PREDICTIVE SECURITY ASSESSMENT IN THE ON-LINE ENVIRONMENT OF POWER SYSTEM OPERATIONS

by

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Abstract

This thesis considers the philosophy of security assessment in power system operations particularly in the on-line environment of the System Control Centre of the power utility.

The essential role of the management of conflicts between security and economy is highlighted and determines the need for on-line predictive security assessment facilities within the overall security management process. The special requirements of such a facility are discussed and existing techniques appraised for their suitability to meet these requirements.

The thesis focusses on the trajectory nature of the problem which is a direct consequence of needing to identify the duration of insecure periods and also the probabilistic nature of the problem which is a direct consequence of working predictively.

A description is given of the development of an original set theory approach to identify insecure MW overload contingencies for the daily trajectory. This is derived in the line flow domain in view of the advantages this gives - particularly the ease with which line flow trajectory information can be used and the explicit line flow criteria which indicate when predicted results are no longer valid and so enable the predictive security assessment results to be used for security constraint set reduction in dispatch. The performance of the method is demonstrated on line flow data from the CEGB system.

The need for predicted estimates of the line flow trajectories is essential for the method and so attention is directed at the uncertainties involved. In particular attention is directed at the dispatch activity component of the uncertainty in generator outputs for a Merit Order type of system.

The thesis presents the results of an original investigation into the effect on line flow uncertainties, of high end cost breakpoints in the otherwise linear cost characteristics of the generators. This results from the use of oil burners to increase the output of a coal fired generator unit. The effect on line flow uncertainty, of the time of day or total load dependence of dispatch activity is also investigated. The results of Monte Carlo simulations are discussed. Finally, an original analytic method is developed to model these effects. The method proceeds by computing the joint probability density functions between generators followed by the computation from these of the means and cross covariances between generators and between generators and nodal demands. These means and covariances are then mapped to the line flows by the dc network model, using a conventional stochastic load flow.

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List of Principal Symbols

P	active power flow (MW)
Q	reactive power flow (MVAR)
$\lvert V \rvert$	voltage magnitude
δ	voltage angle
y	admittance of an individual component
$[Y]$	primitive admittance matrix for a network
$[A]$	branch/node incidence matrix for a network
$[K]$	nodal admittance matrix for a network (dc approximation)
R	line thermal rating
$[H]$	branch/node sensitivity matrix (dc approximation)
df	line outage distribution factor
$f(x)$	probability density function of X
x	a particular value of the random variable X
X	a random variable
$E[X]$	mathematical expectation of X
$Var[X]$	variance of X
$Cov[X,Y]$	covariance of X with Y
σ_x	standard deviation of X
Prob $(X<x)$	probability that X is less than the value x
μ_x	the mean or expectation of X
ϕ	the power balance function
r_{XY}	correlation coefficient between X and Y

Subscripts

Li	the ith line
Gj	the jth generator
Di	the demand at the ith node

Superscripts

T	matrix transpose
M	maximum valve
m	minimum valve
*	complex conjugate

1. General Introduction

This thesis deals with one aspect of secure power system operation; that of predictive security assessment within the on-line environment of the system control centre.

The importance of electricity to the society of an industrialised country has resulted in the need to plan and operate electric power systems which are robust to unexpected events. Examples of such contingent events are:-

- environmental events such as lightning strikes and wind or gale leading to transmission line outages
- component failure events with subsequent outage of associated equipment eg. generator, transformer or busbar
- human error or interference resulting in outage of equipment
- unexpectedly high demands by consumers

The concept of ensuring that the power system is able to continue to supply all its consumers within normal voltage and frequency limits, in spite of such occurrences is known as security.

Two types of security are distinguished - generation security and transmission security. Generation security considers the generator contingency reserve requirements to meet generator loss of availability incidents and demand forecast errors. Transmission security considers the operation of the network and associated plant in the presence of outages of transmission equipment, generator outage events, and changes in flows due to changes in generation and demand distribution.

Security assessment is the process of systematically evaluating the effects on the system of each of a defined set of contingencies and picking out those contingencies which result in any insecure condition. Predictive security assessment is the application of this process to anticipated system conditions in the future.

Insecure conditions are identified as conditions which include direct loss of supply or supply quality to consumers. In addition they include conditions in which equipment damage would occur or in which the system is put into an uncontrollable state as these conditions could subsequently lead to loss of supply or supply quality to consumers.

Currently most security assessment in operations is performed on a deterministic set of contingency cases known as the next contingency set. (Ref. 50). These have been criticised on the basis that these do not take account of the unequal probabilities of the various contingencies nor do they consider the effects of less frequent but more severe contingencies.

Alternative probabilistic approaches, notably the security function method (Ref. 48), have been developed. These take account of the unequal probability of occurrence of each contingency and can produce a ranking of contingencies according to their contribution to overall system risk. However, these approaches have not been implemented because of their lack of modelling for common mode contingency cases and the lack of detailed modelling of generation uncertainty such as that due to dispatch activity.

Outage probabilities have also been considered within the context of deciding between out-of-merit running of generators for security reasons and post-fault load shedding. (Ref. 52). However this approach was restricted to decisions within the dispatch phase and related only to generator dispatch.

The need for predictive security assessment is determined by the need for security constrained unit commitment and dispatching (Ref. 1.). Security constrained unit commitment and dispatching are particularly essential for merit order, steam generator based systems.

This is because of the need to avoid situations in which line outage security cannot be covered because the generators in the affected part of the network (particularly importing areas) are all at full output or because ramp rate limits on the generators prevent them reaching the required output within the required timescale.

The need to decouple security assessment from these in the initial stages of the decision making process arises from the economies that can be achieved by considering the possibilities of corrective action on the network and on the demand and from the possibilities for post-fault corrective action which permit the relaxation of a particular constraint. (Ref. 55).

Predictive security assessment capability is required within the on-line environment in order that system control centre staff may review and revise security plans as circumstances change . The requirements for predictive security assessment within the on-line environment include the systematic analysis of the predicted trajectory, the assessment of the effect of uncertainty in the predicted trajectory, the need for criteria which indicate when predicted results may no longer be valid, the provision of results as rapidly as possible, the provision of a database of predicted generator schedules, nodal demands and network configurations, and the provision of suitable man/machine and software system interfaces. (Ref. 2).

1.1 Objectives of this research

This thesis concentrates on two specific requirements for predictive security assessment. These are the identification of the critical contingencies and their times of occurrence for the daily trajectory and the assessment of the uncertainty in these line flow trajectories due to demand forecast errors and its consequent effects on generator uncertainty through dispatch activity. This research addresses the thermal line overload security problem alone.

The potential of set theory in the line flow domain to identify critical contingencies for the daily trajectory is explored in this research. This is because earlier research has been concentrated in the generator and injection domain and required considerable computational effort to reduce the security constraint set (Refs. 8, 9, 11). It was anticipated that the application of set theory in the line flow domain could reduce the potentially active security constraints by exploiting the correlation between generation and demand which results in line flow ranges that are considerably smaller than implied by treating generators and demand independently. Furthermore, a set theory based approach offered the possibility of precise criteria against which the need to revise predictive security assessment could be judged and so enable the results to be used for contingency selection.

The need in predictive security assessment for predicted line flow trajectories also stimulated an investigation into the effects of uncertainties in those trajectories on the identification process. This investigation concentrates on the effects of cost function breakpoints, corresponding to fuel mix changes, on the uncertainty of marginal generators due to merit order dispatch. Earlier research had concentrated on equal incremental cost dispatch (Refs. 23, 45, 46, 47) and a merit order dispatch without cost breakpoints (Ref. 24). The effect of changes in the marginal generators with demand level is also examined in this research.

Earlier research into the modelling of the uncertainty effects due to merit order dispatch resorted to Monte Carlo simulation (Ref. 24). This research develops an analytic approach to the problem.

1.2 Review of This Thesis

In chapter 2 the concept of security is considered and its role in the on-line environment of the system control centre is described. The essential role of the management of conflicts between security and economy is highlighted. This determines the need for on-line predictive security assessment. The special requirements of such a facility are discussed and existing techniques appraised for their suitability to meet these requirements.

Chapter 3 deals with the trajectory analysis problem that arises from the need to identify, and assess in advance, any periods during which system security is jeopardised. The problem is treated initially as a deterministic one, although in chapter 5 it is treated probabilistically. A mathematical formulation of the problem is given and two solution methods to the problem are compared - solution by a set-theoretic method and solution by multiple power flow methods. The comparative tests are based on recorded on-line data. The analysis is confined to the real power part of the steady state security problem. The set-theoretic method is applied in the line flow domain and the constraints due to normal thermal loadings, line outage contingencies and generator loss contingencies are derived in that domain as is the worst case algorithm.

Chapter 4 is concerned with the problem of modelling the uncertainty of the demand and generation which result from the need to predict these quantities for the daily trajectory. An asymptotic random walk model is discussed for use as a representation of demand estimation errors. The uncertainty in marginal generator outputs introduced by dispatch activity is described and an analytic model of the statistics of such generators is derived which includes the effects of cost breakpoints in the cost function. A short period of the recorded operation of such generators is discussed and Monte Carlo simulation study results are shown which verify the analytic model.

Chapter 5 incorporates the analytic marginal generator uncertainty model of chapter 4 into the worst case algorithm of chapter 3 to yield the Probabilistic Worst Case algorithm. An analytic method to combine the nodal demand and marginal generator statistics and produce the nodal injection covariance matrix is derived. This is then used to obtain the line flow statistics using conventional probability theory. The peak trajectory line flow, to a defined level of confidence, is then obtained by repeated application of this process to the trajectory in conjunction with the Chebyshev inequality. The potentially insecure contingencies are then identified by application of the probabilistic worst case algorithm. The effect of various Monte Carlo simulated dispatch activity strategies on line flow uncertainty are appraised and the Probabilistic Worst Case method is compared with a multiple stochastic outage method. The practical application of this research is described.

Chapter 6 discusses the findings and significance of this research and outlines the scope for further work.

1.3 Original contributions offered by this Research

This thesis offers four original contributions to the understanding of predictive security assessment in on-line power system operations.

These are:-

i. The finding that line flow trajectory information (including its uncertainty) can be effectively used in conjunction with set theory ideas to identify critical MW overload contingencies for the daily trajectory.

ii. The development of this finding to produce the Probabilistic Worst Case algorithm for critical MW overload contingency selection.

iii. The findings that generator cost function breakpoints on a merit order system can significantly affect marginal generator MW uncertainties and consequently line MW flow uncertainties and that the degree of uncertainty will vary with time of day.

iv. The development of an analytic method to model these findings for a merit order system.

2. The Concept and Role of Security

2.1 Introduction

The final responsibility for the safe, secure and economic operation of the power system rests with the staff in the system control centre. Their capability to carry out their control responsibilities relies heavily on the adequacy of the operation plans and guidance provided to them by planning staff. For example planned outages of transmission plant or generation for maintenance work need to be carefully co-ordinated by operational planning staff to avoid unnecessary restriction of economic generation or insecure operation for several hours of the day.

In the event, system conditions may differ from those expected conditions on which the plans were based and there is a need to review and revise security plans in the control room environment. This is the function of on-line security management.

2.2 The Concept of Security

The function of an electric power system is to satisfy the system load requirement as economically as possible and with a reasonable assurance of continuity and quality.

Modern society, because of its pattern of social and working habits, has come to expect that electricity supply should be continuously available on demand.

It is an inherent feature of any practical system, such as a power system, that component parts of that system will fail from time to time. If a system is not designed or operated to continue working successfully in the event of a component failure the system will also fail. It is not physically possible to design or operate a system that can never fail. It is, however, possible to design it so that its probability of failure is reduced either by increasing the reliability of the individual components or by introducing extra components to give redundancy.

For example higher quality materials could be used in the construction of equipment and additional transmission lines could be built along different routes.

It is also possible to operate it so that its probability of failure is reduced. For example planned maintenance at periods of low demand rather than during high demand periods and not during adverse weather conditions, avoiding complex switching sequences when planning corrective control actions, avoiding excessive reliance on newly designed equipment which has no known reliability history and providing generation reserve to cover for outage events or demand forecasting errors.

It is also possible to design and operate a power system so that the failure of components causes only limited system failure rather than complete system failure. For example low frequency relays can be designed into the system to disconnect demand in the event of a falling frequency due to generation outages greater than the generation reserve, and selective load shedding can be carried out to remove overloads on transmission lines and so prevent cascade tripping.

2.3 Definition of Security

A secure supply, from the consumer's point of view, may be defined as one which is continuous and of constant voltage and frequency. Such a definition implies in practice that for the generation/transmission system:-

(i) the frequency of supply must be kept within specified limits,

(ii) system voltages must be constrained to within a range where the tap changers on the bulk supply point transformers can hold the distribution network voltage within its statutory limits,

(iii) no item of equipment should be loaded beyond its normal rating (e.g. short-circuit level, voltage insulation level, cable capacity),

(iv) the system must be dynamically stable,

(v) the system should be secure against the next contingency.

Item v implies that in the event of a breakdown in some item of plant causing a redistribution of generation and/or line flows, conditions i-iv will still be satisfied although short term ratings may now be used in calculating overloads. In addition transient stability must be maintained. The contingencies normally considered include:-

A. The loss of any single transmission circuit.

B. The loss of any two transmission circuits mounted on the same tower.

C. The loss of any single item of plant or equipment. e.g. generator or transformer).

One author (Refs. 49, 50) has defined the concept of security in terms of load and operating constraints. The load constraints are the constraints which define the need to meet the load demand. The operating constraints stipulate the need to use system components within their design limits. The security of the system is then defined as the capability of the system to meet defined contingencies without violating the load and operating constraints. To achieve this the system has to meet security constraints.

By means of these three sets of constraints four main classes of operating states have been defined (Ref. 51).

(i) The normal state in which all three sets of constraints are satisfied.

(ii) The alert state in which the load and operating constraints are satisfied but the security constraints are not.

(iii) The emergency state in which the operating and security constraints are not satisfied and the load constraints may or may not be satisfied.

(iv) The restoration state in which the operating constraints are satisfied but the load and security constraints are not satisfied.

2.4 Secure Operation

Secure operation is defined as the concept of operating the power system so that conditions i-iv are satisfied in pre-contingent conditions and that the occurrence of defined contingent events does not infringe conditions i-iv unless acceptable remedial action has been pre-planned.

In terms of the four operating states secure operation is defined as maintaining the system state within the normal state and only permitting operation within the alert state if acceptable remedial action has been pre-planned.

This ensures that insecure conditions do not occur on the power system as a whole as frequently as the aggregate of component failures. On the CEGB system about 600 transmission circuit failures occur per year on the 400 kV and 275 kV network. Only about 2½% of these lead to any loss of supply (Ref. 4) - in part due to the interconnected system and the use of Delayed Auto Reclose (DAR) and in part due to the secure operation concept.

Within the control room context, secure operation also requires that control staff re-establish secure operating conditions following a first fault, so that a subsequent fault is catered for.

Two types of security are distinguished - generation security and transmission security. Generation security considers the generator contingency reserve requirements to meet generator loss of availability incidents and demand forecast errors. Transmission security considers the operation of the network and associated plant in the presence of outages of transmission equipment, generator outage events, and changes in flows due to changes in generation and demand distribution.

2.5 Defined Contingencies

2.5.1 Contingency Level in Operation

As a general rule the high voltage systems of most electric power systems are operated to an (n-1) contingency level. The (n-1) contingency level is the situation of all possible single circuit outages on an n circuit system. In particular cases planned operation is to an n-level as for example when the magnitude of the demand of a consumer or group of consumers may make it uneconomic to provide security against a fault outage when an outage for maintenance is scheduled. In other cases it is to an (n-2) level or higher level as for example common mode outages such as two circuits strung on the same tower, two circuits in close geographic proximity during storms and multiple circuits connected to the same busbar section in a substation.

(a) n-Level

Widescale operation to an n level is not practiced because situations exist in which the only control action available in response to a line fault is to shed sufficient demand to correct the resulting overload and so prevent a cascade situation developing. This situation particularly applies to importing areas which could therefore have a number of load shedding incidents, which from experience is considered to be unacceptable, unless generation is scheduled in the importing area to cover the loss of any one of the lines feeding the area. Typically the number of faults per year per 100km of 400kV circuit in the UK is 2 or 3 (Ref 4) including those which are restored by DAR - although there is considerable variation in this with geographic location and during storm conditions (Ref 34). In the control phase of operations, it is possible to work to an n level for many contingencies as the capability of post-fault flow control action (without shedding load) to restore overloaded line flows can be monitored.

(b) (n-2) Level

In general, planned operation to an (n-2) level is not practiced because the frequency of occurrence of these contingent events is sufficiently low that the resulting number of times of loss of supply to consumers is, from experience, considered to be acceptable.

Furthermore, in general the costs associated with attempting to operate to an (n-2) level, if it were possible (bearing in mind that the system may have been planned to an (n-1) level), would be high as it would require generation considerably in excess of that required to meet the demand, to be synchronized and part-loaded and hence incur the costs of start-up, and out-of-merit running as well as loss of efficiency costs due to the lack of time for maintenance.

(c) Higher Order Levels

In the control phase of operations a containment philosophy is adopted for these and higher order contingency levels, so that in the event the consequences of these contingencies are prevented from causing a complete system failure. In control room operations the (n-2) level refers to simultaneous outages or to outages which occur sufficiently close together such that remedial action for the first fault has not been completed. This is in contrast to the possibility of concurrent outages in which following the first fault the system operating state is revised so that it is secure to the second fault.

2.5.2 Special Contingencies in Operations

In the control phase of operations situations can arise where the contingent events based upon previous history need to be supplemented by 'on-the-day' events such as the risk of circuit trips due to maintenance work being undertaken on protection equipment or an increased risk of trip due to warnings that a particular piece of equipment is suspect but has not yet actually failed.

2.5.3 Statistical Considerations

A number of methods (e.g Ref. 6, 48, 54) have been developed to take account of the unequal probabilities of the various contingent events their variation with time and the variation of the system state probability with system configuration. These methods forecast the security of the system as a function of time into the future, the aim being to produce a security index, the magnitude of which is used to decide the need or otherwise for corrective action in an objective manner. The steady state security function approach of Ref 6 had the form:-

$$S(t) = \sum P_i(t) \cdot Q_i(t) \qquad (2.1)$$

In the above equation

$P_i(t)$ is the probability of the system being in state i at the time t into the future.

$Q_i(t)$ is the probability of violating the load and/or operating constraints at time t if state i exists.

Here the summation is in theory over all possible system states but in practice over states obtainable from the initial state by some maximum number of contingencies, normally two. The security function, therefore, implicitly defines the contingencies which contribute most to the system risk.

By establishing a maximum tolerable value, which the Security function must not exceed, the security of the system can be appreciated. If the time to perform adequate corrective action is longer than the time period from time now to the time at which the maximum tolerable value is exceeded, the system is insecure and corrective action is needed. This probabilistic approach to contingency events has not been implemented because of its lack of modelling for common mode contingency cases and due to its lack of detailed modelling of uncertainties in generation such as that due to dispatch activity.

In practice, many utilities have adopted a deterministic contingency list known as the next contingency set. This usually includes all possible single circuit line outages, transformers, cables and selected generators and infeeds. Some utilities have also included the more frequent common mode outages in this list e.g. double circuit lines, high risk pairs in particular geographic locations and selected busbar section outages. Specific changes are then made to this list, by control centre staff, in the light of anticipated circumstances for the day ahead.

2.6 Security Rectification

2.6.1 Reasons for Conflicts between Security and Economy

Conflicts arise between security and economy in the operations phase despite the fact that the same contingency level may have been used in planning the system. Such conflicts tend to result in the need to meet security by constraining the optimum economic generation pattern.

System planning tends to examine maximum and minimum demand conditions. However, the transition periods during the day or year can be more exacting from an operational security point of view due to the effect of differential generator costs on line flows.

Delays in commissioning new transmission and generating plant and unforeseen circumstances including changes in factors such as power station fuel costs can affect the security on the day.

In addition, an insecurity lasting for a period of a year or so and disappearing as new facilities are commissioned may be judged in the planning phase uneconomic to cover by advancing commissioning dates. As a result such situations must be handled in the operations phase.

2.6.2 Economic Considerations of Insecure Contingencies

It is possible to include the constraints due to a contingency on the minimum cost operating pattern to produce an operating pattern that is secure for the entire period of insecurity of that contingency. This is known as pre-fault corrective action. A cost could be attached to the security rectification of the contingency as the integrated cost difference over the time period of the insecurity. In practice, this process is complicated by contingencies which cause the same active constraint on generation so that the cost attributable to each contingency would need to be apportioned. For example, the security of an importing group may be jeopardised by the outage of any one of the circuits feeding the group. Each of these insecure outages could be rectified by scheduling extra generation in the importing group.

An alternative strategy is post-fault corrective action although this cannot be described as a general alternative for all contingencies. In this strategy the scope for taking corrective action only in the event of a contingency occurrence is considered. In general, this has the advantage of lower overall costs even though more expensive generators may be used, such as Gas Turbines, because the cost is incurred only when the contingency occurs. The disadvantage of post-fault corrective action is that the system is at increased risk to a second fault as the post-fault action capability has been used up.

2.6.2.1 Pre-fault Corrective Action

For some contingencies pre-fault corrective action is the only possible course of action because of the lack of sufficiently fast post-fault measures or sufficiently adequate post-fault measures or because of plant damage that would occur as a result of the fault. For example transient and voltage stability insecurities tend to require action within timescales that are too short for centrally co-ordinated action in the event; line thermal overloads have sufficient overload time capacity but there may be insufficient post-fault flow capability to reduce the flow to within its continuous rating; and excessive short-circuit levels would damage circuit breakers in the event of a short-circuit fault.

(a) Actions on Generation

For some systems, and in particular merit order, thermal generator based systems, security constrained unit commitment and dispatching is essential in order to avoid situations in which line outage security cannot be covered because the generators in the affected part of the network are all at full output or because ramp rate limits on the generators prevent them reaching the required output within the required timescale.

In addition if storm conditions are predicted, pre-fault corrective action through unit commitment is preferred in order to provide the capability to re-secure the system to a second fault.

(b) Action on the Network

The operation of the system must safeguard plant and consumer equipment from excessively high voltages, large step changes or prolonged operation at excessively low voltages. Thus as well as ensuring that normal voltage profiles can be controlled, adequate MVAr reserves must be available to meet the defined contingencies. System switching of circuits and shunt reactors, operation of synchronous compensators and declutched gas turbines, regulation of generator excitation, transformer tap changing and staggering may all be used to achieve dynamic spare MVArs, without needing to constrain economic generation to provide this spare.

Normally solid substations can be run split in order to reduce the fault levels and change power flows without having to constrain economic generation.

Series reactors and line impedance switching can be used to increase the circuit impedance and so re-distribute the power flows without needing to constrain economic generation.

Circuits can be switched to separate sections of busbar to provide two switch security and so prevent the outage of both circuits in the event of a bus coupler or section switch fault.

(c) Action on the demand

The high voltage transmission system usually feeds lower voltage transmission networks. These lower voltage networks in some cases provide the capability to interconnect two or more substations on the high voltage system, although they are normally operated split to avoid excessive fault levels. The possibility exists of moving the split points within the lower voltage network in order to re-distribute the proportion of demand seen by each of the two high voltage substations.

2.6.2.2 Post-fault Corrective Action

Unforeseen changes to the planned generation and transmission facilities have in some cases led to the situation where rigid adherence to the security criteria can cause severe cost penalties. Increasingly, local automatic control facilities are being employed to overcome these active security constraints. These schemes operate automatically or under operator initiation to eliminate limit violations in the event of the specified contingencies. This avoids the cost penalties associated with before the event corrective action.

These post-fault actions are pre-planned actions for foreseeable defined contingencies. This is in contrast to emergency control actions which are in response to unforeseen contingencies.

(a) Action on Generation

A typical example of post-fault corrective action on generation is where insufficient transmission capacity exists to an in merit station to accomodate its full output and meet stability criteria. To overcome this, selected line trippings are arranged to intertrip some of the generation to ensure post fault recovery. Such an approach is possible because generation is already available to cater for unplanned generator outages. Any requirements to trip more generation than the reserve carried would need to consider the implications of shedding some load as well.

The start-up time for emergency start gas turbine generators (2-5 mins) is sufficiently fast to permit the short-time thermal overload rating of transmission equipment to be fully exploited. Therefore these generators can be used in the event to avoid the pre-fault costs that would otherwise be incurred by security constrained unit commitment and dispatching.

(b) Action on the Network

Situations exist where a network switching corrective action applied prior to the fault occurrence would not rectify the situation but would do so when applied following the fault occurence due to the change in impedances of the circuits relative to each other.

For example one line of a double circuit line which feeds into a substation which is normally split may be overloaded by the outage of the other line. Coupling the substation in the event of either line outage may remove the overload, but coupling the substation before either line outage can result in overloads on both lines as this gives a low impedance path relative to other circuits.

(c) <u>Action on the Demand</u>

In general any action on the demand, such as load transfer, that does not incur loss of load would already have been taken as a virtually zero cost pre-fault corrective action. However, other possibilities exist such as post-fault load reduction by voltage and load reductions by disconnection of non-essential load under load management schemes. As these involve some loss of revenue the cost of this loss of revenue (which will be incurred only if the contingency occurs) needs to be balanced against the cost of committing out of merit generation.

There is also the possibility of selective load shedding in the event and the rotation of such load shedding so that any group of consumers only experience loss of supplies for a short time. In general, as the primary business is electricity supply the cost savings for this option need to be significant and the frequency and duration of loss of load experienced by consumers kept within acceptable limits. In addition it might be judged that the social and political consequences in certain geographical areas such as inner city areas also needs consideration.

There are wide variations in the response by consumers to the loss of supply. These range between classes of consumers (compare a large continuous industrial process with the requirements of a domestic consumer), and are affected by the time of day, the use to which each consumer is putting the electricity when the loss is experienced, to the duration of the loss and to how recently the last loss occurred.

Attempts have been made to put a consumer value on the loss of supplies. One such attempt put this value at 35 times the current cost of a unit of electricity (Ref. 3). However, in practice, the acceptability of load shedding as a planned corrective action is based upon the experience of consumer reaction to it.

2.6.3 Cost/Risk Considerations

The probabilities of the insecure contingencies have an important bearing on the decision between pre-fault and post-fault corrective strategies. The decision in favour of pre-fault action would require the security rectification cost (attributable to such pre-fault action for a specific insecure contingency) to be less than the probabilistically weighted post-fault action cost for that contingency. A general characteristic of such decisions is that all post-fault strategies that do not involve load shedding (but including load management and voltage reductions) have cheaper probabilistically weighted costs than pre-fault actions costs (during normal weather conditions) because of the start-up costs of thermal plant, the fuel cost ratios of plant on practical power systems and the relatively low failure rates of transmission lines. During storm conditions these failure rates increase (Ref. 34) to the extent that the decision favours pre-fault action.

For many post-fault load shedding options the cost of pre-fault action would be cheaper than the probabilistically weighted load shed cost due to the very high costs attached to consumer interruptions. However, the less frequent contingencies may push the decisions in favour of post-fault load shed. (Ref.52).

On the CEGB system, a double circuit outage of lines less than 5km in length is usually relaxed to a single circuit outage risk on this basis except if the resulting insecurity is transient or voltage instability.

2.7 Security Assessment

Security assessment or contingency analysis is the process of systematically evaluating the effects on the system of each of a defined set of contingencies and picking out those contingencies which result in any insecure conditions.

Predictive security assessment is the application of this process to anticipated system conditions in the future.

2.7.1 The Need for Predictive Security Assessment in the On-line Environment

The fundamental need for predictive security assessment is determined by the need for security constrained unit commitment. The need to decouple security assessment from unit commitment in the initial stages of the decision making process arises from the economies that can be achieved from considering the possibilities of corrective action on the network and on the demand and from the possibilities for post-fault corrective action which permit the relaxation of a particular constraint.

The present generation of security assessment facilities have been applied on-line in the system control centre to monitor its security against defined contingencies. In practice the value of the facility is limited as it only advises the control engineer that he is already in an insecure state and does not give him sufficient warning to avoid entering it.

A more complete security management system should include a predictive security assessment facility in order to predict insecurities within timescales that permit actions to avoid moving into insecure states.

A predictive security assessment facility would also assist the post-fault decision process, after any overloads have been rectified, where decisions need to be made about the extent to which the system needs to be secured against any subsequent fault. It would assist the unit commitment phase of operations where decisions need to be made about how much constraint margin to provide for transmission contingencies in case of post-fault flows which could be higher than expected.

2.7.2 Advantages of Predictive Security

In addition to these fundamental needs for predictive security asessment, there are a number of practical advantages. A predictive security assessment facility would assist security monitoring by providing an indication of when actual load and generation patterns are tending towards an insecure situation. This would thereby assist decisions as to whether to take preventive action or to let the situation persist and prepare a post-fault contingency plan for corrective action. Furthermore, the knowledge of what the expected insecurities are provides a degree of robustness to data problems with the current state security assessment facility.

It would assist security constrained dispatching by providing a considerable degree of contingency selection.

2.7.3 Requirements for Predictive Security Assessment

(a) Immediacy

Security management is but one function carried out in a system control centre. Furthermore the progression of time in the on-line environment means that there is always a danger of events overtaking the control engineer before a decision has been made.

These two factors require that the on-line predictive security assessment facility be as automatic as possible in terms of picking up nodal demand predictions, generator schedules, and planned network configurations and as fast as possible in producing results.

(b) Data

A regularly updated database of nodal demand predictions, generator schedules, planned network configurations and defined contingencies is essential for predictive security assessment. The need for security constrained unit commitment dictate that these quantities be available for every half-hour of a 24 hour period and for some peak periods, every 15 minutes.

The prediction of nodal demand data and its variation both with time of day and as a proportion of total demand, is an important pre-requisite for this.

(c) Trajectory

The potential duration for which an insecurity may exist is an important factor in deciding between a pre-fault corrective action strategy or a post-fault corrective action strategy. This is because, with a contingency planning strategy, the system is at an increased risk to the occurrence of a second fault. In addition the start-up costs of steam generators dominate the costs of short duration insecurities.

The time of day that an insecure period starts and ends can also be important for revising the synchronising and desynchronising times of affected generators. These two factors require a predictive security facility that is capable of analysing the security of the expected daily line flow trajectories.

(d) Uncertainty

The need to look ahead carries with it a requirement to consider the uncertainties in the estimated line flow trajectories. Providing an indication of the sensitivity of the insecurities to prediction errors ensures that the control engineer is aware of contingencies that could be insecure if the outturn did not match predictions. An additional advantage is that the degree of uncertainty on a line flow provides an indication of the constraint margin that could be built into the scheduling system.

The line flows at each step of the trajectory are derived from estimates of future nodal demands, generator schedules and network configurations. As each of these future quantities is not known precisely, uncertainty exists. Uncertainty exists in the network configuration due to line outage events and maintenance over-run. Uncertainty exists in the nodal demands because the number and magnitude of consumer loads connected at any future instant is not known. Uncertainty exists in generation schedules partly due to plant failures but also to the need in the event to match the total generation with the total demand.

(e) Plan Review

An unplanned outage of a transmission line is an automatic reason to predictively review the security plan.

However, the impact of a generator loss or an unexpectedly high or low demand gives no such clear indication. A regular predictive review of the security plan is therefore required.

In addition to these needs there are the requirements related to the man/machine interfaces and the software system interfaces. These are not discussed here.

This thesis will concentrate on the requirements for the analysis of the trajectory and the uncertainties in predictions. This will be restricted to the line overload security problem and to the uncertainties in line flows due specifically to demand uncertainty and its effect on generator uncertainty through dispatch activity.

Similar analysis requirements are also necessary for the wider security assessment problem which includes transient stability, voltage insecurity, short-circuit levels etc though these are not dealt with here.

2.8 Review of the Literature and the Current Situation

2.8.1 Deterministic Multiple Load Flows

The existing multiple load flow techniques for steady state dc security assessment (eg Ref 12) have been developed to examine the security of only a single nodal injection set. This corresponds to one point in an n-dimensional space of nodal injections. Predictive security assessment requires consideration of multiple n-dimensional points corresponding to multiple time points in the demand/generation trajectory. The straight forward approach of examining as many time points as possible is computationally excessive for most systems. Furthermore these techniques do not provide any explicit criteria to determine when conditions have deviated or are tending to deviate sufficiently far from the conditions on which security plans were based. This means that the review of the plans can only be achieved by regularly repeating the predictive security assessments. In addition the methods provide no measure of the uncertainties in the insecure post-fault flows.

2.8.2 Contingency selection methods

The performance index (Refs. 15, 16,17) and contingency filtering (Ref. 35) methods of contingency selection would help to reduce the computational time required to assess these multiple points. However they do not provide a measure of the uncertainties in the insecure post-fault flows nor do they provide explicit plan review criteria.

2.8.3 Array Processing

The inherent parallelism in contingency analysis suggests that array processing technology should provide an immediate speed advantage (Ref 36). In practice the advantage is not so marked because of the development of sparsity techniques for contingency analysis. In addition array processing does not obviate the need for uncertainty modelling.

2.8.4 Pattern Recognition

Pattern recognition techniques (eg Ref 18) have been developed to consider a limited number of deterministic points in the demand trajectory. The approach of Ref 18 carries out a large number of off-line security assessment studies for different demand and generation patterns and the results classified into secure and insecure states. These are then used on-line to estimate the classification of the actual state which will in general be slightly different to the states studied off-line. This on-line estimation is achieved by computing the Euclidean distances to all the stored patterns. The unknown pattern is classified as the same as the majority of stored patterns within a threshold distance. If no pattern is found within this threshold then a conventional security assessment is carried out. This approach can lead to misclassification of an insecure state if it is closest to several secure states. Furthermore, a fault resulting in a topology change will require re-classification of the stored patterns. In addition no measure of uncertainty is provided.

2.8.5 Group Constraints

The current predictive security assessment procedure used in the CEGB is based upon the concept of group constraints in which the transfers into groups of substations are compared with the capabilities of circuits into the groups. The definition of each group and hence the circuits which make up the group boundary is based upon the experience of security assessment studies carried out through the various stages of the planning and operational planning of the system. The limiting net flow into a group is determined by:-

1. setting up the study conditions which represent the highest net flow conditions into the group for the day.
2. simulating the outage of each single circuit and double circuit into the group to detect the weakest circuit.
3. computing the change in the net flow required to set the flow on the weakest circuit at its limit. The resulting net flow level is the limiting net flow into the group.

The group limits and the circuits defined by the group boundary are dependent upon the generation and demand distribution within the group. As these are not known precisely uncertainty exists in the group limits and the group boundary definition. These occasionally lead to a critical group being overlooked. In addition, in the event the security of the system needs to be considered on a line flow basis because of the effect of variations in generator and demand distributions.

2.8.6 Stochastic Multiple Load Flows

A considerable amount of research has been directed towards the treatment of uncertainty in load flow methods during the last ten years or so. The techniques developed have been called probabilistic load flows or stochastic load flows. The research effort has been focussed on two main areas - the statistical characteristics of the input data, and the transformation of these, through the load flow equations, to derive the statistical characteristics of the output quantities of line flows and voltages.

2.8.5.1 Statistical Characteristics of Input Data

The determination of the statistical characteristics of the input data is itself a difficult problem. Research effort has been directed towards an understanding of the implications of the statistics of the total demand and nodal demand uncertainties; (Ref . 31) towards the implications of the uncertainties in generation due to losses of availability (e.g.Ref. 40); towards the statistical implications of generator control activity including unit commitment and dispatching; (Refs. 22, 23, 24) and towards the uncertainties in the network configurations due to losses of transmission lines (Refs. 41, 42, 43). This thesis focusses on the effect of generator control activity on generator uncertainty.

Effect of Generator Control Activity

The earliest papers on stochastic and probabilistic methods failed to model the effect of generator control activity. Most papers left the balance of power due to demand uncertainty to be provided by the generation at the slack node (Ref. 28). Other papers spread the uncertainty either equally among the generators (Ref. 27) or in proportion to the generator size (Ref. 44).

Most recent papers have started to consider the implications of generator control activity. Ref. 22 considered the effect of unit commitment. Due to the non-linear dependencies a Monte Carlo approach was used to generate the means, variances and cross covariances of the resulting nodal injections. These were then mapped to line flows by the conventional stochastic load flow techniques. The modelling of the unit commitment was, however, limited to the commitment at the daily peak.

A number of papers have also considered the effect of dispatch activity. All of these except (Ref. 24) have been directed at modelling the effect of dispatch activity on an equal incremental cost type system (Refs. 23, 45, 46, 47). In such a system the cost characteristics of the generators are such that changes in demand are met by several generators changing their output accordingly and in an optimal fashion. The overall effect is that at a given expected demand level the dispatch can be modelled as linear proportional factors and hence linear dependences applied to the marginal generators.

Such an approach is inadequate for a Merit Order type system as this results in non-linear dependences between marginal generators. Ref. 24 has resorted to a Monte Carlo simulation method in order to handle this problem. Such an approach does enable a fully rigorous dispatch formulation. However, the paper simply treated the dispatch problem as one of loading generators from their minimum to their maximum in pure merit order. It neglects the effects of cost breakpoints in the marginal generator cost functions. In addition the method is computationally heavy and has not been applied to the trajectory analysis problem.

2.8.6.2 Transformation of Input Statistics to Output Quantities

The majority of the literature on probabilistic or stochastic load flows has focussed on methods for carrying out the transformation of the input data statistics through the load flow equations to the output statistics. In general the various methods can be distinguished by whether they attempt to transform the probability density functions (Refs 28, 29, 31, 37, 38) or whether they map only the expectations and variances (Refs 27, 25, 30). Some authors have labelled the former methods as probabilistic load flows and the latter as stochastic load flows.

The justification for the need to transform the entire probability density function is usually based on the argument that one is attempting to quantify the probability that a particular variable lies above a certain level or outside a certain range and therefore the tails of the probability density need to be adequately modelled as it is the area under the tails of the function which make up the desired probability - particularly if there is considerable probability mass in this region. The need for accuracy in the tail region is then used to justify the need for modelling the effects of the non-linear equations on the statistical mapping. (Ref. 39).

In this work the expected values and variances are considered sufficient for the predictive security assessment application in the control room as the need is for a measure of the sensitivity of post-fault flows to uncertainties.

These techniques all provide a measure of the uncertainty on the line flows.

However, the computation time required to assess the security of the trajectory using stochastic load flows would be excessive. In addition they do not necessarily provide explicit plan review criteria.

2.8.7 Set Theoretic

The set theory approaches to security assessment aim to characterize the set of steady state secure operating points by defining the set of constraints which bound the secure space. This space is termed the Security Region. This approach is in contrast to the load flow methods which establish the security of a single vector point. The attraction of such an approach is the criteria it provides which indicate when predicted results are no longer valid. This means that the predictions can be used in contingency selection.

The set theory approach was first applied to the problem of determining the maximum interchange capability of a system where the limits were due to overloads of certain lines within that system (Ref. 10). This was developed for a single demand level and did not treat the security of the entire network in the system, nor the effect of unit commitment and dispatch on the security of the system.

This work was extended in (Ref. 8) by treating the generators within the system rather than the nett interchanges as the control variables. The generator maximum and minimum limits were used to reduce the constraint set to those which could be potentially active. In addition a linear programming approach was adopted to further reduce this constraint set by maximising the post-fault flow on each line subject to maintaining all the other constraints.

Ref. 9 extended this work by introducing uncertainty in the nodal demands so that the secure region is now defined by line rating constraints which have been down-rated by the degree of uncertainty in the line flow attributable to the demand uncertainty.

These two approaches tended to produce excessively large numbers of non-sparse constraints to define the security region because of the wide range of values possible on generators and demands between the day and night. By considering several points on the trajectory the range at each point is reduced and hence the potentially active constraints are reduced. This was developed in (Ref. 11). This reference further reduced the potentially active constraint set by defining an ellipse around each trajectory point and identifying which constraints were potentially active within the ellipse. However, the effort required to determine which constraints were potentially active in each ellipse at each trajectory point was too excessive for on-line application.

An area that has not been explored, is the application of set theory within the line flow domain rather than in the generator domain. This could reduce the potentially active constraints by exploiting the correlation between generation and demand which results in line flow ranges that are considerably smaller than implied by treating generators and demand independently.

2.9 Objectives of this Research

The aims of this research project were:-

1. To examine the potential of set theory in the line flow domain (earlier efforts have been developed in the generator and injection domains) for identification of critical contingencies for the daily trajectory. This investigation was limited to the thermal line overload security problem.

2. To investigate the effects of cost breakpoints in generator cost functions on MW line flow uncertainties through economic dispatch activity. Previous research has concentrated on equal incremental cost systems. Only one approach has considered Merit Order systems and this neglected cost breakpoints.

   Previous work on Merit Order systems has resorted to Monte Carlo simulation.
3. To develop an analytic method to model these effects.
4. To develop an algorithm to incorporate line flow uncertainty in the line flow domain contingency selector.

   This work is described in the following chapters.

3. The Trajectory Analysis Problem

3.1 Introduction

From time to time in operation, line or generation outages will cause the system to be operated in an unplanned state. In this situation the security of the system must be re-assessed and schedules adjusted to accommodate the change. The on-line contingency analysis program indicates the insecurities existing on the current faulted system state but gives no indication of the insecurities that could arise in the near future operating states. While this knowledge can be used to take immediate corrective action, information about the duration of the current trouble spots and the location of future trouble spots is necessary to optimise the corrective action.

This requires some form of analysis of the expected trajectory of the individual line flows, which must be capable of use in the on-line control room environment.

3.2 Mathematical Formulation

Consider the real power part of the complex power problem:

$$P = \text{Real } [S]$$

Where P is the real power component and S is the complex power. Given the real power demand versus time trajectory for each nodal demand on the system:

$$P_{Di}(t)$$

where i represents the ith node, so that

$$\sum_{i=1}^{ND} P_{Di}(t) = D(t) \qquad \text{for } 0 \leqslant t \leqslant t_p \quad \text{hours} \tag{3.1}$$

where ND is the number of nodes with demand which is usually a subset of the total number of nodes N,

$D(t)$ is the total system demand versus time trajectory and

t_p is the prediction period in the range 0 to 24 hours.

And given the generation versus time trajectory for each generator on the system (and external transfers if any):

$P_{Gj}(t)$

where j represents the jth generator, so that

$$\sum_{j=1}^{NG} P_{Gi}(t) = G(t) \tag{3.2}$$

where NG is the number of generators on the system,

$G(t)$ is the total generation versus time trajectory.

And given that the real power losses are negligible compared to the overall demand, so that

$$G(t) = D(t) \tag{3.3}$$

Given also the planned transmission network topology with respect to time

$A(t)$

where the bus incidence matrix A is an L X N matrix in which L is the number of branches in the network and N is the number of nodes.

The elements of A are such that

$a_{ij} = 1$ if the ith branch is incident to and oriented away from the jth node

$a_{ij} = -1$ if the ith branch is incident to and oriented towards the jth node

$a_{ij} = 0$ if the ith branch is not incident to the jth node.

The problem is to identify the set (if any) of contingencies which violate any of the security limits:-

$$\left| I_{\ell}^{i}(t) \right| \leqslant I_{\ell}^{M} \qquad \text{for} \quad \ell = 1, L \tag{3.4}$$

where $I_{\ell}^{i}(t)$ is the line current trajectory of line ℓ due to the ith contingency and I_{ℓ}^{M} is the thermal current rating of line ℓ.

3.3 Solution by the Set-Theoretic Approach

Given a set of postulated contingencies, the assessment of steady state security can be posed in two alternative ways: for a fixed pre-contingency steady state, and given that equipment ratings and other operational limits are satisfied, will the operating state resulting from the occurence of any one of the postulated contingencies result in any violation of equipment or operational limits? If the answer is negative, the pre-fault steady state is termed secure.

Alternatively, one can ask: given the set of postulated contingencies, what is the set of all pre-contingency operating states that are secure? The resulting set, which will depend on the network structure, the set of postulated contingencies, and the equipment and operational limits constitutes the steady state security region. The set-theoretic approach characterises this set by systematically mapping the constraints of the post-contingency operating space into the pre-contingency space.

3.3.1 The Injection Domain

The post-contingency states can be mapped into the corresponding pre-contingency states by the steady state-load flow equations. A balanced three-phase power system is assumed, and the transmission system is represented by its positive-phase-sequence network of linear lumped series and shunt branches (Fig. 3.1) which is valid for lines up to about 250 km in length. Nodal analysis, rather than loop analysis, is preferred because of the simplicity of data preparation and the ease with which the bus admittance matrix can be formed and modified for network changes in subsequent cases.

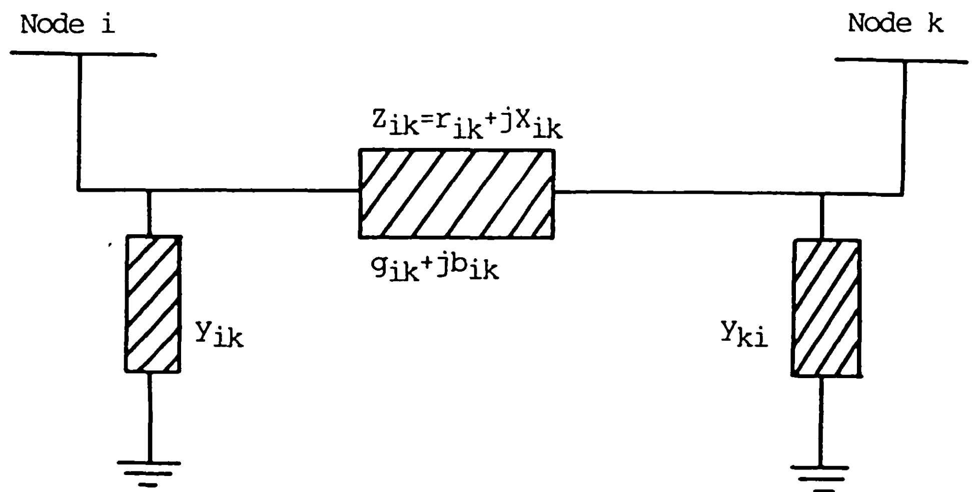
The Steady State Load Flow Equations

By considering Kirchoff's Current Law the complex current $\bar{I}_i$ (superbar indicates a complex quantity) entering bus i of an electrical transmission network can be expressed as:

$$\bar{I}_i = \bar{Y}_{i1}\,\bar{V}_1 + \bar{Y}_{i2}\,\bar{V}_2 + \ldots\ \bar{Y}_{iN}\,\bar{V}_N \tag{3.5}$$

$$= \sum_{j=1}^{N} \bar{Y}_{ij}\ \bar{V}_j$$

where $\bar{Y}_{i1}$, $\bar{Y}_{i2}$ etc. represent the elements of the nodal admittance matrix. In particular, $\bar{Y}_{ii} = G_{ii} + jB_{ii}$ is the sum of the admittances of the branches connected directly to node i; $\bar{Y}_{ij} = G_{ij} + jB_{ij} = Y_{ij}\,e^{j\theta ij}$ are the off-diagonal elements of the nodal admittance matrix and are obtained as the negative of the admittance connecting nodes i and j.



r_{ik}= transmission line resistance

X_{ik}= transmission line reactance

y_{sh}= total line susceptance
(i.e. $y_{sh} = j/X_C$
where $X_C = 1/_{wc}$ the capacitive reactance of line)

$y_{ik} = y_{ki} = y_{sh}/2$

$g_{ik} = r_{ik}/|Z_{ik}|^2$

$b_{ik} = -X_{ik}/|Z_{ik}|^2$

Fig. 3.1 Equivalent π circuit for Transmission Line

$\overline{V}_i = |V_i| \, e^{j\delta i}$ is the complex voltage at node i;

δi being the angle relative to the reference node.

N is the number of busses in the system.

Now:

$$\overline{S}_i = P_i + jQ_i = \overline{V}_i \, \overline{I}_i^* \tag{3.6}$$

where P_i and Q_i represent the net real and net reactive power injections at node i respectively; and an asterisk indicates the complex conjugate.

Substitution of equation (3.5) into equation (3.6) yields:

$$\overline{S}_i = \overline{V}_i \sum_{j=1}^{N} \overline{Y}_{ij}^* \, \overline{V}_j^* \tag{3.7}$$

Equation (3.7) constitutes the general form of the Steady State Load Flow Equations. (Ref 13).

For a given loading condition, each bus in the power system is characterised by the four variables:

$|V_i|$, δ_i, P_i and Q_i

Two of these are specified and the other two must be found.

In a power system of N busses, 2N variables are specified, the remaining 2N unknowns can be determined by solving the system of non-linear equations given by (3.7). Due to the non-linear nature of these equations, there is no assurance that a unique solution can be obtained, but in a normal operating region there is usually one meaningful solution.

In the basic formulation, the busses are divided into three types, depending upon which variables are specified. These are:

(a) A PQ-bus, P_i and Q_i are specified, $|V_i|$ and δ_i are unknown.

(b) A PV-bus, P_i and $|V_i|$ are specified, Q_i and δ_i are unknown.

(c) A slack or reference bus, $|V_i|$ and δ_i are specified, P_i and Q_i are unknown. One slack bus must be given in a power system in order to ensure that the total generation is equal to the total load including transmission losses.

The unknown values of P_i and Q_i and all the power flows in the system are found, when all the bus voltages are known in magnitude and phase. Therefore the basic problem is to find unknown voltage magnitudes and phase angles. A variety of methods exist to solve these equations including the Newton-Raphson technique, the Fast Decoupled and the Second Order method. However, if one is only interested in real power flows a number of simplifying assumptions can be made to equation (3.7) to yield the dc load flow equations.

Separating real and imaginary parts in equation (3.7) and working in polar coordinates we can write for node i.

$$P_i = \text{Real}\left[\sum_{j \in i} |V_i| . |V_j| . |Y_{ij}| . e^{j(-\theta_{ij} - \delta_j + \delta_i)} \right] \tag{3.8}$$

If line resistances are neglected then $\overline{Y_{ij}} = jB_{ij}$ with the additional implication that there are no I^2R losses in the network and we have:-

$$P_i = \sum_{j \in i} |V_i| . |V_j| . B_{ij} \sin (\delta_i - \delta_j) \tag{3.9}$$

In high voltage transmission systems the transmission angles are usually small ($\simeq 10^{o}$) so that the higher order terms in the $\sin(x)$ series expansion can be neglected.

$$P_i = \sum_{j \in i} |V_i| \cdot |V_j| \cdot B_{ij} \quad (\delta_i - \delta_j) \tag{3.10}$$

Under normal operating conditions the voltage magnitudes on a high voltage network remain within $\pm$ 2% to 3% of nominal so that the assumption of 1 p.u. at all nodes is a reasonable one.

Hence we get

$$P_i = \sum_{j \in i} B_{ij} \quad (\delta_i - \delta_j) \tag{3.11}$$

or in matrix form:

$$\left[P_I\right] = \left[K\right] \left[\delta\right] \tag{3.12}$$

where $K_{ii} = \sum_{j \neq i} B_{ij}$ and $K_{ij} = - B_{ij}$

and is a non-singular nodal admittance matrix of dimension N x N.

Computationally, the matrix K can be formed from the matrix product:

$$\left[\begin{array}{c|c} K & \\ \hline & \end{array}\right] = \left[\begin{array}{c|c} A^T & \\ \hline & \end{array}\right] \left[\begin{array}{c|c} Y & \\ \hline & \end{array}\right] \left[\begin{array}{c|c} A & \\ \hline & \end{array}\right] \tag{3.13}$$

with the reference row and column removed.

where $[A]$ is the L x N bus incidence matrix and $[A^T]$ its transpose.

$[Y]$ is the L x L diagonal primitive admittance matrix, whose elements are the admittances of the individual lines. Only if there were magnetic coupling between lines would there be non-zero off-diagonal terms.

In this work the reference node has been taken as a fictitious node outside the system and connected by high impedance lines to the real system. Equation (3.12) provides the mapping between the $[\delta]$ vector space and the $[P_I]$ vector space. We need now to map from the real power flow $[P_L]$ vector space to the $[\delta]$ vector space.

