arXiv:quant-ph/0301148 v1 27 Jan 2003

On the Preparation of Pure States in Resonant Microcavities

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Abstract

We consider the time evolution of the radiation field (R) and a two-level atom (A) in a resonant microcavity in terms of the Jaynes-Cummings model with an initial general pure quantum state for the radiation field. It is then shown, using the Cauchy-Schwarz inequality and also a Poisson resummation technique, that *perfect* coherence of the atom can in general never be achieved. The atom and the radiation field are, however, to a good approximation in a pure state $|\psi\rangle_{A\otimes R} = |\psi\rangle_A \otimes |\psi\rangle_R$ in the middle of what has been traditionally called the "collapse region", independent of the initial state of the atoms, provided that the initial pure state of the radiation field has a photon number probability distribution which is sufficiently peaked and phase differences that do not vary significantly around this peak. An approximative analytic expression for the quantity $\mathrm{Tr}[\,\rho_A^2(t)\,]$, where $\rho_A(t)$ is the reduced density matrix for the atom, is derived. We also show that under quite general circumstances an initial entangled pure state will be disentangled to the pure state $|\psi\rangle_{A\otimes R}$.

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The Jaynes-Cummings (JC) model [1] is one of the simplest but non-trivial examples of two interacting quantum systems. It is an important fundamental theoretical model of the interaction between a two-level atom and a second-quantized single-mode electromagnetic field (for a review see e.g. Ref.[2]). The model is exactly solvable and realizable by experiments e.g. involving the passage of single atoms through a superconducting microwave cavity (see e.g. Refs.[3]). A direct experimental verification of the one-mode field quantization in a microcavity, as assumed in the JC model, has actually been carried out [4]. Experiments on ion-traps [5] provide for an alternative arena in which case JC-like models again are important theoretical tools in describing the relevant physics. Recent developments of nano-electronic devices has also led to an experimental realization of circuits with superconducting Josephson junctions which behaves as two-level systems [6]. Entangling such a device to a resonator naturally leads to a description in terms of an effective JC model (see e.g. Refs.[7]). The results of the present paper to be discussed below may therefore have a quite broad range of potential applications.

In spite of its apparent simplicity, the JC model has led to many non-trivial and unexpected results through the years as e.g. the well-known phenomena of collapses and revivals of the atomic population inversion [8]. In Ref. [9] it was found that the atom is to a good approximation in a pure state $|\psi\rangle_{A\otimes R}$ in the middle of the "collapse region" provided the initial state of the radiation field is a coherent state. It was shown [10] that the existence of this particular pure state can easily be demonstrated in terms of the properties of the von Neumann entropy in quantum mechanics. It has also been shown that the appearance of this pure atomic state is independent of the form of the initial pure atomic state [11]. For mixed states of the radiation field it is also possible to transfer coherence from atomic states to the state of the radiation field [12]. Here we will show that the purification of the atomic state is actually independent of the nature of the initial *pure* state of the radiation field provided that the corresponding number-operator probability distribution is sufficiently peaked and that phase differences of this state are slowly varying around this peak. We will, however, at this moment restrict ourselves to initial states which are not entangled and give some comments on entangled initial states in the end of this paper.

The electromagnetic interaction between a two-level atom, with level separation ω_0, and a single mode of the radiation field in a cavity with frequency ω is described, in the rotating wave approximation, by the JC Hamiltonian [1] (where we take $\hbar = 1$)

$$H = \omega a^\dagger a + \frac{1}{2}\omega_0 \sigma_z + g(a\sigma_+ + a^\dagger \sigma_-) \quad , \tag{1}$$

where the coupling constant g is proportional to the dipole matrix element of the atomic transition. Here we make use of the Pauli matrices to describe the two-level atom and the notation $\sigma_\pm = (\sigma_x \pm i\sigma_y)/2$. The second-quantized single mode electromagnetic field is described in a conventional manner by means of an annihilation (creation) operator a ($a^\dagger$), where we have suppressed the cavity mode labels. For $g = 0$ the atom-field states $|n, s\rangle = |n\rangle \otimes |s\rangle$ are characterized by the quantum numbers $n = 0, 1, \ldots$ of the oscillator and $s = \pm$ for the atomic levels with energies $E_{n,\pm} = \omega n \pm \omega_0/2$.

