Collisional Particle In Cell Modelling Of The Propagation Of Fast Electrons In Solid Density Plasma

Imperial College London

Plasma Physics Group, Blackett Laboratory, Imperial College London, London, SW7 2BZ.

Submitted for the award of Doctor of Philosophy

Rhys David Lloyd

2013
Abstract

This thesis looks at the effects that electron-ion Coulomb collisions have on fast electron transport in solid density plasma. The study of the fast electrons generated in ultra-high intensity laser-plasma interactions is important due to their envisioned use in the fast ignition approach to inertial confinement fusion.

Collisions have been added to the particle-in-cell (PIC) code EPOCH in order to study the propagation of fast electron beams in various solid density targets. By using a collisional PIC model several of the assumptions used in previous studies are not required. The code solves the full Maxwell equations (including the displacement current), does not require assumptions of Ohm’s law and of Spitzer resistivity and does not require the background distributions to be Maxwellian.

The thesis begins with summaries of the background theory and of the previous work performed in this area. The PIC method is then discussed and the way in which collisions were added to EPOCH is outlined. The results from several collisional PIC simulations with different target Z values are then discussed and compared to both collisionless PIC simulation results and hybrid simulation results. The effects of collisions have then been examined by looking into numerous aspects of the simulations that have been performed. Firstly, the generation of fields within the plasma and the subsequent filamentation of the fast electron beam are examined. The effects that the collisions have on the electron distributions within the plasma are then investigated with particular attention given to the divergence of the fast electrons, the energy and momentum distributions of the electrons and the background temperatures within the plasma. Finally, the results of the simulations are used to assess the accuracy of the Spitzer resistivity approximation that is used in hybrid codes.
Declaration of Originality

The work contained in this thesis is entirely the authors unless otherwise cited, acknowledged or considered part of the standard literature of the area.

The collision routine that has been added to the particle-in-cell code EPOCH in order to perform the simulations discussed in this thesis was written by the author. It was written independently of the routine used to perform the simulations discussed in ‘Collisional particle-in-cell modelling of the generation and control of relativistic electron beams produced by ultra-intense laser pulses’ [1].
Copyright Declaration

The copyright of this thesis rests with the author and is made available under a Creative Commons Attribution Non-Commercial No Derivatives licence. Researchers are free to copy, distribute or transmit the thesis on the condition that they attribute it, that they do not use it for commercial purposes and that they do not alter, transform or build upon it. For any reuse or redistribution, researchers must make clear to others the licence terms of this work.
# Contents

1 Introduction ............................................. 16
   1.1 Thesis outline ........................................... 16
   1.2 Laser-Plasma interactions .............................. 20
   1.3 Inertial Confinement Fusion ............................ 22
      1.3.1 The current state of Inertial Confinement Fusion research . . 22
      1.3.2 Inertial Confinement Fusion - Central Hot Spot ignition and Fast Ignition ............................................ 22

2 Laser-Plasma Interactions And Fast Electron Transport 29
   2.1 Electromagnetic fields in plasma .......................... 29
   2.2 The interaction between a single particle and a laser field ........... 31
      2.2.1 The ponderomotive force .................................. 31
      2.2.2 Electron trajectories and momentum relationships ................. 34
   2.3 Laser energy absorption mechanisms ........................ 36
      2.3.1 Inverse Bremsstrahlung ..................................... 36
      2.3.2 Resonance absorption ....................................... 37
      2.3.3 Vacuum heating ............................................. 37
      2.3.4 $\mathbf{J} \times \mathbf{B}$ heating ....................................... 38
6.4.2 Spitzer resistivity test ......................................... 110

7 Collisional And Collisionless PIC Simulation Results .......... 112

7.1 The simulation setup .............................................. 112
7.2 Electric and magnetic fields within the targets .................. 118
7.3 Filamentary structures ............................................ 132
7.4 Fast electron angular distributions ............................. 141
  7.4.1 Fast electron divergence within the target ................. 141
  7.4.2 Initial fast electron divergence ............................. 148
  7.4.3 Fast electron currents ...................................... 153
7.5 Laser energy absorption ........................................... 154
7.6 Energy and momentum distributions .............................. 155
7.7 Background temperatures ......................................... 164
  7.7.1 Lineouts of the background temperature .................... 164
  7.7.2 Spitzer resistivity comparisons ............................. 168
7.8 LSP simulation results ............................................. 178
  7.8.1 Magnetic field comparison .................................. 178
  7.8.2 Temperature comparison ................................... 179

8 Summary Of Results And Future Work Proposals ............... 183

8.1 Summary of results ............................................... 183
8.2 Discussion of further work and computational resources ....... 185
List of Figures

1.1 (a) An artists rendering of a NIF hohlraum. (b) Inside the NIF target chamber. Credit: Lawrence Livermore National Laboratory (https://lasers.llnl.gov) 23

1.2 Diagram showing the fast ignition of an ICF target using a laser beam. 25

1.3 Diagram showing some of the processes involved in the propagation of a fast electron beam through plasma. 27

2.1 The trajectory of an electron in a linearly polarised electromagnetic wave in (a) the wave frame and (b) the electron guiding centre frame. 35

2.2 Magnetic field generation via the thermoelectric effect $\nabla n_e \times \nabla T_e$. 44

3.1 Collision in particle $q \beta$ rest frame 49

5.1 A typical particle in cell code timestep 78

5.2 Apparent shape of computational particle located at $x_i$ with NGP weighting 85

5.3 Cloud in Cell and Particle in Cell viewpoints - 1$^{st}$ order weighting 87

5.4 Cloud in Cell and Particle in Cell viewpoints when using 2$^{nd}$ order weighting 88

5.5 Effective particle shapes for different order weightings 90
5.6 Plots showing the amount of numerical heating when using 2\textsuperscript{nd} order particle weighting. The plots are for tests with grid spacings of (a) $dx = 6.5 \times 10^{-9} m$, (b) $dx = 3.25 \times 10^{-9} m$ and (c) $dx = 1.3 \times 10^{-8} m$. In all of the plots lines are shown which correspond to tests performed with different numbers of particles per cell. ........................................... 92

5.7 Plots showing the amount of numerical heating when using 4\textsuperscript{th} order particle weighting. The plots are for tests with grid spacings of (a) $dx = 6.5 \times 10^{-9} m$, (b) $dx = 3.25 \times 10^{-9} m$ and (c) $dx = 1.3 \times 10^{-8} m$. In all of the plots lines are shown which correspond to tests performed with different numbers of particles per cell. ........................................... 93

6.1 Takizuka and Abé’s random particle reordering ................................. 98

6.2 Takizuka and Abé’s particle pairing (Left: $N_e = N_i$, Right: $N_e > N_i$) 99

6.3 Particle pairing in the EPOCH collision routine ............................... 100

6.4 Rotating the z-axis to be aligned with the momentum vector .............. 102

6.5 Rotation of the momentum due to the binary collision ...................... 103

6.6 Plots showing the theoretical slowing down of a group of particles and the drift velocity resulting from the collision routine for various numbers of particles per cell. The scales are normalised to the initial drift and the collision frequency. ........................................... 108

6.7 (a) As figure 6.6(e) but showing the statistical scatter between three runs with identical initial conditions. (b) The error between code and theory for the four runs. ........................................... 109

6.8 (a) As figure 6.6(e) but showing the results of several simulations performed using different timesteps. (b) The error between code and theory for the timestep comparison results shown in (a). ........... 109

6.9 The measured drift velocity due to an applied electric field throughout collisional PIC simulations with various timesteps (electron-ion collisions only) ........................................... 111
6.10 The measured drift velocity due to an applied electric field throughout a collisional PIC simulation that contains the effects of electron-ion and electron-electron collisions.

7.1 Initial target electron number density.
7.2 Magnetic field (time averaged) z-component at 50 fs.
7.3 Lineouts corresponding to figure 7.2 (a-d) along x=1µm for −1µm<y<1µm.
7.4 Magnetic field (time averaged) z-component at 100 fs.
7.5 Lineouts corresponding to figure 7.4 (a-d) along x=1µm for −1µm<y<1µm.
7.6 Magnetic field (time averaged) z-component at 150 fs.
7.7 Lineouts corresponding to figure 7.6 (a-d) along x=1µm for −1µm<y<1µm.
7.8 Magnetic field (time averaged) z-component at 200 fs.
7.9 Lineouts corresponding to figure 7.8 (a-d) along x=1µm for −1µm<y<1µm.
7.10 Electric field (time averaged) x-component at 50 fs.
7.11 Electric field (time averaged) x-component at 100 fs.
7.12 Electric field (time averaged) x-component at 150 fs.
7.13 Electric field (time averaged) x-component at 200 fs.
7.14 Lineout along y=0 showing the time averaged x-component of the electric field within the target at (a) 40 fs, (b) 50 fs, (c) 60 fs, (d) 70 fs, (e) 80 fs, (f) 90 fs, (g) 100 fs and (h) 150 fs.
7.15 FFT of the time-averaged magnetic field in the collisionless simulation. A $4 \times 4\mu m$ region starting at $x = 1\mu m$ and centred on the $y = 0$ axis has been transformed.
7.16 FFT of the time-averaged magnetic field in the collisional Z=1 simulation. A $4 \times 4\mu m$ region starting at $x = 1\mu m$ and centred on the $y = 0$ axis has been transformed.
7.17 FFT of the time-averaged magnetic field in the collisional Z=3 simulation. A $4 \times 4 \mu m$ region starting at $x = 1 \mu m$ and centred on the $y = 0$ axis has been transformed. ..................................... 137

7.18 FFT of the time-averaged magnetic field in the collisional Z=5 simulation. A $4 \times 4 \mu m$ region starting at $x = 1 \mu m$ and centred on the $y = 0$ axis has been transformed. ..................................... 137

7.19 Maximum RMS magnetic field strength throughout the simulations. The field values shown are the maximum recorded values in the $4 \times 4 \mu m$ region starting at $x = 1 \mu m$ and centred on the $y = 0$ axis. .................................. 140

7.20 Angular distributions of fast electrons with energies greater than 150keV. The plots represent times of (a,b) 50fs, (c,d) 100fs (e,f) 150fs and (g,h) 200fs. Each pair of plots shows the same distribution in regular and polar forms respectively. ..................................... 145

7.21 Angular distributions of fast electrons with energies between 1 and 2 MeV. The plots represent times of (a,b) 50fs, (c,d) 100fs (e,f) 150fs and (g,h) 200fs. Each pair of plots shows the same distribution in regular and polar forms respectively. ..................................... 146

7.22 The divergence half angle of the fast electrons ($\gamma >1.3$) throughout each of the four simulations. .......................................................... 146

7.23 The fast electron flow angle, $\theta$, as a function of $y$. The plots are for 100fs and represent the fast electrons ($\gamma >1.3$) in the following regions: (a) $-1.5\mu m < x < -0.5\mu m$ and (b) $-0.5\mu m < x < 0.5\mu m$ ..................................... 149

7.24 The fast electron flow angle, $\theta$, as a function of $y$ at 200fs. Fast electrons in the region between $-0.5\mu m < x < 0.5\mu m$ are included. ........... 150

7.25 The angular spread ($\Delta \theta$) of the fast electrons as a function of $y$. The plots are for 100fs and represent the following regions (a) $-1.5\mu m < x < -0.5\mu m$ (b) $-0.5\mu m < x < 0.5\mu m$, (c) $-1.5\mu m < x < 0.5\mu m$ (forwards going electrons only) and (d) $-0.5\mu m < x < 0.5\mu m$ (forward going electrons only) ............................... 152
<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.26</td>
<td>Fast electron current densities within the target (averaged over $6\mu m&lt;x&lt;8\mu m$) for (a) 100fs and (b) 200fs</td>
<td>153</td>
</tr>
<tr>
<td>7.27</td>
<td>Time evolution of the total energy of the fast electrons ($\gamma&gt;1.3$). Plot (a) shows the total fast electron energy as a percentage of the energy that has entered the system and plot (b) shows the total fast electron energies normalised to the energy that has entered the system by the end of the plot. The total energy introduced via the laser is also shown.</td>
<td>154</td>
</tr>
<tr>
<td>7.28</td>
<td>Fast electron ($\gamma &gt; 1.3$) number density at 100fs</td>
<td>156</td>
</tr>
<tr>
<td>7.29</td>
<td>Electron $x - p_x$ phase space at 100fs</td>
<td>157</td>
</tr>
<tr>
<td>7.30</td>
<td>Electron energy distributions at 100fs</td>
<td>158</td>
</tr>
<tr>
<td>7.31</td>
<td>Electron energy distributions at 200fs</td>
<td>158</td>
</tr>
<tr>
<td>7.32</td>
<td>Velocity distributions of electrons within a $2\mu m \times 2\mu m$ square centred at $x = 2\mu m$, $y = 0\mu m$ for all four cases at (a) 100fs and (b) 200fs.</td>
<td>160</td>
</tr>
<tr>
<td>7.33</td>
<td>Momentum distributions of the electrons within two $2\mu m \times 2\mu m$ regions at 100fs. The regions are centred at $x = 2\mu m$, $y = 0\mu m$ (front) and at $x = 8\mu m$, $y = 0\mu m$ (back). Each row of three plots shows the same momentum distribution with different scales in order to clearly show the various parts of the distribution.</td>
<td>162</td>
</tr>
<tr>
<td>7.34</td>
<td>Momentum distributions of the electrons within two $2\mu m \times 2\mu m$ regions at 200fs. The regions are centred at $x = 2\mu m$, $y = 0\mu m$ (front) and at $x = 8\mu m$, $y = 0\mu m$ (back). Each row of three plots shows the same momentum distribution with different scales in order to clearly show the various parts of the distribution.</td>
<td>163</td>
</tr>
<tr>
<td>7.35</td>
<td>Background electron temperature lineouts along $y=0$ at 50 fs.</td>
<td>166</td>
</tr>
<tr>
<td>7.36</td>
<td>Background electron temperature lineouts along $y=0$ at 100 fs.</td>
<td>166</td>
</tr>
<tr>
<td>7.37</td>
<td>Background electron temperature lineouts along $y=0$ at 150 fs.</td>
<td>167</td>
</tr>
<tr>
<td>7.38</td>
<td>Background electron temperature lineouts along $y=0$ at 200 fs.</td>
<td>167</td>
</tr>
</tbody>
</table>
7.39 Comparison of the background electron temperatures found from electron energies (left) and from $E/J$ and Spitzer resistivity assumptions (right) at 100fs for (a,b) $Z=1$, (c,d) $Z=3$ and (e,f) $Z=5$. Electrons with energies less than 20keV have been considered as background electrons.

7.40 Comparisons of the background electron temperatures found from electron energies and from $E/J$ and Spitzer resistivity assumptions for $Z=1$ at (a,b) 50fs, (c,d) 100fs and (e,f) 150fs. Electrons with energies less than 20keV have been considered as background electrons.

7.41 Comparisons of the background electron temperatures found from electron energies and from $E/J$ and Spitzer resistivity assumptions for $Z=3$ at (a,b) 50fs, (c,d) 100fs and (e,f) 150fs. Electrons with energies less than 20keV have been considered as background electrons.

7.42 Comparisons of the background electron temperatures found from electron energies and from $E/J$ and Spitzer resistivity assumptions for $Z=5$ at (a,b) 50fs, (c,d) 100fs and (e,f) 150fs. Electrons with energies less than 20keV have been considered as background electrons.

7.43 The factor difference between the resistivity calculated from assuming that $\eta = E/J$ and the resistivity calculated using Spitzer theory (electrons with energies less than 20keV have been considered as background electrons and negative values indicate that the resistivity calculated using the particle data is the larger of the two).

7.44 LSP simulation results showing the z-component magnetic Field. From top to bottom the plots show the cases of $Z=3$, $Z=5$ and $Z=10$ respectively and from left to right the plots are for transverse beam temperatures of 100keV, 200keV and 500keV.

7.45 LSP simulation results showing the background electron temperatures. From top to bottom the plots show the cases of $Z=3$, $Z=5$ and $Z=10$ respectively and from left to right the plots are for transverse beam temperatures of 100keV, 200keV and 500keV.
List of Tables

1 Commonly used symbols (SI units) ................................. 15

5.1 The relationship between the number of particles per cell / grid size
used in a simulation and the amount of numerical heating that occurs in 500fs. The time taken to complete the simulations is also shown. All of the test runs were carried out on a $100 \times 100$ cell grid with periodic boundaries. This table shows the results of tests using $2^{nd}$ order particle weighting. ........................................ 94

5.2 The relationship between the number of particles per cell / grid size
used in a simulation and the amount of numerical heating that occurs in 500fs. The time taken to complete the simulations is also shown. All of the test runs were carried out on a $100 \times 100$ cell grid with periodic boundaries. This table shows the results of tests using $4^{th}$ order particle weighting. ................................. 94
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z</td>
<td>Atomic number</td>
</tr>
<tr>
<td>e</td>
<td>Electron charge</td>
</tr>
<tr>
<td>$m_e$</td>
<td>Electron mass</td>
</tr>
<tr>
<td>$n_e$</td>
<td>Electron number density</td>
</tr>
<tr>
<td>c</td>
<td>Speed of light</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Density</td>
</tr>
<tr>
<td>R</td>
<td>Radius</td>
</tr>
<tr>
<td>v</td>
<td>Velocity</td>
</tr>
<tr>
<td>p</td>
<td>Momentum</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Lorentz factor</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$v/c$</td>
</tr>
<tr>
<td>J</td>
<td>Current density</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Resistivity</td>
</tr>
<tr>
<td>T</td>
<td>Temperature</td>
</tr>
<tr>
<td>$\omega_L$</td>
<td>Laser frequency</td>
</tr>
<tr>
<td>$F_p$</td>
<td>Ponderomotive force</td>
</tr>
<tr>
<td>$\Phi_p$</td>
<td>Ponderomotive potential</td>
</tr>
<tr>
<td>$\omega_p$</td>
<td>Plasma frequency</td>
</tr>
<tr>
<td>$\tau_p$</td>
<td>Plasma period $(2\pi/\omega_p)$</td>
</tr>
<tr>
<td>$n_c$</td>
<td>Critical density</td>
</tr>
<tr>
<td>a</td>
<td>Normalised vector potential</td>
</tr>
<tr>
<td>$a_0$</td>
<td>Peak normalised vector potential</td>
</tr>
<tr>
<td>E</td>
<td>Electric field strength</td>
</tr>
<tr>
<td>B</td>
<td>Magnetic field strength</td>
</tr>
<tr>
<td>A</td>
<td>Magnetic vector potential</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Electric potential</td>
</tr>
<tr>
<td>$\lambda_d$</td>
<td>Debye length</td>
</tr>
<tr>
<td>$b_0$</td>
<td>Impact parameter for $90^\circ$ Coulomb collisions</td>
</tr>
<tr>
<td>$\ln \Lambda$</td>
<td>Coulomb logarithm ($= \ln \frac{\lambda_d}{b_0}$)</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

1.1 Thesis outline

This thesis is dedicated to further understanding the physics arising from the interactions between ultra high intensity lasers and solid targets. We are particularly interested in the effects that electron-ion Coulomb collisions have on the propagation of laser generated fast electrons through solid density plasma. In order to study these effects Coulomb collisions have been added to the particle-in-cell (PIC) code EPOCH and various simulations have been performed.

The accuracy and size of simulations that can be carried out is of course linked to the computing power and resources available to the researcher. In the past PIC simulations have been limited to low-density/high-temperature regimes where collisions are relatively unimportant. However, we are now able to perform particle in cell simulations of solid density plasma (∼1gcc) with temperatures as low as 100eV, a setup that until recently has only been amenable to study via alternative methods. For these high-density/low-temperature cases, Coulomb collisions are expected to become more important and may significantly alter the processes occurring within the plasma.

The inclusion of Coulomb collisions is essentially the same as including resistivity in the model and introduces a source of field generation within the plasma that is neglected in standard collisionless PIC simulations. Although the fast elec-
trons generated by the laser can be treated as being largely collisionless, the fields produced within the plasma can affect their propagation and are therefore required if simulations are to correctly and fully model the underlying physics. Understanding the propagation of these laser produced fast electrons is important to the fast-ignition branch of fusion research, as well as in understanding the physics seen in laser-plasma interaction experiments. Some of the research that has previously been carried out in order to investigate the beams of fast electrons that are produced in high intensity laser-plasma interactions is discussed in chapter 4. Much of the simulation work previously carried out has been performed using the hybrid model approach, in which the fast particle beams are treated kinetically and the background is treated using fluid equations. The reasons why collisional particle in cell simulations are an important supplement to the hybrid simulation results are also discussed in chapter 4.

In order to perform the simulations required to investigate the effects Coulomb collisions have on fast electron transport Coulomb collisions have been added to the PIC code EPOCH (Extendable PIC Open Collaboration H). The simulation setup used corresponds to a $5 \times 10^{19} W cm^{-2}$, $1.05 \mu m$ wavelength laser incident on a plasma slab with a particle density of $3 \times 10^{29} m^{-3}$ ($\sim 1 gcc$). Four simulations have been performed in total; a collisionless simulation and three collisional simulations with $Z$ values of 1, 3 and 5. The results from these simulations are discussed in chapter 7. It is shown that the increase in $Z$ in the collisional cases leads to stronger electric and magnetic fields being recorded within the target and larger scale filaments being seen within the magnetic field plots. The increase in target $Z$ is also seen to lead to a reduction in the divergence angle of the fast electron beam, as well as an increase in the background electron temperatures recorded. For all values of $Z$ the background electron temperatures from the simulations are in good agreement with temperatures calculated by assuming Ohms law and Spitzer resistivity (equations 2.31 and 3.37), as long as the background plasma electron distributions remain Maxwellian. Several plots from hybrid simulations performed using LSP with a similar setup are also shown in chapter 7 and although broadly similar they do show some differences to the collisional PIC code results.
The remainder of this chapter provides an introduction to the fields of laser-plasma interactions and inertial confinement fusion. The importance of improving our understanding of the physics relating to fast electron transport is outlined, with emphasis given to the implications for fusion schemes.

A brief summary of the remaining chapters in this thesis follows:

**Chapter 2**
The theory of laser-plasma interactions is discussed, with emphasis given to the basic properties of fast electron transport.

**Chapter 3**
The theory of Coulomb collisions between particles in a plasma is outlined.

**Chapter 4**
This chapter contains a review of previous work performed in order to study fast electron transport. The reasons why collisional PIC simulations are required in order to supplement previous work performed using hybrid codes is also outlined.

**Chapter 5**
The particle in cell simulation method is outlined. Particular attention is given to the various particle weighting schemes as it is the use of higher order particle weighting within EPOCH that allows for the simulation of solid density plasma without excessive numerical heating.

**Chapter 6**
The way in which Coulomb collisions were added to EPOCH is described.

**Chapter 7**
The data from the four simulations that have been performed is discussed and analysed. Particular attention is given to the fields and associated filaments within the targets. The divergence of the fast electron beam, the energy distributions of the beam and background electrons and the background electron momentum distributions are also examined. The differences between
the collisional PIC results and results from hybrid simulations are also dis-
cussed and the collisional PIC results are used to examine the validity of the
Spitzer resistivity approximation used in hybrid simulations.

Chapter 8
A summary of the results is given and useful future extensions to the work are outlined.
1.2 Laser-Plasma interactions

The field of laser-plasma interactions is an area of modern physics which has been steadily growing for a number of years. It has been a quickly advancing field, both in the experiments being carried out around the world and in the computer simulations being performed. Since the introduction of techniques such as chirped pulse amplification [2] and mode locking, the powers and intensities obtainable using lasers have rapidly increased, leading to new areas of research becoming accessible. Possible applications of high intensity lasers include particle acceleration, fast ion generation, experimental astrophysics and the fast ignition of fusion targets to name but a few. The peak laser intensities obtainable are increasing all the time, with intensities beyond $10^{22} \text{Wcm}^{-2}$ already demonstrated [3]. The main laser at the Central Laser Facility in the UK, Vulcan [4], is capable of reaching petawatt powers and can reach intensities of $10^{21} \text{Wcm}^{-2}$. The electric field corresponding to this intensity is of the order of $10^{14} \text{Vm}^{-1}$. To put this in perspective, the average field strength felt by electrons within atoms is only $E_{av} = 5 \times 10^{11} \text{Vm}^{-1}$ (for ground state hydrogen). Therefore, lasers of these intensities can easily ionise targets, forming plasma which contains electrons that have been accelerated to relativistic velocities. Indeed, lasers with intensities greater than $10^{11} \text{Wcm}^{-2}$ can cause targets to ionise, forming plasma.

In ultra-intense laser fields electrons behave rather differently to how they do in weaker laser beams. A non-relativistic laser beam is characterised by $a_0 < 1$ (where $a_0 \equiv v_0/c = eE_0/m_e\omega_L = eB_0/m_e\omega_L$ is the normalised vector potential). For small values of $a_0$ an electron will oscillate in the laser field with only a small motion in the direction of the laser beam propagation. However, when $a_0 \geq 1$ the electron motion in the direction of the laser propagation will grow larger and as $a_0$ increases further this motion in the laser propagation direction will become dominant. The time average of this oscillatory force is known as the ponderomotive force, $F_p = -m_e c^2 / 4\gamma \nabla a_0^2$ (for linearly polarised light), and it acts to expel particles from regions of higher intensity. In ultra-high intensity laser-plasma interactions the ponderomotive force will produce a beam of fast electrons, moving in the propagation direction of the electromagnetic wave. In simulations as much
as 50% of the incident laser energy is seen to be passed on to electrons, giving rise to electrons with MeV energies [5] which travel largely unimpeded into the target. The energy of these fast electrons is comparable to the ponderomotive potential \( \Phi_p = m_e c^2 \left[ (1 + a_0^2)^{1/2} - 1 \right] \).

The charge separation resulting from the beam of fast electrons moving into the target will cause a thermal return current to form in order to maintain charge neutrality within the target. These two coincident currents are unstable to a variety of instabilities with the fastest growing being the electrostatic two-stream and the electromagnetic Weibel instabilities [6].

The variation between the various scale lengths (such as the Debye length, the collisionless skin depth and the penetration depth of the fast electrons) that are important in fully understanding the various processes involved in fast electron transport result in it being an area of physics that is difficult to accurately model. Several studies have previously been carried out using the hybrid code method and have shown promising results in regards to focusing the fast electron beams. Focussing the fast electrons as they travel through the target is important to fast ignition research as the energy contained in the beam must be deposited into a small enough area for the fusion process to begin.

In this thesis some of the problems faced when attempting to simulate these extreme conditions are discussed. A collisional particle in cell model has been used to look at the propagation of fast electrons and the results have been compared to results produced using the hybrid model method. Although hybrid codes have previously been used to produce useful results and insights, using a collisional PIC model to investigate fast electron transport is an important step in improving our understanding of the underlying physics. This is because the PIC method is largely free of the assumptions that are made when carrying out hybrid simulations. For example, the PIC method solves the full Maxwell equations without neglecting the displacement current (which is needed to correctly model the various instabilities), requires no assumption of Ohm’s law and Spitzer resistivity and allows for background plasma to be non-Maxwellian.

The complex physics involved in the behavior of fast electron beams and the
associated return currents, along with the possible fusion implications, make this an area of physics that is at the forefront of current research.

1.3 Inertial Confinement Fusion

1.3.1 The current state of Inertial Confinement Fusion research

Fusion is seen as the holy grail of electricity sources for the future. As fossil fuels are used up and become more and more expensive alternate sources of electricity become increasingly in demand. There are several possible solutions including renewable energies such as wind, hydro and solar power, as well as using traditional fission power plants. However, fusion is seen as the ultimate aim for future energy production and is expected to be able to provide an abundant supply of clean, safe energy. The idea of creating electricity directly from mass in a fusion power plant is not a new one and scientists have been working towards this goal for many decades. The National Ignition Facility (NIF) in California was built in order to demonstrate ignition and show a gain in energy from a fusion experiment. Scientists are hopeful that such proof of principle experiments will be possible in the near future. Moving on from the proof of principle experiments, the next step will be to turn fusion in to a viable energy source for the future. In Europe for instance, the HiPER (High Power laser Energy Research facility) project has been proposed as a possible next step. The aim of HiPER (or a similar future project) will be to move forwards from the proof of principle experiments and to begin demonstrating the feasibility of commercial laser-fusion reactors.

1.3.2 Inertial Confinement Fusion - Central Hot Spot ignition and Fast Ignition

The desire to make commercial fusion energy a reality is prevalent in of much of the the current research into plasma and high energy density physics. A lot of work has been (and is being) performed in order to further understand the physics in-
Inertial Confinement Fusion

1.3 Inertial Confinement Fusion

Figure 1.1: (a) An artists rendering of a NIF hohlraum. (b) Inside the NIF target chamber. Credit: Lawrence Livermore National Laboratory (https://lasers.llnl.gov)

involved in starting and controlling the fusion process. Perhaps the simplest method of releasing energy from a fusion reaction is by combining deuterium and tritium via the reaction:

\[ D + T = ^4He + n \]  \hspace{1cm} (1.1)

This reaction gives an energy excess of 17.6MeV (14.1MeV with the neutron and 3.5MeV with the \(^4He\)). For the reaction to begin an initial spark is required to ignite the fuel, providing the Lawson criterion \((\langle \rho R \rangle \gtrsim 0.3 \text{gcm}^{-2})\) is met. This condition comes from the fact that the alpha particle reaction product would need to be reabsorbed into the fuel in order to trigger further reactions.

Conventional inertial confinement fusion (ICF) [7] uses spherical fuel capsules which comprise of a hollow shell (~2mm diameter) with a layer of DT ice on the inner surface. The fuel capsules are imploded either directly using lasers (direct drive), or by thermal x-rays created using a hohlraum (indirect drive). An artists impression of a hohlraum is shown in figure 1.1(a). The driver energy for compression is generally in the region of 1-2MJ. The indirect drive method generally results in about 15% of the laser energy being converted into ablation of the capsule, whilst the direct drive method results in around 50% energy conversion. The direct drive approach creates a lower ablation pressure but overall is about twice
as efficient at turning the laser energy into inertial energy within the imploded fuel [8]. Both processes cause the outer surface of the target to be irradiated and result in the target being compressed by ablative processes.

The implosion compresses the DT fuel and results in the formation of a hot spot at the centre of the fuel at the point of stagnation. For ignition to occur within the hot spot a temperature of around 10keV \((1.16 \times 10^8 K)\) and a density-radius product of around \(\langle \rho R \rangle \approx 0.3 gcm^{-2}\) are required [9]. In order to achieve these extreme conditions the fuel needs to be compressed to in excess of 1000 times solid density. If the conditions are all met the hot spot will ignite and fusion will begin to occur. The thermonuclear burn will then propagate out from the hot spot into the rest of the fuel. The compression and heating of the fuel must be performed quickly and accurately as the hot spot only lasts for around 10ps. In conventional ICF (known as Central Hot Spot (CHS) ignition) the fuel is compressed and a hot spot is formed in a single step.

The problem with CHS ignition is that hydrodynamic instabilities (Rayleigh-Taylor) occur easily at both the inner and outer surfaces of the fuel capsule. At the outer surface this is due to the lower density plasma being accelerated into the higher density shell and at the inner surface the collapsing shell will be decelerated by a lower density plasma that is formed from the DT gas inside the capsule [8]. The result of the instability is that the shell can break up before it has fully imploded and that even if the it does undergo enough compression parts of the cooler fuel may enter the hot spot and stop ignition from occurring. To prevent the instability from occurring very high levels of symmetry are needed. The target needs to be made with extremely high precision in order to make sure that it is as close to perfectly spherical as possible, with minimal aberrations to its surfaces. The timing and power of the laser beams used must also be extremely precise. It is relatively simple to make sure that the laser beams all arrive at the same time but it is much harder to make sure that all the beams deliver the same amount of energy. Beam anisotropy can cause uneven compression, setting up the Rayleigh-Taylor instabilities which result in unwanted mixing of the fuel which reduces the heating efficiency.
It has been proposed that a compressed ICF fuel could be ignited using ultra-high intensity lasers [10]. This 'Fast Ignition' (FI) scheme differs from conventional ICF as the process is split into three distinct steps. Firstly, a capsule surrounding the fuel is imploded, forming a high density core. Secondly, a hole is made in the coronal plasma, either via the ponderomotive force [5][11] or by using a gold cone in order to maintain a plasma free channel into the dense core [12][13] (and references within). Finally, the core is ignited using energetic particles generated by an ultra-high intensity laser which is focused directly into the hot core.

