

An Explicit Direct Method for Transient Stability Analysis of Multimachine Power Systems With Nonzero Transfer Conductances

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Abstract—We propose an explicit analytical direct method for the transient stability analysis of multimachine power systems with nonzero transfer conductances. The proposed method addresses two issues. In the first, we study the transient stabilization of the entire power system through excitation control design. To this end, a globally well-defined Lyapunov function is constructed, and a locally well-defined dynamic passivity-based control law is proposed. The closed-loop equilibrium is therefore guaranteed to be locally asymptotically stable. In the second, we study the transient stability property of post-fault initial states. To this end, an optimization-based approach to calculate the critical level set of the proposed Lyapunov function is proposed. This allows to estimate an explicit region of attraction of the closed-loop equilibrium. Therefore, the transient stability property of a post-fault initial state can be directly assessed. A case study on the IEEE 10-machine 39-bus power system, to demonstrate the performance and effectiveness of the proposed direct method, is presented.

Index Terms—Lyapunov’s direct method, optimization, passivity based control, region of attraction, transient stability, transfer conductances.

NOMENCLATURE

δ_i	Rotor angle
δ_{ij}	Angular separation
ω^*	Synchronous angular speed
ω_i	Angular speed
B_{ij}	Transfer susceptance
D_i	Damping coefficient
$E_{fd,i}$	Excitation field voltage (the control signal)
$E'_{q,i}$	Transient voltage along the q -axis
G_{ij}	Transfer conductance
H_i	Inertia time constant
$I_{d,i}$	Stator current along the d -axis
$I_{q,i}$	Stator current along the q -axis

$P_{e,i}$	Active power output
$P_{m,i}$	Mechanical power injection
$T'_{do,i}$	Transient time constant of the d -axis open circuit
V_i	Terminal bus voltage
$X_{d,i}$	Synchronous reactance along the d -axis
$X'_{d,i}$	Transient reactance along the d -axis

I. INTRODUCTION

A. Background

TRANSIENT stability analysis (TSA) is concerned with the ability of a power system to maintain synchronism after being subject to a large transient disturbance, such as a short-circuit fault on the transmission facilities, loss of generation, or loss of large loads [1]. Typically, the fault modifies the topology of the transmission network, forcing the state to diverge from the pre-fault equilibrium. The post-fault initial state is obtained after the fault is cleared. Then, if the post-fault state trajectory converges to a stable equilibrium, the power system is deemed to be transiently stable. Otherwise, the power system loses synchronism which may result in severe consequence such as blackouts. Therefore, a reliable yet efficient method for TSA is of fundamental importance for the secure operation of power systems [1], [2].

Currently, the most widely accepted method for TSA by system operators is through off-line time-domain simulation (TDS) method [3]. This is mainly because the TDS method can provide intuitive results on the state trajectories following a list of contingencies. However, the TDS method has two intrinsic shortcomings. On one hand, the computational cost of the TDS method for TSA is tremendous, since there are typically thousands to millions of contingencies that need to be checked on a regular basis [3], [4]. This raises greater concerns because of the increasing share of inverter-based resources and variable renewable energy sources in low-carbon power systems, for which the transient stability property should be assessed on the basis of more complicated dynamic model and under many more scenarios [5], [6]. On the other hand, the TDS method does not reveal quantitative information on the transient stability margin. Such information is valuable to the systematic guidance for corrective actions and protective relay settings [3]. Therefore, it is of practical importance to take into consideration *direct methods* for reliable and efficient TSA, which avoids computationally expensive simulations for post-fault state trajectories and provides quantitative information on the region of attraction (ROA) of the post-fault stable

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equilibrium [3]. In this regard, the essence of direct TSA is to check whether the post-fault initial state lies inside the ROA of a stable equilibrium.

B. Literature Review

Extensive research work has been conducted towards direct methods for TSA, among which the energy-function-based direct methods have drawn significant attention (see the seminal work [7] and [8]–[21]). Without exception, existing energy-function-based direct methods suffer from an obstacle, i.e., there exists no global analytical energy function for the multimachine power system model with transfer conductances (TCs) included [3], [22]. Note that the existence of TCs in classical power system models arises from small transmission losses and large resistive loads. Hence, the dynamic model of a practical power system inherently includes TCs. This obstacle has resulted in wide uses of numerical energy functions in the design of direct methods for TSA. However, numerical energy functions have no well-defined expression, because their potential-energy-term is path-dependent. Hence, the evaluation of an energy function relies on step-by-step approximation along the state trajectories. If such a way of approximation is not appropriately handled, the numerical energy-function-based methods lead to a misjudgment of transient stability properties [3], [23].

In parallel with the above-mentioned numerical methods, analytical Lyapunov-function-based direct methods for TSA have also been studied (see, e.g., [24]–[30]). These analytical methods are based on Lyapunov stability theory, which allows certifying the stability property of an equilibrium without explicit knowledge of the state trajectories. Furthermore, the level sets of a Lyapunov function can provide an estimation of the ROA of the stable equilibrium [25]. However, it has been a long-standing problem to construct an explicit analytical Lyapunov function for the multimachine power system model with TCs, since 1973 when [24] proposed a rigorous Lyapunov function for the two-machine power system model with TCs. It was conjectured in [24] that this solution could be generalized to the multimachine power system model with TCs. However, follow-up attempts to construct a rigorous Lyapunov function for the multimachine power systems with TCs have been in vain [3], in the sense that the proposed Lyapunov functions are either not well-defined or such that their time-derivatives along trajectories are not strictly non-positive.

Notably, in 2005, [25] has reported a breakthrough in dealing with the TCs for multimachine power systems. This breakthrough has been achieved from a perspective of stabilization of the entire multimachine power system through excitation feedback control design for all synchronous machines. A non-separable form of Lyapunov function has been proposed by introducing a nonstandard cross-term vector. In addition, the interconnection and damping assignment passivity-based control (IDA-PBC) methodology has been applied for the design of excitation feedback control law. This endows the entire closed-loop power system with a port-controlled Hamiltonian structure. Then, it has been proved, using the Implicit Function Theorem, that there exists (local) cross-term vector under the

assumption that the TCs are sufficiently small [25]. It has also been proved that there exists a (locally) well-defined IDA-PBC law ensuring asymptotic stability of the desired equilibrium in the sense of Lyapunov. Furthermore, [25] has claimed that the ROA of the closed-loop equilibrium can be estimated by the largest bounded level set of the non-separable Lyapunov function which contains this equilibrium. Despite the above-mentioned breakthrough, neither an explicit form of Lyapunov function nor an explicit control law has been suggested. Moreover, it is worth noting that the TCs in the dynamic model of a practical power system is far from trivial, which has limited the applicability of the solution proposed in [25].

As a natural extension, [29] has proposed an explicit passivity-based excitation control law, which ensures local asymptotic stability of the closed-loop equilibrium for the multimachine power systems with nonzero TCs. In addition, a locally well-defined Lyapunov function has been proposed. This is mainly achieved by introducing an auxiliary dynamic state which contributes to the explicit derivation of the cross-term vector. However, [29] has not exploited the proposed Lyapunov function for the estimation of ROA. Hence, as a step forward, [30] has exploited the level set of a well-defined Lyapunov function for the estimation of the ROA of the stable equilibrium. It should be pointed out that [30] has overlooked insights into the physical interpretation of the power system model. To be specific, the weighting coefficients of the proposed Lyapunov function do not have clear physical meanings, which leads to the difficulties in tuning control parameters. This may result in non-smooth closed-loop transient performances. Besides, the case study in [30] focuses on the visualization of the ROA by considering numerous randomly generated initial states. However, from a practical perspective, the post-fault initial states should be obtained following a list of contingencies. In this regard, further elaboration on improving closed-loop transient performances as well as applicability of direct methods should be considered for TSA.

C. Contribution

This paper extends the results of [25], [29], [30] in the direction of seeking for an “explicit” analytical solution. To this end, this paper proposes an *explicit* direct method for the TSA of the multimachine power system model with *nonzero* TCs. The contributions and novelties of this work are as follows.

- 1) An explicit globally well-defined Lyapunov function for the flux-decay model of a multimachine power system with nonzero TCs is constructed. First, the proposed Lyapunov function has a prominent advantage over existing analytical Lyapunov functions in that it allows certifying asymptotic stability of a desired equilibrium without the assumption of sufficiently small TCs. Second, the proposed Lyapunov function is inherently better than numerical energy functions in that it provides a direct and consistent assessment of any post-fault initial state, hence avoiding the inevitable approximation error associated to numerical methods. In addition,

the weighting coefficients of the proposed energy-like Lyapunov function are specified with clear physical meanings, which contributes to the tuning of the control parameters and the improvement of closed-loop transient dynamics.