For reasons of simplicity we will now consider a resonant system, i.e. $\omega = \omega_0$. We also assume that initially the atom is in the excited state $|+\rangle$ and the radiation field is in a general pure state $|\gamma\rangle = \sum_{n=0}^{\infty} \sqrt{p_n}\, e^{i\alpha_n} |n\rangle$. Hence, the initial state of the system is $|\psi(0)\rangle = |+\rangle \otimes |\gamma\rangle$. The solution to the Schrödinger equation is then

$$|\psi(t)\rangle = \sum_{n=0}^{\infty} \sqrt{p_n}\, e^{i\alpha_n}\, e^{-i\omega(n+1/2)t} \Big(\cos(gt\sqrt{n+1})|n,+\rangle - i \sin(gt\sqrt{n+1})|n+1,-\rangle \Big) \quad . \tag{2}$$

In order to describe the evolution of the atom alone it is convenient to introduce the reduced density matrix $\rho_A(t) = \mathrm{Tr}_\gamma[\, |\psi(t)\rangle\langle\psi(t)| \,]$, where the trace is over a complete set of radiation field states. One easily finds that

$$\rho_A(t) = p_+(t)|+\rangle\langle+| + p_-(t)|-\rangle\langle-| + c(t)|+\rangle\langle-| + c(t)^*|-\rangle\langle+| \quad . \tag{3}$$

Here $p_+(t) = 1 - p_-(t) = \sum_{n=0}^{\infty}\ p_n \cos^2(gt\sqrt{n+1})$ is the well known form for the revival probability and

$$c(t) \equiv -i \sum_{n=0}^{\infty} \sqrt{p_n p_{n-1}} \cos(gt\sqrt{n+1}) \sin(gt\sqrt{n}) e^{i(\alpha_n - \alpha_{n-1})} e^{-i\omega t} \quad . \tag{4}$$

By straightforward algebra one can now verify that

$$\mathrm{Tr}[\, \rho_\gamma^2(t) \,] = \mathrm{Tr}[\, \rho_A^2(t) \,] = p_+^2(t) + p_-^2(t) + 2|c(t)|^2 \quad , \tag{5}$$

where the radiation field density matrix is $\rho_\gamma(t) = \mathrm{Tr}_A[\, |\psi(t)\rangle\langle\psi(t)| \,]$, and where the trace is over the atomic states. This means that purity occurs in both subsystems at the same time and precisely at the same rate. We notice that, in general, the ω dependence cancels out in $\mathrm{Tr}[\, \rho_\gamma^2(t) \,]$. For a coherent state $|z\rangle$, with $z = |z| \exp(i\alpha)$, we now have that $\alpha_n = n\alpha$ with photon-numbers that are Poisson distributed, i.e. $p_n = \exp(-|z|^2)|z|^{2n}/n!$. The α

dependence will therefore also cancel out in Tr[$\rho_\gamma^2(t)$] for any initial state $|\gamma\rangle$ which has the same phase dependence as a coherent state. By a tedious but straightforward calculation, the identity Tr[$\rho_\gamma^2(t)$] = Tr[$\rho_A^2(t)$] can actually be shown to be valid for any initial pure state of the form $|\psi(0)\rangle = (a|+\rangle + b|-\rangle) \otimes |\gamma\rangle$ with $|a|^2 + |b|^2 = 1$.

