Data Analysis And Modelling For Observations Of Polarisation Of The Microwave Sky

Caroline Natasha Clark

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Supervised by Dr Carlo Contaldi

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Declaration

The work in this thesis is my own and parts that relate to collaborative work are acknowledged and referenced in the relevant section.

Caroline Natasha Clark
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Dedication & Acknowledgements

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To my parents, John and Susan Clark, for the way they have encouraged my academic studies.

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Abstract

The cosmic microwave background (CMB) temperature and polarisation anisotropies contain a wealth of cosmological information concerning the formation and evolution of the universe. Upcoming CMB experiments targeting measurements of the $B$-mode polarisation pattern of the CMB face a major challenge both in terms of experimental design and data analysis due to the small amplitude of the signal and the presence of experimental systematic effects and polarised foregrounds.

This thesis focuses on aspects of preparation for the Spider experiment. Spider is a balloon-borne polarimeter targeting CMB polarisation, it will launch in the Austral summer of 2013 for a long duration flight from Antarctica. It consists of large arrays of 512 detectors in each receiver, creating a large volume of data that is a challenge to analyse, especially when taking into account noise correlations between detectors.

We develop SPIMPI, a mapmaking algorithm for estimating temperature and polarisation maps from Time Ordered Data (TOD). To test the mapmaker, realistic TOD containing signal and noise components are generated from the simulated Spider scan strategy. We use an iterative scheme for solving linear systems (the Preconditioned Conjugate Gradient method) to produce optimal estimates of temperature and polarisation.

We present templates of the intensity and polarisation of emission from two of the main polarised Galactic foregrounds, interstellar dust and synchrotron radiation. We present estimates of the level of polarised foregrounds expected, focusing on high galactic latitudes and patches that will be targeted by upcoming experiments. We describe details of a model for the 3D Galactic magnetic field, examining both large and small scales. We include details of the dust and cosmic ray electron density distributions, grain alignment, the intrinsic polarisation of the emission from an individual grain and details of synchrotron emission mechanisms. We compare the templates with WMAP MCMC best-fit templates for these foreground components.
Contents

Page

Contents 8

List of Figures 10

List of Tables 11

1 Introduction 12
   1.1 Recombination and the CMB . . . . . . . . . . . . . . . . . . . . . 13
   1.2 Big Bang Puzzles . . . . . . . . . . . . . . . . . . . . . . . . . . . . 13
   1.3 Probing Inflation With CMB Polarisation . . . . . . . . . . . . . . 15
   1.4 Polarisation of the CMB . . . . . . . . . . . . . . . . . . . . . . . . 16
      1.4.1 Alternative Motivation for CMB Polarisation Experiments . . 17
      1.4.2 Other Sources of B-modes . . . . . . . . . . . . . . . . . . . 17
   1.5 Stokes Parameters . . . . . . . . . . . . . . . . . . . . . . . . . . . 18
   1.6 $EB$ Decomposition . . . . . . . . . . . . . . . . . . . . . . . . . 19
   1.7 CMB Power Spectra . . . . . . . . . . . . . . . . . . . . . . . . . . 20
   1.8 Standard Model of Cosmology . . . . . . . . . . . . . . . . . . . . . 21
   1.9 Polarisation in Real Space . . . . . . . . . . . . . . . . . . . . . . 21
   1.10 History of CMB Experiments . . . . . . . . . . . . . . . . . . . . . 28
   1.11 Current and Future CMB Experiments . . . . . . . . . . . . . . . . 29
   1.12 Observing from Antarctica . . . . . . . . . . . . . . . . . . . . . . 30
   1.13 Polarised Foregrounds and Galactic Modelling . . . . . . . . . . . . 32
   1.14 Thesis Plan . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 33

2 The SPIDER Experiment 34
   2.1 Introduction . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 34
   2.2 The Receiver . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 35
   2.3 Instrumental Point Spread Function . . . . . . . . . . . . . . . . . 37
   2.4 Scan Strategy . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 37
   2.5 Simulations . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 39
3 SPIMPI: An Algorithm for Massively Parallel Polarised Mapmaking

3.1 Introduction ................................................. 44
3.2 SPIDER Preparation and SPIMPI Development ............... 45
3.3 Solutions to the Mapmaking Problem ......................... 46
3.4 Simulating SPIDER TODs .................................. 47
  3.4.1 Simulating Realistic Noise in the TODs ................. 48
3.5 Reconstruction of I, Q and U Stokes Parameters .......... 49
3.6 The Linear Algebra Approach to Map-Making .............. 50
3.7 The Mapmaking Algorithm: SPIMPI ......................... 52
  3.7.1 Pixel-Pixel Covariance Matrix ....................... 53
  3.7.2 Calculation of the Naive Map ....................... 55
  3.7.3 Steps in Calculation of RHS ($b$) .................. 55
  3.7.4 Iterative Method for Inversion ...................... 57
  3.7.5 Preconditioners .................................. 59
  3.7.6 Chunks of Noise Stationary Data ................... 59
  3.7.7 Noise Power Spectrum Estimation .................. 60
3.8 Gap Filling ................................................. 62
3.9 Memory Requirements ....................................... 62
3.10 Increasing Length of TOD and Number of Detectors ....... 64
3.11 Removal of Noise in the TOD .............................. 69
3.12 Transfer Function ....................................... 70
3.13 Filtering .................................................. 71
3.14 Detector Correlations .................................... 71
3.15 Differencing Timestreams ................................ 76
3.16 Concluding Remarks ..................................... 77

4 Application of SPIMPI to Boomerang 2003 Data .......... 79
4.1 Introduction ................................................. 79
4.2 The B03 Experiment ....................................... 79
4.3 The Receiver .............................................. 80
4.4 Mapmaking ................................................ 81
4.5 Reanalysing 145 GHz Data with SPIMPI .................... 84
  4.5.1 A First Analysis ................................... 84
  4.5.2 Bandpassing the Timestream ....................... 87
  4.5.3 Smoothing the Maps ................................ 91
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.6</td>
<td>Concluding Remarks</td>
<td>91</td>
</tr>
<tr>
<td>5</td>
<td>Galactic Modelling</td>
<td>93</td>
</tr>
<tr>
<td>5.1</td>
<td>Introduction</td>
<td>93</td>
</tr>
<tr>
<td>5.1.1</td>
<td>Galactic Magnetic Field</td>
<td>94</td>
</tr>
<tr>
<td>5.1.2</td>
<td>Total GMF Model</td>
<td>98</td>
</tr>
<tr>
<td>5.2</td>
<td>Interstellar Dust</td>
<td>98</td>
</tr>
<tr>
<td>5.2.1</td>
<td>Dust Total Intensity</td>
<td>98</td>
</tr>
<tr>
<td>5.2.2</td>
<td>Dust Properties</td>
<td>99</td>
</tr>
<tr>
<td>5.3</td>
<td>Synchrotron Radiation</td>
<td>100</td>
</tr>
<tr>
<td>5.3.1</td>
<td>Total Intensity of Synchrotron Radiation</td>
<td>100</td>
</tr>
<tr>
<td>5.3.2</td>
<td>Cosmic Ray Density Distribution</td>
<td>100</td>
</tr>
<tr>
<td>5.4</td>
<td>Concluding Remarks</td>
<td>101</td>
</tr>
<tr>
<td>6</td>
<td>Modelling of Polarized Foregrounds From Interstellar Dust</td>
<td>102</td>
</tr>
<tr>
<td>6.1</td>
<td>Introduction</td>
<td>102</td>
</tr>
<tr>
<td>6.2</td>
<td>Foreground Dust Model</td>
<td>104</td>
</tr>
<tr>
<td>6.2.1</td>
<td>Stokes Parameters</td>
<td>104</td>
</tr>
<tr>
<td>6.3</td>
<td>Scales</td>
<td>107</td>
</tr>
<tr>
<td>6.4</td>
<td>Maps</td>
<td>109</td>
</tr>
<tr>
<td>6.5</td>
<td>Concluding Remarks</td>
<td>109</td>
</tr>
<tr>
<td>7</td>
<td>A Complete Model of Foregrounds on Large Angular Scales</td>
<td>113</td>
</tr>
<tr>
<td>7.1</td>
<td>Introduction</td>
<td>113</td>
</tr>
<tr>
<td>7.2</td>
<td>Polarised Synchrotron Emission</td>
<td>114</td>
</tr>
<tr>
<td>7.2.1</td>
<td>PLANCK Sky Model</td>
<td>117</td>
</tr>
<tr>
<td>7.3</td>
<td>Synchrotron Model</td>
<td>119</td>
</tr>
<tr>
<td>7.3.1</td>
<td>Stokes Parameters</td>
<td>119</td>
</tr>
<tr>
<td>7.4</td>
<td>Maps</td>
<td>120</td>
</tr>
<tr>
<td>7.5</td>
<td>Comparison with WMAP templates</td>
<td>122</td>
</tr>
<tr>
<td>7.6</td>
<td>Foreground amplitudes in sub–orbital sky patches</td>
<td>129</td>
</tr>
<tr>
<td>7.7</td>
<td>Concluding Remarks</td>
<td>132</td>
</tr>
<tr>
<td>8</td>
<td>Conclusions</td>
<td>134</td>
</tr>
<tr>
<td></td>
<td>References</td>
<td>152</td>
</tr>
</tbody>
</table>
# List of Figures

1.1 CMB power spectra from PLANCK best fit cosmological parameters . 22
1.2 Stacked images of hot and coldspots from WMAP data ............... 25
1.3 Temperature - peak polarisation correlation function for hotspots from WMAP data ......................................................... 26
1.4 Stacked images of hot and coldspots from PLANCK data ........... 27
1.5 Status of $TE$, $EE$ and $BB$ measurements from various experiments . 29
1.6 High $\ell$ TE and EE power spectra from PLANCK data ............ 30
1.7 Constraints on $n_s - r$ from PLANCK combined with WMAP BAO and high $\ell$ measurements ........................................... 31

2.1 The SPIDER payload ................................................................. 35
2.2 SPIDER receiver characteristics ........................................... 36
2.3 SPIDER scan region ............................................................... 38
2.4 SPIDER scan profile ............................................................. 39
2.5 Filter transfer function and beam estimates for SPIDER at 150 GHz . 43

3.1 Flowchart of the SPIMPI mapmaking pipeline. ......................... 54
3.2 Convergence of SPIMPI for increasing chunk length ................ 61
3.3 Comparison of input and recovered noise power spectra for one detector 63
3.4 Convergence of SPIMPI for varying TOD length and number of detectors ............................................................ 65
3.5 Comparison of Stokes parameter maps for increasing number of detectors ........................................................... 66
3.6 Comparison of Stokes parameter maps of the input sky with the SPIMPI solution for 24 detectors .................................................. 67
3.7 Stokes parameter maps for 24 detectors with increasing length of timestep .......................................................... 68
3.8 Correlated noise map for analysis of 24 detector TODs over a 1 day scan ................................................................. 69
3.9 Transfer function: 12 detectors, 1 day TOD ............................ 72
3.10 Transfer function: Low noise run, 12 detectors, 1 day TOD ...... 73
List of Tables

3.1 Convergence of SPIMPI with increasing chunk length . . . . . . . . . . 60
4.1 Characteristics of the B03 receiver . . . . . . . . . . . . . . . . . . . . . . 80
7.1 Foreground fits of WMAP MCMC and FGPol template spectra . . . . 127
1 Introduction

The evolution of the world can be compared to a display of fireworks that has just ended; some few red wisps, ashes and smoke. Standing on a cooled cinder, we see the slow fading of the suns, and we try to recall the vanishing brilliance of the origin of the worlds. Lemaître (1951)

This quote by Lemaitre serves well as a description of the study of cosmology, an observational science. From our viewpoint on Earth we look back in time, attempting to infer how the universe began, how it evolved and how structure formed, postulating the laws that govern it and testing them with observations.

Fortunately, although we only have one universe to study, we are able to observe a rich variety of phenomena and recently we have seen a surge in experiments and the amount of cosmological data, a turn of events that demands the development of novel analysis techniques but at the same time opens new windows in our study of the evolution of the universe.

One of several things as yet undiscovered is definitive evidence for the theory of inflation. This theory predicts several possible observable signatures, some of which have been found already. However, the evidence is not yet conclusive and current experiments are searching for a unique signature of this early period of expansion.

To do this, we must achieve new depths in our probes of the cosmic microwave background (CMB). While the temperature fluctuations are of the order $10^{-5}$ K, polarisation fluctuations are 1-2 orders of magnitude lower. This requires a leap forward in technology so to meet the challenge CMB polarimetry has moved from arrays of tens of detectors to large arrays of many hundreds of detectors in a single frequency band. In fact, the polarisation signal we are searching for is much smaller than some of the polarised signals coming from our own Galaxy which are present in the foreground of CMB measurements. Analysis of the polarisation of this cosmic microwave background is a challenging hunt for a tiny signal hidden in noisy observations, this primordial signal is also obscured by the much larger astrophysical foregrounds, a seemingly improbable task motivated by our desire to uncover ‘the vanishing brilliance of the origin of the worlds’.
1.1 Recombination and the CMB

At the epoch of recombination, electrons and protons combine to form neutral Hydrogen. Until this point photons were tightly coupled to electrons and protons, making up the baryon-photon fluid. After recombination, the photons decouple from the matter and freestream through the universe from this last scattering surface. The primordial radiation that decoupled from matter at a redshift of $\approx 1100$ forms this cosmic microwave background radiation field, the CMB. Today it appears as a close to uniform background at radio wavelengths.

The sound horizon at recombination is the distance that the sound waves in this baryon-photon fluid could travel up until the epoch of recombination. This defines a length scale on the surface of last scattering, the angular diameter distance. Combined with the distance to the last scattering surface, we can calculate the angle subtended on the sky by the sound horizon at recombination. Both the angular diameter distance and the distance to the last scattering surface depend on various cosmological parameters. At decoupling, the angular size of the radius of the sound horizon was about $1.2^\circ$.

1.2 Big Bang Puzzles

The CMB is a nearly isotropic Gaussian Random Field. The puzzle of this high level of isotropy leads to the well known horizon problem. The longest wavelength modes only re-entered the horizon long after decoupling. So at the time of decoupling these modes could not have been affected by causal physics that could smooth out the photon temperatures.

This leads us to the necessity of considering new physics to describe why the temperature of the CMB is isotropic to 1 part in $10^5$, when the last scattering surface that we observe contains many regions that were not in causal contact at the time of decoupling when the CMB was produced.

The observed flatness of the universe seems to present a fine tuning problem and is also important to consider, because although not inconsistent with the big bang picture, it is not explained by it. We observe a universe which is very close to flat, which implies that the universe must have been even flatter in the past, having close to zero curvature, because any small deviations from flatness would have been amplified with time.

The horizon and flatness problems could be explained by a period of inflation, accelerated expansion in the early universe. This period of accelerated expansion could be driven by a scalar field. Small perturbations of this scalar field during
inflation generates a spectrum of quantum fluctuations which are stretched to scales larger than the horizon and therefore do not evolve until later in the evolution of the universe when they reenter, these are the perturbations that collapse under gravity to form the structure we observe. Inflation requires an equation of state with negative pressure.

Inflation solves the flatness problem, because acceleration ‘flattens’ the universe, so an initial deviation from flatness is removed and no fine tuning needs to occur. As the comoving Hubble radius decreases during inflation, the observable universe after inflation is smaller than it was before inflation, so it also solves the horizon problem as the scales which could not have been causally connected in the big bang picture would have been in contact before inflation.

Inflation produces a stochastic background of scalar density perturbations and tensor gravitational wave perturbations. The power spectrum of both the scalar and tensor perturbations is close to scale invariant and the amplitudes of each spectra are related to the energy scale and dynamics of inflation.

A nearly scale invariant spectrum of density perturbations as predicted by inflation has been observed in the CMB. However, in order to provide convincing evidence for inflation we need to observe this stochastic gravitational wave background. Measurement of the tensor to scalar ratio $r$ would provide us with the information needed to determine the energy scale of inflation and when combined with measurements of the scalar spectral index, can further constrain inflationary models.

The ratio of the primordial tensor power spectrum to the primordial scalar power spectrum $r$ is given by

$$r = \frac{\Delta^2_t(k_0)}{\Delta^2_R(k_0)}$$

with pivot scale $k_0 = 0.002\, \text{Mpc}^{-1}$ where the primordial tensor and scalar power spectra are given by

$$\Delta^2_t(k) = \Delta^2_t(k_0) \left( \frac{k}{k_0} \right)^{n_t}$$

and

$$\Delta^2_R(k) = \Delta^2_R(k_0) \left( \frac{k}{k_0} \right)^{n_s - 1}$$

for tensor metric perturbations $h_k$ and curvature perturbations $\mathcal{R}_k$. 

14
1.3 Probing Inflation With CMB Polarisation

One key prediction of inflation already mentioned, the prediction of a stochastic gravitational wave background (GWB), would leave an imprint in the polarisation of the CMB. A GWB is one of the few unique predictions of most theories of inflation and would thus be an important discovery in modern cosmology.

Scalar perturbations due to density fluctuations produce only an $E$-mode (gradient type) polarisation of the CMB while tensor perturbations (due to gravitational waves) produce $B$-mode (curl type) and $E$-mode polarisation patterns. While there are other contributions to the $B$-mode power spectrum, on the largest scales $B$-modes would only be produced by a primordial GWB from inflation. The study of $B$-mode polarisation could therefore provide insight into primordial tensor fluctuations.

The detection of this GWB requires measurement of the power spectrum of this ‘$B$-mode’ polarisation, through which we can measure the tensor-to scalar ratio, $r$. The predicted amplitude of the GWB signal is very small and therefore requires a new generation of extremely sensitive experiments. The amplitude of the ‘$B$-mode’ power spectrum is expected to be 1 - 2 orders of magnitude smaller than the temperature power spectrum. The amplitude of $r$ varies significantly between models of inflation; measurement of this parameter therefore offers the possibility of constraining models of inflation. Predictions for the value of $r$ from some of the popular models of inflation are in the range $r \lesssim 0.2$. Detailed discussion of the predictions for a wide range of inflationary models can be found in e.g. Ade et al. (2013f).

Cosmology has made major leaps forward in the last decade with improvements in technology. Detailed measurements have been made of temperature anisotropies in the CMB, which are tiny variations of 1 part in $10^5$. Measuring the ‘$B$-mode’ polarisation signal requires another huge leap forward in technology, with the $B$-mode signal being two orders of magnitude below the level of the temperature anisotropies. We are now at the stage where a GWB could be detected in the next generation of experiments. However, we face a series of challenges in the detection of a primordial GWB. Firstly, the amplitude of this signal is expected to be tiny, so in order to attempt to detect it large arrays of extremely sensitive polarimeters are needed. The primordial signal could also be swamped by later effects such as gravitational lensing of $E$ to $B$ modes.

A huge challenge in detection of such a small signal is that the preparation of these experiments must have extraordinary control over systematic effects. The map-making process must also take into account the problem of jointly estimating
the noise and signal given that the noise component must also be estimated from the data. It must also account for correlations in the noise between possibly hundreds of detectors. There is also the difficulty of the confusion of a primordial signal with polarised Galactic foregrounds such as dust and synchrotron radiation.

### 1.4 Polarisation of the CMB

For radiation incident along the z-axis, if the intensity of radiation in the x and y directions is unequal, then the radiation is polarised. In the case of Thomson scattering of a photon off an electron, only the component of the incident radiation that is transverse to the direction in which the photon is scattered will be transmitted. The component parallel to the outgoing direction is not transmitted. For an electron which sees an isotropic radiation field, clearly no polarisation will be produced by scattering. The simplest example of an anisotropy that produces polarisation is a quadrupolar radiation pattern.

Polarisation is produced during the interval of recombination when both quadrupolar radiation and free electrons are present. As recombination progresses, photons diffuse in a random walk and after sufficient time photons from hot and cold regions have travelled far enough to be scattered off the same electron. Thomson scattering of electrons with the quadrupole moment of the radiation field just before decoupling leads to net linear polarisation of the CMB. This photon diffusion is only possible as the Universe becomes neutral during recombination; as the baryon-photon fluid becomes optically thin during the decoupling of photons from baryons. Once all free electrons have combined with protons, there is no longer an opportunity for Thomson scattering, so no more polarisation is produced after the end of recombination.

There is only a short time interval between the time when photons have diffused a sufficient distance between hot and cold regions and the time when no more Thomson scattering can occur after all photons have combined. This means that only a small amount of polarisation is produced in the CMB. If recombination had occurred instantly, no polarisation would have been produced as photons would not have had time to diffuse from hot and cold regions. These quadrupolar anisotropies are projected onto the polarisation pattern of the CMB.

The quadrupolar radiation (temperature) pattern that produces polarisation creates an anisotropic velocity field that is out of phase with the temperature (density) field. As polarisation depends on the velocity gradient of the photon-baryon fluid, the peaks in the temperature and polarisation power spectra should be out of phase. As electrons and photons are tightly coupled we will have a small quadrupole, lead-
ing to challenges in detecting the resulting small degree of polarisation.

Polarisation can be decomposed into two patterns, $E$- and $B$-modes which are gradient type and curl type fields respectively. Polarisation is described as a headless vector with the length giving the magnitude of the polarisation and the angle giving the axis along which the intensity of the radiation is greater.

1.4.1 Alternative Motivation for CMB Polarisation Experiments

CMB polarisation could also be produced by some other effects such as parity violating physics, which would produce non-zero $TB$ and $EB$ power spectra.

Alternatively, some theories predict either a non-observable level of gravitational waves or the theory does not produce them at all. Decreasing the upper limits on $r$ could probe this possibility.

1.4.2 Other Sources of B-modes

The $B$-mode of polarisation is also produced by a variety of other effects that hamper our ability to detect a primordial $B$-mode component. Later on in the history of the universe, reionisation of hydrogen leads to enhancement of polarisation on large scales. This polarisation can be produced during the epoch of reionisation as neutral hydrogen is ionised by light from the first stars, producing free electrons. Thomson scattering of the quadrupolar radiation field of the CMB off these free electrons creates a polarisation signal. This reionisation bump in the $B$-mode power spectrum is being targeted by some experiments.

Along the line of sight from the last scattering surface, the matter distribution leads to weak gravitational lensing of CMB photons. This lensing converts $E$-modes into $B$-modes, resulting in a large contribution to the observed $B$-mode power spectrum. The level of this lensing effect may limit the possibility of measuring $r$ if it has much larger amplitude than the primordial $B$-mode component. However, this signal is interesting in its own right. Given the sensitivity of this effect to the growth of structure and the geometry of the universe, measurement of this weak gravitational lensing signal can be used to probe a variety of effects including the evolution of structure and the curvature parameter. Lensing affects the smaller angular scales, so generally experiments focus on one regime of angular scales to have greatest sensitivity to a particular source of B-modes.
1.5 Stokes Parameters

The intensity of a radiation field is decomposed into four components, the temperature $T$, two parameters $Q$ and $U$ describing linear polarisation and $V$ which describes circular polarisation. Radiation from the early universe is not expected to have circular polarisation so $V = 0$. The Stokes parameters $I$, $Q$, $U$ and $V$ describe the polarisation state of radiation. As functions of the electric field of the incident radiation they are (see for example Jones et al. (2006))

\[ I = \langle |E_x|^2 \rangle + \langle |E_y|^2 \rangle \]  \hspace{1cm} (1.4)

\[ Q = \langle |E_x|^2 \rangle - \langle |E_y|^2 \rangle \]  \hspace{1cm} (1.5)

\[ U = 2\langle |E_xE_y| \cos(\phi_x - \phi_y) \rangle \]  \hspace{1cm} (1.6)

\[ V = 2\langle |E_xE_y| \sin(\phi_x - \phi_y) \rangle \]  \hspace{1cm} (1.7)

where $\phi_x$ and $\phi_y$ are the phases in the two directions transverse to the direction of propagation of the radiation, these directions are labelled by unit vectors $\hat{x}$ and $\hat{y}$. $E$ is the amplitude of the electric field and the angle brackets indicate time averaging of the electric field.

$E$ and $B$ modes are non-local functions of the CMB polarisation; meaning that these quantities depend on derivatives of $Q \pm iU$. In order to describe the polarisation as a local function, we must use the Stokes parameters. $I$ is the intensity of the radiation, while $Q$ and $U$ give the orientation of the ellipse traced out by the electric field. For CMB photons, $V$ is zero as there’s no relative phase between the polarization directions, $\phi_x - \phi_y = 0$. $Q$ and $U$ describe the linear polarisation along the coordinate axes and axes oriented at $\pm 45^\circ$ to the coordinate axes respectively.

The intensity tensor can be written as

\[ I_{ij} = \frac{1}{2} \begin{pmatrix} T + Q & -U - iV \\ -U + iV & T - Q \end{pmatrix} \]  \hspace{1cm} (1.8)

We can then write the temperature as $T = \frac{1}{3}(I_{11} + I_{22})$ while the $Q$ and $U$ Stokes parameters, describing linear polarisation, are given by $Q = \frac{1}{3}(I_{11} - I_{22})$ and $U = \frac{1}{2}I_{12}$. The polarisation magnitude and angle are given by

\[ P(\theta, \phi) = \left( Q^2 + U^2 \right)^{\frac{1}{2}} \]  \hspace{1cm} (1.9)

\[ \gamma(\theta, \phi) = \frac{1}{2} \arctan \left( \frac{U}{Q} \right). \]  \hspace{1cm} (1.10)
If we rotate in the plane orthogonal to the line of sight \( \hat{n} \) by an angle \( \alpha \) about this axis then \( Q \) and \( U \) transform as

\[
(Q \pm iU)'(\hat{n}) = e^{\mp 2i\alpha}(Q \pm iU)(\hat{n})
\]  

(1.11)

This combination of Stokes parameters is a spin-2 quantity. On the other hand, temperature is a spin-0 quantity as it is invariant under rotation.

We describe fields on the sky using the spherical harmonic functions \( Y_{\ell m}(\theta, \phi) \), which form a complete orthonormal set of basis functions on the sphere. The temperature can be expanded as a sum of the spherical harmonics according to

\[
T(\hat{n}) = \sum_{\ell,m} a_{T,\ell m} Y_{\ell m}(\hat{n})
\]  

(1.12)

where the \( a_{\ell m} \) are the complex spherical harmonic coefficients and should have a Gaussian distribution in the standard inflationary model. \( l \) is the multipole moment and is related to the angular scale on the sky while \( m \) gives the orientation of this mode on the sky. The probability density function (PDF) for the distribution of \( a_{\ell m} \) is

\[
P(a_{\ell m}) = \frac{1}{\sqrt{2\pi C_l}} e^{-a_{\ell m}^2/2C_l}
\]  

(1.13)

where \( C_l \) is the angular power spectrum, (see Section 1.7). The distribution of the \( a_{\ell m} \) thus has a variance of \( C_l \) and a mean of zero, i.e. the \( a_{\ell m} \) are zero mean Gaussian random variables.

