UNIVERSITY OF LONDON

IMPERIAL COLLEGE OF SCIENCE AND TECHNOLOGY
DEPARTMENT OF ELECTRICAL ENGINEERING

POWER SYSTEM GENERATION AND TRANSMISSION PLANNING USING PROBABILITY METHODS

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Thesis submitted for the degree of

Doctor of Philosophy in Engineering.

London, January 1980

ABSTRACT

Power system planners are constantly faced with the problem of finding the optimal way of designing, constructing and operating their systems to satisfy variable consumer demands in an environment of uncertainty.

Of the numerous phases of power system planning, the areas of long term generation planning, reservoir operation planning and transmission system planning have been selected for the studies reported in this thesis. The presence of inherent uncertainty in all these phases is recognised and the use of probabilistic solution methods is emphasised throughout.

The application of long term generation planning procedures to various power systems is illustrated and an interactive solution strategy is discussed. As the computing times for the integer optimisation techniques are generally high, a method of modifying the system constraints to reduce computing times without losing the accuracy of the solution is developed. Reductions of computing times by up to 40% have been realised using this method of modified constraints.

The uncertainty in water inflow to long term storage reservoirs is dealt with by using stochastic dynamic programming techniques. The optimum use of water storage in a mixed hydro-thermal system to reduce thermal power generation is illustrated. Cascaded or parallel operated hydro plants can be simplified to a single equivalent reservoir using the composite reservoir representation.

Adequacy of the transmission system is tested by calculating the reliability parameters at each consumer load point and also calculating overall system performance indices. As these indices are compatible with those used for distribution system reliability evaluation, they form a set of starting values for these studies. The applicability of

the method to medium sized power systems is demonstrated by using the Sri Lanka power network as an example.

Other areas which need to be explored more are identified and suggestions are made for further developments.

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To my parents
relatives & teachers
for their inspiration.

ACKNOWLEDGEMENTS

The work presented in this thesis was carried out under the supervision of Dr B J Cory, DSc(Eng), BSc(Eng), ACGI, CEng, FIEE, MIEEE, Reader in Electrical Engineering, Imperial College of Science and Technology, London. I wish to express my deepest gratitude for his invaluable guidance, constant encouragement and keen interest throughout this project.

I am grateful to the Commonwealth Scholarship Commission in the United Kingdom for the award of a scholarship and the British Council, London, for its administration. I am also grateful to the University of Moratuwa, Sri Lanka, for granting me study leave which made possible to pursue this work.

I wish to express my appreciation to Mr A J McKechnie, Consulting Engineer, Kennedy & Donkin, for providing the data for the case studies, to the colleagues in the Power Systems section and to all those who helped in various ways to make this a success.

Finally I wish to thank my wife for her continued support, especially during the final stages of this work.

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CHAPTER 1

INTRODUCTION

1.1 General Considerations in Power System Planning

The primary requirement in electrical power system planning is the development of a composite generation and transmission system which is capable of economically supplying the consumer at adequate levels of quality and continuity[1]. This implies that power system expansion has to take place in an economically and environmentally acceptable manner whilst satisfying the reliability & other requirements imposed on it[2].

In general, the supply system evolves through the addition of new plant and equipment rather than through a substitution or replacement process[3]. Therefore, decisions regarding system expansion must be based on the associated financial and technical implications evaluated over a long period of time. However, long term studies are obviously affected by a number of factors such as the uncertainty in the size and locations of future load, the possible siting of new power plants, the availability and cost of fuel and the future technology of generation and transmission. These uncertainties do not allow the system planner to produce a strictly optimum expansion plan with a detailed specification of future network configurations. Nevertheless, such a plan is required for the purpose of placing short term reinforcements into a coherent, optimised and long term perspective[4].

The system planner is essentially a decision maker with the prime objective of achieving economic efficiency and reliability in the context of uncertainty. This objective requires the analysis and the use of all relevant information and the scanning of a large number of alternative expansion plans each with their own economic, technical and reliability considerations. The difficulty in selecting the most promising expansion

plan has been eased to a certain extent by the development of large digital computers and suitable mathematical methods. However, it still requires the experience of the system planner in a multitude of areas associated with the construction, operation and analysis of power systems whilst taking into account the latest trends in technological developments.

A comprehensive exposition on power system planning is given by Fernando[5], and only the relevant sections are covered in this Chapter for the work presented in this thesis.

1.2 Economic Coordination of Power Systems

As modern power systems are responsible for the economic conversion of energy sources to electric energy and its economic distribution to the consumers, the effective coordination of various facilities is vital. For example, the combining of loads of diverse characteristics helps to improve the load factors and leads to the exploitation of economies of scale and the pooling of generation reduces the reserve margins required during design and construction and in operation the spinning reserve is minimised.

The load combination has to ensure the maximum exploitation of the most economical resources of energy, the minimisation of transportation losses, the economic and social balance among the several benefits in multipurpose hydro development schemes and the economic use of equipment and personnel. Although this helps to promote the economy of a country and achieves significant conservation of resources, the interaction between decision variables also multiply, increasing the difficulty in evaluating and coordinating system operation.

A carefully planned and constructed generating system with a suitable transmission and distribution network are the basic requirements for proper system coordination and operation. The generating capacity should be capable of supplying the system requirements under conditions of unit

forced outages and unforseen variations in the system load. This should also contain sufficient reserve to permit generating unit preventive maintenance to be carried out simultaneously with adequate system operation. The planned and existing capacity should be capable of satisfying both the system energy and load demands, requiring a suitable mix of generating sources.

The primary function of the transmission network is to transport the energy generated to consumer load points. In addition it may be required to interconnect areas in emergencies and to transport control power. The transmission facilities must be capable of maintaining adequate voltage levels and loadings within the thermal limits of individual circuit elements and within the system stability limits.

In achieving a properly coordinated system, the capability of the generation and transmission system is of no significance if this is not balanced by the distribution system. However, the distribution facilities are generally operated in a radial configuration and therefore result in local curtailment to a limited number of consumers. Bulk supply facilities, on the other hand, often result in system disturbances which affect a large number of consumers on the radial networks. Therefore, in most systems, the design of distribution system is almost entirely decoupled from the transmission system development process. Given a location and the capacity of the terminal station emanating from the bulk supply system, the distribution system design then becomes a separate and independent process[6,7]. Thus, it is necessary to evaluate the performance of the composite generation and transmission system using indices which are compatible and hence can be used as starting values in the design and evaluation of distribution facilities[1].

1.3 Load Forecasting

Power system expansion planning starts with a forecast of anticipated future load requirements. This includes estimates of both demand and energy, essential for the determination of capacity additions to the generation, transmission and distribution systems and the future fuel requirements. The accuracy of the forecast is crucial as it dictates the timing and characteristics of major system expansions and the consequences of over- and under-estimation of load growth are well documented[8,9].

The consumers of electricity can be broadly classified into:

i. residential

ii. commercial and industrial

iii. other (eg. municipalities, state departments).

The residential loads tend to have the most constant annual growth rate and the most seasonal fluctuations and hence are responsible for the seasonal variations in system peak demand. This is due to the widespread use of weather sensitive equipment such as space heaters and air conditioners. Of the high energy devices, refrigeration loads tend to have a constant characteristic as compared to the cyclic load characteristics of dryers and water heaters.

Commercial loads are also characterised by seasonal fluctuations due to the extensive use of air conditioning and space heating equipment. On the other hand, the industrial loads contain little weather dependent variations and are often considered as base loads.

Other loads may have seasonal fluctuations depending on the purpose for which the electric energy is used. In most cases, the growth trend can be considered as stable. However, a decision for example, to electrify the railway network by the state department could lead to an abrupt change in the demand for electric energy.

In addition to the impact of weather on residential and commercial loads, other variations are caused by economic, social and demographic effects[8]. These variations, therefore, favour the analysis of energy generation and hence the fuel requirements and costs to be carried out on a seasonal basis, provided that reasonably reliable long term and seasonal forecasts are available.

1.4 Investment Policies and the Importance of Load Management

The aim of an optimal investment policy is to maximise the net economic benefits to the national economy, subject to all relevant constraints[9]. In general the practicable revenue requirement differs from what is socially acceptable and hence a trade off is often required to ensure that the pricing policy yields revenues sufficient to make a substantial contribution to the self financing of future system expansion.

Rural electrification in developing countries is becoming a major area of investment and is intended to serve both economic and social aims by raising the productivity and living standards at the same time helping to prevent excessive urbanisation[10]. These investments are lumpy, with large initial costs and small economies of scale[9]. The demand, revenue and financial returns tend to be low in the early years. Forecasting to any accuracy is difficult due to the lack of information about consumer requirements. However, there is often a strong response to rural electrification projects indicated by high rates of demand growths, though they start from very low initial levels. This response stems from a wide range of uses of electricity including domestic and agro-industrial demands. Financing of these electrification schemes in developing countries most often requires the capital to be borrowed from international lending organisations. Although this is basically due to the scarcity of capital, an inappropriate tariff structure is partly responsible. A detailed account on setting up of tariffs is beyond the scope of this thesis.

However, the effects of inappropriate tariffs to society at large and to the power industry in particular, and the possibility of the load management through tariffs are discussed below.

Tariffs are expected to reflect the true cost of power supply to the various consumers. However, a low tariff is certain to encourage consumer demand to grow at a faster rate. As demand grows, consumers are, in fact demanding more capital expenditure by the supply industry through increased generation, transmission and distribution capacity. These expansions have to be carried out, often sacrificing the development in other sectors of the economy, unless supply interruptions are preferred. In a developing country where capital is scarce, borrowing from abroad is necessary leading also to spending less on other development projects. Thus, from a national point of view, a faster growth of demand entails increased foreign exchange requirements, interest and repayment burdens causing financial difficulties[9].

If supply interruptions and rationing are not to occur, the pricing policy can be effectively employed to limit demand when capacity limits are strained and immediate reinforcements are not possible. However, as in the case of investment decisions, pricing decisions must also be made in the context of uncertainty. It has been observed[9] that the change of tariff is difficult and can often have major social consequences while the change of tariff structure is more appropriate.

1.5 Generating System Cost Analysis

The costs associated with power supply can be basically divided into capital, maintenance and operating costs. The recent past has witnessed a growing awareness by society to protect environment, and hence the power industry is compelled to invest more on air pollution and water pollution control equipment, thus increasing the capital and operation costs.

According to US sources[11], the power industry accounts for 25% of air pollution and unless appropriate measures are taken, this is expected to rise by as much as 30% towards the year 2000. It is estimated[8] that for a fossil fired unit, the operating cost for pollution control equipment is 10 - 20% of the production cost.

The large quantities of water needed for cooling of thermal plants account for the thermal pollution of rivers and lakes, which has serious and disturbing effects on the ecology by affecting the animal and plant life and the runoffs of rivers[11]. To reduce thermal pollution, closed circuit cooling systems including cooling towers and ponds have become an essential part of the modern power plant. It is estimated[8] that the cost of this equipment is generally about 2 - 10% of the capital cost of the corresponding generating unit.

To avoid long distance transmission, the practice has been to construct large thermal power stations at or near the major load centres. As the industrial load accounts for about 60 - 70% of the total load in a highly developed country, this usually leads to the most densely populated areas receiving the largest share of pollution. Therefore in siting thermal power plants, it is important and necessary to pay more attention to these environmental issues and to consider the power industry as a part of a much larger ecological system, rather than to deal with the planning of the power industry alone[11].

These capital and operating costs and the operating characteristics of the different plant types are the major factors which influence the optimal plant mix in a particular power system. The following sections briefly describe the important characteristics of major energy sources.

1.5.1 Hydro Power Plants

These plants are characterised by a very high capital cost and an operating cost which is almost zero. For a multi dam project the initial costs cover a large part of the civil construction work and the subsequent generating capacity additions can be made at low capital costs. In a multipurpose project, the storage water is mainly used to generate electric power, to regulate river flows and flood control and to irrigate furtile land. The control of water for the optimum benefits is complicated due to these varying requirements and the associated operating constraints.

As the inflows generally show a seasonal variation, the energy availability from these plants are similarly seasonal. These plants are mostly used for intermediate load or peak shaving although base loading is possible if energy is available. The availability of the electrical equipment is very high, in the region of 99%. Depending on the country, the construction times can range between 7 - 10 years. If pumped storage plants are installed they can be used as peaking and regulating plants.

1.5.2 Fossil Fired Power Plants

The fossil fired power plants have a high capital cost and tend to have a high operating cost depending on the type of fossil fuel used and the extent of pollution control associated with it. They are suitable either as intermediate or base load plants. Typical plant availabilities are about 90-95%. The construction times can range from 4 - 7 years.

1.5.3 Nuclear Power Plants

High capital costs and low operating costs are the characteristics of these plants. As the nuclear reactor does not warrant cyclic operation, these plants are only suitable for continuous base load operation. The possible risk of pollution through nuclear waste and radiation are major issues for the rejection of nuclear power plants by the environmental

protection groups. Despite this, most power systems are attracted by the low operating cost and undertake the development of nuclear power especially in the face of apparent instability of the future oil situation. Typical construction time is about 10 years.

1.5.4 Gas Turbine Power Plants

These plants have low capital costs but as they require more refined fuel oil, the operating costs are high. The short construction times of 1 - 2 years make these ideal either to meet unforseen peak demand growth or to meet demand which otherwise cannot be met due to improper planning of generating facilities. Although these plants are designed for peaking and standby purposes, the inadequacy of base load plants in systems have often required these plants to operate continuously at high plant factors. The typical availability is about 85 - 90% and as a consequence of continuous running the availability drops further, demanding more capacity. However, the gas turbine capacity additions must be decided in relation to the relative capacities of other plants in the system as excessive gas turbine capacity can lead to a very uneconomical situation.

1.5.5 Geothermal Power Plants

The exploitation of geothermal energy for electric power generation is a relatively new field. These plants are characterised by high capital costs and almost zero operating costs. The plant availabilities are similar to the fossil fired plants, and geothermal energy is suitable for intermediate and base load operation.

1.5.6 Alternative Energy Sources

Owing to the possible scarcity of fossil fuel in the future and especially after the escalation of fuel oil prices in the early 70's, the emphasis has been on the large scale development of alternative energy sources

for commercial use. Although the use of tidal and wave power for electric power generation is still in the early development stages, wind and solar power are becoming more promising alternative energy sources of the future. The development of solar technology, specially for heating and airconditioning, is expected to have a tremendous impact on load patterns now experienced by supply industries[8].

1.6 Investment Planning

It has been mentioned that the optimum plant mix in a typical power system must be obtained by identifying the complementary functions of different energy sources and the relative capital and operating costs. This requires the scanning of a large number of alternatives with different present and future investment policies and associated constraints. In most practical situations the details are too numerous and hence certain simplifying assumptions and approximate mathematical methods are necessary.

The complexity of the overall power system requires the formation of suitable submodels which can then be analysed in isolation[12]. However, the results thus produced must be compatible for proper coordination to evaluate the performance of the total system. Some of the assumptions and the simplifications used in the mathematical methods can later be removed in a simulation study to reveal a more accurate picture of the system operation.

1.7 Reactive Power Generation Planning

Although much has been said about the planning of generation of active power, little attention has been given to the planning of reactive power generation and its effect on overall system costs[9]. This lack of concern may be attributed to the fact that the cost associated with the generation of reactive power is not well defined[8]. An excitation system is needed even for unity power factor operation, and hence once installed it is

available at no extra cost to produce reactive power.

However, the demand for power at lower power factors requires higher rating of generators and can also lead to additional system energy losses. Although this does not demand higher ratings for the turbines, the larger physical size of the generator corresponds to an increase in the capital cost[8].

Long transmission lines generate large amounts of reactive power during the initial period of operation when not heavily loaded and during off-peak load periods. This problem is compounded by the installation of large generating units with small ranges of reactive power handling capability, implying that less capacity is built into the system to absorb the reactive power produced by the system itself.

Therefore careful consideration must be given in planning the composite generation and transmission system to maintain a proper balance of reactive power capability and an acceptable voltage profile.

1.8 Planning and Investing for Reliability

It is generally agreed that reliability has to be built into the system at the design stage[13], although improvements can be made later by reinforcement at extra cost. However, after a certain level, the increase of reliability of a system can only be effected by increasingly rising cost[14]. Further, the additional benefits often do not justify the additional costs[15].

Although complete redundancy is not practicable, the system reliability is provided by including redundant elements in various sectors of the power system. The degree of redundancy assigned to each sector must reflect the function of that sector within the system and the implications associated with equipment deficiency[7].

The inherent uncertainty associated with the planning of power systems requires the reliability evaluation to be based on probabilistic solution methods. In the area of generating capacity planning, the use of probability techniques is now becoming conventional practice in most countries. However, in the area of transmission and distribution system evaluation, the word "reliability" is most often used in a qualitative rather than a quantitative sense. The basic justification for this approach has been the lack of consistent data and the unavailability of suitable analytical techniques[6,7].

In making investment decisions to enhance reliability of a system the following are important considerations.

It is important to examine whether the consumers are infact willing to pay more for improved reliability or are prepared to accept existing reliability standards. As often there is more than one energy sector where investments are necessary to improve reliability, the sector which has the greatest impact on the society for a given investment must be selected. Further, the continuous improvement of only one sector is very uneconomical with no advantages and a proper balance must exist between all the major sectors of the power system. However, a true appraisal of the cost and worth of reliability[16] to consumers is not easy due to the lack of proper data and to the difficulty in assessing the true response of the consumers.

1.9 Summary of the Study

The foregoing sections serve as a general background to the work presented in this thesis. They have highlighted the presence of uncertainty in all phases of power system planning and the appropriateness and the need for probabilistic solution methods. A short summary of the investigations made during this research and the layout of the thesis is now given.

Chapter 2 is concerned with the planning of long term generation facilities of a hydro-thermal system. The discrete nature of the solution is identified and an optimal way of modifying the system constraints is proposed to increase the efficiency of the integer programming method. Alternative expansion plans are assessed on a probability basis using loss of load probability and expected unserved energy as reliability indices. As the seasonal variations in load demand,energy availability from hydro plants, unit availability, etc. favour a seasonal simulation, operating studies are carried out on a seasonal basis.

The optimal solution is obtained by iterating between optimisation and simulation and Chapter 3 discusses an interactive solution strategy with the effects of system parameters on the final solution. The development of a practical solution from the mathematical optimum is discussed.

In Chapter 4 the problem of optimal operation of a storage reservoir in a mixed hydro-thermal system with the objective of reducing thermal generation costs is discussed. The random nature of the water inflows to the reservoir is treated through the application of stochastic dynamic programming technique.

The use of load flow techniques for the transmission planning problem is discussed in Chapter 5 and a method is presented to evaluate the reliability indices at consumer load points. The method is extended to calculate the overall system performance indices. The reliability indices thus obtained are compatible with those used for distribution system planning. The effectiveness of this method with respect to the conditional probability method is also discussed.

The main conclusions arising from these studies are presented in Chapter 6 and areas for further development are suggested.

1.10 Conclusions

The studies presented in this thesis has been influenced to a certain extent by the characteristics and requirements of the authors' own power system in Sri Lanka. In this system, the generation is mainly by hydro power plants and the major part of the energy is transmitted to load centres using a 132 kV transmission network.

As most of the hydro projects are multipurpose with high priorities for agricultural requirements, the electric power development has become a secondary issue. Once the capital intensive civil construction works are completed, the power plants can be installed at little extra cost. Therefore, at present there is very little activity in the area of generation planning. However, the operation and control of these hydro schemes to realise optimal benefits to the power industry and to the nation as a whole is very important. In this exercise, the use of probabilistic solution techniques is expected to give more realistic results.

The present practice in transmission and distribution system planning is to decide the network additions using heuristic approaches considering the trend and the load growth in different areas. Hence these expansion plans depend heavily on the intuitive considerations and the engineering judgement of the system planners. This has been mainly due to the lack of data and suitable analytical techniques. Therefore it is believed that with the availability of a proper data base, the techniques developed in this thesis will be useful in future planning studies of the Sri Lanka power system.

CHAPTER 2

LONG TERM GENERATION PLANNING USING INTEGER PROGRAMMING AND PROBABILISTIC SIMULATION

2.1 Introduction

The long term generation planning procedures deal with the time horizons in which generation capacity adjustments can be made to keep the probability of supply interruptions due to capacity shortages down to an acceptable level. In order to achieve the high level of reliability, the system must have sufficient reserve capacity to meet the total consumer demand at all times and under all possible contingencies. This could require an uneconomically large reserve on the system capacity which is not practicable. However, an optimal level of reserve capacity for a system can be defined as that capacity whose marginal costs equal the expected social savings from marginal reductions of supply interruptions[9].

With the uncertainties associated with demand forecasts, plant and fuel price forecasts and unit availability forecasts and the ever increasing lead times of construction, sometimes it is necessary to consider changes in investment plans to keep the probability of failure of supply below the agreed level. Five possible ways of achieving this are[8]:

1. Advance the investment program and possibly change the plant mix (eg. gas turbines for oil fired plants).
2. Increase the unit size of new plants planned but not yet under construction. This may not be possible in the case of hydro plants where the plant capacity is governed by the capacity of the reservoir and availability of water.
3. Accelerate the completion of plants not under construction.
4. Build a small new oil burning plant which has a short construction period.

5. Install additional gas turbines. This decision must be taken very carefully considering the relative capacities of other plants already in the system. Otherwise a situation could arise in which the gas turbines have to be base loaded to supply consumer demand.

These planning schedules will naturally shift the system cost structure from the least cost investment policy, whilst maintaining the reliability of supply at the prescribed level.

Alternatively, it is possible to accept a lower reserve margin and an increased probability of failure to meet the peak demand. If this alternative is taken, then saving of additional expenditure on more costly plants and expensive construction programs is obtained at the expense of increased cost of interruptions. This may not be applicable in developed countries where the standard of supply is very high and the industrial productivity and domestic facilities are mostly dependent on the quality of supply. However, in developing countries, since the consumers do not normally enjoy a high standard of electricity supply, an increase in the frequency and the duration of interruptions by a small amount will be of no major consequence. The development will then occur according to the optimal investment policy.

In examining the least cost development of a country's resources the following two points must be taken into consideration.

1. Time dependent (dynamic) optimisation is required.
2. A comprehensive scan of alternative combinations of resources in each time period is needed.

2.2 Balance of Plant Types and Capacities

The first requirement takes into account the effects of the plant mix on the system operation costs over the life span of the plants, which is normally 30 years or more. For example, because of the cost characteristics of different plant types (Section 1.5), the life time savings on system fuel costs of a hydro plant are increased if the fossil plant on the system is extensive. On the other hand, the cost savings are less if nuclear plant is installed. Therefore, a large nuclear power program in future years may shift the present balance towards fossil plants and away from hydro plants.

Similarly, if the system structure is predominantly fossil plant, then the planned additions need to be nuclear or hydro (or both) to save on system fuel costs. This clearly shows that the optimum balance of plant capacities is dependent on both the inherited and the expected structure of the power system.

The second requirement ensures that the alternative energy sources are combined in the most economical way as they have complementary functions in a modern interconnected power system (Section 1.5). Thus the optimum balance of plant capacities in the system at any point in time will depend on the relative capital and operating costs of the alternative energy sources.

These two points therefore require that the investment decisions can be made only after considering the interaction of decision variables at a point in time and over time. Hence the models used in power system planning must be multidimensional and must include past and future evolution of investments[9]. All these requirements can be met by formulating the investment planning problem using Linear Programming (LP) techniques. This fact was recognised as early as in 1957 as the work

carried out by Masse and Gibrat of Electricite de France reported[17]. Since then many different formulations of the problem had been published[18,19,20] with varying degrees of comlexity.

2.3 Mathematical Models in Generating System Planning

Mathematical programming techniques such as linear programming (LP), non-linear programming (non LP), dynamic programming (DP), integer programming(IP) and mixed integer-linear programming (MILP) are nowadays recognised as useful tools in decision making for long term power system expansion. They find extensive uses in the power industry.

However, power system planning is a complex and important issue involving large investments and social and other factors which often cannot be quantified in absolute terms. It is also important to realise that generation planning is one of the most crucial steps in expansion planning and the commitments made at present have a tremendous effect on all other phases of system expansion[8,21]. Further, the solution obtained by mathematical means are limited by the features included in modelling and problem formulation and also by the accuracy of the data available. Therefore practical decisions must not entirely be based on mathematical optima and hence any mathematical programming techniques, to be practicable, must provide facilities for human interaction. This allows the system planner to guide the final solution to a feasible one using personal judgement based on engineering experience. In this exercise however, the studies using mathematical techniques provide valuable background information such as cost sensitivities, effects of uncertainties, the relative importance of different decision variables, etc.

Because of the relative merits of the different programming techniques, one technique may be more suitable for a particular problem than

others. For example, non-LP techniques are widely used[9,20] in investment planning problems involving systems whose plant structure is entirely non-energy limited, such as, nuclear, fossil and gas turbines. Similarly LP techniques find their use in problems involving systems having a mixed structure of energy limited plants such as storage hydro, geothermal and non-energy limited plants. Since this Chapter deals with the planning of systems having hydro plants with seasonal variations in storage and hence energy generation, LP techniques are discussed in detail.

2.3.1 Basic LP Algorithm

The linear programming problem as applied to power system planning can be stated as:

$$\min_{x} \quad c^T x \tag{2.1}$$

subject to

$$Ax \geq b \tag{2.2}$$

$$0 \leq x \leq u \tag{2.3}$$

where

- x - vector of variables
- c - coefficient vector of the objective function
- A - coefficient matrix of the linear structural constraints
- b - vector of right hand values of the constraints
- u - vector of upper bounds on variables.

LP techniques have been successfully utilised for a considerable time in the area of long term planning of power systems. However, the early LP formulations incorporated the operating constraints explicitly and this led to a large growth in model dimensionality with problem size[19] thus preventing their use in real life problems. To over-come

this disadvantage more compact models such as the Z substitute and the improved Z substitute were developed[22,23] taking advantage of particular operating characteristics to reduce the number of constraints. A further improvement has been obtained through the use of combined costs method[19] (CCM) to remove the operating constraints completely from the global optimisation model. The many constraints imposed by the energy aspects are treated through the introduction of starting values for the plant factors. Once a solution is obtained, the system operation can be studied in detail using simulation models (Section 2.4) and the plant factors can be updated if necessary. It is claimed[19] that this formulation reduces the number of constraints by upto one hundredth of that required for early LP models.

Apart from the problem of dimensionality, another important aspect of the mathematical technique is the nature of the problem solution. In generation planning studies, practically significant solutions are essentially discrete, because generation additions at any development stage have to be an integral number of units of specific sizes of particular plant types as determined by the availability of economic and other resources of the country. Hence the normal LP algorithm which gives a non-integer (continuous) solution is not suitable. The discrete nature is best treated through the use of integer programming (IP)[5]. This fact was recognised and great emphasis was placed on the exploitation of the use of IP techniques in generation planning and other related sectors. Some formulations[24] used a mixed integer-linear programming approach whilst the others[5,25] used pure integer programming. The use of integer programming algorithms with 0 - 1 as decision variables are also well established[26].

As mentioned earlier, not only the LP solution but the IP solution too depends on the initial plant factor assumptions and requires

the detailed simulation of operation of the proposed generation system to yield the expected plant factors. These plant factors will be different from those assumed in the original IP formulation but form a better set of values for another IP solution. It is this interactive nature which makes the method very attractive to the practical user since there is the possibility of injecting human judgement between solution steps (Chapter 3).

2.3.2 Problem Formulation Using CCM

It has been noted earlier that an attractive feature of the formulation of the generation planning problem in CCM form is the reduction of the number of constraints which in turn reduces the model dimensionality for a given problem size. In other words, this technique offers facilities to study a system with more units over a longer time horizon for a given problem dimension. This is achieved by assuming a feasible set of values for the plant factors of the non-energy limited (NEL) plants and constraining the operation of energy limited (EL) plants to ensure full exploitation of energy during a given period. The plant factors of EL plants are calculated using the energy availability forecasts. Hence from the CCM point of view, the actual plant factors and the anticipated plant factors for EL plants are identical.

The objective function includes only the total costs present valued at a given discounting rate. The factors such as unit availabilities and operating policies, eg. for the retirement programs of old plants, are represented as constraints. Although, any feasible time period, for eg. half year, 2 years, 5 years and sometimes combinations, can be and have been used[9], yearly subperiods are chosen as a convenient planning interval.

Hence the objective for optimal investment in generation expansion over a planning period of T years is to minimise[25]

$$\sum_{t=1}^{T} \sum_{i=1}^{I} (C_{it} + F_{it}\, f_{it}\, H_t) X_{it} \qquad (2.4)$$

subject to the following constraints.

i. Capacity constraint

$$\sum_{i=1}^{I} a_{it}\, X_{it} \geq D_t\,(1 + d_t) \quad \text{for} \quad t = 1 \text{ to } T \qquad (2.5)$$

ii. Energy constraint

$$\sum_{i=1}^{I} f_{it}\, X_{it}\, H_t \geq D_t\, L_t\, H_t\,(1 + e_t) \quad \text{for } t = 1 \text{ to } T \qquad (2.6)$$

iii. For EL plants

$$f_{it}\, X_{it}\, H_t = \bar{E}_{it} \quad \text{for all EL plants} \qquad (2.7)$$

(This constraint ensures full exploitation of energy from EL plants)

iv. Progressive addition of new plants

$$X_{i(t+1)} - X_{it} \leq \bar{x}_{it} \geq 0 \quad \text{for } i \in \text{new plant over } t=1 \text{ to } (T-1) \qquad (2.8)$$

where $\bar{x}_{it}$ is the maximum permissible capacity addition of a particular plant type i in a particular year t.

v. Progressive retirement of old plants

$$X_{it} - X_{i(t+1)} \leq \bar{x}'_{it} \geq 0 \quad \text{for } i \in \text{old plant over } t=1 \text{ to } (T-1) \qquad (2.9)$$

where $\bar{x}'_{it}$ is the maximum permissible capacity of retirement of a particular plant type i in a particular year t.

vi. Maximum capacities of plants

$$X_{it} \leqslant \overline{X}_{it} \quad \text{for } i \in \text{new plant over } t=t' \text{ to } T \qquad (2.10)$$

where $\overline{X}_{it}$ is the maximum permissible plant capacity at a station in year t and t' is the first interval in which the maximum capacity is specified. This insures that the capacity additions are within the limits imposed by energy, economic or environmental constraints. It gives the additional facility of adding a new plant after a specified time delay, represented by a plant whose capacity during the first n years is zero and its capacity at the (n + 1)th year is the required value.