The current flow through a transmission line can be expressed as:

$$\bar{I}_{ij} = \bar{Y}_{ij} (\bar{V}_i - \bar{V}_j) \tag{3.14}$$

where the variables are defined as in equation (3.5) and the complex power flow through the line as:

$$\begin{aligned} \bar{S}_{ij} &= P_{ij} + jQ_{ij} = \bar{V}_i \bar{I}_{ij}^* \\ &= \bar{V}_i \bar{Y}_{ij}^* (\bar{V}_i^* - \bar{V}_j^*) \end{aligned} \tag{3.15}$$

Considering real power flows only we have:

$$P_{ij} = \text{Real}\left[\bar{V}_i \bar{Y}_{ij}^* (\bar{V}_i^* - \bar{V}_j^*)\right] \tag{3.16}$$

and introducing the assumptions used in deriving the dc load flow equations we get:

$$P_{ij} = B_{ij} (\delta_i - \delta_j) \tag{3.17}$$

Expressing equation (3.17) in matrix form we get:

$$\left[P_\ell \right] = \left[Y_\ell \right] \cdot \left[A \right] \cdot \left[\delta \right] \tag{3.18}$$

Equation (3.18) provides the required mapping. However, since the set of equations (3.12) and (3.18) are linear we can substitute (3.12) into (3.18) directly to give:

$$\left[P_{\ell}\right] = \left[Y_{\ell}\right] \cdot \left[A\right] \cdot \left[K\right]^{-1} \cdot \left[P_I\right] = \left[H\right]\left[P_I\right] \tag{3.19}$$

[H] is an L x N transfer matrix relating branch flows to net bus injections.

Constraint Mapping

Thus from (3.19) we can map directly between the $[P_L]$ vector space and the $[P_I]$ vector space using the linear dc load flow model. The mapping for line outage conditions is also achieved by equation (3.19) but with matrix [H] modified to simulate the line outage condition as described in 3.3.4.2.

Existing literature has defined the secure operating space (sometimes termed a polytope in the hyperspace) of a network either in terms of constraints (sometimes termed hyperplanes in the hyperspace) on the net interchange between areas (Ref. 10) or constraints on generation and demand levels (Refs. 8, 9 and 11) using the mapping provided by (3.19). The secure operating space can also be defined in terms of constraints on the line flow space.

3.3.2 The Line Flow Domain

For a large-scale power system, the number of constraints defining a secure generation and demand level operating space is very large and is of the order of:

2M (L-1)

where M is the number of contingency cases.

Each constraint equation which is in the form of equation (3.19) is non-sparse, and even after deleting redundant constraints, security assessment based upon these is not competitive with conventional multiple load flow methods. The advantages of working in the line flow domain include the ease with which the number of constraints can be dramatically reduced by including line flow trajectory knowledge, the ease of handling line flow uncertainties and the explicit criteria provided for indicating when the prediction becomes invalid.

The constraints that define the steady state security region depend on the security criteria (see 2.3) of the power system. In general these include the following conditions for line overload security:-

(a) Normal Thermal Loading

The power flows in all transmission elements (lines and transformers) should not exceed their thermal ratings in the operating state for the current network.

(b) Line Outage Contingency

The power flows in all transmission elements should not exceed their thermal ratings nor should the system be split if any one of a set of defined line or transformer outages were to occur. These generally include single circuit outages, double circuit outages for lines strung on the same tower and multiple outages which may occur in the event of a loss of a 'Teed' transmission line or a section of busbar in a substation. Only selected bus sections are simulated. These are bus sections that have critical circuits connected to the one section.

(c) Generator Loss Contingency

The power flows in all transmission elements should not exceed their thermal ratings if any one of a defined set of generator outages were to occur, the resulting generation deficiency being allocated to other generators as a function of the frequency response of the remaining generating units or of the response of fast- acting pumped storage generator units.

We now wish to express each of these conditions in terms of constraints on the pre-contingency power flows:

3.3.3 Normal Thermal Loading Constraints

The thermal limits of the transmission elements (lines and transformers) are simply given by the inequality constraints:

$$-R_{\ell} \leq P_{\ell} \leq R_{\ell} \quad ; \quad 1,2...L \qquad (3.20)$$

The emergency line overload ratings or winter or summer ratings could also be included without loss of generality. R_{ℓ} is the thermal rating in MW's of line ℓ . Note that:

$$R_{\ell} = aI_{\ell}^{M}-b \qquad (3.21)$$

Where 'a' is a factor that makes an allowance for the reactive component of the line flow and off-nominal voltages, and 'b' is a factor that makes allowance for the error in real power flow in a dc load flow (Ref. 14).

This approach, of effectively down-rating the line, is entirely valid in the predictive security assessment of a trajectory as the objective is purely to identify the critical outages for closer investigation and so a high degree of accuracy is not required at this stage - the emphasis is on reliability.

The same approach is currently used in practice for the treatment of uncertainty in line flows due to demand prediction uncertainty. However, this does assume some a priori knowledge of the degree of uncertainty on each line flow. Underestimating this would cause potentially critical outages to be missed. Overestimating it could introduce unacceptably large numbers of outages which are not critical. A mathematical model of uncertainty would be desirable. This is examined in chapters 4 and 5.

3.3.4 Line Outage Contingency Constraints

Line outage contingencies include both single branch outages and multiple branch outages acting on the planned network operating arrangement. In order to determine the constraint mapping we first derive the equations relating the post-fault line flows to the pre-fault flows.

3.3.4.1 Line Outage Simulation

There are a variety of approaches for simulating line outages on a network, including:

(a) The elements of the explicit inverse $[K]^{-1}$ could be temporarily modified, to account for line changes, using the Matrix Inversion Lemma (Ref 7).

(b) The original system solution could be temporarily modified to account for matrix changes using the compensation theorem (Ref 12).

(c) A permanent change could be made to the original nodal admittance matrix and the new system state computed from equation (3.19).

(d) Distribution factors could be derived to give the proportion of the pre-fault flow which is distributed to each of the remaining lines.

Each of these will be described in turn.

3.3.4.2 Matrix Inversion Lemma

We require a new solution $[\delta]_{new}$ (or $[P_\ell]_{new}$ for the line flows), following the simulated outage of a line from the system. The Matrix Inversion Lemma for linear systems (Ref 7) is used to amend the known inverse $[K]^{-1}$.

The formula gives:

$$([A] + [C]\ [D]^T)^{-1} \tag{3.22}$$

$$= [A]^{-1} - [A]^{-1}\ [C]\ ([I_m] + [D]^T\ [A]^{-1}\ [C])^{-1}\ [D]^T [A]^{-1}$$

where $[A]$ is an N x N non singular matrix and

$[C]$ and $[D]$ are N x M matrices that give the changes in the elements of $[A]$

$[I_M]$ is an M x M identity matrix.

Thus the problem of inverting

$$[A]^{-1}_{new} = ([A] + [C]\ [D]^T)^{-1} \tag{3.23}$$

simplifies to that of inverting an M x M matrix (particularly when $M \ll N$) provided $[A]^{-1}$ is available.

To use the Lemma in power systems analysis we make

$[A] = [K]$

$[C] = -[a]$ the N x M incident matrix in which M lines are outaged in an N node system.

$[D]^T = [y]\ [a]^T$ in which $[y]$ is the M x M primitive admittance matrix of outaged lines.

Now the original system is:

$$[\delta] = [K]^{-1}\ [P_I] \tag{3.24}$$

and we must find the solution $[\delta]_{new}$ if M lines are opened. The new matrix is $[K]_{new} = [K] - [a]\ [y]\ [a]^T$

Hence from equation (3.22) we have:

$$[K]^{-1}_{new} = [K]^{-1} + [K]^{-1}\ [a]([I_m] - [y]\ [a]^T [K]^{-1} [a])^{-1}\ [y]\ [a]^T\ [K]^{-1} \tag{3.25}$$

Equation (3.25) can be used directly to solve for the new $[\delta]$ as:

$$[\delta]_{new} = [K]^{-1}_{new}\ [P_I] \tag{3.26}$$

Alternatively, expanding equation (3.26) and substituting equation (3.24) the new solution is:

$$[\delta]_{new} = [\delta] + [K]^{-1}\ [a]([I_m] - [y]\ [a]^T\ [K]^{-1}[a])^{-1}\ [y]\ [a]^T\ [\delta] \tag{3.27}$$

i.e. the new solution can be obtained as a correction applied to the original network solution so there is no need to re-factorise the triangular factors of $[K]$. This is the same result as obtained by the compensation theorem approaching the problem from the network viewpoint.

3.3.4.3 Compensation Theorem

The compensation theorem (Ref. 12) states, in effect, that a passive network element can be replaced by an active element that causes the same network response. This means that in the nodal formulation of a network problem a current source can be substituted for a passive branch element without affecting the voltages or currents in the network.

This is a desirable property as it provides a means of avoiding refactorisation of the nodal admittance matrix for each and every contingency case. The derivation of these equations is as follows:

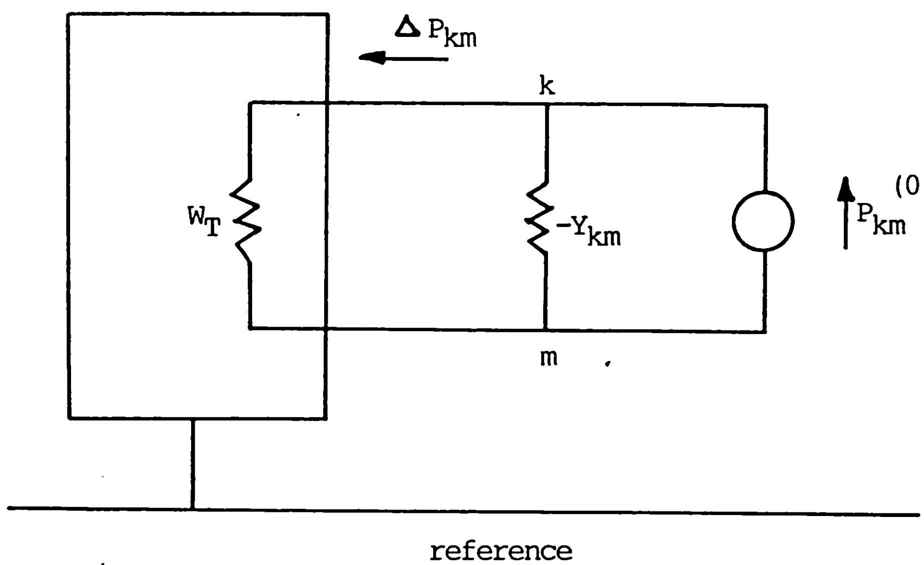
We wish to remove a branch connecting nodes k and m which has a real power flow of P_{km}.

We define an (N x 1) vector as:

$$[a] = \left[\begin{array}{c} 0 \\ 1 \\ 0 \\ \cdot \\ \cdot \\ -1 \\ 0 \end{array}\right] \begin{array}{l} \\ \leftarrow K \\ \\ \\ \\ \leftarrow M \\ \\ \end{array} \qquad (3.28)$$

Let $-y_{km}$ be the admittance of a new branch connected between k and m, in parallel with those already there (Fig. 3.2) where y_{km} is the admittance of the branch whose outage is to be simulated.

Fig. 3.2 NETWORK FOR SINGLE LINE OUTAGE



Then the effect is the same as opening the original branch since the resultant impedance becomes infinite.

Now the pre-outage flow $P^{(0)}_{km}$ in branch k-m, can be distributed to its terminal nodes by making

$$\Delta P_k = P^{(0)}_{km}$$

$$\Delta P_m = -P^{(0)}_{km} \tag{3.29}$$

The part that reaches the original intact network ΔP_{km} can be calculated by solving the equation:

$$\Delta P_{km} = \frac{(^{-1}/y_{km})P^{(0)}_{km}}{W_T + (^{-1}/y_{km})} = \frac{-[a]^T[K]^{-1}[P_I]}{W_T + (^{-1}/y_{km})} \tag{3.30}$$

Where W_T is the Thevenin equivalent impedance shown in (Fig. 3.2). W_T can be obtained as

$$W_T = [a]^T [K]^{-1} [a] \tag{3.31}$$

by considering a source current of 1 p.u. connected between nodes k and m in the intact network. The angle change $[\Delta\delta]$ on the intact network can thus be calculated from equation (3.12) by making

$$[\Delta P] = [a]\Delta P_{km} \tag{3.32}$$

Now from superposition we have:

$$\begin{aligned} [\delta]_{new} &= [\delta] + [\Delta\delta] \qquad (3.33) \\ &= [\delta] + [K]^{-1}[a]\Delta P_{km} \\ &= [\delta] + \\ &\quad [K^{-1}][a](1 - y_{km}[a]^T[K]^{-1}[a])^{-1} y_{km}[a]^T[K]^{-1}[P_I] \end{aligned}$$

which is the same result as that obtained from the Matrix Inversion Lemma.

3.3.4.4 Permanent Changes

The sequential simulation of large numbers of line outage contingencies is not normally performed by making explicit changes to the nodal admittance matrix and subsequently either inverting it or computing its triangular factors. This is simply because of the additional number of numerical calculations required by this approach compared to that of the matrix modification schemes or compensation approach.

3.3.4.5 Distribution Factors

The result derived in equation (3.27) can be put in terms of line flows to give the proportion of the pre-fault flow, of the opened line or lines, which is distributed in each of the remaining lines. These are the distribution factors (Ref 12) which we derive for single and multiple line outages as it is in this form that the constraint mapping is performed in the line flow domain. This calculation need only be carried out once for the given network but since it will be required following a fault outage, some attention to numerical efficiency is justified.

(i) Single Line Outage

We have:-

$$[P_\ell]^{km} = [Y_\ell]^{km} [A]^{km} [\delta]_{new} \qquad (3.34)$$

where $[P_\ell]^{km}$ is the post-fault flow in the network following the outage of line k-m

$[Y_\ell]^{km}$ is the primitive admittance matrix with Y_{km} set to zero

$[A]^{km}$ is the branch incident matrix with the row corresponding to line k-m se* to zero.

Defining equation (3.27) for one line outage and substituting the result into (3.34) we have:

$$[P_\ell]^{km} = [Y_\ell]^{km}\,[A]^{km}\,[\delta] + [Y_\ell]^{km}\,[A]^{km}\,[K]^{-1}\,[a](1-y_{km}\,[a]^T\,[K]^{-1}\,[a])^{-1}y_{km}\,[a]^T\,[\delta]$$

and noting that $P_{km} = y_{km}\,[a]^T\,[\delta]$

and that $[P_\ell] = [Y_\ell]^{km}\,[A]^{km}\,[\delta]$ except for $P_{km} = 0$

we have:

$$[P_\ell]^{km} = [P_\ell] + [DF]^{km}\,P_{km} \tag{3.35}$$

where $[DF]^{km} = [H]^{km}\,[a]\,(1-y_{km}\,[a]^T\,[K]^{-1}[a])^{-1}$ (3.36)

is an L x 1 matrix of the distribution factors of P_{km}.

i.e. for line i-j

$$df^{km}_{ij} = (h_{ij,k} - h_{ij,m})/[1 - y_{km}\,(Z_{kk} + Z_{mm} - 2Z_{km})]$$

The identification of an outage which splits the system can be identified from equation (3.36) as in this situation the Thevenin equivalent impedance equals the impedance of the outaged line. Therefore the denominator term in (3.36) becomes zero and a divide by zero situation arises. This condition can be explicitly checked and identified.

(ii) Simultaneous Multiple Outages

Assume the simultaneous outage of branches k-m and i-j.

We have:

$$[P_\ell]^{km,ij} = [P_\ell]+[H]^{km,ij}[a]\ ([I_m]-[Y]\ [a]^T[K]^{-1}[a])^{-1}\begin{bmatrix} P_{km} \\ P_{ij} \end{bmatrix}$$

$$= [P_\ell] \quad + [DF]^{km,ij}\begin{bmatrix} P_{km} \\ P_{ij} \end{bmatrix} \tag{3.37}$$

Where $[DF]^{km,ij}$ is an (L x 2) matrix and for line r-s

$$[DF]^{km,ij}_{rs} = [(h_{rs,k} - h_{rs,m}),\ (h_{rs,i} - h_{rs,j})] \times \begin{bmatrix} (1-y_{km}\ (Z_{kk}+Z_{mm}-2Z_{km})), & -y_{km}(Z_{ik}-Z_{jk}-Z_{im}+Z_{jm} \\ -y_{ij} - (Z_{ik}-Z_{im}-Z_{jk}+Z_{jm}), & (1-y_{ij}(Z_{ii}+Z_{jj}-2Z_{ij})) \end{bmatrix}^{-1} \tag{3.38}$$

The expressions developed here are perfectly general. For M outages, [DF] becomes a (L x M) matrix, requiring the inversion of an (M x M) matrix which remains trivial for a reasonable number of simultaneous outages.

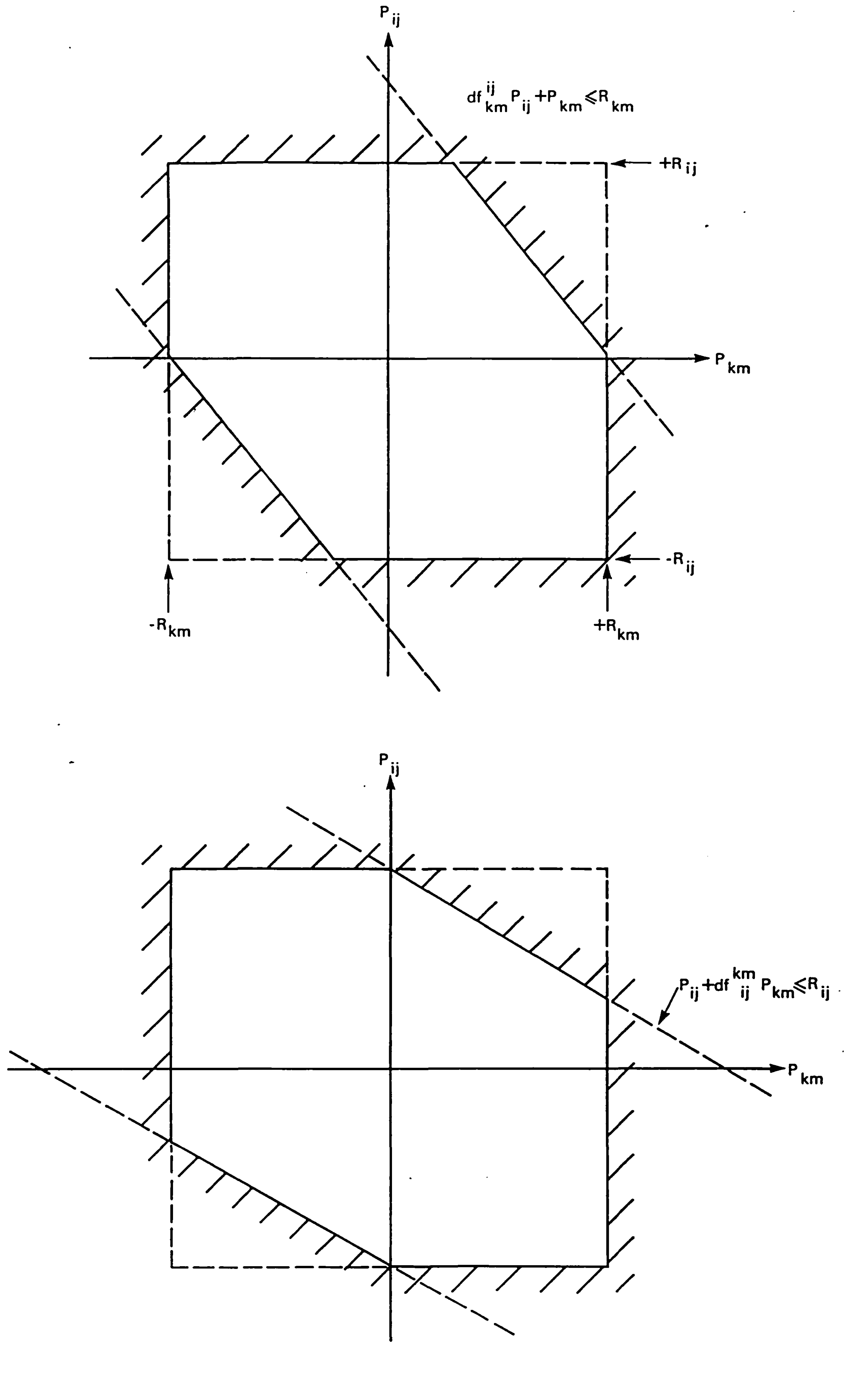
Now for the system to be secure against line overloads in the event of an outage or outages then the post-fault flow on all lines must be less than their thermal ratings, i.e. in the single outage case we have:

$$-[R_\ell] \leqslant [P_\ell] + [DF]^{km}P_{km} \leqslant [R_\ell] \tag{3.39}$$

The set of equations (3.39) are basic in this work as they provide the required constraint mapping in the line flow domain. They say that the flow across line i-j with line k-m out equals the base case flow across line i-j plus a proportion of the base case flow across the removed line. The proportionality constants df^{km} have a magnitude less than or equal to 1 and are also purely network dependent and invariant to injection changes. These factors have been derived by exploiting the linearity of the network equations with regard to MW flows. The derivations make the same assumptions as the dc load flow regarding the network voltage magnitudes and the transmission angles. Their use is therefore only appropriate where the use of a dc load flow is considered adequate. Graphically equations (3.39) and (3.20) appear as in Fig 3.3 for any line pair i-j and k-m. These show a clearly defined secure flow space, for the pre-outage flows on lines i-j and k-m, bounded by the constraints of equations (3.39) and (3.20).

A similar graphical interpretation can be shown for the constraints in the two outage case. However, in this case we have a 3-dimensional space rather than the 2-dimensional space of (Fig. 3.3). Beyond the two outage case we get more than 3 dimensions so we can no longer visualise the secure space in this way, although the analytic equations can be applied to any number of simultaneous outage cases.

Fig.3.3 LINE FLOW DOMAIN - NETWORK CONSTRAINTS



The simulated outage of a 'Teed' circuit, which results in a loss of a loadless node from the mathematical model of the network, could be treated by simulating the simultaneous outage of only two out of the three branches.

Where simulated outages result in demand being lost, eg the outage of a section of busbar resulting in a line outage and a load transformer outage, the generation output changes in accordance with generator governor characteristics. It is usually acceptable in line overload security studies to spread the demand loss among the generators in proportion to their size. The constraints due to such contingencies will be a combination of the constraints discussed in sections 3.3.4.5 and 3.3.5. The derivation of the constraint equations follows the derivations discussed in these sections.

(iii) Sparsity

For large networks many of the distribution factors obtained from equation (3.39) are small as these represent lines that are relatively insensitive to that particular outage.

Those distribution factors that are negligible can be eliminated and enable the benefits of sparsity programming to be obtained. The criteria for this pre-selection is based upon the observation that, from a security point of view, when the transferred power is small compared to the maximum permissible flow on the line to which the power is transferred then this distribution factor can be ignored.

This requires a corporate choice of the level at which a flow is considered small and choice of flow at which a line is considered overloaded.

On this basis the distribution factors which satisfy the following relation can be ignored:

$$|d_{ij}| \leqslant 0.05 \times R_j/R_i \qquad (3.40)$$

where d_{ij} is the ijth element of the distribution factor matrix and R_i, R_j are the thermal ratings of line i and line j, and where small is defined as 5% of the transfer line flow limit.

More factors may be eliminated based upon the sign of the distribution factor and whether or not this causes a flow increase on the transfer line. This selection, however, is flow direction dependent and therefore cannot be computed once only as with the magnitude selection criteria. The benefit in sparsity so obtained would therefore be marginal.

3.3.4.6 Worst Case Algorithm

The elements of this sparse matrix of distribution factors can be used directly to compute the vector of changes in real power flows on each line for each outage, given a vector of line flows.

Traditionally this vector of line flows has been a set of flows which exist simultaneously on the system for the security study. However, as each of the equations in (3.39) are independent, any set of line flows can be used. The simultanaeity condition is not required. Given that we know the range of the flow on each line:

$$[P_{\ell}]^{min} \leqslant [P_{\ell}] \leqslant [P_{\ell}]^{max} \tag{3.41}$$

then each of the summations in (3.39) must also be bounded and we can find:

$$\max \left\{ \left| P^{min}_{ij} + df^{km}_{ij} \cdot P^{min}_{km} \right|, \left| P^{min}_{ij} + df^{km}_{ij} \cdot P^{max}_{km} \right|, \right. \tag{3.43}$$

$$\left. \left| P^{max}_{ij} + df^{km}_{ij} \cdot P^{min}_{km} \right|, \left| P^{max}_{ij} + df^{km}_{ij} \cdot P^{max}_{km} \right| \right\}$$

$$= P^{km,max}_{ij} \quad \text{for all line pairs} \tag{3.42}$$

Now provided that:

$$P^{km,max}_{ij} \leqslant R_{ij} \qquad \text{for all line pairs} \tag{3.43}$$

the system is secure and will be secure for any smaller values of $P^{km,max}_{ij}$. But if for some line outage k-m, equation (3.43) is not completely satisfied then the system is not secure against this outage. However, because the time related behaviour of line flows has been excluded from the analysis, an occasional insecure classification may occur for a line whose peak flow occurs at a different time to the peak flow on the other line and which could therefore be secure. This situation can be explicitly identified with a load flow or by considering the time related behaviour of the line flows concerned. (Note that a 'secure' classification for an 'insecure' outage could never occur).

In graphical terms, the range boundaries on the flow trajectories of lines i-j and k-m are as shown in Fig. 3.4 . The maxima of the vectors

$$P_{ij} + df_{ij}^{km} P_{km} \qquad \text{and} \qquad P_{km} + df_{km}^{ij} P_{ij}$$

correspond to one of the breakpoints, so the maximum magnitude of

P_{ij}^{km} and P_{km}^{ij}

is dependent upon the magnitude and sign of df_{ij}^{km}, P_{ij}^{min}, P_{ij}^{max}, P_{km}^{min},

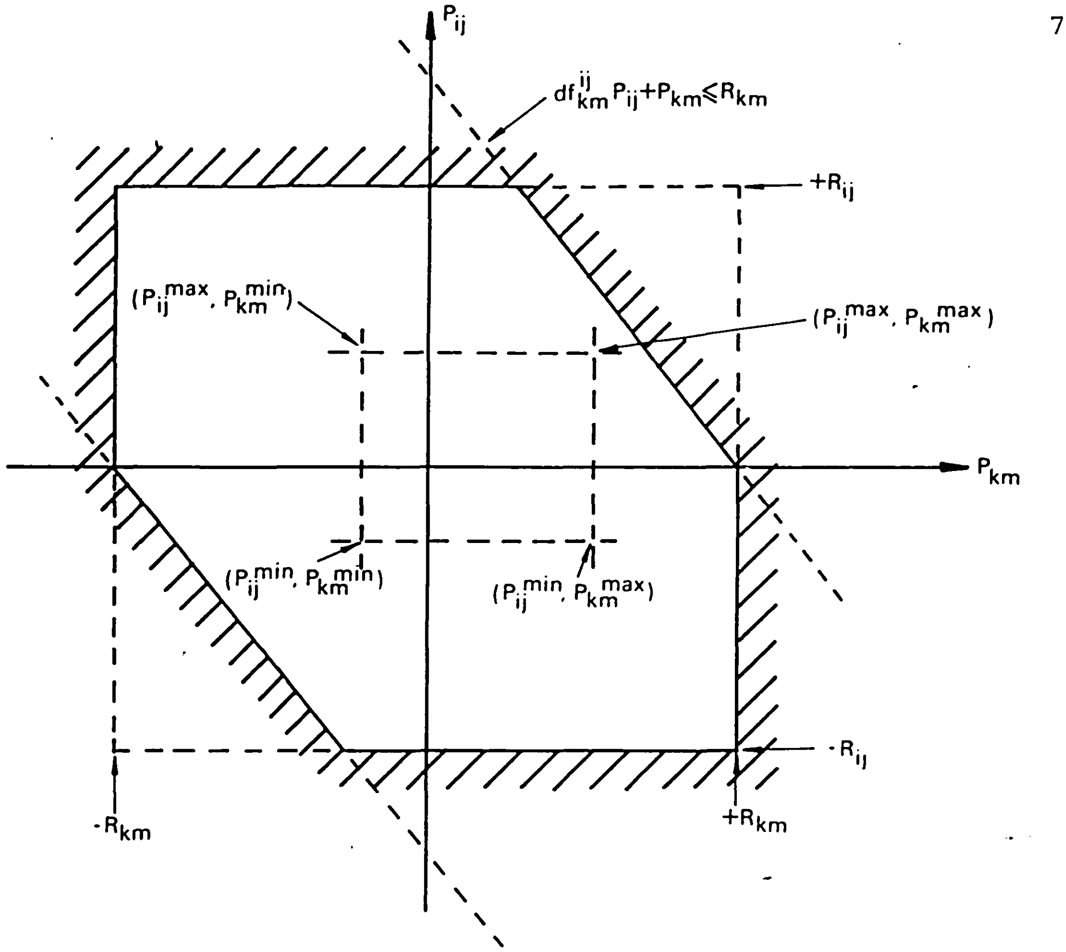
P_{km}^{max} and df_{km}^{ij}.

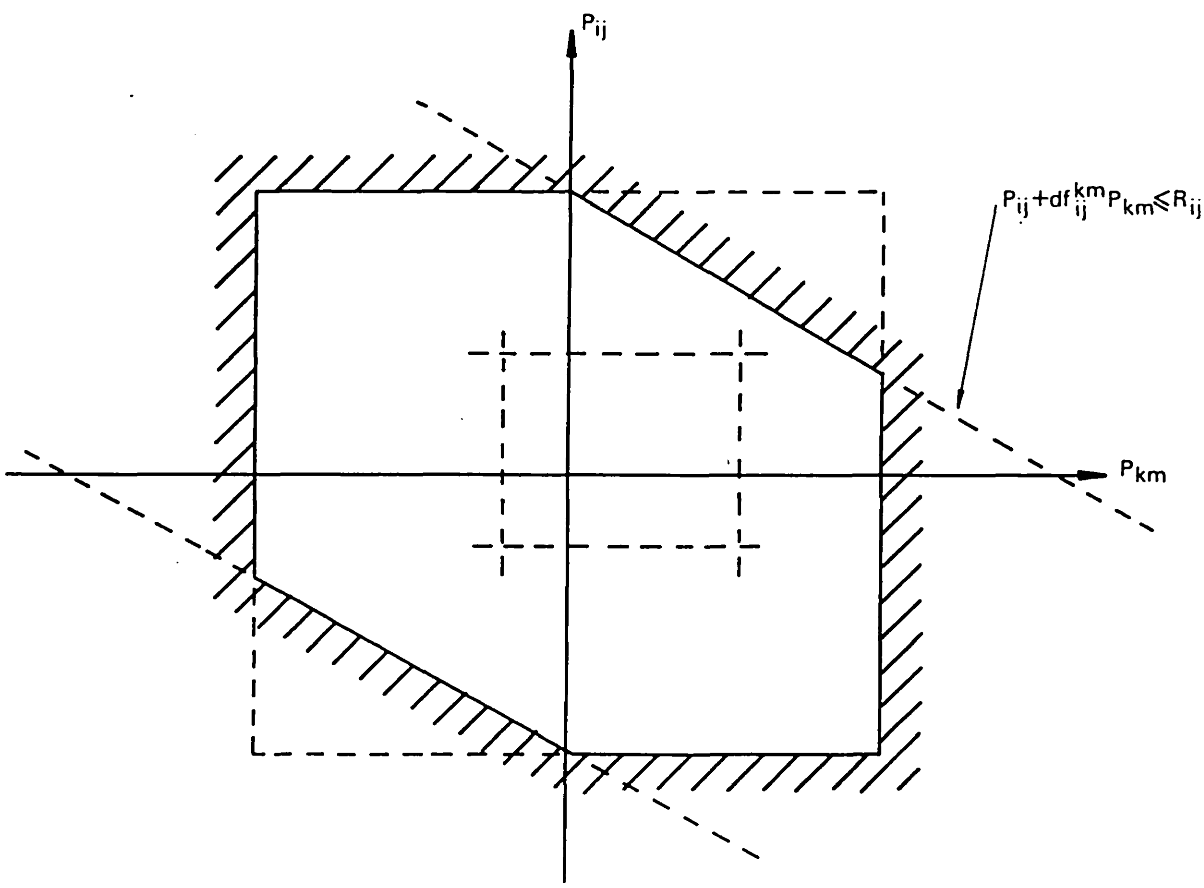
Now provided that these vectors lie wholly within the secure space then line i-j is secure against the outage of line k-m and vice versa.

Thus for trajectory analysis a vector of peak flows is a sufficient test for security. (This also has the merit that under heavy flow conditions the real power flows from the dc model of the system correspond closely to those from an ac model (Ref. 14)).

Furthermore this provides an explicit criteria for determining whether actual conditions have changed sufficiently far from predicted conditions to invalidate earlier predictive studies, i.e. if any of the actual line flows exceed the peak flow considered or if any network topology change occurs then another predictive study is required. The problem now is to estimate the peak trajectory flow for each line in the network. The estimation of these under uncertainty will be treated in detail in the following chapters.

Fig.3.4 LINE FLOW DOMAIN - NETWORK AND TRAJECTORY CONSTRAINTS





3.3.5 Generator Loss Contingency

The constraint mapping for the generator outage contingencies can be derived as follows:

Given that there are NG generator contingencies to be considered, we wish to simulate the sequential outage of each of these.

Consider that the kth generator is forced out, then the generation deficiency is allocated to the other generators according to some pre-determined operating plan in which specific generators have been scheduled to carry the reserve necessary for such generation loss incidents. (Strictly speaking a frequency model of the plant response is required for this). Define these allocation factors for the kth generator as:

$$[\Delta P_G] = [T] \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_k \\ \vdots \\ a_{NG} \end{bmatrix} \cdot P_{Gk} \tag{3.44}$$

and $$\sum_{i=1}^{NG} a_i = 0$$

where P_{Gk} is the pre-outage generation on generator k

a_i is the proportion of P_{Gk} picked up by the ith generator

$[\Delta P_G]$ is the N x 1 vector of changes in nodal generation

$[T]$ is a matrix which maps generators to nodes.

In particular $a_k = -1$

Also if the generator loss is allocated to one fast acting pumped storage plant, say at generator m (increasing generation if in generator mode, decreasing pump load if in pumping mode), then $a_m = +1$.

Now from equation (3.19) we have:

$$[P_\ell]_{new} = [H]\ ([P_I] + [\Delta P_I])$$
$$= [P_\ell] + [H]\ [T]\ [a]_k\ P_{GK}$$
$$= [P_\ell] + [SF]_k\ P_{GK} \quad \text{for } K=1,2....NG \qquad (3.45)$$

i.e. for line i-j the new flow due to outage of the kth generator is:-

$$P^k_{ij,new} = P_{ij} + sf^k_{ij} P_{GK}$$

and for security we have

$$\left| P_{ij} + sf^k_{ij} \cdot P_{GK} \right| \leqslant R_{ij} \qquad (3.46)$$

This defines the secure operating space for the simultaneous flow on line i-j and the generation from the kth generator as shown in Fig. 3.5 and insecure generator outages can be solved using the worst case approach outlined in the preceding section. Note however that the generation shift factors depend upon the pre-determined allocation factors which could change during a trajectory as generators are synchronised and desynchronised from the system. The effect of this would be to introduce additional constraints in the form of (3.46) but with marginally different slopes as shown in Fig. 3.6 .

Fig.3.5 GENERATOR LOSS CONSTRAINTS

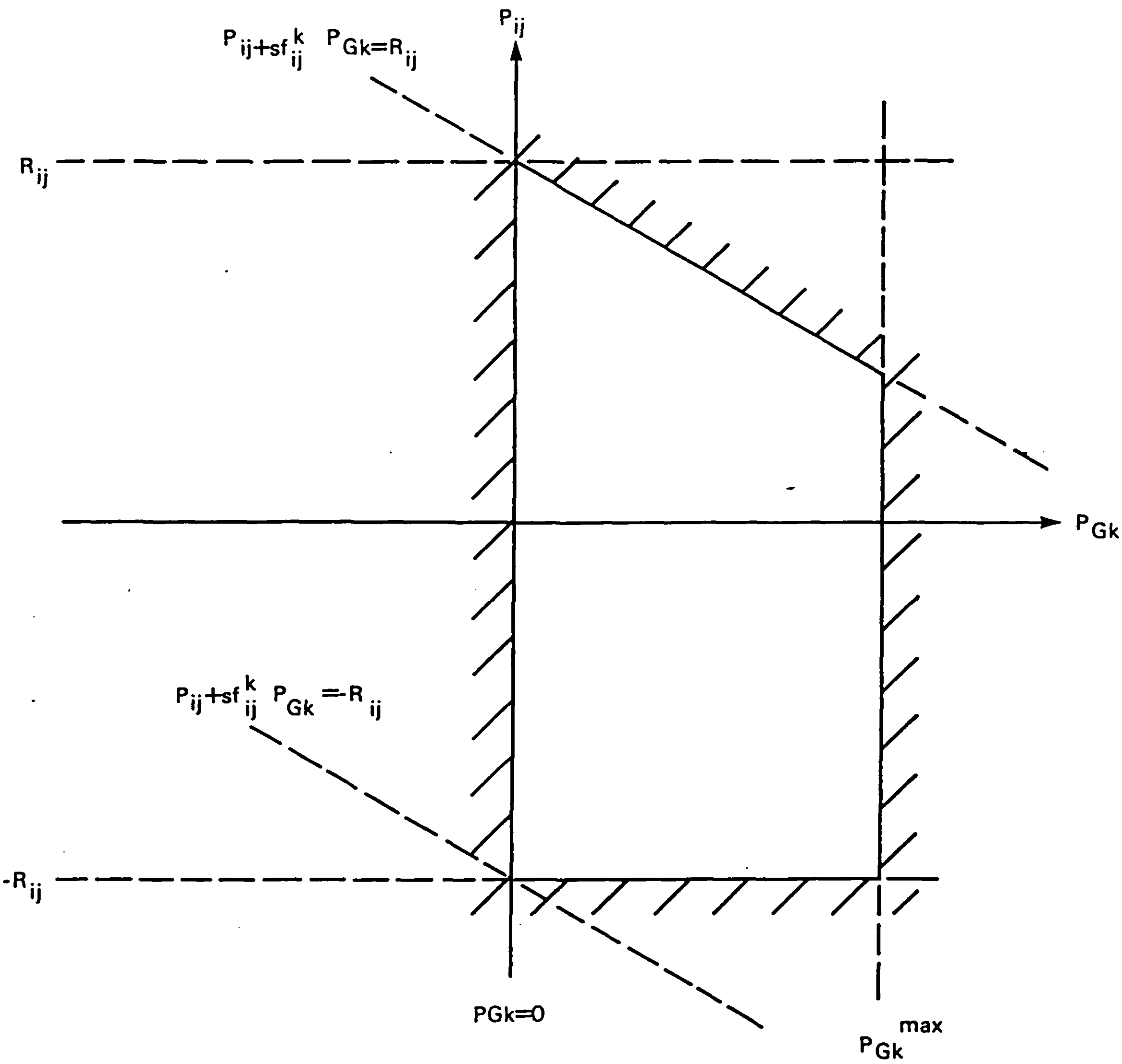
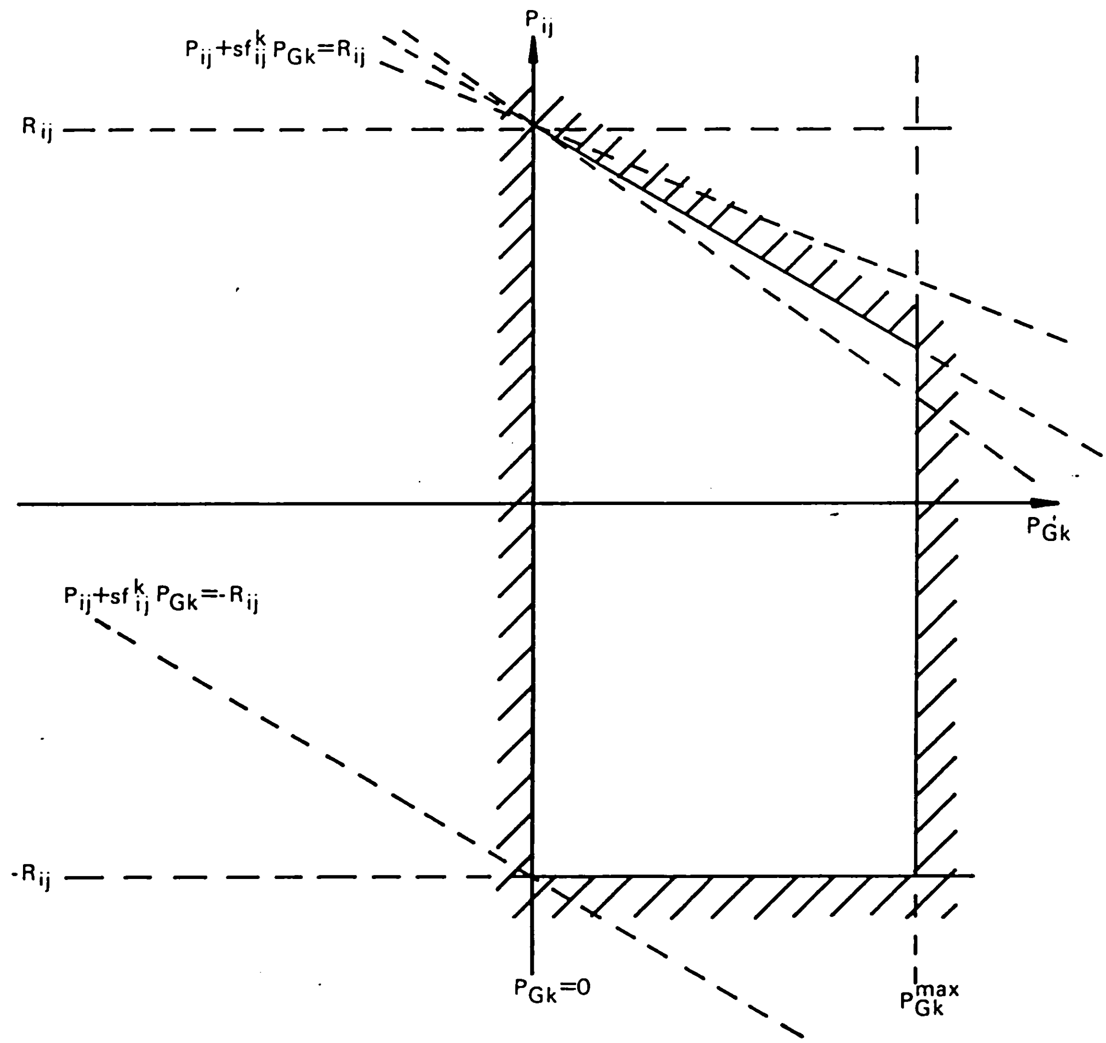


Fig.3.6 GENERATOR LOSS CONSTRAINT - effect of changes in shift factors



The outage of a load transformer which results in a loss of load would be treated in a similar fashion to equation (3.45). The shift factors in this case represent the effect on the line flows of generator output reduction or increased pumped storage pumping load.

3.4 Solution by Multiple Power Flow Methods

In the solution of equation (3.4) by multiple power flow methods, the security asessment of the trajectory basically comprises a base case load flow and sequential outage analysis for each pre-contingency time step on the trajectory. Existing techniques such as the distribution factor method and the dc load flow method with matrix modifications (eg. Ref 12) have all been developed to examine the security of only a single nodal injection set. (This corresponds to one vector in an N-dimensional space of nodal injections). Similarly the critical contingency selection methods or contingency ranking algorithms (Refs. 15,16,17,35,) have all been developed to examine the security of only one single injection set, though somewhat faster.

This is in complete contrast to the pure set-theoretic approach which explicitly defines a secure space and thus also implicitly handles uncertainties though the modification of this in the worst case approach requires some assessment of uncertainty. The straight forward approach of applying each of these multiple load flow methods to the trajectory analysis problem would result in a requirement to solve, in the order of,

$$\frac{T}{\Delta t} \cdot (L+1) \qquad (3.47)$$

load flows,

where T is the total time period covered by the trajectory

Δt is the time interval between successive points on this trajectory

L is the number of contingency cases to be analysed at each point on the trajectory.

This equation implies a uniform sampling rate for the trajectory. The possibility of sampling the demand less frequently during relatively 'steady' demand conditions would not be valid. This is because it assumes that line flows are also relatively steady during steady demand conditions. In practice, relatively small percentage changes in total demand can result in fairly large percentage changes in the output of the marginal generators on a merit order system. This results in large percentage changes in the line flows of lines sensitive to the marginal generators.

The critical factors in equation (3.47) are Δt and T as these determine the number of steady state conditions that must be analysed to rigorously classify the security of the entire trajectory and hence determine the computational workload.

The choice of T is largely determined by the length of time covered by the generator schedule-revision optimisation which may be necessary following a generator loss incident. The scheduling or schedule-revision of steam plant usually covers a 24 hour rolling period due to the influence of constraints such as minimum shut down, minimum on-times and the costs associated with start-up.
Thus T would be of the order of 24 hours.

The choice of Δt is based upon the need to examine security as the load and generation distribution changes with time of day. A typical figure is a period of ½ hour although a period of 15 minutes or less may be required for demand peaks.

3.5 Experimental and Computational Experience

The experimental and computational work presented in this section was designed to test the validity of the theoretical development described in the preceding sections of this chapter. In particular the aims were:

(a) to examine the characteristics of line flow trajectories and the implications of these to trajectory analysis

(b) to measure the performance of the worst case approach (Section 3.3.4.6) in relation to multiple power flow methods (Section 3.4).

This required the development of a test system, the collection of on-line data from this test system, the validation of this data and the development of software to perform worst case, and multiple power flow analysis. This is described in the following three sub-sections, and is followed by the results of the line flow characteristics investigation and the performance indices of the worst case approach.

3.5.1 Development of Test System

The choice of the test system was largely determined by the following criteria:

(a) Ease of access to significant quantities of on-line line flow and generator output data from a real power system is essential for both major objectives.

(b) A significant number of generators covering a broad range of incremental fuel costs and access to the dispatch instructions for these generators is necessary for the work on uncertainty presented in Chapters 4 and 5.

(c) The test system must not be too big in terms of the number of nodes and lines, so that it is not too difficult to manage from a practical viewpoint.

(d) It would be valuable if the 'weak points' and normal flow patterns in the test system were known from experience as this would provide a general 'feel' for the correctness of results produced by the software to be developed, independent of other specific checks.

On this basis, the 20 node, 30 line network of Fig. 3.7 was developed. It is a simplified representation of the southern part of the CEGB 400/275 kV network which is a system of about 220 nodes and 500 lines.

The demand and generation at each node in the test system is the sum of the demand and generation in a corresponding area of the real power system, while the lines in the test system are the equivalents of the real lines joining these areas.

No equivalent was considered necessary for the northern half of the CEGB system as the transfers between the two are relatively small. Any transfers that did exist were allocated as discussed in Section 3.5.2. The real power system data was available from the database of a VAX 11/780 computer located at the National Control Centre and interfaced to the CEGB data acquisition system which provides complete snapshots of all the measurements on the power system every 30 to 45 seconds.

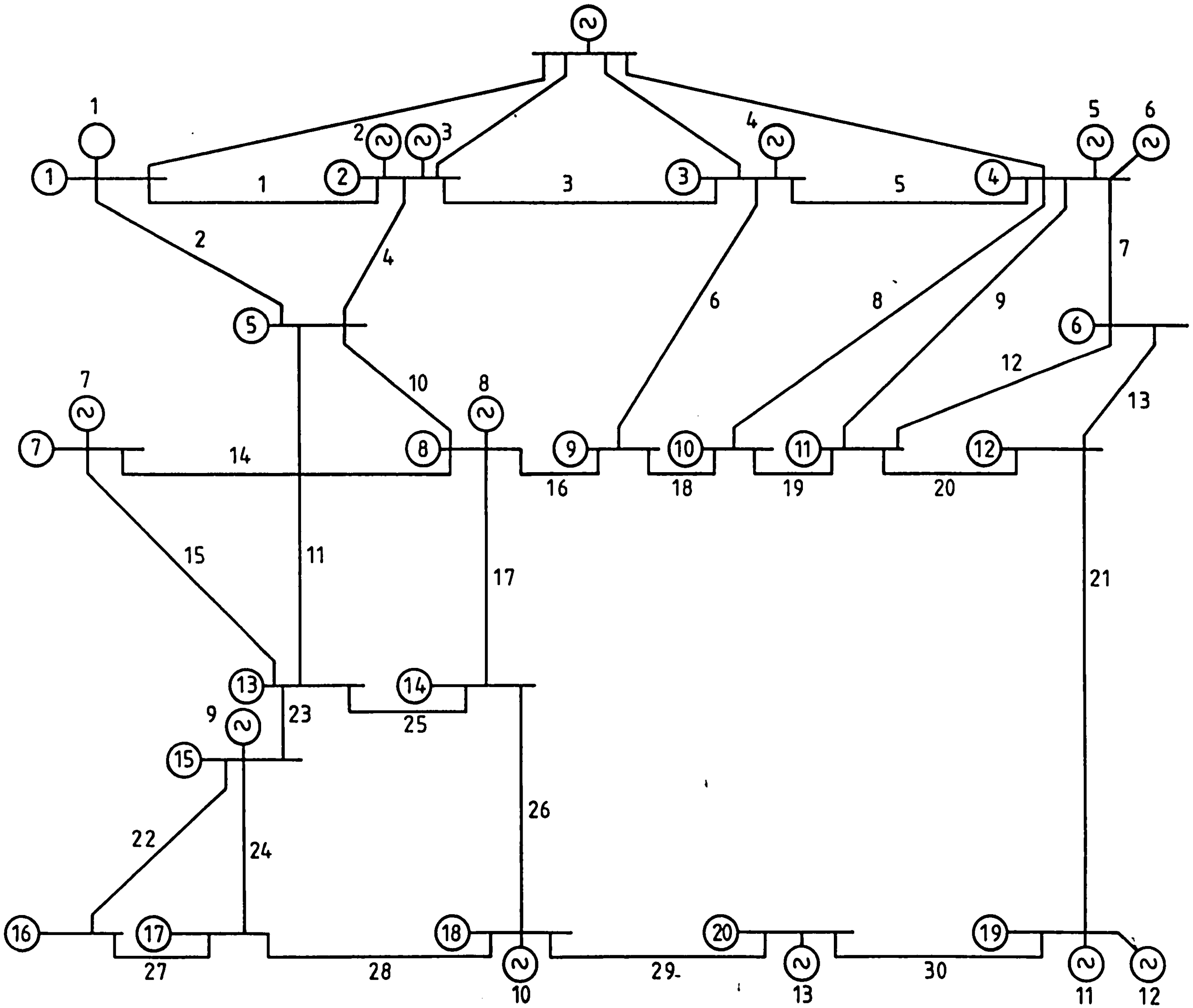


FIG. 3·7 TEST SYSTEM

Table 3.1

Line Number	Nodal Connection		Admittance (on 100 MVA Base)	Line Capacity (MW)
	From	To		
1	1	2	0.9	2200
2	1	5	0.806	2200
3	2	3	1.587	2200
4	2	5	0.769	2200
5	3	4	0.769	2200
6	3	9	0.909	3300
7	4	6	1.205	2600
8	4	10	0.909	3300
9	4	11	0.429	2600
10	5	8	0.476	1100
11	5	14	0.5	1800
12	6	11	1.370	1800
13	6	12	0.356	1300
14	7	8	0.2	1100
15	7	14	0.667	1800
16	8	9	3.704	2200
17	8	15	1.889	2200
18	9	10	4.0	2200
19	10	11	1.389	2200
20	11	12	1.538	2200
21	12	19	0.714	2200
22	13	16	0.513	1800
23	13	14	1.250	1800
24	13	17	2.857	1800
25	14	15	1.333	1100
26	15	18	2.0	2200
27	16	17	0.714	1800
28	17	18	0.592	1800
29	18	20	1.099	2200
30	19	20	0.893	1100
SLACK NODE LINES				
	1	21	0.05	
	2	21	0.05	
	3	21	0.05	
	4	21	0.05	

The on-line data available in this database included MW line flows and generator MW outputs but no explicit measurements of the MW demand on the 400 kV and 275 kV/lower voltage transformers. (These are measured but are not available at this Control Centre). The lumped, equivalent demand at each node in the test system can, however, be obtained from the measured line flows and generation by Kirchoff's current law.

We define P_G, P_D, P_{L1} and P_{L2} etc., as positive when the flow sense is <u>into</u> the substation and negative when it is <u>out of</u> the substation.

Hence we have:

$$P_D = -(P_{L1} + P_{L2}) - P_G \qquad (3.48)$$

where P_D is the lumped equivalent demand at the node considered

P_G is the sum of the measured generation in the area represented by the node

P_{L1} etc., are the measured line flows on the lines connecting this particular area node to the other areas represented by the nodes in the test system.

