycled back to the head of the circuit to reduce the loss of mineral. The grade of the final concentrate from the cleaner of the 5-cell circuit increases by 2.40% compared to the 4-cell Rs circuit at the expense of only 0.02% drop in the recovery.

In the 2PS model, the combination of perfect liberation and the high rate constant for mineral species provides fast and high recovery through only a small number of cells. Therefore the aim of adding more cells is to improve the quality of the concentrate instead of the quantity, i.e. grade over recovery. As the circuit expands to 7 and 8 cells, the optimal layout adopts a scavenger-cleaner cell to further purify the final concentrate. This type of layout referred as rougher-cleaner-scavenger-cleaner layout or ‘RCCS’ (Fig. 4.5).

![Diagram of optimal 7 and 8-cell circuit layout, RCCS, for 2PS model](image)

**Figure 4.5:** Optimal 7 and 8-cell circuit layout, RCCS, for 2PS model

At the end of the rougher bank in the 7-cell circuit, Cell 7 processes the stream which has passed though 4 cells leaving a metal grade of 0.024%, consequently the flowrate of Cell 7’s concentrate is much less and of a lower grade than the roughers at the front of the circuit. To avoid reducing the grade of the final product, Cell 7’s concentrate is combined with the tailings from the cleaner Cell 2 and treated in
the scavenger-cleaner. This provides another 2.46% increase in the grade for 7-cell \( RCC_S \) compared to 6-cell \( RC \) circuit.

For optimal circuits of 9 and 10 cells, a re-cleaner cell is introduced to the circuits for treating the concentrate outflows from the cleaner and the scavenger-cleaner. Again, 9-cell \( RCC_S C_R \) circuit delivered a 2.40% increase in the grade compared to 8-cell \( RCC_S \).

![Diagram of circuit layout](image)

**Figure 4.6:** Optimal 9 and 10-cell circuit layout, \( RCC_S C_R \), for \( 2PS \) model

In summary, as the number of cells increases from 3 units to 10, the recovery and grade of the final concentrate increases alternately, and 4 types of layout are found to be optimal for \( 2PS \) feed model. Due to the ideal liberation assumed for the model, the mineral particles are quickly recovered through a bank of 3 to 5 roughers and the potential of further increasing the recovery by adding more rougher cells is limited. Therefore, the optimisation of the circuit layout for \( 2PS \) is grade improving oriented. The initial optimal \( Rs \) circuits progress to \( RC \), \( RCC_S \) and \( RCC_S C_R \) by mainly adding a cleaner cell in each stage. Within the stages, the optimal circuit expands by increasing the number of cells in the rougher bank.

### 4.1.2 Optimal circuits found using \( 4PS \) model

The same 3 to 10 cell circuit layout optimisation is performed using the \( 4PS \) feed model. In contrast to \( 2PS \), the feed contains particles of two size classes, fine and coarse, of a mass ratio of 1 : 2. To give comparability, the volume based average size in the \( 4PS \) model is the same as in \( 2PS \), but in the coarse class, the fast-floating pure mineral species is substituted with a middlings of 50\%wt in mineral. The
middlings species has a lower rate constant, and its inclusion in the feed significantly reduces recovery of circuits, such that circuit designs for 4PS are relatively recovery improving driven.

The optimal layouts start with a rougher bank that includes a recycle of the concentrate stream from Cell 2 for 3 and 4-cell circuits. This is referred as roughers-with-recycle layout or ‘R’ (Fig. 4.7).

**Figure 4.7:** Optimal 3 and 4-cell circuit layout, R, for 4PS model

Compared to Rs layout, having the concentrate from Cell 2 recycled to the head of the circuit in R trades 0.62% of recovery for 0.84% of grade. The small difference in the grade and recovery for the 3-cell R and Rs circuits gives similar product values: $24.19 \mathrm{t RoM}^{-1}$ and $24.05 \mathrm{t RoM}^{-1}$ respectively.

Unlike the unique case of 2PS, adding more roughers to the R circuit increases recovery but decreases the concentrate grade as seen in Fig. 3.25. This makes improving recovery though longer rougher banks less favourable, such that RC type of layout is found again to be optimal not only for 5 and 6-cell circuits but also up to circuits with 9 cells (Fig. 4.8).

**Figure 4.8:** Optimal 5 to 9-cell circuit layout, RC, for 4PS model

The benefit of having a single cleaner cell decreases as the circuit expands. The amount of mineral fed to the cleaner cell rises as more roughers are added to the circuit, however the performance of the cleaner is not directly proportional to the mineral contained in its feed. In other words, the increase in the recovery of the cleaner concentrate, i.e. the circuit concentrate, is less than the increase in the recovery of the rougher bank. This leads to the addition of a second cleaner for the
circuit containing 10 cells. The new layout is a rougher-cleaner-recleaner circuit or ‘RCC’ (Fig. 4.9).

![Diagram of RCC circuit layout](image)

**Figure 4.9:** Optimal 10-cell circuit layout, $RCC_R$, for 4PS model

The first cleaner is used to treat the concentrate outflows from the first two roughers and the recycled tailings from the re-cleaner, while the concentrate stream from the remaining roughers and the first cleaner are fed to the re-cleaner. This layout produces a concentrate recovering 86.57% of the mineral in the circuit feed, which is in fact the same as the optimal 9-cell $RC$ circuit. However, the extra $C_R$ cell installed and the slight rearrangement of the layout for the 10-cell circuit results in a 1.36% upgrade in the final concentrate grade.

Three types of layout are found to be optimal for the 4PS feed model. The optimal layouts progress from the roughers with recycle $R_R$ to the rougher-cleaner $RC$ layout after the circuit expands to contain more than 4 cells. $RC$ is found to be able to produce the optimal flotation performance for a large range of circuit sizes up to 9 cells, during which the mineral recoveries increase consistently. Compared to the optimal circuits with the 2PS feed model, the rate of mineral recovery is relatively slower in the rougher bank in 4PS, which results in improvements favouring recovery in the circuit optimisation process.

### 4.1.3 Optimal circuits found using 15PS model

The 15PS model describes the particle species through the modelling of particle distribution. This diversity provides a balanced flotation performance as the species of different liberation (the mineral, middlings and gangue) give fast to slow material recovery by true flotation, and the species of different particle size (representative size from 5 to 316µm) give a range of recoveries by entrainment. The
blend of particle species also signifies a greater potential to improve recovery during circuit optimisation, which is similar to the $4PS$ model.

The optimal layout for a 3-cell circuit is found to be the simple $Rs$ layout (Fig. 4.10).

![Figure 4.10: Optimal 3-cell circuit layout, $Rs$, for 15PS model](image)

The $Rs$ layout of 3-cell circuit provides high recovery, which also gives a relatively high gangue content due to the inclusion of particle species of small size class. Therefore a cleaner is added for the 4-cell optimal circuit changing to $RC$ layout (Fig. 4.11).

![Figure 4.11: Optimal 4 to 10-cell circuit layout, $RC$, for 15PS model](image)

In order to recover the more difficult species such as middlings particles with large diameter (low rate constant and high settling velocity), the $RC$ layout is found to be optimal even for the 10-cell circuit, utilising the additional roughers to improve recovery.

Only two layouts, $Rs$ and $RC$, are optimal for 3 to 10-cell circuits with the 15PS feed model. In the feed stream, the mass ratio of the liberated mineral to middlings species is 1.19 : 1 compared to 1 : 2 for $4PS$. More mineral species in the feed means higher recovery in the early concentrate streams due to their higher flotation rate constant, which gives a higher recovery throughout in the graph in Fig. 4.1. Fewer middlings particles means less gangue involuntarily recovered especially through true flotation, which gives a higher grade in the graph. However, this mass ratio of the liberated mineral to middlings species is unique to $P80$ of $78.9\mu m$ (average particle size of $50\mu m$), more middlings will be present in the feed if the $P80$ increases.
4.1.4 Summary

In conclusion, if there is more than one flotation cell in a circuit, the most simple arrangement is to connect cells in series forming a bank of cells. Adding a cell to a circuit to form a bank with an existing cell or to extend an existing bank, and removing a cell from a bank are considered as not changing the type of layout of the circuit. Adding or removing cells in this manner will improve or worsen the mineral recovery of the circuit without causing significant changes in the grade. However, alteration in the layout, such as adding a cleaner cell, can often be seen as a shift in the concentrate grade. In general, to improve recovery cells should be added to a bank, and to improve grade circuit layout should be modified to include cleaner cells.

Circuit layout optimisation is carried out on 3 to 10-cell circuits with the three feed models, $2PS$, $4PS$ and $15PS$. For smaller circuits, layouts consisting only of roughers are found optimal. Roughers ($Rs$) layout is able to produces concentrates of the highest possible mineral recovery, and is optimal for the 3 and 4-cell circuits with the $2PS$ feed model and the 3-cell circuit with $15PS$. Also consisted of only rougher cells, the roughers with recycle ($R_R$) layout provides highest product value for the 3 and 4-cell circuits with the $4PS$ model. As the circuit size expands, cleaner cells are employed to improve the concentrate grade. The roughers only layouts are overtaken by the combination of roughers and a single cleaner. This rougher-cleaner ($RC$) layout is the most common type of optimal layout, it is found to be optimal for the 5 and 6-cell circuits with $2PS$, 5 to 9-cell circuits with $4PS$ and 4 to 10-cell circuits with $15PS$ model. A second cleaner is seen in the 7 and 8-cell optimal $RCC_S$ circuits as a scavenger-cleaner with $2PS$, and in the 10-cell $RCC_R$ circuit as a re-cleaner with $4PS$ model. A third cleaner is added in the 10-cell optimal $RCC_SC_R$ circuit as a re-cleaner with $2PS$.

The optimisation of circuit layout is very sensitive to the feed model. The perfect liberation of mineral from gangue in the simple $2PS$ model provides fast recovery, which in turn limits the room for recovery improvement, such that during the circuit expansion from 3 to 10 cells, the optimal layout changed 3 times to upgrade the concentrate grade through adding cleaner cells. On the other hand, the complex $15PS$ model works well with the $RC$ layout, and changes in circuit size have little effect on the choice of the optimal layout.
4.2 Financial parameters and optimal circuit

During the circuit layout optimisation process, the performance of flotation circuits is measured by the grade and recovery of the concentrate product. A balance between the grade and recovery is provided by the finance-based fitness function, which is essentially a smelter return formula (described in Eq. (3.31) in Subsection 3.4.4 and shown below for quick reference). The circuit with the highest fitness value has the optimal layout. Consequently, the smelter return formula is most direct mechanism that decides which layout is optimal.

\[
Revenue_{\text{circuit}} = M_{\text{conc}} \ p \ (G_{\text{conc}} - u) \ (q - R_{fc}) - M_{\text{conc}} \ T_{rc}
\]

(3.31)

where \(M_{\text{conc}}\) is the circuit concentrate flowrate,

\(p\) is the fraction paid by smelter (97.5%),

\(G_{\text{conc}}\) is the concentrate metal grade,

\(u\) is the grade penalty (1.5%),

\(q\) is the metal price ($8000/t \ metal$),

\(R_{fc}\) is the refining charge ($60/t \ metal$),

\(T_{rc}\) is the treatment charge ($130/t \ concentrate$).

To maximise the \(Revenue_{\text{circuit}}\) for a circuit for fixed number of cells, the layout is altered so that a concentrate of the most profitable combination of grade and recovery can be produced. Therefore, it is of interest to see how grade and recovery is valued in the smelter return formula.

4.2.1 Valuation of grade and recovery

In the formula, the recovery of the metal is encouraged by the product of the first \(M_{\text{conc}}\) (the concentrate mass flowrate), the grade and copper price terms, and the recovery of the gangue is penalised by the product of the second \(M_{\text{conc}}\) and the treatment charge. Here, \(M_{\text{conc}}\) can be expressed in terms of the grade and recovery of the concentrate as shown in Eq. (4.1):

\[
M_{\text{conc}} = \frac{M_{\text{feed}} \ G_{\text{feed}} \ R_{\text{conc}}}{G_{\text{conc}}}
\]

(4.1)
where $G_{feed}$ and $R_{conc}$ are the feed grade and concentrate recovery respectively.

Substituting Eq. (4.1) into the smelter return formula, the product value of the concentrate $Value_{conc}$, dividing the revenue by the feed flowrate, is:

$$Value_{conc} = \frac{G_{feed} R_{conc} [p (G_{conc} - u) (q - Rfc) - Trc]}{G_{conc}}$$

(4.2)

Using this equation, a product value map is drawn in Fig. 4.12 for all the possible concentrate grade and recovery.

Figure 4.12: Effect of the concentrate grade and recovery on the product value

The negative values of the product value are reassigned to zero. The product value increases linearly with the recovery. From the graph, it shows that it is possible to evaluate the trade off for the grade and recovery for a specific product value. This is done by rearranging Eq. (4.2) and substituting in the values for the financial parameters such as $Rfc$ and $Trc$. The result is shown below in Eq. (4.3):

$$R_{conc} = \frac{Value_{conc} G_{conc}}{G_{feed} [p (G_{conc} - u) (q - Rfc) - Trc]}$$

(4.3)

Taking the 6-cell optimal RC circuit for the model 15PS from the previous section as an example, the circuit produces a concentrate of 20.96% copper grade, 86.98% recovery and a value of $28.56(t RoM)^{-1}$. To maintain the same product value,
value, the grade will need to increase 1.46% to compensate a 1% drop in recovery, the recovery, on the other hand, will only need to increase 0.785% to compensate a 1% drop in grade. Therefore for the default values of the financial parameter, the smelter return formula favours the concentrate recovery more than the grade.

To demonstrate the use of the smelter return formula in the optimisation process, the previously found optimal circuit layouts, $Rs$, $RC$ and $RCC_S$, are taken and simulated for circuits containing 3 to 10 cells with 15PS feed model. However, the cleaner cell in the circuit cannot be modelled until there is sufficient material in the feed to the cell, such that $RC$ circuits begin from 4 cells in the circuit and $RCC_S$ begin with 5 cells (at least 3 cells in the rougher bank, the concentrates of which are fed to the cleaner). The performance of these circuits are plotted in Fig. 4.13 and contour lines of equal product value are drawn for the optimal circuits (3-cell $Rs$ circuit and 4 to 10-cell $RC$ circuits).