- 2) An explicit dynamic passivity-based control law is proposed. This endows the closed-loop multimachine power system with a port-controlled Hamiltonian structure and ensures asymptotic stability of the desired equilibrium. This is achieved by including an auxiliary state together with its dynamic law, which forms the major novelty of the proposed method. In addition, the proposed control law enhances the closed-loop transient responses when compared to traditional control schemes such as the automatic voltage regulator (AVR) and the power system stabilizer (PSS).
- 3) An explicit ROA of the closed-loop equilibrium is estimated by exploiting the proposed Lyapunov function. This is achieved through an optimization-based approach which calculates the critical forward-invariant level set of the proposed Lyapunov function. This approach avoids undesired overestimate of the ROA, hence it leads to a reliable assessment of the transient stability properties of post-fault initial states.

D. Organization

The rest of the paper is organized as follows. Section II presents the dynamic model of a multimachine power system with TCs, followed by the problem formulation of direct TSA. Section III presents the first aspect of the proposed direct method, i.e., the construction of Lyapunov function together with the design of excitation control law. Section IV presents the second aspect of the proposed direct method, i.e., the design of an optimization-based approach to calculate the ROA of the stable equilibrium. Then, Section V presents a case study on the IEEE 10-machine 39-bus system, which demonstrates the performance and effectiveness of the proposed explicit direct method. Finally, Section VI draws the conclusion and discusses potential future work.

II. MODEL AND PROBLEM FORMULATION

A. Dynamic Model

Consider a power system comprising of n synchronous machines ($n \in \mathbb{N}$ and $n \geq 3$), and these machines are interconnected through the reduced transmission network including nonzero TCs. Hereinafter, the considered power system is termed as the lossy n -machine power system.

The dynamics of the i^{th} machine ($i \in \mathbb{N}$ and $i \leq n$) is given by the well-known flux-decay model (see, e.g., [31, equation (9)], [32, equation (5.11)], and [33, equation (S1)]):

$$\dot{\delta}_i = \omega_i - \omega^*, \quad (1a)$$

$$\frac{2H_i}{\omega^*} \dot{\omega}_i = -\frac{D_i}{\omega^*} (\omega_i - \omega^*) + P_{m,i} - P_{e,i}, \quad (1b)$$

$$T'_{do,i} \dot{E}'_{q,i} = -E'_{q,i} + (X_{d,i} - X'_{d,i}) I_{d,i} + E_{fd,i}, \quad (1c)$$

for $i = 1, 2, \dots, n$. The dynamic state of the i^{th} machine is denoted by $x_i(t) = [\delta_i(t), \omega_i(t), E'_{q,i}(t)]^\top \in \mathbb{R}^3$, which

consists of the rotor angle $\delta_i(t) \in \mathbb{R}$, the angular speed $\omega_i(t) \in \mathbb{R}_{>0}$, and the q -axis transient voltage $E'_{q,i}(t) \in \mathbb{R}_{>0}$. The system parameters include the synchronous angular speed $\omega^* \in \mathbb{R}_{>0}$, the inertia time constant $H_i \in \mathbb{R}_{>0}$, the damping coefficient $D_i \in \mathbb{R}_{\geq 0}$, the constant mechanical power $P_{m,i} \in \mathbb{R}_{>0}$, the d -axis transient time constant $T'_{do,i} \in \mathbb{R}_{>0}$, the d -axis synchronous reactance $X_{d,i} \in \mathbb{R}_{>0}$, and the d -axis transient reactance $X'_{d,i} \in \mathbb{R}_{>0}$. Note that the control signal of the i^{th} machine is the excitation field voltage $E_{fd,i}(t) \in \mathbb{R}$. The system variables include the active power $P_{e,i}(t) \in \mathbb{R}_{>0}$, and the d -axis current $I_{d,i}(t) \in \mathbb{R}$. The active power is given by:

$$P_{e,i} = E'_{q,i} I_{q,i}, \quad (2)$$

where $I_{q,i}(t) \in \mathbb{R}_{>0}$ is the q -axis current, given by:

$$I_{q,i} = \sum_{j=1}^n E'_{q,j} (G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij}), \quad (3)$$

with $G_{ij} \in \mathbb{R}_{>0}$ being the “nonzero” transfer conductance, and $B_{ij} \in \mathbb{R}$ being the transfer susceptance. The subscript j represents the index of other machines ($j \in \mathbb{N}$ and $j \leq n$). Note that $\delta_{ij}(t) \in \mathbb{R}$ is the angular separation between the i^{th} machine and the j^{th} machine, that is $\delta_{ij} := \delta_i - \delta_j$.

With the above-mentioned specification on each i^{th} synchronous machine, the dynamic state vector of the entire lossy n -machine power system is specified as follows. Note that, for TSA, angular separations are considered. Without loss of generality, the rotor angle of the first synchronous machine is set as the angular reference. The angular separation between the i^{th} machine and the angular reference is denoted as $\delta_{i,1} := \delta_i - \delta_1$, for all i . Note that $\delta_{ij} = \delta_{i,1} - \delta_{j,1}$, for all i and j . Then, the angular separation vector of the entire power system is denoted as $\delta(t) = [\delta_{2,1}(t), \dots, \delta_{n,1}(t)]^\top \in \mathbb{R}^{n-1}$. The angular speed vector is denoted as $\omega(t) = [\omega_1(t), \omega_2(t), \dots, \omega_n(t)]^\top \in \mathbb{R}^n$, and the q -axis transient voltage vector is denoted as $E'_q(t) = [E'_{q,1}(t), E'_{q,2}(t), \dots, E'_{q,n}(t)]^\top \in \mathbb{R}^n$. Therefore, the dynamic state vector of the entire lossy n -machine power system is $x(t) = [\delta(t)^\top, \omega(t)^\top, E'_q(t)^\top]^\top \in \mathbb{R}^{3n-1}$. Note also that the equilibrium of the entire power system is denoted as $x^* = [\delta^{*\top}, \omega^{*\top}, E_q^{*\top}]^\top \in \mathbb{R}^{3n-1}$, where $\delta^* = [\delta_{2,1}^*, \dots, \delta_{n,1}^*]^\top \in \mathbb{R}^{n-1}$, $\omega^* = [\omega_1^*, \omega_2^*, \dots, \omega_n^*]^\top \in \mathbb{R}^n$, and $E_q^* = [E_{q,1}^*, E_{q,2}^*, \dots, E_{q,n}^*]^\top \in \mathbb{R}^n$.

Assumption 1. The system parameters are known. The dynamic states are available for the design of the excitation feedback control law.

The parameters of each synchronous machine are known in practice. The parameters of the transmission network are given by the power system operators. The dynamic states are measurable through measurement devices such as phasor measurement unit (PMU) [34]. In this regard, **Assumption 1** is achievable for TSA.

Assumption 2. The post-fault equilibrium x^* is known and unique.

If the fault is self-cleared without changing the network topology, the post-fault equilibrium is the same as the pre-

fault equilibrium. If the fault is cleared with a change of the network topology, the post-fault equilibrium can be calculated through power flow analysis. Hence, the equilibrium is known. To maintain synchronism, the angular separations for all synchronous machines should be bounded. Typically, $|\delta_{ij}| < \frac{\pi}{2}$ is satisfied in practice, for all i and j . It is therefore reasonable to assume that $\delta_{ij} + 2k\pi$ is equivalent to δ_{ij} , for all i, j and $k \in \mathbb{Z}$. Hence, the equilibrium x^* is said to be unique in the practical range of operation. In this regard, **Assumption 2** is standard and reasonable for TSA.

B. Problem Formulation

The essence of direct TSA is to check if, following a contingency, the post-fault initial state lies inside the ROA of a stable equilibrium. In this regard, the problem of direct TSA can be formulated as follows.

Problem 1. Consider the lossy n -machine power system model (1)-(3) and the post-fault equilibrium x^* . For direct TSA, the following two objectives are to be achieved.

- 1) Design an explicit excitation feedback control law which ensures local asymptotic stability of the closed-loop equilibrium x^* . Construct a well-defined Lyapunov function whose time-derivative along the trajectories is strictly non-positive within a region containing the stable equilibrium x^* .
- 2) Calculate the “critical” level set of the Lyapunov function which provides an estimation of the ROA of the post-fault stable equilibrium x^* . Then, the transient stability property of a post-fault initial state, following a contingency, can be directly assessed.

III. LYAPUNOV-BASED TRANSIENT STABILIZATION

This section presents the first aspect of the proposed direct method, which includes the design of an explicit well-defined Lyapunov function for the lossy n -machine power system and the design of an explicit dynamic version of passivity-based excitation control law [35], [36]. This ensures local asymptotic stability of the closed-loop equilibrium in the sense of Lyapunov.