Errors can arise if not all generator outputs in an area are available or if there are some line flows between areas that are not measured. In practice, all significant generator measurements are available and any small privately owned generation simply leads to a negligible reduction in the demand at that node. There are some lower voltage interconnections, between areas, whose flows are unknown at the National Control Centre but the flows on these can be considered negligible compared to the flows on the 400 and 275 kV lines due to their relative impedances, except possibly under certain outage conditions on the 400 and 275 kV system.

These lumped nodal demands are not strictly necessary for the work in this chapter, as injections are used. However, they are needed for the work in Chapter 4 and they also help in carrying out some validation of the measurements.

The line admittances and ratings of the test system are shown in Table 3.1 on a 100 MVA base. For the purposes of this work the resistances were neglected as they are less than 10% of the reactances.

3.5.2 The On-Line Data and Data Validation

The nodal generation and demand data used in the test system were obtained by recording on-line generator and line flow data from the corresponding part of the real CEGB system. This on-line data was recorded every 15 minutes for 24 hours for each generator and each line flow and the corresponding demand data was derived by applying equations of the form of (3.48) to each node in turn. This was done for ten consecutive days in March and one day in May in order to cover typical weekday, week-end and seasonal demand patterns. No on-line state estimation algorithm was yet available to remove gross measurement errors from the line flow data nor were generator output validation routines available at the time so that some approach to gross measurement errors had to be developed. Previous experience with on-line data error rates and types had indicated that the quality of generator MW output data was high and that gross errors were of the sudden 'spike' type, while line flow data was of lower quality but was again usually a 'spike' type of error. These types of error were individually corrected. Measurement errors due to transducer bias, or drift or due to data skew were expected but were not explicitly corrected.

The data validation was carried out retrospectively and interactively on each 24 hour set of nodal generation and nodal demand data. The first check carried out on individual measurements was to eliminate any 'spike' errors by interpolating between the last correct measurement and the next correct measurement using a simple averaging approach of the form:

$$Z_t = (Z_{t-1} + Z_{t+1})/2 \qquad (3.49)$$

Where Z_t is the measurement spike at time t

Z_{t-1}, Z_{t+1} are the good measurements at time t-1 and t+1.

This form of check was carried out for the loss of a line flow measurement for up to about 8 consecutive samples beyond which the data set was discarded.

A global check on the system of measurements was made by computing:

$$\sum_{i=1}^{20} P_{Ii} = \sum_{i=1}^{20} (P_{Gi} + P_{Di}) \quad \text{(note } P_{Di}\text{'s are negative)} \qquad (3.50)$$

for each sampled set of measurements.

This error is a composite of the transfers with the northern part of the system and such errors as transducer bias, drift etc. It was handled by spreading it equally between the four largest demand nodes on the test system.

A typical partial set of the resulting nodal generation and nodal demand trajectories are shown in Figs. 3.8 and 3.9 . The test system line flow data, corresponding to these nodal demands and generation, were then computed using the dc network model (3.19) of the test system. This produced line flow data for every 15 minutes of the 24 hour period for each line as shown for one of the days in Fig. 3.10 . These sets of validated, recorded on-line data were used for the subsequent tests.

3.5.3 Software Development

The practical work carried out in this chapter required the development and testing of software to carry out the dc load flow, the distribution factor method of multiple power flow analysis, the worst case algorithm and a miscellaneous set of on-line data acquisition routines, data validation routines, graph plotting and histogram routines. All the software was written in FORTRAN 77 as implemented under the VMS operating system (Version 3.2) of the VAX 11/780.

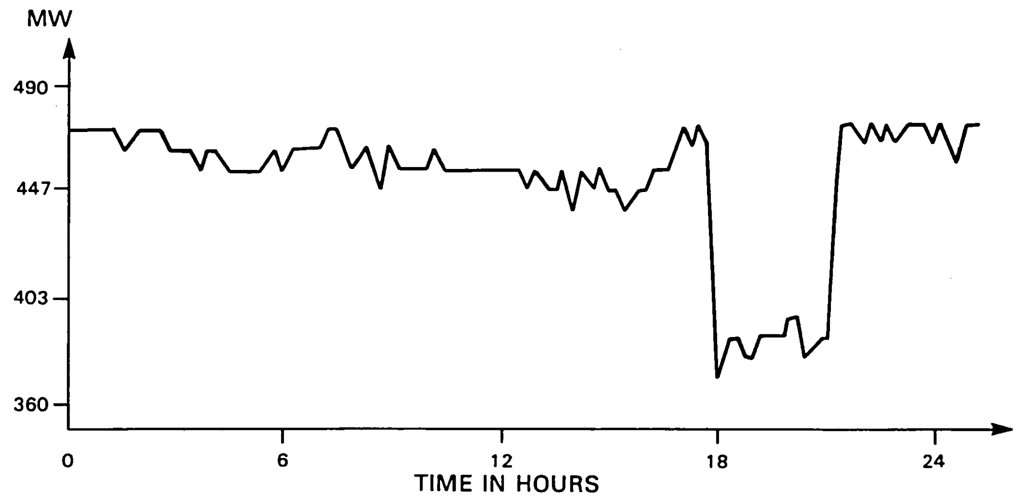
DC Load Flow

(a)

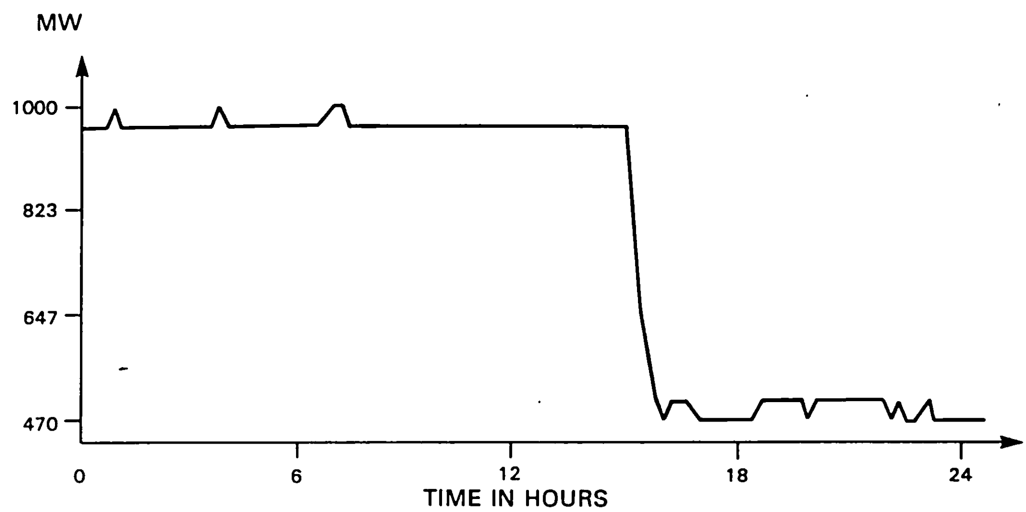
The major components for this are the nodal admittance matrix, the primitive branch admittance matrix, the bus incidence matrix, a matrix multiplication routine and a matrix inversion routine as shown in equation (3.19). The nodal admittance matrix could have been formulated by using equation (3.13) but as its construction was only required once it was constructed manually using the rules:

Fig.3.8 TYPICAL GENERATION TRAJECTORIES

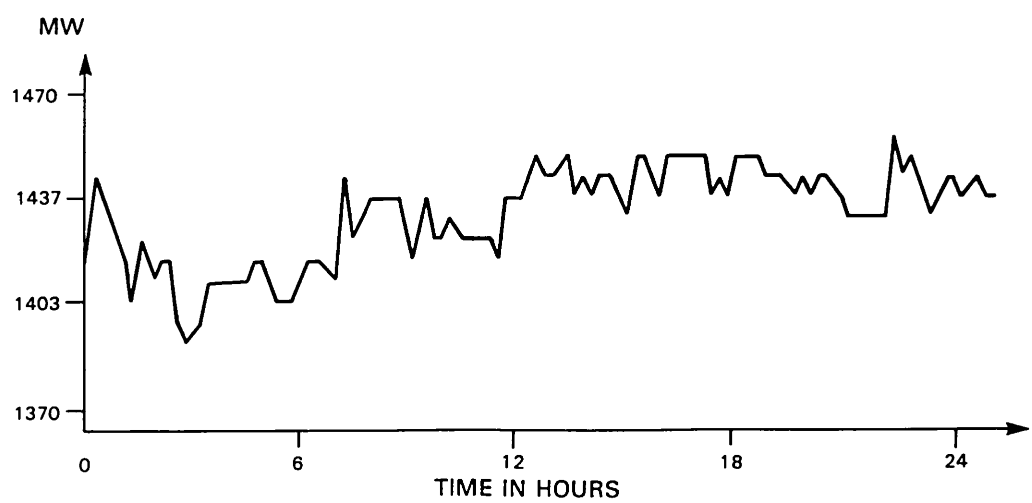
NODE 1 GENERATION PROFILE



NODE 2 GENERATION PROFILE

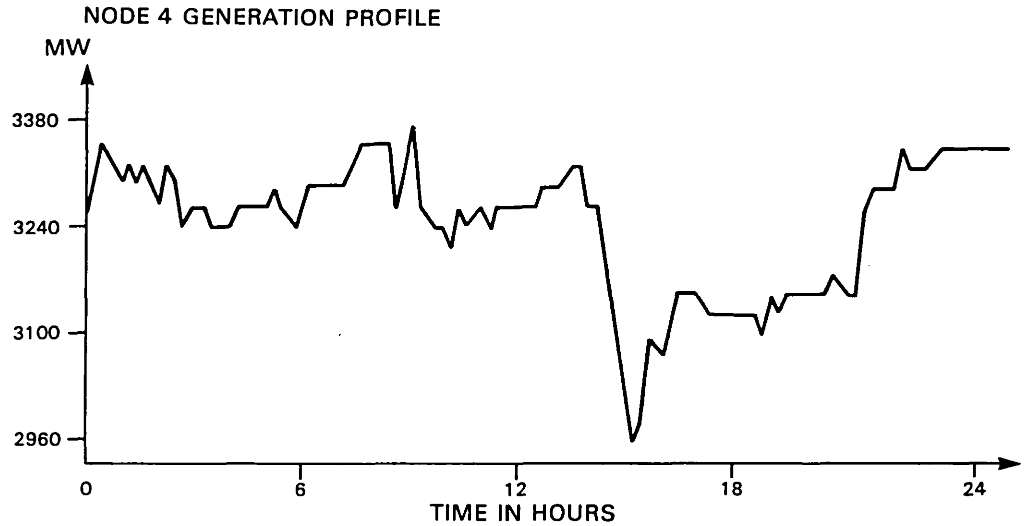


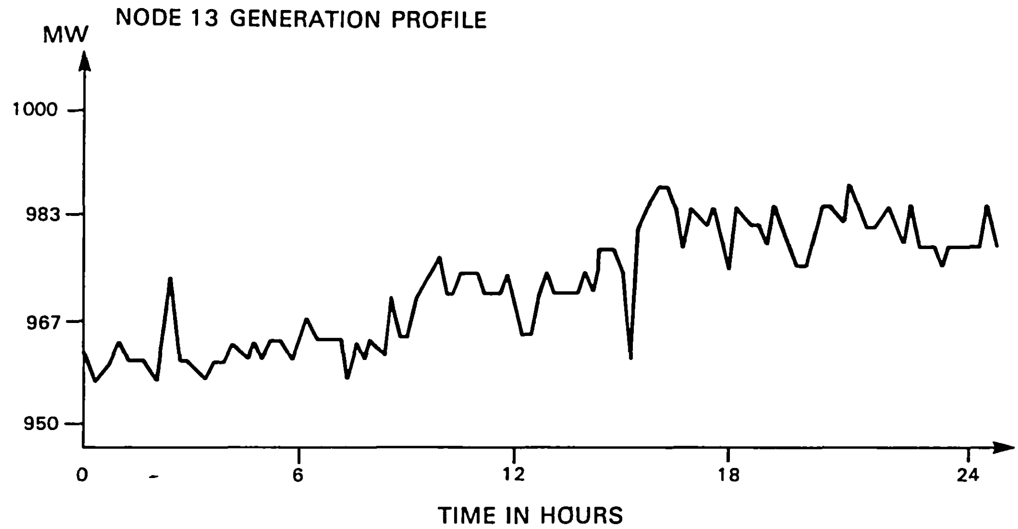
NODE 3 GENERATION PROFILE



(sheet 1 of 2)

Fig.3.8 TYPICAL GENERATION TRAJECTORIES





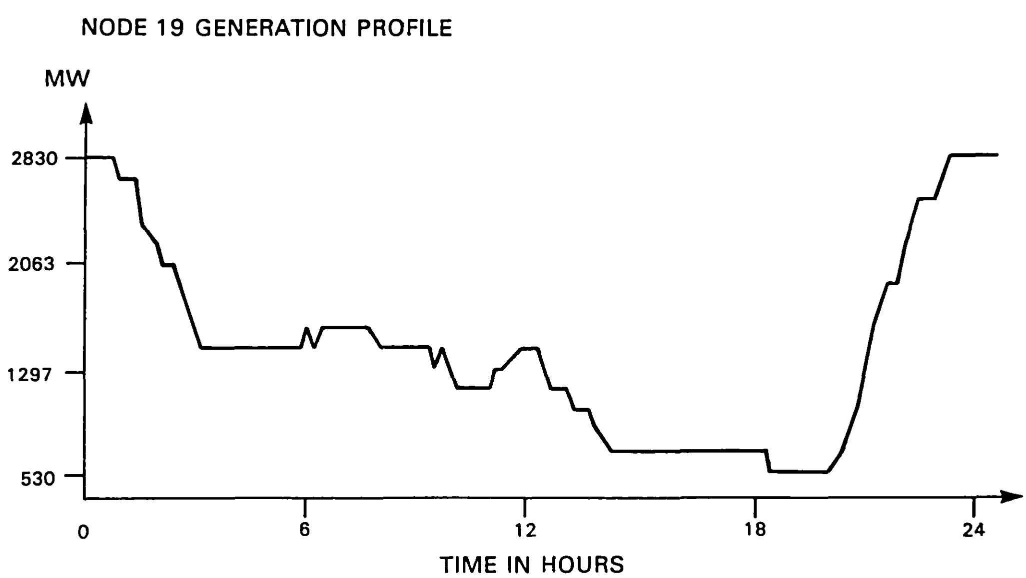
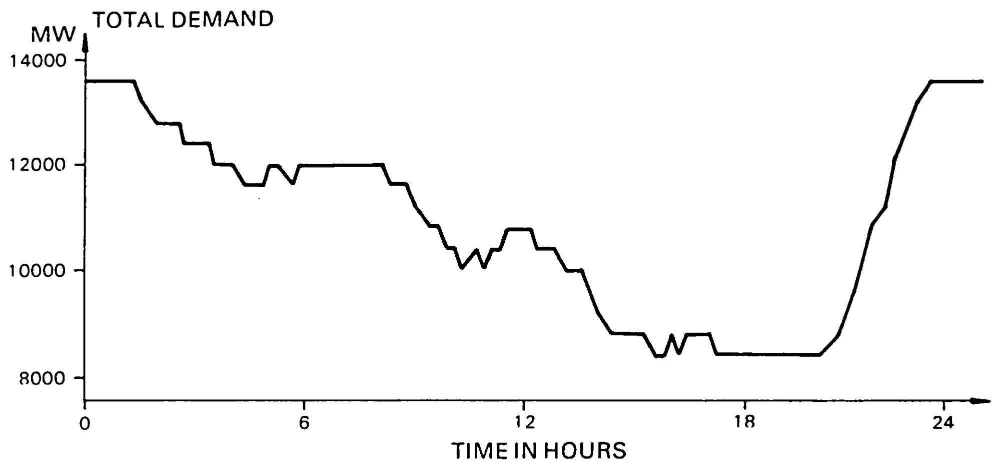
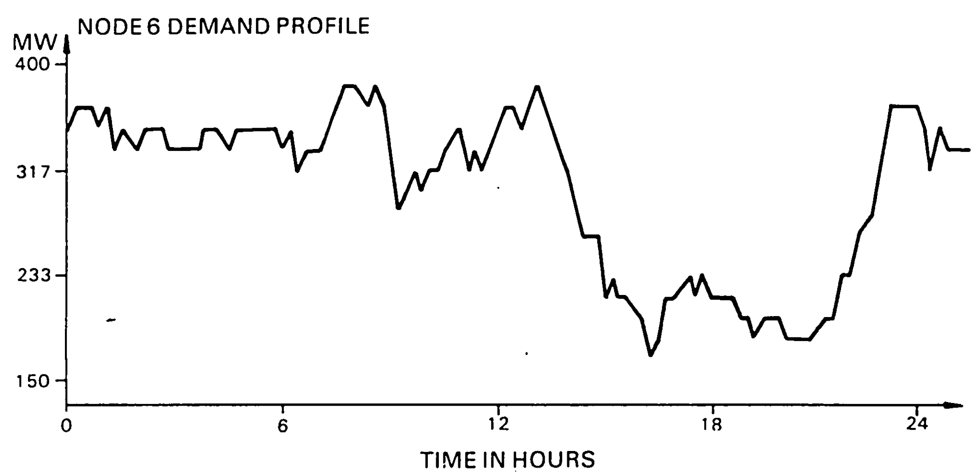


Fig.3.9 TYPICAL TOTAL AND NODAL DEMAND TRAJECTORIES





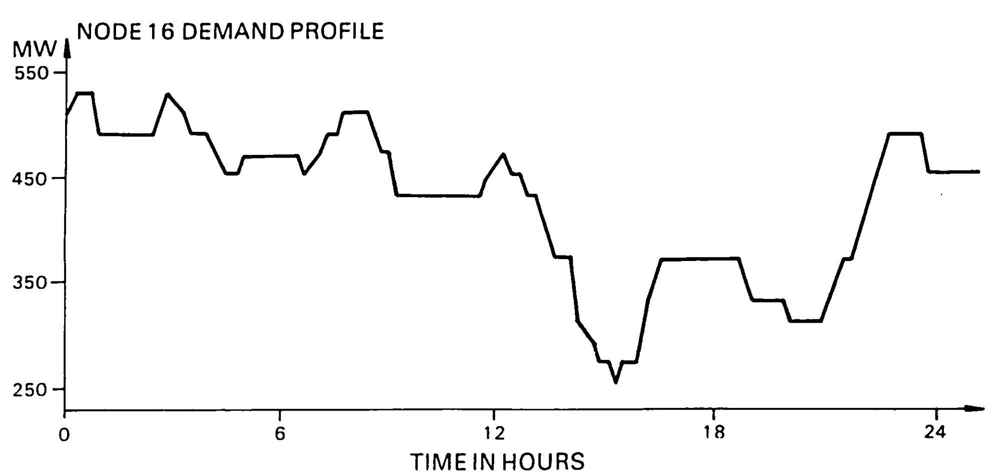
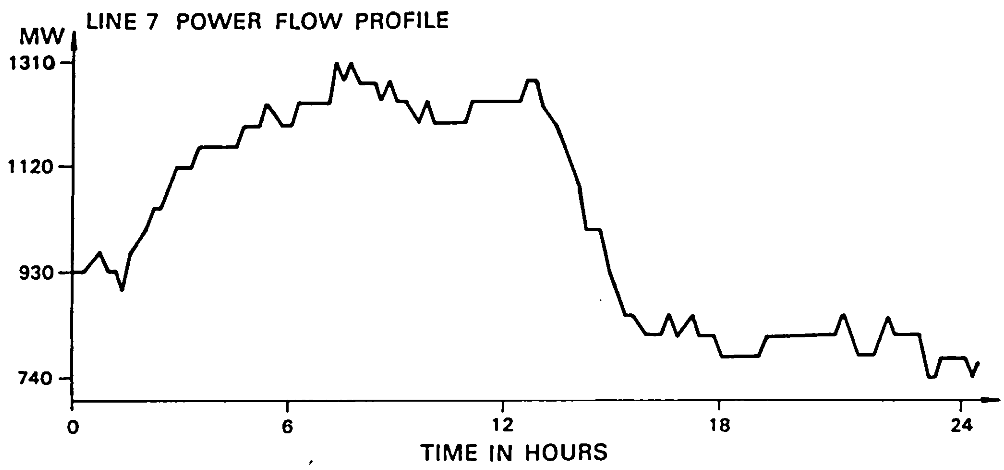
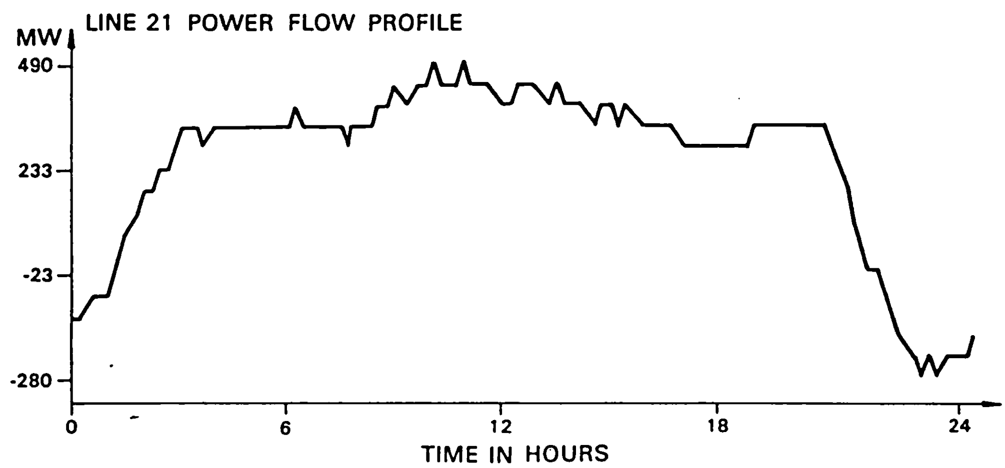
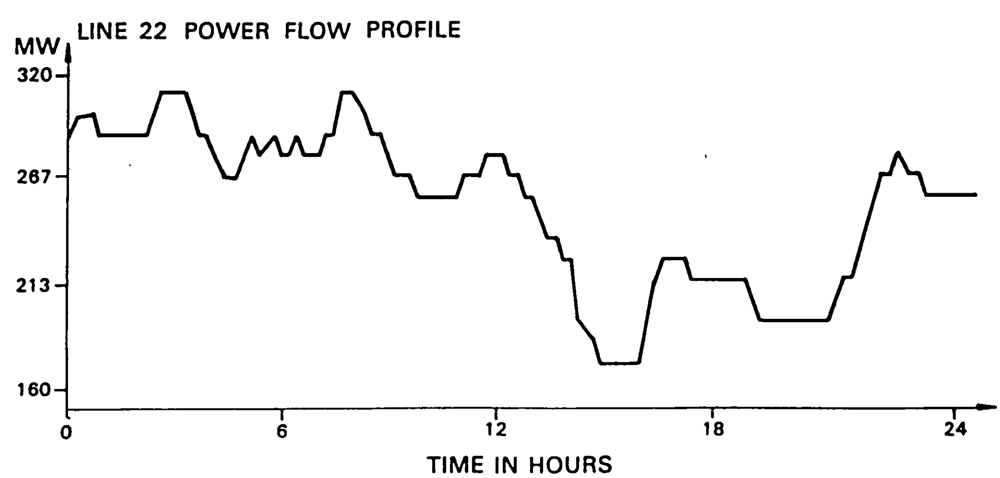


Fig.3.10 TYPICAL LINE FLOW TRAJECTORIES







(1) the diagonal elements are the sum of the admittances of the lines incident to that node

(2) each off-diagonal element is the negative of the admittance of the line between the two nodes represented by that element.

Similarly the branch incident admittance matrix was constructed manually from the bus incidence and branch admittance matrices. A matrix multiplication routine capable of handling up to a 30 x 30 matrix was coded and tested on a 2 x 2 matrix to enable comparison with a manual multiplication. An explicit matrix inverse routine (rather than using triangular factors) was coded and tested by inverting the nodal admittance matrix and then inverting the inverse to check that the original matrix was obtained. No singularity problems or rounding errors were noted with the 32 bit accuracy on the VAX 11/780.

(b) <u>The Distribution Factor Method</u>

Equation (3.36) was implemented for the outage of each line in turn, to construct a 30 x 30 matrix of single line outage distribution factors, and then equation (3.40) was applied to produce the sparse version of this matrix. This produced a matrix with about 55% non-zero elements. No attempt was made to apply sparse programming methods in the use of this matrix (apart from implementing a JUMP instruction when zero element was detected), nor was any consideration given to fast, efficient methods of constructing the matrix of distribution factors, as this was not the prime aim of this research. The distribution factors were tested by comparing the post-fault flows calculated by these with the post-fault flows calculated using the dc load flow with individual lines explicitly removed. No errors were noted.

(c) Worst Case Alogrithm

The major components for this are a maximum/minimum routine and a routine to implement equations (3.42) and (3.43). These were coded and tested as described in (Section 3.5.5).

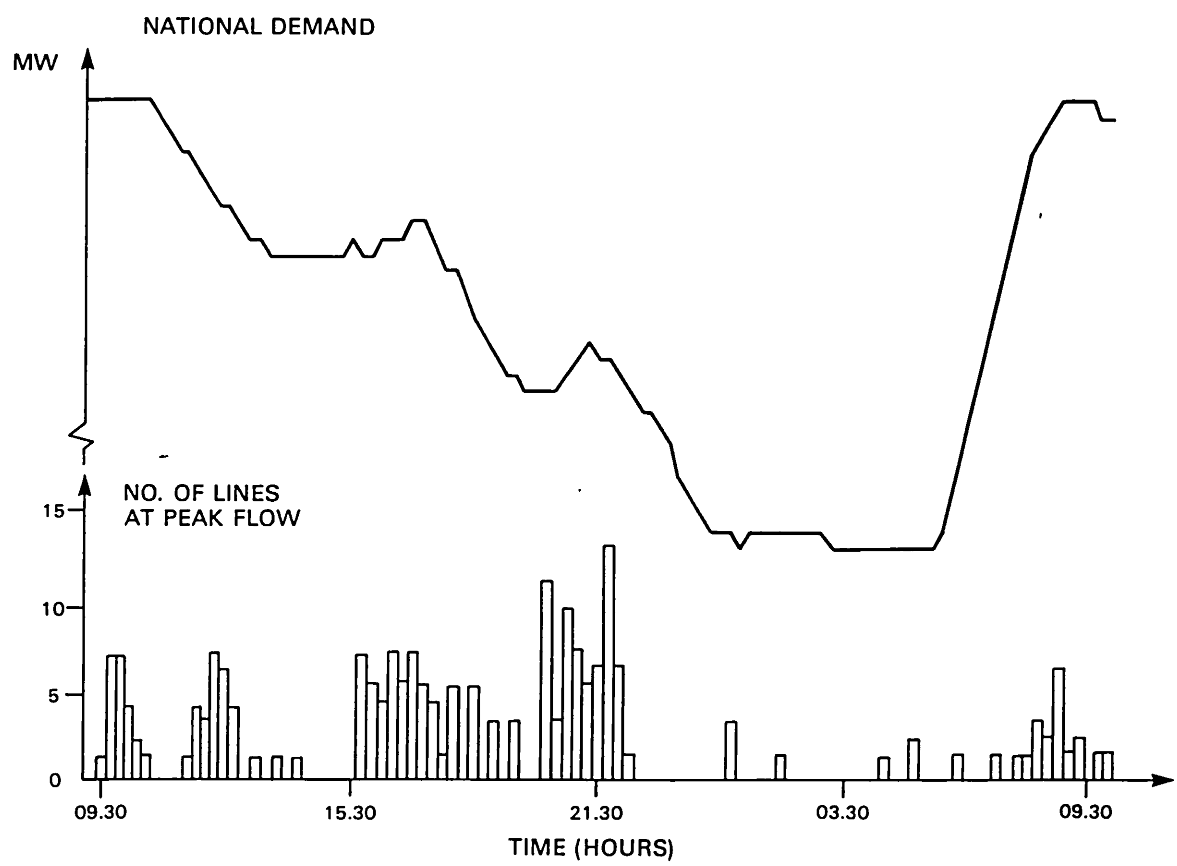
3.5.4 Line Flow Characteristics

Three characteristics of real power system line flows were identified as relevant to the trajectory analysis problem. These were the correlation of line flow peaks with the total demand trajectory; the period of occurence of insecurities, and the correlation of the peak flows of sensitive lines with the line flow trajectory of each simulated outage line.

(a) Correlation of Peak Flows with Demand

Fig. 3.11 shows the plot of a 24 hour total demand trajectory of the test system and a histogram of the time of occurence of the peak line flows (a 5% dead band has been used) on this system. The histogram has been derived by computing the peak flow on each line in the system and then incrementing the histogram count, for each time interval, whenever a line flow magnitude is within 5% of its instantaneous peak. The majority of line flow peaks do not coincide in time with the total demand peak largely because the line flows are predominantly a function of the generation rather than the demand. The range of incremental fuel costs of the generation on the system and their specific geographical locations lead to the specific line flow

Fig.3.11 CORRELATION BETWEEN DEMAND AND PEAK FLOWS



trajectories. Thus in the case of line 21 (Fig. 3.10) the generation at node 19 (Fig. 3.8) has been loaded to meet the peak system demand but as the demand drops off this generation is de-loaded earlier than generation on nodes 1, 2, 3 or 4, because it is more expensive and so the flow on line 21 increases.

There is no significant number of line flow peaks during the early morning rise because the rate of increase of demand in this period of the trajectory is such that individual generator ramp rate capabilities are overriding the fuel cost differences so that generators have to be loaded together.

(b) Occurrence of Insecure Outages

Fig. 3.12 shows the periods of the trajectory when the system is insecure for a particular 24 hour trajectory. This has been derived by applying the distribution factor method to each of the 96 time intervals of the trajectory, i.e. (30 x 96) outage case analyses. In addition to the 3¾ hour insecure period over the demand peak, we have an intermittently insecure period of 2 hours off-the-peak. Thus it would not be adequate to classify the security of the system based purely on the analysis at peak demand conditions. A proper analysis of off-peak conditions is also necessary, particularly at peak line flow transfer conditions.

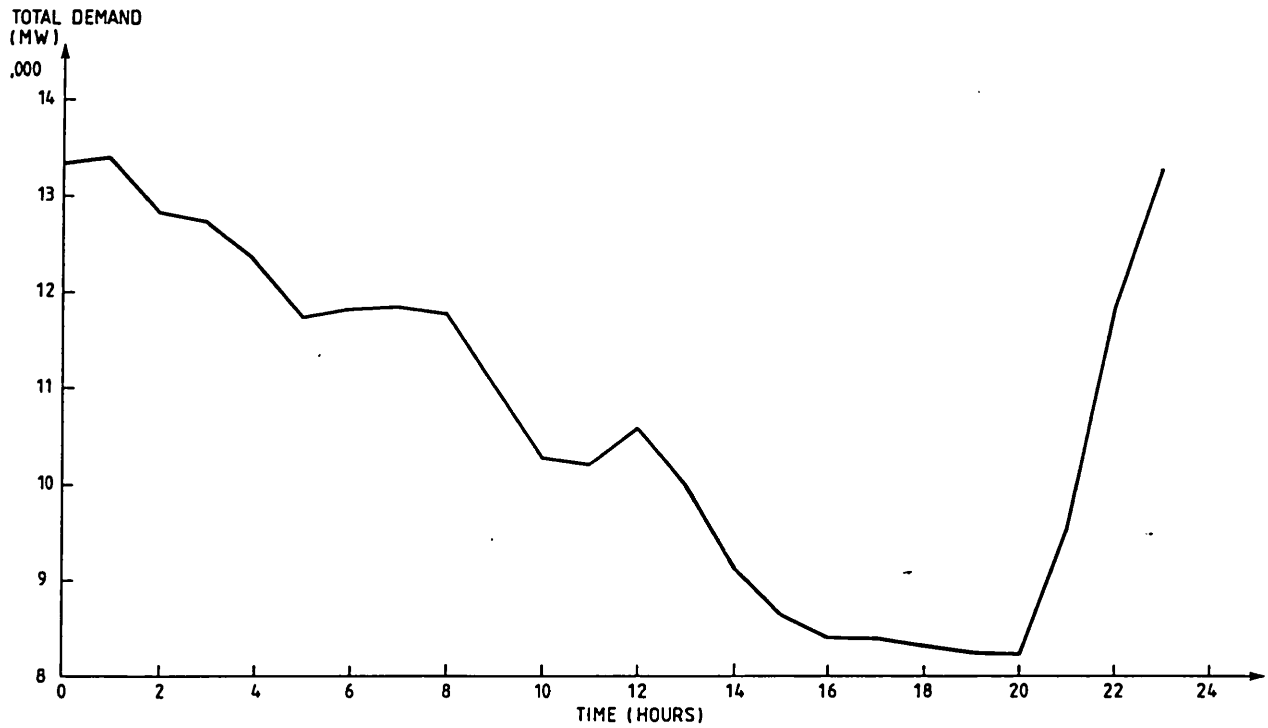
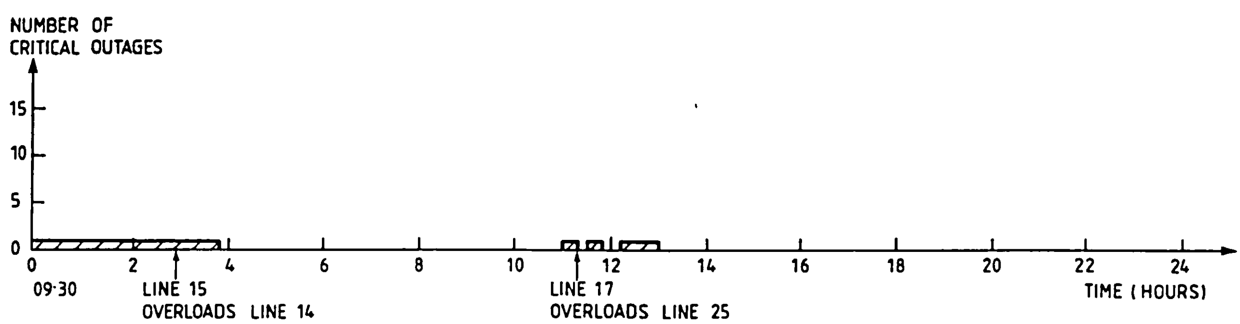


FIG 3.12 POST - FAULT OVERLOAD PERIODS



(c) Correlation of Peak Flows with Outaged Lined Flows

Fig. 3.13 shows the plot of a 24 hour line flow trajectory of the test system and a histogram of the time of occurence of the peak line flows (using a 5% dead band) of those lines which are sensitive to the outage of the line whose trajectory is shown. The sensitive lines are those with non-zero elements in the sparse matrix of distribution factors. The majority of these line flow peaks coincide in time with the peak flow on the outaged line since networks are specifically designed to transfer power from sources of generation to load centres. As such, lines which form parallel paths between generation and load would be expected to exhibit peak flows at approximately concurrent times of day.

There are, however, some line flow peaks which do not coincide with this peak flow. These occurred on lines that had significant flows in both directions as a result of economic dispatch activity. These non-coincident line flow peaks could lead to 'false alarms' in the use of the worst case algorithm as a critical outage selector. The degree to which this may be a problem is investigated in the following section.

3.5.5 Retrospective Performance Indices

In order to measure the performance of the worst case algorithm relative to the multiple power flow methods, some criteria for success or failure had to be defined. These were determind as the Miss Ratio (MR) and the False Alarm Ratio (FAR).

Fig.3.13 CORRELATION BETWEEN PEAK LINE FLOWS

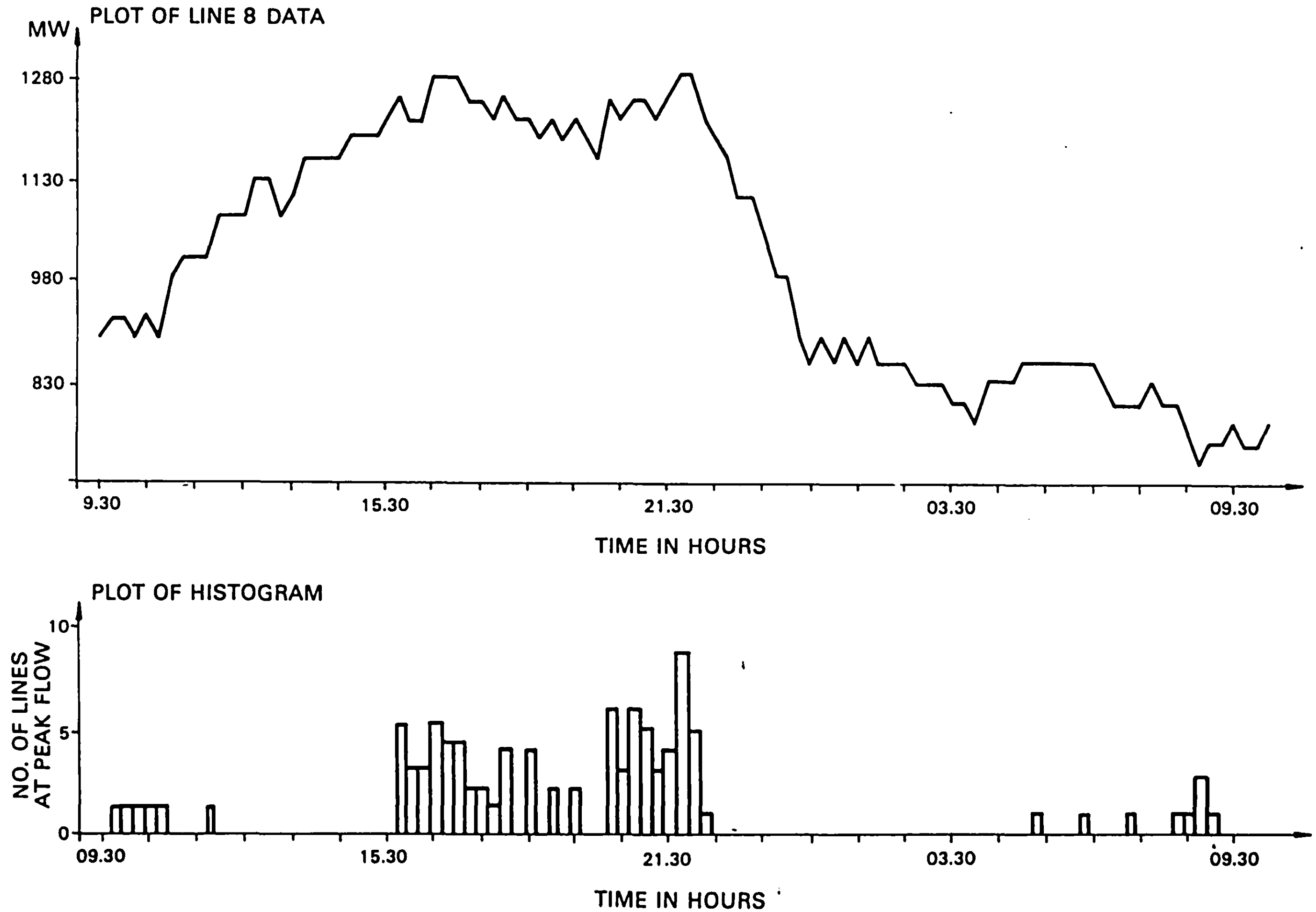
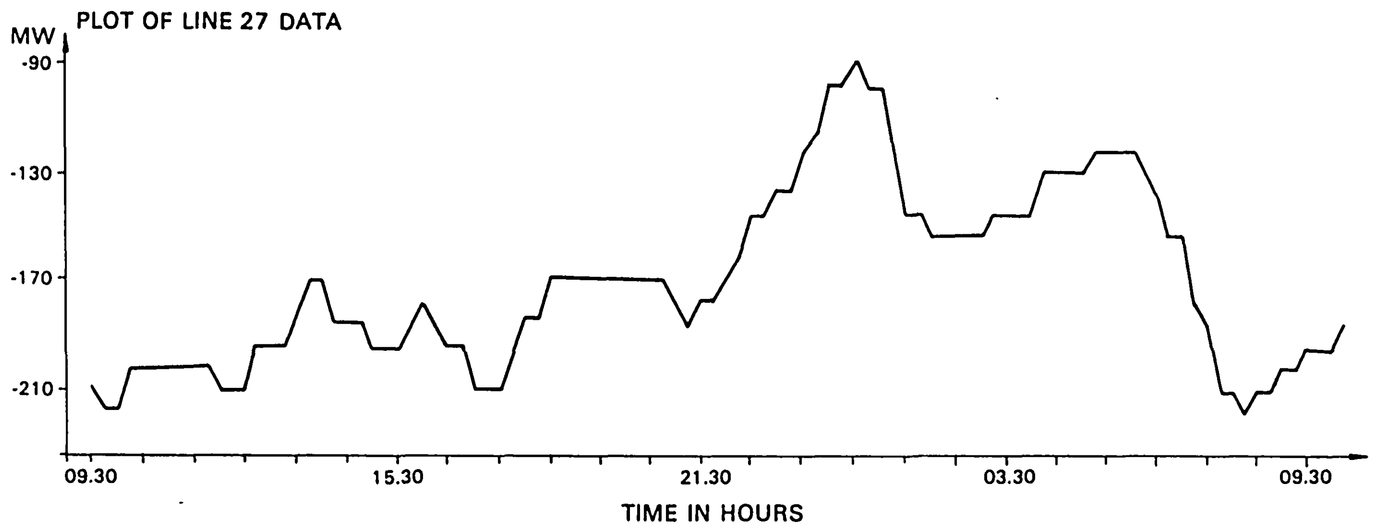
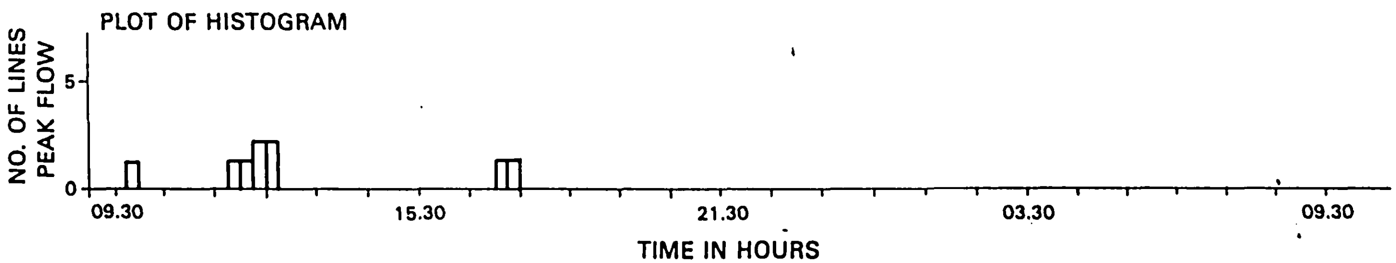


Fig.3.13 CORRELATION BETWEEN PEAK LINE FLOWS





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The success of any trajectory analysis method must be judged on its ability to detect all the critical outages in the defined set of contingencies. Any method that fails to identify all critical outages is unacceptable, although identification of a small number of non-critical outages as possibly critical is acceptable since these can be identified as non-critical during closer study. Accordingly the Miss Ratio and False Alarm Ratio were defined as:

$$MR = \frac{\text{number of missed insecure outages}}{\text{number of truly insecure outages}} \tag{3.51}$$

$$FAR = \frac{\text{number of false insecure outages}}{\text{number of truly secure outages}} \tag{3.52}$$

(In the special cases where the number of truly insecure outages is zero or the number of secure outages is zero, the MR = 0/0 and the FAR = 0/0 are both defined as equal to zero).

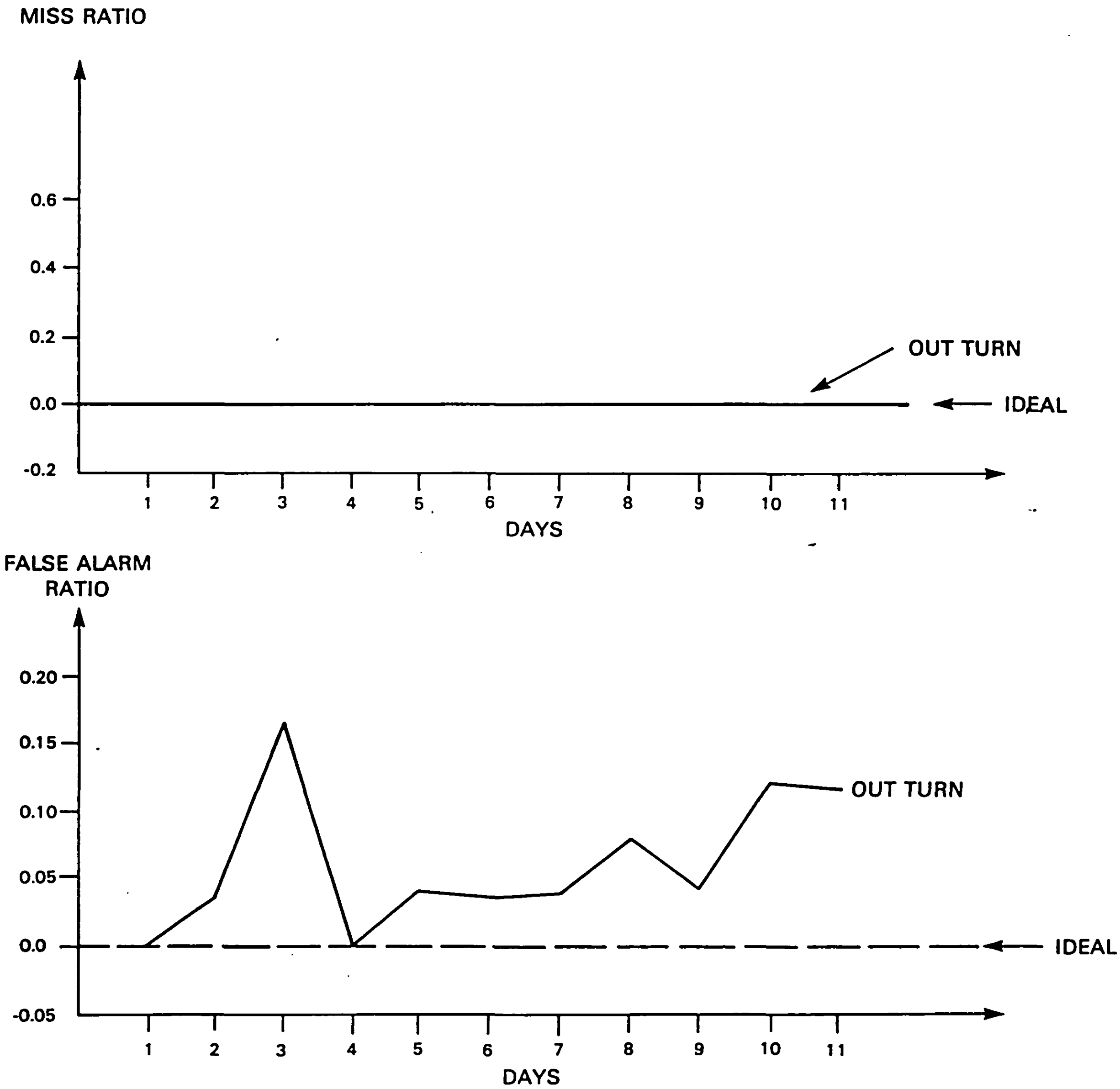
(a) Miss Ratio

The MR should fall in the range:

$$0 \leqslant MR \leqslant 1 \tag{3.54}$$

When it is equal to 0 then no insecure outages have been missed, and when it is equal to 1 then all the insecure outages have been missed. The MR was evaluated for each set of recorded on-line data (Fig. 3.14) using equation (3.51). In each case the number of truly insecure outages was determined by carrying out a multiple power flow analysis of the trajectory using the distribution factor method.

Fig.3.14 RETROSPECTIVE PERFORMANCE INDICES OF WORST CASE ALGORITHM
(evaluations based upon on-line data of one summer day; 10 winter days)



The MR was consistently equal to zero in each case.

(b) False Alarm Ratio

The FAR should fall in the range:

$$0 \leqslant \text{FAR} \leqslant 1 \qquad (3.55)$$

When it is equal to zero means no false insecure outages. When it is equal to one means all secure outages were falsely identified as insecure.

The FAR was evaluated for each set of recorded on-line data (Fig. 3.14) using equation (3.52). These test values range from 0 to 0.167 so that in some cases some outages were falsely labelled as insecure. In each case, however, these falsely labelled outages were entirely due to the effect of non-coincident line flow peaks and this greatly reduced set of contingencies can be simply and explicitly classified using a dc load flow.

3.6 Conclusions

The need for a rigorous trajectory analysis of the power system has been demonstrated in this chapter. The application of set theory to the solution of the trajectory analysis problem has been developed in the line flow domain and the special features of line flow characteristics harnessed to derive the worst case algorithm. The powerful contingency filtering effect of this algorithm has been tested and demonstrated on on-line power system data. The set theory approach using line trajectory information in the line flow domain is a good method for identifying critical single circuit outages for the daily trajectory. This is because it never failed to identify the critical outages and the number of false alarm cases were not significant on the line flow data used for testing single branch outages. The false alarm cases resulted from lines that were sensitive to the simulated outage and which had non-time-coincident peak flows. In particular this occurred on lines that had significant flows in both directions as a result of economic dispatch activity. The worst case algorithm takes no account of the non-time-coincident fact. It is this feature which makes it such a useful contingency selector and yet paradoxically generates the false alarm cases.

The performance of the method in identifying critical multiple circuit outages has not been examined. Chapters 4 and 5 consider the effects of uncertainty on the line flows trajectories and Chapter 5 describes the application of this research.

4. Modelling Uncertainty

4.1 Introduction

The worst case method requires some estimate being made for the near future peak line flows. These estimates could be based on historical peak line flow data; deterministic load flows with some allowances for variations; or stochastic load flows.

The historical data approach would produce reasonable estimates for many of the peak line flows but would fail for some lines under outage conditions where historic data becomes invalid. The load flow approaches both rely on some estimate first being made for the near future nodal demand and generation. Inevitably these forecasts and out-turns will differ.

The deterministic load flow method would therefore have to add some margin to the peak line flow estimates to allow for this variability. This margin would have to be set somewhat arbitrarily to say +100 MW and therefore lead to the possibility of underestimating the peak flows on those lines which exceed this margin.

The statistics of these stochastic errors can, however, be modelled and their effects reflected in the line flows through the stochastic load flow methods. The modelling of uncertainty for this purpose is developed in this chapter and its mapping to the line flows is developed in Chapter 5.

The peak line flow estimates also depend upon the predicted network configuration. Uncertainty exists in this due to line outage events and maintenance over-run. In this research explicit line outage probabilities are not treated. Line outage uncertainty is handled by the use of the 'next contingency set' concept described in Chapter 2 in which each outage is assumed equally likely, at least for the security assessment part of security management but not necessarily for the security rectification part. Maintenance over-run uncertainty is also not treated in this research but could be handled by repeating the predictive security assessment study with the network configuration corresponding to the 'return from maintenance' state delayed in time.

4.2 Mathematical Formulation of the Problem

Given that Kirchoff's law must be obeyed at all times as defined in (3.3) and that we have the planned MW output trajectory for each generator unit:-

$$P^I_{Gj}\ (t) \qquad\qquad \text{for } j = 1,\ \text{NG and } 0 < t < t_p$$

and we have the forecast demand trajectory for each node of the system:-

$$P^F_{Di}\ (t) \qquad\qquad \text{for } i = 1,\ \text{ND and } 0 < t < t_p$$

We have that the out-turn of each generator is:-

$$P_{Gj}(t) = P^I_{Gj}(t) + \Delta P_{Gj}(t) \tag{4.1}$$

And that the out-turn of each nodal demand is:-

$$P_{Di}(t) = P^F_{Di}(t) + \Delta P_{Di}(t) \tag{4.2}$$

Find the statistics of $\Delta P_{Gj}(t)$ and $\Delta P_{Di}(t)$ for each time t, for each generator j and for each node i.