In order to perform a similar expansion for polarisation, we must use the spin-2 weighted spherical harmonics \( sY_{\ell m}(\theta, \phi) \). Using these, the expansions are:

\[
(Q + iU)(\hat{n}) = \sum_{\ell,m} a_{2,\ell m} 2Y_{\ell m}(\hat{n})
\]  

(1.14)

\[
(Q - iU)(\hat{n}) = \sum_{\ell,m} a_{-2,\ell m} -2Y_{\ell m}(\hat{n})
\]  

(1.15)

1.6 \( EB \) Decomposition

The polarisation field of the CMB can be decomposed into ‘electric’ \( E \) (gradient) and ‘magnetic’ \( B \) (curl) modes (see for example Kamionkowski et al. (1997a)). Density perturbations produce only \( E \) modes while tensor perturbations can produce both \( E \) and \( B \) modes.

We can define \( E \) and \( B \) modes, which are independent of the orientation of the
coordinate system, as

\[ E(\hat{n}) = \sum_{\ell, m} a_{E, \ell m} Y_{\ell m}(\hat{n}) \]  
\[ B(\hat{n}) = \sum_{\ell, m} a_{B, \ell m} Y_{\ell m}(\hat{n}) \]

where

\[ a_{E, \ell m} = -(a_{2, \ell m} + a_{-2, \ell m})/2 \]  
\[ a_{B, \ell m} = -(a_{2, \ell m} - a_{-2, \ell m})/2i \]

The advantages of \( E \) and \( B \) modes are that they're scalar (spin 0), real quantities. \( E \) and \( B \) modes also have a distinct parity; under a parity transformation \( E \) is even and \( B \) is odd. This leads to the useful property that certain power spectra vanish.

### 1.7 CMB Power Spectra

If we look at the power spectra of \( T \), \( E \) and \( B \) and their cross-correlations, the variance of the field on a given angular scale is given by

\[ \langle a_{X, \ell m}^* a_{X, \ell' m'} \rangle = C_{\ell}^{XX'} \delta_{\ell \ell'} \delta_{m m'} \]

where \( X = T, B, E \) and the average is over an ensemble of realisations of skies that have the same cosmology. The best estimate of this will be obtained by averaging over \( m \) as we can only observe our own sky.

The correlation between the \( a_{l m} \) is zero unless \( l = l' \) and \( m = m' \) so there is no preferred direction on the sky; this equation is a statement of statistical isotropy.

The \( TT \), \( EE \), \( TE \) and \( BB \) power spectra are given by

\[ \langle a_{T, \ell m}^* a_{T, \ell' m} \rangle = C_{\ell}^{TT} \delta_{\ell \ell'} \delta_{m m'} \]
\[ \langle a_{E, \ell m}^* a_{E, \ell' m} \rangle = C_{\ell}^{EE} \delta_{\ell \ell'} \delta_{m m'} \]
\[ \langle a_{B, \ell m}^* a_{B, \ell' m} \rangle = C_{\ell}^{BB} \delta_{\ell \ell'} \delta_{m m'} \]
\[ \langle a_{T, \ell m}^* a_{E, \ell' m} \rangle = C_{\ell}^{TE} \delta_{\ell \ell'} \delta_{m m'} \]

The shape of the \( BB \) power spectrum has been predicted, however its amplitude depends on the inflationary model and may be unobservable. Certain correlations
vanish due to parity conservation so we have

\[ \langle a_{T, \ell' m'}^* a_{B, \ell m} \rangle = \langle a_{E, \ell' m'}^* a_{B, \ell m} \rangle = 0 \quad (1.25) \]

1.8 Standard Model of Cosmology

CMB data along with a variety of other datasets such as supernova and BAO measurements have allowed us to constrain the parameters of the standard cosmological model, ΛCDM. Remarkably only 6 parameters are needed in this model to fit data from a wide range of measurements. The standard ΛCDM model contains only scalar fluctuations; if we wish to study the possibility for primordial tensor fluctuations we can add this as an additional component whose amplitude is set through the value of \( r \), the ratio of the primordial tensor power spectrum to the primordial scalar power spectrum.

Given initial conditions and values for the cosmological parameters, there are a variety of numerical CMB Boltzmann codes (for example CMBFAST\(^1\) and CAMB\(^2\)) that evolve the initial perturbations using the Boltzmann equations to predict the CMB power spectra defined in Section 1.7. Recently, we have seen both WMAP 9-year and PLANCK data updating constraints on these parameters. In Figure 1.1 we have plotted the \( TT, EE, BB \) and \( TE \) power spectra from the best fit cosmological parameters from PLANCK given in Ade et al. (2013e).

1.9 Polarisation in Real Space

As this work is mainly concerned with the production of real space maps from measurements of temperature and polarisation, it is useful to look at some measurements of polarisation patterns in real space. In particular, the WMAP 7-year paper by Komatsu et al. (2011) stacks the \( I, Q \) and \( U \) Stokes parameter maps around hot and cold spots in the WMAP 7-year V+W band maps in order to look at the polarisation direction and temperature-peak polarisation correlation around hot and cold spots. This stacking allows us to look at an ‘averaged’ polarisation pattern around maxima and minima of the measured CMB maps, which can be compared to the prediction for adiabatic scalar fluctuations. Figure 1.2 shows stacked temperature plots as well as the stacked polarisation information, from WMAP data as well as noiseless simulations.

\(^1\)cmbfast.org
\(^2\)camb.info
Figure 1.1: $TT$ (top left), $EE$ (top right), $BB$ (bottom left) and $TE$ (bottom right) power spectra generated by CAMB for the best fit cosmological parameters from Planck and with a tensor-to-scalar ratio at the upper bound of the Planck + WMAP constraint $r_{0.002} < 0.12$ at 95% CL (Ade et al., 2013e). The black solid line is the total power spectrum (scalar+tensor+lensing) while the red dotted line is the scalar+lensing power spectrum and the blue dashed line is the tensor power spectrum.
The following discussion summarises the results in Section 2.2 of Komatsu et al. (2011). Linear polarisation is generated by Thomson scattering of a quadrupole temperature anisotropy, so polarisation is clearly correlated with temperature. The scalar density (temperature) fluctuations around hot spots create an oscillating pattern directed towards the centre of the maxima or minima of the density field. This creates an $E$-mode pattern as the polarisation pattern is perpendicular to the direction of modulation, resulting in a radial (or $E$-mode) polarisation pattern around the hotspot.

The $E$-mode is correlated with the temperature fluctuation if the modulation of the $E$-mode corresponds to modulation of the temperature. Whether the direction of linear polarisation is parallel or perpendicular to the crest of the temperature fluctuation corresponds to radial polarisation patterns (which corresponds to hotspots) or tangential patterns (corresponding to coldspots) respectively (Crittenden et al., 1995). The $B$-mode is not correlated with temperature, as Thomson scattering cannot generate rotation of an $E$-mode to a $B$-mode (a $45^\circ$ rotation).

Around hot and cold spots the small angle approximation can be used and transforming the Stokes parameters by a rotation through an angle $\phi$ (Kamionkowski et al., 1997b) gives

$$Q_r(\theta) = -Q(\theta) \cos(2\phi) - U(\theta) \sin(2\phi) \quad (1.26)$$

$$U_r(\theta) = Q(\theta) \sin(2\phi) - U(\theta) \cos(2\phi) \quad (1.27)$$

Averaging these functions leads to quantities related to the stacked hotspots, $\langle Q_r(\theta) \rangle$ and $\langle U_r(\theta) \rangle$. These can be related to integrals in harmonic space of $C_{l}^{TT}$ and $C_{l}^{TB}$ respectively. So if parity conservation holds, the map of $\langle U_r(\theta) \rangle$ will be zero (except for in the presence of certain systematic effects). Figure 1.2 shows the polarisation direction around hot and cold spots as well as a map of $\langle Q_r(\theta) \rangle$. The tangential polarisation pattern around cold spots and radial pattern around hot spots is clearly seen.

The angular size of the radius of the horizon at decoupling is $\sim 1.2^\circ$, while the angular size of the radius of the sound horizon is $\sim 0.6^\circ$. The sound horizon is the distance that the sound waves in this baryon-photon fluid could travel before the epoch of recombination. This defines a length scale on the surface of last scattering, the angular diameter distance. Combined with the distance to the last scattering surface we can calculate the angle subtended on the sky. Both the angular diameter distance and distance to the last scattering surface depend on the cosmological parameters.
Different physics on these scales creates different polarisation patterns. On scales separated by more than the horizon size at decoupling (>2.4°) for an overdensity at the centre, \( T < 0 \) and the photon fluid will flow into the gravitational potential well, creating a quadrupolar anisotropy around an electron situated here, leading to a radial pattern, \( Q_r > 0 \). Thus the combination \( TQ_r < 0 \) and we see an anti-correlation. On scales of the angular size of the sound horizon scale, the photon fluid flows towards the potential well at the centre causing compression that increases the temperature. The polarisation pattern is still radial, \( Q_r > 0 \), but as \( T \) becomes positive, the quantity \( TQ_r > 0 \) and we see correlation between these quantities. Lastly on scales around the angular size of the radius of the sound horizon, the direction of the flow reverses which means that the polarisation becomes tangential, \( Q_r < 0 \) and as \( T \) is now positive, \( TQ_r < 0 \) and again we see anti-correlation.

These features are visible in the temperature - peak polarisation correlation \( \langle Q_r(\theta) \rangle \) shown in Figure 1.3. Anti-correlation on scales greater than 2.4°, correlation at scales of 1.2° and anti-correlation on scales of 0.6°. These features correspond visually to maxima and minima in the map of \( Q_r \) in Figure 1.2.

A stunning improvement has been achieved by the low noise measurements of PLANCK with the update to the stacked temperature and polarisation patterns around hotspots and coldspots being shown in Figure 1.4 compared to stacked plots for a ΛCDM realisation from PLANCK’s best fit cosmological parameters.
Figure 1.2: Stacked images of a 5° by 5° patch in temperature (left) and polarisation (right) around hot and cold spots in the WMAP 7-year V+W band maps compared with noiseless simulations. The data has been smoothed to 0.5°. Credit for image: WMAP science team.
Figure 1.3: Temperature - peak polarisation correlation function, $\langle Q_r \rangle(\theta)$, in the WMAP 7-year V+W band maps compared with noiseless simulations. This function is calculated from the stacked maps of hotspots with increasing peak height threshold. The different lines correspond to the beam that is used to smooth the polarisation maps, while the temperature and noiseless simulations are smoothed with a Gaussian beam to $0.5^\circ$. Credit for image: WMAP science team.
Planck Collaboration: The Planck mission

Fig. 27. Stacked maps of the CMB intensity $I$ and polarization $Q_r$ at the position of the temperature extrema, at a common resolution of 30 arcmin. We measure directly the SFR density with around 4 $\sigma$ significance for three redshift bins between $z = 1$ and 7, thus 39

Maps are displayed for CMB temperature cold spots (left) and hot spots (right) for the $I$-band and $Q_r$-band at 857 GHz. Using HI data from three radio telescopes (Parkes, GBT and Epoch of Reionization 857 GHz), we have confirmed that the stacked signal is consistent with the best fit model prediction (bottom row).

The cosmic infrared background (CIB) is a composite signal from star-forming galaxies at $z > 1$ and dust at all redshifts. At multipole $\ell > 100$, the power spectrum is dominated by the CIB, with an amplitude less than 0.2 $\%$ compared to the Galactic dust. At multipole $\ell < 30$, the power spectrum is limited by our ability to separate the CIB from the CMB anisotropies in the Planck HFI maps. Using more refined techniques, we can use auto- and cross-power spectra to constrain the relationship between the clustering of dark matter and the CIB, and probe the clustering properties of galaxies, which in turn can be used to measure the star formation history. According to current models, the dusty star-forming regions in galaxies are tightly correlated with the dark matter distribution of halos hosting CIB sources of log $M_\odot$.

Leveraging the frequency coverage of Planck (217, 353, and 857 GHz), the dominant extragalactic signal is the Planck HFI/IRAS HFI emission, which is considered a reliable tracer of the CMB anisotropies. The conjunction of these two unique probes allows us to measure directly the connection between dark and luminous matter in the high redshift (1.45, 2.0, and 3.0) universe. According to current models, the dusty star-forming regions in galaxies are tightly correlated with the dark matter distribution of halos hosting CIB sources of log $M_\odot$.

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1.10 History of CMB Experiments

The CMB was discovered by Penzias and Wilson in 1965 (Penzias and Wilson, 1965). In 1990 the FIRAS experiment on the COBE satellite measured the spectrum of this radiation field to be an almost perfect black body spectrum (Mather et al., 1994). In 1992 the DMR experiment on COBE produced the first evidence for the imprint of density fluctuations on the CMB (Smoot et al., 1992).

The Boomerang98 flight produced the first maps of the temperature anisotropy on sub-horizon scales ($\approx 1^\circ$) and also detected the first peaks in the power spectrum (Lange et al., 2001), these acoustic oscillations were also found by the experiments TOCO (Miller et al., 2002) and MAXIMA (Lee et al., 1999).

The first statistical detection of CMB polarisation was by DASI (Kovac et al., 2002; Leitch et al., 2005) in 2002 which detected the $E$-mode with 4.9$\sigma$. It was subsequently measured by the CBI (Readhead et al., 2004; Sievers et al., 2007), CAPMAP (Barkats et al., 2005; Bischoff et al., 2008), BOOMERANG (Piacentini et al., 2006; Montroy et al., 2006), MAXIPOL (Wu et al., 2007), WMAP (Page et al., 2007) and QUAD (Brown et al., 2009) experiments which measured the $EE$ and $TE$ power spectra.

BICEP (Yoon et al., 2006) detected the first acoustic peak in the $E$-mode spectrum and provided the best direct constraints on the tensor-to-scalar ratio $r$ to date (Chiang et al., 2010). More recently the QUIET experiment (Samtleben, 2008) has measured the first three peaks of the $E$-mode power spectrum with high signal to noise (QUIET Collaboration, 2010; Araujo et al., 2012).

Currently, the $B$-mode has not been detected but we do have upper limits. The first way of constraining $r$ comes from measurements of the temperature power spectrum when a tensor component is added to the standard $\Lambda$CDM model. The WMAP 7-year results best limit on $r$ comes from WMAP + BAO + supernova measurements, constraining the tensor-to-scalar ratio, $r < 0.2$ at 95% Confidence Limits (CL) (Komatsu et al., 2011). SPT data in combination with measurements of BAO and the Hubble constant constrained $r < 0.17$ at 95% CL (Keisler et al., 2011). The WMAP 9-year results give $r < 0.38$ and a combination of WMAP +ACT+SPT+BAO gives $r < 0.12$ at 95% CL (Hinshaw et al., 2012). PLANCK + WMAP recently constrained $r < 0.12$ at 95% CL, including high $\ell$ measurements from ACT+SPT gives $r < 0.11$.

The second way of constraining $r$ comes from direct measurements of CMB polarisation. The best upper limit from measurements of the CMB $B$-mode polarisation comes from the BICEP experiment, which measured $r < 0.72$ at 95% CL (Chiang et al., 2010). Measurements from a range of experiments are summarised in
Figure 1.5. $TE$, $EE$ and $BB$ measurements by various experiments plotted alongside theoretical $\Lambda$CDM spectra with $r=0.1$ Figure credit: H. C. Chiang et al

1.11 Current and Future CMB Experiments

Some results from the PLANCK satellite have been released in a series of 30 papers on March 21st 2013. Its Low Frequency Instrument (LFI) has 3 frequency bands (30 – 70 GHz) while its High Frequency Instrument (HFI) has 6 bands between 100 – 857 GHz and consists of an array of 52 bolometric detectors. PLANCK has much higher resolution and sensitivity than than its predecessor WMAP, and is able to provide cosmic variance limited measurements for the temperature power spectrum to much smaller angular scales.

Ade et al. (2013d) provides best fits at high $\ell$ values to $\Lambda$CDM EE and TE power spectra from full-sky temperature data and polarisation data from 40% of the sky
outside their mask, with no systematics or foregrounds removed, these spectra are shown in Figure 1.6. The data is plotted in large bins of width $\Delta \ell = 40$. These preliminary results at high $\ell$ values demonstrate Planck’s potential in terms of polarisation.

Ade et al. (2013f) looks at constraints on inflation in the light of the Planck results, providing confidence limit contours in the $n_s$-$r$ parameter space from a combination of Planck data with other datasets. This can be seen in Figure 1.7, the value for $n_s$ is measured to be less than one to a very high level of significance. Planck data is able to rule out some classes of inflationary models and places restrictions on many more models.

There is also a range of ground based experiments targeting polarisation on a variety of angular scales. Experiments involved in this search include SPTpol (McMahon et al., 2009), PIPER (Lazear et al., 2011), ABS (Essinger-Hileman et al., 2010), ACTpol (Niemack et al., 2010), POLAR (Keating et al., 1998), POLARBEAR (The Polarbear Collaboration et al., 2010) and BRAIN (Charlalssier and the BRAIN Collaboration, 2008).

Current balloon-borne experiments, launching from McMurdo, Antarctica, are EBEX (Reichborn-Kjennerud et al., 2010) (launched in December 2012) and SPIDER (Filippini et al., 2010), which will fly in the Austral summer of 2013-2014.

1.12 Observing from Antarctica

Scientific ballooning from Antarctica is an exciting way in which to test the latest advances in polarimetry. Balloon-borne experiments from Antarctica have a range of advantages over other locations. It is particularly suitable for long duration
Figure 1.7: 68\% and 95\% Confidence contours in the $n_s$-$r$ parameter space from Planck combined with WMAP data as well as BAO and high $\ell$ measurements from ACT and SPT. Predictions from various inflationary models are also shown as lines through the parameter space that cover the range of 50-60 e-folds of inflation. Figure credit: Planck science team.

balloon flights, with stable wind currents allowing flights of well over 20 days. In fact, the current record for an LDB flight is held by the super-TIGER experiment, which launched on 8th December 2012 and flew for 55 days, 1 hour, 34 minutes and completed almost three orbits of the pole. LDB flights provide access to angular scales that are not possible for ground based experiments. A large fraction of sky is available for observation that is thought to be quite clean in terms of polarised foregrounds in certain patches.

Scientific ballooning, taking place at an altitude of about 32,000–36,000 m, above the troposphere, also provides a low background noise level for an experiment. In fact, it provides backgrounds very close to what can be achieved in space. It minimises the level of atmospheric fluctuation seen by the telescopes which reduces the measured photon noise and photon loading. In particular, microwave emission from water vapour in the atmosphere makes measurements from the ground very difficult as this vapour is very variable and inhomogeneous. Balloon-borne experiments are above about 99\% of this water vapour.
1.13 Polarised Foregrounds and Galactic Modelling

The presence of foregrounds limits our ability to measure a primordial $B$-mode signal as it places upper bounds on the observable value of the primordial tensor to scalar ratio. In the planning stages of an experiment an accurate prediction of foregrounds is an important consideration when determining scan strategies and the ability to constrain cosmological parameters. Modelling the frequency dependence, morphology and amplitude of these foregrounds is an active area of research. In the near future several experiments will provide new information to guide this modelling. Data from experiments such as SPIDER (Filippini et al., 2010) and PLANCK (The Planck Collaboration, 2006) will be useful for improving knowledge of the polarisation of foreground emission from interstellar dust.

The other important polarised foreground emission, from synchrotron radiation, will dominate at lower frequencies (Page et al., 2007). While it is thought to be negligible at the frequencies of interest to current experiments, it is also important to have models for this emission. Other foregrounds include polarised point sources, anomalous microwave emission possibly due to spinning dust and free-free emission. Recent studies have shown that the polarisation fraction of spinning dust and free-free emission is thought to be negligible, see Macellari et al. (2011), Dickinson et al. (2011) and López-Caraballo et al. (2011). Spinning dust grains are smaller spherical grains; they do not align strongly with the Galactic Magnetic Field as is the case with the larger aspherical dust grains that produce thermal dust emission. Thus emission from spinning dust is not significantly polarised. The scattering direction of free-free emission is random and therefore unpolarised.

Measurement of foreground polarisation could also be useful for studies of the galactic magnetic field on both large scales and small scales where turbulence is important. Currently the amplitude and detailed pattern of this magnetic field is poorly understood, particularly away from the Solar vicinity. Widely used probes of these fields include Faraday rotation measures of pulsars and extra-Galactic radio sources (Haverkorn et al., 2006; Han et al., 2006). Modelling of polarised foregrounds could also be used to provide information on the distribution of dust and cosmic ray electrons in the galaxy when more accurate measurements are made of synchrotron and interstellar dust polarisation.
1.14 Thesis Plan

This thesis focuses on preparation of a map-making algorithm for the SPIDER experiment. The SPIDER experiment is described in Chapter 2. As the first flight of SPIDER was delayed, our focus moved to modelling polarised foregrounds, and this thesis also describes a model of microwave emission from Galactic foregrounds including dust and synchrotron radiation.

In Chapter 3 we present a polarised mapmaker that uses an iterative method, the Preconditioned Conjugate Gradient Method, for estimating $I$, $Q$ and $U$ Stokes parameter maps from time ordered data. To test the mapmaker, realistic datastreams containing signal and noise are generated from simulations of the scan strategy of SPIDER. The map-maker is tested on simulated data in Chapter 3. In Chapter 4 we test the algorithm on data from the BOOMERANG 2003 experiment, recovering maps of the $I$, $Q$ and $U$ Stokes parameter maps that are visually very similar to BOOMERANG03 maps produced by other pipelines.

The other part of this work involves development of realistic estimates of the level of polarised foregrounds expected, with a focus on high galactic latitudes that will be targeted by a range of suborbital experiments.

In Chapter 5 we present details of a 3D model for the Galactic magnetic field, examining both large and small scales and the cosmic ray and electron density distributions. In Chapter 6, we present templates of the intensity and polarisation of emission from one of the main Galactic foregrounds, interstellar dust. As well as the details of the 3D magnetic field model, we include details of the dust density, grain alignment and the intrinsic polarisation of the emission from an individual grain. We present Stokes parameter template maps at 150 GHz and provide an online repository\(^3\) for these and additional maps at frequencies that will be targeted by upcoming experiments such as EBEX, SPIDER and SPTpol. In Chapter 7, we look at the other main polarised Galactic foreground, synchrotron radiation. We include details of the cosmic ray electron density and details of the emission mechanism for synchrotron radiation. We present Stokes parameter template maps at 150 GHz. We compare the dust and synchrotron templates with WMAP MCMC best fit templates for these foreground components.

\(^{3}\)http://www.imperial.ac.uk/people/c.contaldi/fgpol
2 The SPIDER Experiment

2.1 Introduction

This chapter will give an overview of the SPIDER experiment and summarise experimental details that are needed for the development of the mapmaking software. I will refer the reader to publications of the SPIDER collaboration that will provide the details of the design and development of the SPIDER experiment that I do not go into here. An outline of the SPIDER experiment is given in Crill et al. (2008), Filippini et al. (2010) and Runyan et al. (2010).

SPIDER is an Antarctic Long Duration Balloon (LDB) experiment that will produce high resolution maps of the polarisation of the microwave sky with highly sensitive arrays of detectors. It will attempt to probe fundamental physics through detection of the signature of an inflationary epoch. It will also probe the gravitational lensing of CMB polarisation along the line of sight to the last scattering surface (LSS) and the physics of the interstellar medium through measurements of foreground emission from diffuse interstellar dust. It aims to detect or place upper limits on $B$-modes, providing evidence for, or excluding, a range of inflationary models.

SPIDER is a balloon-borne polarimeter targeting CMB polarisation; its first flight will be for about 20-30 days in the Austral summer of 2013-2014 from McMurdo, Antarctica at an altitude of approximately 36,000 m, targeting a large patch of the sky in the Southern hemisphere thought to have relatively low interstellar dust contamination. It will measure in 3 frequency bands, 90 GHz, 150 GHz and 280 GHz over the course of its two science flights. In each frequency band it will have large arrays of 256 optical pixels, each measuring orthogonal polarisations, i.e. 512 bolometric detectors.

SPIDER’s first flight will include Focal Plane Units (FPUs) at 90 GHz and 150 GHz. The second flight includes plans for the 280 GHz FPUs. The bands were chosen to fall between galactic and atmospheric emission lines and its frequency coverage is chosen to allow for the removal of the foreground signal through differences in the spectral signatures of the CMB and foreground emission components.
SPIDER is designed for maximum sensitivity at $\ell \approx 80$ and is predicted to be sensitive down to $r < 0.03$ at $3\sigma$ after one flight if no foregrounds were present or two flights given the presence of foregrounds. It will target the range $10 \lesssim \ell \lesssim 300$ measuring approximately 10% of the sky with less than degree-scale resolution. SPIDER will target slightly smaller scales than PLANCK thereby providing a complimentary probe of the $B$-mode power spectrum. PLANCK will probe the large scale reionisation peak in the polarisation power spectrum. On smaller scales, the amplitude of the power spectra of polarised foregrounds are smaller relative to the primordial signal, so foregrounds will be less important (although they are still the most important factor in the potential for detecting a signal from a primordial gravitational wave background).

2.2 The Receiver

SPIDER’s payload is made up of six telescopes, see Figure 2.1. Pairs of telescopes are oriented at 45° with respect to each other so that $Q$ and $U$ can be measured simultaneously. Relative to the direction in which SPIDER is scanning, the pairs of detectors are oriented at 22.5° and 112.5°. The design of the optics is based on BICEP and BICEP2 designs. These telescopes illuminate focal planes that are made up of antenna-coupled transition-edge sensor (TES) arrays. Each optical pixel of the focal plane unit (FPU) contains two arrays of slot antennas with sensitivity to orthogonal polarisations. An antenna is coupled to a TES by a filter and all these components are etched on a silicon tile, this whole array is read out using SQUIDs. Kuo et al. (2008) describes these antenna coupled TES arrays developed for experiments such as SPIDER, currently the BICEP2 and KECK arrays are also
<table>
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<th>Band Center (GHz)</th>
<th>Bandwidth (GHz)</th>
<th>Beam FWHM (arcmin)</th>
<th>Number of Spatial Pixels</th>
<th>Number of Detectors per FPU</th>
<th>Detector Sensitivity (µK√s)</th>
<th>FPU Sensitivity (µK√s)</th>
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Figure 2.2: Observing bands, pixel count, detector counts, detector and Focal Plane Unit sensitivities. The FPU sensitivity is the detector sensitivity divided by \(\sqrt{N_{\text{det}}}\) assuming a yield of 85%. For 90 and 150 GHz these values were obtained from lab measurements while the 280 GHz values are scaled from BOOMERANG sensitivities.

using this technology, see Orlando et al. (2010) and O’Brien et al. (2012). Figure 2.2 outlines the specifications of each SPIDER receiver.

