



On the uniform torsional rigidity of square concrete-filled steel tubular (CFST) sections

W.F. Zhang^a, L. Gardner^{b,*}, M.A. Wadee^b, K.S. Chen^c, W.Y. Zhao^c

^a School of Civil Engineering & Architecture, Nanjing Institute of Technology, Nanjing, China

^b Department of Civil and Environmental Engineering, Imperial College London, London, UK

^c School of Civil and Architecture Engineering, Northeast Petroleum University, Daqing, China

ARTICLE INFO

Keywords:

Composite construction
Concrete-filled steel tubes
Finite element modelling
Uniform torsional rigidity

ABSTRACT

There are a number of scenarios in which structural members experience torsion, including under the direct application of torsional loading, when transverse loading is applied at an eccentricity to the shear centre and as a second order effect arising from lateral torsional instability. To date, the torsional rigidity of concrete-filled steel tubular (CFST) sections has yet to be fully explored; hence a study into the uniform torsional rigidity of square CFST sections is presented herein. First, the strain energy of square CFST sections is formulated, in which the longitudinal warping displacement is assumed to have an undetermined constant. The undetermined constant is then deduced by means of the principle of minimum strain energy, and thus an analytical expression for the uniform torsional rigidity of square CFST sections is obtained. The accuracy of the derived formula is verified against existing theoretical solutions for simplified scenarios, test data and the results of numerical simulations. Finally, the influence of the key parameters in the derived formula for the torsional rigidity of square CFST sections are analysed, and a simplified design formula is presented.

1. Introduction

Concrete-filled steel tubular (CFST) members offer a number of benefits over bare steel or reinforced concrete elements in terms of structural performance and constructability [1–3]. These benefits have led to broadening practical applications and include (1) the enhanced strength and ductility of the core concrete due to the confining action of the steel tube, (2) the increased resistance to local buckling of the steel tube due to the presence of the core concrete restricting inward deformations, (3) the heat sink effect of the concrete improving the fire resistance of CFST members [4–7] and (4) the improved constructability associated with the steel tube acting as permanent formwork for casting of the core concrete.

It is well established that the confinement effect in CFST members is dependent on the shape of the cross-section, and that square CFST columns generally have slightly lower strength and ductility than equivalent circular CFST columns [2,8]. However, square CFST columns are often preferred for functional reasons, including more practical beam-to-column connections.

Experimental and numerical studies into the behaviour of square CFST members under a range of loading conditions have been presented

in the literature. Uy et al. [9] conducted a series of experiments on square CFST beam-columns, subjected to combinations of loading from pure compression to pure bending. Further experiments, together with numerical simulations, were performed by Uy et al. [10] on square CFST beam-columns using high strength steel sections with yield strengths up to 750 MPa. Interaction formulae for design purposes were developed on the basis of the findings. Varma et al. [11] tested square CFST beam-columns using high strength concrete with cylinder strengths up to 110 MPa. Ouyang and Kwan [12] presented a finite element (FE) study of axially loaded square CFST members, in which the FE model was first validated against test results on CFST specimens collected from the literature covering a wide range of concrete cylinder strengths from 24 to 110 MPa, steel yield strengths from 262 to 835 MPa and steel tube depth-to-thickness ratios from 18 to 102. Zhou et al. [13] presented an experimental study into the mechanical behaviour, particularly the tensile stiffness, of square CFST members under axial tension. It was found that the tensile stiffness of square CFST members was, on average, around 30 % higher than that of the unfilled members. Cai et al. [14] examined experimentally the mechanical performance of square CFST elements under predominantly shear loading, while Li et al. [15] explored the flexural behaviour of square CFSTs with high strength

* Corresponding author.

E-mail address: leroy.gardner@imperial.ac.uk (L. Gardner).

<https://doi.org/10.1016/j.istruc.2022.06.046>

Received 19 May 2021; Received in revised form 17 June 2022; Accepted 18 June 2022

Available online 26 June 2022

2352-0124/© 2022 The Author(s). Published by Elsevier Ltd on behalf of Institution of Structural Engineers. This is an open access article under the CC BY license (<http://creativecommons.org/licenses/by/4.0/>).

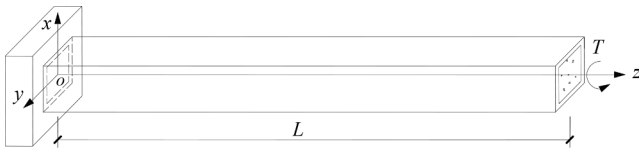


Fig. 1. Square CFST member subjected to torsional moment T .

concrete and high strength square steel. Han et al. [16] studied the torsional behaviour of square and circular CFST subjected to pure torsion using finite element analysis.