Complete statistical information for the random variable ΔP_{Gj} is given by its probability density function:-

$$f(\Delta P_{Gj})$$

where we define the probability that ΔP_{Gj} lies between a and b as:-

$$\text{Prob}(a<\Delta P_{Gj} < b) = \int_a^b f(\Delta P_{Gj})\, d\Delta P_{Gj} \tag{4.3}$$

In particular the expectation and variance of ΔP_{Gj} can be computed as:-

$$\overline{\Delta P}_{Gj} = E\left[\Delta P_{Gj}\right] = \int_{-\infty}^{\infty} \Delta P_{Gj} f(\Delta P_{Gj})\, d\Delta P_{Gj} \tag{4.4}$$

$$\text{Var}\ (\Delta P_{Gj}) = E\left[(\Delta P_{Gj} - \overline{\Delta P}_{Gj})^2\right] \tag{4.5}$$

Similar equationsexist for the random variables ΔP_{Di}. The statistical relationships between the random variable ΔP_{Gj} and other generator units, or nodal demand forecast errors ΔP_{Di} etc. is given by their respective joint probability density function:-

$$f\ (\Delta P_{Gj},\ \Delta P_{Di})$$

where the probability that ΔP_{Gj} lies between a, and b and ΔP_{Di} lies between c, and d is given by:-

$$\text{Prob}(a<\Delta P_{Gj} < b,\ c<\Delta P_{Di} < d)$$

$$= \int_a^b \int_c^d f(\Delta P_{Gj}, \Delta P_{Di})\, d\Delta P_{Gj}\, d\Delta P_{Di} \tag{4.6}$$

In particular the covariance and correlation coefficient are defined as:-

$$\mathrm{Cov}(\Delta P_{Gj}, \Delta P_{Di}) = E\left[(\Delta P_{Gj} - \overline{\Delta P_{Gj}})(\Delta P_{Di} - \overline{\Delta P_{Di}})\right] \tag{4.7}$$

$$r = \mathrm{Cov}(\Delta P_{Gj}, \Delta P_{Di}) / \sqrt{\mathrm{Var}(\Delta P_{Gj}) \times \mathrm{Var}(\Delta P_{Di})} \tag{4.8}$$

We examine the underlying physical processes in order to develop a model of each process from which these statistics can be derived.

4.3 Demand Uncertainty

The total demand on a power system varies in a manner such that it can be expressed as the sum of daily, weekly, and annual cyclic components together with an unpredictable term, which represents the deviations from forecast weather and consumer behaviour.

In the absence of any significant centralised load management facility this unpredictable term should lend itself to an empirical approach to determine its statistical characteristics. In this section we outline the theoretical reasoning behind a commonly used statistical model of total demand uncertainty in order to derive the model of nodal demand uncertainty used in this work.

4.3.1 The Random Walk Model

The statistical representation of the unpredictable total demand fluctuations is based on the asymptotic random walk model (Ref. 19).

The reasoning for such a model is based upon measurements and analysis of short term fluctuations in demand (i.e. upto about 30 minutes lead time) due in the main to consumer load switching and of the longer term fluctuations (i.e. from about 30 minutes to 120 minutes lead time) attributable to consumer response to changing weather conditions.

(a) Short Term Fluctuations

The total demand is subject to changes of given magnitude over successive short time intervals. These changes are equally likely to be increases or decreases and so could represent the resultant of consumers switching loads on or off i.e.:

$$\left.\begin{array}{l}\text{change in total}\\ \text{demand over}\\ \text{short time interval}\end{array}\right) = \sigma_D X(t) \tag{4.9}$$

where X(t) is a random variable of zero mean and unit variance and such that $X(t_1)$ and $X(t_2)$ are uncorrelated wherever $t_1 \neq t_2$. It is the sum of the random variables which represent the individual consumers load switching. As such it would have a Gaussian distribution. σ_D is the rate of switching demand on or off.

The unpredictable change in demand over a longer interval, of length T say, is given as the sum of a large number N of these random variables each describing the variation in demand over interval T/N.

Thus the change ΔD over the period T is given by the equation:-

$$\Delta D = \int_{0}^{T} \sigma_D X(t)dt \qquad (4.10)$$

From the central limit theorem we have that the resulting random variable ΔD will have a Gaussian distribution. Its mean would be zero and its variance given as the sum of the variances of $\sigma_D X(t_1)$, $\sigma_D X(t_2)$ ... etc. is:-

$$\mathrm{Var}(\Delta D) = \sigma_D^2 T \qquad (4.11)$$

i.e. the mean square fluctuation is proportional to the length of the time interval.

Measurements do not indicate any simple relationship between mean square fluctuation and total demand (Ref. 19). They suggest rather that the observed fluctuations in demand are due in the main to variations in a component of demand that does not show large seasonal changes. Such a component is the demand attributable to continuous industrial processes. If the major cause of short-term fluctuations is the variation of a particular component, we would expect the mean square fluctuation to level off, after an initial approximately linear increase with time interval, at a value corresponding to the maximum deviation of this demand from its mean value.

(b) Longer-Term Fluctuations

The study of fluctuations over lead time periods between 30 minutes and 120 minutes (Ref. 19) indicate that the mean square error in demand forecasting increases approximately linearly with lead time over this period and this suggests that a random walk model for this lead time period is also reasonably valid.

For longer time intervals, the mean square fluctuation in demand cannot go on increasing indefinitely and we would expect it to level off over the two hour to twenty-four hour lead time period due to the predictability of the cyclic component of the daily load.

A model which gives this asymptotic behaviour, and the linear rate of increase due to the random walk is:-

$$\mathrm{Var}\ (\Delta D(t)) = \left[aD(t)\right]^2\left[1-e^{-t/T_o}\right] \tag{4.12}$$

where 'a' is the proportion of the total demand at which the mean square error levels off. This value depends on the prediction method.

T_o is the lead time period at which this levelling off becomes significant.

This corresponds to a simple modification to the random walk model of (4.10).

This model is normally associated with the unpredictable part of the total system demand, however, provided that the number of individual loads connected to any one node of the system is large so that (4.9) applies we can assume that the model is equally valid for the unpredictable part of each nodal demand.

The validity of this assumption depends upon the number and relative magnitude of the individual loads served by the node. The forecast errors for the sum of a large number of an equal sized switching loads will have a normal distribution. This can be seen from the application of the central limit theorem or from the result of convolving the on/off distributions of such loads. However, if there is one significantly larger switching load, such as a 40 MW arc furnace, this will tend to dominate the forecast errors for the sum of the loads.

This would produce a double humped demand forecasting error distribution.

In practice a typical sum of the nodal demands served from the CEGB supergrid system are of the order of 200 MW.

Arc furnace loads would not dominate the forecast errors for nodal demand predictions at this level, although during night-time conditions as other loads fall off the arc furnace load could start to affect nodal demand prediction errors.

In this research nodal demand prediction errors have been simulated as normal distributions.

4.3.2 Nodal Demand Correlations

We now have the added complication of correlations between the unpredictable elements of each nodal demand with those of other nodes. The dominant reasons for this correlation are due mainly to common environmental factors such as temperature, wind speed, cloud cover, rain, light conditions etc. and to common social factors such as tv programmes, working habits etc. As these factors are likely to affect all loads in a geographic area of a similar nature in a like manner, a degree of correlation will exist between the demand uncertainty of nodes in the area.

4.4 Generation Uncertainty

The unpredictable element of the total demand on a power system must be met at every instant by the generation on the system in order to maintain the balance of power of (3.3). Mismatches in this power balance result in frequency deviations away from target until a new power balance is achieved due to the demand/frequency sensitivity.

Fortunately, the cyclic components of total demand are much larger in magnitude than the unpredictable term, and it is possible to operate the system by scheduling plant, several hours or even several days in advance, to supply the cylic demand components.

As system load varies from the predicted load, upon which the plant loading schedule is based, control action is required to bring generation and load into balance. Over short time intervals this control action is provided by steam plant free governor response; over longer time intervals from a few minutes upto a couple of hours, the fuel infeed of some plant is varied to match the demand by dispatch control; and over longer periods, dependent upon the precise shape of the daily load curve, unit commitment revisions are carried out.

This control of generator output is required also to compensate for unpredictable deviations from scheduled output on other items on the system such as the whole or partial failure of another generator or some of its station auxilliary equipment; or due to station control difficulties caused other than by equipment failures.

Thus at any instant the actual MW output of a particular generator unit is given by:-

$$P_{Gj}(t) = P^{I}_{Gj}(t) + P^{A}_{Gj}(t) + P^{R}_{Gj}(t) + P^{D}_{Gj}(t) + P^{F}_{Gj}(t) + P^{S}_{Gj}(t) \quad (4.13)$$

or

$$\Delta P_{Gj}(t) = \Delta P^{A}_{Gj}(t) + \Delta P^{R}_{Gj}(t) + \Delta P^{D}_{Gj}(t) + \Delta P^{F}_{Gj}(t) + \Delta P^{S}_{Gj}(t) \quad (4.14)$$

where j indicates the jth generator on the system

P_{Gj} is the actual MW output at time t

P^{I}_{Gj} is the planned MW output as given by a unit commitment instruction made at time t-T.

ΔP^{A}_{Gj} is the change in this planned output due to a change in the availability of the generator which may have occurred at any timesince the prediction was made.

ΔP^{R}_{Gj} is the change in the planned output due to a revision of the unit commitment instruction which may have occurred since the prediction was made.

ΔP^D_{Gj} is the change in the planned output due to an economic dispatch instruction (or accumulated series of instructions) which may have occurred since the most recent unit commitment revision.

ΔP^F_{Gj} is the change in the planned output due to automatic frequency control.

ΔP^S_{Gj} is the MW difference attributable to station control difficulties.

The majority of the literature on the modelling of generator uncertainty for load flow applications has simply spread the total uncertainty in the balance of power equally between all the generators (Ref. 27) or placed it all on the reference node. (Ref. 28).

While this may be adequate for long term planning applications it is most unrealistic in the operations timescale. A number of papers have considered more realistic treatment of this total uncertainty. Ref. 22 considered the effect of unit commitment although this was limited to the commitment at the daily peak. Refs. 23, 45, 46, 47 considered the effect of dispatch activity on equal incremental cost type systems. Ref. 24 considered the effect of dispatch activity on a merit order type system but neglected the effects of breakpoints in the cost functions of the marginal generators and resorted to a Monte Carlo solution method. This research addresses the effects of breakpoints in marginal generator cost functions and develops an analytical method to model these effects.

The modelling of uncertainty due to generator outage contingencies is not examined in this research as this has been treated in Ref. 40 using the convolution of capacity outage probabilities.

4.4.1 Dispatch Activity

We consider the dispatch activity part of the uncertainty in the generator MW output of (4.14). The dispatch activity is a consequence of adjusting the generation on the system as a whole to meet the sum of overall demand uncertainty due to forecasting errors and errors from target. These errors are random variables so that the dispatch activity component of (4.14) will also be a random variable.

Dispatch instructions appear as variations of the MW output of a generator but only within the period that the generator is synchronised and also within its minimum stable generation level and its maximum output usable level, and as minor modifications in synchronising and desynchronising times. Dispatch is thus conditional on the generator being synchronised and available. The probability characteristics attributable to dispatch are the conditional probabilities given that the generator is committed and that it is available.

The dispatch instructions are the visible result of calculations designed to minimize some objective function. Usually this function is the overall cost of production and is thus termed economic dispatch. In order to determine the characteristics of dispatch activity we must examine this objective function. In this work we consider only the process of economic dispatch.

The economic dispatch problem (in which production costs are minimized, neglecting transmission loss costs) can be expressed as:-

Minimize

$$F = \sum_{i=1}^{n} \sum_{j=1}^{NG} f_j(P_{Gj}) \Delta t_i \tag{4.15}$$

subject to

Generator minimum and maximum constraints

$$P_{Gj}^{min} \leqslant P_{Gj} \leqslant P_{Gj}^{max} \tag{4.16}$$

Balance of Power

$$\sum_{j=1}^{NG} P_{Gj,i} = D_i \qquad \text{for } i = 1,n \tag{4.17}$$

Generator rate constraints

$$(P_{Gj}^{i+1} - P_{Gj}^{i})/\Delta t_i \leqslant (dP_{Gj}/dt)^{max} \tag{4.18}$$

Immediate reserve requirements to cover generator loss incidents

$$0 \leqslant S_{Gj} \leqslant S_{Gj}^{max} \tag{4.19}$$

Network and Contingency Constraints

$$I_m^k \leqslant I_m^{max} \qquad \text{for all lines } m = 1,2 \ldots L$$

$$\text{and for all outages } k = 1,\ 2 \ldots L \tag{4.20}$$

which must be satisfied for each time step.

where f_j is the cost function of generator j

$f_j[P_{Gj}]$ is the cost at a particular output level

Δt_i is the ith time step in the n time step optimisation.

The number n and length of each time step Δt_i are variables which are chosen to cover the expected trajectory of the required prediction period.

A variety of solution methods exist to solve (4.15) subject to (4.16) to (4.20). The choice of method depends upon whether the cost function is modelled as a linear function or as a non-linear one. In the linear model case, standard linear programming techniques such as the Revised Simplex method or its dual are considered appropriate (Ref. 21).

4.4.1.1 Generator Cost Functions

The general shape of the cost function for a throttle governed-steam generator in common use on the CEGB system has three cost segments with two cost breakpoints (Fig. 4.1) (Ref. 53). This excludes the minor discontinuities due to mill quantisation effects. The low-end cost segment represents the fuel costs attributable to oil burners in use for flame stabilisation, the mid-range cost segment represents the fuel costs on coal firing alone and the high-end cost segment represents the fuel costs with oil burners in use to enhance the MW output under wet coal conditions or mill unavailability conditions. The cost breakpoints are the MW levels at which the fuel mix changes. This research examined the effects of the high-end cost breakpoints on the uncertainty of marginal generators as it is the output from the high-end cost segment that will be instructed when the system demand has been underestimated and it is the effect of this on line flows that needs to be assessed in predictive security assessment.

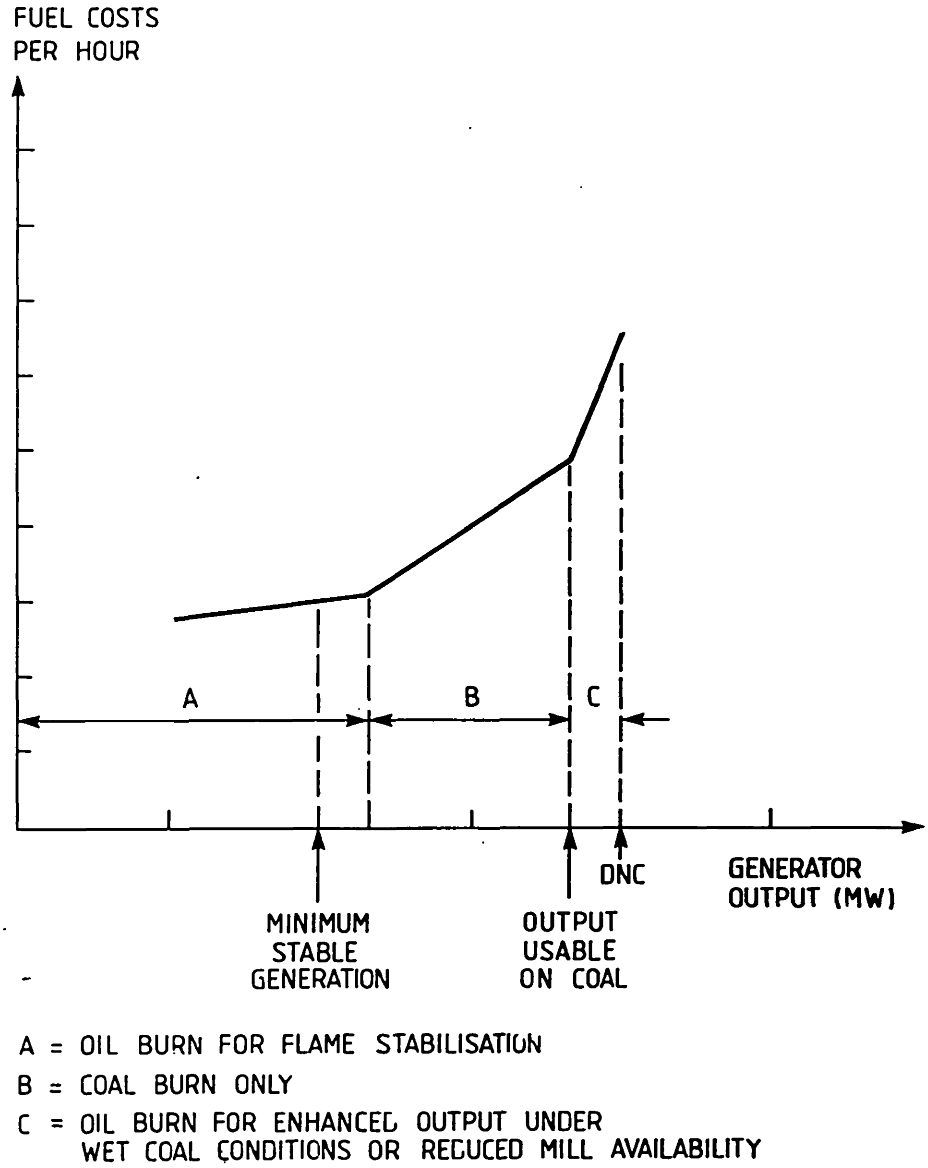


FIG. 4.1 A TYPICAL COAL - FIRED GENERATOR COST FUNCTION
PIECEWISE LINEAR APPROXIMATION OF THREE SEGMENTS

4.4.1.2 Merit Order Dispatch

When the cost function is modelled as linear so that the incremental cost is constant over the entire output range, the solution of (4.15) subject only to (4.16) and (4.17) results in the merit order dispatching method. Each generator is successively loaded to its maximum, starting with the cheapest, until the predicted load is met, so that uncertainties in demand are handled by varying the output of the most expensive generators within their limits.

Similarly when the cost function is modelled as piecewise linear so that the incremental cost is constant over the output range between consecutive cost breakpoints, the solution also results in the merit order dispatching method, with the difference that the same generator will appear several times in the list, each appearance corresponding to a particular segment of its cost function model.

Each generator is successively loaded to the MW level of the cost breakpoint corresponding to a cost segment, starting with the cheapest cost segments and will not be loaded beyond this level until the cost of its next segment becomes the next cheapest generation available.

(a) Effect of Constraints

The effect of the constraints (4.18), (4.19) etc. is to distort this merit order depending upon which generators the constraints are active. However, these would not generally effect the characteristic use of the most expensive plant to handle uncertainties. In the case of an active rate constraint a more expensive generator or generators would be instructed to increase its output simultaneously with the constrained generator in order to meet the anticipated increase in demand within the expected time between the instruction time and the time that the actual output is required.

In the case of an active immediate reserve constraint more expensive standby generation such as gas turbines would be synchronised to satisfy the constraint.

In the case of active network or contingency constraints more expensive generation unaffected by the active constraint(s) would be required to compensate for the constraint restricted generation.

Thus for any given point on the total demand trajectory, we can determine a set of generators that are independent of the demand level given by:

$$P_{Gj} \qquad j = 1, 2 \ldots NI \tag{4.21}$$

and a set of generators that are dependent on the uncertainty of the demand.

$$P_{Gj} \qquad j = NI+1, \ldots\ldots NG \tag{4.22}$$

which satisfy the balance of power (4.17), constraint limits (4.16) and which are dispatched in merit order to meet uncertainties. In this research the merit order distortions due to rate constraints and immediate reserve requirements are not quantified, although their effects could be handled approximately by modifying the merit order. In addition network constraints and contingency constraints will be explictly highlighted in the trajectory analysis so these need not be modelled.

(b) Loading Level

In addition we can determine the loading level for each of these generators based upon their cost functions. In the simple case of no intermediate cost breakpoints all the generators in the independent set will be at their maximum output usable (OU) level; all those generators in the dependent set whose incremental costs are less than that of the system marginal cost will also be at their maximum OU; all those generators in the dependent set whose incremental costs are greater than that of the system marginal cost will be at their minimum stable generation (MSG) level; and the generator whose cost equals the system marginal cost will be somewhere in between its OU and MSG level depending upon the exact demand level. A similiar argument applies to the situation with intermediate cost breakpoints, in which case loading levels also depend upon the cost breakpoints.

(c) MW Transmission losses

Optimising the cost of production, including MW transmission losses, can produce different generator dispatch instructions from those produced by an optimisation which excludes transmission losses. This particularly affects generators which have very similar incremental costs or generators which are remote from the load centres. The effect of transmission losses is to modify the effective incremental cost of a generator by a factor which represents the cost of supplying the extra transmission losses that would be incurred by supplying the demand from that generator rather than from some other generator.

The effect of MW transmission losses on merit order dispatch under uncertainty is not quantified in this research. Its effect could be accommodated approximately by using separate merit orders (modified by transmission loss penalty factors) for different levels of system load.

4.4.1.3 Dispatch Under Uncertainty

We have from equation (4.11) that the total demand uncertainty at any instant has a Gaussian probability density with zero mean and with a variance that is dependent upon the lead time of the prediction and the expected total demand.

The dispatch process must take up any demand uncertainty following the most recent unit commitment. This can be anytime upto about four hours. This is achieved by continuously updating predictive dispatch instructions (as forecast errors reduce closer to the event) and eventually deploying the regulating reserve capacity explicitly provided for this by the unit commitment process.

The overall effect of dispatch activity is the sum of all these instructions for each generator since the most recent unit commitment.

Thus from equation (4.15) the utilisation of the reserve capacity will be determined by the following strategy.

Negative values of ΔP_D (i.e. overestimates of demand at the unit commitment stage) will be taken up by de-loading the system marginal cost generator from its scheduled level to its next cost breakpoint (which may be its MSG level), and then the next most expensive generator and so on until the balance of power is met. Positive values of ΔP_D (i.e. underestimates of demand at the unit commitment stage) will be taken up by increasing the loading on the system marginal cost generator from its scheduled level to its next cost breakpoint level (which may coincide with its maximum output usable level), and then the next cheapest, marginal generator and so on until all the part-loaded steam generators are fully loaded, followed by gas turbine generators loaded to their maximum output in merit order. In the event of these summated reserves being insufficient, the balance of power will need to be achieved by load reduction, initially by voltage reductions and then by explicit load shedding.

The proportion of this regulating reserve held on steam generators as opposed to holding on gas turbines is determined in the unit commitment phase, and depends upon the cost of reserve holding for these two strategies. The cost minimum occurs when the increase in no load costs of holding an additional increment of reserve on steam generators exceeds the cost of the utilisation of the more expensive gas turbines. In practice, the unit commitment process will always provide some part-loaded steam generators on the system because of the combined effect of the facts that the total demand is rarely equal to the sum of the usable output of the connected generation; and the MSG levels are typically 50% of the generator's capacity.

4.4.1.4 - Example

We consider, for simplicity, an example where there are only two marginal steam generators with allocated regulating reserve capacity and each with one intermediate cost breakpoint. The remainder of the reserve capacity needed to make up the demand uncertainty is held on gas turbines although the model can be generalised to any number of marginal generators.

We define:-

(a) the incremental cost segments of the generators as constant and in the cost order of:-

$$\frac{df_{G1}^{(0)}}{dP_{G1}} < \frac{df_{G2}^{(0)}}{dP_{G2}} < \frac{df_{G1}^{(1)}}{dP_{G1}} < \frac{df_{G2}^{(1)}}{dP_{G2}} < \text{(incremental costs of gas turbines)} \qquad (4.23)$$

where the (0) and (1) indicate the cost segments

(b) the minimum and maximum limits as:-

$$P_{G1}^{min} \leqslant P_{G1} \leqslant P_{G1}^{max}$$

$$P_{G2}^{min} \leqslant P_{G2} \leqslant P_{G2}^{max} \tag{4.24}$$

$$0 \leqslant P_{GT1} \leqslant P_{GT1}^{max}$$

etc. for the remaining gas turbines

(c) the cost breakpoints as:-

$$P_{G1}^{BRK} \tag{4.25}$$

$$P_{G2}^{BRK}$$

(d) the regulating reserve capability of each generator as in Fig. 4.2 .

(e) the scheduled levels as:-

$$P_{G1} = P_{G1}^{BRK}$$

$$P_{G2} = P_{G2}^{BRK} \tag{4.26}$$

(in general the schedule level of the system marginal cost segment would only in particular cases be at one of the breakpoints).

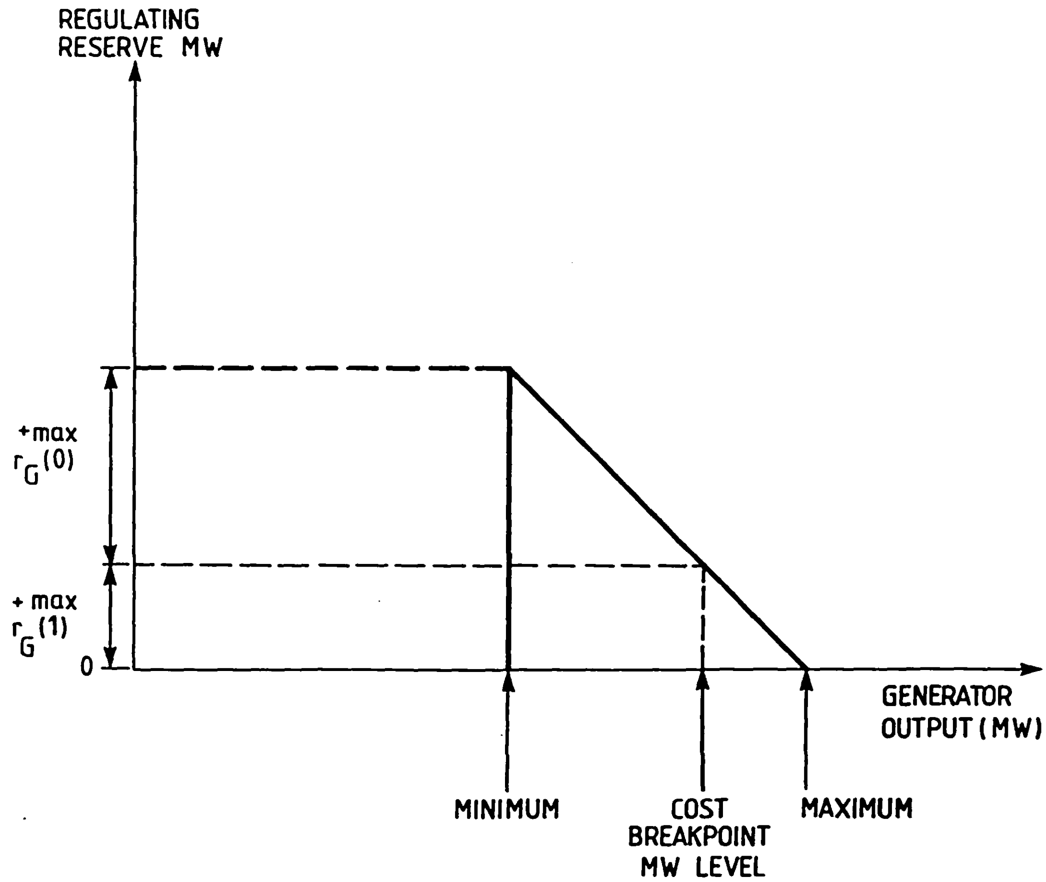


FIG 4.2 POSITIVE DISPATCHABLE RESERVE CAPACITY FOR GENERATOR DISPATCH EXAMPLE

For positive values of ΔP_D:-

The reserve utilisation is defined as:-

$$
\begin{aligned}
PG1 &= P_{G1}^{BRK} + \Delta P_D && \text{if } 0 < \Delta P_D \leqslant R^{+}_{G1(1)} \\
&= P_{G1}^{max} && \text{if } \Delta P_D > R^{+}_{G1(1)} \\
PG2 &= P_{G2}^{BRK} + \Delta P_D - R^{+}_{G1(1)} && \text{if } R^{+}_{G1(1)} < \Delta P_D \leqslant R^{+}_{G2(1)} \\
&= P_{G2}^{max} && \text{if } \Delta P_D > R^{+}_{G2(1)}
\end{aligned}
\tag{4.27}
$$

etc. for each gas turbine.

For negative values of ΔP_D, the reserve utilisation is defined as:

$$
\begin{aligned}
PG2 &= P_{G2}^{BRK} + \Delta P_D && \text{if } R^{-}_{G2(0)} \leqslant \Delta P_D < 0 \\
&= P_{G2}^{min} && \text{if } \Delta P_D < R^{-}_{G2(0)} \\
PG1 &= P_{G1}^{BRK} + \Delta P_D - R^{-}_{G2(0)} && \text{if } R^{-}_{G1(0)} \leqslant \Delta P_D < R^{-}_{G2(0)} \\
&= P_{G1}^{min} && \text{if } \Delta P_D < R^{-}_{G1(0)}
\end{aligned}
\tag{4.28}
$$

etc. for cheaper steam generators in merit order.

where r^{+}_{G} is the positive regulating reserve for an individual generator cost segment.

r^{-}_{G} is the negative regulating reserve for an individual generator cost segment.

R^+_G is the cumulative positive regulating reserve of cheaper marginal generators plus r^+_G

R^-_G is the cumulative negative regulating reserve of more expensive marginal generators plus r^-_G

4.4.2 An Analytic Model of Dispatch Uncertainty

(a) Means and Variances of Marginal Generators

In terms of the probability density of the total demand uncertainty the strategy of (4.27) and (4.28) produces the result shown in Fig. 4.3 so that the probability density for each marginal generator can be given as:-

$$
\begin{aligned}
&\mathrm{Prob}(P_{G1} > P^{max}_{G1}) = 0 \\
&\mathrm{Prob}(P^{BRK}_{G1} = P^{max}_{G1}) = \int_{R^+_{G1(1)}}^{\infty} f(\Delta P_D)\, d\Delta P_D \\
&\mathrm{Prob}(P^{BRK}_{G1} < P_{G1} < P^{max}_{G1}) = \int_{0}^{R^+_{G1(1)}} f(\Delta P_D)\, d\Delta P_D \\
&\mathrm{Prob}(P_{G1} = P^{BRK}_{G1}) = \int_{R^-_{G2(0)}}^{0} f(\Delta P_D)\, d\Delta P_D \qquad (4.29)\\
&\mathrm{Prob}(P^{min}_{G1} < P_{G1} < P^{BRK}_{G1}) = \int_{R^-_{G1(0)}}^{R^-_{G2(0)}} f(\Delta P_D)\, d\Delta P_D
\end{aligned}
$$

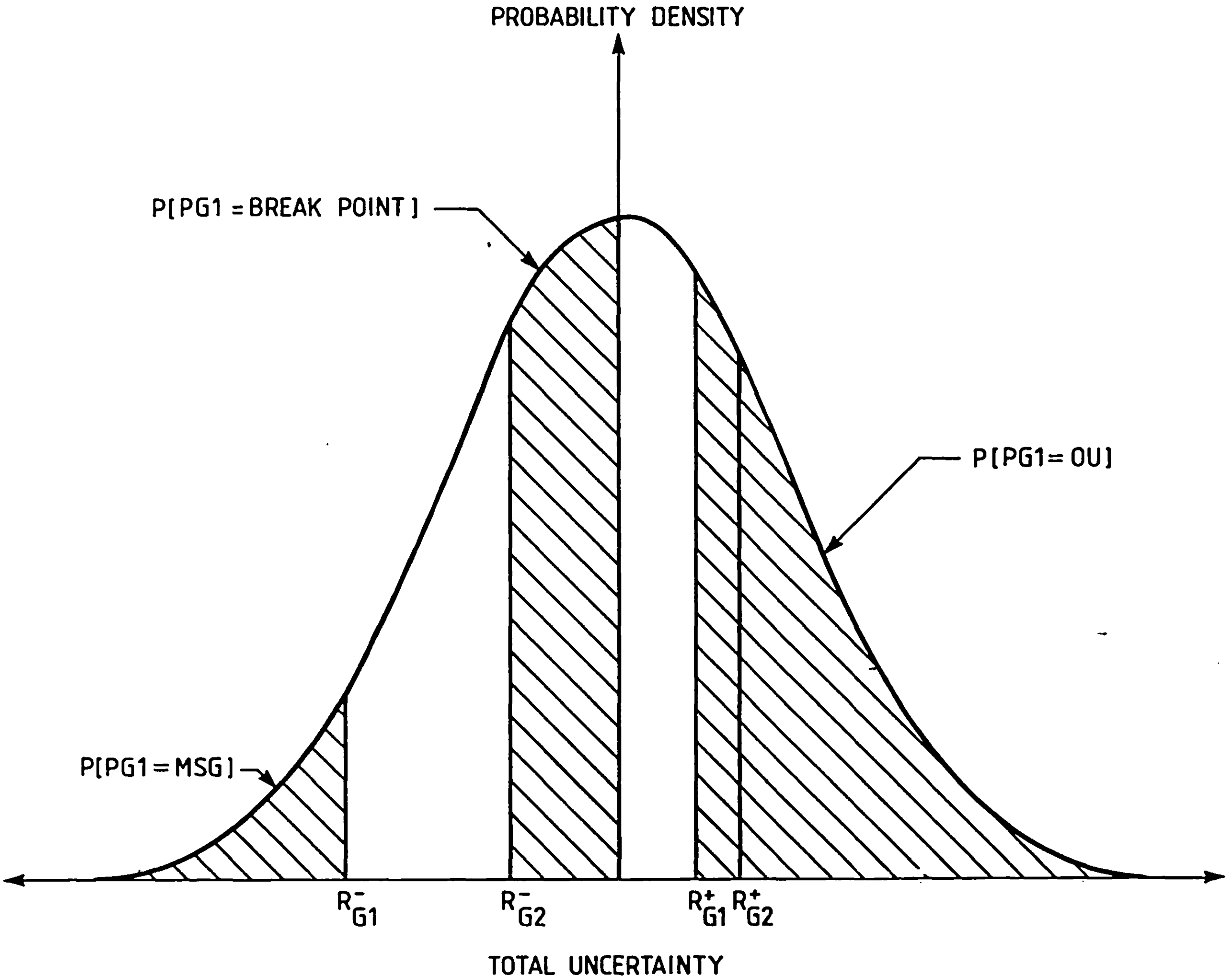


FIG. 4.3 GRAPHICAL VIEW OF METHOD OF COMPUTING MARGINAL GENERATOR PROBABILITY DENSITY AT CARDINAL POINTS

$$\mathrm{Prob}(P_{G1} = P_{G1}^{min}) = \int_{-\infty}^{R^{-}_{G1(0)}} f(\Delta P_D)\, d\Delta P_D$$

$$\mathrm{Prob}(P_{G1} < P_{G1}^{min}) = 0$$

and similarly for the other marginal generators.

Hence we can calculate the expected value and variance of the marginal generators as:-

$$\overline{PG1} = \int_{-\infty}^{\infty} p_{G1} . f(p_{G1}) . dp_{G1} \tag{4.30}$$

$$\mathrm{Var}\ (PG1) = \int_{-\infty}^{\infty} (p_{G1} - p_{G1})^2 . f(p_{G1}) . dp_{G1} \tag{4.31}$$

and by lumping the probability mass of the small effects of the values between the minimum stable generation level and the breakpoint level and the breakpoint level and the maximum output usable level we have that:-

$$\overline{PG1} \simeq p_{G1}^{min} . \int_{-\infty}^{R^{-}_{G1(0)} - \frac{1}{2} r^{-}_{G1(0)}} f(\Delta p_D) . d\Delta p_D$$

$$+ \; p_{G1}^{BRK} . \int_{R^{-}_{G1(0)} - \frac{1}{2} r^{-}_{G1(0)}}^{R^{+}_{G1(1)} - \frac{1}{2} r^{+}_{G1(1)}} f(\Delta p_D) . d\Delta p_D$$

$$+ p_{G1}^{max} \cdot \int_{R^{+}_{G1(1)} - \frac{1}{2} r^{+}_{G1(1)}}^{\infty} f(\Delta p_D).d\Delta p_D \qquad (4.32)$$

and

$$\mathrm{Var}\ (PG1) \simeq (p_{G1}^{min} - \overline{PG1})^2 \cdot \int_{-\infty}^{R^{-}_{G1(0)} - \frac{1}{2} r_{G1(0)}} f(\Delta p_D).d\Delta p_D$$

$$+ (p_{G1}^{BRK} - \overline{PG1})^2 \cdot \int_{R^{-}_{G1(0)} - \frac{1}{2} r^{-}_{G1(0)}}^{R^{+}_{G1(1)} - \frac{1}{2} r^{+}_{G1(1)}} f(\Delta p_D).d\Delta p_D$$

$$(4.33)$$

$$+ (p_{G1}^{max} - \overline{PG1})^2 \cdot \int_{R^{+}_{G1(1)} - \frac{1}{2} r^{+}_{G1(1)}}^{\infty} f(\Delta p_D).d\Delta p_D$$

The values of the probability density functions at these three discrete output levels can be straight forwardly calculated from the normalised probability density of the total demand uncertainty. (See 4.5.2.2(e))

(b) Crosscovariances between Marginal Generators

The crosscovariances between every marginal generator pair may also be computed from the knowledge of the total uncertainty pdf and the regulating reserve capacities of all the marginal generators.

We have, for one such pair, that the crosscovariance is:-

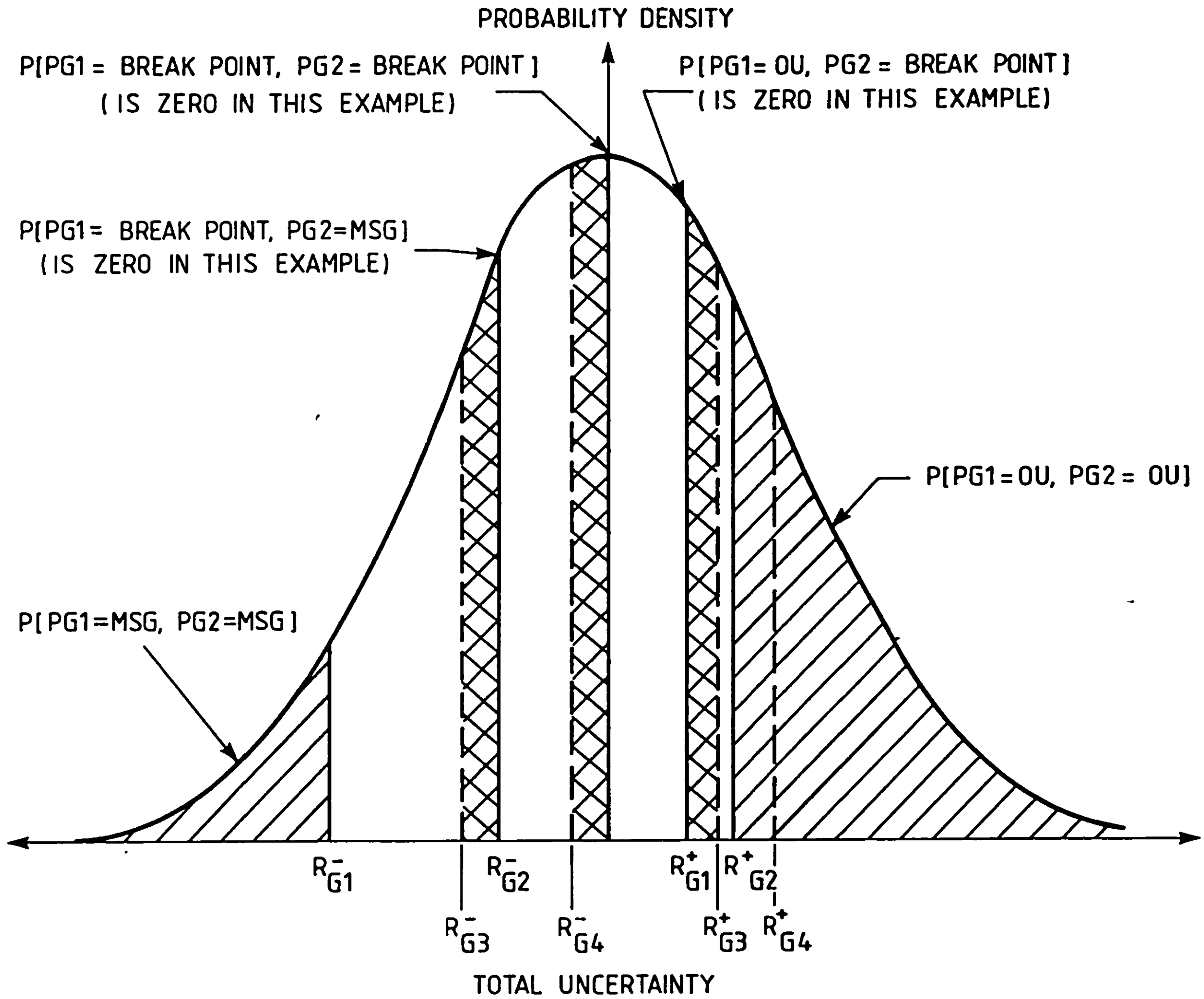
$$\mathrm{Cov}(PG1,PG2) = \int\int_{-\infty}^{\infty} (PG1-\overline{PG1})(PG2-\overline{PG2})f(P_{G1},P_{G2})dP_{G1}dP_{G2} \quad (4.34)$$

where $\overline{P_{G1}}$ is the expected value of PG1

$f(P_{G1}, P_{G2})$ is the joint probability density between P_{G1} and P_{G2}

The joint probability density may be computed directly from the total uncertainty pdf and the marginal generator regulating reserve capacities by integrating the appropriate areas of the total uncertainty pdf.

The nature of this joint pdf is such that most of the probability mass is in fact situated at the discrete points which represent the MSG, cost breakpoint and OU levels. This enables an approximate joint pdf to be computed as the probability mass at these points only, and consequently the crosscovariance computation need only be calculated at these points as well. The computation of this approximate joint pdf can be clearly visualised by considering the density function diagram of Fig. 4.4 . We see that the probability that both generators are at their MSG level simultaneously is the overlapping area of the total uncertainty pdf and so on for the other major points as shown in Fig. 4.4 .



NOTE: G3 AND G4 INTRODUCED TO SHOW HOW CROSS-HATCHED PROBABILITY MASS MAY EXIST

FIG. 4.4 GRAPHICAL VIEW OF METHOD OF COMPUTING MARGINAL GENERATOR JOINT PROBABILITY DENSITIES AT CARDINAL POINTS

As in the means and variances case we can lump the probability mass of the values away from these major points to the probability mass at these points. In the case where the two marginal generators are the same generator we then have the same equation as (4.33) for the autocovariance.

Equations (4.32 to 4.34) constitute the model of uncertainty for the marginal generators of (4.22).

The model for marginal generators can equally be applied to the gas turbine utilisation.

The error introduced by lumping the intermediate probability mass at the cost breakpoints can be compensated by scaling each covariance term by the ratio of the total demand variance and the sum of the generator covariances. This avoids the error that could occur if this approximation were applied to a system where one generator is capable of meeting the entire demand uncertainty.

4.4.3 - Synchronising and Desynchronising Time Uncertainties

The statistical modelling of dispatch activity for the trajectory problem is further complicated by the modifications that occur to the synchronising and desynchronising times of generators (Fig. 4.5) and hence to the time at which the MSG level is achieved.

A complete statistical model of generator dispatch activity would have to take account of these uncertainties as well, by modifying equations (4.27) during the short periods of uncertainty of the synchronising and desynchronising events.

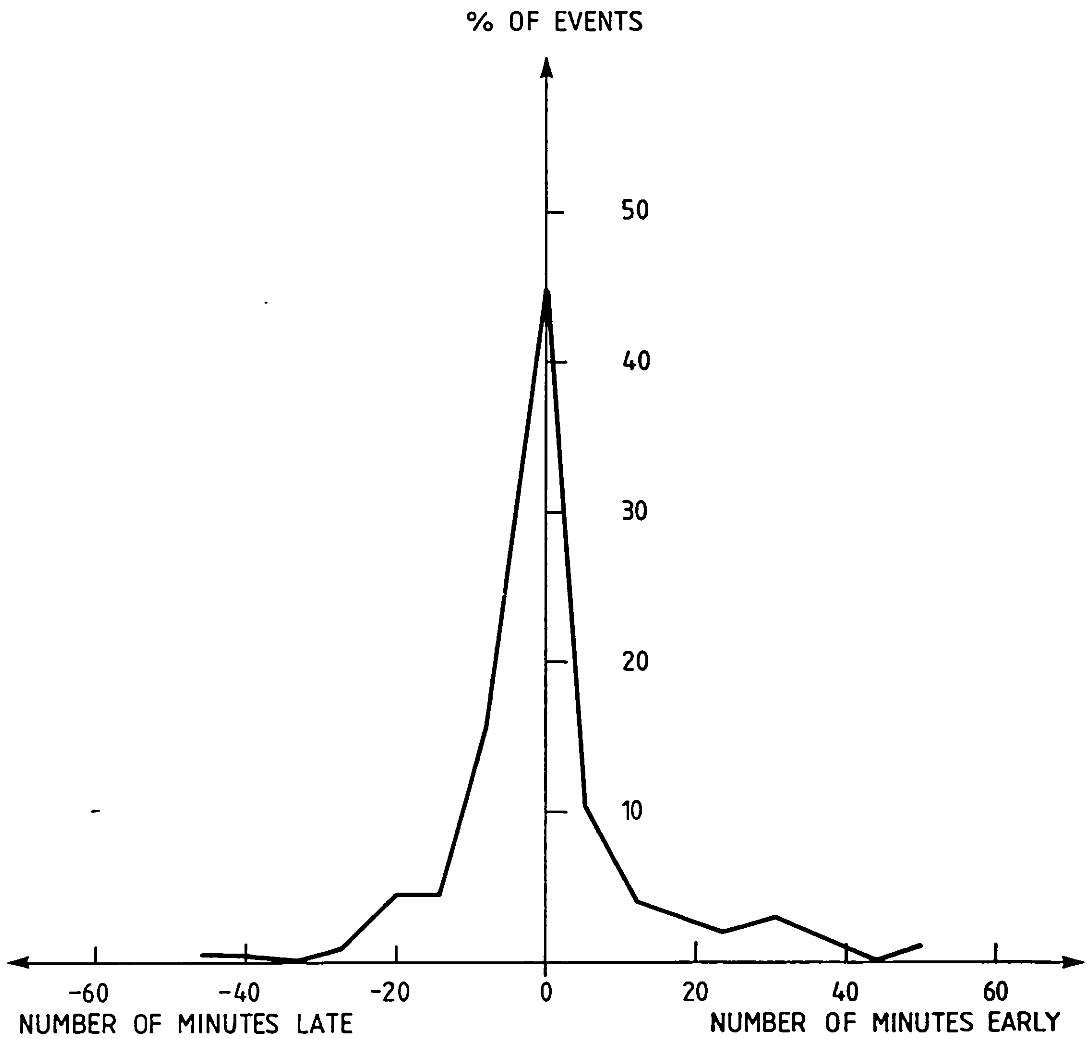


FIG 4.5 UNCERTAINTY OF TIME OF SYNCHRONISING AND DESYNCHRONISING EVENTS

4.5 Experimental and Computational Experience

The experimental and computational work presented in this section was designed to test the validity of the theoretical development related to the dispatch activity aspect of generator uncertainty described in the preceding sections.

In particular the aims were:-

(a) to verify the characteristic use of the marginal plant to meet overall uncertainties in the system and the resulting implication of time-of-day dependent correlations between such generators

(b) to verify the effect of generator minimum and maximum limits and cost breakpoints on the model of dispatch uncertainty as formulated in (4.27).

The general approach adopted was to observe the actual characteristics of generator uncertainties as far as practically possible and to supplement these with Monte Carlo type simulations to examine particular aspects of dispatch activity in greater detail under controlled conditions.

4.5.1 Observation of Generator Uncertainty Characteristics

We have from (4.1) that the uncertainty in the output of any generator at any instant is the difference between the actual MW output of the generator and its planned MW output for that instant. Thus we require access to measured generator outputs and to planned outputs for the corresponding times.

4.5.1.1 Source of data

This data was available in the database of the VAX 11/780 computer provided for a research experiment into the feasibility and benefits of on-line economic dispatch. The actual generator outputs were available as raw data values each updated approximately every 20 to 30 seconds, as validated minute average values or as validated half-hour average values.

The planned generator outputs were available as predicted values for 10 mins. 15 mins., 30 mins., 1 hour, 1½ hours and 2 hours ahead, each prediction being updated every 5 minutes. These values were produced from an on-line dispatch algorithm implemented on the VAX 11/780 computer.

4.5.1.2 Choice of Data Items

The current dispatch method is a manually oriented one in which the operational control of the 50 GW or so of the generation on the CEGB System is devolved to six control centres each of which control the generators in their own geographic area. Each control centre is required to control its generation in an economic manner in order to achieve target inter-Area transfers which are regularly updated by the National Control Centre and which reflect the economics of the marginal generators in each Control Area.

However, for a period of one week these Inter-Area targets were replaced with individual generator output targets produced by the on-line dispatch system, and the Area loading engineers were requested to implement these targets as far as possible. Thus as both the targets and out-turns are known, it should be possible to make meaningful analyses of the differences.

Accordingly the planned generator outputs and the generator outturns for a number of low cost generators and a number of higher cost generators were logged for each half-hour for several hours. The planned outputs at the 10 minute and 2 hour ahead time steps were chosen to observe the effect of changes in total uncertainty on the marginal generator targets. In addition the cost function of each of these generators was logged with any changes to it and to the generator OU's. The total demand forecasts for these timesteps and the demand out-turn were also logged as was the system marginal cost for these timesteps. The period during which data logging was carried out was chosen to be at a time when each of these generators was ready and available for dispatch and not likely to be subject to a unit commitment revision.

The plots of the logged generator out-turns, 10 min. ahead dispatch and 2 hours ahead dispatch are shown in Fig. 4.6 .

4.5.1.3 Data Interpretation

The logged data clearly indicate the use of marginal generators for loading and regulation. In addition it is apparent that these generators are loaded to their appropriate cost breakpoint level.

Another observed characteristic is that revisions of the 2 hour ahead dispatch instructions closer to the event result in changes to the marginal generator instructions to appropriate cost breakpoint levels. Whether or not a particular marginal generator has a target revision depends upon the magnitude of the demand estimation error, the sensitivity of the system marginal cost at this system demand level and the cost of the particular marginal generator relative to the system marginal cost.

Actual generator loadings relative to the 10 minute ahead dispatch revision indicate the use of the marginal generators to pick-up other generator shortfalls, target errors and demand estimation errors.

