Collusion in regulated pluralistic markets

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Abstract

We analyse incentives for cooperative behaviour when heterogeneous providers are faced with regulated prices under yardstick competition. Providers are heterogeneous in the degree to which their interests correspond to those of the regulator, with close correspondence labelled altruism. Deviation of interests may arise as a result of de-nationalisation or when private providers enter predominantly public markets.

We assess how provider strategies and incentives to collude relate to provider characteristics under yardstick competition regulation.

Our results suggest that under the yardstick competition each provider’s choice of cooperative cost is decreasing in the degree of the other provider’s altruism, so a self-interested provider will operate at a lower cost than an altruistic provider. The prospect of defection serves to moderate the chosen level of operating cost. More general results show that collusion is more stable in homogeneous than in heterogeneous markets; in markets served by purely altruistic providers there is no collusion on costs while in markets served by purely self-interested providers there is scope for collusion. Our analysis demonstrates that it is important to consider the composition of the market when designing yardstick competition arrangements.

Keywords: Price regulation; yardstick competition; collusion; altruism.

JEL classification: I1, I18, L33

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1. Introduction

Many countries have introduced some form of yardstick competition in order to regulate prices in contexts where providers face limited competitive pressure. Examples are the maximum price limits each water company may charge its customers in the UK (Ofwat, 1993); price caps imposed by the Federal Energy Regulatory Commission to hold down the wholesale price of natural gas and electricity in interstate commerce in the US (US Department of Energy, 2002); postal tariffs determined by independent regulators in countries such as Germany, the Netherlands and the UK (NERA, 2004); and prospective payment system (PPS) that have been introduced to pay for health care services in many countries (Schreyögg et al., 2006; Ma, 1994).

The fundamental idea behind yardstick competition is that the price (or price cap) faced by each provider is dependent on the actions of all the other providers (Schleifer, 1985; Laffont and Tirole, 1993). According to Schleifer’s rule, the price each provider faces is based on the costs of all other providers in the industry but not its own. This creates strong incentives for cost control: each provider’s cost reducing effort will not be detrimental to the price it faces. A potential drawback with yardstick competition is that providers have an incentive to collude on higher costs, first because they can get a higher price for their services and, second, because they can exert less cost reducing effort, thereby benefiting from slack (Wilson, 1989).

In contexts where there is a large number of providers, this is unlikely to be problematic, mainly because the cost of collusion rises (Pope, 1989). But there is greater potential for cooperative behaviour in contexts where there is a limited number of providers. This is likely for utilities, rail or postal services. But it can also arise in health care, for instance because specialist services (like bone marrow or lung transplantation) are concentrated among a handful of providers or in places such as Northern Ireland or Iceland, which have considered introducing PPS arrangements despite there being fewer than five hospitals in each country.

The incentive to collude with other providers will depend on the objectives of the provider, particularly the extent to which their objectives correspond with those of the price-setting regulator. We use the terms “altruistic” to describe providers that have objectives closely related to those of the regulator and “self-interested” to describe providers whose interests are more divergent from those of the regulator (Rose-Ackerman, 1996; Bozeman, 1984; Rainey et al. 1976). If providers differ in their degree of altruism, they may behave quite differently in response to financial incentives (Aas, 1995). Divergence among providers may arise in
situations where greater plurality of provision is being encouraged. For example, traditionally public (National Health Service) systems such as England, France, Portugal and Italy are encouraging more private sector organisations to enter the health care market (Oliveira and Pinto, 2002; Aballea et al 2006; Levaggi, 2007; Pollock and Godden 2008). Similarly many countries have de-nationalised many services, either wholly or in part. Public providers may have a strong sense of mission, aiming to maximize the well-being of the people they serve (Wilson 1989), just as the regulator would like. But private providers are also accountable to their shareholders, with an interest in profit making. This implies that they have a weaker sense of “public” service mission, and might have objectives that are less closely aligned to those of the regulator (Newhouse, 1970; Hansmann, 1980; Glaeser and Shleifer, 2001).

There are a number of works that have addressed the issue of collusion under yardstick competition (Boardman et al 1986; Tangeras, 2002; Chong and Huet, 2005). Our paper is particularly close to Potters et. al. (2004). The authors present an adapted version of Schleifer’s model (Schleifer, 1985) and test it experimentally in order to explore collusion incentives under different yardstick competition schemes. However the existing literature assumes homogeneous providers. The aim of the paper is to analyse incentives for cooperative behaviour when heterogeneous providers are faced with regulated prices under yardstick competition. We analyse the choice of cost when providers do not co-operate and when they collude, and we consider incentives to defect from the collusion agreement. More general results show that collusion is more stable in homogeneous than in heterogeneous markets, in markets served by purely altruistic providers there is no collusion on costs while in markets served by purely self-interested providers there is scope for collusion. The paper is organized as follows. Section 2 introduces the main assumptions of the model, and considers cooperative behaviour under a yardstick competition model. Section 3 presents summarizes the main results of the paper and section 4 draws the main conclusions.
2. The Model

Consider a market with three types of agent: consumers, providers and a regulatory authority. We consider two providers \( i \) (with \( i = 1, 2 \)) each with its own population of consumers defined geographically, so that each provider is a local monopolist facing a downward-sloping demand curve with \( p_i \) being the price paid by consumers for each unit of service \( q \).

In sectors that provide services of public interest, such as for postal services, utilities, or the healthcare sector consumers may face the full or a partially subsidised price. Under yardstick competition, the regulator establishes a payment that gives the providers incentives to reduce costs. In particular each provider faces a regulated price set beforehand equal to the average (say) of the marginal costs of all the other providers in the market except its own (Shleifer, 1985). We assume that costs are observed by the regulator.

The regulator sets a cap - \( \hat{p} \) - on the price that each provider can charge. Note that this restriction will bind in equilibrium otherwise there would be no need for regulation.

The main objectives of a regulation policy are to promote technical efficiency and allocative efficiency by simulating the outcomes of competitive markets (Laffont and Tirole, 1993). When providers enjoy a degree of monopoly power, they can provide a lower volume of output than they would in a competitive situation and, thereby, secure higher prices. This causes welfare loss. Moreover, monopoly firms lack incentives to be cost efficient, thus undermining technical efficiency.

The utility of provider \( i - U_i \) is a function of the regulated price \( \hat{p}_i \), the marginal costs \( c_i \) and the altruism level \( \alpha_i \). We assume that altruistic and self-interested providers are distinguished by the degree to which they are concerned about consumer surplus, \( \alpha_i \).

\[
CS = \int_{p}^{\infty} q(x)dx
\]  

(1)

This is graphically represented by the area under the demand curve for their services, above their price. Recall that consumer surplus is decreasing in the unit price of the service, so that the greater the degree of altruism, the greater the utility providers derive from lower prices.

We further assume that the provider cares about consumer welfare to some proportion \( \alpha_i \) with \( i = 1, 2 \). Without loss of generality we assume that provider 2 is at least as altruistic as provider 1, i.e. \( \alpha_1 \leq \alpha_2 \). We further assume that providers benefit from slack, i.e. they derive utility from avoiding cost reducing effort (Bradford and Craycraft, 1996; Pope, 1989). The
benefit of slack $S(c_i)$ is an increasing function of cost at a decreasing rate (i.e. $S'(c_i) > 0, S''(c_i) < 0$). Thus the utility of each provider is given by the sum of net revenues, the benefit from slack, and the utility the provider derives from increased consumer welfare,

$$U_i(\hat{p}_i, c_i; \alpha_i) = (\hat{p}_i - c_i)q_i(\hat{p}_i) + S(c_i) + \alpha_i \int_{\hat{p}_i}^{\infty} q(x)d(x) \quad (2)$$

**Assumption 1:** For $i = 1, 2, i \neq -i$ we have: (i) $\partial U_i^2 / \partial c_i^2 < 0$; (ii) $|\partial U_i^2 / \partial c_i^2| > |\partial U_i^2 / \partial c_{-i}|$

Assumption (i) insures that $U_i$ is well behaved and therefore that the second order conditions for a maximum are met (it also ensures that the trace of the Jacobian matrix is negative); (ii) states that the own price effects on marginal utility are of greater magnitude than cross-price effects; (i) and (ii) ensure that the Jacobian determinant is positive that is a sufficient condition for the equilibrium to be stable.

### 2.1 The first best and free price scenario

For comparison purposes we first develop a first best benchmark. Consider a first best scenario by which the regulator can decide on both the price and the cost of each service. In each local market the optimum is then characterized by the pair $\{p_i^*, c_i^*\}$ that maximizes social utilitarian welfare $W(.)$ given by the sum of consumer surplus and the provider's utility\(^5\), i.e.:

$$W(p_i, c_i) = (1 + \alpha_i) \int_{p_i}^{\infty} q(x)d(x) + (p_i - c_i)q(p_i) + S(c_i) \quad (3)$$

with $i = 1, 2$.

Maximizing welfare with respect to price and cost, the social optimum\(^6\) is then given by the first order conditions (FOC henceforth) with respect to the price,

$$(p_i - c_i)q'(p_i) = \alpha_i q(p_i) \quad (4)$$

and with respect to the cost,

---

\(^5\) Note that consumer surplus shows twice in this utilitarian welfare function because some providers are altruistic. We have assumed a utilitarian welfare function as it is commonly used in the literature. Other functional forms would have an impact on our results but that is out of the scope of our analysis.

\(^6\) Social optimum solved in Appendix A1
According to (4) the optimal price should be such that the marginal net revenues due to an increase in the price equal the change in consumer surplus weighed by the altruistic parameter $\alpha_i$. Correspondingly (5) entails that the provider's marginal benefit from slack should be equal to the effect of increased costs on revenues. From (4), the socially optimal price rule can be written as:

$$\frac{p_i - c_i}{p_i} = -\frac{\alpha_i}{|\epsilon_i|}$$  \hspace{1cm} (4a)

With $\epsilon_i$ being the price elasticity.

For $0 < \alpha_i \leq 1$ we have a negative mark-up i.e. $p_i^* < c_i^*$ while for $\alpha_i = 0$ the mark-up is zero, i.e. $p_i^* = c_i^*$. For the existence of an interior solution the condition $\alpha_i < 1 - \frac{q'(p_i)}{S''(c_i)}$ must hold (ensures a negative definite Hessian).

Note that the optimal price differs with the level of altruism, so if $\alpha_1 \leq \alpha_2$ the price for the less altruistic provider is at least the same as the price for the more altruistic provider, i.e. $p_2^* \leq p_1^*$, while by (5) the first best cost of the more altruistic provider is lower than the cost of the least altruistic provider, i.e. $c_2^* \leq c_1^*$ (see Appendix A1).

Before proceeding with the analysis it is useful to evaluate a free price scenario. The following proposition summarizes the results.

**Proposition 1:** In a free price scenario the price and cost of the more altruistic provider are lower than those of the most altruistic provider. Furthermore providers’ prices and costs are higher than in the first best.

**Proof:** Proof in Appendix A2.

According to Proposition 1, in the absence of regulation the provider would optimally price higher than the socially optimal price. Therefore, any price cap regulation will bind in equilibrium.

2.2 The provider's problem

*Non-cooperative solution*
We will analyse two types of games. First we start by describing a setting in which providers strategically choose the cost level in a one shot game. In section 2.3 we characterize a repeated game.

In a one-shot non-cooperative game, each provider \( i \) maximises its utility by choosing the cost \( c_i \) given the price rule to which the regulator will commit. Provider \( i \)’s problem is given by,

\[
\max_{c_i} U_i(\hat{p}_i, c_i; \alpha_i) = (\hat{p}_i - c_i)q_i(\hat{p}_i) + S(c_i) + \alpha_i \int_{\hat{p}_i}^{\infty} q(x)d(x) \tag{6}
\]

Since we are considering a two-agent model, the yardstick rule is such that provider \( i \) faces a price per service that is equal to the competitor’s \((-i)\) marginal cost in providing the same service, i.e. \( \hat{p}_i = c_{-i} \).