Structural members can experience torsion in a number of scenarios, including under the direct application of torsional loading, when transverse loading is applied at an eccentricity to the shear centre and as a second order effect arising from lateral torsional instability. Common examples of where such actions can arise in CFST members include CFST beams that are curved on plan, CFST corner columns of buildings experiencing twisting under wind or earthquake loading and I-beams with CFST compression flanges undergoing lateral torsional buckling [17]. It is clear from the literature that, while there have been many investigations in the structural response of square CFST sections in compression and bending, research into the torsional performance of square CFST sections is limited; this is therefore the focus of the present study. Specifically, an analytical expression for the uniform torsional rigidity of square CFST sections is first derived by means of the principle of minimum strain energy. Emphasis is placed on the uniform torsional rigidity of square CFST sections since it is well established that this is dominant over non-uniform (warping) torsional rigidity in the case of closed sections. The accuracy of the derived formula is then assessed against existing theoretical results and test data, as well as finite element simulations performed herein. Finally, the importance of the constituent components of the theoretical formulation is analysed, and a simplified design formula is presented.

2. Theory of CFST members under uniform torsion

2.1. Problem description

The primary objective of the present study is to examine the response of square CFST members under uniform torsion, as shown in Fig. 1. The following notation is adopted: E_s , G_s and μ_s are the modulus of elasticity, shear modulus and Poisson's ratio of the steel cross-section, and E_c , G_c and μ_c are the modulus of elasticity, shear modulus and Poisson's ratio of the filled concrete. The length of the square CFST member is denoted by

L , while the section depth and the wall thickness of the steel tube are denoted by D and t respectively.

When a square CFST member is subjected to a torsional moment T , the resulting torsional deformation is shown in Fig. 2, in which $\theta(z)$ is the angle of twist of the cross-section. The angle of twist at the tip (i.e. the tip rotation) θ_L of a member of length L is given by:

$$\theta_L = \frac{TL}{(GJ)_k} \quad (1)$$

where $(GJ)_k$ is the torsional rigidity of CFST cross-sections, considering the contributions of both concrete core and the steel tube, to be determined in this study.

2.2. Basic assumptions

The following basic assumptions are made in deriving the torsional rigidity of square CFST sections:

- (1) The cross-sections of the square CFST members do not distort in-plane during twisting, such that every point in the cross-section rotates through the angle $\theta(z)$ about the centre of twist. This assumption is akin to Vlasov's rigid peripheral assumption of thin-walled bars [18]. Therefore, based on this assumption, the transverse deformations at any point on the cross-section of square CFST members can be uniquely determined.
- (2) There is no slip in the longitudinal direction at the interface between the core concrete and the steel tube, i.e. longitudinal warping of the steel and concrete are compatible at the interface during uniform torsion of the square CFST members.
- (3) The square CFST member is subjected to pure torque, that is, the longitudinal warping is not constrained and does not vary along the length of the member.

2.3. Strain energy

2.3.1. Displacement field

Only uniform (so-called 'free') torsion is considered, i.e. the case where the twist rate is constant. According to the first basic assumption, in the coordinate system shown in Fig. 2, the translational displacement of any point on the CFST section along the x and y directions can be expressed as:

$$u(y, z) = -y\theta; \quad v(x, z) = x\theta \quad (2)$$

According to the second basic assumption, the longitudinal

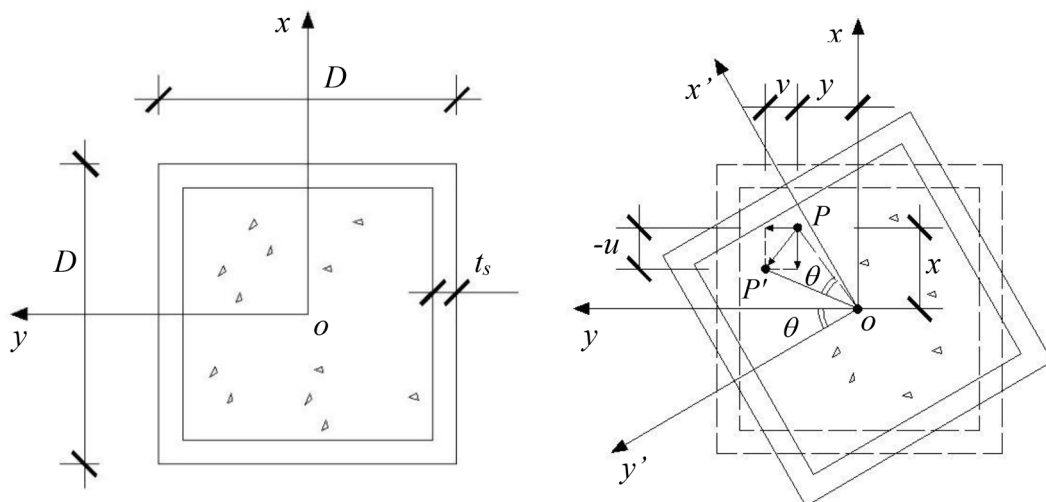


Fig. 2. Torsional deformation of square CFST cross-section.

displacement, i.e. the warping function of any point on the CFST section along the z -axis is:

$$w = A_1 xy(x^2 - y^2) \left(\frac{d\theta}{dz} \right) \quad (3)$$

where A_1 is a generalized coordinate representing the warping constant to be determined by the minimum strain energy principle and $d\theta/dz$ is the rate of twist.

