CONTINUUM MECHANICS APPLIED TO THE TENSION-LEVELLING PROCESS.

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by

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ABSTRACT.

The complexity of the tension-levelling process has hindered the development of an adequate theoretical model for design purposes.Empirical data and operating experience have been utilized to the present date.A brief background to the necessity for tension-levellers is presented together with the elementary analysis proposed by previous authors.

The development of a complete model for the levelling of an elastic non-linear work-hardening material is described, this model requiring numerical analysis by computer.Computerization has allowed the inclusion of the Bauschinger phenomenon by the assumption of a non-reversible model of strain-hardening.The discrepancies between this assumption and the assumption of isotropic hardening are readily apparent and are, to some extent, contrary to those expected.The analysis is directly applicable to material of good shape,the effects of bad shape being discussed.

Sufficient results for initial theoretical verification have been obtained on an instrumented experimental leveller built at Imperial College.The design of this machine and the techniques used for investigation of the relevant variables are described.Comparison between theoretical and practical results for temper rolled mild steel is presented.

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CHAPTER 1 INTRODUCTION.

Developments in the metal user industries are often reflected on the metal producing and rolling industries in the form of more stringent demands for quality and dimensional control.The latter has resulted in the rapid progress of rolling techniques to the extent where Sendzimir and 'constant gap' mills are in wide use.These mills are more rigid than their predecessors and can produce thinner more accurate gauges,aided by widespread introduction of techniques for computer control.Although it is now possible to roll to fine crown and gauge tolerances, the associated strip shape still presents intractable difficulties,especially in the thinner grades ie.<0.25mm.This problem is further aggravated when well shaped strip distorts on subsequent slitting as a result of a high internal stress pattern.

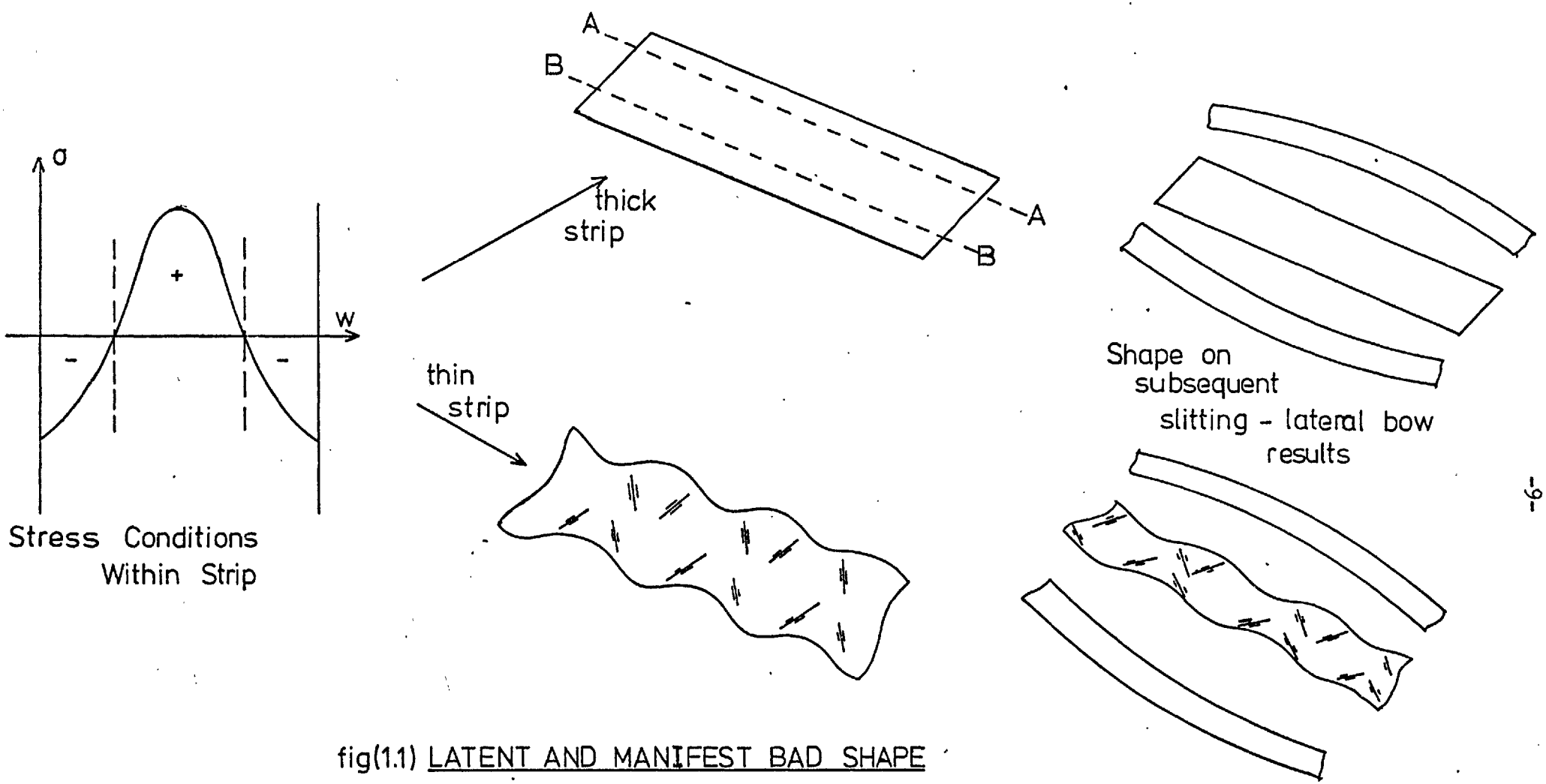
Bad shape may be introduced by a number of processes (Appendix 1), conversely it may be removed by a similar number,the most recent being the tension or roll stretch leveller.The process relies on the small elongation given to a material which is bent under the action of a tensile force.

1.1.0 The Origins of Bad Shape.

Bad shape results from a differential elongation over the width of cold rolled strip (1), which can be accommodated only by the introduction of high internal stress levels or,where conditions allow,by local elastic buckling of the strip.The latter results in the presence of waves or ripples extending along the length of the strip and covering the whole,or part, of the width.Where the section modulus is sufficient to resist buckling, the strip will appear to be stress-free.Subsequent slitting will,however, generally result in lateral bowing of the narrow widths (Fig.1.1).

The nature of the residual stress distributions contributing to bad shape is generally compressive,but the more complex forms of buckling are often the result of shearing forces.Figure 1.2 shows the relationship between the operating stress system and the buckling form where bad shape is manifest (2).The nature of the cold rolling process results in the more common occurrance of the 'long edge' and 'long middle' defects which are the result of a generalised mismatch between the stock and roll gap. Defects such as 'quarter buckle' or 'frilly edge' result from irregularities in comparative crown(6-17) which may be caused by roll wear,uneven temperature profiles, ridges on incoming strip etc. (12,13). These complex effects show in strip with low section moduli.

To control strip shape it is necessary to provide absolute correlation



fig(1.1) LATENT AND MANIFEST BAD SHAPE

between the strip and roll gap profiles during the cold rolling.The intoduction of roll bending techniques and coolant control (15-23) has greatly aided the control of simple shape,but even so the inter-relationship between shape and crown/gauge defects (2,5,6) makes adequate shape control virtually impossible.

The problems relatingto shape control are further discussed in Appendix 1, though it is true to say that even material produced by the most sophisticated rolling lines will, more often than not,require some form of shape correction.

1.2.0 Shape Correction Methods.

A number of methods for removing shape defects are available commercially, each one having its particular merits when suitably applied.The requirements of any shape correcting method are two-fold:-

(1) to remove any manifest effects

(2) to remove any residual stress system that will produce undesirable strip deflections in subsequent generations.

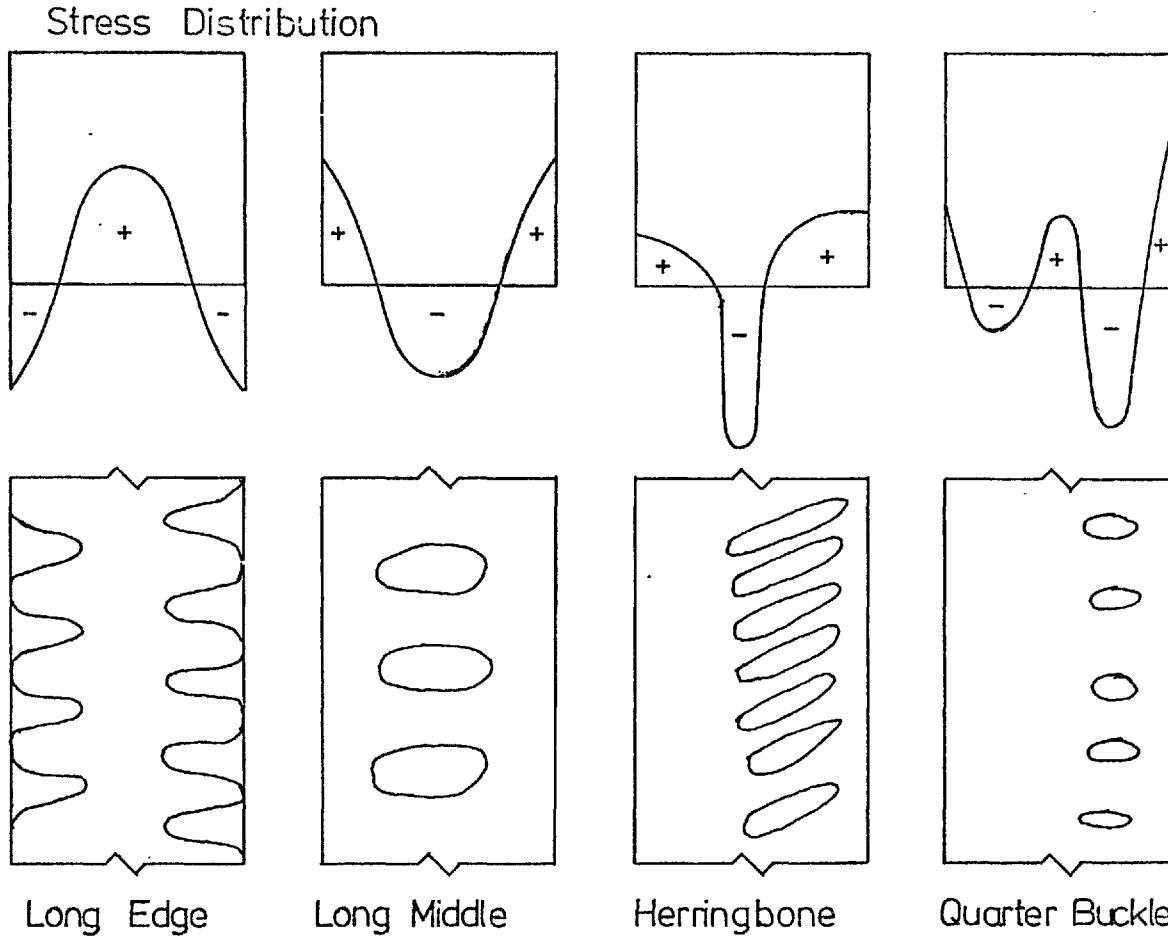
The most common method of stress relief used in metallurgical industries is,perhaps, that of heat treatment.Where strip or sheet products are concerned,this will generally result in undesirable effects on mechanical properties and, will not remove any manifest defects; mechanical methods must therefore be utilized,of which four are the most widely used.

(1) Temper Rolling.

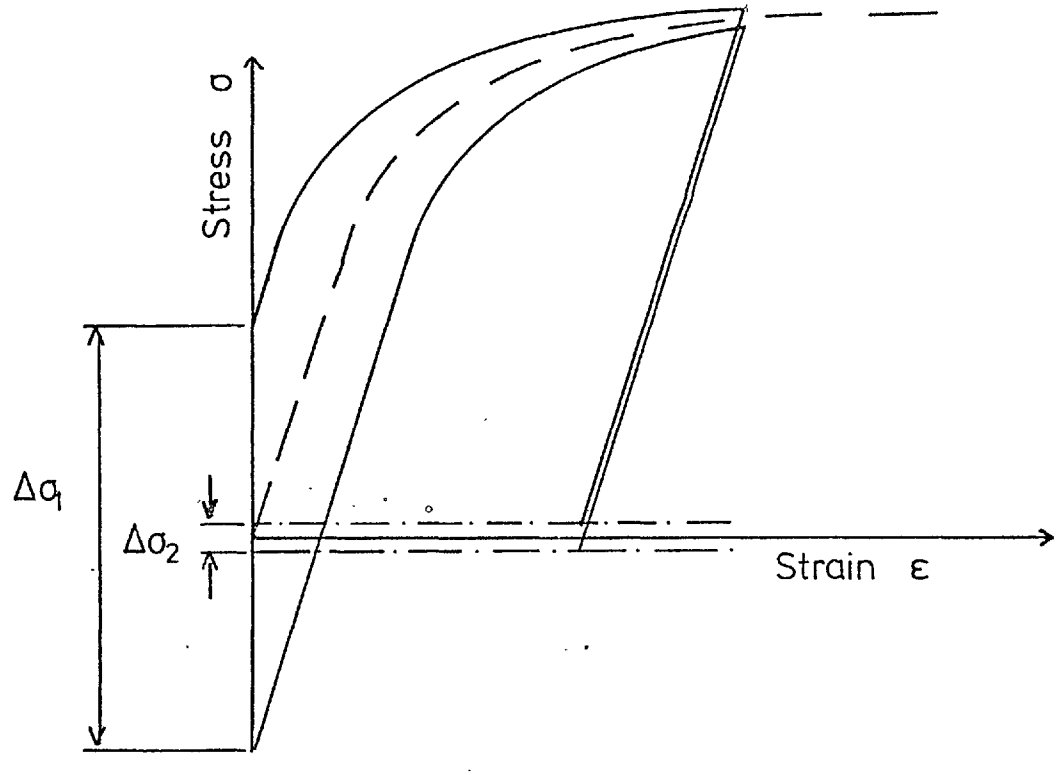
This is essentially a surface rolling process used,primarily,to induce large residual strains within the surface layer.The discontinuous nature of the deformation results in the suppression of the yield point and yield point elongation in annealed mild steels and materials exhibiting similar phenomena (23-32), and provides the main usefulness of the operation.The shape correcting effect of the temper mill is dependent upon the small e given to the material under the high tension conditions pertaining.In general a subsequent process to remove coil set and remaining shape defects is required.

(2) Stretching.

Shape correction by stretching involves the introduction of a small (1% to 3%) elongation into the material by the application of a uniaxial tensile force.This results in plastic yielding of the whole or part,of the cross section.Load concentration occurs in the 'short' regions of the strip width which are thus yielded preferentially.If the strain differential is replaced by the equivalent stress difference ($\Delta\sigma_1$ in Fig 1.3), thus applying equally to the manefest or latent forms of shape defects,straining the complete cross- scction by a suitable amount (ε)



fig(1.2) MANEFEST BUCKLING FORMS



fig(1.3) RESIDUAL STRESS ELIMINATION BY TENSION

results in a greatly reduced stress level on load relaxation($\Delta\sigma_2$).It will be apparent that providing all fibres have the same mechanical properties, the more nearly perfectly plastic the material the smaller will be the residual stress magnitudes.(This statement applies to all mechanical methods of levelling or stress relieving).

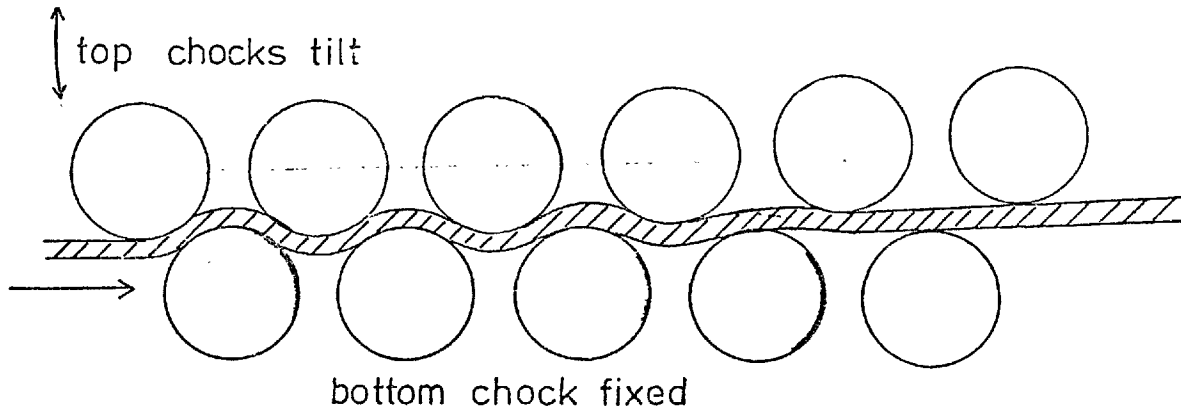
Stretching is performed as a jobbing operation for plate and sheet, the load normally being applied hydraulically via wedge grips, thus entailing material wastage(33).For strip, the straightening force may be applied continuously (34-36) by the use of tension bridles.In both these cases the use of simple tension to induce yielding leaves the process wide open to the problems of strip tearing where edge cracks and other concentration inducers for stress are present.

(3) Roller Levelling.

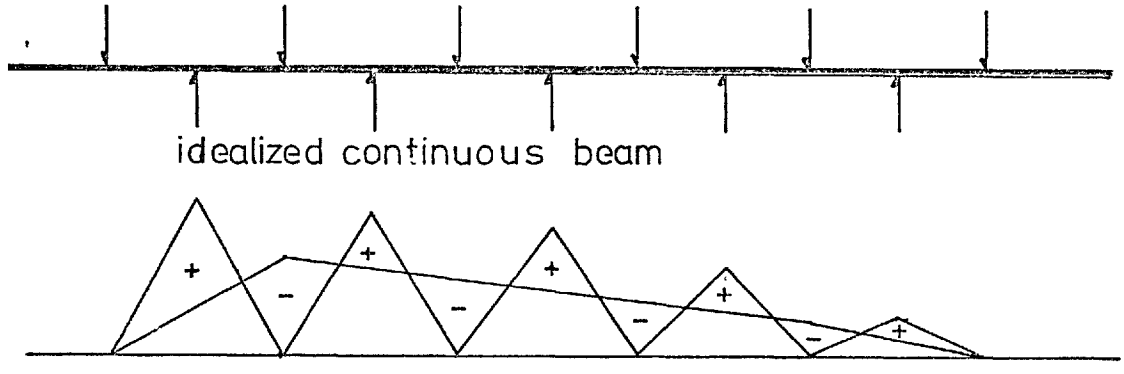
In this,the most widely used shape correction method,longitudinal yielding is again induced,but it is the result of bending strains.The complete cross-section never becoming plastic.The principle of the roller leveller is illustrated in fig.1.4.1,the equalization of strains being attained by subjecting the strip to reversals of stress.Bending moments imposed at each flexure are controlled either by the roller diameter or the roll interlock.The process may thus be idealized as a continuous beam acted upon by alternate positive and negative bending moments of decreasing magnitude (37-44) (Fig 1.4.2).Relief of residual stresses through the strip thickness (ie. coil set) are illustrated in Fig 1.4.3., the analysis simply being extended over the strip width where latent defects prevail.For this case any localized overall internal tensile or compressive force induces a small shift of the neutral axis of bending away from the centre of section,with the effect that a small overall strain is given to that region.

Roller levelling with rigid work rolls is thus only possible where the section modulus is sufficient to prevent localized buckling.Where manifest defects and thin strip are present, simply driving the work rolls may not be sufficient to remove bad shape.The modern roller leveller provides the facility for work roll flexing by means of a split back-up roll design which allows pressure variations along the length of the work roll (Fig 1.4.4).This allows the path length through the machine to be increased for the 'tight'areas,which results in an effective applied tension and thus elongation in those areas.

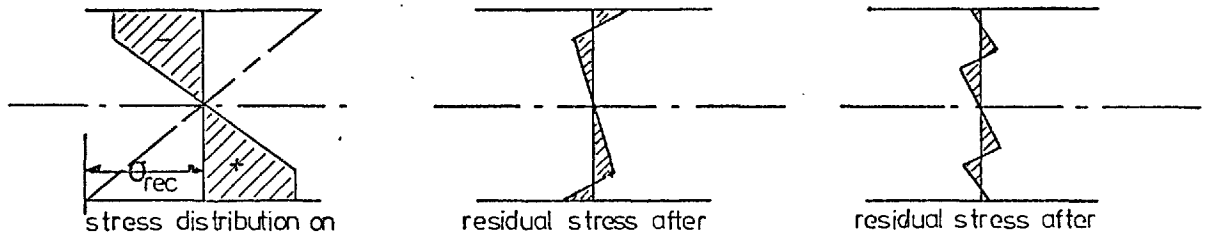
The diameter of the work rolls in the leveller is limited by bearing considerations,these rolls being driven.The state can thus be reached,with



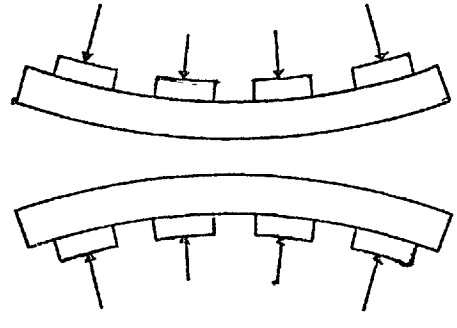
fig(1.4.1)



fig(1.4.2)



fig(1.4.3)



work roll flexing by sectioned back-up rollers

fig (1.4.4)

fig (1.4) THE ROLLER LEVELLER

strip<0.25 mm,where even flexure of the work rolls cannot induce sufficient extension to remove shape defects, the bend radius needing to be less than the roll radius.To overcome this problem many roller levelling lines have been enclosed in tension bridles to provide sufficient tension to induce the required strain.These lines are highly complex and require skilled operation.If tension bridles of sufficient power are installed it is possible to remove the roller drives and the roll flexing system and still produce strip of good shape (45).

(4) Tension or Roll-Stretch Levelling.

This process is thus a derivative of the roller leveller, the work rolls being undriven and unflexed.The applied tension,as well as providing the machine driving force serves three specific purposes

(a) Tension and bending stresses induce surface yielding at larger radii than with pure bending.

(b) The applied force encourages the strip to conform to the roll radius,thus eliminating problems arising from the inherent inflexibility of the strip and the need for control of roll interlock.

(c) Tensile loads concentrate within the 'short' areas of the strip width,leading to preferential deformation of those areas and hence shape correction.

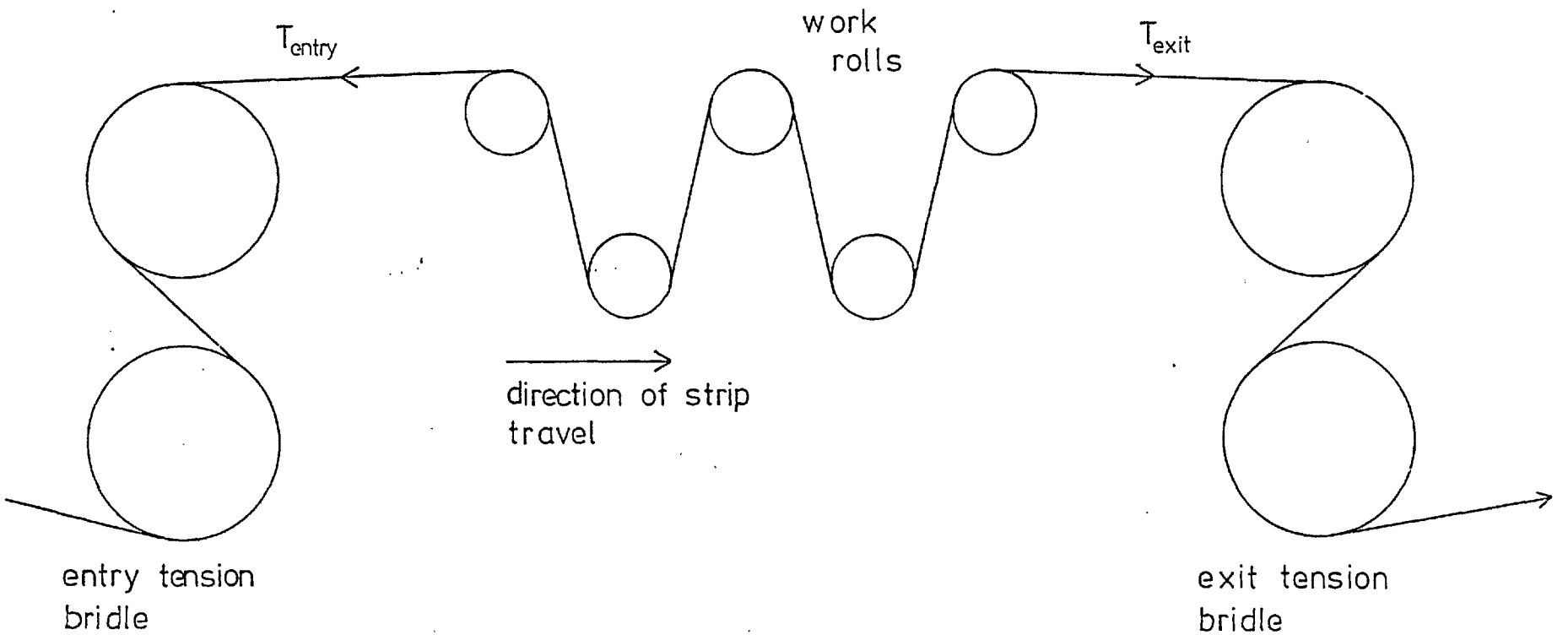
The basis of the tension leveller are shown in Fig.1.5, the work rolls being undriven,fixed and unflexed.Fixing the roll radii to a given value (which may be constant or variable through the machine),leaves the control of the process totally with the applied tension force.Accepting that the overall extension given to an element of strip width is proportional to the roll diameter, it becomes apparent that two methods of control are available:-

(1) The control of the applied tensile force, by the combination of a driven tension bridle at the leveller exit and a braked bridle at entry.

(2) The control of elongation, by means of a mechanically or electrically linked speed differential between exit and entry bridles.The tension level in the system is allowed to find its own equlibrium.

The method of elongation control has definite advantages over the former method in that it can be directly related to incoming strain differentials and will not result in overstraining in materials with a low work-hardening rate.

Since the radius to which the strip is bent is fixed by the roll diameter,the relationship between elongation,roll radius and applied tension



fig(1.5) THE TENSION-LEVELLER (SCHEMATIC)

for any material must be fairly accurately known to allow optimum design of a levelling installation; especially where bridle power requirements are concerned.Once this is achieved the advantages of the tension leveller over conventional levelling, such as higher line speeds (46),simpler operation and improved strip shape,will effect the wide scale installation of these levelling systems.

1.3.0 The Problem.

The development of a suitable theoretical model of tension levelling requires a break down of the process into the basic operations.The exit shape of strip passing through a roller-stretcher is dependent upon:-

(1) the entry shape and internal stress levels.

(2) the imposed overall elongation.

(3) the ratio of the applied tension stress to the mean material flow stress.

(4) the ratio of strip thickness to roll diameter.

(5) the number of reverse loadings.

Normal control of the levelling operation depends upon the tensile forces imposed on the strip,these forces being either directly set or being the result of an elongation setting.

For the general case it may be said that the second variable is the most important since the internal stress levels developed during the process are very much greater than the entry levels and at the same time are more uniform over the strip width. The problem can thus be reduced(in the first instance at least), to the derivation of the relationship between applied load and material elongation for an element of strip width,where machine settings and material properties remain constant.

Once this knowledge is obtained the effects of incoming bad shape may be introduced into the analysis.The increased analytical complexity is, however, unnecessary where industrial design is concerned,and the incidence of bad shape will be analysed only in part in section 3.7.The theoretical derivations of chapter 3 will allow leveller design to discard its hitherto 'experience' basis in favour of a more scientific approach.

CHAPTER 2- THE PROBLEM.

The stress distribution responsible for shape defects,is,to some extent,superfluous to the theoretical problem these stresses being orders of magnitude lower than those encountered during the correction process. The incidence of bad shape or of a transverse stress distribution at the leveller entry simply produces a readjustment of the tensile load over the cross-section.Each element of the width then deforms under its respective external stress level and, as will be seen in section 3.7., strain differeences are very quickly eliminated,(normally at the first roll), and the whole strip width then deforms at uniform applied stress for the bulk of the process.

The assumption that the overall levelling strains can be predicted in the terms of the mean applied stress is reasonably valid therefore, and until section 3.7.,the analysis will be concerned only with a unit width of strip,the applied load distribution being assumed uniform over that width.The problem for analysis is now specified as,'the relationship between applied load and overall extension during reverse loading under tension! Thus the basic problem is simply that of bending under tension. The elongation produced is a function of 1) the applied tensile load, 2) the material characteristics, 3) the bend radius, 4) the strip thickness, 5) the incoming longitudinal stress or strain distribution (through the strip thickness), ie.coil set or normal rolling residuals.

Variables (1),(3) and (4) are well defined but it is necessary to consider (2) and (5) in more detail.The material characteristics are important because the condition of horizontal equlibrium requires that the external tensile force is balanced by the integral of the internal stresses.Internal stresses are in turn dependent upon the bend radius and the applied tension.It is thus essential to provide an accurate mathmatical relationship to describe these stresses,which must include both the elastic and plastic stress relationships.The derivation of the necessary stress/ strain relationships and their use in the process analysis forms an important part of the theoretical considerations.

Variable (5) is relatively unimportant where the first leveller roll is concerned,since the entry stress levels will be minimal when compared with the stresses imposed by bending to a tight radius.Where the second & subsequent rolls are to be considered, the coil set and the bend radii will be of the same order and entry stresses cannot be ignored.

The actual process of bending and unbending strip round a roller may

be separated into three distinct operations, namely

(1) Bending to a roll radius.

(2) Elastic springback,as the bending influence is removed.

(3) Straightening, as the residual strip curvature is removed by the applied tension.

All three of these operations must be analysed as functions of the various process parameters.

With regard to the basic assumption that may be applied to the analysis, since the material is thin in relation to its width,it may be safely assumed that plane strain conditions obtain.The application to thin strip also allows the assumption that stresses tending to compress the fibres are negligible.

2.2.0 Previous Work.

The relatively recent introduction of roller-stretching as a useful production process has resulted in only a limited amount of direct publications.Those that have slipped through the web of industrial security have been concerned mainly with the advantages of tension levelling over conventional processes,the elements of machine design or with the 'hard' facts relating elongation and applied tension for specific machines and materials.(45-53).The few theoretical aspects discussed (45,49,52) are elementary and, in the case of Bland and Alters (45) misleading (unless the addition of tensile and compressive stresses really does increase the compressive magnitude !).

The basic aim of the current project is to develop a suitable theoretical analysis of roll-stretch levelling and a complete review of papers relating to the process is thus unnecessary here.The review paper by Sheppard and Roberts (Appendix 1) does however give a comprehensive picture if required.The relevant aspects to be considered here are three fold,

(1) The analysis of bending and springback under tension.

(2) The analysis of material stress/strain relationships.

(3) The analysis of strip shape.

This last aspect is the subject of numerous papers,although none are directly concerned with the levelling process.The complete review given in Appendix 1 will be used in section 3.7 where shape is considered in more detail.The current review will thus be concerned with the first two aspects only.

2.2.1 Bending under Tension.

Swift (54), over twenty years ago, laid the foundations for the theory of bending in conjunction with longitudinal tension,in a paper devoted to the analysis of the thinning accompanying the drawing of thin strip round a radius.For the analysis he assumed that elastic effects were negligible and considered the case of bending and subsequent straightening for a perfectly plastic and a linearly work-hardening material.The essential feature of the article is the realization that the solution to the problem is dependent upon the balance of the external force by the internal stress variations over the strip thickness.To accomplish this equlibrium it is necessary to have a greater portion of the strip thickness strained in tension than in compression.The resultant stress and strain patterns are illustrated in Fig 2.1, whence

$$2a\sigma_T = \int_{-a}^{+a} \sigma \, dy \qquad \text{----2.2.1}$$

It is noted that a linear strain distribution and thus zero shear stress is assumed.

The effect of adding tension to the simple bend is thus to cause a'shift' of the neutral axis of bending towards the compressive side of the strip. This in turn has three effects:

(1) The actual bend radius is reduced by the amount of shift.

(2) The outer strip surface contains a greater strain magnitude than the inner surface.

(3) An overall mean strip elongation occurs which is equal to the strain at the section centre.

Observations of the internal stress pattern indicates that for satisfaction of equation 2.2.1 the shift of the neutral axis is dependant upon:

(1) The applied load.

(2) The basic material strength ie. yield stress.

(3) The work-hardening nature of the material.

The 'shift' magnitude,and thus the mean strain increases with an increase in (1)and decreases with increases in (2) and (3).

Swift assumed that the material was bent to a radius sufficient to eliminate elastic stresses and,for the case of a perfectly plastic material derived the relationship for the mean longitudinal strain.

$$e = \frac{T}{2R_N\sigma_o} \qquad \text{----2.2.2}$$

where e is the strain pertaining to the central axis and is expressed also as

$$e = \frac{b}{R_N} \qquad \text{----2.2.3}$$

The paper continues, to give an account of the necessity for the tension passing onto the roll (T_0) to be less than that passing off (T_4) as a result of the work done on the sheet. This tension difference leads Swift into an analysis of the strip path over the roll. Figure 2.2 shows this path and illustrates the necessity for the lines of action of T_0 and T_4 about the roller axis to be of differing length. Moment considerations are thus satisfied. Figure 2.2 clearly shows the straightening action of T_4 as the strip passes off the roll. This is analysed by Swift as the reverse of the bending process thus predicting an overall strain of

$$e_{tot} = \frac{T}{R_N \sigma_o} \qquad \text{----2.2.4}$$

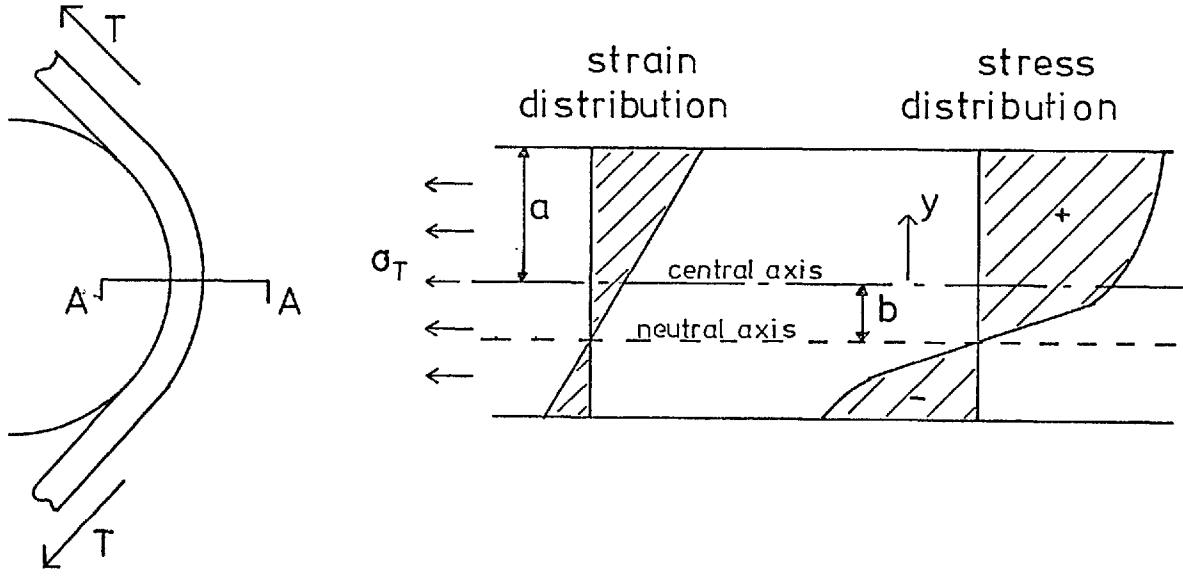
for a perfectly plastic material.

Equation 2.2.4 forms the basis of the theoretical considerations given in the papers by Hilsen, Levy and Polakowski (51,52) and Brock, Bowers and Smith (49). Hilsen et. al. utilize the relationship to define a 'K' factor for the levelling machine where

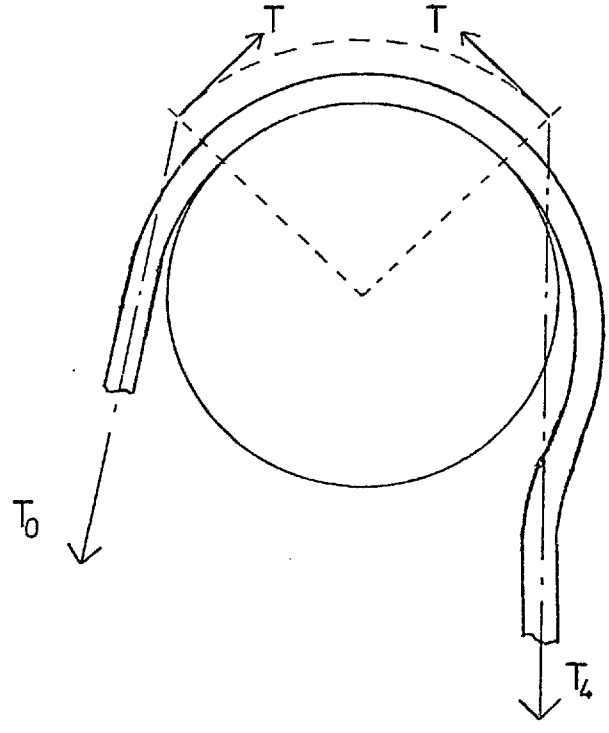
$$K = \frac{T_{EX}}{D_{MIN} \sigma_o} \qquad \text{----2.2.5}$$

K is thus equivalent to half the strain produced over the smallest roll and is plotted against in-process extension for various material/machine combinations. The discussion accompanying these diagrams, although basically correct, is misleading since D_{MIN} relates to the smallest work roll. Admittedly the strain increment obtained at the smallest roll will be the largest, but the statement that, 'for rapidly work hardening materials elongation is confined to this roll,' holds no water. As will be seen in Chapter 3 the relationship between work-hardening, roll diameter and elongation are very complex and no generalization can be made. Equally however, it could be said that under high work-hardening conditions there may be a minimum roll diameter corresponding to maximum straining. In a leveller all rolls will contribute to the extension to a greater or lesser extent.

A further discrepancy in the discussion arises in the consideration



fig(2.1) STRESS AND STRAIN DISTRIBUTIONS DURING BENDING UNDER TENSION



fig(2.2) STRIP PATH OVER A ROLL

of work-softening materials.It must be left until section 3.4.2 to provide a full explanation but,it will often prove the case that a work-softening material will require a higher tension to achieve a given elongation,than a similar work-hardening material.Hilsen et.al. propose that the reverse is always true.

As far as insight into the problem of roll radius attainment is concerned, the paper gives useful illustrations.It is apparent from the results given that although thin strip at high tensions may well conform to the roll at included wrap angles less than 30^{o} thicker strip ie.>0.25mm may require wrap angles of about 90^{o} before conformity is achieved,especially where hard materials are being processed on small diameter rollers. This problem of roll radius attainment is pointed out as one of the 'unknowns' of the tension levelling process.General opinion considers wrap angles of 40^{o} to be adequate for most purposes; the above results however,illustrate the need for a theoretical model for a roll radius attainment.(section 5.1.1).

The same basic equation (2.2.4) again forms the basis of the relationship for total strain proposed by the B.N.F. team (49).Their resultant equation shows an attempt to include elastic effects and is written

$$e_{tot} \leqslant N\left(\frac{T_{mean}}{R_o\,\sigma_o} - \frac{\sigma_o}{E}\right) \qquad \text{----2.2.6}$$

where N equals the number of work rolls of uniform radius R_o.The reasons for the choice of the elastic correction term σ_o/E are unclear.

It is interesting to note that equations2.2 to 2.6 contain no term relating to the strip thickness.Published work to date does, however, provide ample evidence that a definite dependence exists between elongation and strip thickness.Strip thickness,other variables being constant,has two effects.

(1) Increased thickness increases the degree of work-hardening, surface layers containing higher strain magnitudes.

(2) Increased thickness increases the bending moment required for roll conformity.

The first of these effects is the most critical since roll conformity can always be achieved under suitable conditions of applied tension and roll wrap.A generalized theory is thus required which includes the effects of strain-hardening and, to be complete, must include elastic effects.

2.2.2 The Stress/Strain Relationship.

The inclusion of work-hardening properties in any analysis requires a relevant and mathematical description of these properties.It has been found by Swift(55) that the plastic stress/strain relationships for material deforming under uniaxial tension are addequately defined by the equation (over the strain range applicable to tension levelling)

$$\bar{\sigma} = A(B + \bar{\varepsilon}^{p})^{m} \qquad \text{----2.2.8}$$

The assumption of plane-strain, together with the yielding mechanism complicates the longitudinal fibre stress-strain relationships even for those materials that may be assumed elastic perfectly plastic.The most suitable form of yield criterion to be used for a general analysis is that due to von Mises,the results of Siebel (56),Hundy and Green (57), Morrison (58),and Taylor and Quinney (59) demonstrating the close fit with reality obtained by the use of this criterion.The alternative criterion that of Tresca although possibly describing the initial yield of mild steel more accurately is not sufficient for the general case.

Experimental results,notably those of Morrison and Shepherd (58),have provided sufficient evidence to support the use of the Prandtl-Reuss equations for the description of plastic deformation.By utilizing these equations it is possible to determine the stress/strain relationships in the longitudinal direction for the plane-strain assumption.Two methods are available:

(1) by the use of a stiffness coefficient,as proposed by Marçal (60).

(2) by considering the expansion of the yield surface on the deviatoric plane (61,62).

The first method uses the linearity of the incremental form of the von Mises yield criterion and Prandtl-Reuss equations.Matrix analysis is thus possible.This suffers the drawback however,of extreme complexity when considering the reverse loading conditions and lacks easy physical interpretation.

Method two is capable of clear interpretation, solutions having been given by Alexander (61) and Denton (62) for non-,and work-hardening materials respectively.The complexity of the solution given by Denton hinders its application to real problems and a more convenient approach is given in section 3.2.

2.2.3 Elastic Effects.

The acceptance of material elasticity has two results

(1) the complete section can never become plastic.

(2) elastic spring-back occurs on the release of applied bending moments.

The first consideration is self-explanatory. Springback has been analysed for the case of simple bending by many authors (41,63-76). The addition of external tensile forces increases the degree of springback for purely elastic deformation and reduces the effect where plastic deformation is present. Benedyk and Newnham (77) have measured the springback of copper foils (0.051 - 0.089 mm) after bending under tension. The explanatory theory developed does not however, allow for the plane-strain conditions imposed during the test, elastic/plastic effects, or any work-hardening nature of the material.

CHAPTER 3 - PROBLEM ANALYSIS.

3.1.0 Theoretical Assumptions.

3.1.1 Plastic Relationships.

The range of roll diameter:strip thickness ratios used in tension levelling often allows a substantial portion of the strip to remain elastic during bending.A further portion,possibly the bulk of the section,lies within the range of strains equivalent to two or three times the yield strain.Within this range the elastic components of strain are by no means insignificant and a Prandtl-Reuss based analysis is thus necessary.These equations may be written for work-hardening materials as

$$d\varepsilon_i = \frac{d\bar{\sigma}}{\bar{\sigma} H'} \left(\sigma_i - \frac{1}{2}(\sigma_j + \sigma_k)\right) + \frac{1}{E}\left(d\sigma_i - \nu(d\sigma_j + d\sigma_k)\right) \qquad \text{----3.1.1.}$$

$$\text{where } H' = \frac{d\bar{\sigma}}{d\bar{\varepsilon}^p} \qquad \text{----3.1.2.}$$

From the relationship defined in equation 2.2.8.

$$\bar{\sigma} = A(B + \bar{\varepsilon}^p)^m$$

$$\text{thus } H' = \frac{d\bar{\sigma}}{d\bar{\varepsilon}^p} = \frac{mA^{1/m}}{\bar{\sigma}^{\frac{1-m}{m}}} \qquad \text{---- 3.1.3.}$$

The use of equation 2.2.8. to describe the equivalent stress/equivalent strain curve thus allows a simple expression of the work-hardening parameter H' in terms of the current equivalent stress $\bar{\sigma}$.

For this work equation 2.2.8 is modified to

$$\bar{\sigma} = A(B + \bar{\varepsilon})^m \qquad \text{----3.1.4.}$$

It has been found in the current work that the inclusion of elastic effects by the use of $\bar{\varepsilon}$ rather than $\bar{\varepsilon}^p$ in equation 2.2.8. makes no difference to curve fitting accuracy for any given stress/strain curve. The advantages of equation 3.1.4. is that the constants A,B and m may be determined without reference to the elastic modulus of the material.This latter constant is often open to error.

3.1.2 Plane Strain.

Strip width to thickness ratios in industrial levelling applications normally lie within the range 100 to 300.A massive resistance to transverse

straining is therefore present,and the contraction strains necessary to accomodate material elongation occur in the thickness direction.There is little doubt that plane-strain conditions obtain.

3.1.3. Zero Thickness Stress.

The strip thickness is that to which the "thin cylinder"assumption would be given in other contexts.The assumption is most certainly valid, for the springback and straightening operations although strip/roll contact during bending will produce pressure stresses,at least in the lower portion of the strip.Section 5.1.2 will demonstrate the 1% yield stress nature of these stresses,which,is negligible in comparison with the longitudinal stress levels.

3.1.4. Zero Shear Stresses.

The assumption that sections plane before bending remain plane afterwards allows the assumption of a linear strain distribution through the strip thickness.Shear stresses will undoubtably be present as a result of the rapid change of bending moment which occurs as the strip approaches a leveller roll.The level of these stresses is a matter for debate and is considered further in section5.1.4.The analysis would be impossible without the linear strain approximation and the fact that some form of solution is possible greatly outweighs any errors incurred by the assumption.

3.1.5. Isotropy and Hardening Nature.

The material is assumed to be homogeneous and isotropic,although the Bauschinger effect is not ignored because of the interesting effect it has on leveller theory.

Two theoretical interpretations of hardening on reverse loading are available.The model of isotropic work hardening assumes that the yield stress in compression will be equal to the final tensile flow stress during the previous deformation.The kinematic hardening model assumes that the elastic range (tensile to compressive yield) of the material remains constant irrespective of deformation.Tensile plastic straining thus results in a decrease of compressive yield over the virgin state.This second model provides some explanation of the Bauschinger effect.

For simplicity the theoretical analysis is derived initially in terms of the isotropic model, the differences encountered by the inclusion of kinematic hardening characteristics being expounded where necessary.It is assumed throughout that in the virgin state the material processes the same stress/strain relationship in tension and compression.

3.1.6. The Angle of Wrap.

It is assumed in the basic theoretical derivation that the bent strip conforms (instantaneously at least) to the roll radius.The implications of this assumption are discussed later together with an analysis of the conditions of tension and wrap angle necessary to achieve the above under various levelling conditions.

3.1.7. Strain Rate.

It is assumed that the discrepancy between the strain rates present during levelling and those of the preliminary tensile tests is negligible as far as material properties are concerned.

This,and the other assumptions made for theoretical simplicity will be discussed in section 5.1 .

3.2.0. The Stress/Strain Relationships.

Shape correction depends upon the equalization of longitudinal strains. An accurate description of the stress/strain relationship in this direction under the conditions obtained in a leveller will thus allow the problem to be reduced to two rather than three dimensions; transverse stresses finding their own equlibrium and having no effect on the analysis.

The problem is approached by considering the expanding yield surface on the deviatoric plane (61,62,78,79).This method is based upon the vectorial representation of a three dimensional stress by hydrostatic and deviatoric components,Fig 3.1. Bridgeman (81) and many other workers have demonstrated the independence of flow stresses to the hydrostatic pressure,thus allowing the simple consideration of the deviatoric components. The new co-ordinate system,representing these reduced stresses,has its origin at 0' (fig 3.1),the axes projecting onto the deviatoric plane at 120° separation (fig 3.2).This projection is accompanied by a scale reduction factor of $\sqrt{\frac{2}{3}}$ which is normally overcome by increasing the scale by $\sqrt{3/2}$.

The von Mises yield criterion for principal stresses

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2\bar{\sigma}^2 \qquad \text{----3.2.1.}$$

when written in terms of deviatoric stresses becomes

$$\sigma'_1{}^2 + \sigma'_2{}^2 + \sigma'_3{}^2 = \frac{2}{3}\bar{\sigma}^2 \qquad \text{----3.2.2.}$$

and is therefore represented by a circle radius $\sqrt{\frac{2}{3}}\ \bar{\sigma}$ in deviatoric stress space. The multiplication factor $\sqrt{3/2}$ allows the criterion to be plotted as a circle radius. $\bar{\sigma}$ on the deviatoric plane.

Since, by defination, deviatoric stresses, may be written

$$\sigma'_i = \sigma_i - \sigma_m \qquad \text{----3.2.3.}$$

where

$$\sigma_m = (\sigma_1 + \sigma_2 + \sigma_3)/3 \qquad \text{----3.2.4.}$$

then

$$\sigma'_1 + \sigma'_2 + \sigma'_3 = 0 \qquad \text{----3.2.5.}$$

By figure 3.2. we have,

$$\text{AB: BC} = 2:k \qquad \text{----3.2.6.}$$

If one of the stresses, say σ_2 is zero, then from equation 3.2.3.

$$\sigma'_2 = -\sigma_m \qquad \text{----3.2.7.}$$

and by figure 3.2

$$\sigma_1 = \text{O' D and } \sigma_3 = \text{DA} \qquad \text{----3.2.8.}$$

A useful visualization of actual stresses is thus provided.

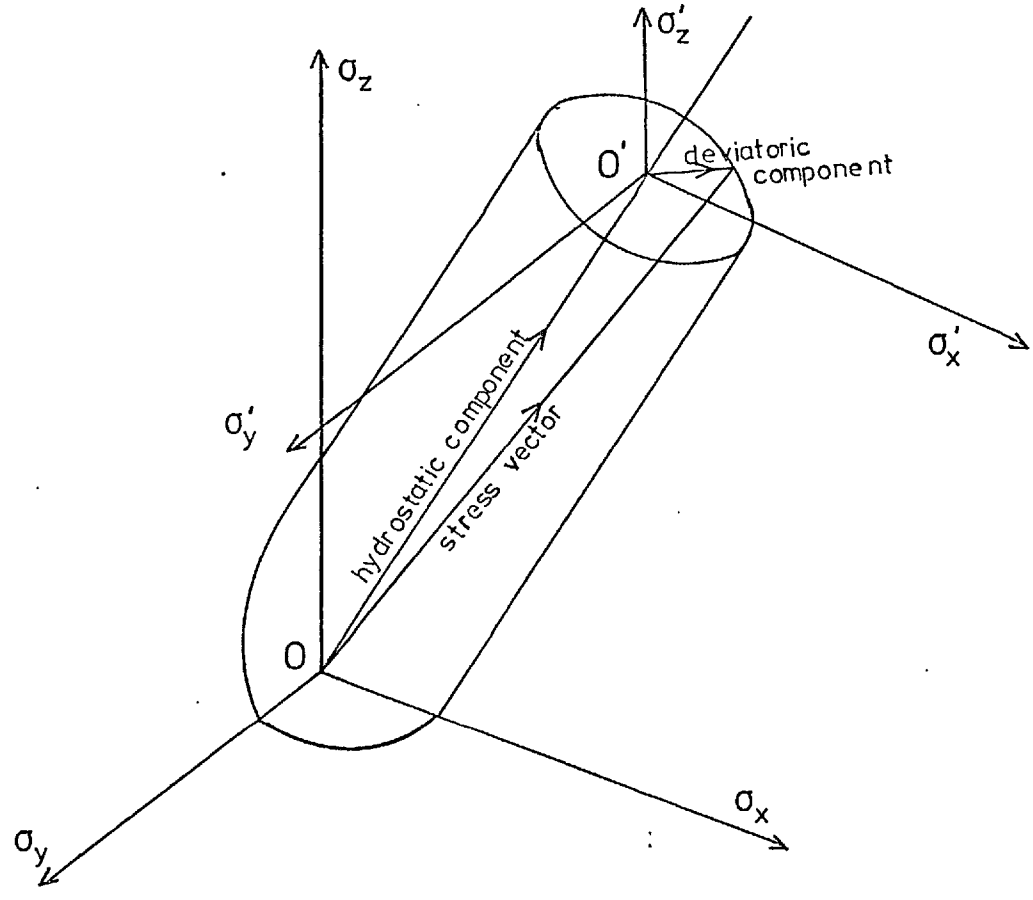
To enable the meaningful plotting of strains on the same diagram, it is convenient to use a multiplication factor such that the stress/strain points coincide in the elastic range(78,79) ie. within the yield locus. By considering a system of pure shear depicted in Fig 3.2

$$\tau_{12} = \sigma_1 = -\sigma_2 \text{ , and } \sigma_3 = 0 \qquad \text{----3.2.9.}$$

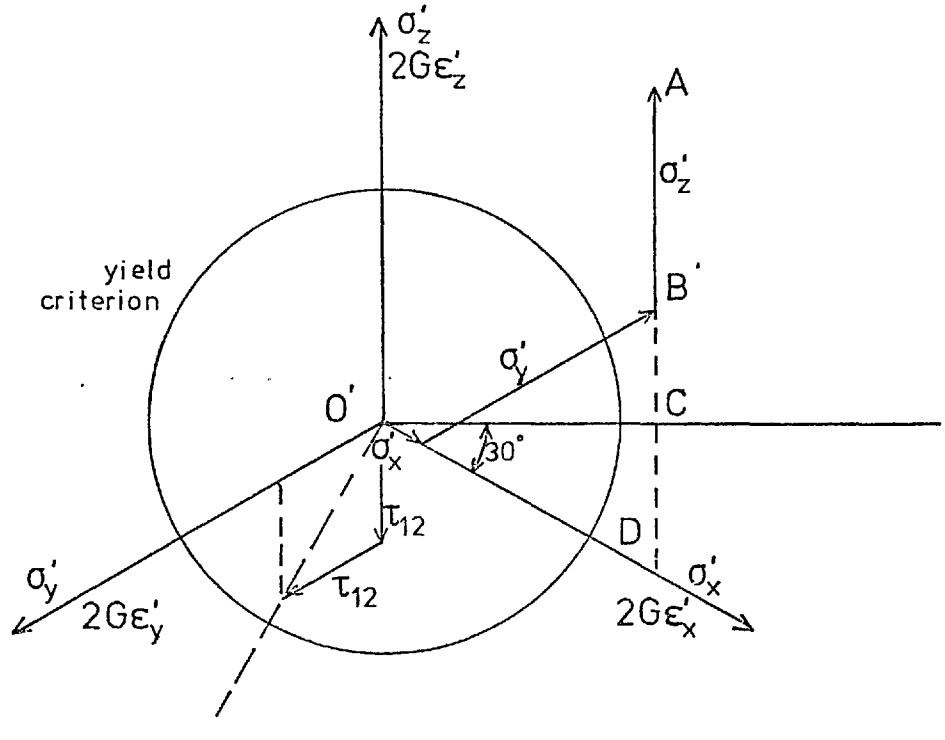
$$\therefore \varepsilon_1 = -\varepsilon_2 = \frac{\tau_{12}}{E}(1+\nu) = \gamma_{12}/2 \qquad \text{----3.2.10.}$$

But

$$\gamma_{12} = \tau_{12}/G \qquad \text{----3.2.11.}$$



fig(3.1) THREE-DIMENSIONAL STRESS SYSTEM



fig(3.2) DEVIATORIC PLANE REPRESENTATION

$$\therefore \quad G = E/2(1+\nu) \qquad \text{----3.2.12.}$$

and

$$\varepsilon \times 2G = \tau \qquad \text{----3.2.13.}$$

A suitable multiplication factor is thus 2G.

For the current topic and using the notation of Fig 3.3, we have the assumptions

(1) the material is homogeneous and isotropic.

(2) plane strain conditions exist.Hence :

$$\varepsilon_z = d\varepsilon_z = 0 \qquad \text{----3.2.14.}$$

(3) zero thickness stress.Hence:

$$\sigma_y = d\sigma_y = 0 \qquad \text{----3.2.15.}$$

(4) the von Mises criterion (3.2.2) is obeyed,thus with the conditions of 3.2.14 and 3.2.15 we may write:

$$\sigma_x^2 + \sigma_z^2 - \sigma_x \sigma_z = \bar{\sigma}^2 \qquad \text{----3.2.16.}$$

3.2.1. Solution- Loading Isotropic Hardening.

The non-hardening case has been solved by Alexander (61),both analytically and geometrically.The possibility of the geometrical construction leads directly to the mathmatical representation of this construction.This was initially analysed for the perfectly plastic case,and the simplicity of the final result indicated the possibility of a similar analysis for non-linear work-hardening materials.The analysis subsequently developed is given below.A shortened form of this derivation forms reference(80) which is included as Appendix 9 .

3.2.1a. Elastic Conditions.

On the application of a simple longitudinal loading force the material initially behaves elasticallly,viz.

$$\sigma_z = \nu\sigma_x \qquad \text{----3.2.17.}$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_z) = \frac{\sigma_x}{E}(1-\nu^2) \qquad \text{----3.2.18.}$$

$$\varepsilon_y = \frac{-\nu}{1-\nu}\varepsilon_x = \frac{-\sigma_x}{E}(\nu+\nu^2) \qquad \text{----3.2.19.}$$

equations 3.2.14 and 3.2.15 being applicable.

3.2.1b. Yield Conditions.

From equation 3.2.16. and 3.2.17. we have at yield

$$\sigma_{x_o}^2 \ (1 - \nu + \nu^2) \quad = \quad \bar{\sigma}_o^2 \qquad \text{----3.2.20.}$$

For a typical value of $\nu = 0.3$,

$$\sigma_{x_o} = 1.125 \ \bar{\sigma}_o \qquad \text{----3.2.21.}$$

$$\sigma_{z_o} = 0.3375 \ \bar{\sigma}_o \qquad \text{----3.2.22.}$$

Similarily $2G\varepsilon_{x_o} = \dfrac{\bar{\sigma}_o \ (1 - \nu)}{\sqrt{1 - \nu + \nu^2}} = 0.7875\bar{\sigma}_o$ ----3.2.23.

and $2G\varepsilon_{y_o} = \dfrac{-\bar{\sigma}_o \ \nu}{\sqrt{1 - \nu + \nu^2}} = 0.3375\bar{\sigma}_o$ ----3.2.24.

Reference to figure 3.4. shows that by equations 3.2.17 and 3.2.24.,the stress and strain paths are linear to yield and make an angle α_o with the longitudinal stress axis,where

$$\cos\alpha_o \quad = \quad (\sigma_{x_o}^2 + \bar{\sigma}_o^2 - \sigma_{z_o}^2) \ /2 \ \sigma_{x_o} \ \bar{\sigma}_o \qquad \text{----3.2.25.}$$

This angle is therefore solely dependent upon the value of Poisson's ratio and, for the general case where $\nu = 0.3$,

$$\cos\alpha_o \quad = \quad 0.9563 \text{ and } \alpha_o \simeq 17^{\circ}. \qquad \text{----3.2.26.}$$

3.2.1c. Plastic Relationships.

After yield the stress/strain relationships are described by the Prandtl-Reuss equations (3.1.1.).From the plane strain assumption (equation 3.2.14) we have from equation 3.1.1.

$$0 = d\varepsilon_z = \frac{d\bar{\sigma}}{\bar{\sigma}H'}\left(\sigma_z - \frac{1}{2}(\sigma_y + \sigma_x)\right) + \frac{1}{E}(d\sigma_z - \nu(d\sigma_y + d\sigma_x)) \qquad \text{----3.2.27.}$$

But $\sigma_y = d\sigma_y = 0$, (equation 3.2.15)

$$\therefore \frac{d\bar{\sigma}}{\bar{\sigma}H'}\left(\sigma_z - \frac{\sigma_x}{2}\right) + \frac{1}{E}(d\sigma_z - \nu \, d\sigma_x) = 0. \qquad \text{----3.2.28.}$$

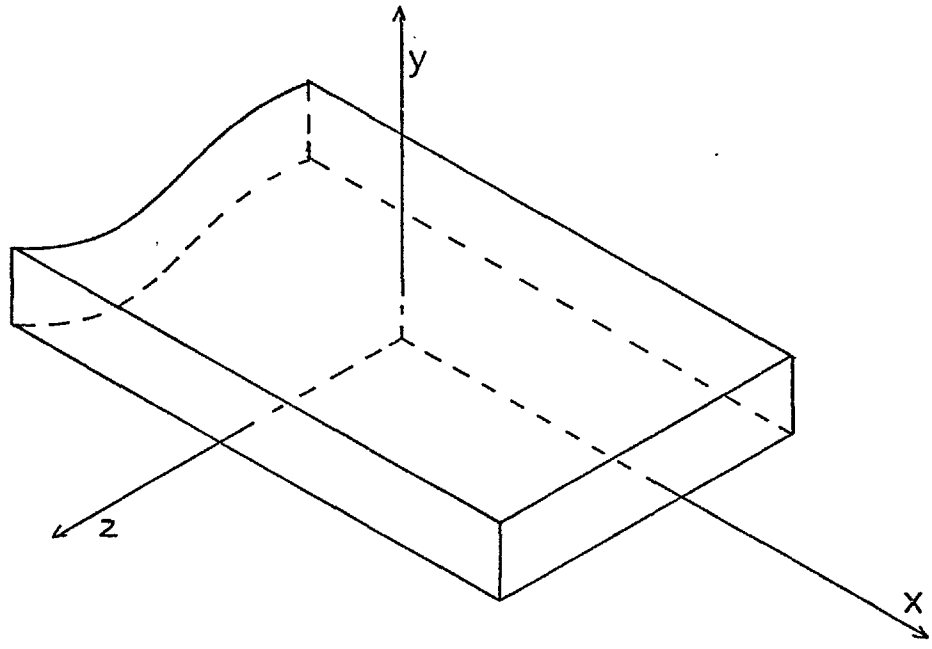


fig (3.3) AXIS DEFINITION

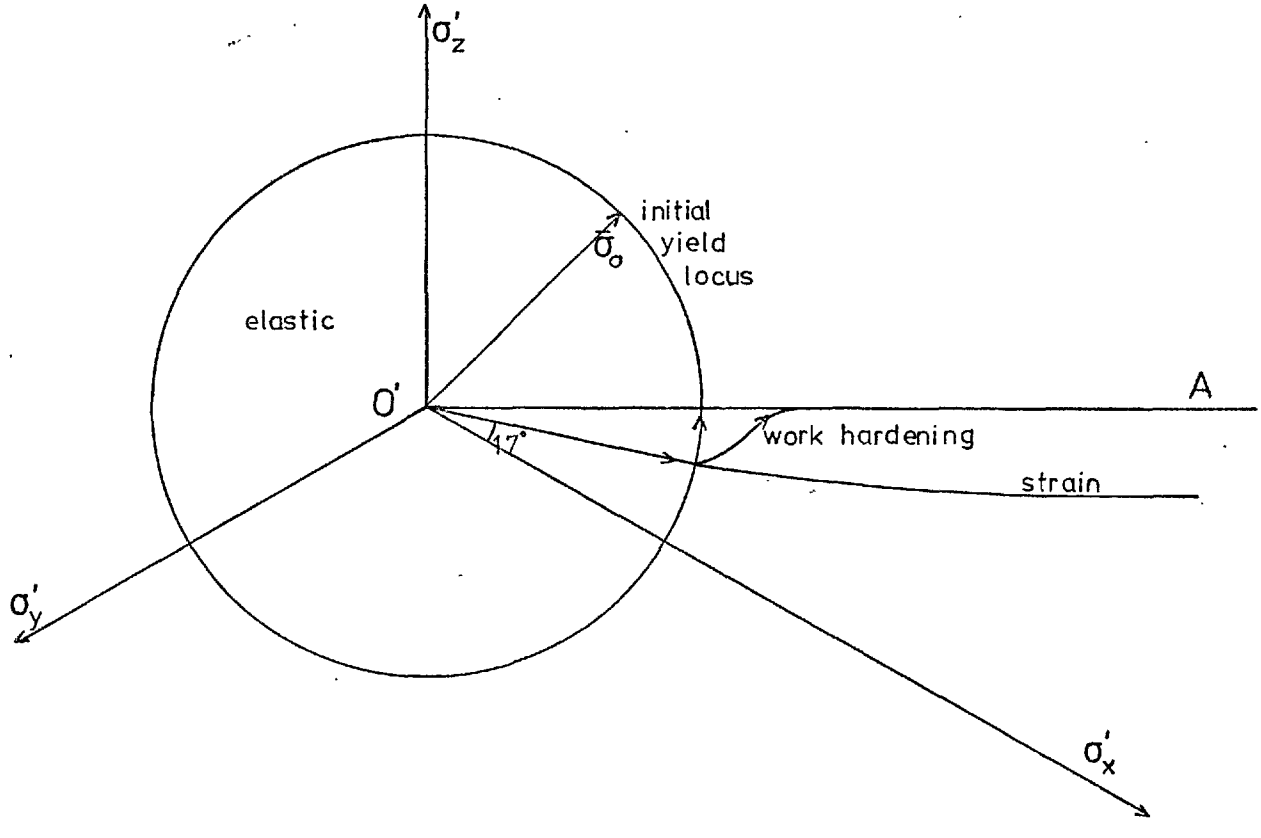


fig (3.4) STRESS PATH—PLANE STRAIN

The first term of equation 3.2.28. represents the plastic component of the strain increment $d\varepsilon_z$, this component being a function of the stress magnitudes.The latter term is the recoverable elastic component, its value being dependent upon the magnitude of the stress increments.The elastic term becomes negligible as deformation progresses and conditions of perfect plasticity prevail.Then

$$\sigma_z = \sigma_x / 2$$

and

$$\varepsilon_y = -\varepsilon_x \qquad \text{----3.2.29.}$$

These conditions correspond to the stress point having a position on the line perpendicular to the z axis.Equation 3.2.28 is automatically satisfied.

For smaller strain values ie. before 3.2.29 is satisfied, the elastic component will be equal and opposite to the plastic.

$$\text{Therefore} \quad d\varepsilon_z^E = -d\varepsilon_z^P \qquad \text{----3.2.30.}$$

For the plastic portion constancy of volume ensures that the deviatoric and total strains are equal.

$$d\varepsilon_z'^P = d\varepsilon_z^P \qquad \text{----3.2.31.}$$

An elastic dilation occurs however and

$$d\varepsilon_z'^E = d\varepsilon_z^E - d\varepsilon_m \qquad \text{----3.2.32.}$$

$$\text{where} \quad d\varepsilon_m = (d\varepsilon_z^E + d\varepsilon_y^E + d\varepsilon_z^E)/3$$

$$= (1 - 2\nu)\, d\sigma_m / E. \qquad \text{----3.2.33.}$$

From these equations we have

$$d\varepsilon_z' = d\varepsilon_z'^E + d\varepsilon_z'^P = \frac{-d\sigma_m}{E}(1-2\nu) \qquad \text{----3.2.34.}$$

$$\therefore d\varepsilon_z' = \frac{(2\nu - 1)}{3E}(d\sigma_x + d\sigma_z)$$

To obtain the conditions of the equation 3.2.29 from those of 3.2.21/22, the stress point must follow the path illustrated in Figure 3.4.If the material were perfectly plastic this path would coincide with the initial yield locus becomming perpendicular to the line O'A.Where expansion of the yield locus occurs,and providing that this expansion continues to fracture,

the stress path becomes asymptotic to and lies on this line.

After the initial yield, the strain path no longer coincides with the stress path and, for this analysis, follows the curve shown in Fig.3.4. This curve is related to the stress path by the Prandtl-Reuss equations and becomes asymptotic to the line O'A.

With the aid of equation 3.2.28 and knowledge of the gradient of the equivalent stress/equivalent strain curve H', the complete stress/strain history for plane strain loading can be determined. The initial yield values $\bar{\sigma}_o$, σ_{x_o}, σ_{z_o}, ε_{x_o} and ε_{y_o} have previously been determined.

Figure 3.5 shows the stress relationship for a stress point P_n (fig3.4), since by equation 3.2.8 total stresses may be plotted when one stress is zero. Hence,

$$\sigma_{x_n} = \bar{\sigma}_n(\cos\alpha_n + \sin\alpha_n/\sqrt{3}) \qquad \text{----3.2.36.}$$

$$\sigma_{z_n} = \frac{2}{\sqrt{3}}\bar{\sigma}_n \sin\alpha_n \qquad \text{----3.2.37.}$$

An equivalent stress increment $d\bar{\sigma}_n$

where $\bar{\sigma}_n = \bar{\sigma}_{n-1} + d\bar{\sigma}_n$ ----3.2.38.

is concurrent with a longitudinal stress increment $d\sigma_{x_n}$

where $\sigma_{x_n} = \sigma_{x_{n-1}} + d\sigma_{x_n}$ ----3.2.39.

From equations 3.2.36/38/39 we may write

$$d\sigma_{x_n} = \bar{\sigma}_{n-1}(\cos\alpha_n + \sin\alpha_n/\sqrt{3} - \cos\alpha_{n-1} - \sin\alpha_{n-1}/\sqrt{3}) + d\bar{\sigma}_n(\cos\alpha_n + \sin\alpha_n/\sqrt{3}) \qquad \text{----3.2.40.}$$

But $\alpha_n = \alpha_{n-1} + d\alpha_n$ ----3.2.41.

and since $\sin(d\alpha_n) \simeq d\alpha_n$ ----3.2.42.

equation 3.2.30 may be rewritten as

$$d\sigma_{x_n} = \bar{\sigma}_{n-1}\, d\alpha_n \left(\frac{\cos(\alpha_{n-1} + \frac{d\alpha_n}{2})}{\sqrt{3}} - \sin(\alpha_{n-1} + \frac{d\alpha_n}{2})\right) + d\bar{\sigma}_n\left(\frac{\cos\alpha_n + \sin\alpha_n}{\sqrt{3}}\right) \qquad \text{----3.2.43.}$$

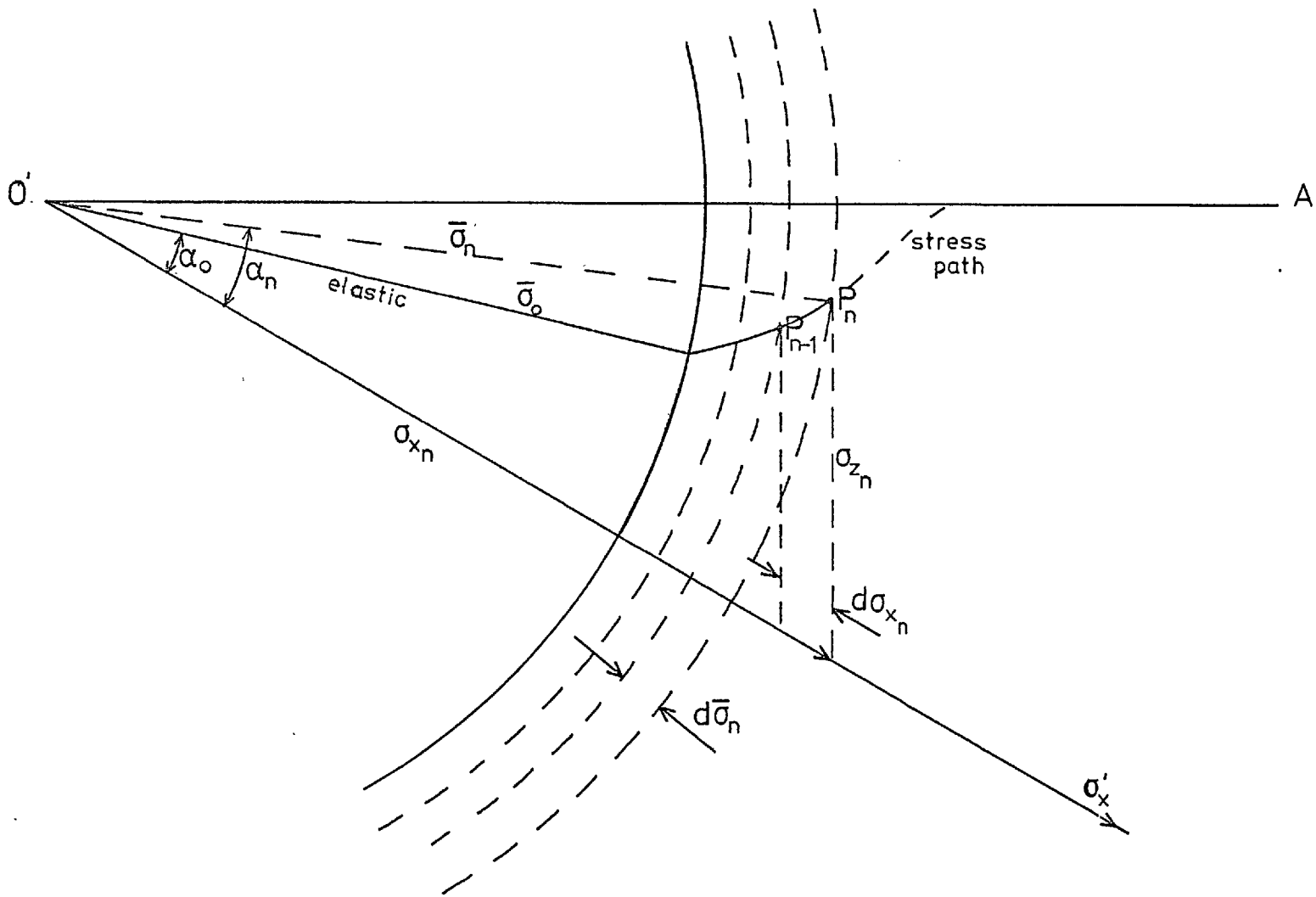


fig (3.5) STRESS RELATIONSHIPS - PLANE STRAIN - WORK-HARDENING

Similarly we may write

$$d\sigma_{z_n} = \bar{\sigma}_{n-1} d\alpha_n \left(\frac{2}{\sqrt{3}} \cos\left(\alpha_{n-1} + \frac{d\alpha_n}{2}\right)\right) + d\bar{\sigma}_n \frac{2}{\sqrt{3}} \sin\alpha_n \qquad \text{----3.2.44.}$$

Substituting equations 3.2.36/37/43/44 into equation 3.2.28 gives

$$d\varepsilon_z = 0 = \frac{d\bar{\sigma}_n}{H'_n}\left(\frac{2}{\sqrt{3}} \sin\alpha_n - \frac{\cos\alpha_n}{2} - \frac{\sin\alpha_n}{2\sqrt{3}}\right) + \frac{\bar{\sigma}_{n-1} d\alpha_n}{E}\left(\frac{2}{\sqrt{3}} \cos\left(\alpha_{n-1} + \frac{d\alpha_n}{2}\right)\right.$$

$$\left. - \frac{\nu}{\sqrt{3}} \cos\left(\alpha_{n-1} + \frac{d\alpha_n}{2}\right) + \nu \sin\left(\alpha_{n-1} + \frac{d\alpha_n}{2}\right)\right) + \frac{d\bar{\sigma}_n}{E}\left(\frac{2}{\sqrt{3}} \sin\alpha_n - \nu\left(\cos\alpha_n + \frac{\sin\alpha_n}{\sqrt{3}}\right)\right)$$

$$\text{----3.2.45.}$$

To continue the analysis we require the relationship between the yield expansion ($d\bar{\sigma}_n$) and the angular increment ($d\alpha_n$). Equation 3.2.45 is rearranged, giving

$$d\bar{\sigma}_n = \frac{\bar{\sigma}_{n-1} d\alpha_n \left(\frac{2-\nu}{\sqrt{3}} \cos(\alpha_{n-1} + d\alpha_n/2) + \sin(\alpha_{n-1} + d\alpha_n/2)\right)}{\frac{E}{2H'_n}\left(\cos\alpha_{n-1} - \sqrt{3}\sin\alpha_{n-1}\right) - \frac{2-\nu}{\sqrt{3}}\sin\alpha_n + \nu\cos\alpha_n}.$$

$$\text{----3.2.46.}$$

It is relatively simple a matter to obtain the relationship between $\bar{\sigma}_n$ and α_n from equation 3.2.46, provided that an iterative technique is utilized to determine H'_n from equation 3.1.3. or 3.1.5. Knowing this relationship the current stresses σ_{x_n} and σ_{z_n} may be determined from equation 3.2.36/37.

The von Mises criterion may be expressed, for $\varepsilon_z = d\varepsilon_z = \sigma_y = d\sigma_y = 0$, in the incremental form as

$$\bar{\sigma} d\bar{\sigma} = \sigma_x d\sigma_x + \sigma_z d\sigma_z - \frac{\sigma_x}{2} d\sigma_z - \frac{\sigma_z}{2} d\sigma_x \qquad \text{----3.2.47.}$$

substitution of the relevant equations into equation 3.2.47 yields the inequality

$$d\bar{\sigma} \neq d\bar{\sigma} + \frac{(d\alpha)^2}{2} \sigma_{n-1} \qquad \text{----3.2.48.}$$

Providing that a sufficiently small value of $d\alpha$ is chosen, the yield criterion will be satisfied throughout.

Consideration of the denominator of equation 3.2.46 will show that at values of α_n approaching 30°, (the limiting value) instability will occur depending upon the current H' and V values. The point of instability effectively represents the limit of the validity of equation 3.1.1.

The object of this analysis is to derive the longitudinal stress strain relationships (σ_x/ε_x). We have so far derived the relationship between σ_x and α. The corresponding strain increment for the deviatoric stress increment $P_{n-1}P_n$ fig 3.5 is shown in fig 3.6. $Q_{n-1}Q_n$. By the Prandtl-Reuss equations this strain increment is the vector sum of the elastic and plastic components of strain. $Q_{n-1}R$ represents the elastic increment $(2Gd\varepsilon'^E)$ and is equal in magnitude and direction to $P_{n-1}P_n$. RQ_n thus represents the plastic increment and is parallel to the instantaneous stress vector $O'P_{n-1}$. The length of this increment is the vector sum of the deviatoric plastic strain increments $d\varepsilon'^P_x$, $d\varepsilon'^P_y$, $d\varepsilon'^P_z$., but

$$d\bar{\varepsilon}^P = \sqrt{\tfrac{4}{3}}\,(d\varepsilon_x'^P{}^2 + d\varepsilon_y'^P{}^2 + d\varepsilon'^P_z{}^2)^{\frac{1}{2}} \qquad \text{----3.2.49.}$$

and the vector RQ_n is therefore of length

$$RQ_n = 3G\, d\bar{\varepsilon}^P_n \qquad \text{----3.2.50.}$$

once the required scaling factors $\sqrt{3/2}$ and 2G have been included.

The strain section of figure 3.6 has been redrawn in fig 3.7 and since $d\varepsilon_z = 0$, the vectors $Q_{n-1}T$ and TQ_n represent the total strain increments $d\varepsilon_x$ and $d\varepsilon_y$ with the scale factor 2G included. SR may be drawn equal and parallel to the incremental increase in equivalent stress $d\bar{\sigma}_n$. For small $d\alpha$ this line is the straight continuation of $Q_nR.Q_{n-1}S$ is of length $\bar{\sigma}_{n-1}d\alpha_n$.

Using the geometry and notation of fig 3.7.

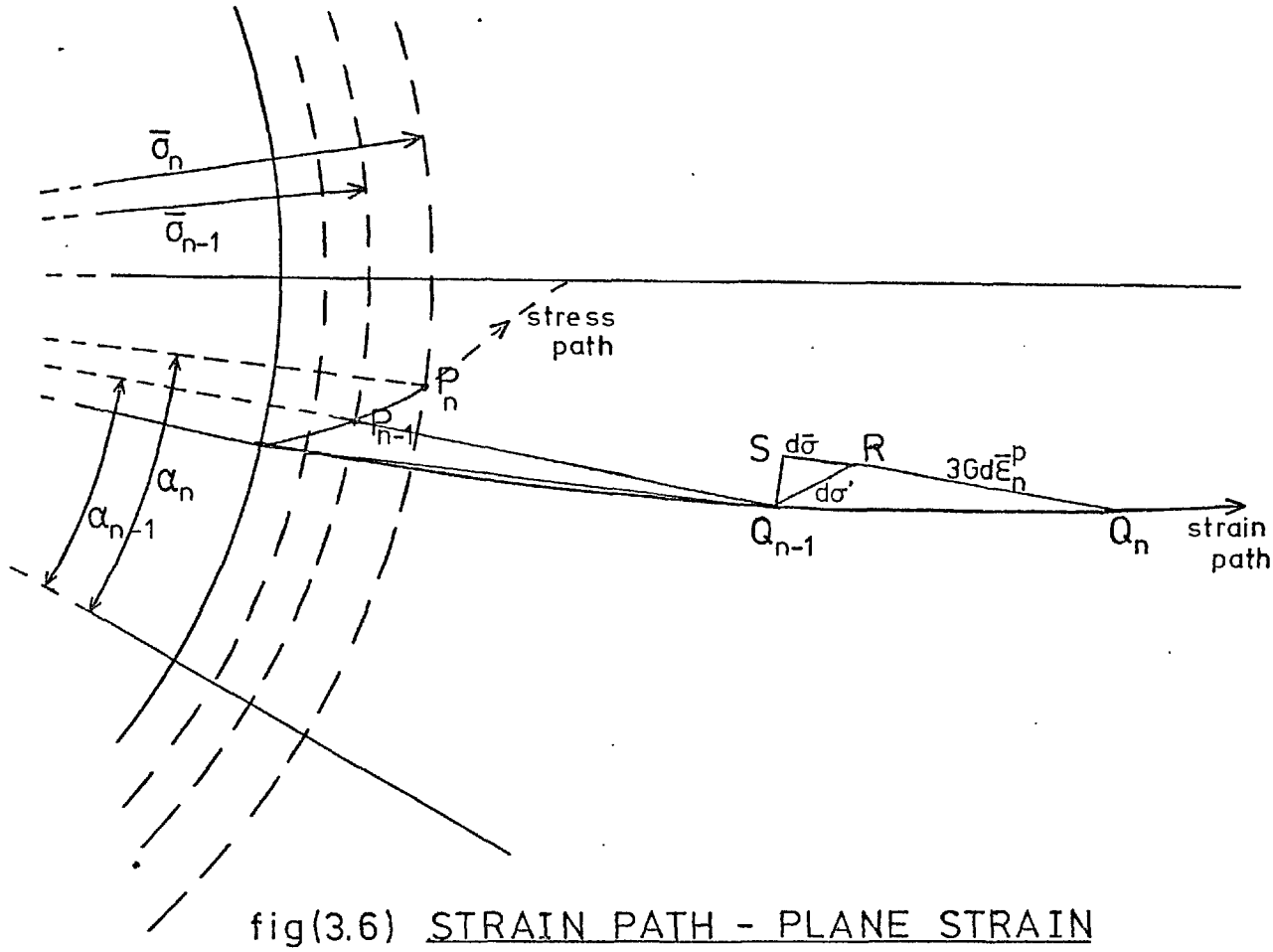
$$SQ_n = d\bar{\sigma}_n + 3G\, d\bar{\varepsilon}^P_n$$

$$= d\bar{\sigma}_n\left(1 + \frac{3G}{H'_n}\right) \qquad \text{----3.2.51.}$$

since $VU = SQ_{n-1}$,

$$VQ_n = \bar{\sigma}_{n-1}d\alpha_n/\tan\,(60 - \alpha_{n-1}) \qquad \text{----3.2.52.}$$

$$\therefore\ Q_{n-1}U = SV = d\bar{\sigma}_n\frac{(1+3G)}{H'_n} - \frac{\bar{\sigma}_{n-1}d\alpha_n}{\tan\,(60-\alpha_{n-1})} \qquad \text{----3.2.53.}$$



fig(3.6) STRAIN PATH - PLANE STRAIN

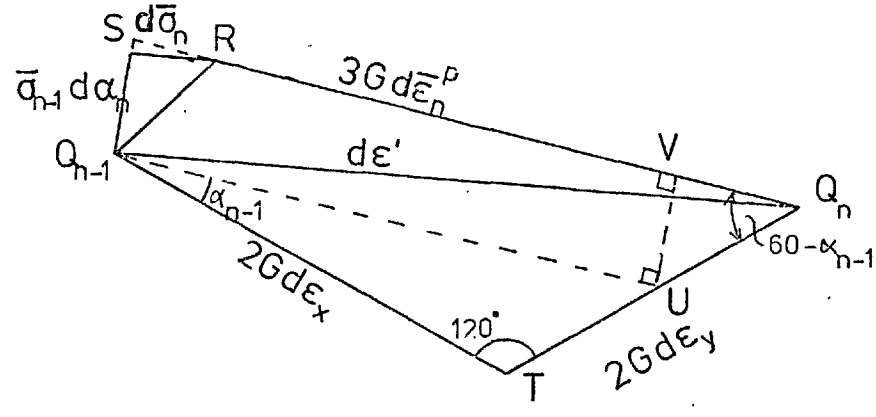


fig (3.7) STRAIN INCREMENT (DETAIL)

$$\text{also} \quad UQ_n = \frac{\bar{\sigma}_{n-1}\, d\alpha_n}{\sin(60-\alpha_{n-1})} \qquad \text{----3.2.54.}$$

Applying the same rule to triangle $Q_{n-1}UT$,

$$\frac{Q_{n-1}U}{\sin 120^\circ} = \frac{TU}{\sin\alpha_{n-1}} = \frac{Q_{n-1}T}{\sin(60^\circ - \alpha_{n-1})} \qquad \text{----3.2.55.}$$

From equations 3.2.53/55 we have

$$2Gd\varepsilon_x = Q_{n-1}U = \left(d\bar{\sigma}_n\left(1 + \frac{3G}{H'_n}\right) - \frac{\bar{\sigma}_{n-1}d\alpha_n}{\tan(60-\alpha_{n-1})}\right)\frac{\sin(60-\alpha_{n-1})}{\sin 60}$$

$$= \frac{2}{\sqrt{3}}\left(d\bar{\sigma}_n\left(1 + \frac{3G}{H'_n}\right)\sin(60-\alpha_{n-1}) - \bar{\sigma}_{n-1}d\alpha_n\cos(60-\alpha_{n-1})\right)$$

$$= d\bar{\sigma}_n\left(1 + \frac{3G}{H'_n}\right)\left(\cos\alpha_{n-1} - \frac{\sin\alpha_{n-1}}{\sqrt{3}}\right) - \bar{\sigma}_{n-1}d\alpha_n\left(\frac{\cos\alpha_{n-1}}{\sqrt{3}} + \sin\alpha_{n-1}\right)$$

----3.2.56

Similarly

$$-2Gd\varepsilon_y = TU + UQ' = \frac{2}{\sqrt{3}}\left(d\bar{\sigma}_n\left(1 + \frac{3G}{H'_n}\right)\sin\alpha_{n-1} + \bar{\sigma}_{n-1}d\alpha_n\cos\alpha_{n-1}\right) \qquad \text{----3.2.57}$$

Once a value of $d\bar{\sigma}_n$ corresponding to the angular increment $d\alpha_n$ has been determined from 3.2.46, equations 3.2.56/57 may be used to plot the corresponding strain increments. The summation of these increments allows the relationship σ_x/ε_x to be determined. The similarity between the form of the $\bar{\sigma}/\bar{\varepsilon}$ and σ_x/ε_x relationship allows the latter to be described by an equation similar to equation 3.1.4. Thus

$$\sigma_x = A'(B' + \varepsilon_x)^{m'} \qquad \text{----3.2.58.}$$

The computer program written to determine the relationship σ_x/ε_x is described and listed in Appendix 2, and includes the program used in fitting the relationships of equations 2.2.8 and 3.2.58. A flow chart for the basic program is given in figure 3.8. The resultant longitudinal stress/strain relationships for three different materials are shown in figure 3.9., together with the equivalent and, transverse stress magnitudes. These relationships are, however, limited in their use because of the uni-directional loading nature assumed in their derivation.

The adjustments required to provide suitable relationships for reverse loading conditions are naturally dependent upon the assumed behaviour

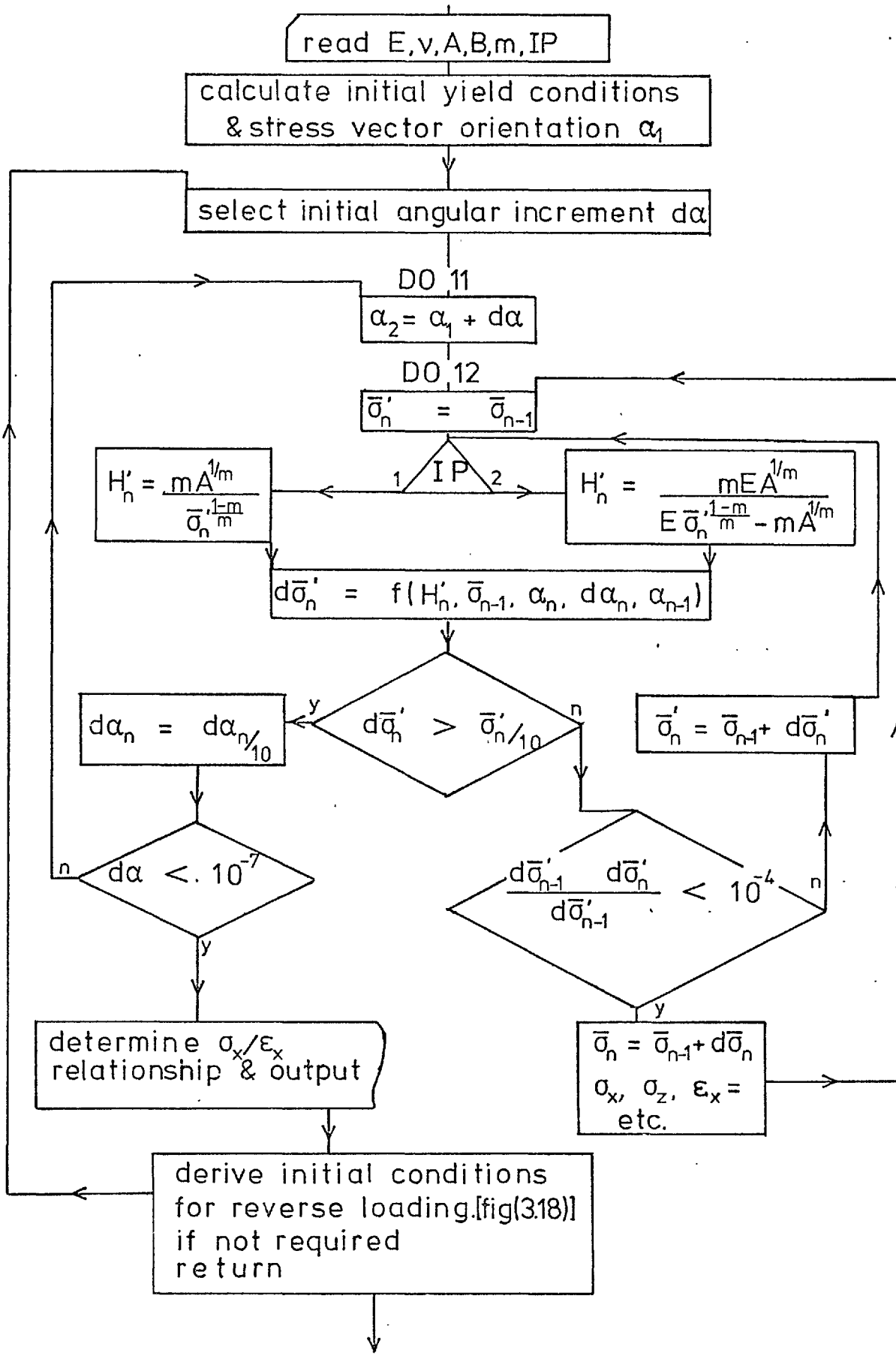
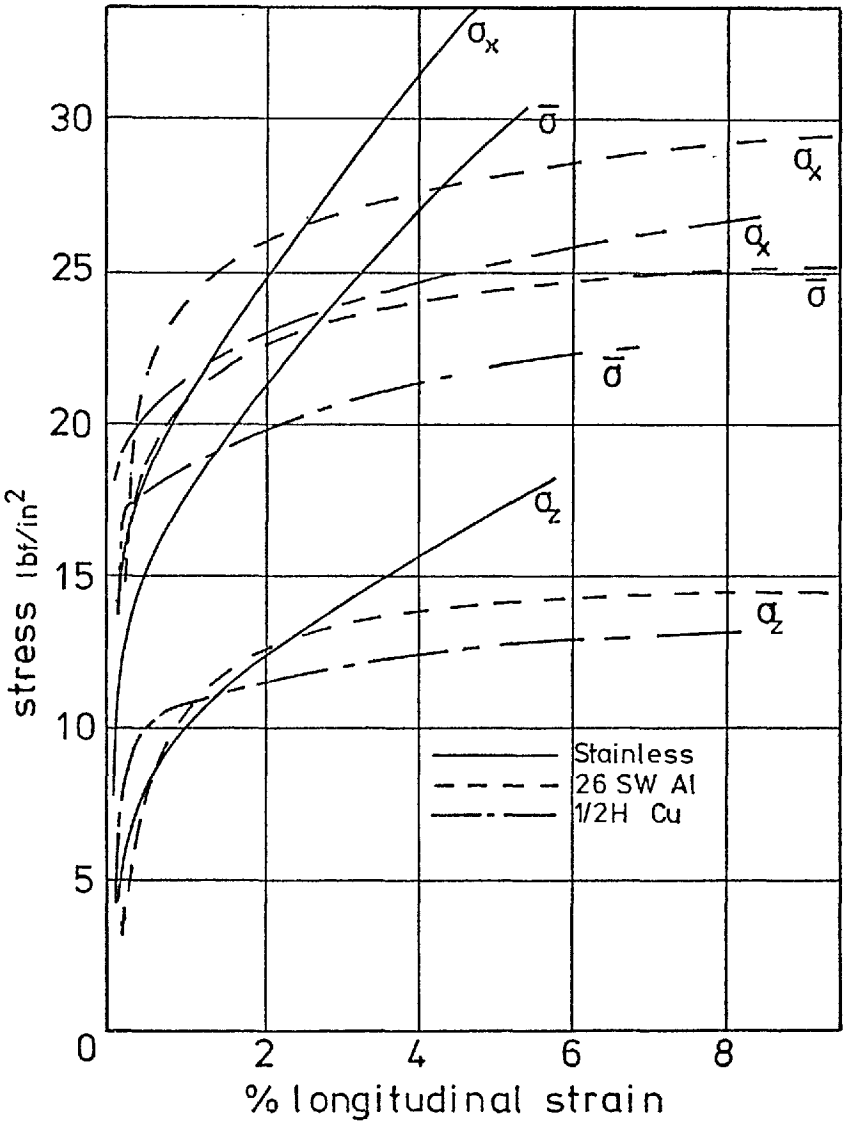


fig (3.8) FLOW CHART - SUBROUTINE HARDEN



fig(3.9) STRESS/STRAIN RELATIONSHIPS DURING PLANE-STRAIN LOADING

on unloading and reloading in the opposite direction.The simplest case of the isotropic hardening model will be considered first.

3.2.2. Reverse Loading - Reversible Strain Hardening.

The assumption of isotropic hardening requires the uniform expansion of the initial yield surface during subsequent plastic flow (fig 3.6),this expansion being independent of the stress vector orientation.The elastic range of the material is correspondingly increased as a result of plastic flow.

On relaxation of the applied stress the material behaves elastically, the elastic relationships being assumed identical to those of the initial elastic loading (section 3.2.1a).The elastic unloading and reverse loading line is thus parallel to the initial loading line on the deviatoric plane and is represented by $P_o' P_1$ in fig 3.10. 'Assumed' is used here since it is well known that the tangent modulus on elastic relaxation is normally less than the loading modulus (E) (eg. ref 82) and the equivalent stress/ equivalent strain relationships will not therefore be the same.Further, the Prandtl-Reuss equations predict a small deviatoric elastic strain increment in the z direction on plastic loading which is recoverable on relaxation.

Reference to Fig 3.10 shows that the elastic stress range before yielding occurs is less for plastically deformed material than for virgin material with an equivalent yield stress,ie the line parallel to $P_o' P_1$ through O'.

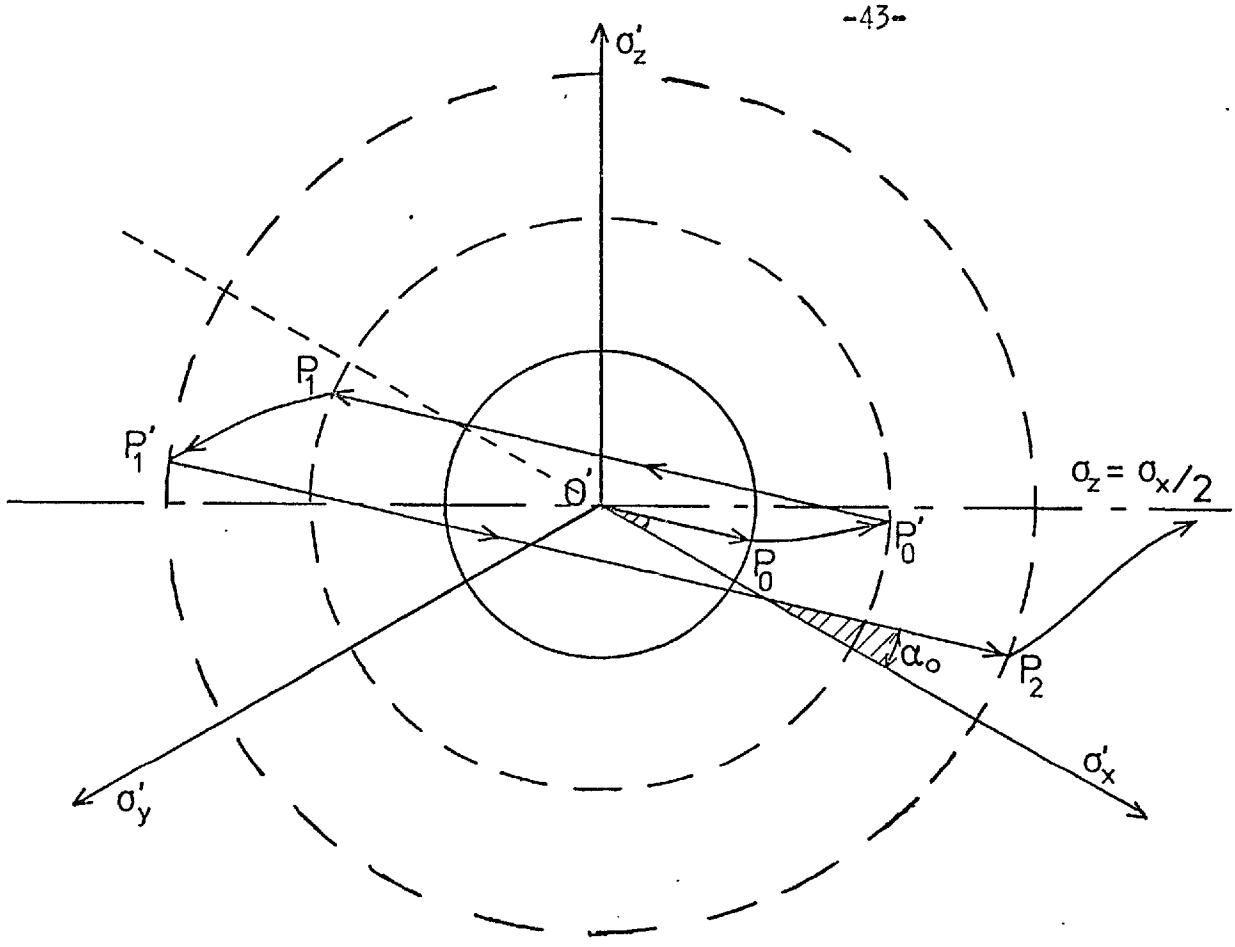
The stress relationships at P_1 are seen to be different from those at P_o. Compressive plastic straining will,however,cause the stress point to again approach the line $\sigma_z = \sigma_x/2$, where the elastic strain increments are negligible.The stress/strain relationships for the loading path P_1P_1' can be determined in an identical manner to that given in Section 3.2.1c, providing that the new yield angle α_{o2} is known.This angle is dependent upon the position of P_1 on the new yield locus,which,in turn,is a function of (1) the degree of prior plastic deformation, and (2) Poisson's ratio for the material.

From Fig 3.11. it can be shown that $P_o' P_1$ makes an angle $(\alpha_{f1} - \alpha_{o1})$ with the horizontal. α_{o2} may thus be expressed as

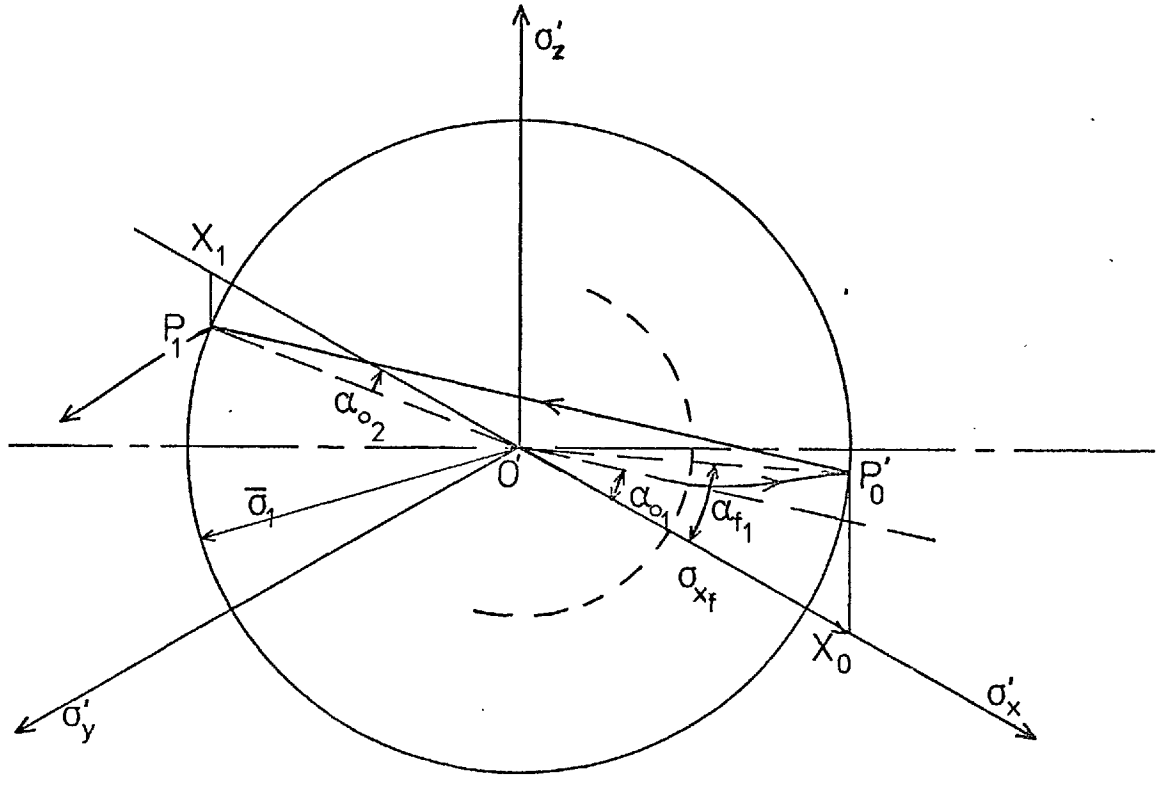
$$\alpha_{o_2} = (30^\circ + \alpha_{f_1} - \alpha_{o_1}) \qquad \text{----3.2.59.}$$

Since

$$P_o' P_1 = 2\bar{\sigma} \cos (\alpha_{f_1} - \alpha_{o_1}) \qquad \text{----3.2.60.}$$



fig(3.10) STRESS PATH - REVERSE LOADING (ISOTROPIC HARDENING)



fig(3.11) REVERSE LOADING (DETAIL)

and

$$\frac{P'_o P_1}{\sin 60} = \frac{X_o X_1}{\sin(120-\alpha_{o1})} \qquad \text{----3.2.61}$$

then

$$\Delta\sigma_x^E = X_o X_1 = \frac{4}{\sqrt{3}}\bar{\sigma}\cos(\alpha_{f1}-\alpha_{o1})\cos(30-\alpha_{o1}) \qquad \text{----3.2.62}$$

but

$$0'X_o = \sigma_{xf} = \bar{\sigma}(\cos\alpha_{f1} + \frac{\sin\alpha_{f1}}{\sqrt{3}}) \qquad \text{----3.2.63}$$

hence

$$\sigma_x^E = \frac{4\,\sigma_{xf}\cos(30-\alpha_{o1})\cos(\alpha_{f1}-\alpha_{o1})}{\sqrt{3}\cos\alpha_{f1} + \sin\alpha_{f1}} \qquad \text{----3.2.64}$$

For the bulk of materials, $\nu \simeq 0.3$ which gives values of α_{o1} of $\simeq 17^\circ$. Fig 3.12 shows a plot of $\Delta\sigma_x^E/\sigma_{xf}$ against $(\alpha_{f1}-\alpha_{o1})$ for $\alpha_{o1} = 17^\circ$. It is seen that the maximum deviation of the initial compressive yield stress from the final tensile flow stress is 10%, which occurs when the purely plastic state is reached during the initial loading. The corresponding elastic strain range to the above stress range is written as

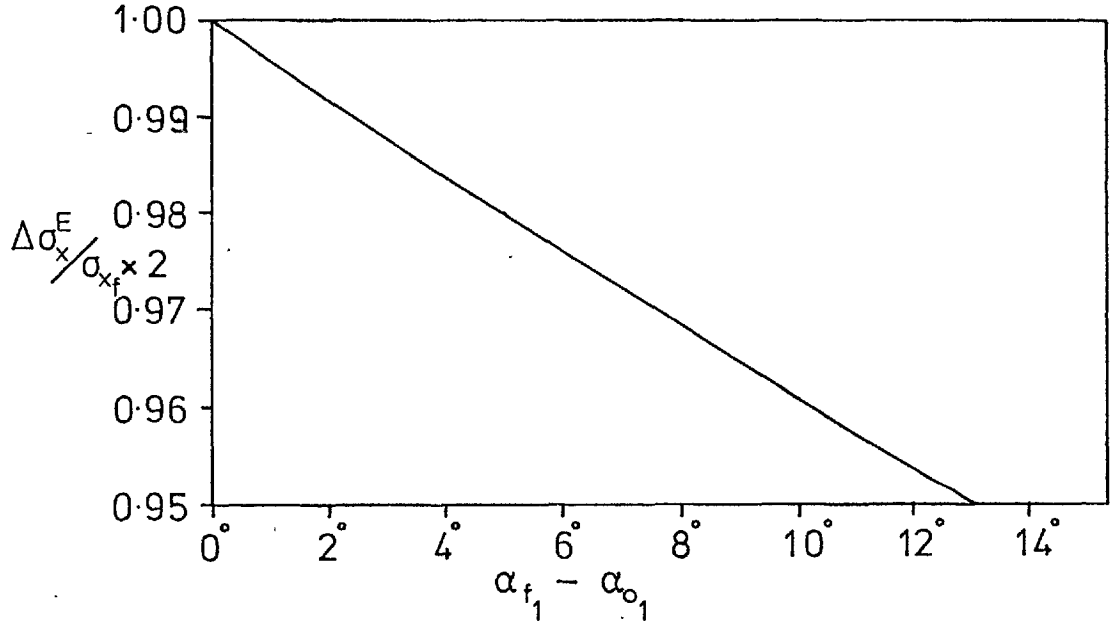
$$\Delta\varepsilon_x^E = \Delta\sigma_x^E\,(1-\nu^2)/E \qquad \text{----3.2.65.}$$

When considering the tension levelling process, where each thickness fibre is strained by differing amounts, it is necessary, in order to reduce errors in the stress/strain relationship, to plot the absolute stress against the total longitudinal strain for a number of stress reversals. The additional stress reversals over the first are analysed in an identical manner to the first. The longitudinal stress/strain relationship is then determined as the mean curve shown in Fig 3.13 where

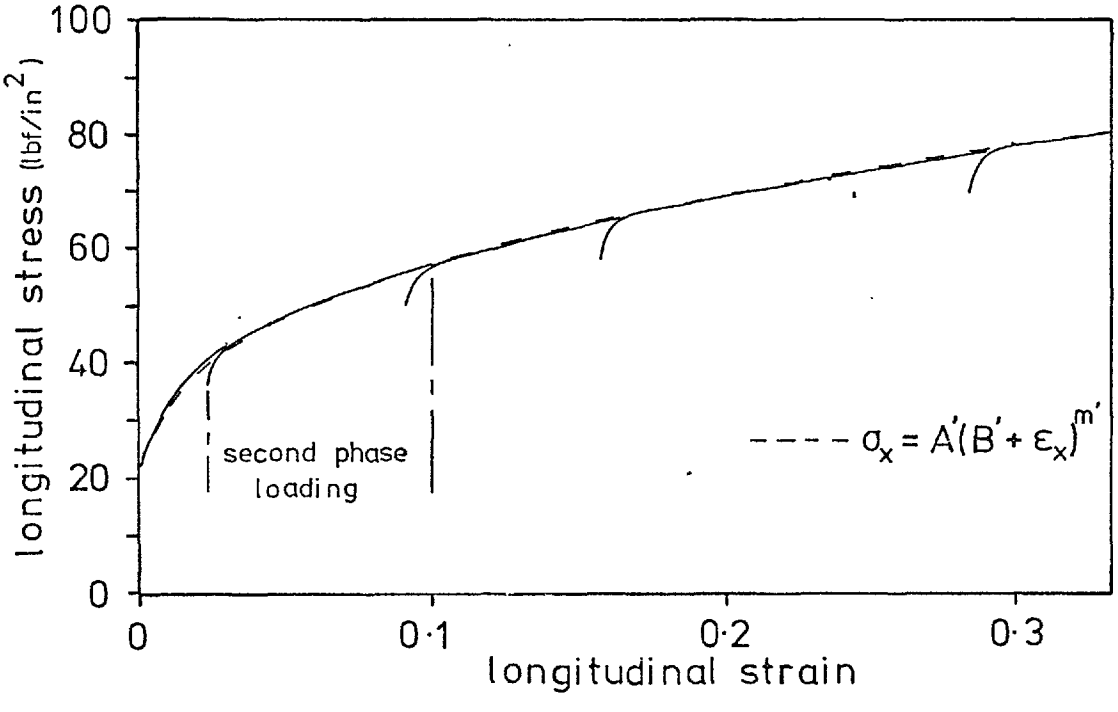
$$\sigma_x = A'(B' + e_x)^{m'} \qquad \text{----3.2.66.}$$

It is sufficient, in deriving the individual loading curves, to take each loading cycle to the maximum value of α_n before instability sets in.

From equations 3.2.64 and 3.2.65 the current elastic strain range is a function of the inclination of the instantaneous stress vector to the initial yield stress vector. A plot of this inclination versus strain for a reversing loading path is illustrated in Fig 3.14. It can be seen



fig(3.12) EFFECT OF DEGREE OF PRIOR STRAINING ON ELASTIC STRESS RANGE



fig(3.13) STRESS/STRAIN RELATIONSHIPS DURING REVERSE LOADING (ISOTROPIC HARDENING)

that unless the actual value can be determined for any fibre at any point through the leveller,(a process which would involve an immense amount of programming), the reduced elastic strain range should in general be utilized,unless the total strain is less than about twice the initial yield strain.

ie $\varepsilon < 2\varepsilon_{x_0}$ ----3.2.67.

If equation 3.2.67 is satisfied the elastic strain range $\Delta\varepsilon_x$ should be given the value

$$\Delta\varepsilon_x = 2|\sigma_x|\ (1 - \nu^2) / E \qquad \text{----3.2.68.}$$

where σ_x = current longitudinal stress level.

3.2.3. Reverse Loading - Kinematic Hardening.

The non-reversible hardening assumption requires that the initial yield envelope does not alter its initial form and orientation during plastic straining.Instead of the expansion assumed in the isotropic model,the yield locus translates in stress space like a rigid body (eg 83,84) (Fig 3.15).The direction of this translation is outwards along the normal to the current stress point.For the conditions under consideration, the direction of translation after first yield is at 17° to the x direction(for $\nu = 0{\cdot}3$).As is indicated by Fig 3.14 however, the stress point rapidly approaches the line $\sigma_z = \sigma_x/2$ and the centre of the yield circle is brought back to this line.As any reverse loading theory is by nature somewhat approximate,it is reasonable to assume that the initial loading relationships can be determined by the equations used in section 3.2.1c but that as straining takes place the centre of the yield locus translates along the line $\sigma_z = \sigma_x/2$. Using this assumption of horizontal translation the stress/strain relationships can be determined by the use of a diagram such as Fig 3.16.

The inclination of the elastic unloading line will again be defined by equation 3.2.59, but subsequent yielding will occur at stresses

$$\sigma_{x_{kin}} = \sigma_{x_{iso}} - \frac{2}{\sqrt{3}} d\bar{\sigma}_{tot} \qquad \text{----3.2.69.}$$

$$\sigma_{z_{kin}} = \sigma_{z_{iso}} - \frac{1}{\sqrt{3}} d\bar{\sigma}_{tot} \qquad \text{----3.2.70.}$$

where $\sigma_{x_{iso}}$ and $\sigma_{z_{iso}}$ are the respective yield stresses for the isotropic case.

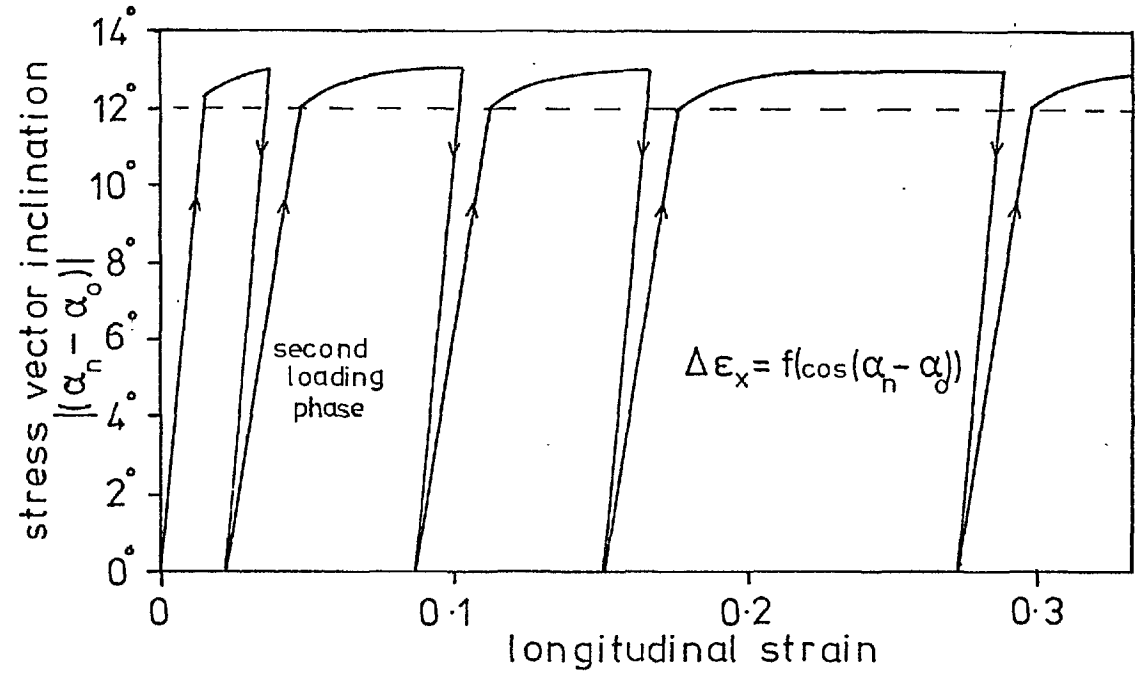


fig (3.14) STRESS VECTOR INCLINATION DURING REVERSE LOADING (ISOTROPIC HARDENING)

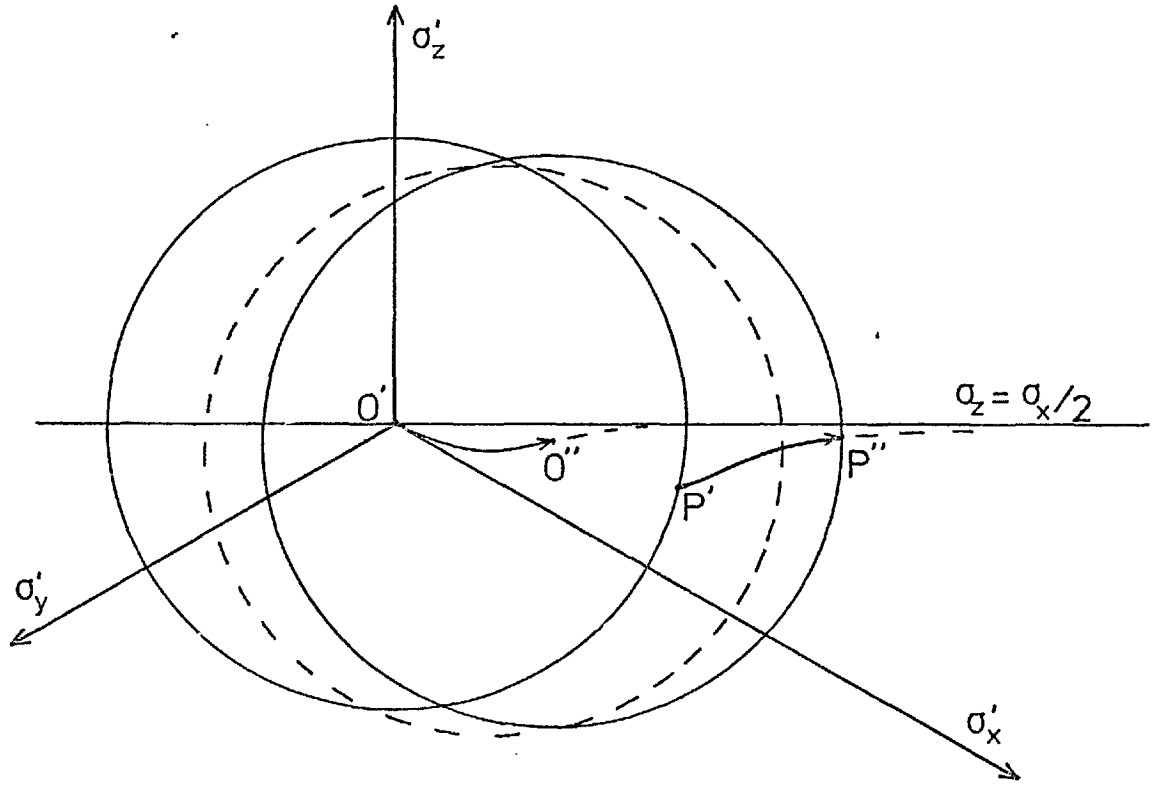


fig (3.15) STRESS PATH – PLANE STRAIN (KINEMATIC HARDENING)

The total increase in the equivalent yield stress during the loading phase being represented by $d\bar{\sigma}_{tot}$.

Depending upon this increase and the value of V for the material, yielding on reverse straining may well occur before the stress magnitudes have reduced to zero (Fig 3.16).The elastic stress and strain ranges in the loading direction are again dependent upon the orientation of the current stress vector and are expressed by equation 3.2.62/65 where the magnitude of $\bar{\sigma}$ remains constant at the initial yield value.

Substitution of equations 3.2.69/70 into the analysis of section 3.2.1c in conjunction with equations 3.2.36 and 3.2.37 again leads to the results for $d\bar{\sigma}_n$, $d\varepsilon_{x_n}$ and $d\varepsilon_{y_n}$ expressed by equations 3.2.46/56/57. The determination of the σ_x/ε_x relationship is thus carried out in exactly the same manner,the necessary adjustments being made for equations 3.2.69/70.

The relationship

$$\sigma_x = A' (B' + e_x)^{m'}$$

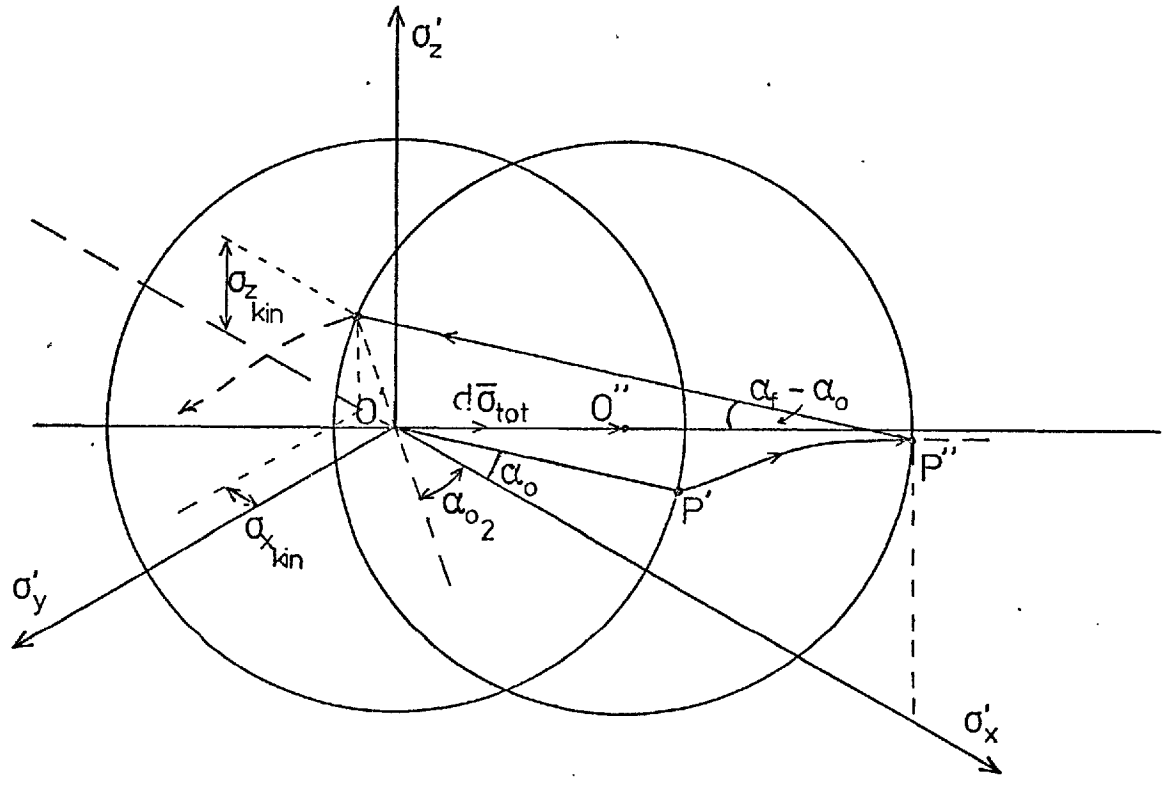
must in this case, be determined in a different manner to that of the isotropic case.The kinematic model assumes that the current degree of work hardening is only a function of the degree of straining beyond yield for each loading phase.The generalized curve must thus be taken as the mean of the individual loading curves, and the stress/strain diagram equivalent to Fig 3.13 for the reversible hardening case, will appear as in Fig 3.17.

The effect of allowing the deriving program to continue to instability for each loading phase is very much more critical than with the isotropic case.Due to the yield locus translation the stress levels on reversal of the plastic straining are entirely dependent upon degree of straining prior to the sign reversal.To provide a suitable mean curve for this case it has been found necessary (for the typical strain values found in the tension leveller), to plot the initial loading curve a number of times equal to the number of reverse loading curves.In this manner a reasonable generalized curve has been developed.

