A CRITICAL ASSESSMENT OF TURBULENT FLOW OVER TEXTURED SUPERHYDROPHOBIC SURFACES

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Declaration of Originality

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Abstract

Over the past century, a sustained effort has been expended on the research and development of surfaces that reduce the amount of drag experienced by a fluid as it passes by, motivated by both environmental and economic savings.

Superhydrophobic surfaces have recently emerged as an attractive means to reduce the levels of skin-friction drag under both laminar and turbulent flow conditions. A superhydrophobic state is attained naturally or synthetically through a combination of surface topology and surface chemistry and can, in some cases, support a free-stress gas-liquid interface. In the presence of bulk fluid motion, the interfaces permit a finite slip velocity which has been credited to the reduction of the average wall shear stress. The fundamental drag reduction mechanism, however, remains unclear.

In order to accurately resolve the full spectrum of turbulent scales, direct numerical simulations of fully turbulent channel flow over superhydrophobic textures at a friction Reynolds number of \( Re_\tau \approx 180 \) were conducted. The instantaneous flow fields were subject to triple decomposition which permits statistical quantities to be accumulated in a phase-averaged form. From these phase-averaged statistics the mean, periodic and stochastic fluid motions can be considered independently. Following a detailed statistical analysis, the contributions of the mean, periodic and stochastic fluid motions towards the local levels of wall shear stress were determined by the derivation and evaluation of an appropriate skin-friction identity.

In addition, a new modification to superhydrophobic surfaces is investigated by means of meandering the surface topology in the streamwise direction. Relative to a streamwise-aligned topology, it was anticipated that superior drag reduction would be achieved due to the addition of an oscillatory spanwise motion to the mean flow.
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Nomenclature

**Acronyms**

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<th>Description</th>
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<tbody>
<tr>
<td>LFC</td>
<td>Laminar flow control</td>
</tr>
<tr>
<td>NASA</td>
<td>National Aeronautics and Space Administration</td>
</tr>
<tr>
<td>TKE</td>
<td>Turbulence kinetic energy</td>
</tr>
<tr>
<td>RANS</td>
<td>Reynolds-Averaged-Navier-Stokes</td>
</tr>
<tr>
<td>DNS</td>
<td>Direct numerical simulation</td>
</tr>
<tr>
<td>SHS</td>
<td>Superhydrophobic surface</td>
</tr>
<tr>
<td>PTFE</td>
<td>Polytetrafluoroethylene</td>
</tr>
<tr>
<td>PIV</td>
<td>Particle image velocimetry</td>
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<tr>
<td>SVD</td>
<td>Singular value decomposition</td>
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<tr>
<td>DR</td>
<td>Drag reduction</td>
</tr>
<tr>
<td>LES</td>
<td>Large eddy simulation</td>
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<tr>
<td>FVM</td>
<td>Finite volume method</td>
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<tr>
<td>TS</td>
<td>Tollmien-Schlichting</td>
</tr>
<tr>
<td>FIK</td>
<td>Fukagata, Iawmoto &amp; Kasagi (2002)</td>
</tr>
<tr>
<td>AIM</td>
<td>Anisotropy invariant map</td>
</tr>
<tr>
<td>PDF</td>
<td>Probability density function</td>
</tr>
<tr>
<td>IB</td>
<td>Immersed boundary</td>
</tr>
<tr>
<td>LSM</td>
<td>Level set method</td>
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### Global notation

<table>
<thead>
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<th>Notation</th>
<th>Description</th>
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<tbody>
<tr>
<td>$(\cdot)_w$</td>
<td>Denotes a quantity at the wall</td>
</tr>
<tr>
<td>$(\cdot)^+$</td>
<td>Denotes a quantity non-dimensionalised with wall (plus) units</td>
</tr>
<tr>
<td>$(\cdot)_i$</td>
<td>Denotes the $i$'th component of a vector quantity</td>
</tr>
<tr>
<td>$(\cdot)_{ij}$</td>
<td>Denotes the $i$'th and $j$'th indices in tensor notation</td>
</tr>
<tr>
<td>$(\langle \cdot \rangle)$</td>
<td>Denotes a phase-averaged quantity</td>
</tr>
<tr>
<td>$(\langle \cdot \rangle)^p$</td>
<td>Denotes a quantity averaged along lines of constant streamwise phase</td>
</tr>
<tr>
<td>$(\langle \cdot \rangle)^zz$</td>
<td>Denotes a quantity averaged across all possible phases</td>
</tr>
<tr>
<td>$(\langle \cdot \rangle)^{ns}$</td>
<td>Denotes a quantity averaged across all no-slip phases</td>
</tr>
<tr>
<td>$(\langle \cdot \rangle)^{fs}$</td>
<td>Denotes a quantity averaged across all free-slip phases</td>
</tr>
<tr>
<td>$(\tilde{\cdot})$</td>
<td>Denotes a periodic quantity</td>
</tr>
<tr>
<td>$(\cdot)'$</td>
<td>Denotes a fluctuating quantity</td>
</tr>
<tr>
<td>$(\cdot)_{rms}$</td>
<td>Denotes a root-mean-squared quantity</td>
</tr>
<tr>
<td>$(\cdot)'_s$</td>
<td>Denotes a slipping quantity</td>
</tr>
<tr>
<td>$(\cdot)'$</td>
<td>Denotes a quantity in wavenumber space</td>
</tr>
<tr>
<td>$(\cdot)^{n-1}$</td>
<td>Denotes a quantity at timestep $n - 1$</td>
</tr>
<tr>
<td>$(\cdot)^n$</td>
<td>Denotes a quantity at timestep $n$</td>
</tr>
<tr>
<td>$(\cdot)^n$</td>
<td>Denotes a quantity at an intermediate timestep</td>
</tr>
<tr>
<td>$(\cdot)^{n+1}$</td>
<td>Denotes a quantity at timestep $n + 1$</td>
</tr>
<tr>
<td>$(\cdot)_{i,j,k}$</td>
<td>Denotes a quantity at the cell-centre</td>
</tr>
<tr>
<td>$(\cdot)_{i\pm\frac{1}{2},j,k}$</td>
<td>Denotes a quantity staggered in the streamwise direction</td>
</tr>
<tr>
<td>$(\cdot)_{i,j\pm\frac{1}{2},k}$</td>
<td>Denotes a quantity staggered in the wall-normal direction</td>
</tr>
<tr>
<td>$(\cdot)_{i,j,k\pm\frac{1}{2}}$</td>
<td>Denotes a quantity staggered in the spanwise direction</td>
</tr>
</tbody>
</table>
Roman symbols

$a_m$ Maximum spanwise acceleration of Stokes boundary layer
$A$ Contact area
$A_z$ Maximum spanwise oscillation amplitude
$b_{ij}$ Turbulence anisotropy tensor
$C_f$ Skin-friction coefficient
$d$ Width of no-slip SHS features
$D_{lam}$ Laminar drag, $\int_A \mu \left. \frac{du}{dy} \right|_w dA$
$DR$ Drag reduction, relative to the reference no-slip case
$E$ Total disturbance energy, $\frac{1}{2} \int_V \left( u'^2 + v'^2 + w'^2 \right) dV$
$g$ Width of free-slip SHS features
$G$ Energy amplification
$i$ Unit complex number, $\sqrt{-1}$
$I$ Identity matrix
$I_b, \Pi_b, \Pi_2$ First, second and third principal invariants of the anisotropy tensor
$k_x, k_z$ Streamwise and spanwise wavenumbers
$k'^2_x, k'^2_z$ Modified streamwise and spanwise wavenumbers
$l_{m}$ Penetration depth of Stokes boundary layer
$l_{\lambda_2}$ Vortex core length
$L_s$ Effective slip length
$L_x, L_y, L_z$ Domain length in the streamwise, wall-normal and spanwise directions
$m$ Mass flow rate
$N_x, N_y, N_z$ Number of mesh-points in the streamwise, wall-normal and spanwise directions
$p$ Pressure
$P_{ij}$ Reynolds stress production tensor
$Q_{ij}$ $Q_{ij}$
$Q_x$ Flow rate per unit width in the streamwise direction
$R$ Divergence-free random noise between $[-1, +1]$
$Re_b$ Reynolds number based on bulk velocity
$Re_{crit}$ Critical Reynolds number
$Re_{cl}$ Reynolds number based on centre-line velocity
$Re_{ij}$ Reynolds stress tensor
$Re_{\nu}$ Reynolds number based on viscous scales
$Re_{\tau}$ Reynolds number based on friction velocity
$R_{uu}$ Autocorrelation of streamwise velocity fluctuations
$s$ Riblet peak-to-peak spacing
$S$ Drag reduction scaling parameter, (Choi et al., 2002)
$S_{ij}$ Symmetric part of velocity gradient tensor
$S$ Surface tensor bounding control volume $V$
$S^\xi, S^\eta, S^\zeta$ Contravariant vectors in general system
$S_\xi, S_\eta, S_\zeta$ Covariant vectors in general system
$t$ Time
$T$ Time period
$T_{int}$ Time interval
$u$ Streamwise velocity component
$u_\tau$ Friction velocity, $\sqrt{\tau_w/\rho}$
$U_b$ Bulk velocity, $\frac{1}{2} \int_0^2 u dy$
$U^\xi, U^\eta, U^\zeta$ Velocity flux vector in general system
$U_e$ Convection velocity of turbulence fluctuations
$V$ Volume
$v$ Wall-normal velocity component
$w$ Spanwise velocity component
$W_m$ Maximum spanwise amplitude of Stokes boundary layer
$W_{th}$ Threshold value of spanwise amplitude of Stokes boundary layer
$x$ Streamwise direction
$y$ Wall-normal direction
$z$ Spanwise direction
Greek symbols

$\beta$  Maximum turning angle of SHS
$\delta$  Channel half-height
$\delta_{ij}$  Kronecker delta
$\delta_{\nu}$  Viscous lengthscale, $\nu/u_z$
$\Delta$  Laplacian operator
$\Delta x, \Delta y, \Delta z$  Mesh spacing in $x$, $y$ and $z$ directions
$\epsilon$  Perturbation amplitude
$\eta$  Wall-normal component in general non-orthogonal coordinate system
$\eta'$  Perturbation wall-normal vorticity, $\partial u'/\partial z - \partial w'/\partial x$
$\lambda_2$  Second largest eigenvalue of the tensor $S^2 + \Omega^2$ (Jeong & Hussain, 1995)
$\lambda_x$  Streamwise wavelength
$\mu$  Dynamic viscosity
$\mu_m$  Micrometer, $1 \times 10^{-6}$ m
$\nabla$  Del operator
$\nu$  Kinematic viscosity
$\omega$  Complex frequency, $ck_x$
$\omega_x, \omega_y, \omega_z$  Vorticity components in $x$, $y$ and $z$ directions
$\Omega_x, \Omega_y, \Omega_z$  Mean vorticity components in $x$, $y$ and $z$ directions
$\Omega_{ij}$  Anti-symmetric velocity gradient tensor
$\phi$  Pseudo-pressure
$\Phi$  Mean streamwise pressure gradient
$\phi_x, \phi_z$  Spatial phase in $x$ and $z$ directions
$\pi$  $= 3.141592654 \ldots$
$\Psi$  SHS solid fraction
$\rho$  Density
$\rho_{c_\beta}$  Vortex core population density
$\sigma_x, \sigma_z$  Disturbance lengthscales in the $x$ and $z$ directions
$\tau_{\mu}$  Viscous shear stress
$\tau_{\text{turb}}$  Turbulent shear stress
$\tau_{\text{tot}}$  Total shear stress
$\theta_C, \theta_N, \theta_W$  Contact angles for Cassie-state, no-slip and Wenzel-state
$\xi$  Streamwise component in general non-orthogonal coordinate system
$\zeta$  Spanwise component in general non-orthogonal coordinate system
Chapter 1

Introduction

1.1 Motivation

With an ever-increasing shortage of energy resources facing the planet today, a sustained effort is required to research, develop and implement technologies aimed at minimising energetic losses, motivated by both environmental and economic savings. In the vast majority of engineering flows, energy is expended due to a phenomenon referred to as drag. Drag is mechanical force that is generated by the interaction and contact of a solid body with a fluid (liquid or gas) which can manifest itself in various forms. Pressure drag, for example, arises due to the shape or form of an object and can be suppressed by reducing cross-sectional area or by appropriate streamlining. The focus of this dissertation, however, is skin-friction drag. Skin-friction drag arises from the interaction between a fluid and its surroundings, be it a micro-channel, an oil pipeline, or the hull of an oceangoing vessel. The majority of engineering flows are turbulent and, relative to laminar flows, experience increased levels of skin-friction. High levels of skin-friction are undesirable, resulting in poor efficiency, high operational costs and increased structural fatigue. Therefore, the ability to manipulate turbulent flow to effect a beneficial change in skin-friction is of immense technological importance and is beneficial towards a sustainable future society.

1.2 Drag reduction

The hydrodynamic drag that exists at a fluid-solid interface is of significance to the design of a broad spectrum of mechanical devices that involve a semi- or fully-bounded fluid flow. In laminar flow without flow separation, viscous effects are the dominant contribution towards drag. For a fluid moving past a impermeable plane solid surface, the local shear stress exerted against the wall is due to the product of the fluid viscosity, \( \mu \), and the wall-normal gradient of the streamwise velocity, \( Du/Dy \). The total laminar drag, \( D_{lam} \), exerted by the solid on the fluid is equal to the
integral of the wall shear stress over the contact area,

\[ D_{lam} = \int_A \mu \frac{du}{dy} \, dA_w. \]  

(1.1)

For a given laminar fluid, equation 1.1 demonstrates that the levels of drag may be reduced by either decreasing the wall shear stress or the contact area between the fluid and the surface. Alternatively, the viscosity of the fluid can be altered by, for example, shifts in temperature or the addition of chemical agents.

For fully developed laminar flow in a channel flow geometry (see figure 1.1) the shear stress is distributed linearly and is greatest at the walls and equal to zero at the channel half-height. If the velocity gradient at the walls is decreased, the shear stress remains linearly distributed but the slope of the shear stress distribution is decreased. For fully developed turbulent channel

Figure 1.1: Schematic of typical channel flow geometry. By computing the bulk velocity, \( U_b = \frac{1}{2\delta} \int_0^{2\delta} U(y) \, Dy \), and the channel half-height, \( \delta \), the bulk Reynolds number can be defined as \( Re_b \equiv \frac{U_b \delta}{\nu} \), where \( \nu \) is the kinematic viscosity.

flow, the transport of momentum due to fluctuating fluid motion dominates, except in the very near-wall region where viscous transport becomes important. Turbulent transport arises due to the development and sustenance of three-dimensional time-dependent motions in the flow, an inherent feature of any turbulent flow. As in the case for laminar flow, the drag experienced at the walls is due to the product of the viscosity and the wall shear stress, integrated across the contact area. The total local shear stress, \( \tau_{tot} \), acting in the streamwise direction in a turbulent flow is comprised of a viscous shear stress and turbulent shear stress,

\[ \tau_{tot} = \mu \frac{d\langle u \rangle}{dy} - \rho \overline{\langle u' v' \rangle}, \]  

(1.2)

where \( \langle \cdot \rangle \) denote an appropriately averaged value. The viscous shear stress, \( \tau_{\mu} \), is the product
of the fluid viscosity, $\mu$, and the streamwise velocity gradient, $d\langle u\rangle/dy$. The turbulent shear stress, $\tau_{\text{turb}}$, is the product of the fluid density, $\rho$, and the covariance of the stochastic velocity components $u'$ in the streamwise ($x$) and $v'$ in the wall-normal ($y$) directions. The term $\langle u'v' \rangle$ is typically referred to as the Reynolds shear stress and is responsible for the distortion of the mean streamwise turbulent velocity profile, relative to the laminar parabolic profile. The wall shear stress normalised by a reference velocity is called a skin-friction coefficient. By evaluating equation 1.2 at the wall and normalising it by the bulk velocity the skin-friction coefficient, $\langle C_f \rangle$, can be written as,

$$\langle C_f \rangle \equiv \frac{\tau|_{w}}{\frac{1}{2} \rho U_b^2},$$

(1.3)

where $\tau|_{w}$ denotes the value at the wall and where $U_b$ denotes the bulk velocity.

The distribution of both the viscous and turbulent shear stresses for a fully developed turbulent channel at bulk Reynolds number $Re_b \equiv U_b \delta / \nu = 2800$ are illustrated in figure 1.2a as a function of the wall-normal position. At the channel half-height ($y = \delta$) both components of shear stress vanish. In the near-wall region ($y \to 0$) the viscous shear stress dominates, but decreases with increasing $y$ whereas the turbulent shear stress becomes more influential. In fully developed turbulent channel flow, the total shear stress varies linearly across the channel, in a similar manner to laminar channel flow. From figure 1.2 it is evident that, close to the wall, the viscosity $\nu$ and wall shear-stress $\tau_w$ are particularly significant parameters. Using these quantities, along with density $\rho$, appropriate viscous scales can be defined that form the relevant velocity and lengthscales for fluid in the near-wall region. The appropriate velocity scale for the
inner-region is referred to as the friction velocity which is defined by,

\[ u_\tau \equiv \sqrt{\frac{\tau_w}{\rho}}, \]  

(1.4)

whereas the appropriate lengthscale is referred to as the viscous lengthscale which is defined by,

\[ \delta_\nu \equiv \nu \sqrt{\frac{\rho}{\tau_w}} = \frac{\nu}{u_\tau}. \]  

(1.5)

The Reynolds number based on the viscous scales is defined by

\[ Re_\nu \equiv \frac{u_\tau \delta_\nu}{\nu}, \]  

(1.6)

which, due to definitions of friction velocity \( u_\tau \) (equation 1.4) and the viscous lengthscale \( \delta_\nu \) (equation 1.5), is identically unity, whereas the friction Reynolds number defined by,

\[ Re_\tau \equiv \frac{u_\tau \delta}{\nu} = \frac{\delta}{\delta_\nu}. \]  

(1.7)

where \( \delta \) is the channel half-height.

At a particular wall-normal distance from the wall, the relative importance of viscous and turbulent processes can be determined by computing,

\[ y^+ \equiv \frac{u_\tau y}{\nu} \]  

(1.8)

where \( y^+ \) is the distance from the wall measured in wall units. Wall units can be used to identify different regions, or layers, in near-wall turbulent flow. Figure 1.2b shows the distribution of the total shear stress for a fully developed turbulent channel as a function of \( y^+ \), where a logarithmic axis has been used to clarify near-wall behaviour. For \( y^+ < 5 \), the turbulent shear stress is negligible compared with the viscous stress. Whereas, for \( y^+ > 30 \), the viscous shear stress is negligible compared to the turbulent shear stress. By minimising the shear stress, the frictional resistance at the wall can be reduced. The aim of flow control strategies, several of which are discussed in the next section, is to reduce drag or skin-friction in both the laminar and turbulent regimes.
1.3 Literature survey

1.3.1 Flow control

The manipulation of a flow field to effect a beneficial change is of immense practical importance. Typically, this involves forcing a fluid from its natural state to an alternate state in order to achieve some engineering goal, a process referred to as flow control. A successful flow control strategy could achieve engineering goals such as enhanced mixing, augmentation of heat transfer, suppression of noise and pollution, increased lift and manoeuvrability, or, the focus of this work, a reduction of turbulent skin-friction drag. The challenge in selecting a flow control scheme is to accomplish a beneficial goal at a minimum cost, without compromising any other goals.

The roots of flow control strategies can be traced back several thousand years, where the accuracy and aerodynamics of prehistoric weaponry were empirically optimised using fin-stabilisers and streamlining. In order to account for the absence of rigorous mathematical theory and sophisticated experimental apparatus, a trial and error approach had to be adopted in order to develop these flow control strategies.

The scientific study of flow control rose to prominence in the early twentieth century, where the application of physical principles and fundamental fluid mechanics became an integral part of their design. Prominent examples of this scientific approach towards flow control included the pioneering work of Prandtl (1904), who proved the validity of his boundary-layer theory by conducting a series of appropriately designed experiments. By applying suction through a narrow slit on one side of a cylinder, Prandtl demonstrated that the boundary layer on the controlled suction side remained attached for longer compared to the boundary layer on the opposite uncontrolled side. As a result of the delayed separation, the drag on the suction side was reduced. Prandtl clarified the behaviour of viscous fluid flow in the vicinity of solid boundaries and his analytical theories, supported by compelling experimental evidence, proved that the resulting boundary layer could be controlled in order to achieve desirable engineering goals and these results represent the origin of the modern-era of flow control.

The research and development of flow control strategies intensified throughout World War II and the Cold War, where militaries demanded shorter response times and increased efficiencies across almost all of their aeronautical and maritime arsenals including aircraft carriers, battleships, submarines, torpedoes, aircraft and missiles. During this period drag reduction by polymers was discovered in the laboratory by Toms (1948), where the addition of a small amount of a long-chained polymer into a turbulent Newtonian solvent (parts per million by weight) was shown to reduce turbulent skin-friction by up to 80%, relative to the regular no-slip channel. The impressive drag reductions offered by the addition of long-chain polymers were successfully field-tested by Sellin and Ollis (1980), whose tests in a 760mm diameter sewer showed a 20%
drag reduction over a pipe with a total length of 8km.

Another notable flow control strategy that emerged during this period was laminar flow control (LFC). The goal of LFC is to force a boundary layer to remain laminar beyond Reynolds numbers which typically characterise the onset of transition to turbulence, resulting in prolonged levels of lower, laminar skin friction. Typically, the laminar run of the boundary layer is lengthened by using wall-normal suction through porous surfaces or arrays of specially designed slots. For example, Braslow et al. (1948) tested a porous bronze aerofoil, with a chord length of 3-ft, in a subsonic wind tunnel. When suction was activated, laminar flow was observed over 83% of the total chord length. In a subsequent study, Braslow et al. (1951) used the same model, but with the suction turned off, and observed laminar flow over only one third of the aerofoil, confirming that the laminar run of the boundary layer was significantly increased in the presence of suction. After several years of development, the National Aeronautics and Space Administration (NASA) flight-tested a purpose-built experimental aircraft, the Northrop X-21A, designed specifically to investigate the industrial viability of LFC. The X-21A program ran from 1960-1965 and more than two-hundred LFC flight-tests were completed from 1963-1965. During the last year of the X-21A flight-tests, laminar flow was realized up to 96, 81, and 59 percent chord for Reynolds numbers (based on free-stream velocity and chord-length) of $2 \times 10^7$, $3 \times 10^7$, and $4 \times 10^7$, respectively. A thorough overview of LFC projects including results from numerous wind-tunnel and full-scale flight-tests spanning some six decades can be found in the NASA review article by Joslin (1998). Although several of the X-21A flight-tests were extremely successful, the suction-slots required to control the boundary layer were susceptible to contamination by the build-up of insects, debris and ice-crystals that often prematurely “tripped” the laminar flow into a fully turbulent state. As a consequence of the laminar-turbulent transition, the levels of skin-friction increased dramatically which, ultimately, led to the X-21A project being discontinued in 1968.

An alternate flow control strategy is to relaminarize an initially turbulent flow. The relaminarization of a turbulent flow could be mistaken for a type of LFC. However, the flow physics of maintaining an initially laminar flow and forcing an initially turbulent flow back into a laminar state are quite different. Although the goal for both LFC and relaminarization flow control strategies is to ensure a laminar state, the energy required to relaminarize a turbulent flow far exceeds the energy required to maintain a laminar flow. Relaminarization can, for example, be achieved by imposing a severe favourable pressure gradient — an approach that has been investigated experimentally by Patel and Head (1968).

There are various ways to classify flow control strategies. One possibility is to consider where the control itself is physically applied — usually either directly at a bounding surface or some distance away from it. For example, surface parameters including wall-normal suction and blowing, temperature and roughness can all influence the flow. Suction and blowing can alter the
shape of the near-wall velocity profile, which could lead to transition or separation or delay these events/states. Temperature can influence the flow through the resulting gradients in viscosity and density. Localised roughness can prematurely “trip” laminar flow into a turbulent state or be designed to energise the flow to delay separation of suppress particular instabilities. Control methods away from the wall include electro- and magneto-hydrodynamic body forces or acoustic forcing.

A second classification procedure is determined by the energy requirements of control schemes. Using this classification, flow control strategies can either be passive, requiring no auxiliary power, or active, which require an external supply of energy. However, before attempting to control a turbulent flow, an understanding of turbulence itself and, in particular, how it behaves in the near-wall region is required.

1.3.2 Wall-bounded turbulent flow

Above a critical Reynolds number, or in the presence of sufficiently strong instabilities, a laminar flow transitions to turbulence. Turbulence is characterised by fluctuating disorderly motions in time and all three spatial directions which span a broad spectrum of time- and length-scales. Many flows in industry and in nature are turbulent. However, not all turbulent flows are alike. For example, a turbulent boundary layer and a turbulent jet display certain differences, which, for the most part, can be credited to the differences of flow geometries. In this work, focus is drawn to the dynamics of turbulence in the proximity of solid boundaries. Near-wall turbulence behaves quite differently compared to, say, the core regions of a turbulent channel flow or at the edge of turbulent boundary layer flow. Although turbulence is characterised by chaotic motions, its spatial structure should not be regarded as being entirely random. This is particularly true in the case of near-wall turbulence, where the stochastic field exhibits clear spatial structures known as coherent structures (Cantwell, 1981). Conceptually, coherent structures are an attractive idea since they break the disorderly, multi-scale turbulent motions into more elementary organised motions which, upon careful interpretation, can be used to obtain physically meaningful results. However, due to the inherent complexity of turbulent flows, rigorous definitions of eddies or coherent structures do not exist. To emphasise this point, consider the following three different definitions of coherent structures, which span two decades,

1. Hussain (1986): “A connected turbulent fluid mass with instantaneously phase-correlated vorticity over its spatial extent... Thus, coherent vorticity is the primary identifier of coherent structures, which have distinct boundaries and independent territories.”

2. Robinson (1991): “A three-dimensional region of the flow over which at least one fundamental flow variable (velocity component, density, temperature, etc.) exhibits significant
correlation with itself of with another variable over a range of space and/or that is significantly larger than the smallest local scales of flow”

3. Adrian (2007): “These motions can be thought of as individual entities if they persist for long times, i.e., if they possess temporal coherence. By virtue of fluid continuity, all motions possess some degree of spatial coherence, so coherence in space is not sufficient to define an organised motion. Only motions that live long enough to catch our eye in a flow visualisation movie and/or contribute significantly to time-averaged statistics of the flow merit the study and attention we apply to organised structures.”

Using the above excerpts, the basic concept of coherent structures can be thought of as localised patches of organised motion in an otherwise unorganised field. Ultimately, a complete understanding of both the kinematic (size, shape, energy, etc..) and dynamic (origin, stability, contribution to statistics, etc...) properties of these motions is desired. The behaviour of coherent structures in the proximity of solid boundaries is of particular interest to the current work, since their modification in this region may help achieve engineering goals such as the reduction of skin-friction. Figure 1.3 shows contours of turbulence kinetic energy for a fully developed turbulent flow at four different wall-normal locations in a plane channel geometry. At the channel half-height, the turbulence kinetic energy is at its lowest and appears to be incoherent. Moving closer to the wall, the turbulence kinetic energy begins to increase. The increase in energy indicates that, relative to the half-height, additional turbulence must be produced in this region. Very close to the wall, the turbulence kinetic energy is predominantly aligned in elongated patterns.

Figure 1.3: Contours of turbulence kinetic energy, $k = 0.5 \langle u'_i u'_j \rangle$, in fully turbulent plane channel flow at four different wall-normal positions that including (a) $y^+=180$; (b) $y^+=100$; (c) $y^+=30$ and (d) $y^+=5$. Contours are clipped at $k \in [0, 0.015]$. 
along the streamwise direction in a clear, regular manner. A taxonomy of coherent structures was provided by Robinson (1991) and presents a useful framework for relating various structural features of turbulence. Included in the original classification of Robinson (1991) were

1. High- and low-speed streaks,
2. Ejections of low-speed fluid outward from the wall, including lifting low-speed streaks,
3. Sweeps of high-speed fluid inward toward the wall, including inrushes from the outer region,
4. Vortical structures of various forms.

Figure 1.4: Coherent structures in a turbulent channel flow at $Re\tau \approx 180$ including low- and high-speed streaks (left) and vortical structures (right) visualised using isosurfaces of $\lambda_2 = -0.03$ (Jeong & Hussain, 1995) including (a) plan-view and (b) side-view. All quantities normalised using bulk velocity, $U_b$.

Near-wall streaks are regions of relatively slow-moving fluid, which convect downstream at approximately half the speed of the time-averaged turbulent flow, and are visualised in figure 1.4a. Numerous flow-visualisation experiments educed low-speed streaks, most notably the hot-wire techniques employed by Kline et al. (1967). In addition, by injecting coloured dye into a turbulent boundary layer, the experimental results of Kline et al. (1967) revealed a characteristic behaviour of the streaks, known as bursting. The bursting process begins with streaks that slowly move away from the wall as they convect downstream, until, at some critical wall-normal position, they move away from the wall in a much more violent fashion. The lifting of low-speed streaks away from the wall is referred to as an ejection. Visual evidence of inclined low-speed streaks are provided in figure 1.4b, and are prominent in the region $0 < y^+ < 50$. Ejected streaks then tend to undergo a rapid oscillatory behaviour and eventually breakdown (i.e. burst) into finer-scale motions.

Mass conservation demands that ejections of low-momentum fluid away from the wall must be balanced by an inrush of high-momentum fluid towards the wall. This complimentary event
was identified in the laboratory by Corino & Brodkey (1969), and is referred to as a *sweep*. The significance of sweeps and ejections is that they represent regions of flow where streamwise and wall-normal velocities are anti-correlated. The time-averaged effect of this anti-correlation gives rise to Reynolds shear stress, which is effectively the averaged effect of turbulent convection. The action of the Reynolds shear stress against the mean flow tends to distort the time-averaged velocity profile in a manner which increases skin-friction, relative to the laminar parabolic profile. Therefore, a common goal in many flow control strategies is to attenuate sweep and ejection events.

Vortical structures are a prominent feature of wall-bounded turbulence. Vortices with an orientation other than wall-normal have the potential to function as a “pump” that transports mass and momentum across the mean velocity gradient (Robinson, 1991) (see figure 1.5). Depending on the sign of vorticity, this pumping motion can be attributed to either sweep or ejection events. Unfortunately, the study of vortex structures in turbulent flows is complicated by the lack of a rigorous definition of a vortex in unsteady, viscous flows. Three-dimensional vortices are extremely difficult to characterise in the laboratory, but the advent of numerically simulated turbulence has permitted advances in this area. However, even though numerical databases of fully resolved three-dimensional turbulent flows have been widely available for some time now (since Kim *et al.* (1986)), both vortex-identification techniques and the definition of a vortex are still matters of debate. Here, vortical structures were identified using the $\lambda_2$ criterion, which was first proposed by Jeong & Hussain (1995). The $\lambda_2$ criterion defines a vortex in terms of the eigenvalues of the symmetric tensor $S^2 + \Omega^2$, where $S$ and $\Omega$ are the symmetric and anti-symmetric parts of the velocity gradient tensor, $\nabla u$. A top view of an isosurface of $\lambda_2 = -0.03$ in the range $0 < y^+ < 60$ is plotted in figure 1.4a. The dominant vortices are predominantly aligned along the streamwise direction. Compared to the low-speed streaks (also included in figure 1.4a), the vortical structures are individually shorter, but tend to tangle with one another to form chains of overlapping vortices that can approach the length of the streaks. A side view of vortical structures, shown

![Figure 1.5: Schematic diagram of counter-rotating vortical structures inducing sweep and ejection events including (a) streamwise-aligned vortical structure and (b) spanwise-aligned vortical structures.](image)
in figure 1.4b, reveals that they are positively inclined in the streamwise direction, in a similar manner to the streaks, which are also included in figure 1.4b.

However, it is important to mention that the interrelationship between streaks and vortices may not be as intuitive as figure 1.4 readily suggests. For example, Chernyshenko et al. (2006) examined the streak-eddy relationship by studying an artificial instantaneous turbulent velocity field described by the following system of equations,

\[
\begin{align*}
\mathbf{u}(x, z) &= \begin{bmatrix} 14.06 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 2.889 \cos(0.02\pi z + s_{\text{rand}}) \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} u_{\text{rand}}(x, z) \\ v_{\text{rand}}(x, z) \\ w_{\text{rand}}(x, z) \end{bmatrix}.
\end{align*}
\] (1.9)

In the above system of equations 1.9, \( u_i = (u, v, w) \) represent the streamwise, wall-normal and spanwise velocity components, respectively, \( x \) and \( z \) are the streamwise and spanwise coordinates, \( s_{\text{rand}} \) is a random quantity uniformly distributed over \([0, 2\pi]\) and the vector field \((u_{\text{rand}}, v_{\text{rand}}, w_{\text{rand}})\) is random, homogeneous and isotropic. The first term on the left-hand side of equation 1.9 represents the average streamwise velocity. The second term represents spanwise variations in the streamwise velocity, which characterise the streaky structures of near-wall turbulent flow (see figure 1.4a). The last term represents the three-dimensional turbulent fluctuations.

Chernyshenko et al. (2006) detected eddy motions on an \( xz \) plane of their synthesised turbulent velocity field (equation 1.9) at \( y^+ = 36 \) (at a friction Reynolds number of \( Re_\tau = 360 \)) by computing the swirling strength parameter,

\[
S_{\text{swirl}} = \left( \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} \right)^2 - 4 \left( \frac{\partial u}{\partial x} \frac{\partial w}{\partial z} - \frac{\partial w}{\partial x} \frac{\partial u}{\partial z} \right),
\] (1.10)

where large negative values of \( S_{\text{swirl}} \) were used to distinguish the eddy structures. After computing \( S_{\text{swirl}} \), it was noted that the eddies had a tendency to align themselves horizontally in the neighbourhood of the streaks (similarly in figure 1.4). However, Chernyshenko et al. (2006) pointed out that the inclination of eddies to align themselves along the edges of streaky structures could be explained by the definition of swirling strength parameter itself, as opposed to a dynamically significant mechanism directly connecting the streaks and vortical structures. Upon substitution of the two-dimensional continuity equation,

\[
\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0,
\] (1.11)
into the swirling parameter equation 1.10, one can write

\[ S_{\text{swirl,2D}} = 4 \left( \frac{\partial u}{\partial x} \right)^2 + 4 \left( \frac{\partial w}{\partial x} \frac{\partial u}{\partial z} \right). \]

(1.12)

Due to the squared term on the right-hand side of equation 1.12, it follows that if \( S_{\text{swirl,2D}} < 0 \) then \((\partial w/\partial x)(\partial u/\partial z) < 0\). Hence, if \( S_{\text{swirl,2D}} < 0 \), an increase in \( |\partial u/\partial z| \) results in an increase in \( |S_{\text{swirl,2D}}| \). In other words, this analysis shows that the observed relation between streaks and vortices may simply be due to a numerical artefact that originates from the definition of the swirling strength parameter (equation 1.12). The studies of Chernyshenko et al. (2006) emphasise that care must be taken when analysing and interpreting instantaneous turbulent velocity fields and, in particular, when trying to understand the dynamics and interplay of near-wall turbulent structures.

However, the basic qualitative similarities between streaks and vortices still raise questions about their origin, interplay and maintenance and has been, and continues to be, thoroughly researched. Several conceptual models describe a regeneration or autonomous cycle (Hamilton et al., 1995; Jimenez & Pinelli, 1999; Schoppa & Hussain, 2002), whereby vortical structures are generated by low-speed streak instability (nonlinear amplification and breakdown), with the new vortical structures regenerating new streaks. A recent linear analysis, however, suggests that streaks can arise from wall-normal velocity perturbations alone and do not require the action of vortical structures (Chernyshenko & Baig, 2005).

Although the analysis of instantaneous turbulent structures is fundamentally important, it is unlikely that a quantitative theory based on the dynamical interactions of a handful of structures which fully describe the physics of near-wall turbulence will be achieved. From an engineering perspective, the averaged effects of instantaneous events prove more useful and hence a statistical description of the turbulent flow is desirable. Various statistical quantities can be used to describe turbulent velocity fields including means, variances, probability density functions, two-point correlations and spectra — each of which can then be related back to the relevant instantaneous motions to provide a more complete understanding. The statistical approach begins with the Navier-Stokes equations,

\[ \frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{1}{Re_\lambda} \frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_j}, \]

and

\[ \frac{\partial u_j}{\partial x_j} = 0. \]
Using the Reynolds decomposition of the instantaneous velocities and pressure

\[ u_i = \langle U_i \rangle + u'_i, \]
\[ p = \langle P \rangle + p', \]

and substituting into the governing equations, and averaging, one recovers the Reynolds-Averaged-Navier-Stokes equations,

\[ \frac{\partial \langle U_i \rangle}{\partial t} + \langle U_j \rangle \frac{\partial \langle U_i \rangle}{\partial x_j} = \frac{\partial \langle P \rangle}{\partial x_i} + \frac{1}{Re_b} \frac{\partial^2 \langle U_i \rangle}{\partial x_j \partial x_j} - \frac{\partial}{\partial x_j} \langle R_{ij} \rangle \]

where the braced term, \( \langle R_{ij} \rangle \), is the Reynolds stress tensor. On one hand, averaging the governing equations means that a significant amount of information which was once known has now been lost, and, on the other hand, new information has been introduced which is now unknown. However, in an attempt to remedy this situation, a dynamical equation for the Reynolds stress tensor can be derived as,

\[ \frac{\partial}{\partial t} \langle u'_i u'_k \rangle + \langle U_j \rangle \frac{\partial}{\partial x_j} \langle u'_i u'_k \rangle = - \langle u'_k u'_j \rangle \frac{\partial \langle U_i \rangle}{\partial x_j} - \langle u'_i u'_j \rangle \frac{\partial \langle U_k \rangle}{\partial x_j} + \langle u'_i \frac{\partial p'}{\partial x_j} \rangle + \langle u'_k \frac{\partial p'}{\partial x_i} \rangle + \frac{1}{Re_b} \frac{\partial^2 \langle u'_i u'_k \rangle}{\partial x_j \partial x_j} - \frac{2}{Re_b} \frac{\partial}{\partial x_j} \langle \partial \langle u'_i \rangle \partial \langle u'_k \rangle \rangle - \frac{\partial}{\partial x_j} \langle u'_k u'_i u'_j \rangle \]

which, altogether, has created six new equations and, after accounting for all tensor symmetries, twenty-two new unknowns. This illustrates the closure problem of turbulence. Basically, because of the nonlinearity of the Navier-Stokes equations, additional unknowns are generated as higher and higher moments are computed.

How then can the turbulence problem be tackled? The Navier-Stokes equations are nonlinear partial differential equations in almost every real engineering flow. The non-linearity is due to convective acceleration, which is an acceleration associated with changes in velocity over position. Therefore, any convective flow, whether laminar or turbulent, will be nonlinear. Under laminar flow conditions, however, various closed-formed solutions of the Navier-Stokes do exist. These include, for example, the boundary layer equations (which are nonlinear), entry flow into a channel, as well as many fully developed laminar flows in canonical flow geometries. Unfortunately, however, closed formed solutions for any turbulent flows do not exist. An experimental approach can be adopted in order to circumvent the turbulence problem, but this relies on intricate and expensive experimental apparatus which often under resolve the smallest, dissipative turbulent scales. A complete description of a turbulent flow, where the both the velocities and pressure are known as a function of space and time, can only be obtained by solving the Navier-Stokes equations numerically. This type of numerical simulation is referred to as direct numerical simulation.
(DNS) and it is this approach that is used throughout this dissertation.

1.3.3 Example of passive control: Riblets

Passive flow control strategies require no auxiliary power and typically rely on topological or morphological changes to bounding surfaces in order to influence the flow. Riblets are a form of passive flow control which can reduce drag by up to 10% (Walsh, 1982). Physically, riblets consist of micro-grooves of the height of the viscous sublayer with triangular or semi-circular cross-section. Practical use of the riblets was demonstrated by the U.S. men’s rowing team at the Los Angeles Olympic Games in 1984, the victorious Stars and Stripes yacht in the 1987 America’s Cup, and on swimsuits used in the Sydney Olympic Games in 2000, worn by many record-breaking athletes (Karniadakis & Choi, 2003). In 1988, a flight test demonstration of riblets was successfully performed. By covering 700m$^2$ of the skin of an A320 passenger airliner, in regions where the flow was turbulent, fuel consumption was reduced by approximately 2%.

Walsh (1982) was among the first to systematically investigate the use of riblets for turbulent drag reduction and showed that skin-friction could be reduced by choosing a nondimensional riblet spacing $s^+ = su_r/\nu$ of approximately 15 wall units. However, a performance degradation was observed when $s^+ > 30$. As well as being sensitive to spacing, an additional off design condition is the alignment of riblets with the local flow direction — the riblet yaw angle. In fact, Gaudet (1987) demonstrated that at a yaw angle of approximately 30 degrees riblets offer no drag reduction, and beyond this angle a drag penalty is incurred due to local boundary layer separation. The near-wall turbulent structures have been investigated in the laboratory by, for example, Walsh (1990) and Tardu (1995). These experimental studies were complemented by DNS conducted by Chu & Karniadakis (1993), Choi et al. (1993) and Goldstein et al. (1995). A

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{riblets.png}
\caption{Schematic diagram of drag reduction and increase mechanisms by riblets: (a) $s^+ \approx 20$ - drag reduction (limited area affected by downwash motion); (b) $s^+ \approx 40$ - drag increase (extensive area affected by downwash motion). Figure and caption adapted from Choi et al. (1993).}
\end{figure}
simple drag reduction mechanism for riblets was proposed by Choi et al. (1993), who investigated two different riblet geometries: one, which reduced drag by 10%, with a narrow spacing of $s^+ \approx 20$ and the other, which increased drag by 6%, with a wider spacing of $s^+ \approx 40$. From the results database of Kim et al. (1986), Choi et al. (1993) estimated that the centres of the streamwise vortices were located on average at $y^+ \approx 20$, and that their average diameter was $d^+ \approx 30$. Since the average diameter of the streamwise vortices above the wall is larger than the spacing of the riblets with $s^+ \approx 20$, the vortices cannot freely move inbetween the riblets and thus only a limited area of the riblet tips is exposed to the downwash of high-momentum fluid (i.e. sweep events) induced by the vortical pumping motions. On the other hand, for the wider spaced riblets with $s^+ \approx 40$, the vortices are free to move deep inside the riblet geometry and a larger surface area is exposed to sweeping motions. A schematic of these two behaviours is provided in figure 1.6. Relative to a planar no-slip wall, the effective surface area for both riblet spacings is increased. In the case of $s^+ \approx 20$, only a small portion of the riblet topology is susceptible to downwash events and therefore skin friction per unit area is reduced. The opposite is true for the $s^+ \approx 40$ case, where the wider spacings expose a higher percentage of surface area to drag-inducing events which are governed by vortical motions.

By examining the effects of riblet spacing and geometry on a turbulent boundary layer, Goldstein & Tuan (1998) used DNS to explain why riblets lost their drag-reducing capacity when spaced to widely. Goldstein & Tuan (1998) suggested that the performance drop-off of wider riblet geometries could be credited to the generation of secondary streamwise vorticity over the riblets, which shed small-scale vortices thus creating extra dissipation.

Contrary to the conclusions of Choi et al. (1993), Mayoral & Jimenez (2011) argued that the worse performance of widely-spaced riblets was not due to vortices becomings lodged in the riblet valleys (see figure 1.6b). Instead, it was argued that the disappearance of a transverse recirculating region in the riblet valleys lead to an increase of the Reynolds stresses at the plane of the riblet tips that increased drag.

In summary, riblets are a well-documented, well-researched passive flow control strategy that have been successfully implemented across a wide range of engineering flows. Depending on operational conditions, their drag reduction ranges from approximately 2% to 10%. A vast amount of effort has been expended in both experimental and numerical environments in order to assess how riblets influence the structure of near-wall turbulence.

### 1.3.4 Example of active control: Spanwise wall movement

Active flow control strategies require an auxiliary power source in order to effect any changes on the flow. The additional power can be used to drive various mechanical devices that induce, for example, local surface deformations (Kang & Choi, 2000) or suction and blowing (Choi et al.,
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A drawback of these active control strategies is that they only affect local regions of the near-wall turbulence at any one time. Practically relevant strategies should aim to encompass extensive portions of the near-wall flow and, ideally, should be global in nature. A demonstrably effective approach is to impose in-plane wall motions, which can be spatially harmonic or uniform, which affects the entire near-wall layer.

Transient reductions in skin-friction were obtained by imposing a near-wall pressure-gradient, which induced spanwise motion on an initially streamwise-aligned boundary layer in the laboratory by Bradshaw & Pontikos (1985). Howard & Sandham (2000) used DNS to investigate the response of a turbulent channel flow at \( Re_\tau = 180 \) subjected to a suddenly imposed spanwise, unidirectional wall motion and demonstrated that the skin-friction reduction could be statistically linked to a suppression of turbulence kinetic energy in the near-wall region. However, the effect persists only for a short time. An analysis of instantaneous flow fields highlighted that the skewing motion weakened and flattened streamwise vortices. A steady-state drag reduction was demonstrated using DNS by Jung et al. (1992), whereby unsteady, high-frequency transverse wall-oscillations were imposed in a turbulent channel flow at \( Re_\tau = 200 \). Jung et al. (1992) imposed the spanwise forcing using

\[
W_{wall} = 0.8 \left( \frac{Q_x}{2\delta} \right) \sin \left[ \frac{2\pi t}{T} \right]
\]

where \( Q_x \) is the (fixed) flow rate per unit width in the streamwise direction, \( \delta \) is the channel half-height and \( T \) is forcing period. The magnitude of drag reduction was found to be strongly dependent on the oscillation frequency. The largest drop obtained was when \( T^+ = 100 \), where \( T^+ = T u_\tau^2/\nu \), where skin-friction reduced by 40% compared to the unforced turbulent channel. At this optimal forcing frequency, the wall-normal, spanwise and Reynolds shear stresses were all suppressed by at least 30%, whereas the streamwise Reynolds stress dropped by only 14%. In addition, isosurfaces of vorticity magnitude confirmed the suppression of near-wall vortical structures. Since the seminal simulations of Jung et al. (1992), much subsequent work, both of computational and experimental nature, has been undertaken in order to investigate the mechanics of drag reduction due to transverse wall oscillations. Previous experimental studies were conducted by Choi & Graham (1998), Di Cicca et al. (2002) and Ricco (2004) who utilized particle-image velocimetry, hydrogen-bubble technique and hot-wire anemometry, respectively, to investigate the effects of wall-oscillation at similar Reynolds numbers. At the optimal forcing frequency, Ricco (2004) reported a drag reduction of roughly 39%, in good agreement with previous numerical predictions of Jung et al. (1992), and his data suggested that the regeneration of turbulence was disrupted due to decreased inter-streak spacing and streak length.

Despite the impressive levels of drag reduction offered by spanwise wall oscillations, which are clearly superior to those offered by, say, riblets, there are major drawbacks concerning the
auxiliary power requirements of such flow control strategies, not to mention the complexity of their implementation and integration in industrial-scale flows. In addition, active flow control techniques would require routine electro-mechanical maintenance checks, retro-fitting and upgrading, all of which come at an additional cost, to ensure their efficient and safe operation. In the extensive parameter study conducted by Quadrio & Ricco (2004), the required power to oscillate the plate against the viscous flow above was often found to exceed the power savings recouped by skin-friction reduction, resulting in a negative net power saving. Out of the thirty-seven cases investigated by Quadrio & Ricco (2004), less than a third returned net power saving which was positive, with the worst-performing case having a power savings of $-716\%$ (minus seven-hundred and sixteen). The maximum net power saving was $7.3\%$, which is in fact worse than the maximum $10\%$ drag-reduction offered by riblets.

1.4 Hydrodynamic effects of textured superhydrophobic surfaces

A variety of plants and animals have textured surfaces on their bodies. Some textured surfaces evolved in order to improve adhesion, whilst others evolved remarkable water-repellent characteristics. The latter effect is referred to as superhydrophobicity, a material property that can be found on, for example, the wings of butterflies, the exo-skeletons of insects, and the leaves of several varieties of plant-life. The last category contains the most famous of all hydrophobic surfaces: the lotus leaf. As water droplets come into contact with a superhydrophobic surface (SHS), they tend to exhibit a characteristic “beading” behaviour, where, instead of spreading out, the droplets “sit up” and appear as near-perfect spheres. To illustrate this point, some images of water droplets in contact with SHS are shown in figure 1.7. As a result of the beading

![Figure 1.7: The Lotus Effect. Water droplet exhibiting superhydrophobic characteristics whilst resting upon a lotus leaf (left) and the exoskeleton of an Army ant (right).](image-url)
behaviour, the wetted area between the surface and the droplet can be extremely small. The reduction of wetted area is an attractive observation from an engineering perspective since, in the presence of bulk fluid motion, it could translate to large-scale skin-friction savings. However, before considering any potential performance savings that SHS may offer, a summary of their structure, properties and manufacture is provided.

### 1.4.1 Superhydrophobic surfaces

A superhydrophobic state is attained naturally or synthetically through a combination of surface topology and surface chemistry. Lotus leaves, for example, have micrometer-sized nubs covered in hydrophobic wax crystalloids (Barthlott & Neinhuis, 1997). For a textured SHS like that of a lotus leaf, water will either fully or partially wet the surface and this depends on the static pressure in the water, the hydrophobicity of the surface, and the texture topology. In the fully wetted scenario, water floods the cavities between neighbouring micro-features and this is referred to as the Wenzel state, after Wenzel (1936). In the partially wetted scenario, the hydrophobicity of the surface prevents the water from penetrating the cavities, resulting in an air-water interface being supported between the micro-features — referred to as the the Cassie state, after Cassie & Baxter (1944). The contact angle, $\theta$, can be used to quantify the degree of hydrophobicity of textures. Typically, droplets on top of SHS textures exhibit contact angles in excess of $150^\circ$. The schematic shown in figure 1.8 demonstrates that largest contact angles, as well as the smallest wetted area, are associated with the Cassie state.

![Figure 1.8: Various wetting scenarios. From left to right the contact angle, $\theta$, increases.](image)

Synthetic materials that exhibit hydrophobic behaviour are commercially available. Examples include Teflon® and Gore-Tex®, both of which attain a hydrophobic state due a chemical coating of Polytetrafluoroethylene (PTFE). However, to obtain a superhydrophobic state, a combination of surface chemistry and surface topology is required. Precision-engineered structured micro-geometries have been manufactured in the laboratory using a variety of techniques. For
example, Woolford et al. (2009b) used photolithographic methods to etch out micro-riblet geometries from silicon wafers. After fabrication the micro-riblets were characterised with scanning electro microscopy to be 15µm high, 8µm wide and spaced 32µm apart. To render the textures superhydrophobic, a coating of Teflon® was applied to each of the etched silicon wafers. Subsequent measurements by Woolford et al. (2009b) computed the contact angle to be in excess of 150°, confirming their superhydrophobicity. The manufacture of SHS in the laboratory is an important step towards their future implementation in industry. Their precision-engineered nature, in terms of feature widths and spacings, have permitted systematic and parametric studies to be conducted.

1.4.2 Modelling superhydrophobic surfaces

To permit the analytical and/or numerical study of SHS textures an appropriate surface model is required in order to enforce boundary conditions. The ideal surface model should be a fair physical representation of the naturally occurring SHS texture, and should be straightforward to implement in an analytical and numerical sense.

For viscous fluids, the no-slip boundary condition states that at a solid boundary, the fluid will have zero velocity relative to the boundary. However, as bulk fluid flows over a SHS texture, the no-slip boundary condition may only be valid locally and depends on how the texture is wetted. In the collapsed Wenzel state (see figure 1.8), the micro-topology becomes submerged and the no-slip boundary condition can be enforced everywhere. However, in the supported Cassie state (see figure 1.8), the no-slip boundary condition only applies to regions where the liquid comes in contact with the micro-features. Between the micro-features, an air-water interface is supported and, in the presence of bulk fluid motion a non-zero slip velocity is obtained.

To date, two surface models have received widespread attention. The first model is the Navier-slip boundary condition (Navier, 1823) which can be written mathematically as

\[ u_s = L_s \frac{\partial u}{\partial y} \mid_w. \]

where subscript \( s \) denotes a slip velocity. In Navier’s model, the slip velocity is proportional to the shear rate at the wall, where the constant of proportionality is the effective slip-length, \( L_s \). Geometrically, the Navier-slip boundary condition can be interpreted as a linear extrapolation of the velocity profile below the wall until the velocity satisfies the no-slip boundary, where the effective slip length \( L_s \) is the wall-normal distance required to satisfy the condition \( u = 0 \). A schematic of the Navier slip boundary condition is provided in figure 1.9. The main advantage of the Navier-slip model is its simplicity, since only the effective slip-length needs to be prescribed in order to enforce the boundary condition. However, the various topologies of physical SHS textures (see figure 1.10) cannot be accounted for by the Navier-slip model. The Navier-slip
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Figure 1.9: Schematic representation of the Navier slip model.

The model is a planar-integrated representation of a general texture topology which, in the presence of bulk fluid motion, admits a finite slip-velocity $\langle U_s \rangle_{xz}$ due to non-zero velocities which can occur at the air-water interfaces.

The second model uses a mixture of no-slip and free-slip boundary conditions to characterise the SHS textures. Philip (1972a) was the first to theoretically study Stokes flow passing over these mixed boundary conditions, although his motivation was to understand the flow through porous media and not necessarily drag reduction. The no-slip boundary condition is enforced at the surface of the micro-features, whereas the free-slip boundary condition is used to represent the air-water interfaces, which are assumed to be stress-free. In addition, the air-water interface is assumed to be flat (no meniscus curvature), so that the modelled SHS texture appears as a plane surface with patterned periodic boundary conditions. Figure 1.10 shows the arrangement of mixed boundary conditions for micro-ridge and micro-post topologies with a feature width $d$ and a feature spacing $g$, which are periodic in both the streamwise ($x$) and spanwise ($z$) directions. Compared to the Navier-slip model, the periodic no-slip and free-slip boundary conditions permit specific surface topologies to be characterised. In addition, the slip velocity is determined from

Figure 1.10: Schematic of mixed no-slip no-stress model applied to micro-ridge (left) and micro-post (right) surface topology.
the flow field solution for a particular texture topology, as opposed to being prescribed as an arbitrary effective slip-length.

Since the air-water interfaces are prescribed as shear-free surfaces, skin-friction can only occur across the micro-features where the flow satisfies the no-slip boundary condition. The ratio of the no-slip area to the total area is referred to as the solid fraction \( \Psi_s \) and, for the ridge and post topologies shown in figure 1.10, can be written as

\[
\Psi_{s,\text{ridges}} = \left(\frac{d}{d + g}\right) \times 100\%
\]

\[
\Psi_{s,\text{posts}} = \left(\frac{d}{d + g}\right)^2 \times 100\%
\]

which gives the percentage area of the total planar area which experiences non-zero skin-friction. As \( d \) tends to zero the solid-fraction increases until the no-slip boundary condition is enforced everywhere and \( \Psi = 1 \). One strategy to enhance drag reduction is to decrease the solid fraction. There is, however, a practical lower limit on the solid fraction. For liquid in the supported Cassie-state, there is a maximum static pressure that can be supported by the air-water interface before it catastrophically fails and the micro-cavities become flooded. At this point the SHS texture reverts to the collapsed Wenzel-state.