The FOC with respect to cost \( \partial U_i/\partial c_i \), is given by:

\[
\frac{\partial U_i}{\partial c_i} = \left( \frac{\partial \hat{p}_i}{\partial c_i} - 1 \right) q(\hat{p}_i) + (\hat{p}_i - c_i)q'(\hat{p}_i) \frac{\partial \hat{p}_i}{\partial c_i} + S'(c_i) - \alpha_i q(\hat{p}_i) \frac{\partial \hat{p}_i}{\partial c_i} = 0 \tag{7}
\]

**Proposition 2**: Under a non-cooperative the equilibrium is such that providers optimally choose the same level of costs, i.e. \( c_1^{nc} = c_2^{nc} = c^{nc} \), the cost does not change with the altruism level. For \( \alpha_1 \leq \alpha_2 \forall \alpha_i \geq 0 \), \( c_1^f \geq c_2^f \geq c_1^{nc} \geq c_2^{nc} \geq c_1^e \geq c_2^e \), while \( p_1^f \geq p_2^f \geq p_1^{nc} = p_2^{nc} \geq p_1^e \geq p_2^e \).

**Proof**: Proof in Appendix A3.

**Corollary 1**: When providers are purely altruistic i.e. \( \alpha_1 = \alpha_2 = 1 \) then \( p^f = c^f = c^{nc} = p^{nc} > c^* > p^* \). When providers are purely self-interested i.e. \( \alpha_1 = \alpha_2 = 0 \), then \( p^f > c^f > c^{nc} = p^{nc} = c^* = p^* \).

**Proof**: Proof in Appendix A3.

The scenario under which providers are purely self-interested \( (\alpha_1 = \alpha_2 = 0) \) is akin to Scheiffer’s (1985) original model and the first best price coincides with the yardstick price the regulator has committed to. It follows that under such regulated price while a provider’s cost reduction leads to a reduced price faced by the other provider, it does not adversely affect its own price. This arrangement gives both providers strong incentives to operate at a socially optimal cost level. Take the more altruistic provider \((i = 2)\), which affords greater weight to consumer surplus. The price this provider faces depends on the costs of the other
provider, implying that the consumer surplus has less influence on its own choice of costs. The opposite rationale holds for the more self-interested provider. These results hold independently of the degree of altruism. Indeed, it is straightforward to show that 
\[ \frac{dc_i^{nc}}{d\alpha_i} = \frac{dc_i^{nc}}{d\alpha_{-i}} = 0 \] (see Appendix A3).
However, for any other levels of altruism the regulated price is no longer set according to the first best price rule (indeed in the first best \( p_i^* < c_i^*, \forall \alpha_i \neq 0, i = 1,2 \)). Consequently as the yardstick price is a weaker regulatory instrument when compared to \( p_i^* \) it follows that providers costs levels will be higher than in the first best, i.e. \( c_1^{nc} = c_2^{nc} > c_i^* > c_2^* \).

**Cooperative solution**

Still on a one shot game, we will now characterize the cooperative solution within which providers maximize their joint utility \( U = \sum_i U_i \). The advantage of agreeing on a strategy is that the providers can avoid “competing” against each other in lowering their production costs. Collusion allows providers to limit their cost reducing effort while receiving a higher price for their services. Offsetting these benefits, there are the negative effects resulting from lower demand as well as reduced consumer surplus (which affects utility in proportion \( \alpha_i \)). Thus, the final outcome will depend on the balance of these effects. Letting the superscript \( c \) indicate the cooperative solution, the following proposition summarises the results in a cooperative scenario.

**Proposition 3**: In a cooperative scenario, for \( \alpha_2 \geq \alpha_1 \) and \( \alpha_i \in [0,1] \) for \( i = \{1,2\} \) the providers’ cost strategies are such that \( c_2^c \geq c_1^c, \forall \alpha_i \in [0,1] i = 1,2 \ and \alpha_1 \leq \alpha_2 \). With asymmetric levels of altruism \( c_2^c > c_1^c > c_i^{nc} > c_i^* \) for \( i = 1,2 \). Providers cost strategies decrease on both levels of altruism (i.e. \( dc_i / d\alpha_{-i} < 0, dc_i / d\alpha_i < 0 \)).

**Proof**: Proof in Appendix A4.

**Corollary 2**: In the case of homogeneous purely altruistic providers, i.e. \( \alpha_2 = \alpha_1 = 1 \) cost strategies are such that \( c_2^c = c_1^c = c_i^{nc} > c_i^* \). In the case of homogeneous purely self-interested providers, i.e. \( \alpha_2 = \alpha_1 = 0 \) it follows \( c_2^c = c_i^c > c_i^{nc} > c_i^* \).

**Proof**: Proof in Appendix A4.

Note that, in a cooperative scenario, providers maximize joint surplus and therefore provider
i’s decision rule displays provider -i’s altruism level. It follows that the providers’ optimization problem is symmetric apart from the differences between providers’ altruistic levels namely \(-\alpha_2 q(c_i)\) and \(-\alpha_i q(c_2)\). This implies that the costs of one provider decrease in relation to the level of altruism displayed by the other.

As in the non-cooperative solution, the more altruistic provider cannot influence the consumer surplus it produces as this depends solely on the cost chosen by the other, less altruistic, provider. It can impact, though, on the other provider’s consumer surplus even if this weighs less in the optimal decision rule. The situation under this yardstick regime is akin to the two providers swapping their roles. Indeed, even though provider 2 is more altruistic than provider 1, a situation of pure collusion is such that provider 1 exhibits the strongest cost response in order to reflect the impact of costs on consumer surplus. In this way provider 2 can afford a higher cost. This higher cost will allow provider 1 a higher yardstick price that will counterbalance the decreased benefit from slackness caused by a lower cost.

For a given \(c'_i\), we note that the cooperative and the non-cooperative best responses of provider \(i\), are given respectively by:

\[
\frac{\partial J_U}{\partial c_i} = -q(c_{-i}) + S'(c_i) + (c_i - c_{-i})q'(c_i) + (1-\alpha_{-i})q(c_i) = 0 \quad i = 1, 2 \tag{8}
\]

and

\[
\frac{\partial U_i}{\partial c_i} = S'(c_i) - q(c_{-i}) = 0 \tag{9}
\]

These expressions differ in the quantity \((c_i - c_{-i})q'(c_i) + (1-\alpha_{-i})q(c_i)\). The term \((1-\alpha_{-i})q(c_i)\) is the net effect that provider \(i\)'s cost directly has on provider \(-i\)'s revenues. The term \((c_i - c_{-i})q'(c_i)\) is the effect of a unit of provider \(i\)'s cost on the joint surplus as determined through the demand function. We note that, with regard to provider 1, the impact is positive because \(c'_1 < c'_2\). Thus we can conclude that, for a given \(c_2\), the cooperative strategy of the more self-interested provider 1 is that it will operate at a higher cost than in the non-cooperative scenario. It can be shown (see Appendix A4) that this result holds also for the more altruistic provider 2.
If the market is served by two purely altruistic providers, the costs will be the same as in the non-cooperative scenario.

To summarise the results, given that the consumer surplus depends on the regulated price and given that the regulatory scheme sets \( \hat{p}_i = c_{-i} \), the maximization of the joint utilities (JU) is such that provider \( i \)'s choice will affect provider \(-i\)'s consumer surplus. It follows that provider \( i \) makes a decision on costs bearing in mind the altruism level of the other provider.

**Defection solution**

This cooperative solution can never be sustainable in a one shot game. Indeed, consider provider \( i \). If this provider defects from the cooperative agreement (considering that provider \(-i\) plays according to the cooperative strategy), then it will revert to behaving according to the best response function as in (9) with the optimal defection cost \( c^d_i \) (where the superscript \( d \) indicates defection) satisfying:

\[
-q(c^c_{-i}) + S'(c^d_i) = 0
\]

**Proposition 4**: Provider \( i \)'s defection cost lies between the optimal non-cooperative and the cooperative strategies i.e. \( c^*_i \leq c^{nc}_i \leq c^d_i \leq c^c_i \). Furthermore \( c^d_2 \leq c^d_i \) for \( \alpha_1 \leq \alpha_2 \) and the defection costing strategies are decreasing with both providers altruism level, i.e. \( \frac{\partial c^d_i}{\partial \alpha_i} < 0 \), \( \frac{\partial c^d_i}{\partial \alpha_{-i}} < 0 \) for \( i = \{1,2\}, -i = \{1,2\}, i \neq -i \).

**Proof**: Proof in Appendix A5.

Intuitively provider \( i \)'s best response is \( c^d_i < c^c_i \), given the choice of the other, and it would still face a higher price and therefore increase its surplus. The provider’s decision is based on the maximization of its own utility and the FOC will coincide with the non-cooperative FOC. However the defection level will differ from the non-cooperative level as provider \(-i\) is still playing the cooperative solution.

Given the latter, since \( q'(\cdot) < 0 \) the negative impact of the yardstick price on providers’ profit through the cost of providing the service (i.e. \( -q(c^c_{-i}) \)) is higher in absolute value for provider 2 (since in the cooperative scenario the demand for this provider is higher than for
provider 1) moreover since \( S''(.) < 0 \) the cost of reducing the slack is lower for provider 2. Therefore these imply that for provider 2 it pays off more to decrease its marginal cost of providing the service (even if that implies a reduction in the benefit from slack) than for provider 1 and therefore the deviation strategies in equilibrium are such that \( c_2^d \leq c_1^d \).

Given that the cooperative costing strategies decrease with the altruism level, then it follows that the cost of providing the service for each provider also increases with the altruism due to an increase in the demand (i.e. \(-q(c_i^e) \) is bigger in absolute value for higher levels of altruism). Therefore both providers will need to deviate further from the cooperative agreement in order to compensate for this impact of increased altruism on the cost of providing the service.

To sum up we have shown that in a one shot game deviation from the cooperative agreement is always profitable and, consequently, collusion is never sustainable. Therefore the one shot Nash equilibrium is non-cooperative. This result is consistent with the findings of the existent literature (Tirole, 1988).

2.3 Repeated game: Incentives to collude

Let us consider a repeated game in which the providers can play grim trigger strategies (Friedman, 1971). At the beginning of each period the two providers choose the cost level and act according to the following trigger strategies. If one of them defects in some period \( t \), by choosing a cost level \( c_i^d \neq c_i^e \), then in any subsequent period the other provider reverts to play her best response to defection from that point onwards. This is a typical "trigger strategy", whereby if a provider deviates from the cooperative agreement all providers revert to the one shot Nash equilibrium from thereon. Therefore, in deciding whether to stick to the cooperative agreement, a provider compares the stream of profits of cooperating \( U_i^c / (1 - \delta_i) \) with the stream of profits obtained by deviating i.e. \( U_i^d + \delta_i U_i^{nc} / (1 - \delta_i) \). It is easy to show that collusion is sustainable for provider \( i \) if and only if \( \delta_i \geq (U_i^d - U_i^c) / (U_i^d - U_i^{nc}) \) where \( U_i^{nc} \) is the equilibrium payoff provider \( i \) receives in the non-cooperative scenario, \( U_i^c \) is the payoff gained in collusion and \( U_i^d \) is the payoff obtained in defection. The outcome depends on the individual discount rate \( \delta_i \in [0,1] \), that represents the extent to which each provider considers short term profits more valuable than profits accrued later in time. The higher the rate the lower is each provider's incentive to collude. Therefore it follows that collusion is
sustainable for \( \delta \geq \delta^* \)
\[
\delta^* = \max\{\delta_i, \delta_{-i}\}
\]

With,
\[
\delta_i \geq \frac{U_i^d - U_i^c}{U_i^d - \gamma_i} \quad \text{and} \quad \delta_{-i} \geq \frac{U_{-i}^d - U_{-i}^c}{U_{-i}^d - \gamma_{-i}}
\]

When the market is served by two purely altruistic providers, i.e. \( \alpha_i \to 1 \) it is easy to see that providers have no incentive to collude since \( U_i^c \to U_i^{nc} \). In fact providers do not have an incentive to deviate from the non-cooperative cost under joint profit maximization.