A. Brief Review of Non-Separable Lyapunov Functions

For stability analysis of the lossy n -machine power system model (1)-(3), the non-separable Lyapunov function has been given in [25, equation (27)]:

$$\begin{aligned} \mathcal{H}_d(\delta, \omega, E'_q) = & \psi(\delta) + \frac{1}{2} \sum_{i=1}^n (\omega_i - \omega^*)^2 \\ & + \frac{1}{2} \sum_{i=1}^n (E'_{q,i} - E'^*_{q,i} \lambda_i(\delta))^2, \end{aligned} \quad (4)$$

where $\psi : \mathbb{R}^n \rightarrow \mathbb{R}_{\geq 0}$ is the potential-energy-like term, and $\lambda : \mathbb{R}^n \rightarrow \mathbb{R}_{\geq 0}^n$ is the cross-term vector.

It has been shown in [25] that the inclusion of the cross-term vector λ allows a free selection of any well-defined potential-energy-like term ψ . By the Implicit Function Theorem, [25] has proved the existence of an excitation feedback control law

such that the closed-loop equilibrium is asymptotically stable with the Lyapunov function (4). However, no explicit form of the cross-term vector has been given.

B. Explicit Design of Lyapunov Functions and Control Laws

The major difficulty in deriving explicit results lies in the calculation of the cross-term vector λ in (4). To overcome this difficulty, we introduce an auxiliary state $\zeta(t) = [\zeta_1(t), \zeta_2(t), \dots, \zeta_n(t)]^\top \in \mathbb{R}^n$. This is expected to provide an estimation of the q -axis current (3), i.e., $\zeta_i(t) \approx I_{q,i}(t) > 0$, for all $t \geq 0$, and for all i . Note that the equilibrium of the auxiliary state is denoted by $\zeta^* = [\zeta_1^*, \zeta_2^*, \dots, \zeta_n^*]^\top \in \mathbb{R}^n$, where $\zeta_i^* := \frac{P_{e,i}}{E'^*_{q,i}} = I_{q,i}^* > 0$, for all i .

As an extension of (4), we propose the augmented non-separable Lyapunov function:

$$\begin{aligned} \mathcal{H}(\delta, \omega, E'_q, \zeta) = & \psi(\delta) + \sum_{i=1}^n \frac{H_i}{\omega^*} (\omega_i - \omega^*)^2 \\ & + \frac{1}{2} \sum_{i=1}^n T'_{do,i} (E'_{q,i} - E'^*_{q,i} \lambda_i(\delta, \zeta))^2 \\ & + \frac{1}{2} \sum_{i=1}^n (I_{q,i} - \zeta_i)^2, \end{aligned} \quad (5)$$

where $\lambda : \mathbb{R}^{n-1} \times \mathbb{R}^n \rightarrow \mathbb{R}_{\geq 0}^n$ is the new cross-term vector. The potential-energy-like term ψ is such that $\psi|_{\delta \neq \delta^*} > 0$, $\psi|_{\delta = \delta^*} = 0$ and $\frac{\partial \psi}{\partial \delta_i}|_{\delta = \delta^*} = 0$, for all i .

The proposed Lyapunov function (5) is explicit in the sense that the potential-energy-like term ψ is free to select and the new cross-term vector can be explicitly derived as a function of the auxiliary state ζ . The design of ζ will be clarified in the following. In addition, the weighting coefficients (such as H_i and $T'_{do,i}$) have clear physical meanings when compared to the Lyapunov function in [29], [30], which contributes to the tuning of control parameters and improves closed-loop transient performances.

For ease of illustration in Lyapunov analysis, we let:

$$E_{fd,i} := E'_{q,i} - (X_{d,i} - X'_{d,i}) I_{d,i} + u_i, \quad (6)$$

where u_i is the control law to be designed. Then, applying (6) to (1c) yields: $T'_{do,i} \dot{E}'_{q,i} = u_i$. With this specification, the time-derivative of (3) along the system solution of (1)-(3) is given by:

$$\begin{aligned} \dot{I}_{q,i} = & \sum_{j=1}^n E'_{q,j} (-G_{ij} \sin \delta_{ij} + B_{ij} \cos \delta_{ij}) (\omega_i - \omega_j) \\ & + \sum_{j=1}^n (G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij}) \frac{u_j}{T'_{do,j}}. \end{aligned} \quad (7)$$

We consider Lyapunov stability analysis as follows. Taking the time-derivative of (5) along the system solution of (1)-(3) yields: $\dot{\mathcal{H}} = \dot{\mathcal{H}}_1 + \dot{\mathcal{H}}_2 + \dot{\mathcal{H}}_3 + \dot{\mathcal{H}}_4$, where

$$\dot{\mathcal{H}}_1 = \sum_{i=1}^n (\omega_i - \omega^*) \left(\frac{\partial \psi}{\partial \delta_i} + P_{m,i} - E'_{q,i} I_{q,i} \right), \quad (8a)$$

$$\dot{\mathcal{H}}_2 = - \sum_{i=1}^n \frac{D_i}{\omega^*} (\omega_i - \omega^*)^2, \quad (8b)$$

$$\dot{\mathcal{H}}_3 = \sum_{i=1}^n (E'_{q,i} - E'^*_{q,i} \lambda_i) (u_i - T'_{do,i} E'^*_{q,i} \dot{\lambda}_i), \quad (8c)$$

$$\dot{\mathcal{H}}_4 = \sum_{i=1}^n (I_{q,i} - \zeta_i) (\dot{I}_{q,i} - \dot{\zeta}_i). \quad (8d)$$

Note that (8a) can be rewritten as: $\dot{\mathcal{H}}_1 = \dot{\mathcal{H}}_{1,1} + \dot{\mathcal{H}}_{1,2} + \dot{\mathcal{H}}_{1,3}$, where

$$\dot{\mathcal{H}}_{1,1} = \sum_{i=1}^n (\omega_i - \omega^*) \left(\frac{\partial \psi}{\partial \delta_i} + P_{m,i} - E'^*_{q,i} \lambda_i \zeta_i \right), \quad (9a)$$

$$\dot{\mathcal{H}}_{1,2} = - \sum_{i=1}^n (\omega_i - \omega^*) E'^*_{q,i} \lambda_i (I_{q,i} - \zeta_i), \quad (9b)$$

$$\dot{\mathcal{H}}_{1,3} = - \sum_{i=1}^n (\omega_i - \omega^*) (E'_{q,i} - E'^*_{q,i} \lambda_i) I_{q,i}. \quad (9c)$$

In what follows, we present three explicit design selections such that, in combination of them, the condition $\dot{\mathcal{H}} \leq 0$ can be satisfied locally around x^* and ζ^* .

1) *Cross-Term Design*: Zeroing the indefinite terms $\dot{\mathcal{H}}_{1,1}$ in (9a) yields the explicit expression for the cross-term:

$$\lambda_i(\delta, \zeta_i) = \frac{\frac{\partial \psi}{\partial \delta_i} + P_{m,i}}{E'^*_{q,i} \zeta_i}. \quad (10)$$

Note that, by definition, $\zeta_i^* = I^*_{q,i} > 0$. Hence, at the equilibrium, the denominator of (10) is positive, i.e., $E'^*_{q,i} I^*_{q,i} = P_{m,i} > 0$. Since $\frac{\partial \psi}{\partial \delta_i}|_{\delta=\delta^*} = 0$, we have $\lambda_i(\delta^*, \zeta_i^*) = 1$, for all i . Note also that we focus on “local” property of the cross-term. Since the auxiliary state is continuous, we have that the denominator of (10) remains locally positive around ζ^* . Therefore, the proposed cross-term (10) is locally well-defined around x^* and ζ^* .

Note that the time-derivative of (10) along the system solution is given by:

$$\dot{\lambda}_i = \sum_{j=1}^n \frac{\partial \lambda_i}{\partial \delta_j} (\omega_j - \omega^*) + \frac{\partial \lambda_i}{\partial \zeta_i} \dot{\zeta}_i, \quad (11)$$

where the dynamic law $\dot{\zeta}_i$ is to be defined in the next step.

2) *Auxiliary State Design*: Consider $\dot{\mathcal{H}}_4$ in (8d) and $\dot{\mathcal{H}}_{1,2}$ in (9b) with common factors $(I_{q,i} - \zeta_i)$, for all i . We design, through interconnection and damping assignment, the dynamic law of the auxiliary state as:

$$\begin{aligned} \dot{\zeta}_i &= \dot{I}_{q,i} - (\omega_i - \omega^*) E'^*_{q,i} \lambda_i \\ &\quad + k_{\zeta,i} (I_{q,i} - \zeta_i), \end{aligned} \quad (12)$$

where $k_{\zeta,i} \in \mathbb{R}_{>0}$ is a tunable coefficient, and $\dot{I}_{q,i}$ is given in (7). The initialization of the auxiliary state is set as its equilibrium, i.e., $\zeta_i(0) := \zeta_i^* = \frac{P_{e,i}}{E'^*_{q,i}}$, for all i . Note that, at the equilibrium x^* , we have $I^*_{q,i} = P_{e,i}/E'^*_{q,i}$ and $\dot{I}_{q,i} = 0$, for all i . In addition, we have $\omega_i = \omega^*$, for all i . Hence, in (12), we have $\dot{\zeta}_i(0) = 0$, for all i . This entails that ζ^* is an equilibrium for (12). Such an initialization ensures that, if the initial state is at the equilibrium, it remains at the equilibrium. In this regard, such a way of initialization is a reasonable choice.