The fast ignition method is predicted to have a maximum gain 2-5 times larger than in CHS ignition. As well as this the fast ignition method has a lower ignition threshold and the most difficult part of conventional ICF (overcoming the symmetry and stability issues involved in the formation of the central hot spot) is removed leaving just the problem of achieving enough compression [8]. The higher gain and lower ignition threshold in the fast ignition scheme are due to the fact that in fast ignition the main fuel density may be lower than is required for CHS ignition. This means that the technical challenge is shifted away from the symmetry and stability issues of conventional ICF to creating short, intense laser pulses, and exploring how energy is transported in the conditions created by lasers of such high intensities.
When first introduced by Tabak et al. in 1994, it was proposed that a powerful pre-pulse laser beam (with a duration of around $10^{-10}$ s) would push radially through the plasma (formed from the pre-compressed target) via the ponderomotive force, creating a channel through to the critical density surface (and beyond due to hole boring). A second more powerful laser would then be focused into this channel (see figure 1.2) where a large amount of the laser energy would be absorbed (typically around 30%), resulting in electrons being thrown into the target with MeV energies [14][15]. This stream of ‘fast’ electrons would then be required to travel into the target and deposit their energy in the hot spot region. Electrons with energies of approximately 1-2 MeV are expected to have mean free paths of the magnitude required for the energy to be deposited into the required hot spot region. The recognised optimum fuel density for fast ignition is $300\text{gcm}^{-3}$ ($3\times10^5\text{kgm}^{-3}$) [8] and a hot spot energy of around 20 kJ is required for the fusion reaction to begin. This energy must be supplied into a volume with a radius of \(~20\mu\text{m}\) at a depth of around 20-40\(\mu\text{m}\) and must be supplied in less than 20\(\text{ps}\) [16].

Since the laser pulse duration is long compared to the plasma period ($\tau_p = \frac{2\pi}{\omega_p}$) the plasma remains close to having charge neutrality throughout this process. This means that the current of the fast electron beam generated by the high intensity laser must be opposed by a return current formed from thermal electrons in the background plasma [17]. This return current will be resistive so a non-uniform electric field will be produced, in turn making a magnetic field, both of which will affect the motion of the electrons. If the target is unable to allow this return current to form a space charge separation will quickly build up and prohibit the fast electrons from moving deeper into the target.

The inhibition of fast electron transport that will occur if there is no charge neutralising return current can be shown with a simple calculation. Using approximate values for the Vulcan PW laser give a power of $10^{15}$ W which may be focused into a 5\(\mu\text{m}\) spot radius. The corresponding fast electron energy is around 10 MeV, assuming that 50% of the laser energy is absorbed into the fast electrons. The current in the fast electron beam may then be estimated by using $J = \frac{I_{\text{abs}}}{\phi_0}$, where $J$ is the current density, $I_{\text{abs}}$ is the absorbed power per unit area and $\phi_0$ is the aver-
age electron energy. This means that the current flowing in the fast electron beam will be approximately 50MA. If we consider the area around the laser focus as a simple capacitor we see that in a time of 0.1ps it would charge up to a voltage of $\sim 10000\text{MeV}$. This is easily enough to prevent our 10MeV electrons from escaping the region immediately behind the laser focus. Indeed, this area would charge up to 10MeV in as little as $0.1\text{fs}$, which is less than the laser period. Therefore for a beam of fast electrons to propagate into a material a return current must be supplied so that charge neutrality is retained. This is an important factor in fast ignition as the inhibition of fast electron transport would mean either that the laser energy would need to be absorbed deeper into the dense core, or that a more powerful laser would be required for the fast electrons to reach the hot spot.

Due to the neutralising return current the fast electron beam may have a cur-
rent far in excess of the Alfvén current [18] \((I_A = 17\beta\gamma, \text{ where } \beta \text{ and } \gamma \text{ are the standard Lorentz factors})\). The main processes involved in the propagation of the fast electron beam and the associated return current are shown in figure 1.3. The two counter streaming beams must be coincident to within about a collisionless skin depth \((c/\omega_p)\), the reasons for which are discussed in chapter 2.4. The collisionless skin depth typically approximates to the laser wavelength, but may become larger when the fast electron beam density drops. The counter streaming electron currents will be unstable to a variety of instabilities, the fastest growing being the transverse Weibel electromagnetic instability and the electrostatic two stream instability [19][20]. The result of these instabilities is that the fast electron beam will break up and form several filaments. These filaments will then undergo complex interactions and can eventually, under certain conditions, join to form a well collimated beam [21][22].

The interaction between ultra-high intensity lasers and solid density plasmas produces a wealth of interesting effects including the production of MeV electrons. The fast electron beam is essentially collisionless and can have a range of hundreds of \(\mu m\), which is in stark contrast to the collisional return current and the cold plasma Debye length which can be shorter than \(10^{-2}\mu m\) in solid density plasma [23]. It is the variation in the magnitudes of the different scale lengths and the links between the various processes that make this an area of physics that is difficult to model in its entirety. The methods and codes used to examine these processes are constantly evolving, but the models still need to be improved further in order to fully account for the variety of complex physical processes that occur in these extreme conditions.
Chapter 2

Laser-Plasma Interactions And Fast Electron Transport

2.1 Electromagnetic fields in plasma

The peak intensities attainable by modern day lasers easily exceeds the threshold required to break materials down into their constituent parts and form plasma. Therefore, when examining laser-solid interactions (for laser intensities higher than \(10^{11} \text{ W cm}^{-2}\)) plasma physics must be used in order to accurately describe the processes that occur.

In a vacuum electromagnetic fields of any size may propagate freely. However, within plasma electromagnetic waves will push particles leading to charge separations. Electromagnetic radiation of a specific frequency is unable to propagate in plasma with a density larger than a corresponding critical density, \(n_c\). This critical density comes from the dispersion relation for electromagnetic waves in plasma [24]:

\[
\omega^2 = \omega_{pe}^2 + c^2 k^2
\]

where \(\omega_{pe}\) is the electron plasma frequency, commonly referred to simply as the plasma frequency. The dispersion relation comes from Maxwell’s equations and
2.1 Electromagnetic fields in plasma

the Lorentz force equation (looking at a non-relativistic case where the $v \times B$ term
is unimportant - see equation 2.8). The frequency of electromagnetic radiation that
can be supported by a plasma is limited by the plasma frequency

$$\omega_{pe} = \sqrt{\frac{n_e e^2}{\epsilon_0 m_e}} \quad (2.2)$$

which arises from looking at the oscillation of electrons around their equilibrium
positions, for which a harmonic oscillator result is formed. If the laser intensity
is large enough to warrant a relativistic treatment of the electrons, the relativistic
electron plasma frequency is obtained by replacing $m_e$ by $\langle \gamma \rangle m_e$ where $\gamma$ is the
usual Lorentz factor. If $\omega_{pe}$ is larger than $\omega_L$ (the laser frequency) the laser will
not be able to propagate into the plasma. By substituting $\omega_{pe} = \omega_L$ we can calcu-
late the critical density that a specific frequency of electromagnetic radiation may
propagate up to:

$$n_c = \frac{\omega_L^2 \epsilon_0 m_e}{e^2} \quad (2.3)$$

For example, a laser with $\omega_L = 2 \times 10^{15}\text{rad/s}$ (which is the case for a laser with a
wavelength of 1$\mu$m) may only propagate up to a critical particle number density
of $1.25 \times 10^{27} \text{m}^{-3}$. A Plasma with a density higher than the critical density is com-
monly referred to as being an over-dense plasma, whereas a plasma whose density
is lower than the critical density is commonly referred to as being an under-dense
plasma.

An important point to come out of this is that in any interaction between a high
intensity laser and a solid target, the actual laser-plasma interaction is confined to
a small area of low density plasma at the front of the target. To look in more detail
at the laser-plasma interaction we must first look at how a laser interacts with a
single charged particle.
2.2 The interaction between a single particle and a laser field

2.2.1 The ponderomotive force

The ponderomotive force acting on particles within a laser field will act to expel the particles from regions of high intensity. For example, consider that an electron oscillating within a laser pulse will move into regions of different intensities as it oscillates. If the electron moves from a region of higher intensity to a region of lower intensity in the first half of its quiver motion it will move down the intensity gradient with a force stronger than the return force it will experience moving up the intensity gradient in the second half of its motion. In a laser-plasma interaction this affect will cause a depletion of electrons from the laser focal region. This will result in an area of lower electron density being built up around the laser focus, until the ponderomotive force is balanced by the force created from the charge separation produced. For laser intensities lower than $10^{18} W cm^{-2}$ the velocity of an electron in an electromagnetic field is non-relativistic and the magnetic component of the force equation may be neglected to first order. Let the electric field given by $E = E(r) \cos(\omega t)$ (in $Vm^{-1}$) act on an electron oscillating about its equilibrium position. By Taylor expanding the electric field, looking at the oscillation velocity to first order and averaging over a laser period we may write the ponderomotive force on the particle as

$$F_p = -\frac{e^2}{2m_e\omega_L^2} \nabla E^2.$$  \hspace{1cm} (2.4)

From equation 2.4 it is immediately clear that the force act to will push electrons (and ions) away from regions of higher intensity. It is convenient here to introduce the normalised vector potential, $a$, which is defined as

$$a = \frac{Ee}{m_e c \omega_L}.$$  \hspace{1cm} (2.5)

The oscillatory velocity of electrons in the laser field becomes relativistic as $a_0$ (the peak normalised vector potential) approaches 1, and becomes strongly relativistic as $a_0$ is further increased. By combining equations 2.4 and 2.5 the ponderomotive
force may be rewritten as
\[ F_p = -\frac{m_e c^2}{2} \nabla a^2. \] (2.6)

It is common to use time averaged values when discussing laser plasma interactions, and time averaging the normalised vector potential yields \( \langle a^2 \rangle = \frac{a_0^2}{2} \) for linearly polarised light and \( \langle a^2 \rangle = a_0^2 \) for circularly polarised light. The ponderomotive potential may also be defined as
\[ \Phi_p = \frac{e^2}{4m_e \omega_L^2} E_0^2. \] (2.7)

However, when the velocity of the particle becomes relativistic the force due to \( B \) can no longer be ignored and the relativistic version of the ponderomotive force must now be considered [26][27][28]. The starting point for looking at the relativistic ponderomotive force is the relativistic Lorentz force equation
\[ \frac{dP}{dt} = q \left( E + v \times B \right) \] (2.8)

where \( P = \gamma mv \) is the relativistic particle momentum, \( E \) and \( B \) are the electric and magnetic fields respectively and \( q \) is the particles charge. By replacing \( E \) and \( B \) using the magnetic vector potential, \( A \), and the electric potential, \( \phi \), defined by
\[ E = -\nabla \phi - \frac{\partial A}{\partial t} \] (2.9)

and
\[ B = \nabla \times A \] (2.10)

the Lorentz force equation may be rewritten as
\[ \frac{dP}{dt} = q \left( -\frac{\partial A}{\partial t} - \nabla \phi + v \times \nabla \times A \right). \] (2.11)

Rearranging and using the convective derivative then yields
\[ \frac{d}{dt} (P + qA) = q((\nabla A) \cdot v - \nabla \phi). \] (2.12)

From left to right, the terms in this equation describe the rate of change of canonical momentum, the coupling of electromagnetic field and current density and the electrostatic force that is built up due to the the charge separation. This equation can then be rewritten in terms of the canonical momentum \( u = P - eA \) to give
\[
\frac{du}{dt} = -\frac{e^2 \nabla A^2}{2\gamma m} - \frac{e \nabla A \cdot u}{m \gamma} + e \nabla \phi. \tag{2.13}
\]

The ponderomotive force on a single electron is given by the first term on the right hand side of equation 2.13. To show this we can look at the time average of this equation assuming a high frequency electromagnetic wave. Averaging over a time period \(\Delta T = \frac{2\pi}{\omega}\) gives

\[
\frac{1}{\Delta T} \int_T^{T+\Delta T} \frac{du}{dt} dt = -\frac{1}{\Delta T} \int_T^{T+\Delta T} \frac{e^2}{2\gamma m} \nabla A^2 dt + \frac{1}{\Delta T} \int_T^{T+\Delta T} e \nabla \phi dt - \frac{e}{m \Delta T} \int_T^{T+\Delta T} \nabla A \cdot \frac{u}{\gamma} dt \tag{2.14}
\]

which can be simplified to

\[
\langle \frac{du}{dt} \rangle = -\langle \frac{e^2}{2\gamma m} \nabla A^2 \rangle + e \langle \nabla \phi \rangle - \frac{e}{m} \nabla \langle A \rangle \cdot \frac{u}{\gamma}\bigg|_T^{T+\Delta T} + \int_T^{T+\Delta T} \frac{e}{m} \langle A \rangle \cdot \frac{u}{\gamma} dt \tag{2.15}
\]

The last two terms in equation 2.15 tend to zero because we have assumed that the envelope of the vector potential has a scale length much longer than the fast oscillations (i.e. \(\langle A \rangle \approx 0\)). For the ponderomotive force on a single particle the electrostatic term may also be neglected, leaving the relativistic ponderomotive force on a single electron to be written as

\[
\langle \frac{du}{dt} \rangle = -\langle \frac{e^2}{2\gamma m} \nabla A^2 \rangle - \frac{m_e c^2}{2 \langle \gamma \rangle} \nabla \langle a^2 \rangle \tag{2.16}
\]

where the final alteration is made by noting that \(a = eA/m_e c\). The time averaging produces a result similar to the non-relativistic case, the difference being the that a factor of \(\langle \gamma \rangle\) has been introduced to the equation. As with the non-relativistic version, the dependence on the square of the charge implies that the force acts to expel both positively and negatively charged particles from regions of higher intensity.
2.2 The interaction between a single particle and a laser field

2.2.2 Electron trajectories and momentum relationships

The trajectories of particles in an intense laser field (where \( a_0 \gtrsim 1 \)) may be analytically calculated using the Lorentz force equation. Consider a plane wave traveling in the x-plane with \( A = A_y \hat{y} \) (so \( \frac{\partial A_y}{\partial y} = 0 \)) interacting with an electron that is initially at rest. Starting with equation (2.12), for a single electron case (meaning \( \nabla \phi = 0 \)) and first considering the \( \hat{y} \) direction, the conservation of canonical momentum

\[
\frac{d}{dt}(P_y - eA) = 0
\]  
(2.17)

may be rearranged to give the first momentum relation

\[
\frac{P_y}{mc} = a. 
\]  
(2.18)

Taking the x-component instead gives

\[
c\frac{dP_x}{dt} = -ce\frac{\partial A_y}{\partial x}v_y
\]  
(2.19)

and combining equation 2.18 with \( P^2 = P_x^2 + P_y^2 = (\gamma^2 - 1)mc^2 \) gives

\[
\frac{P_x^2}{mc^2} = \gamma^2 - 1 - a^2.
\]  
(2.20)

The next step is to take the x-component of the energy equation \( (mc^2 \frac{d\gamma}{dt} = -ev \cdot E) \), yielding

\[
mc^2 \frac{d\gamma}{dt} = -evy \frac{\partial A_y}{\partial t}
\]  
(2.21)

and then to subtract equation 2.19, which gives

\[
c\frac{dP_x}{dt} - mc^2 \frac{d\gamma}{dt} = -ev_y(c \frac{\partial A_y}{\partial x} - \frac{\partial A_y}{\partial t}).
\]  
(2.22)

The right hand side of this equation disappears for a wave of the form \( A_y = A_{y0}e^{i(kx + \omega t)} \). Integrating the left hand side of the equation gives

\[
\frac{P_x}{mc} = \gamma - 1.
\]  
(2.23)

The second momentum relation is now given by eliminating \( \gamma \) form equations 2.20 and 2.23 and can be written as

\[
\frac{P_x}{mc} = \frac{a^2}{2}.
\]  
(2.24)
2.2 The interaction between a single particle and a laser field

Combining the two momentum relationships finally yields

\[ P_x = \frac{P_y^2}{2}, \]  

(2.25)

where the normalisation \( P \rightarrow P/m_e c \) has been used.

By time averaging equation (2.24) we see that the electromagnetic wave will induce a constant drift in the x-direction when \( a_0 \) is much larger than unity. In the lab frame this drift is given by

\[ \langle v_D \rangle = \frac{c a_0^2}{4 + a_0^2} \hat{x}. \]  

(2.26)

To further understand the particle motion we convert the two momentum relations into the wave frame where \( \tau = t - \frac{x}{c} \). The x and y components of an electron’s trajectory in a laser field can then be found by integrating the momentum relations, assuming a wave of the form \( a = a_0 \cos(\omega \tau) \), yielding

\[ y(\tau) = \frac{c a_0}{\omega} \sin(\omega \tau) \]  

(2.27)

\[ x(\tau) = \frac{c a_0^2}{4} \tau + \frac{c a_0^2}{8\omega} \sin(2\omega \tau) \]  

(2.28)

These equations show that there is a constant oscillation in the y-direction and that on top of the constant drift in the x-direction there is an oscillation at twice the laser frequency. This means that the electron trajectory in the electron guiding

Figure 2.1: The trajectory of an electron in a linearly polarised electromagnetic wave in (a) the wave frame and (b) the electron guiding centre frame.
centre frame will be a ‘figure of eight’ motion, whilst in the wave frame it will have this motion with an added constant drift. The trajectories in both frames are shown in figure 2.1.

2.3 Laser energy absorption mechanisms

There are several mechanisms in which laser energy may be absorbed into a plasma and the importance of each depends on the intensity of the laser being used. For moderate laser intensities inverse bremsstrahlung and resonance absorption are the dominant absorption mechanisms, however, as the laser intensity is increased beyond $I\lambda^2_{\mu} > 10^{18} W cm^{-2} \mu m^2$ vacuum and $J \times B$ heating mechanisms become the dominant processes. A review of the main absorption mechanisms occurring in laser-plasma interactions has been produced by Wilks and Kruer [29] and more detail can be found in books such as Kruer’s ‘The Physics Of Laser Plasma Interactions’ [24].

2.3.1 Inverse Bremsstrahlung

Inverse Bremsstrahlung is the process responsible for the majority of the heating occurring in plasmas interacting with lasers of relatively low intensities (i.e. $I\lambda^2_{\mu m} \lesssim 10^{15} W cm^{-2} \mu m^2$) [30]. The heating occurs because the electrons oscillate in the laser field as the laser propagates through the underdense plasma. As they oscillate some of the electrons will undergo collisions with ions, transferring some of the quiver energy into thermal energy which heats up the plasma. The collisions damp the motion of the electrons, hence reducing the energy contained within the laser field. Energy absorption through inverse Bremsstrahlung is more effective for high density, high $Z$ and low temperature plasmas. At higher laser intensities the electron quiver motion becomes large enough that the electrons essentially become collisionless and the importance of inverse Bremsstrahlung heating decreases.
2.3 Laser energy absorption mechanisms

2.3.2 Resonance absorption

Resonance absorption is a collisionless absorption mechanism that results in the laser energy being converted into plasma waves. When a laser is incident at an angle on a plasma with a density profile such that \( \mathbf{E} \cdot \nabla n_e \neq 0 \) the energy of the electromagnetic wave will become coupled with oscillations of the electrons, creating fluctuations in the charge density of the plasma. As previously noted, a laser may not propagate through a plasma beyond the critical density, \( n_c \), however, if the electric field oscillates at an angle \( \theta \) to the target normal the laser will only be able to propagate up to a density of \( n_c \cos^2 \theta \) before being reflected. When the laser light is polarised such that a component of its electric field lies in the direction of the density gradient the electric field component parallel to the density gradient may tunnel through to the critical density and excite plasma waves. These plasma waves will then be damped and a fraction of the lasers energy will be transferred to the plasma [29][31].

2.3.3 Vacuum heating

Vacuum (or Brunel) heating [32] occurs when a linearly polarised laser pulse is incident on an overdense plasma that has a sharp density jump. Unlike resonance absorption vacuum heating occurs due to electrons at the edge of the plasma being ejected into the vacuum by the laser field. As the polarity of the laser field changes these ejected electrons can be be accelerated back towards the plasma with a velocity approximately equal to the quiver velocity. The electric field of the laser pulse will only penetrate a short distance into the overdense plasma (approximately equal to the skin depth, \( c/\omega_p \)) and the accelerated electrons may travel beyond this distance, hence escaping from the effects of the laser field. The accelerated electrons are then free to travel into the overdense plasma until they are stopped by collisions.
2.3.4 $J \times B$ heating

$J \times B$ absorption arises due to the ponderomotive force and occurs when the laser intensity is high enough to make electrons oscillate with relativistic velocities, which causes the magnetic component of the Lorentz force equation describing the electrons motion to become comparable to the electric field component. The result of this is that twice per laser wavelength the magnetic field will drive the oscillating electrons in the laser propagation direction, resulting in bunches of oscillating electrons travelling into the target at a frequency of $2\omega_L$ [33]. The amount of kinetic energy that the electrons receive from the laser is comparable to the ponderomotive potential and the effective temperature of the accelerated electrons scales with the laser intensity as $t_h \propto (I\lambda^2)^{1/3}$ [5].

2.4 Fast Electron Transport

The interactions between ultra intense lasers and dense plasmas gives rise to high energy (MeV) electrons which will propagate through the plasma target. These electrons are thrown into the target twice per laser cycle by the oscillating ponderomotive force and will have a distribution of energies that may be compared to the ponderomotive potential. This ponderomotive scaling of the fast electron energy with laser intensity was found from PIC simulations performed by Wilks et al. [5], although experiments carried out by Beg et al. [34] and theoretical work performed by Haines [35] point towards a slower increase in fast electron temperature. The standard ponderomotive scaling is given by

$$t_h = \sqrt{1 + a_0^2} - 1$$

whilst Haines’ work suggests that the scaling is given by

$$t_h = \sqrt{1 + \sqrt{2}a_0} - 1$$

where $t_h$ is the normalised hot electron temperature $eT_h/m_ec^2$.

Experimental measurements of the hot electron temperature can be made ei-
ther by placing an electron spectrometer behind the target in order to detect the fast electrons that escape, or by measuring x-ray bremsstrahlung emissions. In [34] laser intensities up to $10^{19} \text{Wcm}^{-2}$ are investigated and the characteristic temperature of the fast electrons is determined by measuring the $K_{\alpha}$ x-ray emissions from layered targets. The scaling of the fast electron temperature with laser intensity found is now simply known as Beg’s law, $T_{\text{fast}} \sim (I\lambda^2\text{Wcm}^{-2})^{1/3}$. However, inferring the fast electron distribution and temperature in this manner is not perfect as assumptions must be made such as that the fast electron distribution function is Maxwellian. It is generally accepted that Beg’s scaling is correct for laser intensities up to $10^{19} \text{Wcm}^{-2}$ and that beyond this intensity the ponderomotive scaling rule is followed.

Experiments have shown that high intensity laser-plasma interactions result in a strong absorption of laser energy into a fast electron population, although the exact fraction of the laser energy absorbed into the fast electrons is seen to vary somewhat between experiments. Key et al. [14] observed generation of fast electrons with greater than 30% efficiency and a general trend of an increased amount of absorption with increasing laser intensity. More recently work by Ping et al. [36] shows that this trend continues to laser intensities above $10^{20} \text{Wcm}^{-2}$, with the total absorption reaching 60% for near normal incidence and above 80% at 45° incidence. Ping et al. also discuss PIC simulation results showing that both hole boring and a larger pre-plasma lead to an increased energy absorption.

The fast electrons are accelerated into the target between the target normal and the laser propagation direction at a variety of angles which depend on experimental conditions [36]. Typically the electrons propagate into the target in a cone of approximately 30-40° half angle ([23] and references within). The electrons can travel through a large amount of target material due to their long mean free paths and they often reach the target rear surface. As they leave the target a large positive charge will be created due to charge separation, which in turn can accelerate ions from the rear surface, as well as causing the fast electrons to turn around and travel back into the target. Of course, the fast electrons cannot simply travel through the target without having an effect on the background particle distributions. The fast
2.4 Fast Electron Transport

electrons travel into the target with a current that can exceed the Alfvén current, \( I_A = 17\beta\gamma \), where \( \beta \) and \( \gamma \) are the usual Lorentz factors (the Alfvén current is the maximum current that can flow in a single filament due to the magnetic field around the filament decreasing the Larmor radius of the propagating electrons below the beam width). Taking the basic version of Ohm’s law, the electric field that is set up due to the fast electrons is given by

\[
E = -\eta J_{\text{fast}}
\]

(2.31)

where \( \eta \) is the plasma resistivity and \( J_{\text{fast}} \) indicates the fast electron current density. The charge that the fast electrons carry must be largely neutralised by a return current created from the ‘cold’ background electrons, shown simply by stating the total current as

\[
J_{\text{total}} = J_{\text{fast}} + J_{\text{return}} \approx 0.
\]

(2.32)

We can see that this must indeed be the case by looking at a few simple arguments. Firstly, following the arguments in [37], we may look at the electric field that would grow if there were an imbalance between the two currents. The electric field generated by the imbalance will grow with \( \partial E/\partial t = -j/\varepsilon_0 \). Considering an imbalance of just 1% over 100fs leads to the electric field growing to a value of around \( 5 \times 10^{12} V m^{-1} \) (taking a value for the fast current of \( 5 \times 10^{16} Am^{-2} \) from a simulation result). This field is strong enough to stop a 1MeV electron in less than a micron. A second argument may be made by looking at how the target would charge up if the return current were not supplied. The result of this is discussed in more detail in chapter 1.3.2 but the end result is that if there is no return current a charge large enough to stop the fast electrons would build up very quickly. These two arguments show that the current must either be opposed by an electric field which stops the fast electrons at the front of the target or that the background plasma must supply a thermal return current in order to maintain charge neutrality.

A final argument may be used to discuss how close the fast electron and ther-
mal return currents must be to one another [23]. Consider the situation where an electron current with beam radius $a$ and density $n_e$ is balanced by a return current with beam radius $b$. It is clear that the magnetic field is zero for a radius $r$ that is larger than $b$. By looking at the energy stored in the magnetic field around the beam we see that the return current must flow within about a collisionless skin depth of the forward going current, else the electron beam would not have enough energy to create the required magnetic field.

The above arguments show that locally the two opposing currents must cancel out to a very good approximation. However, there is not exact cancellation between the two currents at every point in space and this discrepancy will lead to the formation of magnetic fields within the target. Combining Maxwell’s equations and equation 2.31 gives

$$\nabla \times \mathbf{B} = \mu_0 (\mathbf{J}_{\text{return}} + \mathbf{J}_{\text{fast}}) + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \quad (2.33)$$

and

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} = -\nabla \times \eta \mathbf{J}_{\text{return}}. \quad (2.34)$$

Combining these and neglecting the displacement current yields an equation for the growth of the magnetic field:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \eta \mathbf{J}_{\text{fast}} - \nabla \times \frac{\eta}{\mu_0} \nabla \times \mathbf{B}. \quad (2.35)$$

Fast electrons in solid targets can have mean free paths of 100's of $\mu m$ and collisional loss times of several ps. Therefore they are largely collisionless meaning that their transport through the target is dominated by the electric and magnetic fields present. Bell et al. [17] have produced a simple 1D, electrostatic model for predicting the depth fast electrons can reach due to the stopping power of the electric field. Davies et al. [38] have also derived expressions for the maximum electric and magnetic fields that can be generated by the fast electrons produced by a given laser intensity if there is no return current supplied. From these they calculate that the effect of the electric field cannot be neglected for laser intensities higher than around $10^{17} W cm^{-2}$ and that the magnetic field cannot be neglected for laser intensities beyond approximately $10^{16} W cm^{-2}$.
The changes in magnetic field strength resulting from the terms in equation 2.35 can act to collimate the fast electrons as they propagate through the target. The first term on the right of equation 2.35 shows that the fast electron beam is a source of magnetic field generation. This term can be split into $\eta \nabla \times J_{\text{fast}}$ and $J_{\text{fast}} \times \nabla \eta$. The first of these two terms will act to create a magnetic field around the beam resulting in collimation of the beam, whilst the second term will create an opposing magnetic field where there are gradients in the resistivity. The effects of these terms are discussed further in chapter 4, when discussing work that has been previously performed using hybrid models in order to simulate fast electron transport. The second term on the right of equation 2.35 is a diffusive term that will lead to the magnetic field being spread with a diffusion coefficient $\eta/\mu_0$. This diffusive term can often be neglected as the characteristic diffusion distance is small for conductors or ionised materials [39]. This also implies that magnetic fields created at the surface of targets will not diffuse into the target meaning the $\nabla \times \eta J_{\text{fast}}$ term will be the primary generator of magnetic field within the target. The magnetic field generation is strongly linked to the resistivity of the target which is in turn linked to the manner in which the target heats up. The heating of the target will generally lower the resistivity which in turn will reduce the magnetic field generation, hence reducing the force acting to collimate the electron beam.

The conditions required for the fast electron beam to be collimated by the magnetic field are discussed by Bell and Kingham [22]. In their work they derive a condition for the collimation of the fast electron beam. The work assumes that the resistive diffusion term in 2.35 may be neglected, that magnetic field generation is given by $\partial B/\partial t = \eta J_{\text{fast}}/R$ and that the ohmic heating of the background is given by $(3/2)ne\partial T/\partial t = \eta J_{\text{fast}}^2$ where the resistivity is found using the Spitzer formula discussed in chapter 3.6. Collimation is found to occur if $R/r_L > \theta_{1/2}^2$, where $R$ is the radius of the beam, $r_L$ is the fast electron Larmor radius and $\theta_{1/2}^2$ is the square of the half angle of the beam divergence. This means that the magnetic field is strong enough to bend the fast electron trajectory through $\theta_{1/2}$ over $R/\theta_{1/2}$, which is the distance over which the fast electron beam radius would otherwise have doubled. Collimation of the fast electron beam occurs if the condition $\Gamma > 1$ is met. If there is substantial resistive heating $\Gamma$ is given by
where $n_{23}$ is the electron density in units of $10^{23} \text{cm}^{-3}$, $T_{511}$ is the fast electron temperature in terms of 511 keV, $R_{\mu m}$ is the beam diameter in microns, $t_{\text{psec}}$ is the time in picoseconds, $P_{\text{TW}}$ is the power in the fast electron beam in TW ($P = \pi I R^2$ where $I$ is the beam intensity given by $I = J_{\text{fast}} T_{\text{fast}}$) and $\theta_{\text{rad}}$ is the half angle in radians. Most dependencies are weak due to the fact that the decrease in resistivity seen as the target heats up will reduce the the electric field required to draw the neutralising return current. Several things to note from this equation are that collimation is greater for weaker beam powers, higher $Z$ targets and for beams with a lower initial divergence. The effects of instabilities were not considered in this work and may adversely affect the collimation of the fast electron beam.

2.5 Magnetic field generation due to the thermoelectric effect

There are several ways in which strong magnetic fields can be produced in laser-plasma interactions, a summary of which may be found in [25]. At the front of the target there will be a very large magnetic field generated around the laser focal spot due to the thermoelectric ($\nabla n_e \times \nabla T_e$) effect which arises due to the $\nabla P_e$ term in Ohm’s law. When a laser interacts with a plasma with a density gradient (as occurs in the ablated plasma in laser-solid target interactions) and a temperature gradient (which occurs due to the heating of the plasma in the area around the laser focal spot) an azimuthal magnetic field is formed as shown in figure 2.2. The fields produced by this effect can be very large, especially when compared to the fields found within the target.

