

Tuning the Resonant Frequency and Damping of an Electromagnetic Energy Harvester Using Power Electronics

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Abstract—In order to maximize power density, the resonant frequency of an energy harvester should be equal to the source excitation frequency and the electrical damping set equal to the parasitic damping. These parameters should be adjustable during device operation because the excitation characteristics can change. This brief presents, for the first time, a power electronic interface that is capable of continual adjustment of the damping and the resonant frequency of an energy harvester by controlling real and reactive power exchange between the electrical and mechanical domains while storing the harvested energy in a battery. The advantages of this technique over previously proposed methods are the precise control over the tuning parameters of the electrical system and integrated rectification within the tuning interface. Experimental results verify the operation, and the prototype system presented can change the resonant frequency of the electro-mechanical system by $\pm 10\%$ and increase the damping by 45%. As the input excitation frequency was swept away from the un-modified resonant frequency of the harvester, the use of the tuning mechanism was shown to increase real power generation by up to 25%. The prototype harvester is capable of generating 100 mW at an excitation frequency of 1.25 Hz.

Index Terms—energy harvesting, pendulum energy harvester, impedance emulator, maximum power transfer.

I. INTRODUCTION

INERTIAL energy harvesters achieve maximum power density when the transducer is operated at resonance and when the electrical damping coefficient D_e is equal to the parasitic mechanical damping D_p or further increased if the motion of the mass is displacement limited [1]. These harvesters are typically modelled using a mass-spring-damper system whose resonant frequency is determined by the proof mass and spring constant [2], [3]. Typically in such systems, the resonant frequency cannot be modified and this reduces the generated power when the source excitation frequency deviates from the transduction mechanism's mechanical resonant frequency.

Fig. 1 shows a typical configuration of an electrical equivalent circuit for an inertial energy harvester using an electro-

magnetic transduction mechanism (represented by a parasitic inductance and resistance, L_{arm} and R_{arm} , respectively, and a transformer). The resistor, inductor and capacitor on the primary-side of the transformer represent parasitic damping D_p , spring constant k , and proof mass, m , respectively. A resistor, R_L , is connected to the secondary windings to form a typical electrical load, and its value determines the electrical damping of the transducer and therefore the generated power for a given source excitation of frequency ω and amplitude Y_0 . Clearly, the addition of the reactive components, L_L and C_L in Fig. 1 is also possible, and the reason for this is explained below.

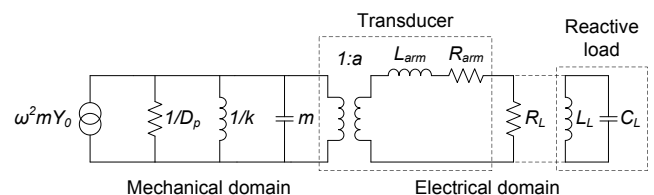


Fig. 1. Energy harvester electromechanical model with a generic electrical load consisting of discrete resistive and reactive components.

Previous attempts in changing the resonant frequency of energy harvesters have involved modifying the mechanical properties of the resonant system, such as the spring constant, as reported in [4]–[6]. This is equivalent to changing the value of the primary-side inductor in the system model in Fig. 1. In [4], Adams *et al.* developed a technique to modify the spring stiffness using an electrostatic comb-drive actuator, and in [5], Ayala *et al.* published results from a harvester with a contactless magnetic tuning mechanism, which modified the tensile strain in a cantilever thereby altering the spring stiffness. Challa *et al.* demonstrated a different configuration of permanent magnets capable of increasing and decreasing the resonant frequency [6]. The main disadvantage of these techniques, which alter the mechanical properties of the system, is the requirement of an actuator to perform the tuning, increasing the system complexity, volume and power overhead.

A different approach is presented in [7] where the mechanical resonant frequency of the harvester is modified by electrical means by changing the secondary-side impedance using reactive loads (L_L and C_L in Fig. 1). As can be seen, by referring these reactive components to the transformer primary, the shunt load inductance increases the system resonant frequency, whereas the load capacitance decreases the system

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resonant frequency because the electrical components are in parallel with the mechanical parameters. The main advantage of this technique is that the complexity is low. However, there are limitations in which a finite number of discrete reactive load components cannot produce a system with continuously variable tuning. In addition, an energy storage element with an appropriate interface is required instead of dissipating energy in a resistive load.