3) *Excitation Control Design*: Consider $\dot{\mathcal{H}}_3$ in (8c) and $\dot{\mathcal{H}}_{1,3}$ in (9c) with common factors $(E'_{q,i} - E'^*_{q,i} \lambda_i)$, for all i . We design, through interconnection and damping assignment, the passivity-based control law as:

$$\begin{aligned} u_i(x, \zeta) &= T'_{do,i} E'^*_{q,i} \dot{\lambda}_i + (\omega_i - \omega^*) I_{q,i} \\ &\quad - k_{u,i} (E'_{q,i} - E'^*_{q,i} \lambda_i), \end{aligned} \quad (13)$$

where $k_{u,i} \in \mathbb{R}_{>0}$ is a tunable coefficient. Now we are at the position to calculate “explicitly” the control law, hence the synthesis to E_{fd} . To this end, we substitute firstly (7) into (12), then (12) into (11), and finally (11) into (13). This allows us to obtain the “explicit” feedback control law:

$$u(x, \zeta) = [\mathcal{I} - \mathcal{M}(\delta, \zeta)]^{-1} \mathcal{N}, \quad (14)$$

where $\mathcal{I} \in \mathbb{R}^{n \times n}$ is the identity matrix, $\mathcal{M} \in \mathbb{R}^{n \times n}$ is a square matrix with the element being:

$$\mathcal{M}_{ij} = T'_{do,i} E'^*_{q,i} \frac{\partial \lambda_i}{\partial \zeta_i} (G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij}) \frac{1}{T'_{do,j}}, \quad (15)$$

and $\mathcal{N} \in \mathbb{R}^{n \times 1}$ is a vector with the element being:

$$\begin{aligned} \mathcal{N}_i &= (\omega_i - \omega^*) I_{q,i} - k_{u,i} (E'_{q,i} - E'^*_{q,i} \lambda_i) \\ &\quad + T'_{do,i} E'^*_{q,i} \sum_{j=1}^n \frac{\partial \lambda_i}{\partial \delta_j} (\omega_j - \omega^*) \\ &\quad + T'_{do,i} E'^*_{q,i} \frac{\partial \lambda_i}{\partial \zeta_i} \left[k_{\zeta,i} (I_{q,i} - \zeta_i) - (\omega_i - \omega^*) E'^*_{q,i} \lambda_i \right. \\ &\quad \left. + \sum_{j=1}^n E'_{q,j} (-G_{ij} \sin \delta_{ij} + B_{ij} \cos \delta_{ij}) (\omega_i - \omega_j) \right]. \end{aligned} \quad (16)$$

Finally, taking (14) into (6) yields the “explicit” excitation controller, for the entire lossy n -machine power system (1)-(3), as:

$$E_{fd} = E'_q - \text{diag}\{X_d - X'_d\} I_d + [\mathcal{I} - \mathcal{M}]^{-1} \mathcal{N}. \quad (17)$$

Note that the determinant of the “index” matrix is positive at the equilibrium, i.e., $\det[\mathcal{I} - \mathcal{M}(\delta^*, \zeta^*)] > 0$. Note again that we focus on “local” stability property of the closed-loop equilibrium. Recall (10), we have $\frac{\partial \lambda_i}{\partial \zeta_i} \propto \frac{1}{\zeta_i^2}$ which is continuous with respect to ζ_i , for all i . Then, by continuity, the determinant of the index matrix remains locally positive. Therefore, the proposed excitation controller (17) is locally well-defined around x^* and ζ^* .

C. Main Results

Now the objective 1) in **Problem 1** has been achieved, and the main results are summarized in the following statement.

Proposition 1. *Consider the lossy n -machine power system (1)-(3) controlled by (17) with (10), (15), (16) and (12). Let **Assumptions 1** and **2** hold. The closed-loop equilibrium $[x^{*\top}, \zeta^{*\top}]^\top$ is guaranteed to be locally asymptotically stable with the proposed Lyapunov function (5).*

Proof. Taking the time-derivative of the Lyapunov function (5) along the solution of (1)-(3) yields:

$$\dot{\mathcal{H}} = \dot{\mathcal{H}}_{1,1} + \dot{\mathcal{H}}_{1,2} + \dot{\mathcal{H}}_{1,3} + \dot{\mathcal{H}}_2 + \dot{\mathcal{H}}_3 + \dot{\mathcal{H}}_4. \quad (18)$$

To begin with, since $D_i \geq 0$, for all i , we have $\dot{\mathcal{H}}_2 \leq 0$. The cross-term in (10) renders $\dot{\mathcal{H}}_{1,1} = 0$. The design of the auxiliary dynamic state in (12) is such that:

$$\dot{\mathcal{H}}_{1,2} + \dot{\mathcal{H}}_4 = -\sum_{i=1}^n k_{\zeta,i} (I_{q,i} - \zeta_i)^2 \leq 0. \quad (19)$$

Finally, the design of the control law (17) is such that:

$$\dot{\mathcal{H}}_{1,3} + \dot{\mathcal{H}}_3 = -\sum_{i=1}^n k_{u,i} (E'_{q,i} - E'^*_{q,i} \lambda_i)^2 \leq 0. \quad (20)$$

As a result, the design selections (10), (12) and (17) are such that:

$$\begin{aligned} \dot{\mathcal{H}} = & -\sum_{i=1}^n \frac{D_i}{\omega^*} (\omega_i - \omega^*)^2 \\ & -\sum_{i=1}^n k_{u,i} (E'_{q,i} - E'^*_{q,i} \lambda_i)^2 \\ & -\sum_{i=1}^n k_{\zeta,i} (I_{q,i} - \zeta_i)^2. \end{aligned} \quad (21)$$

Since $k_{u,i} > 0$, and $k_{\zeta,i} > 0$, for all i , we have $\dot{\mathcal{H}} \leq 0$, from which we conclude that all trajectories of the closed-loop lossy n -machine powers system are locally bounded. Hence, the equilibrium is locally stable.

The attractivity of the equilibrium is analyzed as follows. To begin with, by $\dot{\mathcal{H}} = 0$, we have $\omega_i = \omega^*$, for all i , which entails $P_{e,i} = P_{m,i}$, for all i . By $\dot{\mathcal{H}} = 0$, we also have $E'_{q,i} = E'^*_{q,i} \lambda_i$, for all i . Then, by (2), we have $E'^*_{q,i} \lambda_i I_{q,i} = P_{m,i}$, for all i .

Then, by (10), we have $\frac{\partial \psi}{\partial \delta_i} + P_{m,i} I_{q,i} = P_{m,i}$, for all i . Again, by $\dot{\mathcal{H}} = 0$, we have $I_{q,i} = \zeta_i$, for all i . Then, we have $\frac{\partial \psi}{\partial \delta_i} = 0$, for all i , which entails that $\delta_{ij} = \delta^*_{ij}$, for all i and j . This further entails that $E'_{q,i} = E'^*_{q,i}$, for all i , hence attractivity of x^* . In addition, at x^* , we have $\lambda_i = 1$, for all i , which yields $\frac{P_{m,i}}{E'^*_{q,i} I_{q,i}} = 1$, for all i , hence $I_{q,i} = I^*_{q,i}$, for all i . This in turn entails that $\zeta_i = I^*_{q,i} = \zeta^*_i$, for all i , hence attractivity of ζ^* .

Note that the design selections (10), (12) and (17) are well-defined subject to the “positivity” of the auxiliary state ζ and the “invertability” of the index matrix $[\mathcal{I} - \mathcal{M}(\delta^*, \zeta^*)]$. These two conditions are satisfied locally around x^* and ζ^* . By *LaSalle's invariance principle*, we conclude “local” asymptotic stability of the closed-loop equilibrium $[x^{*\top}, \zeta^{*\top}]^\top$ with the Lyapunov function (5). $\square$

IV. AN OPTIMIZATION-BASED APPROACH FOR THE ESTIMATION OF ROA

This section presents the second aspect of the proposed direct method. To be specific, we propose an optimization-based approach to calculate the critical level set of the proposed Lyapunov function (5), which allows to estimate an explicit ROA of the stable equilibrium $[x^{*\top}, \zeta^{*\top}]^\top$. Then, the transient stability property of a post-fault initial state, following a contingency, can be directly assessed.

