

COLLAPSE OF DAMAGED RING STIFFENED CYLINDERS

by

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To my Parents

ABSTRACT

The damage caused to the ring stiffened legs of offshore platforms due to the berthing impact of a vessel is addressed in this dissertation. Towards this end experimental and numerical investigations are carried out aimed at quantifying the effects of the damage and developing a method for residual strength assessment of the damaged leg.

Experiments are performed on nine multi-bay small-scale ring stiffened cylinders. The models are first subjected to denting through a knife edge indenter representing an idealised side-edge impact of a ship. The models are then tested under axial compression in order to examine the effect of the dent.

Numerical modelling by finite element analyses of both denting and axial compression tests are presented. For the denting process a gap/contact element is developed to model the gradual contact and load transfer from the indenter to the shell. The proposed analysis for the denting process can predict the post-damage state of deformations and stresses which provide an initial condition for the axial compression analysis.

The choice of suitable models is discussed for the analyses and emphasis given to the most important modelling considerations, namely mesh representation and boundary conditions.

Numerical analyses of all damaged experimental models are performed and results correlated with the experiments. Analyses of each intact model are performed to obtain the original strength. By comparing these values with the damaged strengths, estimates of the damage-induced strength reduction are obtained. The effect of residual stresses due to denting is also examined.

Based upon the observations from the experimental testing and relevant numerical analyses, parameters of potential interest are identified and examined further. The numerical models are employed in a study of shells of slender geometry outside the scope of the testing. Various dent levels are examined as well as the possible interaction of the dent-form with the critical failure modes and initial imperfection shapes.

In the light of the results of this study recommendations and guidance on damage assessment is provided.

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NOTATION

A	Fourier coefficient
B	Fourier coefficient
d	Dent depth
D	Diameter
D'	Stress-strain matrix
E	Young's modulus
G	Strain displacement matrix
H	Depth of stiffener
K	Stiffness matrix
k	Local element axis
ℓ	Panel length
L	Total length
m	Number of longitudinal half-waves
n	Number of circumferential waves
NX	Updated coordinate in X-direction
P	Transformation matrix
r	Local element axis
R	Radius
s	Local element axis
S	Surface over which spring is operative
t	Shell thickness
T	Thickness of gap/contact element
TOL	Tolerance for closed gap
w	Imperfection amplitude
X	Global axis
Y	Global axis
Z	Global axis
δ	Displacement matrix
$\Delta R+$	Maximum outward imperfection
$\Delta R-$	Maximum inward imperfection
θ	Angle subtended by half-wave
σ	Stress
σ_d	Collapse stress of dented shell
σ_i	Collapse stress of intact shell
σ_{ip}	Collapse stress of perfect intact shell
σ_o	Gap stress
σ_u	Ultimate stress

σ_y Static compressive yield stress
 σ_{yd} Dynamic tensile yield stress
 σ_{yt} Static tensile yield stress

Subscripts

cr Critical
d Dent inward deflections
do Dent outward deflections
f Flange
i Indentor node
j Shell node
r Local element axis
s Local element axis
k Local element axis
w Web

Abbreviations

C Circumferential
D Dented shell
FE Finite elements
I Intact shell
L Longitudinal
UD Uniform displacements
US Uniform stress
UDR Uniform displacements with rotation

CONTENTS

ABSTRACT	3
ACKNOWLEDGEMENTS	4
NOTATION	5
CONTENTS	7

CHAPTER 1 INTRODUCTION

1.1 Introduction	12
1.2 Aims and Scope of Thesis	13
1.3 Organisation of Thesis	14

CHAPTER 2 LITERATURE REVIEW

2.1 Introduction	16
2.2 Structural Damage in Ship/Platform Collision	17
2.2.1 Risk and Probability of Collision	17
2.2.2 Collision Damage in Ships	18
2.2.3 Collision Damage in Platforms	19
2.2.4 Design Against Collision	20
2.2.5 Lateral Load on Tubular Bracing Members	21
2.2.5.1 Experimental Investigations	22
2.2.5.2 Theoretical Models for Local Denting	24
2.2.5.3 Theoretical Models for Overall Bending	26
2.2.6 Residual Strength of Damaged Tubulars	28
2.2.6.1 Experimental Investigations	29
2.2.6.2 Theoretical Analyses	30
2.2.7 Damage of Ring Stiffened Cylinders	32
2.2.8 Damage of Stringer and Orthogonally Stiffened Cylinders	34
2.2.9 Discussion	35
2.3 Intact Ring Stiffened Cylinders for Offshore Construction	36
2.3.1 Characteristics of Behaviour	36
2.3.1.1 Initial Imperfections	36
2.3.1.2 Residual Stresses	37
2.3.1.3 Failure Modes	37
2.3.2 Experimental Investigations	38
2.3.3 Theoretical Analyses	39

2.3.4 Design Methods	40
2.3.5 Discussion	40
2.4 Relevant Applications of the Finite Element Analysis Method	42
2.4.1 Shell Buckling by Finite Elements	42
2.4.1.1 Shell Finite Element Formulations	42
2.4.1.2 Programs with Shell Buckling Analysis Capabilities	43
2.4.1.3 Shell Buckling Analyses	43
2.4.2 Contact Problems by Finite Elements	45
2.4.3 Discussion	46
2.5 Conclusions	47

CHAPTER 3

EXPERIMENTAL TESTING

3.1 Introduction	48
3.2 Experimental Testing of Series 1 Models	48
3.2.1 Model Geometries	48
3.2.2 Model Fabrication	49
3.2.3 Strain Gauging	50
3.2.4 Material Testing	51
3.2.5 Experimental Procedure	51
3.2.6 Initial Imperfection Measurements	52
3.2.7 Denting Tests	52
3.2.7.1 Test Rig	53
3.2.7.2 Choice of Indentor	53
3.2.7.3 Transducer Arrangement	54
3.2.7.4 Denting Procedure	55
3.2.8 Axial Compression Tests	56
3.2.8.1 Test Rig	56
3.2.8.2 Axial Testing Procedure	56
3.2.8.3 Deflection Measurements	57
3.3 Experimental Results of Series 1 Models	57
3.3.1 Denting Tests	57
3.3.1.1 Load Deflection Response	57
3.3.1.2 Stress Distribution	59
3.3.1.3 Residual Stresses	60
3.3.1.4 Energy Input	61
3.3.2 Initial Imperfections	61
3.3.3 Axial Compression Tests	62
3.3.3.1 Load-Axial Deflection Response	62

3.3.3.2 Stress Distribution	62
3.3.3.3 Failure Modes	64
3.3.4 Discussion	65
3.4 Experimental Testing of Series 2 Models	66
3.4.1 Model Geometries	66
3.4.2 Model Fabrication	67
3.4.3 Strain Gauging	67
3.4.4 Material Testing	67
3.4.5 Experimental Procedure	68
3.4.6 Initial Imperfection Measurements	68
3.4.7 Denting Tests	68
3.4.8 Axial Compression Tests	68
3.5 Experimental Results of Series 2 Models	69
3.5.1 Denting Tests	69
3.5.1.1 Load Deflection Response	69
3.5.1.2 Stress Distribution	69
3.5.1.3 Residual Stresses	70
3.5.1.4 Energy Input	70
3.5.2 Initial Imperfections	71
3.5.3 Axial Compression Tests	71
3.5.3.1 Load-Axial Deflection Response	71
3.5.3.2 Stress Distribution	71
3.5.3.3 Failure Modes	72
3.5.4 Discussion	73
3.6 Conclusions	74

CHAPTER 4

NUMERICAL MODELLING OF DAMAGE TO RING STIFFENED CYLINDERS

4.1 Introduction	187
4.2 A New Gap/Contact Element	188
4.2.1 Element Matrices	188
4.2.2 Gap/Contact Element Constitutive Model	191
4.2.3 Computer Implementation	192
4.3 Finite Element Program FINAS	193
4.4 Finite Element Model for the Denting Process	194
4.4.1 Finite Element Mesh	194
4.4.2 Description of Elements Used	195
4.4.3 Loading and Boundary Conditions	195
4.4.4 Solution Time and Convergence	196

4.5	Correlation of Experimental and Numerical Results	197
4.5.1	Load Dent Deflection Curves	197
4.5.2	Overall Deflection Response	197
4.5.3	Stress Distributions at Peak Load	198
4.5.4	Stress Distributions in the Residual State	198
4.6	Further Numerical Results	199
4.7	Discussion and Conclusions	199

CHAPTER 5

NUMERICAL ANALYSES OF AXIAL COMPRESSION TESTS

5.1	Introduction	209
5.2	Development of Finite Element Model	210
5.2.1	Circumferential and Longitudinal Extent of the Model	210
5.2.2	Loading and Boundary Conditions	211
5.2.3	Mesh Study	212
5.2.4	Final Choice of Finite Element Model	213
5.3	Initial Imperfections	214
5.4	Axial Compression Analyses of Intact Models	214
5.4.1	Collapse Load Predictions	214
5.4.2	Failure Modes	215
5.5	Axial Compression Analyses of Dented Models	216
5.5.1	Load Deflection Response	216
5.5.2	Stress Distributions	217
5.5.3	Failure Modes	218
5.6	Analyses Including Different Boundary Conditions	220
5.6.1	Axial Shortening	220
5.6.2	Collapse Load Predictions	220
5.6.3	Stress Distributions and Failure Modes	221
5.6.4	Discussion of Results	222
5.7	Effect of Residual Stresses	222
5.8	Damage-Induced Strength Reductions	223
5.9	Discussion and Conclusions	225

CHAPTER 6

THE EFFECT OF DIFFERENT PARAMETERS ON THE STRENGTH OF SLENDER RING STIFFENED CYLINDERS

6.1	Introduction	262

6.2	Study on Slender Geometries	263
6.2.1	Geometries Analysed	263
6.2.2	Dent Levels	263
6.2.3	Initial Imperfections	263
6.2.4	Models Employed in the Analyses	264
6.3	Results of Analyses-Shells with R/t of 300	265
6.3.1	Dent Deflections	265
6.3.2	Initially Perfect Shells	265
6.3.2.1	Load Deflection Paths	265
6.3.2.2	Stress Distribution and Failure Modes	266
6.3.2.3	Residual Strength	267
6.3.3	Shells with Axisymmetric Initial Imperfections	268
6.3.3.1	Load Deflection Paths	268
6.3.3.2	Stress Distribution and Failure Modes	268
6.3.3.3	Residual Strength	269
6.3.4	Shells with Periodic Initial Imperfections	269
6.3.4.1	Load Deflection Paths	269
6.3.4.2	Stress Distribution and Failure Modes	270
6.3.4.3	Residual Strength	271
6.4	Results of Analyses-Shells with R/t of 500	271
6.4.1	Dent Deflections	271
6.4.2	Initially Perfect Shells	272
6.4.3	Shells with Axisymmetric Initial Imperfections	273
6.4.4	Shells with Periodic Initial Imperfections	274
6.5	Discussion and Conclusions	274

CHAPTER 7

CONCLUSIONS

7.1	General Conclusions	328
7.1.1	Experiments	328
7.1.2	Numerical Studies	329
7.2	Suggestions for Further Work	331
	REFERENCES	333

CHAPTER 1

INTRODUCTION

1.1 INTRODUCTION

In recent years there has been an increasing need for exploitation of remotely located sources of energy. This has been brought about by the fact that more easily attainable sources are being used up. To this end attention has been focussed on oil-fields located offshore. Advances in offshore technology have enabled oil to be extracted from beneath the seabed. This is effected by constructing platforms over the fields to carry the necessary instrumentation and machinery required for controlling and performing the oil extraction operations.

In shallow waters platforms are fixed on the seabed and are constructed using either steel or concrete. As operations move towards deeper waters, floating production platforms become a more economic proposition, and this is where lightweight steel platforms become attractive.

One of the problems associated with offshore operations is the possibility of structural damage caused to the platform by a ship impact. The likelihood of such an event is becoming more significant with the increasing number of platforms in areas like the North Sea. Many of these platforms lie in major shipping lanes and there is also an increasing number of associated supply vessels. One of the most likely collision events is a supply ship operating in adverse weather conditions impacting on the platform and causing damage in the form of local denting. The environment of the North Sea is particularly hostile for several months of the year. Given the cost of human life and the possible vast financial losses which could occur due to such an event, it is of great importance to incorporate impact requirements into the design. Nevertheless, situations may still arise in which collisions cause significant structural damage. In the actual event of a collision accident it is important to be able to

assess the subsequent integrity of the structure. This enables vital economic and safety decisions to be made regarding the suitability of costly repairs or abandonment of the structure.

The conventional steel jacket platforms employed in relatively shallow waters are built of cylindrical hollow members. The main legs of these platforms are relatively slender stiffened shells while the bracing members, which add to the stability of the structure, are thick-walled tubulars. In the new generation of buoyant platforms, where weight minimisation is important, the design comprises stiffened cylindrical components of even more slender geometry. In a ship collision situation a number of these cylindrical members could be damaged, thus affecting the stability of the structure as a whole.

While the problem of impact damage to tubular members has been addressed before, at the time of initiation of this project, there was little information on the behaviour of stiffened cylinders representative of the main legs of the platforms. Given the primary role that the latter assume in the stability of the structures, especially in the buoyant type of construction, their performance under impact deserved particular attention.

To this end the present investigation has examined the behaviour of ring stiffened shells under impact loading and their subsequent axial load carrying capacity. Geometrical configurations examined cover a wide range of practical geometries spanning both fixed and floating platforms. Appropriate numerical models have been formulated and guidance given for damage assessment.

1.2 AIMS OF THESIS

The primary aim of this thesis is to develop a methodology for assessing the residual strengths of ring stiffened shells including damage in the form of a local dent. Towards this end it is necessary to undertake the following activities:

- To review the current state of knowledge in the field of ship impact damage to offshore structures and other relevant areas.
- To design and perform a series of experiments modelling both the damage process and the subsequent response to axial compressive loading.
- To generate controlled experimental data for validation of numerical models of analysis.

- To employ the nonlinear finite element method in developing suitable models for analysing the experiments.
- To develop a numerical model for predicting the response of a shell to denting and obtaining the residual state of deformations and stresses for a given energy input.
- To modify the existing finite element package to enable the modelling of the gradual contact between the indenter and the shell to be made.
- To develop a model for analysing the dented shell under axial compression.
- To examine the sensitivity of the above model to different modelling considerations and propose a suitable modelling technique.
- To analyse all the experimental models and correlate with the experiments.
- To analyse the intact configurations of the experimental models, thus providing a basis for comparison and quantification of the dent effects.
- To identify areas of particular interest and examine them further using the numerical models developed.
- Through the experimental and numerical studies to generate data covering a wide range of parameters.
- To provide guidance for damage assessment.

1.3 ORGANISATION OF THE THESIS

The general subject of structural damage in ship-platform collisions is introduced in Chapter 2. Relevant work performed to date is also reviewed. The general behaviour of ring stiffened cylinders used in offshore construction is covered in the chapter as well as relevant applications of the finite element method.

The small-scale experimental programme is described and presented in Chapter 3. Two series of tests on multi-bay small-scale models subjected first to denting and subsequently to axial compression have been performed.

Chapter 4 presents the development of the numerical model for the denting of ring stiffened cylinders. It includes the development of a new gap-contact element which enables the modelling of the gradual contact between the indenter and the shell. The model is validated by experimental correlations.

The choice of a suitable numerical model for the axial compression loading of the dented shells is discussed in Chapter 5. The model chosen is used to analyse both dented and intact shells with experimental geometries. The results of the dented model analyses are compared with the experimental data. Comparisons between dented model responses and those of the corresponding intact models are presented which identify and quantify the effects of the dent. Analyses of the experimental models assuming different boundary conditions are presented and discussed. The effects of denting residual stresses are also considered.

In Chapter 6, the proposed numerical models are employed in a systematic study of parameters of potential interest outside the scope of the tests. The study concentrates on shells of more slender geometries and examines the interaction between different levels of denting, critical buckling modes and general initial imperfections. Results are compared with a simple analytical procedure. Based on the results of this study specific recommendations and guidance on residual strength assessment are given.

Chapter 7 summarises the main conclusions of the thesis and suggestions are made for extending the work.

CHAPTER 2

LITERATURE REVIEW

2.1 INTRODUCTION

The problem of damage due to ship collision with offshore platforms is introduced in the first part of this chapter. This problem, which has not been addressed until recent years, follows previous work on ship to ship collision and therefore some reference to this work is also made. The risk and probability of collision is briefly covered and the methods for assessing the overall response of platform and ship involved in a collision event are presented. Research carried out on the behaviour of single elements used in offshore construction, namely tubulars and stiffened cylinders, is presented in more detail. Throughout this review it is clear that although several researchers have addressed the behaviour of tubulars under lateral load and their subsequent load carrying capacity, very little information exists on stiffened cylinders, and especially ring stiffened cylinders, which is the theme of this dissertation.

In the second part of this chapter, the behaviour of intact ring stiffened cylinders is reviewed. This information is directly relevant to this thesis and essential in understanding and quantifying the effects of damage on these cylinders. Reference to this work illustrates some important characteristics of their behaviour and sheds some light on the treatment of the problem under investigation.

In the final part relevant applications of the finite element method, which is the numerical tool used in this dissertation, are presented. In particular applications in shell buckling are discussed as well as the solution of contact problems by finite elements.

2.2 STRUCTURAL DAMAGE IN SHIP/PLATFORM COLLISION

2.2.1 Risk and Probability of Collision

From the point of view of design, in the area of establishing code safety factors and general guidelines for overall design philosophy, a measure of the probability of collision is highly desirable.

The probability of occurrence is affected by several factors: traffic monitoring; navigational aids; field layout; platform topology; design of fendering and other defence systems; size and manoeuvrability of vessels; mooring equipment, etc.

Reference [1] examined the feasibility of predicting ship/platform encounters in the North Sea by using readily available information on shipping movements, recorded incidents and environmental data.

Three groups of ships were identified that might collide with the platforms

- supply boats making approved visits
- vessels waiting nearby
- vessels on passage.

The risks of collision associated with each of the above groups and structures located in different fields were presented. For platforms the risks ranged between 0.05 – 0.15 while for offshore rigs higher values were obtained in the range of 0.32 – 0.37 events per year. It was clear from the results that the colliding vessel is most likely to be small (less than 1000 tons) due to the relatively large numbers of these vessels.

Other attempts at evaluating the probability of collision were reported in [2, 3]. The method outlined in [2] made use of simulation techniques. A total risk assessment was not feasible for the time being due to the scarcity of data concerning collision events. Reference [3] presented the interactive ship sailing simulator, SAILSIM, which had been developed for studying the manoeuvrability of ships and floating structures under the influence of waves, wind, current and limited depth. The program was well suited for studying manoeuvrability of ships in restricted water, or close to offshore loading terminals under different environmental conditions.

A mathematical risk model was used in [4], using input data available from survey information on shipping lane locations, shipping traffic and lane width. Results of this study show that for platforms close to major shipping routes, the risk level was estimated to be of the order of 0.1 to 0.01 per year. This represented a high level of risk, well above most acceptable criteria.

2.2.2 Collision Damage in Ships

Several researchers have investigated the problem of ship collision damage. Some of the more significant work in this field is summarised here. The first attempt towards assessment of ship collision damage was made by Minorsky [5] who developed an empirical relationship between the volume of damaged material and the amount of energy absorbed based on data from real collisions. It was assumed that the energy was absorbed plastically and a linear relationship was found to exist between energy absorbed, E_a , and a resistance factor R_T , which represented the volume of damaged material in the ship.

$$E_a = C_1 R_T + C_2$$

The coefficients C_1 and C_2 are constants. It was found that the method did not represent the low energy region very well. However the method is still widely used, because of its ease of application.

The membrane resistance of the shell plating of the ship, which was not considered in the above analysis, can be a significant factor. An approximate method to account for this effect was proposed in reference [6]. Woisin [7] also modified Minorsky's method to account for membrane effects.

For low energy collisions, Rosenblatt and Son Inc. [8, 9] proposed an approximate damage theory for a tanker. A similar approach was also used by Hysing [10]. The procedure adopted in this method involves the following steps:

- assuming a collapse model
- calculating the load for different indentations
- checking the reaction forces in the neighbouring transverse frames against their buckling load
- and extending the collapse model accordingly.

Amdahl [11] treated the problem of ship resistance to collapse load by introducing conceptual cuts to the bow-section, thus reducing it to an assembly of basic elements. The contributions to the internal energy dissipation from each element were added up and equated to the work done by the external load over specified distances. Analysis methods proposed for estimating the energy absorption of individual elements found in ship construction were reviewed by Ronalds in reference [12].

Valsgard and Petersen presented a method [13] capable of identifying the damage to the striking bow and the struck ship side and predicting the collision force and

energy absorbed by the two ships. The stiffness of the striking bulbous bow was represented as a set of nonlinear springs evaluated using a semi-empirical approach and the struck ship side was simulated by simplified nonlinear finite element procedures. A critical strain level for side shell rupture was also included. Critical speed values were estimated and related to the stiffness of the bow.

2.2.3 Collision Damage in Platforms

The kinetic energy of the ship striking a platform is determined by:

$$E = \frac{1}{2}(m + a)V^2$$

where V is the velocity of the ship just before impact, m is the mass and a is the hydrodynamic added mass. The values most frequently used for a are:

$$\begin{aligned} a &= 0.1m && \text{for head-on collision} \\ a &= 0.4m && \text{for sideways collision.} \end{aligned}$$

The impact energy E_k can be dissipated in the following modes:

- elastic vibration of the structures involved
- plastic deformation of the ship, structural member of the platform and fendering system if present.

The proportion of energy dissipated in the form of elastic vibrations is not significant, as indicated in [14, 15, 1]. In cases of ships striking small steel jacket structures it is possible that a considerable part of the kinetic energy can be absorbed in the elastic mode [16]. For more severe collisions, however, this form of energy dissipation is insignificant. It is therefore generally assumed that the kinetic energy is fully absorbed in plastic deformations of the two structures involved and the fendering system.

Several procedures have been proposed for evaluating the energy that a ship can absorb during impact, as already discussed in previous sections. Some methods proposed for estimation of the collision resistance of platforms are discussed hereafter.

Petersen and Johnsen have dealt with the collision resistance of mobile offshore units. A simplified nonlinear method was proposed in reference [17] based on the following procedure:

- (a) By means of momentum and energy considerations, calculate the energy which might be absorbed during the collision.

- (b) Apply a collision load to the structures and calculate by means of nonlinear methods the resulting damage.
- (c) Calculate the energy absorbed by each of the elements where damage has occurred according to the damage picture obtained in (b).
- (d) Add energy contributions together and compare with available energy in (a). If more energy must be absorbed, increase the load and repeat steps (b), (c) and (d).

Step (b) posed the greatest problem and the following method of analysis was proposed for it. The elements were represented by simplified nonlinear stiffnesses within a space frame computer program. The solution took the form of an iterative procedure where the stress-strain state of each element was checked for each load increment and the stiffness was replaced if the element had passed beyond the linear limit. The effects of local denting of elements, development of plastic hinges, membrane tension yielding and buckling of bracings were all included in the analyses. Examples of jack-up and semi-submersible structures were presented. The method is useful for deciding if a structure will suffer major or just minor damage when hit by a ship. The same method was presented in [18] and demonstrated through examples of impact on jack-up legs and cellular structures.

Davies and Mavrides [19] developed a method by which it is possible to define the timewise variation of the interaction force between ship and structure and get a first indication of the way in which both the ship and structure distort as the collision develops.

Furnes and Amdahl [21, 3] have also studied the ship collisions with offshore platforms, examining the different modes of deformation of ship and platform.

2.2.4 Design Against Collision

From the point of view of design the collision event should be considered in terms of both probability and consequence. As already discussed, insufficient information is available to base the design rules explicitly on such an approach. For this reason, the design guidance as specified by the DnV code [21, 22] is based on design collision situations. The platform should be designed to withstand the following two load cases:

- An operational impact load caused by a supply boat when approaching or leaving the structure.

- An accidental impact load caused by a supply boat out of control, drifting against the structure.

The impact load should be calculated for the maximum authorised vessel, or if no restrictions are specified a 5000 tons displacement vessel should be considered. The impact load calculations should be based on energy distribution between boat, structure and possible fendering system. Force indentation curves are recommended for broadside and bow collisions. The impact energy is calculated as

$$E = \frac{1}{2} (m + a) V^2$$

where m is the mass of the ship and a the added mass equal to $0.4m$ for broadside impact and $0.1m$ for bow and stern collision. V is the impact velocity.

For the operational impact, V should be taken as 0.5m/sec and for accidental impact 2m/sec .

2.2.5 Lateral Load on Tubular Bracing Members

In the following section, research carried out into the behaviour of tubulars under lateral load is reviewed. A number of researchers have looked at the problem with a variation in the load application. For the purposes of offshore application the load is mainly applied through a V-shaped indenter with a tip radius representing the side edge of the ship, or a square indenter modelling the collision of a ship with vertical sides.

The geometries for tubular bracing members are of the range [23]:

$$10 \leq R/t \leq 30$$

$$20 \leq L/R \leq 60$$

and mainly work performed in these regions will be discussed here.

When a tubular is subjected to lateral load, the deformations take the form of:

- a local dent
- overall bending of the member

Experiments have been performed examining the two forms of deformation. These have considered the effects both independently and in combination. On the theoretical side, the problem has been treated at different levels of sophistication varying from simple analytical methods to finite element elasto-plastic analyses.

2.2.5.1 Experimental Investigations

Thomas *et al* [24] investigated the energy absorption of tubes under lateral load with a view to designing them as energy absorption devices mainly for motor vehicles. A number of tubes were tested under the action of transverse loading of either one wedge-shaped indenter or two opposed indentors. The tubes were loaded up to the point of maximum load. The slenderness of the tubes examined was generally lower than would be found in offshore construction, nevertheless the work demonstrates some interesting aspects of their behaviour. In the shorter tubes examined, the deformation was mainly inextensional in the form of hoop bending about generators of the tube. For longer tubes membrane stretching predominates close to the indentors, accompanied by axial bending of the generators. The resulting profile of the upper and lower generators of the tubes could be fitted with reasonable accuracy to an exponential curve having a well defined decay rate.

Tooth and Jones [25], with the view to investigating the plastic collapse of cylindrical pressure vessels supported by rigid saddles, tested a number of vessels loaded through both welded and loose saddles. The geometries of the cylinders tested were in the range

$$24 < R/t < 84$$

which is outside the range of offshore tubulars.

Smith and his co-workers have dealt with the specific problem of tubulars for offshore application, and tested a number of models over the last few years [26, 27, 28, 29]. Throughout their work the tubes were not tested to failure. Only moderate indentations were applied. In reference [26] small scale tests were reported of $\frac{1}{4}$ scale where the load was applied through a 60° steel knife edge with about 3mm tip radius or a semicircular cradle. Both full scale and small scale tests were presented in reference [27], where comparisons between the two show that provided care was taken to simulate full scale geometries and material properties small scale testing can be used confidently in experimenting with tubulars.

Further experimental investigations were undertaken later [29] where different dent shapes and locations, as well as various combinations of bending and denting, were examined. Small scale tubulars were used with D/t 's of 26 and 46 and column slenderness ratios mainly in the imperfection sensitive range $0.8 < \lambda < 1.2$ (where $\lambda = L\sqrt{\sigma_y}/(R\pi\sqrt{E})$). The models were subjected to indentations $d/D < .18$ and bending deformations $\delta/L < .002$. Conditions of simple support were simulated at the ends, offering no resistance to rotations and axial displacements. By varying the load reaction on the opposite side by considering cases with and without a cradle the

local indentation mode was enhanced. In the absence of the cradle a combination of both local and overall modes were observed. Dent profiles obtained by different indentors are presented. It was clear from the tests that a square indentor resulted in the development of two plastic hinges across the tube wall rather than the single hinge given by a sharp indentor. Consequently higher load was required for a square dent than for a sharp dent and the former was associated with load/deflection curves of a somewhat different form including some shape-hardening. Formation of a rounded dent resulted in no well-defined plastic hinges and involved loads only slightly higher than those for a sharp dent.

Rashed *et al* [30] reported on the testing of 21 tubes dented through a knife edge indentor of 5mm nose radius with the back of the tube supported to minimise bending deformations. The dent was applied at $\frac{1}{8}$ of the span to differentiate between buckling collapse and plastic collapse at the dent. Moderate dent levels were applied to the tubes with depths in the range of 0.05 D to 0.13 D. The tube ends were simply supported.

Ueda and Rashed [31] tested a series of 18 tubulars using a similar set-up as Rashed *et al* [30]. Dent depths between 0.05 D – 0.1 D were applied at mid-span through a 25mm radius knife edge indentor.

Cho and Frieze [32] also tested tubulars under dynamic lateral load. Twenty-four small scale tubulars were tested with simply supported roller end conditions. Both denting and overall bending were observed in all tests with local denting preceding the overall bending which was then followed by some additional local denting. Elastic flexural vibrations of the impacted models were apparent in most tests.

Load-deformation characteristics of tubulars have also been examined by Soreide and Amdahl [33, 11] who tested a series of small scale models in the following geometrical range:

$$11 < R/t < 31$$

$$20 < L/R < 40$$

The main aims of their investigation were to examine two effects, namely the influence of membrane forces and dynamic loading. Unlike Smith's experiments [26–29], in this series of tests the cylinders were loaded up to failure. The loading was applied through a square indentor.

Two boundary conditions were considered, the first one allowing free horizontal movement and the second applying full membrane restraint. In both cases the ends were rotationally clamped. In the restrained case the effect of the membrane force was demonstrated by a significant difference in load carrying capacity. More overall ovalisation was also observed.

In the case of the free ends, maximum load was reached at a much smaller indentation and this was attributed firstly to the flattening of the central cross-section, which reduced the plastic section modulus, and secondly to local crippling failure occurring on the compression side of the ends, which reduced the moment capacity.

When full membrane restraint was applied, the load increased continuously until fracture of the material close to the weld occurred.

In the dynamic tests the load was applied at a rate of 54mm/sec as opposed to 0.15mm/sec for static tests. The dynamic rate corresponded to 1 – 2m/sec full scale speed. A 10 percent increase in the load was observed in the dynamic response. The dynamic loading primarily affected the local deformation at the point of impact where the strain rates reached high magnitudes and the increase in load carrying capacity was caused by a rise in the material stress-strain curve.

2.2.5.2 Theoretical Models for Local Denting

The problem of local energy absorption of tubulars has been tackled by different researchers, mainly by developing simple upper bound plastic methods of analyses. Some of these models are reviewed here. These can be divided into two categories, namely ring models and indentation models.

Ring models are applicable when a tube is crushed between rigid plates and indentation models more appropriate for load application through knife edge or short indentors. The second category is more relevant to this dissertation and will be reviewed in detail.

De Runtz and Hodge [34] proposed a ring model for analysing the crushing of tubes between rigid plates. Later Redwood [35] recognised the importance of strain hardening in this type of problem and introduced linear strain hardening behaviour for the tube material. Reid and Reddy [36] further advanced the role of strain hardening in this type of analytical model.

An indentation model was introduced by Thomas [24] for the tube behaviour under two opposed wedge indentors. This was a bow-tie deformation pattern which included eight flattened surfaces, eight lengthwise hinges and one transverse plastic hinge. The work done was assumed to be dissipated in flattening the initially curved tube and in rotating the hinges.

Morris and Caladine [37] developed a mechanism for load applied through a rigid boss. The mechanism comprised two hinge polygons parallel to each other. The inner hinge enclosed the boss through which the load was applied. Straight hinges were assumed where there was only rotation involved. An iteration process was used where for each step the shape of the polygons was optimised and then small rotations applied to obtain the new configuration.

Tooth and Jones [25] presented a rectangular shaped mechanism which applies in the case of a saddle supporting a cylinder.

So far, the indentation models mentioned were not developed for offshore application. Nevertheless it is useful to refer to them since the work done for offshore tubulars, which will be discussed hereafter, follows similar approaches.

De Oliveira [38, 39] proposed a wedge shaped mechanism composed of four identical inclined hinge lines, one central hinge, and two flat triangular surfaces. It follows the same principle as reference [24]. While this formulation applies for the case of a wedge shaped indenter which models impact with the deck edge of a vessel, another formulation was presented for the case of impact with the straight side of a vessel. This included a central flat portion between the two triangular faces.

In reference [40] a mechanism consisting of two rounded hinges was used which divided the cylinder in three parts:

- the undeformed part of the cylinder, which is assumed to remain circular,
- the deforming plasticised region bounded by the two hinges
- and the already deformed rigid region inside the inner hinge line.

Isometric transformations were assumed. It was found that the analysis overestimated the load by a factor of three and this was attributed to the fact that ovalisation of the tube wall was not included.

Soreide and Amdahl [33] in their analysis assumed a simple yield line model similar to the one in reference [38] for straight side impact. However, they extended the method by including the tension work due to elongation of the generators.

- An accidental impact load caused by a supply boat out of control, drifting against the structure.

The impact load should be calculated for the maximum authorised vessel, or if no restrictions are specified a 5000 tons displacement vessel should be considered. The impact load calculations should be based on energy distribution between boat, structure and possible fendering system. Force indentation curves are recommended for broadside and bow collisions. The impact energy is calculated as

$$E = \frac{1}{2} (m + a) V^2$$

where m is the mass of the ship and a the added mass equal to $0.4m$ for broadside impact and $0.1m$ for bow and stern collision. V is the impact velocity.

For the operational impact, V should be taken as 0.5m/sec and for accidental impact 2m/sec .

2.2.5 Lateral Load on Tubular Bracing Members

In the following section, research carried out into the behaviour of tubulars under lateral load is reviewed. A number of researchers have looked at the problem with a variation in the load application. For the purposes of offshore application the load is mainly applied through a V-shaped indenter with a tip radius representing the side edge of the ship, or a square indenter modelling the collision of a ship with vertical sides.

The geometries for tubular bracing members are of the range [23]:

$$10 \leq R/t \leq 30$$

$$20 \leq L/R \leq 60$$

and mainly work performed in these regions will be discussed here.

When a tubular is subjected to lateral load, the deformations take the form of:

- a local dent
- overall bending of the member

Experiments have been performed examining the two forms of deformation. These have considered the effects both independently and in combination. On the theoretical side, the problem has been treated at different levels of sophistication varying from simple analytical methods to finite element elasto-plastic analyses.

The two phases of local denting and global bending were treated separately [40] assuming that first local indentation occurs and then global bending predominates as the increase in local deformations reduces. The switch between the two phases is assumed to occur at the deflection for which the two loads are equal. It is a simple approach but tends to overestimate both the maximum load and the deflection at which it occurs. Certain trends were identified from the investigation of different parameters. The load-carrying capacity reduced as L/R increased, and the load carrying capacity increases with R/t . These trends are comparable to rigid plastic beam behaviour.

A simplified collision dynamics model was also presented in [40] involving translational motion only. Various collision scenarios were examined for stiff or soft, bow, side or stern collision with a tubular brace.

Cho and Frieze [32] proposed very simple upper bound empirical formulae in explicit form which were based on the results of their experimental investigations. Only simply supported roller end conditions were considered.

Ellinas and Walker [41] assumed full flexural restraint at the ends which were free to pull in longitudinally. Regarding the two modes of deformation, local and bending, they assumed the following behavioural sequence. Local denting occurs first, which erodes the stiffness and then bending deformation. When the beam is weak enough to be able to bend plastically in an overall mode, the local denting does not increase any further and the remainder of the energy is absorbed by the reduced section tube deforming plastically in an overall bending mode. In this way the reduction in moment capacity due to local denting is accounted for.

Richards and Andronicou [44] adopted Ellinas and Walker's [41] method and extended this to handle the four different combinations of boundary conditions resulting from flexural and rotational freedom or fixity. Two examples of damaged members reported from offshore platforms were used to illustrate the procedure. An interesting feature was observed in the results for tubulars with axially restrained boundary conditions. The loss of load carrying capacity by the hinge mechanism was compensated for by membrane action to give almost constant lateral force with increasing deformation. Failure was deemed to have occurred when membrane forces reached yield.

Smith *et al* [27] used an incremental finite element analysis based on elasto-plastic beam-column theory, to obtain the overall response of the tubular when subjected to lateral load. The column was represented by an assembly of straight beam elements

which in turn were subdivided into element fibres, thus allowing for progressive development of plasticity. By application and removal of lateral load, the damage deformations and residual stresses were obtained. The effects of different end conditions were also examined.

Soreide and Amdahl [33] modified a simple finite element program IMPACT to take care of local effects such as moment capacity reductions due to indentation and local buckling of the tube wall. The deformation model at the point of impact comprised a flat part and the remaining part had a constant radius. It was assumed that the reduced section has the same area as the intact tube. The program system consists of three parts:

- Finite element beam program with elasto-plastic material behaviour and large deflections incorporated.
- A yield line program for predicting local indentation
- A program for calculating reduced moment capacity due to local deformation at point of impact.

The load was applied in increments with equilibrium iterations performed within each load step.

A full shell analysis using the program TUBBUC was also used in [33]. For this analysis a fine mesh was required in the dent region and a coarser mesh outside this area. Strain hardening was accounted for by combining the sublayer technique with a flow rule of plasticity. Geometric nonlinearity was modelled by an updated Lagrangian formulation.

Results of the analysis give a lower load than the rigid-plastic analysis, as expected, and demonstrate the effect of membrane forces for the clamped end conditions. Although a costly method for design, it can be used for the study of special effects, e.g. the stress distribution at the point of contact and local instability phenomena.

2.2.6 Residual Strength of Damaged Tubulars

In the cases of low energy impact, limited structural damage is likely to occur on the platform. The damage occurring from such situations may not have serious consequences at the time of the accident but may affect the safety of the structure in its ability to withstand extreme loads, thus reducing its functional life. It is therefore important to know the ultimate and post-ultimate strengths of structural components with small damage in order to assess the safety of the platform.

In an impact situation the tubular braces are amongst the main components of jacket construction likely to be damaged. Several models for analysing such tubulars, including moderate levels of damage, have been proposed, accompanied by a number of experimental investigations on the subject. A summary of this work is given in the following sections.

2.2.6.1 Experimental Investigations

Several tests on damaged tubulars were reported by Smith [26, 27]. A series of 16 small scale models were tested [26] in order to check the accuracy of the proposed analysis and provide empirical information on the influence of local instability and damage on collapse and post-collapse behaviour. Both full scale and small scale models were presented in reference [27] from which it is clear that slight lateral bending or denting can cause substantial loss of strength. It was also illustrated that small scale models can be used satisfactorily in these tests provided care is taken in the fabrication procedure.

In a later paper [29] another experimental programme was reported which investigated the following effects:

- influence of dent location on damage effects
- influence of dent shape on damage effects
- effect of various combinations of bending and denting on tube performance.

From the results it is clear that while loss of compressive strength depends critically on the amplitude of bending and dent damage, it is insensitive to damage location. Dent shape does not appear to affect the strength significantly either.

The loss of stiffness caused by damage was small in the initial stage of loading, but upon approaching collapse significant loss of stiffness and rounding of peaks occurred.

A series of 21 dented tubulars were tested under axial compression by Rashed *et al* [30]. Conditions of simple support were applied at the ends of the tubes. End shortening as well as lateral deflections at mid-length of the tube and at $\frac{1}{3}$ of the length (the dent location) were recorded during testing.

Similarly, Taby and Moan [45] tested damaged tubulars including combinations of overall bending and local denting. Valsgard and Foss [46] also reported on the testing of damaged tubulars under axial compression while Ueda and Rashed [31] subjected damaged tubulars to pure bending.

2.2.6.2 Theoretical Analyses

Smith *et al* [26–29] showed how elasto–plastic beam–column or frame analyses can be used to provide reasonable accuracy in estimating the residual stiffness and strength of damaged tubulars. This applies only for tubes where local instability does not occur. The method allows for initial deformations, residual stresses, eccentricity of load and any specified boundary conditions. The tube cross–sections are subdivided into elemental areas, or fibres, in order to represent progressive development of plasticity.

Parametric application of such analyses has led to data curves which give an immediate indication of damage effects. It was shown that while loss of compressive strength depends critically on the amplitude of bending deformation, it is insensitive to the location of the damage along the tube. Where residual stresses were introduced in the analysis, it was found that they have negligible influence on loss of strength and this was attributed to the favourable tensile residual stresses in areas mostly stressed in compression under axial load application.

For approximate representation of dents within the framework of beam–column analysis a simple procedure has been proposed [28]. It was assumed that the dent area follows a bilinear stress–strain response of reduced stiffness and maximum load carrying capacity. The two coefficients of reduced stiffness and maximum load were obtained empirically. Through regression analysis of experimental results [29], a regression formula was obtained giving values for these coefficients.

Richards and Andronicou [44], in their large displacement elasto–plastic analysis, assumed a dent area of elliptical profile along the length of the tubular. The material within the dent zone was assumed to have its elastic modulus and yield stress reduced by a coefficient obtained using Smith's [29] empirical relationship. A half sine wave was included to represent manufacturing irregularity and the damage out of straightness was represented as bilinear on either side of the dent. Results were correlated with comparable values by Smith [29] and Ellinas *et al* [41, 47] and good agreement was obtained. A theoretical model for the analysis of a simply supported tubular member including local denting damage (without bending) subjected to axial compression was presented by Rashed *et al* [30]. Elastic–perfectly plastic material behaviour was assumed. In the initial stages of loading, axial compressive stresses were assumed to develop uniformly in the tube wall with bending stresses superimposed in the dent area. Consequently, yielding occurred in the dent area first. The uniform stress δ_{dp} in the tube at which a plastic hinge line forms at the middle of the dent was calculated. After this point, the stiffness of the dented region was reduced significantly. Any increment in load was assumed to be carried

by the intact part of the shell at an appropriate eccentricity. Maximum compressive stress occurred in the fibres adjacent to the dent and the ultimate strength was considered to be reached when this stress reached yield. It was assumed that the axial load remained constant until a full plastic hinge formed at the critical cross-section. Unloading then occurred, which was governed by plastic mechanism theory. This method may be readily extended to account for combined denting and bending damage and combined loading. A computer program, DENTA, was developed to perform the analysis. Comparisons carried out with the results from their extensive experimental programme showed good agreement.

Ellinas and Walker's [41, 47] analysis was based on Rashed's [30] method, which was further extended to include the effect of bending deformation, together with an additional imperfection factor accounting for other structural defects such as residual stresses. A quadratic equation for the ultimate strength was obtained. Results from this analysis, when compared with experimental and theoretical results by Smith [26, 27], show that Ellinas and Walker's theory provides a safe lower bound.

Ueda and Rashed [31] derived an ultimate strength-indentation relationship for the dented cross-section of a tubular member subjected to axial force and bending moments in two orthogonal directions. This relationship was implemented in an idealised structural unit analysis to account for the effect of damaged members. The concept of a dent plastification stress was assumed when the dent was on the compression side of the member, while on the tension side the dented area was assumed to carry load up to yield. Results of this analysis method were compared with their experimental findings on dented tubulars subjected to bending.

Taby and Moan [45] identified from experimental results three areas which warrant some modifications or extensions to the existing theories.

- The strength of the dented portion of the cross-section depends on R/t .
- First yield is a very conservative collapse criterion.
- Reduced plastic strength occurs due to growth of the dent or ovalisation.

The computer program DENTA [31] was modified to model pin-ended, unstiffened tubulars with damage in the form of a dent and lateral deflection of any size and location along the length of a member. The three above-mentioned areas of potential development were introduced in the program, which comprised three main parts.

- Analytical calculation of the behaviour up to first yield.
- A numerical calculation of the behaviour from first yield up to elasto-plastic instability and/or ultimate load.

- A numerical calculation of the fully plastic post-collapse behaviour.

Comparisons with experimental results showed good agreement. From a parametric investigation performed, it is clear that the dent shape and location is irrelevant while the dent depth and lateral deflection is an important parameter affecting strength.

2.2.7 Damage of Ring Stiffened Cylinders

Smith and Dow [48] examined the effect of damage on ring-stiffened shells subjected to external pressure. They suggested that the form of damage likely to cause most serious loss of strength was that involving bending deformation of ring frames. In order to demonstrate this effect, a finite element analysis was carried out for a typical ring-stiffened cylindrical shell. Damage was assumed to be caused by an external concentrated radial force reacted by an opposing force distributed uniformly round the circumference. Analysis of the cylinder was confined to a single frame with associated strip of shell plating, which provided a conservative response. The frame was divided into a number of frame elements around the circumference and each element into fibres. The load was first applied and then removed in order to simulate the damage process. This was repeated for different levels.

A rigid plastic collapse mechanism of a ring frame was also examined which comprised four plastic hinges around the circumference. Results agreed with the finite element analyses above.

Each damage configuration was subsequently analysed under uniform external pressure using the computer program. Results showed a reduction in collapse strength caused by increasing initial deformation.

Frieze and Sachines [49] used the finite difference method to analyse ring stiffened shells subjected to axial compression, including local damage effects. The damage was assumed to be created by a point load applied at a mid-panel position with the ends of the cylinder free to move longitudinally. The point load was assumed to represent a patch load at that location. In order to generate different extents of damage, the load was reduced to zero at least three times during each loading cycle. The residual deformations were used as input data for axial load analyses. The residual strengths of damaged configurations were thus obtained. The effect of residual stresses was neglected in the analyses. A short parametric study was performed into the effect of damage. It was found that, when $R/t = 100$ and $l/R = 0.25$, an increase in damage distortion from two to six times the shell thickness decreases the compressive strength by only up to 10 percent. However, the

greatest influence appeared to be in the most slender cylinders examined, suggesting that the role of damage assessment will become increasingly important as more slender configurations are adopted in the new generation floating platforms.

Walker and McCall are currently examining the effects of local denting on ring-stiffened cylinders subjected to pressure loading. This work was part of the cohesive buckling programme of research funded by the UK Department of Energy and the Science and Engineering Research Council [50]. Four small scale ring stiffened shells of geometries identical to some of the models tested in this dissertation have been tested under combined hydrostatic and axial loading. Hydrostatic pressure was first applied and increased gradually to a predetermined holding level and then the axial component of force was increased until collapse occurred. Two of the shells buckled before the holding level of pressure was reached.

As part of the above programme Walker and Kwok they have studied the denting process on unstiffened cylinders with a view to extending it to ring-stiffened cylinders. Results from five unstiffened cylinders of identical geometry were reported in reference [51]. The cylinders were dented using a knife edge indenter. A plastic mechanism method was developed based on a diamond shaped mechanism which provides a method for predicting the dent depth for a given applied load. Good agreement was achieved with the geometry tested.

Experimental and analytical comparisons of damage to a main column (ring-stiffened shell) of a semi-submersible offshore rig resulting from a supply ship collision were presented in reference [52]. A model test was performed on a semi-submersible of 1 to 6.25 scale. The thickness of the main legs was increased to account for the stringers present in the prototype, and flat ring stiffeners were included. The collision load was applied statically over four panels, and three different longitudinal locations were examined.

An idealised elasto-plastic solution was presented. The components included in the analysis were the tension strain energy in the plate and the flexural strain energy in the ring stiffeners. The contribution of the membrane forces to energy absorption was shown to be minimal at small indentations but became a major component as indentation increased. The inelastic force-displacement relationship was obtained by performing a finite element analysis of a typical ring. The amount of energy in the elastic range was small compared to the plastic range and once a plastic hinge was formed the load did not increase substantially. Thus it can be assumed that the force-indentation relationship was a straight line. Analytical and experimental results compared fairly well, while the analysis was found to overestimate the strength. This

was attributed to the fact that the method of analysis did not predict localised premature tearing of the shell plate or vertical stringer.

2.2.8 Damage of Stringer and Orthogonally Stiffened Cylinders

Walker and Davies [53] reported on the testing of a stringer stiffened cylinder including a large imperfection at mid-span. The model, stiffened with eight stringers, was given an initial imperfection of twice the shell thickness and then tested under axial compression. A reduction of 10 per cent was observed in the strength.

No other significant research on this problem was undertaken before Ronalds and Dowling [12,54–56] undertook an extensive investigation on the subject. An experimental and theoretical investigation into the effect of damage on orthogonally stiffened cylinders was carried out recently at Imperial College. On the experimental side, a number of cylinders were first subjected to lateral load by a knife edge indenter and subsequently tested under axial compression. The test rigs and instrumentation used were similar to those employed in this dissertation (Chapter 3), since the two programmes were running in parallel.

On the analytical side, a plastic formulation was developed to model the denting process [12,54]. The dented shell was divided into T-sections representing a stringer with its associated plating. The response of this element to a central point load was derived including the effect of the induced membrane tension force. Summing the responses of a series of beams enabled the behaviour of longitudinally stiffened shells to be predicted under knife edge loading perpendicular to the stiffeners. The theory was also extended to cover orthogonally stiffened shells. Good correlation between theory and experiment was achieved.

An analytical formulation was developed by Ronalds and Dowling [55] for the residual strength of the dented cylinder. The dented zone which, suffers large lateral deflections when subjected to axial load, carries very little stress. Hence for the purposes of this formulation it was assumed that the dent area is completely ineffective. The concentrically applied load acts at an eccentricity to the centroid of the effective section, thus producing a non-uniform stress distribution with the maximum occurring adjacent to the dent. Failure was assumed to occur when this maximum stress reached the ultimate stress of an equivalent intact shell. The analysis seems to give good results for the stocky geometries mainly covered by this program, but overestimates the strength for more slender shells [56]. This was attributed to the large outward initial imperfections adjacent to the dent which reduce the load carrying capacity in this critical zone.

Walker and McCall reported on the testing of two orthogonally stiffened shells subjected to hydrostatic pressure and axial loading [50].

2.2.9 Discussion

A substantial amount of work has been done so far in the investigation of the effects of damage on the behaviour of tubular bracing members representative of those found in steel jacket offshore platforms. Several theoretical approaches have been proposed and validated through experimental studies, and can now be confidently used in analyses of actual damage events or for the production of parametric data for design guidance.

For the stiffened legs of offshore platforms little information is, however, available. Work in this field is regarded by this author as essential since damage to a main leg of a platform, as opposed to a brace, could potentially have more serious consequences on the stability of the platform as a whole. In addition, with the new generation of floating platforms, where weight optimisation leads to slender stiffened legs with very few redundancies, the consequences of damage to stiffened cylinders is likely to be even more serious.

This dissertation aims at providing some information in this field by investigating the behaviour of ring stiffened cylinders to collision damage.

2.3 INTACT RING STIFFENED CYLINDERS FOR OFFSHORE CONSTRUCTION

An understanding of the behaviour of intact ring stiffened cylinders used for offshore construction is a necessary pre-requisite for this dissertation. A brief review of the research work in this field, pointing out the most important characteristics of behaviour, will be given hereafter.

Most of the work into the stability of circular cylindrical shells was originally carried out for aircraft and aerospace applications but more recently this has extended to offshore application. The available literature on shells for the aircraft and aerospace industry, although of great value to the understanding of shell behaviour, is not directly applicable to offshore shells. This is mainly due to the fact that previous research concentrated on shells failing elastically while offshore shells fail in the elasto-plastic range. In addition, the different manufacturing processes of the latter produce higher initial imperfections and residual stresses which significantly affect behaviour and strength.

Experimental and theoretical investigations performed on the stability of offshore shells cover both unstiffened and stiffened cylinders subjected to different loading conditions. This review concentrates on ring stiffened cylinders subjected to axial loading only.

2.3.1 Characteristics of Behaviour

2.3.1.1 Initial Imperfections

Initial imperfections present in ring stiffened cylinders are induced due to the fabrication process. The cold-rolling and welding of the shell results in circumferential periodic imperfections while the circumferential welding of panels induces axisymmetric imperfections. The periodic imperfections can significantly affect the strength of cylinders prone to periodic forms of collapse, while the axisymmetric imperfections have an important effect on the strength of cylinders which fail in axisymmetric modes.

Although the effect of initial imperfections on load-carrying capacity has been recognised and investigated by several researchers, their effect has not been completely quantified yet. Hence Design Codes [57–65] are based on empirical lower bounds to available experimental results and specify tolerance requirements on imperfection levels.

2.3.1.2 Residual Stresses

Residual stresses result from the longitudinal and circumferential welding. Additional stresses are introduced during initial cold rolling operations in which the steel plate is cylindrically curved. A parametric study based on a finite element formulation [66] has shown that welding and cold bending residual stresses in ring stiffeners have an appreciable effect on general buckling strength. However, it must be noted that their effect is not additive to that of initial imperfections. Faulkner [67] showed that when imperfection levels are close to tolerance limits, the additional effect of residual stresses is much smaller than when only very small imperfections are present.

2.3.1.3 Failure Modes

The following forms of failure are possible in ring-stiffened cylinders:

1. Panel buckling
2. General instability
3. Ring stiffener tripping failure.

Because of the extreme unloading that may accompany the general instability mode, most codes apply a policy of exclusion by imposing certain requirements on ring stiffener design. Ring stiffener failure, in any case, is unlikely to occur in an axially loaded cylinder. This is attributed to the tensile stresses induced in the ring stiffeners during initial axisymmetric radial expansion of the shells.

Hence, for axial compression, panel buckling is the only relevant form of failure and this can take two different forms:

1. Axisymmetric collapse.
2. Periodic collapse.

Axisymmetric collapse, which occurs with short ring spacing and stocky shells, is in the form of an axisymmetric outward bulge between stiffeners and is sensitive to axisymmetric imperfections. Collapse is induced mainly through material yield and is associated with relatively small losses in peak capacity after collapse.

The periodic form of collapse which is expected in slender shells with widely spaced rings comprises a number of waves around the circumference of the cylinder. Where the periodic mode of failure is applicable, imperfections of the same periodic form can have a significant effect on the strength. Failure occurs mainly through elastic instability with rapid unloading in the post-buckling range.

2.3.2 Experimental Investigations

Particular problems associated with cylinders for offshore construction have initiated a number of experimental programmes in recent years. Reviews on experimental programs looking at both unstiffened and stiffened cylinders under different loading conditions are given in references [68–71]. Only tests on ring stiffened cylinders under axial compression will be listed here.

Miller [72, 73] reported on the testing of large and small scale ring stiffened cylinders conducted by Chicago Bridge and Iron Co (CBI). Forty models were tested with radius to thickness ratios in the range of 242 to 500 and both local and overall buckling modes were observed. Models were fabricated from different grades of steel and it was found that plasticity occurred before collapse in the models of the lowest steel grade.

Norway is another centre where extensive studies on shell buckling have been performed. Odland [74] presented the results of an experimental study on ring stiffened cylinders which were either machined or welded. All models failed by inter-ring panel buckling in an axisymmetric or asymmetric mode. The effect of initial imperfections, which are higher in welded compared to machined models, was illustrated. A difference of up to 30% was observed between the two.

In Britain, a major buckling research programme was sponsored by the Department of Energy and the Science and Engineering Research Council, which included testing of ring stiffened cylinders.

At Imperial College [75, 76] three large scale, three bay models were tested. Initial imperfection measurements were recorded before testing. One of the models failed in an axisymmetric outward bulge. This was the model with the smallest ring spacing. The other two models failed suddenly near the rings at lower collapse loads. Another small scale model on a stocky model which failed in an axisymmetric mode was reported by Tsang *et al* [77]. This was part of an interactive loading series of tests.

More small scale tests were carried out at University College [78, 79]. A series of ten models were reported in which both axisymmetric and asymmetric modes of failure were observed. All models failed near the ends of the shell panel. A high degree of imperfection sensitivity was suggested by the steep unloading path of these models.

The collection and critical evaluation of all available experimental information relating to the behaviour of unstiffened and ring stiffened cylindrical shells were undertaken in reference [80]. Results from aerospace-related research were also included in this study. It was clear from the plots of the different experimental strengths that there is a strength dependency on the parameters ℓ/R and R/t . An increase in ℓ/R reduces the strength of the cylinder. This length dependency was more pronounced for $\ell/R < 0.5$, which is the practical range for offshore construction, and disappears when $\ell/R > 1.0$. The R/t parameter was another important factor, an increase in which reduces the collapse load significantly, as predicted by classical theory.

2.3.3 Theoretical Analyses

Analyses of ring stiffened cylinders can be treated by numerical methods. Both finite element and finite difference computer packages are available which can be used to model this problem. Initial imperfections and residual stresses of any distribution can be readily incorporated in the analyses and can be used to investigate their effects on the strength. However, these methods are costly and other simplified methods of analysis are required to supplement the much needed design information on the different parameters affecting the strength. Relevant work in this field is outlined here.

A large-deflection elasto-plastic finite difference program was developed and used by Harding [81,82] to analyse ring stiffened cylinders of geometries found in fixed and floating platforms. Axial compressive loading was considered as well as combinations of bending and external pressure. Results presented were compared with existing codes of practice and recommendations for design were made based on this comparison and on observations from large-scale tests.

Later the program was modified to include the behaviour of rings as discrete elements [83]. Different geometries and loading combinations were examined and results were correlated with those from experiments and alternative finite element analysis. Some insight into the behaviour of ring stiffeners was given.

Tsang *et al* [71,77] examined the behaviour of ring stiffened cylinders under axial load by the finite element method but their work mainly concentrated on interactive loading.

A reduced stiffness method proposed by Batista and Croll [84] for unstiffened cylinders [84,85] has been extended to provide lower bound to ring stiffened shells [86–88]. The reduced stiffness model is so called because it neglects the membrane

energy in the minimum critical mode. The method predicts lower bound to first material yield and first full plasticity which would be appropriate for allowable stress and ultimate limit state design respectively. Both inter-ring buckling and general instability have been considered and the method seems to correlate well with experiments [89].

The plastic mechanism method is another alternative approach. Andronicou and Walker [90] modified a previous approach for unstiffened cylinders under axial compression [91] in order to study post-buckling behaviour. The method is applicable to cylinders failing in an axisymmetric mode. Later Tsang and Harding [92] modified the above model for panel collapse under interactive loading and proposed a new overall collapse mechanism [93] for pressure dominated loading.

2.3.4 Design Methods

Most design codes determine the shell buckling stresses by applying knock-down factors, accounting for initial imperfections and plasticity, to the classical buckling stresses. The knock-down factors are obtained empirically representing a lower bound to the available experimental results. All codes except ASME [64] apply a policy of exclusion for general instability by specifying certain requirements on ring stiffener design. However, such an approach, which leads to heavy ring stiffeners, may not be the ideal from the point of view of design efficiency.

In an extensive study [80] of all the available experimental results and a critical evaluation of the existing codes, attention was drawn to the variability of available information regarding initial imperfection levels and boundary conditions. The need for a reliable theoretical method to eventually replace the empirically based design procedures was highlighted. The "reduced stiffness" [84–89] method which accounts for the effect of initial imperfections in a rational manner and provides a safe lower bound to test results appears to be a reliable alternative. References [94,95] demonstrate the suitability of the method for design.

2.3.5 Discussion

The behaviour of ring stiffened cylinders under axial compression is marked by their sensitivity to initial imperfections and residual stresses. It has been shown that if initial imperfections in a form similar to the characteristic failure mode are present in the shell then significant reductions in strength can result.

In a damaged ring stiffened cylinder, large initial geometric imperfections concentrated in certain regions accompanied by large residual stresses are to be anticipated. Given the high imperfection sensitivity of these cylinders any attempt to analyse their characteristics in the damaged configuration should take account of these imperfections. In addition, if the damage effects are to be quantified accurately the intact model configuration should be analysed in a comparable manner. The finite element method of analysis, in which the effect of initial imperfections and residual stresses could be directly incorporated, appears to be an appropriate method for achieving the aims of this thesis.

2.4 RELEVANT APPLICATIONS OF THE FINITE ELEMENT METHOD OF ANALYSIS

The finite element method finds its application in a wide range of fields where complexity of behaviour does not allow treatment by closed form theoretical solutions. The main fields of application are solid mechanics both in linear and non-linear phases, fluid mechanics, heat transfer and electro-magnetism. Recent developments of the method as well as advances in computer technology have widened the horizon of application and improved efficiency [96]. As a result, implementation of the method is becoming more and more common practice, especially in the field of structural engineering, for the analysis of complex non-linear structures.

Numerous publications exist on the subject of finite element formulations and the rate at which developments appear is quite remarkable. Some of the applications of the method in fields relevant to the work of this dissertation will be reviewed in the following sections. In particular, the field of shell buckling and the problem of contact between two bodies by finite elements are considered.

2.4.1 Shell Buckling by Finite Elements

2.4.1.1 Shell Finite Element Formulations

Special element formulations have been developed for the analysis of shell structures. Zienkiewicz [96] provided some guidance on the formulation of these elements. Chang *et al* [97] presented a non-linear formulation of a 9-node shell element for large deformation and post-buckling analysis of shells. The formulation was based on an updated Lagrangian description and finite deformation constitutive equations. The post-buckling analysis was conducted via either the modified or full Newton–Raphson procedure with a constant displacement length method when the limit load of the structure is reached. Numerical examples were included to demonstrate the post-buckling analysis procedure. One of the examples considered was a thin cylindrical shell with hinged supports along the longitudinal edges and a concentrated force applied at the crown. The material was assumed to be linear elastic and isotropic. This example portrays a dramatic post-buckling failure and has been adopted by several investigators [98, 99] to demonstrate the usage of thin shell elements and solution algorithms.

Bathe *et al* [100] discussed the requirements of general shell elements and non-linear analysis in practical engineering environments and presented their approach to meet these needs. A formulation of a 4-noded and an 8-noded shell element using mixed interpolation of tensorial components was developed. The applicability of these

elements was illustrated through a number of examples. Lee *et al* [104] presented their study on 9-noded mixed formulation finite elements for thin plates and shells. Bates [102] recently developed an 8-noded shell element for incorporation in the finite element program FINAS [99] which was used for the numerical analyses of this dissertation. A brief description of the element is given in Chapter 4.

2.4.1.2 Programs with Shell Buckling Analysis Capabilities

Several programs are available today with shell buckling analysis capabilities ranging from the very powerful program ADINA [103] to other commercially available packages like ASAS, LUSAS, ANSYS, NFAP, and NONSAP. Other programs also exist which have been developed for research applications into shell buckling. The non-linear finite element program FINAS [99] was developed for this purpose at Imperial College and has been found to be very efficient compared to many other available packages.

2.4.1.3 Shell Buckling Analyses

Finite element analysis of shell structures has been applied in the following fields: aircraft, aerospace and automotive industries and more recently in offshore structures. Applications in the latter group are presented here. The use of non-linear large deflection finite element analysis is well suited to the particular geometries of the stiffened cylindrical shells currently used in offshore platforms which fail by elasto-plastic buckling. The method also lends itself to examination of the effects of initial imperfections and residual stresses which are very important for offshore geometries.

Agelidis and Dowling [69, 104] investigated the buckling of stringer stiffened cylinders subjected to combined axial and pressure loading using the finite element program FINAS [102]. By limiting attention to those geometries in which failure occurs in a local deflection mode, it was possible to reduce the problem size to that of a single stiffener and shell panel. The analysis method was correlated against a series of small and large scale model tests. It was shown that this relatively small finite element model gave very good results for most of the cylinders, except those models that failed in overall modes. Using this finite element model an extensive parametric study was undertaken providing results over a broad range of parameters.

Fahy [70] addressed the problem of buckling of stringer stiffened cylinders subjected to axial compression or combined axial load and end moment. He also adopted the stiffened panel unit within the finite element program FINAS [992]. Elastic and

elasto-plastic analyses were performed and results were compared with experimental data. The elasto-plastic analyses were found to be in good agreement with the experiments. Different forms of initial imperfection were included in the elasto-plastic analyses and geometries with high imperfection sensitivity were identified.

Finite element analyses were used by Tsang [71] to correlate with a series of experiments on ring stiffened cylinders subjected to interactive loading. Both local and overall buckling modes were successfully reproduced by the finite element program FINAS. The importance of boundary conditions and initial imperfections across the whole range of loading interaction was examined. The finite element package was used further in a parametric study of the axial load dominated region. An axisymmetric model was used where possible but in situations where the assumptions of axisymmetry become unrealistic the results were supplemented by an appropriate 3-D model. Initial imperfections, based on experimental measurements, were also incorporated in the analyses.

In reference [77] Tsang *et al* reported on correlations with a series of small scale ring stiffened cylinders subjected to combinations of pressure and axial loading. Two different finite element programs were used to perform the analyses. The first program used was FINAS and only the central panel of each cylinder was analysed, with boundary conditions at the rings of simple and clamped supports. The second program was developed at University College, London. It allowed the large deflection elasto-plastic analysis of the axisymmetric behaviour of cylinders subjected to combined loading. Comparisons between the results of the two finite element programs showed that they produced similar trends as far as general effects of boundary conditions, loading and imperfections are concerned. However differences between the strength predictions and the test results were apparent and probably originated from the modelling of the initial imperfections.

Mousourous [105] investigated the buckling with bending of straight circular cylindrical shells subjected to static and bending moments. This was achieved using the finite element program ABAQUS in conjunction with preprocessing and postprocessing computer programs. The results compared favourably with experimental data. To model the loading an "end rotation" control technique was used which assumed plane sections remain plane at the ends. The technique predicted bending moments and work done on the structure as a function of mean curvature.

As the finite difference method is comparable to the use of the finite element procedure, reference to analyses of shells by this method is also relevant.

Harding *et al* [81–83] analysed ring stiffened cylinders using a large-deflection elasto-plastic finite difference program, as already reported in Section 2.3.3.

Galletley *et al* [106] analysed a series of forty buckling tests on unstiffened welded steel cylindrical shells subjected to combined axial and pressure loading, using BOSOR5 [107]. Ratios between the BOSOR5 predictions and tests were 0.54 to 0.93, since no imperfections were included in the analyses. However, similar forms of interaction curve were obtained with experimental curves exhibiting lower strengths.

Sobel and Newman [108] investigated the plastic buckling of shells under axial compression using the finite difference program STAGS [109].

Almroth *et al* [110] analysed cylinders with two diametrically opposed rectangular cutouts using STAGS. Stress distribution plots showed that stress redistribution occurred towards the intact part of the cylinder. It was found that the effect of the cutout predominated over effects from imperfections in geometry or material properties. Correlations with tests showed good agreement. Cutouts with reinforced edges were found to produce smaller reductions in the strength. In reference [111] circular cutouts were analysed which were shown to produce the same strength reduction as rectangular cutouts of the same width.

Ring stiffened cylinders with large initial imperfections in the form of a local dent were analysed by Frieze *et al* [49] using finite differences and Smith [48] by finite elements, as already mentioned in section 2.2.7.

2.4.2 Contact Problems by Finite Elements

In finite element analysis, the contact between two bodies is a problem which needs special attention. Some attempts to treat this problem are presented here. Chan and Tuba worked on a finite element method for the contact problem of solid bodies [112] and presented applications to turbine blade fastenings [113].

Okamoto and Wakazowa [114] worked on the development of a theoretical method which gave a solution for non-linear contact problems with irreversibility resulting from stick-slip phenomena. The geometrical and statical boundary conditions on contact surfaces were treated as additional conditions which were independent of the stiffness equations. As a result the algorithm of calculation was simplified and only the part of the equilibrium equations relating to the contact surfaces was required to be solved at each step rather than the overall stiffness equations. Furthermore, the magnitude of the load causing a change in the contact condition of one contact node

pair was taken as a load increment. This is analogous to the incremental iterative procedure for elastic-plastic problems. Illustrative examples were presented and comparisons with experiments showed reasonable agreement.

Tsen and Olson [115] illustrated the application of a mixed finite element method to two-dimensional elastic contact problems. In their approach they treated some of the displacement variables as natural boundary conditions in the contact region. In cases where the contact region was independent of the applied loading, an iterative procedure was used to establish the sliding and adhering portions of the contact region. Where the contact region was a function of the applied loading, for example in progressive contact, an incremental formulation was employed to describe the discretised contact stages before invoking the former iterations. Numerical examples were given and results were compared with those from conventional potential energy or displacement finite element methods. One of the examples given was a long solid cylinder under a uniform line load resting on a flat rigid base.

Voyiadjis *et al* [116] investigated the indentation of an elasto-plastic half-space by a spherical rigid indenter using finite elements. The novelty of the computational scheme proposed was its capability of dealing with contact problems of surface pressure which were unknown in advance and which acted over a variable and unknown region of the boundary. This used an incremental iterative procedure of non-linear elasto-plastic analysis. Results were compared with exact small-strain solutions for elastic, rigid-plastic and elastic ideal-plastic half-spaces. In another paper [117] the contact problem of a cylinder embedded in a body was treated.