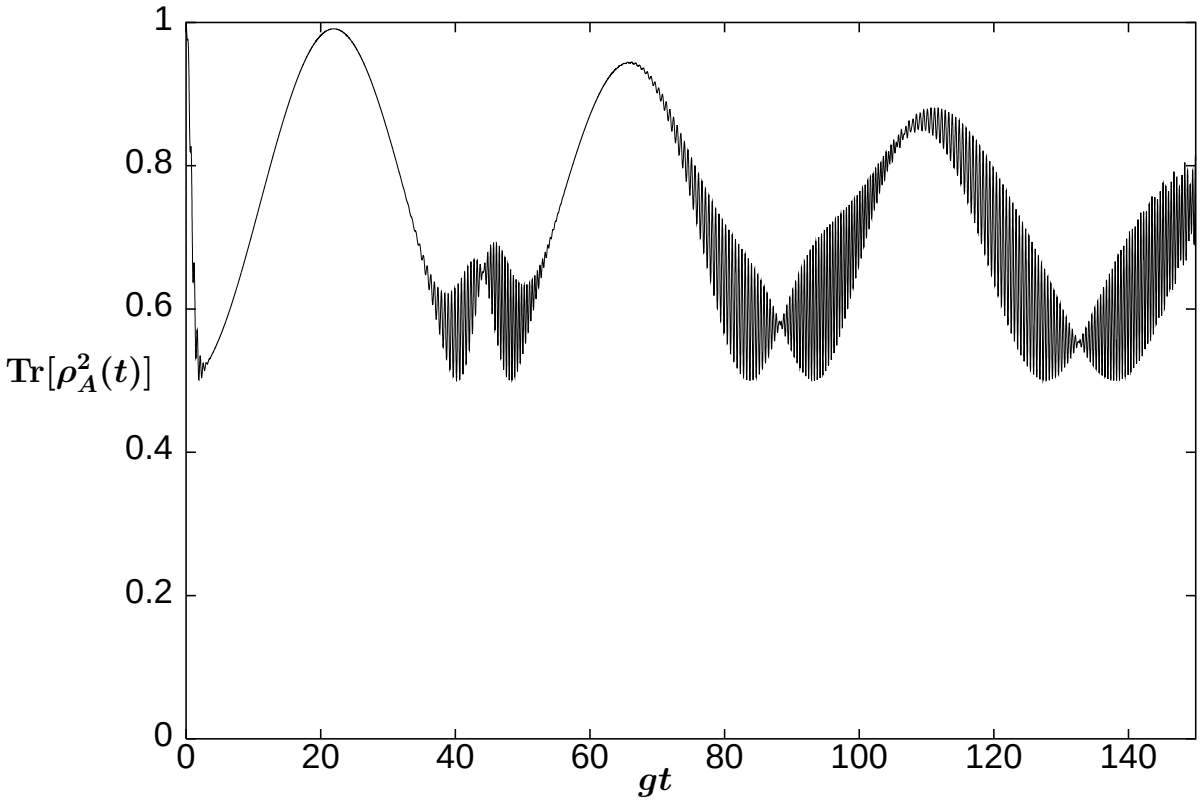

Figure 1: This figure shows Tr[$\rho_A^2(t)$] (= Tr[$\rho_\gamma^2(t)$]) as a function of gt with an initial coherent state of the radiation field with $|\alpha|^2 = 49$ based on the exact numerical results due to Eq. (5). This equation with a Gaussian photon-number distribution p_n, with mean $\bar{n} = |\alpha|^2$ and standard deviation $\sigma_n = \sqrt{\bar{n}}$, describes the data with a very high numerical accuracy. With our choice of parameters and within the accuracy of the figure shown, Eq. (5) and the approximative expression of Tr[$\rho_A^2(t)$] according to Eqs.(14) and (19) agree. As the mean number of photons gets larger Tr[$\rho_A^2(t \simeq t_0 = \pi\sqrt{\bar{n}})$] approaches one at an exponential rate as can be seen from Eq. (18).

The purity of the atomic state can then be determined considering the time evolution of the quantity Tr[$\rho_A^2(t)$][9]. A necessary and sufficient condition for the ensemble to be described in terms of a pure state is then that Tr[$\rho_A^2(t)$] = 1, in which case clearly a state-vector description of each individual system of the ensemble is possible. On the other hand, for a two-level system, a maximally mixed ensemble corresponds to

$\mathrm{Tr}[\,\rho_A^2(t)\,] = 1/2$. Alternatively one may consider the von Neumann entropy of the density matrix $\rho_A(t)$ [10] or simply its eigenvalues [11]. One can also make use of the Schmidt decomposition of composite system (see e.g. Ref.[13]) to reach the same results as given below.

The definition of $c(t)$ according to Eq. (4) above now suggests an immediate interpretation in terms of an l_2 Hilbert space scalar product of two complex vectors a and b with components $a_n = \sqrt{p_n}\cos(gt\sqrt{n+1})\exp(i\alpha_n)$ and $b_n = \sqrt{p_{n-1}}\sin(gt\sqrt{n})\exp(i\alpha_{n-1})$ respectively with $b_0 = 0$. The Cauchy-Schwarz inequality then tells us that $|c(t)|^2 \leq p_+(t)p_-(t)$ with equality if and only if the vectors a and b are parallel, i.e. $a_n = \beta(t)b_n$ where $|\beta(t)|^2 = p_+(t)/p_-(t)$. In general we then see that $1/2 \leq \mathrm{Tr}[\,\rho_A^2(t)\,] \leq 1$. The condition $a_n = \beta(t)b_n$ can be used to find a particular p_n such that $\mathrm{Tr}[\,\rho_A^2(t)\,] = 1$ at a fixed time $t = t_f$. In order to see this let us assume that $\alpha_n = \alpha n + \alpha_0$. At $t = t_f$ we then obtain a recursion formula $p_n\cos^2(gt_f\sqrt{n+1}) = |\beta(t_f)|^2 p_{n-1}\sin^2(gt_f\sqrt{n})$. Depending on the parameter gt_f this recursion formula can be solved. The corresponding probability distribution can now be used as an initial distribution and is then such that $\mathrm{Tr}[\,\rho_A^2(t=t_f)\,] = 1$ by construction. The time parameter t_f is, however, not in general related to any natural revival time of the system.

In order to proceed in a more general setting, we assume that the distribution p_n, with mean $\bar{n}$ and variance σ_n, is peaked around $n = \bar{n}$ or more precisely $\sigma_n/\bar{n} \ll 1$. We observe that σ_n should, however, not be arbitrarily small in order to have a non-zero $c(t)$. An explicit condition on σ_n will be discussed below using Poisson resummation techniques. Following the analysis of Ref.[9], and by considering time-scales t close to half of the first revival time $t_{rev} = 2\pi\sqrt{\bar{n}+1}/g$, i.e. $t \simeq t_0 \equiv t_{rev}/2$, one finds that $\mathrm{Tr}[\,\rho_A^2(t)\,] = 1/2 + 2|c(t)|^2$ using that $p_\pm(t \simeq t_0) = 1/2$. As long as $g|t - t_0| << 2\sqrt{\bar{n}+1}$, we can make use of the fact that $gt\sqrt{n+1} = gt\sqrt{n} + \pi/2$ [9] and we find that $|c(t)|^2 = 1/4$ provided $\sqrt{p_n}\exp(i\alpha_n) = \beta\sqrt{p_{n-1}}\exp(i\alpha_{n-1})$ for all n with a complex phase β ($|\beta| = 1$). This is possible with high accuracy only if *i*) p_n is non-zero (and constant) for a finite range of n as we actually have assumed, and *ii*) that the phases α_n are of the form $\alpha_n = \alpha n + \alpha_0$, i.e. the phase differences $\alpha_n - \alpha_{n-1}$ do not vary significantly around $n = \bar{n}$. Under such circumstances we then see that $\mathrm{Tr}[\,\rho_A^2(t)\,] = 1$ and the atom is in a pure state at $t \simeq t_0$.