In this brief, we present a new approach to developing a tuneable energy harvester by using an H-bridge power electronic interface, which is able to mimic a generalised complex load impedance and is also capable of rectifying the generated electromotive force (EMF) and storing the generated energy into a battery. The rest of the brief is divided into the following sections: a description of the power electronic interface, harvester overview and modelling, simulation results, the system architecture, experimental results and conclusions.

II. HARVESTER POWER ELECTRONICS

A model of the energy harvesting system with an H-bridge interface to the battery is shown in Fig. 2.

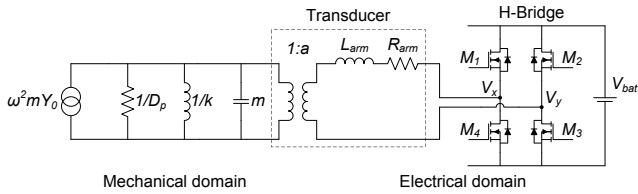


Fig. 2. Electrical equivalent circuit of an electromagnetic energy harvester with an H-bridge interface.

In order for the power electronic interface to be able to mimic the electrical characteristics of an inductor, capacitor or resistor (or a combination of these components to form a generalised load), the terminals of the converter that connect to the transducer must be able to supply and consume reactive power and consume real power. This means that, for any polarity of transducer EMF, the direction and magnitude of the flow of power (between the mechanical system and the battery) must be controlled as a function of the load impedance that the interface circuit is emulating. By choosing the duty cycle and switching sequence of the MOSFETs (M_1 – M_4), the H-bridge is able to perform this task because the bridge can operate as either a buck or boost converter for either polarity of input voltage, where the inductance of the transducer L_{arm} forms the converter inductance in each configuration. Provided the battery voltage exceeds the generated emf from the transducer, operating the bridge in boost mode steps up the voltage from the transducer and pushes energy into the battery. Conversely, operating the H-bridge in buck mode transfers energy from the battery to the mechanical system.

With reference to Fig. 2, if the polarity of transducer voltage is such that $V_x > V_y$, power can be transferred from the transducer to the battery by operating the H-bridge as a boost converter, meaning that M_3 is held permanently on and M_2 is permanently off. M_4 is then pulsed on and off at high frequency and the antiparallel diode of M_1 forms the

freewheeling path in the converter. The duty cycle of M_4 then sets the magnitude of the current flow. In order to transfer energy in the opposite direction, *i.e.* from the battery to the mechanical system, M_2 and M_3 keep the same states but now M_1 is pulsed and the anti-parallel diode of M_4 provides the freewheeling path. Conversely, if $V_y > V_x$, M_4 is held permanently on and M_1 permanently off and M_2 and M_3 control the direction of power flow. The efficiency of the system can be increased by using synchronous rectifiers rather than using the diode freewheeling paths in the MOSFETs.

If the bridge consumes or supplies only reactive power, then the circuit mimics an inductor or capacitor, respectively, which has the effect of increasing or decreasing the system resonant frequency. If the bridge consumes only real power, the circuit emulates a pure resistance and this corresponds to a transfer of energy from the mechanical domain to the battery. By controlling the effective value of this resistance, the electrical damping can be set to maximise the generated power. In the presence of both real and reactive power flows the converter sets both the electrical damping and the resonant frequency. As the switch duty cycles are continuously variable (between 0 and 1), the bridge input impedance can effectively take any value between the limits set by parasitic components in the circuit and is therefore not restricted to specific values, which is the case when using discrete reactive loads, as shown in [7].

III. PENDULUM HARVESTER

A resonant inertial energy harvester was built to validate the concepts presented in this brief. The harvester, as shown in Fig. 3, is a pendulum-type device designed for harvesting energy from the rocking motion that naturally arises in a small boat. The proof mass is a 120° segment of a hollow dural cylinder, freely suspended from a pivot by ball bearings. The proof mass is 18.4 cm in length, with inner and outer radii of 9 cm and 11 cm, respectively. The pendulum motion is coupled, via a rack-and-pinion arrangement, to two brushed dc motors, which are connected in series to form the transducer, and to a third dc motor, which acts as a tachogenerator.