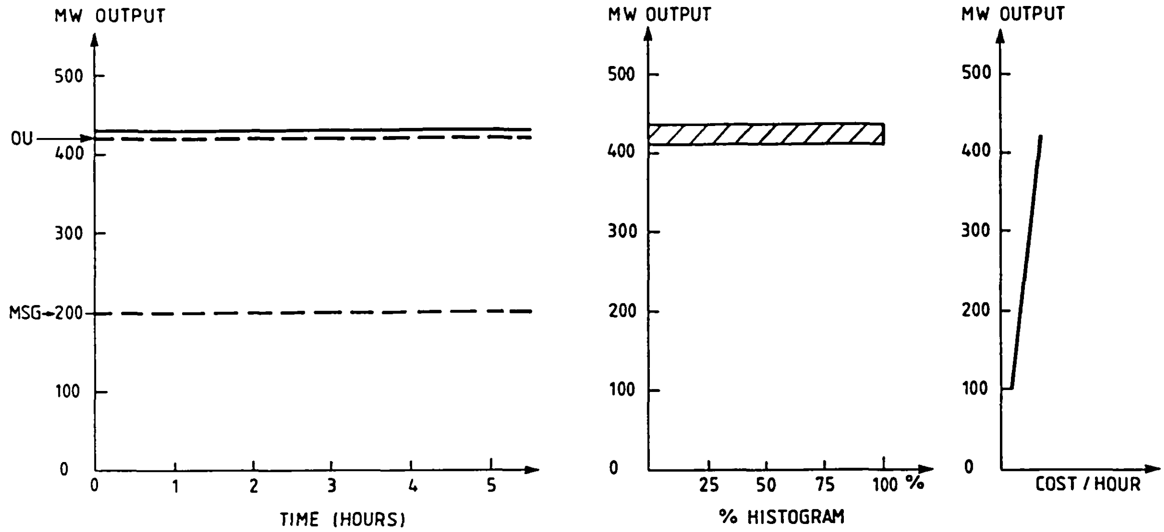


FIG. 4.6 RECORDED GENERATOR DATA-GENERATOR 'A' OUTTURN

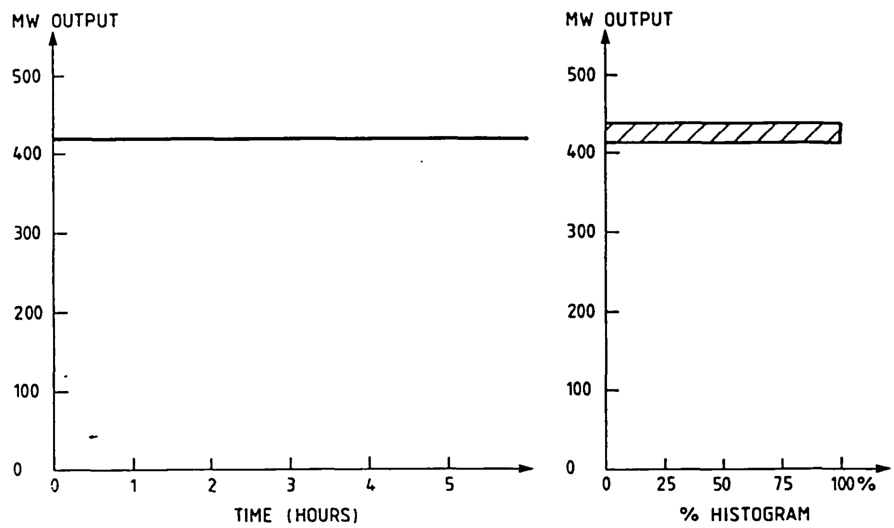


FIG. 4.6 RECORDED GENERATOR DATA - GENERATOR 'A' 10 MIN. AHEAD PREDICTION

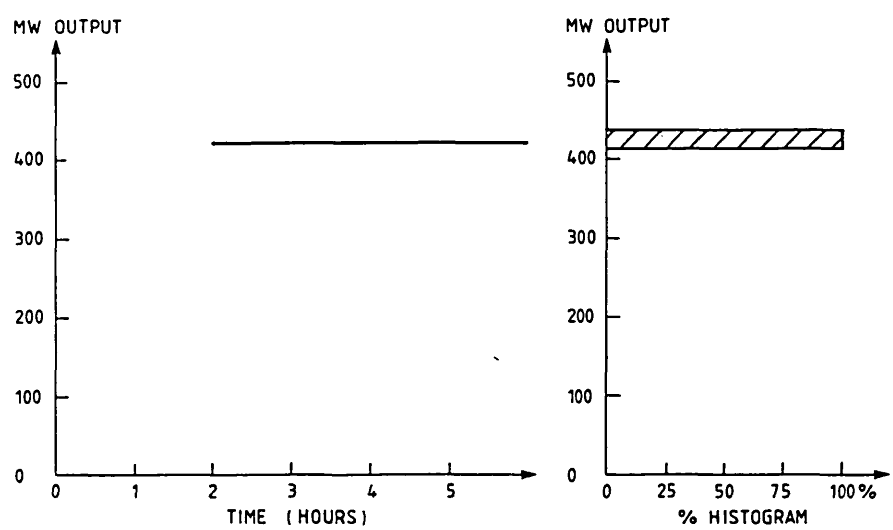


FIG. 4.6 RECORDED GENERATOR DATA - GENERATOR 'A' 2 HOUR AHEAD PREDICTION

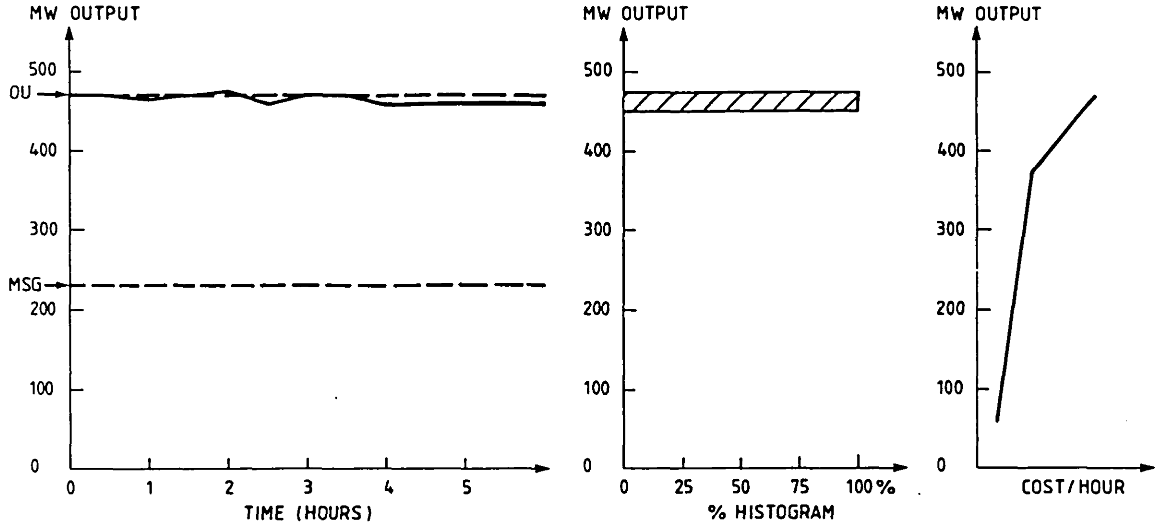


FIG. 4.6 RECORDED GENERATOR DATA – GENERATOR 'B' OUTTURN

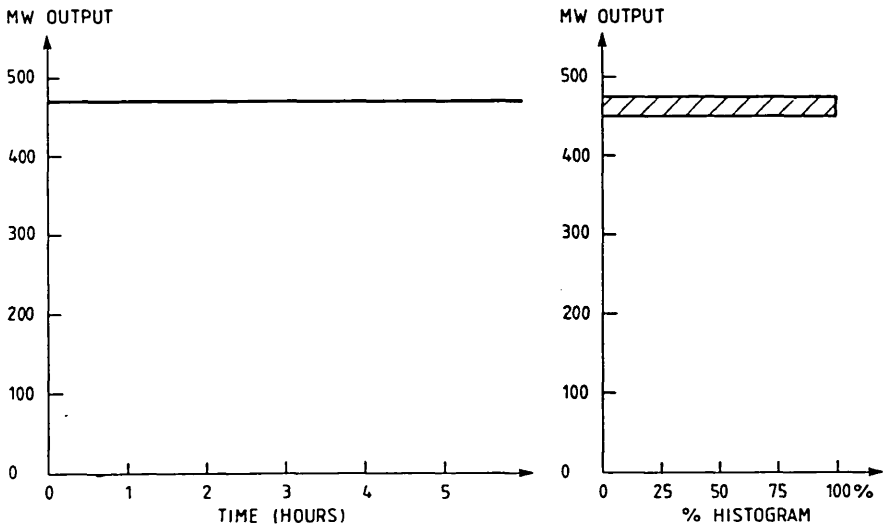


FIG. 4.6 RECORDED GENERATOR DATA – GENERATOR 'B' 10 MIN. AHEAD PREDICTION

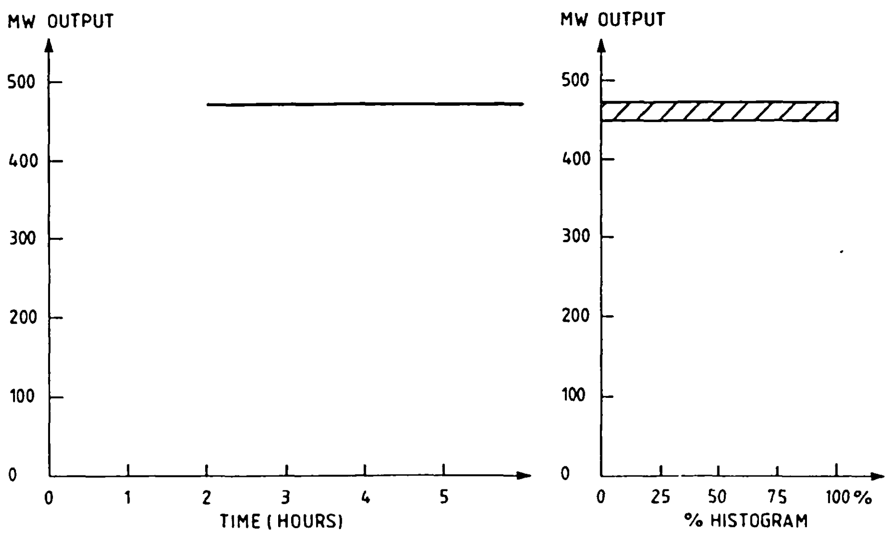


FIG. 4.6 RECORDED GENERATOR DATA – GENERATOR 'B' 2 HOUR AHEAD PREDICTION

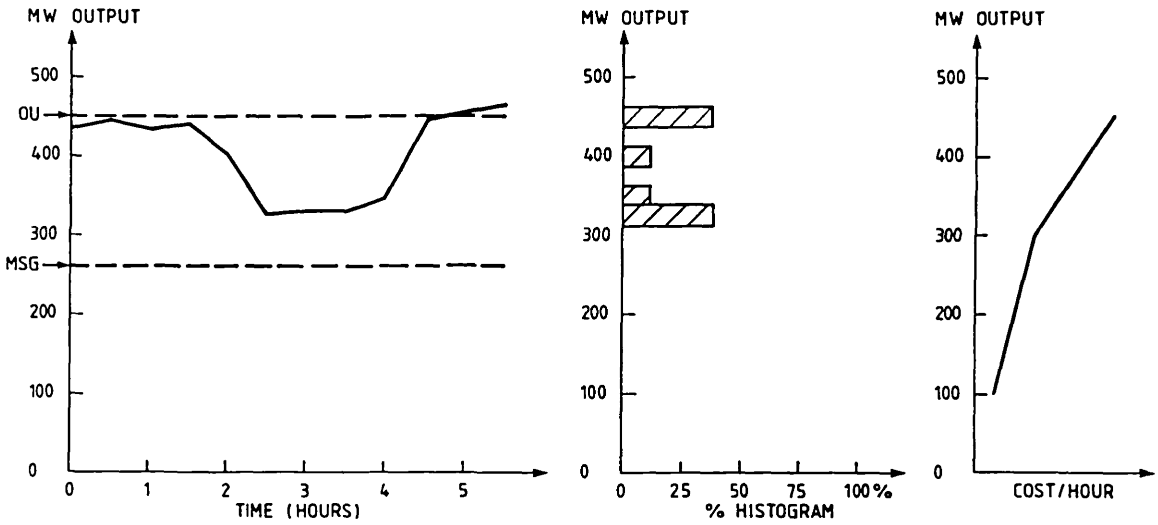


FIG 4.6 RECORDED GENERATOR DATA – GENERATOR 'C' OUTTURN

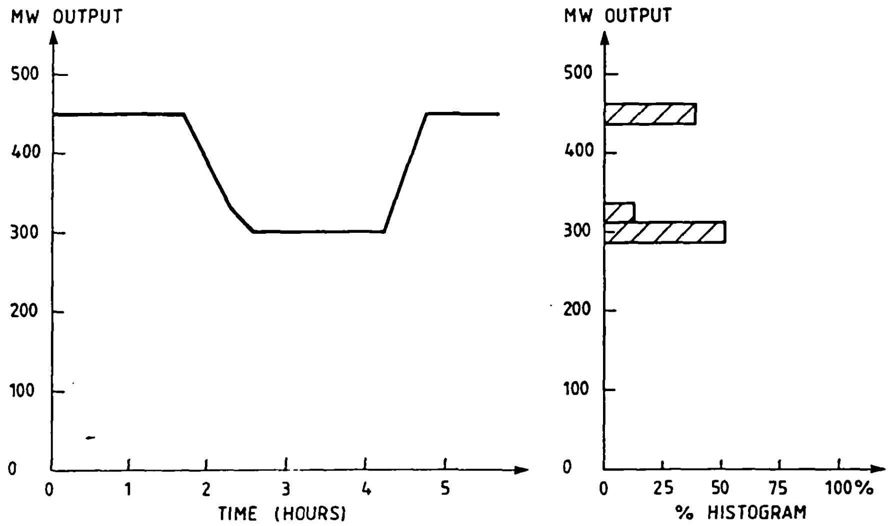


FIG. 4.6 RECORDED GENERATOR DATA GENERATOR 'C' 10 MIN. AHEAD PREDICTION

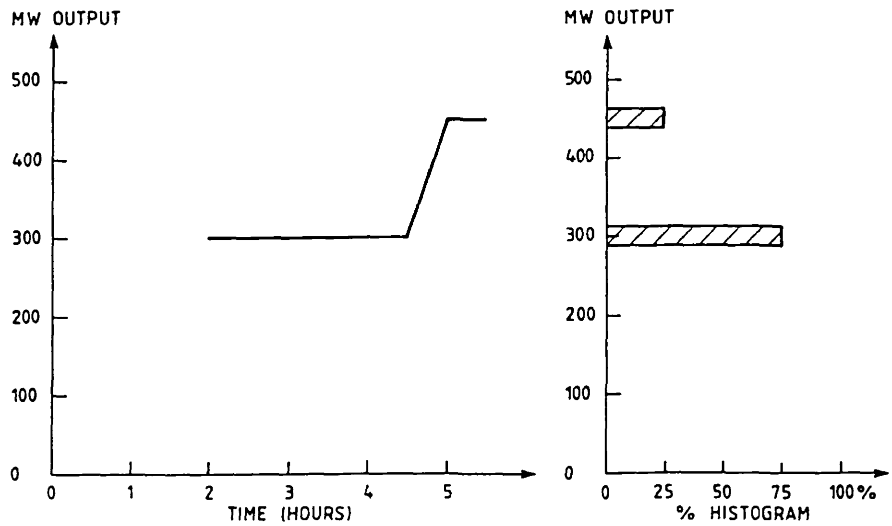


FIG 4.6 RECORDED GENERATOR DATA – GENERATOR 'C' 2 HOUR AHEAD PREDICTION

These features are consistent with the probability model of (4.32) which gives a high probability of finding the output of a marginal generator at either its minimum, its maximum or at one of its cost breakpoint values as a result of dispatch activity.

4.5.2 Simulation of the effect of Demand Uncertainty on Dispatch

4.5.2.1 Basic Approach

One practical way of examining the effect of total demand uncertainty on dispatch is through a Monte Carlo simulation which consists of running and probabilistically processing several thousand cases of individual dispatches where the total demand data are generated by pseudo random numbers. The flow chart for this computational procedure is shown in Fig. 4.7 and required the development of software to produce: Gaussian shaped random numbers with defined mean and variance; economic dispatch instructions for each generator for each value of total demand uncertainty; and a probability density function analyser.

4.5.2.2 Software Development

(a) Random Number Generator

Random sequences of uniformly distributed numbers can be generated by several algorithms. One commonly used algorithm generates the (i+1)th random number x_{i+1} as the remainder in the division:-

$$x_{i+1} = \text{Remainder}\ [\ (a x_i + C)/m\ ] \qquad (4.35)$$

where $a \geqslant 0$

$C < m$

and, a, c, m and x_i are all integers.

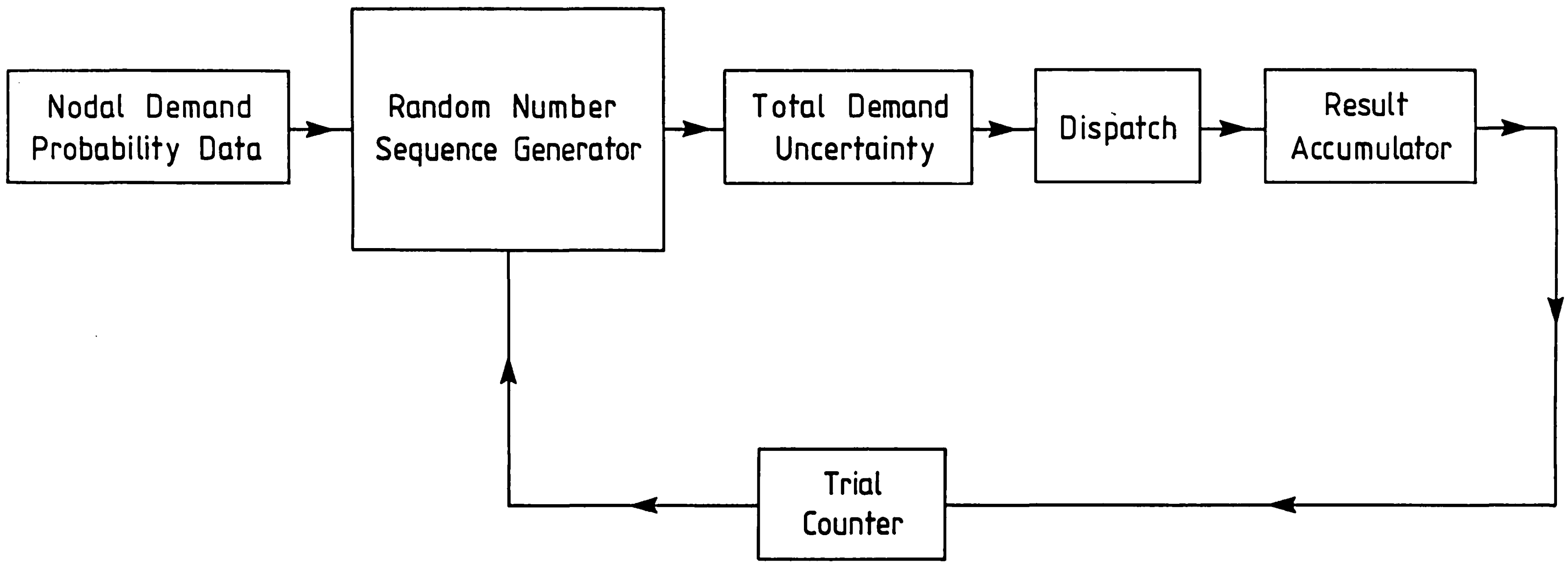


FIG. 4.7 MONTE CARLO SIMULATION FLOW CHART

As the sequence generated by the algorithm repeats itself over a certain period we have a pseudo random number sequence. However, as long as the period is larger than the number of trial cases in the simulation we can consider the sequence to be truly random. The pseudo random number generator available as a FORTRAN library function on the VAX 11/780 was used in this work. This provided a pseudo random number uniformly distributed in the range $0 \leqslant p < 1$. We need to convert this uniform distribution into a Gaussian distribution.

(b) Gaussian distribution shaping

A simple method of obtaining a Gaussian random variable from a uniform random variable is to sum a number of consecutive samples of the uniform random variable. This sumis itself a random variable and we have from the Central Limit Theorem that its distribution is Gaussian.

A more precise method is as follows:-

Consider a random variable T with the distribution function $F_T(t)$.

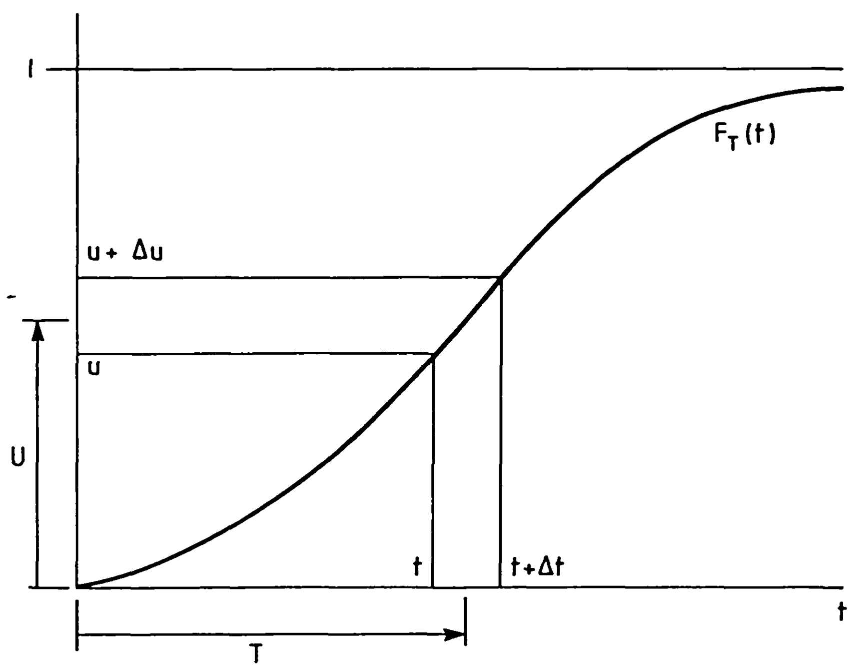
With each value t that T can assume let a value u be associated such that $u = F_T(t)$. This set of u values then defines a random variable U which depends on T, as shown in Fig. 4.8 . Thus:-

$$\text{Prob}(u < U \leqslant u + \Delta u) = \text{Prob}(t < T \leqslant t + \Delta t) \tag{4.36}$$

where, since $F_T(t)$ is the distribution function of T,

$$\text{Prob}(t < T \leqslant t + \Delta t) = F_T(t + \Delta t) - F_T(t) \tag{4.37}$$

FIG. 4.8 MAPPING A UNIFORM RANDOM VARIABLE TO A DIFFERENT DISTRIBUTION



By Fig. 4.8 however, the right-hand side of (4.37) equals Δu and,

therefore, by combining (4.36) and (4.37) one obtains:

$$\text{Prob}(u < U \leqslant u + \Delta u) = \Delta u \qquad (4.38)$$

This result indicates that U has a uniform distribution between 0 and 1

It follows that if one randomly selects a value u from among a set of numbers uniformly distributed in the range (0,1), and computes t from

$$t = F_T^{-1}(u) \qquad (4.39)$$

where $F_T^{-1}(u)$ is the inverse function of $F_T(u)$, the t values will form a set with the distribution function $F_T(t)$.

The approach of (4.39) was already implemented as a FORTRAN library function on the VAX 11/780 using a formula in Ref. 32 to provide the required transformation to a Gaussian distribution with zero mean and unit variance.

(c) Probability Density Function Analyser

A routine to accumulate the sequence of random numbers into their probability density function was required. This was developed as follows:-

For any random variable X, its probability density function is given by:-

$$f(x) \simeq \text{Prob}\left[\frac{x \leqslant X \leqslant x + \Delta x}{\Delta x}\right] \qquad (4.40)$$

Now this represents the frequency with which X falls within a band $x + \Delta x$ for values of x from $-\infty$ to $+\infty$. Accordingly a histogram count normalised by the number of trial cases and divided by the histogram's horizontal resolution will give the probability density function.

The software to generate the Gaussian distributed pseudo random numbers and the PDF analyser was tested by comparing their results with the results from an analytic computation of a Gaussian PDF (Fig. 4.9). The analytic form was calculated as:-

$$f(x) = \frac{1}{\sqrt{2\pi}} \cdot e^{-x^2/2} \qquad (4.41)$$

The results are shown for the probability density of the total demand on the test system, i.e. mean of 13347 MW and standard deviation of 667 MW which were obtained by scaling the standardised Gaussian random numbers as:-

$$x = \sigma z + \mu \qquad (4.42)$$

where x is the scaled variable

z is the standardised variable

μ and σ^2 are the mean and variance of x

The normalisation of the histogram count by its horizontal resolution is important in order to produce the probability density from the probability mass values produced by the histogram count normalised by the total count.

However, the form of the probability density functions of the marginal generators - with the probability mass at discrete points - is such that a probability value for these points is a more accurate representation for the probability density (the probability mass attributable to the rest of the

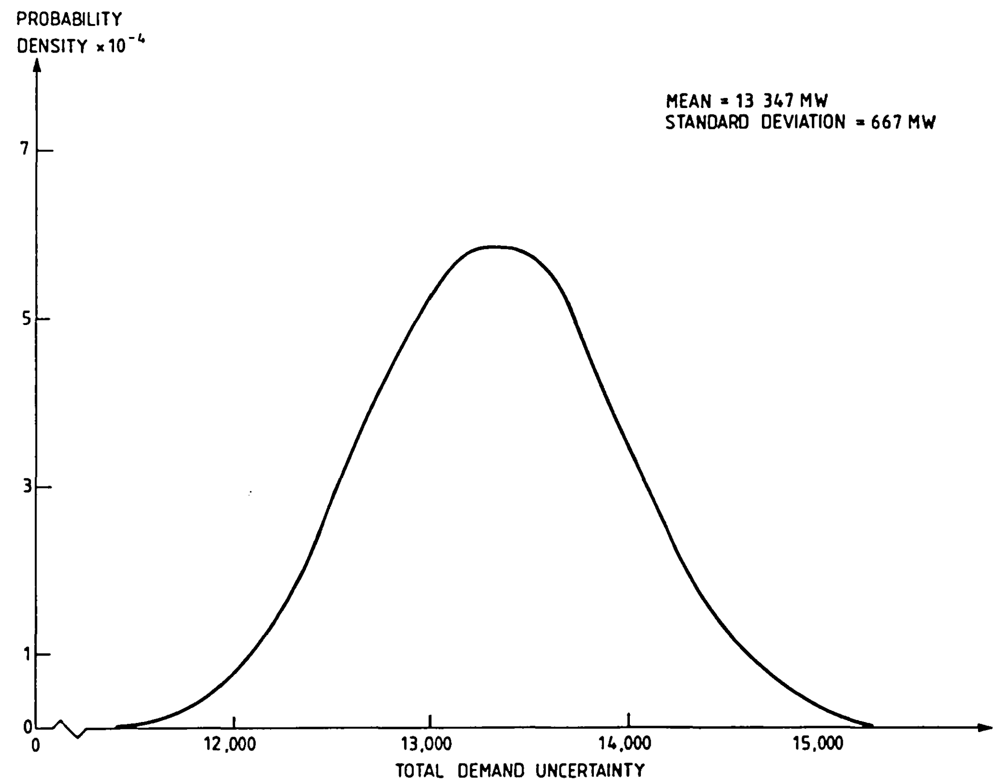


FIG. 4.9 TOTAL DEMAND UNCERTAINTY ANALYTIC PDF

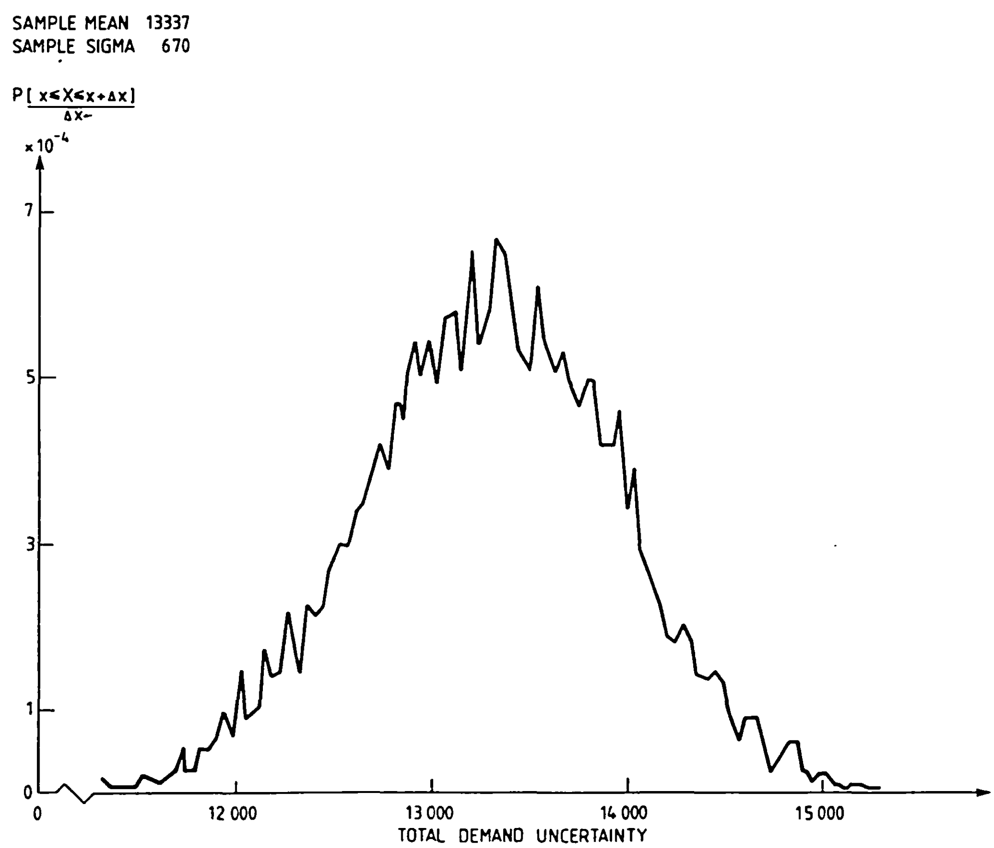


FIG. 4.9 TOTAL DEMAND
PROBABILITY DENSITY
MONTE CARLO SIMULATION 5000 TRIALS

band being negligible). Thus, rather than compute probability density functions, probability mass functions were computed on the basis that valid comparisons be made only at discrete points.

(d) Dispatch Instructions

Dispatch instructions in the form of (4.27) and (4.28) were coded and tested by generating the sequences of dispatch instructions resulting from the random sequence representing total demand uncertainty and then re-constituting the total demand uncertainty back from these instructions. The simulation parameters used are discussed in 4.5.2.3.

(e) Analytical Computation of Joint Probability Densities and Cross Covariances

The calculations were carried out on a cost segment basis and subsequently combined to give generator values. We have:-

$$PG1 = PGX + PGY \qquad (4.43)$$

$$PG2 = PGW + PGZ$$

where PGX, PGY, PGW, PGZ are the outputs of the cost segments of generators 1 and 2.

$$\text{So } E\,[PG1] = E\,[PGX + PGY] = E\,[PGX] + E\,[PGY]$$

$$E\,[PG2] = E\,[PGW] + E\,[PGZ] \qquad (4.44)$$

$$\text{and } \mathrm{Cov}\,(PG1, PG2) = E\,[(PG1 - \overline{PG1})(PG2 - \overline{PG2})]$$

$$= \mathrm{Cov}\,(PGX, PGW) + \mathrm{Cov}\,(PGX, PGZ) + \mathrm{Cov}\,(PGY, PGW) + \mathrm{Cov}\,(PGY, PGZ)$$

These segment means and cross covariances can be computed from the joint probability densities of the cost segments.

These were computed based upon the well known properties of the Gaussian pdf and its integral - the error function:-

$$\mathrm{erf}(x) = \frac{1}{\sqrt{2\pi}} \int_0^x e^{-y^2/2}\, dy \qquad (4.45)$$

This function was computed using the Trapezoidal method. In this method the integral of the exponential function is approximated as the sum of the areas of a number of trapezoids.

ie for a small strip of width h:-

$$\mathrm{erf} = \int_{x_0}^{x_1} y(x)dx \simeq {}^{h}/_{2}(y_0 + y_1) \qquad (4.46)$$

where $h = x_1 - x_0$; $y(x) = e^{-x^2/_2}$

In this work each erf (x) value was computed using 10 strips. We have for generator segment X and generator segment Y, in which Y is more expensive than X, that the joint probability density (as a first approximation) is given by:

$$P(X = XMIN,\ Y = YMIN) \simeq \tfrac{1}{2} - \mathrm{erf}(|\bar{R}_x - \tfrac{1}{2}\bar{r}_x|)$$

$$P(X = XMAX,\ Y = YMIN) \simeq \mathrm{erf}(|\bar{R}_x - \tfrac{1}{2}\bar{r}_x|) - \mathrm{erf}(|\bar{R}_y - \tfrac{1}{2}\bar{r}_y|)$$

$$P(X = XMAX,\ Y = YMAX) \simeq \tfrac{1}{2} + \mathrm{erf}(|\bar{R}_y - \tfrac{1}{2}\bar{r}_y|) \qquad (4.47)$$

These equations apply when both X and Y are in the left hand half of the total uncertainty pdf. Equations of similar form apply when X is inthe left half and Y is in the right half, and when both X and Y are in the right half.

In the case of the system marginal cost generator segment:-

$$P(X = XMIN, X = XMIN) \simeq \tfrac{1}{2} [1 + erf(|r_x^+|) - erf(|r_x^-|)]$$

$$(4.48)$$

$$P(X = XMAX, X = XMIN) = 0$$

$$P(X = XMAX, X = XMAX) \simeq \tfrac{1}{2} [1 + erf(|r_x^-|) - erf(|r_x^+|)]$$

The approximation of lumping the low level probability mass as in (4.47) introduces an error in the total system covariance sum. This error was removed by scaling each segment covariance by

$$CoV (PGX, PGY) = Cov (PGX, PGY) \times COVPD/\Sigma\ COVPG \qquad (4.49)$$

where COVPD is the variance of the total demand uncertainty

Σ COVPG is the variance of the total system generation uncertainty.

4.5.2.3 Simulation Parameters

The test system of (3.5.1) was chosen as it was for this system that the results of this uncertainty modelling were required in the Worst Case Algorithm.

(a) Demand Uncertainty Parameters

The demand uncertainty model of (4.12) was considered to be an appropriate one for each of the nodal demands. The lead time period was taken such that the mean square demand uncertainty error had approximately levelled off in order to account for the total uncertainty that dispatch may handle. On the CEGB system this occurs at a lead time of about 2-3 hours for the total demand and at a level of about 3% of the total demand (Ref. 20), i.e. an rms error of 1000 MW on a 30,000 MW demand.

On this basis the total demand variance and the nodal demand variances for the Test System were computed as:-

$$\sigma_i = \sqrt{\frac{\mu_i}{\mu_T}} \cdot \sigma_T = 5.774 \sqrt{\mu_i} \qquad (4.50)$$

where σ_i^2 is the variance at node i

μ_i is the mean demand at node i

in which we assume that the nodal demand uncertainties are independent,so that

$$\sigma_T^2 = \sum_{i=1}^{20} \sigma_i^2 \qquad (4.51)$$

where σ_T^2 is the variance of the total demand on the Test System.

The mean nodal demands were taken as the corresponding nodal demands recorded at the time of the total demand peak and shown in Fig. 3.9 . These particular values are shown in Table 4.1 .

These values of μ_i and σ_i were used to scale the standardised Gaussian random numbers as in (4.42) to produce random nodal demand values with defined means and variances. Care was taken to screen out values of nodal demand of less than zero. The errors introduced by this screening process are negligible.

Table 4.1

Mean and Standard Deviation of Nodal Demands

Node	Mean (MW)	Standard Deviation (MW)
1	582	139
2	384	113
3	840	167
4	932	176
5	274	95
6	350	108
7	577	138
8	370	111
9	646	146
10	955	178
11	1041	186
12	243	90
13	273	95
14	140	68
15	616	143
16	508	130
17	460	123
18	1383	214
19	2372	281
20	402	114
Total	13348	667

(b) Dispatch Parameters

In the absence of the constraints of (4.18), (4.19) and (4.20), the dispatch problem of (4.15) for demand uncertainty simplifies to equations of the form of (4.27), so that the required parameters are:- the incremental fuel cost order of each segment for each generator; the minimum stable generation, cost breakpoint levels and maximum output usable for each generator; the generator schedule; the model of reserve capability for each generator for its full range of output; and the treatment of demand uncertainty levels greater than the sum of the reserve held on part-loaded generators.

The incremental fuel cost values for each segment and the MSG, cost breakpoint and OU values were derived as lumped equivalents which broadly represent some actual generator data from a CEGB Merit Order listing. The resulting generator cost order is show in Table 4.2 .

The scheduled generator output levels and hence the regulating reserve (using the model of Fig. 4.2) were defined as shown in Table 4.3 for the peak demand condition of the logged test data in Fig. 3.9 . These give a total scheduled positive regulating reserve of 670 MW. Any demand under-estimate level of greater than this was allocated to gas turbines rated at 110 MW and loaded in merit order and assumed to exist at every generation node on the system. This gives a total positive regulating capability of 2100 MW which is just greater than three standard deviations of total demand uncertainty.

The negative regulating reserve for demand over-estimates was defined using the reserve model of Fig. 4.3.

Table 4.2

<u>Generator Cost Order</u> (shown in increasing cost order)

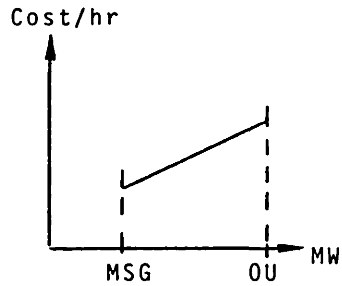
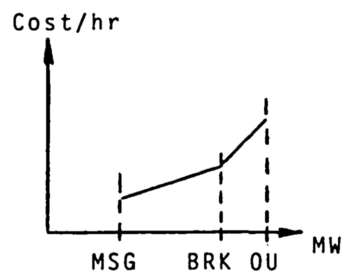
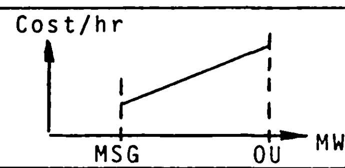
Generator	Nodal Connection	Cost Function	MSG	BRKPT	OU
13	20		200	-	400
9	13		160	-	970
4	3		690	-	1420
5	4		880	-	1900
2	2		240	-	490
6	4		660	-	1400
1	1		180	-	470
3	2		240	-	480
8	8		460	-	920
Cost breakpoint case					
11(L)	19		820	1600	1750
7(L)	7		960	1700	2000
10(L)	18		240	400	500
12(L)	19		600	1200	1320
11(U)	19				
7(U)	7				
10(U)	18				
12(U)	19				
No cost breakpoint case					
11	19		820	-	1750
7	7		960	-	2000
10	18		240	-	500
12	19		600	-	1320

Table 4.3

Scheduled Generator Outputs

Generator	Scheduled Output (MW)	
	Cost Breakpoint	No Cost Breakpoint
1	470	470
2	490	490
3	480	480
4	1420	1420
5	1900	1900
6	1400	1400
7	1700	2000
8	920	920
9	970	970
10	400	500
11	1600	1750
12	1200	650
13	400	400
TGO	13350	13350

(c) Number of Trials

An important parameter in the Monte Carlo simulation approach is the number of trials that are required to provide an accurate estimate of the statistics of the process which is being simulated. Strictly speaking we require an infinite number of trials in order to find the true statistics, so that any number of trials less than this will produce estimates of the true statistics which will differ from the true values by an error which will get smaller as the number of trials increases.

Therefore, we need to determine the number of trials which will give an acceptably small error.

This can be determined by applying the Chebyshev inequality (Ref. 26). We have that:-

$$\text{Prob}\ (|S_n - \mu| \geqslant E) \leqslant \frac{\sigma^2}{nE^2} \tag{4.52}$$

where S_n is the sample mean of the uncertainty in the random variable

μ is its true mean

σ^2 is its true variance

n is the number of trials

i.e. the probability of the sample mean S_n differing from its true mean value by more than an amount E is less than or equal to the right hand side of (4.52) regardless of whether we know the probability distribution of S_n.

We require the error E to be small relative to the size of the changes that we might expect as a result of dispatch activity.

On this basis 5000 trials were found to give satisfactory accuracy. The magnitude of the error for any variable can be obtained, for a given probability level by applying 4.52. For example if we require the probability that S_n lies within E of its true mean to be greater than or equal to 0.99 we get:-

$$E^2 = \sigma^2/(1-0.99)n \tag{4.53}$$

4.5.2.4 Simulation Results

The accumulated results of running 5000 dispatch cases are shown in Figs. 4.9 to 4.11 and Tables 4.4 and 4.5 . The probability density functions for a typical nodal demand uncertainty and for the total demand uncertainty are shown in Figs. 4.10 and 4.9 respectively, while the probability mass functions for the marginal generators are shown in Fig. 4.11 in increasing cost order. These clearly indicate the characteristic probability mass at the MSG, the cost breakpoint value and the OU as predicted by the analytic dispatch uncertainty model of (4.29) and as suggested by the measurement of uncertainty on actual generators in fig. 4.6 .

The analytically computed probability mass at each cardinal point are also shown in Fig. 4.11. The differences between the simulated probability masses and the analytical pdf's at the cardinal points of MSG, cost breakpoint and OU are entirely due to the small but finite value of the probability mass situated away from these discrete values but yet within the count band.

The simulated means and standard deviation for each marginal generator are shown in Table 4.4 and the cross correlations between marginal generators in Table 4.5 together with the joint probability table for one marginal generator.

The expected value of each marginal generator shows a significant shift from its predicted value due to the effect of the probability masses at the MSG and OU values.

We also observe the highly non-linear nature of the joint probability table which, however, yields a fairly high degree of correlation when taken over the full range of a marginal generator.

The corresponding analytic results are also presented in these tables. The means and standard deviations compare reasonably

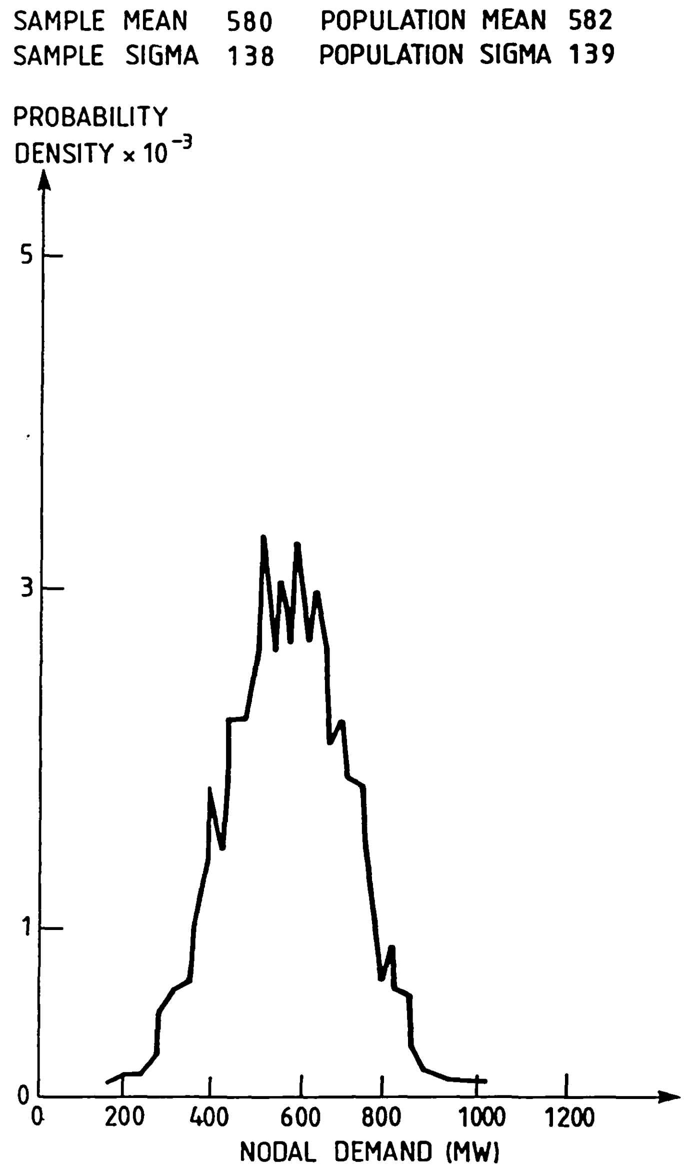


FIG 4.10 TYPICAL NODAL DEMAND UNCERTAINTY
(NODE 1 PROBABILITY DENSITY)

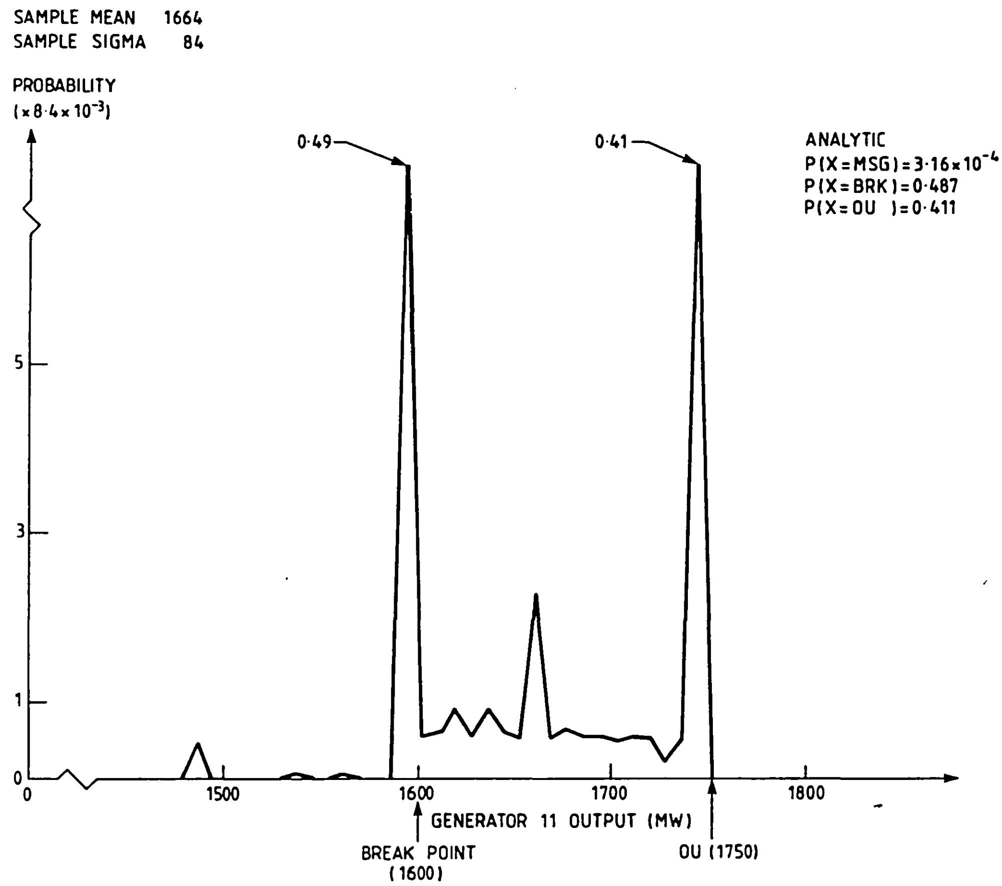


FIG. 4.11 MARGINAL GENERATOR UNCERTAINTY
(DISPATCH SIMULATION WITH COST BREAK POINTS)

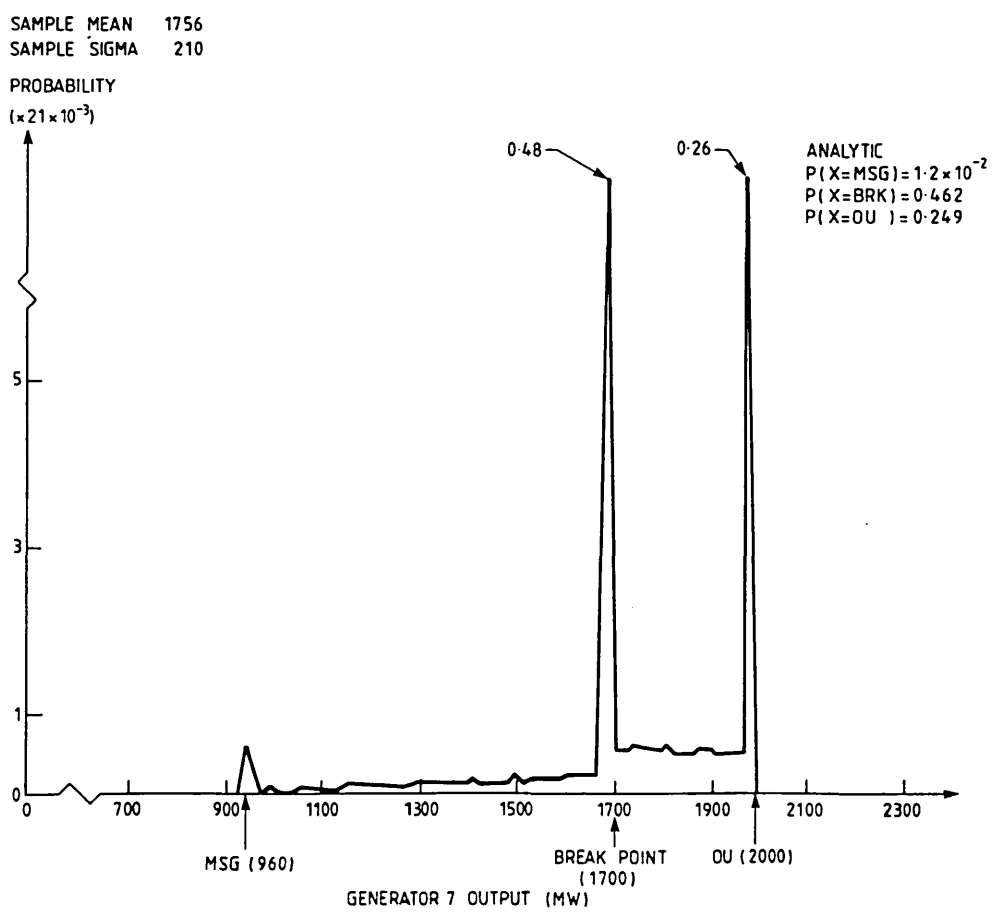


FIG. 4.11 MARGINAL GENERATOR UNCERTAINTY
(DISPATCH SIMULATION WITH COST BREAK POINTS)

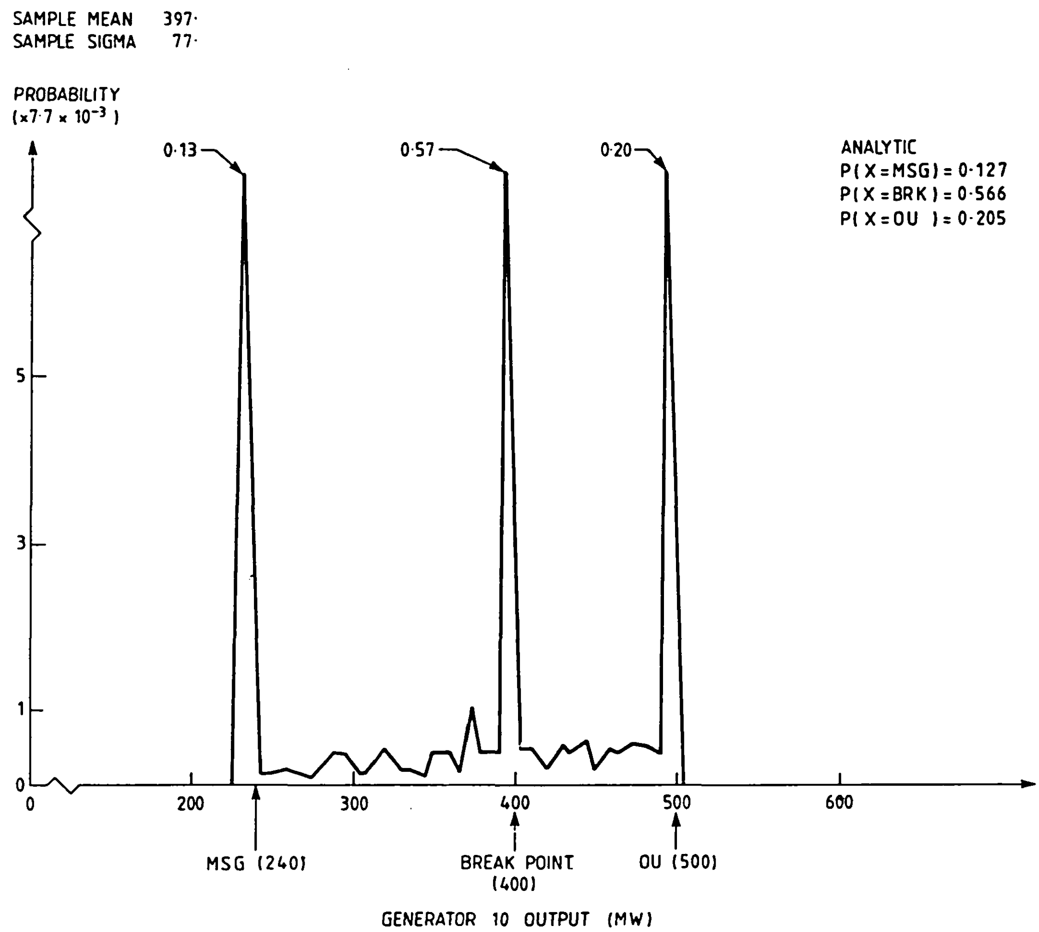


FIG. 4.11 MARGINAL GENERATOR UNCERTAINTY
(DISPATCH SIMULATION WITH COST BREAK POINTS)

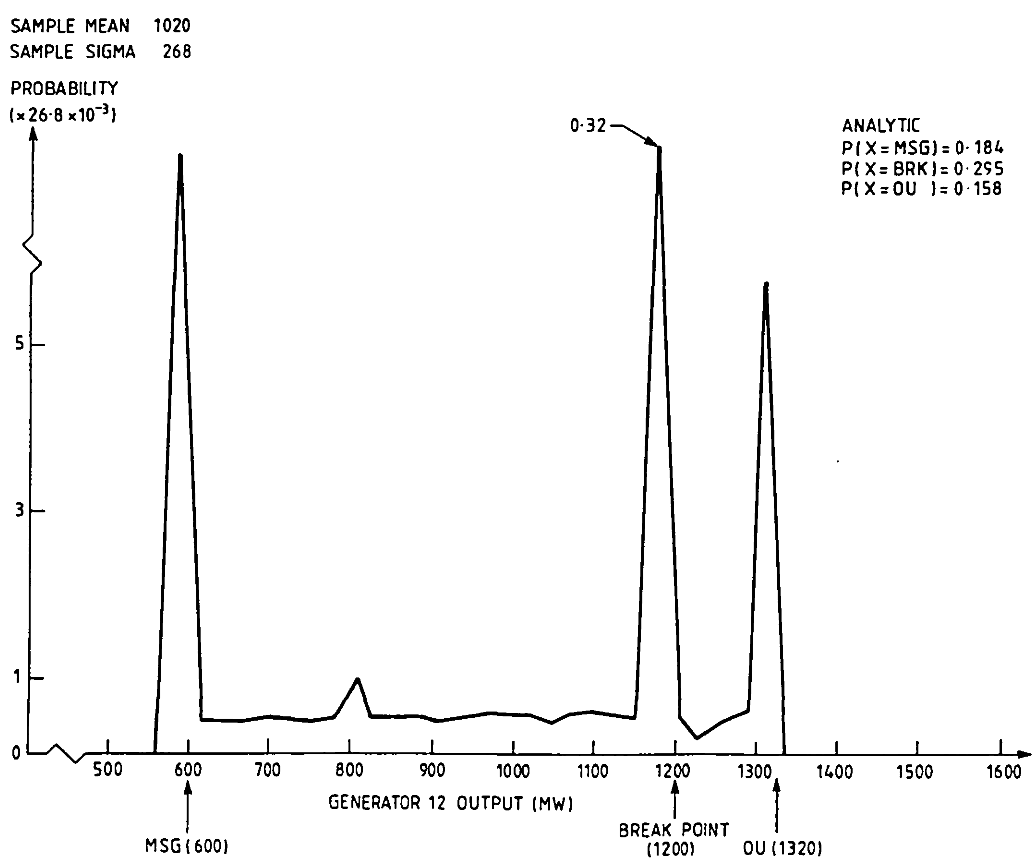


FIG. 4.11 MARGINAL GENERATOR UNCERTAINTY
(DISPATCH SIMULATION WITH COST BREAK POINTS)

Table 4.4

Mean and Standard Deviation of Marginal Generators (Cost Breakpoint Case)

Generator	Mean		Standard Deviation	
	Monte Carlo Simulation	Analytic	Monte Carlo Simulation	Analytic
11	1664	1666	84	84
7	1755	1765	210	220
10	397	398	77	78
12	1019	1026	268	295

Note: Analytic variances scaled by $\sum$ COVPD/ $\sum$COVPG to ensure consistency between total demand variance and total generation variance.