The underlying idea is as follows. The condition $\dot{\mathcal{H}} \leq 0$ is satisfied locally around the equilibrium only if (10) and (14) are well-defined. In other words, the Lyapunov function (5)

remains monotonically decreasing given that the conditions $\zeta > 0$ and $\det[\mathcal{I} - \mathcal{M}] > 0$ are satisfied. By Lyapunov stability theorem, this provides a sufficient condition on the evaluation of the ROA. For direct TSA, we are interested in the “largest” invariant set of (5) which contains the post-fault equilibrium x^* . From such a perspective, the “critical value” of the proposed Lyapunov function can be obtained by solving the following optimization problem:

$$\begin{aligned} & \underset{x, \zeta}{\text{minimize}} \quad \mathcal{H}(x, \zeta), \\ & \text{subject to} \quad \zeta > 0, \\ & \quad \det[\mathcal{I} - \mathcal{M}] = 0. \end{aligned} \quad (22)$$

The objective of (22) is to determine the largest level set of the Lyapunov function (5) which is contained in the “space” defined by $\zeta > 0$, and is “tangent” to the “surface” defined by $\det[\mathcal{I} - \mathcal{M}] = 0$ [30]. Since the Lyapunov function (5) is globally well-defined, the global solution of (22) yields the critical value and it is denoted by $\mathcal{H}_{cr}$. For effectively solving (22), the initial guess is selected as x^* and ζ^* .

On the basis of $\mathcal{H}_{cr}$, the estimation of the ROA of x^* is given by the invariant set:

$$\{x \in \mathbb{R}^{3n-1} : \mathcal{H}(x, \zeta^*) \leq \mathcal{H}_{cr}\}. \quad (23)$$

Now the objective 2) in **Problem 1** has been achieved, and the conclusions are summarized in the following statement.

Theorem 1. *Consider the lossy n -machine power system (1)-(3) controlled with (17). Let the critical value $\mathcal{H}_{cr}$ be obtained by solving (22) with the Lyapunov function (5). Let a post-fault initial state x_c be given following a contingency. If $\mathcal{H}(x_c, \zeta^*) \leq \mathcal{H}_{cr}$, then x_c is transiently stable.*

Proof. Asymptotic stability of the equilibrium is given by **Proposition 1**. For a given post-fault initial state x_c , if $\mathcal{H}(x_c, \zeta^*) \leq \mathcal{H}_{cr}$, then x_c is inside the ROA (23) and the resulting trajectory converges to the stable equilibrium x^* . Therefore, such an initial state x_c is directly assessed to be transiently stable. $\square$

V. CASE STUDY

This section illustrates the performance and effectiveness of the proposed direct method by studying the IEEE 10-machine 39-bus power system, as shown in Fig. 1. The case study consists of two parts. The first part demonstrates the performance of the proposed excitation controller (17) in improving the transient responses of all machines (when compared to traditional AVR and PSS [1]). The second part demonstrates the effectiveness of the proposed direct method in evaluating the critical clearing times under various contingencies.

For the implementation of the proposed direct method, we select the potential-energy-like term as:

$$\psi(\delta) := \frac{1}{2} \sigma \sum_{i=1}^{n-1} \sum_{j=i+1}^n (\delta_{ij} - \delta^*_{ij})^2, \quad (24)$$

where $\sigma \in \mathbb{R}_{>0}$ is a tunable weighting coefficient. Note that the quadratic form in (24) contributes to effectively solving the optimization problem (22).

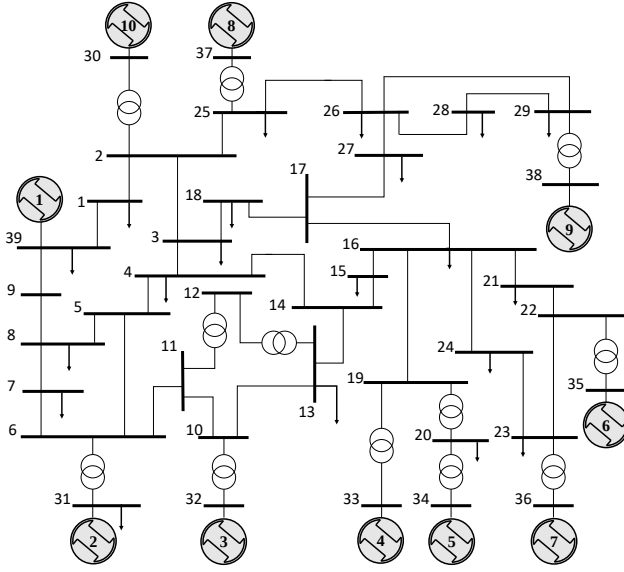


Fig. 1. IEEE 10-machine 39-bus power system [37].

The simulations are set up as follows. The operating condition is determined by the power flow initialization as per the report [37]. For the swing bus connected machine, the mechanical power injection is $P_{m,2} = 5.21 p.u.$ All machine parameters are as per [37]. The synchronous speed is $\omega^* = 120\pi [rad/s]$. The damping coefficient is set as $D_i = 0$ for all machines. For implementation of the proposed excitation controller (17), we select the control parameters $k_{\zeta,i} = 10$, $k_{u,i} = 100$, for all i , and the weighting coefficient $\sigma = 0.5$, respectively. The parameters of AVR and PSS are as per [1, chapter 13]. The control limits are set as $E_{fd,max} = 7.0$ and $E_{fd,min} = -6.4$, as per [1]. The numerical simulation of the transient dynamics is conducted using the MATLAB function “ode45”, with the option “odeset(‘RelTol’,1e-6,‘AbsTol’,1e-6,‘Maxstep’,1e-3)”. The optimization problem (22) is solved by the MATLAB function “fmincon”, with the option of *interior-point algorithm* and the initial point at x^* and ζ^* .

A. Transient Performances

We consider two scenarios to demonstrate the performance of the proposed controller (17) in improving the transient responses. In the first scenario, a three-phase short circuit fault occurs at bus 16 at $t_f = 1 s$, and it is cleared by switching out the faulted line 16-17 after $\Delta t_f = 80 ms$, i.e., at $t_c = 1.08 s$. In the second scenario, a three-phase short circuit fault occurs at bus 2 at $t_f = 1 s$, and it is cleared by switching out the faulted line 2-25 after $\Delta t_f = 80 ms$, i.e., at $t_c = 1.08 s$. The fault admittance is set as $10^6 p.u$ [1].

The transient responses of all machines in these two scenarios are shown in Fig. 2 and Fig. 3, respectively. It can be observed that, given the same prescribed range of control actions, the proposed controller performs better than AVR+PSS in several aspects. First, the proposed controller dampens out the oscillation of angular separations and active powers more effectively. Second, the proposed controller drives the angular speeds back to the synchronous speed. Notably, the value of

TABLE I
THE CCTs OBTAINED BY THE PROPOSED DIRECT METHOD AND BY TDS METHOD, RESPECTIVELY, FOR DIFFERENT CONTINGENCIES SPECIFIED BY FAULTED BUS AND TRIPPED LINES (UNIT: [ms])

#	Faulted Bus	Tripped Line	$\mathcal{H}_{cr}$	Δt_{cr}^{DM}	Δt_{cr}^{TDS}	$\eta(\%)$
1	2	2-3	13.69	139	225	61.8
2	5	5-6	19.08	165	283	58.3
3	6	6-5	19.04	164	273	60.1
4	6	6-11	20.96	173	273	63.4
5	10	10-11	34.00	208	269	77.3
6	10	10-13	14.94	152	262	58.0
7	13	13-14	24.89	186	294	63.3
8	16	16-15	21.27	150	215	69.8
9	16	16-17	39.36	189	235	80.4
10	16	16-21	8.82	107	240	44.6
11	16	16-24	34.30	169	211	80.0
12	21	21-16	9.13	139	283	49.1
13	21	21-22	31.91	223	257	86.8
14	25	25-26	10.15	117	156	75.0
15	26	26-27	10.85	136	211	64.4

Lyapunov function is monotonically decreasing along the post-fault state trajectories. However, these are at the expense of higher fluctuation of the terminal bus voltages, which settles down eventually and converges to an acceptable steady state.

B. Evaluation of Critical Clearing Times

For a specific contingency, the post-fault initial state x_c is determined with the fault-clearing time Δt_f . The critical clearing time (CCT), denoted by Δt_{cr} , is defined as the largest fault-clearing time such that the post-fault system remains transiently stable. Since CCT is an intuitive information to reveal the transient stability property, it is of crucial importance to the practical operation of power systems [1], [3].

We consider 15 randomly selected contingencies, as listed in TABLE I, to demonstrate the effectiveness of the proposed direct method in evaluating the CCTs. In each contingency, we consider a three-phase short circuit fault and specify the faulted bus and the line to be tripped. The fault admittance is set as $10^6 p.u.$ By invoking **Theorem 1**, the post-fault initial states x_c such that $\mathcal{H}(x_c) \leq \mathcal{H}_{cr}$ are directly assessed to be transiently stable. Therefore, the CCT is determined by the fault-clearing time Δt_f such that $\mathcal{H}(x_c(\Delta t_f)) = \mathcal{H}_{cr}$. We take the contingencies #1, #2, #5, #9, #11 and #13, respectively, to illustrate the evaluation process for the CCTs on the basis of $\mathcal{H}_{cr}$. These results are shown in Fig. 6.