Each FPU consists of four detector tiles each with 8 × 8 pixels (150 GHz) or 6 × 6 pixels (90 GHz). This means that the 150 GHz FPU consists of 256 optical pixels that have two bolometers to measure polarised intensity along orthogonal axes, totaling 512 bolometers.

The receivers are inserted into a liquid helium cryostat that cools the FPUs using 3He fridges; the design of this cryogenic system is described in Gudmundsson et al. (2010). The gondola and sun-shielding is shown in Figure 2.1; the gondola is a lightweight carbon-fibre frame that houses the cryostat and connects the experiment to the balloon, the design is similar to the BLAST design (Pascale et al., 2007). The experiment can spin in azimuth and step in elevation in a similar way to the BOOMERANG experiment (Masi et al., 2006), with the telescope pointing kept track of by GPS, star cameras, rate gyroscopes and sun sensors. The expected telescope pointing uncertainty is expected to be around 1′; better than the degree of uncertainty required (10′) to reduce systematics to the target level for SPIDER (Fraisse et al., 2011). BOOMERANG achieved a pointing uncertainty of 2.4′.

SPIDER will use half wave plates (HWPs) (Bryan et al., 2010) to modulate polarisation sensitivity, stepping 22.5° each day. The HWPs are positioned in the telescope insert to allow the polarisation sensitivity of each optical pixel on a focal plane unit to be switched from Q to U while keeping the beam constant thereby modulating only the sky signal, which allows rejection of a variety of systematic effects as has been demonstrated through optical testing in the lab and the operation of HWPs in experiments at the South Pole, in particular the KECK array (Sheehy et al., 2011).
2.3 Instrumental Point Spread Function

In multipole space, an estimate of the beam transfer function $B_\ell^2$ for SPIDER is shown in Figure 2.5. A Gaussian beam is modelled in harmonic space as

$$B_\ell = \exp\left[-2(\ell + \frac{1}{2})^2 \sin^2(\theta_s/2)\right]$$  \hspace{1cm} (2.1)

where $\theta_s = \theta_{\text{FWHM}}/\sqrt{8 \ln 2}$ is a parameter introduced to describe the smoothing of the field by a Gaussian kernel with a Full Width at Half Maximum (FWHM) of $\theta_{\text{FWHM}}$. The beam must be deconvolved from the measurement of the power spectrum $C_\ell$ through

$$C_\ell = \frac{C_\ell^{\text{obs}}}{B_\ell^2 p_\ell^2}$$  \hspace{1cm} (2.2)

where $p_\ell$ is the HEALPIX pixel window function. Expected beam FWHMs in each SPIDER frequency band are shown in Figure 2.2.

In flight, bright compact sources will be observed to keep track of uncertainty in the beam centroids. Beam characterisation in both the near and far field has been looked at to study beam effects. Some far field beam characterisation has been undertaken using a blackbody source located about 30 m from the SPIDER cryostat, this is described in more detail in Trangsrud (2012).

The simulations work looked at the level of false $B$-mode signal from a variety of beam effects, showing as an example that differential beam width and differential ellipticity produce negligible false $B$-mode signals.

2.4 Scan Strategy

SPIDER performs scans in azimuth along with steps in elevation, performing one full scan cycle per day. The HWP is also rotated at a certain time each day to switch $Q$ to $U$ in each receiver. The gondola azimuthal scans have a sinusoidal velocity profile with period of $\sim 45$ s and maximum acceleration of $0.8^\circ/\text{s}^2$ and maximum speeds of $6^\circ/\text{s}$. The SPIDER scans are chosen to remain $90^\circ$ away from the Sun and within a patch of the sky chosen as it has been predicted to have relatively low foreground contamination. The elevation of the instrument is stepped at $1^\circ$ per hour from $28^\circ$ (minimum elevation) at 24:00 local sidereal time (LST) to $40^\circ$ (maximum elevation) at 12:00 LST and back. Synchronising these steps with LST ensures the largest possible range in the angle of declination and therefore the largest field of view. The scan region is shown in Figure 2.4.

The scan strategy is chosen to maximise crosslinking of observations. Through
combining the effects of sky rotation, scan strategy and combination of observations from all detectors on a focal plane unit, good modulation of the angle of orientation of a detector with respect to the sky (denoted by $\psi$) is achieved. However, for pixels where good modulation is not achieved, there will be singularities in the polarisation decorrelation matrix.

The polarisation decorrelation matrix $M_p$ for pixel $p$ is an average over all observations of that pixel (as in Equation 3.4 with $\gamma = 1$). As each observation is made with different orientation of the instrument, this matrix is averaged over all observation angles. This matrix must be inverted for each pixel in the map in order to reconstruct the Stokes parameter maps, see Section 3.5. It is given by an average over all observations falling in a pixel $p$:

$$M_p = \sum_{i\epsilon p} \begin{pmatrix} 1 & c_i & s_i \\ c_i & c_i^2 & s_i c_i \\ s_i c_i & s_i & s_i^2 \end{pmatrix}$$

(2.3)

where $c_i = \cos(2\psi_i)$ and $s_i = \sin(2\psi_i)$ and $\psi_i$ is the detector orientation for observation $i$. Singularities in this matrix are problematic for the inversion of this matrix, which must be carried out in order to estimate the Stokes parameters $I$, $Q$ and $U$ at each pixel $p$.

Fraisie et al. (2011) presents plots of the excess variance in the map caused by anisotropic angular coverage in the $Q$ and $U$ maps. The inverse of the polarisation decorrelation matrix provides an estimate for the signal covariance of pixel $p$. For isotropic observations, averaging over angles means the diagonal of the inverse of

Figure 2.3: The region of sky targeted by Spider in equatorial coordinates, smoothed with a 30’ beam. Left: IRAS satellite data showing the dust emission in this patch, Right: WMAP 94 GHz temperature data in the same area. The Spider patch is outlined in white, Boomerang is the left gray outline while BICEP is the right gray outline. The predicted region of minimum foreground contamination is outlined in black. The south ecliptic pole is highlighted by a star and the south Galactic pole by a cross. Figure credit: Fraisse et al. (2011).
Figure 4: The top plot shows five minute periods every three hours for one detector over a 24 hour period. The fraction of excess variance for one detector after four days is shown in the bottom left and for 512 detectors, a whole focal plane unit, after four days in the bottom right. Figure credit: Fraisse et al. (2011).

Figure 2.4: The top plot shows five minute periods every three hours for one detector over a 24 hour period. The fraction of excess variance for one detector after four days is shown in the bottom left and for 512 detectors, a whole focal plane unit, after four days in the bottom right. Figure credit: Fraisse et al. (2011).

Figure 2.4 from Fraisse et al. (2011) shows that 96% of the Spider patch is observed isotropically and polarisation is reconstructed with less than a percent of excess variance. This figure shows the result from one full focal plane, however in reality we will have pairs of telescopes oriented at 45° to each other to obtain measurements of $Q$ and $U$ simultaneously. This will achieve greater crosslinking and better angular coverage so should reduce the excess variance measured in polarisation due to anisotropic sky coverage even more than is indicated in this figure.

2.5 Simulations

Spider requires characterisation of a variety of systematic effects in order to detect
this incredibly faint signal. An analysis of the systematics that will contribute is
given in MacTavish et al. (2008). Simulations of SPIDER’s observation strategy,
analysis of systematic effects and the ability to reach its science goals are described
in this work. O’Dea et al. (2011a) looked at several systematic effects and also
used a model of thermal emission from interstellar dust to inform the SPIDER scan
strategy. Both these papers were written when SPIDER was planning to launch from
Alice Springs, leading to a very different scan strategy.

The paper by Fraisse et al. (2011) updated the scan strategy to an Antarctic LDB
flight. In terms of systematics, the $B$-mode contamination from systematic effects is
calculated, showing that systematic errors are significantly below the $B$-mode signal
for $r = 0.03$. It also looks at the impact of polarised sidelobes. The paper uses the
model of foreground emission from interstellar dust to show the expected power
spectrum from this foreground in SPIDER’s frequency bands. It estimates that in
the presence of foregrounds SPIDER will constrain $r < 0.03$ at 99% confidence limits
after two flights, or after one flight if no foregrounds were present.

A detailed simulation pipeline has been developed and uncertainties in gain, beam
effects and pointing as well as half-wave plate nonidealities and the impact of the
magnetic field of the Earth on the electronics are systematic effects that have been
looked at through simulations. These simulations measure whether the false $B$-
mode signal from $I \rightarrow Q,U$ or $Q \rightarrow U$ mixing for each of these systematics is
sufficiently below the expected cosmological $B$-mode signal at $\ell = 100$. The target
for SPIDER has been to reduce systematic errors to the level of the cosmological $B$-
mode power spectrum at $\ell = 100$ for $r = 0.03$. This target value for the combination
$\ell(\ell + 1)C_\ell/(2\pi)$ is equal to 43 nK$^2$. The target level of control over systematics and
whether this has been achieved is described in Table 2 of Fraisse et al. (2011). All
effects have been found to be small compared to a primordial signal at $r = 0.03$.

2.6 Transfer Function

Important noise contributions for SPIDER are atmospheric backgrounds (which leads
to a scan synchronous effect), instrumental backgrounds and $1/f$ noise from gain
drifts. The mapmaking is expected to require high pass filtering of the raw TODs
in order to remove the effect of low frequency noise. This filtering leads to a loss
of information on long timescales which can be characterised with the filter transfer
function.

The filter transfer function describes the loss of information as a function of mul-
tipole due to handling of effects such as scan synchronous noise by filtering the
timestreams. This transfer function gives an idea of the loss of spatial modes caused by the filtering that is applied to remove low frequency $1/f$ noise. The filter transfer function $F_{\ell}$ is defined as in Hivon et al. (2002) and calculated from an average of an ensemble of signal only realisations. The measured power spectrum,

\[
\langle C^{\text{obs}}_{\ell} \rangle = F_{\ell} B_{\ell}^{2} p_{\ell}^{2} \langle C_{\ell} \rangle + \langle N^{\text{obs}}_{\ell} \rangle
\]

where $F_{\ell}$ is the filter transfer function, $B_{\ell}$ is the beam function, $p_{\ell}$ is the HEALPIX pixel window function and $N_{\ell}$ is the estimate of the average noise power spectrum. $C_{\ell}$ is the power spectrum from a signal only realisation of the sky.

$F_{\ell}$ is calculated by comparing the power spectrum of the ensemble of signal only CMB realisations that is input to the mapmaker with the power spectrum of the output maps (either the naively coadded map of the data or the iterated PCG solution map). There is no noise in these realisations, so $N_{\ell}$ is zero. The filter transfer function is therefore given by

\[
F_{\ell} = \frac{\langle C^{\text{obs}}_{\ell} \rangle}{B_{\ell}^{2} p_{\ell}^{2} \langle C_{\ell} \rangle}
\]

The estimates for this transfer function from Fraisse et al. (2011) are shown in Figure 2.5 using a least squares mapmaker. The estimate of $F_{\ell}$ is multiplied by the beam $B_{\ell}^{2}$ so that the effect of both the convolution and the filtering on the power spectrum can be seen visually. SPIDER’s large sky coverage increases the range of angular scales we can reconstruct from the data. Analysis of this transfer function has shown that SPIDER has good recovery of a wide range of angular scales, in part due to the low noise environment achieved by stratospheric balloon experiments. There is much less atmospheric noise for balloon-borne experiments compared with ground based experiments, this means the filtering of the data does not need to be as harsh and loss of information is reduced. Compared with the Boomerang03 case, SPIDER has much larger sky coverage and a higher scan rate resulting in better recovery of signal to lower multipoles.

Low frequency $1/f$ detector noise leads to a scan synchronous signal due to long timescale drifts. Measurements of SPIDER’s bolometers have been made with no optical loading, so that photon noise is not present and an estimate of instrumental noise can be obtained. The estimates of the noise power spectrum used in Figure 2.5 are characterised by a $1/f$ knee of $\approx 100$ mHz. The iterative method, which filters the data by an inverse noise kernel that is calculated from this estimate of the noise power spectrum, leads to better recovery of angular scales compared to naive binning where no attempt is made to remove the $1/f$ noise. More recent tests showed that
SPIDER bolometers will have a $1/f$ knee of 25 mHz which will reduce the loss of information on large scales, so this plot gives a pessimistic picture for the final SPIDER filter transfer function.

On the largest angular scales SPIDER will be noise dominated on time scales corresponding to the half period of the azimuthal scan $\sim 20-25s$. This corresponds to frequencies of 40–50 mHz. Therefore Fraisse et al. (2011) looked at two cases for the high pass filter cut-off, corresponding to an optimistic estimate just above the scan frequency at 25 mHz and a worse case estimate at the half period of the scan at 50 mHz, informed by the Boomerang experiment. Clearly the optimistic case leads to less information loss on large scales.

The filter transfer function is calculated from an ensemble of simulations; the uncertainties on this function will be calculated by averaging over the ensemble. As discussed there is a lot of uncertainty in estimating the low frequency noise power spectrum with regard to its knee frequency, resulting in significant uncertainty in the filter transfer function for values of $\ell < 100$. This can be seen in Figure 2.5, with large differences between the red and blue curves corresponding to a factor of two change in the value of the $1/f$ knee.
Figure 2.5: Filter transfer function and beam function estimates for SPIDER at 150 GHz compared to a typical ground based experiment (BICEP) and a typical balloon borne experiment (BOOMERANG03). They are calculated from ensembles of signal realisations with high pass filtering in the time domain and naive binning for the black and blue curves. The red curve is calculated from the output of an iterative mapmaker with high pass filtering and estimation of the detector noise kernel. This figure is taken from Figure 6 of Fraisse et al. (2011).
3 SPIMPI: An Algorithm for Massively Parallel Polarised Mapmaking

3.1 Introduction

One of the aims of the current generation of CMB experiments is to measure $B$-mode polarisation of the CMB. Measurement of this signal is difficult, as the polarisation anisotropies are at least an order of magnitude smaller than the temperature anisotropies across the sky, so an attempt to detect $B$-modes or decrease the upper limits on the $B$-mode signal requires an increase in number and sensitivity of detectors. This problem is complicated as the polarised signal is made up of both CMB and foregrounds (which may dominate over the whole sky in all frequency bands of interest), so as well as reconstruction of the signal, component separation must be carried out to separate the various foregrounds from the CMB. Polarised foregrounds are explained in more detail in Chapters 6 and 7.

Analysis of these experiments involves several stages of data compression. Time Ordered Datastreams (TODs) (of the order of several Terabytes for many experiments) are compressed to form maps (of the order of several Megabytes of data). These maps allow study of foreground contamination and systematics. They are also the subject of studies of the statistical properties of the CMB temperature and polarisation fields.

The data analysis pipeline for an experiment such as SPIDER involves many stages. The raw TODs must first be processed. For example, samples that are contaminated by cosmic ray hits or other effects must be flagged. Variations in calibration of detectors over time can also be corrected for during analysis. The processed TODs are then passed through an iterative mapmaker that simultaneously estimates the signal and the noise in the TOD, a process which must take into account things such as scan synchronous effects and correlations in the noise between detectors on a focal plane unit (FPU) in order to produce maps of the temperature and
polarisation across the measured region of the sky at a range of frequencies.

This data compression provides a computationally tractable way to fit for cosmological parameters from the power spectrum of the map, a process which is too computationally intensive directly from the TODs. This intermediate map-making step involves no loss of information in the maximum likelihood approach and also produces the smallest errors. A further exercise in data compression involves the estimation of power spectra from which cosmological parameters can be inferred, with a temperature or polarisation map of millions of pixels being described by a power spectrum over a much smaller range of multipole moments.

The aim of this work is to develop the procedure for the mapmaking step for the SPIDER experiment, where TODs from a full FPU of detectors will be compressed to form maps of the $I$, $Q$ and $U$ Stokes parameters (corresponding to maps of temperature and linear polarisation) through a massively parallel, optimal polarised map-maker using an iterative approach for joint signal and noise estimation.

The increase in number of detectors in upcoming experiments massively increases the size of the dataset to analyse, especially when correlated noise between detectors is taken into account, which occurs due to a variety of effects including simultaneous readout of a column of detectors on the FPU and atmospheric fluctuations affecting many detectors at once. The analysis of correlations between several hundred detectors is a huge challenge. Unlike the uncorrelated white noise, this correlated noise does not average down as the number of detectors increases, so treatment of the correlations is an important part of the mapmaking process.

### 3.2 SPIDER Preparation and SPIMPI Development

In this Chapter we give a detailed outline of the mapmaking algorithm. SPIDER originally planned to have a test flight in 2009 that was cancelled and was expected to launch a science flight in December 2012 which was also cancelled due to unavoidable delays. These delays meant the mapmaking algorithm development undertaken during the first one and a half years of this PhD was put to one side and focus moved to developing models of polarised foregrounds that form the latter part of the work produced during this PhD. However, we have recently revisited the mapmaking code and looked at applying the mapmaker to a real dataset, from the 2003 flight of Boomerang which provided a nice test of SPIMPI and was quite challenging due to firstly finding the appropriate data and then rediscovering the intricacies of the dataset. We have also begun to look at various tests of the mapmaker in terms of loss of information from filtering the TOD and scaling with number of detectors and
length of timestream. Work on detector correlations and more extensive tests will be undertaken in the near future in the runup to the SPIDER launch in December 2013.

3.3 Solutions to the Mapmaking Problem

Mapmaking involves solving a linear algebra system to obtain a map of sky pixels from a vector of time domain detector samples. Previous experiments with more manageable datasets could tackle the mapmaking problem by direct matrix inversion due to the smaller number of pixels and detectors, for example through Cholesky decomposition of the matrix. There is a large body of literature on approaches to CMB map-making. The linear algebra description of the map-making problem, developed in analysis of COBE DMR differential radiometer data, is described for example in the papers by Wright et al. (1995) and Lineweaver et al. (1994). Also applied to this data, a maximum likelihood approach to CMB mapmaking and power spectrum estimation is described in Gorski (1996) and maps and results are described in Bennett et al. (1996).

In a paper by Tegmark (1997), the various linear and non-linear approaches to map-making were reviewed in terms of loss of cosmological information. The maximum likelihood treatment retains all cosmological information in the TODs, thus providing the best constraints on cosmological parameters. The COBE method was shown to be a natural choice as it does not lose any information in the data compression of TODs to maps and other (e.g Wiener filtered) maps can be computed from the COBE solution.

In the development of methods to take algorithms further for analysis of larger datasets and include polarisation data, many solutions were developed. A comparison of three algorithms was carried out in Poutanen et al. (2006). Two maximum likelihood methods, ROMA (de Gasperis et al., 2005) and MAPCUMBA (Dore et al., 2001) and one destriping method, MADAM (Keihanen et al., 2005) were compared by looking for example at the residual noise in final maps from simulated observations. The procedure for making MAXIMA maps is described in Stompor et al. (2002).

The MADCAP algorithm (Borrill, 1999) was also applied to BOOMERANG and MAXIMA data. MAPCUMBA, described in Dore et al. (2001), uses a multigrid Jacobi method. A maximum likelihood approach to PLANCK data analysis was presented in Natoli et al. (2001). Various other papers focus on joint estimation of the noise power spectrum and maximum likelihood map (see Prunet et al. (2000))
and an application to the 1998 flight of BOOMERANG is shown in Prunet et al. (2001). A method for simultaneous noise and signal estimation was proposed in Ferreira and Jaffe (1999).

More recently, the MADMAP algorithm (Cantalupo et al., 2010) was developed, this is a maximum likelihood approach that is scalable to very large datasets. Sutton et al. (2009) developed a massively parallel destriping code (DESCART) that includes correlated noise between detectors. SANEPIC, developed for BLAST, is described in Patanchon et al. (2007). This joint signal and noise estimation method is similar to the MADCAP (Borrill, 1999) method. However, for applications of MADCAP the correlated noise between detectors was unimportant while in the BLAST case there are strong correlations between timestreams which are handled by the SANEPIC algorithm.

An alternative approach to the maximum likelihood mapmaking methods are destriping methods (see for example Delabrouille (1998) and Maino et al. (2002)) which can produce close to optimal signal estimates. The destriping mapmakers approximate the correlated detector noise and subtract it from the TODs before naively binning into a map which averages down the white noise component. They model the low frequency, correlated component of the noise time streams as a superposition of baselines. An example of a destriping algorithm is the MADAM code which has been updated to include polarisation and is designed for experiments with scan strategies such as PLANCK, see Keihanen et al. (2005) and Keihanen et al. (2010). The amplitudes of the basis functions are determined through a maximum likelihood analysis in which the map and set of baselines are estimated simultaneously. Power spectrum estimation from this approach is described in Poutanen et al. (2004).

3.4 Simulating SPIDER TODs

SPIMPI is tested on simulated timestreams for subsets of detectors of a SPIDER focal plane. Firstly, a pointing solution for the subset of detectors is generated (using SPIDER collaboration pipeline code, flightsim) which creates solutions for right ascension, declination and angle of detector orientation for each detector over a flight of a number of days with a realistic SPIDER scan strategy in the Southern hemisphere and a bolometer sampling rate of 153.996 Hz. In this thesis, we generate pointing information for tens of detectors for up to about 10 days, with a launch at midnight on 14th December 2011 at a latitude of $-77.8^\circ$ and a longitude of $-166.7^\circ$. flightsim uses the information on the scan strategy, such as the elevation range allowed by the experiment and how often we perform steps in elevation, the
position of the sun and the azimuthal acceleration of the gondola in order to generate
the pointing solution for each detector.

Simulated full-sky CMB only temperature and polarisation maps are generated
using *synfast*, a HEALPIX routine for generating realisations of Gaussian Random
Fields (GRFs) on the sphere, characterised by a ΛCDM power spectrum. This CMB
realisation is then scanned with the pointing solution for a realistic SPIDER flight
to create a TOD for each detector.

SPIDER will modulate the polarisation using a rotating half-wave plate, allowing
the Stokes parameters $Q$ and $U$ to be resolved for every detector which will reduce
many systematic effects. The half-wave plate will be rotated by $22.5^\circ$ once per day;
this rotation is also simulated in the TODs.

### 3.4.1 Simulating Realistic Noise in the TODs

Ground based experiments suffer from higher atmospheric noise and larger back-
grounds, leading to lower signal to noise maps than balloon borne experiments are
capable of achieving. However, for stratospheric balloon experiments, the back-
ground noise is more variable, requiring careful modeling of systematics in the plan-
ning stages of the experiment. The noise component of the TODs includes contribu-
tions from unidentified cosmic rays, atmospheric fluctuations and instrumental noise
(including detector noise). Correlations in the noise component between detectors,
induced by the read-out electronics or spatial atmospheric fluctuations across whole
focal planes or subsets of detectors, could be very important for experiments with
large arrays of detectors.

The detectors used by CMB experiments have noise drifts over long timescales,
this correlated noise shows up in the maps as striping along the direction of the
scan. The power spectrum of this correlated noise is given by a $1/f$ component to
the noise power. This $1/f$ component is characterised by its knee frequency and
index.

We add noise to the simulated TODs through generating realisations of a realistic
SPIDER noise power spectrum. The noise power spectrum can be approximated by
the sum of white noise and $1/f$ components given by

$$P(f) = \sigma^2 \left(1 + \frac{f_{\text{knee}}}{f}\right)^\alpha$$

where the index $\alpha = 2$ (this index typically has a value between 1 and 2). The
values of $\sigma$ and $f_{\text{knee}}$ vary between detectors and SPIMPI solves for the noise
power spectrum of each detector individually. Measurements of the detector noise
have been carried out in lab tests to give estimates for these values.

Realisations of this power spectrum are combined with the signal timestream. Noise realisations are generated for each chunk of noise stationary timestream and a buffer of length equal to the correlation scale $\lambda_c$ is added on each side of the chunk. These buffers mean that the Fourier transform is not carried out on a chunk with hard boundaries which would lead to boundary effects. Noise correlated between detectors can be added by statistically correlating these noise realisations, this is currently work in progress, see Section 3.14.

This $1/f$ behaviour means we must taken into account time correlations of the noise. The noise covariance matrix of a given detector is given by $N_{tt'} = \langle n_t n_{t'} \rangle$ where $t$ indicates we are in the time domain and $n_t$ is the estimate of the noise timestream. There are several characteristics of $N$ that are important for analysis. Firstly, that the noise is Gaussian, so that all statistical information is described purely by its second moment, the variance. Secondly, that the data is subdivided into chunks where the noise is stationary so that the matrix $N$ can be approximated as a circulant matrix and is thus diagonal in Fourier space. In order to retain this noise stationarity we must remove small sections of the TOD that are contaminated by things like cosmic ray hits, we fill these gaps with constrained realisations of the noise (see Section 3.8). The correlations between detectors can be added into the combined noise covariance matrix, $N_{iit'} = \langle n_{it} n_{i't'} \rangle$ where $i$ subscripts the detectors from 1 to $N_{\text{det}}$. This is discussed in for example Patanchon et al. (2007).

3.5 Reconstruction of I, Q and U Stokes Parameters

The signal and noise measured by an individual detector is given by:

$$d_i = I + \gamma(Q \cos 2\psi_i + U \sin 2\psi_i) + n_i$$  (3.2)

where $n_i$ is the noise at time $i$, $\gamma$ is a measure of polarisation efficiency and $\psi_i$ is the orientation of the detector on the sky. $\gamma$ measures the cross polar response, the degree to which one polarisation component is contaminated by the orthogonal component. This value will be different for each detector. The signal plus noise in
a given detector, $d_i$, is used to reconstruct $I$, $Q$ and $U$ through this equation:

$$
\begin{pmatrix}
  d_i \\
  d_i \gamma c_i \\
  d_i \gamma s_i
\end{pmatrix}
= M
\begin{pmatrix}
  I \\
  Q \\
  U
\end{pmatrix}
$$

(3.3)

where $c_i = \cos(2\psi_i)$, $s_i = \sin(2\psi_i)$ and $M$ is the polarisation decorrelation matrix (also defined in Equation 2.3). This matrix is an average over all observations of a pixel $p$, given by

$$
M_p = \sum_{i \in p}
\begin{pmatrix}
  1 & \gamma c_i & \gamma s_i \\
  \gamma c_i & \gamma^2 c_i^2 & \gamma^2 s_i c_i \\
  \gamma s_i & \gamma^2 c_i s_i & \gamma^2 s_i^2
\end{pmatrix}
$$

(3.4)

To make maps of temperature and polarisation of the sky from a TOD, the polarisation decorrelation matrix is inverted. $I$, $Q$ and $U$ are then obtained using:

$$
\begin{pmatrix}
  I \\
  Q \\
  U
\end{pmatrix}
= M^{-1}
\begin{pmatrix}
  d_i \\
  d_i \gamma c_i \\
  d_i \gamma s_i
\end{pmatrix}
$$

(3.5)

When multiple detectors are being analysed the polarisation decorrelation matrix becomes a sum over detectors and the variation in polarisation efficiency must be taken into account:

$$
M_p = \sum_{j=1}^{N_{det}} w_j \sum_{i \in p}
\begin{pmatrix}
  1 & \gamma_{j} c_i & \gamma_{j} s_i \\
  \gamma_{j} c_i & \gamma_{j}^2 c_i^2 & \gamma_{j}^2 s_i c_i \\
  \gamma_{j} s_i & \gamma_{j}^2 c_i s_i & \gamma_{j}^2 s_i^2
\end{pmatrix}
$$

(3.6)

where $w_j$ accounts for differences in calibration (or sensitivity) between detectors and $N_{det}$ is the number of detectors.