The problem in CCM formulation had been solved[19] as a LP model giving a continuous solution for the optimal generation capacities. However, it is obvious that most of the time these capacities have no relation to the preferred unit sizes and hence is not physically realizable. An approach which was used in practice in the early days was to round off the solution to the nearest unit sizes and this would definitely shift the solution from the optimal, unless all the possible combinations of preferred unit sizes are scanned.

However, this type of solution could suffice, if the requirements are only for an initial study of an expansion program to be followed by extensive studies using IP techniques. The obvious advantage of using IP is that the unit sizes of different plants can be constrained to their preferred sizes and the optimal set of unit capacities would consist of these.

2.3.3 Branch & Bound Method

The merits and demerits of various IP algorithms such as cutting plane, branch and bound, etc. are well documented[27]. Out of these the branch & bound (BB) class of IP algorithms are very powerful, robust and reliable, and hence are most suitable for applications in generation planning studies. Further, a very convenient property of the BB algorithm is that in addition to the optimal integer solution, it also gives many near optimal integer solutions, which are called 'tails'. These can be used in the simulation to test for operationally better solutions than the optimal. However, one disadvantage of the basic BB techniques used in these studies is that they are very time consuming, generally of the order of hundreds of CP seconds for a practical system. This could be as much as thirty times the time for an ordinary LP solution to the same problem. Consequently, this research has attempted to improve the cost effectiveness of the BB method for the planning problem.

2.3.4 Generalised Branch & Bound (GBB) Method

Recently, a method was proposed by Brocklehurst[28] which effectively reduces the number of nodes to be examined, thereby reducing the number of tails produced in the BB method. This was called the generalised branch and bound (GBB) method and the basic difference between the two methods lies in the method of branching. Instead of branching according to the dichotomies of the form either

$$x_k \geq \gamma \tag{2.11}$$

or

$$x_k \leq \gamma - 1 \tag{2.12}$$

more general dichotomies of the form

$$\alpha^T x \geq \alpha_o \tag{2.13}$$

or

$$\alpha^T x \leq \alpha_o - 1 \tag{2.14}$$

are used in the GBB, where

α is a vector of coefficients and α_o is an integer, both yet to be determined.

Essentially GBB is a BB process in which the constraints of the original problem are replaced by simpler constraints having smaller coefficient values which are approximately parallel to those of the original problem.

In practice a dichotomy $x_k \geq \gamma$ or $x_k \leq \gamma - 1$ may well be effective if one of the new constraints introduced is a fair approximation to a constraint of the Convex Hull of Feasible Integer Points (Appendix A1). In this case the alternative branch is unlikely to contain any feasible integer points with near optimal function values and as a consequence its descendant nodes should be easy to fathom.

It is information about the convex hull or, at least, the constraints defining the optimal vertex of it and its relationship to the constraints of the LP problem, that any new method must seek to find. The nature of the GBB is based on the following important factors:

1. Since it appears to be difficult to deal with the constraints as they stand, it is necessary to find new constraints or to deduce useful information about feasible integer points. The best strategy is to devise some method of approximating the convex hull with constraints which will yield useful information.
2. The new approximation to the optimal vertex should not become flatter than the old vertex.
3. The extra amount of information that has to be stored in order to define a node should not be excessive.

The GBB method is explained more fully in Appendix A1.

2.3.5 Application of the GBB Method to the Problem of Generation Planning

It has been observed[28] that the GBB method can give no further improvement to the ordinary BB method if the coefficients of the constraints involve only 1, 0 or -1. A close examination of the constraint matrix in the generation planning problem reveals that, due to the nature of the generation structure of a practical system (either a generator of a given capacity is installed or not), the coefficients of most of the constraints are either 1 or 0. The energy constraints are, however, different from the above and in general could be written as

$$\sum_{j} \lambda_{jt} X_{jt} \geqslant E_t \qquad \text{for all } t \tag{2.15}$$

where

$0 \leqslant \lambda_{jt} \leqslant 1.0$ is the normalised plant factor

X_{jt} is the capacity of generator j

E_t is the energy demand

j is the plant type index

and t is the time period index .

Therefore it can be seen that the coefficients of all the energy constraints can take any real value between 0 and 1 and hence can be modified using the method of Section 2.3.6 to improve the performance of the method.

The number of energy constraints in a problem is NP where NP is the number of years in the study. The total number of constraints M is always greater than the number of variables N where

$$N = NT \times NP \tag{2.16}$$

and NT is the number of plant types.

i.e. $M > NT \times NP$. (2.17)

From this it is evident that the number of constraints that are available for modification is much less than the total number of constraints. The values for 2 typical problems are shown in Table 2.1, columns (1) to (3).

2.3.6 Modified BB Method

The main idea behind the modification is to obtain equivalent constraints with simple coefficients which are also approximately parallel to the original energy constraints. As was mentioned earlier, this has to be done in such a way that the new constraints exclude comparatively large part of the feasible non-integer region and that they pass through many integer points. Thus a positive integer P_i for each constraint i has to be determined so that

$$\frac{P_i}{\left[\sum_j \delta_{ij}^2\right]^{\frac{1}{2}}} \quad \text{is a maximum.} \tag{2.18}$$

As a convenient alternative to $\left[\sum_j \delta_{ij}^2\right]^{\frac{1}{2}}$ in eqn. (2.18), $\sum_j |\delta_{ij}|$ can be used, where

$$\delta_{ij} = a_{ij} - P_i \alpha_{ij} \tag{2.19}$$

$$\alpha_{ij} = \underline{\text{nint}}\left[\frac{|a_{ij}|}{P_i}\right] \tag{2.20}$$

and nint is the nearest integer to $[\cdot]$.

Then eqn. (2.18) becomes

$$\frac{P_i}{\sum_j |a_{ij} - P_i \alpha_{ij}|} \tag{2.21}$$

For a range of values of P_i, eqns. (2.20) and (2.21) are evaluated and the optimal P_i is obtained which maximises eqn. (2.21). Once this is obtained, the coefficients are divided by P_i and the new coefficients

α_{ij} are found such that eqn. (2.20) is satisfied.

After obtaining the new coefficients the new right hand side constant α_{oi} is found by writing

$$\alpha_{oi} = \alpha_{ij}^{T} x^{c} \tag{2.22}$$

where x^{c} is the LP solution to the problem with the original constraints. Once all the constraints are modified the ordinary BB is used to obtain an integer solution. A simplified flow chart is shown in fig. 2.1.

2.3.7 Results

The modified BB method has been tested using a number of generation planning problems. The results obtained for two typical generation expansion problems are presented in Table 2.1. The extra information required in the computational procedure to obtain these reductions in computing times is negligible. Considering the comparatively few constraints available to be operated on compared to the total number of constraints, these reductions in computing times are however quite attractive.

2.4 Simulation Models in Generating System Planning

It has been previously noted that the decision making in investment planning problems should be based on engineering judgement with computer solution as background information. The optimisation methods such as BB, provide many near optimal solutions which sometimes may prove operationally better than the optimal. Hence the system planner is faced with the task of choosing the best generation expansion plan among the many available, in an environment of uncertainty. These uncertainties creep in owing to the factors which are not easily quantifiable in absolute terms or due to reasons such as questionable accuracy of data available, complexity of modelling limited by the features included in them, etc[20].

In addition to the uncertainty inherent in forecasting future load requirements, the uncertainties associated with

i. unit reliability and maintenance schedules

ii. fuel costs

iii. pollution control costs

iv. construction costs

v. start up costs

vi. availability and cost of captial

must also be included in the evaluation of alternative strategies[8].

There are two approaches to obtain alternative plans for system expansion planning.

1. The planning staff collectively define a number of feasible alternatives on the basis of

    a) load growth

    b) construction time

    c) availability of sites

    d) availability of fuel/water

A good understanding of the existing system, the past experience in operating difficulties, etc., are very valuable and helpful in choosing the right capacity of the right type of plant to be installed at the right time.

2. Semiautomated procedure in which capacities, types of plant and the commissioning dates are determined over the planning horizon using optimisation methods.

As mentioned earlier, the final solution from the analytical method will be limited by the features of the existing and the expected system included in modelling and formulation of the problem.

Once feasible alternatives are formulated, a detailed reliability analysis must be carried out primarily to ensure that these plans satisfy a desired level of reliability. In addition, the performance indices and associated costs of various plans must be compared before the final selection is made.

2.4.1 Reliability Indices

The level of reliability of an electric supply system is usually defined by reliability indices. However as a single index cannot describe completely the reliability of a system, a number of indices have been introduced. These can be broadly classified into the following categories[29].

1. Probability of failure
2. Frequency of occurrence
3. Mean duration of outage
4. Expectation of outage

The calculation of frequencies applies only to repairable components which form most parts of the electric power system. It is also important to note that a single frequency index is not sufficient to describe system reliability and must be accompanied by the duration index.

Some of the widely used indices which belong to the above categories are:

i. Loss of load probability (LOLP)
ii. Expected unserved energy ($E[UE]$)
iii. Expected unserved demand ($E[UD]$)
iv. Frequency & Duration indices (F&D)
v. Load interruption index
vi. Interruption duration index (IDI)

Out of these, the LOLP[30,31,32] and F&D[33,34] indices are the most used in the field of generation planning.

In generation planning calculations it is assumed that the transmission network is perfectly reliable and capable of delivering energy from any generating plant to any load point without impairing the quality of supply. This implies that the system is successfully operating as long as there is sufficient generation to meet the load demand. However, the impetus for development of the F & D method for generation planning has come from the need for generation system reliability techniques comparable with transmission system reliability methods[8].

The above list of indices is by no means complete and in the course of system evolution, many new indices will be defined. In the recent past, the emphasis has been on the development of indices which would faithfully reflect the quality and continuity of supply to the average consumer. It has been pointed out[35] that most of the reliability indices mentioned above do not directly reflect consumer welfare which the supply industry is dedicated to maximise within certain constraints.

In the evaluation of reliability indices for generation expansion planning, the uncertainties in unit availability and load forecasts are included using probabilistic models to represent the stochastic nature of the generation and the load.

2.4.2 Probabilistic Generating Unit Model

The random behaviour of a generating unit during its operating life can be described as a stochastic process which is defined[8] as a process that develops in time in a manner controlled by probabilistic laws. Thus a zero order, discrete state, continuous transition Markov process which has the following properties is used in representing generating units in reliability calculations.

1. The system described can be characterised as being in one of a set of mutually exclusive, collectively exhaustive, discrete

states at any time. In other words the generating unit is assumed to be in either the up or down state, but not in both simultaneously, and hence the states are mutually exclusive and discrete. Further, it is also assumed that for a generating unit the up and down states define the only possible states and hence the states are collectively exhaustive.

2. Changes of states are possible at any time and hence they are a random process.

3. The probability of departure from a state depends only on the current state and is independent of time.

4. The probability of more than one change of state during an appropriately small time interval is negligible.

Two reasons for modelling the random behaviour of a generating unit as a Markov process[8] are:

a. Markov process is usually a fairly accurate model of real life

b. the mathematical description of the process is simple.

2.4.3 Two State Representation of Generating Units

The process described above can be illustrated by a state space diagram as shown in fig. 2.2. However, the life history of a generating unit is usually not as simple as that described by the two state process[29]. In addition to complete outages, they experience partial outages and may continue to operate at a reduced capacity level (derated state). Further, the present day electric supply systems consist of many different types of units, all with their own pecularities. Therefore the models required to represent the stochastic nature of the generating units differ greatly in their sophistication[13,36]. However, it is well established [5,8,25,31,37] that for the accuracy required in

long term planning studies, the two state model representation is quite adequate.

The long term probabilities for unit availability and unavailability can therefore be evaluated by the following expressions.

$$\text{unit availability} \qquad p = \frac{\mu}{\lambda + \mu} \tag{2.23}$$

$$\text{unit unavailability} \qquad q = \frac{\lambda}{\lambda + \mu} \tag{2.24}$$

where λ is the failure rate (transition rate from UP state to DOWN state) and μ is the repair rate (transition rate from DOWN state to UP state).

The traditional term for the unit unavailability is the forced outage rate (FOR). This is infact a misnomer, since the concept is not a rate.

By definition,

$$FOR \triangleq \frac{FOT}{OT + FOT} \tag{2.25}$$

where

OT is the unit operating time

and

FOT is the unit forced outage time.

For a repairable system with exponentially distributed up and down times, the FOR if computed over a long period of time, approaches the long term probability of the unit being unavailable.

i.e. $$FOR = \frac{\lambda}{\lambda + \mu} = q \tag{2.26}$$

2.4.4 Representation of Partial Outages

It has been mentioned earlier that the two state model does not account for partial outages of generating units. Although this is a fairly accurate representation of small or medium size units, large units need to be represented more accurately using multi-derated state models[38].

However, the complexity of computations is increased as the number of transition states are increased.

Alternatively, the forced outage time can be increased by an appropriate amount of time called the 'equivalent forced outage time' (EFOT) which is obtained by the following expression:

Let t_{pi} - the partial outage time in hours

X_{ri} - the corresponding fractional capacity reduction

i - number of occurrences

then
$$EFOT = \sum_i t_{pi} X_{ri} \tag{2.27}$$

Using the EFOT, an equivalent forced outage rate (EFOR) can be defined as

$$EFOR \triangleq \frac{FOT + EFOT}{OT + FOT} \tag{2.28}$$

where the operating time now includes the partial outage times as well. This approach fairly accurately approximates the model for a generating unit with partial outages, to the two state model for a unit with full outages only[29].

2.4.5 Repair and Maintenance

The components in a power system undergo preventive maintenance in addition to the repairs upon failure. Maintenance is usually carried out with the expectation that, the mean time between failures will increase, thus increasing the availability of the component. The maintenance of a repairable component can be modelled as a three state Markov process having normal (U), repair (D) and maintenance (M) states. The state space diagram of such a process is shown in fig. 2.3. If it is assumed that no failure can occur during maintenance and maintenance

is not started during repair, the possible transitions between the states are only those indicated in fig. 2.3.

The long term probabilities of being in these states can be obtained by the following expressions[29].

Probability of being in normal state

$$P_U = \frac{\mu\mu_M}{\mu\mu_M + \lambda\mu_M + \lambda_M\mu} \tag{2.29}$$

Probability of being in repair state

$$P_D = \frac{\lambda\mu_M}{\mu\mu_M + \lambda\mu_M + \lambda_M\mu} \tag{2.30}$$

Probability of being in maintenance state

$$P_M = \frac{\lambda_M\mu}{\mu\mu_M + \lambda\mu_M + \lambda_M\mu} \tag{2.31}$$

If it is assumed that maintenance is performed at weekends and during low load periods, it can be assumed that maintenance state itself does not contribute to system failures. Further, if the period of observation is long enough, the repair times and the operating times are proportional to the corresponding state probabilities. Therefore, the FOR which is defined by eqn. (2.25) can be expressed as

$$\begin{aligned} FOR &= \frac{P_D}{P_U + P_D} \\ &= \frac{\lambda}{\lambda + \mu} \end{aligned} \tag{2.32}$$

which is the same as unavailability q. This shows that the parameter unavailability is a good approximation of the unit failure probability even when preventive maintenance is considered.

2.4.6 The Effective Load Carrying Capability (ELCC) of a Generating Unit

The concept of ELCC[8] is important to understand the implications of the addition of a new unit to improve system reliability. When a new unit is added to a system, the reliability is often improved but as demand grows the reliability begins to decrease. The amount of load growth that can occur before reliability returns to its value before the new unit was installed is less than the capacity of the new unit because the remainder must be held in reserve to maintain the desired reliability level in the face of increased LOLP and $E(UE)$. Therefore the ELCC of a generating unit is that part of capacity that is available to supply increases in demand to maintain system reliability above the prescribed level.

The ELCC of a unit varies with the unit capacity and in a particular situation a unit of capacity X would be more appropriate if it has a ELCC greater than that of a unit of capacity 2X. Since unused capacity means expenditure which does not account for operating revenue, the system planner must avoid expansion plans in which units cannot be effectively utilised to maintain a sound financial posture.

2.4.7 Probabilistic Load Model

The probabilistic load models used in the early loss of load calculations were very simple and as a consequence it was possible only to calculate the probability of failure to meet the load demand. A two level representation of the daily load curve was proposed and used by Ringlee et al[33,34], in the frequency and duration method. Later Billinton and Singh[39,40] proposed the use of load models with multi-level exposure factors. All these are quite refined in comparison to the simple load duration curve, indicating the state of the art in reliability techniques.

The best reliability index (or indices) and hence the best load model to use, depends on the nature of the application. Since a number of valid reliability indices is available they must be evaluated using the right load model and interpretted correctly. However, it has been documented[29,36] that in long term system expansion planning studies, the risk criterion can be expressed in any of the indices with the choice having very little effect on the resulting expansion schedule.

In long term generation planning studies, it is important to determine the probability of the load exceeding a certain generating capacity, for example the installed capacity of a proposed generating system. This overall probability is defined as the loss of load probability and the probabilistic load model that best suits the situation is the one which describes the probability that the load will exceed a certain value.

There are two possible ways of obtaining this load model. If hourly load readings over a time period are used the load model is the well known load duration curve[29,41]. If it is assembled from the daily peaks, with the abscissa indicating the percentage of days when the peak exceeds the amount of load shown by the ordinate, then it is the cumulative curve of daily peak loads. The obvious implication in the latter case is that the peak load of the day would last all day and the variation of load within a day is not recognised. This makes the LOLP obtained by that method a rather crude approximation of the true system failure probability. However, when projecting to the distant future it is not possible to forecast accurately the daily load variations and the accuracy of LOLP obtained using the cumulative load curve would be adequate for the purpose.

The area under the load curve is the energy consumed by the load and transmission system and this also defines the energy to be produced collectively by peaking, midrange and base load units.

2,4.8 Load Duration Curve as a Load Model

In the calculation of LOLP, the generating capacity outage table and the load model are convolved and hence, it is necessary to interchange the axis parameters of the load duration curve and to normalise, thus producing a load probability distribution as shown in fig. 2.4.

It has been recognised that the shape of a typical duration load curve cannot be accurately represented by a single higher order polynomial because of the practical difficulty in obtaining a good fit in the vicinity of the peak demand[25]. Small errors in curve fitting in the peak region of the duration load curve often lead to large errors in the estimated generation from peaking and standby plant, which may result in substantial errors in system operating costs. This problem is overcome by dividing the curve into a suitable number of sectors and fitting polynomials to each sector (fig. 2.5). It is also evident that most systems show a considerable variation in minimum demand over successive years and when the duration load curve is represented on a normalised scale, this increase accounts for the change in load factor with system load growth (fig. 2.6).

In these studies, however, the usual practice is to describe the duration load curve for the first period and to use the same curve for all the other periods. The implication of this is that the extra energy requirements due to change in load factors are not included and the resulting expansion schedule will obviously be incapable of supplying the total demand, without degrading the level of system reliability. One possibility is to include the change in load factor by suitably modifying the coefficients of the curve obtained in the first period. It has been realised that this is particularly easy if the duration load curve can be approximated by a piece-wise linear curve with reasonable accuracy.

2.4.9 Change in Load Factor due to Increase in Minimum Demand

Assuming that the duration load curve for the first period is represented by a piece-wise linear curve, the following method can be used to modify the coefficients to suit the change in load factor.

Let

$$y = a_o + a_1 x \tag{2.33}$$

be the equation of a linear piece for the first period (fig.2.6, x_2 to x_1).

If the change in minimum demand is given by

$$y = \beta x \tag{2.34}$$

where

$$\beta = \frac{y_2}{(x_4 - x_2)}$$

and if the modified coefficients are

$$a_o' \text{ and } a_1'$$

the equation for the next period would be

$$y = a_o' + a_1' x \tag{2.35}$$

where

$$a_o' = \frac{x_4 y_1 - x_3 y_2}{x_4 - x_3}$$

and

$$a_1' = \frac{y_2 - y_1}{x_4 - x_3}$$

but $x_4 = x_2 - y_{2/\beta}$

and $x_3 = x_1 - y_{1/\beta}$

then

$$a_o' = \frac{(x_2 + y_{2/\beta})y_1 - (x_1 + y_{1/\beta})y_2}{(x_2 + y_{2/\beta}) - (x_1 + y_{1/\beta})}$$

which reduces to

$$a_o' = \frac{\beta}{\beta + a_1} a_o \tag{2.36}$$

Similarly

$$a_1' = \frac{y_2 - y_1}{(x_2 + y_{2/\beta}) - (x_1 + y_{1/\beta})}$$

$$= \frac{\beta}{\beta + a_1} a_1 \tag{2.37}$$

The modified limits can slso be found from

$$x_4 = x_2 + y_{2/\beta}$$

By substituting

$$y_2 = a_o + a_1 x_2$$

then

$$x_4 = \frac{(\beta + a_1)\, x_2 + a_o}{\beta} \tag{2.38}$$

It can be seen from the expressions for a_1' and a_o' that it is a tedious process to modify coefficients if higher order polynomials were assumed for the first period.

2.4.10 Seasonal Variation of Parameters

The use of the duration load curve for calculation of performance indices such as operating costs, etc., introduces one important assumption[9]; i.e. the costs and availability of supply depend only upon the magnitude of the load and not on the time at which the load occurs. This assumption is quite accurate for all thermal systems although the plant availability is seasonal because of maintenance schedules. However, this is not valid for a system with hydro plants because of the large variation in energy availability between seasons and hence the operation must be simulated using seasonal duration load curves[8,9].

In addition, various difficulties are presented by the seasonal variation of load demand, avaialbility of plants and cost of fuel. These too favour seasonal simulation of operation of the generation structure.

However, if for reasons such as the unavailability of good data for a seasonal model, etc., an annual model were used, the system operating costs would be overestimated in the filling up periods when hydro is operating on base load and underestimated in the discharge periods when hydro is operating in peak or midrange positions. Although these over- and under-estimates tend to offset each other to a large extent, the seasonal simulation provides a valuable insight into the system operation problem[9].

2.5 Comparison & Evaluation of Reliability Indices

The use of LOLP and $E(UE)$ indices or the frequency and duration indices in the evaluation of generation system reliability is well established[8, 31, 37]. However, it is interesting to discuss the advantages and disadvantages of the two methods before favouring a particular method in these applications.

The origin of the frequency and duration method can be traced back to 1958[42]. In this method, however, the load model was restricted to a single peak value representation and hence this version of the method was not widely used. Ringlee et al[33, 34] developed a recursive approach with a two level representation of daily load curve in which the daily peak load is assumed to exist for a portion of the day designated by the exposure factor e. This provided a partial representation of the daily load variation by recognising that the peak does not last for the entire day, but it does not recognise that the load varies continuously over the day.

Much of the criticism[5,35] of this method has been centred around the question of selecting the exposure factor and the peak level of demand. Although, it is not possible to give an exact analytical approach for selecting the load level at which the exposure factor is calculated, it has been found[43] that the resulting expansion plan is virtually independent of the exposure factor selected. The question of selecting the peak demand becomes important if the daily load curve has a sharp peak with a small duration, but not so if the peak demand is relatively constant over a considerable period thus giving a flat curve easily approximated by a step function. In the former case, it has been said[35] that the approximation of the sharp peak by a broad step curve would unrealistically increase the risk and duration of load shedding. Although the variation in the exposure factor can have a significant effect on the probability of being in a capacity deficient state, it has only a small effect on the frequency of encountering a deficiency condition. However, a figure of 75 - 80% has been suggested[36] as a suitable level of load for selecting an exposure factor in practical studies.

The two level load model also introduces the concept of load model transition rates. The frequency of encountering a generation capacity deficient state is made up of the two components due to load model and generation model transitions. However, the contribution from the generation model is dictated by the system reserve, generating unit failure rates and repair rates. In general, failure rates are quite low and the repair rates relatively high and if the system has a substantial reserve, the component due to the generation system is low. This is the principal reason[36] why both the probability and frequency criteria when used in a generation expansion study, often give the same expansion schedule.

This method has also been criticised[35] on the grounds that it produces two reliability indices rather than one . In this respect, it has been pointed out[5] that these are weak arguments for rejecting the frequency and duration method.

Following the success of the two level load model, other models such as the multi-level exposure factor load model and the continuously varying exposure factor load model were developed. Since they represent the daily load variations more closely than the two level load model, there has been a considerable improvement of the accuracy of reliability indices calculated using them.

It has been said that the LOLP index has gained recognition because it provides a probabilistic figure which can be used in system planning and it is also relatively simple to evaluate. However, the following are some of the points which have been put forward[35] in criticising the use of LOLP method.

1. It gives no indication of the extent of load shedding, its duration or the frequency, to the average consumer.

2. The LOLP in days/year merely indicates the number of days in a year in which there will be generation capacity shortages but the actual frequency of load shedding may be higher than this figure in the case of double peaked daily load curves.

The first point has no significance relevance if the LOLP index is used for the evaluation of alternative expansion plans for the same system. The second argument holds only when the LOLP index has been obtained using the cumulative curve of daily peak loads (Section 2.4.7). As the use of the load duration curve based on the hourly load readings should account for the double peaks in a day, the LOLP would include the effects of these.

2.5.1 Relationship between LOLP and Margin Frequency Indices

It is also interesting to note that under certain assumptions, there exists a definite algebraic relationship between the LOLP and frequency indices which would be of assistance in comparing and contrasting them[37].

Let n_L - expected number of occurrences of load L during the year

$\bar{A}_G$ - corresponding unavailability of generation capacity sufficient to meet load L

then

$$LOLP \triangleq \sum_L n_L \bar{A}_G \qquad (2.39)$$

Similarly, the frequency of zero margin condition can be written as

$$f_m = \sum_L A_L f_G - \bar{A}_{Go} + \frac{1}{365} \sum_L n_L \bar{A}_G + (1-e)\, f_{Go} \quad \text{per day} \qquad (2.40)$$

where

A_L - availability of load L

$\bar{A}_{Go}$ - unavailability of generation during low load period

f_G - frequency of occurrence of generation deficiency to serve load L

f_{Go} - frequency of occurrence of generation deficiency during low load period

e - exposure factor.

In most practical cases, however, the risk during the low load period is very small compared to the peak load period, giving

$$f_{Go} \ll A_L f_G$$

and $$A_{Go} \ll A_G$$

Hence $$f_m \approx \sum_L A_L f_G + \frac{1}{365}(LOLP)$$

It can be shown that

$$A_L = e n_L$$

then

$$f_m \approx e\sum_L n_L f_G + \frac{1}{365}(LOLP)$$

for very small e

$$f_m \approx \frac{LOLP}{365} \quad \text{per day} \tag{2.41}$$

Therefore, on an annual basis, the LOLP index and the frequency index become equivalent.

2.5.2 Energy Indices

The shortcomings of the LOLP and F & D methods have been the reasons for the introduction of new reliability indices such as Interruption Duration Index (IDI)[35], Loss of Energy Index (LEI)[29], etc. Some of these defined as follows.

1. Interruption Duration Index $\triangleq \frac{E(UE)}{D}$ hours/year (2.42)

where

D - annual peak demand in MW

E(UE) - expected unserved energy in MWh

It is claimed[35] that this index is independent of the total energy generation, the effects of variation in the shape of the load curve and the number of consumers and their contribution to maximum demand.

2. Loss of Energy Index $\triangleq \frac{E(UE)}{E}$ (2.43)

where

E - annual energy demand in MWh

It has been said that[35] this index comes nearer than any other single index to assess the true reliability by reflecting the inconvenience and loss to the consumer. However, in the face of changing load curves, especially due to load factor improvement with time, this index become less meaningful[35]. On the other hand, as it assumes higher values for

more serious events than for marginal failures even if their probabilities and frequencies are the same, this provides an 'in depth' measure of reliability and has a more physical significance than the LOLP index[29]. It is also necessary to realise that the true loss of energy cannot be calculated on the basis of the cumulative curve of daily peak loads. It is usual to describe this as a percentage and is then referred to as the Percentage Energy Loss index[35].

These arguments therefore establish the significance of an energy index in addition to the other indices. Hence in these studies, the reliability evaluation has been based on the evaluation of LOLP and E(UE) indices.

2.5.3 Calculation of LOLP & E(UE) Indices

Since the generating plants are not perfectly reliable, those plants at the lower end of the loading order are limited to a load factor less than 100% and the plants higher up in the loading order must correspondingly generate more energy. The information such as the energy generated and the operating time of each plant can be obtained from the plant duration curve[44] which combines the availability of plant in the presence of load. This is essentially a chronological simulation of system operation and one of the great drawbacks of such deterministic simulation techniques is that they have difficulty in accounting for the use of the more expensive peaking units. The reason is that, for a reliable system a positive reserve margin of generation capacity over the expected peak demand is required. A deterministic simulation will always take for granted that the peaking units will never be operated, because by definition there will always be a cheaper unit available to run[15].

Information similar to that contained in the plant duration curve can be obtained by using a probability method to combine the load and unit availability probability distributions. This method is due to Baleriaux

and Jamoulle[45], later reviewed by Booth[44]. For this purpose an equivalent load (sometimes referred to as the combined load[46]) can be defined as

$$P_e \triangleq P_{ld} + P_{lr} + P_{od} + P_{or} \tag{2.44}$$

where

P_e - equivalent system load

P_{ld} - deterministic component of load

P_{lr} - random component of load

P_{od} - deterministic component of unit outages

P_{or} - random component of unit outages.