For both the reverse loading models it is necessary to cycle the basic program for Section 3.2.1c (Fig 3.8), a number of times equal to the number of rollers.At the start of each cycle the new yield conditions are determined on the basis of the analyses of Sections 3.2.2 and 3.2.3. The flow chart for this process is given in Fig 3.18.Fitting the final mean curve to the equation.

$$\sigma_x = A'(B' + e_x)^{m'} \qquad \text{----3.2.71.}$$

is described in Appendix 2, along with the complete program listing.It



fig(3.16) REVERSE LOADING – KINEMATIC HARDENING

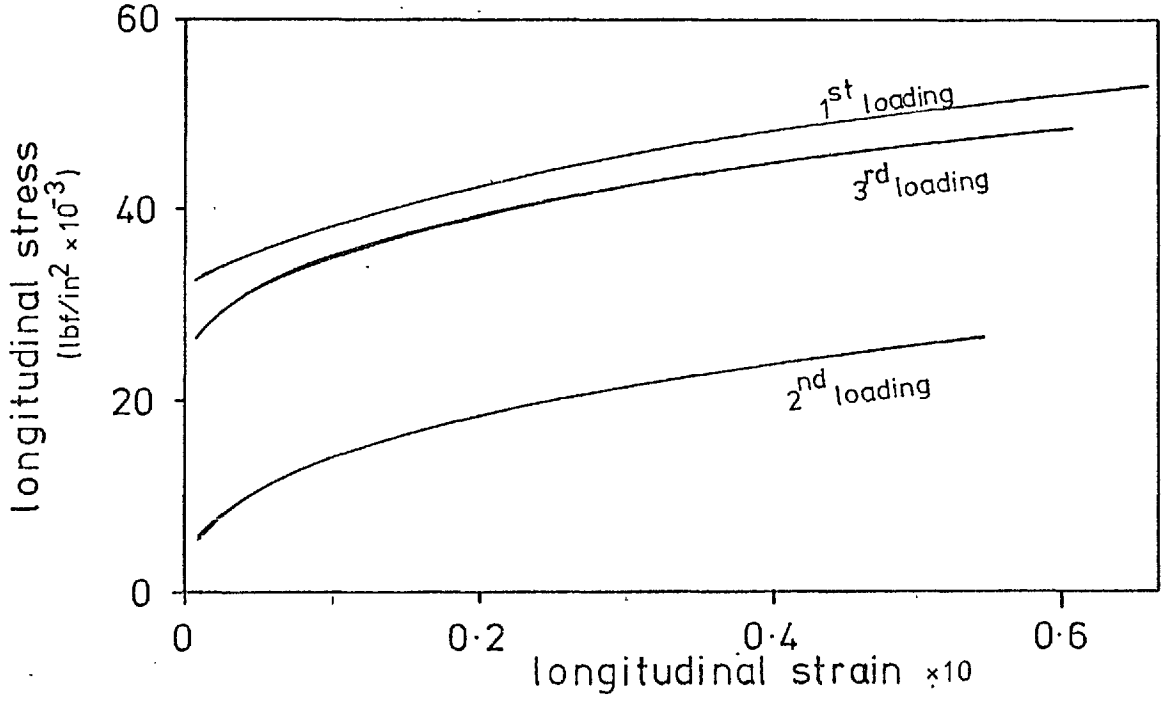


fig (3.17) STRESS/STRAIN RELATIONSHIPS KINEMATIC HARDENING

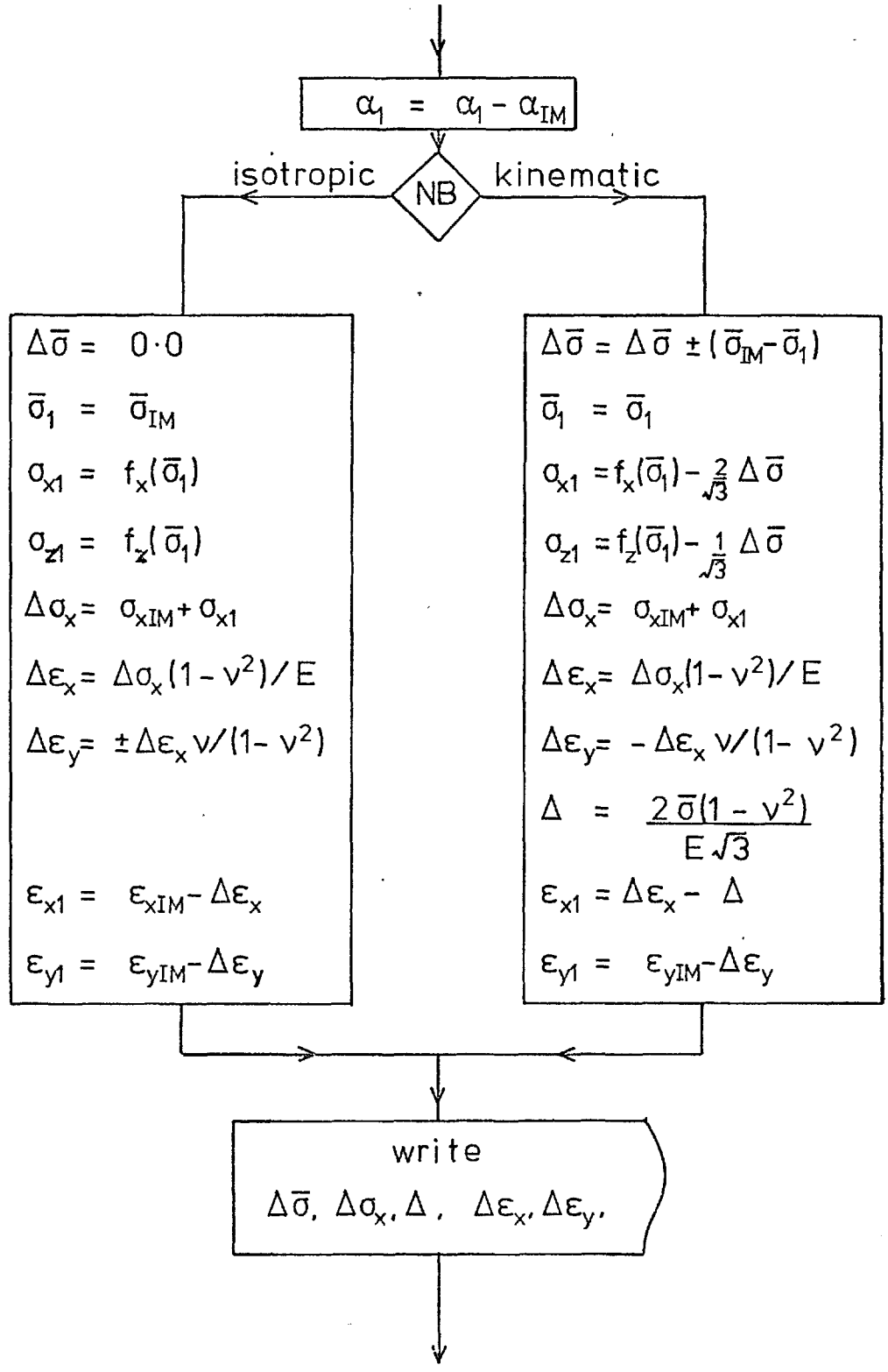


fig (3.18) FLOW CHART - REVERSE LOADING ADJUSTMENTS

is to be noted here that equation 3.2.71 is derived in terms of engineering strains rather than logarithmic strains to provide compatibility with the strain derivation in the theory of the leveller, section 3.3.0.

3.3.0. The Analysis of Bending and Unbending.

In section 3.2.0. we have developed the mean longitudinal stress/strain relationship pertaining to the conditions of tension levelling. The effects of transverse stresses can now be ignored and the analysis developed in two dimensions only. The required analysis is now similar to that developed by Swift (54) with the inclusion of elastic and work-hardening effects.

Figure 3.19 illustrates the process and indicates the zones to be considered. In zone AB the material is subjected to a gradually increasing bending moment until the roll radius is attained at B.

The work needed to bend the material requires that $T_1 > T_o$. If no frictional effects are present then no further deformation occurs over the contact arc BC and $T_2 = T_1$. In reality however friction losses occur at the roller bearings and must be represented by $T_2 > T_1$.

On the immediate loss of roll contact, elastic springback occurs (CD) but since no positive work is done $T_3 \simeq T_2$. During the final straightening in zone DE, (ignoring the slight overstraightening that must occur to return the strip to its nominal path) further work will be done such that $T_4 > T_3$.

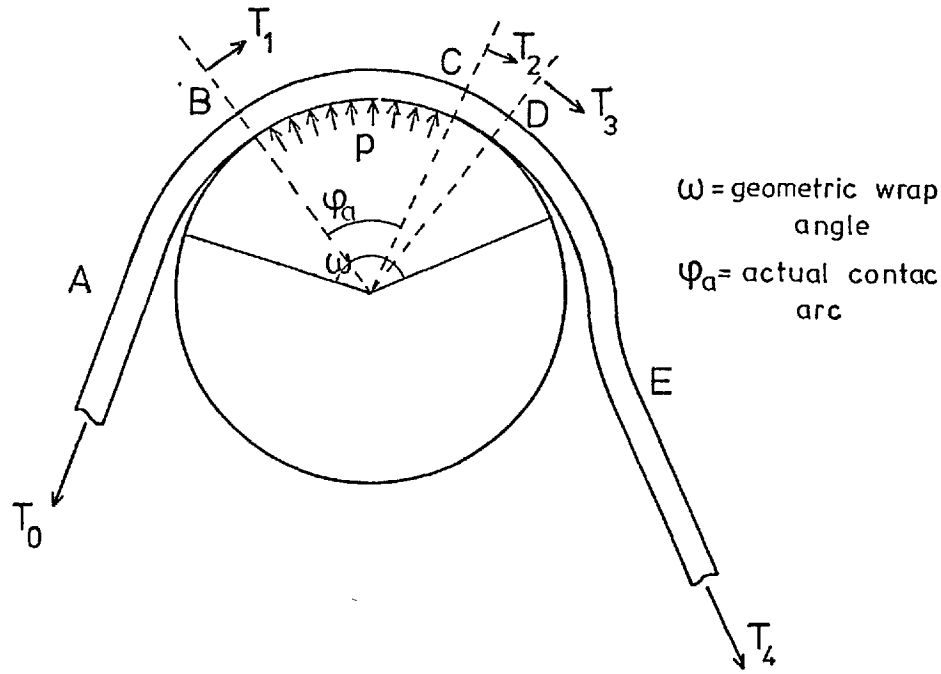
Because of the need for a complex computer program to provide the complete analysis, it is unnecessary to include these tension losses in the theoretical model. Tension increases are easily included at the programming stage, and for the theoretical derivations we shall assume constant applied load T throughout bending and unbending where

$$T = (T_o + T_4)/2. \qquad \text{----3.3.1.}$$

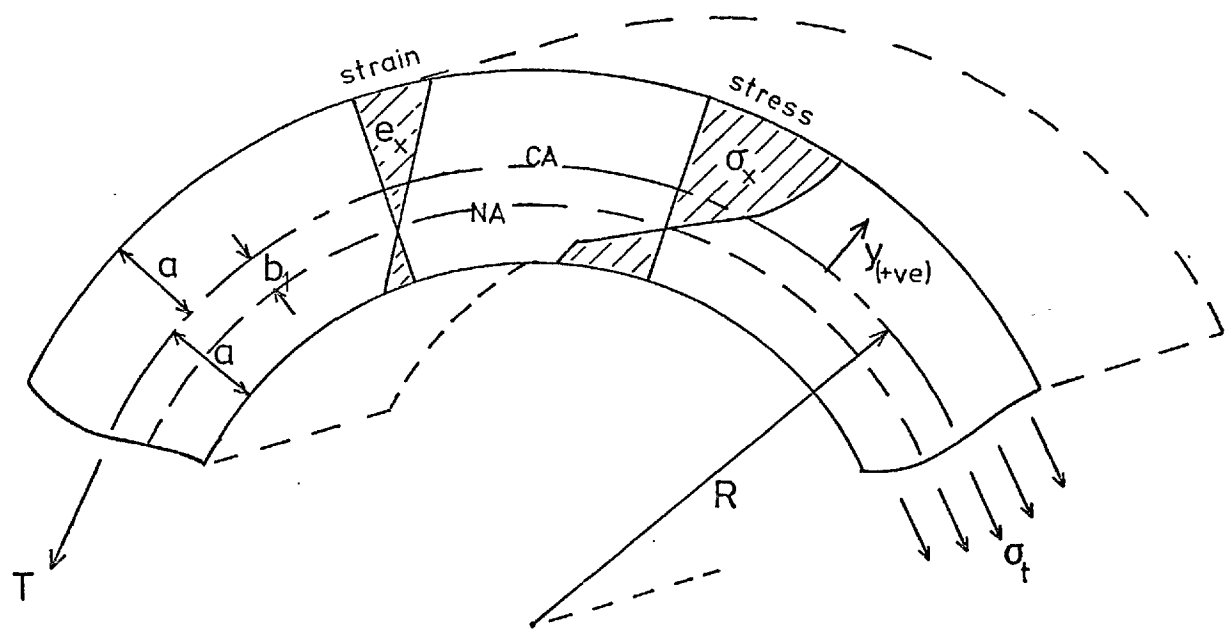
In deriving the basic equations for each operation it is sufficient in the first instance, to consider an elastic, perfectly plastic material. The effects of work-hardening will be considered where necessary.

3.3.1. Bending under Tension.

As indicated by Swift (54) the effect of the additional tensile stresses causes a shift of the neutral axis of bending towards the inside surface of the strip. The stress and strain distributions are as shown in Fig 3.20.



fig(3.19) DEFORMATION ZONES DURING BENDING UNDER TENSION



fig(3.20) BENDING STRESS AND STRAIN DISTRIBUTIONS

By the notation of 3.20, for equlibrium

$$T = 2a\sigma_t = \int_{-a}^{+a}\sigma dy = \int_{-b_1}^{a}\sigma dy + \int_{-a}^{-b_1}\sigma dy \qquad \text{----3.3.2.}$$

where σ_t = mean applied stress / unit width.

On the assumption of zero shear stress, the strain distribution is linear over the cross section and proportional to distance from the neutral axis (y), thus

$$e_y = (y + b_1) / (R - b_1) \qquad \text{----3.3.3.}$$

Providing that the complete relationship between σ and e are known, equation 3.3.2/3.3.3 may be solved for b_1.

Three types of stress pattern (shown in Fig 3.21.) are possible, being dependent upon the degree of bending and the magnitude of the applied tensile stress.

These are :

(1) Purely elastic stresses

(2) Elastic, with tensile yield

(3) Elastic, with both tensile and compressive yield.

It is assumed that conditions of complete tensile yield are never attained under levelling situations.

Since the strain distribution is linear over the section, a proportion of the region of tensile straining ($y = -b_1$ to $y = a-2b_1$) is the mirror image of the compressed region. It is only necessary therefore, to consider the stress relationship in the upper $2b_1^{ths}$ of the thickness for the equlibrium conditions. The stress/strain relationships are :

(1) Elastic

$$\sigma = \frac{E}{(1 - \nu^2)} \ln (1 + e) \qquad \text{----3.3.4.}$$

for plane strain deformation.

(2) Plastic

$$\sigma = \sigma_o \qquad \text{----3.3.5.}$$

where $\sigma_o \simeq \frac{2}{\sqrt{3}} \times$ uniaxial yield stress ($\bar{\sigma}$) for plane strain.

From section 3.2.0. the yield strain under plane strain conditions may be written.

$$e_o = \exp(\bar{\sigma} (1 - \nu^2) / E (1 - \nu + \nu^2)^{\frac{1}{2}}) \qquad \text{----3.3.6.}$$

For an isotropic material this strain is of equal magnitude in tension and compression.

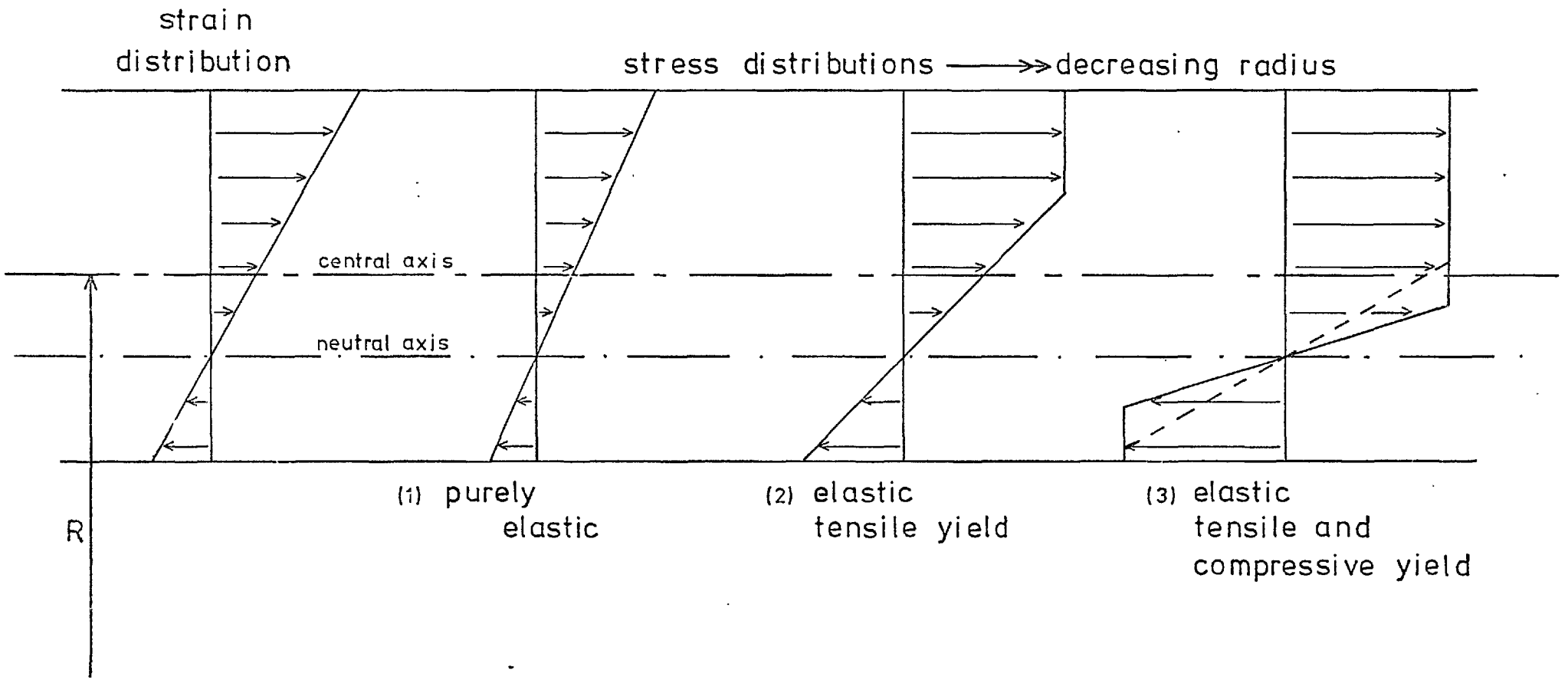


fig (3.21) TYPES OF BENDING-STRESS DISTRIBUTION

ELASTIC, PERFECTLY-PLASTIC MATERIAL

3.3.1.1. Purely Elastic Deformation.

From equations 3.3.2/3/4,

$$2a\sigma_t = \frac{E(R-b_1)}{1-\nu^2.} \int_{\frac{a-b_1}{R-b_1}}^{\frac{a+b_1}{R-b_1}} \ln(1+e)\,de$$

$$= \frac{E(R-b_1)}{1-\nu^2.} \left[(1+e)\ln(1+e)-1-e\right]_{\frac{a-b_1}{R-b_1}}^{\frac{a+b_1}{R-b_1}} \qquad \text{----3.3.7.}$$

For elastic straining $e<0.02$ and

$$\ln(1+e) = e \qquad \text{----3.3.8.}$$

Equation 3.3.7. may thus be written

$$2a\sigma_t = \frac{E(R-b_1)}{1-\nu^2.} \left[\frac{e^2}{2}\right]_{\frac{a-b_1}{R-b_1}}^{\frac{a+b_1}{R-b_1}} = \frac{(4ab_1 \times E)-2b_1E}{(1-\nu^2)(R-b_1)} \qquad \text{----3.3.9.}$$

$$\text{and } b_1 = \frac{R\sigma_t a(1-\nu^2)}{(2a-1)E+\sigma_t(1-\nu^2)a} \qquad \text{----3.3.10.}$$

Equation 3.3.10 is satisfied where

$$\frac{a-b_1}{R-b_1} < \frac{a+b_1}{R-b_1} < e_o \qquad \text{----3.3.11.}$$

3.3.1.2. Elastic Tensile Yielding.

For this case, from equations 3.3.2/3/4/5/8

$$2a\sigma_t = R-b_1\left[\int_{\frac{a-b_1}{R-b_1}}^{e_o} \frac{E\,e\,de}{(1-\nu^2.)} + \int_{e_o}^{\frac{a+b_1}{R-b_1}} \sigma_o\,de\right] \qquad \text{----3.3.12.}$$

The resultant equation for b_1 is complex, the result being applicable where

$$\frac{a-b_1}{R-b_1} < e_o < \frac{a+b_1}{R-b_1} \qquad \text{----3.3.13.}$$

3.3.1.3. Tensile and Compressive Yield.

Here the tensile stress over the required range ($y=a-2b \rightarrow a$) is uniform at σ_o. Thus from equations 3.3.2/5

$$2a\sigma_t = R-b_1 \int_{\frac{a-b_1}{R-b_1}}^{\frac{a+b_1}{R-b_1}} \sigma_o\,de = 2b_1\sigma_o \qquad \text{----3.3.14.}$$

and $b_1 = \frac{a\sigma_t}{\sigma_o}$ 3.3.15.

This is satisfied for

$$e_o < \frac{a-b_1}{R-b_1} < \frac{a+b_1}{R-b_1} \quad \text{----3.3.16.}$$

In deriving the stress system operating under given conditions, it is necessary to solve equations 3.3.10/12/15 for b_1 and substitute into the respective limit equations (3.3.11/13/16) to evaluate the operating conditions.

Reference to equation 3.3.3. will indicate that the mean strip elongation is given by the strain at the central axis (y=0). Thus:

$$e_{mean} = \frac{b_1}{R-b_1} \quad \text{----3.3.17.}$$

Substitution of equations 3.3.15/17 yields

$$e_{mean} = \frac{a\sigma_t}{R\sigma_o - a\sigma_t}. \quad \text{----3.3.18.}$$

and since generally $R\sigma_o >> a\sigma_t$

$$e_{mean} = \frac{a\sigma_t}{R\,\sigma_o} \quad \text{----3.3.19.}$$

Equation 3.3.19 is equivalent to that derived by Swift and, being dependent upon the perfectly plastic assumption with both tensile and compressive yield is limited in its application. This equation, when σ_o is suitably adjusted to account for work-hardening, does however give a useful indication of the strains present during bending under tension.

The inclusion of work-hardening into the analysis requires the replacement of σ_o in equation 3.3.5. by $\sigma = A'(B' + e)^{m'}$. In equation 3.3.14 we thus have,

$$2a\sigma_t = R - b_1 \int A'(B' + e)^{m'}\, de$$

$$= \left[(R - b_1) \frac{A'}{m'} (B' + e)^{m'} \right]_{\frac{a-b_1}{R-b_1}}^{\frac{a+b_1}{R-b_1}} \quad \text{----3.3.20.}$$

For the linear work-hardening case, m' = 1 in equation 3.2.58 and 3.3.20.

$$\therefore 2a\sigma_t = 2A'b_1 \quad \text{----3.3.21.}$$

and

$$e_{mean} \simeq \frac{a\sigma_t}{RA'}. \quad \text{----3.3.22.}$$

This illustrates the dependence of the overall elongation on the basic material strength,not the yield strength where

$$\sigma_o \simeq A'(B' + e_o) \qquad \text{----3.3.23.}$$

for linear work-hardening.$B' \simeq 0.01 - 0.02$ and $e_o \simeq 0.001$, hence A' is 50 to a 100 times the yield stress value for a large number of materials.

3.3.2. Springback Analysis.

Loss of roll contact is concurrent with the elastic relaxation of internal stresses.The degree of this relaxation is such that the internal moment is reduced to zero,whilst the conditions relating to circumferential equlibrium are still maintained.Any residual curvature remaining after the springback operation is immediately removed by the action of the applied tensile forces and will be considered in section 3.3.3.

In the general case, stresses behave elastically on springback,the recovery strains and stresses being proportional and varying linearly with distance from the current neutral axis.The magnitude of these stresses is dependent upon the final value of the internal moment pertaining to the bending operation,strain magnitudes are directly related by the usual elastic relationships.

It will be apparent that with the provision of purely elastic recovery the relaxation axis will coincide with the centroid of section.In this manner the horizontal equlibrium will be maintained,the stress magnitude being defined by the equation

$$M_{rec} = 2 \int_0^a \sigma_{rec}\, y\, dy \qquad \text{----3.3.24.}$$

where σ_{rec} varies linearly with y, and may be written

$$\sigma_{rec} = -\Psi y \qquad \text{----3.3.25.}$$

The distribution of these stresses is shown in Fig 3.22.
From 3.3.24/25 we may write

$$M_{rec} = 2\Psi a^3/3 \qquad \text{----3.3.26.}$$

and hence

$$\sigma_{rec} = \frac{-3M_{rec}y}{2a^3}. \qquad \text{----3.3.27.}$$

The recovery moment is identical in magnitude with the applied moment which is easily determined once the bending stress pattern has been

determined, viz:-

$$M_{app} = \int_{-a}^{+a} \sigma\, y\, dy$$

$$= -\int_{-a}^{-y_{e_o}} \sigma\, y\, dy - \frac{E}{1-\nu^2}\int_{-y_{e_o}}^{-b} e\, y\, dy + \frac{E}{1-\nu^2}\int_{-b}^{y_{e_o}} e\, y\, dy$$

$$+ \int_{y_{e_o}}^{a} \sigma_o y\, dy \qquad \text{----3.3.28.}$$

The recovery strains are related to the recovery stresses by

$$e_{rec} = \exp\left(\frac{\sigma_{rec}(1-\nu^2)}{E}\right) - 1 \qquad \text{----3.3.29.}$$

and when added to the bending strains yield a residual distribution as shown in Fig 3.23. together with the residual stresses.

Reference to Fig 3.22. will show that the recovery axis coincides with the central axis as long as the region of tensile straining on bending is restricted to half the strip thickness.Where a bending distribution, such as that shown in Fig 3.24. is present , recovery will initiate further plastic deformation in the band OT.An effective internal stress loss thus occurs.To restore equlibrium the relaxation axis must undergo a 'shift' (b_2) towards the tensile surface.It is clear that a small shift will readily restore a moderate stress loss.

Recovery stresses must now be written as

$$\sigma_{rec} = -\frac{3M_{app}}{2a^3}(y-b_2) \qquad \text{----3.3.30.}$$

the strains being correspondingly adjusted to give

$$e_{rec} = \exp\left(-\frac{3M(y-b_2)(1-\nu^2)}{2Ea^3}\right) - 1 \qquad \text{----3.3.31.}$$

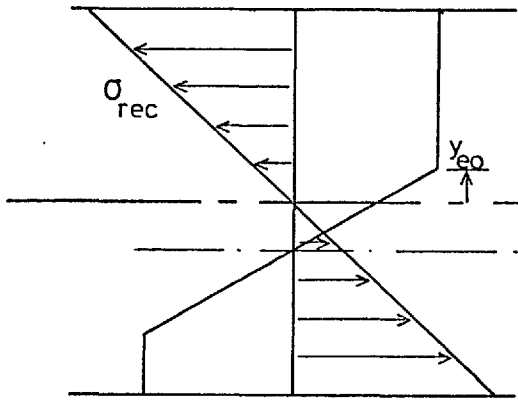
Again as a result of the small magnitude of these strains,engineering strain is equivalent to true strain and equation 3.3.31 reduces to

$$e_{rec} = -\frac{3M(1-\nu^2)(y-b_2)}{2Ea^3} \qquad \text{----3.3.32.}$$

From equations 3.3.3/32 we are thus able to write the residual strain distribution as

$$e_{resid} = \frac{y+b_1}{y-b_1} - \frac{3M(1-\nu^2)(y-b_2)}{2Ea^3}. \qquad \text{----3.3.33.}$$

A residual stress distribution may be similarly obtained by summing equation 3.3.30 and the bending stress levels (Fig 3.25.).



fig(3.22)

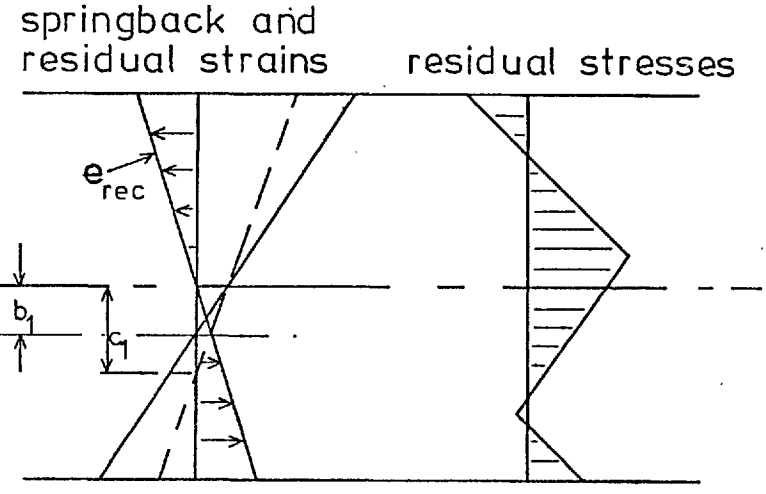


fig (3.23)

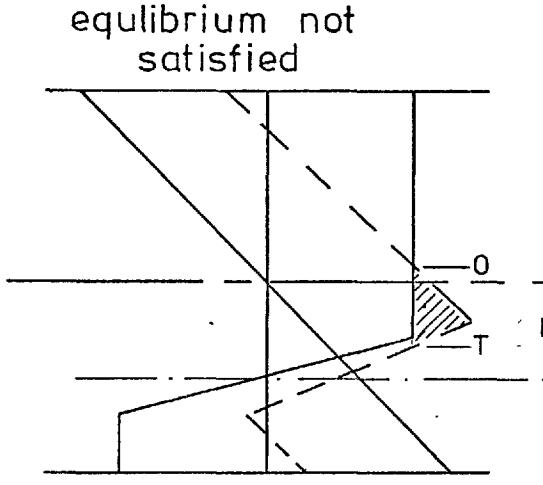


fig (3.24)

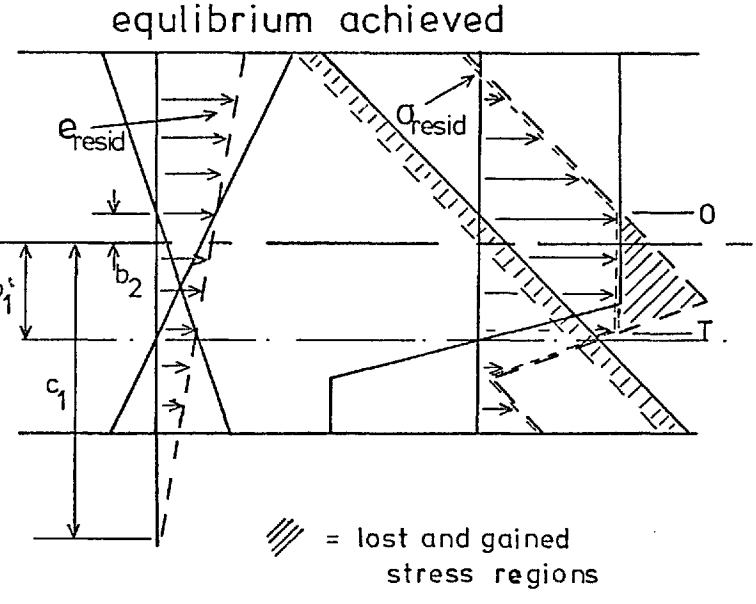


fig (3.25)

At the point $y= -C_1$, (Fig 3.25) the overall strain produced during the process so far is zero.The linear residual strain distribution may thus be expressed as:

$$e_{resid} = \frac{y+c_1}{R_{resid}-C_1} \qquad \text{----3.3.34.}$$

where R_{resid} = residual curvature of central axis.
Substituting $y= -C_1$ into equation 3.3.33. yields :-

$$C_1 = -\frac{(3Mb_2(1-\nu^2) + 2Ea^3 b_1 / (R-b_1))}{3M(1-\nu^2) - 2Ea^3 / (R-b_1)} \qquad \text{----3.3.35.}$$

and substituting $y = 0$ into equation 3.3.33/34 gives:-

$$\frac{C_1}{R_{resid}-C_1} = \frac{b_1}{R-b_1} + \frac{3Mb_2(1-\nu^2)}{2Ea^3} \qquad \text{----3.3.36.}$$

From 3.3.35/36

$$R_{resid} = \frac{2Ea^3(R-b_1)}{(2Ea^3-3M(R-b_1)(1-\nu^2))} + C_1$$

$$= \frac{2Ea^3R + 3Mb_2(R-b_1)(1-\nu^2)}{2Ea^3 + 3M(R-b_1)(1-\nu^2)} \qquad \text{----3.3.37.}$$

It is worthwhile noting here the effect of applied tension on the internal moment due to bending.

For the case of simple elastic bending,

$$e = y/R \qquad \text{----3.3.38.}$$

and thus $$M = 2\int_0^a \sigma y\, dy = \frac{2Ea^3}{3R(1-\nu^2)} \qquad \text{----3.3.39.}$$

With the inclusion of a tensile force, equation 3.3.38 becomes

$$e = \frac{y-b_1}{R-b_1} \qquad \text{----3.3.3.}$$

The bending moment then becomes

$$M = \int_{-a}^{+a} \frac{y+b}{R-b_1} \frac{E}{(1-\nu^2)} y\, dy$$

$$= \frac{2E\,a^3}{3(1-\nu^2)(R-b_1)} \qquad \text{----3.3.40}$$

Comparison of equations 3.3.39/40 shows an increase in the applied bending moment.As is to be expected,substitution of equation 3.3.40 into 3.3.37 yields an infinite residual curvature.

For the case of completely plastic bending, Fig 3.26, the application of tension causes a decrease in internal moment from

$$M_{app} = a^2\sigma_o \qquad \text{----3.3.41.}$$

for pure bending, to

$$M_{app} = (a^2 - b_1^2)\,\sigma_o \qquad \text{----3.3.42.}$$

with applied tension.The residual curvature is thus reduced as a result of reduced springback effects.In any system of bending with applied tension the effect of tension application will be to reduce recovery where plastic straining has occurred, the negative plastic effect being much larger than the positive elastic.

3.3.3. Straightening.

The applied strip tension imposes a straightening moment on the curved strip after springback,causing rotation of the residual strain line to provide uniform strain over the strip thickness.Again, the plastic deformation requires a shift (b_3) of the neutral axis away from the centre line, its direction being that of b_2, Fig 3.27.The straightening strains may be expressed as

$$e_{st} = \frac{(-y+b_3)}{R_{st}-b_3} \times (1+\xi) \qquad \text{----3.3.43.}$$

where R_{st} = effective straightening radius

ξ = residual strain at $y = b_3$.

From Fig 3.27 it is apparent that the magnitude of e_{st} at $y = -c_1$ equals the residual strain at $y = b_3$.

From equations 3.3.34/43.

$$\frac{(c_1 + b_3)}{(R_{resid} - c_1)} = \frac{(c_1 + b_3)(1+\xi)}{(R_{st} - b_3)} \qquad \text{----3.3.44.}$$

$$\therefore \frac{1+\xi}{R_{st}-b_3} = \frac{1}{R_{resid} - c_1} \qquad \text{----3.3.45.}$$

and equation 3.3.43. becomes

$$e_{st} = \frac{-y + b_3}{R_{resid} - c_1} \qquad \text{----3.3.46.}$$

The need for an analysis of the 'effective straightening radius' is eliminated and the straightening strains are predicted from known values. Total strain, resulting from the bending and straightening operation over one roller is written as

$$e_{tot_1} = e_{resid} + e_{st}$$

$$= \frac{y + c_1}{R_{resid} - c_1} + \frac{-y + b_3}{R_{resid} - c_1}$$

$$= (c_1 + b_3) / (R_{resid} - c_1) \qquad \text{----3.3.47.}$$

Determination of b_3 again requires the satisfaction of equilibrium, which in turn requires a knowledge of the stress distribution which is dependent upon b_3.

With the fact in mind that the above analysis must be extended to further rolls, the complex stress pattern after springback makes it worthwhile introducing a numerical technique at this stage.The use of computer techniques allows the straightening stresses to be added to the incomming stress pattern as a hypothetical elastic line:

$$\sigma_{st} = \exp\left(\frac{E}{1-\nu^2}\left(\frac{b_3 - y}{R_{resid} - c_1}\right)\right) - 1 \qquad \text{----3.3.48.}$$

the resultant stress level at any point through the thickness being

$$\sigma_{tot} = \sigma_{bend} + \sigma_{rec} + \sigma_{st} \qquad \text{----3.3.49.}$$

With the computer in mind, the value of σ_{tot} can rapidly be checked at all points for a value of b_3, and reduced to the yield stress value if above.Integration over the section will show if equilibrium obtains. Deviation from this necessitates the adjustment of b_3 to provide a further stress pattern.

The internal stress distribution present after straightening is similar to that shown in Fig 3.28.This distribution is approximately the reverse of Fig 3.21 for bending and, as such, again shows a high dependence upon work-hardening, especially where isotropic hardening properties are present and flow stresses are dependent upon total plastic strain.Material strain hardening properties cause a reduction in b_3 over the perfectly plastic value with a similar reduction in overall strip elongation.

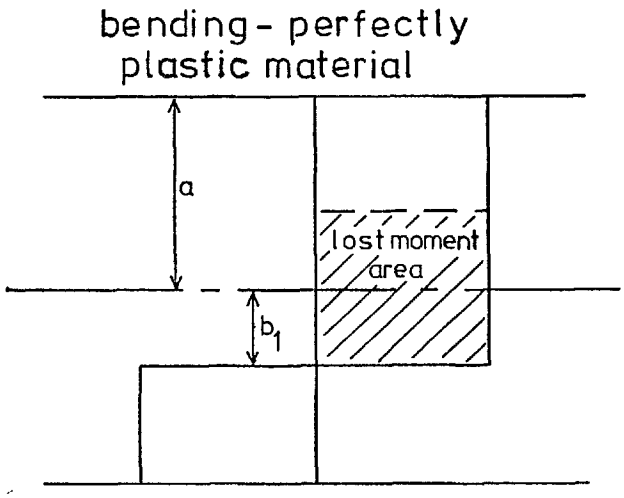


fig (3.26)

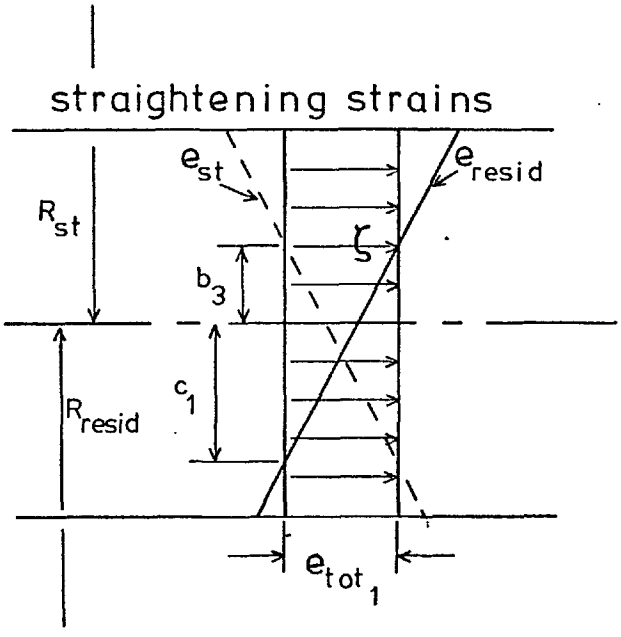


fig (3.27)

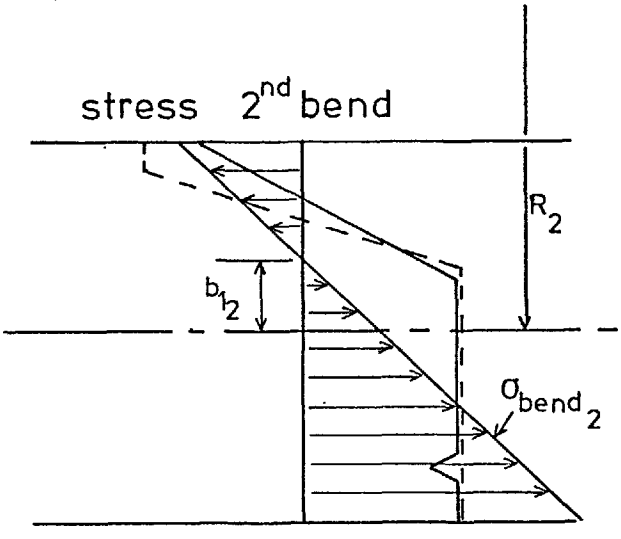


fig (3.29)

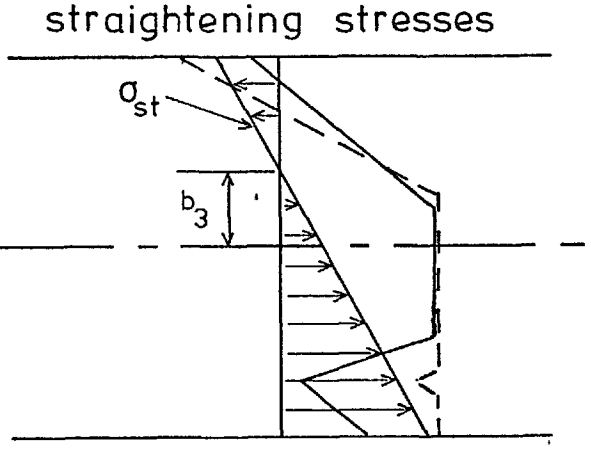


fig (3.28)

— — — = sum stress

The computer program required for the analysis of straightening requires only minor adaptions to enable numerical analysis of bending and springback.Further extension to include the second and subsequent rolls becomes obvious.Work-hardening is also easily included in a computer analysis, it being necessary only to record the total plastic strain for any fibre and determine the current flow stress from equation 3.2.71.

3.3.4. The Second Roll.

After straightening the strip posesses both an overall elongation and an internal stress distribution, which may be complex.The second bend can be treated exactly as the straightening process, and an effective elastic stress distribution added to the residual pattern.Strain during this bend is

$$e_{y_2} = \left((-y + b_{1_2}) / (R_2 - b_{1_2}) \right) X \ (1 + e_{tot_1}) \qquad \text{----3.3.50}$$

the stress distribution to be added being

$$\sigma_2 = \ln (1 + e_{y_2}) \, E / (1 - \nu^2) \qquad \text{----3.3.51.}$$

(Fig 3.29)

The total stress pattern is adjusted where plastic flow occurs and is checked for equlibrium.The determined value of b_{1_2} will be less than b_{1_1} for both the non-hardening and hardening cases, the incoming stress form being little altered by the bend.Springback and straightening are analysed as in sections 3.3.2/3.

3.4.0. Application to Levelling.

3.4.1. The General Program.

Section 3.3. has shown the development of the basic equations for bending and unbending round a free roll and has shown that with suitable stress reductions for plastic flow, each stress increment may be added on an elastic basis.Extension of theory to include any number of rolls thus requires only(1) the generalization of the basic equations.

(2) a routine to determine plastic flow stresses and

(3) a method for stress integration over the strip thickness.

At this stage it is also worth including any initial stress pattern at leveller entry.

To allow suitable numerical analysis, especially where (3) is concerned it is necessary to divide the strip into a number of thickness fibres. Since a Simpson's Rule integration technique is readily adaptable to

computerization,129 fibres were selected, numbers 1 and 129 lying at the strip surface and 65 at the centroidal axis.Fibre 1 is defined as that which comes into contact with the first roll. (Fig 3.30)

A high number of record points is required to provide sufficient accuracy where large stress gradients are present, ie. in the elastic range. Accurate determination of overall internal stress and bending moments are possible with the chosen number of record fibres.

The routine to determine the 'maximum permissible stress' for plastic flow must allow for both the isotropic and kinematic models of hardening. The necessary inclusion of the kinematic model is examined in section 3.4.2., whilst the above routine is explained in section 3.4.3.

3.4.1.1. Initial Conditions.

Unless the levelling installation immediately follows the cold or temper mill, incomming strip will, of necessity, possess a coil set, and in either case will contain residual stresses.Transverse stress patterns have been previously mentioned and will be discussed where necessary.The thickness distribution is that developed by normal rolling processes as illustrated in Fig 3.31.Rolling stresses are, however, small in comparison with those developed during levelling and are secondary to those produced during the removal of coil set, since curvature removal entails at least some plastic straining, with stresses greater than yield.They will therefore, be ignored during this analysis.

During the tension build up at leveller entry, coil set is removed and is replaced by a stress system.This system may be adjusted by passage over bridle rolls.It is assumed however, that the effective coil set after the highest tension bridle roll is known and that this strain distribution in the free state is accompanied by zero stress levels. (A reasonable assumption except for heavily cold rolled strip.) Removal of this coil set is in general coincident with the development of plastic stresses within the strip.(Fig 3.32)

The analysis of coil set removal is similar to that of straightening after bending.Flattening strains are thus represented by

$$e_f = \frac{(\pm y + p)}{R_{curve} + p} \qquad \text{----3.4.1.}$$

The sign of y is dependent upon the direction of the initial coil set. Fig 3.33. The corresponding elastic stress distribution being

$$\sigma_f = \ln (1 + e_f)\, E / (1 - \nu^2) \qquad \text{----3.4.2.}$$

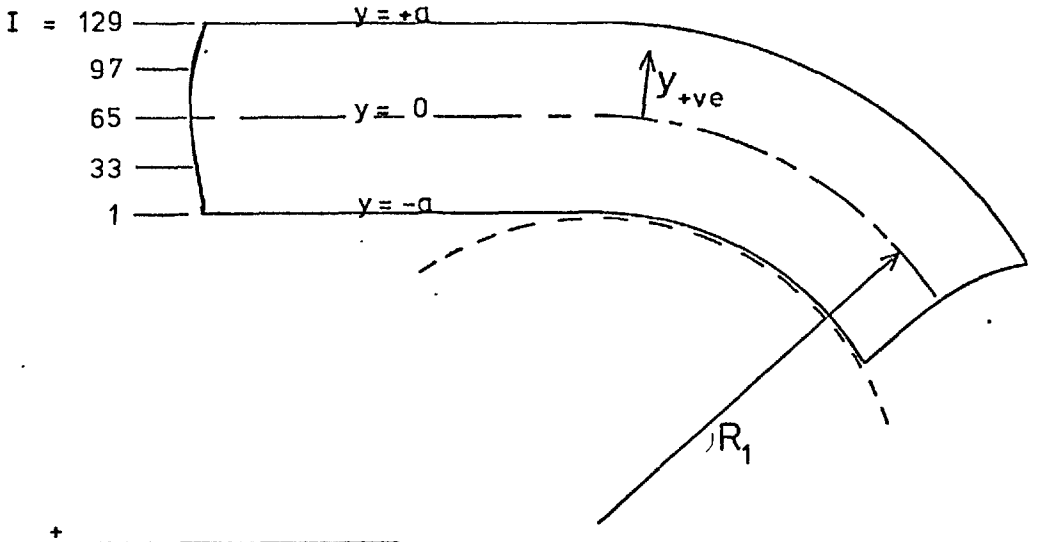


fig (3.30)

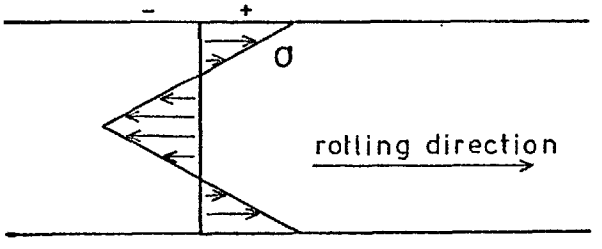


fig (3.31) ROLLING RESIDUALS

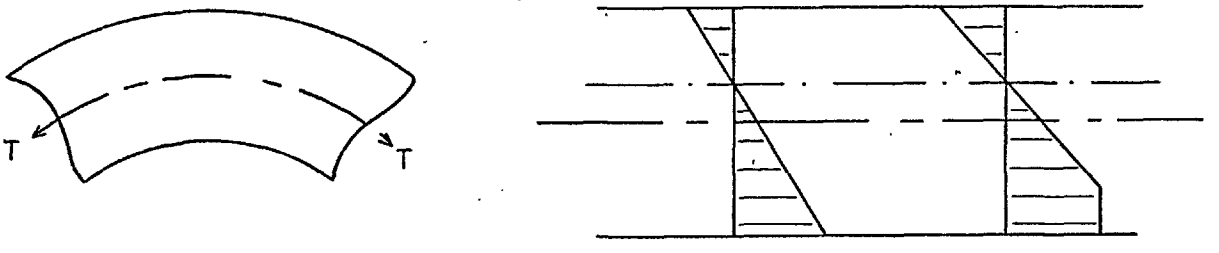


fig (3.32) COIL SET REMOVAL

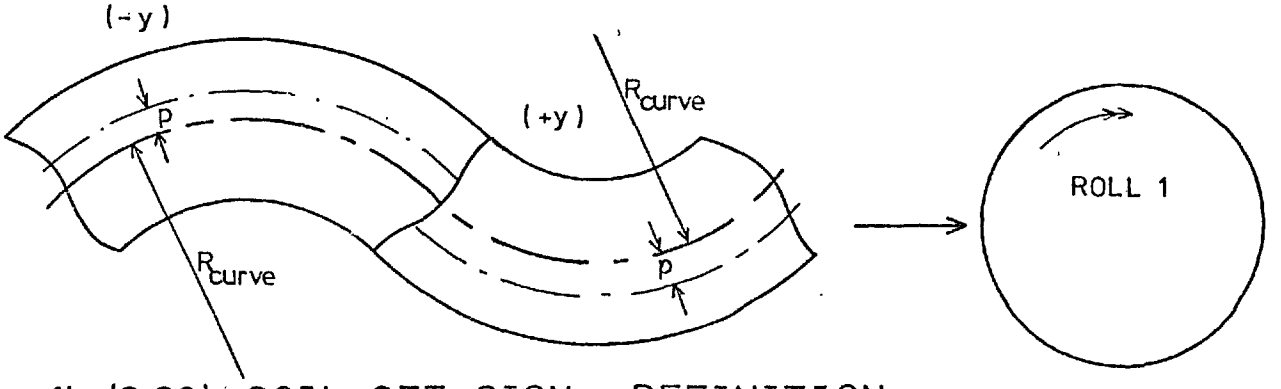


fig (3.33) COIL SET SIGN - DEFINITION

The program operates, as in all incremental operations, by

(1) setting p to zero

(2) evaluating the elastic stress at each fibre.

(3) Adjusting to plastic flow stress where necessary.

(4) Integrating to test equlibrium.

(5) Adjusting p where necessary and repeating

(6) On equlibrium satisfaction, stress and strain values at each fibre are recorded.

3.4.1.2. Roller Conditions.

(1) Bending.

The general equation for strain during bending is

$$e_B = \frac{(- (-1)^n y + {}^{b_1}{}_n)}{(R_n - b_{1_n})} \times (1 + {}^{e_{tot}}y) \qquad \text{----3.4.3.}$$

where n = roll number.

R_n = radius of central axis.

The corresponding elastic stresses are given by 3.4.2, and the determination of b_1 is identical to that of p with the provision that incomming stresses are added.

(2) Springback.

The final stress pattern after bending is integrated to evaluate the internal moment, M_n. Recovery stresses are then determined by

$$\sigma_r = - \frac{3M_n}{2a^3} (y + {}^{b_2}{}_n(-1)^n) \qquad \text{----3.4.4.}$$

the corresponding strains being

$$e_r = \exp(\sigma_r (1- \nu^2) / E) -1 \qquad \text{----3.4.5.}$$

Evaluation of b_2 is as for p and b_1.

(3) Straightening.

From equation 3.3.46, straightening strains are given by

$$e_s = \frac{(y(- 1)^n + b_{3_n})}{(|R_{resid_n}| - c_{1_n})} \qquad \text{----3.4.6.}$$

where R_{resid_n} and c_{1_n} are evaluated from equations 3.3.35 and 3.3.37.

Stresses are again given by equation 3.4.4. and valuation of b_3 is performed as before.

In this manner, by keeping a record of stresses and strains at all fibres, the overall elongation over any number of rolls and under any conditions may be determined.

On tension relaxation at the leveller exit, recovery of the final straightening stresses occurs and a residual curvature develops.This process is treated as for springback after bending,but equilibrium now requires zero overall stress. Thus

$$\sigma_{rf} = \frac{3M_f}{2a^3}(y-b_4 \ (-1)^{nr}) \qquad \text{----3.4.7.}$$

where nr = number of leveller rolls.

Mf = final internal moment.

The corresponding strains are given by equation 3.4.5.

This final strip curvature will in general be adjusted by strip passage over the exit tension bridle.

During passage through the leveller, the material undergoes an overall extension.We are assuming plane strain conditions and thus a corresponding thinning action occurs.The true strain occurring during any incremental process is

$$\varepsilon = \ln (1 + e^{65}) \qquad \text{----3.4.8.}$$

e^{65} = engineering strain at central axis.

With minimal error it can be assumed that each operation occurs at constant volume and the natural thickness change may be written.

$$\ln \frac{t_1}{t_0} = \ln (1 + e^{65}) \qquad \text{----3.4.9.}$$

Hence

$$t_1 = t_o (1 + e^{65}) \qquad \text{----3.4.10.}$$

The method used in obtaining the operating neutral axis 'shift' for any operation is therefore as follows:

(1) From the incomming thickness derive the y values for fibres 1 to 129

ie. $$y_I = -a + \frac{a(I-1)}{65} \qquad \text{----3.4.11.}$$

(2) Set b equal to zero and derive the relevant incremental elastic stress and strain at each fibre.

(3) Add these values to any incomming values.

(4) Utilizing the 'Maximum Stress' routine, determine the operating stress levels.

(5) Integrate stress over the section and check for equilibrium.

(6) a. If equilibrium is attained, calculate any necessary bending moment etc. and output required values.

b. Where force balance is not obtained, adjust b by a suitable increment and return to (2).

The flow diagram for this program (subroutine ROLL) is shown in Fig 3.34, a description and listing being available in Appendix 3.

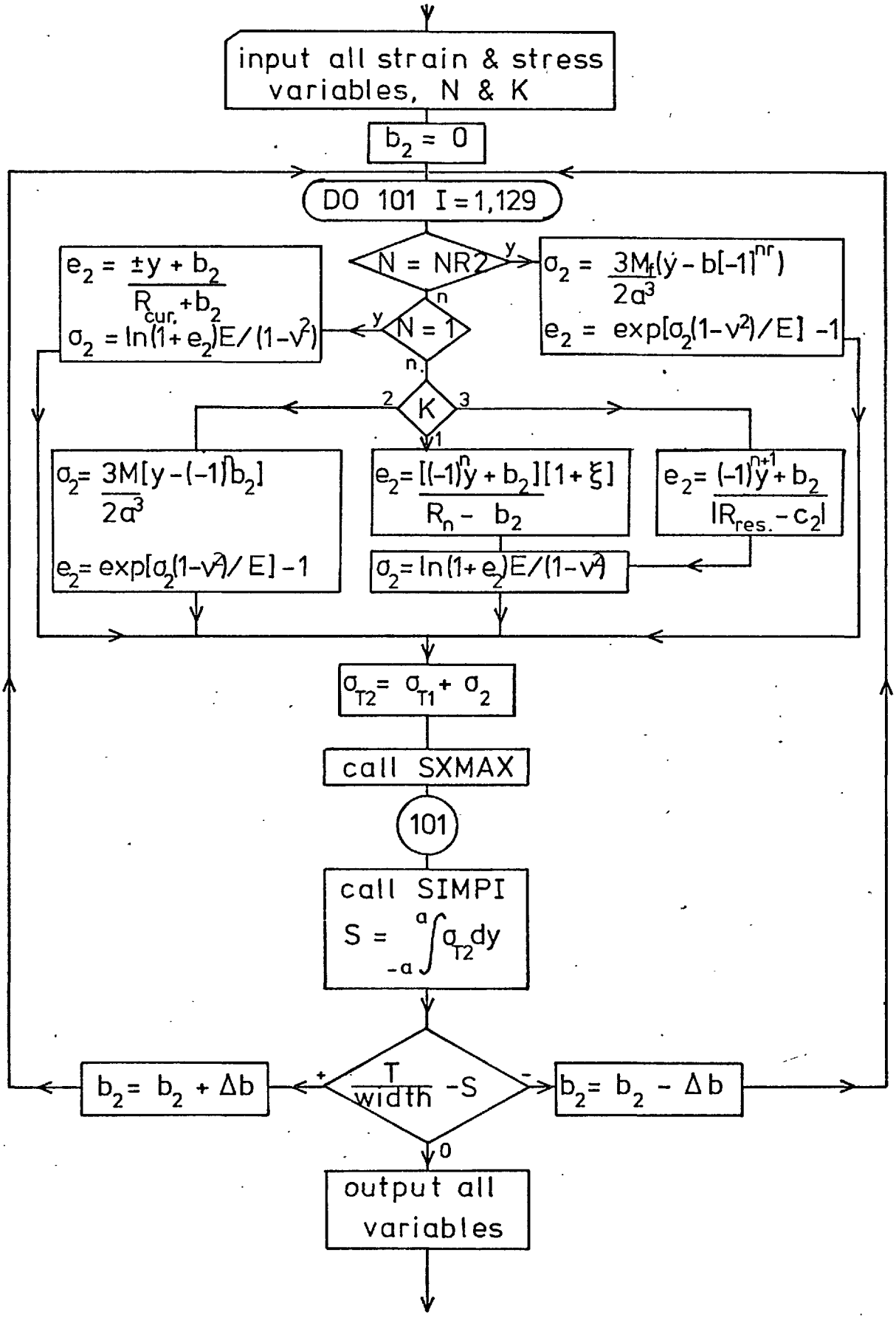
This basic routine is relatively simple and presents no programming difficulties with careful adjustment of b. The routine required to predict the plastic flow stress at any fibre and at any point in the leveller needs more thought however.Incoming stress levels may be either elastic or plastic.The stress increment may be either tensile or compressive.If plastic straining does ensue the strain for initiation is dependent upon the assumed stress/strain relationship, as is the level of plastic stress.

Before presenting the details of this routine, it is necessary to examine the need for more than one model of stress/strain relationship where hardening is included.

3.4.2. Isotropic or Kinematic ?

The discrepancies introduced by the assumption of perfect plasticity have previously been mentioned.The normal assumption of reversible hardening will predict more realistic results, but the reverse loading nature of levelling does not allow the exclusion of the irreversible model of hardening which gives a partial explanation of Bauschinger phenomena.The effects of reverse loading on the material stress/strain curve are shown in Fig 3.35, for a work-hardening material, together with the stress paths predicted by various approximations.

Isotropic hardening predicts yield in compression at the final tensile stress (F).The kinematic model predicts yield at (C). In reality the compressive yield stress will be approximately that at D. Mróz (84) has extended the kinematic model to include the concept of a 'field of work-hardening moduli' Fig 3.36.The complete model assumes the initial yield locus is surrounded by n hypersurfaces which define regions of constant work-hardening moduli.Work-hardening during loading results in the movement of these loci as shown in Fig 3.37.It can be seen that



fig(3.34) FLOW CHART - SUBROUTINE ROLL

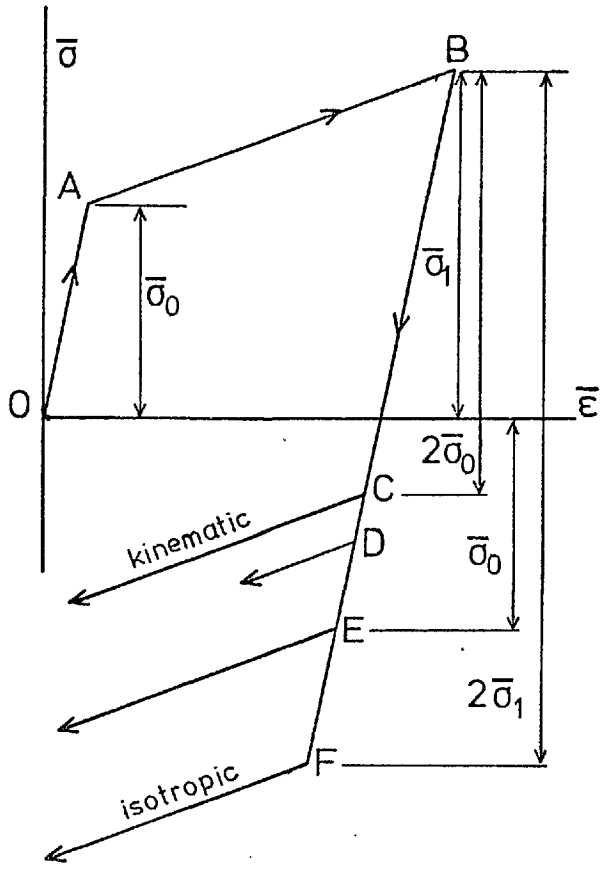


fig (3.35)

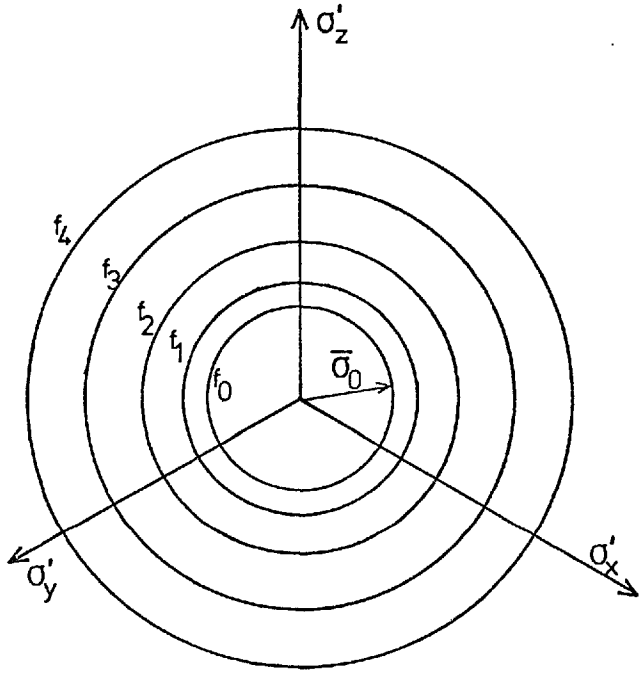


fig (3.36) FIELD OF WORKHARDENING MODULI (MRÓZ)

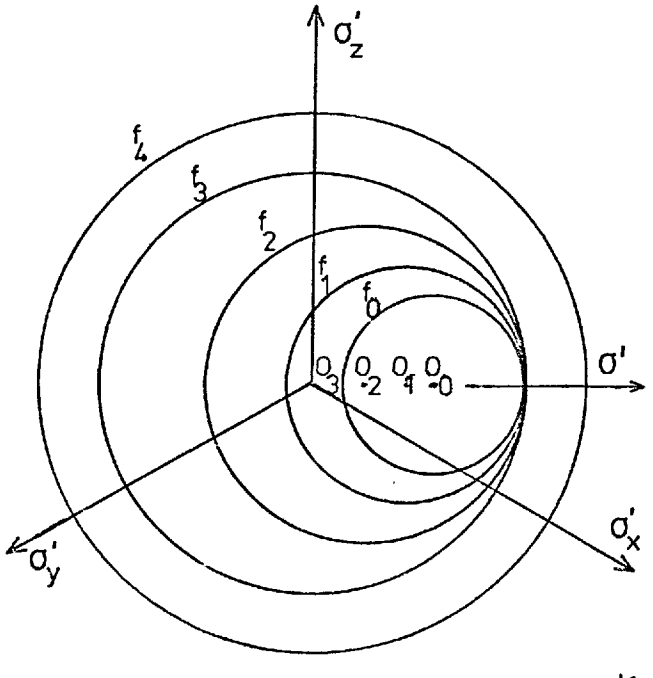


fig (3.37) STRESS SPACE SITUATION DURING LOADING

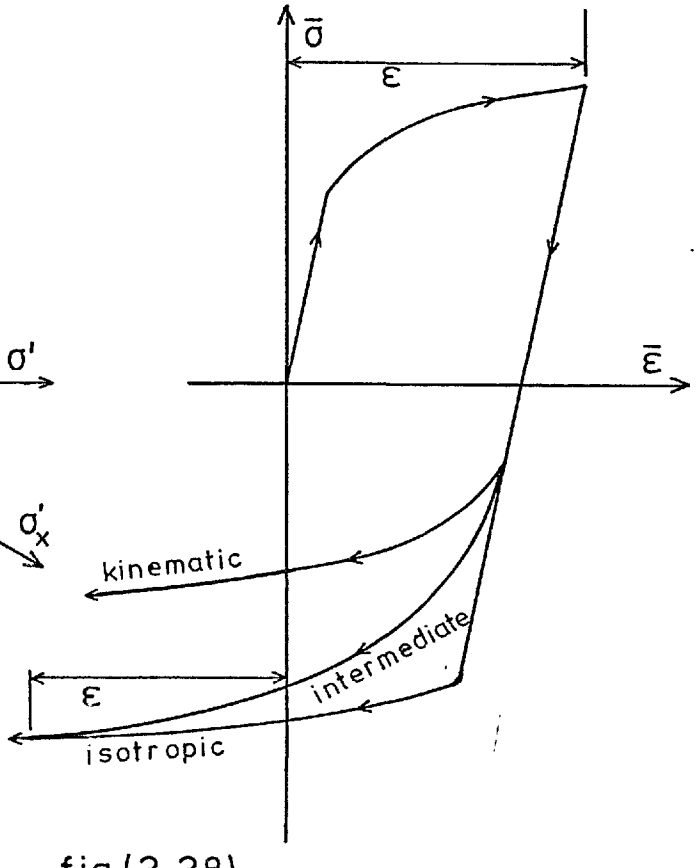


fig (3.38)

unloading and reverse loading will result in an elastic range equivalent to that of the kinematic model, but, on compressive yield, the material is effectively harder than with the irreversible assumption.The comparison between these models is given in Fig 3.38, and it is seen that the combined model most closely approaches reality.This model is equivalent in its results to the analysis given by Wells (83) from the assumption of homogeneous strain and a model of slip based upon reversible, long-range dislocation interactions.

The various hardening assumptions effect the longitudinal stress pattern through the strip thickness in the following manner:

1. Isotropic Hardening.

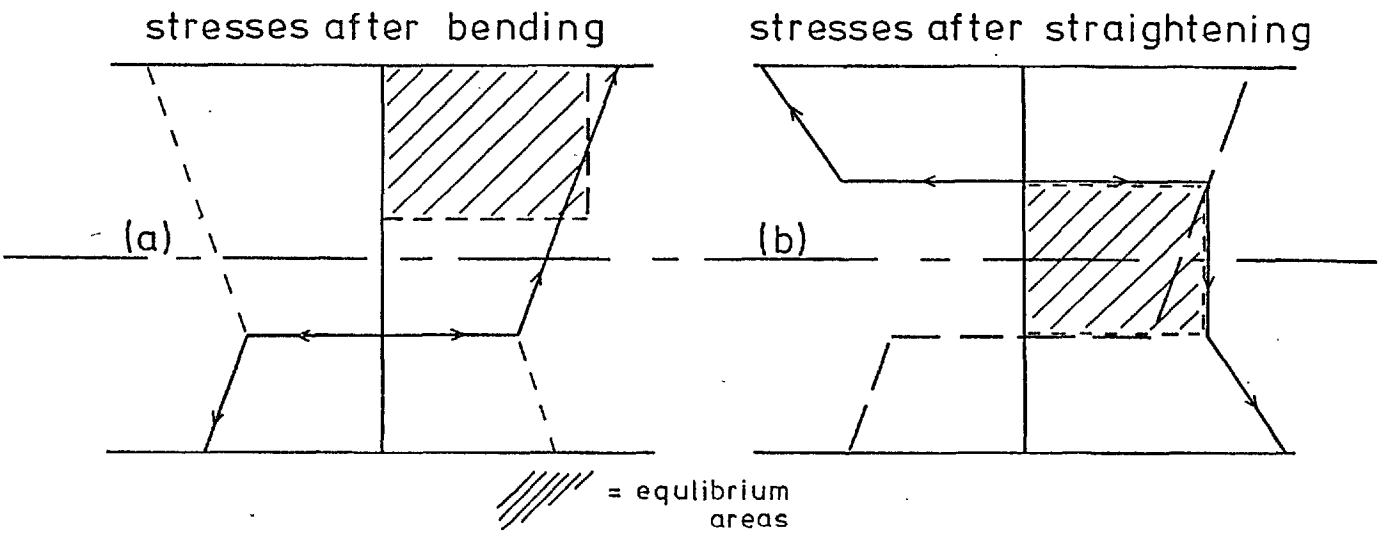
For the purposes of illustration we can assume a linear hardening curve in the first instance.The stresses developed during bending and after straightening are shown in Fig 3.39, the positions of zero stress being approximately equidistant from the centroidal axis.The stress regions required to balance the applied load are seen to be equivalent since the additive work-hardening in the upper and lower regions is equal and of opposite sign.Neglecting elastic effects, the overall strain imposed during straightening is identical to that of bending.

2. Kinematic Hardening.

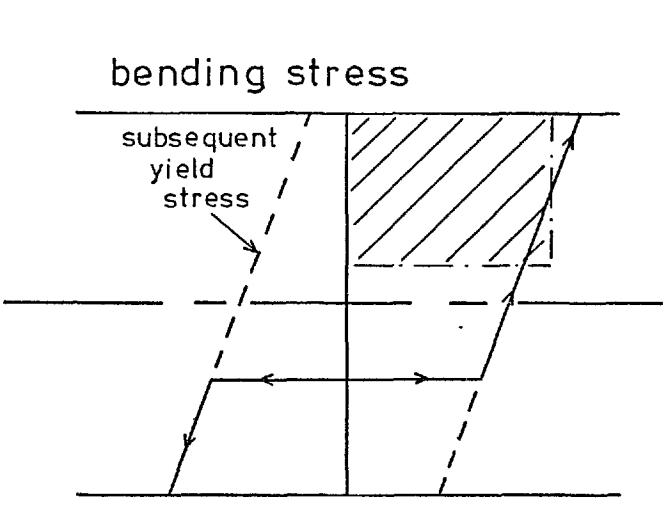
Assuming an identical loading relationship to that of (1), the initial bending distribution is again as in Fig 3.39(a).The constant elastic range produces yield stress values as shown in Fig 3.40.If we now assume the strip to be unbent about the neutral axis shown Fig 3.39 (b), the final stress levels are shown in Fig 3.41. The shaded region is that required for equilibrium, being identical to that of Fig 3.39(b).In this case, however,the unshaded tensile and compressive regions do not cancel and an overbalance of tensile force is present.To provide a return to equilibrium it is necessary for the neutral axis 'shift' on straightening to be less than that for the isotropic case.The overall strain produced during the operation is similarly reduced.

The position of the neutral axis during the second bend will be closer to the central axis than for straightening in both the isotropic and kinematic cases.The decrease in shift will be slightly greater for the isotropic model as a result of greater work-hardening.Even so a greater strain will still occur.Subsequent straightening reverts to the condition of Fig 3.41, the centre line strain being very much less for the kinematic case.

The factors of decreased strain and work-softening appear to contradict for the kinematic case.The explanation is dependent upon the fact that



fig(3.39) ISOTROPIC/LINEAR WORKHARDENING



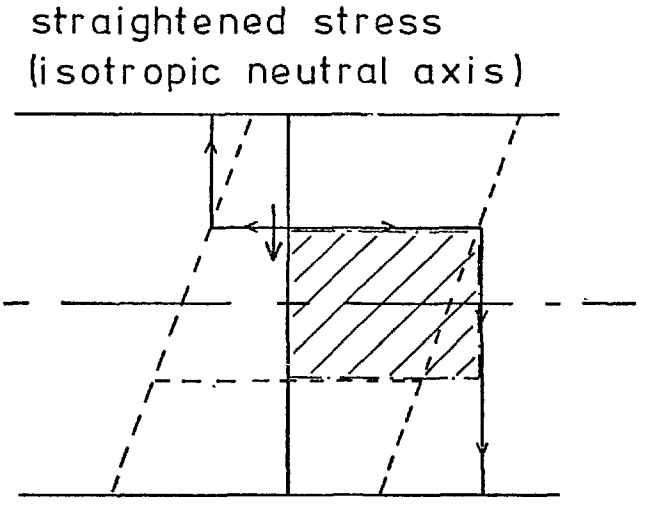


fig (3.40)

fig (3.41)

KINEMATIC/LINEAR WORKHARDENING

the predominating strain type for bending under tension is tensile.This results in the softening effect being more apparent within the compressed zones, which more than accomodates for the tensile softening over the isotropic work-hardening case.Should the applied tension be high enough to cause the bending axis to approach the strip surface, the overall softening may be predominately greater and the overall strain will be increased above that of the reversible work-hardening assumption.(Fig 3.42)

3. Isotropic/Kinematic.

This model reduces to the simple kinematic hypothesis for a linear work-hardening material, and must therefore be considered in terms of a non-linear hardening curve.The effects of bending and unbending are given in Fig 3.43 for a curve of this type

$$\sigma = B + e^{\frac{1}{2}} \qquad \text{----3.4.12.}$$

As indicated by Mróz and Wells, the reverse and subsequent loading curves may be expressed as

$$\sigma = \sigma_{aax} - 2\left(\frac{\Delta e}{2}\right)^{\frac{1}{2}} - 2B \qquad \text{----3.4.13.}$$

where σ_{aax} = maximum attained flow stress.

This equation is derived from the fact that on reverse loading, the relationship between half the stress change and half the strain change is identical to the stress/strain relationship on loading Fig 3.44.

For this model a reduction in strain over the equivalent isotropic hypothsis is seen, this reduction being less than that of the kinematic assumption by virtue of the effective increased hardening.This model is also represented in Fig 3.42.

Choice of the model depends upon the material being considered and it is difficult to deduce the overall differences unless a complete analysis is performed for each.It was deemed easier to include all three models in the stress limiting routine,'SXMAX', than to attempt manual differentiation.

It should be noted here that the Isotropic/Kinematic model was not considered in the derivation of stress/strain relationships in section 3.2. This however was unnecessary because of the direct relationship between the kinematic and above models.For the kinematic model, we may write

$$\sigma = A'(B' + e)^{m'} \qquad \text{----3.4.14.}$$

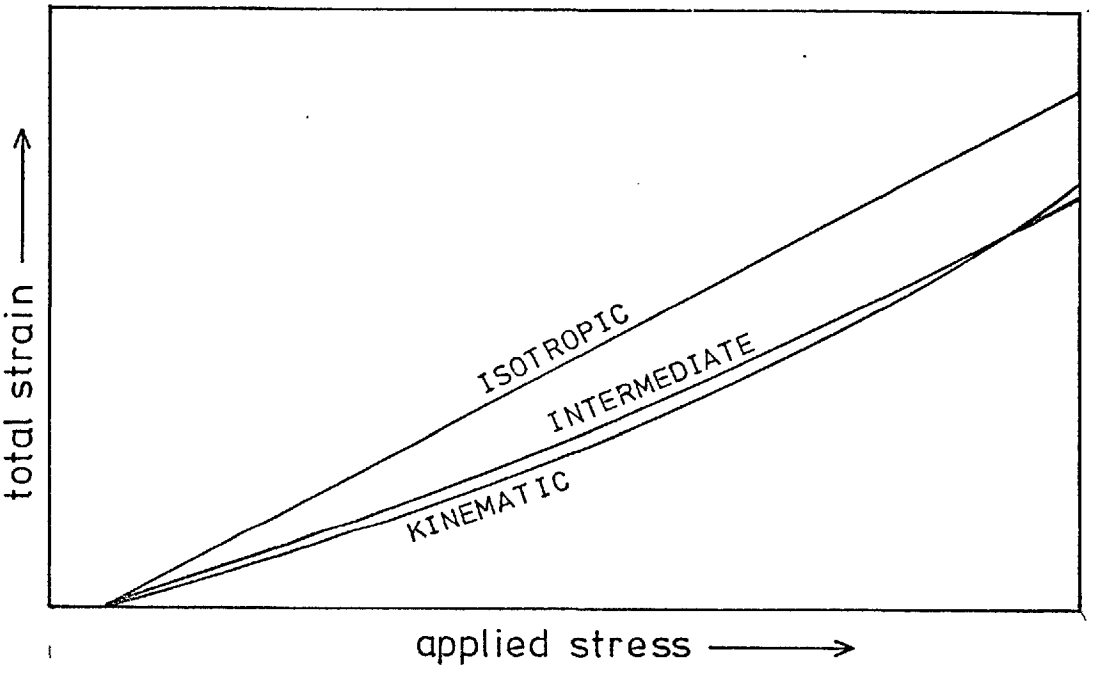


fig (3.42) MODEL COMPARISON - TENSION LEVELLING

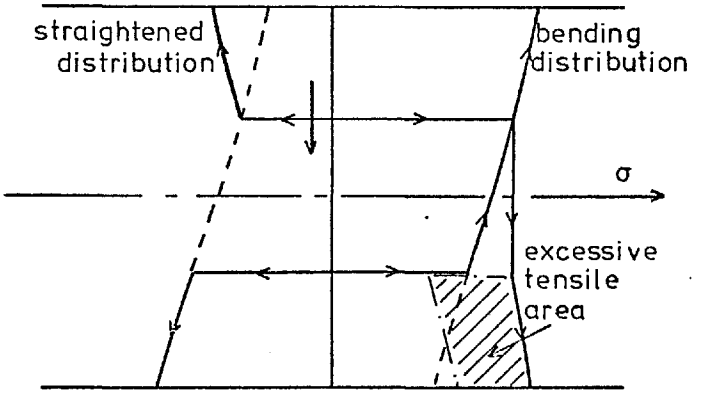


fig (3.43) STRESS FORMS - INTERMEDIATE MODEL

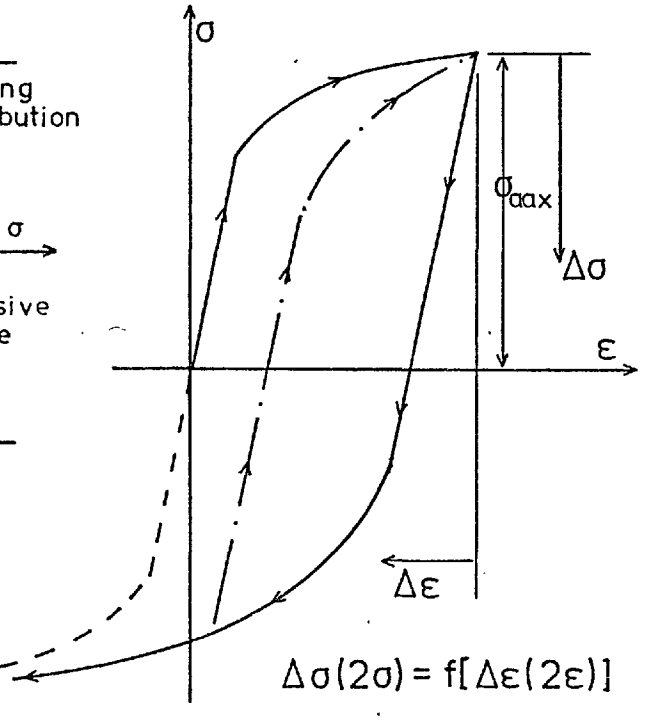


fig (3.44) REVERSE LOADING INTERMEDIATE MODEL

From Mróz's postulation we may write for reverse loading

$$\sigma = \sigma_{aax} - 2A'(B' + \frac{\Delta e}{2})^{m'} \qquad \text{----3.4.15.}$$

where Δe = incremental strain from that at σ_{aax}.
Equation 3.4.15 is easily incorporated into the routine 'SXMAX', thus eliminating the need for further deviatoric plane analysis.

3.4.3. Subroutine SXMAX.

This routine is called once for each thickness fibre each time an elastic stress system is added to a residual system. Its purpose of limiting stress to values predicted by the assumed flow laws is therefore critical and allows small scope for error.

For unyielded fibres the stress level is related to the total strain by

$$\sigma = \ln(1 + e)\, E/(1 - \nu^2) \qquad \text{----3.4.16.}$$

Where yielding has occured it is necessary to

(1) Determine the current strain position.

(2) Determine the direction and extent of the next strain increment.

(3) Determine the strain contributing to hardening. (This is dependent upon the hardening model).

(4) Determine the relevant stress level.

The theoretical basis to the development of this routine is best described in relation to the reversible hardening model. Adjustments for the kinematic models then become readily apparent.

3.4.3.1. Isotropic Hardening.

Taking the most general case, previous strain increments may have resulted in the stress point (Fig 3.45) lying at any one of the positions A, B or C. If a non-directional model is developed, this reduces to points A or B, being plastic and elastic respectively. We must, however, be able to determine which point is involved. Three more strain functions as well as those of the incremental strain e, and the total strain e_{tot} must be utilized.

(1) e_m = maximum strain contributing to hardening. Use of the relationship,

$$\sigma = A'(B' + e)^{m'}$$

requires that the value of e_m is the sum total of the absolute values of plastic strain plus the initial yield strain.

(2) e_E = current length of the elastic strain region.

(3) e_{ab}, where $e_m - e_E < e_{ab} \leqslant e_m$.

e_{ab}, thus defines the position of the strain point with respect to the current yield conditions.Fig 3.46.

For the purposes of this routine we shall define all revelant variables as :

e_1 & e_2 = previous and current fibre strains ($\pm$)
e_{t1}& e_{t2} = previous and current sum strains ($\pm$)
e_{m1}& e_{m2} = " " "maximum" strains (+)
e_{ab1}& e_{ab2} = " " absolute strains (+)
e_{E1}& e_{E2} = " " elastic range (+)

σ = current stress increment on elastic basis ($\pm$).
σ_{t2} = sum stress = $\sigma_{t1} + \sigma$ ($\pm$)
σ_{m2} = current "maximum" permissible stress. (+).

To determine the position of the stress point we may write : (Fig 3.45)

if ($e_{ab1} = e_{m1}$) - strain at A ----3.4.17

if ($e_{ab1} < e_{m1}$) - strain at B ----3.4.18

For a strain point at A three possible conditions are avaliable for the next strain increment.

(1) e_1 and e_2 are of the same sign, therefore strain A→D.

$\dot{e}_{ab2} = e_{m1} + |e_2|$

$e_{m2} = e_{ab2}$

U = 1 ----3.4.19.

(2) e_1 and e_2 are of opposite sign

$|e_2| < e_{E1}$ $\therefore$ A→B.

$\dot{e}_{ab2} = e_{ab1} - |e_2|$

$e_{m2} = e_{m1}$

U = 2 ----3.4.20.

(3) e_1 and e_2 are of opposite sign,

$|e_2| > e_{E1}$ $\therefore$ plastic strain (A→C).

$\dot{e}_{ab2} = e_{m1} + |e_2| - e_{E1}$

$e_{m2} = e_{ab2}$

U = 3. ----3.4.21.

If on the other hand, the previous stress point lies in the elastic zone ie. equation 3.4.18 satisfied, four possibilities exist.We shall assume the stress point reached B (Fig 3.47) via X and A.

(1) e_1 & e_2 are of the same sign.

$$e_{abl} - |e_2| \leqslant e_{ml} - e_{El}$$

No plastic deformation, B⟶B'.

$$e_{ab2} = e_{abl} - |e_2|$$

$$e_{m2} = e_{ml}$$

$$U = 4 \qquad \text{----3.4.22.}$$

(2) e_1 and e_2 are of the same sign.

$$e_{abl} - |e_2| > e_{ml} - e_{E2}$$

Stress path B⟶C and plastic strain

$$e_{ab2} = e_{ml} - (e_{ml} - e_{El}) + (e_{abl} - |e_2|)$$

$$e_{m2} = e_{ab2}$$

$$U = 5 \qquad \text{----3.4.23.}$$

(3) $e_1$and e_2 are of the opposite sign

$$e_{abl} + |e_2| \leqslant e_{ml}$$

No plastic deformation, B⟶B"

$$e_{ab2} = e_{abl} + |e_2|$$

$$e_{m2} = e_{ml}$$

$$U = 6. \qquad \text{----3.4.24.}$$

(4) e_1 and e_2 of opposite sign

$$e_{abl} + |e_2| > e_{ml}$$

Stress path B⟶D.

$$e_{ab2} = e_{abl} + |e_2|$$

$$e_{m2} = e_{ab2}$$

$$U = 7 \qquad \text{----3.4.25.}$$

For all conditions, U = 1 to 7, total strains may be written

$$e_{t2} = e_{t1} + e_2 \qquad \text{----3.4.26.}$$

Providing that two reversals of stress do not occur with the stress remaining in the elastic range the seven equations presented above allow

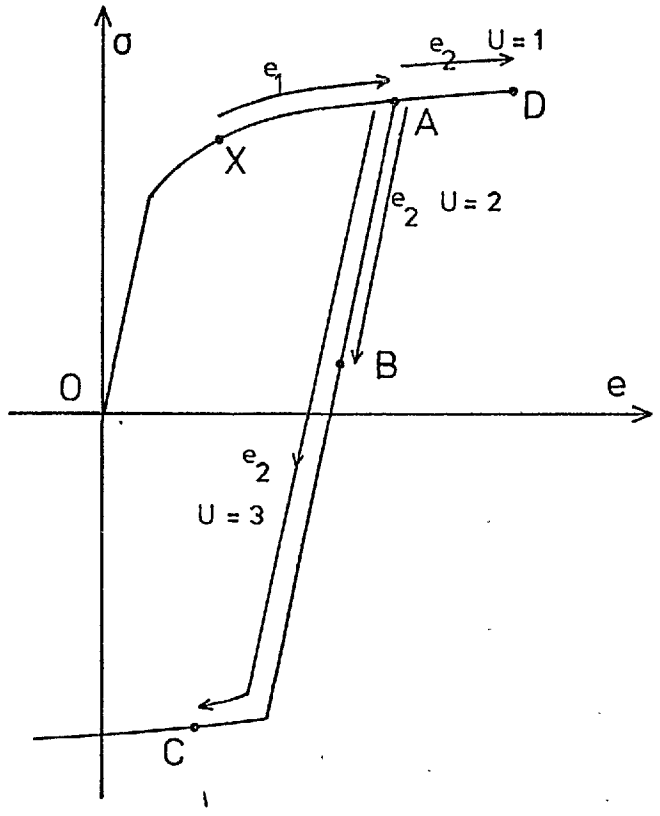


fig (3.45) DEFORMATION MODES FROM PLASTIC RANGE

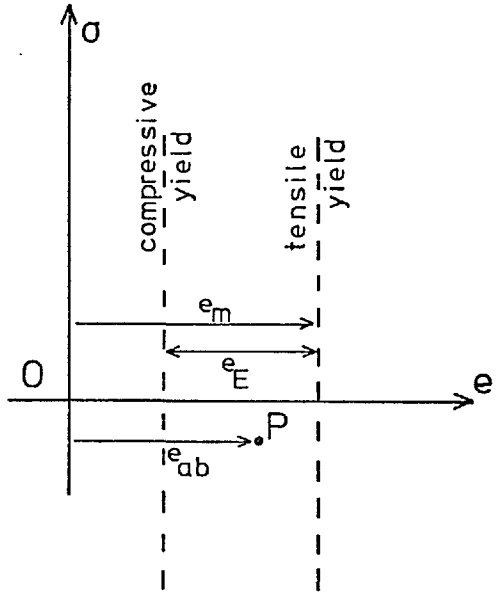


fig (3.46) DEFINITION OF STRAIN PARAMETERS

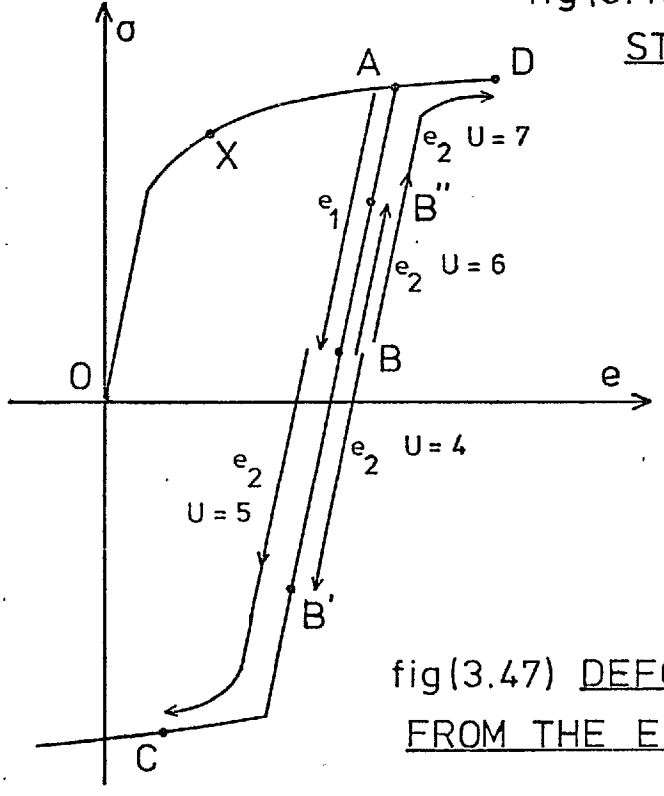


fig (3.47) DEFORMATION MODES FROM THE ELASTIC RANGE

a complete record of the strain contributing to work-hardening.The likelihood of the above conditions occurring in the tension leveller are remote and may safely be ignored.

The program determines which of the above equations applies to the fibre being considered and determines the "maximum permissible stress" for plastic flow from the equation

$$\sigma_{m2} = A' (B' + e_{m2})^{m'} \qquad \text{----3.4.27.}$$

If the sum stress σ_{t2} satisfies the condition.

$$|\sigma_{t2}| \leqslant \sigma_{m2} \qquad \text{----3.4.28.}$$

the stress point lies in the elastic range and the value σ_{t2} is the true value of the current stress level.Where $|\sigma_{t2}| > \sigma_{m2}$, the stress is plastic and the magnitude of σ_{t2} must be reduced to that of σ_{m2}, whilst retaining its original sign.The new value of the elastic range is determined from

$$e_{E2} = 2(\exp(\sigma_{m2}(1-\nu^2)/E) - 1) \qquad \text{----3.4.29.}$$

From the results of section 3.2.2, it is necessary to adjust the value of e_{E2}, where the value of $e_{t2} >$ twice the initial yield strain, to allow for the decrease of $\simeq 10\%$ in elastic strains.

3.4.3.2. Kinematic Hardening.

The irreversible hardening hypothesis predicts a constant elastic strain range and a strain hardening function independent of prior deformation.The permissible level of plastic stress is therefore only a function of the degree of plastic strain which has occurred since plastic straining of the opposite sign. This factor is represented by the parameter e_b Fig 3.48.

As illustrated in Fig 3.48, work-hardening produces a shift (S) of the elastic stress range to one side or other of the zero stress axis.A record of this shift together with a determination of e_b will allow elucidation of the relevant stress magnitudes.The method utilized for pinpointing the stress point in the isotropic case is applicable here and the values of e_t, e_m and e_{ab} must therefore be determined.Equations 3.4.19 to 3.4.25 must however have the additions given below.

$$U = 1, \quad e_{b2} = e_{b1} + |e_2|$$

$$S_2 = S_1 \qquad \text{----3.4.19a.}$$

$$U = 2, \quad e_{b2} = e_{b1}$$

$$S_2 = S_1 \qquad \text{----3.4.20a.}$$

$U = 3, \quad e_{b2} = e_{ab2} - e_{m1}$

$S_2 = \sigma_{m1} - \sigma_o$ ----3.4.21a.

$U = 4, \quad e_{b2} = e_{b1}$

$S_2 = S_1$ ----3.4.22a.

$U = 5, \quad e_{b2} = e_{ab2} - e_{m1}$

$S_2 = \sigma_{m1} - \sigma_o$ ----3.4.23a

$U = 6, \quad e_{b2} = e_{b1}$

$S_2 = S_1$ ----3.4.24a.

$U = 7, \quad e_{b2} = e_{b1} + |e_2|$

$S_2 = S_1$ ----3.4.25a.

where σ_o = initial yield stress under plane strain conditions.

For U = 2,4 and 6 (ie. stresses in the elastic range) stresses are those predicted by the elastic addition.

For U = 1,3 and 5, the "maximum stress" is given by

$$\sigma_{m2} = A'(B' + \frac{eE}{2} + e_{b2})^{m'} - S_2 \quad \text{----3.4.30.}$$

for U = 7,

$$\sigma_{m2} = A'(B' + \frac{eE}{2} + e_{b2})^{m'} + S_2 \quad \text{----3.4.31.}$$

e_E remains constant and is defined by the equations of section 3.2.2. Equation 3.4.31 must be substituted for equation 3.4.30 where U values of 7 and 1, or 6 and 5 are consecutive.

3.4.3.3. Isotropic/Kinematic.

For this model a different stress/strain relationship exists for reverse loading unless the absolute value of the total strain is greater than the previous maximum (normally tensile).Where this condition is satisfied, the original curve is followed.This maximum value is defined as e_{mn}, thus for

$|e_{t2}| > e_{mn1}$ ----3.4.32.

the original curve is followed, Fig 3.49 and a new value of e_{mn} must be established where

$e_{mn2} = |e_{t2}|$ ----3.4.33.

If the above condition is unsatisfied

$e_{mn2} = e_{mn1}$ ----3.4.34.

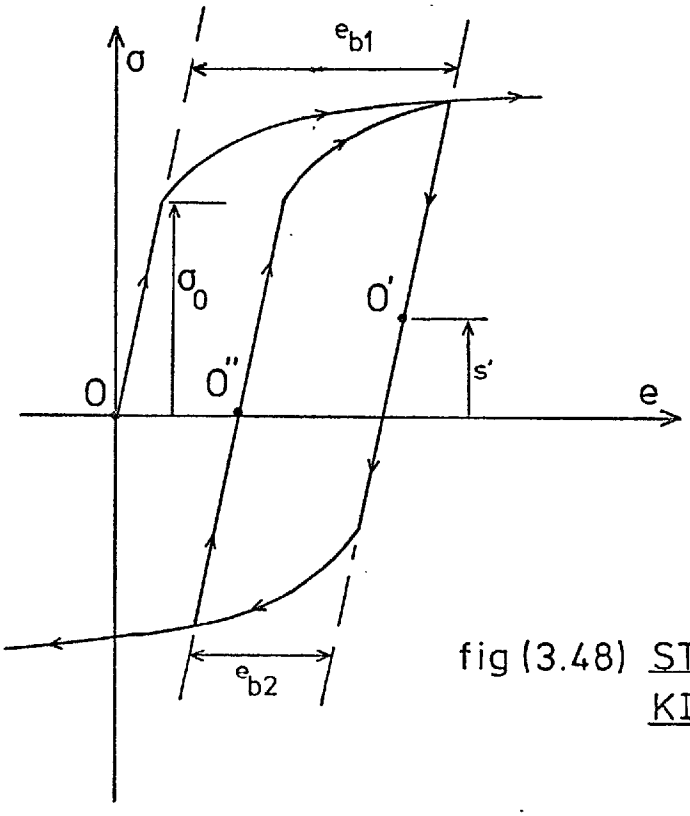


fig (3.48) STRESS REVERSAL KINEMATIC MODEL

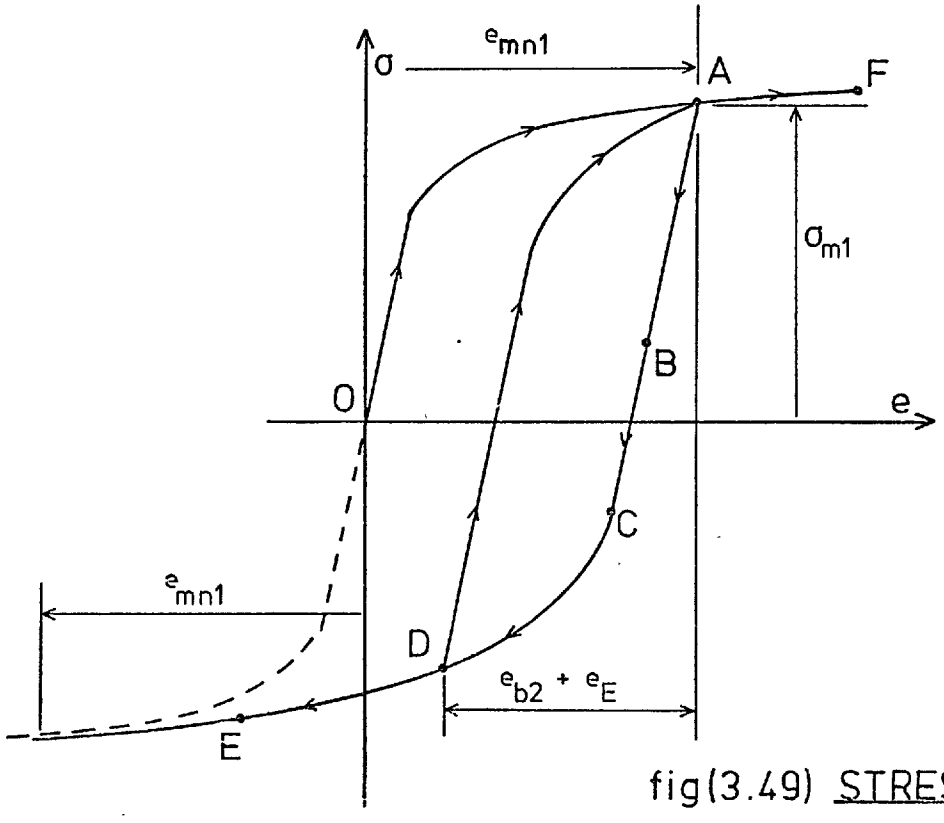


fig (3.49) STRESS REVERSAL ISOTROPIC/KINEMATIC MODEL

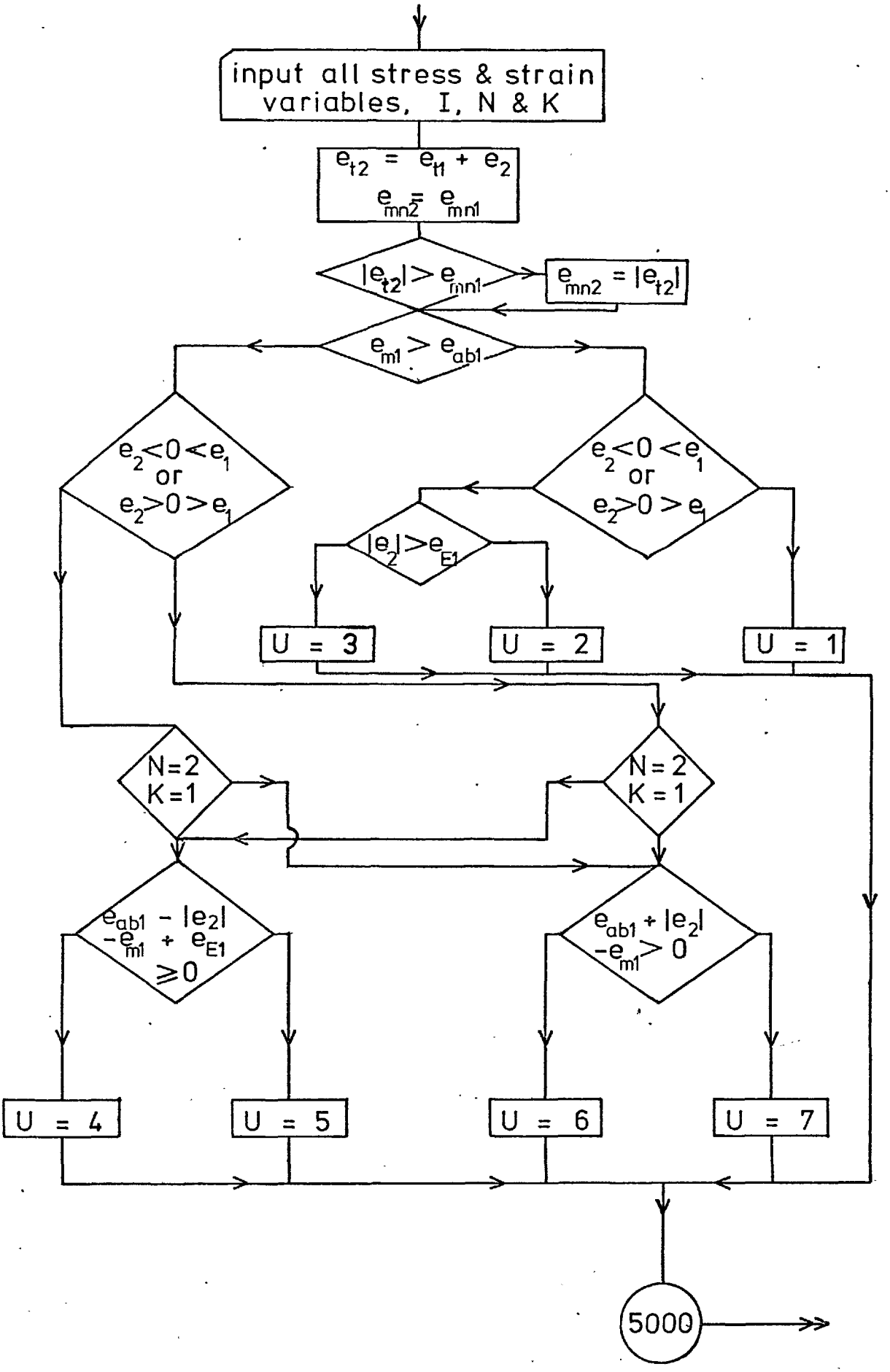


fig (3.50a) FLOW CHART - SUBROUTINE SXMAX

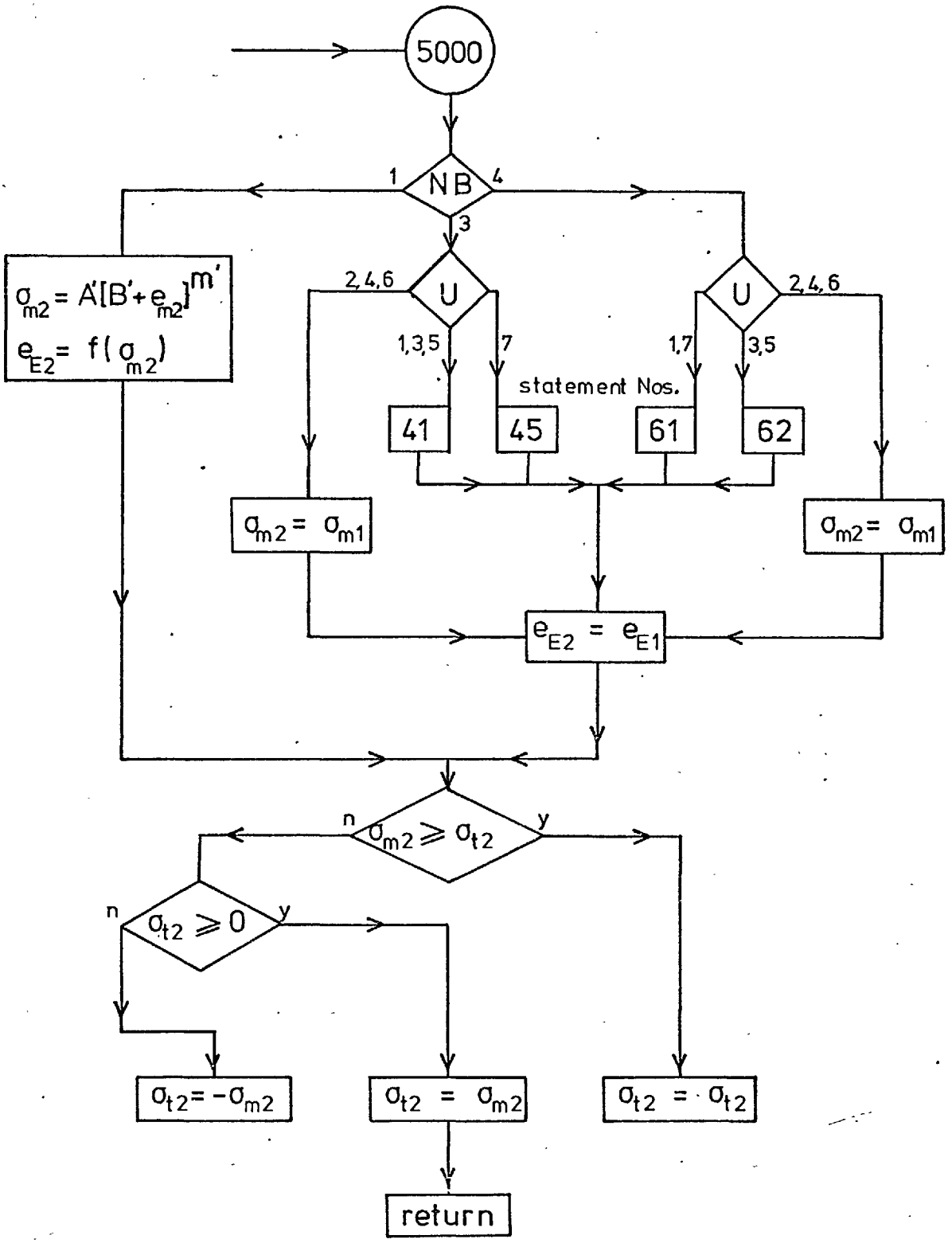

5000
NB
1
4
3
$\sigma_{m2} = A'[B' + e_{m2}]^{m'}$
$e_{E2} = f(\sigma_{m2})$
U
2,4,6
1,3,5
7
U
2,4,6
1,7
3,5
statement Nos.
41
45
61
62
$\sigma_{m2} = \sigma_{m1}$
$\sigma_{m2} = \sigma_{m1}$
$e_{E2} = e_{E1}$
n
$\sigma_{m2} \geq \sigma_{t2}$
y
n
$\sigma_{t2} \geq 0$
y
$\sigma_{t2} = -\sigma_{m2}$
$\sigma_{t2} = \sigma_{m2}$
$\sigma_{t2} = \sigma_{t2}$
return

fig (3.50b)

For U values of 2,4 and 6, stress levels are again within the elastic range.For U equal to 3 or 5,

$$\sigma_{m2} = 2A'(B' + \frac{eE}{2} + \frac{e_{b2}}{2})^{m'} - \sigma_{m1} \qquad \text{----3.4.35.}$$

and represents loading from A to D or B to D.

For U equals 1 or 7, stress D to E or B to D after elastic reversal,

$$\sigma_{m2} = \sigma_{m1} + 2A'(B' + \frac{e_E}{2} + \frac{e_{b2}}{2})^{m'} - 2A'(B' + \frac{e_E}{2} + \frac{e_{b1}}{2})^{m'} \qquad \text{----3.4.36.}$$

Equation 3.4.36 is also applicable to consecutive U values of 6 and 5.

The flow chart for this routine is depicted in Fig 3.50, whilst the solution is listed in Appendix 4.

3.5.0 Tension Effects.

To finalize the program it is necessary to consider the work done by the external forces, so that a prediction of the tension losses and hence power requirements for a particular machine may be made.For a perfectly free roll, no frictional effects are encountered at the roll bearings and any work done serves purely to bend and unbend the strip.Where the roll is not completely free,power is transmitted to the roll to overcome frictional effects.A tension rise (T_1-T_2) (Fig 3.19) thus occurs over the arc of contact resulting in a further overall straining of the strip and subsequent slipping.The belt friction relationships for an extensible belt are thus encountered, the relationship between entry and exit tensions to the arc of contact being given by

$$T_2 = T_1 e^{\mu'\varphi_a} \qquad \text{----3.5.1.}$$

As indicated by Swift (54), where a material of finite thickness is being considered, this equation takes the modified form

$$T_2 = T_1 e^{R_o\mu'\varphi_a/R} \qquad \text{----3.5.2.}$$

The couple supplied to the roll is thus:-

$$\text{transmitted couple} = (T_2-T_1)R \qquad \text{----3.5.3.}$$

Unless this couple is sufficient to overcome the friction couple of the roll system, the strip will slide bodily over the roll.

The friction couple at the work roll may be written

$$\text{friction couple} = (T_o+T_4)\sin\frac{\varphi}{2}\,\mu'R \qquad \text{----3.5.4.}$$

by the notation of Fig 3.19, where μ' is the 'virtual' friction coefficient at the work roll.Where rotation of the roll system occurs equations 3.5.3. and 3.5.4 may be equated, hence

$$T_2 = T_1 + \mu'(T_o + T_4) \sin \frac{\varphi}{2} \qquad \text{---3.5.5.}$$

It will be seen that by means of equations 3.5.2 and 3.5.5. the 'active' arc φ_a may be determined since equation 3.5.2 may be written

$$\frac{R_o}{R}\mu'\varphi_a = \frac{2(T_2 - T_1)}{(T_1 + T_2)} \qquad \text{----3.5.6.}$$

(from the expansion of $\ln\frac{T_2}{T_1}$.)

The tension rises T_o to T_1 and T_2 to T_4 are more complex, being dependent upon the work done on the strip.For any mean increment of strain (e), the work done is related to the tension increase by

$T_1(1 + e) - T_o$ = work done/unit width/unit length. ----3.5.7.

If the tensions T_o and T_1 are to be equal at T, then

$T e$ = work done ----3.5.8.

a condition which is not fulfilled because of the redundant bending deformation.

$$\text{Work/unit volume} = \int_{x_1}^{x_2} \frac{\text{force } dx}{\text{volume}} = \int_{\varepsilon_1}^{\varepsilon_2} \sigma de \qquad \text{----3.5.9.}$$

Therefore Work/unit width/unit length

$$= \int_{-a}^{+a}\int_{\varepsilon_1}^{\varepsilon_2} \sigma dt\, de \qquad \text{----3.5.10.}$$

By determining equation 3.5.9 for each thickness fibre,integrating over the thickness and equating to 3.5.7 the tension rise on bending or straightening may be determined.

Evaluation of 3.5.9 for elastic/plastic deformation is complex. Reverse loading often means that fibres are compressively strained whilst under the action of tensile forces.Work is thus recovered.A program (RWORK) has been written to solve 3.5.9. for any fibre and is described more fully in Appendix 5.The development of this program has been greatly simplified by the presence of the "maximum stress" subroutine, since the extent of the plastic and elastic strains in any

increment is known.The routine is based upon the two basic equations for work done:-

(1) Elastic deformation;

$$\text{work/unit vol} = \frac{E}{2(1-\nu^2)}(\varepsilon_2^2 - \varepsilon_1^2) \qquad \text{----3.5.11.}$$

(2) Plastic deformation (isotropic case,)

$$\text{work/unit vol} = \frac{A'}{(1+m')}\left((B' + e_2)^{1+m'} - (B' + e_1)^{1+m'}\right) \qquad \text{----3.5.12.}$$

where $\sigma = A'(B' + e)^{m'}$

Complications in the use of equation 3.5.12 naturally occur when the kinematic hardening models are utilized.

It is apparent that the tension rise during bending will require that the final bending stress distribution is in equilibrium with T_1 (Fig 3.19).The value fed in to the computer as back tension or passed on from the previous roll is T_o.Determination of T_1 is thus performed by:

(1) assuming bending occurs at $T = T_o$.

(2) determining the work done and hence an initial value of T_1 from equation 3.5.7.

(3) re-evaluate the bending strains for $T = T_1$.

(4) repeat 2 and 3 until negligible change appears in T_1.Two or three iterations are adequate.

This process is again used for $T_2 \longrightarrow T_3$ and $T_3 \longrightarrow T_4$.

During the passage of the strip, over the arc of contact no further bending occurs, but a rise in tensile force must take place to accommodate the bearing friction, equation 3.5.5.To eliminate a large amount of computer time (since T_4 is as yet unknown), it can be reasonably assumed that the work done during straightening is approximately equal to that of bending.

$$\therefore T_1 - T_o \simeq T_4 - T_2 \qquad \text{----3.5.13.}$$

equation 3.5.2. therefore becomes,

$$T_2 = T_1 + \mu'(T_1 + T_2)\sin(\varphi/2) \qquad \text{----3.5.14.}$$

This tension increase, although not altering the degree of bending requires an adjustment of stress over the contact arc.Some strip/roll slip will thus occur.An iterative technique is again used for solution, the stress and strain distributions and tensile load prior to spring-back are consequently known.Friction losses are incorporated in the main program (Appendix 3).

Inclusion of a "work done" routine in the numerical analysis of levelling allows evaluation of both the overall extension and the necessary tension increase as the strip passes through the machine. Figure 3.51 depicts the discrepencies between the use of a simple program which assumes constant tension at the mean value and the complete solution.

The magnitude of the virtual coefficient of bearing friction is dependent upon the roll set-up, roller diameters and bearing types. For the experimental system depicted in Fig 3.52, the large back-up roll compared with the work roll results in a large increase in μ' over the back-up bearing value. The friction couple at the back-up roll is written as

$$L_b = 2T_m \sin\frac{\varphi}{2}\ \frac{\pi}{2}\ R_b\,\mu_1 \qquad \text{----3.5.15.}$$

that due to the intermediate back-up bearings is a function of the resolved force acting on the bearings and is written

$$L_i = 2T_m \sin\frac{\varphi}{2}(\tan\gamma + \tan\beta)\frac{\pi}{2}\,\mu_2\,R_i \qquad \text{----3.5.16.}$$

The sum of equations 3.5.15 and 3.5.16 is the effective friction couple acting at the work roll, and therefore

$$2T_m \sin\frac{\varphi}{2}\,\mu' R_w = 2T_m \sin\frac{\varphi}{2}\ \frac{\pi}{2}\ \mu_1 R_b + 2T_m \sin\frac{\varphi}{2}\ \frac{\pi}{2}(\tan\gamma + \tan\beta)\ \mu_2\ R_i \qquad \text{----3.5.17.}$$

$$\text{and}\quad \mu' = \frac{\pi}{2R_w}\left(\mu_1 R_b + \mu_2\,R_i\,(\tan\gamma + \tan\beta)\right) \qquad \text{----3.5.18.}$$

For the current work, the back-up bearing is of roller type and μ_1 will be $\simeq$ 0.0015 under maximum loading conditions. Since experimentally loads of only one tenth maximum have been used, the acting value will be of the order of μ_1 = 0.003. The intermediate bearings being PTFE bushes will show friction coefficients of approximately μ_2 = 0.04.

For this work R_b = 2.0 ins and R_1 = 0.625 ins, tan β remaining constant at 0.245. Tan γ is dependent upon work roll diameter:

R_w = 0.25 $\rightarrow$ 1.0 ins.

tan γ = 1.02 $\rightarrow$ 0.42

Substitution into equation 3.5.18 yields the following results; work roll diameters of 2.0, 1.0 and 0.5ins give rise to virtual friction coefficients of 0.035, 0.09 and 0.24 respectively.

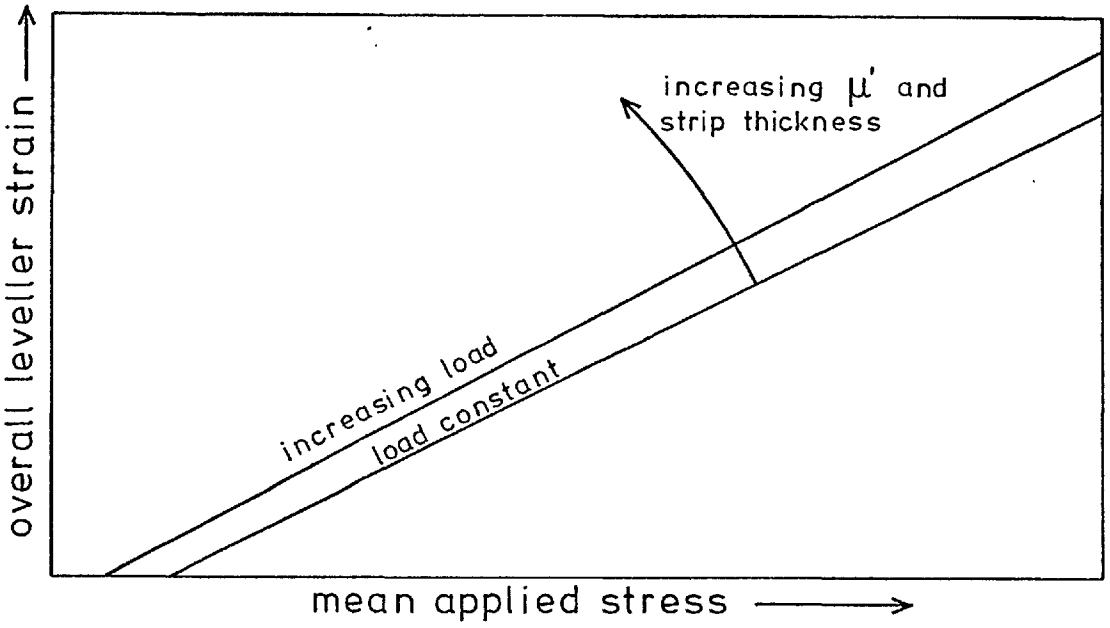


fig (3.51) EFFECT ON THEORETICAL RESULTS OF THE INCLUSION OF 'WORK DONE' EFFECTS

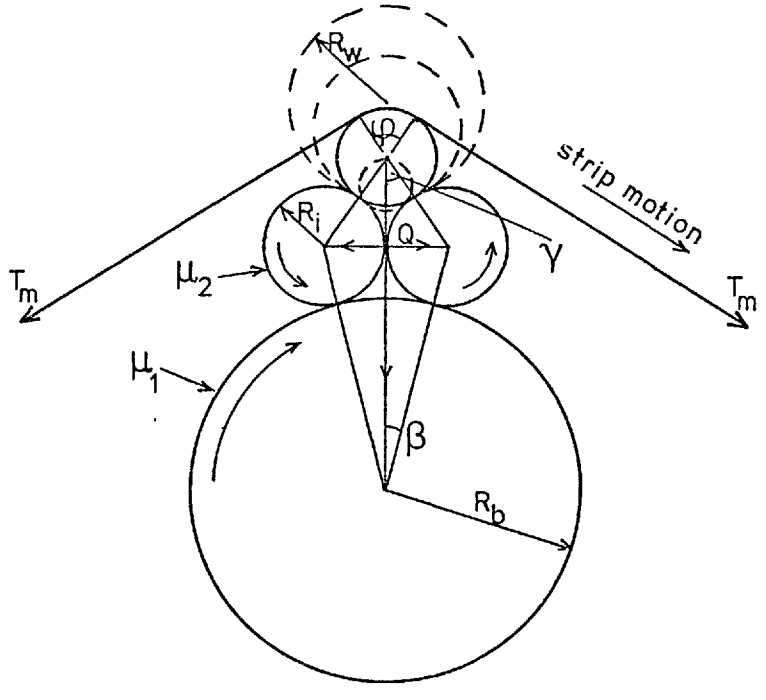


fig (3.52) EXPERIMENTAL ROLL SET-UP (VARIABLE WORK-ROLL DIAMETER)

3.6.0. Process Parameters.

In chapter 3 so far we have developed a vigorous approach to the analysis of the tension leveller. The relative importance of the various process parameters remains to be considered.

From equation 3.3.17, overall strip elongation during bending is written

$$e_{mean} = \frac{b_1}{R - b_1} \qquad \text{----3.6.1.}$$

The shift (b_1) of the neutral axis is a function of the applied tension and material strength, such that

$$b_1 \simeq \frac{a\sigma_t}{\sigma_m} = \frac{T}{2\sigma_m} \qquad \text{----3.6.2.}$$

where T = applied tensile load/unit width.

σ_m = mean flow stress.

In general $\sigma_m R \gg b_1 \sigma_t$ and

$$e_{mean} \simeq \frac{T}{D\sigma_m} \qquad \text{----3,6,3,}$$

If no spring back occurred, the straightening strain would be similar such that the overall leveller strain,

$$e_{tot} \simeq \frac{2NT}{D\sigma_m} \qquad \text{----3.6.4.}$$

an equation which is equivalent to the 'K' parameter of Hilsen et.al(52).

The presence of springback increases the straightening radius and equation 3.6.4 is rewritten

$$e_{tot} \simeq \frac{NT}{D\sigma_m} + \frac{NT}{D'\sigma_m} \qquad \text{---3.6.5.}$$

where $\frac{1}{D'} = \frac{1}{D} - \frac{1}{D \text{ released}}$.

Where the spring-back axis lies on the central axis

$$D \text{ released} = \frac{4Ea^3}{3M_{app}(1-\nu^2)} \qquad \text{----3.6.6.}$$

Neglecting elastic contributions to bending moment

$$M_{app} \simeq \sigma_m(a^2 - b^2) = \sigma_m a^2 \left(1 - \left(\frac{\sigma_t}{\sigma_m}\right)^2\right) \qquad \text{----3.6.7.}$$

Therefore in 3.6.5.

$$e_{tot} \simeq \frac{NT}{D\sigma_m} + \frac{NT}{D\sigma_m} - \frac{NT3(1-\nu^2)\left(1 - \left(\frac{\sigma_t}{\sigma_m}\right)^2\right)}{4Ea} \qquad \text{----3.6.8.}$$

But $T = 2a\sigma_t$

Therefore $e_{tot} \simeq \frac{2NT}{D\sigma_m} - \frac{3N(1-\nu^2)}{2E}\sigma_t\left(1 - \left(\frac{\sigma_t}{\sigma_m}\right)^2\right)$ ----3.6.9.

Under normal leveller conditions $\sigma_t \simeq \sigma_m/3$ and 3.6.9. reduces to

$$e_{tot} \simeq \frac{2NT}{D\sigma_m} - \frac{N\sigma_m}{2E} \qquad \text{----3.6.10.}$$

for narrow strip.

This equation is equivalent to that obtained by Brock et.al.(49), with the provision that spring-back effects are only relevant after bending.This accounts for the factor of two discrepancy between the elastic term denominators.

Providing that an intelligent guess can be made to the value of σ_m, equations 3.6.9. or 3.6.10 give a sensible prediction of controlling parameters of levelling.It is convenient to rewrite 3.6.9. as

$$e_{tot} \simeq NT\left(\frac{1}{R\sigma_m} - \frac{3(1-\nu^2)}{2Et}\left(1 - \frac{\sigma_t^2}{\sigma_m^2}\right)\right) \qquad \text{----3.6.11.}$$

It should be noted that although a strip thickness parameter is missing from the first term of equations 3.6.11, this is, in fact, incorporated in the value of σ_m which is dependent upon the degree of work-hardening and as such is directly related to the roll diameter/strip thickness ratio.

3.7.0. Bad Shape.

The use of a theoretical model for tension levelling which ignores all aspects of bad shape is perhaps, open to question.Length differentials will be discussed in their theoretical aspects in this section but it should be remembered that their inclusion in the analysis on the following principles may not be entirely correct.In this section, it is assumed that the path length of any element of strip width is identical to that of any other element.It follows that any strain differential may be represented by the equivalent stress difference or vice versa.In practise, path differences may occur in the leveller,eg. flexible bridle roll coverings allow bedding in of tight areas, and a strain difference may not produce its equivalent stress difference when tension is applied.

Pearson (4) and others (2,21,23) have discussed the magnitudes of the strain differentials necessary to constitute "bad shape".For a material

to lie within B.S.2L 100; 1961 Class A lateral curvature tolerance, length differentials not exceeding 0.006% /cm are required.This is equivalent to a long edge or middle of 0.3% on one metre wide strip.Very bad shape would correspond to a 1% differential in the same strip.Sivilotti and Johansson (23) quote figures for a maximum off-flattness allowed in aluminium comodities viz:

Mill products	0.1%
Levelled	0.025%
Stretched	0.010%

It can thus be said that the order of strain difference dealt with by tension levellers is 0.5%, which may increase to 1% in very severe conditions.

For the common cases of long edge and long middle the shape of the strip showing manefest defects can within experimental error, be represented by a sine wave Fig 3.53, where

$$\frac{\Delta l}{l} = \frac{L-\lambda}{\lambda} = \left(\frac{A\pi}{2\lambda}\right)^2 \qquad \text{----3.7.1.}$$

Assuming a constant wave length over the strip width and a linear change of amplitude in the same direction, for the case of long edge

$$A_w = A_{max} \times W \qquad \text{----3.7.2.}$$

where W = fraction of half strip width,

hence $$\frac{\Delta l_w}{l} = \frac{\Delta l_{max}}{l} \times W^2 \qquad \text{----3.7.3.}$$

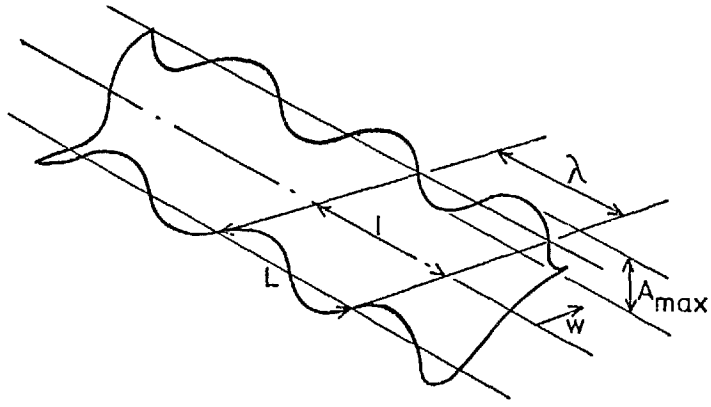
For this case the strain differential increases as the square of the distance from the centre line Fig 3.54.

Sabatini and Yeomans (21), from a theoretical model of strip shape after cold rolling, have derived an expression

$$\frac{\Delta l_w}{l} = PW^2 + QW^4 \qquad \text{----3.7.4.}$$

The length differential is thus a function of the parabolic and quartic terms of W.The relationship is derived from a consideration of the shape of the loaded roll gap and the authors find the quartic term necessary to adequately describe strip shape.Even so some form of plot, such as that in Fig 3.54, may be drawn.

For the case of centre buckle, quarter buckle and frilly edge,Fig 3.54, would assume the form shown in Fig 3.55. Since the tension leveller is basically indifferent to the actual position of the short areas,providing a plot such as that shown in Fig3.55 can be obtained,Fig 3.56 may



fig(3.53) SINUSOIDAL REPRESENTATION OF LONG EDGE

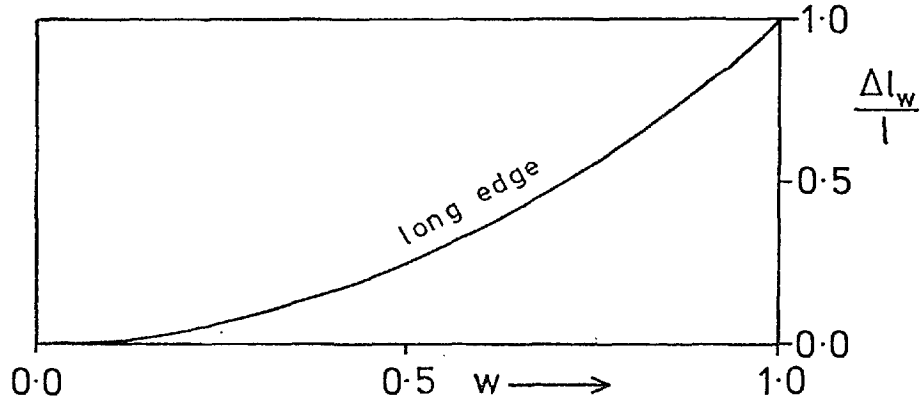
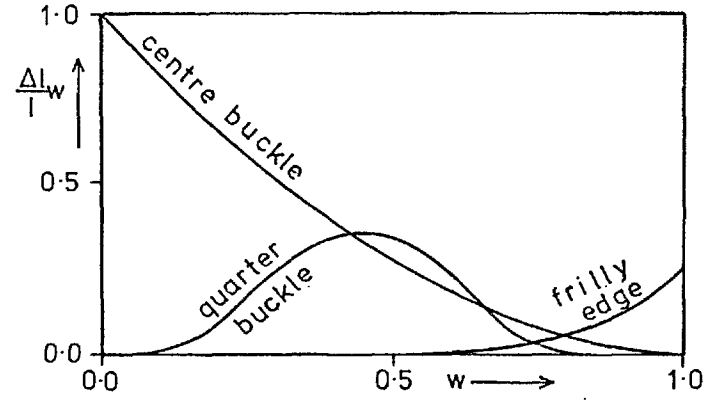


fig (3.54)

fig(3.55) LENGTH DIFFERENTIALS OVER THE STRIP WIDTH



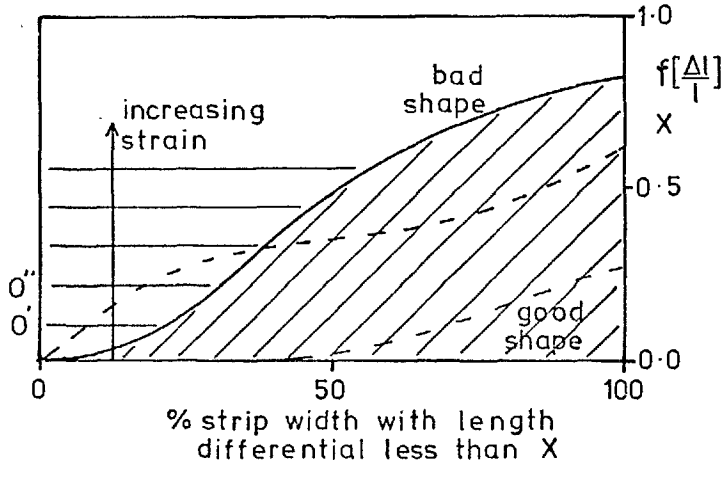


fig (3.56) GENERAL SHAPE DIAGRAM

be drawn.This figure is a plot of the proportion of strip width with shape differential better than a given value.The action of the leveller is to elongate the short regions, thus resulting in a shift of the zero shape axis up the diagram.By following the movement of this axis it is possible to evaluate the process completely.