### 3.6 The Linear Algebra Approach to Map-Making

The rough scheme is as follows. The TOD is modelled as a data vector of dimension $N_{TOD}

$$
d_t = A_{tp} x_p + n_t
$$

(3.7)

where $t$ indicates the temporal domain and $p$ the spatial domain. $n_t$ is the noise vector and $A_{tp} x_p$ is the signal vector. The signal vector is the result of observation of a sky map $x_p$ by an ‘observation matrix’ $A_{tp}$ of dimension $N_{TOD} \times N_{pix}$. The ‘observation matrix’ $A_{tp}$ scans the sky map with a given scan strategy, observing a
pixel \( p \) at time \( t \) and summing this into the TOD sample \( d_t \). Its transpose, \( A_{tp}^T \), the ‘pointing matrix’, maps the time domain elements \( d_t \) into pixels on the sky map \( x_p \).

The observation matrix also encodes the details of the beam properties, containing the value of the beam response in each pixel observed at a given time sample. However, in the case of a symmetrical beam, the observation matrix is much simpler and contains only ones or zeros with one element per row where a pixel is observed. In this case the map solved for is the sky convolved with the experimental beam.

The resolution of the map needs to be high enough to avoid loss of information (as stated in Patanchon et al. (2007) the pixel size should be smaller than \( 1/3 \) of the FWHM of the instrumental beam). One approach to map-making is to solve for \( x_p \) using a linear estimator

\[ \hat{x}_p = W_{pt}d_t \]  

(3.8)

The map is estimated from all detector TODs in an FPU. The size of the dataset leads to computational challenges and noise correlations between detectors must be taken into account. Calculation of the matrix \( W \) is far from trivial due to the difficulty of inverting large matrices.

Taking the log likelihood of Equation 3.7 and maximising with respect to the sky map \( x_p \) (maximising the probability of the estimated map given the data) gives the maximum likelihood estimator for the map \( \hat{x}_p \) with the linear estimator \( W_{pt} \) given by

\[ W = [A^TN^{-1}A]^{-1}A^TN^{-1} \]  

(3.9)

where \( N \) is the noise covariance matrix.

In the case of white noise, \( N \) is diagonal in Fourier space and the matrix \( W \) simplifies to give the coaddition operator \( P = (A^TA)^{-1}A^T \) where \( (A^TA)^{-1} \) is the pixel hit counter. This operation is a simple averaging of the TOD into pixels on the map (the ‘naive map’).

Along the lines of Dore et al. (2001), we perform a change of variable so that instead of solving for the sky map \( \hat{x} \), we solve for the stripes map \( \hat{y} \) where

\[ \hat{y} = \hat{x} - Pd \]  

(3.10)

If we substitute Equation 3.9 into Equation 3.8 and multiply by the pixel-pixel covariance matrix we obtain

\[ A^TN^{-1}A\hat{x} = A^TN^{-1}d \]  

(3.11)

Multiplying each side by the pixel hit counter, and substituting for \( \hat{x} \) using Equa-
In Dore et al. (2001) they show that the right hand side of Equation 3.12 only depends on the noise \( n \) and not the signal \( x \). This means that this change of variable amounts to solving for a ‘stripes map’ which only includes the noise and effects of the scan strategy.

The optimal estimate for the sky map \( \hat{x} \) is then given by the sum of the estimate of the stripes map \( \hat{y} \) (reached via an iterative matrix inversion) and the naively coadded map \( Pd \). I will now describe the estimation of the stripes map through solving Equation 3.12 with an iterative method, the Preconditioned Conjugate Gradient (PCG) method. Equation 3.12 is a general inversion problem of the form

\[
M \hat{y} = b
\]  

(3.13)

Solving this equation for the stripes map \( \hat{y} \) involves iteratively inverting \( A^T N^{-1} A \) (the inverse pixel-pixel covariance matrix) so comparing Equation 3.11 and Equation 3.12 we see that this change of variable doesn’t lead to a difference in the inversion that must be undertaken. However, the right hand side of these two equations is different as the combination \( (d - APd) \) depends only on the noise so does not have large variation as we scan across high or low signal regions. This means that the residual, which is the difference between the right and left sides of Equation 3.12 is better behaved.

3.7 The Mapmaking Algorithm: SPIMPI

SPIMPI (SPIder MPI) is a maximum likelihood mapmaker developed for experiments such as SPIDER where there are several thousand detector timestreams to analyse. The software is written in Fortran and makes use of the Message Passing Interface (MPI) which has been developed for writing massively scalable parallelised code. In the following sections we outline some of the key features of SPIMPI. An overview of the analysis pipeline is given in Figure 3.1.

We have taken an alternative approach to the parallelisation of SPIMPI compared to many other mapmaking algorithms that divide a detector TOD between processors so that each processor analyses a chunk of noise stationary data from all detectors. We give each processor the whole TOD from one detector, all processors then step through the chunks analysing the TODs in parallel.

There are several advantages to this alternative method for parallelisation. Firstly,
it makes it easy to produce maps from any length of timestream (through an option in the parameter file that is passed to the mapmaker). In the alternative approach to parallelisation, analysing shorter lengths of timestream would mean that fewer processors deal with the same amount of information. In our approach, each processor deals with one TOD, so you will still make use of all processors, they will just have less data to analyse so the program will run faster. This makes it easier for producing Stokes maps from shorter sections of timestream. It is also advantageous in outputting other diagnostic maps as each processor is running through the TOD at the same time, so outputting maps such as the number of hits per pixel and the current estimate of the residual are very straightforward in this approach.

One possible disadvantage is the extra time it takes to calculate the $N_{\text{det}}(N_{\text{det}} - 1)/2$ cross power spectra between detector pairs, as information will have to be shared between processors. However, we have not yet looked at the strength of noise correlations for SPIDER or the implementation of this calculation in SPIMPI so it is too early to comment on this in detail (see Section 3.14 for more discussion on noise correlations).

We make use of the HEALPix implementation for pixelisation of the sphere into 12 equal squares. The number of pixels over the full sky is calculated from the length of the sides of these squares through $N_{\text{pix}} = 12N_{\text{side}}^2$. For developing SPIMPI we create TODs from maps at a resolution of $N_{\text{side}} = 1024$ and create final maps at $N_{\text{side}} = 256$.

### 3.7.1 Pixel-Pixel Covariance Matrix

The term in Equation 3.9, $(A^TN^{-1}A)^{-1}$, is the pixel-pixel noise covariance matrix. The inversion of this term is difficult, see for example Patanchon et al. (2007). The PCG method avoids this problem as we never explicitly calculate the inverse pixel-pixel covariance matrix.

If the assumption that the final map estimate contains only Gaussian noise is appropriate, then the pixel-pixel noise covariance matrix encodes the noise variance for a pixel and the correlations with all other pixels of the map. As we only deal with the inverse of this matrix, it would be useful to have an estimate for the pixel-pixel noise covariance matrix. If we assume that the final map will only contain white noise then this estimate can be obtained by inverting only the diagonal elements of this matrix (Patanchon et al., 2007).
Figure 3.1: Flowchart of the SPIMPI mapmaking pipeline.
3.7.2 Calculation of the Naive Map

The optimal estimate of the map $\hat{x}$ is the sum of the stripes map $\hat{y}$ and the naively co-added map $Pd$ which averages all observations of a pixel $p$. Firstly we will describe the calculation of the naive map. To create the naive map, we must sum up the polarisation decorrelation matrix by looping through samples and sharing this information via MPI across all detectors. At the same time a map of the number of observations per pixel is calculated and stored.

The degeneracy of the polarisation decorrelation matrix for pixel $p$ is analysed through looking at the condition number of the matrix (the ratio of its maximum to minimum eigenvalues). If this number is equal to one then the matrix is well conditioned, which means its inverse can be calculated accurately. If this matrix is much larger than one (i.e. the matrix is ill-conditioned) then the matrix is almost singular so its inverse cannot be calculated accurately (this occurs when we do not observe a pixel with isotropic distribution of the detector orientation angle $\psi$). Ill-conditioned pixels are removed from the mapmaking and the gap filling procedure described in Section 3.8 is applied. These pixels are replaced with a constrained noise realisation to retain the Toeplitz property of the noise covariance matrix in the time domain.

For all other pixels, the inverse of the polarisation decorrelation matrix is calculated. From this matrix calculated at each pixel $p$, the naive map is calculated through application of Equation 3.5 where we have also divided by the hits map to perform the averaging.

3.7.3 Steps in Calculation of RHS ($b$)

This Section outlines the calculation of the right hand side of Equation 3.12.

3.7.3.1 Step 1: Low Pass and High Pass Filtering of TODs

A high pass filter is applied to the original TOD to remove low frequencies that are dominated by scan synchronous systematic effects.

3.7.3.2 Step 2: Difference original TOD and scanned naive map

We calculate the quantity $(d - APd)$ which is the difference between the original TOD and the scan of the naive map.
3.7.3.3 Step 3: Calculate $N^{-1}$

This step involves calculation of the inverse noise covariance matrix. This cannot be done in the time domain, given the large number of samples. Under the assumption of noise stationarity then the noise covariance matrix is circulant which means we have a diagonal matrix in Fourier space. Any manipulation of the inverse noise covariance matrix, $N^{-1}$, is therefore done in Fourier space as it is faster computationally to perform a simple filtering of the Fourier Transform of the timestream by the inverse of the noise power spectrum. This avoids having to invert the huge time domain noise covariance matrix $N_{tt}$.

For the approximation of circulant data over a chunk of noise stationary data, the noise covariance matrix depends only on the time interval between elements $t$ and $t'$. This correlation function drops to zero when elements are separated by a time interval greater than a certain correlation length given by $\lambda_c$,

\[
N_{tt'} = \begin{cases} 
C(|t - t'|) & \text{for } |t - t'| < \lambda_c \\
0 & \text{for } |t - t'| > \lambda_c 
\end{cases}
\]  

(3.14)

where $C$ is the correlation function between elements $t$ and $t'$.

In Fourier space, the diagonal elements of $N$ are equal to the power spectrum of the noise. Given that the inverse of a circulant matrix is also circulant, we can write the inverse noise covariance matrix (written in Patanchon et al. (2007)) as

\[
N_{tt'}^{-1} = \tilde{C}(|t - t'|) = \mathcal{F}^{-1} \left[ \frac{1}{P(\omega)} \right] (\Delta t)
\]  

(3.15)

This means that the inverse noise covariance matrix in the time domain comes from an inverse transform of the power spectrum of the noise.

Computationally the Fourier transform is an $O(N_{\text{chunk}} \log N_{\text{kernel}})$ operation where $N_{\text{chunk}}$ is the length of the chunk of TOD over which noise is considered to be stationary and $N_{\text{kernel}}$ is the width of the kernel (the width of the diagonal of the inverse noise covariance matrix).

3.7.3.4 Step 4: Filtering the Differenced TOD by the Noise Estimate

The next step involves the application of $N^{-1}$ to the differenced TOD $d - APd$. Acting with the inverse noise covariance matrix on the differenced TOD, $N^{-1}(d - APd)$, is done in Fourier space as it is easier, as it is a simple multiplication by a diagonal matrix. Specifically, it involves dividing the Fourier transform of the differenced TOD $d - APd$ by the inverse of the noise power spectrum. This is an operation which filters the timestream, creating a timestream of ‘whitened’ noise.
3.7.3.5 Step 5: Scan TOD into Map

The next step is very fast, the application of the pointing matrix $A^T$ in calculation of the product $A^T N^{-1}(d - APd)$, which is simply the mapping of the filtered TOD into pixels on the sky.

3.7.3.6 Step 6: Divide by Pixel Hit Counter

The map from Step 5 is divided by a map of the hits per pixel, $A^T A$.

3.7.4 Iterative Method for Inversion

The next step involves an iterative method for matrix inversion to solve the system given by Equation 3.13 for the stripes map $\hat{y}$. This involves implementing the Preconditioned Conjugate Gradient (PCG) Method.

The PCG method provides fast convergence for solving linear systems through minimisation of the quadratic $\frac{1}{2}y^T My - b^T y + c$. Where $M$ is positive definite, this quadratic is convex and minimisation finds the position where the gradient is zero, which corresponds to solving Equation 3.13. The method searches in conjugate directions for the minimum of this quadratic. As each iteration moves in an orthogonal direction to all previous steps the method results in fast convergence.

It calculates the residual at each step, given by $r = PN^{-1} A\hat{y} - PN^{-1}(d - APd)$. Using the square of the residual $r^T r$ as the criteria for minimisation, it can be seen that this will be a minimum if we have found the maximum likelihood solution.

The PCG method is a widely used algorithm that I will outline here. In the first iteration, we assume that $y_0 = 0$. Therefore, the initial residual

$$r_0 = b - My_0 = b$$

(3.16)

From this, two other arrays are initialised,

$$z_0 = Cr_0$$

(3.17)

where the choice of preconditioner $C$ is given in Section 3.7.5. For the Conjugate Gradient method with no preconditioning, $C$ is omitted. We also have

$$d_0 = z_0$$

(3.18)
We then loop over $n$. We calculate

$$\alpha_n = \frac{z^T_n r_n}{d^T_n M d_n}$$  \hspace{1cm} (3.19)

The next iteration of the stripes map is then

$$y_{n+1} = y_n + \alpha_n d_n$$  \hspace{1cm} (3.20)

the estimate of the residual $r$ is also updated:

$$r_{n+1} = r_n - \alpha_n M d_n$$  \hspace{1cm} (3.21)

and similarly $z$ is updated

$$z_{n+1} = C r_{n+1}$$  \hspace{1cm} (3.22)

A ratio of the previous iteration to the current one gives $\beta$

$$\beta_{n+1} = \frac{z^T_{n+1} r_{n+1}}{z^T_n r_n}$$  \hspace{1cm} (3.23)

from which we update $d$

$$d_{n+1} = z_{n+1} + \beta_{n+1} d_n$$  \hspace{1cm} (3.24)

We then move to the next iteration, $n = n + 1$ moving back to Equation 3.24 and updating all the quantities again. The algorithm has converged once the sum over the square of the residual $r$ (see Equation 3.26) at iteration $n$ divided by the value at iteration $n = 0$ has decreased below a certain tolerance threshold value $T$:

$$\varepsilon = \frac{R_n}{R_0} < T$$  \hspace{1cm} (3.25)

The sum over the square of the residual $r$ at step $n$ is given by

$$R_n = \sqrt{\sum_{j=1}^{N_{\text{maps}}} \sum_{i=1}^{N_{\text{pix}}} r^2_n(i, j)}$$  \hspace{1cm} (3.26)

where the sum is over all $N_{\text{pix}}$ pixels of the $N_{\text{maps}} = 3$ Stokes parameter maps $I$, $Q$ and $U$.

We use the value $T = 10^{-5}$ for all tests of the code presented here. This is more than sufficient for the temperature map to converge, however the Stokes $Q$ and $U$ parameter maps converge much slower to the optimal estimate. When the full focal plane is analysed, this tolerance threshold will be set to a smaller value, as
pixels will be observed with enough coverage in the detector orientation angle $\psi_i$ to measure $Q$ and $U$. For the tests presented here, the small number of detectors we simulate means that there is too much noise in the $Q$ and $U$ maps for them to converge, so setting the tolerance threshold $T$ to a smaller value would not result in any significant improvement in the $Q$ and $U$ estimates. An alternative approach would be to estimate $I$ first as this converges quickly, then solve for $Q$ and $U$. This will be tested in future versions of the code as it may speed up the algorithm.

We can output various diagnostic maps which are updated each iteration, including the map of the residual $r$, the current estimate of the stripes map $\hat{y}$ and the current estimate of the optimal solution $\hat{y} + Pd$.

The noise is resampled for enough iterations to converge, in a joint signal and noise estimation procedure as described in Section 3.7.7. The converged estimate of the noise covariance matrix $N^{-1}$ is then used as the best guess of the correlated noise component.

### 3.7.5 Preconditioners

Through choice of the preconditioner the convergence rate of the method can be increased. Here we use as a preconditioner

$$\sqrt{(A^T A)^{-1}} = \begin{cases} \sqrt{N^{-1}_{\text{hits}}(p)} & \text{for } p = p' \\ 0 & \text{for } p \neq p' \end{cases}$$

(3.27)

where $N_{\text{hits}}(p)$ is the combined number of hits in pixel $p$ across all detectors. An alternative preconditioner would be $(A^T A)^{-1}$ or $(A^T N^{-1} A)^{-1}$ where $N^{-1}$ could be the inverse noise covariance matrix with null off-diagonal elements.

### 3.7.6 Chunks of Noise Stationary Data

The previous steps describe data that is in one continuous stream, in fact data is subdivided into chunks of data over timescales where the noise is considered stationary. For each chunk of data the noise is assumed to be uncorrelated, resulting in the combined noise covariance matrix $N_{ii'}tt'$ across all detectors being block diagonal and each chunk of timestream works with its own small block of $N$ that is inverted separately.

For Boomerang03, the data were divided into 215 chunks of about an hours length. The auto power spectra (and cross spectra in the case of noise correlations between detectors) can be binned logarithmically and stored on disk which can then
Table 3.1: Number of iterations to converge for one continuous chunk of data. The quantity $\varepsilon$ (see Equation 3.25) has decreased to $10^{-5}$ of its original level. These runs were for 12 detectors on 12 CPUs of the Imperial high performance cx1 cluster.

<table>
<thead>
<tr>
<th>Chunk length (hrs)</th>
<th>TOD length (Mb)</th>
<th>No. iterations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>13.86</td>
<td>88</td>
</tr>
<tr>
<td>2</td>
<td>27.62</td>
<td>221</td>
</tr>
<tr>
<td>10</td>
<td>138.60</td>
<td>247</td>
</tr>
<tr>
<td>20</td>
<td>277.19</td>
<td>351</td>
</tr>
</tbody>
</table>

be interpolated when they need to be sampled (Jones et al., 2006).

The length of the chunk sets the length of the noise kernel used to filter the TOD in the calculation of $N^{-1}d$ in Fourier space. Thus the computational time taken for each iteration is $N_{\text{chunk}} \log N_{\text{kernel}}$.

For consideration of noise correlations between different detector timestreams the same approach is taken; we divide all timestreams into chunks, assuming that between chunks there are no correlations, so each block of $N$ encodes the correlations with other detectors for this chunk of timestream.

A comparison of the convergence of SPIMPI as the length of the chunk of timestream analysed increases from 1 hour to 20 hours can be seen in Table 3.1 and Figure 3.2.

### 3.7.7 Noise Power Spectrum Estimation

We don’t have complete knowledge of the noise power spectrum so we reestimate it given the current estimate of the signal. This is done iteratively, with reevaluation of the noise covariance matrix followed by reestimation of the signal. For the first iteration of the mapmaker we will need a realistic estimate for the noise power spectra in order to avoid biases being introduced in the signal power spectrum estimation. For experiments with low signal to noise, a good guess at the noise covariance matrix is to use the power spectrum of the raw signal timestream. When an experiment has high signal to noise, it is more difficult as estimates for the signal and noise must be estimated through a joint iterative procedure.

We resample the noise power spectrum for a number of iterations until it has converged, after which it is frozen in and only the stripes map is updated. To estimate the noise power spectrum, we scan the current iteration of the map estimate $\hat{y} + Pd$ and subtract it from the original TOD to create an estimate of the noise TOD. We then Fourier transform the noise TOD ($n_t$) and average the quantity.
Figure 3.2: Convergence of the PCG algorithm versus number of iterations for varying chunk length, analysing one chunk of length 1 hour (solid line), 2 hours (dotted line), 10 hours (dashed line) and 20 hours (long dashed line). For illustration, the long dash-dotted line shows linear convergence and the short dash-dotted line shows quadratic convergence.
$n^*n$ over bins that are defined logarithmically. When the power spectrum is then interpolated this is also carried out logarithmically. This power spectrum is then used as the filter to generate the update of the ‘stripes map’ $y_{n+1}$.

The recovery of the input power spectrum is an important test of the algorithm. Plots of the input and converged spectra for subset of detectors can be seen in Figure 3.3. This shows good recovery of the input power spectrum, but variation in the final estimate of the noise power spectrum depending on the chunk of timestream analysed.

Where noise is correlated between detectors, all auto and cross power spectra must be calculated at each iteration to form the inverse noise covariance matrix $N^{-1}$. Computationally this can be done in several ways, described in more detail in Section 3.14.

### 3.8 Gap Filling

The continuity of a chunk must be preserved in order to retain noise stationary data. This is important in order to perform the multiplication $N^{-1}d$ in Fourier space, as to have a diagonal matrix in Fourier space requires a circulant matrix in real space. Any gap is damaging to the continuity of the data. Thus gaps in the timestream (which are flagged before being passed to the mapmaker and are due to things such as cosmic rays and changes in scan direction) are filled with constrained realisations of the noise. The ‘constraint’ is that we fill the gap in the bolometer timestream with a realisation of the estimate of the noise power spectrum. Computationally, when we observe a flagged pixel, this sample is directed to a single dummy pixel that is excluded from being reprojected to the map.

Cosmic rays are a potential problem for analysis, they may affect multiple timestreams at once resulting in large sections of data that need to be replaced. The percentage of data flagged for the BOOMERANG03 flight was about 7\%, which was low enough that the impact of the gap-filling procedure was negligible (Jones et al., 2006) while for the BLAST05 flight about 2\% of was removed due to cosmic ray hits in the timestreams (Patanchon et al., 2007).

### 3.9 Memory Requirements

We used between 1 and 40 cores of the Imperial High Performance Cluster cx1 in simulations to date, these are intel Xeon dual core, with 4 MB cache and 8 GB RAM.

The original TOD is written to unformatted textfiles (one textfile per chunk)
Figure 3.3: Comparison of input and recovered noise power spectra from analysis of 1 day long TODs from 12 detectors, each TOD is separated into 24 chunks of an hour of noise stationary data. Here we plot the recovered noise power spectrum for all 24 hour long chunks of timestream for one detector (X1T1R1C8A).
which is then reread when a chunk is analysed so that the TOD doesn’t need to be stored in memory. In terms of memory requirement, we analyse a group of chunks at a time, the size of the group is calculated from the size of all the pointing files that must be read and held in memory for the analysis of each group of chunks. SPIMPI loops over all chunks of the TOD to calculate the number of samples to be analysed and these chunks are then grouped based on the memory available on the machines being used.

The maximum memory that can be allocated depends on the specifications of the cluster. For a typical run on the Imperial high performance cluster to produce results for this thesis where we have analysed anything up to about 10 day long TODs, 7.8 GB was available per node, divided between 4 CPUs per node, each CPU analysing data from one detector. This was more than enough memory for the results presented here.

A 20 day flight with a sample rate of 159 Hz will generate $20 \times 24 \times 60 \times 60 \times 159 \times 8\,\text{bytes} = 2.20\,\text{GB}$ for each array that must be read in (and we need 3 of these for the ra, dec and psi fields). The combined memory requirement across the nodes on the cluster for 512 detectors would be 1.13 TB of data that must be stored per field, with a total of 3.38 TB for three pointing fields. For higher sampling rates, the amount of data needing to be stored in memory would increase beyond the limits of most machines, which is why the ability to analyse the full flight in a series of bunches has been built in.

### 3.10 Increasing Length of TOD and Number of Detectors

This section presents results of analysis of subsets of detectors over a variety of timescales. The convergence versus number of iterations of SPIMPI for the combined temperature and polarisation analysis is shown in Figure 3.4. The solution Stokes parameter maps are shown in Figure 3.5 as the number of detectors increased. For the largest subset of detectors presented here, we show the resulting Stokes parameter maps compared to the input simulation in Figure 3.6. We can assess how well the mapmaking algorithm has done by looking at the difference between the input Stokes parameter maps shown in the left hand column of Figure 3.6 and the output solution maps for a particular run, where we may vary either the number of detectors or length of scan.

In Figure 3.7 we show how we can assess the improvement when increasing the length of timestream analysed from 1 day to 4 days. We look at quantifying the im-
Figure 3.4: Convergence of the PCG algorithm versus number of iterations for varying TOD length and number of detectors. Analysis of 4 detectors (dotted line), 12 detectors (solid line) and 24 detectors (short dashed line) are shown for a 1 day long scan and a longer scan of 4 days is plotted for 24 detectors (long dashed line). For illustration, the long dash-dotted line shows linear convergence and the short dash-dotted line shows quadratic convergence.
Figure 3.5: Comparison of simulations of 1 day’s worth of SPIDER data, assuming that noise was stationary over the whole day (1 chunk) as the number of detectors that are analysed is increased. The sky fraction increases and better crosslinking is achieved. The $Q$ and $U$ Stokes parameter maps were smoothed to $1^\circ$, the same resolution as the input signal maps. This allows comparison of the structure in the $Q$ and $U$ maps as the raw maps are dominated by pixel noise.
Figure 3.6: Comparison of input Stokes parameter maps (left) with a run of 24 detectors over a 1 day period (right). The data is assumed to be stationary over a period of a day so the TOD is analysed in 1 chunk. Both Q and U Stokes parameter maps are smoothed with a $1^\circ$ beam in order to reduce the level of noise in the final Q and U Stokes maps and allow a comparison with the input maps.
Figure 3.7: Comparison of 24 detector TODs over a 1 day period (left) and a 4 day period (right). The data is assumed to be stationary over a period of a day so the 4 day analysis was done in 4 chunks. We show the solution Q Stokes parameter maps smoothed with a 1° beam in the top row. We also plot the difference between the input and solution Stokes maps in the bottom row. The much higher amplitude of the difference maps for the 1 day case along the top and bottom sides of the scan region indicates better recovery of the Q Stokes map over a 4 day period, which is a direct result of better modulation of the detector orientation angle in the 4 day case compared to the 1 day case.
Figure 3.8: Map of the correlated noise (the ‘stripes’ map) for the combination of 24 detector TODs for $I$ Stokes (left) and $Q$ Stokes (right). The length of the scan was 1 day.