Further magnetic fields are also seen to form due to electron currents running along the front of targets. These fields grow for to the same reasons as the fields produced inside the target, as previously discussed in chapter 2.4.
2.6 Instabilities

The propagation of fast electrons within a plasma and the corresponding counter-flowing return current are predominately affected by two instabilities; the electrostatic two stream instability and the electromagnetic Weibel instability.

2.6.1 The two stream instability $k \parallel v$

The first of these is the two stream instability, which occurs when one group of particles is moving relative to another group. For example, let us consider the simple case when electrons in a cold, uniform, unmagnetised plasma have a velocity $v_0$ relative to the ions (which are stationary) [20]. By looking for the growth of electromagnetic waves of the form $E_1 = E e^{i(kx - \omega t)} \hat{x}$ by linearising the equations of motion for each species (and using the continuity equation and Poisson’s equation) yields a longitudinal wave dispersion relation that may be written as
If solved this dispersion relation yields a fourth order equation for \( \omega \) which may be written as \( \omega_j = \text{Re}(\omega_j) + \text{Im}(\omega_j) \). If the imaginary part is zero each root would give a possible oscillation of \( E_1 = E e^{i(kx-\omega_j t)\hat{x}} \) and there would be no growth or damping. However, if some of the roots are complex they will occur in complex conjugate pairs and there will now be a time dependence to the oscillations. The oscillations can now be written as \( E_1 = E e^{i(kx-\text{Re}(\omega_j)t)\hat{x}} e^{\text{Im}(\omega_j)t} \hat{x} \) so positive \( \text{Im}(\omega) \) corresponds to an exponentially growing wave. Because the roots appear in conjugate pairs there will always be an exponentially growing wave if the roots are not entirely real.

This process can be thought of as the opposite of Landau damping. For example, when fast electrons are travelling through a plasma the electron velocity distribution will have a bump on its tail. If a wave has a phase velocity that is in the region where the distribution slopes upwards due to the bump there will be a greater number of particles with a velocity slightly higher than the wave than than there are going slightly slower. This means that more energy is transferred from the particles to the wave than is transferred from the wave to the particles, resulting in the growth of the wave.

### 2.6.2 The Weibel (filamentation) instability \( k \perp v \)

Unlike the two stream instability the filamentation instability is an electromagnetic instability. The simplest description of this instability is to imagine that there are two counter-streaming electron currents in the y-direction, but there is no net current flow. If a field spontaneously arises from background noise of the form \( B = B_z \cos(kx) \) the Lorentz force will bend the counter streaming electrons towards opposite nulls in the magnetic field creating current sheets with the correct phase as to reinforce the initial perturbation in the magnetic field. The filamentation and Weibel instabilities are often used interchangeably in the literature, however the original Weibel instability is for an anisotropic plasma that does not necessarily
have a beam of particles [19]. If the plasma has a temperature that is higher in one direction, transverse waves with wavevectors that are normal to the direction with higher temperature can spontaneously arise in the same manner as outlined above.

The Weibel instability may grow in various directions, although the strongest growth is found for wavevectors normal to the direction of higher temperature. Therefore most studies of this instability have been restricted to waves in this plane only (a through examination of both the filamentation and two-stream instabilities in various directions is given by Bret et al. [40]). It has also been shown that if the electron beam undergoes enough angular scattering the transverse temperature of the beam can become large enough to stabilise the Weibel instability [41][42]. However, this is only true of the collisionless form of the instability. When particle collisions are also accounted for the theory shows that the beam will be unstable to the filamentation instability and there will be small but non-negligible growth rates, regardless of the transverse temperature [43]. Particle collisions also act to move the filaments towards larger wavelengths and can act to either amplify or reduce the growth rate depending on the exact conditions within the plasma [44][45][46]. It has also been shown that the two-stream instability can interplay with the Weibel instability, acting as an effective source of collisions that will drive the Weibel instability, even when the transverse beam temperature would otherwise suppress it [47].

2.7 A more complete Ohm’s law and the Nernst effect

Equation 2.31 is only a simple model of Ohm’s law and the full equation is somewhat more complex. It has been shown that by taking a velocity moment of the Fokker-Planck equation and considering a collision frequency that depends on $v^{-2}$ rather than $v^{-3}$ Ohm’s law may be written exactly as

$$\eta J = \mathbf{E} + \frac{\mathbf{V}_e \times \mathbf{B}}{e} + \frac{\nabla p_e}{n_e e} + \frac{\nabla T_e}{e} + \frac{2}{5} \frac{q_e \times \mathbf{B}}{n_e T_e}$$

(2.38)

where $\mathbf{V}_e$ is the bulk flow velocity and $q_e$ is the heat flow [48][49]. Combining
2.7 A more complete Ohm’s law and the Nernst effect

this with Faraday’s law shows us how this affects the magnetic fields within the plasma. The terms on the right hand side of this equation are, from left to right, the electric field, the advection of B with the centre of mass velocity, the electron pressure gradient responsible for the $\nabla n_e \times \nabla T_e$ magnetic field generation, the thermoelectric term and the Nernst term. The Nernst term originates from the velocity dependence of the collision frequency and states that the magnetic field is convected more by the hot electrons because of their lower collision frequencies and diffusion rates. The result of this is that the magnetic field is convected with a velocity of $V_e + 2q_e/5n_eT_e$, which reflects the preference of convection with the hotter electrons. Although this form of Ohm’s law is calculated by assuming that $\nu \propto v^{-2}$ the Nernst term is similar to a heat flux equation and if the correct $\nu \propto v^{-3}$ dependence is restored the magnetic field will be convected at an even faster rate, indicated by a larger coefficient in front of $q_e \times B$. The relevance of the Nernst effect to fast ignition research is that if it becomes large enough it could cause the magnetic field to be advected away with the heat flow, hence the field inside the target may not persist once the plasma has become heated. The magnetic fields that grow inside the target play an important role in collimating the fast electrons so the removal of these fields would be detrimental to fast ignition scenarios.
Chapter 3

Particle Collisions In Plasmas

In a plasma most collisions between particles result in only small changes to the velocity vectors of the particles. This is a significantly different case to that of an ordinary gas where particle collisions result in the large angle changes typical of random path Brownian motion. The fact that small angle collisions dominate means that we can describe the collisional effects on the motion of particles within a plasma using a Fokker-Planck operator.

The collisional part of the rate of change of the distribution function for particles in a plasma, \( f(v,t) \), may be written as [50]

\[
C(f) = \left. \frac{\partial f(v, t)}{\partial t} \right|_{\text{collisions}} = -\frac{\partial}{\partial v} \cdot \left( \frac{\langle \Delta v \rangle}{\Delta t} f \right) + \frac{1}{2} \frac{\partial^2}{\partial v \partial v} : \left( \frac{\langle \Delta v \Delta v \rangle}{\Delta t} f \right) - \ldots \tag{3.1}
\]

where \( \Delta v \) and \( \langle \Delta v \Delta v \rangle \) are the expectation values relative to the probability distribution, given by

\[
\langle \Delta v \rangle \equiv \int P(v, \Delta v) \Delta v d^3 \Delta v \tag{3.2}
\]

\[
\langle \Delta v \Delta v \rangle \equiv \int P(v, \Delta v) \Delta v \Delta v d^3 \Delta v \tag{3.3}
\]

Equation 3.1 is the basic form of the well known Fokker-Planck collision operator. It is valid when the there is a large probability that \( |\Delta v| \ll |v| \), i.e. in the situation where small angle scattering is more important than large angle scattering events. Higher order terms are neglected in the Fokker-Planck collision operator as the higher order terms are smaller by a factor of \( \ln \Lambda \) (the Coulomb logarithm) and
describe the effects of large angle collisions. $\langle \Delta v \rangle$ is a coefficient of dynamical friction and describes the slowing down of particles due to collisions whilst $\langle (\Delta v)^2 \rangle$ is a diffusion coefficient and describes the spreading of particles in velocity space due to collisions. To calculate these coefficients we must look at the statistics of multiple Coulomb collisions.

### 3.1 Coulomb collisions

In calculating the collision frequencies and coefficients within a plasma we must first look at the dynamics of a single collision between two charged particles. In a Coulomb collision the two interacting particles can be considered to be moving in a plane which is generally referred to as the ‘orbital frame’. Consider a test particle $\alpha$ of charge $q_\alpha$, which moves past particle $\beta$ which has a charge of $q_\beta$. For now particle $\beta$ will be considered to be stationary (a reasonable approximation for an electron-ion Coulomb collision). As long as the impact parameter, $b$, is large, the direction of the velocity, $v$, of particle $\alpha$ will only change by a small angle, $\chi$, resulting in a collision process as shown in figure 3.1.

![Figure 3.1: Collision in particle $q_\beta$ rest frame](image)

Conservation laws can now be used to derive a formula for the angle, $\chi$, that the test particle is deflected through, assuming that there is no radiation generated as the particle accelerates and for now neglecting the recoil of the second particle. We begin by looking at the angular momentum,
\[ |L| = |r \times p| = bm_\alpha v_0 = m_\alpha r^2 \dot{\theta} \] (3.4)

where \( m_\alpha \) is the mass of particle \( \alpha \), \( v_0 \) is the initial velocity and \( \theta \) is the angle as shown in figure 3.1. Putting this into the formula for the change in angular momentum yields

\[
\Delta P = \int_{-\infty}^{+\infty} F \cos \theta dt = \frac{q_\alpha q_\beta}{4\pi \varepsilon_0 r^2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{v^2}{bv_0} d\theta = \frac{q_\alpha q_\beta}{2\pi \varepsilon_0 bv_0} \cos \frac{\chi}{2} \] (3.5)

As there is no overall change in momentum during the collision (\( \Delta P = 0 \)) we may equate the change in angular momentum to the change in linear momentum along the line \( \vec{OD} \), which is given by \( \Delta P = 2m_\alpha v_0 \sin \frac{\chi}{2} \). This leads us to the classical Rutherford formula

\[
\tan \frac{\chi}{2} = \frac{q_\alpha q_\beta}{4\pi \varepsilon_0 m_\alpha v_0^2 b} \] (3.6)

and thus in the small angle limit

\[
\chi = \frac{q_\alpha q_\beta}{2\pi \varepsilon_0 m_\alpha v_0^2 b}. \] (3.7)

Here it is useful to introduce the parameter \( b_0 \); the impact parameter for a 90 degree scattering event, given by

\[
b_0 = \frac{q_\alpha q_\beta}{2\pi \varepsilon_0 m_\alpha v_0^2}. \] (3.8)

The angle \( \chi \) is small as long as \( b \) is much greater than \( b_0 \). In a normal gas, collisions between particles occur as large angle collisions, such that each collision is likely to produce a change in the velocity vector by 90° or more. If the collision time is defined to be the time taken for a change in velocity of 90° or more, a good approximation to this would be the time between collisions: \( t_c = 1/n_\beta v_\alpha \pi b_0^2 \). However, within a plasma this becomes a very poor approximation to the collision time and gives a mean free path far too large [51]. This is due to the long range nature of the Coulomb force, meaning that when two particles pass one another at distances larger than \( b_0 \) the deflection is far from negligible. To the contrary, due to
the much larger number of collisions between particles at large separations these collisions greatly outweigh the effect of large angle collisions. Overall, the long range Coulomb collisions will tend to cancel out due to the random nature of the changes to the velocity vectors of the particles. To follow the motion of the particles a statistical model is therefore required. The model has to look at collisions between impact parameters of a lower limit, given by $b_0$, and an upper limit, given by the Debye length (discussed in chapter 3.2). The Debye length is the distance beyond which a particle's charge will be shielded, meaning a particle's charge will not act to scatter other particles at distances beyond this. The ratio between these two limits, $\Lambda = \frac{\lambda_D}{b_0}$, is much greater than one when the majority of collisions cause small angle deflections. This ratio defines the Coulomb logarithm, $\ln \Lambda$, which is discussed further in chapter 3.3.

3.2 The Debye length

The upper cut off in the Coulomb logarithm is usually taken to be the plasma Debye length. The Debye length refers to the distance around a test charge beyond which the electric field is screened due to mobile charge carriers (i.e. electrons). Particle effects occur most strongly within this distance and collective effects dominate at larger length scales. At distances smaller than the Debye length charge can be non-neutral but at larger distances the charge is screened and the plasma appears to be neutral. The Debye length, $\lambda_D$, for a neutral electron ion plasma may be written as

$$\lambda_D = \sqrt{\frac{\epsilon_0 k_B T_{\text{eff}}}{n_0 e^2}}$$  \hspace{1cm} (3.9)

where $n$ is the density and $T_{\text{eff}}$ is the effective temperature ($\frac{1}{T_{\text{eff}}} = \frac{1}{T_{\text{ion}}} + \frac{1}{T_{\text{electron}}}$).
3.3 The Coulomb logarithm

As previously mentioned, the Coulomb logarithm is the logarithm of the maximum and minimum impact parameters for Coulomb collisions in a plasma. The upper cut-off is generally taken to be the Debye length because beyond this distance the fields will be screened out. The lower cut-off is generally taken as the impact parameter for a 90° collision. The Coulomb logarithm typically has values that are around 10 for a wide range of plasma densities and temperatures (A table showing the values of $\ln \Lambda$ for plasmas with various temperatures and densities may be found in [51]). For most collisions in a plasma to result in small angle collisions the requirement is that $\Lambda \gg 1$. In the relativistic regime the temperatures may be high enough that the uncertainty in the particles position becomes comparable to $b_0$ and this cut-off must be replaced with the de Broglie wavelength. In strong fields it is also necessary to replace the maximum cut-off with the gyroradius.

3.4 Coulomb collisions in the centre of mass frame

The formula for Coulomb collisions remains mostly the same if we allow for arbitrary masses of the two particles and look at the collision in the centre of mass frame, as well as now allowing for the recoil of the second particle. The equations and derivations in this chapter largely follow those given in [50] and [52]. In the centre of mass frame we have the relative velocity, reduced mass and centre of mass defined respectively by

$$u \equiv v_\alpha - v_\beta = \dot{r}_\alpha - \dot{r}_\beta$$  \hspace{1cm} (3.10)$$

$$\mu \equiv \frac{m_\alpha m_\beta}{m_\alpha + m_\beta}$$ \hspace{1cm} (3.11)$$

$$R \equiv \frac{m_\alpha r_\alpha + m_\beta r_\beta}{m_\alpha + m_\beta}.$$ \hspace{1cm} (3.12)$$

The Lagrangian for this system is
\[ L = \frac{m_\alpha r_{\alpha}^2}{2} + \frac{m_\beta r_{\beta}^2}{2} - \frac{q_\alpha q_\beta}{4\pi\epsilon_0 |r_\alpha - r_\beta|} = \frac{(m_\alpha + m_\beta) \dot{R}^2}{2} + \frac{\mu \dot{r}^2}{2} - \frac{q_\alpha q_\beta}{4\pi\epsilon_0 r} \]  

(3.13)

where \( r = r_\alpha - r_\beta \). Putting this into the equation of motion for the Lagrangian, \( \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{R}} \right) = \frac{\partial L}{\partial R} \), shows that \((m_\alpha + m_\beta) \ddot{R} = 0\) since \( L \) is independent of \( R \), implying that the centre of mass moves with a constant speed of \( \dot{R} \). The remaining terms in the Lagrangian are identical to that of the case shown in figure 3.1, where one particle moves in the Coulomb field of the second heavier particle, except that we are now in looking at the reduced mass and relative velocity. Therefore the problem has been reduced to the scattering of particles from a fixed scattering centre and by analogy to equations 3.7 and 3.8 we may now write the centre of mass frame version of the scattering angle of the relative velocity as

\[ \tan \left( \frac{\chi_{rel}}{2} \right) = \frac{q_\alpha q_\beta}{4\pi\epsilon_0 \mu u^2 b} \]  

(3.14)

and the impact parameter as

\[ b_{0,rel} = \frac{q_\alpha q_\beta}{2\pi\epsilon_0 \mu u^2}. \]  

(3.15)

The coordinates in the original frame can be expressed in terms of the centre of mass coordinates via the following:

\[ r_\alpha = R + \frac{\mu}{m_\alpha} r \quad r_\beta = R - \frac{\mu}{m_\beta} r \]  

(3.16)

\[ \Delta v_\alpha = \frac{\mu}{m_\alpha} \Delta u \quad \Delta v_\beta = -\frac{\mu}{m_\beta} \Delta u. \]

In calculating the changes in velocity due to Coulomb collisions it is useful to change the coordinate system to an orthogonal coordinate system \((x, y, z)\) where \( u \) is initially purely in the \( x \) direction. The change in the relative velocity due to a collision is given in this frame as
\[ \Delta u_x = -u (1 - \cos \chi_{rel}) \]  
\[ \Delta u_y = u \sin \chi_{rel} \cos \phi \]  
\[ \Delta u_z = u \sin \chi_{rel} \sin \phi \]

where \( \phi \) is the change in angle perpendicular to the deflection plane. By combining equations 3.16 and 3.17 the change in in velocity during the collision can be calculated as

\[ \Delta v_x = -\frac{\mu}{m_\alpha} u (1 - \cos \chi_{rel}) \approx -\left(1 + \frac{m_\alpha}{m_\beta}\right) \frac{q_\alpha q_\beta}{8\pi^2 \epsilon_0^2 m_\alpha^2 b^2 u^3} \]  
\[ \Delta v_y = \frac{\mu}{m_\alpha} u (\sin \chi_{rel} \cos \phi) \approx \frac{q_\alpha q_\beta \cos \phi}{2\pi \epsilon_0 m_\alpha b u} \]  
\[ \Delta v_z = \frac{\mu}{m_\alpha} u (\sin \chi_{rel} \sin \phi) \approx \frac{q_\alpha q_\beta \sin \phi}{2\pi \epsilon_0 m_\alpha b u}. \]

It would be impossible to follow the motion of a single particle due to many consecutive Coulomb collisions exactly, and in any case the results of such a calculation would be of dubious usefulness. However, we are now in the position to calculate the average change in velocity that Coulomb collisions would cause on an ensemble of particles in a statistical manner.

We can now look at the cumulative effect of collisions between a particle of type \( \alpha \) and particles of type \( \beta \). The number of collisions occurring in a volume \( dV \) and a time \( \Delta t \) may be written as

\[ \Delta t d\theta db d\Phi \int f_\beta(v_\beta) ud^3 v_\beta. \]  

The cross section in this case is therefore given by \( d\sigma = bdbd\theta \) and the volume spanned by the cross section in a time \( \Delta t \) is equal to \( dV = u\Delta td\sigma \) and \( f_\beta(v_\beta) \). This describes the density of particles in phase space, which when integrated over all velocities gives the collision frequency. To find the average changes in velocity in the various directions due to collisions we multiply the change caused by one collision by the number of collisions (i.e. equations 3.18 and 3.19) and integrate over all possible angles and impact parameters. This average change in the velocity of particle \( \alpha \) due to collisions with particles of type \( \beta \) can be written as
\[
\frac{(\Delta v_x)^{12}}{\Delta t} = - \left( 1 + \frac{m_\alpha}{m_\beta} \right) \frac{q_\alpha^2 q_\beta^2 \ln \Lambda}{4 \pi \epsilon_0^2 m_\alpha^2} \int \frac{1}{u^3} f_\beta (v_\beta) \, d^3v_\beta \tag{3.20}
\]

where \( \ln \Lambda \) is the Coulomb logarithm, which appears due to the cutting off of the integral at impact parameters of \( b_{\text{min}} \) and \( \lambda_D \).

The average changes in \( v_y \) and \( v_z \) cancel due to symmetry

\[
\frac{(\Delta v_y)^{12}}{\Delta t} = \frac{(\Delta v_z)^{12}}{\Delta t} = 0 \tag{3.21}
\]

so we therefore look at the average squared changes in velocity in order to describe the \( y \) and \( z \) components

\[
\frac{\langle (\Delta v_y)^2 \rangle^{12}}{\Delta t} = \frac{\langle (\Delta v_z)^2 \rangle^{12}}{\Delta t} = \frac{q_\alpha^2 q_\beta^2 \ln \Lambda}{4 \pi \epsilon_0^2 m_\alpha^2} \int \frac{1}{u} f_\beta (v_\beta) \, d^3v_\beta. \tag{3.22}
\]

The average squared change in \( v_x \) is considered negligible due to the fact that there is no divergent integral over the impact parameters and it is therefore smaller by a factor of \( \ln \Lambda^{-1} \). For the same reason the higher order moments of the Fokker-Planck expansion are also considered to be negligible.

Generalising equations 3.20 and 3.22 to an arbitrary orthogonal coordinate system with unit vectors \( e_k \) yields

\[
\frac{(\Delta v_k)^{12}}{\Delta t} = \langle e_k \cdot \hat{\mathbf{x}} \Delta v_x \rangle^{12} = - \left( 1 + \frac{m_\alpha}{m_\beta} \right) \frac{q_\alpha^2 q_\beta^2 \ln \Lambda}{4 \pi \epsilon_0^2 m_\alpha^2} \int \frac{u_k}{u^3} f_\beta (v_\beta) \, d^3v_\beta \tag{3.23}
\]

and

\[
\frac{(\Delta v_k \Delta v_l)^{12}}{\Delta t} = \langle (e_k \cdot (\hat{\mathbf{y}} \Delta v_y + \hat{\mathbf{z}} \Delta v_z)) e_l \cdot (\hat{\mathbf{y}} \Delta v_y + \hat{\mathbf{z}} \Delta v_z) \rangle^{12} = \frac{q_\alpha^2 q_\beta^2 \ln \Lambda}{4 \pi \epsilon_0^2 m_\alpha^2} \int U_{kl} f_\beta (v_\beta) \, d^3v_\beta \tag{3.24}
\]

where \( U_{kl} \) represents a tensor with components

\[
U_{kl} \equiv \frac{1}{u^3} (u^2 \delta_{kl} - u_k u_l). \tag{3.25}
\]
These two expectation terms are often rewritten in shorthand to emphasise their meanings. Following the notation in Helander and Sigmar [50] we may write these two diffusion coefficients as

\[ A_{k}^{\alpha\beta} \equiv -\frac{\langle \Delta v_{k} \rangle_{\alpha\beta}}{\Delta t} \]  

and

\[ D_{kl}^{\alpha\beta} \equiv \frac{\langle \Delta v_{k} \Delta v_{l} \rangle_{\alpha\beta}}{2\Delta t} . \]  

\( A_{k}^{\alpha\beta} \) represents the rate at which particles in our ensemble are slowed down due to interactions with the particles of type \( \beta \) (the field particles) and was coined the ‘coefficient of dynamical friction’ by Chandrasekhar [53]. Chandrasekhar’s work was initially carried out in order to describe the relaxation of stellar systems by analysing the nature of the forces acting upon stars, but was later shown to be just as useful in describing the relaxation of the charged particles within plasma. \( D_{kl}^{\alpha\beta} \) describes the diffusion of the particles in velocity space.

Inserting these values into the Fokker-Planck equation (equation 3.1) finally yields

\[ C^{\alpha\beta}(f_{\alpha}, f_{\beta}) = \frac{\partial}{\partial v_{k}} \left[ A_{k}^{\alpha\beta} f_{\alpha} + \frac{\partial}{\partial v_{l}} \left( D_{kl}^{\alpha\beta} f_{\alpha} \right) \right] . \]  

(3.28)

The first derivation of the Fokker-Planck collision operator was carried out by Landau [54] and is equivalent to equation 3.28. Landau’s version of the Fokker-Planck collision operator may be written as

\[ C^{\alpha\beta}(f_{\alpha}, f_{\beta}) = \Gamma \frac{\partial}{\partial v_{k}} \int U_{kl} \left[ f_{\beta} \left( \frac{v_{\beta}}{m_{\beta}} \right) \frac{\partial f_{\alpha} \left( v_{\alpha} \right)}{\partial v_{l}} - f_{\alpha} \left( v_{\alpha} \right) \frac{\partial f_{\beta} \left( v_{\beta} \right)}{\partial v'_{l}} \right] d^{3}v_{\beta} \]  

(3.29)

where \( \Gamma = q_{\alpha}^{2} q_{\beta}^{2} \ln \Lambda / 8\pi \epsilon_{0}^{2} m_{\alpha} \). In the collision routine that has been added to EPOCH (described in chapter 6) the random pairing of particles over many timesteps is equivalent to the integration of the distribution function described in equation 3.29 [55].
3.5 Relaxation times

The relaxation times for particles due to small angle Coulomb collisions may be calculated from equation 3.28 or equation 3.29 by assuming that the species that is doing the scattering is Maxwellian [50]. This yields values for the expectation values that may then be separated into several collision frequency equations. Following the notation used in NRL Plasma Formulary the collision frequency equations may be written as [56]:

\[
\frac{dv_\alpha}{dt} = -\nu^\alpha_\beta v_\alpha \\
\frac{d(v_\alpha - \bar{v}_\alpha)}{dt} = \nu^\alpha_\perp v_\alpha^2 \\
\frac{d(v_\alpha - \bar{v}_\alpha)}{dt} = \nu^\alpha_\parallel v_\alpha^2
\]  

where \( v_\alpha = |v_\alpha| \) is again the velocity of the test particles. The coefficients in these equations are given by

\[
\nu^\alpha_\beta = \left(1 + \frac{m_\alpha}{m_\beta}\right) \psi\left(x^{\alpha\beta}\right) \nu^\alpha_\beta_0 \\
\nu^\alpha_\perp = 2 \left[ \left(1 - \frac{1}{2x^{\alpha\beta}}\right) \psi\left(x^{\alpha\beta}\right) + \psi'\left(x^{\alpha\beta}\right) \right] \nu^\alpha_\beta_0 \\
\nu^\alpha_\parallel = \left[ \frac{\psi\left(x^{\alpha\beta}\right)}{x^{\alpha\beta}} \right] \nu^\alpha_\beta_0
\]  

where \( \nu^\alpha_\beta_0 = 4\pi e^2 \alpha^2 e^2 \beta^2 \ln \Lambda_{\alpha\beta}/m_\alpha^2 v_\alpha^3 \) is the standard collision frequency, \( x^{\alpha\beta} = m_\beta v_\alpha^2/2k_BT_\beta \) and \( \psi(x) = \frac{2}{\sqrt{\pi}} \int_0^x t^{1/2}e^{-t}dt \). The above equations are all in cgs units and \( T \) is the temperature in eV. \( \nu_s \) is the slowing down frequency describing the rate at which the velocity of particles of species \( \alpha \) is reduced due to collisions with particles of species \( \beta \), \( \nu^\alpha_\perp \) is the deflection frequency and describes the transverse diffusion of particles of test species \( \alpha \) due to collisions with particles of species \( \beta \) and \( \nu^\alpha_\parallel \) is the collision frequency describing the parallel diffusion of particles of species \( \alpha \) due to collisions with particles of species \( \beta \).
3.6 Spitzer resistivity

The electrical resistivity of a plasma, $\eta$, has been calculated by Spitzer [51] and may be written as

$$\eta = \frac{m_e \nu}{n_e e^2}. \quad (3.36)$$

The value of $\nu$ used in the above equation is obviously important. Spitzer calculated values for the resistivity by integrating the Fokker-Planck equation, assuming that the distribution is Maxwellian. In the Lorentz limit the Spitzer resistivity equation may be rewritten as

$$\eta_L = 3.8 \times 10^3 Z \frac{\ln \Lambda}{T^{3/2}} \text{ohm cm}. \quad (3.37)$$

Of course the actual resistivity in a plasma must also include the effects of electron-electron collisions. The effects of intra species collisions on the resistivity have been calculated by Spitzer and Harm [57] who found that the actual resistivity was given by $\eta = \eta_L/\gamma_E$, where $\gamma_E$ is a constant that varies with the ionic charge, $Z$. When $Z$ is equal to 1, $\gamma_E$ is equal to 0.582. The values of the constant at various other values of $Z$ may be found in books such as ‘Collisional Transport In Magnetized Plasmas’ [50] and ‘Physics of Fully Ionized Gases’ [51].
Chapter 4

Review Of Fast Electron Transport Research

Since the fast ignition method for achieving fusion was proposed by Tabak et al. [10] there has been a renewed interest in the study of energetic electron beams and their associated physics. In this chapter a brief review of several of the key developments in the study of fast electron transport is presented. Firstly, the hybrid simulation model is discussed and some of the results obtained from using this method are outlined. Then, a few of the improvements that have been made to the model over the years are discussed and a few reasons why further improvements are still required are outlined. Much of the recent work that has been carried out in order to understand and control the propagation of fast electron beams is then discussed, with particular attention given to the instabilities affecting the propagation of the fast electron beam and the effects that Coulomb collisions have on the subsequent filamentation of the beam.

4.1 The hybrid model approach

The contrasting range of length scales involved in fast electron transport make it an area of physics that is very computationally expensive to study via simulations. At the same time the complexity of the physics involved in the growth of the various
instabilities makes analytical investigations equally as challenging. The result of this is that in order to study the physics certain assumptions and approximations must be made. Numerous simulation models have been developed in order to study plasma physics and fast electron transport but there is always a trade-off between the number of approximations and assumptions that are made and how computationally demanding the codes are.

The method of using a hybrid code to study the propagation of fast electrons in solid density targets was introduced by Davies et al. [38]. Their original aim was to investigate the transport of fast electrons generated by a 1ps duration, 1µm wavelength laser pulse incident on an aluminium target. The laser had an intensity of $2 \times 10^{18} \text{Wcm}^{-2}$ and focused to a spot diameter of 20µm. The fast electron distribution was found by assuming that 30% of the laser energy is absorbed by the fast electrons and by following the scaling results from Beg et al. [34]. The distribution is assumed to follow $e^{-K/kT}$, where kT is the temperature and K is the kinetic energy of the fast electrons. The fast electrons were then injected into the simulation at a random angle, uniformly distributed in a cone of 20° half angle. In the simulation the fast electrons are treated kinetically using a relativistic Fokker Planck equation (although standard PIC modeling of the fast electrons is also possible). In this original study the changes in the density of the background electrons due to the fast electron beam were not included and the code is therefore limited to the regime where the beam density is much less than the background density. This is a reasonable assumption because the fast electron beam density is comparable to the critical density of the laser used. The ‘cold’ background is treated as a fluid with a simple resistive MHD description. The background response to the fast particles was set so that it obeys

$$E = \eta J_{\text{return}} = -\eta J_{\text{fast}} \quad (4.1)$$

where $\eta$ is the resistivity and $J_{\text{fast}}$ and $J_{\text{return}}$ are the fast and background current densities respectively. This equation assumes that the background current balances the fast electron current. The magnetic field was found using

$$\frac{\partial B}{\partial t} = -\nabla \times E = \nabla \times \eta J_f = \eta \nabla \times J_f + \nabla \eta \times J_f \quad (4.2)$$

where the final term will only contribute when background heating is accounted
4.1 The hybrid model approach

for (as this will lead to gradients in the resistivity). Choosing the correct resistivity to use in these equations is one of the difficulties in using this hybrid approach. In the early work using hybrid codes the resistivity was calculated using fits to experimental data produced by Milchberg et al. [58]. The effect of the displacement current was also neglected in the model and its absence is justified by the argument that it is only important on short time scales whilst the return current is established. Neglecting the displacement current term is valid as long as the electric field varies slowly. Rotational symmetry is also assumed so field values for $E_r$, $E_z$ and $B_\theta$ are used in the simulations. This assumption means that the model does not allow for the growth of instabilities such as the two stream instability. The resistivity in the model changes with the background temperature and the model includes the effects of ohmic and collisional heating on the background, although it is considered to be isotropic and linear. To calculate the change in the background temperature from the energy losses of the fast electrons the heat capacity of aluminium at room temperature was used as a fixed value. This is an approximation as the actual values would be dependent on temperature and are not well known in these extreme cases. As the energy required to release a fourth electron from an aluminium ion is around 120eV this is considered to be a fair approximation for investigating the effects of the variable resistivity at temperatures up to 100eV.