For those post-fault initial states x_c such that $\mathcal{H}(x_c) > \mathcal{H}_{cr}$, their transient stability properties can not be directly predicted. This is due to the conservativeness of the Lyapunov approach for stability analysis. In such cases, TDS is used as an auxiliary tool for further check. We take the TDS-based CCT as a benchmark, and define the efficiency of the direct method (DM)-based CCT as the ratio between the DM-based CCT and the TDS-based CCT, i.e., $\eta := \Delta t_{cr}^{DM} / \Delta t_{cr}^{TDS}$. It is worth

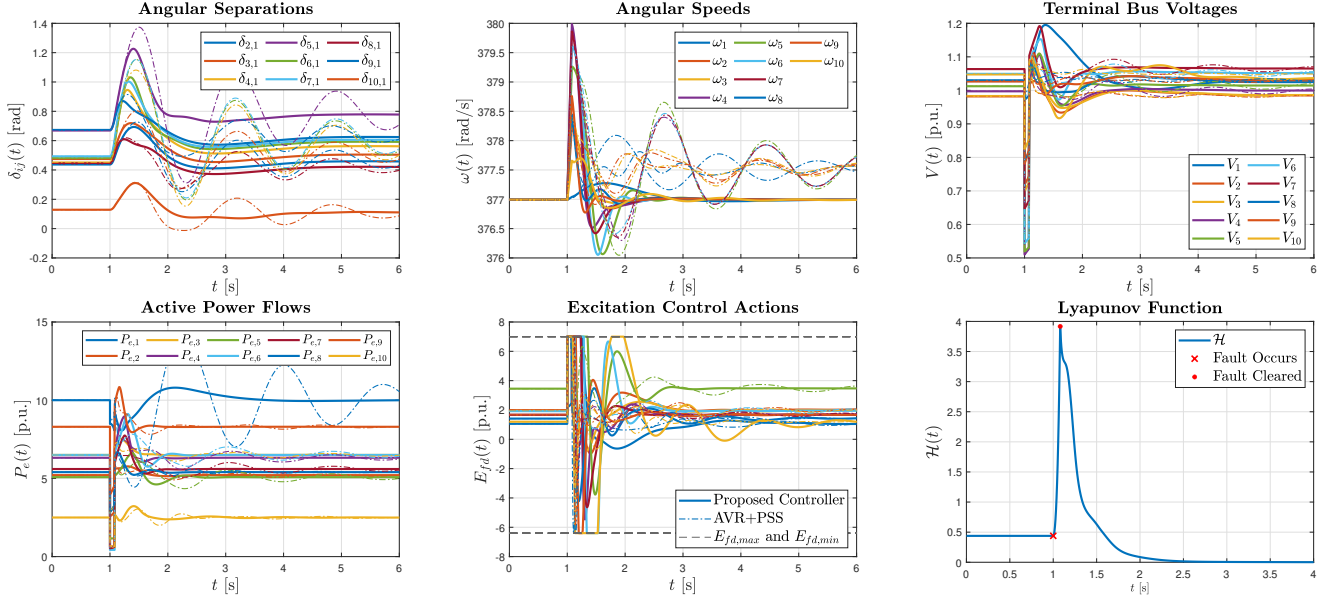


Fig. 2. Transient responses of all machines in the first scenario of subsection V-A, where a three-phase short circuit fault occurs at bus 16 at $t_f = 1$ s and it is cleared by switching out the faulted line 16-17 at $t_c = 1.08$ s. The solid curves are associated to the proposed controller (17), while the dash-dotted curves are associated to AVR+PSS.

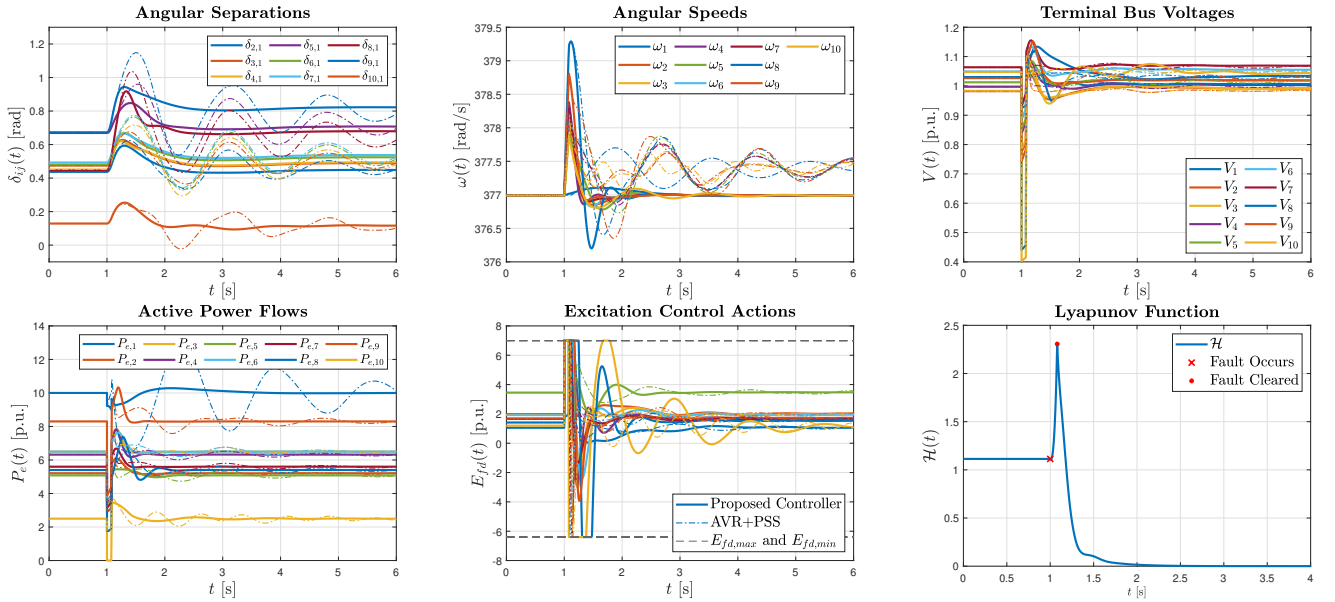


Fig. 3. Transient responses of all machines in the second scenario of subsection V-A, where a three-phase short circuit fault occurs at bus 2 at $t_f = 1$ s and it is cleared by switching out the faulted line 2-25 at $t_c = 1.08$ s. The solid curves are associated to the proposed controller (17), while the dash-dotted curves are associated to AVR+PSS.

mentioning that the efficiency should be within the interval $\eta \in (0, 1]$, and that the closer to 1 the less conservative the result. The results of the evaluated CCTs for all the selected contingencies are listed in TABLE I. It can be observed that all the DM-based CCTs are not larger than the corresponding TDS-based CCTs. Hence, no case of overestimated CCT is encountered, which entails that the proposed direct method leads to secure assessments of the transient stability. In these cases, the efficiency of the proposed direct method for CCTs evaluation is at least 44.6% and up to 86.8%. Such a range of efficiency is promising when compared to some of existing

direct methods (see, e.g., [38, Table II], where the proposed approach is with an average efficiency below 20%). In this regard, the efficiency of the proposed direct method shows its potential in screening out a large portion of post-fault initial states which can be directly assessed as transiently stable. Finally, we consider contingencies #9 and #13 to verify that the post-fault initial states yielded with the DM-based CCT are indeed transiently stable. The transient responses of all machines are shown in Fig. 4 and 5, respectively.