Improvement from 1 day to 4 day maps through looking at the difference between the Stokes parameter maps input to the mapmaker and the output solution maps. The difference maps, shown in the bottom row of Figure 3.7, clearly show much higher amplitude along the top and bottom of the scan region for the 1 day maps. This indicates better recovery of the $Q$ Stokes map when looking at 4 days of data, resulting from better modulation of the detector orientation angle over longer timescales.

3.11 Removal of Noise in the TOD

Correlated low frequency $1/f$ noise can be seen in the maps and we have demonstrated that this can be removed by band-passing the data. The timestreams are dominated by this $1/f$ noise below a certain frequency, so we remove these frequencies with a high pass filter. This scan synchronous noise can be seen in the temperature as well as $Q$ and $U$ maps, the striping it produces can be seen in Figure 3.8.

The estimates of the noise power spectrum reached through the PCG method compared to the input noise power spectrum that went into the simulated TODs can be seen in Figure 3.3 for one example detector of a focal plane. These are the final estimate of the noise power spectra after 8 estimations of the noise power spectra.
3.12 Transfer Function

We plot a simplified version of the full transfer function (which is calculated from an ensemble of simulations) that is given by the power spectrum of the SPIMPI output map (either the naively coadded map or PCG estimate) divided by the power spectrum of the input signal map generated by \textit{synfast} and smoothed to a resolution of 1°. It should be emphasised that the results shown here do not show the ‘full’ transfer function, as they are calculated from only one input realisation of the sky. In this Section we analyse 1 day long TODs from 12 detectors in 1 continuous chunk of data over which we assume the noise is stationary.

The ‘base’ case is when we input a realistic level of noise and signal into the simulated TODs and run it through the mapmaker, see Figure 3.9. The features in this Figure can be better understood by looking at two extreme cases where we amplify the noise and signal components of the simulated TODs.

The first of these we look at is an unrealistic, ‘low noise’, case where the noise timestream going into the simulation is divided by 1000 so that we are looking at something close to a ‘signal’ transfer function. This is shown in Figure 3.10. Clearly, for this case the naive map should be very close to the best estimate of the map and indeed we see that there is almost perfect recovery of information. However, the first estimate for the noise power spectrum has not been changed from the ‘base’ case so clearly the noise estimation procedure will not converge on the correct noise power spectrum resulting in some of the signal being treated as noise. The transfer function for the PCG solution therefore shows that we have filtered some of the signal leading to worse recovery for large scales.

The other case looked at is an unrealistic, ‘low signal’, case where the signal timestream going into the simulation is divided by 1000 in order to look at how the noise behaves. This is shown in Figure 3.11. As the division is carried out in the simulation of the TODs, the power spectrum of the input map is much too large, which is why the transfer function is now much less than one. However, for the $TT$ spectrum, we see the expected exponential increase at high $\ell$ from the noise term that is filtered down in the iterated map.

The behaviour of the transfer function of the naive map at small scales (high $\ell$) in Figure 3.9 is due to the noise component that was added to the timestreams. The increase in the transfer function is explained by looking at the ‘low signal’ case where the timestream only contains noise; the top left panel of Figure 3.11 shows exactly the same increase in the transfer function. By comparing the transfer functions of the naive and iterated maps on small scales for the $TT$ case (top left panel of Figure 3.9), it can be seen that the algorithm effectively removes the noise...
component of the timestream, as it brings the transfer function back to one, which indicates the same amount of power on all scales in the input signal map and output iterated map. For the $EE$ and $BB$ transfer functions it can also be seen that the behaviour on small scales in Figure 3.9 comes from the noise component shown in Figure 3.11.

### 3.13 Filtering

Analysis of Spider data will need accurate removal of systematic effects. It will probably be necessary to filter the raw timestreams before the mapmaking procedure is carried out. Filtering the TODs to remove scan synchronous systematics suppresses both the signal and noise components in the TOD, which decreases the signal to noise ratio on large scales and suppresses the amplitude of the output power spectra. Filtering of low frequency noise leads to larger variance in power spectrum estimates compared to the optimal maximum likelihood approach. It is important to look at the recovery of the signal component for the naive map compared with the maximum likelihood PCG estimate.

Through Monte Carlo generation of signal only realisations the signal transfer function on all angular scales could be calculated; this will be looked at in future work. This transfer function can be deconvolved from the estimated CMB power spectrum of output maps from Spimpi to give the unbiased power spectrum of the CMB signal. These transfer functions could be used to look at $E \rightarrow B$ leakage from the filtering of the TODs as in Sutton et al. (2009).

We have looked at the transfer functions for the $TT$, $EE$, $BB$ and $TE$ power spectra following the bandpassing of the input TODs by a high pass filter of 0.010 Hz and low pass filter of 100 Hz (see Figure 3.12). This was for a 12 detector run over a period of 1 day which was analysed in one continuous chunk. The $TT$ transfer function shows that the iterative procedure recovers information on large angular scales compared to a naive coadding of the data. This filtering has less impact on $EE$ and $BB$ power spectra. as polarisation remains noise dominated, there is little information in these maps on large scales.

### 3.14 Detector Correlations

A huge challenge for massively multi detector experiments is the problem of calculating and analysing the auto- and cross-correlations between detectors. These correlations need to be calculated over each chunk of noise-stationary data. There
Figure 3.9: Transfer function versus angular scale showing the TT (top left), EE (top right), BB (bottom left), TE (bottom right) power spectrum of SP1MPI output maps divided by the power spectrum of the input signal map generated by *synfast* for the naively coadded map (blue) or PCG solution map (black). This was for a 12 detector run over a scan of 1 day analysed in 1 chunk, therefore assuming noise stationarity over the whole day.
Figure 3.10: Transfer function versus angular scale showing the TT (top left), EE (top right), BB (bottom left), TE (bottom right) power spectrum of low noise SPIMPI output maps divided by the power spectrum of the input signal map generated by synfast for the naively coadded map (blue) or PCG solution map (black). This was for a 12 detector run over a scan of 1 day analysed in 1 chunk, therefore assuming noise stationarity over the whole day.
Figure 3.11: Transfer function versus angular scale showing the TT (top left), EE (top right), BB (bottom left), TE (bottom right) power spectrum of low signal SPIMPI output maps divided by the power spectrum of the input signal map generated by synfast for the naively coadded map (blue) or PCG solution map (black). This was for a 12 detector run over a scan of 1 day analysed in 1 chunk, therefore assuming noise stationarity over the whole day.
Figure 3.12: Transfer function following *bandpassing of input TODs: high pass* 0.010 Hz, *low pass* 100 Hz for the TT (top left), EE (top right), BB (bottom left), TE (bottom right) power spectra of SPIMPI output maps divided by the power spectrum of the input signal map generated by *synfast* for the naively coadded map (blue) or PCG solution map (black). This was for a 12 detector run over a scan of 1 day analysed in 1 chunk, therefore assuming noise stationarity over the whole day.
are $N_{\text{det}}(N_{\text{det}} - 1)/2 + N_{\text{det}}$ auto and cross power spectra to calculate, which could lead to problems for experiments with potentially hundreds of correlated detectors.

In testing the ability of SPIMPI to deal with correlated noise, we must first simulate these correlations. The noise timestream is made up of two components, a correlated noise component as well as an uncorrelated Gaussian white noise component seen by each individual detector. Introducing correlations to TOD simulations involves statistically correlating the $1/f$ noise TODs for subsets of detectors and then combining this correlated timestream with the uncorrelated Gaussian white noise realisations for each detector.

Sutton et al. (2009) account for noise correlations between detectors in the DESCART algorithm. The SANEPIC algorithm also describes a method for calculating the cross spectra between different detectors in Patanchon et al. (2007). The calculation of the cross power spectra in SPIMPI will be presented in a future work.

### 3.15 Differencing Timestreams

This Section summarises the work in Jones et al. (2006) on differencing timestreams of PSB pairs. SPIDER’s detectors are arranged in pairs to measure polarisation along orthogonal axes. BOOMERANG03, PLANCK and QUAD all made use of Polarisation Sensitive Bolometers (PSBs), for which the use of difference time-streams between PSB pairs that were oriented in different directions is useful in the removal of scan synchronous effects. A detailed description of the benefits of differencing timestreams in a PSB pair is given in Jones et al. (2006). We would like to look at the effect of differencing timestreams on map estimation for SPIDER.

If we take a sample

\[
s_i = I + \gamma(Q \cos 2\psi_i + U \sin 2\psi_i)
\]

then the sum and difference can be written as

\[
^+s_i = \frac{1}{2}(s^1_i + s^2_i) = I + \frac{1}{2}(^+\alpha_iQ + ^+\beta_iU)
\]

\[
^-s_i = \frac{1}{2}(s^1_i - s^2_i) = \frac{1}{2}(^-\alpha_iQ + ^-\beta_iU)
\]

where 1 and 2 labels the detectors in a pair. The coefficients $\alpha$ and $\beta$ are given by

\[
^\pm\alpha_i = \gamma^1 \cos 2\psi_i^1 \pm \gamma^2 \cos 2\psi_i^2
\]
\[ \pm \beta_i = \gamma^1 \sin 2\psi^1_i \pm \gamma^2 \sin 2\psi^2_i \]  

(3.32)

It can be seen that this summing and differencing of detector TODs in a pair separates the temperature and polarisation components.

Linear combinations of the differenced timestreams are now formed through

\[
\begin{pmatrix}
-s^i\alpha_i \\
-s^i\beta_i
\end{pmatrix} = \frac{1}{2} \begin{pmatrix}
-\alpha^2_i & -\alpha_i\beta_i \\
-\alpha_i\beta_i & -\beta^2_i
\end{pmatrix} \begin{pmatrix}
Q \\
U
\end{pmatrix}
\]  

(3.33)

The polarisation decorrelation matrix becomes

\[
-M_i = \frac{1}{2} \sum_{j}^{N_{\text{pairs}}} w_j \sum_{i\epsilon p} \begin{pmatrix}
-\alpha^2_i & -\alpha_i\beta_i \\
-\alpha_i\beta_i & -\beta^2_i
\end{pmatrix}
\]  

(3.34)

where \( w_j \) is now weight for pair \( j \) and \( N_{\text{pairs}} \) is the number of pairs. \( Q \) and \( U \) can then be substituted into Equation 3.29 to obtain the \( I \) Stokes parameter map.

Including an option in SPIMPI to work from sum and difference TODs is quite straightforward. We have produced naive maps of temperature and polarisation working from sum and difference timestreams. However, there are difficulties with implementing the PCG method computationally in the current version of SPIMPI that have not yet been addressed.

### 3.16 Concluding Remarks

We have developed SPIMPI an algorithm for estimating temperature and polarisation maps from massively multi-detector experiments. This code uses the PCG method, a fast method for iterative inversion of matrices. We have implemented the code in MPI in a novel way, involving the handling of only one detector’s timestream per processor. The code is straightforwardly scalable to any number of detectors when run on a high performance cluster with several hundred cores.

We have implemented maximum likelihood mapmaking with the added option to filter the raw TODs to remove low frequency scan synchronous effects. It has been shown through the filter transfer function that while TOD filtering involves loss of information on large angular scales, the PCG method recovers some of this information.

As we increase the number of detectors, the number of iterations needed for convergence should decrease due to better crosslinking of observations leading to a better conditioned polarisation decorrelation matrix. However, in our current analysis we have not masked the regions around the edges of the map that remain dominated by
noise as the number of detectors increases which may explain why this is not seen
in the plots of convergence shown here.

In a future publication we will describe the analysis of noise correlations between
detectors. We will also look at comparing the results from analysing detector TODs
separately with results from using the difference timestream between pairs of detec-
tors in the hope of better removal of certain scan synchronous effects.

**Acknowledgments**

We acknowledge useful discussions with the SPIDER team and in particular Jeff
Filippini and Becky Tucker for providing realistic noise estimates. We acknowledge
use of the FFTW library and HEALPIX software. Calculations were carried out on
a facility provided by the Imperial College High Performance Computing Service\(^1\).

\(^1\)http://www.imperial.ac.uk/ict/services/teachingandresearchservices/
    highperformancecomputing
4 Application of SPIMPI to Boomerang 2003 Data

4.1 Introduction

This section focuses on applying SPIMPI to data from the 2003 flight of Boomerang (hereafter B03). Boomerang’s first flights in 1997 and 1998 provided the first high signal to noise measurements of temperature anisotropies in the CMB at resolution of less than 1 ° and detected the first peaks of the temperature power spectrum.

After improving the pointing accuracy and upgrading to include polarisation sensitive bolometers (PSBs), B03 also made high signal to noise maps of the temperature anisotropy of the CMB in all frequency bands and made a statistical detection of CMB polarisation in its 145 GHz band.

We neglect some complexities that went into generating these results in order to focus on broader features of the mapmaking algorithm when applied to a real dataset.

4.2 The B03 Experiment

B03 launched on 6th January 2003 near McMurdo in Antarctica, taking data for 11 days. Measurements of the $TT$, $EE$, $BB$ and $TE$ power spectra from this flight can be found in Piacentini et al. (2006), Montroy et al. (2006) and Jones et al. (2006) while the cosmological parameters obtained are described in MacTavish et al. (2006). After 11 days the altitude of the experiment became too low for pointing reconstruction.

The 2003 flight spent 75 hours on a larger region of 3.0% of the sky that was mapped with shallow scans (corresponding to less integration time per pixel) and 125 hours on the smaller region of 0.28% mapped with deep scans (longer integration time per pixel). The size of the patches was roughly 750 and 90 square degrees respectively. These patches were chosen to give the highest signal to noise ratios for $TE$ and $EE$ measurements respectively. However, restrictions on the scan strategy
Table 4.1: Characteristics of the B03 receiver (the noise (NET) column is the average for all detectors in a frequency band). The Table is taken from taken from Masi et al. (2006).

<table>
<thead>
<tr>
<th>Frequency GHz</th>
<th>Bandwidth GHz</th>
<th>No. Detectors</th>
<th>Beam FWHM arcmin</th>
<th>NET$_{\text{CMB}}$ $\mu K \sqrt{s}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>145</td>
<td>45</td>
<td>8</td>
<td>9.95</td>
<td>170</td>
</tr>
<tr>
<td>245</td>
<td>80</td>
<td>4</td>
<td>6.22</td>
<td>320</td>
</tr>
<tr>
<td>345</td>
<td>100</td>
<td>4</td>
<td>6.90</td>
<td>450</td>
</tr>
</tbody>
</table>

lead to some compromise and the size of the shallow region was constrained by the galaxy on one side of the scan region and keeping the telescope pointed away from the sun.

The part of the shallow region used for spectral analysis was 1.8%. For the analysis of the deep region used in polarisation results, measurements where taken from 0.22% of the sky over 6 days in a region centred around Right Ascension 82.5° and Declination −45°.

The scans consist of azimuthal scans of the gondola at fixed elevation for at least an hour with cross linking of observations obtained by sky rotation. The elevation was stepped between observation periods, no more than once an hour.

The bolometer responsivity changed during the flight as a result of changes in temperature and loading. A calibration lamp behind a small hole in the tertiary mirror recorded calibration drifts by flashing at 15 minute intervals. The calibration signal measured by the lamp showed calibration changes of a few percent throughout the B03 flight.

### 4.3 The Receiver

The 2003 flight measured bands centred at 145 GHz, 245 GHz and 345 GHz. B03 had eight pixels with pairs of detectors in each. Four of the pairs where PSBs operating at 145 GHz while the other four were 2-color photometers.

For the reported polarisation spectra, measurements were from the four pairs of PSBs at 145 GHz. The PSBs consisted of a pair of square grids oriented at right angles to measure both components of the electric field for each pixel of the focal plane. All PSBs were in a row at the same elevation.

The other bands used spider web bolometers which consisted of mounted polarising grids at the front of the feed horns, each bolometer was sensitive to one polarisation component. These were separated by 0.5° in elevation from the row of PSBs. Across both rows of detectors, adjacent pixels were separated by 0.5°. See Figure 4.1 for the layout of the focal plane. See Table 4.1 for more details on the
4.4 Mapmaking

Processing of the raw TODs was similar to the procedure that SPIDER data will undergo. This processing involved flagging and filling gaps in the TODs caused by things such as cosmic ray hits, samples contaminated by the calibration lamp and some samples around elevation changes. The next stage was the deconvolution of the combined transfer function from the thermal response of the detector and also from the high pass filter of the readout electronics using pre- or in-flight measurements of these transfer functions.

The CMB dipole has been removed from this data. This shows up as a rise and fall along the scan direction, creating a triangular pattern in the signal measured in the raw TODs, see Figure 4.2 for the measured dipole signal across the scan region.

Masi et al. (2006) outlines the making of polarisation maps from the 2003 flight from the two pipelines that were used, the NA (North American) and IT (Italian) pipelines. Both pipelines achieved consistent results, which indicated that the mapmaking methods were robust to differences between the algorithms.
The pipelines use the iterative Generalised Least Squares (GLS) method described in Chapter 3 to jointly estimate the $I$, $Q$ and $U$ Stokes parameters. They follow the procedure described previously, using the linear estimator given by Equation 3.9 for the estimation of the sky map from a datastream. The IT pipeline solved this linear system using a preconditioned conjugate gradient (PCG) method (de Gasperis et al., 2005) and the NA pipeline used a Jacobi method (Jones et al., 2006).

One difference between the two methods is that the IT pipeline assumed the noise was stationary over the full flight whereas the NA pipeline separates the timestreams into approximately one hour chunks, which means a slightly less accurate noise filter but no assumption of noise stationarity over the whole flight.

Through cross correlation of detector timestreams, it was observed that there was a small correlated noise component between different detectors that was neglected in the mapmaking pipelines but accounted for in power spectrum estimation. Both pipelines assume no noise correlation between detectors; in reality the level of cross correlation was about two orders of magnitude below the auto correlations of the noise timestreams.

Another important difference when comparing with SPIMPI is that the IT pipeline analysed detector timestreams separately while the NA pipeline differedenced
Figure 4.3: Left: The Boomerang collaboration maps of the I, Q and U Stokes parameters for the deep region (left) and the shallow analysis region (right)
the signals of detectors within PSB pairs.

An outline of the IT method is given in Masi et al. (2006). They first produce a naive map by coadding a bandpass filtered TOD into a map, this is subtracted from the original TOD to give an estimate of the noise, from which the noise power spectrum is estimated (this method for joint signal and noise estimation was developed in Ferreira and Jaffe (1999)). This estimate of the power spectrum is then used to generate the next iteration of the signal map. Approximately 5 iterations were needed for convergence of the noise power spectra for B03 data. It was demonstrated that there was no bias on the noise estimation from this procedure through a series of simulations of both signal and noise. This procedure was carried out for each bolometer from which variation in the calibration of each detector was estimated. The ROMA pipeline took about 20 minutes on 128 processors and converged to the required level within about 200 iterations.

The B03 maps for the deep and shallow region can be seen in Figure 4.3, with the masks of the deep and shallow analysis regions applied.

### 4.5 Reanalysing 145 GHz Data with SPIMPI

In order to test SPIMPI, we performed analysis of data from the 145 GHz PSBs of the B03 flight. The data included pointing files for all 8 bolometers along with the measured bolometer TODs. These TODs had undergone the flagging procedure previously described and the dipole and several other artifacts had been removed. The bolometer sampling frequency for B03 was 60 Hz. One feature we do not look into is differences in calibration between detectors.

#### 4.5.1 A First Analysis

Before removal of scan synchronous effects by high pass filtering, the converged maps can be seen in Figure 4.4 compared with the maps produced by the Boomerang collaboration. A large scan synchronous signal is seen although in the centre of the map CMB temperature fluctuations can be seen.

These maps include all eight 145 GHz detectors analysed without differencing of PSB pairs. We do not include variations in calibration of the different detectors and the units of the output maps are in Volts. In order to convert to temperature units, the responsivity of the detectors must be taken into account. We analyse the data in chunks of about an hour long and perform joint iteration on the signal and noise. We estimate the noise power spectrum individually for each chunk and freeze in the
Figure 4.4: $I$, $Q$ and $U$ Stokes parameter maps of the B03 deep region (left) compared with the PCG solution from SPIMPI (right). The iteration involved analysis of 129 chunks of noise stationary data. The noise estimate was reached through 8 iterations before being frozen in.
Figure 4.5: Convergence of $I$, $Q$ and $U$ for all B03 detectors, from analysis of the deep and shallow regions. The convergence is also plotted as the high pass filter applied to the TODs is increased.
noise after 8 iterations, assuming it to have converged. We analyse the deep region in 20 bunches (see Section 3.9).

The residual plotted versus the number of iterations for the combined $I$, $Q$ and $U$ analysis of the deep region separated into 129 chunks is shown in Figure 4.5. SPIMPI converged to $10^{-5}$ of the initial residual within 70 iterations on 8 processors for all runs presented here. This can be compared to the value presented in Masi et al. (2006) for the ROMA pipeline, which took about 20 minutes and 200 iterations on 128 processors to reach the required tolerance. The tolerance level achieved is not reported in this paper, but it seems that the SPIMPI algorithm provides a fast way of analysing BOOMERANG data with minimal computational resources.

Apart from the increase in available computing power that will result in speed up of the algorithm, the improvement in performance when using SPIMPI could be due to several aspects of the algorithm. The main details of the algorithm are similar (using the preconditioned conjugate gradient (PCG) method along with joint signal and noise estimation) however there are clearly differences in implementation that result in a speed up of the algorithm. The clearest difference is the method for parallelisation used in SPIMPI; we handle one TOD per processor so only require 8 processors, while the ROMA pipeline handles one chunk per processor resulting in the use of 128 processors.

Any comparison of the SPIMPI maps with BOOMERANG collaboration maps has been done by eye, as the aim of this Chapter is to show that SPIMPI is able to handle a realistic dataset and to highlight some of the complications (for example scan synchronous effects) that arise when applying mapmaking algorithms to real data. We have not attempted a quantitative comparison of both pipelines, as we have not corrected for differences in calibration between detectors, something which is outside the scope of this work.

### 4.5.2 Bandpassing the Timestream

B03 data required harsh high pass filtering to remove the effects of scan synchronous noise. This high pass filter was set to $\approx 7.5$ times the scan frequency, (Masi et al., 2006). In order to remove the large scan synchronous signal seen in Figure 4.4, the TOD must have a high pass filter applied. This removes frequencies in the TOD below the filter cut-off, which are dominated by noise. By applying a high pass filter, the scan synchronous effect was removed. In Figure 4.6 we look at two cases for the high pass filter, it can be seen that a lot of the noise is removed as the filter cut off frequency is increased.

Similar plots for the shallow region can be seen in Figure 4.7 when a high pass filter
Figure 4.6: $I$ and $Q$ Stokes parameter maps of the B03 deep region (top row) compared with the PCG solution from SPIMPI after a high pass filter of 0.019 Hz was applied (middle row) and 0.040 Hz (bottom row). The iteration involved analysis of 129 chunks of noise stationary data.
Figure 4.7: $I$ and $Q$ Stokes parameter maps of the B03 shallow region (top row) compared with the PCG solution from SPIMPI after a high pass filter of 0.019 Hz was applied (middle row) and 0.040 Hz (bottom row). The iteration involved analysis of 81 chunks of noise stationary data.
Figure 4.8: $I$ and $Q$ Stokes parameter maps of the B03 deep region (top row) compared with the PCG solution from SPIMPI after a high pass filter of 0.040 Hz was applied and the solution map is further smoothed with a Gaussian beam of 1° (bottom row). The iteration involved analysis of 129 chunks of noise stationary data.
is applied. The resulting maps still contain some noise, however CMB temperature fluctuations very similar to the BOOMERANG collaboration maps can be seen.

Convergence for the bandpassed TODs was similar to the analysis with no filtering, see Figure 4.5

4.5.3 Smoothing the Maps

The polarisation measurements of B03 were low signal to noise maps and therefore there is no structure in the observed $Q$ and $U$ maps. To check by eye whether there is any structure, we can smooth the data with a Gaussian beam of $1^\circ$ which will average down the noise.

This is carried out for the maps in Figure 4.8. We see that the smoothed map is dominated by noise. There is higher amplitude around the edges of the map, where there is more noise due to fewer observations.

4.6 Concluding Remarks

In this Chapter, we have analysed data from the deep and shallow regions observed by the B03 experiment. We have looked at temperature and polarisation data from the 145 GHz channel which consisted of 8 polarisation sensitive bolometers oriented to give measurements of polarisation in as many orientations as possible.

The combined analysis of all 8 detectors (with no differencing) converged to $10^{-5}$ of the initial residual within 70 iterations on 8 processors for all runs presented here, so SPIMPI provides a very fast way of analysing data from this experiment.

We have seen that the data includes a large scan synchronous noise component that must be removed through high pass filtering of the timestreams. We have looked at two different levels of filtering and presented the difference visually in the estimated Stokes parameter maps. The results of this filtering have been shown for the deep and shallow regions, producing temperature maps with high signal to noise and polarisation maps dominated by noise as expected. It is clear that we have not removed all the low frequency noise even with the harshest filter used in this discussion. However, we have demonstrated that SPIMPI, developed in the context of the SPIDER experiment, can be used with little modification to analyse data from similar experiments.

It would be interesting to develop this analysis by differencing the datastreams of detectors within PSB pairs, as was done in the NA pipeline. Differencing PSB pairs can help in removal of scan synchronous components seen by detector pairs.
Acknowledgments

We acknowledge the use of data provided by the BOOMERANG collaboration. We acknowledge use of the FFTW library and HEALPIX software. Calculations were carried out on a facility provided by the Imperial College High Performance Computing Service.
5 Galactic Modelling

5.1 Introduction

Foreground emission from within our Galaxy can be polarised due to the presence of the Galactic Magnetic Field. There are two dominant polarised foregrounds: interstellar dust and synchrotron radiation. Mechanisms that align dust grains perpendicularly to the magnetic field leads to net linear polarisation of this emission and synchrotron radiation is also polarised in a direction at right angles to the magnetic field direction. The model presented here predicts the polarisation amplitude and angle based on a chosen GMF model. The choice of total intensity for our templates is external to the model and can be set to reflect any existing template. Given the sensitivity of the foreground emission templates to the underlying GMF, upcoming data will allow for the possibility of constraining the parameters of GMF models and better understanding of the structure of the magnetic field, the dust density and the cosmic ray electron density distribution in our Galaxy.