In the simulations performed it was shown that 90% of the fast electron energy had been lost to the background by the time they had propagated a distance of 600μm, 70% of which is attributed to collisions with the remaining 30% being attributed to the electric field. They show that the electric field is only important only at early times, agreeing with previous theoretical studies [59]. The electric field was seen to lower the maximum energy of the fast electrons before they spread through the target. This energy lost by the fast electrons was found to almost exclusively go towards the heating of the background. The heating near the axis eventually lowers the resistivity which in turn weakens the electromagnetic fields allowing the fast electrons to travel further into the target. The magnetic fields created were found to reduce the spread of the fast electrons, increase the penetration depth of intermediate energy fast electrons and cause the lower energy fast electrons to be reflected. They also noted the importance of the resistivity of the target, noting
that changes in the resistivity significantly effected the field generation within the target. When the target heats up the resistivity behind the focal spot is seen to become low enough that the electric field becomes negligible and the magnetic field becomes fixed. The magnetic field is also seen to be affected by the $\nabla \eta \times J_{fast}$ term, causing the field in the region of peak resistivity to drop and, with time, change sign.

This first hybrid method has since been improved upon so that it also includes the resistive diffusion of the magnetic field [60], although in this work the displacement current was still neglected. In this improved model equations 4.1 and 4.2 become

$$E = -\eta J_{fast} + \frac{\eta}{\mu_0} \nabla \times B$$

(4.3)

and

$$\frac{\partial B}{\partial t} = \nabla \times \eta J_f - \nabla \times \frac{\eta}{\mu_0} \nabla \times B.$$  

(4.4)

In this case the transport of electrons through 70-250$\mu$m thick plastic (CD2) targets was investigated. The fast electron source was set up to be equivalent to the fast electrons produced in experiments carried out using the VULCAN laser (i.e. 20J supplied in 1ps from a laser with an intensity of $10^{19} W cm^{-2}$ and a wavelength of 1$\mu$m). As in the original model, the fast electron distribution is found using Beg’s scaling law to create a fast electron energy distribution going as $e^{-K/kT}$. They assumed 20% absorption and electrons were injected with a half angle of 15°, although they state that their results showed only a weak dependence on the half angle used. In this study the resistivity used in the code was given by a function which starts at $2.3 \times 10^{-6} \Omega m$ and tends towards the Spitzer result at higher temperatures. This lower resistivity limit is matched to values found from experiments (see references within [60] for details) and in the Spitzer resistivity equation $Z \ln \Lambda$ was set to 8.

They found that the radial distribution of fast electrons at the rear surface was peaked at approximately half the laser spot radius. The simulations also showed that the electric field that is initially created within the plasma prevented the propagation of fast electrons, leading to heating of the background and a lowering of the resistivity. In turn these effects lead to a lowering of the electric field which re-
4.1 The hybrid model approach

results in the electrons being able to propagate through a channel of lower resistivity. Regions with a higher electric field initially, such as on axis where the current density is higher, end up with lower electric field values due to the ohmic heating that occurs. The initial radial variation in the electric field quickly generates a negative magnetic field through the $-\eta \partial J_z / \partial r$ term in equation 4.4 which acts to pinch the electron beam. This pinching effect is reduced due to the reduction in the electric field caused by ohmic heating, which results in the magnetic field being reduced through the $-\nabla \eta_r J_z$ term in equation 4.4. Over time this effect will create a positive magnetic field close to the axis. The region where the magnetic field changes direction is another region in which the electron flow could be focused. However, in the simulations the magnetic fields were not seen to grow to strengths where distinct filaments would form. The combination of these processes results in an electron beam that propagates directly through the target, without appreciable spreading out beyond the initial electron source size.

In the simulations strong magnetic fields were also seen to form at the rear of the target, where they act to focus the electron beam on to the back of the target. Davies et al. attribute this effect to the fact that the electrons impacting on the rear surface were predominantly moving radially outwards. Their reasoning follows from the way in which the electrons leaving the rear of the target are reflected back into the target (which occurs within a Debye length due to the electric field that is generated). The magnetic field around the beam of fast electrons which acts to collimate the initial beam has the opposite effect on the electrons that have been reflected at the rear surface, causing their average direction of motion to move even further radially outwards. On top of this the reflected electrons will then generate their own magnetic field which reinforces the initial magnetic field, further pinching the incident electrons and further spreading the reflected electrons. This results in the reflected electrons travelling back into the target in a cone and increases the amount of energy deposited at the rear surface in their simulations. The energy deposited behind the rear surface was seen to be reduced when the resistivity was reduced because the reduction in resistivity means that the magnetic field strength will also be reduced, resulting in a less well defined cone of reflected electrons. The electrons leaving the rear of the target were also seen to set up an electric field
strong enough to ionise the back of the target leading to further heating and to ions
being accelerated.

An example of the further advancement of the hybrid model approach is given
by the code PÂRIS [61], which is a three-dimensional extension to the work per-
formed by Davies et al. The code has been used to look at the resistive filamen-
tation of high intensity electron beams in solid targets and in [61] it was shown
that electron beams with small transverse velocities had emphasised sensitivity to
fragmentation in three-dimensional geometries. The filaments were seen to each
carry currents close to the Alfvén current limit meaning that much larger over-
all beam intensities could be driven in the fragmented system due to the strongly
neutralising return current. The simulations also showed that when the initial di-
vergence of the electron beam was increased the filamentation of the electron beam
was prevented.

Hybrid simulations have also been performed using the code LSP, which is
a commercial code marketed by MRC (Albuquerque), New Mexico, USA. LSP
uses an energy and momentum conserving fluid description of the background
and an implicit particle in cell description for the hot electrons, although the fluid
model still assumes Spitzer resistivity and the ideal gas equation of state. How-
ever, the model includes the displacement current and is therefore able to describe
the various instabilities affecting the electron beam propagation. The code is able
to solve the full Maxwell equations which means that there is potential to carry out
fully integrated simulations of the heating and transport effects occurring in high
density plasmas. Simulations have been performed using LSP in order to inves-
tigate beams of electrons with a temperature of 1MeV and a half angle of around
40° that are injected into plasma targets equivalent to CH (with densities varying
from $0.25 - 4gcm^{-3}$) [23]. The magnetic focusing of the fast electron beam is show
and the transition from a diverging filamented beam to a focused beam at densi-
ties greater than $2gcm^{-3}$ can be seen, which is in agreement with the results of
Kingham and Bell [22].

An interesting recent development in the use these codes is the possibility of
using implicit PIC methods such as in LSP to model the whole laser plasma interac-
4.2 The limitations of the hybrid approach

The hybrid models discussed in the previous section have several drawbacks in the form of assumptions that ideally would not be necessary. Firstly, the fast electron distribution entered into the simulations must be approximated in the hybrid models. Likewise, the background resistivity and heat capacities used within the codes must also be specified despite the fact that none of these variables are accurately known. There is also a limitation due to the fact that the fast electron number density must be much lower than the background number density because the resistivity is assumed to be linear, so independent of the electric field. This is the case when either the drift velocity of the background electrons which carry the return current is small compared to their thermal speed, which is true if the target is
highly collisional, or if there are far more electrons carrying the return current than there are carrying the forwards (fast electron) current. The neglect of the displacement current term is also a possible cause of error in the models as it prevents the Weibel instability from occurring.

The use of a simple version of Ohms law is another possible source of error in hybrid simulations. Extra effects such as the Nernst effect (discussed in chapter 2.7) are not included in the model meaning the advection of the magnetic fields is not being treated in full. The Nernst effect arises from the velocity dependence of the collision frequency and is related to the heat flow which is dominated by the fast electrons and it implies that the magnetic field will be preferentially transported along with the fast electrons. This effect has been seen in experiments (and simulations) where the magnetic field is seen to expand into the target with speeds that are far too fast to be due to the plasma flow velocity but are consistent with calculations of the Nernst velocity [62].

Other possible sources of magnetic field generation such as from changes in the flow velocity of the background electrons (as discussed by Cai et al. [63]) are also not included in the hybrid models. For example, in [63] it is show that magnetic fields can be created which act to collimate the fast electrons even in cases where the hybrid model approach shows the creation of de-collimating fields.

The use of Spitzer resistivity is another possible issue with regards to the accuracy of the hybrid simulations. Work by Sherlock et al. [64] has shown that the return currents are not necessarily given by the standard transport equations at low plasma densities. They conclude that the isotropic part of the background electron distribution can become strongly non-Maxwellian and that pressure inhomogeneities can be set up by Ohmic heating which lead to electric fields being formed which are an order of magnitude larger than those created due to Spitzer transport. The Spitzer equations rely on the perturbations to the background distributions not causing significant deviations away from a Maxwellian distribution. When the background electron density is not much greater than the fast electron density the drift of the background electrons can have a value of $v_d/v_e$ (the current strength parameter defined in [64]) which becomes greater than unity. The rapid
heating in the lower density plasma caused by this current causes pressure gradients that can become the dominant sources of electric field, yielding values an order of magnitude greater than in Spitzer’s theory.

4.3 Recent work on Fast Electron Transport

Detailed reviews describing much of the recent work carried out to improve our understanding of fast electron transport can be found in review papers such as [65], which was produced by Norreys et al. The following sections provide a brief outline of many of the recent studies discussed in [65], as well as of a few other noteworthy developments.

4.3.1 The relation between laser intensity and the divergence angle of the fast electron beam

The effect of laser intensity on the divergence of the fast electrons produced in high intensity laser-plasma interactions has been studied by J.S.Green et al. [66]. In [66] Green et al. discuss experiments in which foil targets (Tu or Cu) were irradiated with 1\(\mu\)m wavelength lasers of various intensities, going up to a maximum intensity of \(4 \times 10^{19} \text{Wcm}^{-2}\). The fast electron beam divergence in the various experiments was determined from \(K_\alpha\) x-ray emission imaging from buried high Z layers and by using optical self-emission imaging techniques to infer the divergence angles of the fast electrons emerging from the rear surface of foil targets. The various methods of detecting the divergence of the fast electrons were consistent in the divergence angles that they yielded. By combining their data for the divergence of the fast electron beam with the results of several other experimental studies (see references within [66]) a clear correlation was found between the divergence angle of the fast electron beam and the laser intensity used. The various experimental measurements were taken using an array of different focal spot conditions which indicates that that the laser intensity itself is the important parameter. This trend was then examined via 2D3V collisionless particle-in-cell modelling performed us-
ing the code OSIRIS. The simulations showed that the spot size used has little effect on the temperature or divergence of the fast electrons, although it was noted that slightly lower temperatures were seen for the smallest spot size tested (which was a focal spot 4µm in diameter).

Two possible reasons for this dependence on laser intensity are given. The first is that it is a result of the rippling of the critical density surface in the laser plasma interaction region. As well as the hole-boring caused by the laser the critical surface is seen to form ripples on the scale of the laser wavelength [5]. In simulations these ripples are seen to grow faster when higher laser intensities are used and hence the laser fields around the ripples could be affecting the fast electron divergence. The second possibility is that the divergence angle is altered by the growth of the filamentation instability. Adam et al. [67] and Ren et al. [68] have performed simulations that suggest that the fast electron divergence is controlled by the deflections of the electrons off of the magnetic fields that are generated due to the filamentation instability. The filamentation instability has previously been shown to be independent of intensity for a beam with a water bag distribution [42]. However, the fast electrons have a wide range of energies so there will be many beam electrons with a Lorentz factor close to one, and a smaller number with large Lorentz factors. If the average Lorentz factor of the beam remains close to one whilst the number of beam particles increases with intensity, the overall beam divergence will increase with intensity.

The increase in beam divergence caused by increasing the laser intensity results in a limit on the maximum laser intensity that is suitable for fast ignition. If no other effects are considered (such as beam collimation due to magnetic field generation) this intensity is found to be around $I\lambda^2 = 5 \times 10^{19} W cm^{-2} \mu m^2$.

### 4.3.2 Reasons For The Beam Divergence

J.C.Adam et al. [67] have studied the divergence of fast electron beams using two and three dimensional particle-in cell simulations. They initially discuss a two dimensional simulation of a plasma with a density of $80n_c$ and an initial temperature
4.3 Recent work on Fast Electron Transport

of 10keV, which is impacted upon by a laser with a spot FWHM of 5µm and an intensity of \(10^{20} \text{W cm}^{-2}\). Their simulation shows that initially (at around 50fs into the simulation) the magnetic field develops filaments with a wavelength comparable to \(c/\omega_p\). These filaments were then seen to merge over time, forming larger filaments as their amplitude increased. The filaments suggest that a Weibel-like instability is developing in the region behind the focal spot. In this region the magnetic field was seen to have peak values of over 20,000T whilst fluctuations of below 1000T were seen in the bulk plasma. They note that the results are independent of the initial plasma temperature and the number of computational particles used because the temperature in the layer of interest was rapidly increased to the order of 100keV. The particles in this region were typically seen to have energies between 100keV to 1MeV. In the simulations the electrons were seen to be injected into the target twice per laser wavelength (corresponding to the \(J \times B\) mechanism) and were deflected as they travelled through the strong magnetic field layer. The simulation results indicate that the divergence of the fast electrons is dominated by the strong fields in this region and that beyond this region the electrons are able to propagate freely through the plasma. This was shown by plotting the angles that the fast electrons were traveling at within various parts of the target. In their simulations electrons with energies greater than 1 MeV were initially seen to be emitted in a cone of around 20° half angle, which rose up to 40° at later times, corresponding to an increase in both the strength and thickness of the magnetic field layer with time.

This is in agreement with the work of Ren et al. [68] in which two dimensional PIC simulations were performed in order to examine a laser with a peak intensity of approximately \(10^{20} \text{W cm}^{-2}\) interacting with 40\(n_e\) plasma targets. They used a large simulation area so that a vacuum buffer could be included in the simulations to ensure that the boundary conditions used did not affect the results. The simulations showed filaments in the particle densities and in the magnetic fields, but the filaments were not seen to coalesce and form larger filaments over time (which has been seen in simulations carried out using smaller simulation areas). The filaments were seen to appear at the shock front created by the laser-plasma interaction and were again attributed to the transverse Weibel instability arising due
4.3 Recent work on Fast Electron Transport

to the counter-propagating fast electrons and charge neutralising return current. When an underdense plasma region was included the laser was seen to filament in this region first which then led to density modulations at the laser plasma interface. These density modulations then acted to focus the laser filaments creating deeper channels. In the region behind the laser focus a region of high density plasma with a sharp boundary was seen. Filaments could also be seen in the electron and ion densities, as well as in the magnetic field. They note that the polarity of the magnetic field that is generated indicates that the current in the high density region was due to the return current electrons. Outside of this dense region the magnetic field was seen to become much smaller and the fast electrons were seen to lose their filamentary structure.

To look into the fact that the electron filaments were not seen to coalesce in the larger simulations Ren et al. performed further simulations using a much smaller simulation area whilst keeping all of the other parameters the same. In the smaller system the filaments in the current and magnetic field had less angular spread and almost all of them were seen to be pointed in the laser propagation direction. This was attributed to the recirculation of electrons caused by using periodic boundary conditions. The electrons were seen to quickly recirculate through the upper and lower boundaries, meaning that on average the current was horizontal at all points. In the larger simulations the electrons may travel sideways and hence the filaments formed could have a greater angular spread. The fact that the filaments are seen to join in simulations of smaller systems (such as in Pukhov’s paper on hole boring [11]) was attributed to the attraction between these parallel current filaments. Three dimensional fluid code simulations carried out by A.Macchi et al. [69] have also shown that the fast electron beam becomes filamented without any coalescence occurring at later times.

4.3.3 Controlling the beam divergence

Once researchers began to gain a better understanding of the electron beam divergence the obvious next step was to investigate ways in which this divergence could be controlled. Early work on controlling the beam divergence was performed
by Campbell et al. using the hybrid code LSP. The simulations they carried out showed that having a vacuum gap or using different materials to make up the target could result in a radial density gradient which would result in the generation of strong fields that act to confine the fast electrons.

Simulations have also been carried out by Robinson and Sherlock in order to investigate a target with a core of higher density than the surrounding material [70], an arrangement that should act to collimate the fast electron beam. Recent experiments on Vulcan PW have also tested this theory showing that the fast electrons are indeed restricted to the sandwiched layer due to the growth of magnetic fields at the boundaries between the layers of differing density [71]. Equation 2.35 states that a magnetic field will be produced that pushes fast electrons towards regions of higher fast electron current density and towards regions of higher resistivity. Simulations containing a higher density sandwiched layer are discussed in [70] and show that this is indeed the case. The collimation occurs due to the $Z$ dependence of the resistivity; as the fast electrons travel into the target a magnetic field will be set up which acts to push the fast electrons into the region of higher resistivity, hence increasing the fast electron current in this region. This process is later reinforced by the generation of magnetic fields that act to push the fast electrons towards regions of higher current density.

Recent work to investigate how these specially engineered targets could be used to control the collimation of fast electrons has been carried out by Hong-bo Cai et al. [63] who have used a collisional PIC model to expand on previous work by Zhou and Wu (see references within [63]). They again found that a sandwiched target that has a core comprised of a different density material can be used to focus the fast electrons. The set-ups that collimated the electron beam involved using a target that had either a high resistivity core with low resistivity surround structure or a low density core with high density surround structure. The former is similar to the results discussed in [70] where the magnetic fields generated by the gradients in resistivity and currents were seen to collimate the fast electrons. However, in the second case the magnetic fields are found to be generated by rapid changes in the flow velocity of the background electrons resulting from the density jump
at the material interface in the transverse direction. This setup was seen to decol-limate the fast electrons in the hybrid simulations discussed in [70] but Hong-bo Cai et al. report the appearance of collimating fields. Their results showed that for targets with a low density core ($5n_c$ surrounded by $200n_c$ materials) the fast electrons are collimated by the fields arising due to differences in the flow velocity of the background electrons between the regions, despite the fact that the resistivity effects still act to decollimate the beam. The collimating magnetic field in this case was seen to reach 100’s of MG and the main benefit of this target design is that the fast electrons are collimated without their energies being reduced due to a high resistivity in the core material.

Various other ideas of ways to control the fast electron divergence have also been devised, including shaping the tip of a cone to create fields that will confine the fast electrons. This has been demonstrated experimentally in a set up where wire targets were attached to the end of cones into which the laser was focused. The fast electrons were confined to the wires and lost transverse momentum as they travelled, resulting in the fast electron divergence at the end of the wire being substantially reduced [72]. It has also been suggested that a strong pre-pulse laser could be used in order to set up a self-generated magnetic field capable of collimating the fast electrons created by the main pulse [73]. This method has the advantage that no extra target engineering is required and has also been experimentally tested (see references in [65]). However, the initial experiments saw no additional collimation of the fast electron beam. Preliminary analysis of these experiments has suggested that pulses with higher contrast ratios are required for this double pulse system to work effectively.

### 4.4 Instabilities affecting Fast Electron Transport

A large amount of work has been performed in order to understand the behavior of fast electron beams travelling through plasma. As previously mentioned, a beam of fast electrons is seen to become filamented as it travels through the background plasma and understanding both how and why this occurs is crucial to fast ignition
4.4 Instabilities affecting Fast Electron Transport

research. This filamentation occurs due to the two-stream and Weibel instabilities, discussed in 2.6. However, there is a complex interplay between these instabilities and the resultant ‘oblique’ mode is also of importance to fast ignition scenarios [74]. A detailed review of the various multidimensional electron beam-plasma instabilities is given by Bret, Gremillet and Dieckmann [75], where much of the previous work examining the various instabilities has been pulled together. The growth rates for the two-stream, filamentation/Weibel and oblique modes in the diluted beam limit ($\alpha \ll 1$) may be written as [75][76][77]:

$$\delta T_S / \omega_p \sim \sqrt{3} \alpha^{\frac{3}{2}} \frac{2^3}{\gamma_b}$$

(4.5)

$$\delta F / \omega_p \sim \beta \left(\frac{\alpha}{\gamma_b}\right)^{\frac{1}{3}}$$

(4.6)

and

$$\delta O / \omega_p \sim \sqrt{3} \frac{2^3}{\gamma_b} \left(\alpha \frac{\gamma_b}{\gamma_b}\right)^{\frac{1}{3}}$$

(4.7)

respectively, where $\alpha$ is the beam to background density ratio, $\beta = v_b/c$ (with $v_b$ being the mean beam velocity) and $\gamma_b = 1/\sqrt{1-\beta^2}$. The so-called ‘oblique’ mode arises from the coupling of the Weibel and two-stream instabilities and is the dominant mode for highly relativistic beams and at low beam to background ratios. Equation 4.6 shows that the filamentation instability is stabilised both at low beam velocities and at strongly relativistic beam velocities.

Various studies have been performed in order to investigate the effects that these different instabilities have under various conditions (examples can be found in [76], [78], [79], and [80]). These studies have resulted in the production of parameter space diagrams indicating the regions where each instability is expected to dominate. Various PIC simulations have also been performed in order to back up the theoretical analyses. The two-stream instability is seen to dominate for non-relativistic cases while the oblique instability is seen to dominate when the beam density is less than roughly half of the background plasma density. It has also been shown that assuming the filamentation instability to be purely transverse can result in the growth rate being overestimated by a factor of approximately $\sqrt{\gamma_b}$.

Due to the obvious implications for fusion research the effects that particle collisions have on the Weibel (or filamentation) instability are of particular interest.
Epperlein et al. [81] investigated the growth rate for the collisional Weibel instability in an overdense plasma slab, finding that there can be significant growth rates ($> 10^9 s^{-1}$) at densities above critical. Epperlein and Bell [82] have also examined the case of a plasma where the equilibrium is a balance of the inward heat flow (resulting from a laser heated corona) and the outward flow of mass and energy associated with the plasma ablation. The equilibrium solution could then be calculated using the approximation of a Spitzer heat flow. Although various extensions to the work would be required in order to apply it to short pulse laser-plasma interactions they make the important point that there can be circumstances where the local (uniform plasma) methods used by various authors may predict incorrect growth rates. Epperlein and Bell’s work points out that the advection of material and magnetic field due to ablation and the Nernst effect can effectively stabilise the growth of the Weibel instability when local theory predicts instabilities with large growth rates.

PIC simulations performed by Wallace et al. [41] have shown that collisions can decrease the growth rate of the Weibel instability below the collisionless value. It was found that when the collision rate was increased beyond the collisionless Weibel growth rate the instability was completely suppressed. Silva et al. [42] have also shown that the transverse Weibel instability can be stabilised by a relatively small transverse electron beam temperature. They found that the threshold for the growth of the transverse Weibel instability can be written as

$$\alpha > \gamma_b \frac{\beta_\perp^2}{\beta_\parallel^2}$$

where $\alpha$ is once again the beam to background density ratio, $\gamma_b$ is the Lorentz factor of the beam and $c\beta_\perp$ and $c\beta_\parallel$ are the perpendicular and parallel beam velocities respectively.

However, the effects that collisions have on the Weibel instability are not simple. Contrary to the results of Wallace et al., a non-relativistic analytical study by Molvig [43] has actually shown that filamentation is assured if collisions are accounted for. Particle collisions are seen to result in a small but non-negligible growth rate, even at temperatures that would otherwise act to suppress the collisionless Weibel instability. PIC simulations performed by Kumar et al. [83] have
also shown that collisions can act to revive the instability rather than suppress it when the transverse beam temperature is large enough to otherwise suppress the collisionless form of the instability. Karmakar et al. [47] have performed two-dimensional PIC simulations in different geometries in order to separately investigate the Weibel instability and the coupled Weibel-two-stream (oblique) instability. Their simulations also show that a transverse temperature will act to suppress the Weibel instability with collisions in the background plasma return current acting to reintroduce the instability. The simulations also involving the two-stream instability did not show the suppression of the Weibel instability at large transverse beam temperatures, leading to the conclusion that the two-stream instability may be treated as a source of effective collisionality which acts to drive the filamentation of the electron beam. Their work implies that in three-dimensional geometries the complete suppression of the filamentation instability will be difficult.

Recent analytical studies by Hao et al. [44][45] have given a physical picture of why collisions have been seen to both enhance and suppress the filamentation instability. They show that collisional effects can suppress the filamentation instability for symmetric or quasi-symmetric counter-streaming currents, but enhance it for asymmetric counter-streaming currents. Fiore et al. [46] have gone on to numerically and theoretically examine the linear stage of the collisional filamentation instability for a fast ignition setup (a fast electron beam and counter streaming return current/cold ion background). Collisions were seen to cause the preferential formation of larger filaments than were seen in the collisionless regime. They also found that the instability could be either enhanced or suppressed depending on the configuration being examined, although collisions were again seen to guarantee that the instability occurs, with the simulations again showing filamentation even when large transverse temperatures were present. The suppression/enhancement of the instability was shown to be dependent on the initial value of $\gamma_b$, with an enhancement to the instability found for cases where $\gamma_b \gtrsim 1.73$ (when a beam to background contrast ratio of 0.1 is assumed). This dependency on the Lorentz factor of the beam electrons was attributed to the slowdown of the beam due to the collisional effects.
Chapter 5

Particle In Cell Codes

The particle in cell simulation method was first introduced around 50 years ago as a means of computationally dealing with complex systems involving large numbers of charged particles. Despite the rapid rise in the processing power of computers over this period computational limitations have meant that researchers have always struggled to simulate regions as large and as detailed as they would ideally like. For example, the particle number density is around $1 \times 10^{27} \text{cm}^{-3}$ for fusion plasmas, $1 \times 10^{18} \text{km}^{-3}$ for space plasmas and $1 \times 10^{12} \text{cm}^{-3}$ for general laboratory plasmas. From these figures it is easy to see that even the most powerful supercomputers will not be processing anywhere near as many particles as there are in real systems for the foreseeable future. However, the particle in cell method allows computational particles to be viewed as representing many real particles and although the amount of computational particles required in simulations can still be very large (and increases with the particle density we wish to simulate), detailed two-dimensional simulations of laser-solid density plasma interactions may now be performed with moderate modern day computer resources.

The method of using a fixed spatial grid to define charge densities, potentials and fields on was being used in Stanford by Buneman and his co-workers by 1963 and the introduction of this method yielded a much faster way of computationally following the motion of large numbers of charged particles. In this ‘particle in cell’ approach each particles charge is interpolated to its nearest grid point (or points
depending on the interpolation method used). Differencing methods may then be used in order to calculate and advance the field values at the gridpoints and these field values can be used in order to calculate the particle accelerations. With the particle accelerations known the particle positions can then be updated using a leapfrog or Runge-Kutta method. Detailed background and examinations of the particle in cell method may be found from numerous sources, including books by Birdsall and Langdon [84] and Hockney and Eastwood [85].

The finite spatial grid on which field values are calculated also plays an important role in reducing the forces between computational particles that are within the same cell. The force between two charged particles gets larger as they move closer together (i.e. from $F \propto q^2/r^2$) and two particles moving past each other at close range will experience strong and quickly changing forces. At the same time, the slow tail off of the Coulomb force causes interactions to occur over large distances which gives rise to the collective behavior of plasmas. It is this physics that the particle in cell method aims to correctly model. Calculating the force that the particles have on one another via the finite spatial grid reduces the value of the Coulomb force at particle separations less than the separations between the gridpoints. This is an important feature because the Coulomb force in simulations is increased at short ranges due to the fact that the simulation particles represent many real particles. The finite grid approach results in the charge of the computational particles being spread over their nearby gridpoints, which changes them from behaving as classical point particles to behaving as finite sized particles. As these finite sized particles begin to overlap the force between them will drop, hence reducing the unwanted (unphysical) effects. At the same time the long range forces remain the same, Coulombic, meaning that the collective behavior we wish to study is retained.

A standard PIC code follows a cycle similar to the one shown in figure 5.1. The particle positions and velocities (along with a specific weighting function as discussed in chapter 5.3) can be used to calculate charge and current densities on the grid positions ($x_i$ and $v_i \rightarrow \rho_J$ and $J_J$, where particle properties are denoted with the subscript ‘$i$’ and grid quantities are denoted by the subscript ‘$J$’). These
Figure 5.1: A typical particle in cell code timestep

may then be used to advance the fields forwards in time and these new fields can be used to calculate the force acting on the particles ($\rho_J$ and $J_J \rightarrow E_J$ and $B_J \rightarrow F_i$). This force can then be used to calculate new particle velocities and hence to move the particles a step forwards in time.

5.1 The particle in cell method

A new 1D2V particle in cell code was created as part of the work that went into this thesis. This section describes the main equations that the code follows in order to simulate laser plasma interactions. The simulation results discussed in chapter 7 were produced with the two-dimensional version of the particle in cell code EPOCH. EPOCH essentially follows the same logic presented here and simply involves retaining more terms in Maxwell’s equations in order to describe the extra spatial dimension involved. A clear and detailed description of how two-
dimensional particle in cell codes work can be found in [86], or in chapter 15 of Birdsall and Langdon [84].

5.1.1 Field advancing

Particle in cell codes work by solving the finite difference versions of Maxwell’s equations in order to advance the electromagnetic fields forwards in time. This section describes the key equations used in order to update the fields. For simplicity, the finite difference equations shown are mostly for a 1D2V case, corresponding to the code that was produced in the initial phase of work on this thesis. The extra terms in the finite difference equations that are used to describe a full two-dimensional (or three-dimensional) simulation are obtained by retaining further terms in the differenced forms of Maxwell’s equations.

The Magnetic field values are advanced forwards in time using Faraday’s law

\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad (5.1) \]

whilst the electric field values are advanced using both the Maxwell-Ampère law

\[ \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \quad (5.2) \]

and Gauss’ equation

\[ \nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}. \quad (5.3) \]

As field values are defined to be on grid positions, the derivatives may be rewritten as difference equations and can then easily be solved by centred differencing, which will be accurate to second order in both space and time. For a 1\frac{1}{2} dimensional case (meaning particles can have velocities in the x and y directions but can only move in the x direction) the equations become

\[ \frac{B_{z,j+\frac{1}{2}}^n - B_{z,j+\frac{1}{2}}^{n-\frac{1}{2}}}{\Delta t} = -\frac{E_{y,j+1}^n - E_{y,j}^n}{\Delta x} \quad (5.4) \]

and
\[ \frac{E_{y,j}^{n+1} - E_{y,j}^n}{\Delta t} = -c^2 \frac{B_{j+\frac{1}{2}}^{n+\frac{1}{2}} - B_{j-\frac{1}{2}}^{n+\frac{1}{2}}}{\Delta x} - \frac{J_{y,j}^{n+\frac{1}{2}}}{\epsilon_0} \]  

(5.5)

where the subscript '\( J \)' denotes the grid position and the superscript '\( n \)' represents the current timestep.

Rearranging these equations yields a set of equations that may be used to advance both the magnetic and electric fields forwards in time. Firstly, the magnetic field is advanced by half a timestep using

\[ B_{z,j+\frac{1}{2}}^n = B_{z,j+\frac{1}{2}}^{n-\frac{1}{2}} - \frac{\Delta t}{2\Delta x} (E_{y,j+1}^n - E_{y,j}^n). \]  

(5.6)

The magnetic field is only advanced half way because it is needed at integer times in order to advance the particles (i.e. at the same time as when the electric field value is known) whilst it is needed at half integer times in order to advance the electric field forwards in time.

With the new magnetic field values known the electric field may then be advanced using the Maxwell-Ampère equation. Rearranging in order to advance \( E_y \) we have

\[ E_{y,j}^{n+1} = E_{y,j}^n - c^2 \frac{\Delta t}{\Delta x} (B_{j+\frac{1}{2}}^{n+\frac{1}{2}} - B_{j-\frac{1}{2}}^{n+\frac{1}{2}}) - \frac{\Delta t J_{y,j}^{n+\frac{1}{2}}}{\epsilon_0} \]  

(5.7)

whilst to advance \( E_x \) in a one-dimensional system we may simply use Gauss’ law

\[ E_{x,j+1}^n = E_{x,j}^n + \Delta x \frac{\rho_{j+\frac{1}{2}}^n}{\epsilon_0}. \]  

(5.8)

In a two dimensional system this simple use of Gauss’ law would not be applicable and the electric field would have to be advanced using

\[ E_{x,k+1}^{n+1} = E_{x,k}^n - c^2 \frac{\Delta t}{\Delta y} (B_{k+\frac{1}{2}}^{n+\frac{1}{2}} - B_{k-\frac{1}{2}}^{n+\frac{1}{2}}) - \frac{\Delta t J_{x,k}^{n+\frac{1}{2}}}{\epsilon_0} \]  

(5.9)

which contains additional dependencies on gridpoints in the y-plane (labeled here by the subscript ‘\( K \)’).