In Fig.1 we illustrate the behaviour of $\mathrm{Tr}[\,\rho_A^2(t)\,]$ in the case of an initial coherent state for the radiation field with $\bar{n} \gg 1$. The distribution p_n can the be approximated with a sharp Gaussian distribution obeying the condition *i*) above with high accuracy. In Fig. 2 we illustrate the behaviour of $\mathrm{Tr}[\,\rho_A^2(t)\,]$ in the case of a pure state with p_n which varies rapidly in terms of a Schrödinger cat state here chosen to be of the form

$|\gamma\rangle = N(|\alpha\rangle + \exp(i\phi)|-\alpha\rangle)$ with $|N|^2 = 1/(2+2\cos(\phi)\exp(-2|\alpha|^2)$ (see e.g. Ref.[15]). For such a state it follows from the analysis above that $|c(t)|$ cannot reach its maximal value $1/2$ as is also clearly exhibited in Fig. 2. The behaviour of Schrödinger cat states at $t = t_0$ including the effects of damping has been studied in more detail in Ref.[16].

It is now clear from the definition Eq. (4) of $c(t)$ that if the phases α_n are rapidly varying functions of n, $c(t)$ will be small or zero. If in particular the phases are random the state of the radiation field will correspond to a mixture instead of a pure state and we have to average over the phases of $\rho_A(t)$ before computing its square. We then obtain $\mathrm{Tr}[\,\rho_A^2(t)\,] = 1 + 2p_+(t)(p_+(t)-1)$ with purity of the atomic state only at times t such that $p_+(t) = 1$ or 0. In such a case we also notice that $\mathrm{Tr}[\,\rho_A^2(t)\,]$ in general is different from $\mathrm{Tr}[\,\rho_\gamma^2(t)\,]$. The purity of the state of the radiation field is therefore required in order to purify a general initial mixture of the atom. For related studies of purification in the JC model see Refs.[17].

The quantity $\mathrm{Tr}[\,\rho_A^2(t)\,]$ can now be re-written in a form where quantum revivals become explicitly by making use of a Poisson summation technique [14]. As above, we consider a probability distribution p_n is peaked around $n = \bar{n}$ with $\sigma_n/\bar{n} \ll 1$. If the phase differences $\alpha_n - \alpha_{n-1}$ are slowly varying functions of n around $n = \bar{n} \gg 1$, i.e. we assume that $\alpha_n \simeq \alpha n + \alpha_0$ for $n \simeq \bar{n}$, we then see that $|c(t)|$ according to Eq. (4) can be written in the form

$$|c(t)| = \sum_{n=0}^{\infty} \sqrt{p_n p_{n-1}}\cos(gt\sqrt{n+1})\sin(gt\sqrt{n}) \approx \frac{1}{2}(S_s(t) - S(t)) \quad , \tag{6}$$

and we can then write $\mathrm{Tr}[\,\rho_A^2(t)\,]$ in the following convenient form

$$\mathrm{Tr}[\,\rho_A^2(t)\,] \approx \frac{1}{2}(1 + S_c^2(t) + S_s^2(t) + S^2(t) - 2S(t)S_s(t)) \quad , \tag{7}$$

where we have defined the functions

$$S_c(t) = \sum_{n=0}^{\infty} p_n \cos(2gt\sqrt{n+1}) \quad , \quad S_s(t) = \sum_{n=0}^{\infty} p_n \sin(2gt\sqrt{n+1}) \quad , \tag{8}$$

and

$$S(t) = \sum_{n=0}^{\infty} p_n \sin(gt/2\sqrt{n+1}) \quad . \tag{9}$$

An exact Poisson resummation technique [14] now enables us to e.g. write

$$S_c(t) + iS_s(t) = \sum_{\nu=-\infty}^{\infty} f_\nu(t) + p_0\frac{e^{2igt}}{2} \quad , \tag{10}$$