By definition, the equivalent load is a function of both demand and generation pattern of the power system. The inclusion of unit outages as a load component can be considered as the self demand of the system for installed capacity. The probability distribution of the equivalent load $F(P_e)$, may be obtained by the convolution of the probability distributions for the components of P_e. This function, shown in fig. 2.8, is called the equivalent load curve (sometimes the combined load duration curve (CLDC)[46]) and relates the combined demand for power to the expected duration of time in a given period during which the combined demand is equalled or exceeded. Therefore this function describes the probability that the combined demand for power exceeds a certain given capacity, for example, the installed capacity (IC). By definition this is the loss of load probability (LOLP).

$$\text{i.e. } LOLP = T \cdot F(P_e)\Big|_{P_e=IC} \tag{2.45}$$

where

T is the duration of period in hours.

The area under the equivalent load curve from P_e = IC to the maximum value of P_e represents the energy deficit of the system

$$\text{i.e. } E(UE) = T \int_{IC}^{\infty} F(P_e) \cdot d(P_e) \qquad (2.46)$$

It is important to realise that irrespective of the size of the system reserve margin, there is always a probability that demand may not be met and hence $E(UE)$ can never be zero although it may approach it asymptotically as further units are added on to the system[15].

The probability distribution relating to the load is specified by the load duration curve and that for the unit forced outages is obtained from a binomial expansion. Since it is possible to store these conveniently in the form of discrete distributions, a discrete convolution technique can be used. Another attractive feature of the use of this technique is the ease with which the equivalent load curve can be modified to calculate the energy generated by each unit of the system[5,44]. This is done by removing units from the system one at a time and obtaining the modified equivalent curve. For example, if the last unit of capacity X_n is removed and the new equivalent load curve is $F_{n-1}(P_e)$,

then

$$G_n = T\, a_n \int_{X_n}^{IC} F_{n-1}(P_e)\, d(P_e) \qquad (2.47)$$

where

G_n - the energy generated by unit n

a_n - availability of unit n.

2.6 Generation System Expansion Cost Analysis

The planning procedure based on the evaluation of reliability level generally produces several alternative expansion plans. It is the task of the system planner to indentify the one plan that produces the most favourable impact on the electricity supply industry as a whole. This involves the assessment of present values of future investments which, in turn, depend on the interest rates, availability of capital, etc. The production cost data required for this assessment is obtained using a simulation technique (Section 2.4) that portrays the way the units will most probaly be operated if the plan is implemented. In addition the decision will also be influenced to a great extent by social, political, enviromental and other such factors which are not possible to quantify in mathematical terms. In this respect, judgement based on the past experience plays a very important role.

2.6.1 Corporate Modelling in Generation Planning

Another approach which is becoming popular among system planners is the use of corporate models[8,47] in financial decision making. This, as the name implies, is a model of the financial structure of a utility which can be used to obtain the impact of a major expansion plan on its financial status. It can be seen from the simplified block diagram of the corporate model shown in fig. 2.9[8] that it is an open loop, multiple input, multiple output system. The major inputs of such a model are:-

i. energy demand forecasts

ii. construction costs for new plants, transmission facilities, etc., for each month

iii. operating costs for each month

iv. purchased energy costs (interconnected systems only)

v. costs associated with customer accounts, sales and administration.

Similarly the major outputs are:-

i. gross income or operating income

ii. retained earnings

iii. earnings per share

iv. outside capital requirement.

Although building a complete corporate model of a utility is not so easy, it is clear that such a planning tool is both convenient and necessary. From the output of the model the system planner can determine

a. how much outside capital is needed and where it can be obtained

b. whether the earnings are maintained at sufficient levels to make a substantial contribution to the self financing of future system expansions.

It should also be clear from fig. 2.9 that excessive interest charges without supporting operating revenue (a typical situation when plants are being constructed but not yet in operation,) can result in low earnings and perhaps no construction capital and hence no expansion. Low earnings can also be created by energy conservation minded consumers as well as by excessively high production costs due to supply and demand fluctuations in the international fuel market.

In fig. 2.9 the major operating expenses are production and purchased energy costs, followed by customer accounts, sales and administration costs. In addition, depreciation, taxes and maintenance are also expenses to be covered by gross income. However, depreciation is not a real expense, except for tax purposes. Therefore it is subtracted to obtain gross income but added again before outside capital requirements are determined.

2.6.2 Probabilistic Production Costing

It has been mentioned earlier that the optimisation model does not incorporate production costs explicitly in its formulation and hence a separate production cost analysis is required. It is also clear from the discussion on present worth analysis and corporate model studies that it is essential to determine the expected energy produced by each unit as well as associated production costs for a particular expansion plan. For these purposes a probabilistic simulation method based on the load energy curve[41] is used. In the simulation of nuclear and hydro (both conventional and pumped storage) plants, difficulties are encountered primarily because of their rigid refuelling and hydrological constraints.

The probabilistic models used to represent generating units and loads in production cost analysis are the same as those for reliability analysis and hence production calculations can be performed as part of the reliability analysis. This reduces the computing time considerably since the time consuming convolution techniques used in the determination of the probability distribution of the equivalent load are not repeated.

However, it should be emphasised that, unlike the case of reliability analysis in which the values of LOLP and $E(UE)$ are independent of the way units are convolved, in production cost analysis, the order in which the units are scheduled is important to obtain accurate production cost values. This order must be such that the corresponding production cost is a minimum. Such a simulation procedure will produce reasonably accurate energy generations, production costs and fuel inventories to be used in selecting the best expansion plan and negotiating fuel contracts.

The integer programming and probabilistic simulation techniques developed in this Chapter have been applied to the planning of generation facilities of a number of power systems. The results and the interactive experience are discussed in Chapter 3. These techniques have also been successfully applied in determining the firm interconnection capacity of a transmission link between two major generation areas[48].

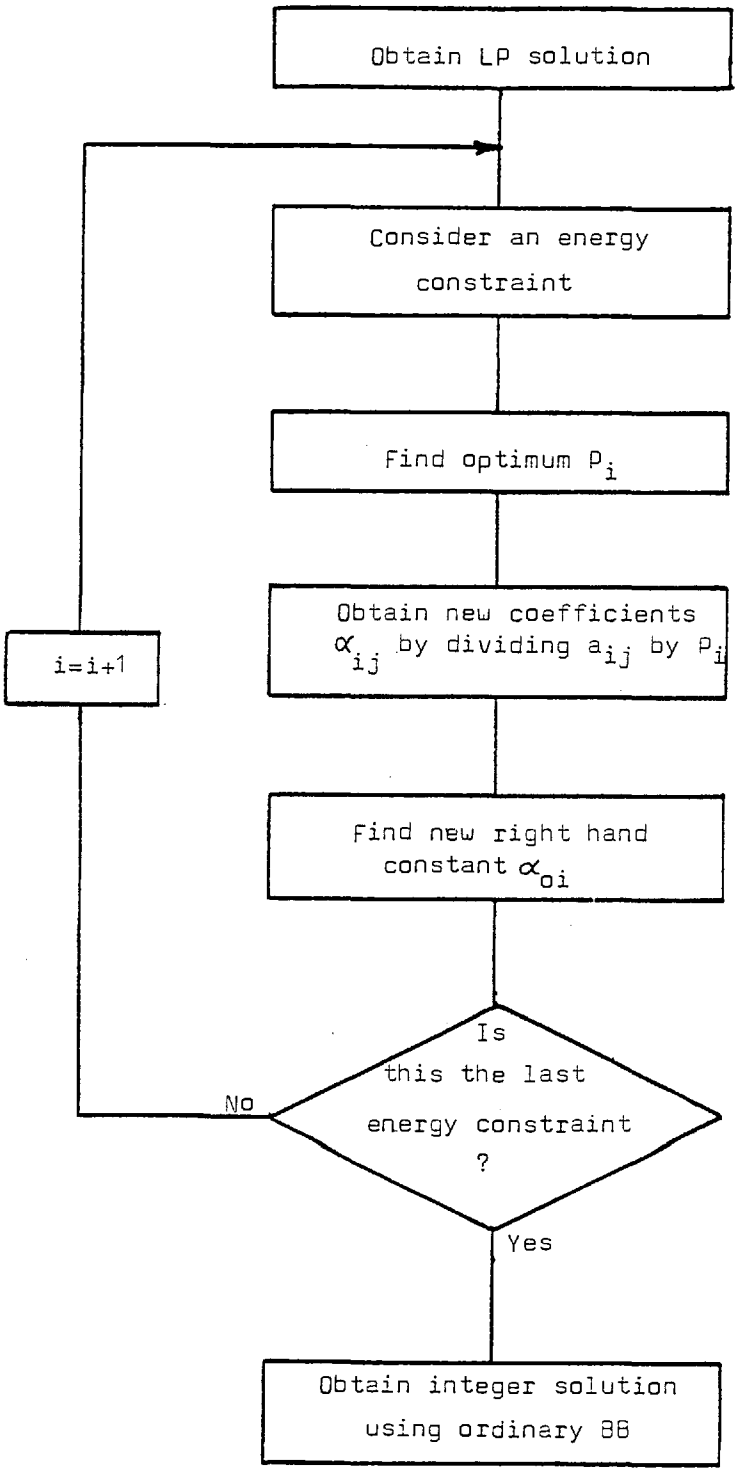

Fig. 2.1 Simplified Flow Chart for Modified BB

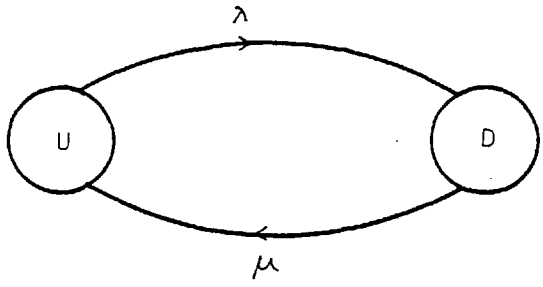

Fig 2.2 Generating Unit State Space Diagram

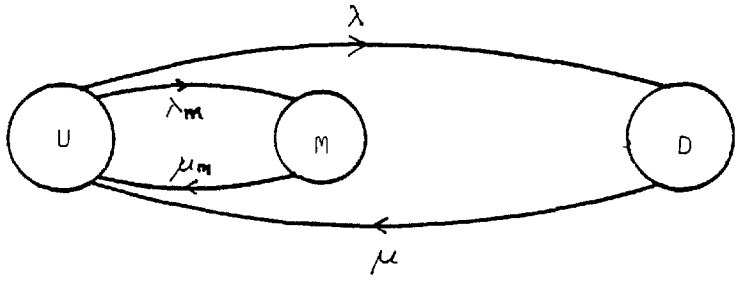

Fig. 2.3 Three State Model to Include Maintenance

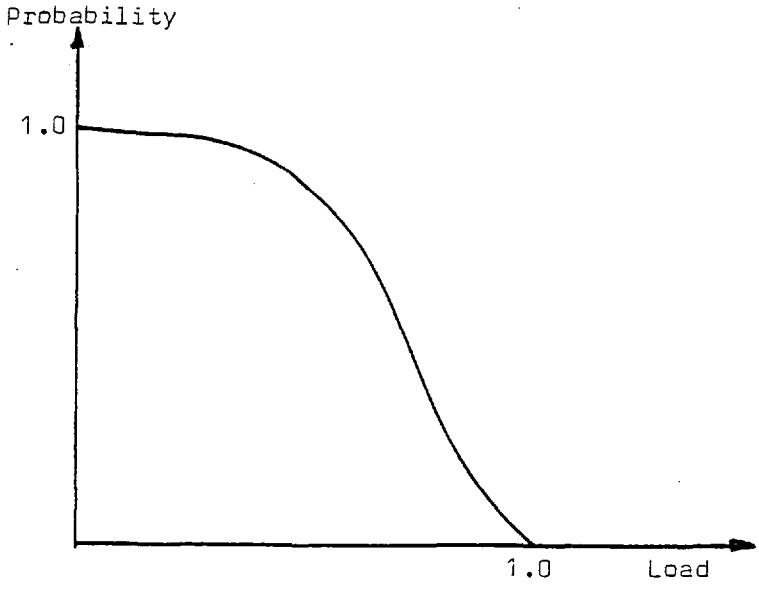

Fig. 2.4 Load Probability Distribution

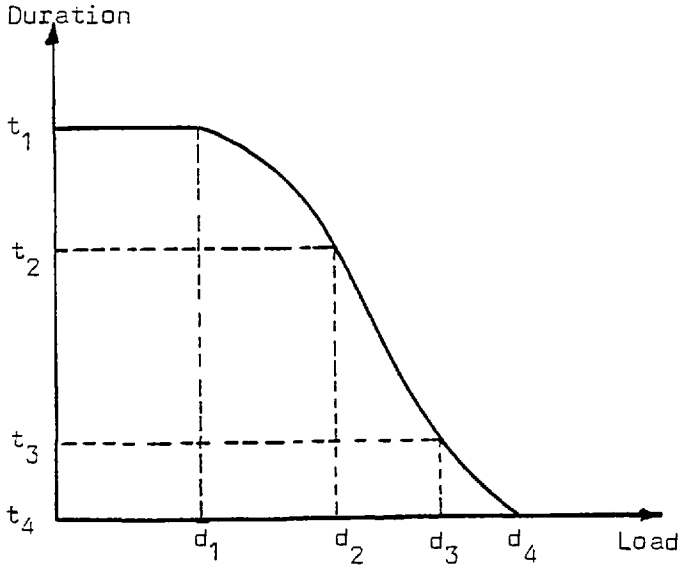

Fig. 2.5 Duration Load Curve

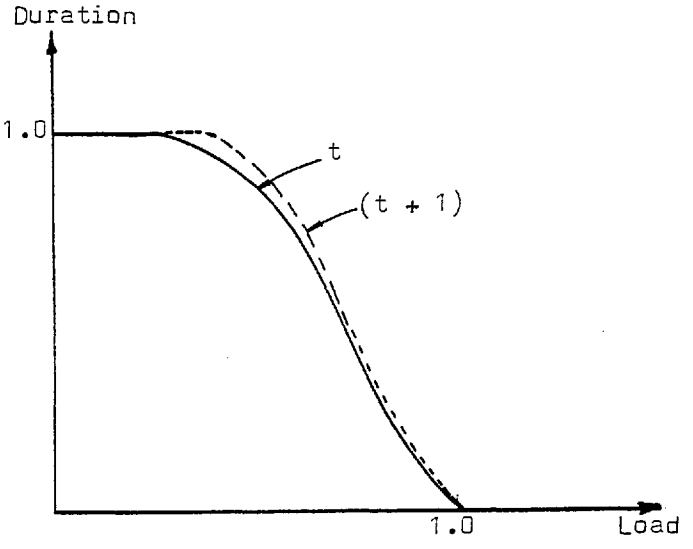

Fig. 2.6 Increase in Minimum Demand from Period t to t+1

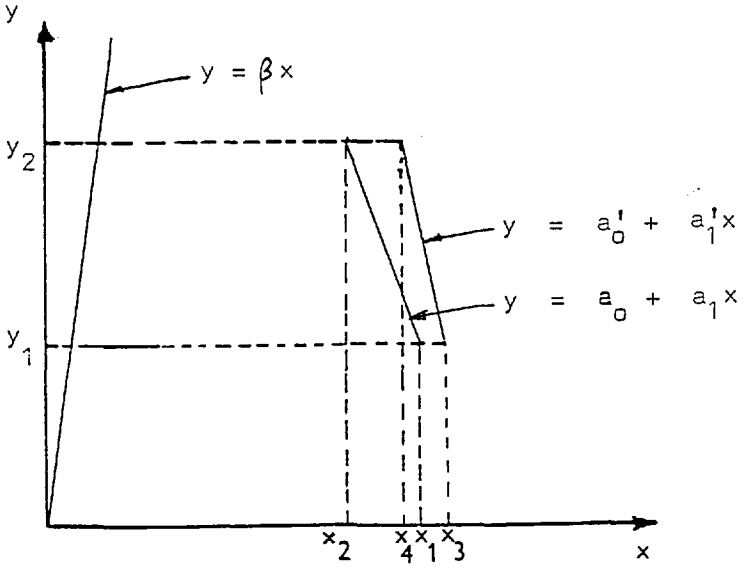

Fig. 2.7 Illustration of the Modification of Coefficients

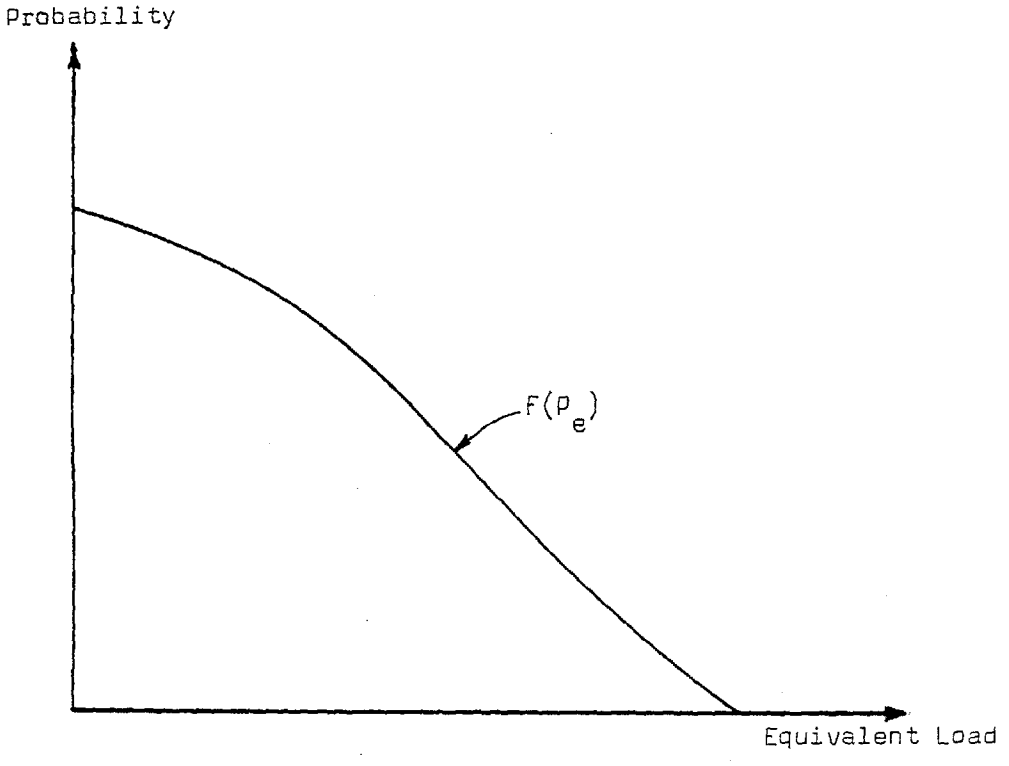

Fig. 2.8 Equivalent Load Curve

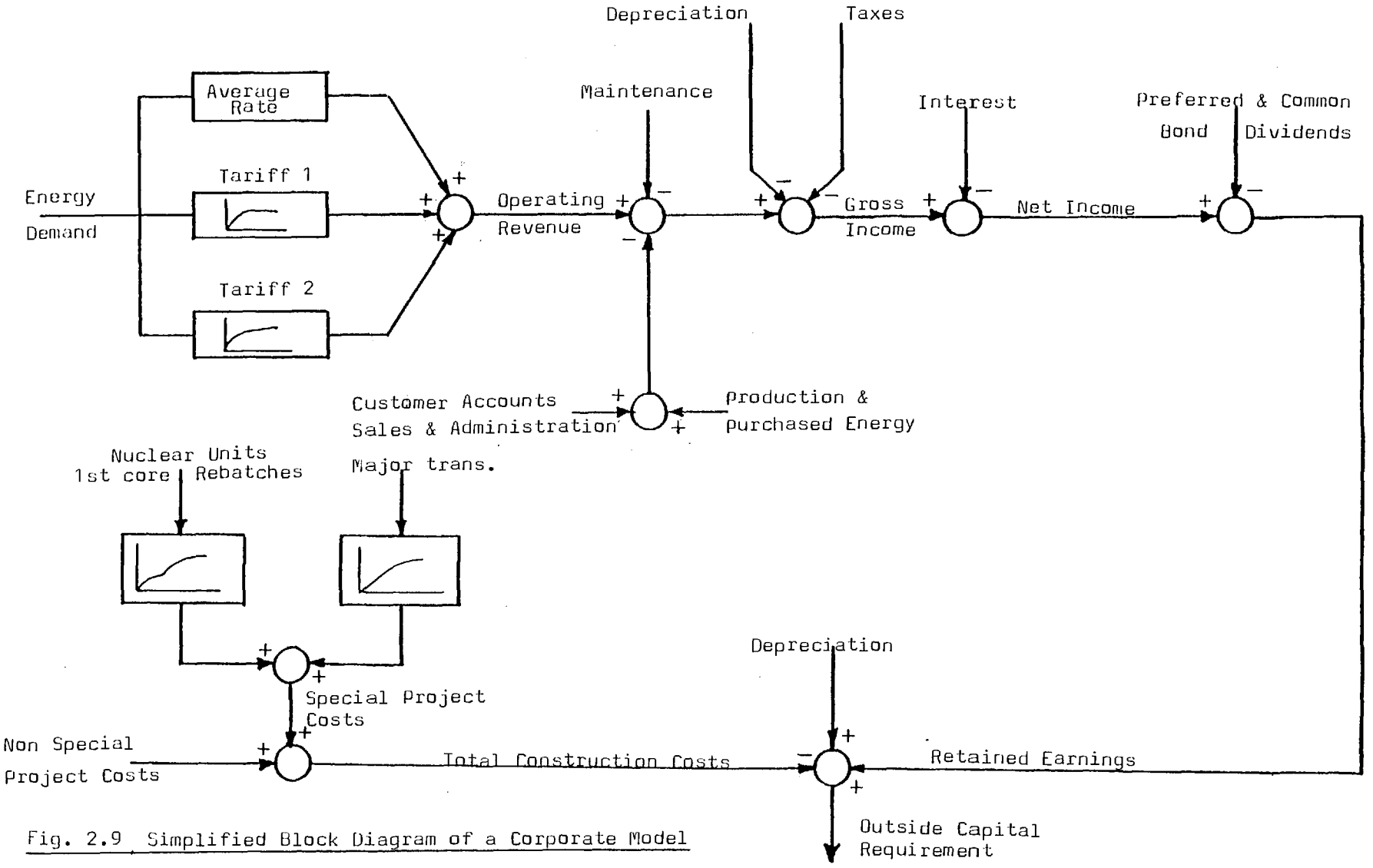

Fig. 2.9 Simplified Block Diagram of a Corporate Model

Table 2.1 Data and results for two typical systems

METHOD	(1) System Size		(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
	No. of Plants	No. of Years*	No. of Variables	No. of Constraints	No. of LP Iterations	Time for LP Solution **	Time for Mod. **	No.of Tails	Time for BB Solution **	Total time for integer Solution **	Reduction in Time **	% Reduction in Time
Ord. BB	8	4	32	36	51	0.783	-	2	4.796	4.796	-	-
Mod. BB	8	4	32	36	51	0.789	0.496	2	3.068	3.564	1.232	25.7
Ord. BB	11	8	88	96	115	4.677	-	5	140.889	140.889	-	-
Mod. BB	11	8	88	96	106	4.176	1.395	5	83.684	85.079	55.810	39.6

* equals no. of energy constraints in problem.

** The computing times are in CDC 6600 seconds.

CHAPTER 3

EXAMPLES & INTERACTIVE EXPERIENCE ON GENERATION PLANNING

The techniques developed for the solution of the long term generation planning problem (Chapter 2) have been applied to the planning of generating systems of varying characteristics, eg. plant mix, system size. This Chapter reports the results of some of these studies and discusses the interactive approach needed in deriving a feasible solution.

3.1 Example 1

This example[25] is concerned with the planning of generation facilities for a medium sized power system in a developing country from 1980 to 1987. In this particular country, as in many tropical countries, the year may be conveniently divided into a wet season and a dry season, each of six months duration.

Analysis of monthly maximum demand data showed a variation within ±10% of the trend, the maximum demand occuring in the dry season. Annual maximum demand was 250 MW in the first year of the study, rising to 510 MW in the eighth year, this growth being equivalent to an annual compound growth rate of over 10%. Separate load duration curves were obtained for each season using half hourly load data.

Plant existing at the first year of the study comprised a mixture of storage hydro, oil fired steam and gas turbine plant, their characteristics being shown in Table 3.1. The choice of new plant is between a hydro project having three dam sites, development of proven geothermal resources and extending an existing steam station. Gas turbines are also an option, especially for providing peaking and standby capacity.

Both hydro and geothermal plant have operating costs that are almost zero, but the seasonal variation in water availability and limitation in reservoir size mean that the energy available from hydro stations

may be limited, particularly during the dry season. It is technically and economically desirable that the geothermal plant be run almost continuously to supply the base load. In this system it is important to note that hydro stations tend to have maximum energy availability in the wet season when the demand for energy is low. However, hydro units are scheduled for maintenance in the dry season when their output is limited by water availability. Thermal and geothermal units are maintained in the wet season when demand is low and hydro energy relatively abundant. Scheduled maintenance accounts for the difference in availability between the wet and dry seasons.

All costs shown in Table 3.1 reflect the cost to the country of resources used in the construction and operation of the projects and in the analysis constant prices are used. A discount rate of 12% is used, which is typical for a country with low per capita income where capital is scarce. A separate cost benefit exercise has shown that the LOLP for generation outages should not rise above 100 hrs/year, although this level of system security is low in comparison to the standards used in developed countries. The demand forecast data is given in Table 3.2.

3.1.1 Interactive Solution Procedure

The solution procedure developed in Chapter 2 takes the form of an iteration between optimisation and probabilistic simulation. It is interesting to note that there is nearly always a tendency for convergence although convergence cannot be guaranteed when iterating between two non-linear routines.

3.1.1.1 Step 1: Optimisation

The initial plant factors assumed in the first optimisation step are shown in Table 3.3. For thermal and gas turbine plants these are obtained from the unit availability studies whereas for hydro plants

these are determined from the energy availability studies. In the first optimisation step a margin of 15% is assumed for power and energy reserves. These reserve margins allow planning of facilities in excess of system demand to have a reasonable reserve on the system to satisfy reliability requirements. It is important to note that as a result of inadequate system reserve capacity, infeasible plant factors can arise in the operation simulation of the generation system. As can be seen later, the interactive nature of the procedure (fig. 3.1) allows the planning engineer to alter these reserve margins to assess the minimum system reserve which would satisfy reliability requirements.

Fernando[5] has proposed a method of automatic adjustment of the reserve margins based on the desired and calculated LOLP and E(UE) respectively. However, as these modifications depend on two predefined constants which have to be supplied by the system planner, direct human interaction is considered to be more appropriate.

The results obtained in this step are shown in Table 3.4 which shows the capacities of various plants needed on the system in each year.

3.1.1.2 Step 2: Simulation

The estimated plant factors for the NEL plants obtained in the first simulation step is shown in Table 3.5. The plant factors for the hydro plants are the same as the initial values assumed (Section 2.3.2). From the LOLPs given in column (3) of Table 3.6, it can be seen that the system reserve is not sufficient to satisfy the required reliability standard of 100 hrs/year in most years. Therefore it is necessary to increase the power reserve margins before entering the next optimisation step. However, it is very important to make certain that there is adequate plant capacity in the proposed plant schedule to choose from, to

meet the extra capacity requirements. Otherwise it will be necessary to consider advancing the construction program of a particular plant (Chapter 2) to satisfy the capacity constraint.

Table 3.6 also gives the total installed capacities, LOLPs and E(UE)s obtained in the first three iterations. Although the generation structure obtained in the optimisation step of iteration 3 satisfies the reliability requirements, it is interesting to note the difference in overall costs obtained in the optimisation and simulation. This is basically due to the fact that in optimisation the costs are calculated using the assumed plant factors whereas in the simulation these are obtained using the actual plant factors. Therefore it is considered appropriate to iterate once more using the latest plant factors to obtain a new generation structure. The resulting generation structure is shown in Table 3.7 and it can be seen that the overall cost is less than the previous iteration. It is also clear from Table 3.7 that as the new plants are constructed, the operation of some of the old plants becomes expensive so that they may be conveniently retired towards the end of the planning period.

The expected plant factors from the simulation step of iteration 4 are shown in Table 3.8. It can be seen that as the new plants are constructed the existing thermal plants and gas turbine plants which are expensive to run are operated only as peaking plants. In addition to the reliability parameters the simulation routine produces the discounted expected operating costs for each plant in each year. These costs and the other results obtained in the simulation phase of iteration 4 are shown in Table 3.9.

In addition to the simulation of annual operation of the generation system given in Table 3.7, a seasonal simulation study was carried out using seasonal unit availability and demand forecast data. From the reliability indices shown in Table 3.10, it can be observed that the

system is more reliable in the wet season than in the dry season. This is largely due to the combined effect of high unit availabilities and plant factors in the wet season.

3.2 Examole 2

This example is concerned with the development of generation facilities for a South American mixed hydro-thermal system for the period from 1978 to 1985[49]. As a consequence of the industrial development and the many uses of electrical energy by the residential and commercial consumers in a rapidly developing economy, the annual growth rate experienced by this country is about 11%. In this study, it was agreed to satisfy a LOLP of 100 hrs/year in the critical period from 1978 - 81 and in the long term to reduce this to 48 hrs/year.

This system considered, consists of 87 existing generation units of different types and sizes and 25 proposed units for system expansion over the planning period of 8 years. These numbers were reduced to 8 and 3 respectively by grouping plant of similar characteristics and the resulting configuration is given in Table 3.11. One of the special features of this system is the large number of constraints on plant capacities in some years, primarily due to the lead times of construction schedules. As none of the existing plants could be extended, Table 3.12 only shows the limits for the proposed plants in each year. Another feature of this system is the use of expensive-to-run gas turbines at high plant factors due to the inadequate base load plant capacity in the system.