[Figure 1 in here]

If providers are homogeneous and self-interested, it follows that since \( \delta_1 = \delta_2 = \delta \) and \( \delta \) decreases with the altruism level. Collusion in such a market is more likely than in a market served by two purely altruistic providers.

In heterogeneous markets collusion stability will depend on \( \delta^* \). Sustainability of collusion in the presence of asymmetric providers depends on the shape of the demand and slack functions. In particular for the more altruistic provider (provider 2) cooperation is profitable only if the benefit from slack is big enough to offset the financial loss and the decrease in consumer surplus that more altruistic firms have to bear in cooperation, i.e. \( U_{2}^{nc} < U_{2}^{c} \) holds if and only if:

\[
( S(\xi_{2}^{nc}) - S(\xi_{2}^{c}) ) \leq (c_{i}^{c} - c_{2}^{c})q(c_{i}^{c}) - \alpha_{2} \int_{\xi_{i}^{nc}}^{\infty} q(x) \, dx + \alpha_{2} \int_{\xi_{i}^{c}}^{\infty} q(x) \, dx \quad (11)
\]

If this condition is not verified, if provider 2 were to collude he would sustain a loss with respect to the non-cooperative strategy in every single period, i.e. \( \delta_2 \geq 1 \) implying that there would be no collusion (see Figure 2).

[Figure 2 in here]

In order to infer how \( \delta_i \) changes according to the level of altruism we performed a comparative static analysis to study how provider \( i \)'s rate changes with their own and the competitors degree of altruism. Proposition 5 summarizes the results.
**Proposition 5:** A provider’s incentive to collude is a decreasing function of its own level of altruism and an increasing function of its competitor’s altruism.

**Proof:** Proof in Appendix A7.

Departing from a homogeneous sector in which providers have the same level of altruism, increasing one provider’s level of altruism decreases its incentive to collude in the market, while increasing its competitor level of altruism increases the incentive to collude. Thus in a heterogeneous market, the altruistic provider’s incentive to collude\(^7\) is lower than it would be in a market where providers have the same level of altruism. Intuitively it might be that homogeneous providers find it easier and more profitable to collude because of their symmetric objectives.

With regards to the more self-interested provider 1, similarly it can be shown that, as provider 2’s degree of altruism increases, provider 1’s rate decreases. Thus in a heterogeneous market, the self-interested provider’s incentive to collude is higher than it would be in a market where providers have the same level of altruism.

Comparing \(\delta_i\) across providers it is easy to show that \(\delta_2 \geq \delta_1\) for \(\alpha_2 \geq \alpha_1\) (see Appendix A7) therefore as long as (11) is verified there is scope for collusion in heterogeneous markets and collusion is less (more) likely when the altruism of the more (less) altruistic provider increases (decreases).

**Corollary 3:** For a given level of \(\alpha_i\), since \(\delta_2 \geq \delta_1\) collusion is more likely to be sustained in homogeneous than in heterogeneous markets.

**Proof:** Proof in Appendix A7.

This result is in line with the existing literature that has shown that asymmetries between providers are an obstacle to collusion (see for e.g. Scherer, 1970; Barla, 2000; Compte and Ray, 2002). Given that in many contexts policy has been to encourage the entry of private providers in traditionally public settings these results suggest that increasing the plurality in

\(^7\) Note that here we are merely referring to the individual incentive to stick with the cooperative agreement rather than the sustainability of collusion as that will depend on the actions of both providers considered simultaneously. That analysis follows in the paper.
service provision renders collusion less likely to occur.
Finally, when comparing pure altruistic homogeneous markets with pure self-interested markets, we notice collusion on costs higher than the non-cooperative costs is more likely in the latter.

3. Conclusion

A potential drawback with yardstick competition regulation is that it might be susceptible to collusion, because by colluding on higher costs, providers may be able to secure a higher price for their services. We find that the incentive will depend on the degree to which provider objectives correspond to those of the regulator under yardstick competition arrangements.

We generalize the literature analysis by allowing for provider heterogeneity in their degree of altruism. By relaxing the assumption of provider homogeneity we are able to explore a fundamental change in the provision of public services where greater plurality is being encouraged. For example, traditionally public health systems such as England, France, Portugal and Italy are encouraging more private sector organisations to enter the health care market. Similarly many countries have de-nationalised many other public services, either wholly or in part. The important qualitatively different results obtained by our framework indicate that market structure should be considered when designing yardstick competition arrangements.

Our analysis demonstrates that it is important to consider the composition of the market when designing yardstick competition arrangements. We show that in markets served by purely altruistic providers there is no collusion on costs while in markets served by purely self-interested providers there is scope for collusion. We show that collusion is more stable in homogeneous than in heterogeneous markets, i.e. departing from a scenario where providers are homogeneous, we find that a change in the altruism of one provider decreases the stability of collusion in a repeated game. To sum-up, the incentives to collude depend on the extent to which providers share similar objectives. With pluralistic markets being encouraged in many countries and sectors of the economy it is increasingly important that provider heterogeneity is taken into account when designing regulatory policies.
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A Appendix

A1 First Best Solution

The solution to the following optimization program,

$$\max_{p, c} W(p_i, c_i) = (1 + \alpha_i) \int_{p_i}^{\infty} q(x)dx + (p_i - c_i)q(p_i) + S(c_i)$$

must satisfy the following FOCs:

$$\frac{\partial W_i}{\partial p_i} = -q(p_i) - \alpha_i q(p_i) + q(p_i) + (p_i - c_i)q'(p_i) = 0$$

$$\frac{\partial W_i}{\partial p_i} = -\alpha_i q(p_i) + (p_i - c_i)q'(p_i) = 0 \quad (A1.1)$$

$$\Rightarrow p_i = c_i + \alpha_i \frac{q(p_i)}{q'(p_i)}$$

$$\frac{\partial W_i}{\partial c_i} = -q(p_i) + S'(c_i) = 0 \quad (A1.2)$$

So that \( p^* \) and \( c^* \) denote the optimal level of price and cost that solve the system of equations defined by (A1.1) and (A1.2).

With some manipulation and dividing both sides of (A1.1) by \( p_i \) we obtain

$$\frac{p_i - c_i}{p_i} = -\frac{\alpha_i}{|\varepsilon_i|} \quad (A1.1a)$$

With \( \varepsilon_i \) the price elasticity. For \( 0 < \alpha_i \leq 1 \) we have a negative mark up \( p_i \leq c_i \).

Note that \( p_i^* < c_i^* \) and for different levels of altruism regulator fixes different prices for different firms, so if \( \alpha_1 \leq \alpha_2 \) we have \( p_2^* \leq p_1^* \) and by equation (A1.2) \( c_2^* \leq c_1^* \).

Indeed, let,

$$F_i \equiv p_i - c_i - \alpha_i \frac{q(p_i)}{q'(p_i)} = 0$$

$$F_2 \equiv -q(p_i) + S'(c_i) = 0$$

Total differentiation leads to:
\[
\begin{align*}
\frac{\partial F_1}{\partial c_i} + \frac{\partial F_1}{\partial p_i} + \frac{\partial F_1}{\partial \alpha_i} d\alpha_i &= 0 \\
\frac{\partial F_2}{\partial c_i} + \frac{\partial F_2}{\partial p_i} + \frac{\partial F_2}{\partial \alpha_i} d\alpha_i &= 0
\end{align*}
\]

In matrix format:
\[
\begin{bmatrix}
\frac{\partial F_1}{\partial c_i} & \frac{\partial F_1}{\partial p_i} \\
\frac{\partial F_2}{\partial c_i} & \frac{\partial F_2}{\partial p_i}
\end{bmatrix}
\begin{bmatrix}
\frac{\partial c_i}{\partial \alpha_i} \\
\frac{\partial p_i}{\partial \alpha_i}
\end{bmatrix}
= \begin{bmatrix}
-\frac{\partial F_1}{\partial \alpha_i} \\
-\frac{\partial F_2}{\partial \alpha_i}
\end{bmatrix}
\]

Using Cramer’s rule we obtain:
\[
\frac{\partial c_i}{\partial \alpha_i} = \frac{A_1}{J}, \quad \frac{\partial p_i}{\partial \alpha_i} = \frac{A_2}{J}
\]

Where:
\[
J = \begin{vmatrix}
\frac{\partial F_1}{\partial c_i} & \frac{\partial F_1}{\partial p_i} \\
\frac{\partial F_2}{\partial c_i} & \frac{\partial F_2}{\partial p_i}
\end{vmatrix}

A_1 = \begin{vmatrix}
\frac{\partial F_1}{\partial c_i} & \frac{\partial F_1}{\partial p_i} \\
\frac{\partial F_2}{\partial c_i} & \frac{\partial F_2}{\partial p_i}
\end{vmatrix}

A_2 = \begin{vmatrix}
\frac{\partial F_1}{\partial c_i} & -\frac{\partial F_1}{\partial \alpha_i} \\
\frac{\partial F_2}{\partial c_i} & -\frac{\partial F_2}{\partial \alpha_i}
\end{vmatrix}
\]

Therefore computing the partial derivatives in A_1, A_2 and J:
\[
\frac{\partial F_1}{\partial p_i} = 1 - \alpha_i \left[ \frac{q'(p_i)^2 - q''(p_i)q(p_i)}{q'(p_i)^2} \right] \leq 1
\]
\[
\frac{\partial F_1}{\partial \alpha_i} = -\frac{q(p_i)}{q'(p_i)} > 0
\]
\[
\frac{\partial F_2}{\partial c_i} = S''(c_i) < 0
\]
\[
\frac{\partial F_2}{\partial p_i} = -q'(p_i) > 0
\]
\[
\frac{\partial F_2}{\partial \alpha_i} = 0
\]

In the case of a linear demand function we have \[
\frac{\partial F_1}{\partial p_i} = 1 - \alpha_i
\]

And
\[
J = q'(p_i) - (1 - \alpha_i)S''(c_i)
\]

From (A.1.2) we know that \(|q'(p_i)| \leq |S''(c_i)|\) therefore if \(\alpha_i < 1 - \frac{q'(p_i)}{S''(c_i)}\) then J is positive. The condition \(\alpha_i < 1 - \frac{q'(p_i)}{S''(c_i)}\) ensures a negative definite Hessian matrix (for the existence of a maximum) and therefore an interior solution.

With \(J > 0\) the sign of \(\frac{\partial c_i}{\partial \alpha_i}, \frac{\partial p_i}{\partial \alpha_i}\) will be determined by the sign of \(A_1\) and \(A_2\):
\[
A_1 = (-\frac{\partial F_1}{\partial \alpha_i} ) (\frac{\partial F_2}{\partial p_i}) - (\frac{\partial F_2}{\partial \alpha_i} ) (\frac{\partial F_1}{\partial c_i}) = -q(p_i) < 0
\]
\[
A_2 = (-\frac{\partial F_1}{\partial c_i} ) (\frac{\partial F_2}{\partial \alpha_i}) - (-\frac{\partial F_1}{\partial \alpha_i} ) (\frac{\partial F_2}{\partial c_i}) = -\frac{q(p_i)}{q'(p_i)} S''(c_i) < 0
\]
Therefore it follows that:

$$\frac{\partial c_i}{\partial \alpha_i} < 0, \frac{\partial p_i}{\partial \alpha_i} < 0$$

therefore for $\alpha_1 \leq \alpha_2$ we have that $p_2^* \leq p_1^*$ and $c_2^* \leq c_1^*$
A2 Free price scenario

**Proof of Proposition 1:** Maximizing

\[ U(p_i, c_i, \alpha_i) = (p_i - c_i)q(p_i) + S(c_i) + \alpha_i \int p \rightarrow \infty q(x)dx \]  

(A2.1)

with respect to \( p_i \) and \( c_i \) the optimal price \( p_i^f \) and cost \( c_i^f \) are the solution for the following FOCs: are:

\[ \frac{\partial U_i}{\partial p_i} = (p_i - c_i)q'(p_i) - \alpha_i q(p_i) + q(p_i) \]  

(A2.2)

\[ \frac{\partial U_i}{\partial c_i} = -q(p_i) + S'(c_i) = 0 \]  

(A2.3)

Rearranging (A2.2):

\[ \frac{p_i - c_i}{p_i} = \frac{1 - \alpha_i}{|\varepsilon_i|} \]  

(A2.2a)

We have a positive mark up, if \( \alpha_i < 1 \) (i.e. \( p_i^f > c_i^f \)); zero mark-up for \( \alpha_i = 1 \) (i.e. \( p_i^f = c_i^f \)). In the latter when \( \alpha_i = 1 \) the free price solution is the same as in the first best solution with zero altruism.