Both the Navier-slip and textured surface models have received widespread attention in the literature and have been implemented in numerous analytical and numerical studies. Advances in nano- and micro-fabrication methods have also permitted the study precision-engineered textured surfaces in the laboratory — under both laminar and turbulent flow conditions.

1.4.3 Experimental studies

The dynamics of water droplets upon SHS textures have been investigated thoroughly. These previous studies have explored the effect of micro-feature orientation, different surface chemistries, and the structural topology (Chen et al., 1999; Youngblood & McCarthy, 1999; Oner & McCarthy, 2000; Bico et al., 2002). Within the last decade, the effects of textured SHS on bulk fluid flow have received considerable attention, in particular the potential for such surfaces to reduce drag in both the laminar and the turbulent regimes.

Prior to the fabrication of precision-engineered SHS textures, a laminar drag reduction of 14% in a circular pipe with a highly water-repellent wall was observed in the laboratory by Watanabe et al. (1996). In this study, two pipe geometries were investigated: a narrow pipe with 6mm diameter and a wide pipe with 12mm diameter and both pipes were 475mm in length. Four pipes were manufactured in total: two of which were rendered water-repellent by the application of a porous hydrophobic silica (about 10\( \mu \)m in thickness), whereas the remaining two pipes were left in an untreated “no-slip” state. To demonstrate that their chemical coating induced a
hydrophobic state, Watanabe et al. (1996) measured the contact angle of a single water droplet on top of the treated wall and it was excess of 150°. The dimensions of the pipes were sufficiently large such that the initially laminar flow underwent a complete transition to turbulence, and the Reynolds number (based on hydraulic diameter, flow rate and cross-sectional area) spanned the range 100 < Re < 10000. By using a pressure transducer, a maximum laminar drag reduction of 14% was observed for the 12mm diameter pipe. The effect of the hydrophobic surface on the fully developed laminar velocity profile was characterised by using a hot-film anemometer and, for a fixed streamwise pressure gradient, the bulk velocity increased due to the finite slip velocity. By visualising the micro-structure of their hydrophobic surface using a scanning electron microscope, Watanabe et al. (1996) highlighted that although the porous structure of their water-repellent wall was not completely understood, it was hypothesised that the air-water interfaces played a central role in both the mechanisms of fluid slip and laminar drag reduction.

Ou et al. (2004) conducted a series of experiments which demonstrated significant drag reduction for a laminar flow of water through a microchannel using precision-engineered SHS textures. Both micro-riblet and micro-post geometries were considered and had feature sizes ranging from 20 to 30µm and feature spacings ranging from 15 to 150µm. The effectiveness of the SHS textures was found to increase linearly with increased feature spacing, and pressure drop reductions of up 40% were reported. In a subsequent study, Ou & Rothstein (2005) used micro-particle image velocimetry (µ-PIV) to characterise the local velocity field in laminar flow in microscale channel with SHS textures that consisted of longitudinally oriented micro-riblets. The µ-PIV resolved the flow below the lengthscale of the periodic microridge surface structure, and the velocity measurements demonstrated that a non-zero slip velocity was obtained across the air-water interface supported between the features. Slip velocities of up to 60% of the average velocity were measured across the air-water interface and the measurements of Ou & Rothstein (2005) demonstrated a direct correlation between the reduction in driving pressure and the increase in the slip velocity.

The level of laminar drag reduction has also been found to depend on the orientation of the micro-features relative to the primary flow direction. For a fixed spacing and feature width, SHS textures aligned parallel to the flow (Ou et al., 2004; Ou & Rothstein, 2005) offer superior performance to SHS textures aligned perpendicular to the flow (Davies et al., 2006; Byun et al., 2008). The wetting state also strongly affects the drag-reducing performance of SHS textures in laminar flow. The results of Woolford et al. (2009a) demonstrated that, for a fixed surface topology, superior drag reduction was obtained when the SHS texture remained in the supported Cassie state, compared to the collapsed Wenzel state.

An alternative use for SHS textures was investigated by Ou et al. (2007), who demonstrated their potential to enhance mixing in laminar flows whilst operating in the supported Cassie state. By aligning a micro-ridge geometry at an oblique angle to the primary flow direction, a helical
secondary flow was produced across the micro-features and the air-water interface, which was shown to efficiently stretch and fold fluid elements and reduce the mixing length by more than an order of magnitude compared to that of a smooth, no-slip microchannel. The mixing effect was maximised for a micro-ridge topology with a feature width and gap spacing of 30\(\mu\)m aligned at 60\(^\circ\) to the primary flow direction. Beyond a yaw angle of approximately 60\(^\circ\), Ou et al. (2007) reported that the mixing length began to increase.

There is also experimental evidence supporting the drag-reducing capabilities of SHS textures in the turbulent flow regime. Using \(\mu\)-PIV, Woolford et al. (2009b) acquired measurements over a range of Reynolds numbers (based on bulk velocity and hydraulic diameter) from 4800 to 10000. SHS textures aligned parallel to the flow resulted in lower streamwise turbulence intensity, lower total turbulent shear stress and lower production of turbulence, and drag was reduced by approximately 11\%. SHS textures aligned perpendicular to the flow resulted in higher streamwise turbulence intensity, greater turbulent shear stress and greater production of turbulence, and increased drag by approximately 7\%. Due to resolution limitations of the \(\mu\)-PIV apparatus, the measurement position closest to the wall made by Woolford et al. (2009b) was \(y^+ \approx 10\). Consequently, quantification of any mean slip velocity occurring on the SHS texture was not possible.

The difficulties in obtaining experimental near-wall measurements in turbulent flows, as well as taking measurements directly at the wall, motivates the use of numerical simulation where the time-dependent three-dimensional flow fields can be assessed in detail. From this data the turbulent flow can be characterised using statistical and instantaneous quantities which will permit an exploration of the fundamental turbulent drag reduction mechanisms.

1.4.4 Numerical simulations

Some numerical results of laminar flow over SHS textures in plane channel geometries were included in the works by Ou et al. (2004) and Ou & Rothstein (2005) which, compared to their experimental results, showed good agreement. However, the choice of adopting a numerical approach is often motivated by the restrictions associated with experimental methods, particularly in the near-wall region. These difficulties are especially relevant to both transitional and turbulent flows, where the velocity- and length-scales often become too small for experimental readings to be considered accurate or to be considered at all. Using a suitably resolved numerical approach, three-dimensional time-dependent flow fields can be acquired, from which statistical and instantaneous quantities can be extracted, which can then be used to complement available experimental data and provide a more complete understanding of fluid flow phenomena.

The effect of SHS textures on stability and transition of a plane channel flow was investigated
using DNS by Min & Kim (2005), who used the Navier slip boundary conditions
\[ u_s = L_s \frac{\partial u}{\partial y} |_w, \quad w_s = L_s \frac{\partial w}{\partial y} |_w \]
to represent the textured surface. In order to delineate the effect of the streamwise and spanwise slip-boundary conditions separately, three different cases were considered: (Case 1), streamwise slip only \((u_s \neq 0, w_s = 0)\); (Case 2), spanwise slip only \((u_s = 0, w_s \neq 0)\); and (Case 3), slip in both directions \((u_s \neq 0, w_s \neq 0)\). For each of the three cases, three different slip lengths were investigated: (1) a small slip length, \(L_s = 0.005\); (2) a medium slip length \(L_s = 0.01\); and (3) a large slip length \(L_s = 0.02\). In all cases, the effective slip lengths in the streamwise and spanwise directions were equal. Min & Kim (2005) investigated the effects of slip-boundary conditions on the transient growth of initial disturbances using the singular value decomposition (SVD) analysis of the linearised Navier-Stokes equations. The maximum transient growth (i.e., the amplification factor for the optimal disturbance) was found to reduce with streamwise slip, indicating that non-normality of the linearised Navier-Stokes operator was reduced with streamwise slip. The opposite behaviour was observed with spanwise slip, which increased the maximum transient growth, indicating that the non-normality of the linearised Navier-Stokes operator was increased. In general, prescribing a spanwise slip-length increased the amplification of optimal disturbances.

To complement their linear stability analysis, Min & Kim (2005) studied the effect of the Navier-slip boundary condition on the transition process using DNS. Qualitatively, the results from the transition simulations were in agreement with the linear stability analysis: streamwise slip delayed transition whereas spanwise slip promoted it.

Min & Kim (2004) used DNS to investigate the effects of SHS textures on turbulent skin-friction in a fully turbulent plane channel flow at \(Re_\tau = 180\) and used to Navier-slip boundary conditions
\[ u_s = L_s \frac{\partial u}{\partial y} |_w, \quad w_s = L_s \frac{\partial w}{\partial y} |_w \]
to represent the influence of the SHS. The effects of streamwise and spanwise slip on the skin-friction coefficient were delineated by considering three cases: (Case 1) streamwise slip only \((u_s \neq 0, w_s = 0)\); (Case 2), spanwise slip only \((u_s = 0, w_s \neq 0)\); and (Case 3), slip in both directions \((u_s \neq 0, w_s \neq 0)\). A total of seven different effective slip-lengths were prescribed, resulting in a total of twenty-one unique parameter combinations. In all cases, the effective slip lengths in the streamwise and spanwise directions were equal. A maximum drag reduction of 29% occurs for an exclusive streamwise slip-length of \(L_s = 0.02\), whereas a maximum drag increase of 26% occurs for an exclusive spanwise slip-length of equal magnitude. In general, the results of Min & Kim (2004) suggest that increasing spanwise slip degrades performance, whereas increasing streamwise slip has favourable impact on performance (see table 1.1). In addition to
computing percentage drag reduction, Min & Kim (2004) accumulated statistics to characterise both the mean and stochastic fields. For exclusive streamwise slip, the mean streamwise velocity profile was affected in two ways. On the one hand, as the slip length was increased, the slip velocity at the wall also increased and, on the other hand, the log-law region was observed to monotonically elevate away from the wall. The latter observation is a common symptom of turbulent flows undergoing drag reduction. The results of Min & Kim (2004) demonstrated that the slip velocities in the streamwise and spanwise directions affected the near-wall turbulence field quite differently, resulting in a drag decrease with the former and drag increase with the latter. A schematic representation of these effects is provided in figure 1.11. As shown in figure 1.11a, wall shear stress is reduced with a finite streamwise slip, and a direct consequence of this is a reduction in drag. As shown in figure 1.11b, the effect of spanwise slip is to strengthen the near-wall streamwise vortices, which results in an indirect drag increase. Min & Kim (2004) likened this drag increase mechanism to the in-phase blowing and suction studied by Choi et al. (1994), which enhanced the strength of the streamwise vortices and, as a consequence, increased the skin-friction drag.

Martell et al. (2009) used DNS to study fully developed turbulent channel flow over SHS textures made up of periodic arrays of micro-post and micro-ridge geometries. The top surface of each microfeature was taken to be no slip, whereas the suspended liquid-gas interface between the microfeatures was simulated as flat and stress-free, an arrangement that is shown graphically in figure 1.10. Only one side of the channel was modeled as superhydrophobic, and the no-slip boundary condition was prescribed on the opposite wall. The DNS were carried out at a fixed pressure gradient corresponding to a regular no-slip turbulent channel flow at a friction Reynolds number of \(Re_f = 180\). In total, Martell et al. (2009) considered seven unique surface topologies and collected statistical quantities that included mean streamwise velocity profiles, slip velocities, Reynolds stress profiles and wall shear stress levels. Some parameters describing the performance of each topology are included in table 1.2. The results included in the table demonstrate that drag

<table>
<thead>
<tr>
<th>(L_s/\delta)</th>
<th>(L_s^+)</th>
<th>Case 1 (\langle U_s^+\rangle^{xz})</th>
<th>-</th>
<th>Case 2 (\langle U_s^+\rangle^{xz})</th>
<th>-</th>
<th>Case 3 (\langle U_s^+\rangle^{xz})</th>
<th>-</th>
</tr>
</thead>
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<tr>
<td>0.0002</td>
<td>0.036</td>
<td>0.035</td>
<td>0</td>
<td>0.035</td>
<td>0</td>
<td>0.035</td>
<td>0</td>
</tr>
<tr>
<td>0.0005</td>
<td>0.089</td>
<td>0.088</td>
<td>0</td>
<td>0.089</td>
<td>0</td>
<td>0.089</td>
<td>0</td>
</tr>
<tr>
<td>0.0010</td>
<td>0.178</td>
<td>0.176</td>
<td>-1</td>
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</tr>
<tr>
<td>0.0020</td>
<td>0.357</td>
<td>0.349</td>
<td>-5</td>
<td>0.355</td>
<td>-1</td>
<td>0.355</td>
<td>-1</td>
</tr>
<tr>
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<td>-10</td>
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<td>-3</td>
<td>0.877</td>
<td>-3</td>
</tr>
<tr>
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<td>1.618</td>
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<td>1.707</td>
<td>-8</td>
<td>1.707</td>
<td>-8</td>
</tr>
<tr>
<td>0.0200</td>
<td>3.566</td>
<td>3.006</td>
<td>-29</td>
<td>3.238</td>
<td>-17</td>
<td>3.238</td>
<td>-17</td>
</tr>
</tbody>
</table>

Table 1.1: Turbulent drag reduction performance of textures investigated by Min & Kim (2004). Note that superscript \(xz\) denotes a planar-averaged quantity.
Chapter 1. Introduction

Figure 1.11: A schematic representation of drag decrease and increase mechanism, adopted from Min & Kim (2004): (a) drag decreases with a streamwise slip (shown in red) due to reduced wall-shear stress; (b) drag increases with a spanwise slip velocity (shown in red) due to a strengthening of the near-wall streamwise vortices.

<table>
<thead>
<tr>
<th>Texture topology</th>
<th>Designation</th>
<th>$Ψ$ (%)</th>
<th>$Δτ_w$ (%)</th>
<th>$ΔU_s$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Micro-ridges</td>
<td>“15µm-15µm”</td>
<td>50.0</td>
<td>24.2</td>
<td>7.1</td>
</tr>
<tr>
<td></td>
<td>“30µm-30µm”</td>
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<td>39.6</td>
<td>9.8</td>
</tr>
<tr>
<td></td>
<td>“30µm-50µm”</td>
<td>37.5</td>
<td>51.8</td>
<td>15.8</td>
</tr>
<tr>
<td></td>
<td>“30µm-90µm”</td>
<td>25.0</td>
<td>64.1</td>
<td>25.3</td>
</tr>
<tr>
<td>Micro-posts</td>
<td>“30µm-30µm”</td>
<td>25.0</td>
<td>47.2</td>
<td>10.5</td>
</tr>
<tr>
<td></td>
<td>“30µm-50µm”</td>
<td>14.1</td>
<td>61.3</td>
<td>19.8</td>
</tr>
<tr>
<td></td>
<td>“30µm-90µm”</td>
<td>6.25</td>
<td>76.6</td>
<td>36.5</td>
</tr>
</tbody>
</table>

Table 1.2: Turbulent drag reduction performance of textures investigated by Martell et al. (2009) including percentage solid fraction, $Ψ$, the percentage reduction in wall shear stress, $Δτ_w$ (%), and percentage streamwise slip velocity normalized by bulk velocity, $ΔU_s$ (%).

reduction performance of the textured surfaces considered by Martell et al. (2009) increases with increasing feature spacing or, equally, decreasing percentage solid fraction $Ψ_s$. For a given feature width and feature spacing, micro-post geometries always outperformed micro-ridge geometries and the performance increase was attributed to an increased slip velocity, which in turn reduced the levels of wall shear stress.

In a subsequent study by Martell et al. (2010), slip velocities, wall shear stresses and Reynolds stresses were determined for a variety of SHS textures at three friction Reynolds numbers $Reτ = \{180, 395, 590\}$. In total, 14 different topologies were investigated. For the largest micro-feature spacing, an average slip velocity over 80% of the bulk velocity was reported, with a corresponding shear stress reduction of over 50%. The worst-performing texture was a micro-ridge topology aligned transversely with the primary flow direction, for which a nominal increase in drag was observed.
Since Martell et al. (2009, 2010) chose to hold the mean streamwise pressure gradient constant, a decrease in shear stress on the textured wall must be balanced by an increase on the top of the wall. Hence, the average shear stress between the two walls is constant. Therefore, Martell et al. (2009, 2010) could choose to scale statistical quantities by either the averaged friction velocity or the local friction velocity that was achieved at the SHS texture. On the one hand, by choosing to scale their results by the local friction velocity, Martell et al. (2010) demonstrated that the profiles of Reynolds shear stress collapsed in the near-wall region for a variety of SHS textures, which suggests that the fundamental structure of turbulence close to the wall does not change. Whereas, on the other hand, choosing to scale their results using the average friction velocity, Martell et al. (2010) demonstrated that the peak value of Reynolds shear stress decreased. To complement their statistical analysis, Martell et al. (2010) also considered the effect of the SHS textures on the instantaneous turbulent structures including streamwise streaks and streamwise vortices. Martell et al. (2010) argued that, at their lowest friction Reynolds number, \( Re_\tau = 180 \), micro-ridge SHS textures had the potential to act in a similar manner to a riblet geometry. This is because the feature spacing and the spacing of counter-rotating vortices (50+ units according to Kim et al. (1986)) are approximately equal, a behaviour that is shown schematically in figure 1.12a. At this Reynolds number, Martell et al. (2010) speculated that the damping of the spanwise motion of streamwise vortices could, in part, be a feature of the drag reduction mechanism. However, for their highest friction Reynolds number, \( Re_\tau = 590 \), the previous reasoning breaks down. This is because although the physical dimensions of the texture remain unchanged, the increase in Reynolds number results in the vortices being much smaller than the micro-features, a behaviour that is shown schematically in figure 1.12b. In general, precise details of the fundamental drag reduction mechanisms due to SHS textures are not provided by Martell et al. (2010). The extent to which the SHS textures affected the near-wall region was also considered by Martell et al. (2010). By plotting the time-averaged streamwise velocity on the \( xz \) plane for a friction Reynolds number \( Re_\tau \approx 180 \), it was concluded that the micro-features affected the near-wall region up to a distance less than or equal to the

Figure 1.12: Schematic representing pairs of counter-rotating vortices for channel flow over ridges at two different Reynolds numbers, adopted from Martell et al. (2010). (a) \( w^+ = g^+ = 33.75 \) at \( Re_\tau \approx 180 \) and (b) \( w^+ = g^+ = 110.62 \) at \( Re_\tau \approx 590 \).
feature spacing, $g^+$. Martell et al. (2010) speculated that the spanwise variation of time-averaged statistics taken over the micro-features would resemble those for a “normal” no-slip wall, and similarly statistics taken across a free-slip gap would be similar to those found above a “normal” free surface. In general, turbulent structures in the channel flow were found to shift closer to SHS textures but otherwise were largely unaffected.

1.5 Current understanding of drag reduction over SHS textures

Much of the current understanding of the turbulent drag reduction mechanisms by SHS textures can be inferred from the results provided by the numerical studies of Martell et al. (2009, 2010) and experimental works by Woolford et al. (2009b) and Daneillo et al. (2009). These previous studies primarily focused on the drag-reducing capabilities of periodic arrays of streamwise-aligned micro-post and micro-ridge texture topologies. A significant amount of time and effort has been devoted to quantify the average slip velocity, $\langle U_s \rangle_{xz}$, that occurs across the SHS textures. The slip velocity can be directly related to the wall shear stress, which can be used to assess the performance of drag-reduction. In general, further decreases in wall shear stress are observed as the percentage solid fraction decreases, which also increases the average slip velocity. The computation of various turbulence statistics including profiles mean streamwise velocity and Reynolds stresses has helped elucidate the response of the turbulent flow field to the presence of the texture. The turbulent statistics collected by Martell et al. (2009); Woolford et al. (2009b); Martell et al. (2010) suggest that the fundamental structure of the turbulent channel flow remains unaltered and that instantaneous turbulent structures simply move towards the SHS textures. Although the study of turbulent flow over SHS textures has received widespread attention within the last five years, a detailed understanding of the fundamental drag reduction mechanisms is yet to been established.

1.5.1 Phase-averaged statistics

The drag reduction mechanisms proposed by Martell et al. (2009, 2010) are based on statistical quantities that were averaged both in time and across the streamwise-spanwise plane. Considering that the SHS textures are comprised of spatially periodic features of width $d$ spaced at a distance $g$, it seems likely that the statistical features of the turbulent velocity field will show a similar
periodicity. Thus, any flow quantity can be decomposed as follows

\[
f(x, t) = \langle f(x) \rangle + f'(x, t)
\]

\[
= \langle F(y) \rangle^{xz} + \hat{f}(x) + f'(x, t)
\]

(1.13)

where any phase-averaged quantity \( \langle f \rangle \) is comprised of a mean component \( \langle F \rangle^{xz} \) that has been averaged across all possible phases and a periodic component \( \hat{f} \) that isolates the phase-dependent information. Valuable information can be gained by choosing to collect statistical quantities in a phase-averaged form by averaging points across the texture that share a common spatial phase. For example, streamwise-aligned micro-ridge topologies could be phase-averaged along the spanwise direction and arithmetically averaged along the homogeneous streamwise direction, whereas micro-post topologies could be phase-averaged along both the streamwise and spanwise directions. By employing phase-averaging techniques, an additional level of statistical detail is obtained which would clarify the phase-variations of various statistical quantities over of no-slip and free-slip phases.

Figure 1.13: Schematic representation of phase-averaging techniques. From left to right, (a) micro-ridges on the \( xz \) plane, the spanwise phase-averaged micro-ridges, and the spanwise phase-averaged streamwise-averaged micro-ridges; (b) shows micro-posts on the \( xz \) plane, spanwise phase-averaged micro-posts, and streamwise-spanwise phase-average micro-posts. Basically, the original SHS texture is reduced to a phase-averaged form, which upon extrusion/tessellation recovers the original topology.
1.5.2 Alternate texture topologies

One way to increase the drag-reducing performance of SHS textures is to decrease the solid-fraction, $\Psi_s$, which allows more of the fluid to slip along the wider air-water interfaces, resulting in a larger average slip velocity and hence a larger decrease in wall shear stress. This approach has been verified both experimentally (Woolford et al., 2009b) and numerically (Martell et al., 2009, 2010). There is, however, a practical lower limit on the solid-fraction, beyond which the air-water interface ruptures and the SHS texture transitions from the Cassie to the Wenzel state and, as a result, significant performance penalties are incurred. Therefore, an alternate way to increase the performance of SHS textures that does not involve increasing the feature spacing is highly desirable.

A logical strategy is to combine the proven drag-reducing capabilities of streamwise-aligned SHS textures with another successful flow control strategy, ideally one which can be easily integrated into the existing streamwise-aligned topologies at minimal additional cost. These design constraints more or less rule out all forms of active flow control, since they require an auxiliary power source to effect any beneficial changes on the surrounding turbulent flow field. This does not, however, rule out an attempt to passively mimic flow physics which are typically associated with active flow control schemes.

A simple geometric modification of the streamwise-aligned SHS textures can be achieved by sinusoidally varying spanwise position of the micro-feature with downstream position. The goal is to add a spanwise velocity component that varies in streamwise direction over the SHS textures, which will potentially incur additional drag-reduction benefits over conventional geometry by analogy with spanwise wall oscillations. The question becomes: for a fixed solid-fraction, does there exist a spatially varying SHS texture that can outperform its regular counterpart?

The concept of streamwise oscillation of spanwise velocity at the wall of a channel or turbulent drag reduction has been studied before and serves as motivation for the current work. Jung et al. (1992) introduced the active spanwise-oscillating wall technique, where the wall of a fully developed turbulent channel flow is subject to alternate harmonic motion in the transverse direction. The boundary condition that defines this forcing method is,

$$w = A_z \sin \left( \frac{2\pi}{T} t \right),$$

where $t$ is time, and $A_z$ and $T$ are the maximum wall oscillation speed and period, respectively. By exploring $A_z$-$T$ parameter space, Jung et al. (1992) observed, by means of DNS, a strong suppression of turbulence in the near-wall region, accompanied by a significant reduction in the mean friction at an optimal, non-dimensional forcing period, $T_{opt}^- = 100$. Viotti et al. (2009) extended the oscillating-wall technique and translated the time-dependent forcing law into a
steady formula. This was achieved by exploiting the convective nature of wall-bounded flows. Kim & Hussain (1993) demonstrated that near the wall, say below $y^+ = 15$, the convection velocity of turbulent fluctuations, $U_c^+$, becomes independent of wall-normal position and remains constant at the value $U_c^+ \approx 10$. By using the simple formula $\lambda_{x,\text{opt}}^+ = U_c^+ T_{\text{opt}}^+$, Viotti et al. (2009) translated the temporal forcing originally studied by Jung et al. (1992) into a spatially oscillating forcing described by,

$$w = A_z \sin \left( \frac{2\pi x}{\lambda_x} \right),$$

where $\lambda_x$ is the forcing wavelength. After exploring $A_z-\lambda_x$ parameter space, thanks to a number of DNS, Viotti et al. (2009) demonstrated a drag reduction of up to 52\% for $(A_z, \lambda_x)^+ = (20, 1250)$. For all the amplitudes considered by Viotti et al. (2009), the forcing wavelength that returned the maximum drag reduction was found to correspond to the optimal period of the oscillating-wall technique converted through $U_c^+$, which confirmed the validity of the analogy between temporal and spatial forcing. In their concluding remarks, Viotti et al. (2009) assessed the practicality of their flow control strategy and deemed it unsuitable for widespread application, but pointed out that the success of their steady control law was an important step towards the realization and design of a passive drag-reducing device which could effect the equivalent flow response. Not only do the results and remarks of Viotti et al. (2009) provide direct motivation for the current work, they have already been used to optimize existing passive flow control strategies. By using Large Eddy Simulation (LES), Peet & Sagauty (2008) reported a 50\% performance improvement for sinusoidally varying riblets, compared to their streamwise-aligned counterparts.

### 1.6 Overview

This dissertation is comprised of six chapters. An overview of turbulent flow, previous flow control strategies, and previous relevant research of skin-friction reduction using SHS textures has been provided in this chapter. Chapter 2 contains computational aspects of the work, including details of the numerical method, accumulation of statistics and derivations of relevant governing equations. Chapter 3 is devoted to validating the simulation methodology. Chapter 4 is a critical assessment of the drag reduction mechanisms for streamwise-aligned SHS textures, and Chapter 5 details the effects of meandering the SHS textures. Chapter 6 offers conclusions and suggestions for future work in the field of skin-friction reduction using SHS textures.
Chapter 2

Computational methodology

This section describes the solution method for the direct numerical simulation of the Navier-Stokes equations. The governing equations are given in section 2.1. The numerical method is described in section 2.2, and includes details of spatial and temporal discretization and the solution of the pressure equation. The boundary conditions which characterise the SRS textures are described in section 2.3. Finally, section 2.4 describes the accumulation of statistical quantities in phase-averaged form and their corresponding governing equations.

2.1 Governing equations

The governing equations for an unsteady, three-dimensional, viscous, incompressible flow are

\[ \frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = - \frac{\partial p}{\partial x_i} + \frac{1}{Re_b} \frac{\partial^2 u_i}{\partial x_j \partial x_j} \]
\[ \frac{\partial u_j}{\partial x_j} = 0. \]

The variables in the governing equations are non-dimensionalised by the bulk velocity \( U_b \) and channel half-height \( \delta \). The non-dimensional variables are defined as

\[ u_i = \frac{u_i^*}{U_b} \quad t = \frac{t^*}{U_b} \quad x_i = \frac{x_i^*}{\delta} \quad p = \frac{p^*}{\rho U_b^2} \quad Re_b = \frac{U_b \delta}{\nu} \]

where the superscript * denotes a dimensional quantity. The subscript index \( i = (1, 2, 3) \) corresponds to the streamwise \( (x_1 = x) \), wall-normal \( (x_2 = y) \) and spanwise directions \( (x_3 = z) \), respectively. The corresponding velocities are in accordance with common convention and are denoted by \( u, v \) and \( w \), respectively. The pressure is denoted by \( p \).


2.2 Numerical method

The accurate and efficient numerical solution of the Navier-Stokes equations is of key importance to the current work. To this end, the governing equations are solved using direct numerical simulation (DNS) — a solution methodology that requires no turbulence to be modelled and therefore grants access to the full range of length and timescales that a turbulent flow naturally possesses. DNS is the only way to obtain a complete description of a turbulent flow, where the flow variables (here, velocity and pressure) vary as a function of both space and time. There are various numerical schemes available to integrate the governing equations both in space and in time. Here, spatial discretization is achieved using a second-order accurate finite volume method and time advancement is achieved using the fractional step algorithm.

2.2.1 Governing equations

The unsteady incompressible Navier-Stokes equation and continuity equation can be written in vector form as,

\[
\frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot \mathbf{u} \mathbf{u} = -\nabla \cdot P \mathbf{I} + \frac{1}{Re_b} \nabla \cdot \nabla \mathbf{u}, \quad (2.2a)
\]

\[
\nabla \cdot \mathbf{u} = 0, \quad (2.2b)
\]

where \( \mathbf{u} = (u, v, w) \), \( \nabla \) denotes the differential operator, \((\cdot)\) denotes a dot product and \( \mathbf{I} \) is the identity matrix. The governing equations are solved between two parallel flat plates of length \( L_x \) and width \( L_z \), that are separated by a distance \( L_y = 2\delta \) in the wall-normal direction, where \( \delta \) is the distance measured from the bottom wall to the channel half-height (see figure 1.1). The computational domain is periodic in both the streamwise \((x)\) and spanwise \((z)\) directions. The governing equations are integrated forward in time using a fractional step method, first devised by Chorin (1967), and are discretized in space using a second-order central-difference finite volume method.

The first step is to manipulate the governing equations in to a form that makes them particularly amenable to the implementation of both the fractional step and finite volume methods. Equation 2.2a and equation 2.2b can be integrated over a fixed time-independent control volume, \( V \), to give,

\[
\iiint_V \frac{\partial \mathbf{u}}{\partial t} dV = -\iiint_V \nabla \cdot \mathbf{uu} dV - \iiint_V \nabla \cdot P \mathbf{I} dV + \frac{1}{Re_b} \iiint_V \nabla \cdot \nabla \mathbf{u} dV, \quad (2.3a)
\]

\[
\iiint_V \nabla \cdot \mathbf{u} dV = 0. \quad (2.3b)
\]

With the aid of Divergence Theorem, \( \iiint_V \nabla \cdot \mathbf{A} dV = \iint_S \mathbf{A} \cdot d\mathbf{S} \), to the right-hand-side of equation
2.3a and equation 2.3b gives,

\[ \iiint_V \frac{\partial \mathbf{u}}{\partial t} \, dV = -\iiint_S \mathbf{u} \cdot d\mathbf{S} - \iiint_S P I \cdot d\mathbf{S} + \frac{1}{Re_b} \iiint_S \nabla \mathbf{u} \cdot d\mathbf{S}, \quad (2.4a) \]

\[ \iiint_S \mathbf{u} \cdot d\mathbf{S} = 0. \quad (2.4b) \]

Equation 2.4a can be rewritten as,

\[ \iiint_V \frac{\partial \mathbf{u}}{\partial t} \, dV = -\iiint_S \mathbf{T} \cdot d\mathbf{S}, \quad (2.5) \]

where \( \mathbf{T} \) is the operator

\[ \mathbf{T} = -\mathbf{u} \mathbf{u} - P I + \frac{1}{Re_b} \left( \nabla \mathbf{u} + (\nabla \mathbf{u})^T \right), \quad (2.6) \]

or, choosing to return to tensor notation, \( \mathbf{T} \) can be expressed as,

\[ \mathbf{T} = -u_i u_j + P \delta_{ij} + \frac{1}{Re_b} \left[ \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right]. \quad (2.7) \]

Next, the details concerning the discretization of the area vector \( \mathbf{S} \), shown above in the momentum integral equation 2.5, are provided.

### 2.2.2 Basic geometric quantities

The governing equations are discretized using a finite volume method (FVM). Originally, the DNS algorithm was discretized using general coordinates following the procedures outlined by Rosenfeld et al. (1991). In the current work, the general coordinate system is unnecessary due to the Cartesian computational geometry (see figure 1.1). However, for the sake of completeness, the spatial discretization procedures used by Rosenfeld et al. (1991) will be outlined and then reduced to an appropriate Cartesian form. Rosenfeld et al. (1991) defined their general non-orthogonal coordinate system \( (\xi, \eta, \zeta) \) as

\[ \mathbf{r} = \mathbf{r}(\xi, \eta, \zeta) \quad (2.8) \]

where \( \mathbf{r} = (x, y, z)^T \) is the Cartesian coordinate system. The computational domain \( (\xi, \eta, \zeta) \) is divided into uniform primary cells with mesh size \( \Delta \xi = \Delta \eta = \Delta \zeta = 1 \). In the finite-volume discretization scheme, the integral governing equations are approximated over the computational volume in physical space. According to Rosenfeld et al. (1991), at an arbitrary point the area
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The tensor is equal to

\[ \mathbf{S} = \left( S^\xi, S^\eta, S^\zeta \right) \]  

(2.9)

where the vector components are defined as

\[ S^q = \frac{\partial r}{\partial (q+1)} \times \frac{\partial r}{\partial (q+2)} \]  

(2.10)

and where \( q = \xi, \eta, \zeta \) are in cyclic order. The vector quantity \( \mathbf{S}^q \) has the magnitude of the area of the face and a direction pointing outwardly normal from it. For the Cartesian case, the evaluation of the vector cross-products in equation 2.10 simplifies to

\[
\left( S^\xi, S^\eta, S^\zeta \right) = \begin{bmatrix}
S^\xi_x & 0 & 0 \\
0 & S^\eta_y & 0 \\
0 & 0 & S^\zeta_z \\
\end{bmatrix},
\]

(2.11)

and a typical computational cell is shown in figure 2.1. The cell-centre is located at \((i, j, k)\), and the cell-faces, \( \mathbf{S}^q \), are located at staggered locations. To ensure each cell is closed, the area vectors \( \mathbf{S}^q \) should satisfy,

\[ \int_S d\mathbf{S} = 0 \]  

(2.12)

or, in discrete form

\[ \sum_q \mathbf{S}^q = 0. \]  

(2.13)

To compute the volume fluxes, \( \mathbf{U}^q = (U^\xi, U^\eta, U^\zeta) \), across the faces of the computational cells, the Cartesian velocities are also stored in a staggered arrangement, which is shown in two-dimensions in figure 2.2. The fluxes can be obtained using the Cartesian velocity components \( \mathbf{u} \) and equation 2.11 to give,

\[
U^\xi = S^\xi \cdot \mathbf{u} = S^\xi_x u \\
U^\eta = S^\eta \cdot \mathbf{u} = S^\eta_y v \\
U^\zeta = S^\zeta \cdot \mathbf{u} = S^\zeta_z w.
\]

(2.14a)  

(2.14b)  

(2.14c)
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Figure 2.1: Typical computational cell showing cell-centre at \((i, j, k)\) and example area vectors \((S^\xi, S^\eta, S^\zeta)\) pointing outwardly normal from cell-faces.

Figure 2.2: Staggered computational mesh in two dimensions showing the pressure, \(P\), stored at cell-centre and streamwise \((u)\) and wall-normal \((v)\) velocities stored at cell-faces.
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The Cartesian velocity components can be recovered from the volume fluxes using the reciprocal base of \( S \) which Rosenfeld et al. (1991) defined as

\[
S_m = \frac{S^{m+1} \times S^{m+2}}{S^m \cdot (S^{m+1} \times S^{m+2})}
\]  

(2.15)

where \( m \) is the cyclic permutation of \((\xi, \eta, \zeta)\). For the Cartesian case, equation 2.15 yields,

\[
(S_\xi, S_\eta, S_\zeta) = \begin{bmatrix} 0 & 0 & \frac{1}{S_\xi} \\ \frac{1}{S_\eta} & 0 & 0 \\ 0 & \frac{1}{S_\zeta} & 0 \end{bmatrix}
\]  

(2.16)

Note that the scalar product of equation 2.11 and equation 2.16 recovers the standard basis vectors that define three-dimensional Euclidean space.

Also, according to Rosenfeld et al. (1991), the volume of each computational cell can be determined using

\[
V_{cell} = \frac{\left( S^\xi_{i-\frac{1}{2},j,k} + S^\xi_{i,j-\frac{1}{2},k} + S^\xi_{i,j,k-\frac{1}{2}} \right)}{3} \cdot \left( r_{i+\frac{1}{2},j+k+\frac{1}{2}} - r_{i-\frac{1}{2},j-k-\frac{1}{2}} \right)
\]  

(2.17)

which, for the Cartesian case, is simply

\[
V_{cell} = \frac{\left( S^\xi_{x,\xi} + S^\eta_{y,\eta} + S^\zeta_{z,\zeta} \right)}{3} = \Delta x \Delta y \Delta z
\]  

(2.18)

which satisfies

\[
\sum_{ncells} V_{cell} = V_{domain}
\]  

(2.19)

where \( ncells \) is the number of primary volumes used to discretize the total computational volume, \( V_{domain} \).

2.2.3 Spatial discretization

Discretization of the integral mass conservation equation (equation 2.4b) over a computational cell yields,

\[
(S^\xi_z u)_{i+\frac{1}{2},j,k} - (S^\xi_z u)_{i-\frac{1}{2},j,k} + (S^\eta_z v)_{i,j+\frac{1}{2},k} - (S^\eta_z v)_{i,j-\frac{1}{2},k} \\
+S^\zeta_z w)_{i,j,k+\frac{1}{2}} - (S^\zeta_z w)_{i,j,k-\frac{1}{2}} = 0
\]  

(2.20)
where each term on the left-hand-side of equation 2.20 approximates the volume flux over the corresponding face. Equation 2.20 states that no mass can be generated in the cell and that the net mass flux over each cell is zero. An equivalent expression to equation 2.20, recast in terms of volume fluxes, can be written as

\[ U_{i+rac{1}{2}, j, k} - U_{i-rac{1}{2}, j, k} + U_{i,j+k+rac{1}{2}} - U_{i,j,k} + U_{i,j,k+rac{1}{2}} - U_{i,j,k-rac{1}{2}} = 0. \] (2.21)

Recall the integral momentum equation 2.5 with Cartesian velocity components as unknown variables,

\[ \iiint_V \frac{\partial u}{\partial t} dV = - \iint_S T \cdot dS. \]

Approximating the above expression over an arbitrary time-constant control volume \( V \) yields,

\[ V \frac{\partial u}{\partial t} = \sum_l S^q \cdot T \] (2.22)

and then premultiplying each side by the area vector \( S^q \) (equation 2.10) yields,

\[ S^q \cdot \left( V \frac{\partial u}{\partial t} \right) = S^q \cdot \left( \sum_l S^q \cdot T \right) \] (2.23)

The bracketed term on right-hand side of the above equation can be written as,

\[ \sum_q S^q \cdot T = \sum_q S^q \cdot \left\{ -uu - PI + \frac{1}{Re_b} \left( \nabla u + (\nabla u)^T \right) \right\} \]

\[ = \sum_q S^q \cdot (-uu) + \sum_q S^q \cdot (-PI) + \sum_q S^q \cdot \left[ \frac{1}{Re_b} \left( \nabla u + (\nabla u)^T \right) \right] \] (2.24)

The three momentum equations should be further discretized over different staggered control volumes and further details can be found in Rosenfeld et al. (1991).

### 2.2.4 Temporal discretization

Recall the integral momentum equation 2.5 with Cartesian velocity components as unknown variables,

\[ \iiint_V \frac{\partial u}{\partial t} dV = - \iint_S T \cdot dS. \]
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Approximating the above expression over an arbitrary time-constant control volume $V$ yields,

$$V \frac{\partial u}{\partial t} = \sum_l S^q \cdot T \quad (2.25)$$

and then premultiplying each side by the area vector $S^q$ (equation 2.10) yields,

$$S^q \cdot \left( V \frac{\partial u}{\partial t} \right) = S^q \cdot \left( \sum_l S^q \cdot T \right) \quad (2.26)$$

which, after using equations 2.14a-2.14c, can be written in terms of fluxes as

$$V \frac{\partial U^q}{\partial t} = H_q + R_q + D_q \quad (2.27)$$

where $H_q$ represents convection terms, $R_q$ represents pressure terms and $D_q$ represents the diffusion terms corresponding to the terms on the right-hand-side of the operator $T$ (see equation 2.6). Next, consider equation 2.27 for the $\xi$ direction,

$$V \frac{\partial U^\xi}{\partial t} = H_\xi + R_\xi + D_\xi. \quad (2.28)$$

The terms on the right-hand-side of the above equation are discretized using a second order explicit Adams-Bashforth scheme for convection terms,

$$H_\xi (U^q) = \frac{1}{2} \left[ 3H_\xi (U^q)^n - H_\xi (U^q)^{n-1} \right]. \quad (2.29)$$

an implicit Euler scheme for pressure terms,

$$R_\xi (P) = R_\xi (P)^{n+1} \quad (2.30)$$

and a Crank-Nicolson scheme for the diffusion terms,

$$D_\xi (U^q) = \frac{1}{2} \left[ D_\xi (U^q)^{n+1} + D_\xi (U^q)^n \right]. \quad (2.31)$$

The temporal discretization form of equation 2.28 can be written as

$$V \left[ \frac{(U^\xi)^{n+1} - (U^\xi)^n}{\Delta t} \right] = \frac{1}{2} \left[ 3H_\xi (U^q)^n - H_\xi (U^q)^{n-1} \right] + R_\xi (P)^{n+1} + \frac{1}{2} \left[ D_\xi (U^\xi)^{n+1} + D_\xi (U^\xi)^n \right]. \quad (2.32)$$
Similarly, for \( \eta \):

\[
V \left[ \frac{(U^\eta)^{n+1} - (U^\eta)^n}{\Delta t} \right] = \frac{1}{2} \left[ 3H_\eta (U^\eta)^n - H_\eta (U^\eta)^{n-1} \right] + R_\eta (P)^{n+1} \\
+ \frac{1}{2} \left[ D_\eta (U^\eta)^{n+1} + D_\eta (U^\eta)^n \right],
\]

and for \( \zeta \):

\[
V \left[ \frac{(U^\zeta)^{n+1} - (U^\zeta)^n}{\Delta t} \right] = \frac{1}{2} \left[ 3H_\zeta (U^\eta)^n - H_\zeta (U^\eta)^{n-1} \right] + R_\zeta (P)^{n+1} \\
+ \frac{1}{2} \left[ D_\zeta (U^\zeta)^{n+1} + D_\zeta (U^\zeta)^n \right].
\]

Next, the above equations 2.32-2.34 are split into two separate equations by the introduction of an intermediate volume flux variable \( \hat{U}^q \) and a pseudo-pressure flux variable \( \phi \). One can interpret the role of pressure in the momentum equations equations as a projection operator which projects an arbitrary vector field onto a divergence-free vector field. Along the \( \xi \) direction, equation 2.32 is split into

\[
V \left[ \frac{(\hat{U}^\xi) - (U^\xi)^n}{\Delta t} \right] = \frac{1}{2} \left[ 3H_\xi (U^\eta)^n - H_\xi (U^\eta)^{n-1} \right] + \frac{1}{2} \left[ D_\xi (\hat{U}^\xi) + D_\xi (U^\xi)^n \right],
\]

(2.35a)

\[
V \left[ \frac{(U^\xi)^{n+1} - (\hat{U}^\xi)}{\Delta t} \right] = R_\xi (\phi)^{n+1}.
\]

(2.35b)

Upon summation of the split equations 2.35a-2.35b, the original equation 2.32 is recovered under the assumption that \( \phi = P \). The validity of this assumption is demonstrated later. Similar split equations can be obtained for \( \eta \):

\[
V \left[ \frac{(\hat{U}^\eta) - (U^\eta)^n}{\Delta t} \right] = \frac{1}{2} \left[ 3H_\eta (U^\eta)^n - H_\eta (U^\eta)^{n-1} \right] + \frac{1}{2} \left[ D_\eta (\hat{U}^\eta) + D_\eta (U^\eta)^n \right],
\]

(2.36a)

\[
V \left[ \frac{(U^\eta)^{n+1} - (\hat{U}^\eta)}{\Delta t} \right] = R_\eta (\phi)^{n+1}.
\]

(2.36b)
and for $\zeta$:

\[
V \left[ \frac{(\hat{U}^\zeta)^{n+1} - (U^\zeta)^n}{\Delta t} \right] = \frac{1}{2} \left[ 3H_\zeta (U^q)^n - H_\zeta (U^q)^{n-1} \right] + \frac{1}{2} \left[ D_\zeta (\hat{U}^\zeta) + D_\zeta (U^\zeta)^n \right]
\] (2.37a)

\[
V \left[ \frac{(U^\zeta)^{n+1} - (\hat{U}^\zeta)}{\Delta t} \right] = R_\zeta (\phi)^{n+1}.
\] (2.37b)

Next, the split equations are recast in a form suitable for the implementation of the fractional step method. For example, along the $\xi$ direction, equation 2.35a is rewritten as

\[
[I - A_\xi] \left[ \hat{u}^\xi - (u^\xi)^n \right] = \frac{\Delta t}{2V} \left[ 3H_\xi (U^q)^n - H_\xi (U^q)^{n-1} \right] + 2A_\xi (u^\xi)^n
\] (2.38)

where $A_\xi = [\Delta t/2V] \frac{D_\xi}{2}$. Similarly along the $\eta$ direction, equation 2.36a is rewritten as

\[
[I - A_\eta] \left[ \hat{u}^\eta - (u^\eta)^n \right] = \frac{\Delta t}{2V} \left[ 3H_\eta (U^q)^n - H_\eta (U^q)^{n-1} \right] + 2A_\eta (u^\eta)^n
\] (2.39)

where $A_\eta = [\Delta t/2V] \frac{D_\eta}{2}$. Finally, along the $\zeta$ direction, equation 2.37a is rewritten as

\[
[I - A_\zeta] \left[ \hat{u}^\zeta - (u^\zeta)^n \right] = \frac{\Delta t}{2V} \left[ 3H_\zeta (U^q)^n - H_\zeta (U^q)^{n-1} \right] + 2A_\zeta (u^\zeta)^n
\] (2.40)

where $A_\zeta = [\Delta t/2V] \frac{D_\zeta}{2}$. Equations 2.38-2.40 are then solved for the intermediate flux field, $\hat{U}^q$, which, in general, is not divergence-free. In order to satisfy the incompressibility constraint at the next time-step, the intermediate flux field, $\hat{U}^q$, is projected onto a divergence-free space by computing the divergence of the pseudo-pressure, $\phi$. The velocity update equations can be obtained by simply rearranging equations 2.35b, 2.36b and 2.37b to yield,

\[
(u^\xi)^{n+1} = \frac{\Delta t}{V} R_\xi (\phi)^{n+1} + \hat{u}^\xi,
\] (2.41a)

\[
(u^\eta)^{n+1} = \frac{\Delta t}{V} R_\eta (\phi)^{n+1} + \hat{u}^\eta,
\] (2.41b)

\[
(u^\zeta)^{n+1} = \frac{\Delta t}{V} R_\zeta (\phi)^{n+1} + \hat{u}^\zeta.
\] (2.41c)

The computation of the pseudo-pressure, $\phi$, is achieved by the solution of a Poisson equation and the details are provided shortly in subsequent subsection.

### 2.2.5 Summary of the fractional step algorithm

The governing equations (see equation 2.2a), are discretized using a second-order Adams-Bashforth scheme for the convective terms and second-order Crank-Nicolson scheme for the diffusive terms,
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giving the discrete system,

\[
\frac{u^{n+1} - u^n}{\Delta t} + \frac{1}{2} \left[ 3H(u^n) - H(u^{n-1}) \right] = -Gp^{n+1} + \frac{1}{2Re_b}L(u^{n+1} + u^n) \quad (2.42a)
\]

\[Du^{n+1} = 0 \quad (2.42b)\]

where \(H\) is the discrete form of the convective term, \(G\) is the discrete gradient operator, \(L\) is the discrete Laplace operator and \(D\) is the discrete divergence. A variant of the fractional-step method, first introduced by Chorin (1967), was used for the time-advancement of the governing equations. The first step of the fractional-step algorithm is to obtain an intermediate velocity field, \(u^*\). The prediction of the intermediate velocity field is achieved by advancing the convective and diffusive terms in the absence of pressure,

\[
\frac{u^* - u^n}{\Delta t} + \frac{1}{2} \left[ 3H(u^n) - H(u^{n-1}) \right] = \frac{1}{2Re_b}L(u^* + u^n). \quad (2.43)
\]

The divergence of the intermediate velocity field does not satisfy the incompressibility constraint. The corrector equation for the next time-step can be written as

\[
u^{n+1} = u^* - \Delta tG(\phi + \Phi) \quad (2.44)
\]

where \(\phi\) represents a pseudo-pressure and \(\Phi\) is a mean streamwise pressure, the latter being required to drive the flow through the channel. An equation for \(\phi\) can be obtained by taking the divergence of equation 2.44 and invoking mass conservation to give

\[
L\phi^{n+1} = \Delta tDu^*. \quad (2.45)
\]

where the relation \(DG\Phi = 0\) was used. Using the solution to the Poisson equation for \(\phi\), the mean streamwise pressure \(\Phi\) can be determined by subtracting \(u^{n+1}\) from both sides of equation 2.44 and invoking mass conservation to give

\[
G\left(\phi^{n+1} + \Phi^{n+1}\right) = \frac{u^* - u^n}{\Delta t} - \nabla \phi^{n+1}. \quad (2.46)
\]

Finally, the pressure field is updated using the pseudo-pressure

\[
p^{n+1} = \phi^{n+1} + \Phi^{n+1} \quad (2.47)
\]

The accuracy of the updated pressure field, shown above in equation 2.47 can be determined by expressing \(p\), which was initially omitted when computing the intermediate velocity field, in terms
of $\phi$, which was introduced as a result of the operator-splitting methods inherent in fractional-step methods. Rearranging equation 2.44 for $u^*$ and substituting the resulting expression into equation 2.43 gives

$$
\frac{u^{n+1} - u^n}{\Delta t} + \frac{1}{2} \left[ 3H(u^n) - H(u^{n-1}) \right] = \left[ -G(\phi^{n+1} + \phi^{n+1}) + \frac{1}{2Re_b} \Delta t LG \phi^{n+1} \right] + \frac{1}{2Re_b} L(u^{n+1} + u^n),
$$

(2.48)

which is equal the original discrete system shown in equation 2.42a, except for the substitution

$$
-Gp^{n+1} = \left[ -G(\phi + \Phi)^{n+1} + \frac{1}{2Re_b} \Delta t LG (\phi + \Phi)^{n+1} \right]
$$

(2.49)

which means that the accuracy of the pressure update, given in equation 2.47, improves by either prescribing a shorter time-step $\Delta t$ or a larger Reynolds number. The latter observation makes the fractional-step algorithm particularly well suited to incompressible engineering flows, which are typically associated with high Reynolds numbers.

In summary, the fractional-step method can be summarised in four sequential steps:

1. Computation of an intermediate velocity field $u^*$, obtained by the time-advancement of the convective and diffusive terms, in the absence of the pressure term.
2. Computation of the divergence of the intermediate velocity field and solving a Poisson equation for the pseudo-pressure, $\phi$.
3. Computation of the mean streamwise pressure gradient $\Phi$ in order to maintain a fixed mass flow rate.
4. Projection of the intermediate velocity field onto a divergence-free space using the gradient of the pseudo pressure and the mean streamwise pressure gradient.

### 2.2.6 Solution of the pressure equation

The solution of the elliptic pressure equation returns a scalar field that is used to project the velocity field on to a divergence-free space at each timestep. The elliptic nature of the pressure equation means that it is computationally expensive to solve, since every point in the three-dimensional computational space is coupled and must be considered simultaneously. Theoretically, this problem can be attributed to pressure waves travelling at an infinite speed throughout an incompressible fluid and thus perturbing the entire system instantaneously. The computational geometry of the current work facilitates the use of Fourier transforms in the streamwise...
and spanwise directions, which are both periodic and have uniform grid-spacing. Basically, the Poisson equation for $\phi$, (see equation 2.45), can be solved using 5 straightforward steps

1. Take a Fourier transform of each line of the grid in the $z$-direction; that is, diagonalise the $\frac{\partial^2 \phi}{\partial z^2}$ part of the operator.

2. Take a Fourier transform of each line of the grid in the $x$-direction; that is, diagonalise the $\frac{\partial^2 \phi}{\partial x^2}$ part of the operator. At this stage the system has been reduced to a set of tri-diagonal matrices, each linear system is associated with one grid line in the $y$-direction.

3. Solve a tri-diagonal matrix for each line of the discrete system.

4. Take an inverse Fourier transform on each line of the grid in $z$-direction.

5. Take an inverse Fourier transform on each line of the grid in $x$-direction.

Mathematically, the Poisson equation can be written as

$$\nabla^2 \phi = f$$

(2.50)

where the right hand side source term is $f = (\nabla \cdot u^*) / \Delta t$. Discretizing equation 2.50 in space results in

$$
\begin{pmatrix}
\frac{\partial^2 \phi}{\partial x^2}, \frac{\partial^2 \phi}{\partial y^2}, \frac{\partial^2 \phi}{\partial z^2}
\end{pmatrix}_{ijk} = f_{ijk},
$$

(2.51)

and taking a Fourier transform in both the streamwise and spanwise directions reduces equation 2.51 to the following tri-diagonal system

$$
\frac{\partial^2 \hat{\phi}_{ijk}}{\partial y^2} - \left( k_x' \right)^2 \hat{\phi}_{ijk} = \hat{f}_{ijk}
$$

(2.52)

where the modified wavenumbers for the central-differencing operators are

$$
k_x'^2 = \frac{2}{\Delta x^2} \left[ 1 - \cos \left( \frac{2\pi i}{N_x} \right) \right]
$$

(2.53a)

$$
k_z'^2 = \frac{2}{\Delta z^2} \left[ 1 - \cos \left( \frac{2\pi k}{N_z} \right) \right]
$$

(2.53b)

where $i \in [0, N_x - 1]$ and $k \in [0, N_z - 1]$. The formulation and solution of a tri-diagonal system shown in equation 2.52 is attractive since large matrices no longer need to be inverted directly or approximately. Instead, low-storage, high-efficiency direct solution algorithms can be adopted.
2.3 Boundary conditions and texture characterisation

The SHS textures are prescribed at both the top \((y = 2\delta)\) and bottom \((y = 0)\) walls (see figure 2.3) and therefore the boundary conditions are symmetric about the channel half-height. The SHS textures are comprised of strips of no-slip and free-stress boundary conditions, which represent the micro-ridges and liquid-gas interfaces, respectively.

Along the tops of the micro-ridges, the no-slip boundary condition is prescribed on both the streamwise and spanwise velocities,

\[
\begin{align*}
    u &= 0, \quad (2.54a) \\
    w &= 0, \quad (2.54b)
\end{align*}
\]

whereas, for the wall-normal velocity, the no-penetration boundary condition is enforced using

\[
\frac{\partial v}{\partial y} = 0. \quad (2.55)
\]

Since \(\partial u/\partial x|_w = \partial w/\partial z|_w = 0\), then, from mass conservation, the additional boundary condition

\[
\frac{\partial v}{\partial y} = 0. \quad (2.56)
\]

is also enforced along the tops of the no-slip micro-features.