The resulting generating structure and the reliability indices after 4 iterations using the reserve margins given in Table 3.13 are shown in Taoles 3.14 and 3.15 respectively. In these calculations a discounting rate of 10% and an inflation rate of 2% have been assumed.

This example was expected to illustrate the effects of lead times of construction programs on the operation of the system. In obtaining the optimal generation structure the increase of reserve margins alone to satisfy reliability requirements would violate the capacity constraints and hence a gas turbine plant which has a short construction period has been proposed to be commissioned in 1979. The short lead time also allows this palnt capacity to be increased from 550 MW in 1979 to 950 MW in 1980 as required by the reliability standards.

From the results of Table 3.14 it can be observed that the most critical year is 1980 as indicated by a LOLP of 115 hrs/year. To satisfy this reliability level the capacity of gas turbine plant NEW must be increased by 400 MW. The construction of hydro plant GURI 2 in 1981 improves the reliability level considerably and the increase of capacity by 800 MW in 1982 allows the retirement of plant UNIT 6. These observations are very useful for the system planner in proposing alternative development strategies using his experience.

For example, if a lower reliability level can be tolerated in 1981, the capacity of gas turbine plant can be limited to 550 MW until the end of planning period, amounting to a considerable saving on the capital cost. On the other hand, if the 950 MW of gas turbine plant capacity is considered essential, then the addition of 800 MW at GURI 2 in 1982 can be delayed until the demand is sufficiently increased. A further alternative which stems up from these observations is that the existing steam plant UNIT 6, although not very reliable and expensive to operate and maintain, can be allowed to continue generation for a few more years delaying the construction of gas turbine plants. These are only a few examples which illustrates the usefulness of the results of optimisation and simulation in system planning.

3.3 Example 3

Based on the experience in the use of these techniques, it has been proposed in reference 49 that it is very desirable and economical to round off plant capacities such that each capacity is a small multiple of the lowest common multiple of integer unit size. This enables considerable savings to be made in computing times and costs associated in preliminary studies. The exact unit capacities can be used in the later stages when the solution begins to converge. This example is chosen to illustrate the effect of the choice of minimum unit size on the computing times.

All the data relevent to this system is available in reference 49. Two solutions obtained using 200 MW and 100 MW (corresponding to the unit sizes of plants NEW 1 and NEW 2) as the minimum unit size are shown in Tables 3.16 and 3.17 respectively. It is clear from these results that as the minimum unit size is increased, the plants having unit capacities lower than this are over-estimated and hence the total cost is much larger than the cost from the LP solution. However, since the number of combinations of plant capacities are less, the solution is obtained in a short time. When the minimum unit size is decreased, the number of combinations and the computing times are increased, but a more realistic planning schedule is obtained. This is indicated by a total cost which is not much different from the LP optimum cost. It can also be seen that, except for the difference in capacities, the two schedules are similar. This is mainly because there are only two proposed palnts in this example. In general, as there are many candidate plants, the planning schedules for different minimum unit sizes will be different.

However, during the initial stages of planning, these solutions are useful in estimating the reserve margins necessary to satisfy reliability requirements. Further, a certain amount of screening is possible by

removing plants which are unlikely to be in the optimum solution, from the proposed list, thereby reducing the number of variables. This screening process helps to obtain a final solution which is more representative of the most promising planning schedule and to achieve considerable savings of computing and engineering time and costs.

3.4 Conclusions

The results presented in this Chapter demonstrate the applicability of the techniques developed in Chapter 2 for the solution of the problem of long term generation planning. However, the successful application of these techniques depends considerably upon the interaction of the system planner between solution steps, in guiding the final solution to a feasible one. In this respect, some knowledge and experience of the system under consideration can be very helpful but useful results can still be obtained by "learning" during the computation process.

The examples have been specially chosen to highlight the effects of certain parameters such as reserve margins, minimum unit size, etc,, which could be altered interactively in deriving results. This also illustrates the flexibility of the use of optimisation and simulation with engineer interaction to yield realistic planting schedules.

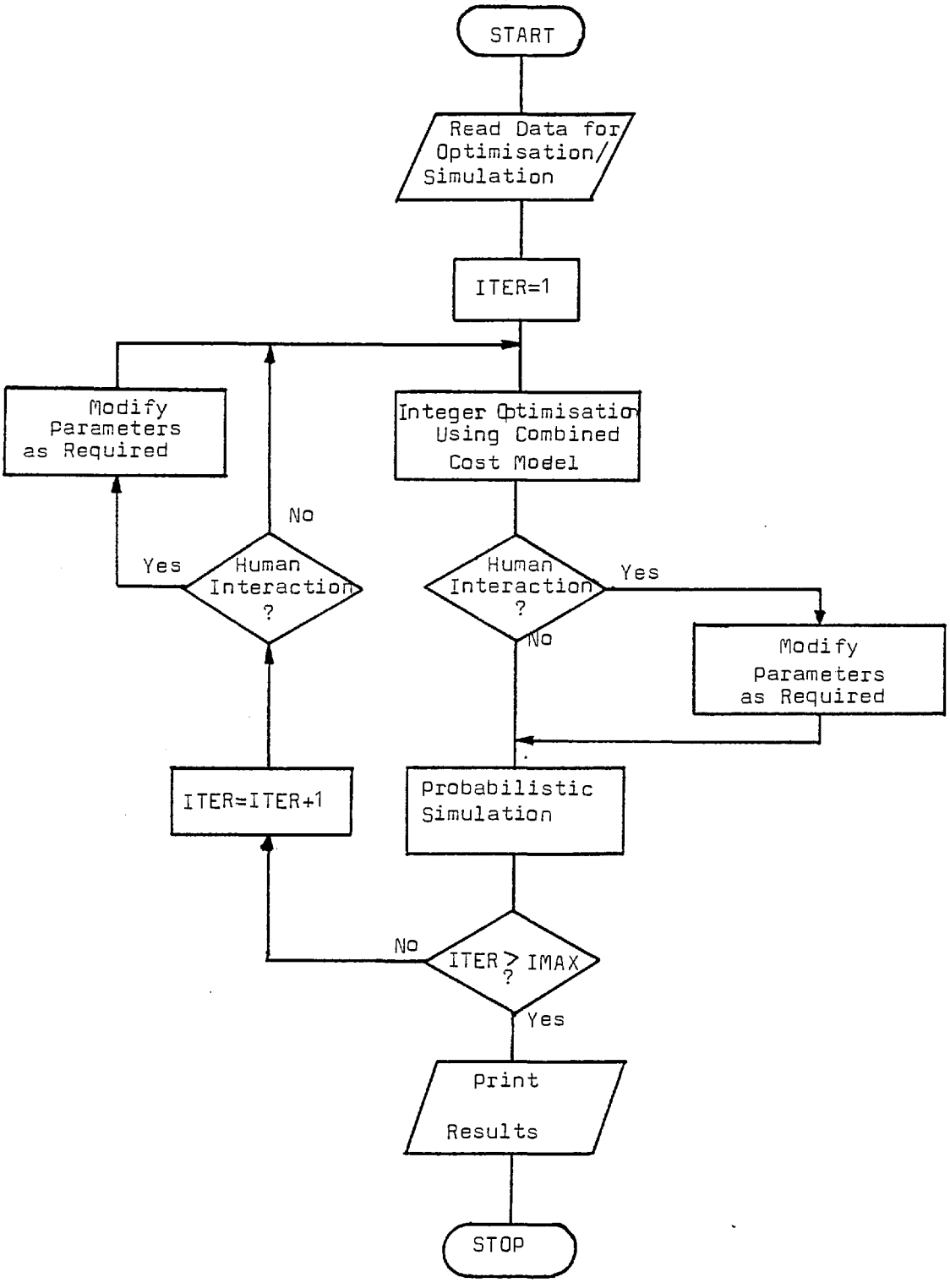

Fig. 3.1 Combined Optimisation and Simulation

Table 3.1 Basic plant data

	Plant Name	Capacity (MW)	Unit Availability (%)			Maximum Load Factor (%)			Operating Cost ($/MWh)	Fixed Cost ($/MW/yr)	Capital Cost ($/kW)	Accounting Life (Yrs)
			Annual	Wet Season	Dry Season	Annual	Wet Season	Dry Season				
Existing Plant	HYDE 1	2 x 25	95	99	91	52	55	49	0	7360	-	40
	HYDE 2	2 x 25	95	99	91	43	46	40	0	7360	-	40
	THME 1	2 x 15	88	86	90	88	86	90	20.0	7570	-	25
	THME2	1 x 45	88	86	91	88	86	91	20.0	7325	-	25
	THME 3	2 x 50	90	85	95	90	85	95	18.4	7960	-	25
	GASTE	3 x 15	86	83	89	87	85	89	48.6	8400	-	15
New Plant	HYDN1	2 x 55	94	99	88	33	40	26	0	4120	1517	40
	HYDN2	2 x 35	94	99	90	52	63	41	0	6990	1037	40
	HYDN3	2 x 25	94	99	90	45	48	42	0	8380	1246	40
	GEOTH	1 x 50	85	82	89	86	82	89	0	7130	1115	30
	THMN1	2 x 50	90	85	95	90	85	95	18.4	7960	402	25
	GASTN	2 x 20	82	80	85	82	80	85	38.6	8400	189	15

Table 3.2 Load forecast data

Year	Annual		Wet Season		Dry Season	
	Maximum Demand (MW)	Energy sent out (GWh)	Maximum Demand (MW)	Energy sent out (GWh)	Maximum Demand (MW)	Energy sent out (GWh)
1980	250	1416.0	233	651.4	250	764.6
1981	280	1580.0	261	726.8	280	853.2
1982	311	1754.0	291	806.8	311	947.2
1983	345	1938.3	322	891.6	345	1046.7
1984	380	2132.0	355	980.7	380	1151.3
1985	420	2345.2	392	1078.8	420	1266.4
1986	462	2579.8	432	1186.7	462	1393.1
1987	510	2838.0	477	1305.5	510	1532.5

Table 3.3 Initial plant factors - 8 planning periods

1980 - 87 assumed for iteration 1

THME3	THMN1	THME2	THME1	GASTN	GASTE	HYDE1	HYDE2	HYDN1	HYDN2	HYDN3	GEOTH
0.90	0.90	0.88	0.88	0.82	0.87	0.52	0.43	0.33	0.52	0.45	0.86

Table 3.4 Generation structure obtained from iteration 1 Step 1: Optimisation

Year	THME3	THMN1	THME2	THME1	GASTN	GASTE	HYDE1	HYDE2	HYDN1	HYDN2	HYDN3	GEOTH
1980	100	-	45	30	-	15	50	50	-	-	25	-
1981	100	-	45	30	-	15	50	50	-	35	25	-
1982	100	-	45	30	-	15	50	50	-	35	25	-
1983	100	-	45	30	-	15	50	50	-	70	25	-
1984	100	-	45	30	-	15	50	50	-	70	25	50
1985	100	-	45	30	-	15	50	50	55	70	25	50
1986	100	-	45	30	-	15	50	50	55	70	50	50
1987	100	50	45	15	20	0	50	50	55	70	50	50

Table 3.5 Estimated plant factors for NEL plants from iteration 1

Step 2: Simulation

Year	THME3	THMN1	THME2	THME1	GASTN	GASTE
1980	0.6203		0.8920	0.0102		0.0213
1981	0.6272		0.8920	0.0076		0.0170
1982	0.8136		0.8920	0.0237		0.0502
1983	0.8443		0.8920	0.0205		0.0429
1984	0.6389		0.8920	0.0167		0.0301
1985	0.7057		0.8920	0.0096		0.0186
1986	0.8568		0.8920	0.0153		0.0277
1987	0.8946	0.5137	0.8920	0.0169	0.0239	0.0

The plant factors for the EL plants are the same as the initial values assumed.

Table 3.6 Results from iterations 1,2 & 3

Year	Iteration 1			Iteration 2			Iteration 3		
	In-stalled capacity (MW)	LOLP (hrs/yr) (3)	E(UE) (GWh)	In-stalled capacity (MW)	LOLP (hrs/yr)	E(UE) (GWh)	In-stalled capacity (MW)	LOLP (hrs/yr)	E(UE) (GWh)
1980	315	58.6	0.897	370	10.1	0.144	320	52.9	0.805
1981	350	44.7	0.674	370	37.9	0,616	370	37.9	0.616
1982	350	154.4	2.772	370	124.9	2.352	390	64.6	1.119
1983	385	139.4	2.492	390	209.7	4.335	415	84.3	1.516
1984	435	113.8	2.140	475	41.0	0.719	450	79.9	1.432
1985	490	68.4	1.270	475	154.8	3.240	500	66.1	1.217
1986	515	113.8	2.245	520	124.6	2.542	545	54.0	0.962
1987	555	162.4	3.568	555	190.3	4.357	580	90.3	1.815
C_o	292,696,640			224,660,380			231,386,910		
C_s	217,630,000			206,630,000			198,230,000		

C_o = Overall cost from optimisation ($)

C_s = Overall cost from simulation ($)

Table 3.7 Generation structure obtained from iteration 4 Step 1: Optimisation

Year	THME3	THMN1	THME2	THME1	GASTN	GASTE	HYDE1	HYDE2	HYDN1	HYDN2	HYDN3	GEOTH
1980	100	-	45	30	-	45	50	50	-	-	-	-
1981	100	-	45	30	-	45	50	50	-	-	-	50
1982	100	-	45	30	-	45	50	50	-	35	-	50
1983	100	-	45	30	20	45	50	50	-	35	-	50
1984	100	-	45	30	20	30	50	50	-	70	-	50
1985	100	50	45	30	20	30	50	50	-	70	-	50
1986	100	50	45	30	40	15	50	50	-	70	50	50
1987	100	100	45	30	40	15	50	50	-	70	50	50

Overall cost = M$ 212.29

Table 3.8 Expected plant factors for NEL plants from iteration 4

Year	THME 3	THMN 1	THME 2	THME 1	GASTN	GASTE
1980	0.8894	-	0.5121	0.0504	-	0.0124
1981	0.8468	-	0.0854	0.0293	-	0.0086
1982	0.8738	-	0.0685	0.0234	-	0.0068
1983	0.9000	-	0.4322	0.0646	0.0271	0.0116
1984	0.9000	-	0.5262	0.0572	0.0248	0.0131
1985	0.9000	0.8386	0.1506	0.0417	0.0195	0.0101
1986	0.9000	0.9000	0.1918	0.0296	0.0103	0.0047
1987	0.9000	0.8017	0.0653	0.0288	0.0103	0.0050

The plant factors for the EL plants are the same as the initial values assumed.

Table 3.9 Results from the simulation in iteration 4

Year	Maximum Power Demand (MW)	Energy Demand (GWh)	Installed Capacity (MW)	LOLP (hrs/yr)	E(UE) (GWh)	Expected Operating Costs (M$)						
						THME3	THMN1	THME2	THME1	GASTN	GASTE	TOTAL
1980	250	1416.0	320	52.9	0.805	14.34	-	4.04	0.26	-	0.24	18.88
1981	280	1580.0	370	37.9	0.616	12.43	-	0.61	0.14	-	0.15	13.33
1982	311	1754.0	405	30.8	0.498	11.63	-	0.45	0.10	-	0.11	12.34
1983	345	1938.3	425	59.0	1.012	10.96	-	2.57	0.26	0.14	0.17	14.10
1984	380	2132.0	445	93.4	1.734	9.98	-	2.85	0.21	0.12	0.11	13.27
1985	420	2345.0	495	76.8	1.454	9.09	4.23	0.74	0.14	0.08	0.08	14.36
1986	462	2579.8	550	48.3	0.849	8.28	4.14	0.86	0.09	0.08	0.02	13.47
1987	510	2838.0	600	52.8	0.967	7.54	6.71	0.27	0.08	0.07	0.02	14.69

Optimum Overall Cost = M$ 198.19

The operating costs for the EL plants are zero

Table 3.10 Results from the seasonal simulation study

Year	Wet seasons		Dry seasons	
	LOLP (hrs/season)	E(UE)(GWh)	LOLP (hrs/season)	E(UE)(GWh)
1980	16.1	0.214	22.7	0.333
1981	13.1	0.212	14.8	0.227
1982	7.8	0.129	14.4	0.228
1983	14.7	0.209	29.2	0.493
1984	18.8	0.264	57.2	1.064
1985	17.1	0.253	39.6	0.730
1986	7.7	0.127	28.2	0.505
1987	9.8	0.142	26.9	0.489

Table 3.11 Basic plant data

	Plant Name	Plant Type	Unit Capacity (MW)	Capital Cost (£/kW)	Fixed Cost (£/Mw/yr)	Operating Cost (£/MWh)	Availability (%)	Accounting Life (Yrs)
Existing Plant	UNIT 3	STEAM	1200	-	4225.00	1.40	89.0	25
	UNIT 6	GAS	950	-	4683.70	2.52	82.6	20
	UNIT 2	STEAM	250	-	10729.30	1.9	99.916	25
	UNIT 1	STEAM	400	-	9362.66	2.0	99.916	25
	UNIT 4	GAS	250	-	4519.69	2.67	83.0	20
	UNIT 5	GAS	100	-	4666.00	1.6	82.0	20
	GURI 1	HYDRO	1900	-	3970.00	-	99.986	30
	MACAG	HYDRO	350	-	8500.00	-	99.987	30
Proposed Plant	TACEN	STEAM	400	1437.83	3250.00	1.40	89.0	25
	GURI 2	HYDRO	200	766.41	2300.00	-	99.98	30
	NEW	GAS	50	2090.00	3125.00	2.7	99.8	20

Table 3.12 Maximum plant capacities in each year (Proposed plants only)

Plant name	1978	1979	1980	1981	1982	1983	1984	1985
TACEN	400	800	800	800	800	800	800	800
GURI 2	-	-	-	1800	3400	4800	4800	4800
NEW	-	550	1000	1000	1000	1000	1000	1000

Table 3.13 Reserve margins

Year	Reserve margin (%)
1978	35
1979	38
1980	30
1981	35
1982	20
1983	20
1984	20
1985	20

Table 3.14 Generation structure from iteration 4

Plant Name	Year							
	1978	1979	1980	1981	1982	1983	1984	1985
UNIT 3	1200	1200	1200	1200	1200	1200	1200	1200
UNIT 6	950	950	950	950	-	-	-	-
TACEN	400	800	800	800	800	800	800	800
UNIT 2	250	250	250	250	250	250	250	250
UNIT 1	400	400	400	-	-	-	-	-
UNIT 4	250	250	250	-	-	-	-	-
UNIT 5	100	100	100	-	-	-	-	-
NEW	-	550	950	950	950	950	950	1000
GURI 1	1900	1900	1900	1900	1900	1900	1900	1900
MACAG	350	350	350	350	350	350	350	350
GURI 2	-	-	-	1800	2600	3400	4000	4800
Total Installed Capacity (MW)	5800	6750	7150	8200	8050	8850	9450	10300
Demand (MW)	4038	4857	5490	6073	6630	7228	7867	8546
LOLP (hrs/yr)	79.3	68.9	115.4	31.0	64.4	35.7	37.1	22.9
E(UE) (GWh)	25.7	22.6	40.2	9.7	19.4	10.7	10.9	6.8

Overall Cost from Optimisation M£ 595.2

Overall cost from Simulation M£ 586.7

Table 3.15 Plant factors from simulation

Year	Plant Name										
	UNIT 3	UNIT 6	TACEN	UNIT 2	UNIT 1	UNIT 4	UNIT 5	NEW	GURI 1	MACAG	GURI 2
1978	0.6539	0.2802	0.0589	0.0331	0.0153	0.0085	0.0060	-	0.7000	0.7000	-
1979	0.6539	0.7310	0.1274	0.0633	0.0363	0.0184	0.0133	0.0084	0.7000	0.7000	-
1980	0.6539	0.9204	0.2735	0.1538	0.0942	0.0533	0.0403	0.0201	0.7000	0.7000	0.7000
1981	0.6539	0.3494	0.0418	0.0170	-	-	-	0.0056	0.7000	0.7000	0.7000
1982	0.2967	-	0.7352	0.0332	-	-	-	0.0123	0.7000	0.7000	0.7000
1983	0.2967	-	0.5320	0.0210	-	-	-	0.0074	0.7000	0.7000	0.7000
1984	0.2967	-	0.5354	0.0206	-	-	-	0.0074	0.7000	0.7000	0.7000
1985	0.2967	-	0.3840	0.0150	-	-	-	0.0051	0.7000	0.7000	0.7000

Table 3.16 Solution 1

Integral unit size = 200 MW

Solution from the 6th tail

Period	OLD 1	OLD 2	OLD 3	OLD 4	NEW 1	NEW 2
1	300	300	200	300	800	-
2	300	300	200	300	1000	-
3	300	300	200	300	1000	600
4	300	300	200	300	1000	1200

CP time for execution = 42.98 sec

IP cost = £ 101,558,885.43

Table 3.17 Solution 2

Integral unit size = 100 MW

Solution from the 11th tail

Period	OLD 1	OLD 2	OLD 3	OLD 4	NEW 1	NEW 2
1	300	300	200	300	800	-
2	300	300	200	300	1000	-
3	300	300	200	300	1000	500
4	300	300	200	300	1000	1100

CP time for execution = 136.66 sec.

IP cost = £ 98,478,124.01

LP optimum cost = £ 96,021,003.37

CHAPTER 4

RESERVOIR OPERATION & CONTROL USING STOCHASTIC DYNAMIC PROGRAMMING

4.1 Introduction

The optimal operation of a long range hydro storage reservoir in a mixed hydro-thermal system is complicated due to the sequential nature of the decision process and the many constraints imposed on the control of water discharge. These constraints arise from a variety of requirements including electric power generation, regulation of river flows in flood control, navigation and irrigation. The decisions are also affected to a great extent by the random nature of the water inflow to the reservoir.

Depending on the system requirements, the objective function can be formulated in many different ways. In a multipurpose hydro scheme, the water has to be used primarily for power generation and irrigation. Although the monetary returns associated with irrigation and navigation are difficult to quantify, the objective is to maximise the net economic return to the national economy as a whole. However, from the point of view of power industry, the hydro power with zero marginal cost can be used not only to satisfy firm load commitments, but also to displace other firm resources or to serve secondary loads. In this repect, the minimisation of energy imports in an interconnected system is an important aspect. Therefore, in formulating the objective in a particular system, the existence of various types of loads has to be identified and the objective function must be based on a proper market structure.

In this formulation, certain simplifying assumptions can be included. For example, in Sri Lanka, the water from multipurpose hydro schemes are used only for electricity generation and irrigation and not for

navigation. Therefore from a national point of view, the objective function will include these two aspects. Further, in most of the existing hydro schemes, the irrigation demands are met by providing a certain minimum downstream water flow. Thus, from the point of view of the economy to the electric power industry, the objective is mainly the optimum use of water for power generation while satisfying a minimum downstream water flow.

The reservoir operation problem can be divided into two, namely, short range and long range operation. The short range problem is concerned with the operation during a short interval, for eg. a week. In this, the variations in reservoir elevations and plant efficiencies can be assumed to have small net effect on the optimum operation. Therefore the objective here is the allocation of load among the hydro and thermal plants, so as to minimise the total fuel cost over a short future time[50].

However, in the long range problem, the current operating decisions have an appreciable influence on long range economy due to cumulative changes in storage elevations and plant efficiencies. Thus, the generation is determined by a combination of factors including reservoir elevation, natural inflows and water discharges. Therefore the objective is the optimum use of water to generate power so as to minimise the operation costs over a long future time interval. The long range optimisation problem is important as the amount of water allocated in the short range operation is determined from this.

The most important requirement of the optimisation procedure is an ability to produce the schedule of water discharge for electric power generation in the present period, and the amount to be stored for the next period, when the storage at the beginning of the period and the power and energy demands are given[9]. As these decisions depend on the

water inflows to the reservoir which usually shows a high variability, the capacity to handle this aspect is an advantage.

The problem of optimisation of water resources has been formulated with many different objective functions[51,52,53,54,55] and both mathematical programming[51,52,53,54,55] and analytical techniques[50,56] have been used. The inflows have been considered as deterministic[56] and stochastic[51,52,53,54]. It has been claimed[51,54] that inspite of the limitations such as large computer time and storage requirements, dynamic programming[57,58] is still one of the most powerful techniques available for water resources systems. Therefore many different formulations of dynamic programming, including forward dynamic programming[54], successive approximations[54], iteration in policy space[54], approximate discrete differential dynamic programming[51] and stochastic dynamic programming[52,53] have been reported.

In the method proposed in this Chapter, the problem is first formulated assuming a deterministic hydrology. The sequential nature of the decision process is dealt with by the application of dynamic programming techniques. Since the initial storage of the reservoir during the first period is not known in advance, the problem is formulated using backward recursion and hence a number of optimal control policies can be obtained for a given range of initial storage values.

The formulation is then extended to include the random variations of the water inflows using stochastic dynamic programming technique. All variables are assumed to be deterministic except the inflow variable, which is represented by its probability distribution.

The algorithm has been applied to a hypothetical system with one reservoir and a hydro plant and a group of base load and peaking thermal plants.

4.2 Formulation of the Problem

The application of optimisation techniques to determine the operation policy of a hydro-thermal system requires a suitable model which adequately represents the reservoir dynamics and the various types of loads[52]. In the formulation presented, it is assumed that the network has only a primary load which is required to serve at all times. To satisfy the demand, power can be drawn from hydro plants and thermal plants. Interconnection facilities are not considered and hence the aspects of energy imports and exports are absent.

4.2.1 The Objective Function

The objective is to obtain the hydro operation schedule which minimises the expected operating costs of the system over a given period of time and guarantees a terminal value for water remaining in the reservoir at the end of the planning horizon, for a given initial reservoir storage capacity. The terminal value for the reservoir capacity is important to ensure that the operating policy does not drawdown the reservoir completely at the end of the planning period.

Assuming zero cost for hydro power generation, the cost C_T associated with meeting a given energy demand can be written as

$$C_T = \sum_{n=1}^{N} (E_b C_b + E_p C_p)_n \qquad (4.1)$$

where

E_b - energy produced by base load thermal plants

E_p - energy produced by peak load thermal plants

C_b - operation cost of base load thermal plants

C_p - operation cost of peak load thermal plants

N - number of periods in the study

Therefore the objective is to minimise the cost of eqn.(4.1) subject to the conditions and constraints imposed by the hydro dynamics of the reservoir, the load demand characteristics of the electrical power system and the capacity constraints of the power plants.

4.2.2 Hydro Operating Policy

The position of the hydro plant in the dispatching schedule determines the amount of energy produced by it and the energy generations from the thermal plants. If the hydro energy available in any period is E_n, this has to be utilised completely in order to minimise fuel cost of thermal plants. Further it must be operated at times of peak demand when the peaking plants are operating and hence the fuel costs on the system are most expensive. Considering the variations in demand between weekdays and weekends in a typical power system, it has been argued[9,10] that, for the optimum benefits, the hydro plant need to be operated at the same place in the dispatching schedule on both week and weekend days.

Therefore the operation of the hydro plant is fixed by its capacity X_n and the available hydro energy E_n (fig. 4.1). The optimality condition can be written as

$$\begin{aligned} e_2 - e_1 &= E_n \\ p_2 - p_1 &= X_n \end{aligned} \qquad (4.2)$$

This optimal operation will ensure that no potential energy from the hydro plant is wasted and that at no time it is operated above its maximum available capacity. However, the available energy E_n for power generation during a particular period depends on the initial storage capacity, water inflow, maximum permissible capacity and the down stream water requirements during that period.

This definition of the hydro operating policy is very convenient in the optimisation problem as the energy requirements from the thermal plants are easily obtained from the load energy curve[41] as illustrated in fig. 4.2. If there are more than one hydro plant, the single plant represents the aggregate characteristics of all the hydro plants[9].

4.2.3 Hydro Reservoir Model

Neglecting evaporation and seepage losses, the difference equation describing the conservation of water can be written as

$$S_{n+1} = S_n + F_n - Q_n - R_n \qquad (4.3)$$

where

S_n - reservoir storage at the beginning of period n

F_n - inflow to the reservoir during period n

Q_n - controllable discharge from the reservoir during period n

R_n - spillage of the reservoir during period n

All the variables in eqn. (4.3) are subject to non-negativity constraints and some are bounded as follows:

i. Inflow constraint

$$0 \leq F_n \qquad (4.4)$$

ii. Discharge constraint

$$0 \leq \underline{Q}_n \leq Q_n \leq \overline{Q}_n \qquad (4.5)$$

The minimum discharge $\underline{Q}_n$ ensures that there is always enough water downstream for irrigation and navigation purposes. The maximum discharge $\overline{Q}_n$ is obtained by considering the possibility of flood in the period due to unexpected rain fall or melting of snowpacks.

iii. Storage constraint

$$0 \leq \underline{S}_{n+1} \leq S_{n+1} \leq \overline{S}_{n+1} \qquad (4.6)$$

where

$\underline{S}_{n+1}$ - minimum storage of the reservoir at the beginning of

period (n + 1) excluding the inflow to the reservoir during period (n + 1). This will ensure sufficient energy availability for power generation even if the actual inflow during period (n + 1) is less than the anticipated inflow.

and $\bar{S}_{n+1}$ - maximum storage of the reservoir at the beginning of period (n + 1) including the inflow to the reservoir during period (n + 1). This ensures that there is sufficient storage capacity to absorb any unexpected inflow due to rainfall or snow melting in the catchment area, thus minimising the risk of flooding.

4.2.4 Hydro Electric Generation

In a given period n, the available hydro plant capacity is specified after deducting the capacities of the units on maintenance (if there are any) from the total installed capacity.