2.3.2. Strain field

According to the theory of elasticity, there are a total of 6 independent strains. According to the well-known strain–displacement equations, these can be expressed thus:

$$\varepsilon_x = \frac{\partial u}{\partial x} = 0; \quad \varepsilon_y = \frac{\partial v}{\partial y} = 0; \quad \gamma_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} = \theta - \theta = 0 \quad (4)$$

$$\varepsilon_z = \frac{\partial w}{\partial z} = A_1 xy(x^2 - y^2) \left(\frac{d^2\theta}{dz^2} \right) \quad (5)$$

$$\gamma_{xz} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} = A_1 \left(\frac{d\theta}{dz} \right) (3x^2y - y^3) - y \left(\frac{d\theta}{dz} \right) \quad (6)$$

$$\gamma_{yz} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} = A_1 \left(\frac{d\theta}{dz} \right) (x^3 - 3xy^2) + x \left(\frac{d\theta}{dz} \right) \quad (7)$$

According to Saint-Venant torsion theory, there are only two non-zero strains (γ_{xz} and γ_{yz}); the remainder, i.e. the first four strains, should be equal to zero. Clearly, according to Eq. (3), to fulfil the condition that $\varepsilon_z = 0$, it follows that:

$$A_1 \neq 0, \quad \left(\frac{d^2\theta}{dz^2} \right) = 0 \quad (8)$$

Upon integrating the second term with respect to z , it follows that $(d\theta/dz)$ is a constant, i.e. the change of the rotation along the length is constant. This is the free torsion situation studied in the current section, since the third basic assumption is satisfied. Henceforth, $d\theta/dz$ is denoted as ϕ .

2.3.3. Constitutive equations

According to the theory of elasticity, the constitutive equations of the square CFST member are:

$$\begin{aligned} \text{Steel: } \tau &= G_s \gamma \\ \text{Concrete: } \tau &= G_c \gamma \end{aligned} \quad (9)$$

2.3.4. Strain energy

The strain energy of the elastic solid considered currently is:

$$U = \frac{1}{2} \iiint_V [G\gamma_{xz}^2 + G\gamma_{yz}^2] dx dy dz \quad (10)$$

since $\varepsilon_x = \varepsilon_y = \varepsilon_z = \gamma_{xy} = 0$. Substituting non-zero strain expressions γ_{xz} and γ_{yz} and using integration to consider the contributions from the steel and core concrete, the following expression is obtained:

$$\begin{aligned} U &= \frac{1}{2} \int_0^L \int_{-D/2}^{D/2} \int_{-D/2}^{D/2} G_s [(A_1 \phi (3x^2y - y^3) - y\phi)^2 + (A_1 \phi (x^3 - 3xy^2) + x\phi)^2] dy dx dz \\ &\quad - \frac{1}{2} \int_0^L \int_{-D/2+t}^{D/2-t} \int_{-D/2+t}^{D/2-t} G_s [(A_1 \phi (3x^2y - y^3) - y\phi)^2 + (A_1 \phi (x^3 - 3xy^2) + x\phi)^2] dy dx dz \\ &\quad - \frac{1}{2} \int_0^L \int_{-D/2+t}^{D/2-t} \int_{-D/2+t}^{D/2-t} G_c [(A_1 \phi (3x^2y - y^3) - y\phi)^2 + (A_1 \phi (x^3 - 3xy^2) + x\phi)^2] dy dx dz \end{aligned} \quad (11)$$

After integration with respect to x and y , the expression becomes:

$$U = \frac{1}{2} \phi^2 \frac{L}{840} \begin{pmatrix} (D-2t)^4 \left(\frac{9(D-2t)^4 A_1^2 -}{28(D-2t)^2 A_1 + 140} \right) G_c + \\ D^4 (9D^4 A_1^2 - 28D^2 A_1 + 140) G_s - \\ (D-2t)^4 \left(\frac{9(D-2t)^4 A_1^2 -}{28(D-2t)^2 A_1 + 140} \right) G_s \end{pmatrix} \quad (12)$$

2.3.5. Principle of minimum strain energy

For the square CFST torsion problem studied currently, the minimum strain energy principle can be stated as seeking the constant A_1 that minimizes the strain energy U given in Eq. (12), thus:

$$\frac{\partial U}{\partial A_1} = 0 \quad (13)$$

which leads to:

$$\frac{1}{840} \phi^2 \begin{pmatrix} (D-2t)^4 (18(D-2t)^4 A_1 - 28(D-2t)^2) G_c + \\ D^4 (18D^4 A_1 - 28D^2) G_s - \\ (D-2t)^4 (18(D-2t)^4 A_1 - 28(D-2t)^2) G_s \end{pmatrix} = 0 \quad (14)$$

from which the following solution for the constant A_1 can be obtained:

$$A_1 = \frac{\frac{1}{30}(D-2t)^6 G_c + \frac{1}{30} D^6 G_s - \frac{1}{30}(D-2t)^6 G_s}{\frac{3}{140}(D-2t)^8 G_c + \frac{3}{140} D^8 G_s - \frac{3}{140}(D-2t)^8 G_s} \quad (15)$$

Substituting this into Eq. (6) and Eq. (7), the shear strains (γ_{xz} and γ_{yz}) can be obtained. By using the constitutive equations, the corresponding shear stresses in the steel and core concrete can be deduced.