Comparing (A2.2a) with (A1.1a) \( \forall \alpha_i, -\frac{\alpha_i}{|\varepsilon_i|} \leq 0 \) and \( \frac{1-\alpha_i}{|\varepsilon_i|} \geq 0, \) it follows that \( p_i^f > p_i^* \).

Moreover given that the mark-up decreases with the altruism level it follows that \( p_1^f \geq p_2^f \) for \( \alpha_1 \leq \alpha_2 \). Furthermore given (A2.3) it follows that \( c_1^f \geq c_2^f \).

Indeed, let,

\[ F_1 = p_i - c_i - \alpha_i \frac{q(p_i)}{q'(p_i)} + \frac{q(p_i)}{q'(p_i)} = 0 \]

\[ F_2 = -q(c_i) + S'(c_i) = 0 \]

Total differentiation leads to:

\[ \begin{cases} \frac{\partial F_1}{\partial c_i} dc_i + \frac{\partial F_1}{\partial p_i} dp_i + \frac{\partial F_1}{\partial \alpha_i} d\alpha_i = 0 \\ \frac{\partial F_2}{\partial c_i} dc_i + \frac{\partial F_2}{\partial p_i} dp_i + \frac{\partial F_2}{\partial \alpha_i} d\alpha_i = 0 \end{cases} \]

\[ \Longrightarrow \begin{cases} \frac{\partial F_1}{\partial c_i} dc_i + \frac{\partial F_1}{\partial p_i} dp_i + \frac{\partial F_1}{\partial \alpha_i} d\alpha_i = -\frac{\partial F_1}{\partial \alpha_i} \\ \frac{\partial F_2}{\partial c_i} dc_i + \frac{\partial F_2}{\partial p_i} dp_i + \frac{\partial F_2}{\partial \alpha_i} d\alpha_i = -\frac{\partial F_2}{\partial \alpha_i} \end{cases} \]

In matrix format:
\[
\begin{bmatrix}
\partial F_i / \partial c_i & \partial F_i / \partial p_i \\
\partial F_2 / \partial c_i & \partial F_2 / \partial p_i \\
\end{bmatrix}
\begin{bmatrix}
\partial c_i / \partial \alpha_i \\
\partial p_i / \partial \alpha_i \\
\end{bmatrix} = \begin{bmatrix}
-\partial F_i / \partial \alpha_i \\
-\partial F_2 / \partial \alpha_i \\
\end{bmatrix}
\]

Using Cramer’s rule we obtain:
\[
\frac{\partial c_i}{\partial \alpha_i} = \frac{A_1}{J}, \quad \frac{\partial p_i}{\partial \alpha_i} = \frac{A_2}{J}
\]

Where:
\[
J = \begin{bmatrix}
\partial F_1 / \partial c_i & \partial F_1 / \partial p_i \\
\partial F_2 / \partial c_i & \partial F_2 / \partial p_i \\
\end{bmatrix}, \quad A_1 = \begin{bmatrix}
-\partial F_i / \partial \alpha_i & \partial F_1 / \partial \alpha_i \\
-\partial F_2 / \partial \alpha_i & \partial F_2 / \partial \alpha_i \\
\end{bmatrix}, \quad A_2 = \begin{bmatrix}
-\partial F_i / \partial \alpha_i & \partial F_1 / \partial \alpha_i \\
-\partial F_2 / \partial \alpha_i & \partial F_2 / \partial \alpha_i \\
\end{bmatrix}
\]

Therefore computing the partial derivatives in \( A_1, A_2 \) and \( J \):
\[
\partial F_i / \partial c_i = -1 < 0
\]
\[
\partial F_i / \partial p_i = 1 - \alpha_i \left( \frac{q'(p_i)^2 - q''(p_i)q(p_i)}{q'(p_i)^2} \right) + \left( \frac{q'(p_i)^2 - q''(p_i)q(p_i)}{q'(p_i)^2} \right)
\]
\[
\partial F_i / \partial \alpha_i = -\frac{q(p_i)}{q'(p_i)} < 0
\]
\[
\partial F_2 / \partial c_i = S''(c_i) < 0
\]
\[
\partial F_2 / \partial p_i = -q'(p_i) > 0
\]
\[
\partial F_2 / \partial \alpha_i = 0
\]

In the case of a linear demand function we have \( \frac{\partial F_i}{\partial p_i} = 2 - \alpha_i \)

And,
\[
J = q'(p_i) - (2 - \alpha_i)S''(c_i)
\]

From (A.1.2) we know that \( q'(p_i) \leq S'(c_i) \), therefore if \( \alpha_i \geq 0 \) \( J \) is positive. With \( J > 0 \) the sign of \( \partial c_i / \partial \alpha_i, \partial p_i / \partial \alpha_i \) will be determined by the sign of \( A_1 \) and \( A_2 \). Since,
\[
A_1 = \left( -\partial F_i / \partial \alpha_i \right) \left( \partial F_2 / \partial p_i \right) - \left( \partial F_2 / \partial \alpha_i \right) \left( \partial F_1 / \partial c_i \right) = -q(p_i) < 0
\]
\[
A_2 = \left( -\partial F_i / \partial \alpha_i \right) \left( \partial F_2 / \partial \alpha_i \right) - \left( -\partial F_1 / \partial \alpha_i \right) \left( \partial F_2 / \partial c_i \right) = -\frac{q(p_i)}{q'(p_i)} S''(c_i) < 0
\]

It follows that:
\[
\partial c_i / \partial \alpha_i < 0, \partial p_i / \partial \alpha_i < 0
\]

Therefore, for \( \alpha_1 \leq \alpha_2 \) we have that \( p^*_2 \leq p^*_1 \) and \( c^*_2 \leq c^*_1 \)

Furthermore since (A2.3) is the same as (A1.2). Knowing that \( p^*_i > p^*_i \) since \( q'(p_i) < 0 \) and \( S''(c_i) < 0 \) it follows from (A2.3) that \( c^*_i > c^*_i \).
A3  Non-Cooperative Scenario: The Provider’s problem

Proof of Proposition 2 and Corollary 1

Given the utility function being maximized (A2.1), the FOCs with respect to cost $c_i$ are given by,

$$\frac{\partial U_i}{\partial c_i} = \left(\frac{\partial \hat{p}_i}{\partial c_i} - 1\right)q(\hat{p}_i) + (\hat{p}_i - c_i)\frac{\partial q(\hat{p}_i)}{\partial \hat{p}_i} + S'(c_i) - \alpha_i q(\hat{p}_i) \frac{\partial \hat{p}_i}{\partial c_i} = 0 \quad \text{(A3.1)}$$

$\forall \alpha_i \in [0,1], i = (1,2)$.

Given the regulatory rule $\hat{p}_i = c_{-i}$ (where $\hat{p}_i$ is the regulated price), (A3.1) becomes,

$$\frac{\partial U_i}{\partial c_i} = -q(c_{-i}) + S'(c_i) = 0 \quad \forall \alpha_i \in [0,1], i = (1,2) \quad \text{(A3.2)}$$

Given assumption 1, for $i,-i = \{1,2\}, i \neq -i$ we have: (i) $\partial U_i^2 / \partial c_i^2 < 0$; (ii) $|\partial U_i^2 / \partial c_i^2| > |\partial U_i^2 / \partial c_{-i} \partial c_{-i}|$, i.e. $|S''(c_i)| < |q'(c_{-i})|$.

From (A3.2) we see that price do not depend on altruism and the price is given under regulated price. Therefore the condition for profit maximization (A3.2) becomes:

$$S'(c_i) = q(c_{-i})$$

We argue that the optimal solution $(c^{nc}_1, c^{nc}_2)$ is the symmetric solution to (A3.2), i.e.

$c^{nc}_i = c^{nc}_{-i} = c^{nc}$.

To prove that there is no asymmetric solution we will look for profitable deviations from the asymmetric equilibrium. Consider the asymmetric solution $c^{nc}_1 > c^{nc}_2 = c^{nc}$.

Suppose that firm 1 decreases its cost by $\Delta c_1$. From (3) we can see that firm 1 by varying $c_1$ in $\Delta c_1$ gains $q(c^{nc}_2)\Delta c_1$ at the cost of $S'(c_1)\Delta c_1$. By the SOCs we know that $S'(c_1) < q(c^{nc}_2)$ therefore it is always profitable for firm 1 to decrease its cost.

Now suppose that firm 1 increases its cost by $\Delta c_1$. From (3) we can see that firm 1 by varying $c_1$ in $\Delta c_1$ gains $S'(c_1)\Delta c_1$ at the cost of $q(c^{nc}_2)\Delta c_1$. Given that by SOCs $S'(c_1) < q(c^{nc}_2)$ then it is not profitable for firm 1 to increase its cost.

Consider the asymmetric solution $c^{nc}_1 < c^{nc}_2 = c^{nc}$. 


Suppose that firm $i$ deviates by decreasing its cost by $\Delta c_i$. From (3) we can see that firm 1 by varying $c_1$ in $\Delta c_1$ gains $q(c_{2\text{nc}}^n)\Delta c_1$ at the cost of $S'(c_1)\Delta c_1$. Now for $c_1^{nc} < c_{2\text{nc}}^{nc} = c_{nc}$ and given the SOCs we know that $S'(c_1) > q(c_{2\text{nc}}^{nc})$ therefore it is never profitable to deviate by decreasing the cost.

Now suppose that firm $i$ increases its cost by $\Delta c_i$. From (3) we can see that by varying $c_1$ in $\Delta c_1$ it gains $S'(c_1)\Delta c_1$ at the cost of $q(c_{2\text{nc}}^{nc})\Delta c_1$. Now for $c_1^{nc} < c_{2\text{nc}}^{nc} = c_{nc}$ and given the SOCs we know that $S'(c_i) > q(c_{2\text{nc}}^{nc})$ therefore it is always profitable to deviate by increasing the cost.

Therefore the only possible solution is the symmetric equilibrium $c_1^{nc} = c_{2\text{nc}}^{nc} = c_{nc}$.

Comparing costs and prices across the different scenarios.

First note that in the non-comparative scenario $c_{nc}$ does not vary with the altruism level (neither the price given the price rule $\hat{p}_i = c_{-i}$) while prices and costs in the first best and free price scenarios (i.e. $c_i^f, c_i^t, p_i^f$ and $p_i^t$) are all decreasing in the level of altruism (see proof above).

**Symmetric case**

For the case of selfish firms, i.e. $\alpha_i = \alpha_{-i} = 0$, then $p_f > c_f > c^* = p^* = p_{nc} = c_{nc}$ so that prices and costs are the same in the first best and non-cooperative scenario and higher in the free price scenario. Indeed, comparing the non-cooperative with the first best solution. We know that for $\alpha_i = 0 \forall i = \{1, 2\}$, from (A1.1) it follows that $p_i^* = c_i^*$. Therefore given that the cost FOC in the non-cooperative scenario (A3.2) is equal to the cost FOC in the first best (A1.2) it follows that $c_i^{nc} = c_i^* = p_i^*$. The remainder inequalities have been demonstrated above.

When $\alpha_i = \alpha_{-i} = 1$, then $p_f = c_f = p_{nc} = c_{nc} > c^* > p^*$, in this case the free price scenario and the non-cooperative scenario under yardstick competition give the same result.