The energy available during period n can be expressed as a function of average head and discharge[51,56,59]. In reference 56, it is assumed that the head of the reservoir is proportional to the storage. However, in an actual reservoir, the relationship between the head and the storage need to be expressed by a higher order polynomial for more accurate results. A typical curve[60] is shown in fig. 4.3 and the parabolic shape confirms the property

$$H \propto \sqrt{S}$$

where H - head

and S - storage

as used in reference 51.

The amount of electrical energy produced from the available hydro energy depends on the conversion efficiency of the turbines and generators.

This in turn depends on the gate openings and the tail race water levels[60]. Therefore, including the turbine-generator efficiency η, the energy generated during period n can be written as

$$E_{gn} = 2.7 \times 10^{-6} Q_n H_n \eta \qquad \text{MWh} \tag{4.7}$$

where Q_n - the discharge in m^3 during period n

H_n - average head in m during period n.

4.3 Computational Procedure

The sequential nature of the decision process is dealt with by solving the functional equation described below using dynamic programming (DP) techniques. This allows the decomposition of the N stage optimisation problem described by eqn. (4.1) into N number of single stage optimisation problems. Since the storage level of the first period is not known in advance and the terminal value of storage capacity in the last period is specified, the problem is formulated as an initial value[57,58] problem and the resulting recursion equation is solved in the backward direction.

4.3.1 Functional Equation

The functional equation for the n^{th} stage can be written as

$$f_n(S_n) = \min_{Q_n} \left[(E_b C_b + E_p C_p)_n + f_{n-1}(S_{n-1}) \right] \tag{4.8}$$

with $f_o(S_o) = 0$ (4.9)

where $f_n(S_n)$ is the minimum cost at stage n

$$S_{n-1} = T_n(S_n , Q_n) \tag{4.10}$$

and T_n is the stage transformation function as given by eqn. (4.3).

4.3.2 Evaluation of the Functional Equation

In order to evaluate eqn. (4.8), it is necessary to obtain the energy produced by the base load and peak load plants respectively, for a given discharge Q_n, satisfying the optimality conditions described by eqn. (4.2) and the load demand of the system. However, the optimal position of the hydro plant in the dispatching schedule is determined by its available capacity and the available hydro energy. Thus there are 3 possible cases to be considered.

4.3.2.1 Hydro Plant Operating in the Peak Load Region

In this case the base load is entirely met by base load thermal plants and the hydro energy is used to displace the expensive peaking plants. The equations describing this situation are:

$$\begin{aligned} p_2 - p_1 &= X_n \\ e_2 - e_1 &= E_{gn} \\ E_p &= (1-e_2) + e_1 - \Phi(X_b) \\ E_b &= \Phi(X_b) \end{aligned} \tag{4.11}$$

where $e = \Phi(x)$ is the higher order polynomial representation of the load energy curve for period n.

4.3.2.2 Hydro Plant Operating in the Base Load Region

The energy availability and the capacity of the hydro plant permit the operation in the base load region thus displacing both the base load and peaking thermal plants. The equations describing this case are:

$$\begin{aligned} p_2 - p_1 &= X_n \\ e_2 - e_1 &= E_{gn} \\ E_p &= 1 - \Phi(X_b + X_n) \\ E_b &= \Phi(X_b + X_n) - E_{gn} \end{aligned} \tag{4.12}$$

4.3.2.3 Hydro Plant Operating in the Base Load & Peak Load Regions

In this case the optimality condition places the hydro plant in a position to displace part of the base load thermal and peaking plants. This is characterised by:

$$\begin{aligned} P_2 - P_1 &= X_n \\ e_2 - e_1 &= E_{gn} \\ E_p &= 1 - e_2 \\ E_b &= 1 - e_1 \\ X_b &= P_1 \\ X_p &= 1 - P_2 \end{aligned} \tag{4.13}$$

In a predominantly hydro system, where base load and intermediate load are met by hydro, and thermal plants are only used for peaking, the situation is characterised by:

$$\begin{aligned} E_p &= 1 - e_2 \\ X_p &= 1 - P_2 \end{aligned} \tag{4.14}$$

In all the above cases, the power and energy capacities must satisfy the following system load conditions.

$$\begin{aligned} X_b + X_p + X_n &= X_L \\ E_b + E_p + E_{gn} &= E_L \end{aligned} \tag{4.15}$$

where X_L and E_L are total system power and energy demands respectively. The risk of capacity shortages can occur if

$$X_b + X_p + X_n < X_L$$

and

$$E_b + E_p + E_{gn} < E_L \tag{4.16}$$

Insufficient hydro plant capacity is indicated by a large forced

discharge and in some cases may justify expansion of hydro plant capacities, if load characteristics demand such additional capacities.

With the amount of energy generated by base load and peak load thermal plants it is now possible to evaluate the functional equation and to obtain optimal discharge schedules.

4.3.3 Stochastic Hydrology

The optimisation problem to determine long term operating policy of a hydro-thermal system presented in the previous sections considered the water inflows to the reservoir to be deterministic. However, the variability of the actual inflows requires this to be considered as a random variable represented by its probability distribution[52,54].

In formulating the stochastic problem, it is assumed that the inflow in each month is statistically independent of the previous inflows. Hence the monthly inflow F_n is characterised by a discrete distribution given by

$$Pr\left[F_n = F_{nj}\right] = P_{nj} > 0 \qquad (4.17)$$

where

$$\sum_{j=1}^{J_n} P_{nj} = 1 \qquad (4.18)$$

and J_n is the number of discrete steps in the inflow distribution for the n^{th} month.

However, in reality there is a significant statistical correlation between inflows of successive months. Therefore it has been suggested[53] to approximate the sequence of inflows by a first order Markov process. This models the inflow in a particular month to depend only on the preceding one[53,61], but the unavailability of proper correlation data in most system favour the use of independent assumption.

Then the functional equation for stochastic dynamic programming (SDP)

can be written as

$$f_n(S_n) = \min_{Q_n} \underset{F_n}{E} \left[(E_b C_b + E_p C_p)_n + f_{n-1}(S_{n-1}) \right] \qquad (4.19)$$

where $f_n(S_n)$ is the minimum expected cost of operation in stage n.

Then the stage transformation equation becomes

$$\begin{aligned} S_{n-1} &= \underset{F_n}{E} \left[S_n + F_n - Q_n - R_n \right] \\ &= \sum_{j=1}^{J_n} P_{nj} \left[S_n + F_{nj} - Q_n - R_n \right] . \end{aligned} \qquad (4.20)$$

4.4 Composite Reservoir Representation

The main difficulties encountered in the application of optimisation techniques based on DP to the problem of multireservoir systems are concerned with the computer limitations. For example, the computing time increases exponentially with the number of state variables[53]. The increased number of state variables is also responsible for the high dimensionality of the problem which prevents the representation of the total system. The composite reservoir representation proposed by Arvanitidis & Rosing[62] becomes useful particularly in the long range operational problem where the sequence of monthly decisions on total hydro generation is of greater economic significance than the allocation of this total among the various hydro plants[53,62].

Using the method of reference 62, the multireservoir hydro system can be transformed to a potential energy model, which in effect receives, stores and releases potential energy. This also represents the main characteristics and the capabilities of the total system. A composite generation function which relates the actual power generated for a given amount of potential energy released, and a statistical model for the potential energy inflow to the reservoir can be obtained as outlined

in this reference. However, the ability to transform the multireservoir system to an equivalent potential energy model is possible only if historical data over a long time period is available. Therefore in this research the composite reservoir representation was not used.

4.5 Application of the Method

The stochastic dynamic programming technique formulated in Section 4.3.3 has been applied to a hypothetical system with one dam and hydro plant and a group of base load and peaking thermal plants. The water inflows to the reservoir in each month is described by a discrete probability distribution. A typical distribution is shown in Table 4.1.

The data used are shown in Table 4.2 and the optimal discharge schedule for an initial water storage capacity of 750×10^6 m^3 is shown in fig. 4.4. One load energy curve, represented by a 5^{th} order polynomial is used for all the periods. A constant turbine-generator efficiency of 88% is assumed in the calculations.

4.6 Conclusions

The problem of long range optimal control and operation of a reservoir in a mixed hydro-thermal system using a SDP method is illustrated in this Chapter. Although the problem is formulated in its most basic form, it is expected to demonstrate the applicability of DP techniques to the control of water resource systems.

In these applications, the analytical methods require sufficient continuity and differentiability conditions on the objective function. The DP techniques on the other hand, provide an extremely powerful and general approach as the non-linearities in the system equations and the objective function can be easily handled. Further, the constraints on both decision and state variables introduce no

difficulties. One of the most attractive features is the adaptability of DP techniques to stochastic control problems without major problems.

In the analysis, a constant efficiency for the turbine-generator system is assumed. However, for a more realistic representation the variation of efficiency with gate openings and the tail race water level can be included. Start-up costs of thermal plants is another aspect that could be included to give a more realsitic optimum, if desired.

The explicit modelling of a multireservoir system is bound to introduce limitations on the computer time and storage. However, this can always be simplified to a single reservoir system provided that suitable historical data is available. The long range optimum control problem is important as the amount of water allocated for hydro generation in the short term is derived from these results.

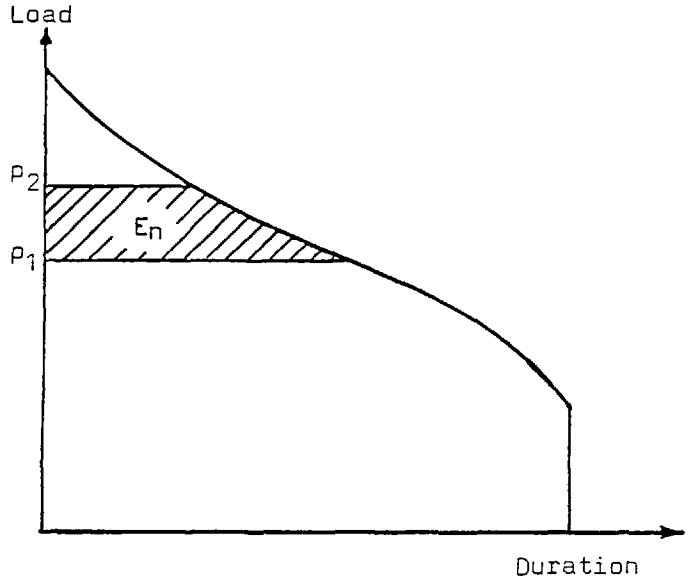

Fig. 4.1 Optimal Position of Hydro Plant in the Dispatching Schedule

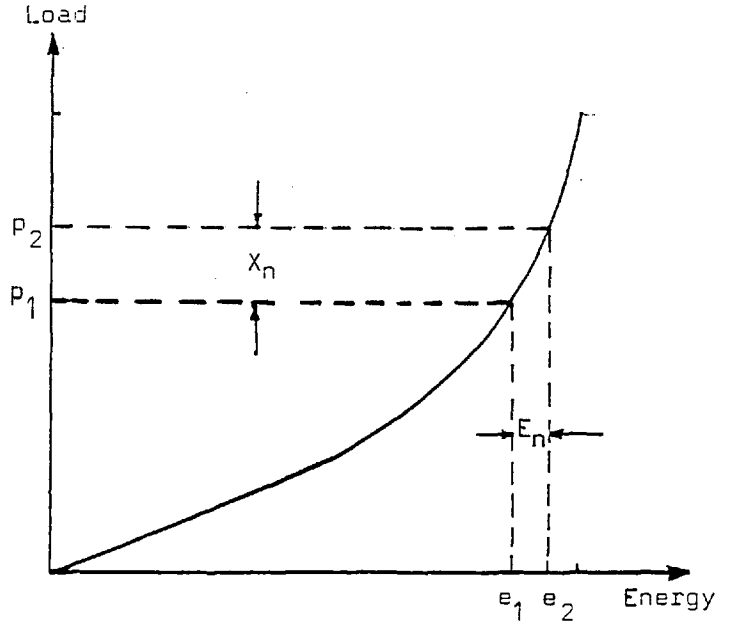

Fig. 4.2 Representation of the Optimal Position of Hydro Plant on the Load Energy Curve

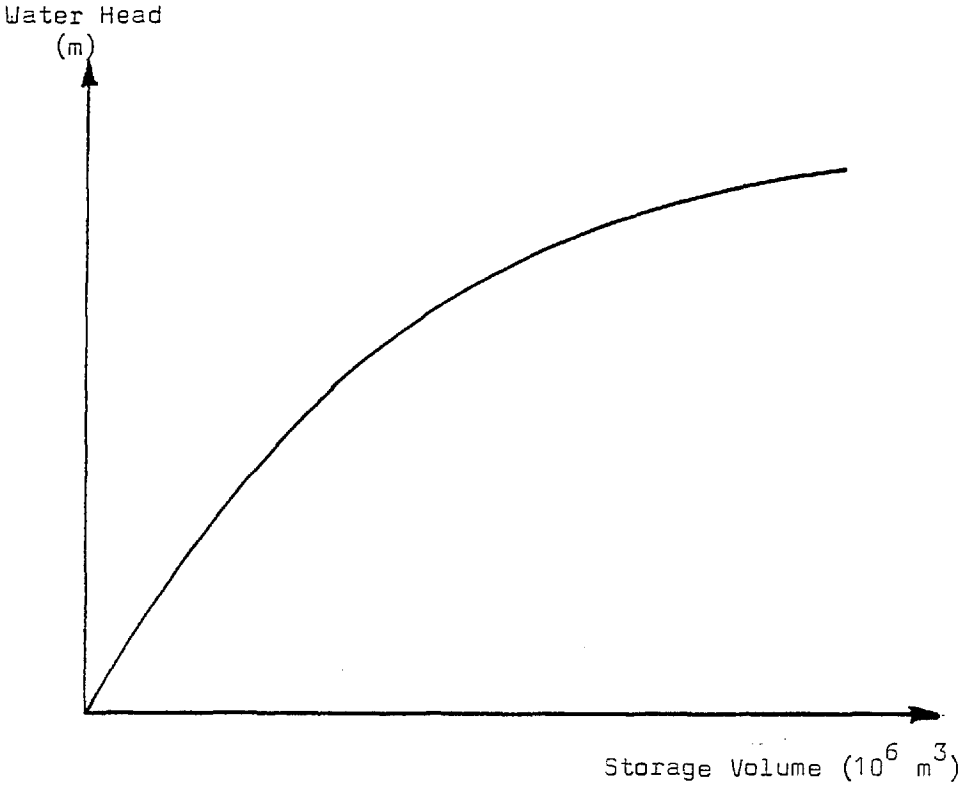

Fig. 4.3 Head-Storage Curve for a Typical Reservoir

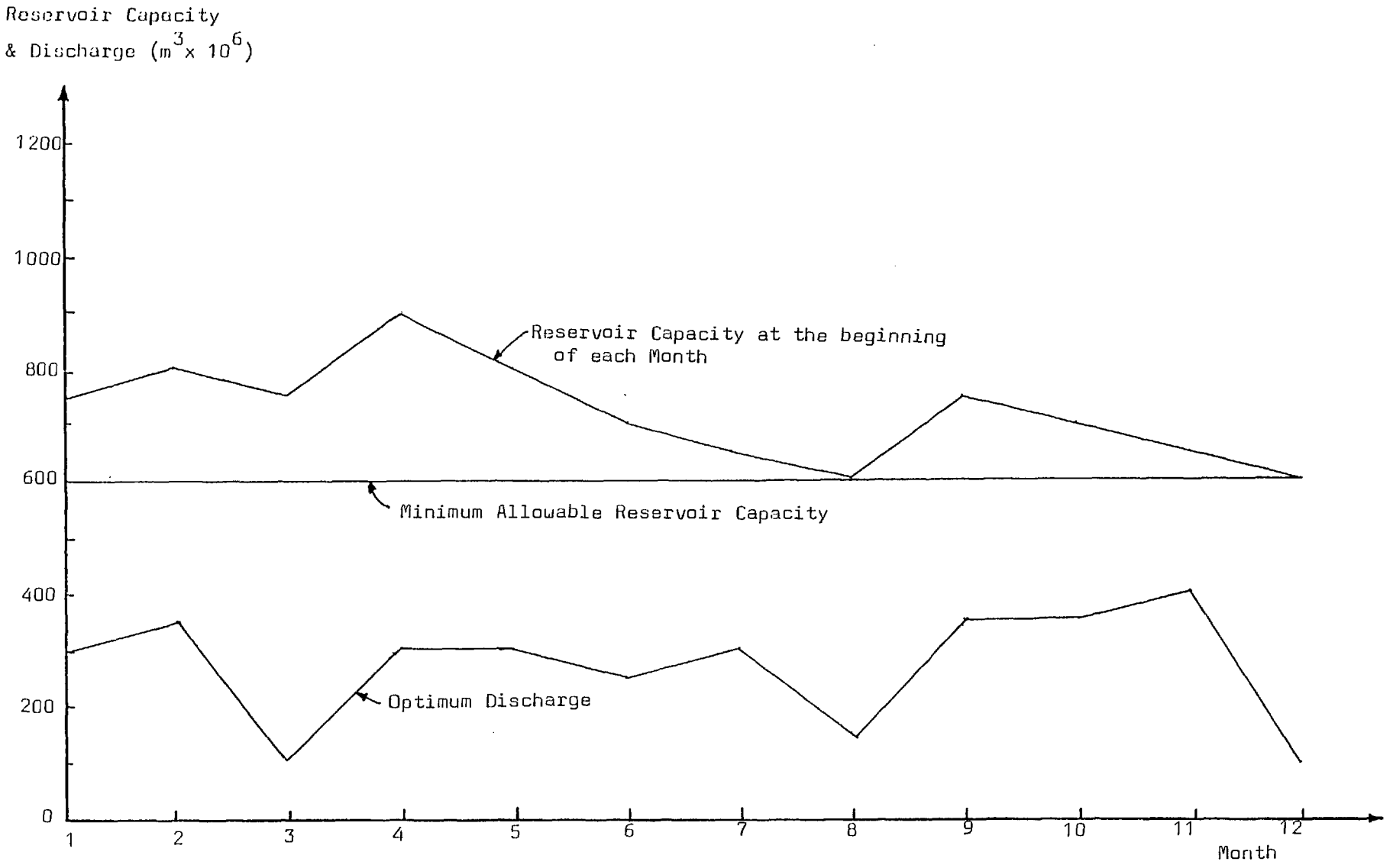

Fig. 4.4 Results for an Initial Storage of 750 x $10^6 m^3$

Table 4.1 Probability distribution of monthly inflows

Water inflow (m^3)	Probability of occurence
10	0.02
30	0.04
60	0.06
90	0.08
120	0.1
180	0.1
290	0.1
380	0.1
520	0.2
660	0.1
700	0.08
920	0.02

Table 4.2 System data for the operation study

Month	1	2	3	4	5	6	7	8	9	10	11	12
Maximum allowable storage at the beginning of the month (10^6 m^3)	1050	1100	1150	1200	1250	1200	1150	1100	1050	1000	1000	1000
Maximum Power Demand (MW)	1100	1150	1200	1075	1100	1175	1200	1250	1250	1150	1100	1050
Maximum Energy Demand (10^3 MWh)	311	315	322	304	308	318	320	345	350	312	308	303
Available Hydro Plant Capacity (MW)	100	100	100	75	75	100	100	100	75	75	100	100
Available Base Load Plant Capacity (MW)	600	600	800	800	800	600	500	800	800	600	800	800

CHAPTER 5

EVALUATION OF RELIABILITY INDICES FOR TRANSMISSION PLANNING

5.1 Introduction

One of the major assumptions made in the area of long term generation planning studies (Chapter 2) is that the transmission system is fully reliable and the consumer interruptions are only due to generator outages. However, the components of a practical transmission network also experience outages and the effects of these on the consumers must be evaluated.

In this evaluation, it is important to note the varying requirements of consumers; for example, an industrial consumer whose production process requires a complicated start up procedure after an interruption is interested in the frequency of service interruptions, whereas a residential/domestic consumer is basically interested in the duration of service interruption. Until the recent past,little effort has been placed on the determination of cost & effects of service interruptions to consumers and the levels of supply reliability have been determined mainly by means of security standards[63]. This has been largely due to difficulties in evaluating reliability of supply at consumer load points in quantitative terms.

A survey carried out in 1970[63] concerning the cost and effects of supply interruptions to industrial consumers revealed that the total cost of interruption can be expressed in terms of a fixed cost dependent upon the frequency of interruptions, and a variable cost proportional to the duration of interruption.

In addition to the varying requirements of consumers, the quality of service offered by the supply utility changes, primarily because of the topology of the power transmission network. That is, a consumer

located near generating centres generally receives a higher quality of supply than a consumer far removed from generation. These disparities therefore, require the determination of load point reliability of a network by two indices, namely, frequency of supply interruption and the duration of supply interruption.

5.1.1 Reliability of Transmission Systems

The "reliability" of the power transmission system can be defined as its ability to transport electric energy of acceptable quality continuously from the generation centres to the load centres in the quantities required. The limits of acceptable quality differ from system to system and are mainly dictated by the statutory regulations governing voltage and frequency limits.

5.1.2 Historical Background

In almost all countries the load demand is growing at a steady rate, so the utilities are continually planning and constructing additions to their power transmission network. In the early days of electricity industry, the system planners simply added more lines to the network as the demand for more and better service increased. However, due to economic and environmental reasons the practice of adding lines to alleviate potential overloads is now unacceptable. It is now imperative that the adequacy of transmission networks at the planning stage are tested by considering the effects of component outages on the power system.

The first approach to this problem in the 1930's had been the use of network analysers to simulate system operation to monitor the operating conditions under all contingencies of interest. Even though limited only to small networks, not only the AC network analyser but also the DC network analyser[64] can be used to obtain both real and reactive power flows and bus voltages in the network under contingency states.

The advent of the digital computer and the development of faster, reliable and more powerful numerical methods[65,66,67,68] to solve the load flow problem, however, marked an important stage in the development of load flow techniques for the evaluation of contingencies in large networks. Thus the adequacy of transmission networks was tested by repeated load flow studies. As these deterministic load flow methods produce results fully compatible with the given data (apart from numerical round off errors), the analysis of the networks over the load forecast error required many load flows for each contingency. The obvious disadvantages of this method are the prohibitively large computer time and the cost associated with these studies.

With the use of reliability criteria in system planning, network methods have evolved (such as cut set and tie set methods[69,70]), in which reliability is based on the continuity of the network. If all components in a transmission system are fully redundant, then reliability is only a matter of continuity with success and failure easily defined. However, as complete component redundancy is not economically feasible, continuity is not an acceptable single criterion[71].

These mathematical procedures do not recognise the fact that all transmission network elements have finite limits of capability and that for proper system operation, the bus voltages must be within given limits. Therefore any procedure which obtains network reliability in quantitative terms must include the load forecast uncertainties, network element outages and the operating limits of system elements. Such a method is suggested in this chapter.

In this method, the frequency and duration indices of supply interruptions are determined on the basis of a supply quality criterion. The load uncertainties are accounted for by using stochastic load flow

techniques (Section 5.4) and the effects of network contingencies are analysed with the use of sensitivity factors (Section 5.5.7). It is assumed that the network can be represented as a stationary random process which possesses the ergodic property of equivalent time and ensemble averages[72]. This implies that the probability of an event represents an estimate of the time that the particular event will exist. The 'expected portion of time' index is beleived to be comparable to the loss of load probability (LOLP) at the bus in question. Although the frequency and duration indices can be used to evaluate the cost of interruptions to consumers, in this thesis, they are used to evaluate the performance and to assess the adequacy of alternative transmission expansion plans.

5.2 Conditional Probability Approach

One method which meets the requirements of Section 5.1 is the conditional probability approach used by Billinton et al[1,73], employing a service quality standard as the reliability criterion[71]. This procedure recognises that under each outage condition there is a maximum load at each bus that can be supplied without violating the service quality standards. The probability that the load will exceed this maximum can be determined from the load probability distribution for the bus in question.

Following reference 73, the reliability indices of interest can be defined as follows.

Probability of service interruption at bus k

$$P_k = \sum_j \left[Pr(B_j)\ Pr(L_j)\right] \tag{5.1}$$

and Expected frequency of service interruption at bus k

$$f_k = \sum_j \left[F(B_j)\ Pr(L_j)\right] \tag{5.2}$$

where

B_j is a specific outage condition in the transmission network

$Pr(B_j)$ is the probability of existence of outage B_j

$Pr(L_j)$ is the conditional probability of load at bus k exceeding the maximum load that can be supplied at that bus during outage B_j

$F(B_j)$ is the frequency of occurrence of outage B_j

$\sum_j$ is the summation over all outage conditions of interest.

The load variation at each bus is represented by a normalised load duration curve, approximated by a single straight line and load flow calculations are performed by dividing the load duration curve into a number of steps. In the analysis, the generation schedules are modified for each outage case. The reliability indices thus evaluated are more accurate as the bus voltages and line loadings are affected by the generation schedule.

The conclusions that can be drawn from these studies[1,73] are that:

1. The risk levels obtained at the various load points in the system in terms of probability and expected frequency of failure give a consistent measure of transmission adequacy.
2. Sufficiently accurate results can be obtained by considering only single independent outages and the outages of higher order contribute negligibly to the total risk.
3. The effects of storms is virtually negligible in major meshed transmission reliability evaluation over relatively long distances as storms become more local in nature[31]. However, if a particular system shows a high failure rate due to

storm associated events, this can be included by considering multiple contingencies.

The disadvantage of this method is that it requires many load flow calculations for each contingency and although this method seems to be acceptable in that it calculates reliability indices, the computational requirements are similar to the early contingency analysis using repeated load flow studies.

5.3 Fast Decoupled Load Flow

Since the method of reference 73 and most other contingency analysis methods used in transmission planning studies are based on a load flow technique, this section briefly outlines the fast decoupled load flow (FDLF) method[68].

The decoupled load flow model is derived by taking the polar power mismatch Newton method as a convenient starting point in which the sparse real Jacobian matrix equation

$$\begin{bmatrix} \Delta P \\ \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & N \\ \\ J & L \end{bmatrix} \begin{bmatrix} \Delta \theta \\ \\ \Delta V/V \end{bmatrix} \tag{5.3}$$

is solved successively.

However, it has been observed that the numerical methods are at their best when physical properties of the system are incorporated in the solution[68]. One such property is the loose physical interaction between real and reactive power flows in a power system. Therefore it is possible to mathematically decouple the MW - θ and MVAR - V calculations by neglecting the off diagonal coupling sub-matrices N and J in eqn. (5.3), giving two separate equations

$$\begin{bmatrix} \Delta P \end{bmatrix} = \begin{bmatrix} H \end{bmatrix} \begin{bmatrix} \Delta \theta \end{bmatrix} \tag{5.4}$$

$$\begin{bmatrix} \Delta Q \end{bmatrix} = \begin{bmatrix} L \end{bmatrix} \begin{bmatrix} \Delta V/V \end{bmatrix} \tag{5.5}$$

where

$$H_{kl} = L_{kl} = V_k V_l (G_{kl} \sin\theta_{kl} - B_{kl} \cos\theta_{kl}) \quad \text{for } k \neq l$$

and

$$H_{kk} = -B_{kk} V_k^2 - Q_k$$

$$L_{kk} = -B_{kk} V_k^2 + Q_k$$

Even though it is possible to solve eqns. (5.4) and (5.5) alternatively by reevaluating and factorising $[H]$ and $[L]$, the following justifiable assumptions are almost always valid in a practical power system.

i.e. for small transmission angles θ_{kl}

$$\cos\theta_{kl} \approx 1$$

$$G_{kl} \sin\theta_{kl} \ll B_{kl}$$

and

$$Q_k \ll B_{kk} V_k^2 \ .$$

Hence eqns. (5.4) and (5.5) can be written as

$$[\Delta P] = [VB'V][\Delta\theta] \tag{5.6}$$

$$[\Delta Q] = [VB''V][\Delta V/V] \tag{5.7}$$

where the elements of $[B']$ and $[B'']$ are essentially elements of $[-B]$.

The final algorithm is obtained by

i. omitting from $[B']$ the representation of those network elements that predominantly affect MVAR flows, i.e. shunt reactances and off nominal in phase transformer taps.

ii. neglecting series resistance in calcualting the elements of $[B']$. This is of minor importance for the base case load flow, but is found to give improved convergence characteristics in contingency analysis.

iii. taking the left hand side V terms from eqns. (5.6) and (5.7) to the left hand side of the equations and removing the influence of MVAR flows on the calculation of $[\Delta\theta]$ in eqn. (5.6)

by setting all right hand side V terms to 1.0 pu. This is possible since the V terms on the left hand side only affect the behaviour of the defining functions and not the coupling.

iv. omitting from $[B'']$ the angle shifting effects of phase shifters.

With these simplifications, the final equations become

$$\left[\frac{\Delta P}{V}\right] = [B'][\Delta\theta] \tag{5.8}$$

$$\left[\frac{\Delta Q}{V}\right] = [B''][\Delta V] \tag{5.9}$$

The matrices $[B']$ and $[B'']$ are real and sparse and have the same structures of $[H]$ and $[L]$ respectively. As they contain only network admittances, they are constant and need to be evaluated and factorised once only at the beginning of the study. For the investigations reported in this thesis, a bifactorisation algorithm[74,75] with a near optimal ordering technique is used to factorise the matrices $[B']$ and $[B'']$.

The $(1\theta, 1V)$ iteration scheme, using the most recent values of voltages to update $[\theta]$ and $[V]$, remains the best iteration scheme for all applications including contingency analysis. Separate convergence tests are used for power mismatches with the criteria

$$\max |\Delta P| \leq C_p$$

and

$$\max |\Delta Q| \leq C_q \quad .$$

It has been found[68] that in contingency analysis, the use of a base case solution as starting values for the outage calculations is on the average distinctly better than using the mormal flat voltage start each time. As the outage calculations need not be carried out to a greater accuracy, further improvemnets of convergence have been obtained by using convergence tolerences of a higher magnitude (normally about 10 times the base case) in the contingency analysis.