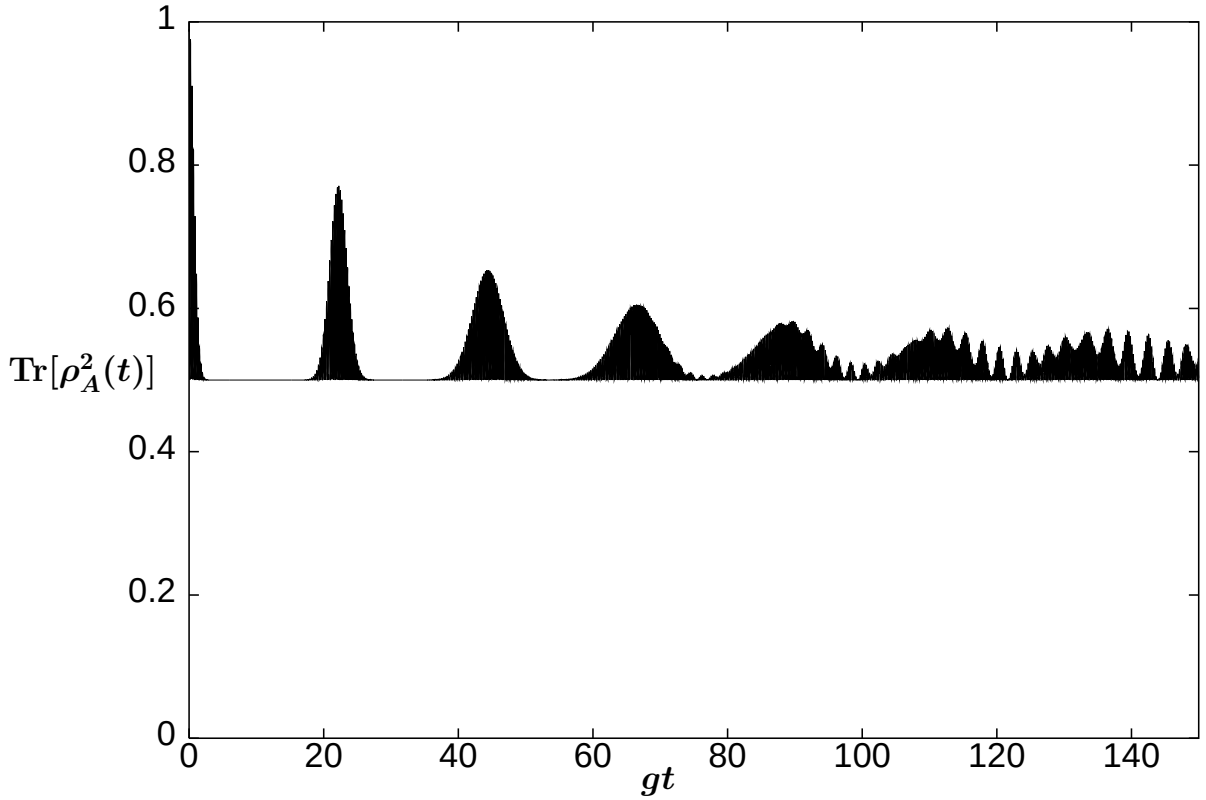

Figure 2: The quantity $\mathrm{Tr}[\,\rho_A^2(t)\,]$ $(=\mathrm{Tr}[\,\rho_\gamma^2(t)\,])$ as a function of gt when the initial state of the radiation field is a Schrödinger cat state where $p_n = |\alpha|^{2n}\,[1+(-1)^n\cos\phi]\,/[\,n!(e^{|\alpha|^2}+e^{-|\alpha|^2}\cos\phi)\,]$ with $|\alpha|^2 = 49$ and $\phi = 0$. Purity of the atom occurs in this case for $gt = 0$ only.

where

$$f_\nu(t) = \int_0^\infty dn\; p(n) e^{2iS_\nu(n)} \quad . \tag{11}$$

Here $p(n)$ is an analytical continuation of p_n and $S_\nu(n) = \pi\nu n - gt\sqrt{n+1}$. If $p(n)$ varies slowly as compared to the variation of $S_\nu(n)$ one can evaluate $f_\nu(t)$ for $\nu \neq 0$ by a stationary phase approximation [14]. For a Gaussian distribution with mean $\bar{n}$ and variance σ_n^2, i.e. $p(n) = \exp(-(n-\bar{n})^2/2\sigma_n^2)/\sqrt{2\pi\sigma_n^2}$, this corresponds to the condition $\sigma_n^2 \gg (\bar{n}+1)/\pi\nu$, provided $\bar{n} \gg 1$. The contribution from $\nu = 0$ can be estimated by again considering a Gaussian distribution with $\bar{n} \gg 1$ in which case one finds that

$$\int_0^\infty dn\; p(n) e^{2igt\sqrt{n+1}} \approx e^{-(gt\sigma_n)^2/2\bar{n}} e^{2igt\sqrt{\bar{n}+1}} \quad . \tag{12}$$

The stationary phase condition on $S_\nu(n)$ now immediately leads to the general revival times $t_{rev} = 2\pi\nu\sqrt{\bar{n}+1}/g$. Applying the same techniques to $S(t)$ according to Eq. (9) leads to revival times which are of the order $\bar{n}$ larger than the those of $S_c(t)$ and $S_s(t)$ and we therefore make the approximation