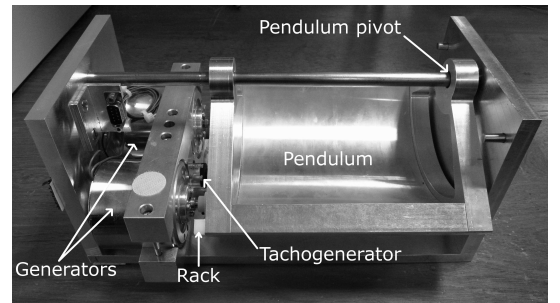


Fig. 3. The prototype pendulum energy harvester with DC generators and tachogenerator connected via a rack and three pinions.

The restoring torque, T_r , for a pendulum is given by:

$$T_r = -mgr \sin(\theta) \quad (1)$$

where m is the proof mass, g is the acceleration due to gravity, r is the distance from the rotation axis to the centre of mass, and θ is the angular displacement.

For small displacements, T_r is approximately linear, and the resonant frequency of the pendulum is expected to be

$$f_n = \frac{1}{2\pi} \sqrt{\frac{mgr}{I}} \quad (2)$$

where I is the moment of inertia referred to the rotation axis.

For the prototype harvester, $m = 2.4$ kg, $I = 0.023$ kgm² and $r = 7.8$ cm; hence, the resonant frequency is expected to be 1.42 Hz. In practice, f_n will be lowered by damping and, at larger amplitudes, by the weakening of the restoring torque due to the $\sin(\theta)$ term. The transducers and tachogenerator will also add some inertia. The natural resonant frequency of the pendulum harvester was measured by observing the voltage on the tachogenerator when the pendulum was released from a displaced position and was found to be 1.2 Hz.

At this stage we make a distinction between linearly and rotationally excited harvesters [8]. The usual harvester model, as represented by Figs. 1 and 2, assumes a reciprocating linear excitation of the proof mass and is the most common type of harvester studied to date. In contrast, the pendulum device is rotationally excited, and as shown in Fig. 4, the frame oscillates in an arc of amplitude Ω_0 at a driving frequency ω . A relative angular velocity between the pendulum and the frame causes the transducer to generate an EMF, driving a current and damping the relative motion with electrical damping constant D_e . Parasitic mechanical losses are represented by D_p . For the rotationally excited pendulum experiencing small angles of θ , the linearised equivalent circuit can be represented by Fig. 2 if the current source has magnitude of $\omega^2 \Omega_0 I$ and the primary-side capacitance has a value of I .

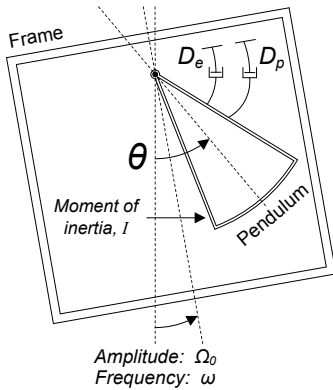


Fig. 4. Mechanical schematic of the pendulum harvester.

IV. PSpice SIMULATIONS

A mixed electromechanical model of the rotationally excited pendulum was constructed in PSpice. The values on the primary-side of the transformer in the model were set to be equivalent to the mechanical parameters described in Sec. III. The turns ratio of the transformer was set based on the dc generator's motor constant and the gear ratio between pinion and rack.

An instantaneous demand current to be drawn from the transducer by the bridge was calculated in the simulation based on the measured EMF of the transducer and the required

bridge input impedance, where the input impedance is set to behave as a parallel RLC circuit. In a practical implementation, the optimal demand current is dependent on the source excitation frequency and bridge input impedance. The former can be obtained by processing the output of one or more accelerometers, and the latter can be adjusted according to precalculated values. Alternatively, a microprocessor can be used to implement a *perturb and observe* algorithm to achieve maximum power transfer. This method may not require any inertial sensors. The demand reference current I_{ref} is therefore set as

$$I_{ref}(t) = \frac{V_{emf}(t)}{R} + C \frac{dV_{emf}(t)}{dt} + \frac{1}{L} \int_0^t V_{emf}(t) dt \quad (3)$$

where V_{emf} is the transducer generated EMF.

