Stock-Bond Correlation: 
Time variation, Predictability & 
Hedging

Farouk Tazdin Jivraj
July 2012

Submitted in partial fulfilment of the requirements for the degree of 
Doctor of Philosophy in Finance for Imperial College London 
and the Diploma of Imperial College London
Declaration

I hereby certify that this thesis constitutes my own work and that all material, which is not my own work, has been properly acknowledged.

Farouk Jivraj
to my family...

*quicumque ego sum quod quicumque ego ero est propter tobus vestrum*

(all that I am and all that I will be is because of all of you)
Acknowledgements

I firstly must thank my supervisor Professor Robert Kosowski. The guidance, support and above all encouragement has benefited not only the research presented in this thesis but also the person I have become through undertaking the journey of a Ph.D. This thesis is the culmination of countless discussions and the endless generation of ideas on ways to tackle problems and take the research forward. Such a process has allowed me to develop my research abilities and instincts. This took time and dedication and I thank Robert deeply for both.

I would also like to thank my second supervisor Dr. Enrico Biffis. This thesis benefited greatly from your persistence in highlighting the research angles that clarified the overall message. I thank Enrico for this and for the encouragement he gave throughout my time here but especially during the last few months of writing up the thesis. I would also like to extend my thanks to Professor Andrea Buraschi, Professor Paolo Zaffaroni, Dr. David McCarthy, Dr. Lara Cathcart, Professor Karim Abadir, Professor Gilles Chemla, Dr. Filippou Papakonstantinou and Dr. Pasquale Della Corte for helpful discussions and suggestions at various stages of the Ph.D.

To my fellow Ph.D. colleagues. Thank you not only for the interesting discussions but more truly for the friendships: Paul Whelan, James Grant, Andrea Carnelli, Worrawat Sitrarukul (Goff), Andres Reibel, Komkrit Ovararin (Owen), Jan Ahmerkamp, Dr. Toluola Lawal, Dr. Nikhil Shenai, Dr. Ewan Mackie, Dr. Nick Baltas, Paul Takla, Saad Badaoui, Lorenzo Pitotti, Dr. Filip Zikes, Dr. Adam Iqbal, Dr. Xianghe Kong and Dr. Leo Evans. Undertaking a Ph.D. can at times be a lonely existence, it has been your friendships at these times that I will remember most. I wish you all the very best and look forward to the lasting friendships in the future.

I must also thank Professor Dorothy Griffiths for the support and lectureship position she extended to me in September 2011 before completion of my Ph.D. My enjoyment with teaching (and research) comes from the belief that “knowledge is power”; where “power” is in the context of one’s mind and how one’s process of thought develops as a result of acquiring that “knowledge”. I must also thank therefore Dr. Jean-Pierre Zigrand and Dr. Stephane Guibaud for my first opportunity as a teaching assistant at the London School of Economics and my supervisor Professor Robert Kosowski for my first opportunity at Imperial College Business School. The two Imperial College Business School prizes for Best Performing Teaching Assistant awarded in 2011 and 2012 are a product of your guidance and advice, and I thank you all for that. I must also thank Gillian Forsyth, Phoebe Voong, Susan Mossey, Wee Ming Lim, Lisa Umenyiora, Rosemarie Brown, Jolante Leonaite and Edina Hamzic-Maguire for course related administrative assistance over the years of teaching.

The Ph.D. was started and has finished during the 2007-2012 financial crisis. It has also witnessed two of the most significant events in my life...the first being the meeting of the love of my life, the second being getting married to the love of my life. To my wife, Afsane (Jetha) Jivraj, I thank you for the love, patience and support that you have shown me during the Ph.D., especially during the last few months. I will always hold dear to me the memories of those times in which I needed you the most, it was your encouragement and character that pushed me through. In the future, I hope to do the same for you as you have done for me.
To my sister, Feiza (Jivraj) Datoo, thank you for the support and encouragement to do my Ph.D. in the first place! Although I believe that such a path has been the most difficult (both intellectually and emotionally), I feel it has held the greatest personal reward. I thank you for the continual believe that you have in me.¹

Finally to my parents, Tazdin and Nargis Jivraj, when I reflect upon the opportunities that you have given me throughout my life, I cannot help but think that I am “standing on the shoulders of giants”. Thank you for the unconditional love, support and encouragement not only throughout the Ph.D. but for all of my endeavors. I hope that I have made you proud and continue to do so.

I warmly dedicate this thesis to all of you.

Farouk Jivraj
18th July 2012

¹I would also like to thank both Areef Datoo and Nisaa Jetha for their support over the years, and an honourable mention goes to my niece Hanna Amara who was born during the final months of this thesis.
Abstract

The correlation between stock and bond markets is of critical importance. Pension funds, mutual funds, institutions and individuals all face an asset allocation decision on the amount of wealth to invest across stock and bond markets. Indeed asset allocation decisions have been shown to account for in excess of 70% of the performance of portfolios (Brinson et al. 1991). Since it is now widely accepted that the correlation between stocks and bonds is subject to fluctuations over time, with the implication that these changes impact portfolio risk and thus investors’ diversification benefits, this thesis looks at three distinct but related topics to do with time variation in stock-bond correlation: contemporaneous changes, predictability and hedging unexpected changes.

The first topic is an empirical examination of the economic mechanisms underlying the contemporaneous time variation in stock-bond correlation. Based on a theoretical framework motivated by the Campbell and Shiller (1988) decomposition to express unexpected stock and bond returns into news components related to macroeconomic fundamentals, time-varying co-movement among these innovations can reveal the macroeconomic drivers of the time-variation in realised second moments of stock and bond returns. Using a novel dataset of macroeconomic analysts’ forecasts, uncertainty in cash flow (corporate profits) and the real short-term interest rate is able to explain a relatively substantial part of the variation in stock volatility. Bond return volatility can be attributed to the uncertainty in inflation and the real short-term interest rate, while the interaction between several of the macroeconomic news components account for a portion of the variation in the covariance between stock and bond returns. Most notably the interaction between cash flow news and real short-term interest rate news is a driver of negative stock-bond correlation.

The second topic is on time-series predictability of realised stock-bond correlation. This is investigated in the context of improving investors’ ex-ante allocation of wealth between stock and bond markets using macroeconomic analysts’ forecasts. In-sample such forecasts display some predictability of the volatility and correlation. Out-of-sample however, analysts’ forecasts are not able to improve investors’ ex-ante allocation. Based on the framework of the global minimum-variance portfolio, net of transaction costs, analyst forecast data does not provide any benefits above historical returns in forming a minimum-variance portfolio. Whilst there are benefits to using such forecasts during the 2008 financial crisis, this is overshadowed by the effectiveness of simply using the realised correlation estimate to form the minimum-variance portfolio.

The third topic investigates stock-bond correlation risk and the importance of unexpected changes in correlation for the asset-liability management mandate of a pension fund. Focusing solely on the role of interest rate risk, liabilities can be thought of as long duration bonds. Since pension funds are typically net long stocks and net short bonds, changes in the correlation between these two asset classes will affect the funding ratio. Empirically this is shown for a stylised pension fund with contributions invested 60/40 across stocks and long-term bonds: The funding ratio decreases when adverse changes to stock-bond correlation occur. A stock-bond correlation swap to hedge against such a risk is therefore naturally motivated. By structuring a stock-bond correlation swap contract, a utility indifference pricing model with stochastic correlation in an incomplete market is developed. The model incorporates the role of pension fund preferences in fairly pricing the swap and leads to several intuitive findings: model-implied quotes of the correlation-swap strike fall within the range of quotes obtained from actual stock-bond correlation swaps; the higher the risk aversion and/or the more important the liabilities are, the higher the correlation swap strike the pension fund would be willing to pay in order to hedge stock-bond correlation risk.
## Contents

<table>
<thead>
<tr>
<th>Acknowledgements</th>
<th>iii</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abstract</td>
<td>v</td>
</tr>
<tr>
<td>Contents</td>
<td>vi</td>
</tr>
<tr>
<td>List of Figures</td>
<td>ix</td>
</tr>
<tr>
<td>List of Tables</td>
<td>x1</td>
</tr>
<tr>
<td><strong>1</strong> Introduction and literature review</td>
<td>1</td>
</tr>
<tr>
<td><strong>2</strong> Changing expectations and the correlation of stocks and bonds</td>
<td>8</td>
</tr>
<tr>
<td>2.1 Introduction</td>
<td>8</td>
</tr>
<tr>
<td>2.2 Theoretical stock-bond correlation</td>
<td>13</td>
</tr>
<tr>
<td>2.2.1 Surprises in stock returns</td>
<td>14</td>
</tr>
<tr>
<td>2.2.2 Surprises in bond returns</td>
<td>15</td>
</tr>
<tr>
<td>2.2.3 Stock-bond correlation</td>
<td>16</td>
</tr>
<tr>
<td>2.2.4 Component expectations</td>
<td>18</td>
</tr>
<tr>
<td>2.3 Data</td>
<td>20</td>
</tr>
<tr>
<td>2.4 Empirical proxies of news components</td>
<td>22</td>
</tr>
<tr>
<td>2.4.1 Cash flow news</td>
<td>23</td>
</tr>
<tr>
<td>2.4.2 Real interest rate news</td>
<td>25</td>
</tr>
<tr>
<td>2.4.3 Excess stock return news</td>
<td>24</td>
</tr>
<tr>
<td>2.4.4 Inflation news</td>
<td>25</td>
</tr>
<tr>
<td>2.4.5 Excess bond return news</td>
<td>26</td>
</tr>
<tr>
<td>2.5 Results</td>
<td>29</td>
</tr>
</tbody>
</table>
2.5.1 Realised second moments .................................................. 26
2.5.2 Descriptive statistics of news components ............................... 27
2.5.3 Unconditional variance decompositions .................................. 30
2.5.4 Conditional (co)variance regressions ..................................... 34
2.6 Robustness ................................................................. 12
2.7 Concluding remarks .......................................................... 14

3 Stock-bond correlation and out-of-sample portfolio performance using analyst forecasts 46
3.1 Introduction ................................................................. 46
3.2 Stock-bond asset allocation .................................................. 50
3.3 Data ............................................................................. 51
3.3.1 Stock and bond returns .................................................. 51
3.3.2 Analyst forecast data ..................................................... 55
3.4 In-sample predictability ....................................................... 60
3.4.1 Volatility ................................................................. 60
3.4.2 Correlation ............................................................... 63
3.5 Out-of-sample predictability .................................................. 64
3.5.1 Portfolio performance metrics ......................................... 64
3.5.2 Statistical significance of performance ............................... 66
3.5.3 Portfolio strategies ....................................................... 67
3.6 Robustness ................................................................. 79
3.6.1 Performance during the 2008 financial crisis ....................... 79
3.7 Concluding remarks .......................................................... 80

4 Pension funds and stock-bond correlation risk: The case for a correlation swap 81
4.1 Introduction ................................................................. 81
4.2 Stock-bond correlation risk: The case for pension funds ................ 89
4.3 Trading stock-bond correlation risk ......................................... 89
4.3.1 Stock-bond correlation swap contract ................................ 95
4.3.2 Realised correlation ..................................................... 97
4.3.3 Swap rate - Implied correlation ..................................... 99
4.4 Empirical support for stochastic correlation ............................. 102
4.4.1 Implied volatility ....................................................... 102
4.4.2 Unspanned volatility risk ............................................. 107
## CONTENTS

4.5 Utility indifference pricing: Stock-bond correlation swap ........................................... 110  
4.5.1 Stock-bond pricing model .................................................................................. 111  
4.5.2 Stock-bond correlation swap pricing model ...................................................... 113  
4.5.3 Monte carlo simulation ..................................................................................... 117  
4.5.4 Parameter values ............................................................................................... 118  
4.6 Results: Implied stock-bond correlation ................................................................. 122  
4.7 Concluding remarks ............................................................................................... 127  

5 Conclusions and future work .................................................................................... 129  

Appendix ......................................................................................................................... 133  
A: Theoretical return expressions .............................................................................. 133  
B: Data construction for Chapter 2 ............................................................................ 135  
C: Construction of news time series proxies .............................................................. 137  
D: Conditional covariance models ............................................................................ 141  
E: Calculation of Treasury bond returns .................................................................. 144  
F: The bond proxies: CMT rates vs CMS rates ......................................................... 145  
G: Implied volatility index construction ................................................................... 146  
H: Optimal mean-variance portfolio with pension liabilities .................................... 147  
I: Discretisation of the SDE ....................................................................................... 148  

Bibliography ...................................................................................................................... 150
# List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Monthly time series of realised second moments of stock and bond returns</td>
<td>9</td>
</tr>
<tr>
<td>2.2</td>
<td>Monthly time series of forecasted versus realised excess stock and bond returns</td>
<td>25</td>
</tr>
<tr>
<td>2.3</td>
<td>Monthly time series of the news components from the Campbell-Shiller decomposition</td>
<td>28</td>
</tr>
<tr>
<td>3.1</td>
<td>Monthly time series of realised volatilities and Sharpe ratios of historically estimated and perfect foresight minimum-variance portfolios</td>
<td>48</td>
</tr>
<tr>
<td>3.2</td>
<td>Monthly time series of realised volatilities and correlation of stock and bond returns</td>
<td>54</td>
</tr>
<tr>
<td>3.3</td>
<td>Monthly time series of the number of complete survey responses</td>
<td>57</td>
</tr>
<tr>
<td>3.4</td>
<td>Monthly time series of mean-consensus and dispersion measures from analyst forecasts</td>
<td>58</td>
</tr>
<tr>
<td>3.5</td>
<td>Contribution to adjusted $R^2$ of the volatility and correlation of stock and bond returns</td>
<td>61</td>
</tr>
<tr>
<td>3.6</td>
<td>S&amp;P500 and 10-year Treasury bond level</td>
<td>68</td>
</tr>
<tr>
<td>3.7</td>
<td>Growth of $1$ invested in selected portfolio strategies net of transaction costs</td>
<td>77</td>
</tr>
<tr>
<td>3.8</td>
<td>Net opportunity cost of constructed portfolios versus the MinVar portfolio</td>
<td>78</td>
</tr>
<tr>
<td>3.9</td>
<td>Net Sharpe ratios of portfolios during 2008 financial crisis</td>
<td>79</td>
</tr>
<tr>
<td>4.1</td>
<td>Over-The-Counter (OTC) correlation swap contract between the S&amp;P500 and 10-year Constant Maturity Swap (CMS) rate.</td>
<td>84</td>
</tr>
<tr>
<td>4.2</td>
<td>Percentage funding ratio at risk from adverse shocks to assets and liabilities of a pension fund</td>
<td>91</td>
</tr>
<tr>
<td>4.3</td>
<td>Daily time series of rolling window correlation between S&amp;P500 returns and 10-year CMS yield changes</td>
<td>98</td>
</tr>
<tr>
<td>4.4</td>
<td>Term structure of implied correlation between S&amp;P500 returns and 10-year CMS yield changes as on 08/04/2011</td>
<td>100</td>
</tr>
<tr>
<td>4.5</td>
<td>Time series of 1-year implied correlation between S&amp;P500 returns and 10-year CMS yield changes</td>
<td>101</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>------------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>4.6</td>
<td>Daily time series of 1-year implied volatility of the S&amp;P500 and 10-year Treasury note</td>
<td>103</td>
</tr>
<tr>
<td>4.7</td>
<td>Daily time series of rolling window correlation between (1-year) implied volatilities of S&amp;P500 and 10-year Treasury note</td>
<td>106</td>
</tr>
<tr>
<td>4.8</td>
<td>Volatility leverage effect within the S&amp;P500 index and 10-year Constant Maturity Treasury yield</td>
<td>120</td>
</tr>
<tr>
<td>4.9</td>
<td>Yield spread of 10-year CMS over 10-year Constant Maturity Treasury</td>
<td>121</td>
</tr>
<tr>
<td>4.10</td>
<td>Stock-bond implied correlation quote</td>
<td>123</td>
</tr>
<tr>
<td>4.11</td>
<td>Stock-bond implied correlation quote sensitivity</td>
<td>125</td>
</tr>
<tr>
<td>F.1</td>
<td>Comparison of the 10-year CMT yield and 10-year CMS rate</td>
<td>146</td>
</tr>
</tbody>
</table>
# List of Tables

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Component forecast source</td>
<td>19</td>
</tr>
<tr>
<td>2.2</td>
<td>Descriptive statistics of the news components</td>
<td>29</td>
</tr>
<tr>
<td>2.3</td>
<td>Unconditional (co)variance decomposition</td>
<td>31</td>
</tr>
<tr>
<td>2.4</td>
<td>Regression of realised stock variance</td>
<td>35</td>
</tr>
<tr>
<td>2.5</td>
<td>Regression of realised bond variance</td>
<td>39</td>
</tr>
<tr>
<td>2.6</td>
<td>Regression of realised stock-bond covariance</td>
<td>40</td>
</tr>
<tr>
<td>2.7</td>
<td>Robustness regressions</td>
<td>43</td>
</tr>
<tr>
<td>3.1</td>
<td>Descriptive statistics of stock and bond returns</td>
<td>53</td>
</tr>
<tr>
<td>3.2</td>
<td>Descriptive statistics of the (monthly) analyst forecasts</td>
<td>59</td>
</tr>
<tr>
<td>3.3</td>
<td>Predictive regressions for second moments of returns using analyst forecast measures</td>
<td>62</td>
</tr>
<tr>
<td>3.4</td>
<td>List of portfolios considered</td>
<td>69</td>
</tr>
<tr>
<td>3.5</td>
<td>Portfolio performance for benchmark and minimum-variance portfolios based on historical (realised) data</td>
<td>72</td>
</tr>
<tr>
<td>3.6</td>
<td>Portfolio performance inclusive of transaction costs for benchmark and minimum-variance portfolios based on historical data</td>
<td>78</td>
</tr>
<tr>
<td>3.7</td>
<td>Portfolio performance for portfolios formed using analyst forecast measures</td>
<td>79</td>
</tr>
<tr>
<td>4.1</td>
<td>Regressions of surplus returns onto realised correlation of stock returns and bond yield changes</td>
<td>93</td>
</tr>
<tr>
<td>4.2</td>
<td>Term structure of implied stock-bond correlation as on 08/04/2011</td>
<td>99</td>
</tr>
<tr>
<td>4.3</td>
<td>Descriptive statistics of the implied volatility time series for S&amp;P 500 and 10-year Treasury note</td>
<td>106</td>
</tr>
<tr>
<td>4.4</td>
<td>Principal components analysis on the term structure of interest rates</td>
<td>108</td>
</tr>
<tr>
<td>4.5</td>
<td>Evidence on the spanning of volatility risk in stock and bond markets</td>
<td>109</td>
</tr>
</tbody>
</table>
# LIST OF TABLES

4.6 Descriptive statistics of S&P500 returns and 10-year Constant Maturity Treasury yield changes .............................................. 119

4.7 Parameter values for stock-bond pricing model .............................................. 119

4.8 Unconditional moments of the yield difference between 10-year Constant Maturity Treasury and 10-year Constant Maturity Swap .............................................. 122

4.9 Portfolio weights for a one-period mean-variance ALM problem without and with access to a stock-bond correlation swap .............................................. 126

C.1 Forecasts of excess stock returns .............................................. 140
Chapter 1

Introduction and literature review

The seminal work of Markowitz [1952] highlighted the use of a mean-variance framework in asset management decisions. A manager who invests across stocks and Treasury bonds should thus not only be concerned about the forecast of returns on these assets over time, but also with the forecast of correlation between stock and bond returns over time. The latter forecast has direct implications on the (expected) diversification benefits of the portfolios formed. Despite this, academia has mainly focused its attention on the contemporaneous changes and predictability (in the time series) of expected returns on stocks and bonds, whilst research into the correlation of stock and bond returns has only come to the forefront in the last 10 years.

Research on the pricing of stocks and bonds has mostly evolved along separate lines. This is surprising from both a theoretical and empirical perspective given there is no obvious reason for market segmentation. There is however a vast amount of work on respectively describing and forecasting stock and bond volatility. For example, Schwert [1989] investigates the affect of macroeconomic volatility together with other variables on stock volatility, while Jones et al. [1998] examine the reaction of macroeconomic news on bond-market volatility. Historically, the independent evolution of research in stock and bond volatility may have been due to a naive assumption that the correlation between stocks and bonds

2By forecasting the correlation I implicitly assume forecasting the covariance and volatilities of stock and bond returns given the definition of correlation as the covariance of returns divided by the respective volatilities.

3There is a plethora of work separately investigating time series predictability of stock returns [Fama and French 1988, Cochrane 1991, Lamont 1998, Lettau and Ludvigson 2001 among others.] and bond returns [Fama and Bliss 1987, Cochrane and Piazzesi 2005, Ludvigson and Ng 2009 among others.]. Those that jointly investigate stock and bond returns are Keim and Stambaugh 1986 who find that variables that reflect the current level of stock and bond prices are able to predict stock and bond returns, Fama and French 1993 find five common (risk) factors (the overall market, size, value, term and default) that are able to explain average returns on stocks and bonds over time; a non-exhaustive list of other works that jointly price stocks and bonds in the time series are by Mamaysky 2002, Bekaert and Grenadier 1999, Bekaert et al. 2010, Koijen et al. 2010 among others.

4A non-exhaustive list of other works on stock volatility include French et al. 1987, Schwert 1990 among others; and on bond volatility include Viceira 2010 among others.
CHAPTER 1. INTRODUCTION AND LITERATURE REVIEW

was constant: For example, Benjamin Graham’s 1949 edition of *The Intelligent Investor* advocated an equally weighted portfolio of stocks and bonds based on the claim that the correlation between stock and bond returns was constant and negative. In subsequent editions of the book however, the suggestion of an equally weighted portfolio is removed with an acknowledgement that the correlation structure between stocks and bonds changes over time.

Now that it is well accepted that stock-bond correlation varies substantially over time with sustained periods of negative correlation ([Gulko 2002], [Connolly et al. 2005]). Academics have found it puzzling to explain and forecast this time variation. This is the focus of this thesis. Generally the research on stock-bond correlation has evolved along three major lines. The first is on studies that have tried to understand the economic linkages driving the co-movement of stock and bond markets. Indeed [Barsky 1989] presents the first theoretical model to investigate the drivers of stock and bond markets, suggesting that low productivity growth and high market risk were likely to lower both corporate profits and the real interest rate, propelling stock and bond prices in opposite directions. This was followed by [Shiller and Beltratti 1992] who investigated the co-movement of stock prices and bond yields using a rational expectations present value model. While [Campbell and Ammer 1993] use a Campbell-Shiller decomposition to highlight the role of several offsetting forces behind stock-bond correlation. These studies all implicitly assume that the correlation is time-invariant however and so essentially study the unconditional determinants of the correlation between stock and bond returns.

More recently the focus has shifted to contemporaneously explaining and forecasting the time variation using either reduced form or more structural models. For example, [Baele et al. 2010] use a flexible approach to model realised correlation of stock and bond returns using a wide range of factors\(^5\) both macroeconomic and non-macro, finding that while macroeconomic factors fail to explain time varying correlations and covariances, liquidity factors seem to be important. However they are unable to explain the change in sign from positive to negative over the current decade. [David and Veronesi 2009] have more success through presenting a structural model to highlight the importance of inflation shocks and earnings shocks in forecasting realised stock-bond correlation.\(^6\) They find that the changing interaction between inflation shocks and earning shocks drives the time variation in stock-bond correlation with

---

\(^5\) More specifically they assume a dynamic factor model for returns, with expected returns being constant and unexpected returns being homoscedastic and diagonal. Time varying correlation thus arises from time varying conditional variances of the shocks to the structural factors motivated to be important for both stock and bond returns. The model-implied correlation is also affected by time varying factor loadings.

\(^6\) In particular they define economic specifications of fundamental factors (among them inflation and earnings) together with a pricing kernel and explicitly solve for stock and bond prices and their volatilities/correlation in closed-form. In estimating their model they assume realised volatility and covariance to be observable processes.
CHAPTER 1. INTRODUCTION AND LITERATURE REVIEW

positive covariance between the shocks leading to negative stock-bond correlation.

D’Addona and Kind [2006] present an affine stock-bond pricing model to investigate how macroeconomic fundamentals can influence a model-implied correlation measure, finding that the volatility of the real interest rate increases the correlation, whilst inflation shocks tend to decrease the correlation. Estimating the model, they compare their in-sample model-implied correlation to realised correlation finding a plausible but imperfect fit. Their attempt to forecast out-of-sample stock-bond correlation is limited however. Campbell et al. [2009b] propose a structural model with time varying covariance between inflation shocks and real shocks that captures the changing covariance between (unexpected) stock and bond returns. However their focus is on explaining the changing risk of nominal bonds.

Bekaert et al. [2010] present a consumption-based stock-bond pricing model with stochastic risk aversion to investigate its importance in time varying stock-bond correlation. However their model overestimates both the unconditional and conditional correlation on average. Hasseltoft [2009] presents a general equilibrium model with time varying first and second moments of consumption growth, inflation and dividend growth combined with Epstein-Zin preferences. In estimating the model, although it produces a time varying model-implied correlation, with respect to realised correlation, it has difficulties in both explaining and forecasting the (realised) correlation, especially the extent of the negative correlation observed over their sample period.

As a result of the limited empirical success of theoretical models in explaining the time variation, especially the occurrence of negative correlation, a second line of research emerged which directly focused on data to identify and understand stylised facts and historical patterns of the time variation. Gulko [2002], Connolly et al. [2005] and Connolly et al. [2007] study the effect of “flight-to-quality” events on stock-bond co-movement. They generally find that rising stock market uncertainty tends to decrease the co-movement between stock and bond markets, establishing that negative stock-bond

---

7Contrary to David and Veronesi [2009] they do not assume that the realised correlation process is observable.

8Specifically, they specify stochastic processes for economic variables and allow for the covariance between shocks to inflation and real variables to vary over time, potentially switching sign. They find that this is key to account for changing covariance between stock and bond returns.

9In their model both expected excess stock and bond returns depend negatively on stochastic risk aversion. This commonality induces the variation in correlation between stock and bond returns.

10By individually modelling consumption growth, inflation and dividend growth within a heteroscedastic specification, time varying volatility and covariances of shocks to the three macro variables allow for time varying conditional correlation between stock and bond returns.

11This is often proxied for by using a market volatility index such as CBOE’s VIX Index, which measures the implied volatility of options on the Standard & Poor’s 500 stock index. It is often called the “fear index” by market practitioners. For more details on the history and purpose of this index see Whaley [2009].
correlation is related to these “flight-to-quality” events, causing a decoupling of the asset classes.\textsuperscript{12}

Fleming et al. \citeyear{1998} develop a stochastic volatility model of speculative trading to highlight the role of revisions in (common) expectations on stock and bond markets and cross-market hedging on volatility linkages across these markets.\textsuperscript{13} They find that if volatility changes across these markets are highly correlated, bonds may not provide the safe haven asset managers are looking for.\textsuperscript{14} Chordia et al. \citeyear{2005} and Goyenko \citeyear{2006} find that stock-bond co-movement is due to time variation in investors’ liquidity needs. Since stock and bond markets are highly integrated, the author’s argue that liquidity has cross-market re-balancing effects, which they attribute to trading activity across these markets. More recently, Viceira \citeyear{2010} find that the yield spread and short term nominal rate are able to forecast bond volatility and stock-bond covariance as such variables are good proxies for business cycle conditions.

The third line of research has been on econometric models to describe and predict the time variation in stock-bond correlation. The development of econometric techniques to model time varying correlation has been a research area unto itself; this development in the literature is well documented by Andersen et al. \citeyear{2006} and Engle \citeyear{2009}. The most notable of these works is that by Engle \citeyear{2002} who presents a flexible multivariate generalised autoregressive conditional heteroscedastic (GARCH) model with dynamic conditional correlation (DCC) estimators.\textsuperscript{15} Its use in modelling time varying stock-bond correlation is presented in Cappiello et al. \citeyear{2006}. Extending the DCC model to allow for asymmetries in the correlation dynamics, they find that both stock and bond returns exhibit asymmetries in conditional correlation with stocks responding more strongly to joint bad news. Similarly De Goeij and Marquering \citeyear{2004}, assuming a conditional covariance matrix that follows a multivariate GARCH process, find that the covariance between stock and bond returns tend to be low after bad news in stock returns and good news in bond returns.

The aim of this thesis is to examine the reasons and risks of the time variation in stock-bond correlation. The following three chapters are distinct and presented independently of each other but are

\textsuperscript{12}Indeed Kodres and Pritsker \citeyear{2002} develops a multi-asset rational expectations model of asset prices to highlight the role on contagion from cross-market re-balancing. Specifically a shock in one market can generate cross-market portfolio re-balancing in non-shocked markets with resulting pricing implications.

\textsuperscript{13}I note that their focus is not directly on stock-bond correlation but on the volatility linkages between stock and bond markets as a result of the information flow within and across these markets. However their findings do have implications on the co-movement of stock and bond markets.

\textsuperscript{14}Typically asset managers shift funds from stocks to bonds when there is increased uncertainty in stock markets \cite{Connolly et al. 2005}, the risk reduction of the new portfolio depends on spill-over effects between the two markets.

\textsuperscript{15}I note however that the most widely used time varying correlation estimators are the moving average and exponentially weighted moving average (EWMA) models as championed by JPMorgan \citeyear{1996}. Their wide use is due to their simplicity but each of the estimators come with inherent “start up” problems as documented in Engle \citeyear{2009}.
CHAPTER 1. INTRODUCTION AND LITERATURE REVIEW

Intrinsically linked and contribute to the above literature in specific ways. Each chapter investigates the time variation from a different angle and the thesis is organised as follows. Chapter 2 starts with the question of “what are the macroeconomic drivers of changes in stock-bond correlation?” Such a question is of great importance in order to understand the link between the macro-economy and stock and bond markets, which have significant implications on asset managers allocation decisions and resulting diversification benefits.

Using a Campbell-Shiller decomposition I express unexpected stock and bond returns into news components related to macroeconomic fundamentals. The variance and covariance of these news components should constitute the variance and covariance of stock and bond returns. I thus attempt to use time-varying co-movement among the innovations to shed light on the economic mechanisms driving the time variation in realised second moments of stock and bond returns.

Based on using survey forecast data together with other forecast sources where survey forecasts are not available for the macroeconomic components of the decomposition, I show that uncertainty in cash flow news and real short-term interest rate news is able to explain the variation in excess stock variance up to an $R^2$ of 16%. The variation in excess bond variance can be attributed to the uncertainty in the long-run inflation rate news and real short-term interest rate news up to an $R^2$ of 16%. As for the covariance between stock and bond returns, through the interaction between several of the macroeconomic news components I account for up to 26% of the variation. I highlight the importance of the interaction between cash flow news and real short-term interest rate news for negative stock-bond correlation. These findings complement those of Campbell and Ammer [1993] and Baele et al. [2010] in the role of cash flow news and inflation rate news on the variation in stock and bond variance; but contradict those of David and Veronesi [2009] on the determinant of stock-bond correlation - they theoretically motivate the interaction between inflation rate news and cash flow news as a driver of negative stock-bond correlation, while I empirically find this to be the interaction between cash flow news and real short-term interest rate news.

Understanding the economic mechanisms driving the time variation although important from the contemporaneous perspective, has greater implications from a predictability perspective. Given the motivation and success of the macroeconomic survey forecasts in explaining a portion of the contemporaneous time variation in Chapter 2, I thus naturally extend the use of these analyst forecasts to examine whether they can improve investors ex-ante allocation of wealth between stocks and bonds in Chapter 3. Li [2002] highlights the economic value of predicting stock-bond correlation out-of-sample but does
so assuming that the volatilities of stocks and bonds is constant. Using the global minimum variance portfolio as a framework, I examine if the information contained in analyst forecasts of macroeconomic variables improves investors’ ex-ante allocation of wealth between stocks and bonds.

In-sample I find that analyst forecasts display some predictability of the volatility and correlation of stock and bond returns. However, out-of-sample, analyst forecasts do not significantly contain information which helps to improve investors’ diversification benefits. The out-of-sample performance is evaluated using five metrics: volatility, Sharpe ratio, certainty-equivalent return, turnover and opportunity cost. For the minimum-variance portfolios formed using analyst forecasts, net of transaction costs, these do not outperform the minimum-variance portfolio based on the sample covariance matrix of historical returns. Robustness checks confirm that simply using the realised estimate of the correlation between stock-bond returns during the 2008 financial crisis to form the minimum-variance portfolio, outperformed the use of analyst forecasts.

In Chapter 3 the out-of-sample benefits of predicting stock-bond correlation is clear. Although the macroeconomic survey forecasts display some out-of-sample power, predictability of the future correlation level will never be perfect. Chapter 4 thus investigates the risk of unexpected changes in stock-bond correlation for a particular type of asset manager with a natural exposure to stock-bond correlation: Pension funds. Such funds have an Asset-Liability Management (ALM) mandate, thus managing their assets in order to meet future liabilities. Since pension fund liabilities are essentially streams of future payments that depend on interest rates among other factors, they can assumed to be highly correlated to long-term bonds. Contributions by participants and the plan sponsor are invested in stocks (among other assets) in order to earn a risk premium by taking some risk so as to meet their liabilities in the long-run.16

Pension funds are thus typically net long stocks and net short bonds. Changes to the correlation between these two asset classes will therefore affect the funding ratio status of these pension funds. I examine this both theoretically and empirically for a stylised pension fund which invests contributions in a 60/40 split across stocks and long-term bonds. I find that the funding ratio is negatively affected when adverse changes to stock-bond correlation occurs.17 Given that pension funds can typically have long

---

16 Practical reasons means that these liabilities cannot be completely hedged by just investing in bonds. One such reason is the supply considerations in the availability of long-duration bonds needed to match the long duration nature of the liabilities. See Chapter 3 for a further discussion.

17 “Adverse” implying a shock to the correlation level in the direction that theoretically decreases the funding ratio. The direction depends on the way that stock-bond correlation is constructed as fully outlined in Chapter 4.
CHAPTER 1. INTRODUCTION AND LITERATURE REVIEW

re-balancing horizons (Hoevenaars et al. [2008]), this makes them particularly susceptible to stock-bond correlation risk.

The need for a method to hedge such a risk of adverse changes to the stock-bond correlation level is therefore apparent for pension funds. One such risk management method is the use of a stock-bond correlation swap. I show in Chapter 4 that such a derivative already exists but little is known about how to price and manage this multi-asset class derivative. I therefore develop a simple stock-bond correlation swap pricing model in the context of (utility) indifference pricing within an incomplete market setting.\textsuperscript{18} Such a formal and intuitive pricing model allows for the role of preferences in fairly pricing a stock-bond correlation swap.

Calibrating the model to historical data I find that the model-implied quotes of implied correlation fall within the range of quotes obtained from actual stock-bond correlation swaps.\textsuperscript{19} In one case, a spread is being charged on the correlation swap strike above its model-implied “fair value”, implying the potential of a premium being added to the “fair value” of the correlation swap. Such an observation is particularly important in the context of the benefits of financial innovations (Henderson and Pearson [2011]).

Although each chapter has its own respective conclusions, I present some final remarks and ideas for future research in Chapter 5. Lastly, the Appendix contains important derivations and additional materials for all of the chapters, the section to which these derivations and additional materials pertain are clearly highlighted within each of the following chapters.

\textsuperscript{18} I specifically investigate unspanned volatility risk in stock and bond markets in Chapter 4. Buraschi and Jackwerth [2001] and Collin-Dufresne and Goldstein [2002] respectively find that there is at least one state variable which drives innovations in equity and interest rate derivatives but does not affect innovations in the underlying stock and bond markets. Thus stocks or bonds do not span equity or interest rate derivatives respectively, and so do not hedge the volatility risk. Although I am unable to perform such an investigation specifically for correlation risk (given the lack of availability of data), from the relationship between volatility and correlation I infer the possibility of unspanned correlation risk. I therefore work in an incomplete market setting.

\textsuperscript{19} Although the market for stock-bond correlation swaps is small and illiquid, I obtained a handful of quotes on the implied stock-bond correlation, with which I compare my model-implied quotes to.
Chapter 2

Changing expectations and the correlation of stocks and bonds

2.1 Introduction

In this chapter, I explore the economic mechanisms driving the time variation in stock-bond correlation using macroeconomic forecasts. The correlation between returns of stocks and bonds and is defined as the covariance of returns between stocks and bonds divided by the respective volatilities. I make this distinction to highlight the importance of the respective components. Figure 2.1 plots the monthly realised volatility, covariance and correlation of the US stock and 10-year US Treasury bond returns. The variation over time of the covariance and volatility of returns, and thus the correlation is clear from this. For instance, in the early 1990’s when the covariance remained fairly constant but volatility changes across these markets were highly correlated; as can be seen from Figure 2.1 the correlation decreases. I therefore focus this study on explaining the time variation in covariance and variances of stock and bond returns.

I use the Campbell-Shiller (1988) decomposition to express unexpected stock and bond returns as components related to economic fundamentals. The decomposition uses an accounting identity to decompose unexpected stock returns into changing expectations (i.e. unexpected values or news) of future real cash flow, future real short-term interest rates and the future excess returns on stocks (stock risk premium). Unexpected bond returns are decomposed into changing expectations of future inflation rates (this determines the real value of the nominal bond payments), future real short-term interest rates and future excess returns on long-term bonds (bond risk premium). Because the variance and covariance in returns
CHAPTER 2. CHANGING EXPECTATIONS AND THE CORRELATION OF STOCKS AND BONDS

Figure 2.1: Monthly time series of realised second moments of stock and bond returns

Panel A plots the monthly time series of realised volatility of stock and bond returns, computed as the annualised standard deviation of daily returns within the month. Panels B and C plot the realised covariance and correlation between stock and bond returns, also computed from daily data within the month. All plots are overlaid with NBER recession bands. Stock returns are based on the value-weighted return index of stocks traded in the NYSE, AMEX and Nasdaq markets from the Centre for Research in Security Prices (CRSP). Bond returns are based on the US 10-year Treasury bond yields obtained from daily off-the-run Treasury yield curves constructed by Gurkaynak et al. [2007].
CHAPTER 2. CHANGING EXPECTATIONS AND THE CORRELATION OF STOCKS AND BONDS

are based on the variance and covariance in the unexpected values, I use time-varying co-movement among the news time series to shed light on the economic mechanisms driving the time variation in realised second moments of stock and bond returns.

Campbell and Ammer [1993] use such a framework within a vector autoregressive (VAR) model, however they estimate their model implicitly assuming that the stock-bond correlation is time-invariant.\textsuperscript{20} Such an assumption is not supported by empirical evidence as shown in Figure 2.1. Also, several studies such as Chen and Zhao [2009] show that use of a VAR model is extremely sensitive to the choice of state variables and can thus lead to differing conclusions depending on the choice of state variables. More recently, Piazzesi and Schneider [2011] show that investors’ actual historical predictions (from survey forecast data) are different from the in-sample statistical predictions obtained from using a VAR model. Thus by using actual historical forecasts rather than statistical forecasts to obtain the expected values of the decomposed components, such a method can better help to understand the true nature of asset pricing puzzles such as the time variation in stock-bond correlation.

The approach of this chapter therefore consists of using forecast data from the BlueChip Economic Indicators (BCEI) survey to obtain a time-series of expected values for the fundamental components of the decomposition, namely cash flow, short-term interest rate and excess bond returns. To the best of my knowledge, this data has not been used before to study stock-bond correlation. Forecasts for the inflation rate are obtained from a structural model developed at the Federal Reserve Bank of Cleveland by Haubrich and Bianco [2010] and forecasts of excess stock returns are obtained by using a predictive linear model on a range of state variables known in the literature to display some predictability for stock returns (Campbell and Shiller [1988], Cochrane and Piazzesi [2005]).

Once I have a time-series of forecasts for the decomposed components together with knowing the realised values, time-series of the unexpected values (news) of these components are easily obtained. I then use the Dynamic Conditional Correlation (DCC) model introduced by Engle [2002] to describe the time-varying co-movement among these news components and look at the extent to which these explain the time variation in the second moments of stock and bond returns.\textsuperscript{21} Lastly, I also compare an unconditional (co)variance decomposition based on the methodology of Campbell and Ammer [1993] using historical data within a VAR model with such a decomposition based on survey data. I note that

\textsuperscript{20}They use a Generalised Method of Moments (GMM) approach correcting for the heteroscedasticity and autocorrelation of the pricing errors but which ultimately assumes a constant variance-covariance matrix of the pricing errors.

\textsuperscript{21}As a robustness check, I also estimate an Exponential Weighted Moving Average (EWMA) model in place of the DCC model to check the significance of the results.
CHAPTER 2. CHANGING EXPECTATIONS AND THE CORRELATION OF STOCKS AND BONDS

whilst I do such a comparison, as commented above, the methodology of Campbell and Ammer [1993] implicitly assumes that stock-bond correlation is constant and so strictly is not a fair comparison with the results based on survey data, where I do not make such an assumption.

The chapter is related to three streams of the literature with three resulting contributions. First, similar to Campbell and Ammer [1993], I employ a dynamic accounting framework using the Campbell-Shiller decomposition in order to directly investigate the role of macroeconomic factors in changing stock and bond prices. Campbell and Ammer [1993] use a VAR model to examine the offsetting effects on the variances and covariance of stock and bond returns. They find that real interest rate shocks or risk premia shocks drive stocks and bonds in the same direction, while shocks to the expected long-term inflation rate move stock and bond returns in opposite directions. However, Campbell and Ammer [1993] focus on unconditional moments and implicitly assume that the stock-bond correlation is constant. I contribute to this strand of literature by using analyst forecasts to generate a time-series of forecasts for the decomposed components of stock and bond returns, and am thus able to investigate the time variation of stock-bond correlation.

Second, this chapter is related to previous work that has attempted to shed light on the underlying economic linkages that tie fundamentals to stock-bond return volatilities and covariances. Barsky [1989] was amongst the first to investigate this by proposing a general equilibrium model to show that unconditional stock-bond covariance is state-dependent. Bekaert et al. [2010] develop a stock-bond pricing model where agents have stochastic risk aversion. Campbell et al. [2009b] propose a model with time varying covariance between inflation and real shocks to capture changing covariance of stock and bond returns. David and Veronesi [2008] theoretically motivate shocks to inflation and earnings as an important driver of second moments of stock and bond returns. Such models have had differing degrees of success in explaining the time variation, especially the occurrence of negative stock-bond correlation. Indeed Baele et al. [2010] find that macroeconomic factors actually contribute very little to the time variation, concluding that “flight-to-liquidity” plays a more important role. The contribution here is to empirically investigate the economic mechanisms that drive changes in the volatility and correlation of stock and bond returns over time.

---

22 He found that low productivity growth and high market risk are likely to lower both corporate profits and the real interest rate, which propels stock and bond prices in opposite directions. Note that similarly to Campbell and Ammer [1993], this study also implicitly assumes that the stock-bond correlation is constant over time.

23 More specifically find that the covariance of inflation with the real economy is a key state variable whose movements can account for the changing covariance of stock and bond returns. They also allow the covariance of inflation with real shocks to switch sign to accommodate negative stock-bond correlation.
Third, econometric tools have been developed to acknowledge and describe the time variation in correlation. Specifically for stock and bond returns, De Goeij and Marquering [2004] and Cappiello et al. [2006] both develop and estimate dynamic correlation models. I contribute to this strand of literature by adopting the Dynamic Conditional Correlation (DCC) model of Engle [2002] to explicitly obtain (co)variance time series of stock and bond return news components, allowing their role in explaining stock-bond correlation to be investigated. Such a tool allows for the work of Viceira [2010] to be extended to explore the time variation in the covariance of the news components of both stock and bond returns, hence directly investigating the role of the interaction of macroeconomic news factors on stock-bond correlation.