2.3.6. Torsional rigidity of square CFST sections

Substituting the constant A_1 , given by Eq. (15), into Eq. (12), we obtain:

$$U = \frac{1}{2} (GJ)_{k,SE} \phi^2 L \quad (16)$$

where $(GJ)_{k,SE}$ is the uniform torsional rigidity derived from the principle of minimum strain energy of a square CFST cross-section, which can be expressed as follows:

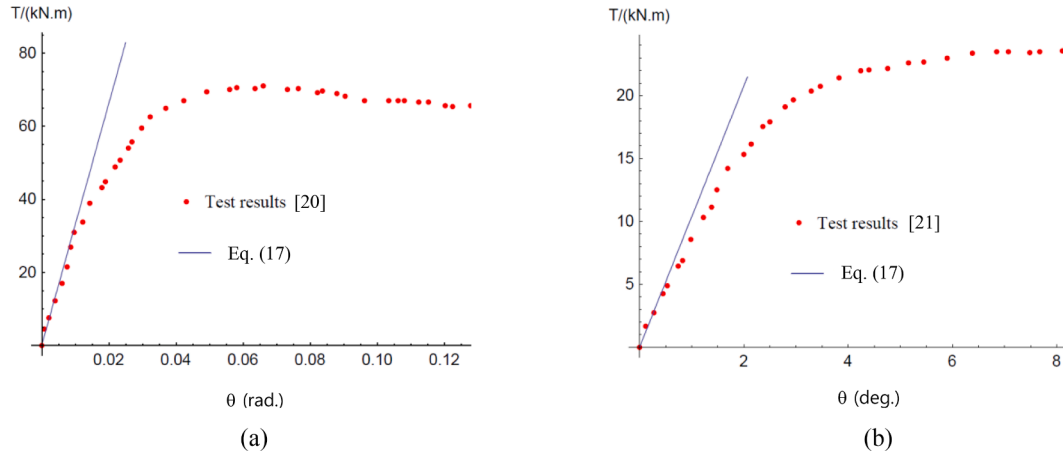
$$(GJ)_{k,SE} = \frac{19D^4 G_s}{135} \Psi(m, \alpha) = 0.1407 D^4 G_c \Psi(m, \alpha) \quad (17)$$

where.

Table 1

Geometric and material properties of CFST members tested in [20,21].

Reference	L (mm)	D (mm)	t (mm)	f_y (MPa)	f_{cu} (MPa)	f_{ck} (MPa)	E_s (GPa)	E_c (GPa)
[20]	1620	200	4.5	261.4	39.0	26.1	206	32.4
[21]	1590	123.5	4.5	274	25.6	17.2	206	28.1

**Fig. 3.** Comparison between test T - θ curve from (a) [20] and (b) [21] and Eq. (17).

$$\Psi(m, \alpha) = \frac{45}{38m} \frac{(-m(1-0.5\alpha)^4 + m + (1-0.5\alpha)^4)}{7(-m(1-0.5\alpha)^6 + m + (1-0.5\alpha)^6)^2} \frac{45(-m(1-0.5\alpha)^4 + m + (1-0.5\alpha)^4) \times (-m(1-0.5\alpha)^8 + m + (1-0.5\alpha)^8) - 7(-m(1-0.5\alpha)^6 + m + (1-0.5\alpha)^6)^2}{7(-m(1-0.5\alpha)^6 + m + (1-0.5\alpha)^6)^2} + 1 \quad (18)$$

is the non-dimensional constant of uniform torsional rigidity of square CFST sections, $m = G_s/G_c$ is the ratio of the shear modulus of steel to that of concrete, $\alpha = A_s/A_c$, referred to as the steel ratio, is the ratio of the cross-sectional area of the steel A_s to the cross-sectional area of the concrete A_c , and can be approximated as $\alpha \approx 4t/D$.

3. Verification of uniform torsional rigidity of square CFST sections

3.1. Theoretical verification

When the steel ratio $\alpha = 0$, Eq. (17) reduces to the uniform torsional rigidity of a square section of a single material (pure concrete), i.e.

$$(GJ)_{k,SE} = \frac{19D^4G_s}{135} \Psi(m, \alpha) = \frac{19D^4G_c}{135} \approx 0.1407D^4G_c \quad (19)$$

The exact solution, as given by Timoshenko and Goodier (1970), is:

$$(GJ)_{k,TG} = 0.1406D^4G_c \quad (20)$$

It can be seen that the difference between the analytical solution (Eq. (19)) and the exact solution (Eq. (20)) is only 0.1%. This shows that the longitudinal displacement mode assumed in this paper is reasonable, and that the principle of minimum strain energy has been suitably applied.