![Figure 4.13: Flotation performance for 3 to 10-cell optimal and suboptimal $Rs$ (circular points), $RC$ (triangular points) and $RCC_S$ (diamond points) circuits with equal product value contours](image)

For circuits of the same number of cells (data points of the same colour in Fig. 4.13), there is no layout that has the grade and recovery both higher than the others. The equal product value contours drawn for the optimal circuits act as Pareto fronts, and it is now clear to see that the optimal changed from $Rs$ for 3-cell circuit to $RC$ for the rest. It is interesting to see that circuits of $Rs$ starts being the optimal layout for 3-cell circuit and gradually moves further away from the Pareto front as the circuit expands, in the meantime, $RCC_S$ moves closer to the
Pareto front produced by $RC$ layout and has the potential to overtake and become the optimal layout if the circuit continues to expand.

### 4.2.2 Effect of financial parameters

There are five financial parameters that govern the smelter return formula, and by increasing or decreasing the value of the parameters, the balance of the grade and the recovery can be shifted. An increase in the copper price, $q$, makes circuits with a higher recovery more favourable, while an increase in the treatment charge, $T_{rc}$, penalises the gangue content in the product and favours circuits with a higher grade.

6-cell $Rs$ and $RC$ circuits with the $15PS$ feed model are used as example. The ratio of the product values of the concentrates from the two circuits is plotted against the financial parameters which are varied between 20% and 180% of their original values in Fig. 4.14.

![Figure 4.14: Effect of financial parameters on layout selection for 6-cell circuit using 15PS feed model](image)

The copper price, grade penalty and treatment charge are the dominating financial parameters, and the effects of refining charge and percentage paid by the smelter are minimal. The value of the ratio remained less than 1 for the range of the financial parameter tested, which means that the $Rs$ layout produces concentrate of a lower value and stays suboptimal compared to $RC$. However, the test is done by varying the financial parameters independently, it is possible for $Rs$ layout to
become optimal if the copper price increases and the grade penalty and treatment charge decrease simultaneously.

### 4.2.3 Financial parameters and optimal circuits selection

From the previous subsection, it is found that varying the value of a financial parameter or a combination of parameters can result in changes in the optimal circuit layout. To investigate such layout change, the financial parameters are grouped so that the smelter return formula can be rearranged to:

\[
Revenue_{circuit} = M_{conc} (f_A G_{conc} - f_B) \quad (4.4)
\]

where \( f_A = p (q - Rfc) \) and \( f_B = u p (q - Rfc) + Trc \).

So that the ratio of the product values of the 6-cell \( Rs \) and \( RC \) circuits from previous example becomes:

\[
\frac{Revenue_{Rs}}{Revenue_{RC}} = \frac{M_{conc,Rs} (f_A G_{conc,Rs} - f_B)}{M_{conc,RC} (f_A G_{conc,RC} - f_B}) \quad (4.5)
\]

When the ratio is greater than 1, \( Rs \) layout is optimal, otherwise \( RC \) is optimal. As a result, critical values can be determined for the consolidated parameters \( f_A \) and \( f_B \) so that both layout are of equal product value and at the Pareto front in terms of the grade and recovery. This is expressed by Eq. (4.6):

\[
(f_B/f_A)_{critical} = \frac{M_{conc,Rs} G_{conc,Rs} - M_{conc,RC} G_{conc,RC}}{M_{conc,Rs} - M_{conc,RC}} \quad (4.6)
\]

The value of the right hand side of Eq. (4.6) depends only on the flotation process and does not vary with changes in the financial parameters. It gives the value of \((f_B/f_A)_{critical}\) which is calculated from the financial parameters \( q, p, u, Rfc \) and \( Trc \). The parameters can be of different sets of values but result in the same \( f_B/f_A \).

\((f_B/f_A)_{critical}\) is analysed for 3 to 10-cell circuit under \( Rs \) and \( RC \) as well as \( RCC_S \) layout, so that a ‘map’ of the layouts with highest product value for specific ranges of \( f_B/f_A \) can be created (Fig. 4.15).
Figure 4.15: 3 to 10-cell circuit layouts comparison with consolidated financial parameters

The original values of the financial parameters gives an $f_B/f_A$ of 0.032, which correspond to the result in Fig. 4.15 showing the change of the optimal $Rs$ 3-cell circuit to $RC$ 4 to 10-cell circuits. It is important to note that this method of using consolidated financial parameters to analyse circuits cannot be used to determine the optimal layout for a circuit, instead, it is used to compare layouts, to determine which is the ‘best’ among the candidates, and to illustrate their tolerance for changes in the smelter contract. In fact, $RCC_S$ layout is taken from results of previous optimisation using $2PS$ feed model, it has not been found as the optimal layout for the region showing in Fig. 4.15. However, $RCC_S$ layout produces a higher value concentrate compared to $RC$ for large circuit with high $f_B/f_A$.

4.2.4 Summary

The financial function provides the balance between grade and recovery, it is the most direct mechanism that affects the choice of the optimal circuit layout. Using the original values of the financial parameters, layout optimisations are relatively emphasised on recovery improving than grade. The study of such balance provides the grounds for drawing equal value contours on the grade and recovery graph, which helps to visualise the selection of optimal circuits.

Suboptimal circuits can become favourable by changing the values of one or more financial parameters. If the treatment charge or grade penalty increases, circuits that produce high grade concentrate such as rougher-cleaner becomes more
financially beneficial; on the other hand, high recovery circuits are preferred if the price of the metal is high.

The financial parameters can be consolidated into a single variable. The critical value of this variable, which is calculated based on the ratio of the flotation performance of two competing layouts, can be used to determine the optimal circuit. An area map of optimal circuits is drawn which shows the conditions when the roughers, rougher-cleaner and rougher-cleaner-scavenger-cleaner layouts are optimal. When the financial parameters are at their original values, it is found that roughers layout ($Rs$) is optimal for the 3-cell circuit, and rougher-cleaner layout ($RC$) is optimal for circuits consist of more than 4 cells.
Chapter 5

Effect of feed conditions on optimal circuit and performance

The importance of the feed model in determining optimal circuit layout and the performance of that layout have been shown, however it is also of interest to predict how a flotation cell and circuit respond to changes in the feed properties.

Flotation performance is governed by material recovery through true flotation and entrainment. Both pulp and froth phase behaviour are strongly dependent on feed properties such as particle size and metal grade. In this chapter, a range of particle sizes and feed grades will be used to test the sensitivity of the optimal circuit layout to these variables and to investigate further the effect of feed properties on the flotation process in a single cell.

The first part of this chapter will begin by discussing the effect of variations in the feed particle size on optimal circuit layout. The discussion will be followed by single-cell analyses of key variables, such as froth recovery and entrainment factor, to study the variation in flotation performance caused by changing particle size. In the second part, analysis is carried out to investigate the effect of feed grade on optimal layout and cell performance.
5.1 Sensitivity to variation in particle size

While the financial function directly influences the optimal circuit layout, it is the flotation models that determine the separation performance, which ultimately governs the financial function. The size of the particles in the feed is a key variable in the froth phase models as it plays a major role in determining the froth recovery and entrainment factor. The effect of the particle size on optimal circuits comprising 6 cells is studied to determine how it in turn affects the optimal circuit layout.

In order to provide comparability between the three feed models, the volume based average particle size $d_{p,\text{mean}}$ is used as benchmark. A single size class is present in the 2PS (two particle species) feed model, for which $d_{p,\text{mean}}$ is taken as the same as the average size; for 4PS and 15PS models, the average size is calculated as the sum of the product of the particle size of each species and its volumetric fraction (Eq. (5.1)).

$$d_{p,\text{mean}} = \frac{\sum (d_{p,x}Q_x)}{\sum Q_x} \quad (5.1)$$

In this case study, the volume based average particle size is varied from 25$\mu$m (50% of the base case value of 50$\mu$m) to 75$\mu$m (150%), and the effects of particle size variation on circuit and cell performance are investigated. Examples of $d_{p,\text{mean}}$ with the particle size settings for each feed model are demonstrated in Table 5.1.

Table 5.1: Average particle size variation and the corresponding sizes for each feed model

<table>
<thead>
<tr>
<th>Volume based average particle size</th>
<th>2PS: two particle species</th>
<th>4PS: four particle species</th>
<th>15PS: fifteen particle species</th>
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</thead>
<tbody>
<tr>
<td>(µm) of base case value</td>
<td>One size (µm)</td>
<td>Fine (µm)</td>
<td>Coarse (µm)</td>
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<td>75.0</td>
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<td>90.0</td>
</tr>
</tbody>
</table>

5.1.1 Optimal circuits for different average particle sizes

The layout optimisation is performed on circuits of 6 cells with a feed particle size from 25 to 75$\mu$m using the three different feed models. The performance of the
optimal circuits in terms of grade and recovery is plotted in Fig. 5.1.

The optimal circuits found for the base case, i.e. at particle size of 50µm, are of RC layout (Rougher-Cleaner layout for all feed models, see Fig. 5.3). These base-case optimisations are essentially repeats of the 6-cell optimisations in Section 4.1, and act as references for comparing the flotation performance with circuits at smaller or larger particle size. The result RC layouts are also the same as the ones that had been reported previously.

From the equations describing the flotation models in Section 3.1, the settling velocity of the solid particles in the Plateau Borders increases exponentially as particle size increases, therefore the larger the particle, the faster it drops out of the froth. This subsequently decreases froth recovery and especially entrainment. As a result, increasing particle size in general reduces the concentrate recovery but lifts the grade, which is seen in Fig. 5.1.

The graph of the 2PS feed model is segmented into three sections, while the trends of the graphs of 4PS and 15PS are continuous. The discontinuity in the 2PS graph, similar to the step changes seen in Section 4.1, indicates changes in the optimal layout. There are two different types of layout found to be optimal for the 2PS model, RR (Roughers with recycle, see Fig. 5.2) and RC (Fig. 5.3), when the feed particle size is varied from 25 to 75µm. The RR type of layout has been seen
earlier in Subsection 4.1.2, and $RC$ is the common optimal layout which is again optimal for $4PS$ and $15PS$ with all particle sizes.

**Figure 5.2:** $R_R$ type circuit layout for $2PS$ model with feed particle size of 25 to $37.5\mu m$

**Figure 5.3:** Optimal $RC$ type circuit layout for $2PS$ model with feed particle size larger than $40\mu m$ and $4PS$ and $4PS$ models with all particle size cases

However, in the graph of the $2PS$ model in Fig. 5.1, the separation of the middle section, for particle sizes of 40 to $47.5\mu m$, from the upper-left section, particle sizes of 50 to $75\mu m$, is not a result of a layout change, but is due to the froth recovery being limited to a maximum of 1 in the first rougher and cleaner cell. In these cells, the solid volume fraction in the Plateau Border is higher compared to other cells in the circuit due to a higher concentration of mineral particles in the feed stream, which results in low settling velocity. The high feed grade also contributes to a high air recovery, which together with low settling velocity gives a higher froth recovery. As a result, the gangue content in the concentrate is relatively higher for the $RC$
circuits with particle size of 40 to 47.5µm, and this is shown as the gap in the graph, while the concentrate grade is lower than would otherwise be expected.

The $R_R$ layouts (shown in Fig. 5.2) are found to be optimal for the lower-right section in the graph of the $2PS$ model (Fig. 5.1), for particle sizes of 25 to 37.5µm. In contrast to the other sections, the trend of the graph here is horizontal. As the feed particle size increases, the recovery decreases with minimal change in the concentrate grade.

There is subtle variation in the optimal layouts in the number of concentrate streams that are recycled to the head of the circuit for this section (25 to 37.5µm for the $2PS$ model), more recycling is favourable for circuits with a finer feed. The optimal layout for a circuit with a feed particle size of 25µm consists of 6 roughers, the concentrate of each cell is recycled to the head of the circuit, with the exception of the first rougher (shown in Fig. 5.2a). This layout is referred as $R_{R5}$ and is found to be optimal after the step increases in the feed particle size up to 35µm. At 35µm, the number of recycled concentrate streams is reduced to two (shown in Fig. 5.2b), and the layout is referred as $R_{R2}$. For a particle size up to 37.5µm, only the concentrate from the second rougher is recycled, leaving the rest exiting the circuit as outputs (shown in Fig. 5.2c), this is referred as $R_{R1}$.

To study the reason for these changes in the optimal layout, the three $R_R$ layouts are tested using the $2PS$ feed model and a particle size of 25 to 40µm. The resulting grade and recovery is plotted in Fig. 5.4.

![Figure 5.4: Comparison of the flotation performance between $2PS$ $R_R$ circuits with feed particle size from 25 to 47.5µm](image-url)
From Fig. 5.4, as the particle size increases, it is clear to see that the concentrate grade for the $R_{R5}$ layout is overtaken by $R_{R2}$ and then $R_{R1}$ at 35 and 37.5 $\mu$m respectively. The increasing trend of the concentrate grade before levelling off for $R_{R2}$ and $R_{R1}$ is again caused by the froth recovery limit in the first rougher. When the particle size is smaller than 35 $\mu$m in $R_{R2}$ and 37.5 $\mu$m in $R_{R1}$, the froth recovery remains at the maximum value of 1 for the mineral species in the first cell. Starting from 25 $\mu$m, as the particle size increases, the gangue entrainment decreases, but there is little change in the minerals recovered through true flotation, this results in the increase in the concentrate grade. After the particle size reaches 35 $\mu$m in $R_{R2}$ and 37.5 $\mu$m in $R_{R1}$, the froth recovery starts to decrease along with the entrainment which provides relatively stable grade, but lower recoveries.

In Fig. 5.4 as well as Fig. 5.1, the difference in particle size between adjacent points is 2.5 $\mu$m. If this step difference is small enough, it may be possible for circuits with different numbers of recycled concentrate streams, such as $R_{R4}$ and $R_{R3}$, to be found as optimal layouts. Despite the difference in the concentrate grade among the $R_R$ circuits, the recoveries are similar for cases with the same feed particle size.

For circuits having feed particle size equal or larger than 40 $\mu$m based on the 2PS model, or circuits using the 4PS and 15PS models, the RC layout is found to be optimal. Smooth graphs are observed in Fig. 5.1, which show the mineral recovery is traded off with concentrate grade as the feed particle size increases.

In general, although the concentrate grade increases as the average feed particle size increases, the decrease in the recovery is more prominent in terms of the product value of the circuit concentrate for all feed models. This is shown in Fig. 5.5.