A recent development in the ADINA finite element code was the addition of an algorithm for handling the contact problem between two or more bodies using Coulomb friction [118]. These bodies may be composed of elements or defined by fixed nodes in space. Bodies may repeatedly come into contact during the analysis. The direction of relative motion between the bodies need not be specified and large relative displacements and sliding between the contact surfaces are allowed. Some applications of this analysis were presented in reference [119].

2.4.3 Discussion

The finite element method has been shown to be a successful numerical tool for the analysis of large displacement elasto-plastic problems such as those found in the field of shell buckling. Initial imperfections can be readily incorporated in the analysis for an evaluation of their effect on behaviour and strength. Based on the experience drawn from previous applications, the method was used in this dissertation for the

analyses of damaged and intact ring stiffened cylinders.

Furthermore, use of the method within the context of contact problem analyses has been presented. The latter is relevant to the problem of indentation of a shell through gradual contact. This problem is tackled numerically in this thesis and a simple gap/contact element has been developed to facilitate the analysis of the denting process.

2.5 CONCLUSIONS

From a study of the available literature it is clear that there is a scarcity of information regarding the behaviour of ring stiffened cylinders for offshore construction subjected to impact damage. There is an increasing interest in this field, enhanced by the new generation of offshore platforms comprising more slender cylindrical members. Given the potential susceptibility of such shells to imperfections, serious consideration of this problem is warranted, and comprises the primary scope of this dissertation.

The complexity of the behaviour apparent from previous studies on intact ring stiffened cylinders and the additional complication of local damage imperfections suggests that a comprehensive investigation of the subject is necessary with the backup of extensive experimental data. For this reason an experimental programme formed a substantial part of this research. The experimental results obtained have been used to verify the finite element modelling, thus enabling further examination of important parameters.

CHAPTER 3

EXPERIMENTAL TESTING

3.1 INTRODUCTION

The experimental programme examining the effect of impact damage to ring-stiffened shells is presented in this chapter [120,121]. Throughout the programme small-scale multi-bay models were tested. A number of preliminary tests have been performed followed by two series of carefully designed and extensively monitored experiments. The experimental models were fabricated using special techniques, so that full scale imperfections were properly reflected in them. Due to the high costs involved in these special fabrication techniques, only a limited number of carefully selected geometrical parameters and dent locations could be tested.

Each model was tested in two parts; first subjected to lateral loading and then tested under axial compression. Initial geometrical imperfections, which were anticipated to play an important role in the cylinder strength, were measured on the models before and after the damage process and subsequently incorporated in the numerical studies performed. Extensive instrumentation was employed for monitoring the tests.

Through this experimental study it was hoped to provide controlled experimental results for ring-stiffened cylinders, spanning both fixed and floating platform geometries, and to indicate some important aspects and trends in their performance in damage situations. The data obtained would also serve to validate the numerical method of analysis.

3.2 EXPERIMENTAL TESTING OF SERIES 1 MODELS

3.2.1 Model Geometries

From previous knowledge of the buckling behaviour of cylinders subjected to axial load it is clear that they are very sensitive to panel imperfections. If the dent is

regarded as an initial panel geometrical imperfection of large magnitude, then it seems important to investigate its effect on the strength of the model. This was the basic reason for the first series of tests.

In choosing the model configuration the following had to be considered. The model had to be sufficiently long in order to reduce the effect of the boundary conditions on its response to lateral loading. With the above considerations in mind and the additional factor of fabrication economy the choice of five panel models was made. Given that the middle panel was dented, the two panels on either side provided enough room for the deflections to develop as they would in the prototype.

The role of the ring stiffeners in this series of tests was thought to be relatively secondary. For this reason they were designed as flats to provide fabrication economy. They did however meet the main inertia requirement within the DnV rules [59].

This test series comprised three preliminary denting tests and five models subjected both to denting and subsequent axial compression.

The geometrical parameters chosen for these models represented both fixed and floating platform geometries used in current practice.

The models used for the initial denting tests were retrieved from a previous test series used for interactive loading documented in reference [77]. They had R/t values of 267 and ring spacing of $0.15R$ for the central panel, while the other panels were shorter and thicker (in the case of the end bays).

The first new model was of similar geometry to the above models. This enabled direct comparison of the dented model test with the axial compression test of the previous series to be made. These tests would provide an experimental pilot study and at the same time provide some validation of the numerical analysis.

The new models were five-bay ring stiffened cylinders of equal panel lengths. Their geometries covered two basic slenderness values (R/t), 267 and 133 and two ring spacings (ℓ/R), 0.25 and 0.5. Exact geometrical parameters are given in Table 1.

3.2.2 Model Fabrication

The ring-stiffened cylinders were made mainly in two stages. The first operation was to make complete unstiffened cylindrical panels of length equal to the spacing

between the ring stiffeners or a ring and end boundary. The second stage was to T-butt weld the stiffener panel assembly from the outside.

The full length of the model was first rolled and T-butt welded on a mandrel and then cut to the required lengths. The flat stiffeners were then made as complete annuli by trepanning out of plate.

Two successive panels with the stiffener in between were welded from the outside. An assembly of thick walled tubes holding the panels and stiffener in place was used. Once one weld was completed the assembly was removed and set up again on the adjacent stiffener with its associated panels.

Heat treatment was then carried out to remove the high residual stresses induced in the model by the fabrication process. During heat treatment the model shape was retained using sand. A tooling ring was positioned in both ends of the cylinder and a fine foundry sand packed inside the model and around the outside in a drum container.

After heat treatment end rings were attached to the model. These consisted of two annular rings which were used to provide a clamped end condition. A sand and araldite resin mix was added to fill the gaps between the shell and the rings leaving 1 mm of the cylinder to project through the composite. The model ends were then lapped to ensure even loading around the circumference of the model.

3.2.3 Strain Gauging

High levels of stress as well as sudden variations in magnitude and sign were anticipated in the dent area during the denting phase and possibly the axial test. It seemed important to capture the development of this stress field around the dent which would provide an insight into the cylinder's response to lateral loading. An overall picture of the axial stress distribution around the model was also desirable to indicate any possible stress redistribution during the failure test and provide an indication of the collapse mechanism.

The first model to be strain-gauged was model CY-1 which was the first of the new models tested. There did not appear to be much point in strain-gauging any of the preliminary models which mainly served as an indication of the load and deflection levels to be encountered. Model CY-1, however, was designed as a pilot test and it was heavily strain-gauged as shown in Figure 3.1. A high concentration of gauges was provided in the dent area where sudden changes in stress distribution were

expected. Circumferential and longitudinal symmetry were assumed around the dent centreline and gauges were therefore concentrated into one quarter of the dent area. Gauges were applied at five vertical sections located between 0° and 45° from the centre of the dent. For measuring stresses at the centre and adjacent midpanel, back to back cross gauges were used at sections 1, 3 and 4. For stresses near the end of the centre panel small linear gauges had to be used because of the small scale of the model. These were applied in pairs to measure both axial and hoop stresses both inside and outside the model. The same type of linear gauges were placed on the outstand of the stiffener to measure hoop compression at sections 1 – 4. At the same locations the axial stress on the panel at the ring intersection was measured by linear gauges. At sections 1 and 2 extra gauges were applied at quarter panel positions but on the other side of the dent lines. In the rest of the sections, 5 – 10, cross gauges at the centre of the middle panel served to measure the stress distribution around the model.

For the rest of the models the number of gauges was reduced. The vertical sections were reduced to six, with cross gauges mainly positioned at midpanel positions. In addition one pair was applied in the adjacent panel and linear gauges used on the stiffener outstand at section 1. This different arrangement is shown in Figure 3.2.

3.2.4 Material Testing

The material used for the models was similar, after heat treatment, in stress-strain characteristic to a normal structural steel and had a significant yield plateau and a straight line elastic response. Tensile tests were carried out in order to obtain its material properties. The tensile specimens were cut from the same sheet of steel as the models. The specimens were subjected to heat treatment with the corresponding model.

For each cylinder four specimens were tested, three of which were cut transverse and the fourth parallel to the direction of rolling. The tests were conducted in a servo-controlled testing machine at an approximately constant rate of elongation. The results are tabulated in Table 3.2. Compressive lower yield stress was assumed to be equal to the static tensile yield stress plus 10N/mm^2 as recommended by TRRL [122].

3.2.5 Experimental Procedure

Each model was subjected to two main tests, the denting and failure test. The denting test was a representation of the damage formation due to side edge impact of

a ship. The failure test was a means of assessing the damage effect on the strength and overall behaviour of the cylinder. An integral part of the testing procedure was the imperfections measurements of the model before and after denting that would be incorporated in numerical analyses of both the intact and dented model configurations.

The main operations performed for the testing of each model were as follows:

- Initial imperfection measurements of the intact model
- Denting test
- Initial imperfection measurements of the dented model
- Axial compression test

The testing procedure as well as the instrumentation employed at different stages of the experiments are described in detail below.

3.2.6 Initial Imperfection Measurements

Initial imperfections of the cylinder before and after denting were measured using forty inductive transducers mounted in the test rig used for the axial compression tests, as shown in Figure 3.3. The transducers were fixed on a ring frame which could be moved vertically or rotated, thus enabling measurements to be taken at different longitudinal and circumferential positions on the models. Datum readings were produced using an accurately machined solid reference cylinder. It was estimated that the readings obtained were accurate within to $\pm 0.05\text{mm}$. A least squares fit method was used to determine deviations from the best fit cylinder representation. Imperfection readings were utilised in subsequent finite element analyses described in Chapters 4 and 5, to represent the initial geometry of the finite element models.

3.2.7 Denting Tests

The objective of these tests was to investigate the response of the cylinders to lateral load from ship impact and relate their behaviour to the associated impact energy and their subsequent strength in the post-damage state. Due to the slow nature of ship impact the damage process has been treated as a static problem by previous investigators. This is also the approach adopted in this study.

In the tests the models were subjected to lateral load through a knife edge indenter. The load was applied statically with a stepwise increase in load. During the denting process applied load, stresses, and displacements were monitored at each increment of load application.

3.2.7.1 Test Rig Arrangement

The preliminary denting tests and that of model CY-1 were performed in an existing column test rig which was modified specifically for the purposes of these experiments. The arrangement is shown in Figure 3.4 where the top platen was modified to carry a knife edge indenter. Below the bottom platen a 5-ton load cell was placed. The models were placed lying horizontally on the bottom platen with the end rings resting on circular cradles of equal diameter. These cradles were in turn fixed on the platen. The latter was raised by a screw jack, thus pushing the model against the indenter with the applied load monitored through the 5-ton load cell.

For the denting tests, no attempt was made to introduce any special end conditions. The clamped end conditions introduced in previous axial tests by Fahy [70], Agelidis [69] and Tsang [71], using steel rings with a sand-resin bonding material were also employed for these tests. It was felt that these would still be appropriate although there was some concern about the lateral loading inducing relative axial shortening which could potentially damage the resin bond. By monitoring the axial shortening of the shell and the end ring displacements separately near the dent region, negligible values of displacement were recorded [123]. This showed that no relative displacement or bending of the end rings occurred. This was mainly because the small dent size imposed restricted the amount of axial shortening.

The same denting rig could not be used for the longer models of series 1, namely CY-4 and CY-5. For these models another rig had to be modified in the same way as the first. The new rig is shown in Figure 3.5. In this rig the indenter was fixed on a cross beam located above the models.

3.2.7.2 Choice of Indenter

A V-shaped line indenter was employed in these tests. This was chosen to represent the side edge impact of a ship. The same form of indenter was originally used by Smith *et al* with a tip radius of 3mm [26] for denting unstiffened tubulars. By scaling from their geometry a 1mm radius was originally thought to be appropriate and the preliminary tests for model CY-1 were carried out with such an indenter. Later a 3mm radius was used to avoid possible damage to the models in the form of tearing arising from a very sharp indent. This modification was justified since the exact shape of the indenter has been shown to have little effect on the behaviour of the cylinder [29].

In addition, the choice of a V-shaped indenter appeared to be favourable for the following reason. The dent resulting from the use of such an indenter happened to be similar in form to the individual buckle of a lobular failure mode. This mode of failure was the critical mode for part of the range of geometries tested in this series. Bearing in mind the extreme imperfection sensitivity of these models it appeared to be important to test the effect of such a damage form on the strength of these cylinders.

3.2.7.3 Transducer Instrumentation

In the pilot test of model CY-1 extensive instrumentation was used to measure panel and ring deflections during denting. Small inductive transducers of 10mm stroke were employed to enable the use of large numbers in close proximity at such a small scale. The radial deflections along the dent line and at two lines close to the ring-stiffeners on either side were monitored using nineteen transducers combined together to form one block as shown in Figures 3.6 and 3.7a. The block was placed inside the model before testing and fixed on the rings in a position where all transducers were just touching the shell. Readings were recorded using a data logger. Figure 3.7b shows model CY-1 with the transducer block set-up ready for testing.

Both radial and rotational deflections of the ring-stiffeners on either side of the dent were monitored. This was thought to be necessary because both forms of deformation were expected. The radial deflections could only be measured from the outside. For this purpose transducers were cantilevered from the end rings and pivoting arms were used to enable measurements to be taken close to the indenter. These transducers are shown in Figure 3.7a and in Figure 3.7b they are fixed on the model.

The rotation of the rings was monitored by transducers fixed on the inside transducers' block. Because of the size of the model, it was not possible to have transducers measuring directly on the stiffeners. This problem was bypassed by introducing a small pivoting angle of equal legs with one leg touching on the edge of the stiffener and the other leg on a transducer. Figures 3.6 and 3.7a show the relevant instrumentation.

The axial shortening of the model was also measured by three external transducers in order to detect any differential shortening during the denting tests. Two of these were located on either side of the load application and the third diametrically opposite the centre of the load application.

From the results of CY-1 it was clear that the rotation of the stiffeners was negligible and only small lateral deflections occurred. In the light of this information the part of the instrumentation associated with the stiffener rotation was removed for the rest of the experiments. A typical arrangement for one of the longest models ($l/R = 0.5$) is shown in Figures 3.8 and 3.9a. The inside block was in this case fixed on two arms which were directly fixed on the bottom platen of the test rig as shown in Figure 3.9b. The block could be moved vertically relative to the supporting arms. This enabled setting of the transducers to the beginning of their range and also resetting the block when the deflections increased beyond the 10mm range.

3.2.7.4 Denting Procedure

The model was placed on the bottom platen of the rig with its axis lying horizontally. The end rings were resting on shallow circular cradles. All transducers, strain gauges and load cells were connected to the logger. The inside transducer block was placed inside the model and adjusted both longitudinally and circumferentially so that the centre transducer was touching the highest mid-panel point. The supporting arms or plates were then clamped in position and vertical adjustment of the block was made, ensuring that all the transducers were reading properly and that not much of their stroke was lost initially. The outside transducers were placed on the model cantilevered from the end rings and resting on 2mm balls stuck on the cylinder at the points where measurements were to be taken. The three outside transducers, measuring axial shortening, were also placed in position. The indenter was finally adjusted so that it was parallel and right above the mid panel line of dent application. A final check on the functioning of the different parts of the instrumentation was made. The lower platen was raised up to the point where the gap between the indenter and the shell closed and at that point the test was ready to start.

Initial readings were recorded and then the load was applied in small increments. The maximum displacement was monitored and steps of 0.2 to 0.5mm were applied. For every load step, load, displacements and strain gauge readings were recorded through the data logging equipment. For some of the models the transducers block had to be reset half way through the test to prevent the transducers going out of range. Load was applied up to a predetermined level and then gradually reduced to zero.

3.2.8 Axial Compression Tests

After the denting tests were completed the models were failed under axial compression. The aim was to examine the overall behaviour of the cylinders in the damaged state and determine their residual axial strength. To achieve the above objectives the models were tested in an axial compression rig by applying the load in small increments. For each load step readings of load, axial and radial deflections and strain-gauge were obtained. A full description of the instrumentation employed and the testing procedure is given below.

3.2.8.1 Test Rig

In the test rig used for the axial loading (Figure 3.3), the models were loaded by applied displacement through rigid platens. A spherical bearing was introduced above the test piece to maintain concentricity of overall load application. Load was applied by a screw jack located beneath the bottom platen. A large cross beam provided the reaction. The load was transferred directly through the load cells placed between the two bottom platens. These were used to monitor total load and had been used in previous tests to check load concentricity. A ring frame carrying forty inductive transducers was mounted on the rig. This was used for measuring radial deflections during the test. The same frame was also used in scanning the initial imperfections before and after denting as explained in a previous section.

3.2.8.2 Axial Testing Procedure

Before placing the model onto the rig all contact surfaces between the model and the different loading plates were thoroughly cleaned for good contact. The model was then located on the rig concentrically in relation to the load cells. This was achieved by checking the end ring position with an accurately fitted circular plate. The three transducers for measuring the axial shortening were placed inside the model. Another circular plate carrying concentrically a thick hollow tube was then lowered onto the model. All the inside transducers were wired to the logger through openings on the top plate and tube (this also explains their use). A final check was made on the functioning of these transducers before another plate, carrying the ball seat at its centre, was placed on top. The model was then raised using the screw jack until the ball seat was just in contact with the cross beam. This was checked by monitoring the load cells. The model was then ready for testing. At this stage initial readings were taken of the load cells, strain gauges and inside transducers. A separate exercise involved taking initial readings of the forty transducers (fixed on the ring frame) at different levels longitudinally on the model.

Load was then applied by the screw jack at a slow rate. Initially increments of 2.5 percent of squash load were applied but as the load reached the expected failure load smaller increments (1.5 percent of squash load) were used. For each load step, loads, strains and axial shortening measurements were recorded. The lateral deflection measurements along the whole length of the cylinder were initially performed at bigger intervals.

3.2.8.3 Deflection Measurements

Both axial and radial displacements were monitored during the axial compression test. Three internal transducers were placed inside the model measuring the relative displacement between the plates above and below the model, thus giving the axial shortening of the cylinder. Readings of these three transducers were taken at every load step.

The transducer frame mounted on the rig was used to measure the radial deflections. This was a time consuming process because it involved moving the frame to a number of longitudinal positions in order to scan the whole cylinder. For this reason a full scan of the deflections was taken every four load steps, and then every two steps as the load approached the predicted collapse load.

3.3 EXPERIMENTAL RESULTS OF SERIES 1 MODELS

3.3.1 Denting Tests

3.3.1.1 Load Deformation Response

The three preliminary denting tests performed on failed models retrieved from a previous testing series [77] will be discussed here, together with the results of model CY-1 and the rest of the series 1 models.

The first three tests, models (P1, P2, P3), which were performed with the minimum of instrumentation, together with the pilot study on CY-1, served to examine the load denting procedure. Load deflection responses of these four models are plotted in Figure 3.10. The three curves corresponding to CY-1, P1 and P2, which had exactly the same geometry, show the load deflection response with varying dent location in relation to the rings. Model CY-1, which was subjected to a mid-panel dent, has the lowest stiffness, while P2, which was subjected to a ring dent, shows the highest stiffness. A quarter panel dent on model P2 produced a stiffness only slightly higher than the mid-panel dent response. The response of model P3 is not directly comparable with the rest because the stiffeners in this model are much more

slender than in the other models. The test on P3 produced a very interesting effect when the model was loaded at mid-panel position to a moderate dent depth. While in the previous models the rings deflected and rotated very little, in model P3 marked rotations were observed in the rings, which also tripped locally at two positions on either side of the dent, as shown in Figure 3.11. This was a result of the lower stiffness and higher slenderness of the stiffener which was taking high outward compression at the locations of the buckles. This outward compression resulted from a local outward deflection of the shell in regions adjacent to the inward deflected dent area.

In the case of the ring dent (model P1) the ring deflected inwards without rotating. The quarter panel dent caused the ring closer to the indenter to deflect by about 50% of the corresponding dent deflections. A maximum rotation of 13° accompanied the deflections while the effect on the other stiffener was negligible. There was little difference between the loads appropriate to mid- and quarter-panel dents (for the larger stiffener) but both were some 25 percent less than that related to the stiffener denting.

In all cases elastic recovery of radial deflections upon unloading resulted in a reduction of some 15 – 20 percent in the peak deflection.

In model CY-1 the effect of the indentation, as seen from the deflection measurements obtained, was very localised with little lateral ring deflection. Figure 3.12 shows that sections outside the indenter contact zone deflected very little from their original positions. The rings deflected radially inwards by very small values but did not rotate. Shortening of the model, which was monitored during the test, was found to be negligible. This was checked to ensure that the normal axial test boundary restraints were sufficient for the denting tests.

Load central deflection curves from the denting tests of models CY-2, 3, 4 and 5 are shown in Figure 3.13. By comparing the responses of these models with that of model CY-1 it is clear that the latter showed a much higher stiffness. At this point it should be noted that the stiffening in CY-1 was much heavier. This geometry was chosen to match previous models P1, P2 and P3 which were designed to ensure centre panel buckling under pressure loading. For this reason the load-deformation results of model CY-1 are not directly comparable to those of the series 1 models.

All the other models, which were designed according to the DnV requirements for axial loading [62], responded in a similar way to denting and some useful comparisons can be drawn from the results. The deflection measurements recorded at different

levels of denting (see Figures 3.14 - 3.17) show that the rings suffered some radial deflection. The deflections of the adjacent rings were in fact around 40% of the maximum radial deflection. This showed that the whole length of the model was effective in this case, unlike model CY-1.

It is clear from the load indentation curves (Figure 3.13) that the lateral stiffness of the panel to indentation was, as expected, influenced by the radius to thickness ratio and the ring spacing. Model CY-3, for example, with the lowest R/t and ℓ/R has the greatest stiffness, while CY-5 with larger values of R/t and ℓ/R has the lowest. These models could be directly compared because they had an equal number of panels. Otherwise this parameter would have had an effect on the load. The stiffness would be expected to reduce with increasing number of panels. This is where comparison with model CY-1 is useful. Its stiffness appears to be around four times higher than that of model CY-2. According to the trend observed between models CY-2 and CY-5 (CY-1, CY-2, CY-5 had the same R/t) the difference between CY-1 and CY-2 should not have been more than say 30 percent. Model CY-1, however, acts as a single panel short cylinder, thus explaining this discrepancy in the loads involved.

The relative axial shortening recorded during denting was of very small magnitude. This suggests that the presence of the end rings and the axial stiffness of the adjacent panels allowed some stress redistribution to occur around the model and prevent significant differential shortening. The values recorded for the bigger dents were, however, larger than originally anticipated.

All models exhibited elastic unloading ranging between 30 to 50 percent of the loaded deflection. The highest unloading is seen in the more slender model CY-5, while CY-3 with the highest lateral stiffness suffered the least unloading.

3.3.1.2 Stress Distribution During Denting

Very high stresses were measured in the dent area indicating high bending and stretching of the region. Axial stress distributions around the centre mid-panel at peak load are plotted for model CY-1 in Figure 3.18a. In this Figure inside and outside stresses are shown separately. Plots of the average stress would conceal the bending effect which is very pronounced in the dent region. Near the centre of the dent high compression was measured on the outside surface and high tension on the inside, which is indicative of the high inward deflections and bending about the indenter lines. Both stresses reached yield and very high strains were recorded during the test. This situation reverses circumferentially away from the centre,

suggesting reverse bending in that region. This is the region associated with outward deflections recorded adjacent to the dent in the tests. These stresses were again fully plastic. Another reversal of stress was observed further away. This must be associated with smaller inward deflections in the area since the recorded stresses are only up to 30 percent of yield. Away from this region very low stresses were measured. This suggests that the stress redistribution occurred within two or three times the width of the dent on either side.

Stresses recorded in the adjacent panel in line with the dent area are shown in Figure 3.19a. Stress values here indicated outward deflections and bending which was observed on the deflection measurements of the series 2 models, where these deflections were monitored. Reversal of the stresses occurred immediately outside the dent as expected. These stress distributions can be summarised in a schematic representation showing the mode of deformation with the associated stress condition (Figure 3.20).

In Figures 3.21 – 3.24 where the stress distributions of models CY-2, 3, 4 and 5 are plotted, similar trends to those observed in model CY-1 can easily be identified. In the first two gauge locations (from 360° towards 0) the two reverse bending situations are obvious. The third bending reversal, as seen in model CY-1, is not clear in these models. This is attributed to the fact that less circumferential positions were monitored in these tests, thus missing the latter point. The stresses in the rest of the circumference are again very small, indicating the very local nature of the response to denting.

3.3.1.3 Residual Stresses

After removing the load, a similar stress distribution to the one recorded at peak load was observed in spite of the partial degree of elastic unloading. These stresses for model CY-1 are plotted in Figures 3.18b and 3.19b. The highest stress unloading seems to occur in areas away from the dent near values of θ of 43° and 315°. where stresses below yield were originally observed at peak load. Where full yield stresses had been recorded at peak no unloading was observed. This, however, may not be a completely true picture of the residual stresses at these locations, but a result of the inability of these highly strained gauges to follow the unloading path. In Chapter 4 the finite element analysis of the denting process suggests that some unloading actually occurred at these gauge locations.

Similar residual stress distributions are plotted in Figures 3.21b, 3.22b, 3.23b, 3.24b for model CY-2, 3, 4 and 5. Full yield stresses are observed inside and immediately

adjacent to the dent zone while smaller stresses of up to around 30 percent of yield are recorded further away from the dent. In the rest of the circumference the stress levels are very small, with maximum values of 5 percent of yield in model CY-3.

3.3.1.4 Energy Absorption During Denting

In order to relate the results of these denting tests to real cases of impact damage and to the code requirements, the energy input during denting has been calculated. This was obtained by integrating the load versus maximum deflection curves. The resulting energy levels are plotted against maximum radial deflection in Figure 3.25. The curves, as expected, show the same trends as those of the load indentation responses, with the least energy being involved in denting the more slender shells. Final energy absorption ranges from 7 to 30 KN mm for this series of tests. These energy levels when scaled to full-scale geometries [120] were found to correspond to operational impact as defined by DnV [21,22].

3.3.2 Initial Imperfections

Initial imperfections of all the models were measured at forty circumferential positions per panel, before and after the denting process. The readings were analysed by a computer program using a least squares fit to determine the deviations from a best fit cylinder. Imperfections were then analysed in terms of the following Fourier series.

$$w_o = \sum_{m=1,2,\dots} \sum_{n=0,1,\dots} [A_{mn} \cos n\theta + B_{mn} \sin n\theta] \sin \left[\frac{m\pi x}{L} \right]$$

The shell imperfection results are summarised in Table 3.3. Deviations from the best fit cylinder for each of the models before and after denting are presented in two ways. The first is in the form of a three dimensional surface of each cylinder unrolled from the weld line. This is for general visual appreciation of the imperfect form. Secondly the imperfections of the forty vertical sections around the cylinder are presented. These results are shown in Figures 3.26 – 3.41 for models CY-2, 3, 4 and 5.

Figures 3.34 – 3.37 of model CY-4, for example, show the relative magnitude of the dent compared to the rest of the imperfections. In all models the inward deflections of the dent were accompanied by outward deflections of much smaller amplitude in a region circumferentially outside the indenter contact zone. These outward deflections can be seen in Figures 3.35 and 3.37 to extend over three or four sections on either side of the dent, while the dent itself appears in four to five sections. The outward deflections resulted from the high hoop compression created along the indenter contact line during denting and also from continuity of slopes in the area.

For mid-panel dented models, the presence of the ring-stiffeners did not prevent the dent from extending to the adjacent panel, but there was a distinct change of slope at the stiffener lines. Much sharper slopes were observed in the dented panel of the more slender models (high R/t) (e.g. CY-5, Figures 3.38 – 3.41), in comparison with the more stocky models (e.g. CY-4). It can be noted that the deflections of the adjacent rings and panels were smaller in the slender shells. The low bending stiffness of the shell plate, resulted in a more localised form of dent.

3.3.3 Axial Compression Tests

3.3.3.1 Load-Axial Deflection Response

In the failure tests under axial compression, the more slender shells (i.e. models CY-1, 2 and 5) exhibited a sudden failure while the stockier ones (i.e. models CY-3 and 4) suffered some significant reduction in stiffness prior to collapse. This was in line with the response expected of equivalent intact models. Load-axial deflection characteristics of these shells are shown in Figures 3.42 – 3.46. Collapse loads are shown in Table 3.1.

3.3.3.2 Stress Distribution

The axial stress distributions during the failure tests are presented in two ways. Firstly the stresses on the inside and outside surfaces at peak load are plotted separately. These plots illustrate the inward and outward bending that occurred in different areas of the model and especially around the dent. In the second type of plots average axial stresses are plotted at different load levels. These show the general axial stress distribution around the models.

As shown in Figure 3.47a of model CY-1, there was a pronounced difference between the inside and outside stresses, not only in the dent area but also in the rest of the model, which indicated high bending. At the centre of the dent and on the outside surface full yield tensile stresses were measured while the inside was in full yield compression, showing inward bending in the area. These stresses reversed outside the dent to represent outward bending. This was due to the presence of outward imperfections associated with the dent formation, as discussed in previous sections. Outward bending was also present in the rest of the circumference with the outside tension stresses reducing to very small values opposite the dent. This outward bending, seen in the intact areas of the shell, reflects the Poisson effect which is particularly pronounced in this narrow ring spacing model. The inside surface yielded in compression for most of the circumference and very high strains were actually recorded during the test, especially around the dent. In the second plot of Figure

3.47b the average stresses around the circumference can be seen. Very small axial stresses were measured in the dent area and this was attributed to the high bending in the region. Outside the dent the average compressive stresses increased rapidly until they reached yield at around 45 degrees from the centre of the dent. These peak stresses were caused by stress redistribution from the weak area of the dent towards the intact part of the shell. Stresses then gradually reduced to around 40 percent of yield diametrically opposite the dent. The general shape of the distribution was a reflection of the small load eccentricity caused by the presence of the weak zone of the dent. This resulted in an overall bending effect on the cylinder causing higher stresses towards the dent and lower diametrically opposite this.

In model CY-2 (Figure 3.48a) inward bending was indicated again at the centre of the dent by the high tension on the inside and compression on the outside surfaces. This changed rapidly outside the dent to illustrate the outward bending which continued all the way around the model, showing again the Poisson effect quite clearly. In the average stress plot (Figure 3.48b) low stress can be seen again through the dent followed by a peak compressive stress outside it. Stresses are then reduced gradually opposite the dent.

In model CY-3 a similar distribution could be seen (Figure 3.49a and b). In these plots, however, higher outward bending was indicated outside the dent (than in CY-2), which reduced the axial load-carrying capacity of the region, and shifted the peak of the average axial stress even further away from the dent (90° from the dent centre). The other feature of Figure 3.49a was that the Poisson effect was less than in model CY-2 which had the same ring spacing. This is justified in terms of the higher thickness of the shell in model CY-3.

In model CY-4 (Figure 3.50a) inward bending of the dent followed by outward deflections outside the dent could be seen, while in the rest of the circumference the shell cross-section was in pure compression. This would be expected in a model with this wider ring spacing. In Figure 3.50b the effect of the outward deflections in the area adjacent to the dent is shown very clearly by the reduction in axial stress at a θ value of about 330° . This caused two peaks in the stress distribution, one on each side of this region.

Model CY-5 (Figures 3.51a and b) developed high bending stresses inside and outside the dent of opposite sign which gradually reduced away from these areas. The average axial stress distribution indicated that little load was transferred through the dent. A peak in the stress can be observed immediately outside the dent area. This stress reduced towards the intact part of the cylinder.

From the results presented above the following common features could be seen in all the models. High bending occurred in the dent area, which allowed very little transfer of axial stress. The overall axial stress distribution illustrated the eccentricity effect of the weak area of the dent. This resulted in higher stresses close to the dent and lower stresses diametrically opposite. High outward bending was indicated outside the dent area, which reduced and shifted the eccentricity/induced peak stresses further away from the dent.

3.3.3.3 Failure Modes

For most of the models, failure in the central panel around the dent was accompanied by coincident failure in other areas of the model. The various modes obtained are described in detail hereafter.

Model CY-1 failed by an outward bulge of the centre panel (Figure 3.52). This is the mode expected in an equivalent intact model. In this case the effect of the dent was insufficient to overcome the strong Poisson effect between the closely spaced rings.

Model CY-2 failed in an outward bulge of the centre panel on one side of the dent. This was again the same mode that would occur in an undented model (Figure 3.54 – 3.56). Figure 3.53 shows the radial deflections of this model at different load levels during the failure test. Deflections recorded at sections around the dent were significantly larger than elsewhere in the model. The dent centreline section (no. 21) and two adjacent sections (nos. 20, 22) deflected inward and about two to three sections on either side (i.e. 17, 18, 19 and 23, 24, 25) deflected outwards. In the case of model CY-2 the development of the outward bulge can also be seen from sections 24 – 33. In the failure deflection plot (Figure) 3.54 this outward bulge spread to sections 34 and 35. This mode was accompanied by a small inclined buckle on the other side of the dent, together with some stiffener twisting and inward bulging in the adjacent panel.

Model CY-3 failed in a lobular form, spreading to the adjacent panel (Figures 3.57 – 3.60). The interesting feature here was that all these small buckles coincided in form with the dent itself, suggesting that the latter was in sympathy with a critical mode-form of the equivalent intact shell. Although the geometry of this model would normally dictate failure in an outward bulge, as for CY-2, there was no evidence of this latter mode. The dent in this case, while not being sympathetic with the normal critical mode as for CY-2, was 65 percent larger than in the latter model and induced failure in a higher order mode which had a comparable critical

load value. The significantly larger magnitude of the dent imperfection was sufficient in this case to induce the lobular failure pattern to develop.

Lobular buckling occurred in models CY-4 and CY-5 as would be expected for equivalent intact models with this wider ring spacing.

In the case of models CY-4 (Figures 3.61 – 3.64) very few buckles occurred in the centre panel on either side of the dent. The majority of them occurred in the outer panel with the first of these buckles developing on a line corresponding with the high stress region to one side of the dent.

3.3.4 Discussion

The results of the panel denting tests showed that the slenderness parameter (R/t), interacts with the ring spacing to radius ratio (l/R) to determine the lateral stiffness of the shell. As expected, a larger R/t value combined with a wider ring spacing gave lower lateral stiffness, highlighting the low resistance of the cylinder to collision load. Another parameter which clearly affects the lateral stiffness is the number of panels in the model. As the number of panels increases the lateral stiffness reduces.

The deflections occurring due to denting were inwards along the indenter contact zone but were accompanied by smaller outward deflections outside the dent in the circumferential direction. High longitudinal stresses occurred around the dent area and while some elastic unloading occurred, a significant level of residual stress remained in areas both longitudinally and circumferentially adjacent to the dent line.

In the axial compression tests the slender shells showed a sudden failure, while the stockier ones suffered some significant reduction in stiffness prior to collapse. This sort of behaviour is in line with equivalent intact shell behaviour.

The failure modes exhibited by the cylinders comprised a primary failure in the dented panel, accompanied by coincident failures in adjacent panels. The primary failure corresponded with the critical mode of an equivalent intact model. The latter observation is true for all models except for one. Model CY-3 failed in a lobular mode, while the expected failure mode was an outward bulge. This is explained in terms of the magnitude of the dent and its non-sympathetic form, which prevented the outward bulge mode developing.

The secondary failures which accompanied the centre panel main collapse mode appeared longitudinally adjacent to the areas of high stress concentrations. This suggests that they were triggered by the high stresses in the area.

The main influence of the dent appears to be in creating an area of weakness within the shell which can carry very little axial load. This causes an overall load eccentricity and results in high stresses outside the dent. These areas of high stress concentrations are responsible for triggering collapse. The exact location and level of these stresses depends on the weakness-induced eccentricity effect of the dent and the stress capacity reduction of the adjacent imperfect areas.

In model CY-5 there may have been an additional effect on the strength. The low lateral stiffness of the shell allowed a more localised form of damage to occur very much in sympathy with the expected lobular buckling mode. This suggests that the dent in conjunction with the adjacent outward imperfections had a further and more direct effect on the strength by acting as an extreme sympathetic imperfection. This imperfection effect of the dent may be more severe outside the range of geometries tested.

3.4 EXPERIMENTAL TESTING OF SERIES 2 MODELS

3.4.1 Model Geometries

Series 2 of the experimental programme was designed to investigate the possible effect of the dent on the ring stiffener stability. Dents were applied at or close to the middle ring, causing bending and possible rotation of the stiffener. Failure of ring stiffened cylinders through ring instability has been observed before under pressure loading. In those cases manufacturing initial imperfections in the rings can have a significant effect on the strength. Stiffener torsional imperfections resulting from impact damage can be much larger than those due to the manufacturing processes, making the stiffener failure mode a possibility even under axial load.

With the above considerations in mind it seemed important to investigate the effect of ring dents and dents close to the ring on the strength of ring stiffened cylinders. This was the purpose of this second series of tests.

Two basic geometries ($R/t = 267$, $l/R = 0.15$ and $R/t = 133$, $l/R = 0.5$) were selected and two models of each were tested. One model of each geometry was subjected to a ring dent and the other to a dent near the ring that would produce higher rotation of the ring. Detailed geometries of the models are tabulated in Table 4.

In these tests it was clear that the stiffener geometry had a primary role, and therefore tee stiffeners, representing normal fabrication, were employed. These were

designed according to DnV specifications, so that the behaviour of stiffeners appropriate to current practice could be established.

The choice of four panel models was made with the intention of applying the dent on the middle stiffener. This provided two panels on either side for longitudinal stress redistribution.

3.4.2 Model Fabrication

The model fabrication procedure followed was similar to that for series 1. The shell was rolled from a steel plate and cut to the right panel length sizes. The stiffeners, however, could not be made in the same way as for series 1, where the rings were trepanned out of a plate. In the current series the stiffeners were of tee-section. The option of welding the flange onto a flat ring (the web) which was originally conceived, was not possible due to the very small size of the stiffener. Hence, machining out of a solid ring was the only feasible option for these small scale models. Each ring was then welded between the two adjacent panels from the outside using T-butt welds. Heat treatment of the cylinders followed to remove the high fabrication induced residual stresses.

3.4.3 Strain Gauging

The stress distribution around the model was of interest for both phases of the testing, with more detail needed in the dent area where sudden changes in stress distribution were anticipated. With this in mind, cross gauges were located at six locations around the circumference, as shown in Figure 3.69. With one exception, these were all located at the same height along the cylinder, i.e. mid-panel below the middle ring where the dent was to be applied. At a second section, shown in the figure, another pair of cross gauges was applied inside and outside the shell at the adjacent mid-panel position. A number of small linear gauges were used on the middle stiffener in order to record the bending and rotational strains.

3.4.4 Material Testing

Four tensile coupons were tested for each model using the same procedure as for series 1. This was described in section 3.2.4 in detail. Results of these tests are tabulated in Table 3.5.

3.4.5 Experimental Procedure

The four basic operations of the experimental procedure were:-

- Initial imperfection measurements of the intact model.
- Denting tests.
- Initial imperfection measurements of the dented model.
- Axial compression test.

This procedure was similar to the one followed in series 1, with some modifications in the instrumentation. These changes are mainly highlighted in the next few sections.

3.4.6 Initial Imperfection Measurements

The transducer frame mounted on the axial rig was again used for measuring the initial imperfections of the models before and after denting. Measurements were taken at eight longitudinal locations per panel and forty circumferential locations. In the case of the near ring dented models an extra longitudinal set of readings was taken at the dent elevation. A least squares method was used to determine the deviations from the best fit cylinder.

3.4.7 Denting Tests

The test rig of series 1 and the 3mm indenter were used for the denting tests. In the light of information obtained from the series 1 tests it was felt important to change the transducer arrangement in order to obtain a more general view of the deflections during the denting. Only the middle panel deflections were monitored in the original arrangement of series 1. Two symmetry lines passing through the centre of the dent were assumed for the ring dents. Hence four rows of transducers were located above the middle ring scanning the deflections of a quarter of the dent as shown in Figure 3.70. For the near ring dents the assumption of longitudinal symmetry about the middle ring was no longer valid. Therefore, the four rows of transducers were rearranged to scan the two centre panels. The maximum depth of the dent was measured by an outside transducer recording the total movement of the indenter.

3.4.8 Axial Compression Tests

The axial compression tests were carried out in the same manner as for the models of series 1. A full description of the test rig, instrumentation and test procedure was

given in Sections 3.2.8.1, 2, 3.

3.5 EXPERIMENTAL RESULTS OF SERIES 2 MODELS

3.5.1 Denting Tests

3.5.1.1 Load Deflection Response

The load deflection responses of the models of series 2 subjected to ring dents are presented in Figure 3.71. The stiffness of CY-7, which was subjected to a near ring dent, was 8 percent lower than CY-6 and both models showed an appreciably greater stiffness than the equivalent mid-panel dented model, CY-4, of series 1. This was attributed to the relative stiffness of the panel and the ring to lateral loading. The same trend was only observed in the linear range of loading of models CY-8 and CY-9. In the post-elastic stage the near ring dented model, CY-9 became stiffer than CY-8. The reason for this is not clear.

Another observation on the results of the denting tests is that all loading curves in Figure 3.71 are similar. For this series of models the bending stiffness of the rings has the greatest influence on behaviour and for the models tested the effective ring inertias including plating were all similar. The shell parameters R/t and particularly ℓ/R had a much lower influence than for series 1.

Detailed radial deflection measurements taken at different levels of denting for models CY-6, 7, 8 and 9 are presented in Figures 3.72 - 3.75 (see also Figure 3.70 for transducer arrangement). It can be seen from the graphs that the deflections were mainly in the form of inward displacements along the indenter contact zone with some outward deflections just beyond the contact area. There were no major differences in the general geometry of the dent shapes between the two series.

In the case of the central ring dents the stiffener did not suffer any significant rotation due to denting. The near ring dents were accompanied by some stiffener rotation with maximum values of 9° and 10° (models CY-7 and CY-9 respectively) in the residual state.

Elastic unloading occurred for all the models of the order 25-30 percent of maximum deflection. Higher unloading was observed in models CY-8 and CY-9 with the higher R/t values.

3.5.1.2 Stress Distribution

Axial stress distributions around the models are plotted in Figures 3.76-3.78 for

models CY-6, 7 and 8 respectively. Stresses for model CY-9 are not presented because there appeared to be an error in the wiring of the gauges during the tests. In model CY-6 (Figure 3.76) tension was observed on the outside and compression on the inside at the position above the centre of the dent. The tensile stress was higher than the compressive stress which indicated outward bending and stretching in the area. Outside the dent the compressive stress on the inside surface peaked to full yield stress while the outside tension reduced. Further away from the dent the stresses reduced to very small values. A similar distribution can be seen in model CY-7 (Figure 3.77). The signs of the stresses observed around the dent area are not comparable to those observed in the series 1 plots (Figures 3.18 and 3.21-3.24) since those were plotted at a different level in relation to the dent. At this point it must be noted that while for series 1 models the stresses were measured close to the dent line, stresses in the second series were measured at a distance equal to half of the ring spacing. It has been shown from measurements taken in model CY-1 (Figures 3.18 and 3.19) that the stress distribution varies along the length of the model. Stresses measured in the adjacent panel of model CY-1, as seen in Figure 3.19, compare well with models CY-6 and 7.

Model CY-8, with narrow ring spacing, suffered much higher stretching above the centre of the dent, in relation to models CY-6 and 7. Both inside and outside stresses reached yield in model CY-8 and this was attributed to the higher level of denting applied to this model in relation to its thickness. The inside stress away from the centre changed from tension to compression and tension again, indicating outward followed by smaller inward deflections away from the dent.

3.5.1.3 Residual Stresses

After the load was removed some unloading occurred but high residual stresses remained in the area, with a distribution similar to the one observed at peak load. Compressive stresses on the inside and tensile on the outside remained after unloading in models CY-6 and 7 (Figures 3.76b, 3.77b). Tensile stresses on both phases were measured above the centre of the dent and lower compressive stresses outside the dent in model CY-8 (Figure 3.78b). Negligible residual stresses were observed in the rest of the circumference for all the models.

3.5.1.4 Energy Input

The energy input during denting was calculated by integrating the load indentation curves. These energy levels are plotted versus indentation in Figure 3.79 for the series 2 tests. As seen on the figure the energies involved in these four models are

very close, following the load deflection response. Again the maximum energy levels were found to relate to the operational impact defined by DnV [21,22].

3.5.2 Initial Imperfections

Imperfection data were analysed by a least squares fit method and expressed in terms of Fourier series. Maximum deviations from the best fit cylinder together with the dominant modes observed are tabulated in Table 3.6 together with DnV tolerances.

Initial imperfections of the models both before and after denting are plotted in Figures 3.80–3.95 on a cylinder by cylinder basis. Both three-dimensional plots and longitudinal sections are presented. All models showed a similar pattern of deflections due to denting. Inward deflections along the circumferential extent of the dent were followed by outward deflections in sections outside the dent. Dent induced imperfections extended beyond the adjacent rings which suffered deflections of about 1/5 of the dented ring.

3.5.3 Axial Compression Tests

3.5.3.1 Load–Axial Deflection

Load axial deflection responses recorded during the failure tests are presented in Figures 3.96–3.99. All models failed abruptly. The more stocky models CY–6 and CY–7, unlike model CY–4 of the first series, did not exhibit any loss of stiffness before failure. This could be attributed to the lower failure load of the former two models which corresponded to load levels on the linear path. Collapse loads of all models are shown in Table 3.4.

3.5.3.2 Stress Distribution

Axial stress distributions are presented in two forms. First, the inside and outside stresses are plotted separately at peak load and then the average axial stresses at different load levels are presented.

In Figure 3.100a of model CY–6, compressive stresses can be seen on the inside and tensile on the outside, indicating outward deflections in the area. This agrees with the deflection measurements as seen in section 20 of Figure 3.104, where large outward deflections were recorded above and below the dent. Outside the dent, higher compressive stresses were recorded on the outside surface than on the inside, indicating small inward deflections which again can be seen in sections 24 and 25 of Figure 3.104. The inside and outside stress distributions in model CY–6 are exactly

the opposite to the stresses measured in other models, for example in CY-5. This is because the stresses refer to different longitudinal levels in relation to the dent. Inward deflections near the dent line change to outward deflections above it and the reverse applies to the adjacent imperfect area.

In the intact area of the cylinder, the load was carried in pure compression across the shell thickness. The average stress distribution (figure 3.100b) was characterised by the low stress through the dent and the peak stress outside this area. This was a result of the load stress shedding from the weak area of the dent to the intact shell. A reduction in the stress occurred farther away from the dent. The shape of the overall distribution showed that the presence of the dent induced an eccentricity of the load creating higher stresses on the dent side. This effect has already been discussed in the context of mid-panel dents of series 1.

Model CY-7 (Figures 3.101a and b) showed a very similar response to model CY-6 (of the same geometry), suggesting that the near ring dent which induced a high rotational imperfection in the stiffeners did not have any significant effect on the behaviour of the model. This is further discussed later in the thesis.

In model CY-8 (Figure 3.102a), low stresses were measured above the dent centre. Stresses increased in compression outside the dent with the inside surface carrying higher compressive stresses. This continued in the intact area of the shell due to the high outward bulging, attributed to the Poisson effect. This model had a very short ring spacing. The average axial stress plot (Figure 3.102b) shows that very little axial stress was carried above the dent area. This changed rapidly outside the dent where a peak stress was observed. Stresses then reduced gradually towards the intact part of the cylinder. The stress distribution was similar for different load levels.

The stress distributions in the models of series 2 were very similar to those for series 1 models with the main features being the very low axial load carried by the dent and an eccentricity effect on the whole distribution. Differences in the observed distributions are due to the fact that the stresses have been plotted at different longitudinal levels in relation to the dent. These results suggest that the large ring imperfections induced by the ring and near-ring dents did not have any dramatically different effect on the behaviour of the models.

3.5.3.3 Failure Modes

All models failed in the mode expected of intact equivalent shells. Models CY-6 and CY-7 exhibited lobular buckling involving the centre stiffener and the adjacent panel

(Figures 3.104–3.106 and 3.108–3.110). In the deflections of CY-6, measured during loading (Figure 3.103), there was some indication of the development of this lobular mode. This was more pronounced in model CY-7 (Figure 3.107).

Models CY-8 and CY-9, with the small ring spacing, failed in an axisymmetric outward bulge. In the case of CY-8, the panels above and below the dent failed on alternate sides of the longitudinal generator through the dent centre line (Figures 3.112–3.114). In CY-9 (Figures 3.116–3.118), only the panel which contained the near ring dent failed by an outward bulge.

In the deflection plots of both CY-8 and CY-9 (Figures 3.111, 3.115), there was clear evidence of the development of the failure mode. It is interesting to see that the dent centre line section 21 and the two adjacent sections (20 and 22) deflect in an axisymmetric mode. Deflections could be seen to pivot around the straight longitudinal generators mentioned previously. It can also be seen that for the models with the narrow ring spacing, the dent line itself also acts as a pivot with deflections above and below the dent in the central panel developing in opposite directions.

This does not occur in the case of models with the wider ring spacing where the dent progressively moves inwards under increasing axial load. The following explanation has been formulated for the above observation. The dent line has a sharper curvature in the case of models with narrow ring spacing, and it is therefore more likely to act as a pivot. Furthermore, the Poisson induced outwards expansion is stronger for these shells, hence resisting the inward deflections and enhancing this rotational effect.

3.5.4 Discussion

The lateral stiffness of the models to ring or near ring dents is mainly determined by the bending stiffness of the ring and its associated plating. Unlike mid-panel dents the radius to thickness and length to radius parameters are not of major importance.

Ring dents result in pure bending of the rings with no rotation, while near-ring dents are associated with bending and some significant rotation of the rings. This lateral rotation of the rings does not, however, develop any further to cause stiffer failure when the cylinder is tested under axial load. Therefore, the presence of this high initial rotation has little effect on the ultimate collapse of the model.

The presence of the dent creates high bending stresses inside and outside the shell. These areas are therefore very weak in terms of axial load-carrying capacity. As a

result a load eccentricity is caused which creates higher stresses in the hemicircle to the side of the dent where collapse is triggered. The exact location of the peak stresses and their magnitude are both affected by the outward deflections developing in the imperfect adjacent areas.

Comparisons with the mid-panel dent effect, as seen in the series 1 results, show no significant difference in the general response to axial loading. This suggests that the dent location does not have a significant effect on the behaviour. Further confirmation of this observation will be provided from the numerical correlations and the evaluation of the actual dent-induced strength reductions.

3.6 CONCLUSIONS

Experimental techniques have been successfully applied in investigating the effects of impact damage in the form of a local dent on the strength of ring stiffened cylinders. Experiments performed on a number of small scale models provided valuable information on the subject and the main conclusions from that work are summarised below.

If a ring stiffened cylinder is dented at a mid-panel position its lateral stiffness is dependent on the slenderness parameter (R/t) and the ring spacing (ℓ/R). The lateral stiffness reduces with increasing R/t and ℓ/R . These parameters are not as important in the case of a ring dent where the bending stiffness of the ring stiffener with the associated plating mainly determine the response. Another parameter which is important is the total length of the model. As the number of panels increases the lateral stiffness is reduced.

The deflected shape takes the form of inward deflections in the dent area, which spread across the whole length of the model. These are accompanied by smaller outward deflections circumferentially outside the dent. The stiffeners adjacent to the dent deflect appreciably but no significant rotation occurs except for near ring dents.

In the axial compression tests, the slender shells showed a sudden failure, while the stockier ones suffered significant reduction in stiffness prior to collapse. This behaviour is in line with the equivalent intact shell behaviour.

The failure modes are characterised by inward buckling inside the dent and outward buckling outside it. The rest of the shell circumference fails in the critical mode of an equivalent intact model. If the dent is non-sympathetic to the natural mode of buckling, it is possible for it to trigger a different mode of collapse; if the dent is

of sufficient size and is in the form of a closely associated critical collapse mode. Stress concentrations caused by the dent also trigger secondary collapses.

The location of the dent in relation to the rings does not have any significant effect on the strength. Axial compression tests on ring dents which included large initial rotations of the stiffener showed no sign of stiffener distress.

The main influence of the dent on the behaviour of the cylinder appears to be in inducing a zone of axial weakness and load eccentricity. These result in peak stresses in areas adjacent to the dent which are responsible for triggering collapse. Numerical studies later in the thesis show that collapse is triggered when these peak stresses are close to the equivalent intact model collapse stress. Their exact location and magnitude appears to be an interaction between the weakness-induced load eccentricity effect of the dent and stress capacity reduction of the imperfect adjacent areas.

If the dent is sympathetic to the natural mode of failure an additional stress reduction effect may be induced by the dent.

Table 3.1: Model geometries and collapse loads – Series 1
(Centre panel dents)

Model	ℓ/R	R/t	R	t	H_w	t_w	d/t	Collapse σ_d/σ_y
CY-1	0.15	267	160	0.6	12.8	1.6	5.7	0.58
CY-2	0.25	267	160	0.6	4.8	0.6	5.5	0.73
CY-3	0.25	133	160	1.2	6.72	0.84	4.5	0.75
CY-4	0.50	133	160	1.2	4.8	0.6	5.6	0.78
CY-5	0.50	267	160	0.6	4.8	0.6	9.0	0.59

Table 3.2: Materials properties – Series 1

Model	σ_{yt} (N/mm ²)		E (kN/mm ²)		σ_{yd} (N/mm ²)	σ_u (N/mm ²)	σ_y (N/mm ²)
	Measured	Average	Measured	Average			
CY-2 L	349	344	220	223	336	413	354
	345		209				
	344		242				
	340		205		326	402	350
CY-3 L	346	341	216	222	332	450	351
	341		214				
	337		237				
	308		207		302	413	318
CY-4 L	332	324	220	219	322	443	334
	325		216				
	325		220				
	312		230		305	423	322
CY-5 L	349	349	231	233	343	417	359
	354		243				
	346		226				
	337		199		336	375	347

Table 3.3: Summary of initial imperfection data for shell panels – Series 1
(Values in mm; DnV tolerances [61] in brackets)

Mode	Perfect radius	Max. Outward reading, Δ_R^+	Max. Inward reading, Δ_R^-	Out of straightness	Dominant imperfection modes	
					m	n
CY-2	159.62	.42	-.47	.23 (.20)	1,3	2,3,4,5
CY-3	160.35	.76	-.80	.30 (.34)	1	2,3,4,14
CY-4	160.28	.39	-.45	.38 (.46)	1	2,3,4,8
CY-5	159.59	.56	-.55	.31 (.33)	1	2,3,4,5

Table 3.4: Model geometries and collapse loads – Series 2
(Ring and near ring dents)

Model	ℓ/R	R/t	R	t	H_f	t_f	H_w	t_w	d/t	Collapse σ_d/σ_y
CY-6	0.5	133	160	1.2	4.0	0.6	3.0	0.6	5.8	0.74
CY-7	0.5	133	160	1.2	4.0	0.6	3.0	0.6	6.2	0.70
CY-8	0.15	267	160	0.6	6.0	0.84	3.0	0.6	10.5	0.58
CY-9	0.15	267	160	0.6	6.0	0.84	3.0	0.6	10.6	0.59

Table 3.5: Materials properties – Series 2

Model	σ_{yt} (N/mm ²)		E (kN/mm ²)		σ_{yd} (N/mm ²)	σ_u (N/mm ²)	σ_y (N/mm ²)
	Measured	Average	Measured	Average			
CY-6 L	341	339	209	203	318	429	349
L	351		201				
L	325		200				
C	334		194		326	439	334
CY-7 L	409	352	208	208	321	433	362
L	345		211				
L	303		205				
C	357		211		357	453	357
CY-8 L	386	376	213	218	358	426	386
CY-9 L	373		221				
L	369		221				
C	354		206		356	427	354

Table 3.6: Summary of initial imperfection data for shell panels – Series 2
(Values in mm; DnV tolerances [61] in brackets)

Mode	Perfect radius	Max. Outward reading, Δ_R^+	Max. Inward reading, Δ_R^-	Out of straightness	Dominant imperfection modes	
					m	n
CY-6	160.39	.58	-.39	.43 (.46)	1	1,2,3,4,
CY-7	160.39	.44	-.79	.31 (.46)	1,2,3	2,3,4,5
CY-8	159.74	.31	-.28	.21 (.20)	1,3	2,3,12,16
CY-9	159.66	.24	-.31	.13 (.20)	1,2	2,4,10,16

STRAIN GAUGE LOCATIONS
MODEL CY-1

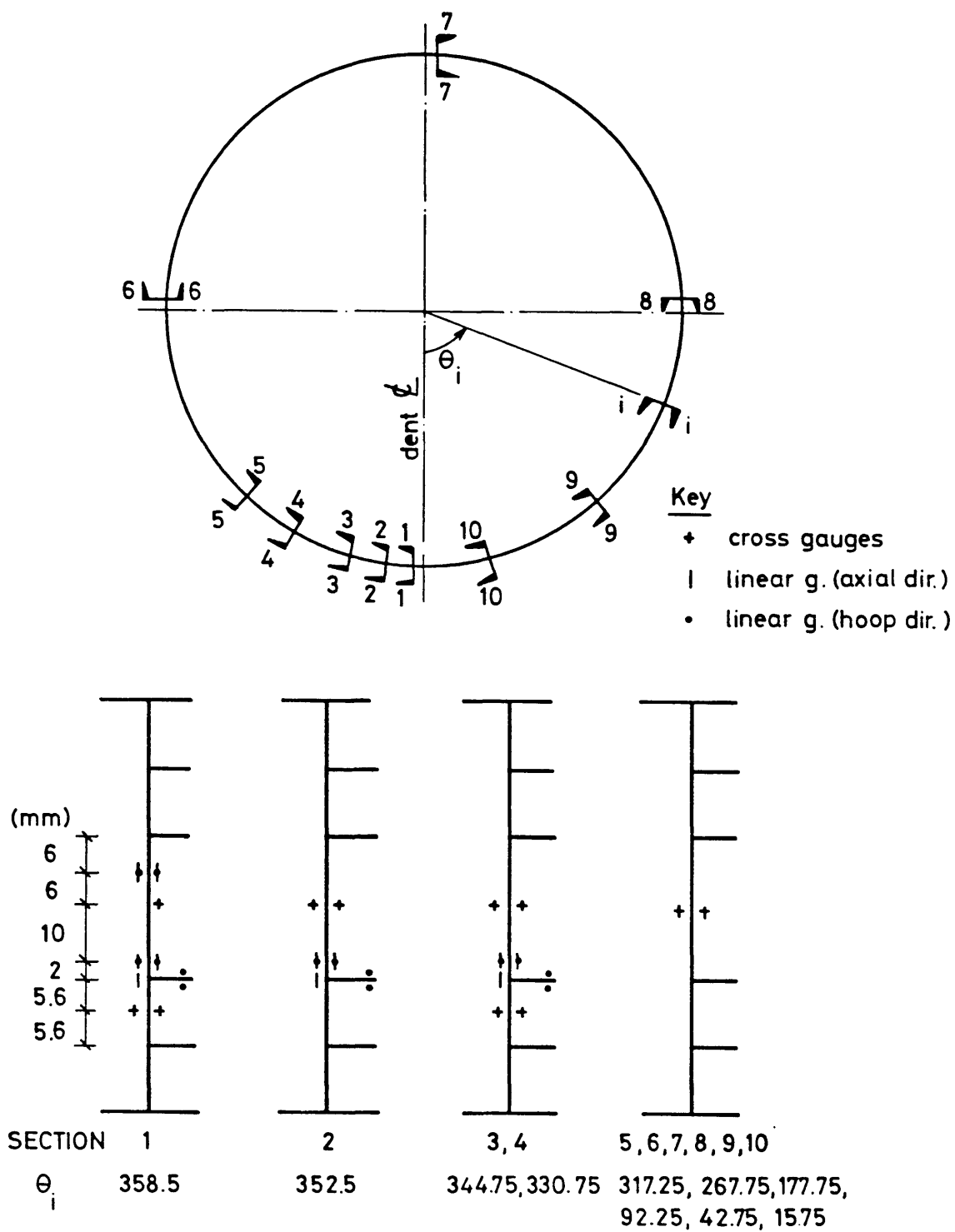


Figure 3.1

STRAIN GAUGE LOCATIONS
SERIES 1

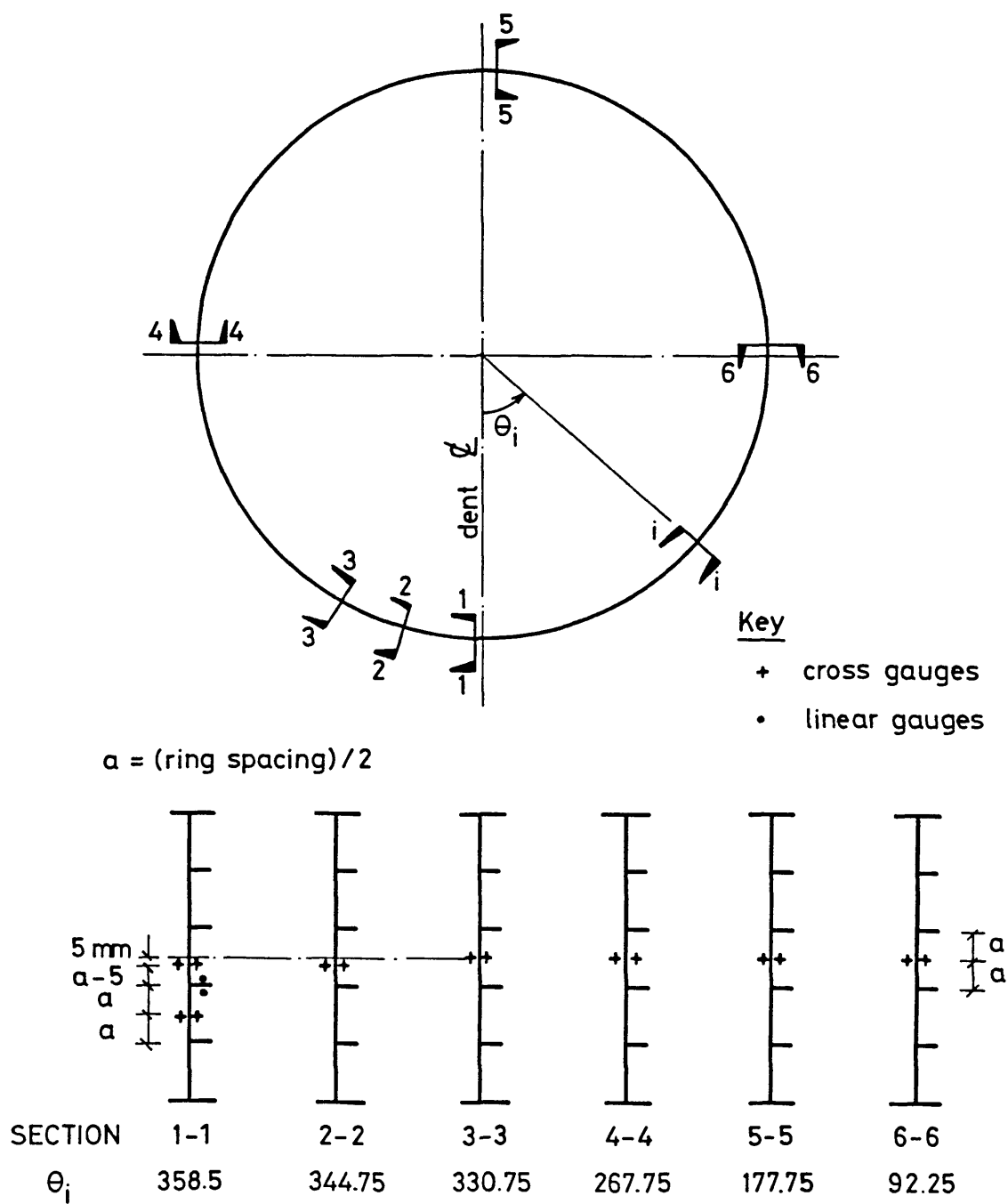


Figure 3.2

AXIAL COMPRESSION RIG

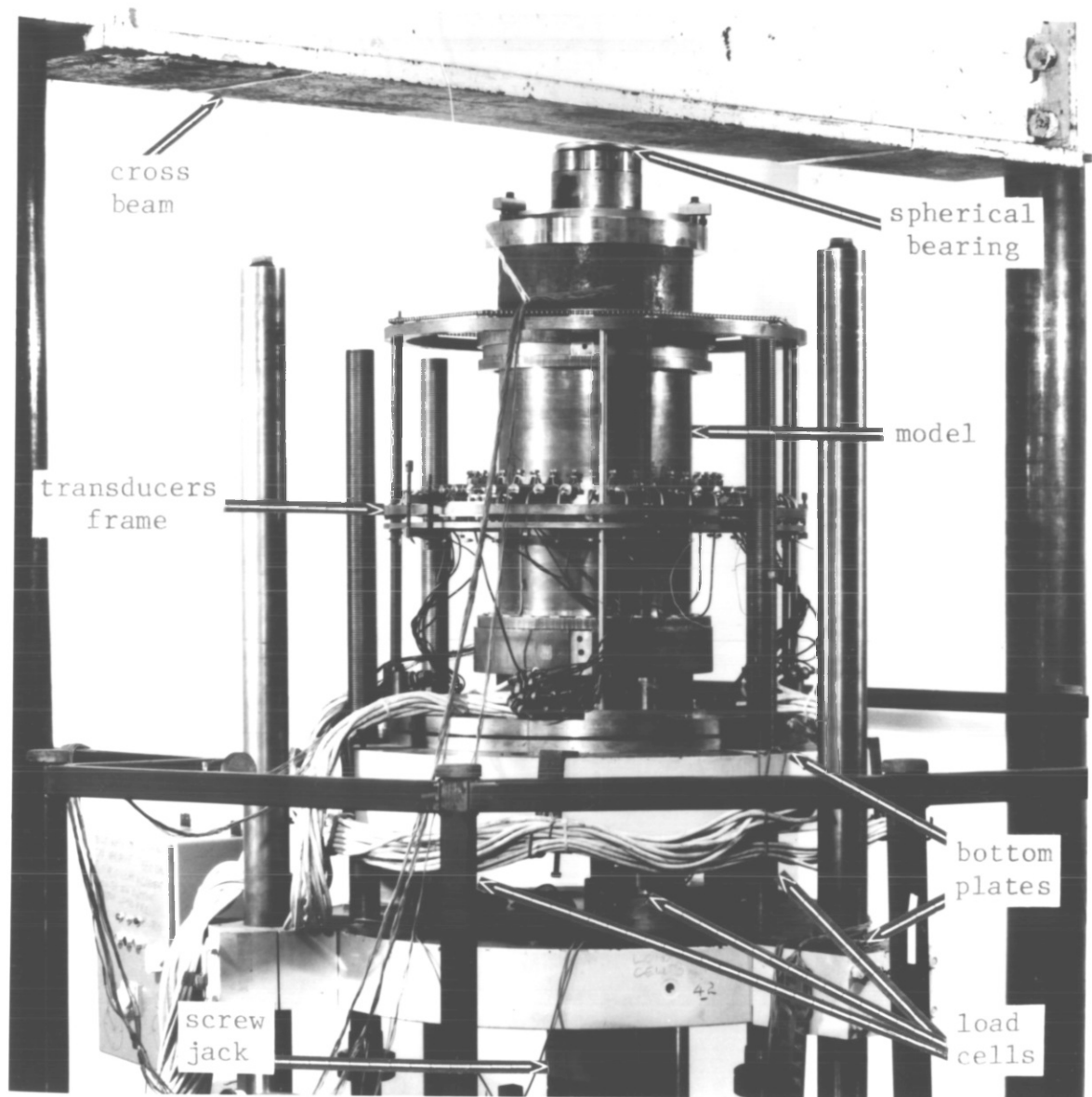


Figure 3.3

TWO VIEWS OF THE DENTING RIG
FOR MODEL CY-1

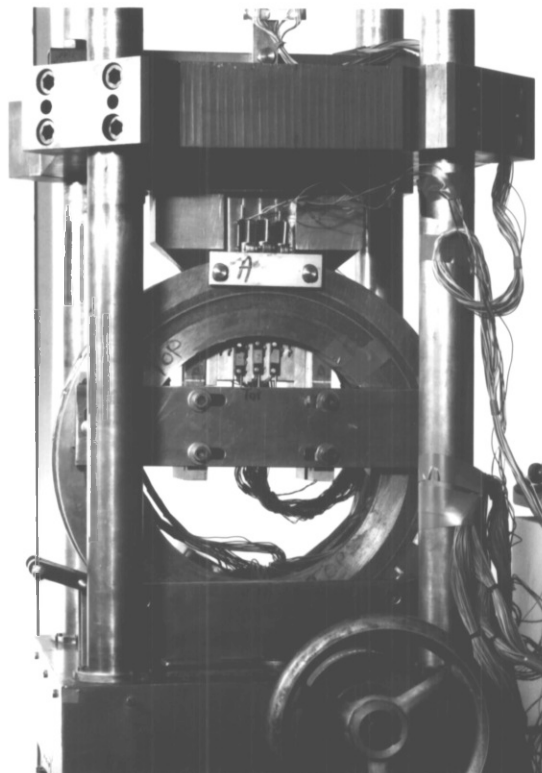
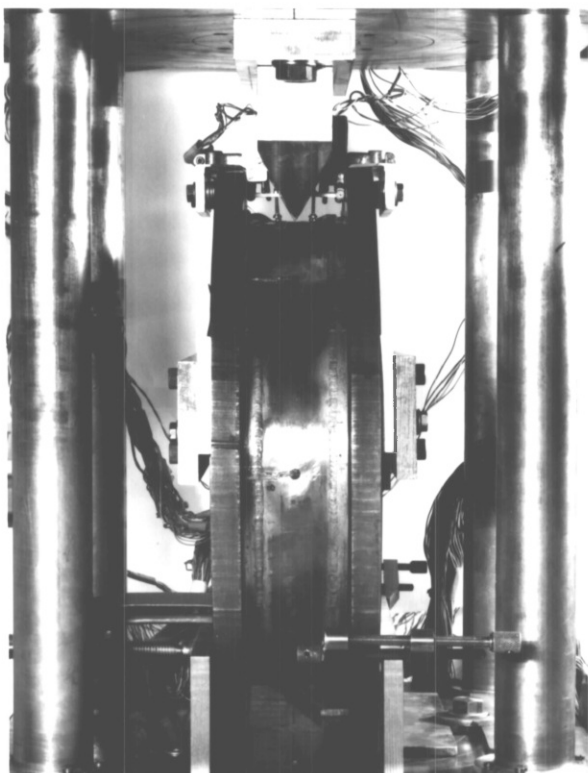