Vlasov [18] attempted to generalize his law of sectorial areas to solve the non-uniform torsion problem of rectangular sections, and derived a

different formula for calculating the uniform torsional constant of the rectangular section with a single material, i.e.,

$$J = \frac{4I_x I_y}{I_x + I_y} \quad (21)$$

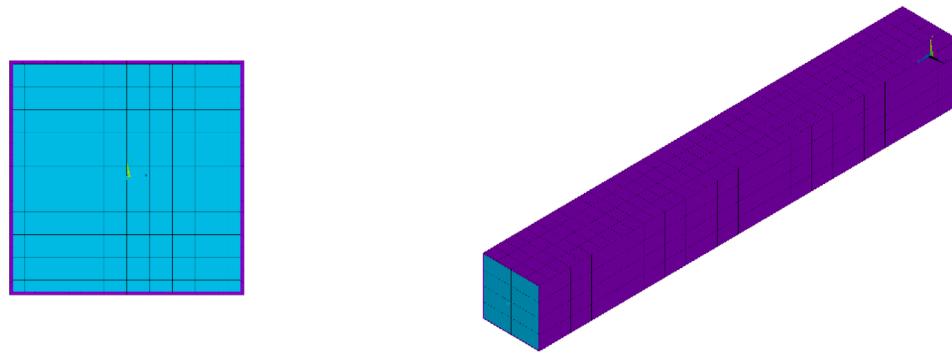
According to this, Vlasov's uniform torsional rigidity for the square cross-section is.

$$(GJ)_{k,V} = 0.1667D^4G_c \quad (22)$$

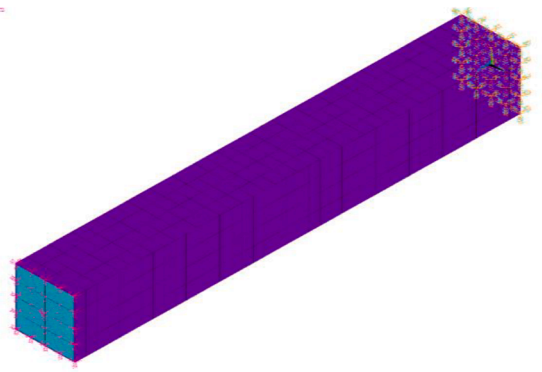
It can be seen that the difference of the uniform torsional rigidity between Vlasov's solution and the exact solution given by Timoshenko and Goodier [19] is 18.6%. Therefore, Vlasov's torsion constant is not sufficiently accurate and is not considered further.

3.2. Experimental verification

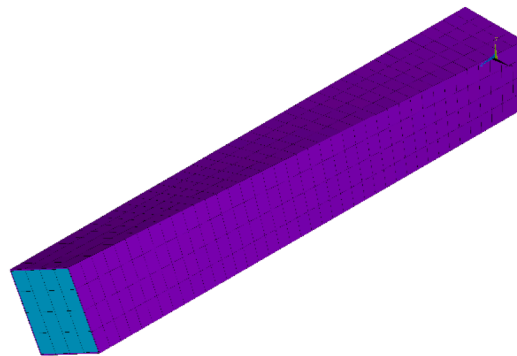
To verify the accuracy of the derived uniform torsional rigidity given in Eq. (17) further, the results of a torsion test conducted by Chen [20] on a square CFST member are considered. The geometric and material properties of the tested member are shown in Table 1, where f_y is the yield strength of the steel, f_{cu} and f_{ck} are the cube and cylinder strength of the concrete, while the full-range moment-rotation (T - θ) curve is shown in Fig. 3. The calculated rigidity of the member, as determined from Eq. (17), is also plotted in Fig. 3(a) (with $G = E/[2(1 + \mu)]$ and $\mu_c = 0.2$ for concrete and $\mu_s = 0.3$ for steel) and can be seen to be in good agreement with the test result in the elastic range (i.e. up to about $T = 40$



(a) FE model mesh



(b) Applied torque at tip and fixed boundary conditions at root of member



(c) Deformation of member under applied torque

Fig. 4. FE model of square CFST member.

kNm for the case shown). Similarly good predictions of torsional rigidity (within about 12%) were achieved following comparisons with the experimental results reported in [21], as shown in Fig. 3(b), the key properties of which are provided in Table 1.

3.3. FE verification

Verification of the derived uniform torsional rigidity given in Eq. (17) was also sought by means of finite element modelling. A series of square CFST members was modelled in ANSYS using SOLID65 and SHELL181 elements, for the core concrete and steel tube and concrete, respectively. An element size of 20 mm (see Fig. 4(a)) in all directions was adopted following a convergence study. The contact elements TAGRE170 and CONTAL173 were used to simulate the contact between the steel and the concrete. Torque was applied by introducing an MASS21 mass element with rotational degrees of freedom at the tips of

the members, while all degrees of freedom of all nodes were restrained at the fixed ends of the members, as shown in Fig. 4(b). An illustration of the deformation of a member under the action of the applied torque is shown in Fig. 4(c).

Twelve square CFST members, consisting of a Q235 steel SHS outer tube ($E_s = 206$ GPa and $\mu_s = 0.3$) filled with C40 concrete ($E_c = 32.4$ GPa and $\mu_c = 0.2$) of different lengths and depths were modelled. Tip rotations were determined under an applied torque $T = 1.0$ Nm using: (1) the FE models to give $\theta_{L,FE}$ and (2) Eq. (1) with the uniform torsional rigidity $(GJ)_k$ given by Eq. (17) to give $\theta_{L,SE}$ (referred to as ‘Theory’). The results are listed in Table 2, including the mean and standard deviation of the predictions, and plotted in Fig. 5. It can be seen that the theoretical tip rotations are in very close agreement with the finite element tip rotations, with the maximum absolute difference being the results being less than 0.5%.