The circuits using 2PS feed model have the concentrates of the highest product value due to the assumption of the perfect liberation of the minerals from gangue. The 15PS feed model has a higher mineral to middlings ratio compared to 4PS which results in a higher product value, but it should be noted that this ratio is governed by the particle distribution, i.e. the value of $P_{80}$, and decreases as the average feed particle size increases. This is shown in the graph as the difference in product value between 4PS and 15PS becomes smaller.

In summary, as the feed particle size increases, the mineral recovery decreases and concentrate grade increases due to reductions in froth recovery and gangue
entrainment. This is shown as smooth decline in the concentrate product value for all 3 feed models. For the feed particle sizes tested, the optimal layout is dominated by the $RC$ type. For relatively small feed particle sizes with the $2PS$ model, the $RR$ layouts are optimal. Though the flotation performance is sensitive to the feed models (the effect of mineral liberation is significant and affects the recovery of the circuits), the type of the optimal layout is resilient to changes in the feed particle size for the cases tested.

### 5.1.2 Effect of average particle size on cell performance

Having observed the effect of particle size on the circuit performance and optimal layout behaviours, it is of interest to study how a single cell reacts to the variation in feed particle size.

Flotation performance for a single cell for the three feed models is plotted in Fig. 5.6, showing the effect of particle size on cell grade and recovery. The particle size range is 25 to 75$\mu m$, and the difference between two adjacent points is 2.5$\mu m$ in the average particle size. The hollowed points are of the base case of 50$\mu m$.

For all feed model cases, the concentrate grade increases with the average particle size while the recovery decreases with size in general. The recoveries and grades of the cells with $2PS$ model are the highest followed by $15PS$ and $4PS$ mainly due the differences in the mineral liberation. More interestingly, $2PS$ and
**Chapter 5 - Effect of feed conditions**

15%  
17%  
19%  
21%  
23%  
50%  
60%  
70%  
80%  
90%  

**Figure 5.6:** Comparison of the flotation performance in a single cell with feed particle size from 25 to 75 µm between the three feed models. 4PS present distinctive horn shaped graphs. These unexpected shapes are due to the presence of a peak in the mineral recovery, which is discussed in later subsections. For a clearer representation, the effects of particle size on the mineral recovery and concentrate grade are analysed separately in Fig. 5.7.

4PS present distinctive horn shaped graphs. These unexpected shapes are due to the presence of a peak in the mineral recovery, which is discussed in later subsections. For a clearer representation, the effects of particle size on the mineral recovery and concentrate grade are analysed separately in Fig. 5.7.

**Figure 5.7:** Comparison of the recovery and the grade in a single cell between the three feed models. Solid points are recoveries, hollow points are grades.

In Fig. 5.7, the data with the solid points are mineral recoveries (on the left-hand axis) for the three feed models, which are plotted against average particle size. The shape of the curves is governed by the recovery of the mineral and middlings through true flotation, and more than 99% of the mineral content in the concentrate
is recovered this way for all feed models. The peaks in the recovery seen in Fig. 5.6 are at 37.5 µm for 2PS model and at 40 µm for 4PS.

The hollowed points show the concentrate grade (on the right-hand axis). The shape of the curves is mainly governed by the entrainment of the gangue. The gangue recovered by entrainment accounts for 68% of the gangue content in the concentrate for the 4PS model. Compared to true flotation, the lower percentage of gangue recovery through entrainment is due to the amount of gangue involuntarily recovered as part of middlings through true flotation. Having less middlings in the feed (the mineral to middlings flowrate ratio is constant 1:2 in 4PS, and 2.2:1 to 0.83:1 with average particle size of 25 to 75 µm in 15PS model), the percentage is increased to 75% for 15PS model. As the 2PS model lacks of middling class, the percentage of gangue recovered through entrainment accounts for 97% of the gangue in the concentrate for this feed model.

The shapes of the grade and recovery curves in Fig. 5.7 are also determined by the solid volume fraction in the Plateau borders. The solid volume fraction is at the centre of the flotation models (it governs various properties of the froth phase and the concentrate, such as the settling velocity of the particles and the volumetric flowrate of the water recovered in the concentrate). The effect of the volume based average particle size on the solid volume fraction is shown in Fig. 5.8.

![Figure 5.8: Comparison of the effect of particle size on solid volume fraction in a single cell for the three feed models](image)

The graphs of solid volume fraction in Fig. 5.8 share strong similarities with the recovery graphs and the mirror images of the grade graphs in Fig. 5.7. The
smooth trend of 15PS model is a result of having fixed particle size classes and shifting the particle distribution instead of altering the particle size of each species.

In order to study the impact of the different methods of modelling the feed stream on the froth behaviour, the effect of the particle size on the flotation performance for each feed model is analysed individually.

**Effect of the feed particle size in 2PS feed model**

Only pure mineral and pure gangue species of the same size are assumed in the feed for the 2PS model. The froth recovery and entrainment factor of the two species are plotted in Fig. 5.9 for the average feed particle size range of 25 to 75µm.

![Figure 5.9: The effects of average particle size on froth recovery and entrainment factor in a single cell for 2PS](image)

The entrainment factor for both mineral and gangue follows a relatively mild logarithmic decrease as the particle size increases. However, the value of the froth recovery is restricted at the maximum of 1 for mineral particles smaller than 35µm and gangue smaller than 45µm, and the value decreases for particle species with larger diameter. This restriction is seen in the first cell in $R_R$ circuits in Subsection 5.1.2 and earlier in Subsection 3.5.6. In the froth-phase flotation models, the froth recovery is inversely related to the settling velocity of the particles in the froth, which is in turn inversely related to the particle size. The froth recovery is the fraction of the material that enters the froth attached to the bubbles that reports to the concentrate either attached or detached, thus the value of which is limited to 1. As
a result, the froth recovery is 1 for small particle sizes then decreases as the particle size increases.

The peak in the mineral recovery observed for the $2PS$ feed model in Fig. 5.7 is caused by the combination of the irregular trend of the froth recovery and the flotation rate constant of the mineral species. As described in Eq. (3.3) in Sub-section 3.1.1, the mineral recovered in the concentrate through true flotation is calculated as the product of the cell volume, rate constant, froth recovery and concentration of the particle species in the tailings. The froth recovery of the mineral particles shown in Fig. 5.9 is plotted together with the rate constant in this feed model in Fig. 5.10.

\[
\text{Figure 5.10: Comparison of the effect of particle size on flotation rate constant and froth recovery for fully liberated mineral particle in } 2PS \text{ model}
\]

The rate constant is determined using the flotation kinetics model, it strongly depends on the degree of mineral liberation and the particle size. In this case where perfect liberation is assumed, the flotation rate constant is only influenced by the size of the particles, and it reaches a maximum at $88 \mu m$. Therefore, for the size range of 25 to $75 \mu m$ in this test, the rate constant increases steadily with the feed particle size as shown in Fig. 5.10.

The mineral recovery is governed by the product of the rate constant and froth recovery of the mineral particles. As shown in Fig. 5.10, having the froth recovery retained at 1 for small particle sizes gives a maximum in the product of the two parameters. This maximum is at particle size of $37.5 \mu m$, and corresponds
to the peak in the 2PS model recovery graph in Fig. 5.7 and the turning point in the grade and recovery graph in Fig. 5.6. The presence of the peak suggests that it is possible to have an optimal particle size for the feed (produce a high value concentrate) at which the value of froth recovery is close to 1 and the rate constant is relatively high, assuming the flotation kinetics model (Subsection 3.1.2) and air recovery model (Subsection 3.1.3) are a good representations of real systems.

**Effect of the feed particle size in 4PS feed model**

In the case of the 4PS model, the volume based average particle size is calculated to provide comparability with 2PS. It averages the sizes in the fine (pure gangue and fully liberated mineral particles) and coarse classes (pure gangue particles and middlings), such that at the lower boundary of the size range studied, 25\(\mu m\), the particles are of 15 and 30\(\mu m\) in the fine and coarse class respectively.

Having the mineral species at smaller particle size results in the froth recovery remaining at the maximum of 1 for the species compared to the coarser middlings. This difference in the froth recovery between the mineral and middlings species is reflected in the shape of the mineral recovery for 4PS in Fig. 5.7 (solid triangular points).

In the graph, the mineral recovery increases with the average particle size when the average size of the feed is smaller than 37.5\(\mu m\). In these fine feed cases, the rate constant is the only variable which increases with the particle size, as the froth recoveries for both mineral and middlings particles remain at 1. The froth recovery of the middlings species starts decreasing as the feed particle size increases from 37.5 to 60\(\mu m\), while the froth recovery of the finer mineral particles remains at 1. This counteracts the increasing value of the rate constant and thus gives a relatively level graph for the mineral recovery. For cases of average particle size of 60 to 75\(\mu m\), the froth recoveries for both mineral and middlings species decrease with increasing size. This dominates the trend of the graph in spite of the increasing rate constant, and results in a reduction in the overall recovery in the concentrate.

The shape of the graph for recovery also depends on the distribution of particles between the fine and coarse classes. The original feed stream in the 4PS model is assumed to contain equal amounts of mineral in fine and coarse classes. As the mineral content in the middlings is 50\%wt, the flowrate of the coarse middlings is
twice as much as of the fine mineral (denoted as $F:C$, which is 1:2). The maximum in mineral recovery lies at $37.5\mu m$ where the froth recovery for the coarse particles starts to decrease from 1. However, if the particle distribution changes and the mineral in the coarse class is considerably less than the mineral in the fine class, it is possible for the overall recovery to continue to increase until the both froth recoveries of mineral and middlings decrease with increasing particle size, which is at $60\mu m$. Therefore, the maximum in recovery can move within the particle size range of $37.5$ to $60\mu m$ when the particle distribution is varied. Three cases of feed with different fine to coarse ratio are tested including the original feed with $F:C$ of 1:2. The single cell recovery is plotted against the average feed particle size for the three cases and is shown in Fig. 5.11.

![Figure 5.11](image.png)

**Figure 5.11:** Comparison of the effect of particle size on the recovery in a single cell for fine to coarse class mass ratio of 1:2, 1:1 and 3:2 in the feed in 4PS model.

The additional two cases have $F:C$ of 1:1 and 3:2, of which the amounts of mineral in fine and coarse classes are no longer equal but of ratios of 2:1 and 3:1 respectively. However it should be noted that the overall amount of mineral in the feed and the feed grade are the same in all cases, only the particle distribution is varied. From Fig. 5.11, as more mineral content is distributed in the fine mineral species, the recovery increases significantly. In addition, the maximum in the recovery moves from $37.5\mu m$ for the original feed which has $F:C$ of 1:2; it passes through $50\mu m$ when less mineral content is in the form of middlings but more is in pure mineral as $F:C$ increases to 1 : 1; and the feed of average particle size of $60\mu m$ recovers the most mineral when $F:C$ equals to 3 : 2. The changes in the material distribution can be interpreted as variations in the comminution process and possible mineral grain
sizes in the ore. The more material in the fine class, the better the mineral liberation.

Compared to $2PS$ feed model, $4PS$ introduces a coarse particle size class, which only contains a middlings species and a gangue species. The inclusion of the middlings results in a lower recovery due to a loss of some of the middlings by its lower rate constant and a lower grade due the gangue recovered involuntarily as part of the middlings. The flotation performance also depends on the particle distribution; more pure mineral and less middlings improves the recovery generally, and the shape of the recovery graph also depends on the amount of material in the fine and coarse classes.

**Effect of the feed particle size in $15PS$ feed model**

The maximum in the mineral recovery is not observed in the $15PS$ model. As $15PS$ has a unique system of describing the materials in the feed stream, it is fundamentally different from the other two models. The use of the Rosin-Rammler particle size distribution model in $15PS$ has an important influence on the flotation performance of the cell, and this is the underlying reason for producing the relatively smooth and uniform grade and recovery graphs. When the average particle size is varied from 25 to $75\mu m$, instead of altering the actual size of each particle species as for in $2PS$ and $4PS$, the value of $P_{80}$ (80% of particle is finer than the specified size) is adjusted accordingly.

Adjustments to $P_{80}$ (to achieve 25 to $75\mu m$ variation in the average particle size) varies the distribution of the particles among the existing size classes while keeping the particle size of the classes fixed (see Fig. 5.12).

Compared to $2PS$ and $4PS$, shifting the particle distribution in the size classes instead of the particle size itself affects the flotation performance differently. From the example given in Fig. 5.9, it is clear to conclude that the particle size has significant impact on the froth recovery and entrainment factor, such that having an ideal situation (fully liberated mineral and gangue with uniform particle size) as in the $2PS$ model is essential for studying a theoretical flotation behaviour. However, the results are less reliable if the aim is to mimic an industrial flotation process. In contrast, the shifting of the particle distribution can be used to accurately reflect
the properties of the feed, which improves the reliability of the model but at the expense of increased complexity and needs of computational power.

In addition, in order to maintain the constant mineral grade of 0.5% within the size classes, the mass fraction of particles in each liberation type is invariant regardless of the change in the value of \( P_{80} \). As a result, shifting of the particle distribution involuntarily changes the ratio of mineral, middlings and gangue particles that is feeding to the circuit Fig. 5.13.

The variation of the feed mass flowrate of the gangue for different average particle size can be ignored (and is not plotted). The mineral particles recovered in
the concentrate is proportional to the feed flowrate of mineral which decreases as the particle size increases. However it is very interesting to see that although the feed flowrate of the middlings is doubled when the average size increased from 25 to 75µm, there is little change in the recovered middlings. This stable middlings recovery, together with the similar decreasing mineral and gangue flow in the concentrate results in the relatively unchanging grade graph in Fig. 5.7.