The empirical investigation uncovers several new results and also confirms several previous findings. In line with existing findings, uncertainty in cash flows and uncertainty in future excess returns are able to explain up to 16% in the monthly time variation of stock variance. A new finding in that the dynamics between these two uncertainties also contributes to stock market volatility. Such an observation seems intuitive considering the large cash flow shocks observed in the recent financial crisis (together with the potential expected returns shocks, although directly unobservable) and the corresponding volatility of the stock markets. As for bond market volatility, uncertainty in the future inflation rate and the real interest rate are able to contemporaneously explain up to 16% in the monthly variation in bond variance. I also (newly) find that the interaction between the uncertainties also plays a significant role in explaining bond variance. Even though the inflation rate over the sample period is relatively low and stable, inflation plays a major role in the volatility of bond returns [Campbell et al. 2009b]. The real interest rate news also appears important since unexpected decreases of the real interest rate during the recent financial would have pushed bond prices up which would have been accompanied by an increase in bond market variance.

Regarding the covariance between stock and bond returns, the uncertainty in cash flow, inflation rate, real interest rate and future excess stock returns can explain up to 26% of the variation in the covariance. I empirically document the importance of three terms in particular in driving the dynamics of stock-bond covariance: the uncertainty between cash flow news and real interest rate news, the uncertainty between future excess stock return news and inflation rate news and the uncertainty between excess stock return news and real interest rate news. Such empirical findings are contrary to David [2010] uses a VAR approach to study the covariance of bond return news components with the short term nominal rate and yield spread. By using excess stock returns as one of the state variables, the role of the short rate and yield spread on stock-bond correlation was also examined. This chapter instead focuses on the interaction between news components to contemporaneously explain the time variation in stock-bond correlation.
and Veronesi [2008] who theoretically motivate the importance in the covariance between cash flow and inflation news in the dynamics of stock-bond correlation. The findings highlight the importance of shocks to the real short-term interest rate. Indeed, recently Viceira [2010] motivates the short-rate as a proxy for the business-cycle and demonstrate its importance for stock-bond correlation.

Given that I adjust the nominal short-rate for inflation to obtain the real short-term interest rate, shocks to the real short-rate thus incorporate shocks to inflation. Therefore, the real short-term interest rate news series not only incorporates revisions to the expected business cycle but also revisions to the expected inflation rate (i.e. the informational content of inflation rate news). Further, I demonstrate that the mechanism by which stock and bond returns become negatively correlated is through uncertainty between cash flow news and real interest rate news. A possible reason being that an unexpected increase in the real short-rate is bad news for bond returns (either from the inflation rate or short-rate being higher than expected), while an unexpected increase in cash flow is good news for stock returns. This is contemporaneously accompanied by an increase in stock prices and a decrease in bond prices (investors sell their bonds in favour of stocks), thus driving the correlation to be negative. The results highlight that macroeconomic factors do have the ability to partially explain the time variation in the second moments of stock and bond returns contrary to the conclusions of Baele et al. [2010]. I thus investigate the economic mechanisms that drive the (co)variance of stock and bond returns.

The rest of the chapter is structured as follows. Section 2.2 presents the theoretical expressions of the second moments of stock and bond returns using the Campbell-Shiller decomposition. Section 2.3 outlines the data sources for our survey forecasts and historical data. Section 2.4 details the method of construction for the component news time series. Discussion of our results and robustness checks are made in Sections 2.5 and 2.6 respectively. Concluding remarks are in Section 3.7.

2.2 Theoretical stock-bond correlation

The decomposition of Campbell and Shiller [1988] and Campbell and Ammer [1993] provides a convenient theoretical framework for the empirical work. Their methodology expresses the innovation to a long-term asset return as the sum of revisions in the expected decomposed components. I show how the conditional stock-bond correlation can be written as various components on stock and bond return innovations.

[25]More precisely they form a model of stock-bond dynamics subject to economic regimes. The uncertainty between cash flow shocks and inflation shocks will then affect the prices of stocks and bonds in differing ways depending on the current economic regime.
CHAPTER 2. CHANGING EXPECTATIONS AND THE CORRELATION OF STOCKS AND BONDS

To keep the focus clear, the aim of this chapter is to investigate the economic mechanisms at work in the time variation of realised second moments of stock and bond returns. I use the framework of the Campbell-Shiller decomposition to (naturally) motivate the choice of economic news components in explaining the time variation. I therefore equate the variance/covariance of unexpected returns (left hand side of Equations (2.7), (2.8) and (2.9)) to be the realised variance/covariance of returns. In this I implicitly assume the expected one-period ahead returns to be constant over the period, thus not allowing for time varying expected returns within the period. As this study is at a monthly frequency, such an assumption seems reasonable given the difficulty in predicting returns over short time horizons.

2.2.1 Surprises in stock returns

Stocks have claim to stochastic cash flows. Appendix A derives the innovation to stock returns as changing expectations of future real dividend growth, future real interest rates and future excess stock returns:

\[
e_{t+1}^S - E_t[e_{t+1}^S] = (E_{t+1} - E_t) \left[ \sum_{j=1}^{\infty} \rho^j \Delta d_{t+j} - \sum_{j=1}^{\infty} \rho^j r_{t+j} - \sum_{j=2}^{\infty} \rho^j e_{t+j}^S \right]
\]

(2.1)

where \(e_t^S\) is the log excess stock return at time \(t\), \(r_t\) is the log real short term interest rate, \(\rho\) is a constant discount factor, \(d_t\) is the log real dividend paid at time \(t\) and \(\Delta\) is the one-period backwards difference. This equation for stock returns relates the unexpected stock return at time \(t + 1\) to changes in rational expectations (otherwise known as surprises, innovations or news) of future dividends, future expected interest rates and future excess returns. Note that \(E_t\) denotes the expectation formed at time \(t\), conditional on an information set that includes the history of stock prices, short term interest rates and dividends up to time \(t\). Equation (2.1) is an identity which is obtained through imposing internal consistency on expectations to rule out the possibility of an asset pricing bubble. I write it more simply as:

\[
\tilde{e}_{t+1}^S = \tilde{S}_{CF,t+1} - \tilde{S}_{r,t+1} - \tilde{S}_{e,t+1}
\]

(2.2)

where the tilde is used to denote a surprise to a component and the subscript is used to denote the

---

26 Similar to Campbell and Ammer [1993] I assume that \(\rho\) is constant but as a robustness check, I change the value of \(\rho\) to examine if the conclusions change as a result of this. I find that they do not.
CHAPTER 2. CHANGING EXPECTATIONS AND THE CORRELATION OF STOCKS AND BONDS

respective components and the time at which the surprise occurs. As an illustrative example, if I assume that the short-term interest rate is deterministic, Equation (2.2) neatly states that if the unexpected return to stocks is positive, then either the expected future dividends must be higher or expected future excess returns are lower, or both. From such an illustrative example, it is clear that the new information used in each time period to update the forecasts of future dividends, short term interest rate and excess returns, is what drives the surprises in stock returns.

It has been suggested that earnings rather than dividends should be used as the appropriate measure of cash flow. Earnings are more stable than dividends, less affected by financial policy and/or share repurchases and that the Modigliani-Miller Proposition on the irrelevance of dividend policy give no theoretical reason to expect managers to pursue any particular dividend policy. I therefore choose to adapt the cash flow component in Equation (2.1) according to:

\[ \tilde{S}_{CF,t+1} = (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j \Delta d_{t+j} = (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j \Delta \epsilon_{t+j} \quad (2.3) \]

where \( \Delta \epsilon_{t+j} \) is the growth in (log) real earnings at time \( t+j \), which I express more simply as:

\[ \Delta \epsilon_{t+j} = \log \left( 1 + \frac{y_{t+j}}{y_{t+j-1}} \right) \quad (2.4) \]

where \( y_t \) is defined as the real earnings at time \( t \). Equation (2.4) therefore implies that innovations to dividend growth and earnings growth over an infinite horizon contain the same information. Using this alternate expression I therefore use forward-looking earnings forecasts to back out a proxy for cash flow news.

2.2.2 Surprises in bond returns

Government bonds are subject to fixed nominal cash flows. I can thus derive an expression for bonds which holds exactly\(^{27}\):

\[ e_{t+1}^{B,(N)} - \mathbb{E}_t[e_{t+1}^{B,(N)}] = (E_{t+1} - E_t) \left[ - \sum_{j=1}^{N} \pi_{t+j} - \sum_{j=1}^{N} \pi_{t+j} - \sum_{j=2}^{N} e_{t+j}^B \right] \quad (2.5) \]

where \( e_t^B \) is the log excess bond return at time \( t \) (similarly defined as the log holding period return

\(^{27}\)See Appendix A for details of the decomposition.
CHAPTER 2. CHANGING EXPECTATIONS AND THE CORRELATION OF STOCKS AND BONDS

in excess of the short term interest rate) and \( \pi_t \) is the inflation rate at time \( t \). Changes in expected inflation alter the real value of the fixed nominal payoff on the bond, so can cause capital gains or losses even if the expected future return on the bond is constant.\(^{28}\) Similarly to above, I write Equation (2.5) more simply as:

\[
\tilde{\epsilon}_{t+1}^B = - \tilde{B}_{\pi,t+1} - \tilde{B}_{r,t+1} - \tilde{B}_{e,t+1}
\]  

(2.6)

If I were to assume \( \tilde{B}_{e,t+1} = 0 \), i.e. that the bond risk premium is constant, I could recover the expectations hypothesis of the term structure. However, I leave Equation (2.6) free from restrictive assumptions and obtain a time series for all of the news components.

2.2.3 Stock-bond correlation

Using Equations (2.2) and (2.6) the conditional variance of excess stock and bond returns can be expressed as:

\[
\text{Var}_t(\tilde{\epsilon}_{t+1}^S) = \text{Var}_t(\tilde{S}_{CF,t+1}) + \text{Var}_t(\tilde{S}_{r,t+1}) + \text{Var}_t(\tilde{S}_{e,t+1})
\]

\[
- 2\text{Cov}_t(\tilde{S}_{CF,t+1}, \tilde{S}_{r,t+1}) - 2\text{Cov}_t(\tilde{S}_{CF,t+1}, \tilde{S}_{e,t+1}) + 2\text{Cov}_t(\tilde{S}_{r,t+1}, \tilde{S}_{e,t+1})
\]  

(2.7)

\[
\text{Var}_t(\tilde{\epsilon}_{t+1}^B) = \text{Var}_t(\tilde{B}_{\pi,t+1}) + \text{Var}_t(\tilde{B}_{r,t+1}) + \text{Var}_t(\tilde{B}_{e,t+1})
\]

\[
+ 2\text{Cov}_t(\tilde{B}_{\pi,t+1}, \tilde{B}_{r,t+1}) + 2\text{Cov}_t(\tilde{B}_{\pi,t+1}, \tilde{B}_{e,t+1}) + 2\text{Cov}_t(\tilde{B}_{r,t+1}, \tilde{B}_{e,t+1})
\]  

(2.8)

I have highlighted the components that were found to be important by Campbell and Ammer \[1993\] (CA93) for stock and bond variance as a basis for comparison with the results of this study. The

\(^{28}\)Note that in this case the maturity of the bond decreases as time passes, so the relevant expectations are taken over the maturity length. The summation is from 1 to \( N \) which represents the maturity of the bond.
CHAPTER 2. CHANGING EXPECTATIONS AND THE CORRELATION OF STOCKS AND BONDS

conditional covariance between excess stock and bond returns can be written as:

\[
\text{Cov}_t(\tilde{e}_{t+1}^S, \tilde{e}_{t+1}^B) = -\text{Cov}_t(\tilde{S}_{CF,t+1}, \tilde{B}_{\pi,t+1}) - \text{Cov}_t(\tilde{S}_{CF,t+1}, \tilde{B}_{e,t+1})
\]

Theoretically motivated by DV08

\[-\text{Cov}_t(\tilde{S}_{CF,t+1}, \tilde{B}_{e,t+1}) + \text{Cov}_t(\tilde{S}_{e,t+1}, \tilde{B}_{\pi,t+1})
+ \text{Cov}_t(\tilde{S}_{e,t+1}, \tilde{B}_{r,t+1}) + \text{Cov}_t(\tilde{S}_{r,t+1}, \tilde{B}_{\pi,t+1})
+ \text{Cov}_t(\tilde{S}_{r,t+1}, \tilde{B}_{e,t+1}) + \text{Cov}_t(\tilde{S}_{e,t+1}, \tilde{B}_{r,t+1})
\]

CA93

The conditional correlation between returns thus being a function of Equations (2.7), (2.8) and (2.9) through:

\[
\text{Corr}_t(\tilde{e}_{t+1}^S, \tilde{e}_{t+1}^B) = \frac{\text{Cov}_t(\tilde{e}_{t+1}^S, \tilde{e}_{t+1}^B)}{\sqrt{\text{Var}_t(\tilde{e}_{t+1}^S)\text{Var}_t(\tilde{e}_{t+1}^B)}}
\]

(2.10)

I again highlight the components in Equation (2.9) found to be important for stock-bond covariance by Campbell and Ammer [1993] (CA93) and David and Veronesi [2008] (DV08). The latter work theoretically motivate the interaction between inflation news and cash flow news as a driver for negative stock-bond correlation. This naturally arises from a positive correlation between inflation and cash flow news given the negative sign on the component, but also intuitively follows from an unexpected increase in long-run inflation being bad news for bond markets while an unexpected increase in cash flow being good news for stock markets, driving a negative correlation between stock and bond returns.

On the other hand, Campbell and Ammer [1993] empirically find a negative correlation between excess stock return news and inflation news as a driver of negative stock-bond correlation. While a positive correlation between future excess stock return news and real interest rate news, and also between future excess stock and bond return news, move stock and bond returns in the same direction.
CHAPTER 2. CHANGING EXPECTATIONS AND THE CORRELATION OF STOCKS AND BONDS

2.2.4 Component expectations

The aim is to explore the economic mechanisms that drive the time variation in second moments of stock and bond returns through examining the time variation in the variance and covariance of the decomposed news components, namely cash flow news, real interest rate news, inflation news and excess stock and bond return news. In order to obtain a time-series of news components, I need a method to obtain a time-series of expected values for these components. There are three potential methods to obtain such expectations, as I outline below.

Model-implied: A common method is to build a predictive model for the variable of interest (either structural or reduced form) using state variables known to exhibit some predictability. After estimating the model based on historical data, the model can then be used to generate expected values of the variable \(N\) periods ahead. For example, one commonly used model is the VAR system as used by Campbell and Ammer [1993] for the Campbell-Shiller style decompositions of Equations (2.2) and (2.6) above. For this particular method however, Welch and Goyal [2008] and Chen and Zhao [2009] find that VAR models are sensitive to the sample period and the choice of state variables, and thus the model’s conclusions can change as a result of the choice of sample period and state variables. Also, in the case of Campbell and Ammer [1993], use of a VAR model implicitly assumes that the correlation between stock and bond returns is constant. Use of such a methodology would thus invalidate a specific study on the time-variation in the correlation.

Market-implied: By using market data it may be possible to back out the “market’s expectation” of the variable. For example, a popular market measure of expected inflation is the break-even inflation rate defined as the difference between equivalent-maturity yields on nominal Treasury bonds and Treasury inflation protected securities (TIPS). The problem with such an approach is that it is often influenced by other factors such as liquidity and risk. For the case of the break-even rate, Campbell et al. [2009a] show that liquidity differences between nominal and TIPS bonds, and an inflation risk premium, bias the level of the expected inflation rate obtained from TIPS. Such a method to generate the expected value of the variable may therefore not provide a precise expectation value for that variable.

Survey data: A recent trend within the literature is to use professional forecast data as a direct measure of the expectation of the variable of interest. Ang et al. [2007] find that professional forecasts significantly outperform time series, Philips curve and term structure models for predicting inflation out of sample. Piazzesi and Schneider [2011] also note that investors’ actual predictions historically are
different from the in-sample predictions generated by statistical models such as the VAR model. They comment that investors ex-ante may not recognise the same patterns observed ex-post with the benefit of hindsight - thus ex-ante survey forecasts provide actual expectations of the variable as opposed to ex-post expectations generated from using a statistical model over the whole sample period of historical data. Professional forecast data therefore may be a promising method to obtain actual expectations of the decomposed components of Equations (2.2) and (2.6) above.

In this study, I use both professional survey forecasts but also model implied forecasts. Whilst I would prefer to just use survey forecasts to obtain the component news time-series, I am restricted by the availability of forecasted variables by surveys. As a result, expectations of cash flow, the short term interest rate and long-term bond returns are obtained from forecasts made by a panel of economists from the BlueChip Economic Indicators (BCEI) survey database on corporate profits, 3-month T-Bills and 10-year Treasury bonds. Expectations of the inflation rate are obtained from a structural model developed at the Federal Reserve Bank of Cleveland by Haubrich and Bianco [2010], which provides monthly forecasts of the inflation rate over the next 10 years.29 Lastly, expectations of excess stock returns are obtained from the well-adopted method of using a predictive regression on a battery of state variables known in the literature to have some predictive power (Fama and Schwert [1977], Fama and French [1988], Campbell and Shiller [1988], Cochrane and Piazzesi [2005]).

Table 2.1: Component forecast source

<table>
<thead>
<tr>
<th>Component</th>
<th>Source of forecast</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash flow</td>
<td>BCEI forecasts of corporate profits</td>
</tr>
<tr>
<td>Real interest rate</td>
<td>BCEI forecasts of the nominal 3-month T-Bill rate less inflation forecast (see below)</td>
</tr>
<tr>
<td>Excess stock returns</td>
<td>Predictive regression on a battery of state variables</td>
</tr>
<tr>
<td>Inflation</td>
<td>Structural model developed by the Cleveland Fed</td>
</tr>
<tr>
<td>Excess bond returns</td>
<td>BCEI forecasts of 10-year Treasury bond yield</td>
</tr>
</tbody>
</table>

29Whilst the BCEI survey database does provide inflation forecasts, I choose to use a model-implied forecast made by the Federal Reserve Bank of Cleveland as it displays good out-of-sample predictability (as documented in Haubrich and Bianco [2010]) and because the model is estimated based on both historical and SPF data.
CHAPTER 2. CHANGING EXPECTATIONS AND THE CORRELATION OF STOCKS AND BONDS

After backing-out the news time-series from the forecasts and actual (realised) values of the components, I employ the Dynamic Conditional Correlation (DCC) model of Engle [2002] to describe the conditional variance and covariance of these news components. I then look to the explanatory power of these conditional second moments of the decomposed news components to explain the time variation in the second moments of stock and bond returns. As noted above, I also perform an unconditional (co)variance decomposition to compare the methodology of Campbell and Ammer [1993] based on using historical data with a VAR model over the sample period with the decomposition based on survey data.

2.3 Data

The study uses monthly data on U.S. stock market returns, U.S. 10-year Treasury bond returns, survey forecasts of corporate profits, the 3-month Treasury bill yield, the 10-year Treasury bond yield and model-implied forecasts of inflation rates and stock returns. I also obtain the actual (realised) time-series of all the forecasted variables above. The data series go from July 1984 to December 2009, a total of 306 observations. I proxy for the U.S. stock market by using the aggregate value-weighted return index of the stocks traded in the NYSE, AMEX and Nasdaq markets from the Centre for Research in Security Prices (CRSP). I proxy for the U.S. bond market by the 10-year Treasury bond, since monetary policy has less of an impact on long-term government bonds than on short-term bonds. The nominal zero-coupon yield for the 10-year bond is obtained from the daily off-the-run Treasury yield curves constructed by Gurkaynak et al. [2007]. I also require the nominal 3-month yield which is obtained from the Federal Reserve System’s H.15 Release. The monthly Treasury yields are observed as of the last trading day of each month.

Survey forecasts for the future level of corporate profits, 3-month nominal yield and the 10-year T-Bond yield come from the BlueChip Economic Indicators (BCEI) database which surveys approximately 50 economists employed by financial institutions, non-financial corporations and research organisations. At the beginning of each month participants forecast future values of various variables for the current calendar year and for the next calendar year. From this I back-out the one-year ahead forecast for the variables. Each month I obtain the ‘consensus’ forecast which is the mean of the participants’ forecast as the 1-year ahead expected value of the level of corporate profits, the 3-month T-Bill yield and the

---

30 As a robustness check I use a Exponentially Weighted Moving Average(EWMA) model to describe the conditional variance and covariance of the news components. The findings remain qualitatively unchanged.

31 Their daily Treasury yield curves are available from 1961 to the present at http://www.federalreserve.gov/econresdata/researchdata.htm.


33 For more details on the procedure see Appendix B.
10-year T-Bond yield.

I note that forecasts of corporate profits from BCEI are forecasts of the level of corporate earnings before-tax with inventory valuation and capital consumption adjustment for the National Income and Product Accounts (NIPA) at the Bureau of Economic Analysis (BEA). This represents an aggregate measure of cash flow to US firms from current production. Although this is based on all US firms that are required to file Federal corporate tax returns and so includes both public and private firms, I use this variable as the proxy for aggregate cash flow instead of the more traditional sources of earnings forecasts from IBES.

Since this survey is not anonymous, the career concerns of the respondents may influence their official stated forecast. I address this concern by comparing the BCEI data to the Survey of Professional Forecasts (SPF) data, which covers a range of forecast horizons that overlap with those obtained from BCEI. I find that the mean and median forecasts from SPF are similar to those from the BCEI. This robustness check is reassuring since Ang et al. [2007] find that forecasts from SPF significantly outperform a variety of other methods for predicting inflation. Since the participants in the BCEI survey have qualifications similar to those of the SPF participants, it is likely that the BCEI forecasts also exhibit these attractive features for corporate profits and interest rate forecasts. As for the differences between these survey databases, the SPF data is based on anonymous survey responses and is quarterly in frequency. However, the main advantage of the BCEI survey data over SPF is based on the procedure of collecting the survey responses. Once responses by survey participants have been collected, the participants are not allowed to change their forecasts, which prevents analysts from updating their forecasts when they see the forecast made by a competitor analyst. Also, one can argue that an anonymous survey is worse given the diminished responsibility for analysts to justify their forecasts when the actual value is published, thereby reducing the pressure for accurate forecasts.

Forecasts of the inflation rate are directly from the Federal Reserve Bank of Cleveland (Cleveland FED) who generate the expected values of the inflation rate through estimating a structural model on the real and nominal term structures. The work by Haubrich and Ritchken [2011] at the Cleveland FED estimate their model using nominal Treasury yields, SPF survey forecasts of inflation and inflation swap rates. I therefore am confident that the expected inflation rate implied by this model is the best of

---

34 This data is obtained directly from the Philadelphia FED.
35 These are obtained from derivative securities known as zero coupon inflation swaps. They are the most liquid inflation derivative contracts and trade in the over-the-counter market.
the survey and market-implied measures of future inflation rates. For this reason I therefore use the model-implied data for the expected inflation measure.\textsuperscript{36}

To obtain a model-implied forecast of stock returns, I use a predictive linear model based on state variables known in the literature to display some predictability. These variables include the dividend yield, term spread, default spread and the Cochrane and Piazzesi [2005] factor. The data to construct these variables was obtained from both CRSP and DataStream. For further details on the data sources and construction, see Appendix B.

2.4 Empirical proxies of news components

I now outline the construction of the news proxies. Note that as I consider a 10-year Treasury bond, the summations in Equation (2.5) pertain to a 10-year horizon ($N = 10$), with the revisions calculated over monthly intervals. Henceforth, to prevent confusion in the notation, $t$ refers to the month in which I take the expectation, whilst $j$ refers to the yearly horizon of the forecast value (expected value) that I require. Using the inflation rate as an example, with $j = 2$, $\pi_{t+12j}$ denotes the 2-year forecast of the annual inflation rate at month $t$. I therefore express the expectation at month $t$ of the total future inflation rate over the next 10-years as: $E_t\left(\sum_{j=1}^{10} \pi_{t+12j}\right)$. This analogy applies similarly to the other expected components that I consider.

Note also that the summations of the components for the innovation of excess stock returns in Equation (2.1) have an infinite horizon. For the purposes of this study, I set the horizon of the summations for Equation (2.1) to be the same as those in Equation (2.5) of $N = 10$. This implicitly assumes that near term revisions carry more weight than long-term revisions, which I believe is not an unreasonable assumption in order to obtain the news time series of each component.

For brevity I outline the detailed construction of the news proxies in Appendix C. Here I present the generalised method for each of the news proxies together with a discussion of the assumptions made to obtain the news proxies.

\textsuperscript{36}The data is available directly from http://www.clevelandfed.org/research/index.cfm.
CHAPTER 2. CHANGING EXPECTATIONS AND THE CORRELATION OF STOCKS AND BONDS

2.4.1 Cash flow news

I construct the cash flow news proxy using the BCEI forecast of corporate profits through:

\[
\tilde{S}_{CF,t+1} = \sum_{j=1}^{10} \rho^j \Delta e_{t+12j|t+1} - \sum_{j=1}^{10} \rho^j \Delta e_{t+12j|t}
\]

where Term A is the total forecast of the growth in corporate profits over the next 10 years from month \(t\), taking the expectations from month \(t+1\), i.e. knowing the actual (realised) corporate profits growth between month \(t\) and \(t+1\); while Term B is the total forecast of the growth in corporate profits over the next 10 years at month \(t\). I assume that \(\rho = 0.96\) in line with the literature.\(^{37}\)

Note that as the BCEI survey database only provides a 1-year horizon forecast of earnings in each month \(t\), I require further forecasts of real earnings growth at longer horizons and thus need to impose some assumptions and structure on the expected real growth in corporate profits at longer horizons. I adopt the method of Pástor et al. \(2008\) to generate such forecasts of the growth in corporate profits over the following 9 years by assuming that the annualised growth in earnings linearly mean-reverts to a steady state over the following 9 years in which the forecasts are being extended. I believe using such a method does not bias the subsequent forecasts in any direction and through assuming a mean-reverting process for the subsequent forecasts, I conservatively extend the forecast horizons.

2.4.2 Real interest rate news

Survey data from BCEI gives the 1-year forecast of the annual average nominal rate of returns on 3-month Treasury bills. Similarly to the approach adopted to extend the horizon of the earnings growth forecasts, I generate forecasts of the annual average rate on these bills over the following 9 years by assuming that the forecasts mean-revert to a steady state over the following periods. Note that the forecast for the real return on 3-month T-bills are defined as the forecast for the nominal return on 3-month T-Bills less the corresponding inflation rate forecast obtained from the Cleveland FED model. I thus define the real interest rate news as:

\[
\tilde{S}_{r,t+1} = \tilde{B}_{r,t+1} = \sum_{j=1}^{10} r_{t+12j|t+1} - \sum_{j=1}^{10} r_{t+12j|t}
\]

where Term C is the expectation from month \(t+1\) of the total future real T-bill rate over the next 10

\(^{37}\)I note that as a robustness check if I instead assume that \(\rho = 1\), I obtain very similar results.
CHAPTER 2. CHANGING EXPECTATIONS AND THE CORRELATION OF STOCKS AND BONDS

years from month $t$, therefore including the realised (annualised) real T-bill rate from month $t$ to $t + 1$;
and Term D is the total forecast of the future T-bill rate over the next 10 years at month $t$.

Note that I set the real interest rate news for stocks and bond to be the same due to the earlier assumption of the horizon for the summations of Equation (2.1) being the same as that for Equation (2.5), of $N = 10$. Indeed if I include $\rho$ in the calculation of real interest rate news for stocks, the correlation between this measure and that above is 0.99. I therefore choose to have the one time series to represent the news to the real interest rate for both stocks and bonds.

2.4.3 Excess stock return news

Using Equation (2.1) I can back out the excess future stock return news from knowing the unexpected stock returns, cash flow and real interest rate news. I obtain unexpected stock returns by using a predictive linear regression to model the 1-year expected excess return at a monthly frequency. I use the usual battery of variables known in the literature to display some predictability, such as the dividend yield, term spread, default spread, lagged returns and nominal return on the 3-month Treasury bills. Cochrane and Piazzesi [2005] also suggest a factor (CP factor henceforth) constructed from a linear combination of forward rates, which seems to have significant predictive power for both future bond and stock returns in their sample period. Interestingly, the significance of the CP factor as a forecasting variable for stock returns has changed since the work by Cochrane and Piazzesi [2005]. See Appendix C for more of a discussion.

Given the actual (realised) excess stock returns, I work out the unexpected excess stock returns series via:

$$e_{t+1}^S = \frac{1}{12} e_{t+1}^S - \frac{11}{12} \mathbb{E}_t(e_{t+12}^S)$$  \hspace{1cm} (2.13)

where $e_{t+1}^S$ is the realised annual excess returns to stocks in month $t + 1$.

Panel A of Figure 2.2 plots the forecasted excess returns on stocks versus the actual returns. Unexpected stock returns is essentially the vertical distance between the realised and forecasted curves. From this returns to stocks are somewhat hard to predict at a monthly frequency using the state variables that I have stated above.

---

38 Annual excess returns means that this is the excess returns over the past 12 months.
CHAPTER 2. CHANGING EXPECTATIONS AND THE CORRELATION OF STOCKS AND BONDS

Figure 2.2: Monthly time series of forecasted versus realised excess stock and bond returns

2.4.4 Inflation news

Model-implied data from the Cleveland FED gives forecasts of the annual rate of inflation over the next 10 years at a monthly frequency. Therefore without the need to impose additional structure to generate longer horizon forecasts, I express the inflation rate news as:

$$\tilde{B}_{\pi,t+1} = E_{t+1} \left( \sum_{j=1}^{10} \pi_{t+12j} \right) - E_t \left( \sum_{j=1}^{10} \pi_{t+12j} \right)$$  \hspace{1cm} (2.15)$$

where Term E is the month $t+1$ expectation of the total future inflation rate over the next 10 years as of month $t$ and Term F is the month $t$ expectation of the total future inflation rate as of month $t$. Thus similar to the previous news definitions, Term E captures the realised (annualised) inflation rate from month $t$ to $t+1$.

I also consider an alternative way to construct inflation rate news as:

$$\tilde{B}_{\pi,t+1} = Inft_{t+1} + \frac{9}{10} InftTotal_{t+1} - Inft_{t} - \frac{9}{10} InftTotal_{t}$$  \hspace{1cm} (2.14)$$

The correlation between these two measures is 0.995. I therefore do not consider this measure going forward.
CHAPTER 2. CHANGING EXPECTATIONS AND THE CORRELATION OF STOCKS AND BONDS

2.4.5 Excess bond return news

I use Equation (2.5) to back out the excess future bond return news from unexpected bond returns, inflation rate and real interest rate news. Instead of obtaining unexpected bond returns through assuming a model for expected excess bond returns as I did for excess stock returns above, I utilise the 1-year forecast of the 10-year T-bond yield from the BCEI forecast survey database. Note that the forecast for the 10-year T-Bond is from a survey question that asks for a constant-maturity Treasury yield expectation. As I proxy for the long-term bond using the 10-year nominal zero coupon bond, to obtain the yield expectation implied by the surveys, I first compute the expected change in the 10-year Treasury bond yield and then add the expected change to the current 10-year zero coupon bond yield, which I denote as $TBond_{1,t}$, that is the 1-year forecast of the nominal return on the 10-year zero-coupon T-Bond at month $t$. I can then express the expected excess bond returns in one year as:

$$E_t (e_{t+12}^B) = TBond_{1,t} - TBill_{1,t}$$ (2.16)

where $TBill_{1,t}$ represents the 1-year forecast of the nominal return on the 3-month T-Bills, also obtained from the BCEI database. Knowing the actual (realised) excess bond returns, I back out the unexpected returns to the 10-year T-Bond through:

$$e_{t+1}^B = \frac{1}{12} e_{t+1}^B - \frac{11}{12} E_t (e_{t+12}^B)$$ (2.17)

where $e_{t+1}^B$ is the realised annual excess returns to the 10-year T-Bond in month $t + 1$.\(^\text{40}\) Panel B of Figure 2.2 plots the forecast of excess bond returns versus the actual returns in that month. The vertical distance between the forecast and the realised values makes up the news. It is clear that the survey forecasts for bond returns is a much better predictor than the model-implied forecasts for stock returns in the sample period. This should theoretically imply that the risk premium to bonds should be somewhat lower that the risk premium to stocks. Of particular interest is to study the role of the news of the risk premiums in explaining the time variation of stock-bond volatility and covariance.

2.5 Results

2.5.1 Realised second moments

Figure 2.1 plots the monthly realised second moments of stock and bond returns. Following the approach of Schwert [1989], I construct these based on daily returns within a month. Denoting $r_{i,t}^S$ and $r_{i,t}^B$ as the

\(^{40}\)Similarly to that above, annual excess returns means the excess returns over the previous 12 months.
CHAPTER 2. CHANGING EXPECTATIONS AND THE CORRELATION OF STOCKS AND BONDS

daily stock and bond returns on day \(i\) in month \(t\) respectively, the realised variance of stock and bond returns are computed as the sum of squared daily returns:\(^{41}\)

\[
\sigma^2_{S,t} = \sum_{i=1}^{N_t} (r_{S,i,t})^2, \quad \sigma^2_{B,t} = \sum_{i=1}^{N_t} (r_{B,i,t})^2
\]

where \(N_t\) is the number of daily returns in month \(t\). The monthly realised covariance between stock and bond returns is obtained from:

\[
\text{Cov}_t(S,B) = \sum_{i=1}^{N_t} r_{S,i,t} r_{B,i,t}
\]

I note the inherent noise in the estimate of monthly variance from just 22 daily observations. Ideally I would employ a method similar to Bollerslev and Zhou \cite{Bollerslev2006} which uses higher-frequency intra-day data to estimate the variance. However intra-day data over the time period that I require for both stocks and bonds is difficult to obtain. I therefore continue to use daily returns to construct the variance and covariance measures.

Panel A of Figure 2.1 shows that the stock market was very volatile during the stock market crash of 1987, during the Asian financial crisis, the Russian government’s debt default and the collapse of the hedge fund Long-Term Capital Management (LTCM) during 1997-1998, the bursting of the internet bubble in 2001 and during the more recent financial crisis in 2007-2009. In contrast looking at panel B the bond market has been relatively more stable. The notable exceptions being the volatility around the stock market crash of 1987, between 1997-2001 and during the recent financial crisis.

Panel C of Figure 2.1 shows that the stock-bond covariance is large in magnitude around increased stock and bond market volatility. Interestingly, before 1998 during periods of increased volatility the covariance remained positive. It is only from 1998 until the end of the sample period that the covariance between stocks and bond becomes negative when financial turmoil occurs. Naturally the same pattern is observable from Panel D for the correlation. It is clear that there is substantial time variation, with most of the movement occurring around the periods of increased stock market volatility.

2.5.2 Descriptive statistics of news components

Figure 2.3 displays the time series of the news components constructed in Section 2.4 and highlights that the forecasting errors fall in a range around zero. This is confirmed by the mean values of the forecasting errors as shown by the descriptive statistics in Table 2.2 implying the use of well performing

\(^{41}\)I do not subtract the sample mean from each daily return to compute monthly second moments as this is a very minor adjustment.
forecasts for the components.

Figure 2.3: Monthly time series of the news components from the Campbell-Shiller decomposition

Table 2.2 reports the correlation matrix of the return components estimated from the news time series over the full sample. Several observations are apparent. First, shocks to excess stock returns and to excess bond returns have a positive correlation of 0.21. The convention that long-term assets tend to move together holds here, which arises from the notion that similar variables are able to forecast both stock and bond returns as shown by Fama and French [1993]. Campbell and Ammer [1993] also find this particular correlation to be 0.20 in their full sample period (1952-1987), however based on their methodology over our sample period, I find this correlation to be 0.04. The difference between this and 0.21 is due to both data and methodology. The former is obtained from survey/model-implied forecasts (directly for the variables) based on the methodology based above, whilst the latter is inferred from applying a VAR model to historical data, which implicitly assumes that stock-bond correlation is constant.

Second, I notice that future excess stock return news has a positive correlation of 0.13 with the news to the future long-run inflation rate. This implies that when investors learn the long run-inflation will be higher than expected, they also tend to learn that stock risk premium (future excess stock returns) will be higher than expected. Assuming that inflation risk is priced into the stock market (Chen et al.)
CHAPTER 2. CHANGING EXPECTATIONS AND THE CORRELATION OF STOCKS AND BONDS

Table 2.2: Descriptive statistics of the news components

Table 2.2 reports the descriptive statistics of the news time series over the period 07/1984 to 12/2009. Panel A shows the sample mean and standard deviation of each of the news series respectively. Panel B gives the correlation between the news series and panel C presents the first 5 lags in the autocorrelation of the news time series.

<table>
<thead>
<tr>
<th>Panel A: Mean &amp; Std Dev</th>
<th>( \tilde{S}_{CF} )</th>
<th>( \tilde{S}<em>{r} ) &amp; ( \tilde{B}</em>{r} )</th>
<th>( \tilde{S}_{e} )</th>
<th>( \tilde{B}_{\pi} )</th>
<th>( \tilde{B}_{e} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>-0.002</td>
<td>0.008</td>
<td>-0.010</td>
<td>-0.002</td>
<td>-0.013</td>
</tr>
<tr>
<td>Std Dev</td>
<td>0.051</td>
<td>0.021</td>
<td>0.071</td>
<td>0.016</td>
<td>0.039</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: Correlation</th>
<th>( \tilde{S}_{CF} )</th>
<th>( \tilde{S}<em>{r} ) &amp; ( \tilde{B}</em>{r} )</th>
<th>( \tilde{S}_{e} )</th>
<th>( \tilde{B}_{\pi} )</th>
<th>( \tilde{B}_{e} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tilde{S}_{CF} )</td>
<td>1.000</td>
<td>0.057</td>
<td>0.659</td>
<td>0.042</td>
<td>0.045</td>
</tr>
<tr>
<td>( \tilde{S}<em>{r} ) &amp; ( \tilde{B}</em>{r} )</td>
<td>1.000</td>
<td>-0.214</td>
<td>-0.695</td>
<td>-0.105</td>
<td></td>
</tr>
<tr>
<td>( \tilde{S}_{e} )</td>
<td>1.000</td>
<td>0.127</td>
<td>0.209</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \tilde{B}_{\pi} )</td>
<td>1.000</td>
<td>-0.381</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \tilde{B}_{e} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel C: Autocorrelation</th>
<th>Lags 1</th>
<th>0.850</th>
<th>0.373</th>
<th>0.477</th>
<th>0.307</th>
<th>0.011</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lags 2</td>
<td>0.765</td>
<td>0.405</td>
<td>0.341</td>
<td>0.183</td>
<td>-0.028</td>
<td></td>
</tr>
<tr>
<td>Lags 3</td>
<td>0.671</td>
<td>0.389</td>
<td>0.335</td>
<td>0.188</td>
<td>0.147</td>
<td></td>
</tr>
<tr>
<td>Lags 4</td>
<td>0.594</td>
<td>0.408</td>
<td>0.333</td>
<td>0.177</td>
<td>0.224</td>
<td></td>
</tr>
<tr>
<td>Lags 5</td>
<td>0.522</td>
<td>0.426</td>
<td>0.254</td>
<td>0.084</td>
<td>0.010</td>
<td></td>
</tr>
</tbody>
</table>

...this increase in the stock risk premium could imply compensation for an investor who is willing to bear inflation risk when holding stocks for the long-run.

Similarly to Campbell and Ammer [1993], excess bond return news and inflation news are negatively correlated; thus when investors learn that the long-run inflation rate will be higher than expected, they learn that future excess bond returns will be lower than expected. Since zero-coupon bonds have fixed nominal payoffs, the capital loss from higher expected inflation will be offset by the capital gains from the lower bond risk premium (expected excess bond returns). This does not necessarily imply that bonds are contemporaneously able to hedge inflation shocks since it depends on the magnitudes of these capital losses and gains.42

Third, innovations to future excess stock and bond returns are negatively correlated to real short-term interest rate news. Contemporaneously risk premia in both markets unexpectedly increase (thus decreasing stock and bond prices) when the real short-rate is lower than expected. [Campbell and Am-

---

42Indeed I find the contemporaneous correlation between the one-period inflation news and unexpected bond returns in the data is -0.10, indicating that bonds are not able to contemporaneously hedge inflation shocks.
mer [1993] also find that shocks to real short-term interest rate drive stock and bond returns in the same direction. This is intuitively clear since news about real interest rates is the only component common to both assets. Indeed Campbell and Ammer [1993] comments that the low correlation between stock and bond returns is due to the low variability of this common component, both observations of which can be confirmed by the results in Table 2.2.

Lastly, the correlation between cash flow news and excess stock return news is strongly positively correlated at 0.66. This is similar to the observations by Vuolteenaho [2002] at a firm-level. Such a result will be useful when looking at which components drive aggregate-level stock returns.

I note that all the news time series appear to be stationary in the sample period. Dickey-Fuller tests and augmented Dickey-Fuller tests with 5 lags reject the unit root hypothesis at the 5% level or better. This suggests that stationary asymptotic distributions are likely to approximate well the finite sample distributions of the coefficients and test statistics for the regressions that are performed below.

Given the short sample period of this study, I choose not to do a subperiod analysis since there is no natural break point and do not wish to data mine the results. Instead, I compare the findings with those of Campbell and Ammer [1993] which was performed over a different sample period. Such an approach will also address any multi-collinearity concerns between the conditional (co)variances of the news time series that I use in the conditional covariance decompositions below.

2.5.3 Unconditional variance decompositions

Table 2.3 reports the (unconditional) variance decomposition for excess stock returns, excess bond returns and the (unconditional) covariance decomposition between excess stock and bond returns based on the unconditional expressions of Equations (2.7), (2.8) and (2.9). The table reports the (co)variances of the different news components that make up the (co)variance equations of excess stock and bond returns. These numbers are normalised by the (co)variance of the return innovation itself, so that the numbers sum to one for each decomposition. I also present the comparative results from Campbell and Ammer [1993] over the same sample period.

2.5.3.1 Variance decomposition for excess stock returns

Panel A reports that the variance of stock returns is mainly attributable to the variance of future excess stock return news followed by the variance of future real cash flow news. The results are the opposite to
Table 2.3: Unconditional (co)variance decomposition

Table 2.3 reports the unconditional second moments of stock and bond returns that can be attributed to the unconditional second moments of the news components based on the Campbell-Shiller decomposition. The table reports the variances and covariances of these components, divided by the (co)variance of unexpected stock and bond returns so that the numbers reported add up to one. Panel A and panel B report the ratios of the stock and bond variance components respectively. Panels C and D report the amount of unconditional stock-bond covariance that can be attributed to the unconditional covariance of the decomposed news components. For comparison, I also present results from using the [Campbell and Ammer] methodology over our sample period (CA methodology).