Along the tops of the interfaces, the free-stress boundary condition is prescribed on both the streamwise and spanwise velocities,

\[
\begin{align*}
    \frac{\partial u}{\partial y} &= 0, \quad (2.57a) \\
    \frac{\partial w}{\partial y} &= 0, \quad (2.57b)
\end{align*}
\]

![Figure 2.3: Schematic of computational geometry with SHS textures on top \((y/\delta = 2)\) and bottom \((y/\delta = 0)\) walls.](image)
and the no-penetration condition on wall-normal velocity, shown above in equation 2.55, is also prescribed. Due to staggered arrangement of the velocities (see figure 2.2), the streamwise and spanwise components do not lie exactly at the wall, thus the boundary conditions cannot be imposed directly, except in the case of no-penetration boundary condition (equation 2.55) for the wall-normal velocity. To enforce the remaining boundary conditions, values are prescribed “below” (at $y/\delta = 0$) and “above” (at $y/\delta = 2$) the SHS textures using halo cells. For example, along the tops of the micro-features, the no-slip boundary condition for the streamwise velocity (equation 2.54a) is enforced using,

$$
\left. u \right|_w = 0.5 \left( \frac{\text{Halo cell}}{u_{j=-1} + u_{j=1}} \right), \\
= 0.5 \left( -u_{j=1} + u_{j=1} \right), \\
= 0, \\
$$

where the value inside the halo cell ($j = -1$) is prescribed to be equal and opposite to value in the first interior cell above the wall ($j = 1$) which ensures that, after interpolating, that the no-slip boundary condition for $u$ is satisfied at ($j = 1/2$) (see figure 2.4). A similar approach is used to enforce the no-slip boundary condition on spanwise velocities. For the wall-normal velocity, the

Figure 2.4: Enforcement of no-slip boundary condition on streamwise velocity $u$ and Neumann boundary condition on wall-normal velocity $v$ using halo cell approach.

no-penetration condition (equation 2.55) can be enforced explicitly since it is defined exactly at the wall. The Neumann condition on $v$ along the top of the no-slip micro-features is enforced using

$$
\left. \frac{\partial v}{\partial y} \right|_w = \frac{\text{Halo cell}}{-v_{j=-1} + v_{j=1}} \\
0 = \frac{-v_{j=-1} + v_{j=1}}{2\Delta y}, \\
v_{j=-1} = v_{j=1},
$$

(2.59)
where subscripts $j = -1$ and $j = 1$ correspond to the halo cell and the first point away from the wall, respectively. Basically, the value inside the halo cell ($j = -1$) is prescribed to be equal to the value in the first interior cell above the wall ($j = 1$), which ensures that the Neumann condition is correctly imposed.

Along the tops of the interfaces, the free-stress boundary condition for the streamwise and spanwise velocities is also enforced using a halo cell approach. For example, the free-stress boundary condition for the spanwise velocity is enforced using,

\[ \frac{\partial w}{\partial y} \bigg|_{w} = \frac{\text{Halo cell}}{2\Delta y} \]

\[ 0 = -\frac{w_{j=-1} + w_{j=1}}{2\Delta y} \]

\[ w_{j=-1} = w_{j=1}, \]

where the value inside the halo cell ($j = -1$) is prescribed to be equal to value in the first interior cell above the wall ($j = 1$) which ensures that the free-stress boundary condition for $w$ is satisfied at ($j = 1/2$). A similar approach is used to enforce the free-stress boundary condition on the streamwise velocity component. The locations of the no-slip boundary conditions for a typical micro-ridge SHS texture are shown in figure 2.5 and highlights their arrangement when complicated slightly by the staggered mesh.

Figure 2.5: Locations of no-slip boundary condition for a streamwise-aligned ridge texture on a staggered mesh.
2.4 Statistical averaging

In the current flow configuration, it is desirable to isolate contributions from the periodic and stochastic motions. Therefore, the spatial analogue of the temporal phase-averaging procedures outlined by Hussain & Reynolds (1970) is adopted,

\[ f(x, t) = \langle F \rangle_{xz} + \hat{f} + f'(x, t) \]

where \( f \) is the instantaneous variable, \( \langle \cdot \rangle \) is the phase average and \( f' \) is the stochastic component.

After phase-averaging equation 2.61, it follows that the periodic component can be isolated according to \( \hat{f} = \langle f \rangle - \langle F \rangle_{xz} \).

Due to the doubly-periodic nature of the current flow configuration, instantaneous variables are phase-averaged in both the streamwise \( (x) \) and spanwise \( (z) \) directions in order to detect harmonic flow features due to the presence of SHS. As a consequence of periodicity, an integer number of SHS features are prescribed in the streamwise and spanwise directions,

\[ k_x = \frac{L_x}{\lambda_x}, \quad k_z = \frac{L_z}{(d + g)} \]

where \((k_x, k_z)\) are the integer wavenumbers in the streamwise and spanwise directions, respectively, and where \(\lambda_x\) denotes the streamwise wavelength. Phase-averaged variables are integrated along lines of constant streamwise phase, denoted by a path \( s(x) \) (---). The slope of path \( s \) is given by \( dx/ds = \cos \theta \).

\[ \langle f (y, z) \rangle_x = \frac{1}{\lambda_x} \int_0^{\lambda_x} \langle f (x, y, z) \rangle dx \]

where \( dx = \cos \theta ds \).
A variable averaged across all phases can be written as
\[
\langle F \rangle_{xz} = \frac{1}{2} \left( \langle f \rangle_{ns} + \langle f \rangle_{fs} \right)
\] (2.64)

with
\[
\langle f (y) \rangle_{ns} = \frac{1}{d} \int_0^d \langle f \rangle_x^z dz, \quad \langle f (y) \rangle_{fs} = \frac{1}{g} \int_0^{d+g} \langle f \rangle_x^z dz,
\] (2.65 a,b)

where superscript ns and fs denote an average over all no-slip and shear-free phases, respectively.

The root-mean-square contribution by the periodic component, \( \hat{f} \), can be computed using,
\[
\hat{f}_{rms} = \sqrt{\frac{1}{(d + g) \lambda_x} \int_0^d \int_0^{d+g} \left( \langle f \rangle - \langle F \rangle_{xz} \right)^2 dx dz},
\] (2.66)

and is a useful measure when flow symmetries would result in arithmetic mean returning zeroes.

Essentially, the phase-averaging procedure rejects the background turbulence and extracts the organised motions from the total signal. The phase-averaged quantities can then be averaged further in the post-processing phase to determine how statistical quantities vary across local portions of the texture. Now that the phase-averaging procedure has been summarised, the next step is to derive the statistical equations that govern the dynamics of the various components that make up the total signal.

### 2.4.1 Dynamical equations for the mean and periodic motions

Incompressible turbulent flow is governed by the Navier-Stokes momentum equations
\[
\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{1}{Re_b} \frac{\partial^2 u_i}{\partial x_j \partial x_j},
\] (2.67)

and the continuity equation,
\[
\frac{\partial u_j}{\partial x_j} = 0.
\] (2.68)
In the current work, DNS allows us to evaluate all terms in the statistical representation of the flow explicitly. Starting with the decomposition for the total velocity and pressure,

$$u_i(x, t) = \langle u_i(x) \rangle + u'_i(x, t)$$ \hspace{1cm} (2.69a)  
$$p(x, t) = \langle p(x) \rangle + p'(x, t),$$ \hspace{1cm} (2.69b)

it is straightforward to show, from equation 2.68, that

$$\frac{\partial \langle U_i \rangle}{\partial x_i} = 0, \quad \frac{\partial u'_i}{\partial x_i} = 0,$$ \hspace{1cm} (2.70)

which ensures that mass is conserved for both the phase-averaged and stochastic fluid motions. The Reynolds-Averaged-Navier-Stokes (RANS) equations were derived by substituting equation 2.69a and equation 2.69b into equation 2.67 and then phase-averaging,

$$\frac{\partial \langle U_i \rangle}{\partial t} + \langle U_j \rangle \frac{\partial \langle U_i \rangle}{\partial x_j} = \frac{\partial \langle P \rangle}{\partial x_i} + \frac{1}{Re_b} \frac{\partial^2 \langle U_i \rangle}{\partial x_j \partial x_j} - \frac{\partial}{\partial x_j} \langle u'_i u'_j \rangle.$$ \hspace{1cm} (2.71)

The phase-averaged RANS equation 2.71 is the same as the governing equation 2.67 for the total velocity, except for the last term which originates from the averaging of the convective term. This term is the divergence of the Reynolds stress tensor, $\langle u'_i u'_j \rangle$. Another form of the RANS equation can be derived by using the triple decomposition for the total velocity and pressure,

$$u_i(x, t) = \langle U_i(y) \rangle + \tilde{u}_i(x) + u'_i(x, t)$$ \hspace{1cm} (2.72a)  
$$p(x, t) = \langle P(y) \rangle + \tilde{p}(x) + p'(x, t).$$ \hspace{1cm} (2.72b)

From equation 2.68, is straightforward to show that

$$\frac{\partial \langle U_i \rangle^{xz}}{\partial x_i} = 0, \quad \frac{\partial \tilde{u}_i}{\partial x_i} = 0, \quad \frac{\partial u'_i}{\partial x_i} = 0,$$ \hspace{1cm} (2.73)

which ensures that mass is conserved for mean, phase-averaged and stochastic fluid motions. The RANS equations are derived by substituting equations 2.72a and 2.72b into equation 2.67 and then phase-averaging to give,

$$\langle U_j \rangle^{xz} \left( \frac{\partial \langle U_i \rangle^{xz}}{\partial x_j} + \frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial}{\partial x_j} \langle u'_i u'_j \rangle \right) = - \left( \frac{\partial \langle P \rangle^{xz}}{\partial x_i} + \frac{\partial \tilde{p}}{\partial x_i} \right) + \frac{1}{Re_b} \left( \frac{\partial^2 \langle U_i \rangle^{xz}}{\partial x_j \partial x_j} + \frac{\partial^2 \tilde{u}_i}{\partial x_j} \right)$$ \hspace{1cm} (2.74)

Averaging equation 2.74 across all phases yields,

$$\langle U_j \rangle^{xz} \frac{\partial \langle U_i \rangle^{xz}}{\partial x_j} = - \frac{\partial \langle P \rangle^{xz}}{\partial x_i} + \frac{1}{Re_b} \frac{\partial^2 \langle U_i \rangle^{xz}}{\partial x_j \partial x_j} - \frac{\partial}{\partial x_j} \left( \langle \tilde{u}_i \tilde{u}_j \rangle^{xz} + \langle u'_i u'_j \rangle^{xz} \right).$$ \hspace{1cm} (2.75)
To isolate the equation for the periodic component \ref{2.75} was subtracted from equation \ref{2.74} to give
\[
\langle U_j \rangle^{zz} \frac{\partial \hat{u}_i}{\partial x_j} + u_j \frac{\partial \langle U_i \rangle^{zz}}{\partial x_j} = -\frac{\partial p_i}{\partial x_i} + \frac{1}{Re_b} \frac{\partial^2 \hat{u}_i}{\partial x_j^2} - \frac{\partial}{\partial x_j} (\langle \hat{u}_i \hat{u}_j \rangle - \langle \hat{u}_i \rangle \langle \hat{u}_j \rangle^{zz} ). \tag{2.76}
\]

Averaging equation \ref{2.76} across all phases equals zero.

\subsection{2.4.2 Dynamical equations for the stochastic component}

To isolate the equation for the stochastic contribution equation \ref{2.74} was subtracted from equation \ref{2.67} to give
\[
\frac{\partial u'_i}{\partial t} + \langle (U_j) \rangle^{zz} + \hat{u}_j \frac{\partial u'_i}{\partial x_j} + u'_j \left( \frac{\partial \langle U_i \rangle^{zz}}{\partial x_j} + \hat{\lambda}_i \right) = -\frac{\partial p'_i}{\partial x_i} + \frac{1}{Re_b} \frac{\partial^2 u'_i}{\partial x_j^2} + \frac{\partial}{\partial x_j} (\langle u'_i u'_j \rangle - u'_i u'_j) . \tag{2.77}
\]

A dynamical equation for the phase-averaged Reynolds stresses can be derived by first multiplying \ref{2.77} by \( u'_k \), and then phase-averaging,
\[
\left\langle u'_k \frac{\partial u'_i}{\partial t} \right\rangle + \langle (U_j) \rangle^{zz} + \hat{u}_j \left\langle u'_k \frac{\partial u'_i}{\partial x_j} \right\rangle + \langle u'_k u'_j \rangle \left( \frac{\partial \langle U_i \rangle^{zz}}{\partial x_j} + \hat{\lambda}_i \right) =
\]
\[
\left\langle -u'_k \frac{\partial p'_i}{\partial x_i} \right\rangle + \frac{1}{Re_b} \left\langle u'_k \frac{\partial^2 u'_i}{\partial x_j^2} \right\rangle + \left\langle u'_k \frac{\partial}{\partial x_j} (-u'_i u'_j) \right\rangle . \tag{2.78}
\]

Rewriting \ref{2.78} with \( i \) and \( k \) reversed, gives
\[
\left\langle u'_i \frac{\partial u'_k}{\partial t} \right\rangle + \langle (U_j) \rangle^{zz} + \hat{u}_j \left\langle u'_i \frac{\partial u'_k}{\partial x_j} \right\rangle + \langle u'_i u'_j \rangle \left( \frac{\partial \langle U_k \rangle^{zz}}{\partial x_j} + \hat{\lambda}_k \right) =
\]
\[
\left\langle -u'_i \frac{\partial p'_k}{\partial x_k} \right\rangle + \frac{1}{Re_b} \left\langle u'_i \frac{\partial^2 u'_k}{\partial x_j^2} \right\rangle + \left\langle u'_i \frac{\partial}{\partial x_j} (-u'_k u'_j) \right\rangle . \tag{2.79}
\]

then adding \ref{2.78} and \ref{2.79} and using the product rules \( \frac{\partial}{\partial x_j} (u'_i u'_k) = \frac{\partial u'_i}{\partial x_j} u'_k + u'_i \frac{\partial u'_i}{\partial x_j} = u'_i u'_k \frac{\partial u'_i}{\partial x_j} + \frac{\partial u'_i}{\partial x_j} u'_k \), where applicable, gives
\[
\frac{\partial}{\partial t} \left\langle u'_i u'_k \right\rangle + \langle (U_j) \rangle^{zz} + \hat{u}_j \frac{\partial}{\partial x_j} \left\langle u'_i u'_k \right\rangle = - \left\langle u'_i u'_k \right\rangle \left( \frac{\partial \langle U_i \rangle^{zz}}{\partial x_j} + \hat{\lambda}_i \right) - \left\langle u'_i u'_j \right\rangle \left( \frac{\partial \langle U_j \rangle^{zz}}{\partial x_j} + \hat{\lambda}_j \right)
\]
\[
\quad + \left\langle u'_i \frac{\partial p'_i}{\partial x_i} \right\rangle + \frac{1}{Re_b} \left\langle u'_i \frac{\partial^2 u'_i}{\partial x_j^2} \right\rangle - \frac{2}{Re_b} \left\langle u'_i \frac{\partial u'_k}{\partial x_j} \right\rangle - \frac{\partial}{\partial x_j} \left\langle u'_i u'_k \right\rangle . \tag{2.80}
\]
which is the Reynolds stress transport equation. The turbulent kinetic energy equation is one half the trace of equation 2.80. Hence, setting \( k = i \) gives,

\[
\frac{\partial \langle k \rangle}{\partial t} + \langle (U_j)^{xz} + \tilde{u}_j \rangle \frac{\partial \langle k \rangle}{\partial x_j} = -\langle u'_i u'_j \rangle \left( \frac{\partial (U_i)^{xz}}{\partial x_j} + \frac{\partial \tilde{u}_i}{\partial x_j} \right) + \left\langle \frac{\partial \tilde{u}'_i}{\partial x_i} \right\rangle \\
+ \frac{1}{Re_b} \frac{\partial^2}{\partial x_j \partial x_j} \langle k \rangle - \frac{1}{Re_b} \left\langle \left( \frac{\partial u'_i}{\partial x_j} \right)^2 \right\rangle - \frac{1}{2} \frac{\partial}{\partial x_j} \langle u'_i u'_i u'_j \rangle.
\]

(2.81)

where \( k \equiv \frac{1}{2} \langle u'_i u'_i \rangle \) is the phase-averaged turbulent kinetic energy (TKE).
Chapter 3

Validation

The purpose of this chapter is to thoroughly validate the temporal and spatial accuracy of the numerical method and averaging procedures described in the previous chapter. Four validation cases were chosen: two transitional and two fully turbulent cases.

Transition describes the physical process by which an initially laminar flow undergoes a sequence of instabilities and state-space changes before finally settling into fully turbulent fluid motion. The instabilities that trigger the transition process can originate from a broad range of sources including infinitesimal, instability waves or high-amplitude perturbations in the form of, for example, vortical disturbances. The unsteady, non-equilibrium nature of transition combined with the broad spectrum of length- and time-scales that must be accurately resolved render the transition process a formidable numerical challenge and hence serves as an excellent validation procedure to evaluate both the temporal and spatial accuracy of the current DNS algorithm. The accurate computation of initial conditions that are used to trigger the transition processes is an additional requirement. Both the natural and bypass transition mechanisms are considered. The results of Sandham & Klesier (1992) are used to validate natural transition to turbulence in plane channel flow, where a superposition of instability waves derived from linear stability theory is prescribed as an initial condition. The results of Henningson et al. (1993) are used to validate the bypass route of transition, where a localised disturbance, derived from inviscid theory, at various amplitudes is used to investigate the linear, non-linear and breakdown stages of the transition process.

The accurate and efficient simulation of fully turbulent flows is important to the current work. Since it is difficult to fully model and characterise the time dependent and chaotic evolution of turbulent flows, a large portion of the run-time is devoted to the accumulation of averaged quantities to facilitate a statistical description of the flow. The results of the seminal simulations by Kim et al. (1986) serve as an excellent database to validate the current DNS algorithm. An exhaustive range of statistical quantities are reproduced and compared to the results of Kim.
et al. (1986). Of equal importance is the accurate implementation of the textured no-slip-no-stress boundary conditions that play a central role throughout this text. The recent simulations conducted by Martell et al. (2009) are selected in order to validate that the textured boundary conditions are implemented correctly.

In all of the validation cases, where appropriate, reference is made to differences in initial condition, numerical method, grid stretching, grid resolution, temporal resolution, sampling period, statistical evaluations, etc... between the specific validation case and the current work. These issues are addressed as fully as possible in order to ensure the most accurate results are obtained.

3.1 Natural transition of channel flow

Natural transition from an initially laminar flow to a fully turbulent state can be characterised by a sequence of spatial changes. The pre-requisite for natural transition in a quiescent laminar flow is the presence of infinitesimal, broad disturbances which can lead to a two-dimensional viscous instability wave, referred to as a Tollmien-Schlichting (TS) wave. As an exponentially unstable TS-wave convects downstream, it can develop spanwise vorticity leading to spanwise variations in the streamwise velocity fluctuations. As the vortices are stretched they break up into smaller structures, which eventually manifest themselves as localised patches of intensely fluctuating flow, or turbulent spots. The spots then entrain the surrounding laminar fluid, continuously growing in volume as they convect downstream, until they coalesce with one another to form a fully turbulent flow. In summary the natural transition process can be described by the following steps:

1. Initially laminar flow, with infinitesimal broadband perturbations.
2. Inception of an exponentially unstable two-dimensional TS-wave.
3. Development of spanwise vorticity perturbations.
4. Vortex breakdown into intense three-dimensional perturbations.
5. Formation of localised turbulent spots and entrainment of surrounding laminar flow.
6. Growth and merging of turbulent spots into fully turbulent flow.

The first three stages are amenable to linear analysis, whereas the final three are nonlinear phenomena. The rich flow physics of the natural transition process clearly pose a much greater challenge than the simulation of steady-state flows (Kleiser & Zang, 1991) and thus presents itself as an appropriate measure of the accuracy of the current DNS algorithm.

In addition to a highly accurate numerical scheme, an appropriate initial condition that satisfies the early stages of the natural transition process must be generated. Following Drazin &
Chapter 3. Validation

Reid (1987), the initial development of instability waves about a parallel, viscous, incompressible mean flow \( (U(y))^{xz} \) can be written in a velocity-vorticity form as

\[
\begin{bmatrix}
\frac{\partial}{\partial t} & \Delta^{-1} \left( - (U)^{xz} \frac{\partial}{\partial x} \Delta + \frac{\partial^2 (U)^{xz}}{\partial y^2} \frac{\partial}{\partial z} + \frac{1}{Re} \Delta^2 \right) & 0 \\
0 & - \Delta^{-1} ( - \langle U \rangle^{xz} \frac{\partial}{\partial x} \Delta + \frac{\partial^2 \langle U \rangle^{xz}}{\partial y^2} ) & - \frac{1}{Re} \Delta \\
\end{bmatrix}
\begin{bmatrix}
\nu' \\
\eta' \\
\end{bmatrix}
= \begin{bmatrix}
\nu' \\
\eta' \\
\end{bmatrix}
\] (3.1)

where \( \Delta \) is the Laplacian operator, \( \Delta^{-1} \) is its inverse and where the perturbation normal vorticity is \( \eta' = \frac{\partial u'}{\partial z} - \frac{\partial w'}{\partial x} \).

The eigenvalue problem associated with equation 3.1 is obtained by taking Fourier transforms in the homogeneous streamwise and spanwise directions, as well as in time:

\[
\begin{bmatrix}
\nu' (x, t) \\
\eta' (x, t) \\
\end{bmatrix}
= \begin{bmatrix}
\hat{\nu}' (y) \\
\hat{\eta}' (y) \\
\end{bmatrix} e^{i(k_x x + k_z z - \omega t + \phi)}.
\] (3.2)

where \( \hat{\cdot} \) denotes a Fourier transformed variable, \( i \) is the unit complex number, \( k_x \) is the streamwise wavenumber, \( k_z \) is the spanwise wavenumber, \( \omega \) is the complex frequency and \( \phi \) is a random phase-shift angle. Substituting equation 3.2 into 3.1 leads to the Orr-Sommerfeld-Squire eigenvalue problem

\[
-i \omega \begin{bmatrix}
\hat{\nu}' \\
\hat{\eta}' \\
\end{bmatrix}
= \begin{bmatrix}
\Delta^{-1} \left( - (U)^{xz} i k_x \Delta + \frac{\partial^2 (U)^{xz}}{\partial y^2} i k_z + \frac{1}{Re} \Delta^2 \right) & 0 \\
0 & - (U)^{xz} i k_z + \frac{1}{Re} \Delta \\
\end{bmatrix}
\begin{bmatrix}
\hat{\nu}' \\
\hat{\eta}' \\
\end{bmatrix}
\] (3.3)

where \( \omega = ck_x \) is the complex frequency and \( c \) is the phase speed.

A spectral collocation technique using a Chebyshev expansion in the inhomogeneous wall-normal direction is used to solve equation 3.3. Details of the collocation method can be found in Schmid & Henningson (2001). For a given Reynolds number and wavenumber pair \((k_x, k_z)\), the solution of the Orr-Sommerfeld-Squire problem yields a discrete spectrum of eigenvalues. Once \( \hat{\nu}' \) and \( \hat{\eta}' \) are known, the streamwise and spanwise wall-normal eigenfunctions can be recovered by using the Fourier-transformed continuity equation,

\[
\begin{align*}
ik_x \hat{u}' + \frac{\partial \hat{\nu}'}{\partial y} + ik_z \hat{w}' &= 0,
\end{align*}
\] (3.4)

together with the Fourier-transformed perturbation wall-normal vorticity equation,

\[
\hat{\eta}' = ik_z \hat{u} - ik_x \hat{w}',
\] (3.5)
which, after the following few manipulations,

\[
\begin{align*}
  ik_x \hat{u}' & = -\frac{\partial \hat{v}'}{\partial y} - ik_z \hat{w}', \\
  ik_x \hat{u}' & = -\frac{\partial \hat{v}'}{\partial y} - ik_z \left( \frac{ik_x \hat{u}' - \hat{\eta}'}{ik_x} \right), \\
  i^2 k_z^2 \hat{u}' & = -ik_x \frac{\partial \hat{v}'}{\partial y} - i^2 k_z^2 \hat{u}' + ik_z \hat{\eta}', \\
  -\hat{w}' (k_x^2 + k_z^2) & = -i \left( k_z \frac{\partial \hat{v}'}{\partial y} - k_z \hat{\eta}' \right),
\end{align*}
\]

yields,

\[
\hat{u}' = \frac{i}{(k_x^2 + k_z^2)} \left( k_x \frac{\partial \hat{v}'}{\partial y} - k_z \hat{\eta}' \right). \tag{3.6}
\]

Similarly, after the following few manipulations,

\[
\begin{align*}
  ik_z \hat{w}' & = -\frac{\partial \hat{v}'}{\partial y} - ik_x \hat{u}', \\
  ik_z \hat{w}' & = -\frac{\partial \hat{v}'}{\partial y} - ik_x \left( \hat{\eta}'' + ik_z \hat{w}' \right), \\
  i^2 k_x^2 \hat{w}' & = -ik_z \frac{\partial \hat{v}'}{\partial y} - ik_x \hat{\eta}' - i^2 k_x^2 \hat{w}'', \\
  -\hat{w}' (k_x^2 + k_z^2) & = -i \left( k_z \frac{\partial \hat{v}'}{\partial y} + k_x \hat{\eta}' \right),
\end{align*}
\]

yields,

\[
\hat{w}' = \frac{i}{(k_x^2 + k_z^2)} \left( k_z \frac{\partial \hat{v}'}{\partial y} + k_x \hat{\eta}' \right). \tag{3.7}
\]

The eigen-analysis only describes the asymptotic stability characteristics, i.e. whether a disturbance will ultimately continue to amplify or decay and vanish due to viscosity. The lowest Reynolds number for which an exponentially unstable eigenvalue exists is referred to as the critical Reynolds number, $Re_{crit}$. Linear theory predicts that $Re_{crit} = 5772$ for plane channel flow, based on the channel half-height and centreline velocity, with wavenumber pair $(k_x, k_z) = (1.02, 0.0)$. However, this result is often contested by transition well below the predicted $Re_{crit}$, a scenario referred to as sub-critical transition. A transient phase of short-time energy amplification has been credited for this sub-critical behaviour, due to the differing decay rates of a superposition of non-orthogonal eigenvectors. It is clear then that a spectral analysis, i.e. a study of the eigenvalues, must be complemented by a study of the transient growth which in channel flow is a vortex-tilting mechanism.

An accurate DNS of the natural route to transition in a channel geometry was performed by Sandham & Klesier (1992). Their numerical method was fully spectral, with the periodic
directions being treated with Fourier expansions and the wall-normal directions with Chebyshev expansion. A second-order Adams-Bashforth method was used to advance the nonlinear terms, together with Crank-Nicolson for the viscous and pressure terms. The dimensions of the box were \( L_x \times L_y \times L_z = 2\pi/1.12 \times 2 \times 2\pi/2.1 \) with a grid resolution of \( N_x \times N_y \times N_z = 160 \times 128 \times 160 \). The Reynolds number was 5000, based on the channel half-height and laminar centreline velocity, and the mass flow rate was held constant throughout the simulation. The initial condition for the three-dimensional velocity field can be written as

\[
\begin{align*}
    u_i (t = 0) &= \langle u_i \rangle + u'_i, \\
    u_i (t = 0) &= \left \langle \left[ \begin{array}{c}
    \langle U \rangle_{x, \text{lam}}^x \\
    0 \\
    0 \\
    \langle u_i \rangle
    \end{array} \right] + \sum_{n=1}^{3} \epsilon_n R \left[ \begin{array}{c}
    \frac{i}{(k_x^2 + k_z^2)} \left( k_x \frac{\partial \hat{v}'_n}{\partial y} - k_z \hat{\eta}'_n \right) \\
    \hat{\eta}'_n \\
    \frac{i}{(k_x^2 + k_z^2)} \left( k_z \frac{\partial \hat{v}'_n}{\partial y} + k_x \hat{\eta}'_n \right) \\
    u'_i
    \end{array} \right] e^{i(k_{x,n} x + k_{z,n} z + \phi_n)} \right \rangle,
\end{align*}
\]

(3.8)

where \( \langle U \rangle_{x, \text{lam}}^x \) represents a fully developed parabolic laminar channel profile, \( \epsilon_n \) is the \( n^{th} \) perturbation amplitude, \( R \) denotes the real part of a complex number and where \( \phi_n \) represents a random phase-shift.

In summary, the initial condition (equation 3.8) used by Sandham & Klesier (1992) is comprised of a laminar parabolic channel profile and a perturbation field that is a superposition of three instability waves. The mode shapes for each of the instability waves are obtained from the solution of the Orr-Sommerfeld-Squire system described in equation 3.3. The three modes are normalised so that the maximum streamwise velocity amplitude is equal to 1, and include a two-dimensional TS-wave (see figure 3.1), with \( (k_{x_1}, k_{z_1}) = (1.12, 0.0) \) and \( \epsilon_1 = 0.0015 \), and a pair of equal and opposite oblique waves (see figure 3.2) with \( (k_{x_2,3}, k_{z_2,3}) = (1.12, \pm 2.1) \) and \( \epsilon_{2,3} = 0.0045 \). A spanwise slice of the three-dimensional initial condition (see the braced term in equation 3.8) positioned at \( z/\delta = L_z/2 \) is provided in figure 3.3.

The two main differences between the current work and the work by Sandham & Klesier (1992) is their spectral treatment of spatial derivatives. For a given computational mesh, spectral methods are typically more accurate than finite-difference approximations. In an attempt to resolve this issue, the guideline by Moin & Mahesh (1998) is adopted and the grid resolution is doubled in each spatial direction to give \( N_x \times N_y \times N_z = 320 \times 256 \times 320 \). Another minor difference is their choice of centreline velocity, \( U_c \), as the reference velocity scale, while the present work uses the bulk velocity, \( U_b \). The discrepancy in velocity scales meant that the Reynolds number and the perturbation amplitude both had to be rescaled. Using the simple relation \( U_b = 2/3U_c \) the Reynolds number, based on bulk velocity, was prescribed as \( Re_b = 3333 \). Accordingly, the original perturbation amplitudes \( \epsilon \) used by Sandham & Klesier (1992) were each increased by a
Chapter 3. Validation

To validate the time-dependence of the transition process, the planar-average of the friction Reynolds number, $Re_{\tau}$, at the bottom wall was monitored and compared to the previous results of Sandham & Klesier (1992). Good agreement between the two data sets was observed and is shown in figure 3.4.

Figure 3.1: Mode shapes corresponding to two-dimensional TS-wave used in the initial condition for validation against Sandham & Klesier (1992) including real (—), imaginary (—–), and absolute (—) components. (a) $\hat{u}'$ component and (b) $\hat{v}'$ component.

factor of $3/2$. 
Figure 3.2: Mode shapes corresponding to three-dimensional oblique wave used in the initial condition for validation against Sandham & Klesier (1992) including real (---), imaginary (- - -), and absolute (—) parts. (a) $\hat{u}'$ component, (b) $\hat{v}'$ component and (c) $\hat{w}'$ component.
Figure 3.3: Spanwise slice of initial three-dimensional perturbation field (see equation 3.8) for validation against Sandham & Klesier (1992) at $z/\delta = L_z/2$. The perturbation field is comprised of a two-dimensional TS-wave and a pair of equal and opposite three-dimensional oblique waves. (a) streamwise, (b) wall-normal and (c) spanwise perturbation velocity fields.

Figure 3.4: Validation of natural transition against Sandham & Klesier (1992). Time-history of planar-averaged friction Reynolds number, $Re_{\tau}$. Current results (—) and original results (○) of Sandham & Klesier (1992) are included.
3.2 Bypass transition in channel flow

In the absence of Tollmien-Schlichting waves, an alternate route to transition is possible and is referred to as bypass transition. The significance of the term “bypass” is that this transition scenario skips, or bypasses, the initial stages of the natural transition process where instability waves are observed to amplify slowly on an exponential timescale. Instead bypass mechanisms exhibit a rapid breakdown to turbulence typically induced by high-amplitude disturbances originating from, for example, wall roughness or high levels of background noise. A mechanism for bypass transition in channel flow was investigated using DNS by Henningson et al. (1993) using a localised disturbance, originally devised by Landahl (1983). The wavepacket consists of two pairs of counter-rotating streamwise vortices and has been generated in the laboratory by displacing a membrane up and down (Breur & Haritonidis, 1990). The equations describing this disturbance can be written as

\[
\begin{align*}
\langle u_i \rangle (t=0) &= \langle u_i \rangle + u'_i, \\
u_i (t=0) &= \begin{bmatrix}
\langle U \rangle_{\text{lam}}^{xz} \\
0 \\
0 \\
\langle u_i \rangle 
\end{bmatrix} + \epsilon \begin{bmatrix}
f \left( \frac{x}{\sigma_x} \right) \\
1 - \left( \frac{x}{\sigma_x} \right)^2 \\
e^{-\left( \frac{x}{\sigma_x} \right)^2 + \left( \frac{z}{\sigma_z} \right)^2} \\
- \frac{\partial f}{\partial y} \left( \frac{x}{\sigma_x} \right) ze^{-\left( \frac{x}{\sigma_x} \right)^2 + \left( \frac{z}{\sigma_z} \right)^2} \\
u'_i
\end{bmatrix},
\end{align*}
\]

where \( f = (1 + y)^p (1 - y)^q \) is a wall-normal distribution function and where \( \sigma_x \) and \( \sigma_z \) are streamwise and spanwise disturbance length scales, respectively. The localised disturbance is visualised in figure 3.5, where contours of the initial wall-normal and spanwise perturbation velocities are provided. Note that the streamwise perturbation velocity is initially zero.

By systematically varying the amplitude of the localised wavepacket, Henningson et al. (1993) studied the linear, non-linear and breakdown stages of bypass transition. Here we consider three perturbation amplitudes: small \((\epsilon/U_b) = 0.00015\), medium \((\epsilon/U_b) = 0.1049\) and large \((\epsilon/U_b) = 0.2907\) which correspond the linear, nonlinear and breakdown stages, respectively. For the breakdown stages, the high-amplitude localised disturbance was evolved in time until a turbulent spot was observed. The broad range of results acquired by Henningson et al. (1993) make it a suitable choice for this validation procedure since it provides a thorough test of the current numerical solution of the Navier-Stokes equations. For the linear and nonlinear calculations the time-evolution of the total disturbance energy, \( E \), was computed using

\[
E (t) = \frac{1}{2} \int_V \left( u'^2 + v'^2 + w'^2 \right) dV,
\]

(3.10)
Figure 3.5: Initial perturbation field for validation against Henningson et al. (1993) including wall-normal (left) and spanwise (right) velocity components on the (a) \(xz\) plane at \(y/\delta = 1\), (b) \(xy\) plane at \(z/\delta = 11\) and (c) \(zy\) plane at \(x/\delta = 22\). Identical contour levels used for each figure.

and the corresponding energy amplification, \(G\), was evaluated as

\[
G(t) = \frac{E(t)}{E(t=0)}.
\] (3.11)

Henningson et al. (1993) adopted a spectral approach to solve the Navier-Stokes equations, with Fourier expansions in the streamwise and spanwise directions, and a Chebyshev expansion in the wall-normal direction. The nonlinear terms were computed in physical space, i.e. treated in a pseudo-spectral manner. Time advancement was implemented by using a third-order Runge-Kutta scheme for the nonlinear terms and a second-order Crank-Nicolson scheme for the linear terms. Details of the domain size and grid resolution are provided in table 3.2. The Reynolds number was 3000 based on the channel half-height and the laminar centreline velocity, and the mass flow rate was held constant throughout the simulation. The discrepancy in velocity scales meant that the Reynolds number and the perturbation amplitude both had to be rescaled. Using the simple relation \(U_b = 2/3U_{cl}\) the Reynolds number, based on bulk velocity, was prescribed.
Chapter 3. Validation

Table 3.1: Discretization details of validation cases including original parameters used by Henningson et al. (1993) and current setup

<table>
<thead>
<tr>
<th>Simulation</th>
<th>Re_cl</th>
<th>Re_b</th>
<th>( \epsilon_{cl} )</th>
<th>( \epsilon_b )</th>
<th>( L_x \times L_y \times L_z )</th>
<th>( N_x \times N_y \times N_z )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Henningson</td>
<td>-</td>
<td>2000</td>
<td>0.0001</td>
<td>-</td>
<td>48 \times 2 \times 24</td>
<td>96 \times 65 \times 96</td>
</tr>
<tr>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.0699</td>
<td>-</td>
<td>48 \times 2 \times 24</td>
<td>96 \times 65 \times 192</td>
</tr>
<tr>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.1398</td>
<td>-</td>
<td>33 \times 2 \times 13</td>
<td>400 \times 121 \times 320</td>
</tr>
<tr>
<td>Current</td>
<td>3000</td>
<td>-</td>
<td>0.00015</td>
<td>-</td>
<td>48 \times 2 \times 24</td>
<td>193 \times 129 \times 193</td>
</tr>
<tr>
<td>Current</td>
<td>-</td>
<td>-</td>
<td>0.1049</td>
<td>-</td>
<td>48 \times 2 \times 24</td>
<td>257 \times 129 \times 385</td>
</tr>
<tr>
<td>Current</td>
<td>-</td>
<td>-</td>
<td>0.2907</td>
<td>-</td>
<td>33 \times 2 \times 13</td>
<td>513 \times 257 \times 513</td>
</tr>
</tbody>
</table>

as \( Re_b = 2000 \). Accordingly, the original perturbation amplitudes \( \epsilon \) used by Henningson et al. (1993) were each increased by a factor of \( 3/2 \).

First, results for the small-amplitude disturbance (\( \epsilon/U_b = 0.0015 \)) are considered. Both the energy amplification, \( G \), and the percentage contribution made by each of the velocity components towards \( G \) are reported. These results are then compared to those by Henningson et al. (1993) and in figure 3.6. The agreement between the two sets is excellent. The initial energy growth is associated with the generation of the normal vorticity perturbations via forcing by the normal velocity, commonly referred to as the lift-up effect. The energy amplification reaches its peak value at \( t \approx 11 \) and subsequently decays due to viscous dissipation. As a result of the lift-up mechanism, streamwise perturbations are generated and quickly become the dominant contribution towards to total disturbance energy, while the wall-normal and spanwise components both decay. For instance, at \( t = 12.5 \) approximately 80% of the total energy resides in the \( u' \)-component. Figure 3.7 shows the temporal evolution of each perturbation velocity component.
Chapter 3. Validation

Figure 3.7: Time-evolution of streamwise (left), wall-normal (centre) and spanwise (right) perturbation velocities for small-amplitude ($\epsilon/U_b = 0.00015$) wavepacket at (a) $t = 0$, (b) $t = 6.6$ and (c) $t = 26.6$.

for the small-amplitude wavepacket. At $t = 0$ (figure 3.7a), only wall-normal and spanwise velocity perturbations are present and correspond to the initial perturbation field shown in figure 3.5. At $t = 6.6$ (figure 3.7b), strong streamwise perturbations have been produced, whereas both wall-normal and spanwise perturbations have decayed. Some time later, at $t = 26.6$ (figure 3.7b), the streamwise perturbations also begin to decay and disperse. An initial production of streamwise velocity perturbations, followed by their decay and dispersion at later times, and a decay in both the wall-normal and spanwise velocities are consistent with the trends shown in figure 3.6b.

Next, results for the moderate-amplitude disturbance are considered. Both the energy amplification, $G$, and the percentage contribution made by each of the velocity components towards $G$ are plotted in figure 3.8. Compared to the results by Henningson et al. (1993), an excellent level of agreement is observed. Unlike the small-amplitude case, energy increases in magnitude and amplifies monotonically. Although the energy growth exhibits no signs of transient behaviour, the componential contributions towards the growth function have the same behaviour
Figure 3.8: Validation against Henningson et al. (1993) for moderate-amplitude ($\epsilon = 0.1049$) disturbance. (a) Energy amplification, $G$, (—). (b) Percentage contribution from $u'$ (—), $v'$ (- - -) and $w'$ (—) towards $G$. Results of Henningson et al. (1993) (◦) are included in both figures.

as the small-amplitude case, with streamwise perturbations dominating at long times. The structures associated with this behaviour are shown in figure 3.12 at $t = \{0, 6.6, 26.6\}$. Compared to the small-amplitude case the elongated streamwise structures are narrower, resulting in sharp spanwise gradients typically associated with the presence of streaky structures.

Finally, results for the high-amplitude ($\epsilon = 0.2907$) disturbance are considered. Henningson et al. (1993) chose not to compute the energy amplification, $G$, for the high-amplitude wavepacket and instead focused on particular flow structures which, ultimately, mature to form a turbulent spot at $t \approx 60$. For example, figure 3.10 compares contours of wall-normal vorticity corresponding to the large-amplitude disturbance at $t = 40$ obtained by the current DNS algorithm and an original image by Henningson et al. (1993). The agreement between the two figures is excellent.

The amplification of wall-normal velocity perturbations and their subsequent variations in the spanwise direction produce streamwise vorticity, defined here as $w'_x \equiv \frac{\partial v'}{\partial z} - \frac{\partial w'}{\partial y}$. Figure 3.11 shows the streamwise vorticity at three $yz$ planes and reveals coherent streamwise vortices in the vicinity of both the top and bottom walls. An original image by Henningson et al. (1993) is also included in figure 3.11 and, compared to the current results, shows an excellent level of agreement. An interaction of the streamwise vortices with the walls is evident, whereby secondary vorticity of the opposite sign is pulled away from the wall. The position of the vortex core appears to vary in the wall-normal direction and as a result of this slight inclination, the streamwise vortices intensify due to the action the mean shear.

The development of a turbulent spot is shown in figure 3.12, where contours of $u'_i$ are plotted at $y/\delta = 0.44$ and $t = \{0, 6.6, 26.6, 64.2\}$. At the final time (see figure 3.12d), fluctuating motions have developed in all three velocity components and suggests that the onset of a fully developed turbulent flow is imminent.
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Figure 3.9: Time-evolution of streamwise (left), wall-normal (centre) and spanwise (right) perturbation velocities at $y/\delta = 0.44$ for moderate-amplitude ($\epsilon/U_b = 0.1049$) wavepacket at (a) $t = 0$, (b) $t = 6.6$ and (c) $t = 26.6$.

Figure 3.10: Normal velocity for large-amplitude ($\epsilon = 0.2907$) disturbance at $t = 40$ and $y/\delta = 0.44$.

Original image from Hemmingson et al. (1993)
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Figure 3.11: Streamwise vorticity for large-amplitude ($\epsilon = 0.2907$) disturbance at $t = 40$ at (a) $x/\delta = 17$, (b) $x/\delta = 20$ and (c) $x/\delta = 20$. (d) shows the original image from Henningson et al. (1993).
Figure 3.12: Time-evolution of streamwise (left), wall-normal (centre) and spanwise (right) perturbation velocities for large-amplitude ($\epsilon/U_b = 0.2907$) wavepacket at $y/\delta = 0.44$ and (a) $t = 0$, (b) $t = 6.6$, (c) $t = 26.6$ and (d) $t = 64.2$. 
3.3 Fully developed turbulent channel flow

Once the transition process is complete, fluid flow enters a fully developed turbulent regime. Turbulence involves a broad spectrum of perturbations with varying length and timescales, making a deterministic description practically impossible. However, the seemingly random structure of turbulence can be broken down into more elementary organised motions that are often referred to as coherent structures. The basic premise is that these structures are spatially and temporally distinguishable from an otherwise disorderly background field. A study of the structures is motivated by the quest to identify order within apparent chaos, to attribute the dynamics of the structures to important turbulence mechanisms and to identify the most important structures with a view to their potential modification and/or control in order to achieve engineering goals. Coherent structures were classified by Robinson (1991) who divided them into eight separate categories. Out of the eight, four structures have particular relevance to the current work:

1. Low- and high-speed streaks in the region $0 < y^+ < 10$.
2. Ejections of low-speed fluid away from the wall.
3. Sweeps of high-speed fluid towards the wall.
4. Vortical structures of various forms.

The streaks are plotted at a wall-normal $y^+ = 8$ in figure 3.13a. Based on two-point correlation data, the length of the streaks in fully developed channel flow is of the order $10^3$ wall units, and they are separated by a spanwise distance of the order of $10^2$ units. The expulsion of low-speed fluid towards the channel centre is referred to as an ejection. Continuity requires that the upwash motion is complemented by an inrush of high-momentum fluid towards the wall, referred to as a sweep event. The streaks have a characteristic behaviour, referred to as bursting, where with increasing downstream distance a streak moves away from the wall, begins to oscillate and then breaks up into finer-scale motions. A side-view of the streaks is shown in figure 3.13b and clearly shows the breakup of streaks during the burst process, the lift-up of low-momentum fluid during the ejection and inrush of high-momentum fluid during the sweep events.

From an engineering perspective, the study of instantaneous snapshots can be insightful but is not sufficient. Reproducible turbulent phenomena can be obtained by adopting a systematic statistical approach which describes the average behaviour of the system over a large time interval or spatial extent. The statistical results can be supplemented with conclusions drawn from instantaneous data to increase our overall understanding. The first fully-resolved direct numerical simulation of turbulent channel flow was conducted by Kim et al. (1986) and their results serve as an excellent resource to validate the current DNS algorithm. Since the remainder of this document is devoted to the analysis of fully turbulent flow, an exhaustive validation procedure is required.
Figure 3.13: Coherent structures in fully developed turbulent channel flow including (a) near-wall streaks visualised by contours of $u$-perturbation velocity at $y^+ \approx 7.5$ and (b) burst, sweeps and ejection events using the same contour levels as in (a).

Kim et al. (1986) recast the Navier-Stokes equations in a velocity-vorticity form before solving them numerically. The governing equations were reduced to a fourth-order equation for the normal velocity, $v$, and a second-order equation of the normal vorticity, $\eta$, and can be written as

$$\frac{\partial}{\partial t} \nabla^2 v = h_v + \frac{1}{Re_b} \nabla^4 v \tag{3.12}$$

$$\frac{\partial}{\partial t} \eta = h_g + \frac{1}{Re_b} \nabla^2 \eta \tag{3.13}$$

where

$$h_v = \frac{\partial}{\partial y} \left( \frac{\partial H_1}{\partial x} + \frac{\partial H_3}{\partial z} \right) + \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right) H_2$$

$$h_g = \frac{\partial H_1}{\partial z} - \frac{\partial H_3}{\partial x}$$

and $H_i$ contains both the convective and mean pressure gradient terms. A reason for choosing to work in a velocity-vorticity framework is that the number of dependent variables ($v, \eta$) is halved into comparison to ($u_i, p$), which reduces computational expense. To compute the spatial derivatives Kim et al. (1986) employed a spectral method: Fourier series in the streamwise and spanwise directions, and Chebyshev polynomial expansion in the normal direction. Gauss-Lobatto offers superior clustering closer to the wall, which is where the increased resolution
is desirable. An additional consideration is that the truncation error associated with spectral methods decreases exponentially with increasing number of meshpoints, whereas the error of finite-difference-based methods decreases algebraically.

Details of the computational mesh used by Kim et al. (1986) and the current mesh are provided in table 3.4.

Prior to the collection of turbulence statistics a suitable fully developed turbulent initial condition is required. This can be achieved, for example, by prescribing unstable linear stability modes or zero-mean high-amplitude broadband disturbances and advancing the field in time. Here the latter approach was chosen, with the initial condition prescribed as

\[
\begin{align*}
    u_i (t = 0) &= \langle u_i \rangle + u_i', \\
    u_i (t = 0) &= \begin{bmatrix}
        \langle U \rangle_{\text{lam}}^{zz} \\
        0 \\
        0 \\
        \langle u_i \rangle
    \end{bmatrix} + \epsilon \begin{bmatrix}
        \langle U \rangle_{\text{lam}}^{zz} \\
        \langle U \rangle_{\text{lam}}^{zz} \\
        \langle U \rangle_{\text{lam}}^{zz} \\
        u_i'
    \end{bmatrix} R 
\end{align*}
\]

where \( R \) is a function that returns divergence-free random noise between \([-1,+1]\). The initial perturbation amplitude was prescribed as \( \epsilon = 0.4 U_b \), which was sufficiently large to ensure that the flow transitions to a turbulent state. The choice to weight the function \( R \) by the local mean velocity ensured that the boundary conditions for the stochastic field were enforced appropriately.

The initial condition in equation 3.14 was advanced in time until a fully developed turbulent state was realised. In order to assess the nature of the flow, a planar average of the friction Reynolds number was monitored in time. A fully developed turbulent field was achieved when the friction Reynolds number fluctuated about some mean value in a statistically stationary manner. A typical time-history of planar-averaged \( Re_\tau \) is shown in figure 3.14 which clearly illustrates this behaviour for \( t U_b / \delta > 100 \). Once the initial condition has reached a statistically stationary state, turbulence statistics are accumulated. The total integration time, \( T_{\text{int}} \), spanned approximately 40 non-dimensional time units \( (t/u_\tau \delta) \), which is roughly four times the sampling duration reported by Kim et al. (1986). The increase in sampling time should go some way to improving the convergence and accuracy of the statistical results.

Plots of the mean streamwise velocity profile scaled by the friction velocity are shown in figure 3.15 a). Excellent agreement is observed between the two data sets. Quantitatively, the
Figure 3.14: Time-history of planar-averaged friction Reynolds number, $Re_{\tau}$, during generation of turbulent initial condition.

maximum absolute error is less than 0.5% for all $y$-locations. Plotting $y$ on a logarithmic scale reveals the “law of the wall” and helps identify the various regions of the turbulent boundary layer structure as, in order of increasing wall-normal position, the viscous sublayer, the buffer layer and the log-law region. Within the viscous sublayer, $y^+ < 5$, viscosity dominates and the velocity profile varies linearly. By the time the log-law region is reached $y^+ \approx 30$ turbulent stresses are the dominant force. This behaviour can be clarified by consideration of the variation of total shear stress,

$$\langle \tau \rangle^{xz} = \frac{1}{Re_b} \frac{\partial (U)^{xz}}{\partial y} - \langle R_{12} \rangle^{xz}$$  

(3.15)

which contains both a viscous and turbulent contribution. The variation of $\langle \tau \rangle^{xz}$ is shown in figure 3.15b and demonstrates that the viscous contribution drops from 100% at the wall, to 50% at $y^+ \approx 12$ and is less than 10% by $y^+ \approx 50$. The peak turbulent contribution occurs at $y^+ \approx 30$, which is within the buffer layer, and at this position accounts for 88% of the total shear stress. The linear profile of the total shear stress confirms that the channel is operating in a fully developed state.

The turbulence kinetic energy profile is shown in figure 3.16a and agrees well with the data from Kim et al. (1986) for all $y$-locations. Peak turbulence kinetic energy is observed in the buffer region at $y^+ \approx 16$ which affirms that vigorous turbulent motions reside in this region. To clarify the dynamics of the turbulence kinetic energy, the evaluation of its budget equation is required. The terms in the budget provide details concerning the production, dissipation and transport of turbulence kinetic energy. The turbulence kinetic energy budget for this fully developed,
streamwise-spanwise homogeneous turbulent flow can be

\[
0 = \left\langle u_i' u_j' \frac{\partial (U_j)}{\partial x_j} \right\rangle_{xz}^{xz} + \left\langle u_i' \frac{\partial y'}{\partial x_i} \right\rangle_{xz}^{xz} + \frac{1}{Re_b} \frac{\partial^2}{\partial x_j \partial x_j} \left\langle k \right\rangle_{xz}^{xz}
\]

\[
- \frac{1}{Re_b} \left\langle \frac{\partial u_i'}{\partial x_j} \frac{\partial u_i'}{\partial x_j} \right\rangle_{xz}^{xz} - \frac{\partial}{\partial x_j} \left\langle k u_j' \right\rangle_{xz}^{xz}.
\]

The budget terms are plotted in figure 3.16b and agrees well with the previous data of Kim et al. (1986). For \( y^+ > 35 \) production and dissipation are equal and opposite. At \( y^+ \approx 12 \) production exceeds dissipation and reaches its maximum value. The surplus energy is therefore transported away by turbulent convection and viscous diffusion terms. As the wall is approached an energy deficit is experienced due to dissipation being larger than production, and as a result the role of transport terms switches and they begin to move energy towards the wall. At the wall itself dissipation is balanced by viscous diffusion.

The production of turbulence kinetic energy is directly related to Reynolds shear stress and hence its accurate computation is of interest. The Reynolds stresses are plotted in figure 3.17 and are in excellent agreement with the data of Kim et al. (1986). Out of the three normal stresses, the streamwise component dominates and reaches its maximum in the buffer layer at \( y^+ \approx 14 \), which explains why peak turbulence kinetic energy is observed in this region. The Reynolds shear stress reaches its peak in the log-law region at \( y^+ \approx 32 \).

A primary question in wall turbulence concerns the mechanisms responsible for creating Reynolds shear stress. The quadrant analysis of the Reynolds shear stress provides detailed
Figure 3.16: Validation against Kim et al. (1986), including a) turbulence kinetic energy profile (—) and b) the terms of the turbulence kinetic energy budget (see equation 3.16). All quantities are scaled using the regular no-slip friction velocity. Results of Kim et al. (1986) represented by symbols.

Figure 3.17: Validation against Kim et al. (1986). Reynolds stresses profiles (—). All quantities are scaled by local friction velocity. Results of Kim et al. (1986) represented by symbols.
information on the contribution by various events occurring in the flow (Willmarth & Lu, 1972; Wallace et al., 1972). The analysis divides the Reynolds shear stress into four categories according the signs of $u'$ and $v'$. The Reynolds shear stress was decomposed as

$$\langle R_{12} \rangle = \sum_{n=1}^{4} \langle Q_{12}^n \rangle$$

where $\langle Q_{12}^n \rangle$ is the contribution made by the $n^{th}$ quadrant. The first quadrant, $(u' > 0$ and $v' > 0)$, contains the upwash of high-momentum fluid; the second quadrant, $(u' < 0$ and $v' > 0)$, contains the upwash of low-momentum fluid; the third quadrant, $(u' < 0$ and $v' < 0)$, contains the downwash of low-momentum fluid; and fourth quadrant, $(u' > 0$ and $v' < 0)$, contains the downwash of high-momentum fluid. Second and fourth quadrants represent ejection and sweep motions, respectively, and contribute to the negative Reynolds shear stress which results in positive production of turbulence kinetic energy. First and third quadrants contribute to the positive Reynolds shear stress which results in negative production of turbulence kinetic energy. Contributions towards Reynolds shear stress from each quadrant normalised by the local mean Reynolds shear stress are plotted in figure 3.18. An excellent level of agreement is observed between current results and the previous results of Kim et al. (1986). Figure 3.18 shows that sweeps / ejections are the dominant negative Reynolds-shear-stress-producing events above / below $y^+ = 15$. Quadrant analysis does not provide the structure of the eddies responsible for the sweep and ejection events, but it can help to evaluate the contributions these events make to total mean values, such as the production of turbulent kinetic energy.