Where latent shape defects are present a knowledge of the stress differentials will allow the evaluation of the shape differentials since $$\Delta\varepsilon = \frac{\Delta\sigma\ (1-\nu^2)}{E} \qquad \text{----3.7.5.}$$

In general the imposition of the levelling tensile force removes any visual effects of manefest bad shape, and replaces these by an elastic stress pattern similar to that of latent bad shape.During bending under tension, the position of the neutral axis and, consequently the mean strain, is dependent upon the overall applied tensile stress.The greater the local applied force the more that element of width will stretch. Since on elastic straining tensile loads localize in the short areas, as bending over the first roll commences, incremental strains will be greater in the shortest incoming element.As this straining occurs however,its plastic nature requires a concurrent redistribution of applied stress.The process then continues until uniform stress level occurs over the whole strip width.

Extension of the basic theory to include bad shape thus requires that the strip be divided into a number of width elements, and that each bending (or staightening) operation be considered incrementally.The method of analysis is thus as follows:

(1) Determine the initial stress and strain distribution at leveller entry.

(2) Sort width zones into batches of equal strain.

(3) Commence bending of each section under their respective stress levels until straining of the shortest batch provides equalization with the next.

(4) Determine the new stress distribution.

(5) Continue bend and (3) and (4) until the roll radius is achieved.

As yet the problems of spring-back and straightening have not been mentioned, but it is evident that the extension of the computer program to include the above would involve great complexity which should be justified.Elementary calculations on the method indicate that all but the worst bad shape is essentially removed during the first bend.Naturally the complete computer program would allow prediction of the lateral

residual stress pattern, but this is dependent upon the work-hardening nature of the material and can be improved little by altering levelling conditions.

It was therefore deemed unnecessary to pursue the analysis to completion, especially since the basic program has not compared with sufficient industrial results to test its application.The method of strain equalization during levelling is, however, illustrated in the following example.

3.7.1. The Bending of rigid, perfectly-plastic strip with Bad Shape.

To illustrate the process we shall consider a piece of strip divided into five elements.It is assumed that the inner three zones show latent shape defects, whilst the outer zones show manifest shape errors.

Incoming (before load application) stress and strain distributions are given in Fig 3.57 which are approximated to that shown in Fig 3.58. Strain differentials are

0.001 in zones 3/2&4.

and

0.0025 in zones 3/1&5.

Assuming material specifications of

$E = 20 \times 10^6$ lbf/in^2

$\sigma_m = 40 \times 10^3$ lbf/in^2

$t = 0.02$ in

stress levels in the centre and immediately adjacent zones will be 13,340 and 6,670 lbf/in^2 respectively.

We shall assume a mean applied tensile load of one quarter yield stress, ie 10×10^3 psi = 200 lbs per section.Total load therefore equals 1000 lbs.

During initial bending the centre three zones take all the applied load, hence

$$\sigma_t = \frac{1000}{3 \times 2 \times 10^{-2}} = 16{,}670 \text{ lbf/in}^2$$

Therefore

$\sigma_{31} = 30{,}010$ lbf/in^2

$\sigma_{21} = \sigma_{41} = 10{,}000$ lbf/in^2

$\sigma_{11} = \sigma_{51} = 0$ ----3.7.6.

For the perfectly plastic case, the displacement of the neutral axis is independent of bend radius and by equation 3.3.15

$$b_{31} = \frac{0.01 \times 30{,}010}{40{,}000.} = 0.0075 \quad \text{----3.7.7.}$$

$$b_{21} = \frac{0.01 \times 10{,}000}{40{,}000} = 0.0025 \qquad \text{----3.7.8.}$$

The initial centre line strain differences between zone 3 and zones 2 and 4, is 0.001. By equation 3.3.17 we may write

$$\frac{b_{31}}{R_1 - b_3} = \frac{b_{21}}{R_1 - b_{21}} + 0.001 \qquad \text{----3.7.9.}$$

where R_1 is the bend radius to provide equal centre line strains. Since $R \gg b_{31}, b_{21}$.

$$0.001R_1 \simeq 0.0075 - 0.0025 = 0.005$$

$$\text{Therefore } R_1 = 5\text{ins.} \qquad \text{----3.7.10}$$

Bending to a 5ins radius thus equalizes strains in the central zones. The centre line strain of zone 3 is therefore

$$\frac{0.0075}{5} = 0.0015 \qquad \text{----3.7.11.}$$

at this radius. The remaining strain differential between the inner and outer zones is 0.001.

The applied stress on the inner three zones now adjusts to the uniform value

$$\sigma_t = \frac{1000}{3 \times 0.02} = 16{,}670 \text{lbf/in}^2 \qquad \text{----3.7.12.}$$

and the neutral axis shift for these sections on further bending is now

$$b_{32} = b_{22} = b_{42} = \frac{0.02 \times 16{,}670}{2 \times 40{,}000} = 0.00417 \qquad \text{----3.7.13.}$$

The total strain distributions now present in sections 2, 3 and 4 and 1 and 5 are shown in Fig 3.59. If we consider further bending of section 3, we must therefore include the current strain at the new neutral axis.

$$\varepsilon_3 (\text{at } b = 0.00417) = \frac{-0.00417 + 0.0075}{5} = 0.00067 \qquad \text{----3.7.14.}$$

Further bending to R_2 causes equalization of all centre line strains and since we are assuming zero applied stress in the outer elements we can write.

$$\frac{b_{32}}{R_2} (1 + 0.00067) + 0.00067 = 0.0025 \qquad \text{----3.7.15.}$$

$$\text{Therefore } R_2 \simeq 2.28\text{ins} \qquad \text{----3.7.16.}$$

Strain distribution in all sections are now identical (Fig 3.60). Checking for surface strains,

$$\text{Zone 3, } \varepsilon_{surf} = \frac{0.00417 + 0.01}{2.28.} + 0.00067 = 0.00689$$

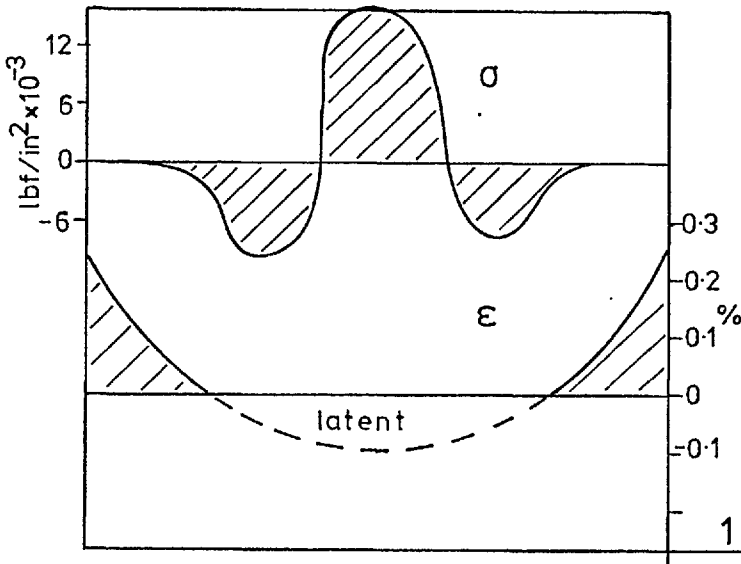


fig (3.57) ENTRY STRESS AND STRAIN LEVELS

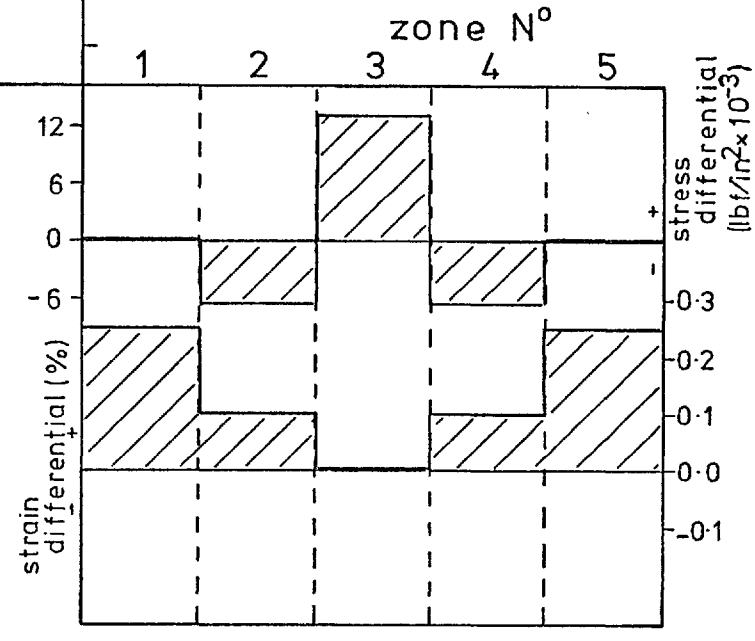


fig (3.58) ASSUMED DISTRIBUTIONS

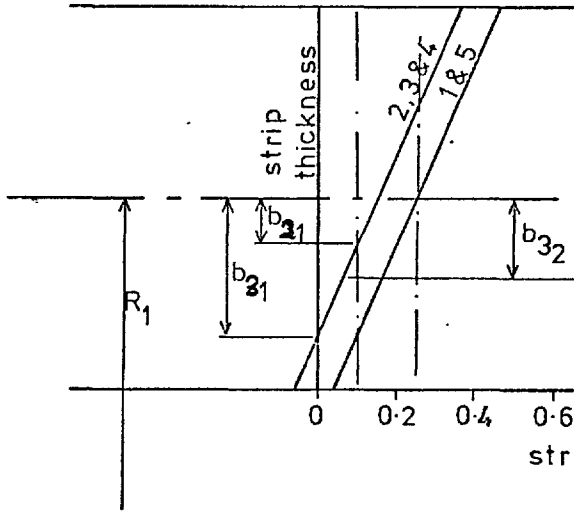


fig (3.59) STRAIN AT END OF FIRST INCREMENT

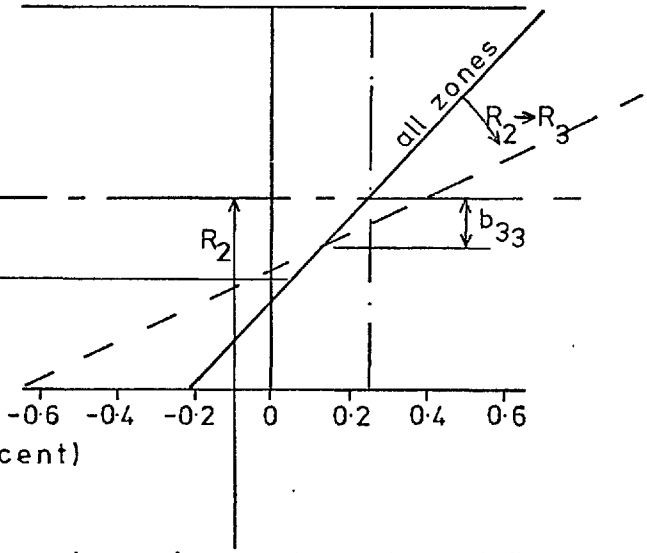


fig (3.60) STRAIN AT END OF SECOND AND THIRD

Zone 5, $\varepsilon_{surf} = \frac{0.01}{2.28} + 0.0025 = 0.00689$ ----3.7.17.

By bending to a radius of 2.28ins, the strain differential of 0.25% has thus been removed.If we say the roll radius is 1 ins, we can analyse the total zone 3 strain by continuing the bending under the now uniform stress of 10,000 lbf/in^2.The incremental neutral axis is now at

$$b_{33} = \frac{0.01 \times 10}{40} = 0.0025 \quad \text{----3.7.18}$$

The current strain at this section is

$$\varepsilon_{tot_{0.0025}} = 0.0025 - \frac{(0.0025)(1.0025)}{2.28}$$

$$= 0.0014 \quad \text{----3.7.19}$$

On bending to $R_3 = 1.0$, we have

$$\varepsilon_{tot3} = 0.0014 + \frac{(0.0025)(1.0014)}{1}$$

$$= 0.0039 \quad \text{----3.7.20.}$$

This example illustrates the effectiveness of the tension leveller in the removal of strain differences.If we had assumed uniform applied stress throughout, the mean strain of zone 3 would have been,

$$\varepsilon_{tot3} = \frac{0.0025}{1} = 0.0025 \quad \text{----3.7.21}$$

and the outer zones would also have strained accordingly.Stress concentration during the initial part of the bend, thus allows preferential elongation of the short areas and even provides considerable elongation, of the long regions over the first roll.Further rolls serve only to provide more working of the material, with the result that the residual stress levels, which will be high for a work-hardening material, where $\frac{1}{4}$% strain differences exist, are very much reduced.

Where buckled,or latently bad shaped strip is being levelled, the "uniform tension" theory of section 3.4.0 will predict a mean strain for the whole strip.Providing the machine is set for an elongation at least equal to the initial strain discrepancy, the basis of bad shape will be removed.On the basis of the example given above, tension levelling will result in a smaller residual stress system than simple stretching, since non-uniform working is given to the thickness of the material and work-hardening discrepancies tend to be eliminated.

CHAPTER 4.
EXPERIMENTAL PROCEDURE.

4.0.0 Scope of Experimental Work.

Chapter 3 has shown the development of a theoretical model for tension -levelling, which should provide sufficiently accurate predictions of overall strain magnitudes under specific leveller conditions.Undoubtedly these predictions will provide better correlation with practise than those of the simple B.N.F. theory, the latter often being 50% in error.To support the derived theory experimental comparisons are required in the form of data derived from actual levelling operations.If theory is to be used for industrial design purposes these results are best obtained from an industrial set-up, although there is much in favour of laboratory results.Time has not allowed a complete experimental investigation under laboratory conditions and although data is currently being obtained from the British Steel Corporation this is unlikely to be available before submission of this work.The results presented here are for skin-passed mild steel only.Unrecorded tests on aluminium do indicate application to non-ferrous materials,although sucess cannot be guaranteed.

Theoretical predictions are based on a knowledge of the stress and strain paths of individual fibres during levelling.The complexity of the stress path eliminates its measurement in the first instance and the experimental rig outlined in the following sections was designed to give a meaningful indication of strain paths.Since it has been assumed that strains vary linearly through the strip thickness the measurement of surface strains provides a suitable basis for theoretical and experimental correlation.

Although mean levelling strains are normally less than 2%,the corresponding surface strains during bending are often of the order of 4%.This latter figure lies outside the range of strains normally considered relevant to strain gauge measurement but it was considered, that with careful choice of gauges and adhesive, representative results of surface strains could be obtained by normal strain gauge techniques.Providing a reasonable correlation exists between measured and predicted surface strains under varying leveller conditions it is fair to say that the derived theory may be used,with care, to predict results in industrial machines.

The experimental precedure was therefore designed to:-

(1) Provide a uniaxial stress/strain curve of the material to be used, thus allowing derivation of the longitudinal stress/strain relationships.

(2) allow measurement of surface and overall strains under known levelling conditions.

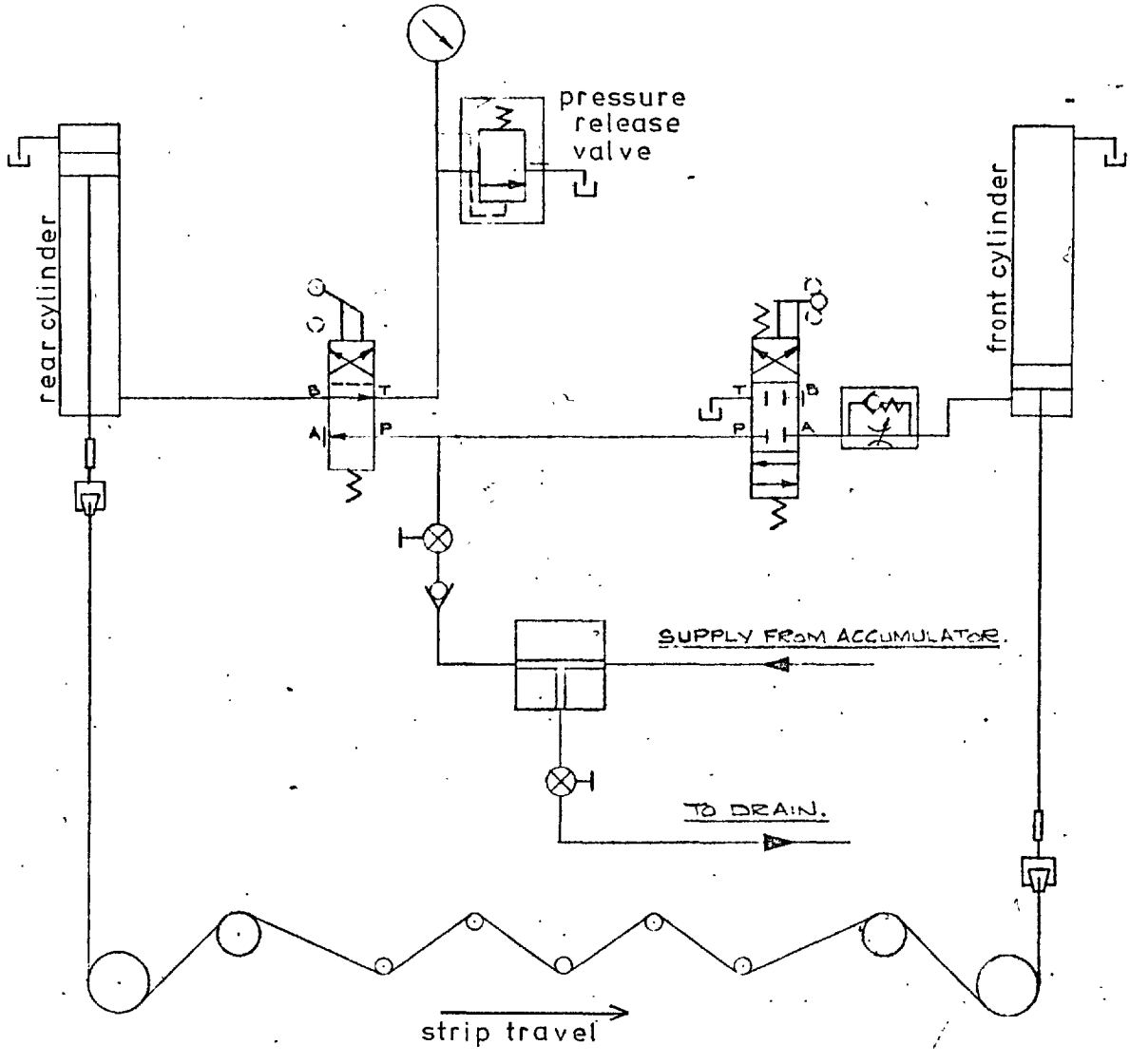
The first factor is straightforward and required the use of standard tensile testing equipment.Results from these tests were fed to a computer program to provide the necessary derivations.The second factor is, however, more complex requiring the design and installation of a small scale,instrumented leveller.This machine was also to be representative of industrial installations.Use of initially " well shaped" material would provide a convenient basis for comparison with theory.

Material containing shape defects has not been utilized to obtain experimental results although test runs have demonstrated the levelling effect of the installation:It is hoped that experiments on buckled strip will be considered in any future work.

4.1.0. Design of the Leveller.

The computer program for levelling, predicts overall strains and load losses for a given applied back load.It was therefore desirable to install a tension controlled machine as opposed to one incorporating elongation control.An existing hydraulic accumulator, capable of supplying oil at a pressure of $20MN/m^2$ to the College's high speed extrusion press required minimal adaption for use as a suitable power supply.The leveller was therefore designed on a"tension control" basis,utilizing hydraulic cylinders as the strip driving mechanism.This was achieved by the use of a 'pulling' cylinder connected directly to the accumulator and a 'tension control' cylinder equiped with a pressure relief valve (range $0.7 \longrightarrow 20MN/m^2$).Strip speed was controlled by adjusting oil flow to the front cylinder.The layout of the machine hydraulics is depicted in Figure 4.1.

Industrially,strip is normally levelled in wide form,(ie. before slitting) and as such,plane strain conditions are likely to ensue.The experimental rig was accordingly designed to take strip up to 200mm wide such that industrial conditions were represented.The load application method eliminated the use of strip in coil form, and man-handling of a cut length into the machine was necessary.With strip of the above width thicknesses greater than 1mm are impossible to use and the control cylinders were thus chosen to enable yielding of 1mm strip in simple



CONTROL DIAGRAM

fig (4.1) HYDRAULIC SUPPLY SYSTEM

tension.(In doing so, adequate power for start-up was ensured).The yield stress assumed in this derivation was typical of mild steels at 280 MN/mm^2, the required'pull' being

$280 \times 0.001 \times 200 = 56$ KN ----4.1.1.

The required cylinder cross-sectioned area was therefore.

$\frac{56}{20} \times 10^{-3} = 2.8. \times 10^{-3}\ m^2$ ----4.1.2.

Cylinders providing the above specifications were accordingly chosen, having a stroke length of 2m and containing a cushioning mechanism at either end.(The cushioning action is essential under high speed conditions !).

The limit of 1mm strip thickness provided a suitable criterion for the choice of work roll diameters to be used.Previous authors have indicated a workable mimimum roll diameter to strip thickness ratio of 40 or 50 to 1.The maximum required work roll diameter was therefore set at 50mm (2ins).By providing for work roll diameters varying from 12.5 to 50mm, diameter to thickness ratios of 25 to 100 could be incorporated at the intermediate strip thickness.

The basic design criterion of applied load and roll diameter capabilities were therefore determined, leaving two further machine variables to be evaluated.These were,

(1) the number of work rolls.

(2) the roll wrap angles.

In determining (1), the work of Brock et. al (49) was considered a convenient basis and five roll stands were included,(although a mimimum of two is really only necessary to provide basic shape correction). Since the effects of roll wrap had not been investigated in anything but a random manner by previous authors,it was necessary to design the roll system such that a practical range of wrap angles could be obtained.The work at B.N.F.M.R.A. was again considered the recommended minimum being about 40°.Hilsen Levy and Polakowski (52) had, however, experimented with wrap angles of 180° and more, using their floating work roll principle and indicated that overall extension is very much dependent upon wrap.The advantages of wrap angles greater than that necessary to promote roll conformity remained elusive however and a maximum wrap of 90° was considered sufficient, a range of 0 to 90 degrees providing ample scope for experiment.

Increasing wrap angle increases the load on a work roll however and deflection often occurs in the vertical plane.The increased tension

loss over the roll, resulting from increased frictional effects also tends to induce lateral roll deflection.It was therefore necessary to provide some form of double back for the work rolls.Double back-up rollers greatly reduce the wrap angles that can be obtained, and experience had shown that bad strip marking ensued with a split back-up system. To provide continuous support the roll arrangement shown in Fig 4.2. was utilized, the large secondary backing roller providing ample vertical support.This roll system made it unnecessary to include work roll bearings and the primary back-up bearings needed to be sufficient only to prevent lateral movement.

It is evident from Fig 4.2. that the primary back-up diameter critically effects the wrap angle attainable in the small work-roll ranges. Fig 4.3. shows a plot of the maximum possible wrap angle for various back-up/work-roll diameter ratios.The limit on the back-up roll size was chosen to provide 40° wrap for the 12.5 mm rolls, this diameter being set at 31.8 mm.

These rolls may still be considered to be fairly small in diameter however, and were thought likely to be subject to excessive roll bending deflections.The reactions in the system are given in Figure 4.2. for a uniform applied tension T.It is evident that deflection of these rolls will occur in an approximately lateral plane and be primarily dependent upon the magnitude of Q.From the geometry of the system (assuming negligible clearance between primary back-ups), Q is related to the applied tension T by

$$Q = T \sin\frac{\varphi}{2}(\tan\gamma + \tan\beta) \qquad \text{----4.1.3.}$$

For a simply supported, uniformly loaded beam, the central deflection is given by the relationship

$$\delta_o = \frac{5}{384}\frac{QL^3}{EI} \qquad \text{----4.1.4.}$$

For the case of the circular beam,

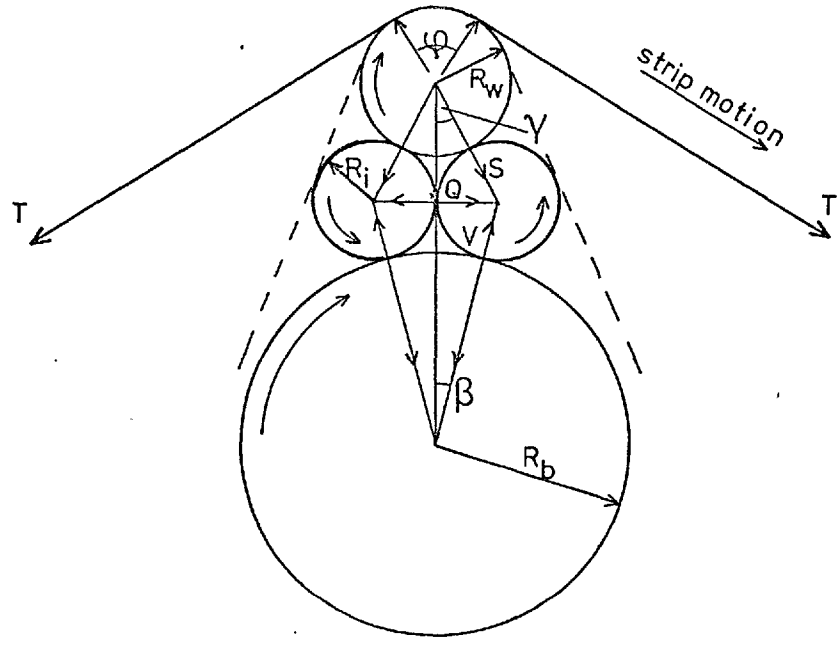
$$I = 0.0491\ D^4 \qquad \text{----4.1.5.}$$

In the present case, $D = 31.8$ mm and hence $I \simeq 4.85 \times 10^{-8}$.
Since $E \simeq 207 \times 10^3$ MN/m^2 and $L = 1.2$m, we may write from equation 4.1.4.

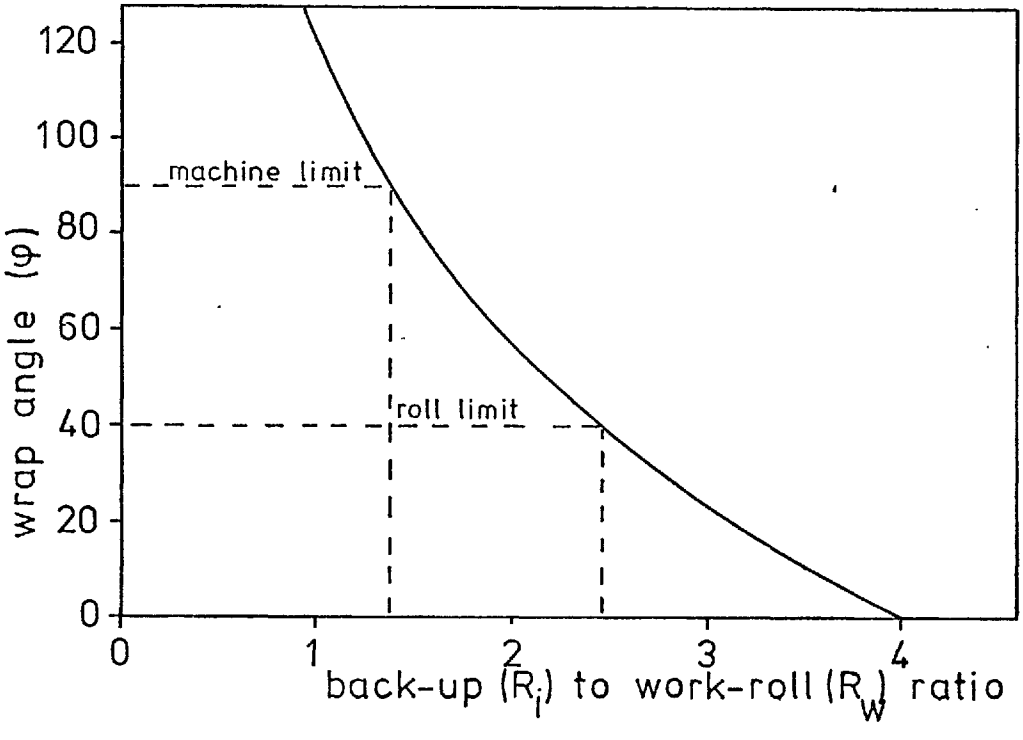
$$\delta_o \simeq 0.104\ Q \text{ metres} \qquad \text{----4.1.6.}$$

where Q has the units MN/m^2.
From equations 4.1.3 and 4.1.6,

$$\delta_o = 0.104\ T \sin\frac{\varphi}{2}(\tan\gamma + \tan\beta) \qquad \text{----4.1.7.}$$



fig(4.2) EXPERIMENTAL ROLL SET-UP



fig(4.3) MAXIMUM ATTAINABLE WRAP ANGLE (φ)

and for $\varphi = 90^{o}$ (the maximum value).

$$\delta_o = 0.0735\ T\ (\tan\gamma + \tan\beta) \qquad \text{----4.1.8.}$$

From Fig 4.2

$$1/\tan\gamma = \left((R_w/R_i)^2 + (2R_w/R_i) \right)^{\frac{1}{2}} \qquad \text{----4.1.9.}$$

and

$$\tan\beta = R_i / \left((R_b + R_i)^2 - R_i^2 \right)^{\frac{1}{2}} \qquad \text{----4.1.10.}$$

Variation in the work-roll diameter from 12.5 to 50 mm gives corresponding variation in $\tan\gamma$ of from 1.0 to 0.4.Tan β is insensitive to the secondary back-up diameter and the latter was chosen at 100 mm to provide negligible deflection under maximum load.With this value and the diameter of the primary back-up rollers at 31.8 mm,

$$\tan\beta = 0.245 \qquad \text{----4.1.11.}$$

From equation 4.1.8 the calculated maximum central intermediate roll deflection is about 4 mm under conditions of full load(corresponding to the smallest work roll).

Conditions of full wrap are never attained with the minimum diameter work-roll however, and with the largest roll ($\tan\gamma = 0.4$)

$$\delta_o = 2.5 \text{ mm} \qquad \text{----4.1.12.}$$

This gives a 5 mm separation of the intermediate back-up rollers.Although this figure appears high, it must be remembered that the interactions between the applied tensile load and the bend radius in general ensure that sufficient elongation is attained with loads of about one tenth maximum.The above figure also ignores the rigidity of the work-roll which prevents settling between the primary back-up rollers.

Undoubtedly some bending of the work roll is to be expected under most conditions, but it was anticipated that the advantages gained by the completely adjustable wrap angle would outweigh any fractionally decreased path lengths through the centre of the leveller.(This has in fact proved the case,buckled edges never having been obtained).

The general arrangement of the final leveller is shown in Fig 4.4, figures 4.5. and 4.6. showing roll-block detail.For convenience the hydraulic cylinders were mounted vertically on two building columns, guide rolls being utilized to transfer the strip from a vertical to a horizontal course.

Roll-block assembly size allowed only a short length of strip to pass completely through the roll system, this length obvoiusly being less than the cylinder stroke.To remove the influence of the pass-line guide rollers on this length they were set 1m from the first and last roll assemblies. Overall a 1m length of strip could pass completely through the roll

system, this being deemed sufficient for experimental purposes.

To provide variable wrap-angle throughout the machine, rolls 1,3 and 5 were made independently adjustable, the remaining two stands being set to the pass-line.Secondary back-up rollers were equipped with roller bearings, the half bearings on the intermediate rolls being of P.T.F.E. and providing adjustment.This bearing system, together with the floating work-roll design ensured minimum friction losses, an essential feature in an experimental rig of this type.

Minimum roll block separation possible was 216 mm and roll interlocks of 230 mm at the 1st, 3rd and 5th stations were provided to enable the utilization of 90° wrap angles at the 2nd, 3rd and 4th stations.This interlock was achieved by the use of a simple screwdown system,shims being used between the roll blocks and roll block guides to provide distance setting.The guides were bolted onto the machine frame and if necessary each roll block could be lifted (by crane) above the pass-line or completely out ,for cleaning.

Figure 4.5 depicts the original work roll design, where these extended into the back-up bearing housing.Lateral movement was prevented by the proximity of the roller bearing housing.This design proved inefficient, however when roll changes were required and the work-rolls were subsequently shortened such that they could slide into place between the bearing blocks.Spacing pieces were then inserted into the back-up housing to prevent lateral movement.Holding plates,attached to the bearing blocks held the work-rolls in place in the absence of strip tension.

The complete machine specifications are retained in the Metallurgy Department of Imperial College, where reference may be made.Other aspects worth noting are

(1) Strip connection to piston rods via serrated jaws acting in wedge grips.These grips were attached to the rods by means of a load cell and universal coupling.

(2) The machine frame was constructed basically from 9" x $3\frac{1}{2}$" channel providing sufficient rigidity to resist localized bending loads.This frame was bolted to the floor between the columns supporting the hydraulic cylinders.

(3) Materials used:

Work and Back-up Rolls - EN. 24.

Guide Rolls - EN 32.

Bearing Housings - EN 8.

Plates 1-5 show the various aspects of the machine as installed, and are self-explanatory.

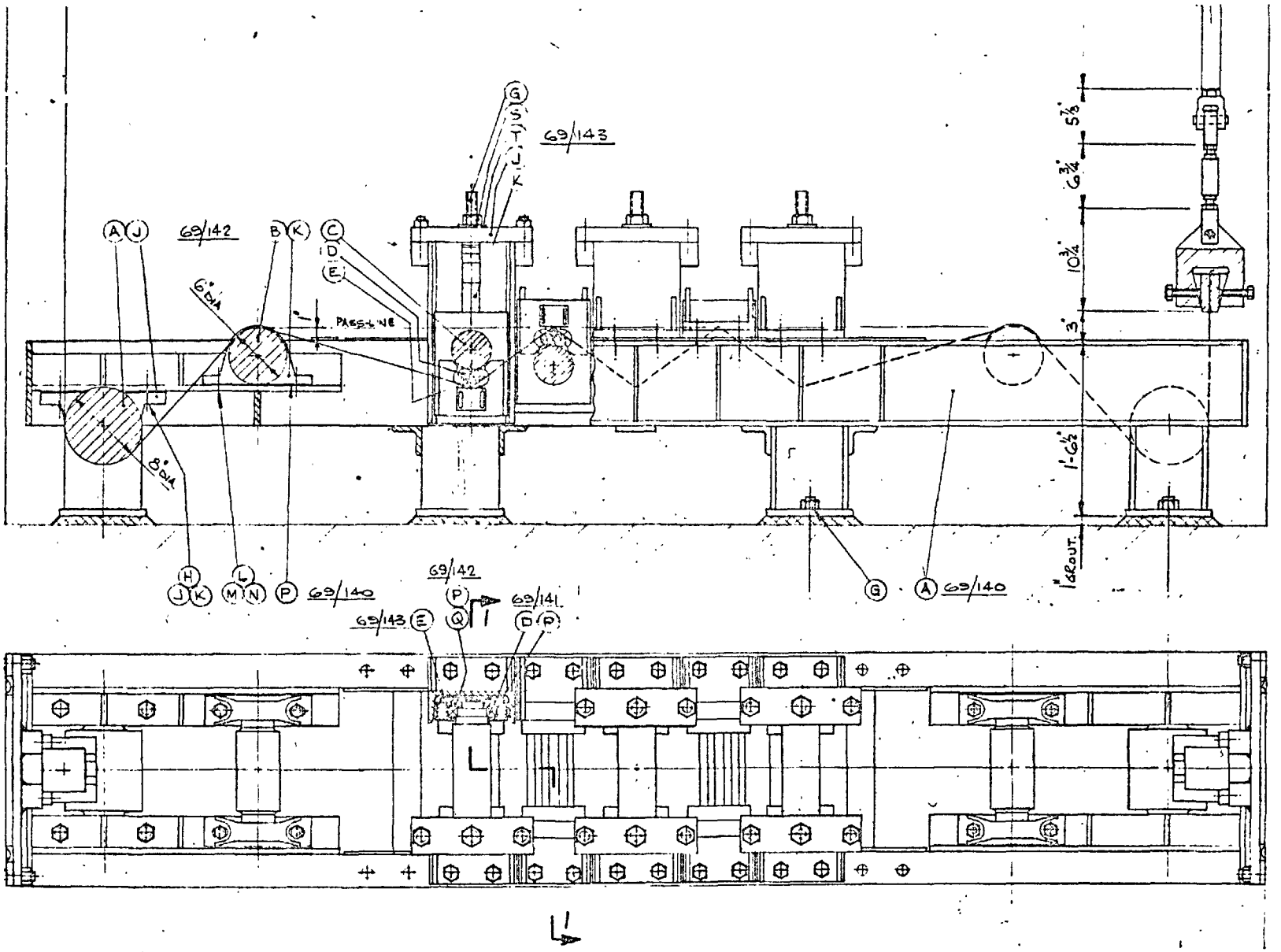


fig (4.4) EXPERIMENTAL LEVELLER - GENERAL ARRANGEMENT

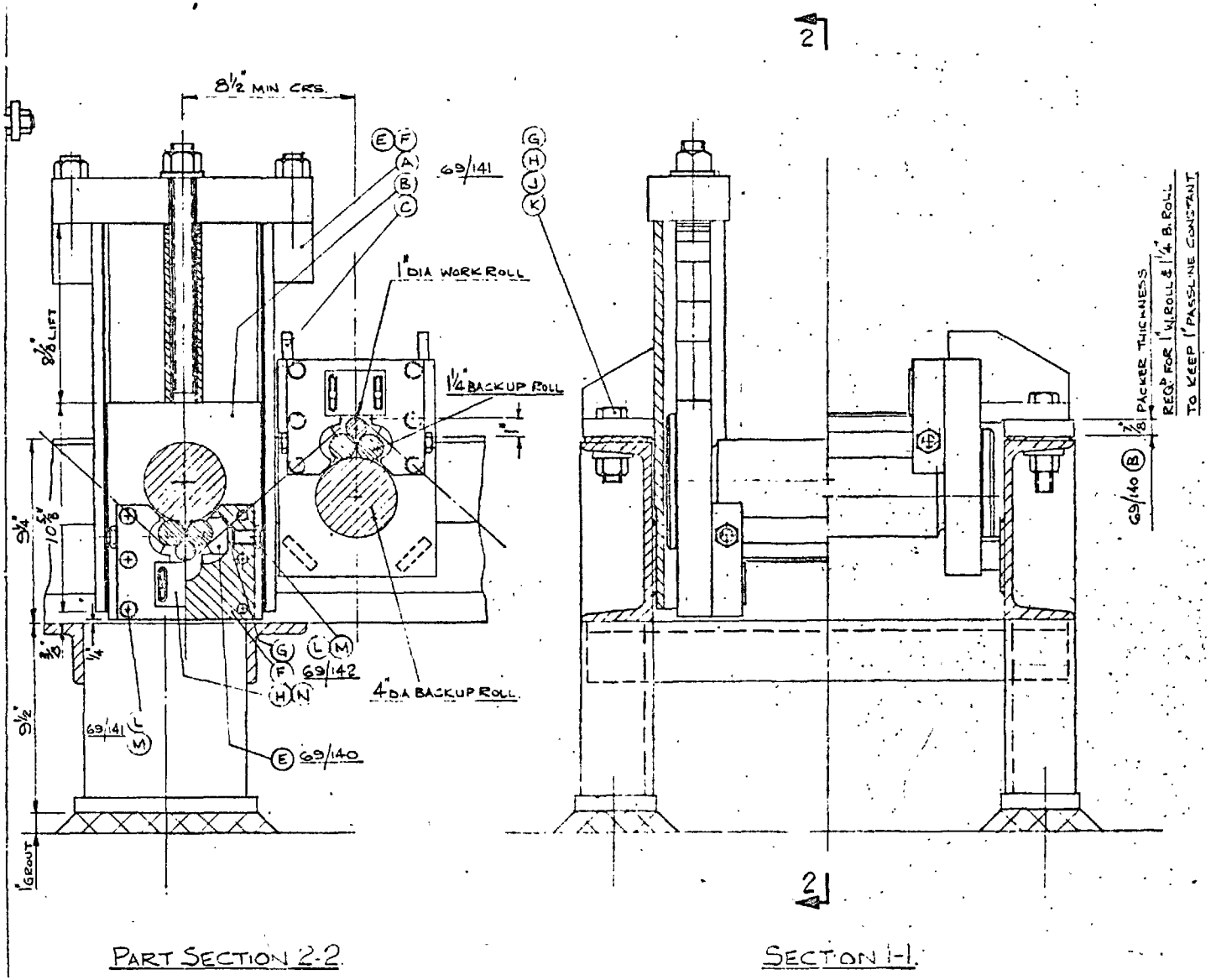


fig (4.5) ROLL-BLOCK DETAIL

fig (4.6) ADJUSTABLE & STATIONARY ROLL-BLOCK MOUNTING

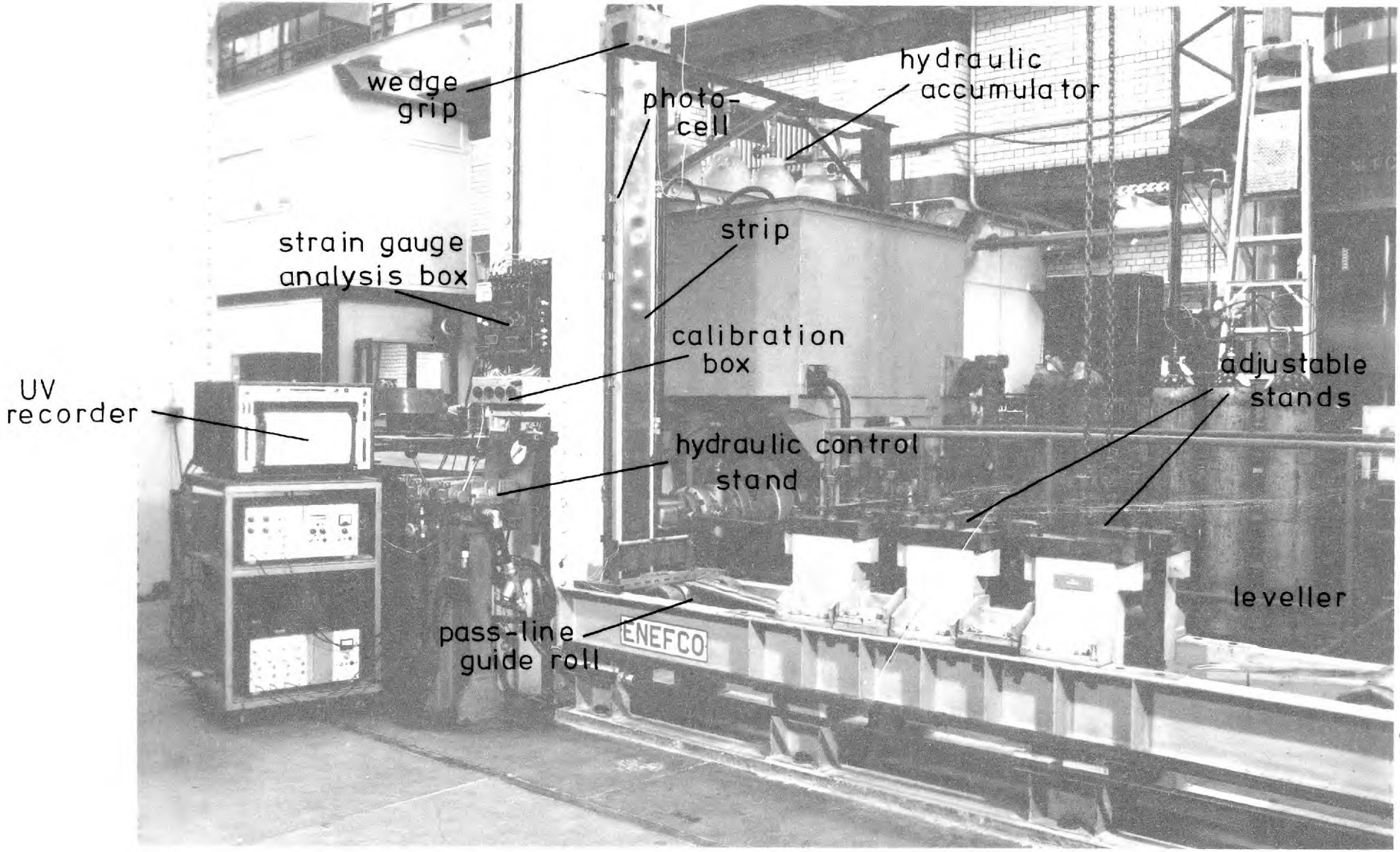


plate (1) EXPERIMENTAL LEVELLER - GENERAL ARRANGEMENT

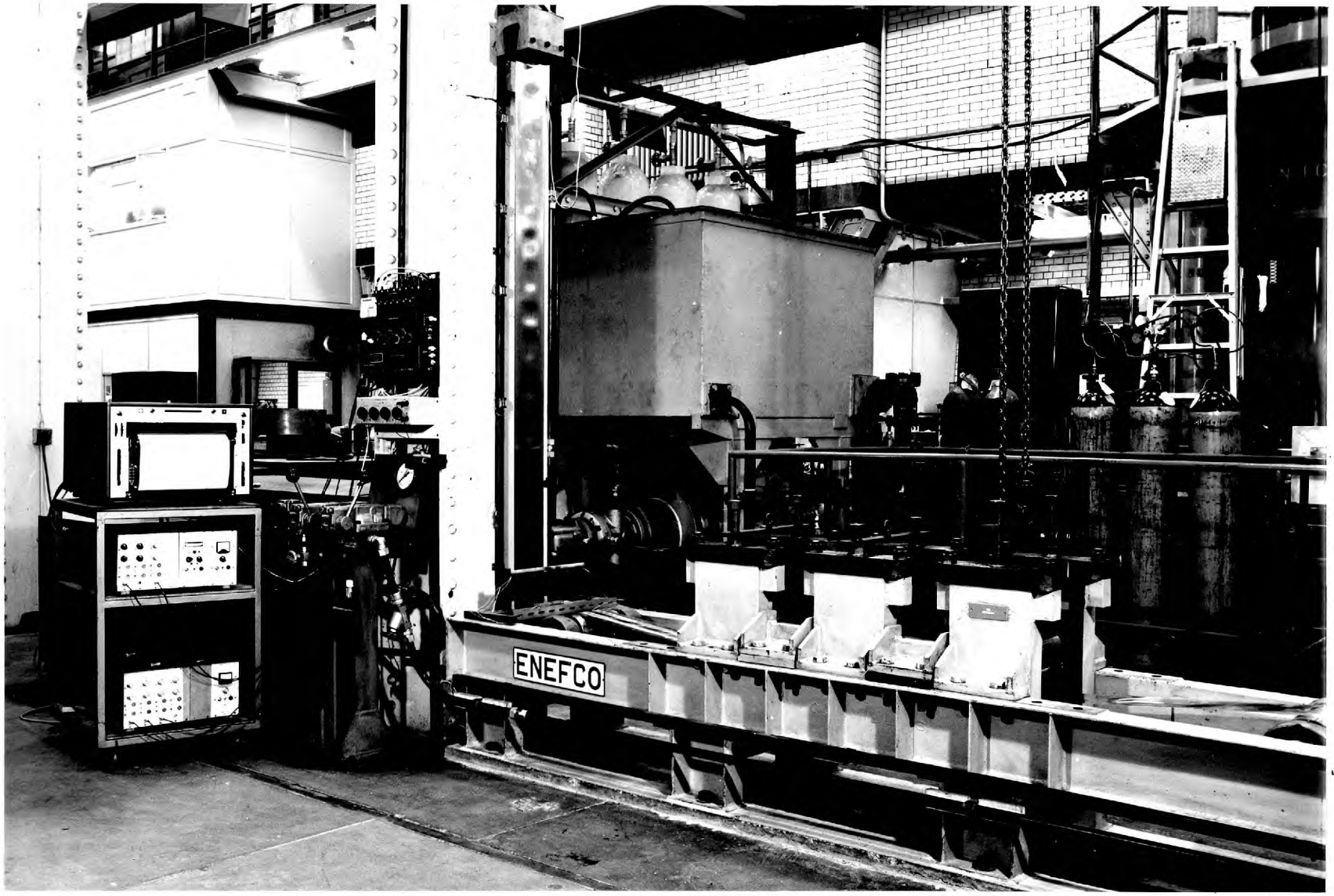


plate (1) EXPERIMENTAL LEVELLER - GENERAL ARRANGEMENT

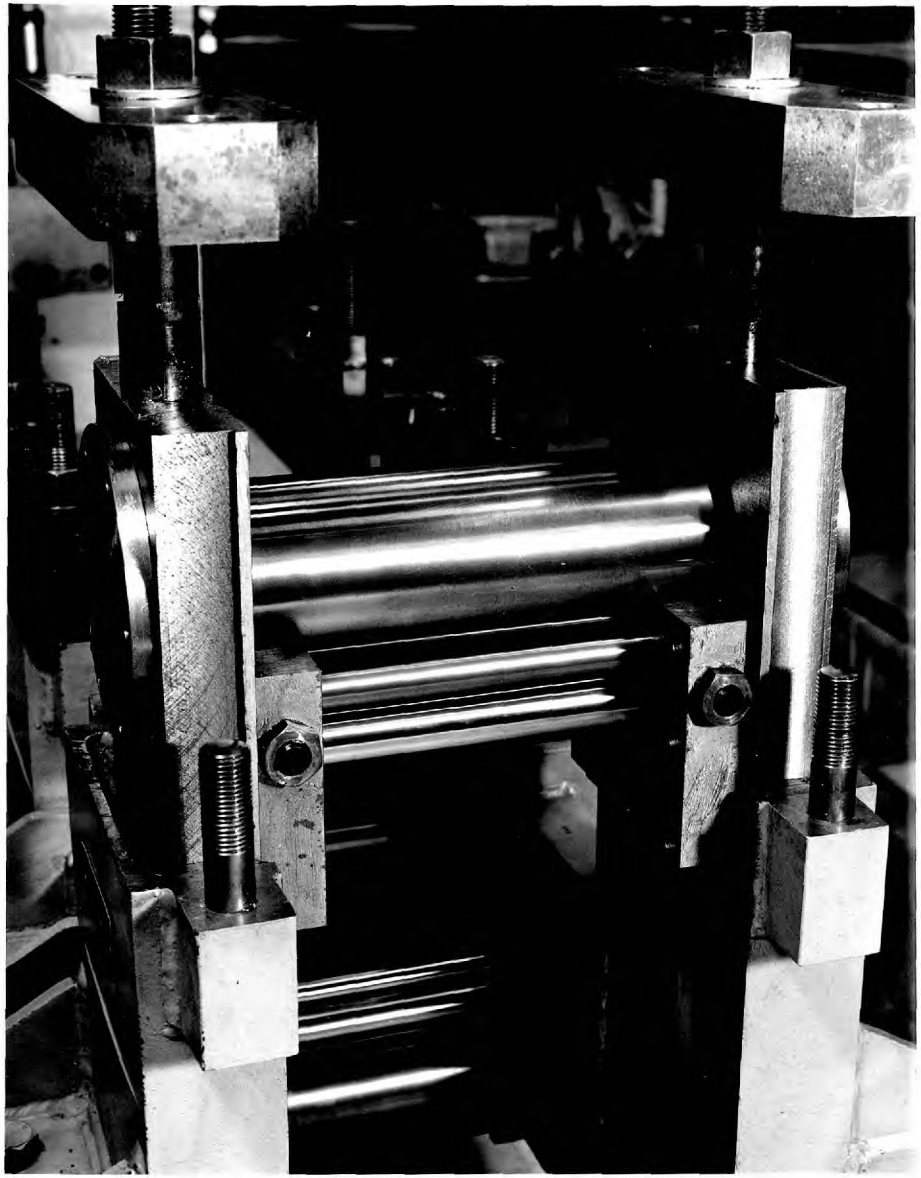


plate (2) WORK-ROLL SYSTEM

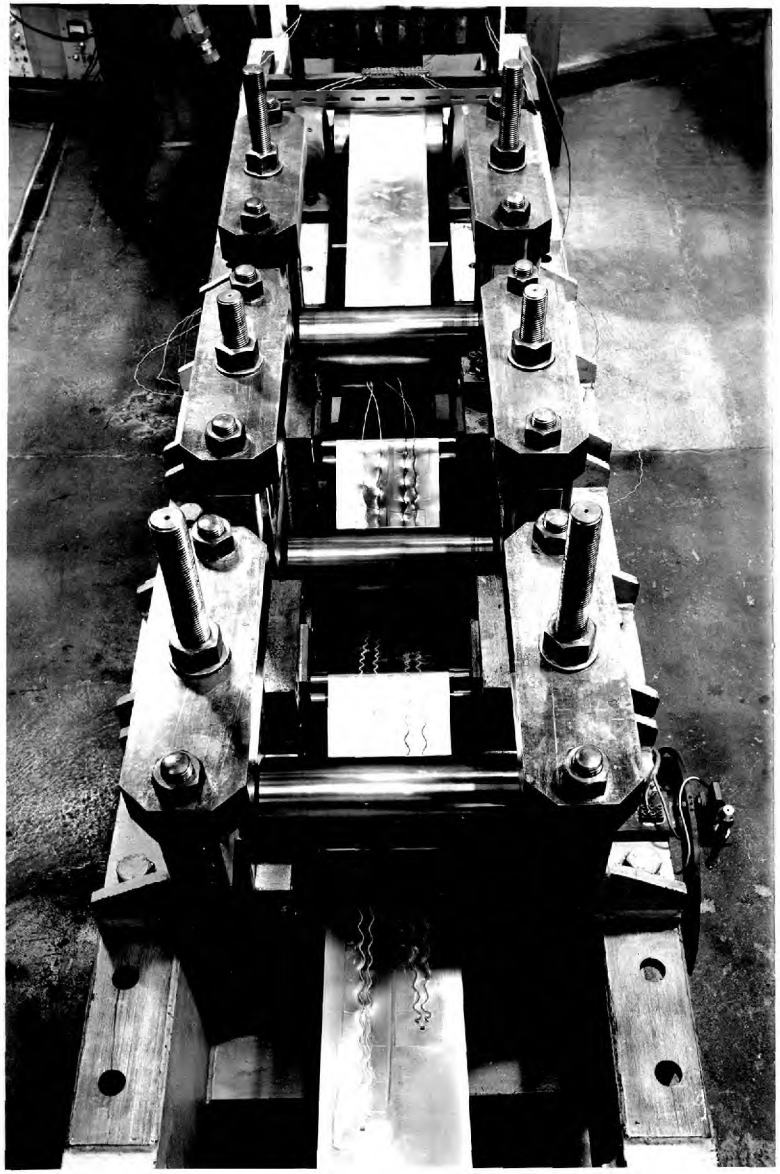
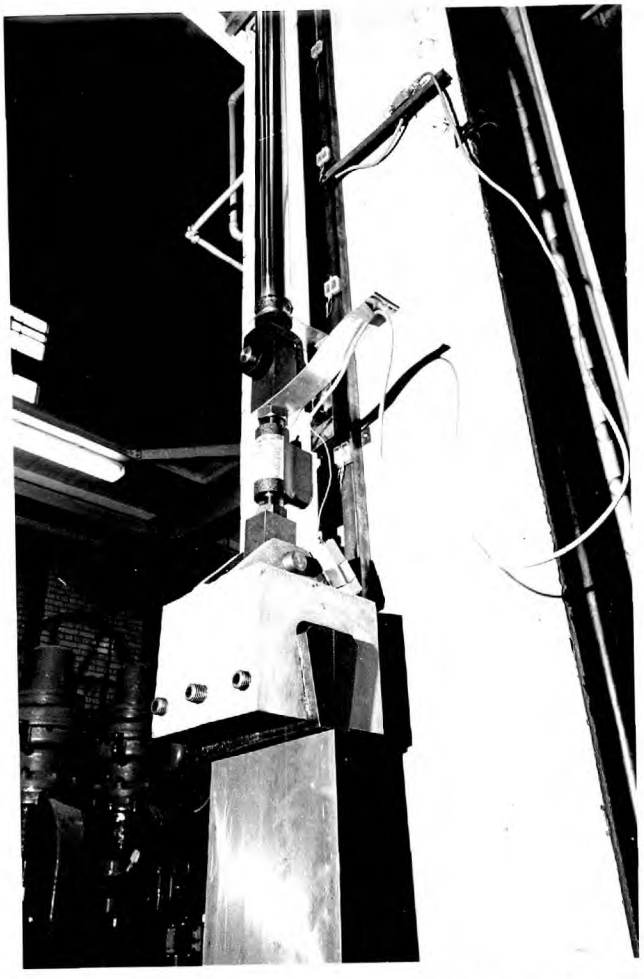


plate (3) TOP VIEW - LEVELLER EXIT



plate(4) LEVELLER ENTRY - SHOWING STRAIN GAUGES ATTACHED AND STRAIN-MEASURING TRAMMEL



plate(5) WEDGE-GRIP SYSTEM & LOAD CELL

4.2.0. Measurement Techniques.

The experimental measurements required for theoretical evaluation were:

(1)Uniaxial stress/strain curve details.

(2) The thickness, width and any curvature of the strip at exit and entry.

(3) Work-roll diameters and wrap angles.

(4) Entry and exit tensile loads.

(5) The surface strain path of strip passing through the leveller together with the resulting 'overall' strip elongation.

(6) Strip spread.

4.2.1.The Uniaxial Stress/Strain Curve.

This relationship was derived from the load/elongation curve via the computer program of Appendix 6, this program also plotted the required relationships and performed a curve-fitting operation.The curve-fit equation has been previously mentioned, being

$$\bar{\sigma} = A(B + \bar{\varepsilon})^m \qquad \text{----4.2.1.}$$

the constants A,B and m being derived for each individual test and for the summation of all test results on the same material.To eliminate the effects of scatter to the greatest extent, the required relationships for any coil were determined from the mean of eight samples taken over the whole length of the coil.

A simple Hounsfield testing machine is insufficiently accurate for the above work and is of limited application in the determination of elastic properties.Load/elongation plots were therefore obtained from the combination of a Denison tensile testing machine and a microformer extensometer, outputs being fed directly to a Baldwin recorder (typeMD-2B)

The available Baldwin extensometer (type PS-5M) was specified for use on round bar up to 12.5 mm diameter, being capable of recording up to 2% strain on the 50.8 mm gauge length.To adapt the extensometer for use with strip specimens ,brass caps were made to fit the gauge screws on one side of the specimen, thus effectively producing a standard flat test-piece extensometer.The test-pieces themselves were obtained by the use of specimen punch (BS.18, 1962.Gauge length 50.8 mm), sheared edges being smoothed with 400 grade silicon carbide paper.Test-pieces were retained in the testing machine by pin grips operating via ball and socket couplings.

Both load and strain measuring systems were accurately calibrated

during the period of testing and were found to produce linear recorder deflections with load and extension.Strain magnification ratios available with the extensometer/recorder combination were, 250,500 and 1000 times any one of these ratios being sufficient for the determination of elastic properties.The width and thickness of the samples were measured by vernier micrometer.

Plate 6 shows the complete testing system, a typical output being clearly visible on the recorder chart.

Since the extensometer was only capable of recording strains of 2% (ie. 0.04ins extension) from one zero setting, it was necessary,(to provide strain records up to about 6% to remove and reset the extensometer twice during each test.Straining was stopped during the required time interval which was found to be sufficiently short to prevent a high degree of stress relaxation.Restraining was found to provide perfect continuation of the load/elongation curve.Resetting the extensometer results in a loss of original material within the gauge length.Providing that straining is always stopped at a measured 2% elongation,correction for loss of material is relatively simple.It is easily shown that on the 1st resetting 0.04in are lost from the gauge length.On the second the total lost material is 0.0808 ins and on the third 0.1224ins.Using the 250 magnification scale where 10ins of chart are equivalent to 2% strain, after resetting we may write,

12in	=	2.408% as opposed to 2.4%
16in	=	3.224% as opposed to 3.2%
20in	=	4.040% as opposed to 4.0%
24in	=	4.872% as opposed to 418%
30in	=	6.120% as opposed to 6.0%
40in	=	8.243% as opposed to 8.0%

Load/elongation values fed into the computer were chosen not to fall within the load relaxation region where true loads must of necessity be estimated.Strain rates during testing were kept reasonably low at 10^{-3} per second in an attempt to reduce relaxation effects on extensometer resetting to a minimum.It was found, that with the particular steel used increased strain rate (up to $7x10^{-3}$ per sec) resulted in little change in strain/stress relationships, any difference lying within the coil scatter-band.

4.2.2. Strip Geometry.

Strip thickness during the Denison tests was taken as the mean of

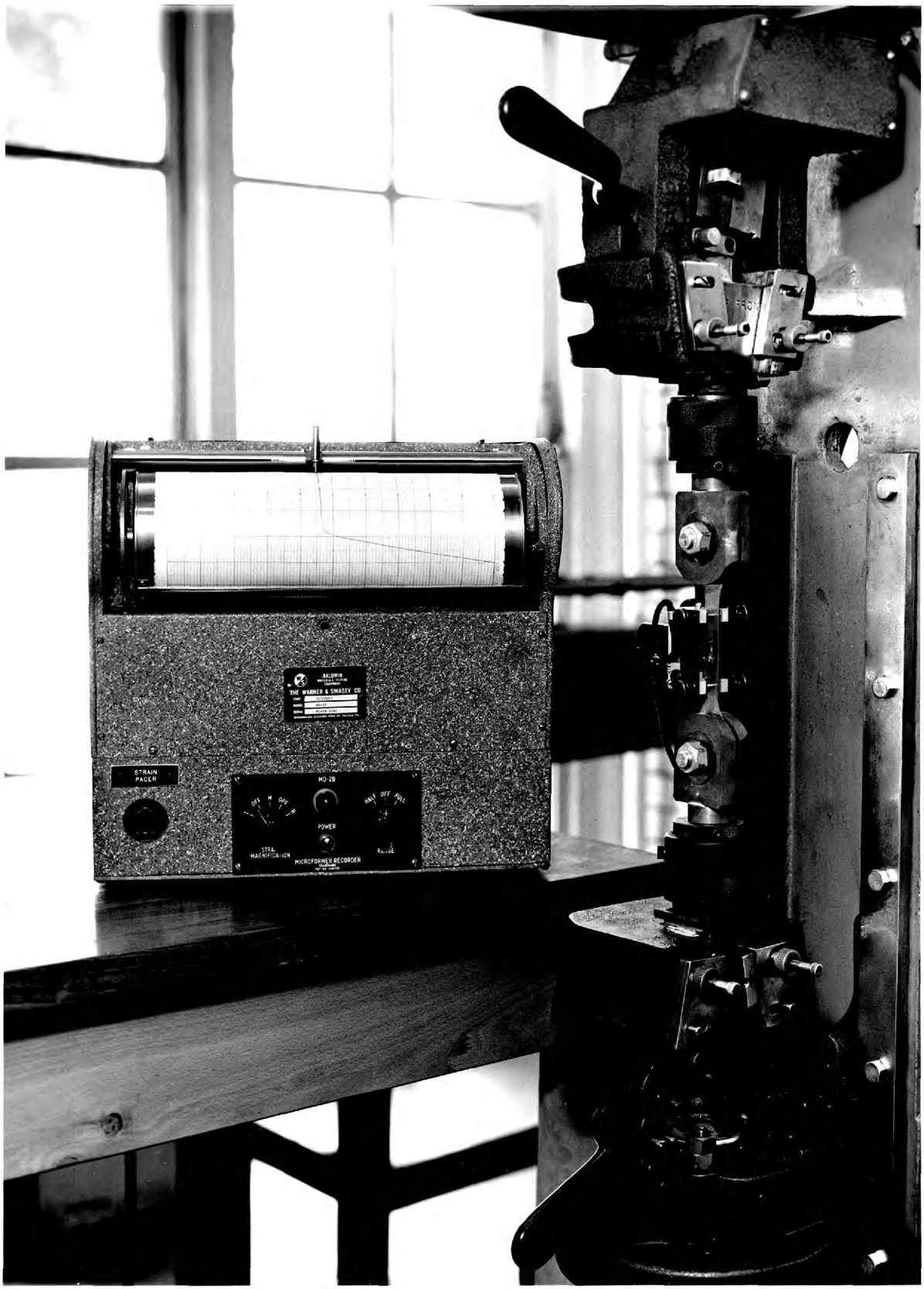


plate (6) MATERIAL TESTING SET-UP, SHOWING BALDWIN EXTENSOMETER & RECORDER

five measurements over the gauge length, the initial width being constant at 15.25 mm.Thickness measurements during leveller tests were more difficult, a large degree of variation being found (0.025 mm) over the strip width and along the strip length in many cases.The recorded value for any test was therefore taken as the mean of eight readings taken at four positions along the 508 mm gauge length (section 4.2.5.).Individual readings were taken one third of the strip width in from each edge.Strip width was measured visually using a steel rule.

A 6.35 m length of strip was required for each experimental run, this length presenting problems for initial curvature measurement.Since the strip was taken from coil it was assumed that only a small variation in curvature occurred and coil set was measured from a 400 mm length taken prior to the test length.Curvature on this length was determined by recording arc height over a given chord length - Plate 7 and Fig 4.7. A consideration of the geometry of Fig 4.7. demonstrates the method where the maximum deflection (h) and the chord length (L) are known.L is fixed as the outside distance between the vertical silver steel pins. From Fig 4.7.

$$R = XZ/2 \sin\theta = L/2 \sin\theta \qquad \text{----4.2.2.}$$

but

$$\tan \hat{OYZ} = ZQ/YQ = L/2h \qquad \text{----4.2.3.}$$

and since

$$\hat{OYZ} = 90 - \frac{\theta}{2} \qquad \text{----4.2.4.}$$

$$\tan \hat{OYZ} = \cot(\theta/2) = L/2h \qquad \text{----4.2.5.}$$

From equations 4.2.5 and 4.2.2 curvature is determined via evaluation of θ. Distance L is adjustable at values of 6,8,10 and 12 ins, the dial gauge being initially zeroed at 0.5 in from the pins.Dial gauge range was 2 ins.

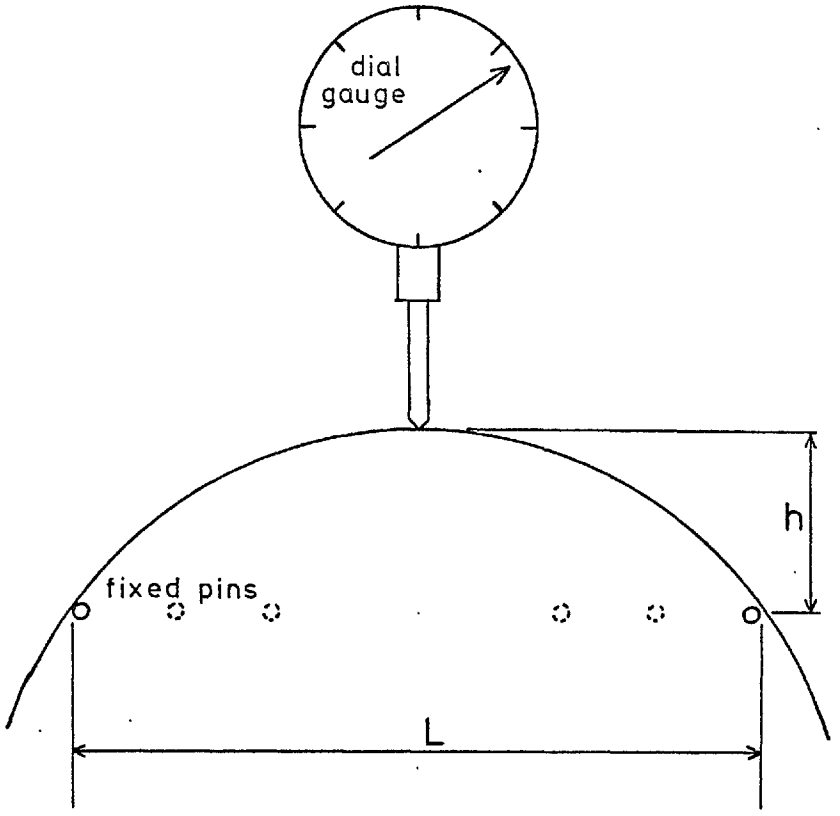
Final strip curvature was measured in the same manner after removal of a 400 mm length of the 508 mm measured gauge length.

4.2.3. Work Roll Interlock and Wrap.

Interlock is measured directly by the shims used in the setting (the latter being evident in Plate 3).Fig 4.8. shows a plot of the shimming required at the adjustable roll stands to attain the pass-line for varying roll diameters.Since slight settling of the rolls occurs under load, these settings are somewhat approximate and the effects of strip thickness are ignored.Nominal wrap angles are easily determined from the



plate (7) CURVATURE MEASUREMENT



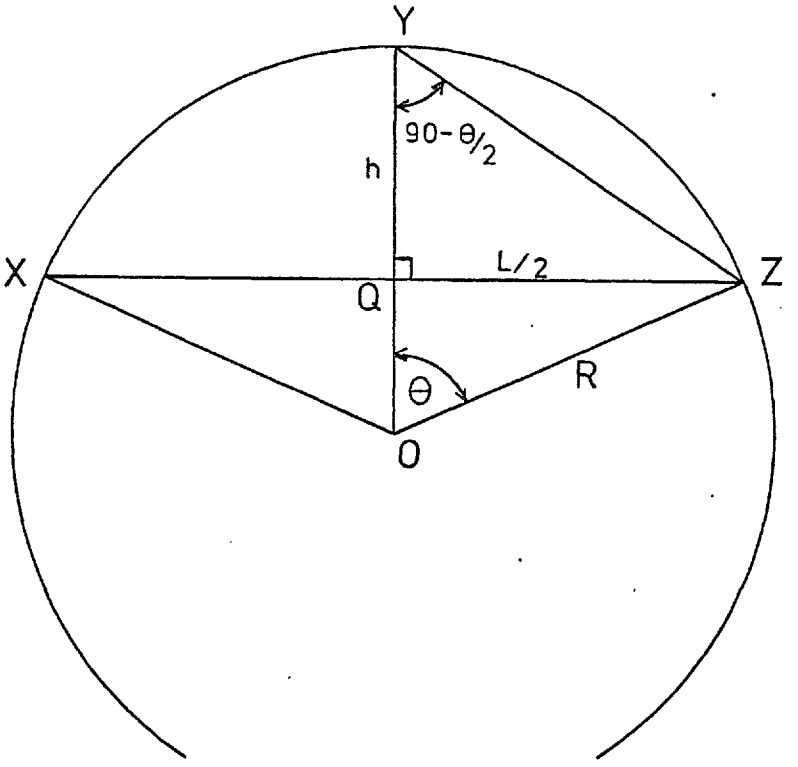


fig (4.7) STRIP CURVATURE DETERMINATION

machine geometry and are given in Fig 4.9.

The finite diameter of the work-rolls results in the actual geometrical wrap angles being greater than the nominal values. Using the notation of Fig 4.10, and assuming work-rolls of equal radius, we may write for the nominal wrap angle,

$$\tan \omega/2 = I/216 \qquad \text{----4.2.6.}$$

where 216 = interstand distance in mm.

The corresponding geometrical wrap angle is written,

$$\varphi/2 = (90 + \varkappa_1) - \varkappa_2 \qquad \text{----4.2.7.}$$

However,

$$\tan\varkappa_1 = (I - D)/216 \qquad \text{----4.2.8.}$$

$$\text{and } \cos\varkappa_2 = D/00' \qquad \text{----4.2.9.}$$

$$\text{since } OX = XO' \qquad \text{----4.2.10.}$$

for constant work-roll diameter. We may also write,

$$00' = \sqrt{216^2 + (I-D)^2} = 216\sqrt{1 + \tan^2 \varkappa_1} \qquad \text{----4.2.11.}$$

From 4.2.9. and 4.2.11. we have

$$\cos \varkappa_2 = D/(216\sqrt{1 + \tan^2\varkappa_1}) = \frac{D \cos\varkappa_1}{216.} \qquad \text{----4.2.12.}$$

Substitution of the various relationships of equations 4.2.6/7/8 and 12 allows the effect of roll diameter on geometrical wrap to be evaluated. The relationship between φ and ω is given in Fig 4.11 for varying roll diameters.

At one stage it was thought possible that an indication of the deviation of the actual strip wrap angle to the geometrical value might be obtained by means of a displacement transducer placed at the geometrical tangency point (Fig 4.12). The gap that may exist between strip and roll at this point is, however, too small to be sucessfully measured by this technique. Although an attempt was made it was therefore soon abandoned.

4.2.4. Entry and Exit Tension.

Tension measurement by previous investigators has been performed by recording roll bearing forces. The use of hydraulic cylinders for load application allows direct recording of front and back strip tensions by load cells positioned between the wedge grips and piston rods. Coutant

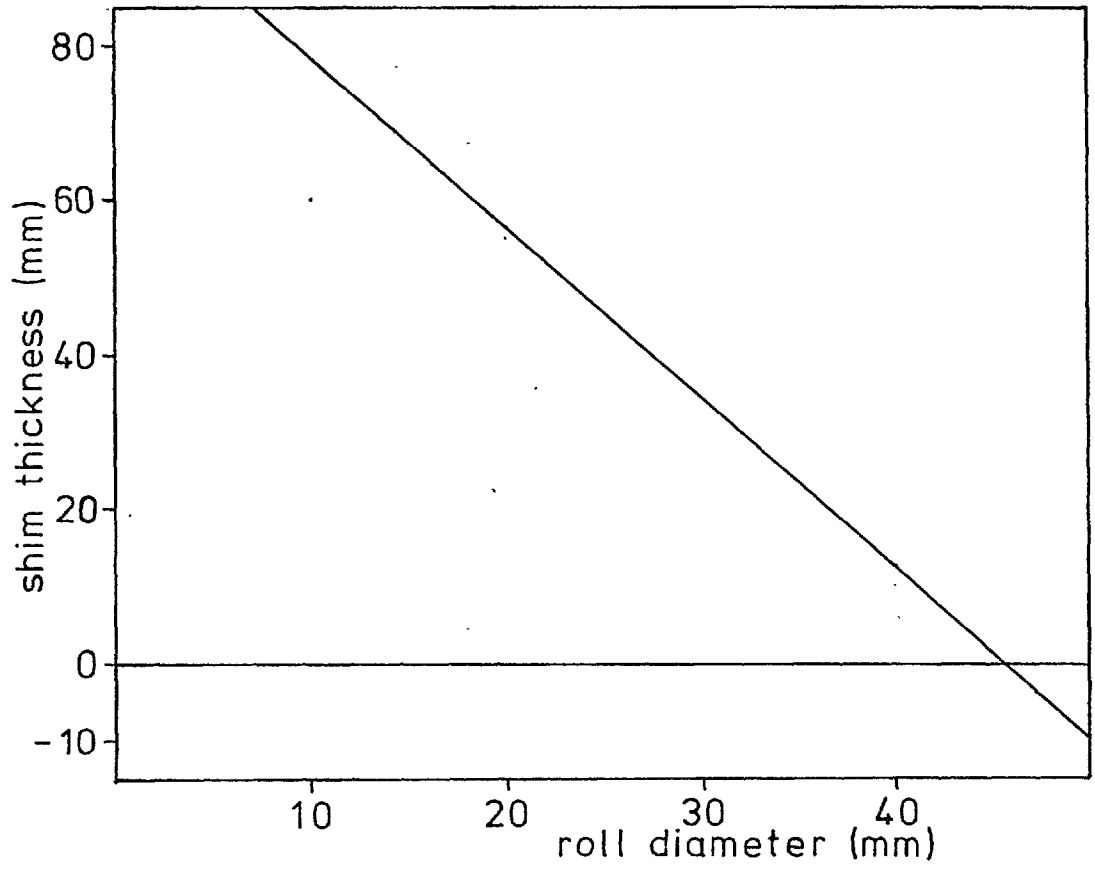
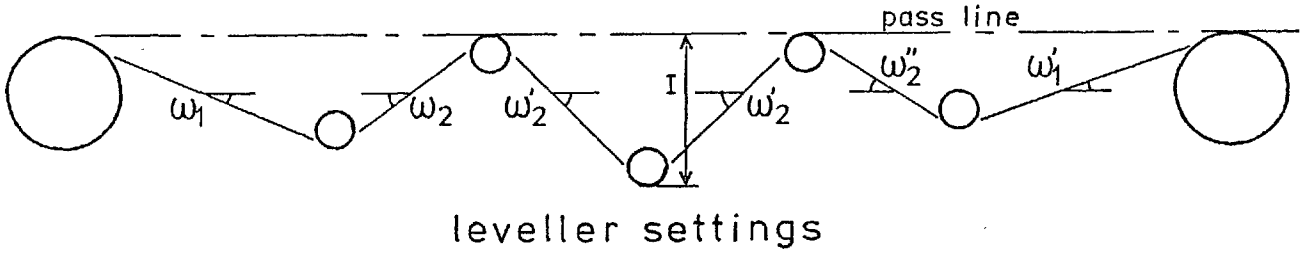


fig (4.8) SHIM SETTINGS FOR ZERO INTERLOCK



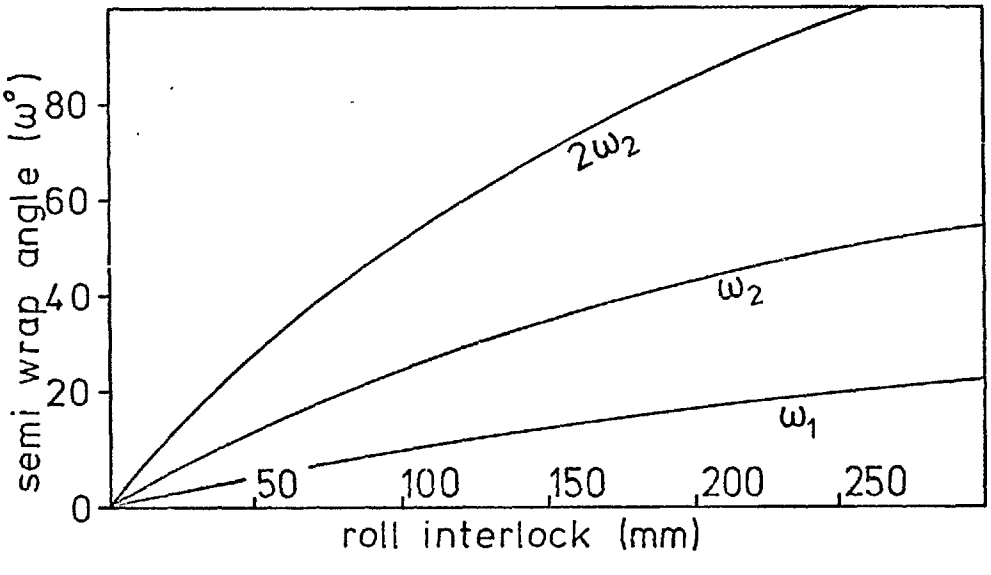


fig (4.9) NOMINAL ROLL WRAP ANGLES

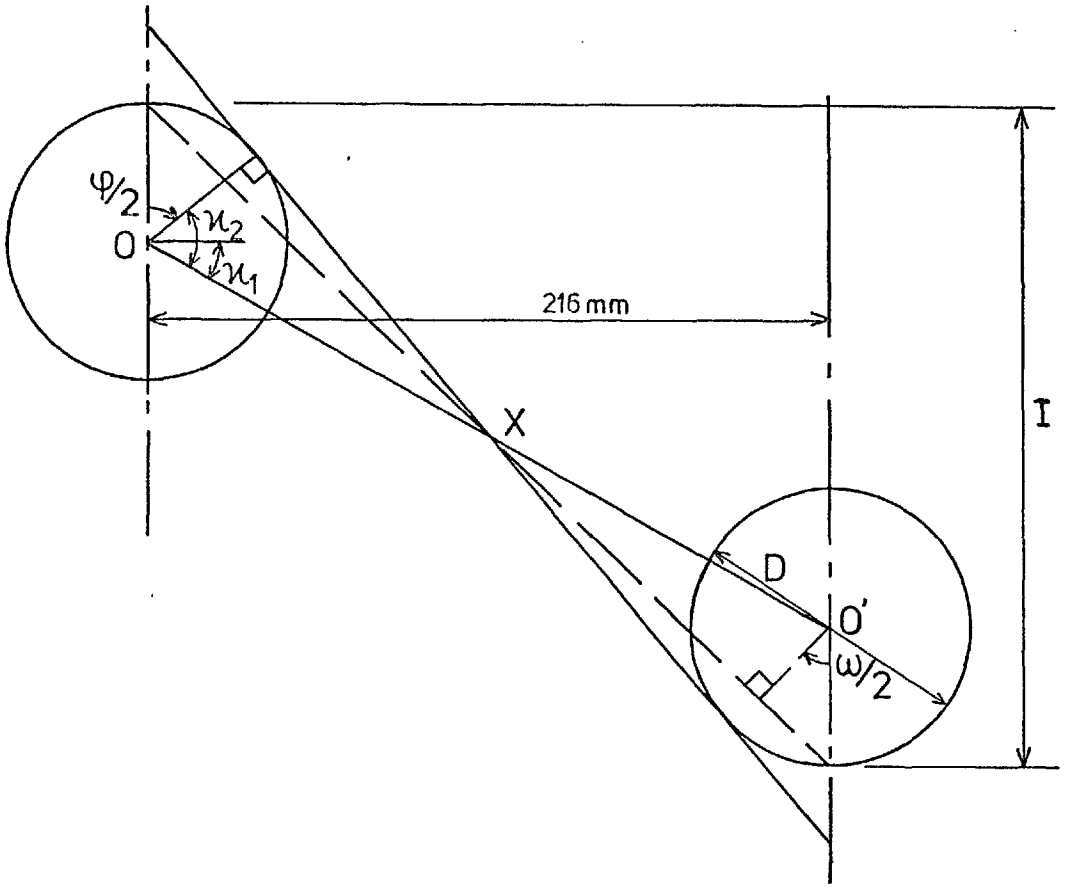


fig (4.10) EFFECT OF FINITE ROLL RADIUS ON GEOMETRIC WRAP ANGLES

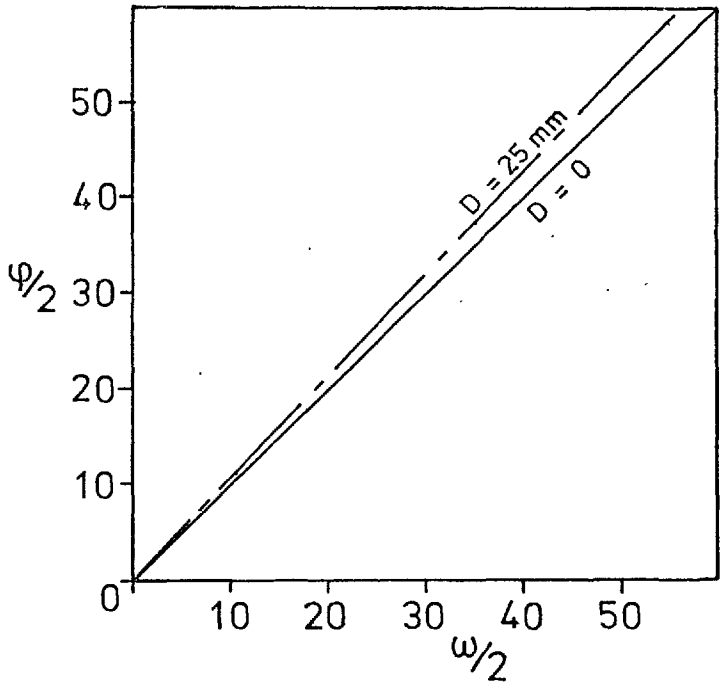


fig (4.11) NOMINAL / GEOMETRICAL WRAP COMPARISON

tensile load cells,(Type E.M. 50 Capacity 10 tons) were used for this purpose, the specification being sufficient to withstand any shock loading conditions encountered..

These load cells were of the standard internal bridge form and were driven at 6 volts from a Southern Instruments Bridge Power Supply (Type M1271).Output was carried to an ultra-violet recorder (Type M1300) via D.C. amplifiers (Type M1266) powered by an M1267 power supply.SM1/M fluid damped galvanometers proved suitable when run at 500 amplification, thus providing 286 mm/mV deflection.

Both load cell circuits were originally calibrated on an Instron tensile testing machine which incorporated an accurate calibration system. After preliminary experimentation however, it was found that some drift had occurred in one cell and recalibration was carried out in-situ by means of a third load cell connected between the wedge-grips and machine frame.The resulting calibration curves are shown in Fig 4.13.

To allow for load losses incurred over the guide rolls, the leveller was run over a test period with all work-rolls raised above the pass-line.Load losses were found to be approximately constant at 225 N, irrespective of applied load.

4.2.5. Strain Measurement.

Overall elongations within the leveller were determined by making two sets of five pock marks over the strip width at approximately 508mm spacing.The machine hydraulics were found to give strip creep under supposedly stationary conditions and by closing the flow control valve and adjusting the pressure relief valve the strip could be maintained at constant load for a considerable period.This facility made it possible to hold the strip under a tension sufficient to ensure flatness when measuring the above gauge lengths.

Gauge lengths at each station over the strip width were determined by the use of a trammel, to which an additional fixed peg had been added (Plate 5).The distance between this and the fixed leg at the opp-osite end was set constant.By means of a 1-2 inch vernier micrometer the fixed peg/adjustable leg distance was accurately measured and the one inch adjustable range allowed direct measurement of strains up to 5%.Gauge length measurements were taken (at constant load) before and after passage of the strip through the leveller, thus allowing the req-uired information to be obtained.It was found that by utilizing this micrometer method strains of 0.1%(corresponding to a 0.5mm increase in

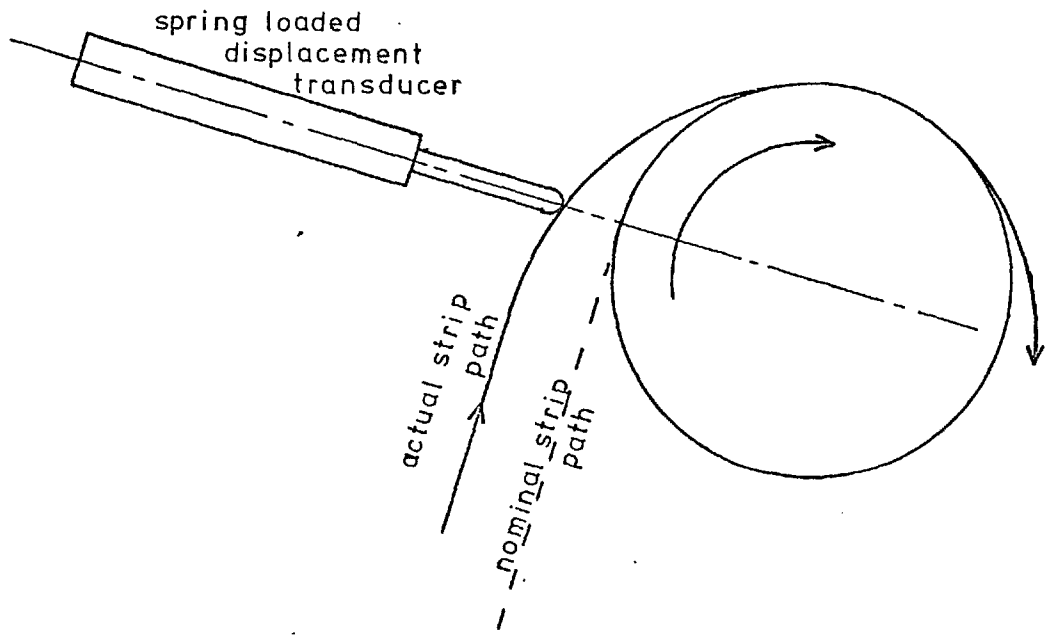


fig (4.12) 'FLY-OFF' TRANSDUCER SETTING

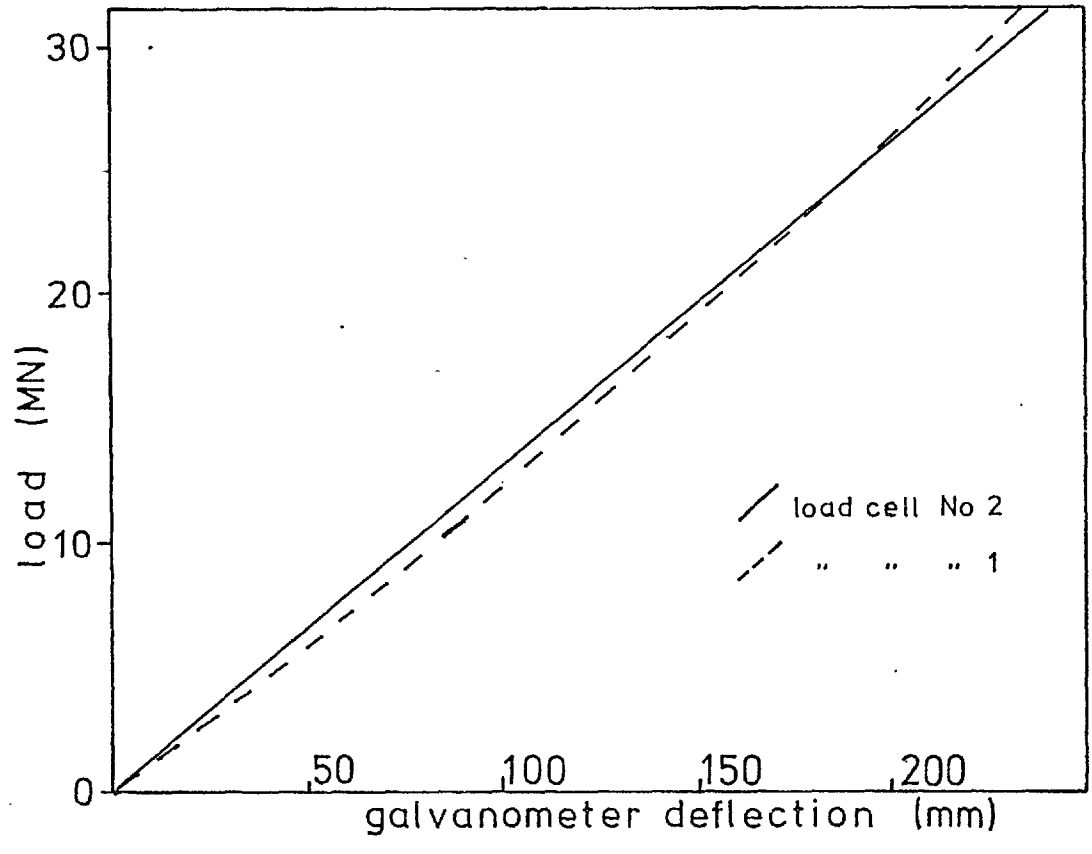


fig (4.13) LOAD CELL CALIBRATION (×500 amp.)

gauge length) could be easily defined after some practise.

Although overall strains posed few measurement difficulties, the determination of incremental strain magnitudes by strain gauges was complex.The initial requirement of such a system was a suitable continuous recording mechanism.

The availability of the ultra-violet recorder and low voltage stabilized power supply made the continuous record of gauge results, concurrent with load values possible.A suitable analysis circuit (in the form of a Wheatstone Bridge) was required.Since a number of parallel circuits were needed a single control box was designed with an eye to the minimization of calibration efforts with gauges of varying gauge factor.Six half-bridge circuits were incorporated in the one box, the circuitry of each bridge system being depicted in Fig 4.14.(All resistances used were of high stability).

These circuits were originally designed to operate with very sensitive (SMI/S), electrically damped galvanometers,(hence the diode protection and output resistance).The purchase of further D.C. amplifiers and SMI/M galvanometers made strain recording simpler and provided a more convenient range of output characteristics.The original circuits were still applicable however.

The resistance change (Δr) of a strain gauge, is dependent upon the strain (ε), the nominal resistance (r) and the gauge factor (f), where

$$\Delta r = \varepsilon r f \qquad \text{----4.2.13.}$$

Where an electric calibration method is to be applied, this is achieved by shunting the fixed half-bridge resistance adjacent to the active gauge.From the notation of Fig 4.14, we may compare the effective resistance change of this gauge (Δr_1) with the corresponding change in the active gauge (Δr_a), hence for similar bridge unbalance,

$$\frac{r_a + \Delta r_a}{r_1} = \frac{r_a}{r_1 - \Delta r_1} \qquad \text{----4.2.14.}$$

hence,

$$\frac{\Delta r_a}{r_a} = \frac{\Delta r_1}{r_1 - \Delta r_1} \qquad \text{----4.2.15.}$$

Comparing this equation with 4.2.13 we obtain

$$\varepsilon f = \frac{\Delta r_a}{r_a} = \frac{\Delta r_1}{r_1 - \Delta r_1} \qquad \text{----4.2.16.}$$

Δr_1 is obtained by shunting r_1 with a calibrating resistance (r_c) and

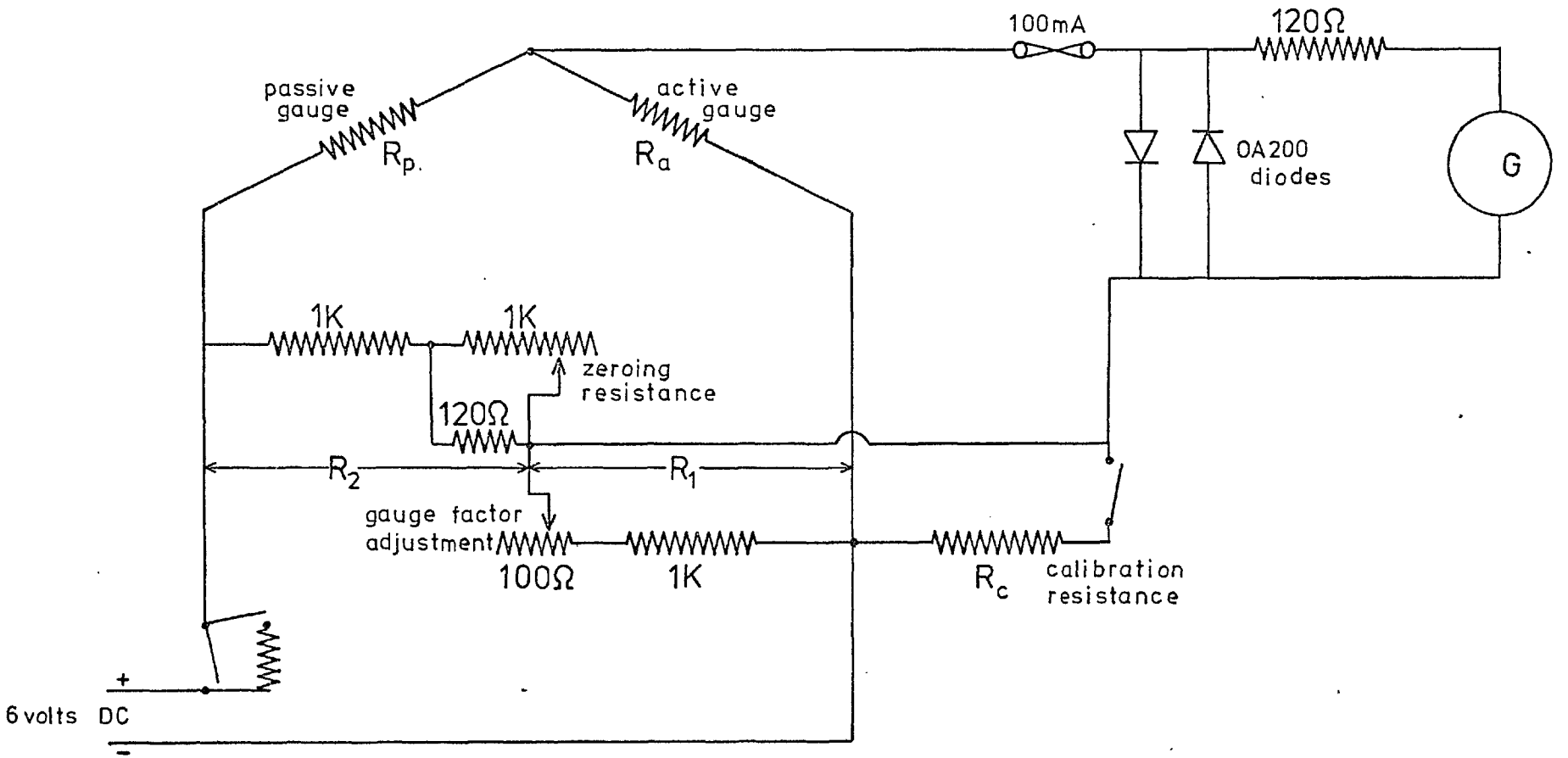


fig (4.14) STRAIN GAUGE OUTPUT CIRCUIT

we may write,

$$\Delta r_1 = r_1 - \frac{r_1 r_c}{r_c + r_1} \qquad \text{----4.2.17.}$$

From equations 4.2.16/17

$$\varepsilon f = \frac{r_1 + r_c}{r_1 r_c}\left(r_1 - \frac{r_1 r_c}{r_1 + r_c}\right) = \frac{r_1}{r_c} \qquad \text{----4.2.18.}$$

To enable the calibrating resistance (r_c) to be independent of f (thus allowing the use of fixed calibration resistances at all gauge factors), the ratio r_1/f must remain constant.In the present system this has been accomplished by utilizing a value of r_1 variable between 1000 and 1100 Ω.This is equivalent to a gauge factor variation of 2.0 to 2.2,(a reasonable range for foil gauge applications).To provide bridge balance, r_2 must necessarily be adjustable, the present system allowing variations from 1000 to 1107 Ω.Observation of Fig 4.14 will indicate that with the circuit design utilized, balancing will be difficult at the low values of gauge factors.This design made use of resistances already present from a previous unsatisfactory system and has proved adequate for experiments conducted so far.These experiements have utilized gauge factors varying from 2.04 to 2.17.

The required values of calibrating resistance for representation of particular strain values can be obtained theoretically from equation 4.2.18.Because of possible junction resistances however calibration resistances for each circuit were obtained experiementally by substitution of a standard strain gauge calibration box for the active gauge.A trial and error technique then determined the necessary calibration values. The slight discrepancies found between each bridge system were eliminated during calibration of the gauge factor dials.

Since experiemental runs were of short duration it was unnecessary to provide dummy temperature compensation gauges on the actual strip surface. Six dummy gauges were therefore attached to a piece of strip steel and used permanently as the fourth bridge resistances.(Plates 1 and 2).On connection of an active gauge into the circuit, the manufacturers gauge factor was selected, the bridge balanced (making use of the voltage divide) and the calibrating resistances switched in, in sequence.Galvanometer deflections represestative of 0 to 5% strain were thus recorded on the output chart.

The strains of 3 to 5% often obtained at the strip surface are beyond the limits of most gauges used for elastic work.High elongation gauges

(capable of withstanding 10% strain) are available(such as Micro-Measurements' EP gauges) but are expensive when the actual passage of a gauge through the leveller is considered.It will be apparent that shear stresses on the gauge as it passes between strip and roll are likely to cause defoliation.The success of any experimental run could not therefore be guaranteed.It was therefore necessary to utilize the cheaper gauges where possible.Manufacturers gauge factors are also somewhat in doubt with high elongation gauges.

As a gauge passes between the strip and roll, pressure stresses are set up,/though foil gauges were therefore preferred their performance in compression is inferior to that of wire gauges.Preliminary experiments, in which transverse pressure was applied to gauges in a Hounsfield testing machine showed a negligible deviation of gauge output with pressure.A full experimental sequence to estimate any effects has not therefore been performed.

Gauges used in experiment were those produced by the Shinkoh and Tokyo Sokki Kenkyujo Companies wherever possible, high elongation gauges being used where high surface strains were anticipated.The large number of tests required, coupled with their short duration necessitated the use of a quick-setting adhesive.Eastman 910 (together with 910 catalyst) has proved suitable, this glue being able to withstand simple tensile strains of approximately 8% when fresh.Attachment to the strip surface was performed according to practise recommended by Micro-Measurements.

One important adaption to this practise was required, however, in that the selotape used in gauge placement was left in position throughout the duration of the test.This selotape layer acted as a lead-in to the strip/roll passage and prevented defoliation to a great extent.Again as a result of the arduous gauge path it was necessary to provide flexibility in the lead wires and complete insulation of jump leads and soldered terminals.Insulation was provided by drawing the short jump leads through the selotape over the gauge (thus giving insulation from the strip) and covering with a further layer to provide insulation from the roll(Fig 4.15).Flexibility in these leads was obtained by crinkling (by the use of tweezers) before connection of the lead wires and selotaping down.The main lead wires (7/0048 PVC) were passed between 12.5mm pitch gear wheels before use to provide a safe working length.The latter were then attached to the strip surface (leading back through the machine) with a further strip of selotape (Fig 4.15).

Plate 5 shows two gauge installations before leveller entry (longitudinal displacement is provided to allow easy differentiation of

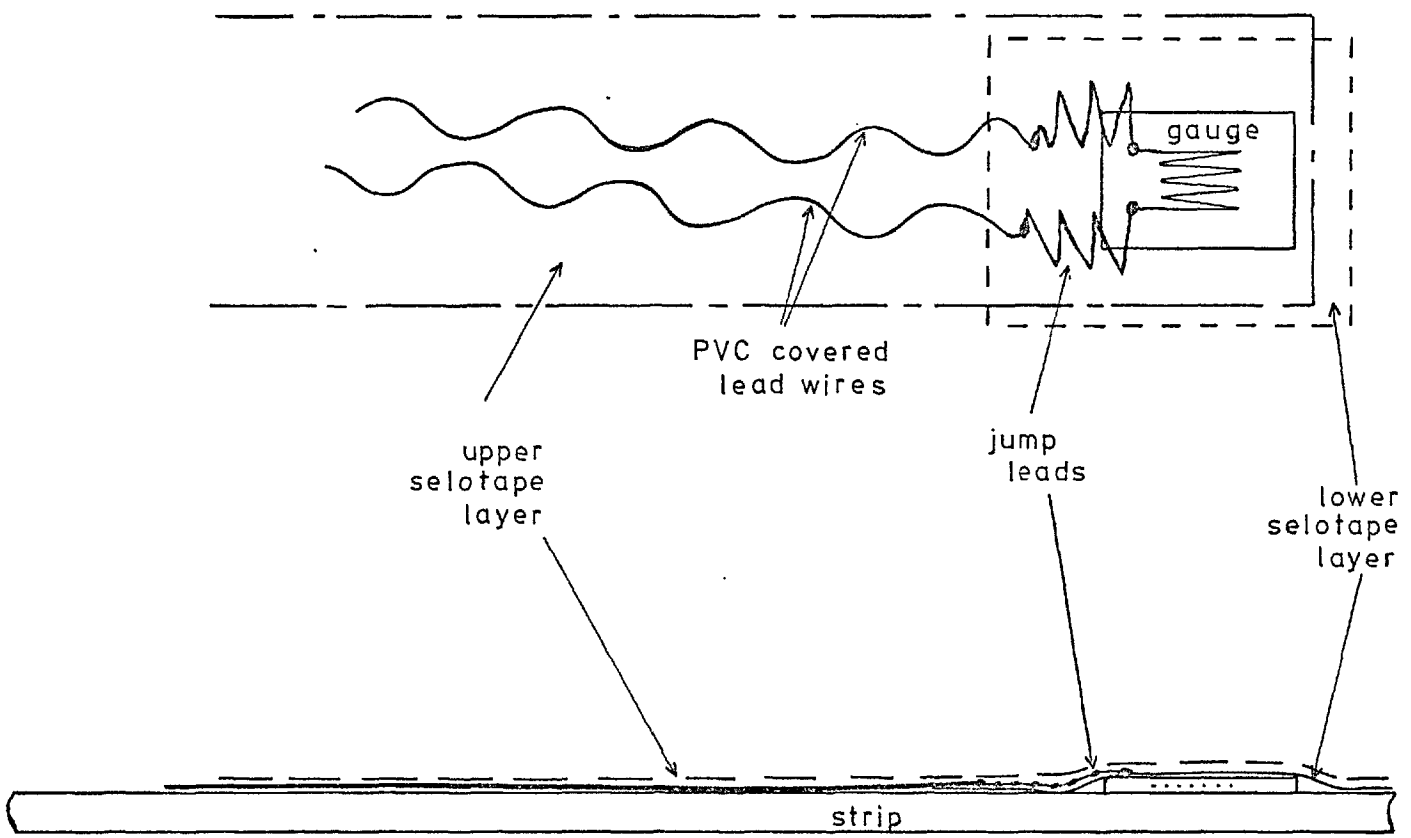


fig (4.15) GAUGE ATTACHMENT

of output traces), together with the gauge adhesive kit and trammel used for overall strain measurement.Plate 2 depicts the same gauges at leveller exit, and, although, they appear satisfactory, failure has occurred in both.

Where gauges are attached to thin strip, (especially with an overlying selotape layer) a considerable thickening effect is experienced. This thickening effect gives rise to an increase in section modulus and correspondingly reduced strain records.It was assumed, in the first instance that the large bulk of surrounding material and the plane strain nature of the test would ensure that strains present in material beneath the gauges were identical to those in other areas.This, however, has proved not to be the case and will be discussed further in Chapter 5.

Figure 4.16 shows a typical output chart for a 3 roll leveller experiment.(This chart has been traced from the ultra violet record paper).It can be seen that once the gauge calibration has been performed bending and straightening strains actually recorded by the gauges are simply determined from deflection measurement.Ignoring for the moment the effects of increased section modulus, strain measurements where the strip is held flat between rolls are identical to those at the surface.Where bending strains are present the displacement of the gauges away from the strip surface results in increased recorded magnitudes than those pertaining to the surface.Since for bending under tension we may write (from equation3.3.3.)

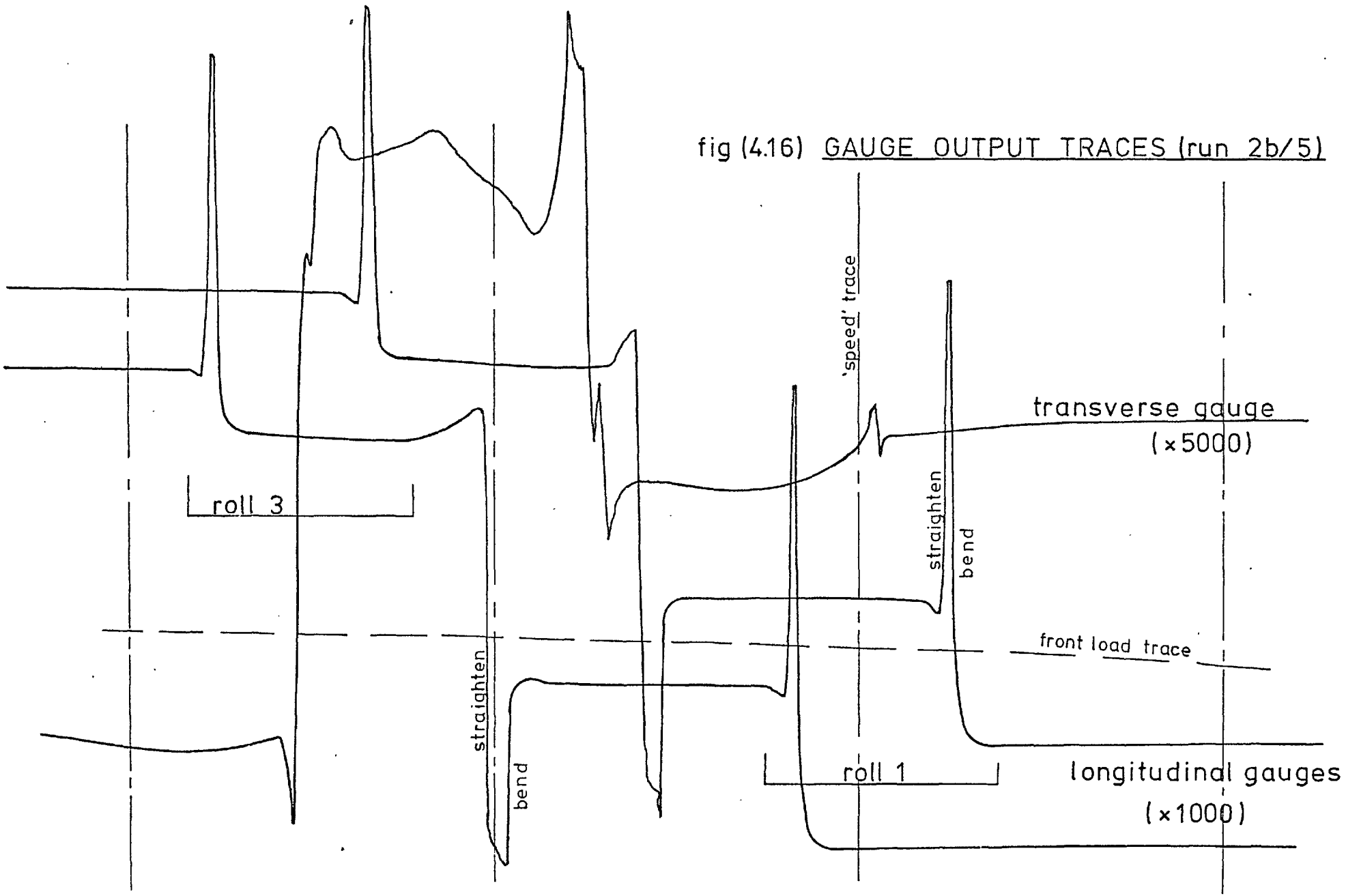
$$e_y = y + b/R-b.$$

it is evident that the order of this over-estimation cannot be evaluated without some quantitative measure of the neutral axis displacement.The theory developed in Chapter 3 does give a good indication of this magnitude however, and gauge results must therefore be considered in conjunction with the theoretical model to provide a quantitative measure of surface strains during bending.

4.2.6. Strip Speed Determination.

The Southern Instruments recorder used for load and strain measurements incorporates an external trigger system, by means of which, an instantaneous trace can be thrown over the chart width.In its usual form this trigger system is activated by the closing of a pair of contacts.Since the trigger can also be connected to a time base, thus giving an accurate record of paper speed, it was thought that a good measure of strip speed could be obtained by situating microswitches at intervals along

fig (4.16) GAUGE OUTPUT TRACES (run 2b/5)



the piston stroke.Accordingly a 3 m length of 25.4 mm square tube was attached to each building column parallel to the gripper jaw path.A 6 mm slot was milled along one face of the tube allowing microswitches to be attached by means of backing plates.The slot allowed continuous adjustment of switch position, whilst the tube served as a shield for the necessary wires.The switches were connected in parellel and positioned at 300 mm intervals along the stroke length.A striker plate was attached to the piston rod immediately above the load cell connection.Unfortunately although the system was sufficiently robust to withstand the lateral movement which occurs during piston travel, oscillations were set up within the switches themselves and could not be damped.Multiple traces resulted on the recorder chart.

Further investigation of the trigger system disclosed that a constant 14 volts output was produced and the current impulse required for activation was not restricted to a zero base level.A suitable photo-electric system was therefore utilized,OCP 71 photo-transistors being mounted in perspex blocks in place of the micro-switches, and a pre-focussed light source replacing the striker plate.The number of photocells required in parallel, and the 14 volts output from the recorder, complicated circuit design, but by a partially trial and error technique a workable circuit was developed incorporating a single amplifying transistor and variable resistances to allow the use of any number of photocells.The circuit diagram for this speed measurement system is given in Fig 4.17.Operation being found faultless under all speed conditions.

4.3.0. Experiemental Investigations.

No recorded work on material posessing shape defects has been performed although the shape correction capability has been demonstrated to the satisfaction of the author.The relatively limited time available after machine installation has necessitated orientation directlytowards theoretical correlation. As such the experiemental investigations do not cover the various metallurgical aspects of levelling (Appendix 1), nor do they provide insight into the parameter relationships required to attain zero strip curvature and minimum stress levels at leveller exit.

For the purposes of commissioning the leveller after installation a 75 Kg coil of CS 2 quality skin passed mild steel was processed, measurements of overall elongation and applied loads being recorded.This commissioning period was utilized to obtain true alignment of all rolls, such that strains were uniform over the strip width.It was found, during

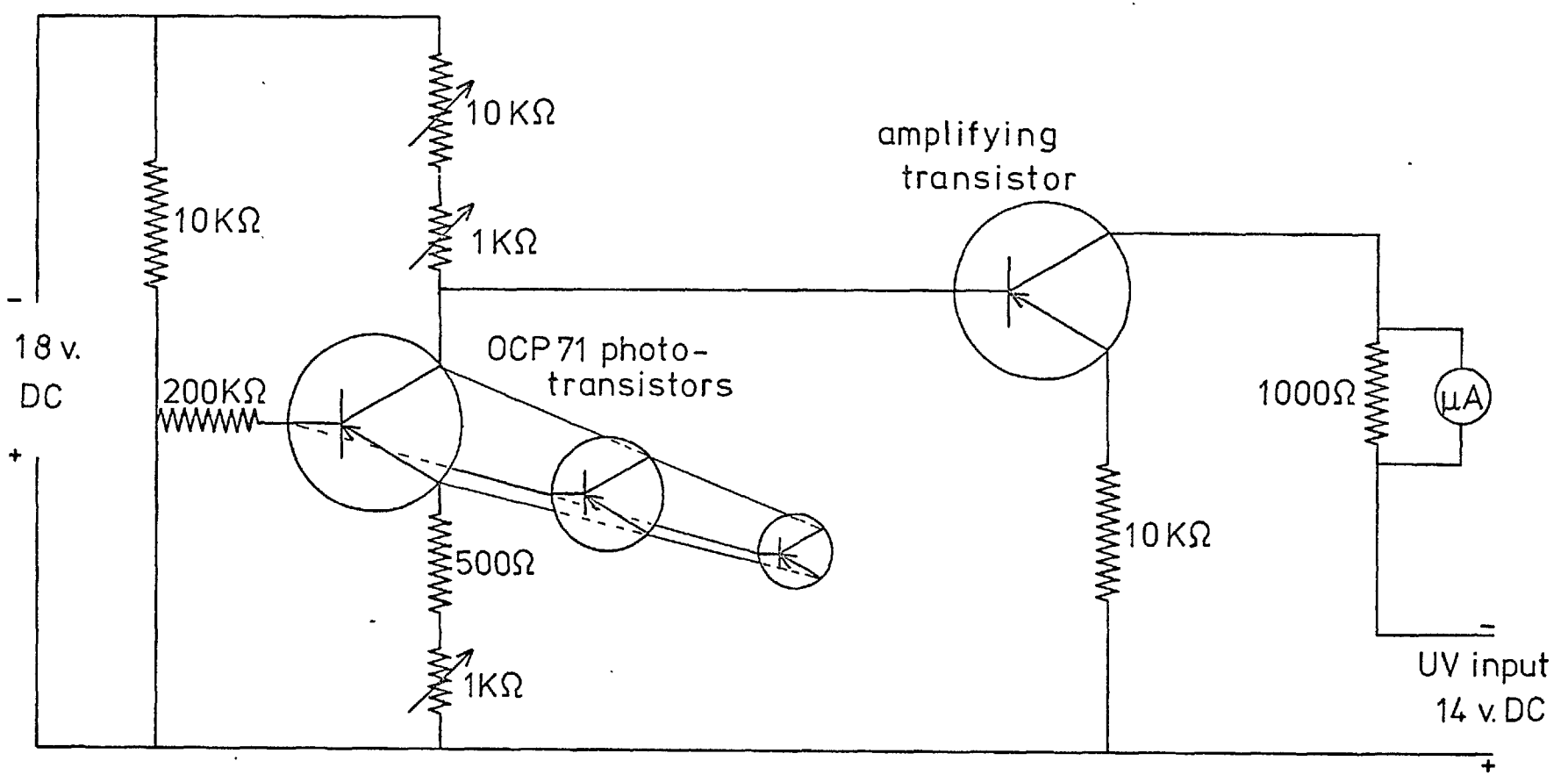


fig (4.17) IRANSISTOR CIRCUIT - STRIP SPEED DETERMINATION

this initial series, that although true alignment of the strip in the wedge grips was difficult to obtain, any deviation from uniform applied load over the strip width was eliminated, before entry of the gauge length into the work-roll system, by extension of material in contact with the rolls during the first take-up of load.The relationship between extension and applied load during this initial series proved similar to that obtained in subsequent experiments.The latter runs served in the developement of the strain gauge techniques used in the following series.

To allow as much isolation of material properties as possible during the true experimental runs, the strip used originated from one 200 mm wide production coil, the composition of which is shown in Table 1.This coil was split and reduced to form gauges of 0.3,0.6,0.9 and 1.2 mm thickness.The final reduction in each case was in the form of a 1½% shin-pass followed by slitting to 170 mm width.This fairly high reduction was utilized to provide relief from strain ageing for a period sufficient for the experimental work.The thickest material proved too heavy for experimental work and the following experimental series were therefore performed on the thinner gauges.

4.3.1. Series 1.

The primary aim of this series was to investigate in some detail the effects of increasing applied load on overall strip elongations.Strain gauge work was continued, however, and the later experiments provided some useful gauge results.Work-roll diameters were kept constant at 25.4 mm, roll interlock also remaining unchanged.Strip of the intermediate thickness was chosen, the mean coil thickness being 0.564 mm, thus providing a roll diameter/strip thickness ratio of 45.

To provide the easiest gauge path possible through the leveller only the centre three roll stands were utilized, rolls one and five being lifted above the pass-line.This action allows three reversals of curvature, gauges coming into contact with a roll only once.A further effect of the three roll set-up is that the total wrap angle is zero, that at the centre roll being exactly twice that at the entry and exit rolls. Although the minimum necessary wrap angle (from gauge consideration) was required at the centre roll, this could only be reduced to 52°,giving 26° at the outer roll stands.This latter figure was considered to be of suitable magnitude for roll radius attainment.

Initially three gauges were attached at equal intervals over the strip-width.After the first few sucessful gauge runs it was found that any

strain differentials that existed over the strip width were linear, thus allowing the use of only two gauges (placed one third of the strip width in from each edge) in subsequent experiments. The 26^{o} minimum wrap angle is equivalent to a 6 mm geometrical contact arc and gauges of 2 mm length were considered sufficiently small to ensure that any flat-topped output peak represented the true bending strains, as opposed to a strain peak travelling through the gauge length.

Again, to ease the burden on the gauges, but also to prevent the introduction of strain rate effects on mechanical properties, strip speed was kept low during the tests. The applied back tension was varied between zero and half the material yield stress, the latter resulting in overall strains more than adequate for industrial purposes.

4.3.2. Series 2.

This series of experiments was designed to reinforce the results of series 1, whilst at the same time providing information as to the effect of wrap angles during levelling.

The high proportion of gauge failures encountered during the series 1 experiments indicated that it would be unwise to increase the number of rolls (thus doubling the chances of defoliation). The three roll set-up was therefore retained and to reduce surface strains to a minimum, whilst maintaining a sufficiently small roll diameter: strip thickness ratio such that wrap effects could be elucidated, work-rolls of 31.5 mm diameter were used. Mean coil thickness was 0.32 mm, the average D/t ratio being 98.5. This increase in ratio over the previous series represented a 50% decrease in surface strains.

Individual experiments were grouped in three sections, the pertinent wrap angles being,

Series 2A ..... $40^{o},80^{o},40^{o}$.
Series 2B ..... $30^{o},60^{o},30^{o}$.
Series 2C ..... $13^{o},26^{o},13^{o}$.

Each section was conducted as series 1, utilizing a range of applied back tensions. During this series only three positions over the strip width were used for overall elongation measurement. Strain gauge practice remained the same however, though the reduced speed utilized (approximately half of that of series 1), coupled with the lower bending strains resulted in a very much greater proportion of sucessful runs.

4.3.3. Series 3.

The results of series 1 and 2 (presented in the next section) indicated that, at least for the material and machine parameters used, good correlation with theory was being obtained.It had become apparent however,that strain gauge results were of limited use.This fact, coupled with the thickness of the remaining material (giving high surface strains), allowed strain gauge measurements to be discarded in favour of total extension measurements.

Three parameters of tension-levelling remained unconsidered, these being,

(1) the number of work-rolls.

(2) the strip thickness/roll diameter ratio.

(3) the inclusion of varying work-roll diameters in the leveller.

It was considered, that providing the results of the first two series, coupled with the results of variations of (1) and (2), the third factor would be adequately described by theory.

Increasing the number of work-rolls to four in this series provided sufficient comparison with series 2 to account for the first factor. Although the quantity of strip available was limited three work-roll/strip thickness ratios were utilized being 56.5,38.8 and 21.2.Strip thickness was 0.92 mm, the roll diameters for the above ratios being 50.8,34.9 and 19.1 mm respectively.Combination of the three series gives a fairly comprehensive coverage of D/t ratios.

Overall strains were again determined as a function of applied load, the vernier technique being utilized.

During all three series of experiments, front and back tensions were recorded.Allowing for work losses at the guide rollers (previously mentioned as approximately constant) the difference between front and back tension values gives a good indication of work losses occurring within the machine.These losses are a function of redundant work done on the strip and bearing friction effects.Subroutine RWORK provides a theoretical interpretation of the first factor, although the second factor remains elusive without a knowledge of bearing friction coefficients. Tension losses were noted however, such that a trial and error computer analysis could be utilized to compare predicted losses at various friction coefficients with those obtained experimentally.This would then provide a suitable basis for discussion of tension losses, when coupled with the analysis of section 3.5.0.

4.4.0. Results.

Results relating to material properties are given in section 4.4.1., whilst those pertaining to leveller tests are presented in the subsequent sections.

4.4.1. Tensile Tests.

Using the equipment and method of section 4.2.1. the equivalent stress/equivalent strain relationships for three materials used were determined. Plots of the derived relationships and the mean curve-fit are given in figures 4.18/19/20, points from only four of the eight tests in each case being plotted.The equations representing these relationships in terms of total equivalent strain were as follows,

Series 1 .... $\bar{\sigma} = 745.0(0.02820 + \bar{\epsilon})^{0.32665.}$ $MN/m^2.$

Series 2 .... $\bar{\sigma} = 662.5(0.01713 + \bar{\epsilon})^{0.29525.}$ $MN/m^2.$

Series 3 .... $\bar{\sigma} = 793.0(0.01628 + \bar{\epsilon})^{0.39285.}$ $MN/m^2.$

The different thermal and working treatments given to the three gauges clearly shows in the values of the derived constants.

Representative values of the elastic moduli obtained during initial loading were,

Series 1 .... $E = 18.2 \times 10^4$ MN/m^2

Series 2 .... $E = 18.6 \times 10^4$ MN/m^2

Series 3 .... $E = 18.3 \times 10^4$ MN/m^2

these values being derived from the slopes over the central portions of the curves, after preloading to approximately three-quarters yield stress.The results of Rolfe et.al., Lubahn and Dalby (82,85-87) have previously indicated a reduction in secant modulus after 1% or so tensile strain.This indeed proved the case, figure 4.21 showing an example loading and reloading curve for a series 2 test piece.The slope of the elastic loading line is seen to drop from 18.2×10^4 MN/m^2 to 16.3×10^4 MN/m^2 on 1% tensile straining, thereafter remaining constant.The gradients of the unloading curves tends to drop more rapidly, settling to a value of approximately 16.2×10^4 MN/m^2.This 10% reduction in the value of the tangent modulus was found in all tests.

Values of Poisson's Ratio for the experimental materials have not been experimentally determined, values of 0.29 to 0.30 being indicated by the suppliers.Theoretical results have been obtained for both these values.

The plane strain, longitudinal stress/strain relationship derived from the program of Appendix 2, assuming an isotropic hardening nature are:-

Series 1 .... $E = 18.2 \times 10^4$ MN/m^2

$\nu = 0.29$ .... $\sigma_x = 838.0(0.02050. + e_x)^{0.3000}$

$\nu = 0.30$ .... $\sigma_x = 836.0(0.02038 + e_x)^{0.29890}$

Series 2 .... $E = 18.6 \times 10^4$MN/m^2

$\nu = 0.29$ .... $\sigma_x = 760.5(0.01254 + e_x)^{0.2771}$

$\nu = 0.30$ .... $\sigma_x = 762.0(0.01276 + e_x)^{0.27810}$

Series 3 .... $E = 18.3 \times 10^4$ MN/m^2

$\nu = 0.29$ .... $\sigma_x = 912.0(0.01229 + e_x)^{0.3707}$

$\nu = 0.30$ .... $\sigma_x = 913.0(0.01241 + e_x)^{0.37115}$

The closeness of these derived relationships to the points of derivation are shown in figures 4.22/23/24 for values of $\nu = 0.29$. Also plotted on these graphs (the lower continuous line) are the curves for the corresponding equivalent stress values. The above results indicate the marginal effect of variations in Poisson's Ratio on the stress/strain relationships.

Although a kinematic hardening model is not applicable to the case of temper rolled mild steel, (the results of Hilsen, Levy and Polakowski 51,52 adequately demonstrate this), the longitudinal stress/strain relationships for a material with an equivalent stress/strain curve as that of the Series 2 material but obeying non-reversible hardening rules, will appear,

$\sigma_x = 720.0(0.00836 + e'_x)^{0.2500}$ ----(Fig 4.25)

for $E = 18.6 \times 10^4$ MN/m^2, $\nu = 0.29$.

For the series 1 case, we may write,

$E = 18.2 \times 10^4$ MN/m^2, $\nu = 0.29$.