Once the electric field has been advanced the magnetic field may then be further advanced. For instance, \( B_z \) may now be advanced by a further half timestep in the same manner as in equation 5.6, obtaining \( B_{z,j+\frac{1}{2}}^n \) from \( B_z^n \) and \( E_y^{n+1} \).
5.1 The particle in cell method

5.1.2 Particle velocity advancing

The particles are advanced using a centred difference version of the Newton-Lorentz force equation of motion when the electric and magnetic fields are both known at the same time. The Force equation can be written as

$$\mathbf{F} = m \frac{d\mathbf{u}}{dt} = q \left( \mathbf{E} + \frac{1}{\gamma} \mathbf{u} \times \mathbf{B} \right),$$

and when rewritten as a difference equation it becomes

$$\frac{\mathbf{u}^{n+1/2} - \mathbf{u}^{n-1/2}}{\Delta t} = \frac{q}{m} (\mathbf{E}^n + \frac{\mathbf{u}^{n+1/2} + \mathbf{u}^{n-1/2}}{2\gamma^n} \times \mathbf{B}_\text{av}^n)$$

(5.11)

where \( \mathbf{u} = \gamma \mathbf{v} \). Note that magnetic field values need to be averaged to the positions where the electric field values are known. To solve equation 5.11 we follow a method from Boris [84][86][87] and separate out the electric and magnetic forces by defining \( \mathbf{u}^- \) and \( \mathbf{u}^+ \) from the following two equations

$$\mathbf{u}^{n-1/2} = \mathbf{u}^- - \frac{q \mathbf{E}^n \Delta t}{2m}$$

(5.12)

$$\mathbf{u}^{n+1/2} = \mathbf{u}^+ + \frac{q \mathbf{E}^n \Delta t}{2m}.$$  (5.13)

Substituting these into equation 5.11 yields a simple rotation with angle \( \theta = -2\tan^{-1} \left( \frac{qB\Delta t}{2\gamma^n m} \right) \), where \( \gamma^n = \left( 1 + \left( \frac{u^-}{c} \right)^2 \right)^{-\frac{1}{2}} \). To obtain \( \mathbf{u}^+ \) we use Buneman’s algorithm which solves the following equations in a computationally efficient way

$$\mathbf{u}' = \mathbf{u}^- + \mathbf{u}^- \times \mathbf{t}$$

(5.14)

$$\mathbf{u}^+ = \mathbf{u}^- + \mathbf{u}' \times \mathbf{s}.$$  (5.15)

In the above equations \( \mathbf{t} = \frac{q B \Delta t}{2\gamma m} \) and \( \mathbf{s} = \frac{2\Delta t}{1+c^2} \). From here, \( u_x^+ \) and \( u_y^+ \) can be substituted back into equation 5.13 to obtain the updated particle velocities \( \mathbf{u}_{x}^{n+1/2} \) and \( \mathbf{u}_{y}^{n+1/2} \).

With the particle velocities now advanced forwards in time the positions of the particles can finally be updated by using the following equation
5.1 The particle in cell method

\[ x^{n+1} = x^n + \frac{u_n^{n+\frac{1}{2}} \Delta t}{\gamma^{n+\frac{1}{2}}}. \]  

(5.16)

5.1.3 Stability of the finite difference method

By looking at how the finite difference versions of Maxwell’s equations behave for electromagnetic waves in a vacuum we can gain insight into how accurate and stable the scheme really is [86]. Substituting fields of the form \( E = E_0 e^{i(k \cdot x - \omega t)} \) and \( B = B_0 e^{i(k \cdot x - \omega t)} \) into the differenced Maxwell equations yields

\[ \Omega^2 = c^2 \kappa^2 \]  

(5.17)

where \( \Omega = \frac{\sin(\frac{\Delta t}{2})}{\Delta t} \) and \( \kappa = \frac{\sin(\frac{k \Delta x}{2})}{\Delta x} \). In the continuum limit \( \Omega \) and \( \kappa \) reduce to \( \omega \) and \( k \), yielding the familiar result of \( \omega = ck \). Errors in \( \omega \) and \( k \) are second order in \( \Delta x \) and in \( \Delta t \). This analysis also introduces a constraint on the timestep used to advance the fields within the code, known as a CFL (Courant-Friedrichs-Lewy) condition [88], which states that a wave must travel less than one grid spacing in one timestep. In a one-dimensional simulation the expression that must be satisfied in order to prevent the system becoming unstable can be written as

\[ \Delta t^2 \leq \frac{\Delta x^2}{c^2}, \]  

(5.18)

whilst in two and three dimensional simulations there will be extra terms in the CFL condition due to the extra dimensions. For a three-dimensional case the condition may be rewritten as

\[ \Delta t^2 < \frac{1}{c^2 \left( \frac{1}{\Delta x^2} + \frac{1}{\Delta y^2} + \frac{1}{\Delta z^2} \right)}. \]  

(5.19)

If this condition is not maintained, the roots of \( \omega \) will become complex when \( k \Delta x \) nears \( \pi \), which can lead to fast and unphysical growth within the system.

To summarise, a basic PIC code has a timestep that will consist of the following steps:

1. Evaluate new particle velocities and positions; \( u^{n-\frac{1}{2}} \rightarrow u^{n+\frac{1}{2}}, x^n \rightarrow x^{n+1} \)
2. Calculate $J$ and $\rho$ at the grid positions; $J_y^{n-\frac{1}{2}} \rightarrow J_y^{n+\frac{1}{2}}$, $\rho^n \rightarrow \rho^{n+1}$

3. Advance the magnetic field by half a timestep; $B_z^n \rightarrow B_z^{n+\frac{1}{2}}$

4. Advance the electric field; $E_z^n \rightarrow E_z^{n+1}$, $E_y^n \rightarrow E_y^{n+1}$

5. Advance the magnetic field by the remaining half timestep; $B_z^{n+\frac{1}{2}} \rightarrow B_z^{n+1}$

5.2 EPOCH

EPOCH (Extendable PIC Open Collaboration H) is a new particle in cell simulation code that is currently being developed by a collaboration between Warwick University, Oxford University and Imperial College (funded by EPSRC). There are 1D, 2D and 3D versions of the code, although the results presented in this thesis have been produced using the 2D version only. The code is roughly based on the code PSC by Hartmut Ruhl and contains similar algorithms for field updating and particle mover routines. The code is fully parallelised (including MPI-IO), is fully relativistic and uses 2nd order particle weighting as standard in order to reduce numerical heating. The code also comes with files allowing the output data files to be read directly into programs including ITT IDL, LLNL VisIt and Mathworks MatLab, although it is also fairly straightforward to create custom readers for the output files (for example, many of the plots in this thesis are created using custom file readers that use PGPLOT FORTRAN libraries). EPOCH has been designed and written in a style that tries to make it as simple as possible to add extra features and expansions, as well as in a way that makes the code usable by people with a wide range of programming abilities.

EPOCH is an excellent choice of code with which to study the propagation of fast electrons in laser-solid density plasma interactions. EPOCH uses 2nd order particle weighting as standard which reduces the self heating within the code when under resolving the Debye length. Indeed, the results presented in chapter 7 have been produced using EPOCH with the 4th order spline weighting turned on, which allows for simulations to be performed with even fewer particles whilst still keeping the self heating within the code at low levels. This higher order par-
Particle weighting within EPOCH allows us to model densities of up to a few gcc at temperatures of around 100eV with moderate computer resources.

5.3 Particle weightings

As mentioned previously, particle in cell codes work by using a finite spatial grid on which all of the field quantities required for the simulation are stored. In any simulation the particle positions and properties must be used to set up field values by assigning a charge and current to each gridpoint, which are then used when calculating the field values at the gridpoints. In turn these fields are advanced forwards in time using Maxwell’s equations and the new field values must be used to push the particles and advance the system forwards in time. Of course, this leaves the issue of how to interpolate the values between the continuous positions of the particles and the discrete positions of the gridpoints of the spatial grid. Although we are free to decide how to interpolate the quantities between the grid and particle positions, it is important that we are consistent within the code and use the same interpolation when moving particle quantities to grid positions and grid quantities to particle positions. Failure to do this will result in unphysical self forces and a breakdown of the conservation of momentum within the simulations performed.

All of the possible weightings that can be used to link the grid quantities and particle properties have one important property in common; the fact that the grid is forced to see particles as having a ‘finite size’ as they move around. The physics seen in the simulations is therefore that of finite sized particles and not of point particles. This removes the effects of encounters between particles at distances less than the grid spacing. For many plasma simulations this is not a problem as these effects are small (for hot, low density plasmas for instance) and the longer range collective effects remain unchanged. However, as is obvious in the context of this thesis, these encounters are not always negligible and are often required in order to get physically correct results out of simulations. This has resulted in the development of methods to put the collisional (sub-grid) particle effects back in to
the particle in cell method, without introducing the massive close range forces that would occur if the particles were to interact through the spatial grid on these short scales.

5.3.1 Zero-order weighting

Following the nomenclature and notations of [84] the simplest form of interpolation is called zero-order weighting (or nearest gridpoint ‘NGP’ weighting). This is when the entirety of the charge of any particular particle is assigned to the gridpoint closest to that particle, i.e. if the grid spacing is defined as $\Delta x$ with gridpoints at $X_j$, the a particle will move from being in one cell to another as it crosses the apparent cell boundary located at $X_j \pm \frac{\Delta x}{2}$. The charge at $X_j$ due to a particle at $x_i$ is therefore

$$q_j = \begin{cases} q_i & X_j - \frac{\Delta x}{2} < x_i \leq X_j + \frac{\Delta x}{2} \\ 0 & x_i \leq X_j - \frac{\Delta x}{2}, x_i > X_j + \frac{\Delta x}{2} \end{cases}$$ (5.20)

which essentially states that to find the charge at a grid point you simply sum the charge of all of the particles between $X_j - \frac{\Delta x}{2}$ and $X_j + \frac{\Delta x}{2}$. This leaves the grid seeing finite sized particles of width $\Delta x$ centred at $X_j$. The grid will then see the particles jump to being centred at $X_{j\pm1}$ as they cross the boundaries half way between the gridpoints, as shown in figure 5.2. The arrow shown in the figure shows how the gridpoint ($X_{j=0}$) sees the whole charge of the particle until it crosses the aforementioned boundaries. The same effect also occurs when updating a particle's position as the particles will only see the force resulting from the fields at their nearest gridpoint.

Figure 5.2: Apparent shape of computational particle located at $x_i$ with NGP weighting
This weighting method is computationally fast as only one gridpoint must be located for each particle and the interpolation is straightforward. However, the jumps in charge and density seen by the grid as a particle moves from one cell to another result in densities and electric fields that are very noisy. The noise in the grid quantities means that this form of weighting is unsuitable for all but the simplest of plasma simulations.

5.3.2 1st order weighting

An improvement to NGP weighting is to interpolate using two gridpoints for each particle. This is first order weighting, commonly referred to as cloud-in-cell (CIC) as coined by Birdsall and Fuss, 1969. The advantage of first-order weighting is that the noise is reduced relative to zero-order weighting. However, this method is more computationally expensive as two gridpoints now need to be accessed for each computational particle (for multidimensional simulations this means two gridpoints in each dimension of the simulation domain). The charge at the two nearest gridpoints is shown in equations 5.21, where $q_j$ represents the charge at gridpoint $X_j$ and $q_c$ is the total charge of the cloud. It is assumed that the particle is centred in cell $j$, hence the gridpoint to the left of the particle is at $X_j$ and the gridpoint to the right is at $X_{j+1}$.

$$q_j = q_c \left( \frac{\Delta x - (x_i - X_j)}{\Delta x} \right) = q_c \left( 1 + \frac{X_j - x_i}{\Delta x} \right)$$

$$q_{j+1} = q_c \left( 1 - \frac{X_{j+1} - x_i}{\Delta x} \right)$$

(5.21)

It is convenient to generalise equations 5.21 by introducing the weighting function $W_J(x_i)$, which describes the fraction of the total charge assigned to any particular gridpoint $X_J$ due to a particle $i$ located at $x_i$ [85]. For our top hat shaped cloud of charge the weighting function is

$$W_J (x_i) = \begin{cases} 
1 - \frac{|X_J - x_i|}{\Delta x} & |X_J - x_i| < \Delta x \\
0 & otherwise
\end{cases}$$

(5.22)
5.3 Particle weightings

The term ‘cloud’ is used because the particles appear to be rigid clouds of uniform charge which are able to move through one another. The clouds charge is then area weighted to the two nearest gridpoints. This means that 1st order weighting causes the effective particle shape seen by the grid to be changed from a rectangle to a triangle of width $2\Delta x$. The cloud-in-cell (CIC) and triangular particle-in-cell (PIC) viewpoints are of course equivalent and are shown in figure 5.3, where the arrows on the particle viewpoint show how the grid charge now slides up and down as a particle moves across a cell boundary.

5.3.3 2nd order weighting

For many plasma simulations 1st order weighting provides the best compromise between accuracy and the time required for the simulation. However, in various branches of current research particle-in-cell codes are being pushed to try and simulate larger regions of space and higher density plasmas. This can result in the grid spacing required having to be larger than the plasma Debye length which leads to numerical heating within the code. This unphysical heating can be suppressed by using higher order particle weightings. 2nd order weighting involves area weight-
Figure 5.4: Cloud in Cell and Particle in Cell viewpoints when using 2nd order weighting

Assigning a cloud’s charge to the gridpoints as is performed in 1st order weighting, however, the shape of the cloud is now triangular like the apparent particle shape from 1st order weighting.

This means that three gridpoints are now required when interpolating the various quantities between the gridpoints and the computational particles. This again means that the computational time required to perform the interpolations is increased. However, the extra smoothing of results and reduction in self-heating allow for simulations to be run with far fewer particles than would be required to maintain a stable system with a lower order weighting. The result of this is that simulations can be performed that would not be feasible using lower order weightings.

Again taking the example of the charge distributed at the gridpoints due to a particle at $x_i$ which resides in the cell $X_J$ results in the fractions of the particles charge at the three gridpoints being given by the following integrals

![Cloud in Cell and Particle in Cell viewpoints when using 2nd order weighting](image-url)
5.3 Particle weightings

\[ q_{J-1} = \frac{1}{\Delta x} \int_{x_i-\Delta x}^{x_i} q_c \left( 1 + \frac{x - x_i}{\Delta x} \right) dx \]  

\[ q_J = \frac{1}{\Delta x} \left[ \int_{x_i}^{x_i+\Delta x} q_c \left( 1 + \frac{x - x_i}{\Delta x} \right) dx + \int_{x_i}^{x_i-\Delta x} q_c \left( 1 - \frac{x - x_i}{\Delta x} \right) dx \right] \]  

\[ q_{J+1} = \frac{1}{\Delta x} \int_{x_{i+1}-\Delta x}^{x_{i+1}} q_c \left( 1 - \frac{x - x_i}{\Delta x} \right) dx \]

By generalising and integrating the above equations we obtain the 2\textsuperscript{nd} order weighting function:

\[ W_J(x_i) = \begin{cases} 
\frac{3}{4} - \frac{|X_J-x_i|^2}{\Delta x^2} & |X_J-x_i| \leq \frac{\Delta x}{2} \\
\frac{1}{2} \left( \frac{3}{2} - \frac{|X_J-x_i|}{\Delta x} \right)^2 & \frac{\Delta x}{2} \leq |X_J-x_i| \leq \frac{3\Delta x}{2} \\
0 & \text{otherwise}
\end{cases} \quad (5.24) \]

This results in the grid seeing a particle with the shape shown in figure 5.4.

With this weighting, when a computational particle is exactly centred on a gridpoint that gridpoint only sees \( \frac{3}{4} \) of the total charge, with \( \frac{1}{8} \) of the charge placed at each of the two adjacent gridpoints.

5.3.4 Higher order weightings

Higher order weightings are seldom used in particle in cell codes due to the increased computational expense of looking up more gridpoints when performing the interpolations. However, it is possible to carry on increasing the order of the weighting by working out the convolution of the NGP weighting function with itself over and over [85]. EPOCH version 2.0 and higher include an option to use 4\textsuperscript{th} order spline particle weighting and the effective particle shape for this weighting is shown in figure 5.5. Tests (described in chapter 5.4) have shown that this higher order weighting has the potential for further reducing the self-heating within the code, allowing simulations to be performed using fewer particles and/or more cells whilst keeping the run times acceptable.
5.4 Numerical heating checks

An important consideration when setting up PIC simulations is to make sure that the setup used is suitably resistant to numerical heating. This is especially true when attempting to simulate solid density cases where it is impractical to fully resolve the Debye length. Because of this several tests have been performed to study how EPOCH behaves when simulating the conditions that are used for the simulations discussed in chapter 7. These tests have been performed using the 2nd order particle weighting that EPOCH uses as standard, as well as for the optional 4th order particle weighting. Several systems have been examined in order to see how much the code heats up under various circumstances. A system of 100 × 100 cells was used in all cases and the number of particles per cell and the size of the cells was varied. For the cases investigated in chapter 7 3072 × 3072 cells are used to simulate a 20 × 20µm area containing a plasma with a peak electron number density of approximately 3 × 10^{29}m^{-3}. This gives a cell size of approximately 6.5 × 10^{-9}m so heating tests were carried out using this cell size, as well as using twice and half this cell size for comparison. The test simulations contained plasma at

![Figure 5.5: Effective particle shapes for different order weightings](image-url)
this constant density throughout the whole simulation region and the boundary conditions were set to be periodic. The timesteps used in the heating tests were set to be $0.9 \times$ the CFL condition for the respective cell size used and the initial temperature was set to 100eV.

Table 5.1 and figure 5.6 show how much the temperature increases due to numerical effects in 500 fs, along with how long the various tests took to run, when the 2nd order particle weighting was used. Obviously, with only a few particles per cell the code rapidly heats up to extremely high temperatures and it is only when we get into hundreds of particles per cell that numerical heating does not quickly heat the plasma. In particle in cell codes the standard weighting used is first order particle weighing. However, for high density/low temperature set-ups this is not at all suitable. Indeed, for the best case shown in the tables (200 particles per cell with the cell size set to $3.25 \times 10^{-9} m$, i.e. half that of the main simulations discussed in chapter 7) using second order weighting results in the electrons heating up to 164eV over 500fs. However if first order particle weighting is used, despite the advantage of taking approximately half the time to run, the code numerically heats the electrons to an average temperature of almost 7.5keV in the same amount of time.

Table 5.2 and figure 5.7 show the results of the same tests when performed using the 4th order particle weighting that is also an option within EPOCH. The potential of using this weighting to combat numerical heating within the code is obvious, with clear reductions in numerical heating seen in all of the cases tested.
Figure 5.6: Plots showing the amount of numerical heating when using 2\textsuperscript{nd} order particle weighting. The plots are for tests with grid spacings of (a) $dx = 6.5 \times 10^{-9} m$, (b) $dx = 3.25 \times 10^{-9} m$ and (c) $dx = 1.3 \times 10^{-8} m$. In all of the plots lines are shown which correspond to tests performed with different numbers of particles per cell.
Figure 5.7: Plots showing the amount of numerical heating when using $4^{th}$ order particle weighting. The plots are for tests with grid spacings of (a) $dx = 6.5 \times 10^{-9} m$, (b) $dx = 3.25 \times 10^{-9} m$ and (c) $dx = 1.3 \times 10^{-8} m$. In all of the plots lines are shown which correspond to tests performed with different numbers of particles per cell.
Table 5.1: The relationship between the number of particles per cell / grid size used in a simulation and the amount of numerical heating that occurs in 500fs. The time taken to complete the simulations is also shown. All of the test runs were carried out on a $100 \times 100$ cell grid with periodic boundaries. This table shows the results of tests using $2^{nd}$ order particle weighting.

<table>
<thead>
<tr>
<th>Cell size</th>
<th>( 6.5 \times 10^{-9}m )</th>
<th>( 3.25 \times 10^{-9}m )</th>
<th>( 1.3 \times 10^{-8}m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T (eV)</td>
<td>Time (mins)</td>
<td>T (eV)</td>
<td>Time (mins)</td>
</tr>
<tr>
<td>10</td>
<td>16700</td>
<td>4395</td>
<td>69564</td>
</tr>
<tr>
<td>25</td>
<td>4528</td>
<td>1673</td>
<td>3884</td>
</tr>
<tr>
<td>50</td>
<td>1111</td>
<td>648</td>
<td>414</td>
</tr>
<tr>
<td>100</td>
<td>396</td>
<td>268</td>
<td>169</td>
</tr>
<tr>
<td>200</td>
<td>225</td>
<td>164</td>
<td>129</td>
</tr>
</tbody>
</table>

Table 5.2: The relationship between the number of particles per cell / grid size used in a simulation and the amount of numerical heating that occurs in 500fs. The time taken to complete the simulations is also shown. All of the test runs were carried out on a $100 \times 100$ cell grid with periodic boundaries. This table shows the results of tests using $4^{th}$ order particle weighting.

<table>
<thead>
<tr>
<th>Cell size</th>
<th>( 6.5 \times 10^{-9}m )</th>
<th>( 3.25 \times 10^{-9}m )</th>
<th>( 1.3 \times 10^{-8}m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T (eV)</td>
<td>Time (mins)</td>
<td>T (eV)</td>
<td>Time (mins)</td>
</tr>
<tr>
<td>10</td>
<td>281</td>
<td>248</td>
<td>193</td>
</tr>
<tr>
<td>25</td>
<td>148</td>
<td>151</td>
<td>125</td>
</tr>
<tr>
<td>50</td>
<td>120</td>
<td>123</td>
<td>112</td>
</tr>
<tr>
<td>100</td>
<td>109</td>
<td>111</td>
<td>105</td>
</tr>
<tr>
<td>200</td>
<td>105</td>
<td>105</td>
<td>103</td>
</tr>
</tbody>
</table>
Chapter 6

Collisions In Particle In Cell Codes

6.1 Why collisions are necessary

If a plasma has a low enough density and a sufficiently high temperature it may reasonably be assumed that it behaves as if it were collisionless. This is the area of plasma physics where PIC codes thrive and they are commonly used to study laser-plasma interactions. PIC codes are able to model the whole process of the laser-plasma interaction self-consistently, including any non-Maxwellian effects and non-linear behaviour. However, at low temperatures and/or high densities collisional effects between particles become more important. Due to the nature of PIC codes (i.e. particles interact with the fixed spatial grid on which field variables are stored, not directly with one another) the interactions between particles at ranges shorter than the grid spacing are not resolved. This means that the accuracy of basic PIC codes is questionable in regimes where collisional effects are important. In order to use PIC codes to examine regimes where collisional effects are important extra algorithms must be added to the codes in order to correctly model these sub-grid collisional effects.
6.2 The history of collisions in PIC codes

The dominant collisional process in hot/dense plasmas is the effect of small-angle Coulomb Collisions and much work has been carried out in order to include these collisional effects in PIC simulations. Shanny et al. have produced a Lorentz gas model for one-dimensional simulations where electron sheets are scattered by small-angle collisions with stationary ions [89]. Oliphant and Nielson went on to produce the first binary collision model in 1970, showing that their model successfully reproduced a numerical solution to the Fokker-Planck equation (equation 3.29) [90]. However, their model uses local collision frequencies that have no dependence on the velocities of individual particles. Takizuka and Abé [55] first proposed the method of grouping particles within each cell to form a binary collision model by a Monte Carlo method for plasma simulations in 1977. By pairing particles in a binary collision model and using collision statistics their collision routine corresponds to an N-body problem, compared to the full treatment of particle collisions which would scale with $N^2$.

Over the years Takizuka and Abé’s routine has been built upon and improved whilst various other methods for including Coulomb collisions in PIC codes have also been devised. Miller and Combi [91] devised an algorithm based on Takizuka and Abé’s method that allows for the use of weighted particles. Sentoku et al. then added to the method by first expanding Takizuka and Abé’s method into the relativistic regime [92] and then by producing a fully relativistic energy-conserving binary collision model using the weighted particle method [93]. Alternate methods include using cumulative scattering angles and having a grid based ‘collision field’. Nanbu and Yonemura [94] came up with a method in which a cumulative scattering angle was calculated, enabling many small angle binary collisions to be grouped together and implemented as a single binary collision with a large scattering angle. The obvious advantage of this method is that the collision routine can be called less frequently, thus increasing the efficiency of the simulations performed. This method is suited to regimes where the required timestep is much less than the collision times within the plasma. The ‘collision field’ method proposed by Jones et al. [95] scatters particles using a force that is calculated at grid positions by inter-
polating various quantities in order to reproduce fluid transport equations. The effect of this collision force is then included in the Lorentz force when updating the particle positions within the code.

A version of the approach of Sentoku et al. has been used for the studies in this thesis. This method was chosen due to the fact that it is fully relativistic, an important consideration in the studies of fast electron transport. For the cases considered in this thesis the limiting factor in setting the timestep for the simulations is actually the collision time. This means that Nanbu’s method (which is also non-relativistic) is not necessary. Also, due to the different populations of electrons (i.e. the thermal background electrons and the fast electron beam) Jones’ method would not be adequate.

6.3 The collision routine added to EPOCH

The collision routine that has been built into EPOCH in order to perform the simulations discussed in this thesis was written by the author independently of the routine used to perform the simulations discussed in [1]. The routine uses a particle paring method similar to the technique described by Takizuka and Abé [55] and uses the improved collision mechanics described by Sentoku and Kemp [93]. In the following sections the collision routine that the author has implemented within EPOCH is described in detail and much of the work contained within the previously mentioned publications is outlined.

The collision routine that has been written in order to perform the simulations discussed in this thesis can be used to include the effects of collisions between any species of particles present within the simulations. However, the routine does not currently account for particles having different weights (i.e. the collision routine assumes that every computational particle represents the same number of real particles). The simulations discussed within this thesis have been performed using electron-ion Coulomb collisions only which means that the assumption that the particles all have equal weights within the collision routine is valid. This is because the collisions of interest are between the electrons and ions within the bulk
plasma and these particles will all have the same weighting in the simulations. If electron-electron collisions were also to be included the collision routine would have to be altered to accommodate particles with different weights. This is due to the fact that the fast electrons generated in the laser-plasma interaction at the front of the target will not have the same weighting as the background electrons within the bulk plasma.

### 6.3.1 Particle pairing

The standard Takizuka and Abé collision routine begins by grouping together all particles within each cell. The next step is to randomly swap the memory addresses for particles within each species (as shown if figure 6.1), followed by pairing up the particles and carrying out the actual collisions. If the number \( N \) of particles of each species is the same every particle is collided once, however, if there are more particles of one species than there are of other \( N_{\text{larger}} - N_{\text{smaller}} \) particles from the species with fewer particles will be scattered twice, as shown in figure 6.2 (for the example shown there are two more electrons than there are ions, hence two of the ions are scattered twice).

![Figure 6.1: Takizuka and Abé’s random particle reordering](image)
One of the novel features of EPOCH is that particles are stored as linked lists and a useful compile time option within EPOCH is to have the code create separate secondary lists which exist for each cell. These secondary lists contain pointers to all of the particles within each individual cell at the current timestep. This allows for a novel way to implement the particle pairing; by making the lists for each particle species circular (i.e. linking the last particle in the list to the first). Rather than randomising all of the particles within a cell we may now simply pick a random starting point on our circular linked list and begin to pair particles from this point, a procedure akin to cutting a pack of cards. A simple example of this is shown in figure 6.3. We start pairing particles by pairing the first particle of the species that has fewer particles (in this case there are four electrons and five ions so we use the electrons) with a particle from a random position on the list containing a larger number of particles (in the example we begin from the third ion in the list). We then move down both lists of particles, pairing them off as we go. As there is one more ion than there are electrons in this case one electron is scattered twice, shown by the blue arrow in the figure. The computational cost of including a collision routine in a PIC code is mostly attributed to the gathering, randomising and pairing of particles within each cell. The method employed within EPOCH is equivalent to Takizuka and Abé’s method, yet we are able to miss out the step of randomising the particle addresses hence saving valuable time in the simulations.
6.3 The collision routine added to EPOCH

6.3.2 Relativistic binary collisions

The actual binary collision for small angle Coulomb scattering calculates the changes to the velocities of the two particles over a timestep $\Delta t$. This basically involves transforming variables into the centre of momentum (CoM) frame and then choosing a scattering angle based on collision statistics. The relative velocity vector is then rotated through the angle calculated. The ideal cell size within which collisional effects are calculated would be equal to the Debye length, so that collisions are calculated within a Debye sphere. However, as noted in [93], the only actual constraints on the cell size are that it must be small enough that there are no major temperature or density gradients across the cell and that it must contain enough particles to give good collision statistics. For each pair of particles the following steps must be performed.

**Transform to the centre of mass reference frame**

Following from [92] we will let the two colliding particles (hereafter referred to as $\alpha$ and $\beta$) have masses $m_\alpha$ and $m_\beta$ and laboratory frame velocities of $v_\alpha$ and $v_\beta$ respectively. The centre of mass (CoM) frame therefore moves with a velocity
6.3 The collision routine added to EPOCH

given by

\[ \mathbf{v}_C = \frac{\gamma_\alpha m_\alpha \mathbf{v}_\alpha + \gamma_\beta m_\beta \mathbf{v}_\beta}{\gamma_\alpha m_\alpha + \gamma_\beta m_\beta} \]  \hspace{1cm} (6.1)

and we can define \( \gamma_C \) as \( \frac{i}{\sqrt{1 - \frac{v_C^2}{c^2}}} \).

The particle velocities must then be transformed into the CoM frame using the following Lorentz transform

\[ \mathbf{v}_{i,C} = \left( \frac{\gamma_C - 1}{\gamma_C} \right) \frac{\mathbf{v}_C \cdot \mathbf{v}_i}{v_C^2} \mathbf{v}_C + \mathbf{v}_i - \gamma_C \mathbf{v}_C \]  \hspace{1cm} (6.2)

where \( i = \alpha, \beta \). The particle momentums in the CoM frame are therefore given by

\[ \mathbf{P}_{\alpha,C} = \gamma_{\alpha,C} m_\alpha \mathbf{v}_{\alpha,C} \text{ and } \mathbf{P}_{\beta,C} = \gamma_{\beta,C} m_\beta \mathbf{v}_{\beta,C} \]  \hspace{1cm} (6.3)

and in the CoM frame the sum of the two momenta is zero (\( \mathbf{P}_{\alpha,C} + \mathbf{P}_{\beta,C} = 0 \)). The collision process is elastic, hence momentum is conserved and in the CoM frame the magnitude of the momentum of each particle doesn’t change

\[ |\mathbf{P}_{\alpha,C}| = |\mathbf{P}_{\beta,C}| = |\mathbf{P}_{\alpha,C}'| = |\mathbf{P}_{\beta,C}'| = P. \]  \hspace{1cm} (6.4)

This means that the collision simply causes a rotation of the particle momenta in the CoM frame as described by Rutherford scattering.

**Rotation to the relative momentum frame**

To carry out the scattering within the code the coordinate system is rotated so that the \( p_z \) axis is aligned with the momentum vector of one of the particles. Operating on \( \mathbf{P}_{\alpha,C} \), the transform required is

\[
\begin{pmatrix}
\cos\theta \cos\phi & \cos\theta \sin\phi & -\sin\theta \\
-\sin\theta & \cos\phi & 0 \\
\sin\theta \cos\phi & \sin\theta \sin\phi & \cos\theta
\end{pmatrix}
\begin{pmatrix}
P_x \\
P_y \\
P_z
\end{pmatrix}
= \begin{pmatrix}
0 \\
0 \\
P
\end{pmatrix}
\]

where the angles \( \theta \) and \( \phi \) are as defined in figure 6.4. The collision is carried out in this frame and then the new momentum is rotated back into the CoM frame with the inverse transform.
The momentum rotation due to the collision

Due to the binary collision between the two particles the magnitude of the relative momentum does not change, only its direction is altered. In the new coordinate system P is aligned with the $p_z$-axis and the collision will change the direction of P through the angles $\Theta$ and $\Phi$, as shown in figure 6.5.