![Figure 3.18: Validation against Kim et al. (1986). Reynolds shear stress contribution from each quadrant normalised by the local mean Reynolds shear stress (—). Results of Kim et al. (1986) represented by symbols.](image)

The viscous sublayer ($0 < y^+ < 5$) is dominated by streaky structures, some of which can act as a source for the production of turbulent kinetic energy via the bursting process (Kim et al.,
Chapter 3. Validation

1971). This process consists of elongated low-speed streaks lifting away from the wall, oscillating and then ultimately breaking up into smaller scale structures. An understanding of the physics and geometry of these streaks is desirable in order to inhibit these drag producing events. A basic geometric feature of streamwise streaks is their transverse spacing and this can be evaluated by computing the spanwise two-point autocorrelation

$$
\langle R_{uu}(\Delta z) \rangle = \int_{-\infty}^{+\infty} \langle u'(z) u'(z + \Delta z) \rangle e^{ikz} dz
$$

(3.18)

Evaluation of equation 3.18 at \(y^+ \approx 10\) is shown in figure 3.19 and shows a very good level of agreement with the results of Kim et al. (1986). The minimum value of \(\langle R_{uu}^{xz} \rangle\) occurs at \(\Delta z^+ \approx 50\) and this provides an estimate of the mean distance separating the low- and high-speed streaks, and then mean spacing between the streaks can be approximated as twice this distance.

![Figure 3.19: Validation against Kim et al. (1986). Two-point autocorrelation \(\langle R_{uu}^{xz} \rangle\) at \(y^+ \approx 10\). Results of Kim et al. (1986) denoted as (o).](image)

Instantaneous snapshots of the stochastic field can be used to identify the coherent motions that, ultimately, are responsible for the trends and behaviours conveyed in statistical quantities. The ubiquitous near-wall structures in turbulent flows are the streamwise streaks and streamwise vortices. The near-wall streaks are visualised in an \(xz\) plane at \(y^+ \approx 10\) in figure 1.4a and reveal an organised array of elongated streamwise structures that are regularly separated in the spanwise direction. Their orientation relative to the wall is visualised in figure 1.4b and clearly show inclined high- and low-speed streaks. In order to distinguish the vortical structures from the background turbulence, a vortex identification method based on \(\lambda_2\) is chosen. Here, \(\lambda_2\) is defined as the second largest eigenvalue of the tensor \(S_{ik}S_{kj} + \Omega_{ik}\Omega_{kj}\), where \(S_{ik}\) is the symmetric component and \(\Omega_{ik}\) is the antisymmetric component of the velocity gradient tensor (Jeong & Hussain, 1995). A top view of instantaneous vortical structures for \(0 < y^+ < 100\) is shown in figure 1.4a.
The dominant vortices are aligned predominantly in the streamwise direction, and appear much shorter than the streamwise-oriented streaks. However, the quasi-streamwise vortices have the tendency to intertwine with their neighbours increasing their effective streamwise extent, with some bunches extending up to 800 wall units. A side view of the instantaneous vortical structures is shown in figure 1.4b and demonstrates that the vortices are inclined in a similar fashion as the orientation of streaks. The geometric similarities of the streaks and vortices strongly hints to a connection between these two coherent structures.

Near-wall vortical structures have been described as “pumps” (Robinson, 1991) that pull (sweep) and push (ejection) momentum towards and away from the wall, depending on their sign of rotation, as shown before in figure 1.5. The interplay between streaks and vortical structures is thought to play a central role in the production and the maintenance of turbulence in shear flows. By conducting a series of numerical experiments using DNS, Jimenez & Pinelli (1999) studied turbulent fluctuations in wall-bounded flows and demonstrated that a cycle exists that is local to the near-wall region and does not depend on the outer flow. The autonomous cycle of near-wall turbulence involves the formation of velocity streaks from the advection of the mean profile by streamwise vortices, and the generation of the vortices from the instability of the streaks. Jimenez & Pinelli (1999) noted that interrupting any of these processes lead to laminarization of the turbulent field. There is, however, some debate on the exact origins of coherent structures in turbulent shear-flow and whether streaks are the antecedent of vortices, or vice-versa, and has been discussed by, for example, Hamilton et al. (1995), Schoppa & Hussain (2001) and Chernyshenko & Baig (2005).
3.4 Turbulent channel flow over a superhydrophobic wall

The first DNS of fully developed turbulent channel flow over a textured superhydrophobic wall was conducted by Martell et al. (2009) at friction Reynolds number of $Re_\tau = 180$ under a constant pressure gradient, corresponding to flow condition investigated by Kim et al. (1986). In the work by Martell et al. (2009), regular arrays of microridge and microposts were considered, with a total of seven different texture topologies that are detailed in table 3.3. Martell et al. chose to prescribe a SHS texture only on the bottom wall, and left the remaining top wall as a simple no-slip surface. The top surface of each microfeature was taken to be a no-slip boundary, and the suspended liquid-gas interface between the microfeatures was simulated as a zero-deflected free-shear boundary condition. Schematic representations of the textures can be found in figure 1.10. For their numerical method, Martell et al. (2009) used a second-order accurate Cartesian staggered mesh method with classical projection for the pressure solution. A second-order accurate, three-step, low-storage Runge-Kutta scheme was used for time advancement. A non-uniform mesh was used in the wall-normal direction, to cluster points near the wall, although the exact details were not provided. The computational domain was a box of size $(L_x \times L_y \times L_z) / \delta = (6 \times 2 \times 3)$. For all ridge simulations, eight ridges and eight gaps were prescribed in the spanwise direction. Likewise, for the post simulations, eight posts were prescribed in the spanwise direction and sixteen posts were prescribed in the streamwise direction. The ridge configurations were referred to by the ratio of their spanwise width to spanwise spacing. For example a ridge which was $30\mu\text{m}$ wide and spaced at $30\mu\text{m}$ was referred to as a ‘30$\mu\text{m}$-30$\mu\text{m}$ ridge’. For all the simulations conducted by Martell et al. (2009), 128 points were used in each spatial direction, except for one thin ‘15$\mu\text{m}$-15$\mu\text{m}$ ridge’ case where the resolution was doubled in each direction. For all the ridge cases, the combined distance spanned by the no-slip micro-feature and the free-shear interface was resolved using 16 grid points, except the

<table>
<thead>
<tr>
<th>Texture topology</th>
<th>Designation</th>
<th>$\Psi_s$ (%)</th>
<th>$\Delta\tau_w$ (%)</th>
<th>$\Delta U_s$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Micro-ridges</td>
<td>“15$\mu\text{m}$-15$\mu\text{m}$”</td>
<td>50.0</td>
<td>24.2</td>
<td>7.1</td>
</tr>
<tr>
<td></td>
<td>“30$\mu\text{m}$-30$\mu\text{m}$”</td>
<td>50.0</td>
<td>39.6</td>
<td>9.8</td>
</tr>
<tr>
<td></td>
<td>“30$\mu\text{m}$-50$\mu\text{m}$”</td>
<td>37.5</td>
<td>51.8</td>
<td>15.8</td>
</tr>
<tr>
<td></td>
<td>“30$\mu\text{m}$-90$\mu\text{m}$”</td>
<td>25.0</td>
<td>64.1</td>
<td>25.3</td>
</tr>
<tr>
<td>Micro-posts</td>
<td>“30$\mu\text{m}$-30$\mu\text{m}$”</td>
<td>25.0</td>
<td>47.2</td>
<td>10.5</td>
</tr>
<tr>
<td></td>
<td>“30$\mu\text{m}$-50$\mu\text{m}$”</td>
<td>14.1</td>
<td>61.3</td>
<td>19.8</td>
</tr>
<tr>
<td></td>
<td>“30$\mu\text{m}$-90$\mu\text{m}$”</td>
<td>6.25</td>
<td>76.6</td>
<td>36.5</td>
</tr>
</tbody>
</table>

Table 3.3: Turbulent drag reduction performance of textures investigated by Martell et al. (2009) including percentage solid fraction, $\Psi_s$, the percentage reduction in wall shear stress, $\Delta\tau_w(\%)$, and percentage streamwise slip velocity normalized by bulk velocity, $\Delta U_s(\%)$. 

(2009)
'15µm-15µm ridge' case which used 32 points. Martell et al. (2009) computed a variety of statistical quantities including slip velocities, mean velocity profiles and Reynolds stress profiles - which were all \( xz \) planar-averaged. Note that this is equivalent to collecting phase-averaged statistics, then averaging the result over all phases and, hence, all phase information is lost.

\[
\text{Re}_\tau \quad L_x \quad L_z \quad \Delta x^+ \quad \Delta y^+ \quad y_{\text{min}} \quad \Delta t^+ \quad T^+_{\text{int}}
\]

<table>
<thead>
<tr>
<th>Case</th>
<th>Re(_\tau)</th>
<th>L(_x)</th>
<th>L(_z)</th>
<th>(\Delta x^+)</th>
<th>(\Delta y^+)</th>
<th>(y_{\text{min}})</th>
<th>(\Delta t^+)</th>
<th>T(^+)_{\text{int}}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Martell et al. (2009)</td>
<td>180</td>
<td>6</td>
<td>3</td>
<td>8.44</td>
<td>4.22</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Current</td>
<td>180</td>
<td>6</td>
<td>3</td>
<td>8.44</td>
<td>4.22</td>
<td>2.95</td>
<td>0.19</td>
<td>0.046</td>
</tr>
</tbody>
</table>

Table 3.4: Discretization details of Martell et al. (2009) and the current work.

To ensure that the current simulations can accurately simulate fully developed turbulent flow over textured surfaces, the results of the '30µm-30µm ridge' case first reported by Martell et al. (2009) were reproduced. The rationale for choosing this particular case is motivated by its geometrical simplicity and its close agreement with 30µm-30µm ridge data from experimental results (Daneillo et al., 2009) at the same Reynolds number, which shows the same shear stress reduction as predicted by DNS. The simulation was performed at a constant mean streamwise pressure gradient corresponding to the reference case of Kim et al. (1986). The pressure gradient can be written in terms of the friction Reynolds number by evaluating the Reynolds-Averaged-Navier-Stokes equations for the streamwise direction at the wall to obtain the expression

\[
- \left. \frac{dP}{dx} \right|_w = \left( \frac{\text{Re}_\tau}{\text{Re}_b} \right)^2.
\]

To generate a suitable turbulent initial condition the same procedure outlined in the previous section was used. In short, a high-amplitude, zero-mean broadband disturbance was superimposed on to a laminar parabolic mean flow. The only difference was that the bottom wall was a textured surface, as opposed to no-slip. A time-history of planar-averaged Re\(_\tau\) of both walls was monitored until the friction Reynolds number stabilised near Re\(_\tau\) = 180 in a statistically stationary manner. Once this state was reached, the flow simulation was advanced an additional 10 non-dimensional time units to ensure that any transient effects had been sufficiently washed away. Once a satisfactory initial condition had been obtained, turbulence statistics were accumulated \( T^+ = Tu^2/\nu \approx 1.6 \times 10^4 \). Plots of mean streamwise velocity profile and Reynolds stresses are shown in figure 3.20. An excellent level of agreement is observed between current results and the previous results of Martell et al. (2009), confirming the accuracy of our computations and statistical analysis in the case of surface texturing.
3.5 Overview

In this section a comprehensive validation procedure has been presented. The current DNS code reproduced previous results with excellent levels of agreement for both transitional and turbulent channel flows. These tests confirm the accuracy and reliability of the numerical scheme, boundary conditions, averaging procedures and post-processing routines which are used throughout the remainder of this document.
Chapter 4

Critical assessment of performance of streamwise-aligned ridges

4.1 Introduction

This chapter contains four sections. This section begins with a critical assessment of the previous DNS of turbulent flow of SHS textures by Martell et al. (2009), followed by details of the current computational setup. Section 4.2 presents the results for mean flow statistics of fully developed turbulent channel flow over a streamwise-aligned micro-ridge SHS texture, including details of primary, secondary and slip fluid motions, followed by turbulence statistics presented in Section 4.3. Finally, Section 4.4 contains the derivation, verification and evaluation of an identity that explains how the primary, secondary and turbulent fluid motions contribute to the levels of turbulent skin friction.

4.1.1 Critical assessment of Martell et al. (2009)

The first DNS of fully developed turbulent flow over textured superhydrophobic surfaces was conducted by Martell et al. (2009). This numerical study was conducted at a friction Reynolds number of $Re_\tau \approx 180$, where the mean streamwise pressure gradient was held constant. In their simulations, Martell et al. (2009) used the mixed no-slip no-stress boundary conditions originally studied by Philip (1972a,b) to characterise regular arrays of micro-ridge and micro-post texture topologies. The top of each micro-feature was represented by the no-slip boundary and the liquid-gas interface between the micro-features was simulated as a flat no-stress boundary. A schematic of the textured surface model can be found in figure 1.10.

Martell et al. (2009) solved the governing Navier-Stokes equations using a second-order accurate Cartesian staggered mesh method with classical projection for the pressure solution. A second-order accurate, three-step, low-storage Runge-Kutta scheme was used to advance the solution in time. The dimensions of the computational box and its spatial discretization, along
Chapter 4. Critical assessment of performance of streamwise-aligned ridges

Table 4.1: Spatial discretization details for DNS studies of turbulent channel flow at $Re_\tau \approx 180$. Kim et al. (1986) and Moser et al. (1999) simulated regular no-slip turbulent channel flow. Min & Kim (2004) simulated turbulent channel flow over a Navier-slip surface. Final column denotes whether mass flow rate ($\dot{m}$) or mean streamwise pressure gradient was held constant.

<table>
<thead>
<tr>
<th>Reference</th>
<th>$(L_x \times L_y \times L_z) / \delta$</th>
<th>$\Delta x^+$</th>
<th>$\Delta z^+$</th>
<th>$\Delta y_{min}^+$</th>
<th>$\Delta y_{max}^+$</th>
<th>$\dot{m} = C$</th>
<th>$\frac{d(P)^{xz}}{dx} = C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kim et al. (1986)</td>
<td>$4\pi \times 2 \times 2\pi$</td>
<td>11.78</td>
<td>7.01</td>
<td>0.05</td>
<td>4.40</td>
<td>$\dot{m} = C$</td>
<td></td>
</tr>
<tr>
<td>Moser et al. (1999)</td>
<td>$4\pi \times 2 \times \frac{4}{3}\pi$</td>
<td>17.70</td>
<td>5.90</td>
<td>0.05</td>
<td>4.40</td>
<td>$\dot{m} = C$</td>
<td></td>
</tr>
<tr>
<td>Min &amp; Kim (2004)</td>
<td>$7 \times 2 \times 3.5$</td>
<td>9.84</td>
<td>4.92</td>
<td>0.30</td>
<td>-</td>
<td>$\dot{m} = C$</td>
<td></td>
</tr>
<tr>
<td>Martell et al. (2009)</td>
<td>$6 \times 2 \times 3$</td>
<td>8.44</td>
<td>4.22</td>
<td>-</td>
<td>-</td>
<td>$\dot{m} = C$</td>
<td></td>
</tr>
</tbody>
</table>

with some other relevant references for comparison, are detailed in table 5.1. The computational mesh employed by Martell et al. (2009) was uniform in the streamwise and spanwise directions and had non-uniform spacing in the wall-normal direction in order to cluster points close to the wall, although precise details of the stretching were not provided.

The boundary conditions in the wall-normal direction were asymmetric, with SHS textures being prescribed at the bottom wall and a regular no-slip surface being prescribed at the top wall (see figure 4.1). Periodic boundary conditions were employed in the streamwise and spanwise directions. Martell et al. (2009) textured the bottom wall with eight ridges (and gaps). Similarly,

Figure 4.1: Schematic of asymmetric channel geometry used by Martell et al. (2009), showing textured bottom wall and a regular no-slip top wall.

a minimum of eight posts in the spanwise direction and sixteen in the streamwise direction, were used in the post simulations. In every case studied by Martell et al. (2009), the distance spanned by the sum of each microfeature spacing, $d$, and the gap between them, $g$, was resolved using eight and sixteen grid points in the streamwise and spanwise direction, respectively.

Both streamwise-aligned ridge and post topologies were considered by Martell et al. (2009). The effect of varying the feature width and spacing was systematically varied to investigate the response of statistical results including slip velocities, wall shear stresses and Reynolds stresses.
In total, seven different textures were considered and their geometric details are provided in table 6.2.3.

<table>
<thead>
<tr>
<th>Topology</th>
<th>Designation</th>
<th>g/d</th>
<th>d/δ</th>
<th>g/δ</th>
<th>d⁺</th>
<th>g⁺</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ridges</td>
<td>“15µm-15µm”</td>
<td>1.00</td>
<td>0.09375</td>
<td>0.09375</td>
<td>16.875</td>
<td>16.875</td>
</tr>
<tr>
<td></td>
<td>“30µm-30µm”</td>
<td>1.00</td>
<td>0.18750</td>
<td>0.18750</td>
<td>33.750</td>
<td>33.750</td>
</tr>
<tr>
<td></td>
<td>“30µm-50µm”</td>
<td>1.67</td>
<td>0.14062</td>
<td>0.23436</td>
<td>25.312</td>
<td>42.187</td>
</tr>
<tr>
<td></td>
<td>“30µm-90µm”</td>
<td>3.00</td>
<td>0.09375</td>
<td>0.28124</td>
<td>16.875</td>
<td>50.625</td>
</tr>
<tr>
<td>Posts</td>
<td>“30µm-30µm”</td>
<td>1.00</td>
<td>0.18750</td>
<td>0.18750</td>
<td>33.750</td>
<td>33.750</td>
</tr>
<tr>
<td></td>
<td>“30µm-50µm”</td>
<td>1.67</td>
<td>0.14072</td>
<td>0.23436</td>
<td>25.312</td>
<td>42.187</td>
</tr>
<tr>
<td></td>
<td>“30µm-90µm”</td>
<td>3.00</td>
<td>0.09375</td>
<td>0.28124</td>
<td>16.875</td>
<td>50.625</td>
</tr>
</tbody>
</table>

Table 4.2: Textures topologies investigated by Martell et al. (2009).

The statistical averaging procedures used by Martell et al. (2009) consisted of two steps. First, a set of temporally averaged velocity and pressure data was accumulated around each feature. Second, the temporal averages were ensemble averaged across all the features returning statistical quantities that were a function of wall-normal position only. The two-step procedure is equivalent to averaging phase-averaged statistics across all possible phases, which is denoted by $\langle \cdot \rangle^{xz}$ using the current notation. Using the notation chosen by Martell et al. (2009), the mean streamwise velocity, for example, was computed using,

$$U(y) = \overline{⟨ \overline{u}^{xz} ⟩},$$

$$= \left( \frac{1}{L_x L_z} \int_0^{L_x} \int_0^{L_z} ⟨ \overline{u}^{xz} ⟩ dx dz, \right)$$

where $⟨ \cdot ⟩$ denotes a temporally averaged variable and where $\langle ⟨ \cdot ⟩ \rangle$ denotes an ensemble average over all the posts of ridges in the simulation. The temporally averaged variables contain phase-dependent information, but this additional level of statistical detail was lost upon ensemble averaging.

Martell et al. (2009) quantified their turbulent drag reduction performance using two integral performance parameters. First, the wall shear stress reduction,

$$\Delta \tau_w = \left[ 1 - \frac{⟨ \overline{\tau_B^{xz}} ⟩}{⟨ \overline{\tau_T^{xz}} ⟩} \right] \times 100,$$

where $⟨ \overline{\tau_w^{xz}} ⟩$ is the average wall shear stress between the top (denoted by superscript $T$) and the bottom (denoted by superscript $B$) walls, $⟨ \overline{\tau_w^{xz}} ⟩ \equiv \left( ⟨ \overline{\tau_B^{xz}} ⟩ + ⟨ \overline{\tau_T^{xz}} ⟩ \right) / 2$. As a consequence of choosing to hold the pressure gradient constant, the average wall shear stress $⟨ \overline{\tau_w^{xz}} ⟩$ within the system remained constant. Therefore, a reduction of shear stress on the bottom wall was balanced
by an increase of shear stress on the top wall. The second integral performance parameter was based on the slip velocity normalised by the bulk velocity and was defined as

$$\Delta U_s = \frac{(U_s)^{xz}}{U_b} \times 100,$$  \hspace{1cm} (4.2)

Martell et al. (2009) demonstrated that micro-posts consistently outperformed micro-ridges, for a fixed feature spacing $g$. The best-performing texture was the “30µm-90µm” posts which achieved a slip velocity in excess of 75% of the bulk velocity, with a corresponding wall shear-stress reduction of $\Delta \tau_w = 36.5\%$. Details of turbulent drag reduction performance for all the textures analysed by Martell et al. (2009) can be found in table 4.3. The superior performance that posts had over ridges was credited to an increased average slip velocity, which resulted in decreased wall shear stress. Martell et al. (2009) pointed out that the levels of shear stress reduction for the “30µm-30µm” ridges were in close agreement (within 7.5%) with the experimental results of Daneillo et al. (2009) for the same texture, who reported $\Delta \tau_w = 10.6$, for the same Reynolds number. This agreement motivates a further analysis of this particular topology, and in fact will become the focus of this chapter.

For the “30µm-30µm” ridges, 50% of the surface area is no-slip and the remaining 50% is no-shear. Therefore, if the no-slip regions were to remain unaffected, drag reduction across the wall can be set to a target of 50%. However, if the no-slip regions were to be affected the average drag across the wall could either increase or decrease. Martell et al. (2009) reported that $\Delta \tau_w \approx 10\%$ for the “30µm-30µm” ridges, which is significantly less than the 50% target. Considering that wall shear stress is, by definition, zero along the free-stress interfaces and make no contribution towards the levels of average wall shear stress, the less than ideal performance must be due to increased wall shear stress across the no-slip features. This observation was hinted at by Martell et al. (2009) who wrote, “While the drag is locally zero on the free surface between the posts or

### Table 4.3: Turbulent drag reduction performance of textures investigated by Martell et al. (2009) including percentage solid fraction, $\Psi_s (%)$, the percentage reduction in wall shear stress, $\Delta \tau_w (%)$, and the percentage slip velocity, $\Delta U_s (%)$, which was normalized by the bulk velocity.

<table>
<thead>
<tr>
<th>Texture topology</th>
<th>Designation</th>
<th>$\Psi_s (%)$</th>
<th>$\Delta \tau_w (%)$</th>
<th>$\Delta U_s (%)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Micro-ridges</td>
<td>“15µm-15µm”</td>
<td>50.0</td>
<td>24.2</td>
<td>7.1</td>
</tr>
<tr>
<td></td>
<td>“30µm-30µm”</td>
<td>50.0</td>
<td>39.6</td>
<td>9.8</td>
</tr>
<tr>
<td></td>
<td>“30µm-50µm”</td>
<td>37.5</td>
<td>51.8</td>
<td>15.8</td>
</tr>
<tr>
<td></td>
<td>“30µm-90µm”</td>
<td>25.0</td>
<td>64.1</td>
<td>25.3</td>
</tr>
<tr>
<td>Micro-posts</td>
<td>“30µm-30µm”</td>
<td>25.0</td>
<td>47.2</td>
<td>10.5</td>
</tr>
<tr>
<td></td>
<td>“30µm-50µm”</td>
<td>14.1</td>
<td>61.3</td>
<td>19.8</td>
</tr>
<tr>
<td></td>
<td>“30µm-90µm”</td>
<td>6.25</td>
<td>76.6</td>
<td>36.5</td>
</tr>
</tbody>
</table>
ridges, it is non-zero over the posts and ridges because of the no-slip boundary condition. The net drag is the sum of these two effects and depends on which effect dominates”.

In summary, Martell et al. (2009) were the first to simulate fully developed turbulent channel flow over SHS textures using DNS. Mixed no-slip no-stress boundary conditions were used to characterise regular arrays of micro-ridge and micro-post texture topologies. The performance of seven different textures were quantified by computing reductions in the average wall shear stress and increases in average streamwise slip velocity. Martell et al. (2009) concluded that for a given ratio of microfeature size to microfeature spacing, micro-posts always outperformed micro-ridges.
4.1.2 Current simulation setup

The previous DNS of Martell et al. (2009) simulated fully developed turbulent channel flow over a SHS texture at a friction Reynolds of $Re_{c} \approx 180$, where the mean streamwise pressure gradient was held constant. Here, particular attention to the “30µm-30µm” ridge texture topology studied by Martell et al. (2009), who highlighted good agreement between its reduction in average shear stress with the experimental results of Daneillo et al. (2009) for the same texture at the same Reynolds number. Although 50% of the “30µm-30µm” ridges is characterised by the free-stress boundary condition, only a 10% decrease in average wall shear stress (relative to the regular no-slip flow) was reported by Martell et al. (2009). The question then becomes: why is the reduction in average wall shear stress significantly less than 50%? The purpose of this chapter is to reveal the flow mechanics responsible for the less than ideal levels of turbulent skin-friction reduction. However, before presenting any statistical results, the differences between the current computational setup and the previous work done by Martell et al. (2009) are explained.

There are three major differences between the current simulations and the previous simulations of Martell et al. (2009). First, SHS textures are prescribed on both the top and bottom walls of the channel (see figure 4.2) in the present study. The choice to symmetrically texture the channel walls is motivated by statistical symmetries about the plane $y = \delta$, which permits statistics collected in the lower and upper halves of the channel to be appropriately ensemble-averaged, and also proved useful when deriving an extension of an identity first obtained by Fukagata et al. (2002) that can be used to distinguish the contributions made by mean and turbulent fluid motions towards the total level of turbulent skin-friction.

Second, the mass flow rate is held constant in the current work, which is in contrast to Martell et al. (2009) who opted to fix the mean streamwise pressure gradient corresponding to that of a regular no-slip channel flow at a friction Reynolds number $Re_{c} \approx 180$. For a fully developed turbulent channel flow, viscous losses at the wall are balanced by a (negative) mean streamwise pressure gradient. The balance between pressure gradient and viscous losses at the wall can be
written as
\[- \frac{d \langle P \rangle_{xz}^p}{dx} \bigg|_w = \nu \frac{\partial \langle U \rangle_{xz}^p}{\partial y} \bigg|_w\]

which shows that for a flow with constant density and viscosity if one were to hold the mean pressure gradient constant, it follows that the mean velocity gradient at the wall must also remain constant. For the axisymmetric boundary conditions used by Martell et al. (2009), the balance between the average mean streamwise pressure gradient and viscous losses at top and bottom walls can be written as

\[C = \frac{1}{2\nu} \left[ \frac{\partial \langle U \rangle_{xz}^p}{\partial y} \bigg|_w^B + \frac{\partial \langle U \rangle_{xz}^p}{\partial y} \bigg|_w^T \right]\]

which explains why when the mean wall shear drops on the bottom (denoted as superscript \(B\)) textured wall the wall shear on the top (denoted as superscript \(T\)) must increase to satisfy the constant (denoted as \(C\)) mean streamwise pressure gradient. It follows that from the definition of friction velocity, \(u_r = \sqrt{\nu \frac{\partial \langle U \rangle_{xz}^p}{\partial y}}\), the average of the friction Reynolds number, \(Re_r = u_r \delta / \nu\), between the top and bottom walls remains fixed in the work done by Martell et al. (2009). In the present work, the mass flow rate is held fixed. Therefore, the natural way to infer a drag reduction in the current work is to monitor the streamwise pressure gradient in time. Since the mass flow rate is constant, a reduction in pressure gradient corresponds to a reduction in friction at the wall.

Third, the grid resolution in the spanwise direction was increased four times, relative to Martell et al. (2009). Therefore, each of the eight ridges in the span is resolved using sixty-four points: thirty-two points across the no-slip ridge and thirty-two points across the free-stress interface. The spacing between adjacent cells above the texture, \(\Delta z^+\), is approximately 1 wall unit in the present calculation which is much finer than various other DNS studies of fully turbulent flow (see table 4.4). However, the present spanwise grid resolution is similar to that used by Choi et al. (1993), who conducted DNS of a fully turbulent channel flow to study the

<table>
<thead>
<tr>
<th>Reference</th>
<th>((L_x \times L_y \times L_z) / \delta)</th>
<th>(\Delta x^+)</th>
<th>(\Delta z^+)</th>
<th>(\Delta y_{min}^+)</th>
<th>(\Delta y_c^+)</th>
<th>Control</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kim et al. (1986)</td>
<td>4(\pi) \times 2 \times 2(\pi)</td>
<td>11.78</td>
<td>7.01</td>
<td>0.05</td>
<td>4.40</td>
<td>None</td>
</tr>
<tr>
<td>Choi et al. (1993)</td>
<td>(\pi) \times 2 \times 0.289(\pi)</td>
<td>35.34</td>
<td>1.28</td>
<td>0.15</td>
<td>7.10</td>
<td>Riblets</td>
</tr>
<tr>
<td>Moser et al. (1999)</td>
<td>4(\pi) \times 2 \times \frac{2}{\pi}</td>
<td>17.70</td>
<td>5.90</td>
<td>0.05</td>
<td>4.40</td>
<td>None</td>
</tr>
<tr>
<td>Min &amp; Kim (2004)</td>
<td>7 \times 2 \times 3.5</td>
<td>9.84</td>
<td>4.92</td>
<td>0.30</td>
<td>-</td>
<td>Navier-slip</td>
</tr>
<tr>
<td>Martell et al. (2009)</td>
<td>6 \times 2 \times 3</td>
<td>8.44</td>
<td>4.22</td>
<td>-</td>
<td>-</td>
<td>Texture</td>
</tr>
<tr>
<td>Current work</td>
<td>6 \times 2 \times 3</td>
<td>8.44</td>
<td>1.05</td>
<td>0.20</td>
<td>4.10</td>
<td>Texture</td>
</tr>
</tbody>
</table>

Table 4.4: Spatial discretization details for DNS studies of turbulent channel flow at \(Re_r \approx 180\).
drag-reducing capabilities of a riblet geometry, noted that thirty-two grid points on each riblet surface were necessary to resolve the high-shear rates near the riblet tips.

Now that the main differences between the current simulations and the previous simulations of Martell et al. (2009) have been addressed, a phase-averaged statistic obtained using the current setup for the “30µm-30µm” ridges is discussed. All statistics shown in the present chapter are obtained from an integration time of $T^+ = Tu^2_1/\nu \approx 1.6 \times 10^4$ with a time step of $\Delta t^+ = 0.046$.

For a fully-developed channel flow with a constant mass flow rate, the mean streamwise pressure gradient is balanced by viscous losses. At the wall, the skin-friction coefficient is defined as

$$\langle C_f \rangle \equiv \frac{\tau_w}{2 \rho U_b^2},$$  \hspace{1cm} (4.3)

where the wall shear stress $\tau_w = \rho \nu \frac{\partial \langle u \rangle}{\partial y}|_w$. Figure 4.3 shows the spanwise distribution of skin-friction across the SHS texture. Surprisingly, the skin-friction is higher for all no-slip phases, relative to the regular no-slip flow. In addition, figure 4.3 shows a dramatic increase in skin-friction as the edges of the feature are approached and is in fact almost four times higher than the minimum skin-friction value, which is found at the centre of the no-slip micro-feature at $\phi_z = \pi/2$. As expected, there is zero skin-friction across all of the free-slip phases. The average effect of the increased skin-friction on the no-slip and the zero skin-friction of the free-slip results in a drag reduction of 11.6%, relative to the reference no-slip case. However, the mechanics of the flow which lead to the spatial distribution of $\langle C_f \rangle^x$ remain unclear.

Furthermore, due to the statistical averaging procedures incorporated by Martell et al. (2009),
the phase-dependence as a function of wall-normal position is also unclear, although, in a subsequent publication Martell et al. (2010) speculated that, “
statistics taken over the ridge will resemble those for a “normal” no-slip wall, and similarly statistics taken over a gap will be similar to those found above a “normal” free surface.” This statement was not substantiated with any quantitative evidence and motivates an in-depth analysis of phase-averaged statistics for both mean and turbulent motions. To this end, the aim of this chapter is to address the following questions:

1. How does the presence of the texture globally affect the mean flow?
2. Can the phase-averaged statistics be used to characterise the mean flow field more completely than Martell et al. (2009, 2010)?
3. How does the presence of the texture globally affect the turbulence statistics?
4. Can the phase-averaged statistics be used to characterise the turbulence statistics more completely than Martell et al. (2009, 2010)?
5. What are the flow physics responsible for the \( \langle C_f \rangle \) distribution shown in figure 4.3?

### 4.2 Mean flow statistics

This section presents the results for the mean fluid motions, which are divided into three separate categories. First, results for the primary flow are reported, which is simply the fluid motion aligned with the primary (streamwise) flow direction. Second, results for the secondary flow are reported, which is periodic fluid motion that occurs in the \( y - \phi_z \) plane perpendicular to the primary flow. Lastly, results for the slip flow are reported, which are characterised by fluid motions that occur on the SHS texture itself.

#### 4.2.1 Primary flow

The phase-averaged momentum equation can be written in tensor form as,

\[
\frac{\partial \langle U_i \rangle}{\partial x_j} = \frac{\partial \langle P \rangle}{\partial x_i} + \frac{1}{Re_b} \frac{\partial^2 \langle U_i \rangle}{\partial x_j \partial x_j} - \frac{\partial}{\partial x_j} \langle R_{ij} \rangle,
\]

where the Reynolds stress tensor is \( \langle R_{ij} \rangle \equiv \langle u_i' u_j' \rangle \). For a streamwise-homogeneous texture, the governing equation for the mean streamwise momentum can be written as

\[
\frac{\partial \langle u \rangle}{\partial y} + \frac{\partial \langle u \rangle}{\partial z} = -\frac{d \langle p \rangle}{dx} + \frac{1}{Re_b} \left( \frac{\partial^2 \langle u \rangle}{\partial y^2} + \frac{\partial^2 \langle u \rangle}{\partial z^2} \right) - \left( \frac{\partial \langle R_{12} \rangle}{\partial y} + \frac{\partial \langle R_{13} \rangle}{\partial z} \right),
\]
which, when averaged across all spanwise phases, gives

$$0 = -\frac{d\langle P \rangle^{xz}}{dx} + \frac{1}{Re_b} \frac{\partial^2 \langle U \rangle^{xz}}{\partial y^2} - \frac{\partial \langle R_{12} \rangle^{xz}}{\partial y}.$$  \hspace{1cm} (4.5)$$

Comparing equation 4.4 to equation 4.5 demonstrates the additional terms that are retained due to the phase-averaging procedures. A total of four terms are recovered when the governing equations are derived in phase-averaged form, which can be classified as three different physical mechanisms. The term on the left-hand side of equation 4.4 represents the convection of the mean flow by a steady, periodic velocity component which occurs on the transverse plane - perpendicular to the primary flow direction. The second term on the right-hand side of equation 4.4 contains a new spanwise diffusion term and the third term on the right-hand side includes an additional spanwise gradient of the cross-flow Reynolds shear stress, $\langle R_{13} \rangle$.

The additional terms present in equation 4.4 will be used to characterise the mean flow and turbulent fields, but first, by using equation 4.5, the global effect (i.e. the averaged effect across all phases) that the texture has on the mean flow field will be demonstrated analytically. First, the mean streamwise velocity $\langle U \rangle^{xz}$ is expanded using a Taylor series such that,

$$\langle U \rangle^{xz} = \left. \langle U \rangle^{xz} \right|_w + \left. \frac{\partial \langle U \rangle^{xz}}{\partial y} \right|_w \left( y + \frac{1}{2!} \left. \frac{\partial^2 \langle U \rangle^{xz}}{\partial y^2} \right|_w \right)^2 + O \left( y^3 \right).$$  \hspace{1cm} (4.6)$$

Next, equation 4.5 is evaluated at the wall and the resulting expression was substituted into equation 4.6 to give

$$\langle U \rangle^{xz} = \left. \langle U \rangle^{xz} \right|_w + \left. \frac{\partial \langle U \rangle^{xz}}{\partial y} \right|_w \left( y + \frac{Re_b}{2} \left. \frac{\partial \langle P \rangle^{xz}}{\partial x} \right|_w \right)^2 + O \left( y^3 \right).$$  \hspace{1cm} (4.7)$$

Then using the definitions for the friction velocity $u_\tau = \left( \tau_w / \rho \right)$ and wall units $y^+ = y_{u_\tau} / \nu$, equation 4.7 can be recast in wall units to obtain

$$\langle U^+ \rangle^{xz} = \left. \langle U_x^+ \rangle^{xz} \right|_{\text{viscous sublayer}} + y^+ - \frac{Re_b^2}{2Re_x^2} \left( -\frac{\partial \langle P \rangle^{xz}}{\partial x} \right) y^{+2} + O \left( y^{+3} \right).$$  \hspace{1cm} (4.8)$$

To zeroth order, equation 4.8 demonstrates that the profile for $\langle U^+ \rangle^{xz}$ is offset by a positive source term: the average slip velocity across the texture, $\langle U_x^+ \rangle^{xz}$. To first order, $\langle U^+ \rangle^{xz}$ varies linearly with $y^+$, which can be used to define viscous sublayer. Above the viscous sublayer, equation 4.8 demonstrates that second-order changes in $y^+$ are sensitive to changes in friction Reynolds number and mean streamwise pressure gradient. Therefore, for drag-reduced flows changes above the viscous sublayer region may be expected.

The mean streamwise velocity profile is shown in figure 4.4a, normalised by the mean $u_\tau$. 

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The equations for the standard law of the wall are also included in figure 4.4a. Compared to the reference case, the structure of the turbulent flow profile agree with the analytical form conveyed by equation 4.8: an observed shift of the viscous sublayer due to wall-slip and a wall-normal elevation of the log-law region. The latter observation is a common observation in drag-reduced flows and has been noted across a broad range of passive (Lumley, 1973; Walsh, 1982; Choi et al., 1993) and active (Jung et al., 1992; Choi et al., 1994; Min & Kim, 2004) flow control strategies. The mean streamwise velocity profile normalised by bulk velocity $U_b$ is shown in figure 4.4b, and shows, relative to the reference case, a streamwise momentum surplus/(deficit) below/(above) $y/\delta \approx 0.19$. The surplus momentum in the near-wall region is due to the slip velocity at the wall, which results in a momentum deficit in the outer flow to ensure the mass flow rate is held constant.

![Figure 4.4: Mean streamwise velocity profiles scaled by (a) local friction velocity $u_\tau$ and (b) bulk velocity $U_b$, for the textured (—) and reference no-slip (↔) case. Law of wall equations in (a) are denoted by (---).

Next, by utilising the phase-averaged statistics, the mean streamwise velocity profile was decomposed as

$$
\langle U \rangle^{xz} = \frac{1}{2} \left( \langle U \rangle^{nz} + \langle U \rangle^{fs} \right).
$$

(4.9)

The results from the equation 4.9 are plotted in figure 4.5a and demonstrate that the statistics become phase-dependent below $y^+ \approx 34$, which is equivalent to the spanwise width of the no-slip and free-slip features. This observation is in agreement with Martell et al. (2010), who remarked that textured surfaces affect the near-wall region up to a distance less than or equal to feature spacing in wall units. For $0 < y^+ < 34$ the phase-averaged profiles differ significantly. Above the free-slip feature, the averaged profile has a momentum surplus, relative to $\langle U \rangle^{xz}$, and as the wall is approached the profile plateaus in order to satisfy the free-slip boundary condition. Above the no-slip feature, the average profile has a momentum deficit, relative to $\langle U \rangle^{xz}$, and at the wall
appears to exhibit a larger gradient than the reference no-slip case of Kim et al. (1986).

To quantify the gradients present in the mean flow, the components of the mean rate of strain tensor,
\[
\langle S_{ij} \rangle = \frac{1}{2} \left[ \frac{\partial \langle u_i \rangle}{\partial x_j} + \frac{\partial \langle u_j \rangle}{\partial x_i} \right],
\]
were evaluated. The increased levels of wall shear are confirmed in figure 4.5b, where \( \langle S_{12} \rangle = \frac{1}{2} \left( \frac{\partial \langle u \rangle}{\partial y} + \frac{\partial \langle v \rangle}{\partial x} \right) \) is plotted. Applying the same decomposition shown in equation 4.9 to \( \langle S_{12} \rangle \) shows that the average shear stress across the no-slip phases, \( \langle S_{12} \rangle^{ns} \), is in excess of the reference no-slip value reported by Kim et al. (1986) for \( y^+ < 4.8 \). At the wall, \( \langle S_{12} \rangle^{ns} \) is 32% higher relative to \( \langle S_{12} \rangle^{xz} \) for the reference case. The physical reasons for this increase in wall shear will be addressed fully in a subsequent section.

To completely characterise the phase-dependence of the streamwise velocity field, contours of \( \langle u^+ \rangle \) are plotted on the transverse \( \phi_z-y \) plane in figure 4.6 a. Far from the wall, above \( y^+ = 34 \), the contours show minimal variation in the spanwise direction. As the wall is approached, at \( y^+ = 15 \), the contours start to exhibit a corrugated pattern with the minimum values of \( \langle u^+ \rangle \) occurring at the central no-slip phase, \( \phi_z = \pi/2 \), and the maximum values occurring at the central free-slip phase, \( \phi_z = 3\pi/2 \). This position of these peak / (trough) values are consistent with the notion of streams momentum surplus / (deficit) occurring in the free-slip / (no-slip) phases, respectively. Moving even closer to the wall, \( y^+ = 3 \), the corrugated behaviour of \( \langle u^+ \rangle \) is accentuated significantly and strong mean spanwise shear layers (or, equivalently, columns of wall-normal vorticity, \( \langle \omega_y \rangle^z = \frac{\partial \langle v \rangle}{\partial z} \)) are present in the near-wall region. The strong spanwise
shear has the potential to influence both the mean flow and the stochastic flow. The effect of mean flow skewing can support steady motions that occur in the plane perpendicular to the primary flow, i.e. a secondary flow. Additionally, the abundance of mean spanwise shear will act to correlate turbulent fluctuations in the transverse plane, a phenomenon which remained undetected by Martell et al. (2009) due to their averaging procedures.

Figure 4.6: Phase-averaged mean streamwise momentum including (a) \( \langle U^+ \rangle \) on the transverse plane for \( 0 < y^+ < 60 \) and (b) spanwise profiles of \( \langle U^+ \rangle \) at \( y^+ = \{34, 15, 3\} \).

Figure 4.7: Phase-averaged primary shear including (a) \( \langle S_{12} \rangle_x \) and (b) \( \langle S_{13} \rangle_x \) on the transverse \( y-\phi_z \) plane for \( 0 < y^+ < 60 \).
To completely characterise the phase-dependence of mean shear in the primary flow, contours of \( \langle S_{12} \rangle^x \) and \( \langle S_{13} \rangle^x \) are plotted in figures 4.7a and 4.7b, respectively. The primary wall-normal shear (figure 4.7a) is highest on the edges of the no-slip micro-features and is symmetric about their centre. Moving away from the wall, the primary wall-normal shear becomes phase-independent at \( y^+ \approx 34 \) and beyond this point it remains non-zero until the channel half-height is encountered. The primary spanwise shear (figure 4.7b) is zero everywhere except at the edges the free-slip bands and is antisymmetric about their centre. The regions of high positive / negative spanwise shear at the edges of the free-slip band protrude over the edges of the neighbouring no-slip band and vanish at \( y^+ \approx 10 \). The notion of high shear at the edges of the no-slip band agree well with the notion of increased skin-friction in the same region (see figure 4.3), but the flow mechanics for this behaviour are yet to be clarified.

### 4.2.2 Secondary flow

The phase-averaged streamwise momentum is shown in equation 4.4 and the terms on the left-hand side of this equation are the focus of this subsection. Physically this term represents the convection of mean streamwise momentum by periodic motions in the cross flow plane.

First, the periodic motions are decomposed using

\[
\langle \tilde{v} \rangle_{xz} = \frac{1}{2} (\langle \tilde{v} \rangle_{ns} + \langle \tilde{v} \rangle_{fs}),
\]

\[
\langle \tilde{w} \rangle_{xz} = \frac{1}{2} (\langle \tilde{w} \rangle_{ns} + \langle \tilde{w} \rangle_{fs}).
\]

It should be noted that \( \langle \tilde{v} \rangle_{xz} = \langle \tilde{w} \rangle_{xz} = 0 \). The results for \( \langle \tilde{v} \rangle_{ns} \) and \( \langle \tilde{v} \rangle_{fs} \) are shown in figure 4.8a. Averaged across the no-slip phases, the signature of \( \langle \tilde{v} \rangle_{ns} \) resembles an upwash motion.

![Figure 4.8](image-url)  
**Figure 4.8:** Profiles of averaged secondary motion including (a) wall-normal velocity profiles of \( \langle \tilde{v} \rangle_{xz} \) (---), \( \langle \tilde{v} \rangle_{ns} \) (---) and \( \langle \tilde{v} \rangle_{fs} \) (---) and (b) root-mean-square measure of spanwise velocity components \( \tilde{w}_{rms}^{xz} \) (---), \( \tilde{w}_{rms}^{ns} \) (---) and \( \tilde{w}_{rms}^{fs} \) (---). All quantities scaled using reference no-slip friction velocity \( u_\tau \).
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\((\langle \tilde{v} \rangle_{ns} > 0)\) and reaches its peak value of at \(y^+ = 13\). Averaged across the free-slip phases, the signature of \(\langle \tilde{v} \rangle_{fs}^\perp\) resembles a downwash motion \((\langle \tilde{v} \rangle_{fs} < 0)\) and is the mirror image of the \(\langle \tilde{v} \rangle_{ns}^\perp\) profile about the line \(\langle \tilde{v} \rangle_{xz} = 0\). Averaged across the no-slip and free-slip phases, the signature of \(\langle \tilde{v} \rangle_{ns}^\perp\) and \(\langle \tilde{v} \rangle_{fs}^\perp\) returned zeros. However, it is unlikely that a \(\tilde{w}\) motion does not exist, since mass conservation for the periodic fluid motion demands that \(\partial \tilde{v} / \partial y = -\partial \tilde{w} / \partial z\). To this end, the root-mean-square of \(\tilde{w}\) over all phases,

\[
\tilde{w}_{rms}^{xz} = \sqrt{\frac{1}{(d+g) \lambda_x} \int_0^{d+g} \int_0^{\lambda_x} \tilde{w}^2 dx dz}, \tag{4.12}
\]

was computed in an attempt to detect spanwise periodic motions and the result is provided in figure 4.8b. Then, to further characterise the spanwise variation of \(\tilde{w}\), the root-mean-square of \(\tilde{w}\) over all no-slip phases was computed using

\[
\tilde{w}_{rms}^{ns} = \sqrt{\frac{1}{d \lambda_x} \int_0^{d} \int_0^{\lambda_x} \tilde{w}^2 dx dz}, \tag{4.13}
\]

and, similarly, the root-mean-square of \(\tilde{w}\) over all free-slip phases was computed using

\[
\tilde{w}_{rms}^{fs} = \sqrt{\frac{1}{g \lambda_x} \int_0^{d+g} \int_0^{\lambda_x} \tilde{w}^2 dx dz}. \tag{4.14}
\]

and profiles of both \(\tilde{w}_{rms}^{ns}\) and \(\tilde{w}_{rms}^{fs}\) are plotted in 4.8b. An average spanwise slip velocity is achieved across the free-slip phases, however its direction is, at present, unknown since it cannot be deduced from root-mean-squared values. For \(y^+ < 4.2\), the profiles exhibit a strong phase dependence suggesting that the most pronounced variations in spanwise motion occur in this region, which contains the viscous sublayer.

The variation in both the wall-normal velocity (figure 4.8a) and the spanwise motion (figure 4.8b) hint at the possible existence of streamwise vortex motions occurring in the transverse plane, since periodic streamwise vorticity can be written as

\[
\tilde{\Omega}_x = \frac{\partial \tilde{w}}{\partial y} - \frac{\partial \tilde{v}}{\partial z}. \tag{4.15}
\]

To investigate this further, vectors of \((\tilde{v}, \tilde{w})\) are plotted on the \(\phi_x-y\) plane in figure 4.9a, superimposed on contours of mean streamwise velocity \(\langle U \rangle^x\) which help to emphasise the three-dimensionality of the mean flow field. The \((\tilde{v}, \tilde{w})\) vectors reveal distinct pairs of counter-rotating streamwise vortices. The vortex cores are situated at \(y^+ ≈ 13\), which coincides with the local minima / maxima shown in figures 4.8a and 4.8b, and have a core-to-core spacing of \(z^+ ≈ 32\).
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Figure 4.9: Secondary flow on the transverse plane including (a) vectors of $\langle \tilde{v}, \tilde{w} \rangle$ superimposed on contours of $\langle u \rangle^+ +$ and (b) contours of periodic streamwise vorticity $\tilde{\Omega}_x = \left( \frac{\partial \tilde{w}}{\partial y} - \frac{\partial \tilde{v}}{\partial z} \right)$. For clarity, only every second vector is plotted.

The upwash and downwash behaviours inferred from figure 4.8a are clearly visible above the no-slip and free-slip phases, respectively. A cross-flow boundary layer forms at the edges of the no-slip region, at $\phi_z = (0, \pi)$, for example, and decelerates as the central no-slip phase $\phi_z = \pi/2$ is approached. The boundary-layer thickens in the wall-normal direction as it decelerates in the spanwise direction and hence there is an upwash of momentum away from the no-slip region. Above the free-slip region, there is a downwash of momentum and a stagnation point on the surface, at the central free-slip phase $\phi_z = 3\pi/2$. The flow accelerates outwards in a symmetric fashion about the stagnation point and this explains why $\langle \tilde{w} \rangle^{\text{ns}}$ and $\langle \tilde{w} \rangle^{\text{fs}}$ vanish. Bradshaw (1987) referred to secondary flows as turbulent flow with “embedded mean streamwise vorticity”, and to illustrate this description contours of periodic streamwise vorticity, $\tilde{\Omega}_x = \frac{\partial \tilde{w}}{\partial y} - \frac{\partial \tilde{v}}{\partial z}$, are visualised in figure 4.9b, along with vectors of $(\tilde{v}, \tilde{w})$. Due to their proximity with the SHS texture below them, the streamwise vortices operate in ground-effect. Distinctly different vorticity dynamics occur across the no-slip and free-slip regions of the texture. Across the no-slip band, the mean streamwise vorticity induces a vorticity layer of the opposite sign below $y^+ < 4$. Streamwise vorticity cannot be induced across the free-slip wall due to the free-stress condition on spanwise velocity and the no-penetration boundary condition on the wall-normal velocity.

Steady secondary flows were formally divided into two categories by Prandtl (1952). Secondary flows of the first kind are derived from mean flow skewing (see figure 4.10a), whereas secondary flows of the second kind (see figure 4.10b) are derived from the inhomogeneity of anisotropic wall turbulence. Theories about the origins of steady secondary flows are typically
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Figure 4.10: Production mechanisms of mean streamwise vorticity via (a) mean flow skewing (see term $P_1$ in equation 4.16) whereby initial wall-normal / (spanwise) vorticity is tilted into streamwise vorticity by wall-normal / (spanwise) primary shear and (b) anisotropies in direct turbulent stresses (see term $P_4$ in equation 4.16).

assembled around the Reynolds-averaged Navier-Stokes equations which, upon appropriate manipulation, can be recast into more useful and/or physically insightful forms. An equation particularly suitable for the analysis of secondary flows can be derived by eliminating the pressure, $p$, between the phase-averaged flow equations in the wall-normal and spanwise directions to obtain

\[
\begin{align*}
\langle U \rangle \frac{\partial \hat{\Omega}_x}{\partial x} + \hat{v} \frac{\partial \hat{\Omega}_x}{\partial y} + \hat{w} \frac{\partial \hat{\Omega}_x}{\partial z} & = \hat{\Omega}_x \frac{\partial \langle U \rangle}{\partial x} + \langle \Omega_y \rangle \frac{\partial \langle U \rangle}{\partial y} + \langle \Omega_z \rangle \frac{\partial \langle U \rangle}{\partial z} \\
& + \frac{1}{Re_b} \nabla^2 \hat{\Omega}_x + \left( \frac{\partial (R_{12})}{\partial z} + \frac{\partial \hat{R}_{13}}{\partial y} \right) + \left( \frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial y^2} \right) \hat{R}_{23} \\
& + \frac{\partial^2}{\partial y \partial z} \left( \langle R_{22} \rangle - \langle R_{33} \rangle \right),
\end{align*}
\]

where the phase-averaged vorticity components are defined as $\Omega \equiv \nabla \times \langle u \rangle$. Each term in equation 4.16 can be classified as a kinematic process. The terms of on the left-hand-side represent the convection of steady streamwise vorticity by the primary and secondary flows. The first term on the right-hand-side represents streamwise vortex stretching. The next two terms represent the generation of streamwise vorticity through the skewing of the mean shear by external body forces or pressure gradients and are associated with secondary flows of the first kind (see figure 4.10a). The next term on the right-hand-side represents the diffusion of vorticity due to viscosity, which acts to decelerate the rotating fluid particles. The three remaining terms in 4.16 only exist if the fluid is turbulent and are the origin of secondary flows of the second kind (see figure 4.10b). For
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the streamwise aligned textures, equation 4.16 simplifies to

\[
\tilde{v} \frac{\partial \tilde{\Omega}_x}{\partial y} + \tilde{w} \frac{\partial \tilde{\Omega}_x}{\partial z} = \frac{1}{Re_b} \nabla^2 \tilde{\Omega}_x + \left( \frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial y^2} \right) \tilde{R}_{23} + \frac{\partial^2}{\partial y \partial z} \left( \langle R_{22} \rangle - \langle R_{33} \rangle \right),
\]

(4.17)

which classifies the streamwise vortices shown in figure 4.9 as a secondary flow of the second kind since the only source of mean streamwise vorticity in equation 4.17 is due to the Reynolds stresses that originate from the stochastic fluctuations. This goes some way to explain why secondary motions do not occur in laminar flow across streamwise-aligned textured surfaces (Philip, 1972a) and probably why the same behaviour is observed between laminar (Choi et al., 1991) and turbulent flow (Choi et al., 1993) over identical streamwise-aligned riblet geometries.

Following the procedures outlined by Perkins (1970), the direction of the secondary vortices can be explained by considering the sign of the production term \(P_{\tilde{\Omega}_x} = \frac{\partial^2}{\partial y \partial z} \left( \langle R_{22} \rangle - \langle R_{33} \rangle \right)\), which can be determined by analysing the limiting behaviour of the anisotropic wall turbulence. Consider the series expansions

\[
\begin{align*}
    u' &= a_1 + b_1 y + c_1 y^2 + O(y^3) \\
    v' &= a_2 + b_2 y + c_2 y^2 + O(y^3) \\
    w' &= a_3 + b_3 y + c_3 y^2 + O(y^3)
\end{align*}
\]

(4.18a–4.18c)

where the coefficients \(a_i, b_i\) and \(c_i\) are zero-mean random variables and, for fully developed turbulent channel flow across textured surfaces statistically independent of \(x, z\) and \(t\). Hence, the asymptotic analysis is valid locally at spanwise phases \(\phi_z = \pi/2\) and \(\phi_z = 3\pi/2\) which correspond to the central phases on the no-slip and free-slip bands, respectively.