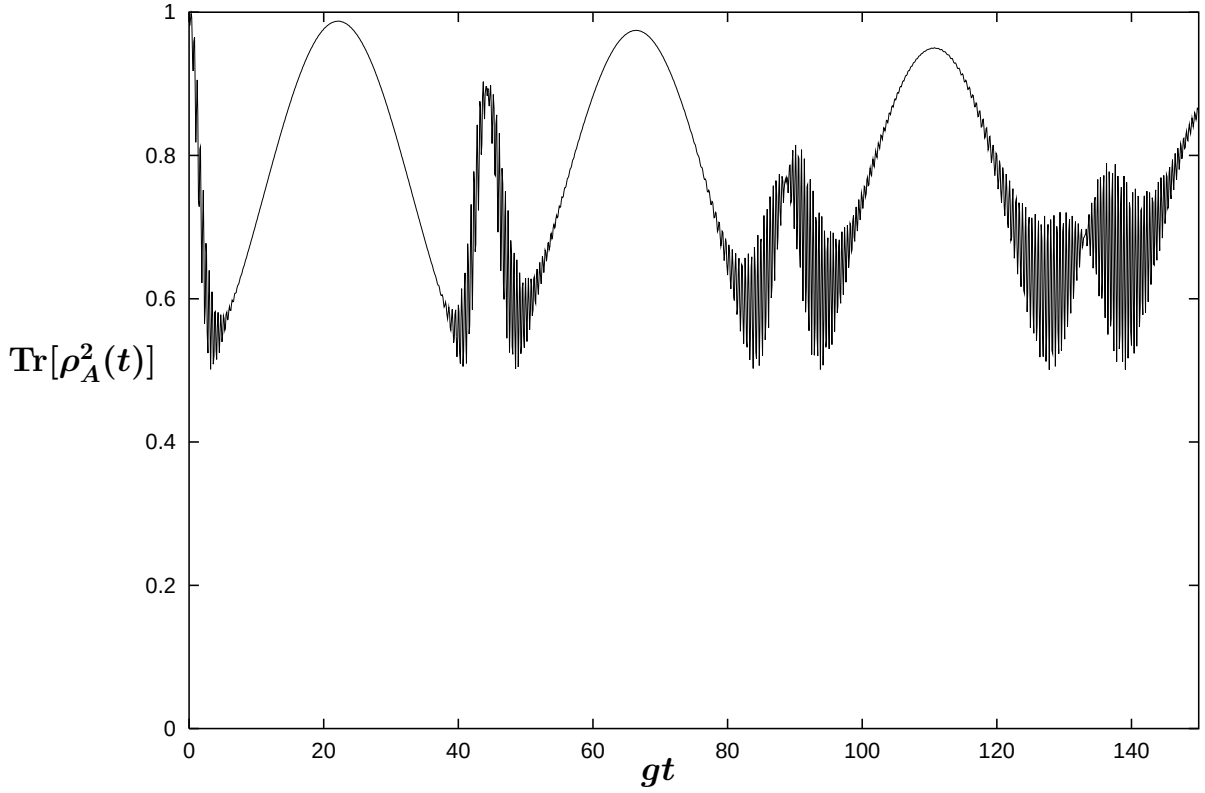

Figure 3: This figure shows illustrates the effect of squeezing on $\mathrm{Tr}[\,\rho_A^2(t)\,]$ $(=\mathrm{Tr}[\,\rho_\gamma^2(t)\,])$ as a function of gt when the initial state of the radiation field is a squeezed coherent state $|\gamma\rangle = S(r)|\alpha\rangle$ with $r = 0.75$ and a real α (≈ 14.72) chosen such that the mean number of photons $\bar{n} = 49$ is the same as in Fig. 1. The photon-number distribution is well approximated by a "squeezed" Gaussian distribution with mean $\bar{n}$ and $\sigma_n^2 = \bar{n}^{0.65}$.

$$\sum_{n=0}^{\infty} p_n \sin\Big(\frac{gt}{2\sqrt{n+1}}\Big) \approx \int_0^{\infty} dn\; p(n) \sin\Big(\frac{gt}{2\sqrt{n+1}}\Big) \approx e^{-(gt\sigma_n)^2/32\bar{n}^3} \sin\Big(\frac{gt}{2\sqrt{\bar{n}+1}}\Big) \quad , \tag{13}$$

where we again have made use of Gaussian distribution and $\bar{n} \gg 1$. The approximations Eqs.(6), (12) and (13) together with the stationary approximation above enables us to write the quantity $\mathrm{Tr}[\,\rho_A^2(t)\,]$ in an analytical form, i.e.

