Transient Liquid Holdup and Drainage Variations in Gravity Dominated Non-Porous and Porous Packed Beds

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ABSTRACT

Transient liquid holdup effects are a crucial aspect of the behaviour of many unsaturated packed beds systems. This study examined both a model system consisting of spherical glass beads and a system containing slightly porous (about 5% water accessible porosity) rock particles. Experiments on different column heights show that the initial wetting front moving through the packed bed takes the form of a soliton or standing wave.

The final drainage of the bed when the liquid addition is turned off shows slightly more complex behaviour than that of the initial wetting of the bed. It was demonstrated that, if the behaviour of the liquid held around the particles is separated from that held within the particles, the same relatively simple model can be used to describe the drainage of both the model glass bead system and the slightly porous ore system despite the apparent differences in their behaviour, such as a much longer time to achieve the steady state, and a markedly different shape to the initial overall saturation versus time curve.

This simple model assumed that, for the liquid between the particles, gravity was the dominant force and that capillarity could be neglected. Neglecting capillarity probably accounts for the slight discrepancy between the experimental and simulated liquid holdup results in the porous ore system at intermediate drainage times.

Keywords: Heap leaching, Liquid drainage, Modelling, Transient liquid holdup, Trickle bed reactors

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1. Introduction

The behaviour of unsaturated gravity driven flow of liquid through packed beds of particles is important to number of different processes ranging from trickle bed reactors (TBRs) to heap leaching.

A trickle bed reactor is a two phase system (liquid and gas) in which fluids usually flow co-currently through a fixed bed of typically porous catalysts or reactant solids (Luciani et al., 2002). The unsteady state operation of TBRs by periodically modulating the liquid or gas supply leads to transient fluid flow characteristics (Khadilkar et al., 1999; Lange et al., 2004; Ayude et al., 2007), which have been reported to increase the reactor performance compared with the steady state flow behaviour (Haure et al., 1989; Lange et al., 1994; Castellari and Haure, 1995; Lange et al., 2004; Tukac et al., 2007).

Heap leaching is another system in which the unsaturated flow behaviour of the liquid through a bed of particles is a crucial aspect of the performance of the system, though they have significant differences compared to trickle bed reactors in terms of the scale and flow rates used (Roman and Bhappu, 1993). This mineral processing technique is used for extraction of base and precious metals from low grade ores by running a leaching solution through an unsaturated bed of ore particles. In heaps, irrigation strategies with alternating solution application periods followed by much longer rest periods has been demonstrated to have the potential to increase the heap performance (Bartlett, 1992).

In this paper the transient variations in the liquid content as the liquid addition rate is varied is studied. In particular the behaviour at relatively low flow rates, when the bed is in the trickle flow or low interaction regime giving droplet and rivulet flow features, is studied. Packed beds consisting of non-porous glass beads as well as slightly porous (approximately 5% porosity) ore particles (a copper ore) are studied in order to ascertain how porosity of the particles affects the behaviour.

Most previous studies in both leaching columns and trickle bed reactors have concentrated on the steady state relationship between the liquid addition rate and liquid holdup as a function of various bed and operating parameters (eg. Yusuf, 1984; Fu and Tan, 1996; Nemec et al., 2001; de Andrade Lima, 2006). While there have been some previous studies into the transient behaviour of these systems (Standish, 1968; Bartlett, 1992; Tukac et al., 2007; Liu et al., 2008, Liu et al., 2009), the descriptions of the behaviour has typically been qualitative in nature, while the aim of this paper is to develop and validate a model that can describe the transient behaviour

of these systems while including the effects of both porosity and the hysteresis observed in these systems.

2. Experimental design

In this study the liquid holdup measurements were performed gravimetrically using a high precision load cell. The gravimetric measurements were compared to an independent measurement based on the volume of drained liquid to obtain uncertainty in the measured holdup values and to verify that the load cell was able to measure the relatively small changes in liquid holdup needed in this work (see Ilankoon and Neethling (2012), for the method and results). It was found out that both methods give virtually identical results and thus providing confidence in the load cell based measurements (Ilankoon and Neethling, 2012, 2013).

The column used is Perspex and has an internal diameter of 243mm. 3 different column heights were used in the study, namely 300, 500 and 800mm. A detailed description of the experimental setup including photos can be found in Ilankoon (2012), Ilankoon and Neethling (2012, 2013).

Mono-dispersed glass spheres of 10 mm and 14 mm were used for the model non-porous system whereas the slightly porous particle system consisted of 8-11.2 mm copper ore particles (average water accessible porosity of 5%) collected from Kennecott Utah Bingham Canyon Mine. The ore particle properties are described in more detail in Ilankoon and Neethling (2013). The experimentally determined average voidage values for the non-porous and porous system were about 40% and 35% respectively. For the ore system this is the external voidage excluding the porosity of the particles, which was about 5%.

Using a novel liquid distributor that allowed even liquid addition over the entire bed surface even at the very low flow rates used (Ilankoon, 2012; Ilankoon and Neethling, 2012), the applied liquid flow rates were 1.26, 2.52, 5.04, 10.08 and 20.16 L/h and the corresponding superficial flow are within the range of 0.0075-0.12 mm/s. Deionised water was used as the liquid for all the experiments in this work.

3. Transients during initial liquid addition

The packed column, which was initially dry, was suspended via a load cell to gravimetrically measure the weight of liquid within the column. In addition, a measuring beaker was kept on top a high precision electronic balance in order to continuously measure the drained liquid weight (see Figure 1). These independent measures of the liquid behaviour within the column were obtained every second until steady state was reached in order to obtain the wetting front

movement. Steady state was indicated by a constant load cell reading over a number of minutes. In addition, the wetting of the columns was filmed through the Perspex walls of the column.

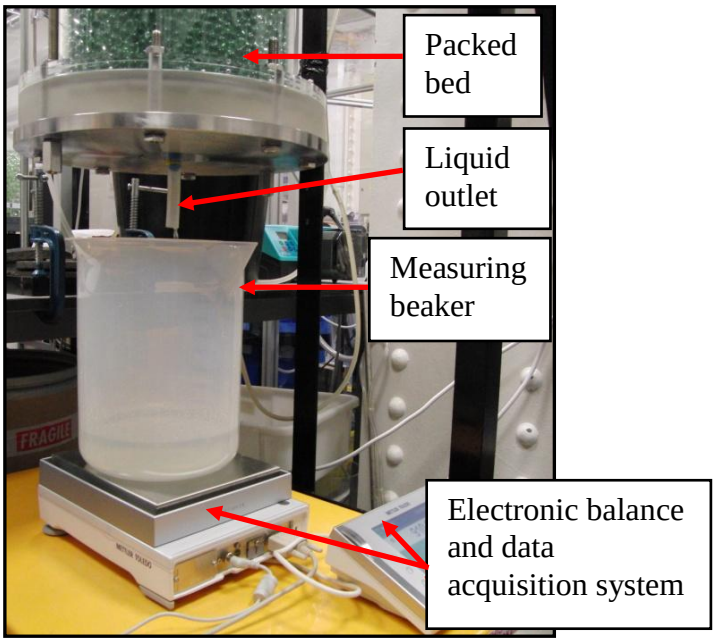


Figure 1: Experimental setup to measure wetting front movement in the model system.