Compared to the other feed models, it is interesting to see the lack of the maximum in the mineral recovery, the maximum is caused by the change in the product of the flotation rate constant and the froth recovery when the particle size is varied. However, the representative sizes of the particle classes do not vary with the particle distribution and the average particle size (P80) in the 15PS model, such that each size class has a constant value for the rate constant. However the froth recovery does decrease with increasing feed particle size. Although froth recovery is a particle species specific variable (unique to each size and liberation class), it is dependent on the overall froth behaviour. Changes in particle distribution affect the density, viscosity, and solid volume fraction of the slurry in the froth, which in turn affect the settling velocity, froth recovery and entrainment factor. However, the variation in these variables caused by the changes in particle distribution is relatively small compared to those caused by the changes in the particle size in 2PS and 4PS model (when the average particle size in the feed varied from 25 to 75µm, the froth recovery for the mineral species in 50 – 100µm size class in 15PS model decreased from 0.49 to 0.36, while in the 2PS model the value decreased from 1 to 0.43). In addition, the 0 – 10µm and 10 – 50µm size classes have small representative sizes of 5 and 22.4µm respectively, this results in having the froth recoveries at the maximum value of 1 regardless of the changes in the particle distribution. As a result, the combination of the constant rate constant and steady froth recovery gives the smooth grade and recovery graph in Fig. 5.6.

Effect of the feed particle size on product value

To balance the grade and recovery of a flotation process, the smelter return formula is used to evaluate the performance and to provide a single value to determine the optimal values of the design variables, such as average particle size in this case. However the financial function does not consider any operating cost of the processes of milling, separation and flotation, which can dominate the potential
profit of selling the concentrate. In addition, as explained in Section 4.2, the smelter return formula favours the recovery over the grade. This can be seen in the graph of 4PS model in Fig. 5.14, where the product value decreases as the size increases for a feed particle size larger than 37.5\( \mu m \) similar to the corresponding recovery graph in Fig. 5.7, despite the increasing grade within the same section.

![Figure 5.14: Comparison of the effect of particle size on product value in a single cell between the three feed models](image)

The lack of middling classes in 2PS model provides a higher possibility for recovering minerals and discarding gangue in the flotation process. As only fully liberated mineral particles are present which have high rate constant, there is better mineral recovery through true flotation and less gangue entrainment. In contrast, cells using 4PS and 15PS feed model produce a concentrate with a lower value. The shape of the product value graph is mainly dominated by the recovery of the mineral through true flotation, which gives the trend for the 2PS and 4PS models.

### 5.1.3 Summary

Although the three feed models are based on the same feed to the flotation circuit at Northparkes Mine, the results from the three feed models show significant differences in the concentrate grade and recovery in a single cell and in optimal circuits when the average particles size is used as the reference and varied from 25 to 75\( \mu m \). In general, the recovery decreases and grade increases as the average particle size increases, and a finer feed produces a higher value concentrate.
6-cell circuits using 2PS are observed to produce both the highest grade and highest recovery concentrates; the inclusion of the middlings in 4PS and 15PS models results in drops in the mineral recovery and concentrate grade. Two circuit layouts, \( R_R \) and \( R_C \), are found to be optimal for the particle size range tested. For a feed with relatively small particle size using 2PS model, the recovery of the optimal \( R_R \) circuit decreases with increasing size and the change in grade is minimal. For a feed with larger particle size using 2PS model and with all sizes tested using 4PS and 15PS models, the concentrate grade of the optimal \( RC \) circuit increases with increasing feed particle size (a result of less gangue entrainment) at the expense of recovery (a result of lower froth recovery).

Similarly, in general the grade increases with particle size in a single cell, and the recovery decreases. However, the effect of particle size on the froth recovery is more prominent and plays a major role in determining the flotation performance. In cases of 2PS and 4PS models, the froth recovery remains at the maximum value of 1 for feed with small particle sizes, as the rate constant is proportional to particle size, this results in improvements in recovery as the particle size increases. After reaching a maximum value in the recovery, the froth recovery starts to decrease with increasing particle size, which gives the trend seen in the 15PS model. This behaviour of the froth recovery with variation particle size is comparable to the experimental results reported in literature (Rahman et al., 2012).

5.2 Sensitivity to variation in feed metal grade

Grade variation in ore bodies is common; often ores from more than one stockpile are mixed together in an effort to minimise fluctuations in the feed grade in the flotation process. Therefore it is important to understand the effect of feed grade variation on the flotation performance and circuit design.

Similar to the previous section, the base case feed grade of 0.5%Cu is varied from 0.1% to 1.0%Cu (20% to 200% of the base case) while keeping the average particle size of the feed constant (50\( \mu m \)), and the effects of feed grade variation on a 6-cell circuit and single cell performance are investigated.

In this study, it is worth noting that in the 4PS and 15PS models, as materials are sorted into two and five size classes respectively, the metal content in the feed
is assumed to be equally spread in each size class. In other words, there is an equal amount of copper in the fine and coarse particle class in the 4PS model, and an equal amount of copper in each of the five particle classes in the 15PS model. Moreover, within each size class in the 15PS model, the metal content is divided between the mineral and middlings species, the ratio of the division is drawn up arbitrarily which relates to the particle size of the class. In the smallest size class of $0 - 10\mu m$, the middlings species is absent which gives a copper mass ratio of $5 : 0$ for the mineral to middlings species. The liberation is assumed to decrease as particle size increases, as a result, there is more middlings and the copper mass ratio decreases to $4 : 1$ in the $10 - 50\mu m$ class. The copper ratio in the mineral to in the middlings species is $3 : 2$ for $50 - 100\mu m$ class, $2 : 3$ for $100 - 200\mu m$ class and finally $1 : 4$ for $200 - 500\mu m$ class (see Table 5.2).

**Table 5.2:** Feed grade variation and the corresponding grades for each feed model

<table>
<thead>
<tr>
<th>Feed metal grade</th>
<th>Metal in 4PS ($t \cdot h^{-1}$)</th>
<th>Metal in 15PS ($t \cdot h^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1% Cu</td>
<td>20%</td>
<td>0.2</td>
</tr>
<tr>
<td>0.5% Cu</td>
<td>100%</td>
<td>1</td>
</tr>
<tr>
<td>1.0% Cu</td>
<td>200%</td>
<td>2</td>
</tr>
</tbody>
</table>

As only two particle species, pure mineral and gangue, of the same size are present in the 2PS model, the amount of the mineral species in the feed responds directly to the variation in the feed grade.

Furthermore, air recovery is assumed to be independent of the circuit feed grade variation between the tests. As the effect of feed grade on air recovery is defined as the ratio of cell feed grade to the circuit, this effect only accounts for the changes in feed grade within a circuit (such as decreases in cell feed grade down a bank) but not the circuit feed grade. As circuit feed grade cannot be controlled easily, experimental analysis linking feed grade to air recovery has not yet been carried out, and this assumption has to be made.

### 5.2.1 Optimal circuits for different feed grade

Circuit layout optimisation is performed on 6-cell circuits with feed grades ranging from 0.1% to 1%Cu using the three feed models. A constant feed particle
size of 50µm is used. The flotation performance of the optimal circuits is plotted in Fig. 5.15. The conditions in the base cases for all the feed models remain unchanged and are of 0.5%Cu feed grade.

![Figure 5.15: Comparison of the flotation performance for optimal 6-cell circuits with feed grade from 0.1% to 1%Cu between the three feed models (the difference in feed grade is 0.05% between adjacent points, and the hollowed ones are of the base case 0.5%)](image)

The difference in the shapes of the grade and recovery graphs between the optimal circuits of different feed models is profound. The recovery increases with the feed grade in general.

![Figure 5.16: Comparison of the concentrate grade for optimal 6-cell circuits between the three feed models](image)

On the other hand, as the feed grade increases, the circuit concentrate grade
(re-plotted against feed grade in Fig. 5.16) is found to decrease in cases using the 2PS and 4PS models, however, the opposite trend is observed for the 15PS model.

The discontinuity in the trend of the graph in Fig. 5.16 indicates the alterations in the layout, such that three different optimal layout are found optimal for circuits using 2PS model. For circuits with feed grades less than 0.15% (the leftmost two data points), the roughers only Rs layout (Fig. 5.17) is able to recover a larger amount of mineral which in turn is found to be optimal.

![Figure 5.17: Optimal Rs type circuit layout for 2PS model with feed grade less than 0.15% Cu](image)

As the feed grade increases up to 0.5% Cu, continuing to use the Rs layout would result in an increase in the recovery but a significant decrease in grade, this prompts the optimal layout to switch to the commonly found optimal RC layout (Fig. 5.18). This RC layout is also optimal for circuits using 4PS and 15PS models for all feed grades tested, such that there is no layout change observed for the complex feed models.

![Figure 5.18: Optimal RC type circuit layout for 2PS model with feed grade between 0.2% and 0.5% Cu](image)

A rougher-cleaner-scavenger-cleaner layout or ‘RCCs’ layout is found optimal for circuits with feed grade of 0.55% and above (see Fig. 5.19, this type of optimal layout was first seen in previous Subsection 4.1.1) for the 2PS model. The last cell in the rougher bank of the previous RC layout is rearranged into a scavenger-cleaner cell to improve the circuit concentrate grade.

The strong effect of feed grade on circuit concentrate grade causes the changes in the optimal layout of the 2PS model. Cleaners are introduced to the circuits.
as feed grade increases to counteract the drops in the concentrate grade. However, despite the fluctuation in the product grades and the changes in the layout, the product value, which acts as a balance between grade and recovery, is seen to increase steadily with feed grade for all three feed models as shown in Fig. 5.20.

The increasing trend of the product value is dominated by the additional metal content present in the feed as the feed grade increases. In comparison, the effect of the differences in the grade and recovery between the feed model has minimal impact on the financial valuation of the circuit performance.

In summary, as the feed metal grade increases, the mineral recovery increases and concentrate grade varies differently according to the feed model and the circuit layout used. Changes in the optimal layout is observed only with 2PS model. Perfect liberation of the mineral from the gangue in this feed model gives rapid
mineral recovery. This results in limited room of improvement in recovery and thus encourages changes in optimal layout. The roughers-only layout is found optimal for circuits with relatively low feed grade. As the feed grade increase, the rougher-cleaner layout and rougher-cleaner-scavenger-cleaner layout become optimal. The rougher-cleaner layout is found optimal throughout the feed grade variation for 4PS and 15PS feed models.

### 5.2.2 Effect of feed metal grade on cell performance

A study of the flotation performance in a single cell is able to show localised effects of the variation of feed grade, which help to understand the trends that are seen in the previous Subsection 5.2.1. The effect of feed grade on cell grade and recovery for the three feed models is shown in Fig. 5.21.

![Figure 5.21: Comparison of the flotation performance in a single cell with feed metal grade from 0.1% to 1% between the three feed models](image)

In this case study, the same variation in the feed grade, from 0.1% to 1%Cu, is used, and the difference between two adjacent points is 0.5%Cu. The average particle size of the feed is 50µm unvaried. The hollowed points are of the base case of 0.05%. It should be noted that as this study is for a single cell, the effect of feed grade on air recovery, which is defined as the ratio of cell feed grade to the circuit, is not taken into account ($f_{FG}$ is kept at 1). In real system, feed grade will affect air recovery, however due to lack of experimental data, it is not possible to model this.
For all feed models, the recovery is dominated by the froth recovery, which increases steadily as the feed grade improves from 0.1% Cu. However, the rate of increase in the froth recovery is dependent on the particle size of the species. For smaller particles such as 30 to 60 \( \mu \)m species in 2PS and 4PS models, the rate of increase is fast and the froth recovery reaches the maximum value around the feed grade of 0.6% Cu. This is shown on the right hand side of the graphs of 2PS and 4PS models in Fig. 5.21. In contrast, the diversity of particle sizes in 15PS model helps to increase the recovery continuously across the whole feed grade range.

The recovery is also affected by the amount of floatable materials in the system. As more floatable materials are in the system when the feed grade increases, the froth becomes more stable by having more particles entering through true flotation, which in turn enhances the recovery. The froth stability is reflected by the solids volume fraction in the concentrate as shown in Fig. 5.22.

![Figure 5.22: Comparison of the effect of feed grade on solid volume fraction in a single cell for the three feed models](image)

In general, the concentrate grade increases as the feed grade increases. The concentrate grade is determined by the amount of gangue recovered, the majority of which is from entrainment. When the feed grade increases, material recovery by both true flotation and entrainment improves, however the former is of greater quantity compared to the latter. As a result, the improvement in mineral recovery (most is by true flotation) is greater than in gangue (most is by entrainment), hence the increasing grade.

The entrained gangue recovery is calculated as the product of the entrainment
factor and water recovery (affected by the froth stability). The relatively high concentrate grades at low feed grade in $2PS$ and $4PS$ models is a result of small values of the entrainment factor. As the entrainment factor is species specific, it is analysed later in sections addressing each feed model. The amounts of water recovered for the three feed models are shown in Fig. 5.23.

![Figure 5.23: Comparison of the effect of feed grade on water recovery in a single cell for the three feed models](image)

The water recoveries in Fig. 5.23 present relatively linear and comparable trends for all feed models, which increase as feed grade increases due to more particles being recovered to the concentrate, with exceptions at the low feed grades of 0.01% and 0.015% Cu.

**Effect of the feed grade in $2PS$ feed model**

The perfect liberation of mineral from gangue accelerates the recovery in the $2PS$ feed model, and the lack of middlings species elevates the concentrate grade as shown in the grade and recovery graph in Fig. 5.21.

In the grade and recovery graph, the increments in the recovery are reduced as the feed grade increases, and after the feed grade reaches 0.7% Cu the recovery remains relatively constant. This is caused by the froth recovery reaching its maximum value of 1 at the high feed grades. The froth recovery variations with the feed grade for the $2PS$ feed models are shown in Fig. 5.24 together with the entrainment factor.
The froth recoveries increase with feed grade, and are limited to 1. This gives the trend observed at high feed grade in the grade and recovery graph. At low feed grade of 0.01% and 0.015% Cu, there is a significant drop in the value of the entrainment factor compared to the relatively consistent trend in froth recovery, which limits the amount of gangue in concentrate and gives the high grade.

As the air rate and particle size remain constant, there is no change in air recovery in all cases. Therefore the froth recovery and entrainment factor are mainly dependent on the settling velocity. The effect of feed grade on settling velocity is shown in Fig. 5.25.
In Fig. 5.25, the settling velocity decreases exponentially with increasing feed grade. From the equations stated in the Methods chapter (Chapter 3), the inverse square-root of the settling velocity is used to calculate the froth recovery and gives it the linear trend, while the high settling velocity at low feed grade to the power of 1.5 is responsible for producing the small entrainment factor seen in Fig. 5.24 (a high settling velocity inhibits entrainment as the particles drop through down the froth faster) and the high concentrate grade seen in Fig. 5.21.