The switches were controlled using a pulse-width modulator (PWM) and, as stated in Sec. II, the bridge can be thought of as operating in four modes depending on the direction in which power should flow and the polarity of the transducer EMF. Here, for the sake of simplicity, the controller designed in PSpice does not differentiate between these four operating modes although the effect of the switching scheme is the same. For each branch of the H-bridge, the high-side switches M_1 and M_2 are operated in antiphase with respect to the low-side switches M_3 and M_4 . When the current error signal is zero, each side of the bridge is operated with a 50% duty cycle. If there is an error, the duty cycle is increased on one side of the bridge and decreased by the same amount on the other. This is the same as breaking up the operation into four modes, but simplifies the controller at the expense of increasing the number of switching operations and thus increasing the switching losses.

The objective of this simulation was to verify that transducer currents could be drawn with variable magnitude and phase, i.e. checking that any complex load can be synthesised by the H-bridge and battery arrangement. In addition, the simulations were also used to demonstrate that a net energy flow could be achieved from the mechanical system into the battery in the presence of losses with real semiconductor device models.

The plots in Fig. 5 show simulation results of the H-bridge extracting real power from the mechanical system into the battery and controlling an exchange of reactive power between the battery and mechanical system. Fig. 5(a) shows the electrical system behaving as a 50 Ω resistor. As expected, the transducer EMF and bridge current are inphase, and thus, instantaneous power into the battery is always positive. Fig. 5(b) shows the bridge emulating a 1000 μ F capacitive load. In this case, predominantly reactive power flows, as can be seen by the alternating sign of the instantaneous power, and the bridge current leads the transducer EMF. In Fig. 5(b), the average power is slightly below zero due to the losses in the bridge.

V. PROTOTYPE SYSTEM ARCHITECTURE

As discussed above, in order to control the input impedance of the converter, the transducer EMF and the input current must be measured. The demand current can be calculated from the transducer EMF based on the required input impedance

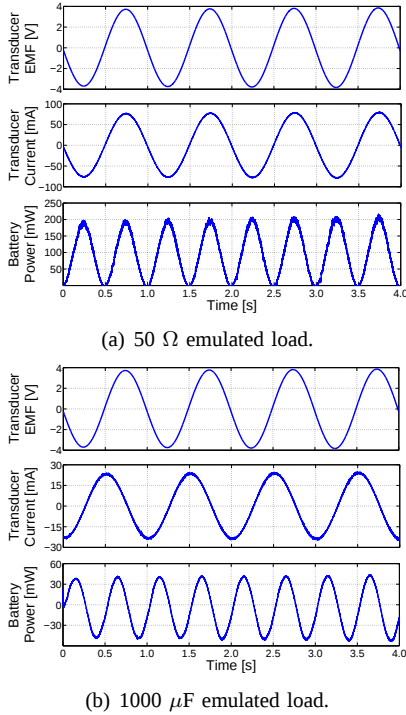


Fig. 5. Simulation of synthesised loads using the H-bridge.

of the converter. A control loop can then be used to ensure that the measured current in the transducer tracks the demand current. However, unlike in the PSpice simulation, it is not possible to directly measure the transducer's generated EMF when a current is flowing because of the transducer's internal impedance. Therefore, the EMF is inferred via a separate tachogenerator, which is mechanically ganged to the generator transducer, as shown in Fig. 6. In this system, the tachogenerator contributes only an additional 2% in volume, 3% in mass and negligible cost over the two dc generators because it does not have to supply power. This means that the overhead of employing this technique is low compared to the ones reported in [4]–[6].