The liquid within the packed bed moves under the influence of both gravitational and capillary/dispersive effects. If the flow is completely dominated by gravity, a sharp wetting front can be expected, while capillary/dispersive will act to spread out the wetting front. The reason why gravity sharpens the front is that the gravity driven flow rate increases with increasing liquid content. This means that, excluding the capillary and dispersive effects, any liquid ahead of the front will be travelling slower than the front and will thus be caught up by the front.

If the gravity effect is strong enough, then the sharpening effect of gravity and the spreading effect of capillarity and other dispersive phenomena will be able to reach an equilibrium that will manifest itself as a standing wave, or soliton, moving through the bed.

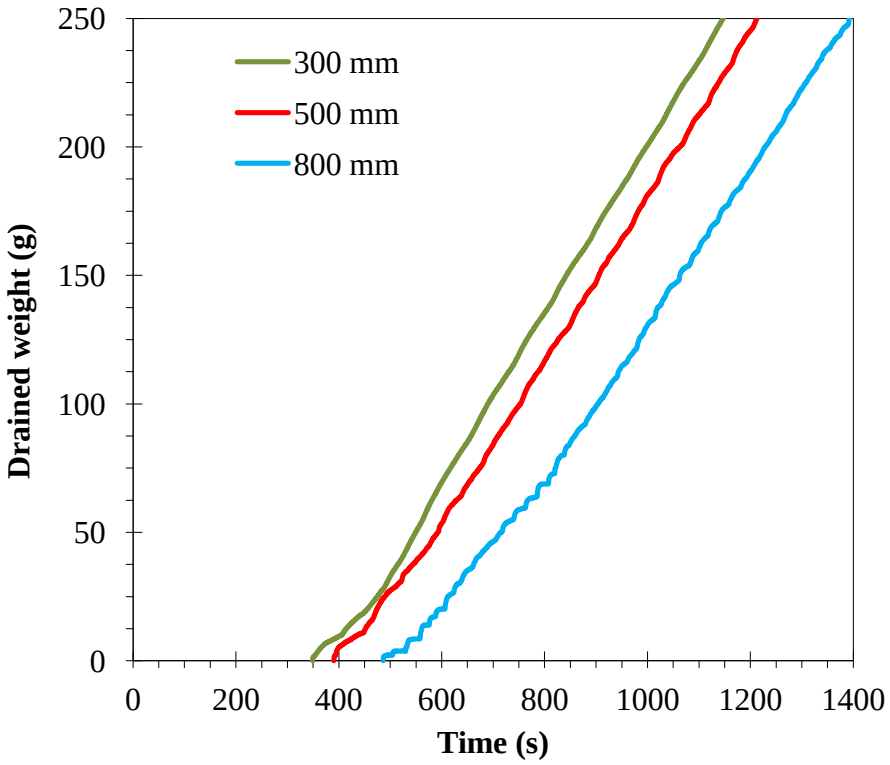
Assuming that the particle bed is uniform in its properties in both the vertical and horizontal directions, once established, the shape and velocity of a soliton moving through the bed should be independent of the position of the soliton within the bed. The existence of a soliton moving through the bed can thus be investigated by examining the liquid flow out of the system as a function of time for different bed heights.

If the liquid is moving through the system as a wetting front there will be an initial time period in which liquid does not drain out of the packed bed followed by a rapid transition region towards a uniform drainage rate (linear increase in drained liquid). If there is a standing wave then, in different column heights, the breakthrough time would change, but the shape of the transition region would be the same.

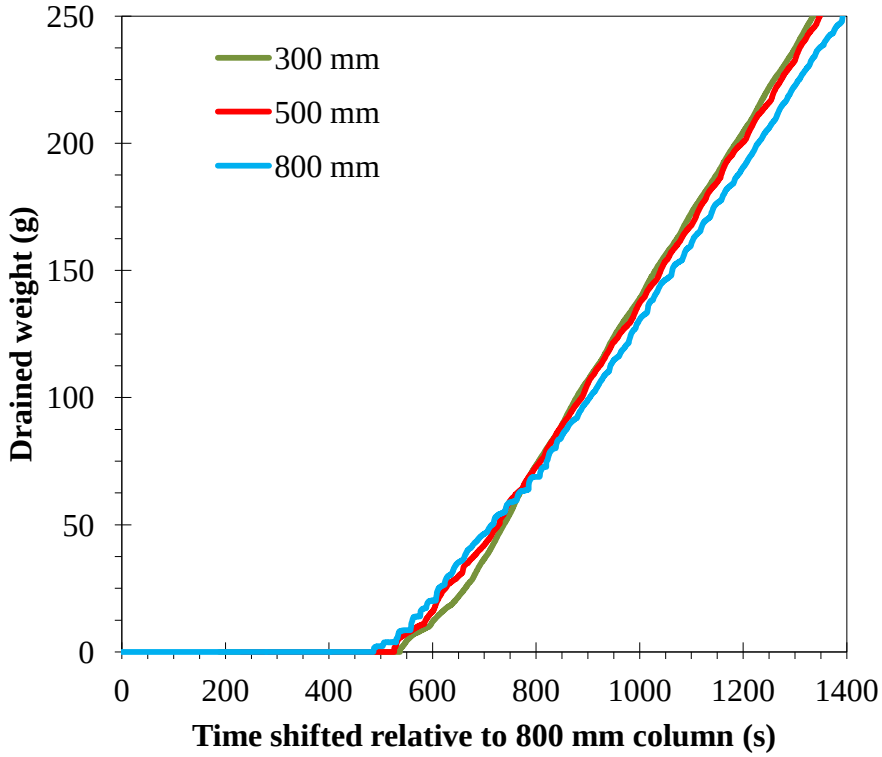
Figures 2a (10mm glass spheres) and 3a (14mm glass spheres) show the variation of the drained liquid weight as a function of the time (time = 0 indicates initial liquid addition) since the start of the liquid addition into the bed. All these experiments were carried out with a superficial liquid velocity of 0.0075 mm/s (1.26 L/h liquid flow rate). The liquid holdup curves for column lengths of 300 mm, 500 mm and 800 mm are shown and the breakthrough time increases with the packed bed height and decreases with the particle size for the same packed bed height. The average gradient of each line after the transition regions in Figures 2a and 3a is 1.19 L/h, which is similar to the input liquid flow rate suggesting that the transition to steady state after the passing of the wetting front is indeed very rapid. The subtle differences in the final slopes are mainly due to small variations in the input liquid flow rate between experiments. The slight fluctuations in the flow are likely to be due to the effect of the peristaltic pump.

In Figures 2b and 3b, it is shown that when the initial liquid holdup curves are shifted onto one another for different column lengths, the transitions are very similar and all very rapid. The shift is achieved by adding a time shift onto the data for the shorter columns. Since, while nominally the same, the flow out of each column is subtly different, the time is shifted by $Z_1/v_1 - Z_2/v_2$, (Z = length of the column, v is the input liquid velocity) where the subscript 1 refers to the 800 mm column and 2 refers to the column being shifted.

All these results indicate that the liquid motion through the bed during the initial wetting takes the form of wetting front, with all the results being consistent with the wetting front taking the form of a solitary wave.