**Effect of the feed grade in 4PS feed model**

In Fig. 5.21, the flotation performance graph of the 4PS feed model is similar to 2PS with a less linear trend. The concentrate recovery increases and the grade fluctuates with the feed grade. Due to the presence of a middlings species, both grade and recovery are lower compared to 2PS.

The majority of mineral and middlings recovered is by true flotation, which consequently links the recovery with the froth recovery. The variation of the recovery is re-plotted in Fig. 5.26 with the froth recoveries of the floatable species against feed grade to illustrate such a linkage.

![Figure 5.26: The effects of feed grade on froth recoveries in a single cell for 4PS with reference to the concentrate recovery](image)

The middlings particles are larger (60µm) than mineral particles (30µm). As smaller particles are more easily suspended in fluid, this results in the mineral having a lower settling velocity compared to the middlings, and in turn higher froth recovery.
The concentrate recovery accounts for both mineral and middlings, which is shown in Fig. 5.26 in the shape of combined froth recoveries of the two species. The gradient of the recovery graph decreases after the froth recovery of the mineral species reaches the maximum of 1, and the gradient is decreases close to 0 when the froth recovery of the middlings reaches the maximum.

The variation in concentrate grade is caused by changes in the gangue recovery. There are two major routes that gangue materials are recovered to the concentrate: one route is being recovered as a part of middlings particles by true flotation, the other is being recovered as pure gangue particles by entrainment. The middlings recovered by true flotation accounts for 20 to 40% of the total gangue as the feed grade varies from 0.1% to 1%Cu (more middlings particles in the feed). The entrainment of pure gangue species can be reflected in the entrainment factor, which is plotted against the feed grade in Fig. 5.27 together with the concentrate grade for comparison.

The gangue particles of 30µm have lower settling velocity and entrainment factor due to the smaller size compared to the gangue of 60µm in diameter. Significant increases in the entrainment factors are found when the feed grade is increased from 0.1% to 0.25%Cu, and the trends levels off at higher feed grades. Similar to the 2PS feed model, this low entrainment, middlings true flotation (shown in Fig. 5.26) and water recovery (shown in Fig. 5.23) at low feed grade is responsible for the initial high concentrate grade observed in Fig. 5.27. As the feed grade increases further, the entrainment factor approach 1 which limits the gangue recovery, while the mineral recovery continues to rise as froth recovery and the amount of mineral material
in the feed increases, and this results in the steady improvement in the concentrate grade seen in Fig. 5.27.

**Effect of the grade size in 15PS feed model**

The results of 15PS feed model shows continuous and relatively uniform trends in Fig. 5.21. In contrast to the other two models, both concentrate grade and recovery increase with feed grade. Small amounts of the middlings species are in the feed of 15PS compared to 4PS, which gives a higher recovery and grade. The uniformity of the trend is the combined result of the complex multi-species particle system. The recovery of particle species by true flotation improves with better liberation (due to higher flotation rate constant for particles with higher mineral content) and smaller particle size (lower settling velocity); the recovery by entrainment increases with smaller particle size. The 15PS feed model consists of 3 liberation classes in each of the 5 size classes, this diversity provides a range of trends in both froth recovery and entrainment factor (see Fig. 5.28), and as a result, the flotation performance avoids being dominated by the behaviours of individual particle species as seen in 2PS and 4PS feed models.

![Graph showing froth recovery and entrainment factor](image)

**Figure 5.28:** The effects of feed grade on froth recoveries of pure mineral species and entrainment factors of pure gangue species in a single cell for 15PS

In Fig. 5.28, the froth recoveries of the pure mineral species show close to linear relationship with feed grade. For mineral particles of 0 to 10µm, the froth
recovery remains at the maximum for all feed grades. As the particle size gets larger, the froth recovery decreases as expected. Compared to the froth recovery, the feed grade and the particle size have the same general effects on the entrainment factor of the pure gangue species. However the graphs of the entrainment factor show a non-linear ‘S’ shape relationship with increasing feed grade.

**Effect of the feed grade on product value**

Through the use of the smelter return formula, the flotation performance, grade and recovery, is translated to a single product value. The effect of feed grade on the product value is shown in Fig. 5.29.

![Graph showing the effect of feed grade on product value](image)

**Figure 5.29:** Comparison of the effect of feed grade on product value in a single cell between the three feed models

The graphs of product value in the single cell bears a strong resemblance to the graphs of product value for optimal 6-cell circuits in Fig. 5.20. The product value shows a direct relationship with the feed grade for all three feed models. A higher quality (grade) of the feed contains more mineral particles which in turn is reflected as higher product values of the concentrate.

**5.2.3 Summary**

The flotation performance in a single cell and in optimal circuits is sensitive to the feed model used when the feed grade is varied from 0.1% to 1%Cu. In general,
both recovery and grade increase as the feed grade increases, and the product value of the concentrate is directly related to the amount of minerals in the feed. The highest grade and recovery concentrates are produced by the circuit and cell using 2PS model; the flotation performance is reduced by the presence of the middlings species in the 4PS and 15PS models.

The fast mineral recovery with perfectly liberated 2PS model results in optimal layout changes. Cleaner cells are introduced to the initial roughers-only circuit improve the concentrate grade as feed grade increases. In contrast, the rougher-cleaner layout is found optimal throughout the feed grade variation for 4PS and 15PS feed models. The type of the optimal layout is resilient to changes in the feed grade if the complex feed model is used.

The study of flotation performance in a single cell shows that as the feed grade increases, the increasing amount of floatable species is reflected in the increasing solids volume fraction in the froth and lowers the particle settling velocity. This results in increases in both froth recovery and entrainment factor. The improvement in mineral recovery by true flotation gives a higher concentrate recovery as feed grade increases, this positive relationship between feed grade and concentrate recovery has also been reported in literature (Schubert et al., 1999). The improvement in recovery is greater than the improvement in gangue recovery by entrainment, hence the generally higher concentrate grade.

The relatively simple feed model, 2PS, is suitable for modelling ideal flotation situations and can provide fast flotation circuit evaluation and optimisation due to the simplified modelling, however this might lead to overvalue the financial potential of the flotation process, especially for a coarser feed. In contrast, 15PS model has more options in modelling a flotation feed stream, which in turn provides more feed specific simulation that can obtain more accurate results. It should be noted that the complexity of the feed and flotation models is translated into computing time when simulating flotation process, and it is important to balance the accuracy of results and model complexity.
Chapter 6

Conclusions and further work

The aims of this project were to develop a system combining detailed physics-based flotation circuit modelling with a robust global optimisation technique to automatically generate optimal circuit layout for any given feed, and to study the effects of key design variables, such as particle size and feed grade, on the flotation performance of a single cell and optimal circuits.

6.1 Modelling and optimisation methods

Flotation circuit optimisation has been studied by researchers for more than three decades. Overall first order kinetics models were commonly used to simulate flotation processes, and relatively small circuits were optimised using various methods which often were inefficient and prone to finding local optima. Recent studies have shown that flotation behaviour in the froth phase plays an important role in determining the performance of the process. Both empirical and physics-based models have since been developed to model the froth phase. Today, advances in optimisation algorithms and computational power has allowed more sophisticated genetic algorithms to be applied to optimising flotation circuit layout. These robust GAs have shown to produce global optimal results.

The main developments in flotation circuit simulation in this project were three fold. Firstly, an empirically derived pulp-phase kinetics model was used to link the rate constant of a particle species to its particle size and degree of liberation also taking the air flow into account. Secondly, physics-based models were
adopted to describe the parameters that governed particle behaviours in the froth phase, such as froth recovery, entrainment factor and water recovery. Furthermore, three models with increasing complexity were used to represent the feed stream; the 2PS model assumed perfect liberation which was commonly seen in early literature, the 4PS model introduced the fine and coarse particle classes, and the most complex 15PS model considered the particle size distribution as well as the variation in liberation with particle size. The combination of feed models, the pulp and froth-phase flotation models formed a circuit simulator which allowed circuits to be accurately modelled with any given feed, without the need of carrying out extensive experiments.

Genetic algorithm has been introduced to solve flotation circuit optimisation problems in recent years, and was used in this project. In contrast to conventional methods, initial guesses of the decision variables were not required, and the values of the objective functions were used in GA instead of the gradients. This allowed the detailed flotation models to be used with significant less computational power during circuit simulation. A smelter return formula balancing the recovery and grade was presented to provide the single value required in the GA.

The flotation models used in the circuit simulator were validated by comparing the modelled performance of a 10-cell bank to the experimental results from Northparkes copper concentration plant where a chalcopyrite and bornite ore was treated. The modelled flotation performance was also found to be sensitive to the feed models used. It was shown that the presence of middlings species in feed models was important, which helped to provide a uniform flotation behaviour and a closer approximation to real flotation processes.

Down the 10-cell bank, the cumulative recovery was shown to increase and the grade to decrease as the feed grade to each subsequent cell decreased. The amount of mineral recovered from simulated circuits was found to be similar to the experimental results. In the froth phase, both froth recovery and entrainment factor decreased down the bank, which were caused by the decreasing air recovery and increasing settling velocity.

The down the bank results demonstrated the variation of key froth phase parameters from cell to cell. Although there were differences between the simulated results and experimental data (which could be caused by the presence of bornite in Northparkes’ feed), the flotation and feed models were considered adequate for the
work on circuit design.

6.2 Parameters affecting flotation performance and optimal layout

Circuit size, financial parameters, particle size and feed grade as key design variables and condition parameters were varied to study their impact on a single cell and optimal circuit performance using all three feed models.

6.2.1 Circuit size

Circuit layout optimisation was carried out on 3 to 10-cell circuits. For smaller circuits, layouts consisting only of roughers were found optimal, these layouts produced concentrates of the highest possible recovery. As the circuit size expands, the use of cleaner cells was shown to improve the concentrate grade. By assuming perfect liberation in the simple 2PS model, the majority of mineral was recovered at the head of the circuit leaving limited room for improvement. This resulted in adding cleaner cells to the circuit to upgrade the concentrate grade. For circuits using the complex 15PS model, minimal change in optimal layout was observed. Overall, the rougher-cleaner was found to be the most common type of optimal layouts.

Furthermore, adding a cell to or removing a cell form a bank were shown to improve or worsen the recovery of the circuit without causing significant changes in the grade. In contrast, altering the layout, such as adding a cleaner cell, was found to cause changes in the final concentrate grade.

6.2.2 Financial parameters

The financial function was used to provide a balance between grade and recovery. The values of the parameters in the financial function were found to direct affect the choice of optimal circuit layout. Circuits containing cleaner cells were preferred when the treatment charge or grade penalty were significantly increased, while roughers only circuits were found to be more financially beneficial if the price of the metal was increased.
A tool has been developed for comparing circuit layouts when variations occur in financial parameters. The critical value of consolidated financial parameters was used to construct an area map of optimal circuits. It was shown that a layout can produce a concentrate with higher profit than the optimal layout if the financial parameters were within a certain preferred range.

### 6.2.3 Particle size

Particle size was found to fundamentally affect both pulp and froth-phase behaviours by governing the rate constant and the settling velocity of particle species. In general, as the average particle size increased, the recoveries of optimal circuits as well as a single cell were found to decrease due to less froth recovery, and the grade to increase due to less entrainment. The froth recovery of smaller particles was at the maximum value and started to decrease with increasing particle size, similar experimental results were reported in literature.

When the average particles size of the feed was varied from 25 to $75\mu m$, the flotation performance was found to exhibit similar trends in the concentrate grade and recovery between the three feed models. In both optimal circuits and a single cell, the results from the $2PS$ model showed the highest grade and highest recovery due its perfect liberation of mineral from gangue. The amount of the middlings in the feed (or the ratio of mineral to middlings) was observed to be one of the main contributors to the differences in flotation performance between $4PS$ and $15PS$ models. The optimal rougher-cleaner layout was found to be commonly occurring throughout the tests and insensitive to the variation in the feed particle size.

### 6.2.4 Feed grade

The amount of floatable species (mineral and middlings) in the feed was reflected in the feed grade, which was shown to indirectly affect the solids volume fraction in the froth. A higher solid content in the froth was shown to hinder the settling of particles, and in turn encouraged froth recovery and entrainment. As the feed grade increased in a single cell, its effect on the froth recovery was found to be greater than on entrainment factor, which resulted in both the recovery and grade to increase with increasing feed grade. However, the overall grade of the concentrate
from optimal circuits varied inconsistently and depended to the feed model and the circuit layout. Changes in the optimal layout was observed with the 2PS model. A roughers-only layout was found to be optimal for circuits with relatively low feed grade, and cleaner cells was used as the feed grade increased. The rougher-cleaner layout was found optimal throughout the feed grade variation for 4PS and 15PS feed models.

6.3 Key findings

A flotation circuit simulator has been developed in this project. It was combined with a genetic algorithm to automatically generate optimal circuit designs. The flotation modelling and optimisation technique were proven to be accurate and robust by comparing to experimental data and brute force optimisation results respectively. Case studies were carried out to investigate the sensitivity of flotation performance and optimal layout to variations in circuit size, financial parameters, feed average particle size and feed grade, and the following conclusions can be drawn:

- Roughers only layouts are optimal for small circuit to maximise recovery, and cleaner cells are introduced to the optimal layouts to improve grade as the circuit size expand. Adding cells to an existing bank will increase recovery, only changing the layout will have a greater impact on grade.

- The unaltered smelter return formula favours improvements in recovery over those in grade. Optimal layouts are specific to the chosen values of the financial parameters, such that suboptimal circuits can become more financially beneficial if there is significant change to the parameters.

- There is a positive relationship between particle size and rate constant, but both froth recovery and entrainment factor decrease with increasing particle size. In general, the recovery decreases and the grade increases as the average feed particle size increases.

- The solids volume fraction increases with increasing feed grade, which lowers the particle settling velocity and in turn increases froth recovery and entrainment factor. Both recovery and grade increases for a single cell, but the grade depends on the feed models and optimal layout for circuits.
• The flotation performance and optimal layouts are strongly dependent on the feed models.

• The optimal circuit layouts are robust and insensitive to variations in particle size and feed grade.

The flotation circuit simulator and genetic algorithm optimisation technique proposed in this project have shown significant improvements compare to the methods in literature. While there is currently no method available that includes detailed froth phase models in circuit design, not only can this system optimise circuit layout for any given feed, but also act as an analytical tool to study the effect of different design variables on flotation circuit performance. However, there is still scope for further development to improve the accuracy and reliability of the flotation models.