Indeed, suppose now that $\alpha_i$ increases from zero such that $\alpha_i > 0 \forall i = \{1, 2\}$. We know that, for $\alpha_i > 0 \forall i = \{1, 2\}$ and from (A1.1) $p_i^* < c_i^*$. For FOC (A1.2) to hold it must follow that $c_i^{nc}\mid_{\alpha_i = 0} > c_i^*\mid_{\alpha_i = 0}$. Since we know that $c_i^*\mid_{\alpha_i = 0} = c_i^{nc}$ it follows that $c_i^{nc} > c_i^*\mid_{\alpha_i = 0}$. 
Comparing the non-cooperative with the free price scenario. We know that for \( \alpha_i = 1 \forall i = \{1, 2\} \), from (A2.2) it follows that \( p_i^f = c_i^f \). Therefore given that the cost FOC in the non-cooperative scenario (A3.2) is equal to the cost FOC in free price scenario (A2.3) it follows that \( c_i^f = p_i^f = c_i^{nc} = p_i^{nc} \). The remainder inequalities have been demonstrated above.

**Asymmetric case**

Finally for \( 0 \leq \alpha_1 \leq \alpha_2 \leq 1 \) we know that:

a. From the non-cooperative scenario (appendix A3) \( p_1^{nc} = p_2^{nc} = c_1^{nc} = c_2^{nc} \)

b. From appendix A1 we know that \( c_i^* \geq c_2^* \)

c. From appendix A2 that \( c_1^f \geq c_2^f \geq c_i^* \geq c_2^* \)

d. Above in this section we have also shown that \( c_1^{nc} = c_2^{nc} \geq c_i^* \geq c_2^* \)

e. For \( \alpha_i < 1 \forall i = \{1, 2\} \), Comparing the free price with the non-cooperative scenario, from (A2.2) it follows that \( p_i^f > c_i^f \). Therefore given that the cost FOC in the non-cooperative scenario (A3.2) is equal to the cost FOC in free price scenario (A2.3) it follows that \( c_1^f > c_2^{nc} = c_2^{nc} = p_1^{nc} = p_2^{nc} \).

f. For \( \alpha_i = 1 \forall i = \{1, 2\} \), as shown above we know that \( c_i^f = p_i^f = c_i^{nc} = p_i^{nc} \).

g. Given c), e) and f) it follows \( c_1^f \geq c_2^f \geq c_1^{nc} = c_2^{nc} = p_1^{nc} = p_2^{nc} \)

h. Finally given d) and g) it follows that \( c_1^f \geq c_2^f \geq c_1^{nc} = c_2^{nc} = p_1^{nc} = p_2^{nc} \geq c_i^* \geq c_2^* \)

Therefore given a)-h) it follows that the costs of the different scenarios are ranked in the following way: \( c_1^f \geq c_2^f \geq c_1^{nc} = c_2^{nc} \geq c_i^* \geq c_2^* \), while the prices ranking is as follows: \( p_1^f \geq p_2^f \geq p_1^{nc} = p_2^{nc} \geq p_i^* \geq p_2^* \).
Comparative statics and equilibrium payoffs

Recall from (A3.2) that the optimal costs must satisfy the FOCs:

\[
\frac{\partial U_i}{\partial c_i} = -q(c_i) + S'(c_i) = 0 \quad \forall \alpha_i \in [0,1], i \neq (1,2)
\]

Let,

\[
F_1 \equiv \frac{\partial U_1}{\partial c_1} = -q(c_2) + S'(c_1) = 0
\]

\[
F_2 \equiv \frac{\partial U_2}{\partial c_2} = -q(c_1) + S'(c_2) = 0
\]

Total differentiation leads to:

\[
\begin{cases}
\frac{\partial F_1}{\partial c_1} dc_1 + \frac{\partial F_1}{\partial c_2} dc_2 + \frac{\partial F_1}{\partial \alpha_1} d\alpha_1 + \frac{\partial F_1}{\partial \alpha_2} d\alpha_2 = 0 \\
\frac{\partial F_2}{\partial c_1} dc_1 + \frac{\partial F_2}{\partial c_2} dc_2 + \frac{\partial F_2}{\partial \alpha_1} d\alpha_1 + \frac{\partial F_2}{\partial \alpha_2} d\alpha_2 = 0
\end{cases}
\Rightarrow
\begin{cases}
\frac{\partial F_1}{\partial c_1} dc_1 + \frac{\partial F_1}{\partial c_2} dc_2 = 0 \\
\frac{\partial F_2}{\partial c_1} dc_1 + \frac{\partial F_2}{\partial c_2} dc_2 = 0
\end{cases}
\]

In matrix format:

\[
\begin{bmatrix}
\frac{\partial F_1}{\partial c_1} & \frac{\partial F_1}{\partial c_2} \\
\frac{\partial F_2}{\partial c_1} & \frac{\partial F_2}{\partial c_2}
\end{bmatrix}
\begin{bmatrix}
\frac{\partial c_1}{\partial \alpha_1} \\
\frac{\partial c_2}{\partial \alpha_1}
\end{bmatrix}
= \begin{bmatrix} A_1 \\ A_2 \end{bmatrix}
\]

Using Cramer’s rule we obtain:

\[
\frac{\partial c_1}{\partial \alpha_1} = \frac{A_1}{J}; \quad \frac{\partial c_2}{\partial \alpha_1} = \frac{A_2}{J}
\]

Where:

\[
J = \begin{vmatrix}
\frac{\partial F_1}{\partial c_1} & \frac{\partial F_1}{\partial c_2} \\
\frac{\partial F_2}{\partial c_1} & \frac{\partial F_2}{\partial c_2}
\end{vmatrix}, \quad A_1 = \begin{vmatrix}
-\frac{\partial F_1}{\partial \alpha_1} & \frac{\partial F_1}{\partial c_2} \\
-\frac{\partial F_2}{\partial \alpha_1} & \frac{\partial F_2}{\partial c_2}
\end{vmatrix}, \quad A_2 = \begin{vmatrix}
\frac{\partial F_1}{\partial c_1} - \frac{\partial F_1}{\partial \alpha_1} \\
\frac{\partial F_2}{\partial c_1} - \frac{\partial F_2}{\partial \alpha_1}
\end{vmatrix}
\]

Therefore computing the partial derivatives in \( A_1, A_2 \) and \( J \):

\[
\frac{\partial F_1}{\partial c_1} = S''(c_1^{nc}) < 0 \\
\frac{\partial F_1}{\partial c_2} = -q(c_2^{nc}) > 0 \\
\frac{\partial F_1}{\partial \alpha_1} = 0 \\
\frac{\partial F_1}{\partial \alpha_2} = 0 \\
\frac{\partial F_2}{\partial c_1} = -q(c_1^{nc}) > 0 \\
\frac{\partial F_2}{\partial c_2} = S''(c_2^{nc}) < 0 \\
\frac{\partial F_2}{\partial \alpha_1} = 0 \\
\frac{\partial F_2}{\partial \alpha_2} = 0
\]

With \( J > 0 \) the sign of \( \frac{\partial c_1}{\partial \alpha_1}, \frac{\partial c_2}{\partial \alpha_1} \) will be determined by the sign of \( A_1 \) and \( A_2 \).

\[
A_1 = (-\frac{\partial F_1}{\partial \alpha_1})(\frac{\partial F_2}{\partial c_2}) - (\frac{\partial F_2}{\partial \alpha_1})(\frac{\partial F_1}{\partial c_2}) = 0 \\
A_2 = (-\frac{\partial F_1}{\partial c_1})(\frac{\partial F_2}{\partial \alpha_1}) + (\frac{\partial F_1}{\partial \alpha_1})(\frac{\partial F_2}{\partial c_1}) = 0
\]
Therefore it follows that:

\[
\frac{\partial c_i^{nc}}{\partial \alpha_i} = \frac{\partial c_2^{nc}}{\partial \alpha_i} = 0
\]  
(A3.3)

**Utility payoffs**

The utility payoff of provider \(i\) as a result of its maximization behaviour is given by,

\[
U_i^{nc} = (c_{-i}^{nc} - c_{i}^{nc})q(c_{-i}^{nc}) + S(c_{i}^{nc}) + \alpha_i \int_{c_{-i}^{nc}}^{\infty} q(x)dx = 
\]

\[
S(c_{i}^{nc}) + \alpha_i \int_{c_{-i}^{nc}}^{\infty} q(x)dx.
\]  
(A3.4)

When \(\alpha_i = \alpha_{-i} = \alpha\), in equilibrium by symmetry \(c_{i}^{nc} = c_{-i}^{nc} = c^{nc}\) and FOC (A3.2) becomes,

\[
\frac{\partial U}{\partial c} = -q(c) + S'(c) = 0
\]  
(A3.5)

Therefore, for both ownership types in equilibrium providers gain the same utility payoff

\[
U^{nc} = S(c^{nc}) + \alpha \int_{c^{nc}}^{\infty} q(x)dx.
\]  
(A3.6)
A4  Cooperative Scenario

Proof of Proposition 3
In a cooperative scenario providers optimally choose $c_i$ by maximizing their joint profits $JU$:

$$\max_{c_i} JU = \sum_{i=1}^{3} \left( (\hat{p}_i - c_i)q(\hat{p}_i) + S(c_i) + \alpha_i \int_{c_i}^{\hat{p}_i} q(x) dx \right)$$

Thus for $\hat{p}_i = c_{-i}$,

$$\max_{c_i, c_{-i}} JU = \sum_{i} \left( (c_{-i} - c_i)q(c_{-i}) + S(c_i) + \alpha_i \int_{c_i}^{\infty} q(x) dx \right)$$

(A4.1)

The FOCs are given by,

$$\frac{\partial JU}{\partial c_i} = -q(c_{-i}) + S'(c_i) + (c_i - c_{-i})q'(c_i) + (1 - \alpha_{-i})q(c_i) = 0 \quad i = 1,2$$

(A4.2)

These conditions can be rewritten as:

$$\frac{\partial JU}{\partial c_2} = (1 - \alpha_1)q(c_2^\epsilon) + S'(c_2^\epsilon) = q(c_1^\epsilon) + (c_2^\epsilon - c_1^\epsilon)q'(c_2^\epsilon)$$\quad (A4.3a)

And

$$\frac{\partial JU}{\partial c_1} = (1 - \alpha_2)q(c_1^\epsilon) + S'(c_1^\epsilon) = q(c_2^\epsilon) + (c_1^\epsilon - c_2^\epsilon)q'(c_1^\epsilon)$$\quad (A4.3b)

Suppose that $c_2^\epsilon < c_1^\epsilon$. Given $S''(c_i) < 0$ and $\alpha_1 \leq \alpha_2 \Rightarrow (1 - \alpha_1)q(c_2^\epsilon) + S'(c_2^\epsilon) > (1 - \alpha_2)q(c_1^\epsilon) + S'(c_1^\epsilon)$.

Therefore,

$$q(c_1^\epsilon) + (c_1^\epsilon - c_2^\epsilon)q'(c_2^\epsilon) > q(c_2^\epsilon) + (c_2^\epsilon - c_1^\epsilon)q'(c_1^\epsilon) \Rightarrow q(c_1^\epsilon) - q(c_2^\epsilon) + (c_1^\epsilon - c_2^\epsilon)|q'(c_2^\epsilon) - q'(c_1^\epsilon)| > 0$$

However for $q'(\cdot) < 0$ and $q''(\cdot) > 0$ this inequality never holds for $c_2^\epsilon < c_1^\epsilon$. Now suppose that $c_2^\epsilon = c_1^\epsilon$. Then rewriting and subtracting (A4.3a) and (A4.3b) we obtain:

$$q(c^\epsilon)(\alpha_2 - \alpha_1) = 0$$

For $q(\cdot) > 0$ and $\alpha_1 \leq \alpha_2$ this can only hold for $\alpha_1 = \alpha_2$. Therefore for $\alpha_1 < \alpha_2$ it must be that $c_2^\epsilon > c_1^\epsilon$. For $\alpha_1 = \alpha_2$ by symmetry $c_2^\epsilon = c_1^\epsilon$.