The model this work is based on was introduced by O’Dea (2009) and first applied in O’Dea et al. (2011b) for the purpose of studying the impact of polarised foregrounds on SPIDER’s ability to detect $B$-mode polarisation. The work in this thesis draws on two papers. Firstly in O’Dea et al. (2012) we use this model to provide specific templates at a range of frequencies that will be targeted by upcoming experiments, a process which involved studying the differences in Galactic Magnetic Field (GMF) models and choosing values of parameters describing the GMF structure and amplitude based on current experimental knowledge. In Clark et al. (2012) the GMF and foreground modelling code is extended to calculate Stokes parameters from synchrotron emission.

In this Chapter we summarise the GMF modelling, based on 3D models for the large scale magnetic field and 1D/3D realisations of a turbulent small scale magnetic field. We also describe the modelling of the dust density and cosmic ray electron density distributions and properties of dust and synchrotron radiation.
5.1.1 Galactic Magnetic Field

Understanding the structure and formation of the GMF is an important area of research that is essential for studies of the evolution of spiral galaxies. One theory for their formation is a dynamo effect resulting from the rotation of the Galaxy in the presence of a ‘seed’ field. There are various possibilities for the ‘seed’ field from which the GMF forms, including a primordial field or a small scale dynamo effect from turbulence. This is a wide area of research, some discussion of the formation of the GMF can be found in Han and Qiao (1994) and Han et al. (2006).

Away from the Galactic center, the Galactic magnetic field is usually considered to have two near-independent components: a large-scale coherent field associated with the Galactic disk, and a small-scale field arising from turbulence in the interstellar plasma sourced by astrophysical events such as supernovae and stellar winds. The most informative probes of these fields are Faraday rotation measures of pulsars and extra-Galactic radio sources (Haverkorn et al., 2006; Han et al., 2006). Whilst there is general agreement that the large-scale field follows a spiral pattern, its detailed structure is still uncertain.

When considering areas of sky at high Galactic latitudes, this uncertainty is unimportant as the dust is concentrated in a thin disk about the Galactic plane, and so we only see emission within around 1 kpc or so of the Sun, a region in which the large-scale field is reasonably well characterised. However, experiments which will target a large fraction of the sky, possibly including part of the Galactic plane, will require a model of the large-scale field structure in the plane.

Attempts have been made to constrain the properties of the magnetic field using CMB polarisation measurements. Jaffe et al. (2010) use a Monte Carlo Markov Chain (MCMC) approach to test components of a 2D Galactic field model using rotation measures and WMAP data in the plane of the Galaxy. Jansson et al. (2009) use rotation measures and WMAP 5 year data to fit for parameters in common 3D models for the Galactic magnetic field. We choose two of the most popular forms for the Galactic magnetic field and study both in the context of polarised dust emission before restricting ourselves to one of these forms when modelling synchrotron emission.
5.1.1.1 Large-Scale Magnetic Field

One popular candidate is the Bi-Symmetric Spiral (BSS) (Han and Qiao, 1994; Sun et al., 2008) which can be written as

\[
B_\rho = -B_0 \cos \left( \Phi + \psi \ln \frac{\rho}{\rho_0} \right) \sin p \cos \chi, \\
B_\phi = -B_0 \cos \left( \Phi + \psi \ln \frac{\rho}{\rho_0} \right) \cos p \cos \chi, \\
B_z = B_0 \sin \chi.
\] (5.1)

Here, \( \rho, \Phi \) and \( z \) are Galacto-centric cylindrical co-ordinates with \( \Phi \), the cylindrical longitude, measured from the direction of the Sun, \( p \) is the pitch angle of the field, \( \psi = 1/\tan p \), \( \rho_0 \) defines the radial scale of the spiral, \( \chi = \chi_0 \tanh(z/z_0) \) parametrizes the amplitude of the \( z \) component and \( z_0 = 1 \) kpc. We use the parameters constrained in Miville-Deschênes et al. (2008): \( p = -8.5 \) degrees, \( \rho_0 = 11 \) kpc and \( \chi_0 = 8 \) degrees, with the field amplitude set to \( B_0 = 3 \mu G \), and take the distance between the Sun and the Galactic center to be 8 kpc. A diagram of the magnetic field orientation and magnitude in the BSS model is shown in the left panel of Figure 5.2 for a slice through the Galactic plane and the left panel of Figure 5.1 shows a side on view towards the Galactic centre.

A number of other magnetic field models have been proposed in the literature. For comparison we also include the Logarithmic Spiral Arm (LSA) model introduced by Page et al. (2007) for use in cleaning of the WMAP data. The model is defined as

\[
B_\rho = -B_0 \sin \left( \psi_0 + \psi_1 \ln \frac{\rho}{\rho_W} \right) \cos \chi, \\
B_\phi = -B_0 \cos \left( \psi_0 + \psi_1 \ln \frac{\rho}{\rho_W} \right) \cos \chi, \\
B_z = B_0 \sin \chi,
\] (5.2)

with parameters obtained by fits to the WMAP K-band field directions; \( \psi_0 = 27 \) degrees, \( \psi_1 = 0.9 \) degrees, and \( \chi \) defined as in the BSS model but with \( \chi_0 = 25 \) degrees. The radial scale is also different in this model with \( \rho_W = 8 \) kpc whereas the scale height is the same as above with \( z_0 = 1 \) kpc. There is no azimuthal dependence in this model. The right panel of Figure 5.2 shows a slice through the Galactic plane for the LSA model while the right panel of Figure 5.1 shows a side on view towards the Galactic centre.

Although both fields are unlikely to provide a full description of our Galaxy (Men
et al., 2008; Sun et al., 2008), they are sufficient for our current purpose as we do not require a precise template of the sky, only a reasonable approximation against which to test foreground separation techniques and the performance of experiments in the presence of systematic effects.

Both magnetic field models assume the field strength $B_0$ is constant although there is weak evidence for some radial dependence (Han et al., 2006). Any such dependence will not affect the polarisation model significantly and the overall radial dependence of the signal is determined by the exponential drop-off in the dust density which modulates the integrand along the line-of-sight. Field reversals may also be present in the spiral arms but, if sharp enough, will not contribute to the signal significantly.

### 5.1.1.2 Small-Scale Galactic Magnetic Field

The turbulent field is somewhat less well understood. When constraining the above large-scale field, Miville-Deschênes et al. (2008) simultaneously fit a small-scale field with best-fit r.m.s amplitude $B_{\text{r.m.s.}} = 1.7 \mu G$. Several different studies agree that the r.m.s. amplitude is similar to the amplitude of the large-scale field in the Solar vicinity (Fosalba et al., 2002; Han et al., 2006), and so here we set $B_{\text{r.m.s.}} = 2 \mu G$. Minter and Spangler (1996) examined the rotation measures of extra-Galactic sources across
Figure 5.2: Cartesian projection of the BSS (left panel) and LSA (right panel) large-scale magnetic field models, showing a slice through the Galactic plane observed along the positive $z$-axis. The filled circle represents the position of the Sun in relation to the Galactic centre. The alignment and magnitude of the magnetic field are shown as headed ticks with the $B_0 = 3 \mu$G scale represented at the top of each panel. The red solid lines are the density contours, in steps of 0.1, of the dust density model $n_d(r, z)$ (see Section 5.2.2) used in the line-of-sight integration. The dust density is normalised to 1 at the Galactic centre.

A small patch of sky and concluded that the data were consistent with Kolmogorov turbulence on scales smaller than 4 pc, assuming a statistically isotropic, homogeneous Gaussian field. On larger scales they found a somewhat flatter energy spectrum with an outer scale of up to 96 pc. Kolmogorov-type spectra up to kilo-parsec scales in the interstellar magnetic field and other interstellar plasma components have also been reported by other studies (Armstrong et al., 1995; Lazarian and Pogosyan, 2000; Cho and Lazarian, 2008).

Kolmogorov turbulence describes the energy distribution among vortices of different size, with the amplitude of the turbulence related to the energy density at that position. Turbulent flow can be viewed as an energy cascade from larger to smaller eddies. At small enough length scales, known as the Kolmogorov length scale, energy is dissipated through viscous dissipation. A Kolmogorov spectrum is proportional to the rate of energy dissipation and the magnitude of the wavevector $k$. Using the Kolmogorov energy spectrum one finds that the power spectrum of a turbulent field is $P(k) \propto k^{-(2+3N_d)/3}$ where $N_d$ is the number of spatial dimensions of the realisation.

We generate realisations of this power spectrum in Fourier space, then apply a Fast Fourier Transform (FFT) to model the three-dimensional turbulent magnetic field.
field in real space. When the line of sight integrals are calculated, the value of the turbulent field is calculated from this 3D Cartesian grid through appropriate coordinate transformations and interpolation. For full sky simulations this box is centred on the Sun, while for smaller patches of sky, this 3D grid is centred between the patch of sky and the Sun.

It is numerically intractable to generate a realisation of this turbulent field in three dimensions at sufficiently high resolution and to accommodate the entire sky, hence we resort to independent one-dimensional realisations along the line-of-sight to each pixel. This model ignores correlations across the sky, but properly incorporates the line-of-sight depolarisation. We choose an injection scale of 100 pc, assume the dissipation scale is small and use the one-dimensional Kolmogorov energy spectral index of $-5/3$.

For smaller patches of the sky, relevant for ground-based observations, a full, three dimensional realisation is feasible together with a higher angular resolution in the line-of-sight integrals.

### 5.1.2 Total GMF Model

We combine the small-scale ($ss$) and large-scale ($ls$) magnetic field values according to

$$
B_r = B_{r,ss} + B_{r,ls},
B_\theta = B_{\theta,ss} + B_{\theta,ls},
B_\phi = B_{\phi,ss} + B_{\phi,ls},
$$

where $r$, $\theta$, and $\phi$ are now Solar-centric spherical polar co-ordinates.

### 5.2 Interstellar Dust

While little data is available on polarisation of thermal dust emission, we do have templates of its total intensity. Modelling thermal dust emission also requires details of grain alignment in a magnetic field and the intrinsic polarisation of the emission from an individual grain of dust.

#### 5.2.1 Dust Total Intensity

Although few data are available regarding the polarised emission from dust, the same is not true of its total intensity. In particular, the IRAS satellite observed
this emission across the sky at 100 µm and 240 µm, close to the peak in the dust emission. By constraining physically-motivated extrapolations of these observations using further data, Finkbeiner et al. (1999, hereafter FDS) provided models of the emission at microwave wavelengths. At 94 GHz, these models have been shown to agree well in terms of morphology with the WMAP observations with some minor structural differences on the Galactic Plane (Gold et al., 2009). However, there are indications that in terms of amplitude the WMAP dust template fit coefficients differ by about 30%. Bennett et al. (2003) suggest that this is possibly due to the degeneracy that exists between the strongly correlated dust and synchrotron emission components in the simultaneous fit of their externally derived template maps to WMAP data.

In the higher frequency bands relevant to experiments observing above \( \sim 90 \text{ GHz} \), data are more limited but agree well with the FDS predictions (Culverhouse et al., 2010; Veneziani et al., 2010). We will use this model (to be precise model number eight in FDS) to trace the total intensity of the dust emission. We exploit the full, 6.1 arcminutes, resolution of the IRAS data by pixelising the dust intensity on healpix (Górski et al., 2005) maps of \( N_{\text{side}}=1024 \).

5.2.2 Dust Properties

We model the large-scale spatial distribution of the dust density, \( n_d \), using a model that was developed in Drimmel and Spergel (2001),

\[
    n_d = n_0 \exp \left( -\frac{\rho}{\rho_d} \right) \sech^2 \left( \frac{z}{z_d} \right). \tag{5.4}
\]

For consistency with the WMAP polarisation analysis (Page et al., 2007), we take the scale height \( z_d = 200 \text{ pc} \) and the scale radius \( \rho_d = 3 \text{ kpc} \). We do not attempt to model the small-scale variations in the dust density and temperature here, which may also affect the polarisation degree and direction. Small-scale variations in the total intensity are included via the FDS model.

The model also requires a description of the physics of grain alignment and of the intrinsic polarisation of the emission from an individual grain. In general these are complex functions of the magnetic field and various properties of the grains. Recently, good progress has been made in describing the details of the alignment using the theory of radiative torques (Lazarian and Hoang, 2007; Hoang and Lazarian, 2008). However, it is still difficult to produce a well-constrained quantitative description to apply to our model (Lazarian and Hoang, 2009).

Instead, we describe the alignment in an integrated manner, without recourse to
the details of a particular physical mechanism. We assume that the polarisation direction is always perpendicular to the component of the magnetic field in the plane of the sky, and that the degree of polarisation depends quadratically on the magnetic field strength. This is similar to the behaviour assumed in Page et al. (2007). We follow this approach in providing our templates and do not attempt to account for any possible misalignment of the axis of orientation of the dust grains with the magnetic field lines, as is done in other work, for example Fauvet, L. et al. (2011).

5.3 Synchrotron Radiation

Despite limited information on the polarisation of synchrotron emission, there are templates of its total intensity. For modelling the distribution of the cosmic ray electron density, we assume that it has the same form as the dust distribution. However, the scale height of the distribution of cosmic ray electrons is quite different, leading to differences in the observed templates of emission.

5.3.1 Total Intensity of Synchrotron Radiation

For synchrotron emission we chose to scale the point source corrected Haslam all-sky survey using a single power law in antenna temperature for simplicity. This scaled map is multiplied by the internally modelled polarisation fraction template to produce Stokes parameter maps with realistic morphology.

Synchrotron emission can be described by a power law in antenna temperature, \( T(\nu) \propto \nu^{\beta_s} \) where \( \beta_s \) is the synchrotron spectral index. The Haslam template in brightness temperature, with a resolution of 0.85°, is scaled to microwave frequencies using a spectral index \( \beta_s = -3 \). Although the map may contain residual contamination by free-free emission (see e.g. Dickinson et al. (2003)) we assume it is dominated by the synchrotron component. The templates can be rescaled using any choice of templates in future.

5.3.2 Cosmic Ray Density Distribution

To model synchrotron emission in Section 7.3.1 a three-dimensional model of the distribution of cosmic rays in the Galaxy is required. The large-scale spatial distribution of the cosmic rays is modelled through its density, \( n_{\text{cr}} \), and is thought to follow the same form as the dust distribution with modified radial and scale heights
(Page et al., 2007),

\[ n_{\text{cr}} = n_0 \exp \left( -\frac{\rho}{\rho_{\text{cr}}} \right) \text{sech}^2 \left( \frac{z}{z_{\text{cr}}} \right). \] (5.5)

where the height and radial scales are set to \( z_{\text{cr}} = 1 \) kpc and \( \rho_{\text{cr}} = 5 \) kpc. These parameter values were chosen for the WMAP analysis of Page et al. (2007) following work by Drimmel and Spergel (2001).

### 5.4 Concluding Remarks

In this Chapter, we have presented two of the common forms of the large scale GMF. We have also given details on the current state of understanding of the small scale turbulent magnetic field due to sources such as interstellar winds and supernova remnants. Upcoming data, including the PLANCK satellite and the SPIDER experiment, will provide more accurate estimates of the intensity and polarisation of emission from Galactic foregrounds, particularly thermal dust emission. This data can be used alongside models such as FGPol to constrain parameters of GMF models and contribute to our understanding of the structure and amplitude of this magnetic field.

While there is little data available on polarisation of Galactic foregrounds, modelling of these foregrounds can make use of existing data and templates of the total intensity of both synchrotron and interstellar dust emission. We have discussed the choice of total intensity templates made for this work. However, this choice is external to the model and can be modified to reflect current knowledge of the total intensity fields. We have also discussed details of the thermal dust and synchrotron emission mechanisms. This includes the current knowledge on the alignment of dust grains with the GMF, how the emission depends on the magnetic field amplitude and the physics of synchrotron emission.

### Acknowledgments

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6 Modelling of Polarized Foregrounds From Interstellar Dust

6.1 Introduction

This Chapter focuses on presenting full-sky templates for foreground emission from interstellar dust. We display templates at frequencies being targeted by EBEX, SPIDER and SPTpol.

Currently operating or upcoming CMB experiments such as EBEX (Reichborn-Kjennerud et al., 2010), SPIDER (Filippini et al., 2010), POLARBEAR (The Polarbear Collaboration et al., 2010), KECK (Sheehy et al., 2011) and ABS (Essinger-Hileman et al., 2010) will routinely reach the sensitivity in polarisation required to detect the curl-type pattern (B-mode) predicted by the simplest models of inflation (Dodelson et al., 2009).

The predicted amplitude of this signal however is comparable or below the predicted signal of foreground polarisation over all observationally relevant frequencies and over most of the sky (Gold et al., 2011). For polarisation in particular the foreground signal is dominated by synchrotron emission at low frequencies ($\lesssim 100$ GHz) and thermal dust emission at high frequencies ($\gtrsim 100$ GHz).

In order to achieve their scientific goals, forthcoming CMB polarisation experiments require in-depth knowledge of this polarised Galactic foreground emission. The PLANCK mission (The Planck Collaboration, 2006) will provide maps of the polarisation of interstellar dust, allowing tests of the structure of the Galactic magnetic field. It should also provide an insight into grain alignment mechanisms.

Of utmost importance will be the accurate separation of foreground emission from the CMB signal. Component separation has been considered in, for example, Brandt et al. (1994); Eriksen et al. (2006); Kogut et al. (2007); Stompor et al. (2009). In order to test and assess methods for separating out the contribution realistic foreground templates will be required. Since however, few data exist at this time at the
observing frequencies in which the upcoming experiments are operating, one must resort to modeling of foreground emission by extrapolating the information from existing data. Furthermore, experiments which observe portions of the sky close to the Galactic plane will find the foreground emission is very bright in comparison to the CMB. The presence of this bright emission in the data may affect the performance of the observation and the analysis strategy an experiment uses. Another important role of foreground modeling, therefore, is informing the planning and proposal stage of any experiment.

Unfortunately, these foregrounds are poorly constrained by current data and poorly understood, particularly above around 90 GHz, where the CMB emission is strongest and where many new CMB experiments will operate. At these frequencies, the foreground emission is expected to be dominated by thermal emission from interstellar dust. A review of the basic physical processes whereby aligned dust grains generate polarisation is given in Lazarian and Cho (2003). Such emission is known to be polarised both through direct measurements (Ponthieu et al., 2005; Kogut et al., 2007; Benoît et al., 2004; Bierman et al., 2011) and through observations of the polarisation of starlight (Heiles, 1996; Fosalba et al., 2002).

This polarisation arises due to the presence of a magnetic field in the Galaxy. The dust grains are generally non-spherical, and preferentially emit radiation polarised along their longest axis. Mechanisms exist which align these grains with this axis perpendicular to the Galactic magnetic field, leading to net linear polarisation.

Polarised foregrounds also include polarised emission from synchrotron. Synchrotron emission, generated by the gyration of cosmic ray electrons in the Galactic magnetic field, is intrinsically polarised and constitutes the main polarised foreground at lower frequencies (Page et al., 2007). However, emission from thermal dust dominates synchrotron emission at higher frequencies and so the first part of this work concentrated on providing templates of emission from only thermal dust.

Foreground radiation also includes free-free and spinning dust emission, however we assume both of these signals to be unpolarised and so do not consider them further here. Evidence for this has emerged recently, with Macellari et al. (2011) showing that free-free emission is unpolarised, setting an upper limit on the free-free polarisation fraction of 3.4% at the 2σ level. They also show that spinning dust emission has a low polarisation fraction. Upper limits on the polarisation fraction of spinning dust emission in molecular clouds have been obtained by Dickinson et al. (2011) and López-Caraballo et al. (2011), who find low levels of polarisation. If there is a similar level at higher Galactic latitude, then this foreground is unimportant in terms of component separation.
Archeops (Benoit et al., 2004) and Bicep (Bierman et al., 2011) have made the highest signal-to-noise maps of the dust polarisation at low Galactic latitudes and have examined the properties of the polarisation fraction and angle. These results cannot be relied upon to calibrate large scale models of the polarisation at higher latitudes since the polarisation properties near the Galactic plane will depend on complex structure that is not included in models such as the one presented in this work.

Here, both polarisation amplitude and angle are modeled internally and our templates are scaled such that the polarisation fraction corresponds to a nominal value when averaged over the maps with the Galaxy masked out. The Archeops and Bicep maps have not been made public and a quantitative comparison of our templates with these observations is not possible. Fauvet, L. et al. (2011) have developed a similar model of both the polarisation of thermal dust and synchrotron radiation and compared with the WMAP K-band and Archeops 353 GHz data, however they have not released any templates based on their model. Dunkley et al. (2008) looked at estimating the level of emission from polarised Galactic foregrounds and its impact on a future satellite mission.

6.2 Foreground Dust Model

The degree and direction of polarisation of the dust emission are highly dependent on the Galactic magnetic field. GMF models are described in Section 5.1.1. As the observed polarisation is the sum of many independent regions along the line-of-sight, it is sensitive to the three-dimensional structure of this magnetic field. Therefore, to proceed we make use of the three-dimensional model of the Galactic magnetic field set up in Chapter 5 and also the other necessary Galactic constituents and physics. Section 5.2.2 describes the details of the dust density distribution and properties of the dust emission. We then set the polarisation degree and direction through the appropriate line-of-sight integrals.

The internally modelled polarisation amplitude and angle must be scaled using external information on the dust total intensity. The choice of dust total intensity template is described in Section 5.2.1.

6.2.1 Stokes Parameters

The Galactic magnetic field on small and large scales as well as how they are combined to form the total magnetic field is described in Chapter 5. The polarisation
at each point along the line-of-sight \( \hat{r} \) is determined by the perpendicular field components, \( B_\theta \) and \( B_\phi \) of the combined small and large scale field.

The Stokes parameters for this model are projected out from our three-dimensional model using the appropriate line-of-sight integrals,

\[
I_{\text{model}}(\theta, \phi) = \epsilon(\nu) \int_0^{r_{\text{max}}} n_d(r) \, dr, \\
Q_{\text{model}}(\theta, \phi) = \epsilon(\nu) p_0 \int_0^{r_{\text{max}}} n_d(r) [B_\phi(r)^2 - B_\theta(r)^2] \, dr, \\
U_{\text{model}}(\theta, \phi) = \epsilon(\nu) p_0 \int_0^{r_{\text{max}}} n_d(r) [2B_\phi(r)B_\theta(r)] \, dr, 
\]

and the normalisation \( p_0 \) is set to reproduce the average polarisation fraction reported by WMAP outside their P06 mask, 3.6% (Kogut et al., 2007). Here, \( \epsilon \) is the emissivity of the dust as a function of frequency, \( \nu \). The dust density \( n_d \) is defined in Section 5.4. Note that we conform to the default convention applied in the healpix\(^1\) package (Górski et al., 2005) regarding the sign of \( U \).

We have chosen the 3.6% average polarisation fraction as a reference value but the templates can be scaled to fit any other preferred value based on more detailed knowledge of the polarisation fraction in smaller patches of the sky. It is also useful to note that since we rescale the \( Q \) and \( U \) components the overall normalisation of the magnetic field model becomes irrelevant. However, the relative contributions from the \( ls \) and \( ss \) components in the field remains as a model parameter.

For the line-of-sight integrals we integrate from zero out to a maximum line-of-sight distance \( r_{\text{max}} \) of 30,000 pc. The integrals are discretised in steps of 0.1 pc. The direction of the lines-of-sight are chosen to coincide with the centre of all healpix pixels at a given \( N_{\text{side}}^p \), where \( N_{\text{side}}^p \) is less than or equal to \( N_{\text{side}} \) of the total intensity template FDS map.

From this model we require maps of the polarisation direction, \( \gamma \), and degree, \( P \), which are given by

\[
P(\theta, \phi) = \frac{(Q_{\text{model}}^2 + U_{\text{model}}^2)^{1/2}}{I_{\text{model}}}, \\
\gamma(\theta, \phi) = \frac{1}{2} \arctan \left( \frac{U_{\text{model}}}{Q_{\text{model}}} \right). 
\]

Figure 6.1 shows maps of \( P \) and \( \gamma \) obtained from a line-of-sight integration at resolution \( N_{\text{side}}^p = 128 \) for the BSS and LSA magnetic field models including a one dimensional, small-scale turbulent component. The turbulent component is seen

\(^1\)See http://healpix.jpl.nasa.gov
Figure 6.1: Polarisation fraction (top) and angle (bottom) in Galactic co-ordinates for our model of thermal dust emission at 150 GHz for the BSS (left column) and LSA (right column). The polarisation angle colour bar ranges from $-\pi/2$ to $\pi/2$. Both models include large ($ls$) and small ($ss$) scale magnetic field components. The $ss$ turbulent component was added in the one dimensional, line-of-sight approximation and can be seen as an uncorrelated noise addition to the coherent $ls$ component. There are significant differences in the morphology of the polarisation fraction between the BSS and LSA models due to the BSS model including the spiral arm structure.

Figure 6.1: Polarisation fraction (top) and angle (bottom) in Galactic co-ordinates for our model of thermal dust emission at 150 GHz for the BSS (left column) and LSA (right column). The polarisation angle colour bar ranges from $-\pi/2$ to $\pi/2$. Both models include large ($ls$) and small ($ss$) scale magnetic field components. The $ss$ turbulent component was added in the one dimensional, line-of-sight approximation and can be seen as an uncorrelated noise addition to the coherent $ls$ component. There are significant differences in the morphology of the polarisation fraction between the BSS and LSA models due to the BSS model including the spiral arm structure.

These maps can be compared to the “geometric suppression” factor shown in the right panel of Figure 8 of Page et al. (2007). There are significant differences between the BSS and LSA field models in the morphology of the polarisation fraction here as an uncorrelated noise contribution to the large-scale correlations induced by the large-scale magnetic field model.

The polarisation fraction and angle plots for the LSA model appear to be ‘smoother’ than the equivalent plots for the BSS model. This is due to the differences in the detailed spiral arm structure of the different field models that is seen in Figures 5.1 and 5.2. The amplitude of the large scale field in the BSS model varies between spiral arms, whereas there is no radial variation in the LSA model. Thus, for the BSS model, the small scale turbulent component (whose r.m.s amplitude does not have a radial dependence) becomes more important relative to the large scale component between the spiral arms. As can be seen in Equation 6.1, the $Q$ and $U$ Stokes maps depend on the combined magnetic field from large and small scale components, so the BSS template maps appear to have more uncorrelated noise, related to this small scale turbulent component.