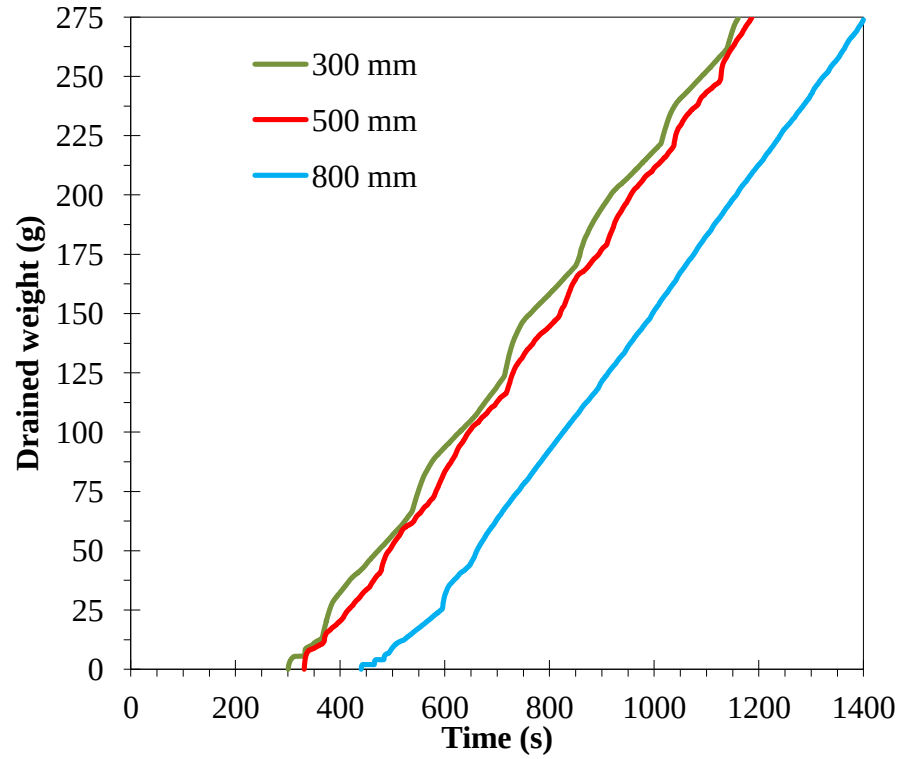


a)

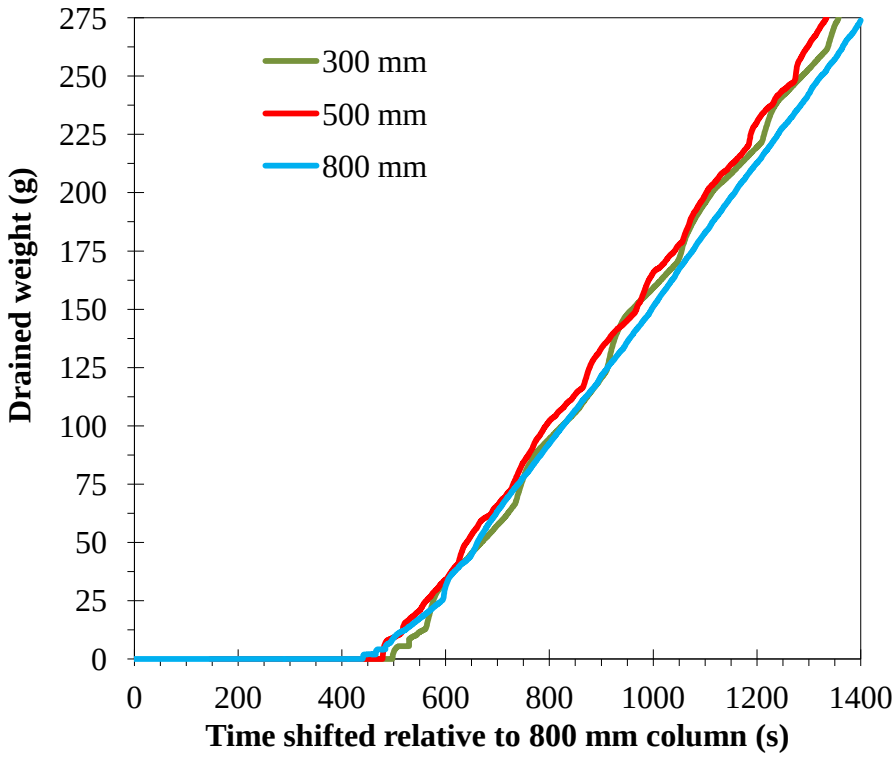


b)

Figure 2: Drained weight of water for 10 mm particles in different column lengths for a superficial velocity of 0.0075 mm/s. a) as a function of time b) as a function of time shifted relative to 800 mm column (time shifting method is described in the text).



a)



b)

Figure 3: Drained weight of water for 14 mm particles in different column lengths for a superficial velocity of 0.0075 mm/s. a) as a function of time b) as a function of time shifted relative to 800 mm column (time shifting method is described in the text).

4. Model for Inter-Particle Liquid Flow

To describe unsaturated flow through a packed bed of non-porous particles, a theoretically based model has been formulated which includes the effect of liquid holdup hysteresis. The following section gives a brief description of this model, though more details can be found in Ilankoon and Neethling (2012).

The two liquid holdups that are important in describing the behaviour of this system are the external liquid holdup at steady state, θ, and the external residual liquid holdup, θ_r, of the system. Definitions of different liquid holdup values are given by de Klerk (2003). The static (truly immobile) and the residual liquid content are not necessarily the same thing since the residual liquid pocket that remains once the liquid has drained could have been involved in the flow.

It was found that rivulet flow behaviour was the dominant liquid flow feature through the particles at low superficial liquid velocities and it can be assumed that the residual liquid holdup consists of the remnants of these rivulets. It can thus be assumed that the residual liquid holdup is proportional to number of rivulets per cross-sectional area ($N_{rivulets}$).

$$\theta_r = A_r N_{rivulets} \qquad (1)$$

where A_r is the average cross-sectional area of the residual liquid in a rivulet.

This average residual area will actually consist of a string of liquid droplets held between particles by capillarity. The size of these residual liquid connections will depend on Bond number, contact angle and particle properties only, and thus A_r is likely to be virtually independent of the flow rate within the system (Ilankoon and Neethling, 2012, 2013).

A relative velocity, v_s^*, (which is proportional to average flow per rivulet, $v_s/N_{rivulets}$, where v_s is the superficial liquid velocity) and a relative holdup, θ^*, (which is proportional to liquid holdup per rivulet, $\theta/N_{rivulets}$) are defined as follows:

$$v_s^* = \frac{v_s}{\theta_r} \qquad (2)$$

$$\theta^* = \frac{\theta}{\theta_r} \qquad (3)$$

The theoretical inter-particle flow model was developed between the relative velocity (v_s^*) and the relative holdup (θ^*) assuming that the cross-sectional shape of the rivulet is reasonably constant (Ilankoon and Neethling, 2012). Full details of this derivation can be found in Ilankoon (2012) including detailed information about the dimensionless drag coefficient and the average cross-sectional area of the residual liquid.

$$v_s^* = K(\theta^* - 1)^2 \qquad (4)$$

where, $K = \rho g A_r / \mu C_{drag}$

where C_{drag} is dimensionless drag coefficient, ρ is density and μ is viscosity of the liquid.

The model gives a squared relationship between v_s^* and $\theta^* - 1$ for the non-porous system, which was indeed found experimentally (see Figure 4). The proportionality K is a function of both particle size and size distribution. While these systems exhibit quite strong hysteresis, there is no hysteresis in the relationship shown in Figure 4. This is because the hysteresis takes the form of changes in the number of flow paths as liquid flow is increased and thus the rescaling of both the velocity and the holdup by the residual liquid content collapses the data and accounts for the hysteresis.