5.4 Stochastic Load Flow (SLF)

The application of stochastic load flow (SLF) methods which take into account the uncertainty of the load forecast in the contingency analysis has the advantage of producing load flow results over the forecast error and eliminates the requirement of repetitive load flow calculations.

In the methods described by Borkowska[76] and Allan et al[77], the load forecast is given by a general probability distribution curve which is subjected to linearised load flow equations to produce complete distribution functions for the line flows and bus voltages. The ability to handle general distribution functions requires convolution procedures[78,79], which, for a large number of input variables may demand uneconomically large computational resources.

A formulation based on the principles of statistical least squares estimation for linear systems was proposed by Dopazo et al[80]. In this formulation the statistics of the input variables are used to calculate the statistics of the state and output variables. As a result of the linearisation procedure and by use of the Central Limit Theorem[81], the output quantities can be taken as normally distributed random variables.

An alternative formulation was proposed by Aboytes and Cory[82], which is also based on the least squares estimator with zero degrees of freedom for which error filtering is impossible. It has been pointed out that in the method by Dopazo et al, the solution is filtered and the variance of output quantities will not correspond to the variances of the input quantities.

5.4.1 Mathematical Model for SLF

The input variables y and the state variables x in a power system can be represented by a set of non-linear equations

$$y = f(x) + \varepsilon \qquad (5.10)$$

where ϵ is a random error vector whose covariance defines the variation of the input variables about their mean values.

Using Taylor's series expansion to linearise eqn. (5.10) around an operating state x_o and neglecting higher order terms

$$y = f(x_o) + J\Delta x + \epsilon \tag{5.11}$$

from which

$$\Delta y = y - f(x_o)$$

$$= J\Delta x + \epsilon \tag{5.12}$$

where J is the Jacobian matrix.

It is always convenient and desirable to have an unbiased estimate[8,82], hence the statistics of the random vector ϵ are specified as

$$E(\epsilon) = 0$$

$$\epsilon \text{ is Gaussian} \tag{5.13}$$

and $$E(\epsilon\epsilon^t) = C_\epsilon$$

where C_ϵ is the covariance matrix of ϵ which gives an indication of the dispersion of the errors around the mean value and E is the expected value operator.

Applying the least squares process to eqn. (5.12) the estimated value $\Delta\hat{x}$ is obtained as

$$\Delta\hat{x} = (J^t C_\epsilon^{-1} J)^{-1} J^t C_\epsilon^{-1} \Delta y \tag{5.14}$$

and $$\hat{x} = x_o + \Delta\hat{x} \tag{5.15}$$

The covariance matrix of $\Delta\hat{x}$ can be written as

$$C_{\Delta\hat{x}} = (J^t C_\epsilon^{-1} J)^{-1} \tag{5.16}$$

Since x_o is not a random variable it is evident that

$$C_{\hat{x}} = C_{\Delta\hat{x}} \ . \tag{5.17}$$

If the relationship of output variables z to state variables x can be expressed as

$$z = g(x) \tag{5.18}$$

then a Taylor's series expansion of eqn. (5.18) around the operating state x_o gives

$$z = g(x) + K\Delta\hat{x} \tag{5.19}$$

where K is the Jacobian matrix of the output variables at the operating point.

The covariance matrix of z can be written as

$$C_z = KC_{\hat{x}}K^t \ . \tag{5.20}$$

5.4.2 Application to the Power Flow Problem

The variables used in the derivation of the mathematical model relates to the variables in the load flow problem in the following way:-

- y - load flow input data, i.e. load bus P & Q and generator bus P & V
- x - bus state variables V and θ
- C_ε - covariance matrix of the errors in input variables y
- J - Jacobian matrix of the load flow equation made up of the partial derivatives of P,Q and V input variables with respect to V and θ state variables.
- z - output variables of the load flow computed from x, such as line flow P and Q and generator bus Q
- K - Jacobian of output variables z with respect to state variables x.

The attractive feature of this method is that it determines the propagation of the errors of input variables to the output variables by starting with a solution of a conventional load flow, followed by a series of non-iterative mathematical calculations. In the method presented by

Dopazo et al[80], it is assumed that C_ε is diagonal, implying that the input variables are independent. This assumption is debated at length in the discussion of reference 80, and it has been pointed out that for long term planning purposes the expectation of P and Q for system peak conditions are not strongly correlated. Further the plant output at peak system conditions at some future year can be considered as normally distributed with a small variance that depends significantly on the distribution of the system peak.

The effects of correlation between load and generations are considered in the formulation by Aboytes and Cory[82] and it is claimed that this makes the solutions obtained more realistic. Although the use of uncorrelated data is not easily justifiable, the lack of correlation data is one of the major problems in analysing a typical power system in stochastic terms.

5.4.3 Extensions of the SLF Technique

Recently two extensions were reported in the literature which are based on the SLF calculations described in Section 5.4.1.

The approach used by Prada & Cory[83] in stochastic security analysis is to assume that the loads can be represented as normal random variables and the propagation of errors to the state and output variables can be obtained by linearising the load flow equations around the estimated state. This is to be obtained using a conventional deterministic load flow technique. The most dangerous line outage conditions are then obtained by calculating the line overload probabilities based on the expected value and the standard deviation of line flows. It is assumed that the output variables are normally distributed. This is valid due to the linearising process and the normal assumption about input variables.

The probability of overload of line m-n due to line p-q out of service is given by[83]

$$p_{mn}^{pq} = 0.5 - \text{erf}\left(\frac{x_{mn}^{pq} - \mu_{mn}^{pq}}{\sigma_{mn}^{pq}}\right) \tag{5.21}$$

where

x_{mn}^{pq} - transmission line maximum flow limit

μ_{mn}^{pq} - expected line flow

σ_{mn}^{pq} - line flow standard deviation

and

$$\text{erf}\left(\frac{x - \mu}{\sigma}\right) \triangleq \int_0^x \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(\xi - \mu)^2}{2\sigma^2}\right) d\xi \tag{5.22}$$

This method is intended for on line short term operation studies and hence one load state with given statistics can be used to obtain the probabilities of line overloads for given line outage conditions.

In the method used by Aboytes[84] for stochastic contingency analysis, a probability is assigned to every contingency and an overall uncertainty interval is obtained due to the combined effect of all the contingencies weighted according to their probabilities. Since this approach considers uncertainties in load and generation as in a stochastic load flow, it is claimed[84] that this technique probabilistically encompass the results of many load flows.

It is assumed that the probability of line outage is known and that every outage is statistically independent thus allowing the overall probability to be evaluated as the product of probabilities. It is mentioned[84] that single contingencies produce results comparable to the results of Monte Carlo simulation techniques.

One of the basic problems associated with the SLF technique outlined so far is that it involves the formulation of load flow and line flow Jacobian matrices. Even if sparsity techniques are used, these matrices can have many non-zero elements which require increased computer storage and time. In this respect the exploitation of symmetry of the Jacobian matrices and the storage of only diagonal terms are mentioned[80]. Further these procedures produce more information than is necessary in long term planning, including the statistics of the output variables.

5.5 Failure of Consumer Load Points

Although the recent SLF methods calculate in probabilistic terms the system conditions due to contingencies, the question of the "probability of failure of consumer load points" remains unanswered.

The method employed in this thesis to calculate the consumer load point failure probabilities is similar to that described by Hayes et al[85]. It is based on the use of SLF techniques and the sensitivity factors relating line flows to injected powers.

This requires the development of models for the representation of system loads, the transmission network and the generation. In the basic formulation, it is assumed that generation is totally available. The load is modelled as a stochastic process representing the uncertainty of load forecasts. The network is modelled as a stochastic process representing the failure uncertainty of network elements.

5.5.1 System Load Model

The system load model is represented as a vector, first order Markov process to include the uncertainties of load forecasting and the sequence of daily peak loads. The daily load cycle is approximated by a square wave representing the daily peak and low load levels. The daily peak

load states L_i, each of mean duration e (<1) days, alternate with periods of low load L_o of duration (1 - e) days. This process is shown in fig. 5.1. The factors which influence the daily peak load states can be stated as weather, day of the week, holidays, etc., and although these cannot be predicted exactly, average values of variations can be obtained[8].

Each load state L_i, $i = 0,1, \ldots\ldots\ldots I$ is defined by a random vector S_i which is given by

$$S_i = P_i + jQ_i$$

$$= (\alpha_i + j\beta_i)\rho_i \qquad i = 0,1, \ldots\ldots I \qquad (5.23)$$

where

ρ_i is a random variable whose characteristics are defined by a particular load state

and

α_i, β_i are deterministic vectors which distribute the system load to invidual buses.

It is assumed in general that the distribution factors α and β are equal for all load states.

i.e.
$$\alpha_i = \alpha_{i-1}$$
$$\beta_i = \beta_{i-1} \qquad i = 1,2, \ldots\ldots I \qquad (5.24)$$

This implies that the individual bus loads are modelled as being perfectly correlated and results from the assumption that external influences will tend to change the loads in the same direction. This assumption is very likely to be as controversial as the independent assumption. However, the reasons in favour of perfect correlation can be stated as:

i. precise correlation data is generally not available

ii. loads of the same type tend to be functions of common random effects such as weather, day of the week, etc.

iii. computational simplicity is achieved.

The load is therefore now a deterministic function of the random variable ρ_i which represents the uncertainty in the load. It is assumed that ρ_i is a normal random variable with mean μ_i and variance σ_i^2 .

Since the process is assumed to be ergodic, the probability of existence of a load state is equal to the average fraction of time spent in that state. The sequence of peak loads is random, however, and the average number of occurrences of each peak load state during the period is assumed to be known. With these assumptions the load model is completed with the following definitions.

n_i - number of occurrences of load state L_i

D - number of days in the study

$$= \sum_{i=1}^{I} n_i \tag{5.25}$$

The probability that the load state L_i exists is

$$Pr(L_i) = \frac{n_i e}{D} \qquad i \neq 0 \tag{5.26}$$

$$Pr(L_o) = 1 - e \tag{5.27}$$

The expected frequency of occurrence of load state L_i is

$$F(L_i) = \frac{n_i}{D} \qquad i \neq 0 \tag{5.28}$$

$$F(L_o) = 1 \tag{5.29}$$

5.5.2 Transmission Network Model

The primary function of a transmission network is to transport energy from a producing node to a consuming node, systematically, adequately, reliably and economically. Therefore the consumers depend very much upon the integrity of the transmission system under possible network contingencies. Although these networks are designed with redundant elements to reduce the possibility of system failure due to loss of a single element, they can be critical during system peak load periods.

The network is modelled as a first order Markov process independent of load which assumes statistically independent, stationary, exponential distributions of network contingencies and repair times. The network state N_o (fig. 5.2) is the state with all components operating normally and the network moves from state N_o to contingency states

$$N_j \ , \quad j = 1,2, \ldots\ldots\ldots J$$

with failure rate λ_j and returns from contingency states to mormal state with repair rate μ_j. The general assumption is made that the failure rate λ_j is a function of operating voltage and length and that the repair rate μ_j is a function of operating voltage only.

The mean duration of contingency states is

$$d_j = \frac{1}{\mu_j} \tag{5.30}$$

and the probability of state N_j existing at some time in the future is

$$Pr(N_j) = \frac{\lambda_j}{\lambda_j + \mu_j} \qquad j \neq 0 \tag{5.31}$$

The expected frequency of occurrence of state N_j is

$$F(N_j) = \mu_j Pr(N_j) \qquad j \neq 0$$

and of state N_o is, (5.32)

$$F(N_o) = \sum_{j=1}^{J} F(N_j) \ .$$

The probability of existence of state N_o is

$$Pr(N_o) = d_o F(N_o) \tag{5.33}$$

where d_o is the mean duration of state N_o.

5.5.3 SLF Using Load Model of Section 5.5.1

The conventional load flow technique (Section 5.3) yields a discrete set of bus voltages and line flows as the solution for a given set of busbar and network conditions. As the load model developed in Section 5.5.1 describes the load by a random variable which has a functional relationship to the system load with known statistics, the statistics of the output variables must be obtained using a SLF technique. Although the SLF techniques described in Section 5.4 produce the statistics of line flows, they do not provide facilities to calculate the consumer load point failure probabilities.

Therefore a SLF model using the load model of Section 5.5.1 which allows the calculation of these probabilities is derived. The load flow eqns. (5.8) and (5.9) provide a point of linearisation for the SLF procedure. The linearisation will be more exact if the full Jacobian matrix is used but the disadvantage is that it needs the calculation and use of more elements than were used in the decoupled load flow equations. Although eqns. (5.8) and (5.9) are very efficient in load flow calculations, they do not provide sufficient accuracy for linearisation. Therefore a set of equations is required which approaches the efficiency of the decoupled equations without losing accuracy compared to the use of the exact Jacobian matrix[86]. This can be achieved by a procedure which recouples the MVAR - θ in the load flow eqn. (5.3) giving

$$\begin{bmatrix} \frac{\Delta P}{V} \\ \\ \frac{\Delta Q}{V} \end{bmatrix} = \begin{bmatrix} B' & 0 \\ \\ J & B'' \end{bmatrix} \begin{bmatrix} \Delta\theta \\ \\ \Delta V \end{bmatrix} \qquad (5.34)$$

where

$$J_{kl} \triangleq -V_l(G_{kl}\cos\theta_{kl} + B_{kl}\sin\theta_{kl})$$

The matrix inversion lemma gives

$$\begin{bmatrix} \Delta\theta \\ \Delta V \end{bmatrix} = \begin{bmatrix} B'^{-1} & 0 \\ -B''^{-1}JB'^{-1} & B''^{-1} \end{bmatrix} \begin{bmatrix} \frac{\Delta P}{V} \\ \frac{\Delta Q}{V} \end{bmatrix} \tag{5.35}$$

from which

$$\left[\Delta\theta\right] = \left[B'^{-1}\right]\left[\frac{\Delta P}{V}\right] \tag{5.36}$$

$$\left[\Delta V\right] = \left[-B''^{-1}JB'^{-1}\right]\left[\frac{\Delta P}{V}\right] + \left[B''^{-1}\right]\left[\frac{\Delta Q}{V}\right] \tag{5.37}$$

If the solution of the discrete load flow equations at the point of linearisation is θ_o and V_o, the stochastic variables can be expressed as

$$\begin{aligned} \theta &= \theta_o + \Delta\theta \\ V &= V_o + \Delta V \\ \rho &= \rho_o + \Delta\rho \end{aligned} \tag{5.38}$$

From eqn. (5.23),

$$P = \alpha\rho = \alpha(\rho_o + \Delta\rho)$$

and

$$Q = \beta\rho = \beta(\rho_o + \Delta\rho)$$

giving

$$\begin{aligned} \Delta P &= \alpha\Delta\rho \\ \text{and} \quad \Delta Q &= \beta\Delta\rho \end{aligned} \tag{5.39}$$

Then

$$\begin{aligned} \left[\Delta\theta\right] &= \left[B'^{-1}\right]\left[\frac{\alpha}{V}\right]\Delta\rho \\ &= \left[Z_1\right]\Delta\rho \end{aligned} \tag{5.40}$$

where

$$\left[Z_1\right] = \left[B'^{-1}\right]\left[\frac{\alpha}{V}\right]$$

and

$$\begin{aligned} \left[\Delta V\right] &= \left\{\left[-B''^{-1}JZ_1\right] + \left[B''^{-1}\right]\left[\frac{\beta}{V}\right]\right\}\Delta\rho \\ &= \left[Z_2\right]\Delta\rho \end{aligned} \tag{5.41}$$

where $$\left[Z_2\right] = -\left[B''^{-1}\right]\left[JZ_1 - \frac{\beta}{V}\right] .$$

5.5.4 Calculation of Probabilities of Bus Voltages Outside Operating Limits

If it is assumed that the bus voltage drops as the system load is increased, the calculation of the probability of bus voltage violating its limits involves the calculation of the probability that the system load ρ exceeds a certain value ρ_c.

Therefore for bus k, if the lower voltage limit is V_{kL}, from eqn. (5.41)

$$\Delta\rho_k = \frac{\Delta V_k}{Z_{2k}}$$

$$= \frac{V_{kL} - V_{ko}}{Z_{2k}} \qquad (5.42)$$

from which

$$\rho_{kc} = \rho_o + \Delta\rho_k$$

$$= \rho_o + \frac{V_{kL} - V_{ko}}{Z_{2k}} \quad . \qquad (5.43)$$

Then the probability of bus under voltage is given by

$$Pr(V_k < V_{kL}) = Pr(\rho > \rho_{kc}) \quad . \qquad (5.44)$$

Similarly the probability of bus voltage violating its upper limit V_{kU} can be calculated from

$$Pr(V_k > V_{kU}) = Pr(\rho < \rho_{kc}) \quad . \qquad (5.45)$$

5.5.5 Calculation of the Probability of Line Overload

The expressions for line real and reactive power flows given by

$$P_{mn} = G_{mn}V_m^2 - V_mV_n(G_{mn}\cos\theta_{mn} + B_{mn}\sin\theta_{mn}) \qquad (5.46)$$

and

$$Q_{mn} = -B_{mn}V_m^2 - V_mV_n(G_{mn}\sin\theta_{mn} - B_{mn}\cos\theta_{mn}) \qquad (5.47)$$

are used to calculate the probabilities of line overloads. For this purpose eqns. (5.46) and (5.47) are linearised around the base case solution P_{mno} and Q_{mno} as follows.

$$P_{mn} = P_{mno} + \Delta P_{mn} \tag{5.48}$$

$$Q_{mn} = Q_{mno} + \Delta Q_{mn} \tag{5.49}$$

From eqns. (5.46) and (5.47)

$$\begin{aligned}\Delta P_{mn} &= \{2G_{mn}V_{mo}Z_{2m} + V_{mo}V_{no}(G_{mn}\sin\theta_{mno} - B_{mn}\cos\theta_{mno})\ (Z_{1m} - Z_{1n}) \\ &\quad -(V_{no}Z_{2m} + V_{mo}Z_{2n})\ (G_{mn}\cos\theta_{mno} + B_{mn}\sin\theta_{mno})\}\Delta\rho \\ &= E_{mn}\,\Delta\rho \end{aligned} \tag{5.50}$$

and

$$\begin{aligned}\Delta Q_{mn} &= \{-2B_{mn}V_{mo}Z_{2m} - V_{mo}V_{no}(G_{mn}\cos\theta_{mno} + B_{mn}\sin\theta_{mno})\ (Z_{1m} - Z_{1n}) \\ &\quad -(V_{no}Z_{2m} + V_{mo}Z_{2n})\ (G_{mn}\sin\theta_{mno} - B_{mn}\cos\theta_{mno})\}\ \Delta\rho \\ &= F_{mn}\,\Delta\rho \end{aligned} \tag{5.51}$$

The power flow in line m-n is

$$\begin{aligned}S_{mn}^2 &= P_{mn}^2 + Q_{mn}^2 \\ &= (P_{mno} + \Delta P_{mn})^2 + (Q_{mno} + \Delta Q_{mn})^2 \\ &= \Delta P_{mn}^2 + \Delta Q_{mn}^2 + 2(P_{mno}\Delta P_{mn} + Q_{mno}\Delta Q_{mn}) + P_{mno}^2 + Q_{mno}^2\end{aligned}$$

substituting for ΔP_{mn} and ΔQ_{mn} in terms of $\Delta\rho$ gives

$$S_{mn}^2 = (E_{mn}^2 + F_{mn}^2)\,\Delta\rho^2 + 2(P_{mno}E_{mn} + Q_{mno}F_{mn})\,\Delta\rho + P_{mno}^2 + Q_{mno}^2 \tag{5.52}$$

If the maximum line capacity is K_{mn}, then the calculation of probability of line overload involves the calculation of the value of ρ which causes the line flow to exceed the maximum capacity.

i.e.
$$S_{mn}^2 > K_{mn}^2 \tag{5.53}$$

or
$$(E_{mn}^2 + F_{mn}^2)\,\Delta\rho^2 + 2(P_{mno}E_{mn} + Q_{mno}F_{mn})\,\Delta\rho + P_{mno}^2 + Q_{mno}^2 - K_{mn}^2 > 0 \tag{5.54}$$

If ρ_L and ρ_U are the values of ρ which satisfy the conditions

$$S_{mn}^2 = K_{mn}^2$$

and $$\rho_L < \rho_U$$

then the probability of line overload is

$$Pr(OL_{mn}) = Pr(\rho < \rho_L) + Pr(\rho > \rho_U). \quad (5.55)$$

The probabilities which have been obtained so far refer to the conditions of network elements resulting from a failure of a particular network element. However, they do not reflect the influence of these failures on the probability of failure of consumer load points. As the function of the transmission network is to successfully transport power from generating centres to load centres, the points of final interest in the long term planning studies are the load buses.

5.5.6 Calculation of Probabilities of Failure of Consumer Load Points

The service interruption at a consumer load point is defined as the condition that a demand exist which cannot be satisfied due to overloaded lines and/or unacceptable bus voltages[85]. Therefore the calculation of probability of failure of consumer load points involves the use of probabilities of line overloads and of bus voltages violating their limits. However, due to the interconnected nature of the modern high voltage transmission networks, only the failure of a set of transmission lines directly contribute to the failure of a consumer load point. Therefore the evaluation of bus failure probabilities requires the identification of those lines which carry a significant part of the load supplied at the bus in question. This can be done by evaluating the sensitivity of power flow in lines to the power demand at a bus[85].

Under steady state conditions, the use of polar power mismatch equations implies that the generators are sources of constant real power (except

for the slack) and incremental reactive power. Therefore in evaluating the line flow to bus demand sensitivity factors, the reactive power equations must be used.

5.5.7 Calculation of Sensitivity Factors

The sensitivity of the reactive power flow in line m-n to the reactive power demand at bus k is determined by the sensitivity factor R_k^{mn}, which is defined by the equation

$$R_k^{mn} = \frac{\partial Q_{mn}}{\partial Q_k} \tag{5.56}$$

But

$$Q_{mn} = Im\left[V_m I_{mn}^*\right] \tag{5.57}$$

$$= Im\left[V_m^2 - V_m V_n(\cos\theta_{mn} + j\sin\theta_{mn})(G_{mn} - jB_{mn})\right]$$

$$= -B_{mn}(V_m^2 - V_m V_n\cos\theta_{mn}) - G_{mn}V_m V_n\sin\theta_{mn} \tag{5.58}$$

For small transmission angles

$$\theta_{mn} \approx 0$$

$$\cos\theta_{mn} \approx 1$$

and $G_{mn}\sin\theta_{mn} \ll B_{mn}$

giving

$$Q_{mn} = -B_{mn}(V_m^2 - V_m V_n) \tag{5.59}$$

from which

$$\frac{\partial Q_{mn}}{\partial Q_k} = -B_{mn}\left[(2V_m - V_n)\frac{\partial V_m}{\partial Q_k} - V_m\frac{\partial V_n}{\partial Q_k}\right] \tag{5.60}$$

Assuming 1.0 pu. voltage for V_m and V_n gives

$$\frac{\partial Q_{mn}}{\partial Q_k} = -B_{mn}\left[\frac{\partial V_m}{\partial Q_k} - \frac{\partial V_n}{\partial Q_k}\right] \tag{5.61}$$

From the decoupled power flow model (Section 5.3)

$$[\partial V] = [B''^{-1}][\partial Q]$$

$$\frac{\partial V_m}{\partial Q_k} = b''^{-1}_{mk} \qquad (5.62)$$

and $$\frac{\partial V_n}{\partial Q_k} = b''^{-1}_{nk}$$

where b''^{-1}_{mk} is the element corresponding to the m^{th} row and k^{th} column of the $[B''^{-1}]$ matrix.

Then

$$R_k^{mn} = -B_{mn}(b''^{-1}_{mk} - b''^{-1}_{nk}) \quad . \qquad (5.63)$$

The relative magnitudes of the sensitivity factors indicate the amount of reactive power carried by the line to a particular load point. If R_k^{mn} is positive then that line is carrying reactive power to the bus and if it is negative then the line is carrying reactive power away from the bus.

With the probabilities evaluated in Sections 5.5.4 and 5.5.5 and the sensitivity factors of Section 5.5.7, the probability of failure can be obtained as follows. Following reference 85,

the probability of failure at bus k is

$$P_k = \frac{\text{expected total duration of supply interruption at bus } k}{\text{total time period}}$$

$$= \sum_{i=0}^{I} \sum_{j=0}^{J} Pr(L_i) Pr(N_j) P_{ijk} \qquad (5.64)$$

where

$Pr(L_i)$ - the probability that load state i exists

$Pr(N_j)$ - the probability that network state j exists

I - number of load states

J - number of network states

P_{ijk} - the conditional probability that the load at bus k will not be satisfied, conditioned on load state i and network state j.

Since satisfying the demand at bus k requires proper bus voltages and availability of all necessary transmission network elements, the value of P_{ijk} is calculated as the maximum of the probabilities of unacceptable bus voltage and overloads of individual transmission elements supplying the demand at bus k. This is analogous to equating the strength of a chain to the strength of the weakest link.

Thus

$$P_{ijk} = \max \left\{ Pr(V_k < V_{kL}), Pr(V_k > V_{kU}), Pr(OL_{mn}^{pq})_D \right\} \tag{5.65}$$

where

$Pr(OL_{mn}^{pq})_D$ is the probability of overload of line m-n due to failure of line p-q

and D is the set of buses which have the largest line flow to bus power sensitivities.

Similarly, the expected frequency of failure at bus k is

$$f_k = \sum_{i=0}^{I} \sum_{j=0}^{J} F_{ij} P_{ijk} \tag{5.66}$$

where F_{ij} is the expected frequency of occurrence of the combined event of load state i and network state j, which can be obtained from the following expressions for various combinations of i and j.

$$F_{ij} = Pr(L_i) Pr(N_j) \left[\frac{1}{e} + \frac{1}{d_j} \right] \qquad i \neq 0,\ j \neq 0 \tag{5.67}$$

$$F_{oj} = Pr(L_o) Pr(N_j) \left[\frac{1}{1 - e} + \frac{1}{d_j} \right] \qquad i = 0,\ j \neq 0 \tag{5.68}$$

$$F_{io} = Pr(L_i) Pr(N_o) \left[\frac{1}{e} + \frac{1}{d_o} \right] \qquad i \neq 0,\ j = 0 \tag{5.69}$$

$$F_{oo} = Pr(L_o) Pr(N_o) \left[\frac{1}{1 - e} + \frac{1}{d_o} \right] \qquad i = 0,\ j = 0 \ . \tag{5.70}$$

5.5.8 Effects of Generator Outages on the Failure of Consumer Load Points

In the analysis so far, it has been assumed that the generating units are fully available and the failures of consumer load points are mainly due to the existence of load states and network states. However, more realistic reliability indices can be obtained by considering the random outages of generating units. The method proposed in this thesis is the representation of a generator connected to the network by a fictitious line of negligible impedance. Further the maximum capacity of the line will be equal to the maximum capacity of the generating unit and the probability of failure of the line will be equal to the forced outage rate of the generating unit. Since the evaluation of reliability indices involves only single contingencies, the generating unit outages can be included in the set of line outages of interest.

5.6 Overall System Performance Indices

The method described in Section 5.5 evaluates the performance indices of consumer load points considering the existence of particular load and network states. However, it is equally important to evaluate the overall system reliability performance indices, namely the overall service failure probability and the expected unserved energy. These two, among other indices, give a reasonable estimate of the adequacy of the composite generation and transmission system.

5.6.1 Calculation of Service Failure Probability

A service failure state is defined (Section 5.5.6) as a condition which causes service interruption at a consumer load point due to overloaded lines and/or unacceptable bus voltages. These interruptions generally result in either complete or partial loss of load at one or more consumer load points, due to the load reductions necessary to alleviate line overloads or inadequate voltages. This implies that for each

contingency state there exists a critical load level (L_c) which, if exceeded, results in a service interruption. Therefore in the evaluation of service failure probability the load can be represented as a two state model as shown in fig. 5.3. The development of such a load model using historical load data is given in reference 87 and the transition rates are given by the following expressions.

$$\mu_{CL} = \frac{n}{D} \cdot T$$

$$\lambda_{CL} = \frac{n}{(1 - D/T)} \qquad (5.71)$$

where

μ_{CL} - rate of return to the load level below critical load

λ_{CL} - rate of entry to the load level above critical load

T - time period

n - number of times the load exceeds the critical load during period T

D - total duration of load in excess of critical load

Using these transition rates, the probability of the load exceeding the critical load level could be obtained as

$$Pr(L_c) = \frac{\lambda_{CL}}{\lambda_{CL} + \mu_{CL}}$$

$$= \frac{D}{T} \qquad (5.72)$$

The service failure probability due to the combined effect of network contingency state j and the corresponding critical load level L_{cj} can be expressed as

$$Pr_j^{sf} = Pr(N_j) Pr(L_{cj}) \qquad (5.73)$$

Therefore the overall system failure probability Pr_T^{sf} can be uritten as

$$Pr_T^{sf} = \sum_j Pr_j^{sf} \qquad (5.74)$$

and the system failure duration

$$T^{sf} = Pr_T^{sf}.T \qquad \text{hrs/year} . \tag{5.75}$$

5.6.2 Calculation of Unserved Energy

In addition to the transition rates, the unserved energy UE_j due to the service interruption caused by network contingency state j can be calculated from the load model as

$$UE_j = \frac{(PD - L_{cj})\ D}{PD.T} \qquad \text{pu.} \tag{5.76}$$

where PD is the peak demand in MW.

Thus the expected unserved energy $E(UE)$ can be written as

$$E(UE) = \sum_j (UE_j.Pr_j^{sf}.PD.T) \qquad \text{MWh/year} . \tag{5.77}$$

If the total energy requirement of the system is E_T MWh/year, then the unserved energy probability

$$Pr(UE) = E(UE)/E_T . \tag{5.78}$$

In the method presented by Marks[88] to evaluate these performance indices, a load flow solution is obtained at the peak load level and if it causes loss of load, load flow solutions are obtained at lower load levels. This process is repeated until the critical load level is reached. As the major portion of this type of stochastic calculation involves time consuming load flow solutions, the requirement of the repeated load flow calculations to obtain the critical load level is unacceptable.