<table>
<thead>
<tr>
<th>Panel A: $\text{Var}(\tilde{e}^S)$</th>
<th>$\text{Var}(\tilde{S}_{CF})$</th>
<th>$\text{Var}(\tilde{S}_r)$</th>
<th>$\text{Var}(\tilde{S}_e)$</th>
<th>$-2\text{Cov}(\tilde{S}_{CF}, \tilde{S}_r)$</th>
<th>$-2\text{Cov}(\tilde{S}_{CF}, \tilde{S}_e)$</th>
<th>$2\text{Cov}(\tilde{S}_r, \tilde{S}_e)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.01</td>
<td>0.17</td>
<td>1.98</td>
<td>-0.05</td>
<td>-1.86</td>
<td>-0.25</td>
</tr>
<tr>
<td>CA methodology</td>
<td>2.06</td>
<td>0.01</td>
<td>0.98</td>
<td>-0.02</td>
<td>-1.82</td>
<td>0.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: $\text{Var}(\tilde{e}^B)$</th>
<th>$\text{Var}(\tilde{B}_x)$</th>
<th>$\text{Var}(\tilde{B}_r)$</th>
<th>$\text{Var}(\tilde{B}_e)$</th>
<th>$2\text{Cov}(\tilde{B}_x, \tilde{B}_r)$</th>
<th>$2\text{Cov}(\tilde{B}_x, \tilde{B}_e)$</th>
<th>$2\text{Cov}(\tilde{B}_r, \tilde{B}_e)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.24</td>
<td>0.39</td>
<td>1.39</td>
<td>-0.43</td>
<td>-0.44</td>
<td>-0.16</td>
</tr>
<tr>
<td>CA methodology</td>
<td>3.96</td>
<td>0.03</td>
<td>0.90</td>
<td>0.00</td>
<td>0.19</td>
<td>-0.04</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel C: $\text{Cov}(\tilde{e}^S, \tilde{e}^B)$</th>
<th>$\text{Cov}(\tilde{e}^S, \tilde{B}_x)$</th>
<th>$\text{Cov}(\tilde{e}^S, \tilde{B}_r)$</th>
<th>$\text{Cov}(\tilde{e}^S, \tilde{B}_e)$</th>
<th>$\text{Cov}(\tilde{e}^B, \tilde{S}_{CF})$</th>
<th>$\text{Cov}(\tilde{e}^B, \tilde{S}_r)$</th>
<th>$\text{Cov}(\tilde{e}^B, \tilde{S}_e)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-0.36</td>
<td>0.17</td>
<td>1.20</td>
<td>0.54</td>
<td>0.32</td>
<td>1.21</td>
</tr>
<tr>
<td>CA methodology</td>
<td>-41.63</td>
<td>0.00</td>
<td>-1.03</td>
<td>-1.60</td>
<td>0.24</td>
<td>0.91</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel D: $\text{Cov}(\tilde{e}^S, \tilde{e}^B)$</th>
<th>$-\text{Cov}(\tilde{S}_{CF}, \tilde{B}_x)$</th>
<th>$-\text{Cov}(\tilde{S}_{CF}, \tilde{B}_r)$</th>
<th>$-\text{Cov}(\tilde{S}_{CF}, \tilde{B}_e)$</th>
<th>$\text{Cov}(\tilde{S}_r, \tilde{B}_x)$</th>
<th>$\text{Cov}(\tilde{S}_r, \tilde{B}_r)$</th>
<th>$\text{Cov}(\tilde{S}_r, \tilde{B}_e)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-0.10</td>
<td>-0.18</td>
<td>-0.26</td>
<td>-0.69</td>
<td>1.27</td>
<td>-0.25</td>
</tr>
<tr>
<td>$\text{Cov}(\tilde{S}_e, \tilde{B}_x)$</td>
<td>$\text{Cov}(\tilde{S}_e, \tilde{B}_r)$</td>
<td>$\text{Cov}(\tilde{S}_e, \tilde{B}_e)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.43</td>
<td>-0.93</td>
<td>1.71</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
those of Campbell and Ammer [1993] in that uncertainty of real cash flow followed by the uncertainty of excess stock returns account for a majority of the total stock variance. The difference in magnitude of the coefficients on the real cash flow component can be explained by the different methodologies used. As for the coefficient on the excess stock return news, this is due to the method in which I obtain excess stock return news for the survey based approach. Since I back-out this news proxy using Equation (2.2) and unexpected (one-period) stock returns, this may exaggerate the effect of excess stock return news in explaining the stock market variance, as can be seen from the coefficient of 1.98.

Regardless of the differences, I note a concern from adopting the Campbell and Ammer [1993] methodology. In their original work, they state that the real interest rate news plays a relatively minor role in explaining the variance of stock returns as even though there is time variation in the ex-ante real short-term interest rate, any changes are largely transitory and thus do not cumulate over time. Therefore in their sample period the expected real interest rate is precisely measured with the variance of the real interest rate news being small. This is surprising given the inflationary environment during their sample period being high and uncertain. I believe such a result is due to their use of a VAR model to generate expectations.

I note that the variance terms of the decomposition sum to greater than 1, this is accommodated by the negative covariance terms of the decomposition. Intuitively, this indicates that the covariance terms which are often overlooked, do have a role to play in explaining the variance of stock returns. I investigate this further within the conditional variance decompositions.

2.5.3.2 Variance decomposition for excess bond returns

The variance decomposition of bond returns in Panel B of Table 2.3 highlights the importance of changing future excess bond returns on the variance of bond returns, accounting for 139%. Changing future real interest rates attribute 39% while the variance of inflation news has a lesser role to play with it accounting for 24% of the variance in bond returns. The large role that the real interest rate news seems to have does not increase the overall variance of bond returns because the real interest rate news is negatively correlated to both excess bond return news and inflation news, the covariance terms of which thus reduce the variance of bond returns. Also the correlation between excess bond return news and inflation news is negative, the covariance of which also having the effect of reducing the bond return variance because the capital loss from higher expected inflation is partly offset by a capital gain from lower expected excess bond returns.
CHAPTER 2. CHANGING EXPECTATIONS AND THE CORRELATION OF STOCKS AND BONDS

These results differ with respect to the role of inflation rate news and real interest rate news compared to the Campbell and Ammer [1993] methodology. The large role that inflation news seems to play is surprising over our sample period given that the inflation rate has been relatively low and stable. Campbell and Ammer [1993] found that over their sample period, between 1952 and 1987, inflation news played a large role, but this makes sense given the inflation rate was high and uncertain over that sample period. I thus believe the difference between the survey forecast and Campbell and Ammer [1993] methodology is due to the implicit assumptions made by use of a VAR model.

As for the real interest rate news, its increased importance from the survey forecasts is due to the expected long-run real interest rate having a persistent effect on changes to bond returns and thus on the volatility of bond returns. Such a result intuitively makes sense over this sample period given the role of the short-rate as a macroeconomic factor for the second moments of stock and bond returns as also shown by Viceira [2010]. I again believe that the difference between Campbell and Ammer [1993] methodology and the survey forecast is due to the VAR model assumptions.

2.5.3.3 Covariance decomposition for excess stock and bond returns

From Panel C of Table 2.3 I report the covariance of unexpected stock returns with each of the decomposed components of unexpected bond returns and vice versa. I find that the covariance between stock and bond returns is determined by the interaction between several offsetting forces. Looking first at unexpected stock returns and the components of unexpected bond returns, I find that the covariation between unexpected stock returns and inflation news mildly drives stock and bond returns in opposite directions. This negative effect is offset by a dominant positive covariance between unexpected stock returns and changing future excess bond returns, driving stocks and bonds in the same direction.

As for how unexpected bond returns and the components of unexpected stock returns affect the stock-bond covariance, I find that the covariation between unexpected bond returns and each of the components drives stock and bond returns in the same direction. The coefficients are similar in magnitude but of an opposite sign to those found by Campbell and Ammer [1993] over their sample period; implying that the role of bonds have changed since the period of 1952 to 1987, as also found by Campbell et al. [2009b]. As for the comparison with historical data based on the Campbell and Ammer [1993] methodology, I find that the coefficients are very different, both in terms of sign and magnitude. Based

\[\text{Campbell and Ammer [1993] describe this presentation of results as: “What would be the covariance of stock and bond returns if one of these asset returns consisted of a single component while the other return were as measured in the data?”}\]
on observations for the stock and bond variance decomposition, I conclude that the differences are due to the implicit assumptions required for the methodology of [Campbell and Ammer 1993].\footnote{I note that while the coefficient on \(\text{Cov}(\tilde{\varepsilon}_S, \tilde{\varepsilon}_B)\) of \(-41.63\) seems like a potential mistake, I have checked the methodology used on the sample period in [Campbell and Ammer 1993] and found similar results. I therefore conclude that this is a real result of the data over the period of 07/1984 to 12/2009.} Whilst it is now commonly accepted that the second moments of stock and bond returns vary over time, using a VAR based model which assumes that this is not the case, to explain what component most explains the (co)variance, now seems inappropriate. I therefore proceed in the rest of the chapter by looking at the conclusions from using the survey based approach, which as discussed above does not make any implicit assumptions of constant stock-bond correlation.

Panel D from Table 2.3 reports the role of covariances between the decomposed components on stock-bond covariance. Such a decomposition is more revealing since I can directly examine the contemporaneous effect of the covariance between the news components on stock-bond covariance. Interestingly, [Campbell et al. 2009b] motivate the covariance between real and inflation rate shocks as the cause of negative correlation between stock and bond returns. The results support this statement with both the covariance of future real cash flow shocks and future real interest rate innovations with inflation shocks being negative, implying that the dynamics between these shocks drive stock and bond returns in opposite directions. I also find that the covariance between real interest rate shocks and excess future stock and bond return shocks play a similar role. Note that [Barsky 1989] highlights the importance of the real short-term interest rate for stock-bond correlation.

The negative covariance causing forces are offset by three large positive forces causing stocks and bonds to move in the same direction: the covariance between real interest rate shocks on stocks and bonds, the covariance between excess future stock return news and inflation news and lastly the covariance between shocks on excess future stock and bond returns. Although there are fewer of these positive forces, they are generally larger in magnitude and thus cause stocks and bond to move in the same direction with more force than those forces that work to decouple the stock and bond markets. These large positive forces could explain the modest but positive correlation between excess stock and bond return news observed from Table 2.2.

### 2.5.4 Conditional (co)variance regressions

In order to get a sense of the contemporaneous role that the news components have in explaining the time variation of the second moments of stock and bond returns according to Equations (2.7), (2.8)
and (2.9), I use the Dynamic Conditional Correlation (DCC) model of Engle [2002] to compute the conditional (co)variances of the news time series. It is estimated in two-stages: The first to estimate the conditional variance of each news component using a univariate GARCH specification. The second to estimate the parameters of the time-varying correlation matrix. I now look to the explanatory power of these conditional second moments of the decomposed news components in explaining the time variation of realised stock and bond return variance and covariance respectively.

I address any multi-collinearity concerns between the conditional (co)variances of the news time series in the regressions below by keeping as large a sample size as possible and thus do not embark on subperiod analysis, relying instead on comparing the results to those of Campbell and Ammer [1993]. I also orthogonalise the conditional covariances which are used in the regressions. This is outlined in more detail below.

2.5.4.1 Variance of excess stock returns

Table 2.4 reports the regression of realised stock market variance on the conditional stock variance news components as specified by Equation (2.7). I perform the regression in such a manner that the coefficients on these components should all be equal to 1. I first perform univariate regressions to determine the importance of the uncertainty in real cash flow, real interest rates and future excess stock returns on stock market variance, and then multivariate regressions with various combinations of the components in Equation (2.7).

Uncertainty in real cash flow explains up to 11% of the time variation in stock market volatility, with a 1% increase in the volatility of real cash flow news leading to an approximate increase of 0.5% in stock market volatility. The coefficient is significant at the 1% level. I also find that the uncertainty in future excess returns is able to explain up to 2% in stock market variance. The increase in stock volatility from a 1% increase in the volatility of future excess returns is approximately 0.2%, with the coefficient again highly significant. In both cases the coefficients are different from the theoretical value of 1. Lastly, uncertainty of the real interest rate seems not to have any explanatory power in a univariate context.

As for the multivariate regressions, I find that the significance of uncertainty in future excess stock returns is driven out by the uncertainty in real cash flow. The variability of the real interest rate news does not have any economic or statistical significance as observed by Campbell and Ammer [1993].

45For more details on the estimation, refer to Appendix D.

35
Table 2.4: Regression of realised stock variance

Table 2.4 reports the coefficients from univariate and multivariate regressions of realised stock variance on the conditional stock variance components:

$$\sigma^2_{S,t} = \alpha + \beta_1 \text{Var}_t(\tilde{S}_{CF,t+1}) + \beta_2 \text{Var}_t(\tilde{S}_{r,t+1}) + \beta_3 \text{Var}_t(\tilde{S}_{e,t+1}) - 2\beta_4 \text{Cov}_t(\tilde{S}_{CF,t+1}, \tilde{S}_{r,t+1}) - 2\beta_5 \text{Cov}_t(\tilde{S}_{CF,t+1}, \tilde{S}_{e,t+1}) + 2\beta_6 \text{Cov}_t(\tilde{S}_{r,t+1}, \tilde{S}_{e,t+1}) + \epsilon_t$$

The coefficients are estimated using OLS based on data from 07/1984 to 12/2009 (306 observations). t-statistics are reported in brackets. The last 3 regressors in regression 9 are residuals after projecting the covariance terms on the individual variance components to reduce the affect of collinearity. *, ** and *** denote statistical significance at 10%, 5% and 1% respectively.

<table>
<thead>
<tr>
<th>Regression</th>
<th>$\alpha$</th>
<th>$\text{Var}(\tilde{S}_{CF})$</th>
<th>$\text{Var}(\tilde{S}_{r})$</th>
<th>$\text{Var}(\tilde{S}_{e})$</th>
<th>$-2\text{Cov}(\tilde{S}<em>{CF}, \tilde{S}</em>{r})$</th>
<th>$-2\text{Cov}(\tilde{S}<em>{CF}, \tilde{S}</em>{e})$</th>
<th>$2\text{Cov}(\tilde{S}<em>{r}, \tilde{S}</em>{e})$</th>
<th>$R^2$</th>
<th>$\text{Adj},R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.00***</td>
<td>0.47***</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.12</td>
<td>0.11</td>
</tr>
<tr>
<td></td>
<td>(3.90)</td>
<td>(6.34)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.00***</td>
<td></td>
<td>-0.59</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>(3.34)</td>
<td>(−0.38)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.00**</td>
<td></td>
<td></td>
<td>0.23***</td>
<td></td>
<td></td>
<td></td>
<td>0.03</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>(2.35)</td>
<td></td>
<td></td>
<td>(2.83)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.00***</td>
<td>0.48***</td>
<td></td>
<td>-1.67</td>
<td></td>
<td></td>
<td></td>
<td>0.12</td>
<td>0.11</td>
</tr>
<tr>
<td></td>
<td>(2.68)</td>
<td>(6.43)</td>
<td></td>
<td>(−1.13)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.00***</td>
<td></td>
<td>-2.31</td>
<td>0.27***</td>
<td></td>
<td></td>
<td></td>
<td>0.03</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>(2.58)</td>
<td></td>
<td>(−1.41)</td>
<td>(3.15)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.00***</td>
<td>0.48***</td>
<td></td>
<td>-0.01</td>
<td></td>
<td></td>
<td></td>
<td>0.12</td>
<td>0.11</td>
</tr>
<tr>
<td></td>
<td>(2.70)</td>
<td>(5.59)</td>
<td></td>
<td>(−0.13)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0.00***</td>
<td>0.47***</td>
<td></td>
<td>-1.79</td>
<td>0.02</td>
<td></td>
<td></td>
<td>0.12</td>
<td>0.11</td>
</tr>
<tr>
<td></td>
<td>(2.58)</td>
<td>(5.51)</td>
<td></td>
<td>(−1.14)</td>
<td>(0.24)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.00**</td>
<td>0.99***</td>
<td>0.31</td>
<td>0.46***</td>
<td>0.94</td>
<td>0.86***</td>
<td>0.99*</td>
<td>0.17</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>(2.07)</td>
<td>(5.89)</td>
<td>(0.16)</td>
<td>(3.33)</td>
<td>(1.21)</td>
<td>(4.06)</td>
<td>(1.74)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>0.00***</td>
<td>0.41***</td>
<td>-1.36</td>
<td>0.01</td>
<td>0.94</td>
<td>0.86***</td>
<td>0.99*</td>
<td>0.17</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>(2.66)</td>
<td>(4.58)</td>
<td>(−0.88)</td>
<td>(0.09)</td>
<td>(1.21)</td>
<td>(4.06)</td>
<td>(1.74)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
CHAPTER 2. CHANGING EXPECTATIONS AND THE CORRELATION OF STOCKS AND BONDS

Regressions 8 and 9 report the estimated coefficients of the regressions of realised stock variance on all the 6 (co)variance components. Since some of these 6 components are mildly correlated and may cause collinearity issues when used as regressands, in regression 9 I use as regressands the 3 conditional variance components and orthogonalised conditional covariance components. Specifically, I project the 3 conditional covariance components on the 3 conditional variance components and use the residuals as the orthogonalised conditional covariance components within the regressions.

Uncertainty of real cash flow remains consistently significant in each regression, with the coefficients always being positive. This highlights the role, both economically and statistically that uncertainty in real cash flow has on stock market variance. This comes at no surprise given the cash flow shocks that were observed during the 2007-2009 financial crisis and the increased stock market volatility during this period. David and Veronesi [2008] among others have established the importance of real cash flow news in explaining and forecasting the variance of stock returns and thus its importance within stock pricing models. Indeed, the work by Bansal et al. [2006] and Hansen et al. [2008] show that in the long-run, cash flow news entirely explains changes in stock prices. The empirical results are in line with these conclusions.

From regression 9 I notice that the covariance between cash flow news and future excess stock return news plays a significant economic and statistical role in explaining the variation in stock market variance. Indeed the $R^2$ increases by 5% between regressions 7 and 9 when I add the covariance terms to the multivariate regressions. Baele et al. [2010] find that non-macroeconomic factors play a more significant role than macroeconomic factors when explaining the variation in stock variance. The findings highlight the contrary. I show that real cash flow news, future excess stock return news and the covariance between the two play an important role in explaining stock market variance. An innovation of this work is highlighting the significance of the covariance terms. I believe that future asset pricing models should therefore include the dynamics of these covariance terms in order to capture the empirical observation of time varying stock volatility.

2.5.4.2 Variance of excess bond returns

Table 2.5 reports the results of the regressions of realised bond variance on the conditional (co)variance of the news components in Equation (2.8). Again the regressions are performed so that the coefficients on these components should all be equal to 1. I immediately notice from regressions 1 to 3 that uncertain

\[46\text{ Also, the importance of cash flow news for stock returns at both the firm and aggregate level has now been well established in the literature (Vuolteenaho 2002 and Chen and Zhao 2009).}\]
certainty in the long-run inflation rate, uncertainty in the long-run real interest rate and future excess bond returns all affect the variance of bond returns both economically and statistically. For the variance of inflation rate news, the coefficient is greater than the theoretical value of 1, implying that a small positive shock to the expected inflation rate has the ability to explain large variation in bond variance.

In the multivariate context, uncertainty of future excess bond returns and long-run inflation news continue to have significant explanatory power, subsuming the role of uncertainty in the real interest rate. In regressions 8 and 9 I include all the covariance terms of Equation (2.8). Comparing regressions 8 and 9, reducing the collinearity helps to identify the true economic scale of the news components in explaining the variation in the variance of bond returns. Regression 9 highlights some interesting results. First, the variance of future excess bond return news is no longer statistically significant. Second, the regression brings out the importance of the uncertainty of the real interest rate for the variation in bond variance. A 1% increase in the volatility of the real interest rate news seems to decrease the volatility of bond returns by around 1%. Third, the importance of the long-run inflation news remains significant in all of the regressions.

Lastly, adding the covariance terms to the regression increases the adjusted $R^2$ to 16%, highlighting the important role that these terms play theoretically in explaining the time variation in bond variance. Indeed, Campbell et al. [2009b] find that the covariance between real and inflation shocks is important in explaining the change in risk premia of nominal bonds. Indeed I find that the covariance between real interest rate news and inflation news is significant both economically (3.04) and statistically (3.98) in explaining the time variation of bond variance.

### 2.5.4.3 Covariance of excess stock and bond returns

Table 2.6 reports the results of the regressions of realised stock-bond covariance on the conditional covariance of the news components from Equation (2.9). Theoretically the coefficients on these components should be equal to 1. From the univariate regressions 4 covariance components display both economic and statistical significance: The covariance between real cash flow news and real interest rate news; the covariance between real interest rate news and future excess bond return news; the covariance between future excess stock return news and inflation news; and the covariance between future excess stock return news and real interest rate news.

From the multivariate regressions, regression 8 is able to account for up to 26% of the variation in stock-bond covariance. Correcting for the collinearity between the regressors in regression 7, the re-
Table 2.5: Regression of realised bond variance

Table 2.5 reports the coefficients from the univariate and multivariate regressions of realised bond variance on conditional bond variance components:

\[ \sigma^2_{B,t} = \alpha + \beta_1 \text{Var}(\tilde{B}_{\pi,t+1}) + \beta_2 \text{Var}(\tilde{B}_{r,t+1}) + \beta_3 \text{Var}(\tilde{B}_{e,t+1}) + 2\beta_4 \text{Cov}(\tilde{B}_{\pi,t+1}, \tilde{B}_{r,t+1}) + 2\beta_5 \text{Cov}(\tilde{B}_{\pi,t+1}, \tilde{B}_{e,t+1}) + 2\beta_6 \text{Cov}(\tilde{B}_{r,t+1}, \tilde{B}_{e,t+1}) + \epsilon_t \]

The coefficients are estimated using OLS based on data from 07/1984 to 12/2009 (306 observations). t-statistics are reported in brackets. The last 3 regressands in regression 9 are residuals after projecting the covariance terms on the individual variance components to reduce the affect of collinearity. *, ** and *** denote statistical significance at 10%, 5% and 1% respectively.

<table>
<thead>
<tr>
<th>Regression</th>
<th>( \alpha )</th>
<th>\text{Var}(\tilde{B}_{\pi})</th>
<th>\text{Var}(\tilde{B}_{r})</th>
<th>\text{Var}(\tilde{B}_{e})</th>
<th>2\text{Cov}(\tilde{B}<em>{\pi}, \tilde{B}</em>{r})</th>
<th>2\text{Cov}(\tilde{B}<em>{\pi}, \tilde{B}</em>{e})</th>
<th>2\text{Cov}(\tilde{B}<em>{r}, \tilde{B}</em>{e})</th>
<th>( R^2 )</th>
<th>( \text{Adj}R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.00***</td>
<td>1.91***</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.09</td>
<td>0.09</td>
</tr>
<tr>
<td>2</td>
<td>0.00***</td>
<td>0.54***</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>3</td>
<td>0.00</td>
<td></td>
<td>0.39***</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.09</td>
<td>0.09</td>
</tr>
<tr>
<td>4</td>
<td>0.00***</td>
<td>2.15***</td>
<td>−0.21</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.09</td>
<td>0.09</td>
</tr>
<tr>
<td>5</td>
<td>0.00</td>
<td>0.23</td>
<td>0.36***</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.10</td>
<td>0.09</td>
</tr>
<tr>
<td>6</td>
<td>−0.00</td>
<td>1.35***</td>
<td>0.28***</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.13</td>
<td>0.13</td>
</tr>
<tr>
<td>7</td>
<td>0.00</td>
<td>1.65***</td>
<td>−0.28</td>
<td>0.29***</td>
<td></td>
<td></td>
<td></td>
<td>0.14</td>
<td>0.13</td>
</tr>
<tr>
<td>8</td>
<td>−0.00</td>
<td>6.41***</td>
<td>1.56***</td>
<td>0.33***</td>
<td>3.04***</td>
<td>1.04***</td>
<td>0.54***</td>
<td>0.18</td>
<td>0.16</td>
</tr>
<tr>
<td>9</td>
<td>0.00***</td>
<td>2.28***</td>
<td>−1.05***</td>
<td>0.11</td>
<td>3.04***</td>
<td>1.04***</td>
<td>0.54***</td>
<td>0.18</td>
<td>0.16</td>
</tr>
</tbody>
</table>
Table 2.6: Regression of realised stock-bond covariance

Table 2.6 reports the coefficients from the univariate and multivariate regressions of realised stock-bond covariance on conditional stock and bond covariance components:

\[
\text{Cov}_t(S, B) = \alpha - \beta_1 \text{Cov}_t(\tilde{S}_{CF,t+1}, \tilde{B}_{\pi,t+1}) - \beta_2 \text{Cov}_t(\tilde{S}_{CF,t+1}, \tilde{B}_{r,t+1}) - \beta_3 \text{Cov}_t(\tilde{S}_{CF,t+1}, \tilde{B}_{e,t+1}) + \beta_4 \text{Cov}_t(\tilde{S}_{CF,t+1}, \tilde{B}_{\pi,t+1}) + \beta_5 \text{Cov}_t(\tilde{S}_{CF,t+1}, \tilde{B}_{r,t+1}) + \beta_6 \text{Cov}_t(\tilde{S}_{CF,t+1}, \tilde{B}_{e,t+1}) + \epsilon_t
\]

The coefficients are estimated using OLS based on data from 07/1984 to 12/2009 (306 observations). t-statistics are reported in brackets. Note that the regressions are performed for all the components listed in the equation above, however only those that displayed any substantial significance, either economic or statistical, in both the univariate and multivariate regressions are displayed in the table. The regressands in regression 8 are the residuals after projecting the covariance terms on other covariance terms that are estimated using the same news time series, to reduce the affect of collinearity. *, ** and *** denote statistical significance at 10%, 5% and 1% respectively.

<table>
<thead>
<tr>
<th>Regression</th>
<th>(\alpha)</th>
<th>-Cov((\tilde{S}<em>{CF}, \tilde{B}</em>{\pi}))</th>
<th>-Cov((\tilde{S}<em>{CF}, \tilde{B}</em>{r}))</th>
<th>Cov((\tilde{S}<em>{\pi}, \tilde{B}</em>{\pi}))</th>
<th>Cov((\tilde{S}<em>{\pi}, \tilde{B}</em>{r}))</th>
<th>Cov((\tilde{S}<em>{e}, \tilde{B}</em>{\pi}))</th>
<th>Cov((\tilde{S}<em>{e}, \tilde{B}</em>{r}))</th>
<th>(R^2)</th>
<th>Adj (R^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.00</td>
<td>0.10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>2</td>
<td>0.00</td>
<td>0.43***</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.04</td>
<td>0.03</td>
</tr>
<tr>
<td>3</td>
<td>-0.00</td>
<td></td>
<td>-0.50</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.02</td>
<td>0.01</td>
</tr>
<tr>
<td>4</td>
<td>-0.00</td>
<td></td>
<td></td>
<td>0.70***</td>
<td></td>
<td></td>
<td></td>
<td>0.07</td>
<td>0.07</td>
</tr>
<tr>
<td>5</td>
<td>-0.00</td>
<td></td>
<td></td>
<td></td>
<td>-0.32***</td>
<td></td>
<td></td>
<td>0.04</td>
<td>0.03</td>
</tr>
<tr>
<td>6</td>
<td>0.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-0.05</td>
<td></td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>7</td>
<td>0.00</td>
<td>2.02***</td>
<td>1.19***</td>
<td>0.38</td>
<td>2.57***</td>
<td>1.05***</td>
<td>0.26**</td>
<td>0.28</td>
<td>0.26</td>
</tr>
<tr>
<td>8</td>
<td>-0.00</td>
<td>-0.37***</td>
<td>2.08***</td>
<td>-0.29</td>
<td>3.78***</td>
<td>-1.61***</td>
<td>-0.18</td>
<td>0.28</td>
<td>0.26</td>
</tr>
</tbody>
</table>
sults of regression 8 show that only three covariance components remain significant for the variation of
stock-bond covariance. The first being the covariance between real cash flow news and real interest rate
news. This comes as a surprise given the significance of cash flow news for stock variance and inflation
rate news for bond variance, together with the theoretical motivation of [David and Veronesi 2008]. I
expected the covariance between cash flow news and inflation rate news to play the most significant
role in the variation of stock-bond covariance. Here however, the role of inflation rate news seems to be
statistically subsumed by real interest rate news.47

This is because of the significance of the short-rate. [Viceira 2010] finds that the short-rate is an
important predictor of stock-bond correlation as it seems to proxy for business-cycle conditions. Given
that real-short term interest rate news is comprised of shocks to the short-rate and shocks to the inflation
rate, the finding of the importance of the real short-term interest rate for the contemporaneous time
variation in stock-bond covariance thus complements the findings of both [Viceira 2010] and captures the
theoretical motivation of [David and Veronesi 2008]. The economic significance of covariance between
cash flow news and real interest rate news is large with a coefficient of 2.08, twice the theoretically value.
This also highlights a potential mechanism for negative stock-bond correlation,48 thus driving stocks
and bonds in opposite directions.

The second component, is the covariance between future excess stock return news and inflation rate
news. The coefficient is 3.78 implying that a small covariation between the news components is able
to explain a large variation in the stock-bond covariance. Theoretically a positive covariance between
inflation rate news and excess future stock return news causes stocks and bonds to move together: A
higher than expected inflation rate (positive inflation news) causes bond yields to increase (and thus
prices to decrease) whilst a higher than expected risk premium to stocks (positive future excess stock
return news) implies stock prices decrease. Empirically I find this to indeed be the case.

Lastly, the covariance between excess future stock return news and real interest rate news is strongly
significant for the variation in stock-bond covariance. Somewhat puzzling is the observation that the
covariance between these news components seems to decrease the covariance between stock and bond
returns. Theoretically, positive shocks to stock risk premium and real short-term interest rate should

47Note however that the covariance of cash flow news and inflation rate news is statistically significant at a 10% level
but the economic significance is counter intuitive given the negative coefficient of −0.37. This implies that the interaction
between cash flow news and inflation rate news actually increases stock-bond correlation.

48This is because I use the term $-\text{cov}(\tilde{S}_{CF}, \tilde{B}_t)$ in the regression. Thus a coefficient of 2.08 implies $-2.08\times \text{cov}(\tilde{S}_{CF}, \tilde{B}_t)$
which naturally means it reduces the value of the dependent variable $\text{cov}(S, B)$ when the covariance between the real
cash flow news and real interest news is positive.
CHAPTER 2. CHANGING EXPECTATIONS AND THE CORRELATION OF STOCKS AND BONDS

cause stock and bond prices to decrease and vice versa, however empirically I observe the opposite: a positive covariance causes stocks and bonds to move in opposing directions. Here it seems that the (unconditional) insight of Campbell and Ammer [1993] in which the covariance of stock and bond returns is made up of several offsetting forces, holds true for the (conditional) time variation in stock-bond covariance.

Note also that Campbell et al. [2009b] motivate the covariance between real and inflation shocks to explain the negative correlation between stock and bond returns. Although theoretically apparent from the decomposition of Equation (2.9), empirically this does not appear to be the case. The only covariance term between real and inflation variables that seems to display some significance is that between future excess stock return news and inflation news. The interaction between these news components actually causes the covariance between stocks and bond to increase. Such a finding therefore supports the scope of “flight-to-safety” and “flight-to-liquidity” in attempting to explain the occurrence of negative stock-bond correlation.

2.6 Robustness

I use the Dynamic Conditional Correlation (DCC) model of Engle [2002] to compute the conditional variance and covariance of the news components for use within the regressions to investigate the role of the interaction between news components in explaining the time variation in second moments of stock and bond returns. I found significant results that were able to be linked to both theoretical and empirical papers on stock-bond correlation. In order to check that these results were not simply an artifact of the DCC model, I also compute the conditional variance and covariance of the news components using the Exponentially Weighted Moving Average (EWMA) (co)variance model as a robustness check. The regression results are reported in Table 2.7 below.

I see that the regression results are fairly similar when using the EWMA model and are encouraged from the finding that the same coefficients are significant both statistically and economically when compared to using the DCC model. Such a robustness check supports the notion of the role of macroeconomic components being able to explain the time variation in the second moments of stock and bond returns.

Interestingly the $R^2$ of the stock variance regression has increased to be above the $R^2$ from the bond variance regression in Table 2.7. David and Veronesi [2008] and Baele et al. [2010] find that economic

49For further details of the model and the estimation methodology, see Appendix D.
Table 2.7: Robustness regressions

Table 2.7 reports the coefficients from the multivariate regressions of realised stock variance, bond variance and stock-bond covariance on Exponentially Weighted Moving Average (EWMA) conditional stock and bond covariance components as a robustness check. The coefficients are estimated using OLS based on data from 07/1984 to 12/2009 (306 observations). t-statistics are reported in brackets. *, ** and *** denote statistical significance at 10%, 5% and 1% respectively.

<table>
<thead>
<tr>
<th>Panel A: Stock variance</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>Var(( \tilde{S}_{CF} ))</td>
<td>Var(( \tilde{S}_{r} ))</td>
<td>Var(( \tilde{S}_{e} ))</td>
<td>-2Cov(( \tilde{S}<em>{CF}, \tilde{S}</em>{r} ))</td>
<td>-2Cov(( \tilde{S}<em>{CF}, \tilde{S}</em>{e} ))</td>
<td>2Cov(( \tilde{S}<em>{r}, \tilde{S}</em>{e} ))</td>
<td>( R^2 )</td>
</tr>
<tr>
<td>0.00***</td>
<td>0.45***</td>
<td>-0.85</td>
<td>-0.11*</td>
<td>0.81***</td>
<td>0.82***</td>
<td>0.62***</td>
<td>0.27</td>
</tr>
<tr>
<td>(5.42)</td>
<td>(4.00)</td>
<td>(-1.28)</td>
<td>(-1.75)</td>
<td>(2.72)</td>
<td>(7.21)</td>
<td>(2.74)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: Bond variance</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>Var(( \tilde{B}_{x} ))</td>
<td>Var(( \tilde{B}_{r} ))</td>
<td>Var(( \tilde{B}_{e} ))</td>
<td>2Cov(( \tilde{B}<em>{x}, \tilde{B}</em>{r} ))</td>
<td>2Cov(( \tilde{B}<em>{x}, \tilde{B}</em>{e} ))</td>
<td>2Cov(( \tilde{B}<em>{r}, \tilde{B}</em>{e} ))</td>
<td>( R^2 )</td>
</tr>
<tr>
<td>0.00***</td>
<td>0.92***</td>
<td>-0.49***</td>
<td>0.05*</td>
<td>1.17***</td>
<td>0.38***</td>
<td>0.20**</td>
<td>0.17</td>
</tr>
<tr>
<td>(7.91)</td>
<td>(3.84)</td>
<td>(-2.62)</td>
<td>(1.92)</td>
<td>(3.54)</td>
<td>(2.87)</td>
<td>(2.47)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel C: Stock-bond covariance</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>-Cov(( \tilde{S}<em>{CF}, \tilde{B}</em>{x} ))</td>
<td>-Cov(( \tilde{S}<em>{CF}, \tilde{B}</em>{r} ))</td>
<td>Cov(( \tilde{S}<em>{r}, \tilde{B}</em>{x} ))</td>
<td>Cov(( \tilde{S}<em>{r}, \tilde{B}</em>{e} ))</td>
<td>Cov(( \tilde{S}<em>{e}, \tilde{B}</em>{x} ))</td>
<td>Cov(( \tilde{S}<em>{e}, \tilde{B}</em>{e} ))</td>
<td>( R^2 )</td>
</tr>
<tr>
<td>-0.00</td>
<td>-0.16*</td>
<td>1.01***</td>
<td>-0.11</td>
<td>1.55***</td>
<td>-0.67***</td>
<td>-0.02</td>
<td>0.25</td>
</tr>
<tr>
<td>(-0.40)</td>
<td>(-1.92)</td>
<td>(5.46)</td>
<td>(-0.66)</td>
<td>(6.75)</td>
<td>(-4.43)</td>
<td>(-0.23)</td>
<td></td>
</tr>
</tbody>
</table>
factor models have a harder time explaining bond volatility than stock volatility, which I observe when using the EWMA model to describe the co-movement between the news time series. I thus highlight the importance of not only the economic variables used in the model used but also the model itself when trying to explain the second moments of stock and bond returns.

2.7 Concluding remarks

In this chapter I conduct an empirical investigation into the time variation of second moments of stock and bond returns. Using a Campbell and Shiller [1988] decomposition I am able to identify and investigate the economic mechanisms on the variation of stock-bond correlation. The first innovation is the use of survey forecasts for the economic components to back out a time series of unexpected values (news) for these components. The second innovation is the use of the DCC model to generate (co)variance time series for the news of the economic components. This allows for regressions of realised second moments on the (co)variances of the decomposed economic news components in order to directly investigate economic mechanisms in explaining the time variation in volatility and correlation of stock and bond returns, this being the third innovation.

From the unconditional variance decomposition I find similar results to Campbell and Ammer [1993]. A large part of the variance of excess stock returns is attributed to changing expectations of future real cash flow followed by changing expectations of future excess stock returns. As for the variance of excess bond returns, it seems that changing expectations of the inflation rate dominate the other components. An important finding is the role of the covariance terms in the decompositions. For both excess stock and bond returns, these covariance components are important contributors of the volatility of returns. The decomposition also reveals that the covariance is determined by the interaction between several offsetting forces.

Complementary to the unconditional decompositions I find that uncertainty of real cash flow and the covariance between real cash flow news and future excess stock return news are all significant for explaining the time variation of stock return variance up to an $R^2$ of 16%. These findings add to the literature that cash flow news obtained from survey forecasts are informative for stock pricing (Pástor et al. [2008]). Variation in bond returns can mainly be explained by the uncertainty in the future inflation rate and the future real short-term interest rate. Another important finding is that the covariance terms also have a significant role in explaining the variation in bond return variance. As for the variation in stock-bond covariance, I report that the covariance of cash flow news and real short-term interest
CHAPTER 2. CHANGING EXPECTATIONS AND THE CORRELATION OF STOCKS AND BONDS

rate news is important for contributing to the occurrence of a negative correlation between stocks and bonds. I note that this study is among the first to demonstrate the informational content of survey forecasts for stock-bond correlation.

The caveats of this work are two fold. The first being the lack of longer range forecasts for the economic components. Since the decompositions of the long-term assets require expected values of the components over a number of horizons, I therefore have to impose some structure in order to generate longer-horizon forecasts. Although every effort was taken to ensure that such a method would only complement the survey forecast data, the possibility of measurement error from using such procedure therefore exists. The second being the use of a conditional volatility model to generate the time varying (co)variances of the news components. Since higher frequency (i.e. daily) forecasts and actual values for the economic components do not exist, I am unable to generate realised news time series from which we can construct the realised monthly (co)variances of the news components. Thus, I am constrained to use a conditional volatility model to generate (co)variance time series of the news components.50

Contrary to Baele et al. [2010] I show that macroeconomic factors are able to explain time variation in the second moments of stock and bond returns. I note that economic models are only as good as the factors that are used in the models. The factors in this study are theoretically motivated and seem to do well at explaining a portion of the time variation in second moments of stock and bond returns, although not all of it. This chapter highlights the role of macroeconomic factors in the time-variation of stock-bond correlation, despite the flight-to-quality/flight-to-liquidity event during the sample period, which would have hampered the effectiveness of the news components in explaining the changing correlation of stock and bond returns.

50As shown, I attempted to address this concern by performing robustness checks of using a different form of conditional volatility model. The conclusions were still justifiable in this case.
Chapter 3

Stock-bond correlation and out-of-sample portfolio performance using analyst forecasts

3.1 Introduction

It is now widely accepted that the correlation between stocks and bonds is subject to fluctuations over time, with the implication that these changes impact portfolio risk and thus investors’ diversification benefits. Of the portfolio allocation process (asset allocation, security selection and market timing), Brinson and Hood [1986] and Brinson et al. [1991] show that the asset allocation stage explains in excess of 70% of the portfolio’s return. Since the optimal allocation of wealth between these asset classes being one of the most important decisions that investors face, this chapter looks at predicting the out-of-sample correlation of stock and bond returns using macroeconomic forecast data from analysts for the optimal stock-bond portfolio.

DeMiguel et al. [2009] show that for a portfolio with a large number of securities, the out-of-sample performance of a (naive) equally weighted (1/N) scheme consistently outperforms the sample-based mean-variance portfolio and its extensions designed to reduce estimation error when using historical (realised) data. More recently, DeMiguel et al. [2010b] and Kostakis et al. [2011] highlight the ad-

\footnote{They further show that for the sample-based mean-variance strategy and its extensions to outperform the 1/N portfolio based on monthly re-balancing, an estimation window of 3000 months for a portfolio with 25 assets is required, increasing to 6000 months for a portfolio with 50 assets. The point being that in-sample improvements in the estimation of the moments of asset return distributions do not necessarily imply out-of-sample improvements.}
vantages of using forward-looking option-implied moments of return distributions to improve portfolio selection compared to use of historical (realised) data. In this chapter, I use analyst forecasts to estimate the second moments of stock and bond returns in determining the optimal portfolio. Comparing this to several methods based on historical data, the main contribution is to investigate the out-of-sample benefits of analysts forecasts to predict the second moments of stock and bond returns.

Although the importance of predicting the correlation between returns in the context of asset allocation is well-acknowledged, the actual out-of-sample ability of predictor variables and/or models is often overlooked or under appreciated. I choose to focus the work on second moment predictability given the known difficulty of estimating the expected returns of assets (Merton [1980]) and because of specific interest in the out-of-sample predictability of various predictors and models for stock-bond correlation. Thus, in the same spirit as Jagannathan and Ma [2003] and DeMiguel et al. [2010b] I focus on obtaining minimum-variance portfolios.

The motivation to attempt to predict the correlation is clear from Figure 3.1. Consider a fund who invests in a minimum-variance fashion across stocks and bonds, who estimates the correlation matrix from historical returns versus a fund with perfect foresight of the correlation process. Panel A of Figure 3.1 reports a reduction in the overall variation in volatility of the “perfect foresight” portfolio from that of the historical estimation portfolio. Panel B compares the Sharpe ratios of these portfolios with those from investing in the individual asset classes themselves. From this the ‘historical estimation’ minimum-variance portfolio results in a lower Sharpe ratio than simply investing in long-term bonds over this sample-period.

Ang et al. [2007] find that analyst forecasts significantly outperform time series, Philips curve and term structure models for predicting inflation out-of-sample. More recently, Piazzesi and Schneider [2011] compare survey forecasts of interest rates with the in-sample predictions found by a vector autoregressive model. They comment that investors ex-ante may not recognise the same patterns that are observed today with the benefit of hindsight together with a statistical model. Therefore, although the latter approach of using a statistical model may display significant in-sample predictability, any out-of-sample advantages may actually come from analyst forecasts who provide subjective forecasts conditional on the current economic state. Indeed, David and Veronesi [2008] show that analysts fore-

\[52\] At any time \( t \) the investor knows perfectly what the volatility/correlation is going to be in the next time period \( t + 1 \) and so adjusts her minimum-variance portfolio at time \( t \) based on the known values of what the volatility and correlation is at time \( t + 1 \). The return of the portfolio is then calculated over the period \( t \) to \( t + 1 \).
Figure 3.1: Monthly time series of realised volatilities and Sharpe ratios of historically estimated and perfect foresight minimum-variance portfolios

Panel A reports the realised monthly volatility from daily return observations on minimum variance portfolios formed on stocks and bonds using a correlation matrix estimated from historical returns versus a portfolio formed in month $t$ using the realised volatility and correlation of returns in month $t + 1$, representing a portfolio formed based on “perfect foresight” of the correlation between returns. Panel B reports the Sharpe ratios of the two portfolios together with those of the individual asset classes themselves. The sample period is from 02/09/1985 to 31/12/2009. Stock and bond return data is outlined in Section 3.3.
CHAPTER 3. STOCK-BOND CORRELATION AND OUT-OF-SAMPLE PORTFOLIO PERFORMANCE USING ANALYST FORECASTS

casts of inflation and earnings significantly predict the stock-bond correlation in-sample. I attempt to establish the effectiveness of such forecasts out of sample.

Several economic factor models have been developed in an attempt to predict the variation in stock-bond correlation. All these works mainly explore the role of real and nominal variables in predicting the second moments of stock and bond returns.\textsuperscript{53} d’Addona and Kind\textsuperscript{2006} motivate the volatility of the real interest rate and inflation shocks for predicting the correlation, but out-of-sample the forecasting ability of their model with these factors is questionable. Li\textsuperscript{2002} highlight the economic value of predicting the stock-bond correlation out-of-sample, however do so whilst assuming that the volatility of stock and bond returns is constant.\textsuperscript{54} Doing so ignores that volatility does indeed change over time and also the effect that volatility spill-over between stock and bond markets has on the correlation process (Fleming et al.\textsuperscript{1998}). Hence is not a fair test on the economic benefits of predicting the stock-bond correlation out-of-sample.

More recently, Viceira\textsuperscript{2010} find significant in-sample predictability of the short-rate and yield-spread for the second moments of stock and bond returns.\textsuperscript{55} However, the out-of-sample performance is not reported. In this chapter I use as forecasting variables those that have been identified as predictors in the literature: David and Veronesi\textsuperscript{2008} use survey-based measures of inflation and earnings to predict realized volatility and covariance between stock and bond returns. I therefore employ forecasts of corporate profits and the consumer price index. Intuitively analyst forecasts of the 10-year Treasury bond rate should have some role in forecasting the comovement of stock and bond markets. Viceira\textsuperscript{2010} motivate the (realised) short-rate for forecasting the second moments of stock and bond returns. I naturally extend such an observation by using analyst forecasts of the 3-month Treasury rate. Lastly, both Li\textsuperscript{2002} and Campbell et al.\textsuperscript{2009b} use real variables in their work on stock-bond correlation. I therefore supplement the corporate profits forecasts with real GDP to predict stock-bond correlation.