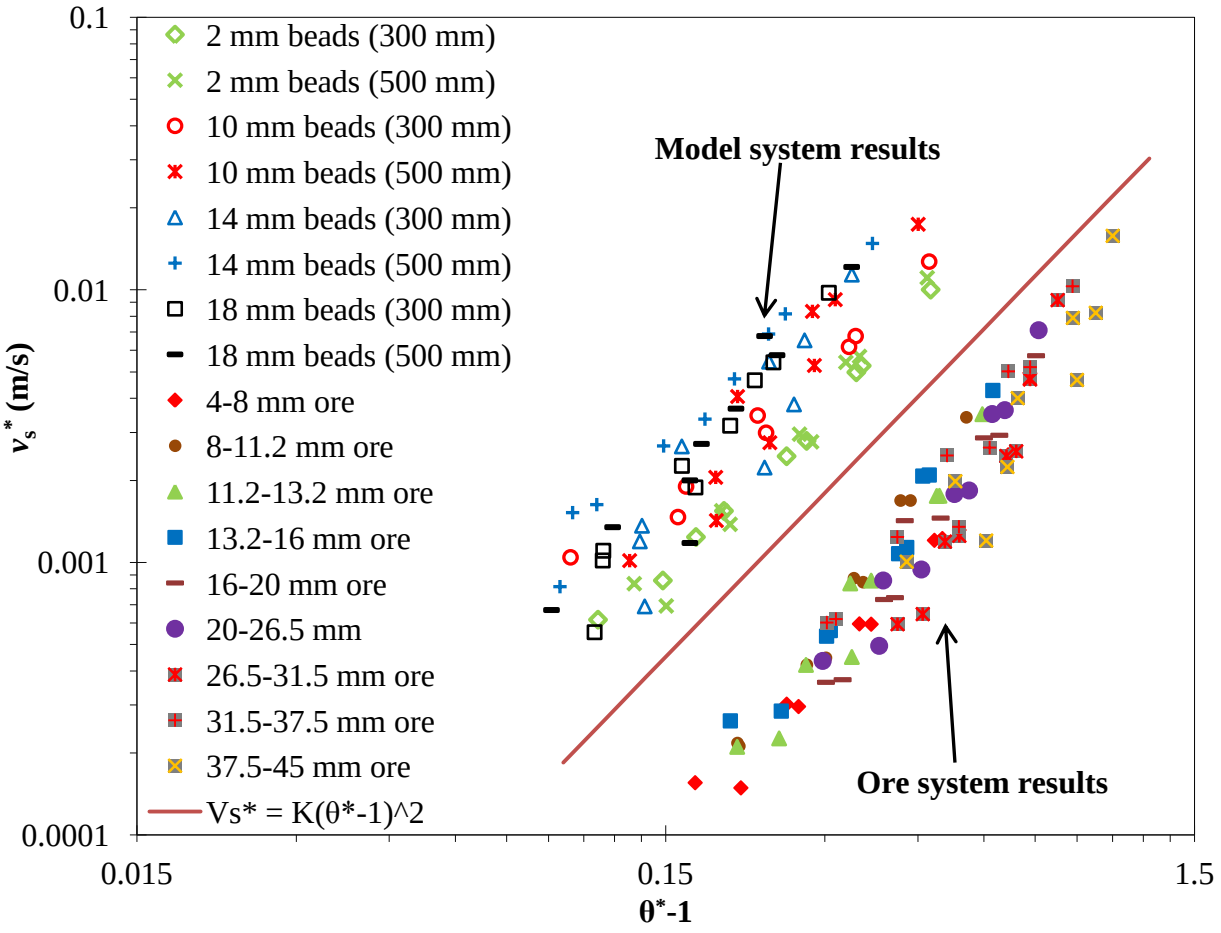


Figure 4: The relationship between the relative flow rate (v_s^*) and the additional liquid content of that rivulet ($\theta^* - 1$) for both model glass bead and slightly porous ore particle systems (after Ilankoon and Neethling, 2013). The line is for a power law exponent of 2, to which all the data is very close to parallel indicating that the theoretical model form is applicable.

5. Drainage behaviour of the porous versus non-porous system

The model described in the previous section was initially developed based on a non-porous glass bead system, but has been shown to be applicable to systems with porous particles when suitable modifications have been made to the model. A brief description of the behaviour and model are given below, though more details can be found in Ilankoon and Neethling (2013).

Unlike the non-porous system, where the system stops dripping and the residual is obtained after only a few minutes in a 300 mm column, the same column filled with porous particles continues to drip for many hours. In both systems there is an initial rapid decrease in the drainage rate, but in the non-porous system the flow out rapidly decreases to zero, while in the slightly porous ore particle system the decrease is not to zero but to a small and near constant flow which persists for many hours until the visible drying of the upper particles approaches the bottom of the column. It can therefore be assumed that this slow flow is associated with the drainage of the liquid within the particles, especially as this flow rate was found to be independent of the superficial liquid velocity of the system.

Within the porous system, the amount of liquid held within the pore spaces of the particles at steady state was slightly lower, but of a similar magnitude to the liquid held between the particles, but the flow associated with this internally held liquid was orders of magnitude lower. This means that to understand and model the liquid motion within the system, the contribution of the liquid around the particles needs to be separated from that within the particles.

To do this, experiments were conducted in which the particles were initially saturated in water for 3-4 days and externally dried using paper towels just before the column is packed (i.e. particles are initially saturated not the packed bed). The difference in the scale of the pores within the particles compared to that around the particles means that as long as there is liquid available in the intra-particle spaces, the particles will remain saturated or very near saturated. Any additional weight above that of these initially saturated particles is thus associated with the liquid held around the particles.

When liquid addition to the column is halted both the inter- and intra-particle liquid begins to drain (though much quicker in the case of the inter-particle liquid). Over a long time period the drainage rate from within the particles is constant. If this drainage rate from within the particles is also assumed to be constant at short times, the amount of liquid held between the particles can be estimated by adding the liquid drained from the particles back onto the measured external liquid content:

$$\theta_{ex} = \theta_{measured} + \frac{v_{end}t}{Z} \qquad (5)$$

where θ_{ex} is estimated external liquid holdup between the particles, $\theta_{measured}$ is the liquid content obtained by subtracting the initial weight of the particle bed containing saturated particles from the subsequent weight of the particle bed, v_{end} is roughly constant flow out of the column at longer times, Z is length of the column and t is the drainage time (see Ilankoon and Neethling, 2013).

This flow through the particles will also eventually drain the residual liquid connections between the particles as well as the particles themselves. Since this rate is constant though, whether it is draining the stationary connections between the particles or the liquid actually within the particles, adding this liquid back onto the total will continue to provide an estimate of the liquid content at the point at which flow between the particles stopped and hence the external residual.

The system thus exhibits two residual liquid contents, a short term residual associated with the cessation in the liquid flow around the particles, $\theta_{r,ex}$, and a much longer term residual associated with the cessation of flow from within the particles. Since the flow model developed in section 4 (equation 4) is for the liquid flow around the particles, the relevant liquid holdups and residuals are the external ones. When these external liquid holdups are used the same model form as that for the non-porous particles is obtained, namely a power law relationship with an exponent of two between v_s^* and $\theta^* - 1$, albeit with a different pre-factor (see Ilankoon and Neethling, 2013 and Figure 4).