The result of the collision is therefore a rotation of P through $(0, 0, p) \rightarrow (P\sin\Theta\cos\Phi, P\sin\Theta\sin\Phi, P\cos\Theta)$. Of course, this is still in the frame where the original relative momentum was aligned with the $p_z$-axis so the new momentum vector must used to find the change in the components of the momentum in the CoM frame. In the CoM frame the transform that has occurred is

$$\begin{pmatrix} \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\theta & \cos\phi & 0 \\ \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \end{pmatrix} \cdot \begin{pmatrix} \Delta P_x \\ \Delta P_y \\ \Delta P_z \end{pmatrix} = \begin{pmatrix} P\sin\Theta\cos\Phi \\ P\sin\Theta\sin\Phi \\ P\cos\Theta \end{pmatrix}$$

which can be solved to give

$$\Delta P_x = \frac{P_x}{P_\perp} P_\perp\sin\Theta\cos\Phi - \frac{P_y}{P_\perp} P_\perp\sin\Theta\sin\Phi - P_x(1 - \cos\Theta) \quad (6.5)$$

$$\Delta P_y = \frac{P_y}{P_\perp} P_\perp\sin\Theta\cos\Phi + \frac{P_x}{P_\perp} P_\perp\sin\Theta\sin\Phi - P_x(1 - \cos\Theta) \quad (6.6)$$

$$\Delta P_z = -P_\perp\sin\Theta\cos\Phi - P_z(1 - \cos\Theta) \quad (6.7)$$
6.3 The collision routine added to EPOCH

Figure 6.5: Rotation of the momentum due to the binary collision

where \( P_\perp = \sqrt{P_x^2 + P_y^2} \). The post collision momenta of the two particles in the CoM frame is then simply given by

\[
P'_{\alpha,C} = P_{\alpha,C} + \Delta P \quad \text{and} \quad P'_{\beta,C} = P_{\beta,C} - \Delta P
\]

(6.8)

where \( \Delta P = (P_x, P_y, P_z) \). Finally, these may be transformed back to the lab frame using the following Lorentz transform

\[
v'_i = \frac{(\gamma_C - 1)\frac{v_C v'_i C}{v'_i C} v_C + v'_{i,C} + \gamma_C v_C}{\gamma_C (1 + \frac{v_C v'_i C}{c^2})}
\]

(6.9)

where \( i = \alpha, \beta \) and \( v_{i,C} \) is found from the momentum using equation 6.3.

6.3.3 Scattering angles

All that remains now is to evaluate the scattering angles \( \Theta \) and \( \Phi \). In the collision routine the angle \( \Theta \) is chosen statistically from a Gaussian distribution in a method equivalent to that stated in Takizuka and Abé’s original paper from 1977. To correctly model the collisions we must calculate the correct variance to use when randomly choosing the angle. A similar derivation has been performed by Sherlock [96], in which the theory is expanded to remove the singularity in the Coulomb Logarithm at 0. The following is a simple description of where the equation for the variance comes from. Takizuka and Abé correctly write the equation as
\[
\langle \delta^2 \rangle = \frac{e_\alpha^2 e_\beta^2 n_L \ln \Lambda}{8\pi e_0^2 m_{\alpha\beta}^2 u^2} \Delta t
\] (6.10)

where $\delta \equiv \tan \frac{\Theta}{2}$, $u$ is the relative velocity, $e_\alpha$ and $e_\beta$ are the particle charges, $m_{\alpha\beta}$ is the reduced mass, $n_L$ is the lower of the particle densities of the different species and the other variables have their usual meanings. We use the lower of the number densities due to the fact that particles from this species will be scattered more than once. The equation calculates $\langle \delta^2 \rangle$, which is the variance that the scattering angle is chosen using. Once the right hand side of this equation has been calculated, the Box-Muller method is utilised in order to generate the required Gaussian random numbers and the scattering angles may then be inferred.

To derive equation 6.10 we begin from the basic equations for a Coulomb collision as described in chapter 3.1. The scattering angle for a Coulomb collision is given by

\[
\tan \left( \frac{\Theta}{2} \right) = \frac{b_0}{2b}
\] (6.11)

where $b$ is the impact parameter (the the shortest distance between the trajectories of the two particles assuming they do not interact) and $b_0$ is the impact parameter required for a 90$^\circ$ collision. If averaged over all collisions $\langle \Theta \rangle = 0$ due to symmetry considerations. The value $\langle \Theta^2 \rangle$ however is finite and gives us the variance for the spread of angles due to all collisions with a specific relative velocity. To calculate the required variance, $\langle \tan^2 (\Theta/2) \rangle$ for this case, we must integrate over all possible impact parameters that collisions can occur over. The number of interactions with an impact parameter between $b$ and $b+db$ occurring in one second is given by

\[
N = 2\pi bdbn_Lu.
\] (6.12)

Note that we are not required to integrate over the velocity distribution as each collision is carried out using a specific relative velocity. Integrating over all possible impact parameters gives us the variance per second:
\[
\langle \tan^2 (\Theta/2) \rangle = \frac{b_0^2}{4b^2} 2\pi db n_b u
\]
\[
= \frac{n v \pi b_0^2}{2} \int_0^{b_{\text{max}}/b_0} \frac{db}{b}
\]
\[
= \frac{n v \pi b_0^2}{2} \ln \left( \frac{b_{\text{max}}}{b_0} \right)
\]
\[
= \frac{e^2 e^2 n_L}{8\pi \epsilon_0^2 m^2_{\alpha\beta} u^3} \ln \Lambda
\]

where we have substituted equation 3.8 for \( b_0 \) and \( \Lambda = b_{\text{max}}/b_0 \), so that \( \ln \Lambda \) is the Coulomb logarithm. For the work in this thesis \( \ln \Lambda \) is set to 10 for all cases, which is an approximate value generally used for all plasmas. This is justified because it is a slowly varying function of the plasma parameters [20]. The variance for the required timestep is then simply given by multiplying by \( \Delta t \), yielding equation 6.10.

It is possible to calculate the Coulomb logarithm on a cell by cell basis rather than assuming a fixed value as has been done in this thesis and in many other studies. Investigating the exact effects this assumption has on the results shown in this thesis would be an interesting further study. Indeed, actually calculating the Coulomb logarithm for the 1gcc plasma at 100eV setup used for the simulations discussed in chapter 7 yields a value close to 3. This means that using a fixed value of 10 is equivalent to artificially increasing the collision frequency (or artificially simulating higher Z materials).

The relativistic alterations to the variance are fairly straightforward to implement. The main difference is that the collision now needs to be defined in the rest frame of the scatterer, meaning that the velocity is required in this frame and that the calculated angle must then be transformed back into the centre of mass frame. The variance in the frame where the scattering particle is stationary is found from the relativistic collision frequency given by [54]

\[
\langle \delta^2 \rangle = \frac{e^2 e^2 n_L \ln \Lambda}{8\pi \epsilon_0^2 p_{rel}^2 v_{rel}} \Delta t. \tag{6.14}
\]

It is important to remember that the relativistic angle is defined in the one particle at rest frame rather than the centre of mass frame. Sentoku et al. have
shown that this angle must be transformed into the centre of mass frame using

$$
tan \Theta_C = \frac{\sin \Theta \gamma_C}{\cos \Theta - \frac{v_C}{v}} \quad (6.15)
$$

where the subscript C again denotes a variable in the CoM frame, $\Theta$ is now the angle in the one particle at rest frame, $v$ is the velocity after scattering in the one particle at rest frame and the other variables are as previously defined. As $v$ is not yet known the pre-collision velocity is used, noting that the velocity is only changed slightly during a small angle collision. This transformation isn’t overly important to the results discussed in chapter 7 due to the fact that only electron-ion collisions are being considered, where $\Theta_C \approx \Theta$. However, in electron-electron collisions this transformation is important because the scattering angle in the centre of mass frame is over two times greater than in the one particle at rest frame.

Finally, the azimuthal angle $\Phi$ is chosen randomly from a uniform distribution between 0 and $2\pi$. In the simulations the value of $\langle \delta^2 \rangle$ may occasionally become greater than 1 and in these rare instances the angle $\Theta$ is simply chosen randomly from a uniform distribution between 0 and $\pi$.

6.4 Testing the collision model

Before using the new collisional version of EPOCH to perform the full scale simulations discussed in the following chapter several tests were performed in order to make sure that the collision routine correctly reproduces the expected physics. The tests were all performed using the two-dimensional version of EPOCH where all boundary conditions were set to be periodic and, as with the simulations discussed in the following chapter, the Coulomb logarithm has been set to 10. The tests discussed in the following sections have been performed using electron-ion Coulomb collisions only unless otherwise stated.
6.4 Testing the collision model

6.4.1 Slowing down collision frequency tests

The collision routine was first tested to make sure that it matches the theoretical slowing down rates for a beam of particles. For these tests the ions were all assumed stationary to begin with and all electrons began drifting in a beam with a velocity of \( \sqrt{k_b T/m} \) (where \( T=1 \times 10^6 \text{ K} \)) in the x-direction. The plasma is a fully ionised hydrogen plasma (\( n_e = n_i \)) with an electron number density of \( 2 \times 10^{27} \). This corresponds to a collision frequency of \( \nu_{ei} \approx 2.7 \times 10^{14} \) and a collision time of \( \tau_{ei} = 1/\nu_{ei} \approx 3.7 \times 10^{-15} \) (\( \nu_{ei} = 1/\tau_{ei} = n e^4 \Lambda/4\pi e_0^2 m_{\alpha\beta} v^3 \)). The average velocity of the beam is reduced as collisions act to isotropise the distribution of particles. Figure 6.6 contains plots showing the average x-component drift velocity throughout several simulations performed using various numbers of particles per cell. Theoretical values calculated using the slowing down frequency \( \nu_s \) (described in chapter 3.5) are also shown on the plots. The plots are normalised to the initial electron drift velocity and the collision frequency.

Obviously, as the number of particles increases the computational results become closer matched to the theoretical fit. It is also worth noting that there can be a degree of statistical variation between runs that are started with the same initial conditions, an example of which is shown in figure 6.7a where four runs with 200 particles per cell are compared. Figure 6.7b shows the difference between theoretical and calculated values for the drift velocity through time. The statistical fluctuations are counteracted by the relaxation due to collisions hence the departures from the theory in the computed results are just fluctuations and the error does not grow with time.

The accuracy of the routine is also influenced by the timestep used in the simulations. Figure 6.8 shows the slowing down of electrons in simulations performed using various numbers of collision times per timestep, along with the deviation from the theoretical values plotted with time. The plots show that as the length of the timestep is increased the fluctuations in the calculated drift velocities deviate further away from the theoretical values. It is noteworthy that the vast reductions in timestep shown only result in small improvements to the accuracy of these test simulations. The number of particles per cell used in the simulations is seen to
Figure 6.6: Plots showing the theoretical slowing down of a group of particles and the drift velocity resulting from the collision routine for various numbers of particles per cell. The scales are normalised to the initial drift and the collision frequency.
6.4 Testing the collision model

Figure 6.7: (a) As figure 6.6(e) but showing the statistical scatter between three runs with identical initial conditions. (b) The error between code and theory for the four runs.

have a larger effect on the accuracy of the collision routine than the actual size of the timestep used.

Figure 6.8: (a) As figure 6.6(e) but showing the results of several simulations performed using different timesteps. (b) The error between code and theory for the timestep comparison results shown in (a).
6.4 Testing the collision model

6.4.2 Spitzer resistivity test

The code was also checked to see if it reproduces the correct drift velocity when a constant electric field is applied to the plasma. Spitzer theory gives values for the resistivity of a plasma (as described in chapter 3.6) and can be used to calculate a current that we can compare our model against.

The setup used for this test is similar to in the previous tests; the electron number density is $2 \times 10^{27} \text{m}^{-3}$, the temperature is $1 \times 10^6 \text{K}$, \( \ln \Lambda = 10 \), the applied field is $1 \times 10^7 \text{Vm}^{-1}$ in the x-direction and only electron-ion collisions are considered so that the drift seen in the simulations should correspond to the Lorentz limit of the current given by Spitzer theory. All other fields in the simulations were clamped to zero to ensure that the simulations were purely a test of the collision routine. One million particles were used for this test due to the fact that the drift velocity is calculated by averaging many positive and negative velocities that are larger than the actual drift. This means that errors due to number of particles used will not be insignificant (i.e. for the setup tested the thermal velocity is approximately $5.5 \times 10^6 \text{ms}^{-1}$ whilst the drift velocity is only approximately $8.2 \times 10^4 \text{ms}^{-1}$). The results from several simulations performed using different timesteps are shown in figure 6.9.

The code accurately produces the correct currents as long as the timestep used is small enough. The code gives a good match to the theory when the collision time is longer than 5 timesteps and gives a good approximation when the collision time is longer than 2.5 timesteps. The current starts to be overestimated by the code when the timestep becomes approximately equal to the collision time. The currents seen in the simulations then continue to increase as the timestep becomes longer than the collision time. It is worth noting that even in cases when the timestep is greater than the collision time and the collision routine is invalid (as it is no longer producing small angle collisions) the currents (although overestimated) do not carry on growing. If this test is carried out with collisions entirely excluded, the drift velocity of the electrons quickly accelerates to the speed of light.

The code has also been tested with electron-electron collisions included to
6.4 Testing the collision model

Figure 6.9: The measured drift velocity due to an applied electric field throughout collisional PIC simulations with various timesteps (electron-ion collisions only)

Figure 6.10: The measured drift velocity due to an applied electric field throughout a collisional PIC simulation that contains the effects of electron-ion and electron-electron collisions

make sure that it still reproduces the expected particle drift. In this test the initial temperature was set to be $10^5 K$, the particle number density was set to $1 \times 10^{26} m^{-3}$, the timestep was set to $1/100$ collision times ($\tau \sim 1 \times 10^{-14}s$) and an electric field of $1 \times 10^7$ was applied. Figure 6.10 shows the outcome of this test and it is again clear that the measured particle drift tends towards the value predicted by Spitzer theory.
Chapter 7

Collisional And Collisionless PIC Simulation Results

7.1 The simulation setup

In this chapter the results of several simulations that have been performed in order to investigate the effects that electron-ion Coulomb collisions have on the propagation of laser generated fast electrons are presented. Four simulations have been performed in total; a collisionless simulation and three collisional simulations with target Z values of 1, 3 and 5. All of the simulations have been performed using the two-dimensional version of the particle in cell code EPOCH, with collisions included as an additional sub-grid parameter as described in the previous chapter. The four simulations are identical apart from the changes to the collisionality of the plasma. The laser used in the simulations has a maximum intensity of $5 \times 10^{19} \text{Wcm}^{-2}$ (which corresponds to $1.94 \times 10^{13} \text{Vm}^{-1}$), a wavelength of $1.05\mu\text{m}$ and it is turned on using a temporal profile given by $\omega^2 t^2 / (10.0 + \omega^2 t^2)$. The laser spot size is given by a Gaussian function centred at zero on the y-axis and has a width of $5\mu\text{m}$ at $1/e^2$. The laser beam is incident normal to the plasma surface and remains on for the entire duration of the simulations.

The plasma itself is assumed to be fully ionised and has a peak density of $1\text{gcm}^{-3}$, which is approximately the density of a CH plastic target. This density
7.1 The simulation setup

Figure 7.1: Initial target electron number density

corresponds to an electron number density of $3.011 \times 10^{29} \text{m}^{-3}$ and is approximately 300 times critical density for the laser intensity used in the simulations. The block of plasma at this peak density is $10 \mu m$ in width and $20 \mu m$ in height (the whole height of the simulation box). The plasma is initially set up to have a two part exponential density ramp at the front of the target in order to allow for a larger interaction region around the critical density. The density ramp has a scale length of $1 \mu m$ in the pre-plasma and of $0.3 \mu m$ nearer to the target surface (it is set using the following equation: $0.1e^{x/1\times10^{-6}} + 0.9e^{x/0.3\times10^{-6}}$). At the rear of the target there is a ramp down in density corresponding to a tanh drop off over $1 \mu m$. The initial target electron number density is shown in figure 7.1 and the ion number density is initially the same. In all of the simulations performed the plasma is set to have an initial temperature of $100 eV$.

The simulation region used is a $20 \mu m \times 20 \mu m$ square which is made up of $3072 \times 3072$ computational cells, which means that the width of each individual cell is $\sim 6 \times 10^{-9} m$. The boundary conditions for the simulations are periodic in the $y$-direction and open in $x$-direction meaning that particles may leave the simu-
7.1 The simulation setup

Simulation via the left and right hand sides of the simulation region. The Debye length for a solid density plasma is measured on the Angstrom scale-length which is significantly smaller than the size of the cells used. This results in the requirement to use the higher order particle weighting employed within EPOCH in order to reduce the numerical heating that occurs in the simulations. The simulations have been performed using a 4th order spline interpolation for particle weightings and $5 \times 10^8$ computational particles in total. Because storing particle data for $5 \times 10^8$ computational particles would be impractical an arbitrary cut-off of 150keV was introduced in order to separate the fast and background electrons within the simulations. In each of the simulations the particle position and velocity data was recorded for all electrons whose energy was greater than 150keV, whilst data was only recorded for 1% of the electrons whose energy was less than 150keV. When examining the fast electrons we are primarily interested in the MeV electrons that are envisioned to be used in fast ignition scenarios, therefore this approximation is very useful because it reduces the amount of data produced from the simulations whilst ensuring that we retain all of the detail on the fast electrons that are of interest. The CFL condition for the set-up used in the simulations actually gives a timestep that is not small enough to accurately measure the collisional effects in the simulations when $Z$ is greater than 1. Therefore the timestep used within the simulations has been set to $7.66 \times 10^{-18}$s (half the CFL condition value) in order to better resolve the collision time. Using this timestep translates to having approximately 22 timesteps per collision time in the $Z=1$ case (where the collision time is defined as used in the collision routine, $\tau_{ei} = \frac{8\pi\epsilon_0^2m_e^2v_t^3}{n_e^4\ln \Lambda}$), approximately 7 timesteps per collision time in the $Z=3$ case and approximately 4.5 timesteps per collision time in the $Z=5$ case. The testing of the collision routine has show that errors appear to be acceptable as long as the collision time is resolved by approximately 5 timesteps, which means that the use of this timestep is definitely pushing the limits of the model in the $Z=5$ case.

It is worth noting that although the collisional simulations that have been performed are generally described as being for the cases of $Z = 1, 3$ and $5$, a fixed Coulomb logarithm of $\ln \Lambda = 10$ is used in all three simulations. Therefore it would be more accurate to describe the three collisional simulations as having fixed $Z \ln \Lambda$
values of 10, 30 and 50. Another important point is that although particle weighting is employed within the simulations in order to handle the vast range of densities present, the collision routine treats every particle as if they had the same weight. As we are only considering the effects of electron-ion Coulomb collisions this is not a problem because the electron-ion collisions of interest are between the background electrons and ions of the main bulk plasma and these particles will all have equivalent weights within the simulations. In order to include the effects of electron-electron Coulomb collisions in future studies the collision routine will need to be altered to account for the various weights of the individual particles. The simulations discussed in this thesis were performed using electron-ion collisions only because these collisions are responsible for adding resistivity into the model and are therefore expected to have the largest effect on the production of fields within the plasma and on the propagation of the fast electrons. Although the original intention was to perform further simulations where electron-electron collisions were also included time constraints have meant that this was not possible. However, by comparing the results of the simulations performed for this thesis with those discussed by Schmitz et al. [1] it is clear that it is indeed the electron-ion Coulomb collisions that have the largest effect on the propagation of the fast electrons.

Each of the simulations was run for a total of 200fs and snapshots of the fields and particle information were taken at 10fs intervals throughout. Time-averaged electric and magnetic field snapshots were also taken at 10fs intervals throughout each of the simulations. The results discussed in this chapter focus on the time averaged field data because the instantaneous fields are too noisy to yield useful information. Much of the discussion of the simulation results that follows is focused on the data for 100fs into the simulations because after this time the results are of less direct relevance to fast ignition scenarios (although the data is obviously still relevant to laser-plasma interaction studies). This is due to the fact that the simulations have been performed using fairly small targets (∼ 10µm across) because larger scale simulations would have required substantially more computational resources. Because of the small target size the fast electrons generated early on in the laser-plasma interaction will have travelled all the way through the
plasma by about 100fs into the simulations. Upon reaching the rear surface the fast electrons will turn around and begin to travel back into the target. When travelling back through the target the fast electrons may themselves supply part of the charge neutralising return current which results in a drop in the strength of the electric field seen within the target. By the end of the simulations there has been enough time for the fast electrons produced at the beginning of the laser-plasma interaction to fully circulate the target. If full scale fast ignition scenarios could be simulated the larger targets would result in this effect occurring at much later times in the simulations.

For fast ignition scenarios the fast electrons generated in the laser-plasma interaction are required to travel deep into the target before depositing their energy into a small ‘hot spot’ region within the compressed fuel [10]. If the fast electrons enter the target with a large divergence angle the amount of energy deposited in the hot spot will be significantly reduced [65]. This means that a well collimated electron beam is crucial in order to ensure that a large amount of energy deposition occurs in the correct region of the target. The divergence of the fast electrons has previously been attributed to the scattering of the electrons by the large fields created by the Weibel-like instability in the region immediately behind the laser interaction region [67]. In this chapter the results of several collisional PIC simulations are presented and compared to collisionless PIC and hybrid simulation results in order to further investigate fast electron generation and transport within solid density plasmas. The benefit of using a collisional PIC code to study fast electron transport is that a PIC code solves the full Maxwell equations (including the displacement current), does not require assumptions of Ohm’s law and of Spitzer resistivity and does not require the background distributions to be Maxwellian. There is also no need for an artificial distinction between beam and background electrons as is made in hybrid simulations. The following sections discuss various aspects of the simulation results, with particular attention given to the aspects affecting fast ignition. Firstly, the the fields seen within the targets and the wavelengths of the filaments arising in the magnetic field plots are examined. Then the energy and momentum distributions of the electrons within the targets are investigated, along with the fast electron divergence near the laser interaction region and
the fast electron currents within the targets. The background electron temperature is then calculated for each of the simulations and these temperatures are compared to temperatures obtained by assuming Spitzer resistivity in order to assess which regions of the simulation area hold true to the Spitzer approximation. Finally, the PIC code results are compared to results from LSP simulations with similar input parameters in order to assess how closely the collisional PIC results agree with the hybrid model results.
7.2 Electric and magnetic fields within the targets

The inclusion of collisions has the effect of introducing resistivity into the PIC model, resulting in the production of stronger electric and magnetic fields within the targets. An electric field grows within the target as the fast electrons travel through it due to the $\partial E/\partial t = -J/\epsilon_0$ term in Ampère’s law. The resistivity introduced by the collisions has the effect of delaying the formation of the return current, which eventually makes $J \sim 0$, resulting in stronger electric field values within the target. The magnetic field growth then follows from $\partial B/\partial t = \nabla \times \eta J_{\text{fast}}$. This equation for magnetic field growth may actually be split into two distinct terms. The first of these terms is $\eta \nabla \times J_{\text{fast}}$, which results in the production of magnetic fields which focus electrons towards regions of higher current density, hence focusing the beam. The second term is $J_{\text{fast}} \times \nabla \eta$, which results in magnetic fields being produced along gradients in the resistivity which act to contain the electrons within regions of higher resistivity.

Plots of the time-averaged z-component magnetic field for each of the four cases are shown for simulation times of 50, 100, 150 and 200 fs in figures 7.2, 7.4, 7.6 and 7.8 respectively. Corresponding lineouts of the time-averaged magnetic field along $x=1 \mu m$ (just behind the peak density surface) are shown in figures 7.3, 7.5, 7.7 and 7.9. The analysis of the magnetic fields seen within the targets has been limited to the z-component of the field because the simulation setup in which the fast electrons and thermal return current flow in the x-y plane primarily results in the generation of magnetic fields in the z-plane. Plots of the x and y components of the magnetic field have been produced but they have not been included in this thesis because they are noisy and no distinctive structure can be seen. The z-component plots, however, clearly show the development of magnetic fields within the target in the collisional cases and the strength of these fields is seen to increase with increasing target density. The plots show clear filaments extending from the laser focal area in the collisional cases which are difficult to make out in the collisionless case. The size of these filaments is seen to vary from around the collisionless skin depth of the beam electrons in the collisionless case, up to around 0.8 $\mu m$ in the Z=5 case. These values for the size of the filaments have been found by perform-
7.2 Electric and magnetic fields within the targets

ing Fourier transforms on a region of the two-dimensional magnetic field plots just inside the main peak density part of the target. The size of the filaments seen in the magnetic field plots for the various cases is discussed in more detail in chapter 7.3.

The time-averaged magnetic field plots for 150fs and 200fs into the simulations show additional filamentation of the magnetic field at the rear surface as the fast electrons begin to reflux back through the target. This effect is also seen in the magnetic field plots from LSP simulations, such as those shown in figure 7.44 and in previous studies such as in [23]. These filaments can be attributed to the fact that once the fast electrons begin to travel back through the target there are two counter-streaming beams of the same density, a regime where the filamentation instability is expected do dominate [75]. This situation is less stable (the growth rate for the filamentation instability is proportional to \((n_1/n_2)^2\) ) hence the field is seen to grow faster here. The filaments near the rear of the target are closer to being purely transverse than those forming as the fast electrons initially enter the target, which is in agreement with work by Bret et al. [76].

We can investigate the effect that the magnetic field filaments will have on the fast electrons that travel across them by considering the Larmor radius of an electron passing through the field. At 100fs into the simulations the strength of the filaments is seen to reach beyond 100T in the collisional cases. For a magnetic field of 100T the Larmor radius of a 1MeV electron is approximately 50µm. This means that the trajectory of the 1MeV electron will not be significantly altered as it passes through the region containing the large magnetic field. An electron would be stopped as it travels through a 100T magnetic field filament with a width of 0.5µm if its velocity was lower than approximately \(1 \times 10^7 ms^{-1}\), which corresponds to electrons with energies less than approximately 300eV. Therefore we would not expect the filaments to significantly alter the flow of the fast electrons within the simulations. Even at later times when the strength of these filaments increases towards 300T, only electrons with energies less than a few keV will be significantly affected.

The collisional simulations also clearly show electric fields within the targets
7.2 Electric and magnetic fields within the targets

that do not appear in the collisionless case. Time-averaged x-component electric field plots for each of the cases are shown in figures 7.10 to 7.13, which again correspond to simulation times of 50, 100, 150 and 200 fs respectively. The increase, and subsequent decrease, in the electric field strength that occurs as the simulations progress can clearly be seen in these plots. The time-averaged electric field that is seen to build up within the targets in the collisional cases reaches values on the order of $10^{10} V m^{-1}$. The peak field values are seen at around 50fs into the simulations (as the fast electrons first enter the bulk plasma) and at this time the electric field in the Z=5 case reaches values in excess of $2.5 \times 10^{10} V m^{-1}$. The increase in electric field strength with target Z can be clearly seen in the plots, whilst the field is seen to remain close to zero in the collisionless case as one would expect. As the fast electrons propagate further into the target the electric field strength immediately begins to decrease due to ohmic heating, leaving weaker electric field values near to the target surface where ohmic heating is greatest.

In addition to looking at the two-dimensional electric field plots it is informative to examine lineouts of the time-averaged x-component electric field along the y=0 axis. Figure 7.14 shows these lineouts at various times throughout each of the simulations. The increase in electric field strength with target Z is more clearly seen in the plots within figure 7.14, as is the subsequent reduction in field strength as the simulations progress. At early times in the simulations the peak value of the electric field can be seen to move deeper into the target with increasing target Z, which is to be expected due to the increased ohmic heating associated with the more collisional cases. The decrease in field strength near the front of the target due to ohmic heating is clear in all three of the collisional cases and a decrease in the field strength near the back of the target is also apparent once the fast electrons begin to recirculate through the target. The decrease in electric field strength due to ohmic heating (i.e. the drop in the electric field strength on the left hand side of the plasma in going from figures 7.14 $(b) \rightarrow (c) \rightarrow (d)$) is an example of how the heating caused by collisions actually acts to reduce the effect of the collisions. The collisions essentially introduce resistivity into the simulations which results in ohmic heating and the growth of the electric fields within the plasma. In turn the ohmic heating acts to lower the resistivity of the plasma, hence reducing the
strength of the electric fields seen. Towards the end of the simulations, when the fast electrons are moving in both the positive and negative x-direction, the strength of the electric field within each of the targets is greatly reduced. As the amplitude of the electric fields continue to decrease the values in all of the simulations are seen to become fairly noisy. This implies that if collisional PIC simulations were to be carried out for longer periods of time than those discussed here, either a better resolution or a larger number of particles per cell would be required in order to more accurately model the electric fields.

We can make a simple estimation of the amount of energy that the fast electrons will lose due to the electric field by integrating the electric field profiles shown in figure 7.14(g), which corresponds to a time of 100fs into the simulations. From this calculation we find that as the fast electrons traverse the 10 µm target they will lose approximately 30keV, 60keV and 80keV for the Z = 1, 3 and 5 collisional cases respectively. Although this calculation is simplistic (as in reality the electric field profile will vary as the electrons move across the target) it gives a rough estimate of how much energy the fast electrons will initially lose as they propagate through the target. The actual time taken for the electrons to traverse the 10 µm target will of course vary with their energy. The fastest electrons (≥500keV) will traverse the target in the region of 30-40fs, whilst slightly less energetic fast electrons will take longer (i.e. ~50fs for 150keV electrons, ~80fs for 50keV electrons and ~120fs for 20keV electrons). Only the lower energy electrons are expected to lose a significant fraction of their energy as they propagate through the target. It is useful to compare the energy the electrons lose due to the electric field with the energy that they will lose directly due to collisions as they traverse the target. For the parameters used in the simulations (and taking Z=1) the MeV electrons which traverse the target in approximately 30fs would lose approximately 1.3keV due to the collisions, whilst the 50keV electrons would lose 3.7keV and the 20keV electrons would lose 5.5keV. These values are notably smaller than the losses due to the electric field which means that in the simulations that have been performed the electric field is the dominant mechanism in reducing the energy of the fast electrons as they traverse the target.
As previously noted, the lineouts in figure 7.14 show that the electric field initially builds up as the fast electrons enter the target, before decreasing in value at the front of the target due to ohmic heating and at the back of the target due to the refluxing electrons. This means that the energy loss due to the electric field is most important when the fast electrons produced early on in the interaction are first travelling into the target. Despite the considerable noise seen in the electric field towards the end of the simulations, integrating the field profiles still shows that the energy losses caused by the field are larger than the losses directly due to collisions. However, by the end of the simulations the losses have reduced to 14keV, 16keV and 23keV for the $Z = 1$, 3 and 5 cases respectively. If the simulations were performed over a longer period of time the field would be expected to decrease further and eventually the losses due to collisions would become the dominant form of energy loss for the fast electron beam. This is in agreement with work by Davies et al [38] who have used a hybrid model to show that the electric field is an important energy loss mechanism, especially early on in the laser-plasma interaction. By using a hybrid model they were able to simulate a much larger target for a longer period of time than is covered by the PIC simulations we have performed. They found that the electric field was indeed important at early times but they were also able to show that the energy losses occurring due to the electric field were reduced to almost nothing by 2ps into the interaction.
7.2 Electric and magnetic fields within the targets

Figure 7.2: Magnetic field (time averaged) z-component at 50 fs

(a) collisionless

(b) collisional Z=1

(c) collisional Z=3

(d) collisional Z=5

Figure 7.3: Lineouts corresponding to figure 7.2 (a-d) along x=1µm for −1µm<y<1µm
7.2 Electric and magnetic fields within the targets

Figure 7.4: Magnetic field (time averaged) z-component at 100 fs

(a) collisionless

(b) collisional Z=1

(c) collisional Z=3

(d) collisional Z=5

Figure 7.5: Lineouts corresponding to figure 7.4 (a-d) along x=1µm for −1µm<y<1µm
7.2 Electric and magnetic fields within the targets

Figure 7.6: Magnetic field (time averaged) z-component at 150 fs

- (a) collisionless
- (b) collisional Z=1
- (c) collisional Z=3
- (d) collisional Z=5

Figure 7.7: Lineouts corresponding to figure 7.6 (a-d) along x=1\(\mu\)m for \(-1\mu\m<y<1\mu\m\)

- (a) collisionless
- (b) collisional Z=1
- (c) collisional Z=3
- (d) collisional Z=5
7.2 Electric and magnetic fields within the targets

Figure 7.8: Magnetic field (time averaged) z-component at 200 fs

(a) collisionless  
(b) collisional Z=1  
(c) collisional Z=3  
(d) collisional Z=5

Figure 7.9: Lineouts corresponding to figure 7.8 (a-d) along x=1\(\mu\)m for \(-1\mu\)m\(\leq y\leq1\mu\)m
7.2 Electric and magnetic fields within the targets

(a) collisionless

(b) collisional Z=1

(c) collisional Z=3

(d) collisional Z=5

Figure 7.10: Electric field (time averaged) x-component at 50 fs
7.2 Electric and magnetic fields within the targets

Figure 7.11: Electric field (time averaged) x-component at 100 fs
7.2 Electric and magnetic fields within the targets

(a) collisionless
(b) collisional Z=1
(c) collisional Z=3
(d) collisional Z=5

Figure 7.12: Electric field (time averaged) x-component at 150 fs
7.2 Electric and magnetic fields within the targets

Figure 7.13: Electric field (time averaged) x-component at 200 fs

(a) collisionless
(b) collisional Z=1
(c) collisional Z=3
(d) collisional Z=5
Figure 7.14: Lineout along y=0 showing the time averaged x-component of the electric field within the target at (a) 40 fs, (b) 50 fs, (c) 60 fs, (d) 70 fs (e) 80 fs (f) 90 fs (g) 100 fs and (h) 150 fs
7.3 Filamentary structures

There are clear differences between the collisionless and collisional time-averaged magnetic field plots shown in chapter 7.2 (figures 7.2 to 7.8). From around 50fs into the simulations the three collisional cases show clear filamentary structures emanating from the laser interaction region, whilst no significant signs of filamentation can be seen in the collisionless plots until much later into the simulation. In order to further examine the filamentary structures Fourier transforms have been performed on a $4 \times 4 \mu m$ region of the magnetic field plots, starting at $x = 1 \mu m$ and centred on the $y = 0$ axis. This has been done for all four cases at times of 100fs and 200fs into the simulations.