The near-wall behaviour of the Reynolds stresses can be obtained from equations 4.18a–4.18c by deriving the phase-averaged products of the series. For \(y = 0\), the no-slip and no-penetration conditions along the micro-features result in coefficients \(a_i = 0\) and \(b_2 = 0\), with the latter being a result of invoking mass conservation. Additionally, for \(y = 0\), the free-slip and impermeability conditions along the interface result in coefficients \(b_1 = b_2 = 0\) and \(a_2 = 0\), respectively. To leading order in \(y\), the limiting behaviour of the Reynolds stresses at the central no-slip phase are

\[
\begin{align*}
    \langle R_{22} \rangle_{x, \phi_z = \pi/2} &= \langle c_2^2 y^4 \rangle_{x, \phi_z = \pi/2} + \cdots, \\
    \langle R_{33} \rangle_{x, \phi_z = \pi/2} &= \langle b_3^2 y^4 \rangle_{x, \phi_z = \pi/2} + \cdots.
\end{align*}
\]

(4.19a–4.19b)
and the Reynolds stresses satisfying at the central free-slip phase are

\[
\langle R_{22} \rangle_{x,\varphi_z=3\pi/2} = \langle b_2^2 y \rangle_{x,\varphi_z=3\pi/2} + \cdots, \quad (4.20a)
\]

\[
\langle R_{33} \rangle_{x,\varphi_z=3\pi/2} = \langle a_3^2 \rangle_{x,\varphi_z=3\pi/2} + \cdots. \quad (4.20b)
\]

Profiles of \( \langle R_{22} \rangle \) and \( \langle R_{33} \rangle \) at discrete phases and averaged across all no-slip and free-slip phases are shown in figure 4.11a and figure 4.11b, respectively, and imply that wall-normal stresses have a weak phase-dependence, whereas spanwise stresses are strongly phase-dependent. Using equations 4.19a-4.19b, equations 4.20a-4.20b and making the approximation \( \frac{\partial}{\partial z} \langle R_{22} \rangle \approx 0 \) the near-wall behaviour of the production term can be written as

\[
P_{\Omega_x} = \frac{\partial^2}{\partial y} (\langle R_{22} \rangle - \langle R_{33} \rangle)
\]

\[
P_{\Omega_x} \approx -\frac{\partial^2}{\partial z} \langle R_{33} \rangle
\]

\[
= -\frac{2}{L_{\phi_z}} \left( \frac{\partial}{\partial y} \langle R_{33} \rangle_{x,\varphi_z=3\pi/2} - \frac{\partial}{\partial y} \langle R_{33} \rangle_{x,\varphi_z=\pi/2} \right) + \cdots
\]

\[
= +\frac{2}{L_{\phi_z}} \langle b_3^2 y \rangle + \cdots \quad (4.21)
\]

which has a positive sign for a fluid particle moving from a no-slip to a free-slip region. For this coordinate system, the positive sign of the production term induced a clockwise rotation, \( \Omega_x > 0 \). The opposite rotation is observed when a fluid particle makes the reverse journey from a free-slip region to a no-slip region. This explains the counter-rotating behaviour of the vortices illustrated in figure 4.9.
4.2.3 Slip flow

To complete the analysis of the mean flow, attention is now turned to the behaviour of the fluid on the SHS texture itself. At the wall, the governing equations for the phase-averaged streamwise momentum can be written as

\[
\frac{\partial \langle u \rangle}{\partial z} \bigg|_{w} = - \frac{d \langle p \rangle}{dx} \bigg|_{w} + \frac{1}{Re_b} \left( \frac{\partial^2 \langle u \rangle}{\partial y^2} + \frac{\partial^2 \langle u \rangle}{\partial z^2} \right) \bigg|_{w} - \frac{\partial \langle R_{13} \rangle}{\partial z} \bigg|_{w},
\]

which shows that primary, secondary and turbulent fluid motions exist on the SHS texture. The term on the left-hand side of equation 4.22 represents spanwise convection of streamwise velocity by the secondary flow. The second and third terms on the right-hand side of equation 4.22 represent viscous diffusion and the gradient of cross-flow Reynolds shear stress, \( \langle R_{13} \rangle \). Averaging equation 4.22 across no-slip phases gives

\[
0 = - \frac{d \langle p \rangle^{ns}}{dx} \bigg|_{w} + \frac{1}{Re_b} \left( \frac{\partial^2 \langle U \rangle^{ns}}{\partial y^2} \right) \bigg|_{w},
\]

which states that, across the no-slip band, viscous losses at the wall are balanced by the mean streamwise pressure gradient - a familiar result from the theory of fully developed turbulent plane channel flow. Inspection of equation 4.23 demonstrates that frictional losses can in no way be associated with fluid slip. Averaged across free-slip phases, equation 4.22 simplifies to

\[
\frac{\partial \langle U \rangle}{\partial z} \bigg|_{fs} = - \frac{d \langle p \rangle^{fs}}{dx} \bigg|_{w} + \frac{1}{Re_b} \left( \frac{\partial^2 \langle U \rangle^{fs}}{\partial y^2} \right) \bigg|_{w} - \frac{\partial \langle R_{13} \rangle^{fs}}{\partial z} \bigg|_{w},
\]

which, compared to equation 4.23, demonstrates that the flow averaged across free-slip phases is more complex and thus warrants further attention.

A vast amount of effort has been devoted to quantify the average streamwise slip velocity, \( \langle U_s \rangle \), for turbulent flow over textured surfaces, both in experimental (Woolford et al., 2009b; Daneillo et al., 2009) and numerical (Martell et al., 2009, 2010) environments. These previous studies demonstrated that higher average slip velocities gave a greater reduction in wall shear stress. The role of spanwise slip over textured surfaces is yet to be determined, but has been linked to increased levels of turbulent skin-friction where the Navier-slip model was adopted (Min & Kim, 2004) to model the texture.

A carpet plot of streamwise and spanwise slip velocities with vectors of \( \langle u_s \rangle, \tilde{w}_s \) is shown in figure 4.12a. The maximum streamwise slip velocity is \( \max (\langle U_s \rangle / U_b) = 0.62 \) and occurs at the
central free-slip phase. The average slip velocity across all phases is \( \max\left(\langle U_s\rangle_{xz}/U_b\right) = 0.27 \). At the edges of the free-slip region, a spanwise slip velocity convects momentum outwards towards the edges of the no-slip region, which is consistent with the orientation of the secondary vortices shown in figure 4.9a. Line profiles of streamwise and spanwise slip velocities are shown in figure 4.12b and figure 4.12c, respectively.

As an aside, under laminar flow conditions the last term on the right-hand side of equation 4.22 is equal to zero and as a result the secondary flow can no longer be supported, since the presence of turbulent stresses is an antecedent for the sustenance of secondary flows of the second kind (see equation 4.17). Therefore, under laminar flow conditions equation 4.22 reduces to

\[
0 = - \frac{d \langle p \rangle}{dx} \bigg|_w + \frac{1}{Re_b} \left( \frac{\partial^2 \langle U_s \rangle}{\partial y^2} + \frac{\partial^2 \langle U_s \rangle}{\partial z^2} \right) \bigg|_w. \quad (4.25)
\]

To complement the current simulations, turbulence was “switched off” by running a laminar
simulation at the same Reynolds number. The streamwise slip velocity profile is included in figure 4.12c. Note that for a streamwise-aligned texture, a spanwise slip velocity can not be supported under laminar flow conditions.

4.2.4 Summary

The mean flow corresponding to the streamwise-aligned texture was characterised in detail and can be classified into three distinct categories:

1. **Primary flow**: which is simply the fluid motion aligned with the primary flow direction.

2. **Secondary flow**: which is periodic fluid motion that occur in the $y$-$\phi_z$ plane perpendicular to primary flow direction.

3. **Slip flow**: which can be characterised by motions that occur on the $\phi_x$-$\phi_z$ plane tangent to the primary motion, located on the SHS texture itself.

For the primary flow, a simple equation for $\langle U^+ \rangle_{xz}$ was derived from a series expansion in order to facilitate an investigation of general structural changes that may be expected in the turbulent mean flow profile. Compared to the reference no-slip case, the two most significant changes were a forward translation of the viscous sublayer, due to an average slip velocity $\langle U^+ \rangle_{xz}$, and an elevation in the log-law — a typical observation of drag reduced flows. Then, by using the decomposition $\langle U^+ \rangle_{xz} = 1/2 \left( \langle U^+ \rangle_{ns} + \langle U^+ \rangle_{fs} \right)$, the wall-normal height to which the presence of the texture was felt by the mean flow was identified to be $y^+ \approx 34$, which is approximately equal to the feature spacing. Next, by decomposing the mean deviatoric rate of strain tensor as $\langle S_{12} \rangle_{xz} = 1/2 \left( \langle S_{12} \rangle_{ns} + \langle S_{12} \rangle_{fs} \right)$ it was demonstrated that the average wall shear stress across the no-slip phases was higher than the reference no-slip value. Finally, to fully characterise the primary flow field, the phase-averaged streamwise velocity, $\langle U \rangle_x^+$, were scrutinised and demonstrated that strong lateral gradients of the primary flow occurred in the spanwise direction for $y^+ < 34$.

For the secondary flow, the relation $\langle \tilde{v} \rangle_{xz} = \langle \tilde{w} \rangle_{xz} = 0$ was confirmed numerically, and hence phase-averaged statistics were utilised in order to characterise the flow occurring on the transverse $y$-$\phi_z$ plane. By decomposing $\langle \tilde{v} \rangle_{xz} = 1/2 \left( \langle \tilde{v} \rangle_{ns} + \langle \tilde{v} \rangle_{fs} \right)$ an averaged upwash / (downwash) signature was detected above the no-slip / (free-slip) phases, respectively. Profiles of $\langle \tilde{w} \rangle_{ns}$ and $\langle \tilde{w} \rangle_{fs}$ returned zero, and this result was remedied by the computing the root-mean-square of $\tilde{w}$ which successfully detected secondary spanwise motions, although provided no information concerning their direction. To fully characterise the secondary flow, vector fields of $(\tilde{v}, \tilde{w})$ were plotted and revealed a striking array of coherent counter-rotating streamwise vortices. The previous upwash / downwash signatures of $\tilde{v}$ were clearly visible in the vector fields. Above the free-slip region, the secondary flow acts to convect momentum towards the edges of the no-slip region in a symmetric fashion (symmetry is observed about the central free-slip phase) via
slipping wall-jet type motions. The lateral wall-jet then encounters the no-slip region, where it essentially becomes a transverse boundary layer. The transverse layer decelerates as the central no-slip phase is approached and hence carries momentum away from the wall as it thickens. In terms of drag reduction, the impingement of the high-momentum wall-jet at the edges of the no-slip band can be interpreted as a performance penalty. On the other hand, the steady upwash of momentum away from the wall can interpreted as a performance gain. The physical origins of the secondary flow were investigated by considering a suitable form of the mean streamwise vorticity equation. A simplification of the governing equations classified the transverse motions as a secondary motion of the second kind where anisotropic turbulent forces are responsible for the sustenance of the secondary motions. Finally, an asymptotic analysis was conducted which confirmed the orientation of the secondary vortices.

For the slip flow, both streamwise and spanwise components of velocity were obtained. By deriving the governing equations for the slip flow, it was shown that spanwise slip velocity only exists in turbulent flows since it is the turbulent stresses that support the existence of transverse secondary motions. This was confirmed by running a complementary laminar simulation. The direction of the spanwise slip velocity was found to be consistent with the orientation of the secondary vortices, and the governing equations admitted a term that physically represents the spanwise convection of mean streamwise momentum by the transverse slip velocity. At present, the exact role of the slip velocities towards the levels of skin-friction remain unclear.
4.3 Turbulence statistics

In turbulent flow, stochastic fluid motions are directly affected by the mean field and vice-versa. In the previous section, mean flow field was characterised in detail including analysis of steady primary, secondary and slipping motions. Each of these three mean flow components has implications for the stochastic field. The purpose of this chapter is to characterise the turbulent field, which essentially maintains itself by extracting energy from the various components of the mean flow field.

The governing equations for the stochastic velocity component was derived and can be written as

\[
\frac{\partial u'_i}{\partial t} + ((U_{j})^{xz} + \bar{u}_j) \frac{\partial u'_i}{\partial x_j} + u'_j \left( \frac{\partial (U_i)^{xz}}{\partial x_j} + \frac{\partial \bar{u}_i}{\partial x_j} \right) = -\frac{\partial p'}{\partial x_i} + \frac{1}{Re_b} \frac{\partial^2 u'_i}{\partial x_j \partial x_j} \]

The first term on the left-hand side of equation 4.26 represents the unsteady nature of the stochastic velocity component. The second term of the left-hand side represents the convection of the turbulent fluctuations by both primary and secondary mean flow motions. The third and final term on the left-hand side represents a production term where, for example, streamwise fluctuations can be produced by the product of the wall-normal perturbations and wall-normal gradients of the mean flow. The first term on the right-hand side of equation 4.26 represents the spatial variations of the stochastic pressure field. The second term on the right-hand side represents the viscous diffusion of turbulence. The final term on the right-hand contains spatial derivatives of the Reynolds stresses tensor and products between the stochastic velocity components.

Computing the phase-average of equation 4.26 returns zero, and hence an alternate approach must be adopted to obtain a statistical description of the stochastic fluid motions. Two common quantities used to statistically describe the turbulent field are the turbulence kinetic energy (TKE) and the components of Reynolds stress tensor, \( \langle R_{ij} \rangle \). In fact, by appropriately manipulating equation 4.26, it is possible to derive equations that govern the Reynolds stresses and TKE. Their evaluation is the focal point of this chapter.

4.3.1 Turbulence kinetic energy

The kinetic energy due to the turbulent motions was computed using

\[
\langle k \rangle = \frac{1}{2} \langle R_{ii} \rangle \quad (4.27)
\]
where \( \langle k \rangle \) is the phase-averaged TKE and \( \langle R_{ii} \rangle \) represents the trace of the Reynolds stress tensor.

The TKE averaged across all phases for the streamwise-aligned ridges is plotted in figure 4.13a, where data from the reference no-slip case is included for comparison. For the textured case, a significant suppression of TKE is observed above \( y^+ \approx 7 \), relative to the reference case. The peak value of TKE for the textured case is 30% lower than the peak value of the reference case and occurs at \( y^+ \approx 13 \), which is 15.3% lower than the position of the peak TKE for the reference case. Below \( y^+ \approx 7 \), increased levels of TKE are observed when comparing the textured case to the reference case. As a result of this near-wall behaviour, the entire viscous sublayer of the textured case is effectively energized, relative to the reference no-slip case. In addition, the TKE persists all the way to the wall where a finite value of \( \langle k^+ \rangle_{y=0} = 1.28 \) is attained which means that turbulence penetrates all the way down to the wall, in an average sense.

\[
\langle k \rangle = \frac{1}{2} (\langle k \rangle_{ns} + \langle k \rangle_{fs})
\]

which helps to clarify the behaviour of the TKE above the no-slip and free-slip phases. Profiles of \( \langle k \rangle_{xz} \), \( \langle k \rangle_{ns} \) and \( \langle k \rangle_{fs} \) are shown in figure 4.14a and attention is drawn to four particular points of interest including the wall, the point where \( \langle k \rangle_{xz} \), \( \langle k \rangle_{ns} \) and \( \langle k \rangle_{fs} \) intersect with one another, the point where TKE reaches its peak value and the point where the TKE becomes phase-independent and varies in the wall-normal direction only. Starting at the wall, it is clear that equation 4.28 reduces to \( \langle k^+ \rangle_{y=0}^{fs} = 2 \langle k^+ \rangle_{y=0}^{xz} \) and this demonstrates that only the TKE above free-slip phases can penetrate all the way to the wall. The profiles of \( \langle k \rangle_{xz} \), \( \langle k \rangle_{ns} \) and \( \langle k \rangle_{fs} \) intersect with one another at \( y^+ \approx 8 \), and beyond this point the inequality \( \langle k \rangle_{ns} \geq \langle k \rangle_{fs} \) is satisfied. Relative to the reference case, the average TKE across no-slip phases is higher for

![Figure 4.13: Turbulence kinetic energy averaged across all phases, \( \langle k \rangle_{xz} \), for textured case (—) and reference no-slip case (—) scaled by (a) reference no-slip friction velocity, \( u_\tau \), and (b) bulk velocity, \( U_b \).](image-url)
Figure 4.14: Decomposition of turbulence kinetic energy averaged across all phases including profiles of $\langle k \rangle^{xz}$ (---), $\langle k \rangle^{ns}$ (---) and $\langle k \rangle^{fs}$ (---) normalised by (a) reference no-slip friction velocity, $u_\tau$, and (b) the bulk velocity, $U_b$. In both figures the data corresponding to the reference no-slip case is denoted using (○).

$y^+ < 7$ suggesting that the turbulence activity is more intense in this region. In the buffer layer, the peak values of TKE are reduced by 24% and 35%, relative the reference case, across the no-slip and free-slip phases, respectively, and both occur at $y^+ \approx 14$. Beyond $y^+ \approx 60$, the profiles of TKE exhibit minimal levels of phase-dependence. To complement figure 4.14a, a plot TKE normalised by bulk velocity is provided in figure 4.14b and demonstrated that TKE remains suppressed in the outer regions of the flow.

To clarify the phase-dependence of TKE, contours of $\langle k^+ \rangle^x$ on the transverse $y-\phi_z$ plane are shown in figure 4.15a. Significant phase-variations of TKE are confined to $y^+ < 60$, consistent with behaviour of $\langle k \rangle^{ns}$ and $\langle k \rangle^{fs}$, shown in figure 4.13a. Away from the wall, across the interval $10 < y^+ < 30$ (which includes both the buffer and log-law regions), vigorous turbulence activity is observed above the no-slip bands and is particularly concentrated across the central no-slip phases. The opposite behaviour is encountered above the free-slip phases, where TKE activity is concentrated at the edges of the free-slip band, and reaches a minimum across the central free-slip phases. Moving closer to the wall, where $0 < y^+ < 10$ (which contains the viscous sublayer), the magnitude of TKE above the no-slip bands reduces dramatically since the no-slip boundary condition must be satisfied. The opposite behaviour is observed above free-slip regions, where TKE penetrates all the way to the wall and achieves its maximum wall value in the vicinity of either edge of the free-slip bands.

The exact spanwise variations of TKE are shown in figure 4.15b where line profiles of $\langle k^+ \rangle^x$ at three particular wall-normal positions are plotted. First, at $y^+ = 13.7$, the TKE profile varies in a sinuous fashion (where the spanwise origin is defined at $\phi_z = 0$) where the peak / trough values occur periodically at $\phi_z = \sum_{n=0}^{\infty} (n + \frac{1}{2}) \pi$, with a peak-to-trough spacing of $\Delta z^+ \approx 33$,
Figure 4.15: Phase-averaged turbulence kinetic energy including (a) contours of $\langle k^+ \rangle^x$ plotted on the $y-\phi_z$ plane and (b) line profiles of $\langle k^+ \rangle^x$ at three particular wall-normal locations $y^+ = \{0, 7.8, 13.7\}$. All quantities scaled by reference no-slip friction velocity, $u_\tau$.

which is equal to the spanwise spacing of the ridges. Second, at $y^+ = 7.8$, the TKE profile varies in a similar fashion, but with the peak / trough values shifted and occurring periodically at $\phi_z = \sum_{n=0}^{\infty} n \frac{\pi}{2}$ across, with a peak-to-trough spacing of $\Delta z^+ \approx 17$. Finally, the spanwise distribution of TKE is considered at the wall. Clearly, TKE equals zero at all no-slip phases, since all fluid motions come to rest. The same cannot be said for the free-slip phases, however, and figure 4.15b demonstrates that, at phases close to the edge of the free-slip band, the TKE is actually higher than the TKE found in the buffer layer. Interestingly, the high levels of TKE at the edges of the free-slip band hint at the possibility of spanwise turbulent momentum transfer towards the neighbouring no-slip phases, which may go some way to explain the increased levels of $\langle C_f \rangle$ at the edges of the no-slip band, shown before in figure 4.3. From the analysis of $\langle k \rangle$, several unanswered questions remain concerning the physical mechanisms:

1. Away from the wall, why is the peak TKE confined to the region above the no-slip phases?

2. What physical mechanisms permit TKE to penetrate all the way to the free-slip region?

3. At the wall, why is TKE maximum at the edges of the free-slip region?
To clarify the dynamical characteristics of the TKE, the governing equation for the phase-averaged TKE was derived and can be written as

\[
\begin{align*}
\text{Mean Convection, } &\langle C_k \rangle = \langle (U_j)^{xz} + \tilde{u}_j \rangle \frac{\partial \langle k \rangle}{\partial x_j} = -\langle u'_i u'_j \rangle \left( \frac{\partial \langle U_i \rangle^{xz}}{\partial x_j} + \frac{\partial \tilde{u}_i}{\partial x_j} \right) + \langle \tilde{u}_i \frac{\partial p'}{\partial x_i} \rangle, \\
\text{Production, } &\langle P_k \rangle = \frac{1}{Re_b} \frac{\partial^2}{\partial x_j \partial x_j} \langle k \rangle - \frac{1}{Re_b} \left( \frac{\partial U_i^{xz}}{\partial x_j} \right)^2 - \frac{\partial}{\partial x_j} \langle ku'_j \rangle, \\
\text{Pressure-diffusion, } &\langle R_k \rangle = \frac{1}{Re_b} \frac{\partial}{\partial x_j} \langle ku'_j \rangle.
\end{align*}
\]

The first term of the left-hand side of equation 4.29 represents the convection of TKE by primary and secondary flows. The second term on the right-hand side of equation 4.29 is the production term, and represents the rate at which kinetic energy is transmitted from the mean flow to the turbulent flow. The third term on the right-hand side is the pressure-diffusion term, which transports turbulence kinetic energy resulting from correlation of pressure and velocity fluctuations. The fourth term on the right-hand side is the viscous diffusion term, which represents the diffusion of TKE caused by the fluid’s molecular viscosity. The fifth term on the right-hand side is the dissipation term, which represents the rate at which TKE is converted into heat (or thermal internal energy). The last term on the right-hand side of equation 4.29 is the turbulent transport term, and represents the rate at which TKE is transported through a fluid by the turbulent fluctuations themselves.

For the reference no-slip case, equation 4.29 reduces to

\[
0 = -\langle R_{12} \rangle^{xz} \frac{\partial \langle U \rangle^{xz}}{\partial y} + \langle v' \frac{\partial p'}{\partial y} \rangle^{xz} + \frac{1}{Re_b} \frac{\partial^2 \langle k \rangle^{xz}}{\partial y^2} - \frac{1}{Re_b} \left( \frac{\partial U_i^{xz}}{\partial x_j} \right)^2 \left( \frac{\partial ku'_j}{\partial x_j} \right)^2 - \frac{1}{2 \partial y} \langle u'_i u'_j v' \rangle^{xz},
\]

which highlights that the production of TKE is restricted to a solitary route via the primary wall-normal shear, \( \frac{\partial (U_j)^{xz}}{\partial y} \), that the TKE can only be transported in the wall-normal direction and that, since the left-hand side is zero, no secondary convection can take place. The numerical evaluation of equation 4.30 is shown in figure 4.16a. Above \( y^+ = 35 \), the production and dissipation terms are equal and opposite. Within the buffer layer, at \( y^+ \approx 12 \), the production term reaches its peak value. Around this peak, production exceeds dissipation, and the surplus energy produced is transported away by the turbulent transport and viscous diffusion terms. Below \( y^+ \approx 3 \),
within the viscous sublayer, the viscous diffusion and dissipation terms are dominant. At the wall equation 4.30 reduces to

\[
\frac{1}{Re_b} \frac{\partial^2 \langle k \rangle}{\partial y^2} = \frac{1}{Re_b} \left( \frac{\partial u_i' \partial x_j}{\partial y} \right)^2_{xz} \quad \text{(Viscous diffusion, } \langle D_k \rangle_{xz}^z \text{)}
\]

\[
\frac{1}{Re_b} \left( \frac{\partial (u_i')^2_{xz}}{\partial y} \right)_{xz} \quad \text{(Dissipation, } \langle \epsilon_k \rangle_{xz}^z \text{)}
\]

which shows that dissipative losses are balanced by positive viscous diffusion towards the wall.

For the streamwise-aligned texture, equation 4.29 reduces to

\[
\left\langle \hat{C}_k \right\rangle^x = \left\langle \hat{P}_k \right\rangle^x + \left\langle \hat{R}_k \right\rangle^x + \left\langle \hat{T}_k \right\rangle^x
\]

\[
\left\langle \hat{P}_k \right\rangle^x = -\left\langle \left( \frac{\partial (U_i')}{\partial y} + \frac{\partial (U_i')}{\partial z} \right) \langle u_i'u_j' \rangle \right\rangle^x + \left\langle \hat{R}_12 \right\rangle \frac{\partial (U)}{\partial y} + \left\langle \hat{R}_{13} \right\rangle \frac{\partial (U)}{\partial z}
\]

\[
\left\langle \hat{R}_k \right\rangle^x = \left\langle \frac{\partial^2 \langle k \rangle}{\partial y^2} + \frac{\partial^2 \langle k \rangle}{\partial z^2} \right\rangle^x + \frac{1}{Re_b} \left( \frac{\partial^2 \langle k \rangle}{\partial x_j^2} \right)^x_{xz} - 1 \frac{1}{Re_b} \left( \frac{\partial u_i'}{\partial x_j} \right)^2_{xz} \quad \text{(Pressure-diffusion, } \langle \epsilon_k \rangle_{xz}^z \text{)}
\]

\[
\left\langle \hat{T}_k \right\rangle^x = -\left\langle \frac{\partial}{\partial y} \langle u_i'u_j' \rangle + \frac{\partial}{\partial z} \langle u_i'u_j' \rangle \right\rangle^x
\]

\[
\left\langle \hat{T}_k \right\rangle^x = \frac{1}{2} \left\langle \frac{\partial}{\partial y} \langle u_i'u_j' \rangle + \frac{\partial}{\partial z} \langle u_i'u_j' \rangle \right\rangle^x
\]

which has several extra terms, compared to the TKE budget in equation 4.30. The term on the left-hand side of equation 4.32 represents the convection of TKE by the secondary flow. The first term on the right-hand side is the production term \( \left\langle P_k \right\rangle^x \) and can be written as

\[
\left\langle P_k \right\rangle^x = -\left\langle \left( \frac{\partial (U_i')}{\partial y} + \frac{\partial (U_i')}{\partial z} \right) \langle u_i'u_j' \rangle \right\rangle^x + \left\langle \hat{R}_12 \right\rangle \frac{\partial (U)}{\partial y} + \left\langle \hat{R}_{13} \right\rangle \frac{\partial (U)}{\partial z}
\]

where the terms have been classified as being either “primary” or “secondary” production terms. The former terms are associated with the production of TKE due to the product of Reynolds stresses and gradients present in the primary flow, whereas the latter terms are associated with the production of TKE due to the product of Reynolds stresses and gradients present in the secondary flow. The pressure-diffusion, viscous diffusion and turbulent transport terms, shown on the right-hand side equation 4.32, all include additional spanwise terms, compared to the reference TKE budget shown in equation 4.30.

First, a direct comparison to the reference TKE budget is made by averaging the terms in equation 4.32 across all phases, and the results are plotted in figure 4.16b. The most striking
Chapter 4. Critical assessment of performance of streamwise-aligned ridges

Figure 4.16: Turbulence kinetic energy budget for (a) reference no-slip case and (b) textured case. All quantities scaled using reference no-slip friction velocity, $u_\tau$.

observation is the near-wall behaviour of the production term, which persists all the way to the wall. In order to better understand the mechanisms responsible for the penetration of the TKE production, the terms on the right-hand side terms of equation 4.33 were evaluated. The production of TKE averaged across all phases, $\langle P_k \rangle^{xz}$, is shown in figure 4.17a, where the data of the reference no-slip case is included for comparison. Above the texture, and relative to the reference case, the peak value of TKE production drops by 35% and shifts 17% closer to the wall, which corresponds to $y^+ = 9.5$. The profiles of $\langle P_k \rangle^{xz}$ intersect at $y^+ = 6.6$, and below this point the production of TKE above the texture exceeds the reference value. In fact, production persists all the way to the wall - which is consistent with the finite value of TKE shown in figure 4.13a.

Next, the production of TKE was decomposed using

$$\langle P_k \rangle^{xz} = \frac{1}{2} \left( \langle P_k \rangle^{ns} + \langle P_k \rangle^{fs} \right).$$

Profiles of $\langle P_k \rangle^{xz}$, $\langle P_k \rangle^{ns}$ and $\langle P_k \rangle^{fs}$ are shown in figure 4.18a and attention is drawn to four
particular points of interest including the wall, the point where $\langle P_k \rangle_{xz}$, $\langle P_k \rangle_{ns}$ and $\langle P_k \rangle_{fs}$ intersect with one another, the point for TKE production reaches it peak value (away from the wall) and the point where the TKE becomes phase-independent and varies in the wall-normal direction only. Starting at the wall, it is clear that equation 4.34 reduces to $\langle P_k \rangle_{fs} = 2 \langle P_k \rangle_{xz}$ and $\langle P_k \rangle_{ns} = \langle P_k \rangle_{xz}$.

this demonstrates that only production of TKE above free-slip phases can penetrate all the way to the wall. The profiles of $\langle P_k \rangle_{xz}$, $\langle P_k \rangle_{ns}$ and $\langle P_k \rangle_{fs}$ intersect with one another at $y^+ \approx 5$, and beyond this point the inequality $\langle P_k \rangle_{ns} \geq \langle P_k \rangle_{fs}$ is satisfied. In the buffer layer, the peak value of $\langle P_k \rangle_{ns}$ is reduced by 11.4% and shifts 24% closer to wall ($y^+ \approx 9.3$) relative to the reference case and the corresponding value of $\langle P_k \rangle_{fs}$ is 53% lower than $\langle P_k \rangle_{ns}$. Profiles of TKE production
become phase-independent beyond \( y^+ > 60 \). The results show that, on average, TKE production penetrates all the way to the free-slip wall and that although the peak value of TKE production is reduced, relative to the reference case, the near-wall production of TKE is enhanced.

Next, equation 4.33 was rewritten as

\[
\langle P_k \rangle^x = \langle P_{k,12} \rangle^x + \langle P_{k,13} \rangle^x + \langle P_{k,22} \rangle^x + \langle P_{k,23} \rangle^x + \langle P_{k,33} \rangle^x
\]

(4.35)

where each term on the right-hand-side is given by

\[
\langle P_{k,12} \rangle^x = - \left\langle \langle R_{12} \rangle \frac{\partial \langle U \rangle}{\partial y} \right\rangle^x,
\]

(4.36a)

\[
\langle P_{k,13} \rangle^x = - \left\langle \langle R_{13} \rangle \frac{\partial \langle U \rangle}{\partial z} \right\rangle^x,
\]

(4.36b)

\[
\langle P_{k,22} \rangle^x = - \left\langle \langle R_{22} \rangle \frac{\partial \tilde{v}}{\partial y} \right\rangle^x,
\]

(4.36c)

\[
\langle P_{k,23} \rangle^x = - \left\langle \langle R_{23} \rangle \left( \frac{\partial \tilde{v}}{\partial y} + \frac{\partial \tilde{w}}{\partial z} \right) \right\rangle^x,
\]

(4.36d)

\[
\langle P_{k,33} \rangle^x = - \left\langle \langle R_{33} \rangle \frac{\partial \tilde{w}}{\partial z} \right\rangle^x.
\]

(4.36e)

Next, the terms described by equations 4.36a-4.36e were computed in root-mean-square form in order to identify the dominant mechanisms responsible for the production of TKE, and the results are shown in figure 4.19. Out of the five terms (equations 4.36a-4.36e) only two make a significant contribution towards the production of TKE. The two dominant terms towards TKE production (equation 4.33) are \( \langle P_{k,12} \rangle \) (equation 4.36a) and \( \langle P_{k,13} \rangle \) (equation 4.36b) and infer
the TKE production can be well approximated as

\[
\langle P_k \rangle_x \approx -\left( \langle R_{12} \rangle \frac{\partial \langle U \rangle}{\partial y} + \langle R_{13} \rangle \frac{\partial \langle U \rangle}{\partial z} \right),
\]

(4.37)

where the three discarded terms (equations 4.36c-4.36e) were found to be at least two (and at most four) orders of magnitude less than the dominant terms. The dominance of these two terms suggests that the primary shear is the driving force behind the production of TKE, whereas the production by secondary gradients is negligible.

To clarify the phase-dependence of TKE production, contours of \( \langle P_k \rangle_{12} \) and \( \langle P_k \rangle_{13} \) are plotted on the transverse \( y-\phi_z \) plane in figure 4.20. Figure 4.20a reveals isolated patches where \( \langle P_k \rangle_{12} > 0 \)

![Figure 4.20](image)

**Figure 4.20:** Contours of the production of phase-averaged turbulence kinetic energy due to terms (a) \( \langle P_{k,12} \rangle \) (equation 4.36a) and (b) \( \langle P_{k,13} \rangle \) (equation 4.36b).

that occur periodically in that span. The patches are elliptical in shape and are centred about \( (y^+, \phi_z) = \left( 11, \sum_{n=0}^{\infty} (n + 1/2) \pi \right) \), with a wall-normal radius of 10 wall units and a spanwise radius of 17 wall units. In between the localised patches, above the free-slip phases, \( \langle P_k \rangle_{12} \approx 0 \) and this is due to the low levels of primary shear in these regions which is insufficient to significantly affect the stochastic fluid motions. Underneath regions where \( \langle P_k \rangle_{12} > 0 \), small pockets where \( \langle P_k \rangle_x < 0 \) also occur periodically in the span. The patches are also elliptical in shape but are centred about \( (y^+, \phi_z) = \left( 2.5, \sum_{n=0}^{\infty} n\pi \right) \), with a wall-normal radius of 1.2 wall units and a spanwise radius of 2 wall units. Figure 4.20b shows protruding lobes where \( \langle P_k \rangle_{13} > 0 \). The lobes start at the edges of free-slip wall, spread outwards and contribute towards the TKE above the no-slip phases. This is an important observation since it demonstrates that the primary spanwise
shear is capable of producing TKE at the edges of the no-slip regions.

4.3.2 Reynolds stresses

The previous analysis demonstrated that the production of TKE can be credited to two mechanisms. On one hand, TKE was produced above the no-slip phases, in regions of strong wall-normal primary shear. On the other hand, TKE was produced at the edges of the free-slip band, in regions of strong spanwise primary shear that originate from the free-slip band. In order to explain this behaviour further, an analysis of the Reynolds stress tensor, \( \langle R_{ij} \rangle \) is required. TKE is defined as half the trace of the Reynolds stress tensor, but an investigation of the off-diagonal components, the Reynolds shear stresses, is also informative. The implications of the Reynolds shear stresses upon the mean flow can be clarified by writing down the RANS equations,

\[
\langle U_j \rangle \frac{\partial \langle U_i \rangle}{\partial x_j} = \frac{\partial \langle P \rangle}{\partial x_i} + \frac{1}{Re_b} \frac{\partial^2 \langle U_i \rangle}{\partial x_j \partial x_j} - \frac{\partial}{\partial x_j} \langle R_{ij} \rangle
\]

which demonstrates that the divergence of the turbulent stresses acts against the mean flow, represented by the last term on the right-hand side of equation 4.38. In the case of plane channel flow, this turbulent force retards the flow in the core regions of the channel and accelerates the flow in the near-wall region (see figure 4.21). Relative to a laminar parabolic profile (where turbulent stresses are always zero), the mean turbulent profile experiences a momentum deficit in the channel centre and a momentum surplus in the near-wall region, which goes some way to explaining the higher levels of wall shear stress typically associated with fully developed turbulent plane channel flow. At present it remains unclear how the divergence of the turbulent stresses

![Figure 4.21](image)

**Figure 4.21:** Distortion of turbulent mean streamwise velocity profile (→) relative to laminar profile (---) due to Reynolds shear stress showing (a) regions of near-wall acceleration (■) and outer deceleration (■) and (b) gradient of the Reynolds shear stress.
affect the mean profile for the textured case and, more importantly, how they contribute towards the local levels of skin-friction along the tops of the micro-features.

The components of Reynolds stress tensor averaged across all phases for the streamwise-aligned ridges are plotted in figure 4.22, where the data of the reference no-slip case is included for comparison. The streamwise component of Reynolds stress, \( \langle R_{11} \rangle^{xz} \), is plotted in figure 4.22a.

![Graphs showing Reynolds stress profiles](image)

Figure 4.22: Reynolds stress profiles averaged across all phases for textured (---) and reference no-slip (----) cases including profiles of (a) streamwise Reynolds stress, \( \langle R_{11} \rangle^{xz} \), (b) wall-normal Reynolds stress, \( \langle R_{22} \rangle^{xz} \), (c) spanwise Reynolds stress \( \langle R_{33} \rangle^{xz} \), and (d) Reynolds shear stress, \( \langle R_{12} \rangle^{xz} \). The two remaining components of \( \langle R_{ij} \rangle^{xz} \) are not included because \( \langle R_{23} \rangle^{xz} = \langle R_{13} \rangle^{xz} = 0 \).

and is suppressed for the textured case beyond \( y^+ > 7 \). In fact, the peak value of \( \langle R_{11} \rangle^{xz} \) is reduced by 32% and is found 16% closer to the wall, at \( y^+ = 11.4 \), when comparing the textured to the reference case. For \( y^+ < 7 \), \( \langle R_{11} \rangle^{xz} \) persists all the wall way to the wall where it attains a value of \( \langle R_{11} \rangle_{y=0}^{xz} = 2.4 \). The wall-normal component of Reynolds stress, \( \langle R_{22} \rangle^{xz} \), is plotted in figure 4.22b and is suppressed for the textured case beyond \( y^+ > 21 \). In fact, the peak value of \( \langle R_{22} \rangle^{xz} \) is by reduced by 19.4% and is found 7% closer to the wall, at \( y^+ = 52.1 \), when comparing the textured to the reference case. Across the interval \( 0 < y^+ < 21 \), \( \langle R_{22} \rangle^{xz} \) is always greater
than the reference value. The spanwise component of Reynolds stress, $\langle R_{33} \rangle^{xz}$, is plotted in figure 4.22c and is suppressed for the textured case beyond $y^+ > 12$. In fact, the peak value of $\langle R_{33} \rangle^{xz}$ is by reduced by 18.6% and is found 21.1% closer to the wall, at $y^+ = 31.1$, when comparing the textured to the reference case. Across the interval $0 < y^+ < 12$, $\langle R_{33} \rangle^{xz}$ is always greater than the reference value and persists all the way to wall, where it attains a value of $\langle R_{33} \rangle_{y=0}^{xz} = 0.17$

The Reynolds shear stress, $\langle R_{12} \rangle^{xz}$, is plotted in figure 4.22d and is suppressed for the textured case for all values of $y^+$. The peak suppression of $\langle R_{12} \rangle^{xz}$ occurs at $y^+ = 31.1$ where the Reynolds shear stress is reduced by 25%, when comparing the textured to the reference case. Note that upon averaging across all phases phases both $\langle R_{13} \rangle^{xz} = \langle R_{23} \rangle^{xz} = 0$.

Next, the Reynolds stress tensor averaged across all phases was decomposed as

$$\langle R_{ij} \rangle^{xz} = \frac{1}{2} \left( \langle R_{ij} \rangle^{ns} + \langle R_{ij} \rangle^{fs} \right)$$

which helps to clarify the average contributions towards the components of $\langle R_{ij} \rangle^{xz}$ made across no-slip and free-slip portions and to identify regions of spanwise phase-dependence. The decomposition of streamwise component of the Reynolds stress tensor, $\langle R_{11} \rangle^{xz}$, is plotted in figure 4.23a and reveals two regions. The first of these regions is confined to $0 < y^+ < 7.8$, where the inequality $\langle R_{11} \rangle^{ns} < \langle R_{11} \rangle^{fs}$ is satisfied. The profiles of $\langle R_{11} \rangle^{xz}$, $\langle R_{11} \rangle^{ns}$ and $\langle R_{11} \rangle^{fs}$ intersect at $y^+ = 7.8$ and beyond this point the inequality $\langle R_{11} \rangle^{ns} > \langle R_{11} \rangle^{fs}$ is satisfied. The second region of phase-dependence occurs across the interval $7.8 < y^+ < 60$, and demonstrates that streamwise Reynolds stress is, on average, stronger above no-slip regions than free-slip regions. Beyond $y^+ > 60$, the profiles of $\langle R_{11} \rangle^{xz}$, $\langle R_{11} \rangle^{ns}$ and $\langle R_{11} \rangle^{fs}$ collapse on top of each other where there are minimal phase-variations. The decomposition of wall-normal component of the Reynolds stress tensor, $\langle R_{22} \rangle^{xz}$, is plotted in figure 4.23b and reveals a single region of weak spanwise-dependence across the interval $0 < y^+ < 20$. Beyond $y^+ > 20$, the profiles of $\langle R_{22} \rangle^{xz}$, $\langle R_{22} \rangle^{ns}$ and $\langle R_{22} \rangle^{fs}$ collapse on top of each other where there are minimal phase-variations. The decomposition of the spanwise component of the Reynolds stress tensor, $\langle R_{33} \rangle^{xz}$, is plotted in figure 4.23c, and compared to streamwise Reynolds stress component, $\langle R_{11} \rangle^{xz}$, exhibits a similar behaviour whereby profiles of $\langle R_{33} \rangle^{ns}$ and $\langle R_{33} \rangle^{fs}$ are phase-dependent across two wall-normal intervals. Their similar behaviour suggests that these two components of the Reynolds stress tensor are somehow linked together, possibly by properties of the mean flow, a matter that will be addressed later in this chapter. The decomposition of the Reynolds shear stress, $\langle R_{12} \rangle^{xz}$, is plotted in figure 4.23d and shows a region of phase dependence across $7.8 < y^+ < 60$.

To clarify the phase-dependence of the direct Reynolds stresses, contours of $\langle R_{ii} \rangle$ on the transverse $y-\phi_z$ plane are shown in figure 4.24. As the preceding analysis of $\langle R_{ii} \rangle^{ns}$ and $\langle R_{ii} \rangle^{fs}$ suggested, only the streamwise and spanwise normal Reynolds stresses exhibit strong phase-dependence. The contours of $\langle R_{11} \rangle$, provided in figure 4.24a, reveal that streamwise turbulent
momentum transfer is at its most intense within an arc-like region which connects a localised patch that spans all no-slip phases, centred at \((y^+, \phi_z) = (15, \phi_z)\), and the edges of the free-slip band. The contours of \(\langle R_{33} \rangle\), provided in figure 4.24c, show that spanwise turbulent momentum transfer is at its most intense in a region above above no-slip phases, centred at \((y^+, \phi_z) = (30, \phi_z)\). The analysis of the normal Reynolds stresses goes some way to explain the behaviours of the turbulence kinetic energy and demonstrates that the majority of energy is due to streamwise momentum transfer, since \(\langle R_{11} \rangle > (\langle R_{22} \rangle + \langle R_{33} \rangle)\). However, it is the Reynolds shear stresses, \(\langle R_{ij} \rangle\), that have a more direct impact on the mean flow and their analysis follows.

To clarify the phase-dependence of the Reynolds shear stresses, contours of \(\langle R_{i\neq j} \rangle\) plotted on the transverse \(y-\phi_z\) plane are shown in figure 4.25. Small pockets where \(\langle R_{12} \rangle^2 > 0\) are visible in figure 4.25a, which are located just above the edges of the interface. The regions where Reynolds shear stress is positive correlate well with the previous negative production of TKE (see figure 4.20a). The positive / (negative) signs of the Reynolds shear stress / (TKE production) can
Figure 4.24: Contours of phase-averaged direct Reynolds stresses on the transverse $y-\phi_z$ plane including a) $\langle R_{11} \rangle^x$, b) $\langle R_{22} \rangle^x$ and c) $\langle R_{33} \rangle^x$. All quantities scaled by reference no-slip friction velocity, $u_\tau$.

be explained by the negative sign on the right-hand side of equation 4.37a. Figure 4.25b shows contours of $\langle R_{23} \rangle^x$ where vectors of $(\tilde{v}, \tilde{w})$ were included to emphasise that where a clockwise / (anti-clockwise) vortex was observed a negative / (positive) patch of $\langle R_{23} \rangle^x$ occurred. In addition, figure 4.25b suggests that $\langle R_{23} \rangle^x$ may be supported solely by the presence of the counter-rotating streamwise vortices since, in the absence of coherent secondary motions, $\langle R_{23} \rangle^x \rightarrow 0$. Contours of $\langle R_{13} \rangle^x$ (figure 4.20c) demonstrate that crossflow Reynolds shear stress occurs at the edges of the interface. The general shape and position of the $\langle R_{13} \rangle^x$ contours in figure 4.20c strongly resemble the production of TKE due to primary shear, as shown previously in figure 4.20b. The lateral force exerted against the mean streamwise velocity by the crossflow Reynolds shear stress
can be written as,

\[ F_z = \frac{1}{d\lambda_x} \int_0^1 \int_0^{\lambda_x} \int_0^{\lambda_x} \frac{\partial \langle R_{13} \rangle}{\partial z} dxdydz, \]

\[ = \frac{1}{d\lambda_x} \int_0^1 \int_0^{\lambda_x} \left[ \langle R_{13} \rangle \right]_0^{\lambda_x} dxdy, \]

which is equivalent to applying Divergence Theorem in one dimension, which states that the divergence within an area is equal to the flux across its bounding curve — in this case, the edges of the no-slip band. Due to the opposite signs of \( \langle R_{13} \rangle \) at the edges of the no-slip band, the force is *additive* and means \( F_z > 0 \). The significance of this result is that the cross-flow Reynolds shear stress has the ability to act against the mean streamwise velocity profile in an analogous manner to the Reynolds shear stress, \( \langle R_{12} \rangle \). At present, it remains unclear if this lateral force contributes towards the local levels of skin-friction in a positive or negative manner.

The components of the Reynolds stress tensor, \( \langle R_{ij} \rangle \), have been quantified and explored...
in some detail, however their precise origin remains unclear. To remedy this, the dynamical equation for the Reynolds stress tensor was derived and can be written as

\[
\frac{\partial}{\partial t} \langle u'_i u'_k \rangle + \left( \langle U_j \rangle \frac{\partial}{\partial x_j} + \tilde{u}_j \right) \frac{\partial}{\partial x_j} \langle u'_i u'_k \rangle = \left( \langle u'_i \frac{\partial \rho'}{\partial x_k} \rangle + \langle u'_k \frac{\partial \rho'}{\partial x_i} \rangle \right) + \left( \langle u'_i \frac{\partial u'_j}{\partial x_k} \rangle + \langle u'_k \frac{\partial u'_j}{\partial x_i} \rangle \right)
\]

\[
- \langle u'_k u'_j \rangle \left( \frac{\partial \langle U_i \rangle}{\partial x_j} + \frac{\partial \tilde{u}_i}{\partial x_j} \right) - \langle u'_i u'_j \rangle \left( \frac{\partial \langle U_k \rangle}{\partial x_j} + \frac{\partial \tilde{u}_k}{\partial x_j} \right)
\]

\[
+ \frac{1}{Re_b} \frac{\partial^2}{\partial x_j \partial x_j} \langle u'_i u'_j \rangle - \frac{2}{Re_b} \frac{\partial u'_i}{\partial x_j} \frac{\partial u'_j}{\partial x_j} - \frac{\partial}{\partial x_j} \langle u'_k u'_j \rangle.
\]

(4.40)

and is referred to as the Reynolds Stress Transport Equation. The first term on the left-hand side of equation 4.40 represents the time rate-of-change of the Reynolds stresses, and is equal to zero for a fully-developed turbulent flow. The second term on the left-hand side represents the steady convection of Reynolds stresses by the primary and secondary flows. The first term on the right-side of equation 4.40 is the correlation between the fluctuating pressure and the fluctuating strain rate and represents a redistributive process whereby energy is filtered off to other Reynolds stress components. The second term represents production of Reynolds stresses by the primary and secondary shear, where Reynolds stresses work against the mean shear to sustain turbulence. The third term represents the diffusion of Reynolds stresses in space. The fourth term represents the viscous dissipation of Reynolds stresses. The fifth and final term on the right-hand side represents the transport of Reynolds stresses by the turbulent fluctuations themselves.

The interchange of momentum from mean flow to stochastic flow is referred to as production. The production of Reynolds stresses is due to an exchange of momentum from the mean flow to the stochastic flow, made possible via the mean shear. The production tensor can be written as

\[
\langle P_{ik} \rangle = -\langle u'_k u'_j \rangle \left( \frac{\partial \langle U_i \rangle}{\partial x_j} + \frac{\partial \tilde{u}_i}{\partial x_j} \right) - \langle u'_i u'_j \rangle \left( \frac{\partial \langle U_k \rangle}{\partial x_j} + \frac{\partial \tilde{u}_k}{\partial x_j} \right).
\]

(4.41)

In the absence of mean shear, it is clear that the right-hand side of equation 4.41 will equal zero, and as a result turbulence simply decays due to viscous dissipation. Thus, an analysis of \( \langle P_{ij} \rangle \) is the natural starting point to better understand previously observed changes the Reynolds stress tensor, \( \langle R_{ij} \rangle \). In addition, an analysis of \( \langle P_{ij} \rangle \) will clarify what role the primary and secondary gradients play in the production process. The diagonal components of \( \langle P_{ij} \rangle \) govern
the production of direct Reynolds stresses. The components of $\langle P_{ii} \rangle$ can be written as

$$
\langle P_{11} \rangle = -2 \left( R_{11} \frac{\partial \langle U \rangle}{\partial x} + R_{12} \frac{\partial \langle U \rangle}{\partial y} + R_{13} \frac{\partial \langle U \rangle}{\partial z} \right),
$$

(4.42a)

$$
\langle P_{22} \rangle = -2 \left( R_{12} \frac{\partial \langle U \rangle}{\partial x} + R_{22} \frac{\partial \langle U \rangle}{\partial y} + R_{23} \frac{\partial \langle U \rangle}{\partial z} \right),
$$

(4.42b)

$$
\langle P_{33} \rangle = -2 \left( R_{13} \frac{\partial \langle U \rangle}{\partial x} + R_{23} \frac{\partial \langle U \rangle}{\partial y} + R_{33} \frac{\partial \langle U \rangle}{\partial z} \right),
$$

(4.42c)

which demonstrates that the production of streamwise Reynolds stress is supported by primary flow gradients, whereas the production of wall-normal and spanwise Reynolds stresses is supported by secondary flow gradients. The off-diagonal components of $\langle P_{ij} \rangle$ govern the production of Reynolds shear stresses and can be written as

$$
\langle P_{12} \rangle = - \left( R_{12} \frac{\partial \langle U \rangle}{\partial x} + R_{22} \frac{\partial \langle U \rangle}{\partial y} + R_{23} \frac{\partial \langle U \rangle}{\partial z} \right) - \left( R_{11} \frac{\partial \langle \tilde{w} \rangle}{\partial x} + R_{12} \frac{\partial \langle \tilde{w} \rangle}{\partial y} + R_{13} \frac{\partial \langle \tilde{w} \rangle}{\partial z} \right),
$$

(4.43a)

$$
\langle P_{13} \rangle = - \left( R_{13} \frac{\partial \langle U \rangle}{\partial x} + R_{23} \frac{\partial \langle U \rangle}{\partial y} + R_{33} \frac{\partial \langle U \rangle}{\partial z} \right) - \left( R_{11} \frac{\partial \langle \tilde{w} \rangle}{\partial x} + R_{12} \frac{\partial \langle \tilde{w} \rangle}{\partial y} + R_{13} \frac{\partial \langle \tilde{w} \rangle}{\partial z} \right),
$$

(4.43b)

$$
\langle P_{23} \rangle = - \left( R_{13} \frac{\partial \langle U \rangle}{\partial x} + R_{23} \frac{\partial \langle U \rangle}{\partial y} + R_{33} \frac{\partial \langle U \rangle}{\partial z} \right) - \left( R_{12} \frac{\partial \langle \tilde{w} \rangle}{\partial x} + R_{22} \frac{\partial \langle \tilde{w} \rangle}{\partial y} + R_{23} \frac{\partial \langle \tilde{w} \rangle}{\partial z} \right),
$$

(4.43c)

which demonstrates that the production of Reynolds shear stresses is due to a mixture of both primary and secondary flow gradients in the primary and secondary flows, with the exception of $\langle P_{23} \rangle$ which only contains secondary flow gradients.

First, $\langle P_{ij} \rangle$ was integrated across all phases to obtain $\langle P_{ij} \rangle^{xz}$ in order to investigate the global response of production to the presence of the texture. Profiles of each component are provided in figure 4.26. The production of streamwise Reynolds stress is shown in figure 4.26a and is simply twice the production term found in the turbulence kinetic energy equation, for both the reference and textured cases. The production of wall-normal and spanwise Reynolds stresses are plotted together in figure 4.26b and, compared to the production of streamwise Reynolds stress
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Figure 4.26: Components of production tensor averaged across all phases for textured (—) and reference no-slip (→), cases including production of (a) streamwise Reynolds stress $\langle P_{11} \rangle^{xz}$, (b) wall-normal Reynolds stress $\langle P_{22} \rangle^{xz}$ and spanwise Reynolds stress $\langle P_{33} \rangle^{xz}$, (c) Reynolds shear-stress $\langle P_{12} \rangle^{xz}$ and (d) cross-flow Reynolds shear stress $\langle P_{23} \rangle^{xz}$ and secondary Reynolds shear stress $\langle P_{23} \rangle^{xz}$. All quantities scaled using reference no-slip friction velocity, $u_\tau$.

(figure 4.26a), are at least two orders of magnitude smaller, which demonstrates that production supported by secondary shear is far weaker than production supported by primary shear. The production of the Reynolds shear stress is shown in figure 4.26c and, relative to the reference case, is reduced across the entire channel half-height, where the peak value is reduced by approximately 29%. The production of the remaining two Reynolds shear stresses, $\langle P_{23} \rangle^{xz}$ and $\langle P_{13} \rangle^{xz}$, were both equal to zero, shown in figure 4.26d.

Next, the root-mean-squared production tensor (equation 4.41) was evaluated using

$$
\langle P_{ij} \rangle^{xz}_{rms} = \sqrt{\frac{1}{(d + g) \lambda_x} \int_0^{d + g} \int_0^{\lambda_x} (\langle P_{ij} \rangle^{xz})^2 \, dx \, dz},
$$

(4.44)
in order to detect any components of $\langle P_{ij} \rangle^{xz}$ that may have cancelled to statistical symmetries.
upon averaging across all phases. Note that, for the reference case, $\langle P_{ij} \rangle_{rms}^{xz} = \langle P_{ij} \rangle^{xz}$, since turbulence statistics for plane channel flow have zero variance in both streamwise and spanwise directions. Profiles of $\langle P_{ij} \rangle_{rms}^{xz}$ are provided in figure 4.27. From the previous analysis, the two dominant components of $\langle P_{ij} \rangle^{xz}$ were found to be $\langle P_{11} \rangle^{xz}$ and $\langle P_{12} \rangle^{xz}$. The current analysis of $\langle P_{ij} \rangle_{rms}^{xz}$ shows that the dominant root-mean-squared components include $\langle P_{11} \rangle_{rms}^{xz}$, $\langle P_{12} \rangle_{rms}^{xz}$ and, an additional third term, $\langle P_{13} \rangle_{rms}^{xz}$. The dominant root-mean-squared values are plotted in figure 4.27a. The three remaining components are plotted in figure 4.27b, where the vertical axis has been normalised by a factor of $10^2$, relative to the vertical axis of figure 4.27a, demonstrating that their relative contribution towards the production of Reynolds stresses is quite small.