6. Modelling of transient liquid drainage behaviour

6.1 The Model

The equations developed in the previous sections describe the relationship between the steady state liquid content and the liquid flux through the system. In order to model the transient behaviour of the system the change in liquid content and flux need to be related, which requires a continuity equation:

$$\frac{\partial \theta_{ex}}{\partial t} = -\nabla F \qquad (6)$$

If residual liquid holdup ($\theta_{r,ex}$) is constant with respect to time and distance and if it is assumed that the only liquid holdup variations are in the vertical direction, equation 6 can be written as follows:

$$\frac{\partial\left(\frac{\theta_{ex}}{\theta_{r,ex}}\right)}{\partial t} = -\frac{\partial}{\partial Z}\left(\frac{F}{\theta_{r,ex}}\right) \qquad (7)$$

In this work it will be assumed that for the liquid between the particles, gravity is the dominant force and that capillarity can be ignored. The validity of this assumption is discussed below. Making this assumption means that equation 4 can be used to describe the flux ($F = v_s$), which, when substituted into equation 7, gives the following relationship (gravity is downwards and K is positive and Z is the distance from the bottom of the column, thus sign changes):

$$\frac{\partial\left(\frac{\theta_{ex}}{\theta_{r,ex}}\right)}{\partial t} = \frac{\partial}{\partial Z}\left[K\left(\frac{\theta_{ex}}{\theta_{r,ex}} - 1\right)^2\right] \qquad (8)$$

For the porous particles the overall flux will include a contribution from the flow within the particles, but this is assumed to be constant at the short times for which this model will be applied and thus does not contribute to the differential. The contribution of the flux within the particles to the flux out of the column is directly added on to the calculated flux, while the predicted liquid content is corrected using equation 5 in order to allow comparisons to be made with the measured results. It has also already been included within the calculation of the residual.

That the effect of capillarity on the flow around the particles will be quite a bit smaller than that of gravity for our experimental system can be demonstrated by the following analysis. The capillary pressures (P_c) will be of the following order (See Appendix A for the derivation):

$$P_c \approx O\left(\frac{\gamma}{r_{ps}}\frac{1}{\sqrt{\theta}}\right) \qquad (9)$$

where r_{ps} is the inter-particle spacing of the system (i.e. the length scale associated with the cross-sectional size of the pore spaces between the particles, which is related to particle size and its distribution and is also a function of packing).

The resultant capillary pressure will be order of tens to hundreds of Pascals since the surface tension of water (γ) is 0.07 N/m, particle spacing is of the order of millimetres and the liquid holdups are a few percent. Thus capillary pressure gradient over the length of the column will be order of 100-1000 Pa/m (column is tens of centimetres tall), which is significantly less than the effect of gravity, which is 10000 Pa/m (i.e. ρg). Thus gravity is the dominant factor in flow of liquid between the particles, with capillarity a small, though not necessarily totally insignificant factor (see below).

This first order PDE (equation 8) is solved numerically using the first order upwind method, which requires an initial condition and a top boundary condition (given that the flow is always

downwards). The top boundary condition is that there is no inflow and the following initial condition holds:

$$\theta/\theta_r\,(0,Z) = \theta_{ss}/\theta_r \qquad (10)$$

where θ_{ss} is the steady state holdup associated with the solution of equation 4 at the flow rate into the column before liquid addition ceased.

6.2. Comparison with Experimental Results

Since the value K is calculated based on experimentally obtained steady and residual holdups and the known liquid flux through the system (initially dry bed of glass beads, for v_s=0.12 mm/s, K=78.5 mm/s and v_s=0.06 mm/s, K=80.8 mm/s), the following simulations are carried out with no additional fitting parameters beyond these measured values. The simulated drainage velocities obtained from the solution of equation 8 were compared to sets of experimental drainage data. Figure 5 shows this behaviour for initial superficial velocities of v_s= 0.12 and v_s= 0.06 mm/s for 10 mm glass beads.

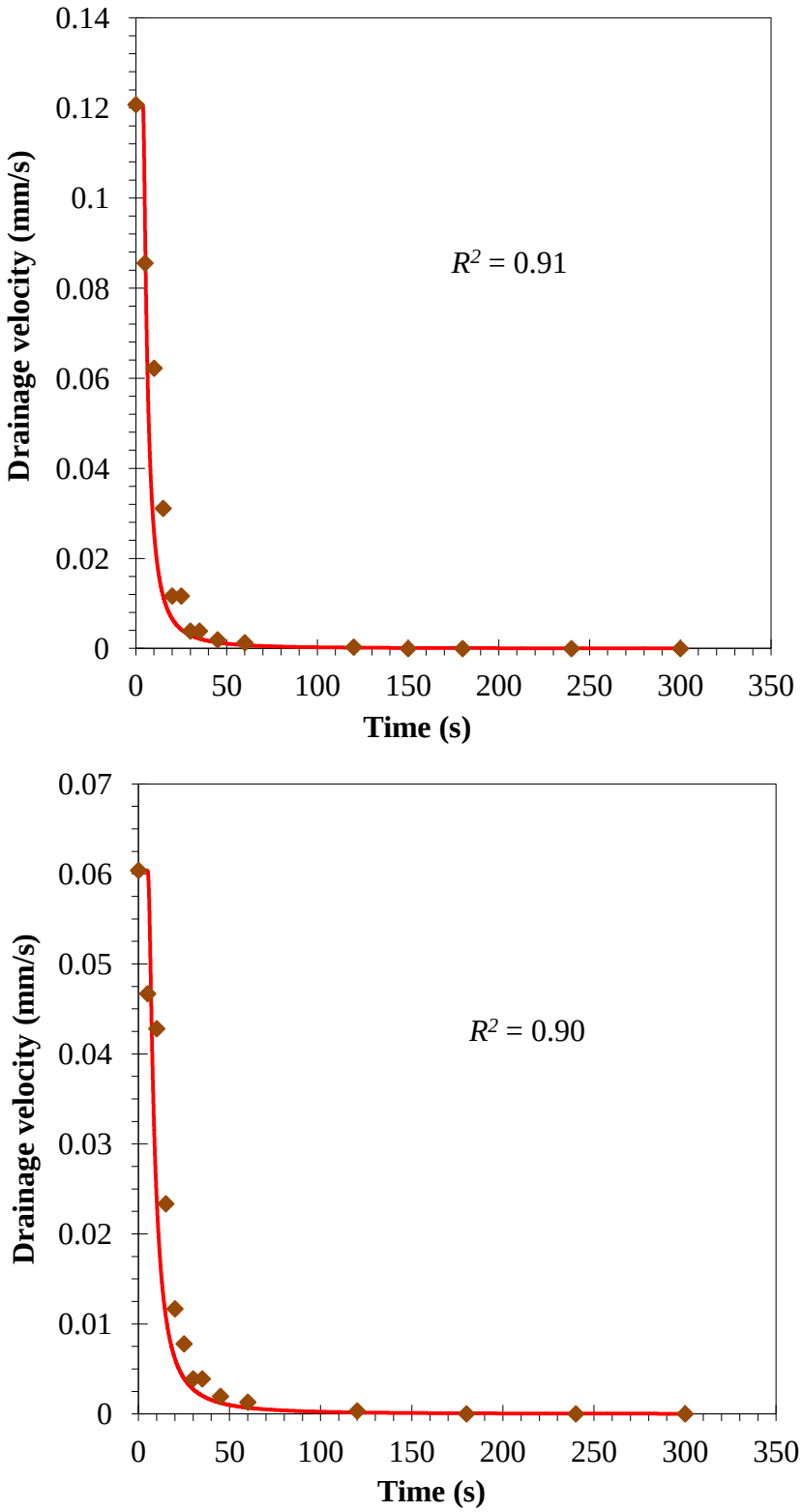


Figure 5: Simulated and experimental drainage curves for different superficial velocities with 10 mm glass beads in 300 mm column (solid line is simulated, diamonds are the experimental results). Top - 0.12 mm/s, bottom - 0.06 mm/s.