Table 2

Comparison of theoretical and FE tip rotations for a series of square CFST members.

No.	L (m)	D (mm)	t (mm)	$(GJ)_k$ (Nm ²)	Theory $\theta_{L,SE}$ (rad)	FE $\theta_{L,FE}$ (rad)	$(\theta_{L,SE}/\theta_{L,FE} - 1) \times 100\%$
1	1.5	100	3.0	389.65×10^3	3.8497×10^{-6}	3.8661×10^{-6}	−0.43
2	1.5	120	3.0	744.73×10^3	2.0142×10^{-6}	2.0120×10^{-6}	0.11
3	1.5	150	3.0	1658.76×10^3	0.9043×10^{-6}	0.9057×10^{-6}	−0.15
4	1.5	200	3.0	4722.10×10^3	0.3177×10^{-6}	0.3162×10^{-6}	0.45
5	2.0	100	3.0	389.65×10^3	5.1329×10^{-6}	5.1557×10^{-6}	−0.44
6	2.0	120	3.0	744.73×10^3	2.6855×10^{-6}	2.6833×10^{-6}	0.08
7	2.0	150	3.0	1658.76×10^3	1.2057×10^{-6}	1.2080×10^{-6}	−0.19
8	2.0	200	3.0	4722.10×10^3	0.4235×10^{-6}	0.4219×10^{-6}	0.39
9	3.0	100	3.0	389.65×10^3	7.6993×10^{-6}	7.7362×10^{-6}	−0.48
10	3.0	120	3.0	744.73×10^3	4.0283×10^{-6}	4.0262×10^{-6}	0.05
11	3.0	150	3.0	1658.76×10^3	1.8086×10^{-6}	1.8129×10^{-6}	−0.24
12	3.0	200	3.0	4722.10×10^3	0.6353×10^{-6}	0.6333×10^{-6}	0.32
Mean							−0.04
St. Dev.							0.33

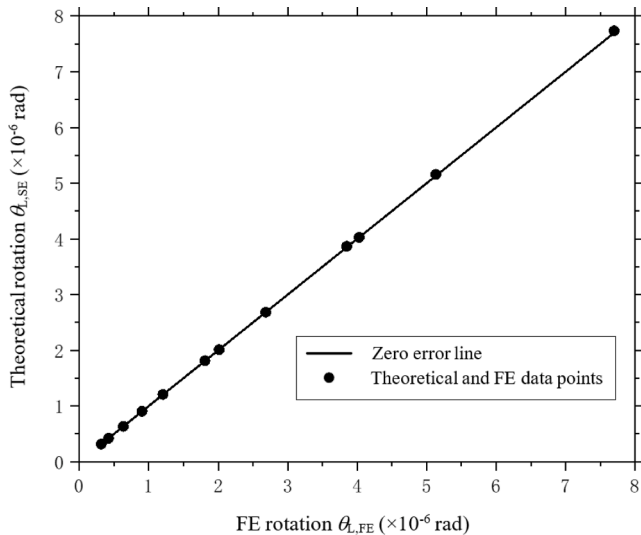


Fig. 5. Comparison of tip rotations for square CFST members determined using FE model and theoretical calculation.

4. Sensitivity analysis and simplified design formula

Although the derived analytical formula, Eq. (17), for the uniform torsional rigidity of square CFST sections given in this paper is very accurate, it is somewhat cumbersome for practical application. To simplify the formula, the influence of two key parameters is assessed in this section and a simplified superposition method for calculating the uniform torsional rigidity of square CFST sections is proposed.

4.1. Parametric sensitivity

The sensitivity of the non-dimensional parameter Ψ , given by Eq. (18), and hence the uniform torsional rigidity of a square CFST section,

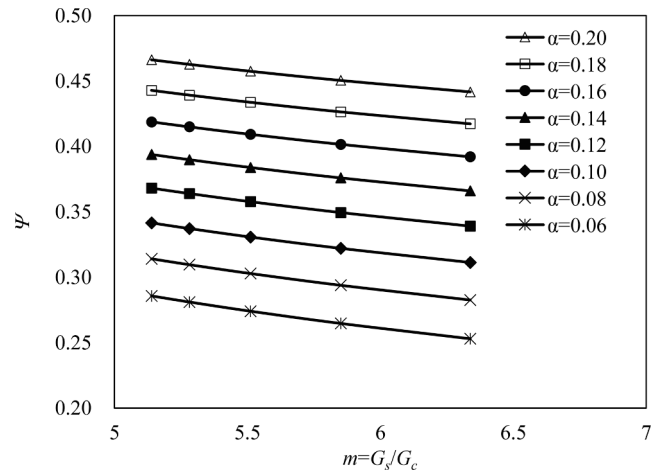


Fig. 6. Variation of $\Psi(m, \alpha)$ with m for a range of α values.