Time-series models have also been developed in an effort to improve the forecast of the second moments of stock and bond returns (De Goeij and Marquering\textsuperscript{2004}, Cappiello et al.\textsuperscript{2006}). To compare the performance of analyst forecasts in predicting the variation in correlation of stock and bond returns, I also employ the Exponentially Weight Moving Average (EWMA) model championed by JPMorgan.

\textsuperscript{53}I also note that there are several papers on explaining the contemporaneous variation in correlation based on economic models (Campbell et al.\textsuperscript{2009b}, Hassellott\textsuperscript{2009}, Yang et al.\textsuperscript{2009}).

\textsuperscript{54}I note that correlation is a function of the covariance of returns and the respective volatilities. Thus assuming that the volatility is constant implicitly assumes that the correlation is solely driven by the covariance of returns.

\textsuperscript{55}Additionally Viceira\textsuperscript{2010} show that the analyst forecasts used by David and Veronesi\textsuperscript{2008} are subsumed in-sample when controlling for the short-rate and yield-spread.
CHAPTER 3. STOCK-BOND CORRELATION AND OUT-OF-SAMPLE PORTFOLIO PERFORMANCE USING ANALYST FORECASTS

1996 and Engle 2002’s Dynamic Conditional Correlation (DCC) model to forecast the correlation out-of-sample.

I find that when using various methods based on historical (realised) data to optimally allocate between stocks and bonds, the best predictor of volatility and correlation is often the previous month’s realised volatility and correlation.\textsuperscript{56} In a similar spirit to the 1/N rule advocated by DeMiguel et al. 2009 for mean-variance investors in stocks, it appears that for minimum-variance investors across stocks and bonds, the best forecast of stock-bond correlation can simply be the previous value of the correlation. As for analysts’ forecasts, I find that while forecasts of the inflation rate, T-bill rate, T-bond rate, change in corporate profits and real GDP, display significant in-sample predictability of the second moment of stock and bond returns, they do not contain out-of-sample benefits when constructing a portfolio based on these forecasts.

Analyst forecasts should provide good out-of-sample performance. In each month $t$, analysts report a forecast for a specific variable conditional on the current and forecasted macroeconomic environment. Therefore such forecasts are projections on the future economic state/ regime of the macroeconomic variables being forecasted. As highlighted by Piazzesi and Schneider 2011, analyst forecasts ex-ante should provide better out-of-sample forecasts than statistical models/methods (based on historical data), which only display significant predictability with the benefit of hindsight i.e. being in-sample. In this chapter I specifically investigate if such macroeconomic projections display predictability for stock-bond correlation out-of-sample in the context of selecting an optimal stock-bond portfolio.

The rest of the chapter is structured as follows. Section 3.2 outlines the asset allocation framework I use to test various methodologies to optimally invest between stocks and bonds. Section 3.3 describes our data set. I demonstrate in-sample predictability of the analyst forecasts in Section 3.4 and the out-of-sample benefits within an asset allocation setting in Section 3.5. In Section 3.6 I perform robustness checks of the out-of-sample predictability results with some concluding remarks in Section 3.7.

3.2 Stock-bond asset allocation

As the focus is on predicting the second moments of stock and bond returns, I use the global minimum variance portfolio as the framework for this chapter. An added incentive in adopting such a framework

\textsuperscript{56}In this chapter I do not study the potential of option-implied volatility/correlation on the allocation between stocks and bonds but highlight this as an avenue for future research.
CHAPTER 3. STOCK-BOND CORRELATION AND OUT-OF-SAMPLE PORTFOLIO PERFORMANCE USING ANALYST FORECASTS

is motivated by the notorious difficulty in estimating expected returns (Merton [1980]) versus the covariance of returns. Under such assumptions I consider an investor whose objective function is fully described by:

$$\min_{\omega_t} \omega_t' \Sigma_t \omega_t,$$  \hspace{1cm} (3.1)

where \(\omega_t \equiv (\omega_t^S, \omega_t^B)'\) is the vector of weights of wealth invested in stocks and bonds respectively, with the portfolio weights summing to one in the constraint of Equation (3.1) and \(\omega_t' \Sigma_t \omega_t\) is the estimated covariance matrix. The solution to the above optimization is \(\omega_{\text{min},t} = \Sigma_t^{-1} \tilde{1}_2 / \tilde{1}_2' \Sigma_t^{-1} \tilde{1}_2\). Decomposing the covariance matrix \(\Sigma\) into volatility and correlation matrices:

$$\Sigma = \text{diag}(\sigma) \Omega \text{diag}(\sigma),$$  \hspace{1cm} (3.2)

where \(\text{diag}(\sigma)\) denotes the diagonal volatility matrix of stock and bond returns, and \(\Omega\) is the correlation matrix. Therefore to obtain the optimal allocation between stocks and bonds from Equation (3.1), I need to estimate forecasts of the volatility and correlation of returns. In this chapter I investigate the benefits of the information contained in analyst forecasts to estimate these second moments of stock and bond returns.

3.3 Data

3.3.1 Stock and bond returns

In order to examine the out-of-sample benefits of using forward-looking analyst forecasts in predicting the volatilities and correlation of stock and bond returns, I use the S&P500 index as the stock index and the 10-year Constant Maturity Treasury bond index. All returns are converted to excess returns using the risk-free rate approximated by the 3-month Treasury Bill rate. I adjust for weekends and holidays in the daily returns calculations.\(^{57}\) Stock data was obtained from CRSP whilst the Treasury bond and bills data were obtained from the Federal Reserve’s H.15 release and covers the period July 2, 1984 to January 29, 2010.

This study is based at a monthly frequency, so I obtain estimates of the monthly realised volatility

\(^{57}\)Appendix E provides full details on the construction of the excess return time series.
CHAPTER 3. STOCK-BOND CORRELATION AND OUT-OF-SAMPLE PORTFOLIO PERFORMANCE USING ANALYST FORECASTS

and correlation at the end of the month $t$ from daily returns in that month:\footnote{I ignore subtracting the monthly sample mean from each daily return as this correction is very minor, however I note that using demeaned returns does not change the conclusions.}

$$\sigma_t^2 = \sum_{i=1}^{N_t} (r_{i,t})^2, \quad \rho_{S,B,t} = \frac{\sum_{i=1}^{N_t} r_{i,t}^S \cdot r_{i,t}^B}{\sigma_{S,t} \cdot \sigma_{B,t}} \quad (3.3)$$

where $N_t$ is the number of daily returns in month $t$, $r_{i,t}^S$ and $r_{i,t}^B$ are the daily stock and bond returns on day $i$ in month $t$.

As Barndorff-Nielsen and Shephard \cite{Barndorff-Nielsen2004} and Andersen et al. \cite{Andersen2005} show, realised second moments represent a model-free consistent estimation of the quadratic return variance under general assumptions based upon arbitrage-free financial markets.\footnote{Specifically their work shows that the theory of quadratic variation implies that these realised measures of second moments converge uniformly to the integrated instantaneous volatility and covariance under weak regularity conditions.} Andersen et al. \cite{Andersen2006} note that the use of realised second moments for estimation and forecasting allows for measurements of the realisations of the latent processes without the need for an explicit model; and also provides a natural benchmark for forecast evaluation purposes. Indeed, Andersen et al. \cite{Andersen2003} find that the use of realised volatility when forecasting future foreign exchange rate volatility outperforms various other models from the literature. I therefore use such an approach to obtain realised volatility and correlation time-series to combine with the analyst forecasts in order to predict the stock-bond correlation out-of-sample.

Table \ref{table:3.1} presents the in-sample statistics of the stock and bond returns.\footnote{The sample period contains two boom and bust periods attributed to the 2001 tech-bubble collapse and 2007 housing crises, together with many other events such Black Monday (1987), both Gulf wars (1990 and 2003), the 1997 Asian crisis, the collapse of Long-Term Capital Management (LCTM) and the Russian financial crisis of 1998 and the terrorist attacks of 11th September 2001.} Interestingly from Panel A, the mean annualised log return on stocks was 7.35\% whilst that on bonds was a larger 8.86\%, with the mean annualised log return on the short-term interest rate (not reported in the table) being 4.38\%. The annualised standard deviation of the log stock returns was 18.40\% and for log bond returns was 7.55\%. Thus, over the sample period the ex-post Sharpe ratio of stocks is 0.16, whilst is 0.59 for bonds. The unconditional correlation of daily stock and bond returns over the sample period is $-0.03$, but as can be seen in Panel B, such a number hides the variation over time of the correlation between stock and bond returns - the standard deviation of realised correlation is 3.4 times its mean, indicating significant variability of the correlation relative to its mean. For stock and bond volatility this ratio is less than 1, which interestingly highlights the greater variability in realised correlation versus realised volatilities. This is visually apparent from Figure \ref{figure:3.2}.
Table 3.1: Descriptive statistics of stock and bond returns

Table 3.1 reports the descriptive statistics of the stock and bond returns over the period 02/07/1984 to 29/01/2010. Panel A reports the sample statistics of the daily log stock and bond returns. Stock returns are based on log returns on the S&P 500 index from CRSP. Bond returns are log returns on the 10-year Constant Maturity Treasury bond index from the Federal Reserve’s H.15 release. Panel B reports the full sample statistics of the monthly realised volatilities and correlation of stock and bond returns. These estimates are obtained using the estimators in Equation (3.3). $\rho(#)\text{ is the autocorrelation coefficient of order } #.$

<table>
<thead>
<tr>
<th>Panel A: Daily stock and bond returns</th>
<th>$r^S$</th>
<th>$r^B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean return (%)</td>
<td>7.35</td>
<td>8.86</td>
</tr>
<tr>
<td>Std Dev (%)</td>
<td>18.40</td>
<td>7.55</td>
</tr>
<tr>
<td>Correlation</td>
<td>−0.03</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: Monthly realised volatilities and correlation</th>
<th>$\sigma_S$</th>
<th>$\sigma_B$</th>
<th>$\rho_{S,B}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean (p.a.)</td>
<td>15.77%</td>
<td>7.24%</td>
<td>0.13</td>
</tr>
<tr>
<td>Std.Dev/Mean</td>
<td>0.64</td>
<td>0.36</td>
<td>3.42</td>
</tr>
<tr>
<td>Correlation with $\sigma_S$</td>
<td>1</td>
<td>0.58</td>
<td>−0.39</td>
</tr>
<tr>
<td>Correlation with $\sigma_B$</td>
<td>0.58</td>
<td>1</td>
<td>−0.17</td>
</tr>
<tr>
<td>$\rho(1)$</td>
<td>0.629</td>
<td>0.547</td>
<td>0.692</td>
</tr>
<tr>
<td>$\rho(2)$</td>
<td>0.514</td>
<td>0.444</td>
<td>0.664</td>
</tr>
<tr>
<td>$\rho(3)$</td>
<td>0.445</td>
<td>0.381</td>
<td>0.641</td>
</tr>
<tr>
<td>$\rho(12)$</td>
<td>0.159</td>
<td>0.227</td>
<td>0.449</td>
</tr>
</tbody>
</table>
Figure 3.2: Monthly time series of realised volatilities and correlation of stock and bond returns

Time series of the monthly realised volatilities and correlation of stock and bond returns over the period 07/1984 to 12/2010. All plots are overlaid with NBER recession bands. Stock returns are based on log returns of the S&P 500 index from CRSP. Bond returns are log returns of the 10-year Constant Maturity Treasury bond index from the Federal Reserve’s H.15 release. These estimates are obtained using the estimators in Equation (3.3).
CHAPTER 3. STOCK-BOND CORRELATION AND OUT-OF-SAMPLE PORTFOLIO PERFORMANCE USING ANALYST FORECASTS

The correlation between the realised second moments shows that the realised volatility across stock and bond markets is highly correlated (0.58). Fleming et al. [1998] attribute such an observation to volatility spill-over between these markets. Interestingly, the correlation between the respective realised volatility and the realised correlation is negative, indicating that the correlation between asset classes decreases when the respective volatility increases. This is especially true for stock market volatility (−0.39). Such an observation is often explained by “flight-to-quality” events when investors re-allocate more of their wealth to bonds in order to reduce their portfolio exposure to the uncertainty in stocks (Connolly et al. [2005]). Lastly, the autocorrelation coefficients for the realised second moments shows that they follow a persistent process which decays slowly.

3.3.2 Analyst forecast data

Analyst forecast data is obtained from the BlueChip Economic Indicators (BCEI) survey database. These are professional survey forecasts made by a panel of economists on a range of economic fundamentals covering nominal, real and monetary variables. At the beginning of each month participants are asked to forecast the future values of these variables for the current calendar year and for the next calendar year. The exact timing within the month of when the surveys are published to subscribers is not available but I use these forecasts at the end of the month in which the survey is conducted. This crucially prevents any potential look-ahead bias arising from overlapping observations of survey responses and returns to stocks and bonds.

Forecasts are available for the period July 1984 to December 2009 which dictates the sample period of the study. I use as forecasting variables those that have been identified as predictors from the literature rather than employ a “fishing” approach by using all of the variables available. David and Veronesi [2008] use survey-based measures of inflation and earnings to predict the realised volatility and covariance between stock and bond returns. I therefore employ the forecasts of corporate profits and the consumer price index in the work. Intuitively analyst forecasts of the 10-year Treasury bond rate should directly have some role in forecasting the co-movement of stock and bond markets. Viceira [2010] motivate the (realised) short-rate for forecasting the second moments of stock and bond returns. I naturally extend such an observation by using analyst forecasts of the 3-month Treasury rate. Lastly, both Li [2002] and Campbell et al. [2009b] use real variables in their work on stock-bond correlation.

Forecasts for nominal variables are of the Consumer Price Index (CPI) and Nominal Gross Domestic Product (GDP). For the real variables they are Real GDP, Disposable Personal Income, Non-residential Fixed Investment, Unemployment, Industrial Production, Corporate Profits, Housing Starts and Automobile Sales. Lastly, the monetary variables forecasted are of the 3-month Treasury rate and the 10-year Treasury rate.
I therefore supplement the corporate profits forecasts with real GDP to predict stock-bond correlation.\textsuperscript{62}

As previously mentioned, two kinds of monthly forecasts are obtained from participants, one for the current calendar year and one for the next calendar year. For instance, in January 2009, participants provide a 12-month forecast for the value of the variable at the end of the current year 2009, and a 24-month forecast for the variable at the end of the next calendar year 2010. For February 2009, the forecast horizon for 2009 is now only 11 months while for 2010 it is 23 months, and so on. From this I obtain a constant one-year forecast for each of the variables via a weighted average of the short and long term forecasts:

\[
C_{t \rightarrow t+12} = \frac{m}{12} C_{t,C} + \frac{12-m}{12} C_{t,N}
\]

where \(C_{t \rightarrow t+12}\) denotes the 12-month forecast/expectation of the variable at month \(t\), \(C_{t,C}\) and \(C_{t,N}\) are the respective expectations of the variable for the current and next calendar year at month \(t\) and \(m\) is the number of remaining months during the current year at month \(t\).

I choose to perform this at the individual level of the forecaster rather than aggregating the short-term and long-term forecasts across the individual forecasters for each variable in month \(t\) and then obtaining the constant horizon forecast for that variable. This approach allows for the response of any forecaster who has not provided both a short-term and long-term forecast to be removed.\textsuperscript{63} Figure 3.3 shows the number of valid forecasts after I apply this correction.\textsuperscript{64} I then obtain the implied one-year forecast for the respective variable for each forecaster from Equation (3.4) in each month \(t\) and take the cross-sectional mean of these implied forecasts as the mean-consensus one-year forecast for that variable in month \(t\).

David and Veronesi \textsuperscript{2008} show that the dispersion in analyst forecasts of the inflation rate\textsuperscript{65} positively (and significantly) forecasts the realised covariance between stock and bond returns. In this spirit, I also construct dispersion measures of the variables by taking the cross-sectional mean-absolute-deviation (MAD)\textsuperscript{66} of the implied one-year forecasts from Equation (3.4) in each month \(t\).

\textsuperscript{62}An important side note is that the analyst forecasts from BCEI of corporate profits, real GDP and CPI are for the change in the level of these variables (the latter thus being a forecast of the inflation rate), whereas for the 3-month Treasury rate and the 10-year Treasury note the forecasts are of the actual level.

\textsuperscript{63}I do this to reduce the unwanted variation in the forecasted signal coming from a respondent who has only reported a forecast for one of the forecast dates.

\textsuperscript{64}I include this figure to highlight that I have a sufficient number of analyst forecasts in any given month for the identified variables.

\textsuperscript{65}This was constructed as the cross-sectional standard deviation of the individual inflation rate forecasts.

\textsuperscript{66}I use MAD instead of the standard deviation as it is a robust statistic which is more resilient to outliers in the cross-sectional data than the standard deviation.
Figure 3.3: Monthly time series of the number of complete survey responses

Panels A to D plot the time-series of the number of complete survey responses (both short-term and long-term forecasts) for corporate profits, CPI, 3-month Treasury rate and the 10-year Treasury note. Although un reported here, a similar time-series is also obtained for forecasts of real GDP. Survey data is from the BlueChip Economic Indicators (BCEI) database for the period 06/1984 to 12/2009. Note that the lowest number of complete responses for any of the forecasted variables during the sample period was 32.
CHAPTER 3. STOCK-BOND CORRELATION AND OUT-OF-SAMPLE PORTFOLIO PERFORMANCE USING ANALYST FORECASTS

Figure 3.4: Monthly time series of mean-consensus and dispersion measures from analyst forecasts

Time-series of the information obtained from analyst forecasts of the change in the level of corporate profits, real GDP and CPI; and for the level of the 10-year Treasury bond yield and 3-month Treasury rate over the period 06/1984 to 12/2009. Panel A plots the cross-sectional mean ($C_t$) of analyst forecasts whilst Panel B plots the cross-sectional mean-absolute deviation ($D_t$) of analyst forecasts.

Figure 3.4 plots the monthly time-series of the mean-consensus and dispersion measures. Panel A shows that mean-consensus for all variables except corporate profits remains fairly stable with abrupt changes occurring around crises or recessionary periods. The mean-consensus of the change in corporate profits has more variability highlighting the economically interesting period in which I conduct this study. Panel B shows that the average level of dispersion has decreased over the sample period with pronounced spikes again occurring around crises or recessionary periods. This highlights that analysts have become more unanimous in their forecasts over the sample period. Summary statistics for the mean-consensus and dispersion measures are given in Table 3.2.

Although the objective in this chapter is to predict the next month’s volatility and correlation, I use a constructed 1-year forecast from the analyst forecast data so as to remove the seasonality that exists in the reported forecasts. Ideally I would want a 1-month horizon forecast, however as I show in the next section, the 1-year horizon forecast displays significant in-sample predictability for the (1-month...
Table 3.2 reports the descriptive statistics of the monthly forecast measures of the long rate ($LR$), short rate ($SR$), change in corporate profits ($CP$), inflation rate ($INF$) and change in real GDP ($RGDP$) obtained from analyst forecasts for the period 06/1984 to 12/2009. Panel A reports the sample statistics of the mean consensus forecasts ($C_t$), taken as the cross-sectional mean of the forecasts in each month $t$. Panel B reports the sample statistics of the dispersion in forecasts ($D_t$), taken as the cross-sectional mean-absolute-deviation (MAD) of the forecasts in each month $t$. Panel C reports the correlation between the mean consensus and dispersion measures. The analyst forecasts are from the BlueChip Economic Indicators (BCEI) survey database.

### Panel A: Mean consensus measures

<table>
<thead>
<tr>
<th></th>
<th>$C_{LR}^t$</th>
<th>$C_{SR}^t$</th>
<th>$C_{CP}^t$</th>
<th>$C_{INF}^t$</th>
<th>$C_{RGDP}^t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>6.895</td>
<td>4.753</td>
<td>5.777</td>
<td>3.008</td>
<td>2.581</td>
</tr>
<tr>
<td>Std Dev</td>
<td>2.305</td>
<td>2.088</td>
<td>4.900</td>
<td>1.011</td>
<td>1.035</td>
</tr>
<tr>
<td>ACF(1)</td>
<td>0.979</td>
<td>0.977</td>
<td>0.968</td>
<td>0.977</td>
<td>0.971</td>
</tr>
<tr>
<td>ACF(2)</td>
<td>0.957</td>
<td>0.949</td>
<td>0.914</td>
<td>0.935</td>
<td>0.913</td>
</tr>
<tr>
<td>ACF(3)</td>
<td>0.934</td>
<td>0.917</td>
<td>0.847</td>
<td>0.881</td>
<td>0.839</td>
</tr>
<tr>
<td>ACF(12)</td>
<td>0.753</td>
<td>0.608</td>
<td>0.394</td>
<td>0.491</td>
<td>0.315</td>
</tr>
</tbody>
</table>

### Panel B: Dispersion measures

<table>
<thead>
<tr>
<th></th>
<th>$D_{LR}^t$</th>
<th>$D_{SR}^t$</th>
<th>$D_{CP}^t$</th>
<th>$D_{INF}^t$</th>
<th>$D_{RGDP}^t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.277</td>
<td>0.272</td>
<td>0.292</td>
<td>0.267</td>
<td>0.320</td>
</tr>
<tr>
<td>Std Dev</td>
<td>0.108</td>
<td>0.112</td>
<td>0.090</td>
<td>0.078</td>
<td>0.133</td>
</tr>
<tr>
<td>ACF(1)</td>
<td>0.851</td>
<td>0.906</td>
<td>0.882</td>
<td>0.846</td>
<td>0.929</td>
</tr>
<tr>
<td>ACF(2)</td>
<td>0.756</td>
<td>0.806</td>
<td>0.759</td>
<td>0.720</td>
<td>0.842</td>
</tr>
<tr>
<td>ACF(3)</td>
<td>0.666</td>
<td>0.717</td>
<td>0.622</td>
<td>0.597</td>
<td>0.738</td>
</tr>
<tr>
<td>ACF(12)</td>
<td>0.503</td>
<td>0.506</td>
<td>0.482</td>
<td>0.387</td>
<td>0.685</td>
</tr>
</tbody>
</table>

### Panel C: Correlation between measures

<table>
<thead>
<tr>
<th></th>
<th>$C_{LR}^t$</th>
<th>$C_{SR}^t$</th>
<th>$C_{CP}^t$</th>
<th>$C_{INF}^t$</th>
<th>$C_{RGDP}^t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_{LR}^t$</td>
<td>0.695</td>
<td>0.536</td>
<td>0.045</td>
<td>0.451</td>
<td>0.125</td>
</tr>
<tr>
<td>$D_{SR}^t$</td>
<td>0.791</td>
<td>0.625</td>
<td>0.030</td>
<td>0.609</td>
<td>0.083</td>
</tr>
<tr>
<td>$D_{CP}^t$</td>
<td>0.308</td>
<td>0.032</td>
<td>-0.066</td>
<td>0.162</td>
<td>-0.255</td>
</tr>
<tr>
<td>$D_{INF}^t$</td>
<td>0.216</td>
<td>0.105</td>
<td>-0.359</td>
<td>0.058</td>
<td>-0.405</td>
</tr>
<tr>
<td>$D_{RGDP}^t$</td>
<td>0.498</td>
<td>0.315</td>
<td>-0.276</td>
<td>0.376</td>
<td>-0.294</td>
</tr>
</tbody>
</table>
forward) volatility and correlation of stock and bond returns. As a robustness check for the out-of-sample performance of portfolio formed on analyst forecasts, I also test such portfolios with different holding periods. This allows for the optimal out-of-sample forecast horizon using the 1-year horizon analyst forecast data to be determined.

### 3.4 In-sample predictability

Section 3.2 shows that I require a forecast estimate of the volatility and correlation in order to obtain the weights for the minimum-variance portfolio. Based on the decomposition of Equation (3.2) I forecast the volatility and correlation of stock and bond returns using linear predictive models based on the informational content of analyst forecasts. This allows for a direct comparison of the benefits of using analyst forecasts in optimal stock-bond portfolio selection.

I first look at the predictability of the mean consensus and dispersion measures from analyst forecasts on the second moments of stock and bond returns in sample. Since in-sample predictability does not imply out-of-sample benefits, the in-sample regressions will provide some comparative insight when I look to the out-of-sample performance of portfolios formed using analyst forecasts in Section 3.5.

#### 3.4.1 Volatility

For volatility and correlation I run the regression:

$$ SM_{t+1} = \tilde{\alpha} + \tilde{\beta}SM_{t} + \tilde{\gamma}C_{t} + \tilde{\delta}D_{t} + \epsilon_{t+1} $$  (3.5)

where SM is the respective monthly realised second moment of stock and bond returns estimated from Equation (3.3), $C_{t}$ is the collection of mean-consensus measures and $D_{t}$ the collection of dispersion measures.

The first regression of Panel A shows that the mean-consensus of the short-rate and change in corporate profits together with the dispersion in the inflation rate significantly forecast stock return volatility with an $R^2$ of 20.75%. Once I control for the lagged value of realised volatility in the second regression, the expected short-rate and dispersion in inflation are subsumed by the significance of the lagged value. Only the expected change in corporate profits survives with the $R^2$ increasing by 20% from the first regression. The persistence in stock volatility is apparent from the last regression in Panel A, with the

---

67 This is expected since volatility is a persistent process as observed from Table 3.1. This is also the motivation behind the numerous Generalised AutoRegressive Conditional Heteroscedastic (GARCH) type models.
lagged value being both economically and statistically significant, subsuming the role of all but one of the analyst forecast measures.

From Panel B of Table 3.3 analyst forecast measures seem to predict bond volatility much better than stock volatility excluding the lagged value as an independent variable (27% vs 21%). The mean-consensus of the long-rate, short-rate, change in corporate profits and change in real GDP are all economically and statistically significant together with the dispersion in the inflation rate for predicting bond realised volatility. This latter observation is consistent with [David and Veronesi 2008](#) who find dispersion in analyst forecasts of inflation to be significant for bond volatility. Interestingly, as the expected level of the short-rate increases (also if the expected change in corporate profits increases), bond volatility decreases in the next monthly period.

**Figure 3.5: Contribution to adjusted $R^2$ of the volatility and correlation of stock and bond returns**

Additional contribution of the (aggregate) mean consensus, dispersion and lagged value to the adjusted $R^2$ from a predictive regression of the second moments of stock and bond returns on the mean consensus and dispersion proxies controlling for the lagged value. Construction of the proxies are based on analyst forecasts of the inflation rate, corporate profits, real GDP, the 3-month Treasury rate and 10-year Treasury note yield.

Contrary to stock volatility, the second regression in Panel B where I include the lagged value of bond volatility, the $R^2$ only increases by 7% to 36% with the mean consensus of the long-rate, short-rate,
Table 3.3: Predictive regressions for second moments of returns using analyst forecast measures

Table 3.3 reports the 1-month horizon in-sample predictive regressions of stock return volatility, bond return volatility and stock-bond return correlation onto a constant and standardised measures of the mean consensus ($C_t$) and dispersion ($D_t$) in analyst forecasts of the long rate ($LR$), short rate ($SR$), change in corporate profits ($CP$), inflation rate ($INF$) and change in real GDP ($RGDP$) for the period 07/1984 - 01/2010. The table reports coefficient estimates with t-statistics in brackets and the adjusted $R^2$ of the regression. The critical value of the t-statistic at which the null hypothesis of $\beta = 0$ is rejected at a significance level of 5% is $|t| > 1.96$.

<table>
<thead>
<tr>
<th>Lagged</th>
<th>$C_{t-1}^{LR}$ (t-stat)</th>
<th>$C_{t-1}^{SR}$ (t-stat)</th>
<th>$C_{t-1}^{CP}$ (t-stat)</th>
<th>$C_{t-1}^{INF}$ (t-stat)</th>
<th>$C_{t-1}^{RGDP}$ (t-stat)</th>
<th>$D_{t-1}^{LR}$ (t-stat)</th>
<th>$D_{t-1}^{SR}$ (t-stat)</th>
<th>$D_{t-1}^{CP}$ (t-stat)</th>
<th>$D_{t-1}^{INF}$ (t-stat)</th>
<th>$D_{t-1}^{RGDP}$ (t-stat)</th>
<th>$Adj R^2(%)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A: Stock volatility</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$-$0.39</td>
<td>$-$0.87</td>
<td>$-$1.15</td>
<td>0.42</td>
<td>0.61</td>
<td>$-$0.10</td>
<td>$-$0.19</td>
<td>$-$0.18</td>
<td>0.68</td>
<td>0.50</td>
<td>20.75</td>
<td></td>
</tr>
<tr>
<td>($-$0.83)</td>
<td>($-$2.06)</td>
<td>($-$3.55)</td>
<td>(1.48)</td>
<td>(1.00)</td>
<td>($-$0.38)</td>
<td>($-$0.61)</td>
<td>($-$0.69)</td>
<td>(3.14)</td>
<td>(1.64)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.53</td>
<td>0.06</td>
<td>$-$0.54</td>
<td>$-$0.60</td>
<td>0.28</td>
<td>0.44</td>
<td>$-$0.24</td>
<td>$-$0.21</td>
<td>$-$0.25</td>
<td>0.35</td>
<td>0.37</td>
<td>41.54</td>
</tr>
<tr>
<td>(10.29)</td>
<td>(0.15)</td>
<td>($-$2.11)</td>
<td>(1.12)</td>
<td>(1.66)</td>
<td>($-$1.03)</td>
<td>($-$0.80)</td>
<td>($-$1.10)</td>
<td>(1.86)</td>
<td>(1.44)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Panel B: Bond volatility</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.33</td>
<td>$-$0.53</td>
<td>$-$0.28</td>
<td>$-$0.11</td>
<td>0.18</td>
<td>$-$0.00</td>
<td>0.10</td>
<td>$-$0.12</td>
<td>0.26</td>
<td>0.02</td>
<td>27.29</td>
<td></td>
</tr>
<tr>
<td>(2.74)</td>
<td>($-$5.03)</td>
<td>($-$3.51)</td>
<td>($-$1.50)</td>
<td>(2.30)</td>
<td>($-$0.02)</td>
<td>(1.32)</td>
<td>($-$1.79)</td>
<td>(4.84)</td>
<td>(0.29)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.36</td>
<td>0.24</td>
<td>$-$0.34</td>
<td>$-$0.20</td>
<td>$-$0.06</td>
<td>0.14</td>
<td>$-$0.03</td>
<td>0.05</td>
<td>$-$0.07</td>
<td>0.18</td>
<td>0.00</td>
<td>35.73</td>
</tr>
<tr>
<td>(6.30)</td>
<td>(2.14)</td>
<td>($-$3.35)</td>
<td>($-$2.60)</td>
<td>($-$0.93)</td>
<td>(1.92)</td>
<td>($-$0.47)</td>
<td>(0.65)</td>
<td>($-$1.14)</td>
<td>(3.55)</td>
<td>(0.03)</td>
<td></td>
</tr>
<tr>
<td>Panel C: Stock-Bond correlation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.28</td>
<td>0.12</td>
<td>0.12</td>
<td>$-$0.02</td>
<td>$-$0.12</td>
<td>0.05</td>
<td>$-$0.16</td>
<td>0.01</td>
<td>$-$0.04</td>
<td>$-$0.01</td>
<td>41.77</td>
<td></td>
</tr>
<tr>
<td>(4.62)</td>
<td>(2.27)</td>
<td>(2.96)</td>
<td>($-$0.59)</td>
<td>($-$2.98)</td>
<td>(1.31)</td>
<td>($-$4.03)</td>
<td>(0.18)</td>
<td>($-$1.46)</td>
<td>($-$0.24)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.46</td>
<td>0.14</td>
<td>0.06</td>
<td>0.06</td>
<td>$-$0.01</td>
<td>$-$0.06</td>
<td>0.03</td>
<td>$-$0.08</td>
<td>0.01</td>
<td>$-$0.03</td>
<td>$-$0.01</td>
<td>53.43</td>
</tr>
<tr>
<td>(8.65)</td>
<td>(2.51)</td>
<td>(1.23)</td>
<td>(1.65)</td>
<td>($-$0.17)</td>
<td>($-$1.71)</td>
<td>(1.05)</td>
<td>($-$2.24)</td>
<td>(0.36)</td>
<td>($-$1.15)</td>
<td>($-$0.22)</td>
<td></td>
</tr>
</tbody>
</table>
CHAPTER 3. STOCK-BOND CORRELATION AND OUT-OF-SAMPLE PORTFOLIO PERFORMANCE USING ANALYST FORECASTS

change in corporate profits and dispersion in the inflation rate all remaining statistically significant. Forecasts of the economic variables that I motivate for stock-bond pricing thus have a more significant role in predicting bond volatility than stock volatility, even after I include the lagged value in the regressions. This is clear from Figure 3.5 with the aggregated effect of the mean-consensus and dispersion measures displaying a larger contribution to the adjusted $R^2$ of the prediction of bond volatility than to that of stock volatility.

3.4.2 Correlation

Li [2002] develops a model using the volatility of (realised) macroeconomic factors to predict stock-bond correlation. He finds that the volatility of expected inflation and the real interest rate are able to predict stock-bond correlation with an $R^2$ of 28% on monthly data over the period 1980-2001. Using a similar approach based on analyst forecast data of macroeconomic factors, I am able to forecast stock-bond correlation with an $R^2$ of 53% over a much larger sample period. Panel C of Table 3.3 show that the mean-consensus of the long-rate, short-rate, change in corporate profits and in real GDP all significantly predict the correlation whilst only the dispersion in the short-rate displays some significant predictability. Contrary to the findings of both Li [2002] and David and Veronesi [2008], I do not observe that the dispersion in expected inflation plays a role in predicting stock-bond correlation.

More recently, Viceira [2010] show that such inflation uncertainty proxies are subsumed by the yield-spread and short-rate, since these are better proxies for business conditions and inflation/economic uncertainty respectively. Therefore I posit from the results that analyst forecasts of the short-rate (and yield-spread) potentially offer a better proxy of the (forward-looking) economic uncertainty. Indeed in the next section, I test the out-of-sample performance of the portfolios formed using analyst forecasts versus the optimal portfolio formed using the (realised) yield spread and short-rate as predictors.68

The second regression of Panel C shows that inclusion of the lagged value of the correlation subdues the significance in the forecasting power of the analyst forecasts, for both the mean-consensus and dispersion measures. Only two predictors remain significant, the mean-consensus of the long rate and the dispersion of the short rate. Figure 3.5 also highlights this with the further observation that the (aggregate) mean-consensus from the analyst forecasts contribute more to the adjusted $R^2$ that the dispersion

---

68 In results unreported, but available from the author upon request, I perform further regressions to that of Panel C, the first being with a constant, the lagged value and the (realised) yield-spread and short-rate as independent variables. In agreement with Viceira [2010] I indeed find that these proxies of business conditions/uncertainty do significantly predict the correlation. After adding the mean-consensus and dispersion measures, the significance of the yield-spread and short-rate are dramatically subsumed with the mean-consensus measures and in particular the dispersion of the short-rate all remaining significant.
measures in predicting the 1-month ahead correlation of stock and bond returns.

In-sample, at least for the case of stock volatility and stock-bond correlation, I find that the historical lagged value seems to subsume the informational content of the analyst forecasts within the predictability regressions. I proceed by investigating if forecasting the monthly volatility and correlation of stock and bond returns using the previous month’s realised estimate or other approaches based on historical realised data, produces superior out-of-sample portfolio performance when compared to the use of the information contained in analyst forecasts when estimating the volatility and correlation.

3.5 Out-of-sample predictability

3.5.1 Portfolio performance metrics

To evaluate the out-of-sample performance I calculate the portfolio mean, standard deviation, Sharpe ratio, turnover, certainty-equivalent and opportunity cost (defined below). This provides standardised metrics to compare the out-of-sample performance of using the forward-looking analyst forecasts versus using realised backward-looking historical data to predict the volatilities and correlation of stocks and bonds returns in constructing the optimal portfolio.

Following the approach of [DeMiguel et al., 2010b], in each month \( t \), the portfolio weights \( \omega^k_t \) are determined for each respective strategy \( k \) via Equation (3.1). A “rolling window” approach is used to obtain estimates for the volatilities and correlation for \( t + 1 \). I denote the length of this window as \( \tau < T \), where \( T \) is the total number of returns in the sample period. I use an estimation window of \( \tau = 60 \) observations which corresponds to 5 years worth of monthly data.\(^69\) Using the data (which could either be returns, realised volatility and/or realised correlation data) over the estimation window \( \tau \), I compute the portfolio weights for each respective strategy.

The corresponding portfolios are then formed and the out-of-sample monthly portfolio return over the period \([t, t + 1]\) is calculated: \( r_{t+1}^k = \omega_t^k r_{t+1} \), where \( r_{t+1} \) denotes excess stock and bond returns over month \( t \) to \( t + 1 \). I re-balance the portfolio every month by rolling the estimation window to include the data for the next month and dropping the data for the earliest month. I continue to do this until the end of the data set is reached and thus obtain a \( T - \tau \) time-series of monthly out-of-sample portfolio

\(^69\)As highlight by [Andersen et al., 2006], rolling samples of five-years of monthly data is commonly used to estimate time-varying covariances. I note that the analysis is repeated using an estimation window of \( \tau = 120 \), with the resulting conclusions remaining the same as for the case of \( \tau = 60 \).
returns based on any given strategy $k$. I then calculate the performance metrics of the mean, standard deviation and Sharpe ratio as follows:

$$
\mu_k = \frac{1}{T - \tau} \sum_{t=\tau}^{T-1} r_{t+1}^k \tag{3.6}
$$

$$
\sigma_k = \left[ \left( \frac{1}{T - \tau - 1} \sum_{t=\tau}^{T-1} (r_{t+1}^k - \mu_k)^2 \right) \right]^{\frac{1}{2}} \tag{3.7}
$$

$$
SR_k = \frac{\mu_k}{\sigma_k} \tag{3.8}
$$

I follow the method of DeMiguel et al. [2009] to calculate the portfolio turnover. For any strategy $k$, turnover is defined as the average absolute change in the weights over the $T - \tau - 1$ rebalancing dates in time and across the $N$ available assets, which is two here. Essentially this is the average percentage of wealth traded at the re-balancing intervals:

$$
PT_k = \frac{1}{T - \tau - 1} \sum_{t=\tau}^{T-1} \sum_{j=1}^{N} (|\omega_{j,t+1}^k - \omega_{j,t+1}^k|) \tag{3.9}
$$

where $\omega_{j,t+1}^k$ denotes the portfolio weight in asset $j$ before rebalancing at time $t+1$, while $\omega_{j,t+1}^k$ is the desired portfolio weight in the same asset after rebalancing at time $t+1$. Consider this for example for the $1/N$ strategy, where 50% of wealth is invested in stocks and 50% in bonds. Thus, $\omega_{j,t} = \omega_{j,t+1} = 1/N$ but $\omega_{j,t+1}$ may be different than $1/N$ from the change in price of asset $j$ between $t$ and $t+1$.

I also consider the certainty-equivalent ($ce_k^*$) and opportunity cost ($OppCost_k^*$) in addition to the Sharpe ratio as the latter only considers the mean and volatility of returns whilst the former considers higher order moments of the portfolio return. I define the certainty-equivalent as:

$$
ce_k^* = \mu_k - \frac{\gamma}{2} (\sigma_k)^2 \tag{3.10}
$$

where $\mu_k$ and $(\sigma_k)^2$ are the mean and variance of the out-of-sample excess returns for strategy $k$ and $\gamma$ is the risk aversion, which I assume to be $\gamma = 1$. Certainty equivalent is defined as the riskless return that an investor is willing to accept rather than investing in the higher return but risky strategy $k$. Thus a larger certainty equivalent implies that it takes a larger riskless return to entice the investor to forego the higher risk but much higher return strategy.

\footnote{\textit{Strictly Equation (3.10) refers to the level of the expected utility of a mean-variance investor which can be shown to be approximately the certainty equivalent of an investor with quadratic utility. However, following common practice I also interpret it as the certainty equivalent for strategy $k$.}}
I also obtain a measure of the economic value of stock-bond correlation timing, similar in spirit to Fleming et al. [2001]. Therefore, following Kostakis et al. [2011] I define an opportunity cost (OppCost$^k$) as the return that needs to be added (or subtracted) to the returns of the minimum-variance portfolio (based on historical data) so that the investor is indifferent in utility terms from the returns of strategy $k$:

$$\left(1 - \frac{T-\tau}{T-\tau} \sum_{t=\tau}^{T-1} U(r_{t+1}^{\text{MinVar}} + \text{OppCost}^k)\right) = \left(1 - \frac{T-\tau}{T-\tau} \sum_{t=\tau}^{T-1} U(r_{t+1}^k)\right)$$

(3.11)

This allows the economic significance of the difference in the performance of other strategies from the minimum variance strategy based on historical data to be assessed. Thus when the opportunity cost is positive (negative) the investor is economically better (worse) off by using the comparison strategy rather than the historical minimum variance method to form their portfolio.

For completeness I also compute the above performance metrics net of transaction costs. I work out the portfolio performance net of transaction costs for each strategy $k$ as:

$$r_{t+1}^{k,\text{net}} = \left(1 + r_{t+1}^k\right)\left(1 - c \times \sum_{j=1}^{N} \left(\left|\omega_{j,t+1}^k - \omega_{j,t}^k\right|\right)\right) - 1$$

(3.12)

where $c$ is the proportional transaction cost, whereby $c \times \sum_{j=1}^{N} \left(\left|\omega_{j,t+1}^k - \omega_{j,t}^k\right|\right)$ is the transaction cost of rebalancing the portfolio at $t + 1$. For $c$ we assume 50 basis points for each transaction.\footnote{Such an assumption is obtained from market practitioners who trade stock and bond index linked products.}

3.5.2 Statistical significance of performance

I test the statistical significance of the performance of the various portfolios. More specifically, I wish to test the significance of the difference in volatility, Sharpe ratio and certainty equivalent of the particular portfolios from that of the benchmark portfolio. DeMiguel et al. [2009] use a parametric Sharpe ratio test by proposed by Jobson and Korkie [1981] after making the correction pointed out in Memmel [2003].

This test statistic however assumes that returns are distributed independently and identically (IID) over time with a normal distribution. Such assumptions are typically violated in time-series data with the resulting test statistic being misleading.\footnote{Ledoit and Wolf [2008] investigate such a concern by proposing bootstrapping methods to test the differences in portfolio performance.}

In order to address such a concern, Politis and Romano [1994] and DeMiguel et al. [2010b] adopt a
nonparametric bootstrap approach which I also adopt here. Such a method does not require any assumption about the distribution of portfolio returns and provides a p-value inferred from the sample distribution. I test the null hypothesis that the volatility, Sharpe ratio and certainty equivalent of a particular portfolio strategy \( k \) is worse than that of the benchmark portfolio \( bm \):

\[
H_0 : \hat{\sigma}^{bm} - \hat{\sigma}^k \leq 0 \tag{3.13}
\]

\[
H_0 : \tilde{S}R^k - \tilde{S}R^{bm} \leq 0 \tag{3.14}
\]

\[
H_0 : \hat{ce}^k - \hat{ce}^{bm} \leq 0 \tag{3.15}
\]

Following the methodology of DeMiguel et al. [2010b] I construct a one-sided confidence interval for the difference using the stationary bootstrap of Politis and Romano [1994] where I obtain \( B \) pairs of size \( T - \tau \) of the portfolio returns by blockwise resampling with replacement. I use \( B = 10,000 \) bootstrap re-samples with an expected block size equal to 20. Denoting the empirical distribution function of the \( B \) bootstrap pairs corresponding to the difference in the volatility, Sharpe ratio or certainty equivalent of portfolio \( k \) from the benchmark portfolio \( bm \) as \( \hat{F} \), the one-side p-value for the null hypotheses is \( \hat{p} = \hat{F}(0) \) for each of the performance measures. This p-value represents the probability of obtaining a test statistic at least as small as the current observed value, assuming that the null hypothesis is true. I therefore reject the null hypothesis of the volatility, Sharpe ratio or certainty equivalent being worse (lower) than that of the benchmark portfolio when the p-value is small, implying that the result is statistically significant.

### 3.5.3 Portfolio strategies

I consider various portfolio strategies in order to compare the performance of using analyst forecasts to predict the stock-bond volatility and correlation out-of-sample: benchmark portfolios, historical portfolios and analyst forecast portfolios. The initial estimation window \( \tau \) is from July 1984 to July 1989 and thus the out-of-sample portfolio performance is over the period from August 1989 to December 2009 (245 monthly observations) with monthly re-balancing. Figure 3.6 plots the level of the S&P 500 index and the 10-year Treasury bond index over the sample period. It is clear that the portfolio allocation problem is conducted over an economically interesting sample period (as previously highlighted), posing the ultimate test for the analyst forecasts as predictor variables.