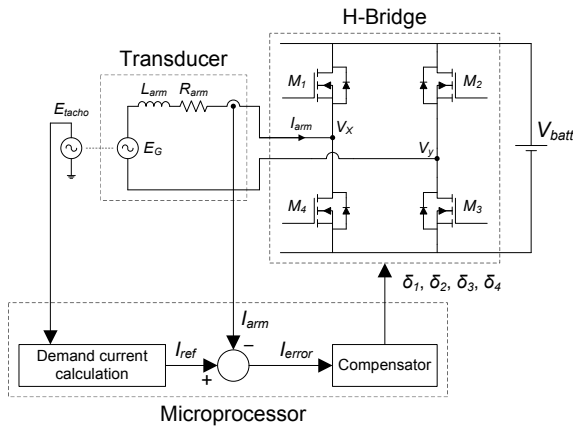


Fig. 6. Schematic of the energy harvesting electronics.

In this implementation, a PIC18F2431 microprocessor samples the tachogenerator EMF and the bridge current through a current transducer, and the microprocessor calculates a

demand current for the bridge every 0.83 ms using (3). A digital proportional-integral-derivative compensator is then closed around the system to ensure that the measured current tracks the calculated demand current. Modification of the bridge current is performed through operation of the switches of the H-bridge, as was done in the PSpice simulations. The bridge MOSFETs (FDN339AN from Fairchild with an $R_{ds(on)}$ of 35 m Ω) were driven by two L6387 low- and high-side gate drivers from STMicroelectronics. The chosen dc generators, when configured in series, have a combined L_{arm} and R_{arm} of 10 mH and 15.6 Ω , respectively. Consequently, in order to keep the transducer ripple current to around 10% of the expected maximum value of around 75 mA [Fig. 5(a)] with a maximum inductor voltage of $V_{batt} = 6$ V, a PWM switching frequency of around 40 kHz is required.

Fig. 7 shows the measured transducer EMF and current waveforms for the prototype setup where the bridge is configured to behave as a pure 1000 μ F capacitor. The current amplitude and phase (relative to the transducer EMF) are as expected for this load, within 5% experimental error.

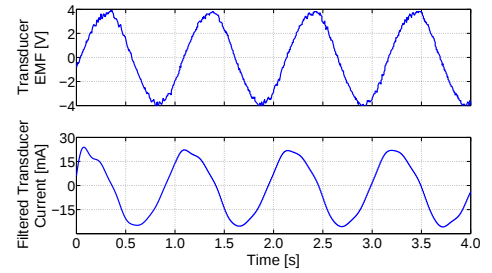


Fig. 7. Measured waveforms from the prototype pendulum harvester, with the H-bridge emulating a 1000 μ F capacitor.

VI. EXPERIMENTAL RESULTS

Having validated the bridge circuit, the complete experimental system was then tested on a laboratory rocking table at different excitation frequencies at a fixed amplitude. The resonant frequency of the system for different synthesised reactive loads was obtained by sweeping the excitation frequency and looking for the peak in the response of the integral of the tachogenerator EMF (corresponding to pendulum position). Changes in system damping through the use of a synthesised resistive load were analysed by normalising the pendulum maximum position with a given load to the open-circuit case.

Fig. 8 shows that the system damping changes as expected with load conductance. As the connected conductance increases, increased current flows in the transducer, causing the pendulum relative motion to decrease. If negative conductance is added, the bridge supplies power to the mechanical system and the normalised pendulum displacement increases.

Fig. 9 shows similar changes to the resonant frequency, i.e., for addition of a positive and negative susceptance, the system's unmodified resonant frequency (1.17 Hz) can be decreased and increased, respectively. A $\pm 10\%$ change in resonant frequency was achieved before the bridge current saturates due to R_{arm} , which is comparable to previously reported results [2].

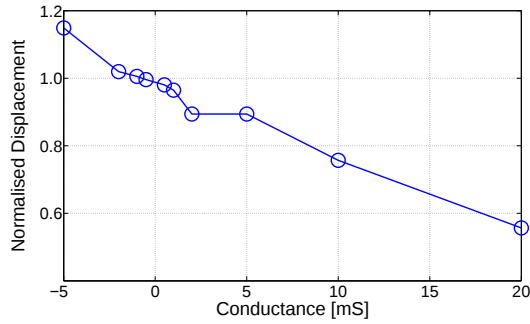


Fig. 8. Observed modification in damping through changes in normalised displacement with the H-bridge emulating different conductances.