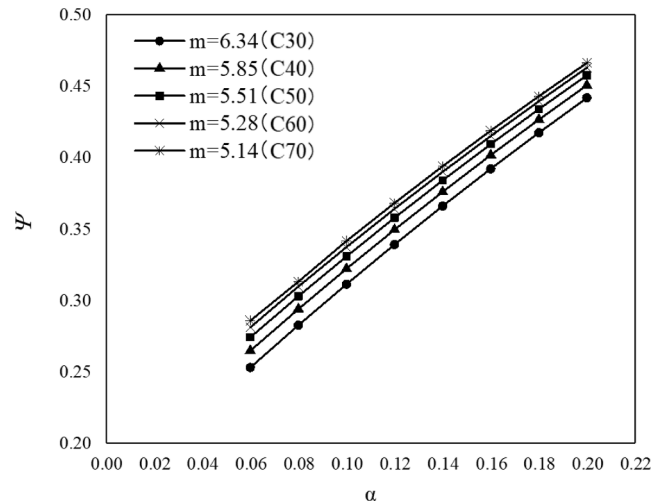


Fig. 7. Variation of $\Psi(m, \alpha)$ with α for a range of m values.

to variation in the steel ratio α and the ratio of the shear modulus of steel to that of concrete m is considered. The shear modulus of steel is given by $G_s = E_s / [2(1 + \mu_s)]$, which, for a value of $E_s = 210$ GPa and $\mu_s = 0.3$, corresponds to a value of $G_s \approx 81$ GPa. The shear modulus of concrete G_c typically lies in the range 12.5 GPa ~ 15.8 GPa, depending on the grade of concrete. Consequently, m is essentially related only to the grade of concrete, and its value ranges between approximately 5.1 ~ 6.4.

Fig. 6 shows a graph of the relationship between Ψ and m for different steel ratios α . It is observed that: (1) as α changes, the trend of the Ψ - m relationship remains similar, indicating that the relationship between Ψ and α is approximately linear and (2) as m increases, Ψ decreases. Fig. 7 shows the Ψ - α relationship for different values of m . It can be seen that: (1) as m changes, the trend of the Ψ - α relationship also remains similar indicating that the relationship between Ψ and m is approximately linear and (2) as α increases, Ψ also increases. Overall, it may be concluded that Ψ varies approximately linearly with both α and m , and that the influence of the steel ratio α is more significant.

4.2. Simplified design formula

Given that it has been demonstrated that the relationships between Ψ and both m and α are approximately linear, the principle of superposition may be invoked. Hence, the uniform torsional rigidity of a square CFST section also can be expressed as:

Table 3

Comparison of tip rotations obtained from simplified superposition method and finite element method.

No.	Concrete grade	Steel grade	L (m)	D (mm)	t (mm)	α	$\theta_{L,FE} (\times 10^{-6}$ rad)	$\theta_{L,SE} (\times 10^{-6}$ rad)	$\theta_{L,[16]} (\times 10^{-6}$ rad)	$(\theta_{L,SE}/\theta_{L,FE} - 1) \times$ 100%	$(\theta_{L,[16]}/\theta_{L,FE} - 1) \times$ 100%
1	C40	Q235	2.0	100	3.0	0.120	5.1949	4.9117	8.2910	−5.45	59.6
2			2.0	120	3.0	0.100	2.7109	2.5796	4.5165	−4.85	66.6
3			2.0	150	3.0	0.080	1.2135	1.1637	2.1230	−4.11	75.0
4			2.0	200	3.0	0.060	0.4249	0.4112	0.7886	−3.21	85.6
5	C50	Q345	2.0	100	3.0	0.120	5.0686	4.7743	7.9881	−5.81	57.6
6			2.0	120	3.0	0.100	2.6374	2.5012	4.3571	−5.17	65.2
7			2.0	150	3.0	0.080	1.1765	1.1249	2.0533	−4.39	74.5
8			2.0	200	3.0	0.060	0.4101	0.3960	0.7659	−3.44	86.8
9	C60	Q390	2.0	100	3.0	0.120	4.9781	4.6762	7.8947	−6.06	58.6
10			2.0	120	3.0	0.100	2.5849	2.4455	4.2663	−5.39	65.1
11			2.0	150	3.0	0.080	1.1502	1.0974	1.9909	−4.59	73.1
12			2.0	200	3.0	0.060	0.3997	0.3853	0.7350	−3.61	83.9
Mean										−4.67	71.0
St. Dev.										0.94	10.5

$$\begin{aligned}
 (GJ)_{k,Sup} &= (GJ)_s + (GJ)_c \\
 &= \left[2G_s \frac{D^2(D-2t)^2}{D+(D-2t)} t + 2G_s \left(\frac{D^3}{3} + \frac{(D-2t)^3}{3} \right) \right] + 0.1406G_c(D-2t)^4
 \end{aligned} \quad (23)$$

where, the first term is the torsional rigidity of a thin-walled square steel tube $(GJ)_s$, and the second term is the torsional rigidity of a solid square concrete core $(GJ)_c$.

To verify the applicability of this simplified design formula, twelve square CFST members, with different values of the ratios m and α , were considered. The properties of the members are listed in Table 3. The tip rotations of the members under an applied torque $T = 1.0$ Nm were calculated according to the above simplified superposition method and using the FE model. The results are presented and compared in the last three columns of Table 3, where close agreement between those obtained from the simplified superposition method and the FE model can be seen, with the difference ranging between -3.21% and -6.06% . Hence, it may be concluded that the simplified expression provides an acceptable approximation for the torsional rigidity of a square CFST cross-section. The model presented in [16], for which the calculated rotations are denoted θ_L [16], leads to safe-sided, but consistently somewhat conservative, predictions of torsional rigidity. The model [16] has been shown however to provide accurate overall predictions of the torsional response of CFST members across the full nonlinear deformation range.