From the above analysis of $\langle P_{ij} \rangle^{xz}$ and $\langle P_{ij} \rangle_{rms}^{xz}$ the dominant terms of Reynolds stress production have been identified as $\langle P_{11} \rangle$, $\langle P_{12} \rangle$ and $\langle P_{13} \rangle$, the last of which was only detectable in root-mean-squared form. For streamwise homogeneous flows, equations 4.42a, 4.43a and 4.43c...
can be written as

\[
\langle P_{11}\rangle_x = -2\left< \frac{\partial \langle U \rangle}{\partial y} \right>_x - 2\left< \frac{\partial \langle U \rangle}{\partial z} \right>_x,
\]

\[
\langle P_{12}\rangle_x = -\left< \frac{\partial \langle U \rangle}{\partial y} \right>_x - \left< \frac{\partial \langle U \rangle}{\partial z} \right>_x - \left< \langle R_{11} \rangle \frac{\partial \langle \tilde{v} \rangle}{\partial x} + \langle R_{12} \rangle \frac{\partial \langle \tilde{v} \rangle}{\partial y} + \langle R_{13} \rangle \frac{\partial \langle \tilde{v} \rangle}{\partial z} \right>_x,
\]

\[
\langle P_{13}\rangle_x = -\left< \frac{\partial \langle U \rangle}{\partial y} \right>_x - \left< \frac{\partial \langle U \rangle}{\partial z} \right>_x - \left< \langle R_{11} \rangle \frac{\partial \langle \tilde{w} \rangle}{\partial x} + \langle R_{12} \rangle \frac{\partial \langle \tilde{w} \rangle}{\partial y} + \langle R_{13} \rangle \frac{\partial \langle \tilde{w} \rangle}{\partial z} \right>_x.
\]

For the reference no-slip case, the only non-zero terms in the above system of equations are the first terms on the right-hand of equation 4.45a and 4.45c, since the primary flow only varies in the wall-normal direction and no secondary motions can be supported. For the streamwise-aligned texture, all the terms on right-hand side of equations 4.45a-4.45c are non-zero.

In order to characterise the primary production of Reynolds shear stresses above the streamwise-aligned texture, each term on the right-hand side of equations 4.45a-4.45c was evaluated, excluding the last terms on the right-hand side of equation 4.45c and equation 4.45c which can be classified as secondary production terms. Figure 4.28a shows the primary production of \(\langle P_{11}\rangle_x\), \(\langle P_{12}\rangle_x\) and \(\langle P_{13}\rangle_x\). In general, high levels of production are observed in two regions. The first region is above the micro-feature, where strong production of streamwise Reynolds stress, \(\langle P_{11,2}\rangle_x\), and Reynolds shear stress, \(\langle P_{12,2}\rangle_x\) are both observed. The former component is closely related to the turbulence kinetic energy, whereas the latter component is related to the wall-normal force that distorts the mean streamwise velocity profile, relative to the laminar profile. The second region of high production can be found near the edges of the interface, and here attention is drawn to the production of crossflow Reynolds shear stress, \(\langle P_{13,2}\rangle_x\), which is related to the production of a turbulent force that can distort the mean streamwise velocity profile in the lateral direction.
4.3.3 **Summary**

The turbulent flow above the streamwise-aligned texture was characterised using two separate statistical measures:

1. *Turbulence kinetic energy*: a scalar measure of turbulence activity, defined as half the trace of the Reynolds stress tensor.

2. *Reynolds stresses*: the time-averaged effect of turbulent convection, which act directly on the mean flow via the divergence of the Reynolds stress tensor.

The presence of the texture had a dual effect on the turbulence kinetic energy, relative to the reference no-slip flow. On the one hand, the TKE was suppressed by 30% at a wall-normal height of \( y^+ \approx 13 \). Whilst on the other hand, the TKE significantly increased for \( y^+ < 7 \) which means that the viscous sublayer becomes energised. By decomposing the TKE using...
\( \langle k \rangle^{xz} = 1/2 \left( \langle k \rangle^{ns} + \langle k \rangle^{fs} \right) \) it was demonstrated that TKE was phase-dependent below a wall-normal height of \( y^+ \approx 60 \) and that \( \langle k \rangle^{fs} \) penetrated all the way to the interface of the SHS texture. Inspection of the phase-averaged TKE demonstrated that intense turbulence activity occurred above the no-slip region at at wall-normal position of \( y^+ \approx 13 \) and at the edges of the interface. The dynamical characteristics of TKE were evaluated by computing the turbulence kinetic energy budget, where focus was drawn to the production term — which persisted all the way to wall. By appropriately decomposing the production of TKE and studying its phase-dependence, it was demonstrated that the intense pockets of TKE above the no-slip micro-feature were supported by the primary wall-normal shear and that the penetration of TKE towards the interface was supported by the primary spanwise shear.

Due to the definition of TKE, an analysis of the components of the Reynolds stress tensor was required. Averaged across all phases, each of the direct Reynolds stresses was suppressed away from the wall and increased close to the wall, which agreed well with the preceding analysis of TKE. A peak suppression of approximately 25% was noted for the Reynolds shear stress. By decomposing the Reynolds stresses using \( \langle R_{ij} \rangle^{xz} = 1/2 \left( \langle R_{ij} \rangle^{ns} + \langle R_{ij} \rangle^{fs} \right) \) it was demonstrated that both \( \langle R_{11} \rangle^{fs} \) and \( \langle R_{33} \rangle^{fs} \) penetrated all the way the interface of the SHS texture. In order to better understand the mechanisms responsible for the production of Reynolds stresses, the production term from the Reynolds Stress Transport Equation was considered. Production mechanisms were split into two categories: primary shear production and secondary shear production. The latter production mechanism was found to be weak in comparison to the former.
4.4 Skin-friction identity

It is well known that the frictional drag of a turbulent channel flow is usually much higher than that of a laminar one. A quantitative relation which explained this observation was derived by Fukagata et al. (2002), herein referred to as the FIK identity, which provided details of various contributions towards the turbulent skin friction coefficient \( \langle C_f \rangle \). The FIK identity demonstrated that increased levels of turbulent drag could be explained by the weighted effect of the Reynolds shear stress distribution. In addition, the FIK identity has proved to be a useful tool which is well suited to the analysis of a range of DNS-based flow control studies including the drag-reducing benefits of wall compliance (Fukagata et al., 2008), suction and blowing (Kametani & Fukagata, 2011), surface-heating or cooling (Kametani & Fukagata, 2012), to study flows in the absence of control including flow over a cylinder (Monte et al., 2011) and geometrically complex surfaces (Peet & Sagaut, 2009). Here, the objective is to obtain a modified version of the FIK identity in order to better understand why the frictional drag across the no-slip phases of the streamwise-aligned texture is higher than the reference case.

Following the procedures outlined by Fukagata et al. (2002), the starting point for the derivation of the modified FIK identity is the streamwise component of the Reynolds-Averaged-Navier-Stokes equations,

\[
- \frac{\partial \langle p \rangle}{\partial x} = \frac{\partial}{\partial y} \left( \langle u \rangle \langle v \rangle + \langle R_{12} \rangle - \frac{1}{Re_b} \frac{\partial \langle u \rangle}{\partial y} \right) + \frac{\partial}{\partial x} \left( \langle u \rangle \langle u \rangle + \langle R_{11} \rangle - \frac{1}{Re_b} \frac{\partial \langle u \rangle}{\partial x} \right) + \langle I_x \rangle + \frac{\partial}{\partial z} \left( \langle u \rangle \langle w \rangle + \langle R_{13} \rangle - \frac{1}{Re_b} \frac{\partial \langle u \rangle}{\partial z} \right),
\]

(4.46)

which can be integrated with respect to \( y \) from the wall to channel half-height to give

\[
\int_0^1 - \frac{\partial \langle p \rangle}{\partial x} dy = \int_0^1 \frac{\partial}{\partial y} \left( \langle u \rangle \langle v \rangle + \langle R_{12} \rangle - \frac{1}{Re_b} \frac{\partial \langle u \rangle}{\partial y} \right) dy + \int_0^1 \langle I_x \rangle dy + \int_0^1 \langle I_z \rangle dy.
\]

(4.47)

By enforcing the no-penetration boundary condition on \( \langle v \rangle \) and exploiting statistical symmetries at the channel half-height, equation 4.47 can be evaluated and rewritten as

\[
- \frac{\partial \langle p \rangle}{\partial x} = \frac{1}{Re_b} \left. \frac{\partial \langle u \rangle}{\partial y} \right|_w + \int_0^1 \langle I_x \rangle dy + \int_0^1 \langle I_z \rangle dy.
\]

(4.48)

Next, the mean streamwise pressure gradient term was eliminated by substituting equation 4.48
into equation 4.47 to give,

\[
\frac{1}{Re_b} \frac{\partial \langle u \rangle}{\partial y} \bigg|_w = \frac{\partial}{\partial y} \left( \langle u \rangle \langle v \rangle + \langle R_{12} \rangle - \frac{1}{Re_b} \frac{\partial \langle u \rangle}{\partial y} \right) + \langle I_x \rangle - \frac{1}{0} \langle I_x \rangle dy + \langle I_z \rangle - \frac{1}{0} \langle I_z \rangle dy.
\] (4.49)

which, by using the definition of skin-friction, \( \langle C_f \rangle \equiv \frac{2}{Re_b \langle U_b^2 \rangle} \frac{\partial \langle u \rangle}{\partial y} \bigg|_w \), can be rewritten as

\[
\frac{\langle U_b^2 \rangle \langle C_f \rangle_{FIK}}{2} = \frac{\partial}{\partial y} \left( \langle u \rangle \langle v \rangle + \langle R_{12} \rangle - \frac{1}{Re_b} \frac{\partial \langle u \rangle}{\partial y} \right) + \left( \langle I_x \rangle - \langle I_x \rangle \right) dy + \left( \langle I_z \rangle - \langle I_z \rangle \right) dy.
\] (4.50)

where the overbar denotes local bulk mean quantities, defined as \( \langle I_j \rangle = \frac{1}{\int_0^y} \langle I_j \rangle dy \). The relation for contributions of different dynamical effects to the local skin friction coefficient can be obtained by applying a triple integration, i.e. \( \int^y_0 \int^y_0 \int^y_0 \), to equation 4.50. The first integration with respect to \( y \) essentially gives the force balance, similar to the familiar linear relation for stresses, and can be written as,

\[
\frac{\langle U_b^2 \rangle \langle C_f \rangle_{FIK}}{2} \left( y - 1 \right) = \langle u \rangle \langle v \rangle + \langle R_{12} \rangle - \frac{1}{Re_b} \frac{\partial \langle u \rangle}{\partial y} + \int^y_0 \left( \langle I_x \rangle - \langle I_x \rangle \right) dy + \int^y_0 \left( \langle I_z \rangle - \langle I_z \rangle \right) dy.
\] (4.51)

The second integration of equation 4.51 with respect to \( y \) essentially leads to the mean velocity profile and upon integration can be written as,

\[
\frac{\langle U_b^2 \rangle \langle C_f \rangle_{FIK}}{4} \left( y^2 - 2y \right) = \int^y_0 \langle u \rangle \langle v \rangle dy + \int^y_0 \langle R_{12} \rangle dy - \frac{1}{Re_b} \left( \langle u \rangle - \langle U_s \rangle \right)
+ \int^y_0 \int^y_0 \left( \langle I_x \rangle - \langle I_x \rangle \right) dy dy + \int^y_0 \int^y_0 \left( \langle I_z \rangle - \langle I_z \rangle \right) dy dy.
\] (4.52)

The third and final integration of equation 4.52 with respect to \( y \) leads to the mass flow rate and can be written as,

\[
-\frac{1}{6} \langle U_b^2 \rangle \langle C_f \rangle_{FIK} = \frac{1}{Re_b} \left( \langle U_s \rangle - \langle U_b \rangle \right) + \int^y_0 \int^y_0 \langle u \rangle \langle v \rangle dy dy + \int^y_0 \int^y_0 \langle R_{12} \rangle dy dy
+ \int^y_0 \int^y_0 \int^y_0 \left( \langle I_x \rangle - \langle I_x \rangle \right) dy dy dy + \int^y_0 \int^y_0 \int^y_0 \left( \langle I_z \rangle - \langle I_z \rangle \right) dy dy dy.
\] (4.53)
which can be rearranged for \(C_f\) to give,

\[
\langle C_f \rangle_{FIK} = \frac{6}{(U_b^2) Re_b} (\langle U_b \rangle - \langle U_s \rangle) - \frac{6}{(U_b^2)} \int_0^1 y (u) \langle v \rangle \, dy \, dy + \frac{6}{(U_b^2)} \int_0^1 y \langle -R_{12} \rangle \, dy \, dy \\
- \frac{6}{(U_b^2)} \int_0^1 \int_0^1 \int_0^1 (\langle I_x \rangle - \langle I_x \rangle) \, dy \, dy \, dy + \frac{6}{(U_b^2)} \int_0^1 \int_0^1 \int_0^1 (\langle I_z \rangle - \langle I_z \rangle) \, dy \, dy \, dy.
\]

(4.54)

Next, the multiple integrations on the right-hand side of equation 4.54 are transformed to single integrations by application of integration by parts. These transformations help simplify the final form of the FIK identity, and clarify the physical meaning of particular terms. For a dummy phase-averaged variable, say \(\langle f \rangle\), double integrals were transformed using,

\[
\int_0^1 \left( \int_0^y \langle f \rangle \, dy \right) \, dy = \left[ \int_0^y \langle f \rangle \, dy \right]_0^1 - \int_0^1 y \langle f \rangle \, dy = \int_0^1 (1 - y) \langle f \rangle \, dy,
\]

Triple integrals were transformed by first introducing an additional dummy phase-averaged variable, say \(\langle g \rangle\), which was defined as \(\langle g \rangle = \int_0^y \langle f \rangle \, dy\). Using the expression for \(\langle g \rangle\) and the results from the transformation of double integrals, the triple integrals were transformed using

\[
\int_0^1 \left( \int_0^y \int_0^y \langle g \rangle \, dy \right) \, dy = \left[ \int_0^y \int_0^y \langle g \rangle \, dy \right]_0^1 - \int_0^1 \int_0^y \langle g \rangle \, dy \, dy \\
= \int_0^1 (1 - y) \int_0^y \langle f \rangle \, dy \, dy \\
= \int_0^1 \left( \int_0^y \langle f \rangle \, dy \right) \, dy - \int_0^1 \int_0^y \langle f \rangle \, dy \, dy \\
= \int_0^1 (1 - y) \langle f \rangle \, dy - \left[ \int_0^y \langle f \rangle \, dy \right]_0^1 - \int_0^1 \frac{y^2}{2} \langle f \rangle \, dy \\
= \int_0^1 (1 - y) \langle f \rangle \, dy - \frac{1}{2} \int_0^1 (1 - y^2) \langle f \rangle \, dy \\
= \frac{1}{2} \int_0^1 (1 - y)^2 \langle f \rangle \, dy.
\]
Using the above integral transforms, equation 4.54 can be rewritten as

\[
\langle C_f \rangle_{\text{FIK}} = \frac{6}{\langle U_b^2 \rangle \text{Re}_b} (\langle U_b \rangle - \langle U_s \rangle) - \frac{6}{\langle U_b^2 \rangle} \int_0^1 (1 - y) \langle u \rangle \langle v \rangle \, dy + \frac{6}{\langle U_b^2 \rangle} \int_0^1 (1 - y) \langle -R_{12} \rangle \, dy
\]

\[
\quad \quad - \frac{3}{\langle U_b^2 \rangle} \int_0^1 (1 - y)^2 \left( \langle I_x \rangle - \langle I_z \rangle \right) \, dy - \frac{3}{\langle U_b^2 \rangle} \int_0^1 (1 - y)^2 \left( \langle I_z \rangle - \langle I_z \rangle \right) \, dy.
\]

(4.55)

Further simplifications of the right-hand side of equation 4.55 are possible. First, by substituting \( \langle I_x \rangle \) and \( \langle I_z \rangle \) (from equation 4.46) the above expression can be expanded to give,

\[
\langle C_f \rangle_{\text{FIK}} = \frac{6}{\langle U_b^2 \rangle \text{Re}_b} (\langle U_b \rangle - \langle U_s \rangle) - \frac{6}{\langle U_b^2 \rangle} \int_0^1 (1 - y) \langle u \rangle \langle v \rangle \, dy + \frac{6}{\langle U_b^2 \rangle} \int_0^1 (1 - y) \langle -R_{12} \rangle \, dy \]

\[
\quad \quad - \frac{3}{\langle U_b^2 \rangle} \int_0^1 (1 - y)^2 \partial_x \left( \langle u \rangle \langle u \rangle \right) \, dy - \frac{3}{\langle U_b^2 \rangle} \int_0^1 (1 - y)^2 \partial_x \left( \langle R_{11} \rangle - \langle R_{11} \rangle \right) \, dy
\]

\[
\quad \quad - \frac{3}{\langle U_b^2 \rangle} \int_0^1 (1 - y)^2 \partial_z \left( \langle w \rangle - \langle w \rangle \right) \, dy - \frac{3}{\langle U_b^2 \rangle \text{Re}_b} \int_0^1 (1 - y)^2 \partial_z^2 \left( \langle w \rangle - \langle w \rangle \right) \, dy.
\]

(4.56)

Next, the three braced terms shown above are grouped to form \( A = A_1 + A_2 + A_3 \) gives,

\[
A = - \frac{6}{\langle U_b^2 \rangle} \int_0^1 (1 - y) \langle u \rangle \langle v \rangle \, dy - \frac{3}{\langle U_b^2 \rangle} \int_0^1 (1 - y)^2 \partial_x \left( \langle u \rangle \langle u \rangle - \langle u \rangle \langle u \rangle \right) \, dy
\]

\[
\quad \quad - \frac{3}{\langle U_b^2 \rangle} \int_0^1 (1 - y)^2 \partial_z \left( \langle w \rangle - \langle w \rangle \right) \, dy,
\]

(4.57)

which, by using the relation,

\[
\frac{\partial}{\partial x} \left( \langle u \rangle \langle u \rangle - \langle u \rangle \langle u \rangle \right) + \frac{\partial}{\partial z} \left( \langle u \rangle \langle w \rangle - \langle u \rangle \langle w \rangle \right) = \left( \langle u \rangle \frac{\partial \langle u \rangle}{\partial x} - \langle u \rangle \frac{\partial \langle u \rangle}{\partial x} \right) + \left( \langle v \rangle \frac{\partial \langle w \rangle}{\partial y} - \langle v \rangle \frac{\partial \langle u \rangle}{\partial y} \right)
\]

\[
\quad \quad + \left( \langle w \rangle \frac{\partial \langle u \rangle}{\partial z} - \langle w \rangle \frac{\partial \langle u \rangle}{\partial z} \right) - \frac{\partial}{\partial y} \left( \langle u \rangle \langle w \rangle - \langle u \rangle \langle w \rangle \right),
\]
means that the $A$ can be rewritten as

$$A = - \frac{6}{(U_d^2)} \int_0^1 (1-y) \langle u \rangle \langle v \rangle dy + \frac{3}{(U_d^2)} \int_0^1 (1-y)^2 \frac{\partial}{\partial y} \left( \langle u \rangle \langle v \rangle - \langle u \rangle \langle v \rangle \right) dy$$

$$- \frac{3}{(U_d^2)} \int_0^1 (1-y)^2 \left( \langle v \rangle \frac{\partial \langle u \rangle}{\partial y} - \langle v \rangle \frac{\partial \langle u \rangle}{\partial y} \right) dy - \frac{3}{(U_d^2)} \int_0^1 (1-y)^2 \left( \langle u \rangle \frac{\partial \langle u \rangle}{\partial x} - \langle u \rangle \frac{\partial \langle u \rangle}{\partial x} \right)$$

$$- \frac{3}{(U_d^2)} \int_0^1 (1-y)^2 \left( \langle w \rangle \frac{\partial \langle u \rangle}{\partial z} - \langle w \rangle \frac{\partial \langle u \rangle}{\partial z} \right) dy,$$  

(4.58)

Integration by parts can be used to rewrite the braced term $A_4$ as

$$A_4 = \left[ (1-y)^2 \left( \langle u \rangle \langle v \rangle - \langle u \rangle \langle v \rangle \right) \right]_0^1 + 2 \int_0^1 (1-y) \left( \langle u \rangle \langle v \rangle - \langle u \rangle \langle v \rangle \right) dy$$

$$= - \left[ (1-y)^2 \langle u \rangle \langle v \rangle \right]_0^1 + 2 \int_0^1 (1-y) \left( \langle u \rangle \langle v \rangle - \langle u \rangle \langle v \rangle \right) dy$$

$$= - \left( 0 - \langle u \rangle \langle v \rangle \right) + 2 \int_0^1 (1-y) \left( \langle u \rangle \langle v \rangle - \langle u \rangle \langle v \rangle \right) dy$$

$$= \langle u \rangle \langle v \rangle - 2 \langle u \rangle \langle v \rangle \left[ y - \frac{y^2}{2} \right]_0^1 + 2 \int_0^1 (1-y) \langle u \rangle \langle v \rangle dy$$

$$= 2 \int_0^1 (1-y) \langle u \rangle \langle v \rangle dy,$$  

(4.59)

and this result can be used to rewrite the term $A$, shown in equation 4.58, as

$$A = - \frac{3}{(U_d^2)} \int_0^1 (1-y)^2 \left( \langle v \rangle \frac{\partial \langle u \rangle}{\partial y} - \langle v \rangle \frac{\partial \langle u \rangle}{\partial y} \right) dy - \frac{3}{(U_d^2)} \int_0^1 (1-y)^2 \left( \langle u \rangle \frac{\partial \langle u \rangle}{\partial x} - \langle u \rangle \frac{\partial \langle u \rangle}{\partial x} \right)$$

$$- \frac{3}{(U_d^2)} \int_0^1 (1-y)^2 \left( \langle w \rangle \frac{\partial \langle u \rangle}{\partial z} - \langle w \rangle \frac{\partial \langle u \rangle}{\partial z} \right) dy,$$  

(4.60)
Substituting the above expression for $A$ into equation 4.56 gives

$$
\langle C_f \rangle_{FIK} = \frac{6}{\langle U_b \rangle \text{Re}_b} (\langle U_b \rangle - \langle U_s \rangle) + \frac{6}{\langle U_b \rangle^2} \int_0^1 (1 - y) (-R_{12}) \, dy
$$

\[ - \frac{3}{\langle U_b \rangle^2} \int_0^1 (1 - y)^2 \left( \langle v \rangle \frac{\partial \langle u \rangle}{\partial y} - \langle v \rangle \frac{\partial \langle u \rangle}{\partial y} \right) \, dy \]

\[ - \frac{3}{\langle U_b \rangle^2} \int_0^1 (1 - y)^2 \left( \langle u \rangle \frac{\partial \langle u \rangle}{\partial y} - \langle u \rangle \frac{\partial \langle u \rangle}{\partial y} \right) \, dy \]

\[ - \frac{3}{\langle U_b \rangle} \int_0^1 (1 - y)^2 \left( \langle u \rangle \frac{\partial \langle u \rangle}{\partial x} + \langle v \rangle \frac{\partial \langle u \rangle}{\partial y} + \langle w \rangle \frac{\partial \langle u \rangle}{\partial z} \right) \, dy \]

\[ - \frac{3}{\langle U_b \rangle^2} \int_0^1 (1 - y)^2 \left( \frac{\partial^2}{\partial x^2} \langle u \rangle \right) \, dy \]

Introducing the notation $\overline{f} = \langle f \rangle - \langle \overline{f} \rangle$, which represents how each phase varies relative to a local bulk mean quantity, simplifies equation 4.61 to

$$
\langle C_f \rangle_{FIK} = \frac{6}{\langle U_b \rangle \text{Re}_b} \left( 1 - \frac{\langle U_s \rangle}{\langle U_b \rangle} \right) + \frac{6}{\langle U_b \rangle^2} \int_0^1 (1 - y) (-R_{12}) \, dy
$$

\[ - \frac{3}{\langle U_b \rangle^2} \int_0^1 (1 - y)^2 \left( \langle u \rangle \frac{\partial \langle u \rangle}{\partial x} + \langle v \rangle \frac{\partial \langle u \rangle}{\partial y} + \langle w \rangle \frac{\partial \langle u \rangle}{\partial z} \right) \, dy \]

\[ - \frac{3}{\langle U_b \rangle} \int_0^1 (1 - y)^2 \left( \langle u \rangle \frac{\partial \langle u \rangle}{\partial x} \right) \, dy \]

\[ - \frac{3}{\langle U_b \rangle^2} \int_0^1 (1 - y)^2 \left( \frac{\partial^2}{\partial x^2} \langle u \rangle \right) \, dy \]

\[ - \frac{3}{\langle U_b \rangle^2} \int_0^1 (1 - y)^2 \left( \frac{\partial^2}{\partial y^2} \langle u \rangle \right) \, dy \]

\[ - \frac{3}{\langle U_b \rangle^2} \int_0^1 (1 - y)^2 \left( \frac{\partial^2}{\partial z^2} \langle u \rangle \right) \, dy. \]
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Using these results for terms \( A_5 \) and \( A_6 \), the phase-averaged FIK identity can be written as,

\[
\langle C_f \rangle_{\text{FIK}} = \left( \begin{array}{c}
\text{Laminar contribution, } \langle C_f \rangle_{\text{lam}} \\
\text{Streamwise convection contribution, } \langle C_f \rangle_{\text{s}} \\
\text{Wall-normal convection contribution, } \langle C_f \rangle_{\text{v}} \\
\text{Wall-normal convection contribution, } \langle C_f \rangle_{\text{w}} \\
\text{(R_{11}) contribution, } \langle C_f \rangle_{R_{11}} \\
\text{(R_{13}) contribution, } \langle C_f \rangle_{R_{13}} \\
\text{Streamwise diffusion contribution, } \langle C_f \rangle_{\text{sd}f} \\
\text{Spanwise diffusion contribution, } \langle C_f \rangle_{\text{sd}f} \\
\end{array} \right)
\]

The above equation indicates that the phase-averaged skin friction coefficient, \( \langle C_f \rangle_{\text{FIK}} \), can be decomposed into nine different contributions. The first term, \( \langle C_f \rangle_{\text{lam}} \), is identical to the well-known laminar solution plus the effect of the local slip velocity. The second term, \( \langle C_f \rangle_{R_{12}} \), represents the contribution made by the Reynolds shear stress. The third term, \( \langle C_f \rangle_{\text{s}} \), represents the streamwise convection of the primary flow by the primary flow itself. The fourth term, \( \langle C_f \rangle_{\text{v}} \), represents the wall-normal convection of the primary flow by the secondary flow. The fifth term, \( \langle C_f \rangle_{\text{w}} \), represents the spanwise convection of the primary flow by the secondary flow. The sixth term, \( \langle C_f \rangle_{R_{11}} \), represents a streamwise force due to axial gradients of streamwise Reynolds stress. The seventh term, \( \langle C_f \rangle_{R_{13}} \), represents a spanwise force due to lateral gradients of the spanwise Reynolds stress. The eighth term, \( \langle C_f \rangle_{\text{sd}f} \), represents the streamwise diffusion of the primary flow. The ninth and final term, \( \langle C_f \rangle_{\text{sd}f} \), represents the lateral diffusion of primary flow.

The skin-friction coefficient can be integrated across all phases and decomposed as

\[
\langle C_f \rangle_{\text{FIK}}^{xz} = \frac{1}{2} \left( \langle C_f \rangle_{\text{ns}} + \langle C_f \rangle_{\text{s}} \right)_{\text{FIK}}.
\]  

(4.64)

Due to the free-stress boundary condition enforced on \( \langle u \rangle \), equation 4.64 reduces to \( \langle C_f \rangle_{\text{FIK}}^{xz} = \frac{1}{2} \langle C_f \rangle_{\text{FIK}}^{ns} \), since \( \langle C_f \rangle_{\text{FIK}}^{s} = 0 \) at any point on the interface. An expression for \( \langle C_f \rangle_{\text{FIK}}^{ns} \) can be
obtained by integrating equation 4.62 across all no-slip phases to give,

\[
\langle C_f \rangle_{FIK}^{\text{ns}} = \langle C_f \rangle_{\text{lam}}^{\text{ns}} + \langle C_f \rangle_{R12}^{\text{ns}} - \left( \langle C_f \rangle_{(u)}^{\text{ns}} + \langle C_f \rangle_{(\bar{v})}^{\text{ns}} + \langle C_f \rangle_{(\bar{w})}^{\text{ns}} \right) + \langle C_f \rangle_{R13}^{\text{ns}} + \langle C_f \rangle_{\text{zdff}}^{\text{ns}} \tag{4.65}
\]

where each of the seven terms on the right-hand-side are given by

\[
\langle C_f \rangle_{\text{lam}}^{\text{ns}} = \frac{6}{(U_b) Re_b}, \tag{4.66a}
\]

\[
\langle C_f \rangle_{R12}^{\text{ns}} = \frac{6}{\lambda_x d (U_b^2)} \int_0^1 (1-y) \int_0^{(u)} \langle -R_{12} \rangle dzdxdy, \tag{4.66b}
\]

\[
\langle C_f \rangle_{(u)}^{\text{ns}} = \frac{3}{\lambda_x d (U_b^2)} \int_0^1 (1-y)^2 \int_0^{(u)} \langle \bar{u} \rangle \frac{\partial \langle u \rangle}{\partial x} dzdxdy, \tag{4.66c}
\]

\[
\langle C_f \rangle_{(\bar{v})}^{\text{ns}} = \frac{3}{\lambda_x d (U_b^2)} \int_0^1 (1-y)^2 \int_0^{(\bar{v})} \langle \bar{v} \rangle \frac{\partial \langle u \rangle}{\partial y} dzdxdy, \tag{4.66d}
\]

\[
\langle C_f \rangle_{(\bar{w})}^{\text{ns}} = \frac{3}{\lambda_x d (U_b^2)} \int_0^1 (1-y)^2 \int_0^{(\bar{w})} \langle \bar{w} \rangle \frac{\partial \langle u \rangle}{\partial z} dzdxdy, \tag{4.66e}
\]

\[
\langle C_f \rangle_{R13}^{\text{ns}} = \frac{3}{\lambda_x d (U_b^2)} \int_0^1 (1-y)^2 \int_0^{[R_{13}]} \frac{1}{Re_b} \frac{\partial^2 \langle u \rangle}{\partial z^2} dzdxdy, \tag{4.66f}
\]

\[
\langle C_f \rangle_{\text{zdff}}^{\text{ns}} = \frac{3}{\lambda_x d (U_b^2)} \int_0^1 (1-y)^2 \int_0^{[R_{11}]} \frac{1}{Re_b} \frac{\partial^2 \langle u \rangle}{\partial x^2} dzdxdy, \tag{4.66g}
\]

which, compared to the right-hand side of the phase-averaged FIK identity (equation 4.63), has two less terms. The two missing terms are \(\langle C_f \rangle_{(R_{11})}^{\text{ns}}\) and \(\langle C_f \rangle_{\text{zdff}}^{\text{ns}}\). Integrating their sum in the \(L_{\phi_x}\) direction, gives

\[
\langle C_f \rangle_{(R_{11})}^{\text{ns}} + \langle C_f \rangle_{\text{zdff}}^{\text{ns}} = \frac{3}{\lambda_x d (U_b^2)} \int_0^1 (1-y)^2 \int_0^{(R_{11})} \left( \frac{\partial \langle R_{11} \rangle}{\partial x} + \frac{\partial^2 \langle u \rangle}{\partial x^2} \right) dzdxdy,
\]

\[
= \frac{3}{\lambda_x d (U_b^2)} \int_0^1 (1-y)^2 \int_0^{(R_{11})} \left[ \frac{\partial \langle R_{11} \rangle}{\partial x} + \frac{1}{Re_b} \left[ \frac{\partial \langle u \rangle}{\partial x} \right] \right] dzdxdy = 0,
\]

due to the periodic boundary conditions in the streamwise direction. The spanwise integral of
the second last term on the right-hand side of equation 4.65 was simplified by using

\[
\langle C_f \rangle_{R_{13}} = \frac{3}{\lambda_x d \langle U_b \rangle} \int_0^1 (1 - y)^2 \int_0^{\lambda_y} \int_0^d \left( \frac{\partial \langle R_{13} \rangle}{\partial z} \right) \, dz \, dx \, dy
\]

which is equivalent to applying Divergence Theorem in one dimension, which states that the divergence within an area is equal to the flux across its bounding curve — in this case, the edges of the no-slip band.

In order to verify the accuracy of the phase-averaged FIK identity its estimate for the phase-averaged skin-friction \( \langle C_f \rangle_{FIK} \) (equation 4.63) was compared to the value obtained directly from the definition of phase-averaged skin-friction (see equation 4.3) which can be evaluated by simply computing the wall-normal derivative of the phase-averaged mean streamwise velocity profile at the wall. The phase-averaged FIK identity was evaluated in finite-difference form using a second-order accurate differencing stencil for both first- and second-order derivatives. The close agreement between \( \langle C_f \rangle_{FIK} \) and \( \langle C_f \rangle \) is shown in figure 4.29a and verifies that the phase-averaged FIK identity can faithfully reproduce the skin-friction for all spanwise no-slip phases. As an additional verification, the phase-averaged skin-friction coefficient was compared under laminar flow conditions and an excellent level of agreement was again observed (see figure 4.29b).

Next, each term in phase-averaged FIK identity (equation 4.63) was evaluated numerically,
and then integrated across all no-slip phases to obtain the various contributions towards \( \langle C_f \rangle_{ns} \) (equation 4.65). A breakdown of \( \langle C_f \rangle_{ns} \) for case T00 (under both laminar and turbulent flow conditions) is shown in figure 4.30, where the data from the reference no-slip case is included for comparison. The numerical values (to within four significant figures) of each term on the right-hand side of equation 4.65 are detailed in table 4.5 and the percentage contributions of each term towards \( \langle C_f \rangle_{ns} \) are detailed in table 4.6.

The numerical evaluation of the skin-friction identity quantifies the various contributions made by primary, secondary and turbulent motions towards the friction at the no-slip wall. Terms with a positive sign can be regarded as performance penalties, whereas terms with a negative sign can be regarded as performance gains. The primary flow makes two distinct contributions to skin-friction through the laminar term, \( \langle C_f \rangle_{ns}^{lam} \), and the spanwise diffusion term, \( \langle C_f \rangle_{ns}^{zdff} \). The former term, derived from laminar theory, is always positive (for attached flows), and remains constant for both the reference and textured cases (under both laminar and turbulent flow conditions) since mass flow rate is held fixed. The latter term represents a performance penalty incurred by the spanwise diffusion of high-momentum fluid above the free-slip band to no-slip regions, where there is low-momentum fluid. The secondary flow makes two contributions to skin-friction for the textured case through the terms \( \langle C_f \rangle_{ns}^{v} \) and \( \langle C_f \rangle_{ns}^{w} \). The former term is the only negative contribution towards skin-friction and represents the steady wall-normal convection of streamwise momentum away from the no-slip wall by the secondary motion. The latter term represents a performance penalty that is incurred by the spanwise convection of streamwise momentum towards the edges of the no-slip band. The signs of both these terms agree with the orientation of the secondary flow, discussed previously in the mean flow section and shown in figure 4.9.

Figure 4.30: Contribution of each term of the averaged FIK identity (equation 4.65) towards turbulent skin-friction coefficient averaged across all no-slip phases, \( \langle C_f \rangle_{ns} \), for reference no-slip turbulent flow (■), streamwise-aligned texture under turbulent (□) and laminar (■) flow conditions. Note that each term has been multiplied by a factor of \( 10^3 \).
The stochastic field also makes two contributions to skin-friction through the terms \( \langle C_f \rangle_{R_{12}}^{ns} \) and \( \langle C_f \rangle_{R_{13}}^{ns} \). Relative to the reference case, the performance penalty incurred by the former term is reduced by 21% for the textured case. The latter term is interpreted here as a performance penalty due to lateral turbulent momentum transfer due spanwise gradients in the Reynolds shear stress.

<table>
<thead>
<tr>
<th>Case</th>
<th>( \langle C_f \rangle_{\text{lam}}^{ns} )</th>
<th>( \langle C_f \rangle_{R_{12}}^{ns} )</th>
<th>( \langle C_f \rangle_{(u)}^{ns} )</th>
<th>( \langle C_f \rangle_{\tilde{v}}^{ns} )</th>
<th>( \langle C_f \rangle_{\tilde{w}}^{ns} )</th>
<th>( \langle C_f \rangle_{R_{13}}^{ns} )</th>
<th>( \langle C_f \rangle_{\text{zff}}^{ns} )</th>
<th>( \langle C_f \rangle_{\text{FIK}}^{ns} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_{\text{ref}} )</td>
<td>2.142</td>
<td>5.916</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>8.058</td>
</tr>
<tr>
<td>( T_{00} )</td>
<td>2.142</td>
<td>4.655</td>
<td>0.000</td>
<td>-2.705</td>
<td>0.723</td>
<td>4.126</td>
<td>3.541</td>
<td>12.48</td>
</tr>
<tr>
<td>( T_{\text{lam}}^{ref} )</td>
<td>2.142</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>2.142</td>
</tr>
<tr>
<td>( T_{\text{lam}}^{00} )</td>
<td>2.142</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>1.697</td>
<td>3.839</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.5: Contribution of each term of the averaged FIK identity (equation 4.65) towards turbulent skin-friction coefficient averaged across all no-slip phases, \( \langle C_f \rangle^{ns} \), for cases \( T_{\text{ref}} \) and \( T_{00} \) under both fully developed turbulent and laminar flow conditions, where the latter is denoted by superscript “\( \text{lam} \)”. Note that each term has been multiplied by a factor of \( 10^3 \).

<table>
<thead>
<tr>
<th>Case</th>
<th>( \langle C_f \rangle_{\text{lam}}^{ns} )</th>
<th>( \langle C_f \rangle_{R_{12}}^{ns} )</th>
<th>( \langle C_f \rangle_{(u)}^{ns} )</th>
<th>( \langle C_f \rangle_{\tilde{v}}^{ns} )</th>
<th>( \langle C_f \rangle_{\tilde{w}}^{ns} )</th>
<th>( \langle C_f \rangle_{R_{13}}^{ns} )</th>
<th>( \langle C_f \rangle_{\text{zff}}^{ns} )</th>
<th>( \sum(%) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_{\text{ref}} )</td>
<td>26.58</td>
<td>73.41</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>99.98</td>
</tr>
<tr>
<td>( T_{00} )</td>
<td>17.16</td>
<td>37.29</td>
<td>0.000</td>
<td>-21.67</td>
<td>5.793</td>
<td>33.06</td>
<td>28.37</td>
<td>100.0</td>
</tr>
<tr>
<td>( T_{\text{lam}}^{ref} )</td>
<td>55.79</td>
<td>0.000</td>
<td>0.000</td>
<td>-21.67</td>
<td>5.793</td>
<td>33.06</td>
<td>44.20</td>
<td>99.99</td>
</tr>
<tr>
<td>( T_{\text{lam}}^{00} )</td>
<td>100.0</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>100.0</td>
</tr>
</tbody>
</table>

Table 4.6: Percentage contribution of each term of the averaged FIK identity (equation 4.65) towards turbulent skin-friction coefficient averaged across all no-slip phases, \( \langle C_f \rangle^{ns} \), for cases \( T_{\text{ref}} \) and \( T_{00} \) under both turbulent and laminar flow conditions, where the latter is denoted by superscript “\( \text{lam} \)”.

### 4.4.1 Summary

In this section a useful mathematical tool was derived, verified and evaluated in order to quantify the contributions of mean, secondary and turbulent motions towards the local levels of skin-friction. The expression is an extension of the FIK identity, first introduced by Fukagata et al. (2002), that takes into account the phase-averaging procedures used throughout this dissertation. Relative to the reference no-slip flow, the skin-friction at the surface of the SHS suffers three additional performance penalties: (i) spanwise convection of primary fluid by spanwise secondary motion; (ii) a lateral force exerted by turbulent shear stress and (iii) spanwise diffusion of primary flow. On the other hand, the streamwise-aligned SHS appears to favourably influence skin-friction in two ways: (i) by suppressing the Reynolds shear stress contribution and (ii) by the wall-normal...
convection of primary flow by wall-normal secondary motion.
Chapter 5

The effect of spatial modulation on texture performance

5.1 Introduction

This chapter contains five sections. This section provides the motivation for a simple geometric modification to the streamwise-aligned micro-ridges by means of a sinusoidal streamwise modulation, characterised by a spanwise amplitude, $A_z$, and a streamwise wavelength, $\lambda_x$. Section 5.2 details the results from a parametric study in $A_z$-$\lambda_x$ space of various spatially varying SHS textures and their drag-reducing potential is quantified using various integral performance parameters. Sections 5.3 and 5.4 are dedicated to statistical analysis of mean and stochastic fluid motions, respectively. Lastly, Section 5.5 investigated changes to the structure of the turbulent flow field.

5.1.1 The need for new texture topologies

One proven strategy to improve the drag-reducing performance of streamwise-aligned SHS textures is to reduce their percentage solid-fraction, $\Psi_s$, (Martell et al., 2009, 2010). The increased spacing between the neighbouring micro-feature permits a larger portion of the fluid to slip over the shear-free interfaces, which increases the average slip velocity, $\langle U_s \rangle_{xz}$, which decreases average wall shear stress, $\langle \tau_w \rangle_{xz}$. There is, however, a lower limit on the solid-fraction, and beyond this limit the air-water will catastrophically rupture causing the SHS texture to transition from the Cassie state to the Wenzel state (see figure 5.1). The question then becomes: For a fixed solid-fraction, does there exist a surface topologies that can outperform its streamwise-aligned counterpart?

Previous work, both experimental and computational in nature, has demonstrated that the orientation of the micro-features relative to the primary flow can have a profound effect on the performance of SHS textures — under both laminar and turbulent flow conditions. For example,
Chapter 5. The effect of spatial modulation on texture performance

Figure 5.1: Schematic representations of a SHS texture operating in (a) the Cassie state, where the solid fraction is sufficiently large to support the air-water interfaces and (b) the Wenzel state, where the solid fraction becomes too small and the air-water interfaces collapse, submerging the micro-features in liquid.

Ou et al. (2007) used microparticle image velocimetry (µ-PIV) to demonstrate that micro-ridges yawed relative to a primary laminar flow produced a helical secondary flow that enhanced mixing, relative to the reference no-slip laminar flow. In a later study, Martell et al. (2010) used DNS to show that any turbulent skin-friction reductions gained by their streamwise-aligned “30µm-30µm” micro-ridges were completely lost by aligning the exact same surface topology perpendicular to the primary flow direction.

To date, most studies have focused on SHS textures that are characterised by periodic arrays of streamwise-aligned micro-ridge and micro-post topologies. The number of new surface topologies, whose drag-reducing capabilities are yet to be explored, is practically unlimited. Rather than testing new topologies at random, it seems logical to try to somehow incorporate the benefits of another proven flow control strategy into the design of a new SHS texture. A study of the available literature suggests that inducing some sort of streamwise-variation of spanwise motion may be potentially beneficial.

5.1.2 The potential benefits of spanwise motion

Spanwise wall-based flow control strategies are concerned with a class of forcing methods designed to favourably alter turbulent flow by the introduction of external action in the lateral (z) direction. For example, the benefits of a temporally harmonic spanwise shear layer in the near-wall region of turbulent flow are well-documented. Jung et al. (1992) were the first to study the effects of a spanwise-oscillating wall on a fully developed turbulent channel flow at $Re_{τ} = 200$ by using DNS. The law that defined this forcing method can be written as

$$w = A_z \sin(\omega t)$$

(5.1)
where \( t \) is time, and \( A_z \) and \( \omega = 2\pi/T \) are the oscillation amplitude and frequency, respectively. By conducting a series of appropriately designed numerical experiments, Jung et al. (1992) studied periods of oscillation, \( T^+ = Tu^2_2/\nu \), ranging from \( 25 < T^+ < 500 \) and noted that oscillations at \( T^+ = 100 \) produced the most effective suppression of turbulence which yielded a drag reduction of 40%, relative to the reference no-slip flow. Much subsequent work, both experimental and computational, which documents the benefits of oscillatory spanwise wall motion, has been undertaken since the DNS of Jung et al. (1992). In a related study, Akhavan et al. (1993) discussed the influence that the spanwise forcing had on the near-wall turbulent structures, highlighting that the suppression of wall-normal momentum transfer disrupted the regeneration process of streaks and vortices. The disruption of the regeneration process was later supported by the experimental results of Laadhari et al. (1994). Further experimental studies that confirmed the drag-reducing capabilities of the wall-oscillation technique were conducted by Di Cicca et al. (2002), Choi (2002) and Ricco (2004), the first characterised the flow field by means of particle-image velocimetry, the second used a hydrogen-bubble technique and the third used hot-wire anemometry, with all three studies reporting a drag reduction of at least 40%. The investigation of Quadrio & Ricco (2004) provided a more complete assessment of the oscillating wall as a drag reduction technique, where the energetic benefit due to the skin-friction savings were compared to the energetic cost of moving the walls against the viscous fluid. In their DNS, Quadrio & Ricco (2004) defined the net percentage power saving, \( \%P_{\text{net}} \), due to spanwise wall-oscillation as,

\[
\%P_{\text{net}} = \%P_{\text{sav}} + \%P_{\text{req}}. \tag{5.2}
\]

where \( \%P_{\text{sav}} \) and \( \%P_{\text{req}} \) represents the percentage power saved due to reductions in skin-friction and the (negative) power required to move the plate against the viscosity of the fluid, respectively. The main objective of the simulations conducted by Quadrio & Ricco (2004) was to obtain a comprehensive database of the drag-reduction as a function of the parameters which defined the oscillatory motion of the wall, namely the maximum wall velocity, \( A_z \), the oscillation period, \( T \), and the corresponding maximum wall displacement, \( D_z = A_z T/\pi \). The database included results from thirty-seven separate DNS which spanned a wide range of oscillation amplitudes \( (0 < A_z^+ < 27) \) and oscillation periods \( (0 < T^+ < 750) \). Out of the thirty-seven cases reported, only ten returned a positive net power savings which, according to equation 5.2, infers that \( \%P_{\text{sav}} > \%|P_{\text{req}}| \). The largest percentage power saving due to reductions in wall shear stress reported by Quadrio & Ricco (2004) was \( \%P_{\text{sav}} = 44.7 \) for actuation parameters \((A_z, T, D_z)^+ = (27, 100, 859)\) but the power required to oscillate the plate against the viscous flow was found to be \( \%P_{\text{req}} = -407.1 \) — approximately an order of magnitude more than \( \%P_{\text{sav}} \). The highest net power saving was observed with actuation parameters \((A_z, T, D_z)^+ = (4.5, 125, 179)\) and returned a net power saving of \( \%P_{\text{net}} = 7.3 \). In addition, the power budget calculations of
Quadrio & Ricco (2004) disregarded mechanical losses, which would be unavoidable in a real-word situation, and would further increase power requirements. The advantages and disadvantages of the oscillating-wall technique are therefore obvious. On the one hand, the oscillating-wall technique offers significant skin-friction savings that motivate its practical implementation, and thanks to its open-loop nature, this active control strategy requires no additional sensors or actuators in order to operate. On the other hand, the unsteady nature of the oscillating-wall technique requires moving parts whose own power requirements often cancel out power savings made by reductions in skin-friction.

The problems associated with the temporal dependence of the wall-oscillation technique motivates the conversion of the time-dependent wall forcing into its stationary counterpart to a spatial sinusoidal distribution of spanwise velocity over a particular streamwise wavelength, $\lambda_x$. To this end, Quadrio et al. (2009) generalised the temporal wall-oscillation technique (equation 5.1), to account for both temporal and spatial variations of spanwise velocity. Quadrio et al. (2009) defined the spatio-temporal control law as

$$w = A_z \sin (k_x x - \omega t) \tag{5.3}$$

where $k_x = 2\pi/\lambda_x$ and $\omega = 2\pi/T$ are the streamwise wavenumber and frequency, respectively. For the spatio-temporal forcing law (equation 5.3 figure 5.2a), the wave of spanwise velocity can move backwards or forwards in the streamwise direction, with a phase speed $c = \omega/k_x$. The temporal forcing law (equation 5.1 and figure 5.2b), can be recovered from equation 5.3 by setting $k_x = 0$, which represents a wave travelling at an infinite phase speed, whereas a steady-state control law can be isolated by setting $\omega = 0$, which represents a stationary wave of spanwise velocity that varies sinusoidally with the streamwise direction. Upon setting $\omega = 0$, equation 5.3 simplifies to

$$w = A_z \sin \left( \frac{2\pi}{\lambda_x} x \right) \tag{5.4}$$

which is the spatial analogue of the temporal forcing law described in equation 5.1 and is shown schematically in figure 5.2c. The space-time conversion of the oscillating-wall technique was studied by Viotti et al. (2009), who exploited the convective nature of turbulence in the near-wall region to study the drag-reducing capabilities of the steady forcing law expressed by equation 5.4. Some years prior to simulations of Viotti et al. (2009), Kim & Hussain (1993) conducted a DNS of fully developed turbulent channel flow at $Re_x = 180$ which demonstrated that although at the wall the mean streamwise velocity profile tends towards zero, the convection velocity of the turbulence fluctuations, $U^{+}_c$, became independent of wall-normal distance below $y^+ \approx 15$ and remained at a constant value of $U^{+}_c \approx 10$. By choosing $U^{+}_c$ as a velocity scale, Viotti et al. (2009)
hypothesised that the spatial forcing law would modify the turbulent flow in a similar manner to the oscillating wall and that the optimal period $T_{\text{opt}}$ for temporal forcing law (equation 5.1) could be converted into an optimal streamwise wavelength $\lambda_{x,\text{opt}}$ through the relation

$$\lambda_{x,\text{opt}}^+ = U_x^+ T_{\text{opt}}^+. \quad (5.5)$$

Upon application of equation 5.5, and in analogy to the oscillating wall technique that yields maximum drag reduction at an oscillation period $T^+$ across the interval $100 < T_{\text{opt}}^+ < 125$, the DNS results of Viotti et al. (2009) quantitatively demonstrated that the optimal wavelength for their streamwise oscillating spanwise velocity component was in the range $1000 < \lambda_{x,\text{opt}}^+ < 1250$ - verifying their initial hypothesis. In total, Viotti et al. (2009) considered forty-one unique $(A_z, \lambda_x)$ parameter combinations, where a maximum drag reduction of 52% was observed for $(A_z, \lambda_x)^+ = (20, 1250)$, which is in excess of the maximum drag reduction of 44.7% reported by Quadrio & Ricco (2004) for the unsteady forcing. Although the wall remains stationary for the steady forcing law expressed by equation 5.4, an external power source is still required to induce the spanwise motion. To assess the net power savings of the steady forcing law, Viotti et al. (2009) carried out similar power budget calculations to those originally devised by Quadrio & Ricco (2004). Compared to the time-dependent forcing, which returned a maximum net power
saving of $%P_{net} = 7.3$ with forcing parameters $(A_z, T)^+ = (4.5, 125)$, the results of Viotti et al. (2009) demonstrated that the steady state forcing returned a maximum net power saving of $%P_{net} = 23$ with forcing parameters $(A_z, \lambda_x)^+ = (6, 1250)$. In addition, Viotti et al. (2009) reported that positive $P_{net}$ existed for larger amplitudes ($A^+ = 12$) for the steady forcing law, whereas for equivalent amplitudes, the unsteady forcing law would typically return an energetic loss in excess of 25%. Despite its superior performance over the unsteady forcing law, the suitability of the steady forcing law as practical drag reduction device is queried in the concluding remarks made by Viotti et al. (2009), who go on to say that the “design of a steady control law is one important step toward the realization of passive drag-reducing device”. This observation motivates the search for a new texture topology, whose spatial structure is somehow related to the steady forcing law, which, for a fixed solid fraction $\Psi_s$, could potentially outperform its regular counterpart by passively mimicking the flow physics associated with active flow control strategies.

5.1.3 Spatially varying textures

The spatially varying textures can be generated from the streamwise-aligned textures by varying the spanwise position of the microfeatures in the streamwise direction. The variation of the SHS textures in the streamwise direction can be characterised by a streamwise wavelength $\lambda_x$ and a spanwise amplitude $A_z$. The spatially varying shape can be described by the following equation

$$\zeta(x) = A_z \sin\left(\frac{2\pi}{\lambda_x}x\right),$$

(5.6)

where $\zeta$ is the deviation of the the spanwise coordinate of the texture surface from the corresponding coordinate of the straight riblet. The maximum waveslope, $\beta$, of the spatially varying riblets was computed by taking the derivative of equation 5.6 with respect to $x$ to obtain,

$$\beta = 2\pi \left(\frac{A_z}{\lambda_x}\right).$$

(5.7)

The modulation of the streamwise-aligned ridges, expressed by equation 5.6, is shown graphically in figure 5.3.

In total, thirty-six separate texture topologies were investigated. To clarify the effects of modulation on turbulent skin-friction, the percentage solid fraction was held constant at $\Psi_s = 50\%$ for all of the textures considered in this parametric study. This ensures that the area corresponding to the no-slip micro-features remains constant for all of the meandering textures. There are both upper and lower limits on which portion of the $A_z-\lambda_x$ parameter space that can be considered. The lower limit on $A_z$ is determined by the spanwise grid resolution ($\Delta z^+ \approx 1$), which sets the minimum oscillation amplitude to be $A_{z,\text{min}}^+ = \Delta z^+$, whereas the upper limit of
Figure 5.3: Schematic representation of texture modulation including the streamwise wavelength, \( \lambda_x \), the spanwise oscillation amplitude, \( A_z \), and the waveslope angle, \( \beta \).

is determined by the sum of feature spacing and feature width, \((d + g)^+ = 67.5\) due to phase-averaging procedures. Owing to the periodic boundary conditions employed in the homogeneous directions, an integer number of texture wavelengths must be contained within the domain length, \( L_x = k_x \lambda_x \), where \( k_x \) is the streamwise wavenumber. The streamwise wavenumber, \( k_x \), must be sufficiently large to ensure that the streamwise extent of the computational domain is long enough to accommodate the largest coherent structures \( (\Delta x^+ \approx 10^3) \) to avoid any recycling effects due to periodicity, and the grid spacing must be fine enough to resolve the smallest turbulent scales.

\[
\begin{array}{cccccccc}
 k_x & \lambda_x/\delta & L_x/\delta & \lambda^+_x & L^+_x & \Delta x^+ & \Delta z^+ & \Delta y^+_{min} & \Delta y^+_{max} \\
 8 & 1.5 & 12.0 & 270 & 2160 & 8.44 & 1.06 & 0.20 & 4.10 \\
 4 & 3.0 & 12.0 & 540 & 2160 & 8.44 & 1.06 & 0.20 & 4.10 \\
 4 & 4.5 & 13.5 & 810 & 2430 & 8.44 & 1.06 & 0.20 & 4.10 \\
 2 & 6.0 & 12.0 & 1080 & 2160 & 8.44 & 1.06 & 0.20 & 4.10 \\
 2 & 7.5 & 15.0 & 1350 & 2700 & 8.44 & 1.06 & 0.20 & 4.10 \\
 2 & 9.0 & 18.0 & 1620 & 3240 & 8.44 & 1.06 & 0.20 & 4.10 \\
\end{array}
\]

Table 5.1: Spatial discretization details for DNS studies of turbulent channel flow at \( Re_\tau \approx 180 \) for spatially varying SHS textures.