$$\mathrm{Tr}[\,\rho_A^2(t)\,] \approx \frac{1}{2} + \frac{1}{2}\; e^{-(gt\sigma_n)^2/16\bar{n}^3} \sin^2\Big(\frac{gt}{2\sqrt{\bar{n}+1}}\Big) + \sum_{\nu=0}^{\infty} w_\nu(t)\;, \tag{14}$$

where

$$w_0(t) = \frac{1}{2}\Big[\; e^{-(gt\sigma_n)^2/\bar{n}} - 2\, e^{-(gt\sigma_n)^2(1+1/16\bar{n}^2)/2\bar{n}} \sin\Big(\frac{gt}{2\sqrt{\bar{n}+1}}\Big) \sin(2gt\sqrt{\bar{n}+1})\Big] \;, \tag{15}$$

and for $\nu \geq 1$

$$\begin{aligned} w_\nu(t) \;&=\; \frac{1}{2}\Big[\; \Big(\frac{gt}{\pi\sqrt{2\nu^3}}\Big)^2 \; p^2\Big(\,(\frac{gt}{2\pi\nu}\,)^2\Big) \\ &-\; 2\,\frac{gt}{\pi\sqrt{2\nu^3}}\; p\Big(\,(\frac{gt}{2\pi\nu}\,)^2\Big)\; e^{-(gt\sigma_n)^2/32\bar{n}^3} \sin\Big(\frac{gt}{2\sqrt{\bar{n}+1}}\Big) \sin\Big(\frac{(gt)^2}{2\pi\nu} - \frac{\pi}{4}\Big)\;\Big]\;. \end{aligned} \tag{16}$$

Apart from the approximations used in order to obtain Eqs.(12)-(13), the Poisson resummed expression for Tr[$\rho_A^2(t)$] as given by Eq. (14) with Eqs.(15)-(16) is valid for any distribution $p(n)$ provided that $\bar{n} \gg 1$ and

$$(\bar{n}+1)/\nu\pi \ll \sigma_n^2 \ll \bar{n}^2 \quad . \tag{17}$$

With this condition, which is always satisfied for a Poissonian distribution, we have also verified through numerical calculations that the various approximations leading to Eq. (14) describe a much larger class of p_n-distributions than just Gaussian ones. For $\bar{n} \gg 1$ we can now extract the leading behaviour of Tr[$\rho_A^2(t)$] close to $t = t_{rev}/2$, i.e.

$$\mathrm{Tr}[\,\rho_A^2(t \simeq \frac{1}{2}t_{rev})] \simeq \frac{1}{2}\Big(1 + e^{-\sigma_n^2\pi^2/16\bar{n}^2} + e^{-\sigma_n^2\pi^2} - 2e^{-\sigma_n^2\pi^2/2}\sin(2\pi\bar{n})\Big) \quad , \tag{18}$$

which show the exponential approach to purity. These results shows that the quantity Tr[$\rho_A^2(t)$] exhibits revivals not only for an initial coherent state, as one is lead to believe from Ref. [9], but for any initial state of the radiation field in the form of a pure state $|\gamma\rangle = \sum_{n=0}^{\infty}\sqrt{p_n}\,e^{i\alpha_n}|n\rangle$ with a sufficiently peaked probability distribution p_n, satisfying Eq. (17), and with phase differences $\alpha_n - \alpha_{n-1}$ varying slowly around this peak.

In particular, if the radiation field is initially in a coherent state with average photon number $\bar{n} = |\alpha|^2$ as in Ref. [9], then p_n is Poisson distributed and reduces to the Gaussian distribution with variance $\sigma_n^2 = \bar{n}$ if $\bar{n} \gg 1$. In this case, Eq. (16) can be reduced to

$$\begin{aligned} w_\nu(t) &= \frac{1}{2}\Big[\,\frac{(gt)^2}{4\pi^3\nu^3\bar{n}}\,e^{-\frac{(gt-g\bar{t}_\nu)^2}{\pi^2\nu^2}} \\ &- \frac{gt}{\sqrt{\pi^3\nu^3\bar{n}}}\,e^{-\frac{1}{8}(\frac{gt}{2|\alpha|^2})^2}e^{-\frac{(gt-g\bar{t}_\nu)^2}{2\pi^2\nu^2}}\sin\Big(\frac{gt}{2\sqrt{|\alpha|^2}}\Big)\sin\Big(\frac{(gt)^2}{2\pi\nu} - \frac{\pi}{4}\Big)\,\Big]\,, \end{aligned} \tag{19}$$

where

$$g\bar{t}_\nu = 2\pi\nu\sqrt{\bar{n}} \quad . \tag{20}$$

As shown in Fig. 1 the analytical form Eq. (14) with Eqs.(15) and (19) describes the actual from of Tr[$\rho_A^2(t)$] with a very high numerical accuracy. If we make the photon-number distribution p_n more peaked as compared to e.g. a Poissonian distribution we now see from Eq. (14) that the approach to purity at later times $t_{rev}/2 + 2\pi k$, with $k = 1, 2, ...$, will be more visible. In Fig. 3 we illustrate this feature for a squeezed coherent state $|\gamma\rangle = S(r)|\alpha\rangle$ with the same mean-value of photons as in Fig. 1. Here we consider a squeezing operator $S(r) = \exp(r(a^2 - a^{\dagger 2})/2)$ and α real leading to the photon-number distribution [18]

$$p_n = \frac{\tanh(r)^n}{n!2^n\cosh(r)}\exp\Big(-|\alpha|^2(1-\tanh(r))\Big)H_n\Big(\frac{\alpha}{\sqrt{\sinh(2r)}}\Big)^2 \quad , \tag{21}$$

where H_n is a Hermite polynomial. In this case the Poisson resummed expression Eq. (14) reproduces the exact answer, within the numerical accuracy of Fig. 3, by making use of the fact that the photon-number distribution is well approximated by a "squeezed" Gaussian distribution with mean $\bar{n}$ and $\sigma_n^2 = \bar{n}^{0.65}$ except for times close to t_0. For the parameters chosen the inequality Eq. (17) actually fails for $\nu = 1$. Evaluating the integral Eq. (11) more carefully for $\nu = 1$, which e.g. can be done analytically by making use of a Gaussian approximation for p_n, one restores the agreement between the exact answer and the approximation Eq. (14) with high numerical accuracy.