$\sigma_x = 840.0(0.01959 + e'_x)^{0.3000}$ ----(Fig 4.26)

and for $\nu = 0.3$

$\sigma_x = 804.0(0.01552 + e'_x)^{0.27595}$

To enable the effects of possible errors in the determination of Young's Modulus to be visualized, Series 2 programs were run incorporating values of E of 16.55 and 20.68×10^4 MN/m^2. ($\nu = 0.29$). The derived isotropic relationships are

$E = 16.6, \sigma_x = 762.0(0.01247 + e_x)^{0.2772}$

$E = 18.6, \sigma_x = 762.0(0.01254 + e_x)^{0.2771}$

$E = 20.7, \sigma_x = 761.0(0.01255 + e_x)^{0.2767}$

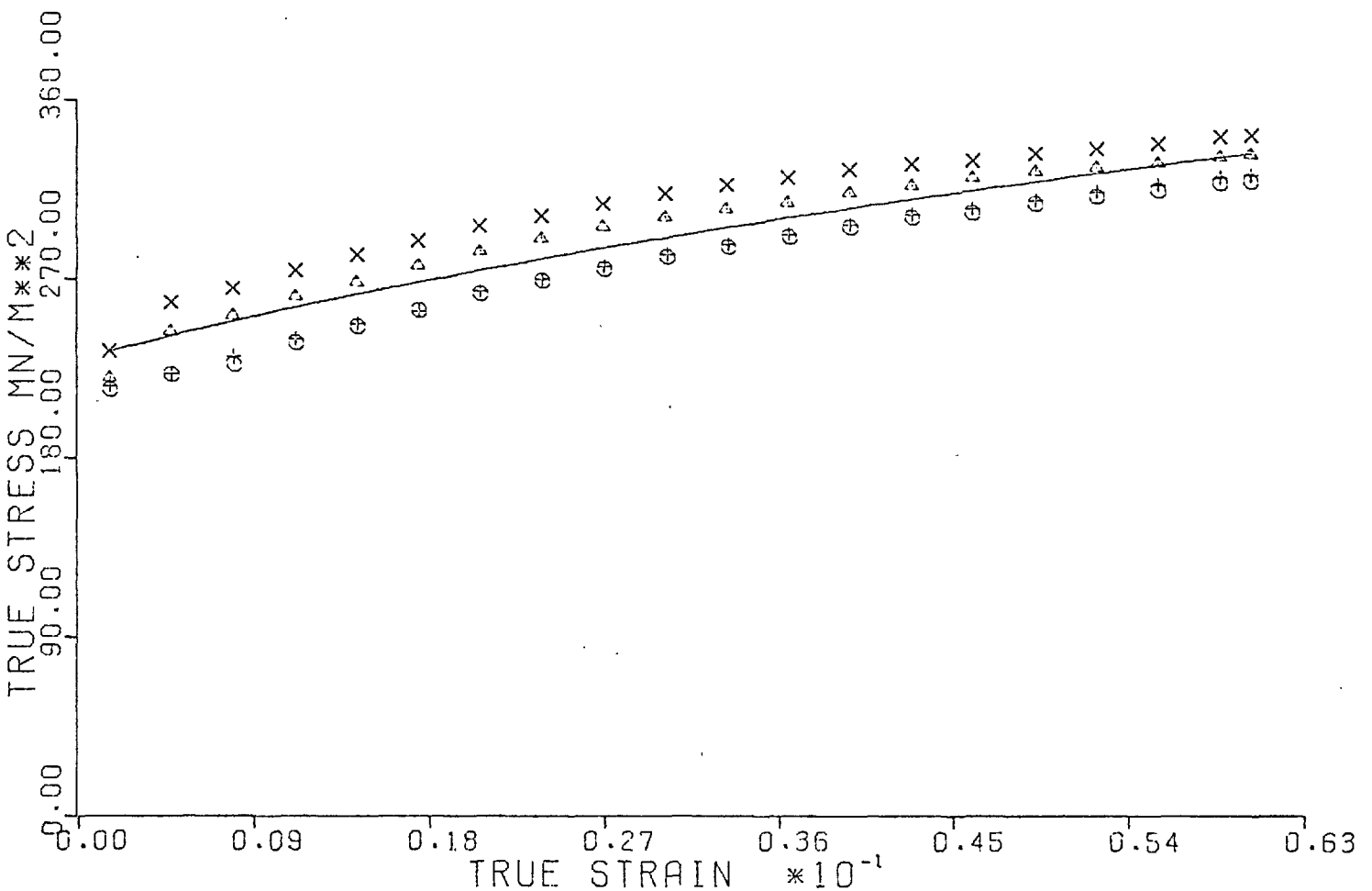
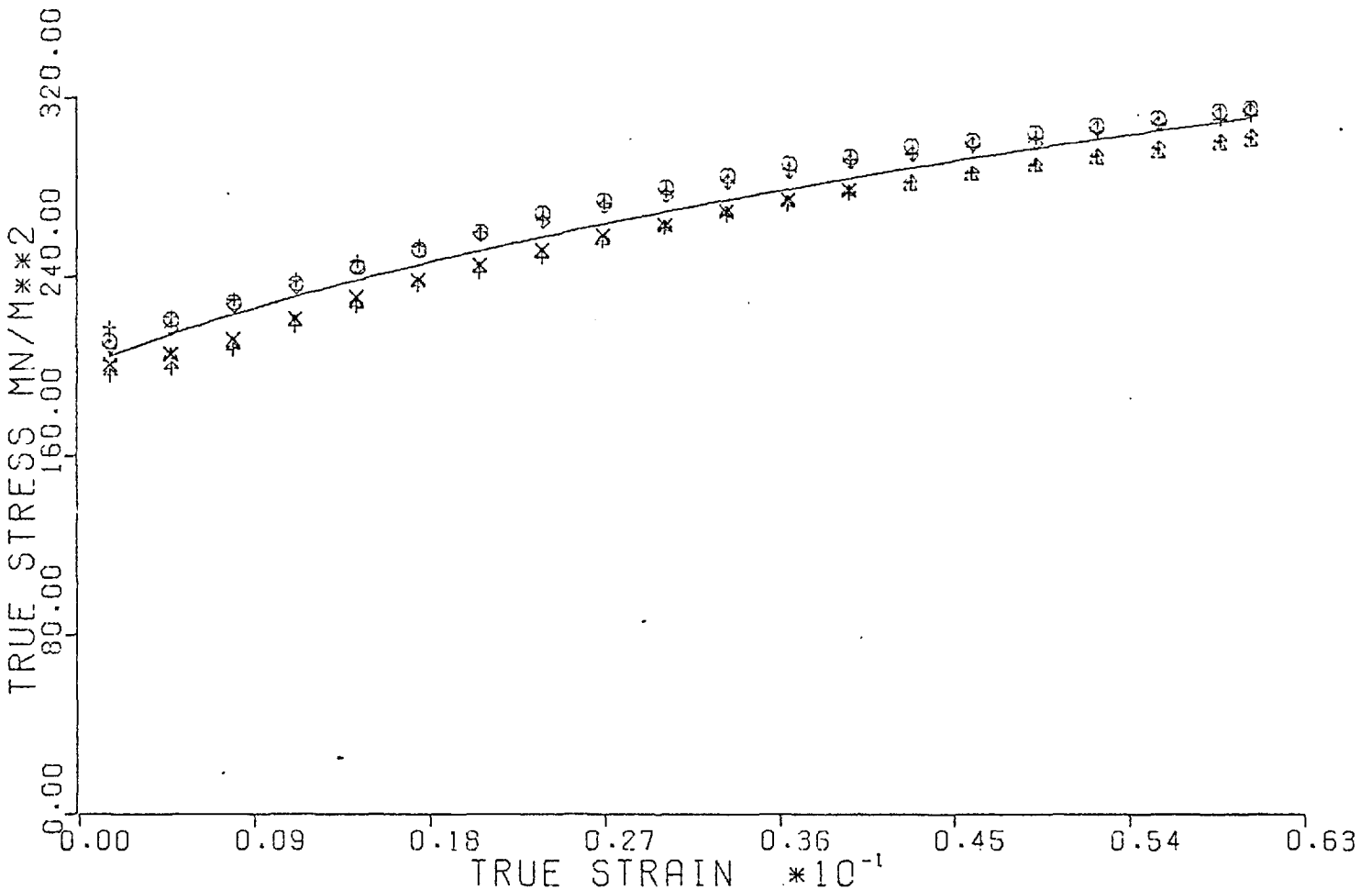


fig (4.18) EQUIVALENT STRESS/STRAIN RELATIONSHIP (SERIES 1)



fig(4.19) EQUIVALENT STRESS/STRAIN RELATIONSHIP (SERIES 2)

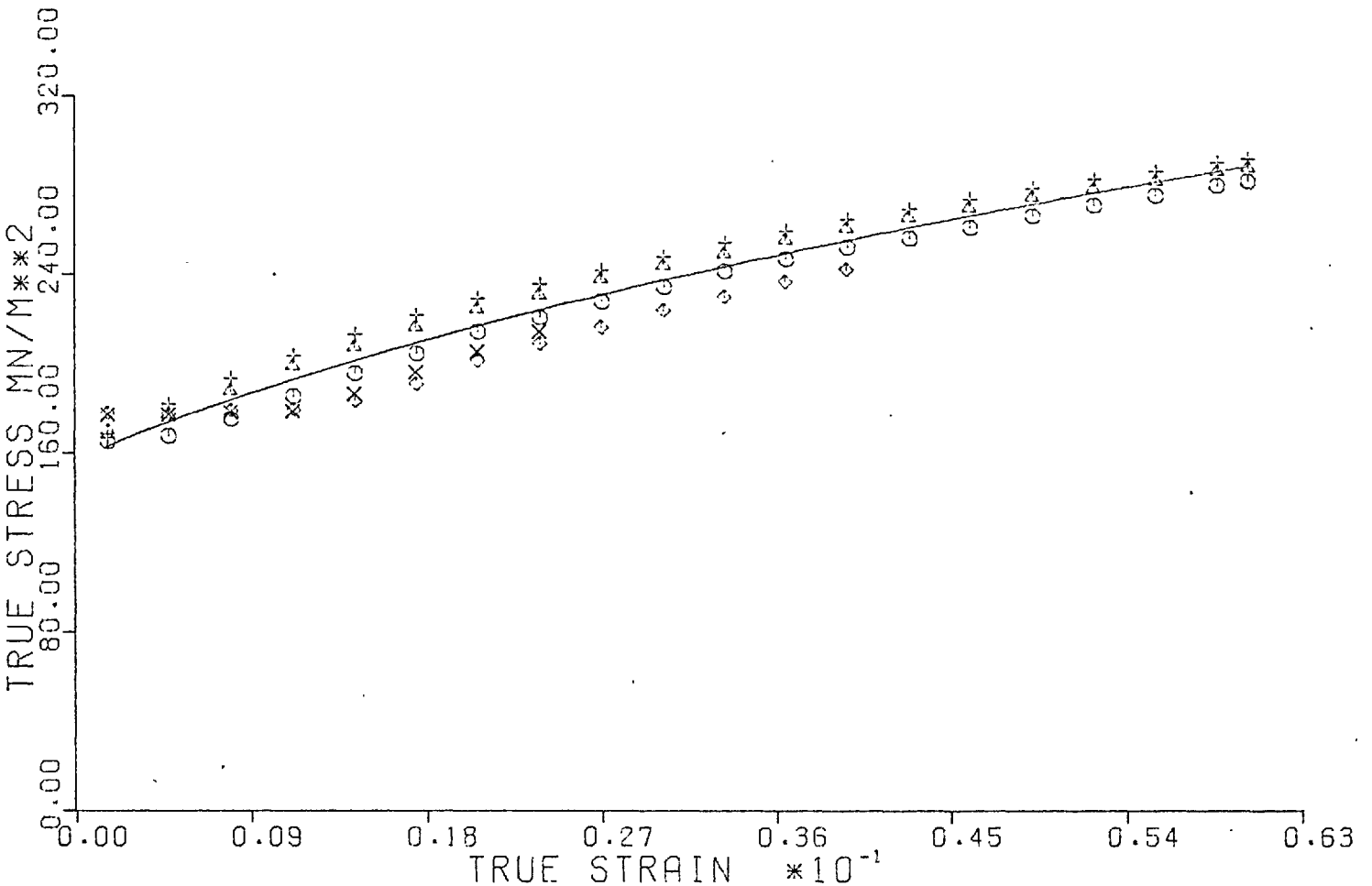


fig (4.20) EQUIVALENT STRESS/STRAIN RELATIONSHIP (SERIES 3)

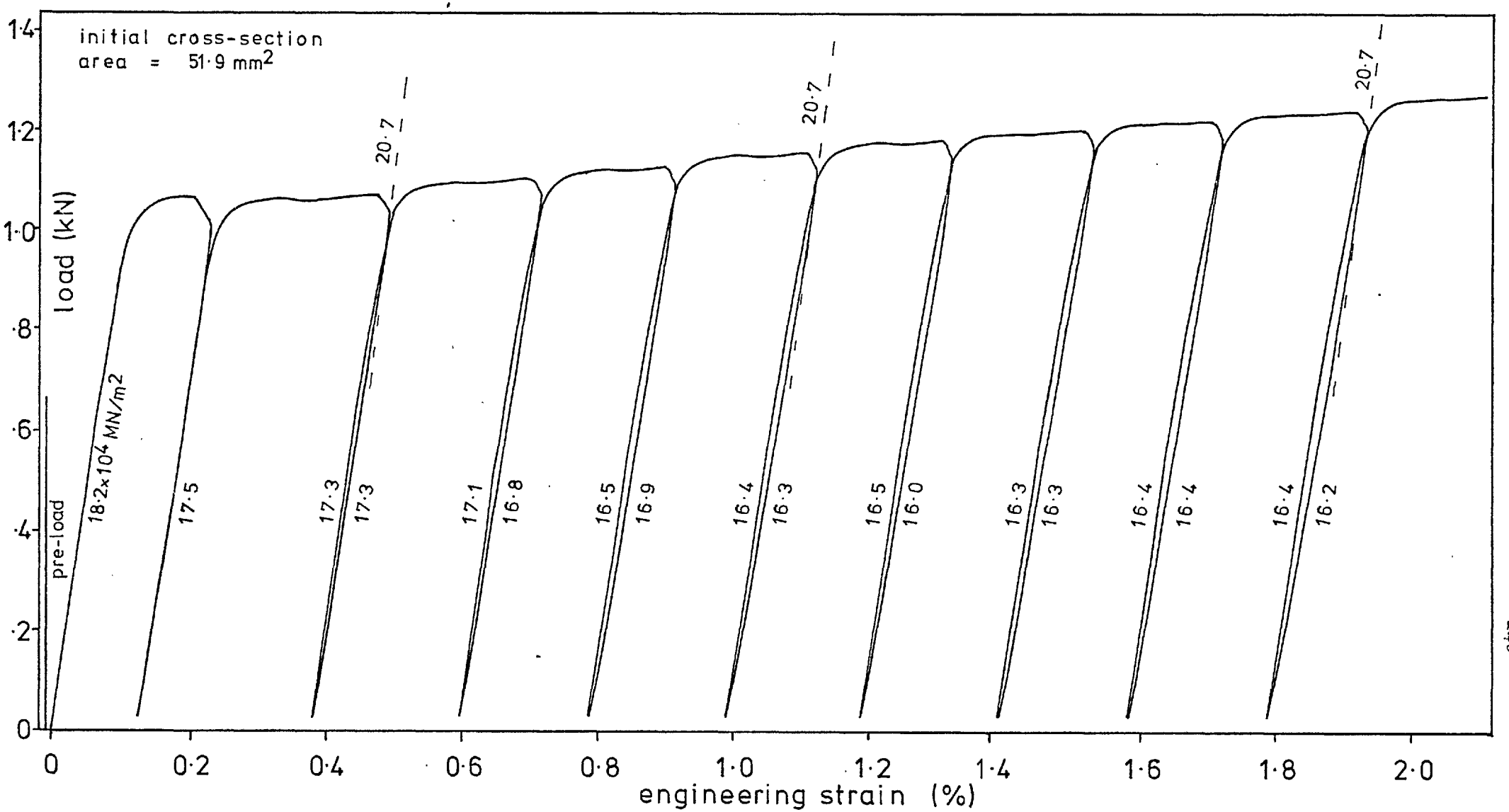


fig (4.21) MODULUS REDUCTION WITH STRAIN

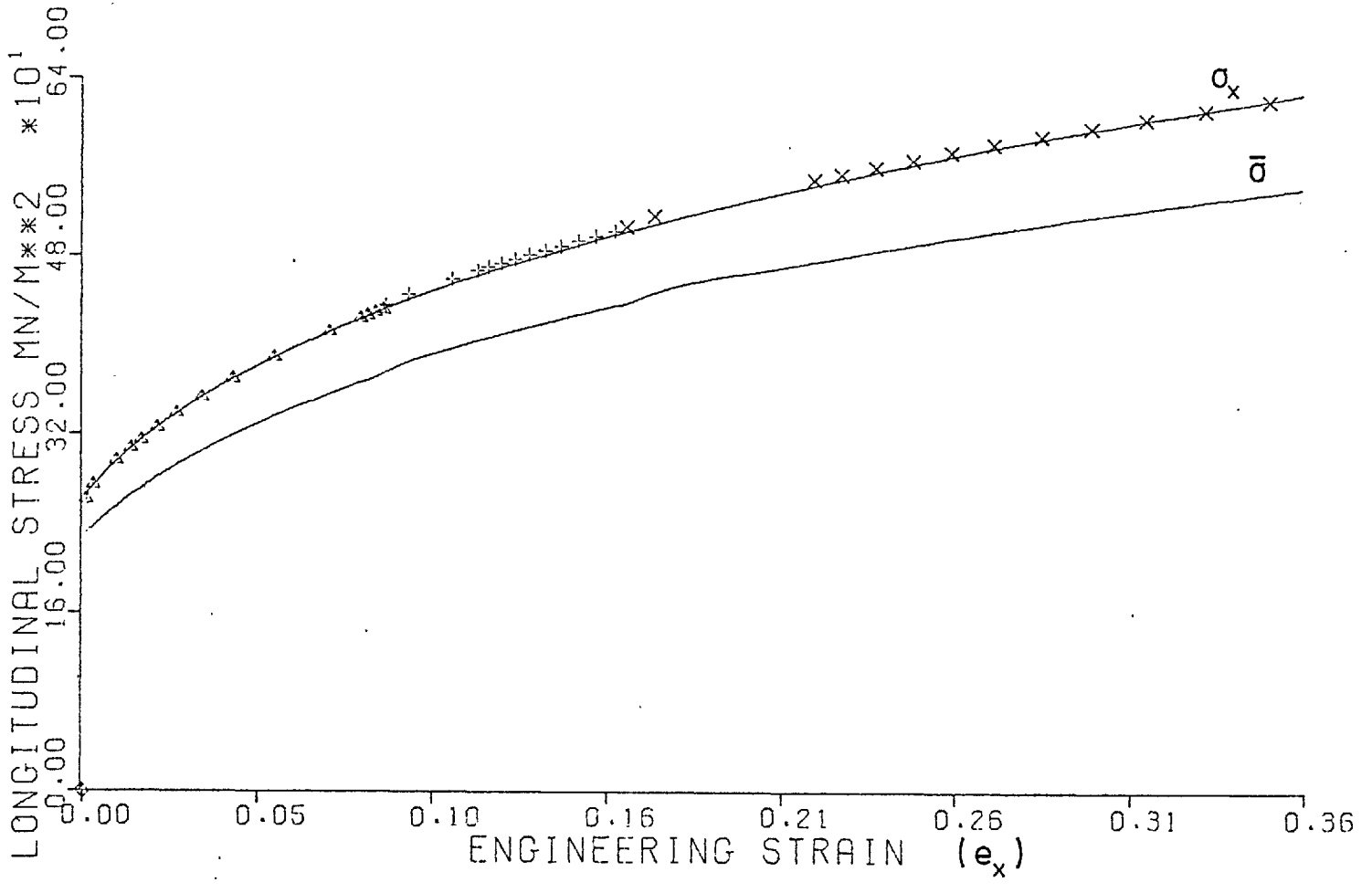


fig (4.22) LONGITUDINAL RELATIONSHIPS (SERIES 1 - ISOTROPIC)

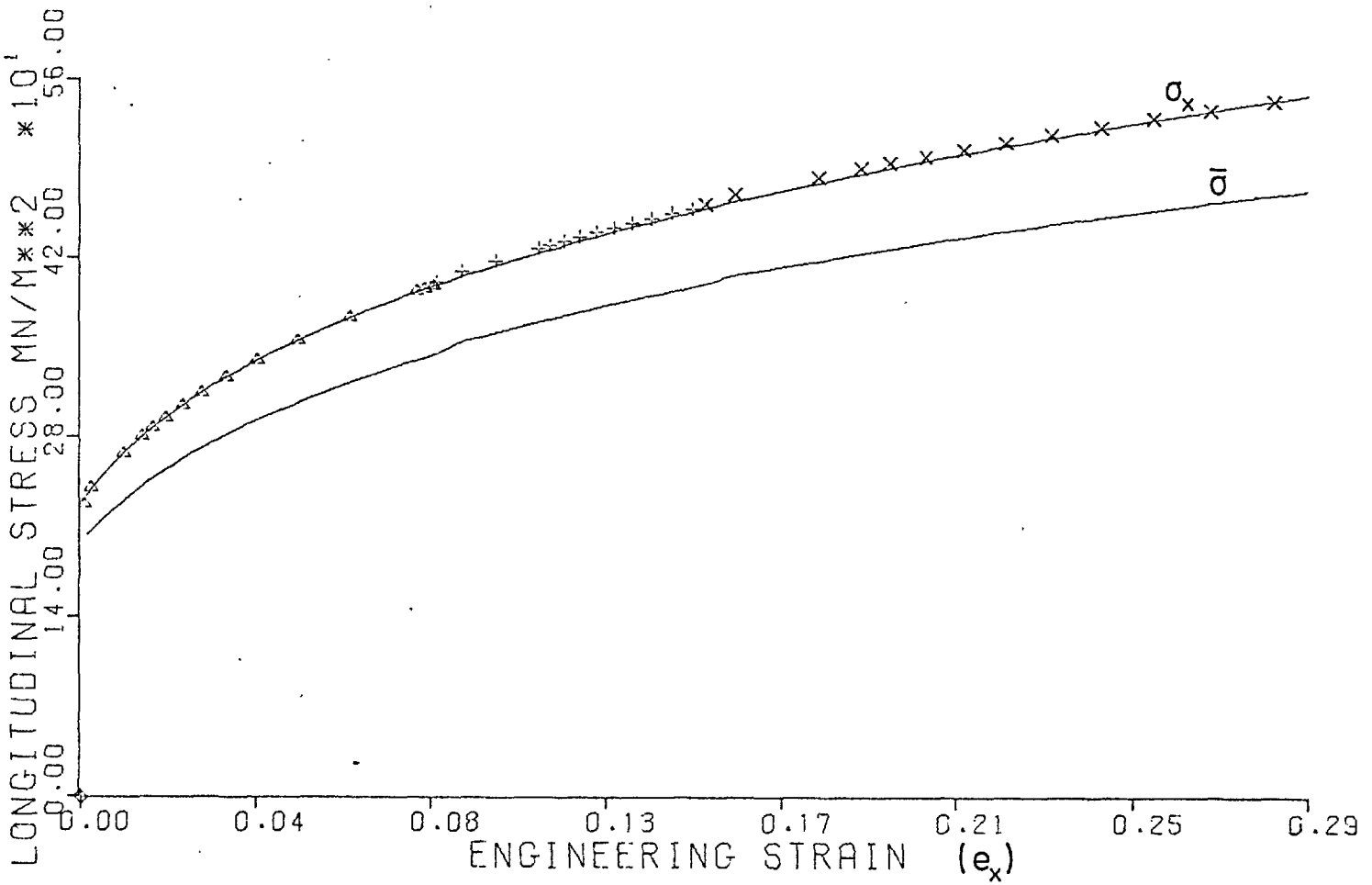


fig (4.23) LONGITUDINAL RELATIONSHIPS (SERIES 2 - ISOTROPIC)

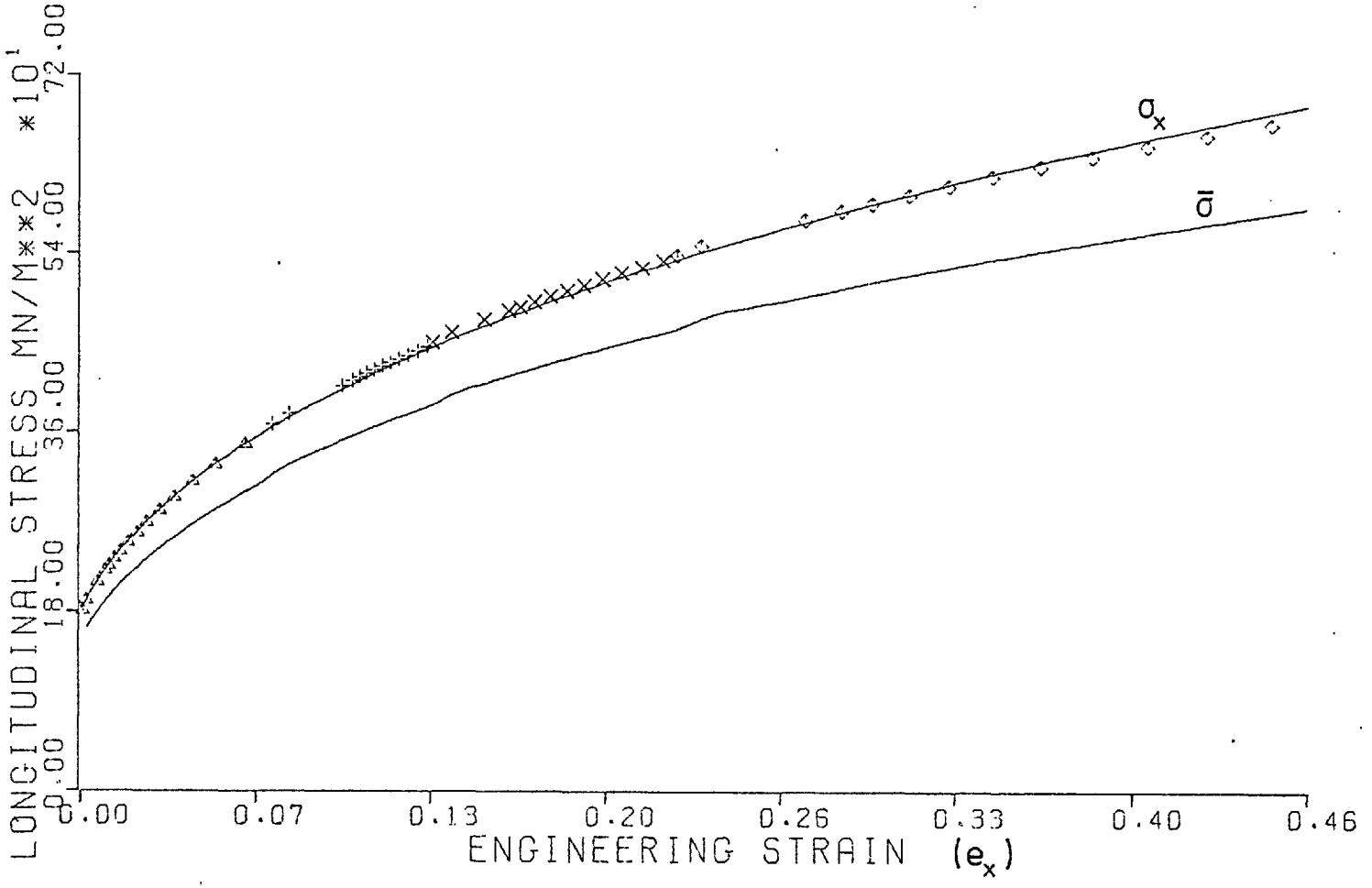


fig (4.24) LONGITUDINAL RELATIONSHIPS (SERIES 3 - ISOTROPIC)

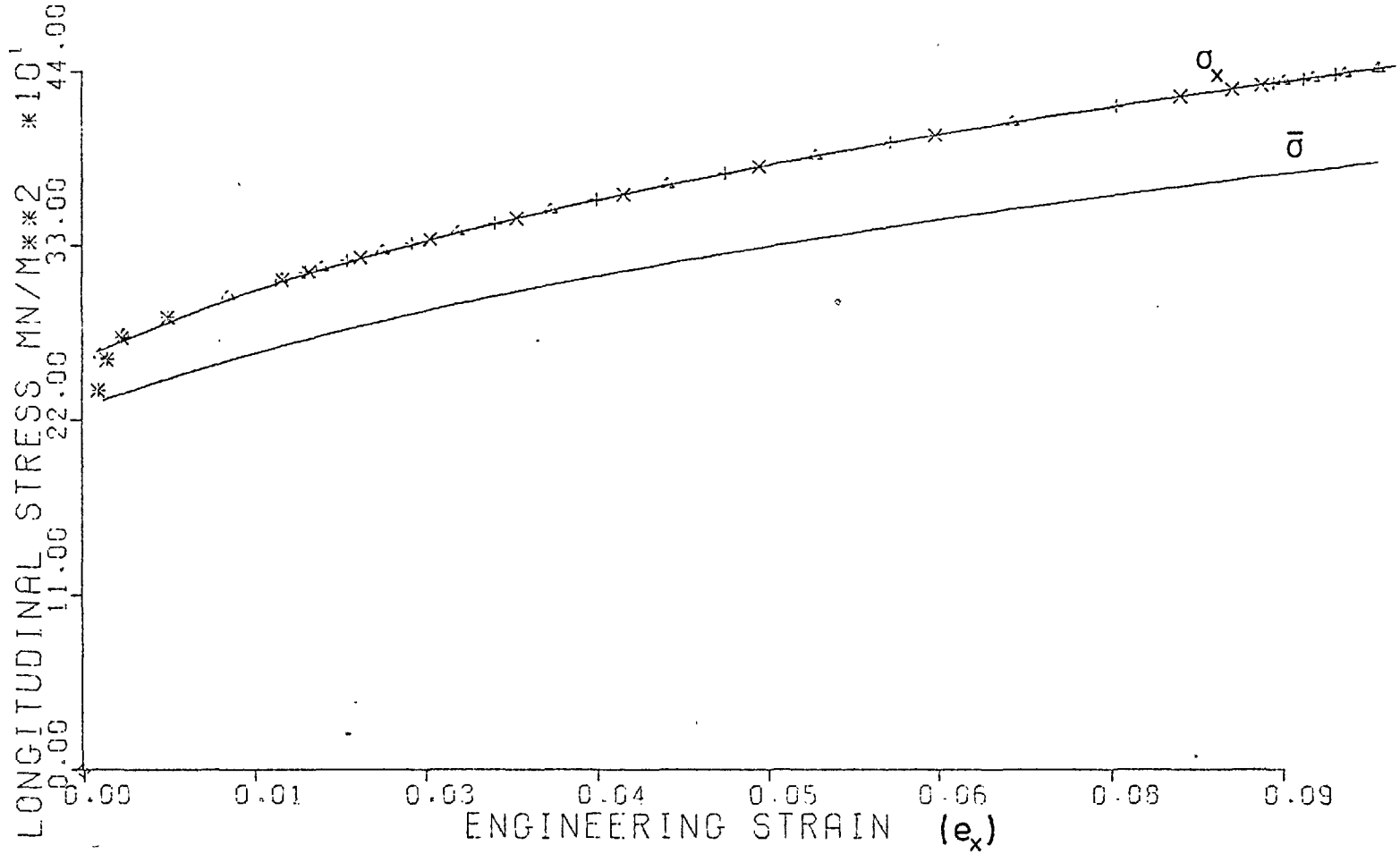


fig (4.25) LONGITUDINAL RELATIONSHIPS (SERIES 1 - KINEMATIC)

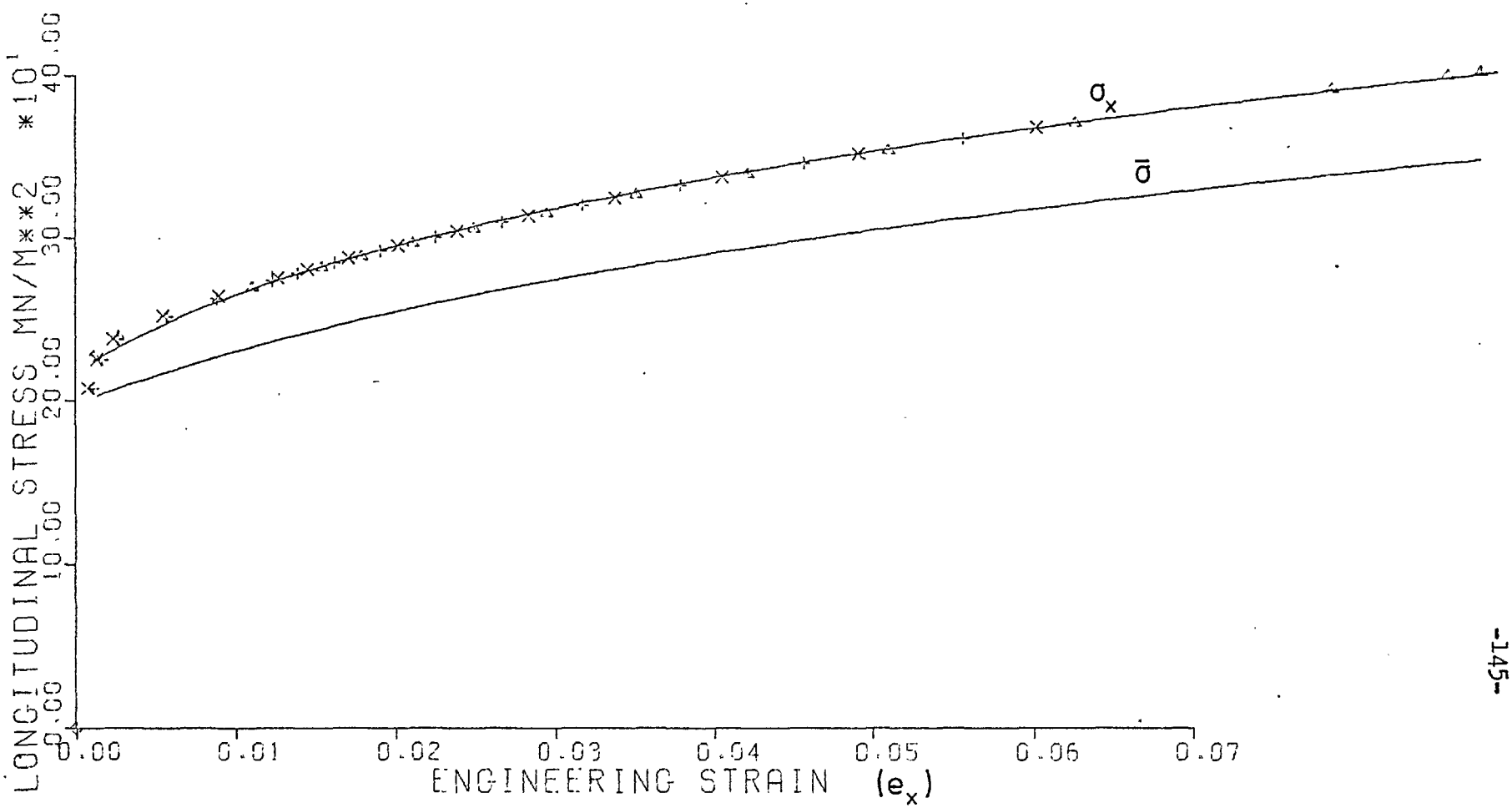


fig (4.26) LONGITUDINAL RELATIONSHIPS (SERIES 2 - KINEMATIC)

It was shown in Fig 3.13 that on load reversal a tail is formed in the longitudinal stress/strain relationships.The inclusion of points lying on this tail for curve-fitting purposes resulted in instabilities in program execution .It was therefore necessary to ignore these tails (represented by the first five points utilized for curve-fit purposes on the second and subsequent loading cycles) to obtain meaningful relationships.The positions of these tail-off regions occur at the discontinuities visible in the equivalent stress curves.The inclusion of the tail during initial loading ensures a representative equation.Tail-off regions are, of course, of no consequence in the cases where kinematic hardening is assumed.

An example computer output for the derivation of the longitudinal stress/strain relationships is to be found in Appendix 7.

4.4.2. Leveller Results.

It is convenient to include both experimental and theoretical results in this section, thus allowing direct comparison plots to be drawn from the outset.The important results to be considered are,

(1) Overall elongation vs.Mean applied load.

(2) Load losses.

(3) Incremental surface strains.

(4) Residual curvature at leveller exit.

Factors (1),(2) and (4) are independent of the sucess of any gauge results during a run.Even though adjustments to gauge results are required (to account for section modulus and bending displacement effects) the results presented in this section will indicate strain values actually recorded by the gauges.The discrepancies between these results and true surface strain readings must be discussed in the light of theoretical accuracy and will therefore remain unconsidered until Chapter 5.

Theoretical results (tabulated and plotted with experimental results), have been determined by use of the complete computer program with the relevant machine and material data.It will be noted that all computer plots are linear, this being due to (a) the nature of the theory, and (b) the use of a mean strip thickness in computer calculations.This latter factor results in small errors in theoretical/practical discrepancies where bending strains are considered, the magnitude of these errors (with the small strip thickness scatter obtained) is such that they can be ignored.It was found that the inclusion of initial strip curvature had negligible effect on computer results and this has consequently been ignored for the purposes of theoretical analysis.

It should be noted once more that all theoretical plots assume the attainment of the roll radius, irrespective of wrap angle and applied load. Example computer outputs for both a constant applied load assumption and a 'full' analysis are given in Appendix 7.

4.4.2.1. Series 1.

Process Parameters:

Strip Width = 171.5mm/strip thickness = 0.564mm (mean)

Initial strip curvature = 350 - 520mm radius.

D/t ratio = 45

Strip speed = 3 - 8m/minute.

Applied back loads = 6 - 57 kN/m width.

Machine Settings:

Roll No.	1	2	3	4	5
Diameter(mm)		25.4	25.4	25.4	
Interlock(mm)	-	0	-100	0	-
Nominal Wrap	-	25°	-50°	25°	-
Geometric Wrap	-	26°	-52°	26°	-

Relevant theoretical and practical results are given in tables 2&3(Appendix8) Figure 4.2.7 shows a plot of measured overall strains against the mean applied tensile load.Superimposed on this graph are various theoretical lines, representing theoretical derivations.Figure 4.2.8 shows the magnitudes of machine load losses as a function of applied back tension. Relevant theoretical curves are again superimposed.Figs 4.29/30/31 show the relevant strain gauge results in graphical form.

4.4.2.2. Series 2.

Parameters:

Strip Width = 171.5mm/strip thickness = 0.32mm.

Initial curvature = 415 - 478mm.

D/t ratio = 98.5.

Strip speed = 3m/min.

Applied back loads = 2 - 63 kN/m width.

Machine Settings: Series 2A.

Roll No.	1	2	3	4	5
Roll Diameter	-	31.5	31.5	31.5	-
Interlock	-	0	-178	0	-
Nominal Wrap	-	39.5	-79.0	39.5	-
Geometric Wrap	-	41.1	-82.2	41.1.	-

Series 2B.

Roll No.	1	2	3	4	5
Roll Diameter	-	31.5	31.5	31.5	-
Interlock	-	0	-127	0	-
Nominal Wrap	-	30.5	-61.0	30.5	-
Geometric Wrap	-	31.6	-63.2	31.6	-

Series 2C.

Roll No.	1	2	3	4	5
Roll Diameter	-	31.5	31.5	31.5	-
Interlock	-	0	-51	0	-
Nominal Wrap	-	13.25	-26.5	13.25	-
Geometric Wrap	-	13.50	-27.0	13.50	-

Relevant results are again tabulated in Appendix 8 (tables 4&5), these results being plotted in Figs 4.32 to 4.41.

4.4.2.3. Series 3.

Process Parameters:

Strip Width = 171.5mm/strip thickness = 0.90mm.

Initial strip curvature = 260-350 mm.

D/t ratios = 56.5,38.8,21.2.

Strip speed = 3m/min.

Applied back loads = 2 - 72kN/m width.

Machine Settings:

Series 3A.

Roll No.	1	2	3	4	5
Roll Diameter	50.8	50.8	50.8	50.8	-
Interlock	-76.2	0	-127.0	0	-
Nominal Wrap	-25.3	50.0	-61.0	30.5	-
Geometric Wrap	-26.0	52.5	-64.5	32.25	-

Series 3B.

Roll No.	1	2	3	4	5
Roll Diameter	34.9	34.9	34.9	34.9	-
Interlock	-76.2	0	-127.0	0	-
Nominal Wrap	-25.3	50.0	-61.0	30.5	-
Geometric Wrap	-25.8	51.6	-63.3	31.7	-

Series 3C.

Roll No.	1	2	3	4	5
Roll Diameter	19.1	19.1	19.1	19.1	-
Interlock	-76.2	0	-127.0	0	-
Nominal Wrap	-25.3	50.0	-61.0	30.5	-
Geometric Wrap	-25.5	50.8	-62.2	31.1	-

Results are tabulated in Appendix 8 (tables 6 & 7).Since no strain gauge measurements were taken during this series, only two comparison plots can be drawn up.Figure 4.42 shows the elongation/mean load relationships together with various theoretical derivations.Figure 4.43 indicates the relative magnitudes of load losses experienced during this series of experiments.

4.4.3. Modulus Effects.

The discrepancies found between gauge and vernier measurements of overall elongation (Figs 5.15 and 5.16) necessitated comparison experiments under simple tensile conditions.This was performed for the series 2 material since the % increase in section modulus due to gauge attachment is greatest with the thinnest material.Direct comparison between gauge strain and extensometer strain was obtained by attaching gauges of standard tensile specimens and comparing the respective outputs.Since three types of gauge had been used and selotaping technique varied slightly over the experimental period, a total of eighteen tests were made, six for each gauge type.Two tests were performed with (a) no selotape, (b) one layer, (c) two layers.

The results from these tests, shown in Fig 4.44, have allowed multiplication factors to be obtained for the correction of gauge strains under tensile conditions.It can be seen from Fig 4.44 that the magnitude of this factor - although constant for any installation over the strain range - is dependent upon gauge type and attachment technique, increasing the amount of selotape naturally increasing the section modulus.

4.4.4. Residual Curvature.

Figure 4.45 shows plots of residual strip curvatures at leveller exit for series 1 and 3A, the corresponding theoretical predictions being superimposed.The large discrepancies are self-evident, but are to be expected since slight over straightening is required to bring the strip back to its mean path.The tabulated data show similar discrepancies for all other experiments.

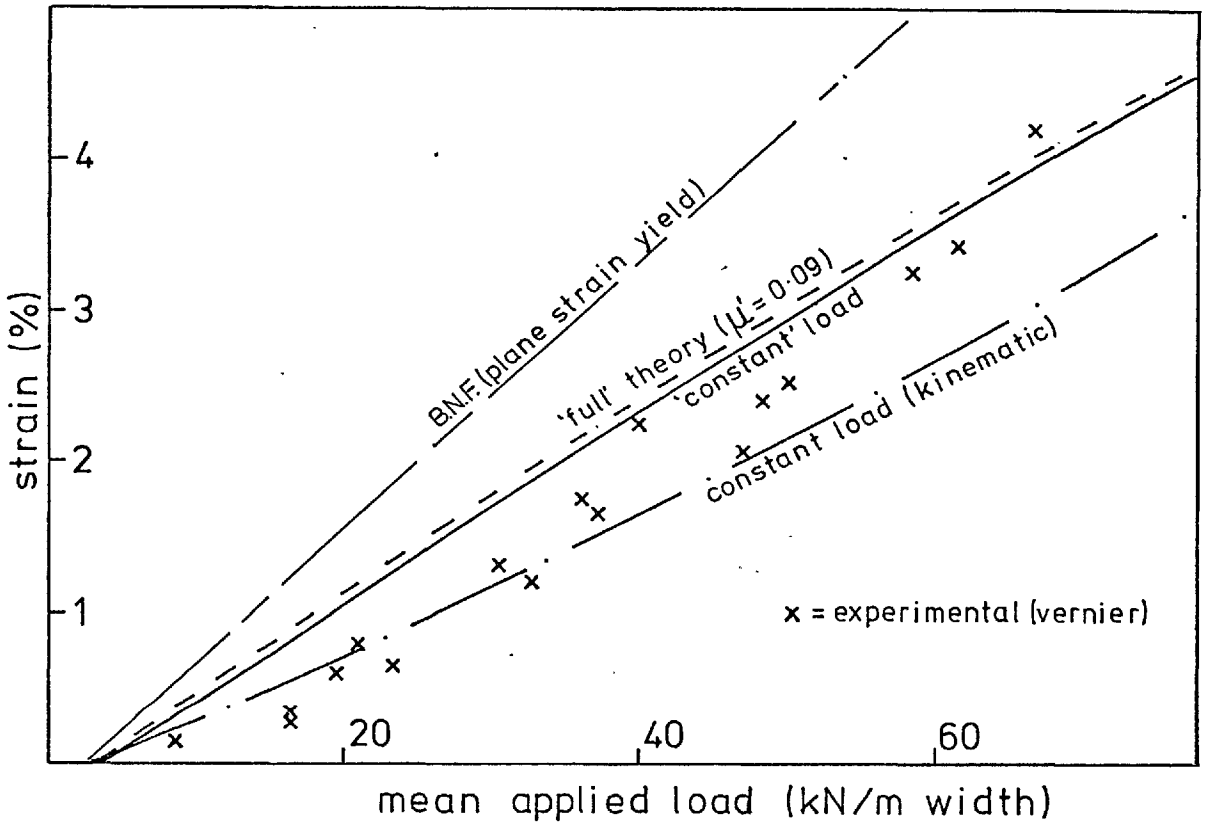


fig (4.27) SERIES 1 - OVERALL STRAIN vs. APPLIED LOAD

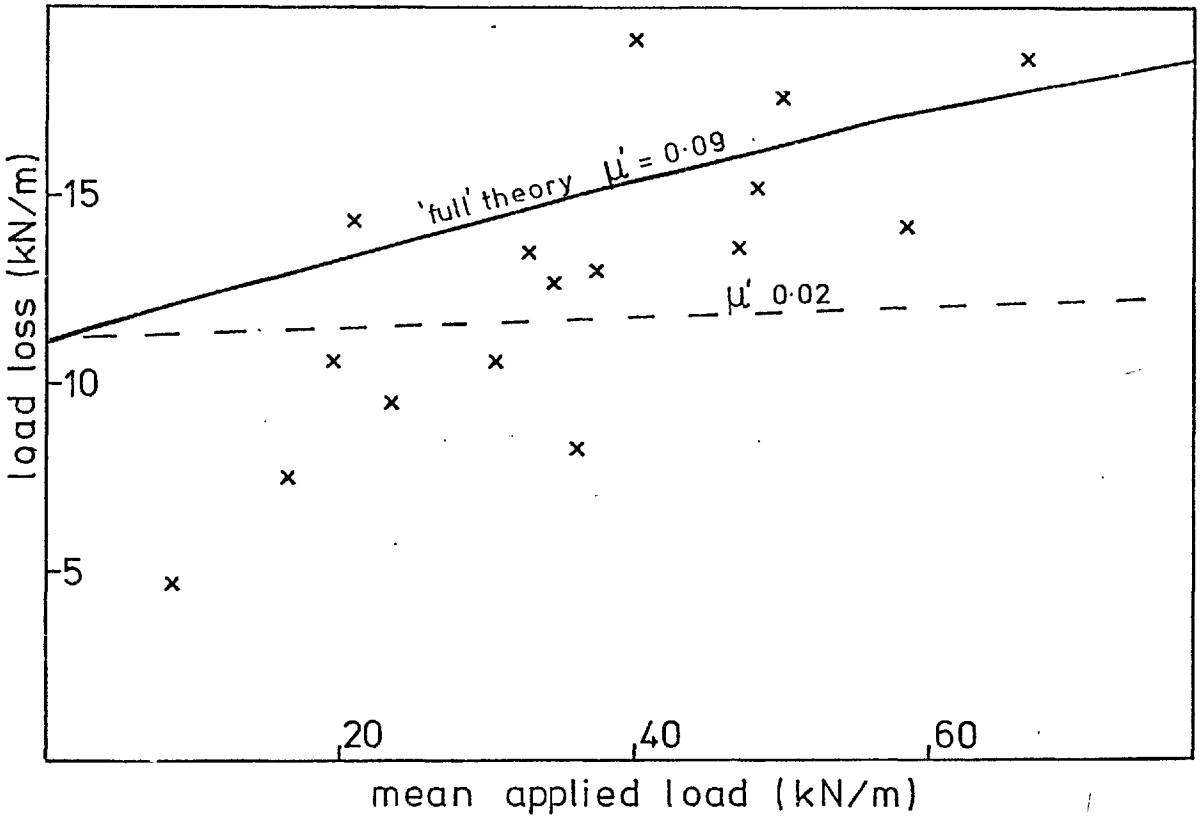
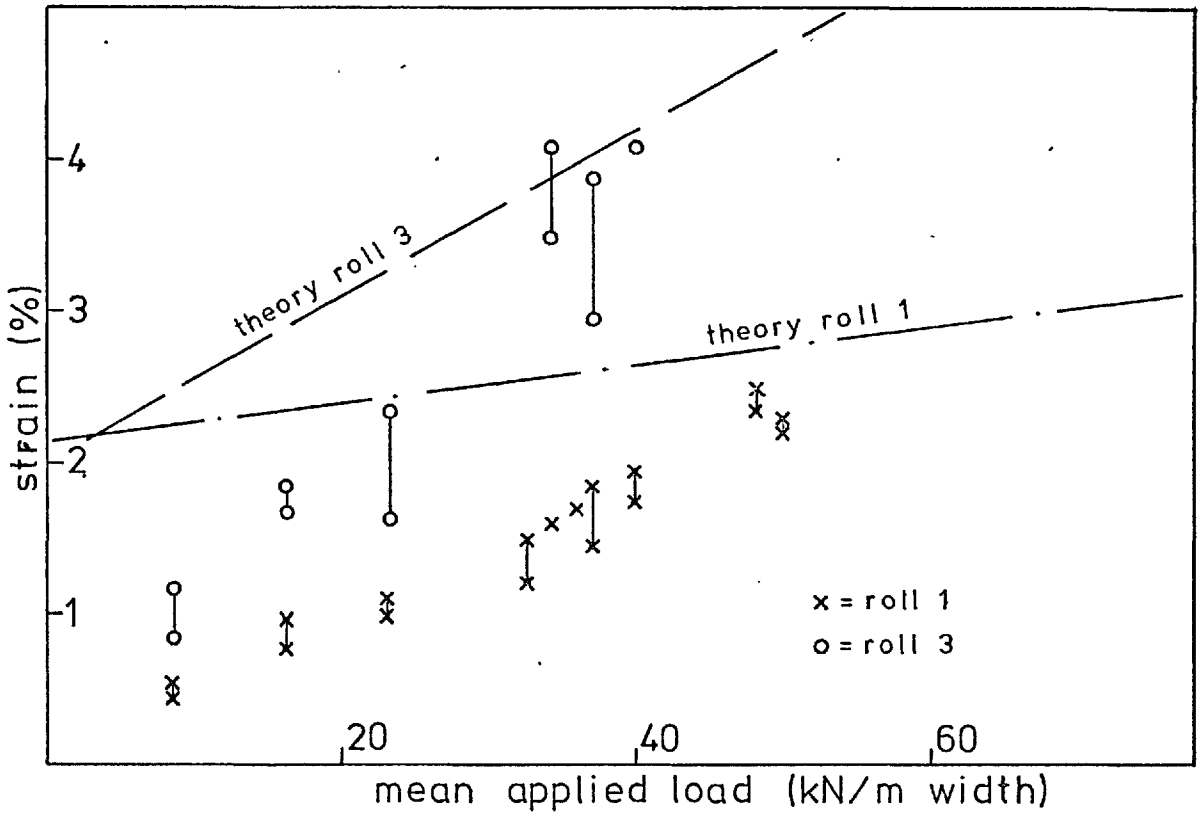


fig (4.28) SERIES 1 - LOAD LOSS vs. APPLIED LOAD



fig(4.29) SERIES 1 - GAUGE READINGS / ROLLS 1 & 3 DURING BENDING

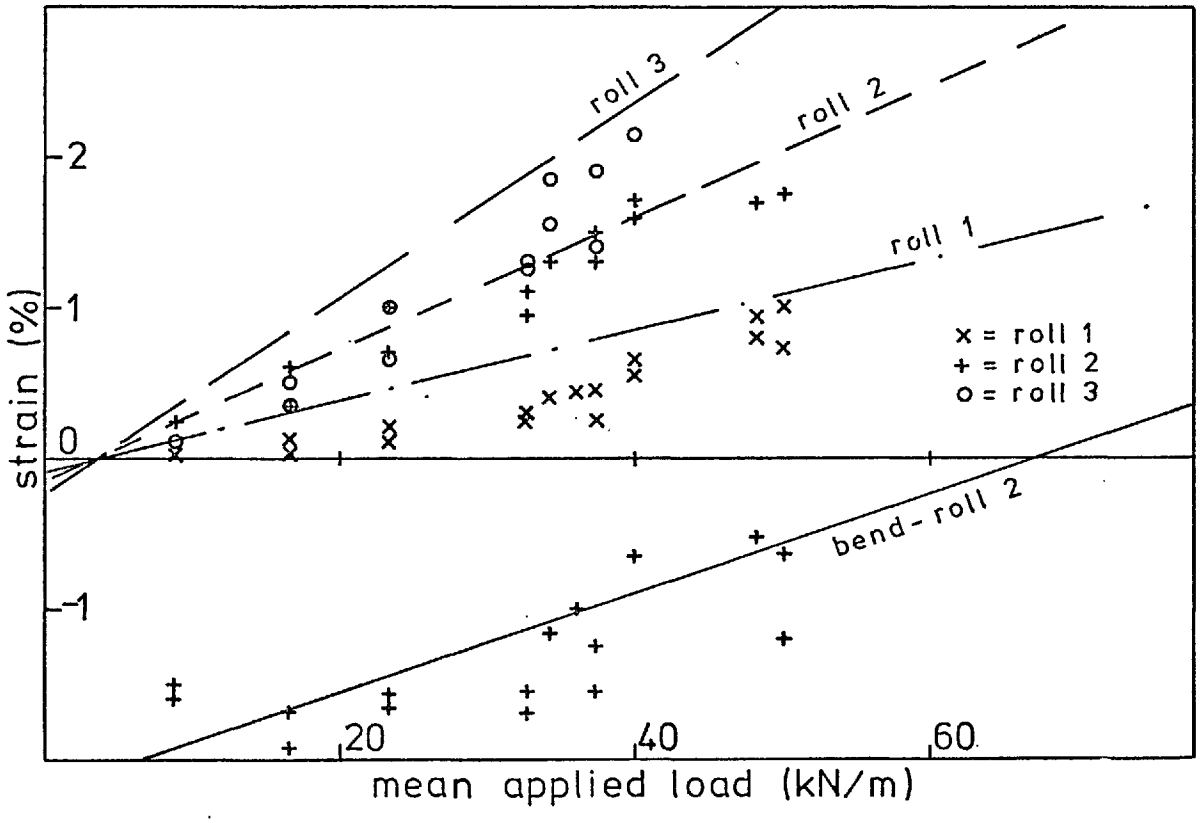


fig (4.30) SERIES 1 - GAUGE READINGS AFTER STRAIGHTENING & ROLL 2 DURING BENDING

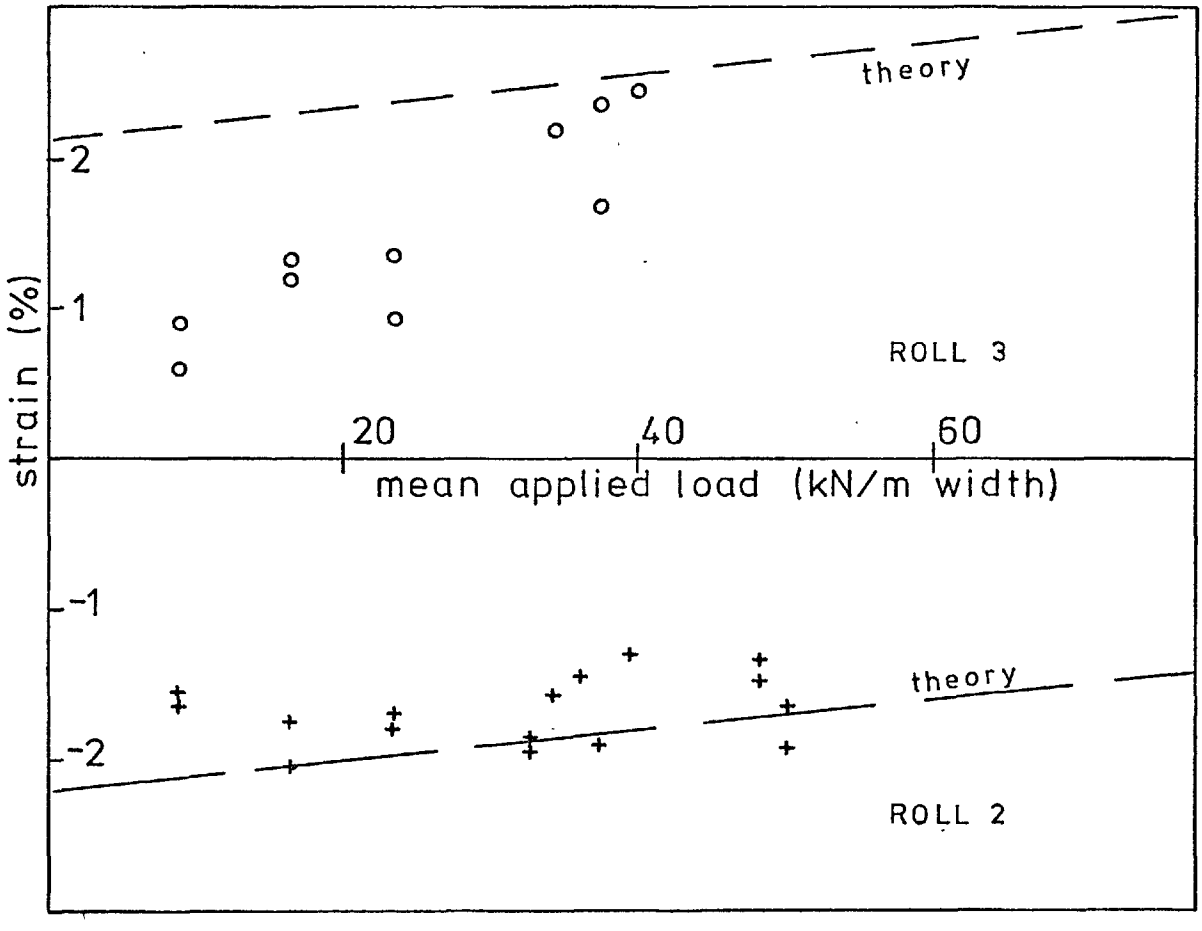
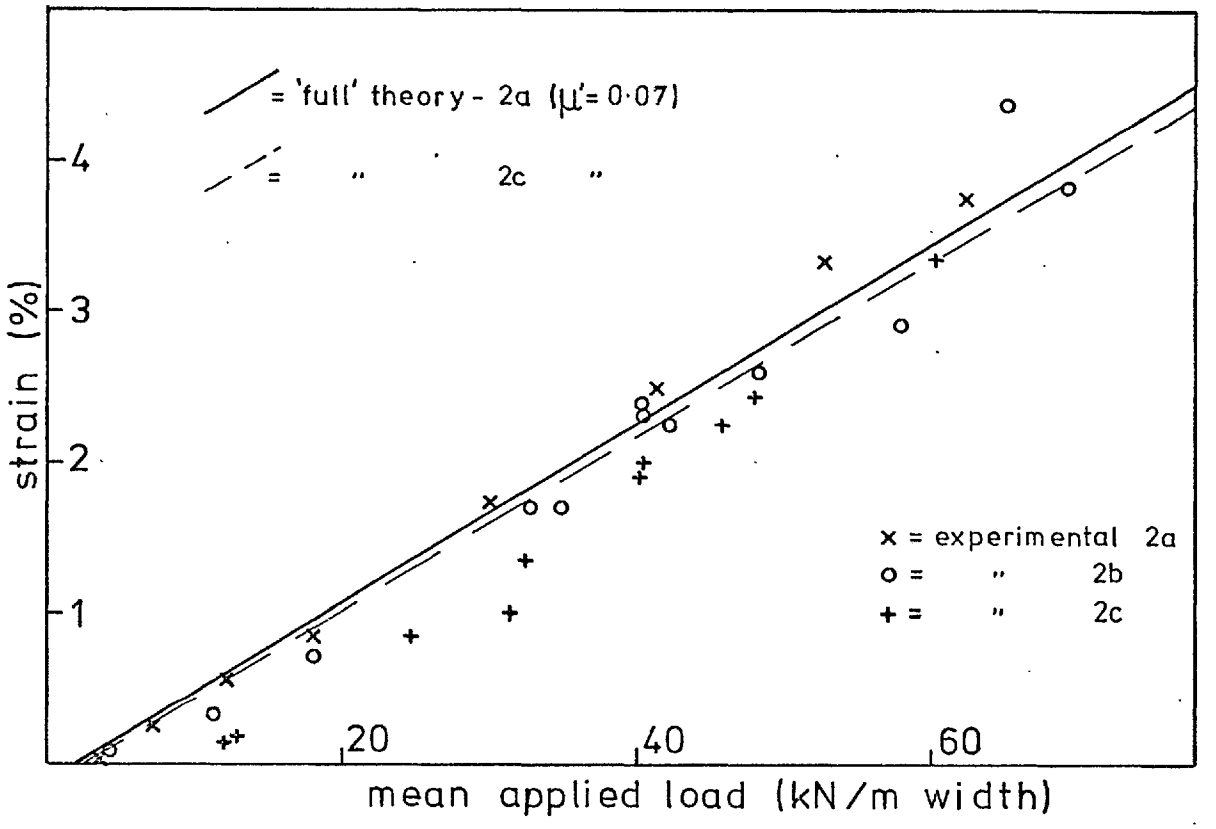
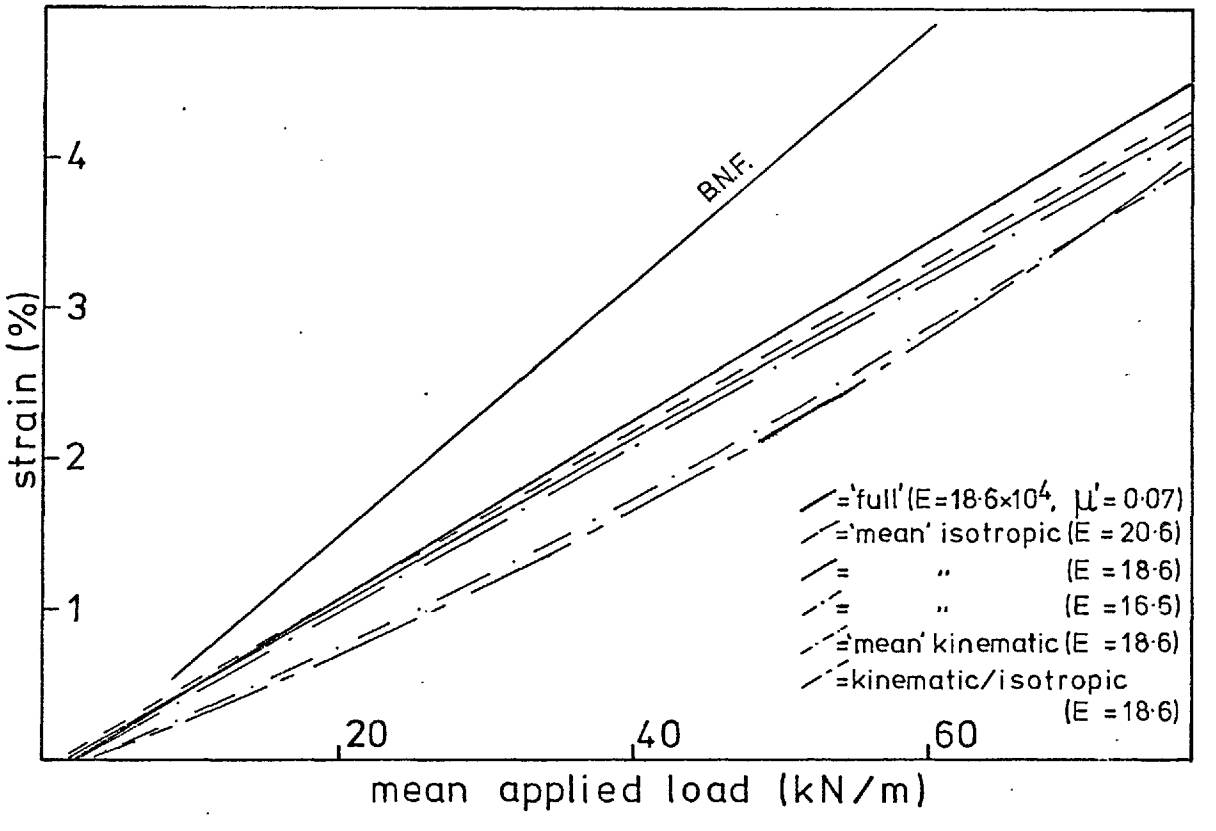


fig (4.31) SERIES 1 - INCREMENTAL GAUGE STRAINS ON BENDING OVER ROLLS 2 & 3



fig(4.32) SERIES 2 - OVERALL EXTENSION vs. APPLIED LOAD (vernier measurements)



fig(4.33) SERIES 2 - THEORY COMPARISON (2a)

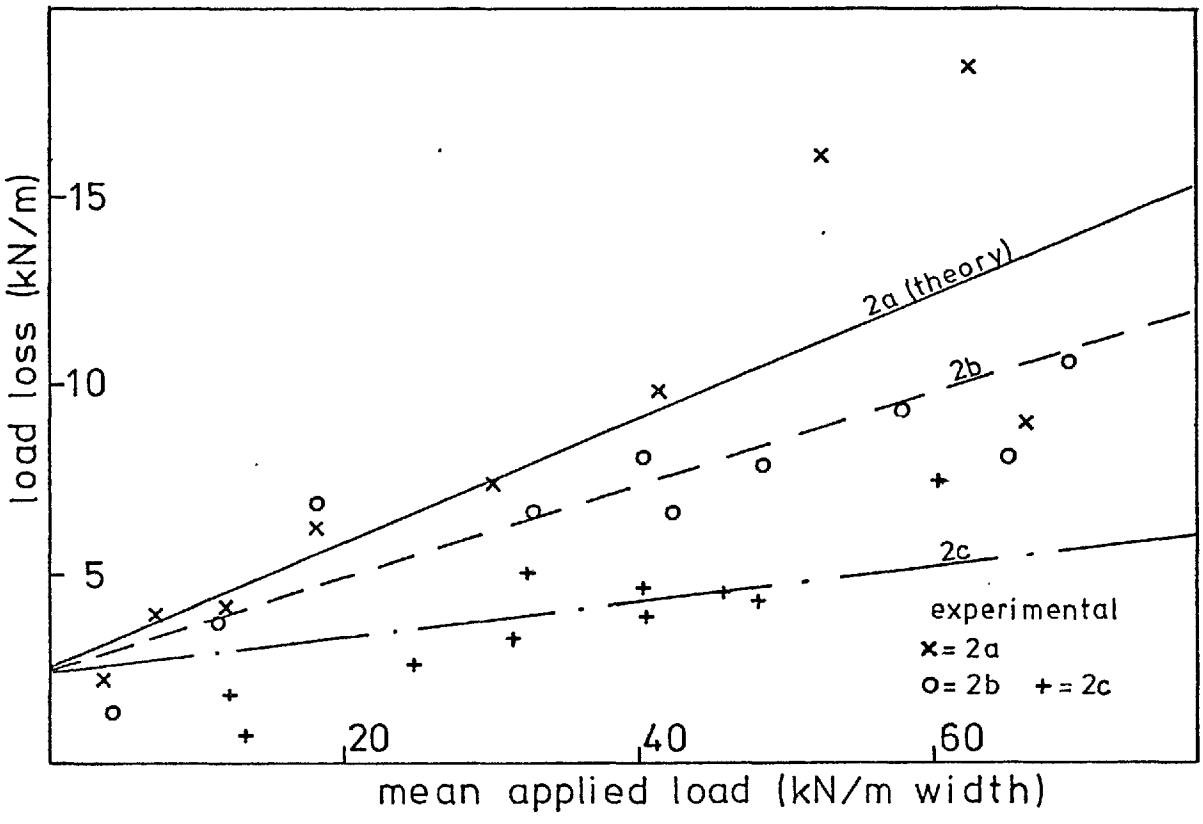


fig (4.34) SERIES 2 - LOAD LOSSES

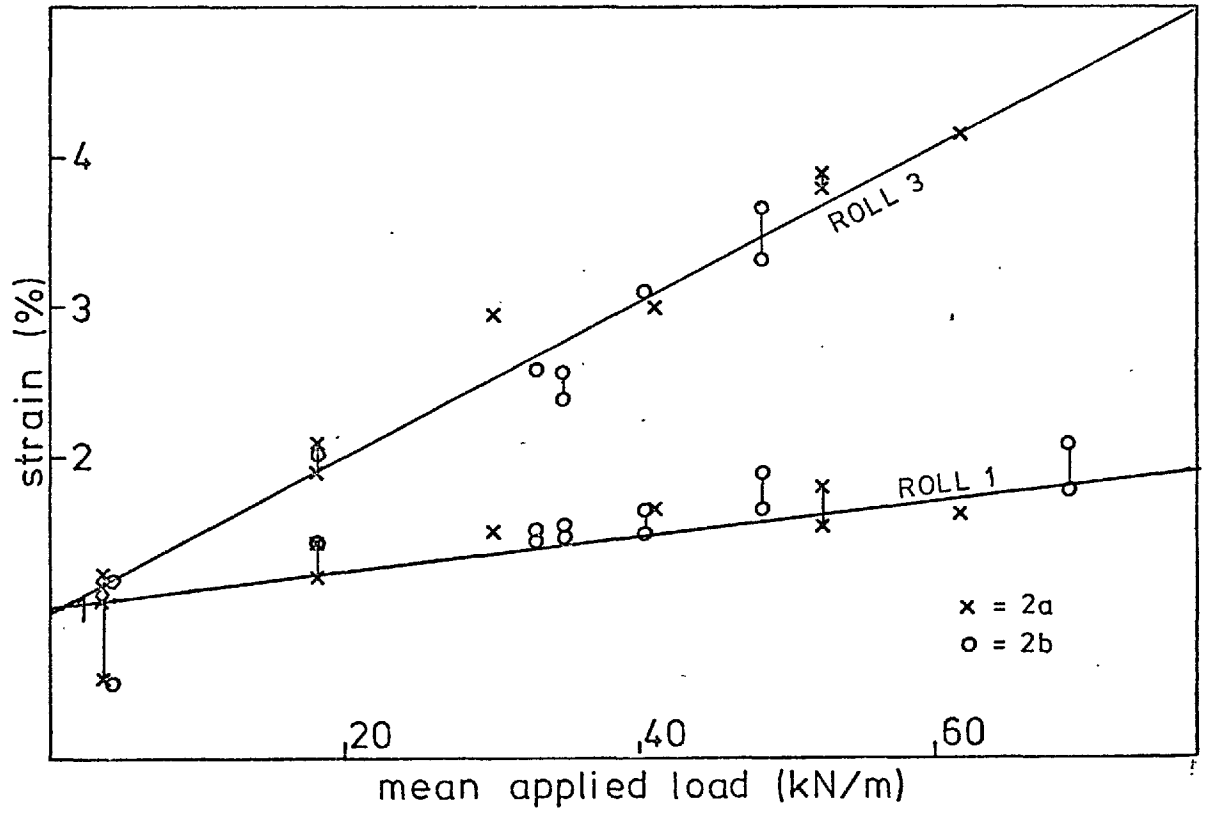


fig (4.35) SERIES 2a/2b - TOTAL GAUGE STRAINS DURING BENDING (ROLLS 1&3)

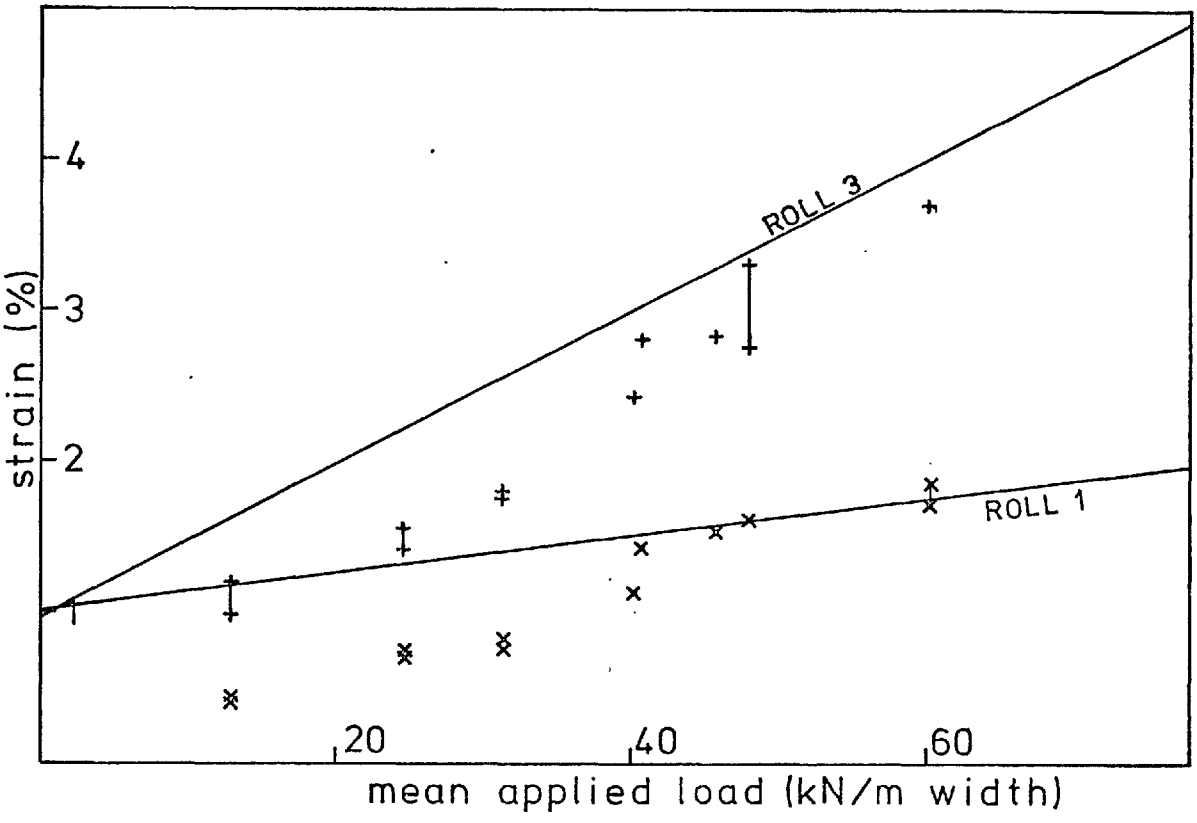


fig (4.36) SERIES 2c - TOTAL GAUGE STRAINS DURING BENDING (ROLLS 1 & 3)

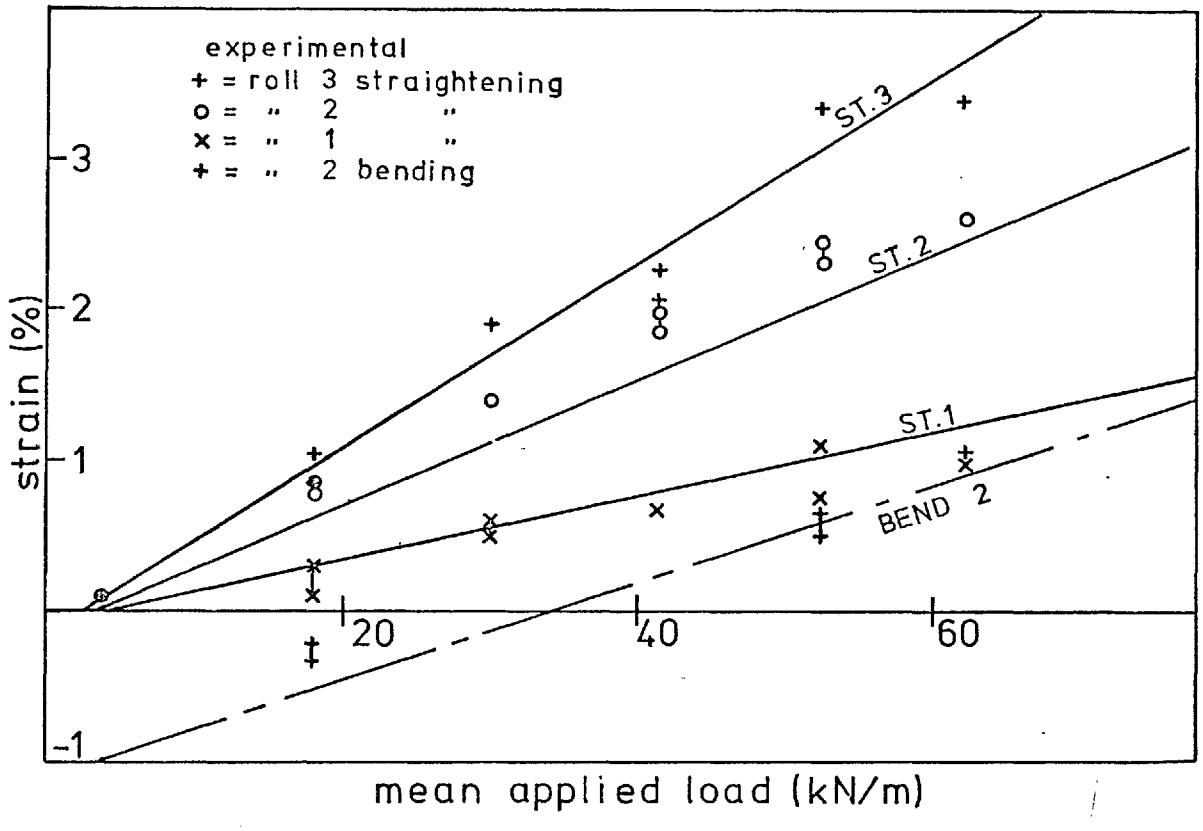
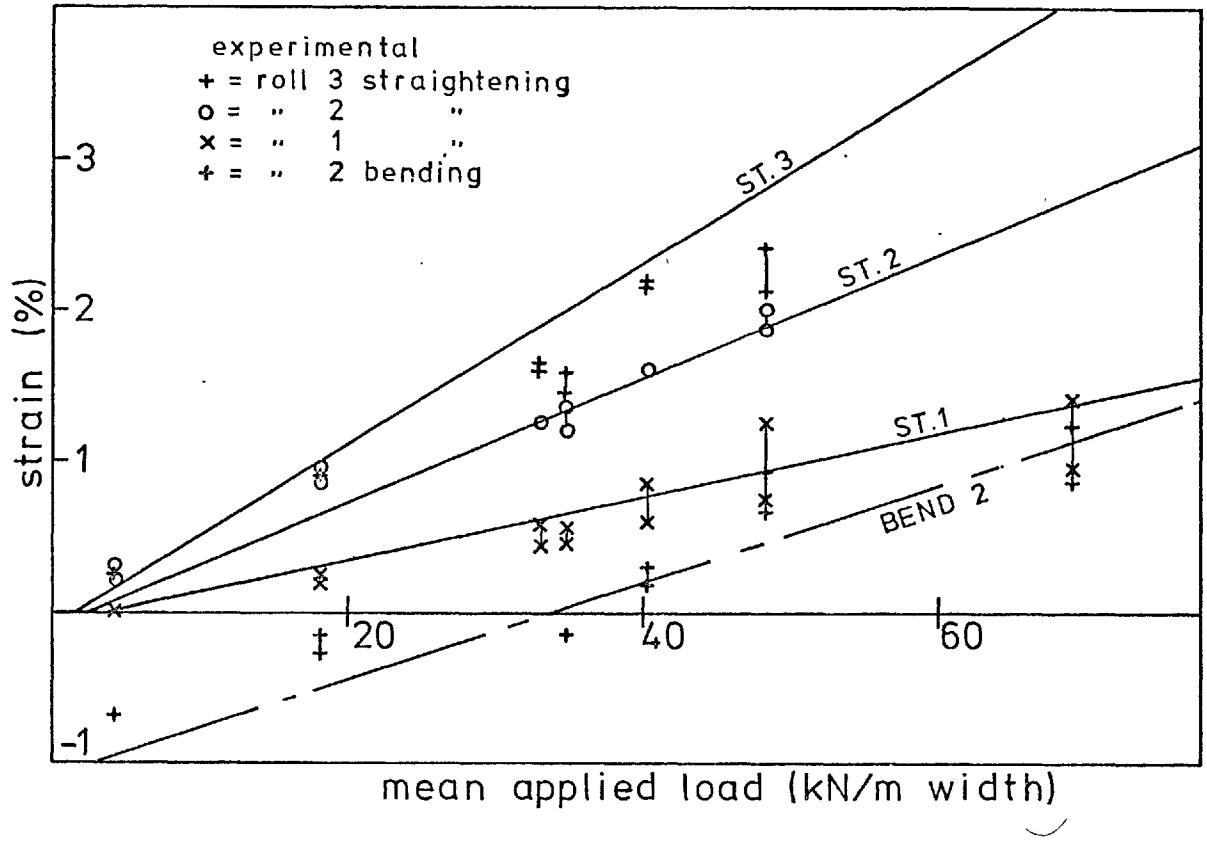
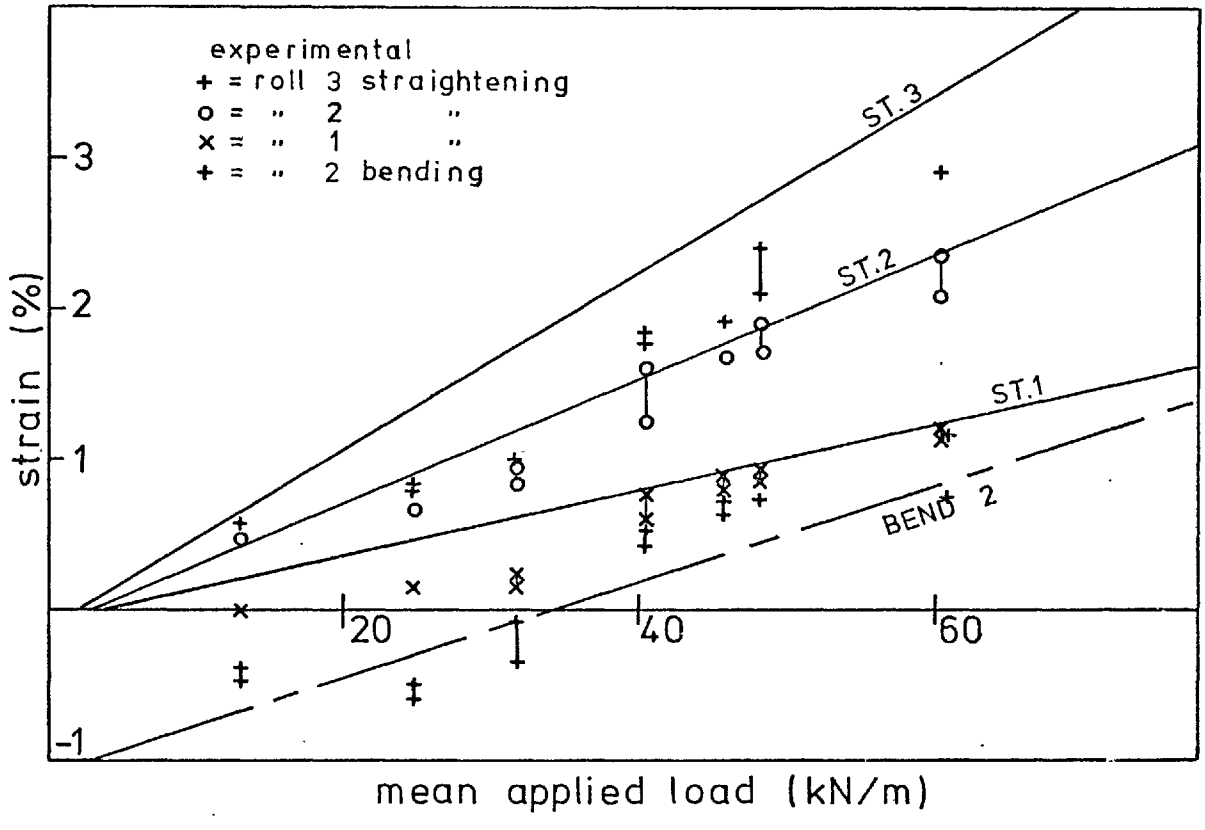


fig (4.37) SERIES 2a - TOTAL GAUGE STRAINS AFTER STRAIGHTENING & ROLL 2 DURING BENDING



fig(4.38) SERIES 2b - TOTAL GAUGE STRAINS AFTER STRAIGHTENING & ROLL 2 DURING BENDING



fig(4.39) SERIES 2c - TOTAL GAUGE STRAINS AFTER STRAIGHTENING & ROLL 2 DURING BENDING

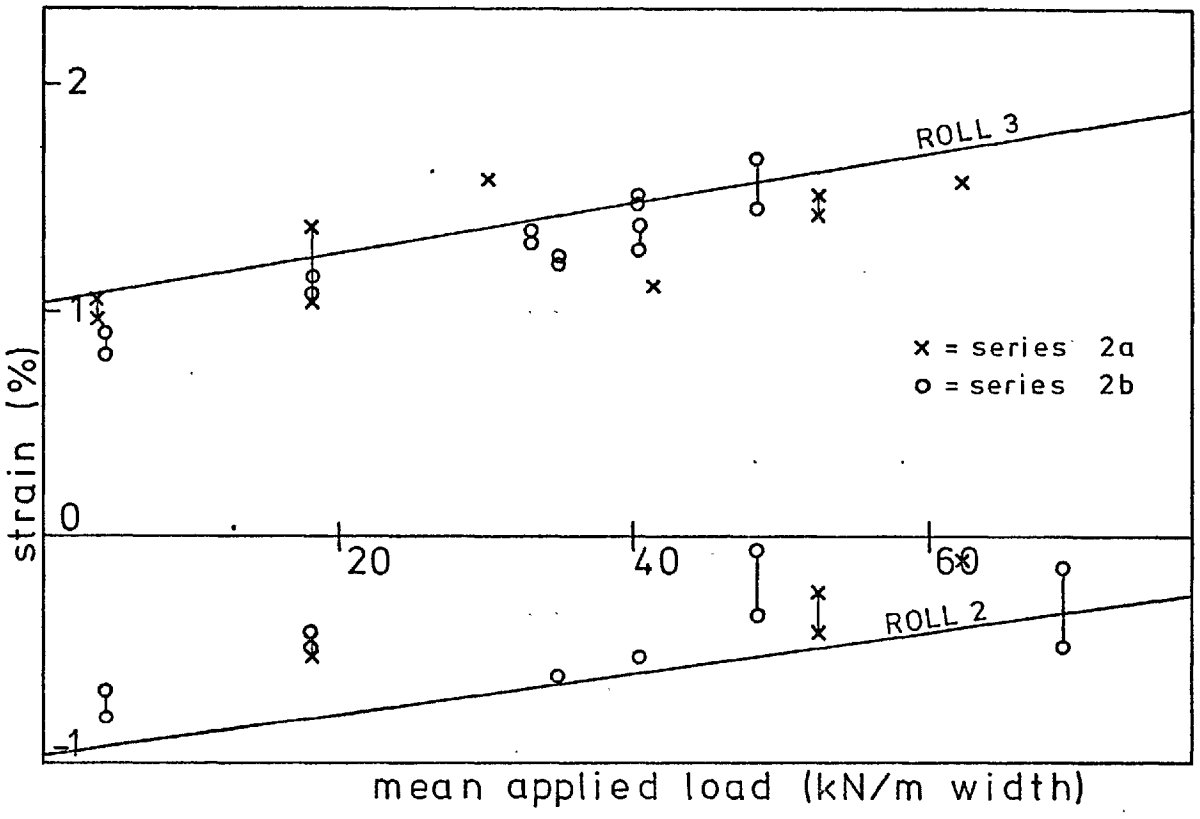


fig (4.40) SERIES 2a/2b - INCREMENTAL GAUGE STRAINS DURING 2nd & 3rd BENDS

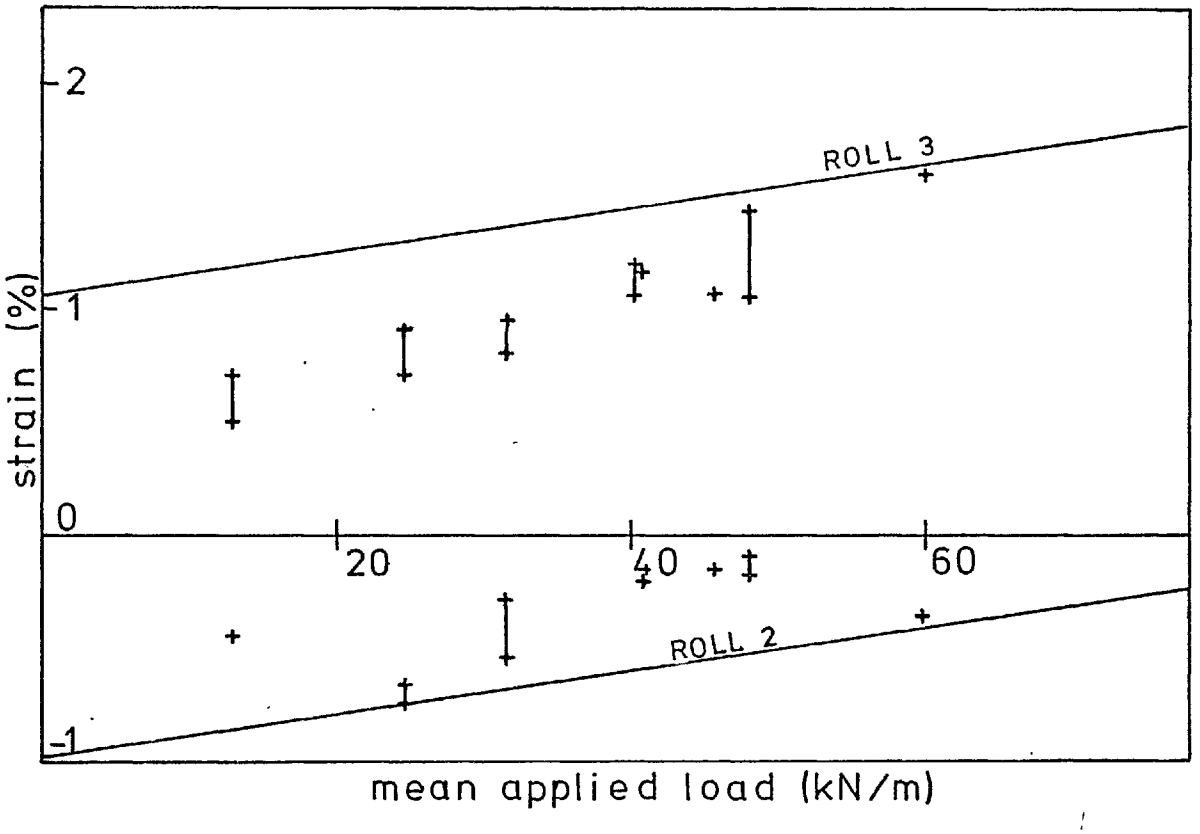


fig (4.41) SERIES 2c - INCREMENTAL GAUGE STRAINS DURING 2nd & 3rd BENDS

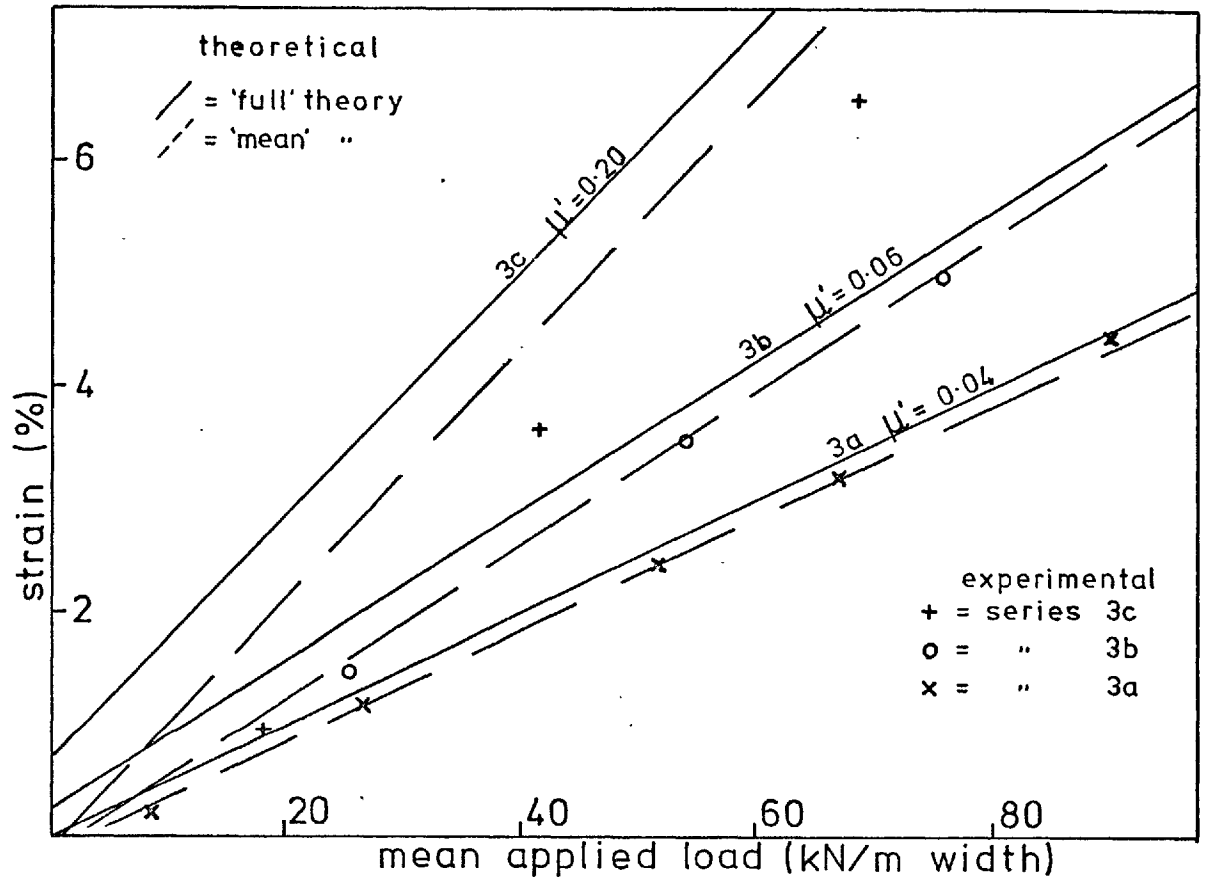


fig (4.42) SERIES 3 - OVERALL EXTENSION vs. APPLIED LOAD

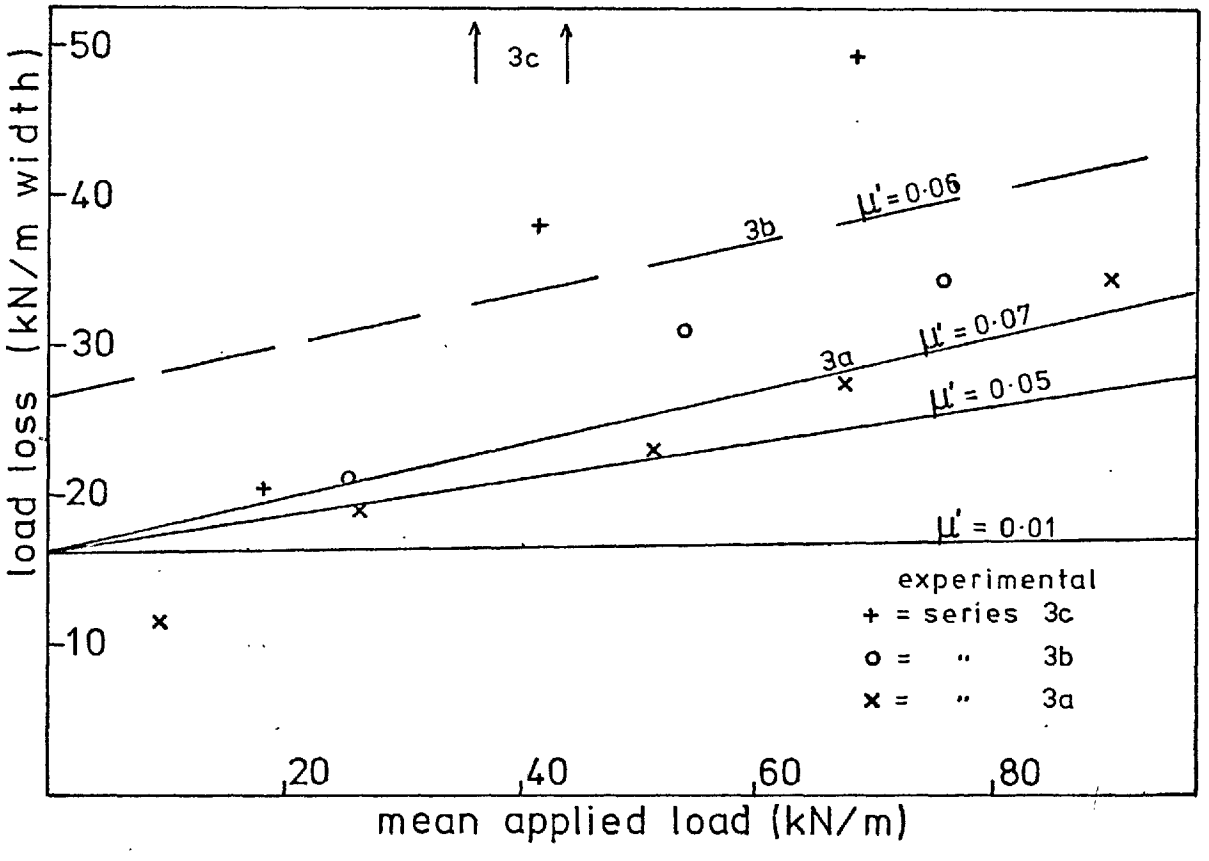


fig (4.43) SERIES 3 - LOAD LOSSES

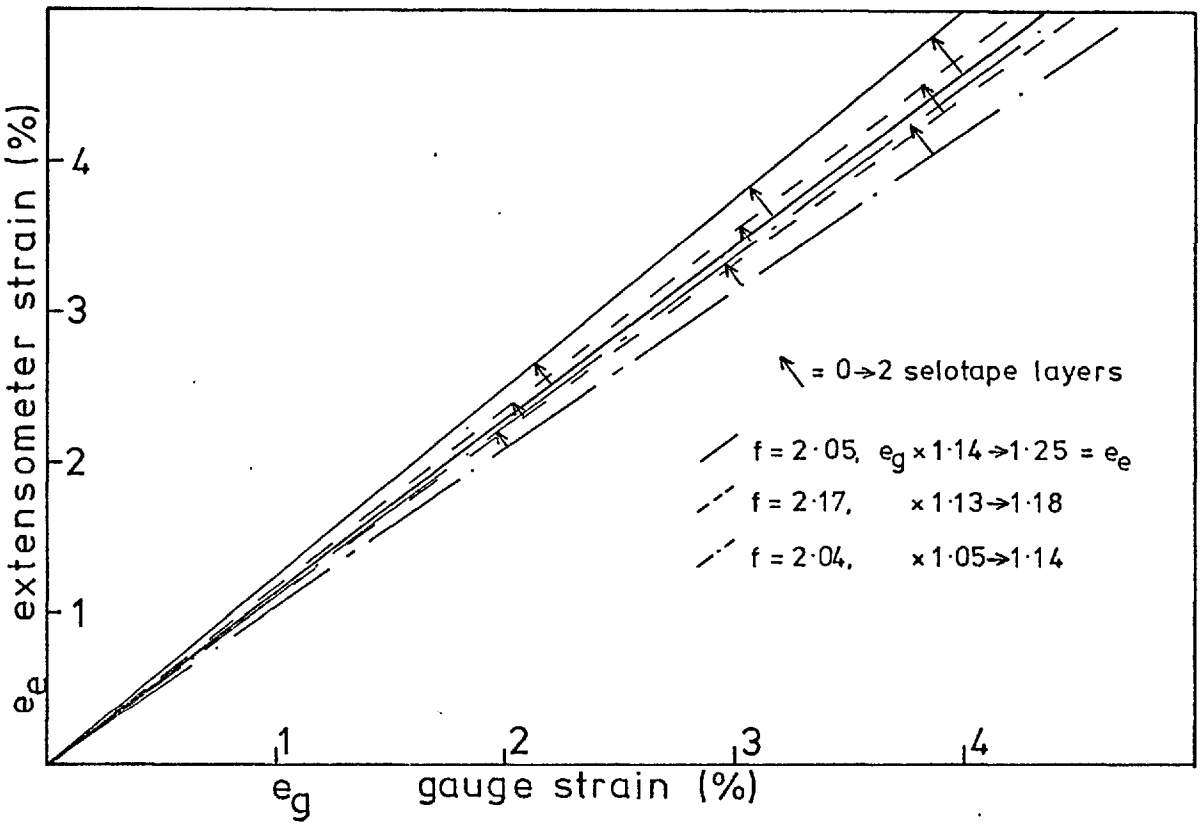


fig (4.44) MODULUS EFFECTS ON GAUGE STRAIN

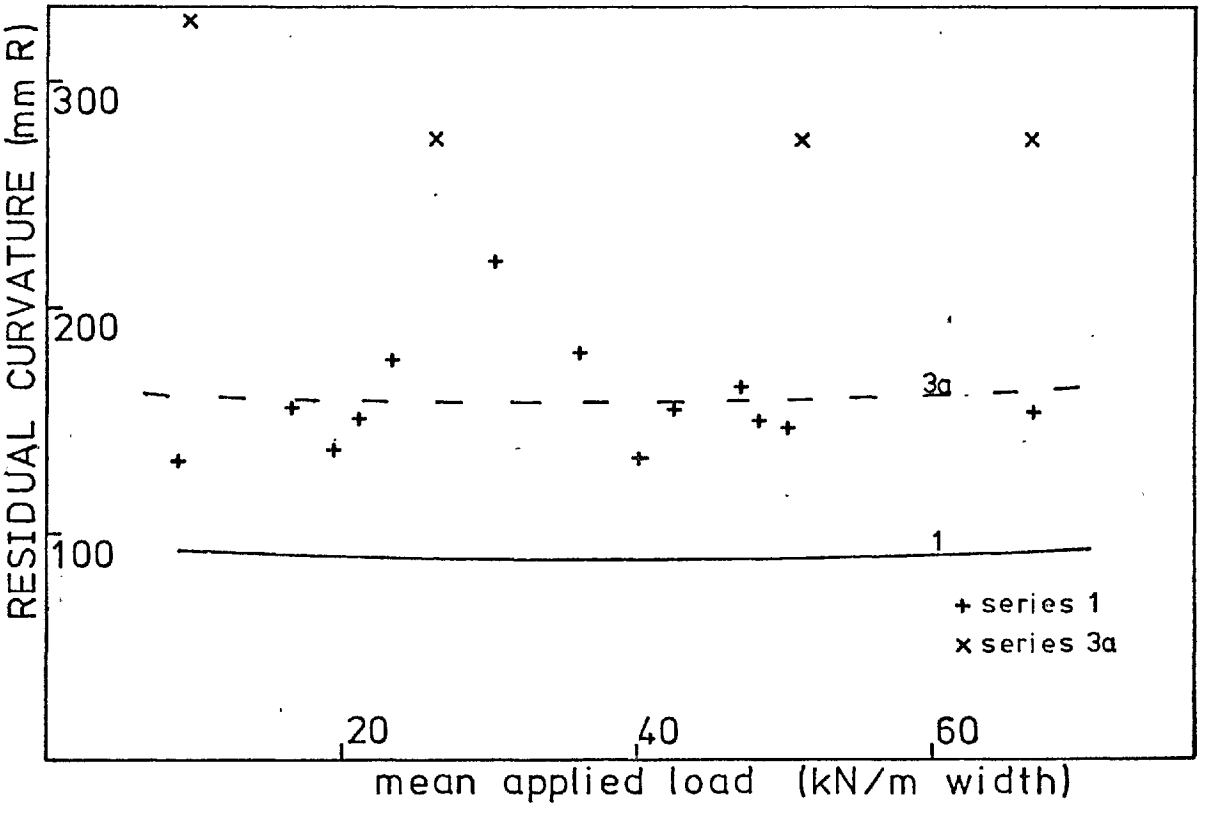


fig (4.45) RESIDUAL STRIP CURVATURES

CHAPTER 5.
DISCUSSION.

5.0.0. Topics for Consideration.

Before embarking on a full discussion of the experimental work of Chapter 4,it is necessary to consider in some detail the various assumptions utilized in the theoretical derivations.The validity of the assumptionswill be demonstrated in Section 5.1, reference being made where necessary, to the experimental results of section 4.4.The correlation between theoretical and practical results is considered in section 5.4.

5.1.0. Discussion of Theoretical Assumptions.

5.1.1. The Angle of Wrap.

Figure 4.2~~7~~ adequately demonstrates that under the particular conditions of the series 1 experiments, the overall elongation values lie below those theoretically where low applied tensions are present.This, coupled with the results of Hilsen and Polakowski (51,52) demonstrates the effect of wrap angle on levelling results.In the theory developed so far the effects of tension losses due to working of the strip,and bearing friction effects,have been included but it has been assumed in all cases, that the strip conforms, momentarily at least, to the roll radius.Where this is the case in fact, such as the series 2A and 3A experiments, the theoretical model has sufficient correlation with practise to be suitable for design purposes.Where the roll radius is not attained however, at least 50% error in theoretical predictions may result.

The factors to be considered for roll conformity are:

(1) Roll diameter/strip thickness ratio.

(2) Wrap angle (ie the angle subtended by the radii of the nominal points of roll contact.).

(3) Applied tension.

(4) Material strength.

An increase in the first three factors aids roll conformity, whereas an increase in material strength serves only to hinder.Unless a suitable combination of the above parameters exists, it is possible that point contact only may occur between strip and roll.The actual bend radius will thus be larger than that of the roll and will be unknown.Errors in theoretical predictions will then ensue.

If theoretical predictions are to be utilized, the only variable which may be altered is that of the wrap angle.A method is therefore required to predict the angle of wrap necessary to provide roll conformity.Admittedly increasing the wrap angle to a high value eg.90^{o} will, in most cases provide radius attainment,but in doing so high friction losses will ensue and roll bending effects will be exaggerated.

From the discussion at an I.M.E. conference on Tension Levelling in April 1971, it was apparent that although attempts had been made to analyse the above problem these had all failed on the grounds of insolubility.The following analysis provides a suitable criterion for the prediction of required wrap angles, this theory being developed after various attempts had been made to analyse the problem with the aid of continuous beam theory.

Figure 5.1. shows the actual path of strip passing over a roll in a leveller.Swift (54) has previously pointed out that the lines of action of the forces T_o and T_4 must necessarily be different, to provide the balance of moments about the roller axis.The moment arm FO for T_0 is thus greater than that (OG) for T_4, resulting in the actual strip path shown in the figure.These lines of action provide the necessary bending couple at B and C to keep the strip on the roll and also ensure a steadily increasing moment as the strip approaches the roll as well as the straightening couple on leaving.

The condition for roll conformity is that the applied bending moment at some stage is sufficient to balance the internal moment for strip bent to the roll radius.We can assume that the theory developed previously defines this moment.The applied straightening moment is derived from the maximum deflection of the unstraightened strip away from the line of action of the front tension T_4.The presence of this straightening moment requires a small degree of over-straightening of the strip.

In Fig 5.1, A and D represent the geometrical tangency points, long range elastic effects preventing roll contact before B and after C.We can assume that spring-back occurs immediately at C and that strip retains the spring-back radius until the point of maximum straightening moment (E) is reached.

Using the notation of Fig 5.1.

$$T_0(R_0 + \gamma) = T_4(R_0 - \beta) \qquad \text{----5.1.1.}$$

where R_0 = roll radius.

The moment required to bend the strip is M_B, whilst that to straighten is M_{ST} and written

$$M_{ST} = T_4(\beta+\gamma) \qquad \text{----5.1.2.}$$

From triangles O'E δ" and OD δ'

$$\frac{R_{resid}}{\text{Cos}\,\varphi'} - \frac{R_o}{\text{Cos}\,\varphi'} - (R_{resid} - R_o) = \frac{\gamma+\beta}{\text{Cos}\,\varphi'} \qquad \text{----5.1.3.}$$

$$\text{giving Cos}\,\varphi' = 1 - \frac{\gamma+\beta}{R_{resid} - R_o} \qquad \text{----5.1.4.}$$

From triangles OG δ and OFS

$$\frac{\text{Cos}\,\varphi'}{\text{Cos}\,\varphi''} = \frac{R_o - \beta}{R_o + \alpha} \qquad \text{----5.1.5.}$$

Since SB = c δ

A positive bending moment acts over the arc FH and we are thus interested in finding the point of maximum applied bending moment within this region. Over the arc of contact BC, pressure forces will exist between the strip and roll and are assumed to be of constant value p. To simplify the analysis the arc FH may be projected on to the line FG and will be of length FI where,

$$FI = (R_o+\alpha)\,\text{Sin}\frac{(\varphi)}{2} + \left((R_o-\beta)\,\text{Sin}\frac{\varphi}{2} - \beta(2R_o-\beta)\text{Cos}\frac{(\varphi)}{2}\right)$$

----5.1.6.

for small α and β

The reduced system can be treated as a simple beam, loaded over part of its length with a uniform pressure force p (Fig 5.2).

The resultant bending moment at any point over BC is

$$M_{APP} = T_0\,\text{Sin}\frac{\varphi}{2}x - \frac{P(x')^2}{2} \qquad \text{----5.1.7.}$$

for normal beam theory

$$\text{where } x' = x - \left((R_o+\alpha)\text{Sin}\frac{\varphi}{2} - R_o\text{Sin}\left(\frac{\varphi}{2} - \varphi''\right)\right) \qquad \text{----5.1.8.}$$

At x' = 0, ie. the initial contact point

$$x = (R_o+\alpha)\text{Sin}\frac{\varphi}{2} - R_o\text{Sin}\left(\frac{\varphi}{2}-\varphi''\right) \qquad \text{----5.1.9.}$$

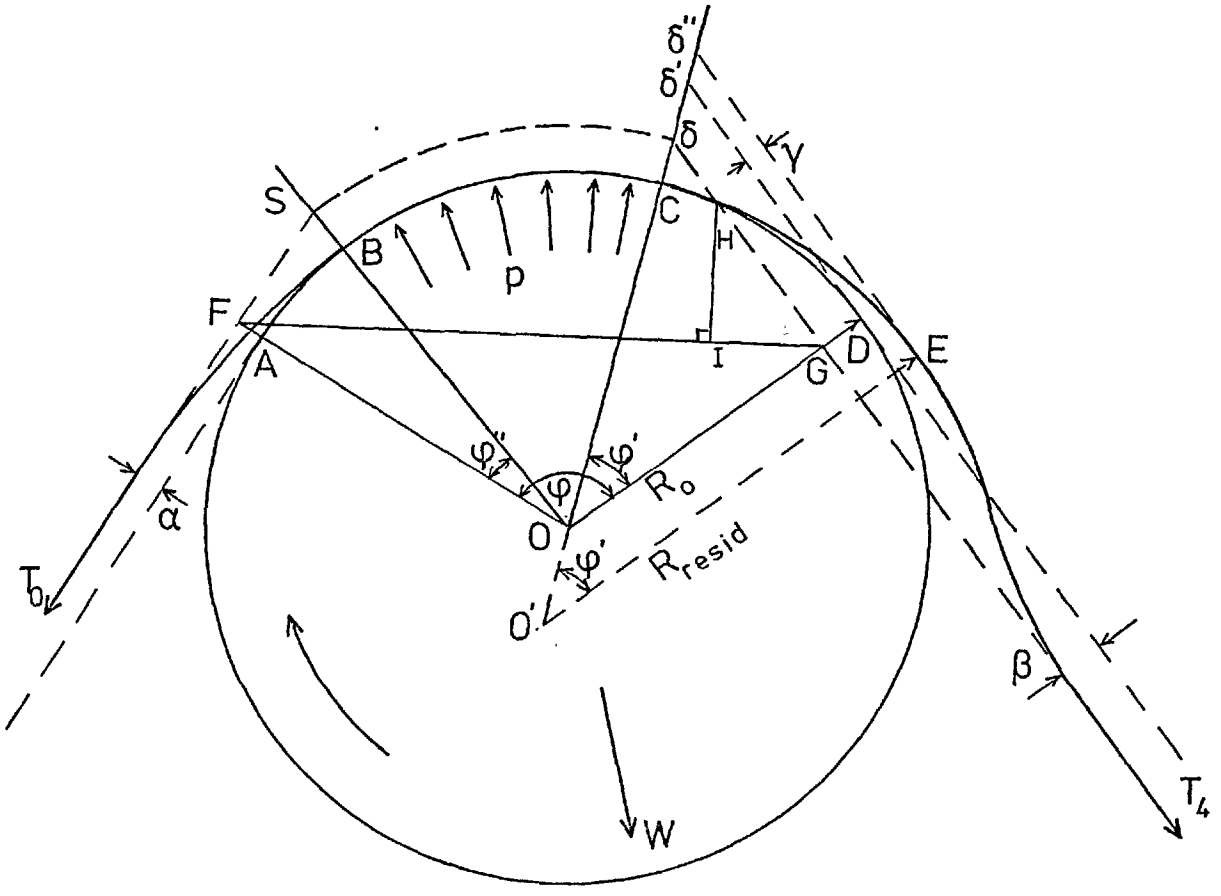
and from 5.17 and 5.19

$$M_{APP} = T_0\text{Sin}\frac{\varphi}{2}\left((R_o+\alpha)\text{Sin}\frac{\varphi}{2} - R_o\text{Sin}\left(\frac{\varphi}{2}-\varphi''\right)\right) \qquad \text{----5.1.10.}$$

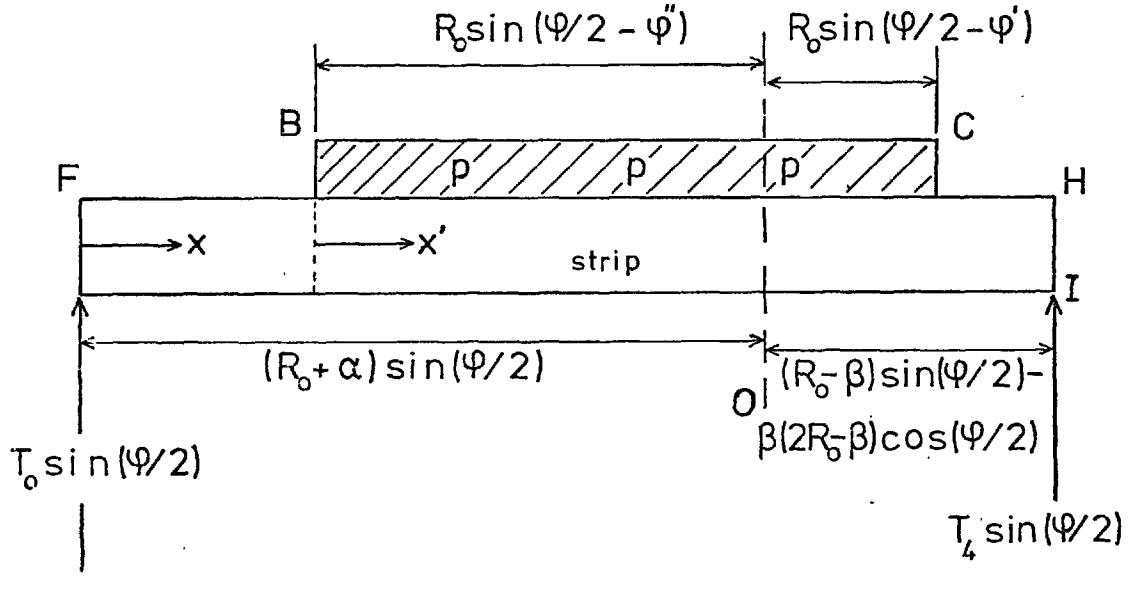
Similarly at the end of the contact arc

$$M_{APP} = T_4\text{Sin}\frac{\varphi}{2}\left((R_o-\beta)\text{Sin}\frac{\varphi}{2} - R_o\text{Sin}\left(\frac{\varphi}{2}-\varphi'\right) - \beta(2R_o-\beta)\text{Cos}\frac{\varphi}{2}\right)$$

----5.1.11.



fig(5.1) STRIP PATH OVER A ROLL (DETAIL)



fig(5.2) SIMPLE BEAM EQUIVALENT

Equations 5.1.10 and 5.1.11 may be equated and if we include equation 5.1.1, we get:

$$T_0 \mathrm{Sin}\left(\frac{\varphi}{2} - \varphi''\right) = T_4\left(\mathrm{Sin}\left(\frac{\varphi}{2} - \varphi'\right) + \frac{\beta(2R_o - \beta)\mathrm{Cos}\frac{\varphi}{2}}{R_o}\right) \quad \text{----5.1.12.}$$

If the arc BC is finite (ie. pressure forces exist) then over this arc the roll radius must have been attained.We are thus interested in the conditions where

$$\varphi' + \varphi'' \leqslant \varphi \quad \text{----5.1.13.}$$

The solution of equation 5.1.1-5.1.13 is possible when suitable values of T_0, T_4, M_B and M_{ST} are known.The complete solution is unnecessary because any machine is designed to process a range of materials and frictional effects are often indeterminate.An approximate solution would therefore be adequate.

To obtain such a solution, we can assume that the front and back tensile forces are equal at their mean value.Then from 5.1.1.

$$\alpha = \beta = 0 \quad \text{----5.1.14.}$$

and in equation 5.1.5

$$\varphi' = \varphi'' \quad \text{----5.1.15.}$$

The pressure region is thus central on the arc of contact.
Substituting 5.1.14 into 5.1.10 and 5.1.11 gives:

$$M_{APP} = T_{mean} R_o\left(\mathrm{Sin}\,\frac{2\varphi}{2} - \mathrm{Sin}\,\frac{\varphi}{2}\mathrm{Sin}\left(\frac{\varphi}{2} - \varphi'\right)\right) \quad \text{----5.1.16.}$$

For the case in which the roll radius is instantaneously attained with point contact:

$$\varphi' = \varphi'' = \frac{\varphi}{2} \quad \text{----5.1.17.}$$

and from 5.1.16 and 5.1.17

$$M_{APP} = T_{mean} R_o\left(\frac{1 - \mathrm{Cos}\,\varphi}{2}\right) \quad \text{----5.1.18.}$$

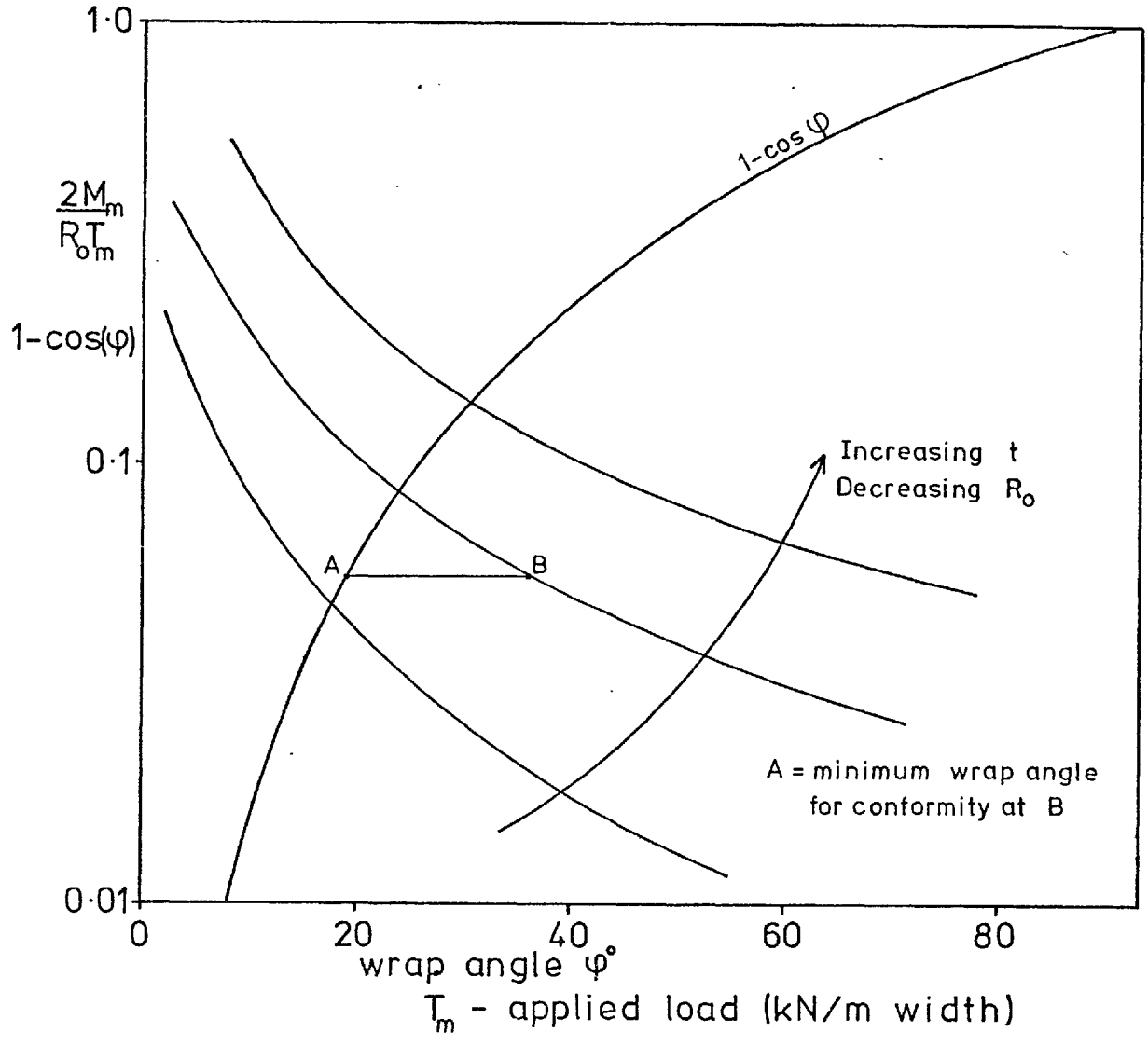
but for conformity

$$M_{APP} = M_B \quad 5.1.19.$$

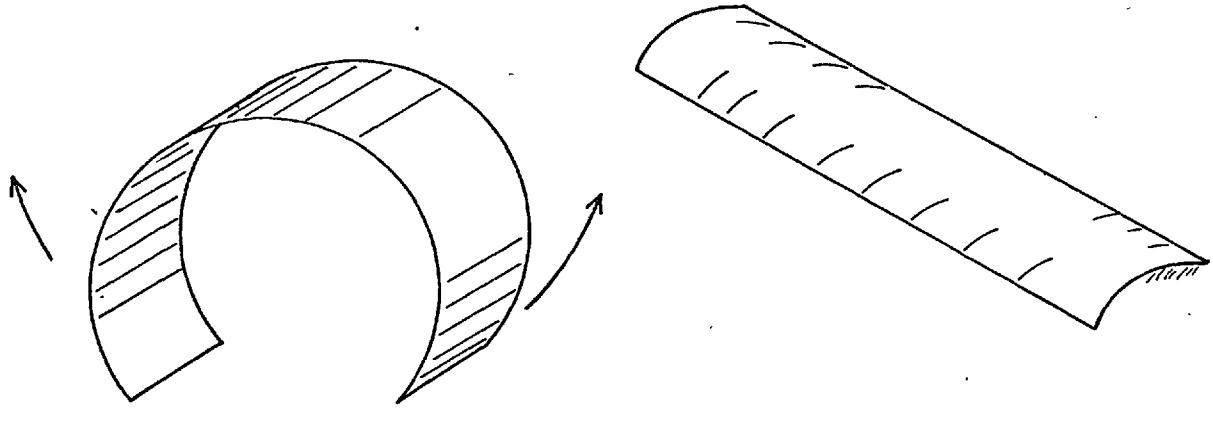
so that we can write

$$(1 - \mathrm{Cos}\,\varphi) \geqslant \frac{2M_B}{R_o T} \quad \text{----5.1.20.}$$

A series of plots may be drawn for any particular material and thickness from which the necessary wrap angle at any particular tension can be ascertained (Fig 5.3).



fig(5.3) REQUIREMENTS FOR RADIUS ATTAINMENT



fig(5.4) ELASTIC ANTICLASTIC CURVATURE

The effect of the various process parameters on the attainment of wrap may be evaluated by returning to the rigid, perfectly-plastic model. Here,

$$M_B = \frac{\sigma_y t^2}{4}\left(1 - \left(\frac{\sigma_T}{\sigma_y}\right)^2\right) \qquad \text{----5.1.21.}$$

where

$$\sigma_T < \sigma_y/3$$

$$M_B \simeq \frac{\sigma_y t^2}{4} \qquad \text{----5.1.22.}$$

From 5.1.20 and 5.1.22, we can write

$$(1 - \cos\varphi) \geqslant \frac{\sigma_y t^2}{2R_o T} \qquad \text{----5.1.23.}$$

and from elementary theory

$$e_{TOT} = \frac{NT}{R\sigma_y} \qquad \text{----5.1.24.}$$

hence,

$$(1 - \cos\varphi) > 2\left(\frac{t}{D}\right)^2 \frac{N}{e_{TOT}} \qquad \text{----5.1.25.}$$

Equation 5.1.25 gives an approximate relationship between the required wrap angle, the thickness/diameter ratio and the required strain. As an elementary example we may take the case of a four roll leveller with a D/t ratio of 40. From equation 5.1.25,

$$1 - \cos\varphi \geqslant \frac{1}{800} \times \frac{4}{e_{TOT}} \qquad \text{5.1.26.}$$

and hence for conformity

$$\cos\varphi \leqslant 1 - \frac{0.005}{e_{TOT}} \qquad \text{----5.1.27.}$$

From this last equation, the wrap angles necessary to obtain the theoretical extension at the theoretical applied loads is simply determined,

eg. e_{TOT} %	$\varphi_{min.}$
0.5	90°
1.0	60°
2.0	41°

The higher wrap angle required for the lower extensions, although appearing in error, is the consequence of the low theoretical tensions

required.Unless suitable geometrical wrap angles are provided, therefore a levelling installation is effectively uncontrollable, except by experience.As can be seen from the above example this lack of control will be more prevalent in the low elongation regions.

5.1.2. Pressure Forces.

The presence of pressure forces at the strip/roll interface has been accepted in section 5.1.1.Since only one face of the strip is in metal/metal contact and the strip is thin, a large thickness stress gradient must exist within the strip, stresses being zero at the free surface. As a result of this the general assumption of zero thickness stress is reasonably valid especially when the complete picture of bending and unbending is considered, where on spring-back and straightening no surface contact is present.It is necessary, however, to deduce the relative magnitudes of pressure stresses operating so that the deviation from true representation of the relationships derived in 3.2.0. is known.

On the assumption that the contact arc is centred about the middle of the geometrical wrap arc, we have from equation 5.1.16

$$\frac{2M_{app}}{T_m R} = (1-\text{Cos}\varphi) - 2\text{Sin}\frac{\varphi}{2}\,\text{Sin}\frac{\varphi_A}{2} \qquad \text{----5.1.28.}$$

where $\varphi_A = \varphi - 2\varphi' =$ actual contact arc.

Taking the result of the example for φ for 2% strain, we see that 41 degrees is necessary to attain the roll radius.If we assume that a 60 degree geometrical arc is available, what pressure stresses are present? From equation 5.1.28 we have,

$$0.25 = (1-\text{Cos } 60) - 2\text{Sin } 30\ \text{Sin}\frac{\varphi_A}{2}$$

$$= 0.5 - \text{Sin}\frac{\varphi_A}{2}$$

Therefore $\text{Sin}\frac{\varphi_A}{2} = 0.25$

Therefore $\varphi_A = 29^\circ = 0.485\varphi.$

If a 90° geometrical wrap is present,

$\varphi_A = 64^\circ = 0.712\varphi.$

The actual contact arc therefore increases more rapidly than the increase in geometrical arc,

ie. $\varphi^\circ > \varphi - \varphi_A$ ----5.1.29.

where $\varphi^\circ = \varphi$ at $\varphi_A = 0.$

From figure 5.1.

$$W = 2T_m \sin \varphi/2 \qquad \text{----5.1.30.}$$

This may be equated to the overall pressure force, such that,

$$W = P_{tot} = 2\int_0^{\varphi_A/2} p \cos \frac{\varphi_A}{2} \, R_o \, d\left(\frac{\varphi_A}{2}\right)$$

$$= 2pR_o \sin \frac{A}{2} \qquad \text{----5.1.31.}$$

From 5.1.30/31

$$p = \frac{T_m \sin \varphi/2}{R_o \sin \varphi_A/2} \qquad \text{----5.1.32.}$$

Where point loading occurs, the pressure over the line of contact is infinitely high and would thus nullify the assumption of zero thickness stress. In general, contact does occur over a finite arc length. The substitution of the results of the above example into equation 5.1.32 will yield an order of magnitude result.