Figure 3.4

TWO VIEWS OF THE DENTING RIG

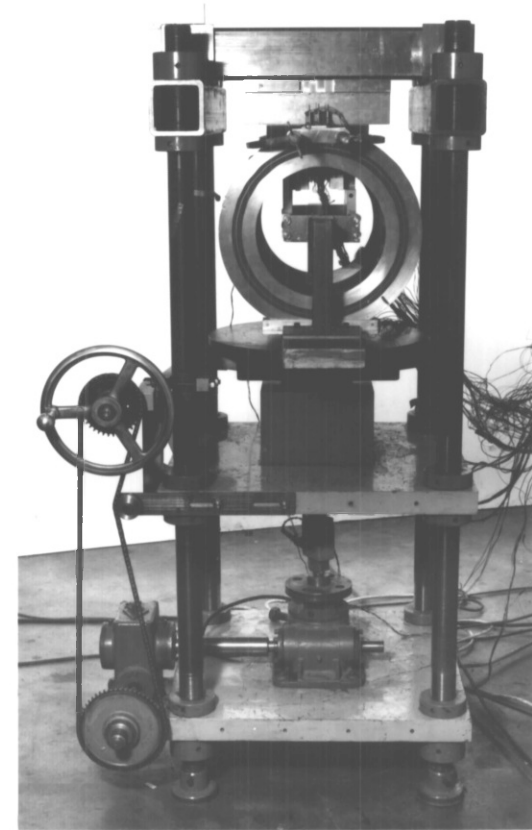
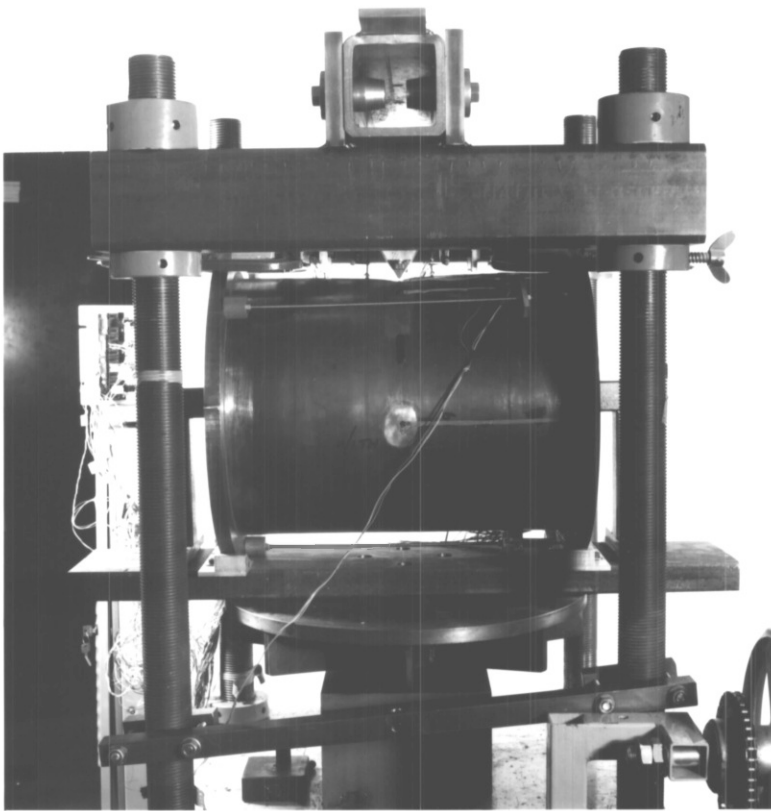


Figure 3.5

TRANSDUCERS LOCATIONS FOR DENTING TEST MODEL CY-1

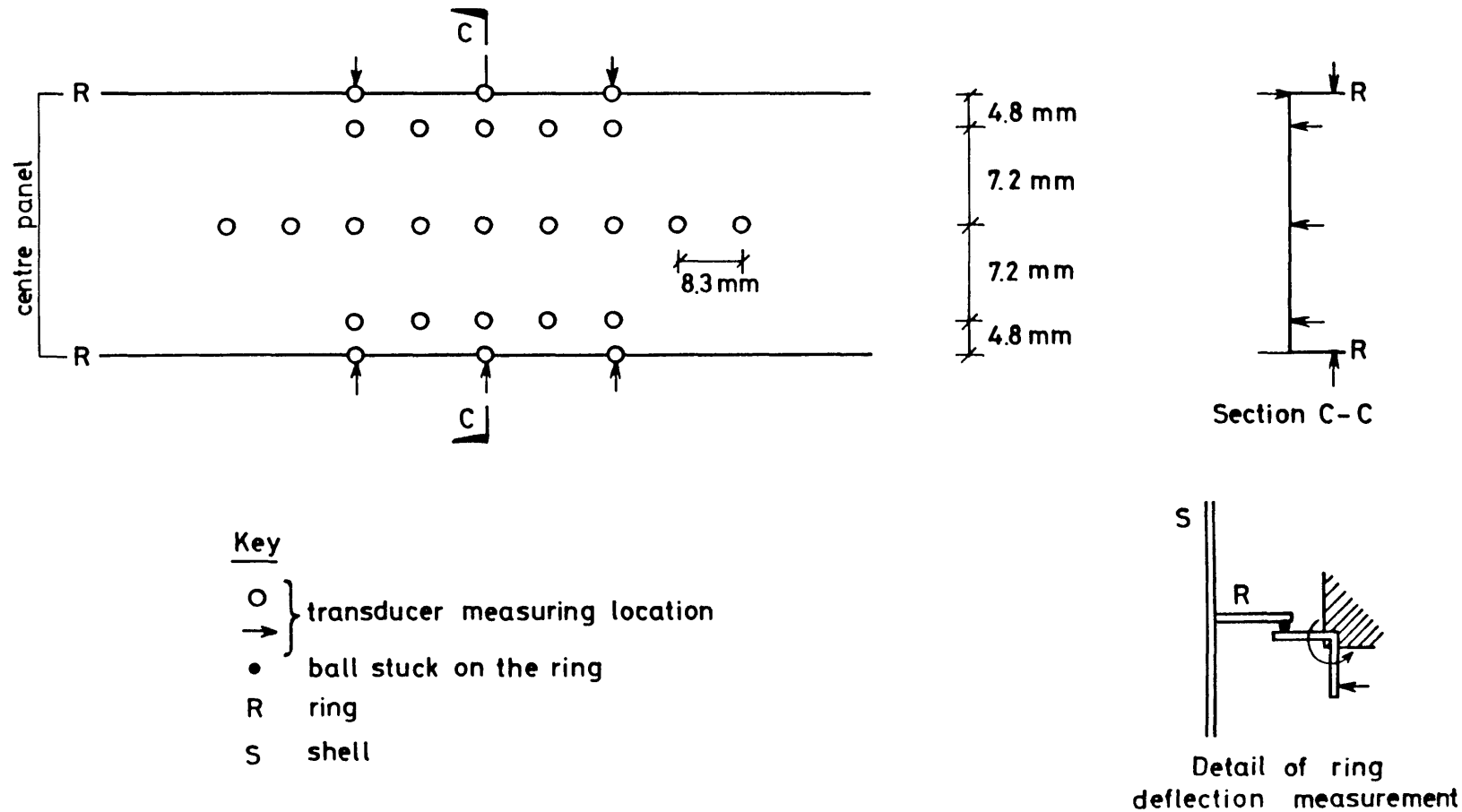
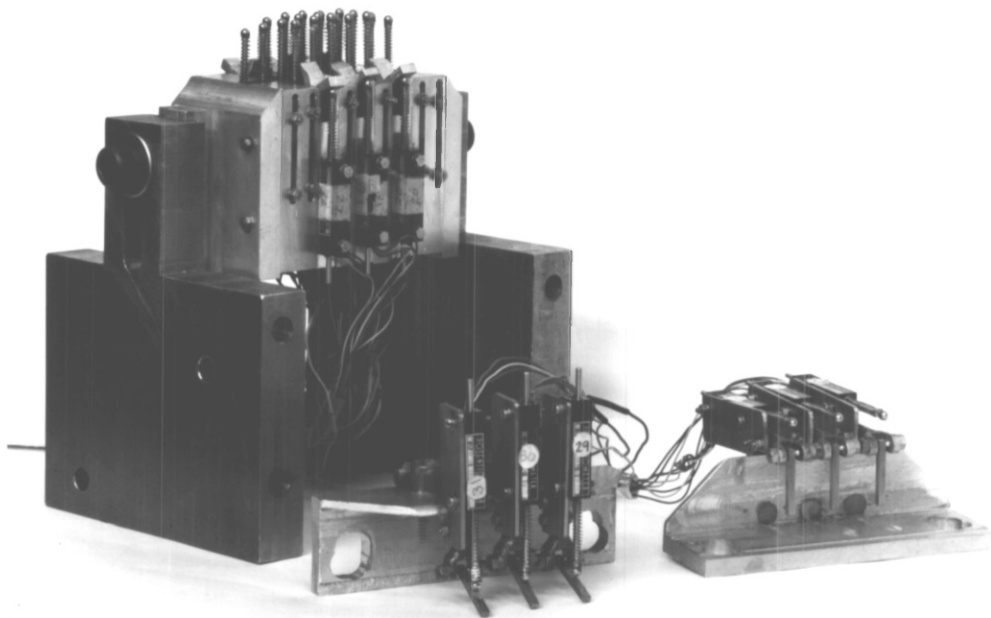
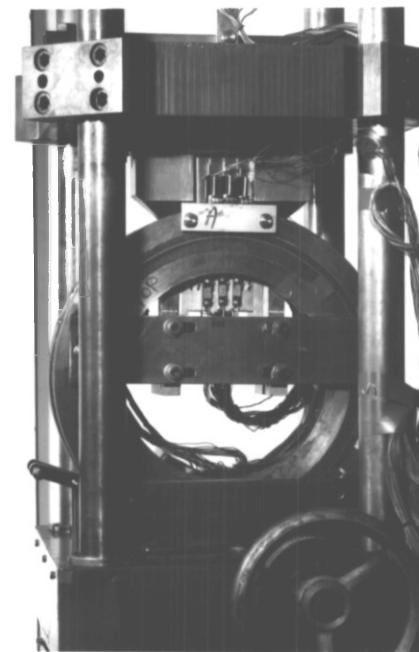


Figure 3.6

TRANSDUCERS INSTRUMENTATION FOR DENTING TEST
MODEL CY-1



(a) TRANSDUCERS BLOCKS



(b) ON THE RIG

TRANSDUCERS LOCATIONS FOR DENTING TESTS SERIES 1

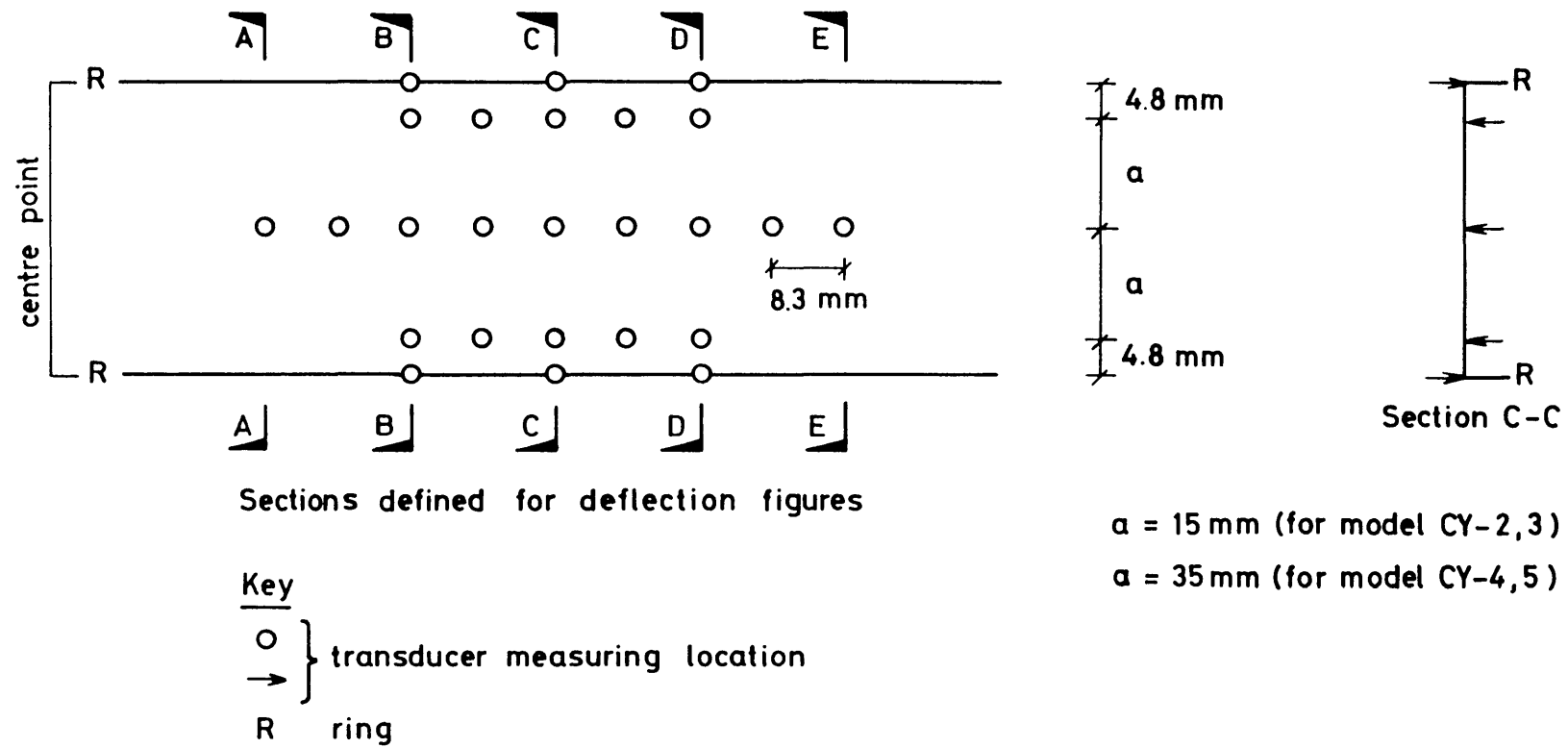
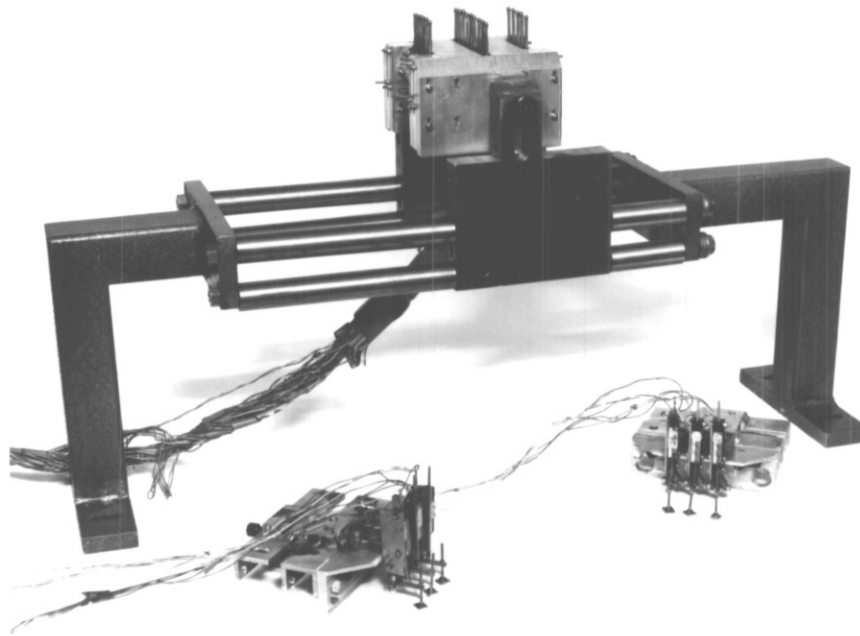


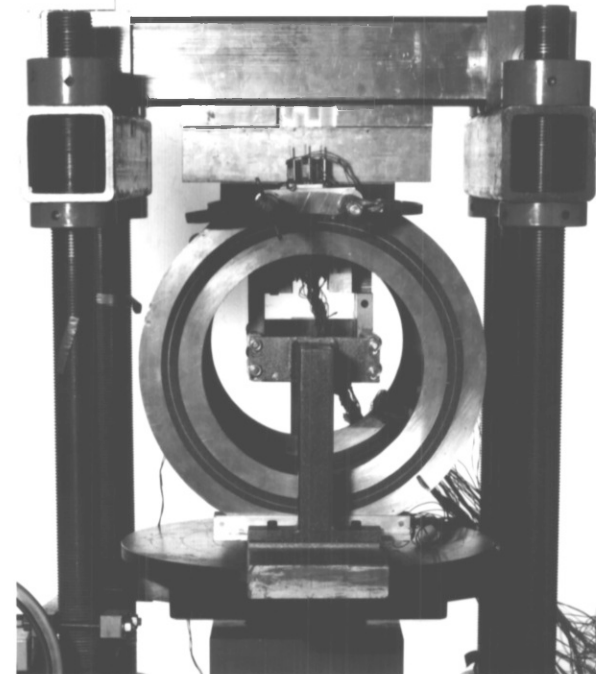
Figure 3.8

TRANSDUCERS INSTRUMENTATION FOR DENTING TESTS

SERIES 1



(a) TRANSDUCERS BLOCKS



(b) ON THE RIG

Figure 3.9

LOCATION OF DENT	TEST	R/t	L/R	t	$(d/t)_s$	t_s
— MID-PANEL	CY-1	167	0.15	0.6	8	1.6
- - - RING DENT	P1	"	"	"	"	"
- - - - QUARTER-PANEL	P2	"	"	"	"	"
- · - · - MID-PANEL	P3	"	"	"	16	0.6

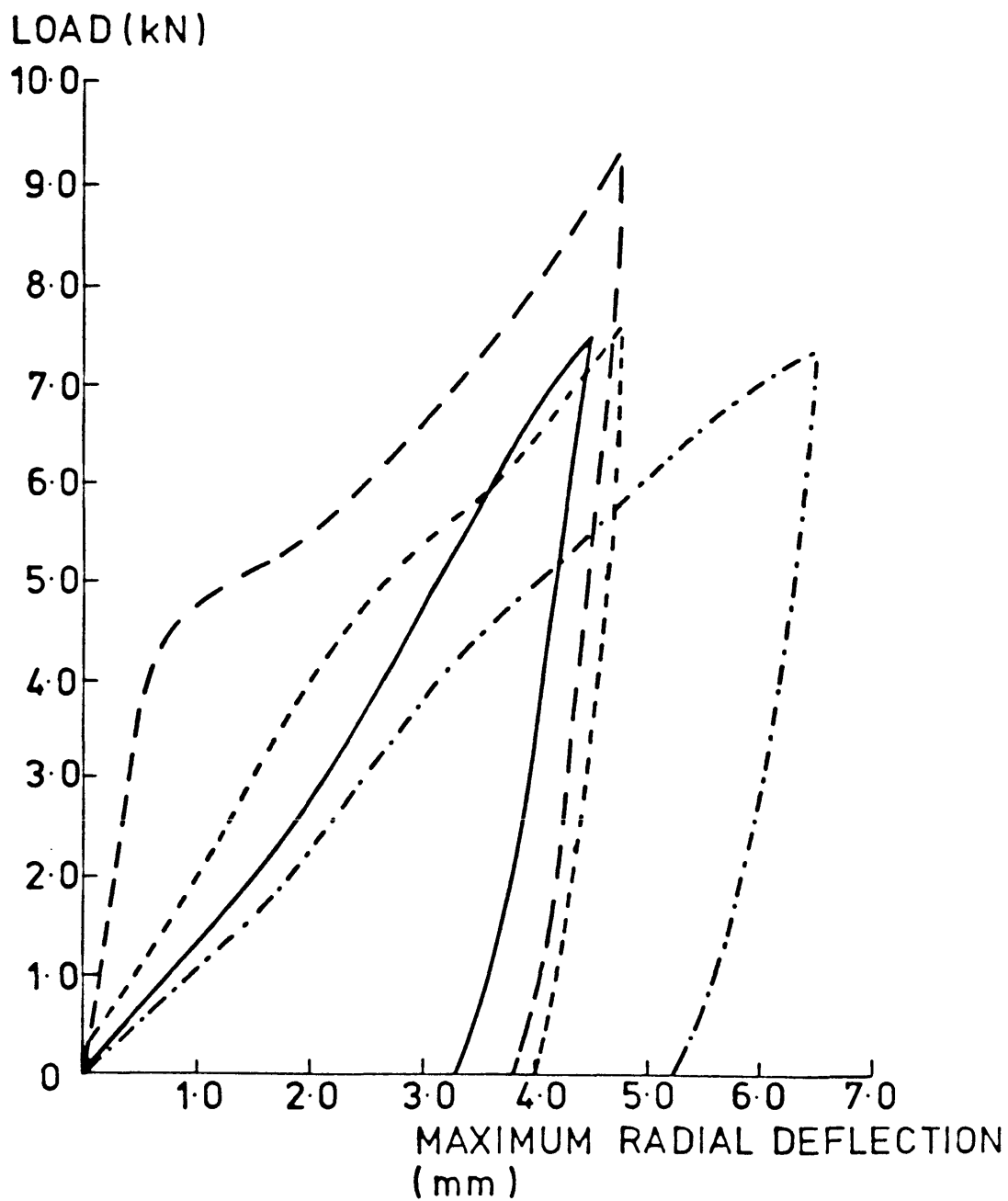


Figure 3.10

DENTED MODEL P3
INSIDE VIEW



Figure 3.11

RESIDUAL DEFORMATIONS FOR MODEL CY-1

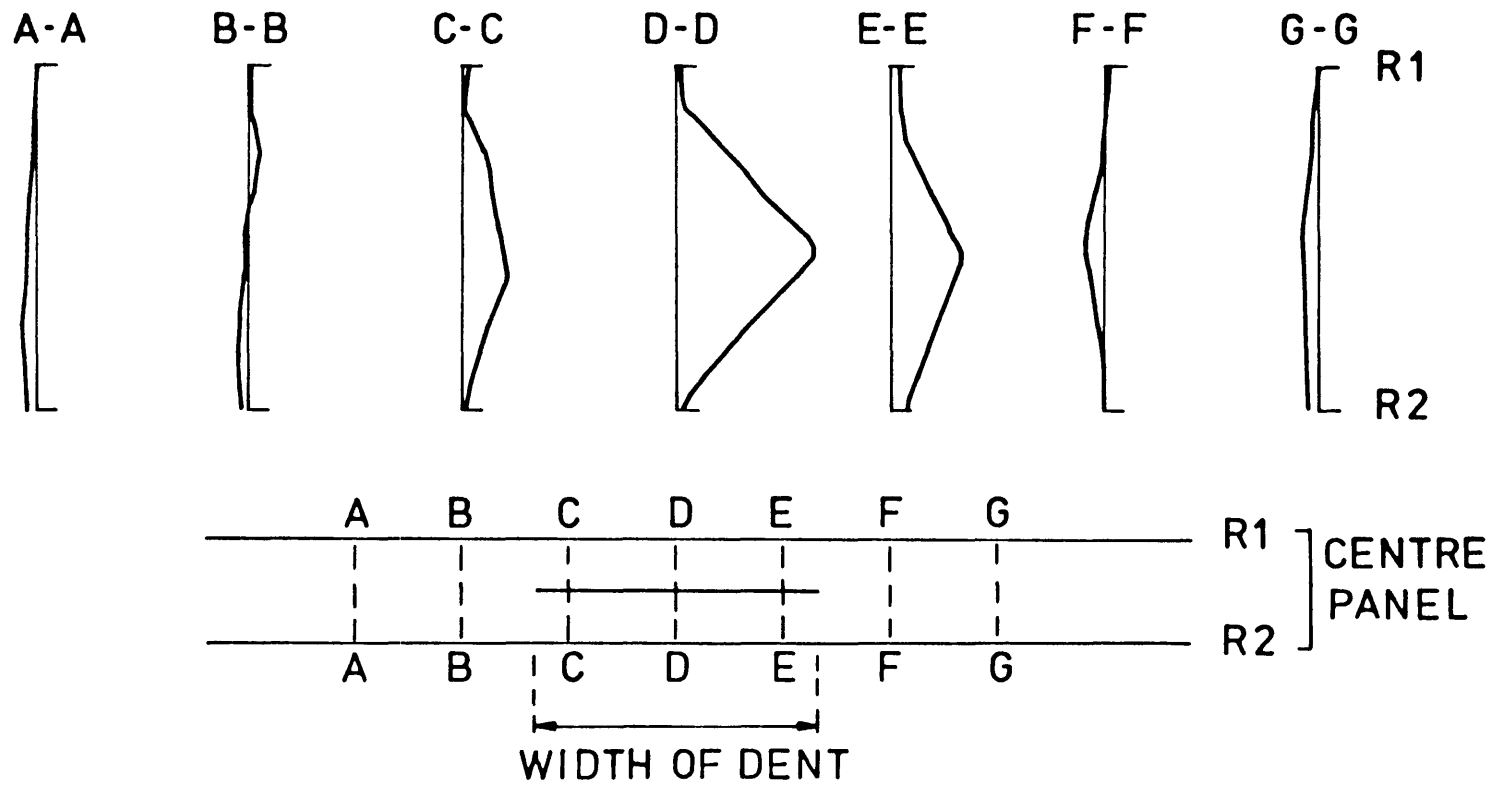
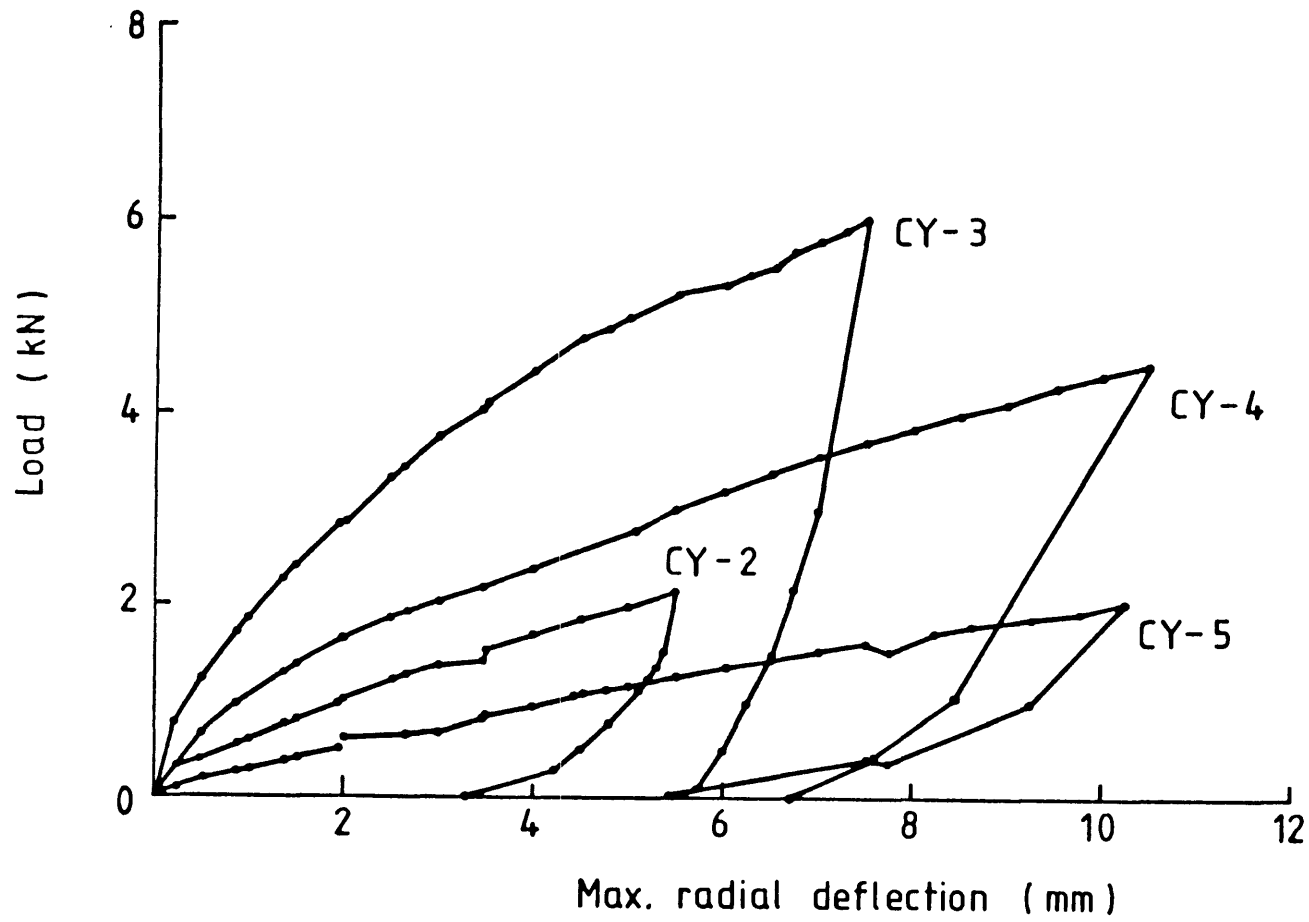


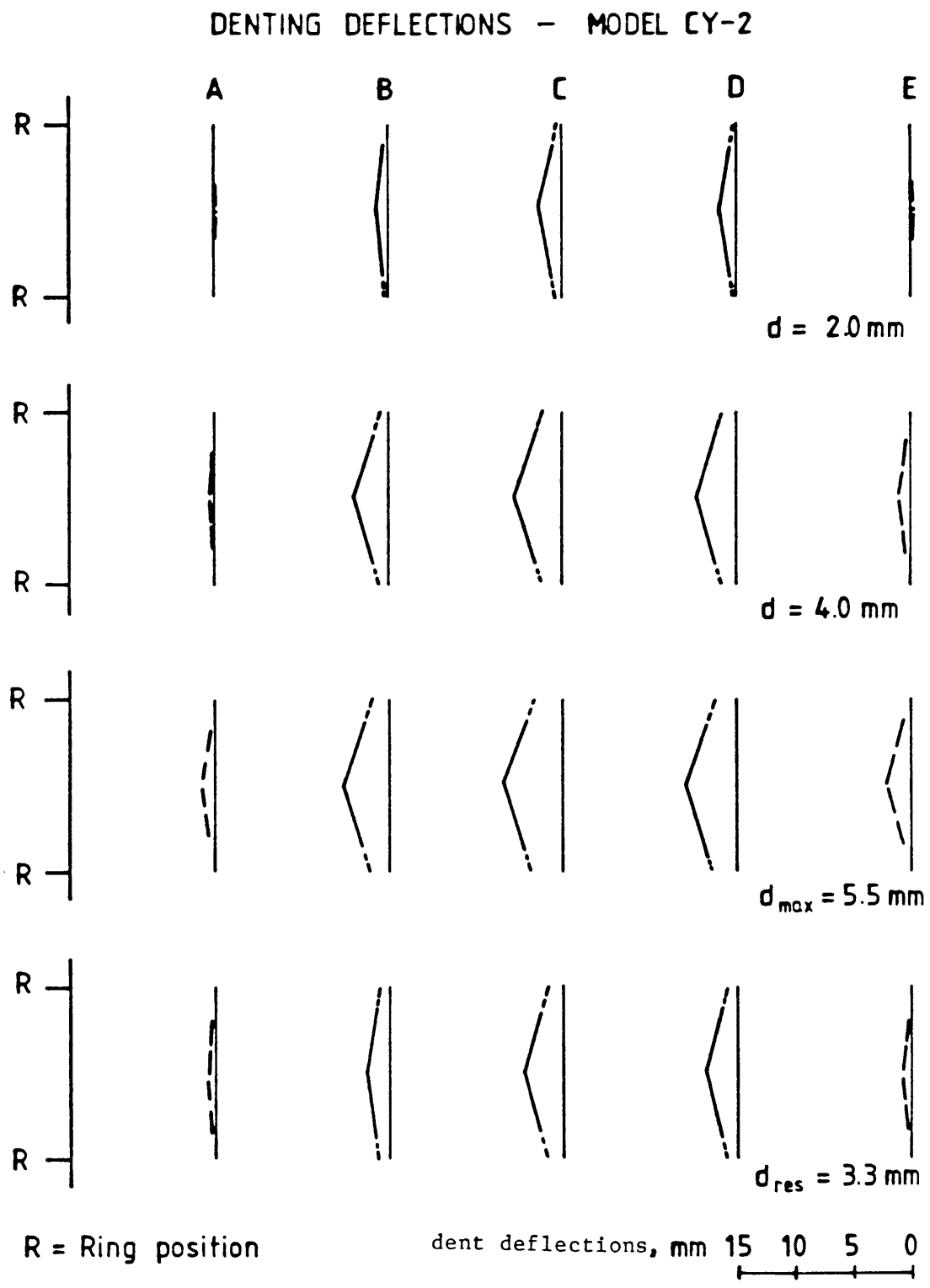
Figure 3.12

LOAD DENTING DEFLECTIONS - SERIES 1



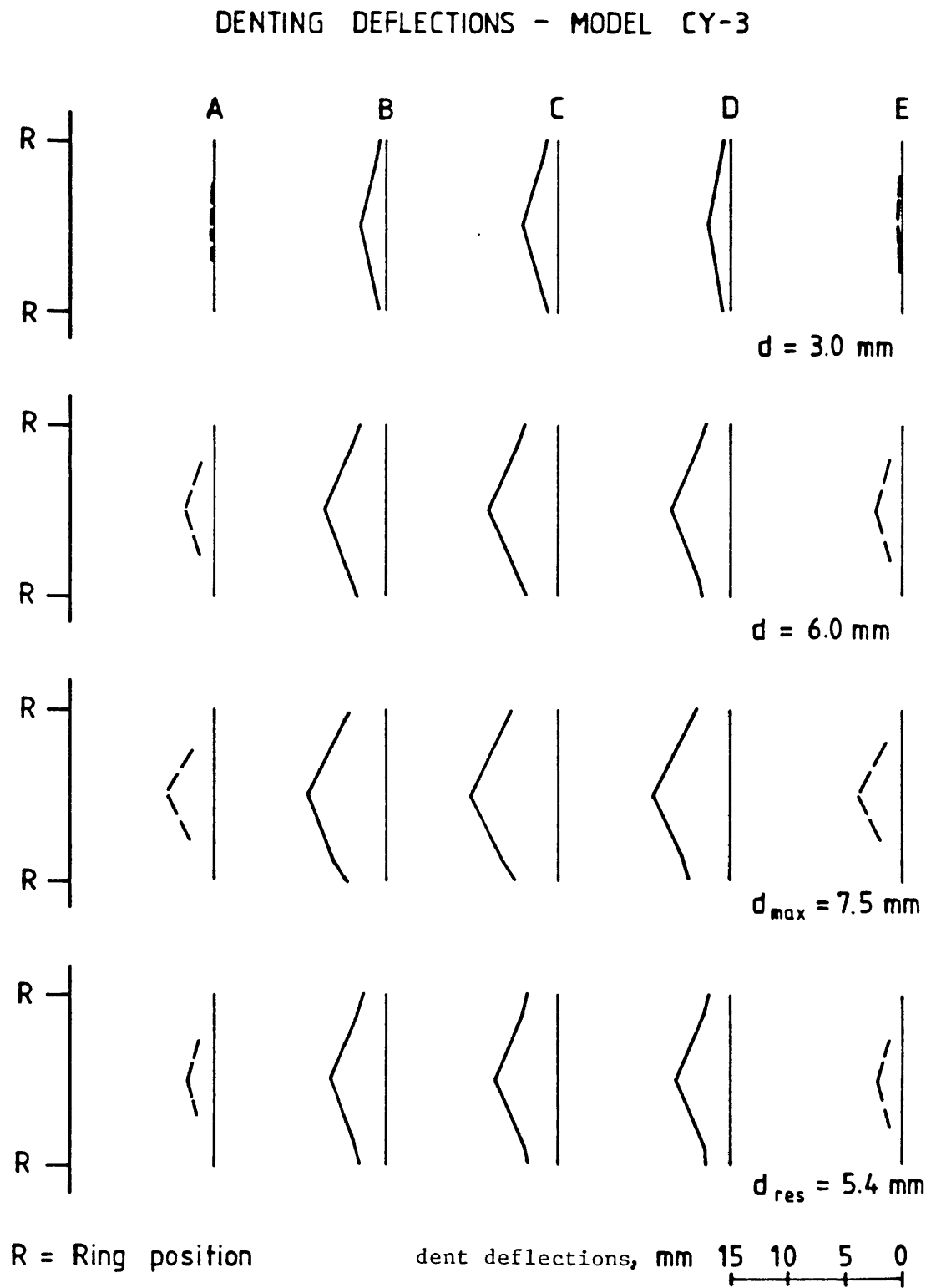
Model	l/R	R/t
CY-2	0.25	267
CY-3	0.25	133
CY-4	0.50	133
CY-5	0.50	267

Figure 3.13



For sections, see Fig. 3.8

Figure 3.14



For sections, see Fig. 3.8

Figure 3.15

DENTING DEFLECTIONS - MODEL CY-4

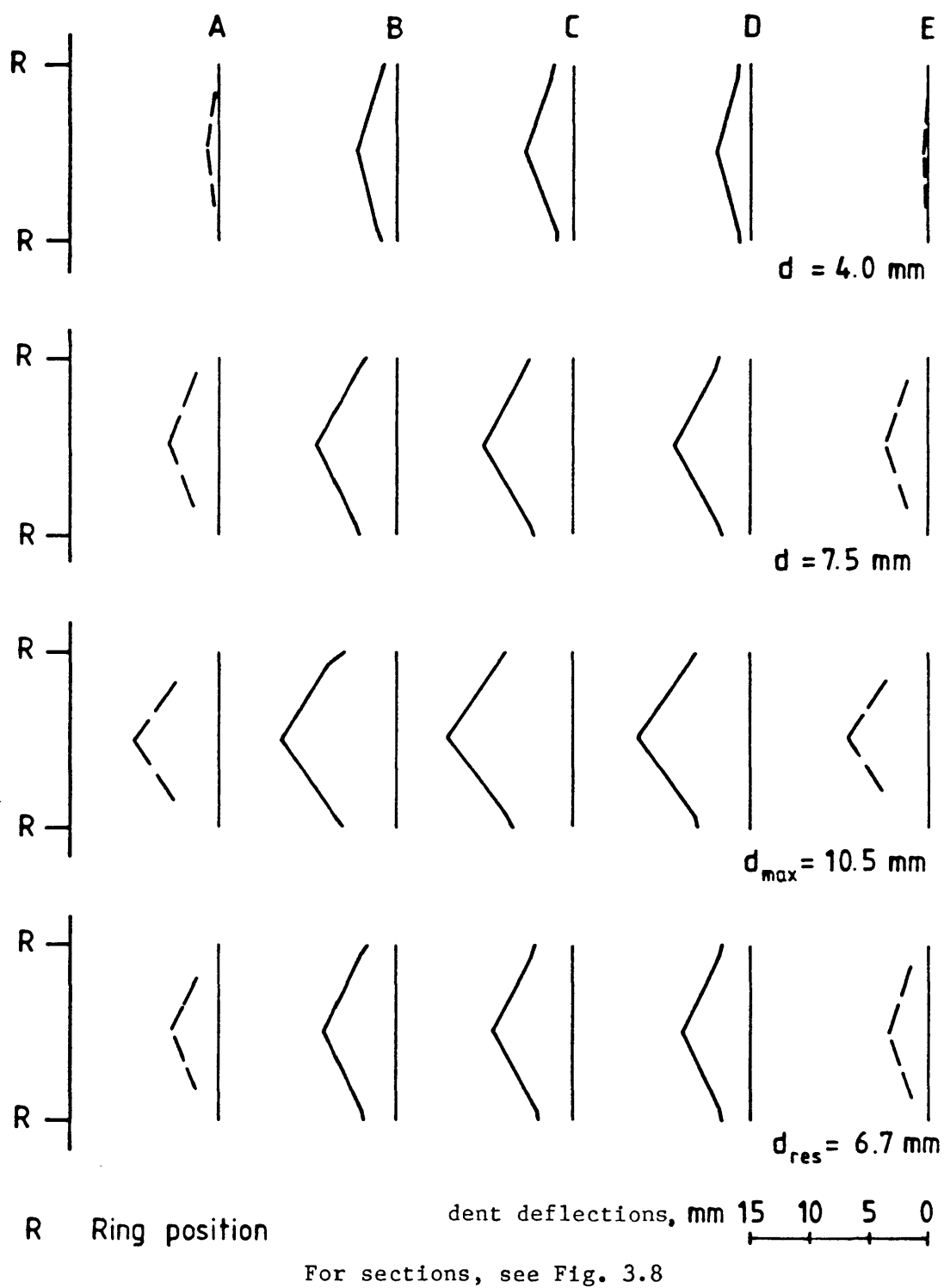
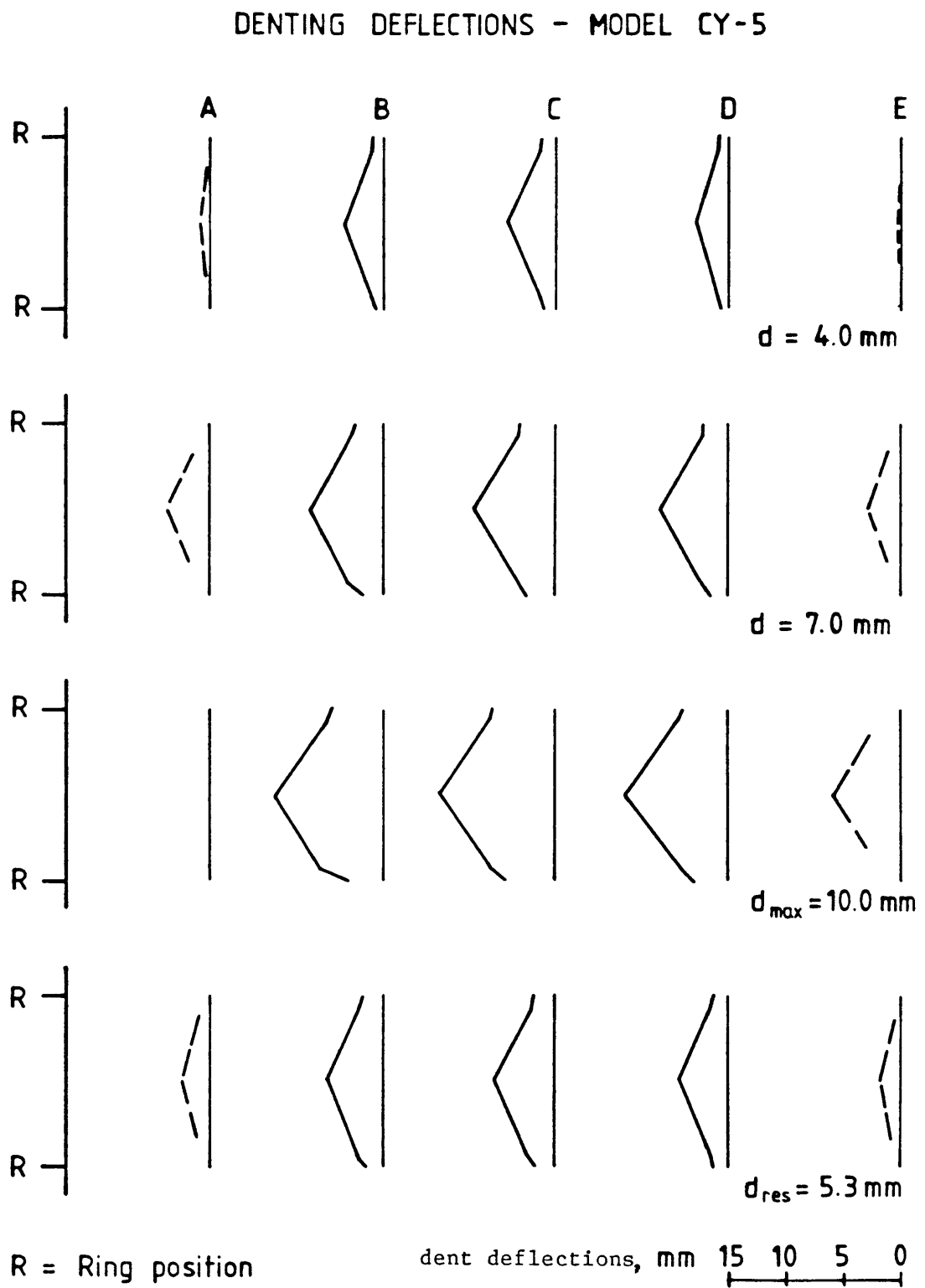
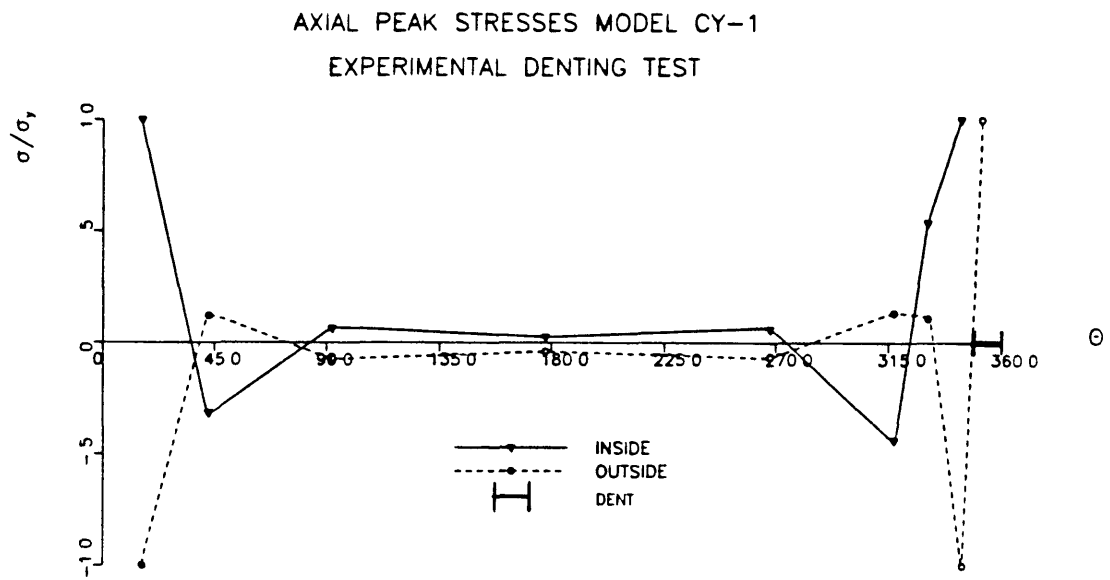


Figure 3.16

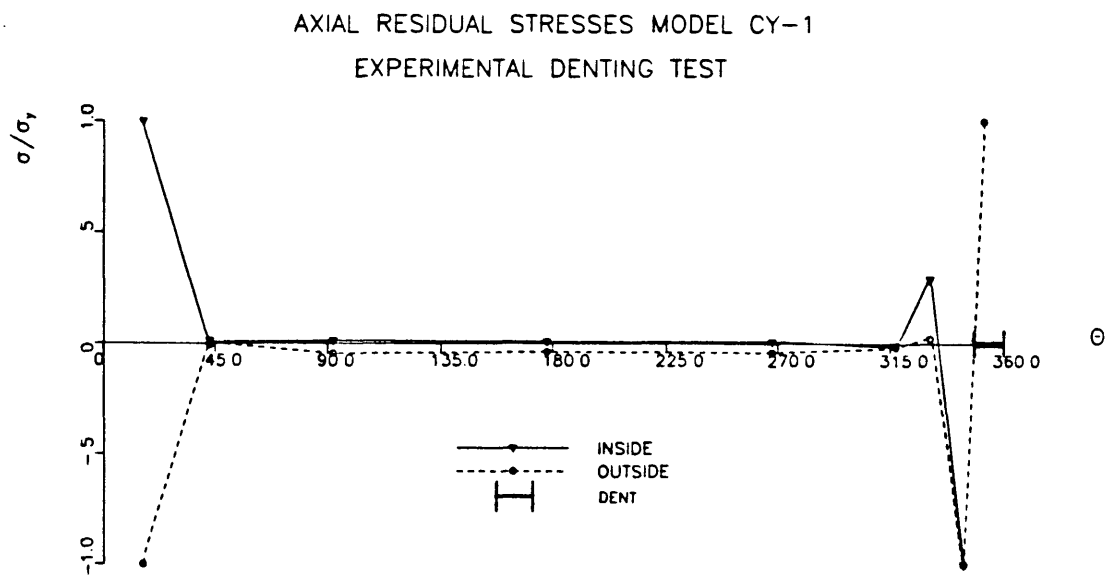


For sections, see Fig. 3.8

Figure 3.17



(a)



(b)

Figure 3.18

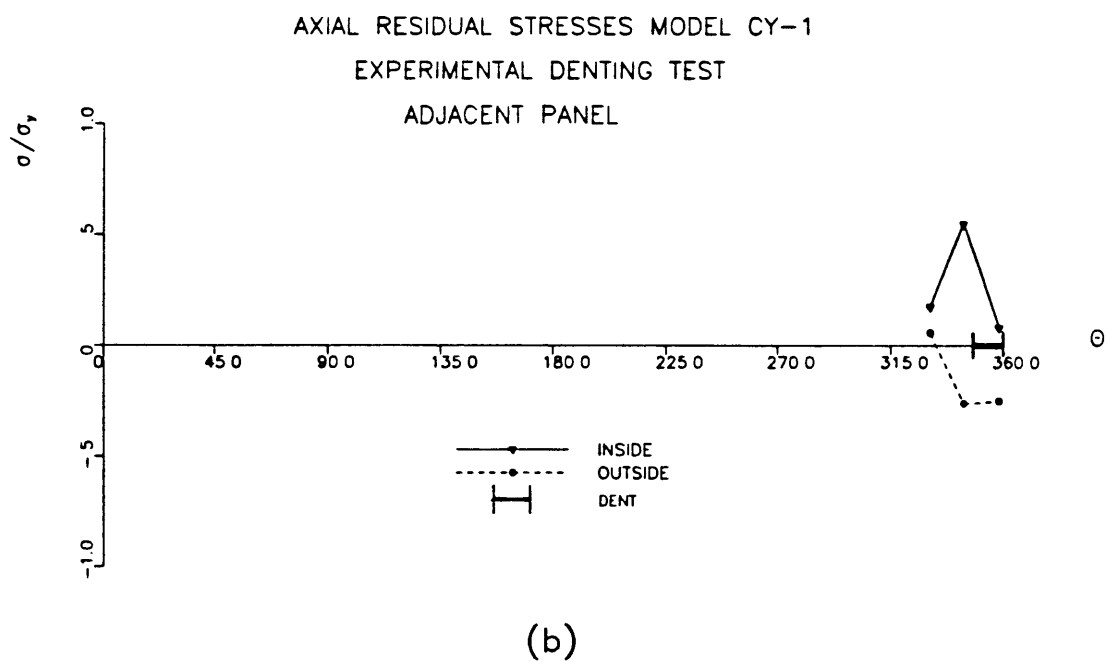
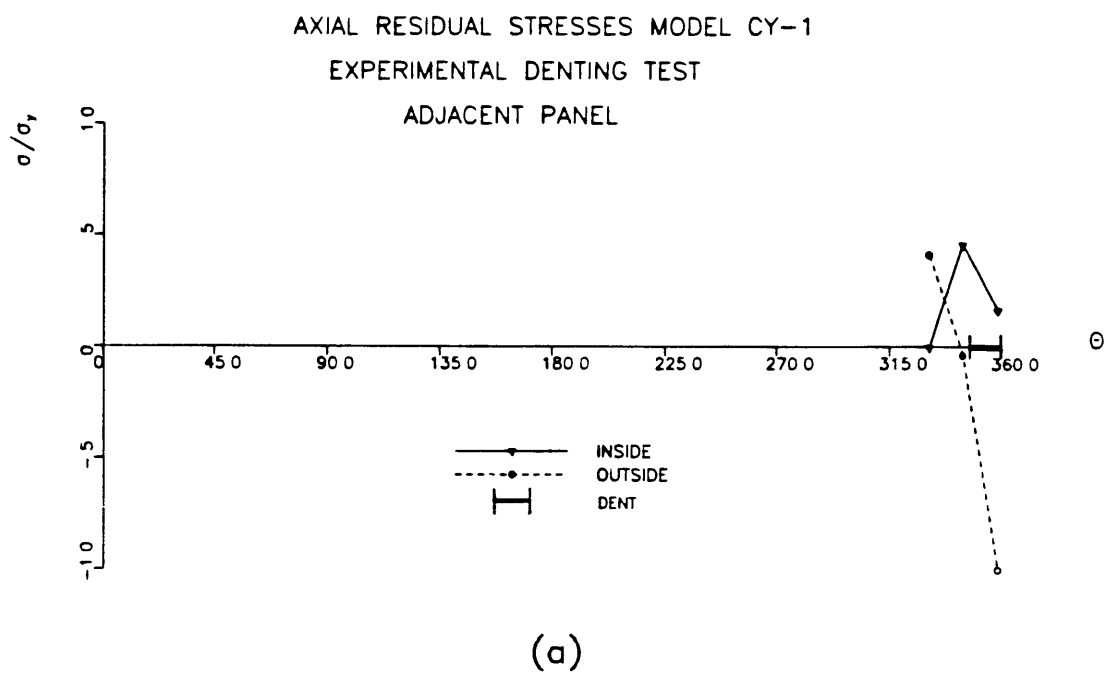


Figure 3.19

AXIAL STRESS PATTERN DURING DENTING
MODEL CY-1

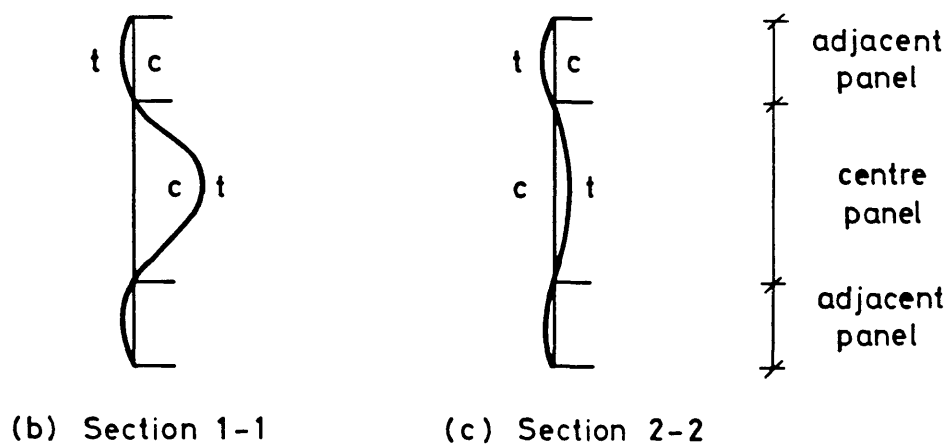
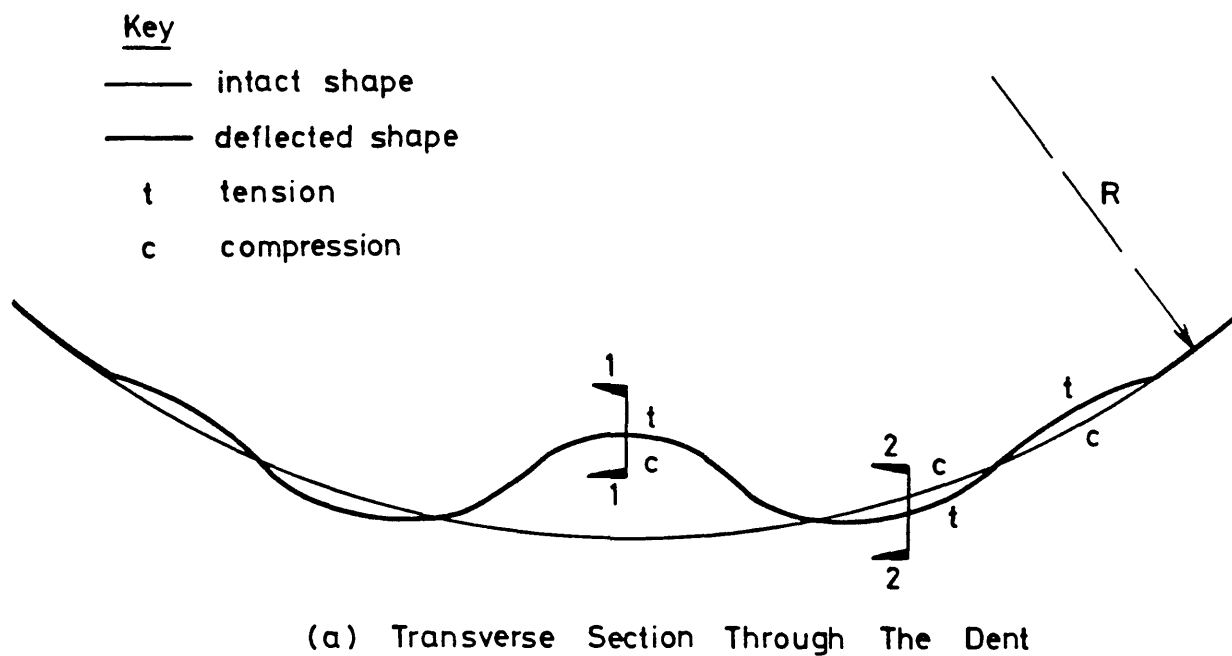


Figure 3.20

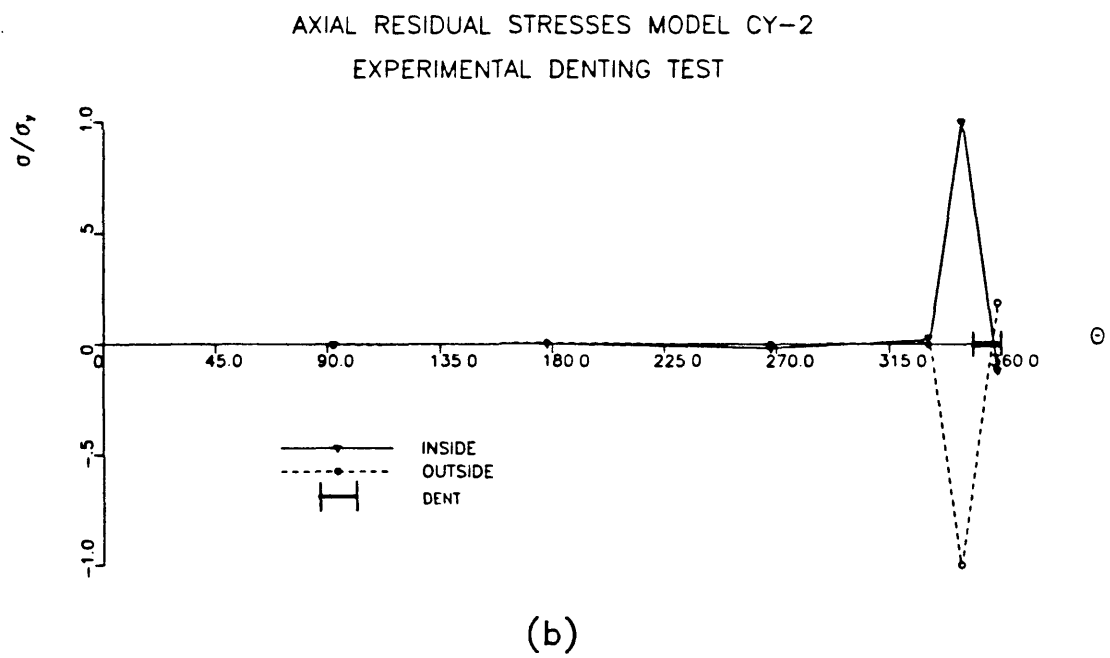
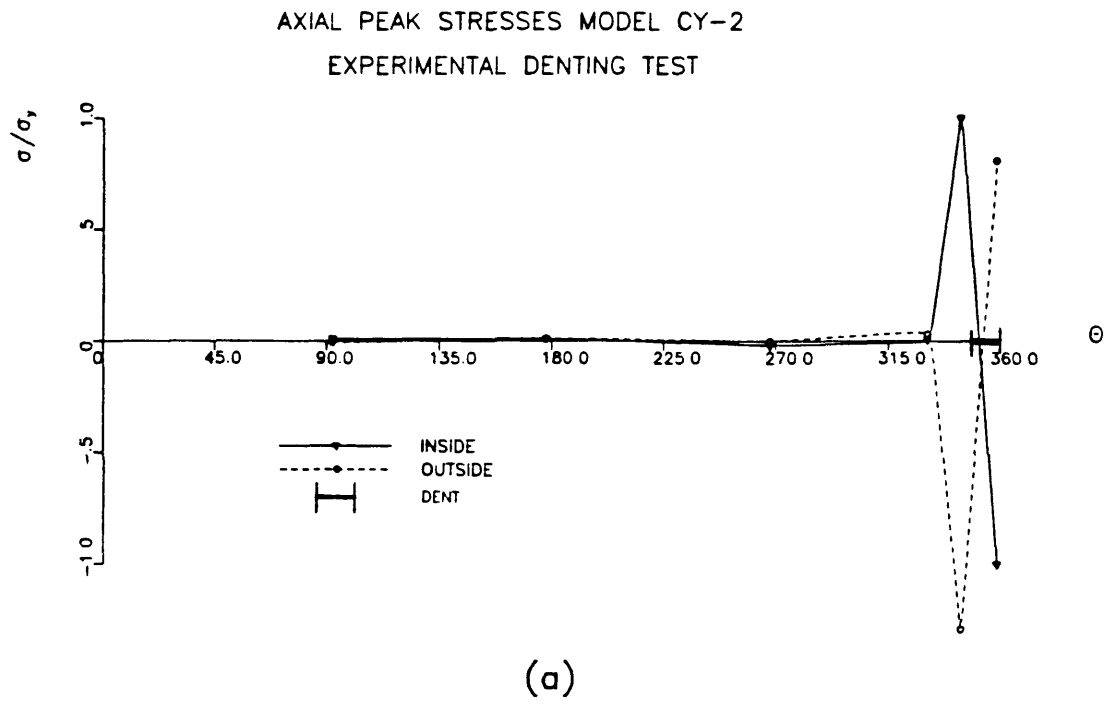
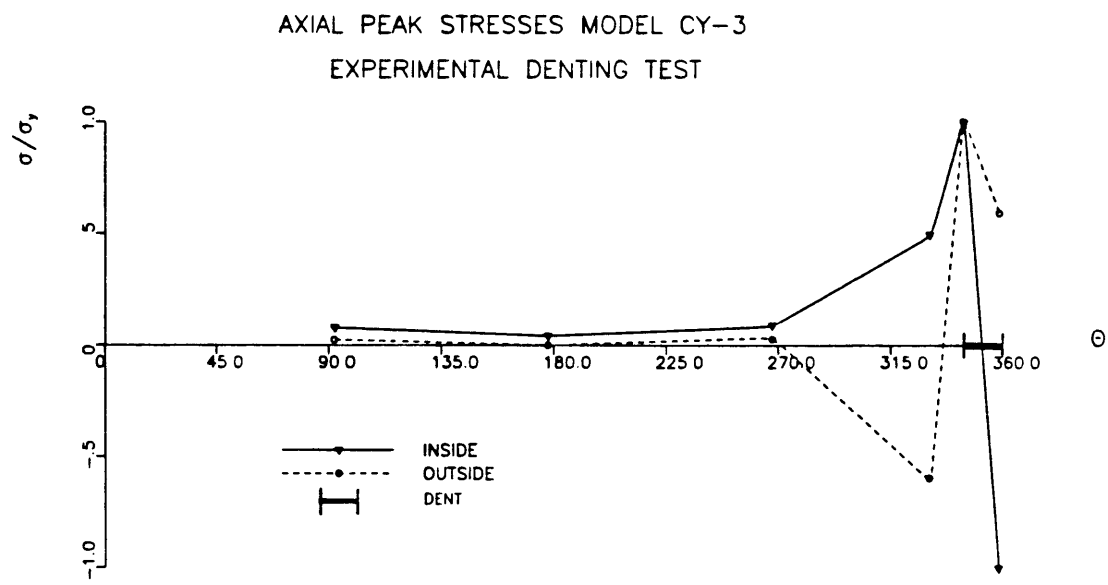
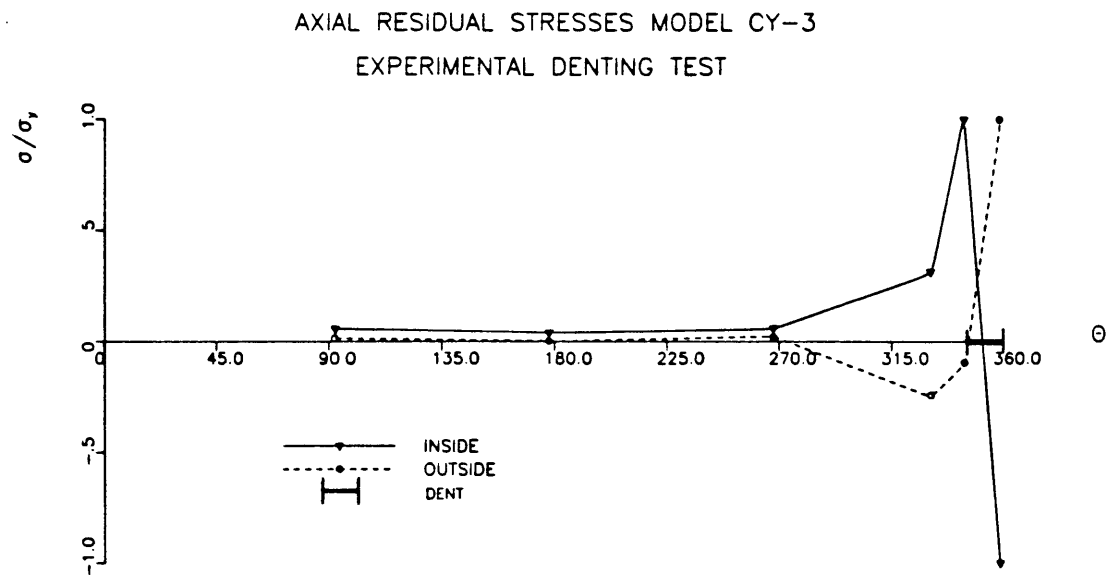