These maps can be compared to the “geometric suppression” factor shown in the right panel of Figure 8 of Page et al. (2007). There are significant differences between the BSS and LSA field models in the morphology of the polarisation fraction
on the sky. The difference is greatest towards the Galactic centre and bulge and the Galactic anti-centre which coincides with a spiral arm. The LSA model does not include any azimuthal dependence and as such does not model any modulation of the magnetic field strength between spiral arms. In addition, the pitch angle of the LSA model, as fit to the WMAP data, is very low and this leads to a very mild dependence of the field alignment in the radial direction. These differences lead to a significantly simpler polarisation structure in the LSA model than in the BSS case which models the spiral arm structure explicitly.

The final dust model at frequency $\nu$ can be written as

$$
I_{\nu}^{\text{dust}}(\theta, \phi) = I_{\text{FDS}}(\theta, \phi),
$$

$$
Q_{\nu}^{\text{dust}}(\theta, \phi) = I_{\text{FDS}}(\theta, \phi) P(\theta, \phi) \cos\left(2\gamma(\theta, \phi)\right),
$$

$$
U_{\nu}^{\text{dust}}(\theta, \phi) = I_{\text{FDS}}(\theta, \phi) P(\theta, \phi) \sin\left(2\gamma(\theta, \phi)\right),
$$

(6.3)

where $I_{\text{FDS}}$ is the total intensity FDS prediction.

Our final product is a template foreground map with small-scale structure modeled by the FDS predictions in the total intensity but with polarisation fraction and angle determined internally by our magnetic field model and line-of-sight integrals.

An alternative approach taken by Page et al. (2007) is to replace $\gamma$ with a map $\gamma_{\text{dust}} = \gamma_* + \pi/2$ where $\gamma_*$ is a smoothed map of observed starlight polarisation directions. This approach, however, is limited by the resolution of the starlight data with only 1578 observations scattered around the sky. It also requires a large smoothing kernel of approximately 10 degrees in size and limits the application of any template derived in this way to very large scales on the sky, corresponding to angular multipoles $\ell \lesssim 15$, and Galactic latitudes $|b| > 10^\circ$.

### 6.3 Scales

It is important to consider the range in angular scales our model is valid for. All our maps are pixelised at $N_{\text{side}} = 1024$, this ensures that the small-scale structure in the FDS prediction is oversampled since the IRAS resolution translates into a limit in angular multipoles of roughly $\ell_{\text{FDS}} \sim 1700$ and the HEALPIX pixel smoothing scale is $\ell_{\text{pix}} \sim 4 N_{\text{side}}$. The overall, effective resolution of our templates is therefore limited by the angular resolution of our line-of-sight coverage which is set by the HEALPIX resolution $N_{\text{side}}^{\text{P}}$.

For the full-sky maps presented here and made available publicly we have chosen $N_{\text{side}}^{\text{P}} = 128$ which corresponds to a limit of roughly $\ell \sim 500$ in multipole space.
We also show in our example maps a small patch prediction with $N_{\text{side}} = 1024$ (see Section 7.4) which again oversamples the resolution given by the FDS templates.

It is also important to consider physically relevant scales that enable the interpretation of the structure in our templates. The most important of these is the injection scale for the turbulent, small-scale component of the magnetic field. We have set this to 100 pc. To obtain a rough estimate of the angular scales at which this physical scale becomes important we can use a “dust weighted” distance measure $\langle r \rangle = \int r n_d(r) \, dr / \int n_d(r) \, dr \sim 7000$ pc for a mid-Galactic latitude line-of-sight. This can be used to place the angular multipole scale of injection at $\ell_{\text{inj}} \sim 220$ or roughly 1 degree. Beyond these scales the stochastic, turbulent component begins to dominate the structure in the polarisation and the model is only a statistical description of the real sky on these scales. An exhaustive exploration of foreground effects on scales below a degree would therefore require a Monte Carlo approach.
6.4 Maps

We show a selection of template, full-sky maps at 150 GHz in Figure 6.2. $I$, $Q$, and $U$ Stokes parameters are shown for both BSS and LSA derived templates (other frequencies are available on the on-line repository). The maps are of thermodynamic CMB temperature in $\mu$K units and are shown in Galactic co-ordinates\(^2\).

As detailed above, the $Q$ and $U$ components have been normalised such that the average polarisation fraction outside the area defined by the WMAP $P06$ mask is 3.6\%. The resolution of the healpix maps is $N_{\text{side}} = 1024$ but the polarisation information is based on a line-of-sight integral at an angular resolution of $N_{P\text{side}} = 128$.

The maps have been obtained by the line-of-sight integration of a magnetic field model that includes a small-scale turbulent realisation only along the line-of-sight direction, ie. our “one dimensional” approximation. Whilst computationally intensive, “3D”, full-sky maps that include a full three dimensional realisation of the turbulent component can be obtained, if required, with computation times of the order of 10 days. However we show results for a smaller $120 \times 120$ degree patch in the southern Galactic hemisphere in Figure 6.3. These maps were obtained using a full three dimensional realisation at an angular line-of-sight resolution of $N_{P\text{side}} = 1024$ and are compared with the same patch in the full-sky “1D” maps. The difference between the two is most clearly seen in comparing the polarisation angle which is uncorrelated with the FDS intensity template. The full three dimensional case contains correlated structure on smaller scales due to the coloured power spectrum of the realisation. In contrast the one dimensional case is uncorrelated on small scales whilst preserving the large-scale correlations induced by the fixed, large-scale magnetic field model. Tailored, high-resolution, “3D” realisations of small patches such as those shown in Figure 6.3 are most useful for sub-orbital experiments that can only observe a limited fraction of the sky.

6.5 Concluding Remarks

We have described a model of the polarised foreground that we expect to observe due to emission from dust within our Galaxy. The model uses a three-dimensional model of the Galactic magnetic field and dust field and integrates along the line-of-sight to each healpix pixel to obtain a polarisation amplitude and angle. This information

\(^2\)Care must be taken in rotating Stokes parameters into other co-ordinate systems such as ecliptic and we have provided rotated maps on the on-line repository since most applications will simulate observations in this frame.
Figure 6.3: Gnomic projection of the polarisation amplitude and angle in a $75 \times 75$ degree patch in the southern Galactic hemisphere. The left column shows the amplitude and angle from a line-of-sight integration including a full three dimensional realisation of the small-scale turbulent magnetic field model. $N_{\text{side}}^P = 1024$ was used to calculate the “3D” maps but only for lines-of-sight corresponding to pixels inside the patch. The right column shows the same area from the full-sky templates with $N_{\text{side}}^P = 128$. The full-sky maps used a one dimensional realisation of the turbulent component along the line-of-sight to speed up the computation. The absence of correlated small-scale structure and lower angular resolution of polarisation information in the “1D” case is clearly seen when comparing maps.
is combined with total intensity, FDS derived template maps at different frequencies to obtain a complete, polarisation template of foreground emission by interstellar dust.

We have concentrated on two popular models for the structure of the large-scale structure of the Galactic magnetic field, namely, the BSS and LSA models. The parameters for the BSS model have been calibrated directly from measurements of the strength of the Galactic magnetic field. In the LSA case we have employed the parameters obtained by Page et al. (2007) in fitting to the WMAP observations. We calculate the polarisation alignment internally to our model in both cases since there is not sufficient external information on polarisation angles at resolutions relevant in this work. Some differences exist between the BSS and LSA derived templates but these are mostly at low Galactic latitudes away from the Galactic centre and as such experiments targeting small areas at high Galactic latitudes will not be sensitive to the differences. The differences do indicate however that a more accurate model of the Galactic magnetic field is required to produce realistic polarisation templates for low Galactic latitudes. In the future, the Planck mission will provide an important test of Galactic magnetic field models through detailed characterisation of galactic foregrounds. Planck’s frequency range is well chosen to accurately separate the thermal dust emission. In further work we will look at updating FGPol based on the recently released total intensity templates from Planck data.

We have developed a one dimensional approximation of the stochastic, turbulent, small-scale component of the field for obtaining full-sky templates. A full three dimensional realisation of the turbulent component can be used to obtain higher resolution templates for smaller patches of the sky.

In the next Chapter we look at extending the model to include synchrotron emission to form a complete picture of foreground emission relevant for polarisation experiments. Other developments will be required to increase the fidelity of the templates on small scales. These include the addition of a stochastic, small-scale density field to model small-scale structure in the density. In full “3D” calculations this will require the generation of an additional three dimensional, turbulence realisation which is correlated to the small-scale magnetic field. In addition, it would be useful to develop a simple model for the correlation of both realisations with the small-scale structure in the FDS derived total intensity templates.

There is significant freedom in the parameters defining the small-scale structure in the templates. Experiments targeting small angular scales over small patches of the sky will be most sensitive to variations in the parameters and also to the stochasticity of the structure in the templates. Further Monte Carlo explorations of the variation
in the maps is therefore warranted to quantify the impact of foregrounds on future sub-orbital experiments. As part of future work we will generate large ensembles of random realisations of the templates on small patches of the sky for the purpose of Monte Carlo studies.

The maps obtained from this model are available for use and can be downloaded from an on-line repository³.

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³http://www.imperial.ac.uk/people/c.contaldi/fgpol
7 A Complete Model of Foregrounds on Large Angular Scales

7.1 Introduction

This Chapter presents templates for polarised emission from Galactic foregrounds at frequencies relevant to Cosmic Microwave Background (CMB) polarisation experiments through using the Galactic Magnetic Field (GMF) modelling set up in Chapter 5 with a focus on large scales. This work extends the results of O’Dea et al. (2012) (hereafter FGPolI) by including polarised synchrotron radiation as a source of foreground emission and then compares the dust and synchrotron templates with available data.

The polarisation direction and fraction in this calculation are based solely on the underlying choice of GMF model and therefore provide an independent prediction for the polarisation signal on large scales. Templates of polarised foregrounds may be of use when forecasting effective experimental sensitivity. In turn, as measurements of the CMB polarisation over large fractions of the sky become routine, this model will allow for the data to constrain parameters in the, as yet, not well understood form of the GMF. Template foreground maps at a range of frequencies can be downloaded from the on-line repository\(^1\).

Given the high levels of polarised foregrounds the mission planning for these and future experiments requires a detailed study of sky coverage to optimise sensitivity to the $B$-mode signal. The impact of the trade off between larger sky coverage, depth of observation and the ‘cleanliness’ of or lack of Galactic foregrounds in a patch of sky must all be considered. Regardless of how clean the final observed sky patch, some level of foreground removal will be required for all experiments. Realistic foreground templates based on models and/or observations of the polarisation direction and amplitude of foregrounds are very useful when carrying out this work.

\(^1\)http://www.imperial.ac.uk/people/c.contaldi/fgpol
However, reliable templates of polarised foregrounds at the frequencies relevant to CMB observations have been hard to come by and only recently, with Wilkinson Microwave Anisotropy Probe (WMAP) K-band observations have reliable estimates of synchrotron polarisation on large angular scales been made.

Little other polarization data exists at the frequencies of interest for future CMB experiments (with frequency bands ranging from 90 GHz up to 450 GHz), hence it is necessary to model foreground emission by extrapolating the information from existing data. This paper builds on previous work presented in FGPolI, which described a model for foreground emission due to interstellar dust in the Galaxy. The dust model was introduced by O’Dea (2009) and first applied in O’Dea et al. (2011a) for the purpose of studying the impact of polarised foreground dust on SPIDER’s ability to detect B-mode polarisation. FGPolI gives a detailed explanation of the dust model and presents a number of full-sky template maps at various frequencies. A complete model of polarised foreground emission must also include the effect of synchrotron emission. This is particularly important for low frequency observations i.e. below the CMB ‘sweet-spot’ at 100 GHz. This will be the focus of the work reported here. Additional components due to spinning dust and free-free emission are not thought to give a significant signal in polarisation and are omitted in our modelling (see for example Macellari et al. (2011) and López-Caraballo et al. (2011)).

Synchrotron emission, generated by the gyration of cosmic ray electrons in the Galactic magnetic field (GMF), is intrinsically polarised and constitutes the main polarised foreground at lower frequencies (Page et al., 2007). While the emission from thermal dust is expected to be higher than synchrotron emission above around 90 GHz, the signal from synchrotron is also not negligible. With the addition of synchrotron, this paper provides a complete model of polarised microwave foreground emission on large angular scales. Here a detailed explanation of the synchrotron model is given and full-sky template maps are presented. The model includes the 3D description of the Galactic magnetic field on both large and small spatial scales described in Section 5.1.1 of Chapter 5. Both polarisation amplitude and angle are modelled internally and the templates are scaled such that the polarisation amplitude corresponds to a nominal value when averaged over the maps.

7.2 Polarised Synchrotron Emission

Diffuse synchrotron emission is one of the dominant Galactic foregrounds for CMB observations. The synchrotron radiation arises when electrons with large relativistic energies are accelerated in the GMF. The frequency dependence of synchrotron
emission depends on the energy spectrum of these cosmic-ray electrons, as well as the intensity of the GMF (see e.g. Rybicki and Lightman (1979)). An ensemble of relativistic electrons with a power law distribution in energy produces a synchrotron emission spectrum that is another power law (Longair (1994)). As mentioned in Section 5.3.1, synchrotron emission is described by a power law in antenna temperature, \( T(\nu) \propto \nu^{\beta_s} \) where \( \beta_s \) is the synchrotron spectral index. In intensity, the frequency dependence of synchrotron emission is given by \( I(\nu) \propto \nu^{\beta_s+2} \) using the relation

\[
I(\nu) = \frac{2k\nu^2}{c^2}T(\nu) \tag{7.1}
\]

At GeV energies, where radio synchrotron emission peaks, the index of the power law is expected to have a range between \( \beta_s \sim -3.5 \rightarrow -2.5 \) (Rybicki and Lightman, 1979) in inferred antenna temperature or, equivalently, \( \alpha \sim -1.5 \rightarrow -0.5 \) in specific intensity where \( \alpha = \beta_s + 2 \). Since the spectral index has been seen to vary with position across the sky (Hinshaw et al., 2007), such a power law in antenna temperature describing synchrotron emission is only an approximation. In addition, the highest energy electrons lose energy more quickly resulting in a gradual steepening in the power law index at the higher frequencies (Bennett et al., 2003).

At microwave frequencies polarised foreground emission is dominated by polarised synchrotron and thermal dust that are both sensitive to the coherent GMF (Page et al., 2007). The dominant emission at lower frequencies is polarised synchrotron radiation. WMAP measurements from its lower frequency bands provide important constraints on polarised synchrotron emission. Synchrotron emission is linearly polarised with direction perpendicular to the projection of the GMF on the plane of the sky (see for example Rybicki and Lightman (1979)). The degree of synchrotron polarisation depends greatly on position on the sky and observing frequency. Changes in magnetic field direction along the line-of-sight leads to a depolarization effect, reducing the fractional polarisation degree of synchrotron emission. At frequencies lower than \( \sim 1 \) GHz, depolarization is significant and hence synchrotron polarisation is as low as few tens of percent (Spoelstra, 1984). At CMB frequencies, depolarisation is minimal, with the degree of synchrotron polarisation being as high as 30 to 50% in some galactic structure.

Free-free emission and spinning dust are also thought to contribute to the foreground signal in total intensity over a range of frequencies. For example, anomalous microwave emission around 20 GHz has been found in WMAP data, with suggestions that this is more likely due to spinning dust emission than a flat synchrotron component (Peel et al., 2011). However spinning dust and free-free emission are not thought to be significantly polarised and their impact on final estimates of e.g. the
tensor-to-scalar ratio $r$ is minimal (Armitage-Caplan et al., 2012) at about the 1% level.

Aside from the spatial dependence of the polarisation angle and amplitude, significant uncertainties remain in the frequency modelling of synchrotron intensity. For example, Bennett et al. (2003) argue that at higher Galactic latitudes (in the halo) the spectral index $\beta_s \sim -3$ while in the Galactic plane (near star forming regions) $\beta_s \sim -2.5$. This results in differences in the observed structure between WMAP K-band at 23 GHz and the Haslam map at 408 MHz (Haslam et al., 1982) as regions with flatter spectral index become more important at higher frequencies. However Miville-Deschenes et al. (2008) find a lower range of variation of the spectral index. This work focuses only on the polarisation fraction and orientation due to the assumed GMF model. We assume a simple frequency scaling of the Haslam template with a single spectral index multiplying the internally modelled polarisation fraction to provide morphologically realistic templates. A more detailed, possibly pixel dependent, frequency rescaling can always be introduced by rescaling the template obtained.

At present the best template of polarised Galactic synchrotron emission is that provided by the WMAP K-band (23 GHz) whilst the intensity has been well measured (free from CMB contamination) at 408 MHz by the Haslam all sky survey. As detailed below we use the Haslam maps to introduce detailed morphology in our templates since the WMAP Maximum Entropy Method (MEM) maps still contain a significant noise residual due to CMB contamination at the smoothing scale adopted ($1^\circ$). We compare the templates obtained here with the WMAP synchrotron and dust maps obtained through their Monte Carlo Markov Chain (MCMC) best-fit procedure (Gold et al., 2011). Similar templates (Fauvet et al., 2010) have been compared with ARCHEOPS maps over a limited fraction of the sky at 353 GHz (Benoît et al., 2004) but these maps are not publicly available.

A number of other studies aimed at modelling foreground emission at microwave frequencies have been carried out (see e.g. Fauvet et al. (2010), Page et al. (2007)). An extensive study modelling different foregrounds in both intensity and polarisation over a large range in angular scales is the PLANCK Sky Model (PSM) (Delabrouille et al., 2012)\(^2\) which we describe in more detail in the next Section.

\(^2\)The PSM description was released in the interim between FGPolI and this work. The PSM template and model are available on a restricted basis and detailed comparisons will be described in a future publication.
7.2.1 Planck Sky Model

The Planck Sky Model (PSM) (Delabrouille et al., 2012) includes detailed modelling of Galactic diffuse emission, including synchrotron and thermal dust emission as well as free-free, spinning dust and CO lines. It also includes information on Galactic HII regions, extragalactic radio sources and several other sources of emission.

Similarly to the FGPol model, the PSM uses the Haslam template (see Section 5.3.1) for its synchrotron intensity template. The default PSM synchrotron template extrapolates the Haslam template to other frequencies using the spectral index map from Miville-Deschenes et al. (2008) with no spectral steepening at higher frequencies, however the model also provide options for using a constant spectral index of $\beta_s = -3$ or any other spectral index map.

On scales smaller than $1^\circ$, fluctuations are added to the synchrotron intensity and spectral index maps for cases where the required resolution is greater than the resolution of the Haslam template. As in FGPol, the PSM uses FDS templates (see Section 5.2.1) as its prediction for the intensity of thermal dust emission.

The PSM also focuses on polarised emission from the two dominant components (synchrotron and thermal dust emission), assuming as in the FGPol model that emission from free-free and spinning dust is only weakly polarised.

For the prediction of polarised emission from synchrotron radiation, rather than using a model of the Galactic magnetic field (GMF), the PSM extrapolates the WMAP 23 GHz $Q$ and $U$ maps with the same spectral index as the intensity template. In doing this the PSM takes the approach of trying to produce templates that reproduce observed structure on the sky as best as possible, rather than trying to predict polarised emission from a given GMF model. Additional fluctuations on scales smaller than $1^\circ$ are again added as described above.

For its prediction of polarised emission from thermal dust, the PSM takes a complicated approach. Instead of the FGPol approach of GMF modelling, the PSM again chooses to use available data on synchrotron emission, assuming that this polarised emission is correlated with the polarised emission from thermal dust. The PSM uses the 23 GHz WMAP and 408 MHz Haslam data to produce maps of the geometric depolarisation factor $g_s$ and polarisation direction $\gamma_s$ smoothed to $3^\circ$, where

$$g_s = \frac{(Q^2_{WMAP} + U^2_{WMAP})^{1/2}}{f_s I_{Hasl}(23/0.408)^{-3}},$$

$$\gamma_s = \frac{1}{2} \arctan \left( \frac{U_{WMAP}}{Q_{WMAP}} \right)$$

(7.2)
and the intrinsic polarisation fraction \( f_s = \frac{3(p + 1)}{(3p + 7)} \) (see Delabrouille et al. (2012)); this has the value \( f_s = 0.75 \) for \( p = 3 \). This provides the structure on scales between 3° and 20°, while again on scales smaller than 3°, additional fluctuations are included as described above.

In order to predict dust template maps of \( g_d \) and \( \gamma_d \), the PSM must then correct \( g_s \) and \( \gamma_s \) for large scale differences in structure between the two emission mechanisms. These differences occur as a result of the different scale heights in the dust and cosmic ray electron density distributions and the differences in dependence on the magnetic field in the Stokes integrals. The PSM predicts Stokes parameter maps for synchrotron and dust emission using a 3D model of the Galactic magnetic field (GMF) and expressions for the distribution of dust and cosmic ray electrons in a similar way to FGPol. However, the differences between the dust and synchrotron emission from this 3D GMF modelling are only used to provide structure on scales larger than 20° in their templates, as a way of correcting the maps of \( g_s \) and \( \gamma_s \) obtained from the WMAP 23 GHz data.

While the GMF, dust density and cosmic ray electron density modelling is similar to the FGPol model, the PSM only uses this model on large angular scales. As discussed above, the PSM prediction on intermediate scales comes from mainly WMAP data; this is a significant difference between the two models. One of the benefits of FGPol is that the prediction for the polarisation angle and degree comes from a purely internal model of the GMF and dust or cosmic ray electron density. This means that FGPol is perhaps better for the purposes of fitting parameters of the GMF to data (for example from PLANCK).

Comparing Figures 17 and 19 in Delabrouille et al. (2012), the maps of polarisation fraction and angle generated by the PSM are almost identical for dust and synchrotron, as they have used WMAP synchrotron measurements to provide the structure in both templates. However, the benefit of the PSM is that it tries to reproduce observed structures on the sky. The \( Q \) and \( U \) maps appear quite different for dust and synchrotron due to the differences in the total intensity templates used for the dust and synchrotron predictions.

Another difference between the models is that the turbulent component added to the PSM to model geometrical depolarisation is simulated independently on each line-of-sight, so it does not include any correlation across the sky. In FGPol, this small scale turbulence can be simulated in three dimensions, thus incorporating correlations in the small scale turbulent component across the sky.
7.3 Synchrotron Model

The predicted degree and direction of polarisation of synchrotron emission depends on the GMF model used. Here we limit the choice to the Logarithmic Spiral Arm (LSA) model introduced by Page et al. (2007) for use in modelling the WMAP data. The model is defined in Section 5.1.1.

Although we focus on large angular scales we also include a small-scale random component in our GMF model by adding a realisation of a Kolmogorov turbulence field with a one-dimensional Kolmogorov energy spectral index of \(-5/3\). An injection scale of 100 pc is chosen for the turbulent realisation with a negligibly small dissipation scale compared to the resolution scale. The small scale field is also described in Section 5.1.1.2.

The predicted emission also depends on the density distribution of cosmic ray electrons responsible for this emission. The cosmic ray density distribution is described in Section 5.3.2. We then set the polarisation degree and direction through the appropriate line-of-sight integrals. The internally modelled polarisation amplitude and angle must be scaled using external information on the synchrotron total intensity. The choice of total intensity template is described in Section 5.3.1.

7.3.1 Stokes Parameters

The direction and degree of polarisation from synchrotron emission are highly dependent on the Galactic magnetic field. To model these we integrate along lines-of-sight using the GMF outlined in section 5.1.1.

The full-sky maps presented here were obtained using a one-dimensional realisation of the small-scale turbulent field. When producing smaller patches that require much fewer lines-of-sight at a given resolution we model the small scale turbulence as a full three-dimensional random realisation which preserves the spatial correlations implied by the Kolmogorov spectrum.

The polarisation at each point along the line-of-sight \(\hat{r}\) is determined by the perpendicular field components, \(B_\theta\) and \(B_\phi\), see Chapter 5.

The Stokes parameters for the synchrotron model are then projected out from the
three-dimensional model using the appropriate line-of-sight integrals,

\[ I_{\text{model}}(\theta, \phi) = \epsilon(\nu) \int_0^{r_{\text{max}}} n_{\text{cr}}(r)(B_\phi(r)^2 + B_\theta(r)^2) \, dr, \]

\[ Q_{\text{model}}(\theta, \phi) = \epsilon(\nu) p \int_0^{r_{\text{max}}} n_{\text{cr}}(r) \frac{(B_\phi(r)^2 - B_\theta(r)^2) B_r^2}{B^2} \, dr, \]

\[ U_{\text{model}}(\theta, \phi) = \epsilon(\nu) p \int_0^{r_{\text{max}}} n_{\text{cr}}(r) \frac{2B_\phi(r)B_\theta(r)B_r^2}{B^2} \, dr, \]

(7.3)

where \( B^2 = B_r^2 + B_\phi^2 + B_\theta^2 \) and \( \epsilon \) is the emissivity as a function of frequency, \( \nu \). The cosmic ray electron density \( n_{\text{cr}} \) is defined in Section 5.3.2. As with the dust templates, we conform to the default convention applied in the HEALPIX\(^3\) package (Górski et al., 2005) regarding the sign of \( U \).

Having computed the line-of-sight integrals for the Stokes parameters we calculate maps of the polarisation direction, \( \gamma \), and degree, \( P \), given by

\[ P(\theta, \phi) = \sqrt{\frac{Q_{\text{model}}^2 + U_{\text{model}}^2}{I_{\text{model}}}}, \]

\[ \gamma(\theta, \phi) = \frac{1}{2} \arctan \left( \frac{U_{\text{model}}}{Q_{\text{model}}} \right). \]

(7.4)

The final synchrotron template at frequency \( \nu \) is then obtained by scaling with the Haslam template

\[ I_\nu^{\text{sync}}(\theta, \phi) = I_\nu^{\text{Has}}(\theta, \phi), \]

\[ Q_\nu^{\text{sync}}(\theta, \phi) = I_\nu^{\text{Has}}(\theta, \phi) P(\theta, \phi) \cos(2\gamma(\theta, \phi)), \]

\[ U_\nu^{\text{sync}}(\theta, \phi) = I_\nu^{\text{Has}}(\theta, \phi) P(\theta, \phi) \sin(2\gamma(\theta, \phi)), \]

(7.5)

where \( I_\nu^{\text{Has}} \) is the total intensity of the Haslam map extrapolated to frequency \( \nu \).

### 7.4 Maps

Figure 7.1 shows \( Q \) and \( U \) Stokes parameter maps at 23 GHz for the whole sky arising from the model with their amplitudes scaled such that the polarisation amplitude corresponds to that of the WMAP counterpart (also shown) when averaged over the maps. The morphology of the polarisation agrees well with the observations with the most visible difference being on scales of a few degrees where the WMAP estimates are dominated by residual noise.