Figures 7.15 to 7.18 show the Fourier transforms at 100fs and 200fs into each of the simulations. In the collisionless case the filaments in the magnetic field plots are not as obvious as they are in the collisional cases, however, they are clearly visible in the transforms shown in figure 7.15. At 100fs there are a variety of wavelengths present in the collisionless case, ranging from around $0.1 \mu m$ to $0.25 \mu m$. This is of the order of the collisionless skin depth of the beam electrons, $c/\omega_{pb}$, which is $0.168 \mu m$ for the densities considered here ($\omega_{pb}$ is the plasma frequency of the beam electrons and is calculated here by assuming that the beam density is roughly equal to the critical density of the laser used). This is in good agreement with Silva et al. [42] who have predicted that the collisionless Weibel instability will result in filaments with a size on the order of the collisionless skin depth of the beam electrons. By 200fs into the simulation the peaks have begun to merge, and are now mainly focused in a region corresponding to wavelengths of around $0.2 \mu m$ in the transverse direction. This peak wavelength is seen at a slight angle in the $k_x - k_y$ plane, showing that the modes are not entirely transverse, in agreement with Bret et al. [97] who have predicted that the maximum growth rate is at oblique $k$. A collisionless simulation that was run for a significantly longer period of time is discussed in Schmitz et al. [1] and apart from this longer duration the simulation is identical to the collisionless simulation discussed here. At a time of 400fs into the simulation a dominant wavelength of $0.37 \mu m$ ($2.2 c/\omega_{pb}$) could be seen and by this time there was a clear tilt in the peaks seen in the $k_x - k_y$ plane.
The collisional simulations also contain the physics required to model the resistive (or collisional) Weibel instability and the magnetic field plots for these cases display a stronger generation of filaments within the plasma. The addition of collisions is also seen to cause the largest peaks in k-space to move to smaller values of $k_x$ (i.e. the instability is more nearly transverse), although there are numerous other peaks seen with non zero values of $k_x$. At 100fs the $Z = 1, 3$ and 5 cases all show filamentary structures, with an increase in wavelength (decrease in k) seen as $Z$ is increased. The transform for the $Z = 1$ case at 100fs (figure 7.16) has a maximum corresponding to a transverse wavelength of around $0.32\mu m$, although there are a number of further peaks going out to wavelengths of around $0.13\mu m$. At 200fs the predominant wavelength remains but it has shifted slightly to centre on approximately $0.36\mu m$. The $Z = 3$ case (figure 7.17) contains a main peak corresponding to a transverse wavelength of around $0.43\mu m$ at 100fs, although there are also smaller peaks visible with wavelengths similar to those seen in the $Z=1$ case. At 200fs the peak at $0.43\mu m$ remains, but a peak corresponding to a wavelength of $0.25\mu m$ has now become the strongest. In the $Z = 5$ case at 100fs (figure 7.18) the strongest peak corresponds to a wavelength of $0.8\mu m$, with further peaks seen corresponding to wavelengths going down to around $0.15\mu m$. At 200fs the strong wavelength at $0.8\mu m$ remains but the other (smaller) peaks are now seen to be clustered near to the $k_x = 0$ axis and have wavelengths going down from $0.8\mu m$ to approximately $0.13\mu m$.

Although the Fourier transform plots are quite noisy, it is clear that the introduction of electron-ion Coulomb collisions leads to the formation of larger filaments than are seen in the collisionless case. It is also clear that as the collisionality of the plasma in the simulations is increased the size of the filaments also increases. The wavelength of the strongest filaments at 100fs (before refluxing electrons will have reached the area we are investigating) increases from between $0.1\mu m$ and $0.25\mu m$ in the collisionless case to $0.32\mu m$, $0.43\mu m$ and $0.8\mu m$ in the $Z=1, 3$ and 5 cases respectively. Assuming that the filament wavelength in the collisionless case is $0.175\mu m$ results in the wavelengths increasing by a factor of approximately 1.8, 2.5 and 4.5 in the collisional cases with $Z$ values of 1, 3 and 5 respectively.
7.3 Filamentary structures

It is interesting to note that in a collisional $Z = 5$ simulation that is discussed in Schmitz et al. [1] a peak wavelength of close to $0.8\mu m$ was also seen. This simulation was the same as the one discussed here, apart from the fact that electron-electron collisions were also included. However, in this case the wavelength is seen to persist for the whole simulation time and there is no sign of the shorter wavelength structures that are seen in the results presented here. This implies that although the electron-ion collisions play an important role in driving the instability towards larger wavelengths, the electron-electron collisions play an important role in the removal of the shorter wavelength structures seen in the simulations.

The trend in the size of the filaments seen in the four simulations is in agreement with the results of Fiore et al. [46] who have previously shown that the instability is shifted towards larger wavelengths when the effects of collisions are included. However, the results presented by Fiore et al. indicate that if the filaments seen in the simulations discussed here were purely due to the Weibel instability, in going from our collisionless to $Z=5$ case the wavelength of the instability would increase by a factor of about 2. Despite the fact that there is considerable noise in figures 7.15 to 7.18, and a large range of wavelengths may be seen in each, it is clear that a larger shift in wavelength has occurred in our simulations. Both the simulations discussed here and those discussed in Schmitz et al. [1], where electron-electron collisions were also included, show a shift in wavelength by a factor of closer to 5 when collisions are accounted for. However, there are several differences between the simulations discussed here and the results in [46] that may explain why the shift in wavelength seen is different. The simulations discussed here model the entire laser-plasma interaction in the longitudinal plane (i.e. the plane containing the propagating fast electron beam), with the peak plasma density being 300 times the laser critical density. In contrast, the theory in [46] is based on an electron beam with a waterbag distribution, and the resultant dispersion relation is solved numerically whilst assuming a beam to background density ratio of 0.1. The simulations performed by Fiore et al. were carried out using transverse geometry (i.e. the plane perpendicular to the propagating beam electrons) in order to examine the Weibel instability, whilst the geometry of the simulations we have performed means that they also contain the effects of the two-stream instability,
resulting in the oblique mode being dominant. The large fast electron divergence angles that arise in the simulations discussed here (see chapter 7.4) could be another reason for the difference, as the theory presented by Fiore et al. assumes the non-energy-conserving Krook collision operator which is known to be unreliable for large transverse temperatures [46].
7.3 Filamentary structures

Figure 7.15: FFT of the time-averaged magnetic field in the collisionless simulation. A $4 \times 4 \mu m$ region starting at $x = 1 \mu m$ and centred on the $y = 0$ axis has been transformed.

Figure 7.16: FFT of the time-averaged magnetic field in the collisional $Z=1$ simulation. A $4 \times 4 \mu m$ region starting at $x = 1 \mu m$ and centred on the $y = 0$ axis has been transformed.
Figure 7.17: FFT of the time-averaged magnetic field in the collisional Z=3 simulation. A $4 \times 4 \mu m$ region starting at $x = 1 \mu m$ and centred on the $y = 0$ axis has been transformed.

Figure 7.18: FFT of the time-averaged magnetic field in the collisional Z=5 simulation. A $4 \times 4 \mu m$ region starting at $x = 1 \mu m$ and centred on the $y = 0$ axis has been transformed.
It is also interesting to look at the effect that the transverse beam temperature would have on the collisionless Weibel instability. As mentioned in chapter 4.4, Silva et al. have shown that the filamentation instability may be stabilised by a relatively small transverse temperature in the electron beam. The instability threshold they derived may be written as \( \alpha > \gamma_b \beta^2 \), where \( \alpha \) is the beam to background density ratio, \( \gamma_b \) is the Lorentz factor of the beam electrons, \( c\beta_\parallel \) is the longitudinal beam velocity and \( c\beta_\perp \) is the transverse beam velocity.

We can make an estimation of transverse beam velocity by using the result derived from the relativistic Fokker-Planck equation by Davies et al. [38] when examining high current relativistic electron beams:

\[
\langle \Delta \theta^2 \rangle = \left( \frac{Z^2 n e^4 \gamma m}{2 \pi \epsilon_0^2 \Lambda} \log \Lambda \right) t = \nu t. \tag{7.1}
\]

This equation shows that even a beam that initially has no transverse velocity will acquire a transverse temperature which will increase linearly with time [15]. Following Evans [15] we may then say that if the beam is initially relativistic, the normalised transverse velocity, \( \beta_\perp \), is equal to \( \sqrt{\nu t} \) and therefore \( \beta_\perp = \langle \Delta \theta^2 \rangle \). In figure 7.25 the angular spread (\( \Delta \theta \)) is shown for for two different slices along the y-direction of the various simulations that have been performed. The angular divergence is seen to dip to just below 20° on axis at \( y = 0 \mu m \) and increases to around 70° for electrons travelling out towards the simulation boundaries at \( y = \pm 10 \mu m \).

In contrast to this, putting numbers into the equation for the threshold of the instability shows that angles much smaller than these would result in the instability being suppressed. For example, using \( \alpha = 1/100 \) and \( \gamma_b \sim 5 \) yields that a divergence of 2.5° will be enough to suppress the instability. This divergence is much smaller than is seen in any simulations that have been performed which means that the purely collisionless form of the Weibel instability should be suppressed within the bulk plasma in all four cases.

There are several possible reasons why filaments can be seen in the collisionless simulation despite that fact that we would expect the purely collisionless form of the Weibel instability to be suppressed. Although the above calculation shows that the Weibel instability should be suppressed within the bulk solid density plasma, near the laser-plasma interaction region the electron density will be close to that of
7.3 Filamentary structures

the relativistically corrected critical density, which is significantly lower than solid density. Therefore the instability seen in the collisionless simulation could have been seeded in the lower density plasma and then convected into the higher density plasma by the laser generated fast electrons. Another important point is that there is always a finite collision frequency in standard collisionless PIC simulations due to numerical effects. The use of finite sized particles in PIC simulations (rather than point particles) greatly reduces the effect of the ‘numerical collisions’ but the effect is not entirely suppressed [98][99]. These ‘numerical collisions’ have previously been seen to have a significant effect in simulations that have a small number of particles per cell and could therefore also be helping to seed the instability seen in the collisionless simulation. Finally, the geometry of the simulations that have been performed means that we are actually viewing the interplay between the Weibel and two-stream instabilities as opposed to a purely Weibel instability. This ‘oblique’ mode has previously been seen to be more resistant to high transverse beam temperatures and it has previously been suggested that the two-stream instability can be treated as a source of effective collisionality which acts to drive the filamentation of the fast electron beam [47].

It is also worth noting that the divergence angles calculated from the simulations are appreciably larger than the real divergence at a single point in space. As mentioned in [1], this is due to the strong correlation between the measured angle and the distance from the y=0 axis; the real ‘emittance’ of the fast electron source will be less than the angular divergence measured from the simulation results would suggest.

Figure 7.19 shows the maximum magnetic field recorded within the region of the targets that the Fourier transforms have been performed on throughout each of the simulations. The values shown on the plot are actually rms field values that have been calculated by averaging the data over regions of approximately $c/2\omega_{p,\text{beam}}$ in size. All four cases show an increase in the strength of the magnetic field recorded within the target as the simulations progress, with slightly weaker fields recorded in the collisionless case. The magnetic fields are seen to carry on increasing for the duration of the simulations which suggests that the instabilities are
not suppressed at any point during the simulations. This agrees with the Fourier transform plots in which the peaks are seen to be considerably stronger in the plots for 200fs than they are in the plots for 100fs into the simulations. However, the simulations that have been performed are fairly noisy so the maximum magnetic field within the target is not necessarily a good indication of the behaviour of the instabilities. Looking at other details such as how the rms magnetic field changes with respect to the variance yields noisy results so it is difficult to draw further conclusions with regards to whether the increased heating in the collisional cases acts to suppress the further growth of the Weibel instability. More detailed simulations would be very useful as more accurate field data would allow for a more detailed examination of how the instabilities develop throughout the simulations.

Figure 7.19: Maximum RMS magnetic field strength throughout the simulations. The field values shown are the maximum recorded values in the $4 \times 4 \mu m$ region starting at $x = 1 \mu m$ and centred on the $y = 0$ axis.
7.4 Fast electron angular distributions

7.4.1 Fast electron divergence within the target

We now turn our attention to examining how the collisions affect the distribution of fast electrons within the target. The angular distributions of the fast electrons at various points throughout each of the simulations are shown in figures 7.20 and 7.21. The plots within figure 7.20 are made up of all electrons with a Lorentz factor of 1.3 (150 keV) or higher, while the distributions shown on the plots in figure 7.21 are similar but only contain electrons that have energies between 1 and 2 MeV. In the simulations, 150 keV is used as an arbitrary cut-off in order to differentiate between the fast and background electrons; positional and momentum data was recorded for all electrons with energies greater than this value so that the fast electrons could be examined in as much detail as possible. Both figures contain plots showing the distributions in regular and polar forms and all of the plots have been made using the details of electrons found in the range $1 \mu m \leq x \leq 6 \mu m$. In order to make the plots the weighting for each electron in this region has been added to 1 of 360 bins depending on the components of its velocity.

From the plots in figure 7.20 it is clear that there are fewer fast electrons in the collisional cases. At 50fs into the simulations, when the fast electrons are starting to enter the region we are investigating, the distributions with and without collisions are very similar, with a slight decrease in the number of particles seen with increasing target Z. By 100fs into the simulations the effects of the collisions have become clearer. Plots (c) and (d) of figure 7.20 show the distributions at this time, which corresponds to the when the initial fast electrons are reaching the rear of the target where they are turned around by the strong fields that form. However, at this time no refluxing fast electrons will have entered the region of the target we are examining. The plots show a slight increase in the peak number densities recorded and a narrowing of the overall distributions as the target Z is increased. The ‘wings’ that can be seen in the collisionless fast electron angular distribution at this point in the simulation are also seen to be suppressed by the inclusion of collisions. The changes in the fast electron distributions when collisions are ac-
counted for can be attributed to the global magnetic field within the target upon which the filamentary structures are superimposed (as shown in figure 7.4). The global magnetic field is seen to become stronger as the target Z value is increased and it acts to collimate the fast electrons as they travel through the target, resulting in the narrowing of the angular fast electron distributions in the collisional cases. The loss of the wings seen in the collisionless fast electron distribution can be attributed to the strong magnetic fields that grow in the density ramp at the front of the plasma (which can also be seen in the collisional magnetic field plots within figure 7.4). As discussed in chapter 7.4.2 and in Schmitz et al. [1], the fast electrons initially traveling at wider angles will have their trajectories bent inwards by the large fields that grow in the density ramp at the front of the target in the collisional cases.

The plots showing the distributions later on in the simulations also contain the effects of the recirculating electrons. As the simulations progress the narrowing of the of the distributions in the collisional cases becomes clearer. However, although there is a relative increase in the number of electrons seen traveling at small divergence angles in the collisional cases there is no absolute increase seen. There is also a clear V-shape seen in the electrons traveling in the negative x-direction in the collisional cases. This reduction in the number of electrons travelling back into the target along the y=0 axis can be attributed to the strong magnetic fields forming near this axis at the rear of the target. Davies et al [60] have also reported a magnetic focusing effect that occurs at the rear surface of a target that may contribute to the distributions seen. They noted that as the electrons turn around at the rear surface of a target they will reinforce the magnetic fields already present within the target, focusing the forward going electrons and causing the returning electrons to spread farther from the target normal.

The distributions of the 1-2 MeV electrons shown in figure 7.21 show very similar trends to those seen when examining all of the fast electrons. Although the results are noisier than when looking at all of the fast electrons the distributions are again seen to narrow as expected in the collisional cases. The numbers of fast electrons traveling into the target with small divergence angles are also fairly con-
sistent across each of the four cases.
7.4 Fast electron angular distributions

(a) 

(b) 

(c) 

(d) 

(e) 

(f)
Figure 7.20: Angular distributions of fast electrons with energies greater than 150keV. The plots represent times of (a,b) 50fs, (c,d) 100fs, (e,f) 150fs and (g,h) 200fs. Each pair of plots shows the same distribution in regular and polar forms respectively.
7.4 Fast electron angular distributions

Figure 7.21: Angular distributions of fast electrons with energies between 1 and 2 MeV. The plots represent times of (a,b) 50fs, (c,d) 100fs (e,f) 150fs and (g,h) 200fs. Each pair of plots shows the same distribution in regular and polar forms respectively.

Figure 7.22: The divergence half angle of the fast electrons ($\gamma > 1.3$) throughout each of the four simulations.
The half angles for the various cases (calculated from the dataset that the plots in figure 7.20 are made from and defined as being the angle at which the number of electrons is half of the peak value) are plotted through time in figure 7.22. The half angles in the four cases are strongly correlated as the fast electrons first enter the target, but then quickly start to diverge as magnetic fields grow within the target. After around 60fs into the simulations the half angles for all four cases carry on increasing, but this increase is less in the collisional simulations with a higher target $Z$ leading to lower half angles. The angles in all cases begin to level off after around 100fs and by the end of the simulations the half angles have settled to approximately 85° in the collisionless case and 80°, 70° and 65° in the collisional $Z = 1, 3$ and 5 cases respectively. These angles are larger than the 30-40 degree half angles seen in experiments [66][100], however, the decrease in angle seen in the collisional cases implies that the generally large divergence angles found from collisionless PIC simulations may be unduly negative in regards to the potential focusing of the fast electron beam.

The relative increase in the number of fast electrons traveling at small angles when collisions are accounted for indicates that additional collimation of the fast electron beam is occurring in the collisional simulations. This additional collimation is caused by the global magnetic field upon which the filamentary structures are superimposed, as can be seen in figures 7.2 to 7.8. The strength of the global magnetic fields within the target increases with the target $Z$ which results in the lower divergence angles in the more collisional cases. To examine this further we can look at how much we would expect the fast electrons to be collimated by the magnetic fields seen in the simulations. Following the argument in [22] which states that collimation occurs if $R/r_L > \theta_{1/2}^2$ (where $R$ is the beam radius, $r_L$ is the Larmor radius and $\theta_{1/2}$ is the half angle divergence) we can estimate whether the magnetic field is strong enough to significantly collimate the fast electron beam. This equation simply states that the beam is collimated if the distance in which the magnetic field would bend an electron through $\theta_{1/2}$ is shorter than the distance over which the width of the beam radius would approximately double. To check this we require a value for the global magnetic field. Taking lineouts of the magnetic field along $x=1\mu m$ at 100fs (and using a Bézier curve fit in order to further
smooth the data) yields field values of approximately 40, 70, and 100 T for the Z = 1, 3 and 5 cases respectively. Taking a 1 MeV electron then yields a Larmor radius of roughly 100µm when Z=1, 70µm when Z=3 and 50µm when Z=5. By combining these values with the above equation and the half angle values shown in figure 7.22 (taken as approximately 80°, 70° and 65° for the Z = 1, 3 and 5 cases respectively) we can see that the field is not strong enough to significantly collimate the electrons. We can make an approximation of \( \Gamma \), the collimation factor discussed in chapter 2.4, for the second half of the simulations when the divergence angles are roughly constant. The values obtained for \( \Gamma \) are approximately 0.06, 0.1 and 0.14 for the Z=1, 3 and 5 cases respectively. Clearly we would not expect the cases simulated to exhibit full collimation, as is indeed the case, although some collimation can be made out in the collisional cases. Further simulations performed with a set up leading to a larger value of \( \Gamma \) would be very useful for comparing collisional PIC and hybrid code results. However, such simulations would require substantially more computing resources than have been available for this study.

### 7.4.2 Initial fast electron divergence

The initial divergence of the fast electrons may be investigated by examining the the average and the standard deviation \( \langle (\Delta \theta)^2 \rangle = \langle \theta^2 \rangle - \langle \theta \rangle^2 \) of the fast electron dispersion angles directly behind the laser interaction region, as has been discussed in [1]. The plots in figure 7.23 show the average flow angle \( \langle \theta \rangle \) of the fast electrons as a function of y in two slices near the laser interaction region at 100fs into the simulations. In these plots electrons have again been classified as fast electrons if their energy is greater than the arbitrary value of 150keV. The first of the slices is positioned so that it is towards the back of the large magnetic fields generated by the Weibel-like instability that occurs behind the laser interaction region, whilst the second slice is located behind this region and contains the large magnetic fields that are seen to develop within the steep density ramp of the pre-plasma.

At 100fs into the simulations the slice immediately behind the laser-plasma interaction region (figure 7.23(a)) shows only a small decrease in angle \( \langle \theta \rangle \) in going from the collisionless to collisional cases. However, once we move beyond this re-
7.4 Fast electron angular distributions

region into the second slice (figure 7.23(b)) the effect of the collisions becomes slightly clearer. The collisional cases exhibit a clear reduction in the fast electron flow angles and this reduction becomes more pronounced as the target Z is increased. Reductions of over 10° can be seen between the collisionless and Z=5 cases. The reason for this reduction is that the collisional simulations contain a focusing magnetic field in the density gradient of the pre-plasma (which can be seen in figures 7.2 to 7.8). This field will alter the trajectories of the fast electrons travelling at large angles to the target normal, hence reducing the average flow angles at larger values of y. The small decrease seen in figure 7.23(a) is due to a small amount of the collimating field also being present in this slice. This reduction in flow angle due to collimating fields in the pre-plasma also explains the suppression of the ‘wings’ on the collisionless fast electron angular distributions shown in figure 7.20. This effect is also discussed in a study by Schmitz et al. [1] where the effects of electron-electron collisions were also included. The addition of electron-electron collisions in the simulation model does not alter the results seen.

![Figure 7.23: The fast electron flow angle, θ, as a function of y. The plots are for 100fs and represent the fast electrons (γ>1.3) in the following regions: (a) -1.5µm<x<-0.5µm and (b) -0.5µm<x<0.5µm](image)

The flow angles of the fast electrons within the second slice across the target at 200fs into the simulations can be seen in figure 7.24. By this point the laser has bored through the pre-plasma and the second slice now contains the large magnetic fields caused by the Weibel-like instabilities behind the laser interaction region. As in the plots for 100fs, the flow angle has a strong dependence on y. If
the scattering of the fast electrons by these large magnetic fields was responsible for the dispersion of the fast electron beam (as has been previously suggested by Adam et al. [67]) there would be a flat region on the plots of $\langle \theta \rangle$ around $y=0$ because the scattering would act to reduce the average flow angles. However, this effect is not seen in figures 7.23 or 7.24, further backing up the results discussed by Schmitz et al. [1]. The plots indicate that the scattering of the fast electrons by these fields is not as important as previously suggested. It is interesting to note that the correlation between flow angle and target $Z$ is far less prominent at 200fs into the simulations. This can be attributed to the fact that the critical density surface has moved forwards whilst the collimating fields in the pre-plasma have remained in the same place, as can be seen in the magnetic field plots shown in figures 7.2 to 7.8. This means that only the electrons traveling at large angles to the target normal or those generated far away from the $y=0$ axis will experience the full effect of these fields. Therefore, at the simulations progress less of the fast electrons will be affected by the fields to the extent that they would have been at earlier times.

Figure 7.24: The fast electron flow angle, $\theta$, as a function of $y$ at 200fs. Fast electrons in the region between $-0.5 \mu m < x < 0.5 \mu m$ are included.

In order to further examine the initial scattering of the fast electrons we will now examine the dispersion of the fast electrons ($\Delta \theta$) as a function of $y$ for the two slices previously discussed. The dispersion of the fast electrons within each of the slices at 100fs into the simulations can be seen in figure 7.25. The figure contains four plots; plots (a) and (b) show the dispersion for all of the fast electrons contained within the two slices whilst plots (c) and (d) are similar but only show the dispersion of the forwards going electrons. Figure 7.25(a) shows that
both the collisionless and collisional cases contain a region of large $\Delta \theta$ in the first slice, implying that the Weibel-like instability behind the laser interaction region does indeed cause a large amount of dispersion. The size of the region containing large values of $\Delta \theta$ is comparable to the size of the laser focal spot. However, in the second slice (which is further away from the region containing the strong magnetic fields) the spread of fast electrons is seen to be lowest on the $y=0$ axis and the transverse temperature of the beam is higher away from the axis. This can be seen in figure 7.25(b) which shows the values of $\Delta \theta$ across the target for the second slice at 100fs. Initially the results for the two slices appear contradictory, however, if we only look at the fast electrons moving towards the target (i.e. only those with $p_x > 0$, as shown in figure 7.25(c)) we see that the values of $\Delta \theta$ in the first slice are substantially reduced compared to when all fast electrons are considered. Therefore there is a lower transverse temperature on axis even in the region containing the strong magnetic fields and the majority of the large dispersion seen in figure 7.25(a) is due to the electrons that are traveling away from the target.

There is still a small increase in $\Delta \theta$ seen around the $y=0$ axis in figure 7.25(c) which is not seen when electron-electron collisions are also included [1]. This is possibly due to the slight differences in the positions of the slabs used in the two studies as the results presented here contain slightly more of the strong magnetic fields than the results discussed in [1]. Debayle et al. [101] have previously found that the scattering of the fast electrons by these strong magnetic fields has a large effect on the divergence of the fast electrons and that the curvature of the critical density surface plays a much smaller role. The results of the simulations discussed here (as well as those discussed in [1]) indicate that the magnetic fields do not represent the dominant mechanism affecting the divergence of the fast electrons. Instead, the divergence of the fast electrons is seen to be strongly correlated with the distance from the $y=0$ axis, with fast electrons generated further away from the axis having larger transverse velocities. This implies that the dominant factor affecting the divergence of the fast electrons is actually the curvature of the critical density surface in the laser-plasma interaction region.
Figure 7.25: The angular spread ($\Delta \theta$) of the fast electrons as a function of $y$. The plots are for 100fs and represent the following regions (a) $-1.5 \mu m < x < -0.5 \mu m$ (b) $-0.5 \mu m < x < 0.5 \mu m$, (c) $-1.5 \mu m < x < -0.5 \mu m$ (forwards going electrons only) and (d) $-0.5 \mu m < x < 0.5 \mu m$ (forward going electrons only).
7.4.3 Fast electron currents

It is also possible to examine the currents carried by the forwards going fast electrons throughout each of the simulations. Figure 7.26 shows the x-component fast electron current density at 100fs and 200fs into the simulations where the currents shown are an average of the current densities in the region between $6\mu m$ and $8\mu m$ behind the peak target density at $x=0$. At 100fs into the simulations the collisional cases show a slight decrease in the current densities seen at large values of $y$ and are relatively more collimated, however, these differences are subtle. The slight changes are consistent with the angular divergence data shown in figure 7.20 where a reduction in the number of electrons with large divergence angles was seen when collisions were accounted for. The reduction in the currents flowing at large values of $y$ in the collisional cases can again be attributed to the magnetic fields generated in the steep density ramp behind the laser-plasma interaction region in these cases. By 200fs into the simulations there is a much clearer reduction in the fast electron currents in the collisional cases, as can be seen in figure 7.26(b). Although the currents in the collisional cases do show signs of collimation the collimation effect is not enough to compensate for the overall reduction in current density compared to the collisionless values.

![Figure 7.26: Fast electron current densities within the target (averaged over $6\mu m < x < 8\mu m$) for (a) 100fs and (b) 200fs](image-url)
7.5 Laser energy absorption

In all of the simulations performed approximately 45% of the incident laser energy is transferred to the fast electrons, which is in good agreement with previous work by Ping et al. [36]. Figure 7.27(a) shows the total energy of the fast electrons as a fraction of the energy that has entered the simulation via the laser, whilst figure 7.27(b) shows the total fast electron energy normalised to the amount of energy that has entered the system by the end of the plot. The slow increase in absorption seen in figure 7.27(a) is simply due to the time it takes for the laser to reach the plasma target. In figure 7.27(b) the energy entering the system via the laser (the light blue line) has been shifted slightly so that it coincides with the initial fast electron energy increase. The total energy of the fast electrons shown on the plots corresponds to the energy of the electrons with a Lorentz factor of 1.3 or higher (150keV or more) and a slight drop in energy is seen as the target Z is increased. The slightly lower energies seen in the collisional cases results from the fact that the fast electrons lose energy due to the effects of the collisions. This will result in some of the lower energy fast electrons dropping below the $\gamma = 1.3$ threshold, meaning they will no longer contribute to the total fast electron energies shown in the plots.

Figure 7.27: Time evolution of the total energy of the fast electrons ($\gamma > 1.3$). Plot (a) shows the total fast electron energy as a percentage of the energy that has entered the system and plot (b) shows the total fast electron energies normalised to the energy that has entered the system by the end of the plot. The total energy introduced via the laser is also shown.
7.6 Energy and momentum distributions

The fast electron number densities for the collisionless and three collisional cases at 100fs may be seen in figure 7.28, where electrons are again defined as being fast electrons when their Lorentz factor is greater than 1.3. This time corresponds to when the initial fast electrons have reached the rear of the target, turned around, and are beginning to travel back through. All four plots clearly show the fast electrons traveling into the target in waves generated twice per laser period. However, the plots are not time averaged and not much difference can be seen between the various cases. The corresponding electron $x-p_x$ phase space for the various cases (where $p_x$ has been normalised to $p_x/m_e c$) can be seen in figure 7.29. There is no obvious difference in the number of electrons with large momentums in the various simulations, although there is a visible increase in the number of electrons close to the $p_x = 0$ axis in the more collisional cases.