Figure 3.21



(a)



(b)

Figure 3.22

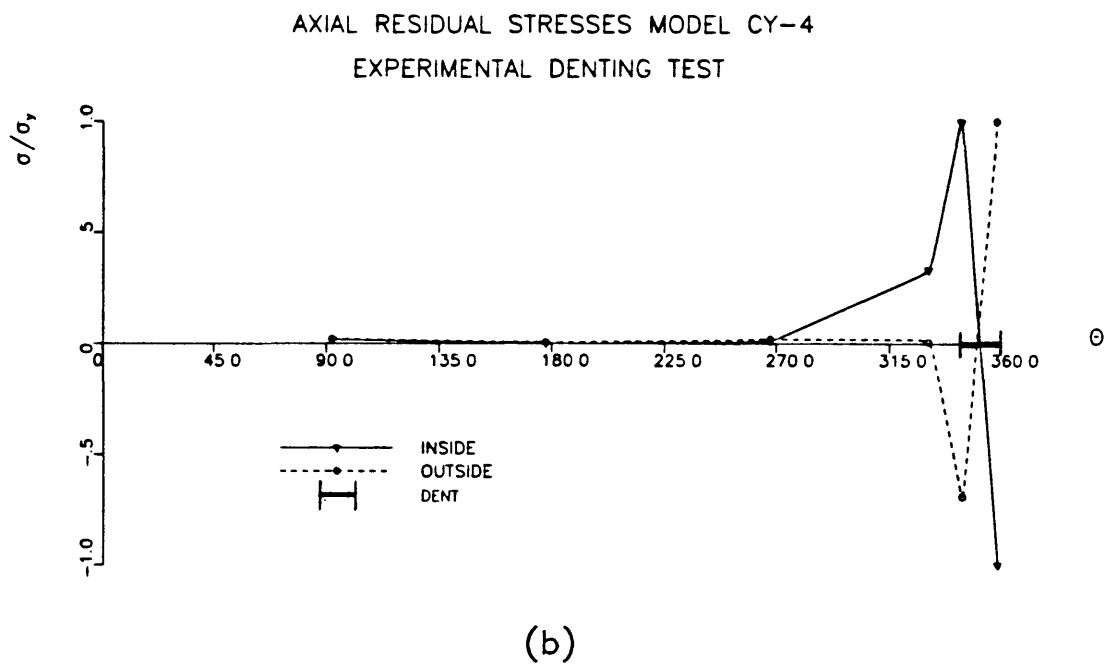
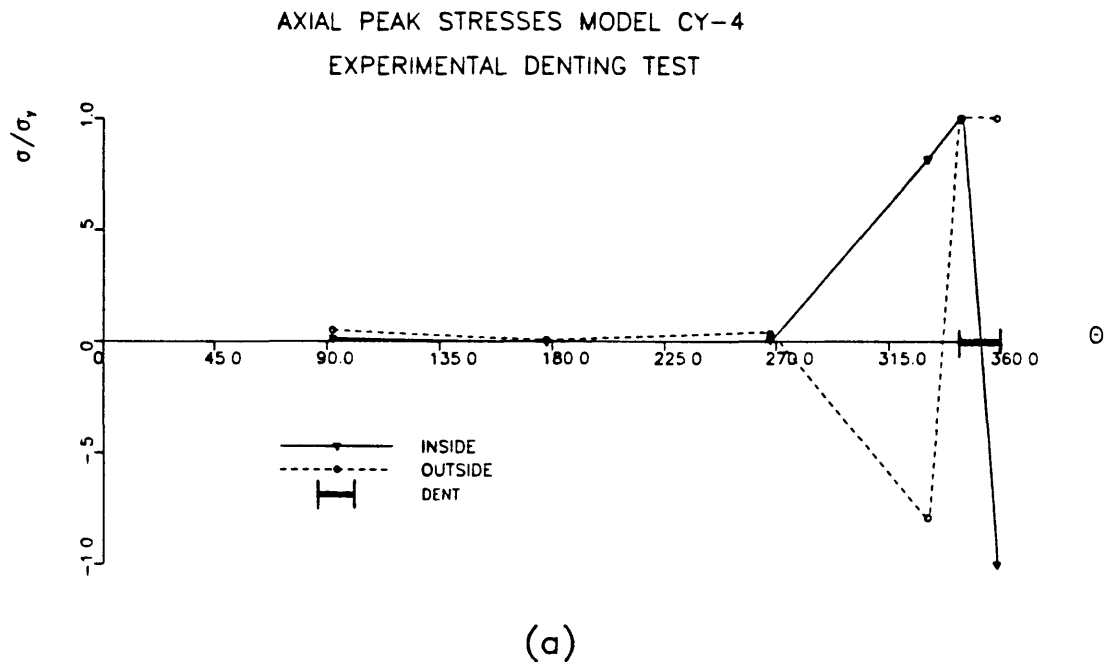


Figure 3.23

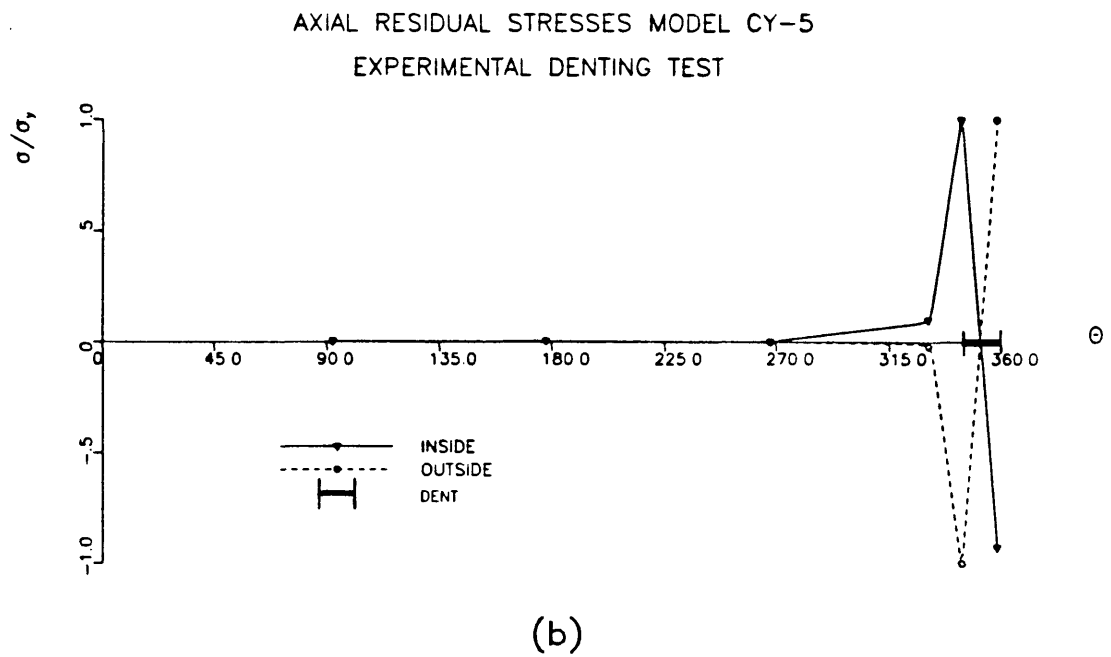
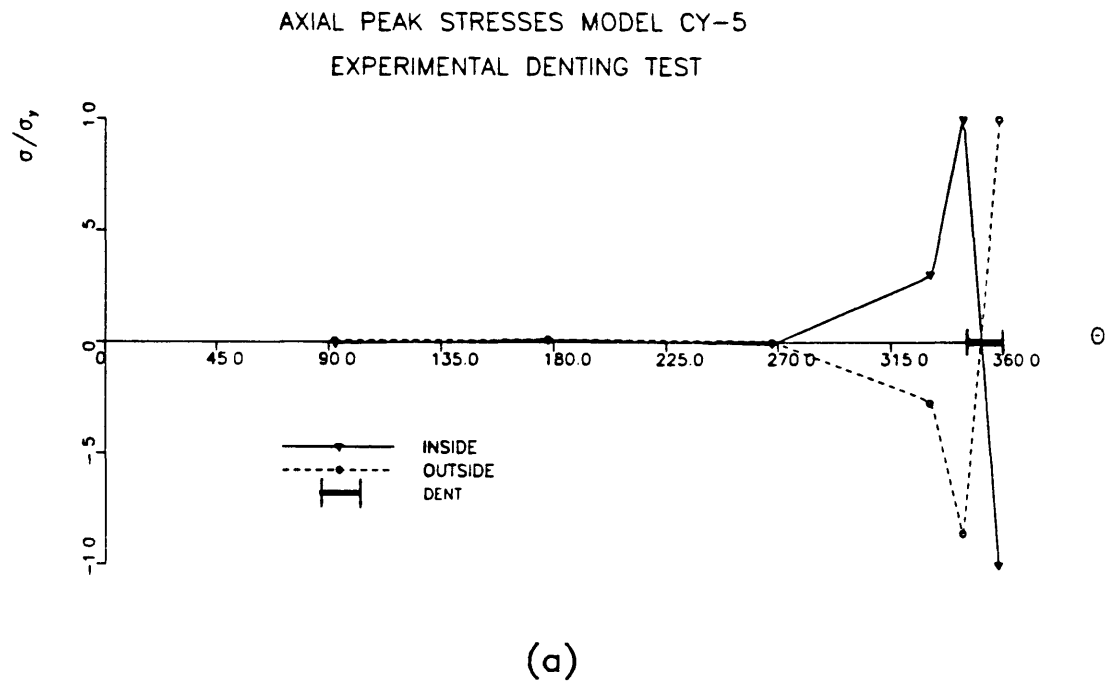


Figure 3.24

ENERGY ABSORPTION DURING DENTING
SERIES 1

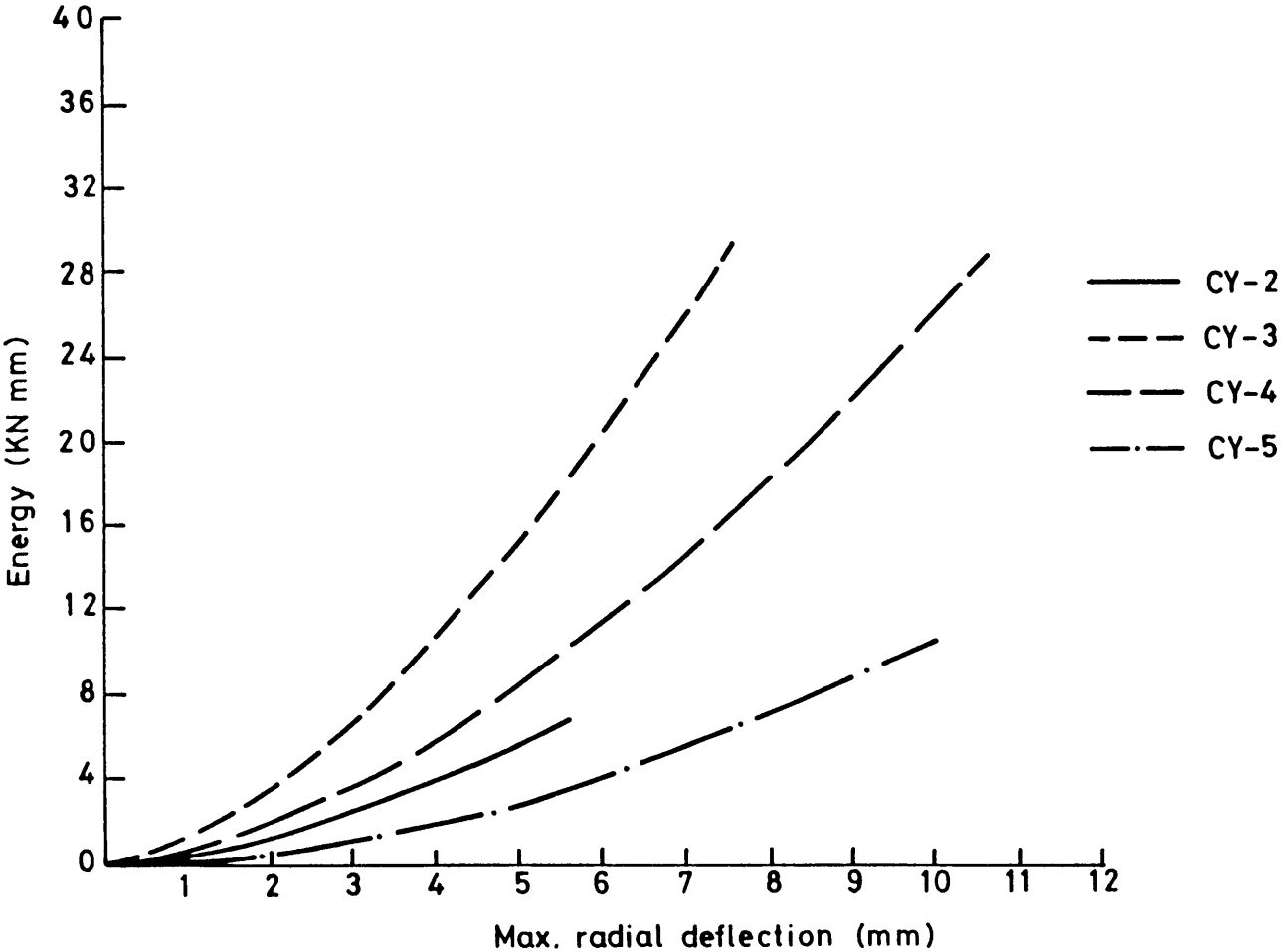


Figure 3.25

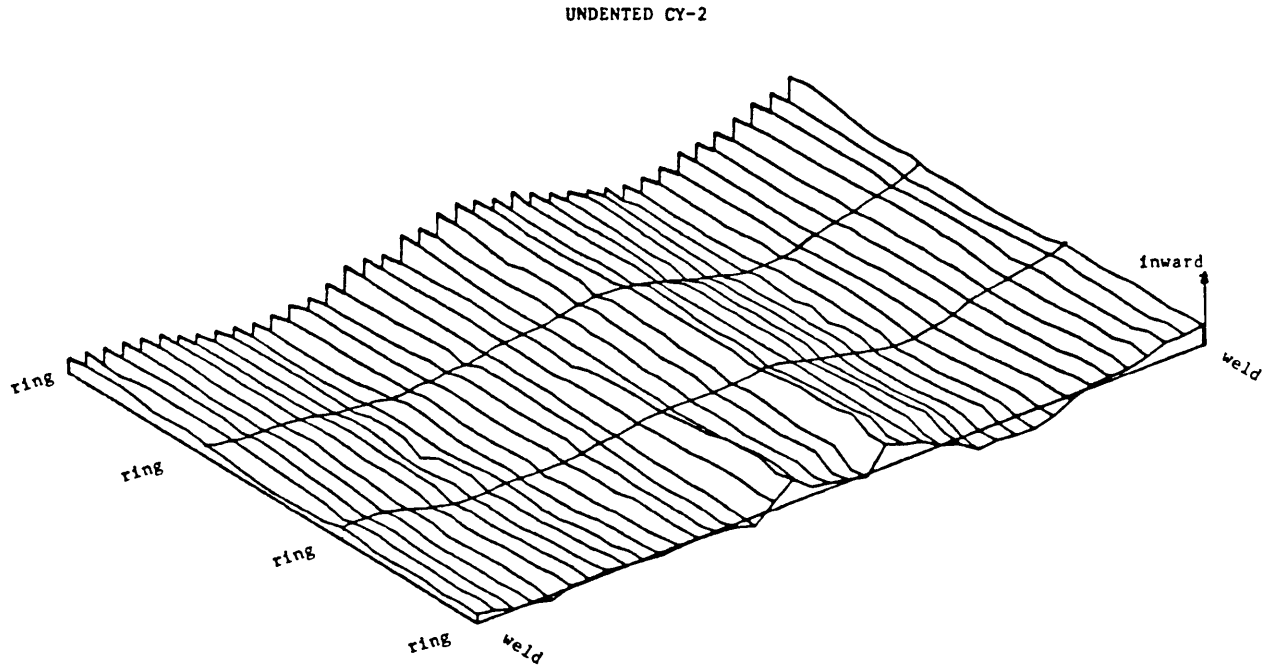


Figure 3.26

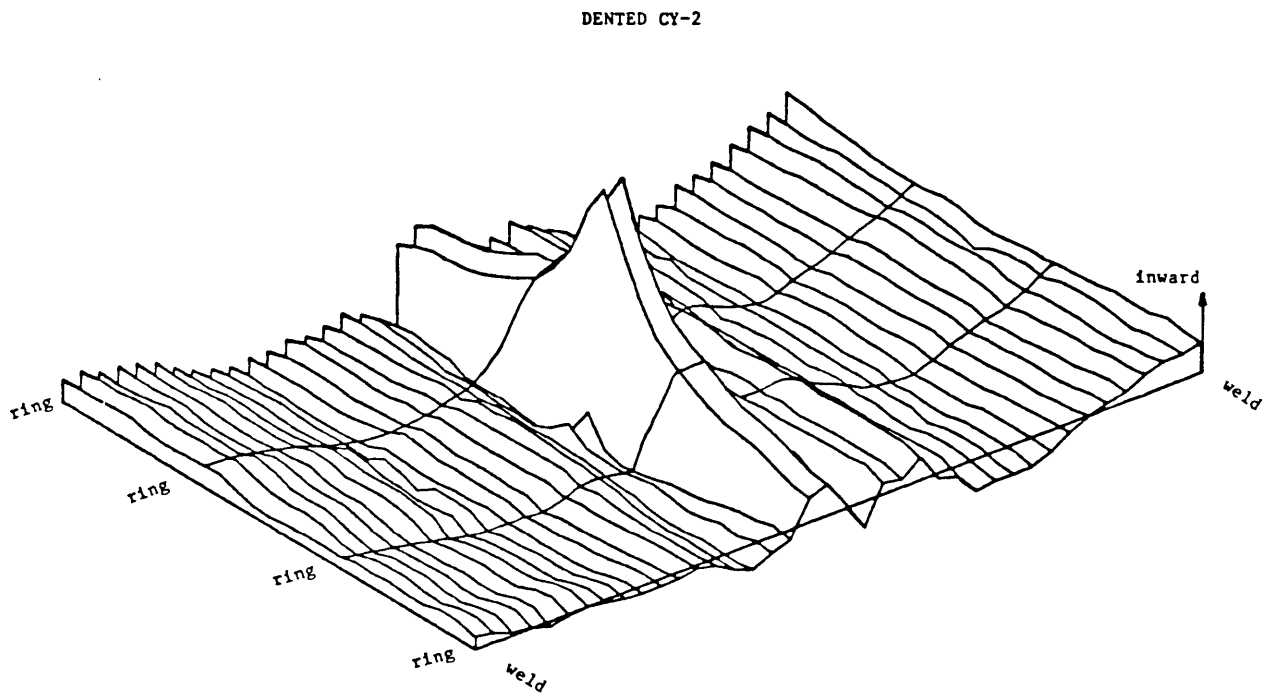


Figure 3.27

MODEL CY-2
IMPERFECTIONS BEFORE DENTING

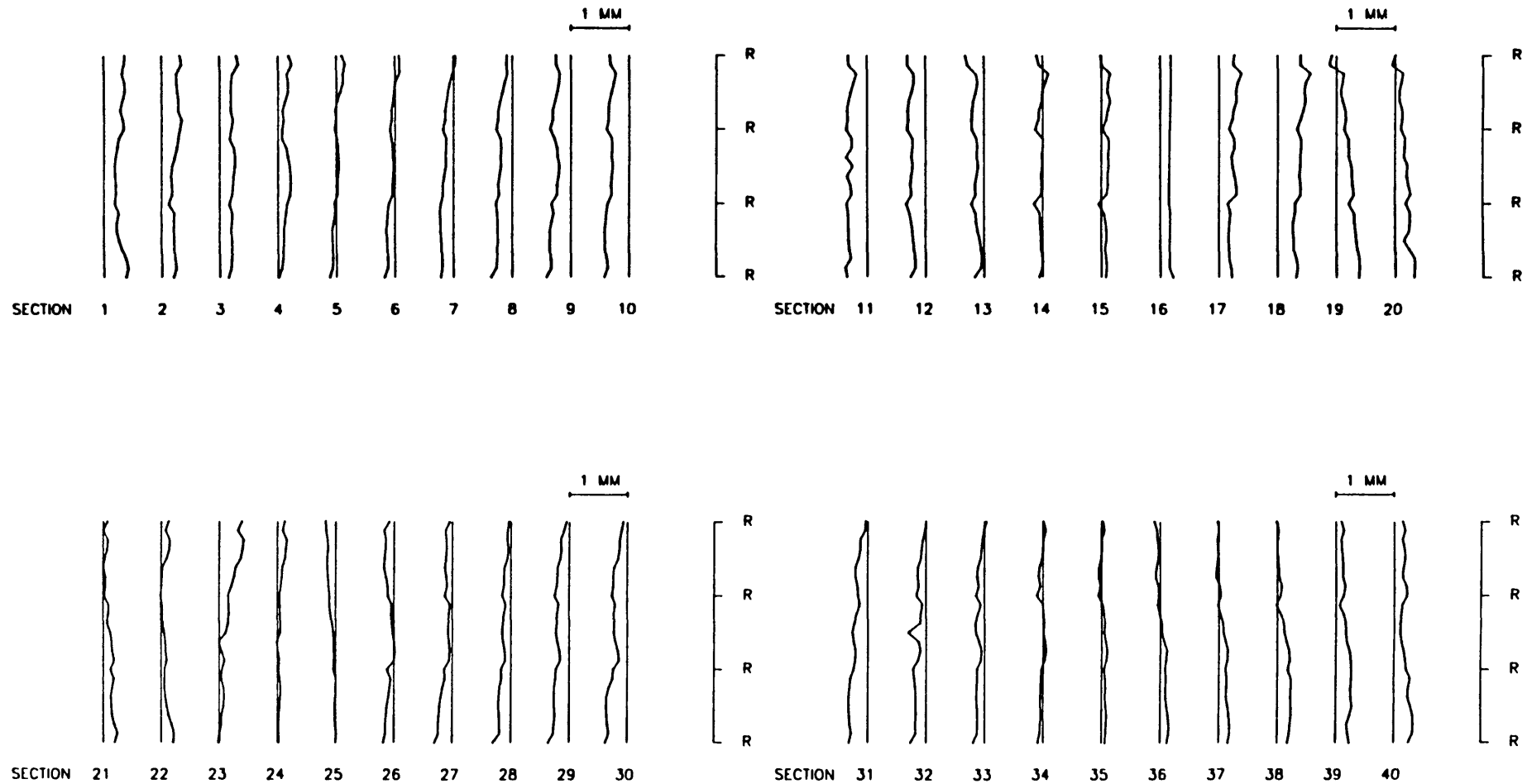


Figure 3.28

MODEL CY-2
IMPERFECTIONS AFTER DENTING

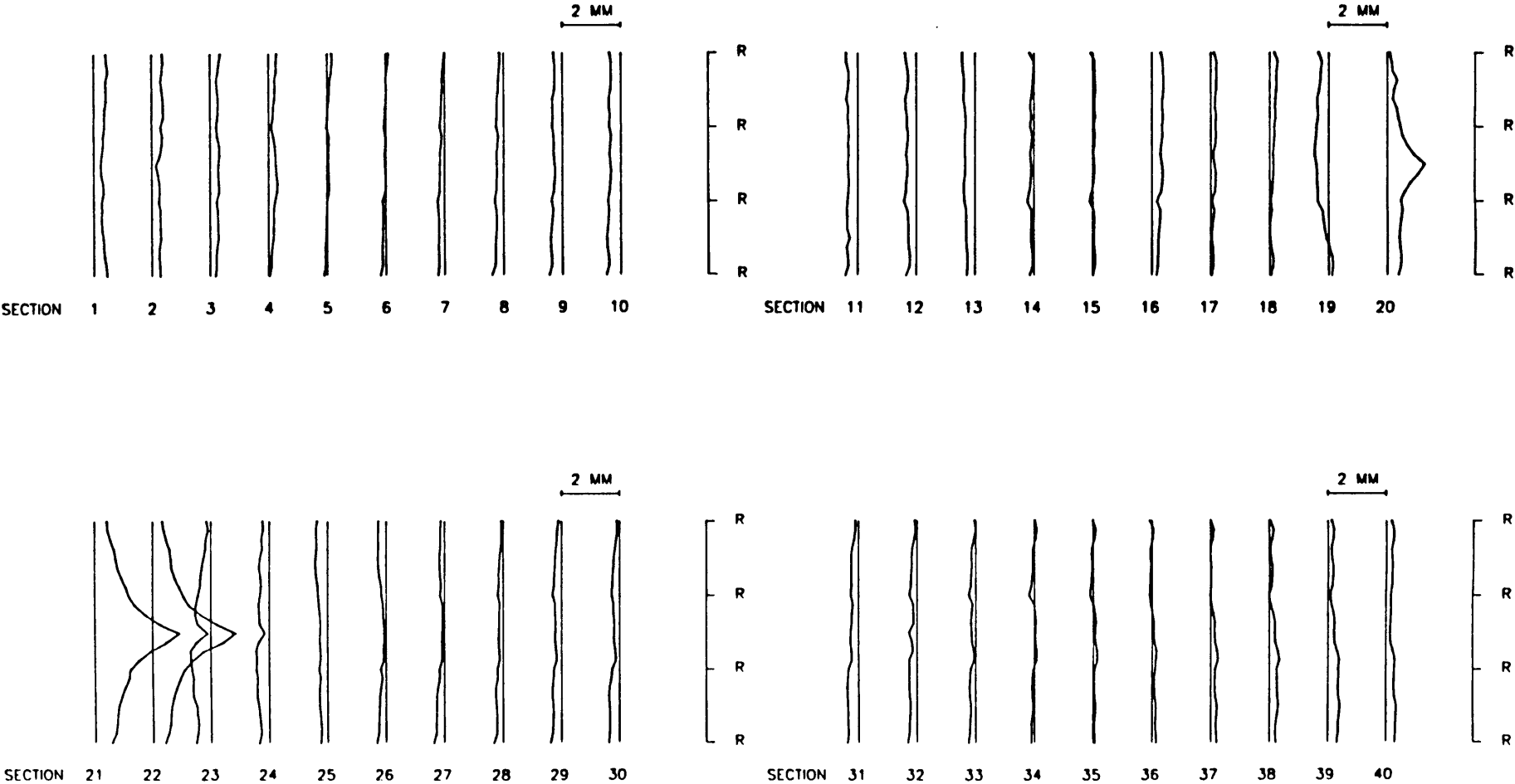


Figure 3.29

UNDENTED CY-3

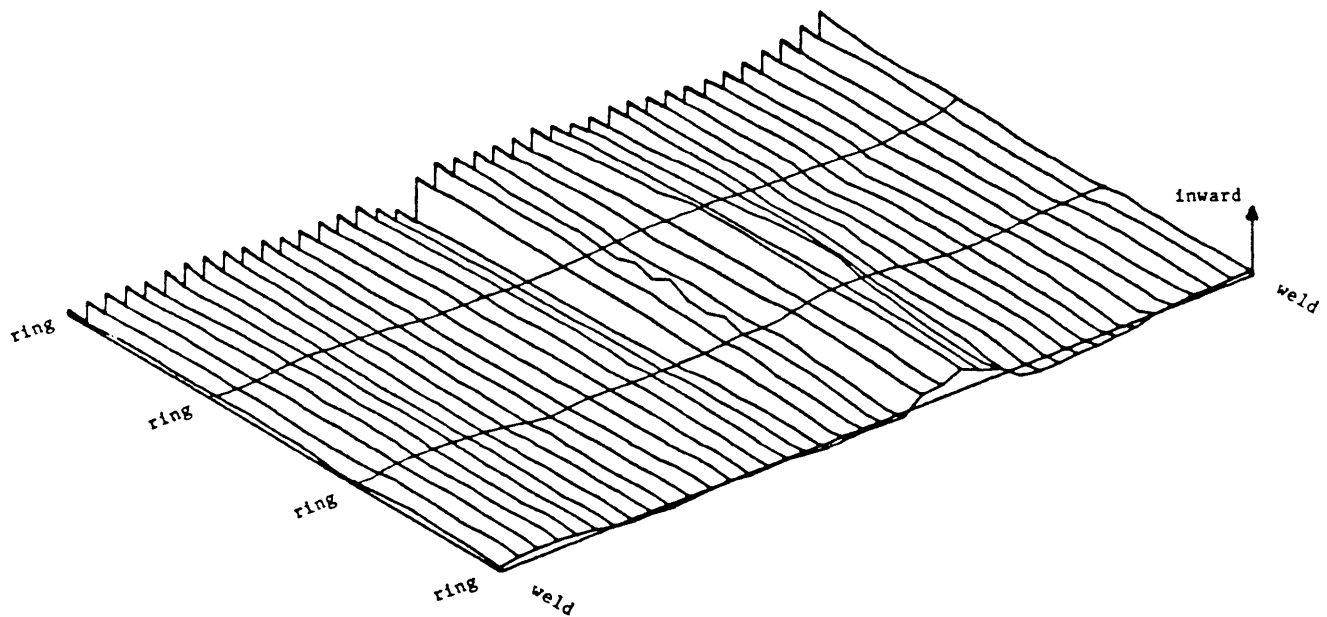


Figure 3.30

DENTED CY-3

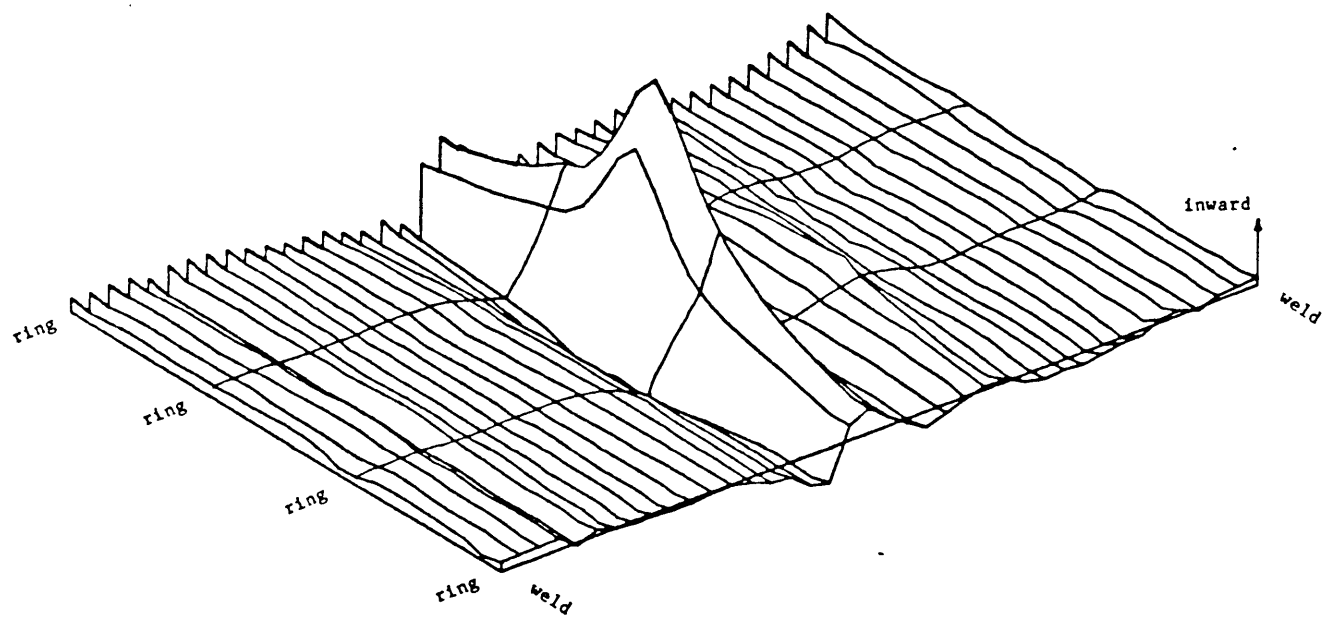


Figure 3.31

MODEL CY-3
IMPERFECTIONS BEFORE DENTING

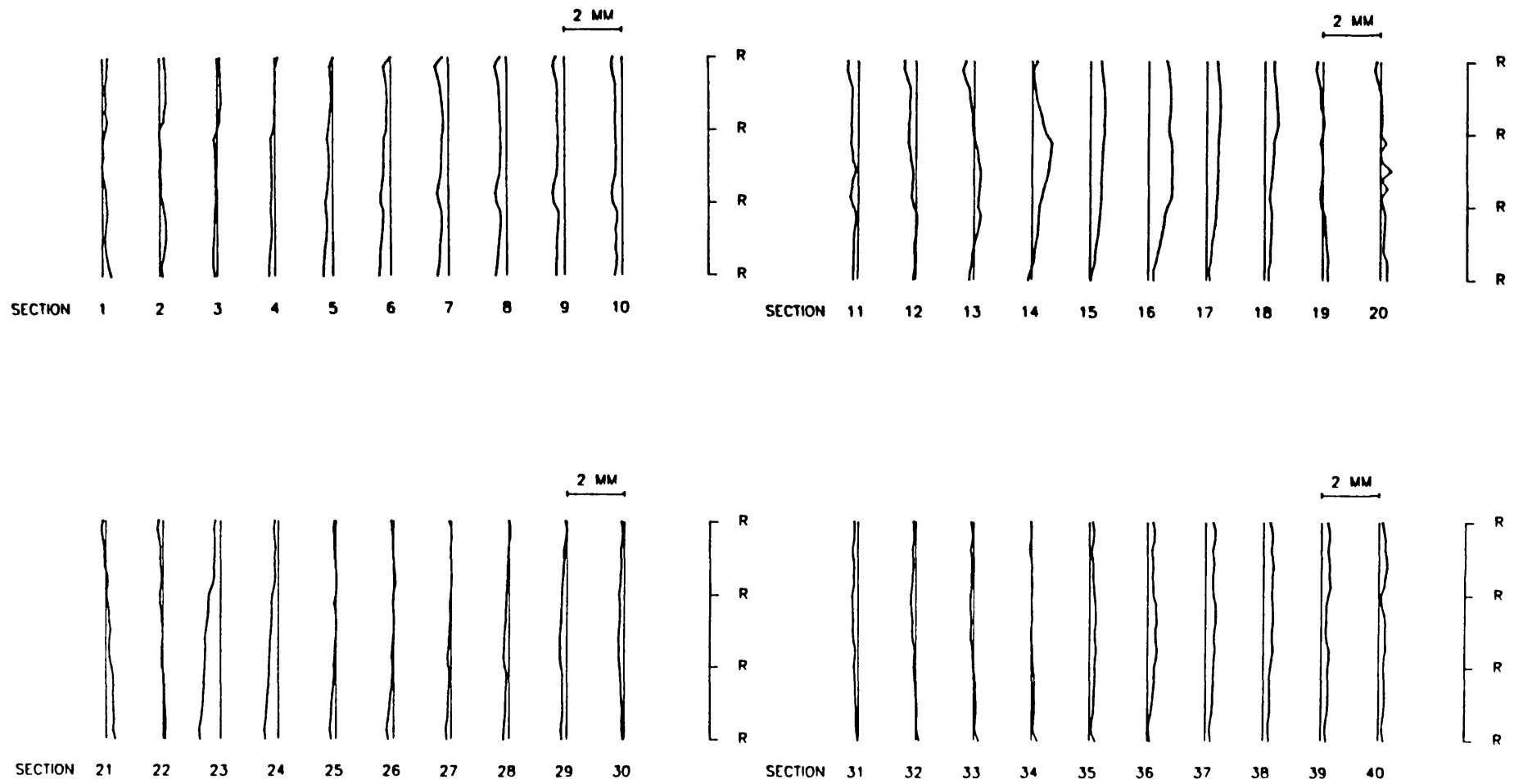


Figure 3.32

IMPERFECTIONS AFTER DENTING

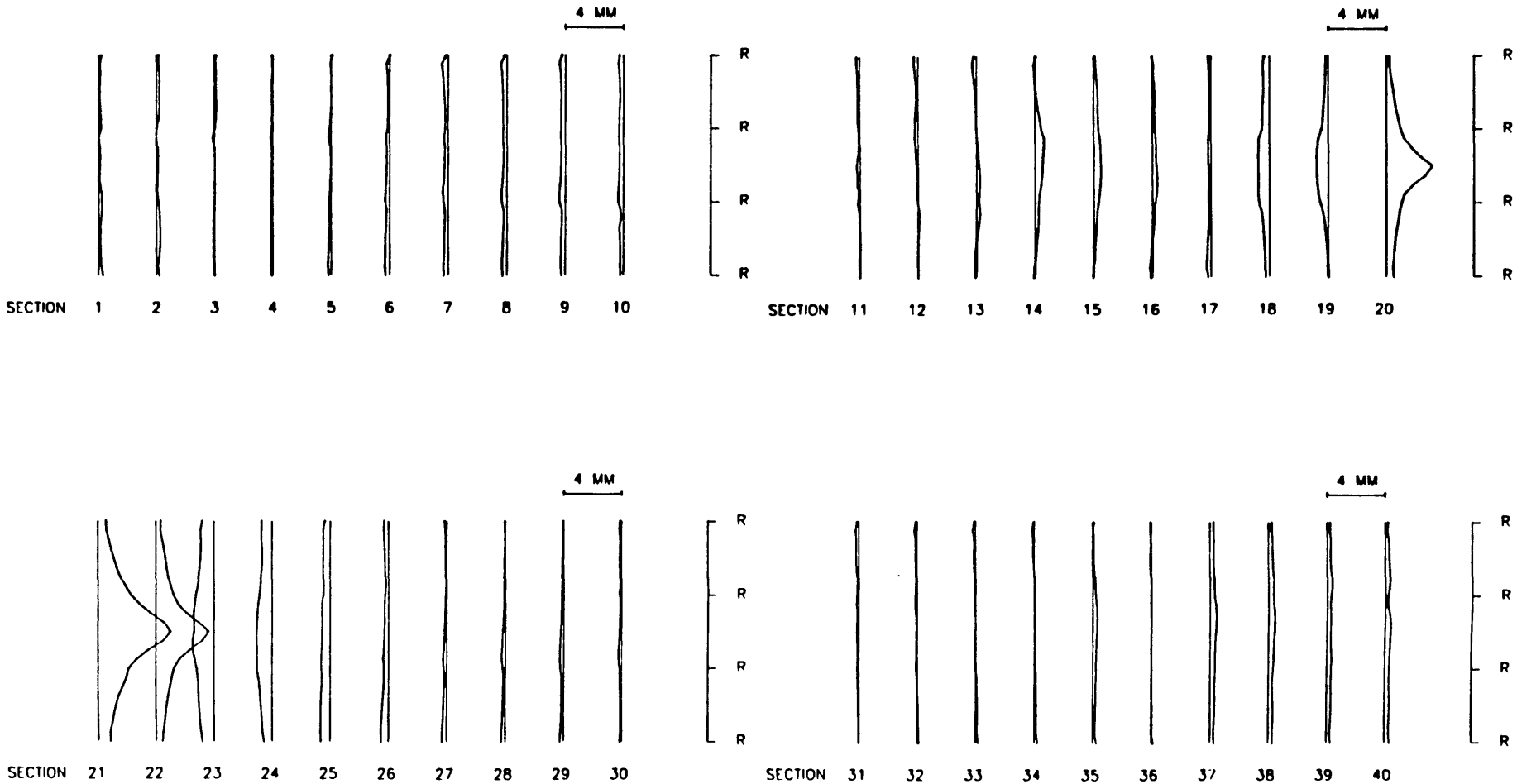


Figure 3.33

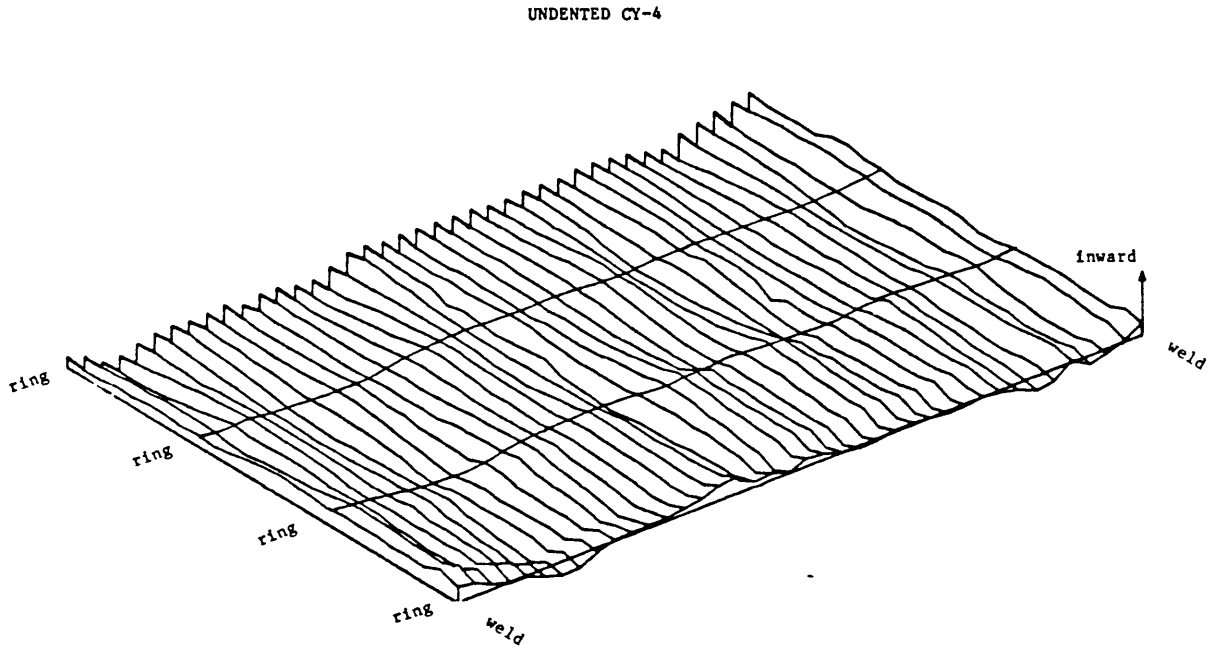


Figure 3.34

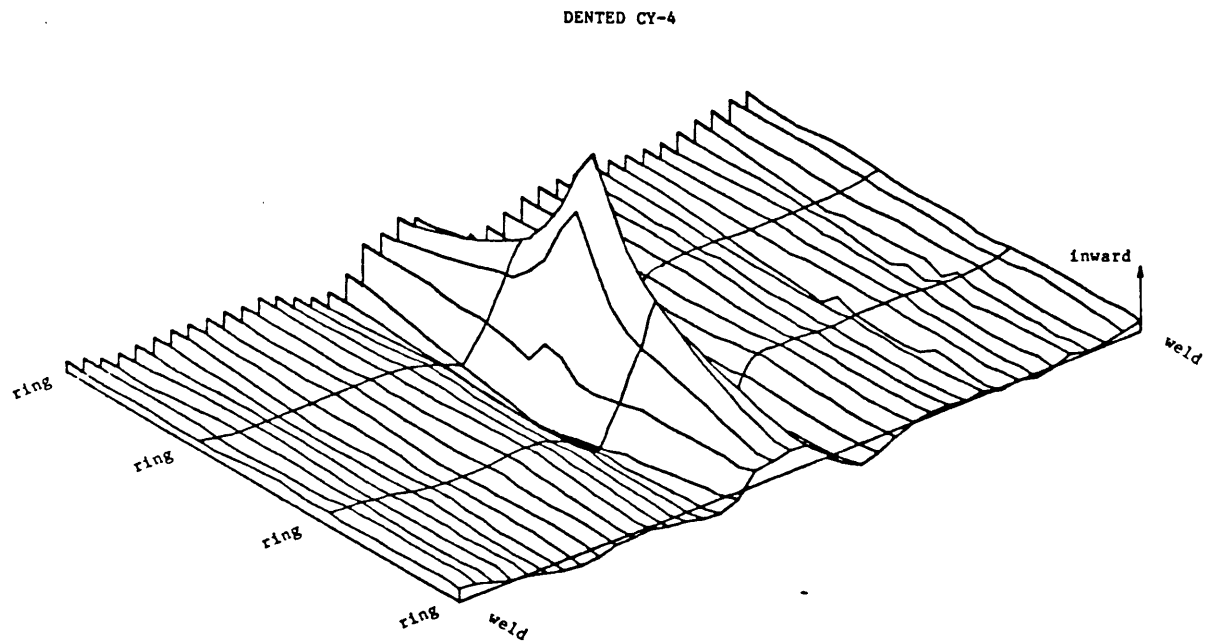


Figure 3.35

MODEL CY-4
IMPERFECTIONS BEFORE DENTING

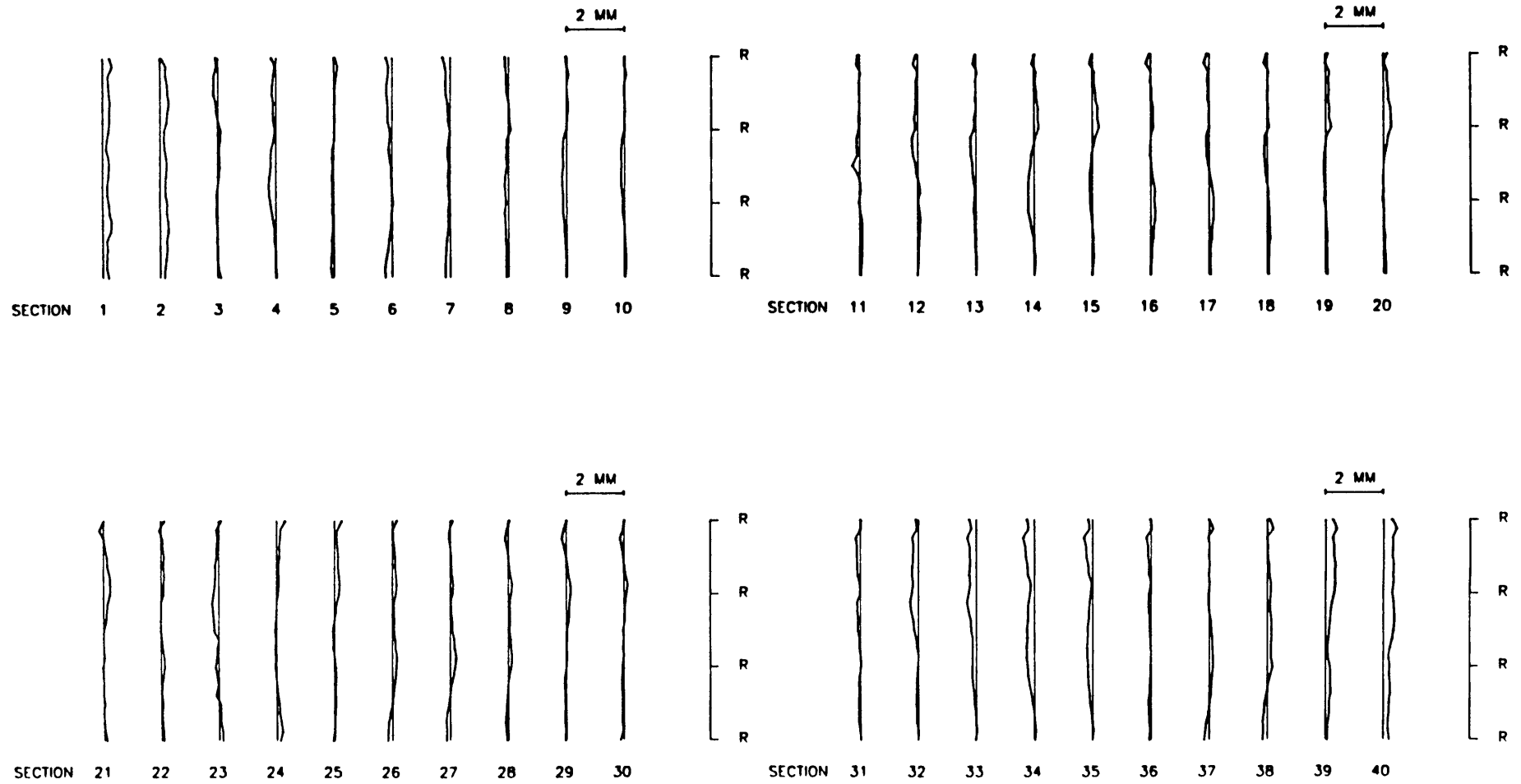


Figure 3.36

MODEL CY--4

IMPERFECTIONS AFTER DENTING

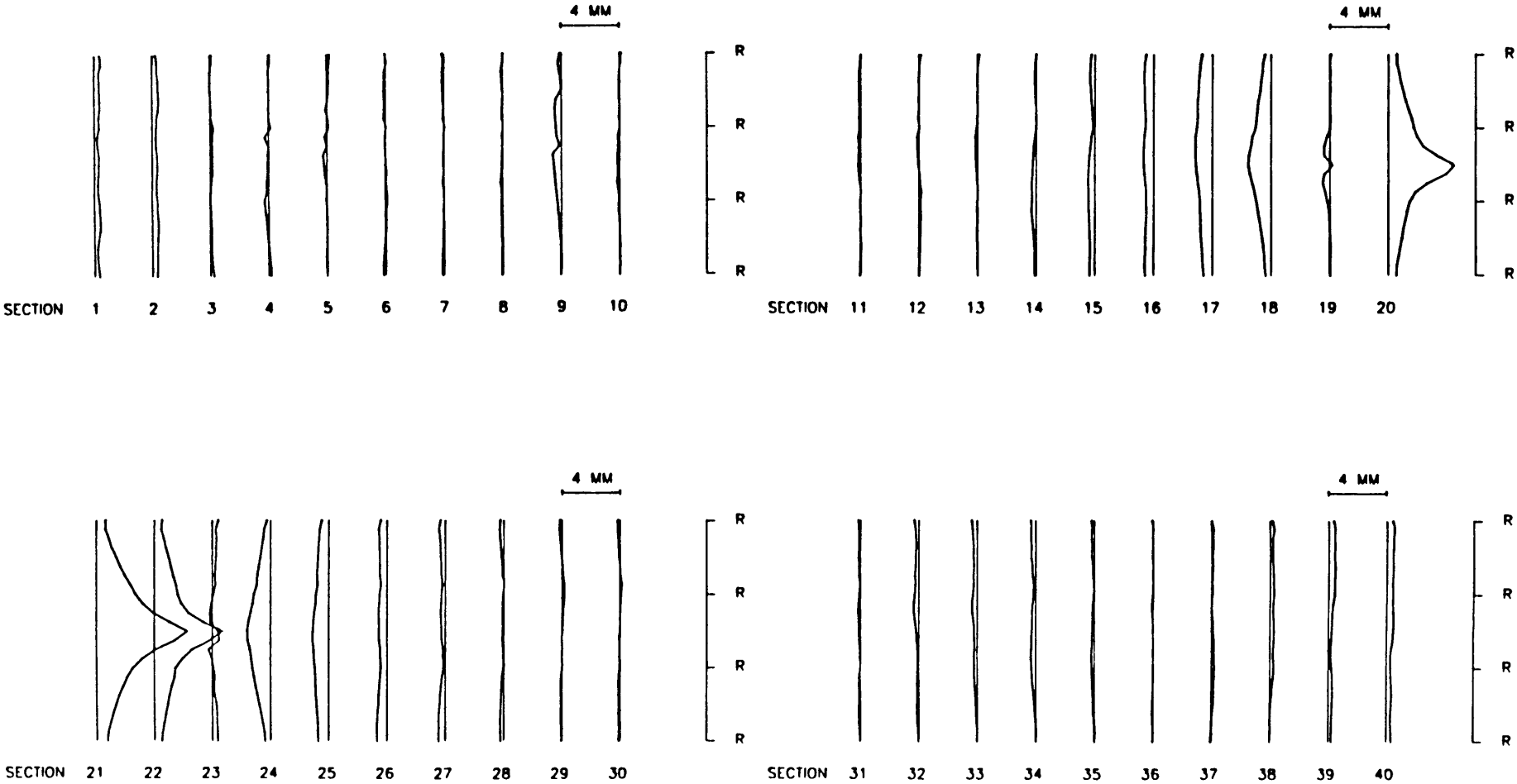


Figure 3.37

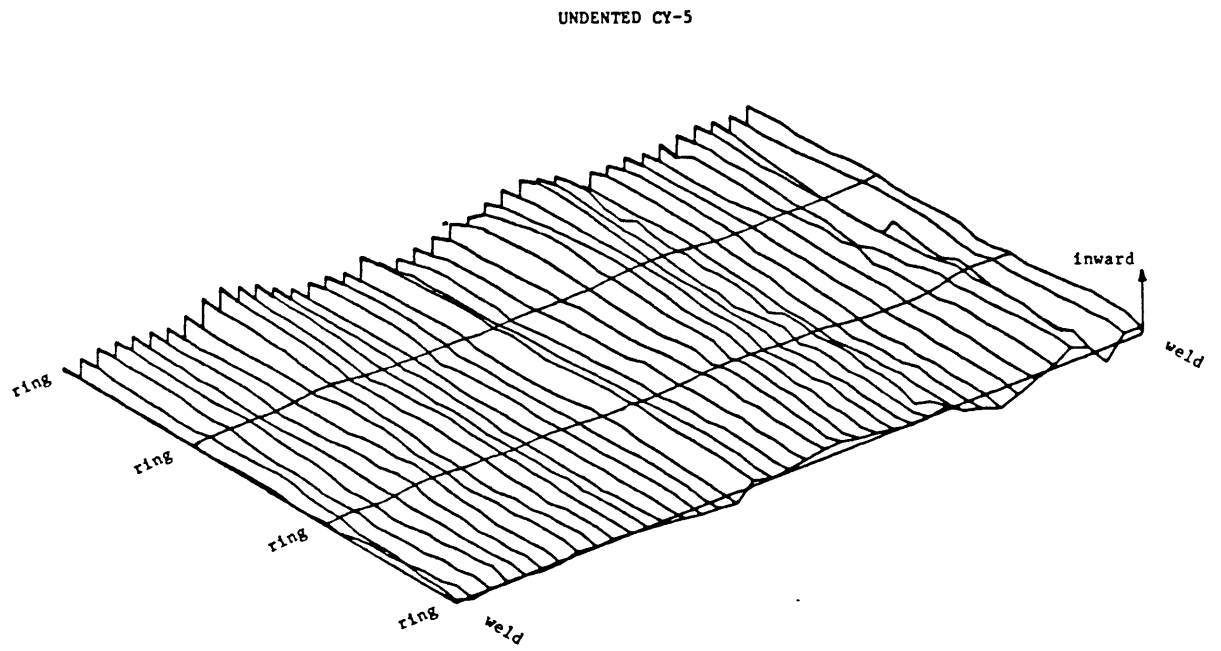


Figure 3.38

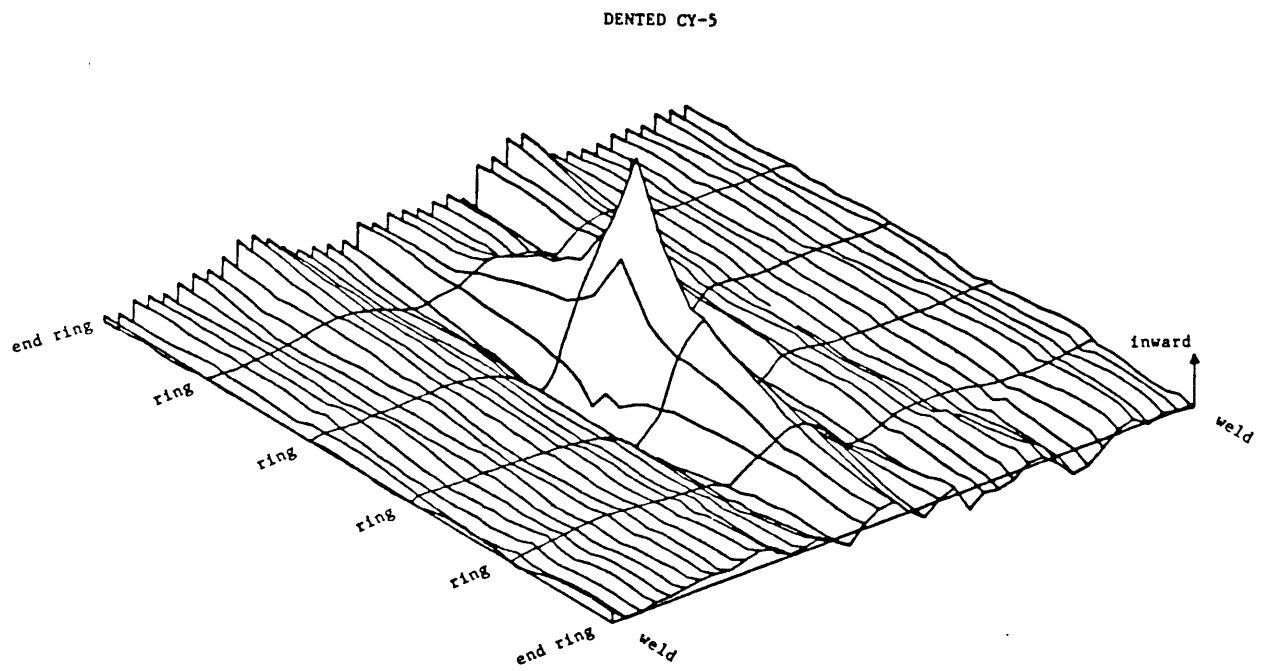


Figure 3.39

MODEL CY-5
IMPERFECTIONS BEFORE DENTING

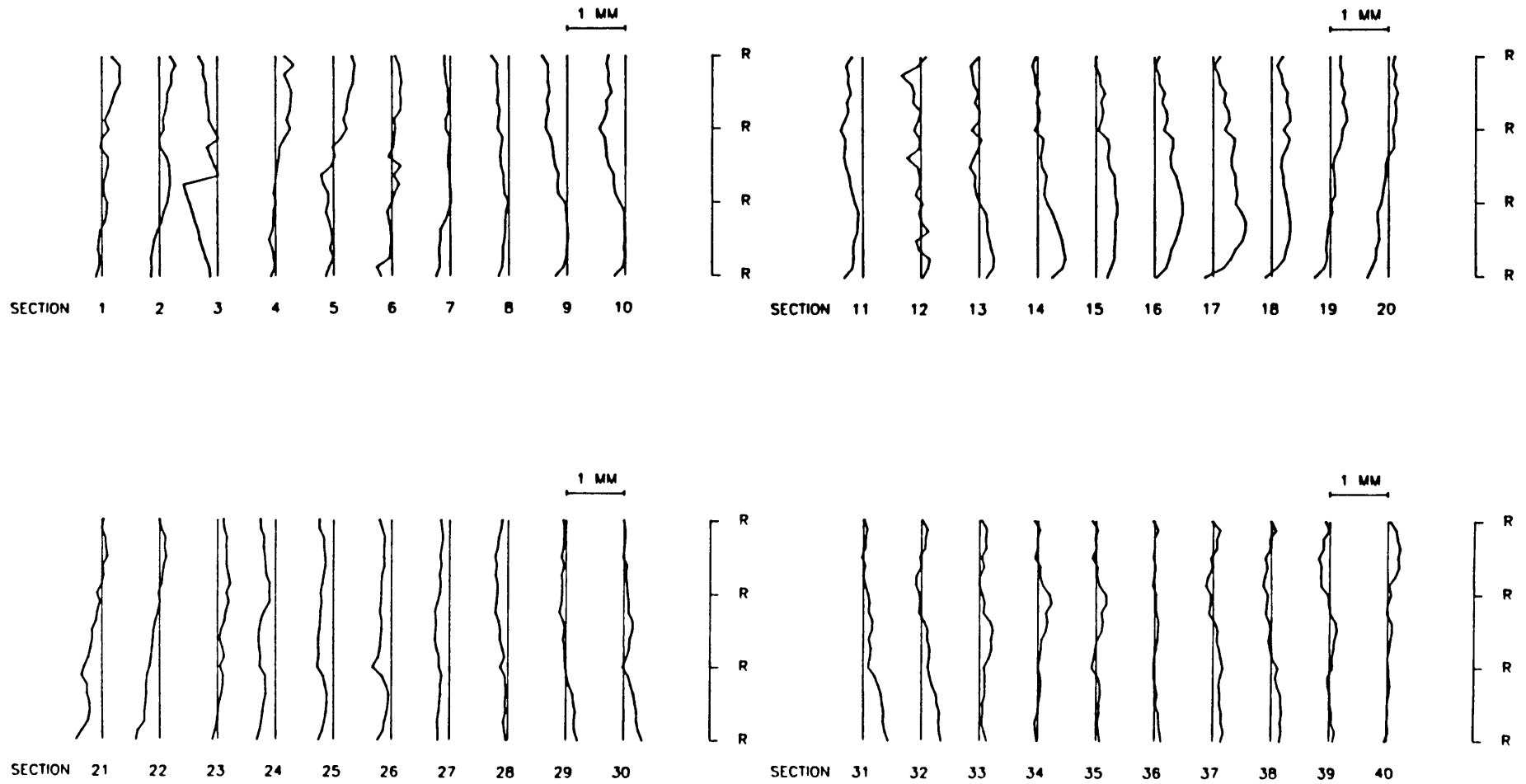


Figure 3.40

MODEL CY-5
IMPERFECTIONS AFTER DENTING

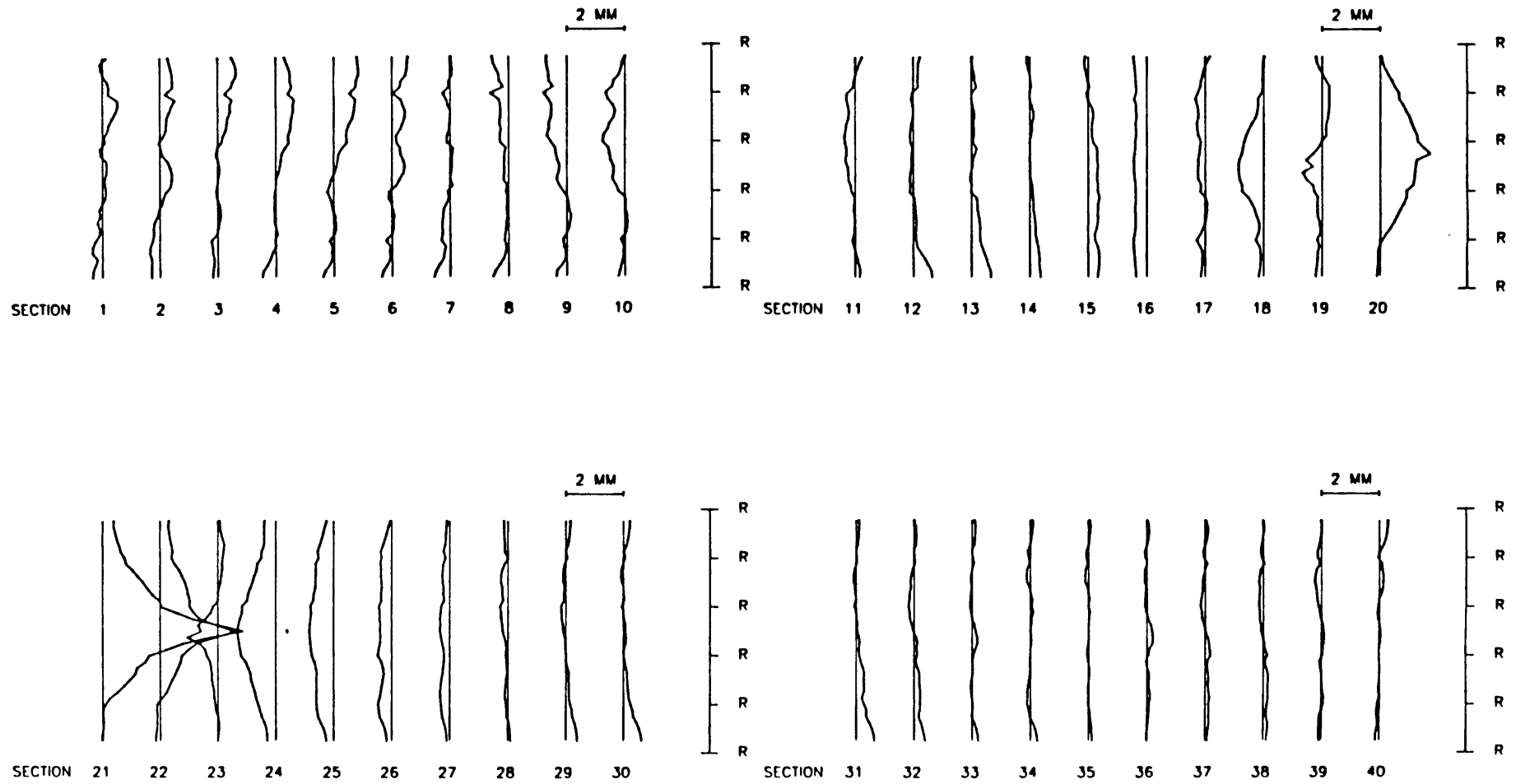


Figure 3.41

LOAD DEFLECTION CURVE (CY-1)

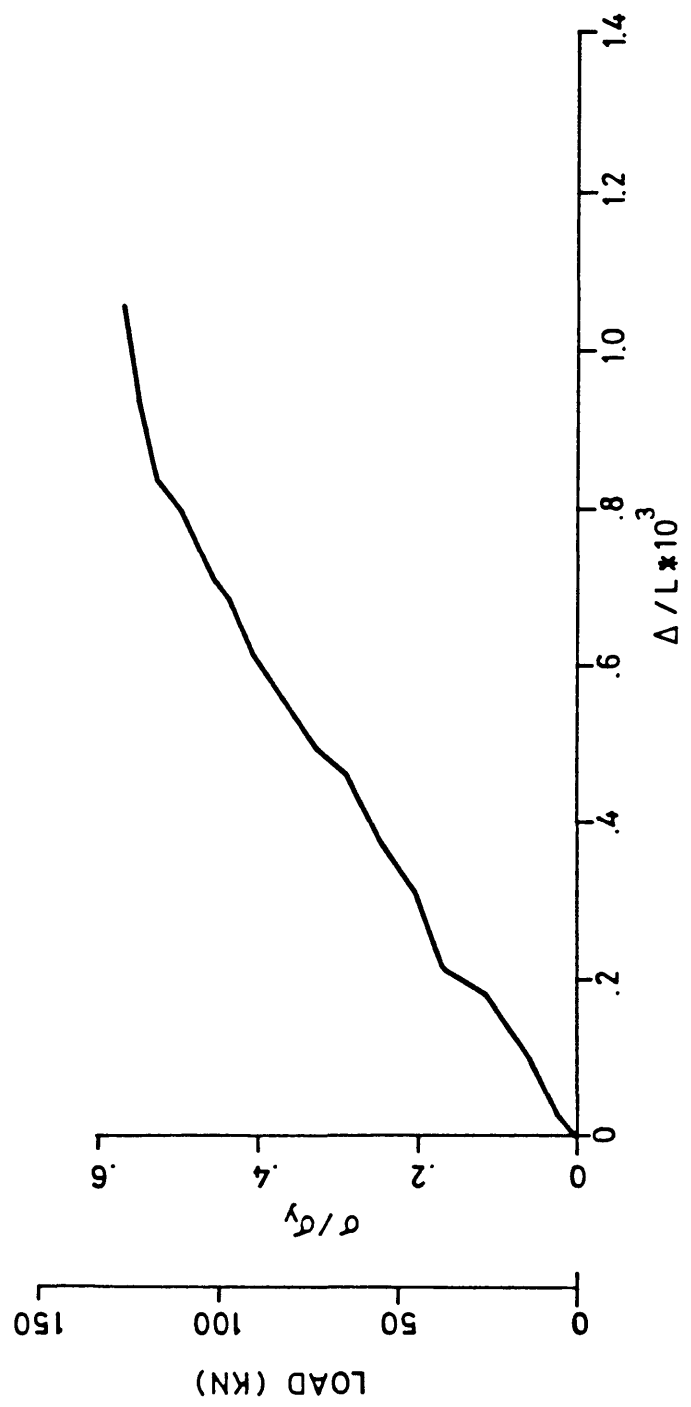


Figure 3.42

LOAD DEFLECTION CURVE (CY-2)