The list of portfolios considered is in Table 3.4. I describe these portfolios in more detail below together with the results of the out-of-sample performance of these portfolios.
CHAPTER 3. STOCK-BOND CORRELATION AND OUT-OF-SAMPLE PORTFOLIO PERFORMANCE USING ANALYST FORECASTS

Figure 3.6: S&P500 and 10-year Treasury bond level


3.5.3.1 Benchmark portfolios

I use the conditional minimum-variance portfolio as the benchmark portfolio. This is based on estimating the “sample covariance” matrix of monthly returns in the estimation window $\tau$. Such a method is the simplest approach to take into account changing volatilities and correlation of stock and bond returns. I also note that the comparative performance across portfolios should primarily focus on the volatilities of these portfolios given the use of the minimum-variance (MinVar) portfolio as the benchmark.

The performance of the MinVar portfolio is listed at the top of Tables 3.5, 3.6 and 3.7 which also report the performance of the various strategies listed in Table 3.4. Over the sample period, the Sharpe ratio of the MinVar (0.417) portfolio is larger than for investing just in the S&P500 (0.134) but not for investing solely in 10-year Treasury bonds (0.432); as for the volatility, the MinVar portfolio (6.875) is lower than for both stocks and bonds. This highlights the importance of obtaining good forecasts of the volatility/correlation when investing across stocks and bonds, especially during crises when a “flight-to-safety” effect aids positive returns to long-term bonds (Connolly et al. 2005). To emphasize
Table 3.4: List of portfolios considered

Table 3.4 lists the various portfolio strategies that I consider. The last column of the table gives the abbreviation that I use to refer to the strategies in the following tables.

<table>
<thead>
<tr>
<th>No.</th>
<th>Portfolio strategy ($k$)</th>
<th>Abbreviation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Benchmark portfolios</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Unconditional minimum-variance portfolio</td>
<td>MinVar</td>
</tr>
<tr>
<td><strong>Panel B: Minimum-variance portfolios based on historical monthly data</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Shorts-constrained</td>
<td>MinVarNoShorts</td>
</tr>
<tr>
<td>3</td>
<td>Shrinkage estimation (Ledoit and Wolf [2008])</td>
<td>MinVarShrinkage</td>
</tr>
<tr>
<td>4</td>
<td>No correlation (set to zero)</td>
<td>MinVarNoCorr</td>
</tr>
<tr>
<td>5</td>
<td>Previous realised monthly covariance</td>
<td>MinVarPrevVolCorrEst</td>
</tr>
<tr>
<td>6</td>
<td>Forecast of realised monthly covariance</td>
<td>MinVarVolCorrEst</td>
</tr>
<tr>
<td>7</td>
<td>EWMA model (JPMorgan [1996])</td>
<td>MinVarEWMA</td>
</tr>
<tr>
<td>8</td>
<td>DCC model (Engle [2002])</td>
<td>MinVarDCC</td>
</tr>
<tr>
<td>9</td>
<td>Short rate and yield spread with lagged values</td>
<td>MinVarSRYS</td>
</tr>
<tr>
<td><strong>Panel C: Minimum-variance portfolios based on analyst forecasts</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Mean-consensus with lagged values</td>
<td>MinVarC$_{t}$</td>
</tr>
<tr>
<td>11</td>
<td>Mean-consensus without lagged values</td>
<td>MinVarC$_{t}$NoPrev</td>
</tr>
<tr>
<td>12</td>
<td>Dispersion with lagged values</td>
<td>MinVarD$_{t}$</td>
</tr>
<tr>
<td>13</td>
<td>Dispersion without lagged values</td>
<td>MinVarD$_{t}$NoPrev</td>
</tr>
<tr>
<td>14</td>
<td>Mean-consensus and dispersion with lagged values</td>
<td>MinVarC$<em>{t}$D$</em>{t}$</td>
</tr>
<tr>
<td>15</td>
<td>Mean-consensus and dispersion without lagged values</td>
<td>MinVarC$<em>{t}$D$</em>{t}$NoPrev</td>
</tr>
</tbody>
</table>
CHAPTER 3. STOCK-BOND CORRELATION AND OUT-OF-SAMPLE PORTFOLIO PERFORMANCE USING ANALYST FORECASTS

this point, from Figure 3.1, perfect foresight of the correlation process over this sample period would have resulted in a portfolio Sharpe ratio of 0.773.

3.5.3.2 Minimum-variance portfolios based on historical data

I adopt several other methods which have been proposed in the literature to improve the out-of-sample performance of the minimum-variance portfolio based on estimating the sample covariance matrix. The first being the shortsale-constrained minimum-variance portfolio (MinVarNoShorts), where I compute the portfolio weights according to Equation (3.1) imposing the additional constraint that all the weights have to be positive.\(^73\) Second, to improve the behaviour of the covariance matrix I follow the method of Ledoit and Wolf [2003] who find that shrinkage of the covariance matrix before applying any optimisation can lead to significantly lower out-of-sample volatility (MinVarShrinkage). Another method as suggested by DeMiguel et al. [2010b] is to set the correlation to zero and just obtain estimates of the volatilities (MinVarNoCorr).\(^74\)

Third, based on the conclusions from the in-sample predictability regressions where the lagged value of volatility/correlation is the most significant and important predictor of the next period’s volatility/correlation (see Table 3.3 and Figure 3.5): I look at the out-of-sample performance of using the past month’s realised volatility/correlation estimate (MinVarPrevVolCorrEst); and also the portfolio performance from forecasting the next period’s volatility/correlation based on estimated coefficients from a predictive linear regression solely on the realised volatility/correlation over the estimation window \(\tau\) (MinVarVolCorrEst).\(^75\)

Fourth, I use two time-series models to forecast the covariance matrix based on historical (monthly) returns over the estimation window \(\tau\). The first being the exponentially-weighted moving average (EWMA) model, championed by RiskMetrics (JPMorgan [1996]), which applies more weight to the most recent observations when estimating the covariance matrix (MinVarEWMA).\(^76\) The second being

\(^73\)Jagannathan and Ma [2003] show that constructing such a portfolio (albeit with a large number of assets within the portfolio) can reduce the overall risk of the estimated optimal portfolio.

\(^74\)Such a portfolio is also investigated by Elton et al. [2006].

\(^75\)More specifically I perform the regression:

\[
SM_t = \alpha + \beta SM_{t-1} + \epsilon_t
\]

where \(SM\) is the respective monthly realised second moment of stock and bond returns estimated from Equation (3.3). I then use the coefficients to forecast the respective second moment at \(t + 1\):

\[
SM_{t+1} = \alpha + \beta SM_t
\]

\(^76\)Such a model is arguable the most commonly used tool among finance practitioners for estimating time-varying covariance matrices. I define the EWMA model by:

\[
\Sigma_{t+1} = \lambda \Sigma_t + (1 - \lambda) r_t r_t'
\]

where \(\lambda\) is the decay rate which I choose to be 0.96 as recommended by RiskMetrics for use on monthly data.
the Dynamic Conditional Correlation (DCC) model of Engle [2002] which provides a more sophisticated estimation of the covariance matrix allowing for empirically observed mean-reversion in volatility and correlation that the EWMA model does not (MinVarDCC).  

Lastly, Viceira [2010] highlight the strong in-sample predictive content of the short-rate and yield-spread for stock-bond correlation. I therefore also form portfolios in a similar manner to footnote 71 with the short-rate and yield-spread as additional predictors together with the lagged value of the realised second moment and test the out-of-sample portfolio performance (MinVarSRYS).

Tables 3.5 and 3.6 report the performance of the portfolios constructed based on historical (realised) monthly data without and with transaction costs respectively. Encouragingly, the volatility of the MinVarNoCorr portfolio highlights that it is better to attempt to predict the correlation even if the prediction may not be the most accurate, rather than not at all. Counter-intuitively though, the Sharpe ratio of the portfolio is better than the MinVar portfolio, even being the best portfolio in term of Sharpe ratio, net of transaction costs. However I note that the unconditional correlation between stock and bond (monthly) returns over the portfolio performance sample period is 0.06, so setting the correlation to zero ex-ante, implicitly predicts the average correlation over the sample period. As a result, this portfolio performs well with low turnover.

From Tables 3.5 and 3.6 only the MinVarNoShorts portfolio has a significantly lower out-of-sample volatility than the MinVar portfolio, whilst the performance of the MinVarSRYS portfolio is the worst of the historical portfolios. Note also that the MinVarShrinkage portfolio performs similarly to the MinVar portfolio. This is because as commented by Ledoit and Wolf [2003], such a method improves the covariance estimation where the number of assets under consideration is large relative to the number of observations of returns on those assets. I have the opposite case: few assets with a large number of observations.

Pre-transaction costs, the portfolios formed using historical (realised) data, the MinVarPrevVolCorrEst and MinVarVolCorrEst portfolios are the best performing in terms of Sharpe ratio and certainty equivalent. Between the two, the MinVarPrevVolCorrEst has the higher Sharpe ratio of 0.485 but this comes with a larger portfolio volatility (6.85%) and a very large turnover: on average 28% of wealth is

---

77The DCC model is a simplified multivariate GARCH model which has the flexibility of a univariate GARCH model together with a parsimonious correlation specification without the traditional computation difficulties associated with multivariate GARCH models. For more details on model structure and estimation I refer the reader to Appendix B. After estimating the parameters required for the model, I use these parameters in a similar way to Equation (3.16) to predict the covariance matrix in the next period.
Table 3.5: Portfolio performance for benchmark and minimum-variance portfolios based on historical (realised) data

Table 3.5 reports the out-of-sample portfolio performance for the benchmark portfolios and the minimum-variance portfolios formed using historical monthly data as listed in Table 3.4 for the period 31/08/1989 to 31/12/2009. I report the annualised standard deviation $\sigma^k$, Sharpe ratio $SR^k$ and certainty equivalent $ce^k$ of monthly returns of the portfolios together with the portfolio turnover $PT^k$ and opportunity cost as defined in Section 3.5.1. I also report a p-value in parentheses below the standard deviation, Sharpe ratio and certainty equivalent measures, the null hypothesis being that the respective performance measures are statistically significantly different from that of the benchmark portfolio.

<table>
<thead>
<tr>
<th>$k$</th>
<th>$\sigma^k$</th>
<th>$SR^k$</th>
<th>$ce^k$</th>
<th>$PT^k$</th>
<th>OppCost$^k$(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MinVar</td>
<td>6.875</td>
<td>0.417</td>
<td>0.026</td>
<td>0.034</td>
<td>–</td>
</tr>
<tr>
<td>MinVarNoShorts</td>
<td>6.870</td>
<td>0.419</td>
<td>0.026</td>
<td>0.033</td>
<td>0.010</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.108)</td>
<td>(0.137)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MinVarShrinkage</td>
<td>6.875</td>
<td>0.417</td>
<td>0.026</td>
<td>0.034</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>(0.461)</td>
<td>(0.434)</td>
<td>(0.478)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MinVarNoCorr</td>
<td>6.994</td>
<td>0.451</td>
<td>0.030</td>
<td>0.030</td>
<td>0.277</td>
</tr>
<tr>
<td></td>
<td>(0.933)</td>
<td>(0.238)</td>
<td>(0.196)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MinVarPrevVolCorrEst</td>
<td>6.849</td>
<td>0.485</td>
<td>0.031</td>
<td>0.281</td>
<td>0.458</td>
</tr>
<tr>
<td></td>
<td>(0.430)</td>
<td>(0.201)</td>
<td>(0.199)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MinVarVolCorrEst</td>
<td>6.766</td>
<td>0.483</td>
<td>0.030</td>
<td>0.101</td>
<td>0.404</td>
</tr>
<tr>
<td></td>
<td>(0.278)</td>
<td>(0.116)</td>
<td>(0.136)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MinVarEWMA</td>
<td>6.758</td>
<td>0.449</td>
<td>0.028</td>
<td>0.038</td>
<td>0.174</td>
</tr>
<tr>
<td></td>
<td>(0.100)</td>
<td>(0.133)</td>
<td>(0.189)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MinVarDCC</td>
<td>6.780</td>
<td>0.416</td>
<td>0.026</td>
<td>0.085</td>
<td>−0.041</td>
</tr>
<tr>
<td></td>
<td>(0.314)</td>
<td>(0.521)</td>
<td>(0.545)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MinVarSRYS</td>
<td>7.053</td>
<td>0.405</td>
<td>0.026</td>
<td>0.036</td>
<td>−0.044</td>
</tr>
<tr>
<td></td>
<td>(0.864)</td>
<td>(0.561)</td>
<td>(0.527)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 3.6: Portfolio performance inclusive of transaction costs for benchmark and minimum-variance portfolios based on historical data

Table 3.6 reports the out-of-sample portfolio performance for the benchmark portfolios and the minimum-variance portfolios formed using historical monthly data as listed in Table 3.4 for the period 31/08/1989 to 31/12/2009 inclusive of transaction costs. I report the annualised standard deviation $\sigma^k$, Sharpe ratio $SR^k$ and certainty equivalent $ce^k$ of monthly returns of the portfolios together with the portfolio turnover $PT^k$ and opportunity cost as defined in Section 3.5.1. I also report a p-value in parentheses below the standard deviation, Sharpe ratio and certainty equivalent measures, the null hypothesis being that the respective performance measures are statistically significantly different from that of the benchmark portfolio inclusive of transaction costs.

<table>
<thead>
<tr>
<th>$k$</th>
<th>$\sigma^k$</th>
<th>$SR^k$</th>
<th>$ce^k$</th>
<th>$PT^k$</th>
<th>OppCost$^k$(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MinVar</td>
<td>6.876</td>
<td>0.388</td>
<td>0.024</td>
<td>0.034</td>
<td>-</td>
</tr>
<tr>
<td>MinVarNoShorts</td>
<td>6.870</td>
<td>0.390</td>
<td>0.024</td>
<td>0.033</td>
<td>0.015</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.049)</td>
<td>(0.060)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MinVarShrinkage</td>
<td>6.876</td>
<td>0.388</td>
<td>0.024</td>
<td>0.034</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>(0.762)</td>
<td>(0.235)</td>
<td>(0.234)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MinVarNoCorr</td>
<td>7.000</td>
<td>0.425</td>
<td>0.027</td>
<td>0.030</td>
<td>0.303</td>
</tr>
<tr>
<td></td>
<td>(0.939)</td>
<td>(0.216)</td>
<td>(0.180)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MinVarPrevVolCorrEst</td>
<td>6.820</td>
<td>0.241</td>
<td>0.014</td>
<td>0.281</td>
<td>-1.017</td>
</tr>
<tr>
<td></td>
<td>(0.377)</td>
<td>(0.970)</td>
<td>(0.969)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MinVarVolCorrEst</td>
<td>6.764</td>
<td>0.394</td>
<td>0.024</td>
<td>0.101</td>
<td>0.011</td>
</tr>
<tr>
<td></td>
<td>(0.276)</td>
<td>(0.468)</td>
<td>(0.496)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MinVarEWMA</td>
<td>6.755</td>
<td>0.415</td>
<td>0.026</td>
<td>0.038</td>
<td>0.149</td>
</tr>
<tr>
<td></td>
<td>(0.091)</td>
<td>(0.172)</td>
<td>(0.220)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MinVarDCC</td>
<td>6.773</td>
<td>0.342</td>
<td>0.021</td>
<td>0.085</td>
<td>-0.343</td>
</tr>
<tr>
<td></td>
<td>(0.303)</td>
<td>(0.802)</td>
<td>(0.823)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MinVarSRYS</td>
<td>7.060</td>
<td>0.372</td>
<td>0.024</td>
<td>0.036</td>
<td>-0.054</td>
</tr>
<tr>
<td></td>
<td>(0.873)</td>
<td>(0.565)</td>
<td>(0.523)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
traded each month. This means net of transaction costs, this portfolio has a Sharpe ratio of 0.241. In comparison the MinVarVolCorrEst portfolio has a lower volatility (6.76%), only slightly lower Sharpe ratio (0.483) but a significantly lower turnover of 10%. However, both pre and post transaction costs, both portfolios are not significantly better than the MinVar portfolio.

Time-series models developed to forecast time-varying correlation (EWMA and DCC models) do well compared to using the previous realised estimate of correlation at lowering the volatility of the portfolios. The volatility of the MinVarEWMA portfolio (6.76%) is the lowest of all the historical (data) formed portfolios and also significantly lower than that of the benchmark portfolio at a 10% level. This is also the case net of transaction costs. However, this does not necessarily translate into the highest Sharpe ratios of the historical portfolios. Surprisingly, as for the MinVarDCC portfolio, pre-transaction costs, it does not provide any additional benefit compared to simply using the previous month’s realised values of volatility and correlation. The DCC model is supposed to be a parsimonious way to capture the variation in correlation. Cappiello et al. [2006] highlight the in-sample strengths of the DCC model to analyse asymmetric dynamics between stock and bond returns. Net of transaction costs, whilst it performs better than the MinVarPrevVolCorrEst portfolio (given its high turnover), it still underperforms the MinVarEWMA portfolio and has a negative opportunity cost.

3.5.3.3 Minimum-variance portfolios based on analyst forecasts

I wish to test if the information contained in analyst forecasts embed forward-looking information on the economic regimes which are able to capture the variation in the second moment of stock and bond returns. To test this I form minimum-variance portfolios based on analyst forecasts through predictability regressions over the estimation window $\tau$. For example, to construct a portfolio based on all the mean-consensus measures of analyst forecasts for the motivated macroeconomic variables (MinVar$C_t$), I forecast the volatility and correlation via:

$$SM_{t+1} = \hat{\alpha} + \hat{\beta}_1 SM_t + \hat{\beta}_2 C_t$$

(3.17)

where $SM$ is the respective monthly realised second moment of stock and bond returns estimated from Equation 3.3; and $\hat{\alpha}$, $\hat{\beta}_1$ and $\hat{\beta}_2$ for each respective second moment is estimated from a regression over the estimation window $\tau$. This provides a forecast of the volatility and correlation at time $t + 1$ to use

---

78 This is an artefact of the forecasting regression approach which naturally smooths out the estimate of the covariance matrix compared to simply using the previous realised estimate of the volatility/correlation.
in Equation (3.1) and obtain the portfolio weights. This method of forecasting based on a simple factor model allows for a transparent framework to measure the out-of-sample benefits of analyst forecasts versus other methods and factors in predicting the second moments of stock and bond returns for asset allocation.

The various portfolios that I construct based on analyst forecasts are listed in Panel C of Table 3.4. As previously mentioned, \( C \) implies the mean-consensus of analyst forecasts while \( D \) denotes the dispersion in the forecasts for the macroeconomic variables. I construct portfolios based on the analyst forecasts including the lagged value of the realised second moment as an independent variable (\( \text{MinVar}_C, \text{MinVar}_D \) and \( \text{MinVar}_C, \text{MinVar}_D \)).

Table 3.7 reports the performance of the portfolios based on the use of analyst forecasts without and with transaction costs included. Whilst the \( \text{MinVar}_C, \text{MinVar}_D \) portfolios have lower volatilities than the \( \text{MinVar} \) portfolio, these are not significantly lower. Pre-transaction costs, the \( \text{MinVar}_D \) portfolio has the highest Sharpe ratio (0.490) of all the portfolios formed, however significant only to a 15% level. Net of transaction costs, given a turnover of 11.3%, the Sharpe ratio becomes 0.380, which is less than that of the \( \text{MinVar} \) portfolio.

This result could be due to the differing contributions that the mean-consensus and dispersion measures have on predicting the second moment of stock and bond returns as shown by Figure 3.5. I therefore also construct portfolios based on both measures. The performance of the \( \text{MinVar}_C, \text{MinVar}_D \) portfolio is not encouraging. It has a much larger volatility than that of the \( \text{MinVar} \) portfolio and although its Sharpe ratio is larger than the benchmark portfolio, it is not statistically larger. When taking transaction costs into account, given a high turnover of 16.8%, its Sharpe ratio (0.21) becomes substantially less than that of the \( \text{MinVar} \) portfolio. Similar conclusions are reached for the certainty equivalent.

Comparing the performance of these portfolios to those in Table 3.5 constructed based on historical data, there does not appear to be out-of-sample benefits of using analyst forecasts to predict stock-bond correlation. This can be seen from Figure 3.7 which plots the net growth of $1 invested in selected portfolio strategies over the out-of-sample period. The \( \text{MinVarNoCorr} \) and \( \text{MinVarEWMA} \) portfolios perform best, net of transaction costs. This is even clearer when looking at the economic value of timing the volatility and correlation. Figure 3.8 highlights this opportunity cost across the portfolio strategies.

\[ \text{In fact contrary to the in-sample results, the out-of-sample performance of the portfolios formed on the mean-consensus measures did worse than those formed on the dispersion measures. In-sample the mean-consensus measures seemed to contribute more than the dispersion measures in predicting the volatilities and correlation.} \]
Table 3.7: Portfolio performance for portfolios formed using analyst forecast measures

Table 3.7 reports the out-of-sample portfolio performance for the portfolios formed using analyst forecasts as listed in Table 3.4 for the period 31/08/1989 to 31/12/2009, both without and with transaction costs included. I report the annualised standard deviation $\sigma^k$, Sharpe ratio $SR^k$ and certainty equivalent $CE^k$ of monthly returns of the portfolios together with the portfolio turnover $PT^k$ and opportunity cost as defined in Section 3.5.1. I also report the p-values in parentheses below the standard deviation, Sharpe ratio and certainty equivalent measures, the null hypothesis being that the respective performance measures are statistically significantly different from that of the benchmark portfolios.

<table>
<thead>
<tr>
<th>$k$</th>
<th>$\sigma^k$</th>
<th>$SR^k$</th>
<th>$CE^k$</th>
<th>$PT^k$</th>
<th>OppCost</th>
</tr>
</thead>
<tbody>
<tr>
<td>MinVar</td>
<td>6.875</td>
<td>0.417</td>
<td>0.026</td>
<td>0.034</td>
<td>–</td>
</tr>
<tr>
<td>MinVarC_t</td>
<td>6.810</td>
<td>0.422</td>
<td>0.026</td>
<td>0.104</td>
<td>0.008</td>
</tr>
<tr>
<td></td>
<td>(0.308)</td>
<td>(0.475)</td>
<td>(0.490)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MinVarD_t</td>
<td>6.724</td>
<td>0.490</td>
<td>0.030</td>
<td>0.113</td>
<td>0.369</td>
</tr>
<tr>
<td></td>
<td>(0.200)</td>
<td>(0.147)</td>
<td>(0.173)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MinVarC_tD_t</td>
<td>6.988</td>
<td>0.465</td>
<td>0.030</td>
<td>0.168</td>
<td>0.372</td>
</tr>
<tr>
<td></td>
<td>(0.743)</td>
<td>(0.252)</td>
<td>(0.220)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>With Transaction Costs</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MinVar</td>
<td>6.876</td>
<td>0.388</td>
<td>0.024</td>
<td>0.034</td>
<td>–</td>
</tr>
<tr>
<td>MinVarC_t</td>
<td>6.811</td>
<td>0.333</td>
<td>0.020</td>
<td>0.104</td>
<td>−0.395</td>
</tr>
<tr>
<td></td>
<td>(0.318)</td>
<td>(0.804)</td>
<td>(0.816)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MinVarD_t</td>
<td>6.723</td>
<td>0.380</td>
<td>0.023</td>
<td>0.113</td>
<td>−0.099</td>
</tr>
<tr>
<td></td>
<td>(0.191)</td>
<td>(0.560)</td>
<td>(0.599)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MinVarC_tD_t</td>
<td>7.000</td>
<td>0.321</td>
<td>0.020</td>
<td>0.168</td>
<td>−0.426</td>
</tr>
<tr>
<td></td>
<td>(0.763)</td>
<td>(0.833)</td>
<td>(0.810)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 3.7: Growth of $1 invested in selected portfolio strategies net of transaction costs

The growth of $1 invested in selected portfolio strategies over the period 31/08/1989 to 31/12/2009. This represents the evolution of the Net Asset Value (NAV) for the respective strategies inclusive of transaction costs. The plot is overlaid with NBER recession bands.
CHAPTER 3. STOCK-BOND CORRELATION AND OUT-OF-SAMPLE PORTFOLIO PERFORMANCE USING ANALYST FORECASTS

considered. For the MinVar portfolio - an investor is economically 0.43% annually better off by simply investing in a MinVar portfolio.

**Figure 3.8: Net opportunity cost of constructed portfolios versus the MinVar portfolio**

Opportunity cost net of transaction costs for selected portfolios formed using historical and analyst forecast data versus the MinVar portfolio as defined by Equation (3.11). Such a measure represents the economic significance of the difference in the performance of other strategies from the MinVar strategy. When the opportunity cost is positive (negative) the investor is economically better (worse) off by using the comparison strategy rather than the MinVar portfolio.

These results show that analyst forecasts do not provide out-of-sample benefits for forecasting stock-bond correlation compared to several methods based on using historical (realised) data. Whilst Piazzesi and Schneider [2011] find that analyst forecasts ex-ante provide better out-of-sample forecasts than statistical models/methods (based on historical data), which only display significant predictability with the benefit of hindsight i.e. being in-sample, for the term structure of interest rates. I do not find the same conclusion for analyst forecasts on macroeconomic variables for predicting stock-bond correlation, out-of-sample.
3.6 Robustness

I report the results of several additional portfolios that I have run to test the robustness of the findings.

3.6.1 Performance during the 2008 financial crisis

I analyse the out-of-sample portfolio performance, net of transaction costs, for the different portfolios during the financial market crisis of 2008 in Figure 3.9. Similar to the findings of DeMiguel et al. [2010a], the minimum-variance portfolio has a negative Sharpe ratio. This comes at no surprise given the extreme volatility in financial markets during 2008. The negative Sharpe ratio of the MinVarNoCorr portfolio highlights the benefit of predicting the correlation during volatile market periods rather than simply ignoring it. Interestingly, it seems that the best strategy during this period, net of transaction costs, would have been to simply use the last realised value of volatility and correlation to form the optimal minimum-variance portfolio.

Figure 3.9: Net Sharpe ratios of portfolios during 2008 financial crisis

Out-of-sample Sharpe ratio net of transaction costs during the 2008 financial crisis of various stock-bond portfolios formed using historical (realised) data and analyst forecast data.
CHAPTER 3. STOCK-BOND CORRELATION AND OUT-OF-SAMPLE PORTFOLIO PERFORMANCE USING ANALYST FORECASTS

Figure 3.9 also highlights the parsimonious nature of the DCC model for predicting the volatility and correlation during turbulent times, with the MinVarDCC model performing better than the MinVarEWMA portfolio. Lastly, whilst it appears that the information contained in analyst forecasts did provide some benefits during the financial crisis, it is clear that simply using the realised value of the volatility/correlation would serve an optimal portfolio best during financial turmoil.

3.7 Concluding remarks

This chapter investigates the out-of-sample benefits of the information contained within analyst forecasts for predicting the second moments of stock and bond returns within a static asset allocation setting to calculate the optimal stock-bond portfolio. The motivation for doing so being two fold. The first based on the importance of predicting the out-of-sample correlation between stock and bond returns for optimal allocation of investors’ wealth between the asset classes. The second being the use of the information contained within analyst forecasts on macroeconomic factors known to display some predictability of the second moments of stock and bond returns.

A hypothesis of analyst forecasts being able to improve investors’ allocation between stocks and bonds over methods based on historical (realised) data is investigated. Intuitively, such forecasts should contain embedded expectations of the variable being forecast conditional on the time $t$ economic state of the world. Such forecasts for the inflation rate in particular have been shown to display better out-of-sample properties than other model-based methods for predicting inflation \cite{Ang2007}. In this chapter I apply analyst forecasts of the long rate, short rate, change in corporate profits, inflation rate and change in real GDP obtained from the BCEI survey database to predict the second moments of stock and bond returns. In-sample I find that such forecasts of macroeconomic variables display varying degrees of predictability for the volatility and correlation of returns.

Out-of-sample I test the predictability of these analyst forecasts through an optimal portfolio choice problem: I find that analyst forecasts are not able to significantly improve the allocation of minimum-variance portfolios formed using these forecasts. Robustness checks also highlight that during financial crisis, the out-of-sample benefits from using the information contained in analyst forecasts are overshadowed by simply using the realised value of volatility and correlation to form the optimal minimum-variance portfolio. Unlike the findings of \cite{Piazzesi2011} for the term structure of interest rates, I do not find any out-of-sample benefits in using analysts’ forecasts to predict stock-bond correlation.
Chapter 4

Pension funds and stock-bond correlation risk: The case for a correlation swap

4.1 Introduction

Stock-bond correlation risk is of great importance to pension funds. With an estimated $29.9 trillion of assets under management (AuM) at pension funds globally as at the end of 2010,\(^80\) such a risk, especially during financial crises plays a significant role in asset/liability management\(^81\). Typically pension funds are net long stocks and net short bonds - this latter net position being from the liabilities which are essentially streams of future payments that depend on interest rates, inflation and mortality/longevity rates, and can be proxied by long duration bonds;\(^82\) the former net position being from contributions invested in stocks. Given the empirical evidence (Connolly et al. [2005]) that during financial crises the correlation between stock returns and bond yield changes increase, known as the “flight-to-quality” effect, where investors sell stocks to buy Treasury bonds. The liabilities of the pension fund increase in value (decreasing bond yields) whilst its assets decrease in value (decreasing stock prices) creating an asset/liability mismatch.\(^83\)

---


\(^81\) Specifically stock-bond correlation is a contentious parameter in the Solvency II rules for pension funds.

\(^82\) Indeed durations can often be so long that this consideration for Liability Driven Investment (LDI) strategies is non-trivial. As the supply of long dated zero-coupon bonds is small, it is quite common for LDI focused investors to add forward starting swaps to a more liquid shorter duration bond portfolio in order to match the duration of the liabilities. See the presentation “When Bonds are not enough” by AXA: [http://en.wikipedia.org/wiki/Inflation-indexed-bonds](http://en.wikipedia.org/wiki/Inflation-indexed-bonds).

\(^83\) Such a risk should be considered in the context of the regulation around pension-plan sponsors to either recognise losses and/or contribute capital to underfunded pension schemes.
I note that the liabilities of pension funds typically cannot be hedged by just investing in bonds. This is for a variety of practical reasons: As noted in Footnote 82, supply considerations need to be taken into account when trying to match the long dated nature of the durations of the liabilities. These supply concerns also apply to Treasury Inflation Protected Securities (TIPS) which pension funds may invest in to hedge the inflation risk present in their liabilities. As of 2008, only $1.5 trillion of inflation-linked bonds were in issue by Governments in the international debt market, this is dwarfed by the amount of assets under management at pension funds, all of which need to be managed in order to meet future liabilities. Lastly, mortality/longevity and other actuarial risks present in the liabilities of pension funds cannot be naturally hedged by bonds/TIPS. For all these reasons pension fund managers typically look to stocks for two reasons, the large amount of capacity available and as a potential way to hedge the actuarial risks.

I also address the theoretical concern from the Modigliani-Miller theorem on why pension funds invest in stocks even though their liabilities are bond like. Pension funds are subject to differing tax treatment and often substantial regulatory requirements violating the assumptions of the theorem. As such, pension funds do need to consider other types of securities such as stocks in order to match their bond-like liabilities. Ultimately, although the theoretical need for pension funds to invest into stocks can be questioned, shows on average 60% of corporation pension fund portfolios are invested in stocks. This makes stock-bond correlation risk of practical concern.

This chapter specifically investigates stock-bond correlation risk and the importance of unexpected changes in the correlation for the Asset Liability Management (ALM) mandate of pension funds. Such a risk is of considerable concern to pension funds given the widely acknowledged time-variation in stock-bond correlation. Despite this, the risk of changes in stock-bond correlation has remained unexplored in the ALM literature. In this work I address this risk in the context of a structured product such as a multi-asset class derivative. For a pension fund manager whose objective is to reduce the probability of an underfunded pension scheme, such instruments that hedge against the mismatch risk between assets and liabilities should form an important part of a pension fund’s portfolio. Indeed such a derivative already exists in the form of a

---

84 TIPS are inflation-linked bonds where the principal and coupon payments are indexed to the inflation rate. The history of and discussion on such securities is in Campbell et al. [2009b].
86 Indeed for such a reason there is increasing interest in the use of longevity swaps to hedge mortality risk exposure inherent in pension fund liabilities (Billis and Blake [2009]).
CHAPTER 4. PENSION FUNDS AND STOCK-BOND CORRELATION RISK: THE CASE FOR A CORRELATION SWAP

stock-bond correlation swap. However little is known about how to price and manage this multi-asset class derivative. It is this that I consider in this chapter.

The importance of correlation risk within a stock market context has been explored by Driessen et al. [2009b].

Equity correlation swaps to manage this risk are widely known about and were actively traded between 2005-2008. However, as for the importance of correlation risk across stocks and bonds, the literature is more sparse. Buraschi et al. [2010] indirectly addresses the issue within a portfolio choice setting finding that a significant hedging demand exists for the correlation risk between stocks and bonds.

If the correlation between stocks and bonds were completely predictable, a pension fund manager with an ALM mandate would be able to adjust her portfolio today to take account of the change in correlation tomorrow. Although Viceira [2010] among others find common factors that display some predictability of the second moments of stocks and bonds, out-of-sample, any predictability of the correlation will not be perfect. Hence, as the pension fund has an exposure to stock-bond correlation, it will therefore require a way to hedge itself from unexpected changes in the correlation. A correlation swap provides a way of doing this, paying out on the difference between the swap rate (i.e. the fixed leg) and the realised correlation.

Such stock-bond correlation swap contracts already exist, Figure 4.1 highlights one such contract.

Figure 4.1 shows the price in correlation points of an Over-The-Counter (OTC) correlation swap contract between S&P 500 returns and yield changes on the 10-year Constant Maturity Swap (CMS) rate. The swap rate at the end of May for the swap expiring in December 2012 (approximate expiry of two and half years) is 31%. An investor could have thus bought/sold a fixed correlation of 31% to go long/short the realised correlation in December 2012. That such a product already exists means that a market for exposure to stock-bond correlation exists. By taking a position in this product, the investor is exposed

---

87 The dispersion trade is often performed in an equity market context to capitalise on the overpricing of index options relative to individual options (Deng [2008]). Recently, Buraschi et al. [2009] and Driessen et al. [2009b] have explained the profits to such a strategy through the presence of a correlation risk premium within stock index options. Intuitively, index options are thus expensive unlike individual options as they offer a valuable hedge against correlation increases and insures against the risk of a loss in diversification benefits.

88 See the online sponsored statement by Barclays Capital: “Equity correlation - explaining the investment opportunity”.

89 See the 2005 technical note by JPMorgan on “Correlation Vehicles.”

90 The investment bank which provided this quote wishes to remain anonymous. In this chapter I present as much data on stock-bond correlation swap quotes that I was allowed to use for research purposes. The market for stock-bond correlation swaps is small and neither the standardization nor liquidity exists (yet) in these products. Hence the correlation swaps tend to trade at customised transactions.

91 The CMS index is similar to the Constant Maturity Treasury (CMT) bond index in that it provides exposure to long dated interest rates - see Appendix C for a comparison. The reason yield changes are adopted as opposed to returns is because calculating the returns on a constant maturity index requires a model to adjust for the constant maturity aspect of the index (De Goeij and Marquering [2004]).
CHAPTER 4. PENSION FUNDS AND STOCK-BOND CORRELATION RISK: THE CASE FOR A CORRELATION SWAP

Figure 4.1: Over-The-Counter (OTC) correlation swap contract between the S&P500 and 10-year Constant Maturity Swap (CMS) rate.

This quote was made at the end of May 2010 for a stock-bond correlation swap maturing in December 2012. The correlation is between returns on the S&P500 and yield changes on the 10-year CMS. The bid quote of 31% represents an implied correlation of 0.31. Thus an investor can pay a fixed correlation of 31% to receive exposure to floating stock-bond correlation. Such a product may be offered by the seller as they expect the correlation to revert from the 30% level as of May 2010, so that they will profit from the payoff where \((31 - \text{realised}) > 0\). It appears from this figure that the implied correlation is set to equal the realised correlation value (this being subject to the method used to calculated the realised correlation as discussed in Section 4.3.2). If the realised correlation increases above 31% at maturity, the seller will payout to the correlation swap buyer. Thus this structured product acts like insurance against increases in stock-bond correlation for the swap buyer.
CHAPTER 4. PENSION FUNDS AND STOCK-BOND CORRELATION RISK: THE CASE FOR A CORRELATION SWAP

to both changes in the market’s expectation of future stock-bond correlation and the ‘cashflow hedge’ from the net correlation exposure at each payment date. 92

The actual market for stock-bond correlation swaps is OTC, relatively new, small and illiquid. I posit this is because the swap has not been well documented with a formal and transparent valuation method. As such the product typically trades at customised transactions, being priced at inception and valued by the participants of the swap. However, even in the absence of a liquid market for these correlation swaps, several interesting research avenues arise with particular implications for pensions funds:

First, in the context of structuring a stock-bond correlation swap contract, I start by motivating both theoretically and empirically the importance of correlation risk between stock returns and bond yield changes on the surplus return of a stylised pension fund.93 Empirically I not only find that positive changes in correlation are significantly related to contemporaneous decreases in surplus returns, but that an equivalent positive shock (in magnitude) to correlation decreases surplus returns by more the longer the horizon of surplus returns. Given that pension funds can typically have long rebalancing horizons (Hoevenaars et al. [2008]), this makes them particular susceptible to stock-bond correlation risk.

Second, by comparing the differing methods to trade correlation risk, I motivate the need for a stock-bond correlation swap given that a combined stock-bond index with actively traded options does not exist. This makes the dispersion trading strategy (Driessen et al. [2009b]), a typical method for trading correlation risk, redundant for the case across stocks and bonds. The development of a pricing model requires assumptions about the stochastic data generating process. Therefore, I proceed by first examining in detail the structure of a stock-bond correlation swap contract, before diving into developing a pricing model. Given the difficulty of obtaining implied stock-bond correlation data,94 I empirically analyse the closest readily available driver of implied correlation: the respective implied volatilities of stock and bond. Such an approach is justified by the work of Connolly et al. [2005], Chordia et al. [2005] and Fleming et al. [1998] who find evidence of volatility linkages between stock and bond markets which

---

92 More specifically, the mark to market process of the swap comes from these two sources. The former being the impact of the swap on the hedger’s (pension fund) balance sheet through changes in market conditions (either from the evolution of the market’s expectation of future stock-bond correlation and/or changes in interest rates. For example, even if future swap payments are expected to provide a good hedge against correlation risk, if interest rates increase, the present value of the expected net payments decrease thereby weakening the hedger’s position); the latter being from the difference between the (current) realised correlation and the pre-set swap (rate) payment at each payment date generating either a cash inflow or outflow to the hedger. Ignoring basis risk (which is the case for bespoke products), these net cashflows represent the hedging potential of the swap to the correlation exposure in operation.

93 Surplus return is defined below but conceptually takes into account both the return of the assets and liabilities of the pension fund.

94 This difficulty ultimately arises from the lack of traded options on a combined stock-bond index. This is fully discussed in Section 4.3.
are also drivers of the correlation between these markets.\footnote{I note that these studies focus on the link of realised volatility and realised correlation of stock and bond markets.}

Using the model-free methodology of \cite{Britten-Jones2000} I back out the 1-year implied volatility from options on the S&P500 index and 10-year Treasury bonds and find important time-varying linkages between the implied volatility of stock and bond markets which would naturally affect implied stock-bond correlation. Highlighting unspanned volatility risk in both stock \cite{Buraschi2001} and bond markets \cite{Andersen2010}, where there is at least one state variable which drives innovations in equity and interest rate derivatives but which does not drive changes in underlying stock or bond markets, I infer the possibility of unspanned correlation risk. This is of both theoretical and practical importance as it implies that the underlying securities cannot be used to hedge the correlation risk between stock and bond markets, suggesting the presence of an incomplete market.\footnote{Markets are said to be dynamically complete if any contingent claim can be replicated by some self-financing strategy using existing traded securities. For example, if uncertainty is driven by $N$ Wiener processes, one needs $N$ risky assets in addition to the money market account for markets to be complete.} This not only motivates the need for a stochastic correlation model to describe the interaction between stock and bond markets but also that of a derivative on stock-bond correlation.

Indeed, to hedge volatility risk one needs derivatives with pay-offs that depend on the variances of the underlying security, such as variance swaps, which are readily traded for stock and bond indices \cite{Carr2009,Mueller2012}. To hedge stock-bond correlation risk, a stock-bond correlation swap is thus needed making it a non-redundant security and one which may form an important part of an optimal stock-bond portfolio. Since such securities are not readily available, I thus work in an incomplete market setting.

Third, motivated by these considerations, I propose a stock-bond pricing model under the historical measure $\mathbb{P}$ using a Wishart framework \cite{Fonseca2007} to jointly model stock prices and bond yields which simultaneously incorporate both stochastic volatility and stochastic correlation. This is done in an incomplete market setting as there are only 2 risky assets available for investment with the covariance matrix dynamics depending on 3 independent Brownian shocks. In a complete market, any contingent claim can be replicated through a self-financing trading strategy, whereby no-arbitrage dictates that the price of the contingent claim would be equal to the initial wealth required to form the replicating portfolio. In this case there exists an unique Radon-Nikodym derivative and by the Fundamental Theorem of Asset Pricing (FTAP) the no-arbitrage price of the contingent claim can be obtained by discounting the expected payoff under what is known as the pricing measure $\mathbb{Q}$. If the
discounting is done using the money market account, this pricing measure is known as the risk-neutral measure.

Given the incomplete market setting I thus assume that the stock-bond correlation swap payoff cannot be replicated through a self-financing trading strategy, which rules out use of the Fundamental Theorem of Asset Pricing (FTAP). I therefore use a utility indifference pricing approach (Davis [1997], Henderson and Hobson [2004]), which takes into account the role of preferences in the pricing of securities. Indeed there is growing interest in the use of indifference pricing for derivatives in incomplete markets (Carmona [2009]). More specifically, under the condition of the stock-bond correlation swap having zero net market value at inception, I obtain the swap rate at inception through assuming that a pension fund (who is incentivised to buy the swap in the first place) is no worse off in expected utility terms than it would have been without the swap. This is the concept of utility indifference pricing. In this work I focus on the demand side of the correlation swap, assuming that the supply side is given. I thus obtain bid quotes of the swap rate. Repeating this procedure for the counter-party (supplier of the correlation swap) would give rise to bid-ask swap rates. In conversations with investment banks who provide and manage stock-bond correlation swaps (hedge supplier), the problem is in identifying a natural supplier of stock-bond correlation. Although this is an interesting research question in itself, I focus on the demand side given the clear motivation and need of stock-bond correlation swaps for pension funds.97

Assuming that pension funds exhibit mean-variance preferences on their surplus return I develop the first, to the best of my knowledge, formal stock-bond correlation swap pricing model in a transparent and intuitive manner. Calibrating the model to historical data I find that the model-implied quotes fall within the range of quotes obtained from market participants such as Figure 4.1. In one case, it appears that a premium is being charged on the correlation swap strike above it’s model-implied “fair value”. Such an observation is important in the context of the benefits of financial innovation (Henderson and Pearson [2011]).

The model also captures the theoretical behaviour of pension funds through the specified preferences, i.e. a pension fund with a higher risk aversion parameter would be willing to enter the correlation swap at a higher price in order to hedge stock-bond correlation risk. This is also the case when the liabilities become more important within the ALM mandate. I also highlight the importance of estimation of the

---

97Intuition would suggest that Sovereign Wealth funds who are typically thought of as long stocks and bonds, may be a source of natural supply. Also, if there were a premium for stock-bond correlation, sophisticated hedge funds may also take interest in supply this correlation to the market place. I leave such a question to be considered for future research.
CHAPTER 4. PENSION FUNDS AND STOCK-BOND CORRELATION RISK: THE CASE FOR A CORRELATION SWAP

model and in particular certain parameters which directly affect the evolution of the correlation process within the model.