The differences between the collisionless and collisional cases are highlighted more effectively by looking at the energy and momentum distributions of the electrons within the targets in more detail. The electron energy distributions at times of 100fs and 200fs can be seen in figures 7.30 and 7.31 respectively. These distributions have been made using the details of electrons between $x = 1\mu m$ and $x = 9\mu m$ so that surface effects were not included (i.e. electrons within 1$\mu m$ of either the front or rear surface of the target have been excluded). The three plots in each figure show (a) the energy distribution up to 20MeV, (b) the energy distribution up to 1MeV and (c) the energy distribution up to 20keV (different numbers of energy bins have been used to create each of the plots so scales on the plots are not directly comparable). The increased noise below 150keV (which can be clearly seen in plot (b)) occurs due to the fact that data was only recorded for a fraction of the electrons with energies below this threshold.

The plots showing the distributions up to 20MeV again show that there is very little difference between the collisionless and collisional cases when looking at the high energy electrons. At 100fs into the simulations the whole energy distribution shown in figure 7.30(a) is not strictly thermal. However, there is a region from...
7.6 Energy and momentum distributions

(a) collisionless

(b) collisional Z=1

(c) collisional Z=3

(d) collisional Z=5

Figure 7.28: Fast electron ($\gamma > 1.3$) number density at 100fs
7.6 Energy and momentum distributions

(a) collisionless

(b) collisional Z=1

(c) collisional Z=3

(d) collisional Z=5

Figure 7.29: Electron $x - p_x$ phase space at 100fs
approximately 2MeV up until the distribution begins to tail off just after 10MeV where the distribution appears closer to what would be expected from a thermal distribution. This more thermal region has a temperature of approximately 2.4MeV, which is in good agreement with the temperature of 2.6MeV predicted by the standard ponderomotive scaling law (equation 2.29).

![Figure 7.30: Electron energy distributions at 100fs](image)

![Figure 7.31: Electron energy distributions at 200fs](image)

It is at the lower end of the energy spectrum (shown in figures 7.30(c) and 7.31(c)) where the differences between the various cases become apparent and to give perspective the energy distribution at the start of the simulations is also shown on these plots. At 100fs the lower energy electrons are seen to have gained energy in all cases, but the increase in energy is much larger in the collisional cases and can be seen to increase further as the target Z is increased. This increase in the energy of the collisional background electrons is mainly due to the ohmic heating.
7.6 Energy and momentum distributions

The plot for 200fs will have been affected by the refluxing electrons so is not a direct measure of the distribution of the laser generated fast electrons. By this time there is a smaller difference between the three collisional cases but all three show a large deviation from the collisionless distribution.

The effects of collisions on the cold electrons can be further examined by looking at the momentum distributions of the electrons within the targets. To this end the momentum distributions have been calculated for electrons within two 2µm × 2µm regions of the target. The regions investigated are centred on the y=0 axis and at x values of 2µm and 8µm respectively. The electron momentum distributions within each of these two regions are shown in figures 7.33 and 7.34. Figure 7.33 shows the distributions at 100fs into each of the simulations, whilst figure 7.34 shows the distributions at 200fs into the simulations. Each of the figures contains three plots for each of the four simulations that have been performed and these plots each show the same momentum distribution but have different scales in order to better highlight the various parts of the distributions.

In the plots for 100fs all four cases show that the background distributions remain close to being locally Maxwellian. This is best illustrated in plots (a), (d), (g) and (j) and a clear increase in thermal temperature with target Z may also be seen in these plots. The momentum distributions for the region near the front of the target are seen to be shifted slightly due to the return current flow. Figure 7.32 shows velocity distributions for the electrons within the region near the front of the targets and from the data shown in this figure the velocity shift due to the thermal return current is found to be between -0.006c and -0.008c for all four simulations. The increased thermal temperature in the collisional cases may also be more clearly seen in figure 7.32. Sherlock et al. [64] have previously reported highly non-thermal return currents in simulations looking at targets with densities of around 20n_c. The simulations discussed here, which have a peak plasma density of 300n_c, yielding a much smaller beam to background ratio, show that the background distributions remain close to being locally Maxwellian but the centre of the distribution is shifted slightly due to the return current.

Although the central region of the plots in figure 7.33 are seen to be largely
Maxwellian the distinction between beam and background particles is not absolute. The electron momentum distributions within the region near the front of the target have a clear tail from where the background distribution is mixing with higher energy electrons that are slowing down. However, the momentum distributions for the region near the rear of the target are perhaps more informative in this matter. In this region the background electron momentum distribution is clearly Maxwellian and a separate ‘bump’ can be seen at higher energies corresponding to the fast electrons. It is worth noting that without detailed particle tracking in the simulations we can not definitively say whether these particles are ‘runaways’ from the bulk distribution or are relaxing back into the distribution from the beam electrons. However, the fact that there is a distinct gap between the two distributions implies that they are indeed fast electrons that are losing energy. This is backed up by the fact that the Dreicer field for the parameters used in the simulations is approximately $10^{12} V m^{-1}$. This is considerably higher than the electric field strength seen in any of the simulations so we would not expect the distributions to deviate from a Maxwellian due to runaway electrons. As the collisionality is increased (i.e. in moving from plots (b)→(e)→(h)→(k)) we can clearly see that the electrons in the higher energy part of the distribution are losing more energy. This results in the forwards going ($p_x > 0$) part of the high energy electron momentum distribution moving closer to the Maxwellian background electron distribution as
Z is increased. In the region near the front of the target this blurring of the distributions is much more advanced, however, at this point in the simulations the relaxing higher energy electrons do not have a large effect on the overall Maxwellian nature of the background electrons.

By 200fs into the simulations the mixing of the background and fast electron distributions has become more advanced. The distributions can be seen in figure 7.34 and it is again clear that the background is hotter in the higher Z cases. The distributions are no longer typical Maxwellians and the distributions near the front of the target in particular are clearly skewed due to the higher energy electrons relaxing back into the main distribution. The distributions near the rear of the target are closer to being Maxwellian, but they now look similar to how the distributions near the front of the target looked earlier on in the simulations. The momentum distributions shown in figures 7.33 and 7.34, along with the energy distributions shown in figures 7.30 and 7.31, indicate that there is no simple distinction between the beam and background electrons within the simulations. Although the momentum distributions show that early on in the simulations such a distinction may be made, as the simulations progress the electron distributions become more and more non-Maxwellian.
Figure 7.33: Momentum distributions of the electrons within two $2\mu m \times 2\mu m$ regions at 100fs. The regions are centred at $x = 2\mu m$, $y = 0\mu m$ (front) and at $x = 8\mu m$, $y = 0\mu m$ (back). Each row of three plots shows the same momentum distribution with different scales in order to clearly show the various parts of the distribution.
Figure 7.34: Momentum distributions of the electrons within two 2µm x 2µm regions at 200fs. The regions are centred at $x = 2\mu m$, $y = 0\mu m$ (front) and at $x = 8\mu m$, $y = 0\mu m$ (back). Each row of three plots shows the same momentum distribution with different scales in order to clearly show the various parts of the distribution.
7.7 Background temperatures

In the previous section it was shown that the background electron distributions remained close to being locally Maxwellian for at least the first 100fs or so of the simulations that have been performed. Therefore we will now turn our attention to further examining the background electron temperatures throughout the various simulations.

7.7.1 Lineouts of the background temperature

Acquiring an accurate value for the background temperature in a PIC simulation is difficult due to the fact that all of the simulation particles are treated in the same way. This means that there is no simple distinction that can be made between background and beam electrons as would be made when looking at the data from hybrid simulations. Figures 7.35 through 7.38 show lineouts of the background electron temperatures along the y=0 axis at simulation times of 50fs, 100fs, 150fs and 200fs. The plots show the background temperatures calculated by assuming that $T \sim \frac{2}{3} \langle E_{\text{kinetic}} \rangle$, where $\langle E_{\text{kinetic}} \rangle$ has been calculated from all electrons whose energies are below various artificial cut-offs between the beam and background. Although in the previous section we have discussed how there is no clear distinction between the beam and background electrons later on in the simulations, examining these artificial cut-offs is still informative as it will give a clearer picture of when and where this distinction may no longer be made. The plots are restricted to only showing the temperature within the targets because the distributions will be strongly non-Maxwellian outside of the bulk plasma. The cut-offs in energy used are 5, 10, 20, 50 and 150 keV and we can examine the differences between the temperatures calculated using the various cut-offs in order to see if the temperatures calculated can be considered as truly thermal.

At 50fs and 100fs into the simulations the temperatures calculated using the various energy cut-offs give are very similar. Only the lineouts made using a 150keV energy cut-off show any major deviation from the rest, which is not unexpected given such a high energy cut-off. All of the plots show a degree of noise in
the temperature lineout made using 150keV as an energy cut-off because a small number of high energy electrons can have a large influence on the temperatures calculated. The agreement between the temperatures calculated using different cut-offs implies that for the first half of the simulation the background temperatures are indeed thermal. A clear jump in temperature can be seen between the collisionless and collisional cases (due to the introduction of ohmic heating) and there is then a further increase visible between the collisional cases as the target Z is increased.

As discussed in chapter 7.6, later on in the simulations the background electron distributions become more non-Maxwellian as the distinction between between the beam and background electrons becomes less clear. By 150fs the temperatures near the front of the target have risen substantially and the temperatures calculated using the different energy cut-offs have begun to diverge. Although the electron distributions are starting to go slightly non-Maxwellian by this point in the simulations the discrepancies between the various temperature lineouts shown on the plots will also be affected by the fact that the average electron energies near the front of the target are becoming a sizable amount of the lower energy cut-offs used. The plots for 150fs into the simulations show that a higher energy cut-off results in a higher temperature being recorded near the front of the target. However, beyond around $3\mu m$ into the target the temperatures calculated using the various energy cut-offs are generally still in agreement. By 200fs the variation in background temperature with the chosen energy cut-off is even more apparent, although all of the lineouts apart from the one made using the lowest cut-off tested are still in agreement from around $3 - 4\mu m$ into the target (which is not unexpected as the lowest cut-off tested is 5keV and the temperatures recorded near the front of the target in the other cases actually become larger than this towards the end of the simulations).
Figure 7.35: Background electron temperature lineouts along y=0 at 50 fs.

Figure 7.36: Background electron temperature lineouts along y=0 at 100 fs.
Figure 7.37: Background electron temperature lineouts along y=0 at 150 fs.

Figure 7.38: Background electron temperature lineouts along y=0 at 200 fs.
7.7 Background temperatures

7.7.2 Spitzer resistivity comparisons

PIC simulations treat the energetic beam particles and the thermal background particles in the same way. Although this makes it difficult to diagnose the background temperature of the plasma, this lack of distinction is also one of the major benefits of the PIC method. In hybrid codes the background particles are treated separately as a fluid with a simple MHD description. This fluid background then responds to the fast electrons by assuming that \( E = \eta J_{\text{return}} = -\eta J_{\text{fast}} \), where the resistivity is assumed to follow the Spitzer result discussed in chapter 3.6. The background plasma electrons have been seen to remain close to being locally Maxwellian for the first 100fs of the simulations that have been performed and the background plasma temperature has also been shown to be well defined up until to this point. After 100fs the background plasma distributions become more non-Maxwellian as the fast electron beam and background electron populations become less distinct. We will now discuss how background temperature and return current values calculated from the particle data can be used along with the averaged electric field data in order to examine how well the Spitzer resistivity approximation used in hybrid codes fits the PIC results. This study is obviously limited to the three collisional simulations because the collisionless PIC simulation lacks the required resistive effects.

To assess the accuracy of the Spitzer approximation the temperatures found from the particle energies may be compared to temperatures found by looking at the currents and fields within the plasma. We can obtain a temperature from \( E \) and \( J \) by assuming that \( E = \eta J_{\text{return}} \) and then combining the calculated value of \( \eta \) with the Spitzer equation which relates resistivity and temperature (equation 3.37). This has been done by breaking the entire simulation domain into \( 1\mu m \times 1\mu m \) blocks and calculating the average electric field, return current and temperature (found from the standard method of assuming that \( T \sim 2/3 \langle E_{\text{kinetic}} \rangle \)) within each of these blocks. Electrons have been considered to have been part of the background plasma if their energy was less than 20keV, a cut-off that has been chosen based on the results discussed in the previous section. This energy cut-off gives a good compromise between the noise introduced by using higher energy cut-offs and
the fact that the lower energy cut-offs were actually too low to fully encapsulate the background electron distribution as it significantly heats up towards the end of the simulations. Although the plots that follow show the whole $20\,\mu m \times 20\,\mu m$ simulation area, the areas from $x=-7.5\,\mu m$ to $x=0\,\mu m$ and from $x=10\,\mu m$ to $x=12.5\,\mu m$ are outside of the main target and we would not expect the distributions in these regions to be Maxwellian. Therefore the temperatures recorded for these regions, both from the background particle data and from the Spitzer approximation, are not expected to be accurate and it is no surprise that the the temperatures found from the two methods disagree in these regions.

The temperatures found from the two methods at 100fs into the simulations can be seen in figure 7.39. The left hand plots show the temperatures calculated using the particle data and the right hand plots show the temperatures calculated using the electric field and return current values, along with the Spitzer resistivity equation. The plots show that within the target there is a good agreement between the temperatures found from the particle data and from the Spitzer equation. However, the plots showing the temperatures calculated using the Spitzer resistivity equation are noisier and show slightly lower temperatures near the laser interaction region.

A more detailed comparison of the temperatures may be seen in figures 7.40, 7.41 and 7.42, which correspond to the Z=1, Z=3 and Z=5 cases respectively. Each figure contains plots showing shows the factor difference (left hand plots) and absolute difference (right hand plots) between the temperatures found from the particle data and from the Spitzer equation at times of 50fs, 100fs and 150fs into the simulations. The factor difference has been calculated as $\frac{\max(T_{\text{particles}}, T_{\text{Spitzer}})}{\min(T_{\text{particles}}, T_{\text{Spitzer}})}$ and has been multiplied by $-1$ wherever the Spitzer temperature is the hotter of the two so that the plots clearly show which of the temperatures is higher. Within the target ($0\,\mu m < x < 10\,\mu m$) there is generally a good agreement between the two temperatures in the regions where the fast electrons have propagated and there is little difference between the results for the various Z cases, with only the plots for 150fs showing any notable differences.

The plots for 50fs (plots (a) and (b) of figures 7.40, 7.41 and 7.42) show that
the temperatures are in good agreement where expected; within a hemispherical region indicating how far the initial fast electrons have propagated into the target. There are parts of this hemispherical shape where the agreement is not as good (particularly near the boundary to the region where the fast electrons have not yet propagated) but the agreement is generally within a factor of 2. Some of the slight differences between the temperatures can be explained by noting that the temperatures calculated using the particle energies have used instantaneous data, whilst the temperatures inferred from $E/J$ have been calculated using time-averaged field data (centred on the time that the particle data corresponds to). Because of this we would expect there to be some discrepancy between the two temperatures, particularly at the fast electron propagation front.

At 100fs, when the fast electrons will have had time to flood the whole target, we see a much better correlation between the two temperatures. The relation is much closer to being 1:1 throughout the whole target, with any deviations seen still being within a factor of approximately 2. Despite the good agreement the $Z=1, 3$ and 5 plots for 100fs all show that the temperatures calculated from the particle data are slightly larger than those calculated using the Spitzer equation near the laser interaction region (this is clearest in plot (d) of figures 7.40, 7.41 and 7.42). This is consistent with the momentum distributions shown in figure 7.33 which show that the background distribution is slightly more non-Maxwellian near the front of the target. By 150fs into the simulations it is much clearer that the correspondence between the two temperatures is breaking down. At this point the temperatures calculated using the electron energies are several times larger near the laser interaction region, whilst the temperatures calculated from the Spitzer equation are generally larger near the edges of the target. The increased level of noise seen in going from the plots for 100fs to the plots for 150fs corresponds to the initial fast electrons travelling back into the target. As is shown in figure 7.14, the electric field within the target starts to get noisy as the fast electrons travel back through, meaning that the temperature calculated from the Spitzer resistivity equation will also become less accurate. As well as this the background distributions are seen to become more non-Maxwellian later on in the simulations so it is not surprising that the temperatures calculated using the two methods diverge as
the simulations progress. The large difference in temperatures at the front of the target corresponds to the fact that the background electron distributions are seen to become more non-Maxwellian at the front of the target where the background heating is greatest, as shown in plots 7.33 and 7.33.

As well as looking at the temperatures found from the two methods we can perform a similar comparison by comparing values for the actual resistivity within the targets. These values are again found by assuming that $\eta = E/J$ and by using the Spitzer resistivity equation, although in this case the Spitzer resistivity equation is used to calculate resistivity values from the temperatures found from the particle data. Figure 7.43 contains plots showing the factor difference between the two resistivity values for each of the simulations at 50fs, 100fs and 150fs. The plots are obviously very similar to the temperature comparison plots shown in figures 7.40-7.42 and despite the obvious noise the plots again show that there is a good agreement where the background remains Maxwellian and the electric field and return current values remain well defined. The plots for 50fs show that the two resistivities generally agree to within approximately 50% in the region where the fast electrons have propagated and an electric field has been set up. At 100fs there is a clearer agreement between the two resistivity values with the relation being close to 1:1 throughout much of the target. However, there is a slightly weaker agreement between the two values at the front of the target near the laser focal region where the temperature is highest and at the edges of the target where there will be reflected electrons travelling back into the target. As with the temperature data, at 150fs into the simulations the relation between the two resistivities appears to be breaking down. This again corresponds to the background electron distributions becoming more non-Maxwellian as the simulations progress, meaning that the Spitzer resistivity equation becomes less valid.

Before the background electron distributions become non-Maxwellian (and the electric field values begin to weaken and become noisy) there is a good agreement between the temperatures and resistivities calculated from the two methods. Naturally, this agreement only holds within the targets ($0\mu m \leq x \leq 10\mu m$) because the distributions are strongly non-Maxwellian outside of this range. The fact that
there is a good agreement implies that the Spitzer approximation is indeed valid within the target, as long as the background plasma remains Maxwellian and the electric field and return current remain well defined. As the simulations progress these rules are broken, resulting in the divergence of the temperatures and resistivities calculated using the two methods. Due to the noise in the simulations we have had to average the data into fairly large regions (1\( \mu \text{m} \times 1\mu \text{m} \) squares) in order to perform these comparisons. Although the distributions become more non-Maxwellian as the simulations progress the recirculating electrons will also have effected these results. The electrons travelling back into the target reduce the need for the thermal background to supply the return current and greatly reduce the electric field strength within the target, leaving a noisy signal. Performing further simulations using more particles and a larger simulation area would reduce both of these effects and should therefore provide a more accurate description of where and why the Spitzer resistivity approximation breaks down.
Figure 7.39: Comparison of the background electron temperatures found from electron energies (left) and from $E/J$ and Spitzer resistivity assumptions (right) at 100fs for (a,b) $Z=1$, (c,d) $Z=3$ and (e,f) $Z=5$. Electrons with energies less than 20keV have been considered as background electrons.
7.7 Background temperatures

Figure 7.40: Comparisons of the background electron temperatures found from electron energies and from $E/J$ and Spitzer resistivity assumptions for $Z=1$ at (a,b) 50fs, (c,d) 100fs and (e,f) 150fs. Electrons with energies less than 20keV have been considered as background electrons.
7.7 Background temperatures

Figure 7.41: Comparisons of the background electron temperatures found from electron energies and from $E/J$ and Spitzer resistivity assumptions for $Z=3$ at (a,b) 50fs, (c,d) 100fs and (e,f) 150fs. Electrons with energies less than 20keV have been considered as background electrons.
7.7 Background temperatures

(a) Factor difference in temperatures (negative numbers simply imply that it is the Spitzer temperature that is the hotter of the two)

(b) Temperature difference (eV)

(c) Factor difference in temperatures (negative numbers simply imply that it is the Spitzer temperature that is the hotter of the two)

(d) Temperature difference (eV)

(e) Factor difference in temperatures (negative numbers simply imply that it is the Spitzer temperature that is the hotter of the two)

(f) Temperature difference (eV)

Figure 7.42: Comparisons of the background electron temperatures found from electron energies and from E/J and Spitzer resistivity assumptions for Z=5 at (a,b) 50fs, (c,d) 100fs and (e,f) 150fs. Electrons with energies less than 20keV have been considered as background electrons.
Figure 7.43: The factor difference between the resistivity calculated from assuming that $\eta = E/J$ and the resistivity calculated using Spitzer theory (electrons with energies less than 20keV have been considered as background electrons and negative values indicate that the resistivity calculated using the particle data is the larger of the two)
Several simulations have also been carried out using the hybrid code LSP in order to assess how similar the hybrid results are to those from the collisional PIC simulations (LSP is a commercial code marketed by MRC (Albuquerque), New Mexico, USA). Exact comparisons between the two simulation methods are difficult due to the nature of the hybrid method in which a beam of energetic electrons with specified parameters is injected directly into the plasma target. Because it is difficult to exactly convert the setup and results of the PIC simulations into parameters that can be used to set up LSP simulations we have performed several LSP runs for comparison. The LSP simulations that have been performed have a $20\mu m \times 20\mu m$ simulation region containing a plasma of the same density as used in the PIC simulations. LSP simulations were then performed for target Z values of 3, 5 and 10 and for each of these cases a set of simulations have been performed where the transverse fast electron temperature has been set to 100keV, 200keV and 500keV. The PIC simulations we have performed appear to correspond to the transverse fast electron temperature being between 200keV and 500keV. However, there is not a simple correspondence between angular distributions seen in the PIC results and the distributions injected into the targets in the LSP simulations.

The results of the LSP simulations are summarised in figures 7.44 and 7.45. Figure 7.44 contains plots of the z-component magnetic field for the various runs and figure 7.45 shows the background electron temperature for the various runs.

### 7.8.1 Magnetic field comparison

The plots from LSP show several differences to those obtained from the collisional PIC simulations. Figure 7.44 contains plots showing the z-component magnetic field in the various LSP simulations and it is clear that the filaments seen in the plots are stronger and more pronounced in the higher density / lower transverse beam temperature cases. The LSP results show markedly clearer filamentation than the PIC results presented in section 7.2 but the global magnetic fields seen in the LSP and collisional PIC results are broadly comparable. This means that the
extra physics included in the collisional PIC simulations is unlikely to adversely affect magnetic collimation techniques.

It is clear from the LSP results that the initial choice of transverse beam temperature has a very large effect on the results of the simulations. The filamentation occurring in the LSP simulations is clearly suppressed as the transverse beam temperature is increased, whilst none of the cases show clear signs of filamentation near the front of the target. The lack of filaments at the front of the target can be attributed to the nature of the electron source in the hybrid simulations. The PIC results do show signs of filamentation at the front of the target, but these filaments clearly originate near the laser interaction region prior to the fast electrons reaching the main part of the target. Results from fully integrated LSP simulations discussed in [23] do contain the features seen in the collisional PIC simulations, but further work is required in order to fully examine the differences between the collisional PIC results and the results from the various types of hybrid simulation.

### 7.8.2 Temperature comparison

Two dimensional temperature maps for the various LSP simulations are shown in figure 7.45. The temperatures are clearly much lower than those seen in the PIC results at the end of the simulations - even the collisionless PIC simulation records a higher background temperature than the highest density LSP simulation performed. However, we must be careful how we compare the PIC results to the LSP data.

The temperature lineouts in chapter 7.7.1 show that as the electrons reach the rear of the target the background temperatures recorded in the collisional PIC simulations are around 500eV in the centre of the target and are in excess of 1keV at the front of the target. This is in agreement with the collisional PIC results discussed in Schmitz et al. [1] where electron-electron collisions were also included. These temperatures are also similar to the temperatures seen in the LSP results shown in figure 7.45, although the PIC simulations all contain a slightly hotter region at the front of the plasma near to the laser focal spot. By the end of the PIC simulations
the calculated background temperatures have risen considerably higher than those seen in the hybrid results (see figure 7.38) which is not surprising because the distinction between the beam and background particles in the PIC simulations breaks down as the simulations progress. However, the LSP results shown are not really comparable to the PIC results because although the data shown in figure 7.45 is for close to 200fs into the LSP simulations, the targets used in the LSP simulations are actually twice the size of the targets used in the PIC simulations.

Further PIC and hybrid simulation results showing a more exact comparison would obviously be beneficial for examining the differences between the two types of model. However, the results we have obtained do highlight some interesting features. The hybrid results are seen to have lower background temperatures than the collisional PIC results, particularly at the front of the target near the laser interaction region where we have seen that the background plasma becomes non-Maxwellian in the PIC simulations. The LSP results also show clearer filamentation, although they lack signs of the filamentation that is seen at the front of the target in the PIC simulations. This filamentation in the PIC results occurs due to the fact that the PIC simulations model the whole laser-plasma interaction and do not rely on using a prescribed fast electron source term. Fully integrated LSP simulations discussed in [23] contain fields much more akin to what we have seen in the collisional PIC results but further simulations will be required in order to provide a more meaningful comparison.
Figure 7.44: LSP simulation results showing the z-component magnetic Field. From top to bottom the plots show the cases of Z=3, Z=5 and Z=10 respectively and from left to right the plots are for transverse beam temperatures of 100keV, 200keV and 500keV.
Figure 7.45: LSP simulation results showing the background electron temperatures. From top to bottom the plots show the cases of $Z=3$, $Z=5$ and $Z=10$ respectively and from left to right the plots are for transverse beam temperatures of 100keV, 200keV and 500keV.
Chapter 8

Summary Of Results And Future Work Proposals

8.1 Summary of results

The work discussed in this thesis has been carried out in order to further our knowledge of fast electron transport and to improve and validate the codes that are used in order to study plasma physics. A collision routine has been written for the particle in cell code EPOCH and this collisional version of EPOCH has been used in order to examine the effects that electron-ion Coulomb collisions have on the transport of the fast electrons generated when solid density plasma targets are illuminated by ultra-intense lasers. The electron density in the simulations was equivalent to $1\, g\, cm^{-3}$, which correspond to $300\, n_c$ for the laser intensity used. In studying the effects of the collisions we have examined the generation of fields within the plasma and the subsequent filamentation of the fast electron beam. We have also examined the effects that the collisions have on the electron distributions within the plasma, in particular looking at the divergence of the fast electrons, the energy and momentum distributions of the electrons and the background temperatures within the plasma.

The collisional PIC simulations that have been performed show that as the target $Z$ is increased the fields within the plasma become stronger, the filamentation
within the target is pushed to larger wavelengths, the divergence angle of the fast electrons decreases and the temperature within the plasma is increased. The electric field produced within the plasma when collisions are included is seen to build as the fast electrons enter the target, before decreasing due to ohmic heating and the refluxing fast electrons. Despite the large drop in the electric field strength by the end of the simulations the field is still seen to play a larger role in the slowing of the fast electrons than the scattering caused directly by collisions. The size of the filaments within the targets is seen to increase from around $c/\omega_p$ (of the beam electrons) in the collisionless case to approximately $0.8\mu m$ in our Z=5 case, which is a larger increase than is expected based on previous studies. There are also strong signs of filamentation in the collisionless simulation, although the large fast electron divergence implies that the purely collisionless Weibel instability should be suppressed. The various simulations showed no signs of the instability being suppressed as the maximum magnetic field within the targets was seen to carry on increasing for the entire duration of all of the simulations performed. We have also found no signs that the dispersion of the fast electrons is due to the large fields generated by the Weibel-like instability occurring directly behind the laser interaction region as has previously been suggested. Instead we find that the angular spread of the fast electrons is due to the curvature of the critical density surface in the laser-plasma interaction region. The background electron distributions are seen to initially be locally Maxwellian, but as the simulations progress this distinction between beam and background electrons begins to break down. By comparing the resistivities (and background temperatures) found from the particle energies with the corresponding values found from the assumption of $E = \eta J_{\text{return}}$ and of Spitzer resistivity we have shown that these assumptions are indeed valid as long as the electric field and return current within the target remain well defined and the background electron distributions remain locally Maxwellian. As the simulations progress and the background electron distributions become more non-Maxwellian the temperatures and resistivities found using the two different methods are seen to become less correlated. Finally, we have compared the collisional PIC results with several LSP simulations and despite the difficulty in comparing the data from the two types of model it is clear that the collisional PIC results contain signifi-
cantly larger temperatures and more pronounced filamentation at the front of the
targets. However, this is one area where further comparisons would be highly
beneficial.

8.2 Discussion of further work and computational resources

This study has been motivated by the need to move towards simulating the whole
laser-plasma-transport process in a consistent and assumption free way. This type
of work is important in order to further understand the physics occurring and
to validate the assumptions used in other models. Although the simulations we
have performed are largely free of assumptions there are a few places in which
our collisional PIC model could be improved further. The simulations discussed
in this thesis only contain the effects of electron-ion Coulomb collisions and future
work should also contain the effects of intra-species collisions (similar simulations
including the effects of electron-electron collisions have now been performed and
are discussed in [1]). The Coulomb logarithm is also fixed in our simulations and
calculating this on a cell by cell basis would be another step towards performing
fully self consistent simulations.

The higher order particle weightings that may be used within EPOCH have
shown great promise in reducing the number of computational particles required
in order to suppress numerical heating in simulations with large target densities.
However, a large number of particles are still required in order to accurately model
the collisions and to stop the fields within the plasma from becoming too noisy.
Even the \(5 \times 10^8\) computational particles used in the simulations we have per-
formed results in the field data becoming very noisy towards the end of the sim-
ulations. Further simulations performed using more particles would therefore be
beneficial and would allow for a more detailed examination of the effects of the
fields at later times. Simulations of a larger target would also be highly beneficial.
The small (20\(\mu m\) high 10\(\mu m\) wide) target used in our simulations means that the
results are very quickly contaminated by electrons travelling over the periodic sim-
8.2 Discussion of further work and computational resources

ulation boundaries in the y-plane and by the electrons that are reflected back into
the target upon reaching the rear surface. A larger simulation region would mean
that these effects would not occur until later on in the simulations. In particular,
increasing the simulation size to $20\mu m \times 20\mu m$ would allow for better comparisons
with the results of hybrid simulations performed by Evans [23]. Extra code diag-
nostics would also be useful for any further simulations that may be performed.
For example, particle tracking would be useful for investigating how the particles
are affected by the various fields within the plasma and for separating out the fast
and background electron distributions. Time averaged particle information could
also yield interesting data for looking at the fast electron currents and densities
within the targets.

The simulations we have performed took several days to run using 256 cores
on the CX1 cluster at Imperial College. The length of the simulations scales roughly
linearly with the number of computational particles used and it is possible to run
simulations using more than 256 cores on CX1, meaning that performing more
detailed simulations at Imperial College is far from an impossibility. The major
limitation of the simulations we have performed is the 100eV starting tempera-
ture, which is significantly higher than we would ideally like. Due to the fact that
the collision time must be resolved it will be very computationally expensive to
lower the starting temperature of the simulations ($\tau \propto T^{3/2}$). Simulations using a
higher target density and a larger simulation region would also be highly desir-
able. Increasing the density of the plasma and increasing the size of the simulation
region are not as computationally expensive as lowering the initial temperature
but they will of course both lead to longer simulation times. The number of com-
putational particles required in the simulations will increase in proportion to the
number of cells used but perhaps more importantly the collision time is inversely
proportional to the density of the target. The simulations we have performed are
right on the boundary of where the collision time trumps the CFL condition in
being the limiting factor on the size of the timestep used in the simulations. This
means that in moving to higher densities or lower temperatures the timestep used
in the simulations will have to be reduced, significantly increasing the amount of
time that the simulations will take to run. It is worth noting that the Coulomb
logarithm was fixed at 10 for the simulations that have been performed, although a more realistic figure would be in the range of 2 to 3. Therefore adding the code required to dynamically calculate this value would relax the timestep constraint for a plasma with a given target Z and number density (although this reduction to \( \ln \Lambda \) is equivalent to simulating a lower density plasma).

Obviously, for looking at the whole picture detailed three-dimensional simulations would be highly desirable in order to ensure that the various instabilities and field effects are correctly modelled. However, the computational resources required for these kind of studies mean that there are very few places in the world where they could currently be performed.
Bibliography