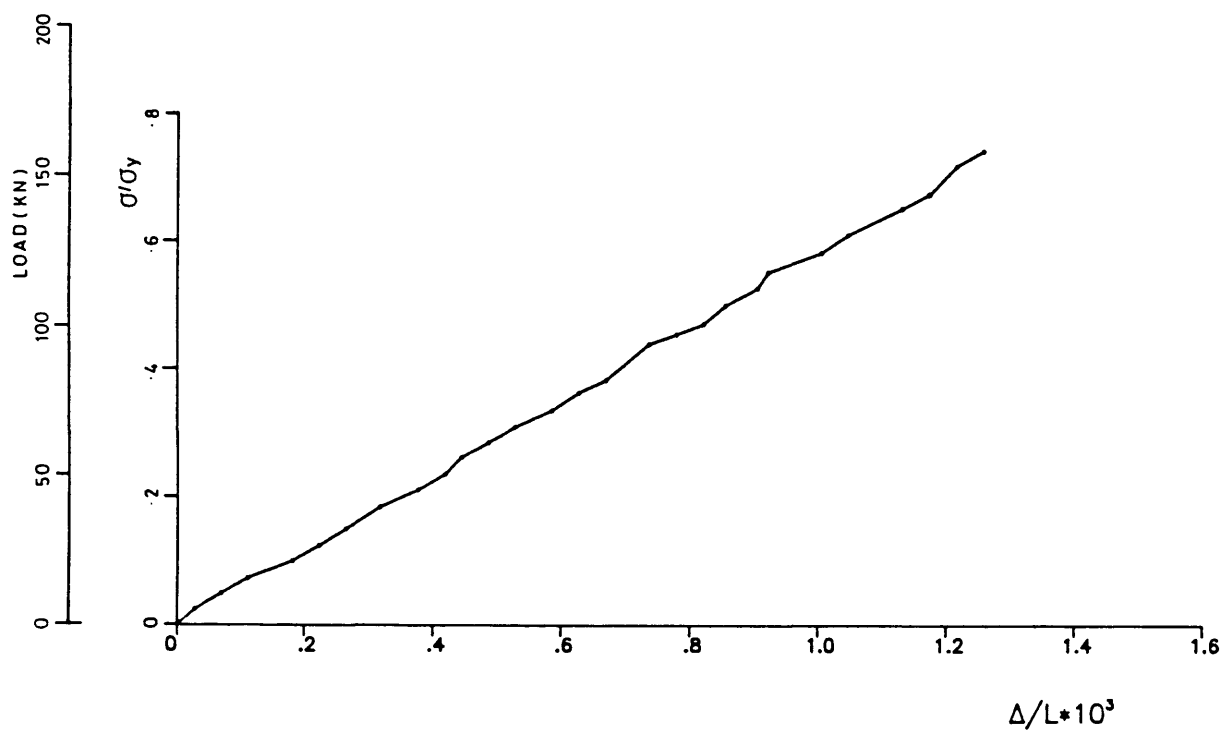


Figure 3.43

LOAD DEFLECTION CURVE (CY-3)

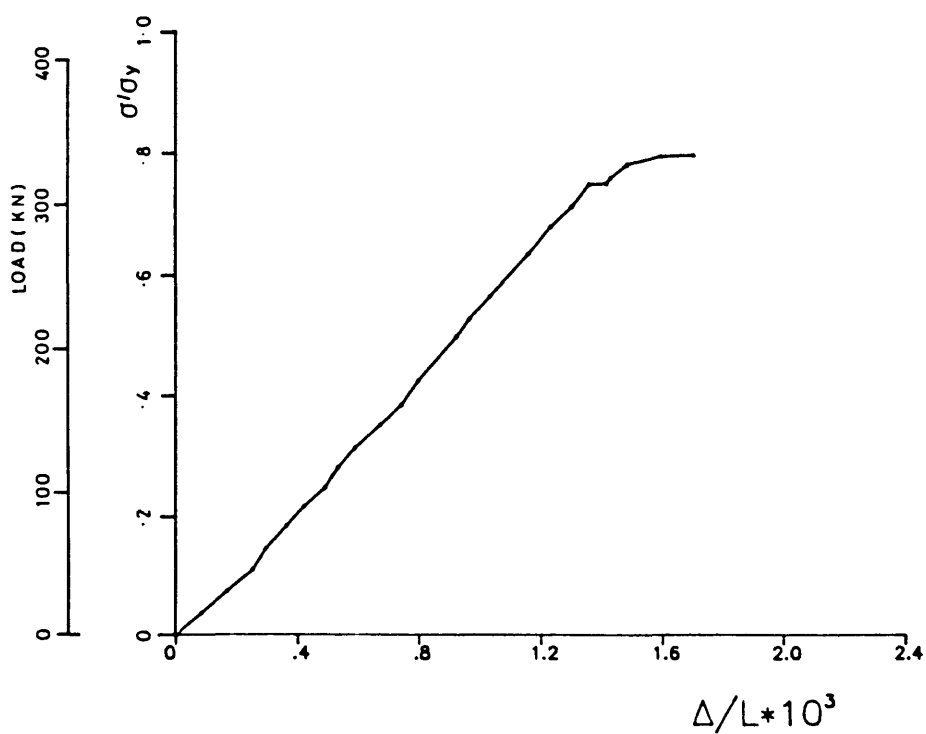


Figure 3.44

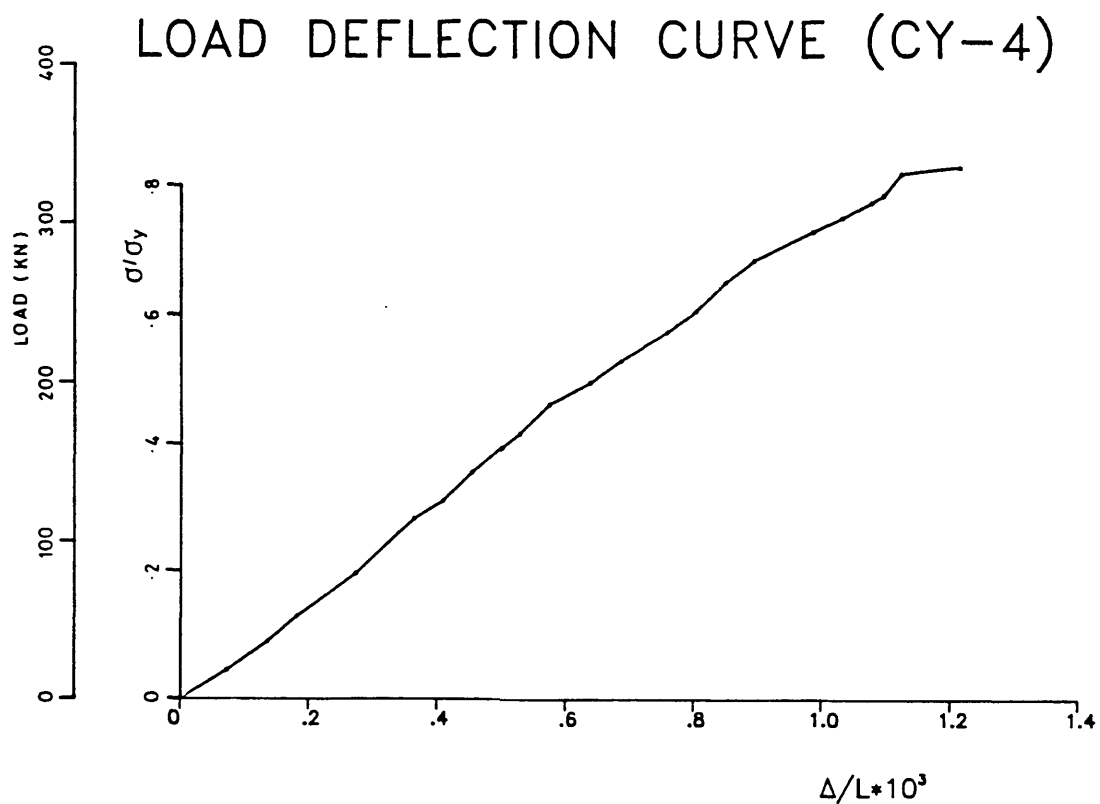


Figure 3.45

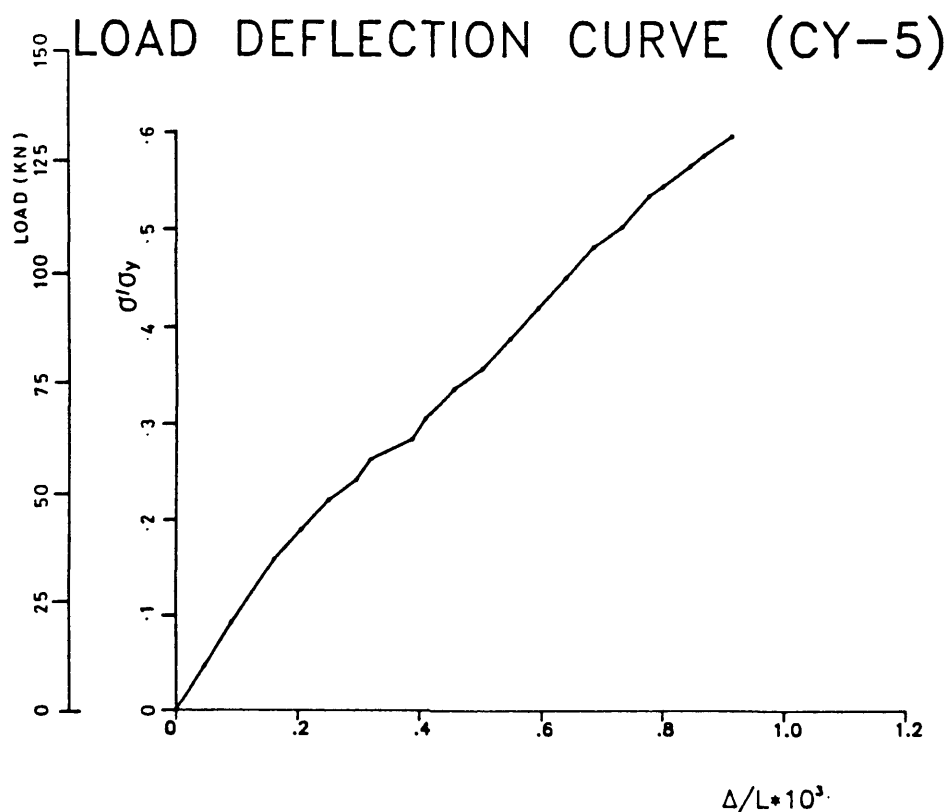
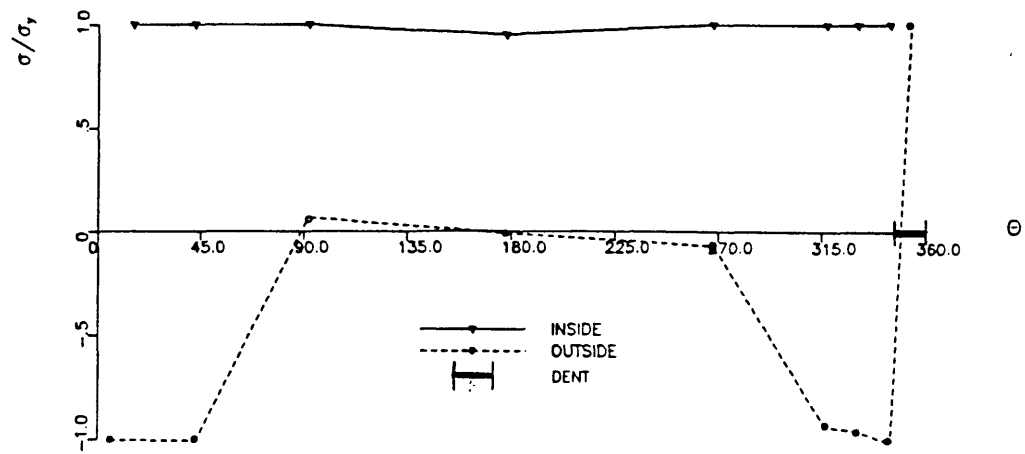


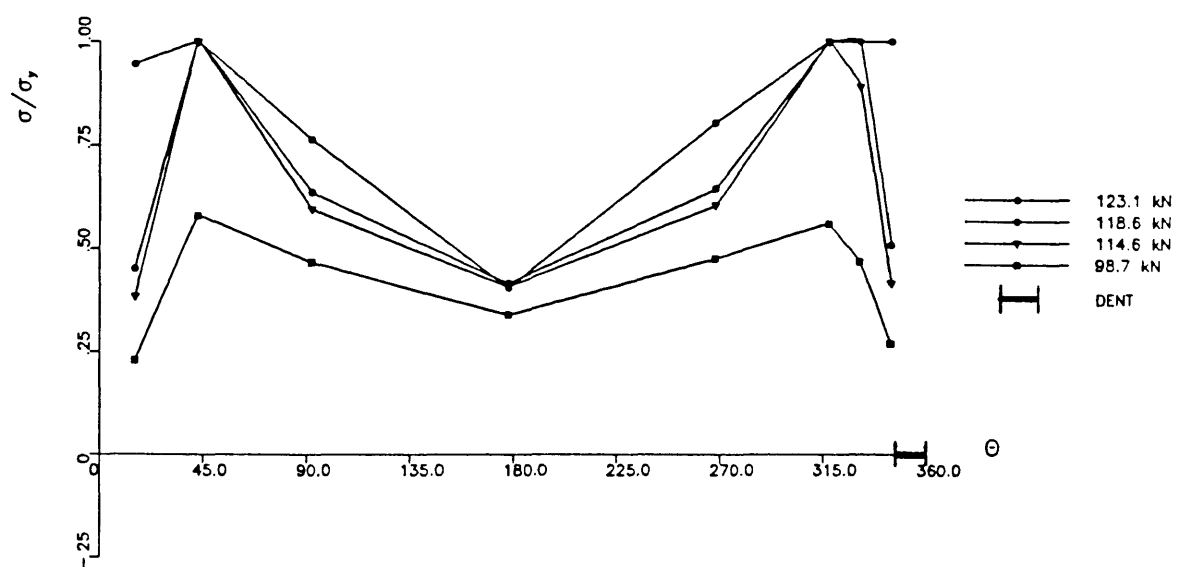
Figure 3.46

AXIAL STRESSES AT PEAK LOAD MODEL CY-1
EXPERIMENTAL AXIAL TEST



(a)

AVERAGE AXIAL STRESSES MODEL CY-1
EXPERIMENTAL AXIAL TEST



(b)

Figure 3.47

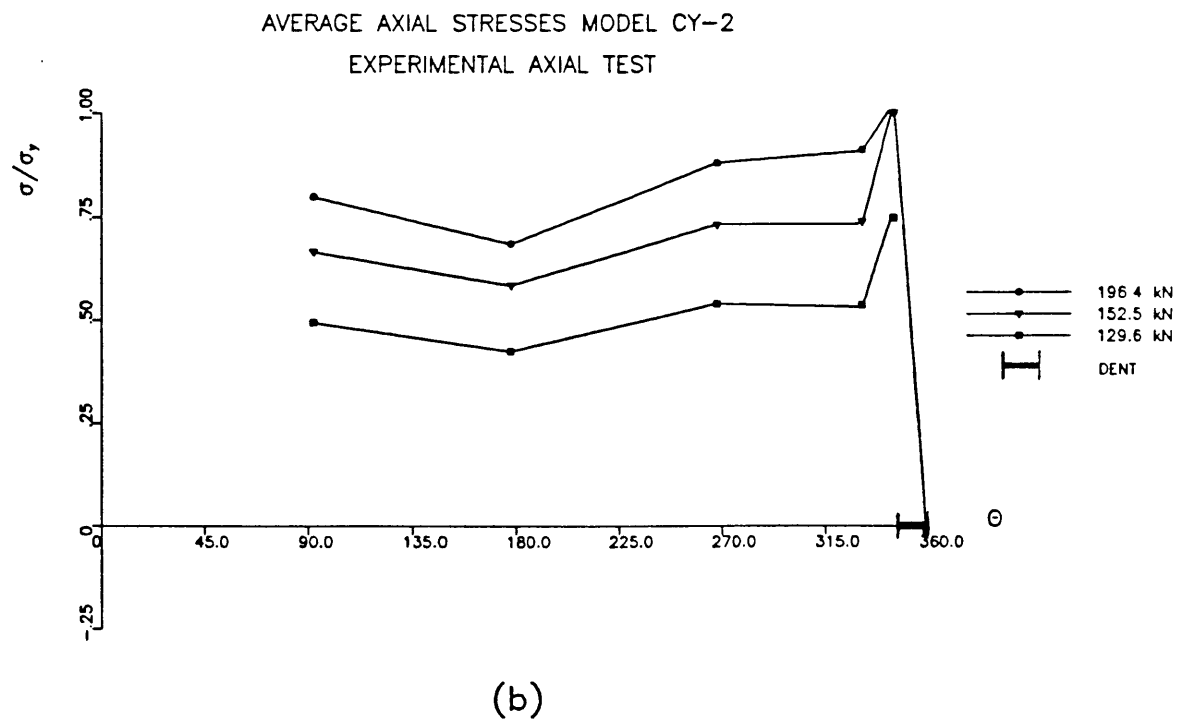
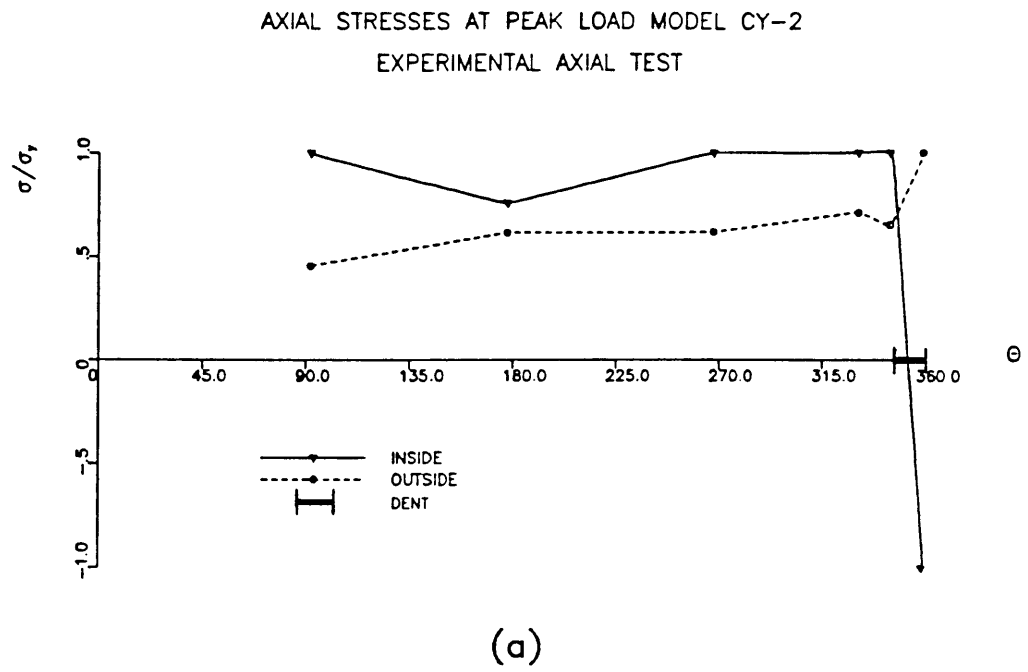
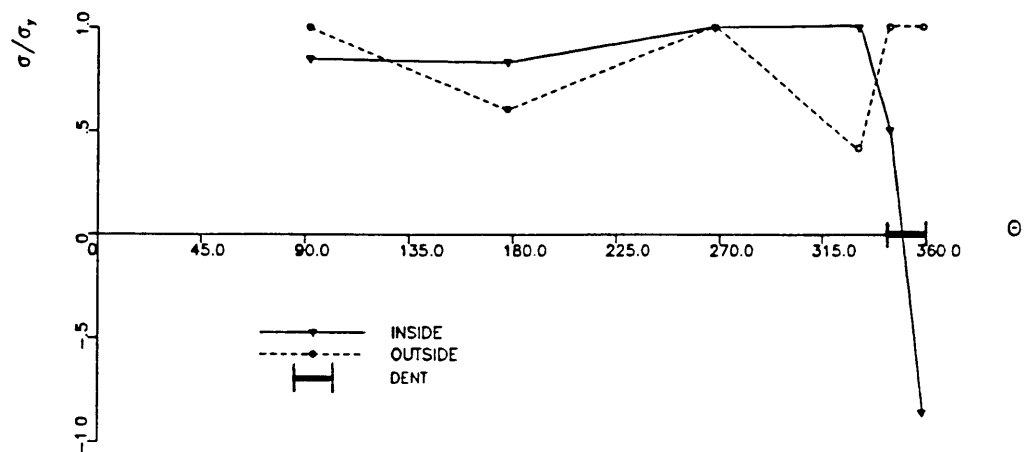


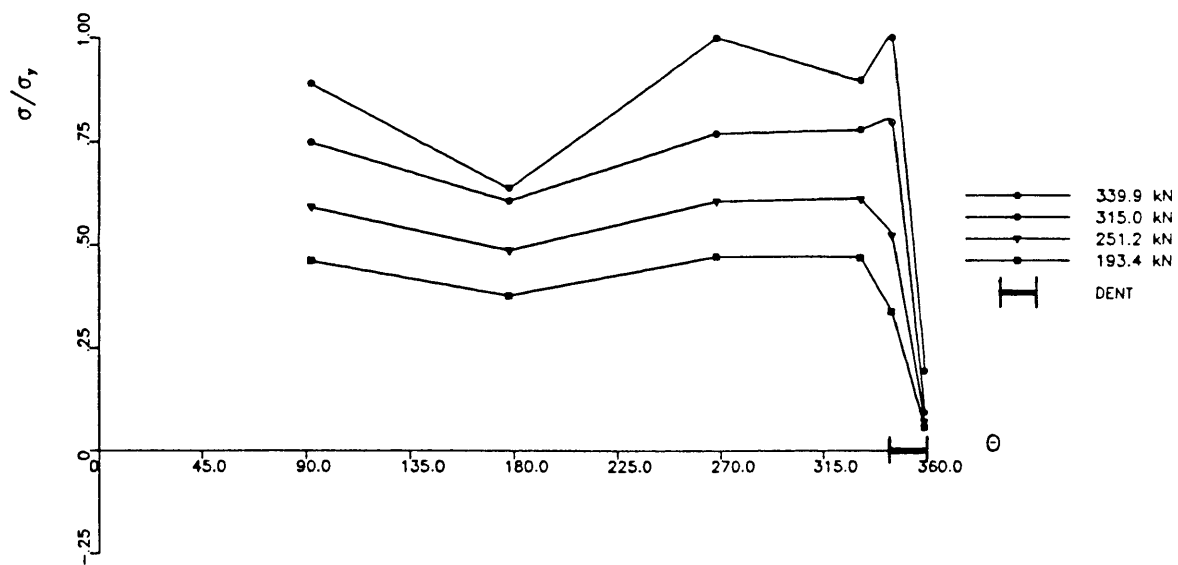
Figure 3.48

AXIAL STRESSES AT PEAK LOAD MODEL CY-3
EXPERIMENTAL AXIAL TEST



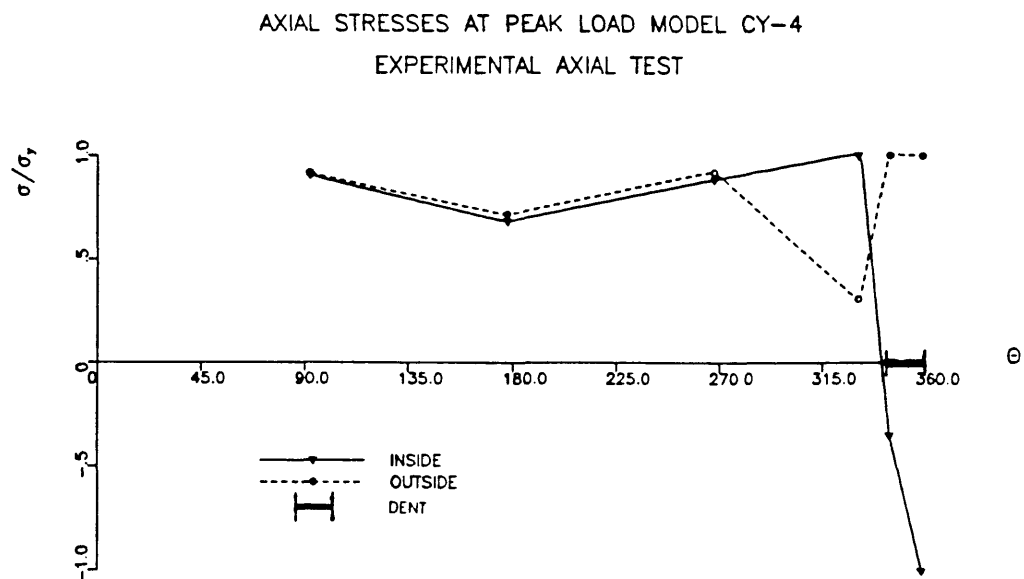
(a)

AVERAGE AXIAL STRESSES MODEL CY-3
EXPERIMENTAL AXIAL TEST

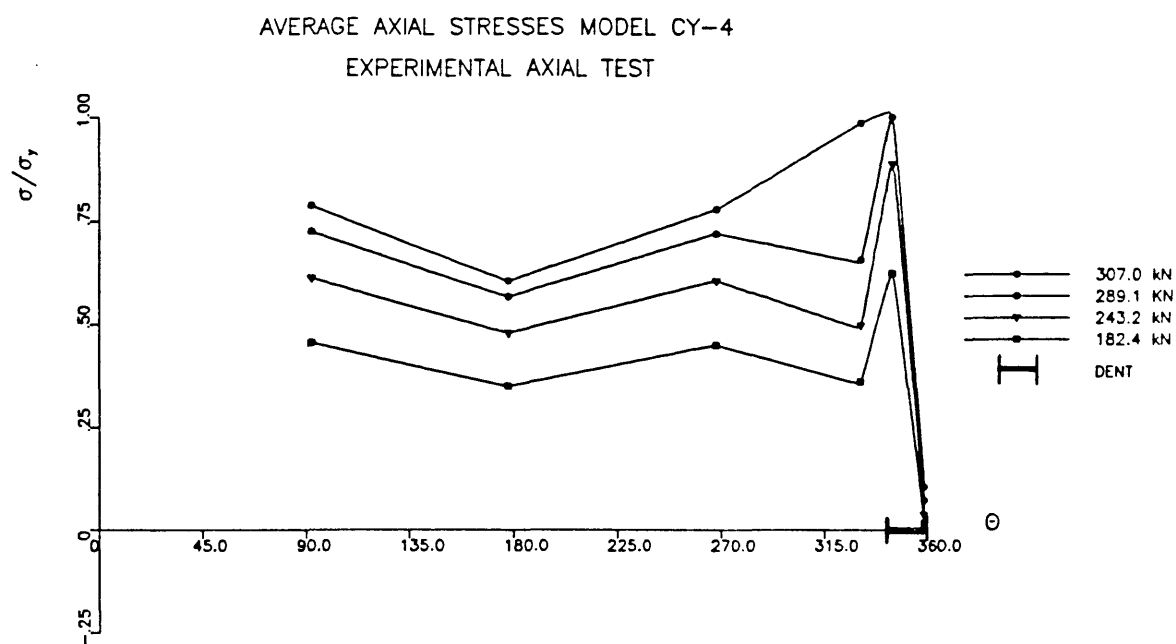


(b)

Figure 3.49

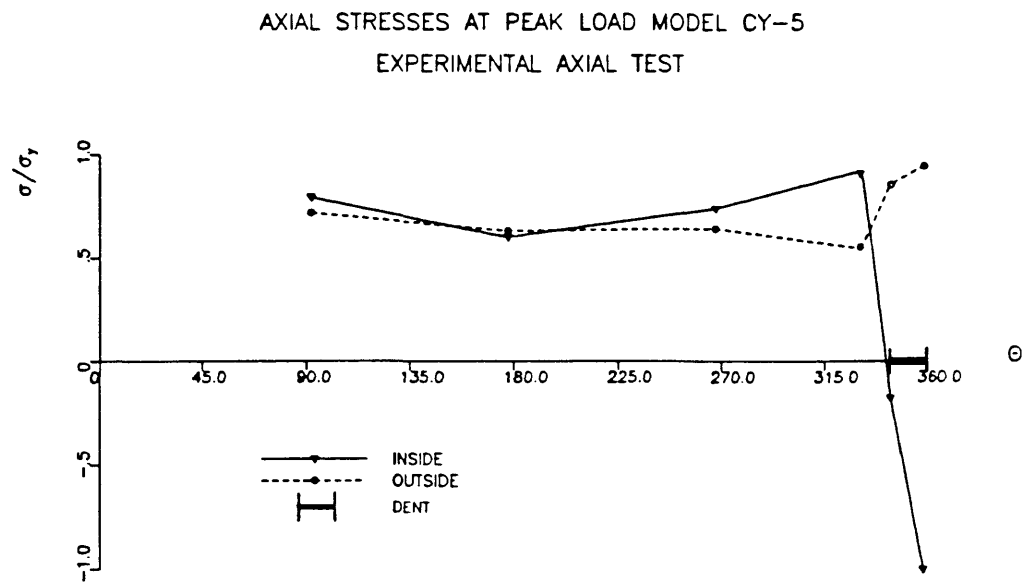


(a)

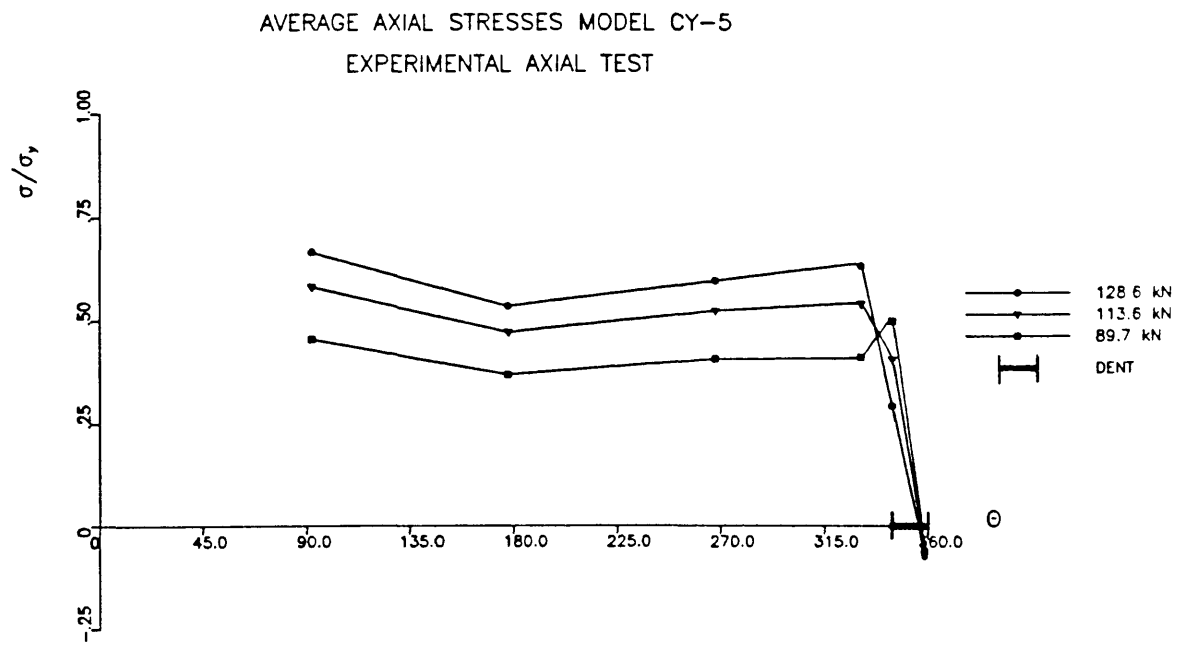


(b)

Figure 3.50



(a)



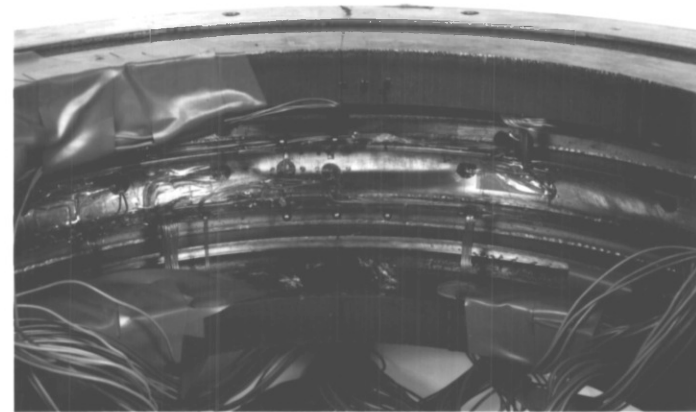
(b)

Figure 3.51

CY-1 BUCKLED SHAPE



OUTSIDE VIEW



INSIDE VIEW

Figure 3.52

MODEL CY-2
DEFLECTIONS DURING FAILURE TEST

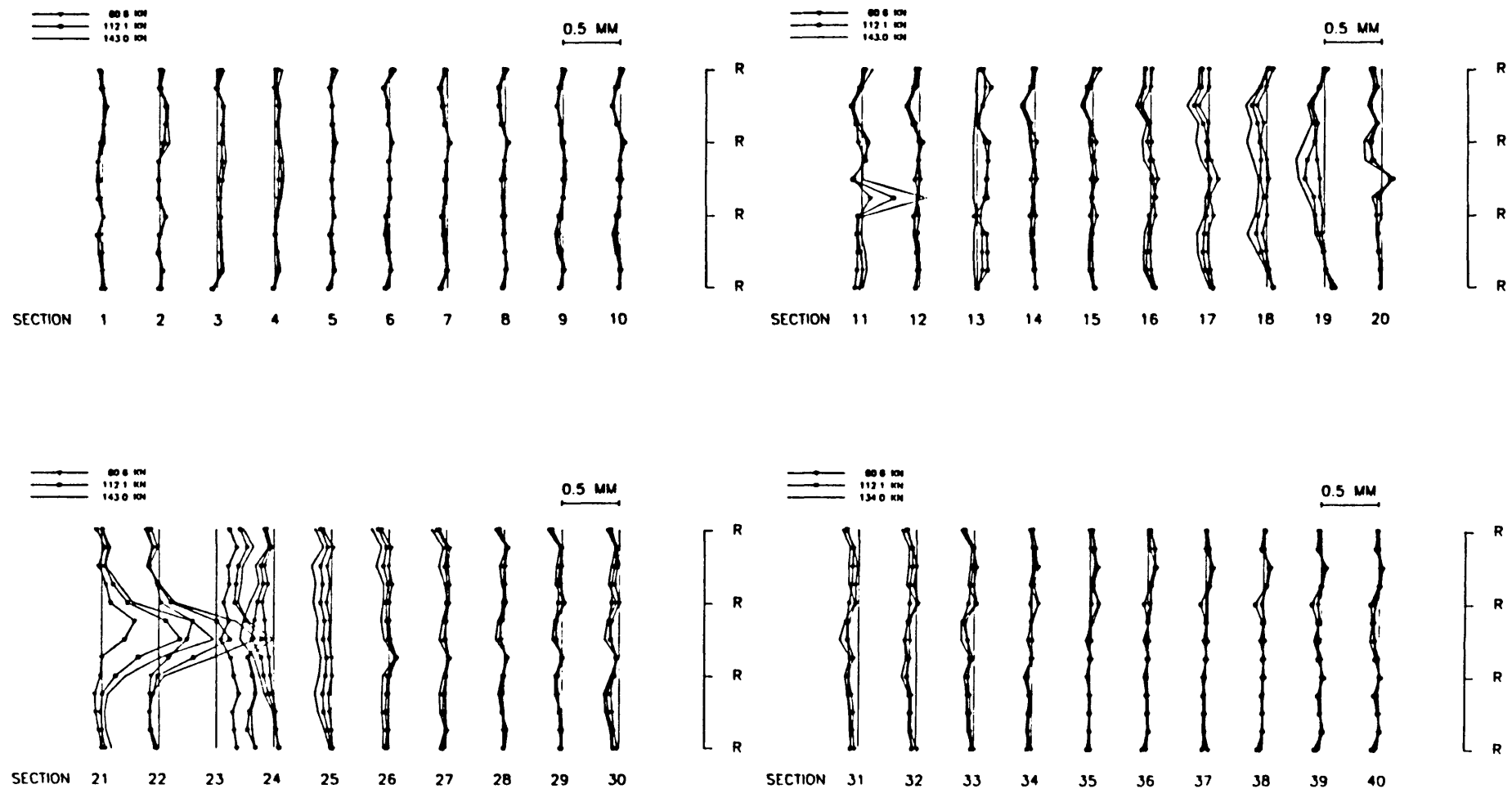


Figure 3.53

MODEL CY-2
DEFLECTIONS AFTER FAILURE

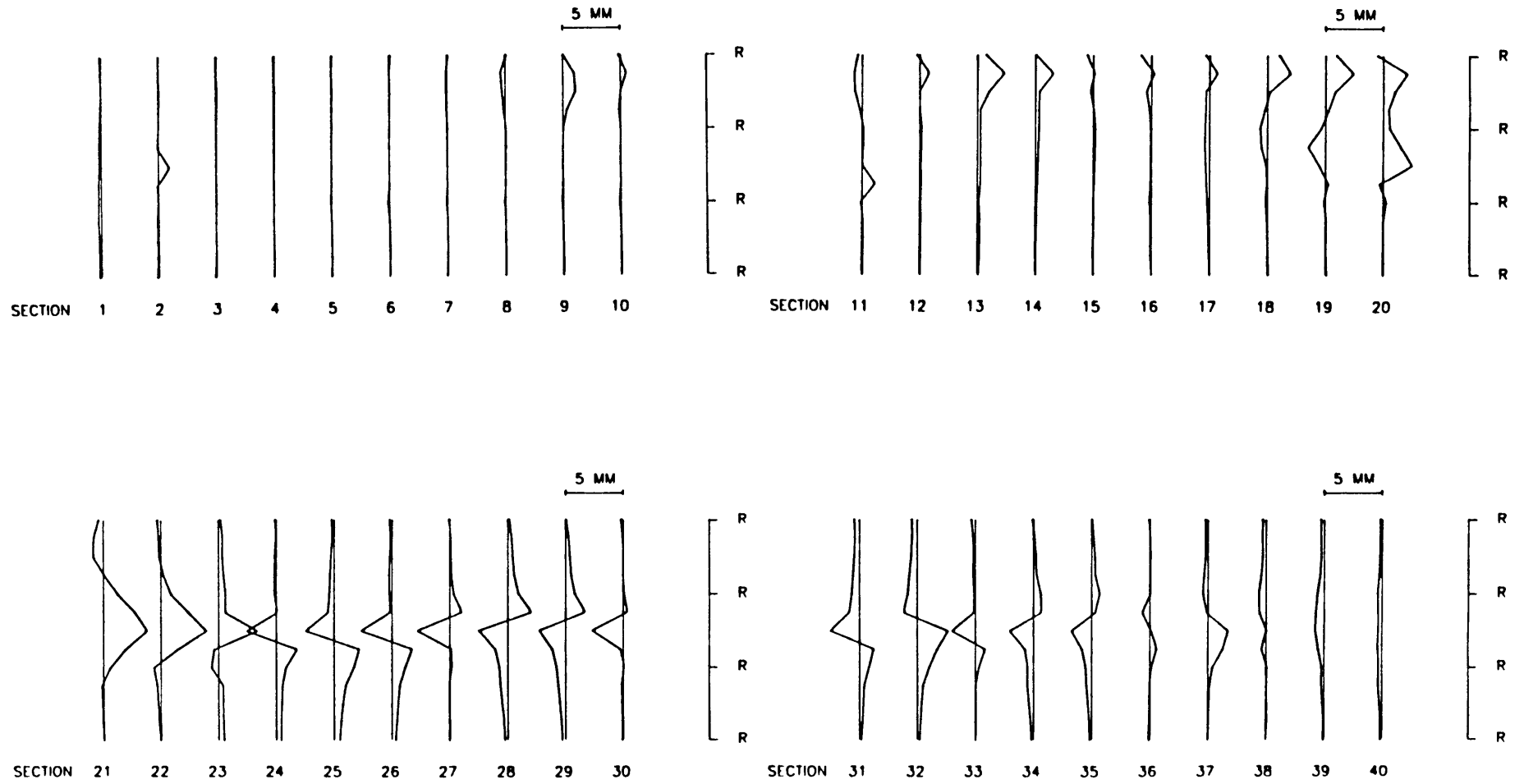


Figure 3.54

FAILURE MODE CY-2

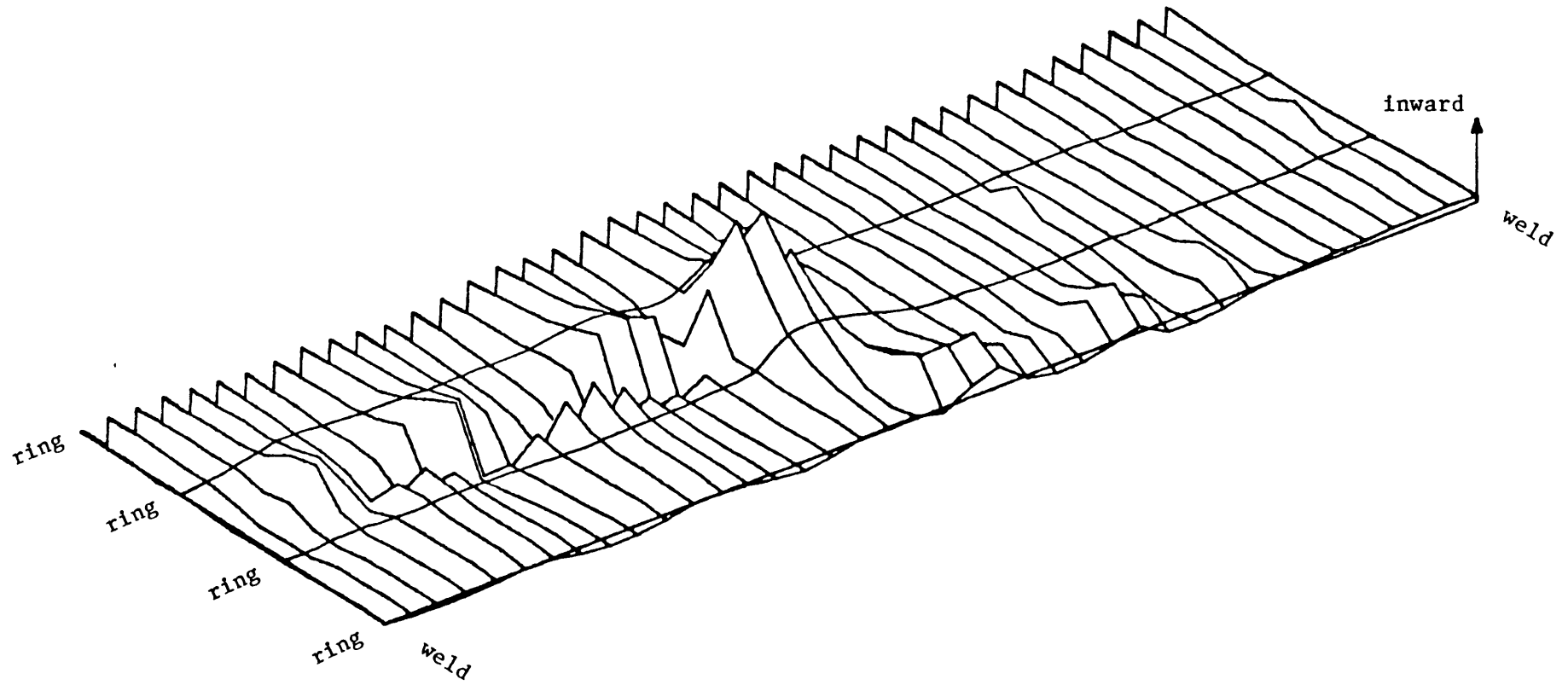
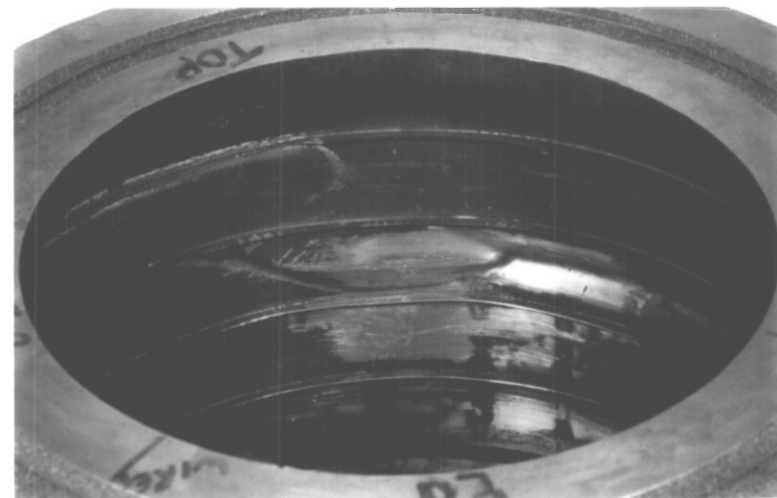


Figure 3.55

CY-2 BUCKLED SHAPE



OUTSIDE VIEW



INSIDE VIEW

Figure 3.56

MODEL CY-3
DEFLECTIONS DURING FAILURE TEST

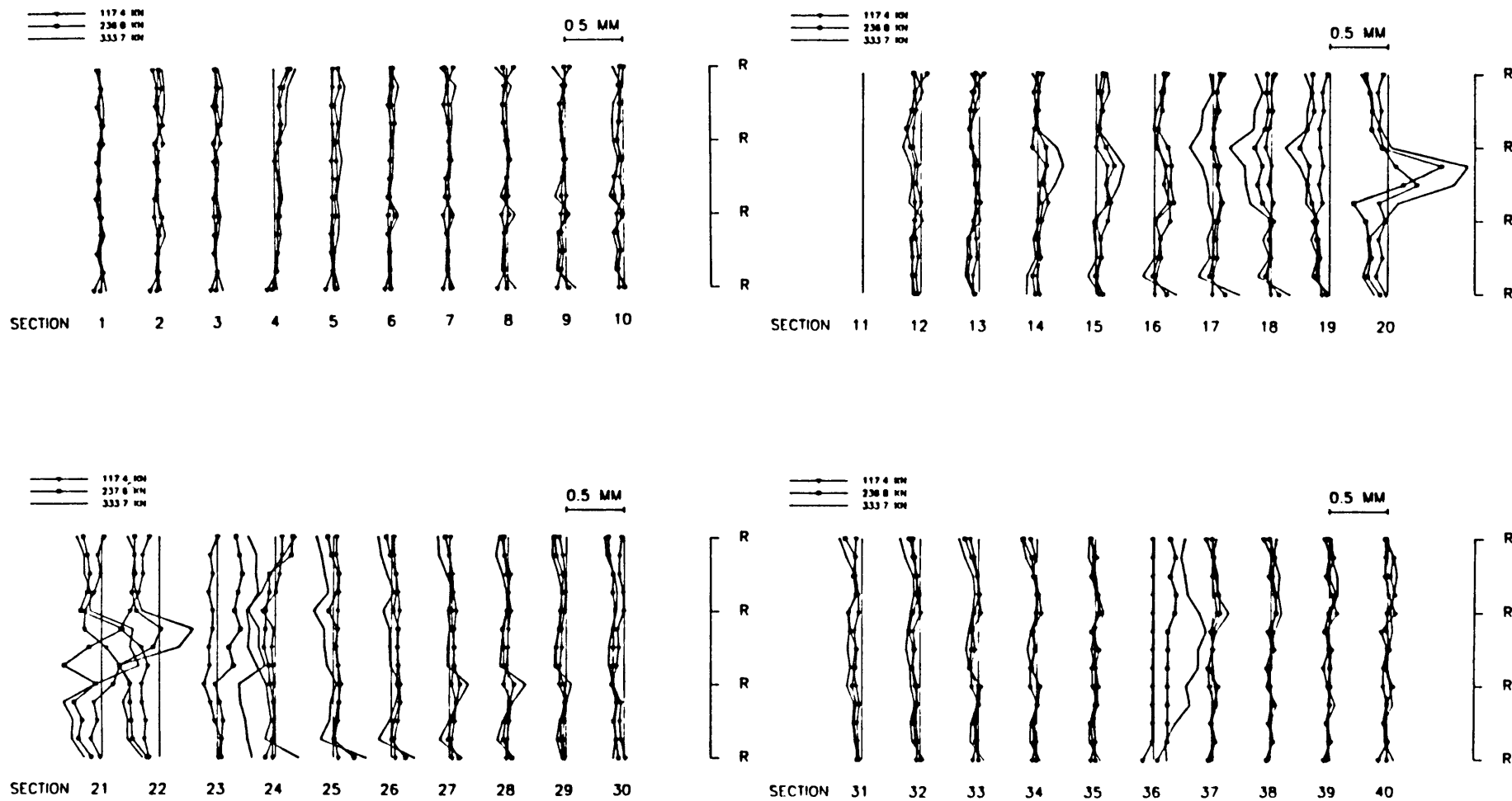


Figure 3.57

MODEL CY-3
DEFLECTIONS AFTER FAILURE

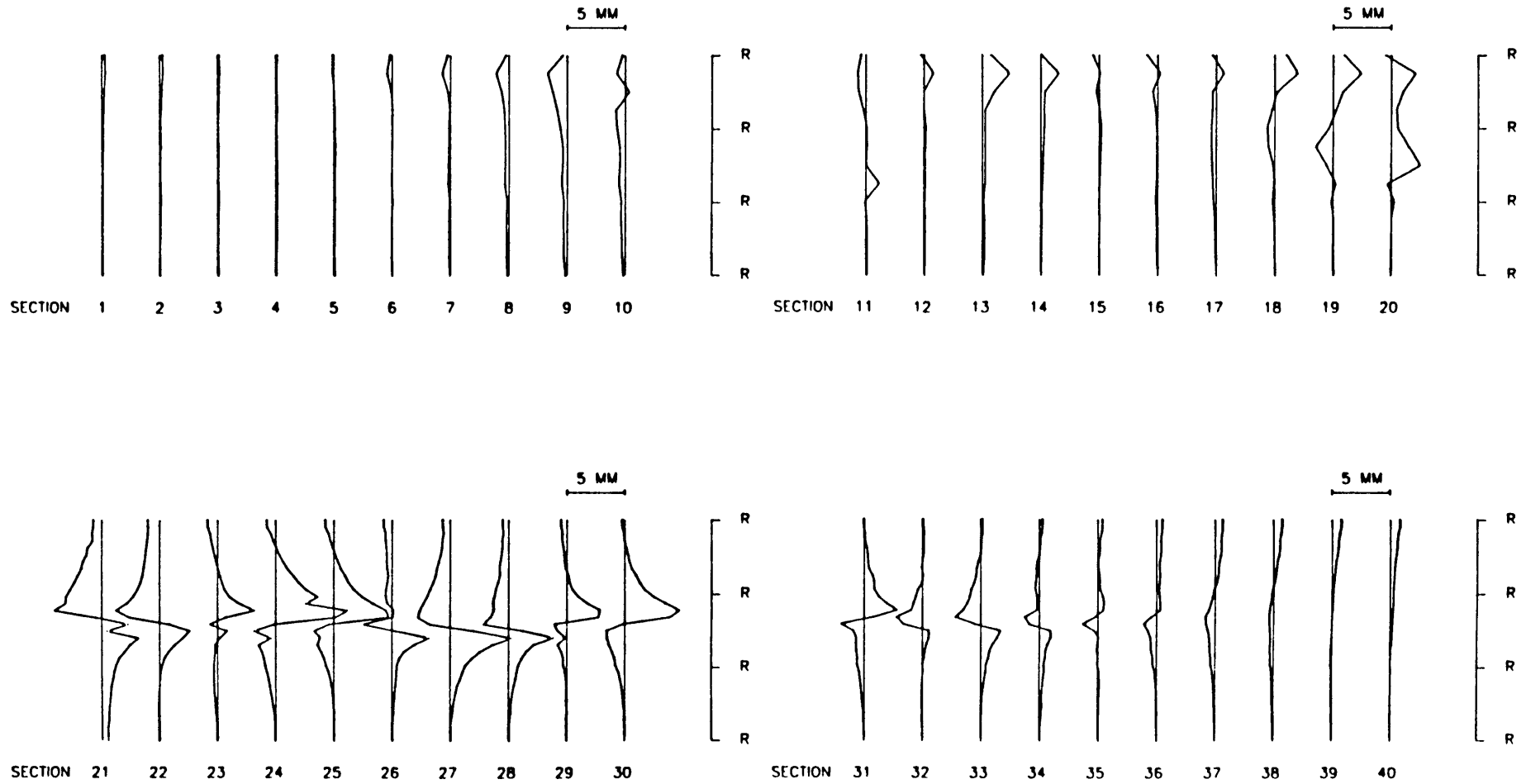


Figure 3.58

FAILURE MODE CY-3

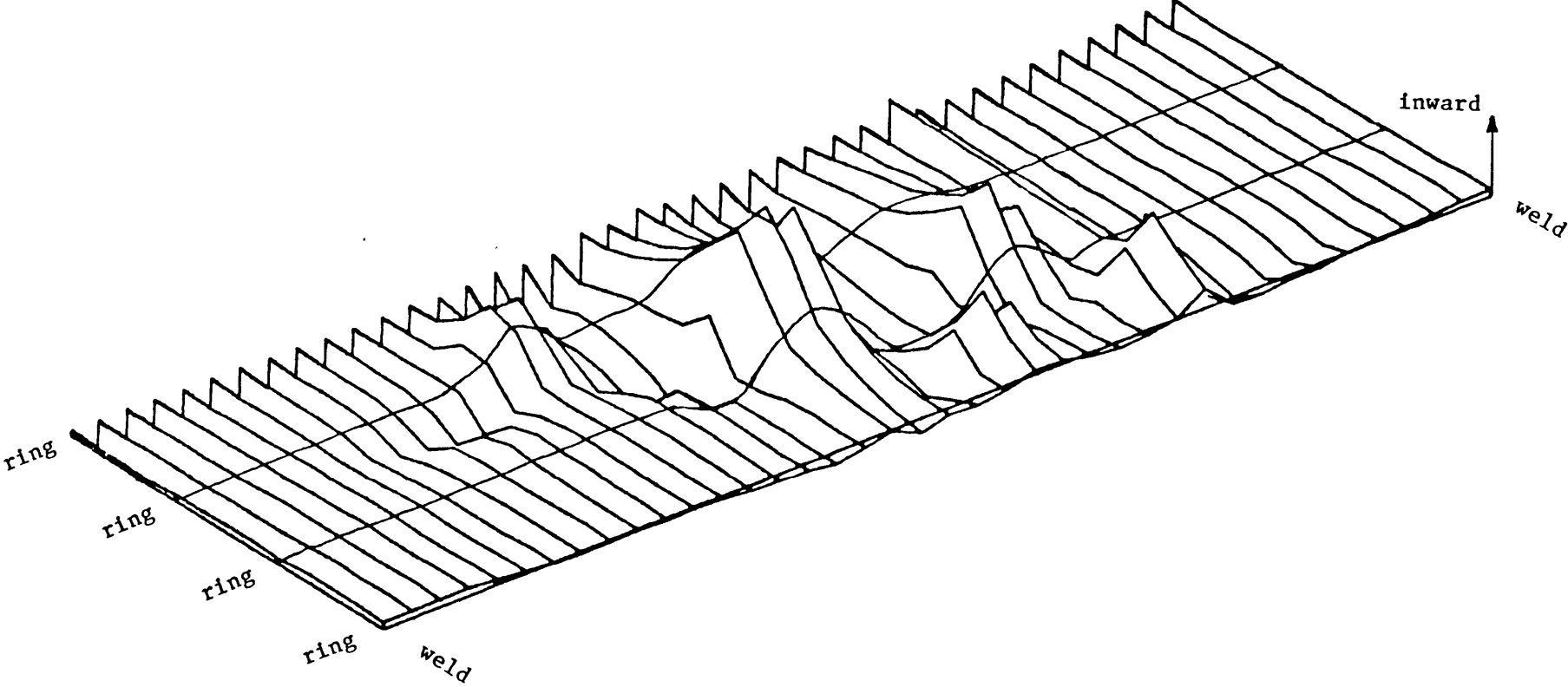
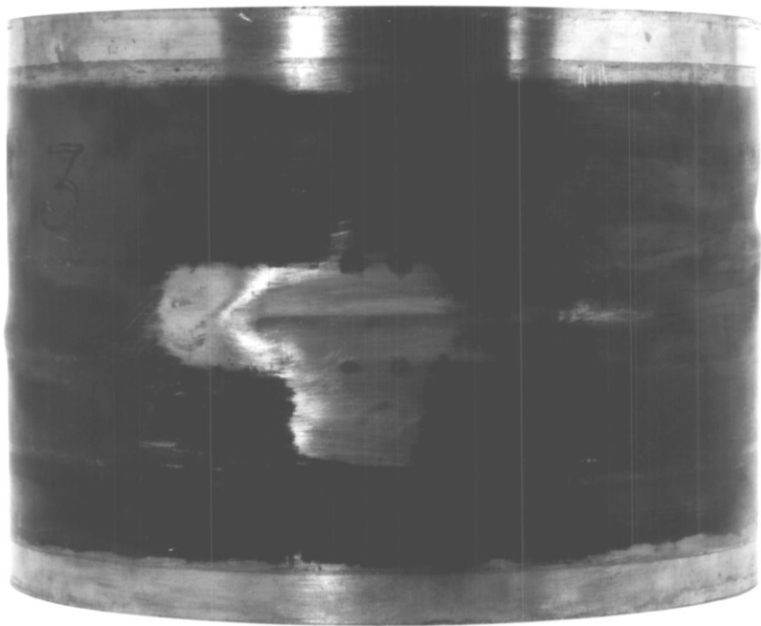
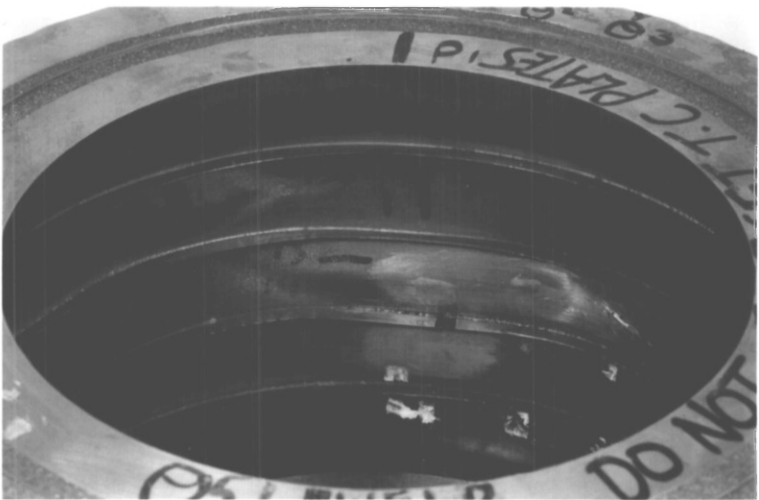