The contribution of this chapter is fourfold. First, I explore the specific role of stock-bond correlation risk on a pension fund’s surplus return, highlighting the need for a way to hedge such a risk within the context of an ALM mandate. Although there are various papers (Van Binsbergen and Brandt [2007]) and research notes (GSAM [2007]) on ALM risks, to the best of my knowledge I am the first to highlight the role of stock-bond correlation risk on the ALM mandate. Second, I empirically document as best as possible, the potential of unspanned stochastic correlation across stocks and bonds. Although unspanned stochastic volatility in equity markets (Buraschi and Jackwerth [2001]) and bond markets (Andersen and Benzoni [2010]) has been investigated, I look at the relationship of these implied volatilities in the context of the dynamics of implied stock-bond correlation.

Fourth, I present a formal yet intuitive stock-bond correlation swap pricing model which captures dynamic correlation between stocks and bond within an incomplete market setting. The literature on option pricing when correlations are stochastic (Fonseca et al. [2007], Branger and Muck [2012]) and on equity correlation swap pricing (Bossu [2007]) is fairly new but developing rapidly. I contribute to these two streams of literature by focusing specifically on modelling stochastic stock-bond correlation together with the pricing of a stock-bond correlation swap. To the best of my knowledge, this is the first of such a pricing model within both an academic and practitioner setting. Lastly, the work provides a method for pension funds to assess the stock-bond correlation risk that they are exposed to and obtain what the fair price should be should they wish to hedge their exposure to correlation risk with a stock-bond correlation swap.

The rest of the chapter is organised as follows. Section 4.2 motivates the importance of stock-bond correlation risk for pension funds. This lead onto Section 4.3 where I look in detail about the methods of trading stock-bond correlation risk and specifically at structuring a stock-bond correlation swap. Section 4.4 documents the need to apply stochastic correlation modeling across stock and bond markets by empirically analysing the closest obtainable driver of implied correlation: the respective implied volatilities in stock and bond markets. Section 4.5 outlines the stock-bond correlation swap pricing methodology while the results of this pricing model are discussed in Section 4.6. Some concluding remarks are made in Section 4.7.
CHAPTER 4. PENSION FUNDS AND STOCK-BOND CORRELATION RISK: THE CASE FOR A CORRELATION SWAP

4.2 Stock-bond correlation risk: The case for pension funds

Asset Liability Management (ALM) should be at the core of the risk management practices of pension funds. The 2007-2008 financial crisis highlighted the risk of underfunded pension plans. A recent article in the Economist “A trillion here, $500 billion there”, October 15th 2011, reports the massive deficits in funding ratios of pensions funds in the U.S as well as the U.K with the average being between 72% and 83%. This has increased the focus on ways to manage the mismatch between assets and liabilities. In this section I investigate the nature of stock-bond correlation on ALM for a stylised pension fund.

Consider a defined-benefit (DB) pension scheme, which promises its members a pre-defined pension benefit when they reach retirement. Liabilities of the pension scheme typically depend on past and expected future earnings of participants, interest rates, inflation rates, plan demographics and mortality rates. In this chapter I focus on the role of interest rate risk on managing the pension fund’s liabilities and thus do not consider actuarial or inflationary risks. Thus I assume liabilities are highly correlated to long-term bonds with a fixed duration of 20 years. Pension funds invest contributions by participants and the plan sponsor to meet their liabilities in the long-run and thus typically take some risk in order to earn a risk premium. Here I assume that the contributions are invested in a 60/40 split across stocks and long-term bonds.

The funding ratio is defined as the ratio of assets ($A$) to liabilities ($L$). I model the market value of liabilities as an amount due in 20 years discounted by an interest rate assumed to be from a 20 year zero-coupon bond. I then set the (current) value of assets to equal that of the market value of liabilities. Assuming that the value of assets is made up of 60% in stocks and 40% in 10-year bonds, I calculate the future value of these bonds using the same interest rate (thus implicitly assuming a flat term structure for illustrative purposes). I then apply a negative shock of 10% to the value of stocks, the level of the interest rate and then to both at the same time. Figure 4.2 reports the percentage decrease in the funding ratio from the initial level of 1 (when $A = L$) as a result of the adverse shocks to the factors affecting the value of assets and liabilities. The percentage decrease in funding ratio from the “correlated (or simultaneous) negative shocks” to stocks and interest rates highlights the importance of

---

98 Another article “Heads I win, tails you pay”, January 17th 2012 in the Economist reports the move by pension funds to invest in riskier asset classes such as private equity in an attempt to reduce the expected short-fall between liabilities and assets. The validity of such a move by pension funds to allocate their assets to alternative (potentially riskier) asset classes is investigated by Hoevenaars et al. [2008].

99 I also assume away the role of funding risk (Inkmann and Blake [2007]) to keep the focus clear.

100 This subjective ratio is obtained from the work of Rauh [2009] who find this approximate ratio in a large cross-section of pension plans.

101 A 10% negative shock to the level of interest rates implies that the bonds in the asset portfolio increase in value, however so does the value of liabilities.
CHAPTER 4. PENSION FUNDS AND STOCK-BOND CORRELATION RISK: THE CASE FOR A CORRELATION SWAP

Figure 4.2: Percentage funding ratio at risk from adverse shocks to assets and liabilities of a pension fund

This figure reports the percentage decrease in the funding ratio from the initial level of 1 (when $A = L$) as a result of the adverse shocks to the factors affecting the value of assets and liabilities, more specifically a 10% (negative) shock to the value of stocks, the level of interest rates and to both at the same time, which I define as a “correlated shock”.

hedging such “correlated adverse shocks”.

To put the above example into context, I outline the theoretical possibilities of the role of correlation shocks between stock returns and interest rate changes on the ALM mandate of a pension fund. To simplify the example, I assume that the assets consist of 100% in stocks. The correlation shocks can be either positive or negative. For the case of positive correlation shocks (case $P$), there are two possibilities $P_1$ and $P_2$ with differing outcomes:

- Case $P_1$: Positive return to stocks and positive change in interest rates → Assets increase whilst liabilities decrease, thus the funding ratio increases.

- Case $P_2$: Negative return to stocks and negative change in interest rates → Assets decrease whilst liabilities increase. The outcome of such a possibility is shown in Figure 4.2 thus funding ratio decreases.

Whilst for the case of negative correlation shock (case $N$) between stock returns and interest rate changes, there are again two possibilities:

102 This naturally presents the most extreme case of a pension fund’s exposure to changes in correlation across stocks and bonds. Adding long-term bonds to the asset portfolio will naturally mitigate stock-bond correlation risk, the degree of which will depend on the amount of long-term bonds added and the nature of these bonds. Despite this, it is clear that any allocation of pension fund assets to stocks, should be accompanied with concern on the potential impact of stock-bond correlation risk on the ALM mandate.
CHAPTER 4. PENSION FUNDS AND STOCK-BOND CORRELATION RISK: THE CASE FOR A CORRELATION SWAP

• Case N1: Positive return to stocks and negative change in interest rates → Both assets and liabilities increase in value, the magnitude of which depends on the size of the stock return/interest rate change. The affect of a negative correlation shock on the funding ratio is unclear.

• Case N2: Negative return to stocks and positive change in interest rates → Both assets and liabilities decrease in value, the magnitude of which depends on the size of the stock return/interest rate change. The affect of a negative correlation shock on the funding ratio is unclear.

This outlines the broad picture on the role of correlation shocks for the ALM status of the pension fund. I note that the above theoretical possibilities should be discussed in the context of time variation in correlation. For example, if the current correlation level is $-0.80$, and there is a positive shock of 0.6 resulting in a correlation level of $-0.20$. Although on average stock returns and interest changes are still negatively correlated, the new correlation level implies either a simultaneous positive or simultaneous negative observation of stock returns and interest rate changes for the strong negative correlation of $-0.80$ to increase to $-0.20$. If the positive correlation shock is a result of the simultaneous negative observations of stock returns and interest rate changes (case $P2$), the increasing risk to the ALM mandate is quite clear. It is this state of the world that positive correlation shocks pose a risk for pension funds.

I next consider if the correlation risk is exhibited empirically in the data. Following Sharpe and Tint [1990] I define surplus of a pension fund, which is closely related to the funding ratio as:

$$SP_t = A_t - \tilde{k}L_t$$

(4.1)

where $A_t$ and $L_t$ are the value of assets and liabilities at time $t$ respectively and $\tilde{k}$ measures the importance that the pension fund attaches to the value of liabilities. I further define return on the surplus ($R_{SP,t}$) at time $t$ as:

$$R_{SP,t}(\tilde{k}) = \frac{SP_t(\tilde{k}) - SP_{t-1}(\tilde{k})}{A_{t-1}}$$

(4.2)

$$= \frac{A_t - A_{t-1}}{A_{t-1}} - \tilde{k} \cdot \frac{1}{FR_{t-1}} \cdot \frac{L_t - L_{t-1}}{L_{t-1}}$$

$$= R_{A,t} - \frac{\tilde{k}}{FR_{t-1}} R_{L,t}$$

where $R_{A,t}$ and $R_{L,t}$ are the return on assets and liabilities respectively at time $t$ and $FR_{t-1}$ is the funding ratio at time $t-1$. The ratio of the importance parameter to the funding ratio at the end of the

103 This is conceptually similar to the funding ratio return introduced by Leibowitz et al. [1994].
CHAPTER 4. PENSION FUNDS AND STOCK-BOND CORRELATION RISK: THE CASE FOR A CORRELATION SWAP

Previous period implies that the actual importance of the liabilities reduces when the asset value exceeds the value of liabilities. I simplify this ratio to \( k = \bar{k}/FR_{t-1} \) and set \( k \in [0, 1] \). With \( k \rightarrow 1 \), I interpret Equation (4.2) as a pension fund mandate, whereas \( k \rightarrow 0 \) can be thought of as the asset-only case of a mutual fund. I thus keep this parameter in the analysis to allow for comparison between stylised pension funds and mutual funds.

Using the above 60/40 split across stocks and long-term bonds, I assume the asset portfolio consists of the S&P500 and 10-year Constant Maturity Treasury bonds. Liability returns are based on the 30-year Constant Maturity Treasury bond index. Describing the liabilities using a constant maturity index implies that the pension fund is in a stationary state. This assumes that the distribution of the age cohorts and the build-up pension rights per cohort are time invariant and that the inflow from contributions equal current payments plus the net present value of new liabilities.

Based on monthly observations from 1st January 1990 to 31st August 2011 (260 observations), I perform the regression:

\[
R_{SP,t-n,t}(k) = \alpha + \beta RC_{t-n,t}(S,B) + \epsilon_t
\]

where \( n \) denotes the horizon of the (overlapping) regression (i.e. monthly, quarterly, semi-annually and annually), thus \( R_{SP,t-n,t} \) is the surplus return and \( RC_{t-n,t}(S,B) \) is the realised correlation on stocks and bonds between the end of the month \( t - n \) and end of the month \( t \). Realised correlation is estimated from daily returns on the S&P500 and daily yield changes on the 10-year CMS index via:

\[
RC_{t-n,t}(S,B) = \frac{\sum_{i=1}^{N_t} R_{S,i} \Delta Y_{B,i}}{\sqrt{\sum_{i=1}^{N_t} R_{S,i}^2 \sqrt{\sum_{i=1}^{N_t} \Delta Y_{B,i}^2}}}
\]

\[
R_{S,i} = \frac{S_i - S_{i-1}}{S_{i-1}}, \quad \Delta Y_{B,i} = \frac{Y_i - Y_{i-1}}{Y_{i-1}}
\]

where \( N_t \) is the number of trading days between the end of month \( t - n \) and end of month \( t \), \( R_{S,i} \) and

---

104 Returns on the S&P500 are calculated in the usual way. Returns on the Constant Maturity Treasury bond indexes are calculated using the methodology of De Goeij and Marquering [2004] as outlined in Appendix E.

105 The Constant Maturity Treasury indexes represent hypothetical bond contracts with semi-annual coupon payments (see Appendix F). I use the 30-year Constant Maturity Treasury bond index to model liabilities as it will have a duration comparable to the previous assumption of 20 years.
CHAPTER 4. PENSION FUNDS AND STOCK-BOND CORRELATION RISK: THE CASE FOR A CORRELATION SWAP

\( \Delta Y_{B,i} \) are daily S&P500 returns and 10-year CMS yield changes respectively on day \( i \).\(^{106}\)\(^{107}\)

Table 4.1: Regressions of surplus returns onto realised correlation of stock returns and bond yield changes

Table 4.1 reports monthly overlapping regressions of surplus returns onto a constant and the realised correlation between S&P500 returns and 10-year CMS yield changes over the period 01/01/90 to 31/08/11 (260 observations) at differing horizons \( n \). I repeat the regression for two values of the liability importance parameter \( k \) to compare the role of stock-bond correlation on the returns of a stylised pension fund \( (k = 1) \) and an asset-only mutual fund \( (k = 0) \). For each regression the table reports coefficient estimates, Newey-West corrected t-statistics and the adjusted \( R^2 \) of the regression. The lag order of the Newey-West correction is equal to the horizon (in months) minus one. *, ** and *** denote statistical significance at 10%, 5% and 1% respectively.

<table>
<thead>
<tr>
<th>Panel A: Mutual fund mandate</th>
<th>( n )</th>
<th>( \beta )</th>
<th>t-stat</th>
<th>Adj ( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.01</td>
<td>-1.60</td>
<td>0.01</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>-0.02</td>
<td>-1.26</td>
<td>0.01</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>-0.04</td>
<td>-1.36</td>
<td>0.03</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>-0.11</td>
<td>-1.59</td>
<td>0.08</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: Pension fund mandate</th>
<th>( n )</th>
<th>( \beta )</th>
<th>t-stat</th>
<th>Adj ( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.02***</td>
<td>-3.08</td>
<td>0.03</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>-0.04**</td>
<td>-2.56</td>
<td>0.04</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>-0.08**</td>
<td>-2.40</td>
<td>0.06</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>-0.16**</td>
<td>-2.35</td>
<td>0.12</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.1 presents the regressions results of surplus returns for two different values of \( k \), at differing horizons onto the realised correlation between stock returns and yield changes. For the pension fund mandate, at all horizons, positive changes in stock-bond correlation are significantly related to negative changes in the surplus return. The magnitude of the \( \beta \) coefficient decreases with horizon, implying that the negative relationship between surplus returns and stock-bond correlation becomes greater at longer horizons - exactly the type of investment horizons that pension funds have. The increasing \( R^2 \)'s with horizon complements this intuition in that a larger proportion of the changes in surplus returns can be explained by changes in stock-bond correlation as the investment horizon increases. The insight from this is that an unexpected increase in stock-bond correlation can significantly increase the risk of a pension plans ALM mandate.

\(^{106}\)Note that I use the 10-year CMS rate and thus yield changes as opposed to returns to estimate realised correlation between stocks and bonds in order to follow “market convention” with respect to the current structure of existing stock-bond correlation sensitive products as seen from Figure 4.1.

\(^{107}\)Note also that derivative dealers tend to use the CMS rate instead of the Constant Maturity Treasury rate due to the flexibility and popularity of the swap market. Swap rates more closely reflect their cost of capital and so has created a central role for swap rates as market interest rate indicators. This is discussed further in Appendix F.
CHAPTER 4. PENSION FUNDS AND STOCK-BOND CORRELATION RISK: THE CASE FOR A CORRELATION SWAP

Although similar conclusions for the (long-only) mutual fund mandate can be drawn, these conclusions are not statistically significant over the sample period. Stock-bond correlation risk would be more significant for mutual funds and hedge funds that engage in long-short stock-bond strategies versus long-only strategies as highlighted in Buraschi et al. [2012].

Here I present a stylized example of how a pension fund can assess the stock-bond correlation risk it is exposed to. Traditionally the risk drivers of a pension fund’s surplus or funding ratio did not include stock-bond correlation risk as the correlation was typically assumed to be constant. Given the now widely accepted time-variation in stock-bond correlation together with the change in the regulatory landscape for DB pension plans, I thus look at how a pension fund can protect itself from stock-bond correlation risk. This involves specifically investigating ways to trade stock-bond correlation.

4.3 Trading stock-bond correlation risk

There are two principal vehicles for trading correlation risk in the literature: The dispersion trade and correlation swaps,¹⁰⁸ both of which have been investigated within stock markets but not so across stock and bond markets.

The dispersion trade consists of taking opposing positions in the volatility of an index and the volatilities of its constituents. Such a strategy is motivated in Driessen et al. [2009b]. It is typically executed by taking a short position in an index option together with long positions in options on the individual constituents of that index.¹⁰⁹ It is common for these index and individual options to be straddles, strangles or variance swaps rather than plain vanilla options in order to delta-hedge the strategy. However it is clear that even though the exposure to other risks (e.g. volatility (vega), convexity (gamma) and time (theta)) from the long leg of the strategy on individual options will potentially be offset by the short leg on the index option, the degree of offsetting will crucially depend on the weighting chosen for the constituent legs of the trade. This implies that the dispersion trade will not only have exposure to correlation but also to other factors.¹¹⁰

¹⁰⁸See the 2005 technical note by JPMorgan on “Correlation Vehicles”.
¹⁰⁹Such a trade generates a short correlation exposure. If one wanted to obtain a long position on correlation, they would simply perform the opposite trades.
¹¹⁰Although pure exposure to correlation is not achievable through such a static strategy, the other exposures can be hedged to some extent by altering the weighting and/or employing dynamic hedging strategies. Buraschi et al. [2009], Deng [2008] and Driessen et al. [2009b] explore some of the different weighting schemes and methodologies for managing the additional exposures within a dispersion trading strategy.
CHAPTER 4. PENSION FUNDS AND STOCK-BOND CORRELATION RISK: THE CASE FOR A CORRELATION SWAP

Correlation swaps on the other hand, allow for a pure exposure to correlation risk with no dynamic hedging/replication required. Such instruments are over-the-counter (OTC) contracts which pay the difference between (a pre-agreed estimate of) the realised correlation and a correlation swap rate (known as the strike of the swap contract) locked in at inception. Such contracts are often used to obtain a longer-dated exposure to correlation than can be obtained using the dispersion strategy and are more commonly found in equity markets.\footnote{Shedding the correlation ‘axe’, Risk magazine, May 2004 (http://www.risk.net/risk-magazine/feature/1506427/shedding-correlation-axe).} The fact that these swaps give investors a pure exposure to correlation based on a fixed analytical formula for the pay-out, without the need to hedge any other exposures, makes correlation swaps an attractive product for pension funds and other investors that wish to simply manage their correlation exposure without being exposed to additional risk factors.

Within a stock market context the most common vehicle for trading correlation risk has historically been the dispersion trade\footnote{For an example of work that uses equity correlation swap data, see Buraschi et al. [2012].} (Driessen et al. [2009b]). Thus although correlation swaps are the easiest and most direct method of trading correlation, they are less liquid than using the dispersion trade route and thus harder to mark to market.\footnote{For an example of work that uses equity correlation swap data, see Buraschi et al. [2012].} For the case of trading stock-bond correlation risk, the dispersion trading method is not available. The main problem is that there exists no combined stock-bond index which options are actively traded on. Therefore the most important leg of the dispersion trade which enables exposure to the correlation cannot be implemented. In order to hedge changes in stock-bond correlation, I thus examine the structuring of a stock-bond correlation swap.

4.3.1 Stock-bond correlation swap contract

Correlation swaps are essentially a collection of forward contracts in which one counterparty agrees to pay the other a notional amount times the difference between a fixed level and realised level of correlation. In this section I structure a stock-bond correlation swap in line with Figure 4.1 which gives investors direct exposure to the average correlation between stock returns and bond yield changes over a pre-agreed time horizon. For a pension fund such a swap will ensure that any further increases in correlation will mitigate the threat of correlation risk on the funding ratio of the pension plan.

As discussed in Buraschi et al. [2012] the payoff to the long side of the swap is formally defined as the difference between the realised correlation over the life of the contract and the swap strike:

\[
C_T = N(RC_{1,T} - K_C) \tag{4.6}
\]
where $C_T$ denotes the payoff of the swap at maturity $T$, $N$ is the contract notional which converts the correlation difference into a dollar payoff, $RC_{t,T}$ is the realised correlation between stock returns and bond yield changes (defined below) between time $t$ and $T$ and $K_{C,t}$ is the swap rate (strike of the swap) agreed upon at inception of the contract $t$. The swap strike is the level of correlation that can be bought or sold by the investor and is typically scaled by a factor of 100%, i.e. a correlation strike 0.31 will be quoted as 31% as in Figure 4.1. If therefore the realised correlation in December 2012 for this contract turns out to be 0.55 between S&P500 returns and 10-year CMS yield changes with a notional of $100,000, an investor who is long the correlation swap would make a profit of $(0.55 - 0.31) \times 100,000 = \$24,000$.

Denoting $CS_0$ as the market value of the swap at inception:\footnote{In this chapter I ignore the impact of collateralization on the swap and the costs associated with this. See Johannes and Sundaresan [2007] for more information on this.}

$$CS_0 = N\mathbb{E}^Q \left[ \exp(-\int_0^T r_s ds) (RC_{0,T} - K_C) \right]$$

(4.7)

$$= N\mathbb{E}^Q \left[ \exp(-\int_0^T r_s ds) RC_{0,T} \right] - NB(0,T)K_C$$

(4.8)

with the conditional expectation taken under a pricing measure $\mathbb{Q}$, which I take as given for now, I add more structure to this later; and $B(0,T)$ denoting the price at inception of a zero-coupon bond with maturity $T$. By construction, the swap has zero net market value at entry, so setting $CS_0 = 0$, the swap rate can be written as:

$$K_C = \mathbb{E}^Q_{\text{Term A}} \left[ RC_{0,T} \right] + B(0,T)^{-1} \mathbb{Cov}_Q \left( \exp(-\int_0^T r_s ds), RC_{0,T} \right)$$

\text{Term B}$$

(4.9)

Equation (4.9) states that ignoring stock-bond correlation risk aversion, the swap rate depends on the expectation of future realised correlation under the pricing measure and on the degree of covariation between the floating leg of the swap and the term structure of interest rates. The swap rate is commonly known as the implied correlation, implied referring to the market’s expectation of future realised correlation. This can be seen more directly when it is assumed that Term B above is equal to zero.\footnote{Such an assumption can be made when the floating leg of the swap is uncorrelated with bond market returns, then by no arbitrage the swap rate should equal the market’s expectation of the future value of the floating leg.} The swap rate would then be equal to Term A, the correlation implied by market expectation.\footnote{In other words, correlation implied by the expectation under the pricing measure.} Such an assumption would be inconsistent here as the floating leg of the swap is composed of bond market yield changes which is strongly (negatively) correlated to bond market returns. Going forward therefore, I
use the term implied correlation synonymously for swap rate and vice versa.

I note that correlation swaps give an observable correlation factor though somewhat indirectly. A direct indication of correlation would be to observe the price of a standardised (with respect to constituents and time to maturity) correlation swap, and to infer the history of correlation expectations implied in these prices. In an equity market context, such a method is used by Buraschi et al. [2012] to construct a time-series of implied correlation for the S&P 500. Unfortunately as for the stock-bond correlation swap, neither the standardization nor liquidity exists yet in these products to obtain an implied stock-bond correlation time-series. This means that although actual stock-bond correlation swaps allows for pure exposure to stock-bond correlation risk, they typically trade at customised transactions (OTC and in non-standardized terms). As such, they are priced at inception and valued by the participants of the swap.

4.3.2 Realised correlation

Typically the procedure for calculating realised correlation is specified in the derivative contract and includes details about the source and observation frequency of the underlying assets, any additional factors which need consideration in the calculation and the exact method of calculating the correlation. Although I obtained a quote for an actual stock-bond correlation swap contract, one of the details that I was not able to obtain was the exact method for calculating realised correlation. I therefore look to the equity correlation swap literature (Bossu [2006]) which highlights two main methods of calculating the realised correlation.

The first method being an equally weighted scheme on the underlying constituent assets in the basket; and the second method being a weighted scheme where the weights sum to 1. Index equity correlation swaps tend to use a weighted scheme, these are typically the index weights as of the trade date, i.e. the stock quantities that a portfolio manager would invest in to track one unit of the equity index. As there is no combined traded stock-bond index to refer to, I choose the former method to obtain a measure of realised stock-bond correlation. Observing the underlying price of the S&P500 index and the 10-year CMS yield at regular intervals (in this work I use a daily interval,\textsuperscript{116} i.e. \( S_0, S_1, \ldots, S_i, \ldots, S_n \) and \( Y_0, Y_1, \ldots, Y_i, \ldots, Y_n \), with \( t_{i+1} - t_i = \Delta t = \frac{T-t_i}{n} \) and \( t = t_0 < t_1 < \cdots < t_n = T \) being the partition of the time interval \([t, T]\) into \( n \) equal segments of \( \Delta t \). I can then define the log return on stocks and log

---

\textsuperscript{116} Broadie and Jain [2010] investigates the effect of discrete sampling frequencies on the calculation of fair variance swap strikes (comparing across different models of the underlying evolution of the asset price) and finds the effect of differing sampling frequencies is small.
yield changes on bonds respectively as:

\[ r^S_i = \log(S_t^i / S_{t-1}^i), \quad r^B_i = \log(Y_t^i / Y_{t-1}^i), \quad i = 1, \ldots, n. \]  \hspace{1cm} (4.10)

where \( n \) is the number of return observations. The equally weighted realised correlation between stock returns and bond yield changes from \( t \) to \( T \) can then be defined as:

\[ RC_{t,T} = \frac{\sum_{i=1}^n r^S_i r^B_i}{\sqrt{\left(\sum_{i=1}^n (r^S_i)^2\right) \left(\sum_{i=1}^n (r^B_i)^2\right)}} \]  \hspace{1cm} (4.11)

This definition of realised correlation differs from the sample correlation in that the sample average is not subtracted from each observation. At a daily frequency it is reasonable to posit that the sample average is approximately zero, thus the realised correlation is close to the sample correlation.

**Figure 4.3: Daily time series of rolling window correlation between S&P500 returns and 10-year CMS yield changes**

This figure reports the 1-year and 2-year rolling window correlation between daily returns on the S&P500 and daily yield changes of the 10-year Constant Maturity Swap (CMS) over the period 01/01/1990 to 30/12/2011. S&P500 data was obtained from DataStream and 10-year CMS data from the Federal Reserve System’s H.15 Release (http://www.federalreserve.gov/releases/h15/data.htm).

Figure 4.3 is an attempt using the method above to reconstruct the realised correlation chart of Figure 4.1. Although the dynamics of the charts are the same, the level of the correlation can be seen to be...
slightly different in various places. This highlights the importance of contract specifications and bid-ask spreads when attempting to replicate the realised correlation of an existing correlation swap contract.

### 4.3.3 Swap rate - Implied correlation

In an equity market context, Driessen et al. [2009a] show that it is possible to obtain the implied correlation of a stock index such as the S&P 500 using option prices on the index and on individual constituents of the index respectively. This is because option prices encode the risk-neutral expected variance of the return on the underlying asset over the life of the option. As index variance is a weighted average of individual variances and covariance terms, they are able to infer the option-implied average correlation between the index constituents over the option’s life.

Bossu [2005] observes that for the equity correlation swap market, the correlation swap has in fact tended to trade around the levels of correlation realised by the relevant basket of underlyings, as opposed to trading around implied levels of correlation inferred from index options (Driessen et al. [2009a]). This is because of the direct nature of the correlation exposure since index option implied correlation often contains other risk exposures. Although implied correlation can also be obtained from the quoted strike of correlation swaps, such quotes can differ substantially from index implied correlation levels as a result of market appetite and difficulties arising from hedging and replication.

As a combined stock-bond index does not exist, let alone actively traded options on such an index, the method of backing out the index implied stock-bond correlation is not available. Instead, the implied correlation for stock-bond correlation swaps must come directly from quoted correlation swaps. Table 4.2 presents an example of this for a term structure of stock-bond implied correlation, on just one trading day, 8th April 2011. Although the data is limited, by providing a term structure of implied correlation, the data shows further evidence of a market appetite for stock-bond correlation exposure at varying horizons.

### Table 4.2: Term structure of implied stock-bond correlation as on 08/04/2011

Table 4.2 reports the implied stock-bond correlation obtained from OTC correlation swaps on 08/04/2011. The investment bank who provided this data wishes to remain anonymous.

<table>
<thead>
<tr>
<th>Horizon (years)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Implied correlation</td>
<td>0.25</td>
<td>0.3</td>
<td>0.31</td>
<td>0.32</td>
<td>0.33</td>
</tr>
</tbody>
</table>

117 The investment bank which provided these quotes wishes to remain anonymous.
CHAPTER 4. PENSION FUNDS AND STOCK-BOND CORRELATION RISK: THE CASE FOR A CORRELATION SWAP

Figure 4.4 plots the data from Table 4.2 together with the historical realised correlation with respect to the horizon length. For example at time $t$, in this case 8th April 2011, for the 3 year estimate, I calculate the historical realised correlation using the past $(3 \times 252)$ 756 trading days based on daily data. As mentioned above such a method is found to closely match the implied correlation within equity correlation swaps and so I use the same method to compare with the few data points I have for stock-bond correlation swap quotes.

Figure 4.4: Term structure of implied correlation between S&P500 returns and 10-year CMS yield changes as on 08/04/2011

Implied correlation quote data is obtained directly from stock-bond correlation swaps. The investment bank who provides these quotes wishes to remain anonymous. The historical realised correlation is from estimating the historical correlation based on a window of daily data whose length is the same as the respective horizon.

Although the estimates are of a similar magnitude to those from the quotes (0.2-0.4), the near term estimates are higher than those quoted from the swap market. One potential explanation for this pattern is the turbulence in equity markets caused by the uncertainty in future world growth combined with the earthquake and resulting tsunami in Japan on the 11th of March 2011.\footnote{See the article a \emph{A flight to safety} on 15th March 2011 in the Financial Times (FT).}

Figure 4.5 presents a timeseries of 1-year implied correlation quotes obtained directly from stock-bond
Figure 4.5: Time series of 1-year implied correlation between S&P500 returns and 10-year CMS yield changes

Time series of 1-year implied correlation (bid) quotes obtained directly from stock-bond correlation swaps at a quarterly frequency from Q4-2011 to Q4-2012. The grey region between the upper and lower bounds of the bid quotes reflect the range of swap rates from this contract over this time period. These quotes were obtained from an investment bank which wishes to remain anonymous.
correlation swaps. The grey region between the upper and lower bounds of the bid quotes reflect the range of swap rates from this contract over this time period. This is due to the nature of OTC swap contracts in that they typically trade at customised transactions. Thus the range of swap rates could reflect differing risk aversion towards stock-bond correlation risk from institutions that wish to enter such a swap contract. I also note that the range seems to have some time variation in both size and level, which likely reflects both changes in risk aversion of the investors together with changes in the interacting dynamics of the S&P500 index and 10-year CMS index. As the implied correlation should capture the changing price of insurance against unexpected changes in the correlation. This raises the question of what the fair price is for protection from the effect of stock-bond correlation risk? Such a question is an important concern for pension funds who are most likely to require such protection.

4.4 Empirical support for stochastic correlation

Given the inherent difficulty in obtaining stock-bond implied correlation data together with the illiquidity of quotes that were able to be obtained, I look to infer the empirical behaviour of stock-bond correlation risk. Implied correlation should naturally be a function of the implied volatilities of the S&P500 and 10-year CMS yield. Given the availability of data to obtain implied volatility on stocks and bonds, I therefore analyse the behaviour of these implied volatilities as an imperfect proxy for stock-bond correlation risk. Such an approach can be justified based on the findings of several papers (Connolly et al. [2005], Chordia et al. [2005]) on the volatility linkages between stock and bond markets. Such linkages are either due to commonality in the information set that simultaneously affect expectations in both markets, i.e. macroeconomic variables (Campbell and Ammer [1993]); or from information spillover caused by cross-market hedging, where a shock to one market is transferred to another market via trading activity (Fleming et al. [1998]). Based on this intuition I therefore posit that factors which affect stock and bond volatility also affect stock-bond correlation due to common information, information spillover and/or hedging effects.

4.4.1 Implied volatility

Following standard practice I use options to back out a proxy for expected volatility under the risk-neutral measure. I use the model free-methodology proposed by Britten-Jones and Neuberger [2000] and Jiang and Tian [2005] which calculates a daily implied volatility from the entire set of option prices on

---

119 For instance from the contact at the investment bank wish provided the quotes, the lower (level) bid range in Q4-2011 was justified as due to the strong supply of stock-bond correlation as a result of the second wave of quantitative easing (QE2) in which the Federal Reserve bought $600 billion of Treasury securities by the end of the second quarter of 2011.
CHAPTER 4. PENSION FUNDS AND STOCK-BOND CORRELATION RISK: THE CASE FOR A CORRELATION SWAP

The methodology is established in the equity markets to produce the “new” VIX (Carr and Wu [2006]) - an implied volatility index for the S&P 500. Such an index captures the implied volatility of a synthetically created ATM option with a constant maturity of 30 days.

I apply the same methodology to construct both the S&P 500 and the 10-year Treasury bond implied volatility indices but for a constant maturity of 365 days. For brevity I outline the methodology in detail in Appendix G.

The data consists of daily market closing prices for the S&P 500 and 10-year Treasury note options traded on the CBOE. I obtain this from OptionMetrics and the sample period is from 4th January 1996 to 31st December 2009, which is a total of 3651 daily observations.

Figure 4.6 plots the implied volatility for the S&P 500 and 10-year Treasury yield. These represent the risk-neutral expectation of the (annualised) volatility on the underlying stock index and interest rate series over the next 365-days. Changes in the implied volatility theoretically affect changes in the implied correlation and it is clear from the figure that there are pronounced changes in both implied volatility series over the sample period. The series are similar in magnitude up until the start of the 2007-2010 financial crisis. The sharp increase in the level and subsequent variation of the implied volatility of the 10-year Treasury yield is likely due to the monetary and fiscal policy decisions during the financial crisis resulting in (repeated) intervention in the Treasury bond markets by the U.S. Federal Reserve.

Table 4.3 reports descriptive statistics for the implied volatility time series. In addition to the observations made from Figure 4.6 here I observe that both series are highly persistent, reflecting the persistence of the underlying stock and bond volatility processes, and both show positive skewness and excess kurtosis.

Co-movement of the implied volatilities will affect the implied correlation and it is clear from Figure 4.6 that there are times when the implied volatility on both stocks and bonds increase simultaneously (1999, 2003, 2007-2008). Indeed, Fleming et al. [1998] investigate volatility spillover between stock and bond

\[\text{120}\text{Such a method avoids use of a model such as the Black and Scholes [1973] formula for equity options and the Black [1976] formula for Treasury options, both of which assume that volatility is constant which is inconsistent with the objective of analysing the risk of volatility changes.}\]

\[\text{121}\text{I use options on the 10-year Treasury note yield (ticker: TNX) as this is the data that is available. Given the very high correlation between 10-year Constant Maturity Treasury and 10-year Constant Maturity Swap yields as shown in Appendix F, I assume the conclusions made from the implied volatility of the 10-year Constant Maturity Treasury also apply for the 10-year CMS.}\]
Figure 4.6: Daily time series of 1-year implied volatility of the S&P500 and 10-year Treasury note

This is obtained using the methodology outlined in Appendix G using daily market closing prices for the S&P500 and 10-year Treasury note options traded on the Chicago Board Options Exchange (CBOE) from OptionMetrics. The sample period is from 4th January 1996 to 31st December 2009, which is a total of 3651 daily observations.
CHAPTER 4. PENSION FUNDS AND STOCK-BOND CORRELATION RISK: THE CASE FOR A CORRELATION SWAP

Table 4.3: Descriptive statistics of the implied volatility time series for S&P 500 and 10-year Treasury note

Table 4.3 reports the summary statistics of the level and changes of the implied \( (IV) \) volatility time series for both stocks and bonds. The sample period is from 4th January 1996 to 31st December 2009 based on daily data which excludes holidays and weekends, which gives 3651 observations in total. Note that \( \rho(1) \) is the coefficient of an AR(1) model with a constant.

### Panel A: Levels of the volatility time series

<table>
<thead>
<tr>
<th></th>
<th>( IV_{S&amp;P500} )</th>
<th>( IV_{10YTBond} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.2057</td>
<td>0.2483</td>
</tr>
<tr>
<td>Median</td>
<td>0.2058</td>
<td>0.2061</td>
</tr>
<tr>
<td>Min</td>
<td>0.1159</td>
<td>0.1111</td>
</tr>
<tr>
<td>Max</td>
<td>0.4842</td>
<td>0.9657</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.0580</td>
<td>0.1197</td>
</tr>
<tr>
<td>Skewness</td>
<td>1.1077</td>
<td>2.2882</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>5.0208</td>
<td>9.8854</td>
</tr>
<tr>
<td>( \rho(1) )</td>
<td>0.9958</td>
<td>0.9903</td>
</tr>
</tbody>
</table>

### Panel B: Changes of the volatility time series

<table>
<thead>
<tr>
<th></th>
<th>( IV_{S&amp;P500} )</th>
<th>( IV_{10YTBond} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.0000</td>
<td>0.0001</td>
</tr>
<tr>
<td>Median</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>Min</td>
<td>-0.0424</td>
<td>-0.4263</td>
</tr>
<tr>
<td>Max</td>
<td>0.0440</td>
<td>0.3755</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.0052</td>
<td>0.0171</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.4295</td>
<td>-1.9048</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>13.8318</td>
<td>253.4940</td>
</tr>
<tr>
<td>( \rho(1) )</td>
<td>-0.0724</td>
<td>-0.1299</td>
</tr>
</tbody>
</table>
Figure 4.7: Daily time series of rolling window correlation between (1-year) implied volatilities of S&P500 and 10-year Treasury note

Correlation between implied volatilities of S&P500 and 10-year Treasury note using a rolling window of 252 trading days (1 year).
markets based on historical volatility, here I show that this may also be the case for implied volatilities. The unconditional correlation between the two time series is 0.66 but this hides the substantial time variation between the implied volatilities as can be seen in Figure 4.7.

I next try to establish if the implied volatilities contain important unspanned components. This is of theoretical and practical concern as it may help to obtain some intuition on whether the underlying securities can be used to hedge the correlation risk between stock and bond markets. If I find that there is at least one state variable that drives innovations in the implied volatilities, assuming that this state variable also drives innovations in implied correlation (Connolly et al. [2005], Chordia et al. [2005], Fleming et al. [1998]), but which does not affect innovations in stock and bond markets, this suggests the presence an incomplete market. Thus, unspanned volatility (and correlation) raises the possibility that a derivative on stock-bond correlation would constitute an important component of an optimal stock-bond portfolio.

4.4.2 Unspanned volatility risk

Within equity markets it is common to assume that volatility risk cannot be hedged by trading in the underlying stock alone (Heston [1993], Buraschi and Jackwerth [2001]). Stock options are therefore not redundant securities. This has also been shown to be the case for fixed income markets (Collin-Dufresne and Goldstein [2002], Li and Zhao [2006] and Andersen and Benzoni [2010]): There is at least one state variable which drives innovations in interest rate derivatives but does not affect innovations in the underlying bond market. Thus bonds do not span interest rate derivatives and so cannot hedge interest rate volatility risk.

I investigate this jointly for stocks and bonds using the implied volatility time series. In order to factorise the information contained in bond prices, I perform principal components analysis to obtain the orthogonal factors that describe changes in the term structure of interest rates. Table 4.4 reports that the first four principal components can explain 99.9% of the variation in bond yields. Inspecting the eigenvectors one can interpret the first three principal components as the level, slope and curvature factors (Litterman and Scheinkman [1991]).

I regress the implied volatility of stocks and bonds on to factors driving the yield curve and directly on stock returns. Table 4.5 reports the findings. Panel A shows that only 46% of the variation in the

---

122 This also follows naturally from the definition of correlation as covariance divided by the respective volatilities.
Table 4.4: Principal components analysis on the term structure of interest rates

Table 4.4 reports the principal components analysis of the levels and changes in zero-coupon bond yields for ten different maturities (1-, 2-, 3-, 4-, 5-, 6-, 7-, 8-, 9-, 10-year). The sample period is from 4th January 1996 to 31st December 2009 based on daily data which excludes holidays and weekends, which gives 3651 observations in total. The numbers reported are the percentages of the variations of the levels and changes of bond yields explained by their first four principal components.

<table>
<thead>
<tr>
<th>Principal Component</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level</td>
<td>95.97%</td>
<td>3.91%</td>
<td>0.11%</td>
<td>0.01%</td>
</tr>
<tr>
<td>Change</td>
<td>93.50%</td>
<td>5.53%</td>
<td>0.78%</td>
<td>0.17%</td>
</tr>
</tbody>
</table>

The level of implied stock volatility is able to be explained by factors driving the yield curve and by S&P500 returns. Interestingly, returns to the S&P 500 significantly affect the implied volatility level together with the slope and curvature factors from the yield curve. Evidence of the leverage effect within implied stock volatility is apparent from the negative coefficient on S&P500 returns.

As for implied bond volatility, Panel A shows that only 74% of the variation in the level is able to be explained by the term structure factors that are able to explain up to 99.99% variation in yields. Such a result is also found by Heidari and Wu [2001] who are able to explain around 60% of the cross-sectional variability in bond option implied volatility from the level, slope and curvature factors. Such a finding implies that interest rate volatility is not fully explained by the information contained in the yield curve. Similarly to Collin-Dufresne and Goldstein [2002] I observe that volatility is negatively related to the level (holding slope and curvature constant) - when interest rates decrease, implied volatility increases, implying a similar “leverage effect” in bond markets.

I also investigate the determinants of changes in implied volatility. Panel B shows that for bonds, almost none of the variation in changes to implied volatility can be traced back to the variation in changes to term structure factors or stock returns, empirically implying the presence of stochastic volatility in bond markets. As for the variation of implied stock volatility, this is significantly related to changes in stock returns explaining up to 45% of the variation in changes to implied stock volatility. However a larger portion of the variation is unexplained by the term structure factors and stock returns, thus motivating stochastic volatility in stock returns. Such findings confirm those in the literature.

With empirical evidence on unspanned stochasticity of implied volatility in stock and bond markets, together with the relationship between volatility and correlation, I posit unspanned stochasticity of implied correlation between stock and bond markets, thus highlighting a market incompleteness. If the
Table 4.5: Evidence on the spanning of volatility risk in stock and bond markets

Table 4.5 reports OLS estimates for regressions of the level and changes in implied volatility of stocks and bonds on the information contained in prices/yields for the S&P500 and fixed income markets. Panels A and B use factors obtained from principal components analysis on the term structure of interest rates (see Table 4.4). The sample period is from 04/01/1996 to 31/12/2009 based on daily data which excludes holidays and weekends, which gives 3651 observations in total (this is 3650 observations for the regression involving changes in the implied volatility). The coefficient t-ratios are in brackets below the point estimates and are corrected for autocorrelation using Newey-West with a lag order of 252. *, ** and *** denote statistical significance at 10%, 5% and 1% respectively.