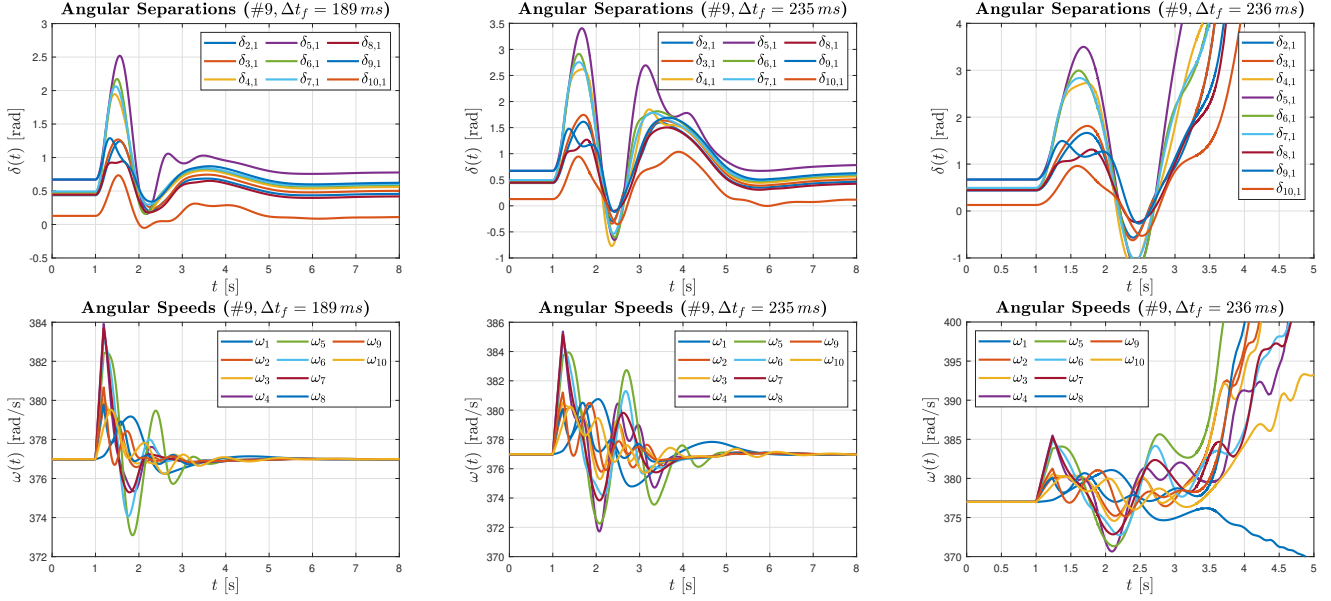


Fig. 4. Transient responses of the angular separations and angular speeds of all machines in the contingency #9, where a three-phase short circuit fault occurs at bus 16 at $t_f = 1$ s and is then cleared by switching out the faulted line 16-17 after the DM-based CCT $\Delta t_f = 189$ ms, the TDS-based CCT $\Delta t_f = 235$ ms, and $\Delta t_f = 236$ ms, respectively.

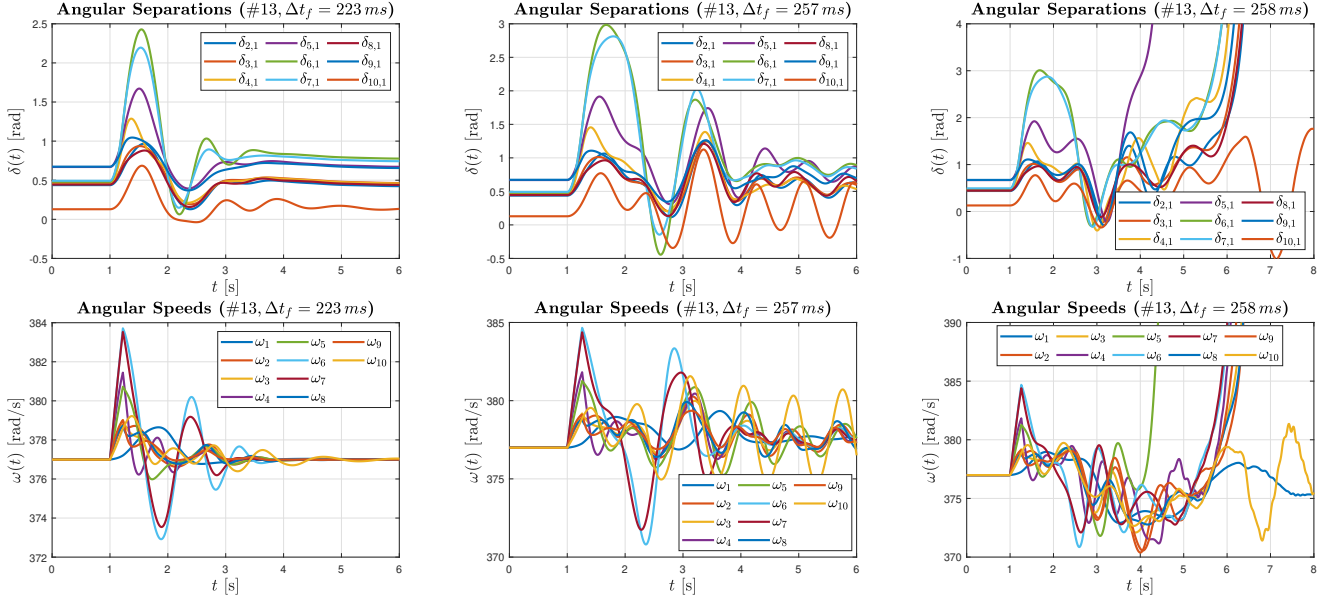


Fig. 5. Transient responses of the angular separations and angular speeds of all machines in the contingency #13, where a three-phase short circuit fault occurs at bus 21 at $t_f = 1$ s and is then cleared by switching out the faulted line 21-22 after the DM-based CCT $\Delta t_f = 223$ ms, the TDS-based CCT $\Delta t_f = 257$ ms, and $\Delta t_f = 258$ ms, respectively.

VI. CONCLUSION AND FUTURE WORK

This paper has proposed an explicit direct method for the TSA of the multimachine power systems with nonzero TCs, which consists of two aspects. In the first, a globally well-defined Lyapunov function for the lossy multimachine power system model has been constructed, and a locally well-defined excitation controller has been proposed. This ensures local asymptotic stability of the closed-loop equilibrium for the entire power system. In the second, an optimization-based approach to calculate the critical level set of the proposed

Lyapunov function has been proposed. This allows to estimate an explicit ROA of the stable equilibrium. Therefore, by checking if a given initial state is inside the ROA, the transient stability property of such a post-fault initial state can be directly assessed.

The performance of the proposed excitation controller has been illustrated in the case study. It is shown that the proposed controller improves closed-loop transient response when compared to traditional AVR and PSS. The effectiveness of the proposed direct method has been demonstrated through the evaluation of CTTs under various contingencies. The

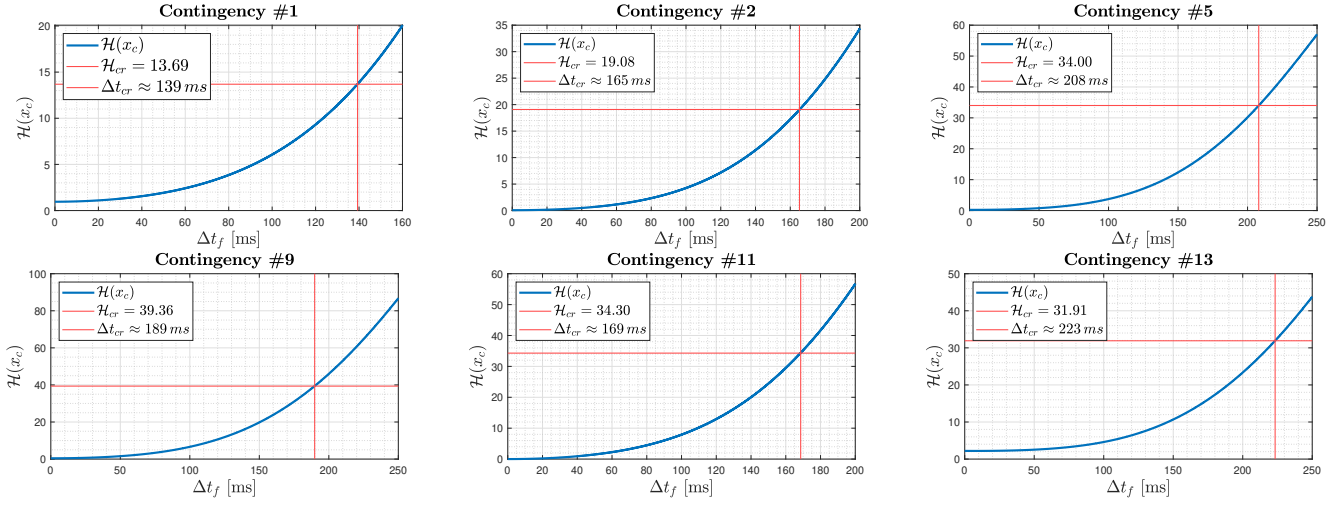


Fig. 6. Evaluations of the CCTs on the basis of $\mathcal{H}_{cr}$ for the contingencies #1, #2, #5, #9, #11 and #13, respectively, as listed in TABLE I. By **Theorem 1**, the post-fault initial states such that $\mathcal{H}(x_c) \leq \mathcal{H}_{cr}$ are directly assessed to be transiently stable.

efficiency of the results is promising, in the sense that it is better than several existing approaches. No overestimated case is encountered. This shows the potential of the proposed direct method in screening out a large portion of post-fault initial states which can be directly assessed as transiently stable. Therefore, the proposed explicit direct method is beneficial to achieving online TSA.