6.4 Further work

6.4.1 Methods

This thesis presents a robust method to simulate flotation circuit and optimise the layout. Various models have developed based on the available experimental data and findings from literature. Thus, there lies potential to adjust and improve these models.

In the flotation kinetic modelling, the effect of particle size on rate constants of fully liberated mineral particles is modelled based on the data from five experiments reported in the literature. These experiments were carried out by floating chalcopyrite and pyrite conditioned with different collectors (KAX and DTP) separately. The average values of the data are used to fit the empirical model. This model governs the variation in the selective mass transfer of particle species from the pulp to froth due to the differences in particle size. As the result, the model has a greater effect on the results from 15PS feed model (5 particle size classes included) compared to the two simpler models. Therefore, the sensitivity of flotation performance of a cell and a circuit to the rate constant model need to be analysed. Series of experiments need to be carried out to provide a thorough understanding of the effect of particle size on flotation kinetics.
Froth stability has been recognised as one of the key factors affects the performance of a flotation process. It is represented by the air recovery in this project. An arbitrary quadratic relationship between superficial air velocity and air recovery is used based on the available data. Although effects of particle size and feed grade on air recovery have been implemented, the physics behind these effects is not yet fully understood. Thorough experiments need to be carried out to provide insights on improving the preliminary relationships used in the air recovery model.

The flotation process is simulated through performing iteration of the flotation models which is centred around the solids volume fraction. The solids volume fractions in the Plateau borders, in the froth and in the concentrate are assume to be equal. This is necessary in the current method to solve the mass balance in the flotation cell, however the water content in the bubble lamellae is ignored in the model and the solids volume fraction is assumed to be constant throughout the froth, which in reality would result in different values of the solids volume fraction. Experiments and detailed froth-phase modelling could be carried out to find the potential link in the solids volume fraction at the different positions in the flotation cell.

This flotation circuit simulator and GA optimisation technique are designed with versatility and flexibility in mind, such that new development of the flotation models and algorithms can be easily implemented to suit the need of different flotation problems.

6.4.2 Analysis

Results from the flotation circuit simulator have been compared to experimental data to analyse the flotation behaviours and performance down a 10 cell bank. Due to the lack of details in the plant data such the feed mineralogy, differences are seen between the simulated and experimental results. With the aim of further testing and validating the simulator, a lab scale flotation cell need to be built. Series of experiments with possible variations in the feed particle size and grade can then be carried out using this cell, and the results can be compared with the ones from the simulator to validate the single cell model.

The flotation models in the simulator contain detailed modelling of both pulp and froth phase, which are strongly affected by the parameters in the single cell
model. This study focuses on the sensitivity of flotation systems to the feed particle size and grade. It is of interest to carry out further investigations on the effects of other parameters such as feed flowrate and air rate, and to identify their potential in improving flotation performance.

The flotation cells used in this work are identical. However, cells are often found to be of difference sizes in an industrial plant to better accommodate their purposes, such as larger cells are often used at the front of a rougher bank. Large cells result in less froth surface area and thus more stable froth especially when treating low-grade ores. Therefore it is important to examine the effect of cell size on flotation performance. However, changing the size of the cell will affect the hydrodynamics in side. In the pulp phase, the flow and turbulence is created by agitation and dependent on the size of the impeller, the speed of rotation and the geometry of the cell. Two flotation cells of different size is likely to have different flow patterns even if every piece of equipment is made to scale, this causes variations in the rate constants (the flow of water and air affects the probability of particle-bubble collision, adhesion and detachment). In the froth phase, bubbles coalesce and water drains through the froth, which are dependent on the geometry of the froth layer. The coalescence and drainage are unlikely to be linearly related to the cell size. As the result, experiments need to be designed to study the effect of cell size on these flotation behaviours. After the relationships between cell size and flotation parameters are drawn up, optimisation of cell size can be implemented into the circuit design.
Nomenclature

\(a_A, a_B, a_C\)  \(a\)  \(A\)  \(A_{cell}\)  \(a\)  \(B\)  \(a\)  \(C\)  \(C\)  
Constants in the air recovery model  
Cross-sectional area of the flotation cell  
Material concentration  
Constants in the rate constant model  
Viscous drag coefficient in the Plateau borders  
Bubble size at zero superficial air velocity  
Axial dispersion coefficient in the froth  
Constants for interface bubble size  
Interface bubble size  
Overflowing bubble size  
Particle size (diameter of the particle)  
Volume based average particle size  
Entrainment factor  
Froth stability factor  
Fraction of material that becomes detached from the vanishing interface during a coalescence event  
Substitutes for financial parameters  
Effect of the feed metal grade on the air recovery  
Effect of the feed particle size on the air recovery  
Metal grade  
Gravitational acceleration  
Froth height from the pulp-froth interface to the bursting surface  
Flotation rate constant  
Flotation rate constant of fully liberated mineral particles  
Balance between gravity and viscosity  
Degree of liberation (mineral mass fraction)  
Mass flowrate
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<th>Symbol</th>
<th>Description</th>
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<tr>
<td>$m$</td>
<td>Particle size distribution - spread</td>
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<tr>
<td>$n$</td>
<td>Number of cells in the flotation circuit</td>
</tr>
<tr>
<td>$p$</td>
<td>Fraction paid by smelter</td>
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<tr>
<td>$P_{80}$</td>
<td>Particle size distribution - size</td>
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<tr>
<td>$P_a$</td>
<td>Probability of adhesion</td>
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<tr>
<td>$P_c$</td>
<td>Probability of particle-bubble collision</td>
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<tr>
<td>$P_d$</td>
<td>Probability of detachment</td>
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<tr>
<td>$P_s$</td>
<td>Probability of successfully recovery</td>
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<td>$P_e$</td>
<td>Peclet number</td>
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<td>Flotatbility</td>
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<td>Refining charge</td>
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<td>$T_{rc}$</td>
<td>Treatment charge</td>
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<td>Superficial air velocity</td>
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<td>$v_{set}$</td>
<td>Particle settling velocity</td>
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<tr>
<td>$v_{term}$</td>
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**Greek symbols**

- $\alpha$ : Air recovery (apparent)
- $\alpha_{\text{max}}$ : Maximum value of the air recovery base on plant data
- $\lambda$ : Length of Plateau borders per volume of froth
- $\mu$ : Viscosity
- $\pi$ : Pi, 3.14159
- $\rho$ : Density
- $\phi$ : Volumetric fraction of solids in the Plateau borders, assumed to be the same as in the concentrate
**Subscript**

- *cell*: Relating to flotation cells
- *circuit*: Relating to the flotation circuit
- *conc*: Concentrate stream
- *feed*: Feed stream
- *froth*: Froth phase
- *pulp*: Pulp phase
- *slurry*: Slurry in the Plateau borders
- *solids*: Solid materials excluding water
- *tail*: Tailings stream
- *x*: Particle species $x$
References


REFERENCES


REFERENCES


Appendix

One refereed publication has been produced from this work, published in Chemical Engineering Science

“Determining flotation circuit layout using genetic algorithms with pulp and froth models”
Determining flotation circuit layout using genetic algorithms with pulp and froth models

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A U T H O R - H I G H L I G H T S

- A flotation circuit simulator has been developed considering pulp and froth phases.
- The simulator is combined with a genetic algorithm to optimise circuit layout.
- The optimal circuit layout for 3–8 cells has been determined.

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Circuit layout

A B S T R A C T

Mineral separation by froth flotation is carried out industrially by large and complex circuit arrangements, the layouts of which are based on flotation kinetics, modelling and experience. Flotation cell performance, in terms of concentrate grade and mineral recovery, can be predicted using kinetic models to describe particle recovery in the pulp phase and physics-based models to describe the behaviour in the froth phase. Linking several cells using this modelling approach allows the performance of flotation circuits to be determined, and, together with a suitable optimisation algorithm, for the optimal layout of the circuit to be established. This paper combines the powerful genetic algorithm optimisation methodology with pulp and froth modelling in each flotation cell to determine the optimal layout for flotation circuits comprising up to eight cells.

Results showed that for three cells, the optimal circuit, that is, the circuit which resulted in the most profitable combination of grade and recovery, was for the cells to be arranged in series. For circuits of four to eight cells, however, the addition of a cleaner cell was shown to yield a higher revenue.

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1. Introduction

Froth flotation is a physicochemical separation of valuable minerals from gangue (waste material) which makes use of their different surface properties. The process is performed in an aerated tank (a flotation cell), where finely ground ore particles are mixed with water and reagents to render the valuable mineral hydrophobic. When air is bubbled into the tank, hydrophobic minerals attach to the air bubbles and form a froth, which overflows the top of the cell and is collected as the mineral-rich concentrate, while the remaining pulp is removed as tailings.

Complete separation of valuable minerals from gangue is not achievable in a single cell due to the physical nature of the separation, therefore cells are connected in series to form linked banks of cells (a flotation circuit) to improve the overall performance. A typical industrial froth flotation circuit employs between twenty and more than one hundred individual cells (Bourke, 2002), and the separation efficiency of a flotation plant is heavily dependent on its circuit configuration layout (Mendez et al., 2009).

1.1. Flotation circuit modelling

There are currently two major approaches to flotation circuit design and optimisation, both of which are based on well-established models to predict flotation performance (Mendez et al., 2009).

The first approach describes the flotation kinetics in terms of a first-order rate process (Mehrotra and Kapur, 1974; Dey et al., 1989; Goria et al., 2005b). The earliest example of this approach was described by Sutherland (1948) based on hydrodynamics of a single particle. This first-order rate flotation model is widely used in industry. It predicts the recovery of the floated species from the residence time (Woodburn et al., 1965), and the experimentally determined flotation rate constant.
The second approach (Franzidis and Manlapig, 1999; Gorain et al., 1998, 1999; Runge et al., 1998; Savassi et al., 1998) considers the recovery of the floated species as a combination of true flotation (the attachment of materials to air bubbles) and entrainment (the suspension of materials in the water between bubbles in the froth). The recovery by true flotation resembles the first order rate model in the previous approach, together with empirical froth phase relationships that account for the froth recovery and entrainment effects. The true flotation component considers the flotation rate constant to be a function of bubble surface area flux and the floatability of the system (Gorain et al., 1997); the rate constant is later combined with the froth recovery factor. The recovery by entrainment is proportional to the water recovery and inversely proportional to the particle size (Smith and Warren, 1989), and is determined by combining the water recovery and a size classification function.

Obtaining a robust flotation model is the first step towards finding the optimal circuit design. While the first order rate constant approach is widely considered to be appropriate for modelling the pulp phase (Guria et al., 2005a), it is insufficient to describe the mechanisms occurring in the froth phase. Recent research addressing specifically the physics of particle and water behaviour in the froth phase has resulted in fundamentally derived models for entrainment, froth recovery and water recovery (Nerthling, 2008; Nerthling and Gilbers, 2002a, 2009; Nerthling et al., 2003).

This paper introduces a simulator based on these models, in parallel with inter-cell mass balances, to simulate flotation circuit performance. This allows the effects of design and operating variables on the optimal circuit layout and on the performance of that circuit to be investigated.

1.2. Conventional circuit optimisation

A optimum search method is required to explore all the possible circuit configurations in order to find the layout that yields the “best” performance in some criteria, e.g. the highest concentrate grade and recovery. Flotation circuit optimisation has been studied using various approaches with the majority adopting conventional optimisation techniques (Yingling, 1993; Mendez et al., 2009). These methods can be classified into three major groups, calculus-based, enumerative, and random.

Mehrotra and Kapur (1974) used calculus-based methods for the circuit optimisation. The methods use either Indirect or Direct search. Indirect search methods seek loci optima by solving a nonlinear set of equations. The equations result from assigning zero to the gradient of the objective function. An optimal position can be found by limiting the search to the positions with gradients of zero in all directions. Direct search methods (hill climbing), on the other hand, pursue local optima by following the function in the direction related to the local gradient. However, both methods only hunt for the optimal point in the vicinity of the present position, which depends on the initial value when addressing a multi-optimum problem. The result is not guaranteed to be the global optimum, but local optima can serve as design guides (Mehrotra and Kapur, 1974).

Enumerative search evaluates every possible location in the search space (possible circuit configurations) and is therefore inefficient (Goldberg, 1989). Yingling (1990) applied the method to a sequence of small scale searches, and Schena et al. (1996) used it to secure the global optimal circuit design with a financial objective function. This method is limited to relatively small problems.

Random search algorithms evaluate the performance of a circuit at a random location in the search space, and the result is retained until it is succeeded by a better circuit at another random location. The method was chosen by Kapur et al. (1991) to optimise feed parameters in flotation circuits. However, random search also has low efficiency.

1.3. Mathematical programming

By using superstructures to represent possible arrangements of a flotation circuit, it is possible to formulate the optimisation problem in a mathematical programming manner, which can be solved using linear programming (LP), mixed integer non-linear programming (MINLP) and mixed integer linear programming (MILP). However, detailed flotation models such as inclusion of froth-phase modelling are difficult to be incorporated into the programming problem.

Yingling (1993) classified early work in flotation circuit layout optimisation into two groups. The first group used the first-order flotation rate modelling approach and conventional optimisation methods. The second group used structure parameters and enhancement factors instead of a direct flotation model to describe the separation process in a flotation circuit. The flotation model or industrial data was implemented through bounds on the enhancement factors which affect the material outputs from a flotation bank. These bounds, mass balances and constraints together with an objective function, described the flotation process in terms of linear equations and inequalities. Optimal layouts were produced by using LP to solve the linearised optimisation problem.

The use of a financial objective function by Schena et al. (1996, 1997) resulted in bringing various non-linearities into the circuit layout optimisation problem. The non-linear constrained optimisation problem was decomposed into sequential LP sub-problems which were solved using MINLP (Schena et al., 1997). Cisternas et al. (2004, 2006) developed two hierarchical superstructures to represent a flotation circuit. This breaking down of the optimisation problem facilitated the linearisation of the financial objective function using a Taylor series, such that MILP could be used to optimise the circuit layout. Through the use of MINLP and MILP, a diversity of arrangements and equipments such as banks with different numbers of cells, regrinding units and flotation columns was allowed to be considered in flotation circuit designs.