With a view to reducing the number of load flow solutions required, it has been assumed in reference 88 that if a network state does not cause loss of service at peak load, it will not cause loss of service at lower load levels. To obtain further reductions it has been proposed[88] to establish a cut-off value for the probability of occurrence of outages. This screening procedure eliminates those contingencies whose

probability of occurrence is extremely small. However, the choice of the cut-off value is dependent on the system under study and a reasonable choice would be the value of probability which is small enough not to affect the results significantly but large enough to produce reasonable computing times.

5.6.3 Calculation of the Critical Load Using the Proposed Method

The attractive feature of the method proposed in this thesis is that the critical load level in the majority of contingencies could be obtained by repeating the load flow calculations only once. The computational procedure is as follows.

If a line is overloaded at the peak load level, the load must be reduced until the line flow is equal to its limit. Since the system load is represented by a random variable ρ (Section 5.5.1), the critical value of system load could be obtained by calculating the critical value ρ_c straight from eqn. (5.54). However, due to the linearisation errors, this value of ρ_c will be different from the true value of ρ_c. Hence the load flow calculation must be repeated once more to establish a more accurate critical load level. If the critical load level ρ_c is not very much different (say 10%) from the peak load, then the critical load obtained from eqn. (5.54) will be reasonably accurate and the load flow calculation need not be repeated. However, if a particular contingency causes a severe overload which requires a considerable reduction of system load, the procedure will require an additional load flow solution.

5.7 Application of the Method and Results

The method proposed in this Chapter is evaluated using the IEEE 14 Bus Test Power System and the 40 Bus Proposed Sri Lanka Power System for 1985.

5.7.1 IEEE 14 Bus Test Power System

The line diagram of the IEEE 14 bus system is shown in fig. 5.4. The busbar and network element data of this system is given in reference 89. The outage data is obtained from reference 1 and given in Tables 5.1 and 5.2. The voltage limits are assumed to be 0.97 and 1.05 pu.

The busbar failure probabilities and the failure frequencies for all the load busbars in the system are shown in columns(2)and(3)of Table 5.3. Columns(4)and(5)of the same Table show the results extracted from reference 1.

5.7.2 40 Bus Sri Lanka Power System

The points of generation, loads and the high voltage transmission network for the year 1985 are shown in fig. 5.5. The load demand is to be met entirely by hydro power because

a. there are no proposed thermal power projects to be commissioned before 1985.

b. the existing 50 MW thermal power station has a very low efficiency and is generally not operated. Hence it is not included in the generation schedule.

Since most of the hydro power projects are concentrated in the central part of the country and the major load centres are distant from the generation, the transmission system consists of many relatively long lines. Although there are some 66 kV lines and some 132 kV lines operating at 66 kV, the major transmission voltage by the year 1985 is believed to be 132 kV.

The line diagram of the proposed transmission network is shown in fig. 5.6, and the line data is available in reference 90. Table 5.4 lists all the busbar names and the generating capacities of power plants. Since the failure and repair data for the transmission lines and transformers

are not available, the values given in Table 5.5 are assumed. The voltage limits are assumed to be 0.95 and 1.10 pu. The results obtained are given in Table 5.6.

5.8 Conclusions

The methods reported in references 1 and 88 to calculate load point reliability indices and overall system reliability indices need the repeated evaluation of the load flow solution for each contingency and hence the computing times are generally high. The method proposed in this Chapter needs only an additional load flow calculation in most cases to obtain these indices and hence is believed to be more practical.

The single step representation of load in reference 1 assumes that the load remains constant at the maximum level throughout the period and as a consequence the indices calculated tend to be higher than those obtained using the multistep load method. From the results of Table 5.3, it can be seen that the performance indices obtained by the proposed method are acceptable when compared to the method of reference 1. However, an exact comparison of the numerical values could not be made as some of the data used for the two studies may be different. For example, the line capacities which have a great impact on the performance indices are not available in reference 1 and hence the values given in reference 83 have been used.

As there are no outage data available for the Sri Lanka power network, values have been assumed and hence the performance indices obtained do not necessarily indicate the true reliability of the network. However, this study is important in that it establishes the applicability of the method to medium sized power systems.

Although a direct comparison of computing times is not possible it can be seen that for a critical load of 75% of the maximum load, both the

conditional probability approach and the method by Marks would require repeating the load flow 4 times, if 10% load steps are used, whereas the proposed method obtains this in at most 2 steps. This is equivalent to a time saving of 2 load flow solutions (or in the average case, about 6-8 load flow iterations).

In the analysis, perfect correlation of load is assumed primarily due to lack of correlation data. This is not a limitation of the method and further investigations could be made using the correlation data, if avaialble. The method can also be extended to include the evaluation of multiple contingencies.

One of the important applications of the method is its use as a planning tool. The idea of sensitivity factors should be further explored and an automatic method of proposing reinforcements to improve the reliability of a particular busbar should be developed.

The reliability indices at individual bulk supply points can also be used to evaluate the expected cost of supply interruptions to consumers by identifying the type of consumers served by the particular busbar and associating with it a cost of supply interruptions. Further, these indices are compatible with those presently used for distribution system reliability evaluation. Therefore these are valuable as starting values in evaluating the distribution system supplying the individual consumers.

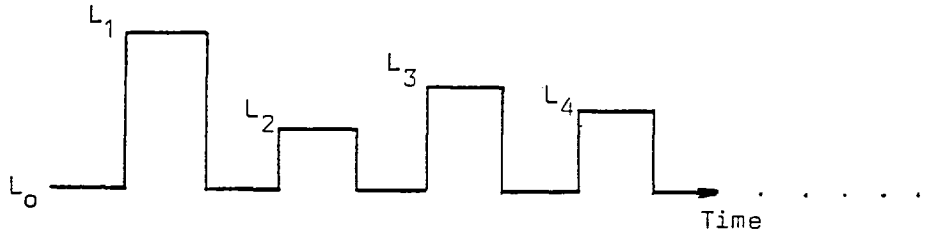

Fig. 5.1 Variation of Daily Load with Time

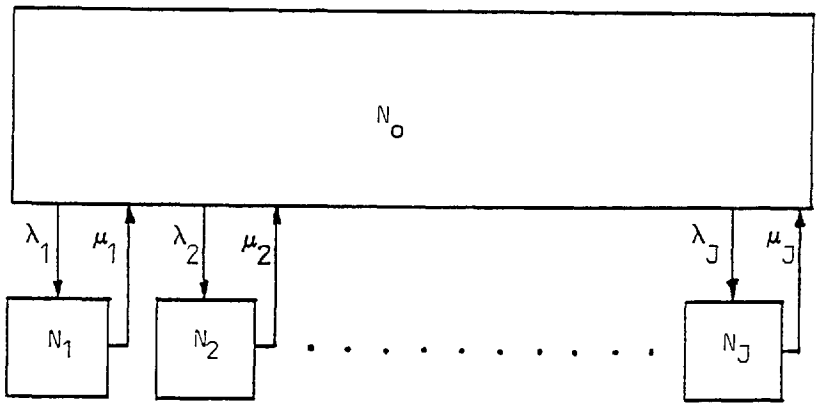

Fig. 5.2 Network State Transition Diagram

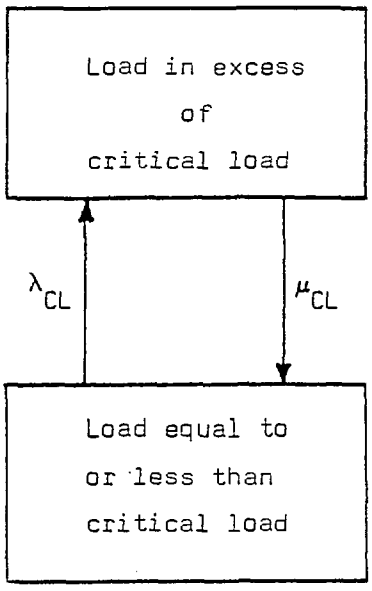

Fig. 5.3 Two State Load Model

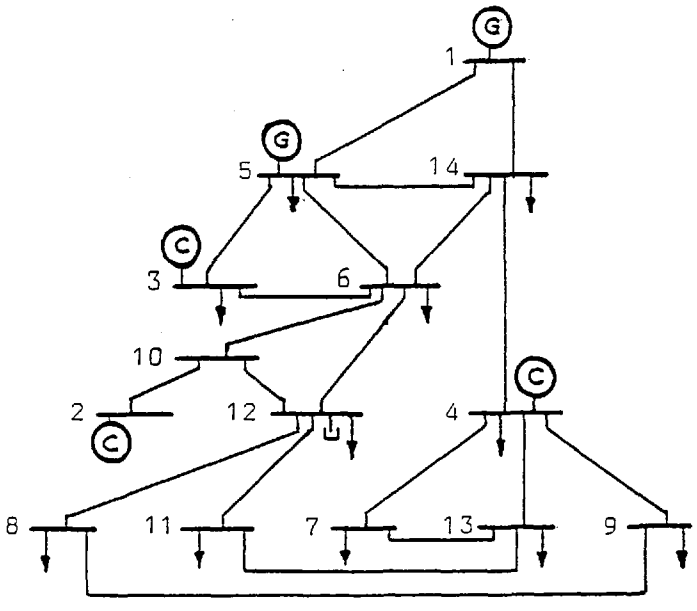

Fig. 5.4 IEEE 14 Bus Test Power System

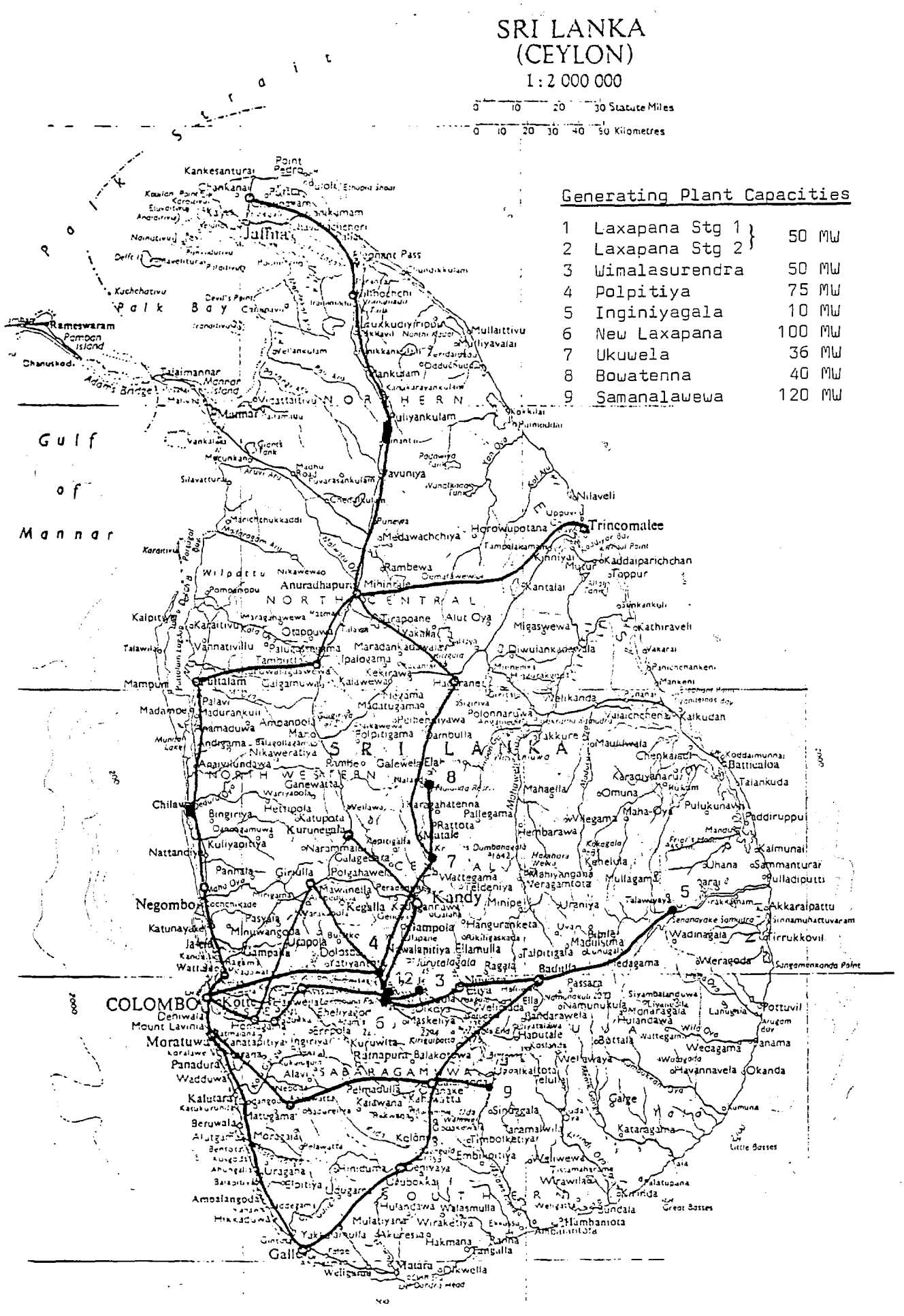

Fig. 5.5 Proposed Generation & Transmission Network for 1985

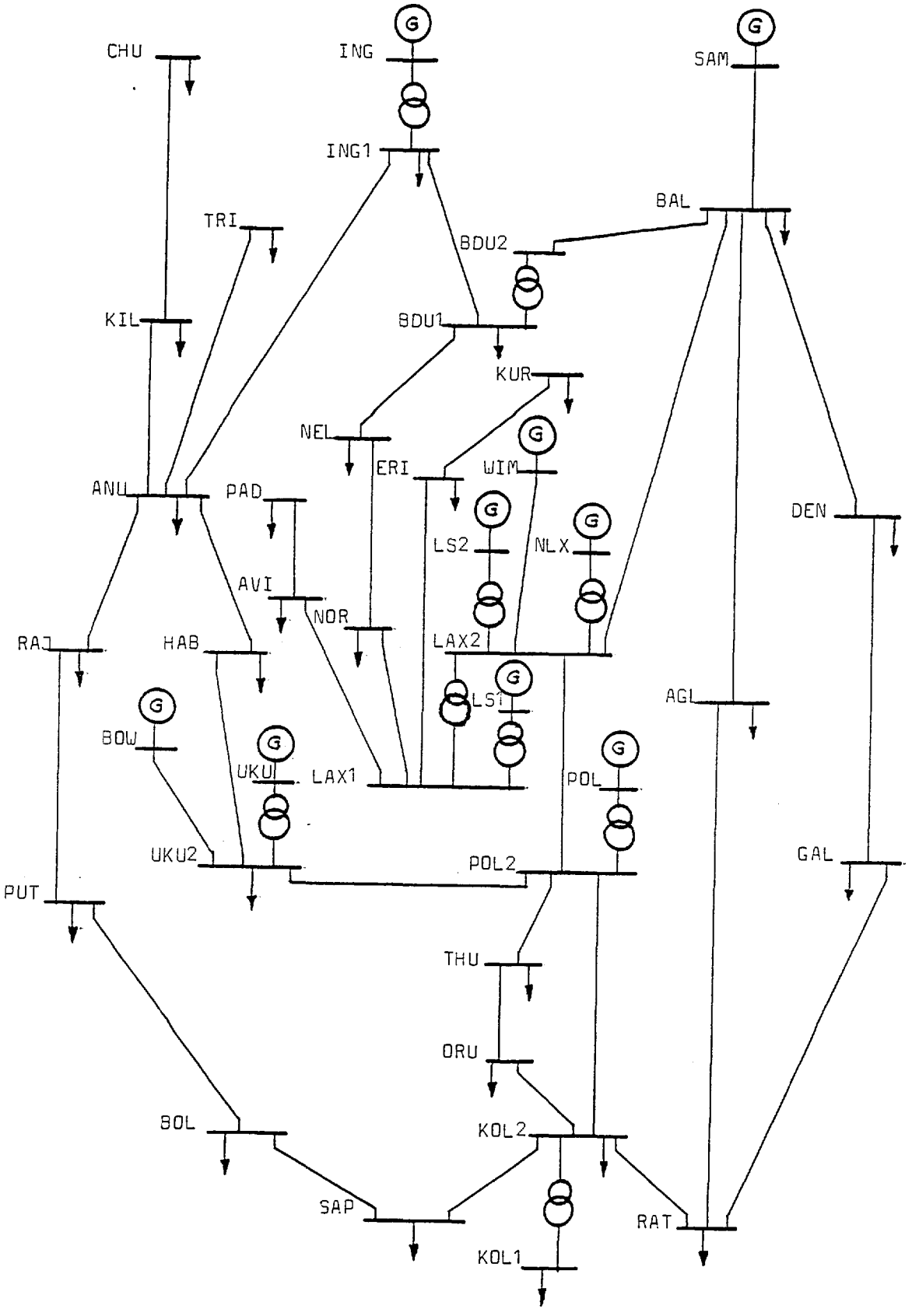

Fig. 5.6 Proposed Transmission Network for Sri Lanka Power System (1985)

Table 5.1 Line outage data

Line No.	SBUS	EBUS	λ (Failures/yr)	Repair Duration (hrs)	Probability of Failure $\times 10^{-3}$
1	1	5	1.06116	10	1.21
2	1	14	2.0	10	2.28
4	3	5	1.7752	10	2.02
5	3	6	1.53363	10	1.75
6	4	7	1.47008	10	1.68
7	4	9	1.14303	10	1.30
8	4	13	0.74864	10	0.85
9	4	14	0.04	336	1.53
10	5	6	1.58106	10	1.81
11	5	14	1.55918	10	1.78
13	6	12	0.04	336	1.53
14	6	14	0.3376	10	0.43
15	7	13	1.14867	10	1.31
16	8	9	1.10379	10	1.26
17	8	12	0.48560	10	0.55
19	11	12	1.55382	10	1.77
20	11	13	2.0	10	2.28

Table 5.2 Generator outage data

Bus No.	Unit No.	Unit Capacity (MW)	λ (Failures/yr)	Repair Duration (hrs)	Forced Outage Rate
1	1	10	2.0	89.4	0.02
	2 - 5	20	2.0	89.4	0.02
	6 -10	30	2.5	108.4	0.03
	11	40	3.0	121.7	0.04
5	12-14	40	3.0	121.7	0.04

Table 5.3 Load busbar failure probabilities and frequencies

IEEE 14 Bus Test System

Bus No.	(2) Failure Probability $\times 10^{-3}$	(3) Failure Frequency (occ/yr)	(4) Failure Probability $\times 10^{-3}$	(5) Failure Frequency (occ/yr)
3	0	0	0	0
4	0	0	0	0
5	0	0	0	0
6	1.16	1.4492	2.62	2.4325
7	4.61	5.7240	2.45	1.4381
8	4.19	5.2441	7.52	5.3253
9	4.63	5.7903	3.28	2.2151
11	1.47	1.8310	1.38	10.3370
12	4.50	5.6430	6.99	4.8319
13	4.43	5.5337	2.99	1.9472
14	1.90	2.3731	1.46	1.3583

Expected Unserved Energy = 3932.27 MWh/year

Service Failure Probability = 0.136

Table 5.4 Proposed Sri Lanka Power System For 1985

Busbar Identification

Bus	Code	Bus Name	Generating Capacity (MW)
1	SAM	SAMANALAWEWA	120
2	POL	POLPITIYA	75
3	LS1	LAXAPANA STAGE 1	25
4	LS2	LAXAPANA STAGE 2	25
5	UKU	UKUWELA	36
6	WIM	WIMALA SURENDRA	50
7	NLX	NEW LAXAPANA	100
8	BOW	BOWATENNA	40
9	ING	INGINIYAGALA	10
10	LAX2	LAXAPANA 132	-
11	POL2	POLPITIYA 132	-
12	UKU2	UKUWELA 132	-
13	KOL2	KOLONNAWA 132	-
14	KOL1	KOLONNAWA 66	-
15	LAX1	LAXAPANA 66	-
16	ORU	ORUWALA	-
17	THU	THULHIRIYA	-
18	GAL	GALLE	-
19	DEN	DENIYAYA	-
20	BAL	BALANGODA	-
21	RAT	RATMALANA	-
22	AGL	AGALAWATTE	-
23	BDU2	BADULLA 132	-
24	BDU1	BADULLA 66	-
25	ING1	INGINIYAGALA 66	-
26	NEL	NUWARAELIYA	-
27	NOR	NORTON	-
28	ERI	ERIYAGAMA	-
29	KUR	KURUNEGALA	-
30	SAP	SAPUGASKANDE	-
31	BOL	BOLAWATTE	-
32	PUT	PUTLAM	-

Table 5.4 continued

33	RAJ	RAJANGANA	-
34	ANU	ANURADHAPURA	-
35	TRI	TRINCOMALEE	-
36	KIL	KILINOCHCHI	-
37	AVI	AVISSAWELLA	-
38	PAD	PADUKKA	-
39	HAB	HABARANA	-
40	CHU	CHUNNAKAM	-

Table 5.5 Line and transformer outage data

40 Bus Sri Lanka System

Component	Failure Rate (failures/yr)	Repair Duration (hrs)	Probability of failure $\times 10^{-3}$
132 kV Transmission Line	0.02482/mile	15.0	0.0425/mile
132 kV Transformer	0.02	50.0	0.12

Table 5.6 Load busbar failure probabilities and frequencies

40 Bus Sri Lanka Power System

Bus No.	Code	Failure Probability x 10^{-3}	Failure Frequency (occ/yr)
10	LAX2	0.11	0.14038
11	POL2	0.09	0.11404
12	UKU2	1.31	1.65364
13	KOL2	0.10	0.12376
14	KOL1	0.04	0.04925
15	LAX1	0.00	0.00028
16	ORU	0.09	0.12086
17	THU	0.09	0.11161
18	GAL	0.15	0.18874
19	DEN	0.16	0.20297
20	BAL	0.11	0.13943
21	RAT	0.11	0.13749
22	AGL	0.12	0.15259
23	BDU2	0.09	0.11413
24	BDU1	0.04	0.05358
25	ING1	0.24	0.29956
26	NEL	0.00	0.00137
27	NOR	0.00	0.00218
28	ERI	0.00	0.00028
29	KUR	0.00	0.00028
30	SAP	0.11	0.13691
31	BOL	0.15	0.19607
32	PUT	0.44	0.57095
33	RAJ	1.65	2.08789
34	ANU	3.76	4.73376
35	TRI	4.47	5.62514
36	KIL	4.48	5.64019
37	AVI	0.00	0.00028
38	PAD	0.00	0.00028
39	HAB	3.23	4.06881
40	CHU	3.58	4.51146

Expected Unserved Energy = 1435.52 MWh/year
Service Failure Probability = 0.422×10^{-3} .

CHAPTER 6

CONCLUSIONS

6.1 The Role of the System Planner

The most important task of the system planner is to identify 'the' expansion plan among the many alternatives, that best suits the socio-economic development of the country under consideration at the same time as satisfying the varying requirements of the consumers. In this exercise, the results of analytical and simulation methods are of great assistance.

The uncertainties associated with future events are not easy to predict and hence these solution methods should use probabilistic modelling. However, the accurate modelling of actual systems is limited by social, political and economic facrors, some of which are difficult to quantify in absolute terms. Therefore the results of any mathematical method can only be a function of the accuracy of including various features in modelling the actual system. This requires that the final decision involving large investments which have considerable impact on the economy, especially in a developing country, cannot be entirely based on mathematical optima.

However, it must be appreciated that without the assistance of these mathematical methods it is practically impossible to evaluate the vast number of variations possible in a modern economy. Further these results are very valuable as useful background information to the system planner when deriving feasible expansion plans by intuition and experience. Consequently the final solution, which is usually a refinement of a mathematical optimum, will be more realistic than a solution obtained using these techniques alone. In this area the simulation and marginal analysis methods find important applications.

The intuitive considerations of the system planner in refining a solution requires a good understanding of the conflicts of various system parameters on the consumer requirements. It is also necessary to ensure a proper balance between demand growth and the energy supply development rate as otherwise over-investment causes unnecessary redundancy and hence under-utilisation of facilities, whilst under-investment causes inadequate facilities and unavoidable service interruptions. Mathematical methods provide the facility to investigate the effects and sensitivity of these parameters on the consumers. Hence the interactive nature of the solution procedure is very important and of considerable advantage to the system planner who is performing an important but difficult task.

6.2 Summary of the Thesis and Contributions

The theme of the thesis has been the use of probabilistic solution methods in three important areas of power system planning, namely, long term generation planning, reservoir operation planning and transmission planning.

The application of the probability methods to generation planning problems is discussed in Chapter 2. The necessity of producing consumer oriented reliability indices is emphasised and the importance of energy based reliability indices is discussed. The use of expected unserved energy as an indicator of reliability is favoured for reasons stated in Chapter 2. It has been recognised that the accuracy of the solutions depends upon the shape of the load duration curve which shows considerable variations over the years, especially after the implementation of load management concepts stemming from the fuel price increases in the early 70's. Further, the system growth is responsible for the increase in annual system demand and must be accurately represented. A technique is presented which modifies the shape of the load duration curve using a piece-wise linear representation to ensure greater accuracy.

Due to the discrete nature of system additions at any development stage, any planning method to be practical must produce compatible results. In this respect the use of integer programming techniques to determine optimum expansion plans is very important. However, it has been observed that ordinary integer programming techniques in these applications are not very efficient but the physical characteristics of certain system constraints can be utilised to modify the coefficients in an optimal way to produce results more efficiently. The modified method has been tested and considerable savings of computing times have been shown to be possible.

In Chapter 3 an interactive solution strategy which iterates between optimisation and simulation is discussed with particular reference to the effect of certain system parameters on the solution. The interactive alteration of these parameters allows the system planner to evolve a final solution which is more in line with the socio-economic development of the country. It is also shown, by taking representative examples, how the system planner can inject intuitive considerations based on engineering judgement to refine a solution when employing a mathematical method. Two case studies which have been specially selected to highlight the interactive solution strategy are presented.

The problem of optimal use of water storage in a hydro-thermal system is outlined in Chapter 4. In the formulation the objective has been to reduce the cost of thermal power generation necessary to satisfy a given load demand. The probabilistic nature of the water inflows to the reservoir is recognised and a stochastic model is used to obtain optimal discharges in each period using dynamic programming. Usually a typical hydro network consists of many cascaded or parallel operated hydro plants and poses difficulties in the analysis when using dynamic programming. Here the composite reservoir representation can be used

to simplify the linked plants to a single equivalent which receives, stores and releases an equivalent amount of potential energy. However, due to the unavailability of necessary historical data for a multireservoir system, the application of this technique to a system with one reservoir is illustrated.

The application of probability methods in transmission planning problems has been in evaluating reliability of consumer load points. Once the unreliable load points are identified, the system expansion necessary to improve these reliabilities can be determined. The improvements must be able to enhance the system reliability to a level compatible with the state of development of the country at a reasonable cost.

The application of a stochastic load flow technique to the problem of evaluating reliability indices of a typical power system is demonstrated in Chapter 5 by taking the Sri Lanka network as an example. The method proposed is more efficient than the conditional probability approach and other reported methods because the load flow is not repeated many times for each contingency. The method is extended to evaluate the expected unserved energy and the probability of service failure of the composite generation and transmission system.

One of the important concepts in maintaining the reliability of the power network is to ensure that any single outage does not lead to cascaded outages and the redundancy of the system should be such that the system can withstand certain credible outages until corrective actions are taken. In this respect the use of automatic load shedding and generation rescheduling are two important issues. However, in the evaluation of transmission reliability, only load shedding is considered and the rescheduling of generation after a service failure is not included. The incorporation of generation rescheduling could make the results more realistic.

Voltage reductions are being used in most countries to maintain supply during emergency situations. However this should not be considered as a design criterion and may be used perhaps as a last resort in system operation.

The reliability indices obtained in Chapter 5 can be used to evaluate the costs and effects of service interruption to the consumers. However, it is important to realise that after a certain point, the improvement of the system reliability can only be affected at an increasingly rising incremental cost. Therefore the reliability parameters can be used in a cost-benefit analysis to obtain the optimum level of reliability.

The original contributions made in this thesis are:

1. The investigation of the problem of long term generation planning with particular reference to the application of integer programming and probabilistic simulation techniques. Extensions have been made in the area of integer optimisation through the development of a method of modifying constraints to reduce computing times.

2. The examination of the interactive solution strategy used in the generation planning problem and the impact of various parameters on the solution procedure. Taking two case studies as examples, the interactive procedure for refining the final solution is also shown.

3. The use of stochastic dynamic programming for the problem of reservoir operation in a hydro-thermal power system.

4. The investigation of the use of load flow and stochastic load flow techniques in the area of transmission planning and the use of a SLF based reliability algorithm which is more efficient in computer use than the conditional probability

approach. The method includes the calculation of overall system performance indices and its applicability to a medium sized power system is demonstrated.

6.3 Suggestions for Further Development

The research reported in this thesis has aimed at developing efficient and accurate solution methods for system planning. However the results depend very much on the complexity of modelling. As the system planner gets more insight into the problem some of the limitations in modelling can be removed, thereby approaching to closer representation of the actual system. Therefore it is felt that the following areas need to be further explored to gain a better understanding of the implications of the system parameters on the optimum expansion plans:-

1. The extension of stochastic dynamic programming techniques for multireservoir systems using composite reservoir representation.
2. Formulation of the optimal water usage problem to optimise different objectives, eg. minimum spill-over a period, maximum irrigation return.
3. In the transmission reliability evaluation, perfect correlation of load data has been assumed. Although this is not a limitations of the method, the lack of correlation data has been the reason for the assumption. If avilable, proper correlation data could give more realistic results.
4. Generation reschedulling after a failure of a network element should be incorporated into the transmission reliability evaluation process.