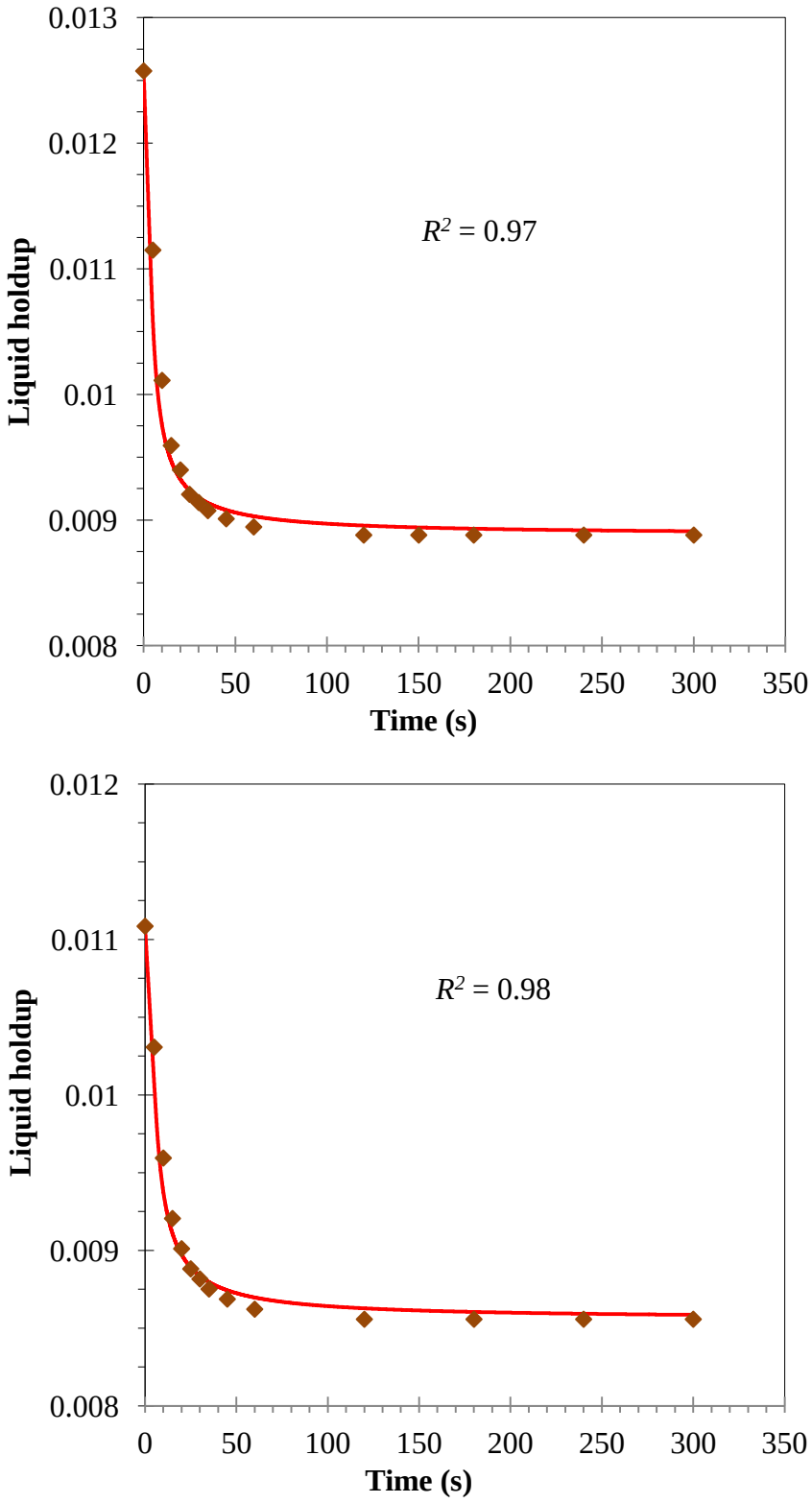


Figure 6: Simulated and experimental liquid contents during the drainage for different superficial velocities with 10 mm glass beads in 300 mm column (solid line is simulated, diamonds are the experimental results). Top - 0.12 mm/s, bottom - 0.06 mm/s.

In addition, Figure 6 shows gravimetrically measured average liquid holdup values (liquid volume/reactor volume) with the simulated liquid contents for 10 mm particles at same superficial liquid velocities in Figure 5. There are high R^2 values for the comparisons presented in Figures 5 and 6 despite the lack of any adjustable parameters beyond the inputted steady state behaviour. This gives us confidence as to the validity of the proposed form of the model.

For each set of experimental drainage curves in the ore system, the simulated drainage velocities were obtained using the numerical solution of equation 8. As stated previously, the flux due to flow through the particles, v_{end}, was added onto that obtained from the simulations, which are only for the flow around the particles.

The comparison of the drainage velocity in the 300 mm ore system with 8-11.2 mm particles with the corresponding simulations are shown in Figure 7 for initial superficial velocities of 0.12 and 0.03 mm/s. Since these simulations only consider gravity, the flow out of the column is simply a function of liquid content at the bottom of the column and not gradients in this liquid content, through which capillarity would act. A good fit to the experimental results can be seen, with very high R^2 values obtained despite the lack of adjustable fitting parameters.

In Figure 8 the simulated and experimentally obtained liquid holdup values are plotted. It can be seen that at short and long times there is good agreement between the simulated and measured liquid contents, but that at intermediate times there is a slight discrepancy between the results. It is surmised that this discrepancy is because the effect of capillarity is ignored. As there is no liquid addition into the column, the liquid content is, of course, directly related to the integral of the liquid flow out of the column. In Figure 9 simulations of the liquid content profile with height within the column are presented at a number of different time points. Note that Figure 8 plots the liquid holdup minus the holdup of the initially saturated ore particles, while Figure 9 is a simulation of the external liquid holdup. These values will differ due to the liquid lost from the pore spaces within the particles. This discrepancy can be estimated using equation 5.

Capillarity will always act down gradients of liquid content. As there is no gradient in the liquid content at the bottom of the column at short time periods, the flow out of the column during this initial period will not be influenced by capillarity. Eventually, though, the reduction in liquid content will hit the bottom of the column, accompanied by a gradient in the liquid content. Since the liquid content decreases with height in the column, the effect of capillarity would be to decrease the flow out of the column compared to the case of gravity acting along. This is consistent with what is seen in Figure 8, where the predicted liquid content (which does not

include the effect of capillarity) is lower than the experimentally measured one. At higher times the gradient decreases and thus so will the influence of capillarity.