Table 4.5

Marginal Generator Cross Correlations (Cost Breakpoint Case)

Monte Carlo Simulation

Generator	7	10	11	12
7	1.000	0.837	0.762	0.736
10	0.837	1.000	0.627	0.811
11	0.762	0.627	1.000	0.741
12	0.736	0.811	0.741	1.000

Analytic

Generator	7	10	11	12
7	1.000	0.697	0.618	0.564
10	0.697	1.000	0.570	0.704
11	0.618	0.570	1.000	0.625
12	0.564	0.704	0.625	1.000

Joint Probability Table (Monte Carlo Simulation)

7 \ 10	0.	50	100	150	200	250	300	350	400	450	500	550	f(7)
2200	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
2100	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
2000	0.	0.	0.	0.	0.	0.	0.	0.	4.	2.	21.	0.	27.
1900	0.	0.	0.	0.	0.	0.	0.	0.	5.	0.	0.	0.	5.
1800	0.	0.	0.	0.	0.	0.	0.	0.	6.	0.	0.	0.	6.
1700	0.	0.	0.	0.	0.	3.	2.	2.	44.	0.	0.	0.	51.
1600	0.	0.	0.	0.	0.	3.	0.	0.	0.	0.	0.	0.	3.
1500	0.	0.	0.	0.	0.	2.	0.	0.	0.	0.	0.	0.	2.
1400	0.	0.	0.	0.	0.	2.	0.	0.	0.	0.	0.	0.	2.
1300	0.	0.	0.	0.	0.	1.	0.	0.	0.	0.	0.	0.	1.
1200	0.	0.	0.	0.	0.	1.	0.	0.	0.	0.	0.	0.	1.
1100	0.	0.	0.	0.	0.	1.	0.	0.	0.	0.	0.	0.	1.
1000	0.	0.	0.	0.	0.	2.	0.	0.	0.	0.	0.	0.	2.
900	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
f(10)	0.	0.	0.	0.	0.	15	2	2	59	2	21	0	101

well, although the cross correlations less so. The need for any greater accuracy (achievable by more accurate modelling of the joint pdf's) can only be determined by the context of their use in Chapter 5.

The equivalent results for the case where no cost breakpoints exist are shown in Figs. 4.12 and Tables 4.6 and 4.7.

Correlations between nodal demand and total demand uncertainties; marginal generators and nodal demands; and marginal generators and total demand were also simulated and the results presented in Table 4.8, together with the joint probability table of these in Tables 4.9, 4.10 and 4.11.

The form of the joint probability table of Table 4.11 clearly shows that the correlation is non-linear in part, but that a linear correlation approximation over the full generator output range could be reasonable. The correlation factors taken over this entire range are shown for both total demand and nodal demands.

The above results apply only to the peak demand condition which was simulated. However, the same model applies to any demand level - the only differences being that the group of marginal generators will not be the same and the total demand variance will vary with demand level.

Thus the uncertainty model of each generator will vary with its cost position relative to the other generators on the system. This effect is shown in the work presented in the next chapter.

4.6 Conclusions

This chapter has examined, by means of empirical measurement and Monte Carlo simulation, the uncertainties associated with the dispatch activity part of the total uncertainty associated with any one generator.

That part of dispatch activity which is due to the need to compensate for demand uncertainty has been investigated for a merit order, steambased generation system. Comparisons have been carried out between merit order dispatch with and without breakpoints in the generator cost functions. The significance of the differences are discussed in Chapter 6.

An analytical model, for the uncertainty on marginal generators due to dispatch,has been developed and compared with Monte Carlo simulations.

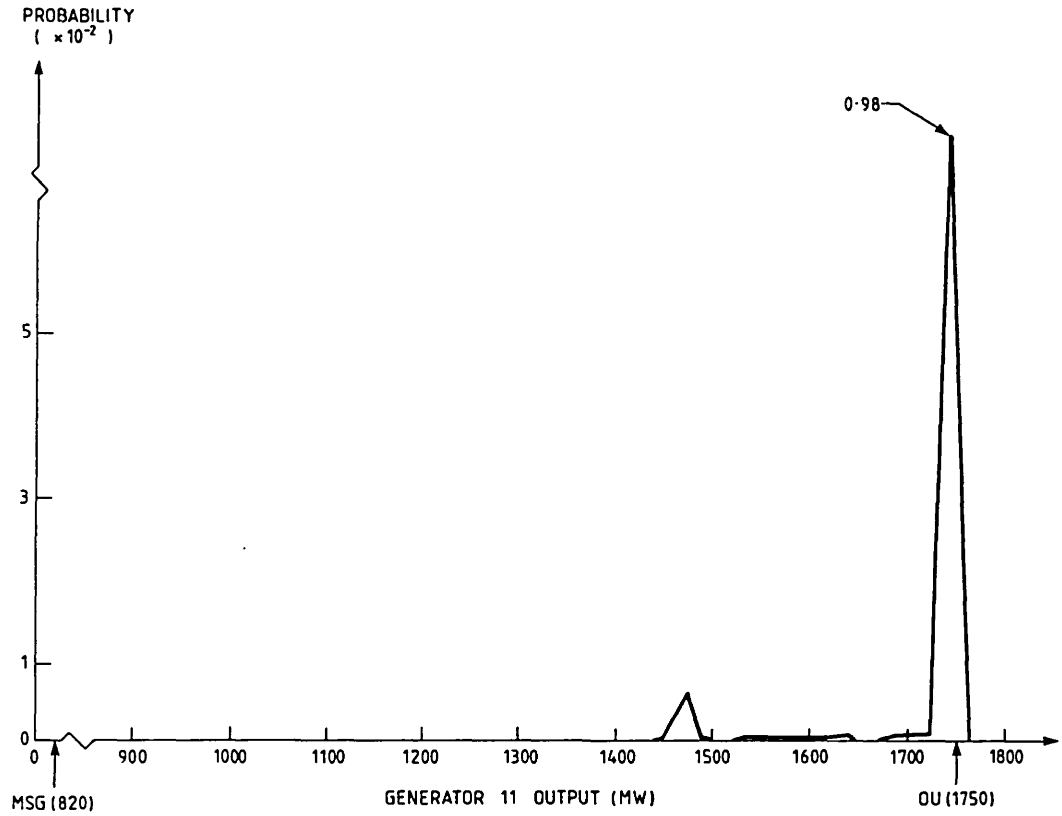


FIG. 4.12 MARGINAL GENERATOR UNCERTAINTY
(DISPATCH SIMULATION WITHOUT COST BREAK POINTS)

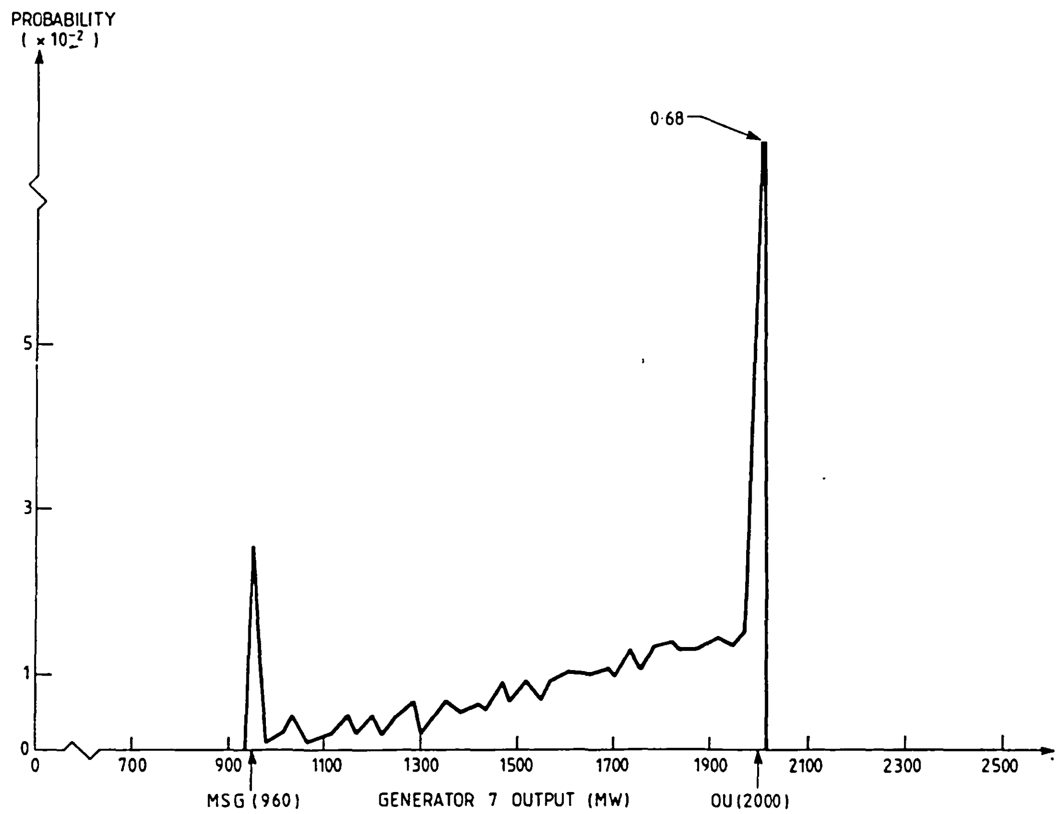


FIG. 4.12 MARGINAL GENERATOR UNCERTAINTY
(DISPATCH SIMULATION WITHOUT COST BREAK POINTS)

SHEET 1 OF 2

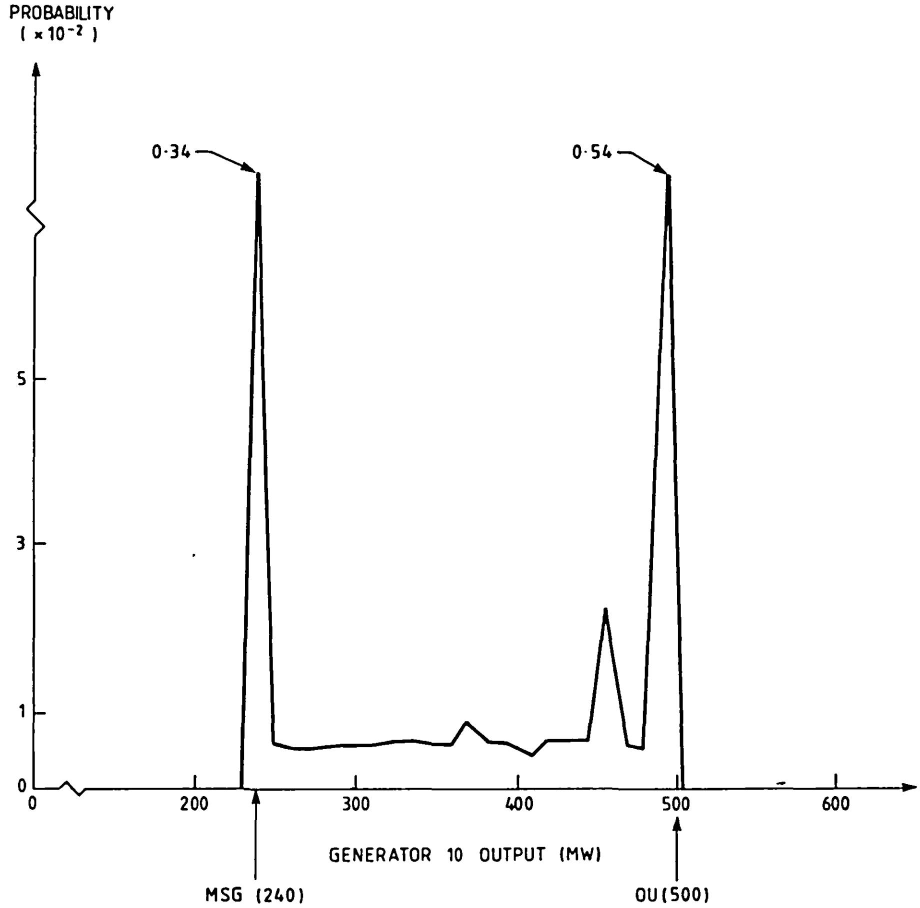


FIG. 4.12 MARGINAL GENERATOR UNCERTAINTY
(DISPATCH SIMULATION WITHOUT COST BREAK POINTS)

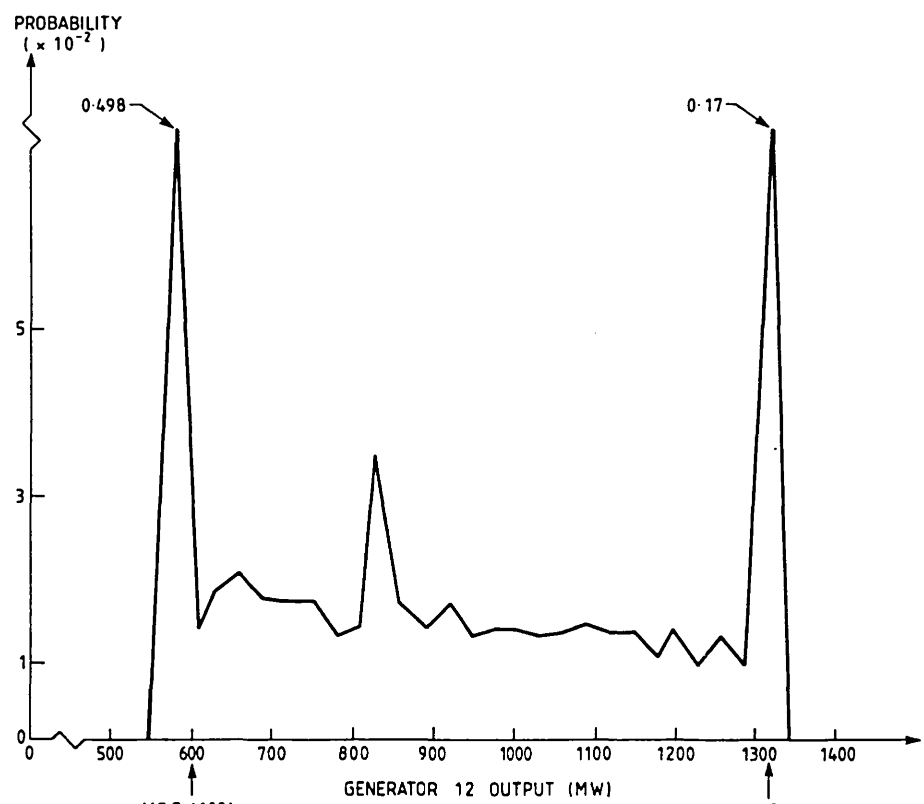


FIG. 4.12 MARGINAL GENERATOR UNCERTAINTY
(DISPATCH SIMULATION WITHOUT COST BREAK POINTS)

Table 4.6

Mean and Standard Deviation of Marginal Generators
(No Cost Breakpoint Case)

Generator	Mean		Standard Deviation	
	Monte Carlo Simulation	Analytic	Monte Carlo Simulation	Analytic
11	1744	1747	52	48
7	1861	1889	266	289
10	396	398	120	114
12	833	847	284	308

Note: Analytic variances scaled by $\sum$ COVPD/ $\sum$ COVPG to ensure consistency between total demand variance and total generation variance.

Table 4.7

Marginal Generator Cross Correlations
(No Cost Breakpoint Case)

Monte Carlo Simulation

Generator	7	10	11	12
7	1.000	0.673	0.377	0.425
10	0.673	1.000	0.145	0.704
11	0.377	0.145	1.000	0.091
12	0.425	0.704	0.091	1.000

Analytic

Generator	7	10	11	12
7	1.000	0.429	0.165	0.246
10	0.429	1.000	0.071	0.573
11	0.165	0.071	1.000	0.041
12	0.246	0.573	0.041	1.000

Table 4.8

Demand and Marginal Generator Cross Correlations

Nodal demand correlations with total demand and marginal generators

(Monte Carlo Simulation)

Node	Total	Generator			
		7	10	11	12
1	0.223	0.206	0.200	0.180	0.210
2	0.172	0.149	0.149	0.133	0.155
3	0.245	0.211	0.217	0.198	0.218
4	0.258	0.215	0.226	0.203	0.237
5	0.115	0.095	0.105	0.092	0.108
6	0.160	0.132	0.142	0.128	0.138
7	0.199	0.168	0.160	0.161	0.175
8	0.165	0.144	0.124	0.139	0.149
9	0.246	0.218	0.217	0.189	0.220
10	0.284	0.254	0.252	0.224	0.260
11	0.271	0.246	0.235	0.214	0.244
12	0.147	0.133	0.131	0.124	0.136
13	0.154	0.136	0.129	0.139	0.140
14	0.111	0.092	0.110	0.086	0.116
15	0.225	0.200	0.206	0.178	0.198
16	0.230	0.209	0.203	0.192	0.201
17	0.191	0.172	0.174	0.171	0.184
18	0.307	0.280	0.274	0.247	0.282
19	0.415	0.367	0.367	0.325	0.368
20	0.160	0.154	0.142	0.138	0.148

Marginal generator correlation with total demand

Generator	Total
7	0.884
10	0.879
11	0.803
12	0.904

Table 4.9

Joint Probability Table - Nodal Demand v Total Demand Uncertainty

T \ N	0	100	200	300	400	500	600	700	800	900	f(T)
-2250	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.2
-1750	0.0	0.0	0.0	0.1	0.3	0.4	0.2	0.1	0.0	0.0	1.1
-1250	0.0	0.0	0.1	0.4	1.0	1.7	1.5	0.8	0.3	0.0	5.8
- 750	0.0	0.0	0.1	1.0	2.8	4.3	4.4	2.3	0.7	0.0	15.6
- 250	0.0	0.0	0.1	1.5	3.9	6.6	7.4	5.2	1.8	0.0	26.6
250	0.0	0.0	0.1	0.7	3.0	6.4	7.6	6.0	2.7	0.0	26.5
750	0.0	0.0	0.1	0.3	1.5	3.2	4.6	4.1	1.7	0.0	15.4
1250	0.0	0.0	0.0	0.1	0.3	0.9	1.4	1.5	0.7	0.0	4.9
1750	0.0	0.0	0.0	0.0	0.1	0.2	0.2	0.2	0.2	0.0	1.0
2250	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
f(N)	0.0	0.1	0.6	4.1	13.0	23.7	27.3	20.3	8.2	0.0	97.2

Table 4.10

Joint Probability Table - Marginal Generator v Nodal Demand

		MSG							BRK			MAX			
D \ 7	900	1000	1100	1200	1300	1400	1500	1600	1700	1800	1900	2000	2100	2200	f(D)
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
100	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1
200	0.0	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.2	0.0	0.0	0.1	0.0	0.0	0.6
300	0.0	0.2	0.1	0.1	0.1	0.1	0.1	0.2	2.6	0.2	0.1	0.4	0.0	0.0	4.1
400	0.0	0.4	0.1	0.2	0.2	0.4	0.4	0.5	7.1	0.7	0.5	2.5	0.0	0.0	13.0
500	0.0	0.6	0.3	0.3	0.4	0.5	0.4	0.3	12.7	1.2	1.3	5.3	0.0	0.0	23.8
600	0.0	0.3	0.2	0.4	0.5	0.3	0.6	0.6	14.0	1.6	1.0	7.7	0.0	0.0	27.3
700	0.0	0.1	0.1	0.1	0.2	0.4	0.3	0.3	9.5	1.2	1.3	6.9	0.0	0.0	20.3
800	0.0	0.1	0.0	0.0	0.1	0.1	0.1	0.1	3.4	0.6	0.6	3.1	0.0	0.0	8.2
900	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
f(7)	0.0	1.8	0.8	1.2	1.4	1.8	1.9	2.5	49.6	5.6	4.8	26.0	0.0	0.0	97.4

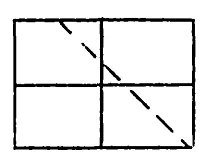
Note: the total does not equal 100% due to the loss of the upper tail region of f(D) beyond 850 MW which the JPDF routine did not accumulate.

Table 4.11

Joint Probability Table - Marginal Generator v Total Demand Uncertainty

D \ 7	900 (MSG)	1000	1100	1200	1300	1400	1500	1600	1700 (BRK)	1800	1900	2000 (MAX)	2100	2200	f(D)
-2000		0.2													0.2
-1800		0.3													0.3
-1600		0.7													0.7
-1400		0.7	0.6												1.3
-1200			0.2	1.3	0.8										2.3
-1000					0.6	1.9	1.0								3.5
- 800							0.9	2.6	1.8						5.3
- 600									7.4						7.4
- 400									9.6						9.6
- 200									11.1						11.1
0									12.0						12.0
200									8.3	3.0					11.3
400										2.8	5.0	2.4			10.2
600												9.1			9.1
800												6.5			6.5
1000												4.0			4.0
1200												2.5			2.5
1400												1.4			1.4
1600												0.9			0.9
1800												0.4			0.4
2000												0.1			0.1
f(7)	0	1.9	0.8	1.3	1.4	1.9	1.9	2.6	50.2	5.8	5.0	27.3	0	0	100.1

(Note: off-diagonal terms due to quantisation levels of joint pdf analyser)



5. The Probabilistic Trajectory Problem

5.1 Introduction

In any area where predictive analysis is carried out some assessment must be made of how sensitive the results of such predictive work are to uncertainties in the input parameters.

In the worst case approach it would be possible to determine the 'distance' of the worst case corner point from each line outage constraint and to rank these in increasing order of 'distance' magnitude. This would indicate the next nearest insecure line outages. However, as the flows follow a trajectory pattern it may be that the nearest line outage constraints are not appropriate.

An alternative approach would be to map the uncertainty of input parameters to the line flows and hence determine the robustness of the predictive results. This latter approach is explored in this chapter.

5.2 Mathematical Formulation

Recall that the line power flows, based on the dc load flow model, are linear functions of the net injection at each node given by:-

$$[P_L] = [H]\ [P_I] = [H]\ ([P_G] - [P_D]) \qquad (5.1)$$

where each matrix or column vector is defined as in equation (3.19) and in which each of the parameters [H], $[P_G]$ and $[P_D]$ are assumed to be specified constants.

Consider the situation in which each of these parameters is perturbed by an amount $[\Delta H]$, $[\Delta P_G]$ and $[\Delta P_D]$ (where ΔP_{Gi} may be the resultant perturbation if more than one generator is connected at node i) so that:-

$$[P_L + \Delta P_L] = [H + \Delta H] \; ([P_G + \Delta P_G] - [P_D + \Delta P_D]) \qquad (5.2)$$

i.e. for a given perturbation of each parameter we can compute the resulting change in the line flow vector. Now in predictive applications each of these perturbations are random variables and can only be described in terms of a probability of holding a particular value. These probability densities for generation and demand have been discussed in Chapter 4.

The probability density of $[\Delta H]$, which represents the change in the network due to the occurrence of any transmission equipment contingency is in practice truncated to one which represents the probability of the various network states due to a defined set of contingencies as discussed in Chapter 2.

Thus equation (5.2) represents the general uncertainty problem (for real power flows) when all the parameters are assumed to be perturbed. In the trajectory analysis application we need to solve for $[\Delta P_L]$ at each trajectory time step.

So, the problem is: given the predicted nodal demand trajectories; the predicted generator output trajectories; the predicted network configuration and its planned changes with time; and the defined contingency set as in Chapter 3 and given the nodal demand prediction uncertainty statistics and the individual generator uncertainty statistics for the respective trajectories as in Chapter 4, then find the sub-set (if any) of contingencies which would result in the insecure system conditions of equation (3.4).

5.3 Probabilistic Worst Case Analysis

Recall from Chapter 3 that we require the maximum and minimum values of the line flow trajectory for each line in the system and from these we wish to calculate the maximum absolute line flow value for each post-contingency line flow.

5.3.1 Peak Flow Under Uncertainty

The first problem is to determine the pre-outage range of each line flow trajectory under uncertain conditions. In the deterministic Worst Case Analysis this was determined by carrying out a deterministic load flow for each time step on the trajectory and testing for the maximum and minimum values for each line flow as we stepped through the trajectory. In an uncertain environment we must also test that the sum of the expected line flow peak plus the uncertainty associated with the flow at that peak is in fact greater than the sum of any off-peak flow level of that line plus its associated uncertainty as the uncertainty may be greater at some off-peak flow level than at the peak. This test must also be applied to the minimum flow conditions.

$$\text{ie find} \quad \left.\begin{array}{l} \text{Max } (\overline{P_{ij}} + C\sigma_{ij}) \\ \text{Min } (\overline{P_{ij}} - C\sigma_{ij}) \\ \end{array}\right) \text{ evaluated at all timesteps.} \qquad (5.3)$$

where $\overline{P_{ij}}$ is the expected line flow value

σ_{ij} is the standard deviation of its uncertainty

C gives the desired confidence level

When we are dealing with a random variable we cannot explicitly state a range within which the random variable will lie without also stating the probability with which we expect to find the random variable in that range. If we were to calculate the probability density functions of the line flow variables for each time step on the trajectory then we could explicitly calculate the probability of each line flow variable being greater than a certain value and the probability of it being less than a certain value by integrating the area under the appropriate part of the appropriate probability density functions.

However, we can also make an estimate (from the mean and variance of a random variable) of the probability that a line flow lies outside a given range without knowing the probability density function. This follows from the Chebyshev inequality (Ref. 26). We have for any random variable X:-

$$\sigma^2 = E[(X-\mu)^2] = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx \tag{5.4}$$

where σ^2 is the variance

$f(x)$ is the probability density function

μ is the mean

Since the integrand is nonnegative, the value of the integral can only decrease when the range of integration is diminished. Thus

$$\sigma^2 \geqslant \int_{x<\mu-E}^{x>\mu+E} (x-\mu)^2.f(x)\,dx \geqslant \int_{x<\mu-E}^{x>\mu+E} E^2.f(x)\,dx = E^2 \int_{x<\mu-E}^{x>\mu+E} f(x)\,dx \tag{5.5}$$

But the last integral is equal to 1-Prob $(\mu - E < X < \mu + E)$

Hence

$$\text{Prob}\,(\mu - E < X < \mu + E) \geqslant 1 - \sigma^2/E^2 \tag{5.6}$$

or with $E = C\sigma$

$$\text{Prob}\ (\mu - c\sigma < X < \mu + c\sigma) \geqslant 1 - 1/c^2 \tag{5.7}$$

i.e. the probability that X will lie within C standard deviations of its mean is greater than or equal to $1 - 1/C^2$.

As a first step towards determining the range of each uncertain line flow trajectory we compute the statistics of the line flow uncertainties.

5.3.2 Transformation of Generation and Demand Uncertainties To Line Flow Uncertainties

The outage of any piece of transmission equipment represents a discontinuous change of state for the topology of the system so that the matrix [H] is a discrete random variable. Because of this, it is possible to consider each topology state individually and the results for the various network states summated in proportion to the probability of each network state, if required.

First we consider (5.2) for a fixed network state:-

$$[P_L + \Delta P_L] = [H]\ ([P_G + \Delta P_G] - [P_D + \Delta P_D]) \tag{5.8}$$

5.3.2.1 Treatment of Reference Node

In a power system network, not all of the net injections in (5.8) can be specified independently. At least one injection must be dependent in order to preserve the power balance between generation and demand as stated explicitly in Kirchoff's current law

$$S - \sum_{i=1}^{N} (P_{Gi} - P_{Di}) = 0 \tag{5.9}$$

where S is the injection required to ensure the power balance of equation (5.9).

In the deterministic dc load flow this condition is explicitly arranged by correctly specifying the values for nodal generation and demand. The reference or slack node in this case is purely a reference from which the relative voltage angles of the other nodes in the system are measured, and which makes the admittance matrix non-singular. There is no injection of power at this node. In a deterministic load flow in which losses are not neglected the losses are unknown prior to the load flow so that an exact balance of power cannot be pre-specified. In this case the reference node will have some power injection (as determined by the algorithm) to compensate for these losses.

In a probabilistic dc load flow the correct specification of each nodal generation and demand value prior to the study would require knowledge of how generation deficiencies or surplusses are handled in the control room phase of operations.

The proper way of maintaining the balance of power is to model this activity as carried out in the control room.

Then we have:-

$$[P_L + \Delta P_L] = [H]\ ([P_G + \Delta P_G + \phi(s)] - [P_D + \Delta P_D]) \qquad (5.10)$$

where $[\phi(S)]$, the elements of which were discussed in Chapter 4, is an Nxl matrix which indicates how the balance of power is to be maintained and:-

$$\sum_{i=1}^{N} \phi_i(S) = S \qquad (5.11)$$

In this research the uncertainty of generators in response to demand uncertainty alone is considered.

So equation (5.10) becomes:-

$$[P_L + \Delta P_L] = [H]\,([P_G + \phi(S)] - [P_D + \Delta P_D]) \qquad (5.12)$$

5.3.2.2 Random Variable Analysis

If we separate (5.12) into deterministic and probabilistic parts we get:-

$$[P_L] + [\Delta P_L] = [H]\,([P_G] - [P_D]) + [H]\,([\phi(S)] - [\Delta P_D]) \qquad (5.13)$$

from which we see that the random variable representing the uncertainty in any line flow prediction ΔP_{Lj} is a function of a sequence of random variables.

We have from (Ref. 26) that the probability density of ΔP_{Lj} is then given by differentiating with respect to ΔP_{Lj} the multiple integral over the appropriate region of the joint probability density function of the sequence of random variables integrated with respect to each of the random variables:-

$$F(\Delta p_{Lj}) = \mathrm{Prob}(\Delta P_{Lj} \leqslant \Delta p_{Lj})$$

$$= \int_R \!\cdots\! \int f(\phi_1(S), \ldots \phi_N(S), \Delta p_{D1}, \ldots \Delta p_{DN})\, d\phi_1(S) \ldots d\phi_N(S)\, d\Delta p_{D1} \, d\Delta p_{DN} \qquad (5.14)$$

where $F(\Delta P_{Lj})$ is the distribution function of ΔP_{Lj}

$f(\;\;)$ is the joint density of the sequence of random variables

R is the region for which $h_{j1}\phi_1(S) + h_{j2}\phi_2(S) + \ldots \leqslant \Delta p_{Lj}$; h_{ji} is the appropriate element of the matrix [H] for line j.

Early approaches modelled $\phi(S)$ as lumped at the reference node and all nodal demand uncertainties independent so that the joint probability density of the nodal uncertainties is the product of the individual probability densities:-

$$f(h_{j1}\Delta p_{D1}, \ldots) = f(h_{j1}\Delta p_{D1}) \times f(h_{j2}\Delta p_{D2}) \times \ldots \text{etc} \tag{5.15}$$

This enabled solution of (5.14) by a straight forward multiple convolution approach (Ref. 28).

$$f(\Delta p_{Lj}) = f(h_{j1}\Delta p_{D1}) * f(h_{j2}\Delta p_{D2}) * \ldots \tag{5.16}$$

where the * symbol indicates mathematical convolution.

The number of arithmetic computations involved in this convolution can be reduced by carrying out the work in the frequency domain. This is because the mathematical convolution of a signal is equivalent to multiplication of the Fourier Transform of that signal (Ref. 38).

$$\text{F.T.}\left[f(\Delta p_{Lj})\right] = \text{F.T.}\left[f(h_{j1}\Delta p_{D1})\right] \times \text{F.T.}\left[f(h_{j2}\Delta p_{D2})\right] \times \ldots \tag{5.17}$$

Later papers handled linear dependences between random variables by separating dependent and independent parts and summating the dependent parts to give a new independent random variable which could then be convoluted with the other independent variables (Ref. 33).

More recent work started to consider the balance of power implications for the computation of probability density functions as described in section 2.8.5.1.

Some approaches have avoided the problems of calculating explicit probability density functions by restricting attention to the mean and variance parameters of the random variables. (Ref. 27) We have that the mean of the jth line flow variable is given by:-

$$\begin{aligned}\overline{\Delta P_{Lj}} = E[\Delta P_{Lj}] &= \int_{-\infty}^{\infty}\!\!\cdots\!\int (h_{j1}\Delta P_{D1} + h_{j2}\Delta P_{D2} + \cdots)\, f(h_{j1}\Delta P_{D1}, \ldots)\, dh_{j1}\Delta P_{D1} \cdots \\ &= h_{j1}\Delta P_{D1} f(h_{j1}\Delta P_{D1})\, dh_{j1}\Delta P_{D1} + \cdots \\ &= h_{j1}\Delta P_{D1} f(\Delta P_{D1})\, d\Delta P_{D1} + \cdots \\ &= h_{j1}E[\Delta P_{D1}] + h_{j2}E[\Delta P_{D2}] + \cdots \end{aligned} \quad (5.18)$$

where E is the mathematical expectation operator.

In vector terms

$$E\,[\Delta P_L] = [\overline{\Delta P_L}] = [H]\,[\overline{\Delta P_I}] \quad (5.19)$$

where $[\Delta P_I]$ is the vector of net injection perturbations at each node due to the demand connected at each node.

While the variance of the jth line flow variable is given as the jth diagonal element of the line covariance matrix $[C_{PL}]$:-

$$[C_{PL}] = E[([\Delta P_L] - [\overline{\Delta P_L}])\,([\Delta P_L] - [\overline{\Delta P_L}])^T] \quad (5.20)$$

which gives

$$[C_{PL}] = [H]\ [C_{PI}]\ [H]^T \qquad (5.21)$$

where T indicates matrix transposition and

$[C_{PI}]$ is the covariance matrix of the nodal injections.

Thus from (5.13) and (5.19) we have that the mean of the line flow including uncertainty is given by:-

$$E\big([P_L] + [\Delta P_L]\big) = [P_L] + [H]\ [\overline{\Delta P_I}] \qquad (5.22)$$

with a variance as given by (5.21)

In the absence of any model for the balance of generation and demand, $-\phi(S)$ is effectively maintained at the reference node. If the nodal injections are assumed to be independent the variance of the injection at the reference node can become very large. Several authors have proposed including some degree of correlation between power injections to overcome this problem, (Ref.25) although the basis on which these correlations could be derived was lacking. The model of dispatch developed in Chapter 4 explicitly provides the mean and variance of the generation involved in the balance of power and also enables computation of the correlation factors between the injections.

We have that

$$[\Delta P_I] = [\phi(S)] - [\Delta P_D] \qquad (5.23)$$

So that

$$[\overline{\Delta P_I}] = [\overline{\phi(S)}] - [\overline{\Delta P_D}] \tag{5.24}$$

and $$[C_{PI}] = [C_\phi] + [C_{PD}] - [Cov(\phi,PD)] - [Cov(\phi,PD)]^T \tag{5.25}$$

where $[C_{PI}]$ is the covariance matrix of nodal injection uncertainties

$[C_{PD}]$ is the covariance matrix of nodal demand uncertainties.

$[Cov(\phi,PD)]$ is the covariance matrix of nodal control activity uncertainties with nodal demand uncertainties.

Structurally these matrices are as follows:-

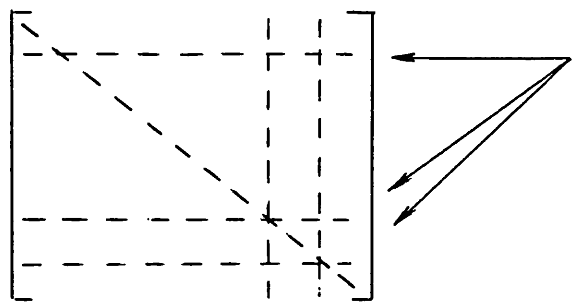
$\underline{[C_{PD}]}$

This is an NxN matrix in which each diagonal element corresponds to the variance of the nodal demand prediction uncertainty of the node represented by that element and where the off-diagonal elements are zero unless there is some correlation between the prediction uncertainties associated with the corresponding pair of nodes.

$\underline{[C_\phi]}$

The elements of this NxN matrix are the variances and crosscovariances of the marginal generation on the system. Thus the elements which correspond to the baseload generation will be zero.

$[C_\phi]$ =



nodes with marginal generation connected

(5.26)

The elements of this matrix are derived directly from the dispatch activity model of Chapter 4 with appropriate nodal mapping.

When more than one marginal generator is connected to the same node, the resulting element is given by:-

$$\phi_i = \phi_k + \phi_m \qquad (5.27)$$

$$\phi_j = \phi_r + \phi_s$$

$$\mathrm{Cov}(\phi_i,\phi_j) = \mathrm{Cov}(\phi_k,\phi_r) + \mathrm{Cov}(\phi_k,\phi_s)$$

$$+ \mathrm{Cov}(\phi_m,\phi_r) + \mathrm{Cov}(\phi_m,\phi_s)$$

where ϕ_k is the random variable representing the uncertainty on an individual marginal generator.

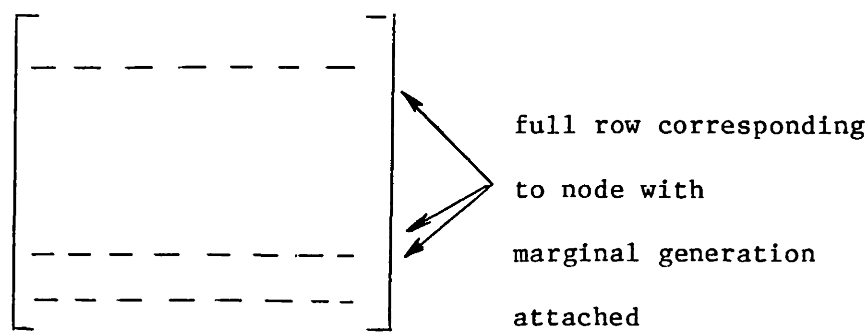
ϕ_i is the random variable representing the uncertainty of the generation on an individual node.

[Cov(ϕ,P_D)]

The elements of this NxN matrix are the crosscovariances of the marginal generation with the nodal demand uncertainty. The balance of power dispatch activity is dependent upon the total uncertainty which in turn is dependent upon the nodal demand uncertainties, so there will be some degree of correlation between marginal generators and nodal demand uncertainty.

This leads to a matrix structure of:-

$[\mathrm{Cov}(\phi,PD)] =$



(5.28)

A rigorous computation of the elements of this matrix would require the computation of the joint pdf's between each marginal generator and nodal demand. The form of this joint pdf is shown in Table 4.10 and would be somewhat difficult to compute. However, as a first approximation we can compute the elements of the matrix as:-

$$\sigma_{ij} = r_{ij}\sigma_i\sigma_j = (\sigma_j^2/\sigma_T)\left(\sum_{k\in i} \sigma_k\right) \qquad (5.29)$$

where σ_j^2 is the variance of the demand uncertainty at node j
σ_T^2 is the variance of the total demand uncertainty on the system; r_{ij} is the correlation between the uncertainty of marginal generation connected at node i with the uncertainty of demand connected at node j.

To derive (5.29) we have made the assumptions that:-

(a) the marginal generation uncertainty is linearly correlated with total demand uncertainty

(b) the uncertainty of nodal demands are independent.

These assumptions enable us to write:-

$$r_{ij} = r_{jT} \tag{5.30}$$

$$r_{jT} = \sigma_j/\sigma_T$$

where r_{jT} is the correlation between the uncertainty in nodal demand and the uncertainty in total demand.

These follow from the following derivations:-

We have that the crosscovariance between the uncertainty of an individual marginal generator and nodal demand uncertainty is:-

$$\mathrm{Cov}(\phi_k, PD_j) = E[(\phi_k - \overline{\phi}_k)(PD_j - \overline{PD}_j)] \tag{5.31}$$

now assuming linear correlation between this marginal generator uncertainty and total uncertainty, we can write:-

$$\phi_k = a\ T \tag{5.32}$$

Substitution of this into (5.31) yields

$$\mathrm{Cov}(\phi_k, PD_j) = a\ \mathrm{Cov}(PD_j, T) \tag{5.33}$$

And since

$$\mathrm{Cov}(\phi_k, PD_j) = r_{kj}\sigma_k\sigma_j \tag{5.34}$$

and

$$Cov(PD_j, T) = r_{jT}\sigma_j\sigma_T \tag{5.35}$$

We get

$$r_{kj} = r_{jT} \tag{5.36}$$

We also have that:

$$Cov(PD_j, T) = E[PD_j - \overline{PD}_j)(T - \overline{T})] \tag{5.37}$$

Now by splitting T into the sum of nodal demands we get:-

$$Cov(PD_j, T) = E\left[(PD_j - \overline{PD}_j)(PD_j - \overline{PD}_j)\right] \tag{5.38}$$

+ the sum of nodal cross covariance terms.

Hence

$$r_{jT} = (\sigma^2_{PDj} + \sum \text{nodal cross covariance terms}) / \sigma_{PDj}\sigma_T \tag{5.39}$$

The cross covariance terms could be simply included if they were known, but on the assumption of independence they are zero, so we have equation (5.30).

The error introduced by the linear correlation assumption, between marginal generators and total uncertainty, can be subsequently compensated by observing that the sum of the covariances in each row are the covariance between that marginal generator and total uncertainty. Hence, they should equal the sum of the covariances of the same row of the [Cϕ] matrix. This can be achieved by scaling each element in a given row by:-

$$\sum_{j=1}^{N} \mathrm{COV}(\phi_i, \phi_j) \Big/ \sum_{j=1}^{N} \mathrm{COV}(\phi_i, PD_j) \qquad (5.40)$$

Thus by applying (5.19) and (5.21) with substitutions as in (5.24) and (5.25) we can determine the shift in the mean line flows and the variance of the line flows due to the uncertainties in generation and demand. This shift in mean line flow due to uncertainty can then be added to the deterministic line flow value as in (5.22) and by application of (5.7) we can determine the maximum and minimum line flow values for a particular confidence level for any timestep on the trajectory. Finally, by applying (5.3) we can determine the range of each trajectory line flow under uncertainty. We now need to consider the problem of transforming these to post-contingency line flows.

5.3.3 Transformation of Pre-Contingency Line Flow Uncertainties To Post-Contingency Line Flow Uncertainties

Consider the flow on line k-m following the outage of line i-j when the precise pre-outage flow on both lines is uncertain.

let $X = \Delta P_{ij}$ the uncertain part of the pre-outage flow on line i-j

(5.41)

$Y = \Delta P_{km}$ the uncertain part of the pre-outage flow on line k-m

$Z = \Delta P^{ij}_{km}$ the uncertain part of the post-outage flow on line k-m with line i-j out.

$a = df^{ij}_{km}$ the proportion of the pre-outage flow on line i-j that is transferred to line k-m following outage of line i-j.

now X and Y are random variables, so that

$$Z = aX+Y \tag{5.42}$$

is also a random variable.

The mean of Z is given by:-

$$\overline{Z} = E[Z] = a\,\overline{X} + \overline{Y} \tag{5.43}$$

where $\overline{X}$ is the mean of X

$\overline{Y}$ is the mean of Y

And the variance of Z is given by:-

$$\mathrm{Var}(Z) = E[(Z - \overline{Z})^2] = a^2\,\mathrm{Var}(X) + \mathrm{Var}(Y) + a\,\mathrm{Cov}(X,Y)$$

$$+ a\,\mathrm{Cov}(Y,X) \tag{5.44}$$

where Var(X) is the variance of X

Var(Y) is the variance of Y

Cov(X,Y) is the covariance of X with Y

and is given as:-

$$\mathrm{Cov}(X,Y) = \mathrm{Cov}(Y,X) = r_{XY}\cdot\sqrt{\mathrm{Var}(X)}\cdot\sqrt{\mathrm{Var}(Y)} \tag{5.45}$$

where r_{xy} is the correlation between X and Y

Now from Chebyshev's Inequality (5.7) we have that the maximum post-outage line flow Z with a degree of confidence of greater than $1- 1/C^2$ can be found as:-

$$\text{Max } [Z] = \overline{Z} + c\sqrt{\text{Var}(Z)} \qquad (5.46)$$

Now in the Worst Case approach the maximum line flows must be evaluated at each of the corner points formed by the intersection of the maximum and minimum ranges of X and Y, which implies that the correlation coefficient is at its extreme values of either +1 or -1.

So we have from (5.44) that:-

$$\text{Var}(Z) = \pm\, a\sqrt{\text{Var}(X)} \pm \sqrt{\text{Var}(Y)} \qquad (5.47)$$

Consequently the confidence level in the range of each pre-outage line flow is also the level for the post-outage line flow range. This will in fact give pessimistic post-outage flow ranges as in practice the line flow correlations will not be perfectly correlated.

Thus we can re-write the deterministic worst case equation of Chapter 3 as:-

$$\text{Max}[\,|a.P_{ij}^{max} + P_{km}^{max}|, |a.P_{ij}^{max} + P_{km}^{min}|, |aP_{ij}^{min} + P_{km}^{max}|,$$
$$|a.P_{ij}^{min} + P_{km}^{min}|\,] \leqslant R_{km} \qquad (5.48)$$

where

$$P_{ij}^{max} = \max[P_{ij} + \overline{\Delta P}_{ij} + c\sqrt{Var(\Delta P_{ij})}]\ )$$

$$P_{km}^{max} = \max[P_{km} + \overline{\Delta P}_{km} + c\sqrt{Var(\Delta P_{km})}]\ ) \qquad \text{evaluated}$$

$$\text{over the}$$

$$P_{ij}^{min} = \min[P_{ij} + \overline{\Delta P}_{ij} - c\sqrt{Var(\Delta P_{ij})}]\ ) \qquad \text{trajectory}$$

$$P_{km}^{min} = \min[P_{km} + \overline{\Delta P}_{km} - C\sqrt{Var(\Delta P_{km})}]) \qquad (5.49)$$

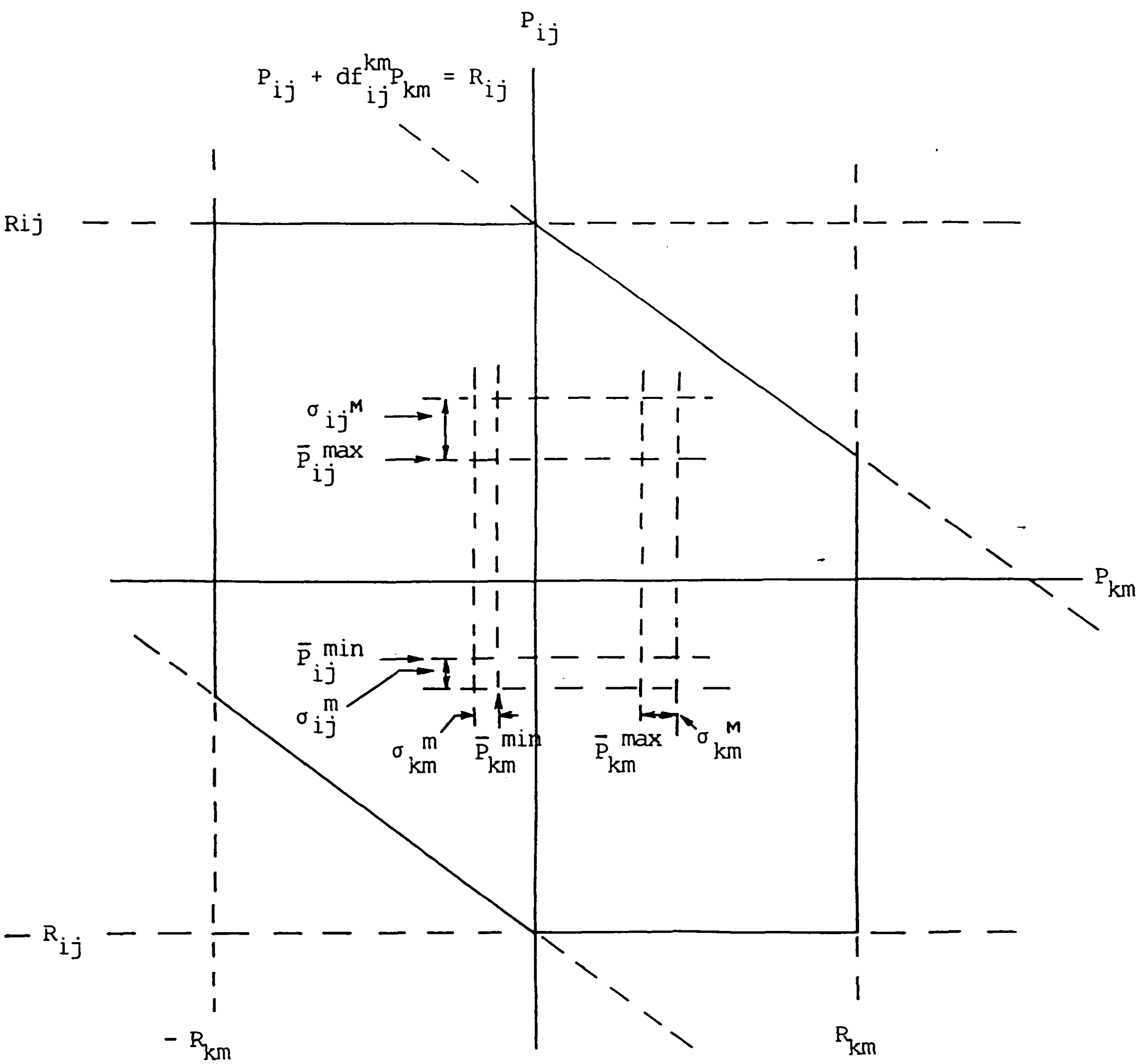
This is shown graphically in Fig. 5.1 .

5.3.3.1 Elimination of False Alarm Contingencies

Any false alarm contingencies generated by the Worst Case assumptions of perfect correlation between line flows could be straightforwardly eliminated by carrying out a stochastic load flow analysis for each of this reduced subset of contingencies.

5.4 Multiple Stochastic Outage Analysis

A stochastic load flow in the form of (5.21) and (5.22) could be carried out for each time step of the trajectory and for each contingency case at each of these time steps simply by modifying the parameters of [H] as discussed in Chapter 3. This was the approach adopted by (Ref. 30) - although no attempt was made to model dispatch activity. The possibility of choosing timesteps at intervals such that the three standard deviations of the total demand uncertainty at each timestep just overlap would not be valid due to the strong effect of unit commitment and dispatch strategies on the line flow uncertainties in the intervening time interval. Dispatching and unit commitment affect the expected value and variance of marginal



LINE FLOW DOMAIN - NETWORK AND
UNCERTAIN TRAJECTORY CONSTRAINTS

Fig. 5.1

generators which in turn affect the expected values and variances of line flows. The generators that are marginal depend upon the total demand level.