From 5.1.32 and 5.1.24

$$p \simeq \frac{\varepsilon \sigma_m}{N} \frac{\sin \varphi/2}{\sin \varphi_A/2} \qquad \text{----5.1.33.}$$

For $\varphi = 60^\circ$, $\varphi_A = 29^\circ$ for 2% strain. For the four roll system

$$p \simeq \frac{0.02\sigma_m}{4} \frac{0.5}{0.25.} = 0.01 \, \sigma_m \qquad \text{----5.1.34.}$$

For $\varphi = 90^\circ$ $\varphi_A = 64^\circ$

Therefore

$$p \simeq \frac{0.02 \, \sigma_m}{4} \frac{0.707}{0.530} = 0.007 \, \sigma_m \qquad \text{----5.1.35.}$$

If we now consider a further example with a geometrical angle of 45° the corresponding true wrap is 6.4°. Resultant pressure forces are thus,

$$p \simeq 0.034 \, \sigma_m \qquad \text{----5.1.36.}$$

It may be said that providing roll conformity is obtained over an arc of 5°, then pressure stresses are negligible. Where the roll radius is not attained, pressure stresses are likely to be sufficiently high to induce errors from the use of the stress/strain relationships developed in 3.2.0. The general theory is not applicable under these conditions and its use for the derivation load/elongation relationships for estimated load radii is not recommended.

5.1.3. The Plane Strain Assumption.

Results to date , both published and unpublished point to the validity of this assumption.Reference to the literature will, however, show frequent discussion on the problem of anti-elastic curvature.

Any strip that possesses a coil set and is then unbent will show an elastic cross bowing (Fig 5.4) unless the supports are sufficiently close together.The first situation is that at the leveller exit whilst the latter is obtained between leveller rolls.Where deformation actually occurs the rigidity of the supporting system ensures that plane strain conditions obtain.It is unnecessary to attempt removal of elastic, anti-elastic curvature in the leveller, since the exit bridle rolls are unlikely to be sufficiently large to prevent the introduction of plastic straining as the strip passes through the bridle.

Plastic,anti-elastic curvature often shows up on leveller strip, but can be explained completely in terms of roll deflection phenomenon. As a point of interest, if we turn our attention to equations 3.2.28 and 3.2.30 it is seen that the zero transverse strain increment is composed of a plastic increment and an elastic increment of opposite sign.The elastic increment is theoretically recoverable on stress relaxation, and will thus leave a residual transverse plastic strain which will vary through the strip thickness, after bending.

Although this strain is of small magnitude (of the order of $\frac{1}{2}\%$ of the longitudinal strain) uneven working between the upper and lower strip surfaces may result in a plastic cross bowing.This result is of course dependent upon the complete acceptance of the Prandtl-Reuss equation as written.Since the equality of the elastic and plastic componants of equation 3.2.28 forms the basis of the analysis of the longitudinal stress/strain relationships, this side effect should not be omitted.

5.1.4. Shear Stresses.

The linear strain distribution assumed throughout the theoretical analysis is totally dependent upon the assumption of zero shear stress. The presence of shear stresses resulting from the bending operation will not affect the longitudinal stress levels, but will cause a distortion of transverse planes which will no longer remain plane(Fig5.5).

It is well known that for a rectangular section, shear stress is distributed evenly over the width and varies approximately parabolically through the thickness.The maximum value occurs at the neutral axis and,

for simple bending, is 1.5 times the mean value.This latter figure may be expressed in terms of the rate of change of bending moment, where,

$$\bar{\tau} = \frac{dM}{dx} \cdot \frac{1}{t} \qquad \text{----5.1.37.}$$

where M = moment/unit width.

The application of tension to the system causes a redistribution of shear stress from the simple bending condition.With the notation of Chapter 3 for the half strip thickness and neutral axis displacement we may write for simple bending,

$$\tau = \frac{3}{4} \frac{dM}{dx} \frac{(a^2 - y^2)}{a^3} \qquad \text{----5.1.38.}$$

The mean shear stress being denoted by:

$$\bar{\tau} = \frac{1}{2a} \frac{dM}{dx} \qquad \text{----5.1.39.}$$

and

$$\tau_{max} = \frac{3}{4a} \frac{dM}{dx}. \qquad \text{----5.1.40.}$$

The displacement b of the neutral axis on the addition of applied tension requires the adjustment of equation 5.1.38 such that

$$\tau = \frac{3}{4} \frac{dM}{dx} \left(\frac{(a+b)^2 - y^2}{a(a^2+3b^2)} \right) \qquad \text{----5.1.41.}$$

the mean shear stress now being,

$$\bar{\tau} = \frac{1}{2a} \frac{dM}{dx} \frac{(a^2 + 3ab)}{(a^2 + 3b^2)} \qquad \text{----5.1.42.}$$

and,

$$\tau_{max} = \frac{3}{4a} \frac{dM}{dx} \frac{(a+b)^2}{(a^2 + 3b^2)} \qquad \text{----5.1.43.}$$

Differentiation of either 5.1.42 or 5.1.43 and equation to zero yields the result that shear stresses are at their maximum at neutral axis displacements of one sixth the strip thickness.This is equivalent in the case of a perfectly plastic material to an applied load of exactly one third of yield stress.The actual values of the maximum shear stress being 1.33 times that in simple bending, and the maximum value of the mean shear stress being 1.5 times the equivalent simple bending value.The maximum shear stress τ_{MAX} with applied tension is therefore 1.33 times the mean value.

The application of tension to a system of bending is therefore seen to exaggerate shear stress effects, especially under the industrial conditions of one third yield stress applied tension.Even though the

overall level is increased however, the deviation over the section is reduced and the plane sections assumption has greater validity.Since the shear stress and strain magnitudes are of similar values but opposite signs during bending and unbending, the overall effect of shearing forces contributing to material elongation will be negligible.

It is worth making some estimate of the shearing forces present however, these being relatively simple once the rates of change of bending moment have been established.This may be performed by forming theoretical plots of internal moment vs radius of curvature for particular values of applied tension.Plots of surface strain corresponding to these radii may be imposed on the same diagram,Fig 5.6.The values of surface strain (determined from strain gauge measurements) may also be plotted against distance from roll contact Fig 5.7.Correlation of Figs 5.6 and 5.7, and equation 5.1.42 then provides a reasonable representation of the acting levels of shear stress.

From Figure 5.7 (which is an exact replica of the output plot obtained from run 2b7 and load varying from 1100 to 1420 lbf over the strip width, measurement shows moment build -up occurs in about 0.3 to 0.5 cm = 0.50 to 0.83 ins with paper and strip speed accounted for. From Fig 5.6 it can be seen that the internal moment developed is of the order of 1.5 lbf-in/in width of strip.Comparison of Figs 5.6 and 5.7 shows that moment build-up is approximately linear during bending, and over the first roll we would expect:

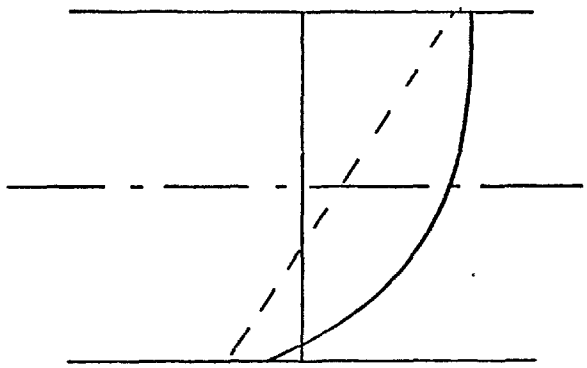
$dM/dx = 2$ to 3.

On pulling straight however, the change in internal moment is approximately twice that of the initial bend and this occurs in a similar distance. dM/dx is thus of the order of 4 6.

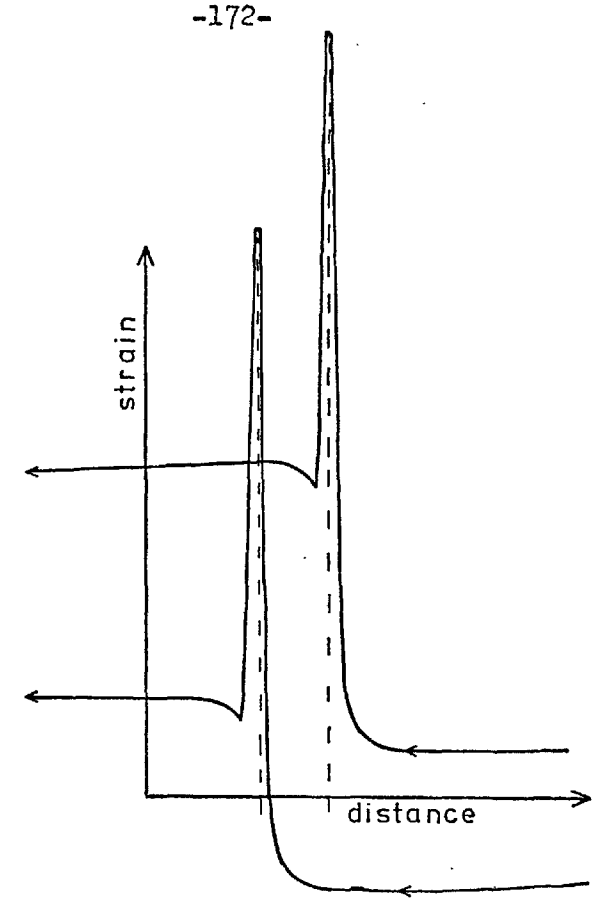
In the above case strip thickness was $\simeq$ 0.0129 ins, the derived value for b at 1200 lbsf load being 0.0023 ins.The situation experimentally therefore corresponds to the worst shear stress conditions. From equation 5.1.43 we may write

$$\tau_{max} = \frac{3}{4} \times \frac{1}{0.0065} \times 5 \times \frac{0.775}{0.580} = 770 \text{lbf/ins}^2 \ (5.3 \text{ MN/m}^2)$$

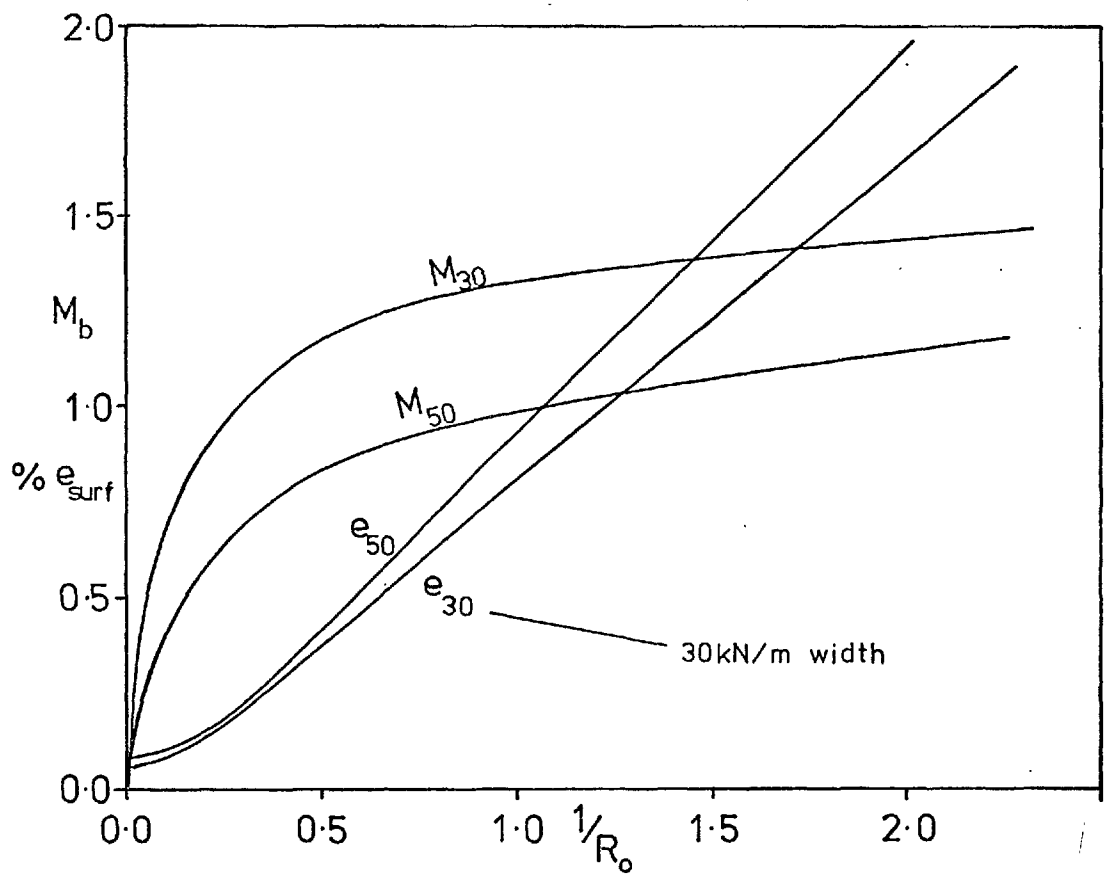
As the applied tension is increased, so the bending and straightening occurs in a reduced distance.However at the same time the internal moment is reduced (Fig 5.6) and shear stress levels remain approximately constant over a range of applied tensions.It is reasonable to assume therefore that plane sections do remain plane throughout the process since shear stresses are at least an order of magnitude less than normal stresses.As previously mentioned during tension levelling, the general



fig(5.5) EFFECT OF SHEAR STRESS ON BENDING STRAIN DISTRIBUTION (exaggerated)



fig(5.7) GAUGE RESULTS



fig(5.6) MOMENT-CURVATURE RELATIONSHIPS

tendency is for section deformed during passage over one roll to be straightened over the next.

5.1.5. Strain Rate Effects.

It has been previously stated (section 5.1.4) that strains during bending and unbending take place over a distance of 0.5 - 1.0 ins.In order to be able to relate directly to stress strain curves obtained in simple tension to those pertaining to leveller conditions it is necessary to estimate any discrepancies in strain rate.

We can reasonably say that the maximum surface strains occurring over the above length interval are approximately 2%.At maximum,strains are therefore 0.04/ins.During the experimental series, strip was pulled through the leveller at $\simeq$3m/min $\equiv$ 2in/sec giving strain rates of 0.08/sec maximum.Industrially this figure may be increased by a factor of 100 giving rates of the order of 10/sec maximum.

The results of various investigations (eg.88,89) tends to indicate that the discrepancy between the rate used in experiment and those of the tensile tests (10^{-3}/sec)has minimal effect on the plastic stress/strain relationships.The industrial rate lies in the doubtful range where strain rate may, or may not be important,being dependent upon material.It would be advisable,for industrial applications to perform preliminary tensile tests at rates greater than unity.

5.1.6 Leveller Theory.

The theoretical derivations of sections 3.3 and 3.4 have assumed that the operating stress level in any fibre is a function of the initial and final strain levels in any unit operation.In doing so the movement of the incremental neutral axis through the strip thickness has been ignored and the assumption must therefore be qualified.Resultant errors will occur where plastic straining takes place during this travel which is unaccounted for by consideration of the final position of the neutral axis only.

To fully evaluate the problem would entail the determination of say bending strains by a number of increments,effecting an increase in the complexity of the determining program which itself would need adequate justification.The relative magnitude of errors incorporated by the simple theory can however,be pictured fairly easily.

During the initial leveller bend,the neutral axis of bending moves into the strip thickness along the bend radius.Decreasing this radius

causes closer approach of the neutral axis to the central axis (Fig 5.8) until tensile yielding counteracts this movement by stress loss in the tensile zone.The neutral axis then settles to its final position.Since plastic yielding retards the movement of the neutral axis,it is impossible for regions of the strip thickness to be plastically strained and subsequently left in a state of elastic or plastic stress of the opposite sign.An incremental model would therefore be of no consequence for the first bend.

The inclusion of work-losses in this analysis does,however,complicate matters in that strip passage round the roll is associated with a tensile load increase.This results in a shift of the zero stress axis towards the centre of curvature (Fig 5.9) and it is apparent that some plastically compressed zones will subsequently become elastic.Within these zones the simple theory will therefore result in an under-prediction of strains contributing to work-hardening.Unless friction losses are large this effect can again be considered negligible since the percentage discrepancy in neutral axis position is only half the percentage increase in load for a perfectly plastic material and is even less when work-hardening is taken into account.

With springback,straightening and any subsequent bending operations the problem must be analysed on a different basis,since the incremental neutral axis initially resides at the centre of section,moving only when equlibrium conditions are violated.Figure 5.10 illustrates the position during springback,a similar situation existing during straightening but with greatly increased stress levels.It is again evident that the possibility exists for zones initially compressed to be subsequently strained in tension.Depending upon the magnitude of the total shift,this compression may be plastic and an over-prediction of neutral axis shift will be presented by the 'overall' theory.Materials do not, however,work-harden sufficiently rapidly to present serious problems and the effect may again be neglected.

It is reasonable to say that tensile load alterations have a more pronounced effect on the above than curvature changes.The resultant errors in a program designed to evaluate the levelling of buckled strip might therefore be large.As analysis for buckled strip would, as is described in section 3.7 be on an incremental basis initially and errors are automatically accounted for.

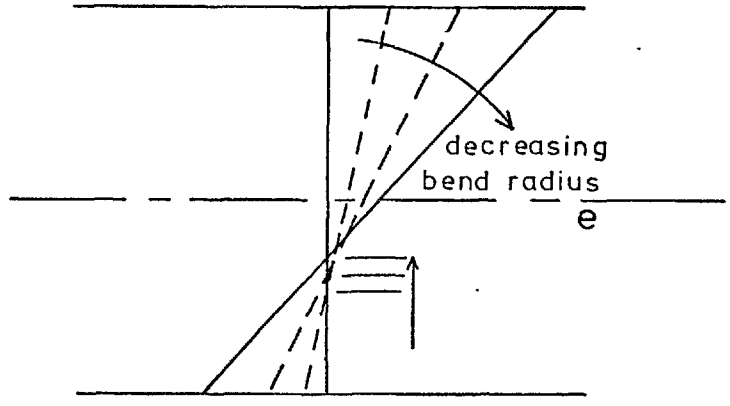


fig (5.8) INCREMENTAL STRAIN DISTRIBUTIONS

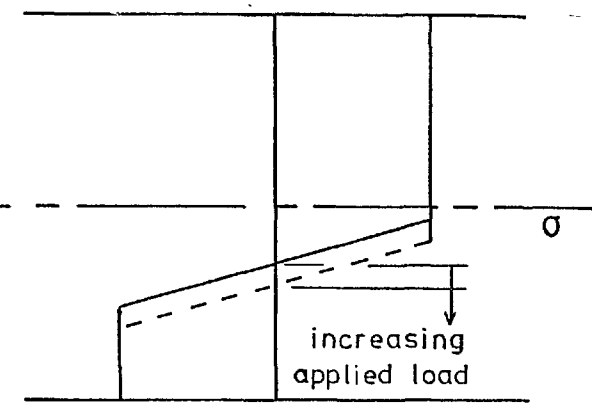


fig (5.9) INCREMENTAL STRESS DISTRIBUTIONS

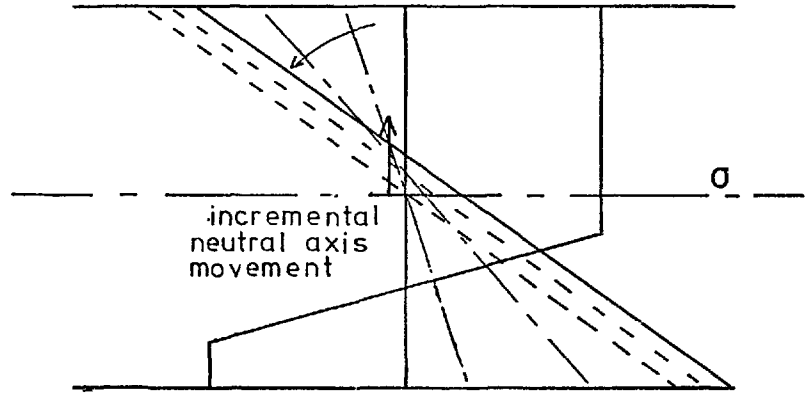


fig (5.10) INCREMENTAL STRAIGHTENING DISTRIBUTIONS

5.2.0 Theoretical Results.

The theoretical model developed is generally applicable and has provided, as well as a simple 'constant load' theory for isotropic work-hardening materials,analyses for materials showing marked Bauschinger effects and for leveller load losses.The discussion of section 5.1.1. also allows a prediction of necessary wrap angles.

Although the applied load/overall elongation predictions are the most important as far as levelling is concerned,all leveller theories provide a good measure of the degree of work-hardening that has taken place during levelling and an indication of the residual stress levels at leveller exit.The hardening produced gives a good indication of the best hardening assumption to use for any material,although a few test runs will provide results sufficient to select the required model from elongation results.The magnitude of predicted residual stresses provides a basis upon which subsequent material treatment can be determined.

Figure 4.27 ,4.33, and 4.42, together with the computer outputs of appendix 7 provide any necessary theoretical comparisons,figure 4.33 having been drawn solely for this purpose.

5.2.1. Hardening Model.

Choice of hardening model is seen to be critical,since the incorporation of a Bauschinger analysis results in a 20 - 30% reduction in predicted strain levels at any particular applied load whilst maintaining minimal work-hardening.The assumption of the Mróz model provides similar results to the simple kinematic,although predicted strains are greater as a result of the increased work-hardening.The outcome of this large discrepancy between isotropic and kinematic hardening models is that care should be taken in defining levelling conditions for materials displaying work-softening.A 50% discrepancy exists between this model and the elementary B.N.F. theory and therefore totally excludes the use of the latter.

5.2.2. Tension Loss Effects.

A prediction of machine work losses requires an estimation of the 'virtual' friction coefficients at the work rolls.For the experimental leveller these have been determined by the analysis of section 3.5.0 and are given as

Series 1 - $\mu' = 0.09$

Series 2 - $\mu' = 0.07$

Series 3a - μ' = 0.04
Series 3b - μ' = 0.06
Series 3c - μ' = 0.24

values depending upon work roll diameters.

The inclusion of work losses is seen to result in an increase in overall elongation at any applied load.This is to be expected since the effects of workhardening reduce with distance through the leveller.Discrepancies between the 'constant load' and 'full' analyses for each roll are given in Fig 5.11,strains at the first being lower by the 'full' theory and greater at the last.An increase in the value of the virtual friction coefficient increases the discrepancy (Fig 4.42),and similarly increases the tension losses during the levelling (Fig 4.43).The base level seen in Fig 4.43 represents the work actually needed to deform the strip.Errors in the definition of the virtual friction coefficient will be of little importance as far as elongations are concerned and will have only a small effect on tension losses, this latter being well within design limits.

Choice of model depends very much on individual requirements.The 'constant load' analysis is likely to be sufficient for the estimation, of leveller strains although tension losses cannot be predicted.Where thin material is being processed however, the work needed to deform the strip is minimal and machine losses can be equated to bearing losses.A simple calculation will provide the required results.

5.2.3. Youngs' Modulus & Poissons' Ratio errors.

In most instances small errors will occur in the choice of E and V values.To incorporate these effects programs have been run with different values of E and V for the same material and leveller conditions,the results being tabulated in Appendix 8.As shown in Fig 4.33 the effect of a reduction in Youngs' modulus is to reduce leveller strain as a consequence of increased spring-back effects.The effect of a variation in V has not been plotted since an increase from 0.29 to 0.30 causes minimal alteration in results.Table 4 shows that an increase in V does not correspond directly to an increase in E (plane strain condition $E' = (E/1-V^2)$)- Poissons' ratio having a more pronounced effect upon longitudinal stress/strain relationships.

The effects of typical errors in the measurement of elastic constants can with reasonable certainty be assumed negligible.

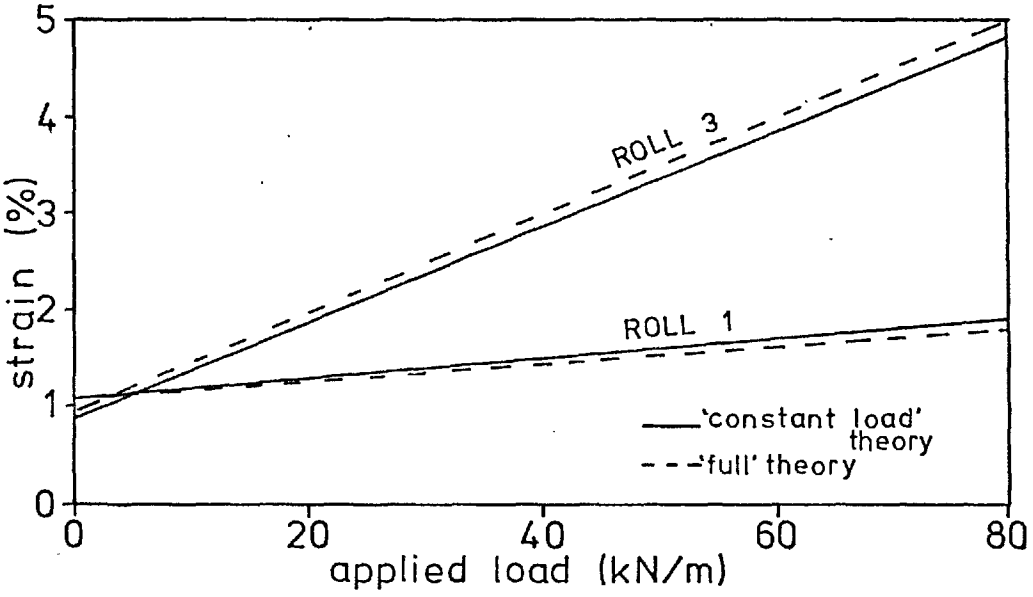


fig (5.11) THEORY COMPARISON - TOTAL SURFACE STRAINS DURING BENDING (SERIES 2a)

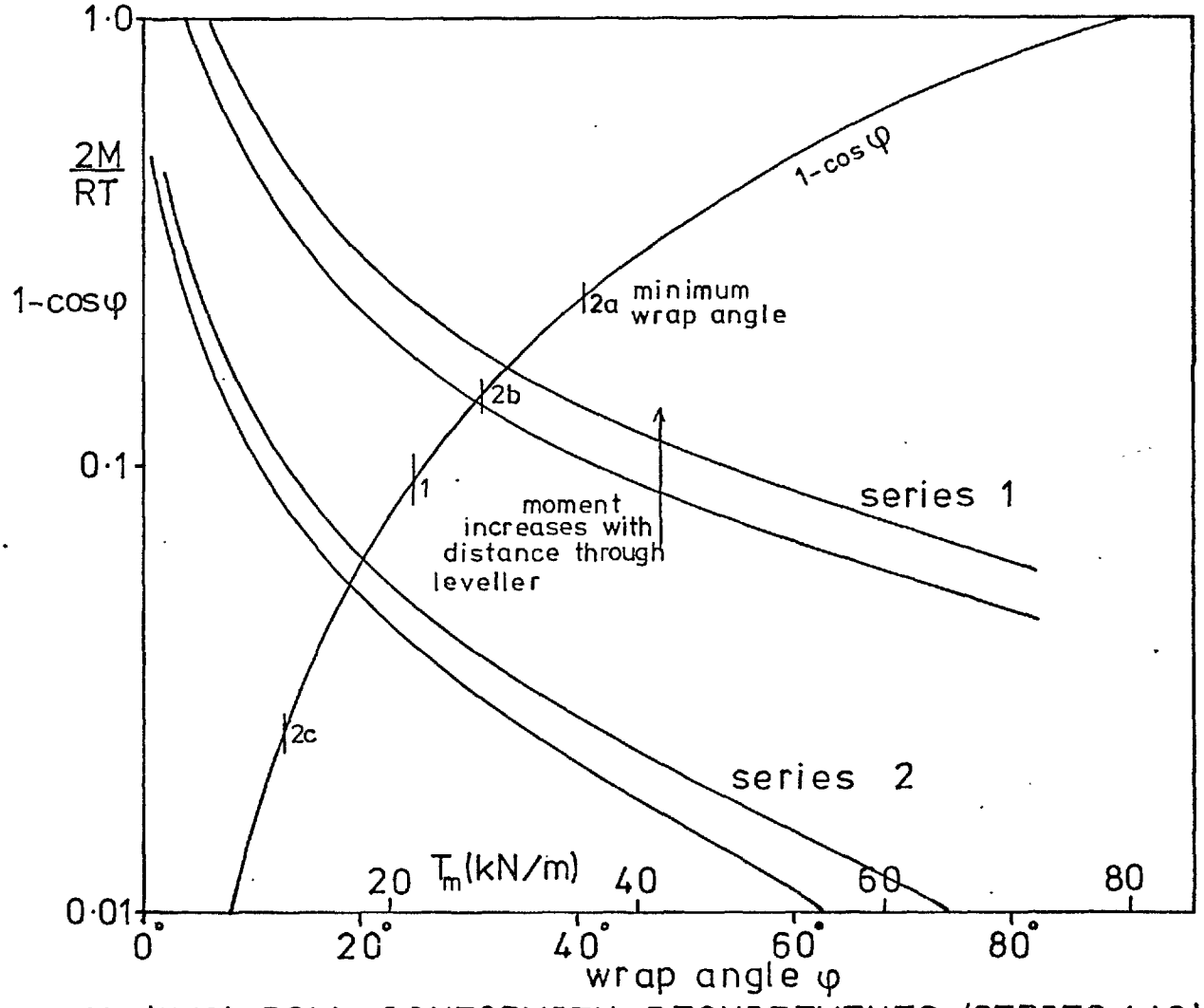


fig (5.12) ROLL CONFORMITY REQUIREMENTS (SERIES 1/2)

5.2.4. Roll radius attainment.

All computer results provide a prediction of the necessary internal moments at roll conformity.From these predictions and the analysis of section 5.11 the wrap angles necessary to provide conformity at any applied load (or the reverse) may be estimated.Utilizing a plot of the type shown in Fig 5.3 the relevant relationships for the three experimental series may be determined.These are shown in Figs 5.12 and 5.13, the relevant wrap angles being superimposed on the $(1 - \cos\varphi)$ line.The necessary loads for conformity at the roll of minimum wrap are given below,

Series 1 - 50kN/m
Series 2a - 4kN/m
Series 2b - 7kN/m
Series 2c - 38kN/m
Series 3a - 34kN/m - 1st roll critical.
Series 3b - 60kN/m - 4th roll critical.
Series 3c - 80kN/m
40kN/m for centre rolls.

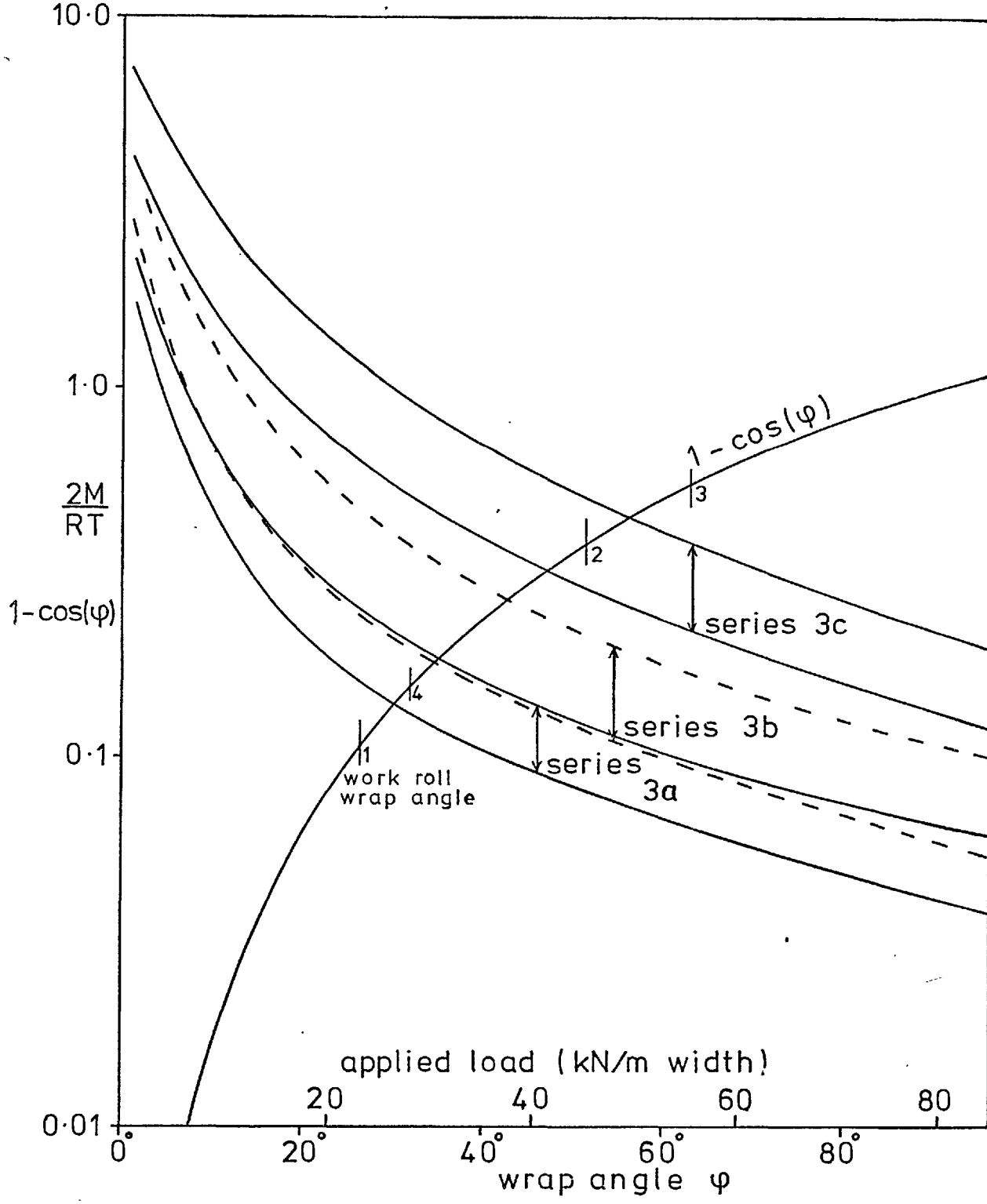
The magnitudes of the required loads for conformity were considered to be lower than these values before the development of the analysis.The theoretical representation would have provided a better basis for choice of machine settings had it been available and would have allowed a better coverage of process variables than has been achieved.It is evident that from the above results roll conformity was not attained during the bulk of the experimental runs of series 1 and 3.

5.2.5. Elementary Theory (B.N.F.)

The elementary model presented in section 3.6.0 is shown in Figs 4.27 and 4.33 by the B.N.F. lines.These lines were derived using the plane strain longitudinal yield stress equation 3.6.10.The discrepancies between these lines and the more accurate theory requires no discussion.Careful choice of a stress value to be used in place of this yield stress produces overall load/elongation results almost coincident with the work-hardening theory.This value must be derived by guesswork and the elementary theory is not suitable for the prediction of results.(90).

5.3.0. Practical Results.

Before embarking on a theoretical/practical comparison it is necessary to consider the experimental results as plotted in some detail.



fig(5.13) ROLL CONFORMITY REQUIREMENTS (SERIES 3)

5.3.1. Overall Elongation Plots.

The elongation values plotted in figures 4.27/33/42 are those determined by the vernier-trammel method.Errors in strain measurement are at maximum equivalent to 0.05% strain.Load measurements are also of good accuracy,readings being taken after 1m travel in all cases,such that the gauge length was completely within the work-roll system.Friction losses over the entry and exit guide rolls are irrelevant where the mean load is required.The degree of scatter seen appears therefore to be a function of the variation of material properties through the coils although the mechanics of the system cannot be ignored,(gauge length deformation occurring immediately after start-up).

The straight line plots obtained for the series 2a,2b and 3a runs indicate that where the roll radius is infact attained,overall strain varies linearly with applied load.(This of course is dependent upon the hardening nature of the material).The curves resulting from the series 1 and 2c experiments provide clear evidence of the effect of wrap angle. Tension levels less than those required for conformity result in low elongations.The results for the series 3b and 3c experiments show a similar effect although the limited number of results make discussion dubious.

5.3.2. Gauge Results.

The strain values plotted in all figures for gauge results are the actual measured gauge strains.Discrepancies between these and surface strains in the bulk of the material are to be expected for the two reasons previously mentioned.

(1.) Modulus Effects.

Figures 5.14 and 5.15 show a comparison between overall gauge strains and measured strip elongations for the series 1 and 2 experiments.The general effect is for gauge strains to be low except at low applied stress levels where the reverse occurs.

Low gauge readings are to be expected on the basis of Fig 4.44.Direct substitution of scaling factors cannot be applied however,because of the bending nature of the strain distributions during levelling.It will be noted from the figures 5.14 and 5.15 that considerable scatter occurs in the gauge/vernier correlation.The tendency is however for an increase in the required scaling factor with increasing overall strain,an expected result since gauge stresses increase with neutral axis displacement and therefore have an increased effect on neutral axis shift.

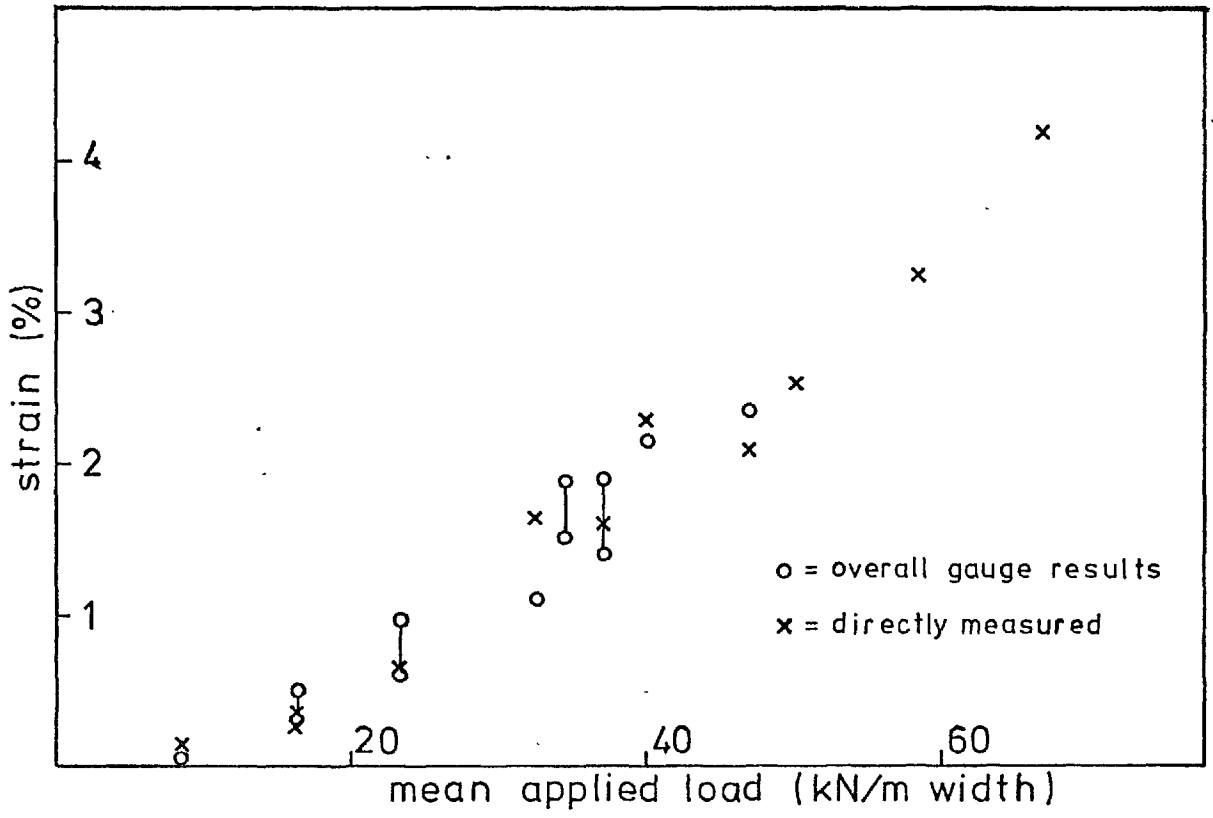


fig (5.14) GAUGE/VERNIER COMPARISON (SERIES 1)

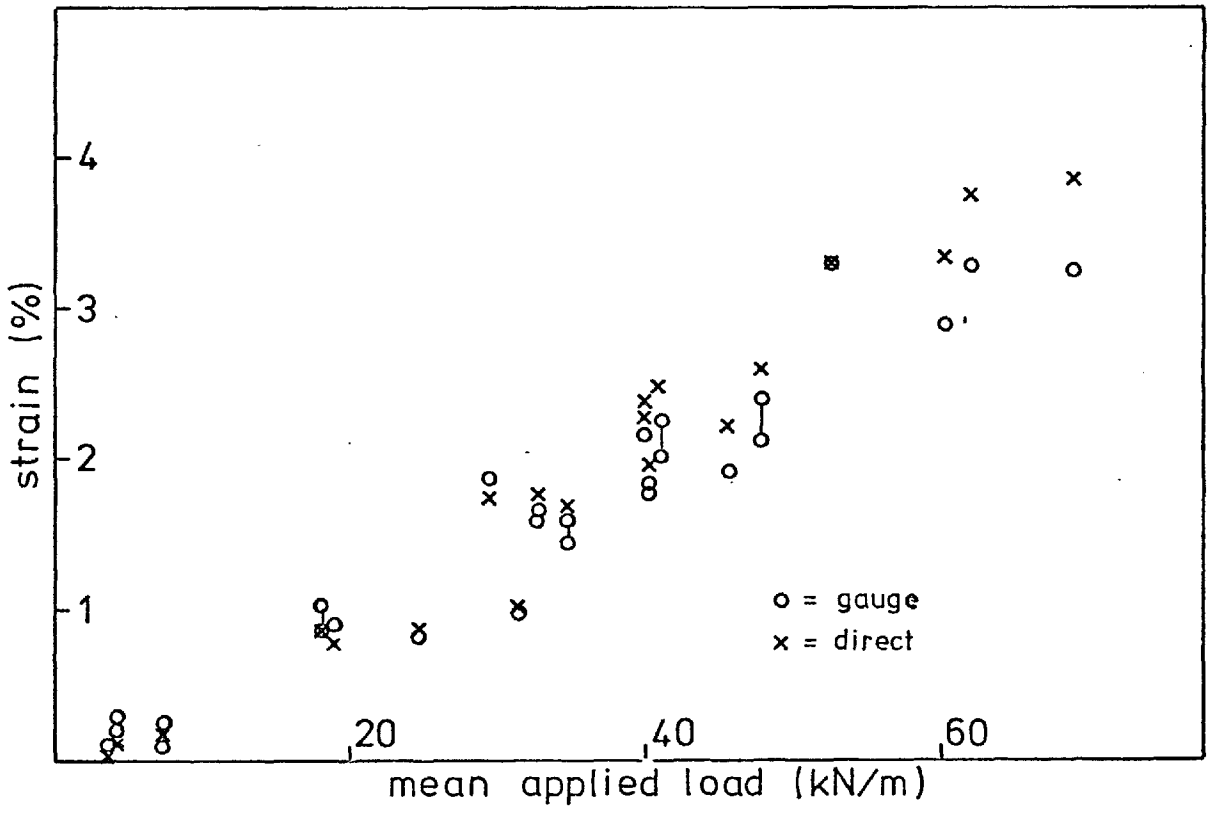


fig (5.15) GAUGE/VERNIER COMPARISON (SERIES 2)

For gauge results after straightening (ie.between roll stands),Figs 4.30,4.37,4.38 and 4.39,any discrepancy between bulk surface strains and gauge readings is due solely to the modulus effect,strain distribution being unifrom throughout the strip thickness.Where bending strains are present however,a further error is introduced.

(2) Gauge Displacement Effects.

It was noted in section 4.2.5. that the physical situation of a gauge will result in an overprediction of strain magnitudes during bending.

From the relevant equations of chapter three, the strain recorded by a gauge on an outside bend is given as

$$e_g = \frac{a + b + j}{R - b} \qquad \text{----5.3.1.}$$

where j = displacement of gauge grid from strip surface, and for an inside bend

$$e'_g = \frac{-(a + j) + b}{R + j' - b} \qquad \text{----5.3.2.}$$

where j' = thickness of gauge installation.

From the experimental results of section 4.4.3,during which micrometer measurements of gauge installation thicknesses were determined,gauge + adhesive thicknesses were found to be

f = 2.04 - 0.0013in
f = 2.06 - 0.0018in
f = 2.17 - 0.0015in ----5.3.3.

(results being given in inches to provide direct comparison with computer results which are in the same units).

Gauge thicknesses were determined as 0.0008,0.0013 and 0.0010 respectively,indicating a mean adhesive thickness of 0.0005in.Assuming the gauge grid to be at the centre of the gauge a mean value of

j = 0.0010 in

may be assumed.Selotape thickness was moderately uniform at 0.0024 in providing values of j of

j' = 0.0040 - 1 selotape layer.
j' = 0.0064 - 2 selotape layers ----5.3.4.

These latter values are negligible when compared with roll radii greater than 0.5in and will therefore be ignored.

To derive the magnitude of the discrepancies between gauge and surface strains during bending as a result of the displacement effect, it is necessary to provide representative values for the neutral axis displacement b. The value of a is simply half the strip thickness.

For the series 2 experiments

$a = 0.00645$ in ----5.3.5.

From the 'constant load' theory neutral axis displacements during bending are given for the series 2 experiments as

Applied Load (kN/m)	$b_{roll\ 1}$	$b_{roll\ 2}$	$b_{roll\ 3}$
5.2	0.00033	0.00017	0.00010
25.9	0.00167	0.00165	0.00163
51.9	0.00334	0.00324	0.00304
77.8	0.00502	0.00461	0.00434

A few elementary calculations on the basis of these results and equations 5.3.1. and 5.3.2. will demonstrate the magnitude of the discrepancies in strain values.

For the case of the outside bend, equation 5.3.1. is applicable and corresponds to bending over roll 1 and 3.(Figs 4.35/36 and 4/40/41.)

Comparison of results for j = 0 and j = 0.001 will provide the percentage error in gauge readings.The denominator is approximately constant and can be ignored.

(a) $T = 0.$ $e_g = f\ (0.00645 + 0.001)$
$e_s = f(0.00645)$
ie 15.5% increase.

(b) $T = 25.9$ $e_g = f(0.00645 + 0.001 + 0.0016)$
$e_s = f(0.00645 + 0.0016)$
ie 12.3% increase

(c) $T = 51.9$ $e_g = f(0.00745 + 0.00319)$
$e_s = f(0.00645 + 0.00319)$
ie 10.4% increase.

(d) $T = 77.8$ $e_g = f(0.00745 + 0.00468)$
$e_s = f(0.00645 + 0.00468)$
ie 9.0% increase.

(neutral axis displacements taken as mean of rolls 1 and 3).

For the case of the inside bend the denominator may again be ignored and numerator comparison made in terms of the neutral axis displacement at the second roll.Results are derived from equation 5.3.2 and are applicable to Figs 4.40/41.

(a) T = 0 e_g = f(-0.00745)
e_s = f(-0.00645)
ie 15.5% increase.

(b) T = 25.9 e_g = f(-0.00745 + 0.00165)
e_s = f(-0.00645 + 0.00165)
ie 20.8% increase.

(c) T = 51.9 e_g = f(-0.00745 + 0.00324)
e_s = f(-0.00645 + 0.00324)
ie 33.2% increase.

(d) T = 77.8 e_g = f(-0.00745 + 0.00461)
e_s = f(-0.00645 + 0.00461)
ie 54.4% increase.

Similar effects,although of greatly decreased magnitude because of the increased strip thickness,are applicable to figures 4.29 and 4.31 for the series 1 experiments.

5.4.0 Theoretical/Experimental Correlation.

5.4.1. Total Leveller Elongation and Load Loss.

The close correlation between total leveller strains and predicted values is amply demonstrated by figures 4.27,4.32 and 4.42.Exact correlation is only to be expected where wrap angles are sufficient to provide conformity (section 5.2.4),the discrepancies between theoretical and practical results for series 1,2c,3b and 3c being adequately explained in terms of this phenomenon.It does however appear that the loads required are slightly under-estimated by the approximate derivation of section 5.1.1, a result of thegreater strip/roll contact assumed by the simple derivation.The usefulness of the derivation in predicting minimum required wrap angles is however, well illustrated.

The effect of reduced wrap is also demonstrated in the correlation obtained between the theoretical and practical predictions of load losses. Again where roll radius attainment is achieved (series 2a,2b and 3a) predicted values of load losses and seem to coincide fairly well with measured values (figs 4.34. and 4.43).

Although a considerable amount of scatter is apparent in the results it is evident that the load loss measurements for other experimental series approach the theoretical predictions at the higher applied loads.

The validity of the effective friction coefficient (μ') derivation of section 3.5.0 is amply demonstrated,the derived values for each series providing the closest fit with experiment.Scatter in these load loss plots is most probably a function of load losses over the entry and exit guide roll,the constant subtraction factor which has been utilized showing some variation during its derivation.

The strength of the load loss correlation and closer fit attained between overall strain measurements and predictions with the 'full' theory,indicate that the inclusion of work losses in the analysis is beneficial.Figure 4.43 shows that work losses due to redundant material deformation are important in thicker strip and the assumption that load losses are solely a function of friction effects is insufficient for design purposes.Use of the 'constant load' theory should therefore be confined to roll diameter/strip thickness ratios no less than 100.

The increase in number of leveller rolls between series 2 and 3 provides sufficient evidence, in the first instance,for the extension of the theory to any number of rolls.The desirability of a large number of work rolls is doubted however,since experiment and theory amply demonstrate that sufficient material extension can be obtained with no more than four rolls,whilst maintaining a reasonable applied stress level.

Figure 4.32 demonstrates that minimal advantage is to be gained by increasing the wrap angle above that required for conformity.This serves only to increase load losses (Fig 4.34),but it should be noted that adjustment of wrap angles is desirable since by the control of these values any roll bending effects can also be controlled.High wrap angles often result in long edge at leveller exit.The use of low wrap angles (ie. where conformity is not obtained) is not to be recommended where the theoretical model is to be used for the prediction of levelling strains.

5.4.2. Gauge Results.

Gauge results must be discussed in the light of the error values developed in section 5.3.2, since the values plotted in figures 4.29/30/31/35/36/37/38/39/40/41 are not typical of the surface strains obtained in the bulk of the material.

The over-prediction of total strains at low loads is the result of the modulus effect,coupled with the problem of wrap angle attainment. A reduced neutral axis shift occurs where the gauge installation is on the outside of the bend,whilst an increased shift occurs when the reverse is true.In both the series 1 and series 2 experiments,wrap angles

at the outer rolls are only half that of the centre roll and bend radii at low loads are not equal to the outer roll radii.The effect of this apparently results in the increased shift at the centre roll more than compensating for the shifts at the outer rolls and a localized strain greater than the mean strain of the material strain results.

As the applied load is increased compressive stresses within the gauge at the centre roll are reduced, whilst at the same time the roll radius is attained at the outer rolls.The resultant effect is to produce strains within the strip beneath the gauge less than those of the material bulk.The above effect was indeed shown to be the case,since at low loads negative overall strains often showed up on the gauge output plots at the first and last positions.

Gauge results for the first experimental series (Figs 4.29/30/31) are seen to be similar to those of series 2c (Figs 4.36/39/41).It has already been demonstrated that roll radii are not attained (at the 1st and 3rd rolls)during these experiments and discussion of gauge/theory results is the best considered in terms of the series 2a and 2b experiments.

5.4.2.1 Bending - Series 2a/2b.

Theoretical comparisons during bending are best discussed in terms of the incremental bending strains.For roll 1, this value is equivalent to the total gauge strains depicted in Fig 4.35.For the second and third rolls incremental strains are plotted in Fig 4.40.A fairly close correlation is derived between theoretical predictions and gauge strains.This is very much the result of chance,it therefore appears that the modulus and gauge displacement effects are of equal and opposite sign for this system.

The relative magnitude of the gauge displacement effect has been shown to be 15.5 to 9.0% of surface strain for outside bends (from zero to approximately yield stress) and it is thus expected that a similar proportional decrease in surface strains will result from the modulus effect.Inner bends providing a 15.5 to 54.4% increase.To justify the theoretical model on an incremental basis an estimate of the modulus effect is therefore required.A full analysis would require a knowledge of the stress/strain relationships for a gauge installation, this however is unavailable, but a simple approximation serves as an illustration.

Applying the simple B.N.F. formula to the overall strain results for series 2a and 2b,yields a value of the working material flow stress of 51,000 lbf/in^2 (these units will be utilized to provide correlation

with theoretical neutral axis displacements).

From equation 3.6.2.

$$b = \frac{T}{2\sigma_m} \qquad \text{----5.4.1.}$$

and at 77.8 kN/m width (445 lbf/in) the neutral axis shift may be written

$$b = \frac{445}{2X51,000} = 0.00436 \text{ in.} \qquad \text{----5.4.2.}$$

For series 2,

$$a = 0.00645 \text{ in.}$$

If we assume that at zero applied load the total load taken by the gauge is L_g, we may write

$$L_g + 2b_1\sigma_m = 0 \qquad \text{----5.4.3.}$$

$$\text{and } b_1 = \frac{-L_g \text{ in.}}{2X51,000} \qquad \text{----5.4.4.}$$

From Figure 4.40 the gauge strain is 1.7 L_g at 77.8 kN/m applied load. Assuming linear gauge installation stress/strain relationships, we can write,

$$1.7\ L_g + 2b_2 51,000 = 445(\text{lbf/in}) \qquad \text{----5.4.5.}$$

From 5.4.4. and 5.4.5,

$$-1.7b_1 + b_2 = \frac{445}{10200} = 0.00436 \qquad \text{----5.4.6.}$$

However from section 5.3.2 the required decrease in surface strain at 445 lbf/in applied load is $\simeq 8.3\%$ to accomodate the 9% increase due to the gauge displacement effect.

From equation 5.4.2, theoretical surface strain may be written,

$$e_s = f(0.00645 + 0.00436) \qquad \text{----5.4.7.}$$

An 8.3% reduction therefore requires a reduction of 0.00090 in the displacement b_2

$$\text{Therefore } b_2 = 0.00436 - 0.0009$$

$$= 0.00346 \text{ in} \qquad \text{----5.4.8.}$$

From 5.4.6 and 5.4.8

$$b_1 = \frac{-0.0009}{1.7} = 0.00053 \text{ in} \qquad \text{----5.4.9.}$$

Modulus effects at zero applied load therefore result in an 8.2% decrease in surface strain.This is to be compared with the 13.5% decrease required to cancel the displacement effect of 15.5%.To provide this 13.5% decrease the equivalent value of b_1 must be 0.00087 in. the corresponding multiplication factor for the gauge load being 1.04 over the strain

range rather than 1.7.This former value is credible since gauge installation cannot really be considered to deform elastically.

Although the foregoing example is somewhat limited in its application the result does indicate that the cancellation of modulus and displacement effects is possible.Allowing for an error margin - which will be no greater than the scatter in gauge results,it is reasonable to say that during bending gauge results do in fact represent surface strains in the bulk of the material.

A similar analysis may be performed for the second bend and assuming that the stress level within the gauge is again constant with strain (as predicted by the first example) a 35% decrease in compressive surface strain at 445 lbf/in applied load (to nullify the 54% increase resulting from the displacement effect results in an 11% decrease at zero load. This is to be compared with the predicted value of 13.5% necessary to accomodate the 15.5% displacement effect at zero load.It can be reasonably assumed that the series 1 gauge strains are also representative over roll two, Fig 4.31.This figure, together with figure 4.40 again demonstrates the theoretical/practical correlation.

5.4.2.2 Straightening.

Gauge readings after each straightening (eg.figs 4.37 and 4.38) are seen to show some correlation with theoretical predictions.Gauge readings here are identical to surface strain levels,the discrepancies between theory and practise resulting from modulus effects during the bending and straightening operations.As expected measured results at the first and third rolls tend to be low as a result of the modulus effect during the straightening increment over-compensating for the effect during bending,(straightening as for roll 2 bending). The reverse is true for the roll 2 straightening operation.The latter results in strains after the second roll being underpredicted by theory,providing that roll one strains are not reduced by any great amount.

Considered as a whole however, the general trend of the curves of figs 4.30/37/38/39 upholds theoretical predictions.

5.5.0 Transverse Material Strains.

During a number of the series 2 experiments gauges were attached transversly to the strip, outputs being run at five times the amplification used for the longitudinal traces.Results for these tests showed a large amount of scatter,due primarily to the development of anti-elastic

curvature between roll stands,and are not presented.

Transverse contraction strains did however occur during levelling over three rolls,this contraction being of the order of 5% of the total longitudinal strain.Although an overall contraction was measured, transverse expansion was found to occur during straining over the second roll.An output trace for a transverse gauge is presented in figure 4.16,together with the longitudinal traces for run 2b5.From this and similar traces, it is evident that the development of transverse strains is greatly influenced by previous strain history.

As expected transverse strains in the increment are of opposite sign to incremental longitudinal strains, but their magnitudes show no definite order.It may be said however, that the magnitude of transverse strain increments is very much a function of the change in longitudinal stress at the surface.This is evident from the magnitudes of the transverse strains which occur during bending at the 2nd and 3rd rolls, the internal stress distribution (see Appendix 7) is altered only marginaly during this bending increment.

Again,gauge results are subject to the errors of section 5.3.2 but as has been discussed incremental bending strains are representative of surface strains.Gauge representation is only in error in-between roll stands and during straightening.This modulus effect may well account for the transverse expansion recorded after the second roll.

Whatever the mode of transverse strain development however, the results of these tests tend to show that deformation during tension-levelling approaches plane strain conditions.

5.6.0. Overstraightening.

Reference to section 5.1.1 and figure 5.1. will indicate the need for some over-straightening of the strip as it leaves the roll.Only if this occurs can the material be straightened and brought onto its nominal path.The longitudinal gauge traces presented in Fig 4.16 show that this does,in fact occur in practise and is represented by the discontinuity in the traces at the end of the straightening process.This effect cannot be included in a theoretical model until a complete analysis of the strip path round a roll has been derived.The effects on the theoretical basis are easily envisaged however.

The magnitude of this effect is seen to be small by fig 4.16.The small degree of restraightening required to bring the strip to its nominal path is not sufficient to induce plastic deformation.The outcome of this

that stresses within the strip during passage between rolls will be elastic with a corresponding reduction in internal moment over that predicted by the proposed theory.Since the subsequent bending operation occurs in the same direction as the major straightening after the previous bend errors resulting in the theoretical predictions will be minimal if not non-existent.One major discrepancy between theory and practise will ensue however, in that predicted curvatures after stress relaxation will be greater than those found in practise.This is amply demonstrated in fig 4.45, similar results being obtained from all other experimental runs. The scatter in these results is a consequence of the long path length between the last work roll and the exit guide roll.Under these conditions straightening appears to be a lazy operation,final curvature measurements depending largely upon the final position of the measured gauge length.

As far as predicted residual stress levels are concerened,these will be representative of those present practically, the discrepancy only being apparent in the predicted residual curvature and the strain distribution.

5.7.0. Conclusions.

Although gauge results require a considerable amount of interpretation, it may be said that these, coupled with the comparison elongation measurement by vernier,provide a good verification for the developed theoretical model of tension-levelling.The proposed method for the evaluation of minimum wrap angles has also been shown to provide results adequate for industrial design purposes.Choice of model for the prediction of levelling strains depends largely upon computer time available economically.The incorporated analysis of the required tension increase through the leveller,has distinct advantages over the 'constant load' model where predictions of load differentials are required.Unless this load differential is high however, the 'constant load' theory will adequately predict leveller strains providing that roll conformity is achieved.

The comparison given between the isotropic and kinematic hardening models provides food for thought.The discrepancies between these models no doubt explains in part the reported discrepancies between the levelling characteristics of ferrous and non-ferrous materials.A material which work-softens will most certainly require a higher load than its isotropic hardening equivalent to achieve the same strain level at similar leveller settings.The exit material will still be softer in the former case.

The method developed for the determination of longitudinal stress/strain relationships under plane strain conditions is basically simple and allows a useful prediction of behaviour under reverseloading conditions. This analysis is by no means limited to use in tension-levelling theory and it is hoped that it may be applied with equal sucess to other processes The same applies equally to the computer subroutines SXMAX and RWORK. These routines,coupled with the stress/strain relationship equations 3.2.58 or 3.2.71 for isotropic or kinematic hardening materials provide simple analyses of elastic/plastic deformation in real materials.It is hoped that the type of computer analysis utilized in this work will be substituted for the approximate analytical approaches developed so far for so many deformation processes.SXMAX is of special relevance here,its use in its present or a modified form eliminating the necessity for complex analytical description of plastic deformation.Numerical techniques are easily applied with modern computer facilities.

Adaption of program LEVEL 4 and its associated subroutine ROLL to provide numerical analysis of the roller-levelling process is a logical extension of the program for tension-levelling predictions.Combination of the programs would provide a complete analysis of the tension-

levelling and subsequent roller-levelling operations.A numerical representation of roller-levelling in isolation will however provide a useful extension to reference[41] enabling roll settings to be correctly adjusted for real materials,where hardening properties are definately non-linear.

To fully verify tension-levelling theory a more complete experimental investigation is required,incorporating a greater range of materials, number of work-rolls and roll diameter to strip thickness ratios.On the industrial scale, the British Steel Corporation are currently collecting data from a fully instrumented leveller, and it is hoped that the correlation between these and equivalent theoretical predictions will be included in the back pocket of this thesis.

There is no evidence to suggest that the proliferation of roll assemblies in industrial tension levellers yields an equivalent benefit in shape correction. The results indicate that up to $\frac{1}{4}\%$ elongation can be obtained over one roll, $\frac{1}{2}\%$ over two rolls, etc. For strip classified by industrial concerns as being of bad shape, elongation of $\frac{3}{4}\%$ will produce total correction and in most cases elongations required will be less than this figure. It therefore becomes obvious that many commercial machines incorporate up to six redundant roll assemblies.

APPENDICES.

APPENDIX 1.

A1.0. Shape Correction in Strip and Sheet.

The following pages of this appendix contain the revised form of a paper submitted for publication as a'Metallurgical Review'.The problems relating to bad shape and its correction are considered in their general terms, although some emphasis has been placed on the tension levelling process.

SHAPE CONTROL AND CORRECTION IN STRIP AND SHEET

1. Introduction

The majority of metals are processed into sheet form and their commercial success in this form depends upon their mechanical properties, their surface finish, the accuracy of gauge and the preciseness of the shape. For some domestic applications, such as Aluminium baking foil, the surface finish and properties sell the product and the preciseness of shape and gauge are not of primary import. However, for many applications the strip may not be commercially viable unless all the above factors are perfect. Typical examples would include steel or aluminium strip which is subsequently embossed or painted and copper or bronze strip produced for certain electrical contact applications.

However it is difficult to measure shape or 'flatness' and no British Standards exist to assist the manufacturer (except in the specific case of the aircraft industry); the product may be rejected because of slight differences in reflectivity noted by the customer or even because hard strip 'clicks' when lightly pressed on a marking table. Clearly there is a need for a shapemeter and standards to work to for each particular application. There is also little agreement within the metal processing industries on the methods of achieving correct shape. There are two main schools of thought, those who maintain that shape should be controlled and corrected at the cold rolling stage by variation of roll profile and those who are convinced that this is impossible and that some form of shape correction is inevitable.

The object of this review is to investigate the way in which strip shape may be controlled at the cold or hot mill or corrected after the rolling operation. In addition it will be necessary to consider the more fundamental aspects of the subject such as the origin of defects, definitiens of poor shape, formation of residual stresses, expansion of the material yield envelope and the effect of any process on the metallurgical properties.

It will be convenient to consider these facets under specific sections and it will perhaps be useful to summarise consumer requirements, bad shape phenomena, origin of defects in the processes which follow. Bad shape phenomena and origin of defects will be covered as they affect each section, thus the separate headings will be Shape at the Hot Mill, Shape at the Cold Mill, Shape Measurement, Shape Correction and Theory and Metallurgical Considerations.

II Summary

(a) Consumer Requirements for Strip and Sheet Products

During the last decade development in the sheet and strip consumer industries has been reflected in the metal rolling industries by continuous demand for more precise thinner gauge with better shape and surface quality. This has resulted in rapid progress in metal rolling techniques with most producers incorporating gauge and tension control into their cold rolling process. There has also been the development of more complex advanced mills such as the 'Szendzimir' and 'constant gap' mills which incorporate smaller work rolls with variable camber (by means of roll bending), cluster rolls and pre-stressed and hence variable stiffness housings. However where vast capital expenditure is involved, industry must to some extent remain conservative and relatively few rolling mills incorporate these recent developments. The problem of rolling to gauge has also been aided by the rapid advance of computer controlled rolling mills and hence dimensional tolerance even in the thinnest grades is no longer a real problem. The control of strip shape, although helped by techniques such as roll bending still presents intractable difficulties, especially in the thinner gauges (i.e. below 0.020"). The problem is further aggravated when good shaped strip produces distorted material after the subsequent slitting operation.

(b) Bad Shape Phenomenon

Bad shape is the result of differential elongation across the width of rolled strip[1]. These differences can only be accommodated by large internal stresses within the strip which, when the section modulus of the strip allows, may cause local elastic buckling. If the section modulus is sufficiently stiff to resist buckling, the strip may appear to have good shape, the latent bad shape being revealed by lateral bowing on subsequent slitting. In the case of strip having insufficient strength to resist the forces imposed the bad shape is immediately manifest in the form of waves or ripples extending along the length of the strip and covering the whole or a part of the width. These two forms of bad shape, 'latent' and 'manifest' are illustrated in fig. 1.

The residual stresses giving rise to these shape deficiencies may be shear or compressive in nature, the former giving rise to buckles diagonally disposed across the strip, the latter being the more common defect and producing buckles parallel to the

strip width. The stress distributions associated with these basic defects have been given by Wistreich[2] and are shown in fig. 2.

Long edge and long middle arise from fairly elementary stress patterns so it is not surprising that they are most commonly observed. As the strip thickness decreases so does the latent stress capacity and hence manifest shape defects are more often observed, very often in the more complex forms such as 'quarter buckle' and 'herring bone'.

(c) Origin of Defects

Internal stresses producing bad shape are the result of differential elongation across the width of the strip during cold rolling[1]-[5]. This is caused by local mismatch between the strip and roll-gap profiles under load. Large edge and centre buckle (long middle) are a result of differences in crown and camber between strip and roll whereas the more complex stress systems are set up by irregularities in comparative crown[6]-[17] caused by roll wear, uneven temperature profiles, ridges on incoming strip etc.[12],[13].

(d) Shape Control

To produce finished strip of good shape it is necessary to ensure that the strip crown rolled in the hot finishing train is correctly correlated with the roll gap profiles in the cold mills. Until recently there was little that could be done to correct mismatch at the cold mill due primarily to the inter-relationship between shape, crown and gauge defects[2],[5],[6]. However the developments in back-up and work roll bending techniques now offer camber control and hence roll profile variation during rolling.

(e) Shape Correction

A number of methods are available by which bad shape may be corrected after the cold rolling operation. These are:-

1. Temper rolling
2. Simple stretching
3. Roller levelling
4. Tension or roller-stretch levelling.

Temper rolling, a surface rolling process, is utilised to induce surface compressive stresses in steel and thus obtain yield point suppression. The shape correcting effect of the temper mill results from a combination of these stresses coupled with small elongation usually combined with relatively high front and back tension.

Correction by stretching is a jobbing operation usually performed by hydraulic machinery in which the strip or plate is subjected to uni-axial tension sufficient to cause yielding over all or part of the cross-section. In this process stress concentration occurs in 'short' areas which are thus strained preferentially to 'long' areas thus producing shape correction.

Roller levelling also induces longitudinal yielding but in this operation the stresses are set up by simple bending and the cross-section does not in general become completely plastic. Bending moments of alternating sign and decreasing magnitude are applied resulting in a large reduction of the internal stress levels.

The tension or roll-stretch levelling process is a combination of roller levelling and stretching, the application of tension allowing a relatively larger roll diameter to be used than in the roller levelling process and the reverse bending allows a lower tensile load than in the simple stretching process. The process is thus particularly suitable for thin strip where the reduction of tensile load reduces the risk of plastic instability but the tensile load is still high enough to help 'persuade' the strip to conform to the roll radius.

(f) Theoretical Basis of Shape Correction Processes

In shape correction processes the elastic and plastic strains are of the same magnitude. The simplifying assumption made in most metal forming analysis of a rigid perfectly plastic material cannot therefore be made. An elastic-plastic work-hardening material must be considered and calculations either using the stiffness coefficient method or deviatoric plane geometry must be attempted.

(g) Metallurgical Considerations

Since levelling processes produce only small total elongations, basic mechanical properties are in general little affected. Some materials may however display a pronounced Bauschinger effect which could result in work-softening. Other materials such as some mild steels exhibit yield point elongation and surface marking in the form of Luders bends may appear after processing, thus rendering the product unsaleable.

III Shape at the Hot Mill

To maintain good strip shape it is essential that crown at entry to the cold mill is compatible with the camber and other mill variables. Crown control at the hot mill is thus of primary import. It is affected and controlled by:-

1. Initial roll camber
2. Roll thermal camber
3. Roll deflection
4. Roll wear
5. Material structure
6. Strip temperature profile.

The strip profile most widely accepted for cold rolling is the symmetrical convex type[6],[8],[10],[14],[15] since this permits heavier cold reductions without the shape difficulties associated with flat strip. In addition the tensile stresses at the edges of the strip are reduced and the strip is less likely to suffer from 'edge cracking'. Concave and wedge profiles (see fig. 3) persist through cold rolling and will result in the shape irregularities we have already discussed.

The method by which a convex profile is produced has altered with mill design[11],[12]. Early hot mills having small diameter work and back-up rolls or, perhaps, even being of 2-4 design, were subject to large roll deflections. They were thus designed with convex cambered rolls to limit the resultant positive strip camber. The increased roll diameter and stiffness of the modern hot rolling mill has restricted roll bending effects and provided much better crown control. Flat or even concave cambered rolls are now being used.

Roll design provides rolls with camber suitable for deflection conditions for given temperature, load and material conditions. The camber experienced by any strip element deviates from this ideal because of (a) differential thermal effects, (b) load variation due to reduction variation, variation in strip width or differing mechanical properties, and (c) roll wear. Careful attention to rolling schedules (e.g. Coffin rolling - the rolling of medium, wide and narrow widths in that order) can provide maximum relief from these problems but it is clear that it is necessary to plan for their existence.

Dealing first with thermal effects, Larke[6] has suggested

that roll expansion along the barrel may be represented by a parabolic law but the results of Sibakin et al.[7](fig4) and of other investigators indicate a more or less uniform expansion over the roll centre when rolling wide strip and suggest that the width of this section of roll is proportional to the ratio Roll barrel length/Strip width. Sibakin also suggests that the distribution of expansion along the roll barrel is proportional to the temperature gradient but as in the most simple case we can obviously express the expansion of an increment dx of roll barrel by

$$\Delta dx = \alpha \Delta T \, dx$$

then one must question this statement. The Peak magnitude of the temperature is a function of the rolling schedule and can be controlled to some extent during processing by variations in the roll coolant system. Control in this way is however rather insensitive and although important cannot influence roll camber quickly enough to provide correction of local defects.

When considering the ferrous onlays work roll wear seems to be unavoidable and is greatest at the contact between roll and strip edge[6],[15]. Hence an initially uniform concave cambered roll changes first to flat and then to irregular convex after 600-700 tons of steel output (fig. 5). Large irregularities in wear location on mill rolls are closely related to gauge peaks in hot rolled material caused by areas of fine grained strip which generally will offer increased resistance to deformation [8],[9],[12],[13]. These heterogeneities in grain size can be due to differential cooling of the strip during descaling or at the first stand of the finishing train. They could also occur if strip is kept 'waiting' between roughing and finishing trains. After hot rolling these gauge peaks tend to cool differentially and produce coarse grains after re-crystallisation which are then softer than the bulk of the material resulting in preferential elongation on subsequent cold rolling. This defect known as 'ridging' can be controlled only by careful attention to strip temperatures.

Banding of hot mill rolls is reported to be caused by surface abrasion by iron oxides. This abrasion and the fact that this rapid temperature cycling produces thermal fatigue leads to rapid deterioration of roll surface quality and subsequent loss of strip shape[9],[10]. Abrasion problems are exaggerated at temperatures above 900°C because magnetite and haematite are much harder than the low temperature wustite.

This combination of roll wear, thermal effects and varying loads and deflection make it essential that some form of shape control should be attempted at the hot mill. To this end considerable research effort is at the moment being expended on roll bending methods of crown control[15]-[23]. The principle is to control shape by modifying crown profile achieved by elastic bending of the rolls. Three roll bending techniques are possible as illustrated in fig. 6. Shohet and Townsend[15] provide a rigorous mathematical model of the roll bending process evaluating the performance of the three 'set-ups' illustrated. The JWN and JBB methods cause a decrease in strip crown when loads are applied to separate the roll chocks. The JWB system counteracts work roll bending by using the rigidity of the back-up rolls to alter work roll camber. The JWN and JWB systems alter the load distribution in the same way and are far more effective than the JBB system where the effect is transmitted via the back-up rolls. The JWB system has the added advantage of maintaining the same housing load and hence producing much less gauge variation. Shohet describes two parameters necessary to describe the process as:-

(a) Crown sensitivity - the relationship between crown and roll bending force

(b) Gauge interference - the relationship between screwdown movement required to correct gauge and roll bending force.

Both of these may be approximated to linear relationships. Crown sensitivity is dependent upon mill width, strip width, mill camber and reduction. Variation is predominantly affected by strip width. Sensitivity increases rapidly with increasing width[15] and mill scheduling involving alternate narrow-wide material would be more beneficial than current practice were these roll-bending techniques introduced to large scale production. The general effectiveness of roll bending is clearly illustrated by Stone[22] who indicates that back-up roll bending alone can almost completely nullify work roll bending even under the most arduous conditions (to within 2%).

In conclusion to this section we must stress the importance of shape at an early stage in the production schedule. It is at this point in the process that irrevocable dimensions are created in the strip. It is the future cold rolling of these mistakes which creates bad shape. These remarks could also apply of course to the primary cold reduction of some copper alloys.

IV Shape at the Cold Mill

Spread during the cold rolling process is small and thus the emerging strip profile is essentially controlled by the hot strip profile presented to the cold mill. The crown correction possible at this stage is thus limited by the interaction between gauge, crown and shape. Sibakin has defined the crown (gauge relationship and the gauge shape relationship has been given by Wistreich[2] and others[21,24.24A]. The crown/gauge relationship being based on no lateral spread conditions, is constant[6]. Although this would appear to be logical O'Connor and Wienstein[25] have shown that spread of small elements can occur when no total spread of the material is obtained. Thus the crown/gauge relationship need not remain constant. There is also evidence that some flattening occurs in the temper rolling process.

The gauge/shape relationship depends upon the extent of roll-bending which is a function of roll load; hence an increase in roll load to correct a gauge error results in shape error because of the change in roll profile. The only methods available to effect gauge or shape changes are to alter the roll gap profile by means of control of back tension. At the moment shape control as practised is almost entirely by tension adjustment because of the relatively few mills equipped with roll bending apparatus. This tension control is unattractive because of the increased incidence of edge cracking which accompanies it. This is due to the tensile stresses set up at the edge of the strip, a problem investigated by Ellis[26] and others[27]-[29].

A novel means of shape control was presented in detail by Polakowski[30] and was based on an original idea put forward by Ungerer[31] which involved a low rigidity work-roll. Shape control difficulties in the normal rolling mill arise because the work rolls just are not flexible enough to match the profile of incoming strip. Thus the proposal is for a small diameter work roll backed up by staggered rolls of large diameter allowing the roll load to be distributed over the complete strip width (fig. 7). Polakowski's analysis shows the need for some rigidity in the system to avoid over-rolling at the strip edges which occurs if the rolls are completely flexible. As well as the fact that problems exist in converting this idea into a production unit, the inter-relationship between gauge and shape would mean that such a rolling mill would present gauge problems.

A logical development of this flexible roll system is the present day Sendzimir Mill[32,33] which utilizes sectional eccentric back-up roll bearings which give camber control across the strip width thus allowing shape and crown control. This has been combined with back up roll bending in the latest Sendzimir mills and enables strip to be processed to close gauge and shape tolerances. A further advantage of these small work roll (or cluster) mills is that smaller contact area reduces roll load and hence roll flattening and the cluster assembly prevents roll distortion in the rolling plane. It is thus much easier to roll harder materials to good shape and close tolerance on this type of rolling mill.

Thus cold mills are being developed which can help to correct the inadequacies of the hot mill. But it is not possible at this stage to eliminate mistakes made at the hot mill and hence bad shaped strip is produced.

In general all shape problems discussed under III and IV are exaggerated when rolling thin strip (i.e. 0.010"). This is because fractional errors in roll gap profile increase with strip thickness decrease producing larger internal stresses. As the strip becomes thinner it is of course in a worse position to withstand this treatment without buckling. Hence shape problems become more acute when rolling thin foils and tinplate gauges. This is to some extent alleviated by the increased roll flattening which obtains in these processes.

V SHAPE MEASUREMENT

We have seen in sections III and IV how rolling mills have developed and are being developed to enable some form of continuous shape control to be practised. The measurement of shape, essential for efficient operation of such mills, has still not been developed into a suitable production technique. Pearson[4] was the first to suggest that a shape meter must be developed. His instrument measured the speed differential between rollers contacting the strip across the width. This instrument could of course measure in-process shape but difficulty would no doubt be experienced by the rollers marking the stock. Wistreich[2] describes an instrument being developed by B.I.S.R.A. which measures the deflection of strip under the influence of tension by the application of a transverse point load. The deflection is a function of the total tensile strip

and thus gives an indication of residual stress and hence shape. Barreto and Hillier[7] studied this method and suggested that the transverse deflection at the strip centre would be given by

$$\frac{\text{strip deflection } (\delta)}{\text{length between supports}} = \frac{K\ (\text{point load})}{\text{Tensile load}}$$

where K = shape coefficient.

The value of K would be dependent upon the general form of the stress distribution within the strip and quoted the following values

	Stress distribution	K
1.	Uniform	0.97
2.	Long edge	0.74
3.	Centre buckle	2.35

Elementary considerations can show us that the constant K is unlikely to be independent of the experimental conditions in this way and hence these results must be deemed unsuitable.

Ward[32] describes an instrument utilizing Wistreich's principle and using a number of detection heads and a non-constant force.

ASEA and Alcan[33,34] have developed a stressometer which consists of a roll which is equipped with embedded 'pressductors'. The pressductor is essentially a transformer having a load bearing core. Alleviation of the mechanical load on the core results in a change in the coupling between the windings because of the variation in magnetic permeability. The stressometer is placed between the mill and the exit tension bridle with a wrap angle of 30°-40° and hence measures the tension and residual stress. The equipment must be combined with roll bending and variable roll coolant if all the defects shown in fig. 1 are to be dealt with.

B.I.S.R.A. have developed a magnetic shape meter[35] which utilizes the screening effect of stressed materials to magnetic flux. A number of sensing heads are positioned across the strip width and a magnetic flux applied to the strip. The output voltage then becomes a function of the stress within that zone. The meter is claimed to be able to measure a shape differential ($\Delta\ell/\ell$) of 10^{-5} per cm of width at speeds of 2000 ft/min. It could thus be incorporated in existing control systems provided that no ferrous material is being processed. The advantage of this meter is that it is independent of applied tension and is a non-contact instrument.

Another non-contact instrument has been developed by the Jones and Laughlin Graham Laboratories.[36] The meter is of the stress transducer type and hence is suitable only for ferromagnetic materials. The meter is independent of stress sensor-strip air gap and measurement is dependent on longitudinal permeability differences. This is achieved by establishing two perpendicular flux loops passing through a common secondary coil in opposite directions. Modificatinns to this basic design effectively eliminate air gap fluctuations and can also be adjusted to compensate for strip anisotropy.

VI SHAPE CORRECTION METHODS

Sections III, IV and V have indicated that with careful control of processing on modern plant it is possible to achieve a high degree of flatness in rolled products. Nevertheless most cold rolled material is subjected to further processing which generally incorporates residual stress reduction processes. This may be necessary for reasons other than bad shape because roll offsets, small differences in roll diameter and inclined pass lines together with coil curvature may result in permanent 'set' of the material [37]. This is particularly prevalent in those materials having low Young's moduli and low resistance to plastic deformation. It may also be necessary to reduce the residual stresses normally produced by correct rolling procedures[37,38] and a levelling operation may be used to achieve this (figure 8).

It should also be recognised that shape errors are frequently introduced after the rolling process during annealing or galvanizing where uneven heating or non-uniform stress conditions may prevail.

There are a number of methods by which residual stress reduction resulting in shape correction may occur:-

a) Temper Rolling

Temper rolling produces a work-hardened surface layer on the strip due to the large residual strains obtaining in that region. The core of the strip is discontinuously deformed which results in the suppression of the yield point and yield point elongation in mild steels and those non-ferrous alloys which exhibit similar phenomena[39-48]. The heterogeneous nature of the deformation process also delays the return of the yield point by a strain aging process.

This suppression of the yield point is a basic requirement of the temper rolling process for mild steels although the essentially surface deformation of the strip can apply some shape correction. This correction is enhanced by increasing the number of stands in the temper mill and hence applying lighter reductions. This is especially important in the thinner gauges (< .020) commonly used as stock in the tin-plate industry. Increasing the number of mill stands has the added advantages of allowing higher tensions to be applied and also of enabling the strip to be exposed to differing roll crowns.

It is still normal to subject strip to a roller levelling operation after the temper rolling process to remove coil set although the shape correcting facility of this process on thin gauged strip is minimal. Inevitably some strip emerges from the temper mill with shape defects of the type illustrated in fig. 1 and these can only be removed by a process which must involve substantial applied tension.

b) Stretch Levelling

This method is theoretically the simplest way of removing shape defects. The strip is gripped at either end[49] and subjected to uni-axial tension which yields part or all of the cross-section. The theoretical section VII reveals that strain equalization occurs thus reducing residual stresses to an acceptable level. Alternatively tension bridles[50-52] may be used to continuously process strip but in this case the maximum permanent extension of the strip is confined to ½%-1% which is only sufficient to remove the less serious defects as the yield strain is only just exceeded (i.e. aluminium and steel having typical yield stresses of 30,000 p.s.i. and 80,000 p.s.i. have yield strains of 0.25% and 0.4% respectively). Guttering of thin strip is prevented in both processes by ensuring a stretch span no greater than ⅔ times the width[52] . (This guttering is the result of lateral instability setting in due to the longitudinal tension.)

When stretching annealed mild steel problems are encountered because of Luders band formation. Several authors[51-54] have indicated that if stretching is continued then these bands may be eliminated. This of course will produce a less ductile material as well as risking plastic instability. A further method of eliminating stretcher-strain markings would be to use an extremely slow strain-rate. Fig. 9 clearly shows the dependence of the

yield point elongation phenomenon on strain rate but to utilize this in production would be too expensive except for those products where there is no alternative.

A tempering effect is of course observable in the stretching process (this is so in any deformation process involving plastic and elastic strains of the same order of magnitude). However the stretching process has advantages because the product will show less directionality and greater ductility than its rolled counterpart.

The deformation processes occurring in the two types of stretching process are quite different. During uniaxial stretching the stress distribution is uniform over any element of strip width and at the end of the process is uniform throughout the strip. The short parts are thus permanently elongated in preference to the larger parts resulting in a flat sheet. AEI-Herr[55] who manufacture tension bridles reason that since the strip is in equilibrium between the tension bridles then the velocity of all points must be the same and hence the strip elongation must occur over the final section of the arc of contact of the last drag bridle roll (fig. 10). Certainly the strip will be subjected to the greatest force over this area so that this reasoning would appear correct. It should not be overlooked that in this process the bending stresses will contribute to the elongation of the strip. A disadvantage of the tension bridle process is that as all the work takes place over the final roll, this roll is subjected to excessive wear leading to production down-time.

The big disadvantages of these stretch-levelling processes are the batch nature of the uni-axial process and the risk of instability in both processes. This limits the degree of bad shape which can be accommodated.

c) Contact-Bend-Stretch Rolling (C.B.S. Rolling)

C.B.S. rolling has been developed by the G.E.C. Co.[56,] during the last ten years specifically for the processing of thin gauge materials (.0002 - 0.150"). During the process material is deformed by compression whilst under the influence of bending and tensile stresses (fig. 11). The bend roll is held in equilibrium by the strip tension and hence control of reduction is effected by control of strip tension which is adjusted by a speed differential between rolls A and B. Deformation occurs at the bite points

between the contact and bend rolls and is dependent upon the strip tension, the bend roll radius and the toggle angle (see fig. 11). Plastic instability is again the limiting mechanism of the process but it is reported that high reductions are possible using this technique (i.e. stainless steel from 0.05 to 004" in eight passes with no anneal). Sections BC and DE (fig. 12) represent the roll bites where most of the material thinning occurs and at each bite a difference in peripheral speeds exists because of the strains imposed upon the strip. This indicates that in the process there are large areas of slip and unless suitable precautions are taken re roll surfaces, then strip defects will result. The overwhelming advantages of the process are the negligible roll bending due to the floating work roll principle which also ensures automatic gauge control as the roll will move into or out of the roll gap as the gauge varies.

d) Roller Levelling

Almost all finished strip products have been subjected to a roller levelling operation at some stage in processing. The process again equalizes strains but in this case does so by bending only. The strip is subjected to stress reversals by bending to alternate concave and convex profiles. The process is continuous, the strip being bent by rollers.

The proportion of the strip thickness which becomes plastic depends on the ratio of strip thickness to bend radius. The bend radius may be controlled by the roller diameter but more often it is the roll interlock which determines this parameter. It is the wrap angle and material properties which determine which of these deformation modes apply. In general then the process may be idealized as a continuous beam acted upon by alternate positive and negative bending moments of decreasing magnitude[57-64] (fig. 13). The magnitude of the applied moment is determined by the roller interlock which is gradually decreased as the material passes through the machine. The mechanism of residual stress reduction is dealt with in the theoretical section and is shown in fig. 14.

Roller levelling can be effective as a tempering process; indeed before the introduction of temper rolling, it was a common method used to remove the yield point elongation. The elongation obtainable is limited because after 4-5% elongation fluting occurs which spoils the appearance of the product.

Since roller levelling is a pure bending operation it is only possible to correct relatively small shape errors and it is usually confined to products which may reveal latent bad shape in subsequent processing. The majority of levellers are equipped with rigid rollers although recent designs incorporate rollers flexed by back-up discs which can preferentially treat sections of the material across the width. This alters the basic process because tension is introduced into the short areas by the restraining effect of the surrounding material (this may also be the case when manifestly bad shaped material is processed in a non-flexible type machine). Unfortunately this generally causes local wear on the work rolls resulting in surface blemishes on the processed material.

The roller leveller becomes ineffective when the minimum bend radius obtainable (i.e. the work roll radius) is too large to cause plastic yielding in a sufficient portion of the strip cross-section. The minimum thickness which can be levelled on a given machine may be calculated from simple bending theory if we assume that it is necessary to yield say 85% of the cross-section of the strip. (If we consider a mild steel having a yield strength of 60,000 p.s.i., the minimum strip which may be successfully processed becomes 0.020" for a 1.5" diameter roll, and 0.013" for a 1" diameter roll.) A further limit must be placed on machine design because the roll diameter is in general restricted by bearing performance and roll wear. The bearing capacity normally limits the effective line speed to 600 f.p.m.

Levellers may be of single or multi-pass design, the former are used in those industries where high outputs are required and the multi-pass designs are more commonly found in low output industries such as fabrication shops or ship yards where material of widely varying gauges and shape are used; strictly speaking, these machines tend to process plate rather than strip.

The majority of single-pass type levellers are equipped with 9-13 rollers and have single tilt mechanisms; this means that the roll interlock is gradually reduced as the product passes through the machine. Small variations in plate or strip thickness can be dealt with without adjusting the rolls. A number of modifications to the basic design are possible, the most common being the use of backed up rollers when processing wide strip. This is

necessary to avoid marking the strip (regency bands) and 4 and 6 Hi levellers are employed. A double tilt design of leveller is claimed to even the work load on the rollers and whilst this claim is undoubtedly correct, it can easily be shown that with this arrangement the final stress levels will be higher[61].

Multi-pass levellers are of much simpler design, usually having 7 rollers. The three bottom rollers are fixed and the complete top assembly of 4 rollers is adjustable vertically so that varying gauge of strip or plate may be processed. The roll interlock is the same in each span and consequently more than one pass is necessary to flatten the product.

In conclusion we may say that roller levelling is a well established process which can successfully deal with plate and strip down to a thickness of say 0.020". On thinner gauges it is difficult to obtain sufficient plastic deformation to reduce successfully the residual stress levels to acceptable limits.

e) Tension and Roll-Stretch Levelling

In section VI(e) we have seen that in the roller leveller it is difficult to ensure that the strip under process conforms to the roll radius. In addition the minimum roll diameter is limited and hence so is the amount of plastic deformation which can be applied to thin strip. The logical development of the roller leveller is therefore to apply tension thus obtaining the plastic work required; this will also ensure that the material conforms to the roll radius. Thus we can see three specific advantages in introducing tension:

i) Tension plus bending stresses induce surface yielding at very much larger radii allowing the use of larger work rolls and eliminating bearing problems. If we consider a comparable example as in (d) and using the same material properties, we find that the application of a tensile stress of 30,000 p.s.i. produces yield in 85% of a material cross-section of 0.0026" thickness using a 1" diameter work roll.

ii) Tension stresses ensure that the strip conforms to the roll radius provided a certain threshold wrap angle is exceeded. This eliminates problems arising from the inherent flexibility of the strip.

iii) On application, tensile loads will concentrate in the 'short' areas of the strip width and preferential deformation will occur in these areas. Shape correction is thus more easily achieved in the tension leveller.

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A history of the tension levelling process has been given by Bland and Alters[65] who record that the first tension levelling line was installed in 1938. Developments since that time have culminated in lines capable of processing strip up to 6'0" wide and at speeds up to 1000 ft/min. In general these lines use a flexed roller design allowing lateral pressure variation to be imposed upon the strip. The tension is usually provided by uncoilers and coiler or by tension bridles. Thus some of the disadvantages of the roller levelling process are also inherent in this process, viz: minimum roll diameters are still limited by roll bending effects and bearing problems. Lateral bending of the roll also limits the magnitude of tension applicable. In many cases the levellers are powered and elongation takes place within the machine this means that extension varies with material and the rapid wear of the rolls leads to a deterioration in the surface finish of the material.

Many of the tension lines in existence cannot provide sufficient plastic deformation to function correctly. In general this is because they have been designed simply to add tension to the roll levelling process and the roll diameter is still not small enough to cope with thin strip. Roller levellers are designed to give accurate control of roll intermesh which in that process controls residual stress relief. In the tension levelling process the strip conforms to the roll radius and to a large extent roll interlock is irrelevant. Hence in general roller levellers are not suited to the tension levelling process and indeed tension levellers could be of much simpler design.

In 1961 the British Non-Ferrous Metals Research Association[66,67] constructed a simple machine which they called the roller stretcher (fig. 15) designed for the treatment of .005"-.025" strip. Work rolls were $\frac{3}{8}$"-$\frac{1}{2}$" in diameter being supported on interlocking back-up rolls allowing an arc of contact of 40°. The work rolls were undriven, the material being pulled through by the difference in front and back tension, supplied by a coiler and a pressure brake pad. The five roll leveller imposed an anti-elastic curvature on the finished strip and this and a longitudinal curvature had to be removed by three leveller rolls at the exit of the machine. Since work roll radius and intermesh were fixed, the elongation of the strip was proportional to strip tension and experiments on bronze, brass, aluminium and steel strip confirmed

this relationship for speeds up to 140 ft/min. There were no significant changes in mechanical properties during the process although a slight deterioration in surface finish was observed. This design of machine with back-up rollers in only partial contact with the rolls inevitably leads to poor surface quality for the reasons outlined in the roller levelling section. It was found that elongations of 2% were sufficient to remove even the worst bad shape and tensile stresses of $\frac{1}{4}$-$\frac{3}{4}$ yield were necessary to achieve this. They considered that an optimum roll diameter equal to 40 times the nominal strip thickness existed but this conclusion must be subject to some criticism because factors such as roll interlock, wrap angle and tension must influence this ratio. As one would expect, stretch leveller breaks occurred at the surface of annealed mild steel being the result of discontinuous yielding. The results indicated that small variations in strip thickness had negligible effect on final extension which seems rather surprising. However one should remember that short areas of the cross-section are subjected to all the tension-pull for most of the process so that the extension is a function primarily of the number of such areas and their thickness. The specific strip thickness may therefore be largely irrelevant.

Kusakabe and Kirasama[68] used a similar pull-through type machine for experiments involving ferrous metals. The machine also allowed tilting of the upper work roll block and hence intermesh variation. These workers found that coil set (and hence anti-elastic curvature whilst the strip was under tension) increased with increasing angle of wrap. Increasing tension as well as producing greater elongation produced a similar result. The coil set could be controlled by suitable adjustment of the exit and entry roll intermesh. The explanation for this is not given nor is it obvious. The most likely solution is that both intermesh and increased tension have the effect of ensuring that the edges of the strip conform to the work roll radius.

Bushinski, Dion and Polakowski[69] have described the design of a roll stretch leveller having a floating work roll (fig. 16) which removes the restrictions imposed by bearings on the work roll diameter. Work rolls as small as $\frac{1}{4}$" are held in equilibrium by the strip tension and adequate backing-up rolls ensure, claim the authors, no deflection. The stability in the lateral direction must however be questioned and this is indeed a common cause of

malfunction in this type of machine. Polakowski found that strip elongation is a function of cumulative wrap angle as well as tension and roll diameter (fig.17). Their experiments however really only indicate that elongation is a function of the number of rolls which is not really surprising. They indicate that a minimum tension is necessary to produce flattening and these results are supported by further work by Hilsen, Levy and Polakowski[70,71].

The effect of single roll wrap angle on the tension levelling process is not yet clear although Sheppard and Roberts[72] suggest that it is a threshold wrap angle which is important. This is the wrap angle which will ensure that the material is bent to the roll radius. The loss of tension throughout the leveller must obviously increase with increasing wrap angle and this in turn must affect the levelling characteristics.

The results of all experiments so far completed indicate the effectiveness of the roll stretch process in eliminating manifest bad shape in thin strip. The BNF type design is limited because split back-up design introduces tram lining (a surface marking of the strip). Designs overcoming this problem are commercially available[55 73, 74] but the provision of a really rigid work roll still presents problems.

However it is the general case that at the present time fixed roll radius and hence elongation is dependent upon the tensile forces imposed:-

i) tension control in which elongation is allowed to find its own value.

ii) elongation control which is applied by synchronising the exit and entry bridles to give a fixed elongation.

The former method is less economical and is more complex as a control mechanism. Plastic instability could more easily occur. Generally a tension bridle is situated on the exit side of the leveller and a brake or drag generator controls the tension on the ingoing side.

The latter method gives a pre-set elongation and has advantages over the tension control machines. There is less risk of plastic instability and the system is more sensitive especially when processing with low work-hardening characteristics.

The roll-stretch-levelling process has therefore many advantages over conventional levellers and the limitations imposed by power, roll deflection, etc., will no doubt be overcome, probably by some form of floating roll design. The major advantage of the process is its ability to operate at much higher speeds than conventional levellers. A speed of 5000 ft/min[55] has been claimed but in the author's opinion this does seem an exaggerated claim.

VI THEORETICAL CONSIDERATIONS

a) General Comments

Shape correcting processes as we have seen involve strains which are of the order of 1%-5%. Thus the elastic strains can be the same magnitude as the plastic strains and cannot be ignored in analysis. The general three-dimensional elastic-plastic problem remains insoluble and hence certain assumptions must be made to facilitate solution. In shape correction processes we are usually dealing with strip which is wide in comparison to thickness ($\frac{\text{width}}{\text{thickness}} > 10$) and hence we can assume that plain strain conditions obtain. We may assume therefore that $\varepsilon_z = 0$. Since we are considering thin strip which in all processes has at least three surfaces force-free, we may further assume that $\sigma_y = 0$.

It is well known that the stress tensor may be resolved into hydrostatic and deviatoric components and that the hydrostatic component has no significant effect on the yield envelope. When dealing with elastic-plastic problems it is necessary only to consider the deviatoric stresses which can be represented graphically on the deviatoric plane. It is usual to assume the V n Mises yield criterion to apply and this projects on to the deviatoric plane as a circle radius $\sqrt{\frac{2}{3}}\,Y$. As the three axes on the deviatoric plane project as $\sqrt{\frac{2}{3}}\,\sigma_x'$, $\sqrt{\frac{2}{3}}\,\sigma_y'$ and $\sqrt{\frac{2}{3}}\,\sigma_z'$, the yield radius is usually plotted as equal to the yield stress in simple tension and the axis scaled up accordingly.

Hill[75] proposes the use of the Prandtl-Reuss equations for those cases where the principal axis of stress and strain are coincident and the material work hardens. These are:-

$$d\varepsilon_{ij} = \frac{3\sigma_{ij}'\, d\sigma'}{2\,\bar{\sigma}\,H'} + \frac{d\,\sigma_{ij}'}{2G}$$

These equations may be represented graphically on the deviatoric plane and the reader is referred to the books by Hill[75] or Ford[76] which give fine explanation.

The compressibility equation written in deviatoric form

$$\varepsilon_x' + \varepsilon_y' + \varepsilon_z' = 0$$

is the other fundamental law of mathematical plasticity necessary to effect a solution.

The restraints imposed upon the stress-strain curve by the assumptions above result in a revised $\sigma_x - \varepsilon_x$ relationship. This is the relationship which determines residual stress relaxation in all shape correction processes. Sheppard and Roberts[77] have shown how this relationship may be derived. It is necessary for a work hardening material to trace the expansion of the yield envelope as the material is processes and by using the above formulae and the graphical representation in fig.18 , the expansion $d\bar{\sigma}$ may be related to the current yield stress, Young's modulus, Poisson's ratio and the prevailing work hardening parameter by the equation

$$d\bar{\sigma}_{n-1} = \frac{\sigma_{n-1} d\alpha \left[\frac{2-\nu}{\sqrt{3}} \cos(\alpha_{n-1} + \frac{d\alpha}{2}) + \nu \sin(\alpha_{n-1} + \frac{d\alpha}{2})\right]}{\frac{E}{2H'}(\cos\alpha_{n-1} - \sqrt{3}\sin\alpha_{n-1}) - \frac{2\nu}{\sqrt{3}}(\sin\alpha_n + \nu\cos\alpha_n)}$$

It is necessary to trace each incremental expansion by computer and hence transform our normal tensile stress-strain curve. The iteration is comparatively straightforward, provided that the $H'/\bar{\sigma}$ relationship for simple tension is known.

Attainment of good shape depends upon complete equalization of residual stress throughout the material. In general certain portions of the strip must be strained more than others and thus assuming an homogenous isotropic material work hardening will ensure that this total equalization cannot take place. The greater the work hardening index n in the stress strain relationship (i.e. $\sigma_x = A(B + \varepsilon_x)^n$ the more difficult will be shape correction. Thus some annealed stainless steel sheets will be very much more difficult to process than will, say, a 40% worked commercially pure aluminium.

One more general property of metals is necessary for the solution of residual stress problems. It must be assumed that

on unloading after processing, each fibre will trace a path on the stress-strain curve parallel to the elastic loading path.

Some comments must be made on the assumptions made in all the theories:-

1) In general the assumption of plane strain has been documented in many processes to be valid providing the width/thickness ratio is greater than 10. For shape correction of thin strip this ratio is usually exceeded.

2) The incompressibility relationship when applied to materials which have been reduced by greater than 20% from the ingot stage is generally accepted.

3) Although in some shape correction processes the rolls will exert a pressure σ_y on one surface of the strip, the stress on the opposite surface is zero. There is evidence that in similar cases (for example thin shells) this assumption does not lead to gross errors.

4) The assumption of a Von Mises material has been proved the best yield criterion available.

5) A further assumption is that sections plane before bending remain plane afterwards. For the geometry under consideration the work of Denton[78] shows this to be reasonable.

6) In some theories a perfectly plastic materis is assumed. It has been demonstrated that for these processes the elastic and plastic stresses are comparable. Sheppard and Roberts[77] show that this assumption can lead to serious errors.

b) Stretching

All residual stress systems consist of tensile and compressive stresses so balanced that they satisfy the equilibrium conditions. This usually means that the internal stresses must summate to zero bending moment and force. Fig. 19 shows a residual stress system typical of the type which causes edge buckle & indicates how the application of uniaxial tension can remove this defect. Forrest[53] and Martin[49] have previously discussed this mode of stress relief. When the compressive and tensile regions are both strained sufficiently to deform them plastically, the internal stress variation is reduced to a small proportion of its original value. The material on unloading will recover elastically thus retaining this small differential in stress. If the initial stresses in the centre of the strip and at the edge are σ_c

and σ_s then on straining to say 1.4% the stresses become σ_{c1} and σ_{s1} which on elastic recovery become σ_{c2} and σ_{s2} corresponding to a permanent strain of about 1%. The stress differential between the centre of the strip and the edge is thus considerably reduced and bad shape may be reduced in this way. A similar argument may be applied to strip having a permanent coil set where the stress differential exists throughout the strip thickness. It is clear however that unless the material is non-work-hardening then the curvature cannot be completely removed.

It is well known that sudden changes in cross-section produce stress concentrations. Hence in the stretching operation it is essential that edge cracked or faulty strip is trimmed and corrected before stretching otherwise the concentration of stress around these irregularities may tear the strip apart.

c) Roller Levelling

It has been shown[61] that the roller levelling process can be analogued as a continuous beam, the interlock of the rollers giving the central deflection and the roller pitch giving the span. Stress distributions in elastic/plastic beams have been investigated by many authors and a number of analyses have been proposed[57, 61-63].

Siebel[62], Lawson[57] and Hayashi[63] have all analysed the roller levelling process using an elastic perfectly plastic model which, as already indicated, can lead to errors of a considerable magnitude. It is especially important to note that even for mild steels the assumption of perfect plasticity will not be satisfactory when processing under conditions where the total applied strain is only two or three times the yield strain. Bending moments, roll loads and power consumption will all be underestimated using these analyses with obvious results. However, bearing in mind the above considerations, it is worth noting the formulae given by Lawson for power consumption in a roller leveller having n rollers as

$$P = bt\sigma_y w \sum_{2}^{n-2} \varepsilon_r \left(1 - \frac{\varepsilon_y}{\varepsilon_r}\right)^2$$

Sheppard and Roberts[77] have indicated that this result should be increased by about 12% to give a workable consumption figure.

Sheppard[61] has analysed the problem for a work-hardening material using a bi-linear stress-strain model. The analysis clearly indicates the way in which the residual stress system in

the strip is modified and continuously reduced by the cyclic bending stresses imposed. The analysis may be criticised because no allowance is made for increase in yield as the strip progresses through the machine but it is shown that for a number of materials the Bauschinger effect (which we shall discuss in VII) annihilates the work-hardening of the material to give a constant yield stress. Hilsen and Polakowski[70] have in fact observed work softening effects at low strains ($<1.0\%$) in the tension levelling process which is not subject to such complete reversals of stress. Sheppard derives the following formulae for the residual curvature of a narrow beam after processing:

$$R_{residual} = \frac{Et}{2\sigma_y}\left[\{\phi\}_{\lambda_1} - \{\phi\}_{\lambda_2} + \{\phi\}_{\lambda_3} \cdots + (-1)^{n+1}\{\phi\}_{\lambda_n}\right]^{-1}$$

where $$\{\phi\}_{\lambda_n} = \left[1 = b\left(\frac{2y}{\lambda_1 t} - 1\right) - \left(\frac{M_n}{M_y}\frac{2y}{t}\right)\right]$$

The bending moment at any roller$_n$ is given by:

$$\frac{M}{M_y} = 2\left[1 - \frac{\sigma_y R_n}{Et}\right]$$

These equations will give an accurate assessment of bending moments together with stress values and residual curvatures and may be utilized to relate applied loads to deflection and residual curvature. The equations derived for narrow beams must be corrected for plane strain conditions. The moment curvature diagram (Fig.20) illustrates how the roller levelling process may be controlled by adjusting the applied bending moment over the first roller. General industrial practice involves adjustment to the moment-curvature relationship over all the rolls by tilting the upper chock assembly and is thus a more complex procedure.

d) Tension and Roll Stretch Levelling

The analysis of the tension levelling process again reveals a dependence on bending theory. However in this process the material is constrained to conform to the roll radius so that an analogy to the beam problem is not valid. The problem must be solved by considering the bending of strip round a roll under superimposed tension under plane strain conditions. As in the roller leveller stress reversals will occur and the Bauschinger phenomenon may affect mathematical considerations. However it is generally

the case that work-hardening does occur and hence the problem must be solved by applying the mathematical theory of plasticity summarised in (a).

The problem of bending with superimposed tension is one which is also important in the deep drawing industry and consequently the 'simple' theory has been well examined. Swift[79,80] investigated the thinning producing in strip wrapped around a roller under superimposed tension and developed a theoretical analysis for a linear work hardening material. He applied his logic to both bending and unbending conditions. The analysis cannot be generally applied since it was assumed (not unreasonable for the case of deep-drawing) that the complete cross-section became plastic; also the analysis was for narrow beam conditions. The analysis further ignored the intermediate springback stage between bending and unbending which can produce erroneous results in the residual stress calculation. Nevertheless, the general principles expounded provide a sound basis for a complex leveller theory which can give a true indication of the neutral axis 'shift' when bending and tension stresses are superimposed.

Bland and Aters[65] have published a totally unacceptable elementary theory for the tension levelling process in which they assume the neutral axis to remain on the strip centreline, the strip to be narrow, an elastic perfectly plastic material and that the centre of the strip remains elastic even although it is necessary for a permanent elongation of the strip to occur.

Kusakabe and Hirasawa[68] suggest that to obtain the same straightening effect in differing materials exhibiting identical bad shape characteristics, it is only necessary to apply curvature such that the ratio $\frac{\text{Applied Curvature}}{\text{Curvature to produce Yield}}$ remains constant. This conclusion would be of some value if the varying parameter in the process were roll diameter, but the control in this case is exercised by elongation or by the tension level. The paper adds little to the understanding of the mechanics of roll-stretch levelling.

The calculation of residual stresses and springbacks in sheet metal under simple bending conditions has been investigated by a number of authors[61, 81-94]. Queener and De Angelis[94] provide the most correct theoretical analysis for materials exhibiting work hardening and strain under plane strain conditions. They make the usual assumptions and use a stress-strain relationship

$\sigma = A\ e^{m}$ (although it is difficult to fit this law to real materials) Their expression for residual curvature then becomes:

$$\frac{R_{roll} + \frac{t}{2}}{R_{residual}} = \left\{1 - \frac{3A(1-\nu^2)}{E(2+m)(\frac{3}{4})^{1+\frac{m}{2}}}\right\}\left\{\frac{2R_{roll}}{t}\right\}^{1-n} + \left\{(\frac{2R_{roll}}{t})(\frac{A}{E})^{\frac{m}{1-m}}\right\}^{3}\left\{\frac{3(1-\nu^2)^{3+m}}{(2+m)(\frac{3}{4})^{\frac{1+m}{2}}(1-\nu+\nu^2)^{\frac{1+m}{2}}} - \frac{(1-\nu^2)^3}{(1-\nu+\nu^2)^{3/2}}\right\}$$

This expression gives residual curvatures which substantially agree with their experimental results.

Benedyk and Newham[95] extend the simple springback theory and consider the effect of bending and tension and point out that the addition of tension reduces springback effects. The theory developed makes no allowance for the plane strain conditions imposed in their experimental work (the width/thickness ratio varied from 300 to 500), nor did it include work hardening effects . though the materials processed were copper alloys exhibiting little work-hardening.

Alexander[96] has analysed the elastic-plastic springback of wide plate. He assumes work hardening and uses the incremental form of the Prandtl-Riuss equations to develop a graphical solution in the deviatoric plane. For the perfectly plastic case it is also possible to obtain an analytical solution giving:-

$$E(\varepsilon_x - \varepsilon_{x_o}) = (1-2\nu)\sigma_2 + \frac{\sigma\sqrt{3}}{4}\log\left\{\frac{\frac{2}{\sqrt{3}}\sigma + \sigma_x}{\frac{2}{\sqrt{3}}\sigma - \sigma_x}\right\} - \sigma\left\{\frac{\nu(1-2\nu)}{\sqrt{1-\nu+\nu^2}}\right.$$

$$\left. + \frac{\sqrt{3}}{4}\log\frac{\frac{2}{\sqrt{3}} + \frac{1}{\sqrt{1-\nu+\nu^2}}}{\frac{2}{\sqrt{3}} - \frac{1}{\sqrt{1-\nu+\nu^2}}}\right.$$

The residual stress systems may be determined on the deviatoric plane (fig. 21) for both the simple tension and simple bending cases. The paper is fundamental to the subject and was

published considerably earlier than other references.

Denton[97] extends Alexander's work by considering the work-hardening case and concludes that a graphical solution on the deviatoric plane is impractical because it is a trial and error method and extremely time consuming. He develops equations on the deviatoric plane which are completed by computer programmed but the equations are mathematically non-descriptive and there is really little to recommend this work.

Marcal[98] criticizes Denton's work claiming that the complexity of his equations and computer techniques are overcome by a stiffness method[99] giving a numerical solution of the Prandtl-Riuss equations together with yield criteria. The method utilizes the fact that in incremental form these equations are linear and can be expressed in matrix form. It is then only necessary to invert the matrix to obtain a solution. The major disadvantage of this method is that it requires excessive computer time for materials exhibiting little work hardening and cannot deal with perfectly plastic materials. The deviatoric plane also enables the metallurgist to obtain a visual indication of progressive deformation.

The modified stress-strain relationship obtained by the method[78] described in VI(a) can be used in conjunction with formulae developed by Sheppard and Roberts[100] to calculate the extension during tension levelling. The mechanics of tension levelling are simple (see fig. 26) for the non-work-hardening case. The material may be considered to be successively straightened, bent, relaxed and re-straightened over each roll and the cumulative effect of such stress cycles calculated. Work-hardening however necessitates the consideration of each longitudinal fibre separately and the strip must be divided into such small elements and the computation entrusted to computer. The reference describes how this is achieved and gives examples of experimental and computer results. These results are in close agreement.

The determination of residual stresses by experimental methods has been dealt with in an excellent 'Metallurgical Review' by Denton[101] and will not be discussed further.

e) Shape

The theoretical treatment of strip which exhibits bad initial shape has received little attention. It has been shown earlier that differential elongation over the width of the strip is the

basic mechanism of correction and several authors[57,78, 100] point out that the residual stresses are to some extent masked by the stresses imposed at the start of the correction process. However it would be straightforward to incorporate the initial strip conditions into formulae provided those conditions can be defined.

The shape criteria must therefore be reduced to a mathematical model expressed in relative longitudinal strains or stresses across the strip width. Pearson[4] has suggested such a model to describe edge and centre buckle effects. He suggests that these defects may be represented by a sinusoidal relationship, fig 22 pronounced bad shape being represented by a linear variation in amplitude from edge to middle and variation in phase lag across the strip width. This method permits the comparison of actual arc length with the stress free length of strip and hence shape may be expressed as a function of wave length, amplitude and stress free length. The relationship becomes:

$$L = \Psi \left[1 + \left(\frac{a\pi}{2\Psi} \right)^2 \right]$$

For the simple case shape may be expressed as the fractional difference between edge and middle so that

$$\text{Shape} = \frac{L - \Psi}{\Psi} = \left(\frac{a\pi}{2\Psi} \right)^2 = \frac{\Delta L}{L}$$

The values of a and Ψ are dependent upon material stiffness in the lateral and longitudinal directions, material thickness and the strip width (w) over which differential elongation occurs. Pearson defines bad shape as the fractional difference in length per unit width:

$$S = \frac{1}{w} \left(\frac{a\pi}{2\Psi} \right)^2$$

Where w is less than half the strip width this shape parameter may be measured because it is approximately equal to the longitudinal curvature of narrow bands of slit strip.

f) Theoretical Conclusions

The theories developed to date are able to predict the final stress and strain conditions in initially flat strip or strip which has a simple curvature at the onset of processing. Some of these theories may be extended to cover strip which has initial bad shape and the shape metres described under section V should ensure that the necessary information becomes available. We shall see in Section VII that the Bauschinger effect, strain rate effects and strip

directionality can further modify stress-strain relationships and the present state of mathematical development cannot process these deviations in full.

VII METALLURGICAL CONSIDERATIONS

Modern metallurgy and metallurgical courses should equip the metallurgist to deal with all of the previous sections; in this section, however, we shall deal with those aspects which come under the envelope of classical metallurgy.

Cold deformation of materials will affect their mechanical strength and in general their surface finish. Shape correction processes will affect these parameters particularly if the material exhibits a discontinuous yield or has a large initial grain size, the Bauschinger effect (which reduces yield stress) and the strain rates imposed by the process will be the factors which cause greatest variation of properties.

As might be expected in the processes which have low strain rates continuous material yield properties and negligible Bauschinger effect the change in mechanical properties correspond to those obtained for a similar elongation in the simple tensile test[66,68,70,71] The reduction of internal stresses may however increase the useful ductility when compared with test results. The reason for this is that in roller or tension levelling the overall strain contributing to work hardening is greatest at the surface of the specimen and the tensile test will measure only the mean ductility. The greatest effect on such materials is that the surface reflectivity is slightly reduced rendering the material unsuitable for those applications demanding a highly polished finish[66-71]. A large initial grain size can produce 'orange peel' effect[66,68] and discontinuous backing up of the work rolls may result in a defect known as 'regency stripes' or 'tram lines'. This latter problem may be eliminated by the use of well lubricated full width back-up rolls, six-high design or floating work roll techniques[69-71, 102].

In the theoretical section we have assumed that negative and positive stressing have a cumulative effect on work-hardening. This is not always strictly correct because several authors have recorded work-softening when processing black plate, galvanised sheet and hard rolled strip. The kinematic straining effects are quite different from the simple isotropic model because the Bauschinger effect may predominate.

The Bauschinger effect which reduces yield stress on stress reversal is due to the fact that on positive straining dislocations pile up on slip planes producing a back stress which on subsequent negative straining aid the deformation.

The elongation and residual stress correction achieved in roller and tension levelling has been shown to be dependent upon loading conditions which in general subject the strip to reversals of stress. The Bauschinger effect will thus result in work softening and it would be expected that an overall increase in strain would result. In fact this does not occur[71] and it will be useful to explain this phenomenon by summarising the various metallurgical models:

(1) Isotropic Hardening[103-105].

Stress increases due to hardening effect act similarly in tension and compression to produce a cumulative effect. Similar overall strains will therefore predominate during bending and unbending at each roll.

(2) Basic Kinematic Model[103-107]

Stress reversals result in reduction in the yield stress. The centre portion of the strip during tension or roll-stretch levelling is subjected to tensile stress at all time. Thus the stress loss occurs in those surface portions of the strip which experienced compressive stress. The relative stress list is thus confined to those areas and a smaller 'shift' of the neutral axis than predicted by theory will occur both during bending and unbending. The elongation at each roll, being directly proportional to neutral axis shift will therefore be less.

(3) Kinematic/Isotropic Model[104, 105, 108]

This model suggests that in reversed loading a decreased flow stress will occur provided that the applied strain is not greater than twice the material pre-strain. It also predicts that the work-hardening index is increased within this strain band. Similar reasoning to (2) will apply but the increased work-hardening rate will reduce the effect over simple theoretical predictions.

For constant overall strain in a levelling operation the applied stress (and hence loads) will be greatest for the simple kinematic model and least for the isotropic model. Real materials will be more likely to concur with model (3) and must produce variation with simple theories. It should also be noted that the results of

Rolfe et al[109] indicate that the Bauschinger effect is dependent upon the pre-straining conditions, the effect being much larger for example under plane-stress conditions than under plane-strain conditions. This is of course because of the restraints imposed by the plane-strain process.

The reasoning given above explains the decrease in overall strain reported by Hilsen et al[70 71] when tension levelling various materials and the increased tension required over that predicted for perfectly plastic materials.

The other predominant metallurgical phenomenon is the dependence of material behaviour over the strain-rate imposed during processing. For a roll diameter/thickness ratio of 40 surface strains will be about 4% and will take place over a distance very nearly equal to the roll radius. For rolls varying from ½"-1" average surface strain rates will thus be 4-2sec^{-1} and mean strain rates 1-0.5 sec^{-1}. As standard tensile tests are conducted at strain rates 10^{-2}sec^{-1}[110-117] some modification of the yield curve may be expected. Roberts[110] indicates that for low carbon steel the increase in strain rate from 10^{-2}-1 sec^{-1} results in a 40% increase in yield stress. The strain rate will of course vary as the strip bends around the roll being considerably greater at entry than at exit so that the controlling rate will be lower than the average ratio given above. We may therefore regretfully conclude that in the absence of experimental measurement these efforts must remain a matter for conjecture but that the general result must be to counteract the Bauschinger effect.

The tension levelling of steel strip is not at the moment a common process in that industry. However roller levelling is applied to almost all strip produced and the effect on surface finish when processing materials with a sharp yield point is an important metallurgical phenomenon. The sharp yield is characteristic of practically all grades of rimmed, capped and commercially produced steels in the hot rolled as annealed condition[5] unless care is taken to remove carbon and nitrogen atoms from the solid solution. These atoms pin dislocations and subsequent straining requires a stress great enough to cause nucleation of further mobile dislocations (i.e. the upper yield stress). Rapid dislocation multiplication at this high stress level results in a yield drop because then a lower stress is

required to produce deformation. This mechanism leading to the plastic instability is known as yield point elongation.

This instability propagates through the material as Luders bands, more commonly referred to as stretcher strains. The Luders front is the region between yielded and unyielded material and is about one grain in thickness. In conditions where a number of fronts exist (due to dislocation nucleation at a number of high strain regions) variations in lower yield stress can be detected as fronts combine and are further nucleated.

Thus yield drop and yield point elongation are functions of dislocation dynamics[118-124] and the shape of the stress-strain curve is rate dependent. Butler[120] shows that in the simple tensile test:-

$$\log \bar{V}_L \propto \frac{1}{\sigma_L}$$

and

$$\epsilon_L = \frac{V_c}{n \bar{V}_L}$$

so that $\sigma_L \propto \log \epsilon_L$

Hence an increase in strain rate produces an increase in lower yield stress and in yield point elongation. Grain size[124-127] also has an important role in yield point elongation since grain size influences the yield flow stress and hence the front velocity. Thus fine grains give rise to high yield elongations.

There is an extensive literature on discontinuous yielding[39-48, 102, 128, 129] in steel and the effect of residual stresses, deformation mechanisms and strain aging on yield behaviour. Much of this work considers the effect of temper rolling and roller levelling because stretcher strains are unacceptable where good surface finish is required. Verduzco and Polakowski[102] have extended Butler's analysis to show the interaction between yield phenomena and roller flexing in the tension levelling process. They indicate quite logically that this spacing of surface markings is dependent upon roll diameter, line speed, wrap angle and grain size. The parameter exercising the greatest control is the roll diameter/thickness ratio, the Luders band frequency increasing as this ratio decreases. This of course is because of the larger stress levels existing at low diameter/thickness ratios which induce yield at

more frequent intervals. This 'fluting' effect is explained by the elastic deflection of the strip away from its nominal path (fig. 27) which leads to localized yielding along a transverse line. The band frequency is also a function of strain rate because at some threshold speed the Luders front velocity is unable to keep up with the strain rate and new fronts must be nucleated in proportion to the line speed. Increased wrap angle also increases the frequency of Luders bends suggesting that in the work reported a further deviation from roll radius occurred at high wrap angle. Alternatively this may be explained by the greater tension which increases exponentially around the roll.

Luders band formation during roller levelling and the subsequent surface deterioration was the reason for temper rolling replacing this process as a means of yield point suppression. This can be achieved by a light skin pass using reductions of 0.5 - 1.25% and because of the highly polished rolls produces an excellent surface finish. The mechanism of suppression in temper rolling has been extensively examined (39-48) and it is accepted that during alight skin pass alternating bands of yielded and unyielded material form along the strip length but only in subcutaneous fibres. This inhomogeneous deformation means that on subsequent straining Luders band nucleation occurs at the undeformed/deformed interfaces and does not affect the surface appearances of the sheet. Thus temper rolled materisl is not subject to characteristic strain marking, an effect which the ageing process will remove. Butler and Wilson[44] give the limiting reductions for this process as 0.5 - 1.0% if work hardening is not to occur.

Thus temper rolled material with the yield point masked in both the longitudinal and transverse directions is ideal for further forming processes (e.g. the forming of automobile bodies) but in rimmed and capped steels storage at room temperature results in strain ageing which restores the yield point elongation characteristic. (This is due to segregation of carbon and nitrogen interstitials to disbcations). Directionality has a pronounced effect on the ageing rate[130], the return of the yield point elongation being much more rapid in the transverse than the longitudinal direction for these materials subjected to small reductions and the opposite for more heavily rolled materials.

The explanation of this orientation dependence is explained by the distribution of tesselated stress within the material; these high localized stress concentrations affect the load/elongation curve because the stress levels must be sufficientto overcome the effects of dislocation priming. The flow stresses involved will thus be a function both of the directionality and the ageing characteristics of the material. For any material Hundy[42,43] has shown that the ageing necessary to promote the yield point return is dependent upon the mechanics and extent of pre-straining. He shows that light roller levelling which just corrects the initial stress distribution promotes a rapid yield point return whilst heavy roller levelling introducing a new macro-stress distribution delays the return of the yield point.

We may therefore conclude that the greatest unknown metallurgical factors in the tension levelling process are the effects of work softening and Luders band formation and this should be a fertile research area. For non-ferrous materials the 'orange-peel' defect and its relation to shape correction processes requires analysis. From the information available, the ageing characteristics of most ferrous and non-ferrous alloys should be predictable.