Figure 3.59

CY-3 BUCKLED SHAPE



OUTSIDE VIEW



INSIDE VIEW

Figure 3.60

MODEL CY-4
DEFLECTIONS DURING FAILURE TEST

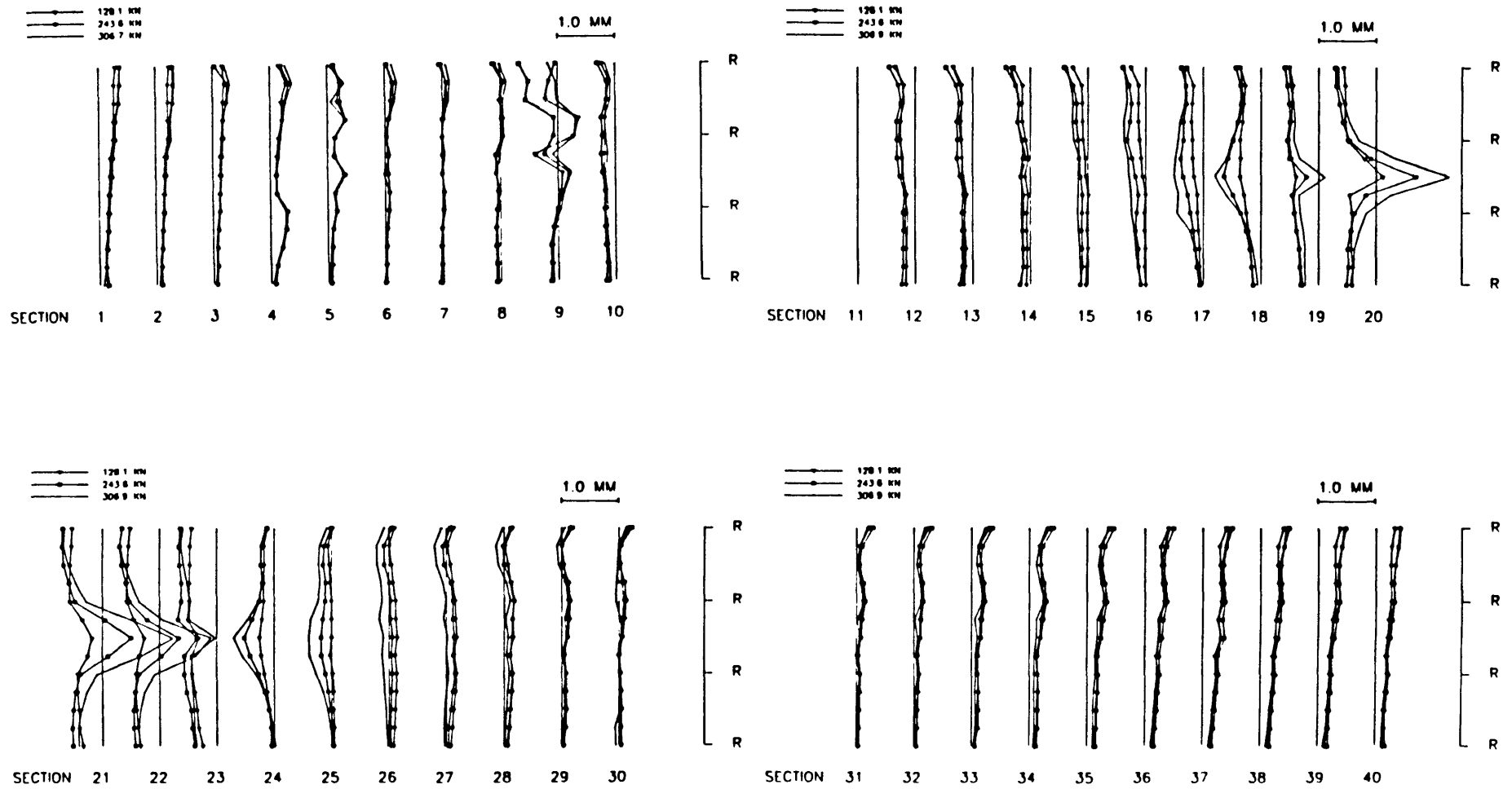


Figure 3.61

MODEL CY-4
DEFLECTIONS AFTER FAILURE

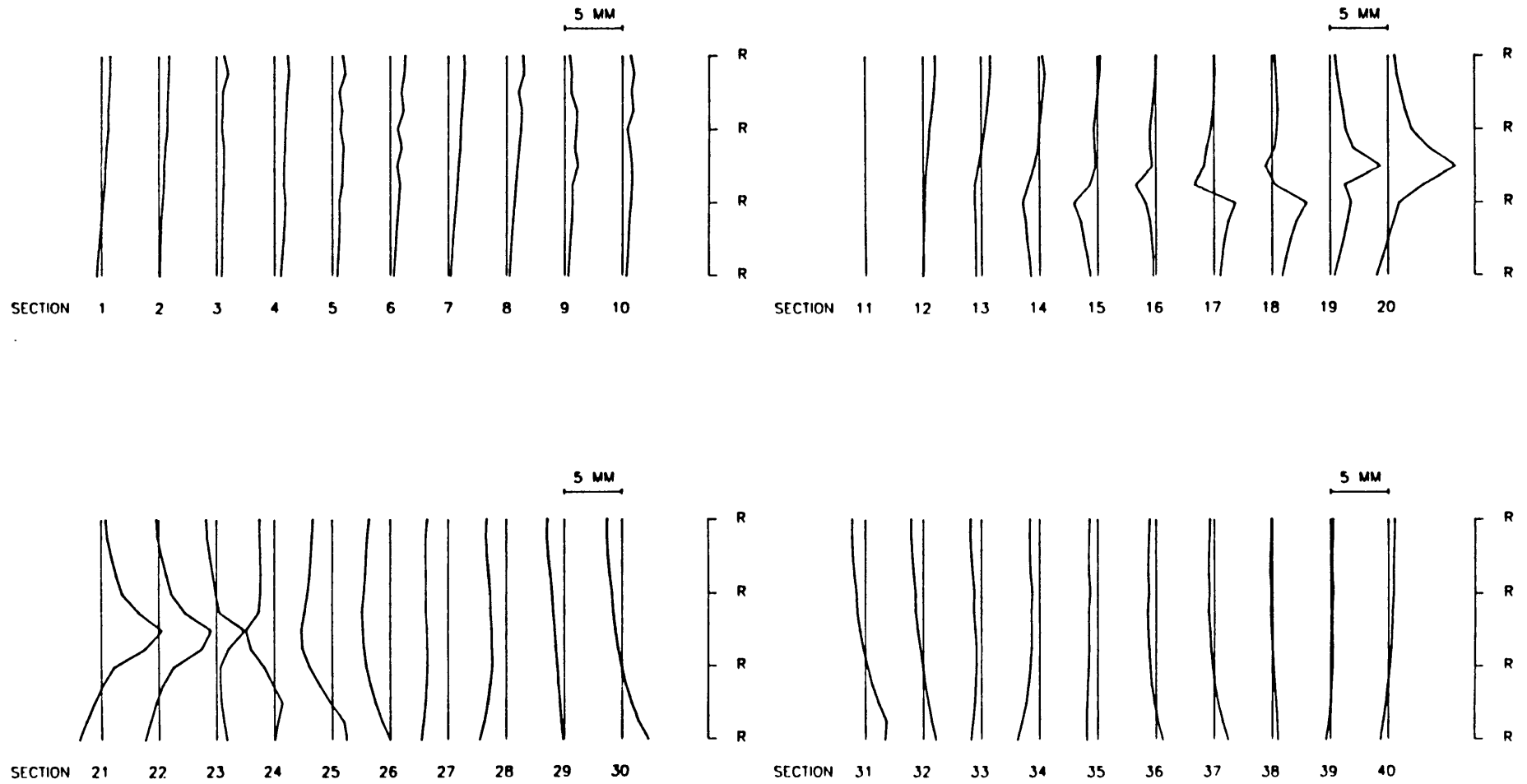


Figure 3.62

FAILURE MODE CY-4

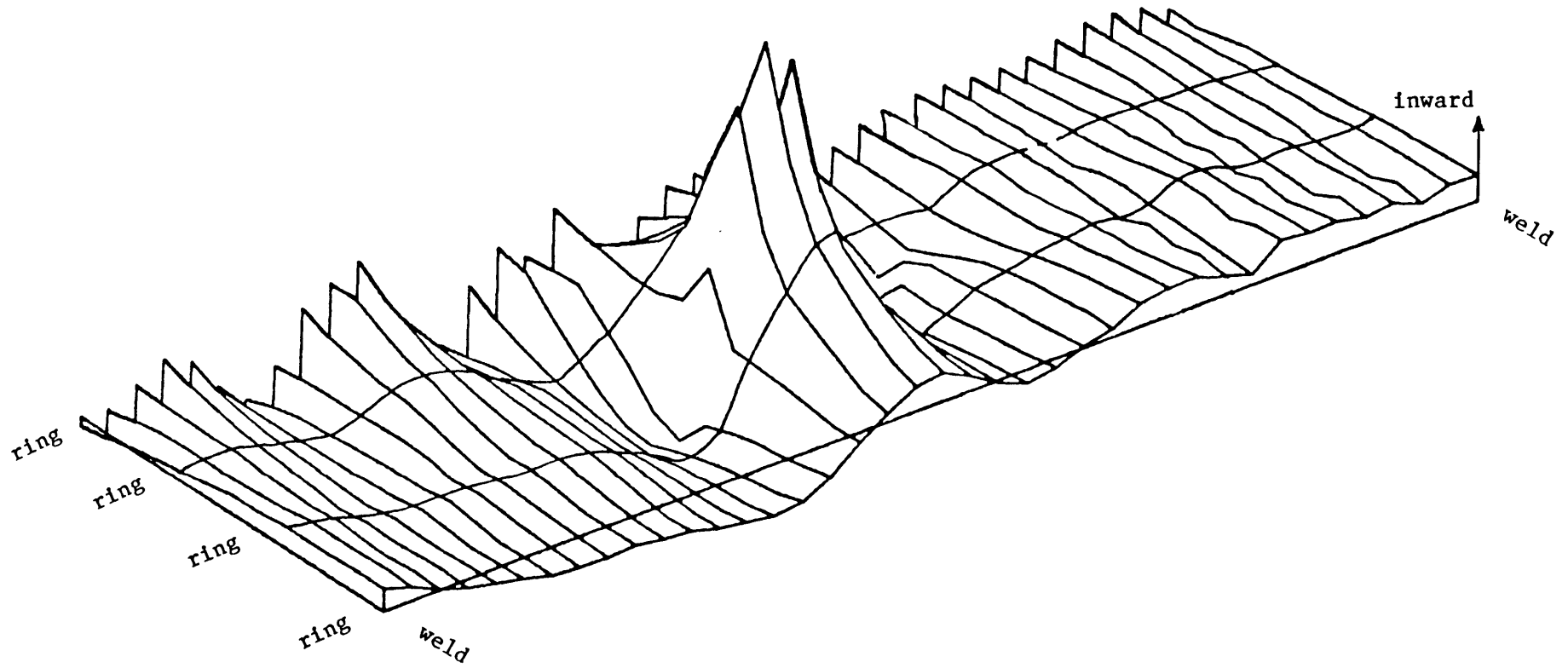


Figure 3.63

CY-4 BUCKLED SHAPE



Figure 3.64

MODEL CY-5
DEFLECTIONS DURING FAILURE TEST

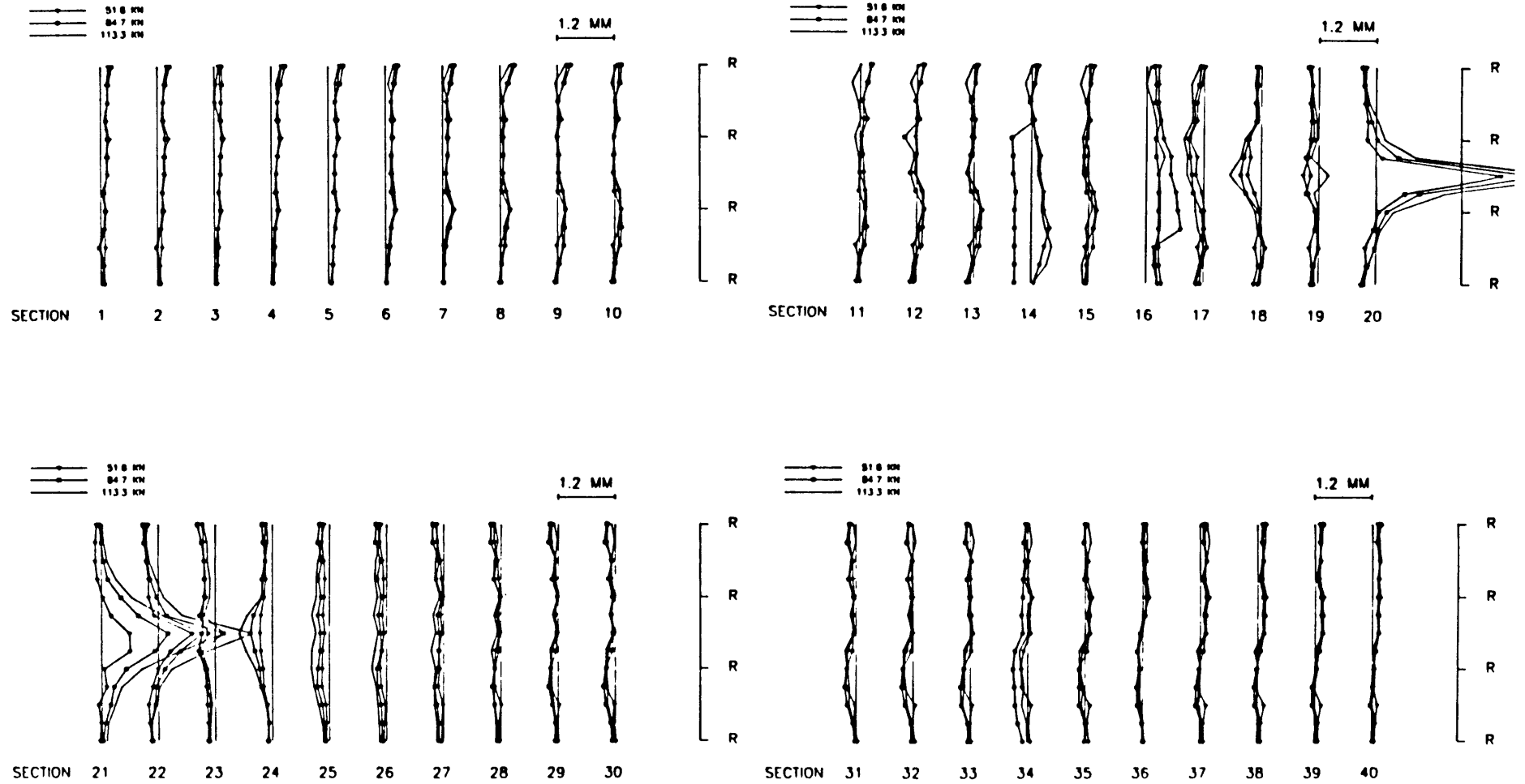


Figure 3.65

MODEL CY-5
DEFLECTIONS AFTER FAILURE

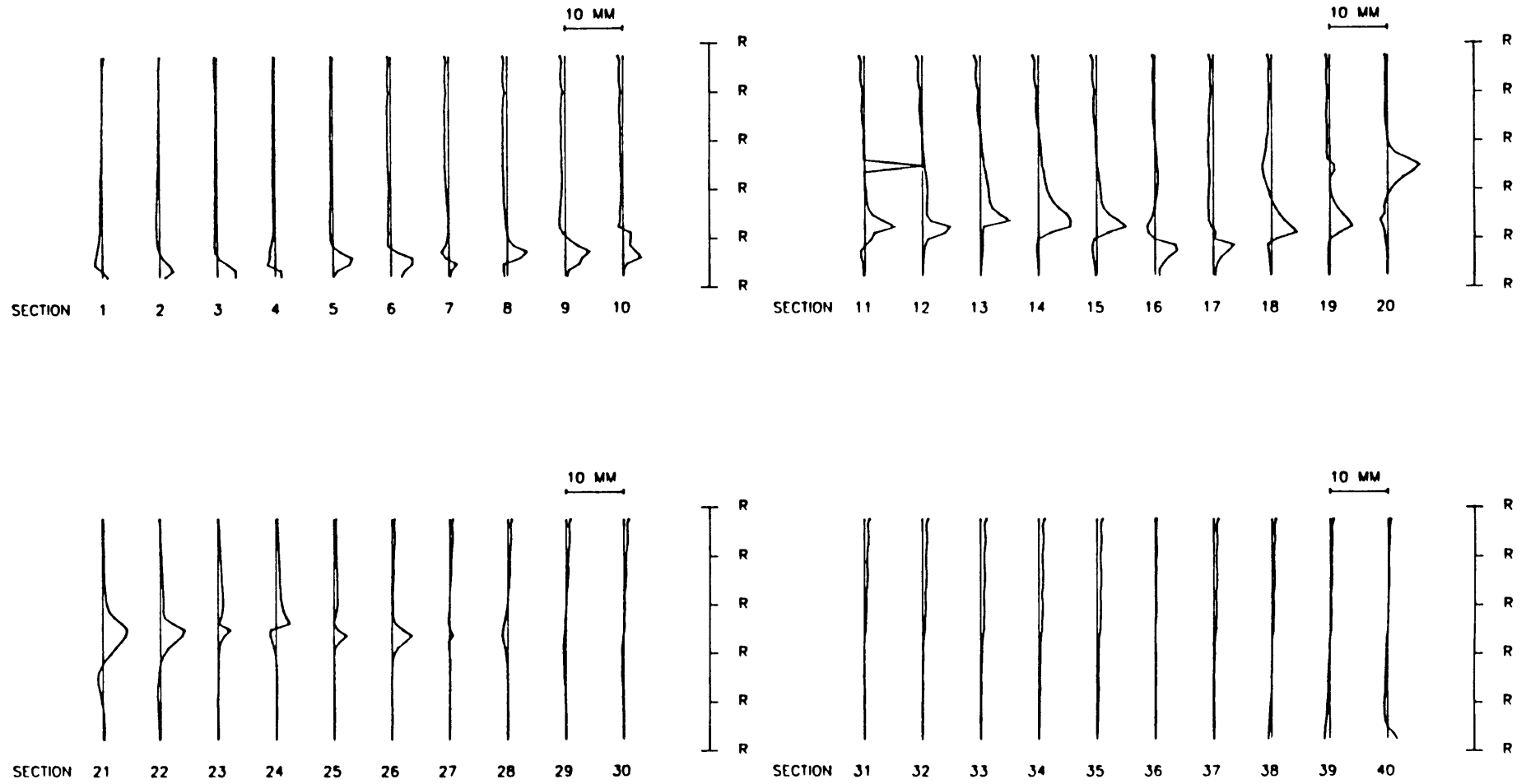


Figure 3.66

FAILURE MODE CY-5

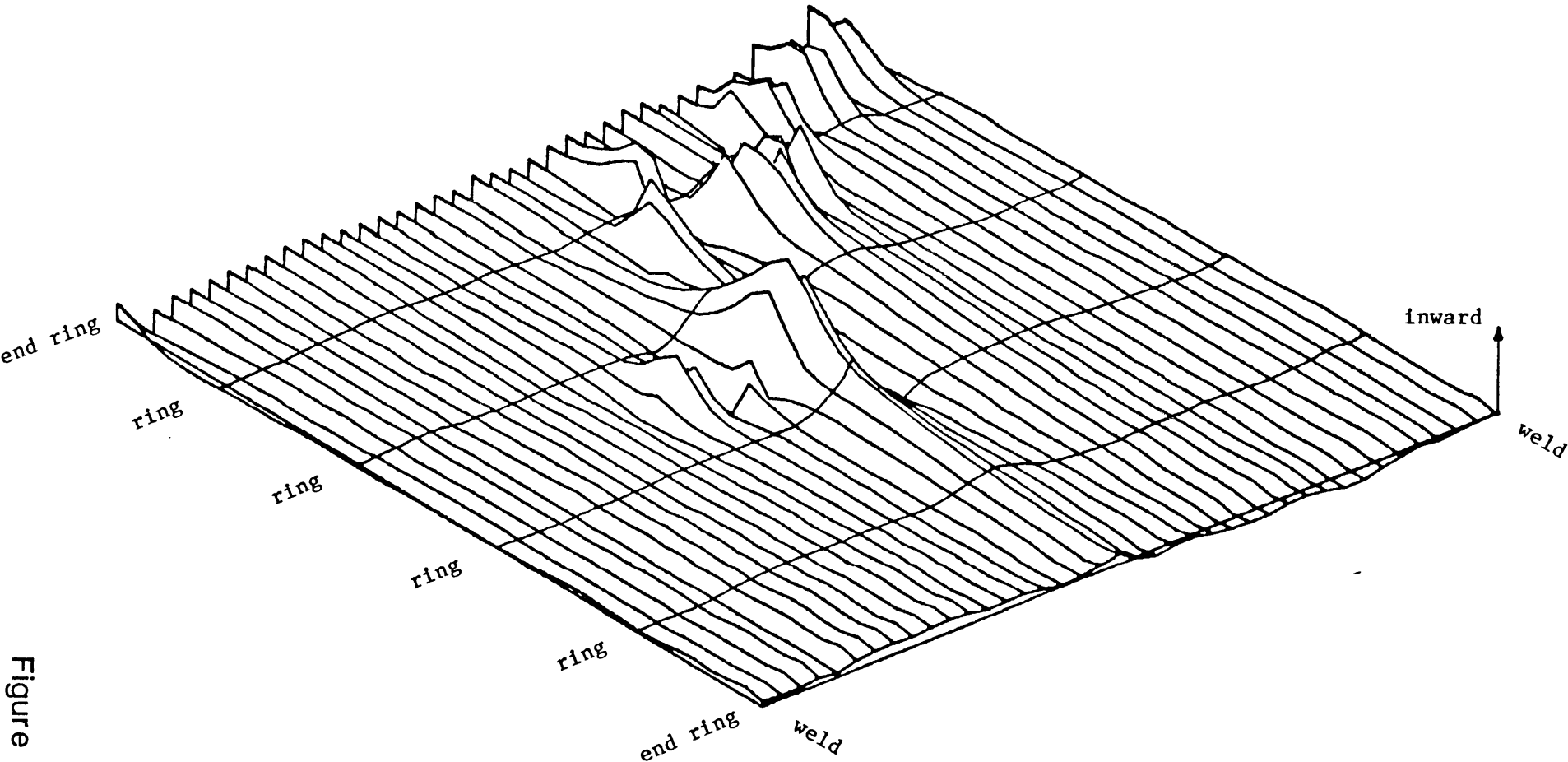


Figure 3.67

CY-5 BUCKLED SHAPE



Figure 3.68

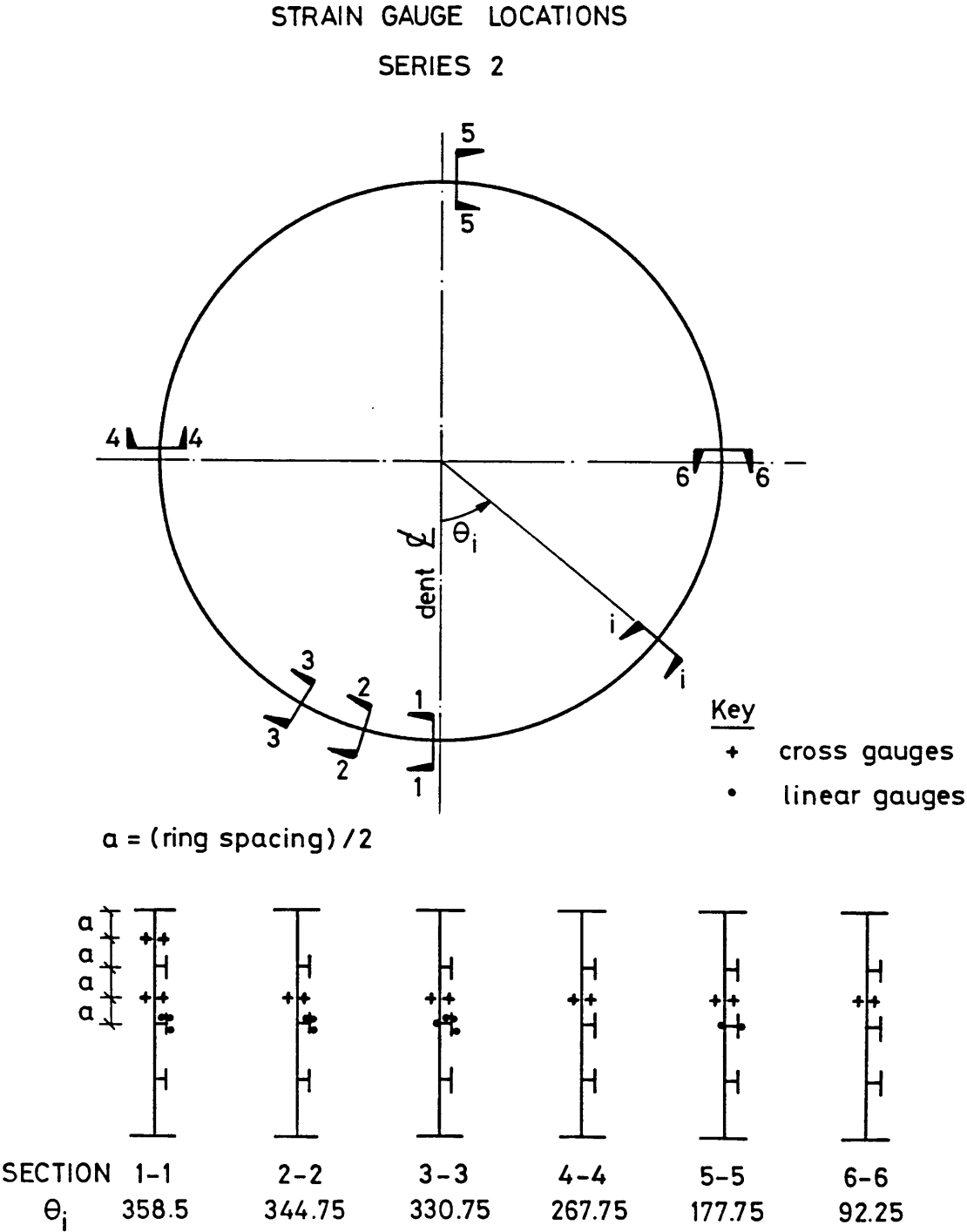
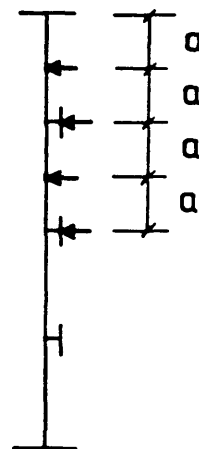
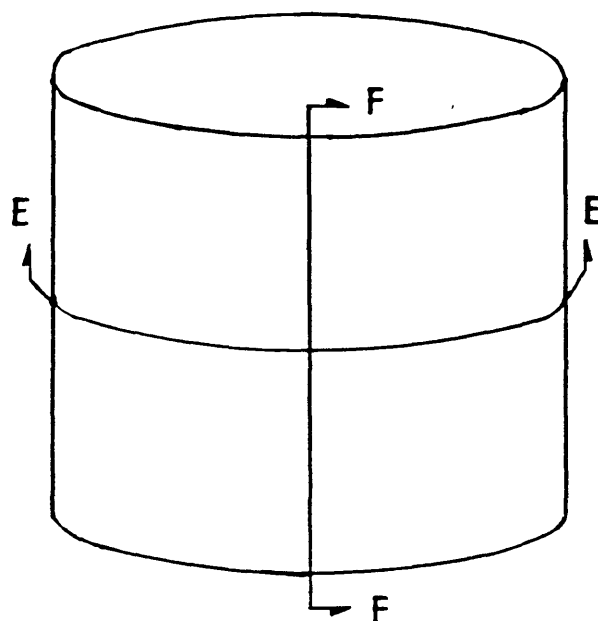


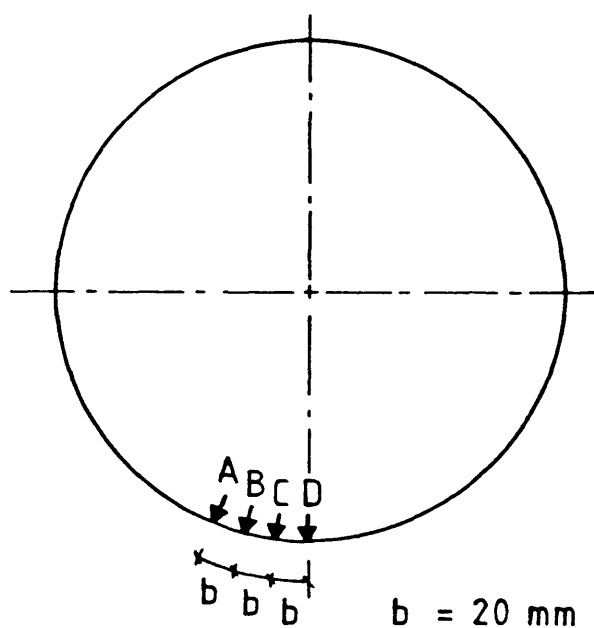
Figure 3.69

TRANSDUCER LOCATIONS FOR DENTING TEST SERIES 2



SECTION F-F

$$a = (\text{ring spacing}) / 2$$



$$b = 20 \text{ mm}$$

SECTION E-E

→ Transducer locations

E-E } Sections through
F-F } dent centreline

Figure 3.70

LOAD DENTING DEFLECTIONS - SERIES 2

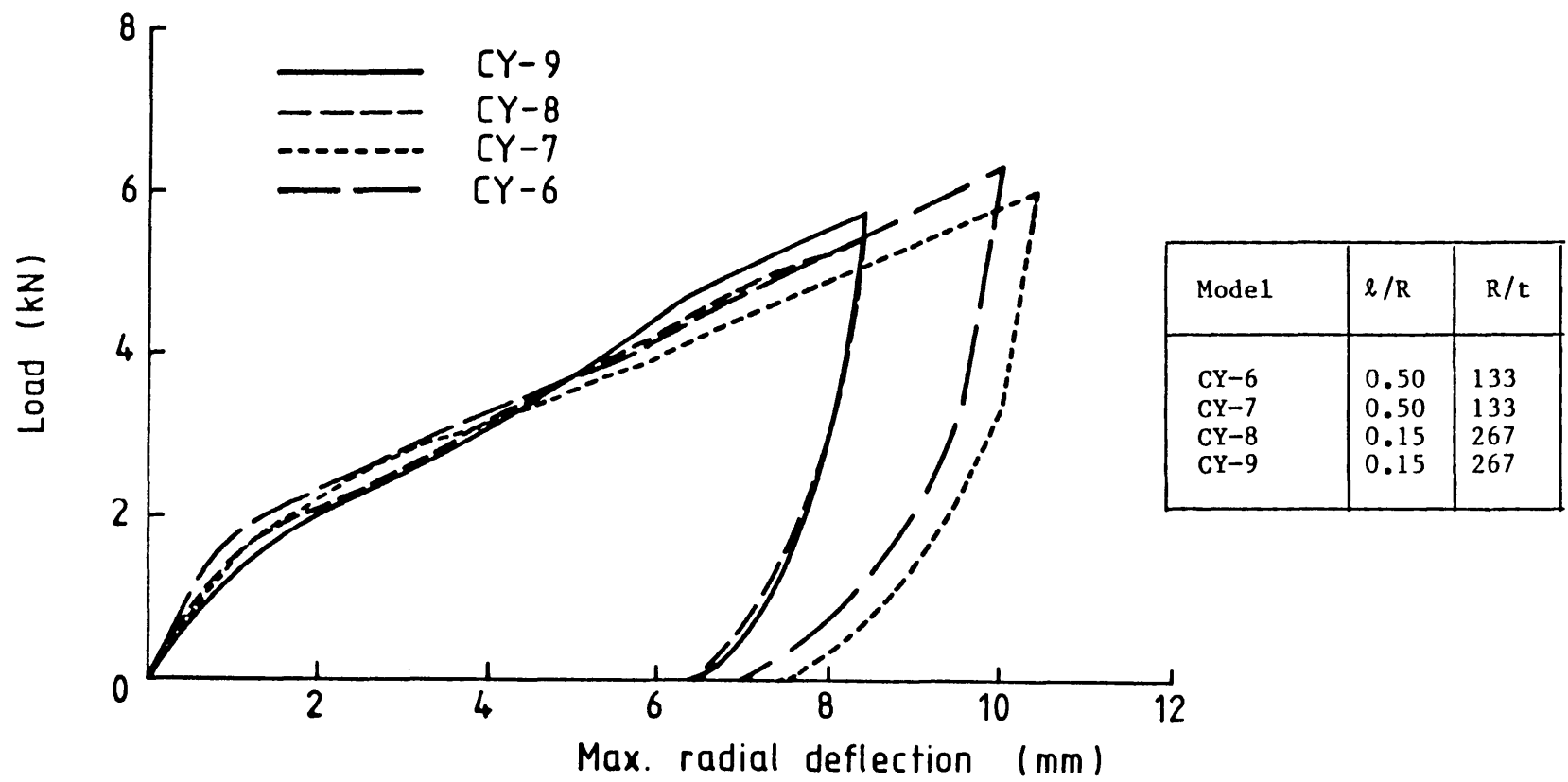
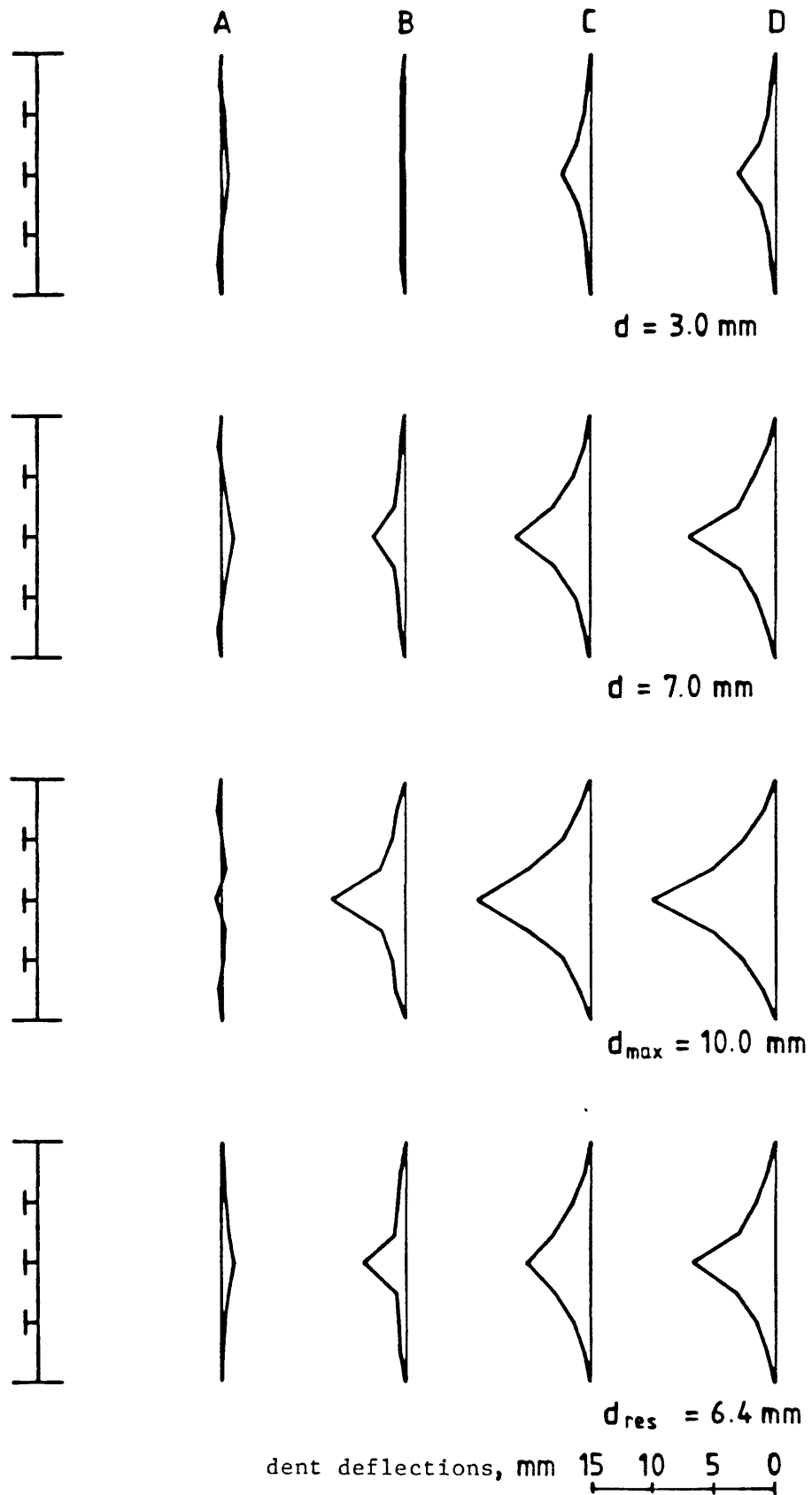


Figure 3.71

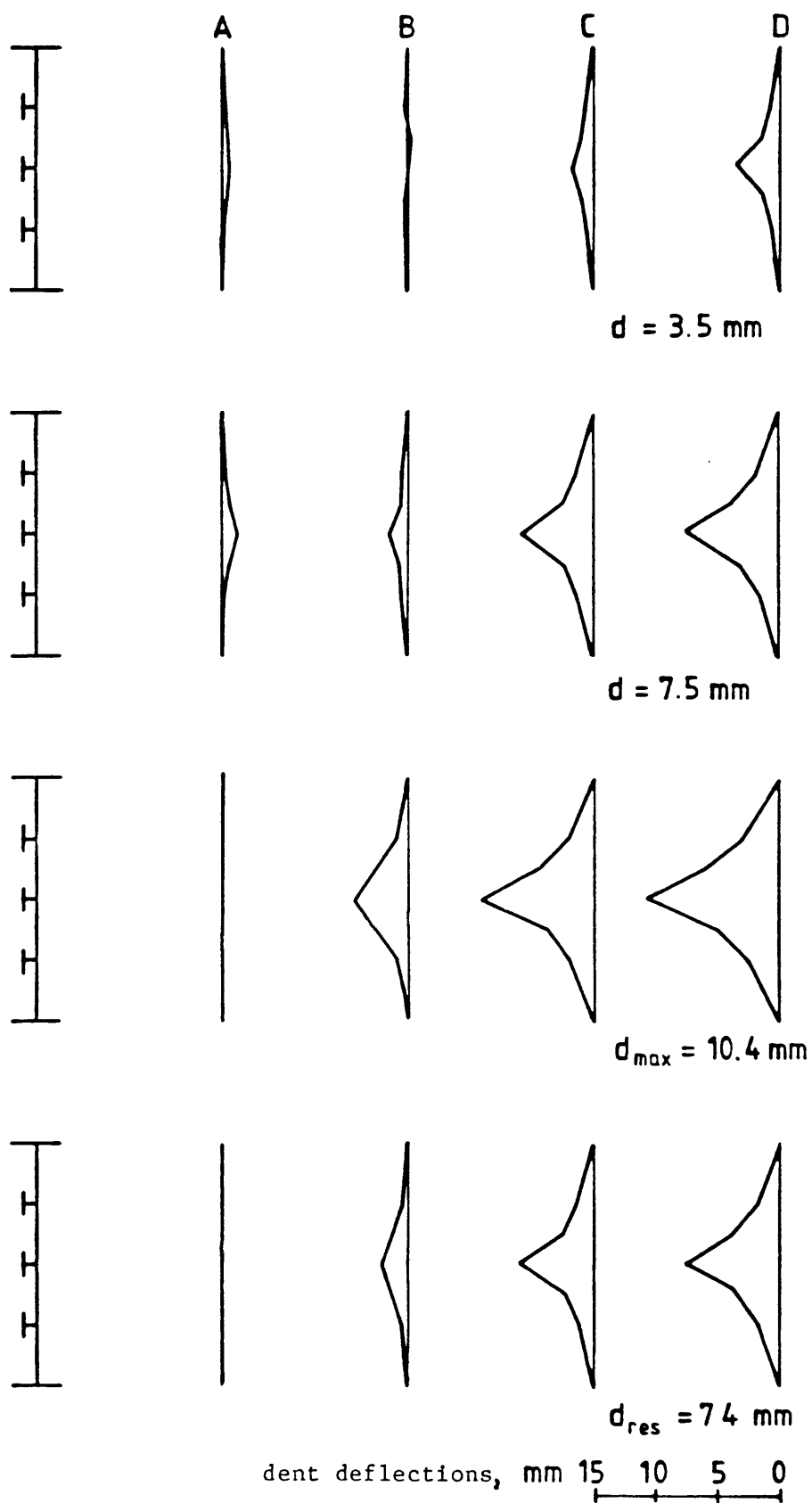
DENTING DEFLECTIONS - MODEL CY-6



For sections, see Fig. 3.70

Figure 3.72

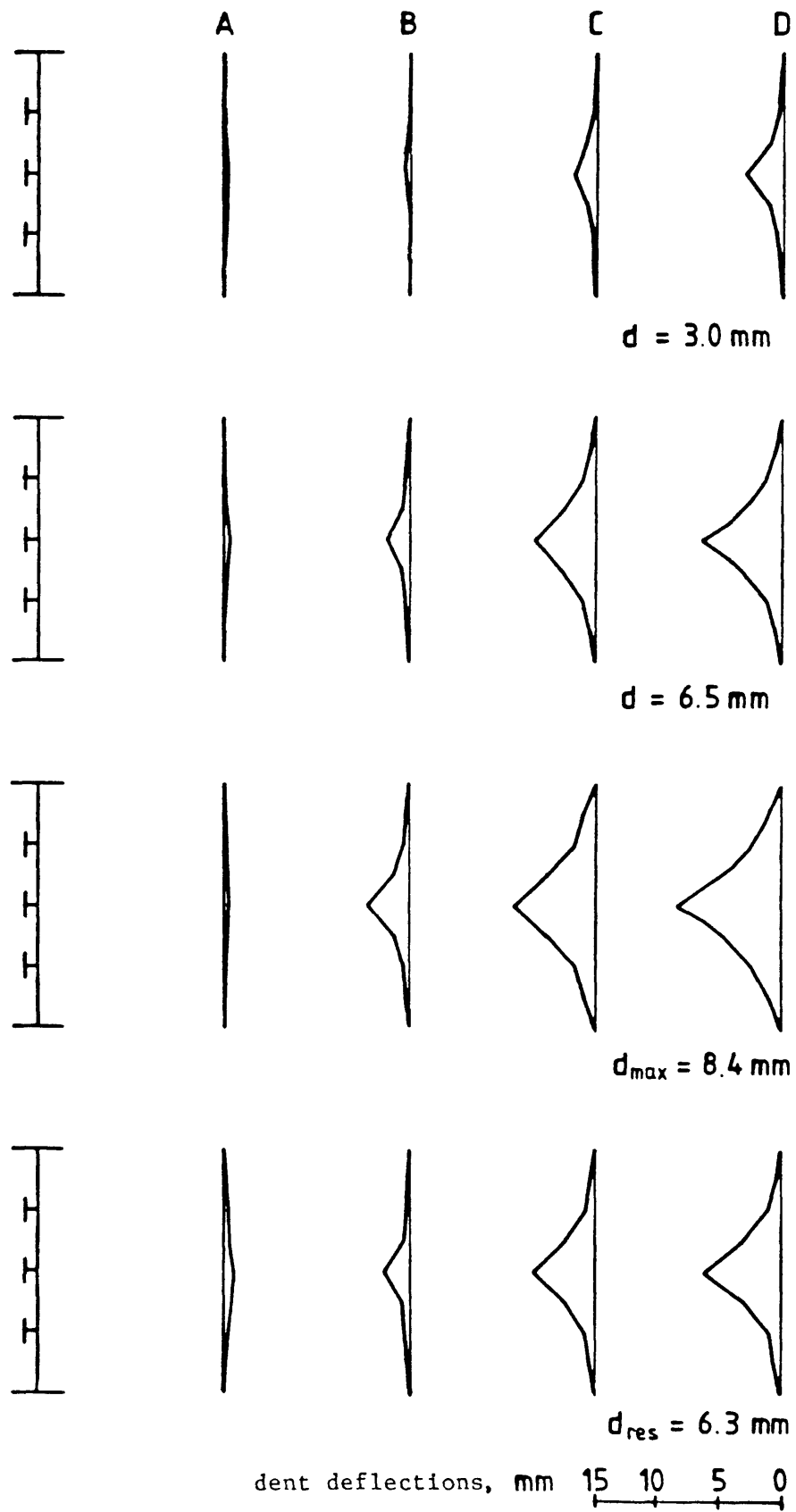
DENTING DEFLECTIONS- MODEL CY-7



For sections, see Fig. 3.70

Figure 3.73

DENTING DEFLECTIONS- MODEL CY-8



For sections, see Fig. 3.70

Figure 3.74

DENTING DEFLECTIONS - MODEL CY-9

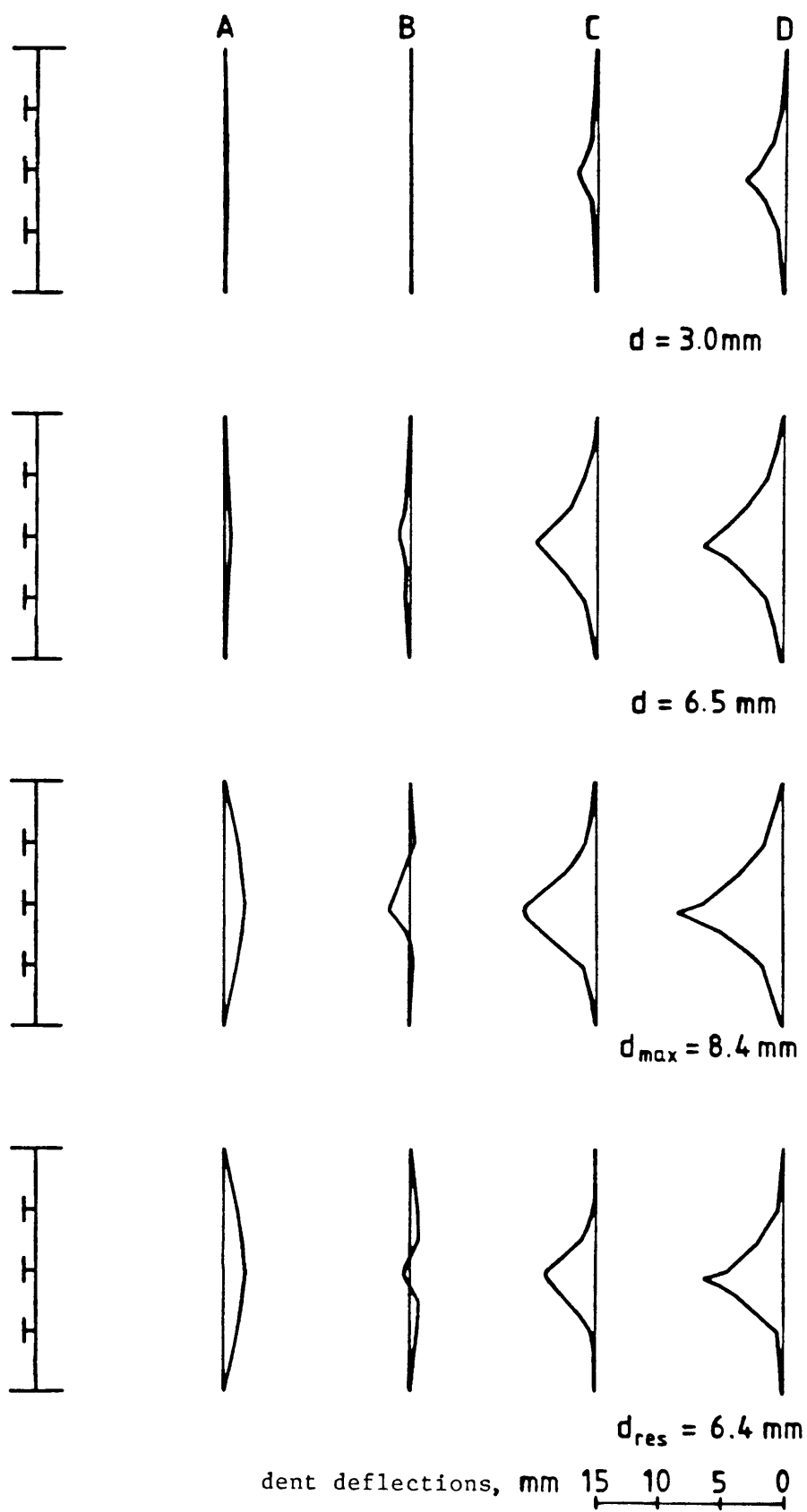
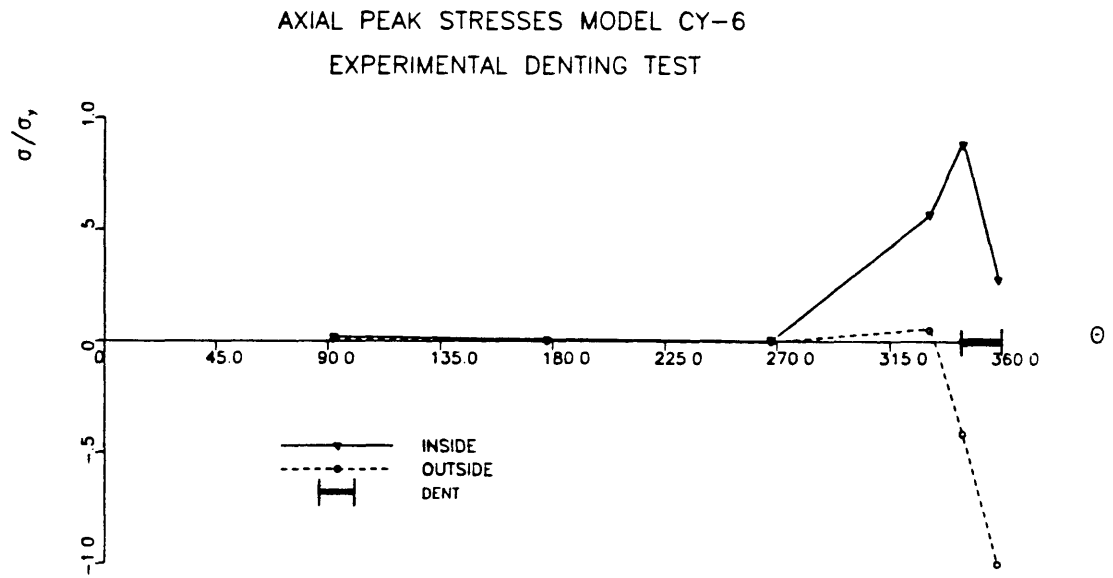
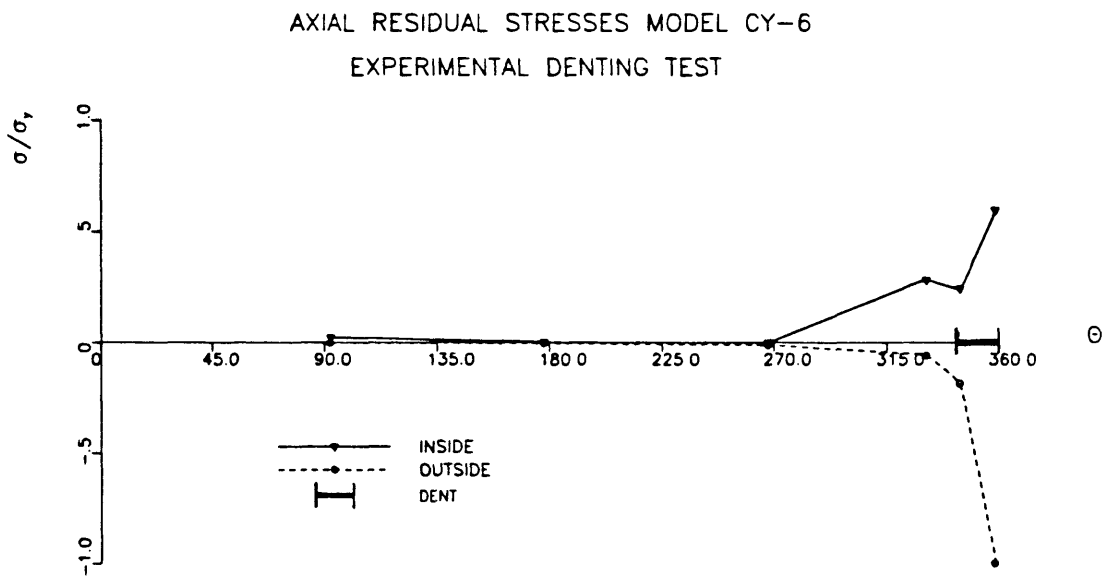


Figure 3.75



(a)



(b)

Figure 3.76

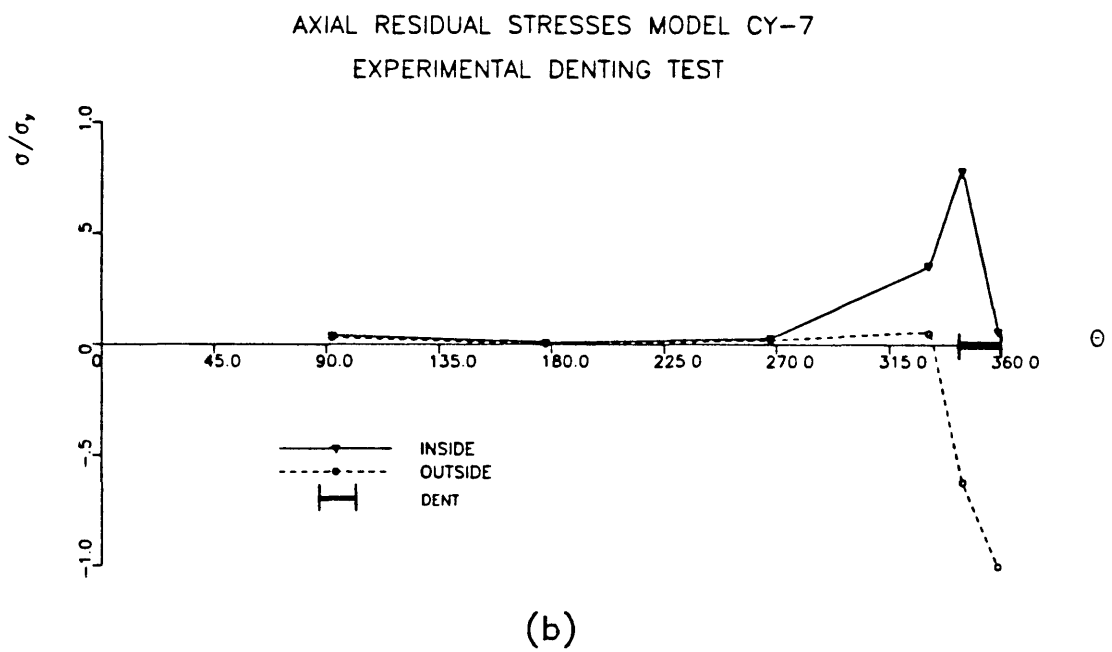
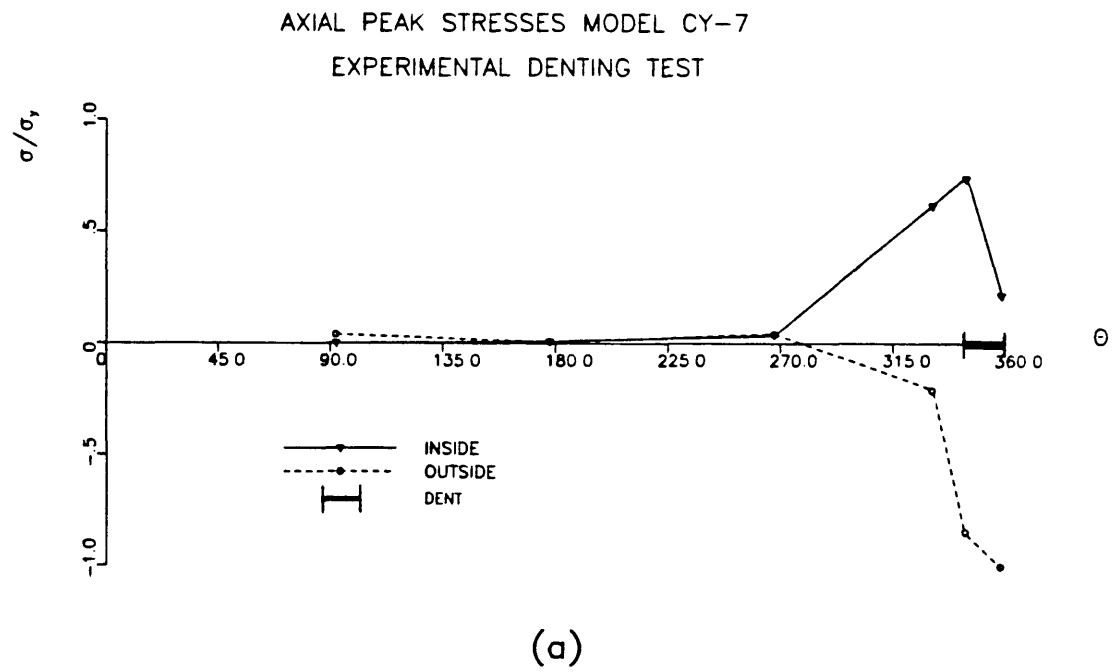


Figure 3.77

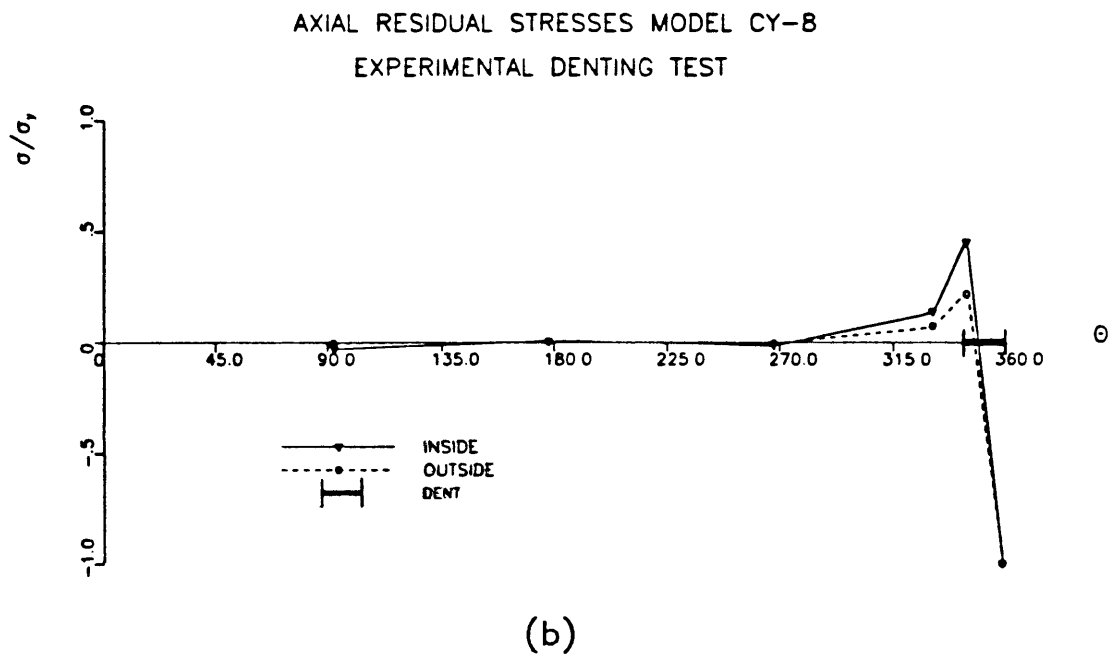
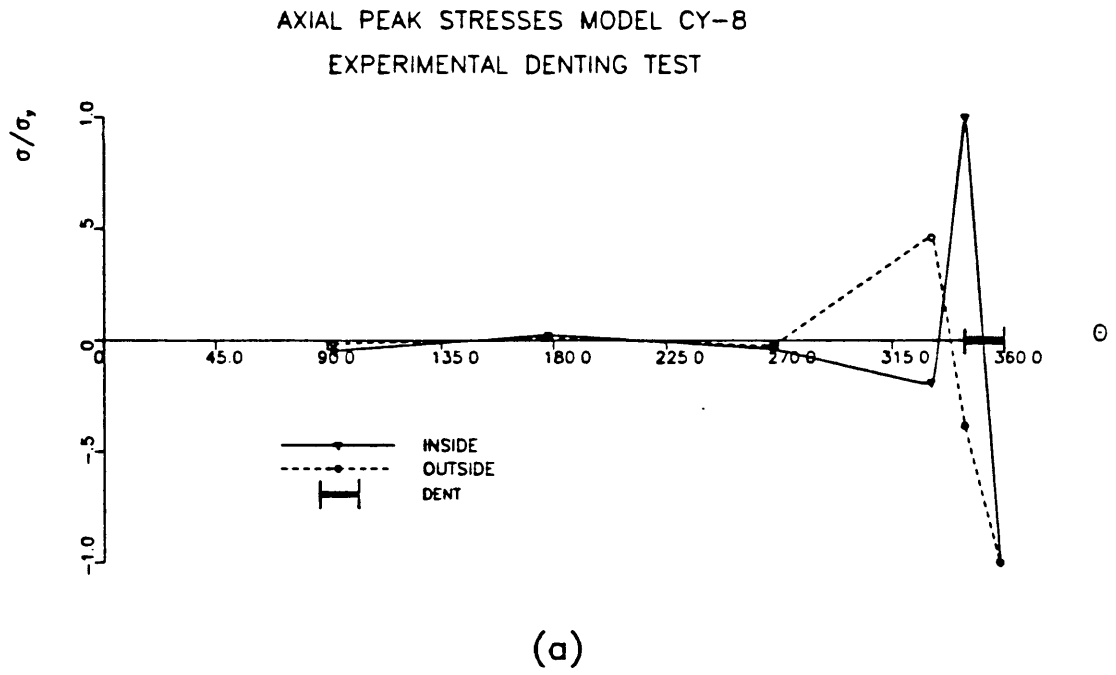