Where we are concerned only with real power flows, an alternative approach to that of modifying [H] for each contingency followed by computation of line flow statistics from generation and demand statistics as in (5.21) and (5.22) would be to compute the post-contingency line flow statistics directly from the pre-contingency line flow statistics using the distribution factors of Chapter 3.

Consider the post-outage flow on line k-m due to the outage of line i-j.

We have:-

$$P_{km}{}^{ij} + \Delta P_{km}{}^{ij} = a(P_{ij} + \Delta P_{ij}) + (P_{km} + \Delta P_{km}) \qquad (5.50)$$

The mean of the post-outage flow is:-

$$\overline{\Delta P_{km}}{}^{ij} = a\overline{\Delta P}_{ij} + \overline{\Delta P}_{km} \qquad (5.51)$$

and the variance is:-

$$\mathrm{Var}(\Delta P_{km}{}^{ij}) = \mathrm{Var}(\Delta P_{ij}) + \mathrm{Var}(\Delta P_{km}) + a\,\mathrm{Cov}(\Delta P_{ij}, \Delta P_{km}) + a\,\mathrm{Cov}(\Delta P_{km}, \Delta P_{ij}) \qquad (5.52)$$

The variance of ΔP_{ij} and ΔP_{km} are the ijth and kmth diagonal elements of the line covariance matrix $[C_{PL}]$ of (5.21).

Thus, having computed $[C_{PL}]$ as in (5.21) we can compute $\mathrm{Var}(\Delta P_{km}^{ij})$ directly, and in conjunction with (5.51) and the deterministic post-outage line flow values we can determine the post-outage line flows under uncertainty for any desired confidence level.

5.5 Experimental and Computational Experience

The experimental and computational work presented in this section was designed to test the validity of the theoretical development described in the preceding sections of this chapter. In particular the objectives were:-

(a) to assess the effect of balance of power dispatch activity on line flow uncertainty

(b) to examine the errors introduced by the assumptions made about cross correlations in Section 5.3.2

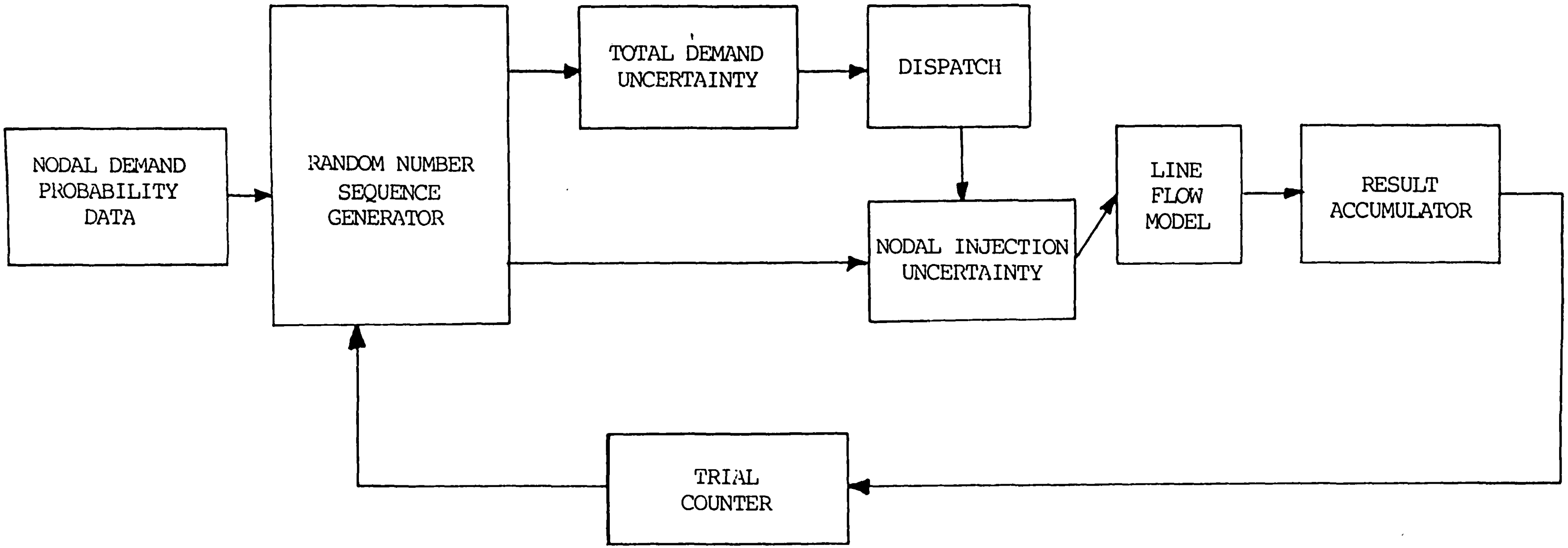
(c) to compare the probabilistic Worst Case algorithm with a Multiple Stochastic Outage method in trajectory analysis.

These required the development of software to perform Monte Carlo type simulations of line flow uncertainty; Stochastic Load flow including a dispatch activity model; Probabilistic Worst Case algorithm; and Stochastic Outage load flows including dispatch activity.

5.5.1 Software Development

Monte Carlo Line Flow Simulator

The Monte Carlo simulation software developed and described in Chapter 4 was extended to include the dc load flow of equation (3.19). A flow chart of the process is shown in Fig. 5.2.



MONTE CARLO SIMULATION FLOW CHART

Fig. 5.2

The software was tested by generating random sequences of line flow values and then summating the line flows around selected demand-only nodes to re-constitute the nodal demand at these nodes for comparison with the input nodal demands.

Stochastic Load Flow Incorporating Dispatch Activity

The routine developed in Chapter 4 to compute the means and cross covariances of individual generator uncertainties due to dispatch activity was used to calculate these parameters for the marginal generators.

These values were used as inputs to a routine which first computed the matrix $[P_L]$ from equation (5.22); then the matrix $[C_{PI}]$ based upon equations (5.25), (5.27) and (5.29) and finally computed the variance of the line flows from equations (5.21). This was tested by comparison with Monte Carlo type simulations as described in Section 5.5.3.

Probabilistic Worst Case Algorithm

The basic equation of the probabilistic worst case algorithm (5.48) was coded. The input variables to this viz; the mean and standard deviation of each line flow corresponding to both the maximum and minimum peak line flows under uncertainty were determined by application of the above Stochastic Load Flow routine to each time step in turn and testing for the trajectory maximum and minimum values. The confidence level was also coded as an input variable.

Stochastic Outage Analysis

The means and variances of the pre-outage line flow uncertainties produced by the above Stochastic Load Flow routine were used as input variables to a routine which computed the post-outage line flow uncertainty statistics using equations (5.51) and (5.52) in conjunctior with the deterministic outage flow. Testing was carried out by comparing post-outage line flow parameters from this routine with the parameters computed from the above Stochastic load flow when the matrix [H] was explicitly modified.

5.5.2 The effects of Dispatch Activity models on Line Flows

The need for a model of dispatch activity in stochastic line flow analysis depends upon the magnitude of the errors introduced to the line flow statistics by not modelling dispatch activity. If these errors are acceptably small for the particular application -in this case probabilistic trajectory analysis then no model is required.

A Monte Carlo type simulation approach was chosen for this assessment. The details of this are described in Section 5.5.1.

Four different models of dispatch activity were investigated:-

(1) The model of dispatch activity (including cost breakpoints) discussed in Chapter 4 and described by equations (4.27) and (4.28). This was taken as the baseline model for further comparative work as it most closely resembles ideal dispatch activity for a lossless, true merit order cost system.

(2) As the baseline model but without intermediate cost break points.

(3) A model of dispatch activity in which all the balance of power is taken up by the slack bus.
This slack bus activity model would only be a realistic model on a lossless, true merit order system in which there was only one marginal generator which was also capable of varying its output to meet the entire range of balance of power requirements without hitting its maximum or minimum generation output level. This could be practical where the total uncertainty is small compared to the generator capacity.

(4) A model of dispatch activity in which the balance of power is achieved by spreading the imbalance equally over all generators without taking account of generator maximum and minimum limits. This could be practical where the total uncertainty is large compared to generator capacities.

The means and the variances produced by the Monte Carlo simulation for the flow on each line in the network are given in Table 5.1 . The most obvious shortcomings of models (3) and (4) are the errors in the line flow means and variances for line flows that are sensitive to nodes with marginal generation. The baseline control activity model introduces mean line flows which are significantly shifted away from deterministically predicted values due to the effect of the limits on the output of marginal generators. This is not reproduced in the cases in which models (3) and (4) are employed. The line flow variance figures produced by models (3) and (4) are also significantly different from the baseline values. Model (3) overestimates the variance on lines sensitive to the slack and underestimates it for lines sensitive to the true marginal generation. Model (4) underestimates the variance on lines sensitive to the true marginal generation.

Models (3) and (4) are in fact the control activity models adopted for most existing probabilistic and stochastic load flow methods and these are therefore seen to be inadequate for the probabilistic trajectory analysis application of a true merit order cost generation system.

The importance of modelling the cost break point characteristic is also clearly identified by the differences between the base-line model and the no cost break point case in the line flows

Table 5.1 Sheet 1 of 2

Effect of Dispatch Strategy on Line Flows

LINE	EXPECTED FLOW			
	(1) (Baseline)	(2)	(3)	(4)
1	-241	-237	-247	-247
2	157	153	147	147
3	8	19	-16	-16
4	355	348	351	351
5	-102	-90	-120	-119
6	695	694	680	681
7	923	937	903	904
8	874	868	874	874
9	468	474	459	459
10	119	122	108	109
11	122	107	118	119
12	445	448	437	438
13	128	138	115	115
14	273	303	258	257
15	907	984	866	865
16	213	250	199	197
17	733	729	719	721
18	262	298	233	232
19	179	208	149	150
20	52	91	6	7
21	-63	-14	-122	-120
22	294	296	293	293
23	-366	-384	-350	-351
24	770	787	755	755
25	525	569	495	495
26	643	682	599	600
27	-217	-216	-219	-219
28	91	110	75	75
29	-250	-193	-307	-307
30	253	196	310	310

(1) = Uncertainty dispatched to marginal generators (cost breakpoints) in merit order.
(2) = Uncertainty dispatched to marginal generators (no cost breakpoints) in merit order.
(3) = Uncertainty dispatched to slack generator only.
(4) = Uncertainty dispatched to all generators equally.

Table 5.1

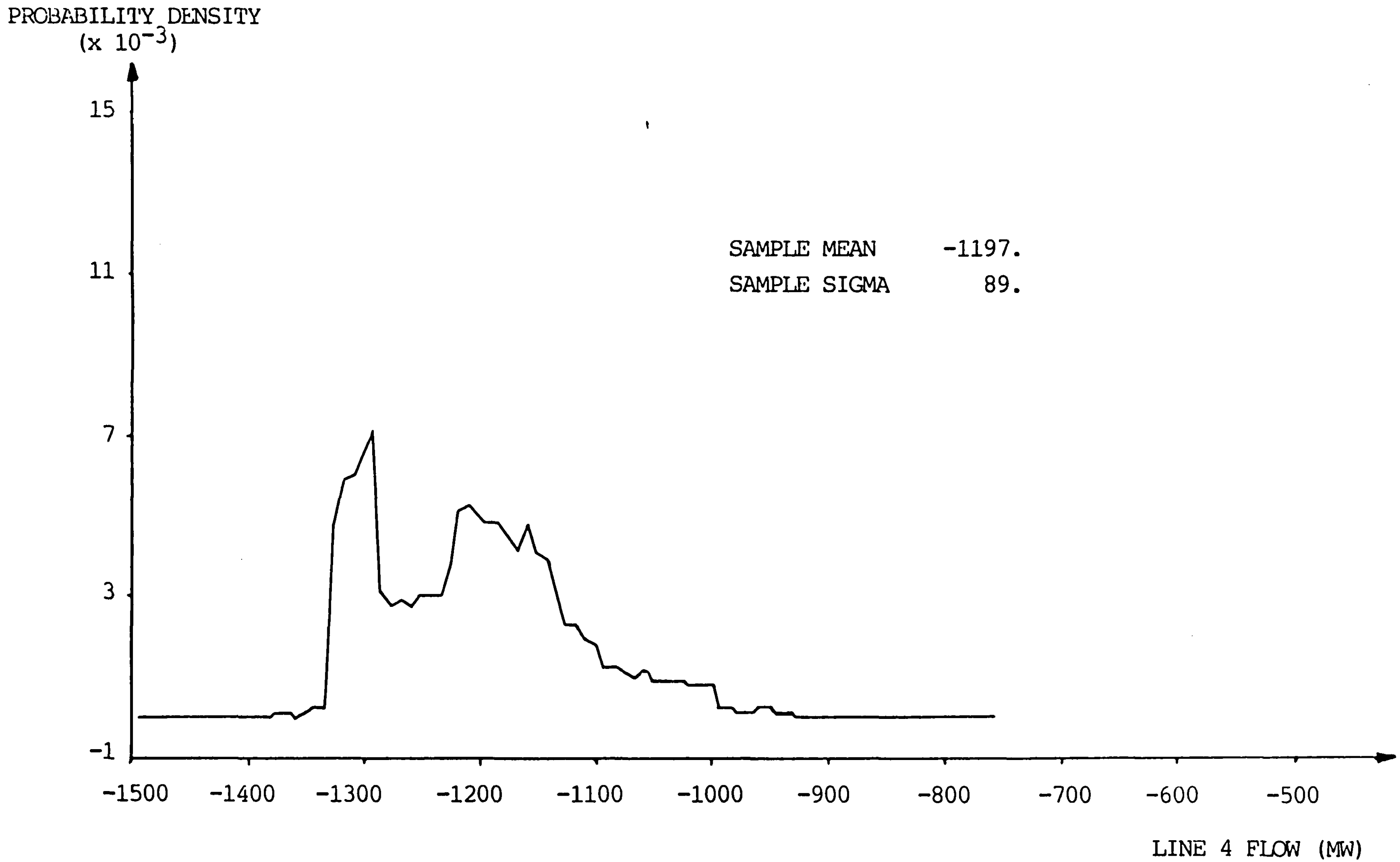
Effect of Dispatch Strategy on Line Flows

LINE	STANDARD DEVIATION OF FLOW			
	(1) (Baseline)	(2)	(3)	(4)
1	74	74	78	73
2	75	76	126	73
3	118	118	145	122
4	57	59	93	57
5	89	89	113	89
6	94	94	172	96
7	82	81	117	85
8	70	71	96	73
9	38	37	57	39
10	54	53	86	53
11	85	90	126	76
12	66	65	77	66
13	43	42	45	35
14	68	83	45	43
15	169	205	101	102
16	202	216	201	183
17	170	175	203	160
18	178	186	172	176
19	124	125	126	120
20	158	151	146	125
21	199	189	174	148
22	57	57	58	57
23	128	130	137	120
24	81	83	86	85
25	107	121	94	95
26	167	172	189	154
27	74	74	73	73
28	93	95	89	89
29	176	175	158	153
30	176	171	145	145

(1) = Uncertainty dispatched to marginal generators (cost breakpoints) in merit order.
(2) = Uncertainty dispatched to marginal generators (no cost breakpoints) in merit order.
(3) = Uncertainty dispatched to slack generator only.
(4) = Uncertainty dispatched to all generators equally.

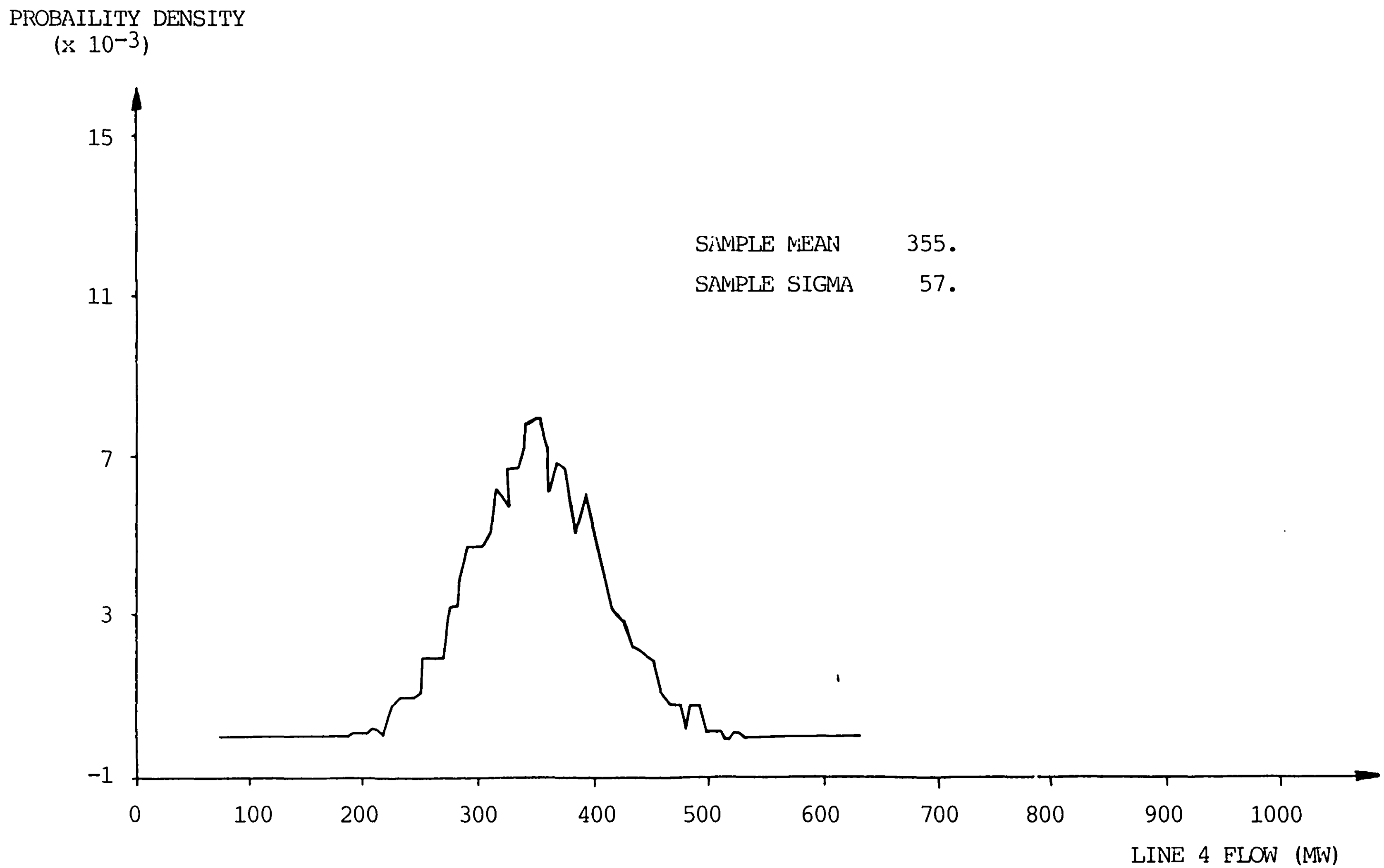
for lines sensitive to the marginal generators. These are due to the considerable change in marginal generator variances as a result of the merit order revisions due to assuming no cost breakpoints.

A further aspect of interest with dispatch activity, is the effect of this on the probability density of the line flows. In order to investigate this effect the probability densities of each line flow were produced from the baseline Monte Carlo simulations and compared with those produced by the same Monte Carlo simulation but without mapping the component of nodal demands across to the line flows. The line flow probability densities of this generation only case Fig. 5.3 are of a highly non-Gaussian form but when the demand component case is included Fig. 5.4 an approximately Gaussian form is produced. This is because the effect of the marginal generator probability mass on line flow pdf's for lines connected directly to that marginal generator depends upon the mean and variance of the demand local to that marginal generator in relation to the size of the generator and the proportion of probability mass at each cost breakpoint. For lines further out into the system the size of the mean and variance of the aggregate demands start to dominate. On the basis that nodal demand forecast errors are Gaussian distributed this will result in line flow uncertainties with approximately Gaussian distributions. While this is not a general conclusion due to some degree of non-Gaussian probability mass on lines connected directly to nodes with marginal generation - it does indicate the approximate validity of the use of the Central Limit Theorem to argue that MW line flow probability densities are approximately Gaussian on a linear system. The implication of this is that the degree of confidence with which a line flow will fall within a given band is significantly better than is apparent from the Chebyshev inequality (Table 5.2).



MONTE CARLO LINE FLOW SIMULATION
- effect of generation only
(cost breakpoint case)

Fig. 5.3



MONTE CARLO LINE FLOW SIMULATION
-demand and generation together
(cost breakpoint case)

Fig. 5.4

Table 5.2

Degree of Confidence
Chebyshev v Gaussian

	Chebyshev			Gaussian		
	σ	2σ	3σ	σ	2σ	3σ
Degree of Confidence	0%	75%	88.89%	68.27%	95.45%	99.73%

5.5.3 Errors introduced by approximations used in analytic model of control activity

In addition to any errors introduced by not modelling a particular aspect of a process, errors can also be introduced by approximations made in the solution method used for the model of the process. In general the magnitude of the accumulated errors due to modelling and solution method approximations should be less than 10% of the magnitude of the line flow means and variances required.

The magnitude of the accumulated errors introduced by the approximations involved in determining the means and cross covariances of the marginal generators (4.32) and (4.33); and the cross covariances of nodal marginal generation with nodal demand (5.29) were examined. This was carried out by comparing the line flow means and variances produced by the baseline Monte Carlo simulation with the line flow means and variances produced by the analytic method of equations (4.32), (4.33), (5.22) and (5.21) with (5.25). The results are shown in Tables 5.3 and 5.4.

The worst error occurred on line 15 in table 5.4. This shows a standard deviation of 217MW versus 205MW on an expected flow of 1003MW versus 984MW. This line is connected directly to one of the marginal generators and the error is due to the residual error from the approximation of lumping all the intermediate probability mass for that generator at its breakpoint values. This error could be reduced by computing the exact pdf's for the marginal generators. However, this degree of error on flows of this magnitude was not considered significant for the predictive security assessment application.

Table 5.3

Comparison between Monte Carlo simulation of line flows and Analytic computation of line flow statistics including control activity model.
(Cost breakpoint case)

Line	Baseline Monte Carlo		Analytic (including Control Activity)	
	Mean (MW)	Standard Deviation (MW)	Mean (MW)	Standard Deviation (MW)
1	-241	74	-242	74
2	157	75	154	74
3	8	118	6	118
4	355	57	354	56
5	-102	89	-103	89
6	695	94	692	91
7	923	82	921	83
8	874	70	873	70
9	468	38	467	38
10	119	54	117	52
11	122	85	118	84
12	445	66	445	66
13	128	43	127	44
14	273	68	276	70
15	907	169	914	174
16	213	202	216	203
17	733	170	728	171
18	262	178	262	181
19	179	124	179	125
20	52	158	49	161
21	-63	199	-66	203
22	294	57	293	57
23	-366	128	-363	128
24	770	81	768	83
25	525	107	528	110
26	643	167	640	170
27	-217	74	-215	74
28	91	93	93	94
29	-250	176	-252	182
30	253	176	254	184

Table 5.4

Comparison between Monte Carlo simulation of line flows and Analytic computation of line flow statistics including control activity model.
(No cost breakpoint case)

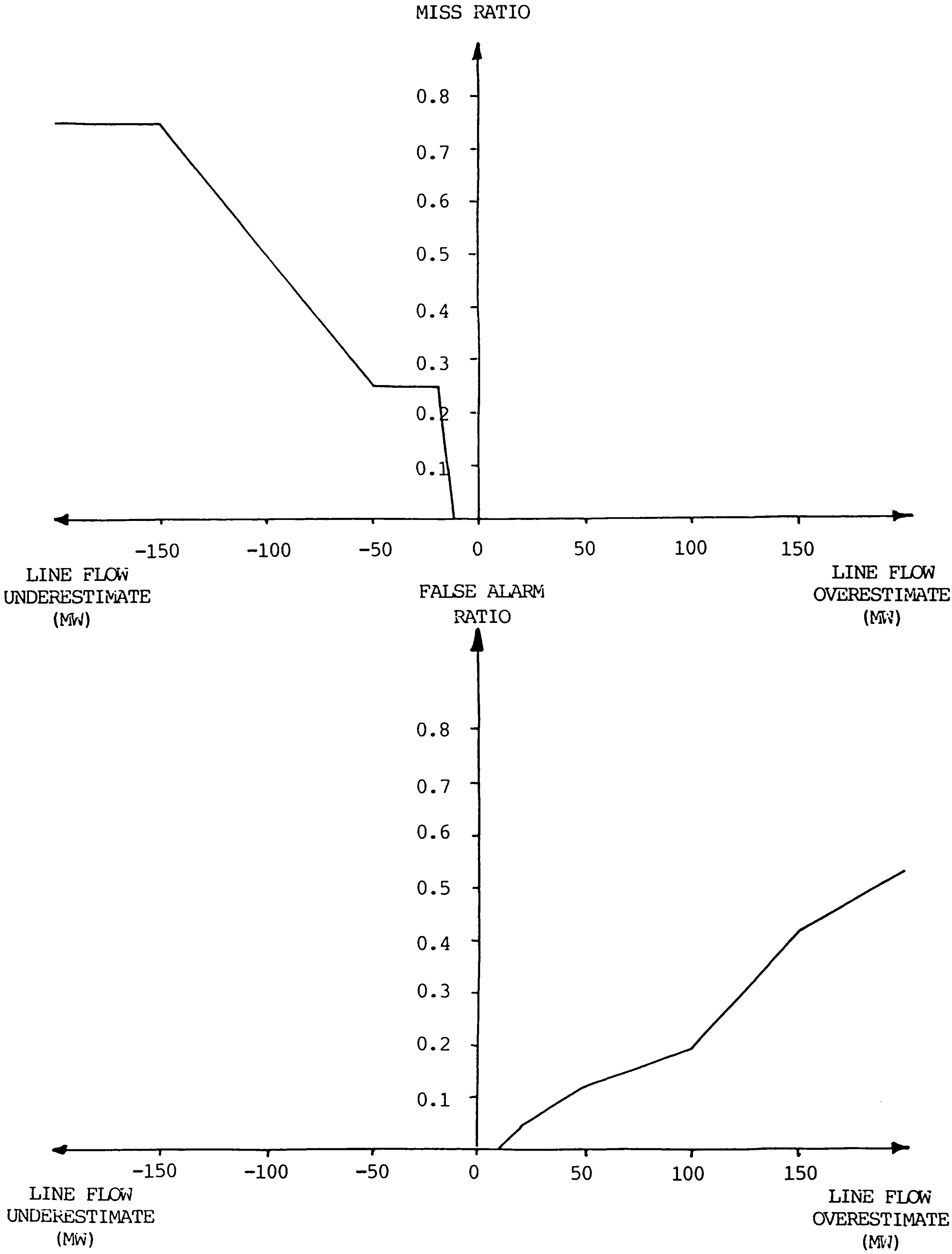
Line	Baseline Monte Carlo		Analytic (including Control Activity)	
	Mean (MW)	Standard Deviation (MW)	Mean (MW)	Standard Deviation (MW)
1	-237	74	-239	74
2	153	76	145	76
3	19	118	14	116
4	348	59	342	59
5	-90	89	-94	89
6	694	94	684	91
7	937	81	931	81
8	868	71	863	71
9	474	37	471	37
10	122	53	117	52
11	107	90	96	91
12	448	65	446	66
13	138	42	135	42
14	303	83	311	87
15	984	205	1003	217
16	250	216	263	216
17	729	175	716	176
18	298	186	301	187
19	208	125	208	125
20	91	151	83	154
21	-14	189	-24	193
22	296	57	294	57
23	-384	130	-382	131
24	787	83	786	87
25	569	121	576	128
26	682	172	677	181
27	-216	74	-214	74
28	110	95	112	98
29	-193	175	-197	173
30	196	171	199	172

5.5.4 Comparison between Probabilistic Worst Case Approach and Multiple Stochastic Outage Approach

In order to gain a general feel for the sensitivity of the Probabilistic Worst Case method to the flow uncertainties, this algorithm was run for a range of line flow uncertainties of from -200 MW to +200 MW about the deterministically predicted maximum and minimum line flow trajectory values and compared with the insecure outages identified by a deterministic multiple outage analysis. The results for the Miss Ratio and False Alarm Ratio from this test are plotted in Fig. 5.5. These clearly indicate the importance of not underestimating the line flow peak trajectory values nor overestimating them if the Miss Ratio is to be maintained at zero and the False Alarm Ratio maintained as close to zero as possible. Though not apparent from these plots it should be noted that the general level of loading on the system also influences the sensitivity of these parameters - the False Alarm Ratio on a lightly loaded system would be somewhat insensitive while on a heavily loaded one it would be quite sensitive due to the degree of line loadings.

A precise comparison of the Probabilistic Worst Case method with a multiple stochastic outage method was carried out by evaluating the Miss Ratio and False Alarm Ratio for a complete one day trajectory with defined statistical parameters.

The test system used was that of Chapter 3 and the cost order of the generators was as defined in Chapter 4. The generation data logged and shown in Fig. 3.8 of Chapter 3 broadly indicated the unit commitment. Scheduled generator values for each time step were derived using the cost order in pure merit order. The corresponding recorded nodal demand data shown in Fig. 3.9 of Chapter 3 was used as a basis to provide the mean nodal demand values for each time step of the trajectory and the variance of each nodal demand was defined as in (4.50).



SENSITIVITY OF MR AND FAR TO PEAK LINE FLOW ESTIMATE

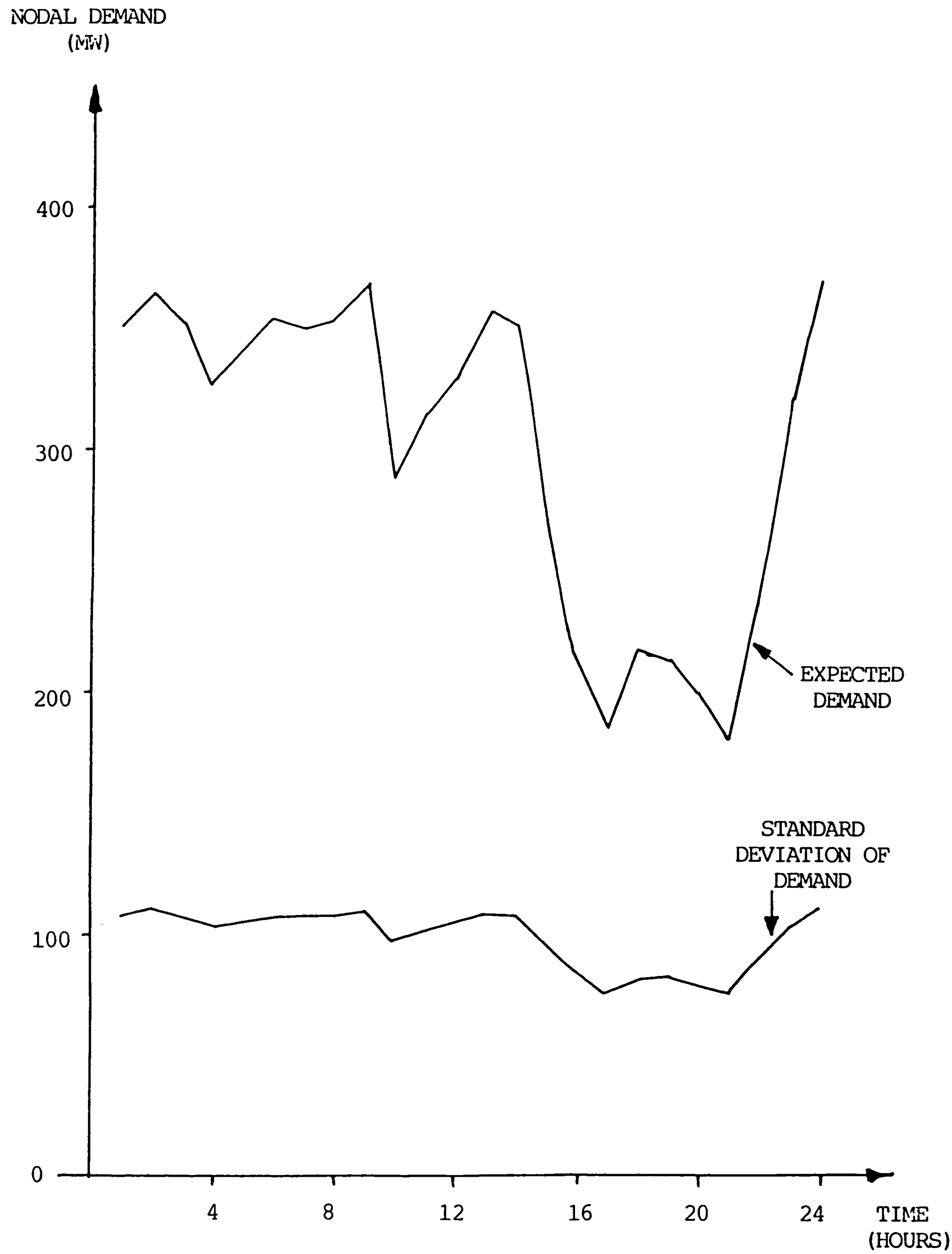
Fig. 5.5

An example of these nodal demand means and variances is shown in Fig. 5.6, while Fig. 5.7 shows the resulting total demand trajectory mean and its variance. The uncertainties have been evaluated for the lead time period from 0 up to 24 hours ahead. The lead time period from 0 to 2 hours ahead should exhibit an increasing uncertainty of total demand with lead time as discussed in Chapter 4. However, this aspect was not modelled. The uncertainty was modelled as that existing when it had substantially levelled off. The effect of neglecting this early lead time dependence was not considered significant for the purposes of this research as it would tend to give pessimistic results. However it is important in practice as it would effect the need for security rectification.

The application of the analytic dispatch activity routine of Chapter 4 produced the means and variances of the nodal generation. These are shown in Fig . 5.8 for the trajectory of one of the generators. This shows a significant change in the standard deviation of this marginal generator with time of day. The standard deviation has increased from 80MW to 340MW over a period when its expected value has decreased from 1670 MW to 1290MW.

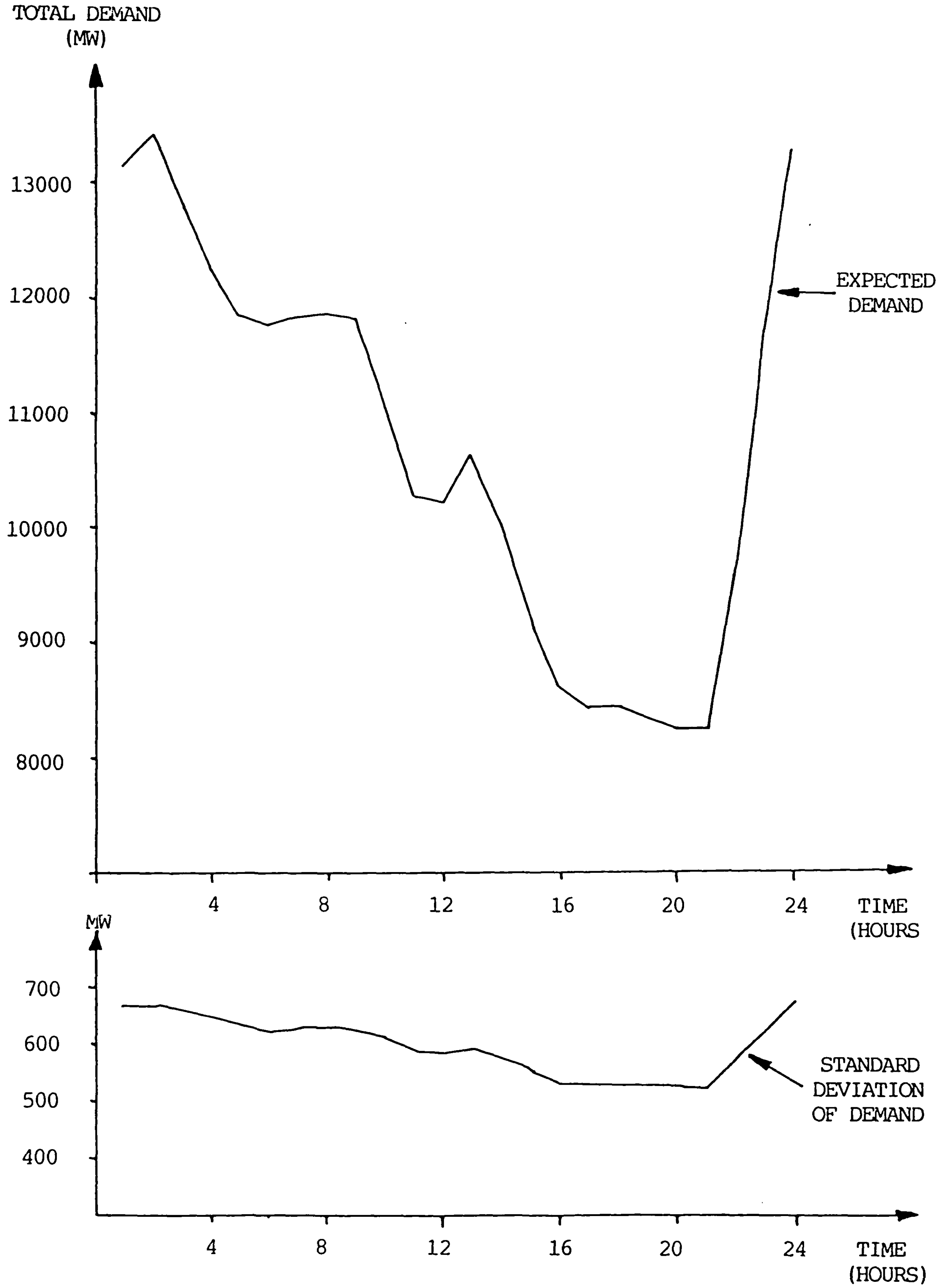
This effect is due to the change in the position in the merit order of this generator relative to the demand. It indicates that generator uncertainties cannot be allocated based purely on generator size, nor can uncertainties be allocated by considering only one point in time.

The stochastic load flow routine incorporating dispatch activity was then used to produce the line flow means and variances for the trajectory. An example is shown Fig. 5.9. The deterministically computed values are also shown for comparison.



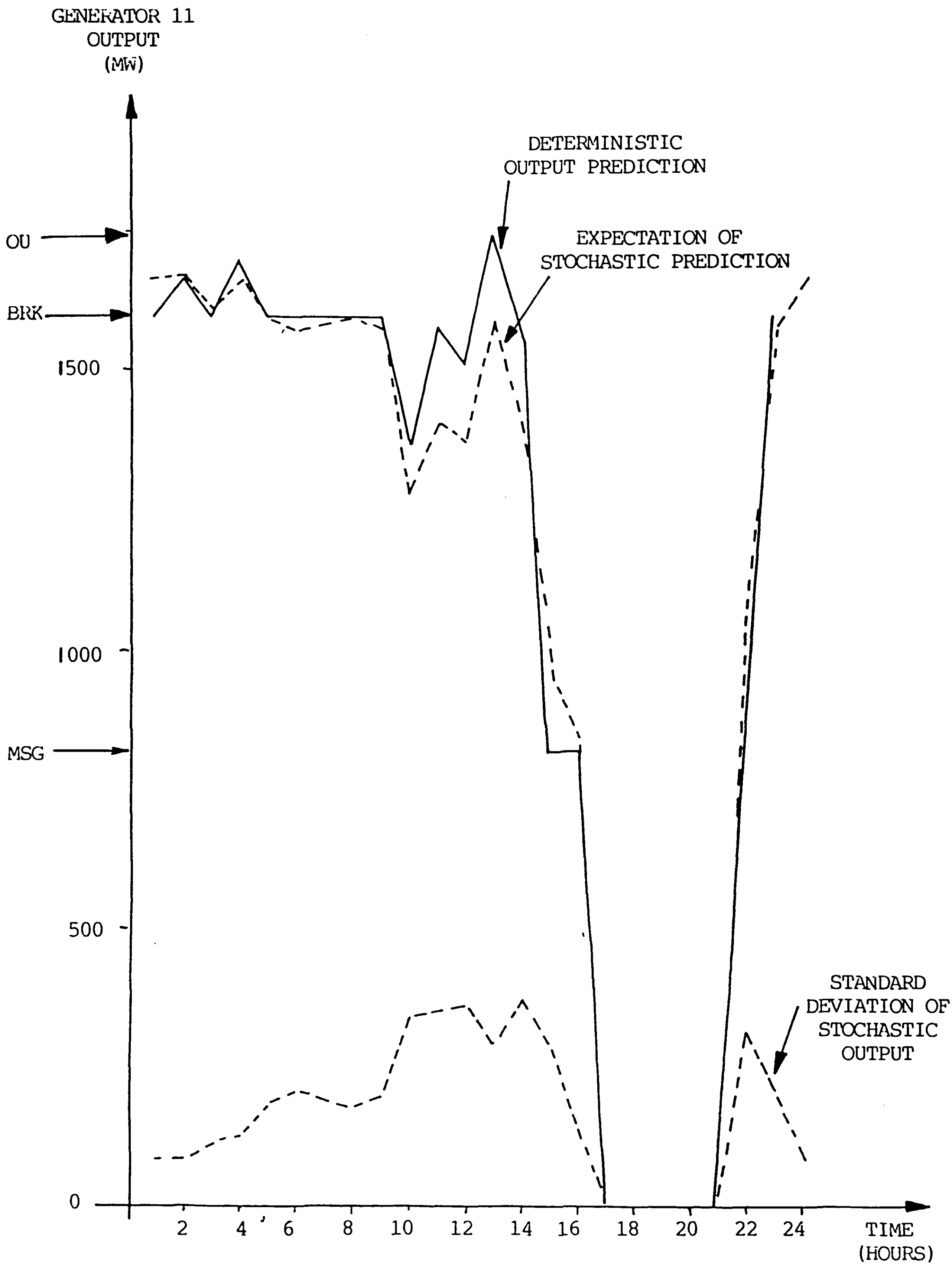
DEMAND TRAJECTORY OF NODE 6

Fig. 5.6



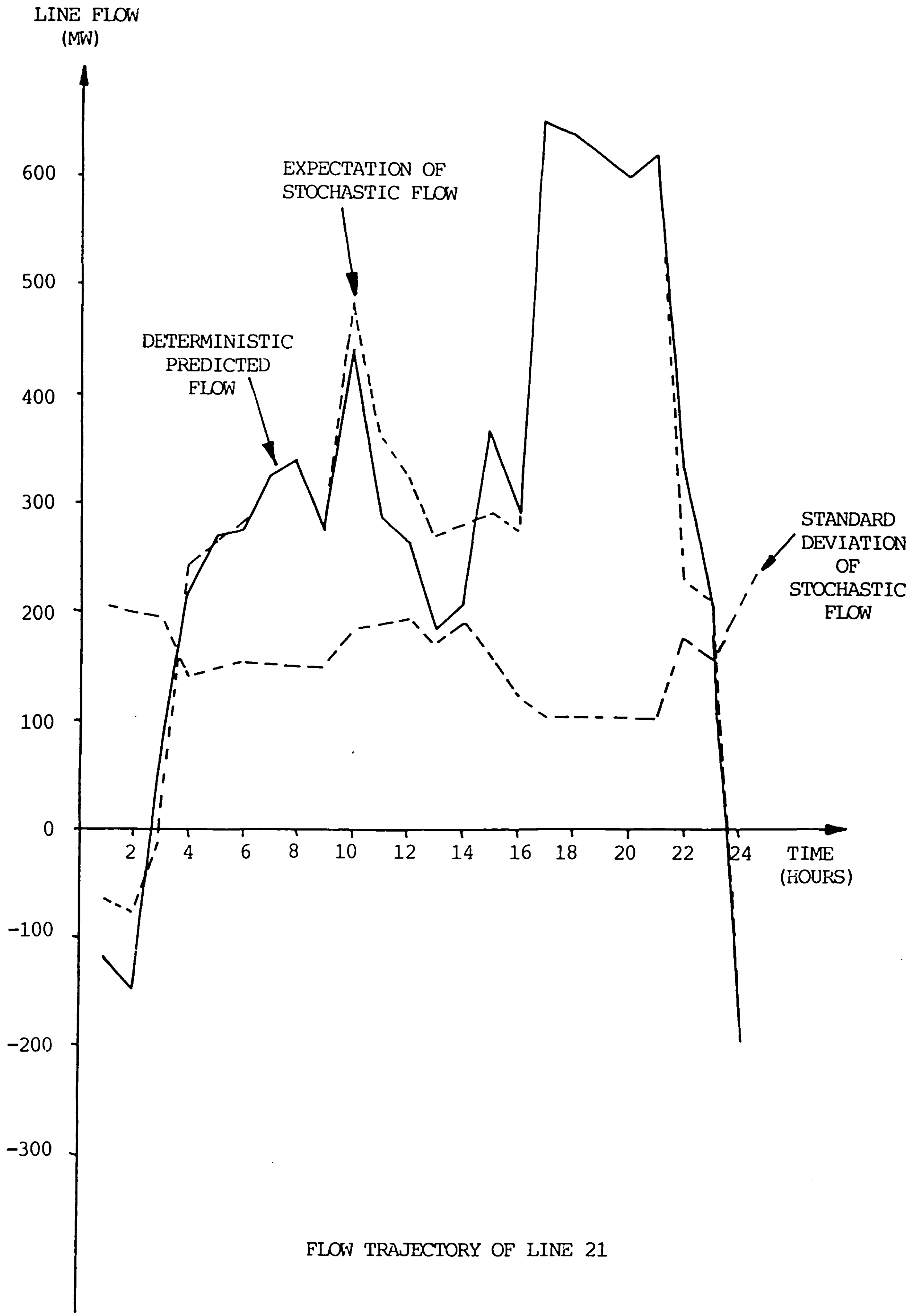
TOTAL DEMAND TRAJECTORY

Fig. 5.7



MW OUTPUT TRAJECTORY OF GENERATOR 11

Fig. 5.8



FLOW TRAJECTORY OF LINE 21

Fig. 5.9

The effect of merit order dispatch activity on line flow uncertainty is to introduce a shift in the expected flow on a line when compared with the deterministically predicted flow. In Fig.5.9 a shift from 185MW to 270MW is apparent at time step 13. The effect of merit order dispatch activity on marginal generator uncertainty is also reflected in line flow uncertainty. In Fig. 5.9 the standard deviation of the line flow varies from 100MW to 205MW on an expected flow ranging between - 180MW and +650MW. The greatest variance does not coincide in time with the greatest expected flow. This means that the peak flow on a line (including uncertainty) need not necessarily occur at the same time as the predicted peak flow. This requires a stochastic model of this time of day dependence effect.

The multiple stochastic outage method was carried out on this same basic data for each time step. The insecure outages and their time of occurrence identified by this method are shown in Fig. 5.10 and the details for two particular insecurities in Fig. 5.11. The diagram shows an increase in the number of critical line outages with uncertainty level. More critical outages are possible at the three standard deviation level than at the one standard deviation level. This indicates the dependence of which lines are critical outages to the degree of loading on the network. The three standard deviation case represents a more heavily loaded network than the one standard deviation case.