Comparative Statics
Proceeding with comparative statics let:
\[ F_1 = \frac{\partial JU_1}{\partial c_1} = (1 - \alpha_1)q(c_1') + S'(c_1') = q(c_1') + (c_2 - c_1)q'(c_1') = 0 \]
\[ F_2 = \frac{\partial JU_2}{\partial c_2} = (1 - \alpha_2)q(c_2') + S'(c_2') = q(c_2') + (c_1 - c_2)q'(c_2') = 0 \]
Total differentiation leads to:
\[
\begin{bmatrix}
\frac{\partial F_1}{\partial c_1}dc_1 + \frac{\partial F_1}{\partial c_2}dc_2 + \frac{\partial F_1}{\partial \alpha_1}d\alpha_1 + \frac{\partial F_1}{\partial \alpha_2}d\alpha_2 = 0 \\
\frac{\partial F_2}{\partial c_1}dc_1 + \frac{\partial F_2}{\partial c_2}dc_2 + \frac{\partial F_2}{\partial \alpha_1}d\alpha_1 + \frac{\partial F_2}{\partial \alpha_2}d\alpha_2 = 0
\end{bmatrix} \iff \begin{bmatrix}
\frac{\partial F_1}{\partial c_1}dc_1 + \frac{\partial F_1}{\partial c_2}dc_2 = -\frac{\partial F_1}{\partial \alpha_1}d\alpha_1 \\
\frac{\partial F_2}{\partial c_1}dc_1 + \frac{\partial F_2}{\partial c_2}dc_2 = -\frac{\partial F_2}{\partial \alpha_1}d\alpha_1
\end{bmatrix}
\]
In matrix format:
\[
\begin{bmatrix}
\frac{\partial F_1}{\partial c_1} & \frac{\partial F_1}{\partial c_2} \\
\frac{\partial F_2}{\partial c_1} & \frac{\partial F_2}{\partial c_2}
\end{bmatrix}
\begin{bmatrix}
\frac{\partial c_1}{\partial \alpha_1} \\
\frac{\partial c_2}{\partial \alpha_1}
\end{bmatrix} = \begin{bmatrix}
\frac{\partial F_1}{\partial \alpha_1} \\
\frac{\partial F_2}{\partial \alpha_1}
\end{bmatrix}
\]
Using Cramer’s rule we obtain:
\[
\frac{\partial c_1}{\partial \alpha_1} = \frac{A_1}{J} \quad \frac{\partial c_2}{\partial \alpha_1} = \frac{A_2}{J}
\]
Where:
\[
J = \begin{vmatrix}
\frac{\partial F_1}{\partial c_1} & \frac{\partial F_1}{\partial c_2} \\
\frac{\partial F_2}{\partial c_1} & \frac{\partial F_2}{\partial c_2}
\end{vmatrix} \quad A_1 = \begin{vmatrix}
\frac{\partial F_1}{\partial \alpha_1} & \frac{\partial F_1}{\partial c_2} \\
\frac{\partial F_2}{\partial \alpha_1} & \frac{\partial F_2}{\partial c_2}
\end{vmatrix} \quad A_2 = \begin{vmatrix}
\frac{\partial F_1}{\partial \alpha_1} & -\frac{\partial F_1}{\partial \alpha_1} \\
\frac{\partial F_2}{\partial \alpha_1} & -\frac{\partial F_2}{\partial \alpha_1}
\end{vmatrix}
\]
Therefore computing the partial derivatives in \(A_1, A_2\) and \(J\):
\[
\frac{\partial F_1}{\partial c_1} = S''(c_1) + q'(c_1) + (1 - \alpha_2)q'(c_1') + (c_1 - c_2)q''(c_1') < 0 \\
\frac{\partial F_1}{\partial c_2} = -q'(c_1) - q'(c_2) \\
\frac{\partial F_1}{\partial \alpha_1} = 0 \\
\frac{\partial F_1}{\partial \alpha_2} = -q'(c_1) - q'(c_2) \\
\frac{\partial F_2}{\partial c_1} = S''(c_2) + q'(c_2) + (1 - \alpha_1)q'(c_2') + (c_2 - c_1)q''(c_2') < 0 \\
\frac{\partial F_2}{\partial c_2} = -q'(c_1) - q'(c_2) \\
\frac{\partial F_2}{\partial \alpha_1} = -q'(c_2) < 0 \\
\frac{\partial F_2}{\partial \alpha_2} = -q'(c_2) < 0
\]
\[
J = \left[ S''(c_2) + q'(c_2) + (1 - \alpha_1)q'(c_2') \right] \cdot \left[ S''(c_1) + q'(c_1) + (1 - \alpha_2)q'(c_1') \right] - \left[ -q'(c_1) - q'(c_2) \right]^2 > 0
\]
Under the assumption \(1 q'(p_1) \leq S'(c_i)\) and linear demand function \(J > 0\) so the sign of \(\partial c_i/\partial \alpha_1, \partial c_2/\partial \alpha_1\) will be determined by the sign of \(A_1\) and \(A_2\):
\[
A_1 = \left( -\frac{\partial F_1}{\partial \alpha_1} \right) + \left( \frac{\partial F_2}{\partial \alpha_1} \right) < 0 \\
A_2 = \left( \frac{\partial F_1}{\partial \alpha_1} \right) + \left( \frac{\partial F_2}{\partial \alpha_1} \right) < 0
\]
For a linear demand function \(A_2 < 0\) and \(A_1 < 0\). It follows that:
\[
\frac{\partial c_i}{\partial \alpha_1} < 0, \frac{\partial c_i}{\partial \alpha_2} < 0 \quad (A4.4a)
\]
Analogously for \(\alpha_2\):
\[
\frac{\partial c_i}{\partial \alpha_2} < 0, \frac{\partial c_i}{\partial \alpha_2} < 0 \quad (A4.4b)
\]
Proof or Corollary 2

In order to show $c^*_2 \geq c^*_1 > c^{nc}$, consider (A3.8b) for $i = 1$ and rewrite it as

$$\frac{\partial JU}{\partial c_i} = -q(c^*_i) + S'(c^*_i) + (c^*_i - c^*_2)q'(c^*_2) + (1 - \alpha_2)q(c^*_i) = 0$$

Comparing with the non-cooperative FOC the latter has an extra term: $(c^*_1 - c^*_2)q'(c^*_2) + (1 - \alpha_2)q(c^*_i)$. As in equilibrium $c^*_2 \geq c^*_1$ it follows that $(c^*_1 - c^*_2)q'(c^*_2) + (1 - \alpha_2)q(c^*_i) > 0$. Consequently $c^*_1 > c^{nc} \geq c*$. As $c^*_2 \geq c^*_1 \Rightarrow c^*_2 > c^{nc} \geq c* \forall \alpha_i \in [0,1]$. 

In the symmetric case, when the providers have the same level of altruism, i.e. when $\alpha_1 = \alpha_2 = \alpha$, in equilibrium by symmetry, $c_1 = c_2 = c^c$, (A4.2) becomes,

$$\frac{\partial JU}{\partial c} = S'(c^c) - \alpha q(c^c) = 0 \quad (A4.2a)$$

Also, in the symmetric case, when $\alpha \rightarrow 1$ (A4.2a) becomes the same as (A3.2), therefore $c^c \rightarrow c^{nc}$.

Now consider $\alpha_1 = \alpha_2 = 1$. The FOC in the cooperative scenario becomes $-q(c^*_i) + S'(c^*_i) + (c^*_i - c^*_2)q'(c^*_2) = 0$. By symmetry $c^*_2 = c^*_1 = c^c$ that is the solution to the FOC that simplifies to $-q(c^c) + S'(c^c) = 0$. Comparing the latter with the (A3.5) it follows that $c^c = c^{nc} > c*$. 

Consider now $\alpha_1 = \alpha_2 = 0$. Since $c^*_2 = c^*_1 = c^c$ the FOCs in the cooperative scenario (A4.3a) and (A4.3b) become $S'(c^c) = 0$. Comparing the latter with (A3.5) since $q(c) < 0$ it follows that $c^c > c^{nc} > c*$. 

Comparing the cooperative solution with the with the first best solution for the symmetric case, analysing the first best FOC $\frac{\partial W}{\partial c_i} = -q(p_i) + S'(c_i) = 0$ we have that:

$-q(p^*) + S'(c^*_i) = -q(c^c) + S'(c^c) = 0 \Leftrightarrow S'(c^*_i) - S'(c^c) = q(p^*) - q(c^c)$ as in the first best case the price is below the marginal cost $c*$ then $q(p^*) - q(c^c) > 0 \Rightarrow S'(c^*_i) - S'(c^c) > 0$. As $S''(.) < 0 \Rightarrow c^c > c*$. 

For the symmetric case $\alpha_1 = \alpha_2 = \alpha$, proceeding with some comparative static analysis, consider the FOC (A4.2) by the implicit function theorem $\frac{dc^c}{d\alpha} = -\frac{\partial^2 JU}{\partial c^c \partial \alpha}$. Differentiating (A4.2) for $c^c$ and $\alpha$ we find that $\frac{\partial^2 JU}{\partial c^c} \frac{\partial c^c}{d\alpha} = -q(c)$ and...
\[ \frac{\partial^2 JU}{\partial^2 c^e} = S''(c) - \alpha q'(c) \]. Given that by the second order conditions the latter is negative then it follows that

\[ \frac{dc^e}{d\alpha} = -\frac{q(c)}{S''(c) - \alpha q'(c)} < 0 \]  

(A4.5)

i.e., the higher is the providers' altruism level the lower is the collusive cost.

**Utility Payoffs**

Substituting the optimal cost strategies on the utility function the utility payoff earned by provider \( i \) under collusion is given by,

\[ U_i^c = (c_i^e - c_i^c)q(c_i^e) + S(c_i^e) + \alpha \int_{c_i^e}^{\infty} q(x)dx \]  

(A4.6)

In the symmetric case for homogeneous providers i.e. \( \alpha_1 = \alpha_2 = \alpha \), the utility payoff is given by,

\[ U^c = S(c^e) + \alpha \int_{c^e}^{\infty} q(x)dx \]  

(A4.7)
A5  Defection Scenario

Proof of Proposition 4

The defecting provider \(i\) will revert to behaving accordingly to the best response function as in (A3.2)

\[-q(c_{i}^{e}) + S'(c_{i}) = 0 \tag{A5.1}\]

In the non-cooperative scenario we know that

\[-q(c^{nc}) + S'(c^{nc}) = 0 \Rightarrow S'(c^{nc}) - S'(c_{i}^{d}) = q(c^{nc}) - q(c_{i}^{e}) . \]

With \(c_{i}^{e} > c^{nc}\) and \(q'(.) < 0\) then

\[q'(c^{nc}) - q'(c_{i}^{d}) > 0 \Rightarrow S'(c^{nc}) - S'(c_{i}^{d}) > 0 . \]

As \(S''(.) < 0\) \(\Rightarrow c^{nc} < c_{i}^{d}\).

Consider now,

\[
\begin{cases}
- q(c_{i}^{e}) + S'(c_{i}^{d}) = 0 \\
- q(c_{i}^{e}) + S'(c_{i}^{d}) = 0
\end{cases}
\]

\[\Rightarrow -q(c_{i}^{e}) + q(c_{i}^{e}) + S'(c_{i}^{d}) - S'(c_{i}^{d}) = 0\]

Given \(q'(c_{i}) < 0\), it follows that \(-q(c_{i}^{e}) + q(c_{i}^{e}) > 0\), then it must be \(S'(c_{i}^{d}) - S'(c_{i}^{d}) < 0\). As \(S''(.) < 0\) it follows that \(c_{i}^{d} < c_{i}^{d}\) must hold true.

Consider equation (A5.1) evaluated at \(c_{i}^{e}\) and \(c_{i}^{e}\) and compare it with the FOC in collusion (A4.2). For \(i = 1\), the last two terms in (A4.2) are positive, thus it must be

\[-q(c_{i}^{e}) + S'(c_{i}^{d}) < 0\]

for the FOC to be satisfied. It follows that the cost chosen in defection is less than the cooperation cost. Thus the defection cost falls within, \(c_{1}^{nc} < c_{1}^{d} < c_{1}^{c}\).