Figure 3.78

ENERGY ABSORPTION DURING DENTING
SERIES 2

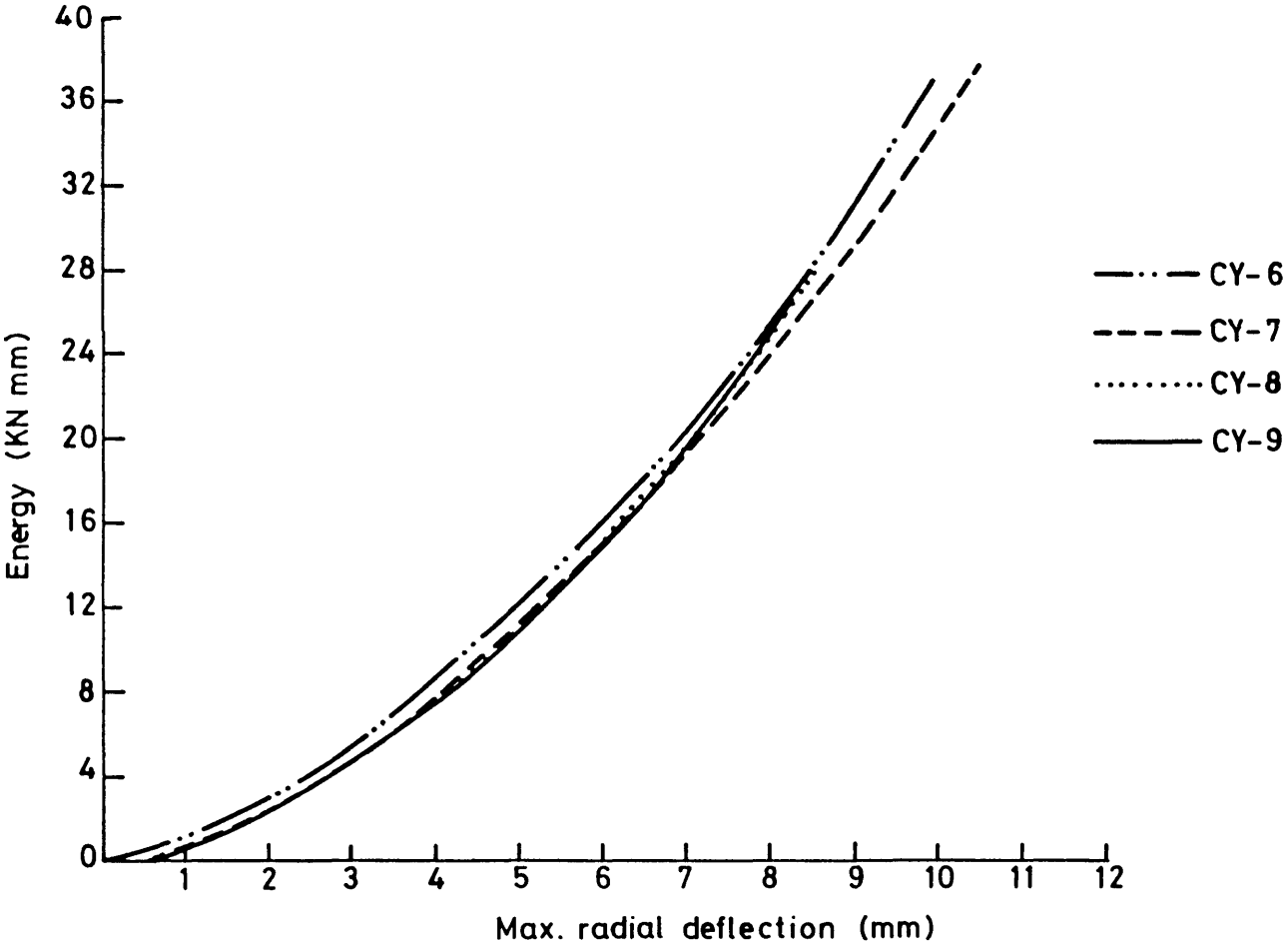


Figure 3.79

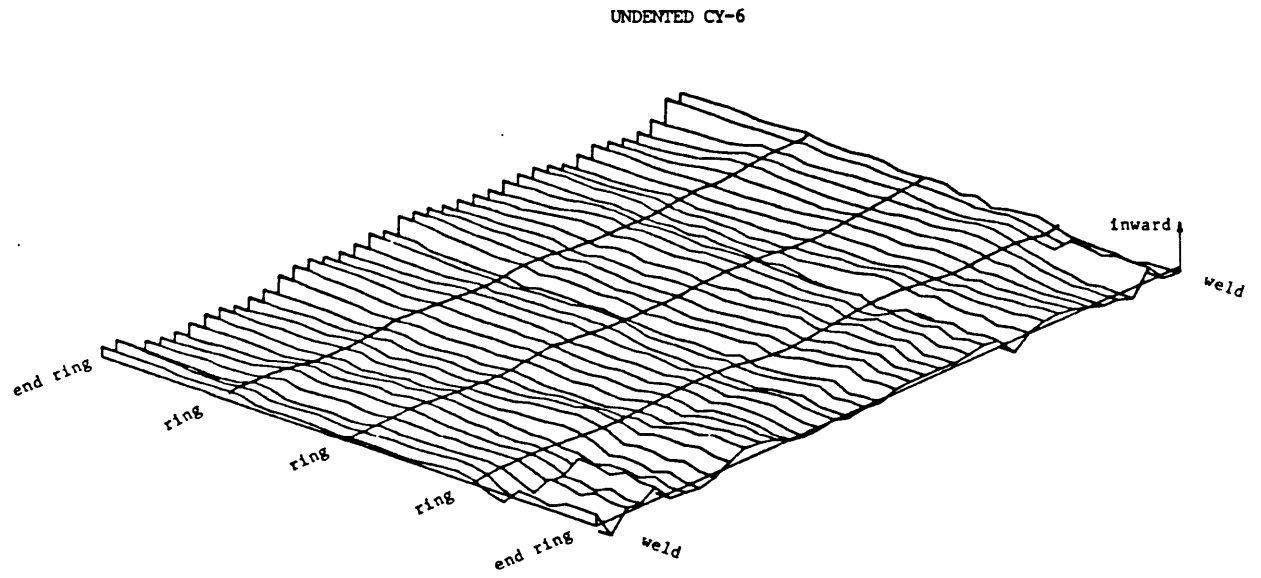


Figure 3.80

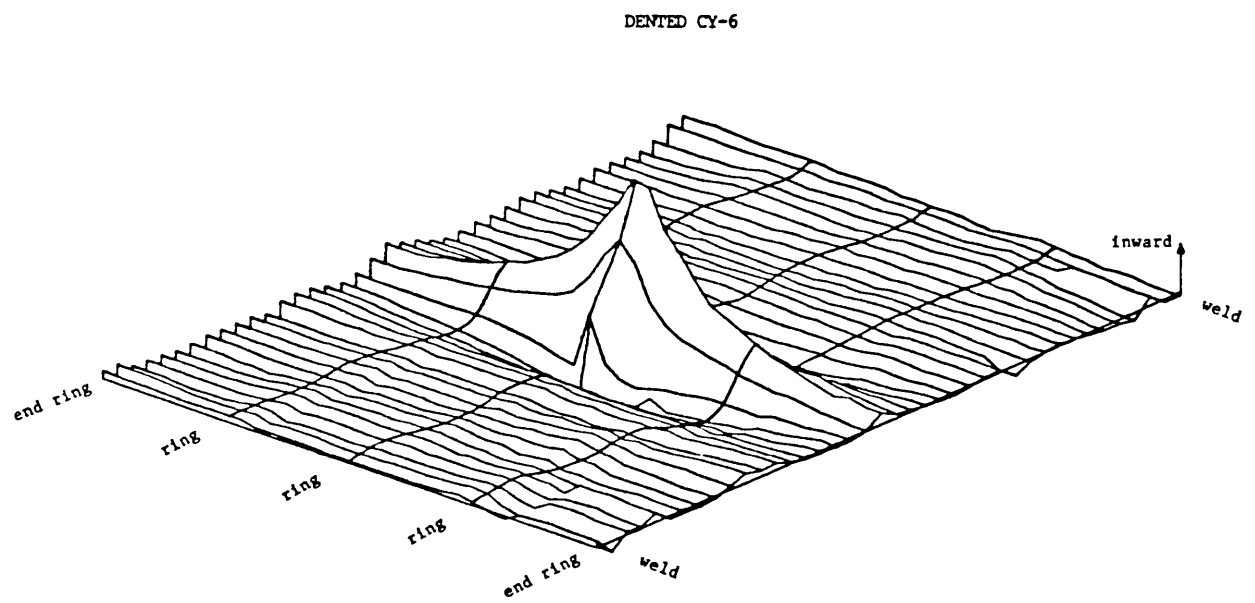


Figure 3.81

MODEL CY-6
IMPERFECTIONS BEFORE DENTING

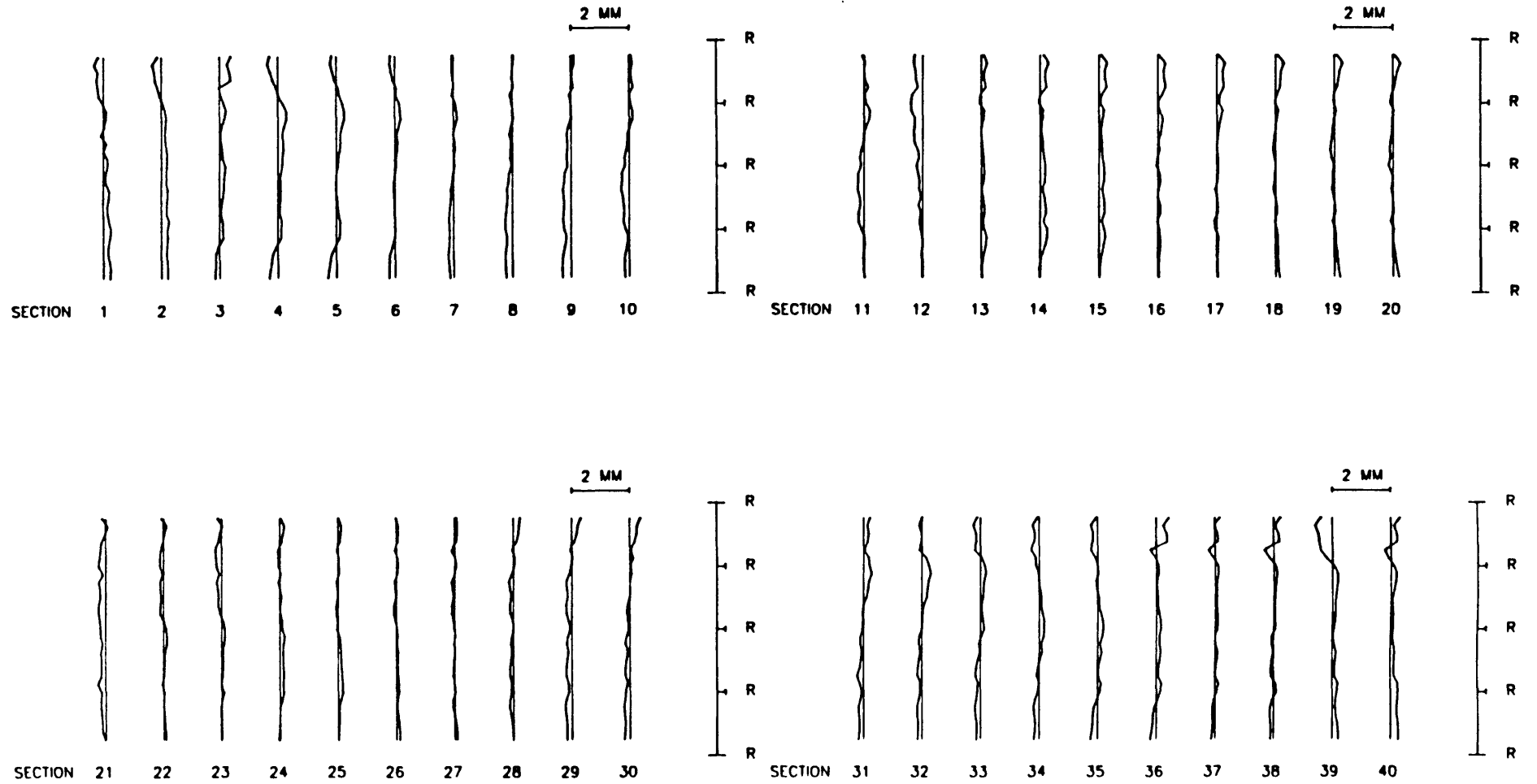


Figure 3.82

MODEL CY-6
IMPERFECTIONS AFTER DENTING

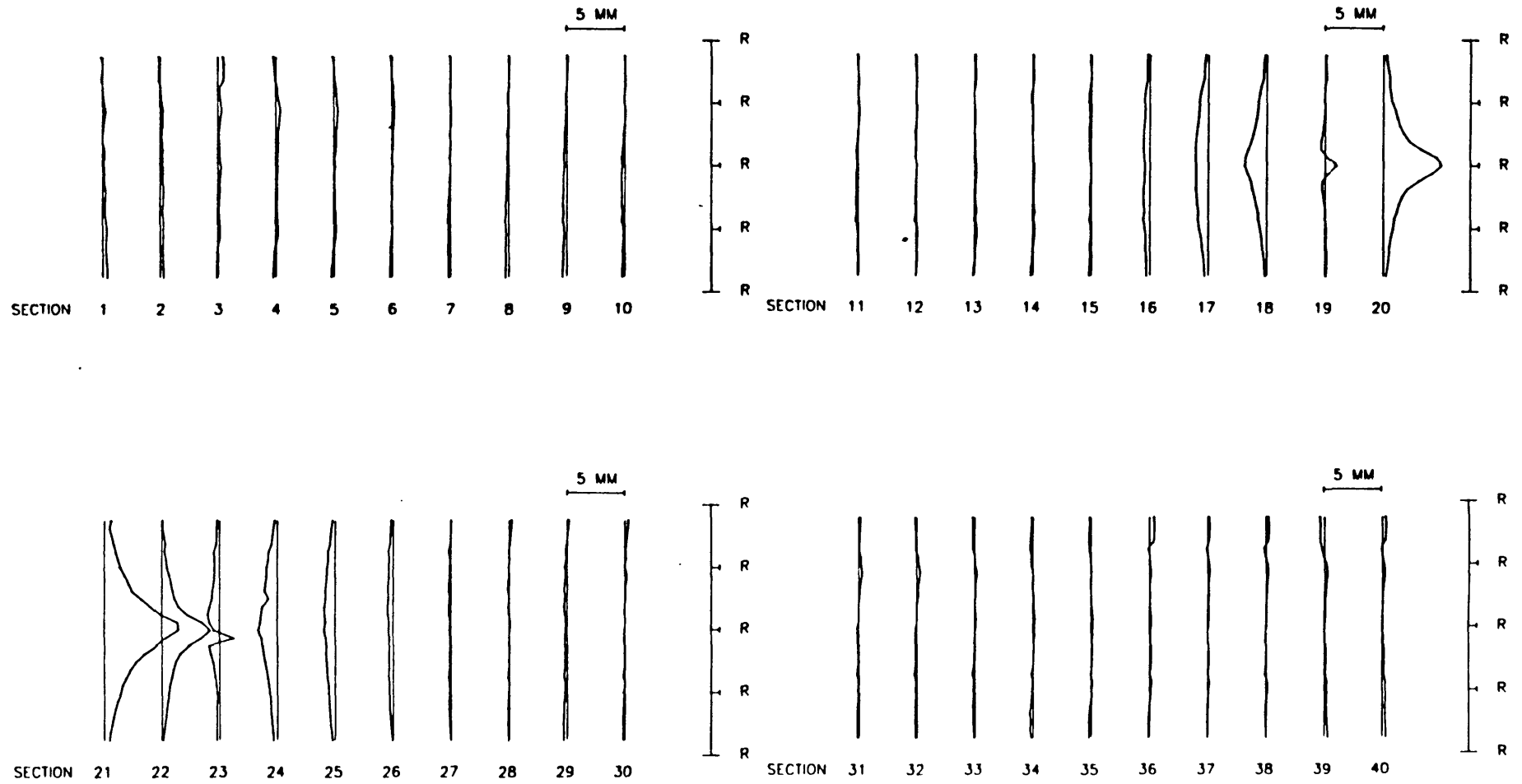


Figure 3.83

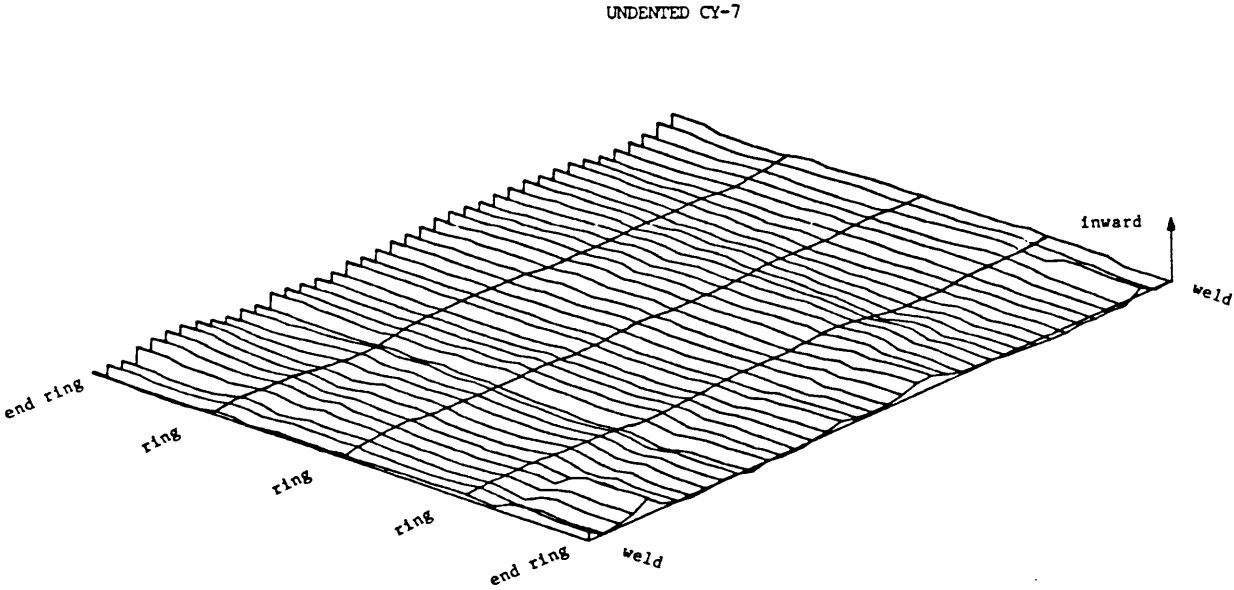


Figure 3.84

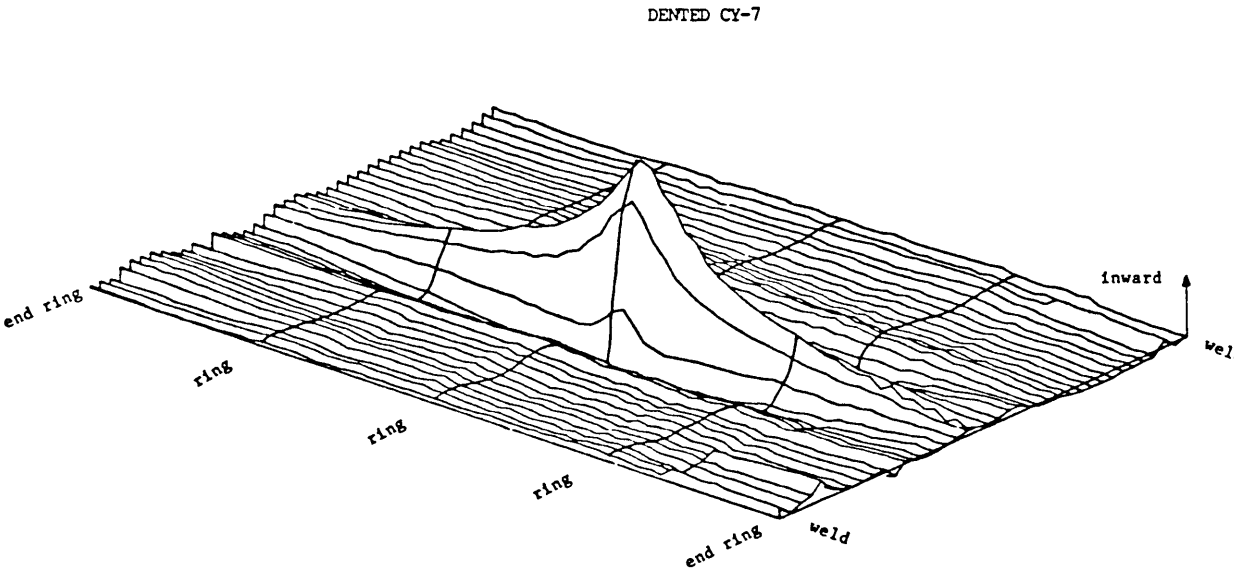


Figure 3.85

MODEL CY-7
IMPERFECTIONS BEFORE DENTING

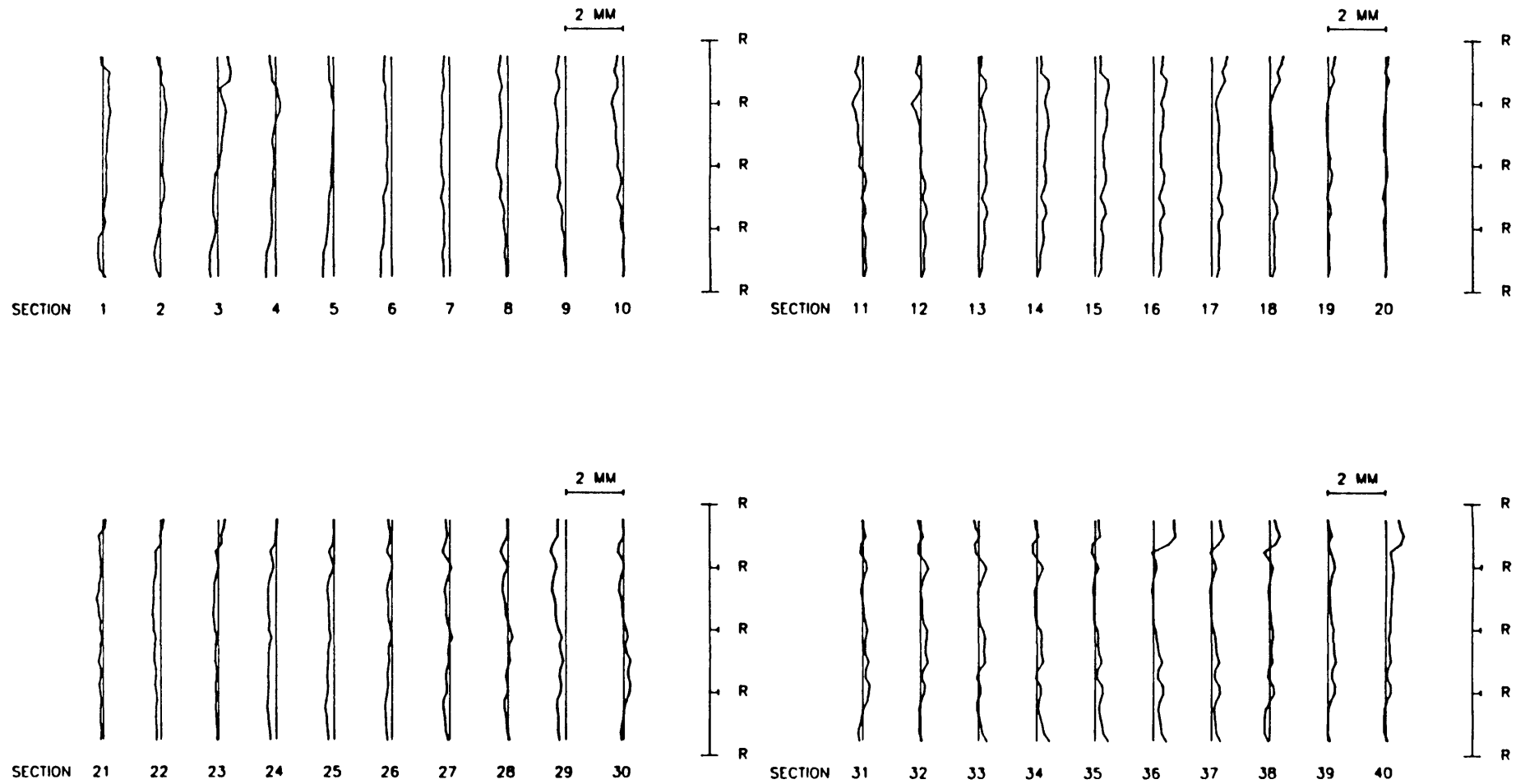


Figure 3.86

MODEL CY-7
IMPERFECTIONS AFTER DENTING

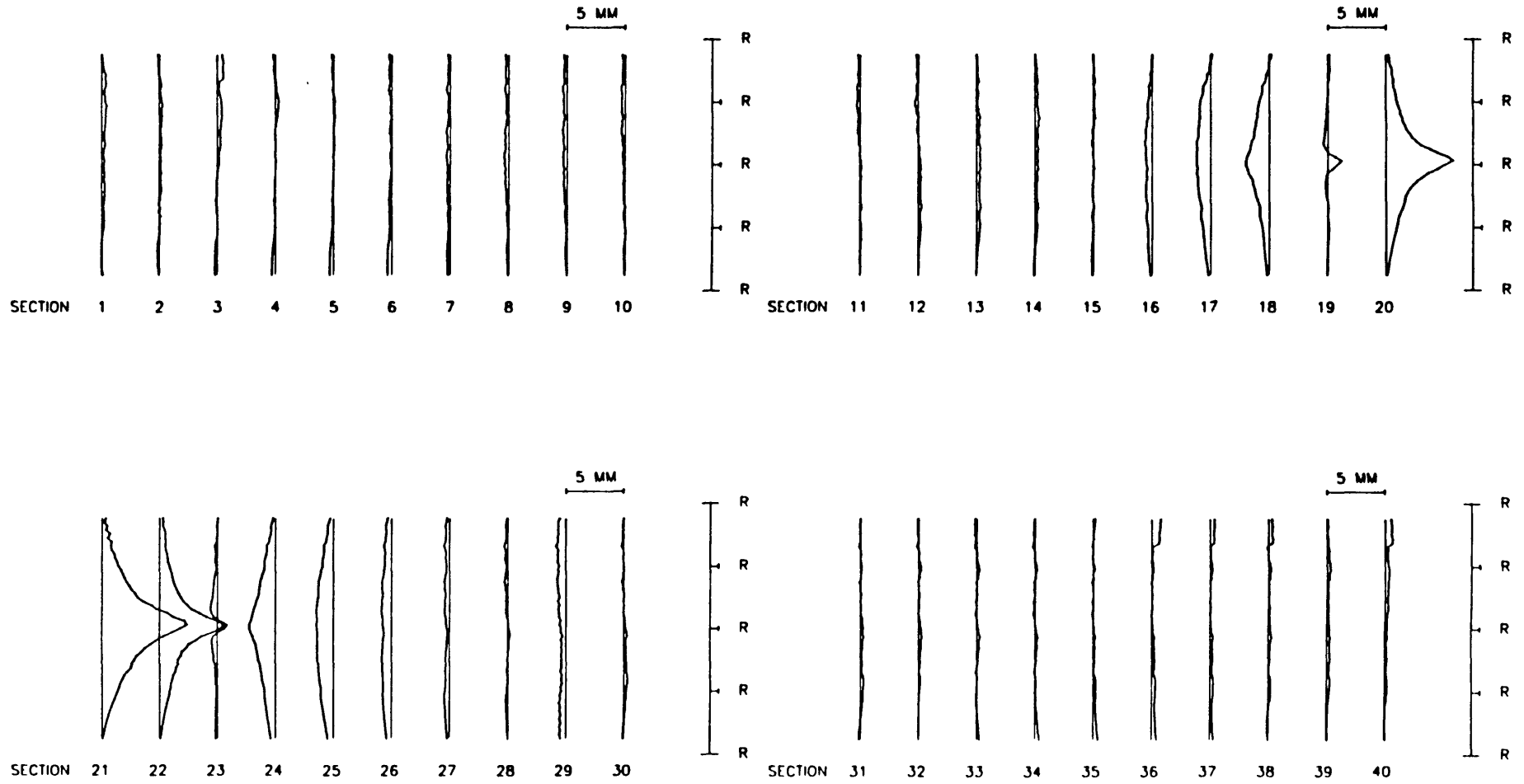


Figure 3.87

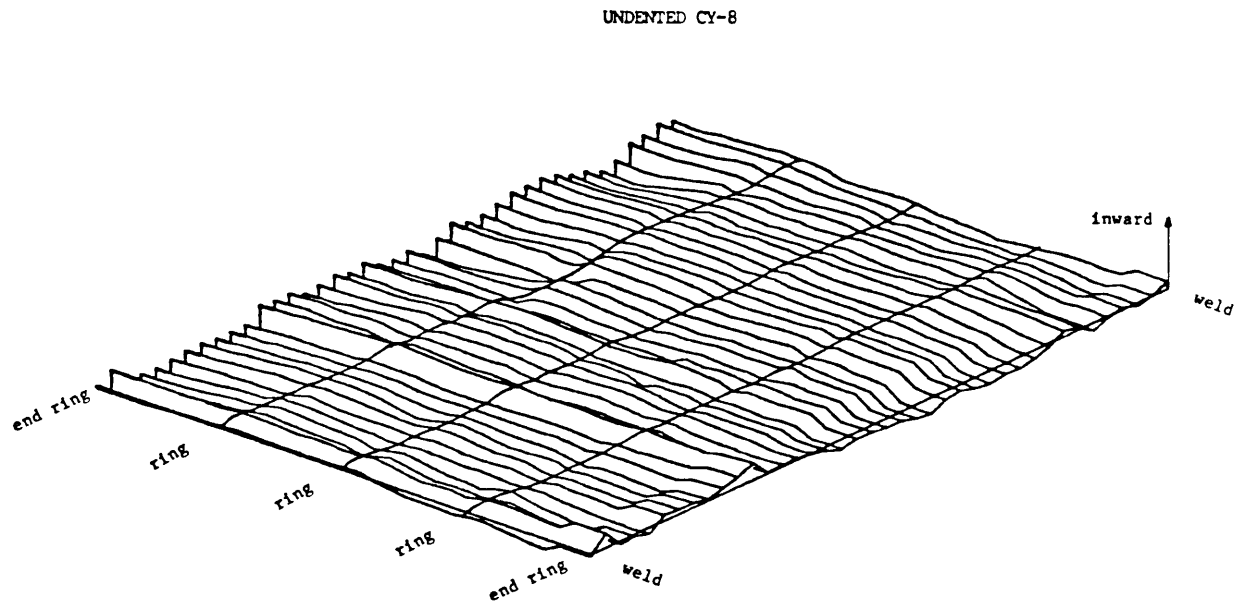


Figure 3.88

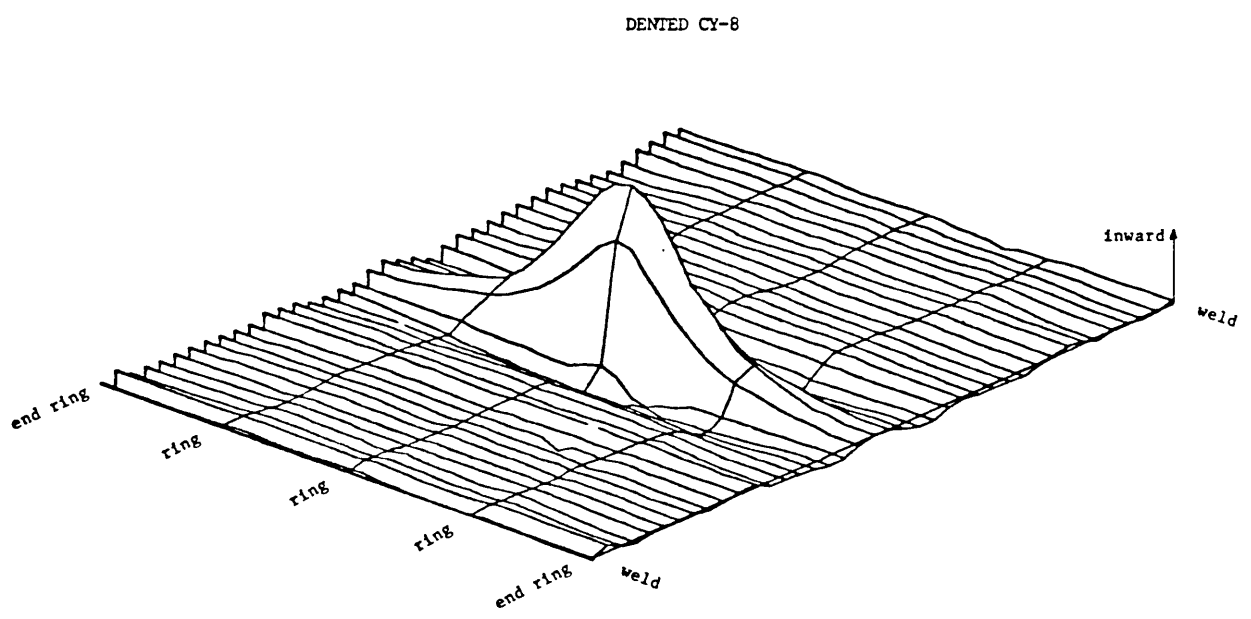


Figure 3.89

MODEL CY-8
IMPERFECTIONS BEFORE DENTING

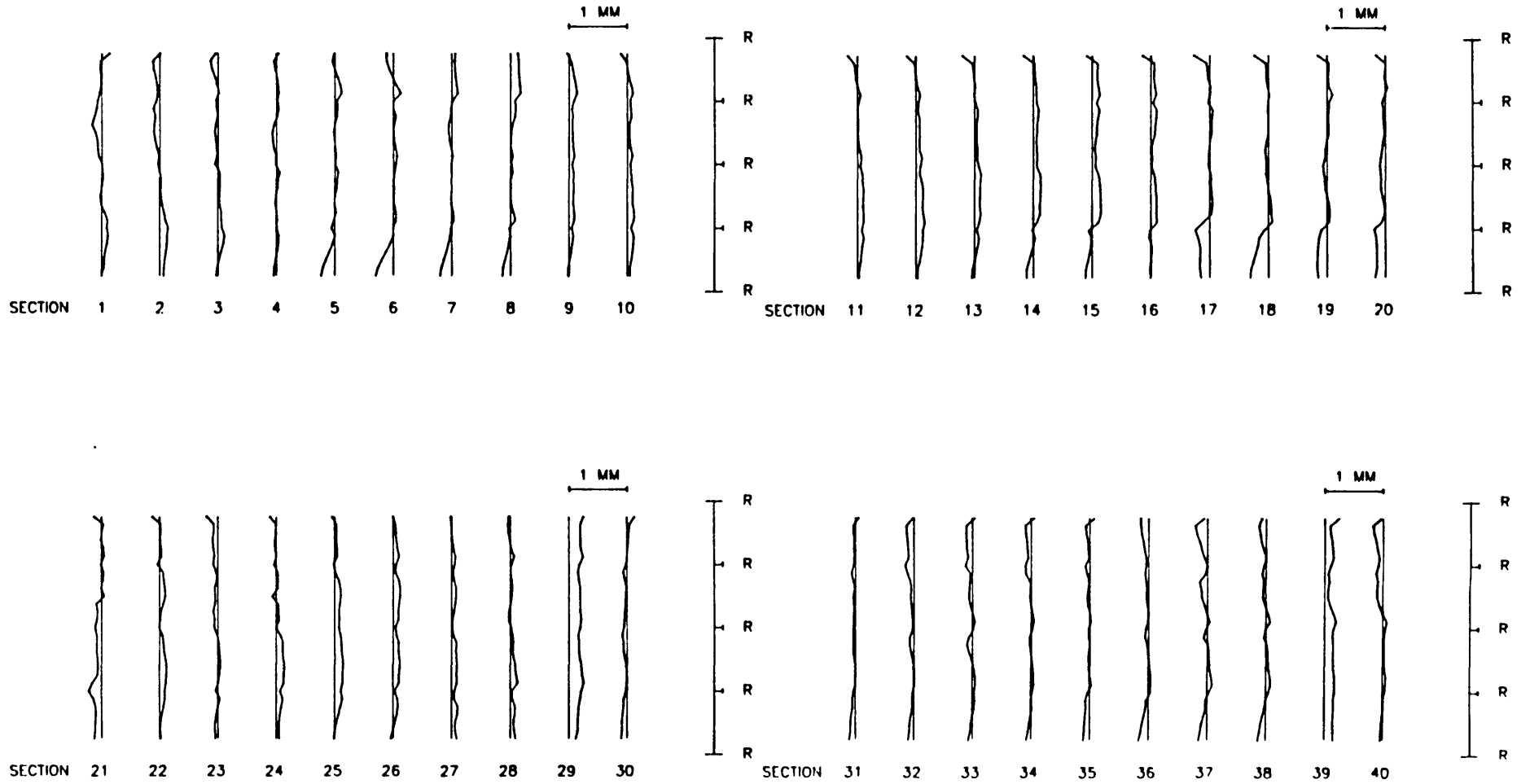


Figure 3.90

MODEL CY-8
IMPERFECTIONS AFTER DENTING

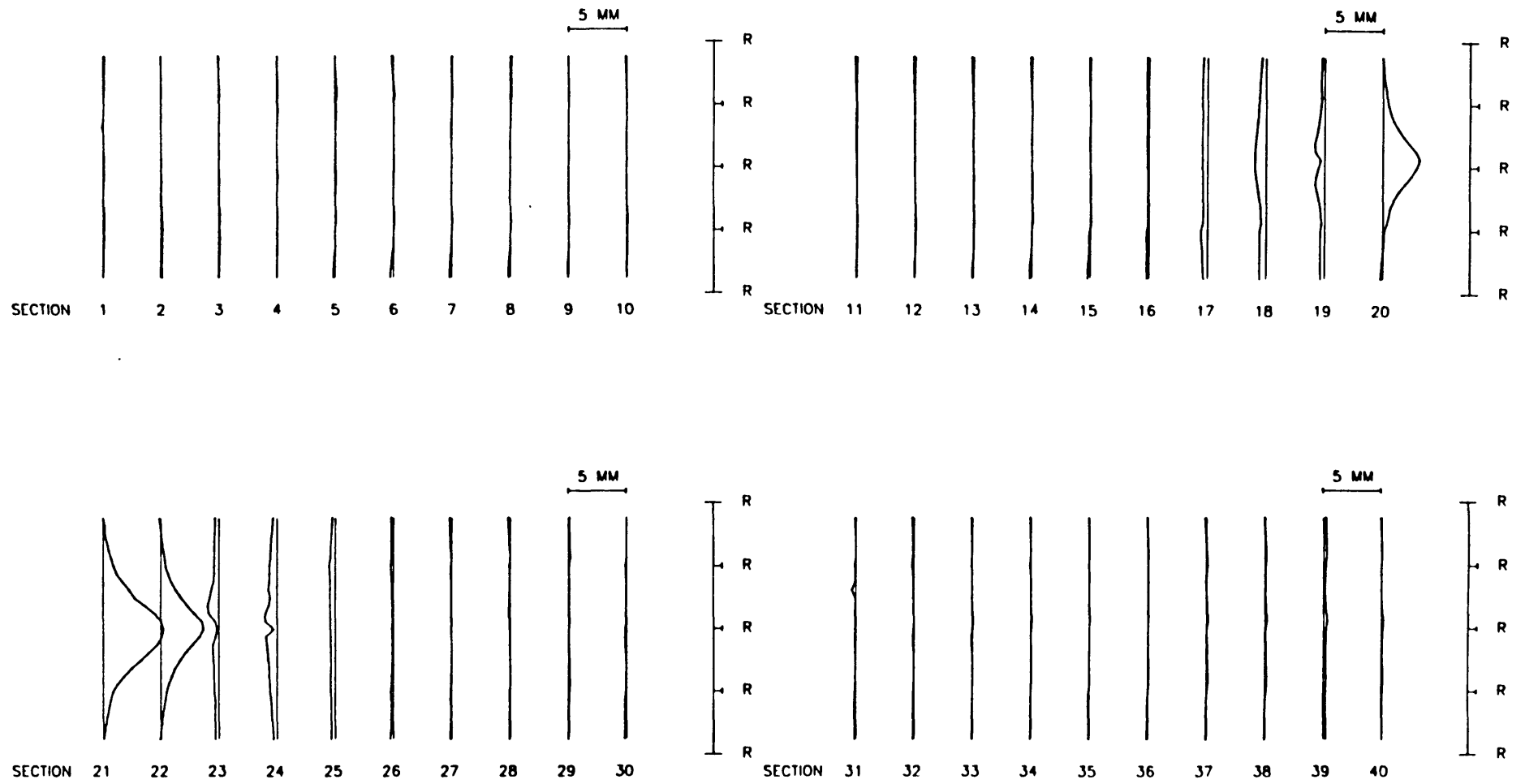


Figure 3.91

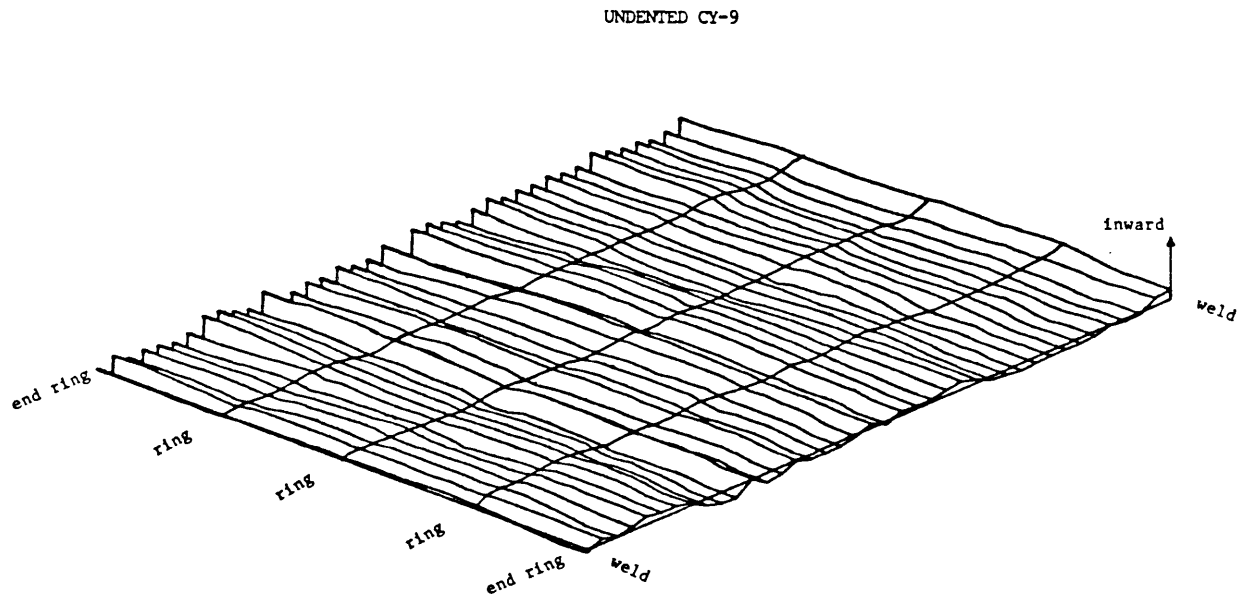


Figure 3.92

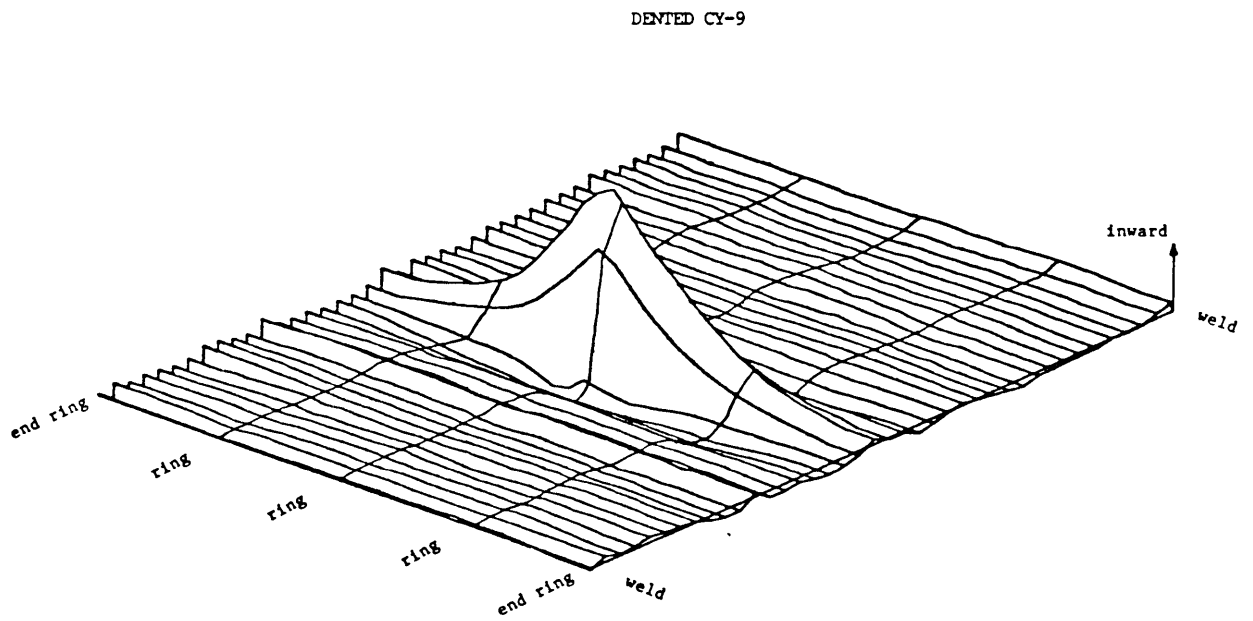


Figure 3.93

MODEL CY-9
IMPERFECTIONS BEFORE DENTING

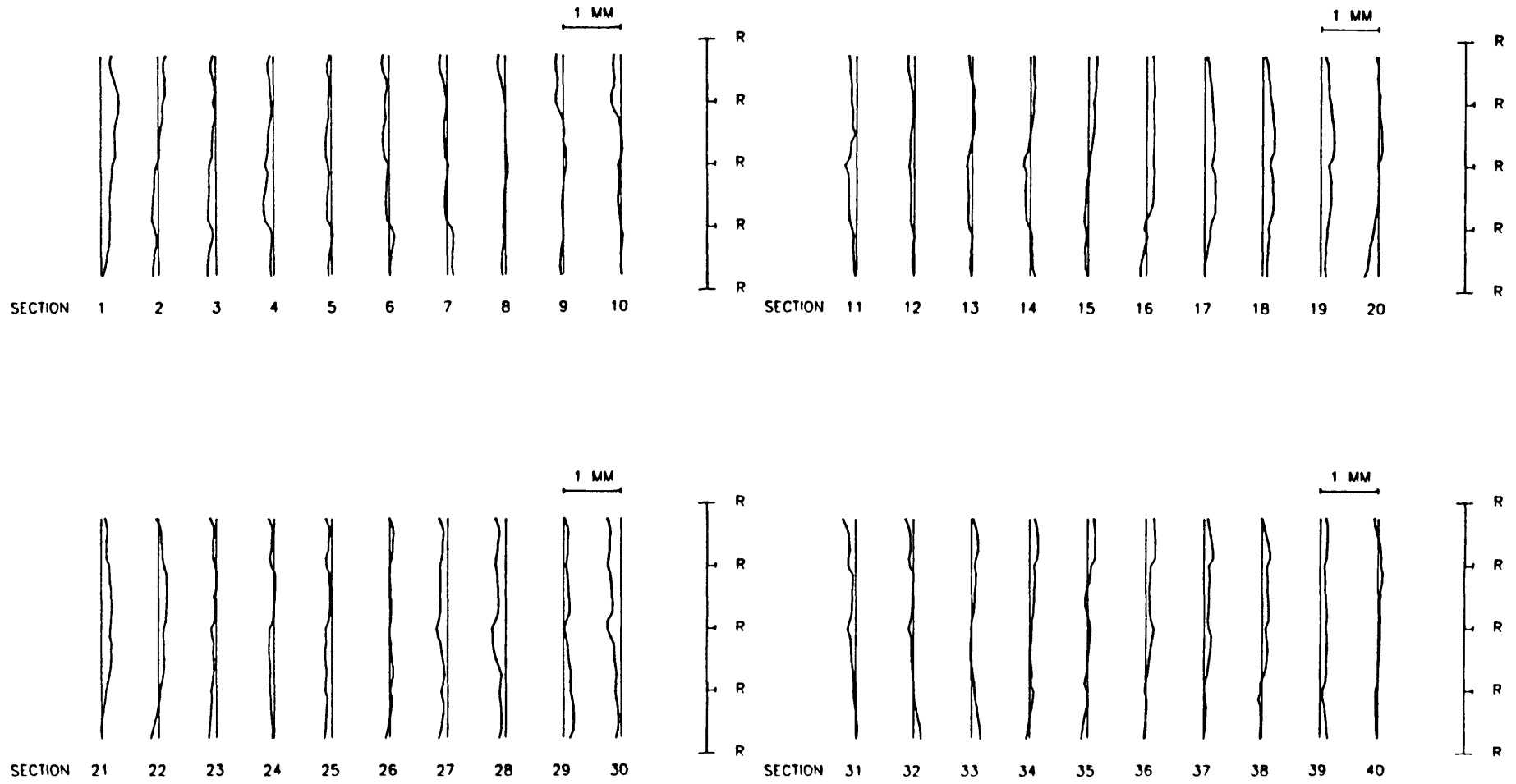


Figure 3.94

MODEL CY-9
IMPERFECTIONS AFTER DENTING

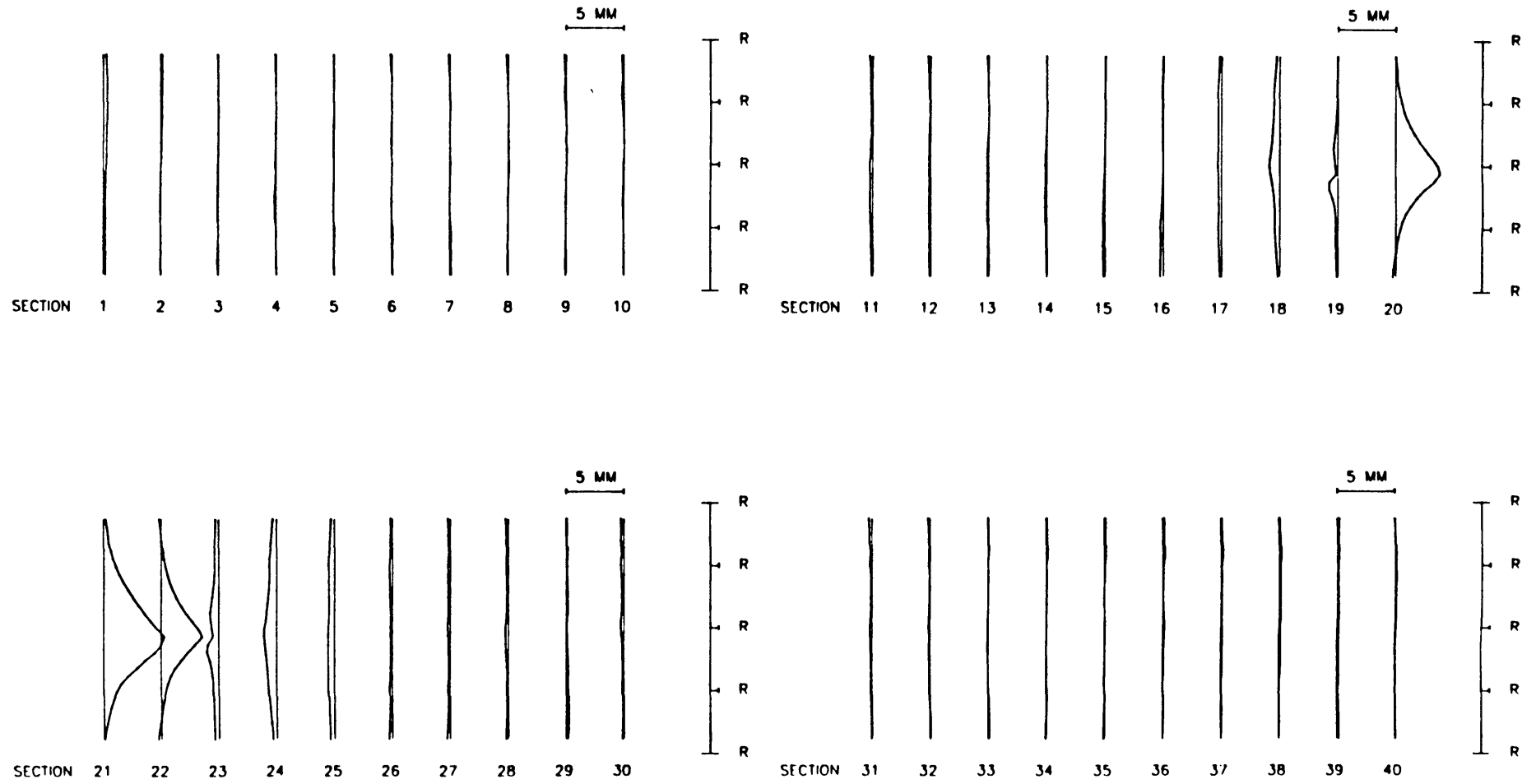


Figure 3.95

LOAD DEFLECTION CURVE (CY-6)

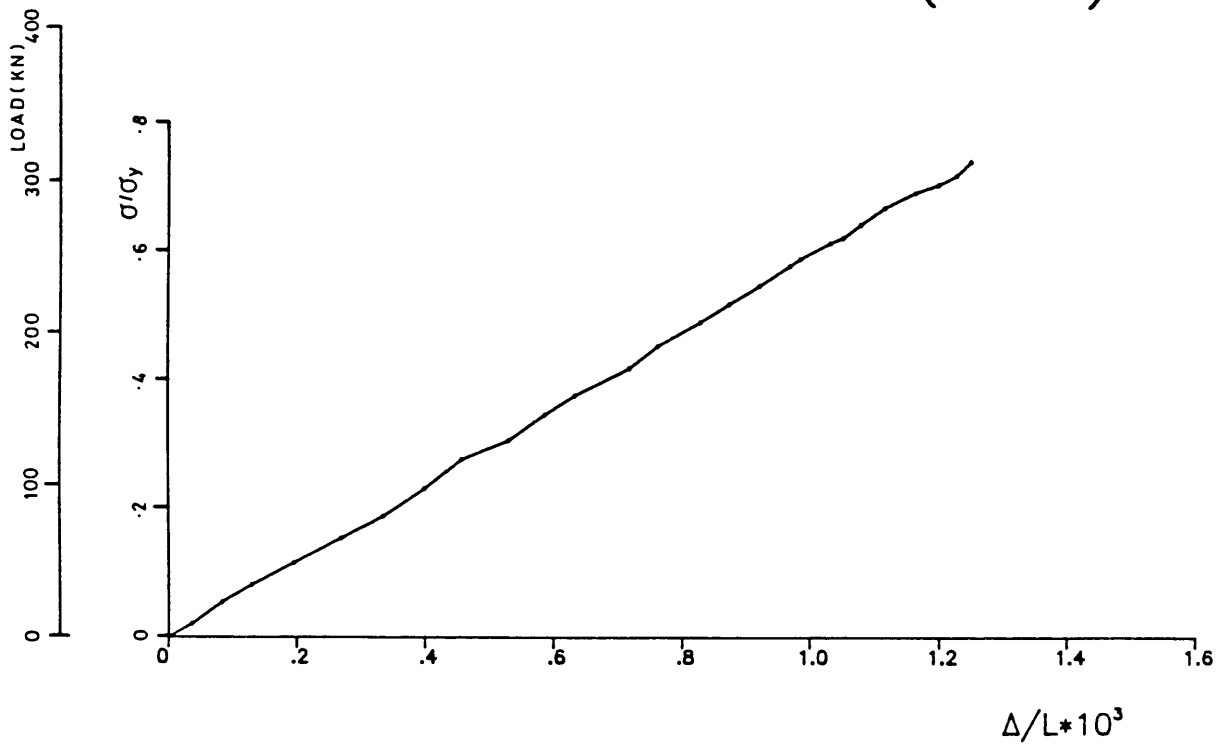


Figure 3.96

LOAD DEFLECTION CURVE (CY-7)

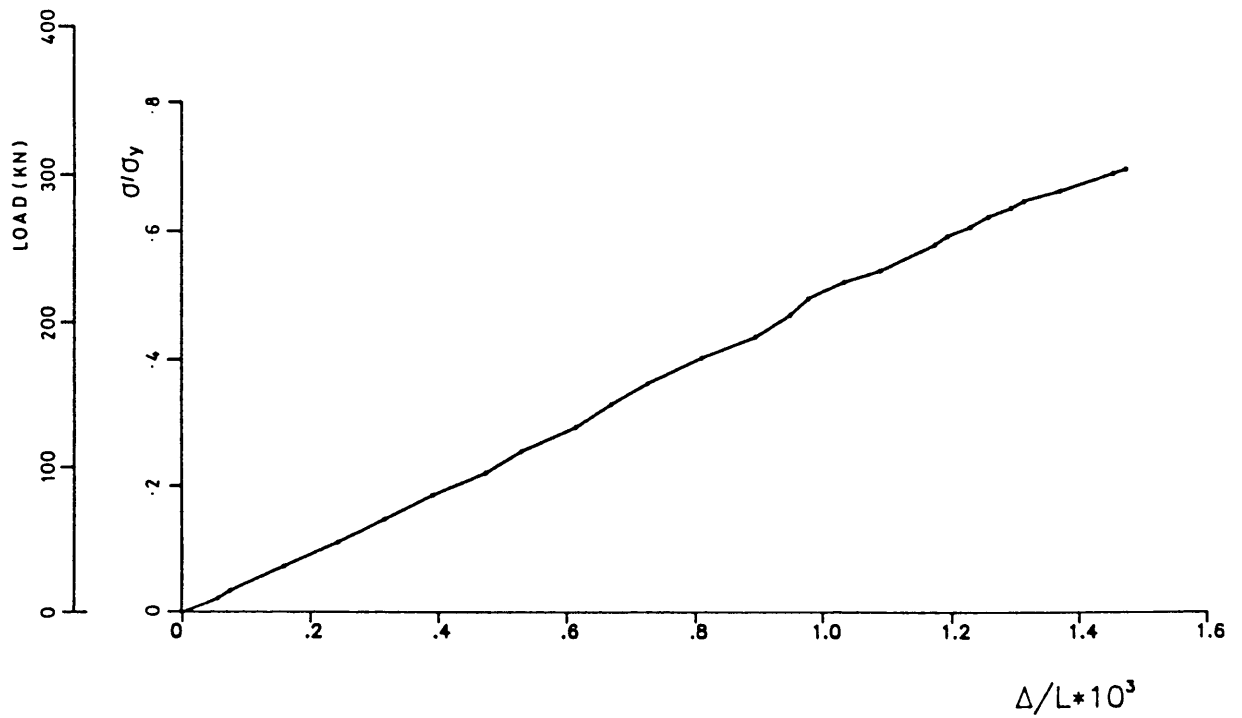


Figure 3.97

LOAD DEFLECTION CURVE (CY-8)

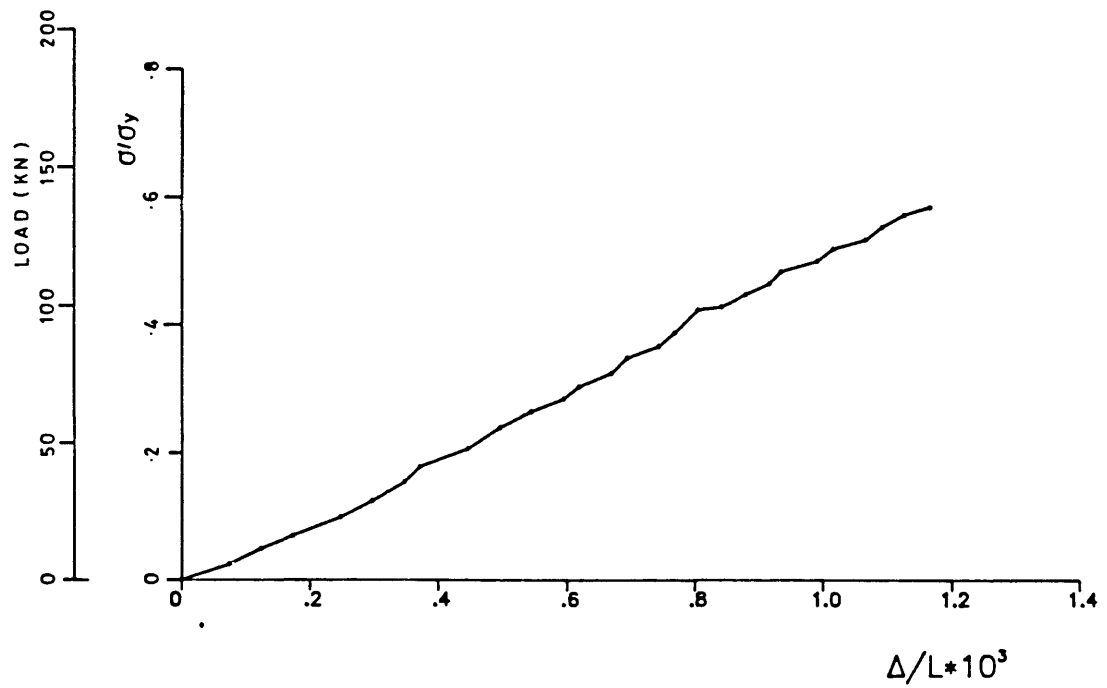


Figure 3.98

LOAD DEFLECTION CURVE (CY-9)

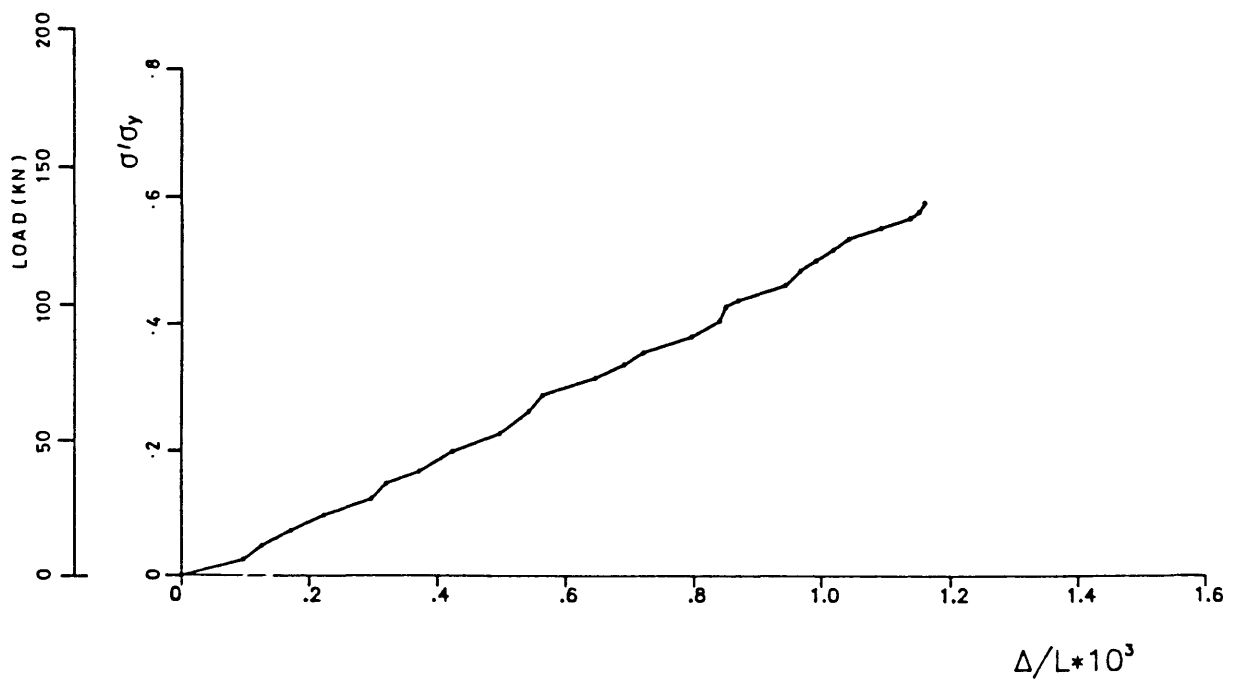


Figure 3.99

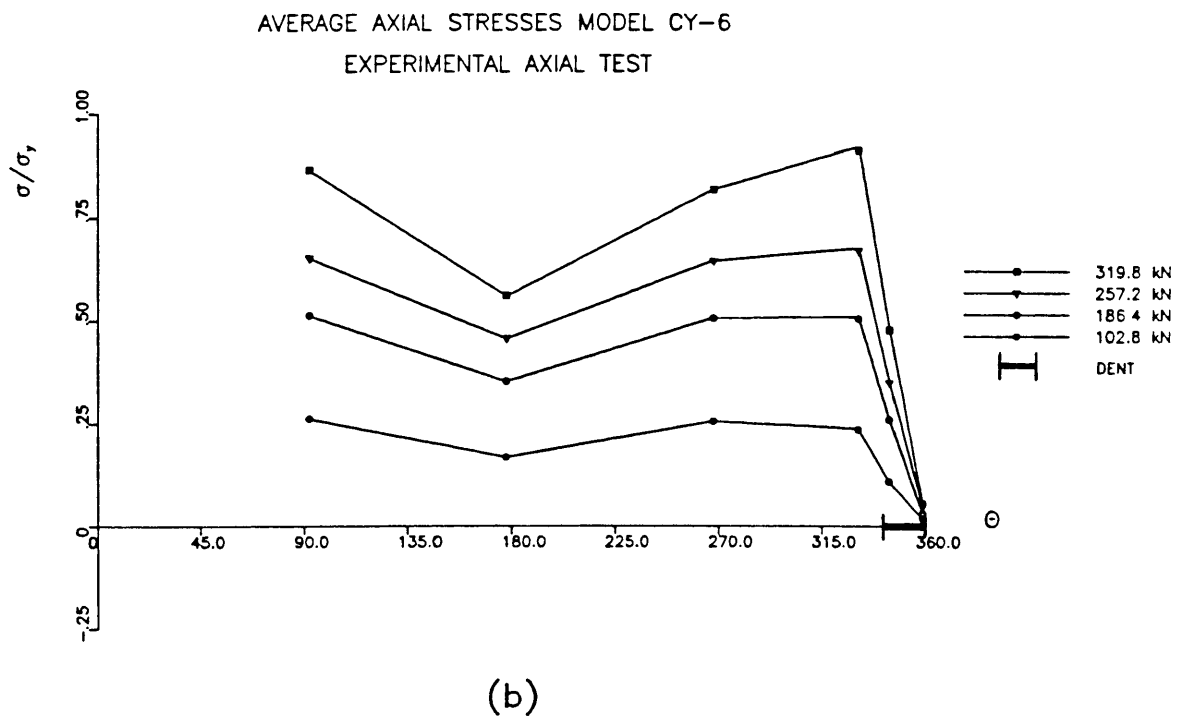
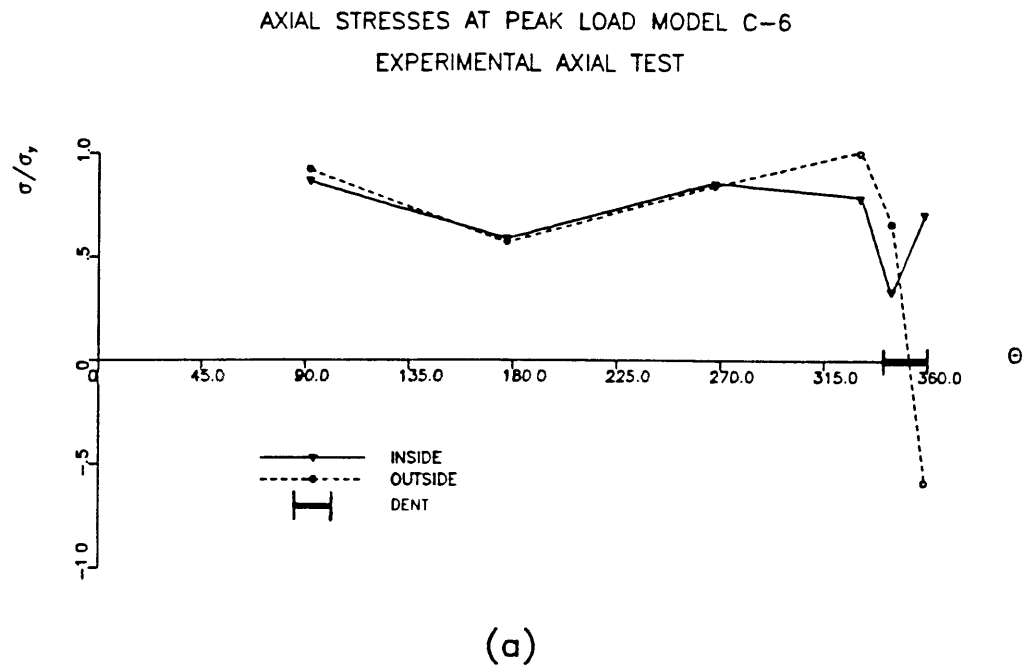


Figure 3.100

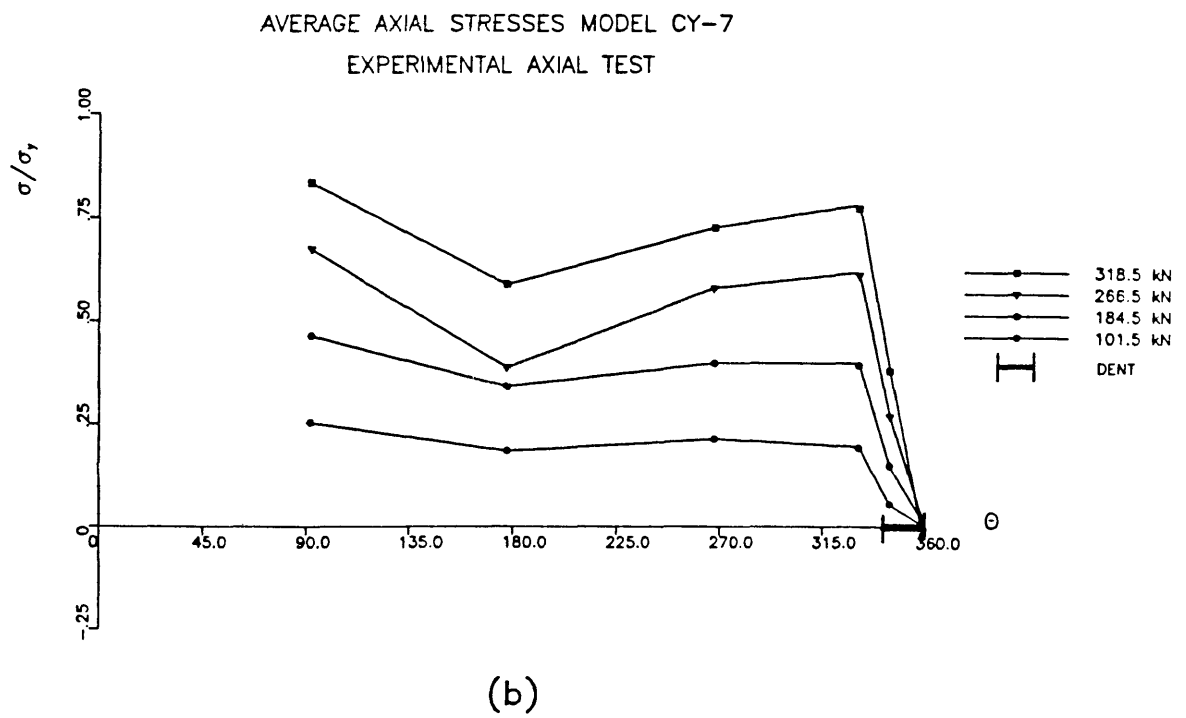
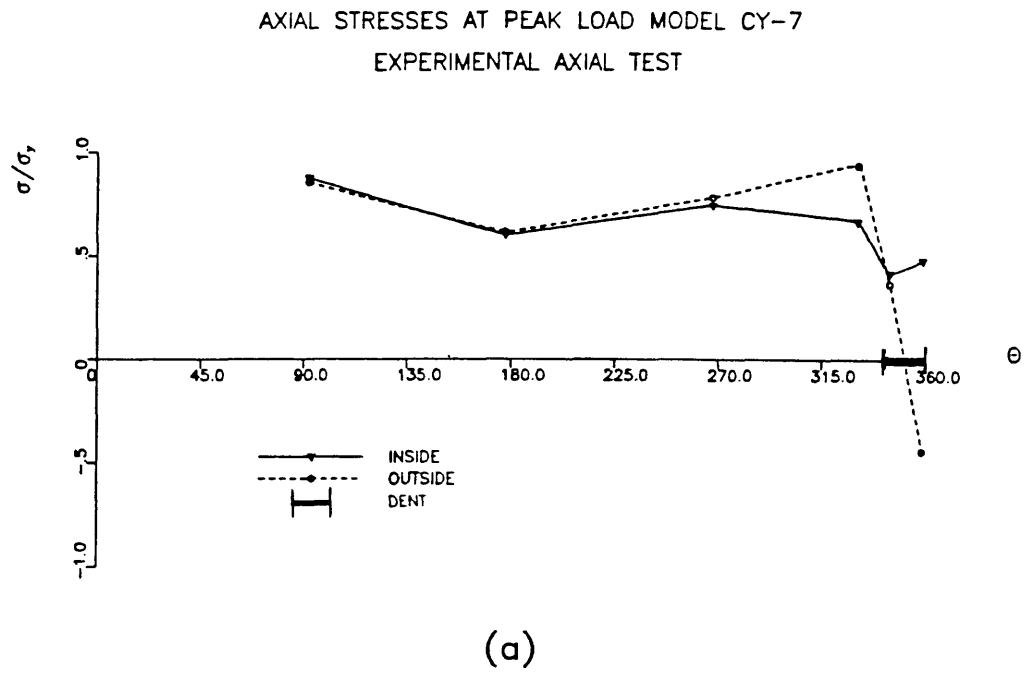


Figure 3.101

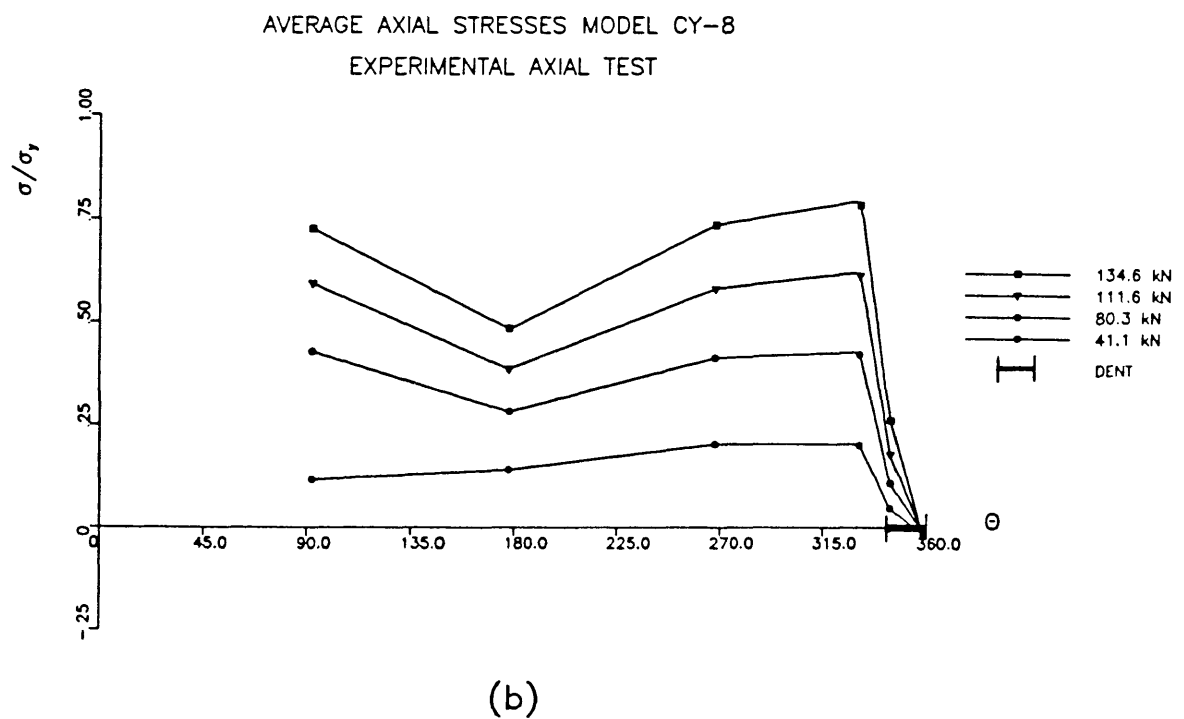
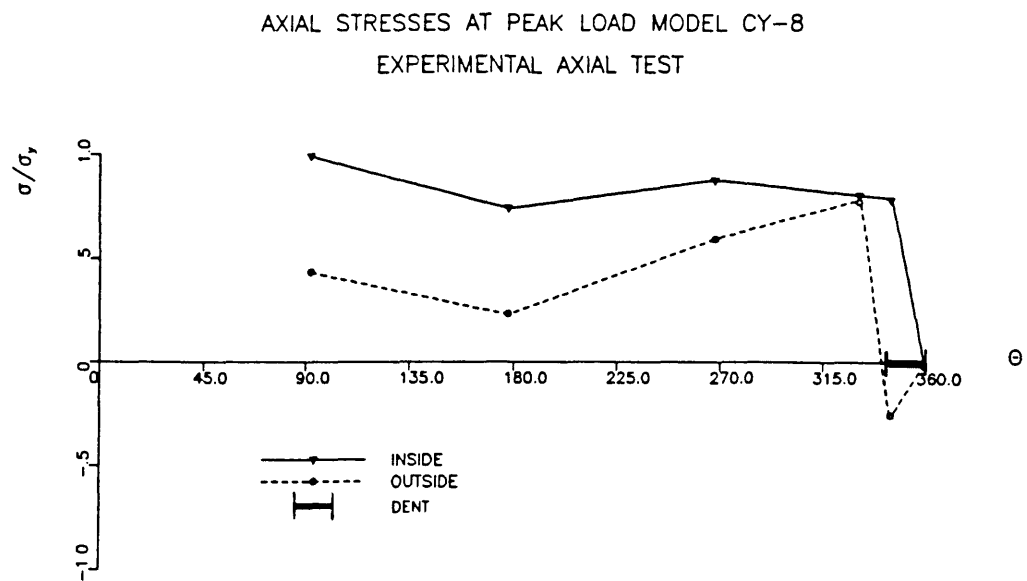