The insecure outages identified by the Probabilistic Worst Case method were then used to calculate the Miss Ratio and False Alarm Ratio values plotted in Fig. 5.12 for flows in the range up to one standard deviation, two standard deviations and three standard deviations. These show that the method never missed any critical outages which is an essential requirement. The FAR is seen to

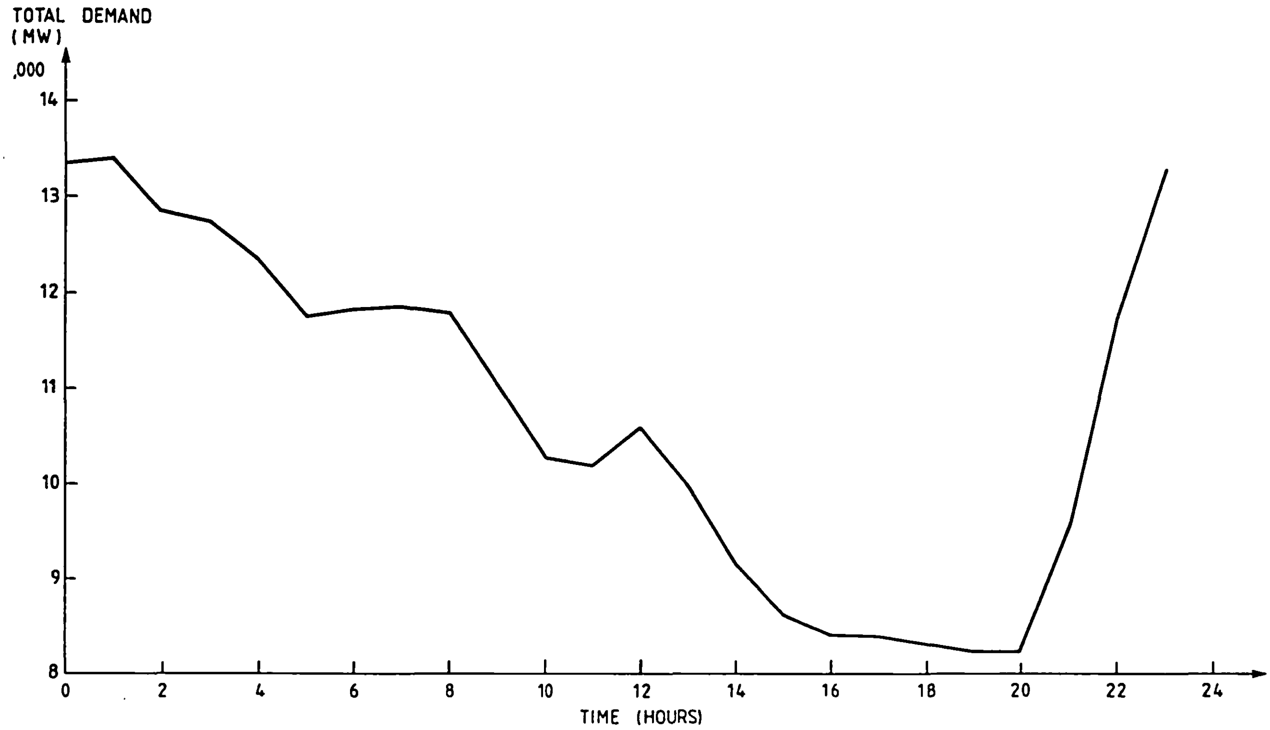
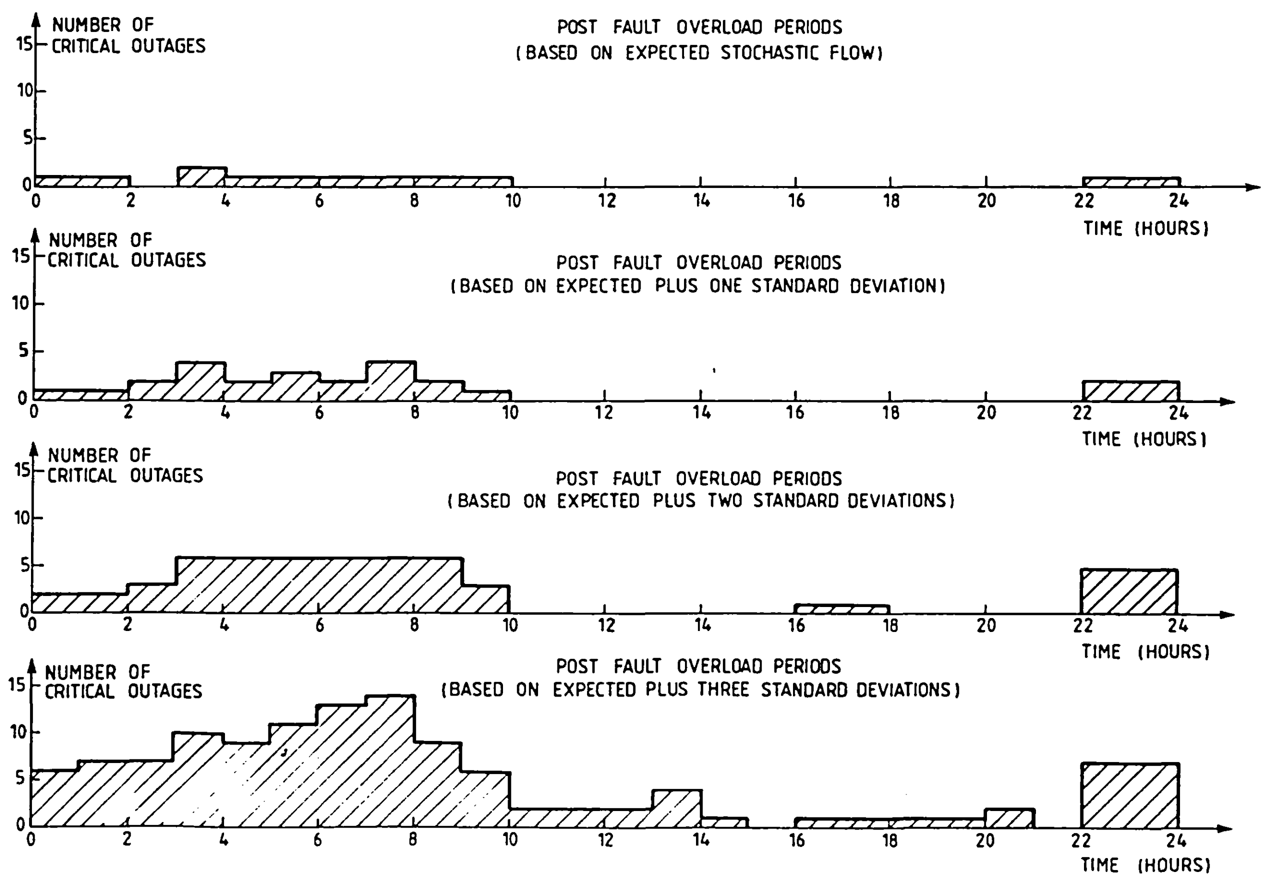
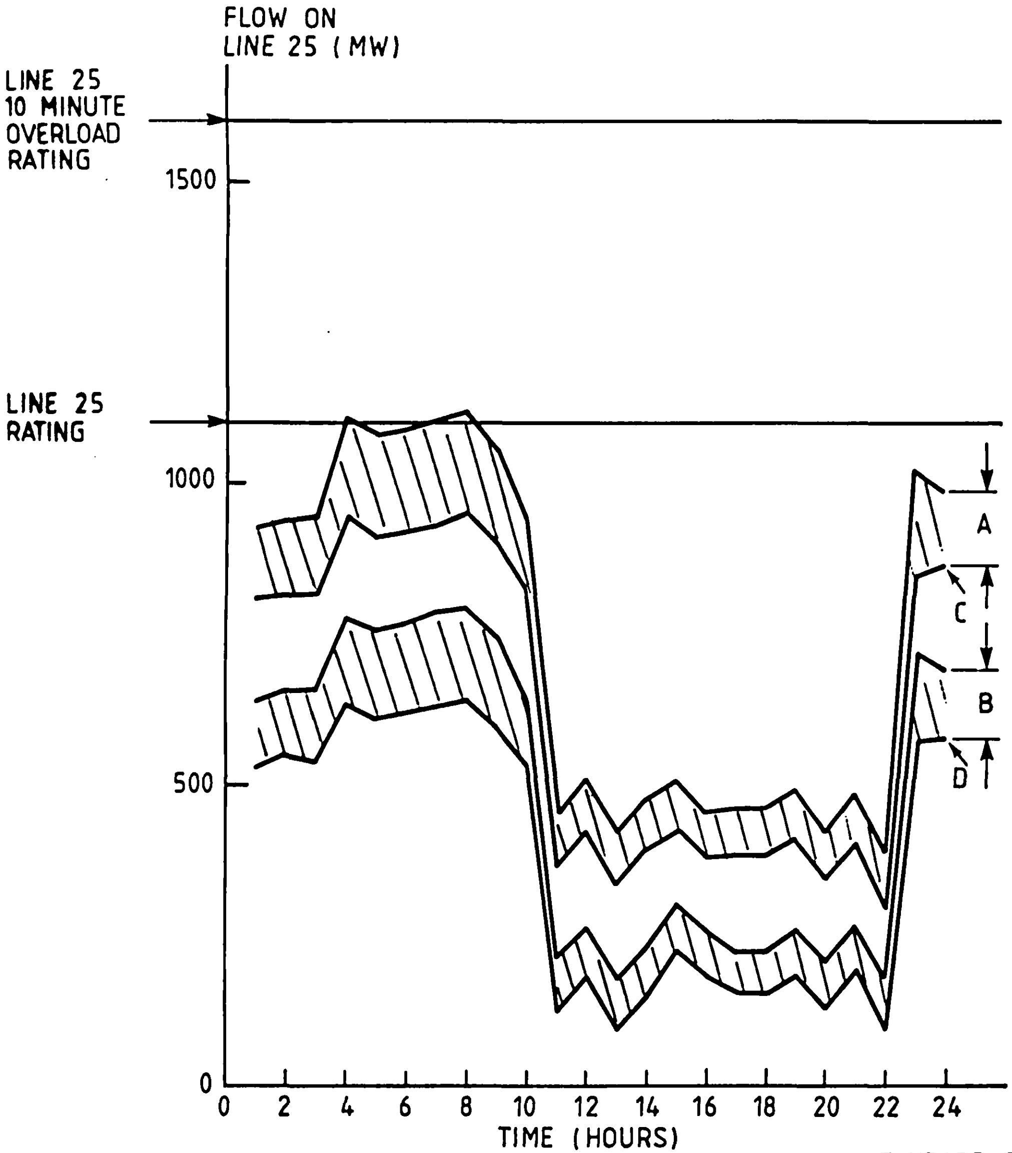
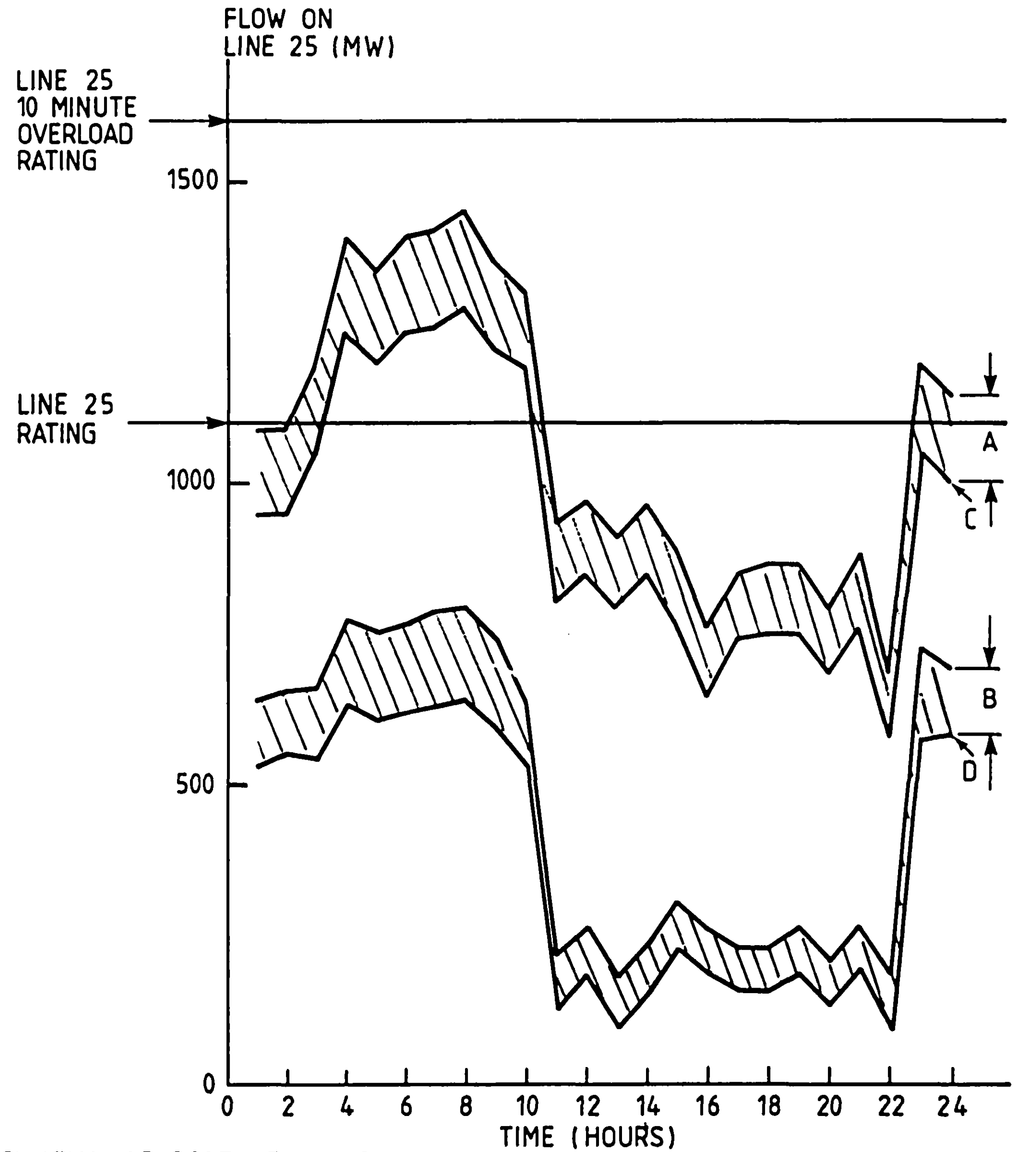


FIG. 5.10 POST - FAULT OVERLOAD PERIODS





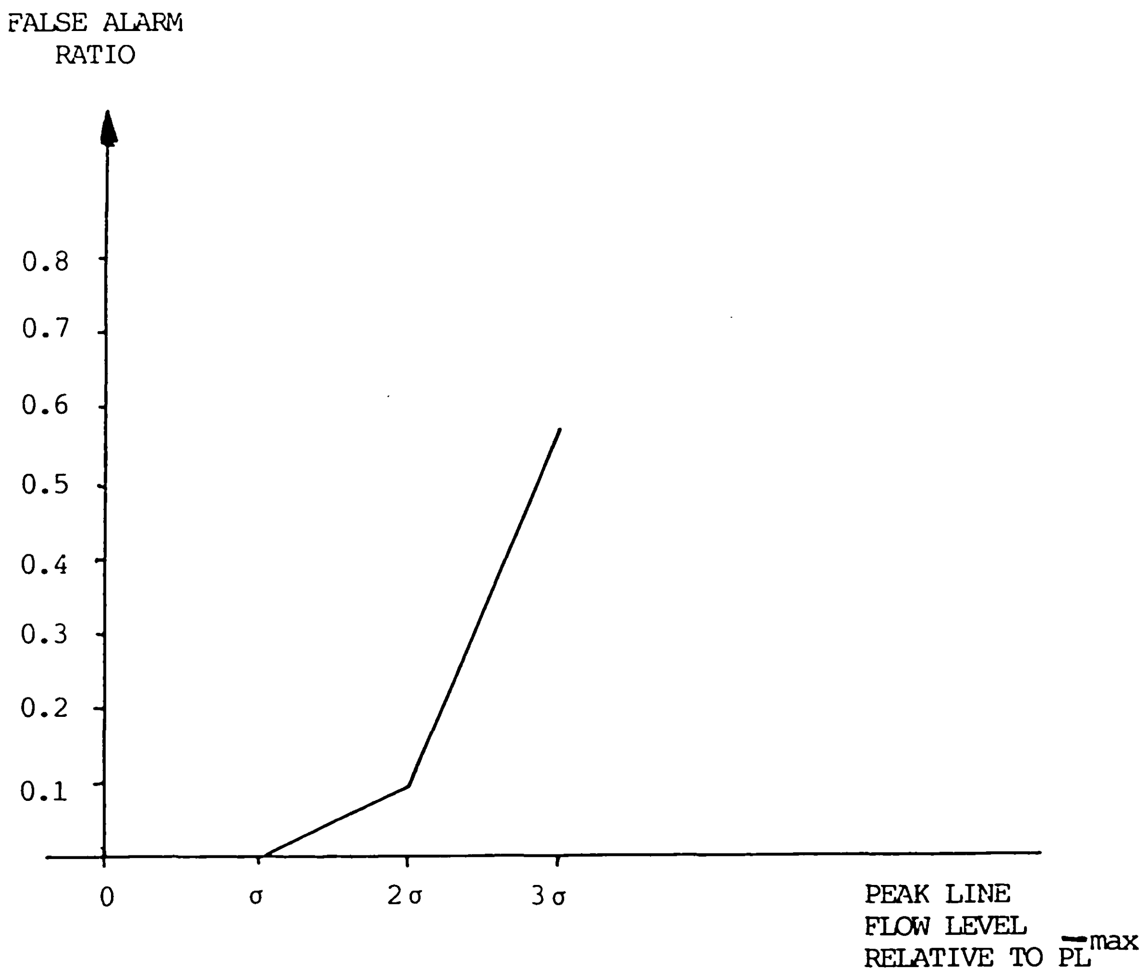


A – STANDARD DEVIATION OF POST – FAULT FLOW
B – STANDARD DEVIATION OF PRE – FAULT FLOW
C – POST – FAULT FLOW ON LINE 25
D – PRE – FAULT FLOW ON LINE 25

FIG. 5.11 FLOW ON LINE 25 WITH LINE 24 FAULTED

FIG 5.11 FLOW ON LINE 25 WITH LINE 17 FAULTED

MISS RATIO = ZERO FOR ALL CASES



COMPARISON BETWEEN PROBABILISTIC WORST CASE METHOD
AND MULTIPLE STOCHASTIC OUTAGE METHOD

Fig. 5.12

increase with uncertainty level due to the effect of neglecting correlations between line flow uncertainties on a more heavily loaded system. The FAR at the three standard deviation level corresponds to a situation in which the true number of critical outages is 16, while the probabilistic worst case method identified 24 out of the 30 lines on the system. At first sight the probabilistic worst case method would appear to be a rather poor contingency filter in heavily loaded conditions. However, as each contingency identified by it only resulted in apparent insecurities on a few other lines (typically from 1 to 3 out of the other 29) the method does dramatically reduce the number of possible security constraints and does, therefore, act as an effective security constraint filter.

5.6 Practical Application

The probabilistic worst case algorithm developed in the research has application to a number of practical problems in the management of security in the on-line environment. These include critical transmission security constraint selection for dispatching and transmission security constraint margins for unit commitment.

5.6.1 Transmission Security Constraint Selection

Fig. 5.13 illustrates the capacity of predictive security assessment to reduce dramatically the number of transmission security constraints that need to be considered in dispatch. This shows that during the next 24 hour period the demand and generation distributions will be such that only 4 security constraints could be active when flows are taken as within one standard deviation of the expected flows. This is out of a possible (Lx (L-1)) single circuit security constraints. In order to take advantage of this, additional criteria need to be specified to the dispatch algorithm to indicate when the active security constraint subset needs to be augmented.

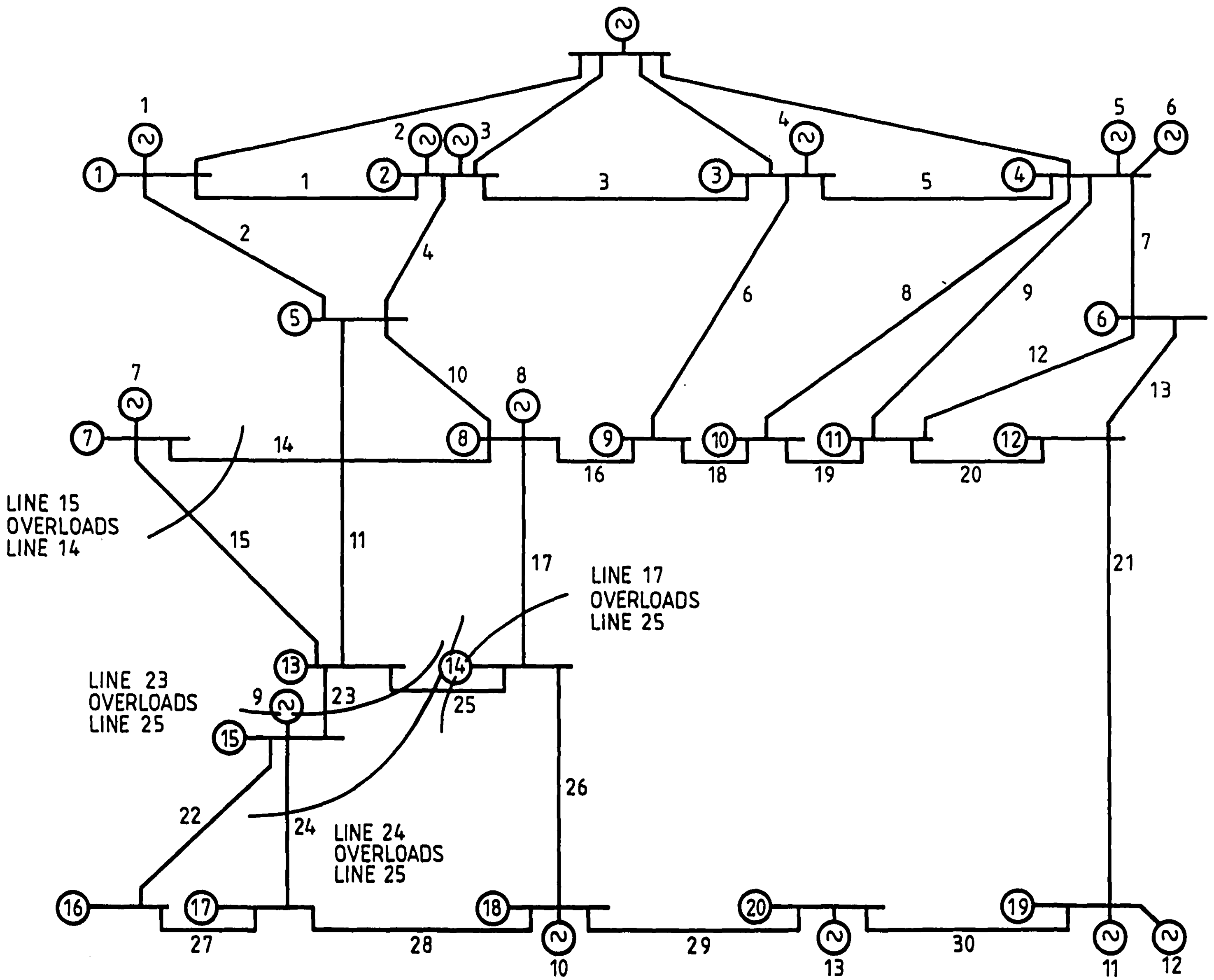


FIG. 5.13 CRITICAL LINE PAIRS
(LINE RATING < POST FAULT FLOW + ONE STANDARD DEVIATION)

The probabilistic worst case algorithm provides such criteria in the form of maximum and minimum line flows.

The implementation of the method could be along the following lines:-

i. obtain an initially feasible solution to the dispatch problem without transmission security constraints.

ii. determine a set of active and near-active transmission security constraints for the network configuration using the probabilistic worst case algorithm with maximum and minimum flows set at within one standard deviation of the expected flow. Where network switching is planned for maintenance outages or for voltage control the equivalent information should also be determined for the resulting network configuration.

iii. decide which of the constraints in this reduced subset may be rendered inactive by pre-fault network switching and load transfers. (The possibilities for this are discussed in section 2.6.2.1 but the means for automating the process are not treated in this research.). If this results in a planned change to the network configuration repeat step (ii). If not continue to step (iv).

iv. decide which of the constraints can be relaxed because acceptable post-fault flow control is available. (The possibilities for this are discussed in section 2.6.2.2 but the means for automating the process are not treated in this research.)

v. obtain a new feasible solution to the dispatch problem including the remainder of this reduced subset of security constraints.

vi. check that the consequent lines flows do not exceed the maximum and minimum flows used in (ii). If any of them do exceed the peak flow to one standard deviation but are less than the peak flow to two standard deviations, augment the subset to include the constraints which fall within the two standard deviation band and repeat step (v) and step (vi) with the exception that the maximum and minimum flow checks should be made against the two standard deviation peak flows.

If any exceed the two standard deviation peak flows but are less than the three standard deviations peak flows augment the subset to include the constraints which fall within the three standard deviation band and if any exceed the three standard deviation peak flows include the security constraints specific to these lines.

vii. check that the actions specified in (iv) are still sufficient. If not, include these particular constraints and repeat from step (v). Otherwise the system is secure.

Steps (i) and (ii) need only be carried out once a day or following an unplanned line outage. Steps (iii) and (iv) can be monitored by a security monitoring program and steps (v) and (vi) carried out within the dispatch algorithm.

5.6.2 Transmission Security Constraint Margins

This is the problem of how to ensure that sufficient, but not excessive, generation is available in each importing area of a network to meet the transmission import constraints into these areas when the flow into these importing areas is uncertain. On a merit order type of system this has to be carried out in the unit commitment phase of operations as the economics of the operation of such generators could lead to the situation in which all the synchronised generators in an importing area are at their maximum and all increases in load in that area are met by increasing the flow into the area. If such flows exceed the post-outage flow capacity into the area, a line fault could result in the need to disconnect demand in the area unless sufficient emergency Gas Turbines were also available. This problem is less likely to happen in an equal incremental cost type of system as this tends to provide considerably more part-loaded generation on the system.

Current practice in the CEGB achieves this by:-

(a) calculating the group constraint limit as describe in section 2.8.5.

(b) making an estimate, based on experience, of the uncertainty in the demand in the group corresponding to maximum flow conditions into the group.

(c) down-rating the group constraint limit figure by an amount equal to the uncertainty in the demand.

This ensures that the unit commitment program provides sufficient generation in the importing area to cover for demands upto the expected demand plus the uncertainty level. This is achieved because the economic generation is constrained by more than required for the maximum expected post-fault line flow level so that the extra out-of-merit generation is available to handle post-fault flows which are greater than expected.

The magnitude of the down-rating follows from considering what revised group constraint limit would ensure that the maximum economic mean flow exceeds this limit by an amount equal to the difference between the true limit and the flow corresponding to the demand plus uncertanity at the maximum expected flow condition.

So $\quad E[PL] - R1 = E[PL] + \sigma_D - R$

and hence $R1 = R-\sigma_D$ (5.53)

where $E[PL]$ is the maximum expected group import

$R1$ is the revised group constraint limit

R is the true group constraint limit

σ_D is the standard deviation of the group demand uncertainty at the time of maximum expected group import.

The degree of uncertainty to be considered ie whether one, two or three standard deviations, depends upon the extra cost in providing for the uncertainty, the failure rates of the lines feeding the group and the consumer costs if load shedding has to be used. (see section 2.6.3).

The limitations in this current pactice are due to the limitations in the group constraint method (see section 2.8.5) and that no indication of the duration of the maximum flow is provided. This is significant as the start-up costs of steam generators can dominate the costs of short duration insecurities.

The probabilistic worst case algorithm developed in this research overcomes these limitations. Its implementation could be along the following lines:-

i. obtain an initially feasible solution to the unit commitment problem without transmission security constraints.

ii. as steps ii and iii in section 5.6.1 to identify a reduced subset of active and potentially active security constraints.

iii. neglect the insecurities in this set if the magnitude and duration of the overload is within the overload time duration capability of the line.

iv. decide which of the constraints can be relaxed because acceptable post-fault flow control is available on the network or on the demand.

v. calculate, for each remaining insecurity, the post-fault effective generation reserve that would be available to correct for it at each timestep.

This could be calculated as:-

$$\overline{S}_{Li} = \sum_{j=1}^{n} h_{ij}(P_{Gj}^{M} - \overline{P}_{Gj}) = \sum_{j=1}^{n} h_{ij}\overline{S}_{Gj} \tag{5.54}$$

where $\overline{S}_{Li}$ is the mean post-fault effective generation reserve for the ith insecure line; h_{ij} is the post-fault sensitivity of line i to generation at node j, and only the negative factors have been taken as these give the generators at which increased output will reduce the line flow.

P_{Gj}^{M} is the maximum output of the jth generator.

$\overline{P}_{Gj}$ the mean output of the jth generator.

The stochastic mean generation is used, rather than the deterministic prediction, to account for those situations where in the event there may not be any reserve as the generator has been dispatched to its maximum.

vi. From these values compute the expected post fault minimum flow for each post-fault overloaded line as:-

$$E[P_{Li}^{pfm}] = E[P_{Li}^{pf}] - \overline{S_{Li}} \tag{5.55}$$

This follows from:-

$$P_{Li}^{pfm} = \sum_{j=1} h_{ij} \left(P_{Dj} - P_{Gj}^{M}\right) \tag{5.56}$$

$$= \sum_{j=1} h_{ij}\left(P_{Dj} - [P_{Gj} + S_{Gj}]\right)$$

$$= P_{Li}^{pf} - \sum_{j=1} h_{ij}\, S_{Gj} \tag{5.57}$$

in which the effective spare has been calculated as a function of demand uncertainty and dispatch uncertainty alone. Generator unavailability uncertainty has not been included.

vii. Now calculate the variance of the post-fault minimum flow for each post-fault overloaded line from (5.56) as:

$$\mathrm{Var}[P_{Li}^{pfm}] = h_{ij}^{2}\ \mathrm{Var}(P_{Dj}) \tag{5.58}$$

since $\mathrm{Var}[P_{Gj}^{M}] = 0$

In general $\mathrm{Var}[P_{Li}^{pfm}] \neq \mathrm{Var}[P_{Li}^{pf}]$

as $Var[P_{Li}^{pf}]$ is a composite of the demand uncertainty of the importing area and the dispatch uncertainty of generators in the importing area. In the particular case where there are no marginal generators in the importing are they will be equal.

viii. neglect those insecurities where the expected post-fault minimum flow on a post-fault overloaded line is less than the line rating minus the standard deviation (or desired uncertainty level) of the post-fault minimum flow.

$$E[P_{Li}^{pfm}] \lesssim R_{Li} - \sigma_{Li} \tag{5.59}$$

where $\sigma_{Li}^{2} = Var[P_{Li}^{pfm}]$

In these cases the amount of generation reserve available is such that the insecurity can be covered in the dispatching phase at least upto the defined uncertainty level.

ix. for the remaining insecure set, down-rate the line rating of each insecure line to:-

$$R'_{Li} = R_{Li} - \sigma_{Li} \tag{5.60}$$

x. Feed these down-rated line ratings into the unit commitment program as transmission security constraints:-

$$R'_{Li} \geqslant \left| \sum_{j=1}^{n} h_{ij} \, (P_{Gj} - P_{Dj}) \right| \tag{5.61}$$

Where R'_{Li} is the down-rated line rating of line i

h_{ij} is the post-fault sensitivity of line i to the jth nodal injection.

xi. Obtain a new feasible solution to the unit commitment problem including this reduced subset of security constraints.

xii. as step (vi) in section 5.6.1

xiii. check that the post-fault actions on the network and demand specified in (iv) are still sufficient and that the post-fault action on the generation implied by (viii) is also still sufficient. If not, include these particular constraints and repeat from step (xi). Otherwise the system is secure.

5.7 Conclusions

The sensitivity of trajectory analysis to uncertainties in input parameters has been examined in this chapter. Earlier research in probabilistic and stochastic load flow methods failed to model the effect of dispatch activity (including cost breakpoints) on line flow uncertainty, and the effect of this on time of day dependence.

The effects of neglecting this were investigated and significant errors were found to be introduced in lines sensitive to the marginal generation.

An analytic method was developed to map dispatch activity effects into line flow uncertainties. This was compared with Monte Carlo type simulations and found to provide adequate accuracy for this trajectory analysis application.

A Probabilistic Worst Case algorithm was also developed and compared with a multiple stochastic outage method. The assessment criteria of Miss Ratio and False Alarm Ratio indicated the features of this algorithm. The applications of the algorithm to transmission security constraint selection for dispatching and to transmission security constraint margins for unit commitment are described.

6. Conclusions

This thesis has concentrated on two specific requirements for predictive security assessment in the on-line environment of power system operations. These are the identification of the critical contingencies and their time of occurrence for the daily trajectory and the assessment of the uncertainty in these line flow trajectories due to demand forecast errors and the consequent effects on generator uncertainty through dispatch activity. The research described in this thesis has been restricted to the thermal line overload security problem.

Predictive security assessment is necessary for security constrained unit commitment and dispatching, particularly on merit order, thermal systems, so as to ensure that the security of importing groups of the network can be achieved. It is necessary in the initial decision stages, to decouple predictive security assessment from unit commitment and dispatch in order to make the economies that are possible from considering corrective action on the network and on the demand and from the possibilities for post-fault corrective action which permit the relaxation of a particular security constraint.

The findings and the significance of this research are summarised below.

6.1 The Findings and Significance of this Research

6.1.1 Critical Line Outage Identification

A set theoretic approach has been developed to automatically select the critical line outages and the insecure lines for the entire daily line flow trajectories including their uncertainties. The identification of the critical line outages and the insecure lines is based upon the use of set theory in the line flow domain. This is in contrast to the existing set theoretic approaches which are based in the generator or injection domain.

The application of the set theoretic approach in the line flow domain was found to dramatically reduce the potentially active constraints when tested on several days line flow data recorded on the CEGB system and applied to a test system which broadly represented the southern half of the CEGB system. The constraint reduction was achieved by exploiting the line flow trajectory knowledge which in turn exploits the correlation between generation and demand trajectories. This results in line flow ranges that are considerablysmaller than implied by treating the generation and demandtrajectories independently. The degree of constraint reduction achieved was typically from 870 (ie L x (L-1) on the test system) potential security constraints to in the region of upto ten active security constraints. This potential for constraint reduction in the final phase of operations is a general characteristic of power system security in this phase. This results from the work carried out in the planning and operational planning phases to ensure that, as far as practically possible, economic generation is not restricted due to transmission limitations.

The method requires the prediction of pre-fault maximum and minimum line flows for the daily trajectory of each line. For each line pair in a system it is possible to derive equations which determine boundaries on the pre-fault flows of those lines which define the secure flows on that line pair. The maximum and minimum pre-fault line flow values can be input to these equations to test which, if any, of the bounds are exceeded. If for a particular line pair none of the bounds are exceeded, this indicates that all combinations of flows on that line pair will be secure provided that neither flow exceeds the predicted maximum or minimum values tested for that network configuration. Such a strategy can never fail to identify a potentially critical outage although it can signal some potentially critical outages which are not critical because of the non-time-coincidence of their peak flows. This aspect was examined in terms of a Miss Ratio (MR) and a False Alarm Ratio (FAR). In the tests on recorded data the method never failed to identify potentially critical outages (MR = 0) although it did occasionally signal some non-critical outages. The maximum FAR experienced was 0.167. These were all due to lines that were both sensitive to the outage and which had non-time-coincident peak flows. In particular this occurred on lines that had significant flows in both directions as a result of economic dispatch activity. The number of such potentially critical but non-critical outages was small and the method acted as an effective critical outage filter. The potentially critical but non-critical outages being identified in subsequent analysis using a dc load flow.

The need to consider several points on the demand trajectory was examined. It was found that the time of maximum line flows did not coincide with the time of maximum demand nor did all line flows peak at the same time. This is due to the effects on the line flows of the cost differences between generators. It is therefore necessary to find the maximum and minimum flows for each line by computing the line flow trajectories from generation schedules and demand trajectories.

The need for predicted estimates of the line flow trajectories is essential for predictive security assessment. Predictions are uncertain because of the uncertainty in the network configuration the nodal demands and the generation schedules.

The effect of uncertain line flow trajectories on the identification of critical outages was examined. It was found that underestimates of peak line flows led to miss ratios of greater than zero while overestimates of peak flows led to greater False Alarm ratios. This indicates that the outages that are critical depends upon the predicted conditions. This is in common with any predictive security assessment scheme. It requires that predictive security assessment should indicate the sensitivity of critical outages to line flow uncertainties.

In this research this aspect was treated by separately indicating those security constraints which were active based upon expected peak flow conditions and upon conditions within one standard deviation of expected peak flows, two standard deviations and three standard deviations. Stochastic error bands were used rather than pessimistic pre-set error bands as pre-set error bands assume some a priori knowledge of the magnitudes of the uncertainty on peak line flows. The uncertainty in line flows due to dispatch activity was found to vary with time of day and from line to line. The degree of variation depended upon the sensitivity of the line to the marginal generation.

In the studies on the test system carried out in this research, using broadly representative CEGB data, the standard deviation of uncertainty for a particular line varied from 100 MW to 205 MW on a flow ranging between -180 MW and +650 MW. The greatest uncertainty did not coincide in time with the greatest expected flow. This was due to dispatch activity on the marginal generation. The significance of this is that the peak flows on a line including uncertainty need not necessarily occur at the same time as the predicted peak flow. This implies that sensitivity studies at the predicted peak flow would not necessarily indicate all the potentially active constraints.

The use of stochastic error bands with set theory in the line flow domain enables the potentially active constraint set defined by each band to be brought into use only when actual line flows are found to be exceeding the flows defined by that band. This avoids the need to repeat predictive security assessment whenever an actual flow exceeds a particular level.

In addition it provides precise criteria against which the need to augment the active constraint set can be assessed. This is significant as it therefore enables the predictive security assessment results to be used for contingency selection.

The approach to critical line outage identification developed in this research exploits the linearity of the network equations with regard to MW flows. As such the method is appropriate wherever the application of a dc load flow is considered adequate. This is because it makes the same assumptions as the dc load flow regarding the network voltage magnitudes and the transmission angles.

6.1.2 Treatment of Uncertainty

6.1.2.1 Uncertain Elements

On-line predictive security assessment requires estimates to be made of nodal demands, generator schedules and network configurations for several time steps starting from the time that the study is initiated upto 24 hours ahead. The number of timesteps and the time horizon is determined largely by the need for security constrained scheduling.

As each of these future quantities is not known precisely, uncertainty exists. Uncertainty exists in the network configuration due to line outage events and maintenance over-run. Uncertainty exists in the nodal demands because the number and magnitude of consumer loads connected at any future instant is not known. Uncertainty exists in generation schedules due to plant failures and to the need in the event to match the total generation with the total demand.

As system load varies from the predicted load, upon which the plant loading schedule is based, control action is required to bring generation and load into balance. Over short-time intervals this control action is provided by steam plant free governor response; over longer time intervals from a few minutes to a couple of hours, the fuel infeed of some plant is varied to match the demand by dispatch control; and over longer periods, dependent upon the precise shape of the daily load curve, unit commitment revisions are carried out.

Thus the uncertainty in total demand is reflected in the uncertainty of generator schedules through the mechanisms of unit commitment revisions, dispatch activity and frequency control. This research concentrates on the dispatch activity component.

6.1.2.2 Dispatch Activity

Dispatch instructions appear as variations of the MW output of a generator but only within the period that the generator is synchronised and also within its minimum stable generation level and its maximum output level. Dispatch is thus conditional upon the generator being synchronised and available.

The dispatch instructions are the visible result of calculations designed to minimize the overall cost of production.

This cost minimisation process was found to determine the uncertainty charateristic of the generators.

Earlier research on generator uncertainty due to dispatch activity had concentrated on dispatch on equal incremental cost systems. One piece of research had considered the effect of dispatch on generator uncertainty on a merit order system. This did not consider the effect of cost breakpoints in generator cost functions on the uncertainty nor did it consider the effects of changes in marginal generators with demand level. In addition it resorted to Monte Carlo simulation to model the uncertainty. This thesis examined both of these points and developed an analytic method to model the effects.

6.1.2.3 Generator Cost Break Points

The general shape of the cost function for a throttle governed steam generator in common use on the CEGB system has three cost segments with two cost breakpoints. (This excludes the minor discontinuities due to mill quantisation effects). The low end cost segment represents the fuel costs attributable to oil burners in use for flame stabilisation, the mid range cost segment represents the fuel costs on coal firing alone and the high end cost segment represents the fuel costs with oil burners in use to enhance the MW output under wet coal conditions or mill unavailability conditions.

The cost breakpoints are the MW levels at which the fuel mix changes.

Monte Carlo type simulations of dispatch were carried out on a test system with data that was broadly representative of generators on the CEGB system. These studies indicated a high probability of finding the output of a marginal generator at one of its cost breakpoint values. These studies were also borne out by observation of actual generator loading on the CEGB system. The magnitude of the probability mass at each cost breakpoint was found to depend upon the position of the generator cost segment in the merit order, the predicted demand, the uncertainty in the demand and the size of the generator relative to the demand uncertainty. The magnitude of the effect of transmission loss penalty factors on the incremental costs was not examined, nor that of rate constraints and reserve constraints although their effects could be handled approximately by modifying the merit order to account for them. Active network and security constraints also distort the pure merit order. However, in the predictive security assessment application these will be explicitly highlighted and need not be modelled in this context.

The Monte Carlo studies showed significant differences between the expected values (upto 18% (833 MW versus 1019 MW)) and standard deviations (upto 56% (77 MW versus 120 MW)) of the marginal generators for cases where cost breakpoints were simulated and where they were not. These were due to the effect of the probability mass at the cost breakpoints.

6.1.2.4 Demand Level Dependence

An additional effect of merit order dispatching is that the marginal generators change with demand level and hence time of day. As a result the uncertainty due to dispatch, for a particular generator, will vary with time of day. In the studies in this research, using representative data, the standard deviation was found to increase from 80 MW to 340 MW for one marginal generator even though the expected value decreased from 1670 MW to 1290 MW in the same period. This was due to the change in its position in the merit order relative to the demand. It indicates that generator uncertainties cannot be allocated based purely on generator size. This significant variation in generator uncertainty is reflected in line flow uncertainty and should therefore be modelled in predictive security assessment.

6.1.2.5 Effect of dispatch activity on line flow uncertainties

Monte Carlo simulation studies of the effects on line flow uncertainty of various ways of modelling dispatch activity showed that these were significant for the generator sizes found on large systems and the degree of uncertainty experienced in lead times upto 24 hours ahead.

Strategies which put all the uncertainty on the slack generator tended to overestimate the variance on lines sensitive to the slack and underestimate it for lines sensitive to the true marginal generation. The strategy of spreading the uncertainty over all generators tended to underestimate the variance on lines sensitive to the true marginal generation.

A strategy in which the uncertainty is dispatched to the marginal generators in merit order, but where the merit order has been revised to represent the average incremental cost of each generator cost function, produced differences of upto 8% (907 MW versus 984 MW) in the expected value and of upto 21% (169 MW versus 205 MW) in the standard deviations of line flows for lines sensitive to the marginal generators. This was due to the considerable change in marginal generator variance produced by revising the position in the merit order of one its segments.

An accurate uncertainty model of the high-end cost segment is particularly significant from the predictive security assessment viewpoint. This is because it is the output in this cost segment that will be instructed when the system demand has been underestimated and it is the effect of this on postfault flows that needs to be assessed.

6.1.2.6 Analytic Model

The Monte Carlo approach to generator uncertainty, resulting from merit order dispatch, used in an earlier piece of research was not considered appropriate for on-line application. An analytic method was developed.

(a) Generator Model

The basic approach used was to compute the joint probability density function of each generator segment pair; then to compute the expected value for each generator segment; and finally to compute the covariance of each segment and the crosscovariance of each pair.

Exact joint probability density functions could be computed using this method, however, a considerable speed improvement and ease of programming was achieved by noting that most of the probability mass for each segment exists at its extremes. Consequently the small probability mass for interim MW levels can be divided and lumped at the extremes.

The error that this approximation produced in the covariance results was then compensated by scaling each generator covariance term by the ratio of the total demand variance and the sum of the generator covariances. This avoided the error that could occur if this approximation were applied to a system where one generator is capable of meeting the entire demand uncertainty.

(b) Transformation to line flows

Having determined the statistics of the marginal generation it was necessary to determine the resulting line flow statistics. Existing approaches using standard probability theory provided the relationship for a dc load flow model. However, these were lacking in any treatment of how to obtain the various elements in the covariance matrices.

The cross covariance components between generators and the shift in their expected values, due to merit order dispatch, were provided by the analytic generator model. To complete the model the cross covariances between marginal generators and nodal demands were also required.

An exact computation of this would require the calculation of the joint probability density function of each of these variables. In this research it was found that a satisfactory result was achieved by initially assuming linear correlation between marginal generator uncertainty and total demand uncertainty to compute the cross covariance between marginal generators and nodal demands. The error introduced by this approximation was subsequently compensated by observing that the sum of the covariances in each row of the matrix of generator/nodal demand covariances is the covariance between that marginal generator and total demand uncertainty. Hence they should equal the sum of the covariances of the same row of the generator cross covariance matrix. Therefore the error was compensated by scaling each generator/nodal demand row term by the ratio of these sums.

The accuracy of the analytic model was assessed by comparing its results with those of a Monte Carlo simulation. The worst difference found was a standard deviation of 217 MW versus 205 MW on an expected flow of 1003 MW versus 984 MW . This was on a line directly connected to a marginal generator and the error is due to the residual error from the approximation of lumping all intermediate probability mass for that generator at its cost breakpoints. This degree of error on flows of this magnitude was not considered significant for the predictive security assessment application.

6.2 Original Contributions

This thesis offers four original contributions to the understanding of predictive security assessment in on-line power system operations. These are:-

(i) The finding that line flow trajectory information (including its uncertainty) can be effectively used in conjunction with set theory ideas to identify critical MW overload contingencies for the daily trajectory.

(ii) The development of this finding to produce the Probabilistic Worst Case algorithm for critical MW overload contingency selection.

(iii) The findings that generator cost function breakpoints on a merit order
system can significantly affect marginal generator MW uncertainties and that the degree of uncertainty will vary with time of day.

(iv) The development of an analytical method to model these findings for a merit order system.

6.3 Recommendations for Use

The techniques for predictive security assessment developed in this research have application to a number of practical problems in the management of security in the on-line environment. These include critical transmission security constraint selection for dispatching and transmission security constraint margins for unit commitment.

6.3.1 Transmission Security Constraint Selection

This thesis has demonstrated the capacity of predictive security assessment to reduce dramatically the number of transmission security constraints that need to be considered in unit commitment or dispatch.

This is a general characteristic of power system security in the on-line environment. However, in order to take advantage of this, additional criteria need to be specified to the dispatch or unit commitment to indicate when the active constraint set needs to be augmented. The set theoretic approach provides such criteria in the form of daily maximum and minimum expected line flows.

6.3.2 Transmission Security Constraint Margins

This is the problem of how to ensure that sufficient, but not excessive, generation is available in each importing area of a network to meet the transmission import constraints when the flow into them is uncertain. On a merit order type system this has to be carried out in the unit commitment phase as the economics of the operation of such generators could lead to the situation in which all the synchronised generators in an importing area are at their maximum and all increases in load in that area are met by increasing the flow into the area. If such flows exceed the post-fault flow capacity into the area a line fault could result in the need to disconnect demand in the area unless sufficient post-fault flow control capability is available.

The type of information produced by the predictive security assessment described in this research can help in this decision. This is because it defines which lines are likely to be overloaded and by what outages, together with the uncertainties in the post-fault flows, the duration of the insecure period and the effect of changing demand and generation patterns.

6.4 Scope for Further Work

A part of this research has dealt specifically with a quantitative assessment of the effects of generator cost function breakpoints on the uncertainty of marginal generators due to dispatch activity and the consequent effects on line flow uncertainty. Further research could usefully be carried out to quantify the effects on generator uncertainties of transmission loss penalty factors, rate constraints and reserve constraints due to dispatch, the effects of unit commitment covering the whole day, the effects of advancing or delaying synchronising and desynchronising times and the effects of frequency control. Research could also be directed to quantifying the effect on steam generator uncertainties of pump storage generators where the utilisation of these is often constrained by energy limitations.

References

1. B E Murray, W S Farrant
'The role of on-line security assessment in operation'
Paper 103-04, IFAC/CIGRE Symposium on Control Applications for Power System Security, Florence, September 1983.

2. W S Farrant, B J Cory and B E Murray
'Security Management in the on-line environment'
IEE Second International Conference on Power System Monitoring and Control, Durham, July 1986.

3. A J Walker
'The Monetary Values that Consumers place on an Interruption of Electricity Supply'.
Electricity Council Engineering Department Report No. D1/SD/P8, August 1981.

4. C.E.G.B. Operations Department
'C.E.G.B. Supergrid Fault Statistics - Annual Report, 1978/9'
C.E.G.B. Operations Department Report No. 0(OS)87, March 1980.

5. C.E.G.B. Operations Department
'Operational Standards of Security of Supply'
C.E.G.B. Operations Memorandum No. 3, Issue No. 2, July 1978.

6. A D Patton
'Short-term Reliability Calculation'
IEEE PAS, Vol. 89, No. 4, April 1970

7. A S Householder
'The Theory of Matrices in Numerical Analysis'
(Book) Blaisdell, New York.

8. E Hnyilicza, S T Y Lee, F C Schweppe
'Steady-State Security Regions: Set-Theoretic Approach'
Proceedings of PICA, pp. 347-355, New Orleans 1975.

9. R Fischl, G C Ejebe, J A DeMaio
'The Identification of Power System Steady-State Security Regions Under Load Uncertainty'
Paper No. A76 495-2, IEEE PES Summer Meeting, Portland, Oregon, July 1976.

10. G L Landgre, S W Anderson
'Simultaneous Power Interchange Capability Analysis'
Paper No. T73 075-9, IEEE PES Winter Meeting, New York, January 1973.

11. M H Banakar, F D Galiana
'Power System Security Corridors Concept and Computation'
Paper No. 81 SM 344-1, IEEE PES Summer Meeting, Portland, Oregon, July 1981.

12. M K Enns, J J Quada, B Sackett
'Fast Linear Contingency Analysis'
Paper No. 81 SM 406-8, IEEE PES Summer Meeting, Portland, Oregon, July 1981.

13. O I Elgerd

'Electric Energy Systems Theory: An Introduction'

(Book) McGraw-Hill, 1971.

14. U G Knight

'Power Systems Engineering and Mathematics'

(Book) Pergamon Press, 1972.

15. G D Irisarri, A M Sasson

'An automatic Contingency selection method for on-line security analysis' IEEE PAS, Vol. 100, No. 4, April 1981.

16. R Wasley and M Daneshdoost

'Identification and Ranking of Critical Contingencies in dependent variable space'

IEEE PAS, Vol. PAS-102, No. 4, April 1983.

17. T A Mikolinna, B F Wollenberg

'An Advanced Contingency Selection Algorithm'

Paper No. 80 SM 592-6, IEEE PES Summer Meeting, Minneapolis, July 1980.

18. Y H Pao et al

'Feasibility of Using Associative Memories for Static Security Assessment of Power System Overloads'

EPRI FInal Report EL-2343, April 1982.

19. C J Myerscough
'Unpredictable Fluctuations in System Loading and Frequency'
C.E.G.B. Report No. PL-ST/38/75, October 1975.

20. W D Laing
'The Load Dispatch Project - an interim report on the demand prediction facility'
C.E.G.B. Research Memorandum No. TPRD/L/CI 267/M83, October 1983.

21. M J H Sterling
'Power System Control'
(Book) Peter Peregrinus Limited, 1978.

22. K W Edwin, D Konig
'The effect of unit commitment in stochastic and probabilistic load flow' Proceedings of the 7th PSCC, Lausanne, July 1981.

23. G J Anders
'Modelling operator action to balance the system in probabilistic load-flow computations'
Electrical Power and Energy Systems, Vol. 4, No. 3, July 1982.

24. A M Leite da Silva, V L Arienti, R N Allan 'Probabilistic Load Flow Considering Dependence Between Input Nodal Powers'
IEEE PAS Vol. 103, No. 6, June 1984.

25. F Aboytes, B J Cory
'An Alternative Formulation of the Stochastic Load Flow Method'
Proceedings of PICA, 1975

26. A Papoulis
'Probability, Random Variables and Stochastic Processes'
(Book) McGraw-Hill, 1965

27. J F Dopazo, O A Klitin, A M Sasson
'Stochastic Load Flows'
'IEEE PAS, Vol. 94, No. 2, March/April 1975.

28. B Borkowska
'Probabilistic Load Flow'
Paper T73 485-0, IEEE PES Summer Meeting, Vancouver, July 1973.

29. R N Allan, C H Grigg, M R G Al-Shakarchi
'Numerical Techniques in Probabilistic Load Flow Problems'
International Journal for Numerical Methods in Engineering,
Vol. 10, 1976.

30. R B Prada, B J Cory
'Stochastic Security Analysis'
14th U.P.E.C., Loughborough, 1979

31. P W Sauer, G T Heydt
'A Generalized Stochastic Power Flow Algorithm'
Paper No. A 78 544-9, IEEE PES Summer Meeting, Los Angeles,
July 1978.

32. M Abramowitz, I A Stegun

'Handbook of Mathematical Functions' p.933 formula 26.2.23

(Book) Dover, 1964

33. R N Allan, M R G Al-Shakarchi

'Linear dependence between nodal powers in probabilistic a.c. load flow' Proceedings IEE, Vol. 124, No. 6, June 1977.

34. K R Jarrett and A R Pearson

'Bunching of supergrid overhead line faults caused by adverse environmental conditions'

CEGB Report No. PL-SP/3/80, March 1981.

35. F D Galiana

'Bound estimates of the severity of line outages in power system contingency analysis and ranking'

Paper No. 84 WM 043-6 IEEE PES Winter Meeting, Dallas, February 1984.

36. Pottle et al

'Technology Assessment Study of near term computer capabilities and their impact on power flow and stability simulation programs'

EPRI EL-946, 1978.

37. R N Allan, B Borkowska and C H Grigg

'Probabilistic analysis of power flows'

Proc. IEE, Vol. 121, No. 12, December 1974.

38. R N Allan, A M Leite da Silva and R C Burchett
'Evaluation methods and accuracy in probabilistic load flow solutions'
IEEE PAS, Vol. PAS-100, No. 5, May 1981.

39. R N Allan and A M Leite da Silva
'Probabilistic load flow using multilinearisations'
IEE Proc., Vol. 128, Part C, No. 5, September 1981.

40. R N Allan, C H Grigg, D A Newey and R F Simmons
'Probabilstic power-flow techniques extended and applied to operational decision making'
Proc. IEE, Vol. 123, No. 12, December 1976.

41. R N Allan, C H Grigg and J A Prato-Garcia
'Effect of Network Outages in Probabilistic Load Flow Analysis'
Paper No. A79 032-4, IEEE PES Winter Meeting, New York, February 1979.

42. A M Leite da Silva, R N Allan, S M Soares and V L Arienti
'Probabilistic load flow considering network outages'
IEE Proceedings, Vol. 132, Part C, No. 3, May 1985

43. F Aboytes
'Stochastic Contingency Analysis'
IEEE PAS, Vol. PAS-97, No. 2, March/April 1978

44. D Sjelvgren

'Application of Mathematical Programming to Electric Power System Expansion Planning'

PhD thesis, Dept. of Electric Power Systems Engineering, The Royal Institute of Technology, Stockholm 1976.

45. M Brucoli, F Torelli and R Napoli

'Quadratic probabilistic load flow with linearly modelled dispatch'

International Journal of Electrical Power and Energy Systems, Vol. 7, No. 3, July 1985.

46. A P Meliopoulos, A G Bakirtzis and R Kovacs

'Power System Reliability evaluation using stochastic load flows'

IEEE PAS, Vol. PAS-103, No. 5, May 1984.

47. G L Viviani and G T Heydt

'Stochastic Optimal Energy Dispatch'

IEEE PAS, Vol. PAS-100, No. 7, July 1981.

48. A D Patton

'A Probability method for bulk power system security assessment - Basic Concepts'

Proceedings of PICA, Boston, May 1971.

49. T E Dy Liacco

'The Adaptive Reliability Control System'

Paper No. 31 TP 66-524, IEEE PES Summer Meeting, New Orleans, June 1966.

50. T E Dy Liacco

'Real Time Computer Control of Power Systems'

Proc. of the IEEE, Vol. 62, No. 7, July 1974.

51. J Bubenko, E Paulsson, D Sjelvgren

'Security Assessment for Power System Operation'

COPOS, Sao Paulo, Brasil, August 1975.

52. A K E Ababneh and B Fox

'Economic Dispatch with Probabilistic Transmission Security Constraints' 21st UPEC, Imperial College, April 1986.

53. S R Browning, P H Plumptre, H R W Lewis and P G Lucas

'Development of the generator ordering and loading program (GOAL)'

20th UPEC, Huddersfield Polytechnic, April 1985.

54. F F Wu

'A unified approach to security assessment'

Paper 102-02, IFAC/CIGRE Symposium on Control Applications for Power System Security, Florence, September 1983.

55. A Monticelli, M V Pereira and S Granville

'Security Constrained Optimal Power Flow with post-contingency corrective rescheduling'.

Paper No. 86 WM 085-5, IEEE PES Winter Meeting, New York, February 1986.