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J.I.S.I. 201, 764 (1963)
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J.I.S.I. 206, 911 (1968)

fig (1) —— see fig (1.1) of script
fig (2) —— see fig (1.2) of script

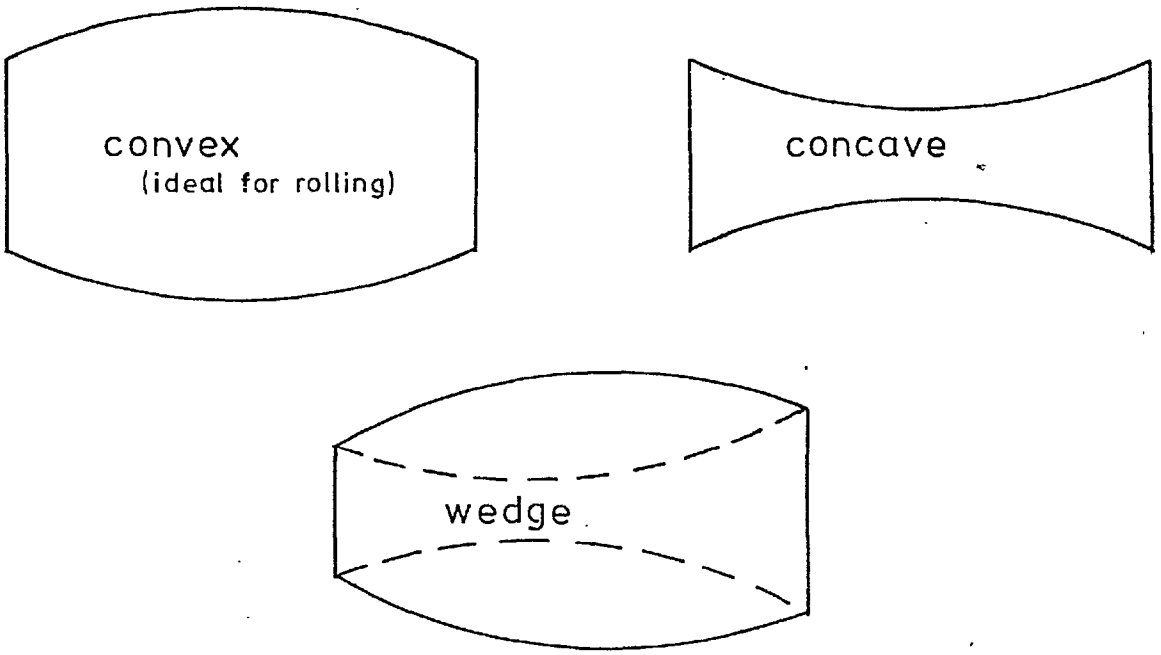


fig (3) STRIP PROFILES

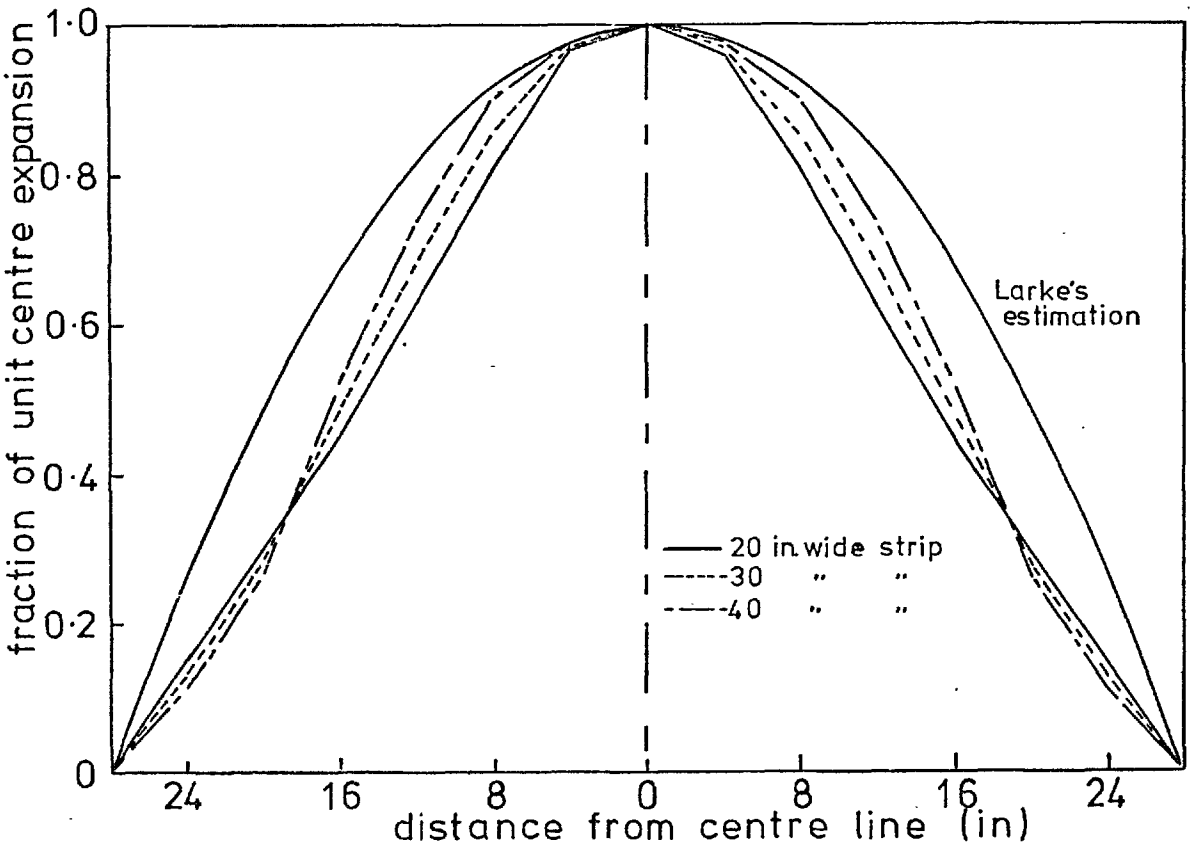


fig (4) THERMAL EXPANSION PROFILES (7)

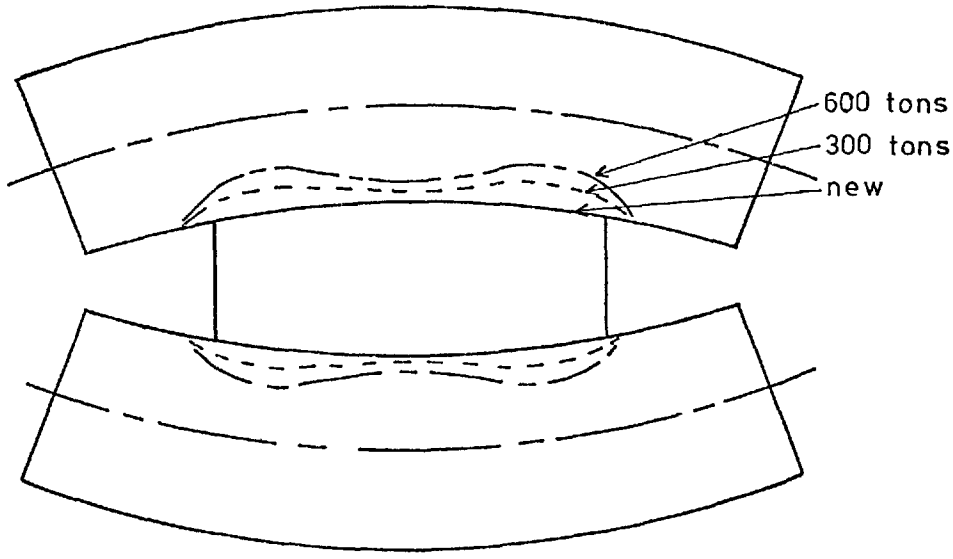


fig (5) ROLL WEAR PROFILES

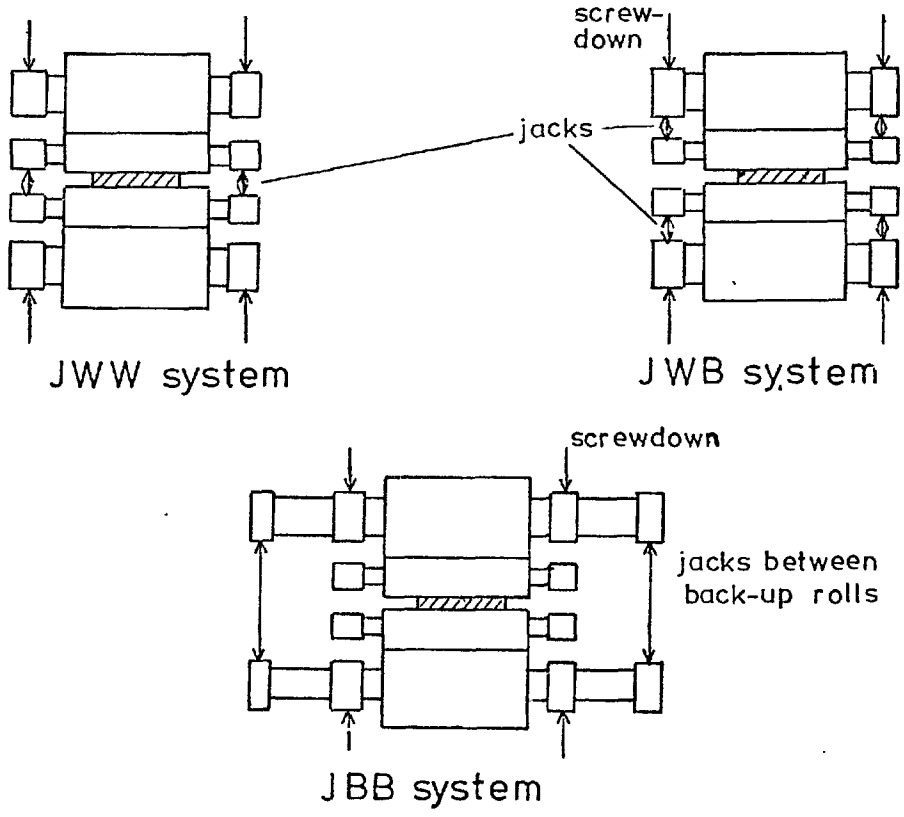


fig (6) ROLL BENDING TECHNIQUES

fig (7) —— equivalent of fig (1.4.4) of script

fig (8) —— see fig (3.31) of script

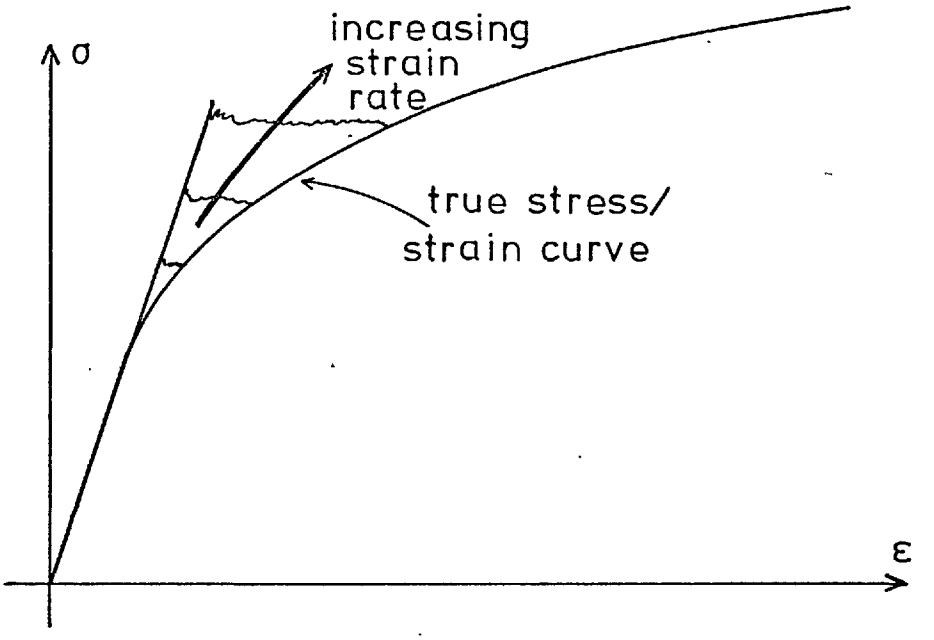


fig (9) RATE EFFECT ON YIELD POINT-ELONGATION

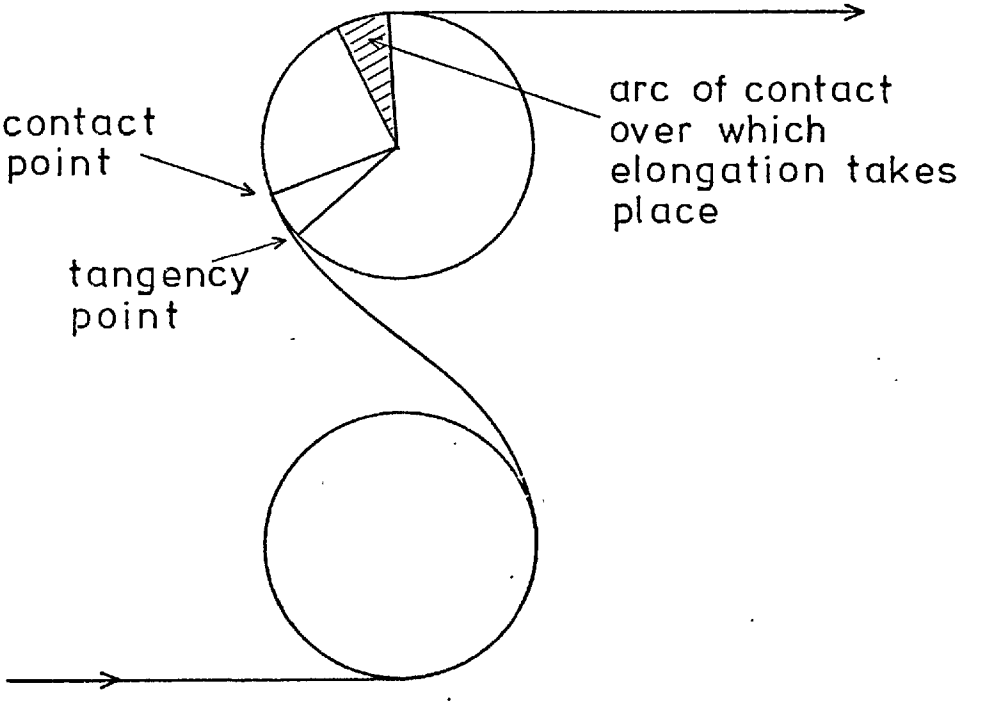


fig (10) ELONGATION POSITION - CONTINUOUS STRETCHER LEVELLING

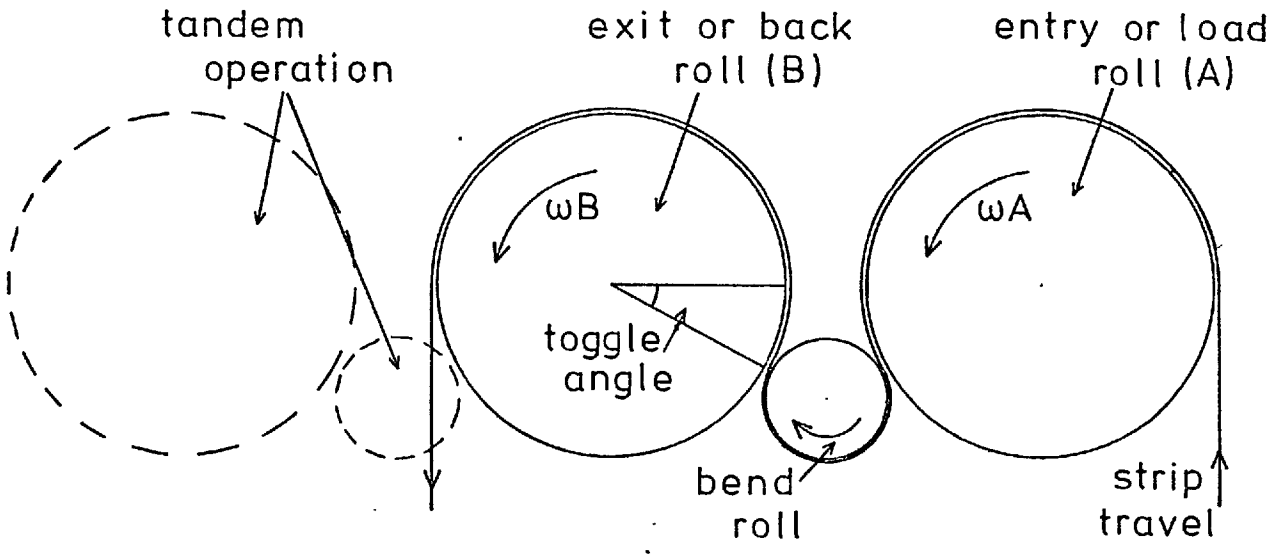


fig (11) THE C.B.S. SYSTEM

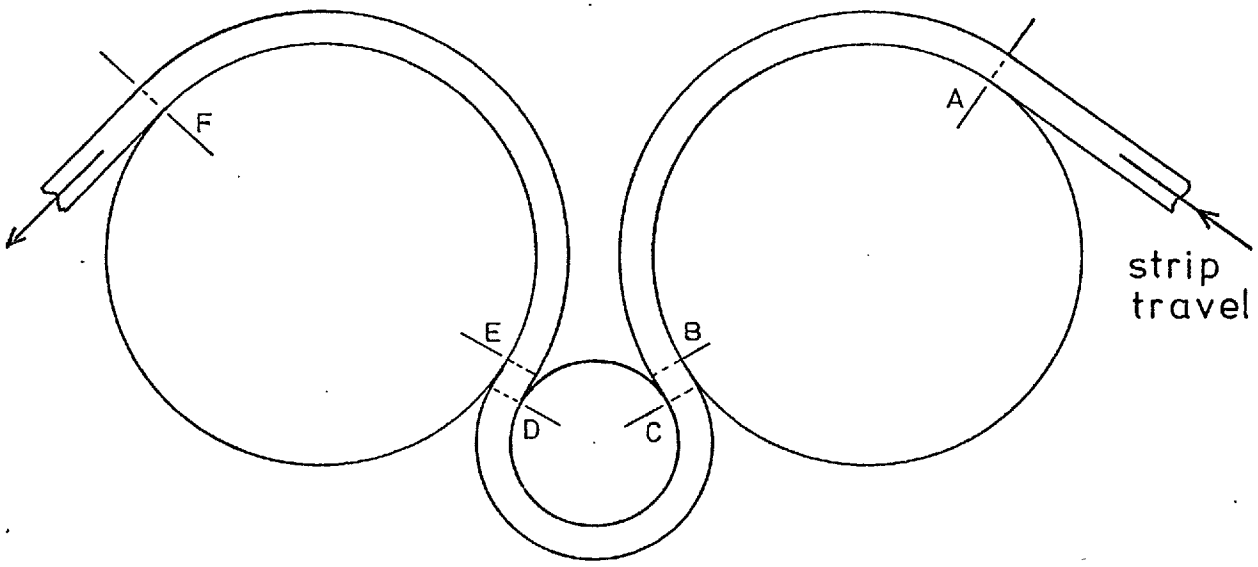


fig (12) ANALYSIS OF C.B.S. ROLLING

fig (13) see fig (1.4.1/2) of script

fig (14) see fig (1.4.3) of script

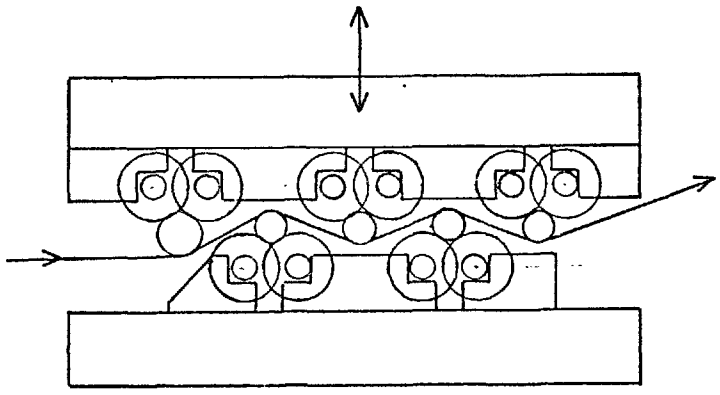


fig (15) THE B.N.F.M.R.A. LEVELLER

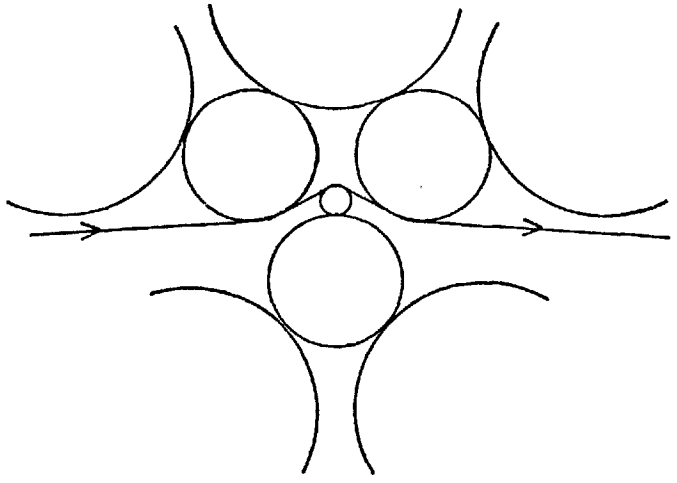


fig (16) POLAKOWSKI'S FLOATING WORK ROLL

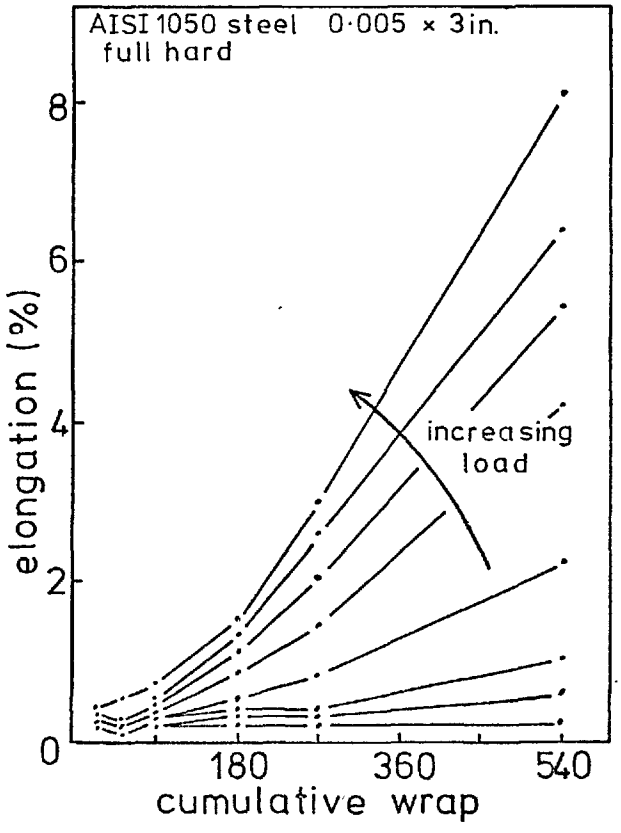


fig (17) EFFECT OF TOTAL WRAP ON LEVELLER EXTENSION

fig (18) —— see fig (3.6) of script

fig (19) —— see fig (1.3) of script

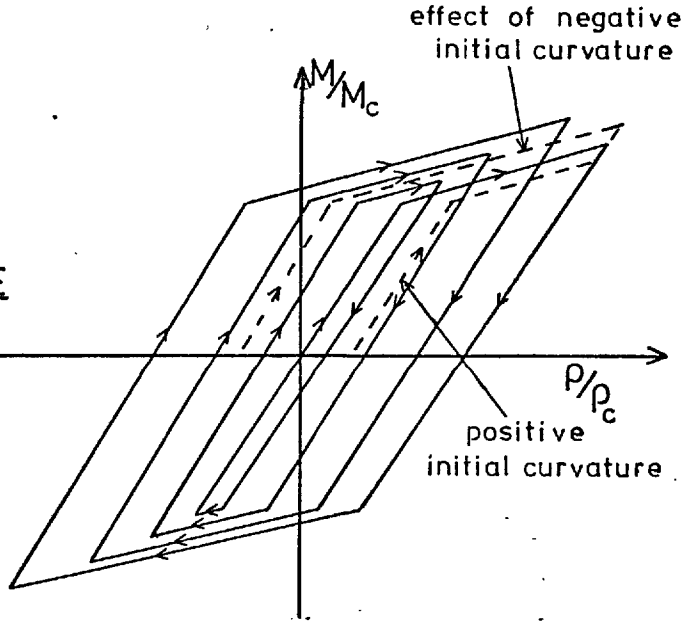


fig (20) MOMENT/CURVATURE DIAGRAM FOR THE ROLLER LEVELLER

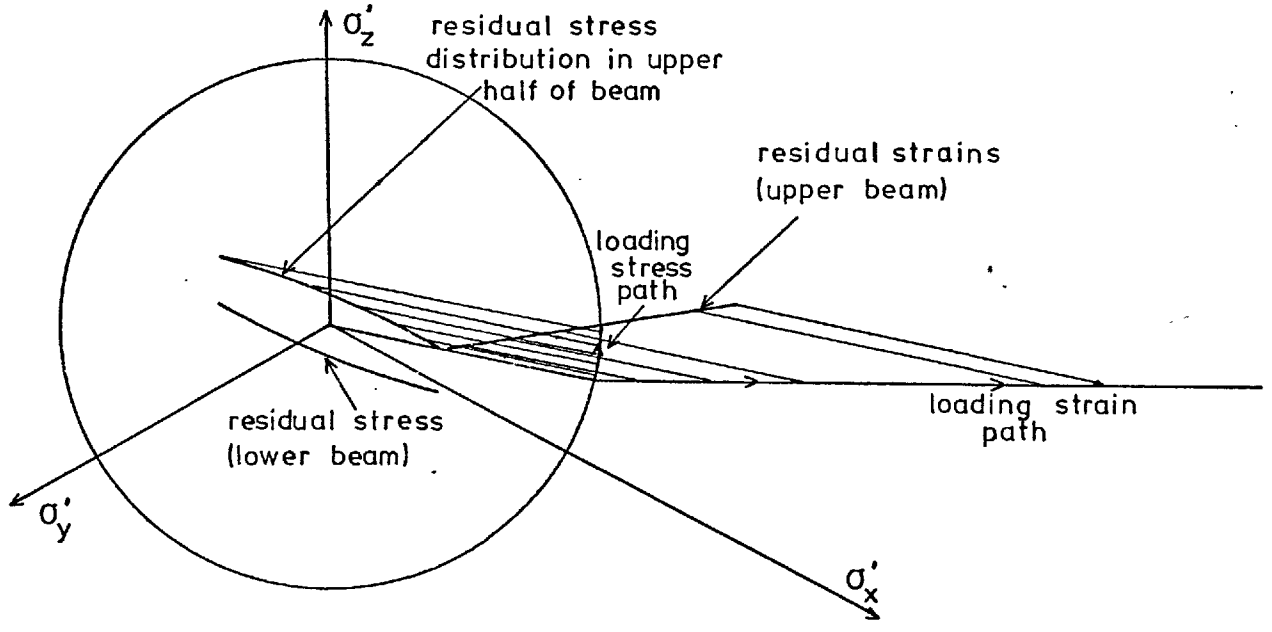


fig (21) GEOMETRICAL SOLUTION - PLANE STRAIN BENDING OF A PERFECTLY PLASTIC BEAM

fig (22) —— see fig (3.53) of script

APPENDIX 2.

A2.0. Stress/Strain Relationships.

The solution of equation 3.2.46 by computer requires only adequate control of the increment of stress vector orientation and an iterative technique for the determination of the operating hardening modulus (H').The derived relationship between $d\bar{\sigma}_n$ and $d\alpha_n$, allows the formulation of the relevant stress/strain relationships by direct application in equations 3.2.36/37/56/57.Input to the program requires only the elastic and plastic material constants (E,ν,A,B&m) and the relevant form of the plastic equation.Program output is simplified by calls to subroutines ORDER and FOLYFT (the latter being a standard POLYFT routine utilized to the 1st order only) which fit the longitudinal stress/strain relationship to equation 3.2.58.Output is therefore in the form of the constants A',B' and m' of equation 3.2.58.The program is further extended to provide computer plots of longitudinal and the corresponding equivalent stress levels vs longitudinal strain.

A2.1. Subroutine HARDEN.

Flow charts have been given in Figs 3.8 and 3.18 for the longitudinal stress/strain derivations.Program operation is as follows:

(1) From the input relationship

$$\bar{\sigma} = A(B + \bar{\varepsilon})^m \qquad \text{----A2.2.1.}$$

or

$$\bar{\sigma} = A(B + \bar{\varepsilon}^p)^m \qquad \text{----A2.2.2.}$$

the initial material yield stress is determined.If equation A2.2.2is applicable we may write,

$$\bar{\sigma}_o = AB^m \qquad \text{----A2.2.3.}$$

Where the equivalent stress is a function of the total equivalent strain however a Newton-Raphson technique is utilized since

$$\bar{\sigma}_o = A(B + \bar{\sigma}_o/E)^m \qquad \text{----A2.2.4.}$$

We may therefore write,

$$F(\bar{\sigma}_o) = (\bar{\sigma}_o/E) + B - (\bar{\sigma}_o/A)^{1/m} \qquad \text{----A2.2.5.}$$

and

$$F'(\bar{\sigma}_o) = (1/E) - (\bar{\sigma}_o^{(1-m)/m}/\, mA^{1/m}) \qquad \text{----A2.2.6.}$$

By standard Newton-Raphson technique

$$\bar{\sigma}_{o2} = \bar{\sigma}_{o1} - (F(\bar{\sigma}_{o1})/F'(\bar{\sigma}_{o1})) \qquad \text{----A2.2.7.}$$

Two or three iterations are sufficient for this process.

(2) The corresponding values of the normal stresses and strains

at yield and the yield orientation of the stress vector, are determined from the elastic relationships (equations 3.2.17 to 3.2.25).

(3) Knowing the above values at yield the incremental solution of equation 3.2.46 is commenced, the angular increment ($d\alpha_n$) being set to one degree initially.Computation of the equivalent stress increment commences at statement 3000.

Since H'_n is a function of $\bar{\sigma}_n$ (equation 3.1.3 and 3.1.5) and $\bar{\sigma}_n$ is unknown, an iterative technique is used, utilizing the value of H'_{n-1}(corresponding to $\bar{\sigma}_{n-1}$) initially, to provide simultaneous evaluation of $d\bar{\sigma}_n$ and H'_n (equation 3.2.46).The relationship between H'_n and $\bar{\sigma}_n$ is again dependent upon the choice of input type and we may write,

(a) A2.2.1. utilized

$$H'_n = mEA^{1/m}/(E\bar{\sigma}^{(1-m)/m} - mA^{\frac{1}{m}}) \qquad \text{----A2.2.8.}$$

(b) A2.2.2. utilized.

$$H'_n = mA^{1/m}/(\bar{\sigma}^{(1-m)/m}) \qquad \text{----A2.2.9.}$$

Using the value of H'_{n-1} a value of $d\bar{\sigma}_n$ (DCB(N)) is evaluated which is in error.Determination of the new value of H',(corresponding to $\bar{\sigma}_n = \bar{\sigma}_{n-1} + d\bar{\sigma}_n$) allows re-evaluation, of DCB(N).The process is self-converging, three to five iterations being sufficient to provide less than 0.1% alteration in $d\bar{\sigma}_n$.The final value is transferred from the evaluation loop (DSB(N)) and the program continues.

(4) From this value the respective values of the equivalent stress and the normal stresses and strains are determined (statement 13).

(5) A further increment is now taken and the process (3 and 4) repeated until program instability sets in.(equation 3.2.46).A factor of ten reduction is then taken in the angular increment ,and the process continued until this increment is less than 10^{-5} degrees.This very small value allows close approach to the stress path asymptote, but also requires that double precision calculations be utilized in the evaluation of the required relationships.

(6) When this final point is reached any required relationships are determined (statement 16) and the results listed.

One loading cycle has been completed.

(7) Preparations for the reverse loading cycle are now made (statements 3010 and 3020) the form of these preparations depending upon the choice of hardening model (ie. isotropic or kinematic).These preparations take the form of the evaluation of the new yield point values.(equations 3.2.59 to 3.2.70).

(8) The program is recycled for a number of times equal to the number of leveller rolls and the curve-fitting routine is then called (statement 3001).

(9) The constants derived by the curve-fitting routine are utilized in the preparation of a comparison of derived longitudinal stress values with those given by the equation constants.Results are those plotted.

A typical complete output listing is to be found in Appendix 7.

A2.2. Subroutine ORDER.

This routine correlates the longitudinal stress/strain arrays with the equation,

$$\sigma_x = A'(B' + e_x)^{m'} \qquad \text{----A2.3.1.}$$

A complex multiple-regression technique is not required since equation A2.3.1. may be rewritten in linear form, hence

$$\sigma_x'' = (\sigma_x)^{1/m'} = A'^{(1/m')}(B' + e_x) \qquad \text{----A2.3.2.}$$

Knowing that m' will lie in the range 0.0 to 1.0 for all materials the program operates by regressing on m' and utilizes a standard linear curve-fitting routine to evaluate the corresponding values of A' and B'.The criterion chosen for the 'best-fit' equation is that of maximum correlation coefficient, the value of this factor being returned at each step by FOLYFT.

Since, values of σ_x'' at values of m' approaching zero will be excessively large, a non-dimensional variable is set up internally where,

$$SXD(I) = (\sigma_x(I) / \sigma(1))^{1/m'} \qquad \text{----A2.3.3.}$$

At values of $1/m' \simeq 20$ this parameter is well within computer capacity.Variations in m' are obtained by use of the equation

$$m'_2 = m'_1 + \frac{(-1)^{J+1}}{10^J \times 2} \qquad \text{----A2.3.4.}$$

J is kept constant at 1 until increasing m' from 0.05 results in a decrease in correlation coefficient.J is then increased by 1 and the process continued.This results in rapid convergence to the maximum correlation point.J being increased to a maximum of 5.

Calls to FOLYFT return an expression of the form,

$$SXD(I) = C0 + Ce_x(I) \qquad \text{----A2.3.5.}$$

The constants A' & B ' are readily determined since from equations A2.3.2./3/5

$B' = CO/C$ ----A2.3.6.

$A' = C^{m'}\sigma_x(1)$ ----A2.3.7.

This routine is used in similar forms in the evaluations of the constants A,B and m in equations A2.2.1 and A2.2.2.In this capacity the routine is called from program TSTRES (Appendix 6) which derives the required equivalent stress/equivalent strain array from input tensile test data.

A2.3. Program Nomenclature.

Only the important program variables will be listed here, other variables are utilized for ease of programming and are self-evident.

A2.3.1. Subroutine HARDEN.

A .... stress/strain input and output constant (A,A').

ALP(I), DALP .... stress vector orientation (α_i) and its increment ($d\alpha_i$).

ALPO .... stress vector orientation at yield (α_o).

ALPI .... stress vector orientation (radian measure).

B .... stress/strain input and output constant (b,B').

CB(N),DCB(N) .... equivalent stress and increment in iterative loop. ($\bar{\sigma}_n$, $d\bar{\sigma}_n$).

D .... as prefix, represents incremental value.

E .... Youngs' Modulus.

EX(I), DEX(I),EXENG(I) .... longitudinal true strain (ε_x), increment ($d\varepsilon_x$) and engineering equivalent (e_x).

EXDIF .... strain range, reverse loading.

EY(I),DEY(I),EYENG(I) .... thickness true strain(ε_y), increment($d\varepsilon_y$) and engineering equivalent (e_y).

EYDIF .... strain range, reverse loading.

EZE(I),DEZE(I) .... transverse elastic true strain (ε_z^E) and increment ($d\varepsilon_z^E$).

ENIKS(INO) .... longitudinal engineering strain array utilized for curve-fitting purposes.

H(N) .... hardening modulus (H'_n).

I .... integer count (n).

IP .... defines input equation type,
IP = 1 for equation A2.2.1.
IP = 2 for equation A2.2.2.

IM .... number of derived stress/strain points in each loading cycle.

INO .... total number of derived stress/strain points used for curve-fitting.

N .... integer count..

NU .... number of loading cycle + 1.

NUN .... number of derived stresspoints used for curve-fitting in each loading cycle.

NB .... 1 for isotropic hardening.

NB .... = 2 for kinematic hardening.

QMU .... Peissons ratio (ν).

RM .... constant m,m' in input/output equation.

SB(1),DSB(1) .... equivalent stress ($\bar{\sigma}_n$) and increment ($d\bar{\sigma}_n$).

SBDIFF .... shift of yield locus,origin on kinematic reverse loading.

SBIKS(INO) .... equivalent stress-- plotting array.

SX(I)DSX(I) .... longitudinal stress (σ_{x_n}) and increment ($d\sigma_{x_n}$).

SXMAX(I) .... longitudinal stress under perfectly plastic conditions.

$$(\sigma_x = \frac{2}{\sqrt{3}} \bar{\sigma})$$

SXCALC(I) .... longitudinal stress from output equation.

SXDIF .... longitudinal elastic stress range on reverse loading.

SNIKS (ENO) .... longitudinal stress array used for curve-fitting.

SZ(I), DSZ(I) .... transverse stress (σ_z) and increment ($d\sigma_z$).

A2.3.2. Subroutine Order.

A1 .... constant A'

AMIN .... maximum correlation coefficient.

B 1 .... constant B'.

CO,CC .... constants of equation A2.3.5. (CO,C).

DVAB (M) .... = STDV(M) .... correlation coefficient.

POWER .... = RM .... output constant m'.

SXD(I) .... dimensionless stress parameter for curve-fitting purposes.

```
C.....................................................................
      SUBROUTINE HARDEN(E,QMU,NR)
C*****PROGRAM TO DETERMINE THE STRESS/STRAIN RELATIONSHIPS IN THE
C*****PRINCIPAL DIRECTIONS FOR A GIVEN WORK HARDENING MATERIAL UNDER
C*****PLANE STRAIN CONDITIONS.          THE WORK HARDENING NATURE OF AN
C*****-Y REAL MATERIAL MAY BE REPRESENTED BY A FUNCTION OF THE FORM
C*****SIGMA(BAR) =  A(B+ETA(BAR)/PLASTIC)**M      WHERE.................
C*****A = ......BASIC MATERIAL STRENGTH CONSTANT
C*****B = ......STRAIN CONSTANT DEPENDING ON PREVIOUS DEFORMATION
C*****   HISTORY OF THE MATERIAL
C*****M = .....WORK HARDENING PARAMETER
C*****IP = 1 FOR TRUE STRAIN RELATIONSHIP AND 2 FOR TRUE STRAIN PLASTIC
C*****THESE CONSTANTS CAN BE DETERMINED FROM THE UNI-AXIAL
C*****STRESS/STRAIN CURVE OF THE MATERIAL
C*****
      DIMENSION SB(150),SX(150),SZ(150),EX(150),EY(150),
     A DSB(150),DSX(150),DSZ(150),DEX(150),DEY(150),
     B          EXENG(150),CB(150),DCB(150),H(150),HARD(150),EYENG(150),
     C SXMAX(150),SBUSBO(150),SXUSBO(150),SZUSBO(150),SXUSXM(150),
     D SZ2USX(150),DIFF(150),EZE(150),DEZE(150)
      DIMENSION C(10),SXCALC(300),SNIKS(300),ENIKS(300),NUN(10)
      DIMENSION DU(300),SBIKS(300)
      DOUBLE PRECISION ALPD,DALP,ALPI,ALPMI,CI,SI,CMI,SMI,SQ,
     A ALP(150),ALPM(150),DE,DQMU,DH,DOCB
      COMMON/C4/A,B,RM,IP,NNB,SIGMA
      COMMON/C8/IPLIT,ILIT
C*****
C*****READ IN THE MATERIAL CONSTANTS.............
C*****E = YOUNGS MODULUS
C*****QMU = POISSONS RATIO
C*****A = STRENGTH CONSTANT
C*****B = PREVIOUS STRAIN CONSTANT
C*****RM = WORK HARDENING INDEX
      WRITE(6,1005)
 1000 FORMAT(1H1)
 1003 FORMAT(///////)
 1005 FORMAT(1HO)
      WRITE (6,1001)E,QMU,A,B,RM
 1001 FORMAT( 5X,4HE = ,F10.1,2X,6HQMU = ,F5.3,2X,4HA = ,F11.3,
     A 2X,4HB = ,F5.3,2X,5HRM = ,F5.3)
      WRITE(6,86)A,B,RM
   86 FORMAT(1HO,5X,11HSIGMABAR = ,F7.0,1H(,F7.6,12H + ETABAR)**,F7.5)
C*****CALCULATE THE INITIAL YIELD CONDITIONS.............
C*****
      GO TO(110,111,111,111),NNB
  110 NB = 1
      GO TO 112
  111 NB = 2
  112 NU = 2
      NUN(1) = 0
      INO = 0
      I = 2
      SB(1) = SIGMA
      DSB(1) = 0.0
      SX(1) = SB(1)/SQRT(1.0-QMU+QMU**2)
      SZ(1) = QMU*SX(1)
      EX(1) = SX(1)*(1.0-QMU**2)/E
      EY(1) = -(EX(1)*QMU)/(1.0-QMU)
      EXENG(1) = EXP(EX(1))-1.0
```

```
      EYENG(1) = EXP(EY(1))-1.0
      SALP = SZ(1)*SIN(3.1416/3.0)/(SB(1))
      ALP(1) = ARSIN(SALP)*360.0/(2.0*3.1416)
      ALPO = ALP(1)
      DEZE(1) = 0.0
      EZE(1) = 0.0
      SBDIFF = 0.0
C*****
C*****SELECT AN ANGULAR INCREMENT ON THE DEVIATORIC PLANE OF ...ALPD...
C*****BEGIN COMPUTATIONS FOR THE INCREMENT IN EQUIVALENT YIELD STRESS...
C*****.....DSB(I)
C*****
 3000 ALPD = 1.0
      DO 11 I = 2,150
      IM = I-1
 59   DALP = (ALPD*2.0*3.1416)/360.0
      ALP(I) = ALP(I-1) + ALPD
      ALPM(I) = ALP(I-1) + ALPD/2.0
      ALPI = (ALP(I)*2.0*3.1416)/360.0
      ALPMI =(ALPM(I)*2.0*3.1416)/360.0
      CI   = DCOS(ALPI)
      CMI  = DCOS(ALPMI)
      SI   = DSIN(ALPI)
      SMI  = DSIN(ALPMI)
      SQ   = DSQRT(3.0D0)
C*****
C*****FROM THE DIFFERENTIAL OF THE PLASTIC STRESS/STRAIN CURVE
C*****DETERMINE THE HARDENING MODULUS AT THE INITIAL YIELD AND
C*****BY A SIMPLE ITERATION TECHNIQUE DETERMINE THE CORRESPONDING
C*****  DSB(I) AND H(I)
C*****
      F = 100000.0
      NN = 3
      DO 4000 N=2,60
 4000 CB(N) = 0.0
      DO 12  N = 2,40
      CB(1) = SB(I-1)
      CBD = CB(N-I)/1000.0
      AD = A/1000.0
      GO TO(100,101)IP
  100 H(N) =((AD**(1.0/RM))/(((CBD**((1.0-RM)/RM))/RM)-((AD**(1.0/RM))/
     A(E*((1000.0**((1.0-RM)/RM))/(1000.0**(1.0/RM))))))))
     B *((1000.0**(1.0/RM))/(1000.0**((1.0-RM)/RM)))
      GO TO 102
  101 H(N) = RM*((AD**(1.0/RM))/(CBD**((1.0-RM)/RM)))*
     A ((1000.0**(1.0/RM))/(1000.0**((1.0-RM)/RM)))
  102 DE = E
      DQMU = QMU
      DH = H(N)
      DOCB = CB(N-1)
      DCB(N) = (DOCB*DALP*((2.0D0*CMI/SQ)+(DQMU*(SMI-CMI/SQ))))/
     A (((DE*(CI-SQ*SI))/(2.0D0*DH))-(2.0D0*SI/SQ)+(DQMU*(CI+SI/SQ)))
      IF (DCB(N).GT.(CB(N-1)/10.0)) GO TO 50
C*****
C*****  INSTABILITY OCCURS AS THE ANGLE APPROACHES A VALUE OF ABOUT
C*****30 DEGREES, THE ACTUAL POSITION DEPENDING ON THE CORRESPONDING
C*****VALUE OF H(I). EJECT WHEN THIS OCCOURS.
C*****
      IF((((DE*(CI-SQ*SI))/(2.0D0*DH))+(DQMU*(CI+SI/SQ))).LT.(2.0D0
```

```
     A    *SI/SQ)) GO TO 50
          GO TO 52
C*****IF EJECTION IS OCCURRING BECAUSE OF A TOO GAREAT AN ANGULAR
C*****INCREMENT DECREASE THE INCREMENT UNTIL SLOW APPROACH TO THE
C*****LIMIT OCCURS
 50       ALPD = ALPD/10.0
          IF(ALPD-0.1D-7)68,68,59
 68       WRITE(6,51) ALP(I)
 51       FORMAT(1X,35HDENOMINATOR HAS GONE NEGATIVE      ,8HALP(I) =,
     A 2X,D17.10)
          GO TO 16
 52       IF(ABS(F-DCB(N))/DCB(N).LT.0.001) GO TO 13
          IF(N-2)54,54,53
 53       NN=N
          IF(ABS(DCB(NN-2)-DCB(NN))/DCB(NN).LT.0.001) GO TO 13
 54       F = DCB(N)
          CB(N) = CB(I)+DCB(N)
          IF(60-N)15,15,12
 15       WRITE(6,1501) I
 1501     FORMAT(1H0,3X,I4,8HLOOP/END)
          GO TO 13
 12       CONTINUE
C*****TAKE THE FINAL RESULT OF THE ITERATION DO LOOP AND PROCEED
C*****WITH THE MAIN PROGRAM
 13       DSB(I) = DCB(N)
          HARD(I) = H(N)
          SB(I) = SB(I-1) + DSB(I)
          SX(I) = SB(I)*(CI+SI/SQ) -(SBDIFF*2./SQ)
          SZ(I) =(SB(I)*2.0*SI/SQ)-(SBDIFF/SQ)
          G = E/(2.0*(1.0+QMU))
          DEX(I) = ((DSB(I)*(1.0+(3.0*G/HARD(I)))*(CI-(SI/SQ)))
     A -(SB(I-1)*DALP*((CMI/SQ)+SMI)))/(2.0*G)
          DEY(I) = -((DSB(I)*(1.0+(3.0*G/HARD(I)))*2.0*SI/SQ)
     A +(SB(I-1)*DALP*2.0*CMI/SQ))/(2.0*G)
          GO TO(4010,4011)NB
 4010     DEY(I) = DEY(I)*((-1.)**NU)
 4011     EX(I) = EX(I-1) + DEX(I)
          EX(I) = EX(I-1) + DEX(I)
          EY(I) = EY(I-1) + DEY(I)
          EXENG(I) = EXP(EX(I))-1.0
          EYENG(I) = EXP(EY(I))-1.0
          DEZE(I) = (DSB(I)/(2.0*HARD(I)))*(CI-(SI*SQ))*((-1.)**NU)
          EZE(I) = EZE(I-1)+DEZE(I)
          K = N
          IF (I.GT.2) GO TO 22
          WRITE(6,1005)
          WRITE (6,1400) I,ALP(I)
 1400     FORMAT(5X,2HI=,     I3,4X,11HALP(I) =    ,D17.10)
          WRITE (6,1450)
 1450     FORMAT(22X,1HN,13X,5HCB(N),15X,6HDCB(N),16X,4HH(N))
          WRITE (6,1500) (N,CB(N),DCB(N),H(N),N=1,K)
 1500     FORMAT(20X,I3,10X,F10.3,10X,F10.3,10X,F10.1)
 22       IF (ALP(I)-28.990)19,20,20
 20       ALPD = 0.1
          IF (ALP(I)-29.890)19,21,21
 21       ALPD = 0.01
 19       IF(ALP(I).GE.30.0) GO TO 16
 11       CONTINUE
 16       DO 14 I = 1,IM
```

```
      SXMAX(I) = SB(I)*2.0/SQ
      SBUSBO(I) = SB(I)/SB(1)
      SXUSBO(I) = SX(I)/SB(1)
      SZUSBO(I) = SZ(I)/SB(1)
      SXUSXM(I) = SX(I)/SXMAX(I)
      DIFF(I)   = 2.0*(1.0-SXUSXM(I))/SQ
      SZ2USX(I) = SZ(I)*2.0/SX(I)
14    CONTINUE
      WRITE(6,1005)
      WRITE (6,1700)
1700 FORMAT(6X,1HI, 6X,6HALP(I),12X,5HSB(I),7X,5HSX(I),7X,5HSZ(I),
     A 9X,5HEX(I),6X,8HEXENG(I),3X,5HEY(I),7X,8HEYENG(I))
      WRITE (6,1800)(I,ALP(I),SB(I),SX(I),SZ(I),EX(I),EXENG(I),EY(I),
     A EYENG(I),I = 1,IM,25)
1800 FORMAT(5X,I3,2X,D17.10,2X,F10.3,2X,F10.3,2X,F10.3,2X,F10.6,
     A 2X,F10.6,2X,F10.6,2X,F10.6)
      IF(NU.GT.2) GO TO 3040
      WRITE(6,2000)
2000 FORMAT(  /13X,5HSB(I),3X,9HSBUSBO(I),4X,5HSX(I),5X,8HSXMAX(I),
     A 2X,9HSXUSXM(I),1X,9HSXUSBO(I),1X,9HSZUSBO(I),1X,9HSZ2USX(I),
     B 1X,7HDIFF(I),3X,6HEZE(I))
      WRITE(6,1900)(I,SB(I),SBUSBO(I),SX(I),SXMAX(I),SXUSXM(I),
     A SXUSBO(I),SZUSBO(I),SZ2USX(I),DIFF(I),EZE(I),I=1,IM,25)
1900 FORMAT(5X,I3,2X,F10.3,2X,F7.4,2X,F10.3,2X,F10.3,2X,F7.4,2X,F7.4,
     A 3X,F7.4,3X,F7.4,2X,F7.4,1X,F10.6)
3040 DO 3030 I=1,IM,5
      IF(NB.EQ.1.AND.I.LT.21.AND.INO.GT.4) GO TO 3030
      INO = INO+1
      SNIKS(INO) = SX(I)+(SBDIFF*2./SQ)
      SBIKS(INO) = SB(I)
      ENIKS(INO) = EXENG(I)
3030 CONTINUE
      NUN(NU) = INO
      IPLUG = (NR+1)*((I-6)/5)
      IF(NB.EQ.2.AND.NU.EQ.0.AND.INO.LT.IPLUG) GO TO 3040
      IF(NU.EQ.NR+1) GO TO 3001
      NU = NU+1
      GO TO(3010,3020)NB
3010 ALP(I) = (2.*ALP0)-ALP(IM)
      SBDIFF = 0.0
      SB(1) = SB(IM)
      DSB(1) = 0.0
      ALPI = (ALP(1)*2.*3.1416)/360.0
      CI = DCOS(ALPI)
      SI = DSIN(ALPI)
      SX(1) = SB(1)*(CI+SI/SQ)
      SZ(1) = SB(1)*2.*SI/SQ
      SALP = SZ(1)*SIN(3.1416/3.)/(SB(1))
      SXDIF = SX(IM)+SX(1)
      EXDIF = SXDIF*(1.-QMU**2)/E
      EYDIF =-((EXDIF*QMU)/(1.-QMU))*((-1.)**(NU-1))
      EX(1) = EX(IM)-EXDIF
      EXENG(1) = EXP(EX(1))-1.
      EY(1) = EY(IM)-EYDIF
      EYENG(1) = EXP(EY(1))-1.
      EZE(1) = EZE(IM)
      WRITE(6,1005)
      WRITE(6,1002) SBDIFF,SXDIF,EZE(1),EXBD0,EXDIF,EYDIF
      IF(MOD(NU,2).EQ.0) GO TO 3000
```

```
      WRITE(6,1000)
      WRITE(6,1003)
      GO TO 3000
3020  SBDIFF = SBDIFF-((SB(IM)-SB(1))*((-1.)**NU))
      ALP(1) = (2.*ALPO)-ALP(IM)
      SB(1) = SB(1)
      DSB(1) = 0.0
      ALPI = (ALP(1)*2.*3.1416)/360.0
      CI = DCOS(ALPI)
      SI = DSIN(ALPI)
      SX(1) = SB(1)*(CI+SI/SQ) -(SBDIFF*2./SQ)
      SZ(1) = SB(1)*2.*SI/SQ -(SBDIFF/SQ)
      SALP = 2.*SI*SIN(3.1416/3.)/SQ
      SXDIF = SX(IM)+SX(1)
      EXDIF = SXDIF*(1.-QMU**2)/E
      EYDIF = -(EXDIF*QMU)/(1.-QMU)
      EXBDO =((SB(1)/SQ)*(1.-QMU**2)/E)*2.
      EX(1) = EXDIF-EXBDO
      EXENG(1) = EXP(EX(1))-1.
      EY(1) = EY(IM)-EYDIF
      EYENG(1) = EXP(EY(1))-1.
      EZE(1) = EZE(IM)
      WRITE(6,1005)
      WRITE(6,1002) SBDIFF,SXDIF,EZE(1),EXBDO,EXDIF,EYDIF
1002  FORMAT( 5X,9HSBDIFF = ,E11.4,2X,8HSXDIF = ,E11.4,2X,8HEZE(I)= ,
     B          E11.4/5X,
     A 9H EXBDO = ,E11.4,2X,8HEXDIF = ,E11.4,2X,8HEYDIF = ,E11.4)
      IF(MOD(NU,2).EQ.0) GO TO 3000
      WRITE(6,1000)
      WRITE(6,1003)
      GO TO 3000
3001  CALL ORDER(SNIKS,ENIKS,INO,A,B,RM,1,NUN,NB)
      DO 23 I=1,INO
      IF((B+ENIKS(I)).LE.0.0) GO TO 25
      SXCALC(I) = A*((B+ENIKS(I))**RM)
      DU(I) = SNIKS(I)-SXCALC(I)
      GO TO 23
   25 SXCALC(I) = 0.0
      DU(I) = 0.0
   23 CONTINUE
      WRITE(6,26)
   26 FORMAT(1H0,11X,11HTRUE STRESS,11X,11HTRUE STRESS,15X,3HDIF,15X,
     A11HTRUE STRAIN,2X/14X,6HACTUAL,15X,10HCALCULATED)
      WRITE(6,24)(SNIKS(I),SXCALC(I),DU(I),ENIKS(I),I=1,INO,5)
   24 FORMAT(11X,E12.5,10X,E12.5,10X,E12.5,10X,E12.5)
      GO TO(6000,6001),ILIT
6001  CALL FACTOR(1.0)
      CALL PLOT(3.0,15.0,-3)
      SXCALC(INO+1) = 0.0
      ENIKS(INO+1) = 0.0
      SXCALC(INO+2) = (IFIX(SXCALC(INO)*6.895/40000.)+1)*10.
      YSCALE = 1.0/SXCALC(INO+2)
      ENIKS(INO+2) = (IFIX(ENIKS(INO)*1000./7.)+1)/1000.
      XSCALE = 1.0/ENIKS(INO+2)
      CALL AXIS(0.0,0.0,18HENGINEERING STRAIN,-18,7.0,0.0,ENIKS(INO+1),
     A ENIKS(INO+2))
      CALL AXIS(0.0,0.0,27HLONGITUDINAL STRESS MN/M**2,27,4.0,90.0,
     A SXCALC(INO+1),SXCALC(INO+2))
      DO 6013 I=1,INO
```

```
      SXCALC(I) = SXCALC(I)*6.895/1000.
      SBIKS(I) = SBIKS(I)*6.895/1000.
6013  SNIKS(I) = SNIKS(I)*6.895/1000.
      GO TO (6010,6011),NB
6010  CALL ARKIST(ENIKS,SXCALC,1,INO,10,XSCALE,YSCALE,0.0,0.0,2,1)
      GO TO 6012
6011  CALL ARKIST(ENIKS,SXCALC,1,NUN(2),10,XSCALE,YSCALE,0.0,0.0,2,1)
6012  DO 6002 NU =2,5
      J =0
      NP = NUN(NU-1)+1
      NPP = NUN(NU)
      DO 6003 I=NP,NPP
      J=J+1
      EX(J) = ENIKS(I)
6003  SX(J) =SNIKS(I)
      SX(J+1) = SXCALC(INO+1)
      EX(J+1) = ENIKS(INO+1)
      SX(J+2) = SXCALC(INO+2)
      EX(J+2) = ENIKS(INO+2)
6002  CALL LINE(EX,SX,J,1,-2,NU)
      CALL ARKIST(ENIKS,SBIKS ,1,INO,10,XSCALE,YSCALE,0.0,0.0,2,1)
      CALL PLOT(9.0,-15.0,-3)
      CALL FACTOR(0.5)
6000  RETURN
      END
```

```
C.....
      SUBROUTINE ORDER(SX,EXENG,IX,A1,B1,POWER,IRM,NUN,NB)
      DIMENSION SX(300),EXENG(300),STDV(1000),SXD(150),EXGGG(150),
     A DVAB(1000),C(10),NUN(10)
      IM = IX
      JM = 1
      IF(NB.EQ.1) IM = NUN(JM+1)-NUN(JM)
      AMIN = 0.0
      RM = (FLOAT(IRM)/20.0)-0.05
      J = 1
   60 IF(J.EQ.5) GO TO 68
      DO 62 M=1,1000
      RM = RM+((-1.0)**(J+1))/((10.0**J)*2.0)
      SXDTOT = 0.0
      DO 64 I=1,IM
      IJ = I
      IF(NB.EQ.1) IJ = NUN(JM)+I
      SXD(I) = (SX(IJ)/SX(1))**(1.0/RM)
      EXGGG(I) = EXENG(IJ)
      SXDTOT = SXDTOT+SXD(I)
   64 CONTINUE
      SXMEAN = SXDTOT/(FLOAT(IM))
      KOR = 1
      CALL FOLYFT(EXGGG,SXD,IM,C,C0,KOR,6,M,STDV)
      DVAB(M) = STDV(M)
      IF(AMIN-DVAB(M))66,70,70
   66 AMIN = DVAB(M)
      POWER = RM
      C00 = C0
      CC = C(1)
      AMINN = AMIN
   62 CONTINUE
   70 J = J+1
      GO TO 60
   68 B1 = C00/CC
      A1 = (CC**POWER)*SX(1)
      WRITE(6,1000)
 1000 FORMAT(1H1)
      WRITE(6,1003)
 1003 FORMAT(////////)
      WRITE(6,86) A1,B1,POWER
   86 FORMAT(1H0,5X,9HSIGMAX = ,F7.0,1H(,F7.6,10H + ETAX)**,F7.5)
      RETURN
      END
```

```
C.....
      SUBROUTINE FOLYFT(X,Y,N,C,CO,KOR,NOUT,MX,STDV)
      DIMENSION X(150),Y(150),A(10),C(10),STDV(1000)
      DIMENSION  SUMX(200),SMYX(100), AMEANX(100)
      KOR = 1
      PTS= N
      NT = NOUT
      KTOR= 2*KOR
      DO  1  I=1,KOR
      SMYX(I)= 0.0
    1 AMEANX(I)= 0.0
      DO 110 I=1,10
      A(I) = 0.0
      C(I) = 0.0
  110 CONTINUE
      DO  2  I=1,KTOR
    2 SUMX(I)= 0.0
      SUMY= 0.0
      XMAX= X(1)
      DO  100  I=2,N
      IF( X(I) -XMAX) 100,101,101
  101 XMAX= X(I)
  100 CONTINUE
      DO  102  I=1,N
  102 X(I)= X(I)/XMAX
      DO  3 J=1,KTOR
      DO  3 I=1,N
    3 SUMX(J)= SUMX(J) +X(I)**J
      DO  4  I=1,N
    4 SUMY= SUMY+ Y(I)
      AMEANY= SUMY/PTS
      DO  6 J=1,KOR
      AMEANX(J)= SUMX (J)/PTS
      DO  6 I=1,N
    6 SMYX(J)= SMYX(J) +Y(I)*X(I)**J
      DO  8 I=1,KOR
      C(I)= SMYX(I) -PTS*AMEANX(I)*AMEANY
      DO  8 J=1,KOR
      K= I+J
      IJ =(J-1)*KOR +I
    8 A(IJ) =SUMX(K) -PTS* AMEANX(I)*AMEANX(J)
      DO  11 I=2,KOR
      III=(I-1)*KOR + I
   11 A(III) = A(III)/ A(1)
      DO  12 J=2,KOR
      KM= J-1
      DO 14 I=J,KOR
      AP1= 0.0
      DO114 K=1,KM
      IK =(K-1)* KOR+ I
      KJ =(J-1)* KOR+ K
  114 AP1 = AP1 + A(IK) *A(KJ)
      IJ =(J-1)* KOR+ I
   14 A(IJ) = A(IJ) -AP1
      JP= J+1
      IF (JP- KOR) 444, 444, 445
  444 DO 16 I=JP,KOR
      AP1= 0.0
      DO116 K=1,KM
```

```
      JK =(K-1)* KOR + J
      KI =(I-1)* KOR + K
  116 AP1 =AP1 +A(JK) *A(KI)
      JI =(I-1)* KOR + J
      JJ =(J-1)* KOR + J
   16 A(JI) = (A(JI) -AP1)/A(JJ)
  445 DUMMY= 0.0
   12 CONTINUE
      C(1) =C(1)/A(1)
      DO  18 I=2,KOR
      AP1= 0.0
      IM=I-1
      DO 118 K=1,IM
      IK =(K-1)* KOR + I
  118 AP1=AP1 +A(IK) *C(K)
      II =(I-1)* KOR + I
   18 C(I) =(C(I)- AP1) / A(II)
      KORM= KOR-1
      IF (KORM) 122, 123,122
  122 DO  21 I=1,KORM
      AP1= 0.0
      M= KOR-I
      MP= M+1
      DO 121 K=MP,KOR
      MK =(K-1)* KOR +M
  121 AP1 =AP1 + A(MK)* C(K)
   21 C(M) =C(M) -AP1
  123 AP1= 0.0
      DO 24  I=1,KOR
   24 AP1 =AP1 +AMEANX(I) *C(I)
      CO  = AMEANY -AP1
      IF (NT)  777,778,778
  777 DO 779 I = 1,KOR
  779 C(I) = C(I)/XMAX**I
      RETURN
  778 SRES = 0.0
      YADIF2 = 0.0
      YCDIF2 = 0.0
      DO  77 I=1,N
      SMOOTH= CO
      DO  27 J=1,KOR
   27 SMOOTH =SMOOTH +  C(J)*X(I)**J
      RES= Y(I)-SMOOTH
      SRES= SRES +RES**2
      X(I)= X(I)*XMAX
      YADIF2 = YADIF2 +((Y(I)-AMEANY)**2)
      YCDIF2 = YCDIF2 +((SMOOTH-AMEANY)**2)
   77 CONTINUE
      STDV(MX) = SQRT(YCDIF2/YADIF2)
      DO 28 I=1,KOR
   28 C(I)   =C(I)   /XMAX**I
   35 RETURN
      END
```

APPENDIX 3.

A3.0. PROGRAM Design.

The program for the numerical analysis of the tension-levelling process is divided into two main units.The first (LEVEL 4) is a control routine, whilst the second (SUBROUTINE ROLL) provides the necessary theoretical link by incorporating the relevant equations of sections 3.3 and 3.4.These routines make use of the subroutines listed in Appendices 2,4 and 5 where necessary.

PROGRAM LEVEL 4 performs the laborious operations necessary in any full program and as such provides the input/output links and controls the use of the mass storage to be utilized in subsequent plotting.The routine pin-points the position in the strip being considered and having determined the relevant tensile load at that position calls ROLL to complete the analysis.Any number of the data units can be processed by the program, input stress/strain relationships being processed by subroutine HARDEN if required, and CALCOMP plots of stress and strain levels at each roll being a further optional feature.A maximum of eight work rolls can be incorporated in the present program, although adjustment of dimension statements would allow any number to be considered but the necessary core storage would be increased above the present value of 35K.

Subroutine ROLL determines (with the aid of subroutine SXMAX) the relevant stress and strain levels at a point in the leveller as defined by the equations of sections 3.3 and 3.4.Neutral axis displacements are naturally evaluated by this subprogram and internal moments and strip curvatures are determined where required.Results are returned to the main program for printing.

It is to be noted that all routines are written in FORTRAN IV, running time on a CDC 6600 computer being approximately 35 seconds per roll, where constant applied load is assumed.The incorporation of work losses increases this value to 80 seconds per roll.It is not recommended that a program of this type be run on any computer slower than a CDC 6400 or equivalent, the time factor being immense.Where computer costs are important care should be taken in the choice of input data, thus providing the minimum possible necessary to adequately define a levelling system.

A3.1. Program LEVEL 4.

A flow chart and listing for this routine are given at the end of this section.

The input data required is read in from three cards :-

(1) Material constants (E, V,A,B and m), relationship to which A,B and m are applicable.

ie. IP = 1 for equation A2.2.1.

IP = 2 for equation A2.2.2.

and hardening mode

NB = 1 for isotropic

NB = 2 for kinematic.

NB = 3 for isotropic/kinematic.

(2) Strip geometry and machine variables (excluding roll diameter).

(3) Effective bearing friction coefficient, roll diameters and respective wrap angles.

The program utilizes one roll unit to evaluate stress conditions on initial flattening, and one for final tension relaxation, the number of program units is therefore equal to the number of rolls plus two (NR2).Each roll unit (denoted by N) is further subdivided into,

(a) bending (K=1)

(b) spring-back (K=2)

(c) straightening (K=3)

up to and including statement number 2000 (card 94), the routine performs necessary 'setting-up' operations (eg. plane strain relationships - statement 4).The functional program commences at card 95, after which to 'N' and 'K' loops are initiated.Redundant K values for N=1 or N=NR2 result in loop termination.

Entry to the inner loop (card 101) initiates stress and strain level determination at a leveller unit.Initially the 'y' positions of the 129 thickness points are determined (statement number 2001) from the strip thickness carried over from the previous loop.The tension force (TLOAD) is set equal to the exit tension of the previous loop and subroutine ROLL is called.ROLL returns the relevant stress and strain levels.

If work losses are to be incorporated subroutine RWORK (Appendix 5) is then called and returns the work done on the strip.If K=1,(ie. the strip is being bent) any bearing friction effects are estimated (equation 3.5.14 - statement number 808), and the loop output tension determined. This output tension is compared with the value utilized in the determination of the stress and strain levels.Less than 0.1% difference allows program continuation.A value greater than this results in a return to

the call to ROLL, the value of TLOAD now being set to the new value. The process is repeated until the above condition is satisfied.Necessary results are then listed.

Statements 98 to 122 write these results onto disc storage and transfer the results into the locations formerly occupied by the results of the previous loop .These values are utilized by the next loop.For programming simplicity, two dimensional arrays are utilized for variable definition, taking the form X(I,M).The 'I' value depicts the strip thickness position and is therefore at a maximum at 129.The 'M' value is set to 2 whilst results are being determined, the values for M=1 being those of the previous loop.

Before returning to loop entry, strip thinning is estimated on the assumption of plane strain incompressibility,the exit thickness being determined from the entry value and mean strip elongation.

On completion of all necessary loops, subroutine DRAW is called(where required) which utilizes the values stored on disc to produce output plots such as those given in Appendix 7.This plotting facility proved to be a good de-bugging method for subroutine SXMAX,errors in this subroutine resulting in clearly visible stress discontinuities in output plots.

A3.2. Subroutine ROLL.

Contol of subroutine ROLL is by 'N' and 'K' values passed over from LEVEL 4.From these values ROLL selects the required theoretical equations for the leveller position under consideration.A simple iterative technique is used in the determination of neutral axes, displacements as shown in the flow chart given in Figure 3.3.4.

K values of 1 or 2 with N equal to 1, result in return to the main program, whilst a K value of 3 selects the initial flattening equations. At the other end of the program, an N value of NR2 with a K value of 1, causes selection of the tension relaxation equations, higher K values being redundant.At intermediate values of N, the relevant equations defined by the K values (section A3.1) are utilized.

With any entry to the program, the incremental neutral axis displacement (BY(N,K)) is initially set to zero & stress and strain increments determined at each of the 129 thickness positions.Each stress value is scrutinized by subroutine SXMAX for validity under plastic flow conditions, necessary adjustments being made by this routine as described in Section 3.4.3..

The total stresses relevant to the zero neutral axis shift are integrated by a call to subroutine SIMPI which performs a standard Simpsons' Integration over the section (hence the choice of 129 thickness positions, which allows numerical integration to high accuracy, the 7 half divisions available often being unnecessary).

The applied load equivalent to the internal stress levels is returned to ROLL where a comparison is made with the actual applied load stored in TLOAD.Should these values coincide, the program continues, in all other cases the value in BY(N,K) is adjusted accordingly.Integral tension less than the applied load causes a shift in the positive y direction, the reverse also being true.The initial shift increments are set at ten thickness divisions.

Repetition of the process eventually results in a change in direction of the additional shift increment,indicating that the required total shift value has been surpassed.This results in a 90% reduction in the shift increment, continuation of the process again resulting in a further reverse in increment direction.A further 90% reduction is taken and convergence to the required total shift position is approached.The iterative process is terminated when the discrepancy between the two tensile loads results in incremental neutral axis shifts less than 0.1% of a thickness division.

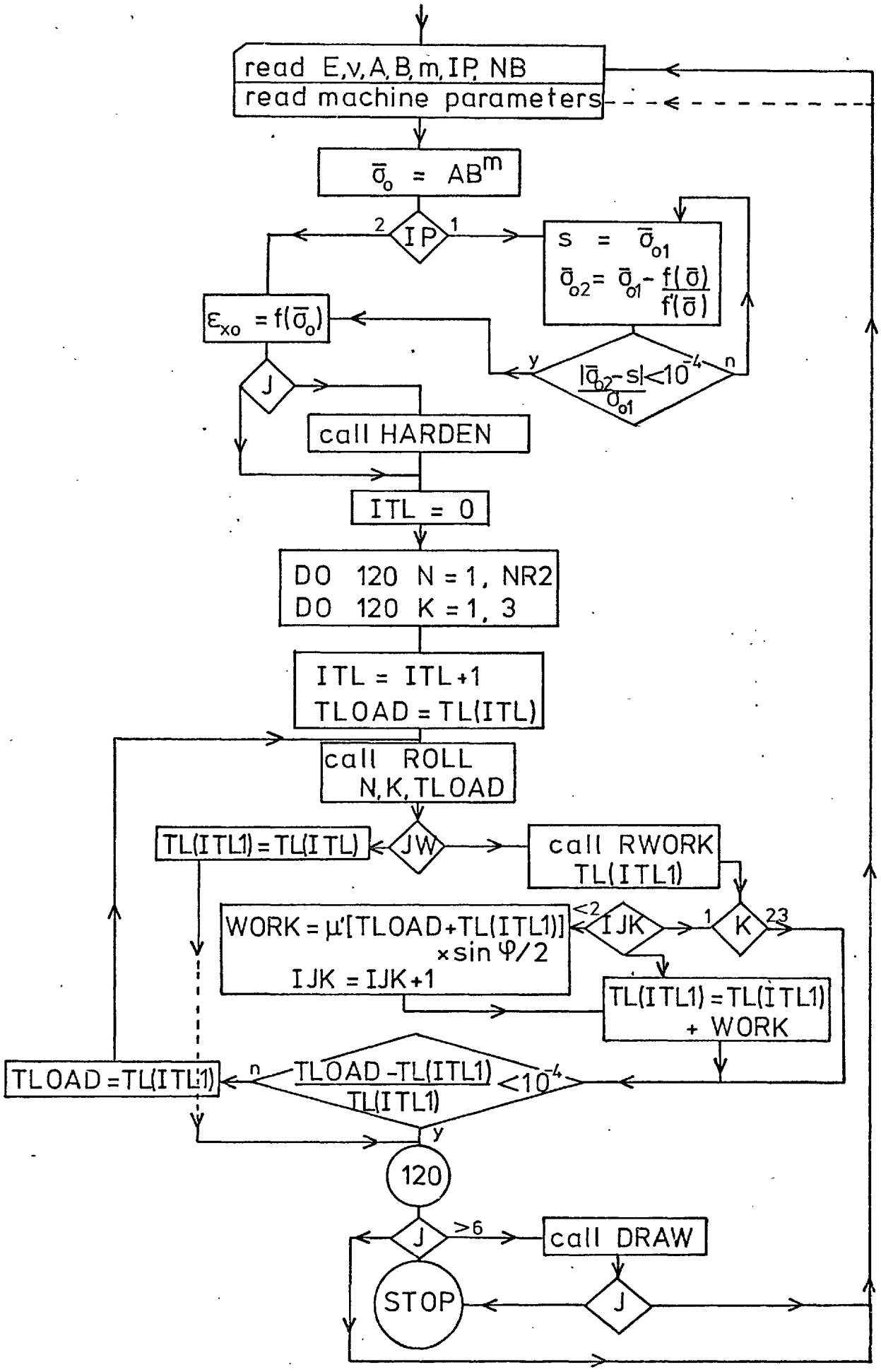
On attainment of this condition the current stress and strain levels are known and the internal moment is determined by a further call to subroutine SIMPI.Integration of different variables by this routine is simplified by the external function (F) utilized by SIMPI.Moments are determined by multiplication of stress levels by y positions before integration.Adjustment of one variable (JJ) only is needed providing the relevant information is contained in function F and'Simpsons' routine is therefore used for the determination of (a) section stress (b) internal moment and (c) section work by use of values of JJ of 1,2 and 3.

If K equals 2 (ie,spring-back has been evaluated), the residual curvature is required for use in the straightening equations.In this case this value is determined before return to the main program.

A3.3. Program Nomenclature

Subroutines - LEVEL 4, ROLL,SXMAX,RWORK,SIMPI and DRAW and function F.

For ease of routine correlation variable names have been standardized use of COMMON statements being made for transfer of variables where applicable.



FLOW CHART - LEVEL 4 (BASIC)

A .... stress/strain relationship constant (A').

AA .... half strip thickness, adjusted by strip thinning.

A/A .... original half strip thickness (CALCOMP plots are performed on this basis).

AINC .... incremental neutral axis shift.

AWORK(I,J) .... work done in any operation, at one thickness position.

AWRAP(N) .... wrap angle (degrees) at roll N-1.

ALP .... orientation of elastic stress path to x axis- deviatoric plane.

B .... stress/strain relationship constant (B').

BB .... strip width.

BY(N,K) .... total neutral axis shift at position (N,K).

BMOM(N,K) .... internal bending moment.

COEFRI .... effective bearing friction coefficient at all rolls.

CY(N,K) .... position of zero total strain axis at position (N,K).

D(N) .... diameter of roll N-1.

DOWN .... lower integration limit.

DRAW .... plotting routine name.

E .... Young's modulus.

EKT .... reduced elastic strain range.

EM .... Young's modulus X 10^{-6}.

EMU .... conversion constant - elastic strain to work done($xE/(2(1-\nu^2))$)

ET .... half elastic strain range at initial yield.

ET1,ET2,ET3,ET4,ET5 .... true strain functions utilized in subroutine RWORK.

ETAXO .... initial yield strain.

ETBU .... strain equivalent to yield locus shift (S).

ETT .... initial longitudinal yield stress from plane strain equation (σ_{x_o}).

ETX(I,J) .... longitudinal strain increment (e).

ETXAB(I,J) .... strain parameter (e_{ab}).

ETXBU(I,J) .... degree of plastic strain (e_b).

ETXE(I,J) .... elastic strain range (e_E).

ETXEZ .... half elastic strain range.

ETXE2 .... strain function utilized in subroutine RWORK.

ETXLMN .... incremental increase in maximum strain level(isotropic/kinematic model).

ETXM(I,J) .... maximum strain contributing to work hardening (e_m).

ETXMN(I,J) .... maximum total strain (e_{mn}).

ETXT(I,J) .... total strain (e_T).

F .... external function name.

FSIG,FDSIG .... function and differential used in Newton-Raphson determination of initial yield stress (LEVEL 4).

HARDEN .... subroutine name (plane strain stress/strain derivation).

HSTR .... applied load per unit width.

I .... thickness position indication.

II .... number of thickness positions (129).

ICU .... 1 for equivalent stress/strain input.
2 for longitudinal stress/strain input.

IJK .... control parameter.

ILIT .... 2 for stress/strain relationship plot.

INDP .... mass storage control variable.

IP .... 1 for input equation A2.2.1.
2 for input equation A2.2.2.

IPLIT .... 1 for no output plot.
2 for output plot.

ITL .... load position indication - operation entry.

ITLI .... load position indicator operation exit.

J .... program control parameter

J = 1,2,3,7,8,9, for HARDEN call.

J = 7,8,9,10,11,12 for output plot.

J = 1,4,7,10 for program end.

J = 2,5,8,11 for new material variables.

J = 3,6,9,12 for new leveller variables.

JF .... 1 for no bearing friction losses.
2 for bearing friction losses.

JW .... 1 for no material work losses.
2 for material work losses.

K .... deformation type indicator.
1 for bending.
2 for spring-back.
3 for straightening.

KOUNT .... mass storage control variable.

LOOP .... neutral axis position loop count.

MSER .... series name

N .... roll number indicator.

N = 1 .... initial flattening.

N=NR2 .... final stress relaxation.

N-1 = roll number.

NB .... hardening type indicator.

1 for isotropic model.

2 for kinematic model.

3 for kinematic with isotropic hardening charactistics.

4 for isotropic/kinematic model.

NBAU(I,J) .... strain increment type number (U).

NBAUSH .... strain type number where adjustment is required.

NPRNT .... number of series results plotted.

NR/1/2 .... number of rolls plus 1 or 2.

NRLAST .... number of rolls plus 1 in last plotted series.

NSERIE .... series name and number.

ORDER .... curve-fitting routine name.

QMU .... Poissons' Ratio (ν).

R(N) .... roll radius.

RM .... stress/strain relationship hardening index.

ROLL .... subroutine name.

ROM .... hardening index - continuing analyses.

PRESID(N,K) .... residual curvature.

RWORK .... material work determination subprogram name.

SIGD(I,J) .... yield locus shift (s) on kinematic hardening.

SIGMA .... initial equivalent yield stress.

SIMPI .... integration routine.

SOP .... storage variable,Newto-Raphson yield stress determination.

SMEAN .... mean applied load.

SMX(I,J) .... stress increment (σ).

SMXM(I,J) .... maximum stress level in plastic range (σ_m).

SMXT(I,J) .... total stress level (σ_T).

SXMAX .... subroutine name.

SUS .... mean applied stress as a proportion of the longitudinal yield stress.

THICK .... strip thickness. (t).

TL(ITL) .... applied load (T).

TLDIF .... load increase due to work done on material.

TLOAD .... applied load for call to ROLL.

TLOST .... total load increase through leveller.

TMEAN .... mean applied leveller load.

UP .... upper integration limit.
VAR(I) .... mass storage variable.
WORK(I,J) .... elemental work value.
WFACT .... frictional load loss.
Y(I) .... strip thickness position (y).

```
      PROGRAM LEVEL4(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE1,
     A TAPE25,TAPE27,TAPE11)
      DIMENSION AWRAP(10),VAR(129),IND(271)
      COMMON/C1/ETX(129,2),ETXT(129,2),ETXAB(129,2),ETXM(129,2),
     A ETXE(129,2),SMX(129,2),SMXT(129,2),SMXM(129,2)
      COMMON/C2/R(10),D(10),TL(20),BY(10,3),CY(10,3),RRESID(10,3),
     A BMOM(10,3),WORK(10,3)
      COMMON/C3/E,QMU,AA,BB,TLOAD,NR,NR1,NR2
      COMMON/C4/A,B,RM,IP,NB,SIGMA
      COMMON/C5/Y(130),AWORK(129,2),ETAXO
      COMMON/C6/ETXBU(129,2),SIGD(129,2),ETXMN(129,2)
      COMMON/C7/NBAU(129,2),ET,ETT
      COMMON/C8/IPLIT,ILIT
      CALL OPENMS(11,IND,271,0)
      READ(5,899) IPLIT,MSER
  899 FORMAT(I1,1X,A8)
      WRITE(6,1000)
      WRITE(6,1003)
      WRITE(6,1001)MSER
      GO TO (2,140)IPLIT
  140 CALL START
      CALL FACTOR(0.5)
    2 READ(5,900)EM,QMU,A,B,RM,IP,NB,ICU,ILIT
      ROM = RM
    3 READ(5,901)NSERIE,J,JW,JF,THICK,WIDTH,TL(1),D(1),NR
      NR1 = NR+1
      NR2 = NR+2
      READ(5,902)COEFRI,(D(I),AWRAP(I),I=2,NR1)
  900 FORMAT(F5.2,5X,F6.4,5X,F8.1,5X,F7.5,4X,F7.5,4X,I1,2X,I1,2X,I1,2X,I
     A1)
  901 FORMAT(A7,2X,I2,2X,I2,1X,I1,3X,F6.4,2X,F5.3,2X,F7.1,2X,F5.1,2X,I2)
  902 FORMAT(F5.3,5(F6.3,1X,F5.1)/5X,3(F6.3,1X,F5.1))
      WRITE(6,1000)
      WRITE(6,1003)
      NRLAST = 1
      NPRNT = 1
      D(NR2) = 0.0
      E = EM*0.1E+7
      AA = THICK/2.0
      A1A = AA
      BB = WIDTH
      IF(ROM.NE.RM) GO TO 701
      IF(B.GT.0.0) GO TO 700
      SIGMA = A
      GO TO 701
  700 SIGMA = A*(B**RM)
      WRITE(6,8000) SIGMA
      IF(IP.GT.1) GO TO 701
      DO 702 IS=1,50
      GO TO (704,705),ICU
  704 FSIG = (SIGMA/E)+B-((SIGMA/A)**(1./RM))
      FDSIG = (1./E)-  ((1./(RM*(A**(1./RM))))*(SIGMA**((1.-RM)/RM)))
      GO TO 706
  705 FSIG = (SIGMA*(1.-QMU**2)/E)+B-((SIGMA/A)**(1./RM))
      FDSIG = ((1.-QMU**2)/E)-((1./(RM*(A**(1./RM))))*(SIGMA**((1.-RM)/
     A RM)))
  706 SOP = SIGMA
      SIGMA = SIGMA-(FSIG/FDSIG)
      WRITE(6,8000) SIGMA
```

```
 8000 FORMAT(5X,8HSIGMA = ,E12.5)
      IF(ABS(SIGMA-SOP)/SIGMA.LT.0.001) GO TO 701
  702 CONTINUE
  701 ETAXO = SIGMA*(1.0-QMU**2)/(E*SQRT(1.0-QMU+QMU**2))
      IF(ICU.EQ.1) GO TO 703
      ETAXO = SIGMA*(1.-QMU**2)/E
C***** SET UP REQUIRED STRESS/STRAIN CURVE FOR THE MATERIAL
  703 GO TO (4,4,4,5,5,5,4,4,4,5,5,5)J
    4 CALL HARDEN(E,QMU,NR)
    5 II = 2**7+1
      WRITE(6,1000)
      WRITE(6,1003)
 1000 FORMAT(1H1)
 1003 FORMAT(///////)
 1004 FORMAT(//)
      WRITE(6,1001)NSERIE
 1001 FORMAT(11X,A8///)
      WRITE(6,1002) E,QMU,A,B,IP,RM,AA,BB,NR,TL(1),COEFRI,J,JW,JF,NB,
     A (I,D(I),I,AWRAP(I),I=1,NR1)
 1002 FORMAT(1H0,10X,4HE = ,F10.1,2X,6HQMU = ,F5.3,2X,4HA = ,F9.1,2X,
     A 4HB = ,F5.3, 8X,5HIP = ,I1, 4X,5HRM = ,F5.3,11X//11X,5HAA = ,F6.4
     B,5X,5HBB = ,F5.2, 3X,5HNR = ,I2, 8X,8HTL(1) = ,F7.1,2X,9HCOEFRI =
     C,F5.3,5X//11X,4HJ = ,I4, 8X,5HJW = ,I3, 5X,5HJF = ,I3, 7X,5HNB = ,
     D I3,5X,//
     D (11X,2HD(,I1,4H) = ,F7.3,6X,6HAWRAP(,I1,4H) = ,F5.1,6X))
C***** SET UP INITIAL CONDITIONS BEFORE ENTERING LEVELLER
      DO 2000 N=1,NR2
      AWRAP(N) = AWRAP(N)*2.0*3.1416/360.0
      DO 2000 K=1,3
      BY(N,K) = 0.0
      CY(N,K) = 0.0
      BMOM(N,K) = 0.0
      RRESID(N,K) = 0.0
      WORK(N,K) = 0.0
 2000 CONTINUE
      ITL = 0
      KOUNT = 0
      DO 120 N=1,NR2
      NN = N-1
      IF(N.NE.1) WRITE(6,1000)
      IF(N.NE.1) WRITE(6,1003)
      DO 120 K=1,3
      R(N) = (D(N)*0.5)+AA
      DO 2001 I=1,II
 2001 Y(I) = (-AA)+(2.0*FLOAT(I-1)*AA)/FLOAT(II-1)
      KOUNT = KOUNT + 1
      IF(N.EQ.1.AND.K.EQ.1) GO TO 120
      IF(N.EQ.1.AND.K.EQ.2) GO TO 120
      IF(N.EQ.NR2.AND.K.EQ.2) GO TO 807
      ITL = ITL+1
      TLOAD = TL(ITL)
      IF(N.EQ.NR2) TLOAD = 0.0
      ITL1 = ITL+1
      IJK = 0
  800 CALL ROLL(II,N,K)
      IF(N.EQ.NR2) GO TO 804
      GO TO (802,801)JW
  801 CALL RWORK(II,N,K,ITL)
      IF(K.NE.1.OR.N.EQ.1) GO TO 806
```

```
      IF(N.EQ.NR2) GO TO 806
      IF(IJK.GT.0) GO TO 808
      TLDIF = TL(ITL1)-TL(ITL)
  808 WFACT = COEFRI*(TL(ITL)+TLOAD+TLDIF)*SIN(AWRAP(N)/2.)
      IJK = IJK+1
      IF(JF.EQ.1) GO TO 806
      TL(ITL1) = TL(ITL1) + WFACT
  806 IF(ABS((TLOAD-TL(ITL1))/TL(ITL1)).LT.0.001) GO TO 803
      TLOAD = TL(ITL1)
      GO TO 800
  802 TL(ITL1) = TL(ITL)
      GO TO 803
  804 TL(ITL1) = TLOAD
  803 CONTINUE
      WRITE(6,1005)
 1005 FORMAT(1H0)
      WRITE(6,1010)NN,R(N),K,BY(N,K),CY(N,K),BMOM(N,K),RRESID(N,K),
     A WORK(N,K),TL(ITL),TL(ITL1)
 1010 FORMAT(10X,10HROLL NO = ,I2,8X,16HROLL RADIUS    = ,F5.3,2X,
     A16HROLL POSITION = ,I1//11X,10HBY(N,K) = ,F7.5,3X,10HCY(N,K) = ,
     B F7.5,6X,12HBMOM(N,K) = ,F10.4//11X,14HRRESID(N,K) = ,F6.2,1X,12H
     CWORK DONE = ,F10.3,3X,13HTENSION IN = ,F8.2,4X,14HTENSION OUT = ,
     D F8.2)
      WRITE(6,1011)
 1011 FORMAT(/15X,4HY(I),6X,3HETX,3X,4HETXT,3X,5HETXAB,2X,5HETXBU,2X,
     A4HETXM,3X,4HETXE,5X,4HSIGD, 6X,3HSMX, 7X,4HSMXT,6X,4HSMXM,4X,
     B5HAWORK)
      WRITE(6,1012)(Y(I),ETX(I,2),ETXT(I,2),ETXAB(I,2),ETXBU(I,2),ETXM(I
     A,2),ETXE(I,2),SIGD(I,2)  ,SMX(I,2),SMXT(I,2),SMXM(I,2),AWORK(I,2),
     BI=1,II,32)
 1012 FORMAT(11X,E11.4,F7.4,F7.4,F7.4,F7.4,F7.4,F7.4,F7.4,1X,F8.1,1X,F9.0,1X,
     A F9.0,1X,F9.0,1X,F7.2)
      M = 1
   98 DO 100 INDPA =1,9
      DO 99 I=1,II
      GO TO (80,81,82,83,84,85,86,87,88,89)INDPA
   80 VAR(I) = ETX(I,M)
      GO TO 99
   81 VAR(I) = ETXT(I,M)
      GO TO 99
   82 VAR(I) = ETXAB(I,M)
      GO TO 99
   83 VAR(I) = ETXM(I,M)
      GO TO 99
   84 VAR(I) = ETXE(I,M)
      GO TO 99
   85 VAR(I) = SMX(I,M)
      GO TO 99
   86 VAR(I) = SMXT(I,M) *6.895/1000.
      GO TO 99
   87 VAR(I) = SMXM(I,M)
      GO TO 99
   88 VAR(I) = AWORK(I,M)
      GO TO 99
   89 VAR(I) = ETXBU(I,M)
   99 CONTINUE
      INDP = INDPA+(9*(KOUNT-2))
      CALL WRITMS(11,VAR,129,INDP)
  100 CONTINUE
```

```
      IF(N.EQ.NR2.AND.M.EQ.1) GO TO 97
      GO TO 807
   97 M = 2
      KOUNT = KOUNT + 1
      GO TO 98
  807 IF(M.GT.1) GO TO 130
      DO 122 I=1,II
      ETX(I,1) = ETX(I,2)
      ETXT(I,1) = ETXT(I,2)
      ETXAB(I,1) = ETXAB(I,2)
      ETXM(I,1) = ETXM(I,2)
      ETXE(I,1) = ETXE(I,2)
      ETXBU(I,1) = ETXBU(I,2)
      SMX(I,1) = SMX(I,2)
      SMXT(I,1) = SMXT(I,2)
      SMXM(I,1) = SMXM(I,2)
      AWORK(I,1) = AWORK(I,2)
      SIGD(I,1) = SIGD(I,2)
      NBAU(I,1) = NBAU(I,2)
      ETXMN(I,1) = ETXMN(I,2)
  122 CONTINUE
      AA = AA/(1.0+ETX(65,1))
  120 CONTINUE
  130 TLOST = TL(ITL)-TL(1)
      TMEAN = (TL(ITL)+TL(1))/2.
      ETOT = ETXT(65,2)
      SMEAN = TMEAN/(WIDTH*THICK)
      SUS = SMEAN*SQRT(1.-QMU+QMU**2)/SIGMA
      WRITE(6,1006)TMEAN,TLOST,ETOT,SUS
 1006 FORMAT(///12X,12HMEAN LOAD = ,F8.2,4X,12HLOAD LOST = ,F8.2//12X,
     A 15HTOTAL STRAIN = ,F7.4,1X,3HAT ,F5.3,1X,12HYIELD STRESS)
      GO TO(1,2,3,1,2,3,131,131,131,131,131,131)J
  131 AA = A1A
      CALL DRAW(Y,II,NPRNT,NRLAST)
      IF(MOD(NR1,2))3000,3000,3001
 3000 CALL PLOT(14.0,-33.0,-3)
      GO TO 3002
 3001 CALL PLOT(14.0,-5.0,-3)
 3002 NPRNT = NPRNT+1
      NRLAST = NR1
      GO TO (1,2,3,1,2,3,1,2,3,1,2,3)J
    1 GO TO (141,142)IPLIT
  142 CALL FACTOR(1.0)
      CALL ENPLOT(7.0)
  141 STOP
      END
```

```
C.....
      SUBROUTINE ROLL(II,N,K)
      COMMON/C1/ETX(129,2),ETXT(129,2),ETXAB(129,2),ETXM(129,2),
     A ETXE(129,2),SMX(129,2),SMXT(129,2),SMXM(129,2)
      COMMON/C2/R(10),D(10),TL(20),BY(10,3),CY(10,3),RRESID(10,3),
     A BMOM(10,3),WORK(10,3)
      COMMON/C3/E,QMU,AA,BB,TLOAD,NR,NR1,NR2
      COMMON/C4/A,B,RM,IP,NB,SIGMA
      COMMON/C5/Y(130),AWORK(129,2),ETAXO
      COMMON/C6/ETXBU(129,2),SIGD(129,2),ETXMN(129,2)
      EXTERNAL F
      UP = AA
      DOWN = -AA
      JJ = 1
      KOUNT = 0
      LOOP = 0
      AINC =  (AA*10.)/FLOAT(II-1)
      HSTR = TLOAD/BB
      IF(N.GT.1) GO TO 99
      DO 98 L=1,2
      DO 98 I=1,II
      ETX(I,L) = 0.0
      ETXT(I,L) = 0.0
      ETXAB(I,L) = 0.0
      ETXBU(I,L) = 0.0
      ETXMN(I,L) = 0.0
      SMX(I,L) = 0.0
      SMXT(I,L) = 0.0
      SIGD(I,L) = 0.0
      SMXM(I,L) = A*((B+(EXP(ETAXO)-1.))**RM)
      ETXM(I,L) = EXP(SMXM(I,1)*(1.0-QMU**2)/E)-1.0
      ETXE(I,L) = 2.0*ETXM(I,1)
   98 CONTINUE
      IF(K.EQ.1.OR.K.EQ.2) GO TO 190
   99 BY(N,K) = 0.0
  102 DO 101 I=1,II
      IF(N.EQ.NR2) GO TO 85
      IF(N.EQ.1) GO TO 80
      GO TO (90,110,130)K
   80 IF(R(N).EQ.AA) GO TO 191
      IF(R(N))81,81,82
C***** INITIAL CURVATURE IN OPPOSITE DIRECTION TO FIRST BEND *****
   81 ETX(I,2) = (Y(I)+BY(N,K))/(R(N)+BY(N,K))
      SMX(I,2) = ALOG(1.+ETX(I,2))*E/(1.-QMU**2)
      GO TO 100
C***** FOR STRAIN IN SAME DIRECTION AS FIRST BEND *****
   82 ETX(I,2) = (-Y(I)+BY(N,K))/(R(N)+BY(N,K))
      SMX(I,2) = ALOG(1.+ETX(I,2))*E/(1.-QMU**2)
      GO TO 100
   85 ND = N-1
      IF(K.EQ.2.OR.K.EQ.3) GO TO 190
      SMX(I,2) =  -((3.*BMOM(ND,3))/(2.*AA**3))*(Y(I)+(((-1.)**NR1)*BY(N
     A ,K)))
      ETX(I,2) = EXP(SMX(I,2)*(1.-QMU**2)/E)-1.
      GO TO 100
C***** BEND CONDITIONS *****
   90 ETX(I,2) = ((((-1.)**N)*Y(I))+BY(N,K))*(1.+ETXT(I,1))/(R(N)-BY(N,K
     A))
      SMX(I,2) = ALOG(1.+ETX(I,2))*E/(1.-QMU**2)
```

```
      GO TO 100
C***** SPRINGBACK CONDITIONS *****
  110 SMX(I,2) =-((3.*BMOM(N,1))/(2.*AA**3))*(Y(I)-(((-1.)**N)*BY(N,K)))
      ETX(I,2) = EXP(SMX(I,2)*(1.-QMU**2)/E)-1.
      GO TO 100
C***** STRAIGHTENING CONDITIONS *****
  130 ETX(I,2) = (((-1.)**(N+1))*Y(I)+BY(N,K))/(ABS(RRESID(N,2))-
     A CY(N,2))
      SMX(I,2) = ALOG(1.+ETX(I,2))*E/(1.-QMU**2)
  100 SMXT(I,2) = SMXT(I,1)+SMX(I,2)
      IF(I.GT.129) WRITE(6,206)
  206 FORMAT(1H0,10HKNICKERS  )
      IF(I.GT.129) GO TO 101
      CALL SXMAX(I,N,K)
  101 CONTINUE
      CALL SIMPI(F,1,N,K,-AA,AA,0.001,7,SI1,S,INTNO,IER)
C***** DETERMINE WHETHER HORIZONTAL EQUILIBRIUM IS MAINTAINED *****
      LOOP = LOOP+1
      IF(LOOP.EQ.1000) GO TO 203
      IF(TLOAD/BB-S)104,106,108
  104 IF(KOUNT.EQ.2) AINC = AINC/10.0
      BY(N,K) = BY(N,K)-AINC
      IF(AINC.LT.AA/(10.0**6)) GO TO 106
      KOUNT = 1
      GO TO 102
  108 IF(KOUNT.EQ.1) AINC = AINC/10.0
      BY(N,K) = BY(N,K)+AINC
      IF(AINC.LT.AA/(10.0**6)) GO TO 106
      KOUNT = 2
      GO TO 102
C***** CORRECT BY(N,K) HAS NOW BEEN DETERMINED , AND THUS THE CORRECT
C***** VALUES OF STRESS AND STRAIN ARE KNOWN...
  106 CONTINUE
      GO TO 205
  203 WRITE(6,204)
  204 FORMAT(1H0,25H1000 LOOPS EXCEEDED     )
C***** CALCULATE BENDING MOMENT FOR FUTURE USE.....
  205 CALL SIMPI(F,2,N,K,-AA,AA,0.001,7,SI1,S,INTNO,IER)
      BMOM(N,K) = S
      GO TO (301,302,302)K
  301 ND = N-1
      KD = 3
      GO TO 303
  302 ND = N
      KD = K-1
  303 IF(K.EQ.3) GO TO 190
C***** CALCULATE RESIDUAL CURVATURE AND CY(N,K) FOR SPRINGBACK...
      CY(N,K) = (((((-1.)**N)*Y(II))*ETXT(1,2))-((((-1.)**N)*Y(1))*
     A ETXT(II,2)))/(ETXT(II,2)-ETXT(1,2))
      RRESID(N,K) =((((((-1.)**N)*Y(1))+CY(N,K))/ETXT(1,2))+CY(N,K))
     A *((-1.)**N)
      IF(CY(N,K).LT.0.0) RRESID(N,K) = RRESID(N,K)-(2.*CY(N,K)*((-1.)
     A **N))
  190 RETURN
  191 DO 192 I=1,II
      SMX(I,2) = HSTR/(2.*AA)
      SMXT(I,2) = SMX(I,2)
      ETX(I,2) = EXP(SMX(I,2)*(1.-QMU**2)/E)-1.
      ETXT(I,2) = ETX(I,2)
```

```
      ETXBU(I,2) = 0.0
      ETXMN(I,2) = 0.0
  192 ETXAB(I,2) = ETX(I,2)
      RETURN
      END
```

```
C.....
      SUBROUTINE SIMPI(F,JJ,NT,K,A,B,DEL,IMAX,SI1,S,N,IER)
      SI1=.0
      S=.0
      N=0
      BA=B-A
      IF (BA)20,19,20
   19 IER=1
      RETURN
   20 IF(DEL)22,22,23
   22 IER=2
      RETURN
   23 IF(IMAX-1)24,24,25
   24 IER=3
      RETURN
C
C     COMPUTE SIGMA(1)
C
   25 X=BA/2.+A
      NHALF=1
      SUMK=F(X,JJ,NT,K)*BA*2./3.
      S=SUMK+(F(A,JJ,NT,K)+F(B,JJ,NT,K))*BA/6.
C
C     DIVIDE (A,B) INTO 2,4,...,2**I INTERVALS,
C     COMPUTE SIGMA(2),SIGMA(4),...,SIGMA(I)
C
      DO 28 I=2,IMAX
      SI1=S
      S=(S-SUMK/2.)/2.
      NHALF=NHALF*2
      ANHLF=NHALF
      FRSTX=A+(BA/ANHLF)/2.
      SUMK=F(FRSTX,JJ,NT,K)
      XK=FRSTX
      KLAST=NHALF-1
      FINC=BA/ANHLF
      DO 26 KN=1,KLAST
      XK=XK+FINC
   26 SUMK=SUMK+F(XK,JJ,NT,K)
      SUMK=SUMK*2.*BA/(3.*ANHLF)
      S=S+SUMK
C
C     COMPARES THE I-TH AND(I-1)ST RESULTS
C
   27 IF( ABS(S-SI1)- ABS(DEL*S)) 29,28,28
   28 CONTINUE
      IER=4
      GO TO 30
   29 IER=0
   30 N=2*NHALF
      RETURN
      END
```

```
C.....
      FUNCTION F(X,JJ,N,K)
      COMMON/C1/ETX(129,2),ETXT(129,2),ETXAB(129,2),ETXM(129,2),
     A ETXE(129,2),SMX(129,2),SMXT(129,2),SMXM(129,2)
      COMMON/C3/E,QMU,AA,BB,TLOAD,NR,NR1,NR2
      COMMON/C5/Y(130),AWORK(129,2),ETAXO
      II = (2**7)+1
      DELT = 2.*AA/FLOAT(II-1)
      I = 1.+(X+AA)/DELT
      GO TO(601,602,603)JJ
  601 F = SMXT(I,2)
      RETURN
  602 F = SMXT(I,2)*Y(I)
      RETURN
  603 F = AWORK(I,2)
      RETURN
      END
```

```
C.....
      SUBROUTINE DRAW(YPOS,II,NPRNT,NRLAST)
      DIMENSION X(131),Y(131),YPOS(130)
      DIMENSION ITIT(3),IFIN(5),ITIP(3),XX(131),YY(131),IFIP(5)
      COMMON/C3/E,QMU,AQ,BB,TLOAD,NR,NR1,NR2
    9 CALL PLOT(10.0,47.0,-3)
      AA = AQ*25.4
      DO 10 I=1,II
   10 Y(I) = (-AA)+(2.*FLOAT(I-1)*AA)/FLOAT(II-1)
      Y(130) = -(AA*(10.0**2))
      Y(131) = AA*(10.0**2)/4.0
      KOUNT = 0
      DO 100 N=1,NR1
      CALL AXIS( 0.0, -4.0,    49H                                    STRIP THICK
     ANESS MM*10,    49,8.0,90.0,Y(130),Y(131))
      X(130) = -0.06
      X(131) = 0.01
      CALL AXIS( -6.0,0.0,30HLOG. ,STRAIN                          ,-30,12.0,0.0
     A,X(130),X(131))
      CALL PLOT(-6.0,-4.0,-3)
      YY(130) = 0.0
      XX(130) = 0.0
      YY(131) = Y(131)/(10.0**2)
      XX(131) = X(131)
      DO 40 K =1,3
      KOUNT = KOUNT + 1
      IF(N.EQ.1.AND.K.EQ.1.OR.N.EQ.1.AND.K.EQ.2) GO TO 40
      INDP = 2 + (9*(KOUNT-1))
      CALL READMS(11,XX,129,INDP)
      DO 30 I =1,II
      YY(I) = Y(I)-(Y(130)/(10.0**2))
   30 XX(I) = XX(I)-X(130)
      CALL LINE(XX,YY,129,1,5,K+1)
   40 CONTINUE
      CALL PLOT(0.0,0.0,3)
      CALL PLOT(12.0,0.0,2)
      CALL PLOT(0.0,8.0,3)
      CALL PLOT(12.0,8.0,2)
      DATA(ITIP(I),I=1,3)/30HSTRAIN DISTRIBUTION OVER ROLL /
      CALL SYMBOL(2.5,8.2,0.21,ITIP,0.0,30)
      A = N-1
      CALL NUMBER(9.2,8.2,0.21,A,0.0,-1)
      CALL PLOT(6.0,-6.0,-3)
      CALL AXIS( 0.0, -4.0,    49H                                    STRIP THICK
     ANESS MM*10,    49,8.0,90.0,Y(130),Y(131))
      X(130) =-720.
      X(131) = 120.
      CALL AXIS(-6.0,0.0,38HSTRESS MN/M**2                          ,
     A -38,12.0,0.0,X(130),X(131))
      CALL PLOT(-6.0,-4.0,-3)
      XX(131) = X(131)
      KOUNT = KOUNT-3
      DO 70 K=1,3
      KOUNT = KOUNT + 1
      IF(N.EQ.1.AND.K.EQ.1.OR.N.EQ.1.AND.K.EQ.2) GO TO 70
      INDP = 7 + (9*(KOUNT-1))
      CALL READMS(11,XX,129,INDP)
      DO 60 I=1,II
   60 XX(I) = XX(I)-X(130)
```

```
      CALL LINE(XX,YY,129,1,5,K+1)
   70 CONTINUE
      CALL PLOT(0.0,0.0,3)
      CALL PLOT(12.0,0.0,2)
      CALL PLOT(0.0,8.0,3)
      CALL PLOT(12.0,8.0,2)
      DATA(ITIT(I),I=1,3)/30HSTRESS DISTRIBUTION OVER ROLL /
      CALL SYMBOL(2.5,8.2,0.21,ITIT,0.0,30)
      CALL NUMBER(9.2,8.2,0.21,A,0.0,-1)
      IF(MOD(N,2))90,90,80
   80 CALL PLOT(6.0,-14.0,-3)
      GO TO 100
   90 CALL PLOT(26.0,42.0,-3)
  100 CONTINUE
      KOUNT = KOUNT + 1
      CALL AXIS( 0.0, -4.0,    49H                                    STRIP THICK
     ANESS MM*10,    49,8.0,90.0,Y(130),Y(131))
      INDP = 2 + (9*(KOUNT-1))
      CALL READMS(11,X,129,INDP)
      X(130) = -0.06
      X(131) = 0.01
      CALL AXIS( -6.0,0.0,30HLOG. STRAIN                          ,-30,12.0,0.0
     A,X(130),X(131))
      CALL PLOT(-6.0,-4.0,-3)
      XX(131) = X(131)
      DO 150 I=1,II
  150 XX(I) = X(I)-X(130)
      CALL LINE(XX,YY,129,1,5,5)
      CALL PLOT(0.0,0.0,3)
      CALL PLOT(12.0,0.0,2)
      CALL PLOT(0.0,8.0,3)
      CALL PLOT(12.0,8.0,2)
      DATA(IFIP(I),I=1,5)/45HSTRAIN DISTRIBUTION AFTER TENSION RELAXATIO
     AN /
      CALL SYMBOL(1.25,8.2,0.21,IFIP,0.0,45)
      CALL PLOT(6.0,-6.0,-3)
      CALL AXIS( 0.0, -4.0,    49H                                    STRIP THICK
     ANESS MM*10,    49,8.0,90.0,Y(130),Y(131))
      INDP = 7 + (9*(KOUNT-1))
      CALL READMS(11,X,129,INDP)
      X(130) =-720.
      X(131) = 120.
      CALL AXIS(-6.0,0.0,38HSTRESS MN/M**2                          ,
     A -38,12.0,0.0,X(130),X(131))
      CALL PLOT(-6.0,-4.0,-3)
      XX(131) = X(131)
      DO 170 I=1,II
  170 XX(I) = X(I)-X(130)
      CALL LINE(XX,YY,129,1,5,10)
      CALL PLOT(0.0,0.0,3)
      CALL PLOT(12.0,0.0,2)
      CALL PLOT(0.0,8.0,3)
      CALL PLOT(12.0,8.0,2)
      DATA(IFIN(I),I=1,5)/45HSTRESS DISTRIBUTION AFTER TENSION RELAXATIO
     AN /
      CALL SYMBOL(1.25,8.2,0.21,IFIN,0.0,45)
      RETURN
      END
```

APPENDIX 4.

A4.0. Subroutine SXMAX.

the mode of operation has been fully described previously (section 3.4.3.) the flow chart being given in Fig 3.50.The routine is scanned once for each stress level determination and therefore represents a large error source should discrepancies be present in its construction. This construction is however, elementary, no 'do-loops' or calls to other routines being present.It was found that 'de-bugging' of the routine was most easily performed on the basis of theory output plots, errors being evident as large stress discontinuities.

Care should be taken in the use of this routine for other systems, since instabilities do exist under certain circumstances.Application to the tension levelling process has proved fortunate in that stress reversals within the elastic range are seldom consecutive.Should this occur in bulk - (as it would for use in roller-levelling analysis) errors in predicted stress levels and equilibrium conditions will result.

```
C.....
      SUBROUTINE SXMAX(I,N,K)
      COMMON/C1/ETX(129,2),ETXT(129,2),ETXAB(129,2),ETXM(129,2),
     A ETXE(129,2),SMX(129,2),SMXT(129,2),SMXM(129,2)
      COMMON/C3/E,QMU,AA,BB,TLOAD,NR,NR1,NR2
      COMMON/C4/A,B,RM,IP,NB,SIGMA
      COMMON/C5/Y(130),AWORK(129,2),ETAXO
      COMMON/C6/ETXBU(129,2),SIGD(129,2),ETXMN(129,2)
      COMMON/C7/NBAU(129,2),ET,ETT
      ETT = A*((B+(EXP(ETAXO)-1.))**RM)
      ET = EXP(ETT*(1.0-QMU**2)/E)-1.
      SALP = (SQRT(3.)*QMU/(2.*SQRT(1.-QMU+QMU**2)))*SIN(3.1416/3.)
      ALP = ARSIN(SALP)*180./3.1416
      TIP = (30.-ALP)*3.1416/180.
      TKP = ETT*(COS(TIP)**2)
      EKT = EXP(TKP*(1.-QMU**2)/E)-1.
C***** DETERMINE SUM STRAIN *****
      ETXT(I,2) = ETXT(I,1) + ETX(I,2)
      ETXMN(I,2) = ETXMN(I,1)
      IF(ABS(ETXT(I,2)).GT.ABS(ETXMN(I,1))) ETXMN(I,2) = ABS(ETXT(I,2))
C***** DETERMINE WHETHER LAST STRAIN WAS PLASTIC *****
      IF(ETXM(I,1)-ETXAB(I,1))16,16,18
C***** IF STRAIN AT MMAX IS NEXT INCREMENT IN SAME DIRECTION *****
   16 IF(ETX(I,2).LT.0.0.AND.ETX(I,1).GT.0.0.OR.ETX(I,2).GT.0.0.
     A AND.ETX(I,1).LT.0.0) GO TO 20
C***** IF STRAIN IS IN SAME DIRECTION *****
      ETXAB(I,2) = ETXM(I,1)+ABS(ETX(I,2))
      ETXBU(I,2) = ETXBU(I,1) + ABS(ETX(I,2))
      ETXM(I,2) = ETXAB(I,2)
      SIGD(I,2) = SIGD(I,1)
      NBAU(I,2) = 1
      GO TO 5000
C***** FOR OPPOSITE DIRECTION ..IE..ENTERING ELASTIC ZONE *****
   20 IF(ABS(ETX(I,2))-ETXE(I,1))22,22,24
C***** ELASTIC RANGE NOT EXCEEDED *****
   22 ETXAB(I,2) = ETXAB(I,1)-ABS(ETX(I,2))
      ETXBU(I,2) = ETXBU(I,1)
      ETXM(I,2) = ETXM(I,1)
      SIGD(I,2) = SIGD(I,1)
      NBAU(I,2) = 2
      GO TO 5000
C***** ELASTIC RANGE EXCEEDED *****
   24 ETXAB(I,2) = ETXM(I,1)+ABS(ETX(I,2))-ETXE(I,1)
      ETXBU(I,2) = ETXAB(I,2)-ETXM(I,1)
      ETXM(I,2) = ETXAB(I,2)
      SIGD(I,2) = SMXM(I,1)-ETT
      NBAU(I,2) = 3
      GO TO 5000
C***** FOR STRAIN IN ELASTIC ZONE, TEST FOR DIRECTION OF STRAIN *****
   18 IF(ETX(I,2).LT.0.0.AND.ETX(I,1).GE.0.0.OR.ETX(I,2).GT.0.0.
     A AND.ETX(I,1).LE.0.0) GO TO 26
C***** FOR SAME DIRECTION OF STRAIN,  TEST FOR PLASTIC DEFORMATION *****
      IF(N.EQ.2.AND.K.EQ.1) GO TO 27
   19 IF((ETXAB(I,1)-ABS(ETX(I,2)))-(ETXM(I,1)-ETXE(I,1)))30,28,28
C***** FOR NO PLASTIC DEFORMATION *****
   28 ETXAB(I,2) = ETXAB(I,1)-ABS(ETX(I,2))
      IF(ETXT(I,2).GT.ETXAB(I,2)) GO TO 27
      ETXBU(I,2) = ETXBU(I,1)
      ETXM(I,2) = ETXM(I,1)
```

```
      SIGD(I,2) = SIGD(I,1)
      NBAU(I,2) = 4
      GO TO 5000
C***** FOR PLASTIC DEFORMATION *****
   30 ETXAB(I,2) = ETXM(I,1)+(ETXM(I,1)-ETXE(I,1)-(ETXAB(I,1)-ABS(ETX(I,
     A 2))))
      IF(ETXT(I,2).GT.ETXAB(I,2)) GO TO 27
      ETXBU(I,2) = ETXAB(I,2)-ETXM(I,1)
      ETXM(I,2) = ETXAB(I,2)
      SIGD(I,2) = SMXM(I,1)-ETT
      NBAU(I,2) = 5
      GO TO 5000
C***** FOR OPPOSITE DIRECTION OF STRAIN *****
C***** TEST TO DETERMINE WHETHER PLASTIC DEFORMATION OCCURS *****
   26 IF(N.EQ.2.AND.K.EQ.1) GO TO 19
   27 IF(ETXAB(I,1)+ABS(ETX(I,2))-ETXM(I,1))32,32,34
C***** FOR NO PLASTIC DEFORMATION *****
   32 ETXAB(I,2) = ETXAB(I,1)+ABS(ETX(I,2))
      ETXBU(I,2) = ETXBU(I,1)
      ETXM(I,2) = ETXM(I,1)
      SIGD(I,2) = SIGD(I,1)
      NBAU(I,2) = 6
      GO TO 5000
C***** FOR PLASTIC DEFORMATION OCCURRING *****
   34 ETXAB(I,2) = ETXAB(I,1)+ABS(ETX(I,2))
      ETXBU(I,2) = ETXBU(I,1)+ABS(ETX(I,2))
      ETXM(I,2) = ETXAB(I,2)
      SIGD(I,2) = SIGD(I,1)
      NBAU(I,2) = 7
      GO TO 5000
C***** KNOWING ETXM(I,2) WE NOW REQUIRE SMXM(I,2) *****
C***** TEST TO DETERMINE WHETHER PLASTIC DEFORMATION HAS EVER OCCURRED *
 5000 IF(NB.GT.1) GO TO 40
      IF(ETXM(I,2)-ET)5001,5001,5002
 5001 SMXM(I,2) = A*((B+(EXP(ETAXO)-1.))**RM)
      GO TO 5005
 5002 SMXM(I,2) = A*((B+ETXM(I,2))**RM)
 5005 IF(ETXM(I,1).GT.(2.*ET)) GO TO 505
      ETXE(I,2)= 2.*(EXP(SMXM(I,2)*(1.0-QMU**2)/E)-1.)
      GO TO 44
  505 SPLUG = SMXM(I,2)*(COS(TIP)**2)
      ETXE(I,2) = 2.*(EXP(SPLUG*(1.-QMU**2)/E)-1.)
      GO TO 44
   40 NBAUSH = NBAU(I,2)
      IF(NBAU(I,2).EQ.1.AND.NBAU(I,1).EQ.7.AND.N.NE.2)NBAUSH = 7
      IF(NBAU(I,2).EQ.5.AND.NBAU(I,1).EQ.6) NBAUSH = 7.
      IF(NB.GT.2) GO TO 50
      GO TO(41,42,41,42,41,42,45)NBAUSH
   41 SMXM(I,2) = A*((B+ET+ETXBU(I,2))**RM) - SIGD(I,2)
      GO TO 43
   45 SMXM(I,2) = A*((B+ET+ETXBU(I,2))**RM) + SIGD(I,2)
      GO TO 43
   50 IF(NB.GT.3) GO TO 60
      GO TO(51,42,51,42,51,42,52)NBAUSH
   51 SMXM(I,2) = A*(((B+ETXM(I,2))**RM)-((B+ETXM(I,1))**RM))+ETT
     A -SIGD(I,2)
      GO TO 43
   52 SMXM(I,2) = A*(((B+ETXM(I,2))**RM)-((B+ETXM(I,1))**RM))+ETT
     A +SIGD(I,2)
```

```
      GO TO 43
   60 IF(ABS(ETXT(I,2)).GT.ABS(ETXMN(I,1))) GO TO 63
      GO TO(61,42,62,42,62,42,61)NBAUSH
   61 SMXM(I,2) = SMXM(I,1)+(2.*A*(((B+((ETXE(I,1)+ETXBU(I,2))/2.))**RM)
     A -((B+((ETXE(I,1)+ETXBU(I,1))/2.))**RM)))
      GO TO 43
   62 SMXM(I,2) = (2.*A*((B+((ETXE(I,1)+ETXBU(I,2))/2.))**RM))-SMXM(I,1)
      GO TO 43
   63 SMXM(I,2) = A*((B+ABS(ETXMN(I,2)))**RM)
      GO TO 43
   42 SMXM(I,2) = SMXM(I,1)
   43 ETXE(I,2) =2.*ET
      IF(ETXM(I,1).GT.2.*ET) ETXE(I,2) =    2.*EKT
   44 IF(SMXM(I,2)-ABS(SMXT(I,2)))5006,5007,5007
 5006 IF(SMXT(I,2))5008,5008,5009
 5008 SMXT(I,2) = -SMXM(I,2)
      GO TO 5010
 5009 SMXT(I,2) = SMXM(I,2)
      GO TO 5010
 5007 SMXT(I,2) = SMXT(I,2)
 5010 RETURN
      END
```

APPENDIX 5.

A5.0. 'Work done' evaluation.

Tension losses resulting from the work done on the strip during any operation are readily determined by equation 3.5.7.The evaluation of this equation requires a quantitative measure of the internal work, which in turn must be derived by numerical integration of the work done at each thickness element.This latter process is moderately complex since fibres are often strained compressively whilst under the action of tensile forces and work is therefore gained.The movement of the elastic origin in the case of kinematic work-hardening further complicates the analysis.

The prior development of subroutine SXMAX has, however,greatly aided the development of subroutine RWORK, since the relative magnitudes of elastic and plastic deformation are accurately defined by this routine. It was therefore only necessary to reconstruct subroutine SXMAX on a 'work done' basis, utilizing equations 3.5.11 and 3.5.12 with approximate adjustment to 3.5.12 whose kinematic hardening models were considered.

For simplicity elastic work functions are considered in isolation to subsequent plastic work.The former will therefore be considered first, using as a basis the 'U' numbers (1 to 7) developed in section 3.4.3.

A5.1. Elastic components.

From equation 3.5.11, we have that

$$\text{work}^E = \frac{E\ (\varepsilon_2^2 - \varepsilon_1^2)}{2(1-\nu^2).} \qquad \text{----A5.1.1.}$$

It will be apparent, however, that simple substitution into this equation will not yield a zero result for reverse loading over the complete elastic range.The elastic range must therefore be divided into positive and negative stress regions.

For the case of U = 1, no elastic straining occurs, and we may write,

$$\text{work}^E = 0.0 \qquad \text{----A5.1.2.}$$

For U = 2,ie. reverse loading into the elastic range, the work done is negative to the zero stress point and positive there-after.Taking the isotropic case initially,(Fig A5.1.).

(a) For strains $e_2 > e_{El}/2$.

$$\text{work}^E = \frac{E\ ((\ln(e_{ml}-(e_{El}/2)-e_{ABl}+1))^2-(\ln(\frac{e_{El}}{2}+1))^2)}{2(1-\nu^2)} \qquad \text{----A5.1.3.}$$

(b) For $e_2 < e_{E1}/2$.

$$work^E = \frac{E}{2(1-\nu^2)}\left((\ln(e_{AB2}-e_{m1}+\frac{e_{E1}}{2}+1))^2-(\ln(\frac{e_{E1}}{2}+1))^2\right). \quad \text{----A5.1.4.}$$

by the notation of Figs A5.1 and A5.2.

For future simplicity we may utilize the strain functions listed in the program, but to make the analysis generally applicable to kinematic hardening models we may substitute for $e_{E1}/2$

$$e'_{E1} = \frac{e_{E1}}{2} + \frac{S_2(1-\nu^2)}{E} \quad \text{-----A5.1.5. (Fig A5.2.).}$$

the second term accounting for the shift (S_2) of the centre of the elastic strain range away from the zero stress axis. Under isotropic hardening conditions,

$$e'_{E1} = e_{E1}/2 \quad \text{----A5.1.6.}$$

The required strain functions are:-

$$ETXE2 = (\ln(e'_{E1}+1))^2 \quad \text{----A5.1.7.}$$

$$ET1 = (\ln(e_{m1}-e'_{E1}-e_{AB1}+1))^2 \quad \text{----A5.1.8.}$$

$$ET2 = (\ln(e_{m1}-e'_{E1}-e_{AB2}+1))^2 \quad \text{----A5.1.9.}$$

$$ET3 = (\ln(e_{AB1}-e_{m1}+e'_{E1}+1))^2 \quad \text{----A5.1.10.}$$

$$ET4 = (\ln(e_{AB2}-e_{m1}+e'_{E1}+1))^2 \quad \text{----A5.1.11.}$$

$$ET5 = (\ln(e_{E1}-e'_{E1}+1))^2 \quad \text{----A5.1.12.}$$

$$EMU = E/2(1-\nu^2) \quad \text{----A5.1.13.}$$

Rewriting equations A5.1.3 and A5.1.4 we have

U = 2.

(a) $work^E = EMU(ET4-ETXE2)$ ----A5.14.

(b) $work^E = EMU(ET2-ETXE2)$ ----A5.1.15.

For U = 3. ie. reverse loading with both elastic and plastic deformation.

$work^E = EMU(ET5-ETXE2)$ ----A5.1.16.

this being zero for zero yield origin shift.

For U = 4. three possible values exist, depending upon the starting and finishing positions within the elastic strain range.

(a) $e_{m1}-e'_{E1} < e_{AB2} < e_{AB1}$.

$work^E = EMU(ET4 - ET3)$ ----A5.1.17.

(b) $e_{AB2} < e_{m1} - e'_{E1} < e_{AB1}$

$work^E = EMU(ET2 - ET3)$ ----A5.1.18.

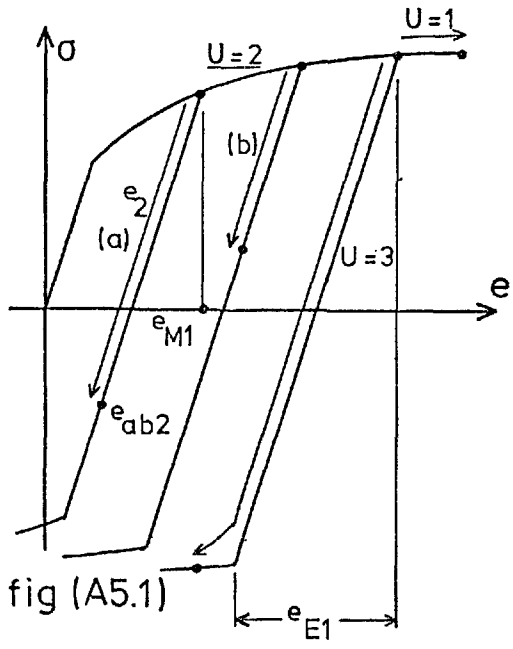


fig (A5.1)

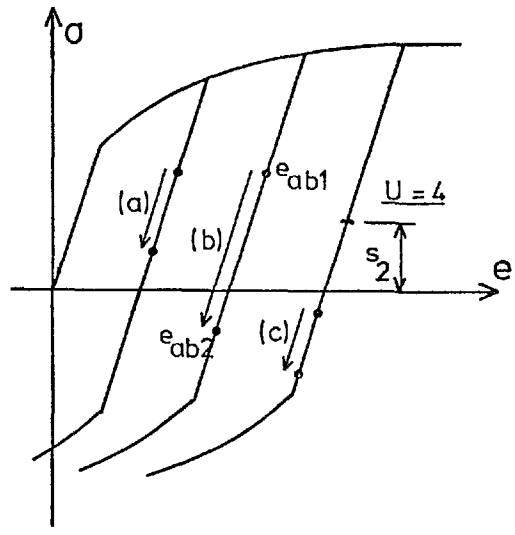


fig (A5.2)

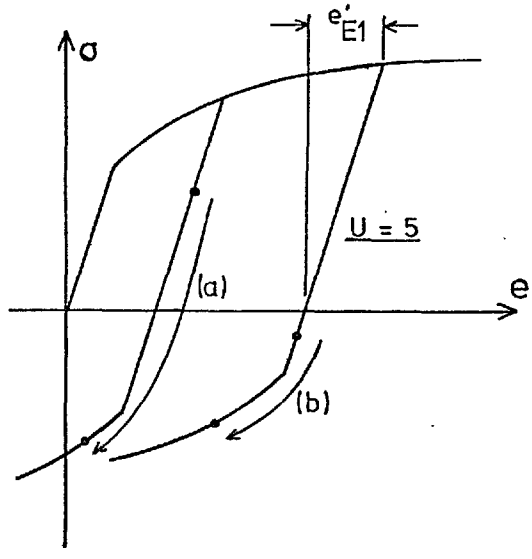


fig (A5.3)

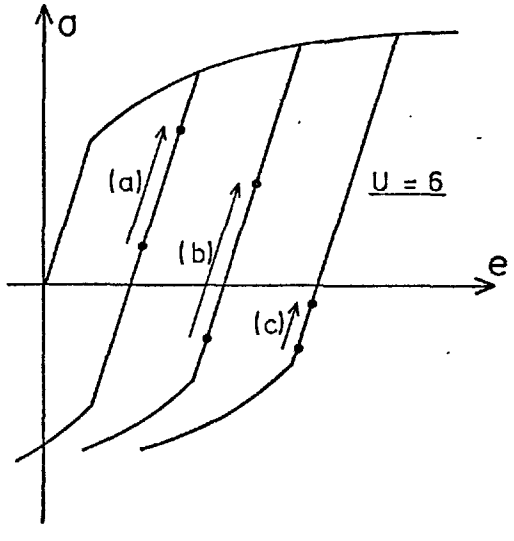


fig (A5.4)

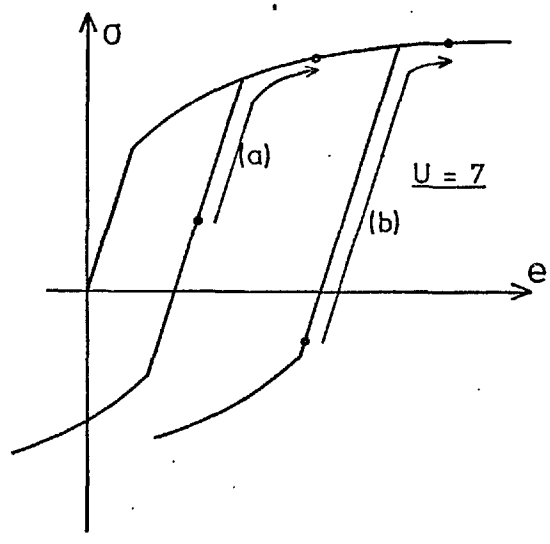


fig (A5.5)

(c) $e_{AB2} < e_{AB1} < e_{m1} - e'_{E1}$.

$$work^E = EMU(ET2 - ET1) \qquad \text{----A5.1.19.}$$

(Fig A5.2.).

For U = 5, two possibilities exist, again being dependent upon starting strain positions.

(a) $e_{m1} - e'_{E1} < e_{AB1}$

$$work^E = EMU(ET5 - ET3) \qquad \text{----A5.1.20.}$$

(b) $e_{AB1} < e_{m1} - e'_{E1}$

$$work^E = EMU(ET5 - ET1) \qquad \text{----A5.1.21.}$$

(Fig A5.3.).

For U = 6.

(a) $e_{m1} - e'_{E1} < e_{AB1} < e_{AB2}$

$$work^E = EMU(ET4 - ET3) \qquad \text{----A5.1.22.}$$

(b) $e_{AB1} < e_{m1} - e'_{E1} < e_{AB2}$

$$work^E = EMU(ET4 - ET1) \qquad \text{----A5.1.23.}$$

(c) $e_{AB1} < e_{AB2} < e_{m1} - e'_{E1}$

$$work^E = EMU(ET2 - ET1) \qquad \text{----A5.1.24.}$$

(Fig A5.4.).

For U = 7.

(a) $e_{m1} - e'_{E1} < e_{AB1} < e_{AB2}$

$$work^E = EMU(ETXE2 - ET3) \qquad \text{----A5.1.25.}$$

(b) $e_{AB1} < e_{m1} - e'_{E1} < e_{AB2}$

$$work^E = EMU(ETXE2 - ET1) \qquad \text{----A5.1.26.}$$

(Fig A5.5.).

Equations A5.1.14 to A5.1.26 provide complete coverage of all possible strain situations during elastic deformation.

A5.2. Plastic Components.

For U values of 1,3,5 and 7 a plastic work component must be added to the elastic component.

With the case of isotropic hardening the additional work is simply derived from equation 3.5.12, and we may write,

$$work^P = \frac{A'}{(1+m')}\left((B' + e_{AB2})^{1+m'} - (B' + e_{m1})^{1+m'}\right) \qquad \text{----A5.2.1.}$$

Equations for the kinematic cases will be more complex, however, since the stress/strain relationships have been derived for use with the variable e_B. This parameter reduces to zero at each change of direction of plastic stress.

With the simple kinematic case, equations 3.4.30 and 3.4.31 have been derived for the definition of the 'maximum permissible stress'.From equation 3.4.30, the work integral for U = 1 may be written.

$$work^P = \frac{A'}{1+m'}\left((B' + e_{B2} + e''_{E1})^{1+m'} - (B' + e_{B1} + e''_{E1})^{1+m'}\right) - S_2(e_{B2} - e_{B1}) \qquad \text{----A5.2.2.}$$

where $e''_{E1} = e_{E1}/2$

for U = 3 or 5, from equation 3.4.30,

$$work^P = \frac{A'}{1+m'}\left((B' + e_{B2} + e''_{E1})^{1+m'} - (B' + e''_{E1})^{1+m'}\right) - S_2(e_{B2}) \qquad \text{----A5.2.3.}$$

For U = 7, from equation 3.4.31 (or where U = 7 and 1, or 6 and 5 are consecutive).

$$work^P = \frac{A'}{1+m'.}\left((B' + e_{B2} + e''_{E1})^{1+m'} - (B' + e''_{E1})^{1+m'}\right) + S_2(e_{B2}). \qquad \text{----A5.2.4.}$$

Again total fibre work is derived by the addition of elastic and plastic components.

For the isotropic/kinematic assumption the integrals of equations 3.4.35 and 3.4.36 are utilized.

For U = 3,5.

$$work^P = \frac{4A'}{1+m'}\left((B' + e''_{E1} + e_{B2}/2)^{1+m'} - (B' + e''_{E1})^{1+m'}\right) - (\sigma_{m1} e_{B2}) \qquad \text{----A5.2.5.}$$

For U = 1,7.

$$work^P = \frac{4A'}{1+m'}\left((B' + e''_{E1} + e_{B2}/2)^{1+m'} - (B' + e''_{E1} + e_{B1}/2)^{1+m'}\right) + (\sigma_{m1} - 2A'(B' + e''_{E1} + e_{B1}/2)^{m'})e_{B2} \qquad \text{----A5.2.6.}$$

Equations A5.2.5 and A5.2.6 only apply where $|e_{T2}| < e_{MN1}$ (equation 3.4.32).If this condition is not satisfied the basic stress strain curve is followed and the plastic work function is composed of two parts.

(i) that portion obeying the reduced stress/strain relationship of equation 3.4.15 up to the strain e_{MN1}.

(ii) the subsequent portion which obeys the relationship of equation 3.4.14 from e_{MN1} to e_{T2}.