5. Conclusions

The uniform torsional rigidity of square concrete-filled steel tubular (CFST) sections has been studied in this paper, and the following conclusions can be drawn:

- (1) An analytical formula for calculating the uniform torsional rigidity of square CFST sections has been derived and shown to provide very accurate predictions (within 0.5%) of numerical data. Accurate prediction of previous experimental results was also obtained.
- (2) The uniform torsional rigidity has been shown to vary in an approximately linear fashion with both the steel ratio α and the ratio of the shear modulus of steel to concrete m , while the influence of the steel ratio α is more significant.

- (3) Following from (2), the principle of superposition has provided an acceptable simplification for the uniform torsional rigidity of square CFST cross-sections.
- (4) The findings presented herein could be combined with those presented in [16] to provide a very accurate predictive model for the full range elastic–plastic torsion moment–deformation response of CFST members.
- (5) The presented work can be extended to study the uniform torsional rigidity of CFST members of other cross-section shapes. Further experimental verification should also be sought.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgements

The authors would like to thank the National Natural Science Foundation of China (Grant numbers 51178087 and 51578120) and the Research Foundation of Nanjing Institute of Technology (Grant number 2016087) for their financial support of this work.

References

- [1] Han LH, Li W, Bjorhovde R. Developments and advanced applications of concrete-filled steel tubular (CFST) structures—Members. *J Constr Steel Res* 2014;100: 211–28.
- [2] Zhong ST. Concrete-filled steel tube structures. Harbin, China: Heilongjiang Technology Press; 1994. In Chinese.
- [3] Han LH. Concrete-Filled Steel Tubular Structures—Theory and Practice (Third edition). China Science Press; 2016. (In Chinese).
- [4] Zhong ST. Unified theory of concrete filled steel tube. Beijing, China: Tsinghua University Press; 2006. In Chinese.
- [5] Espinos A, Gardner L, Romero ML, Hospitaler A. Fire behaviour of concrete filled elliptical steel columns. *Thin-Walled Structures* 2011;49(2):239–55.
- [6] Yang H, Lui F, Gardner L. Performance of concrete-filled RHS columns exposed to fire on 3 sides. *Eng Struct* 2013;56:1986–2004.
- [7] Liu F, Wang Y, Gardner L, Varma AH. Experimental and numerical studies of reinforced concrete columns confined by circular steel tubes exposed to fire. *J Struct Eng, ASCE* 2019;145(11):04019130.
- [8] Schneider SP. Axially loaded concrete-filled steel tubes. *J Struct Eng, ASCE* 1998; 124(10):1125–38.
- [9] Uy B. Strength of concrete filled steel box columns incorporating local buckling. *J Struct Eng, ASCE* 2000;126(3):341–52.

- [10] Uy B. Strength of short concrete filled high strength steel box columns. *J Constr Steel Res* 2001;51(2):113–34.
- [11] Varma AH, Ricles JM, Sause R, Lu LW. Experimental behaviour of high strength square concrete-filled steel tube beam-columns. *J Struct Eng, ASCE* 2002;128(3): 309–18.
- [12] Ouyang Y, Kwan AKH. Finite element analysis of square concrete-filled steel tube (CFST) columns under axial compressive load. *Eng Struct* 2018;156:443–59.
- [13] Zhou M, Fan JS, Tao MX, Nie JG. Experimental study on the tensile behavior of square concrete-filled steel tubes. *J Constr Steel Res* 2016;121:202–15.
- [14] Cai J, Liang SW, Lin H. Experimental study on shear resistance performance of concrete filled square steel tubular columns. *J Shenzhen University Science and Engineering* 2012;29(3):189–94.
- [15] Li GC, Liu D, Yang ZJ, Zhang CY. Flexural behavior of high strength concrete filled high strength square steel tube. *J Constr Steel Res* 2017;128:732–44.
- [16] Han L-H, Yao G-H, Tao Z. Performance of concrete-filled thin-walled steel tubes under pure torsion. *Thin-Walled Structures* 2007;45(1):24–36.
- [17] Al-Dujele R, Cashell KA, Afshan S. Flexural behaviour of concrete filled tubular flange girders. *J Constr Steel Res* 2018;151:263–79.
- [18] Vlasov VZ. *Thin-walled elastic beams*. Jerusalem: Israel Program for Scientific Translations 1961.
- [19] Timoshenko SP, Goodier JN. *Theory of Elasticity*. 3rd Edition, 1970, McGraw-Hill, New York.
- [20] Chen YW. *Research on torsion and section behavior of concrete-filled steel tubular columns*. Taiwan Central University, 2003.
- [21] Kitada T, Nakai H. Experimental study on ultimate strength of concrete-filled square steel short members subjected to compression or torsion. *Proceedings of the Third International Conference on Steel Concrete Composite Structures*. Fukuoka, Japan, 1991.