Here, the streamwise resolution is held constant at \( \Delta x^+ = 8.44 \) for all texture topologies and the streamwise length of the computational box size was systematically varied for different streamwise wavelengths \( \lambda_x \). A total of six spanwise amplitudes and six streamwise wavelengths were considered, which can be written in vector form as:

\[
\begin{align*}
\mathbf{A}_z^+ &= [8.4, 16.9, 25.3, 33.7, 42.2, 50.6]^+ \\
\mathbf{\lambda}_x^+ &= [270, 540, 810, 1080, 1350, 1620]^+
\end{align*}
\]

Additional spatial discretization details are provided in table 5.1 and details of each of the
spatially varying textures can be found in table 5.2.

Table 5.2: Texture topologies

<table>
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<tr>
<th>$T_{ij}$</th>
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5.2 Performance study

5.2.1 Introduction

The purpose of this section is to assess the performance of spatially varying textures. For each texture the percentage drag reductions relative to both the reference no-slip and streamwise-aligned texture cases are computed. After general performance trends are identified, the texture with optimal parameters \((A_z, \lambda_x)_{opt}\) is highlighted and becomes the focus for the remainder of this chapter.

5.2.2 Integral performance parameters

For a constant mass flow rate, changes in turbulent skin-friction can be inferred by monitoring the mean streamwise pressure gradient, \(-\frac{\partial \langle P \rangle}{\partial x}\). Relative to the reference no-slip case, the drag reduction offered by a particular texture topology, \(DR\), was computed using

\[
DR = \left( \frac{-\frac{\partial \langle P \rangle}{\partial x} - \left(\frac{\partial \langle P \rangle}{\partial x}\right)_{ref}}{-\frac{\partial \langle P \rangle}{\partial x}} \right) \times 100 \quad (5.9)
\]

where the subscript \(\text{ref}\) denotes the value corresponding to the reference no-slip case.

The drag-reducing performance of the spatially varying SHS textures relative to the streamwise-aligned textures, \(\Delta DR\), was computed using

\[
\Delta DR = \left( \frac{DR - DR_{T00}}{DR_{T00}} \right) \times 100 \quad (5.10)
\]

where \(DR_{T00}\) corresponds to the percentage drag reduction offered by the streamwise-aligned ridges.

The percentage change in mean streamwise slip velocity, relative to streamwise-aligned SHS texture, is computed as

\[
\Delta \langle U_s \rangle_{xz} = \left( \frac{\langle U_s \rangle_{xz} - \langle U_s \rangle_{T00}^{xz}}{\langle U_s \rangle_{T00}^{xz}} \right) \times 100. \quad (5.11)
\]

5.2.3 Drag reduction

Drag reduction as a function of spanwise oscillation amplitude is plotted in figure 5.4a. In general, texture performance worsens with increasing spanwise amplitude. Drag reduction as a function of streamwise wavelength is plotted in figure 5.4b. In general, texture performance improves with increasing streamwise wavelength. However, it is expected that as \(\lambda_x \to \infty\), meandering
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Textures will offer no performance benefit, relative to the streamwise-aligned texture. For the parameter space investigated here, the optimal texture was found to have modulation parameters $(A_z, \lambda_x)_{\text{opt}} = (16.88, 1080)$.

Figure 5.4: Drag reduction, $DR$, plotted as a function of spanwise oscillation amplitude, $A_z^+$, with $\lambda_x^+ = \{270 (■), 540 (□), 810 (■), 1080 (●), 1350 (○), 1620 (•)\}$ (a) and (b) streamwise wavelength, $\lambda_x^+$, with $A_z^+ = \{8.438 (■), 16.88 (□), 25.31 (■), 33.75 (●), 42.19 (○), 50.62 (•)\}$ s. In both figures, data points that fall within the shaded region have superior drag reduction performance, relative to the streamwise-aligned ridges.

Figure 5.5: Integral performance parameters plotted as a function of maximum turning angle, $\beta$, including (a) percentage drag reduction relative to case $T_0$, $\Delta DR$, and (b) percentage change of mean streamwise slip velocity, $\Delta \langle U_s \rangle^{xz}$. Groups of data points correspond to $\lambda_x^+ = \{270 (■), 540 (□), 810 (■), 1080 (●), 1350 (○), 1620 (•)\}$. In both figures, data points that fall within the shaded region have superior drag reduction performance, relative to the streamwise-aligned ridges.
The scattered DR data shown in figure 5.4 collapses when it is plotted as a function of the waveslope, $\beta$, as shown in figure 5.5. Relative to the streamwise-aligned texture, drag reduction degrades beyond a critical turning angle $\beta_{\text{crit}} \approx 17^\circ$. For $\beta > \beta_{\text{crit}}$ the texture performance appears to decrease log-linearly. For all the meandering textures considered here, the mean slip velocity is reduced, compared to the streamwise-aligned textures.

The results from this parametric study show that, relative to the streamwise-aligned textures, the worst-performing meandering textures increase the drag by over 40%. The best-performing meandering texture decreases drag by a further 18%, relative to the streamwise-aligned case, and now becomes the focus of the remainder of this chapter. Herein, for the purpose of brevity, the streamwise-aligned texture is referred as “texture $T_{00}$” and the optimal meandering texture will be referred to as “texture $T_{11}$”.

5.3 Mean flow statistics

This section presents the results for the mean fluid motions, which are divided into three separate categories. First, results for the primary flow are reported, which is simply the fluid motion aligned with the primary (streamwise) flow direction. Second, results for the secondary flow are reported, which is periodic fluid motion that occurs in the $y - \phi_z$ plane perpendicular to the primary flow. Lastly, results for the slip flow are reported, which are characterised by fluid motions that occur on the SHS texture itself. After the components of mean flow field have been characterised in detail, a quantitative comparison between the spanwise periodic fluid motions above texture $T_{11}$ and spanwise motions induced by the oscillating wall technique is presented in order to determine whether the active flow physics of the latter flow control strategy have been effectively mimicked by the present passive SHS textures.

5.3.1 Primary flow

The governing equation for the mean streamwise momentum was derived in phase-averaged form and can be written as

$$
\langle u \rangle \frac{\partial \langle u \rangle}{\partial x} + \langle v \rangle \frac{\partial \langle u \rangle}{\partial y} + \langle w \rangle \frac{\partial \langle u \rangle}{\partial z} = \frac{d \langle p \rangle}{dx} + \frac{1}{Re_b} \left( \frac{\partial^2 \langle u \rangle}{\partial x^2} + \frac{\partial^2 \langle u \rangle}{\partial y^2} + \frac{\partial^2 \langle u \rangle}{\partial z^2} \right) - \left( \frac{\partial \langle R_{11} \rangle}{\partial x} + \frac{\partial \langle R_{12} \rangle}{\partial y} + \frac{\partial \langle R_{13} \rangle}{\partial z} \right). \quad (5.12)
$$
which, when averaged across all phases, yields

$$0 = - \frac{d\langle P \rangle_{xz}}{dx} + \frac{1}{Re_b} \frac{\partial^2 \langle U \rangle_{xz}}{\partial y^2} - \frac{\partial \langle R_{12} \rangle_{xz}}{\partial y}. \quad (5.13)$$

Comparison of equation 5.13 and equation 5.12 shows that several additional terms are retained as a result of the phase-averaging procedures. A total of seven additional terms are retained when the governing equations are derived in phase-averaged form, each of which can be classified as a kinematic process. The first term on the left-hand side of equation 5.12 represents the convection of the primary flow by itself, which only exists for streamwise-meandering textures. The next two terms on the left-hand side of equation 5.12 represents the convection of primary flow the steady periodic velocity components which occur on the transverse plane - perpendicular to the primary flow direction. The first term on the right-hand side of equation 5.12 represents streamwise gradient of the the pressure, which includes the mean streamwise pressure gradient required to hold the mass flow rate constant. The second term on the right-hand side of equation 5.12 represents the diffusion of primary flow in all three spatial directions. The last term on the right-hand-side of equation 5.12 represents the forces exerted against the primary flow by the stochastic flow, through the gradients of turbulent stresses.

Figure 5.6a shows the mean streamwise velocity profiles for texture $T_{11}$, texture $T_{00}$ and the reference no-slip case. Each velocity profile in figure 5.6 was averaged across all phases and then normalised by its own friction velocity, $u_\tau$, so that structural changes to the turbulent channel profile could be compared in a law of the wall format. By comparing the profiles of $\langle U^+ \rangle_{xz}$

![Figure 5.6: Mean streamwise velocity profiles for the reference case (---), texture $T_{00}$ (—) and texture $T_{11}$ (---). Quantities are scaled by a) local friction velocity $u_\tau$ and b) bulk velocity $U_b$. Law of the wall quantities (---) including viscous sublayer, $\langle U \rangle_{xz} = \langle U_s \rangle_{xz} + y^+$, and log-law region, $\langle U^+ \rangle_{xz} = A \log (y^+) + B$ are included for comparison.](image-url)
for cases $T_{11}$ and $T_{00}$ in figure 5.6a two conclusions can be drawn. First, relative to texture $T_{00}$, the average slip velocity for texture $T_{11}$ is smaller. Second, the log-law region for texture $T_{11}$ is elevated further from the wall and is higher, relative to texture $T_{00}$. This is a significant result because previous studies of turbulent flow over SHS textures using DNS (Martell et al., 2009, 2010) have suggested that reductions in wall shear-stress increase with increasing average slip velocity $\langle U_x \rangle$. The current results contradict these performance trends since there is an elevation / (decrease) in the log-law region / (mean streamwise slip velocity) for texture $T_{11}$, relative to texture $T_{00}$.

### 5.3.2 Secondary flow

The phase-averaged streamwise momentum is shown in equation 5.12 and it is braced second and third terms on the left-hand side of this equation that are the focus of this subsection. Physically these terms represents the convection of primary flow by periodic motions in the plane perpendicular to the primary flow direction.

Profiles of the wall-normal periodic velocity component averaged across no-slip and free-slip phases (computed using equations 4.11a and 4.11b, respectively) are shown in figure 5.7a. Averaged across the no-slip phases, the signature of $\langle \tilde{v} \rangle^{ns}$ for texture $T_{11}$ and $T_{00}$ resembles an upwash motion ($\langle \tilde{v} \rangle^{ns} > 0$). Relative to texture $T_{00}$, the peak wall-normal velocity for $T_{11}$ is suppressed for all wall-normal positions and its peak value is reduced by approximately 37%. Averaged across the free-slip phases, the signature of $\langle \tilde{v} \rangle^{fs}$ resembles a downwash motion ($\langle \tilde{v} \rangle^{fs} < 0$) and is the mirror image of the $\langle \tilde{v} \rangle^{ns}$ profile about $\langle \tilde{v} \rangle^{xz} = 0$. Again, relative to texture $T_{00}$, the suction behaviour is suppressed for texture $T_{11}$.

Profiles of the root-mean-squared spanwise periodic velocity component averaged across all phases, all no-slip and all free-slip phases (computed using equations 4.12, 4.13 and 4.14, respectively) are shown in figure 5.7b and shows that, in general, the transverse motions are amplified for texture $T_{11}$, relative to texture $T_{00}$, particularly in the near-wall region where $y^+ < 10$.

To characterise the secondary flow further, vectors of $(\tilde{v}, \tilde{w})$ are plotted on the $y-\phi_z$ plane at eight separate streamwise stations, with the first and last four shown in figures 5.8 and 5.9, respectively. The rationale of plotting the secondary flow patterns at discrete streamwise stations is to reveal any variations in the streamwise direction of the spanwise velocity component that the previous root-mean-squared values were unable to detect. Inspection of the vectors of $(\tilde{v}, \tilde{w})$ in figures 5.8 and 5.9, suggest a “tilting” of the secondary flow, compared to case $T_{00}$ (see figure 4.9), coherent secondary motions are only visible at particular streamwise stations. From an inspection of the secondary flow patterns for the optimal meandering texture $T_{11}$, it can be concluded that, compared to the streamwise-aligned texture $T_{00}$, the strength of the secondary spanwise motions have been significantly increased (see figures 5.7a-b) and that its orientation...
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Figure 5.7: Profiles of secondary motions for texture $T_{00}$ (—,—) and texture $T_{11}$ (—,—,—). Wall-normal secondary motions are shown in (a) including the average across all phases (—,—,—), the average across no-slip phases (—,—,—) and the average across free-slip phases (—,—,—). The root-mean-square profiles of spanwise secondary motion are shown in (b) where the same averaged quantities are plotted and the colour scheme used in (a) is employed.

varies periodically with the streamwise direction (figures 5.8-5.9). The streamwise variation of periodic spanwise motion suggests that a “Stokes-type” motion is induced by texture $T_{11}$. 
Figure 5.8: Secondary currents for texture T11 on the transverse plane including vectors of \((\tilde{v}, \tilde{w})\) superimposed on contours of \(\langle U \rangle / U_b\). To emphasise its streamwise dependence, the secondary flow is visualised at four separate streamwise stations (see top panel). For clarity, only every second \((\tilde{v}, \tilde{w})\) vector is plotted.
Figure 5.9: Secondary currents for texture $T_{11}$ on the transverse plane including vectors of $(\tilde{v}, \tilde{w})$ superimposed on contours of $\langle U \rangle / U_b$. To emphasise its streamwise dependence, the secondary flow is visualised at four separate streamwise stations (see top panel). For clarity, only every second $(\tilde{v}, \tilde{w})$ vector is plotted.
For the streamwise-aligned texture $T_{00}$, the secondary flow was classified as the second kind, since streamwise vorticity was produced as a result of anisotropy of the direct turbulent stresses in both the wall-normal and spanwise directions (see the term labelled $P_4$ in equation 4.16). The production and sustenance of secondary motions due to stochastic fluid motions for case $T_{00}$ was verified numerically by simulating laminar flow in an identical geometry for which no secondary motion was observed (i.e. $\tilde{u}_i = 0$). There are, however, particular texture topologies for which secondary flows have been observed under laminar flow conditions. By conducting a series of suitably designed $\mu$-PIV experiments, Ou et al. (2007) visualised a helical secondary flow above a micro-ridge texture that was aligned at an oblique angle relative to the primary flow direction. The secondary flow reported by Ou et al. (2007) was not formally classified using Prandtl’s categorisation, but it follows that it must be of the first kind, since under laminar flow conditions equation 4.16 simplifies to,

\[
\begin{align*}
\langle U \rangle \frac{\partial \tilde{\Omega}_x}{\partial x} + \tilde{v} \frac{\partial \tilde{\Omega}_x}{\partial y} + \tilde{w} \frac{\partial \tilde{\Omega}_x}{\partial z} &= \tilde{\Omega}_x \frac{\partial \langle U \rangle}{\partial x} + \frac{1}{Re_b} \nabla^2 \tilde{\Omega}_x \\
&+ \tilde{\Omega}_y \frac{\partial \langle U \rangle}{\partial y} + \tilde{\Omega}_z \frac{\partial \langle U \rangle}{\partial z}
\end{align*}
\]

which shows that mean streamwise vorticity can only be produced by skewing of the mean flow, the term labelled $P_1$. Therefore, secondary flows are expected to occur above spatially varying textures under laminar flow conditions. To verify this, laminar flow at a bulk Reynolds number of $Re_b = 2800$ was simulated over texture $T_{11}$ until a steady state was reached. The secondary motions above texture $T_{11}$ under laminar flow conditions are shown in figure 5.10, where vectors of $(\tilde{v}, \tilde{w})$ are superimposed on contours of mean streamwise vorticity, $\tilde{\Omega}_x$. In order to emphasise its streamwise dependence, the secondary flow is plotted at two separate streamwise stations. At the first streamwise station (figure 5.10a) localised patches of $\tilde{\Omega}_x > 0$ occur periodically across the texture, and the $(\tilde{v}, \tilde{w})$ vectors reveal vortices that rotate in a clockwise manner in the same region. The opposite behaviour is observed at the second, downstream station (figure 5.10b) where patches of $\tilde{\Omega}_x < 0$ and vortices that rotate in an anticlockwise manner occur in the same region.

The secondary flows corresponding to both texture $T_{00}$ and $T_{11}$ can be summarised as follows. For laminar flow conditions no secondary motions can be supported in the presence of texture $T_{00}$, whereas a secondary flow of the first kind can be supported in the presence of texture $T_{11}$ due to skewing of the primary flow. For turbulent flow conditions secondary motions of the second kind can be supported in the presence of texture $T_{00}$, whereas a mixed secondary flow comprised of first and second type motions is supported in the presence of texture $T_{11}$. 

\[
\begin{align*}
\langle U \rangle \frac{\partial \tilde{\Omega}_x}{\partial x} + \tilde{v} \frac{\partial \tilde{\Omega}_x}{\partial y} + \tilde{w} \frac{\partial \tilde{\Omega}_x}{\partial z} &= \tilde{\Omega}_x \frac{\partial \langle U \rangle}{\partial x} + \frac{1}{Re_b} \nabla^2 \tilde{\Omega}_x \\
&+ \tilde{\Omega}_y \frac{\partial \langle U \rangle}{\partial y} + \tilde{\Omega}_z \frac{\partial \langle U \rangle}{\partial z}
\end{align*}
\]
Figure 5.10: Secondary flow on the transverse plane including vectors of (\(\tilde{v}, \tilde{w}\)) with contours of mean streamwise vorticity \(\langle \Omega_x \rangle\) for texture T_{11} under laminar flow conditions. In order to emphasise its streamwise dependence, the secondary flow is visualised at two streamwise stations (see top panel). For clarity, only every second vector is plotted.

### 5.3.3 Slip flow

To complete the analysis of the mean flow, attention is now turned to the behaviour of the fluid on the SHS texture itself. At the wall, the governing equations for the phase-averaged streamwise momentum can be written as

\[
\left( \begin{array}{c} \langle u \rangle \\ \frac{\partial \langle u \rangle}{\partial x} \bigg|_w \\ \frac{\partial \langle u \rangle}{\partial z} \bigg|_w \end{array} \right) + \left( \begin{array}{c} \frac{\partial \langle v \rangle}{\partial x} \bigg|_w \\ \frac{\partial \langle v \rangle}{\partial z} \bigg|_w \\ \frac{\partial \langle w \rangle}{\partial x} \bigg|_w \\ \frac{\partial \langle w \rangle}{\partial z} \bigg|_w \end{array} \right) = -\frac{d \langle p \rangle}{dx} \bigg|_w + \frac{1}{\text{Re}_b} \left( \frac{\partial^2 \langle u \rangle}{\partial x^2} + \frac{\partial^2 \langle u \rangle}{\partial y^2} + \frac{\partial^2 \langle u \rangle}{\partial z^2} \right) \bigg|_w + \left( \frac{\partial \langle R_{11} \rangle}{\partial x} + \frac{\partial \langle R_{13} \rangle}{\partial z} \right) \bigg|_w, \tag{5.15}
\]

which shows that primary, secondary and turbulent fluid motions all contribute towards streamwise slip velocities on the SHS textures. The first term of the left-hand side of equation 5.15
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represents the convection of primary flow by itself. The second term on the left-hand side of equation 5.15 represents the spanwise convection of primary flow. The second and third terms on the right-hand side of equation 5.15 represent the viscous diffusion of primary flow and the effects of turbulent forces, respectively.

Similarly, the governing equation for the phase-averaged spanwise slip velocity can be written as

\[
\langle w \rangle \frac{\partial \tilde{w}}{\partial x} \bigg|_w + \tilde{w} \frac{\partial \langle w \rangle}{\partial z} \bigg|_w = -\frac{d\tilde{p}}{dz} \bigg|_w + \frac{1}{Re_b} \left( \frac{\partial^2 \tilde{w}}{\partial x^2} + \frac{\partial^2 \tilde{w}}{\partial y^2} + \frac{\partial^2 \tilde{w}}{\partial z^2} \right) \bigg|_w - \left( \frac{\partial \langle R_{13} \rangle}{\partial x} + \frac{\partial \langle R_{33} \rangle}{\partial z} \right) \bigg|_w, \tag{5.16}
\]

which shows that primary, secondary and turbulent fluid motions also all contribute towards spanwise slip velocities on the SHS textures. The first term of the left-hand side of equation 5.16 represents the convection of spanwise secondary flow by the primary flow. The second term on the left-hand side of equation 5.16 represents the spanwise convection of the spanwise secondary flow by itself. The second and third terms on the right-hand side of equation 5.16 represent the viscous diffusion of spanwise secondary flow and the effects of turbulent forces, respectively.

Contours of the streamwise and spanwise slip velocity components for textures $T_{11}$ and $T_{00}$ on the phase-averaged $\phi_x-\phi_z$ plane are plotted in figure 5.11. Figure 5.11a shows both components of the slip velocity over the meandering texture, whereas figure 5.11b shows identical contours on the “aligned” $\hat{\phi}_x-\phi_z$ plane, where the phase-alignment in the streamwise direction (denoted by $\hat{\cdot}$) is achieved by aligning points that share a common spanwise phase in the streamwise direction. The advantage of the realigned contours shown in figure 5.11b is that they can be more easily compared to the results for the streamwise-aligned texture, which are provided in figure 5.11c. Upon comparison of figure 5.11b to figure 5.11c, the effects of spatial modulation on both components of slip velocity become clear. Relative to the streamwise aligned texture, the streamwise slip velocity is generally reduced across all phases for the meandering texture. The opposite trend is observed for the spanwise slip velocity, where it is enhanced for the meandering texture, relative to the streamwise-aligned texture. Both these observations support previous results concerning the primary and secondary fluid motions. The mean streamwise velocity profile demonstrated that average slip velocity across phases was lower for the meandering texture (see figure 5.6a), and the magnitude of root-mean-squared spanwise periodic motions at the wall increased for the meandering texture (see figure 5.7b). The streamwise variation of spanwise slip velocity is clearly visible in the right-hand portion of figure 5.11b, which suggests that the flow
physics of the active flow control law originally devised by Viotti et al. (2009),

\[ w = A_z \sin \left( \frac{2\pi}{\lambda_x} x \right), \]

has been successively mimicked using the passive SHS textures (see figure 5.2c).

By integrating the contours in figure 5.11b and figure 5.11c in the streamwise direction, the slip velocity profiles as a function of spanwise phase were obtained, and are shown in figure 5.12. The profiles of streamwise slip velocity (figure 5.12a) confirm that it is reduced for the meandering texture, relative to the streamwise-aligned texture. The averaged peak value of streamwise slip velocity is reduced by approximately 13%. Due to the streamwise variation of spanwise slip velocity, its streamwise-averaged profile suggests that spanwise slip velocity is weaker. The reduction of spanwise slip velocity may be true in the streamwise-averaged sense, but the contours of spanwise slip velocity (figures 5.11a-b) clearly show that its local levels typically exceed the values corresponding to the streamwise-aligned texture (figure 5.11c).
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Figure 5.11: Contours of streamwise slip velocity, $\langle u_s \rangle$, and spanwise slip velocity, $\tilde{w}_s$ with superimposed vectors of $(\langle u_s \rangle, \tilde{w}_s)$ for texture T11 on the (a) $\phi_x$-$\phi_z$ and (b) resorted $\hat{\phi}_x$-$\phi_z$ plane and (c) texture T00 on the $\phi_x$-$\phi_z$ plane. Both components of slip velocity have been normalised by bulk velocity, $U_b$. For clarity, only every sixteenth and second vector in the streamwise and spanwise directions, respectively, are plotted.
Figure 5.12: Streamwise-averaged slip velocity profiles for texture $T_{00}$ (---) and $T_{11}$ (−−−) including (a) streamwise and (b) spanwise slip velocity profiles.
5.3.4 A comparison to spanwise wall-oscillation techniques

The purpose of this subsection is to compare the strength of spanwise secondary motions induced by the meandering textures, to the strength of a typical harmonic shear-layer that varies in the spanwise direction that induces large-scale drag reductions. If their magnitudes are comparable, the superior performance of texture $T_{11}$, relative to texture $T_{00}$, could potentially be credited to a “Stokes-type” mechanism.

Jung et al. (1992) studied the spanwise-oscillating wall technique numerically using DNS, where the wall of a fully developed turbulent channel flow at $Re_\tau = 200$ was subjected to an alternating harmonic motion in the spanwise direction. At appropriate forcing conditions, the flow over the oscillating wall can be regarded as a superposition of two simpler flows, i.e. a fully developed turbulent channel flow in the streamwise direction and an oscillating laminar boundary-layer in the transverse direction.

Under laminar flow conditions, the governing equations for the spanwise momentum can be written as,

$$\frac{\partial w}{\partial t} = \nu \frac{\partial^2 w}{\partial y^2} \tag{5.17}$$

The boundary conditions for equation 5.17 are that the fluid is initially at rest, the velocity is bounded at infinity, and the no-slip condition applies at the plate surface:

$$w(y, t = 0) = 0 \quad (5.18a)$$
$$w(y \to \infty, t) < \infty \quad (5.18b)$$
$$w(y = 0, t) = W_m \sin \left( \frac{2\pi t}{T} \right) \quad (5.18c)$$

where $W_m$ is the maximum wall-oscillation amplitude. The governing equation for Stokes second problem (equation 5.17) is linear, and its solution can be expressed as a sum of a transient and a steady-state solution. It is the latter steady-state solution which is of interest here, since the statistical quantities discussed throughout this chapter are integrated in time. Upon examination of the boundary conditions (equations 5.18a-5.18c), the maximum spanwise velocity $W_m$ and oscillation period $T$ are the relevant velocity and time scales, respectively, which can be used to define the new non-dimensional variables,

$$W_{Stks} = \frac{w}{W_m} \quad (5.19a)$$
$$T_{Stks} = \frac{2\pi t}{T} \quad (5.19b)$$

The viscosity $\nu$ shown in the governing equation for Stokes second problem (equation 5.17) can
be eliminated by forming a similarity variable,

\[ Y_{Stks} = \frac{y}{\sqrt{(\nu T_{osc}/2\pi)}} \]  

(5.20)

where \( Y_{Stks} \) is a non-dimensional wall-normal distance. Using equations 5.19a-5.20, the governing equation for Stokes second problem (equation 5.17) can be written in the following non-dimensional form,

\[
\left[ \frac{W_m}{T/2\pi} \right] \frac{\partial W_{Stks}}{\partial T_{Stks}} - \nu \left[ \left( \frac{W_m}{\nu T/2\pi} \right) \frac{\partial^2 W_{Stks}}{\partial Y_{Stks}^2} \right] = 0
\]

\[
\frac{\partial W_{Stks}}{\partial T_{Stks}} - \frac{\partial^2 W_{Stks}}{\partial Y_{Stks}^2} = 0
\]

(5.21)

with non-dimensional boundary conditions,

\[
W_{Stks} (Y_{Stks}, T_{Stks} = 0) = 0,
\]

(5.22a)

\[
W_{Stks} (Y_{Stks} \rightarrow \infty, T_{Stks}) = 0,
\]

(5.22b)

\[
W_{Stks} (Y_{Stks} = 0, T_{Stks}) = \sin (T_{Stks}).
\]

(5.22c)

Using the above boundary conditions (equations 5.22a-5.22c) the solution to equation 5.21 can be written as

\[
W_{Stks} = \exp \left(-\frac{Y_{Stks}}{\sqrt{2}}\right) \sin \left(T_{Stks} - \frac{Y_{Stks}}{\sqrt{2}}\right)
\]

(5.23)

which is the solution to the second Stokes problem. Equation 5.23 represents a harmonic span-wise motion that is damped in the wall-normal \( y \)-direction and its solution is provided in figure 5.13. The solution to Stokes second problem (equation 5.23) can be recast in term of outer variables by using the channel half-height \( \delta \) and bulk velocity \( U_b \) to obtain,

\[
w(y, t; W_m, T) = W_m \exp \left(-\sqrt{\frac{Re_b \pi}{T}} y\right) \sin \left(\frac{2\pi}{T} t - \sqrt{\frac{Re_b \pi}{T}} y\right),
\]

(5.24)

then, by using the relations \( T^+ = Tu^2/\nu \), \( y^+ = u_r y/\nu \) and \( W^+ = W/u_r \), equation 5.24 can be recast in wall units as

\[
w^+(y^+, t^+; W_m^+, T^+) = W_m^+ \exp \left(-y^+ \sqrt{\frac{\pi}{T^+}}\right) \sin \left(\frac{2\pi}{T^+} t^+ - y^+ \sqrt{\frac{\pi}{T^+}}\right),
\]

(5.25)

which shows that the thickness of the Stokes boundary layer is determined by the oscillation period, \( T^+ \). Figure 5.14 compares profiles of \( w^+ \) for two different oscillation periods, \( T^+ = 100 \) (figure 5.14a) and \( T^+ = 400 \) (figure 5.14b), and demonstrates that Stokes boundary layer is
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Figure 5.13: Stokes boundary layer solution. The wall oscillates according to $W_{\text{Stks}} = \sin T_{\text{Stks}}$, where $T_{\text{Stks}} \in [0, \pi]$. From left to right, the time-period of consecutive profiles are $T_{\text{Stks}} \in \left\{ \frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{3\pi}{4}, \frac{7\pi}{8}, \pi \right\}$.

thicker for the latter, large oscillation period.

Following the procedures first outlined by Choi et al. (2002) (and later by Quadrio & Ricco (2004)), who investigated the conditions required for spanwise wall-oscillation to induce a skin-friction saving in turbulent flows, a parameter can be derived that scales linearly with drag reduction. This parameter depends on the period of oscillation and the maximum spanwise wall velocity and can be derived using physical arguments associated to the interaction between the near-wall turbulence and spanwise laminar Stokes layer. The resultant analytical expression can be used to determine the smallest values of the oscillation parameters which assure a non-zero drag reduction effect. Starting with equation 5.25, the spanwise acceleration of the Stokes boundary layer can be obtained by differentiating equation 5.25 with respect to $t^+$ to obtain,

$$\frac{\partial w^+}{\partial t^+} = \frac{2\pi W_m^+}{T^+} \exp \left( -y^+ \sqrt{\pi/T^+} \right) \cos \left( \frac{2\pi}{T^+} t^+ - y^+ \sqrt{\frac{\pi}{T^+}} \right),$$

(5.26)

with the corresponding maximum spanwise acceleration, $a_m^+$, at a wall-normal distance $y^+ = l_m^+$ being

$$a_m^+ = \frac{2\pi W_m^+}{T^+} \exp \left( -l_m^+ \sqrt{\pi/T^+} \right).$$

(5.27)

A suitable length scale, which represents the penetration depth of the Stokes boundary layer into the turbulent flow, is identified by the requirement that the maximum oscillating velocity at a distance $y^+ = l_m^+$ from the wall has a magnitude higher than a threshold velocity $W_{th}^+$, that represents a value typical of the spanwise turbulent fluctuations. An expression for $l_m^+$ can be
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Figure 5.14: Stokes boundary layer for (a) optimal forcing parameters \((T, W_m)^+_\text{opt} = (100, 12)\) and (b) non-optimal forcing parameters \((T, W_m)^+ = (400, 12)\) at \(Re_\tau = 180\), demonstrating the increased thickness of the Stokes boundary layer with increased oscillation period.

obtained by setting \(w^+ = W_{th}^+\) in equation 5.25 to obtain,

\[
\frac{W_{th}^+}{W_m^+} = \exp\left(-l_m^+ \sqrt{\pi/T^+}\right), \\
\ln\left(\frac{W_{th}^+}{W_m^+}\right) = (-l_m^+ \sqrt{\pi/T^+}), \\
-l_m^+ = \frac{T^+}{\pi} \ln\left(\frac{W_m^+}{W_{th}^+}\right), \\
l_m^+ = \frac{T^+}{\pi} \ln\left(\frac{W_{th}^+}{W_m^+}\right). 
\] (5.28)

The two quantities \(a_m^+\) (equation 5.27) and \(l_a^+\) (equation 5.28) can be used to form a third group,

\[
S^+ = \frac{a_m^+ l_a^+}{W_m^+}, \\
= \frac{2\pi}{T^+} \exp\left(-l_a^+ \sqrt{\pi/T^+}\right) \frac{T^+}{\pi} \ln\left(\frac{W_m^+}{W_{th}^+}\right), \\
= \sqrt{\frac{4\pi^2}{T^+}} \frac{T^+}{\pi} \exp\left(-l_a^+ \sqrt{\pi/T^+}\right) \ln\left(\frac{W_m^+}{W_{th}^+}\right), \\
= 2 \sqrt{\frac{\pi}{T^+}} \exp\left(-l_a^+ \sqrt{\pi/T^+}\right) \ln\left(\frac{W_m^+}{W_{th}^+}\right). 
\] (5.29)

where \(S^+\) is referred to as the “drag reduction scaling parameter”.

The idea that a finite intensity of spanwise forcing is needed to affect any beneficial change on the level of turbulent skin friction is contained within the definition of \(S^+\), where a threshold velocity \(W_{th}^+\) is included (see equation 5.29). By appropriately selecting the threshold value of spanwise velocity \(W_{th}^+\), one could estimate the minimum spanwise velocity \(W_{m, \text{min}}^+\) required to
affect any beneficial changes of the turbulent flow. Quadrio & Ricco (2004) chose $W_{th}^+$ to be the maximum root-mean-square value of the turbulent spanwise velocity fluctuations of the reference no-slip turbulent flow, which, according to the results of Kim et al. (1986), is $w_{rms}^+ \approx 1.1$. In their previous DNS studies, Quadrio & Ricco (2004) reported a skin-friction reduction of by less than 3% for their lowest spanwise amplitude of $W_m^+ = 1.5$.

Of particular interest to the current work are the findings of Viotti et al. (2009), who examined the drag-reducing capacity of the spatial analogue of the temporal Stokes layer (TSL) (shown in figures 5.13 and 5.14), herein referred to as the spatial Stokes layer (SSL). In this study, a steady wall motion (described by equation 5.4 and shown in figure 5.2c) was considered and power savings in excess of 40% were reported for optimal forcing parameters $(A_z, \lambda_x)_{opt}^+ = (12, 1000)$. However, as the spanwise oscillation was reduced to $A_z^+ = 1$, a power saving of less than 2% was reported for the same optimal streamwise wavelength, $\lambda_x^+_{opt} = 1000$.

Motivated by the findings of Viotti et al. (2009), Chernyshenko (2013) devised a new idea for turbulent skin-friction reduction by modifying the shape of a flat solid wall to be of the form

$$ h(x, z) = H \sin \left( \frac{2\pi}{\lambda_x^+} x^+ + \frac{2\pi}{\lambda_z^+} z^+ \right) $$ (5.30)

where $H$ is the wall-normal height required to generate spanwise velocity of the order of 2 walls units. The idea was to create different spanwise pressure gradients on the upstream and downstream sides of the smooth waves, in order to laterally displace fluid in analogy to the spanwise wall velocity shown in figure 5.2c. Chernyshenko (2013) reported a power saving of approximately 2.4% across for streamwise wavelength $\lambda_x^+ = 1520$.

The question then becomes: are the spanwise motions induced by the texture $T_{11}$ sufficiently strong to generate drag-reducing spanwise motions? If the answer is “yes”, then the superior drag reduction of texture $T_{11}$, relative to texture $T_{00}$, could potentially be credited to a “Stokes-type” motion. If the answer is “no”, then the superior drag reduction of texture $T_{11}$, relative to texture $T_{00}$, can not be credited to a “Stokes-type” motion and must due to some other mechanism. The maximum value of spanwise velocity at the wall was found to be $\hat{w}_{s,max}^+ = \pm 0.21$, which is an order of magnitude less than the minimum spanwise velocity considered by Quadrio & Ricco (2004), Viotti et al. (2009) and Chernyshenko (2013). In addition, the values of $\{\hat{w}^{xz}_r, \hat{w}^{ns}_r, \hat{w}^{sf}_r\}$ shown in figure 5.7b show that the root-mean-squared magnitude of spanwise motions are insufficient to affect any large-scale benefits on the turbulent flow.

**5.4 Turbulence statistics**

In this section focus is drawn to an extended version of the FIK identity (Fukagata et al., 2002) which quantifies the contributions made by primary, secondary and turbulent fluid motions.
towards the levels of skin-friction. For an account of turbulence statistics (TKE and Reynolds stresses) the reader is referred to appendix A.

5.4.1 Skin-friction identity

The FIK identity was introduced in the previous chapter (see section 4.4), where details of its derivation and evaluation for both the reference no-slip case and streamwise-aligned texture $T_{00}$ were reported. Here, the objective is to repeat this analysis for the optimal meandering texture $T_{11}$, and to detail the effects of the spatial modulation of the phase-averaged skin-friction coefficient, relative to the streamwise-aligned texture.

The accuracy of the phase-averaged FIK identity (equation 4.63) in estimating the streamwise-averaged skin-friction coefficient, $\langle C_f \rangle^x$, is shown in figure 5.15. Excellent agreement across all spanwise phases is achieved - for both turbulent and laminar conditions.

![Figure 5.15: Streamwise-averaged skin-friction coefficient as a function of spanwise no-slip phases $\phi_z/2\pi \in [0.5, 1.0]$, which was computed from the phase-averaged wall shear stress $\langle C_f \rangle^x$ (---) and from the FIK identity $\langle C_f \rangle^x_{FIK}$ (□) for a) fully-developed turbulent flow and b) laminar flow conditions. Uncontrolled skin-friction coefficient $\langle C_f \rangle^x_{ns}$ (--) from Kim et al. (1986) is included in both plots.](image)

Each of the terms on the right-hand side of no-slip averaged FIK identity (equation 4.65) were evaluated numerically and are plotted in figure 5.16a. The numerical values of each term of the averaged FIK identity (equation 4.65) are provided in table 5.3, and are reported to four significant figures. Upon inspection of figure 5.16a, the most severe performance penalty inflicted as a result of meandering the SHS texture is the third term on the right hand side of no-slip-averaged FIK identity (equation 4.65), $\langle C_f \rangle^x_{ns}$. The term $\langle C_f \rangle^x_{ns}$ can be interpreted as the effect of convection of primary flow by itself and represents the deceleration of streamwise momentum above the no-slip micro-features. The only other apparent performance penalty due incurred as a consequence of meandering the SHS texture is manifest in the fourth term on the
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Contribution to $C_f^{ns}$

Figure 5.16: Contribution of each term of the averaged FIK identity (equation 4.65) towards turbulent skin-friction coefficient averaged across all no-slip phases, $\langle C_f \rangle^{ns}$, for reference no-slip case (■), $T_{00}$ (□) and $T_{11}$ (■) for (a) fully developed turbulent flow and (b) laminar flow. Note that each term has been multiplied by a factor of $1 \times 10^3$.

To complete the analysis the skin-friction coefficient, the distribution of each term of the phase-averaged FIK identity across the $\phi_x-\phi_z$ plane is considered. Each term of the phase-averaged FIK identity is plotted in figure 5.12, where a comparison of case $T_{00}$ (left-hand-side
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Table 5.3: Contribution of each term of the averaged FIK identity (equation 4.65) towards turbulent skin-friction coefficient averaged across all no-slip phases, $\langle C_f \rangle_{ns}$, for cases $T_{ref}$, $T_{00}$ and $T_{11}$ under both fully developed turbulent and laminar flow conditions, where the latter is denoted by superscript “lam”. Note that each term has been multiplied by a factor of $10^3$

<table>
<thead>
<tr>
<th>Case</th>
<th>$\langle C_f \rangle_{lam}^{\text{ns}}$ (%)</th>
<th>$\langle C_f \rangle_{R_{12}}^{\text{ns}}$ (%)</th>
<th>$\langle C_f \rangle_{(u)}^{\text{ns}}$ (%)</th>
<th>$\langle C_f \rangle_{\bar{v}}^{\text{ns}}$ (%)</th>
<th>$\langle C_f \rangle_{\bar{w}}^{\text{ns}}$ (%)</th>
<th>$\langle C_f \rangle_{R_{13}}^{\text{ns}}$ (%)</th>
<th>$\langle C_f \rangle_{\text{zdff}}^{\text{ns}}$ (%)</th>
<th>$\sum$ (%)</th>
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<td>$T_{ref}$</td>
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<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
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<tr>
<td>$T_{00}$</td>
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<td>37.29</td>
<td>0.000</td>
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<td>5.793</td>
<td>33.06</td>
<td>28.37</td>
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<td>0.000</td>
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</tr>
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<td>0.000</td>
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<td>0.000</td>
<td>34.94</td>
<td>99.98</td>
</tr>
</tbody>
</table>

Table 5.4: Percentage contribution of each term of the averaged FIK identity (equation 4.65) towards turbulent skin-friction coefficient averaged across all no-slip phases, $\langle C_f \rangle_{ns}^{\text{ns}}$, for cases $T_{ref}$, $T_{00}$ and $T_{11}$ under both fully developed turbulent and laminar flow conditions, where the latter is denoted by superscript “lam”.

<table>
<thead>
<tr>
<th>Case</th>
<th>$\langle C_f \rangle_{lam}^{\text{ns}}$ (%)</th>
<th>$\langle C_f \rangle_{R_{12}}^{\text{ns}}$ (%)</th>
<th>$\langle C_f \rangle_{(u)}^{\text{ns}}$ (%)</th>
<th>$\langle C_f \rangle_{\bar{v}}^{\text{ns}}$ (%)</th>
<th>$\langle C_f \rangle_{\bar{w}}^{\text{ns}}$ (%)</th>
<th>$\langle C_f \rangle_{R_{13}}^{\text{ns}}$ (%)</th>
<th>$\langle C_f \rangle_{\text{zdff}}^{\text{ns}}$ (%)</th>
<th>$\sum$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{ref}$</td>
<td>26.57</td>
<td>73.41</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>99.98</td>
</tr>
<tr>
<td>$T_{00}$</td>
<td>17.16</td>
<td>37.29</td>
<td>0.000</td>
<td>-21.67</td>
<td>5.793</td>
<td>33.06</td>
<td>28.37</td>
<td>100.0</td>
</tr>
<tr>
<td>$T_{11}$</td>
<td>18.30</td>
<td>36.13</td>
<td>24.87</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>100.0</td>
</tr>
<tr>
<td>$T_{lam}_{ref}$</td>
<td>100.0</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>100.0</td>
</tr>
<tr>
<td>$T_{lam}_{00}$</td>
<td>55.79</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>99.99</td>
</tr>
<tr>
<td>$T_{lam}_{11}$</td>
<td>55.78</td>
<td>0.000</td>
<td>18.09</td>
<td>-9.322</td>
<td>0.494</td>
<td>0.000</td>
<td>34.94</td>
<td>99.98</td>
</tr>
</tbody>
</table>

carpet plot) and $T_{11}$ (right-hand-side carpet plot) is made. Contours of the phase-averaged skin-friction coefficient, $\langle C_f \rangle_{\text{FIK}}$, are shown in figure 5.17a and reveal regions of high wall-shear that are confined to the edges of the micro-features. These localised regions of increased skin-friction are symmetric about the mid no-slip phase, $\phi_z = \pi/2$, and are homogeneous in the streamwise direction for case $T_{00}$, whereas, for case $T_{11}$ vary with streamwise position and are asymmetric in the spanwise direction. The preceding analysis of the averaged FIK identity identified the term $\langle C_f \rangle_{(u)}^{\text{ns}}$ as the most severe performance penalty incurred as a result of meandering the texture, which can be interpreted as the streamwise convection of primary flow by itself, and accounted for almost a third of the total skin-friction, $\langle C_f \rangle_{\text{ns}}^{\text{ns}}$. Upon comparing the phase-averaged skin-friction coefficient, $\langle C_f \rangle$, shown in figure 5.17a and the term $\langle C_f \rangle_{(u)}^{\text{ns}}$, shown in figure 5.17d, a strong spatial correlation is observed between the local patches of increased skin-friction and local deceleration of the primary flow along the edges of the no-slip band. The localised patches of decelerating flow occur as a result of the high-momentum “slipping” fluid above free-slip phases suddenly encountering a no-slip surface, which pins the fluid to the wall, and therefore exerts a
force against which acts in the opposite direction to the bulk fluid motion and tends to decelerate the flow. The term $\langle C_f \rangle^\text{ns}_{\langle u \rangle}$ is computed as the weighted integral of the product between the primary flow, $\langle u \rangle$, and its streamwise gradient, $\partial \langle u \rangle / \partial x$. Since $\langle u \rangle > 0$ for all $y$, indicating no reversal of mean flow, the product of $\langle u \rangle$ and $\partial \langle u \rangle / \partial x$ is negative in regions of streamwise deceleration, which means that the term $\langle C_f \rangle^\text{ns}_{\langle u \rangle}$ will be negative. Multiplication by the additional negative on the right-hand side of the phase-averaged FIK identity (see equation 4.63) means that the term $\langle C_f \rangle^\text{ns}_{\langle u \rangle}$ makes a positive contribution towards the skin-friction coefficient, which explains why this particular term incurs a significant performance penalty. As a result of the decelerating primary flow along the edges of the micro-features, gradients of the secondary flow must also exist in this region to satisfy the continuity of the steady fluid motions. The preceding analysis the averaged FIK identity for case T11 demonstrated that the terms associated with wall-normal secondary motions, $\langle C_f \rangle_{\tilde{v}}$, and spanwise secondary motions, $\langle C_f \rangle_{\tilde{w}}$, accounted for approximately 25% and 3% of the total skin-friction coefficient, respectively, and hence attention is paid to the former, more significant term. The term $\langle C_f \rangle_{\tilde{v}}$ is computed as the weighted integral effect of the product between wall-normal secondary motion, $\tilde{v}$, and the gradient of the primary flow in the wall-normal direction, $\partial \langle u \rangle / \partial y$. Since throughout this work $\partial \langle u \rangle / \partial y > 0$ for all $y$, indicating that there is no primary flow separation, any sign change of the term $\langle C_f \rangle_{\tilde{v}}$ must be due to changes in $\tilde{v}$. Comparing the terms $\langle C_f \rangle^\text{ns}_{\langle u \rangle}$, shown in figure 5.17d, and $\langle C_f \rangle_{\tilde{v}}$, shown in figure 5.17e, it is clear that the deceleration of primary flow at the edges of the micro-features is complemented by a positive upward motion of wall-normal secondary motion, which, as previously mentioned, is required in order to satisfy the continuity of mean fluid motions. The product of the wall-normal secondary motion, $\tilde{v}$, and the primary shear, $\partial \langle u \rangle / \partial y$, means that the term $\langle C_f \rangle_{\tilde{v}}$ is itself positive. Multiplication by the negative sign on the right-hand-side of the phase-averaged FIK identity (equation 4.63) means that the term $\langle C_f \rangle_{\tilde{v}}$ makes a negative contribution towards the skin-friction coefficient, which explains why this particular term is beneficial to performance. On the other hand, as a result of meandering the texture, the performance benefit of the term $\langle C_f \rangle_{\tilde{v}}$ is reduced by approximately 16%, relative the to the streamwise-aligned case. This loss of performance can be explained by the local streamwise deceleration of the primary flow along the edges of meandering micro-features, which disrupts the secondary motions close to the texture.
Figure 5.17: Contribution of each term of the phase-averaged FIK identity (equation 4.63) towards skin-friction coefficient $\langle C_f \rangle$ including (a) skin-friction coefficient $\langle C_f \rangle$; (b) laminar contribution $\langle C_{f,\text{lam}} \rangle$; (c) Reynolds shear stress $\langle C_{f,R_{12}} \rangle$; (d) streamwise convection of primary flow by itself $\langle C_{f,u} \rangle$; (e) wall-normal convection of primary flow by wall-normal secondary flow $\langle C_{f,v} \rangle$; (f) spanwise convection of primary flow by spanwise secondary flow $\langle C_{f,w} \rangle$; (g) cross flow Reynolds shear stress $\langle C_{f,R_{13}} \rangle$; and (h) viscous spanwise
5.5 Turbulence structure

The purpose of this section is to investigate how the fundamental structure of near-wall turbulence is altered by the presence of SHS textures and to relate any changes to previously reported statistical measures. The effect of SHS textures on Reynolds-shear-stress-producing events are investigated using the method of quadrant analysis. Then the effect of SHS textures on vortical structures in the near-wall region are studied by using the $\lambda_2$ criterion. The effect of SHS textures on the anisotropy of turbulence is considered by computing anisotropy invariant maps (AIM), which provide a supplementary view of the impact of SHS textures on the structure of near-wall turbulence — for details the reader is referred to appendix B.

5.5.1 Reynolds shear stress: sweeps and ejections

In the previous section the various contributions towards the skin-friction coefficient made by primary, secondary and turbulent fluid motions were clarified by a numerical evaluation of the phase-averaged FIK identity (see equation 4.63). Upon inspection of the terms on the right-hand-side of the FIK identity it is clear that turbulent fluid motions incur two performance penalties. The first penalty can be credited to the interaction of Reynolds shear stress with the wall, as shown in equation 4.66b. The second performance penalty can be credited to the crossflow Reynolds shear-stress at the edges of the micro-ridges, as shown in equation 4.66b.

The evaluation of the FIK identity demonstrated that texture $T_{11}$ was more effective at suppressing the contribution towards skin-friction from both $\langle C_f^{\text{ns}} \rangle_{R_{12}}$ and $\langle C_f^{\text{ns}} \rangle_{R_{13}}$, relative to the texture $T_{00}$. The remainder of this subsection is therefore motivated by gaining a further understanding of the fundamental turbulent fluid motions responsible for the production of Reynolds shear stress and how texture $T_{11}$ suppresses them more effectively than texture $T_{00}$. To this end, the method of quadrant analysis is adopted which is used to decompose both the Reynolds shear stress and the crossflow Reynolds shear stress in order to distinguish flow events that contribute towards their production.

It has been known now for several decades that the near-wall region of turbulent flow appears to exhibit various coherent motions (Cantwell, 1981). Visual investigations of the wall region of turbulent flow conducted by Kline et al. (1967) and Corino & Brodkey (1969), for example, highlighted the existence of surprisingly well-organised spatially and temporally dependent motions within the viscous sublayer which lead to the formation of low-speed streaks. In addition, Kline et al. (1967) were among the first to speculate that the series of events whereby streaks first slowly lift-up away from the wall, undergo a violent oscillation, before finally ‘bursting’ into smaller-scale incoherent motions played a central role of the production of new turbulence as well as the transfer of momentum towards the wall below. The ‘bursting’ process was subsequently studied by Kim et al. (1971), who employed hot-wire techniques to demonstrate that although...
intermittent, the bursts occurred at a well-defined mean frequency - an instability process that was credited to sustenance of the fluctuations by an energy exchange from the mean flow. In an attempt to gain deeper physical insight into the production of Reynolds shear stress in the near wall region of fully developed turbulent channel flow, Willmarth & Lu (1972) applied the method of quadrant analysis to their hot-film measurements. Willmarth & Lu (1972) classified the instantaneous product signal \( u'v' \) in to four categories according to the sign of its components \( u' \) and \( v' \), which are detailed in table 5.5. Out of the four categories, Willmarth & Lu (1972) classified the second and fourth quadrant events (referred to as ejections and sweeps, respectively) accounted for approximately 70% of Reynolds shear stress in the near-wall region at \( y^+ \approx 15 \). In a later study, Wallace & Brodkey (1977) complemented the classic quadrant analysis technique by computing the probability density functions (PDF) of the \( u' \) and \( v' \) fluctuating components on the \( u'-v' \) plane at nine positions across a fully developed turbulent channel. Wallace & Brodkey (1977) demonstrated that the most probable pairs of velocities did not coincide with the pairs of velocities that give the largest contribution to Reynolds stress, and thus concluded that the principal source of Reynolds stress were high-amplitude events that were observed less frequently. Although the quadrant analysis techniques introduced by Willmarth & Lu (1972) confirmed and quantified the central role that streamwise streaks play in the structure and production of turbulence in the near-wall region, the details of streak formation remained unclear. In attempt to clarify the origins of streaks, Blackwelder & Eckelmann (1979) used hot-film sensors and flush-mounted wall elements to compute the streamwise and spanwise velocity components, and their gradients normal to the wall, to study the vortex structures associated with the bursting phenomenon. By using quadrant probability analysis techniques, Blackwelder & Eckelmann (1979) indicated that pairs of counter-rotating streamwise vortices occur frequently in the near wall region of a turbulent shear flow which “pumped” fluid away from the wall to form a low-speed streak between them. Unfortunately, however, this theory of streak formation did not explain the origin of the vortices, but did highlight the importance of the interaction of coherent structures in the near-wall region of a turbulent shear flow.

At sufficiently high Reynolds numbers, the boundary-layer region in bounded turbulent shear flows is characterised by very small spatial scales and considerable difficulties are encountered.
in attempting to study its structure experimentally. For example, Blackwelder & Eckelmann (1979) noted that in order to accurately capture the spatial structure of near-wall turbulence, intricate arrays of flush-mounted sensors would be required, whereas unobtrusive methods, such as laser-Doppler anemometer, were susceptible to under-resolving the small-scales of turbulence. The advent of sufficient computational power in the 1980s circumvented experimental restrictions and paved the way for fully-resolved direct simulations of near-wall turbulence (Kim et al., 1986; Spalart, 1988) that effectively verified previous experimental results (Kline et al., 1967; Corino & Brodkey, 1969; Kim et al., 1971; Willmarth & Lu, 1972; Wallace & Brodkey, 1977; Blackwelder & Eckelmann, 1979), as well as granting access to fully-resolved, three-dimensional time-dependent velocity and pressure fields. Since the seminal DNS studies of Kim et al. (1986) and Spalart (1988) the near-wall region of turbulent flow has been studied intensively in order to better understand the origins of coherent structures (Schoppa & Hussain, 2002; Chernyshenko & Baig, 2005), their mutual interaction (Jimenez & Hussain, 1994; Kravchenko et al., 1993; Jeong et al., 1997) and their regeneration (Hamilton et al., 1995; Jimenez & Pinelli, 1999).