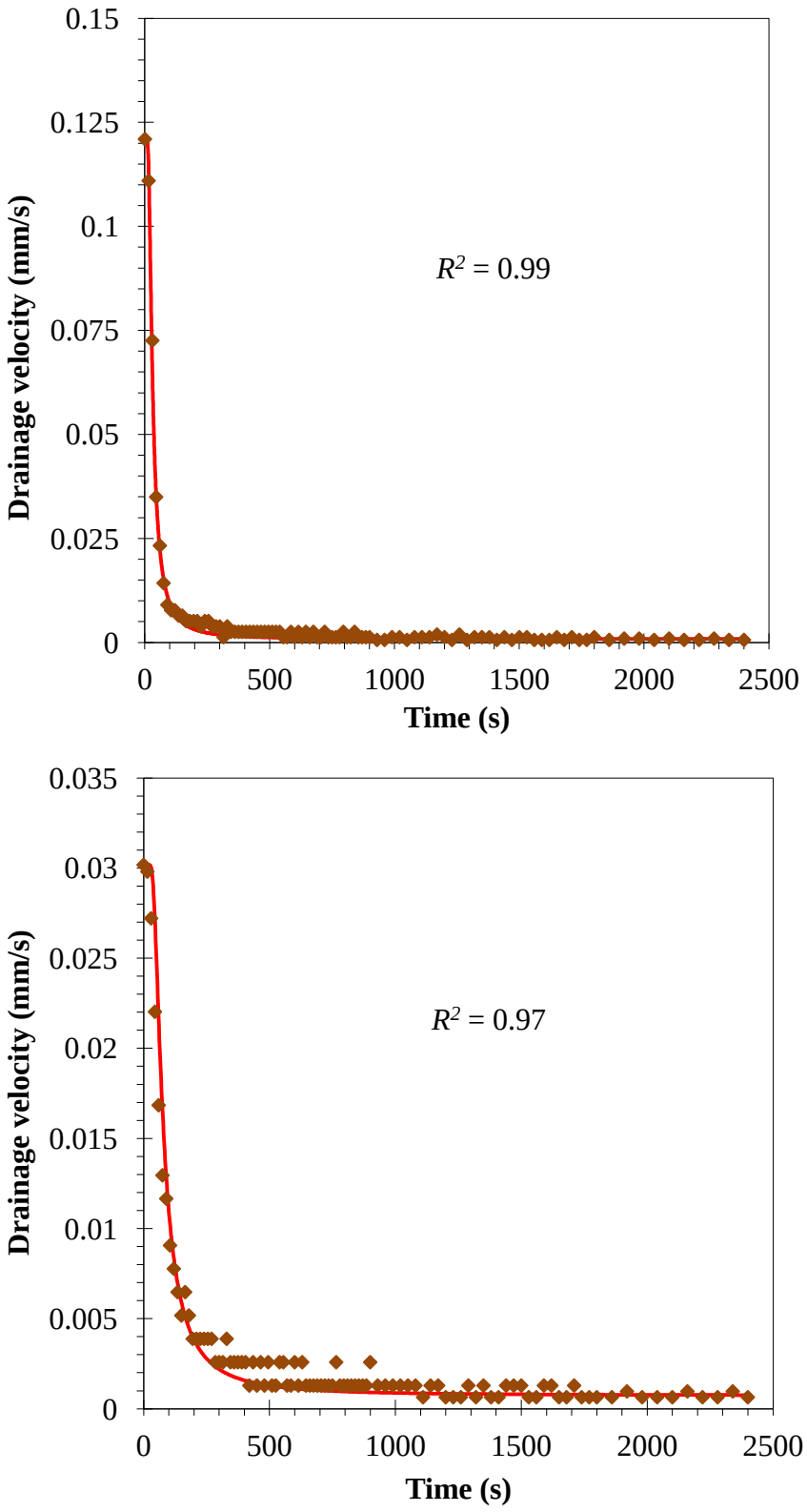


Figure 7: Simulated and experimental drainage curves for different superficial velocities with 8-11.2 mm ore particles in 300 mm column (solid line is simulated, diamonds are the experimental results). Top - 0.12 mm/s, bottom - 0.03 mm/s.

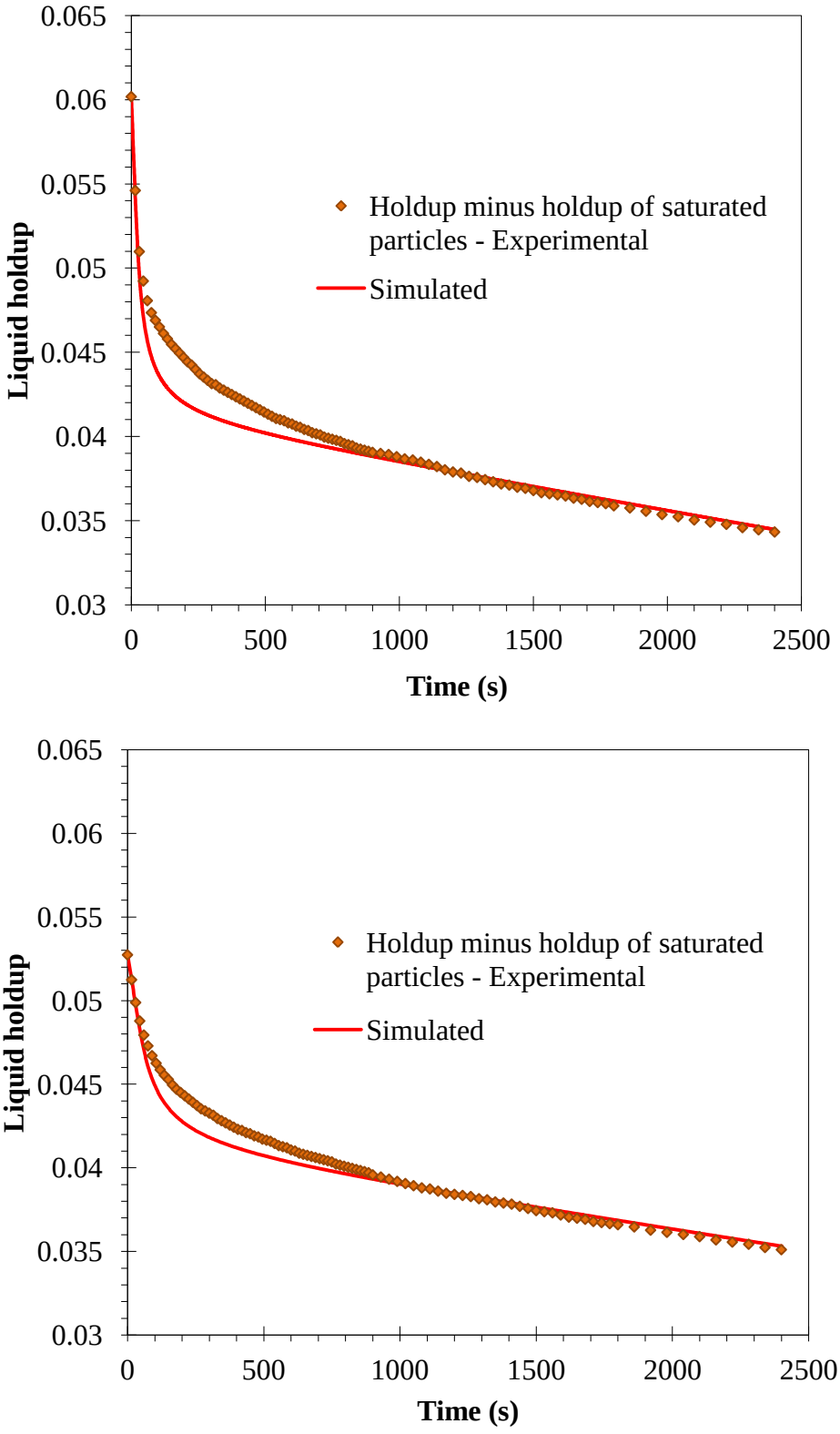


Figure 8: Simulated and experimental liquid contents during the drainage for different superficial velocities with 8-11.2 mm ore particles in 300 mm column. Top - 0.12 mm/s, bottom - 0.03 mm/s.