Several future work can be investigated. The parameter uncertainties can be considered in the stability analysis to enhance the robustness of the proposed controller. The formulation of the optimization-based approach deserves a further investigation to improve the efficiency in obtaining the critical value. The selection of the potential-energy-like term can also be investigated to reduce the conservativeness of the proposed direct method.

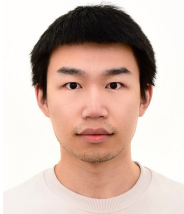
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REFERENCES

- [1] P. Kundur, N. J. Balu, and M. G. Lauby, *Power System Stability and Control*. McGraw-hill New York, 1994.
- [2] N. Hatziaargyriou, J. Milanovic, C. Rahmann, *et al.*, "Definition and classification of power system stability – revisited & extended," *IEEE Transactions on Power Systems*, vol. 36, no. 4, pp. 3271–3281, 2021.
- [3] H.-D. Chiang, *Direct Methods for Stability Analysis of Electric Power Systems: Theoretical Foundation, BCU Methodologies, and Applications*. John Wiley & Sons, 2011.
- [4] T. L. Vu, S. M. Al Araifi, M. S. El Moursi, and K. Turitsyn, "Toward simulation-free estimation of critical clearing time," *IEEE Transactions on Power Systems*, vol. 31, no. 6, pp. 4722–4731, 2016.
- [5] Y. Gu and T. C. Green, "Power system stability with a high penetration of inverter-based resources," *Proceedings of the IEEE*, vol. 111, no. 7, pp. 832–853, 2023.
- [6] B. Chaudhuri, D. Ramasubramanian, J. Matevosyan, *et al.*, "Rebalancing needs and services for future grids: System needs and service provisions with increasing shares of inverter-based resources," *IEEE Power and Energy Magazine*, vol. 22, no. 2, pp. 30–41, 2024.
- [7] P. C. Magnusson, "The transient-energy method of calculating stability," *Transactions of the American Institute of Electrical Engineers*, vol. 66, no. 1, pp. 747–755, 1947.
- [8] P. D. Aylett, "The energy-integral criterion of transient stability limits of power systems," *Proceedings of the IEE-Part C: Monographs*, vol. 105, no. 8, pp. 527–536, 1958.
- [9] A. H. El-abiad and K. Nagappan, "Transient stability regions of multimachine power systems," *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-85, no. 2, pp. 169–179, 1966.
- [10] T. Athay, R. Podmore, and S. Virmani, "A practical method for the direct analysis of transient stability," *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-98, no. 2, pp. 573–584, 1979.
- [11] H.-D. Chiang, M. Hirsch, and F. Wu, "Stability regions of nonlinear autonomous dynamical systems," *IEEE Transactions on Automatic Control*, vol. 33, no. 1, pp. 16–27, 1988.
- [12] Y. Xue, T. Van Cutsem, and M. Ribbens-Pavella, "Extended equal area criterion justifications, generalizations, applications," *IEEE Transactions on Power Systems*, vol. 4, no. 1, pp. 44–52, 1989.
- [13] H.-D. Chiang, F. Wu, and P. Varaiya, "A BCU method for direct analysis of power system transient stability," *IEEE Transactions on Power Systems*, vol. 9, no. 3, pp. 1194–1208, 1994.
- [14] N. Bretas and L. Alberto, "Lyapunov function for power systems with transfer conductances: Extension of the invariance principle," *IEEE Transactions on Power Systems*, vol. 18, no. 2, pp. 769–777, 2003.
- [15] T. L. Vu and K. Turitsyn, "Lyapunov functions family approach to transient stability assessment," *IEEE Transactions on Power Systems*, vol. 31, no. 2, pp. 1269–1277, 2016.
- [16] T. L. Vu and K. Turitsyn, "A framework for robust assessment of power grid stability and resiliency," *IEEE Transactions on Automatic Control*, vol. 62, no. 3, pp. 1165–1177, 2017.
- [17] S. M. Al Araifi, M. S. E. Moursi, and S. M. Djouadi, "Individual functions method for power system transient stability assessment," *IEEE Transactions on Power Systems*, vol. 35, no. 2, pp. 1264–1273, 2020.
- [18] W. Kang, K. Sun, and L. Xu, "Data-driven computational methods for the domain of attraction and Zubov's equation," *IEEE Transactions on Automatic Control*, vol. 69, no. 3, pp. 1600–1611, 2024.
- [19] P. Yang, F. Liu, W. Wei, and Z. Wang, "Approaching the transient stability boundary of a power system: Theory and applications," *IEEE Transactions on Automation Science and Engineering*, vol. 20, no. 4, pp. 2268–2279, 2023.
- [20] S. Alaraifi, S. Djouadi, and M. S. E. Moursi, "Domain of stability characterization for power systems: A novel individual invariance method," *IEEE Transactions on Power Systems*, vol. 39, no. 1, pp. 14–27, 2024.
- [21] Q.-H. Wu, Y. Lin, C. Hong, Y. Su, T. Wen, and Y. Liu, "Transient stability analysis of large-scale power systems: A survey," *CSEE Journal of Power and Energy Systems*, vol. 9, no. 4, pp. 1284–1300, 2023.
- [22] N. Narasimhamurthi, "On the existence of energy function for power systems with transmission losses," *IEEE Transactions on Circuits and Systems*, vol. 31, no. 2, pp. 199–203, 1984.

- [23] A. Llamas, J. De La Ree Lopez, L. Mili, A. Phadke, and J. Thorp, "Clarifications of the BCU method for transient stability analysis," *IEEE Transactions on Power Systems*, vol. 10, no. 1, pp. 210–219, 1995.
- [24] M. Pai and P. Murthy, "On Lyapunov functions for power systems with transfer conductances," *IEEE Transactions on Automatic Control*, vol. 18, no. 2, pp. 181–183, 1973.
- [25] R. Ortega, M. Galaz, A. Astolfi, Y. Sun, and T. Shen, "Transient stabilization of multimachine power systems with nontrivial transfer conductances," *IEEE Transactions on Automatic Control*, vol. 50, no. 1, pp. 60–75, 2005.
- [26] M. Anghel, F. Milano, and A. Papachristodoulou, "Algorithmic construction of Lyapunov functions for power system stability analysis," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 60, no. 9, pp. 2533–2546, 2013.
- [27] D. Casagrande, A. Astolfi, R. Ortega, and D. Langarica, "A solution to the problem of transient stability of multimachine power systems," in *2012 IEEE 51st IEEE Conference on Decision and Control (CDC)*, 2012, pp. 1703–1708.
- [28] J. Gao, B. Chaudhuri, and A. Astolfi, "A direct bounded control method for transient stability assessment," *IFAC-PapersOnLine*, vol. 54, no. 19, pp. 294–301, 2021.
- [29] J. Gao, B. Chaudhuri, and A. Astolfi, "Lyapunov-based transient stability analysis," in *2022 IEEE 61st Conference on Decision and Control (CDC)*, 2022, pp. 5099–5104.
- [30] J. Gao, B. Chaudhuri, and A. Astolfi, "An optimization-based method for transient stability assessment," in *2023 62nd IEEE Conference on Decision and Control (CDC)*, 2023, pp. 4586–4591.
- [31] H.-D. Chang, C.-C. Chu, and G. Cauley, "Direct stability analysis of electric power systems using energy functions: Theory, applications, and perspective," *Proceedings of the IEEE*, vol. 83, no. 11, pp. 1497–1529, 1995.
- [32] T. W. Stegink, C. De Persis, and A. J. Van Der Schaft, "An energy-based analysis of reduced-order models of (networked) synchronous machines," *Mathematical and Computer Modelling of Dynamical Systems*, vol. 25, no. 1, pp. 1–39, 2019.
- [33] F. Dörfler and D. Groß, "Control of low-inertia power systems," *Annual Review of Control, Robotics, and Autonomous Systems*, vol. 6, no. 1, pp. 415–445, 2023.
- [34] J. Yan, C.-C. Liu, and U. Vaidya, "PMU-based monitoring of rotor angle dynamics," *IEEE Transactions on Power Systems*, vol. 26, no. 4, pp. 2125–2133, 2011.
- [35] R. Ortega, A. Van Der Schaft, B. Maschke, and G. Escobar, "Interconnection and damping assignment passivity-based control of port-controlled Hamiltonian systems," *Automatica*, vol. 38, no. 4, pp. 585–596, 2002.
- [36] A. Van der Schaft, *L2-Gain and Passivity Techniques in Nonlinear Control*. Springer, 2000.
- [37] I. A. Hiskens, "IEEE PES task force on benchmark systems for stability controls," *Technical Report*, 2013.
- [38] Z. Qiu, C. Duan, W. Yao, P. Zeng, and L. Jiang, "Adaptive Lyapunov function method for power system transient stability analysis," *IEEE Transactions on Power Systems*, vol. 38, no. 4, pp. 3331–3344, 2023.



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