We may therefore write

U = 1,7

$$\text{work}^P = \frac{4A'}{1+m'}\left((B' + e''_{E1} + (e_{B2} - e_{MN2} + e_{MN1})^{1+m'}/2\right) - (B' + e''_{E1} + e_{B1}/2)^{1+m'}) + (\sigma_{M1} - 2A'(B' + e''_{E1} + e_{B1}/2))(e_{MN2} - e_{MN1}) + \frac{A'}{1+m'}((B' + e_{MN2})^{1+m'} - (B' + e_{MN1})^{1+m'}) \qquad \text{----A5.2.7.}$$

For U = 3,5

$$\text{work}^P = \frac{4A'}{1+m'.}\left((B' + e''_{E1} + (e_{B2} - e_{MN2} + e_{MN1})/2)^{1+m'} - (B' + e''_{E1})^{1+m'}\right) - \sigma_{m1}(e_{MN2} - e_{MN1}) + \frac{A'}{1+m'.}((B' + e_{MN2})^{1+m'} - (B' + e_{MN1})^{1+m'}) \qquad \text{----A5.2.8.}$$

```
C.....
      SUBROUTINE RWORK(II,N,K,ITL)
      COMMON/C1/ETX(129,2),ETXT(129,2),ETXAB(129,2),ETXM(129,2),
     A ETXE(129,2),SMX(129,2),SMXT(129,2),SMXM(129,2)
      COMMON/C2/R(10),D(10),TL(20),BY(10,3),CY(10,3),RRESID(10,3),
     A BMOM(10,3),WORK(10,3)
      COMMON/C3/E,QMU,AA,BB,TLOAD,NR,NR1,NR2
      COMMON/C4/A,B,RM,IP,NB,SIGMA
      COMMON/C5/Y(130),AWORK(129,2),ETAXO
      COMMON/C6/ETXBU(129,2),SIGD(129,2),ETXMN(129,2)
      COMMON/C7/NBAU(129,2),ET,ETT
      EXTERNAL F
      DO 10 I=1,II
      ETBU = SIGD(I,2)*(1.-QMU**2)/E
      ETXEZ = ETXE(I,1)/2.
      IF(NB.GT.1) ETXEZ = ETXEZ+ETBU
      ETXE2 = (ALOG(1.+ETXEZ))**2
      ET1 = (ALOG(ETXM(I,1)-ETXEZ-ETXAB(I,1)+1.))**2
      ET2 = (ALOG(ETXM(I,1)-ETXEZ-ETXAB(I,2)+1.))**2
      ET3 = (ALOG(ETXAB(I,1)-ETXM(I,1)+ETXEZ+1.))**2
      ET4 = (ALOG(ETXAB(I,2)-ETXM(I,1)+ETXEZ+1.))**2
      ET5 = (ALOG(ETXE(I,1)-ETXEZ+1.))**2
      EMU = E/(2.*(1.-QMU**2))
C***** IF STRAIN IS AT MAX *****
      IF(ETXM(I,1)-ETXAB(I,1))100,100,103
C***** TEST FOR DIRECTION OF NEXT INCREMENT *****
  100 IF(ETX(I,2).LT.0.0.AND.ETX(I,1).GT.0.0.OR.ETX(I,2).GT.0.0.AND.
     A ETX(I,1).LT.0.0) GO TO 101
C***** FOR SAME STRAIN DIRECTION ***** I.E. PLASTIC YIELDING *****
      AWORK(I,2) = 0.0
      GO TO 5
C***** FOR OPPOSITE STRAIN DIRECTION ** TEST WHETHER YIELDING OCCURS ***
  101 IF(ETXM(I,2).GT.ETXM(I,1)) GO TO 102
      IF(ABS(ETX(I,2)).LT.ETXEZ) GO TO 110
      AWORK(I,2) = EMU*(ET2-ETXE2)
      GO TO 10
  110 AWORK(I,2) = EMU*(ET4-ETXE2)
      GO TO 10
C***** FOR INCIDENCE OF PLASTIC YIELDING *****
  102 AWORK(I,2) = EMU*(ET5-ETXE2)
      GO TO 5
C***** FOR STRAINS INITIALLY IN THE ELASTIC ZONE TEST TO DETERMINE
C       DIRECTION OF NEXT STRAIN INCREMENT *****
  103 IF(ETX(I,2).LT.0.0.AND.ETX(I,1).GE.0.0.OR.ETX(I,2).GT.0.0.AND.
     A ETX(I,1).LE.0.0) GO TO 104
C***** FOR SAME STRAIN DIRECTION, TEST FOR YIELDING *****
      IF(N.EQ.2.AND.K.EQ.1) GO TO 21
   20 IF(ETXM(I,2).GT.ETXM(I,1)) GO TO 108
      IF((ETXM(I,1)-ETXEZ).GT.ETXAB(I,1)) GO TO 107
      IF((ETXM(I,1)-ETXEZ).GT.ETXAB(I,2)) GO TO 106
  105 AWORK(I,2) = EMU*(ET4-ET3)
      GO TO 10
  106 AWORK(I,2) = EMU*(ET2-ET3)
      GO TO 10
  107 AWORK(I,2) = EMU*(ET2-ET1)
      GO TO 10
  108 IF((ETXM(I,1)-ETXEZ).GT.ETXAB(I,1)) GO TO 109
      AWORK(I,2) = EMU*(ET5-ET3)
      GO TO 5
```

```
  109 AWORK(I,2) = EMU*(ET5-ET1)
      GO TO 5
C***** FOR OPPOSITE STRAIN DIRECTION, TEST WHETHER YIELDING OCCURS *****
  104 IF(N.EQ.2.AND.K.EQ.1) GO TO 20
   21 IF(ETXM(I,2).GT.ETXM(I,1)) GO TO 114
C***** FOR NO PLASTIC YIELDING *****
      IF((ETXM(I,1)-ETXEZ).GT.ETXAB(I,1)) GO TO 112
  111 AWORK(I,2) = EMU*(ET4-ET3)
      GO TO 10
  112 IF((ETXM(I,1)-ETXEZ).GT.ETXAB(I,2)) GO TO 113
      AWORK(I,2) = EMU*(ET4-ET1)
      GO TO 10
  113 AWORK(I,2) = EMU*(ET2-ET1)
      GO TO 10
C***** FOR INCIDENCE OF YIELDING *****.
  114 IF((ETXM(I,1)-ETXEZ).GT.ETXAB(I,1)) GO TO 115
      AWORK(I,2) = EMU*(ETXE2-ET3)
      GO TO 5
  115 AWORK(I,2) = EMU*(ETXE2-ET1)
      GO TO 5
    5 IF(NB.GT.1) GO TO 6
      AWORK(I,2) = AWORK(I,2) + ((A/(RM+1.))*(((B+ETXAB(I,2))**
     A (RM+1.))-((B+ETXM(I,1))**(RM+1.))))
      GO TO 10
    6 NBAUSH = NBAU(I,2)
      IF(NBAU(I,2).EQ.1.AND.NBAU(I,1).EQ.7.AND.N.NE.2)NBAUSH = 7
      IF(NBAU(I,2).EQ.5.AND.NBAU(I,1).EQ.6) NBAUSH = 7
      IF(NB.GT.2) GO TO 11
      GO TO(7,10,8,10,8,10,9)NBAUSH
    7  AWORK(I,2) = AWORK(I,2)+((A/(RM+1.))*(((B+ETXBU(I,2)+(ETXE(I,1)/2
     A.))**(RM+1.))-((B+ETXBU(I,1)+(ETXE(I,1)/2.))**(RM+1.))))
     B -SIGD(I,2)*(ETXBU(I,2)-ETXBU(I,1))
      GO TO 10
    8  AWORK(I,2) = AWORK(I,2)+((A/(RM+1.))*(((B+ETXBU(I,2)+(ETXE(I,1)/2
     A.))**(RM+1.))-((B+(ETXE(I,1)/2.))**(RM+1.))))
     B -SIGD(I,2)*ETXBU(I,2)
      GO TO 10
    9  AWORK(I,2) = AWORK(I,2)+((A/(RM+1.))*(((B+ETXBU(I,2)+(ETXE(I,1)/2
     A.))**(RM+1.))-((B+(ETXE(I,1)/2.))**(RM+1.))))
     B +SIGD(I,2)*ETXBU(I,2)
      GO TO 10
   11 IF(NB.GT.3) GO TO 15
      GO TO(12,10,13,10,13,10,14)NBAUSH
   12 AWORK(I,2) = AWORK(I,2)+((A/(RM+1.))*(((B+ETXM(I,2))**(RM+1.))
     A -((B+ETXM(I,1))**(RM+1.)))) +(ETT - (A*((B+ETXM(I,1))**RM))
     B-SIGD(I,2))*(ETXBU(I,2)-ETXBU(I,1))
      GO TO 10
   13 AWORK(I,2) = AWORK(I,2)+((A/(RM+1.))*(((B+ETXM(I,2))**(RM+1.))
     A -((B+ETXM(I,1))**(RM+1.)))) +(ETT - (A*((B+ETXM(I,1))**RM))
     B-SIGD(I,2))*(ETXBU(I,2))
      GO TO 10
   14 AWORK(I,2) = AWORK(I,2)+((A/(RM+1.))*(((B+ETXM(I,2))**(RM+1.))
     A -((B+ETXM(I,1))**(RM+1.)))) +(ETT - (A*((B+ETXM(I,1))**RM))
     B+SIGD(I,2))*(ETXBU(I,2))
      GO TO 10
   15 IF(ABS(ETXT(I,2)).GT.ABS(ETXMN(I,1))) GO TO 18
      GO TO(16,10,17,10,17,10,16)NBAUSH
   16 AWORK(I,2) = AWORK(I,2)+ ((4.*A/(RM+1.))*(((B+(ETXE(I,1)/2.)+
     A (ETXBU(I,2)/2.))**(RM+1.))-((B+(ETXE(I,1)+ETXBU(I,1))/2.)
```

```
     B **(RM+1.)))))+
     C ((SMXM(I,1)-(2.*A*((B+(ETXE(I,1)+ETXBU(I,1))/2.)**(RM    ))))
     D *ETXBU(I,2))
       GO TO 10
  17 AWORK(I,2) = AWORK(I,2)+ ((4.*A/(RM+1.))*(((B+(ETXE(I,1)/2.)+
     A (ETXBU(I,2)/2.))**(RM+1.))-((B+(ETXE(I,1)/2.))**(RM+1.))))  -
     B (SMXM(I,1)*ETXBU(I,2))
       GO TO 10
  18 ETXLMN = ETXMN(I,2)-ETXMN(I,1)
       GO TO (19,10,22,10,22,10,19)NBAUSH
  19 AWORK(I,2) = AWORK(I,2)+ ((4.*A/(RM+1.))*(((B+(ETXE(I,1)/2.)+
     A (ETXLMN     /2.))**(RM+1.))-((B+(ETXE(I,1)+ETXBU(I,1))/2.)
     B **(RM+1.)))))+
     C ((SMXM(I,1)-(2.*A*((B+(ETXE(I,1)+ETXBU(I,1))/2.)**(RM.    ))))
     D *ETXLMN)
       GO TO 23
  22 AWORK(I,2) = AWORK(I,2)+ ((4.*A/(RM+1.))*(((B+(ETXE(I,1)/2.)+
     A (ETXLMN     /2.))**(RM+1.))-((B+(ETXE(I,1)/2.))**(RM+1.))))  -
     B (SMXM(I,1)*ETXLMN)
  23 AWORK(I,2) = AWORK(I,2)+((A/(RM+1.))*((((B+ETXMN(I,2))**(RM+1.))-
     A ((B+ETXMN(I,1))**(RM+1.))))
  10 CONTINUE
       CALL SIMPI(F,3,N,K,-AA,AA,0.001,7,SI1,S,INTNO,IER)
       WORK(N,K) = S*BB
       TL(ITL+1) = (WORK(N,K)+TL(ITL))/(1.+ETX(65,2))
       RETURN
       END
```

APPENDIX 6.

A6.0 Equivalent Stress/Equivalent Strain Relationships.

Derivation of the above relationships from tensile test data (NT=1) and their equation to the formulae.

$$\bar{\sigma} = A(B + \bar{\varepsilon})^m \qquad \text{----A6.0.1.}$$

or

$$\bar{\sigma} = A(B + \bar{\varepsilon}^P)^m \qquad \text{----A6.0.2.}$$

is entrusted to a computer program TSTRES which calls subroutines ORDER and FOLYFT for curve-fitting purposes. Input values required are the load/elongation arrays, specimen dimensions and the various scale factors.

A6.1. Program TSTRES.

The program as written extends the above functions to provide curve fits for true stress/true strain or true stress/true plastic strain data (NT = 2 or 3) and also provides plotting facilities for the output relationships. For all cases input arrays take the form of distance measurements along the respective axes (y,x), a zero point being allowed for (y_o, x_o).

For the case of load/elongation plots, true stress/true strain values are determined by normal methods viz

$$\sigma_{nominal} = (y-y_o)\ \frac{\text{load scale factor.}}{\text{initial X section area.}} \qquad \text{----A6.1.1.}$$

$$e = (x-x_o)\ \frac{\text{strain scale factor}}{\text{initial gauge length.}} \qquad \text{----A6.1.2.}$$

Unixial stress data are identical to equivalent stress data, and we may write,

$$\bar{\varepsilon} = \ln(1+e) \qquad \text{----A6.1.3.}$$

$$\bar{\sigma} = \sigma_{nominal}(1+e) \qquad \text{----A6.1.4.}$$

If a fit to equation A2.0.2 is required the equivalent plastic strain may be written,

$$\bar{\varepsilon}^P = \bar{\varepsilon} - (\bar{\sigma}/E) \qquad \text{----A6.1.5.}$$

This latter relationship requires an accurate record of Young's modulus and as such, equation A6.0.2 is not considered as adaptable as equation A6.0.1.

The resultant $\bar{\sigma}/\bar{\varepsilon}$ arrays are processed by subroutine ORDER, which returns the required constants of the relevant equation. Any number of data arrays may, however, be utilized and summed, thereby giving a mean curve fit for any particular coil etc. This latter operation was found necessary for the experimental materials used in the current work.

Figs 4.18,4.19 and 4.20 have been derived in this manner, the coil scatter bands being seen to be considerable.

A6.2. Program Nomenclature.

A1 .... output constant A.

AO .... initial cross-sectional area.

AMINN .... curve-fit correlation coefficient.

B1 .... output constant B.

CORMAX .... maximum correlation coefficient.

CORP(IRM) .... curve-fit correlation coefficient.

DIF(I) .... discrepancy between true stresses as measured and determined from output equation.

E .... Young's Modulus.

EE(I) .... curve-fit strain array.

EENG(I) .... engineering strain (e).

EMAX .... maximum true strain.

EP(I) .... true plastic strain ($\bar{\varepsilon}^{P}$).

EPS(I) .... true plastic strain array for mean curve-fit.

ET(I) .... true strain ($\bar{\varepsilon}$).

ETS(I) .... true strain array for mean curve-fit.

GLENG .... test piece gauge length.

GLOT .... plotting routine name.

I .... input array index.

IETA .... curve-fit type index.

IETA = 1 for equation A6.0.1.

IETA = 2 for equation A6.0.2.

IRM .... control variable for initial value of hardening index.

INO .... total array length for mean curve-fit arrays.

J .... program control parameter.

LOAD(I) .... load or stress input array (y).

LSF .... load scale factor.

MN .... number of data groups used for mean curve fit + 1.

NBA .... plot control parameter.

1 for no output plot

2 for plot.

NNO .... data series number.

NRE(MN) .... total number of data points.

NREAD .... data points in one group.

NY .... input data type.

NT = 1 for load/elongation.

NT = 2 for true stress/true strain.

NT = 3 for true stress/true plastic strain.

NTYPE .... load/elongation origin type.

POWER .... output constant m.

S(I) .... nominal stress ($\sigma_{nominal}$).

SLSF .... extension characteristic of tensile machine (Hounsfield).

SN .... data series name.

ST(I) .... true stress ($\bar{\sigma}$).

STCALC(I) .... true stress determined by output equation.

STMAX .... maximum value of true stress.

STRAIN(I) .... strain input array.(x).

STRSF .... strain scale factor.

ZLOAD .... zero load reading (y_o).

ZSTR .... zero strain reading (x_o).

```
      PROGRAM TSTRES (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE1,TAPE25
     A,TAPE27)
C*****
C*****..........PROGRAM TO TRANSFORM TEH LOAD/ELONGATION..(NT=1)..CURVE
C*****  OR TRUESTRESS/TRUE STRAIN..(NT=2)..CURVE, OR TRUE STRESS/TRUE
C*****  PLASTIC STRAIN CURVE FOR ANY MATERIAL, INTO THE CLOSEST FITTING
C*****  CURVE OF THE FORM..SIGMA = A(B+ETABARP)**N.....................
C*****   THE CONTROLLING CRITERION FOR THE CHIOCE OF CURVE IS THE
C*****  MINIMUM ATTAINABLE CORRELATION COEFFICIENT.....................
C*****
      REAL LOAD,LSF
      DIMENSION LOAD(30),STRAIN(30),S(30),ST(30),EENG(30),ET(30),EP(30),
     A CORR(50),SU(300),STCALC(300),ETS(300),EPS(300),EE(300),DIF(300),
     B NRE(10)
C*****
C*****   READ IN CONTROLLING VARIABLES....AO=INITIAL XSECTIONAL AREA
C*****  ....GLENG=GUAGE LENGTH....LSF=LOAD SCALE FACTOR....STRSF=STRAIN
C*****  SCALE FACTOR....ZLOAD=ZERO LOAD....ZSTR=ZERO STRAIN....NREAD=NO
C*****  OF STRESS/STRAIN READINGS....NT=TYPE OF DATA INPUT....J IS NOT
C*****  EQUAL TO 1 FOR THE LAST DATA SERIES.
C*****  NTYPE = 1, FOR HOUNDSFIELD, AND 2 FOR DENISON
      READ(5,200)NBA
  200 FORMAT(I1)
      NNO = 0
  101 INO = 0
      MN = 1
      NRE(1) = 0
      NNO = NNO+1
   10 READ(5,1) AO,GLENG,LSF,STRSF,SLSF,ZLOAD,ZSTR,E,NREAD,NT,J,NTYPE,SN
    1 FORMAT(F7.5,1X,F5.3,1X,F9.4,1X,F9.4,1X,F9.4,1X,F7.3,1X,F7.3,1X,
     AF7.3,1X,I3,1X,I1,1X,I1,1X,I1,A3)
      E = E*(10.0**6)
      MN = MN+1
      NRE(MN) = NREAD+NRE(MN-1)
C*****  READ IN INPUT ARRAY....
      READ(5,2) (LOAD(I),STRAIN(I),I=1,NREAD)
    2 FORMAT (5(F7.3,1X,F7.3,1X))
      IF(NT-2)6,7,8
C*****
C*****  FROM LOAD/ELONGATION CALCULATE TRUE STRESS/TRUE STRAIN
C*****
    6 DO 5 I=1,NREAD
      S(I) = (LOAD(I)-ZLOAD)*LSF/AO
      GO TO(64,62)NTYPE
   64 EENG(I) = (STRAIN(I)-ZSTR-(LOAD(I)/SLSF))*STRSF/GLENG
      IF(I.GT.1) GO TO 61
      EG1 = S(1)/E
      EG1 = EG1/(1.-EG1)
      EG2 = EENG(1)-EG1
   61 EENG(I) = EENG(I)-EG2
      GO TO 63
   62 EENG(I) = (STRAIN(I)-ZSTR)*STRSF/GLENG
   63 ET(I) = ALOG(1.0+EENG(I))
      ST(I) = S(I)*(1.0+EENG(I))
      EP(I) = ET(I)-(ST(I)/E)
    5 CONTINUE
      GO TO 9
C*****
C*****  FROM TRUE STRESS/TRUE STRAIN CALCULATE TRUE STRESS/TRUE PLASTIC
```

```
C*****  STRAIN
C*****
    7 DO 11 I=1,NREAD
      S(I) = 0.0
      EENG(I) = 0.0
      ST(I) = (LOAD(I)-ZLOAD)*LSF/AO
      ET(I) = (STRAIN(I)-ZSTR)*STRSF/GLENG
      E = ST(1)/ET(1)
      EP(I) = ET(I)-(ST(I)/E)
   11 CONTINUE
      GO TO 9
C*****
C*****   FROM TRUE STRESS/TRUE PLASTIC STRAIN
C*****
    8 DO 12 I=1,NREAD
      S(I) = 0.0
      EENG(I) = 0.0
      ET(I) = 0.0
      ST(I) = (LOAD(I)-ZLOAD)*LSF/AO
      EP(I) = (STRAIN(I)-ZSTR)*STRSF/GLENG
      ET(I) = EP(I)+(ST(I)/E)
   12 CONTINUE
    9 WRITE(6,20)SN
   20 FORMAT(1H1, 46H  BASIC STRESS/STRAIN ANALYSIS.... MATERIAL...,A3)
      WRITE(6,21)AO,GLENG,LSF,STRSF,ZLOAD,ZSTR,NREAD,NT
   21 FORMAT(1H0, 1X,15HINITIAL AREA = ,F7.5,2X,15HGAUGE LENGTH = ,F5.3,
     A  2X,15HLOAD SCALE =    ,F9.4,2X,17HSTRAIN SCALE =    ,F6.4,4X//
     B  2X,12HZERO LOAD = ,F7.3, 5X,14HZERO STRAIN = ,F6.3,2X,
     C 14HNO READINGS = ,I3, 9X,13HCURVE TYPE = ,I1)
      WRITE(6,4)E
    4 FORMAT(/ 2X,4HE = ,E12.5)
      WRITE(6,60)
   60 FORMAT(/ 2X,22HIETA=1 FOR TRUE STRAIN// 2X,30HIETA=2 FOR TRUE STRA
     AIN PLASTIC)
      DO 100 I=1,NREAD
      INO = INO+1
      SU(INO) = ST(I)
      ETS(INO) = ET(I)
      EPS(INO) = EP(I)
  100 CONTINUE
      IETA = 1
      IRM = 1
      DO 102 I=1,NREAD
      GO TO(103,104),IETA
  103 EE(I) = ETS(I)
      GO TO 102
  104 EE(I) = EPS(I)
  102 CONTINUE
      CALL ORDER(ST,EE,NREAD,A1,B1,POWER,IRM,AMINN)
      DO 98 I=1,NREAD
      STCALC(I) = A1*((B1+EE(I))**POWER)
   98 DIF(I) = ST(I)-STCALC(I)
      WRITE(6,27)IRM,IETA
      WRITE(6,22)
   22 FORMAT(///4X,7HNOMINAL,8X,4HTRUE,7X,11HENGINEERING,6X,4HTRUE,
     A 7X,11HTRUE STRAIN,3X,11HTRUE STRESS,1X/
     B 5X,6HSTRESS,7X,6HSTRESS,8X,6HSTRAIN,8X,6HSTRAIN,8X,7HPLASTIC,
     C 6X,10HCALCULATED,6X,3HDIF)
      WRITE(6,3)(S(I),ST(I),EENG(I),ET(I),EP(I),STCALC(I),DIF(I),I=1,
```

```
     A NREAD)
    3 FORMAT(1X,E12.5,2X,E12.5,2X,E12.5,2X,E12.5,2X,E12.5,2X,E12.5,2X,
     A E12.5)
       IF(J.NE.1) GO TO 99
       GO TO 10
C*****
C*****  CALL CURVE FITTING ROUTINE....
C*****
   99 DO 19 IETA = 1,1
      CORMAX = 0.0
      MAX = 0
      DO 18 IRM = 1,1
      DO 50 I=1,INO
      GO TO (51,52),IETA
   51 EE(I) = ETS(I)
      GO TO 50
   52 EE(I) = EPS(I)
   50 CONTINUE
      CALL ORDER(SU,EE,INO,A1,B1,POWER,IRM,AMINN)
      CORR(IRM) = AMINN
C*****
C*****  CALCULATE AND WRITE ACTUAL AND CALCULATED RESULTS....
C*****
      EMAX = 0.0
      STMAX = 0.0
      DO 14 I=1,INO
      IF(B1+EE(I))15,16,16
   15 STCALC(I) = 0.0
      GO TO 14
   16 STCALC(I) = A1*((B1+EE(I))**POWER)
      IF(SU(I).GT.STMAX) STMAX = SU(I)
      IF(EE(I).GT.EMAX)EMAX = EE(I)
   14 DIF(I) = SU(I)-STCALC(I)
      WRITE(6,30)
   30 FORMAT(1H1)
      WRITE(6,27)IRM,IETA
   27 FORMAT(1H0,5X,6HIRM = ,I3,5X,7HIETA = ,I2)
      WRITE(6,24)
   24 FORMAT(1H0,11X,11HTRUE STRESS,11X,11HTRUE STRESS,15X,3HDIF,15X,
     A11HTRUE STRAIN,2X/14X,6HACTUAL,15X,10HCALCULATED,35X,9H(PLASTIC))
      WRITE(6,23)(SU(I),STCALC(I),DIF(I),EE(I),I=1,INO)
   23 FORMAT(11X,E12.5,10X,E12.5,10X,E12.5,10X,E12.5)
      IF(CORMAX-CORR(IRM))26,28,18
   26 CORMAX = CORR(IRM)
      MAX = IRM
      GO TO 18
   28 WRITE(6,25)MAX,IRM
   25 FORMAT(1H0,10X,6HIRM = ,I3,2X,10HAND IRM = ,I3,14HARE EQUIVALENT)
   18 CONTINUE
      WRITE(6,29)MAX,CORMAX
   29 FORMAT(///7X,48HMAXIMUM CORRELATION COEFFICIENT OCCURS AT IRM = ,I
     A3,5X,8HVALUE = ,E15.8)
   19 CONTINUE
      IF(NBA.LT.2) GO TO 1000
      CALL GLOT(SU,STCALC,EE,NNO,MN,INO,STMAX,EMAX,NRE)
 1000 IF(J.EQ.0) GO TO 101
      IF(NBA.LT.2) GO TO 1001
      CALL ENPLOT(12.0)
 1001 STOP
```

```
C*****
C*****
      SUBROUTINE ORDER(SX,EXENG,IM,A1,B1,POWER,IRM,AMINN)
      DIMENSION SX(300),EXENG(300),C(10),SXD(300),STDV(1000),DVAB(1000)
      AMIN = 0.0
      RM = (FLOAT(IRM)/20.0)-0.05
      J = 1
   60 IF(J.EQ.5) GO TO 68
      DO 62 M=1,1000
      RM = RM+((-1.0)**(J+1))/((10.0**J)*2.0)
      SXDTOT = 0.0
      DO 64 I=1,IM
      SXD(I) = (SX(I)/SX(1))**(1.0/RM)
      SXDTOT = SXDTOT+SXD(I)
   64 CONTINUE
      SXMEAN = SXDTOT/(FLOAT(IM))
      KOR = 1
      CALL FOLYFT(EXENG,SXD,IM,C,CO,KOR,6,M,STDV)
      DVAB(M) = STDV(M)
      IF(AMIN-DVAB(M))66,70,70
   66 AMIN = DVAB(M)
      POWER = RM
      COO = CO
      CC = C(1)
      AMINN = AMIN
   62 CONTINUE
   70 J = J+1
      AMIN = DVAB(M)
      GO TO 60
   68 B1 = COO/CC
      A1 = (CC**POWER)*SX(1)
      WRITE(6,86) A1,B1,POWER
   86 FORMAT(1H0,5X,9HSIGMAX = ,F7.0,1H(,F7.6,10H + ETAX)**,F7.5)
      RETURN
      END
```

```
      SUBROUTINE GLOT(SU,STCALC,EE,NNO,MN,INO,STMAX,EMAX,NRE)
      DIMENSION SU(300),STCALC(300),EE(300),NRE(10)
      DIMENSION X(40),Y(40)
      IF(NNO.GT.1) GO TO 1
      CALL START
    1 GO TO(75,76,76,77,76,76),NNO
   75 CALL PLOT(3.0,22.0,-3)
      GO TO 78
   76 CALL PLOT(0.0,-9.0,-3)
      GO TO 78
   77 CALL PLOT(10.0,18.0,-3)
   78 NPO = NRE(2)
      DO 71 I=1,INO
      SU(I) = SU(I)*6.895/1000.
   71 STCALC(I) = STCALC(I)*6.895/1000.
      WRITE(6,1000)MN
 1000 FORMAT(10X,I2)
      WRITE(6,1001)(NRE(N),N=1,MN)
 1001 FORMAT(10X,10(I2,2X))
      INO1 = INO+1
      INO2 = INO+2
      SU(INO1) = 0.0
      EE(INO1) = 0.0
      SU(INO2) = (IFIX(STMAX*6.895/40000.)+1)*10.
      EE(INO2) = (IFIX(EMAX*1000./7.)+1)/1000.
      XSCALE = 1.0/EE(INO2)
      YSCALE = 1.0/SU(INO2)
      CALL AXIS(0.0,0.0,11HTRUE STRAIN,-11,7.0,0.0,EE(INO1),EE(INO2))
      CALL AXIS(0.0,0.0,19HTRUE STRESS MN/M**2,19,4.0,90.0,SU(INO1),
     A SU(INO2))
      CALL ARKIST(EE,STCALC,1,NRE(2),10,XSCALE,YSCALE,0.0,0.0,2,1)
      DO 70 I=2,MN
      NE = NRE(I-1)+1
      NO = ((NRE(I)-(NRE(I-1)+1))/2)+1
      IF(MOD(I,2).EQ.0) GO TO 67
      DO 69 J=1,NO
      JJ = NRE(I)+1-J
      JI = NRE(I-1)+J
      SUE = SU(JJ)
      EEE = EE(JJ)
      SU(JJ) = SU(JI)
      EE(JJ) = EE(JI)
      SU(JI) = SUE
   69 EE(JI) = EEE
   67 NMN = NRE(I)-NRE(I-1)
      DO 68 J=1,NMN
      JJ = NRE(I-1)+J
      X(J) = EE(JJ)
   68 Y(J) = SU(JJ)
      N1 = NMN+1
      N2 = NMN+2
      X(N1) = 0.0
      Y(N1) = 0.0
      X(N2) = EE(INO2)
      Y(N2) = SU(INO2)
      WRITE(6,1002)NMN
 1002 FORMAT(10X,I3)
   70 CALL LINE(X,Y,NMN,1,-1,I-1)
      RETURN
```

APPENDIX 7.

A7.0. Example Computer Output.

The presentation of all computer outputs is unnecessary, relevant results from LEVEL 4 being tabulated and plotted, whilst Figs 4.18/19/ and 20 adequately demonstrate program TSTRES.

Three general types of output are attained from LEVEL 4, depending upon the required analysis type.The computer outputs given in this Appendix are relevant to the series 2 material and are considered representative of industrial applications,utilizing mean applied stress levels of 0.30 to 0.35 times yield stress and roll diameter to strip thickness ratios of 100, resulting in approximately 1% overall strain.

The three sections of results assume the following specification for a three roll leveller with 1.24in diameter rolls, with 6.75in wide strip 0.0129in thick, no initial curvature being included.

A7.1. (1) Isotropic hardening (the first four pages giving the plane strain stress/strain relationship derivation.

(2) Constant applied load at 1000 lbf.

(It is to be noted that all stress levels are in lbf/in^2 and all length units are in inches).

Plots of stress (SMXT) and strain (ETXT) levels at each roll are to be found in Fig A7.1. (a to e).Where,

Δ = bending distribution.

$+$ = spring-back distributions.

$\times$ = straightening distributions.

Roll 0 refers to the initial tensile force application.

A7.2. (1) Isotropic hardening.

(2) Work losses are included, the effective bearing friction coefficient being set at 0.08 with wrap angles of 40°,80° and 40° (series 2A).

No output plot provided, this being indistinguishable from that of section A7.1.

A7.3. (1) Simple kinematic hardening (the first three pages giving stress/strain derivation.

(2) Constant applied load as for section A7.1.

An output plot is given in Fig A7.2 and should be compared with that of A7.1.The lower stress and strain levels with the kinematic hardening

assumption are clearly evident.A further point of note is the stress discontinuity visible in the roll 3 plots which results from continued deformation within the elastic range, and has been caused by the use of equation 3.4.30 rather than 3.4.31.This type of error is inevitable (as has been previously considered), but it is of small magnitude and can therefore be safely neglected.

```
SIGMA =    .28917E+05
SIGMA =    .29461E+05     Newton - Raphson solution
SIGMA =    .29449E+05
```

E = 27000000.0 QMU = .290 A = 96101.000 B = .017 RM = .295

SIGMABAR = 96101.(.017130 + ETABAR)** .29530

I= 2 ALP(I) = .1736958843D+02 First iteration

N	CB(N)	DCB(N)	H(N)
1	29449.161	0.000	0.0
2	29492.749	43.588	485865.9
3	29492.656	43.496	484123.4
4	0.000	43.496	484127.1

I	ALP(I)	SB(I)	SX(I)	SZ(I)	EX(I)	EXENG(I)	EY(I)	EYENG(I)
1	.1636958843D+02	29449.161	33047.250	9583.702	.001121	.001122	-.000458	-.000458
26	.2983158843D+02	35114.424	40546.470	20170.065	.014349	.014453	-.013400	-.013310
51	.2985658843D+02	37294.696	43064.070	21438.732	.020867	.021086	-.019859	-.019663
76	.2988158843D+02	40337.291	46577.392	23205.382	.031615	.032120	-.030525	-.030064
101	.2990658843D+02	44472.994	51352.922	25604.009	.049674	.050929	-.048472	-.047316
126	.2992978843D+02	50086.789	57835.199	28856.283	.081478	.084889	-.080124	-.076998

	SB(I)	SBUSB0(I)	SX(I)	SXMAX(I)	SXUSXM(I)	SXUSB0(I)	SZUSB0(I)	SZ2USX(I)	DIFF(I)	EZE(I)
1	29449.161	1.0000	33047.250	34004.962	.9718	1.1222	.3254	.5800	.0325	0.000000
26	35114.424	1.1924	40546.470	40546.645	1.0000	1.3768	.6849	.9949	.0000	.000313
51	37294.696	1.2664	43064.070	43064.205	1.0000	1.4623	.7280	.9957	.0000	.000333
76	40337.291	1.3697	46577.392	46577.492	1.0000	1.5816	.7880	.9964	.0000	.000361
101	44472.994	1.5102	51352.922	51352.990	1.0000	1.7438	.8694	.9972	.0000	.000398
126	50086.789	1.7008	57835.199	57835.242	1.0000	1.9639	.9799	.9979	.0000	.000449

SBDIFF = 0.E+00 SXDIF = .1107E+06 EZE(I)= .4552E-03
EXBDO = 0.E+00 EXDIF = .3755E-02 EYDIF = -.1534E-02

Second Loading Cycle

I= 2 ALP(I) = .3807088432D+01

N	CB(N)	DCB(N)	H(N)
1	50742.278	0.000	0.0
2	50752.005	9.727	130910.7
3	0.000	9.725	130850.6

I	ALP(I)	SB(I)	SX(I)	SZ(I)	EX(I)	EXENG(I)	EY(I)	EYENG(I)
1	.2807088432D+01	50742.278	52116.121	2869.459	.082022	.085479	-.082872	-.079531
26	.2780708843D+02	51433.545	59346.851	27705.421	.087067	.090905	-.078386	-.075393
51	.2994618843D+02	55906.355	64555.069	32225.096	.121709	.129425	-.043840	-.042893
76	.2994868843D+02	57001.361	65819.476	32858.760	.131117	.140101	-.034462	-.033875
101	.2995118843D+02	58197.986	67201.221	33551.102	.141903	.152465	-.023708	-.023429
126	.2995368843D+02	59485.557	68687.983	34295.983	.154116	.166626	-.011530	-.011464

SBDIFF = 0.E+00 SXDIF = .1326E+06 EZE(I)= -.5440E-03

EXBDO = 0.E+00 EXDIF = .4496E-02 EYDIF = .1837E-02

Third

I= 2 ALP(I) = .3783188432D+01

N	CB(N)	DCB(N)	H(N)
1	60760.818	0.000	0.0
2	60768.382	7.564	85015.2
3	0.000	7.562	84989.9

I	ALP(I)	SB(I)	SX(I)	SZ(I)	EX(I)	EXENG(I)	EY(I)	EYENG(I)
1	.2783188432D+01	60760.818	62392.533	3406.773	.162353	.176275	-.000667	-.000667
26	.2778318843D+02	61296.301	70725.905	32992.019	.168255	.183239	-.005979	-.005961
51	.2996098843D+02	64048.195	73956.468	36934.703	.198862	.220014	-.036471	-.035814
76	.2996348843D+02	65682.112	75843.155	37879.802	.218418	.244107	-.055983	-.054444
101	.2996598843D+02	67655.328	78121.630	39020.737	.243636	.275880	-.081148	-.077942
126	.2996848843D+02	69847.028	80652.389	40287.865	.273782	.314928	-.111235	-.105271

SIGMAX = 115096.(.016411 + ETAX)** .30000

TRUE STRESS ACTUAL	TRUE STRESS CALCULATED	DIF	TRUE STRAIN
.33047E+05	.34215E+05	-.11675E+04	.11217E-02
.40546E+05	.40541E+05	.55200E+01	.14453E-01
.43064E+05	.42979E+05	.84684E+02	.21086E-01
.46577E+05	.46437E+05	.14012E+03	.32120E-01
.51353E+05	.51232E+05	.12093E+03	.50929E-01
.57835E+05	.57909E+05	-.73383E+02	.84889E-01
.58533E+05	.58490E+05	.42539E+02	.88322E-01
.64338E+05	.64361E+05	-.23369E+02	.12765E+00
.65555E+05	.65692E+05	-.13665E+03	.13782E+00
.66917E+05	.67190E+05	-.27309E+03	.14986E+00
.68382E+05	.68813E+05	-.43184E+03	.16363E+00
.69962E+05	.70580E+05	-.61794E+03	.17951E+00
.73372E+05	.73995E+05	-.62302E+03	.21292E+00
.74995E+05	.75885E+05	-.88971E+03	.23304E+00
.77182E+05	.78462E+05	-.12795E+04	.26241E+00
.79607E+05	.81363E+05	-.17567E+04	.29829E+00
.82312E+05	.84664E+05	-.23513E+04	.34289E+00

SER/2.M

E = 27000000.0 QMU = .290 A = 115096.0 B = .016 IP = 1 RM = .300

AA = .0065 BB = 6.75 NR = 3 TL(1) = 1000.0 COEFRI = *.000

J = 3 JW = 1 JF = 1 NB = 1

D(1) = 0.000 AWRAP(1) = 0.0
D(2) = 1.240 AWRAP(2) = 40.0
D(3) = 1.240 AWRAP(3) = 80.0
D(4) = 1.240 AWRAP(4) = 40.0

ROLL NO = 0 ROLL RADIUS = .006 ROLL POSITION = 3

BY(N,K) = 0.00000 CY(N,K) = 0.00000 BMOM(N,K) = 0.0000

RRESID(N,K) = 0.00 WORK DONE = 0.000 TENSION IN = 1000.00 TENSION OUT = 1000.00

Y(I)	ETX	ETXT	ETXAB	ETXBU	ETXM	ETXE	SIGD	SMX	SMXT	SMXM	AWORK
-.6450E-02	.0004	.0004	.0004	0.0000	.0012	.0023	0.0	11484.	11484.	34215.	0.00
-.3225E-02	.0004	.0004	.0004	0.0000	.0012	.0023	0.0	11484.	11484.	34215.	0.00
0.E+00	.0004	.0004	.0004	0.0000	.0012	.0023	0.0	11484.	11484.	34215.	0.00
.3225E-02	.0004	.0004	.0004	0.0000	.0012	.0023	0.0	11484.	11484.	34215.	0.00
.6450E-02	.0004	.0004	.0004	0.0000	.0012	.0023	0.0	11484.	11484.	34215.	0.00

ROLL NO = 1 ROLL RADIUS = .626 ROLL POSITION = 1

BY(N,K) = .00167 CY(N,K) = .00191 BMOM(N,K) = 1.4110

RRESID(N,K) = .63 WORK DONE = 0.000 TENSION IN = 1000.00 TENSION OUT = 1000.00

Y(I)	ETX	ETXT	ETXAB	ETXBU	ETXM	ETXE	SIGD	SMX	SMXT	SMXM	AWORK
-.6447E-02	-.0077	-.0073	.0073	.0061	.0073	.0025	0.0	-226568.	-37442.	37442.	0.00
-.3224E-02	-.0025	-.0021	.0021	.0009	.0021	.0024	0.0	-73626.	-34779.	34779.	0.00
0.E+00	.0027	.0031	.0031	.0027	.0031	.0024	0.0	78527.	35307.	35307.	0.00
.3224E-02	.0078	.0082	.0082	.0078	.0082	.0026	0.0	229899.	37888.	37888.	0.00
.6447E-02	.0130	.0134	.0134	.0130	.0134	.0027	0.0	380497.	40113.	40113.	0.00

ROLL NO = 1 ROLL RADIUS = .626 ROLL POSITION = 2

BY(N,K) = .00007 CY(N,K) = .00231 BMOM(N,K) = .0038

RRESID(N,K) = .75 WORK DONE = 0.000 TENSION IN = 1000.00 TENSION OUT = 1000.00

Y(I)	ETX	ETXT	ETXAB	ETXBU	ETXM	ETXE	SIGD	SMX	SMXT	SMXM	AWORK
-.6430E-02	.0018	-.0055	.0055	.0061	.0073	.0024	0.0	51782.	14340.	37442.	0.00
-.3215E-02	.0009	-.0012	.0012	.0009	.0021	.0024	0.0	26189.	-8591.	34779.	0.00
0.E+00	.0000	.0031	.0031	.0027	.0031	.0022	0.0	595.	35318.	35318.	0.00
.3215E-02	-.0008	.0074	.0074	.0078	.0082	.0024	0.0	-24998.	12889.	37888.	0.00
.6430E-02	-.0017	.0117	.0117	.0130	.0134	.0025	0.0	-50592.	-10479.	40113.	0.00

ROLL NO = 1 ROLL RADIUS = .626 ROLL POSITION = 3

BY(N,K) = .00153 CY(N,K) = 0.00000 BMOM(N,K) = -1.4594

RRESID(N,K) = 0.00 WORK DONE = 0.000 TENSION IN = 1000.00 TENSION OUT = 1000.00

Y(I)	ETX	ETXT	ETXAB	ETXBU	ETXM	ETXE	SIGD	SMX	SMXT	SMXM	AWORK
-.6430E-02	.0106	.0051	.0173	.0100	.0173	.0026	3227.4	311861.	41632.	41632.	0.00
-.3215E-02	.0063	.0051	.0070	.0049	.0070	.0025	564.6	186356.	37303.	37303.	0.00
0.E+00	.0020	.0051	.0051	.0047	.0051	.0023	0.0	60316.	36392.	36392.	0.00
.3215E-02	-.0022	.0051	.0089	.0007	.0089	.0024	3672.9	-66266.	-38214.	38214.	0.00
.6430E-02	-.0065	.0051	.0191	.0057	.0191	.0027	5898.5	-193394.	-42289.	42289.	0.00

ROLL NO = 2 ROLL RADIUS = .626 ROLL POSITION = 1

BY(N,K) = .00165 CY(N,K) = .00484 BMOM(N,K) = -1.5834

RRESID(N,K) = -.63 WORK DONE = 0.000 TENSION IN = 1000.00 TENSION OUT = 1000.00

Y(I)	ETX	ETXT	ETXAB	ETXBU	ETXM	ETXE	SIGD	SMX	SMXT	SMXM	AWORK
-.6417E-02	.0130	.0181	.0303	.0230	.0303	.0029	3227.4	380126.	45904.	45904.	0.00
-.3209E-02	.0078	.0129	.0148	.0127	.0148	.0026	564.6	229523.	40674.	40674.	0.00
0.E+00	.0027	.0078	.0078	.0074	.0078	.0024	0.0	78146.	37684.	37684.	0.00
.3209E-02	-.0025	.0026	.0114	.0032	.0114	.0025	3672.9	-74011.	-39311.	39311.	0.00
.6417E-02	-.0077	-.0025	.0268	.0134	.0268	.0028	5898.5	-226958.	-44843.	44843.	0.00

ROLL NO = 2 ROLL RADIUS = .626 ROLL POSITION = 2

BY(N,K) = .00013 CY(N,K) = .00599 BMOM(N,K) = -.0198

RRESID(N,K) = -.77 WORK DONE = 0.000 TENSION IN = 1000.00 TENSION OUT = 1000.00

Y(I)	ETX	ETXT	ETXAB	ETXBU	ETXM	ETXE	SIGD	SMX	SMXT	SMXM	AWORK
-.6400E-02	-.0019	.0162	.0284	.0230	.0303	.0029	3227.4	-56845.	-10941.	45904.	0.00
-.3200E-02	-.0009	.0120	.0138	.0127	.0148	.0026	564.6	-27852.	12822.	40674.	0.00
0.E+00	.0000	.0078	.0078	.0074	.0078	.0024	0.0	1141.	37702.	37702.	0.00
.3200E-02	.0010	.0036	.0104	.0032	.0114	.0025	3672.9	30133.	-9177.	39311.	0.00
.6400E-02	.0020	-.0005	.0248	.0134	.0268	.0028	5898.5	59126.	14284.	44843.	0.00

ROLL NO = 2 ROLL RADIUS = .626 ROLL POSITION = 3

BY(N,K) = .00133 CY(N,K) = 0.00000 BMOM(N,K) = 1.6190

RRESID(N,K) = 0.00 WORK DONE = 0.000 TENSION IN = 1000.00 TENSION OUT = 1000.00

Y(I)	ETX	ETXT	ETXAB	ETXBU	ETXM	ETXE	SIGD	SMX	SMXT	SMXM	AWORK
-.6400E-02	-.0066	.0096	.0360	.0057	.0360	.0030	11688.9	-195980.	-47509.	47509.	0.00
-.3200E-02	-.0024	.0095	.0156	.0008	.0156	.0026	6458.8	-72251.	-40998.	40998.	0.00
0.E+00	.0017	.0095	.0095	.0092	.0095	.0024	0.0	50961.	38490.	38490.	0.00
.3200E-02	.0059	.0095	.0159	.0045	.0159	.0026	5095.9	173659.	41103.	41103.	0.00
.6400E-02	.0101	.0096	.0361	.0093	.0361	.0030	10627.8	295850.	47538.	47538.	0.00

ROLL NO = 3 ROLL RADIUS = .626 ROLL POSITICN = 1

BY(N,K) = .00163 CY(N,K) = .00755 BMOM(N,K) = 1.7294

RRESID(N,K) = .63 WORK DONE = 0.000 TENSION IN = 1000.00 TENSION OUT = 1000.00

Y(I)	ETX	ETXT	ETXAB	ETXBU	ETXM	ETXE	SIGD	SMX	SMXT	SMXM	AWORK
-.6389E-02	-.0077	.0019	.0436	.0134	.0436	.0031	11688.9	-227326.	-49501.	49501.	0.00
-.3194E-02	-.0025	.0070	.0181	.0034	.0181	.0026	6458.8	-74378.	-41940.	41940.	0.00
0.E+00	.0026	.0122	.0122	.0118	.0122	.0025	0.0	77781.	39626.	39626.	0.00
.3194E-02	.0078	.0174	.0237	.0123	.0237	.0028	5095.9	229158.	43859.	43859.	0.00
.6389E-02	.0130	.0225	.0490	.0222	.0490	.0032	10627.8	379763.	50794.	50794.	0.00

ROLL NO = 3 ROLL RADIUS = .626 ROLL POSITION = 2

BY(N,K) = .00009 CY(N,K) = .00955 BMOM(N,K) = .0068

RRESID(N,K) = .79 WORK DONE = 0.000 TENSION IN = 1000.00 TENSION OUT = 1000.00

Y(I)	ETX	ETXT	ETXAB	ETXBU	ETXM	ETXE	SIGD	SMX	SMXT	SMXM	AWORK
-.6372E-02	.0022	.0041	.0414	.0134	.0436	.0031	11688.9	64813.	15313.	49501.	0.00
-.3186E-02	.0011	.0081	.0170	.0034	.0181	.0026	6458.8	32868.	-9072.	41940.	0.00
0.E+00	.0000	.0122	.0122	.0118	.0122	.0025	0.0	923.	39639.	39639.	0.00
.3186E-02	-.0011	.0163	.0227	.0123	.0237	.0028	5095.9	-31022.	12837.	43859.	0.00
.6372E-02	-.0021	.0204	.0469	.0222	.0490	.0032	10627.8	-62967.	-12174.	50794.	0.00

ROLL NO = 3 ROLL RADIUS = .626 ROLL POSITION = 3

BY(N,K) = .00123 CY(N,K) = 0.00000 BMOM(N,K) = -1.7427

RRESID(N,K) = 0.00 WORK DONE = 0.000 TENSION IN = 1000.00 TENSION OUT = 1000.00

Y(I)	ETX	ETXT	ETXAB	ETXBU	ETXM	ETXE	SIGD	SMX	SMXT	SMXM	AWORK
-.6372E-02	.0097	.0138	.0525	.0088	.0525	.0032	15285.8	285412.	51578.	51578.	0.00
-.3186E-02	.0057	.0138	.0223	.0041	.0223	.0027	7725.2	166116.	43385.	43385.	0.00
0.E+00	.0016	.0138	.0138	.0134	.0138	.0025	0.0	46336.	40280.	40280.	0.00
.3186E-02	-.0025	.0138	.0245	.0008	.0245	.0028	9644.0	-73934.	-44120.	44120.	0.00
.6372E-02	-.0066	.0138	.0546	.0055	.0546	.0033	16578.9	-194696.	-52044.	52044.	0.00

ROLL NO = 4 ROLL RADIUS = .006 ROLL POSITION = 1

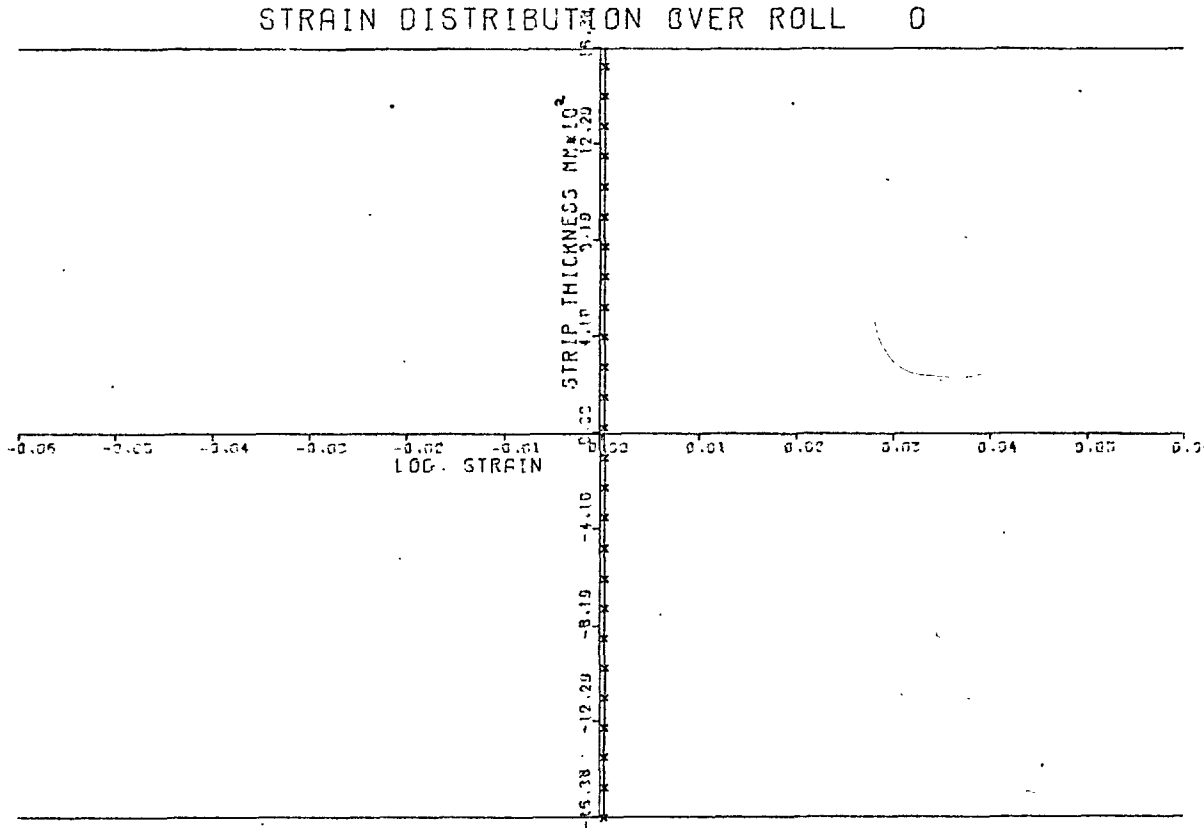
BY(N,K) = -.00113 CY(N,K) = -.03896 BMCM(N,K) = -.0033

RRESID(N,K) = 2.87 WORK DONE = 0.000 TENSION IN = 1000.00 TENSION OUT = 0.00

Y(I)	ETX	ETXT	ETXAB	ETXBU	ETXM	ETXE	SIGD	SMX	SMXT	SMXM	AWORK
-.6362E-02	-.0026	.0112	.0499	.0088	.0525	.0032	15285.8	-76069.	-24490.	51578.	0.00
-.3181E-02	-.0015	.0123	.0208	.0041	.0223	.0027	7725.2	-43774.	-389.	43385.	0.00
0.E+00	-.0004	.0134	.0134	.0134	.0138	.0025	0.0	-11479.	28801.	40280.	0.00
.3181E-02	.0007	.0145	.0238	.0008	.0245	.0028	9644.0	20815.	-23305.	44120.	0.00
.6362E-02	.0018	.0156	.0527	.0055	.0546	.0033	16578.9	53110.	1066.	52044.	0.00

MEAN LOAD = 1000.00 LOAD LOST = 0.00

TOTAL STRAIN = .0134 AT .348 YIELD STRESS


STRAIN DISTRIBUTION OVER ROLL 0
STRIP THICKNESS MM*10**2
12.29
8.19
4.10
0.00
-4.10
-8.19
-12.29
-16.38
-0.05
-0.04
-0.03
-0.02
-0.01
0.00
0.01
0.02
0.03
0.04
0.05
LOG. STRAIN

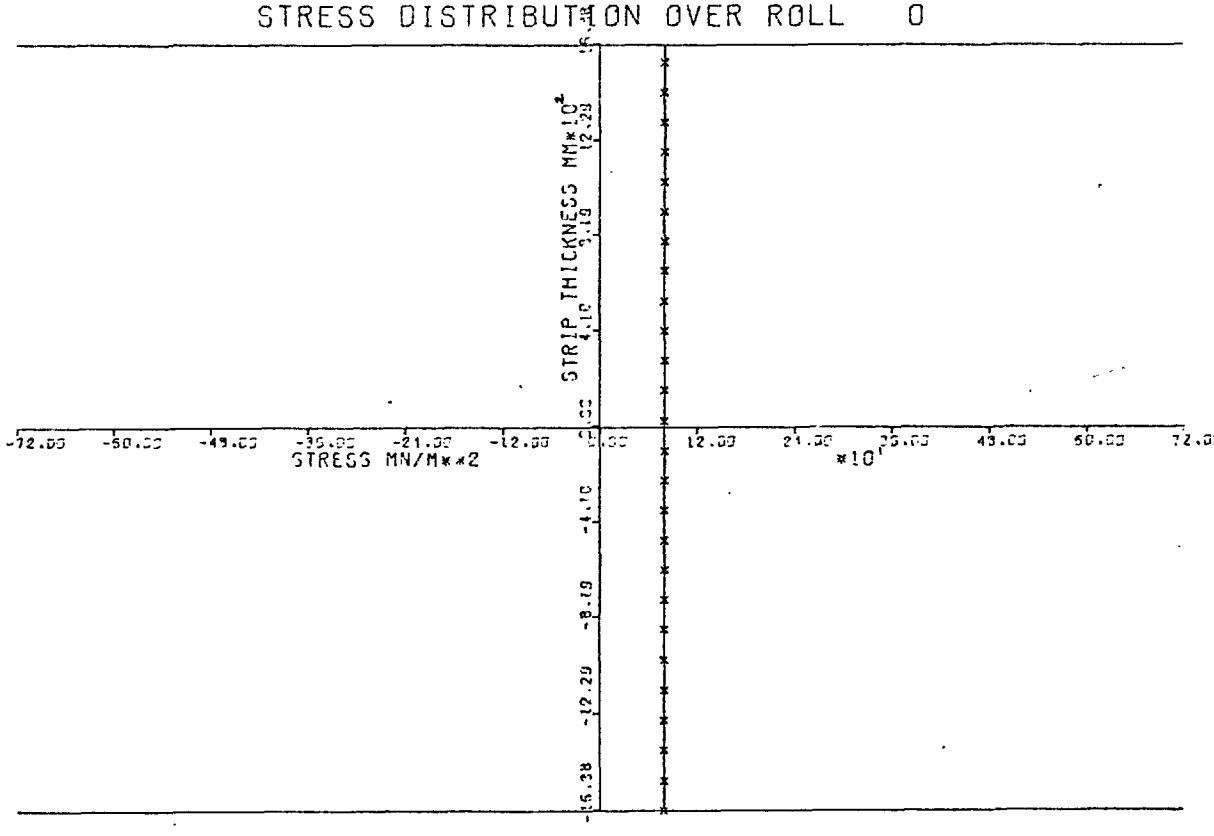
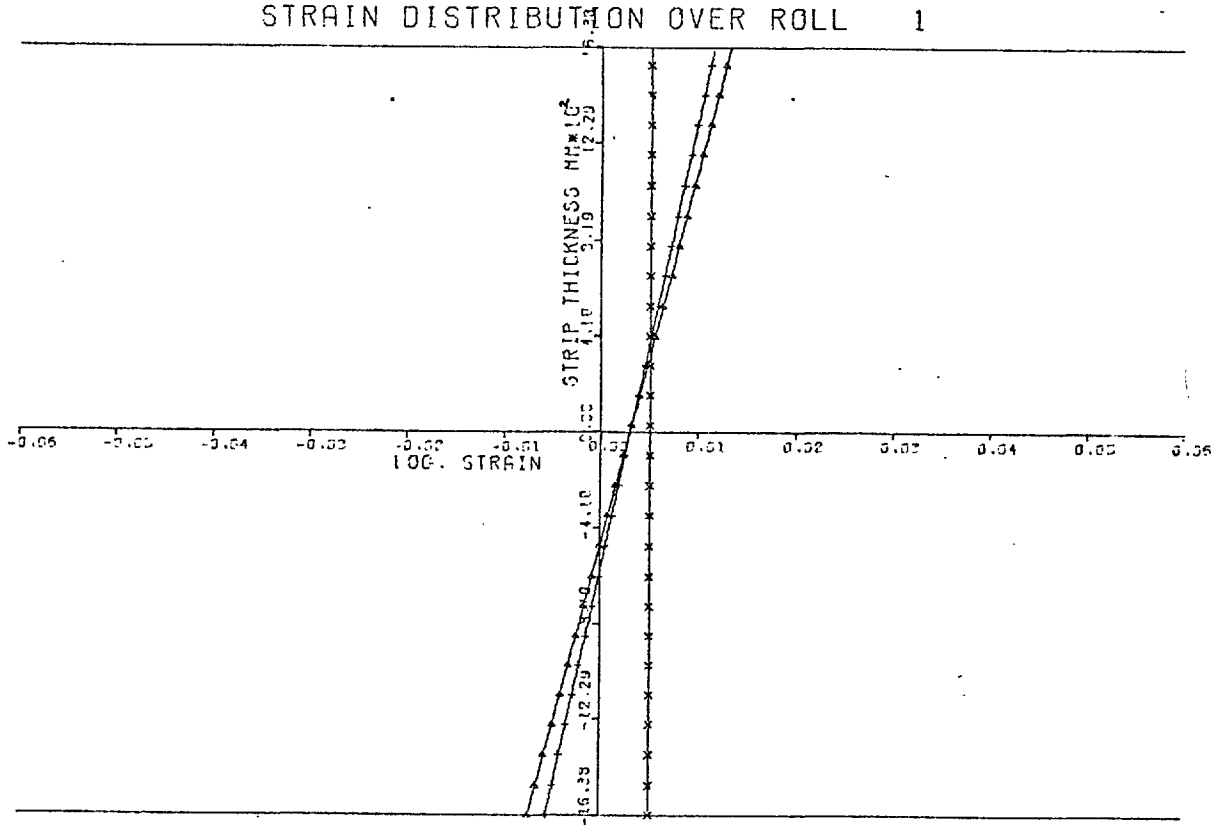

STRESS DISTRIBUTION OVER ROLL 0
STRIP THICKNESS MM*10**2
12.29
8.19
4.10
0.00
-4.10
-8.19
-12.29
-16.38
-72.00
-60.00
-48.00
-36.00
-24.00
-12.00
0.00
12.00
24.00
36.00
48.00
60.00
72.00
STRESS MN/M**2
*10**1

fig (A7.1a)


STRAIN DISTRIBUTION OVER ROLL 1
STRIP THICKNESS MM*10
LOG. STRAIN

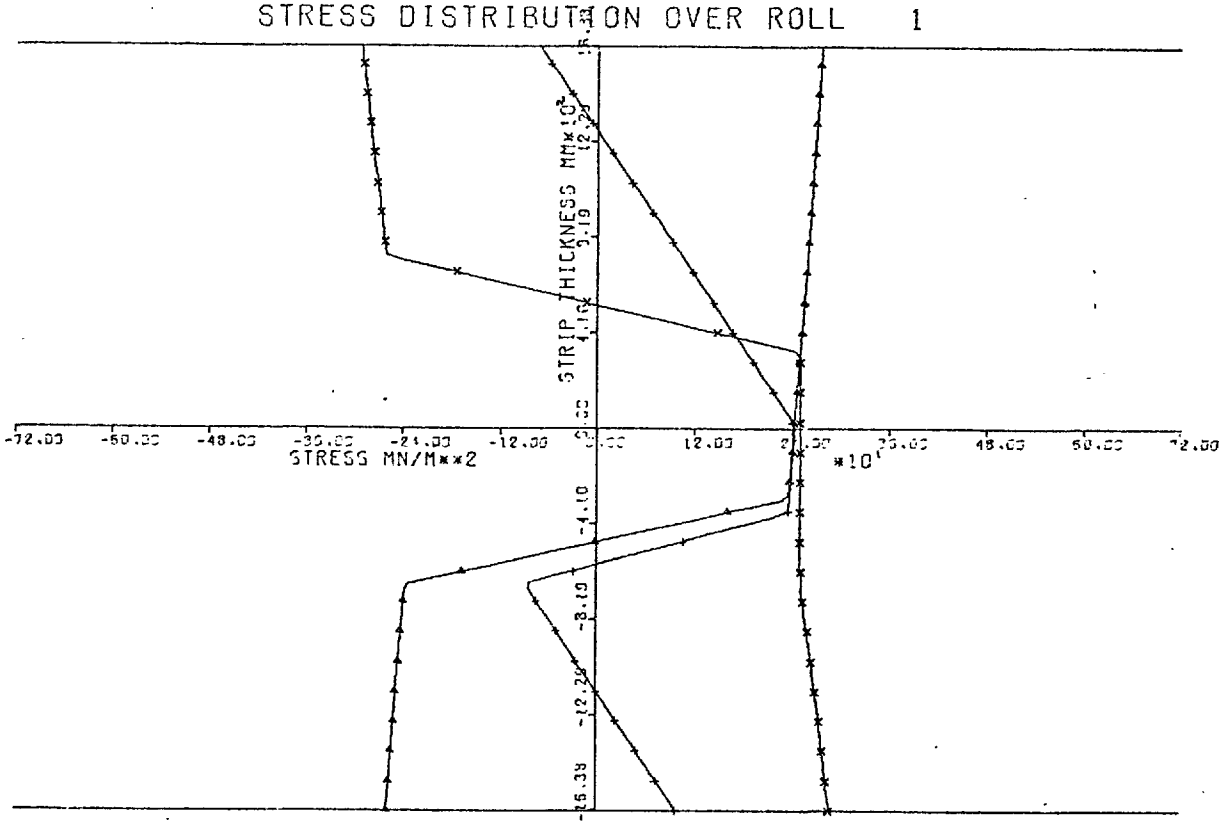
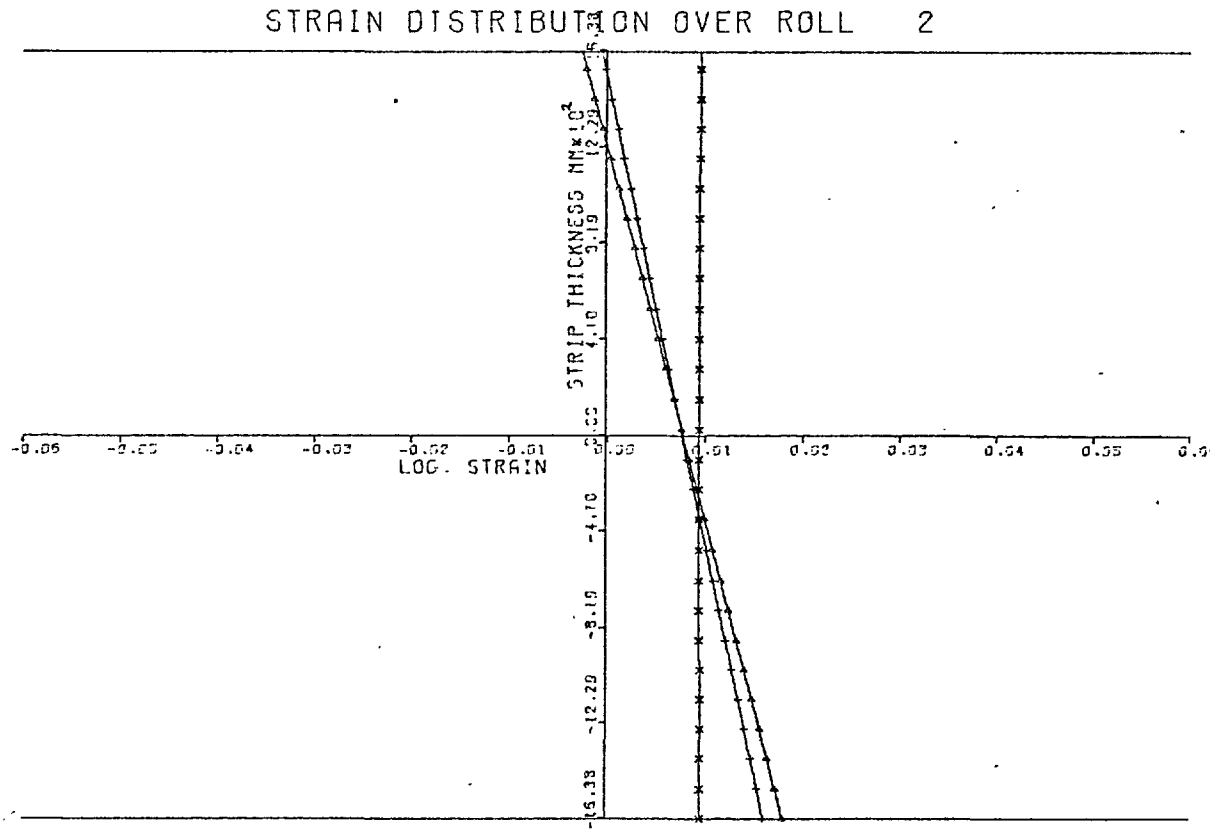

STRESS DISTRIBUTION OVER ROLL 1
STRIP THICKNESS MM*10
STRESS MN/M**2
*10

fig (A7.1b)


STRAIN DISTRIBUTION OVER ROLL 2
STRIP THICKNESS MM*10³
LOG. STRAIN

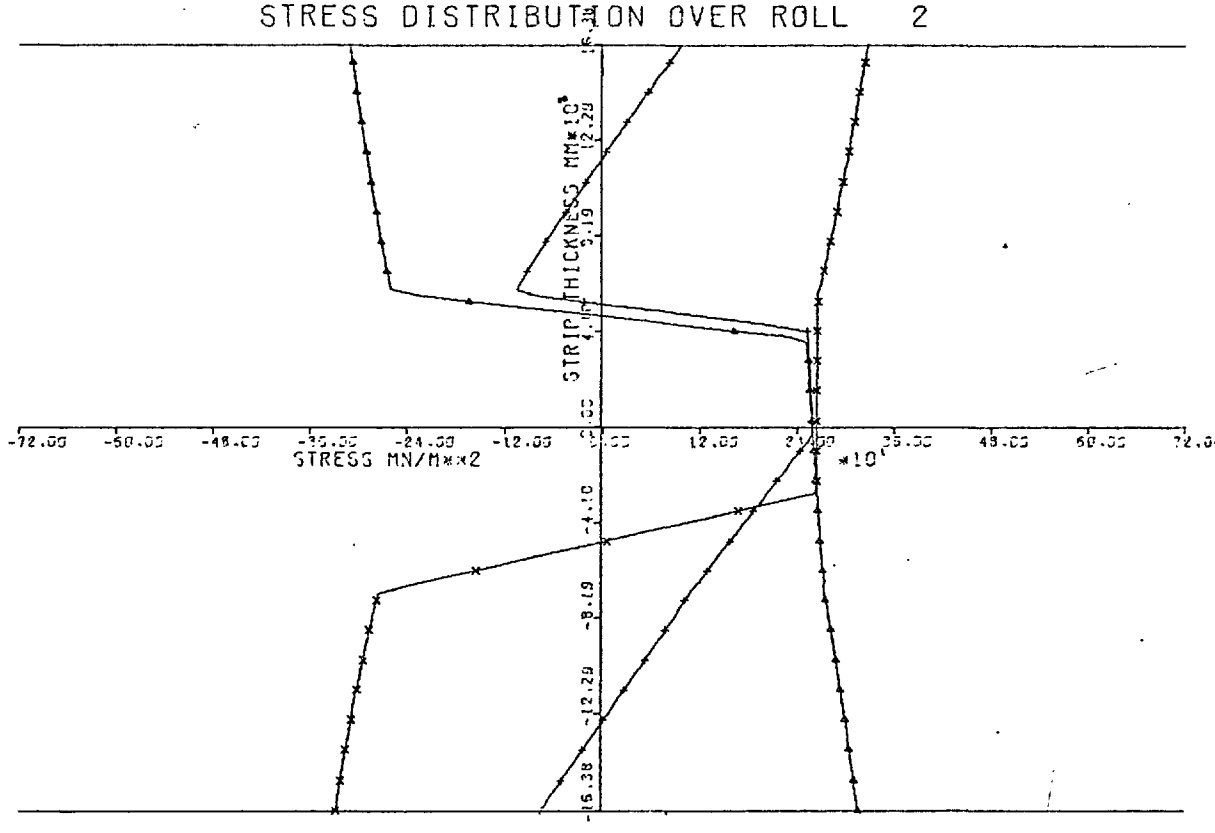
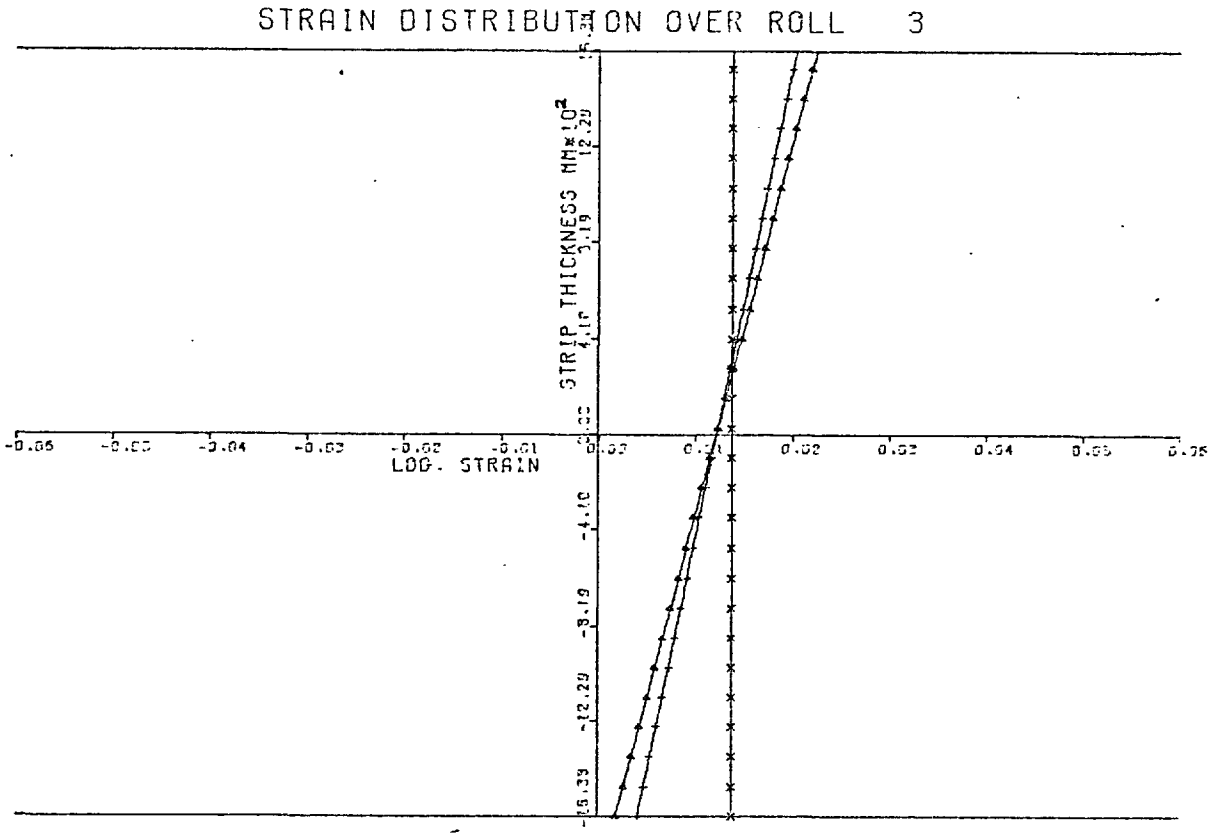

STRESS DISTRIBUTION OVER ROLL 2
STRIP THICKNESS MM*10³
STRESS MN/M**2
*10¹

fig (A7.1c)


STRAIN DISTRIBUTION OVER ROLL 3
STRIP THICKNESS MM*10²
LOG. STRAIN

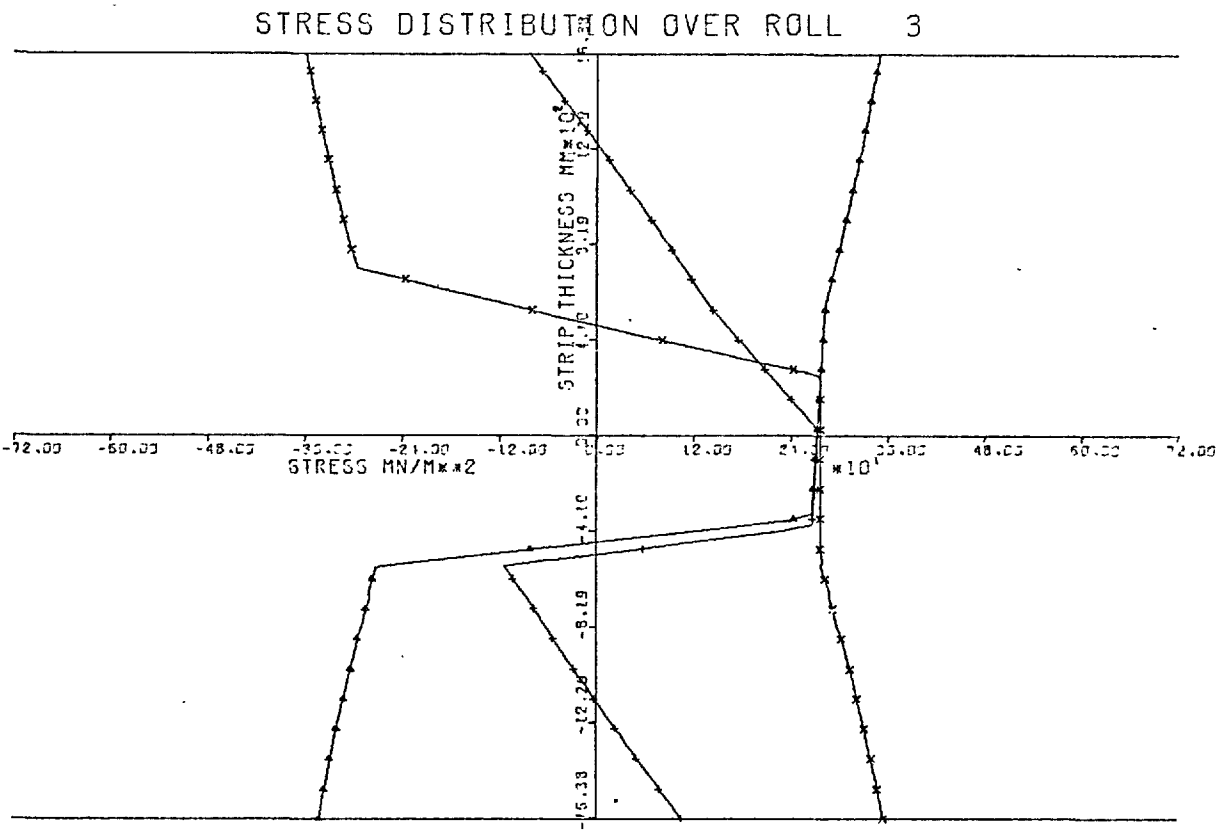
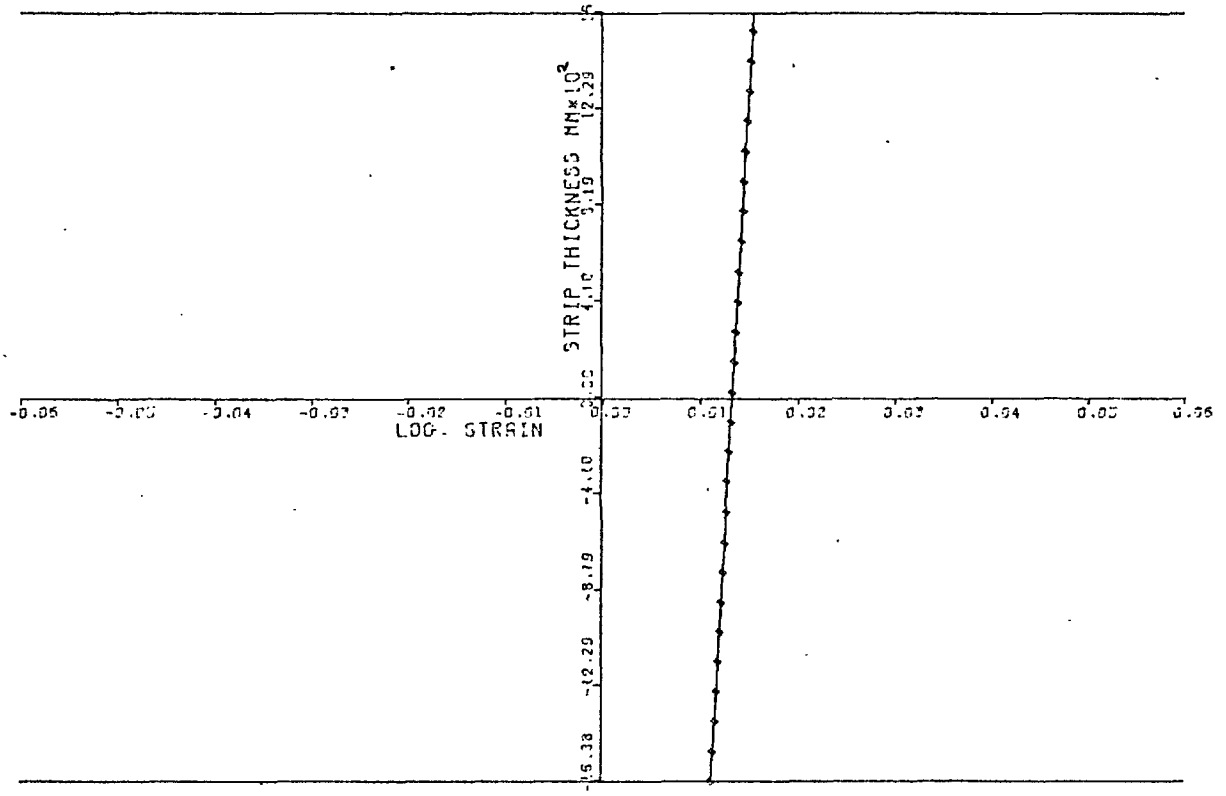

STRESS DISTRIBUTION OVER ROLL 3
STRIP THICKNESS MM*10²
STRESS MN/M**2
*10¹

fig (A7.1d)

STRAIN DISTRIBUTION AFTER TENSION RELAXATION



STRESS DISTRIBUTION AFTER TENSION RELAXATION

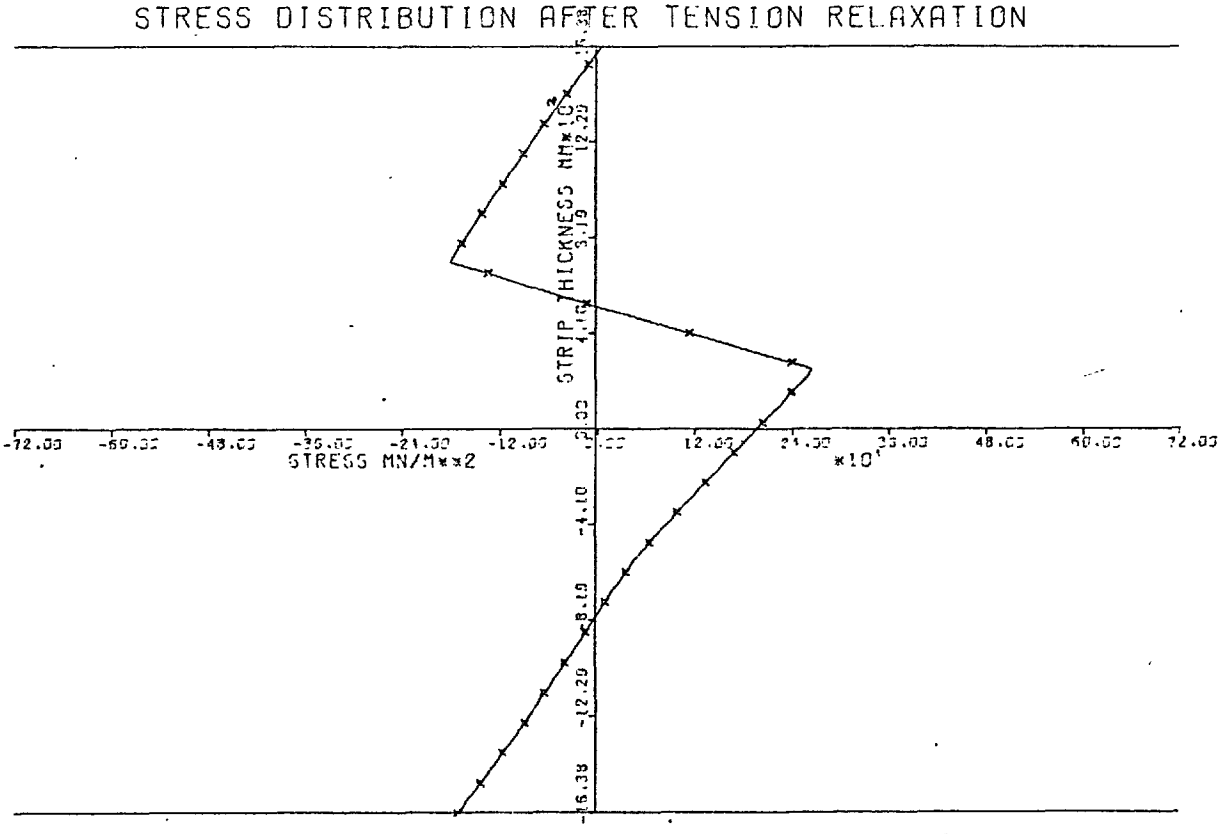


fig (A7.1e)

SER/2AW

E = 27u00000.0 QMU = .290 A = 115096.0 B = .016 IP = 1 RM = .300

AA = .0065 BB = 6.75 NR = 3 TL(1) = 800.0 COEFRI = .080

J = 6 JW = 2 JF = 2 NB = 1

D(1) =	0.000	AWRAP(1) =	0.0
D(2) =	1.240	AWRAP(2) =	40.0
D(3) =	1.240	AWRAP(3) =	80.0
D(4) =	1.240	AWRAP(4) =	40.0

ROLL NO. = 0 ROLL RADIUS = .006 ROLL POSITION = 3

BY(N,K) = 0.00000 CY(N,K) = 0.00000 BMOM(N,K) = 0.0000

RRESID(N,K) = 0.00 WORK DONE = .125 TENSION IN = 800.00 TENSION OUT = 799.88

Y(I)	ETX	ETXT	ETXAB	ETXBU	ETXM	ETXE	SIGD	SMX	SMXT	SMXM	AWORK
-.6450E-02	.0003	.0003	.0003	0.0000	.0012	.0023	0.0	9187.	9187.	34215.	1.43
-.3225E-02	.0003	.0003	.0003	0.0000	.0012	.0023	0.0	9187.	9187.	34215.	1.43
0.E+00	.0003	.0003	.0003	0.0000	.0012	.0023	0.0	9187.	9187.	34215.	1.43
.3225E-02	.0003	.0003	.0003	0.0000	.0012	.0023	0.0	9187.	9187.	34215.	1.43
.6450E-02	.0003	.0003	.0003	0.0000	.0012	.0023	0.0	9187.	9187.	34215.	1.43

ROLL NO = 1 ROLL RADIUS = .626 ROLL POSITION = 1

BY(N,K) = .00144 CY(N,K) = .00164 BMOM(N,K) = 1.4474

RRESID(N,K) = .63 WORK DONE = 15.278 TENSION IN = 799.88 TENSION OUT = 859.03

Y(I)	ETX	ETXT	ETXAB	ETXBU	ETXM	ETXE	SIGD	SMX	SMXT	SMXM	AWORK
-.6448E-02	-.0080	-.0077	.0077	.0065	.0077	.0026	0.0	-237042.	-37645.	37645.	253.74
-.3224E-02	-.0028	-.0025	.0025	.0014	.0025	.0024	0.0	-84100.	-35021.	35021.	66.07
0.E+00	.0023	.0026	.0026	.0023	.0026	.0024	0.0	68053.	35069.	35069.	69.08
.3224E-02	.0075	.0078	.0078	.0075	.0078	.0026	0.0	219425.	37685.	37685.	256.97
.6448E-02	.0126	.0129	.0129	.0126	.0129	.0027	0.0	370023.	39935.	39935.	457.36

ROLL NO = 1 ROLL RADIUS = .626 ROLL POSITION = 2

BY(N,K) = .00004 CY(N,K) = .00199 BMOM(N,K) = -.0010

RRESID(N,K) = .76 WORK DONE = -1.334 TENSION IN = 859.03 TENSION OUT = 857.69

Y(I)	ETX	ETXT	ETXAB	ETXBU	ETXM	ETXE	SIGD	SMX	SMXT	SMXM	AWORK
-.6433E-02	.0018	-.0059	.0059	.0065	.0077	.0024	0.0	52783.	15138.	37645.	-20.14
-.3217E-02	.0009	-.0016	.0016	.0014	.0025	.0022	0.0	26553.	-8468.	35021.	-19.58
0.E+00	.0000	.0026	.0026	.0023	.0026	.0022	0.0	322.	35075.	35075.	.38
.3217E-02	-.0009	.0069	.0069	.0075	.0078	.0024	0.0	-25908.	11777.	37685.	-21.72
.6433E-02	-.0018	.0112	.0112	.0126	.0129	.0025	0.0	-52139.	-12203.	39935.	-24.56

ROLL NO = 1 ROLL RADIUS = .626 ROLL POSITION = 3

BY(N,K) = .00126 CY(N,K) = 0.00000 BMOM(N,K) = -1.4931

RRESID(N,K) = 0.00 WORK DONE = 12.918 TENSION IN = 857.69 TENSION OUT = 869.15

Y(I)	ETX	ETXT	ETXAB	ETXBU	ETXM	ETXE	SIGD	SMX	SMXT	SMXM	AWORK
-.6433E-02	.0102	.0043	.0173	.0096	.0173	.0026	3430.3	299654.	41644.	41644.	398.20
-.3217E-02	.0059	.0043	.0072	.0046	.0072	.0023	806.3	174784.	37404.	37404.	185.69
0.E+00	.0017	.0043	.0043	.0040	.0043	.0023	0.0	49382.	35974.	35974.	59.57
.3217E-02	-.0026	.0043	.0089	.0011	.0089	.0024	3470.4	-76555.	-38194.	38194.	61.18
.6433E-02	-.0069	.0043	.0191	.0061	.0191	.0027	5720.7	-203033.	-42271.	42271.	271.09

ROLL NO = 2 ROLL RADIUS = .626 ROLL PCSITION = 1

BY(N,K) = .00182 CY(N,K) = .00450 BMCM(N,K) = -1.5905

RRESID(N,K) = -.63 WORK DONE = 19.373 TENSION IN = 869.15 TENSION OUT = 981.97

Y(I)	ETX	ETXT	ETXAB	ETXBU	ETXM	ETXE	SIGD	SMX	SMXT	SMXM	AWORK
-.6422E-02	.0133	.0176	.0306	.0229	.0306	.0029	3430.3	388035.	45994.	45994.	581.71
-.3211E-02	.0081	.0124	.0153	.0127	.0153	.0026	806.3	237433.	40861.	40861.	316.93
0.E+00	.0029	.0072	.0072	.0069	.0072	.0024	0.0	86057.	37427.	37427.	107.33
.3211E-02	-.0022	.0021	.0111	.0033	.0111	.0025	3470.4	-66099.	-39178.	39178.	86.66
.6422E-02	-.0074	-.0031	.0265	.0135	.0265	.0028	5720.7	-219046.	-44744.	44744.	322.29

ROLL NO = 2 ROLL RADIUS = .626 ROLL POSITION = 2

BY(N,K) = .00012 CY(N,K) = .00558 BMCM(N,K) = -.0176

RRESID(N,K) = -.77 WORK DONE = -1.396 TENSION IN = 981.97 TENSION OUT = 980.53

Y(I)	ETX	ETXT	ETXAB	ETXBU	ETXM	ETXE	SIGD	SMX	SMXT	SMXM	AWORK
-.6404E-02	-.0019	.0156	.0287	.0229	.0306	.0029	3430.3	-57134.	-11141.	45994.	-27.15
-.3202E-02	-.0010	.0114	.0143	.0127	.0153	.0026	806.3	-28043.	12818.	40861.	-22.61
0.E+00	.0000	.0073	.0073	.0070	.0073	.0024	0.0	1048.	37444.	37444.	1.33
.3202E-02	.0010	.0031	.0101	.0033	.0111	.0025	3470.4	30139.	-9039.	39178.	-21.65
.6404E-02	.0020	-.0011	.0245	.0135	.0265	.0028	5720.7	59230.	14486.	44744.	-23.66

ROLL NO = 2 ROLL RADIUS = .626 ROLL POSITION = 3

BY(N,K) = .00134 CY(N,K) = 0.00000 BMOM(N,K) = 1.6281

RRESID(N,K) = 0.00 WORK DONE = 13.954 TENSION IN = 980.53 TENSION OUT = 992.76

Y(I)	ETX	ETXT	ETXAB	ETXBU	ETXM	ETXE	SIGD	SMX	SMXT	SMXM	AWORK
-.6403E-02	-.0066	.0090	.0362	.0057	.0362	.0030	11778.7	-195510.	-47589.	47589.	291.92
-.3202E-02	-.0024	.0090	.0161	.0008	.0161	.0026	6646.2	-71840.	-41175.	41175.	56.18
0.E+00	.0017	.0090	.0090	.0087	.0090	.0024	0.0	51314.	38250.	38250.	65.94
.3202E-02	.0059	.0090	.0156	.0045	.0156	.0026	4963.3	173955.	40991.	40991.	201.32
.6403E-02	.0101	.0090	.0358	.0093	.0358	.0030	10529.1	296088.	47457.	47457.	452.46

ROLL NO = 3 ROLL RADIUS = .626 ROLL POSITION = 1

BY(N,K) = .00188 CY(N,K) = .00746 BMOM(N,K) = 1.7168

RRESID(N,K) = .63 WORK DONE = 21.343 TENSION IN = 992.76 TENSION OUT = 1067.92

Y(I)	ETX	ETXT	ETXAB	ETXBU	ETXM	ETXE	SIGD	SMX	SMXT	SMXM	AWORK
-.6392E-02	-.0073	.0017	.0435	.0130	.0435	.0031	11778.7	-215789.	-49477.	49477.	354.07
-.3196E-02	-.0021	.0069	.0182	.0029	.0182	.0026	6646.2	-62841.	-41966.	41966.	88.53
0.E+00	.0030	.0120	.0120	.0117	.0120	.0025	0.0	89317.	39565.	39565.	118.09
.3196E-02	.0082	.0172	.0238	.0127	.0238	.0028	4963.3	240693.	43892.	43892.	348.26
.6392E-02	.0134	.0224	.0491	.0227	.0491	.0032	10529.1	391298.	50817.	50817.	657.17

ROLL NO = 3 ROLL RADIUS = .626 ROLL POSITION = 2

BY(N,K) = .00011 CY(N,K) = .00942 BMOM(N,K) = .0053

RRESID(N,K) = .79 WORK DONE = -1.623 TENSION IN = 1067.92 TENSION OUT = 1066.25

Y(I)	ETX	ETXT	ETXAB	ETXBU	ETXM	ETXE	SIGD	SMX	SMXT	SMXM	AWORK
-.6373E-02	.0022	.0039	.0414	.0130	.0435	.0031	11778.7	64514.	15037.	49477.	-29.57
-.3186E-02	.0011	.0080	.0171	.0029	.0182	.0026	6646.2	32811.	-9156.	41966.	-24.95
0.E+00	.0000	.0121	.0121	.0118	.0121	.0025	0.0	1167.	39581.	39581.	1.49
.3186E-02	-.0010	.0162	.0228	.0127	.0238	.0028	4963.3	-30596.	13296.	43892.	-26.26
.6373E-02	-.0021	.0203	.0470	.0227	.0491	.0032	10529.1	-62300.	-11483.	50817.	-33.58

ROLL NO = 3 ROLL RADIUS = .626 ROLL POSITION = 3

BY(N,K) = .00137 CY(N,K) = 0.00000 BMOM(N,K) = -1.7083

RRESID(N,K) = 0.00 WORK DONE = 14.443 TENSION IN = 1066.25 TENSION OUT = 1078.80

Y(I)	ETX	ETXT	ETXAB	ETXBU	ETXM	ETXE	SIGD	SMX	SMXT	SMXM	AWORK
-.6373E-02	.0099	.0138	.0526	.0090	.0526	.0032	15262.3	291470.	51602.	51602.	485.54
-.3186E-02	.0058	.0138	.0225	.0043	.0225	.0027	7751.4	171900.	43475.	43475.	209.81
0.E+00	.0018	.0138	.0138	.0135	.0138	.0025	0.0	51843.	40299.	40299.	70.31
.3186E-02	-.0023	.0138	.0244	.0006	.0244	.0028	9677.1	-68705.	-44090.	44090.	53.01
.6373E-02	-.0064	.0138	.0545	.0053	.0545	.0033	16601.8	-189748.	-52024.	52024.	307.93

ROLL NO = 4 ROLL RADIUS = .006 ROLL POSITION = 1

BY(N,K) = -.00125 CY(N,K) = -.03979 BMOM(N,K) = -.0123

RRESID(N,K) = 2.92 WORK DONE = 0.000 TENSION IN = 1078.80 TENSION OUT = 0.00

Y(I)	ETX	ETXT	ETXAB	ETXBU	ETXM	ETXE	SIGD	SMX	SMXT	SMXM	AWORK
-.6361E-02	-.0026	.0113	.0500	.0090	.0526	.0032	15262.3	-75724.	-24122.	51602.	485.54
-.3181E-02	-.0015	.0124	.0211	.0043	.0225	.0027	7751.4	-44064.	-589.	43475.	209.81
0.E+00	-.0004	.0134	.0134	.0135	.0138	.0025	0.0	-12404.	27896.	40299.	70.31
.3181E-02	.0007	.0145	.0238	.0006	.0244	.0028	9677.1	19256.	-24834.	44090.	53.01
.6361E-02	.0017	.0156	.0527	.0053	.0545	.0033	16601.8	50916.	-1107.	52024.	307.93

MEAN LOAD = 939.40 LOAD LOST = 278.80

TOTAL STRAIN = .0134 AT .326 YIELD STRESS

SIGMA = .28917E+05
SIGMA = .29461E+05
SIGMA = .29449E+05

E = 27000000.0 QMU = .290 A = 96101.000 B = .017 RM = .295

SIGMABAR = 96101.(.017130 + ETABAR)** .29530

I= 2 ALP(I) = .17369588430+02

N	CB(N)	DCB(N)	H(N)
1	29449.161	0.000	0.0
2	29492.749	43.588	485865.9
3	29492.656	43.496	484123.4
4	0.000	43.496	484127.1

I	ALP(I)	SB(I)	SX(I)	SZ(I)	EX(I)	EXENG(I)	EY(I)	EYENG(I)
1	.16369588430+02	29449.161	33047.250	9583.702	.001121	.001122	-.000458	-.000458
26	.29831588430+02	35114.424	40546.470	20170.065	.014349	.014453	-.013400	-.013310
51	.29856588430+02	37294.696	43064.070	21438.732	.020867	.021086	-.019859	-.019663
76	.29881588430+02	40337.291	46577.392	23205.382	.031615	.032120	-.030525	-.030064
101	.29906588430+02	44472.994	51352.922	25604.009	.049674	.050929	-.048472	-.047316
126	.29929788430+02	50086.789	57835.199	28856.283	.081478	.084889	-.080124	-.076998

	SB(I)	SBUSB0(I)	SX(I)	SXMAX(I)	SXUSXM(I)	SXUSB0(I)	SZUSB0(I)	SZUSX(I)	DIFF(I)	EZE(I)
1	29449.161	1.0000	33047.250	34004.962	.9718	1.1222	.3254	.5860	.0325	0.000000
26	35114.424	1.1924	40546.470	40546.645	1.0000	1.3768	.6849	.9949	.0000	.000313
51	37294.696	1.2664	43064.070	43064.205	1.0000	1.4623	.7280	.9957	.0000	.000333
76	40337.291	1.3697	46577.392	46577.492	1.0000	1.5816	.7880	.9964	.0000	.000361
101	44472.994	1.5102	51352.922	51352.990	1.0000	1.7438	.8694	.9972	.0000	.000398
126	50086.789	1.7008	57835.199	57835.242	1.0000	1.9639	.9799	.9979	.0000	.000449

SBDIFF = .2129E+05 SXDIF = .6425E+05 EZE(I)= .4552E-03
EXBDO = .1154E-02 EXDIF = .2180E-02 EYDIF = -.8902E-03

I= 2 ALP(I) = .3807088432D+01

N	CB(N)	DCB(N)	H(N)
1	29449.161	0.000	0.0
2	29469.975	20.814	485865.9
3	0.000	20.793	485032.7

I	ALP(I)	SB(I)	SX(I)	SZ(I)	EX(I)	EXENG(I)	EY(I)	EYENG(I)
1	.2807088432D+01	29449.161	5659.320	-10628.247	.001026	.001027	-.083516	-.080123
26	.2780708843D+02	30927.689	11098.893	4366.059	.004075	.004084	-.086244	-.082630
51	.2982108843D+02	34386.187	15118.382	7451.859	.012233	.012309	-.094290	-.089982
76	.2984608843D+02	36238.317	17257.079	8531.238	.017449	.017602	-.099456	-.094670
101	.2987108843D+02	38955.750	20394.938	10109.869	.026343	.026693	-.108276	-.102620
126	.2989608843D+02	42562.953	24560.210	12202.965	.040667	.041506	-.122503	-.115297

SBDIFF = .3550E+04 SXDIF = .5606E+05 EZE(I)= -.2308E-03
EXBDO = .1154E-02 EXDIF = .1902E-02 EYDIF = -.7767E-03

I= 2 ALP(I) = .3820088432D+01

N	CB(N)	DCB(N)	H(N)
1	29449.161	0.000	0.0
2	29469.985	20.825	485865.9
3	0.000	20.804	485032.3

I	ALP(I)	SB(I)	SX(I)	SZ(I)	EX(I)	EXENG(I)	EY(I)	EYENG(I)
1	.2820088432D+01	29449.161	26151.118	-376.404	.000748	.000748	-.144759	-.134769
26	.2782008843D+02	30931.605	31591.993	14619.472	.003805	.003813	-.147495	-.137133
51	.2982508843D+02	34615.667	35871.642	17830.189	.012576	.012656	-.156148	-.144568
76	.2985008843D+02	36626.905	38194.061	19001.243	.018360	.018529	-.161877	-.149454
101	.2987508843D+02	39462.417	41468.264	20648.147	.027907	.028300	-.171348	-.157471
126	.2990008843D+02	43256.616	45849.460	22849.353	.043511	.044472	-.186850	-.170432

SIGMAX = 104373.(.008356 + ETAX)** .25000

TRUE STRESS ACTUAL	TRUE STRESS CALCULATED	DIF	TRUE STRAIN
.33047E+05	.32566E+05	.48117E+03	.11217E-02
.40546E+05	.40561E+05	-.14964E+02	.14453E-01
.43064E+05	.43235E+05	-.17053E+03	.21086E-01
.46577E+05	.46815E+05	-.23796E+03	.32120E-01
.51353E+05	.51502E+05	-.14898E+03	.50929E-01
.57835E+05	.57676E+05	.15936E+03	.84889E-01
.30246E+05	.32484E+05	-.22376E+04	.10266E-02
.35686E+05	.34857E+05	.82896E+03	.40836E-02
.39706E+05	.39573E+05	.13289E+03	.12309E-01
.41844E+05	.41894E+05	-.50173E+02	.17602E-01
.44982E+05	.45160E+05	-.17810E+03	.26693E-01
.49147E+05	.49321E+05	-.17330E+03	.41506E-01
.30250E+05	.32241E+05	-.19905E+04	.74834E-03
.35691E+05	.34666E+05	.10252E+04	.38127E-02
.39971E+05	.39738E+05	.23266E+03	.12656E-01
.42293E+05	.42264E+05	.29258E+02	.18529E-01
.45567E+05	.45669E+05	-.10202E+03	.28300E-01
.49948E+05	.50038E+05	-.90009E+02	.44472E-01

SER/2KM

E = 27000000.0 QMU = .290 A = 104372.8 B = .008 IP = 1 RM = .250

AA = .0065 BB = 6.75 NR = 3 TL(1) = 1000.0 COEFRI = *.000

J = 3 JW = 1 JF = 1 NB = 2

D(1) =	0.000	AWRAP(1) =	0.0
D(2) =	1.240	AWRAP(2) =	40.0
D(3) =	1.240	AWRAP(3) =	80.0
D(4) =	1.240	AWRAP(4) =	40.0

ROLL NO = 0 ROLL RADIUS = .006 ROLL POSITION = 3

BY(N,K) = 0.00000 CY(N,K) = 0.00000 BMOM(N,K) = 0.0000

RRESID(N,K) = 0.00 WORK DONE = 0.000 TENSION IN = 1000.00 TENSION OUT = 1000.00

Y(I)	ETX	ETXT	ETXAB	ETXBU	ETXM	ETXE	SIGD	SMX	SMXT	SMXM	AWORK
-.6450E-02	.0004	.0004	.0004	0.0000	.0011	.0022	0.0	11484.	11484.	32566.	737.78
-.3225E-02	.0004	.0004	.0004	0.0000	.0011	.0022	0.0	11484.	11484.	32566.	559.51
0.E+00	.0004	.0004	.0004	0.0000	.0011	.0022	0.0	11484.	11484.	32566.	328.04
.3225E-02	.0004	.0004	.0004	0.0000	.0011	.0022	0.0	11484.	11484.	32566.	89.84
.6450E-02	.0004	.0004	.0004	0.0000	.0011	.0022	0.0	11484.	11484.	32566.	35.58

values retained from A7.2

ROLL NO = 1 ROLL RADIUS = .626 ROLL POSITION = 1

BY(N,K) = .00164 CY(N,K) = .00188 BMOM(N,K) = 1.4035

RRESID(N,K) = .63 WORK DONE = 0.000 TENSION IN = 1000.00 TENSION OUT = 1000.00

Y(I)	ETX	ETXT	ETXAB	ETXBU	ETXM	ETXE	SIGD	SMX	SMXT	SMXM	AWORK
-.6447E-02	-.0077	-.0073	.0073	.0062	.0073	.0022	0.0	-227974.	-36928.	36928.	737.78
-.3224E-02	-.0025	-.0022	.0022	.0010	.0022	.0022	0.0	-75031.	-33417.	33417.	559.51
0.E+00	.0026	.0030	.0030	.0026	.0030	.0022	0.0	77122.	34603.	34603.	328.04
.3224E-02	.0078	.0082	.0082	.0078	.0082	.0022	0.0	228494.	37821.	37821.	89.84
.6447E-02	.0129	.0133	.0133	.0129	.0133	.0022	0.0	379092.	40380.	40380.	35.58

ROLL NO = 1 ROLL RADIUS = .626 ROLL POSITION = 2

BY(N,K) = .00004 CY(N,K) = .00226 BMOM(N,K) = .0106

RRESID(N,K) = .75 WORK DONE = 0.000 TENSION IN = 1000.00 TENSION OUT = 1000.00

Y(I)	ETX	ETXT	ETXAB	ETXBU	ETXM	ETXE	SIGD	SMX	SMXT	SMXM	AWORK
-.6431E-02	.0017	-.0056	.0056	.0062	.0073	.0020	0.0	51213.	14285.	36928.	737.78
-.3215E-02	.0009	-.0013	.0013	.0010	.0022	.0022	0.0	25759.	-7659.	33417.	559.51
0.E+00	.0000	.0030	.0030	.0026	.0030	.0020	0.0	305.	34610.	34610.	328.04
.3215E-02	-.0009	.0073	.0073	.0078	.0082	.0020	0.0	-25149.	12672.	37821.	89.84
.6431E-02	-.0017	.0116	.0116	.0129	.0133	.0020	0.0	-50603.	-10223.	40380.	35.58

ROLL NO = 1 ROLL RADIUS = .626 ROLL POSITION = 3

BY(N,K) = .00116 CY(N,K) = 0.00000 BMOM(N,K) = -1.2255

RRESID(N,K) = 0.00 WORK DONE = 0.000 TENSION IN = 1000.00 TENSION OUT = 1000.00

Y(I)	ETX	ETXT	ETXAB	ETXBU	ETXM	ETXE	SIGD	SMX	SMXT	SMXM	AWORK
-.6431E-02	.0101	.0046	.0172	.0098	.0172	.0020	4361.9	297677.	34543.	34543.	737.78
-.3215E-02	.0059	.0046	.0067	.0045	.0067	.0022	851.4	171980.	35035.	35035.	559.51
0.E+00	.0016	.0046	.0046	.0042	.0046	.0020	0.0	45744.	35672.	35672.	328.04
.3215E-02	-.0027	.0046	.0097	.0016	.0097	.0020	5255.4	-81035.	-28557.	28557.	89.84
.6431E-02	-.0070	.0046	.0200	.0067	.0200	.0020	7814.3	-208361.	-29407.	29407.	35.58

ROLL NO = 2 ROLL RADIUS = .626 ROLL POSITION = 1

BY(N,K) = .00129 CY(N,K) = .00414 BMOM(N,K) = -1.3869

RRESID(N,K) = -.63 WORK DONE = 0.000 TENSION IN = 1000.00 TENSION OUT = 1000.00

Y(I)	ETX	ETXT	ETXAB	ETXBU	ETXM	ETXE	SIGD	SMX	SMXT	SMXM	AWORK
-.6421E-02	.0124	.0170	.0296	.0222	.0296	.0020	4361.9	363190.	39679.	39679.	737.78
-.3210E-02	.0072	.0118	.0139	.0118	.0139	.0020	851.4	212587.	38981.	38981.	559.51
0.E+00	.0021	.0067	.0067	.0063	.0067	.0020	0.0	61211.	36959.	36959.	328.04
.3210E-02	-.0031	.0015	.0128	.0046	.0128	.0020	5255.4	-90947.	-30707.	30707.	89.84
.6421E-02	-.0082	-.0037	.0283	.0150	.0283	.0020	7814.3	-243894.	-33443.	33443.	35.58

ROLL NO = 2 ROLL RADIUS = .626 ROLL POSITION = 2

BY(N,K) = .00007 CY(N,K) = .00497 BMOM(N,K) = -.0066

RRESID(N,K) = -.75 WORK DONE = 0.000 TENSION IN = 1000.00 TENSION OUT = 1000.00

Y(I)	ETX	ETXT	ETXAB	ETXBU	ETXM	ETXE	SIGD	SMX	SMXT	SMXM	AWORK
-.6407E-02	-.0017	.0153	.0279	.0222	.0296	.0020	4361.9	-50092.	-10413.	39679.	737.78
-.3204E-02	-.0008	.0110	.0131	.0118	.0139	.0020	851.4	-24754.	14227.	38981.	559.51
0.E+00	.0000	.0067	.0067	.0063	.0067	.0020	0.0	583.	36971.	36971.	328.04
.3204E-02	.0009	.0024	.0119	.0046	.0128	.0020	5255.4	25920.	-4787.	30707.	89.84
.6407E-02	.0017	-.0019	.0265	.0150	.0283	.0020	7814.3	51257.	17814.	33443.	35.58

ROLL NO = 2 ROLL RADIUS = .626 ROLL POSITION = 3

BY(N,K) = .00077 CY(N,K) = 0.00000 BMOM(N,K) = 1.2978

RRESID(N,K) = 0.00 WORK DONE = 0.000 TENSION IN = 1000.00 TENSION OUT = 1000.00

Y(I)	ETX	ETXT	ETXAB	ETXBU	ETXM	ETXE	SIGD	SMX	SMXT	SMXM	AWORK
-.6407E-02	-.0076	.0077	.0368	.0072	.0368	.0020	7112.7	-223818.	-30395.	30395.	737.78
-.3204E-02	-.0033	.0077	.0160	.0021	.0160	.0020	6415.3	-96388.	-27779.	27779.	559.51
0.E+00	.0010	.0077	.0077	.0073	.0077	.0020	0.0	30493.	37564.	37564.	328.04
.3204E-02	.0053	.0077	.0170	.0042	.0170	.0020	-1858.9	156830.	37522.	37522.	89.84
.6407E-02	.0096	.0077	.0376	.0093	.0376	.0020	876.6	282629.	37766.	37766.	35.58

ROLL NO = 3 ROLL RADIUS = .626 ROLL POSITION = 1

BY(N,K) = .00109 CY(N,K) = .00587 BMOM(N,K) = 1.4467

RRESID(N,K) = .63 WORK DONE = 0.000 TENSION IN = 1000.00 TENSION OUT = 1000.00

Y(I)	ETX	ETXT	ETXAB	ETXBU	ETXM	ETXE	SIGD	SMX	SMXT	SMXM	AWORK
-.6401E-02	-.0086	-.0009	.0453	.0158	.0453	.0020	7112.7	-253581.	-34490.	34490.	737.78
-.3200E-02	-.0034	.0043	.0194	.0055	.0194	.0020	6415.3	-100632.	-30067.	30067.	559.51
0.E+00	.0017	.0095	.0095	.0091	.0095	.0020	0.0	51527.	38507.	38507.	328.04
.3200E-02	.0069	.0146	.0239	.0111	.0239	.0020	-1858.9	202905.	41370.	41370.	89.84
.6401E-02	.0121	.0198	.0497	.0214	.0497	.0020	876.6	353511.	42867.	42867.	35.58

ROLL NO = 3 ROLL RADIUS = .626 ROLL POSITION = 2

BY(N,K) = .00001 CY(N,K) = .00710 BMOM(N,K) = .0124

RRESID(N,K) = .76 WORK DONE = 0.000 TENSION IN = 1000.00 TENSION OUT = 1000.00

Y(I)	ETX	ETXT	ETXAB	ETXBU	ETXM	ETXE	SIGD	SMX	SMXT	SMXM	AWORK
-.6389E-02	.0018	.0010	.0435	.0158	.0453	.0020	7112.7	53206.	18716.	34490.	737.78
-.3195E-02	.0009	.0052	.0185	.0055	.0194	.0020	6415.3	26628.	-3439.	30067.	559.51
0.E+00	.0000	.0095	.0095	.0091	.0095	.0020	0.0	50.	38508.	38508.	328.04
.3195E-02	-.0009	.0137	.0230	.0111	.0239	.0020	-1858.9	-26528.	14843.	41370.	89.84
.6389E-02	-.0018	.0180	.0479	.0214	.0497	.0020	876.6	-53105.	-10238.	42867.	35.58

ROLL NO = 3 ROLL RADIUS = .626 ROLL POSITION = 3

BY(N,K) = .00053 CY(N,K) = 0.00000 BMOM(N,K) = -1.2588

RRESID(N,K) = 0.00 WORK DONE = 0.000 TENSION IN = 1000.00 TENSION OUT = 1000.00

Y(I)	ETX	ETXT	ETXAB	ETXBU	ETXM	ETXE	SIGD	SMX	SMXT	SMXM	AWORK
-.6389E-02	.0092	.0102	.0543	.0090	.0543	.0020	1924.0	270365.	36536.	36536.	737.78
-.3195E-02	.0050	.0102	.0232	.0038	.0232	.0020	-2498.7	145795.	37929.	37929.	559.51
0.E+00	.0007	.0102	.0102	.0098	.0102	.0020	0.0	20696.	38868.	38868.	328.04
.3195E-02	-.0036	.0102	.0263	.0024	.0263	.0020	8804.2	-104936.	-25646.	25646.	89.84
.6389E-02	-.0078	.0102	.0572	.0076	.0572	.0020	10301.1	-231105.	-27401.	27401.	35.58

ROLL NO = 4 ROLL RADIUS = .006 ROLL POSITION = 1

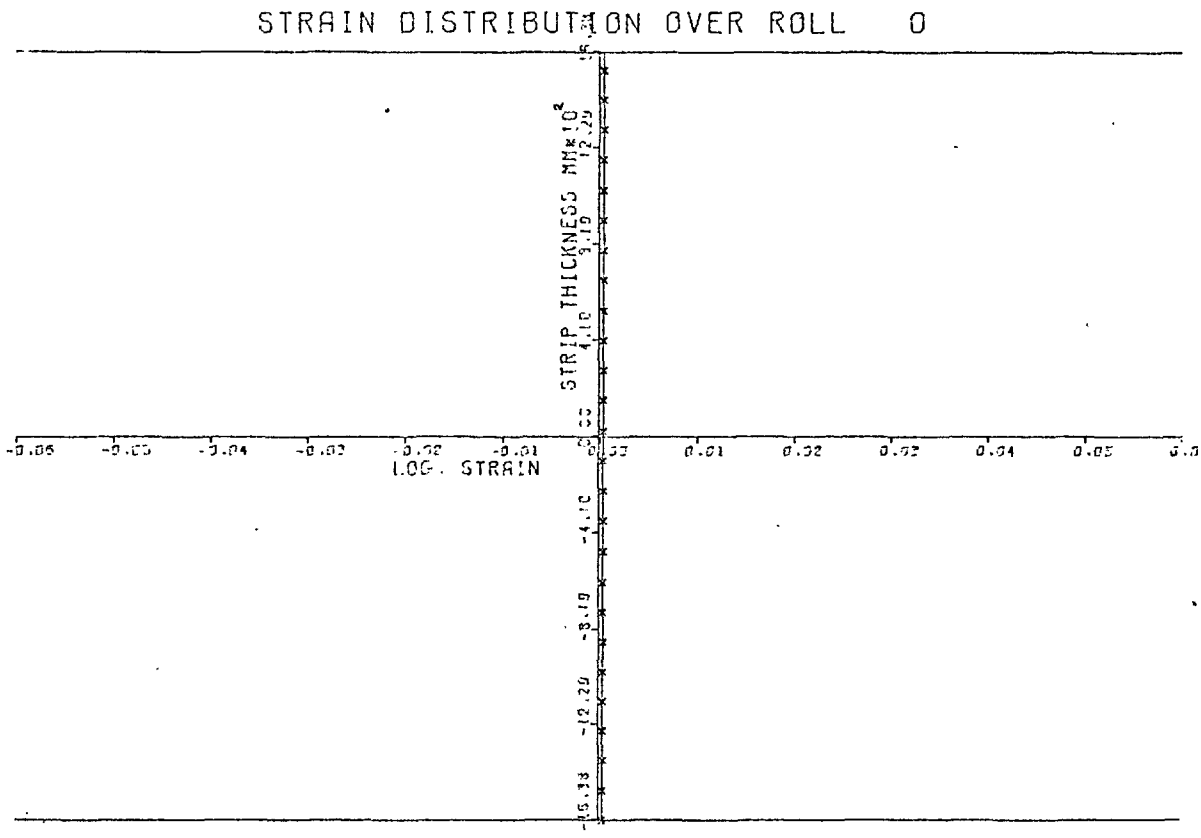
BY(N,K) = -.00160 CY(N,K) = -.03972 BMCM(N,K) = -.0046

RRESID(N,K) = 4.03 WORK DONE = 0.000 TENSION IN = 1000.00 TENSION OUT = 0.00

Y(I)	ETX	ETXT	ETXAB	ETXBU	ETXM	ETXE	SIGO	SMX	SMXT	SMXM	AWORK
-.6385E-02	-.0020	.0082	.0523	.0090	.0543	.0020	1924.0	-57944.	-21408.	36536.	737.78
-.3192E-02	-.0012	.0090	.0220	.0038	.0232	.0020	-2498.7	-34785.	3143.	37929.	559.51
0.E+00	-.0004	.0098	.0098	.0098	.0102	.0020	0.0	-11627.	27240.	38868.	328.04
.3192E-02	.0004	.0106	.0259	.0024	.0263	.0020	8804.2	11531.	-14115.	25646.	89.84
.6385E-02	.0012	.0113	.0561	.0076	.0572	.0020	10301.1	34689.	7289.	27401.	35.58

MEAN LOAD = 1000.00 LOAD LOST = 0.00

TOTAL STRAIN = .0098 AT .348 YIELD STRESS


STRAIN DISTRIBUTION OVER ROLL 0
STRIP THICKNESS MM*10²
LOG. STRAIN

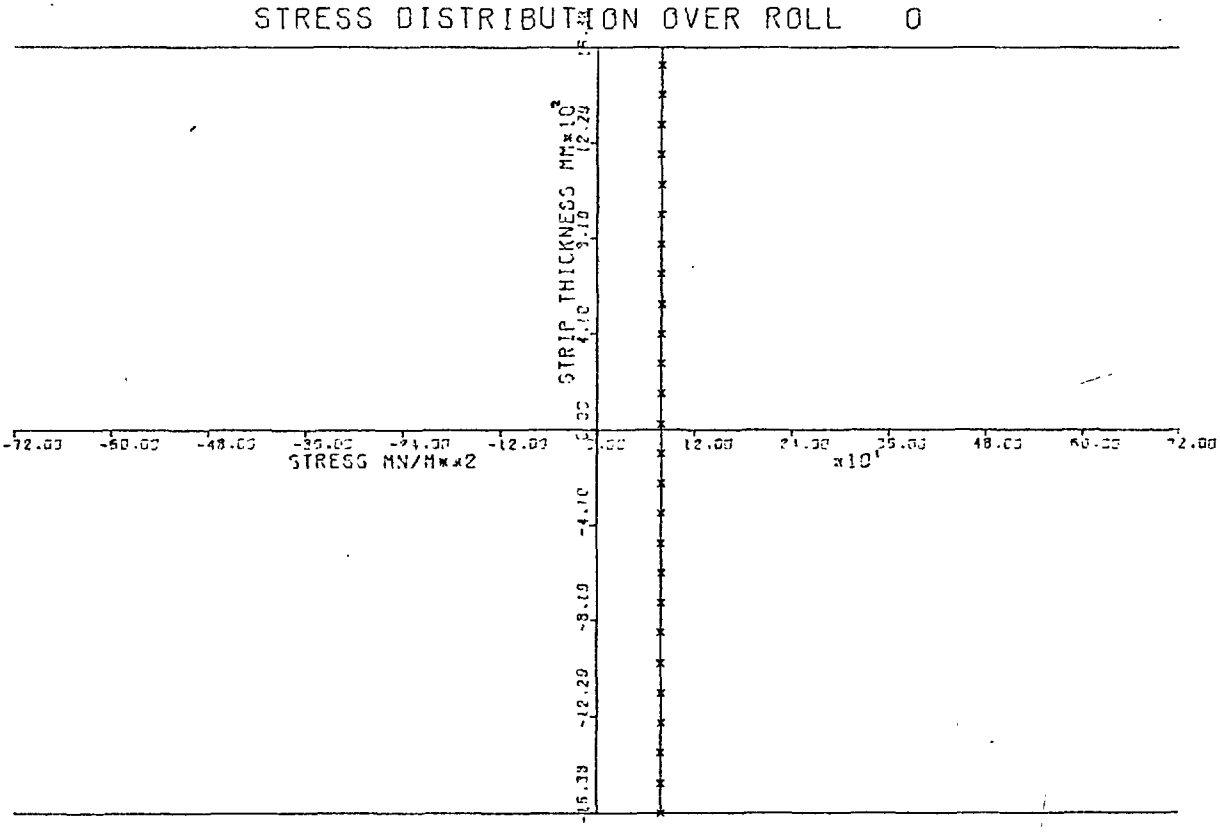
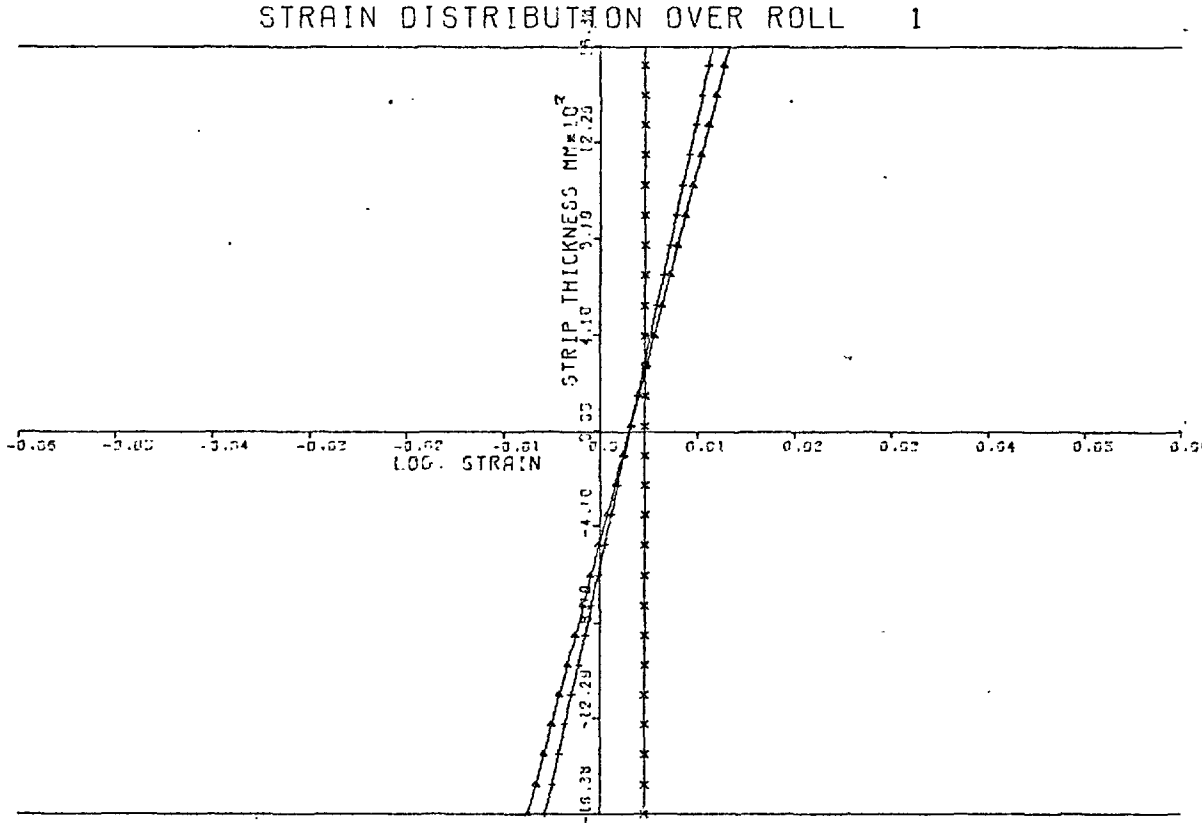

STRESS DISTRIBUTION OVER ROLL 0
STRIP THICKNESS MM*10²
STRESS MN/M**2
x10¹

fig (A7.2a)