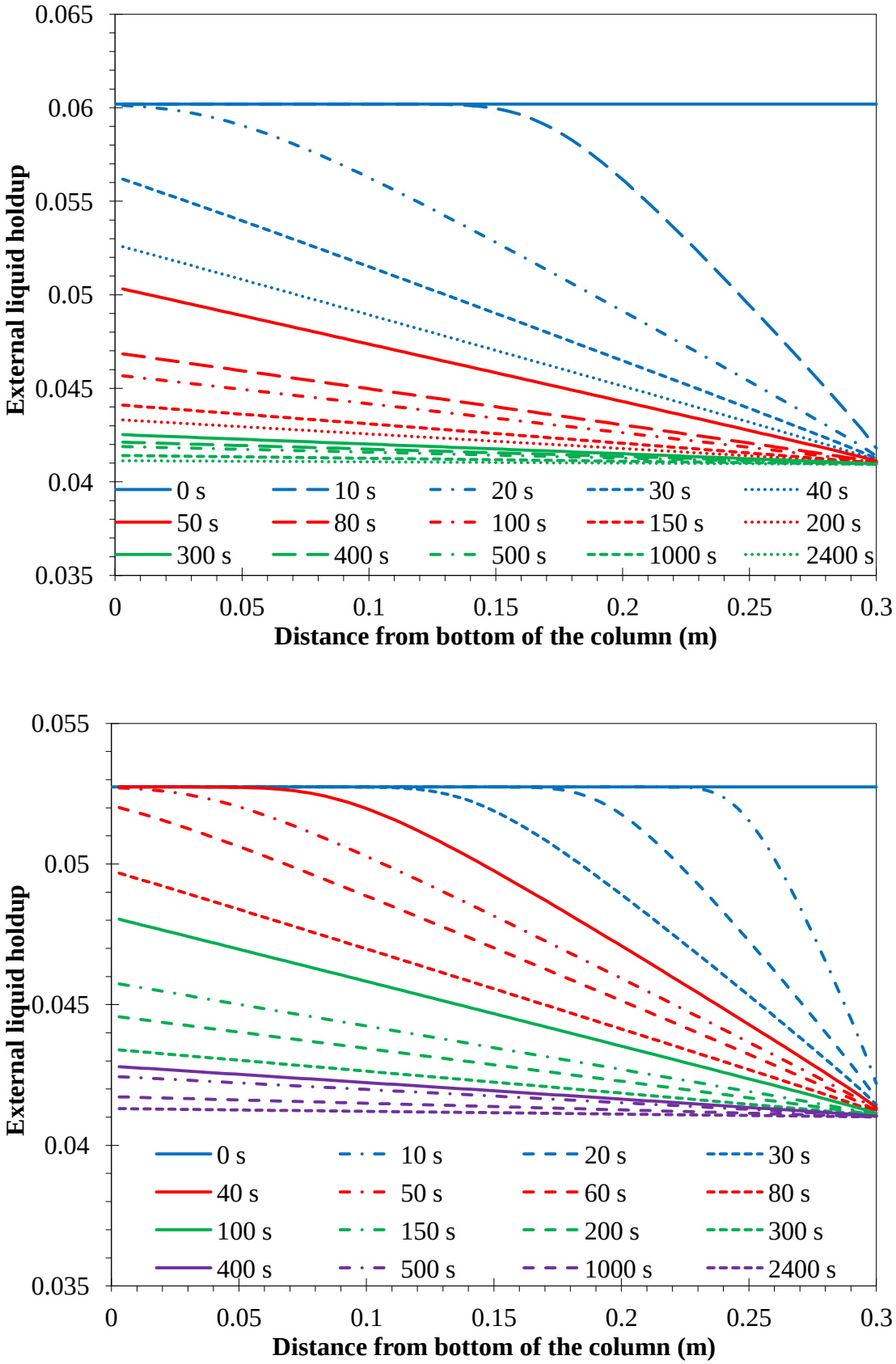


Figure 9: External liquid content of 8-11.2 mm ore particles in 0.3 m column as a function of column height for the superficial velocities of: Top - 0.12 mm/s, bottom - 0.03 mm/s.

7. Conclusions

This paper has shown that the initial wetting behaviour for the packed bed is relatively simple and takes the form of a soliton or standing wave. The drying behaviour of the bed when the liquid addition is stopped is slightly more complex, with distinct differences in the behaviour of the non-porous glass bead system compared to the slightly porous ore system.

When the effect of the liquid held within the particles is separated from that held around the particles, this paper has shown that the same relatively simple model can be used to describe the behaviour of both systems, with very good agreement between the simulated and measured average liquid holdup and liquid flow rate out of the bottom of the column.

The model used assumes that, for the liquid held between the particles, the effect of gravity on the drainage is stronger than that of capillarity. The generally good correlation between the experimental and simulated results indicates that this is a reasonable assumption, though the slight discrepancy in the average liquid holdup values for the porous system at intermediate times could be due to the effect of capillarity as the discrepancy is consistent with the effect that capillarity is likely to have on the system.

These results show that the model form presented in this paper, together with the experimental procedure for obtaining the model parameters allows not only the steady state, but also transient features of the drainage in these systems to be predicted with little need for complex model calibration. This is important in industrial applications where not only the steady state behaviour, but also the transients impact performance, such as the initial wetting and final drain down behaviour in heap leaching.

Appendix A: Derivation of a relationship for the capillary pressure

The dependency of the capillary pressure on the channel size and liquid content can be estimated as follows. This estimate of the order of magnitude of the capillary force is done by assuming an idealised geometry in which the liquid is held in the corner of drainage channels between the particles.

The capillary pressure is given by:

$$P_c = -\frac{\gamma}{r_c} \qquad \text{(A.1)}$$

where γ is the surface tension of the liquid and r_c is the radius of the curvature. Note that as the water is generally a wetting phase, the capillary pressure is negative as interface will curve into the liquid phase (i.e. water lower pressure than air).

$$r_c \propto \sqrt{A'} \tag{A.2}$$

where A' is the area of the channel between the particles occupied by liquid.

Since the total area of the channel will be proportional to the square of the particle spacing, r_{ps}, and the liquid holdup (θ) is proportional to the ratio of the area occupied by liquid to the total area:

$$\theta \propto \frac{A'}{r_{ps}^2} \tag{A.3}$$

Using equations A.2 and A.3:

$$\theta \propto \frac{{r_c}^2}{r_{ps}^2} \tag{A.4}$$

Now the relationship for r_c becomes:

$$r_c \propto r_{ps}\sqrt{\theta} \tag{A.5}$$

Using equations A.1 and A.5, the relationship for the capillary pressure is:

$$P_c \propto \frac{\gamma}{r_{ps}} \frac{1}{\sqrt{\theta}} \tag{A.6}$$

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