APPENDIX A1

BRANCH & BOUND METHODS

A1.1 Definitions

The following are some of the definitions of the terminology used in integer programming techniques:

A Node is a vertex of the tree formed by partitioning (or branching) the original LP feasible space.

A node is Terminal when it is shown that none of its descendants can lead to an improved integer solution because,

i. the solution is integer feasible at that node

or ii. the solution is infeasible

or iii. the objective function is greater than that of a known integer solution.

A branch is Fathomed when all its descendant nodes are shown to be terminal.

A Tail is a node at which the integer solution is feasible. The optimum tail is the one for which the objective function is minimum.

A1.2 Integer Programming Problem

The pure integer programming problem can be stated as:

$$\min_x \quad c^T x \tag{A1.1}$$

subject to

$$Ax \geqslant b \tag{A1.2}$$

$$0 \leqslant x \leqslant u \tag{A1.3}$$

$$x \quad \text{integers of a given step size} \tag{A1.4}$$

where

x - vector of variables

c - coefficient vector of the objective function

A - coefficient matrix of the linear structural constraints

b - vector of right hand values of the constraints

u - vector of upper bounds on variables.

The linear equality and non-equality constraints define a feasible region in the n-dimensional Euclidean Space of which the coordinates are the variables x. The optimisation of linear functions over convex polytopes as defined by these constraints pose no computational problems.

Inside this polytope is a smaller convex polytope which contains the feasible integer points, each vertex of which is a feasible integer point and is known as the Convex Hull of Feasible Integer Points. The two-dimensional case is shown in fig. A1.1.

However, none of the nice mathematical properties which depend on the convexity of the feasible region carry over to the class of integer programming problems[27] and hence special methods must be used to solve them. Out of the many methods which are reported in the literature, Branch & Bound[27] is a very reliable method.

A1.3 Branch & Bound Method

In the BB algorithm, the convex feasible region of the LP is partitioned or branched into convex subsets and a lower bound on the function is obtained for each subset in the partition. If a solution satisfying the integer constraints can be found which has a function value less than the lower bound on all the subsets, then it is the optimum integer solution. The usual practice is to carry out the branching sequentially and a tree graph is formed to represent the subsets for which a bound is obtained.

A1.3.1 Method of Branching

The method of branching and the determination of bounds is based on the original Land & Doig algorithm[91]. In this method, before branching the feasible region, the LP solution is obtained and the variables which have to take discrete values are examined. If they are all at genuinely feasible integer values, then the LP solution is also the integer solution. Otherwise one variable x_k, which is not at an integer value is selected according to the penalty criteria used by Little et al[92]. In general, due to the convexity of the LP feasible region, the value of the function will increase not merely monotonically but at an increasing rate as x_k is fixed at values further from its value at the LP solution. The one-dimensional case is shown in fig. A1.2.

Obviously, the integer values immediately above and below the optimum value will have the lowest bounds on the function. Of the two, let $x_k = \gamma$ give the lower bound. Then the feasible region can be conveniently partitioned into the following mutually exclusive and exhaustive convex sets.

$$\begin{aligned} &\text{Set A :} && x_k \leqslant \gamma - 1 \\ &\text{Set B :} && \gamma - 1 < x_k < \gamma \\ &\text{Set C :} && x_k = \gamma \\ &\text{Set D :} && \gamma < x_k < \gamma + 1 \\ &\text{Set E :} && x_k \geqslant \gamma + 1 \end{aligned} \qquad \text{(A1.5)}$$

Sets B and D need not be considered as there can be no feasible integer solutions in them. This leaves only three sets to be considered and sets A, C and E will be referred to as left hand branch, main branch and right hand branch respectively.

Once the branches are obtained, a new LP is defined with the additional constraint $x_k = \gamma$. The situation now is similar to the original LP

problem, except that one variable is definitely fixed at an integer value. This makes the existing LP solution infeasible and a new LP solution is obtained. If the solution is still not integer feasible, another variable is selected for branching.

When the main branch is completed, the left and right hand branches are investigated for a better integer solution which has the lowest bound among all the feasible integer solutions.

A1.4 Generalised Branch & Bound Method

The basic difference between the ordinary BB and the Generalised Branch & Bound[28] (GBB) lies in the method of branching. Instead of branching according to the dichotomies of the form either

$$x_k \geq \gamma \tag{A1.6}$$

or

$$x_k \leq \gamma - 1 \tag{A1.7}$$

more general dichotomies of the form

$$\alpha^T x \geq \alpha_o \tag{A1.8}$$

or

$$\alpha^T x \leq \alpha_o - 1 \tag{A1.9}$$

are used in the GBB, where

α is a vector of coefficients and α_o is an integer, both yet to be determined.

Essentially GBB is a BB process in which the constraints of the original problem are replaced by simpler constraints having smaller coefficient values which are also approximately parallel to those of the original problem.

In practice a dichotomy $x_k \geq \gamma$ or $x_k \leq \gamma - 1$ may well be effective if one of the new constraints introduced is a fair approximation to a constraint of the Convex Hull of Feasible Integer Points. In this case the alternative branch is unlikely to contain any feasible integer points

with near optimal function values and as a consequence its descendant nodes should be easy to fathom[28].

It is information about the convex hull or, at least, the constraints defining the optimal vertex of it and its relationship to the constraints of the LP problem, that any new method must seek to find.

The nature of the GBB algorithm is based on the following important factors:-

i. since it appears to be difficult to deal with the constraints as they stand, it is necessary to find new constraints or to deduce useful information about feasible integer points. The best strategy is to devise some way of approximating the convex hull with constraints which will yield useful information.

ii. the new approximation to the optimal vertex should not become flatter than the old vertex.

iii. the extra amount of information that has to be stored in order to define a node should not be excessive.

A1.4.1 Selecting a Constraint

A constraint i for which

$$\frac{\mu_i}{\left[\sum_j a_{ij}^2\right]^{\frac{1}{2}}} \qquad \text{(A1.10)}$$

is the largest is selected as the best to branch, where μ_i is the multiplier of the constraint active at the LP solution and a_{ij} are the coefficients of the constraints.

A1.4.2 Approximating the Constraint

The original coefficients are approximated by finding the new coefficients which will make the constraints simpler and also approximately parallel to the original ones. The new coefficients are obtained by dividing the old coefficients by a certain quantity P determined as described below.

The choice of the divisor P is influenced by two considerations:

i. it is desirable to maximise the distance between the constraints

$$\alpha^T x \geq \alpha_o \qquad \text{and} \qquad \alpha^T x \leq \alpha_o - 1 \ ,$$

since this is a rough measure of the feasible space excluded by the dichotomy. This gap is proportional to

$$\frac{1}{\left[\Sigma\ \alpha_j^2\right]^{\frac{1}{2}}} \tag{A1.11}$$

Let δ be the difference between the actual coefficient and the product of P and the approximated coefficient.

i.e. $$\delta_j = a_j - P\alpha_j \tag{A1.12}$$

where $$\alpha_j = \text{nint}\left[\frac{|a_j|}{P}\right] \tag{A1.13}$$

and nint is the nearest integer to $[\cdot]$.

Substituting for α_j in eqn. (A1.11),

$$\frac{1}{\left[\Sigma\alpha_j^2\right]^{\frac{1}{2}}} = \frac{P}{\left[\Sigma\ a_j^2 - 2\Sigma\, a_j\ \delta_j + \Sigma\delta_j^2\right]^{\frac{1}{2}}}$$

and for small values of δ_j

$$\frac{1}{\left[\Sigma\alpha_j^2\right]^{\frac{1}{2}}} \simeq \frac{P}{\left[\Sigma\, a_j^2\right]^{\frac{1}{2}}} \ . \tag{A1.14}$$

ii. the new constraint should be, as nearly as possible, parallel to the old constraint, since this will:

a) keep the vertex sharp

b) be more likely to make the node corresponding to $\alpha^T x \leqslant \alpha_o - 1$ terminal

because the increase in the value of the objective function can be expected to depend upon the distance from the present LP optimum to the intersection of the new plane with the feasible space. This distance also depends on the distance of the plane from the LP solution and the angle between the original and the derived constraint.

The angle between two constraints of the form $a^T x=b$ and $\alpha^T x=\alpha_o$ is given by

$$\cos\theta = \frac{\Sigma a_j \alpha_j}{\left[\Sigma a_j^2 \Sigma \alpha_j^2\right]^{\frac{1}{2}}} \qquad \text{(A1.15)}$$

A convenient approximation to θ may be found by writing

$$\alpha_j = (a_j - \delta_j)/p$$

i.e.

$$\cos\theta = \frac{\Sigma a_j (a_j - \delta_j)/p}{\left[\Sigma a_j^2 \Sigma (a_j - \delta_j)^2/p^2\right]^{\frac{1}{2}}}$$

$$= \frac{1 - \dfrac{\Sigma a_j \delta_j}{\Sigma a_j^2}}{\left[1 - 2\dfrac{\Sigma a_j \delta_j}{\Sigma a_j^2} + \dfrac{\Sigma \delta_j^2}{\Sigma a_j^2}\right]^{\frac{1}{2}}}$$

$$= \frac{1 - s}{(1 - 2s + t)^{\frac{1}{2}}} \qquad \text{(A1.16)}$$

where

$$s = \frac{\Sigma a_j \delta_j}{\Sigma a_j^2} \quad \text{and} \quad t = \frac{\Sigma \delta_j^2}{\Sigma a_j^2} \quad .$$

From eqn. (A1.16), $\tan\theta$ could be written as

$$\tan\theta = \frac{(t - s^2)^{\frac{1}{2}}}{(1 - s)} = \frac{[t(1 - s^2/t)]^{\frac{1}{2}}}{(1 - s)}$$

For the values of a_j and δ_j which are of interest to this problem, the values of s and t are always likely to be small.

i.e. $1 \gg s$ and $1 \gg s^2/t$ and therefore

$$\tan\theta \simeq t^{\frac{1}{2}} .$$

For small angles, $\theta \simeq \tan\theta$

so $$\theta \simeq t^{\frac{1}{2}} \simeq \frac{\left[\Sigma \delta_j^2\right]^{\frac{1}{2}}}{\left[\Sigma a_j^2\right]^{\frac{1}{2}}} \quad . \qquad \text{(A1.17)}$$

To maximise the distance between constraints (eqn. (A1.14)) and to minimise the angle between them (eqn. (A1.17)) simultaneously, the value of P_i for constraint i should be chosen in such a way that

$$\frac{P_i}{\left[\Sigma_j \delta_{ij}^2\right]^{\frac{1}{2}}} \quad \text{is a maximum.} \qquad \text{(A1.18)}$$

Now from eqns. (A1.10) and (A1.18), the best constraint to branch on is the one for which

$$\frac{\mu_i P_i}{\left[\Sigma_j a_{ij}^2 \; \Sigma_j \delta_{ij}^2\right]^{\frac{1}{2}}} \qquad \text{(A1.19)}$$

is the greatest.

However, for the majority of problems, the effectiveness of the GBB is shown to be relatively insensitive to the choice of the constraint and the divisor[28].

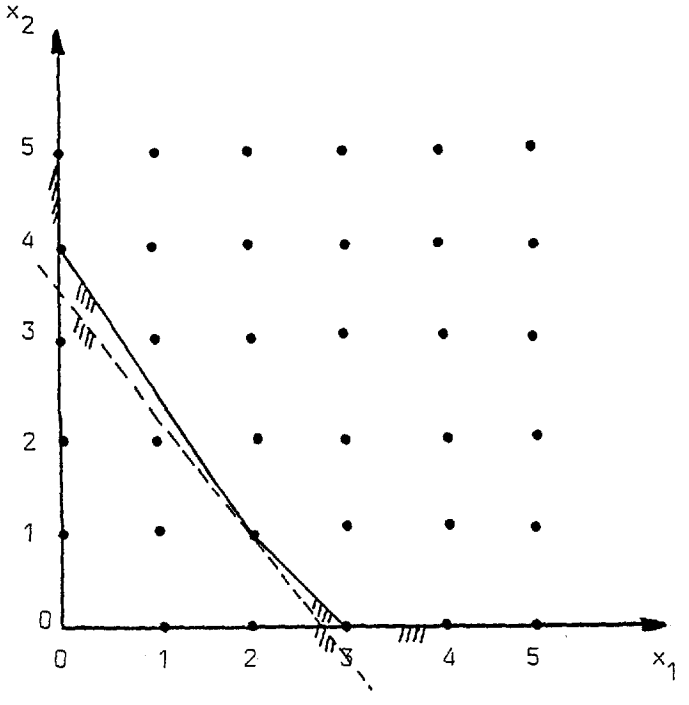

Fig. A1.1 Convex Polytope (----) and the Convex Hull of Feasible Integer Points(——)

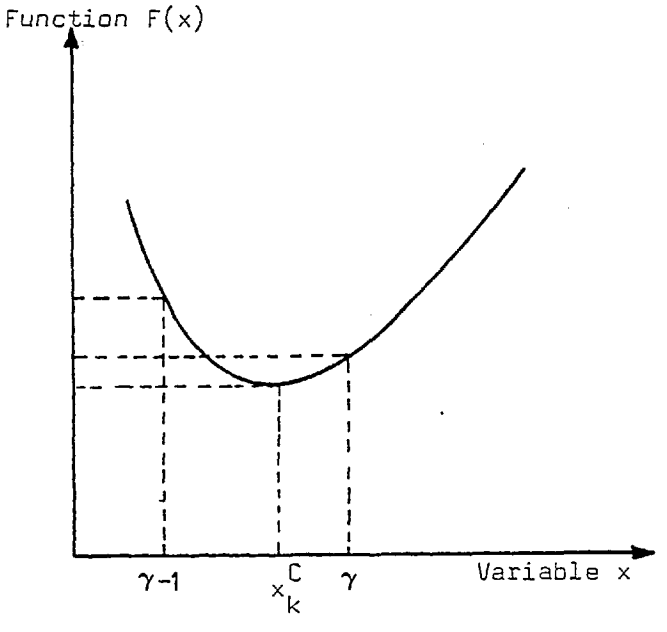

Fig. A1.2 Variation of F(x) with x

REFERENCES

1. Billinton, R., Medicherla, T.K.P. and Sachdev, M.S. 'Composite Generation & Transmission System Reliability Evaluation', Paper A 78 237-0, IEEE PES Winter Meeting, New York, January/February 1978.

2. AIEE Committee Report 'System Planning Practices', Trans. AIEE, Vol. PAS 74, 1955, pp 896-900.

3. Knight, U.G., Booth, R.R., Mallard, S.A. and Lewis, D.M. 'Computers in Power System Planning', Proc. IEEE, Vol. 62, No. 7, 1974, pp 872-883.

4. Manzoni, G., Paris, L. and Valtorta, M. 'Power System Planning Practice in Italy', Trans. IEEE, Vol. PAS 98, 1979, pp 689-699.

5. Fernando, P.N. 'The Planning of Electrical Power System Generation and Transmission Using Integer Programming and Reliability Concepts', PhD Thesis, University of London, 1976.

6. Billinton, R. 'Reliability of Power Supply Systems', Openning address to the IEE Conference on Reliability of Power Supply Systems, London, February 1977.

7. Billinton, R. 'Reliability of Power Supplies', IEE Journ. Electronics & Power, Vol. 24, No. 4, April 1978, pp 307-310.

8. Sullivan, R.L. 'Power System Planning', Mc Graw Hill, 1977.

9. Turvey, R. and Anderson, D. 'Electricity Economics - Essays and Case Studies', Johns Hopkins, 1977.

10. Webb, M.G. 'Power Sector Planning Mannual', Overseas Development Administration, June 1979.

11. Venikov, V.A. 'Cybernetics in Electric Power Systems', Mir Publishers, 1978.

12. Sjelvgren, D. 'Application of Mathematical Programming to Electric Power System Expansion Planning', PhD Thesis, Stockholm, 1976.

13. Singh, C. and Billinton, R. 'System Reliability Modelling & Evaluation', Hutchinson, 1977.

14. Khatib, H. 'The Economics of Reliability in Electrical Power Systems', Technicopy Limited, 1978.

15. Telson, M.L. 'The Economics of Alternative Levels of Reliability for Electric Power Generation Systems', Bell Journal of Economics, 1975, pp 679-694.

16. Shipley, R.B., Patton, A.D. and Denison, J.S. 'Power Reliability Cost vs Worth', Trans. IEEE, Vol. PAS 91, 1972, pp 2204-2212.

17. Masse , P. and Gibrat, R. 'Application of Linear Programming to Investments in the Electric Power Industry', Management Science, Vol. 3, 1957, pp 149-166.

18. Rhode, F.G, and Kalas, P.J. 'Economic Optimisation and Evaluation of Hydroelectric Power Systems', Water Power & Dam Construction, April 1975, pp 145-151.

19. Beglari, F. and Laughton, M.A. 'The Combined Costs Method for Optimal Economic Planning of an Electrical Power System', Trans. IEEE, Vol. PAS 94, 1975, pp 1935-1941.

20. Knight, U.G. 'Power Systems Engineering and Mathematics', Pergamon Press, 1972.

21. Irisarri, G. and El Abiad, A,H. 'Generation Planning Using Improved Measure and Worth of Reliability', Paper A 76 387-1, IEEE PES Summer Meeting, Portland, July 1976.

22. Adams, N., Beglari, F., Laughton, M.A. and Mitra, G. 'Mathematical Programming Systems in Electrical Power Generation, Transmission and Distribution Planning', Proc. 4th PSCC, Grenoble,1972.

23. Berrie, T.W. and Anderson, D. 'Power System Planning, Development Programs and Project Selection - A Discussion of Methods', Proc. 4th PSCC, Grenoble, 1972.

24. Rhode, F.G. and Zoller, E.Ch. 'Mixed Integer LP Model for Electric Power System Planning', Proc. Int. Conf. on Systems Modelling in Developing Countries, Bangkok, May 1978, pp 571-587.

25. Fernando, P.N., Induruwa, A.S., Cory, B.J. and McKechnie, A.J. 'Further Developments in Generation Planning Using Integer Programming', Proc. 6th PSCC, Darmstadt, August 1978, pp 12-21.

26. Gately, D. 'Investment Planning for the Electric Power Industry: An Integer Programming Approach', Report 7035, Department of Economics, University of Western Ontario, 1970.

27. Land, A.H. and Powell, S. 'Fortran Codes for Mathematical Programming', John Wiley, 1973.

28. Brocklehurst, E.R. 'Generalised Branch & Bound: A Method for Integer Linear Programs', Proc. 7th Int. Conf. on Operational Research, Japan, July 1975, pp 17-23.

29. Endrenyi, J. 'Reliability Modelling in Electric Power Systems', John Wiley, 1978.

30. AIEE Committee Report 'Application of Probability Methods to Generating Capacity Problems', Trans. AIEE, Vol. PAS 79, 1960, pp 1165-1182.

31. Billinton, R. 'Power System Reliability Evaluation', Gordon & Breach, 1970.

32. Billinton, R. 'Bibliography on the Application of Probability Methods in Power System Reliability Evaluation', Trans. IEEE, Vol. PAS 91, 1972, pp 649-660.

33. Hall, J.D., Ringlee, R.J. and Wood, A.J. 'Frequency and Duration Methods for Power System Reliability Calculations. Part I: Generation System Model', Trans. IEEE, Vol. PAS 87, 1968, pp 1787-1796.

34. Ringlee, R.J. and Wood, A.J. 'Frequency and Duration Methods for Power System Reliability Calculations. Part II: Demand Model and Capacity Reserve Model', Trans. IEEE, Vol. PAS 88, 1969, pp 375-388.

35. Khatib, H. 'Reliability of Generation Indices - A New Generation Reliability Index', Paper A 76 321-0, IEEE PES Summer Meeting, Portland, July 1976.

36. Billinton, R. and Endrenyi, J. 'Load Modelling in Power System Reliability Evaluation', Paper A 76 305-3, IEEE PES Summer Meeting, Portland, July 1976.

37. Billinton, R., Ringlee, R.J. and Wood, A.J. 'Power System Reliability Calculations', MIT Press, 1973.

38. Dhar, S.B. 'Generator Reliability Modelling for Static & Spinning Reserve Planning', Paper A 76 433-3, IEEE PES Summer Meeting, Portland, July 1976.

39. Billinton, R. and Singh, C. 'System Load Representation in Generating Capacity Reliability Studies. Part I: Model Formulation and Analysis', Trans. IEEE, Vol. PAS 91, 1972, pp 2125-2132.

40. Billinton, R. and Singh, C. 'System Load Representation in Generating Capacity Reliability Studies. Part II: Applications & Extensions', Trans. IEEE Vol. PAS 91, 1972, pp 2133-2143.

41. Boyer, G.C. 'Save Hours of Calculation by Using Energy Load and Load Duration Curves', Power, June 1947, pp 79-82.

42. Halperin, H. and Adler, H.A. 'Determination of Reserve Generating Capability', Trans. AIEE, Vol. PAS 77, 1958, pp 530-544.

43. Singh, C. and Billinton, R. 'Practical Application of the Frequency and Duration Method of Generating Capacity Evaluation', Proc. 7th PICA Conf., May 1971.

44. Booth, R.R. 'Power System Simulation Model Based on Probability Analysis', Trans. IEEE, Vol. PAS 91 1972, pp 62-69.

45. Baleriaux, H., Jamoulle, E. and Guertechin, Fr. Linard de. 'Simulation de l'exploitation d'un parc de machines thermiques de production d'electricite couple a des stations de pomage', Revue E, Vol. 5, No. 7, 1967, pp 3-24.

46. Vardi, J., Zahavi, J. and Avi-Itzhak, B. 'Reliability and Sensitivity Analysis in Power Systems', Paper A 76 346-7, IEEE PES Summer Meeting, Portland, July 1976.

47. Sager, M.A. and Wood, A.J. 'Corporate Model Programs for System Planning Evaluations', Trans. IEEE, Vol. PAS 91, 1972, pp 1079-1084.

48. Jaime, J.E. 'Generation Expansion of the Venezuelan System Using an Integer Program with Probabilistic Simulation', MSc Dissertation, University of London, 1978.

49. Operating Mannual 'A Computer Program for Generation Planning Using Mixed Integer Programming and Probabilistic Simulation', Kennedy & Donkin, August 1979.

50. Cypser, R.J. 'The Optimum Use of Water Storage in Hydro-Thermal Electric Systems', DSc Thesis, Massachusetts Institute of Technology, 1953.

51. Jamshidi, M. and Heidari, M. 'Modelling and Optimisation of Khuzestan Water Resources System', Proc. 6th IFAC Congress, Massachusetts, 1975.

52. Davis, R.E. and Pronovost, R. 'Two Stochastic Dynamic Programming Procedures for Long Term Reservoir Management', Paper C 72 493-5, IEEE PES Summer Meeting, San Francisco, July 1972.

53. Arvanitidis, N.V. and Rosing, J. 'Optimal Operation of Multireservoir Systems Using a Composite Representation', Trans. IEEE, Vol. PAS 89, 1970, pp 327-335.

54. Larson, R.E. and Keckler, W.G. 'Applications of Dynamic Programming to the Control of Water Resource Systems', Automatica, Vol. 5, 1969, pp 15-26.

55. Lindqvist, J. 'Operation of a Hydrothermal Electric System: A Multi-stage Decision Process', Trans. AIEE, Vol. PAS 81, 1962, pp 1-7.

56. Davis, R. 'Computational Procedure for Optimal Scheduling of Hydro-electric Power Generation - Deterministic Hydrology', Report, System Control Inc., California, July 1970.

57. Bellman, R. 'Dynamic Programming', Princeton University Press, 1957.

58. Nemhauser, G.L. 'Introduction to Dynamic Programming', John Wiley, 1966.

59. Bonaert, A.P. and Koivo, A.J. 'Computational Solution for Integrated Hydro Thermal Power Scheduling by Decomposition', Proc. 8th PICA Conf., 1973, pp 91-99.

60. Valvis, P.Z. and Th. Mathioudis, M. 'A New Approach to Hydroelectric Energy Studies', Water Power, August 1974, pp 288-295.

61. Nicholson, T.A.J. 'Optimisation in Industry', Longman, 1971.

62. Arvanitidis, N.V. and Rosing, J. 'Composite Representation of a Multireservoir Hydroelectric Power System', Trans. IEEE, Vol. PAS 89, 1970,pp 319-325.

63. Jackson, A.F. and Salvage, B. 'Costs of Electricity-Supply Interruptions to Industrial Consumers', Proc. IEE, Vol. 121, December 1974, pp 1575-1576.

64. Karunaratne, S., Sriyananda, H., Lucas, J.R. and Induruwa, A.S. 'The DC Network Analyser: An Effective Instrument for the Teaching of the Load Flow Problem', IJEEE, Vol. 15, No. 4, October 1978, pp 351-359.

65. Tinney, W.F. and Hart, C.E. 'Power Flow Solution by Newton's Method', Trans. IEEE, Vol. PAS 86, No. 11, 1967, pp 1449-1460.

66. Uemura, K. 'Power Flow Solutions by a Z-Matrix Type Method and its Application to Contingency Evaluation', Proc. 7th PICA Conf., May 1971.

67. Stott, B. 'Decoupled Newton Load Flow', Trans. IEEE, Vol. PAS 91, No. 5, 1972, pp 1955-1959.

68. Stott, B. and Alsac, O. 'Fast Decoupled Load Flow', Trans. IEEE, Vol. PAS 93, No. 3, 1974, pp 859-869.

69. Stanton, K.N. 'Reliability Analysis for Power System Applications', Trans. IEEE, Vol. PAS 88, No. 4, 1969, pp 431-437.

70. Allan, R.N., Billinton, R. and De Oliveira, M.F. 'An Efficient Algorithm for Deducing the Minimal Cuts and Reliability Indices of a General Network Configuration', Trans. IEEE, Vol. R 25, No. 4, 1976, pp 226-233.

71. Billinton, R. 'Composite System Reliability Evaluation', Trans. IEEE, Vol. PAS 88, No. 4, 1969, pp 276-281.

72. Cox, D.R. and Miller, H.D. 'The Theory of Stochastic Processes', Chapman and Hall, 1972.

73. Billinton, R. and Bhavaraju, M.P. 'Transmission Planning Using a Reliability Criterion, Part I: A Reliability Criterion', Trans. IEEE, Vol. PAS 89, No. 1, 1970, pp 28-34.

74. Brameller, A., Allan, R.N. and Hamam, Y.M. 'Sparsity', Pitman Publishing, 1976.

75. Zollenkoff, K. 'Bi-Factorisation-Basic Computational Algorithm and Programming Techniques', Large Sparse sets of Linear Equations, Academic Press, 1971.

76. Borkowska, B. 'Probabilistic Load Flow', Trans. IEEE, Vol. PAS 93, No. 3, 1974, pp 752-759.

77. Allan, R.N. and Al-Shakarchi, M.R.G. 'Probabilistic AC Load Flows', Proc. IEE, Vol. 123, June 1976, pp 531-536.

78. Gimenez, J.L. 'Power System Design for Varying Loads', MSc Dissertation, University of London, October 1977.

79. Allan, R.N., Grigg, C.H. and Al-Shakarchi, M.R.G. 'Numerical

Techniques in probabilistic Load Flow Problems', Int. Journ. for Num. Mtds. in Eng., Vol. 10, 1976, pp 853-860.

80. Dopazo, J.F., Kiltin, O.A. and Sasson, A.M. 'Stochastic Load Flows', Trans. IEEE, Vol. PAS 94, No. 2, 1975, pp 299-309.

81. Papoulis, A. 'Probability, Random Variables, and Stochastic Processes', Mc Graw Hill, 1965.

82. Aboytes, F. and Cory, B.J. 'An Alternative Formulation of the Stochastic Load Flow Method', Proc. 9th PICA conf., New Orleans, June 1975, pp 209-215.

83. Prada, R.B. and Cory, B.J. 'Stochastic Security Analysis', Proc. 14th UPEC, Loughborough, April 1979.

84. Aboytes, F. 'Stochastic Contingency Analysis', Trans. IEEE, Vol. PAS 97, No. 2, 1978, pp 335-341.

85. Hayes, T.P., Webb, R.P. and Day, J.T. 'A Quantitative Method for Transmission System Reliability Calculations', Paper A 78 566-2, IEEE PES Summer Meeting, Los Angeles, July 1978.

86. Hayes, T. 'Stochastic Load Flow Using Partially Coupled Equations', Proc. Control of Power System Conf., Texas, March 1977, pp 135-138.

87. Christiaanse, W.R. 'Reliability Calculations Including the Effects of Overloads and Maintenance', Trans. IEEE, Vol. PAS 90, 1971, pp 1664-1677.

88. Marks, G.E. 'A Method of Combining High Speed Contingency Load Flow Analysis with Stochastic Probability Methods to Calculate a Quantitative Measure of Overall Power System Reliability', Paper A 78 053-1, IEEE PES Winter Meeting, New York, January/February 1978.

89. Freris, L.L. and Sasson, A.M. 'Investigation of the Load Flow Problem', Proc. IEE, Vol. 115, October 1968, pp 1459-1470.

90. Karunaratne, S. Sriyananda, H., Lucas J.R., Induruwa, A.S. and Perera, B.S.P. 'A Fast Decoupled Sparsity Oriented Algorithm for the Solution of the Load Flow Problem in Electric Power Systems', Trans. IE (Sri Lanka), 1976, pp 36-44.

91. Land, A.H. and Doig, A.G. 'An Automatic Method of Solving Discrete Programming Problems', Econometrica, 28, No. 3, July 1960, pp 497-520.

92. Little, J.D.C., Murty, K.G., Sweeney, D.W. and Karel, C. 'An Algorithm for the Travelling Salesman Problem', Operations Research, 11, 1963, pp 972-989.