Here, the goal is to explain how SHS textures reduce the contribution of Reynolds shear stress to turbulent skin-friction through the analysis of coherent motions in the near-wall region, whose integral effect are manifest in the statistical description of turbulence. The method of quadrant analysis divides the Reynolds shear stress into four categories according to the signs of $u'$ and $v'$. The first quadrant $\langle Q \rangle_1 (u' > 0 \text{ and } v' > 0)$ contains outward motion of high-speed fluid; the second quadrant $\langle Q \rangle_2 (u' < 0 \text{ and } v' > 0)$ contains outward motion of low-speed fluid referred to as ejections; the third quadrant $\langle Q \rangle_3 (u' < 0 \text{ and } v' < 0)$ contains inward motion of low-speed fluid; and the fourth quadrant $\langle Q \rangle_4 (u' > 0 \text{ and } v' < 0)$ contains inward motion of high-speed fluid referred to as sweeps. Here, quadrants $\langle Q \rangle_1$ and $\langle Q \rangle_3$ contribute to the positive Reynolds shear stress (which can be credited to the negative production of TKE), whereas quadrants $\langle Q \rangle_2$ and $\langle Q \rangle_4$ contribute to the negative Reynolds shear stress (which can be credited to the positive production of TKE). From the perspective of skin-friction savings, the sweep events are of particular interest since the impingement of high-momentum fluid on to a no-slip surface can produce localised patches of high shear-rates which, ultimately, lead to high levels of skin-friction. In fact, the presence of coherent structures and their interaction with the wall is one plausible explanation to why turbulent flows experience higher skin friction than their laminar counterparts. Interrupting the interactions between coherent structures and the wall therefore forms a general strategy to achieve skin-friction savings, and various active and passive flow control methods have been applied in order to achieve this reduction.
Quadrant analysis relies on the Reynolds shear-stress, $\langle R_{12} \rangle$, being decomposed as

$$\langle R_{12} \rangle = \sum_{n=1}^{4} \langle Q_{12} \rangle_n$$

(5.31)

where $\langle Q_{12} \rangle_n$ represents the $n^{th}$ phase-averaged quadrant, each of which was computed as,

$$\langle Q_{12} \rangle_1 = \langle u'(x,t) v'(x,t) I_u^+ I_v^+ \rangle$$

(5.32a)

$$\langle Q_{12} \rangle_2 = \langle u'(x,t) v'(x,t) I_u^- I_v^+ \rangle$$

(5.32b)

$$\langle Q_{12} \rangle_3 = \langle u'(x,t) v'(x,t) I_u^- I_v^- \rangle$$

(5.32c)

$$\langle Q_{12} \rangle_4 = \langle u'(x,t) v'(x,t) I_u^+ I_v^- \rangle$$

(5.32d)

where $I_u^+$ and $I_u^-$ are indicator functions for the positive and negative streamwise velocity fluctuations, respectively, and can be defined as,

$$I_u^+(x,t) = \begin{cases} 
1 & \text{if } u'(x,t) > 0 \\
0 & \text{otherwise}
\end{cases}$$

$$I_v^-(x,t) = \begin{cases} 
1 & \text{if } v'(x,t) < 0 \\
0 & \text{otherwise}
\end{cases}$$

and where $I_v^+$ and $I_v^-$ are indicator functions for the positive and negative wall-normal fluctuations, respectively, and can be defined as,

$$I_v^+(x,t) = \begin{cases} 
1 & \text{if } v'(x,t) > 0 \\
0 & \text{otherwise}
\end{cases}$$

$$I_v^-(x,t) = \begin{cases} 
1 & \text{if } v'(x,t) < 0 \\
0 & \text{otherwise}
\end{cases}$$

Since the primary interest of this section is to evaluate the contributions of coherent structures towards turbulent skin-friction, each quadrant was decomposed as,

$$\langle Q_{12} \rangle_{zz}^{ns} = \frac{1}{2} \left( \langle Q_{12} \rangle_{ns}^{+} + \langle Q_{12} \rangle_{ns}^{-} \right)$$

(5.33)

which isolates the contribution from each quadrant towards the Reynolds shear stress above the no-slip micro-features. Figure 5.18 shows the contributions towards Reynolds shear stress from each quadrant plotted as a function of the wall-normal coordinate. In the presence of SHS textures, the contributions from $\langle Q_2 \rangle^{ns}$ and $\langle Q_4 \rangle^{ns}$ towards the overall Reynolds shear stress are
significantly reduced. In addition, the results from the quadrant analysis show that texture T_{11} is more effective at suppressing sweep and ejection events than texture T_{00}. The Reynolds shear stress from \langle Q_1 \rangle^{ns} shows a slight increase for texture T_{00} in the region where y^+ < 15 which contributes towards the negative production of Reynolds shear stress. Considering that the

\begin{equation}
\langle R_{13} \rangle = \sum_{n=1}^{4} \langle Q_{n}^{13} \rangle
\end{equation}

SHS textures modify the positive Reynolds-shear-stress-producing events and do not appreciably change the first- and third-quadrant events, it may be deduced that the SHS textures are most effective in modifying the organised motion associated with the streamwise vortices.

For texture T_{00}, the contribution of crossflow Reynolds shear stress, \langle R_{13} \rangle, accounts for over 30% of the total skin-friction, whereas, for texture T_{11}, accounts for less than 10% (see table 5.4). In order to better understand why the contribution of Reynolds shear stress towards skin-friction is reduced significantly as a result of the meandering texture, relative to the streamwise-aligned texture, the changes in crossflow Reynolds shear stress were investigated using a quadrant analysis approach. The classic quadrant analysis technique, first introduced by Willmarth & Lu (1972), decomposed the Reynolds shear stress, \langle R_{12} \rangle, into four separate categories according to product sign of u' and v'. Here, identical procedures are applied to the cross Reynolds shear stress, \langle R_{13} \rangle, which was decomposed into four separate categories according to the product sign of u' and w'. The crossflow Reynolds shear-stress, \langle R_{13} \rangle, was decomposed as

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure5_18.png}
\caption{Contributions to the Reynolds shear stress from each quadrant including reference no-slip case (\textdagger), case T_{00} (\textdaggerdbl) and case T_{11} (\textdoublespace). All profiles have been normalised by the reference no-slip friction velocity.}
\end{figure}
where \( \langle Q_{13} \rangle_n \) represents the \( n^{th} \) phase-averaged quadrant, each of which was computed as,

\[
\langle Q_{13} \rangle_1 = \langle u'(x,t) w'(x,t) I_u^+ I_w^+ \rangle \\
\langle Q_{13} \rangle_2 = \langle u'(x,t) w'(x,t) I_u^- I_w^+ \rangle \\
\langle Q_{13} \rangle_3 = \langle u'(x,t) w'(x,t) I_u^- I_w^- \rangle \\
\langle Q_{13} \rangle_4 = \langle u'(x,t) w'(x,t) I_u^+ I_w^- \rangle
\]

where \( I_u^+ \) and \( I_w^+ \) are indicator functions for the positive and negative spanwise velocity fluctuations, respectively, and can be defined as,

\[
I_u^+ (x,t) = \begin{cases} 
1 & \text{if } u'(x,t) > 0 \\
0 & \text{otherwise}
\end{cases}
\]

\[
I_w^+ (x,t) = \begin{cases} 
1 & \text{if } w'(x,t) > 0 \\
0 & \text{otherwise}
\end{cases}
\]

The method of quadrant analysis divides the crossflow Reynolds shear stress into four categories according to the signs of \( u' \) and \( w' \). The first quadrant \( \langle Q_{13} \rangle_1 (u' > 0 \text{ and } w' > 0) \) contains positive lateral motion of high-speed fluid; the second quadrant \( \langle Q_{13} \rangle_2 (u' < 0 \text{ and } w' > 0) \) contains positive lateral motion of low-speed fluid; the third quadrant \( \langle Q_{13} \rangle_3 (u' < 0 \text{ and } w' < 0) \) contains negative lateral motion of low-speed fluid; and the fourth quadrant \( \langle Q_{13} \rangle_4 (u' > 0 \text{ and } w' < 0) \) contains positive lateral motion of high-speed fluid. Here, quadrants \( \langle Q_{13} \rangle_1 \) and \( \langle Q_{13} \rangle_3 \) contribute to the positive Reynolds shear stress (which can be credited to the negative production of TKE), whereas quadrants \( \langle Q_{13} \rangle_2 \) and \( \langle Q_{13} \rangle_4 \) contribute to the negative Reynolds shear stress (which can be credited to the positive production of TKE).

A quadrant analysis of the phase-averaged crossflow Reynolds shear stress at the edge \((z = d)\) of the SHS texture was performed in order to better understand how texture T\(_{11}\) suppresses its contribution towards skin-friction, compared to texture T\(_{00}\). Figure 5.19 shows the contributions towards crossflow Reynolds shear stress form each quadrant plotted as a function of the wall-normal coordinate. For the reference no-slip case, the summation of the contributions across the four quadrants equals zero, whereas, for both textured cases, it is non-zero, which is a consequence of the additional gradients present in both the primary and secondary mean flow that act to correlate the \( u' \) and \( w' \) fluctuations. Comparing the results of texture T\(_{11}\) relative to texture T\(_{00}\), it is clear that large reductions occur in both the second and fourth quadrants. In fact, the peak value in the fourth quadrant drops by almost 50\% and its position moves further from the SHS texture itself. Therefore, the events that produce crossflow Reynolds shear stress near the edges of the texture are significantly suppressed and explain why its contribution.
towards skin-friction is reduced.

5.5.2 Vortical structures

In the presence of SHS textures, an analysis of the terms in the phase-averaged FIK identity demonstrated that the turbulent contributions towards the skin-friction coefficient were suppressed, with the meandering texture T_{11} outperforming its streamwise-aligned counterpart. Thus it can be inferred that turbulent motions are suppressed in regions in response to the SHS textures. Moreover, it is necessary to understand the effect of the turbulence structures on the skin-friction. In this subsection, the effect of the SHS textures on the vortical structures is discussed.

Over the past four decades, several types of coherent structures have been proposed in order to explain experimentally observed flow phenomena. A colourful description of coherent structures in turbulent boundary layers was provided by Fielder (1988), who wrote: “Thus, when studying the literature on boundary layers, one is soon lost in a zoo of structures, e.g. horseshoe- and hairpin-eddies, pancake- and surfboard-eddies, typical eddies, vortex rings, mushroom-eddies, arrowhead eddies, etc...” Robinson (1991) detailed a taxonomy of turbulent structures which included low-speed streaks, sweeps, ejections and vortical structures - which are the focus of this chapter. Before going any further, it is important to make a distinction between vortices and vorticity. Vorticity is mathematically defined at a point \( \omega = \nabla \times \mathbf{u} \), whereas a vortex requires a
description that incorporates coherence over an area or a volume in space. There is often a weak correlation between regions of strong vorticity and actual vortices, with this discrepancy being especially true in the near-wall region of reference no-slip flow, where, although strong vorticity is generated as a consequence of the no-slip boundary, very few, if any, vortices are typically observed in this region.

Robinson (1991) pointed out that any vortex with an orientation other than wall normal has the potential to function as a "pump" that can transport momentum across the mean velocity gradient. Depending on the sign of vorticity, this pumping motion has the potential to induce sweep events or ejection events (see figure 1.5) and therefore the following analysis of vortical structures can be used to better understand the dynamics of Reynolds-stress-producing events, which make a direct contribution towards the levels of skin-friction. Experimental acquisition of instantaneous velocity and vorticity field data of three-dimensional flows is often restricted due to the limitations of measurement technology. For example, past studies that used hot-wire anemometry to study the coherent flow patterns in turbulent flow (Townsend, 1979) were restricted to measurements across the two-dimensional $x$-$y$ plane, leaving their three-dimensional behaviour unclear. Thus, in order to circumvent the apparent restrictions of experimental methods, a database of fully resolved three-dimensional flow fields obtained via DNS can be used instead. Adopting a numerical approach has facilitated the study of vortices including studies of the relationship between streamwise vortices and skin-friction (Kravchenko et al., 1993), the dynamical characteristics of coherent structures near the wall of a fully developed turbulent channel flow (Jeong et al., 1997) and theories about their generation and self-maintenance (Hamilton et al., 1995; Jimenez & Pinelli, 1999; Schoppa & Hussain, 2002). Here, the objective is to investigate the response of vortical structures to the presence of SHS textures and, in particular, to highlight the significant differences between texture $T_{11}$ and texture $T_{00}$ in an attempt to explain the superior performance of the former SHS texture.

Since it is the structure of turbulence that is of interest here, a time-sequence of instantaneous fields of fully developed turbulent flow were acquired. For each instantaneous snapshot, the fluctuating motions were isolated using,

$$u'_i(x,t) = u_i(x,t) - \left( \langle U_i(y) \rangle x + \tilde{u}_i(x) \right),$$

which ensured that both primary and secondary mean fluid motions were removed. Figure 5.20 shows a typical snapshot of instantaneous turbulence kinetic energy, $k = \frac{1}{2} u'_i u'_i$, and emphasises the complexity of turbulent flow, particularly in the near-wall region - which is where the most vigorous turbulent activity occurs. The next question is, how many instantaneous fields are required? Jeong et al. (1997) studied coherent structures near the wall in a fully developed reference no-slip turbulence channel flow using a data set made up of 7 (seven) instantaneous,
Chapter 5. The effect of spatial modulation on texture performance

Figure 5.20: Contours of instantaneous turbulence kinetic energy, \( k = \frac{1}{2} \langle u'_i u'_i \rangle \), for fully developed turbulent channel flow including (a) side-view on the \( x-y \) plane, (b) plan view on the \( x-z \) plane at \( y/\delta \approx 0.015 \) and (c) transverse view on the \( y-z \) plane. In all figures, the contours of instantaneous turbulence kinetic energy have been clipped to \( k/U_b \in [0, 0.015] \).

fully developed turbulent flow fields that were separated by a time interval of \( T^+ = T \bar{u}_r^2/\nu = 30 \). Here, the data sets for both the reference no-slip and controlled cases contained approximately 350 (three-hundred and fifty) instantaneous fields, each of which were separated by a time interval of \( T^+ \approx 30 \). Following the acquisition of an appropriate data set, a vortex identification based on \( \lambda_2 \) was employed to distinguish the vortical structures from the turbulent motions. Here, \( \lambda_2 \) is defined as the second largest eigenvalue of the tensor \( \langle S_{ij} S_{jk} \rangle + \langle \Omega_{ik} \Omega_{kj} \rangle \), and where \( \langle S_{ij} \rangle \) is the symmetric component and \( \langle \Omega_{ij} \rangle \) is the antisymmetric component of the velocity gradient tensor (Jeong & Hussain, 1995).

The instantaneous three-dimensional vortical structures are visualised in figure 5.21 and appear to be least populous above texture \( T_{11} \). In addition, for both the reference no-slip and controlled cases, the vast majority of the vortical structures seem to be roughly aligned in the streamwise direction. The streamwise extent of individual vortical structures appears, at first sight, to be significantly less than streamwise extent of a typical low-speed streak (\( \Delta x^+ \approx 1000 \)),
however, the vortical structures do appear to interact with one another, often overlapping to form chain-like structures that can approach the length of the streamwise streaks. The reduction of vortical structures above the textured surfaces serves as empirical evidence that suggests that turbulence structures are reduced. To make certain, however, that the population of vortical structures is reduced in the presence of SHS textures, the empirical evidence offered by figure 5.21 was corroborated with appropriate statistical evidence.

Statistical measures describing the population of the vortical structures were obtained using a suite of purposely designed post-processing algorithms. The primary goal of these algorithms was to identify the vortex cores from each instantaneous snapshot using the $\lambda_2$ criterion and then to store each vortex core as an individual object, so that further statistical techniques could be applied. For each instantaneous snapshot, the basic steps of the vortex-core extraction algorithm can be summarised as (i) identification of vortex cores using the $\lambda_2$ criterion; (ii) locating the regional minima of each vortex core; (iii) dilation of each regional minima using a morphological structuring element; (iv) connection of the dilated minima by sweeping through successive $y$-$z$ planes and (v) storing each connected object individually. After sweeping through successive $y$-$z$ planes, the connected, dilated vortex cores are skeletonized. A topological skeleton of an arbitrary shape can be considered a “thin” version of that shape that is equidistant to its boundaries. Here, skeletonization is achieved by calculating the central point of each transverse
cross-section along the streamwise extent of each dilated vortex core. As a result, the topological skeleton of each dilated vortex core resembles a “wire frame” of unit height and unit width, whilst retaining its extent along the streamwise direction. By using the local coordinates of each vortex core it is straightforward to compute statistical quantities describing their size, orientation and distribution. The tail of each vortex core is defined at the point \((x_t, y_t, z_t)\), whereas its head is defined at \((x_h, y_h, z_h)\). The length of each vortex core was computed as,

\[
l_{\lambda_2} = \sqrt{(x_h^+ - x_t^+)^2 + (y_h^+ - y_t^+)^2 + (z_h^+ - z_t^+)^2}. \tag{5.36}
\]

Following the eduction criteria devised by Jeong et al. (1997), vortical structures that did not satisfy \(l_{\lambda_2} > 150\) were discarded. In order to corroborate the empirical evidence provided in figure 5.21 a statistical measure of vortex core population was evaluated. The population density of vortical structures, \(\rho_{\lambda_2}\), was computed as

\[
\rho_{\lambda_2} = \frac{N_{\lambda_2}}{N_{\text{subdom}} V_{\text{subdom}}} \tag{5.37}
\]

where \(N_{\lambda_2}\) is the number of cores that satisfied \(l_{\lambda_2} > 150\), \(N_{\text{subdom}}\) is the number of instantaneous snapshots and where \(V_{\text{subdom}}\) is the volume of the snapshot. The vortex core population density was computed for the reference no-slip case and both the textured cases and the results are provided in table 5.5.2. The change in population density, relative to the reference no-slip case, was computed as

\[
\Delta \rho_{\lambda_2} = \frac{\rho_{\lambda_2} - \rho_{\lambda_2,\text{ref}}}{\rho_{\lambda_2,\text{ref}}} \times 100 \tag{5.38}
\]

where subscript ref indicates a value corresponding to the reference no-slip case. In summary, relative to the reference case, the population density of vortex cores above texture \(T_{00}\) is reduced by approximately 28%, whereas, for texture \(T_{11}\), it is reduced by a further 12%. This trend corroborates the empirical evidence provided in figure 5.21 and confirms that texture \(T_{11}\) is most effective at suppressing vortical structures.

<table>
<thead>
<tr>
<th>Case</th>
<th>(N_{\lambda_2})</th>
<th>(N_{\text{subdom}})</th>
<th>(V_{\text{subdom}} (\delta^3))</th>
<th>(\rho_{\lambda_2})</th>
<th>(\Delta \rho_{\lambda_2} (%))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T_{\text{ref}})</td>
<td>4480</td>
<td>358</td>
<td>18</td>
<td>0.695</td>
<td>0.00</td>
</tr>
<tr>
<td>(T_{00})</td>
<td>3212</td>
<td>358</td>
<td>18</td>
<td>0.498</td>
<td>-28.3</td>
</tr>
<tr>
<td>(T_{11})</td>
<td>4495</td>
<td>201</td>
<td>54</td>
<td>0.414</td>
<td>-40.4</td>
</tr>
</tbody>
</table>

Table 5.6: Vortex core population density computed across the interval \(0 < y^+ < 180\).

This analysis has demonstrated that in the presence of SHS textures, the population density of near-wall vortical structures in turbulent flow is reduced, relative to the reference no-slip case. In addition, it was demonstrated that spatially meandering textures reduced the population density of vortex cores more than their streamwise-aligned counterparts. As the number of vortex cores is
reduced, so to is their integral “pumping power” (Robinson, 1991) which implies that both sweep and ejection events should be attenuated, which is supported by the quadrant analysis in the previous subsection. Reducing the population density of vortical structures suppresses sweep and ejection events, which attenuates the production of Reynolds shear stress and therefore reduces its contribution towards skin-friction.
Chapter 6

Conclusions and future work

This dissertation contained six chapters. An overview of turbulent flow, previous flow control strategies, and previous relevant research of skin-friction reduction using SHS textures was provided in Chapter 1. Chapter 2 contained the computational aspects of the work, including details of the numerical method, accumulation of statistics and derivations of relevant governing equations. Chapter 3 was devoted to the validation of the simulation methodology. Chapter 4 concerned a critical assessment of the drag reduction mechanisms for streamwise-aligned SHS textures, and Chapter 5 detailed the drag-reducing benefits of meandering SHS textures, relative to their streamwise-aligned counterparts. This chapter offers conclusions and suggestions for future work in the field of skin-friction reduction using SHS textures.

6.1 Conclusions

This dissertation was motivated by the suppression of frictional losses in turbulent channel flow using SHS textures. Martell et al. (2009) were the first to study the drag-reducing benefits of SHS textures in a fully developed channel flow at $Re_{\tau} \approx 180$ using DNS. The top of each micro-feature was represented by the no-slip boundary condition and the liquid-gas interface between the micro-features was simulated as a flat no-stress boundary condition. Martell et al. (2009) reported that by decreasing the solid fraction, $\Psi_s$, one could increase the average wall shear-stress reduction. Throughout this work, however, the solid fraction was held constant at $\Psi_s = 50\%$. The choice to hold the solid-fraction constant was motivated by the “30µm-30µm” ridges studied by Martell et al. (2009), whose average wall shear stress reduction agreed well with the previous experimental data of Daneillo et al. (2009). For the “30µm-30µm” ridges, 50% of the area is no-shear, yet only a 10% wall shear stress reduction was reported by Martell et al. (2009). It was concluded, therefore, that this less than ideal performance must be due to a local increase of skin-friction across the no-slip micro-features.

In order to study the local levels of skin-friction across the “30µm-30µm” ridges, a triple
decomposition of the instantaneous flow field was employed. The advantage of using the triple decomposition was statistics could be collected in a phase-averaged form capable of detecting spatial variations in statistical quantities across the texture. In particular, one can isolate the periodic motions induced by the presence of the texture. Following a detailed statistical analysis of the mean and turbulent fluid motions above the “30µm-30µm” ridges the contributions of primary, secondary and turbulent motions towards the local levels of skin-friction were determined by deriving and evaluating an extended version of the FIK identity (Fukagata et al., 2002).

Martell et al. (2009) suggested that wall shear stress could be reduced further by increasing the average streamwise slip velocity, achieved by increasing the solid-fraction, $\Psi_s$. There is, however, a practical lower limit on the solid-fraction beyond which the air-water interface between micro-features ruptures which results in severe performance penalty. Therefore, an alternative texture topology that does not require the solid-fraction to be altered in order to increase skin-friction savings would be desirable. To this end, the performance of sinusoidally varying micro-ridge texture topologies were investigated. The study of meandering textures was motivated by the active flow control studies of Viotti et al. (2009), where a streamwise variation of spanwise wall velocity achieved large scale drag reductions. The sinusoidal variation of the SHS textures was characterised by a streamwise wavelength, $\lambda_x$, and a spanwise oscillation amplitude, $A_z$.

By conducting a parametric study in $A_z$-$\lambda_x$ space, it was demonstrated that certain textures with ($A_z, \lambda_x$) combinations could out-perform their streamwise-aligned counterpart by up to 20%. A statistical analysis of the best-performing texture then followed. An analysis of spanwise secondary motions demonstrated that the passive meandering texture successfully mimicked the active flow control physics previously investigated by Viotti et al. (2009). However, spanwise motions were found to be too weak to affect any benefit on the turbulent flow. To this end, the contributions from the primary, secondary and turbulent fluid motions were determined using the phase-averaged FIK identity and the performance advantage of the meandering texture were clarified. Finally, the effect of the SHS textures on the turbulence structure was considered by using quadrant analysis and the $\lambda_2$ vortex identification criteria.

### 6.2 Outlook and future work

The purpose of this section is to outline and motivate future work with the aim to further the understanding of the mechanisms by which SHS texture suppress frictional losses associated with fully developed turbulent flow. In total, five self-contained pieces of work are proposed - three of which can be carried out using the present computational procedures and two of which would require development of the numerical methodology. First, a simple extension of the current work is discussed which includes the provision of additional statistical and instantaneous quantities aimed to complement to content of this document. Second, the investigation of alternate texture
topologies is proposed. Third, it is proposed that the drag-reducing capabilities of SHS textures be quantified at higher Reynolds numbers. Fourth, higher-order surface models that incorporate the effects of free-surface curvature are proposed. Finally, the possibility of a two-phase direct numerical simulation is discussed.

6.2.1 Extension of the current work

By simply using the current computational setup, any number of simulations of fully developed flow over SHS textures could be conducted. However, any extension of the current work should, preferably, complement the current results in manner that improves the fundamental understanding of turbulent drag reduction mechanisms due to SHS textures, as opposed to conducting extensive and computationally expensive parametric studies which focus on global properties rather than the underlying fluid mechanics mechanisms.

To explain how the SHS textures force the flow to end up in a low-drag equilibrium state, the provision of complete budgets of the Reynolds stresses formed with the stochastic components, following their derivation from the triple decomposition of velocity, would be desirable. Inspection of individual budgets would allow the identification of important interactions within and between different stress components. This would, in particular, help elucidate the fundamental turbulence mechanisms responsible for changes in each of the components of the Reynolds stress tensor.

6.2.2 Alternate texture topologies

The vast majority of effort has been devoted to the study of streamwise-aligned micro-ridge and micro-post topologies, using both experimental (Ou et al., 2004; Ou & Rothstein, 2005; Daneillo et al., 2009; Woolford et al., 2009a) and numerical (Martell et al., 2009, 2010) approaches. The choice to study fluid flow around streamwise-aligned topologies is motivated by their geometric simplicity, which makes them amenable to efficient, cost-effective manufacture in the laboratory and makes their numerical implementation straightforward. There have been, however, some exceptions to the study of streamwise-aligned textures that include the experimental study of obliquely-aligned micro-ridges (Ou et al., 2007), the study of transversely-aligned micro-ridges using both experimental (Byun et al., 2008; Woolford et al., 2009b) and numerical (Martell et al., 2010) methods, and the spatially meandering textures which have been investigated throughout this dissertation.

Using the current no-slip free-slip surface model, the range of texture topologies that could be investigated is potentially unlimited. This dissertation focused on two distinct texture topologies: streamwise-aligned micro-ridges and sinusoidally varying micro-ridges. In each of these texture topologies the solid fraction was held constant at $\Psi_s = 50\%$. 

1. By holding the micro-ridge width $d$ constant, what effect does increasing/decreasing the micro-ridge spacing $g$ have on drag-reduction?

2. By holding the micro-ridge spacing $g$ constant, what effect does increasing/decreasing the micro-ridge width $d$ have of drag-reduction?

3. By holding the solid-fraction $\Psi_s$ constant, what effect does simultaneously increasing/decreasing the micro-ridge width and micro-ridge spacing have on drag-reduction?

Previous work has demonstrated that the drag-reducing performance of SHS textures increases with decreasing solid fraction $\Psi_s$ for both laminar (Ou et al., 2004; Ou & Rothstein, 2005; Davies et al., 2006; Woolford et al., 2009a) and turbulent flows (Woolford et al., 2009b; Martell et al., 2009; Daneillo et al., 2009; Martell et al., 2010). This increase in performance has been credited to an increase in mean streamwise slip velocity $\langle U_s \rangle_x$, which in turn decreases the gradient of the mean velocity profile at the wall. Here we have shown that slip-velocity has an indirect effect in the drag reduction mechanisms - since the fluid only slips along the free-slip regions which, by definition, make exactly zero contribution towards skin-friction. Therefore using the methods used throughout this dissertation, a clearer understanding of the fundamental drag reduction mechanism for wide range of textures could be achieved.

### 6.2.3 Effect of Reynolds number

Investigating the drag-reducing capabilities of SHS textures at high Reynolds numbers is strongly motivated by the commercial application of SHS textures. Engineering flows typically experience variations in geometry, temperature and flow rate - all of which directly affect the Reynolds number. Textures that offer drag reduction of a range of Reynolds numbers are therefore particularly desirable, as well textures that perform well at higher Reynolds numbers - which are typically associated with engineering flows.

Throughout this dissertation the bulk Reynolds number was held fixed at a friction Reynolds number of $Re_\tau \approx 180$. The choice of $Re_\tau \approx 180$ was motivated by the seminal DNS of Kim et al. (1986), which serves as an excellent validation case and can be used to benchmark SHS texture performance. Some years later, Moser et al. (1999) performed a DNS of turbulent channel flow at three different friction Reynolds numbers that included $Re_\tau = \{180, 395, 590\}$. Moser et al. (1999) accumulated various statistical measures including mean profiles, Reynolds stresses, autocorrelations, energy spectra and TKE budgets, all of which are freely available online at http://turbulence.ices.utexas.edu/MKM_1999.html. This online database serves as an excellent resource to validate future simulations of fully developed turbulent flow at increased Reynolds numbers and as a benchmark from which texture performance can be assessed. The effect of Reynolds number on SHS texture performance has been investigated using DNS by Martell et al.
(2010), who selected the three aforementioned friction Reynolds numbers $Re_\tau = 180$, $Re_\tau = 395$ and $Re_\tau = 590$. There are, however, some concerns about the computational approach adopted by Martell et al. (2010). Their $Re_\tau \approx 180$ cases had $128^3$ grid points for each simulation. Their $Re_\tau \approx 395$ cases had $256^3$ grid points, and the $Re_\tau = 590$ cases had $512^3$ grid points per simulation. For all of the simulations Martell et al. (2010) employed a uniform mesh in all three directions. The choice to cluster grid points in the near-wall region is customary when adopting a DNS approach, and as a result only two grid points are contained with the viscous sublayer $0 < y^+ < 5$ for each of the Reynolds numbers. Insufficient spatial resolution negates the use of DNS. An additional cause for concern in the results of Martell et al. (2010) are the unsmooth regions present in the Reynolds stress profiles for the $Re_\tau = 590$ cases which, as noted by the authors, are caused as a result of insufficient temporal averaging.

### 6.2.4 Effect of free-surface curvature

Throughout this dissertation the surface model used to represent the SHS textures resemble the situation studied theoretically by Philip (1972a), who derived analytical solutions for Stokes flow over mixed no-slip free-slip plane boundaries. Here, the top surface of each microfeature is taken to be a no-slip boundary, and the suspended airwater interface between the microfeatures is simulated as a flat free-slip boundary. The free-slip boundary was assumed to experience no out-of-plane deflection - an assumption which is contested by both theoretical and experimental evidence. Under laminar flow conditions, Ou et al. (2004) acquired the profile of the air-water
interface suspended between microposts \( d = 30 \mu m \) wide and spaced \( w = 30 \mu m \) apart and demonstrated that the interface “drooped” into the gas-filled micro-cavity below in a symmetric, convex manner where the maximum deflection, \( \delta^* \), was found to increase linearly as the pressure drop along the channel was raised. In addition, Ou et al. (2004) demonstrated that the deflection of the free surface, \( \delta^* (z) \), could be approximated by the equation traditionally used to describe the deflection of an elastic beam under a uniformly distributed load,

\[
\delta^* (z) = \alpha \left( z^4 - 2wz^3 + w^3 z \right), \quad (6.1)
\]

and by appropriately choosing the coefficient of proportionality, \( \alpha \), a good quantitative fit to their experimental data was observed.

Incorporating free surface curvature into the existing surface model would be straightforward. The no-slip regions would remain perfectly flat, whereas transverse out-of-plane deflection could be prescribed across the free-slip regions, simply by using equation 6.1. The effects of both constant convex and constant concave curvature (see figure 6.1) could be considered and their effects on texture performance assessed. As a consequence of the interfacial curvature, the channel

\[\text{Figure 6.1: Schematic of mixed no-slip no-shear model applied to micro-ridge topology}
\]

\[\text{including experiencing (a) no-out-plane deflection, (b) concave deflection and (c) convex deflection.}\]

walls would no longer be planar and would not conform to a Cartesian mesh that was employed throughout this dissertation. There are, however, alternative computational strategies which can be employed that can tackle complex surface topologies - some of which retain the Cartesian mesh and others that do not.

For complex geometries, one of the most difficult and time-consuming processes in the numerical simulation of fluid flow is the generation of suitable computational mesh around the object being modelled. The task of grid generation and discretization of the governing equations can be greatly simplified by adopting an immersed boundary (IB) method. IB methods typically employ structured, non-conforming grids and enforce the boundary conditions by altering the Navier-Stokes equations in the near-wall region. Using an IB approach simplifies grid generation, the discretization of the governing equations, and circumvents the problems associated with grid
quality (i.e. non-orthogonality) that exist with boundary-fitted grid techniques. In addition, the IB approach also allows the use of a structured grid solver, which are particularly amenable to highly efficient discrete solution algorithms and scalability on high performance computing facilities, in comparison to unstructured grid methods.

An alternative to an IB method would be to mesh the deflected SHS texture using a boundary-fitted grid. A boundary fitted grid would conform to the either the convex or concave corrugations present due to free-surface curvature on the SHS textures and can be generated using a variety of mathematical techniques. For example, Bechert & Bartenwerfer (1989) theoretically investigated the viscous sublayer region of a turbulent boundary layer on a surface with riblets. By consideration of the first Navier-Stoke equation,

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} + \frac{\partial p}{\partial x} = \frac{1}{Re} \nabla^2 u,$$  \hspace{1cm} (6.2)

Bechert & Bartenwerfer (1989) assumed a flat-plate boundary-layer flow which was steady, without a mean streamwise pressure gradient and neglected the convective terms to obtain,

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0,$$  \hspace{1cm} (6.3)

which is a Laplace equation for the velocity $u$. Bechert & Bartenwerfer (1989) solved equation 6.3 using conformal mapping to obtain *isotaches* (lines of constant velocity) to characterise the flow-field around the riblet geometries in the near-wall region. In a later study, Choi *et al.* (1993) used the isotaches of Bechert & Bartenwerfer (1989) to mesh a riblet geometry which was then used in a DNS of fully developed turbulent flow. A similar approach could be adopted for the SHS textures with interface deflection and would only requires minor changes to the numerical method.

### 6.2.5 Two-phase simulations

Throughout this dissertation the SHS textures were represented using a surface model originally inspired by the theoretical studies of Philip (1972a), by which alternating regions of no-slip and
free-slip boundary conditions were used to respectively represent the top of the micro-features and the air-water interfaces formed between them. A single-phase DNS was then conducted in order to simulate fully developed turbulent flow over this surface model.

In reality, however, the flow over a SHS texture is a multi-phase problem. The air-water meniscus is formed as a result of air-bubbles trapped between the roughness elements. The exclusion of the vapor phase in the current simulations could potentially prevent a thorough understanding of drag reduction over SHS textures from being developed and motivates the simulation of two-phase flow over SHS textures. In a two-phase flow simulation an accurate description of the interface position and evolution is required. Volume of Fluid (VOF) (Hirt & Nichols, 1981) and Level Set Methods (LSM) (Sussman et al., 1994) are two common interface tacking methods which could be incorporated into the current DNS code.

The LSM outlined by Sussman et al. (1994) has been applied to both the motion of air bubbles in water and falling water droplets in air and the corresponding density ratio of 1000 : 1 is perfectly-suited to future, two-phase simulations.
Appendices
Appendix A

Turbulence kinetic energy and Reynolds stresses

A.1 Turbulence kinetic energy

Figure A.1 shows profiles of turbulent kinetic energy for texture $T_{11}$, texture $T_{00}$ and the uncontrolled case which includes results from the decomposition

$$\langle k \rangle^{xz} = \frac{1}{2} \left( \langle k \rangle^{ns} + \langle k \rangle^{fs} \right)$$

In general, the effect of meandering the texture appears to be a suppression of TKE for $y^+ < 10$. This suppression occurs for all the averaged components of TKE, $\{\langle k \rangle^{xz}, \langle k \rangle^{ns}, \langle k \rangle^{fs}\}$, when comparing texture $T_{11}$ to texture $T_{00}$ and suggests that turbulence activity is attenuated in the near-wall region.

Figure A.1: Decomposition of TKE across all phases including profiles of $\{\langle k \rangle^{xz}, \langle k \rangle^{ns}, \langle k \rangle^{fs}\}$ for texture $T_{11}$ $\{-\,-\,-\,-\,-\,-\}$ and texture $T_{00}$ $\{-\,-\,-\}$ normalised by (a) uncontrolled friction velocity, $u_\tau$, and (b) the bulk velocity, $U_b$. In both figures the data corresponding to the uncontrolled case is denoted using $\circ$. 
A.2 Reynolds stresses

Figure A.2 shows profiles the Reynolds stress tensor for texture T_{11}, texture T_{00} and the uncontrolled case which includes results from the decomposition

\[ \langle R_{ij} \rangle^{xz} = \frac{1}{2} \left( \langle R_{ij} \rangle^{ns} + \langle R_{ij} \rangle^{fs} \right) \]

From this decomposition it was noted that \( \langle R_{13} \rangle^{ns} = \langle R_{13} \rangle^{fs} = 0 \) and \( \langle R_{23} \rangle^{ns} = \langle R_{23} \rangle^{fs} = 0 \).

The streamwise Reynolds stress (figure A.2a) is suppressed in the near wall region for texture T_{11}, relative to texture T_{00}, which agrees well with figure A.1, where a near-wall suppression of TKE was also observed. Both the wall-normal (figure A.2b) and spanwise (figure A.2c) components of Reynolds stress are suppressed for all wall-normal positions when comparing the results of
texture $T_{11}$, relative to texture $T_{00}$. A further reduction in Reynolds shear stress (figure A.2d) is also observed for texture $T_{11}$, relative to texture $T_{00}$. 
Appendix B

Anisotropy Invariant Maps

In this appendix a supplementary view of the impact of SHS textures on turbulent flow is explored by considering the anisotropic nature of turbulence in the near-wall region. A general relationship between drag reduction and the anisotropy of turbulence was put forward by Frohnapfel et al. (2007b), who, after surveying a variety of passive flow control strategies including rigid fibres (Paschkewitz et al., 2004), long-chain polymers (Jovanovic et al., 2006), ultra-thin streamwise-aligned riblets (Lammers et al., 2006) and streamwise-aligned micro-grooves (Frohnapfel et al., 2007a), credited their drag-reducing capabilities to a dramatic increase in the anisotropy of near-wall turbulence, relative to the uncontrolled case. Forcing turbulence to a particular anisotropic state that potentially guarantees drag reduction is a particularly desirable design criteria and motivates both the development of current and future flow control strategies. The goal of this subsection, therefore, is to assess the anisotropic state of turbulence in the proximity of SHS textures and to determine whether or not the findings are in agreement with previously proposed trends.

The anisotropy of turbulence can be quantified by using,

\[ \langle b_{ij} \rangle = \frac{\langle R_{ij} \rangle}{2\langle k \rangle} - \frac{1}{3} \delta_{ij} \]  

(B.1)

where \( \langle b_{ij} \rangle \) and \( \delta_{ij} \) represent the anisotropy tensor and the Kronecker delta function, respectively. For flows with non-zero mean shear the dominant anisotropy is manifest as turbulent shear stresses (\( \langle R_{ij} \rangle \neq 0 \)) which arise as a consequence of correlations between turbulent fluid motions. For wall-bounded flows, which is the case here, the turbulent fluid motions typically experience the strongest mean shear in near-wall region, and the weakest mean shear in the vicinity of the channel half-height. Therefore, it seems reasonable to assume that turbulence in the near-wall region should exhibit a high degree of anisotropy, whereas turbulence in vicinity of the channel half-height should tend towards an isotropic state. Isotropic turbulence is characterised by velocity fluctuations that are independent of the axis of reference, i.e. invariant to axis rotation.
and/or reflection and in such a situation, where no gradients of the mean velocity exist, there can be no turbulent shear stresses \( \langle R_{ij} \rangle \equiv 0 \), and consequently the the anisotropy tensor becomes

\[
\langle b_{ij} \rangle = \begin{bmatrix}
\frac{\langle R_{11} \rangle}{2\langle k \rangle} - \frac{1}{3} & 0 & 0 \\
0 & \frac{\langle R_{22} \rangle}{2\langle k \rangle} - \frac{1}{3} & 0 \\
0 & 0 & \frac{\langle R_{33} \rangle}{2\langle k \rangle} - \frac{1}{3}
\end{bmatrix} = 0.
\]

where the definition of TKE, \( \langle k \rangle = 0.5 \langle R_{ii} \rangle \), and the fact that, \( \langle R_{11} \rangle = \langle R_{22} \rangle = \langle R_{33} \rangle \) for isotropic turbulence were used. Therefore, the anisotropy tensor essentially measures the departure of \( \langle R_{ij} \rangle \) from the isotropic state.

Studies of homogeneous turbulence in the absence of external forces have demonstrated that initially anisotropic turbulence is naturally inclined to revert to an isotropic state. The process of turbulence driving itself towards an isotropic state was observed by, for example, Mills & Corrsin (1959) who investigated the response of turbulence produced by a heated grid in a downstream axisymmetric contraction. Mills & Corrsin (1959) reported that the effect of the contraction increased \( \langle R_{22} \rangle \) and \( \langle R_{33} \rangle \) relative to \( \langle R_{11} \rangle \), which suggested that nearly isotropic turbulence behind the grid (where \( \langle R_{22} \rangle \) and \( \langle R_{33} \rangle \) are both less than \( \langle R_{11} \rangle \)) could be obtained by a suitably small amount of contraction prior to a straight duct. The phrase “return to isotropy” was coined by Lumley & Newman (1977) who used freely available experimental data, including the hot-wire measurements of Mills & Corrsin (1959), to verify their derivation of three invariant functions that were used to classify the various anisotropic states of turbulence.

As an interesting aside, the return to isotropy paradigm proves to be particularly challenging from a turbulence modelling perspective. The celebrated \( k-\epsilon \) model, for example, closes the Reynolds Stress Transport Equation by representing the turbulence as kinetic energy, which is a scalar, and therefore cannot account for any anisotropies. A shortcoming to representing the turbulent velocity fluctuations solely by kinetic energy is that sometimes very different component energies are produced, implying that \( \langle R_{11} \rangle \neq \langle R_{22} \rangle \neq \langle R_{33} \rangle \), which means that any directional preference (i.e. the anisotropy) of the turbulent velocity fluctuations cannot be captured. However, no turbulence models are required since the DNS methods used throughout this dissertation fully resolve all the scales of turbulence and therefore pave the way for an accurate analysis of turbulence anisotropy.

By manipulating the anisotropy tensor (see equation B.3) in an appropriate manner, it is possible to determine the physical bounds of turbulence for both the isotropic and anisotropic
Appendix B. Anisotropy Invariant Maps

states. The trace-free condition $\langle b_{ii} \rangle = 0$ shows the anisotropy tensor to be of the form

$$\langle b_{ii} \rangle = \begin{bmatrix} \langle b_{11} \rangle & 0 & 0 \\ 0 & \langle b_{22} \rangle & 0 \\ 0 & 0 & -(\langle b_{11} \rangle + \langle b_{22} \rangle) \end{bmatrix}$$

(B.2)

which represents turbulence in the isotropic state, where it has a complete lack of any directional preference.

Three useful markers for characterising the anisotropy of turbulence are the one-component (1C), two-component (2C) and axisymmetric states. In the 1C state, turbulence has one non-zero component, $\langle R_{11} \rangle \neq 0$, and two components equal to zero, $\langle R_{22} \rangle = \langle R_{33} \rangle = 0$, which means that, from the definition of turbulence kinetic energy, $\langle R_{11} \rangle / \langle k \rangle = 2$. Whereas, in the 2C state, turbulence has two non-zero components, $\langle R_{22} \rangle \neq 0$ and $\langle R_{33} \rangle \neq 0$, and one component equal to zero, $\langle R_{11} \rangle = 0$. From the first component of the anisotropy tensor, $\langle b_{11} \rangle = \langle R_{11} \rangle / 2 \langle k \rangle - 1/3$, and since $\langle R_{11} \rangle / \langle k \rangle \geq 0$, the inequality $0 \leq \langle R_{11} \rangle / \langle k \rangle \leq 2$ holds which implies that $-1/3 \leq \langle b_{11} \rangle \leq 2/3$. The 2C state if defined when $\langle b_{11} \rangle = -1/3$, whilst the 1C state defined when $\langle b_{11} \rangle = 2/3$.

The remaining special state of turbulence is referred to as the axisymmetric state, which can written as $\langle b_{11} \rangle = \langle b_{22} \rangle$, which means the anisotropy tensor can be written as

$$\langle b_{ii} \rangle = \begin{bmatrix} \langle b_{11} \rangle & 0 & 0 \\ 0 & \langle b_{11} \rangle & 0 \\ 0 & 0 & -2 \langle b_{11} \rangle \end{bmatrix}.$$  

(B.3)

Depending on the sign of $\langle b_{11} \rangle$, turbulence can take two distinct forms in the axisymmetric state. If $\langle b_{11} \rangle < 0$, then it follows that from the anisotropy tensor both $\langle R_{11} \rangle < 2/3 \langle k \rangle$ and $\langle R_{22} \rangle < 2/3 \langle k \rangle$, whilst the remaining component satisfies the inequality $\langle R_{33} \rangle > 2/3 \langle k \rangle$. Under these conditions turbulence undergoes axisymmetric expansion, whereby such anisotropy could, for example, be produced as a result of expansion in the plane and compression along the remaining axis. The opposite case is referred to as axisymmetric contraction, where $\langle b_{11} \rangle > 0$, where turbulence undergoes compression in two directions and extension in the remaining direction.

The state of turbulence can be characterised further by evaluating the principal invariants of the anisotropy tensor. The principal invariants can be obtained using the Cayley-Hamilton theorem which states that in $n$ dimensions, $\langle b_{ij} \rangle^n$ is a linear combination of lower powers of $\langle b_{ij} \rangle$: in three dimensions, where $n = 3$, the explicit formula according to the Cayley-Hamilton theorem is

$$\langle b_{ij} \rangle^3 = (III_b) \delta_{ij} - (II_b) \langle b_{ij} \rangle + (I_b) (\langle b_{ij} \rangle)^2,$$  

(B.4)
where $\langle b_{ij} \rangle^3$ is shorthand for the matrix product $\langle b_{ij} \rangle \langle b_{jk} \rangle \langle b_{kl} \rangle$, $\langle b_{ij} \rangle^2$ is shorthand for the matrix product $\langle b_{ij} \rangle \langle b_{ji} \rangle$ and where the first, second and third principal invariants are given by

\begin{align*}
\langle I_b \rangle &= \langle b_{ii} \rangle, \\
\langle II_b \rangle &= -\frac{1}{2} \left( \langle b_{ii} \rangle^2 - \langle I_b \rangle^2 \right), \\
\langle III_b \rangle &= \frac{1}{3} \left( \langle b_{ii} \rangle^3 + 3 \langle I_b \rangle \langle b_{ii} \rangle - 2 \langle b_{ii} \rangle^2 \langle b_{ii} \rangle \right).
\end{align*}

Therefore, an evaluation of the principal invariants of the trace-free tensor, $\langle b_{ii} \rangle$, show in equation B.3, gives

\begin{align*}
\langle I_b \rangle &= 0, \\
\langle II_b \rangle &= -\left( \langle b_{11} \rangle^2 + \langle b_{22} \rangle^2 + \langle b_{11} \rangle \langle b_{22} \rangle \right), \\
\langle III_b \rangle &= \langle b_{11} \rangle \langle II_b \rangle + \langle b_{11} \rangle^3.
\end{align*}

For the axisymmetric state of turbulence, where it can be assumed that $\langle b_{11} \rangle = \langle b_{22} \rangle$, the principal invariants (equations B.6a-B.6c) are $\langle II_b \rangle = -3 \langle b_{11} \rangle^2$ and $\langle III_b \rangle = -2 \langle b_{11} \rangle^3$, or, by choosing to express the third principal invariant in terms of the second principal invariant, $\langle III_b \rangle = \pm 2 \left( \langle II_b \rangle / 3 \right)^{3/2}$, where the positive sign corresponds to axisymmetric expansion and the negative sign to axisymmetric contraction. For the 2C state, where $\langle b_{11} \rangle = -\frac{1}{3}$, it follows that $\langle III_b \rangle = -\frac{1}{3} \langle II_b \rangle - \frac{1}{27}$, whereas for the 1C state, where $\langle b_{11} \rangle = \frac{2}{3}$, it follows that $\langle III_b \rangle = \frac{2}{3} \langle II_b \rangle + \frac{8}{27}$. Figure B.1 shows a schematic of the possible values of $\langle II_b \rangle$ and $\langle III_b \rangle$ with corresponding states of the turbulence: all turbulence must fall within the shaded area. The left and right curves boundaries correspond to the axisymmetric state of turbulence. The elbow on the left corresponds to the vanishing of the axial component, leaving behind isotropic two-dimensional turbulence. Along the diagonal straight the turbulence is two-dimensional, until at the summit one of the components vanishes, leaving one-dimensional turbulence; a state which may also be reached from the right-hand-side axisymmetric state, which corresponds to the vanishing of both transverse components.

Before evaluating the AIM in order to assess the state of turbulence in proximity of SHS textures, the limiting behaviour of turbulence can be predicted by carrying out a perturbation analysis. The asymptotic analysis can be performed for the fluctuating velocity components in proximity of a no-slip and free-slip boundary, in an attempt to distinguish limiting behaviour of turbulence above the SHS texture micro-features and free-shear interfaces, respectively, and then these resulted can be complemented by evaluating the AIM. The fluctuating components,
Appendix B. Anisotropy Invariant Maps

Figure B.1: Plots of the “turbulence triangle”, also referred to as the “Lumley triangle”, after Lumley & Newman (1977) including (a) the limiting values of the second and third invariants for turbulence, which must exist within the area delimited by the shading. Curve A represents the 2C state, defined by \( \langle III_b \rangle = -\frac{1}{3} \langle II_b \rangle - 1/27 \); curve B represents the 1C state, defined by \( \langle III_b \rangle = \frac{2}{3} \langle II_b \rangle + 8/27 \); curve C represents the axisymmetric expansion state, defined \( \langle III_b \rangle = +2 (\langle II_b \rangle / 3)^{3/2} \) and curve D represents the axisymmetric contraction state, defined by \( \langle III_b \rangle = -2 (\langle II_b \rangle / 3)^{3/2} \). Panel (b) shows schematics of the various anisotropic states of turbulence.

The fluctuating velocities, \( u'_i \), can be expanded using Taylor series of the form,

\[
\begin{align*}
    u' &= a_1 + b_1 y + c_1 y^2 + \ldots \quad (B.7a) \\
    v' &= a_2 + b_2 y + c_2 y^2 + \ldots \quad (B.7b) \\
    w' &= a_3 + b_3 y + c_3 y^2 + \ldots \quad (B.7c)
\end{align*}
\]

where the coefficients are zero-mean random variables, and, for fully developed channel flow are statistically independent of \( x, z \) and \( t \). The limiting behaviour of the fluctuating velocities above the micro-features and free-slip interfaces can be obtained from the expansions (equations B.7a-B.7c) by substituting the no- and free-slip boundary conditions, respectively. The no-slip condition yields \( u' = a_1 = 0 \) and \( w' = a_3 = 0 \); and the no-penetration condition yields \( v' = a_2 = 0 \). An additional consequence of the no-slip condition is that the derivatives \( (\partial u'/\partial x)_{y=0} \) and \( (\partial u'/\partial z)_{y=0} \) are both equal to zero, which, from continuity, yields \( (\partial v'/\partial y)_{y=0} = b_2 = 0 \).

\[
\begin{align*}
    u' &= b_1 y + c_1 y^2 + \ldots \quad (B.8a) \\
    v' &= c_2 y^2 + \ldots \quad (B.8b) \\
    w' &= b_3 y + c_3 y^2 + \ldots \quad (B.8c)
\end{align*}
\]

The significance of the coefficient \( b_2 \) being equal to zero is that, in close proximity to the wall,
turbulence exists in the 2C state. The free-slip condition yields \((\partial u'/\partial y)_{y=0} = b_1 = 0\) and 
\((\partial w'/\partial y)_{y=0} = b_3 = 0\); and the no-penetration condition yields 
\(v' = a_2 = 0\).

\[
\begin{align*}
u' - u'_s &= c_1 y^2 + \ldots & \text{(B.9a)} \\
v' &= b_2 y + c_2 y^2 + \ldots & \text{(B.9b)} \\
w' - w'_s &= c_3 y^2 + \ldots & \text{(B.9c)}
\end{align*}
\]

The significance \(b_2\) being the first non-zero component that varies with \(y\) is that, in close proximity to the wall, turbulence exists a 1C state. Note that the streamwise and spanwise slip velocities have been moved to the left-hand-side of equation B.9a and equation B.9c, respectively, since neither are a function of the wall-normal direction. It is reasonable, therefore, to assume that when averaged across all phases, the anisotropic nature of turbulence in the near-wall region lies somewhere between the 1C and 2C states.

Figure B.2 shows the AIM averaged across all phases for the uncontrolled case, texture \(T_{00}\) and texture \(T_{11}\). For the uncontrolled case, the turbulence very close to the wall exists in a 2C state - which is in agreement with the previous perturbation analysis for the limiting behaviour of velocity fluctuations above a no-slip flat plate. Moving away from the wall and through the viscous sublayer, the uncontrolled turbulent flow approaches a 1C state, before relaxing towards an isotropic state as the channel half-height is approached. Relative to the uncontrolled case, the turbulence in the near-wall region for both the textured cases approaches the 1C state. This shift from the 2C towards the 1C state could potentially be an artifact of the differing local anisotropy of turbulence above the no-slip and free-slip boundaries, the effects of which have been have been averaged across all phases.

In order to clarify this point, the invariants were decomposed as

\[
\begin{align*}
\langle II_b \rangle^{xz} &= \frac{1}{2} \left( \langle II_b \rangle^{ns} + \langle II_b \rangle^{fs} \right) & \text{(B.10a)} \\
\langle III_b \rangle^{xz} &= \frac{1}{2} \left( \langle III_b \rangle^{ns} + \langle III_b \rangle^{fs} \right) & \text{(B.10b)}
\end{align*}
\]

which permits the analysis of AIM above the the micro-feature and interfaces.

Figure B.3 shows the AIM which has been integrated across all no-slip phases for both texture \(T_{00}\) and texture \(T_{11}\). Upon inspection, it is clear that in close proximity to the SHS texture micro-features, the turbulence exists in a 2C state, which confirms the validity of preceding asymptotic analysis for turbulence just above a no-slip boundary (see equations B.8a-B.8c). In fact, the no-slip-averaged AIM for both textures behave in a similar fashion to that of the uncontrolled flow (see figure B.2) for all wall-normal positions, whereby very close to the no-slip micro-features, the turbulence exists in a 2C state, before approaching a 1C state in the viscous sublayer, and then finally relaxing towards an isotropic state as the channel half-height is approached. Comparing
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Figure B.2: Anisotropy invariant maps (AIM) for turbulence including (a) limiting states, (b) uncontrolled flow, (c) average across all phases for texture $T_{00}$ and (d) average across all phases for texture $T_{11}$.

Figure B.2: Anisotropy invariant maps (AIM) for turbulence including (a) limiting states, (b) uncontrolled flow, (c) average across all phases for texture $T_{00}$ and (d) average across all phases for texture $T_{11}$.

the no-slip-averaged AIM of texture $T_{11}$ to texture $T_{00}$, suggests that the near-wall turbulence of the former exists in a slightly more $1C$ state.

Compared to the no-slip-averaged AIM (see figure B.3), the AIM averaged across all free-slip phases appear to be strikingly different - as shown in figure B.4. Very close to the interfaces of the SHS textures, turbulence exists in a highly anisotropic $1C$ state - which, again, confirms the validity of the preceding asymptotic analysis for velocity fluctuations just above a free-slip boundary (see equations B.9a-B.9c). Following its $1C$ state as $y^+ \to 0$, the turbulence simply relaxes towards an isotropic state as the channel half-height is approached.

The decomposition of the AIM above no- and free-slip phases (see equations B.10a-B.10b) has helped to delineate the anisotropic nature of turbulence above SHS texture micro-features and interfaces, respectively. Frohnapfel et al. (2007b) suggested that forcing near-wall turbulence towards the anisotropic $1C$ state is a general strategy to achieve drag reduction. Here, however, the $1C$ state is only achieved locally above the free-slip interfaces. The concept of localised anisotropy has been observed in geometrically comparable passive flow control techniques. For
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Figure B.3: Anisotropy invariant maps (AIM) for turbulence above no-slip SHS texture micro-features for (a) texture T_{00} and (b) texture T_{11}.

Figure B.4: Anisotropy invariant maps (AIM) for turbulence above free-slip SHS texture interfaces for (a) texture T_{00} and (b) texture T_{11}.

example, Lammers et al. (2006) used lattice-Boltzmann methods to simulate a turbulent channel flow over ultra-thin “knife-blade” streamwise-aligned riblets, where just above a riblet and at a point equidistant between two riblets the turbulence existed in the 2C and 1C state, respectively.

In summary, this subsection provides an alternate perspective on the effects of SHS textures on the structure of near-wall turbulence by mapping the turbulence to a functional space, first devised by Lumley & Newman (1977), referred to as the anisotropy invariant map (AIM). Although the average levels of anisotropy are increased above SHS textures, relative to the uncontrolled case, the decomposition of the AIM across no- and free-slip phases demonstrated that a high degree of anisotropy is only achieved locally above the free-slip phases.
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