Panel A: \[ IV_t = \alpha + \beta_1 PC1_t + \beta_2 PC2_t + \beta_3 PC3_t + \beta_4 PC4_t + \beta_5 p^S&P500_t + \epsilon_t \]

<table>
<thead>
<tr>
<th></th>
<th>( \alpha )</th>
<th>( \beta_1 )</th>
<th>( \beta_2 )</th>
<th>( \beta_3 )</th>
<th>( \beta_4 )</th>
<th>( \beta_5 )</th>
<th>( R^2 )</th>
<th>( AdjR^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( IV_t^{S&amp;P500} )</td>
<td>0.21***</td>
<td>-0.01***</td>
<td>0.02*</td>
<td>0.26***</td>
<td>-0.01</td>
<td>-0.19***</td>
<td>0.46</td>
<td>0.46</td>
</tr>
<tr>
<td></td>
<td>(23.43)</td>
<td>(-2.43)</td>
<td>(1.71)</td>
<td>(5.96)</td>
<td>(-0.03)</td>
<td>(-5.23)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( IV_t^{10Y Bonds} )</td>
<td>0.25***</td>
<td>-0.03***</td>
<td>0.05***</td>
<td>0.23***</td>
<td>0.38</td>
<td>0.17***</td>
<td>0.74</td>
<td>0.74</td>
</tr>
<tr>
<td></td>
<td>(25.90)</td>
<td>(-6.57)</td>
<td>(3.84)</td>
<td>(3.82)</td>
<td>(1.34)</td>
<td>(3.80)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Panel B: \[ \Delta IV_t = \alpha + \beta_1 \Delta PC1_t + \beta_2 \Delta PC2_t + \beta_3 \Delta PC3_t + \beta_4 \Delta PC4_t + \beta_5 \Delta p^S&P500_t + \epsilon_t \]

<table>
<thead>
<tr>
<th></th>
<th>( \alpha )</th>
<th>( \beta_1 )</th>
<th>( \beta_2 )</th>
<th>( \beta_3 )</th>
<th>( \beta_4 )</th>
<th>( \beta_5 )</th>
<th>( R^2 )</th>
<th>( AdjR^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta IV_t^{S&amp;P500} )</td>
<td>0.00</td>
<td>-0.00***</td>
<td>0.00***</td>
<td>0.00*</td>
<td>-0.00</td>
<td>-0.17***</td>
<td>0.45</td>
<td>0.45</td>
</tr>
<tr>
<td></td>
<td>(0.50)</td>
<td>(-4.40)</td>
<td>(2.81)</td>
<td>(1.74)</td>
<td>(-1.64)</td>
<td>(-11.65)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Delta IV_t^{10Y Bonds} )</td>
<td>0.00</td>
<td>-0.00**</td>
<td>-0.00</td>
<td>-0.00</td>
<td>-0.00</td>
<td>0.002</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>(0.67)</td>
<td>(-2.56)</td>
<td>(-1.12)</td>
<td>(-0.47)</td>
<td>(-1.00)</td>
<td>(1.09)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
CHAPTER 4. PENSION FUNDS AND STOCK-BOND CORRELATION RISK: THE CASE FOR A CORRELATION SWAP

market were complete, any contingent claim could be exactly replicated through a self-financing trading strategy. This means that by no arbitrage, the price of that contingent claim would be equal to the initial wealth required to form the replicating portfolio. In such a case there exists an unique equivalent martingale measure from the fundamental theorem of asset pricing. The contingent claim could therefore also be expressed as the expected discounted payoff, where the expectation is taken under the uniquely defined martingale measure, known as the pricing measure.\(^{123}\)

An incomplete market means that exact replication is impossible implying that no unique martingale measure exists resulting in infinitely many different potential prices which would all still be consistent with no-arbitrage. In this case as Davis [1997] points out: “no preference independent pricing formula is possible”. Pricing a stock-bond correlation swap is therefore not a straight forward exercise of risk-neutral pricing. Instead I look to utility indifference pricing to establish the fair price of a stock-bond correlation swap.

4.5 Utility indifference pricing: Stock-bond correlation swap

Section 4.2 highlights that stock-bond correlation risk is a relevant concern for pension funds. I have also shown that such a risk is a source of market incompleteness when considering multiple asset classes. In order to address the question of the fair price of a stock-bond correlation swap, I therefore look to a utility based asset pricing model in the context of a pension fund within an ALM framework. I note that by using such a setting, I focus on the buyer of the correlation swap and assume a readily available and willing seller of the swap.\(^{124}\)

Given the difficulties discussed in Section 4.3 for obtaining implied stock-bond correlation, I present a simple yet parsimonious framework to model stock-bond (stochastic) correlation and to price a correlation swap (thus obtain implied correlation quotes for such correlation swaps) based on the preferences of a pension fund who is particularly incentivized to purchase a stock-bond correlation swap in order to manage the funding ratio risk. This is, to the best of my knowledge, the first formal pricing model for a stock-bond correlation swap.

I acknowledge that in this utility indifference setting the price of the correlation swap will essentially

\(^{123}\)This is because such a measure would change the stochastic correlation process (from which \(RC_{t,T}\) would be sampled from) into a martingale which can then be discounted using the chosen numeraire. If this numeraire is the money market account (risk-free rate) then the pricing measure is also called the risk-neutral measure.

\(^{124}\)I make such an assumption to narrow the focus of the work for the case of pension funds.
CHAPTER 4. PENSION FUNDS AND STOCK-BOND CORRELATION RISK: THE CASE FOR A CORRELATION SWAP

depend on the current state of the balance sheet of the pension fund and its risk aversion. If the correlation swap were able to be replicated then an objective market price would be obtainable, however this is not possible and therefore I need to assume a representative balance sheet (preferences) and risk aversion within the utility indifference framework. Anecdotal evidence suggests that in practice the current method used to price a stock-bond correlation swap is to look at the realised correlation and add a spread - to adjust for the risk aversion of the market-maker, which is not balance sheet specific. However, this simple method to price a swap is due to the lack of a formalised alternative. Adopting an utility indifference method is an attempt to present a formalised alternative in the context of an incomplete market as investigated above.

Given this first formalised attempt I make several simplifications: I focus on pricing a 1-year maturity stock-bond correlation with the net cashflow only exchanged at maturity. This is achieved within a utility-indifference pricing setting by forming a static portfolio today which is held for 1-year, which is the maturity of the swap, without any rebalancing of the portfolio. This also allows for the mark to market/model considerations of holding a correlation swap over multiple cashflow payment dates to be pacified. Secondly, I opt to directly model the Constant Maturity Treasury yield as opposed to creating a complete term structure model, as the purpose here is not to predict future yields, but to investigate the optimal behaviour of a pension fund manager with and without access to a correlation swap, and the price it would pay for the correlation swap under a reasonable description of a time-varying investment opportunity set.

4.5.1 Stock-bond pricing model

To price a stock-bond correlation swap a joint stock-bond pricing model which incorporates stochastic volatility and correlation is required. For such a model the trade off between complexity and tractability needs to be considered. Complexity refers to the ability of the model to capture the characteristics of the asset price dynamics, whereas tractability is associated with the number of model parameters, calibration and also on numerical procedures required to eventually price the stock-bond correlation swap.

I therefore adopt a Wishart process as in Fonseca et al. [2007], who were the first to present a tractable model that allows for non-trivial stochastic volatility of asset returns and stochastic correlation of cross-
sectional asset returns. Such processes were originally introduced by Bru [1991] and fall within the class of affine factor models as originally defined in Duffie and Kan [1996]. The Wishart process is a direct multivariate extension of the CIR process Cox et al. [1985] which is used with the Heston [1993] stochastic volatility model. Given the extension from the Heston model, the multiple asset setup is also analytically tractable and by providing explicit parametric restrictions it can be assured that the variance-covariance matrix remains positive definite, a key property needed when modeling stochastic dynamic correlations.

Denoting $S_t$ as the level of the S&P500 and $Y_t$ as the 10-year Constant Maturity Treasury yield, I assume a continuous-time frictionless economy on a finite time horizon $[0, T]$ that the joint-dynamics of the asset values $X_t = [S_t, Y_t]'$ in a standard probability space $(\Omega, \mathcal{F}, \mathbb{P})$ under the historical measure $\mathbb{P}$ is given by the bivariate stochastic differential equation:

$$dX_t = \text{diag}[X_t] \left( \mu dt + \sqrt{\Sigma_t} dZ_t \right) \quad (4.12)$$

where $\mu$ is a constant vector of mean changes, $Z_t$ is a standard two-dimensional Brownian motion and $\sqrt{\Sigma_t}$ is the positive square root of the conditional covariance matrix of returns. Thus the evolution of stock price/yield change is conditionally Gaussian whilst I assume that the instantaneous variance-covariance matrix follows a Wishart process:

$$d\Sigma_t = (\Omega\Omega' + M\Sigma_t + \Sigma_t M') dt + \sqrt{\Sigma_t} dW_t Q + Q'(dW_t)' \sqrt{\Sigma_t} \quad (4.13)$$

where $\Omega, M, Q$ are two-dimensional matrices with $\Omega$ invertible and $W_t$ is a $2 \times 2$ standard Brownian motion. Matrix $M$ is assumed to be negative semi-definite to ensure stationarity and the mean reversion of $\Sigma$ which is typically observed in the data for volatility and correlation. As noted in Buraschi et al. [2010], if $\Omega' \gg Q'Q$ then $\Sigma$ is a well defined covariance matrix process. In addition to this, $\Omega$ in Equation (4.13) should satisfy:

$$\Omega \Omega' = \beta Q'Q, \quad \beta > 1 \quad (4.14)$$

to ensure $\Sigma_t$ follows a Wishart distribution (Bru [1991]). $Q$ is the volatility of volatility matrix and determines the co-volatility features of the stochastic variance-covariance matrix.

Finally, to allow for the possibility of volatility and correlation “leverage” effects (Black [1976] and Roll [1988] respectively), which has important implications within optimal portfolio choice, I allow for a
non-zero instantaneous correlation between innovations in returns/yield changes and innovations in the instantaneous variance-covariance process. Specifically, the standard Brownian motion $Z_t$ in Equation (4.12) is defined as:

$$dZ_t = dW_t \rho + \sqrt{(1 - \rho') \rho} dB_t$$  \hspace{1cm} (4.15)

where $B_t$ is a standard two-dimensional Brownian motion independent of $W_t$, $\rho$ is a vector of correlation parameters where $\rho_i \in [-1, 1]$ for $i = 1, 2$ and $\rho' \rho \leq 1$. These correlation parameters allow for a flexible description of the leverage in volatilities and correlation of the return/yield change process in Equation (4.12). It is thus reasonable to expect that these correlations are needed in order to fully capture the role of dynamic stochastic correlation between the stock and bond markets.

4.5.2 Stock-bond correlation swap pricing model

Since only 2 risky assets are available for investment and with the covariance matrix dynamics of Equation (4.13) depending on 3 independent Brownian motions, the market is said to be incomplete since there is no specification of the model that allows the number of risky securities to match the number of independent risks. So although variance swaps readily exist on the S&P500 (Leippold et al. [2007]) and on Treasury notes (Mueller et al. [2012]), a stock-bond correlation swap which is not readily available constitutes an important security to span the state space generated by the variances and covariances and thus would complete the market.

I therefore approach the pricing a stock-bond correlation swap from a utility indifference perspective (Henderson and Hobson [2004]). Such a framework considers the utility of a particular agent, in this case a pension fund, who is incentivised to buy a stock-bond correlation swap to increase the utility of its final wealth. First I define the price of a stock-bond correlation swap under the real-world measure $P$ as:

$$CS_{t,T} = \mathbb{E}_t^P \left[ dQ \frac{dP}{dQ} \exp(- \int_t^T r_s ds) N(RC_{t,T} - K_C) \right]$$  \hspace{1cm} (4.16)

where $dQ \frac{dP}{dQ}$ denotes the Radon-Nikodym (RN) derivative. The swap rate or strike $K_C$ is set so that the present value of the swap at the beginning of the contract, time $t = 0$, is zero:

$$CS_{0,T} = \mathbb{E}_0^P \left[ dQ \frac{dP}{dQ} \exp(- \int_0^T r_s ds) N(RC_{0,T} - K_C) \right] = 0$$  \hspace{1cm} (4.17)

Using the utility indifference framework I define the “indifference price” of the swap as the swap rate or
"implied correlation" at $t = 0$ for the right to receive the claim at maturity $t = T$, such that the pension fund is no worse off in expected utility terms than it would have been without the claim. Taking into account all the possible trading strategies that a pension fund can adopt ($\omega \in \Omega$), I define $u(R_{SP,T}^{\text{without}})$ as the maximum utility that can be achieved at time $T$ without the derivative:

$$u(R_{SP,T}^{\text{without}}) = \sup_{\omega \in \Omega} \mathbb{E}_0 \left( u \left( \int_0^T \omega \left( \frac{dS}{S_0} \frac{dP}{P_0} \right) - k \int_0^T R_{L,s} ds \right) \right)$$ (4.18)

where $\left( \frac{dS}{S_0}, \frac{dP}{P_0} \right)$ is a vector of (stochastic) returns on stocks and bonds. I also define the maximum utility when adding a stock-bond correlation swap to the asset portfolio:

$$u(R_{SP,T}^{\text{with}}) = \sup_{\omega \in \Omega} \mathbb{E}_0 \left( u \left( \int_0^T \omega \left( \frac{dS}{S_0} \frac{dP}{P_0} \frac{dCS}{CS_1} \right) - k \int_0^T R_{L,s} ds \right) \right)$$ (4.19)

where the returns on the correlation swap are calculated based on an investment of $1$ in the swap which pays $RC_{0,T} - K_C$ at time $T$. In concentrating on pricing a 1-year stock-bond correlation swap I simplify the problem in so much that the set of possible trading strategies is restricted to a static portfolio formed at time 0 and held until time $T$ or 1-year, the maturity of the option. Although such an assumption may seem restrictive, pension fund’s typically rebalance their asset portfolios relatively infrequently, thus such an assumption is closely connected with industry practice. Thus, the pension fund optimises its asset portfolio at time 0 in order to maximise its expected surplus return at time $T$, with no trading of any assets in between these times. I thus write the surplus return with and without the correlation swap at time $T$ as:

$$R_{SP,T}^{\text{without}} = R_{f,T} + \omega' \left( \begin{pmatrix} R_{S,T} \\ R_{B,T} \end{pmatrix} - R_{f,T} \hat{1}_n \right) - kR_{L,T}$$ (4.20)

$$R_{SP,T}^{\text{with}} = R_{f,T} + \omega' \left( \begin{pmatrix} R_{S,T} \\ R_{B,T} \\ R_{CS,T} \end{pmatrix} - R_{f,T} \hat{1}_n \right) - kR_{L,T}$$ (4.21)

where $\hat{1}_n$ is an $n$-dimensional vector of ones, $R_{f,T}, R_{S,T}, R_{B,T}, R_{L,T}$ and $R_{CS,T}$ are the returns on a risk-free 1-year Treasury bill, the S&P500 index, the 10-year Constant Maturity Treasury index, liabilities
CHAPTER 4. PENSION FUNDS AND STOCK-BOND CORRELATION RISK: THE CASE FOR A CORRELATION SWAP

and on the correlation swap respectively as defined by:

\[ R_{f,T} = \exp(y_{1,0}) - 1 \]  
\[ R_{S,T} = \frac{S_T - S_0}{S_0} \]  
\[ R_{B,T} = \frac{\exp(-9y_{10,T})}{\exp(-10y_{10,0})} - 1 \]

where \( y_{m,t} \) is the log yield of the \( m \)-year maturity bond at time \( t \). To obtain the return on liabilities I use a fixed duration of 30 years with the (modelled) 10-year Constant Maturity Treasury yield instead of additionally modeling the 30-year Constant Maturity Treasury yield. This is done for simplicity as the dynamics of the 10-year and 30-year Constant Maturity Treasury yields are very similar, the correlation being 0.98 in the sample period used in Section 4.2. The return on liabilities is thus:

\[ R_{L,T} = \frac{\exp(-29y_{10,T})}{\exp(-30y_{10,0})} - 1 \]

In addition to the comments made in Section 4.2 on the assumptions of modeling the liabilities of a pension fund, I offer several further comments here. Firstly, the liabilities in Equation (4.25) follow a stationary stochastic process. This could be easily extended to include a deterministic time trend to capture demographic factors specific to the pension fund. I assume that the liabilities are driven by only one of the interest rate risk factors: changes in the level of long-term interest rates. Litterman and Scheinkman [1991] show that as duration lengthens, level effects become more pronounced; thus the dominant source of interest rate risk for (long-duration) pension liabilities is the impact of changes in the level of interest rates. This is particularly important as big gains in hedging are likely to come from managing the impact of changes in the level of rates.

As for the returns on the correlation swap, following Buraschi et al. [2012] I define this as the returns on a $1 capital allocated to the swap which pays \( (RC_{0,T} - K_C) \) at time \( T \) based on a notional of $1/correlation point:

\[ R_{CS,T} = \frac{1 \times (RC_{0,T} - K_C)}{1} \]

where \( K_C \) is the implied correlation at \( t = 0 \) when buying the stock-bond correlation swap. \( RC_{0,T} \) is calculated as in Equation (4.11) over the period \([0, T]\) where in line with Section 4.3.2 I use the 10-year Constant Maturity Treasury yield.

127I note that the results do not vastly change if either the 30-year Constant Maturity Treasury yield is used instead of the 10-year Constant Maturity Treasury yield, or if the 30-year Constant Maturity Treasury yield is modelled in addition to the 10-year yield within Equations (4.12) and (4.13). This highlights the robust nature of the method to price the stock-bond correlation swap.
CMS rate as the bond leg of the swap, which is modelled as:

\[ Y_{10,t}^{CMS} = Y_{10,t} + \Delta_t \]  (4.27)

Appendix F compares the dynamics of the Constant Maturity Treasury and Constant Maturity Swap yield highlighting that the difference between them reflects the cost of funding together with liquidity, taxation and regulation issues (Hull et al. [2004], Duffee [1996], Reinhart and Sack [2002]). As will be qualified below, I assume \( \Delta_t \) to be constant.

Finally, I obtain the “implied correlation” from equating Equations (4.21) and (4.20):

\[
\max_\omega E_0 [u(R_{SP,T}^{\text{without}})] = \max_\omega E_0 [u(R_{SP,T}^{\text{with}})]
\]  (4.28)

Assuming that the pension fund has a mean-variance preference on its surplus return at time \( T \):

\[
E_0 [u(R_{SP,T})] = \max_\omega \ E_0 [R_{SP,T}] - \frac{1}{2} \gamma \text{Var}_0 [R_{SP,T}]
\text{ s.t. } \omega \hat{1}_n = 1
\]  (4.29)

where \( \gamma \) denotes the risk aversion of the pension fund. I impose that the weights on the assets that the pension fund can trade sum to one. The optimal portfolio weights as derived in Appendix F can then be expressed as:

\[
\omega = \frac{1}{\gamma} \Sigma_A^{-1} (\mu_{A,T} - \lambda \hat{1}_n + \gamma k \Sigma_L)
\]  (4.30)

where \( \Sigma_A \) is a \( n \)-dimensional covariance matrix of asset returns, \( \sigma_L^2 \) is the variance of liability returns, \( \Sigma_L \) is a \( n \)-dimensional vector of covariances between the return on assets and liabilities, \( \mu_{A,T} \) is a \( n \)-dimensional vector of expected returns on the assets and \( \hat{1} \) is a \( n \)-dimensional vector of ones. The zero-beta return is denoted as \( \lambda \) and is defined as:

\[
\lambda = \frac{\hat{1}_n \Sigma_A^{-1} \mu_{A,T} + \gamma k \hat{1}_n \Sigma_A^{-1} \Sigma_L - \gamma}{\hat{1}_n \Sigma_A^{-1} \hat{1}_n}
\]  (4.31)

The first two terms in Equation (4.30) describe the asset-only optimal portfolio weights, whilst the last term describes the liability hedging demand which arises from the covariance between the returns on the assets and liabilities. If the return on assets and liabilities were independent (thus uncorrelated), the optimal portfolio would be the weights for the asset-only portfolio. Otherwise, a positive correlation between an asset and the liability return will increase the weight of that asset relative to the asset-only
case to decrease the volatility of the surplus return and thus minimise mismatch risk.

4.5.3 Monte carlo simulation

I simulate the dynamics of Equations (4.12) and (4.13) in a Monte Carlo framework using an Euler discretisation method. This is outlined below for the time step $t \to \Delta t$ where $\Delta t$ represents one day as such observations are required to compute the realised correlation leg of the correlation swap as shown in Equation (4.11).

4.5.3.1 The variance process

For the variance process I use the full truncation scheme in order to ensure that the approximated process $\Sigma_t$ remains positive-definite:

$$\Sigma_{t+\Delta t} = \Sigma_t + (\beta Q' Q + M(\Sigma_t)^+ + (\Sigma_t)^+ M')\Delta t + \sqrt{(\Sigma_t)^+ Q G \sqrt{\Delta t} + Q' G' \sqrt{(\Sigma_t)^+ \Delta t}} \, (4.32)$$

where $G$ is a two-dimensional matrix of independent Gaussian random variables and the positive part of the symmetric matrix $(\Sigma)^+$ is defined as:

**Definition** If $\Sigma$ belongs to the set of real symmetric square matrices, spectral theorem allows us to write:

$$\Sigma = V \times \text{diag}(\lambda_1, ..., \lambda_n) \times V'$$

$$(\Sigma)^+ = V \times \text{diag}(\lambda_1^+, ..., \lambda_n^+) \times V'$$

where $(\lambda_n)^+$ are the absolute values of the eigenvalues of the process to ensure consistency of the variance-covariance matrix.

4.5.3.2 The stock price/bond yield process

For the S&P500 price and 10-year Constant Maturity Treasury yield I obtain:

$$\log X_{t+\Delta t} = \log X_t + \left(\mu - \frac{1}{4} \text{diag}(\Sigma_{t+\Delta t} + \Sigma_t)\right) \Delta t$$

$$+ \sqrt{\Sigma_t \Delta t G \rho + \frac{\Delta t}{2} \sqrt{(\Sigma_{t+\Delta t} + \Sigma_t) Z \sqrt{(1 - \rho^2)}}} \, (4.33)$$

where $G$ is the same as defined above and $Z$ is a two-dimensional vector of independent Gaussian random variables.
variables. The complete derivation is in Appendix I.

4.5.4 Parameter values

To demonstrate the simulation results for the stock-bond correlation swap pricing model, parameters for Equations (4.33) and (4.32) are chosen based on empirical distributions in the data and intuition from the existent literature on Wishart processes. As I require the model to simulate daily observations for calculation of the realised leg of the correlation swap, I calibrate the model to daily data.\footnote{The difficulty of estimating a discretised version of a multivariate stochastic volatility model such as the Wishart model I have employed is outlined in Da Fonseca et al. [2007]. Also Buraschi et al. [2010] find that calibrating their Wishart model to daily data using Generalised Method of Moments (GMM) leads to a rejection of their model. They find that jumps in the time series of returns and realised second moments are responsible for this rejection. I leave such an investigation of the estimation of the model and possible extension of the model to a matrix valued affine jump diffusion model for future research.}

As a first step I obtain a daily (conditional) time series of realised second moments for the log price/yield processes using the Dynamic Conditional Correlation (DCC) model of Engle [2002].\footnote{See Appendix E for details on estimation of the DCC model.}

As the intent is to price a 1-year stock-bond correlation swap, I use one years worth of daily data of the S&P500 and 10-year Constant Maturity Treasury from 8th April 2010 to 8th April 2011 (262 observations). Table 4.6 presents the descriptive statistics of returns/yield changes, volatilities and correlations for the sample data.

The daily unconditional (annualised) mean of S&P500 returns is 13% while 10-year Constant Maturity Treasury yield changes is -2%. Interestingly, the 10-year Constant Maturity Treasury yield has a higher volatility and volatility of volatility than the S&P500 (this is probably due to interventions in bond markets during the year by the Federal Reserve), with the unconditional sample correlation between stock returns and bond yield changes at 43% with a standard deviation of 21%.

Table 4.7 presents the parameter values used in the simulation. Here I closely follow the intuition in Da Fonseca et al. [2007]. \( \mu \) is the annualised mean of log returns/yield changes on the S&P500 and 10-year Constant Maturity Treasury bond respectively over the sample period. \( \rho \) is set to the unconditional correlation between log returns/yield changes and the respective conditional variance for each asset over the sample period. The volatility leverage effect seems to be more apparent in stock markets than in bond markets as can be seen in Figure 4.8.

The initial variance-covariance matrix \( \Sigma_0 \) is set as the (annualised) conditional variance-covariance matrix as of 8th April 2011 while \( \beta \) is chosen as in Buraschi et al. [2010] to ensure \( \Sigma_t \) follows a Wishart
CHAPTER 4. PENSION FUNDS AND STOCK-BOND CORRELATION RISK: THE CASE FOR A CORRELATION SWAP

Table 4.6: Descriptive statistics of S&P500 returns and 10-year Constant Maturity Treasury yield changes

Table 4.6 reports the unconditional (annualised) moments of S&P500 returns and 10-year Constant Maturity Treasury (CMT) yield changes/returns over the period 08/04/2010 to 08/04/2011 based on daily observations. The unconditional volatility of volatility, mean of correlation and volatility of correlation are estimated as the unconditional volatility/correlation of the conditional volatility/correlation estimated using Engle [2002] Dynamic Conditional Correlation (DCC) model. Note that \( \rho(1) \) is the coefficient of an AR(1) model with a constant.

<table>
<thead>
<tr>
<th>Panel A: Returns and volatility</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>S&amp;P500</td>
<td>10year CMT</td>
</tr>
<tr>
<td>Yield changes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.127</td>
<td>-0.019</td>
</tr>
<tr>
<td>Volatility</td>
<td>0.175</td>
<td>0.343</td>
</tr>
<tr>
<td>Volatility of volatility</td>
<td>0.054</td>
<td>0.066</td>
</tr>
<tr>
<td>Panel B: Correlation</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>S&amp;P500 returns &amp; 10year CMT yield changes</td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.428</td>
<td></td>
</tr>
<tr>
<td>Volatility</td>
<td>0.212</td>
<td></td>
</tr>
<tr>
<td>( \rho(1) )</td>
<td>0.994</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.7: Parameter values for stock-bond pricing model

Table 4.7 reports the parameter matrices \( M, Q, \Sigma_0, \Sigma_\infty \), vectors \( \mu, \rho, X_0 \) and scalars \( \beta, r \) for the stock-bond pricing model of Equations (4.33) and (4.32). Parameters are chosen based on intuition from historical data and the existent literature on Wishart processes. For the historical data, I use time series of S&P500 returns, 10-year Constant Maturity Treasury bond yield changes and their (conditional) realised variance-covariances obtained using the DCC model of Engle [2002], over the period 08/04/2010 to 08/04/2011 based on daily observations.

<table>
<thead>
<tr>
<th>( \mu )</th>
<th>( \rho )</th>
<th>( M )</th>
<th>( Q )</th>
<th>( \beta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.127</td>
<td>-0.038</td>
<td>-3.44</td>
<td>1.17</td>
<td>0.1235</td>
</tr>
<tr>
<td>-0.019</td>
<td>0.107</td>
<td>0.22</td>
<td>-2.87</td>
<td>0.0053</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( X_0 )</th>
<th>( \Sigma_0 )</th>
<th>( \Sigma_\infty )</th>
<th>( r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1328.17</td>
<td>0.0156</td>
<td>0.0153</td>
<td>0.0307</td>
</tr>
</tbody>
</table>
| 3.68      | 0.0153         | 0.0567            | 0.0250 | 0.0026
CHAPTER 4. PENSION FUNDS AND STOCK-BOND CORRELATION RISK: THE CASE FOR A CORRELATION SWAP

Figure 4.8: Volatility leverage effect within the S&P500 index and 10-year Constant Maturity Treasury yield

Panel A plots the leverage effect on the S&P500 whilst Panel B plots the effect for the 10-year Constant Maturity Treasury yield over the period 08/04/2010 to 08/04/2011 based on daily observations. The conditional volatility is estimated using Engle’s DCC model.
CHAPTER 4. PENSION FUNDS AND STOCK-BOND CORRELATION RISK: THE CASE FOR A CORRELATION SWAP

distribution. The parameter $M$ is hard to interpret directly given the multiplicity of roles in the overall model. I therefore choose an $M$ which is consistent with daily data from Da Fonseca et al. [2007]. The $Q$ matrix follows by inverting a relation which links $Q$ to $M, \Sigma_\infty$ and $\beta$:

$$Q'Q = -\frac{1}{\beta}(M\Sigma_\infty + \Sigma_\infty M')$$

(4.34)

where $\Sigma_\infty$ is the annualised mean of the conditional variance-covariance over the sample period. As highlighted in Da Fonseca et al. [2007], obtaining $Q$ in this way ensures stationarity of the correlation process. Lastly $r$ is set to the yield on the 1-year Treasury bill as on 8th April 2011.

Figure 4.9: Yield spread of 10-year CMS over 10-year Constant Maturity Treasury

Panel A plots the 10-year Constant Maturity Treasury yield vs the 10-year Constant Maturity Swap yield and Panel B plots the spread of the Constant Maturity Swap yield over the Constant Maturity Treasury yield over the period 08/04/2010 to 08/04/2011 based on daily observations.

As for the relationship between the 10-year Constant Maturity Treasury and 10-year Constant Maturity Swap in Equation (4.27), I assume above the $\Delta_t$ to be constant. From Figure 4.9 such an assumption does not seem unreasonable over the sample period given the small variation of the yield difference

---

130 Da Fonseca et al. [2007] comment that $\beta$ also impacts the mean reversion and variance of the correlation process: the higher $\beta$ is, the lower the persistence and variance of the correlation process. Setting $\beta = 10$ seems reasonable given the persistence of the conditional correlation process as show in Table 4.6. Also, Da Fonseca et al. [2007] find a $\beta$ of approximately 10 when estimating a similar model across international stock markets.
between the 10-year CMS and 10-year Constant Maturity Treasury rate. This is confirmed by Table 4.8 which shows that the unconditional volatility of the yield difference over the sample period is 6%. I therefore calibrate $\Delta_t$ to be the unconditional mean of the yield difference, 0.05%.

Table 4.8: Unconditional moments of the yield difference between 10-year Constant Maturity Treasury and 10-year Constant Maturity Swap

Table 4.8 reports the unconditional moments of the difference between the 10-year Constant Maturity Swap and 10-year Constant Maturity Treasury yield over the period 08/04/2010 to 08/04/2011 based on daily observations.

<table>
<thead>
<tr>
<th></th>
<th>$Y_{CMS}^{10,t} - Y_{10,t}^{10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.0005</td>
</tr>
<tr>
<td>Volatility</td>
<td>0.062</td>
</tr>
</tbody>
</table>

Finally, after estimating the stock-bond pricing model, I use the Monte Carlo simulation processes combined with the methodology outlined above to obtain the “indifference implied correlation price” at time $t = 0$ that a pension fund would accept for the right to receive the stock-bond correlation swap claim at time $t = T$.

4.6 Results: Implied stock-bond correlation

I run 100,000 simulations with $\Delta t = 1/252$. Figure 4.10 plots the obtained implied correlation quote versus $k$, importance of liabilities parameter and $\gamma$, risk aversion of the pension fund.

From this figure, it is intuitively clear that as risk aversion ($\gamma$) increases, the implied correlation bid price increases demonstrating that a pension fund would be willing to enter the stock-bond correlation swap contract at a higher swap strike, which is effectively a higher “price”. This is because positive correlation shocks will affect the pension fund’s utility of terminal surplus return more for highly risk-averse managers, implying that such managers would be willing to enter a stock-bond correlation swap contract at a higher strike in order to hedge the stock-bond correlation risk.

It is also clear that when $\gamma$ is large (in this case greater than 10), as $k \to 1$, i.e. the liabilities become more important, the pension fund manager would be willing to pay a higher swap strike. Note that as I impose the constraint that the weights on the assets must sum to one, when $k = 0$, which corresponds to the asset-only case, the implied correlation quote does not increase as $\gamma$ increases due
CHAPTER 4. PENSION FUNDS AND STOCK-BOND CORRELATION RISK: THE CASE FOR A CORRELATION SWAP

Figure 4.10: Stock-bond implied correlation quote

This figure plots the implied correlation quote for a stock-bond correlation swap versus $k$, the importance of liabilities parameter and $\gamma$, risk aversion of the pension fund. The “price” of the correlation swap is obtained using a Monte Carlo simulation of the stock-bond pricing model, Equations (4.33) and (4.32) using the parameters in Table 4.7.
CHAPTER 4. PENSION FUNDS AND STOCK-BOND CORRELATION RISK: THE CASE FOR A CORRELATION SWAP

to the mutual fund theorem - the composition of the risky portfolio does not change with \( \gamma \), only the amount that one invests in the risky portfolio. This is the same across different \( \gamma \)'s for the asset-only case given the constraint in that no wealth is invested in the risk-free asset.

The model-implied correlation bid quotes should be interpreted in the context of the assumptions made. Therefore, although these quotes and those obtained directly from correlation swaps cannot be strictly compared, I do so here for illustrative purposes in order to highlight areas of concern when developing a formal pricing model to match market prices, even though such market prices are based on a thin amount of trading. Noting that the model-implied correlation for a 1-year swap on the 8th of April 2011 for \( k = 1 \) and \( \gamma = 30 \) to be 0.42, Table 4.2 and Figure 4.4 show this to be 0.25, obtained directly from quotes on stock-bond correlation swaps. According to the model-implied quote, this implies that either the pension fund would be willing to pay more to enter the correlation swap contract, or that the contract quote reflects that the pension fund manager is either more risk-seeking in behaviour and/or has different preferences to those described by the mean-variance framework used here.

The difference between the model-implied quote and that given in Table 4.2 highlights the need for a formal, transparent pricing methodology. Examining the time series of 1-year implied correlation quotes in Figure 4.5 obtained from a different source to those quotes in Table 4.2, the range of bid quotes at the beginning of April 2011 was 0.4-0.6. This is much higher than the specific quote of 0.25 and the model-implied quote lies at the bottom of this range. In such a case it would appear that a spread is being charged on the correlation swap strike above it’s model-implied “fair value”.

As already noted, such observations cannot be taken too literally given the informal calibration performed. Indeed Figure 4.11 shows the sensitivity of the model-implied correlation quotes to the parameter \( \beta \). Da Fonseca et al. [2007] state that this parameter in particular impacts the mean reversion and variance of the correlation process with a higher \( \beta \) decreasing the persistence and variance of the correlation process. Figure 4.11 compares the model-implied quotes for a \( \beta = 2 \) and \( \beta = 100 \). It is clear that a lower \( \beta \) increases the range of possible model-implied correlation swap quotes which follows from the wider range of realised correlation values possible given the higher variance of the correlation process. Indeed, with a \( \beta = 2 \), the model-implied correlation quotes range from 0.26 to 0.40. This highlights the importance of the estimation methodology of the model and in particular the parameter \( \beta^{131} \). Such concerns are important for assessing if model-implied quotes indicate a positive premium on

---

131 As already noted, given the difficulties associated with estimation, this is left for future research.
CHAPTER 4. PENSION FUNDS AND STOCK-BOND CORRELATION RISK: THE CASE FOR A CORRELATION SWAP

Figure 4.11: Stock-bond implied correlation quote sensitivity

This figure plots the implied correlation quote versus $k$, the importance of liabilities parameter and $\gamma$, risk aversion of the pension fund for two different $\beta$ values. $\beta$ impacts the mean reversion and variance of the correlation process in Equation (4.13). The strike of the correlation swap is obtained using a Monte Carlo simulation of the stock-bond pricing model, Equations (4.33) and (4.32) using the parameters in Table 4.7 (except for the $\beta$ value).
CHAPTER 4. PENSION FUNDS AND STOCK-BOND CORRELATION RISK: THE CASE FOR A CORRELATION SWAP

quoted implied correlation strikes from swaps.

Table 4.9: Portfolio weights for a one-period mean-variance ALM problem without and with access to a stock-bond correlation swap

Table 4.9 reports the asset portfolio weights as calculated by Equation (4.30) for differing values of $k$, the importance of liabilities and $\gamma$, the risk-aversion of the pension fund. Panel A reports the weights for when investable assets consist only of stocks and bonds. Panel B for reports the weights when the investable assets also include the stock-bond correlation swap.

<table>
<thead>
<tr>
<th>$k$</th>
<th>$\gamma = 5$</th>
<th>$\gamma = 30$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>0.5</td>
</tr>
<tr>
<td>Panel A: Without stock-bond correlation swap</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stocks</td>
<td>0.415</td>
<td>0.371</td>
</tr>
<tr>
<td>Bonds</td>
<td>0.585</td>
<td>0.629</td>
</tr>
<tr>
<td>Panel B: With stock-bond correlation swap</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stocks</td>
<td>0.401</td>
<td>0.360</td>
</tr>
<tr>
<td>Bonds</td>
<td>0.568</td>
<td>0.612</td>
</tr>
<tr>
<td>Correlation swap</td>
<td>0.031</td>
<td>0.028</td>
</tr>
</tbody>
</table>

Table 4.9 reports the portfolio weights on investable assets that the pension fund manager has available for ALM when these assets also include the stock-bond correlation swap. These weights are obtained using Equation (4.30). Panel A reports the weights for the case without the correlation swap. These weights are intuitively clear. For the asset-only case ($k = 0$), the risky portfolio is comprised of 59% bonds and 41% stocks. Such an observation follows from the sample period descriptive statistics in Table 4.6 to which I calibrate the model - although bonds have a lower return than stocks, they also have a much lower volatility. Now as liabilities become more important, the weight placed on bonds increases, which is due to bonds matching the risk profile of the liabilities more closely than that of stocks as shown by Nijman and Swinkels [2003]. This is more pronounced when $\gamma$ is larger.

With the addition of the stock-bond correlation swap to the asset portfolio, Panel B of Table 4.9 shows that a percentage of the assets are also invested into the correlation swap. This occurs even for the asset-only case of $k = 0$. Counter-intuitively as $k \to 1$, the weight on the stock-bond correlation swap actually decreases slightly. This is due to a similar observation as from the weights in Panel A: As the importance of liabilities increases, mean-variance preferences increases the weight on long-term bonds as they have a similar risk profile to the liabilities, this naturally reduces the exposure to stock-bond correlation risk given that the weight is taken from stocks to invest more into bonds, causing the need
for the correlation swap to decrease, and hence the weight on it to decrease.

Such findings highlights the need for a formal pricing method as I have presented above to justify and obtain implied correlation swap quotes in a fair and transparent manner. Although stock-bond correlation swaps are currently OTC and therefore typically trade at customised transactions, the pricing methodology presented above allows for certain customisation based around the risk aversion of the pension fund \( \gamma \) and liability importance \( k \). Given the lack of a stock-bond index, this simple yet intuitive pricing methodology that I have developed here allows for a pension fund to assess its exposure to stock-bond correlation risk and the fair price that it should pay in order to hedge against this correlation risk using a stock-bond correlation swap.

### 4.7 Concluding remarks

This chapter investigates the role of stock-bond correlation risk within a pension fund context of ALM. Motivating the importance of managing the mismatch between assets and liabilities, I define the surplus return for a stylised pension fund and empirically demonstrate the role of changes in the correlation of stock returns and bond yield changes on changes in surplus return: positive shocks to correlation are empirically related to negative shocks in surplus return, the (absolute) magnitude of which increases with longer horizons. Thus for a pension fund asset portfolio constructed based on the expected correlation between stocks and bonds within an ALM setting, unexpected increases in the correlation can significantly increase the risk of an underfunded pension plan.

A stock-bond correlation swap would provide a hedge against such a risk for a pension fund. I show that such a swap is currently the only method of trading stock-bond correlation given the lack of a combined stock-bond index on which options are actively traded on. Specific demand for a stock-bond correlation swap can be seen from the rarity of implied correlation quotes obtained (with some difficulty) from two investment banks. These quotes are shown in Table 4.2 and Figures 4.1, 4.4 and 4.5. However, the nature of the quotes reflect that such stock-bond correlation swaps typically trade at customised transactions. This raises the question of what is the fair “price” of a stock-bond correlation swap? Such a question determines the desirability of adding this swap to the pension fund’s asset portfolio.

I therefore develop the first, to the best of my knowledge, formal but simple pricing model for a stock-bond correlation swap which is both transparent and intuitive. I propose a stock-bond pricing model directly on the price of the S&amp;P500 and the yield of the 10-year Constant Maturity Treasury bond for...
simplicity. Given the source of market incompleteness from the lack of tradable assets on stock-bond correlation I therefore approach the pricing of a stock-bond correlation swap using a utility indifference approach. This is where the (specified) expected utility of the pension fund is matched between the case of an asset portfolio with and without a correlation swap in order to back out an indifference price that the pension fund manager would be willing to pay to hold the correlation swap within her portfolio.

I find that the model-implied quotes fall within the range of quotes obtained from traded correlation swaps quotes obtained from various sources. It also naturally captures the intuitive behaviour of a pension fund through the specified preferences. For instance, a pension fund with a higher risk aversion parameter $\gamma$ would be willing to enter the correlation swap with a higher strike in order to hedge stock-bond correlation risk within its ALM mandate. This is also the case when the liabilities become more importance within the ALM mandate framework that I use. I also highlight the importance of estimation of the model and in particular the parameter $\beta$, which describes the persistence and variance of the correlation process within the model, for the model-implied quote that is obtained.

Finally, I posit that in order for the stock-bond correlation swap market to really take off, it is necessary to attract new investors. An index-based stock-bond correlation swap which has a structure that is more familiar to capital market investors would facilitate this. This chapter highlights the need for the development of such a stock-bond (correlation) index in the future.
Chapter 5
Conclusions and future work

In this thesis I focus on three aspects of time-varying stock-bond correlation: contemporaneous time variation, predicting the time variation and hedging the risk of unexpected changes. Here I present suggestions for future work on each chapter and broadly discuss the implications of the respective chapter conclusions for the overall conclusion of the thesis.

In Chapter 2 I conduct an empirical investigation into the contemporaneous time variation of stock-bond correlation. Using the framework of a Campbell and Shiller [1988] decomposition I use survey forecasts for the macroeconomic components to back out a time series of unexpected values for these components. Using a conditional (co)variance model I then obtain (co)variance time series for the news components allowing for the economic mechanisms on the variation of realised stock-bond correlation to be investigated. An extension to the research would be to develop a different proxy for the (co)variance of the news time series by taking the cross-sectional (co)variance of the unexpected values for each of the analysts’ forecasts for each respective economic component. The explanatory power that such a measure would have in comparison to the measures developed in Chapter 2 within the regressions that I perform would be very interesting, especially for the periods of negative correlation. Also, given the inherent noise in the estimation of the realised volatility and correlation of stock and bond returns, I suggest the use of intra-day data in order to obtain a more accurate estimate of realised second moments.\footnote{The issue of estimation error is well acknowledged given that correlation is directly unobservable and differing methods for estimating the statistic are numerous and continuing to develop (Ait-Sahalia et al. [2011], Brandt and Diebold [2006]).}

Chapter 3 investigates the out-of-sample benefits of the information contained within analyst forecasts for predicting the second moments of stock and bond returns within an asset allocation setting. There are several possible extensions for future research. The first being a direct extension of this work using forecast data from the Survey of Professional Forecasters (SPF), who provide analyst forecasts on similar macro variables to those employed in Chapter 3 based at a quarterly frequency. Secondly,
CHAPTER 5. CONCLUSIONS AND FUTURE WORK

the battery of models in the literature developed specifically to predict the time variation in the second moments of stock and bond returns (Baele et al. [2010], Hasseltoft [2009]) should be implemented in this portfolio allocation context to compare it against both portfolios formed using historical (realised) and analyst forecast data.

I note that regime switching models have gained some popularity but with mixed success in the stock-bond literature: Guidolin and Timmermann [2006] use such tools to document that the monthly correlations between UK stock and bond returns are positive and significant in normal and bull states, while they are negative in bear states. In an asset allocation context this is studied by Ang and Bekaert [2002]. One suggestion for future work would be to estimate the transition probabilities of such regime switching models using analyst forecast data - given the potential out-of-sample benefits displayed.

Lastly, I have focused on the use of analyst forecasts for stock-bond portfolio choice in a single-period framework. It would be interesting to extend this to a multi-period setting to consider how analyst forecasts can be utilized for the optimal stock-bond portfolio under both time-varying expected returns (Brennan et al. [1997]) and time-varying second moments (Buraschi et al. [2010]). This is of particular importance as Campbell et al. [2002] has shown that persistent changes in expected returns and second moments of those returns can lead long-term investors to optimally follow differing investment policies compared to the myopic portfolio formed by short-term investors.

This is intuitively clear as a risk averse long-term investor wants to invest in assets that help hedge against time variation in the investment opportunity set. Faced with the risk of correlation changing between stocks and bonds, long-term investors may therefore tilt their portfolios towards bonds, which appear to become safer (less correlated with stock returns) at the peak of business cycles (Campbell et al. [2009b]). Thus forecasts from analysts may help, out-of-sample, to predict the state of the business cycle; thereby providing long-term investors with a better information set on when to tilt their portfolios to hedge stock-bond correlation risk that may be inherent within their portfolios.