With regards to provider 2, rewrite FOC (A3.8) as:

\[\frac{\partial JU}{\partial c_{1}} = -\alpha_{2} q(c_{1}) + S'(c_{1}) + (c_{1} - c_{2}) q'(c_{1}) + q(c_{1}) - q(c_{2}) = 0\]

\[\frac{\partial JU}{\partial c_{2}} = -\alpha_{1} q(c_{2}) + S'(c_{2}) + (c_{2} - c_{1}) q'(c_{2}) + q(c_{2}) - q(c_{1}) = 0\]

As \(c_{2} > c_{1}\) and \(q'(.) < 0\) then \((c_{1} - c_{2}) q'(c_{1}) + q(c_{1}) - q(c_{2}) > 0\). For \(\partial JU / \partial c_{1} = 0\) to be verified it must be that \(-\alpha_{2} q(c_{1}) + S'(c_{1}) < 0\). Analogously, as \((c_{2} - c_{1}) q'(c_{2}) + q(c_{2}) - q(c_{1}) < 0\). For \(\partial JU / \partial c_{2} = 0\) then \(-\alpha_{1} q(c_{2}) + S'(c_{2}) > 0\).

As \(\alpha_{2} \leq 1\) then \(-\alpha_{2} q(c_{1}) + S'(c_{1}) < 0 \Leftrightarrow S'(c_{1}) < q(c_{1})\). Also as \(S'(.)\) is decreasing for \(c_{2} > c_{1}\) \(\Rightarrow S'(c_{2}) < S'(c_{1}) < q(c_{1})\). Therefore if \(S'(c_{2}) - q(c_{i}^{e}) < 0 \Rightarrow c_{2}^{d} < c_{2}^{e}\).

It is now possible to rank all the costs under the yardstick regulation,

\[c_{2}^{d} < c_{1}^{d} \leq c_{1}^{nc} = c_{2}^{nc} \leq c_{2}^{d} \leq c_{1}^{c} < c_{2}^{c}\]

Comparative Statics

Recall from (A5.1) that the optimal costs must satisfy the FOCs:

\[\frac{\partial U_{i}}{\partial c_{i}} = -q(c_{i}^{e}) + S'(c_{i}^{d}) = 0 \quad \forall \alpha_{i} \in [0,1], i = (1,2)\]
Let

\[ F_i = \frac{\partial U_i}{\partial c_i} = -q(c^e_2) + S'(c^d_1) = 0 \]

\[ F_2 = \frac{\partial U_2}{\partial c_2} = -q(c^e_1) + S'(c^d_2) = 0 \]

We know from equation A.3.9 that \( \partial c^e_i / \partial \alpha_i \leq 0 \) so Total differentiation leads to:

\[
\begin{bmatrix}
\frac{\partial F_1}{\partial c_1} dc_1 + \frac{\partial F_1}{\partial c_2} dc_2 + \frac{\partial F_1}{\partial \alpha_1} d\alpha_1 + \frac{\partial F_1}{\partial \alpha_2} d\alpha_2 = 0 \\
\frac{\partial F_2}{\partial c_1} dc_1 + \frac{\partial F_2}{\partial c_2} dc_2 + \frac{\partial F_2}{\partial \alpha_1} d\alpha_1 + \frac{\partial F_2}{\partial \alpha_2} d\alpha_2 = 0
\end{bmatrix} \Leftrightarrow \begin{bmatrix}
\frac{\partial F_1}{\partial c_1} + \frac{\partial F_1}{\partial c_2} = -\frac{\partial F_1}{\partial \alpha_1} \\
\frac{\partial F_2}{\partial c_1} + \frac{\partial F_2}{\partial c_2} = -\frac{\partial F_2}{\partial \alpha_1}
\end{bmatrix}
\]

In matrix format:

\[
\begin{bmatrix}
\partial F_1 / \partial c_1 & \partial F_1 / \partial c_2 \\
\partial F_2 / \partial c_1 & \partial F_2 / \partial c_2
\end{bmatrix}
\begin{bmatrix}
\partial c_1 / \partial \alpha_1 \\
\partial c_2 / \partial \alpha_1
\end{bmatrix} = \begin{bmatrix}
-\partial F_1 / \partial \alpha_1 \\
-\partial F_2 / \partial \alpha_1
\end{bmatrix}
\]

Using Cramer’s rule we obtain:

\[
\frac{\partial c_1}{\partial \alpha_1} = \frac{A_1}{J}; \quad \frac{\partial c_2}{\partial \alpha_1} = \frac{A_2}{J}
\]

Where:

\[
J = \begin{bmatrix}
\frac{\partial F_1}{\partial c_1} & \frac{\partial F_1}{\partial c_2} \\
\frac{\partial F_2}{\partial c_1} & \frac{\partial F_2}{\partial c_2}
\end{bmatrix}
\]

\[
A_1 = \begin{bmatrix}
-\partial F_1 / \partial \alpha_1 & \partial F_1 / \partial c_2 \\
-\partial F_2 / \partial \alpha_1 & \partial F_2 / \partial c_2
\end{bmatrix}
\]

\[
A_2 = \begin{bmatrix}
\partial F_1 / \partial c_1 & -\partial F_1 / \partial \alpha_1 \\
\partial F_2 / \partial c_1 & -\partial F_2 / \partial \alpha_1
\end{bmatrix}
\]

We know from equation A.3.9 that \( \partial c^e_i / \partial \alpha_i \leq 0 \), therefore computing the partial derivatives in \( A_1, A_2 \) and \( J \):

\[
\partial F_1 / \partial c_1 = S''(c^d_1) < 0 \\
\partial F_1 / \partial c_2 = -q'(c^e_2) > 0 \\
\partial F_1 / \partial \alpha_1 = -q'(c^e_2) \frac{\partial c^e_2}{\partial \alpha_1} < 0 \\
\partial F_1 / \partial \alpha_2 = -q'(c^e_2) > 0 \\
\partial F_2 / \partial c_1 = -q'(c^e_1) > 0 \\
\partial F_2 / \partial c_2 = S''(c^d_2) < 0 \\
\partial F_2 / \partial \alpha_1 = -q'(c^e_1) \frac{\partial c^e_1}{\partial \alpha_1} < 0 \\
\partial F_2 / \partial \alpha_2 = -q'(c^e_1) > 0
\]

With \( J > 0 \) the sign of \( \partial c^e_1 / \partial \alpha_1, \partial c^e_2 / \partial \alpha_1 \) will be determined by the sign of \( A_1 \) and \( A_2 \):

\[
A_1 = (-\partial F_1 / \partial \alpha_1)(\partial F_2 / \partial c_2) - (\partial F_2 / \partial \alpha_1)(\partial F_1 / \partial c_2) < 0 \\
A_2 = (-\partial F_1 / \partial c_1)(\partial F_2 / \partial \alpha_1) + (\partial F_1 / \partial \alpha_1)(\partial F_2 / \partial c_1) < 0
\]

Therefore it follows that:

\[
\partial c^d_1 / \partial \alpha_1 < 0, \partial c^d_2 / \partial \alpha_1 < 0
\]

**Utility payoffs**

The utility payoff provider \( i \) earns in defection is given by

\[
U^d_i = (c^e_i - c^d_i)q(c^e_i) + S(c^d_1) + \int_{c^d_i}^{\infty} g(x)dx
\]

(A5.3)
When the providers have the same level of altruism, i.e. \( \alpha_i = \alpha_{-i} = \alpha \), by symmetry the cooperative solution is such that \( c^c_i = c^{c-} = c^c \) and the defection cost \( c^{d_i} = c^{d-} = c^d \) is the solution to \(-q(c^c) + S'(c^c) = 0\). If we evaluate this at \( c = c^c \) and compare it with FOC (A4.2a), we obtain \(-q(c^c) + S'(c^c) \leq 0\). As \( S'(\cdot) \) is a decreasing function, therefore \( c^d < c^c \). Also \(-q(c^c) + S'(c^*) > 0\), then \( c^d > c^* \).

The utility payoff both providers earn is given by:

\[
U^d = (c^c - c^d)q(c^c) + S(c^d) + \alpha \int_{c^c}^{c^d} g(x)dx
\]  

(A5.4)
A6 Payoff Utility Ranking

**Rank of utility payoffs under the different scenarios**

**Symmetric altruism**
Suppose \( \alpha_1 = \alpha_2 = 0 \) the payoff functions at the optimum are:

\[
U^{nc}_{1,\alpha_1=0} = S(c^{nc}) 
\]  

(A6.1)

\[
U^{c}_{1,\alpha_1=0} = S(c^c) 
\]  

(A6.2)

\[
U^{d}_{1,\alpha_1=0} = (c^c_i - c^d_i)q(c^c_i) + S(c^d) 
\]  

(A6.3)

Therefore it follows that: \( U^{nc}_{1} \leq U^c_{1} \leq U^d_{1} \).

Now we consider the case in which \( \alpha_i = \alpha_i = 1 \). We have shown before if \( \alpha_i \to 1 \) then \( c^c_i \to c^{nc}_i \) (see proof of Proposition 3) and at \( c^{nc}_i \) there are no profitable deviations. Therefore it follows that:

\[
U^c_{1} = U^{nc}_{1} = U^d_{1} = S(c^{nc}_i) + \int_{c^{nc}_i}^{\infty} q(x) \, dx 
\]  

(A6.4)

**Asymmetric altruism**

**Case 1**: \( \alpha_1 = 0 \) and \( \alpha_2 = 1 \)

The utility payoffs for these levels of altruism are:

\[
U^{nc}_{1,\alpha_1=0} = S(c^{nc}) 
\]  

(A6.5)

\[
U^{c}_{1,\alpha_1=0} = (c^c_1 - c^d_1)q(c^c_1) + S(c^c_1) 
\]  

(A6.6)

\[
U^{d}_{1,\alpha_1=0} = (c^c_2 - c^d_2)q(c^c_2) + S(c^d_1) 
\]  

(A6.7)

\[
U^{nc}_{2,\alpha_2=1} = S(c^{nc} + \int_{c^{nc}}^{\infty} q(x) \, dx 
\]  

(A6.8)

\[
U^{c}_{2,\alpha_2=1} = (c^c_1 - c^d_2)q(c^c_1) + S(c^c_2) + \int_{c^{nc}}^{\infty} q(x) \, dx 
\]  

(A6.9)

\[
U^{d}_{2,\alpha_2=1} = (c^c_1 - c^d_2)q(c^c_2) + S(c^d_1) + \int_{c^{nc}}^{\infty} q(x) \, dx 
\]  

(A6.10)

Since \( c^{nc}_1 < c^c_1 < c^c_2 \) then \( S(c^c_1) > S(c^{nc}) \) and \( (c^c_1 - c^d_1)q(c^c_1) > 0 \), implying that \( U^{nc}_{1} < U^c_{1} \).

Since \( c^{nc}_1 < c^c_1 < c^c_2 \) analysing (A6.6) and (A6.7) it follows that \( U^c_{1} < U^d_{1} \). Therefore: \( U^{nc}_{1} < U^c_{1} < U^d_{1} \).

Subtracting equation (A6.8) to (A6.9) we obtain:

\[
U^{nc}_{2,\alpha_2=1} - U^{c}_{2,\alpha_2=1} = -(c^c_1 - c^d_2)q(c^c_1) + (S(c^{nc}) - S(c^c_2)) + \int_{c^{nc}}^{\infty} q(x) \, dx - \int_{c^{nc}}^{\infty} q(x) \, dx 
\]

Recall that \( c^c_{1nc} = c^{nc}_2 < c^c_2 < c^c_1 < c^c_2 \). The term \( -(c^c_1 - c^d_2)q(c^c_1) > 0 \) is positive for \( \alpha_2 > \alpha_1 \). Furthermore since \( c^{nc}_2 < c^c_2 \) (see proof of Proposition 3) the term \( \int_{c^{nc}}^{\infty} q(x) \, dx - \int_{c^{nc}}^{\infty} q(x) \, dx > 0 \) is positive since the consumer surplus is bigger in the non-cooperative
game. Finally, since $c_2^{nc} < c_2^c$ the term $(S(c^{nc}) - S(c_2^c)) < 0$ is negative since the slackness is bigger in the cooperative than in the non-cooperative scenario.