\(^{3}\)See http://healpix.jpl.nasa.gov
Figure 7.1: From top to bottom, $Q$ and $U$ Stokes parameter template maps, displayed in Galactic co-ordinates for left: FGPol model of synchrotron emission at 23 GHz for the LSA GMF model. These were generated at $N_{\text{side}} = 1024$ map resolution based on $N_{\text{side}}^P = 128$ line-of-sight resolution. The maps are smoothed to $1^\circ$ and downgraded to $N_{\text{side}} = 64$. right: WMAP MCMC synchrotron map for comparison. Units are $\mu K$ antenna temperature.
The resolution of the HEALPix maps is $N_{\text{side}} = 1024$ but the polarisation information is based on a line-of-sight integral at an angular resolution of $N_{\text{side}}^P = 128$, corresponding to roughly $\ell \sim 500$ in multipole space. We integrate along lines-of-sight to the centre of all HEALPix pixels at a given $N_{\text{side}}^P$ from zero out to a maximum distance $r_{\text{max}}$ of 30,000 pc, with discretisation steps of 0.1 pc. $N_{\text{side}}^P$ is less than or equal to $N_{\text{side}}$ of the total intensity Haslam map.

The LSA model is used for the Galactic magnetic field model with the same parameters as in FGPol. The small scale field is modeled as Kolmogorov turbulence in 1D for large patches of sky, with a power spectrum of $P(k) \propto k^{-(2+3N_d)/3}$ where $N_d$ is the number of spatial dimensions of the realisation and $k$ is the magnitude of the wavevector. All full-sky templates presented here make use of the 1D approximation along the line-of-sight for the small scale turbulent component of the GMF. Small patch templates discussed below are produced with full 3D realisations of the field in the (smaller) volume probed by the reduced coverage.

Figure 7.2 shows maps of $P$ and $\gamma$ obtained using this choice of resolution and modelling of large and small scales. For comparison, maps of $P$ and $\gamma$ for the WMAP 23 GHz MCMC template are also plotted. Differences between the templates and observations are mostly due to noise but there are also obvious differences in the morphology along the galactic plane and around the largest Galactic features such as the Galactic centre and North and South Galactic Spurs. Some of these differences are related to our choice of total intensity template which uses the Haslam maps at 408 MHz. A comparison between the scaled Haslam map and synchrotron templates obtain via the differencing of WMAP $K$ and $Ka$ bands were discussed in Gold et al. (2011). Below we quantitatively compare the broad features of both synchrotron and dust full-sky templates with the corresponding WMAP MCMC best-fit maps.

### 7.5 Comparison with WMAP templates

The WMAP satellite observations provide full-sky maps of temperature and polarisation in five frequency bands between 23 GHz and 94 GHz (Jarosik et al., 2011). The polarisation maps contain important information on Galactic foreground emission and hence provide an important test of our model of Galactic synchrotron radiation. Synchrotron radiation dominates the measured signal in the lower frequency bands, and we use the best fit WMAP 23 GHz synchrotron templates generated by

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4Figure 7.1 can be compared with Figure 2 in Fauvet et al. (2010)
5http://lambda.gsfc.nasa.gov
Figure 7.2: Polarisation amplitude \( P = \sqrt{Q^2 + U^2} \) in antenna temperature \((\mu K)\) (top) and angle (bottom) in Galactic co-ordinates for the synchrotron emission template (LSA GMF model) at 23 GHz (left) and for WMAP 23 GHz (right). The polarisation angle colour coding ranges from 0 to 180 degrees.
an MCMC fit for comparison with our model. Thermal dust emission dominates higher frequency bands, and for comparison with our dust model we use the 94 GHz dust templates generated from the MCMC best fit values.

We use the WMAP MCMC ‘base’ fit which includes three power law foregrounds: dust, synchrotron and free-free emission as well as a contribution from CMB. These maps are smoothed at a scale of $1^\circ$ and have been downgraded to $N_{\text{side}} = 64$ before the MCMC fit is carried out. The WMAP team performed a combined MCMC fit to their five bands at a resolution of $N_{\text{side}} = 64$. The pixel noise is calculated at $N_{\text{side}} = 512$ and downgraded to $N_{\text{side}} = 64$.

FGPOL templates are normalised to the polarisation amplitude $P = \sqrt{Q^2 + U^2}$ of the full sky WMAP MCMC template with no masking or smoothing. The model maps are generated at $N_{\text{side}} = 1024$ for the large scale resolution and $N_{\text{side}}^P = 128$ for the small scale line-of-sight resolution, then smoothed with a Gaussian beam of $1^\circ$ and degraded to $N_{\text{side}} = 64$.

Angular power spectra $C_{\ell}^{XX}$ for $XX \equiv TT, EE$, and $BB$ of both templates were calculated after masking with the P06 polarisation mask (Page et al., 2007) combined with a mask of pixels flagged by the WMAP MCMC process. The spectra are corrected for sky fraction $f_{\text{sky}}$ effects and for their respective pixel and beam smoothing functions.

We then fit for a power law in multipole $\ell$ with an additional white noise component

$$\frac{\ell(\ell + 1)}{2\pi} C_\ell = A\ell^m + \ell(\ell + 1)N^2$$

(7.6)

where $A$ is the amplitude of the foreground component, $m$ is the index and $N$ is the noise amplitude. This procedure is similar to that carried out by Gold et al. (2011). Although the scatter at large angular scales is non-Gaussian and somewhat correlated by the sky cut we adopt a very simple assumption for sample variance in the power spectra by disregarding correlations between multipoles and assuming a Gaussian scatter given by the sample variance for each $C_\ell^{XX}$. The WMAP derived fits also make use of diagonal Fisher error values.

The $EE$ and $BB$ power spectra from the FGPol model and WMAP templates can be seen in Figure 7.3 along with the WMAP MCMC power law plus white noise fits plotted as dotted lines. The WMAP MCMC templates contain a significant white noise component on scales of about a degree which is not present in our model templates. This is reflected in the differences in the power spectra as the FGPol dust power spectrum is flatter, while the power spectra of the WMAP MCMC maps show a steep increase at higher $\ell$ due to this white noise component. In terms of
Figure 7.3: EE (left) and BB (right) power spectra for the WMAP MCMC synchrotron 23 GHz map (black line) and dust 94 GHz map (blue line) compared to the FGPol synchrotron model (green line) and dust model (red line). The WMAP 7-year best fit power law plus white noise fit is also shown for the synchrotron template (black dotted line) and dust template (blue dotted line). The polarisation amplitude of the model templates is matched to the WMAP MCMC templates as described in the text. These were calculated from maps masked with a union of the WMAP P06 mask and the MCMC flagged pixels.
the scaling of the dust and synchrotron foreground components, it can be seen that the scaling of the FGPol and WMAP MCMC power spectra agree quite well at low \( \ell \).

Figure 7 of Gold et al. (2011) also shows the power spectra of WMAP MCMC maps along with their 3-year and 7-year foreground fits. Comparing the WMAP 3-year and 7-year fits, it can be seen that the dust fits changed slightly while the synchrotron fits remained almost the same, highlighting the uncertainty in foreground separation using WMAP data. One point to note is that the WMAP 7-year \( EE \) spectrum fit (given by the dotted line in Figure 7.3) seems to be slightly higher than suggested by the data points at the lowest \( \ell \) values. This may explain the difference in scaling of foreground components between the FGPol model and WMAP MCMC templates.

The power spectrum fitting allows us to quantify the scaling of the angular power spectrum for both templates as a function of multipole \( \ell \) whilst allowing for any residual noise and/or pixelisation effects. They can also be used as a quick guide for the level of foreground contamination at different frequencies on large angular scales either on the full sky or on small patches. Figure 7.4 shows the resulting power law fits in both \( C_{\ell}^{EE} \) and \( C_{\ell}^{BB} \) for the P06 masked FGPol and WMAP MCMC synchrotron templates at a frequency of 23 GHz. Also included are the results of the same procedure applied to the dust FGPol and WMAP MCMC templates at 94 GHz.

The fit values, excluding the noise amplitudes, can be found in Table 7.1. The synchrotron templates agree well with the WMAP MCMC maps in both amplitude and angular dependence whereas there are significant differences between the FGPol dust template and the WMAP MCMC map at 94 GHz.

We also attempt to quantify the level of correlation between the WMAP MCMC maps and FGPol templates. We do this using two separate measures. The first analyses the level of pixel-to-pixel correlation between maps calculated for the area of the sky outside a given galactic latitude cut. The correlation coefficient \( R \) is given by

\[
R(\theta_g) = \frac{\sum_p (W_p - \bar{W}_p)(F_p - \bar{F}_p)}{\sqrt{\sum_p (W_p - \bar{W}_p)^2 \sum_p (F_p - \bar{F}_p)^2}}.
\]  

(7.7)

where \( W \) and \( F \) are the \( I, Q, \) or \( U \) Stokes values of the WMAP and FGPol maps respectively and the index \( p \) sums over all pixels outside the cut at latitude \( \pm \theta_g \). The result, for both dust and synchrotron \( Q \) and \( U \) Stokes parameters is shown in Figure 7.5. The analysis shows that both dust and synchrotron templates are highly
Table 7.1: Foreground power law + white noise fits of WMAP MCMC and FGPol template spectra outside the combination of P06 mask and MCMC flagged pixels. There is good agreement in both $EE$ and $BB$ for synchrotron between the FGPol template and the WMAP MCMC synchrotron component map. The FGPol dust template shows a significantly shallower spectrum than the WMAP MCMC component map indicating relatively more structure at large angular scales.

<table>
<thead>
<tr>
<th>Component</th>
<th>$A \ [\mu K^2]$</th>
<th>$m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>WMAP MCMC</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Synchrotron $EE$</td>
<td>$306 \pm 95$</td>
<td>$-0.91 \pm 0.11$</td>
</tr>
<tr>
<td>Synchrotron $BB$</td>
<td>$144 \pm 44$</td>
<td>$-0.87 \pm 0.12$</td>
</tr>
<tr>
<td>Dust $EE$</td>
<td>$12.9 \pm 6.4$</td>
<td>$-1.06 \pm 0.24$</td>
</tr>
<tr>
<td>Dust $BB$</td>
<td>$6.12 \pm 3.7$</td>
<td>$-0.83 \pm 0.29$</td>
</tr>
<tr>
<td>FGPol</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Synchrotron $EE$</td>
<td>$343 \pm 91$</td>
<td>$-1.05 \pm 0.09$</td>
</tr>
<tr>
<td>Synchrotron $BB$</td>
<td>$110 \pm 29$</td>
<td>$-0.73 \pm 0.08$</td>
</tr>
<tr>
<td>Dust $EE$</td>
<td>$1.38 \pm 0.33$</td>
<td>$-0.12 \pm 0.07$</td>
</tr>
<tr>
<td>Dust $BB$</td>
<td>$1.70 \pm 0.37$</td>
<td>$-0.22 \pm 0.06$</td>
</tr>
</tbody>
</table>

Figure 7.4: $EE$ (left) and $BB$ (right) fits to the foreground angular power spectra for the WMAP MCMC synchrotron 23 GHz and dust 94 GHz maps (solid line) compared to the FGPol synchrotron model and dust model (dashed line). The polarisation amplitude of the model templates is matched to the WMAP MCMC templates as described in the text. The fit values used are given in Table 7.1. These were calculated from maps masked with a union of the WMAP P06 mask and the MCMC flagged pixels.
Figure 7.5: Comparison of the correlation coefficient $\mathcal{R}$ between the FGPol maps and WMAP best–fit templates for both synchrotron and dust. In order to look at the correlation on large scales we smooth these maps to a resolution of $10^\circ$. The correlation is calculated by including areas outside a range of cuts in Galactic latitude. The trend shows that both dust and synchrotron models are highly correlated with the best-fit maps at high Galactic latitudes whilst the correlation falls rapidly at low galactic latitudes for the dust comparison. The WMAP best–fit dust map however has an even larger noise residual than the synchrotron map and this is expected to reduce the correlation significantly.

We also look at the correlation in terms of scatter of the pixel values in $Q$ and $U$ Stokes parameters for the WMAP MCMC maps versus the FGPol templates in both dust and synchrotron. All pixels outside the P06 mask and MCMC flagged pixels are included and the scatter density is shown in Figure 7.6 as two contours encompassing 68% and 95% of pixels. We only show the synchrotron correlation density since the dust one is found to be dominated by the larger WMAP variance.
Figure 7.6: Comparison of the scatter between the FGPol maps and WMAP best-fit templates for synchrotron. This is done for the full-sky templates including pixels outside the combination of the P06 polarisation mask and WMAP MCMC flagged pixels. The dust correlation density is omitted as it is dominated by the variance in the WMAP map due to residual noise.

due to residual noise and is not informative.

7.6 Foreground amplitudes in sub-orbital sky patches

We also examine the amplitude of foreground contamination in smaller sky areas being targeted by a sample of three currently operating or planned sub-orbital experiments; EBEX (Reichborn-Kjennerud et al., 2010), Spider (Filippini et al., 2010) and the Bicep2 and Keck (Orlando et al., 2010) arrays which observe the same field. Angular resolution and sensitivity for the three experiments are varied but they are all targeting the detection of $BB$ power either on large angular scales that are free of lensing effects or, as in the case of EBEX, on smaller angular scales where the lensing effect dominates the $BB$ signal.

We generate high resolution templates with full three-dimensional modelling of the turbulent small-scale GMF over the regions of expected coverage for the three experiments. For the dust templates, the $Q$ and $U$ components are normalised so
that the average polarisation fraction outside the area defined by the WMAP P06 mask is 3.6\%. The coverage areas are outlined in Figure 7.7 with SPIDER targeting the largest area with $f_{\text{sky}} \sim 0.1$, EBEX targeting the smallest patch contained in the SPIDER area with $f_{\text{sky}} \sim 0.01$ and Bicep2/Keck targeting the southern most patch with $f_{\text{sky}} \sim 0.03$.

In order to compare the relative contamination by foregrounds in relation to the relevant signal we analyse the angular power spectrum $C_{\ell}^{BB}$ for both dust and synchrotron. Due to the uncertainties involved in predicting the small scale signal we focus on large scales only with templates smoothed to a common resolution of 1° and rely on extrapolating a power law to scales larger than $\ell \sim 200$ in comparing with the expected signal.

The analysis on small areas of the sky such as these is complicated by the significant correlation induced by the cut on spherical harmonic coefficients. The high level of correlation would result in significant biases if the same power law fitting procedure as used in Section 7.5 were carried out. To avoid this problem we estimate the overall amplitude of foreground contamination by averaging in pixel space assuming a fixed power law in $\ell$ corresponding to our previous near full-sky analysis.

In practice we calculate the variance in both $Q$ and $U$ for each patch and assume
Figure 7.8: Foreground amplitude for synchrotron (left panel) and dust (right panel) calculated from patches targeted by various suborbital experiments compared with theoretical EE and BB spectra for $r = 0.1$. The index of the power law used is from the corresponding near full sky power spectrum fits. Also shown are the near full sky best fit spectra for the FGPol synchrotron template from Table 7.1 along with best fit spectra to the dust templates with $Q$ and $U$ normalised so that the polarisation fraction is 3.6% outside the WMAP P06 mask. The amplitudes were calculated from our FGPol dust and synchrotron templates at 150 GHz calculated from maps generated at $N_{\text{side}} = 1024$ for the large scale resolution and $N_{\text{side}} = 128$ for the small scale line-of-sight resolution, which are then smoothed to 1° and downgraded to $N_{\text{side}} = 64$. Units are $\mu K_{\text{CMB}}$.

A relation between the variance and angular power spectrum of the form

$$\sigma^2 = \frac{1}{4\pi} \sum_{\ell=2}^{\ell_{\text{max}}} (2\ell + 1) C_\ell B_\ell^2(\theta_s),$$

(7.8)

with the signal angular power spectrum modelled as $C_\ell = A \ell^m$ in accordance with (7.6) and with index $m$ set to the corresponding near full-sky best-fit value (see Table 7.1). We take $\ell_{\text{max}} = 128$ and model the smoothing $B_\ell$ applied to the templates as a Gaussian beam with FWHM 1° multiplied by the pixel window function at the working HEALPIX resolution $N_{\text{side}} = 64$. We then invert the relation (7.8) to obtain an ‘average’ polarisation angular power spectrum amplitude $A$, effectively assuming that power is equally distributed between $EE$ and $BB$. 

131
The results are summarised in Figure 7.8 for a single reference frequency of 150 GHz as this is being included as an observing frequency in all experiments being considered. The model power spectra for each patch are shown in thermodynamic temperature in order to compare directly with the expected $BB$ signal for a tensor-to-scalar ratio $r = 0.1$. Both primordial and lensing contribution to the $BB$ signal are shown.

The amplitude of foreground contamination varies by roughly an order of magnitude between the area targeted by different experiments. In particular the area targeted by EBEX seems to be very clean with the foreground signal reduced by an order of magnitude compared to the areas targeted by SPIDER and BICEP2/Keck. This agrees visually with the impression given in Figure 7.7.

### 7.7 Concluding Remarks

We have presented templates for polarised emission from synchrotron radiation within our Galaxy using a 3D model of the Galactic Magnetic Field (GMF) and cosmic ray density distribution. From this model, maps of polarisation amplitude and angle are calculated which are then combined with total intensity measurements from the Haslam 408 MHz all-sky radio continuum survey to provide template maps.

We have compared the FGPol templates obtained from this model with data from the WMAP satellite for both synchrotron and dust emission. We find that the synchrotron template agrees qualitatively with the observations whereas comparison of the dust template is complicated by the large residuals present in the WMAP estimates.

We have also looked at foreground contamination levels in patches that will be targeted by upcoming experiments and found that our model predicts significant differences of up to an order of magnitude in the foreground contamination of different patches. The level of contamination will dominate the ability of various experiments to achieve their target sensitivity with respect to the $B$-mode signal being searched for.

Current data is not conclusive regarding the scaling of synchrotron emission with frequency, however there are suggestions of a steepening in the synchrotron spectral index with frequency that is not currently included in this model; this will affect the amplitude of synchrotron templates presented here. The structure in these templates depends on details of the magnetic field model; there remains uncertainty in both the large-scale GMF and the power spectrum of the small-scale turbulent component that will result in differences in the predicted power spectra from these
templates. These templates also depend on the model for the dust and cosmic ray electron density distributions.

As more polarisation data becomes available the comparison between the model and observations will become more quantitatively precise. In particular the recent PLANCK data release will provide maps of the intensity of thermal dust emission and we will be able to refine our model based on them. Indeed, in future, it should be possible to learn much about the GMF itself by fitting the (many) model parameters to actual data. This will shed light on many aspects of our Galaxy’s physical model that are still poorly understood.

Acknowledgments

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8 Conclusions

From the groundbreaking discoveries of the COBE experiment in the early 1990’s the field of cosmic microwave background (CMB) analysis has advanced rapidly. We now have accurate characterisation of temperature anisotropies over a large range of angular scales and high significance measurements of the values of many cosmological parameters. The CMB is also linearly polarised and the anisotropies of this polarisation field provides a new way of accessing early universe physics. Polarisation can be decomposed into electric $E$ and magnetic $B$ type patterns. The $E$-mode of polarisation of the CMB has been observed by many experiments and the hunt now centres around the search for the $B$-mode polarisation signal.

The main theme of this thesis surrounds challenges of the data analysis for experiments targeting the tiny amplitude $B$-mode polarisation pattern from a gravitational wave background predicted by many inflationary models. The primordial signal is dominated on the largest angular scales by the polarised Galactic foreground emission. As such, the development of models for these foregrounds is an essential part of CMB polarisation analysis. The small amplitude of the signal also means that we require large arrays of extremely sensitive polarimeters in order to increase the signal to noise ratio of measurements of CMB polarisation.

An important area of research in observational cosmology stems from the massive increase in the size of datasets that are being collected. This thesis describes one such challenge, the analysis of correlated datastreams from hundreds of extremely sensitive polarimeters.

We have presented SPIMPI, an algorithm written in Fortran and parallelised using MPI, that is capable of analysing massively multidetector datasets. Mapmaking for CMB experiments amounts to estimation of a sky map given the time ordered data (TOD) from hundreds of detectors. We iteratively solve this linear system using the Preconditioned Conjugate Gradient method. As we do not have accurate estimates of the noise component of the TOD we must also estimate the noise power spectrum of the data using a joint iterative procedure.

We have tested this algorithm with simulated SPIDER datasets and looked at several properties of the mapmaker. In particular, we have looked at the effect of
filtering the TOD to remove low frequencies that are dominated by noise. Following this filtering operation, we have looked at the filter transfer function for the naively coadded data, which shows the loss of information compared to the power of the input simulation as a function of angular scale. We have shown that we lose information on large angular scales through this filtering, but that information is recovered through the maximum likelihood method for estimation of the signal map from the TOD. We would like to study this further, through Monte Carlo generation of signal only realisations and calculation of the ensemble average of the transfer function. This transfer function will be used to deconvolve final estimates of the CMB power spectra.

SPIMPI has also been tested on data from the 2003 flight of the Boomerang experiment. This experiment made statistical detections of the EE and TE power spectra and high signal to noise maps of the temperature anisotropies. We have analysed data from eight polarisation sensitive bolometers at 145 GHz and shown the importance of removal of scan synchronous noise through application of high pass filters to the TOD. SPIMPI converged quickly for analysis of both the deep and shallow regions targeted by Boomerang using 8 processors on the Imperial High Performance cluster.

We have also looked at how well the algorithm recovers the input noise power spectrum, this performs quite well however further analysis must be carried out to determine if the estimate of the noise power spectrum has converged after 8 iterations.

We have provided predictions from and built on a model of polarised foreground emission developed by Daniel O’Dea, focusing on high galactic latitudes that are of interest to upcoming suborbital experiments. We have looked at the two dominant polarised foregrounds, interstellar dust emission and synchrotron radiation. This model includes a description of the Galactic Magnetic Field (GMF), including both its large scale structure and statistical realisations of its small scale turbulent field generated by events such as interstellar winds and supernova remnants. Along with details of the physics of the emission mechanisms for dust and synchrotron radiation, we include details of the dust and cosmic ray electron density distributions.

By integrating along lines of sight from our position in the Galaxy to pixels on the sky, templates of the polarisation amplitude and direction from this GMF model can be estimated. External information comes into the model through scaling by total intensity templates for dust and synchrotron emission. These external templates can be changed based on current measurements and we intend to update FGPol based on results from the Planck satellite on interstellar dust emission.
Alternative models (including the PLANCK Sky Model) also provide estimates for this polarised emission. In the near future, both PLANCK data as well as data from other experiments such as SPIDER will provide more accurate templates of the total intensity and polarisation of foreground emission. Using new measurements of this emission along with models such as FGPol, parameters of the Galactic magnetic field model and Galactic constituents such as the dust and cosmic ray electron density distributions could be constrained.

In this work we also compare the FGPol model to WMAP MCMC templates of dust and synchrotron emission. We have found that the synchrotron templates agree qualitatively with the WMAP templates however for the dust templates our analysis was limited by the accuracy of the WMAP template.

This model can be used to predict the amplitude of foreground emission in patches that will be targeted by suborbital experiments. We have seen that the amplitude of the synchrotron emission is several of orders of magnitude smaller than the $r = 0.1$ $BB$ power spectrum at $\ell = 100$ for templates at a frequency of 150 GHz. However, the amplitude of the dust emission is comparable to the cosmological signal. This result highlights the importance of accurate predictions of foreground emission for experiments such as SPIDER. The predictions for the amplitude of foregrounds in patches targeted by several experiments indicate that the observed amplitude of foregrounds could vary dramatically depending on the region of sky observed.

In the recent PLANCK papers, maps from all HFI channels have been released, this wide range of high frequency channels is dominated by emission from interstellar dust. These maps, shown in Figure 8.1, show the potential for accurate characterisation of the frequency dependence and amplitude of thermal dust emission. PLANCK has produced intensity maps of Galactic foreground components from a low resolution analysis, maps of low frequency emission including synchrotron, anomalous and free-free emission, CO emission and dust emission can be seen in Figure 8.2.

The small amplitude of the primordial $B$-mode signal requires reduction of experimental systematics as well as accurate removal of the much larger foreground signal. In this thesis we have looked at tackling two important challenges for analysis of CMB polarisation data. Firstly, the problem of modelling the emission from two of the dominant polarised Galactic foregrounds, interstellar dust and synchrotron emission. We will update our model of polarised emission in the light of new data from the PLANCK experiment. Secondly, we have developed a mapmaker for analysis of data from SPIDER. In the months leading up to the launch of SPIDER we will develop the mapmaker and look at the problem of correlated detector noise.
Planck Collaboration: Planck-HFI calibration & mapmaking

Figure 19. Signal (left), hit counts (center) and half differences between maps built with only the first and second half of each ring (right) for all HFI frequencies. The last column shows the half ring difference maps, scaled by the square root of the number of TOI samples, which largely removes this correlation. For the two highest frequencies, the differences present residual stripes and signal artefacts, at a low level (below 1\%) with respect to the sky signal. The differences maps have been degraded to the $N_{\text{side}} = 128$ HEALPix resolution.

This calibration is performed through a ring by ring template fit. Its limitations are consequences of the non-ideal behaviour of the ADC from the bolometer read-out electronics. Tiny deviations from linearity in these devices are causing apparent variation of the detector chain with time, which we have been addressing using effective gain correction, $\text{bogopix}$. We showed that this scheme was able to reduce the apparent gain variation in time from 1 to 2\% to lower than 0.3\% by studying the residual of the map reconstruction with time. Higher order signal distortions induced by this systematic effect prevent us from using the precise, orbital dipole based, calibration scheme presented in Tristram et al. (2011).

Correcting for these ADC non linearities should be made prior to any data reduction step. It requires precise measurements of each ADC response, which is currently taking place using warm data. First tests of corrective software are also under way, with promising results.

The calibration for the 545 and 857 GHz is performed by comparing Uranus and Neptune flux densities with models of their emissivities. We had to switch to this scheme due to apparent systematic effects in the FIRAS spectra we used in the HFI Early Data release. At those frequencies, gain variations are lower than the other systematic calibration uncertainties.

We revised our zero level setting method, which now relies on the CIB monopole and the zero of the Galactic emission defined as zero dust emission for a null HI column density.

The statistical uncertainty of the calibration is negligible for all frequencies with respect to the systematic uncertainties. The systematic uncertainty has been evaluated using several methods, presented in Sect. 7.

Figure 8.1: PLANCK HFI intensity maps across its frequency range from 100 GHz to 857 GHz. Figure taken from Ade et al. (2013b). Credit for image: PLANCK science team.
Figure 8.2: PLANCK foreground maps from a low resolution analysis, showing low frequency emission including synchrotron, anomalous and free-free emission (top), CO emission (middle) and dust emission (bottom). Figure taken from Ade et al. (2013c). Credit for image: PLANCK science team.
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148