STRAIN DISTRIBUTION OVER ROLL 1
STRIP THICKNESS MM*10**2
LOG. STRAIN

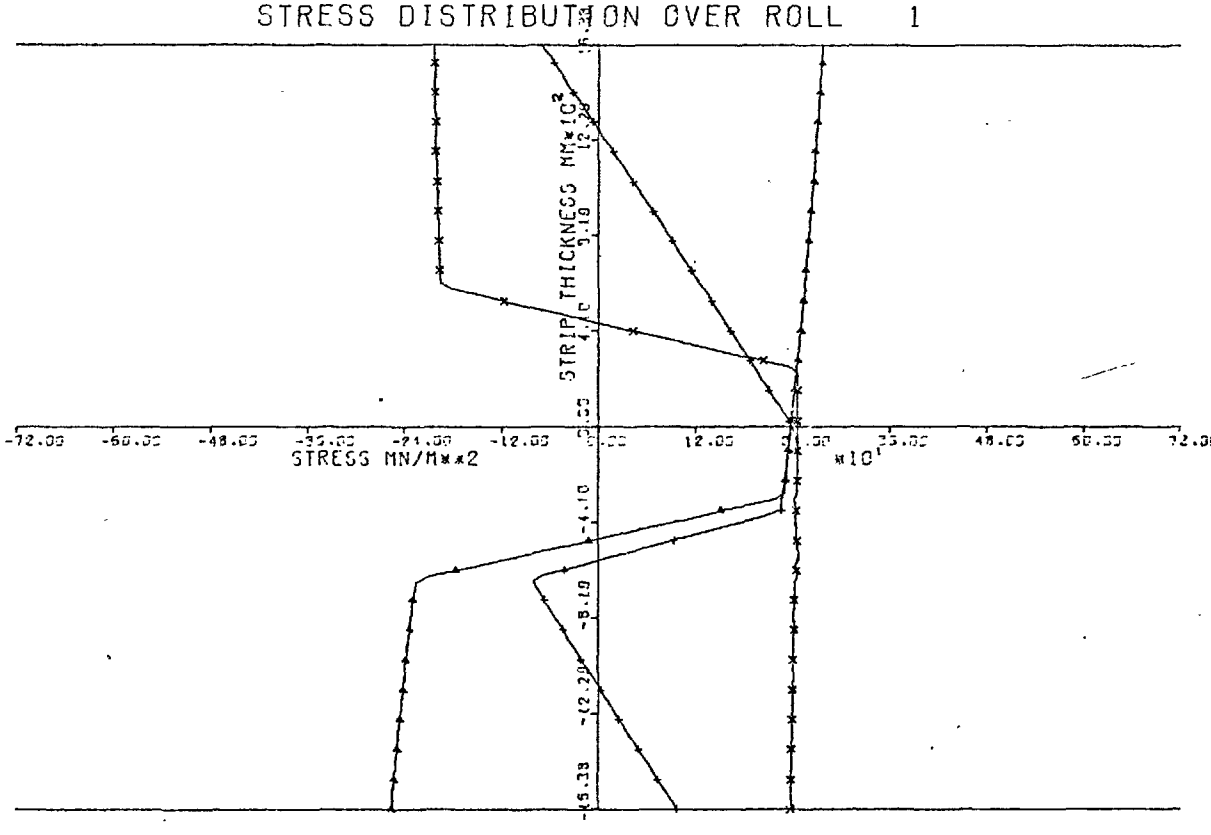
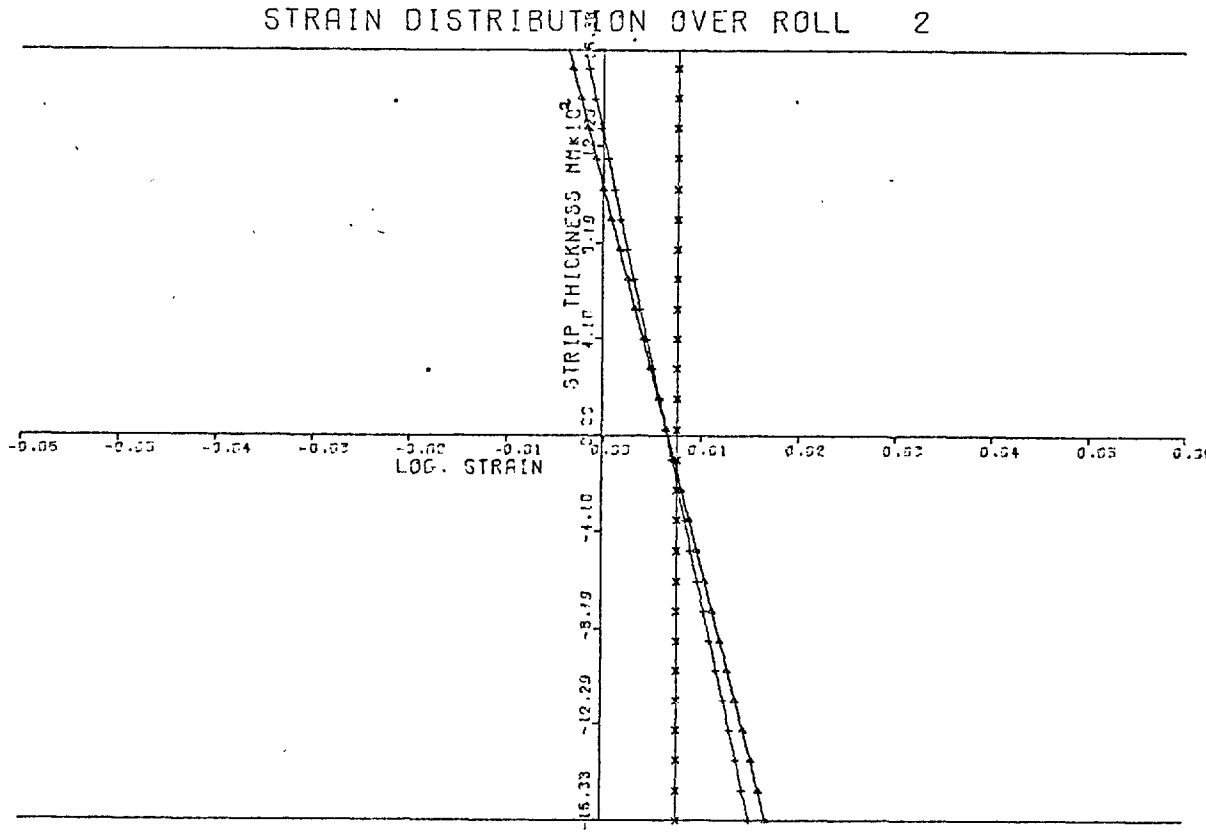

STRESS DISTRIBUTION OVER ROLL 1
STRIP THICKNESS MM*10**2
STRESS MN/M**2
*10**1

fig (A7.2b)


STRAIN DISTRIBUTION OVER ROLL 2
STRIP THICKNESS MM*10**2
LOG. STRAIN

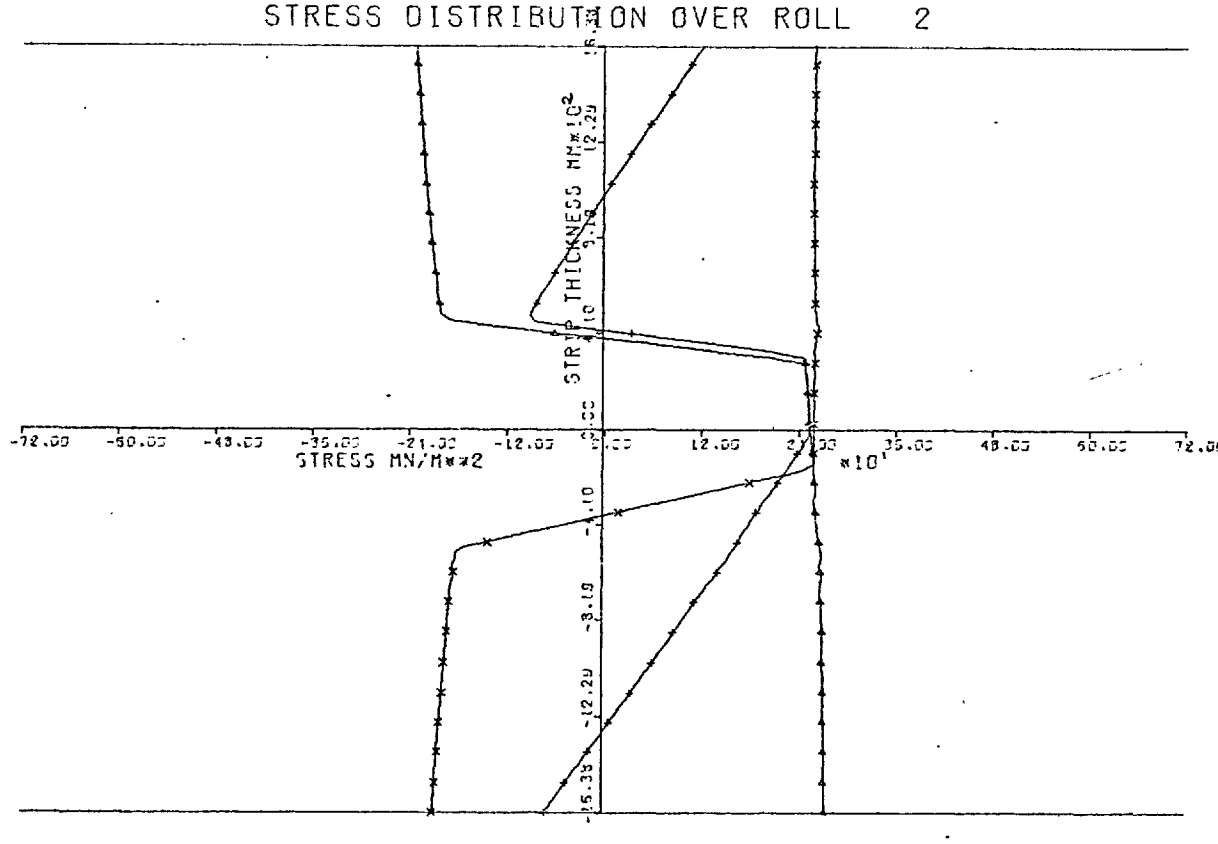
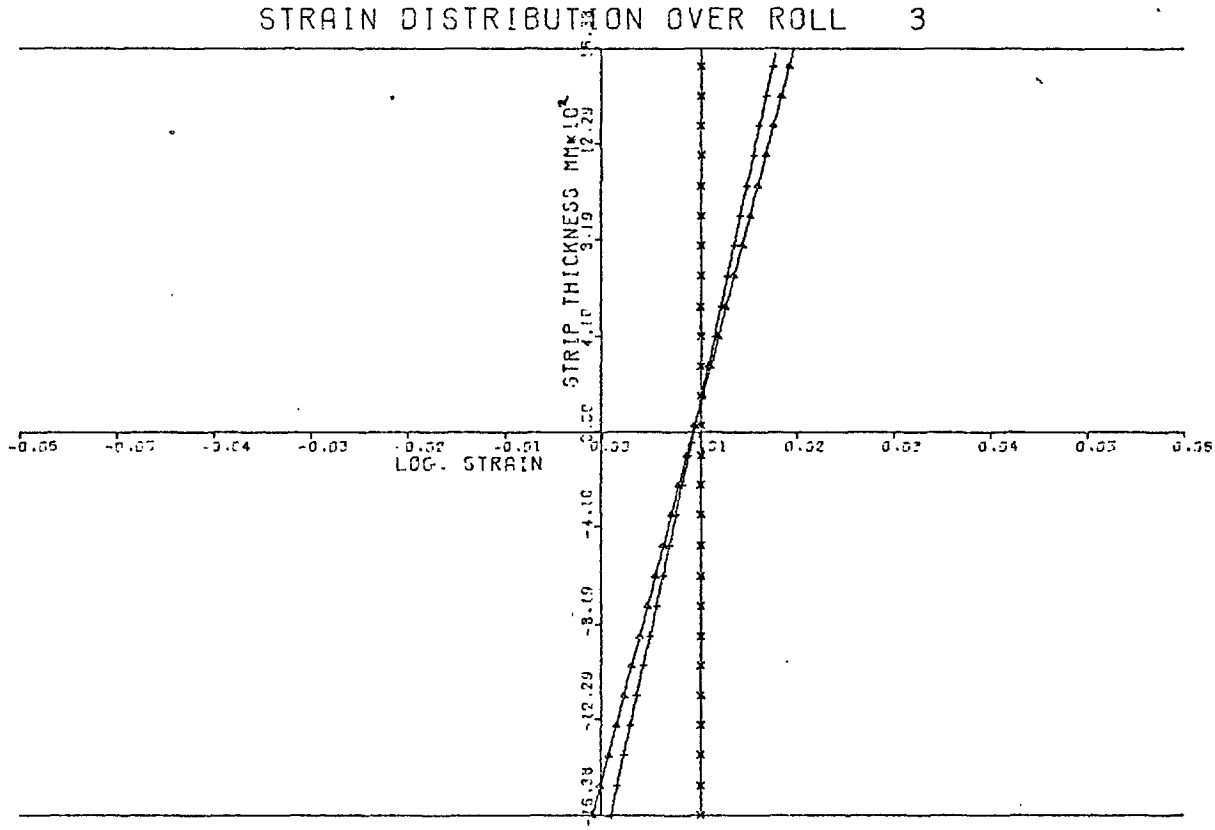

STRESS DISTRIBUTION OVER ROLL 2
STRIP THICKNESS MM*10**2
STRESS MN/M**2
*10**1

fig (A7. 2c)


STRAIN DISTRIBUTION OVER ROLL 3
STRIP THICKNESS MM*10**2
LOG. STRAIN

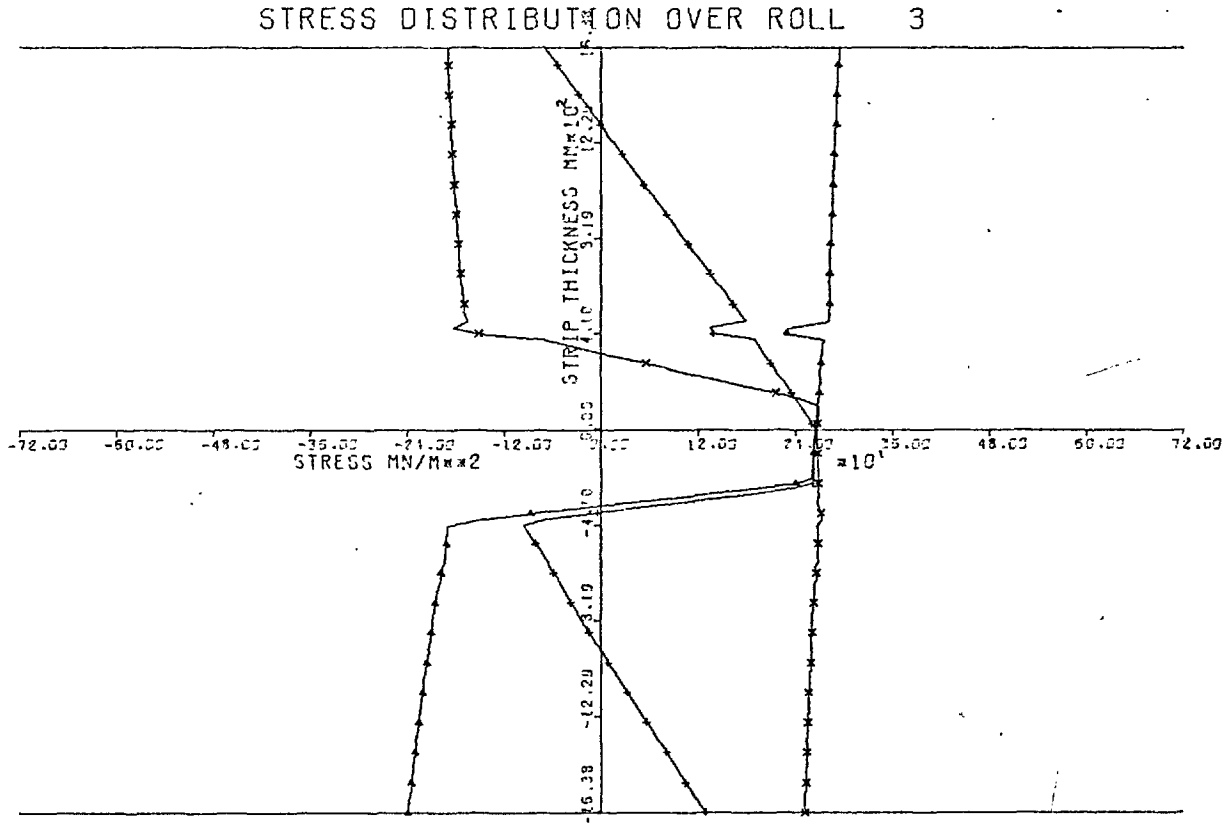
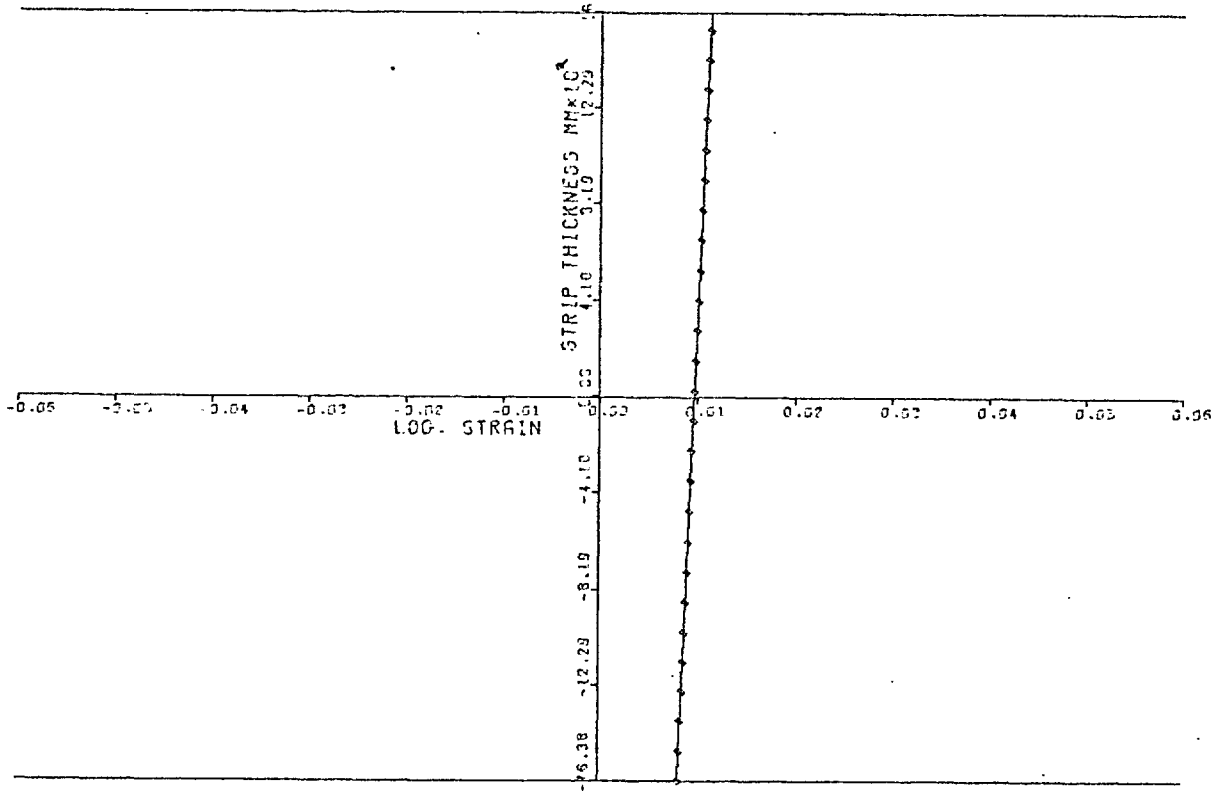

STRESS DISTRIBUTION OVER ROLL 3
STRIP THICKNESS MM*10**2
STRESS MN/M**2
*10**1

fig (A7.2d)

STRAIN DISTRIBUTION AFTER TENSION RELAXATION



STRESS DISTRIBUTION AFTER TENSION RELAXATION

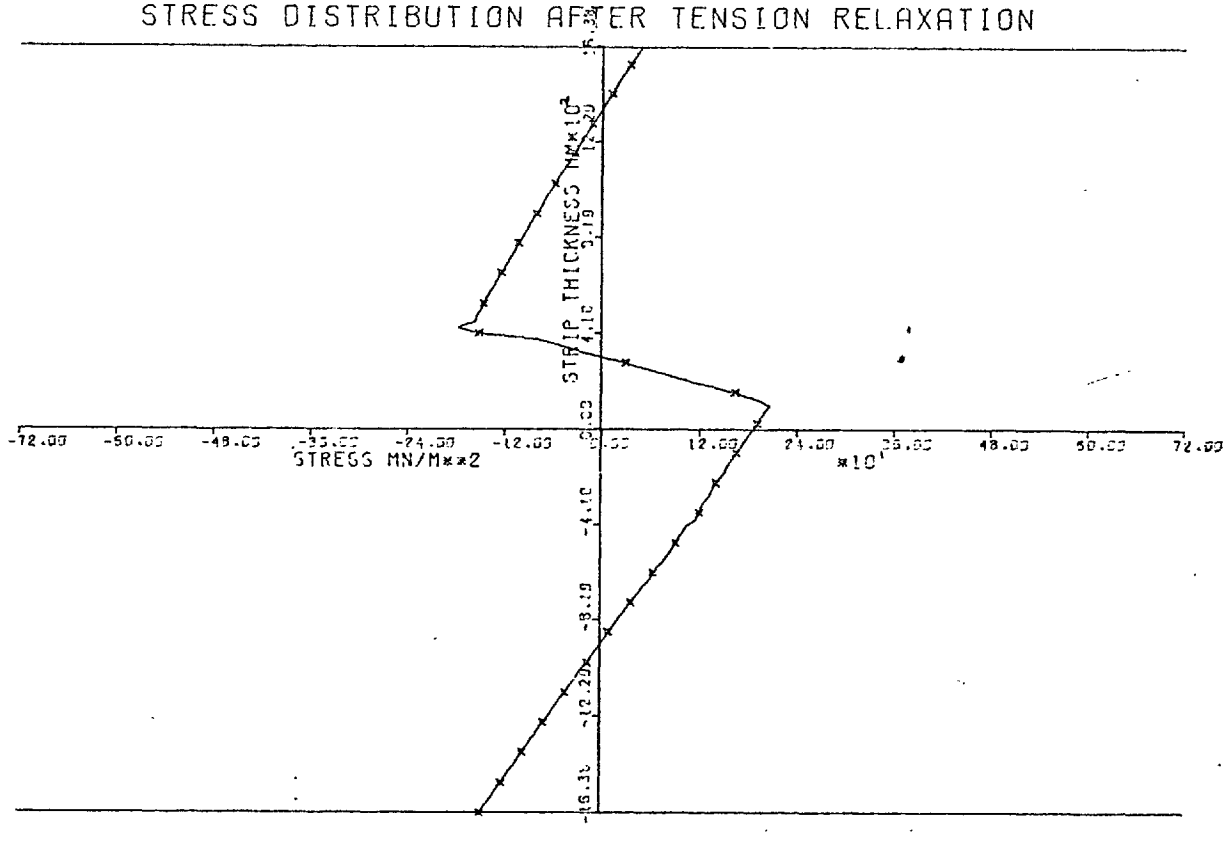


fig (A7.2e)

APPENDIX 8.

A8.0. Tabulated Data.

The theoretical and experimental results plotted in Chapter 4, are tabulated in this section, for numerical comparison. Theoretical strain readings are those predicted at the upper strip surface, the total leveller strain being the mean for the strip thickness. Practical results are those actually recorded by the strain gauges.

A8.1. Title Nomenclature.

Model.	-	MEAN	Uniform load assumed.
		FULL	'Work done' effects included.
		ISO	Isotropic hardening model.
		KIN	Simple kinematic hardening model.
		MRÓZ.	Combined hardening model.

E	Youngs Modulus ($MN/m^2 \times 10^{-4}$).
V	Poisson's Ratio.
t	Strip thickness.(mm).
L_m	Mean Applied Load (kN/m width).
L_L	Load Loss (kN/m width).
R_1	Curvature radius at leveller entry (mm).
R_2	Curvature radius at leveller exit (mm).
e_{tot}	Predicted leveller strain.
e_{TV}	Measured total strain (vernier method).
e_{TG}	Measured total strain (strain gauge).
e_{B1}	Surface strain during 1st bend (%).
e_{S1}	Surface strain after first straightening (%).
de_{B2}	Incremental strain - second bend (%).
e_{B2}	Surface strain - second bend (%).
e_{S2}	Straightening strain - second bend (%).
$de_{B3}, e_{B3}, e_{S3},$	- - third bend.(%).
e_{B4}, e_{S4}	- - fourth bend (%).
V	strip velocity (mm/sec.).

TABLE 1. Material Composition.

B.S.1449.	CR2/4 range.	
C - 0.306	Si - 0.01	Mn - 0.404/0.401
S - 0.025/0.071	P - 0.017/0.022	N - 0.00/ 0.002
Ni - 0.00	Cr - 0.01/0.014	Mo - 0.006
Trace - 0.01		

TABLE 2. Series 1. Theory.

Model	E	ν	t	L_m	L_L	R_1	R_2	e_{tot}	e_{B1}	e_{S1}	de_{B2}	e_{B2}	e_{S2}	de_{B3}	e_{B3}	e_{S3}
MEAN	18.2	0.30	0.56	5.2	0	-	92.0	0.11	2.23	0.07	-2.13	-2.06	0.10	2.18	2.28	0.12
ISOTROPIC				20.8	0	-	90.0	1.10	2.42	0.41	-1.94	-1.53	0.77	2.38	3.16	1.12
				36.4	0	-	90.5	2.09	2.60	0.79	-1.79	-1.01	1.47	2.55	4.02	2.13
				52.0	0	-	91.0	3.05	2.80	1.13	-1.63	-0.49	2.15	2.70	4.85	3.10
				67.6	0	-	92.5	3.98	2.99	1.51	-1.49	0.02	2.81	2.84	5.65	4.04
FULL	18.2	0.30	0.56	14.5	11.4	-	90.5	0.77	2.29	0.17	-1.98	-1.81	0.44	2.39	2.84	0.79
ISOTROPIC				34.8	11.8	-	90.3	2.09	2.54	0.66	-1.78	-1.12	1.37	2.60	3.97	2.13
				61.2	12.7	-	92.7	3.87	2.89	1.35	-1.50	-0.15	2.63	2.89	5.52	3.93
FULL	18.2	0.29	0.56	15.4	13.3	-	89.4	0.86	2.29	0.18	-1.96	-1.78	0.48	2.42	2.91	0.88
ISOTROPIC				37.2	16.5	-	89.9	2.32	2.55	0.69	-1.73	-1.03	1.49	2.67	4.16	2.36
μ'=0.09				68.0	20.5	-	94.5	4.25	2.91	1.39	-1.40	-0.01	2.84	2.99	5.83	4.32
FULL	18.2	0.29	0.56	14.2	10.9	-	143.5	0.48	2.28	0.13	-202	-1.90	0.29	2.33	2.62	0.49
KIN				36.3	14.4	-	141.3	1.55	2.54	0.57	-1.82	-1.25	1.11	2.49	3.60	1.59
μ'=0.09				68.2	20.6	-	141.5	3.19	2.91	1.23	-1.53	-0.30	2.31	2.74	5.05	3.26

TABLE 3.Series 1. Practical.

Run No.	t	v	R_1	R_2	L_m	L_L	e_{Tv}	e_{TG}	e_{B1}	e_{S1}	de_{B2}	e_{B2}	e_{S2}	de_{B3}	e_{B3}	e_{S3}
1a 1.	0.546	125	522	180	36.2	8.3	1.74	-	1.72	0.43	-1.44	-1.01	-	-	-	-
2.	0.552	125	508	221	30.6	10.7	1.30	-	1.46	0.33	-1.57	-1.24	-	-	-	-
3.	0.574	-	-	-	34.4	12.7	-	1.50	1.62	0.40	-1.57	-1.17	1.32	2.18	3.50	1.52
								1.84	1.60	0.40	-0.99	-0.59	1.86	2.32	4.18	1.88
4.	0.552	15	470	134	40.2	19.1	2.24	2.15	1.96	0.64	-1.29	-0.65	1.71	2.45	4.16	2.16
									1.73	0.56	-0.77	-0.21	1.59	-	-	-
5.	0.559	45	452	133	8.7	4.7	0.14	0.03	0.46	0.03	-1.53	-1.50	0.26	0.91	1.17	0.11
								0.02	0.56	0.03	-1.63	-1.60	0.24	0.59	0.83	0.05
5a	0.559	475	-	165	47.0	13.6	2.06	-	-	-	-	-	-	-	-	-
6.	0.569	22	457	136	19.6	10.8	0.60	-	1.02	0.16	-1.17	-1.01	-	-	-	-
7.	0.548	26	457	150	20.9	14.3	0.81	-	0.92	-	-	-	-	-	-	-
8.	0.554	130	442	-	16.5	7.5	0.28	-	-	-	-	-	-	-	-	-
8a	0.554	130	-	155	16.5	7.5	0.35	0.28	0.77	0.05	-1.72	-1.67	0.34	1.33	1.67	0.32
								0.36	0.81	0.13	-1.93	-1.80	0.55	1.29	1.84	0.49
								0.40	0.97	0.09	-2.04	-1.95	0.61	1.18	1.79	-.49
9.	0.554	111	432	154	37.3	12.7	1.64	1.82	1.84	0.46	-1.92	-1.26	1.49	2.42	3.91	1.90
								1.24	1.61	0.34	-1.88	-1.54	1.29	1.67	2.96	1.39
									1.45	0.25	-	-	-	-	-	-
10.	0.564	73	432	150	48.5	15.2	2.37	-	2.49	0.94	01.33	-0.53	1.69	-	-	-
								-	2.37	0.81	-1.47	-0.52	-	-	-	-
								-	2.34	0.81	-	-	-	-	-	-

TABLE 3 Contd.

Run No.	t	v	R_1	R_2	L_m	L_L	e_{Tv}	e_{TG}	e_{B1}	e_{S1}	de_{B2}	e_{B2}	e_{S2}	de_{B3}	e_{B3}	e_{S3}
11.	0.544	60	390	178	23.4	9.5	0.64	0.49	1.00	0.20	-	-	-	-	-	-
								0.47	1.11	0.13	-1.70	-1.57	0.71	0.92	1.63	0.61
								0.86	1.12	0.12	-1.78	-1.66	1.02	1.34	2.36	0.99
12.	0.554	31	387	153	66.9	18.6	4.20	-	-	-	-	-	-	-	-	-
13.	0.574	52	358	-	32.8	13.5	1.20	1.10	1.49	0.29	-1.90	-1.61	0.95	-	-	1.23
								1.13	1.36	0.26	-1.95	-1.69	1.12	-	-	1.28
								-	1.20	0.30	-1.85	-1.55	-	-	-	-
14.	0.576	52	360	148	50.2	17.6	2.51	-	2.20	0.72	-1.92	-1.20	1.74	-	-	-
								-	2.27	0.77	-	-	-	-	-	-
								-	-	1.02	-1.64	-0.62	-	-	-	-
15.	0.584	66	-	-	58.6	14.2	3.26	-	-	-	-	-	-	-	-	-
16.	0.572	61	-	-	61.6	11.4	3.36	-	-	-	-	-	-	-	-	-

TABLE 4. Series 2. Theory.

Model	E	ν	t	L_m	L_L	R_1	R_2	e_{tot}	e_{B1}	e_{S1}	de_{B2}	e_{B2}	e_{S2}	de_{B3}	e_{B3}	e_{S3}
MEAN	18.6	0.3	0.328	5.2	0	-	71.4	0.11	1.09	0.07	-1.00	-0.93	0.10	1.05	1.15	0.12
ISO				26.0	0	-	73.0	1.35	1.34	0.50	-0.75	-0.25	0.96	1.30	2.26	1.39
				52.0	0	-	86.1	2.80	1.65	1.09	-0.54	0.55	2.00	1.53	3.53	2.88
				78.0	0	-	124.6	4.22	1.97	1.65	-0.27	1.38	3.05	1.75	4.80	4.34
MEAN	18.6	0.29	0.328	5.2	0	-	71.4	0.10	1.09	0.07	-1.00	-0.93	0.09	1.05	1.14	0.11
ISO				26.0	0	-	73.0	1.34	1.34	0.51	-0.77	-0.25	0.96	1.30	2.25	1.38
				52.0	0	-	85.6	2.83	1.65	1.09	-0.51	0.58	2.03	1.53	3.56	2.91
				78.0	0	-	124.0	4.22	1.96	1.66	-0.28	1.37	3.05	1.76	4.80	4.35
MEAN	18.6	0.29	0.328	5.2	0	-	104.3	0.06	1.08	0.05	-1.02	-0.97	0.06	1.03	1.10	0.07
KIN				26.0	0	-	102.4	0.98	1.33	0.46	-0.82	-0.37	0.77	1.21	1.98	1.02
				52.0	0	-	109.1	2.30	1.65	1.04	-0.63	0.41	1.77	1.40	3.17	2.38
				78.0	0	-	158.5	4.05	1.97	1.65	-0.32	1.33	2.98	1.69	4.67	4.18
MEAN	18.6	0.29	0.328	5.2	0	-	103.1	0.08	1.09	0.06	-1.01	-0.95	0.08	1.04	1.12	0.09
MRÓZ				26.0	0	-	100.2	1.05	1.34	0.47	-0.82	-0.36	0.78	1.27	2.06	1.09
				52.0	0	-	107.0	2.37	1.65	1.04	-0.60	0.45	1.81	1.41	3.22	2.45
				78.0	0	-	138.8	3.95	1.97	1.65	-0.32	1.33	2.95	1.67	4.62	4.08
MEAN	16.6	0.29	0.328	26.0	0	-	65.6	1.30	1.34	0.50	-0.77	-0.26	0.94	1.30	2.23	1.35
ISO				78.0	0	-	114.2	4.16	1.97	1.64	-0.28	1.36	3.02	1.76	4.77	4.30

TABLE 4 Contd.

Model	E	V	t	L_m	L_L	R_1	R_2	e_{tot}	e_{B1}	e_{S1}	de_{B2}	e_{B2}	e_{S2}	de_{B3}	e_{B3}	e_{S3}
MEAN	20.7	0.29	0.328	26.0	0	-	80.4	1.39	1.34	0.53	-0.76	-0.24	1.00	1.30	2.28	1.42
ISO				78.0	0	-	135.7	4.27	1.97	1.67	-0.28	1.39	3.08	1.75	4.83	4.38
2(a)FULL	18.6	0.29	0.328	20.5	5.9	-	72.2	1.10	1.25	0.35	-0.79	-0.43	0.74	1.29	2.03	1.14
ISO				64.5	13.2	-	111.3	3.68	1.76	1.29	-0.31	0.97	2.56	1.73	4.29	3.79
μ'=0.07.																
2(b)FULL	18.6	0.29	0.328	20.0	5.0	-	71.9	1.06	1.25	0.35	-0.80	-0.45	0.72	1.28	1.99	1.09
ISO				64.0	10.3	-	105.4	3.54	1.75	1.28	-0.37	0.91	2.48	1.69	4.17	3.64.
μ'=0.07.																
2(c)FULL	18.6	0.29	0.328	19.2	3.3	-	71.4	0.98	1.25	0.34	-0.81	-0.47	0.68	1.25	1.93	1.02
ISO				60.5	5.2	-	96.5	3.35	1.74	1.25	-0.41	0.84	2.37	1.64	4.01	3.44
μ'=0.07.																

TABLE 5. Series 2. Practical.

Run.No.	t	v	R_1	R_2	L_m	L_L	e_{Tv}	e_{TG}	e_{B1}	e_{S1}	de_{B2}	e_{B2}	e_{S2}	de_{B3}	e_{B3}	e_{S3}
2a 1.	0.315	54	477	124	3.72	2.2	0.03	0.10	1.09	-	-	-	0.10	1.05	1.15	0.11
								-	0.76	-	-	-	0.12	1.01	1.13	-
								0.10	0.53	-	-	-	0.27	0.95	1.22	0.11
2	0.322	56	445	120	18.1	6.2	0.81	0.81	1.20	0.12	-0.45	-0.33	0.74	1.37	2.11	0.86
								0.99	1.43	0.32	-0.53	-0.21	0.86	1.03	1.89	1.04
3	0.328	56	457	146	52.6	16.1	3.32	3.15	1.54	0.74	-0.25	0.49	2.46	1.42	3.88	3.30
								-	1.78	1.09	-0.42	0.67	2.32	1.49	3.81	-
4	0.318	51	477	167	6214	18.4	3.74	3.18	1.62	0.93	-0.11	0.82	2.59	1.56	4.15	3.28
5	0.322	96	463	-	66.3	9.0	6.25	-	3.30	2.41	-0.06	2.35	4.18	1.64	5.82	4.98
								-	3.59	2.88	-0.26	2.62	4.12	1.80	5.92	5.02
6	0.312	46	470	140	30.3	7.4	1.73	1.84	1.50	0.59	-	-	1.41	1.57	2.98	1.88
								-	1.51	0.52	-	-	1.37	-	-	-
7	0.318	47	470	133	41.2	9.8	2.48	2.18	1.67	0.85	-	-	1.97	-	-	2.24
								1.93	1.66	0.83	-	-	1.86	1.11	2.97	2.02
8	0.346	59	467	130	7.1	4.0	0.26	-	-	-	-	-	-	-	-	-
9	0.346	56	462	129	12.1	4.1	0.56	-	-	-	-	-	-	-	-	-

TABLE 5 Contd.

Run.No.	t	v	R_1	R_2	L_m	L_L	e_{Tv}	e_{TG}	e_{B1}	e_{S1}	de_{B2}	e_{B2}	e_{S2}	de_{B3}	e_{B3}	e_{S3}
2b.1.	0.330	47	430	123	4.2	1.3	0.08	0.18	0.53	-0.01	-0.80	-0.81	0.22	0.79	1.01	0.19
								0.26	0.48	-0.02	-0.67	-0.69	0.31	0.88	1.19	0.27
2	0.330	54	430	121	18.0	6.9	0.72	0.87	1.42	0.23	-0.50	-0.27	0.96	1.07	2.03	0.89
								0.84	1.42	0.20	-0.34	-0.14	0.86	1.15	2.01	0.86
3	0.335	43	432	139	48.2	7.9	2.61	2.04	1.65	0.72	-0.06	0.66	1.86	1.44	3.30	2.12
								2.30	1.89	1.27	-0.35	0.92	2.00	1.66	3.66	2.41
4	0.353	56	430	159	69.3	10.5	3.82	-	1.78	0.95	-0.12	0.83	-	-	-	-
								-	2.10	1.40	-0.38	1.02	2.45	0.96	3.14	-
5	0.327	58	430	130	34.8	15.9	1.69	1.52	1.53	0.55	-	-	1.34	1.23	2.57	1.58
								1.37	1.46	0.46	-0.62	-0.16	1.18	1.20	2.38	1.43
6	0.327	53	424	135	40.3	8.1	2.39	2.06	1.51	0.61	-	-	1.64	1.46	3.10	2.15
								2.10	1.64	0.85	-0.53	0.32	1.62	1.50	3.12	2.18
7	0.318	59	432	131	40.4	8.2	2.33	2.04	1.55	0.76	-0.53	0.23	1.78	1.27	3.05	2.14
								2.09	1.66	0.90	-	-	1.65	1.38	3.03	2.18
8	0.318	56	426	130	32.9	6.7	1.71	1.49	1.45	0.43	-	-	1.26	1.34	2.60	1.58
								1.55	1.51	0.57	0.03	0.60	1.27	1.30	2.57	1.64
9	0.320	53	424	144	42.0	6.6	2.25	-	-	-	-	-	-	-	-	-
10	0.330	54	419	150	57.9	9.3	2.89	-	-	-	-	-	-	-	-	-
11	0.335	54	425	233	65.1	8.1	4.37	-	-	-	-	-	-	-	-	-
12	0.361	71	414	130	11.3	3.7	0.32	-	-	-	-	-	-	-	-	-

TABLE 5.Contd.

Run.No.	t	v	R_1	R_2	L_m	L_L	e_{Tv}	e_{TG}	e_{B1}	e_{S1}	de_{B2}	e_{B2}	e_{S2}	de_{B3}	e_{B3}	e_{S3}
2c.1.	0.353	-	465	133	31.4	3.3	1.01	0.95	0.83	0.22	-0.29	-0.07	0.95	0.79	1.74	0.99
								0.90	0.76	0.16	-0.54	-0.38	0.84	0.94	1.78	0.97
2	0.320	-	-	167	60.6	7.5	3.35	2.76	1.72	1.12	-0.39	0.73	2.08	1.60	3.68	2.89
								-	1.85	1.20	-0.06	1.14	2.34	-	-	-
3	0.320	53	460	165	13.1	0.7	0.18	0.49	0.42	0.05	-0.44	-0.39	0.47	0.50	0.97	0.53
								0.52	0.39	0.00	-0.47	-0.47	0.49	0.70	1.19	0.55
4	0.322	55	462	150	48.0	4.3	2.43	2.33	1.58	0.86	-0.11	0.75	1.89	1.43	3.32	2.40
								2.03	1.61	0.92	-0.18	0.74	1.70	1.05	2.75	2.10
5	0.320	47	457	135	45.8	4.5	2.26	1.84	1.50	0.78	-0.16	0.62	1.67	1.06	2.73	1.92
								1.90	1.56	0.89	-0.14	0.75	1.66	1.05	2.71	1.93
6	0.318	50	452	140	40.7	3.9	2.03	1.78	1.42	0.70	-0.22	0.48	1.63	1.17	2.80	1.89
								-	1.44	0.74	-0.15	0.59	1.55	-	-	-
7	0.312	55	452	130	24.7	2.6	0.85	0.72	0.75	0.16	-0.67	-0.51	0.71	0.71	1.42	0.77
								0.78	0.70	0.15	-0.74	-0.59	0.64	0.91	1.55	0.83
8	0.315	43	447	133	40.3	4.6	1.98	1.68	1.22	0.58	-	-	1.37	1.04	2.41	1.75
								1.76	1.24	0.63	-	-	1.26	1.19	2.45	1.83
9	0.320	66	437	136	32.6	5.0	1.35	-	-	-	-	-	-	-	-	-
10	0.346	50	442	184	12.2	1.8	0.17	-	-	-	-	-	-	-	-	-

TABLE 6. Series 3. Theory.

Model	E	V	t	L_m	L_L	R_1	R_2	e_T	e_{B1}	e_{S1}	e_{B2}	e_{S2}	e_{B3}	e_{S3}	e_{B4}	e_{S4}
3(a)	18.3	0.30	0.905	5.2	0	-	168	0.09	1.79	0.05	-1.68	0.07	1.83	0.09	-1.65	0.10
MEAN ISO				91.0	0	-	160	4.35	2.48	1.36	0.157	2.45	4.74	3.44	2.21	4.40
3(b)	18.3	0.30	0.905	5.2	0	-	142	0.20	2.58	0.07	-2.42	0.12	2.68	0.16	-2.33	0.20
MEAN ISO				78.0	0	-	133	5.18	3.36	1.59	-0.30	2.90	6.06	4.10	2.16	5.22
3(c)	18.3	0.30	0.905	5.2	0	-	111	4.68	4.62	0.14	-4.33	0.25	4.88	0.38	-4.09	4.71
MEAN ISO				65.0	0	-	100	7.15	5.61	2.15	-1.56	3.97	9.36	5.63	1.85	7.19
3(a)FULL ISO μ'=0.05	18.3	0.30	0.905	18.0	18.5	-	163	0.88	1.83	0.12	-1.51	0.31	2.22	0.57	-0.98	0.90
				39.6	21.2	-	160	2.03	2.00	0.46	-1.04	0.94	3.00	1.48	0.05	2.06
3(a)FULL ISO μ'=0.07	18.3	0.29	0.905	28.2	21.6	-	161	1.47	1.91	0.27	-1.29	0.61	2.60	1.02	-0.46	1.49
				72.5	29.2	-	161	3.67	2.24	0.92	-0.37	1.81	4.09	2.76	1.54	3.72
3(b)FULL ISO μ'=0.06	18.3	0.29	0.905	34.0	33.0	-	137	2.50	2.74	0.43	-1.83	0.99	3.90	1.70	-0.38	2.54
				77.8	39.9	-	136	5.40	3.18	1.30	-0.61	2.58	5.87	4.00	2.23	5.46

TABLE 7.Series 3. Practical.

Run.No.	t	v	R_1	R_2	L_m	L_L	e_{Tv}
2a.1.	0.903	54	351	276	26.5	19.1	1.19
2	0.917	47	351	286	90.2	34.4	4.45
3	0.903	48	348	276	51.6	23.0	2.41
4	0.900	44	343	276	66.9	27.5	3.20
5	0.917	60	318	326	9.5	11.5	0.20
2b.1.	0.889	48	307	226	25.4	21.1	1.44
2	0.897	48	297	225	53.8	31.0	3.46
3	0.895	53	280	223	75.7	33.9	4.95
2c.1.	0.910	60	269	241	18.2	20.4	0.92
2	0.910	43	259	300	68.5	49.2	6.54
3	0.908	48	226	249	41.6	38.1	3.60

APPENDIX 9.

A9.0. Stress/Strain Relationships for Strip-Shape Correction Processes.

This paper gives a concise picture of the theoretical analysis of section 3.2.0. and demonstrates its usefulness by reference to the problem of spring-back radii after bending. No reference to stress/strain behavior under reverse loading conditions is given.

Stress/Strain Relationships for Strip-Shape Correction Processes

T. Sheppard and J. M. Roberts

The processing of thin strip takes place under conditions that impose restraints on the material such that plane strain obtains. In most shape-correction processes the application of high longitudinal stresses, coupled with low normal stresses, further influences the stress/strain relationships. Moreover, the elastic and plastic strains in these processes are of the same magnitude; hence, to omit the elastic part may lead to large errors. Calculations reported to date ignore the influence of constraints or of work-hardening, or both. The paper describes a method of obtaining accurate stress/strain relationships for these processes and demonstrates that for the simple case of plastic bending the radii after springback may be calculated with much greater accuracy.

List of Symbols

x, y, z	= rectangular Cartesian coordinates
E	= Young's modulus
ν	= Poisson's ratio
G	= modulus of rigidity
H'	= current gradient of the equivalent stress/plastic strain curve
M	= internal bending moment
R	= bend radius
R'	= springback radius
t	= thickness of strip
w	= width of strip
$\bar{\varepsilon}$	= equivalent strain
$\bar{\varepsilon}^P$	= equivalent plastic strain
ε_s	= springback strain
$\varepsilon_x, \varepsilon_y, \varepsilon_z$	= normal true strains
$\varepsilon_x', \varepsilon_y', \varepsilon_z'$	= deviatoric strains
$d\varepsilon_x, d\varepsilon_y, d\varepsilon_z$	= incremental values of strains
$d\varepsilon_x', d\varepsilon_y', d\varepsilon_z'$	= incremental values of deviatoric strains
$d\bar{\varepsilon}^P$	= incremental equivalent plastic strain
α	= angle of inclination of stress vector to the x axis
$d\alpha$	= incremental deviation of angle
$\bar{\sigma}$	= current equivalent yield stress
$\sigma_x, \sigma_y, \sigma_z$	= normal stresses
$\sigma_x', \sigma_y', \sigma_z'$	= deviatoric stresses
$d\bar{\sigma}$	= incremental equivalent yield stress
$d\sigma'$	= incremental total deviatoric stress
$d\sigma_x, d\sigma_y, d\sigma_z$	= incremental values of stresses
$d\sigma_x', d\sigma_y', d\sigma_z'$	= incremental values of deviatoric stresses
A, B, m	= constants in the stress/strain relationships.

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Developments in the sheet and strip consumer industries have resulted in continuous demands for more precise thinner gauges with better shape and surface quality. It is almost impossible to remove at the cold mill the bad shape originating at the hot mill; indeed, the resulting mismatch of roll/strip profile will, in the majority of cases, aggravate the problem. Consequently, some form of shape-correction process is usually necessary after completion of rolling. For plate and "thick" strip this may take the form of roller levelling or stretcher levelling, and for thin strip it would be necessary to resort to tension levelling.

In all these processes the strip is so constrained that no spread occurs across the width and hence plane-strain conditions obtain. However, this constraint, together with the yielding mechanism, complicates the fibre stress/strain relationship in the longitudinal direction even for those materials that can be considered elastic perfectly plastic. Bad shape correction relies upon an equalization of longitudinal stresses for success and hence the first step when considering shape-correction mechanics must be to ascertain the longitudinal stress/strain relationship.

There are two methods by which this problem may be attacked: (*a*) by the use of the stiffness coefficient approach, as proposed by Marcal;[1] (*b*) by considering the expansion of the yield surface on the deviatoric plane. Method (*a*) suffers the drawback that a physical interpretation of results is difficult and, perhaps more important, the method demands an advanced knowledge of matrix algebra which many people do not possess. Method (*b*) is capable of a clear physical interpretation and requires no more computation than the matrix method.

The solution for a non-work-hardening material has been given by Alexander,[2] who applied the results to the simple bending problem. Denton[3] has investigated the work-hardening case but his equations are extremely lengthy and complex.

General Theoretical Considerations and Assumptions

The method is that described in detail by Hill[4] and has as its basis the fact that any three-dimensional stress system can be represented vectorially and can also be represented by hydro-

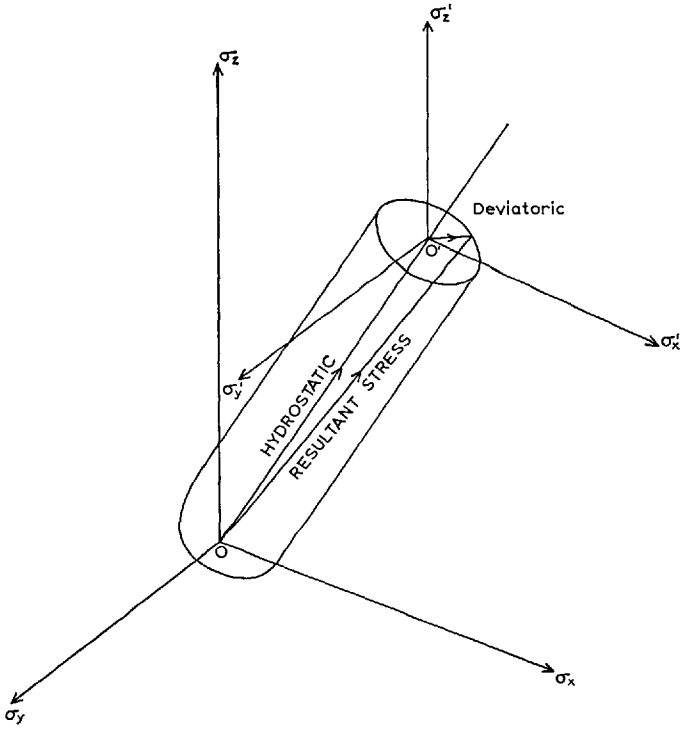


Fig. 1 *Three-dimensional stress system.*

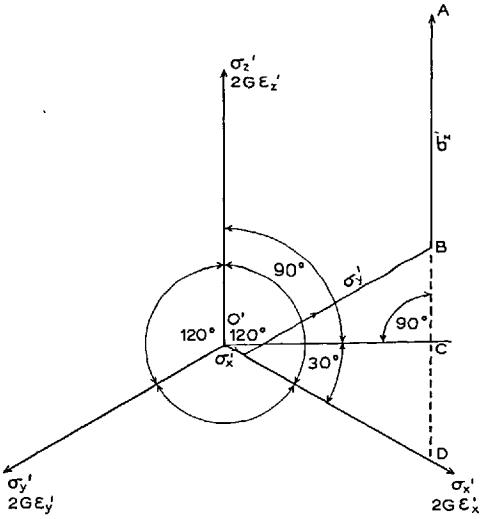


Fig. 2 *Hill's construction.*

static and deviatoric components, as shown in Fig. 1. Bridgman and many other workers have demonstrated that the yield envelope, and hence plastic flow, is independent of hydrostatic pressure; hence, it is sufficient to consider only the deviatoric component, as it is this component which causes plastic flow. The new co-ordinate system representing reduced or deviatoric stresses has its origin at point O′ and the three axes will project into the deviatoric plane at equal angles of 120°, as shown in Fig. 2. A vector component σ_x' will project into the plane as $\sqrt{2/3}\,\sigma_x'$ and since it is more convenient to plot σ_x' the scale of the diagram is multiplied by $\sqrt{3/2}$. This also has the fortunate result of allowing the von Mises yield criterion to be represented by a circle of radius $\bar{\sigma}$, the yield stress in simple tension. Hill has shown that since $\sigma_x' + \sigma_y' + \sigma_z' = 0$, then referring to Fig. 2 AB : BC = 2 : 1. In the event of one of the stresses, say σ_y, being zero, then OD = σ_x and AD = σ_z, which gives a useful visualization of actual stress values. We are now in a position to consider the assumptions necessary to apply the theory to shape-correction problems. The assumptions made will be:

(1) The material is homogeneous and isotropic giving a uniform expansion of the yield surface.

(2) Plane-strain conditions exist. Hence

$$\varepsilon_z = d\varepsilon_z = 0$$

(3) Since the *t* : *w* ratio is small

$$\sigma_y = d\sigma_y = 0$$

Development of the Stress/Strain Relationship

Elastic Relationships

The case of elastic straining is well known and can be summarized as follows

As
$$\varepsilon_z = \sigma_y = 0$$
$$\sigma_z = \nu\,\sigma_x \qquad \ldots(1)$$

$$\varepsilon_x = \frac{\sigma_x}{E}[1 - \nu^2] \qquad \ldots(2)$$

$$\varepsilon_y = -\frac{\nu}{1-\nu}\,\varepsilon_x \qquad \ldots(3)$$

Plastic Relationships

The material is assumed to obey the Prandtl–Reuss equations

$$d\varepsilon_x = \frac{d\bar{\sigma}}{\bar{\sigma}H'}[\sigma_x - \tfrac{1}{2}(\sigma_y + \sigma_z)] + \frac{1}{E}[d\sigma_x - \nu(d\sigma_y + d\sigma_z)]$$

$$d\varepsilon_y = \frac{d\bar{\sigma}}{\bar{\sigma}H'}[\sigma_y - \tfrac{1}{2}(\sigma_x + \sigma_z)] + \frac{1}{E}[d\sigma_y - \nu(d\sigma_x + d\sigma_z)]$$

$$d\varepsilon_z = \frac{d\bar{\sigma}}{\bar{\sigma}H'}[\sigma_z - \tfrac{1}{2}(\sigma_x + \sigma_y)] + \frac{1}{E}[d\sigma_z - \nu(d\sigma_x + d\sigma_y)]$$

$$\ldots(4)$$

For convenience, these equations may be written in deviatoric form

$$d\varepsilon_x' = \frac{3\sigma_x'\,d\bar{\sigma}}{2\bar{\sigma}\,H'} + \frac{d\sigma_x'}{2G} \qquad \ldots(5)$$

The geometric representation on the deviatoric plane is given in Fig. 3. OP and OQ represent the current deviatoric stress and strain in the planes of deviatoric stress and strain. OP represents the elastic strain recoverable by complete

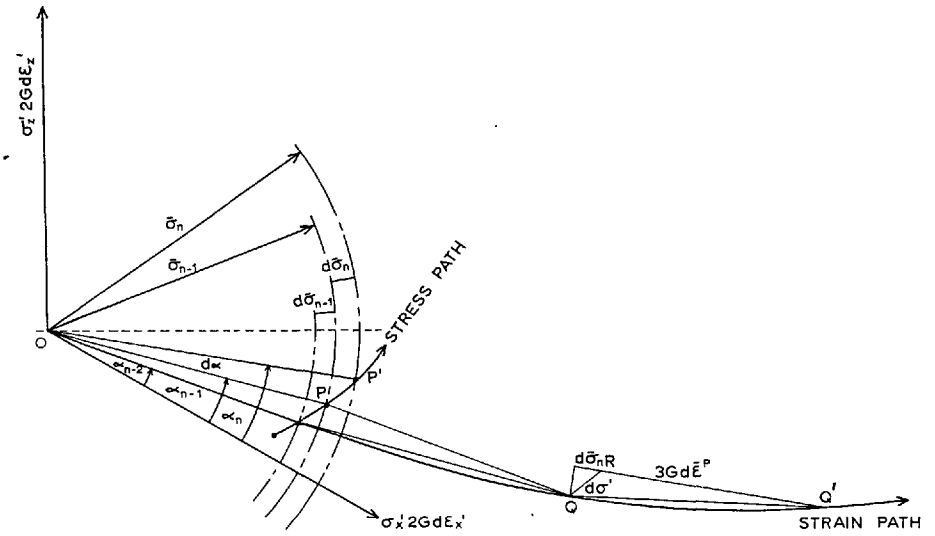


Fig. 3 Deviatoric plane stress/strain path.

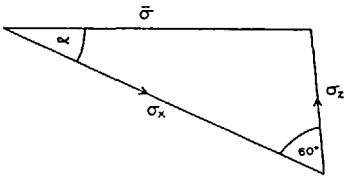


Fig. 4 Deviatoric plane (detail).

unloading. PQ therefore represents the total plastic strain. If PP′ is an increment of stress during subsequent plastic flow, then QQ′ is the corresponding increment of strain. QR is the elastic component of this strain and is equal to the stress increment PP′ and parallel to it. To achieve this equality it is necessary to multiply the strain axis by $2G$. The plastic component RQ′ must be parallel to the instantaneous-stress vector OP′.

The problem is tackled by considering the expansion $d\bar{\sigma}$ of the yield envelope by an incremental change in inclination $d\alpha$ of the instantaneous stress vector OP. Fig. 3 gives the nomenclature used, and Fig. 4 the relationship between the applied stresses σ_x and σ_z and the current yield stress $\bar{\sigma}$.

Fig. 5 shows the strain increments for a stress-vector movement around the deviatoric plane of $d\alpha$. ED is equal and parallel to the incremental increase in yield surface, AD is the elastic strain movement, and DB the plastic strain increment parallel to the stress vector $\bar{\sigma}_{n-1}$. As $\varepsilon_z = 0$ for plane strain, AC and CB represent the incremental changes in the actual strains $2Gd\varepsilon_x$ and $2Gd\varepsilon_y$.

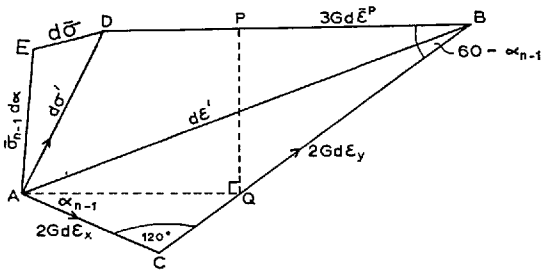


Fig. 5 Deviatoric plane (detail).

Hence, from Figs. 4 and 5

$$\sigma_x = \bar{\sigma}\,(\cos\alpha + \sin\alpha/\sqrt{3}) \qquad \ldots(6)$$

$$\sigma_z = \frac{2}{\sqrt{3}}\,\bar{\sigma}\sin\alpha \qquad \ldots(7)$$

Hence

$$\sigma_{x_n} = (\bar{\sigma}_{n-1} + d\bar{\sigma}_n)\left(\cos\alpha_n + \frac{\sin\alpha_n}{\sqrt{3}}\right)$$

and

$$\sigma_{x_{n-1}} = (\bar{\sigma}_{n-1})\left(\cos\alpha_{n-1} + \frac{\sin\alpha_{n-1}}{\sqrt{3}}\right)$$

but

$$d\sigma_{x_n} = \sigma_{x_n} - \sigma_{x_{n-1}}$$

so that

$$d\sigma_{x_n} = \bar{\sigma}_{n-1}\left(\cos\alpha_n + \frac{\sin\alpha_n}{\sqrt{3}} - \cos\alpha_{n-1} - \frac{\sin\alpha_{n-1}}{\sqrt{3}}\right) + d\bar{\sigma}_n\left(\cos\alpha_n + \frac{\sin\alpha_n}{\sqrt{3}}\right)$$

but

$$\alpha_n - \alpha_{n-1} = d\alpha$$

thus

$$d\sigma_{x_n} = \bar{\sigma}_{n-1}\,d\alpha\left\{\cos\left(\frac{\alpha_{n-1} + \frac{d\alpha}{2}}{\sqrt{3}}\right) - \sin\left(\alpha_{n-1} + \frac{d\alpha}{2}\right)\right\} + d\bar{\sigma}_n\left\{\cos\alpha_n + \frac{\sin\alpha_n}{\sqrt{3}}\right\} \qquad \ldots(8)$$

Similarly

$$d\sigma_{z_n} = \bar{\sigma}_{n-1}\,d\alpha\,\frac{2}{\sqrt{3}}\cos\left(\alpha_{n-1} + \frac{d\alpha}{2}\right) + d\bar{\sigma}_n\,\frac{2}{\sqrt{3}}\sin\alpha_n \qquad \ldots(9)$$

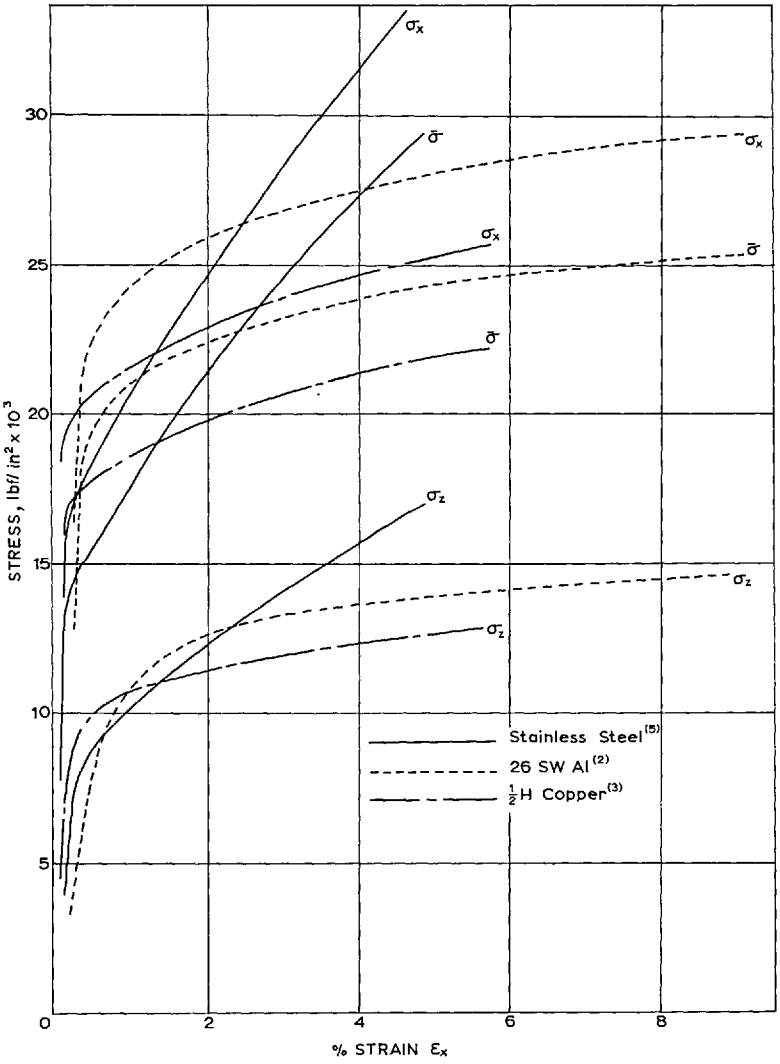


Fig. 6 Stress/strain curves.
(1 lbf/in² = 6894·76 N/m²).

The Prandtl–Reuss equations (4) give

$$d\varepsilon_z = 0 = \frac{d\bar{\sigma}}{\bar{\sigma}H'}\left[\sigma_z - \tfrac{1}{2}\sigma_x\right] + \frac{1}{E}[d\sigma_z - \nu d\sigma_x]$$

Substituting (6)–(9) into this equation yields

$$d\bar{\sigma}_n = \frac{\bar{\sigma}_{n-1}\, d\alpha\left[\dfrac{2-\nu}{\sqrt{3}}\cos\left(\alpha_{n-1}+\dfrac{d\alpha}{2}\right) + \nu\sin\left(\alpha_{n-1}+\dfrac{d\alpha}{2}\right)\right]}{\dfrac{E}{2H'}(\cos\alpha_n - \sqrt{3}\sin\alpha_n) - \dfrac{2-\nu}{\sqrt{3}}\sin\alpha_n + \nu\cos\alpha_n} \quad \ldots(10)$$

Referring once more to Fig. 5 and applying the sine rule to triangles PBQ and ACQ and noting that AQ = EB − PB, we find that

$$2G\, d\varepsilon_x = \frac{2}{\sqrt{3}}\left[d\bar{\sigma}\left(1+\frac{3G}{H'}\right)\sin(60-\alpha_n) - \bar{\sigma}_{n-1}\, d\alpha\cos(60-\alpha_{n-1})\right] \quad \ldots(11)$$

also

$$-2G d\varepsilon_y = \mathrm{CQ} + \mathrm{QB} = \frac{2}{\sqrt{3}}\, d\bar{\sigma}\left(1+\frac{3G}{H'}\right)\sin\alpha_{n-1} + \frac{2}{\sqrt{3}}\bar{\sigma}_{n-1}\, d\alpha\cos\alpha_{n-1} \quad \ldots(12)$$

The true stress/true plastic strain relationship for any material may over a limited strain interval be represented by the formula

$$\bar{\sigma} = A(B + \bar{\varepsilon}^P)^m \quad \ldots(13)$$

The gradient of this curve may thus be determined in terms of the equivalent stress since

$$H' = \frac{d\bar{\sigma}}{d\bar{\varepsilon}^P} = mA(B+\bar{\varepsilon}^P)^{m-1} \quad \ldots(14)$$

and from (13) and (14)

$$H_n' = \frac{mA^{1/m}}{(\bar{\sigma}_n)^{1-m/m}} \quad \ldots(15)$$

Using a regression technique the constants A, B, and m may be determined from the true stress/true strain graph for any real material. Good correlation can be obtained between actual graphs and equation (13), especially if the total strains are $\gtrsim 5\%$. In most corrective forming processes strains larger than this do not obtain.

When H' is not constant (i.e. a real material where work-hardening is non-linear), a simple iteration technique allows the radius of the yield locus to be obtained in terms of the angle of inclination of the instantaneous stress vector α_n. From this relationship and equations (6) and (11) the effective longitudinal stress/strain relationship for a real material may be obtained. The correlation between these and uniaxial stress/strain plots is indicated in Fig. 6 for 26SW aluminium, ½H copper, and stainless steel.

The longitudinal stresses resulting from a plane-strain bending or pulling operation can be calculated to be ~ 13% higher than in the narrow-beam case. The transverse stresses approach a value that is half the longitudinal stress as the beam becomes completely plastic. The use of the above equations, together with an accurate stress/strain model, enables the correlation between theory and practice to be 20% more accurate than with the simple model. The importance of this point when considering loading and power requirements is obvious. Equally important is the fact that transverse stresses and springback may now be more correctly determined.

Consider for example the simple bending and springback of 26 SW aluminium. The stress/strain curve given by Alexander[2] may be represented as

$$\bar{\sigma} = A(B + \bar{\varepsilon}^P)^m$$

where $A = 27{\cdot}01$, $B = 0{\cdot}001\ 757$, and $m = 0{\cdot}074\ 05$

thus $\sigma_x = 31{\cdot}68\ (-\ 0{\cdot}002\ 2 + \varepsilon_x)^{0{\cdot}077\ 85}$

For a 0·02 in- (0·5 mm) thick strip bent to a radius of 1 in (25·4 mm) springback radii for (*i*) a perfectly plastic narrow beam; (*ii*) a perfectly plastic wide beam; and (*iii*) a work-hardening wide beam, may be determined.

For simple bending

$$\varepsilon_x = \frac{y}{R} = \frac{0{\cdot}01}{1} \times 100 = 1\%\ \text{strain}$$

Assuming, as Alexander does, that the material is perfectly plastic with a yield stress of 17 tonf/in² (262 MN/m²) (E = 4730 tonf/in² (7·3 × 10^4 MN/m²) and $\nu = 0{\cdot}3$), yield occurs at a distance $\sigma_x R/E = 0{\cdot}003\ 594$ in (0·09 mm) from the neutral axis for a narrow beam and

$$\frac{\bar{\sigma}}{\sqrt{1 - \nu + \nu^2}} \times \frac{1 - \nu^2}{E} \times R = 0{\cdot}003\ 679 \text{ in the plane-strain}$$

case. The internal bending moments (M) for the three cases under consideration are:

(i) *Narrow Beam*

$$\sigma_x = E \times \varepsilon_x \qquad \varepsilon_x > \varepsilon_y$$

$$\sigma_x = 17\ \text{tonf/in}^2\ (265\ \text{MN/m}^2) \qquad \varepsilon_x > \varepsilon_y$$

$$M = 2\int_0^{0{\cdot}003\ 594} \frac{4730}{1{\cdot}0}\, y^2\, dy + 2\int_{0{\cdot}003\ 594}^{0{\cdot}01} 17\, y\, dy$$

$$= 1{\cdot}627 \times 10^{-3}\ \text{tonf in (0·4 Nm)}$$

(ii) *Wide Beam : Perfectly Plastic*

We can utilize the result obtained by Alexander in Fig. 16 of his paper,[2] giving

$$M = 1{\cdot}82 = 10^{-3}\ \text{tonf in (0·455 Nm)}$$

(iii) *Wide Beam : Work-Hardening*

$$\sigma_x = 31{\cdot}68\ (-\ 0{\cdot}002\ 207 + \varepsilon_x)^{0{\cdot}077\ 85}$$

$$M = 2\int_0^{0{\cdot}003\ 679} \frac{4730}{0{\cdot}91}\, y^2\, dy + 2\int_{0{\cdot}003\ 679}^{0{\cdot}10} 31{\cdot}68\left(-\ 0{\cdot}0022 + \frac{y}{R}\right)^{0{\cdot}077\ 85} y\, dy$$

$$= 1{\cdot}98 \times 10^{-3}\ \text{tonf in (0·495 Nm).}$$

The springback radii and bending moments are directly related because the springback strain may be written

$$\varepsilon_s = -\frac{3M}{2Ea^3}\, y \quad \text{for a narrow beam}$$

Thus

$$\varepsilon_{\text{total}} = \frac{y}{R} - \frac{M_y}{2Ea^3} = \frac{y}{R'}$$

$$R' = \frac{2Ea^3R}{2Ea^3 - 3MR}$$

For a wide beam the springback strain may be written

$$\varepsilon_s = -\frac{3M(1 - \nu^2)}{2Ea^3}\, y$$

and hence

$$R' = \frac{2Ea^3R}{2Ea^3 - 3MR(1 - \nu^2)}$$

For the three cases considered above this gives final radii of:

(i) For a narrow beam	$R' = 2{\cdot}0666$ in (52·5 mm)
(ii) For a perfectly plastic wide beam	$R' = 2{\cdot}106$ in (53·5 mm)
(iii) For a work-hardening wide beam	$R' = 2{\cdot}333$ in (59·3 mm)

The effect of the simplifying assumptions thus leads to errors of ~ 10%.

Conclusions

Equations (8)–(10) describe the stress/strain behaviour of strain-hardening materials under plane-strain loading conditions in which one of the stresses can also be assumed zero. Such conditions are commonly found in shape-correction processes (i.e. roller levelling, tension levelling, stretching, &c.).

It has been demonstrated for the case of simple bending that the simplifying assumptions generally made lead to errors in the region of 10%. The method can be used to determine residual stresses and curvatures in most operations involving thin wide strip.

The sequence of operations to obtain such a solution is:

(1) Plot a stress/strain graph for simple tension.
(2) Using a regression technique obtain constants A, B, and m.
(3) Hence determine the equivalent stress/equivalent strain relationship.
(4) Calculate the $H'/d\bar{\sigma}$ relationship.
(5) By an iteration technique obtain initial $d\bar{\sigma}$, dx, dy, $d\bar{\sigma}_x$, $d\bar{\sigma}_y$.
(6) Obtain instantaneous-stress relationships.
(7) Plot relationships of interest.
(8) Apply equilibrium equations to obtain radial stresses, bending moments, &c.

Acknowledgement

The work was carried out under a programme designed to investigate the tension-levelling process, supported by the British Steel Corporation (South Wales Group). Thanks are due to Professor J. M. Alexander of the Department of Mechanical Engineering, Imperial College of Science and Technology, London, for helpful discussions.

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APPENDIX 10.

A10.0. On the Mechanics of the Tension-Levelling Process.

The pages of this appendix contain a copy of the proof of a paper to be published in J.I.M. in November 1971. The paper presents in simple terms the basics of the theoretical analysis developed in this thesis.

On The Mechanics of the Tension-Levelling Process

J. M. Roberts and T. Sheppard

A theory is developed to predict the stresses and strains imposed on sheet metal during processing in the tension-levelling machine. A computer programme has been written to aid calculations and results are presented. An experimental machine has been developed and computer results show good correlation with experimental data.

List of Symbols

x, y, z	= rectangular Cartesian coordinates x = strip length y = strip thickness
a	= half strip thickness
β_j	= displacement of neutral axis on bending at j^{th} roll
$\beta' j'$	= displacement of zero strain point at j^{th} roll
γ_j	= displacement of recovery axis at j^{th} roll
δ_j	= displacement of straightening axis at j^{th} roll
E	= Young's modulus
F	= applied compressive force
I	= second moment of area
l	= length of material
L	= applied load : unit width
M	= internal bending moment
R	= bend radius of central axis
e_x	= longitudinal engineering strain
$\bar{\varepsilon}$	= equivalent strain
ε_x	= longitudinal strain
ε_T	= applied tensile strain
ε_j	= overall strain at j^{th} roll
$\bar{\sigma}_0$	= equivalent yield stress
σ_{x0}	= longitudinal yield stress under plane-strain conditions
σ_x	= longitudinal stress
σ_{rec}	= longitudinal recovery stress
σ_T	= nominal applied tensile stress

The bad shape resulting from differential elongation of parts of rolled strip was, until comparatively recently, accepted by both producer and consumer. As the strip becomes thinner the buckling-resistance decreases, so that the residual stresses manifest themselves in various shape defects. These defects have been specified by Wistreich[1] (Fig. 1). Euler's formula

$$F = \frac{\pi^2 EI}{l^2}$$

indicates that instability is dependent upon the elastic modulus of the material and hence buckling-resistance is likely to be lower in non-ferrous materials. However, the applied force resulting from internal stresses will be dependent upon the magnitude of these stresses and non-ferrous materials in hard temper and ferrous materials having a high yield locus will thus be highly likely to exhibit shape defects. This is because the higher the value of the yield locus, the steeper will be the stress gradients in a fixed width of material.

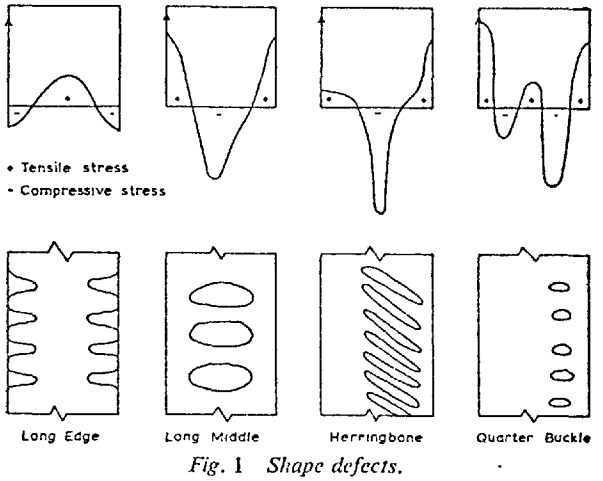


Fig. 1 Shape defects.

Shape correction of strip may be achieved by roller-levelling unless the strip is so thin that the deflection caused by the rollers is insufficient to produce appreciable yielding of the cross-section. The mechanics of this process have been investigated by Sheppard,[2] who gives solutions for residual stress levels in a width increment for a linear work-hardening material. He also points out that for this process, where complete stress reversal obtains, the Bauschinger effect and work-hardening are complementary and no overall hardening of the material occurs during the operation. In general, the strip does not conform to the roll radius and the deformed radius of the stock is determined by the roll interlock. Another process by which plate and sheet may be straightened is uniaxial stretching,[3] the disadvantage of this process being that high stress levels are involved giving rise to an inherent risk of plastic instability.

Manuscript received 19 March 1971; in final form 7 June 1971. T. Sheppard, M.Sc., B.Sc., C.Eng., M.I.Mech.E., A.I.M., is a Lecturer in Mechanical Metallurgy and J. M. Roberts, B.Sc.Eng., A.R.S.M., is a Research Student in the Department of Metallurgy, Imperial College of Science and Technology, London.

The tension-levelling process attempts to overcome the individual difficulties of the two processes by combining them in such a way that thin strip is bent around a roll under superimposed tension (Fig. 2(*a*)). The strip is then bent and stretched in the reverse direction and the process may be repeated many times. This results in material elongation at comparatively lower values of applied tension ($\sim \frac{1}{3}$ of the yield stress). In general, some form of roller-levelling in which the neutral axis remains on the centre of the strip will be necessary to straighten the strip finally.

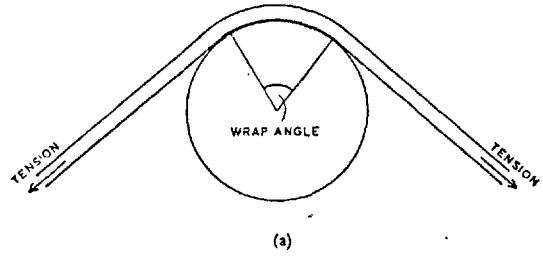


Fig. 2(a) *Bending under superimposed tension.*

Tension-levelling was initially applied in the aluminium industry and it was in this field that it was first used as a full-scale technological process.[4] Increasingly stringent consumer demands in the steel industry have recently resulted in a new interest in the process as applied to steel strip. There are, it should be noted, metallurgical difficulties associated with steel processing that do not often arise in the non-ferrous industries, especially the discontinuous yield phenomenon which occurs at the magnitudes of deformation prevailing in the tension-levelling process and manifests itself as Lüders markings.

It will be obvious that the relative magnitudes of the bending and tension stresses involved will play an important part, both in machine design and product flatness. At the present time roll diameter is determined by the minimum strip thickness and maximum hardness and the tension level by the maximum cross-section and hardness. Tension must also depend upon the degree of shape correction required (i.e. the more prevalent the buckles, the greater the tension required). This is illustrated in Fig. 2(*b*) by reference to a simple tension diagram. The tensile load is usually applied by bridles and while it is essential that enough tension be imposed, it is equally important that the expense of large-scale machinery be justified. The interrelation of bending and tensile stresses will affect both the stability of the process and the metallurgical features previously mentioned.

This paper illustrates the basic mechanism of deformation within the tension-leveller with respect to applied and residual stress and strain distribution.

Assumptions

The material is assumed to be isotropic with uniform expansion of the yield locus and to have the same yield stress in tension and compression. As compressive stresses greater than yield are unlikely to exist in a substantial proportion of fibres, the Bauschinger effect is ignored.

Since tension-levelling is applied to thin wide strip the width : thickness ratio is large and thus plane-strain conditions are assumed to prevail. As the material is thin the stresses tending to compress the fibres (σ_y) may be considered to be zero.

We shall further assume that sections plane before the process remain plane afterwards. In general, the strip thickness : bend radius is small so that this is a valid assumption. This leads to a linear strain distribution across the strip.

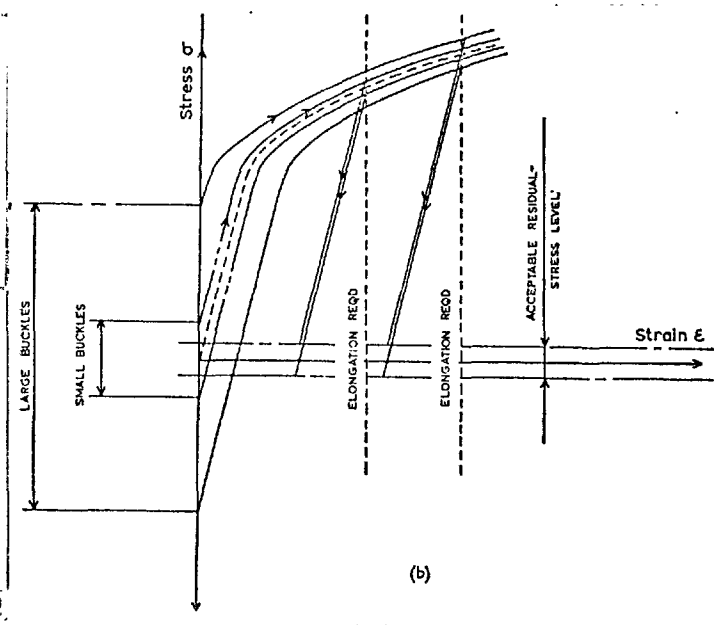


Fig. 2(b) *Elimination of residual stresses by tension.*

Finally, we should assume that the tensile force is supported by a stress that is uniform across the strip width. We shall also ignore the tension losses that occur throughout the machine. The first of these assumptions is clearly not valid, because short areas of the strip are elongated in preference to long areas. However, it is the elongation of these short areas in which we are interested and any analysis assuming uniform tension will give this elongation.

The Theory of Tension-Levelling

The precise shape of the stress distribution across the strip section will depend upon the elastic–plastic stress/strain relationship in the longitudinal direction. A suitable method of determining this parameter for an elastic non-linear work-hardening material has been given by Sheppard and Roberts[5] and will not be discussed further; the computer results cited later do, however, make use of this method. To illustrate the mechanism of strip extension it will be sufficient to assume an " accepted " elastic–perfectly plastic curve and to point out the differences in calculation necessary when considering a work-hardening material.

The applied tension in the process encourages the strip to conform to the roll radius; the effect of roll interlock is thus minimal. The problem is thus the bending of strip around a roll while under the influence of tension, a problem dealt with for a linear work-hardening material by Swift.[6] The application of tensile forces produces a " shift " of the neutral axis away from the centroidal axis of the strip, the offset being determined by a consideration of free equilibrium.

Fig. 3 illustrates the case where the neutral axis has moved a distance β from the centroid of area. The longitudinal strain at a distance y from the central axis is thus given by

$$\varepsilon_x = \frac{y + \beta}{R - \beta} \qquad \ldots (1)$$

In general, in the tension-levelling process the strains involved are sufficient to cause yielding both at the tensile and at the compressive surface. The stress distribution is shown in Fig. 4(*a*). Applying a force balance over a unit width of section the " shift " may be determined by the equation

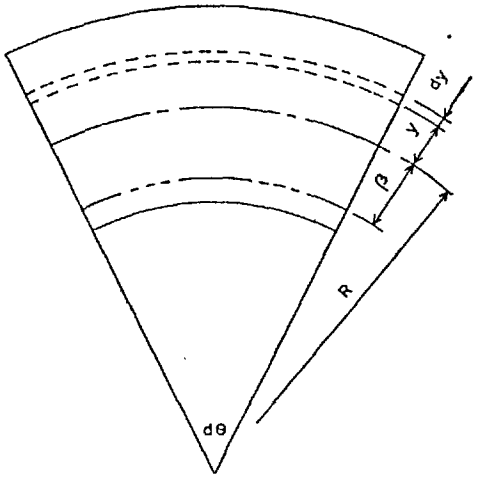


Fig. 3 Elemental section of deformed strip.

$$2\,a\,\sigma_T = \int_{-\beta}^{a} \sigma_x\,dy - \int_{-a}^{-\beta} \sigma_x\,dy \qquad \ldots (2)$$

For the general case where yielding occurs at both surfaces we can thus write

$$\beta = \frac{a\,\sigma_T}{\sigma_{x0}} \qquad \ldots (3)$$

The neutral-axis displacement is thus directly proportional to the applied load and inversely proportional to the material yield stress. To move the neutral axis outside the strip cross-section the applied tension must give rise to a stress greater than the yield stress. This is not actually achieved in practice because of inadequate friction between strip and bridle roll and also because tensions of this magnitude would lead to an unstable process.

The strain distribution is linear, as shown in Fig. 4(*b*).

The application to a work-hardening material would result in the shift of the neutral axis being slightly reduced.

On leaving the roll the bending moment on the strip is released and the stresses in the strip are sufficient only to balance the applied tension σ_T, which pulls the material straight. It is convenient to consider this as consisting of two operations: recovery and straightening. The sequence is illustrated in Fig. 5.

The loss of roll contact is equivalent to the application of an elastic bending moment equal in magnitude and opposite in sign to the elastic–plastic moment experienced by the strip on the roll. That the stresses behave elastically on recovery is a well-known fact and hence the stresses must vary linearly with distance from the centroidal axis.

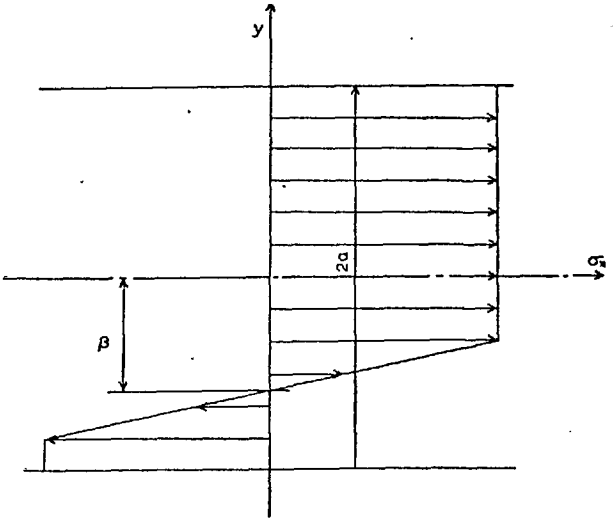


Fig. 4 (a)
Stress over 1st roll

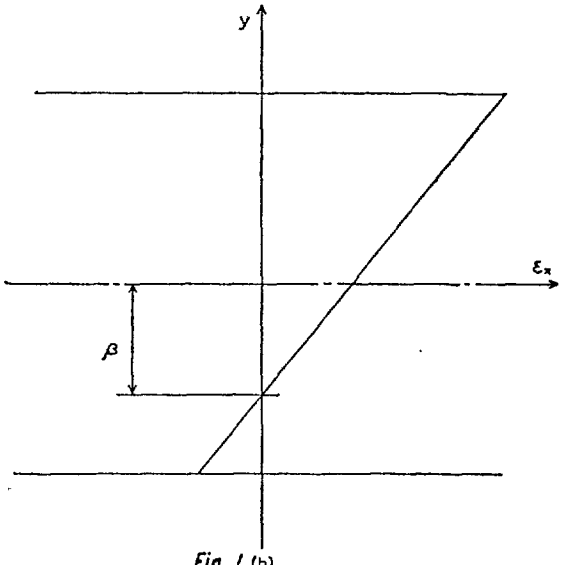


Fig. 4 (b)
Strain over 1st roll

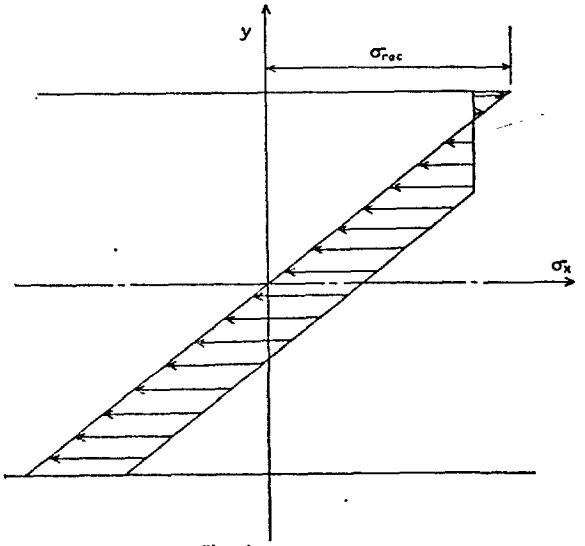


Fig. 4 (c)
Stress over 1st roll. Small applied tension

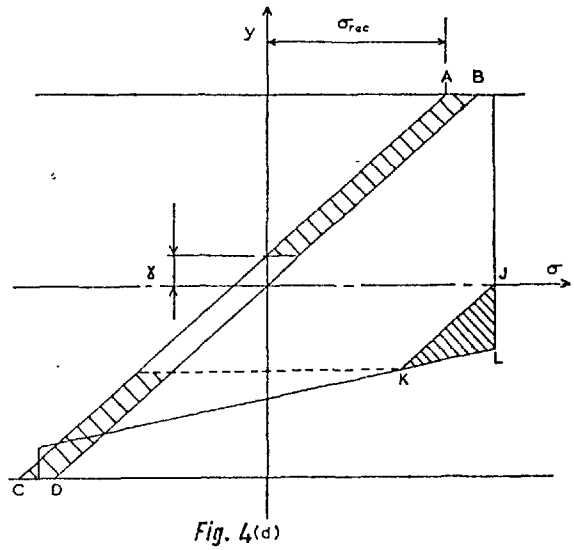


Fig. 4(d)
1st roll springback

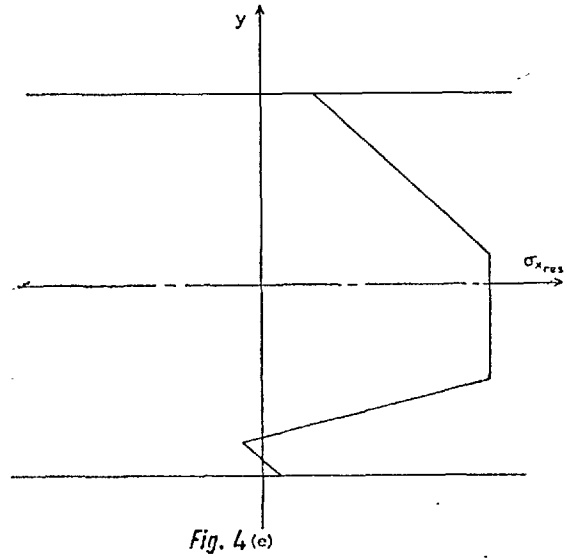


Fig. 4(e)
Residual stresses after 1st roll

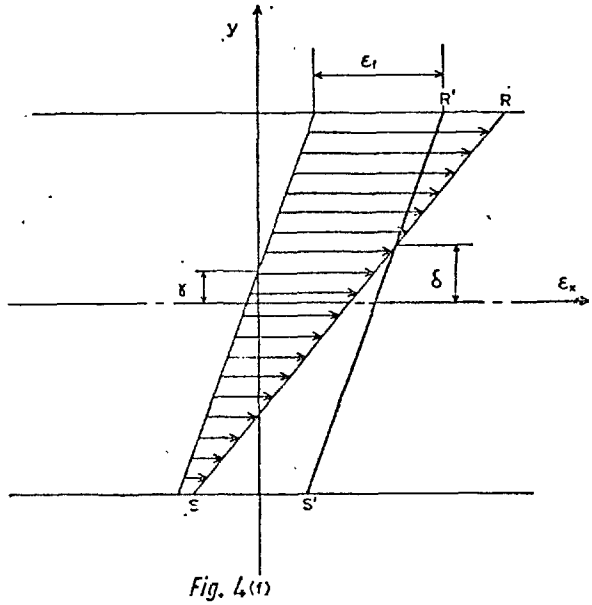


Fig. 4(f)
Strains after springback

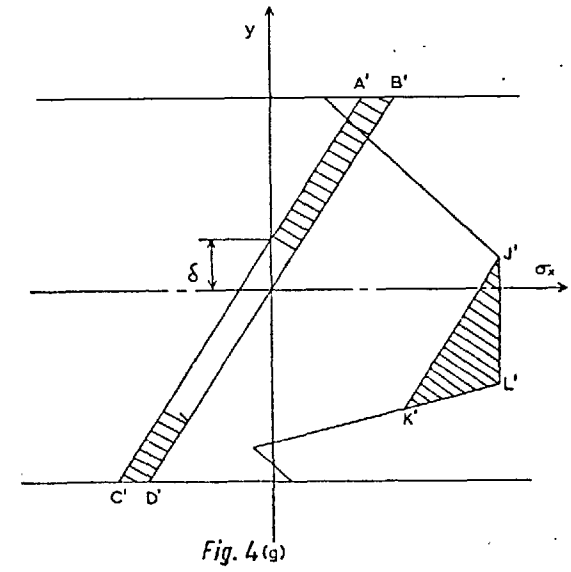


Fig. 4(g)
Stress during straightening after the 1st roll

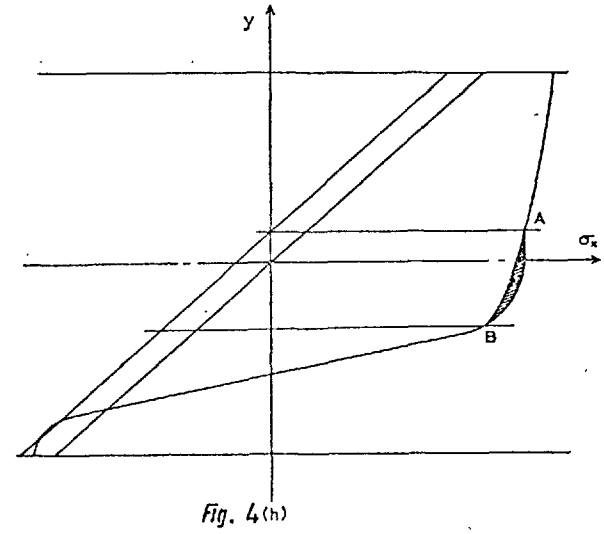


Fig. 4(h)
Effect of work-hardening

Figs. (a)–(h). Stress and strain disrtibutions during tension levelling.

The recovery stresses (σ_{rec} in Fig. 4(c)) can be calculated by equating the recovery moment to the applied moment. Thus

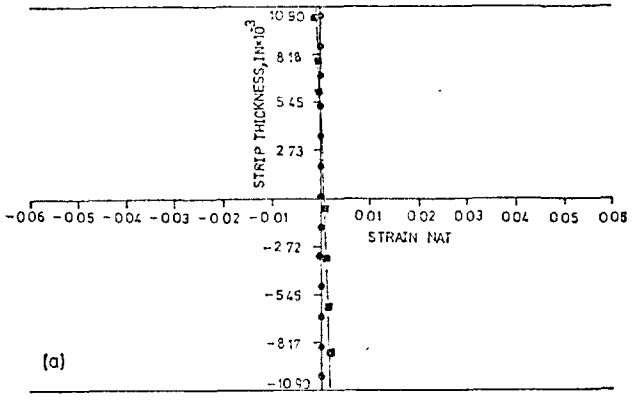
$$-\tfrac{2}{3}\,\sigma_{rec}\frac{a^3}{y} = \int_{-\beta}^{\alpha}\sigma_\chi\, y\, dy + \int_{-a}^{-\beta}\sigma_x\, y\, dy = M$$

or

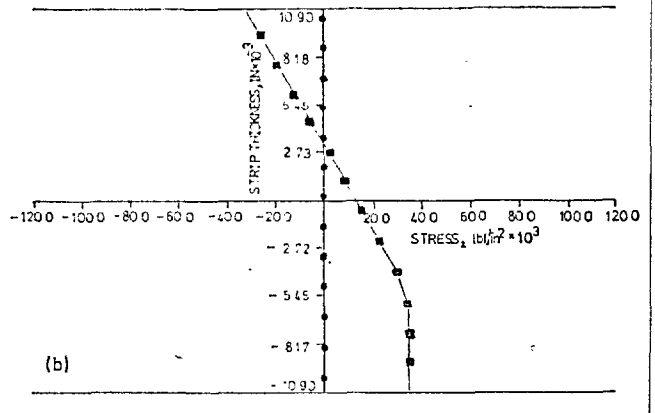
$$\sigma_{rec} = \left(-\frac{3M}{2a^3}\right)y \qquad \ldots (4)$$

The balance of longitudinal forces must be maintained during the spring-back operation and, provided that no plastic deformation occurs, then spring-back is about the centroid of area. This condition obtains in all cases where the initial tensile yield does not extend to more than half the cross-section (Fig. 4(c)).

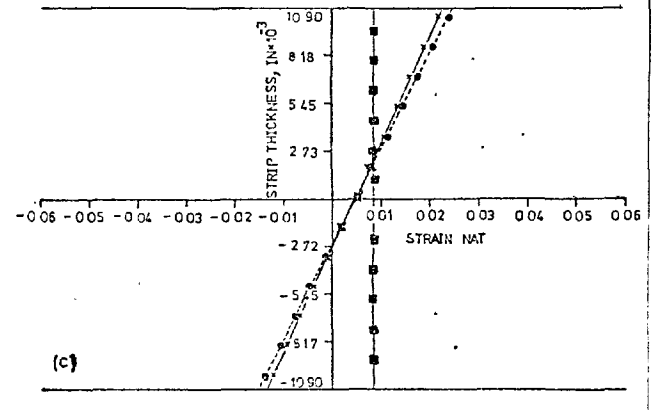
STRAIN DISTRIBUTION ON INITIAL FLATTENING



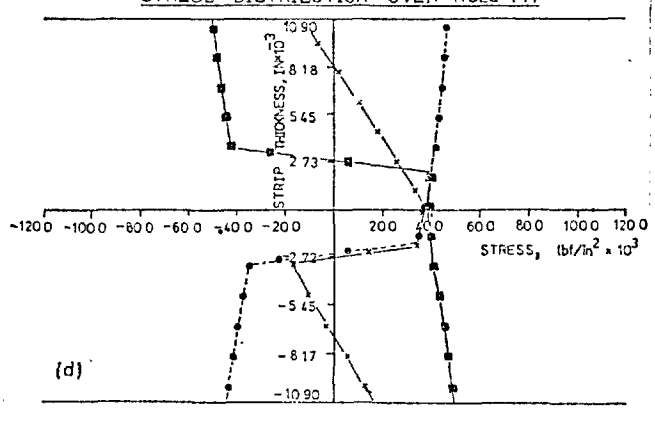
STRESS DISTRIBUTION ON INITIAL FLATTENING



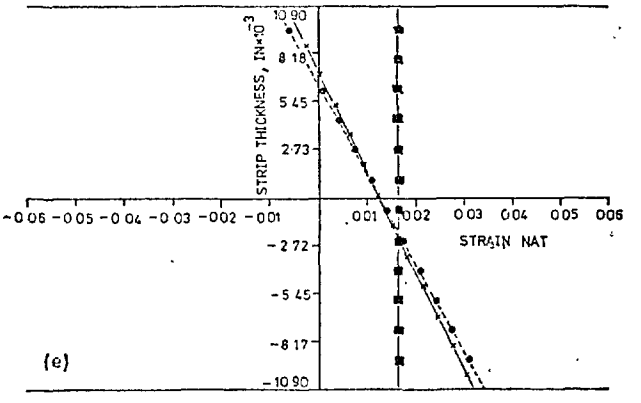
STRAIN DISTRIBUTION OVER ROLL (1)



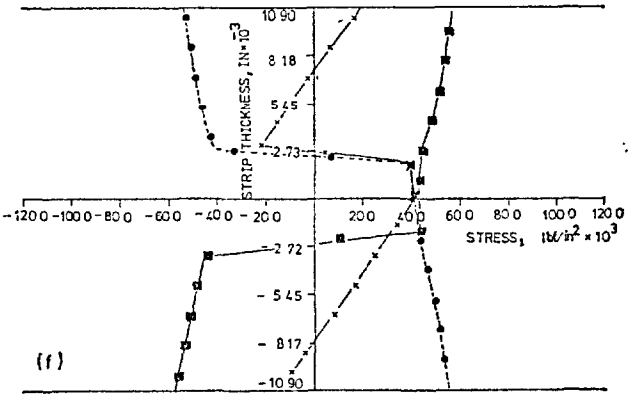
STRESS DISTRIBUTION OVER ROLL (1)



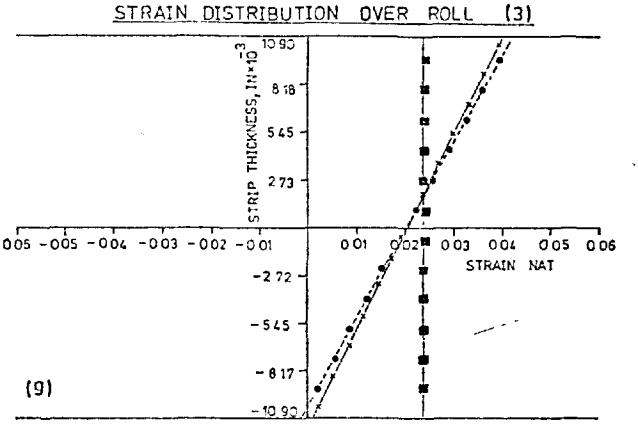
STRAIN DISTRIBUTION OVER ROLL (2)



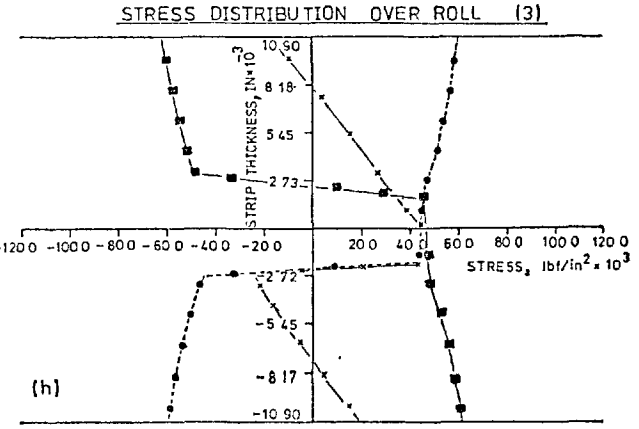
STRESS DISTRIBUTION OVER ROLL (2)

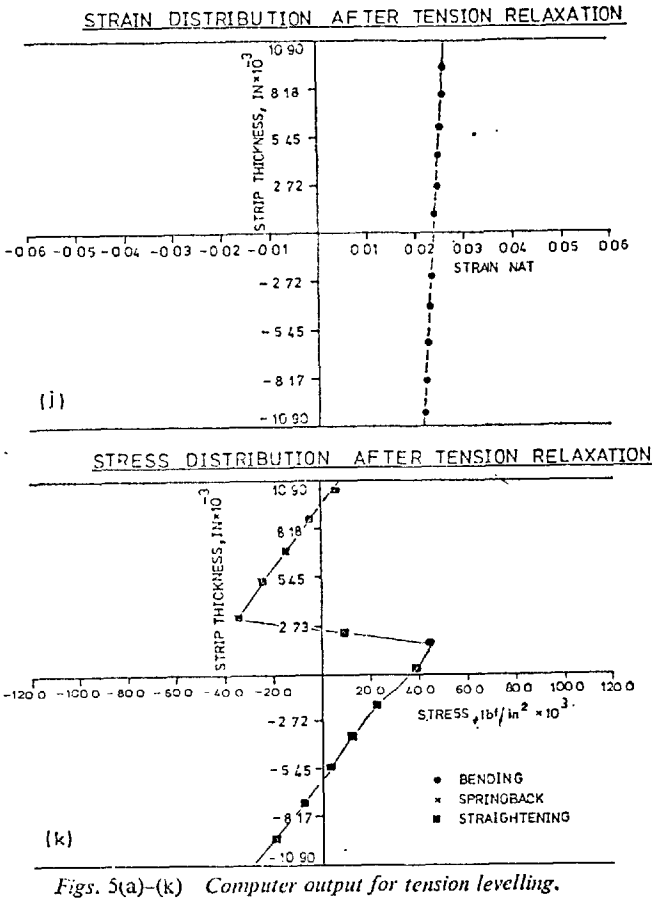


STRAIN DISTRIBUTION OVER ROLL (3)



STRESS DISTRIBUTION OVER ROLL (3)





Figs. 5(a)–(k) *Computer output for tension levelling.*

However, in the more general case where tensile yield has exceeded 50% (Fig. 4(*d*)), then a recovery stress distribution passing through the central axis will result in an unbalance of longitudinal forces due to the loss of region JKL when integrating the internal stresses. The total force loss is equivalent to area JKL. This must be balanced by a move of the recovery axis a distance γ such that

$$\text{area JKL} = \text{area ABCD}$$

Spring-back stress may now be written

$$\sigma_{rec} = -\frac{3M}{2a^3}(y - \gamma)$$

A small increase in the value of γ produces a greater increase in ABCD than in JKL and hence only a small " neutral recovery axis shift " is in general required to provide equilibrium conditions. The final residual stress distribution is shown in Fig. 4(*e*) and gives zero internal moment with longitudinal force equilibrium. The resulting strain distribution is given in Fig. 4(*f*) and may be written

$$\varepsilon_{res} = \frac{y + b}{R - b} - \left[\exp\left\{\frac{3M(1 - \nu^2)(y - \gamma)}{2\,\varepsilon a^3}\right\} - 1\right]$$

and the residual curvature by

$$R_{res} = \frac{2\,E\,a^3\,(R - b)}{2\,Ea^3 - 3M\,(R - b)\,(1 - \nu^2)} + \beta'$$

$$= \frac{2Ea^3R + 3M\gamma(R - b)(1 - \nu^2)}{2Ea^3 + 3M(R - b)(1 - \nu^2)}$$

This residual curvature is immediately removed by the applied tension force and the straightened strip is thus subject to a bending moment that further modifies the stress system. The effect of this applied bending moment is to rotate the line RS in Fig. 4(*f*) about a point distant δ from the neutral axis, such that the strains become uniform across the section.

If yielding does not occur, then clearly the stress distribution is modified by the superposition of a linear stress distribution symmetrical about the centroid of area. If, however, yielding takes place, then the same conditions as those governing initial recovery must be satisfied (i.e. force equilibrium) and once again a " shift " of the neutral axis occurs. This is shown in Fig. 4(*g*) by the distance δ, which is calculated by equating areas J'K'L' to area A'B'C'D'. The equations describing the stress distributions now become somewhat complex, even for the simple non-work-hardening case.

Bending over the second roll is analysed in exactly the same way, the governing conditions being the applied strain distribution and the horizontal equilibrium requirement. Over the second roll the strain may be described by

$$\varepsilon_x = \frac{(-y + \beta_2)}{R - \beta_2}(1 + \varepsilon_1)$$

where ε_1 is the overall strain in the deformation process over the first roll.

Although the value of β_2 is of the same order as β_1, the increased yielding due to residual stresses existing from the previous bend will result in a slight increase in magnitude of this parameter. During the straightening operations the reverse occurs (i.e. the " shifts " of the recovery axis are reduced).

The stress and strain distributions resulting from the second tension bend are resolved in a similar manner to the first bend.

The main difference between the calculations necessary for work-hardening and non-work-hardening materials is that in section AB in Fig. 4(*h*) a greater yield stress will be permissible because of the increase in strain recovery.

The sections of Fig. 4 have not allowed for an initial residual stress distribution in the sheet, which in any case would not affect the calculations and would confuse the fundamental concepts involved.

In practice, these calculations must be entrusted to a digital computer. The expansion of the yield envelope for a work-hardening material has been dealt with elsewhere by Sheppard and Roberts.[5] The neutral-axis displacements must be determined by an iterative technique, thus allowing the computation of stress and strain distributions. In many ways the calculation is easier for a work-hardening material where the stress/strain curve may be represented by a continuous, rather than a discontinuous function.

Thus, the analysis ensures that with a knowledge of strip thickness, applied tension, and roller diameter the extension through the process may be calculated. Fig. 5 shows the computer output obtained when processing.

Roll 0 refers to the initial removal of coil set, roll 1 to the deformation process on the 1st roll, &c.

The elongation over a roll is given by the ε_f in Fig. 4(*f*) minus the applied tensile strain ε_T and this must be correlated with the total extension required to straighten the strip. This is reported to be 0·8%–1·6% for low-carbon strip.[7,8]

Experimental Work

To test the validity of the equations presented, a programme of experimental work has been performed. An experimental five-roll tension-leveller was built and is shown diagrammatically in Fig. 6. Tension and back tension are applied by hydraulic cylinders clamped to the building columns of the laboratory. The guide rolls are 10 in (250 mm) in dia., which is large enough to ensure that no plastic deformation ensues as the material is bent over them. The work-roll assembly is such that any size of roll up to 2 in (50 mm) dia. may be accepted, but to check these equations 1 in (25 mm) dia. rolls were used and the material bent over only three of the five available rolls. The leveller will accept strip up to a maximum width of 8 in (200 mm).

The strip processed was 6·75 in (177 mm) wide and 0·022 in (0·55 mm) thick, giving a roll diameter : strip thickness ratio of ~ 45. The mild-steel strip was annealed and given a light skin pass by the British Steel Corporation (South Wales Group) so that the assumption that the initial residual curvature could be calculated from strip curvature would be valid. This also removed the discontinuous yield. The strip was of good shape with Young's modulus E having a value of 30×10^6 lbf/in^2 (2×10^5 MN/m^2) and the yield stress 30 000 lbf/in^2 (200 MN/m^2); these values were obtained on a standard Hounsfield tensometer. The computer programme gave the following stress/strain relationships

$$\bar{\sigma} = 81740\ (0{\cdot}0090 + \bar{\varepsilon})^{0{\cdot}211}$$
$$\sigma_x = 94720\ (0{\cdot}0059 + \varepsilon_x)^{0{\cdot}203}$$

The tensile load applied to the strip was measured by Coutant tensile load cells (EM 50, capacity 10^T) positioned as shown in Fig. 6. The output was traced on an ultraviolet recorder via Southern Instrument amplifiers (M1300 Recorder, M1266 D.C. Amplifier). Strain measurement through the leveller was by Shinkoh foil-gauges (2 × 2 mm). Eastman 910 cement proved a suitable bonding medium as long as the strains did not exceed 4%. During passage through the leveller the gauges pass between strip and roll and pressure and friction forces tend to tear the gauge from the strip. The foil-gauges ensure that the pressure effect is of little importance and the defoliation problem has been solved by fixing adhesive tape to the strip to act as a " lead-in ". Gauge leads must also be passed through the machine and to provide the necessary working length the wire is first inserted between ½ in- (12·5 mm) pitch gear wheels and then fixed by adhesive tape to the strip surface. The gauge output was again fed via a calibration circuit and amplifier to the ultraviolet recorder.

Overall extension through the leveller was measured by scribing two lines on the strip surface of 500 mm gauge-length. The lines were scribed and measured accurately by the Vernier technique.

Strip speed was measured by phototransistors (Mullard OCP 71) mounted at 6 in (150 mm) intervals on the columns holding the hydraulic cylinders. A focused light source was mounted on the gripper jaws and produced a trace on an ultraviolet recorder as it passed each transistor. The speed could thus be evaluated as long as the speed of the ultraviolet paper was recorded. For the experimental results given below, the strip speed was 5 ft/min (0·025 m/s), although the experimental apparatus is capable of speeds of up to 3000 ft/min.

Initial and final coil sets were determined by measurement of the central deflection over a given arc length.

Results and Discussion

The results are presented in graphical form in Figs. 7–10, which also indicate the theoretical results obtained from the computer programme.

Fig. 7 illustrates the effect on overall strains produced by varying the mean tension force. Both measured and strain-gauge results are plotted together with theoretical values, assuming a strip thickness of 0·022 in. There is an approximately linear relationship between mean tension and overall strain and the predicted values can be seen to be high for low tension values. We will return to this point later.

Fig. 8 shows the bending and straightening strains over the first roll and here it is apparent that the theoretical values overestimate the strains recorded. Fig. 9 indicates that over the second roll the theoretical and practical values are in agreement and Fig. 10 that over the third roll the predicted values are again higher.

It is thus clear that, where the tension force is low, the theoretical equations presented overestimate the strain in the strip. This discrepancy is gradually reduced, so that at higher tension values there is good agreement between theory and experiment. If we consider the strain-gauge results presented in Figs. 8–10 it can be seen that strain discrepancies are greatest over the first roll, least on the second, and increase again on the third roll. The corresponding wrap angles are 26, 52, and 26°).

This indicates that the wrap angles on the first and third rolls were, in general, insufficient to force the strip to bend to the true roll radius and to test this premise wax impressions of the strip were made at tensile loads varying from 1000 lbf (4500 N) to 2000 lbf (9000 N). The curvature of the impressions was determined and this indicated that actual curvatures of the strip were 1·5–2·0 times greater than the roll radius.

The theoretical values plotted are based upon initially flat strip (i.e. no coil set), 0·022 in thick and 6·75 in wide, and some experimental variation can be expected because of the effect of variation in coil set and gauge.

Thus, we can conclude that there is a " threshold " wrap angle beyond which experimental and predicted results will agree. This angle will depend on strip width, thickness, roll diameter, and the tension load applied.

It may, however, also be concluded that the experimental results obtained indicate that the values of strain experienced by the strip surface during levelling correspond closely to theoretical values. Thus, the model developed will predict stress, residual stress, and strain values during the tension-levelling of metal strip. This analysis applied to specific industrial processing could be invaluable in predetermining the reaction of tension, bending, and material properties.

Acknowledgement

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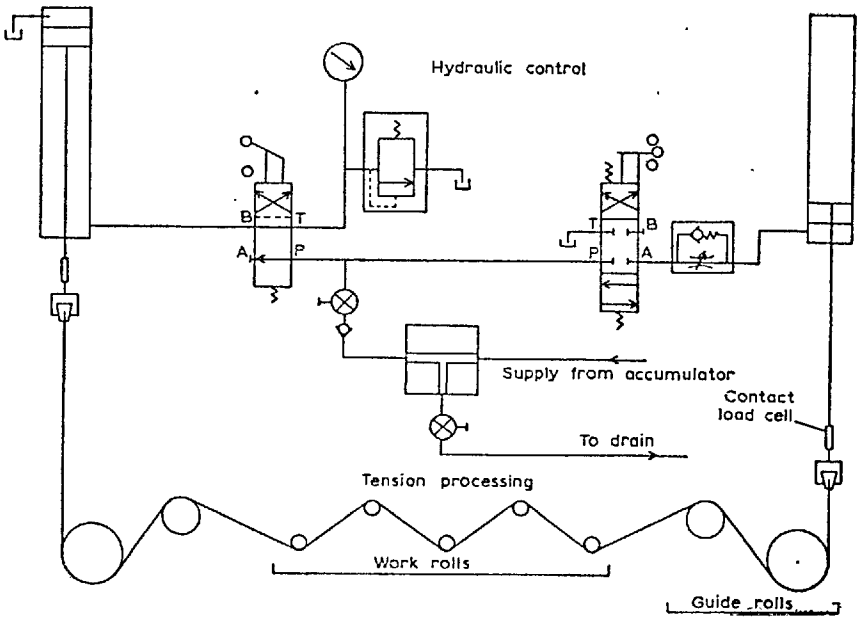


Fig. 6 Schematic diagram of apparatus.

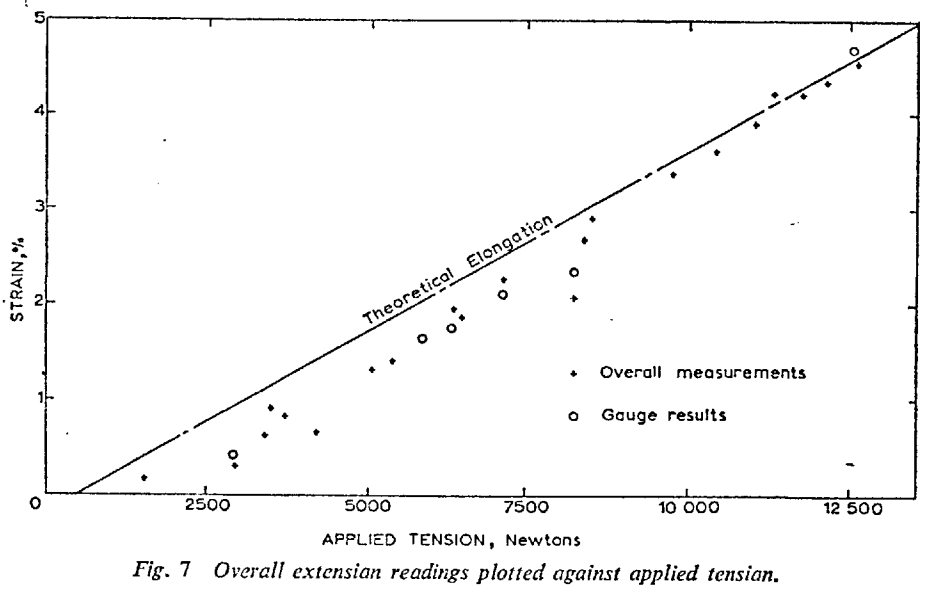


Fig. 7 Overall extension readings plotted against applied tension.

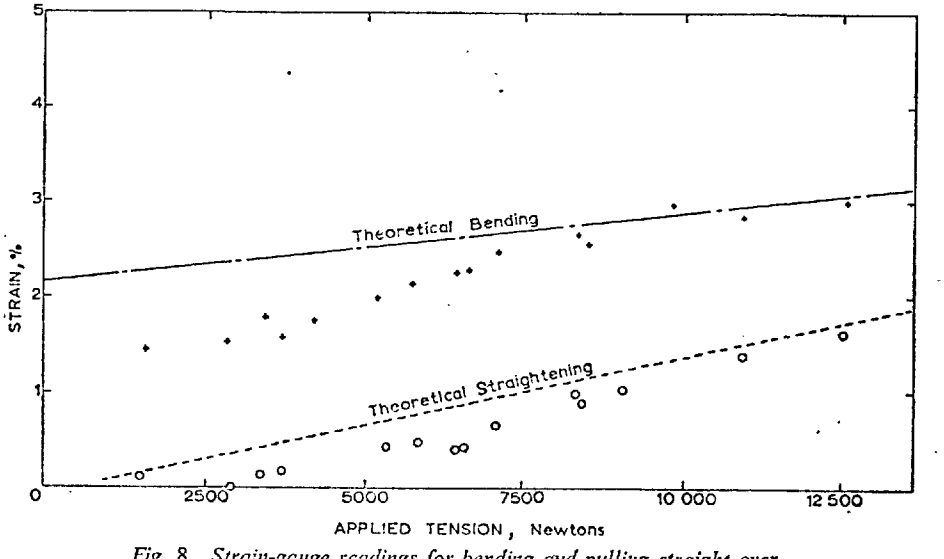


Fig. 8 Strain-gauge readings for bending and pulling straight over roll 1.

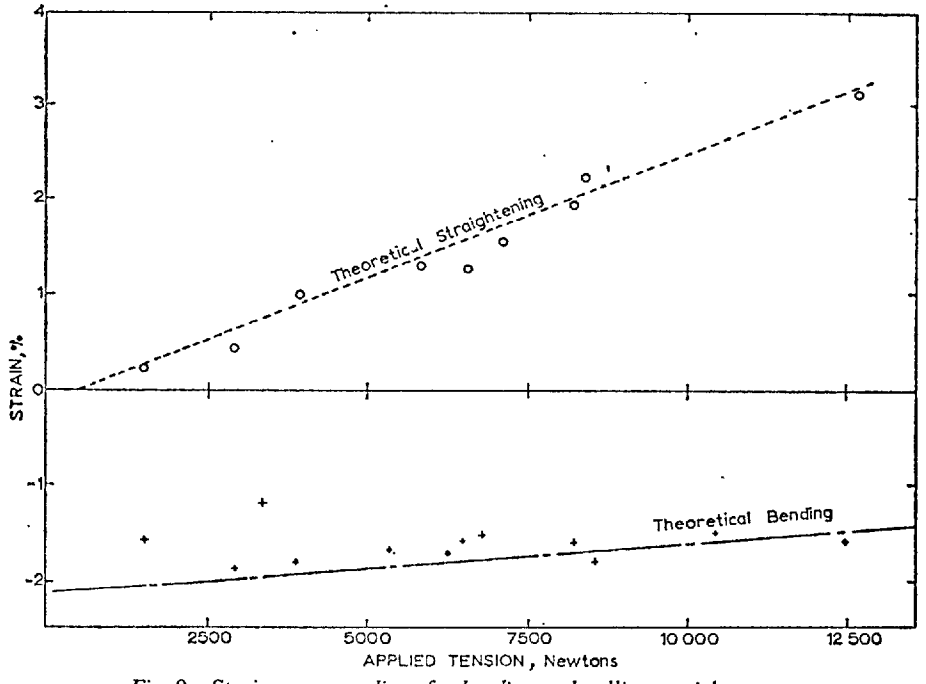


Fig. 9 Strain-gauge readings for bending and pulling straight over roll 2.

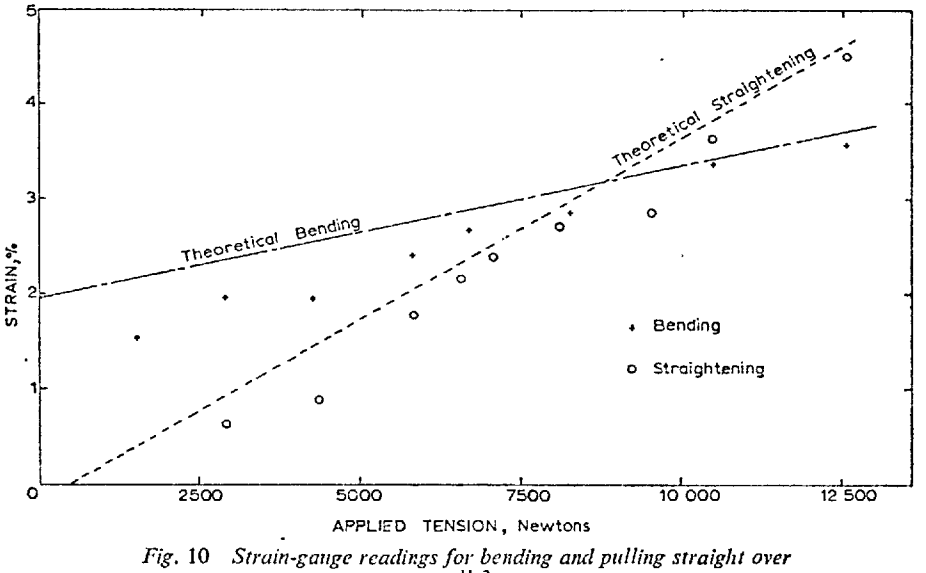


Fig. 10 Strain-gauge readings for bending and pulling straight over roll 3.

APPENDIX 11.

A11.0. Nomenclature.

A list of the main symbols used in the text is given here. It will be found that some symbols have been used to describe different variables, the required defination will, however, be evident in the text.

Symbol	Definition.
A	Equivalent stress/strain relationship constant.
A	Sine wave amplitude (section 3.7.0.)
A'	Longitudinal stress/strain relationship constant.
A_{max}	Maximum sine wave amplitude.
A_w	Amplitude at distance w from the strip centre.
a	Half strip thickness.
B	Equivalent stress/strain relationship constant.
B'	Longitudinal " " " "
b	Neutral axis shift from central axis.
b_1	Incremental neutral axis shift during bending.
b_2	" " " " " spring-back.
b_3	" " " " " straightening.
c_1	Distance of zero strain axis from central axis.
D	Work roll diameter.
D_{min}	Minimum roll diameter in leveller.
$D_{released}$	Effective released diameter during spring-back.
D'	Residual strip curvature diameter.
d	As a prefix - increment of main symbol.
E	Young's Modulus of elasticity.
e	Engineering strain and incremental fibre strain (longitudinal direction).
e_{ab}	Absolute strain, (where $e_m - e_E < e_{ab} < e_m$).
e_b	Bending strain.
e_b	Incremental plastic strain contributing to work-hardening - kinematic model.
e_E	Elastic strain range.
e'_E	Semi elastic strain range (kinematic models).
e_f	Flattening strain.
e_g	Gauge strain.
e_m	Maximum strain contributing to work-hardening (isotropis model).
e_{mean}	Central axis strain.

e_{mn}	Maximum total strain in either tension or compression (isotropic/kinematic model).
e_o	Longitudinal yield strain.
e_{rec}	Recovery strain.
e_{resid}	Residual strain.
e_s	Surface strain.
e_{st}	Straightening strain.
$e_t = e_{tot}$	Total strain at one roll or through leveller.
e_y	Strain increment at distance y from the central axis.
f	Gauge factor.
G	Shear modulus.
H'	Slope of the equivalent stress/equivalent strain curve in the plastic region.
h	Arc height.
I	Moment of inertia.
j	Gauge grid/strip surface distance.
j'	Gauge installation thickness.
L	Length of element of strip.Roll length.Chord length.
l	Length of shortest element of strip.
Δl	Length differential - bad shape.
M	Bending moment.
M_{app}	Externally applied moment.
M_f	Internal moment at leveller exit.
M_n	Internal moment due to bending at roll N.
M_{rec}	Recovery moment.
m	Hardening index,equivalent stress/strain relationship.
m'	" " longitudinal " " "
N	Number of leveller rolls or roll number as subscript.
n	Subscript for stress point definition.
P	Stress constant.
P,P',P"	Stress point.
P	Neutral axis shift during flattening.
p	Pressure stress.
Q	Stress constant.
Q	Intermediate back-up roll bearing force.
R	Central axis bend radius.
R_b	Secondary back-up roll radius.
R_i	Intermediate back-up roll radius.
R_n	Neutral axis bend radius.
R_o	Roll radius.

R_{resid}	Residual curvature radius after recovery (central axis)
R_{st}	Effective straightening radius of central axis.
r	Gauge resistance.
r_a	Active gauge.
Δr_a	Active gauge resistance charge on straining.
r_c	Calibrating resistance.
r_l	Gauge factor control resistance.
Δr_l	Resistance charge equivalent to Δr_a by the inclusion of r_c in parallel.
S	Work roll/intermediate back-up contact force.
s	Longitudinal stress level at the mid-point of the elastic range.
T	Applied tensile load per unit width.
T_{ex}	Applied load at leveller exit.
T_m	Mean applied load at work roll.
t	Strip thickness.
t_o	Entry thickness.
t_1	Exit thickness.
V	Primary/secondary back-up roll contact force.
W	Secondary back-up roll bearing force.
w	Strip width co-ordinate (bad shape).
x,y,z	Rectangular Cartesian co-ordinate.
x	Longitudinal strip axis.
y	Thickness axis.
z	Transverse axis.

Greek Symbols.

α	Angular orientation of stress vector to longitudinal deviatoric stress axis.
α_f	Orientation at the end of a loading cycle.
α_o	Orientation at yield.
γ,β	Orientation of roll centres in the experimental roll system.
γ	Shear modulus.
δ_o	Roll deflection at centre of span.
ε	True strain.
$\bar{\varepsilon}$	Equivalent strain.
$\bar{\varepsilon}^P$	Equivalent plastic strain.

$\Delta\varepsilon$	Bad shape strain differential.
ε_{surf}	Surface strain.
ε_{m}	Elastic hydrostatic strain.
ε_{tot}	Total strain.
$\varepsilon_x, \varepsilon_y, \varepsilon_z,$	Normal strains.
$\varepsilon_{xo}, \varepsilon_{yo}, \varepsilon_{zo},$	Normal yield strains.
$\varepsilon'^{E}_x, \varepsilon'^{E}_y, \varepsilon'^{E}_z,$	Elastic deviatoric components.
$\varepsilon_x'^{P}, \varepsilon_y'^{P}, \varepsilon_z'^{P},$	Plastic deviatoric components.
$\varepsilon_x^{E}, \varepsilon_y^{E}, \varepsilon_z^{E},$	Elastic components of normal strain.
$\Delta\varepsilon_x^{E}$	Reverse loading longitudinal elastic strain range.
ξ	Residual strip strain at $y = b_3$
λ	Sine wave, wave length.
μ	Friction coefficient.
μ'	Effective bearing friction coefficient at work roll.
ν	Poissons ratio.
σ	True stress.
$\Delta\sigma$	Stress differential - bad shape.
$\bar{\sigma}$	Equivalent stress.
$\bar{\sigma}_o$	Equivalent yield stress.
$d\bar{\sigma}_{tot}$	Yield locus translation.
σ_{aax}	Previous maximum flow stress.
σ_{bend}	Bending stress.
σ_f	Flattening stress.
σ_m	Maximum permissible flow stress.
σ_m	Hydrostatic stress.
σ_o	Longitudinal yield stress.
σ_{rec}	Recovery stresses.
σ_{resid}	Residual stress after recovery.
σ_{rf}	Final recovery stress at leveller exit.
σ_{st}	Straightening stress.
σ_T	Applied tensile stress.
$\sigma_x, \sigma_y, \sigma_z$	Normal stresses.
$\sigma_x', \sigma_y', \sigma_z'$	Deviatoric stresses.
$\sigma_{xo}, \sigma_{yo}, \sigma_{zo}$	Normal yield stresses.
$\sigma_{xiso}, \sigma_{xkin}$	Yield stress on reverse loading.
$\sigma_{ziso}, \sigma_{zkin}$	" " " " "

σ_{xF}	Final longitudinal pre-load stress.
$\Delta\sigma_x^E$	Longitudinal elastic stress range on reverse loading.
σ_y	Yield stress.
τ	Shear stress.
$\bar{\tau}$	Mean shear stress.
τ_{max}	Maximum shear stress.
φ	Geometrical wrap angle.
φ_a	Actual wrap angle
φ' and φ''	Actual/geometrical wrap discrepancies.
ψ	Recovery stress/thickness position constant.
ω	Nominal wrap angle.

APPENDIX 12.

A.12.0. References.

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