Therefore it follows that $U_2^{nc} < U_2^c$ if and only if:

$$
(S(c_2^{nc}) - S(c_2^c)) \leq (c_1^c - c_2^c)q(c_1^c) - \int_{c_1^{nc}}^{\infty} q(x) \, dx + \int_{c_1^c}^{\infty} q(x) \, dx
$$

(A6.11)

A more general formulation of equation A6.11 for $0 \leq \alpha_2 \leq 1$ is

$$
(S(c_2^{nc}) - S(c_2^c)) \leq (c_1^c - c_2^c)q(c_1^c) - \alpha_2 \int_{c_1^{nc}}^{\infty} q(x) \, dx + \alpha_2 \int_{c_1^c}^{\infty} q(x) \, dx
$$

(A6.11a)

Therefore for firm 2 cooperation is profitable only if we assume that the benefit from slack is big enough to offset the financial loose and the decrease in consumer surplus that more altruistic firms have to bear in cooperation. So $U_2^c > U_2^{nc}$, only if equation (A6.11) holds, otherwise $U_2^c < U_2^{nc}$.

For the less altruistic firm, i.e. firm 1 condition (A6.11) is easier to hold since $(c_1^c - c_2^c)q(c_1^c) > 0$. 
Proof of propositions 5 and Corollary 3

Let $\delta^*$ denote the discount rate above which collusion is sustainable such that:

$$\delta^* = \max\{\delta_i, \delta_{-i}\}$$

With

$$\delta_i \geq \frac{U_i^d - U_i^c}{U_i^d - U_i^{nc}} \quad \delta_{-i} \geq \frac{U_{-i}^d - U_{-i}^c}{U_{-i}^d - U_{-i}^{nc}}$$

Consider firm $i$. In order to understand how the each firm collusive behaviour varies with the level of altruism we need to assess the $\Delta \delta_i$ given $\Delta \alpha_i > 0$. Consider the payoff utilities:

$$U_i^{nc} = S(c_i^{cn}) + \alpha_i \int_{c_i^{nc}}^{\infty} q(x) \, dx$$  \hspace{1cm} (A7.1)

$$U_i^c = (c_i^c - c_i^e)q(c_i^c) + S(c_i^c) + \alpha_i \int_{c_i^c}^{\infty} q(x) \, dx$$  \hspace{1cm} (A7.2)

$$U_i^d = (c_i^d - c_i^a)q(c_i^d) + S(c_i^d) + \alpha_i \int_{c_i^a}^{\infty} q(x) \, dx$$  \hspace{1cm} (A7.3)

Derivation with respect to $\alpha_i$:

$$\frac{\partial U_i^c}{\partial \alpha_i} = \left( \frac{\partial c_i^c}{\partial \alpha_i} - \frac{\partial c_i^e}{\partial \alpha_i} \right) q(c_i^c) + (c_i^c - c_i^e)q'(c_i^c) \frac{\partial c_i^c}{\partial \alpha_i} + S'(c_i^c) \frac{\partial c_i^e}{\partial \alpha_i} + \int_{c_i^c}^{\infty} q(x) \, dx$$

$$- \alpha_i q(c_i^c) \frac{\partial c_i^c}{\partial \alpha_i}$$  \hspace{1cm} (A7.4)

$$\frac{\partial U_i^d}{\partial \alpha_i} = \left( \frac{\partial c_i^d}{\partial \alpha_i} - \frac{\partial c_i^a}{\partial \alpha_i} \right) q(c_i^d) + (c_i^d - c_i^a)q'(c_i^d) \frac{\partial c_i^d}{\partial \alpha_i} + S'(c_i^d) \frac{\partial c_i^a}{\partial \alpha_i} + \int_{c_i^a}^{\infty} q(x) \, dx$$

$$- \alpha_i q(c_i^d) \frac{\partial c_i^d}{\partial \alpha_i}$$  \hspace{1cm} (A7.5)

Consider for now the symmetric case $\alpha_i = \alpha_{-i} = 0$. As seen above the payoff utilities can be ranked as: $U_i^{nc} \leq U_i^c \leq U_i^d$ (see Graph 1).

Now consider the case for which $\alpha_i = \alpha_{-i} = 1$, as we have shown before if $\alpha_i \to 1 \Rightarrow c_i^c \to c_i^{nc}$ implying $U_i^c = U_i^{nc} = U_i^d$.

Therefore (A7.4), (A7.5) and (A7.6) can be written as:

$$\frac{\partial U_i^{nc}}{\partial \alpha_i} = + \int_{c_i^{nc}}^{\infty} q(x) \, dx$$  \hspace{1cm} (A7.4a)
\[
\frac{\partial U_i^c}{\partial \alpha_i} = \left( \frac{\partial c_{e_i}^c}{\partial \alpha_i} - \alpha_i \frac{\partial c_{e_i}^c}{\partial \alpha_i} \right) q(c_{e_i}^c) + (c_{e_i}^c - c_i^e) q'(c_{e_i}^c) \frac{\partial c_{e_i}^c}{\partial \alpha_i} + \frac{\partial c_i^e}{\partial \alpha_i} \left( S'(c_i^e) - q(c_{e_i}^c) \right) \\
+ \int_{c_{e_i}^c}^{\infty} q(x) \, dx
\]  
(A7.5a)

\[
\frac{\partial U_i^d}{\partial \alpha_i} = \left( \frac{\partial c_{e_i}^c}{\partial \alpha_i} - \alpha_i \frac{\partial c_{e_i}^c}{\partial \alpha_i} \right) q(c_{e_i}^d) + (c_{e_i}^c - c_i^d) q'(c_{e_i}^c) \frac{\partial c_{e_i}^c}{\partial \alpha_i} + \frac{\partial c_i^d}{\partial \alpha_i} \left( S'(c_i^d) - q(c_{e_i}^c) \right) \\
+ \int_{c_{e_i}^c}^{\infty} q(x) \, dx
\]  
(A7.6a)

Consider (A7.4a) and (A7.5a). We know that for \( \alpha_i = 0 \), \( U_i^{nc} < U_i^c \) also that \( \alpha_i \rightarrow 1 \Rightarrow U_i^c \rightarrow U_i^{nc} \) therefore it must hold that \( \frac{\partial U_i^c}{\partial \alpha_i} \leq \frac{\partial U_i^{nc}}{\partial \alpha_i} \) meaning that \( U_i^{nc} \) grows faster than \( U_i^c \).

Therefore since for \( \alpha_i \rightarrow 1 \Rightarrow c_i^c \rightarrow c_i^{nc} \) it follows that \( \frac{\partial U_i^c}{\partial \alpha_i} \rightarrow \frac{\partial U_i^{nc}}{\partial \alpha_i} \) (see Graph 1).

If \( U_i^{nc} \) increases faster than \( U_i^c \) the it follows that \( \delta_i \) increases with \( \alpha_i \), i.e. for firm \( i \) cooperation becomes more difficult for higher altruism levels.

Consider now the impact of \( \Delta \alpha_i > 0 \) on \( \delta_{-i} \). Consider the payoff utilities for firm \(-i\)

\[
U_i^{nc} = S(c_{e_i}^{nc}) + \alpha_i \int_{c_{e_i}^{nc}}^{\infty} q(x) \, dx
\]  
(A7.7)

\[
U_i^c = (c_i^c - c_{e_i}^c) q(c_i^c) + S(c_{e_i}^c) + \alpha_{-i} \int_{c_{e_i}^c}^{\infty} q(x) \, dx
\]  
(A7.8)

\[
U_i^d = (c_i^d - c_{e_i}^d) q(c_i^d) + S(c_{e_i}^d) + \alpha_{-i} \int_{c_{e_i}^d}^{\infty} q(x) \, dx
\]  
(A7.9)

Derivation with respect to \( \alpha_i \):

\[
\frac{\partial U_i^{nc}}{\partial \alpha_i} = 0
\]  
(A7.10)

\[
\frac{\partial U_i^c}{\partial \alpha_i} = \left( \frac{\partial c_i^c}{\partial \alpha_i} - \alpha_i \frac{\partial c_{e_i}^c}{\partial \alpha_i} \right) q(c_i^c) + (c_i^c - c_{e_i}^c) q'(c_i^c) \frac{\partial c_i^c}{\partial \alpha_i} + S'(c_i^c) \frac{\partial c_i^c}{\partial \alpha_i} - \alpha_i q(c_i^c) \frac{\partial c_i^c}{\partial \alpha_i}
\]  
(A7.11)

\[
\frac{\partial U_i^d}{\partial \alpha_i} = \left( \frac{\partial c_i^d}{\partial \alpha_i} - \alpha_i \frac{\partial c_{e_i}^d}{\partial \alpha_i} \right) q(c_i^d) + (c_i^c - c_{e_i}^d) q'(c_i^c) \frac{\partial c_i^d}{\partial \alpha_i} + S'(c_i^d) \frac{\partial c_i^d}{\partial \alpha_i} - \alpha_i q(c_i^c) \frac{\partial c_i^d}{\partial \alpha_i}
\]  
(A7.12)

It is straightforward to see that \( \frac{\partial U_i^c}{\partial \alpha_i} \geq \frac{\partial U_i^{nc}}{\partial \alpha_i} \), i.e. when the opponent’s altruism increases the firm cooperation utility payoff also increases. Since \( \frac{\partial U_i^{nc}}{\partial \alpha_i} = 0 \) it follows that \( \delta_{-i} \) decreases with the opponent’s altruism level.

Summarizing, we have shown that \( \Delta \alpha_i > 0 \) leads to a decrease in \( \delta_{-i} \) and an increase in \( \delta_i \). Since:
\[ \delta^* = \max\{\delta_l, \delta_{-l}\} \]

We need to assess \( \max\{\delta_l, \delta_{-l}\} \). If \( \alpha_l = \alpha_{-l} \) we know that \( \delta_l = \delta_{-l} \). Departing from the symmetric case, consider a positive increase in firm’s 2 altruism level, i.e. \( \Delta \alpha_2 \). As shown above this will increase \( \delta_2 \) and decrease \( \delta_1 \), implying that \( \delta_2 \geq \delta_1 \).

Still, departing from the symmetric case, consider a decrease in firm’s 1 altruism level, i.e. \( \Delta \alpha_1 < 0 \). As shown above this will increase \( \delta_2 \) and decrease \( \delta_1 \), implying that \( \delta_2 \geq \delta_1 \).

Therefore we conclude that \( \delta^* = \delta_2 \) implying that \( \delta^*|_{\alpha_1 = \alpha_2} \leq \delta^*|_{\alpha_1' < \alpha_2} \) for \( \alpha_1 \geq \alpha_1' \) and \( \delta^*|_{\alpha_1 = \alpha_2} \leq \delta^*|_{\alpha_1 < \alpha_2} \) for \( \alpha_2 \leq \alpha_2' \).

Note that departing from a symmetric case an increase in \( \alpha_1 \) and a decrease in \( \alpha_2 \) are not feasible since in our model \( \alpha_1 \leq \alpha_2 \).
B  Figures

Figure 1- Homogeneous Providers

\[
(c^e_i - c^d_i)q(c^e_i) + S(c^d_i)
\]

\[
S(c^e) \quad S(c^{ne})
\]

\[ a_i \quad 1 \]

Figure 2- Asymmetric Providers

\[
S(c^e) - (S(c^{ne}) + (c^e_i - c^e_i)q(c^e_i)) = a_i \int_{c^e}^{c^d} q(x) \, dx - a_i \int_{c^e}^{c^d} q(x) \, dx
\]