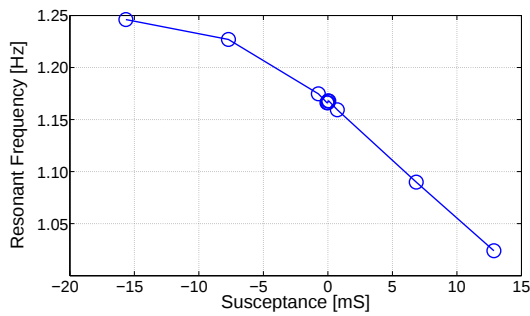


Fig. 9. Observed modification in resonant frequency with the H-bridge emulating different susceptances.

The output power improvement as a result of the tuning mechanism, is shown in Fig. 10 where a synthesised load consisting of a $150\ \Omega$ resistance and capacitances of 0, $300\ \mu\text{F}$ and $-300\ \mu\text{F}$ was used. Negative values of capacitance are used to compare the effects of increasing and decreasing the H-bridge reactance. They are possible because the values are simply used in the mathematical calculation of (3).

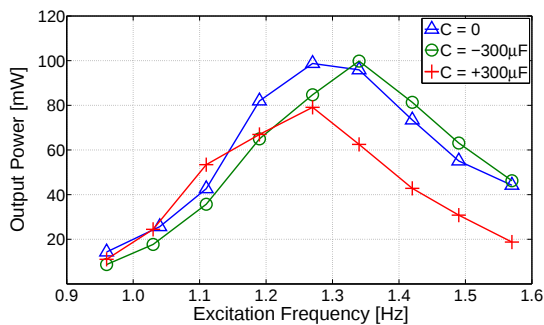


Fig. 10. Power output of the prototype harvester with $150\ \Omega$ real load impedance and different parallel reactances.

For the excitation amplitude used in testing, a maximum power of 100 mW was generated at resonance (1.27 Hz) with a damping corresponding to the bridge synthesising a $150\ \Omega$ real impedance. Adding a synthesised $-300\ \mu\text{F}$ capacitance increases the output power at frequencies beyond ~ 1.34 Hz due to the expected increase in resonant frequency. In contrast, with a synthesised capacitance of $300\ \mu\text{F}$, the output power is greater than the purely resistive case at frequencies below ~ 1.13 Hz. At 1.1 Hz, the increase in real power output with the addition of the synthesised $300\ \mu\text{F}$ capacitor is around

25%, confirming that the system can be used to substantially increase the power output of energy harvesters. The peak power at resonance is lower when the $300\ \mu\text{F}$ capacitor is added because of increased losses associated with the reactive current flow, and also because of the cubic relationship between power and excitation frequency, which is inherent to inertial harvesters [1]; with the $-300\ \mu\text{F}$ capacitor, these two effects oppose one another with the result that the peak power is similar to that with $C = 0$.

VII. CONCLUSION

Tuning of resonant frequency and damping is necessary if inertial energy harvesters are to be successfully deployed in environments where vibration frequencies change over time. Previous demonstrations of resonant frequency tuning have used mechanical means or have added discrete reactive loads. The former requires an additional actuator, whereas the latter does not allow continuously variable tuning.

This prototype system has demonstrated, for the first time, a technique using a power electronic solution to tuning energy harvesting devices over a continuous range, which also includes rectification and the capability to interface to a battery. An H-bridge interface capable of synthesising a range of complex load impedance values was designed, simulated and tested as an interface to an inertial pendulum harvester using brushed dc motors as the transducer. The control system was constructed using a low-power microprocessor and the power devices were chosen to achieve efficient operation. Further minimisation of losses and controller overhead would be possible using higher quality generators and a custom control application-specific integrated circuit.

Experimental results show that a $\pm 10\%$ deviation from the transducer's unmodified mechanical resonance was achieved, enabling the energy harvester to operate over an increased frequency range with an increase in real output power of up to 25%. The components required to implement this tuning system consume negligible mass and volume.

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