1.4. Genetic algorithms and flotation circuit optimisation

In recent years, a new global optimisation technique, genetic algorithm (GA), has been developed (Holland, 1975; Goldberg, 1989) as a powerful search tool. The technique is based on the mechanics of natural selection and natural genetics. It uses the concept of survival of the fittest, in combination with randomised structure information exchange to mimic the process of natural evolution (Goldberg, 1989). In contrast to conventional searching algorithms, GA starts with a pool of randomly generated initial guesses of the decision variables (cell linkage) which spreads over the search space. In addition, values of the objective functions are used in GA instead of their gradients, and as a result the complexity of modelling is reduced (Guria et al., 2005a). The efficiency of the algorithm is dramatically improved (fitness guided search) compared to random search. Although GA has been used widely in a number of disciplines, it is a relatively new optimisation search algorithm for flotation circuit optimisation.

Applications of GAs to flotation optimisation were initially proposed by Guria et al. (2005a,b, 2006). These studies used the first order flotation rate approach developed by Mehrotra and Kapur (1974) to model the flotation process. The GA was applied to the problem to evolve from an initial population of circuit designs. The authors were able to solve for multi-objectives with a jumping gene adaptation to improve the algorithm efficiency. However, the flotation model did not account for entrainment nor the effects of operating conditions, such as air flowrate. The model also lacked
a solid demonstration that the optimal result was the global solution. More recently, GA was employed with additional process-based constraints by Ghobadi et al. (2011). Entrainment was considered in addition to first order kinetics. The scale of the process in the studies from both Guria and Ghobadi was relatively small (2–4 cell/bank circuits). These studies were focused mainly on solving optimisation problems rather than investigating the effects of the underpinning operating variables.

This paper adopts a robust GA and integrates it with both pulp-phase and froth-phase models to describe the steady-state flotation system. This algorithm and modelling system are used to generate the optimal flotation circuit design for a specified feed.

2. Flotation model

In froth flotation, particles can be recovered to the concentrate attached to bubbles (valuable particles) or unattached in the channels between the bubbles (valuable and gangue). Unattached particles can be those entrained into the froth from the pulp (valuable and gangue), or those which detached from bubbles during coalescence and bursting (valuable).

The flotation simulator described in this section takes into account different routes to recovery, classifying particles that are attached at the pulp–froth interface and subsequently recovered to the concentrate (either attached or detached) as being recovered by true flotation, and those that are non-selectively recovered as entrained. These are combined with a pulp-phase flotation rate model and mass balances to simulate the performance of each cell, and of the circuit.

2.1. Material mass balance

The mass balance for a single cell matches the sum of the mass flows (product of the volumetric flowrate $Q$ and concentration $C$) of a material species $x$ in the concentrate (conc) and tailings (tail) to the feed (feed):

$$Q_x C_{\text{con}} + Q_x C_{\text{tail}} = Q_{\text{feed}} C_{\text{feed}}$$

Valuable material in the concentrate can be recovered by both true flotation and entrainment. The true flotation term is governed by the apparent flotation rate, which is described in terms of specific rate constant $k_x$ and froth recovery factor $R_f$. The recovery of material through true flotation in a flotation cell at steady-state is often predicted in terms of a CSTR (continuous stirred tank reactor) model (Guria et al., 2005b). Defining $M$ as the mass flow of material and $V_{\text{pul}}$ as the volume of the pulp phase in the flotation cell, rearranging Eq. (4), the concentration of species $x$ in the tailings and concentrate, $C_{\text{tail}}$, and $C_{\text{conc}}$, can be expressed as

$$C_{\text{conc}} = \frac{Q_{\text{conc}} C_{\text{con}}}{V_{\text{cell}} k_x R_f + Q_{\text{conc}} E_{\text{conc}} + Q_{\text{tail}} C_{\text{tail}}}$$

$$C_{\text{tail}} = \frac{V_{\text{cell}} k_x R_f + Q_{\text{conc}} E_{\text{conc}} + Q_{\text{tail}} C_{\text{tail}}}{Q_{\text{conc}}}$$

2.2. True flotation recovery

The true flotation recovery model accounts for the material entering the froth attached to bubbles, that is recovered either attached or unattached. Froth recovery is defined as the fraction of particles that are recovered in this way, and it is determined using Eqs. (7) and (8). These equations include the effects of superficial air velocity $v_{\text{froth}}$, particle settling velocity $v_{\text{set}}$, and air recovery $\alpha$ (the fraction of air entering a flotation cell that overflows the cell lip as unburst bubbles) (Neethling and Cilliers, 2008).

$$v_{\text{froth}} = \frac{g (\rho_x - \rho_{\text{water}}) \mu_{\text{water}}^2}{18 \rho_{\text{water}}}$$

$$v_{\text{set}} = \frac{g \rho_x (1 - \phi) \mu_{\text{water}}^2}{\phi}$$

Here $f$ is the fraction of material that becomes detached from the vanishing interface during a coalescence event, $v_{\text{froth}}$ is the bubble size at the pulp–froth interface and $f_{\text{rel}}$ is the bubble size overflowing the cell lip.

For air recoveries lower than 50%, the froth recovery is highly dependent on air recovery. In this study, $f$ was assumed to be unity, that is, when bubbles coalesce, all the material is detached from the coalescing surface. The particle size $d_p$ is also important in the froth recovery; it contributes to the particle terminal velocity $v_{\text{set}}$, and determines whether a particle will be carried upwards in the froth and collected to the concentrate (Neethling and Cilliers, 2002b).

$$\rho_{\text{water}} = \rho_{\text{water}} \exp \left(\frac{2.5}{1 - 0.609 \phi} \right)$$

The volumetric fraction of solids, and the viscosity of the slurry, in both the froth and the concentrate are assumed to be the same as $\phi$ and $\mu_{\text{water}}$. Ideally, the froth and concentrate would take account of the particles which are attached to the bubble film as well as those in the Plateau borders. However, without performing detailed modelling of the froth, it is not possible to predict the precise values of the parameters in the froth and in the concentrate.

2.3. Entrained flotation recovery

Entrained is the process by which unattached particles enter the froth at the pulp–froth interface in the channels between the bubbles (Plateau borders) and are recovered to the concentrate. It has been found that the entrained material recovery is proportional...
to the water recovered to the concentrate (Engelbrecht and Woodburn, 1975; Tjahar, 1981).

The water recovery depends on the liquid behaviour in Plateau borders and is governed by gravity, viscous drag and capillary suction in the froth (Neethling et al., 2003)

\[ \text{if } a < \frac{1}{2}, \quad Q_{\text{water}} = \frac{A_{\text{cell}}v_1^2}{k_1}(1-a) \]  
\[ \text{if } a > \frac{1}{2}, \quad Q_{\text{water}} = \frac{A_{\text{cell}}v_1^2}{4k_1} \]  

where \( A_{\text{cell}} \) is the cross-section area of the flotation cell, and \( \lambda \) is the length of Plateau borders per volume of froth (Neethling et al., 2003) where the bubbles are assumed to be Kelvin cells

\[ \lambda = \frac{6.815}{A_{\text{cell}}} \]  

The constant \( k_1 \) in the water recovery model represents the balance between gravity and viscosity (Neethling et al., 2003)

\[ k_1 = \frac{\rho_{\text{slurry}} g}{3\xi_{\text{water}}} \]  

where \( c_{gb} \) is the viscous drag coefficient in the Plateau borders. Its value is taken to be 49 due to the immiscible interface in flotation froth (Neethling et al., 2003). \( \rho_{\text{water}} \) is the density of the concentrate

\[ \rho_{\text{water}} = \rho_{\text{slurry}} + (1-\phi)\rho_{\text{water}} \]  

To quantify the entrained material using the water recovery, the entrainment factor is introduced to incorporate the effects of liquid motion, particle settling and particle dispersion (Neethling and Cilliers, 2009).

\[ \text{if } a < \frac{1}{2}, \quad \text{Ent}_t = \exp \left( \frac{v_1^2\lambda_{\text{froth}}}{D_{\text{axial}}\sqrt{\rho_1}(1-a)} \right) \]  
\[ \text{if } a > \frac{1}{2}, \quad \text{Ent}_t = \exp \left( \frac{2v_1^2\lambda_{\text{froth}}}{D_{\text{axial}}\sqrt{\rho_1}} \right) \]  

where \( \lambda_{\text{froth}} \) is the froth height from the pulp–froth interface to the bursting surface, and \( D_{\text{axial}} \) is the axial dispersion coefficient. \( D_{\text{axial}} \) has been found to be a function of relative liquid velocity and the Peclet number \( Pe \), and this relationship is simplified to Neethling and Cilliers (2009)

\[ D_{\text{axial}} = \frac{v_1^5}{\sqrt{k_1\left(\frac{3}{2} - \frac{1}{\rho_1}\right)}} \]  

where \( Pe \) is taken to be 0.15 (Lee et al., 2005).

2.4 Integrated cell and circuit model

In order to simulate the flotation process in a cell and determine its concentrate and tailings flowrate and composition, the models described previously are combined and solved for a given feed. An iterative approach is taken to solving the equations, and this is shown schematically in Fig. 1.

The properties of the feed are given (flowrate, density, grade and particle size), and an initial value for the volumetric solids fraction of the concentrate is taken as an input. The density of the solids in the concentrate is assumed to be the same as that for the feed. This allows the density and viscosity of the concentrate to be evaluated using Eqs. (16) and (11) respectively (Step 1 in Fig. 1). Knowing the individual density of each solid species and the particle size from the given feed, the particle settling velocity is predicted using the Stokes’ Law in Eqs. (9) and (10). The gravity and viscosity balance are calculated using Eq. (15) (Step 2).

Subsequently the volumetric concentrate flowrate, froth recovery and axial dispersion coefficient are found using Eqs. (12)/(13), (7)/(8) and (19) according to the air recovery value (Step 3). Air recovery has been shown to vary down a bank of cells (Smith et al., 2008; Hadler et al., 2010), which has been linked to changes in the amount of froth stabilizing particles. A model linking air recovery to the concentration of froth stabilizing material has therefore been included in the simulator. Next, by performing a mass balance for the flotation cell and using Eq. (17) or (18), the volumetric tailings flowrate and the entrainment factor are obtained (Step 4). Therefore the concentrations of solid species in the tailings and concentrate streams are determined by performing the mass balances in Eqs. (5) and (6) (Step 5). Finally, knowing the concentrations in the concentrate stream allows the initially guessed volumetric solids fraction and assumed solids density to be recalculated for the next iteration cycle (Step 6). The iteration is repeated until the volumetric solids fraction converges.

After the first cell in a flotation circuit is modelled using the method above, the resulting concentrate and tailings are either new feeds to downstream or a part of the circuit output. The iterative flotation modelling is repeated for the remaining cells in the circuit, and by summing the total concentrate and tailings of the circuit output, the overall grade and recovery are determined.

The improvement of this modelling approach is the inclusion of the detailed froth models. This improvement allow the effects of design and operating variables such as cell dimensions and air inlet to be taken into account in the flotation performance, which in turn produces accurate and realistic results. However, in contrast to the first order rate modelling approach, this iterative modelling requires relatively intensive computation. The more cells are in a circuit, the longer it takes to model. In addition, the detailed froth models require more inputs such as bubble sizes and froth height, which can be difficult to measure or estimate.

3. Circuit layout optimisation

A typical froth flotation circuit consists of four to eight flotation banks, with five to twenty cells within each bank (Bourke, 2002). The possible number of different circuit configurations is extremely large.

Every cell outputs two product streams; concentrate and tailings. In a flotation system consisting of \( n \) cells, each cell’s output stream can be connected to one of \( n \) destinations, every other cell \((n - 1)\) and the circuit output \((1)\) which excludes self-recycles. It is fruitless to direct the tailings to the same destination
as the concentrate. As a result, there are \( n \times (n - 1) \) ways to connect a cell in the system. Taking all cells into account, the number of different configurations rises exponentially to \( (n - 1)! \). However, a circuit must have at least two product streams, which means that at least one concentrate and one tailings from any two cells are taken to be the products, so the number of configurations is corrected to \( (n - 1)! / C_2(n - 1)! \). The feed stream can be introduced to any cell in the circuit, which increases the number of configurations by a factor of \( n \times (n - 1)! - 1 \). This suggests that even for a 5-cell circuit, there are 800,000 possible circuit arrangements (see Fig. 2), although not all are valid and many are equivalent to one another.

Most of the possible designs are suboptimal or invalid as a result of technical faults or illogical configuration (e.g., recycling the tailings of the scavenger to the cleaner). Consequently, adopting an optimisation method is critical to first screen out the flawed designs, and to generate the optimum design which satisfies both technical and economic constraints.

3.1. Genetic algorithm methodology

In order for the genetic algorithm methodology to be applied to flotation circuits, cell linkages (i.e., the destination of output streams) must be coded as a finite-length string. A population of strings representing flotation circuit layouts is first generated (parent population); for each of these a single fitness value of performance is determined (based on efficiency or profit, for example). Daughter strings of the same population size are randomly selected from the original strings. It is a fitness proportionate selection (roulette wheel selection), as the greater the fitness of the original string, the more likely it is to be selected as a daughter. This means that the best strings are expected to be selected more than once in the process. To encourage the survival of the circuit layouts which yield better performance, the current best string (highest fitness) is always included as a daughter. After obtaining the daughter population, every two strings are mated, so that the genetic information between two randomly chosen string positions is swapped. This process is termed 'crossover', hence the daughter strings now contain features from both of their parents. For every parameter, there is a slight possibility that it undergoes mutation. The value of the mutating bit is randomly reassigned during this process. Although this minor change appears insignificant, the operation is an efficient means of preventing unification of the whole population and avoiding being trapped at only local optima. The fitness value is re-evaluated for every daughter string to enable the selection process to take place again. One selection-crossover-evaluation cycle is referred to as one generation, and the cycle is repeated until a designated number of generations are reached or the fitness satisfies minimum criteria (e.g., minimum concentrate grade) (Fig. 3). The final daughter string is the solution to the optimisation problem. For in depth discussions on GAs see Goldberg (1989) and Mitchell (1998).

3.2. The string representation of a circuit

A flotation circuit configuration is coded in a systematic way to represent the linkages of every cell. Instead of a binary form, a decimal digit string is used to symbolise the connections between cells for a straightforward and concise representation. Cells in an \( n \)-cell circuit are numbered from 1 to \( n \). In the decimal string, each digit represents a stream in the circuit. The position of the digit indicates the origin of the stream and whether this stream is a concentrate or tailings; the value of the digit denotes the destination cell of the stream.