With the importance of hedging stock-bond correlation risk in mind, Chapter 4 directly focuses on the role of this risk for a particular type of long-term investor in the context of structuring and pricing a swap contract to hedge against the exposure. In developing a stock-bond correlation swap pricing model, simplifications were made which leaves many avenues for future work. The first extension would

---

133 These states are exogenously determined rather than driven by some economic variables
be to allow for pricing correlation swaps with longer maturities. Such an extension would have to take into account the ability of a pension fund to re-balance its asset portfolio before maturity of the swap and thus requires the use of dynamic programming methods. The mark to market/model process also needs to be taken into account in this case. Indeed it would be interesting to compare the re-pricing of the 1-year correlation swap allowing for monthly re-balancing of the portfolio to the implied correlation quotes obtained in Chapter 4.

Mean-variance preferences were used to keep the framework as intuitive as possible and also as such preferences are used in an insurance risk pricing context. Whether such preferences are adequate to describe the behaviour of pension funds is an interesting question in itself. Another extension would be to investigate the role of differing preferences on the model-implied correlation quote. Also, pension plans are typically bound by ex-ante risk constraints such as Value-at-Risk, short-sale and maximum weight constraints and also ex-post risk constraints such as the requirement to obtain additional contributions from the plan sponsor when the plan becomes underfunded. Such realistic constraints for a pension fund will affect the expected utility and thus the model-implied correlation quote.

Including the role of inflation risk on pension liabilities would be another interesting extension. The impact on the model-implied correlation quote would be of great interest given the importance of inflation (shocks) in the time variation of stock-bond correlation [David and Veronesi [2009], Campbell et al. [2009b] et al.). Also, as previously commented, an efficient estimation methodology for the developed model needs to be investigated in order to usefully implement this model and obtain model-implied quotes which can directly be compared with implied correlation quotes from OTC correlation swaps.

This thesis investigates the time variation in stock-bond correlation. As discussed throughout, the literature has been mixed on the macroeconomic drivers in the time variation, especially for the occurrence of negative stock-bond correlation. The (conceptual) contribution of Chapters 2 and 3 in providing a better understanding of these macroeconomic forces in the time series patterns of stock-bond correlation is clear. While the contribution of 4 in highlighting the role of changing stock-bond correlation on surplus returns of pension funds is also clear. An interesting next step would be to investigate the role of stock-bond correlation risk on expected (surplus) returns. Such an investigation would raise the possibility of stock-bond correlation risk premia. Indeed Chapter 2 highlights the role of macroeconomic risks in affecting the time variation; and it is this interaction between the time variation and macroeconomic risks that the discussion of a stock-bond correlation risk premium should be framed (Cochrane 2001).
CHAPTER 5. CONCLUSIONS AND FUTURE WORK

As the availability of data is a limiting factor in empirically investigating this, the discussion of a stock-bond correlation risk premium needs to be theoretical. The need for a stock-bond structural model which endogenously allows for a stock-bond correlation risk premium is thus clear and the next natural step in developing the research from this thesis. This would provide the framework for a discussion on the cost of insuring against bad stock-bond correlation states. Chapter 4 shows that such a state of the world to be of significant concern to a particular type of asset manager. Thus a structural discussion on the possible existence of a stock-bond correlation risk premium will have broad but important implications for stock-bond asset managers. It is my hope that the insights from this thesis aid such a suggestion for future work on the time variation of stock-bond correlation.

\footnote{This being until the demand and liquidity of stock-bond correlation swaps or another structured product with exposure to stock-bond correlation allows for an implied stock-bond correlation to be consistently obtained. This is fully discussed in Chapter 3.}

\footnote{I note however an analogy to that discussed in Buraschi et al. \citeyear{Buraschi2012} in that the risk of (unexpected) changes in stock-bond correlation can affect all types of stock-bond portfolios. This can come from combined long/short positions in stocks and bonds together with positions in options on stocks and bonds.}
Appendix

A: Theoretical return expressions

Surprises in stock returns

Using the well-known identity expression of stock returns:

\[ R^S_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t} \]  \hspace{1cm} (A.1)

where \( R^S_{t+1} \) is the holding period return for stocks from period \( t \) to \( t + 1 \), \( P_{t+1} \) is the price and \( D_{t+1} \) is the dividend paid, all at time \( t + 1 \). Rearranging the equation above, taking logs and then log-linearizing I can approximately write the continuously compounded returns as:

\[ r^S_{t+1} \approx k + \rho (p_{t+1} - d_{t+1}) + (d_{t+1} - d_t) - (p_t - d_t) \]  \hspace{1cm} (A.2)

\[ \approx k + \rho (p_{t+1} - d_{t+1}) + \Delta d_{t+1} - (p_t - d_t) \]  \hspace{1cm} (A.3)

where lowercase letters refer to logs, \( k \) is a constant of linearization, \( \rho \) is a discount factor which is slightly below 1 and \( \Delta \) represents a one-period backward difference. Log-linearization is a type of series expansion where the log of the sum is approximately a weighted average of the log of the components of the sum. The approximation is good if I assume that the log dividend-price ratio does not follow an explosive process so that I can impose the terminal condition \( \lim_{j \to \infty} \rho^j (p_{t+j} - d_{t+j}) = 0 \). Rearranging the equation in terms of \((p_t - d_t)\) to solve the equation forward and taking expectations at time \( t \) I obtain:

\[ p_t - d_t \approx k \frac{1}{1 - \rho} + \mathbb{E}_t \sum_{j=1}^\infty \rho^{j-1} (\Delta d_{t+j} - r^S_{t+j}) \]  \hspace{1cm} (A.4)

Note that \( \mathbb{E}_t \) denotes the expectation formed at the end of period \( t \) conditional on the information set.
APPENDIX

that includes the history of stock prices and dividends up to period \( t \). I get returns from price changes or dividends. Using the above equation I can therefore relate changes in returns to changes in rational expectations of future dividend growth and future stock returns. Ignoring the constant, thus treating the variables as deviations from the mean, I define in general the unexpected return as:

\[
r_{t+1} - E_t[r_{t+1}] = E_{t+1}[p_{t+1}] - E_t[p_{t+1}]
\]

This is simply just taking expectations with respect to the information set in the previous period so as to obtain the one-step ahead predictor of prices. Substituting into the equation above I obtain:

\[
r^S_{t+1} - E_t[r^S_{t+1}] = (E_{t+1} - E_t) \left[ \sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j} - \sum_{j=2}^{\infty} \rho^{j-1} r^S_{t+j} \right]
\]

Decomposing stock returns into excess stock returns and the short-term interest rate I can write:

\[
r^S_{t+1} = e^S_{t+1} + r_{t+1}
\]

where \( e^S_{t+1} \) is the log real excess stock returns and \( r_{t+1} \) is the log real short term interest rate. Combining with Equation (A.6) leads to:

\[
e^S_{t+1} - E_t[e^S_{t+1}] = (E_{t+1} - E_t) \left[ \sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j} - \sum_{j=1}^{\infty} \rho^{j-1} r_{t+j} - \sum_{j=2}^{\infty} \rho^{j-1} e^S_{t+j} \right]
\]

**Surprises in Bond Returns**

Defining the log price of an nominal N-period zero-coupon bond at time \( t \) as \( p^N_t \), in a similar spirit to that above, the log holding period return on such a bond, which is held from \( t \) to \( t + 1 \) can be written as:

\[
r^{(N)}_{t+1} = p^{(N)}_{t+1} - p^N_t
\]

This can be thought as a difference equation in the log bond price. Solving this equation forward to the maturity date of the bond, where at maturity the bond price is at par (which I set equal to 1 so that the log of the price is zero, i.e. \( p^{(0)}_{t+N} = 0 \), I obtain:
This equation holds ex-post but can also hold ex-ante. Taking expectations at time $t$:

$$p_t^{(N)} = -E_t \sum_{j=1}^{N} r_t^{(N+1-j)}$$  \hspace{1cm} (A.12)

where I obtain an expression that states that the log price of an N-period zero-coupon bond at time $t$ is a sum of the expected future returns. Substituting this into Equation (A.5) I can write:

$$r_t^{(N)} + 1 - E_t[r_t^{(N)}] = -\sum_{j=2}^{N} (E_{t+1} - E_t)r_t^{(N+1-j)}$$  \hspace{1cm} (A.13)

This equation states that as the nominal returns to a zero-coupon bond are known over the life of the bond, any unexpected nominal return gains that I see today must be offset by decreases in expected future nominal returns, and vice versa. As I shall be comparing the returns on stocks to those on bonds, I need to work with real returns. As above, I further decompose bond returns into excess bond returns and a short-term interest rate. I therefore write:

$$e_t^B = r_t^{(N)} - \pi_t^{t+1} - r_t^{t+1}$$  \hspace{1cm} (A.14)

where $\pi_t^{t+1}$ is the inflation rate at time $t + 1$ and as above, $r_t^{t+1}$ is the log real short term interest rate. Substituting this into Equation (A.13) I obtain:

$$e_t^B - E_t[e_t^B] = (E_{t+1} - E_t) \left[ -\sum_{j=1}^{N} \pi_t^{t+j} - \sum_{j=1}^{N} r_t^{t+j} - \sum_{j=2}^{N} e_t^{t+j} \right]$$  \hspace{1cm} (A.15)

**B: Data construction for Chapter 2**

The data runs from July 1984 to December 2009 since this is the period in which I have forecast data from the BlueChip Economic Indicators (BCEI) database. As one of the innovations to the literature, I use a monthly frequency in the paper, whilst most other empirical work studying stock-bond correlation use quarterly data.
APPENDIX

Stock data

Stock market index: I use the CRSP value-weighted market index comprising of the stocks traded in
the NYSE, AMEX and NASDAQ as the market portfolio. Excess returns are defined as returns over
the past 12 months less the rolling 3-month Treasury Bill yield over the same holding period.

Earnings: These are from the National Income and Product Account (NIPA) tables (Table 1.12, line
13). I use corporate earnings before-tax with IVA and CCadj. This data is quarterly so to construct
monthly earnings data I use a linear interpolation scheme.

Bond data

Bond market index: I use the monthly 10-year zero-coupon bond yield from the daily off-the-run Treas-
sury yield curves constructed by Gurkaynak et al. [2007] which is available from the Fed web page. If
I let $e^{(n)}_{t+1}$ denote the continuously compounded log excess return on an $n$ year discount bond in period
$t + 1$. Bond excess returns are then defined as $e^{(n)}_{t+1} = r^{(n)}_{t+1} - y^{(1)}_{t}$, where $r^{(n)}_{t+1}$ is the log holding period
return from buying an $n$ year bond at time $t$ and selling it at $t + 1$ as an $n - 1$ year bond. $y^{(1)}_{t}$ is the
log yield on a rolling 3-month Treasury bill held for one year.

Cochrane and Piazzesi [2005] factor: Following their procedure, I construct 1 through 5 year forward
rates from the nominal bond yields, as well as 2 through 5 year excess returns. I then regress the average
of the 2 through 5 year excess return on a constant, on the one year yield and the 2 through 5 year
forward rates. The CP factor is then the fitted value of this regression.

Survey forecasts of the decomposed components

The BlueChip Economic Indicators (BCEI) database provides survey forecasts on an individual level
of various macroeconomic variables. Two kinds of monthly forecasts are obtained from participants,
one for the current calendar year and one for the next calendar year. For instance, in January 2009,
participants provide a 12-month forecast for the value of the macro-variable at the end of the current
year 2009, and a 24-month forecast for the variable at the end of the next calendar year 2010. Thus,
for February 2009, the forecast horizon for 2009 is now only 11 months while for 2010 it is 23 months,
and so on. In order to obtain a constant and consistent time-series of expected values, the forecast for
the current year and the next year are weighted together to create a rolling constant horizon 12-month
### APPENDIX

forecast:

\[ E_{t \rightarrow t+12} = \frac{m}{12} E_{t,C} + \frac{12 - m}{12} E_{t,N} \]  \quad (B.1)

where \( E_{t \rightarrow t+12} \) denotes the 12-month forecast/expectation of the variable at time \( t \), \( E_{t,C} \) and \( E_{t,N} \) are the respective expectations of the variable for the current and next year at time \( t \) and \( m \) is the number of remaining months during the current year. For each year being forecasted, 24 forecasts with horizons varying from 1 month to 24 months are made. The constant horizon forecast that I extract from the data therefore displays seasonality. To mitigate this problem of seasonality, I adjust the series with a X-12 ARIMA filter.\(^{136}\)

### C: Construction of news time series proxies

#### Cash flow news

I construct the cash flow news proxy using the BCEI forecast of corporate profits. Denoting the 1-year forecast of real earnings at month \( t \) as:

\[ E_{t+12|t} = \frac{CP_{1t}}{(1 + Inf_{1t})} \] \quad (C.1)

where \( CP_{1t} \) is the month \( t \) 1-year forecast of annualised corporate profits (earnings) from the BCEI survey and \( Inf_{1t} \) is the month \( t \) 1-year forecast of the annualised inflation rate from the Cleveland FED model. Subsequently, I express the 1-year forecast for real earnings growth in month \( t \) as:

\[ \Delta e_{t+12|t} = \log \left( 1 + \frac{E_{t+12|t}}{E_t} \right) \] \quad (C.2)

where \( E_t \) is the actual value of real annualised corporate earnings at month \( t \) defined as \( \frac{CP_{0t}}{1 + Inf_{0t}} \), with \( Inf_{0t} \) being the realised annualised inflation rate and \( CP_{0t} \) being the realised annualised corporate profits, both at month \( t \). From Equation 2.3 it is clear that I require further forecasts of real earnings growth at longer horizons. Since this data is not available from the BCEI database, I need a method to generate forecasts of the annualised growth in corporate profits over the following 9 years. Using a similar methodology to Pástor et al. [2008] I assume that the annualised growth in earnings linearly mean-reverts to a steady state over the following 9 years in which the forecasts are being extended. I believe that by using such a method I do not bias the subsequent forecasts in any direction and through assuming a mean-reverting process for the subsequent forecasts, I conservatively extended the forecast.

\(^{136}\)See [http://www.census.gov/srd/www/x12a](http://www.census.gov/srd/www/x12a) for more details.
APPENDIX

horizons. The steady state of earnings growth \( g^{\Delta e} \) is computed as the rolling average of the growth in real (annualised) corporate profits starting from 1948 to the prior month in which I require the expected value of the level of corporate profits. The subsequent forecasts can thus be expressed as:

\[
\Delta e_{t+12j|t} = \Delta e_{t+12j-12|t} + \frac{g^{\Delta e}}{9} \Delta e_{t+12|t} \quad \text{where} \quad j = 2, \ldots, 10.
\]  
(C.3)

I can then express the total forecast of the growth in corporate profits over the next 10 years at month \( t \) as:

\[
\mathbb{E}_t \left( \sum_{j=1}^{10} \rho^j \Delta e_{t+12j} \right) = \sum_{j=1}^{10} \rho^j \Delta e_{t+12j|t}
\]  
(C.4)

I assume that \( \rho = 0.96 \) in line with the literature\(^{137}\). In order to obtain the unexpected value (news) to cash flow, I obtain the total forecast of the growth in corporate profits over the next 10 years from month \( t \), taking the expectations from month \( t + 1 \), i.e. knowing the actual (realised) corporate profits growth between month \( t \) and \( t + 1 \):

\[
\Delta e_{t+12|t+1} = \log \left( 1 + \frac{E_{t+1}}{E_t} \right)
\]  
(C.5)

\[
\Delta e_{t+24|t+1} = \log \left( 1 + \frac{E_{t+12|t+1}}{E_{t+1}} \right)
\]  
(C.6)

\[
\Delta e_{t+12j|t+1} = \Delta e_{t+12j-12|t+1} + \frac{g^{\Delta e}_{t+1} - \Delta e_{t+24|t+1}}{8} \quad \text{where} \quad j = 3, \ldots, 10.
\]  
(C.7)

I therefore obtain the cash flow news time series through:

\[
\tilde{S}_{CF,t+1} = \sum_{j=1}^{10} \rho^j \Delta e_{t+12j|t+1} - \sum_{j=1}^{10} \rho^j \Delta e_{t+12j|t}
\]  
(C.8)

It is clear that the approach above is limited by the nature of the survey forecast data which provides at each month \( t \) only a forecast of earnings at a 1-year horizon. This method necessarily imposes some assumptions and structure on the expected real growth in corporate profits at longer horizons which I believe compliments the 1-year rolling forecast.

**Real interest rate news**

For the case of the 3-month T-Bills, I denote \( TBill_{0t} \) as the actual (realised) annual nominal rate of return on the T-Bills at month \( t \) and \( TBill_{1t} \) as the 1-year forecast of the nominal return on the 3-month

\(^{137}\)I note that as a robustness check if I instead assume that \( \rho = 1 \), I obtain very similar results.
APPENDIX

T-Bills. Note that that the forecast for the real return on 3-month T-bills are defined as the forecast for the nominal return on 3-month T-Bills less the corresponding inflation rate forecast obtained from the Cleveland FED model, which is shown clearly below. The steady state value $g^{TBill}$ is computed as the rolling average of the real 3-month T-bill rate starting from 1925 to the prior month in which I require the expected value of the T-bill rate. Mathematically:

\[ r_{t+12|t} = TBill_{t+1} - Inf_{t} \]  
\[ r_{t+12j|t} = r_{t+12j-12|t} + \frac{g^{TBill} - r_{t+12|t}}{9} \text{ where } j = 2, \ldots, 10. \]

where $Inf_{t}$ is the month $t$ 1-year forecast of the annualised inflation rate. The total forecast of the future T-bill rate over the next 10 years at month $t$ can therefore be expressed as:

\[ E_{t} \left( \sum_{j=1}^{10} r_{t+12j} \right) = \sum_{j=1}^{10} r_{t+12j|t} \]  

Taking expectations from the following month, $t+1$, when the information from month $t$ to $t+1$ becomes available, I write:

\[ r_{t+12|t+1} = TBill_{0t+1} - Inf_{0t+1} \]  
\[ r_{t+24|t+1} = TBill_{1t+1} - Inf_{1t+1} \]  
\[ r_{t+12j|t+1} = r_{t+12j-12|t+1} + \frac{g^{TBill}_{t+1} - r_{t+24|t+1}}{8} \text{ where } j = 3, \ldots, 10. \]

where $Inf_{0t+1}$ is the realised annualised inflation rate at month $t+1$. Therefore, obtaining the expectation from month $t+1$ of the total future T-bill rate over the next 10 years from month $t$:

\[ E_{t+1} \left( \sum_{j=1}^{10} r_{t+12j} \right) = \sum_{j=1}^{10} r_{t+12j|t+1} \]  

The real interest rate news for both stocks and bonds can be constructed as:

\[ \tilde{S}_{r,t+1} = \tilde{B}_{r,t+1} = \sum_{j=1}^{10} r_{t+12j|t+1} - \sum_{j=1}^{10} r_{t+12j|t} \]  

**Excess stock return news**

Table [C.1] evaluates the forecast of excess stock returns. I only include those variables from the list above that show reasonable significance in the regressions. I run the predictive regression of the 1-year ahead...
APPENDIX

annualised excess stock returns on the dividend yield $d/p$, term spread $y^{(5)} - y^{(1)}$ and the CP-factor $\gamma^T f$. Dividend yield is defined as the log dividend payments over the past year minus the log price level; the term spread is defined as the difference in yields between the 5 year and 1-year zero coupon bond; and the CP factor is a single tent-shaped linear combination of forward rates. I perform the regression:

$$
\epsilon_{t+12}^S = \alpha + \beta_1 d_p + \beta_2 \left( y^{(5)} - y^{(1)} \right) + \beta_3 \left( \gamma^T f \right) + \epsilon_{t+12}
$$

where $t + 12$ monthly observations ahead implies the 1-year ahead excess stock return.$^{138}$

Table C.1: Forecasts of excess stock returns

<table>
<thead>
<tr>
<th>Regression</th>
<th>$d/p$ (t-stat)</th>
<th>$y^{(5)} - y^{(1)}$ (t-stat)</th>
<th>$\gamma^T f$ (t-stat)</th>
<th>AdjR$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.83 (1.78)</td>
<td></td>
<td></td>
<td>0.07</td>
</tr>
<tr>
<td>2</td>
<td>3.94 (1.46)</td>
<td>2.11 (1.05)</td>
<td>0.97 (0.63)</td>
<td>0.04</td>
</tr>
<tr>
<td>3</td>
<td>3.94 (1.46)</td>
<td>2.06 (0.88)</td>
<td>1.12 (0.67)</td>
<td>0.04</td>
</tr>
<tr>
<td>4</td>
<td>3.94 (1.46)</td>
<td>2.06 (0.88)</td>
<td>1.12 (0.67)</td>
<td>0.04</td>
</tr>
<tr>
<td>5</td>
<td>3.94 (1.46)</td>
<td>2.06 (0.88)</td>
<td>1.12 (0.67)</td>
<td>0.04</td>
</tr>
<tr>
<td>6</td>
<td>3.94 (1.46)</td>
<td>2.06 (0.88)</td>
<td>1.12 (0.67)</td>
<td>0.04</td>
</tr>
<tr>
<td>7</td>
<td>3.94 (1.46)</td>
<td>2.06 (0.88)</td>
<td>1.12 (0.67)</td>
<td>0.04</td>
</tr>
</tbody>
</table>

Regressions 1, 2 and 4 from Table C.1 display the well-known result of the dividend yield and term spread forecasting different components of returns, since the coefficients are relatively unchanged in multiple regressions and with the $R^2$ increasing. I also notice from regression 5 that the term spread is driven out by the CP-factor, which is plausible since the information content of the CP-factor subsumes that of the term spread, since both are from zero-coupon bond yields. However, it seems as if the recent financial crisis has restored the forecastability of the dividend yield since in regressions 6 and 7 it subsumes the CP-factor. I use the coefficients from regression 4 to produce a model-implied expectation for excess stock returns 1 year from now.

$^{138}$ I note that the (restricted) CP-factor was constructed using the Fama-Bliss zero coupon bond yields obtained from CRSP from January 1964 to November 2009 to recover the well-known tent-shape for the coefficients on the respective combination of forward rates. Only using data from June 1984 to November 2009 would not produce the tent shaped structure.
APPENDIX

Inflation news

Denoting $Inf_{1t}$ as the 1-year forecast of the annual inflation rate at month $t$, with the number changing depending on the horizon of the forecast, I express the average forecasted (expected) future inflation rate over the next 10 years at month $t$ as:

$$InfTotal_{10t} = \frac{1}{10}(Inf_{1t} + Inf_{2t} + Inf_{3t} + Inf_{4t} + Inf_{5t} + Inf_{6t} + Inf_{7t} + Inf_{8t} + Inf_{9t} + Inf_{10t})$$ (C.18)

Naturally I can write:

$$E_t \left( \sum_{j=1}^{10} \pi_{t+12j} \right) = 10InfTotal_{10t}$$ (C.19)

From Equation (2.5), I also need the total inflation rate over the next 10 years from month $t$ but taking expectations from month $t + 1$: $E_{t+1} \left( \sum_{j=1}^{10} \pi_{t+12j} \right)$. This implies that I have one month of realised information available, so that I know the realised inflation rate from month $t$ to month $t + 1$, which I denote as $Inf_{0t+1}$. Therefore, taking expectations from month $t + 1$ of the total future inflation rate over the next 10 years from month $t$:

$$E_{t+1} \left( \sum_{j=1}^{10} \pi_{t+12j} \right) = 10 \left( \frac{1}{12}Inf_{0t+1} + \frac{11}{12}InfTotal_{10t+1} \right)$$ (C.20)

I therefore express the news to the inflation rate as:

$$\tilde{B}_{\pi,t+1} = E_{t+1} \left( \sum_{j=1}^{10} \pi_{t+12j} \right) - E_t \left( \sum_{j=1}^{10} \pi_{t+12j} \right)$$ (C.21)

D: Conditional covariance models

Dynamic Conditional Correlation (DCC) model

The Dynamic Conditional Correlation (DCC) model proposed by Engle [2002] is a generalisation of the Constant Conditional Correlation (CCC) model by Bollerslev [1990]. It is thus a simplified multivariate Generalised AutoRegressive Conditional Heteroscedasticity (GARCH) model which has the flexibility of a univariate GARCH model together with a parsimonious correlation specification without having the traditional computational difficulties associated with multivariate GARCH models.

139 This is an annual rate and is constructed as the year-on-year percentage change in the CPI level.
Assuming a random variable $x_t$ as an $n$-dimensional multivariate normal process with zero mean and variance-covariance matrix $H_t$, I write:

$$x_t \mid F_{t-1} \sim \mathcal{N}(0, H_t) \quad (D.1)$$

I can also write this in terms of a “mean equation”:

$$x_t \equiv \sqrt{H_t} \epsilon_t \quad (D.2)$$

where $\epsilon_t \sim \mathcal{N}(0, 1)$ which are the standardised normally distributed disturbances. I express the variance-covariance matrix as:

$$H_t = E_{t-1}(x_t x_t') = D_t R_t D_t \quad (D.3)$$

where $D_t = diag(\sqrt{H_t})$ whose diagonal elements are the time-varying standard deviation of the residuals of the mean equation for each of the $n$ processes which I assume all respectively follow a GARCH(1,1) model:

$$H_{t,ii} = E_{t-1}(x_{i,t}^2) = \omega_i + \alpha_i x_{i,t-1}^2 + \beta_i H_{t-1,ii} \quad (D.4)$$

After estimating this model to obtain the conditional variance for each process, the standardised residuals are then defined by:

$$\epsilon_t = D_t^{-1} y_t \quad (D.5)$$

In contrast to the CCC model the correlation matrix $R_t = E_{t-1}(\epsilon_t \epsilon_t')$ is now allowed to be time-dependent. Thus a quasi-correlation matrix for the standardised residuals is proposed as a stochastic process for a matrix $Q$ that is an approximation to the correlation matrix. I use a mean-reverting model for the correlation process analogous to the GARCH(1,1) process:

$$Q_t = \bar{R} + \alpha(\epsilon_{t-1} \epsilon_{t-1}' - \bar{R}) + \beta(Q_{t-1} - \bar{R}) \quad (D.6)$$

where $\bar{R} \equiv \frac{1}{T} \Sigma_{t=1}^T \epsilon_t \epsilon_t'$ is the unconditional correlation of the standardised residuals. Thus, the conditional correlation depends on the common GARCH parameters $\alpha$ and $\beta$ and on the unconditional correlation between the standardised residuals. The matrix $Q$ is guaranteed to be positive definite if $\alpha$, $\beta$ and $(1 - \alpha - \beta)$ are all positive and if the initial value, $Q_1$ is positive definite. This is because each subsequent value of $Q$ is a weighted average of positive semi-definite and positive-definite matrices, and thus it is positive-definite. This produces a process for the matrix $Q$ that delivers a positive-definite
APPENDIX

quasi-correlation matrix for each time period. It does not ensure however that this is a conventional correlation matrix. Thus to convert these Q processes into correlations, it is re-scaled according to:

\[ R_t = \sqrt{\text{diag}(Q_t)}Q_t\sqrt{\text{diag}(Q_t)} \]  

(D.7)

In order to estimate the variance and correlation parameters, a Maximum Likelihood Estimation (MLE) method can be employed. Such a technique uses trial and error to determine the optimal parameter value that maximises the likelihood of the data to occur for the particular model. I can write the log-likelihood for the data set \( \{x_1,...x_T\} \) as:

\[ L = -\frac{1}{2} \sum_{t=1}^{T} \left( n \log(2\pi) + \log|D_tR_tD_t| + x_t'D_t^{-1}R_t^{-1}D_t^{-1}x_t \right) \]  

(D.8)

\[ L = -\frac{1}{2} \sum_{t=1}^{T} \left( n \log(2\pi) + 2\log|D_t| + \log|R_t| + \epsilon_t'R_t^{-1}\epsilon_t \right) \]  

(D.9)

\[ L = -\frac{1}{2} \sum_{t=1}^{T} \left( n \log(2\pi) + x_t'D_t^{-2}x_t - \epsilon_t'R_t^{-1}\epsilon_t + \log|R_t| + \epsilon_t'R_t^{-1}\epsilon_t \right) \]  

(D.10)

As outlined in [Engle 2009], the log-likelihood can simply be maximised with respect to all the parameters in the model. However the log-likelihood can also be decomposed into two parts. The first containing the variance parameters and the data; the second containing the correlation parameters and the standardised residuals:

\[ L = L_{vol} + L_{corr} \]  

(D.12)

\[ L_{vol} = -\frac{1}{2} \sum_{t=1}^{T} \left( n \log(2\pi) + 2\log|D_t| + x_t'D_t^{-2}x_t \right) \]  

(D.13)

\[ L_{corr} = -\frac{1}{2} \sum_{t=1}^{T} \left( \log|R_t| + \epsilon_t'R_t^{-1}\epsilon_t - \epsilon_t'\epsilon_r \right) \]  

(D.14)

This can then be estimated using a two-step procedure. The first-step is to maximise the variance part of the likelihood function, \( L_{vol} \), by computing the univariate GARCH models for each of the series and taking the sum of these likelihood functions. The second-step is to then take the standardised residuals from the first-step and maximise the correlation log-likelihood function, \( L_{corr} \), with respect to the correlation parameters. The term \( \epsilon_t'\epsilon_t \) can be ignored as it does not depend on the parameters being optimised.
**APPENDIX**

**Exponentially Weighted Moving Average (EWMA) model**

An Exponentially Weighted Moving Average (EWMA) model can also be used for the conditional variance-covariance matrix. Assuming the same distribution as above for the random variable $x \sim \mathcal{N}(0, H_t)$, I define the EWMA variance-covariance model as:

$$H_t = \lambda H_{t-1} + (1 - \lambda)x_{t-1}x_{t-1}'$$  \hspace{1cm} (D.15)

$$\lambda > 0$$  \hspace{1cm} (D.16)

where $\lambda$ is the weight assigned to the lagged variance-covariance matrix; it is also known as a decay rate since the weight assigned to the $x^2$ terms decline exponentially as one moves back through time.

Note that the same value of $\lambda$ should be used for both the variance and covariance series in order to ensure consistency of the combined variance-covariance matrix. I similarly estimate $\lambda$ using the MLE technique. The log-likelihood can thus similarly be written as:

$$L = -\frac{1}{2} \sum_{t=1}^{T} \left( n \log(2\pi) + \log|H_t| + x_t' H_t^{-1} x_t \right)$$  \hspace{1cm} (D.17)

In the usual fashion as above, I determine the value of $\lambda$ using an iterative procedure to maximize this log-likelihood.

**E: Calculation of Treasury bond returns**

I calculate returns from the 10-year Constant Maturity Treasury (CMT) yield using the method of De Goeij and Marquering [2004]. Their method is based on a hypothetical bond contract, with semi-annual coupon payments, the coupon being half the stated CMT yield. Under this assumption, the price of the bond at the beginning of the holding period is equal to its face value. They then calculate an end-of-period price of the bond using the next-period’s yield augmented with the accrued interest rate:

$$P_{t+1,n-hd} = \sum_{i=1}^{2n-1} \frac{1}{2} y_{t,n} \left( 1 + \frac{1}{2} y_{t+1,n} \right)^i + \frac{1}{2} y_{t,n} \frac{1}{(1 + \frac{1}{2} y_{t+1,n})^{2n}} + \frac{hd}{365} y_{t,n}$$  \hspace{1cm} (E.1)

where $P_{t+1,n-hd}$ is the end-of-period price of the bond with $hd$ being the number of day the bond is held for, $n$ is the (constant) maturity in years of the bond and $y_{t,n}$ is the yield of the $n$-year bond at time $t$. Essentially the method is based upon on buying the $n$-year maturity bond at par and then selling this bond in the next period $t+1$ (after $hd$ days). The amount obtained from liquidating this position is
APPENDIX

then reinvested in a new $n$-year maturity par bond and so on. The $hd$-return is then calculated as:

$$r_{t+1}^{n-yearCMT} = P_{t+1,n-hd} - 1$$  \hspace{1cm} (E.2)

**F: The bond proxies: Constant Maturity Treasury (CMT) rates vs Constant Maturity Swap (CMS) rates**

A Constant Maturity Treasury (CMT henceforth) rate is defined as the par yield of a Treasury bond with a specified/fixed maturity. The $m$-year CMT index thus reflects what the par yield should be on an $m$-year maturity Treasury bond. Note that Treasury bills, securities with maturities of 0.25, 0.5 and 1 year, are issued as discount securities\(^{140}\) whilst Treasury notes and bonds, securities with maturities greater than 1 year, are coupon bearing securities with payments semi-annually. Given that each Treasury security has a specific time to maturity, the $m$-year CMT index is constructed by determining (at any point in time) the yield on the $m$-year maturity treasury security; if there are no treasury issues with the appropriate maturity, the value of the index is determined by interpolating between the yields of other treasury issues. Typically, CMT indexes exist for maturities ranging from 1 month to 30 years.

A $m$-year swap rate is the average of the fixed rate that a swap market maker is prepared to pay in exchange for receiving LIBOR\(^{141}\) (swap bid rate) within a plain vanilla fixed-for-floating $m$-year swap;\(^{142}\) and the fixed rate that the market maker is prepared to receive in return for paying LIBOR (swap offer rate) for the same swap structure. Since the value of such a swap can be characterized as the difference between the value of a fixed-rate bond and a floating-rate bond with the swap being worth zero at inception, swap rates thus define a set of par yield bonds. A Constant Maturity Swap (CMS henceforth) rate can therefore be defined as the par yield on bonds with a specified/fixed maturities.

Thus the CMT and CMS indices both allow for a flexible and market efficient access to long dated interest rates. The difference between them should reflect the cost of funding, together with liquidity, taxation and regulation issues\(^{[Hull et al. 2004\), [Duffee 1996\], [Reinhart and Sack 2002\). Derivative traders tend to use the swap curve in their pricing models because they consider swap rates to more closely reflect their opportunity cost of capital and not the equivalent maturity Treasury instrument. Also, the swap market’s increased popularity and liquidity relative to the Treasury’s benchmark rates\(^{\text{\[Hull et al. 2004\)\), [Duffee 1996\], [Reinhart and Sack 2002\)\).

\(^{140}\)Such securities only pay out the notional amount at maturity and therefore sell for less than their par value.

\(^{141}\)London Inter-Bank Offered Rate is the interest rate that the largest financial institutions charge each other for loans.

\(^{142}\)Where fixed payments are made every 6 months in exchange for floating LIBOR payments every 3 months up until maturity at $m$-years.
has created a central role for swap rates as market interest rate indicators.

**Figure F.1: Comparison of the 10-year CMT yield and 10-year CMS rate**

This figure directly compares the 10-year CMT yield and 10-year CMS rate over the period 2nd January 1989 to 30th December 2012.

![CMT vs CMS yield graph](image)

Figure F.1 shows that the CMT yield does tend to be slightly depressed relative to the CMS rate, however the two series are highly correlated with an unconditional correlation of 0.99 over the sample period.

**G: Implied volatility index construction**

For each of the underlying assets respectively, I firstly choose the two option contract series with the nearest expiration to 365 days. Note that only OTM options with a non-zero bid and ask price are used in the calculation. Second, for each option series selected, I calculate the mid-quote \( Q(K, T) \). Third, the forward level of the underlying asset is determined at the strike price at which the absolute difference between call and put prices is smallest. This forward level \( F_{t,T} \) is determined for each expiration under consideration via the put-call parity relation:

\[
F_{t,T} = e^{r(T-t)} (C_{t,T} - P_{t,T}) + K \tag{G.1}
\]
where \(C_{t,T}\) and \(P_{t,T}\) is the call and put price at time \(t\) for options which mature at \(T\). Fourth, I apply Equation (G.2) below on each of the two series with the mid-quotes that I have generated and come up with two index values, one for each expiration:

\[
\sigma_T^2 = \frac{2}{T-t} \sum_i \left( \frac{\Delta K_i}{K_i^2} e^{r(T-t)} Q(K_i, T) \right) - \frac{1}{T} \left( \frac{F_{t,T}}{K_0} - 1 \right)^2 \tag{G.2}
\]

where \(T\) is the time to maturity of the options involved in the calculation, \(F_{t,T}\) is the forward level at time \(t\) derived from the option prices, \(K_0\) is the first strike below \(F_{t,T}\), \(K_i\) is the strike price of the \(i^{th}\) OTM option (call if \(K_i > K_0\), put if \(K_i < K_0\); both put and call if \(K_i = K_0\), \(\Delta K_i\) is the half interval between the strike prices \(K_{i+1}\) and \(K_{i-1}\), \(r\) is the time \(t\) risk-free interest rate and \(Q(K_i)\) is the average of the quoted bid-ask spread (mid-quote) for each option with strike \(K_i\). Fifth, I interpolate between the two index values to obtain an index value with 365 days expiration. I use the same interpolation formula that the Chicago Board Options Exchange (CBOE) uses for the case of the VIX. The implied volatility is thus the square root of this multiplied by 100:

\[
IV_t = 100 \times \sqrt{T_1 \sigma_1^2 T_2 - 365 \left( T_2 - T_1 \right) + T_2 \sigma_2^2 \left( \frac{365 - T_1}{T_2 - T_1} \right)} \tag{G.3}
\]

where \(T_1\) and \(T_2\) denote the two maturity dates and \(IV_t\) is the 365-day implied volatility at time \(t\).

Implementation of the methodology assumes a very liquid market, however due to the low liquidity of particularly the bond options the conditions required by the method are not met for the construction of the indices on some of the days. For these particular days I use the value obtained for the previous trading day.

**H: Optimal mean-variance portfolio with pension liabilities**

Starting from the mean-variance optimisation problem of Equation (4.29) restated below:

\[
E_0 [u(R_{SP,T})] = \max_{\omega} \quad E_0 [R_{SP,T}] - \frac{1}{2}\gamma \text{Var}_0 [R_{SP,T}]
\]

s.t. \(\omega' \hat{1}_n = 1\)

(H.1)

with the expected surplus return at time \(T\) expressed as:

\[
E_0 [R_{SP,T}] = R_{f,T} + E_0 \left[ \omega' (R_{A,T} - R_{f,T}\hat{1}_n) - kR_{L,T} \right] = R_{f,T} + \omega' \mu_{A,T} - k\mu_{L,T} \tag{H.2}
\]
where additionally I define $R_{A,T}$ as a $n$-dimensional vector of returns on $n$ assets that the pension fund can invest in, with expected asset portfolio return of $E[(R_{A,T} - R_{f,T}\hat{1}_n)] = \mu_{A,T}$ and expected liability return of $E[R_{L,T}] = \mu_{L,T}$. As for the variance of the surplus return at time $T$:

$$\text{Var}_0[R_{SP,T}] = \text{Var}_0\left(\omega'(R_{A,T} - R_{f,T}\hat{1}_n)\right) + k^2\text{Var}_0\left(\omega'(R_{A,T} - R_{f,T}\hat{1}_n), -kR_{L,T}\right)$$

$$= \omega'\Sigma_A\omega + k^2\sigma_L^2 - 2kw'\Sigma_L$$

(H.3)

where $\Sigma_A$ is the $n \times n$ covariance matrix of asset returns, $\sigma_L^2$ is the variance of liability returns and $\Sigma_L$ is a $n$-dimensional vector of covariances between the returns on assets and liabilities.

The objective function $L$ can then be defined as:

$$L(\omega, \lambda) = R_{f,T} + \omega'\mu_{A,T} - k\mu_{L,T} - \frac{1}{2}\gamma\left(\omega'\Sigma_A\omega + k^2\sigma_L^2 - 2kw'\Sigma_L\right) - \lambda(\omega'\hat{1}_n - 1)$$

(H.4)

Maximising the objective function $L$ leads to the first order conditions:

$$\frac{\partial L}{\partial \omega} = \mu_{A,T} - \gamma\Sigma_A\omega + \gamma k\Sigma_L - \lambda\hat{1}_n = 0$$

(H.5)

$$\frac{\partial L}{\partial \lambda} = \omega'\hat{1}_n - 1 = 0$$

(H.6)

Solving the above equations the optimal portfolio weights $\omega$ can be expressed as:

$$\omega = \frac{1}{\gamma}\Sigma_A^{-1}(\mu_{A,T} + \gamma k\Sigma_L - \lambda\hat{1}_n)$$

(H.7)

with the zero-beta return $\lambda$ as:

$$\lambda = \frac{\hat{1}_n'\Sigma_A^{-1}\mu_{A,T} + \gamma k\hat{1}_n'\Sigma_A^{-1}\Sigma_L - \gamma}{\hat{1}_n'\Sigma_A^{-1}\hat{1}_n}$$

(H.8)

I: Discretisation of the stock price/ bond yield SDE

Diffusing the log of the spot price/yield and applying Ito’s Lemma I can write:

$$d(\log X_t) = \left(\mu_t - \frac{1}{2}\text{diag}(\Sigma_t)\right) dt + \sqrt{\Sigma_t} \left(dW_t + \sqrt{(1 - \rho^2)} dB_t\right)$$

(I.1)
Then integrating over the interval $[t, t + \Delta t]$ gives:

$$
\int_t^{t+\Delta t} d \left( \log [X_t] \right) = \int_t^{t+\Delta t} \left( \mu_s - \frac{1}{2} \text{diag}(\Sigma_s) \right) ds + \int_t^{t+\Delta t} \sqrt{\Sigma_s} dW_s \rho \\
+ \int_t^{t+\Delta t} \sqrt{\Sigma_s} dB_s \sqrt{1 - \rho^2} (I.2)
$$

When approximating the above integrals I choose to use the predictor-corrector scheme for the discretisation as it is known to provide better results than the Euler scheme at low additional computational cost. The idea of the predictor-corrector scheme is first to handle the time integral in a centered manner. Assuming that $\mu_t$ remains constant:

$$
\int_t^{t+\Delta t} \left( \mu_s - \frac{1}{2} \text{diag}(\Sigma_s) \right) ds \approx \Delta t \left( \mu - \frac{1}{4} \text{diag}(\Sigma_{t+\Delta t} + \Sigma_t) \right) (I.3)
$$

I use the usual Euler approximation for the stochastic integral against the Brownian $W$:

$$
\int_t^{t+\Delta t} \sqrt{\Sigma_s} dW_s \rho \approx \sqrt{\Sigma_t} \Delta t G \rho (I.4)
$$

where $G$ is a $2 \times 2$ matrix whose elements are independent Gaussian random variables. For the last integral, given that $B$ is independent of $\Sigma$, the stochastic integral with respect to $B$ is (conditionally) Gaussian with zero mean and variance:

$$
\int_t^{t+\Delta t} \sqrt{\Sigma_s} dB_s \sqrt{1 - \rho^2} \approx \sqrt{\Delta t} \sqrt{\Sigma_t} \Delta t + \Sigma_t) Z \sqrt{1 - \rho^2} (I.5)
$$

where $Z$ is a 2-dimensional vector of Gaussian innovations independent of $G$. I thus obtain the following scheme for the stock price/bond yield process:

$$
\log X_{t+\Delta t} - \log X_t = \left( \mu - \frac{1}{4} \text{diag}(\Sigma_{t+\Delta t} + \Sigma_t) \right) \Delta t \\
+ \sqrt{\Sigma_t} \Delta t G \rho + \sqrt{\Delta t} \sqrt{\Sigma_{t+\Delta t} + \Sigma_t) Z \sqrt{1 - \rho^2} (I.6)
$$
Bibliography


BIBLIOGRAPHY


N. Branger and M. Muck. Keep on smiling? the pricing of quanto options when all covariances are stochastic. *Journal of Banking & Finance*, 2012. 88


BIBLIOGRAPHY


J.Y. Campbell, A. Sunderam, and L.M. Viceira. Inflation bets or deflation hedges? The changing risks of nominal bonds. *NBER working paper*, 2009b. 3, 11, 12, 33, 34, 38, 42, 49, 55, 82, 130, 131

BIBLIOGRAPHY


153
BIBLIOGRAPHY


BIBLIOGRAPHY


BIBLIOGRAPHY


BIBLIOGRAPHY


C. Memmel. Performance hypothesis testing with the sharpe ratio. SSRN, 2003. 66


BIBLIOGRAPHY