In concluding we would like to comment on the nature of pure state $|\psi\rangle_{A\otimes R}$ at $t \simeq t_0 = t_{rev}/2 = \pi\sqrt{\bar{n}+1}/g$. We again assume that $\bar{n} \gg 1$ and that the conditions Eq. (17) are fulfilled for any probability distribution p_n under consideration. As an initial state we consider a general pure entangled state, i.e.

$$|\psi\rangle = a|+\rangle \otimes |\gamma_+\rangle + b|-\rangle \otimes |\gamma_-\rangle \quad , \tag{22}$$

where $|a|^2 + |b|^2 = 1$ and $|\gamma_\pm\rangle = \sum_{n=0}^{\infty} \sqrt{p_n^\pm}\, e^{i\alpha_n^\pm}|n\rangle$. As long as $|t - t_0| << 2\sqrt{\bar{n}+1}/g$ we again make use of the fact that $gt\sqrt{n+1} = gt\sqrt{n} + \pi/2$ [9] and we find that

$$|\psi(t \simeq t_0)\rangle \simeq (|+\rangle + |-\rangle e^{i\omega t_0} e^{-i\Delta\alpha}) \otimes |\gamma(t \simeq t_0)\rangle \quad , \tag{23}$$

with $\Delta\alpha = \Delta\alpha^\pm(\bar{n})$, where $\Delta\alpha^\pm(n) \equiv \alpha_n^\pm - \alpha_{n-1}^\pm$, and

$$|\gamma(t)\rangle = -\sum_{n=0}^{\infty} e^{-i\omega(n+1/2)t}\Big(ae^{i\alpha_n^+}\sqrt{p_n^+}\sin(gt\sqrt{n}) + ibe^{i\alpha_{n+1}^-}\sqrt{p_{n+1}^-}\cos(gt\sqrt{n})\Big)|n\rangle \quad . \tag{24}$$

In order to obtain Eq. (23) we only assumed that the distributions $p_n^\pm$ have their main contribution at the same $n = \bar{n}$ around which $\Delta\alpha^\pm(n)$ are assumed to be slowly varying and equal when evaluated at $n = \bar{n}$. Apart from the phase factor $e^{i\omega t_0}$, Eq. (23) agrees with the result of Ref.[9]. With regard to experimental realizations it is, however, important to realize that the relative phase in Eq. (23) depends on both $\Delta\alpha$ and ωt_0. The JC-model therefore predicts disentanglement at $t \simeq t_{rev}/2$ to the pure atomic state $|\psi\rangle_A = (|+\rangle + |-\rangle e^{i\omega t_0} e^{-i\Delta\alpha})/\sqrt{2}$ fairly independent of the nature of the initial pure entangled state.

In summary, we have shown that *exact* coherence of the atom is in general never regained for a JC model with a general initial pure quantum state $|\gamma\rangle$ of the radiation field. One can, however, come arbitrarily close to a disentangled pure atomic state $|\psi\rangle_{A\otimes R}$ when $|\gamma\rangle$ has a general form $\sum_{n=0}^{\infty} \sqrt{p_n}\, e^{i\alpha_n}|n\rangle$ provided that the probability distribution p_n is sufficiently peaked around its mean value $\bar{n} \gg 1$ and that the phases differences

$\alpha_n - \alpha_{n-1}$ do not vary significantly around this peak. Under such conditions we have then derived an approximative analytical expression for the quantity Tr[$\rho_A^2(t)$]. Hence, the quantity Tr[$\rho_A^2(t)$] exhibits revivals not only for an initial coherent state of the radiation field, as one is lead to believe by the results of Ref. [9], but actually for a very general set of pures of the radiation field. By making use of the analysis of Ref.[11] one can, in a straightforward manner, make use of the methods above and verify that this actually remains true independent of the initial state of the atom. We have also seen that under quite general circumstances an initial pure entangled state also leads to the same purification of the atomic state.

ACKNOWLEDGMENT

This work was supported in part by the UK Engineering and Physical Science Research Council, by the European Union IHP Network "QUEST" and the IST Network QUBIT. P.K.R. acknowledges support by the Research Council of Norway under contract no. 151565/432. B.-S.S. wishes to thank NorFA for support and Göran Wendin and the Department of Microelectronics and Nanoscience for hospitality.

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