As seen from Fig. 4, the first digit block in the string representation denotes the destination of the circuit feed (the cell which the external feed is connected to), and every two blocks followed are the destinations of the concentrate and tailings stream from Cell 1 to Cell \( n \). The streams which exit the circuit are assigned with ‘0’. An example circuit is shown below in Fig. 5, it is coded as ‘1 2 3 0 1 2 4 2 0’. The exiting horizontal arrows from the sides of the cells represent tailings outflows, while vertical arrows from the bottom of the cells represent concentrate outflows. The circuit feed is connected to Cell 1 which begins the circuit string representation with ‘1’; it is followed by Cell 1’s concentrate destination ‘2’ and tailings destination ‘3’; the next pair of the digit represents Cell 2’s concentrate and tailings destination of ‘0’ and ‘1’ respectively. In the same manner, the rest of cells are concatenated into the string to give ‘1 2 3 0 1 2 4 2 0’.

3.3. GA parameters

When implementing a GA, various algorithm settings, such as population size, GA cycle length, crossover rate and mutation possibility must be decided. The reliability of the result is heavily

![Figure 2](image-url) Number of possible configurations for circuits of different size.

![Figure 3](image-url) The GA cycle for circuit performance optimisation.

![Figure 4](image-url) Possible destinations of streams.

![Figure 5](image-url) A simple circuit configuration coded as ‘1 2 3 0 1 2 4 2 0’.
dependent on the values set for these parameters, which are interrelated. Although the parameter settings are specific to the optimisation problem being considered, many researchers use those settings that have been successful in previous studies (Mitchell, 1998).

In the algorithm used in this paper, the crossover rate was set to 1. Post-crossover circuits were checked against the validation criteria. The crossover was reversed if the process had been attempted five times and the circuits remained flawed. This uncrossed portion of the population decreased the apparent crossover rate to approximately 0.7. Having a crossover rate lower than 1 helps preventing a premature convergence of the result due to allele loss, which in turn is caused by discarding high performance circuits faster than crossover can produce improvements.

The mutation rate is used to prevent unification of the population at local optima and maintain genetic diversity. In the later GA cycles, however, it is desirable for convergence to arise as the optimal solution can be delayed or partially prevented if a large number of mutations occurs. According to previous works (Fogarty, 1989; Bäck, 1993), the effect of mutation rate on the efficiency of the algorithm is dependent on the nature of the optimisation problem (e.g. finite or infinite search space; whether the starting parent population is a matrix of all zeros or random digits) and fitness function, such that it is arguable whether the implementation of a time-dependent variation of the mutation rate can accelerate the optimisation. The mutation rate was kept at 0.005 in this study.

De Jong (1975) and later Schaffer et al. (1989) performed a systematic investigation on the effect of population size and GA cycle length on the algorithm performance. It was found that the most efficient population size was 20–100 individuals, the mutation rate was 0.001–0.01.

Although these parameters were proven reliable and efficient in these studies, the complexity of the optimisation problem differs from one problem to the next. Therefore a range of population size and GA cycle length were tested.

3.4. Circuit validation rules

The random circuit generated in the parent population at the beginning of the GA and the daughter circuits evolved after crossover and mutation are validated against several criteria. This is done to eliminate the designs with technical faults or unreasonable linkages, so that a design is only valid if there are

- no cell self-recycles;
- at least one feed to every cell;
- at least one concentrate and one tailings stream from the cells that exit the circuit as products;
- the concentrate and tailings streams from the first cell, to which the circuit feed is introduced, do not both exit the circuit as products;
- the concentrate and tailings from a single cell do not flow to the same destination;

The above criteria do not guarantee a valid circuit and thus the final check is to see if the iteratively solved mass balance converges, which it must for a valid circuit.

3.5. Circuit optimisation fitness function

After every valid circuit design is simulated, a finance-based fitness function is used to evaluate the circuit performance by providing a single quantitative value, therefore enabling the comparison of different circuit configurations. The fitness function and revenue calculation used in this study are given in Eqs. (20) and (21).

\[
\text{Circuit Fitness} = \frac{\text{Revenue}_{\text{max}} - \text{Revenue}_{\text{current}}}{\text{Revenue}_{\text{max}}} (20)
\]

The circuit revenues are calculated using the financial function (a net smelter return formula) suggested by Schena et al. (1996)

\[
\text{Revenue}_{\text{current}} = \text{Revenue}_{\text{max}} \cdot \frac{\text{Gconc}_{\text{current}}}{\text{Gconc}_{\text{max}}} - \text{q} \cdot 10^6 \cdot (1 - \text{frc}_{\text{max}}) - \text{Trc} \cdot \text{Cconc}_{\text{max}} (21)
\]

where \(M_{\text{conc}}\), is the circuit concentrate flowrate, \(p\) is the fraction paid by smelter, \(C_{\text{conc}}\) is the concentrate metal grade, \(u\) is the grade penalty, \(q\) is the metal price, \(fsc\) is the refinery charge, \(Tnc\) is the treatment charge and \(Revenue_{\text{max}}\) is the minimum possible profit observed when 50% of minerals and gangue in the circuit feed are recovered in the final concentrate (if less mineral or more gangue is recovered, the tailings becomes more profitable), whereas \(Revenue_{\text{max}}\) is the maximum profit observed when all the minerals in the circuit feed are recovered in the final concentrate without any of the gangue.

The circuit with the highest fitness value among the randomly generated strings is recorded as the current optimal circuit design, and it is overwritten by new optimal circuit designs as the GA cycle progresses. At the end of the cycle, the circuit yielding the highest fitness function will be, therefore, the optimal circuit for a given set of conditions.

The improvement of the average fitness of the population through GA cycles follows a logarithmic shape, the tail of which levels off at the fitness value of the optimal circuit. Fig. 6 shows a typical progress of the average fitness of a 6-cell circuit optimisation problem. The population size set for the example in Fig. 6 was 1296, 576 GA cycles were carried out, and the test was repeated 20 times.

4. Sensitivity of GA parameters

Due to the probabilistic nature of GA, the final result may be suboptimal if the population size or the evolution cycles used are insufficient. It is therefore necessary to investigate the reliability of the final result, and to determine the optimal settings of the GA parameters.

4.1. Comparison of GA with brute force optimisation

There is a finite number of ways of arranging the flotation cells in a circuit. For flotation circuits with a relatively small number of cells (i.e. fewer than 7 cells), every possible circuit configuration can be tested in order to determine the optimal circuit design. This brute-force enumerative method (BF) enables the optimal circuit generated by the GA to be validated. It was found that the circuit with the highest fitness value (the optimal circuit with best
5. Results and discussion

5.1. Feed and plant conditions

To determine the impact of the number of cells on circuit layout and flotation performance, data from a copper concentrator was used to test the combined flotation simulator-GA methodology, details of which are given in Tables 1 and 2. The feed stream contains two particle species of 40 μm in diameter, pure mineral and pure gangue. Additionally, the following assumptions were made:

Additionally, the following assumptions were made:

- the mineral was perfectly liberated from the gangue;
- the flotation cells employed in the circuits were identical;
- the operating conditions across the circuit were the same (constant froth depth, bubble sizes and air flowrate).

Because the flotation models used in the simulator are mostly species specific, it is possible for more species with imperfect liberation and different sizes to be included in the feed. The difference in liberation between species can be seen by variances in density, metal content and flotation rate constant.

Although the air flowrate was not varied from cell to cell, the differences in the feed composition and flowrate were reflected in the differences in the solid fraction in the concentrate, ϕ. Through the iteration cycle in the simulator (Fig. 1), this difference resulted in variations in both the froth recovery, Rf, and the entrainment factor, Ent. By performing circuit simulation, it was found that Rf and Ent decreased along a flotation bank, which corresponded to experimental results in the literature (Savassi, 1998).

5.2. Effect of number of cells

The optimisation was performed on circuits with 3–8 flotation cells. Fig. 5a shows the optimal configuration for a 3-cell circuit. The circuit is a bank of cells within which the concentrate streams from every cell report to the final concentrate stream. A
the Revenue circuit product value of $0.17/mt RoM from 67.81% to 70.08%. This was also shown as an increase in the circuit dropped from 11.92% to 11.65%, but the suboptimal circuit to the optimal, the final cell is recycled to the head of the circuit. The differences in the structure of the suboptimal circuit is illustrated in Fig. 8b; the concentrate of the final cell is recycled to the head of the circuit.

In contrast to the 3-cell circuit, the optimal configuration for a 4-cell circuit (Fig. 9) changed from the "straight" bank to a circuit with a cleaner cell (rougher-cleaner), into which the concentrate from the three rougher cells was directed. The tailings from this cleaner cell were recycled to the head of the circuit. Up to 8 cells were simulated; however no major change in the structure of the optimal configuration was observed. For each additional cell, the new units were connected to the rear of the circuit, so that the stream which reported to the circuit tailings previously can be further processed.

The final grade and recovery values for the optimal circuits are shown in Fig. 10. The bottom left data point represents the optimal 3-cell configuration. The effect of the addition of the cleaner cell can be observed in the increase in the final concentrate grade, while the addition of the cells results in an improvement in overall recovery. Although the circuit structure of the optimal 4 to 8-cell circuits differed from the 3-cell circuit, the difference was not reflected as a significant difference in the value of the product. Fig. 11 shows a smooth increasing trend of the product value for the optimal circuits.

It should be noted that there were several assumptions in these simulations, such as the air flowrate and the rate constant remaining constant in all cells; in reality, these would vary around the circuit. The plant on which this data was based, however, contains 8 cells in the rougher-scavenger bank and a cleaner circuit and is therefore not dissimilar to the layout determined using the GA. Furthermore, the plant grade and recovery was 33% and 89% respectively during the period that the feed data was taken, therefore yielding similar results to those predicted by the simulator. The lower grade and higher recovery from the simulations compared to the plant measurements is likely to be due to the small layout difference between the two circuits.

Comparison with related work

Previously published applications of GA to flotation circuit layout design and optimisation have considered up to 4 cells/banks only (Guria et al., 2005a,b, 2006; Ghobadi et al., 2011). Two representative optimal 4-cell circuits determined in these studies are shown in Fig. 12a and b. It should be noted that these layouts from literature were optimised for a small feed [0.68t h\(^{-1}\) (Guria et al., 2006), 100t h\(^{-1}\) (Ghobadi et al., 2011)] with relatively high grade [24.5%–25% mineral, or 8.5%–8.7% metal (chalcopyrite)]. The flotation cell sizes and operating conditions were also different from the values used in this paper.

To compare the flotation performance of the circuits shown in Fig. 12a and b to the optimal 4 cell layout determined in this study, the feed and operating conditions and cell dimensions from the plant data (Tables 1 and 2) were used in the configurations from the previous studies, and these circuits were modelled with the flotation simulator. It should also be noted that the operating conditions, such as air flowrates, were the same as those used in this study. A comparison of the final grades and recoveries is shown in Fig. 13.

From the figure, the optimal circuit generated in the present study produces the concentrate with the highest grade and recovery among the three designs, such that the circuit layouts from literature were suboptimal in performance when used for the flotation process in this paper. This is the result of the fact that...
circuits were optimised for treating different quality and quantity of feed in each study. In addition, various constraints and objectives were set up to maintain or maximise key variables to favourable values. In the work by Guria et al. (2006), the mineral grade of the final concentrate was retained at 55%, the residence time of each cell was kept under 20 min and the material recovery was maximised in the optimisation problem which obtained the layout in Fig. 12a. Ghobadi et al. (2011) maintained a higher mineral grade of 65% and maximised the separation efficiency. However in this study, fewer constraints were implemented. Every cell was preset to be 50 m² in volume, the product grade and recovery were automatically balanced by the net smelter return formula and maximised by the GA to generate an optimal circuit with the highest product value. The optimal circuit is sensitive and is likely to be unique to the values set in the fitness function, which are dependent on the specifics of the smelter contract.

6. Conclusions

A flotation simulator considering both pulp and froth phase behaviour has been combined with genetic algorithms to allow optimal circuit layout to be studied. In the pulp phase, first order rate constants were considered, while in the froth phase, models that predict froth recovery and entrainment were included, all of which are dependent on air recovery.

Preliminary investigation of the GA parameters showed that the minimum cycle number and number of generations was 0.1% of the total possible circuit configurations. This robust optimisation system was used to generate designs for circuits with 3–8 cells. The optimal circuit layout progressed from a straight-bank structure for 3-cell circuit to a cleaner with recycled tailings for circuits with 4–8 cells. Froth recovery and entrainment factor were found to decrease down a flotation bank.

The optimal 4-cell circuit layout was compared to two layouts suggested in prior work, where three circuits were modelled using the flotation simulator and the data from the copper concentrator. This robust optimisation system was used to generate designs for the optimal 4-cell circuit and literature designs.

Fig. 13. Comparison of the final grade and recovery for the optimal 4-cell circuit and literature designs.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>A_{cfl}</td>
<td>cross-sectional area of the flotation cell</td>
</tr>
<tr>
<td>C</td>
<td>material concentration</td>
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C_p viscous drag coefficient in the Plateau borders
D_{aflu} axial dispersion coefficient in the froth
f | fraction of material that becomes detached from the vanishing interface during a coalescence event
G | metal grade
h | gravitational acceleration
h_{froth} froth height from the pulp–froth interface to the bursting surface
k | flotation rate constant
k_t balance between gravity and viscosity
M | mass flowrate
n | number of cells in the flotation circuit
p | fraction paid by smelter
Pe | Peclet number
Q | volumetric flowrate
q | metal price
R | refined charge
R_{fc} | froth recovery
R_{sm} | interface bubble size
R_{ovf} | overflowing bubble size
t | treatment charge
u | grade penalty in the smelter contract
V_{vol} | volume of the pulp phase in the flotation cell
V_{set} | superficial gas velocity
V_{sett} | particle settling velocity
V_{term} | particle terminal velocity

Subscripts

feed feed stream
conc concentrate stream
solids solid particles
slurry slurry in the Plateau borders
tail tailings stream
x xth particle species

Greek symbols

α | air recovery
λ | length of Plateau borders per volume of froth
μ | viscosity
ρ | density
φ | volumetric fraction of solids in the Plateau borders of the froth

References