Figure 3.102

MODEL CY-6
DEFLECTIONS DURING FAILURE TEST

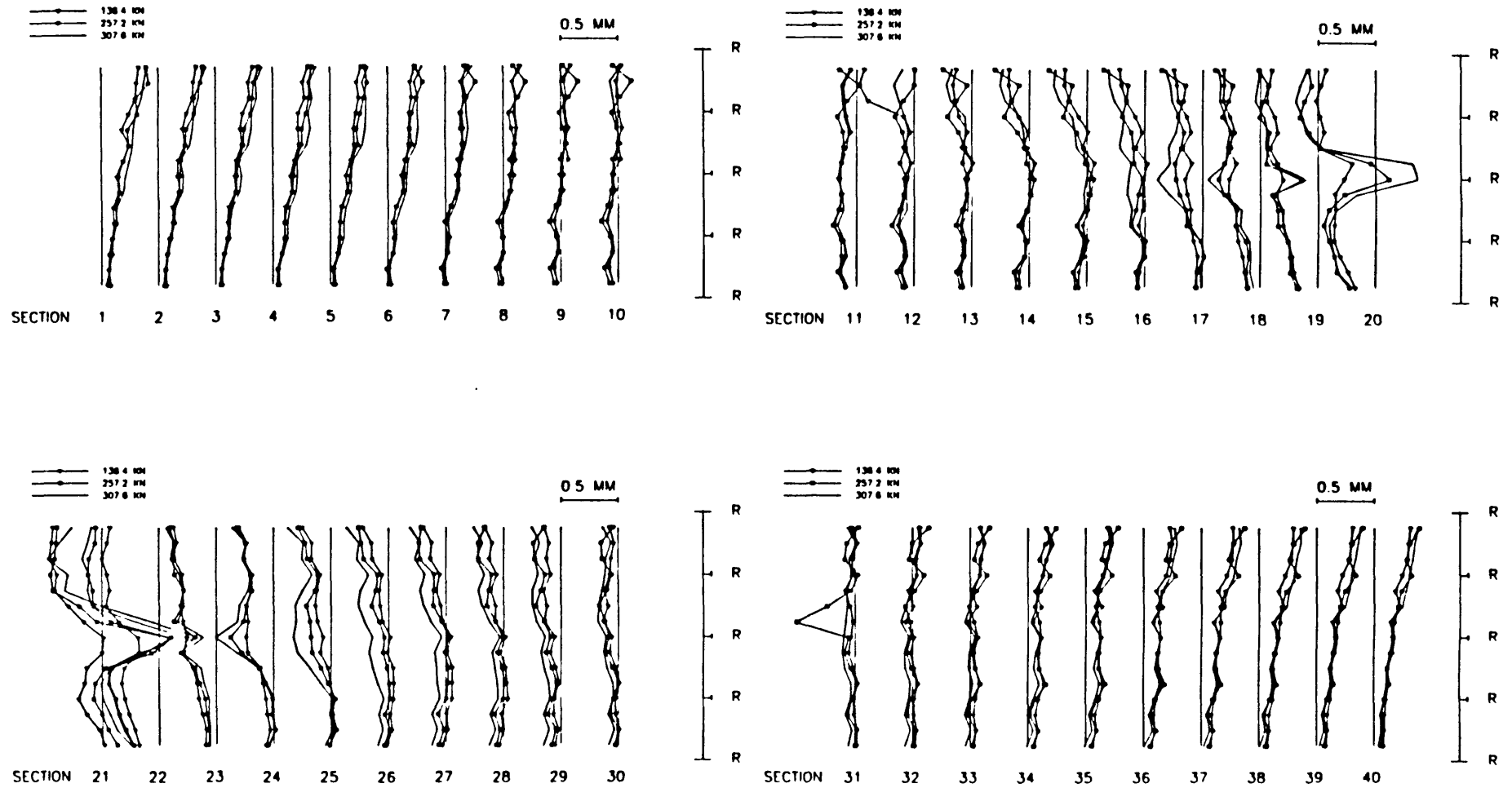


Figure 3.103

MODEL CY-6 DEFLECTIONS AFTER FAILURE

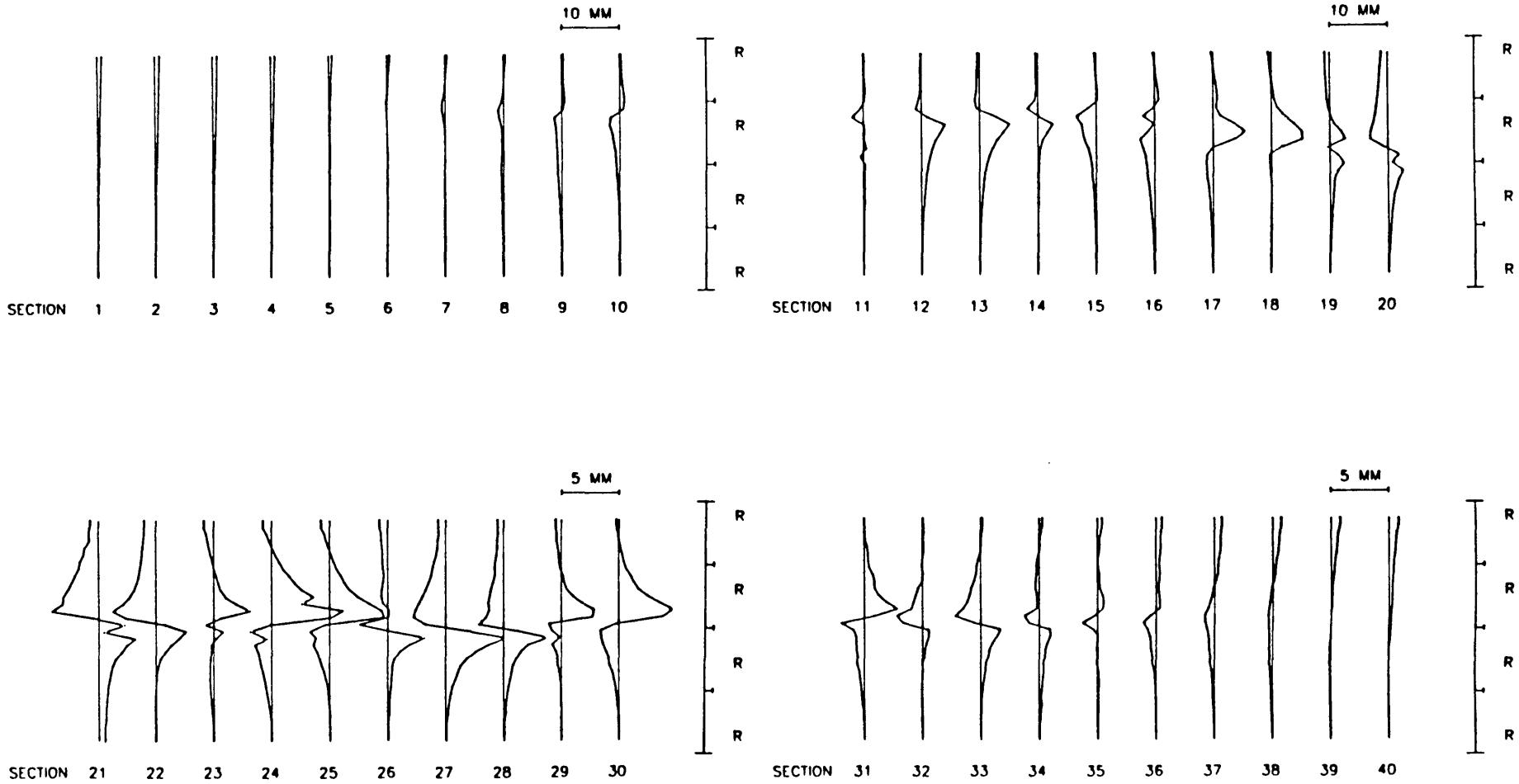


Figure 3.104

FAILURE MODE CY-6

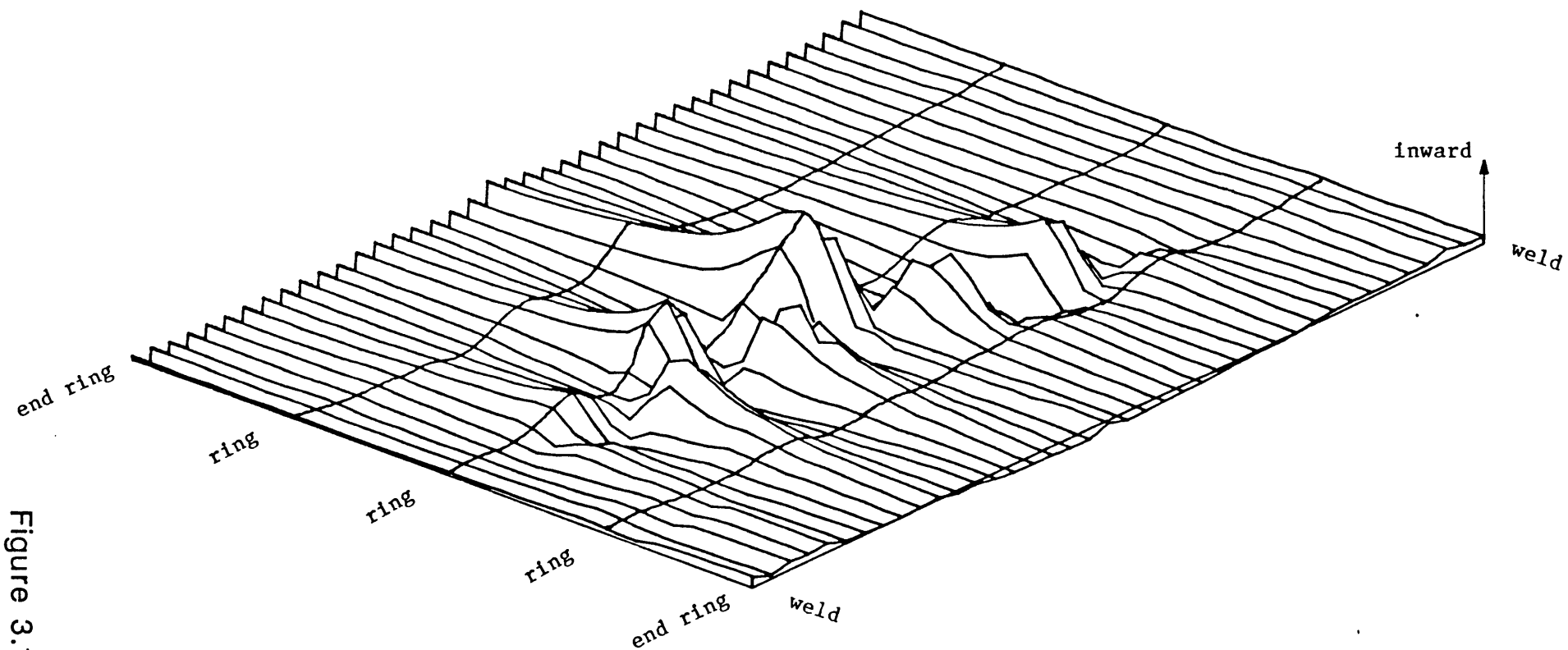
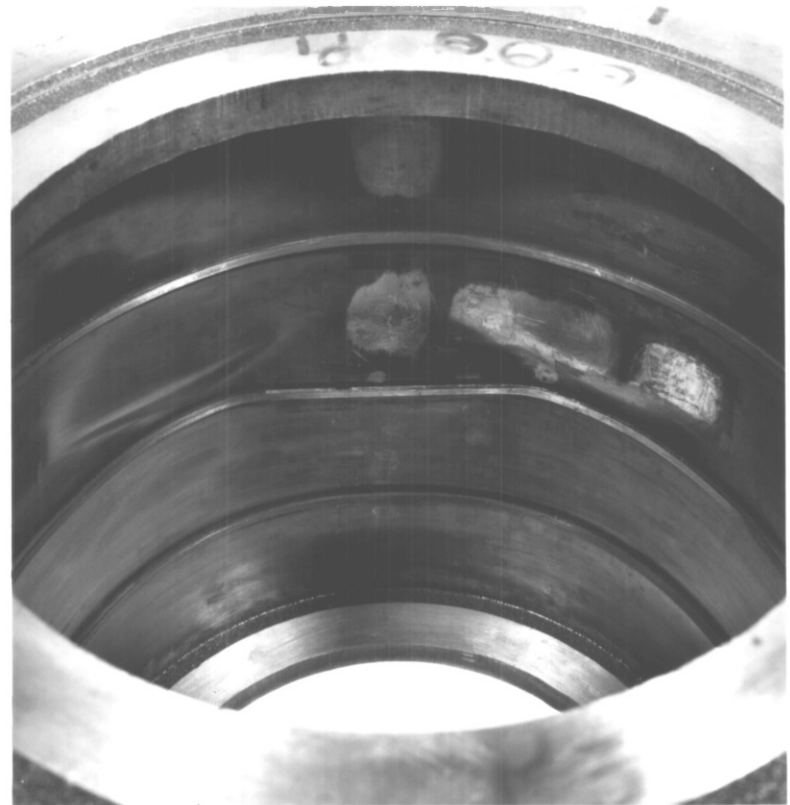


Figure 3.105

CY-6 BUCKLED SHAPE



OUTSIDE VIEW



INSIDE VIEW

Figure 3.106

MODEL CY-7 DEFLECTIONS DURING FAILURE TEST

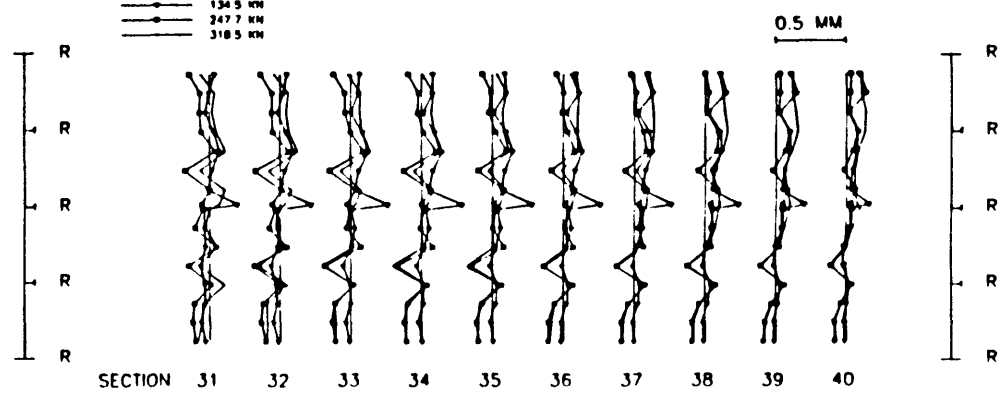
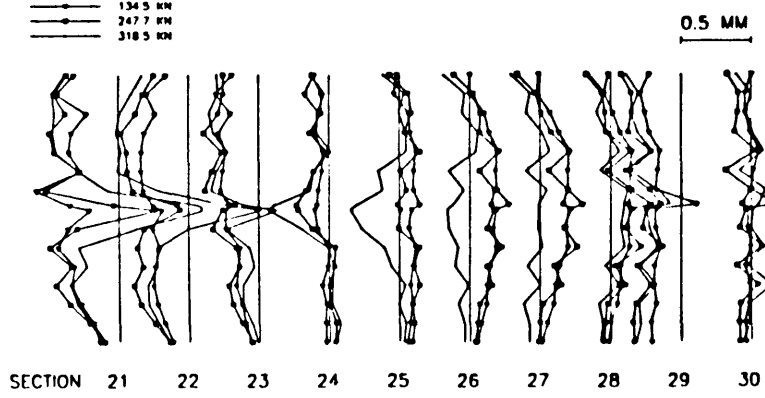
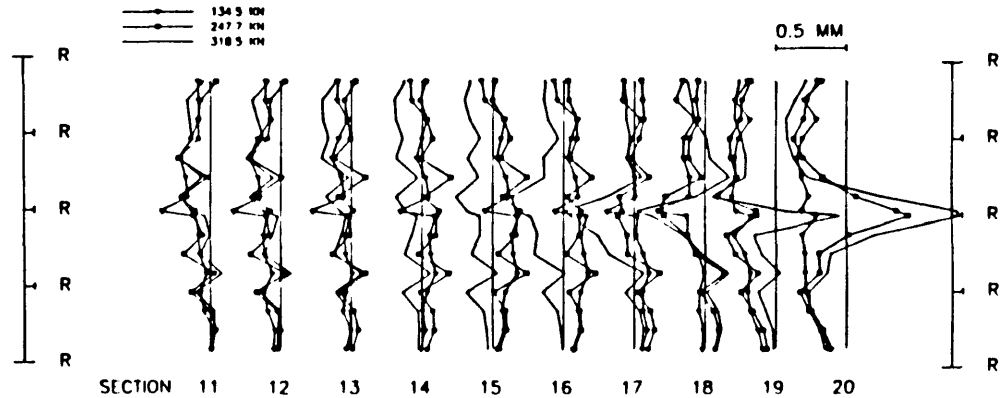
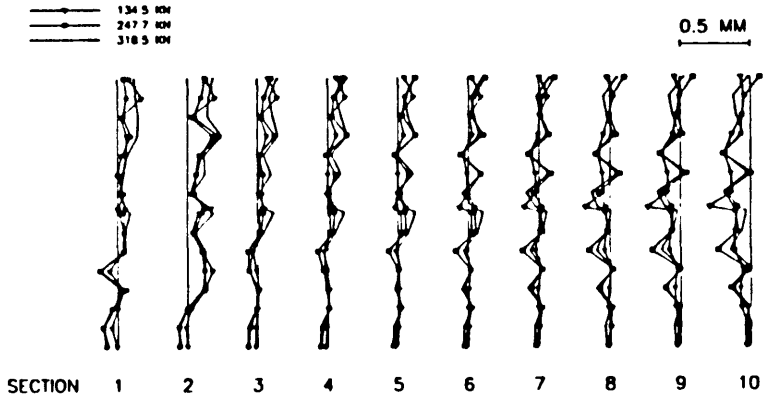


Figure 3.107

MODEL CY-7
DEFLECTIONS AFTER FAILURE

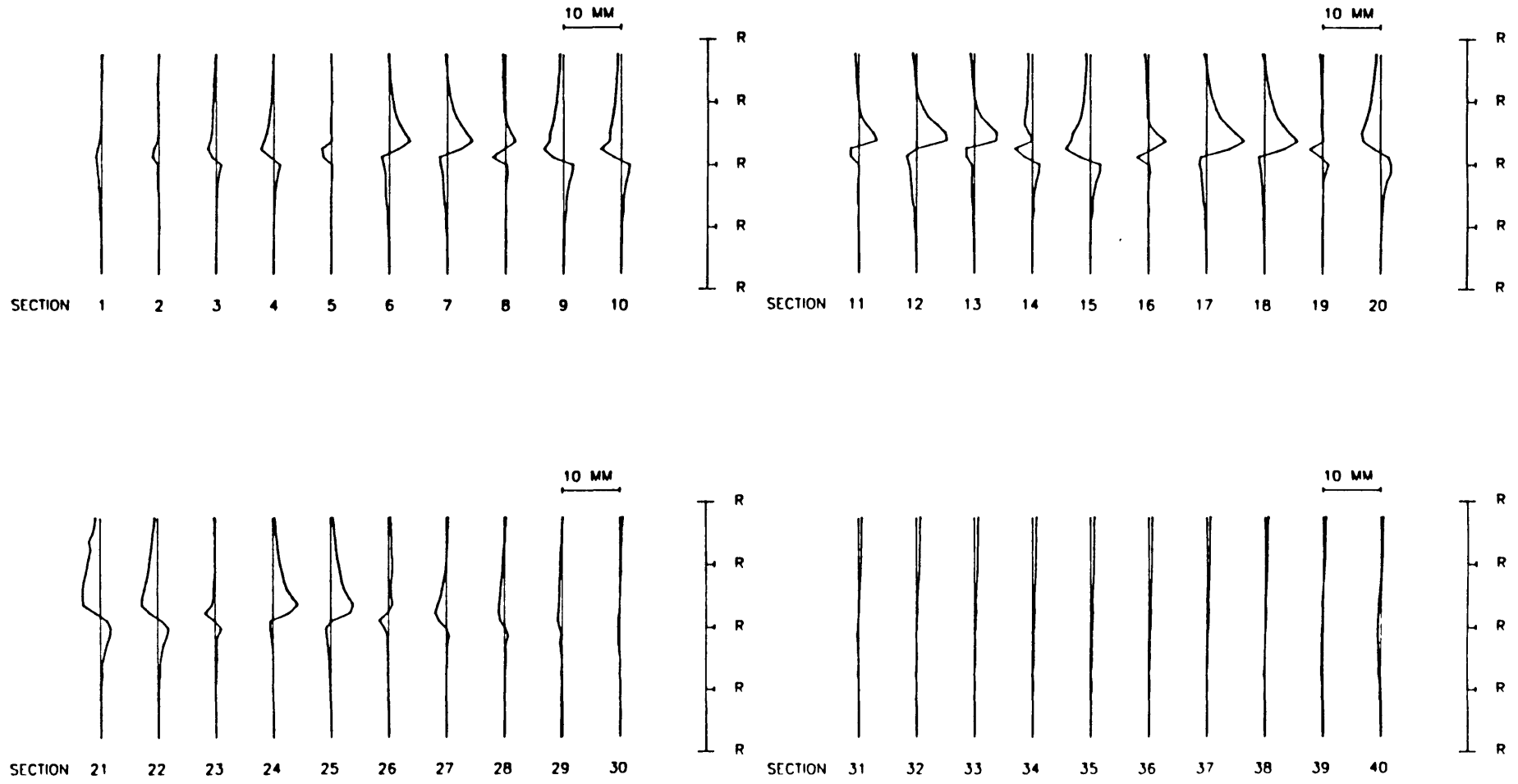


Figure 3.108

FAILURE MODE CY-7

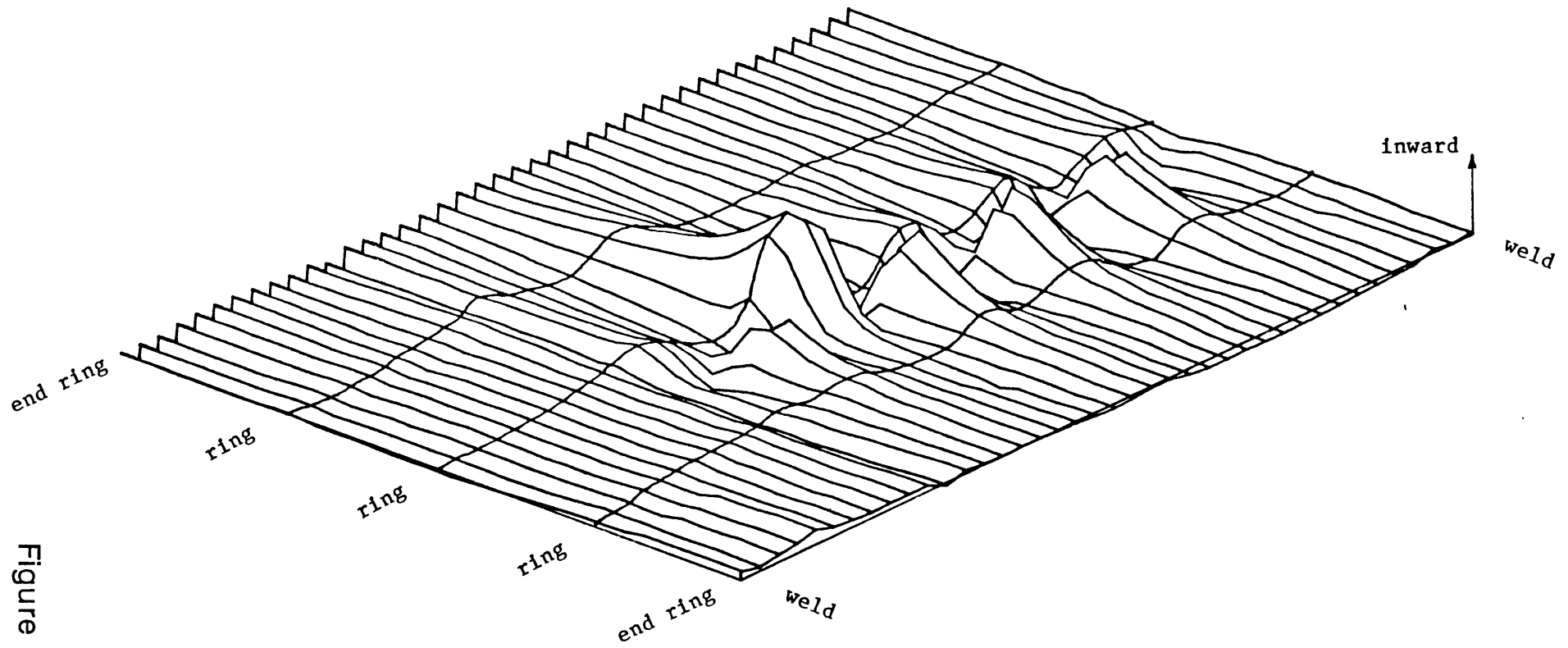
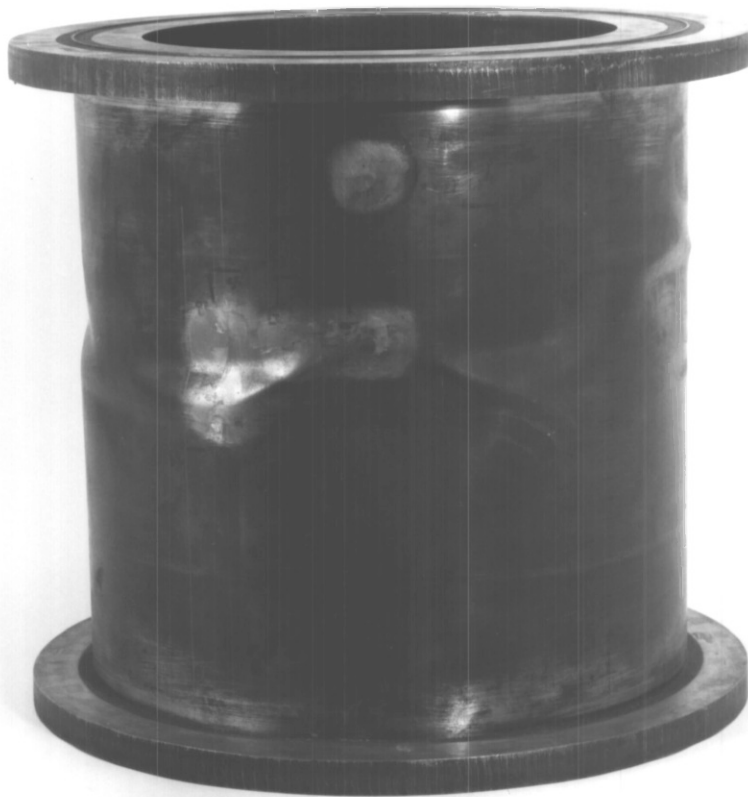
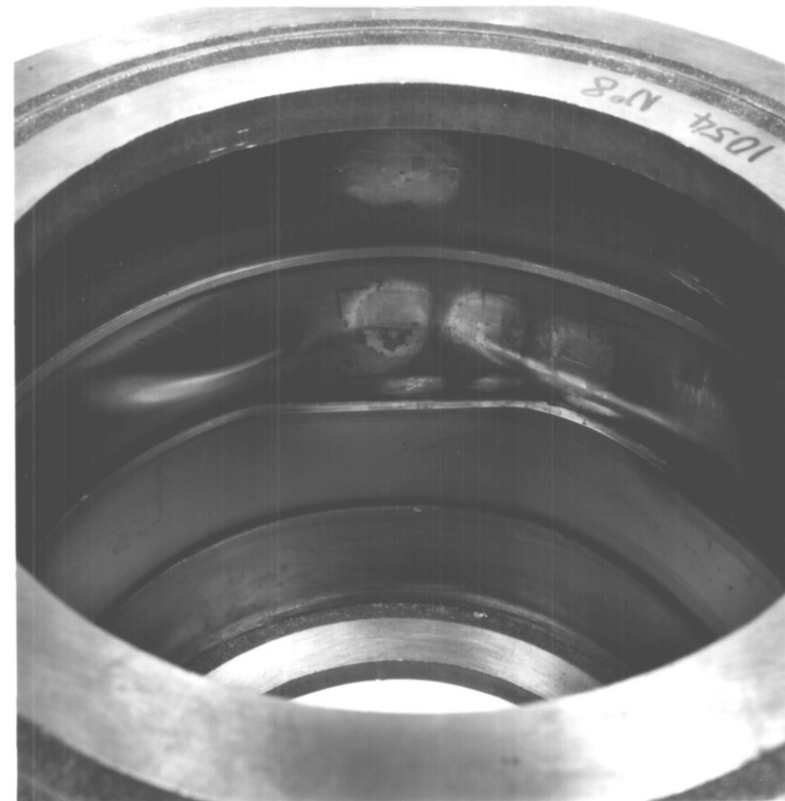


Figure 3.109

CY-7 BUCKLED SHAPE



OUTSIDE VIEW



INSIDE VIEW

MODEL CY-8
DEFLECTIONS DURING FAILURE TEST

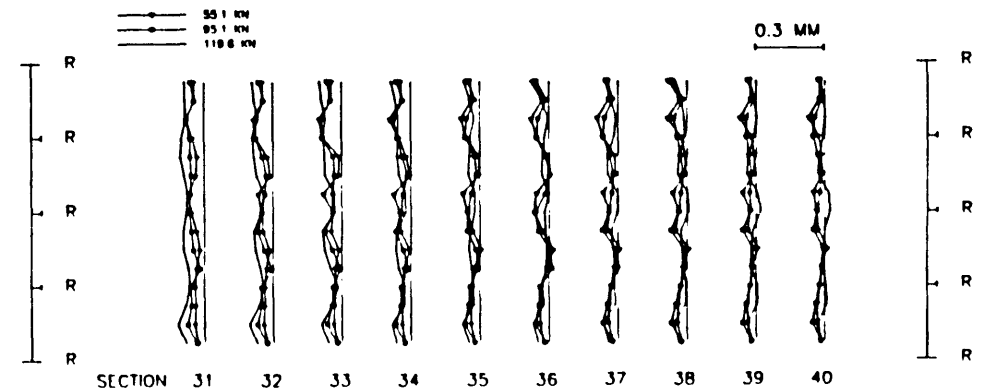
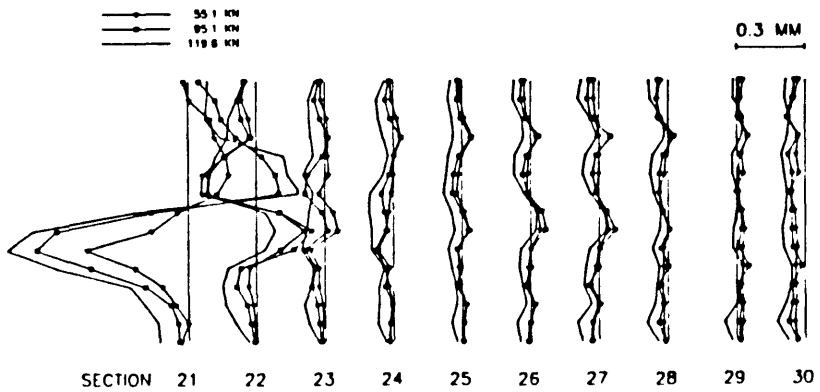
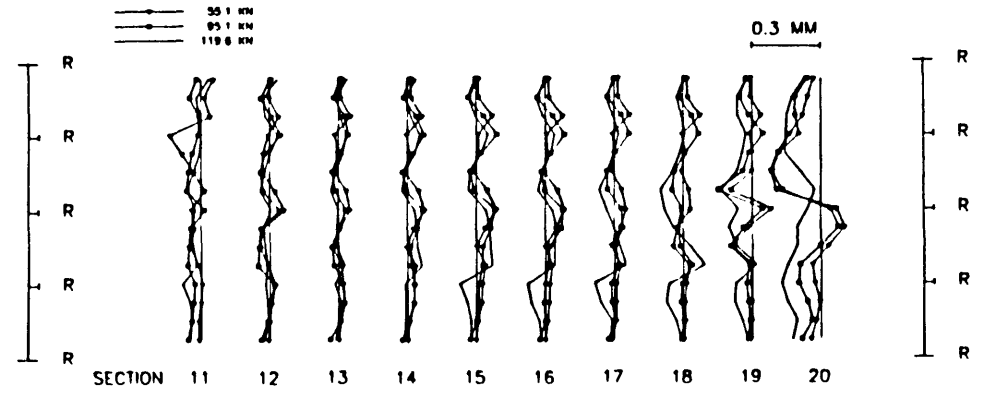
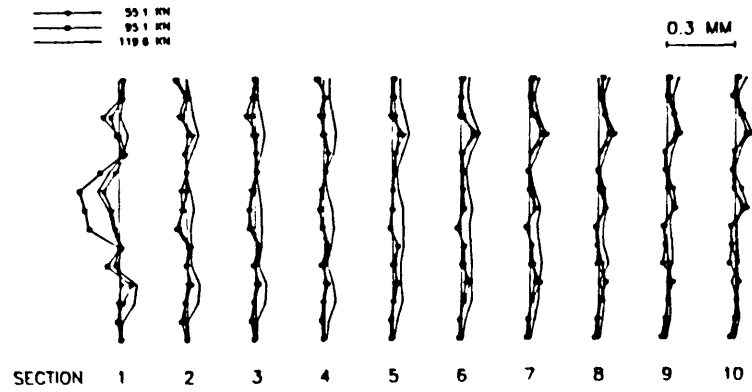


Figure 3.111

MODEL CY-8
DEFLECTIONS AFTER FAILURE

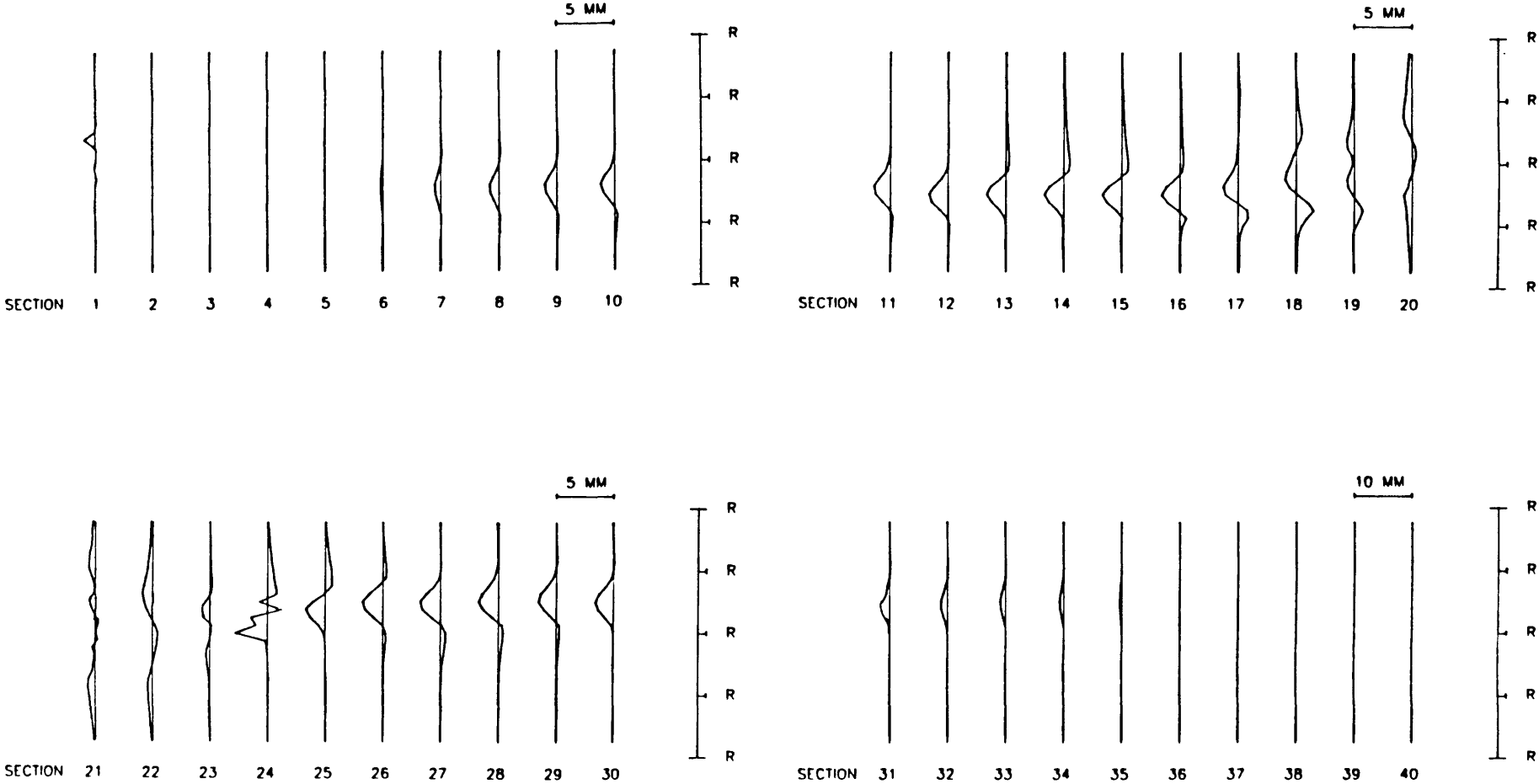


Figure 3.112

FAILURE MODE CY-8

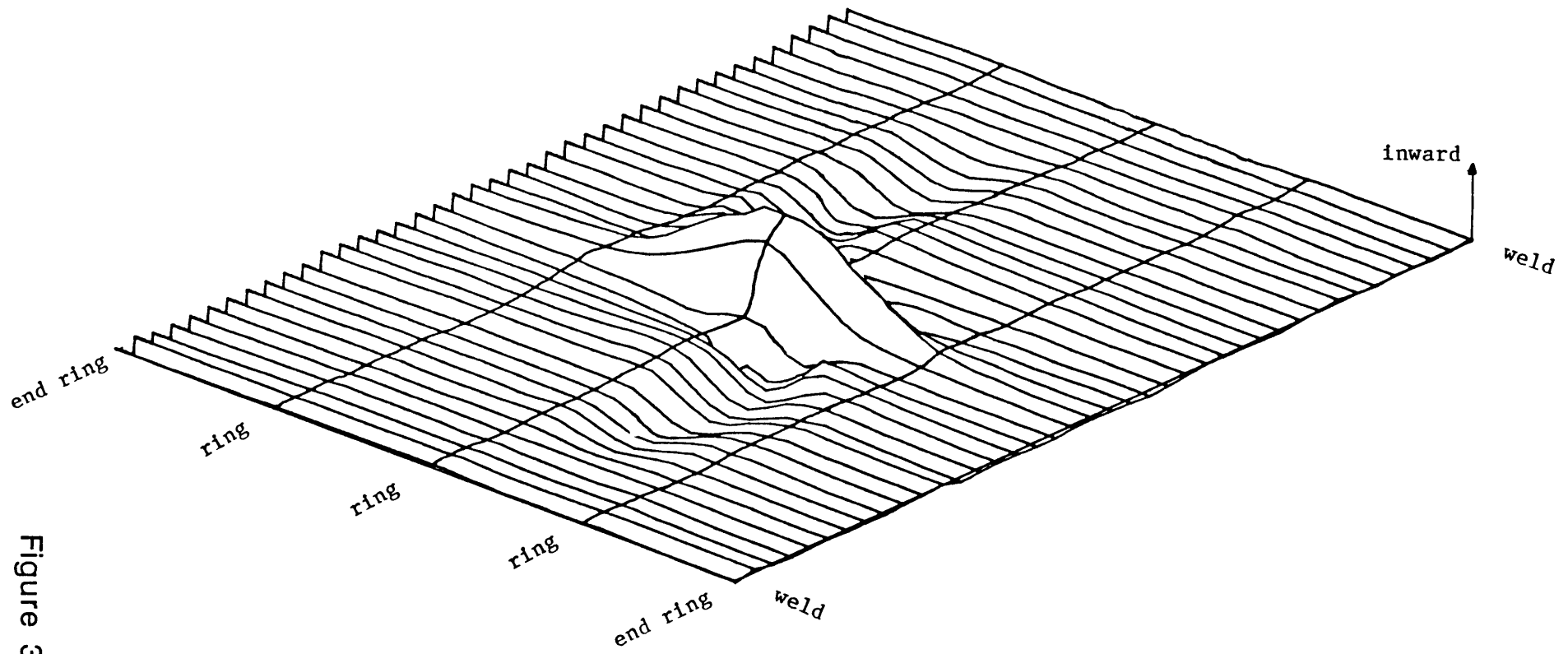
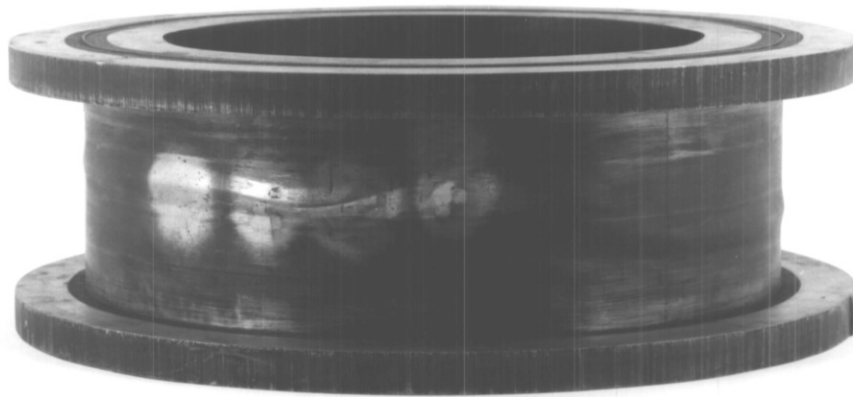
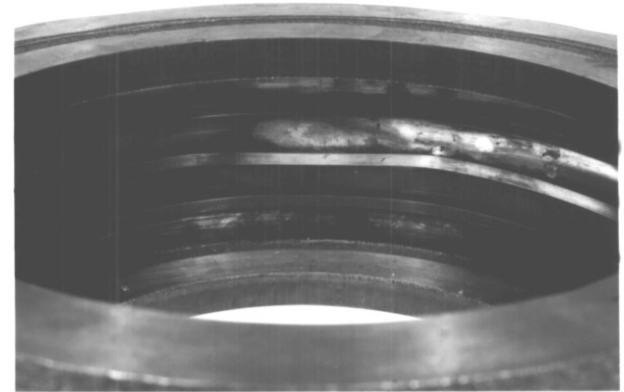


Figure 3.113

CY-8 BUCKLED SHAPE



OUTSIDE VIEW



INSIDE VIEW

Figure 3.114

MODEL CY-9
DEFLECTIONS DURING FAILURE TEST

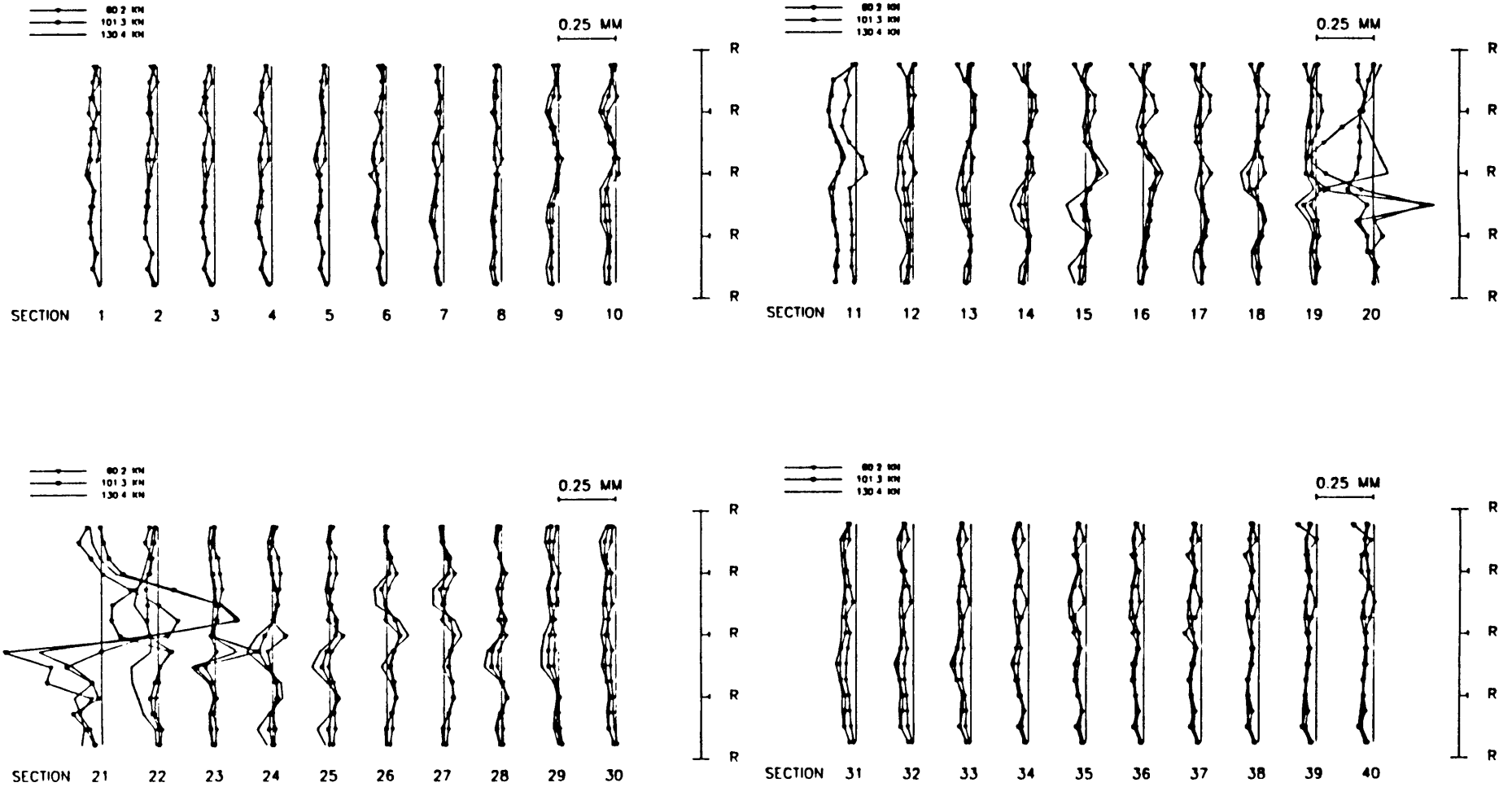


Figure 3.115

MODEL CY-9
DEFLECTIONS AFTER FAILURE

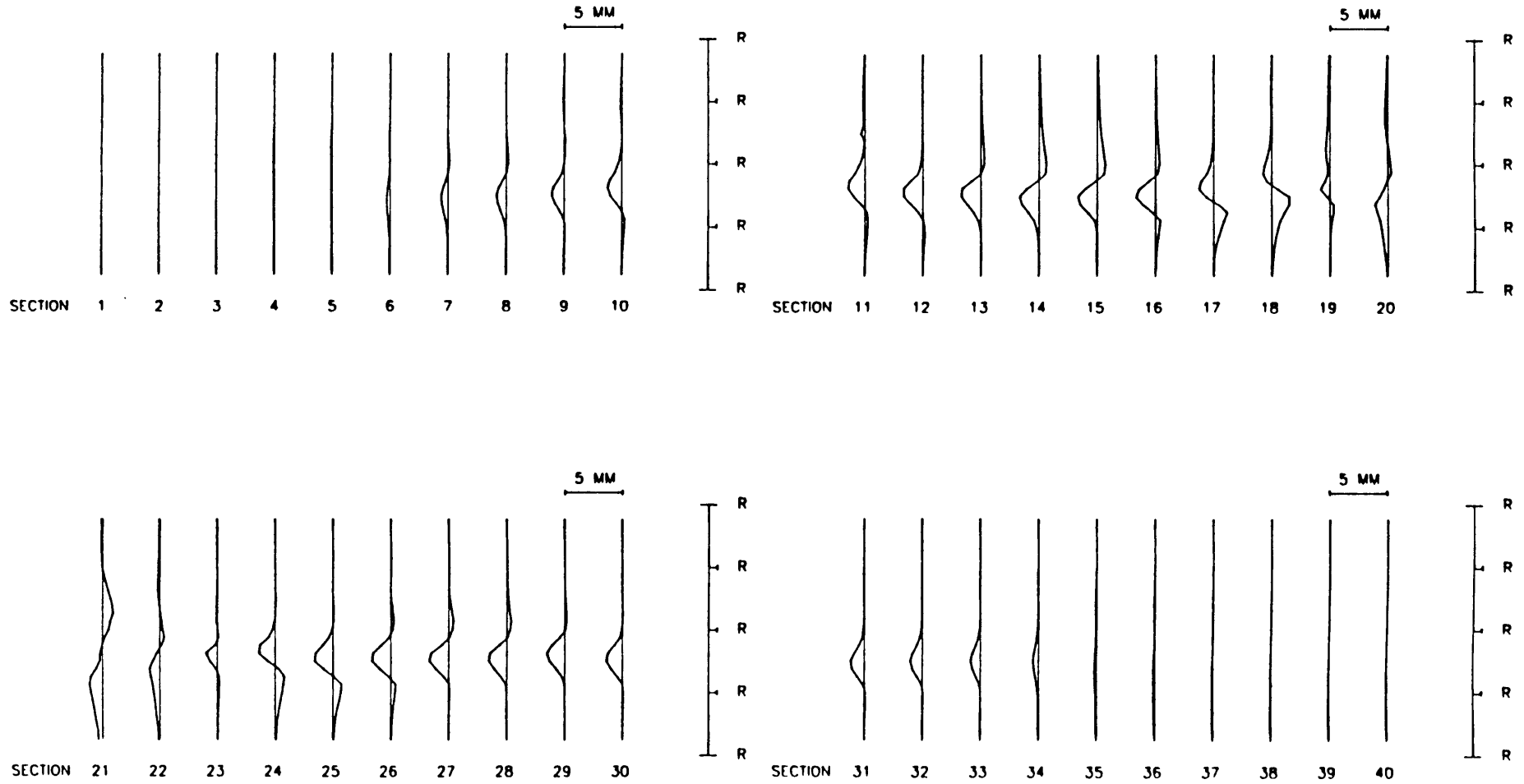


Figure 3.116

FAILURE MODE CY-9

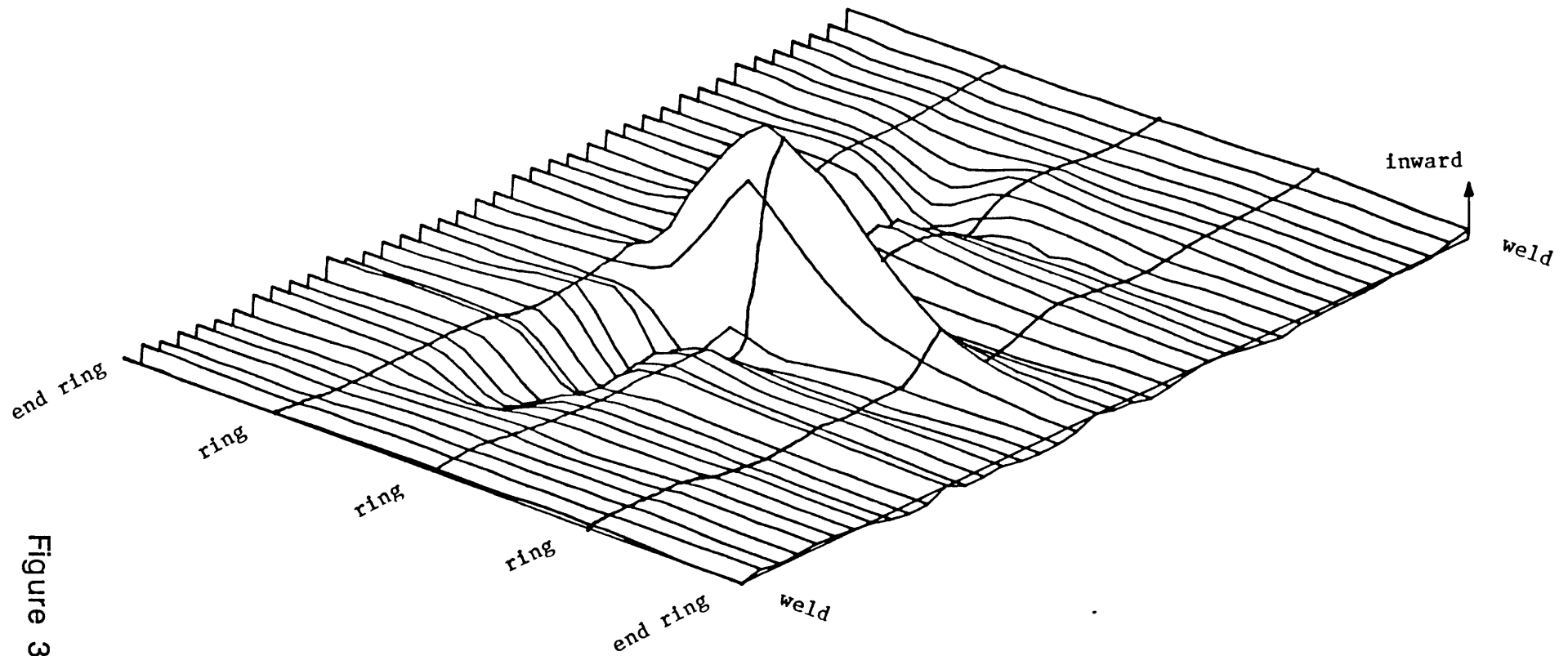
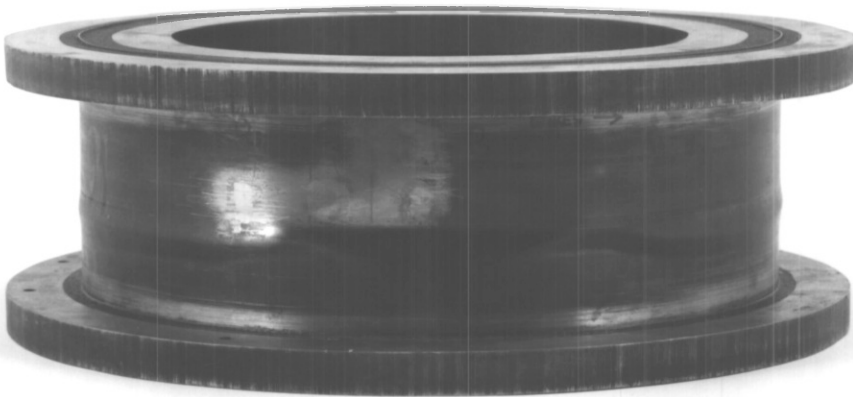
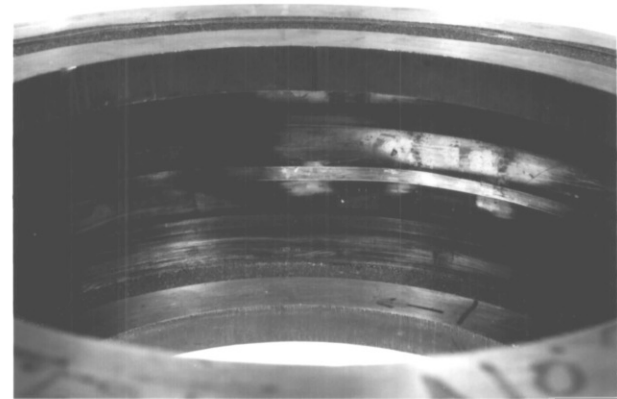


Figure 3.117

CY-9 BUCKLED SHAPE



OUTSIDE VIEW



INSIDE VIEW

Figure 3.118

CHAPTER 4

NUMERICAL MODELLING OF DAMAGE TO RING STIFFENED CYLINDERS

4.1 INTRODUCTION

A model for analysing the denting process was needed. When correlated with experiments such a model could be used for predicting the residual damaged state of ring stiffened cylinders subjected to impact situations other than those considered in the experimental series described in this thesis.

The plastic method of analysis is the main technique that has been applied so far for the solution of such problems. Examples of such applications are the plastic mechanism method by Walker and Kwok [51], based on a diamond shaped mechanism used to examine the denting of unstiffened cylinders. Ronalds and Dowling solved the problem of denting of orthogonally stiffened cylinders [54] by developing a plastic formulation which looked at the beam column behaviour of the stiffeners. Similar principles have been applied previously in the analysis of tubulars subjected to denting [33, 38, 39, 41]. Frieze and Sachinis [49] attempted to solve the problem of damage to ring stiffened cylinders by the finite difference method of analysis. The load, however, was applied as a point load and only a load versus central deflection curve for a single geometry was presented. No information regarding overall deflections and stresses was given in their papers.

It was clear from previous work that there was little information regarding the detailed response of a ring stiffened cylinder to lateral load. In the case of cylindrical shells the development of local deformations inside and outside the dent area have a potentially important effect on the residual strength of the cylinder. The residual stresses resulting from denting may also have an effect on the strength. The work presented in this chapter is intended to provide information on the distribution of these residual deformations and stresses for further examination of their effects on the strength.

A numerical model is presented which predicts the loading history and residual state caused by denting [124]. The development of a gap/contact finite element is described which enables modelling of the progressive contact between the indenter and the shell. The numerical model is used to provide a correlation with the experimental data, thus validating the numerical model.

4.2 A NEW GAP/CONTACT ELEMENT

When modelling the lateral load applied to the shell, complications arise from the fact that the area of application of the load increases with increasing deformations. This means that the loading function on the cylinder is unknown, and hence that the cylinder cannot be analysed in isolation. Both the shell and the indenter have to be modelled. The finite element package FINAS [99] employed in this work had to be modified to include an interface element joining nodes on the shell to corresponding nodes on the indenter. Appropriate penalty functions have been introduced in the formulation which produce the effect of a gap/contact element; a detailed description of the element follows:

4.2.1 Element Matrices

The relative displacements of a triaxial gap element in the local axis system are given by:

$$\begin{aligned}\Delta r &= r_j - r_i \\ \Delta s &= s_j - s_i \\ \Delta k &= k_j - k_i\end{aligned}\tag{4.1}$$

where r , s and k are the local element axes and i and j are the two nodes to be linked together. The strain vector can be written as:

$$\begin{aligned}\epsilon_r &= \Delta r/T \\ \epsilon_s &= \Delta s/T \\ \epsilon_k &= \Delta k/T\end{aligned}\tag{4.2}$$

assuming that T is the thickness of the gap/contact element in all three directions. Equation (4.2) can be written as:

$$\begin{bmatrix} \epsilon_r \\ \epsilon_s \\ \epsilon_k \end{bmatrix} = 1/T \begin{bmatrix} -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_i \\ s_i \\ k_i \\ r_j \\ s_j \\ k_j \end{bmatrix} \quad (4.3)$$

or $\epsilon = G \delta$

The stress-strain transformation is given by:

$$\begin{bmatrix} \sigma_r \\ \sigma_s \\ \sigma_k \end{bmatrix} = \begin{bmatrix} d_{rr} & d_{rs} & d_{rk} \\ d_{sr} & d_{ss} & d_{sk} \\ d_{kr} & d_{ks} & d_{kk} \end{bmatrix} \begin{bmatrix} \epsilon_r \\ \epsilon_s \\ \epsilon_k \end{bmatrix} \quad (4.4)$$

or $\sigma = D' \epsilon$

The stiffness matrix is then given by the following expression:

$$K_{rsk} = \int_V G^T D' G dV \quad (4.5)$$

This operation gives the matrix

$$K_{rsk} = V \begin{bmatrix} d_{rr} & d_{rs} & d_{rk} & -d_{rr} & -d_{rs} & -d_{rk} \\ d_{sr} & d_{ss} & d_{sk} & -d_{sr} & -d_{ss} & -d_{sk} \\ d_{kr} & d_{ks} & d_{kk} & -d_{kr} & -d_{ks} & -d_{kk} \\ -d_{rr} & -d_{rs} & -d_{rk} & d_{rr} & d_{rs} & d_{rk} \\ -d_{sr} & -d_{ss} & -d_{sk} & d_{sr} & d_{ss} & d_{sk} \\ -d_{kr} & -d_{ks} & -d_{kk} & d_{kr} & d_{ks} & d_{kk} \end{bmatrix} \quad (4.6)$$

which is the general expression for an inclined triaxial two-noded gap contact element representing a dilatent interface. The dilatency effect is only significant for rough irregular surfaces, such as jointed rock (Jaegar and Cook [125]), and can be neglected for steel/steel and steel/concrete interfaces, hence the mixed terms in (4.6) can be omitted.

Furthermore, for an orthogonal interface coincident with the rsk set of axes, the rotation $P^T K P$ produces an identical matrix to (4.6) but in the XYZ coordinate

system.

A further simplification can be made regarding the Z-displacements since the dent line can be treated as a symmetry line as explained in the next section. Hence DZ vanishes and ϵ_z is also zero. This can be allowed for by deleting the third and sixth rows and columns in (4.6). This reduces the stiffness matrix to a 4X4 one.

Following the above discussion the ensuing stiffness matrix can be written as:

$$K_{xyz} = S/T^2 \begin{bmatrix} d_{xx} & 0 & -d_{xx} & 0 \\ 0 & d_{yy} & 0 & -d_{yy} \\ -d_{xx} & 0 & d_{xx} & 0 \\ 0 & -d_{yy} & 0 & d_{yy} \end{bmatrix} \quad (4.7)$$

where S is the surface over which the spring is operative.

This 2-D version of the matrix is identical to that which would be obtained from the formulation given by Ghaboussi, Wilson and Isenberg [126], if the hoop stiffness coefficient is set to zero. This points to a possible explanation for the singularity encountered by Ghaboussi *et al* for vertical contact surfaces. It is suggested by the present author that the hoop term in the Ghaboussi *et al* formulation represents the difference in hoop stiffness between the two ends of the element. The formulation given above is for a point contact and hence encounters no such difficulty.

The final step in the formulation is to avoid the definition of an element thickness (T), which can cause problems upon closure of the gap. This can be achieved by multiplying equation (4.3) by the thickness (T), which then gives a stress displacement transformation. Such an expression is not only more convenient, but is also more significant in contact problems where experimental readings are often given in terms of the displacements relative to the thickness between the two surfaces. The ensuing expression for the stiffness matrix is similar to (4.7) but with no thickness term.

A corresponding formulation has been used for bond-slip problems of composite structures by Elnashai, Carroll and Dowling [127]. It has also been used for modelling of flexible web-flange connections in box-girder construction and soil structure interaction. For the present application, the most significant parameter is the stiffness coefficient in the direction normal to the interface (X-direction) between the indenter and the shell. The stiffness in the other direction is given a near zero value. The normal stiffness is defined in the next section.

4.2.2 Gap/Contact Element Constitutive Model

For the purpose of the present study a number of assumptions were made which enabled the problem to be treated in a simple way. The simplifications were based on the following points:

1. The body of the indenter through which the lateral load was applied to the shell was free to move in the loading direction only (X-direction). Consequently the main component of deflection was in this direction with values in the Y-direction being of a different order.
2. Deflections in the Z-direction (the longitudinal direction of the shell) were symmetric about the dent line and hence at the dent line the Z-displacements were zero.

The new element is based on the interface element originally developed by Elnashai [128] to model bond-slip between concrete and steel tube in composite tubular connections. For the present application the most significant parameter is the stiffness coefficient in the direction of loading. By introducing appropriate penalty functions on this stiffness coefficient, the two distinct phases of gap and contact between the two nodes could be modelled. Figures 4.1a and 4.1b show the characteristics of the element in the principal direction (X), which are explained in detail below.

1. Gap phase: The spring is assumed to have a negligibly small stiffness while there is a gap between the two nodes (Figure 4.1a). Hence it transfers a very small load to the shell (Figure 4.1b) and it is in effect acting as a "gap" element.
2. Contact phase: When the displaced co-ordinates of the two nodes differ by less than the specified tolerance of .0001mm, i.e. when the gap is sufficiently small to be regarded as closed, the spring stiffness in the X-direction (the loading direction) is multiplied by a sufficiently large value to represent near infinite stiffness. This provides an essentially infinite contact stiffness in relation to the lateral stiffness of the shell (Figure 4.1a). As a result any subsequent increase in loading produces negligible relative displacement between the nodes in contact, which therefore remain in contact as loading increases further. Load is now transferred onto the shell (Figure 4.1b) and the spring essentially acts as a "contact" element. The number of nodes in contact progressively increases as the loading history progresses.

There is an initial error introduced by the small initial stiffness of the spring, resulting in a gap stress of σ_0 but this stress is negligible compared to the stress transferred to the shell on contact.

In order to be able to switch between the gap and contact phases, the following conditions were introduced in the computer implementation.

The updated co-ordinates of the two nodes in the X-direction are calculated and checked against the defined tolerance. If X_i and X_j are the initial X-co-ordinates of indenter and shell nodes respectively, while NX_i and NX_j are the updated coordinates and $X_i > X_j$ at zero load, then:

1. If $NX_i - NX_j > TOL$ -gap phase
-zero stiffness
2. If $NX_i - NX_j < TOL$ -contact phase
-infinite stiffness

The second condition takes care of two things. Firstly, when the gap is very small the contact phase is switched on. In addition it takes care of any crossing that may occur between the two nodes before the contact phase is switched on. This can take place since the relative displacement steps are normally bigger than the tolerance. No significant overlap, however, occurs because as soon as crossing is detected the contact conditions are switched on, which pull the two nodes together, resulting in negligible gaps of the order of .0001mm.

No attempt is made to control the stiffness in the Y-direction, which is kept at a very low value throughout loading. Hence the formulation allows some slip to occur in that direction, which was also possible in the test, but resulting deflections are insignificant as discussed previously.

4.2.3 Computer Implementation

The main steps involved in the calculations are given by the following.

1. Input material and geometric properties.
2. Set-up shape function weighting to each element.
3. Calculate initial and current distance between nodes.
4. Check for two conditions.

- a. If distance between the nodes smaller than tolerance
 - b. If nodes crossed.
5. Calculate stresses using secant parameters.
6. From stresses calculate equivalent nodal loads.
7. Generate tangent stiffness matrix.
8. Repeat 1. through 7. until convergence is satisfied.

4.3 FINITE ELEMENT PROGRAM FINAS

A brief description of the finite element program 'FINAS' [99] employed in this work is given below. This program was developed at Imperial College as part of the SERC offshore research package. It is particularly suited to elasto-plastic non-linear geometric solutions of arbitrarily deformed and loaded plates and shells. The main features of the program are described below:

Input phase: The input data is given in free format. The program offers some automatic mesh generation facilities for regular straight, circular and parabolic regions.

Element library: In addition to isoparametric membrane, axisymmetric and 3-D continuum elements, the program has a doubly curved shell and a fully compatible 3-nodal isoparametric beam element. The shell and beam elements are ideally suited for the modelling of thin and thick shells and stiffened cylinders.

Constitutive relations: The following material models can be used with any of the above elements.

- Linear elastic
- Elasto-plastic analysis (Ivanov single layer yield condition)
- Elasto-plastic multilayer (von Mises yield condition)

There is also a choice of two formulations for large displacements, namely:

- Total Lagrangian, where all variables are referred to the initial geometry.
- Updated Lagrangian, where all variables are referred to the last known equilibrium configuration.

Assembly and solutions: The solution of the ensuing simultaneous equations is based on Gauss elimination, and is not restricted to positive definite matrices. There is also a band width optimiser (Gibbs-Poole-Stockmeyer algorithm), which has been shown to be superior to other techniques (e.g. Guthill-McKee, Levy, Rosen..etc.).

Non-linear solution strategies: Any combination of Newton-Raphson, modified Newton-Raphson and KT_{01} (up dating at the beginning of the step and after the first iteration) can be used. There is also an option for automatic switching from one strategy to another depending on the number of iterations in the previous load step.

4.4 FINITE ELEMENT MODEL FOR THE DENTING PROCESS

4.4.1 Finite Element Mesh

In choosing an appropriate mesh for the analysis, the following important aspects were considered. It was clear from the experiments that the entire length of the models was affected by the denting process. This suggested that the whole length of the cylinders would need to be analysed. In addition, a degree of mesh refinement was needed, especially in the area of the dent, to model accurately sudden changes in deflections and stresses.

One cylinder was chosen, CY-8, to examine in detail the progress of the denting process.

To reduce the size of the problem two symmetry planes were introduced. Both longitudinal and circumferential symmetry were assumed through the centre of the dent, as shown in Figure 4.2. This reduced the problem significantly since only a quarter of the cylinder (ABCD) need be analysed.

Two panels were modelled starting from the dented ring. In the first panel four shell elements were used in the longitudinal direction and thirteen in the circumferential direction. Mesh refinement was applied circumferentially in the dent area where large variations in deflection and stresses were anticipated and had been monitored during the experiment. In the second panel the same circumferential refinement was applied but only two elements were used longitudinally. The ring stiffeners were modelled by beam elements.

The indenter was modelled using four beam elements of very high stiffness. The first node on the indenter was common with the first shell element node. This point represented the first point of contact. There was no requirement for a gap/contact element at this location. The rest of the beam indenter nodes were joined with the corresponding shell nodes by gap/contact elements.

4.4.2 Description of Elements Used

The doubly curved shell element available in the finite element program "FINAS" was used to model the panels and the fully compatible (3-noded isoparametric) beam element to represent the ring stiffeners. The latter was also used to model the indenter.

The shell elements are 8-noded of the isoparametric type. They have six degrees of freedom per node and allow both smooth and discontinuous shells to be analysed. Transverse shear stresses are taken into account, hence the elements are equally suited to thin or thick shell analyses. A penalty function procedure is used to avoid "shear locking" in thin shells which can be a problem in elements incorporating transverse shear stresses.

The stiffener element is 3-noded isoparametric. It uses thin-walled beam theory and includes shear deformation. It is capable of representing any open cross-section composed of up to 20 linear segments. In stiffened shell problems the beam and shell elements share the same nodes. A seventh degree of freedom, the warping distortion of the beam, is included at the nodes where the beam is present.

The element formulations are based on a nonlinear strain displacement relationship and therefore include large displacement/rotation effects. Material nonlinearity is based on the elasto-plastic multilayer technique and uses the von Mises yield criterion. The element stiffness matrix is evaluated by Gauss numerical integration at up to ten quadrature points in each direction, hence variable accuracy can be used.

Detailed description of the gap/contact elements joining the indenter and the shell was given above.

4.4.3 Loading and Boundary Conditions on the Finite Element Model

Load was applied in the X-direction and was modelled as uniformly distributed on the indenter. The indenter itself was allowed to move in the X-direction only, which was the case in the experiment. The load distribution in the indenter was not significant because of the high indenter stiffness relative to the shell.

On three boundary lines of the cylinder AB, CD, and AD, symmetry conditions were applied. The top end conditions, however, proved to be more complicated.

As already discussed in a previous section, the end rings of the experimental models did not provide clearly defined boundary conditions. Both bending of the end rings and possibly some limited bond slip between the shell and the resin-sand mix could have occurred. Consequently only upper and lower bound solutions to the test could be obtained.

The two idealised boundary conditions considered were the following:

- Upper bound solution given by assuming that the end rings were rigidly fixed to the cylinder. Beam elements were used to model the end rings.
- Lower bound solution given by no end rings attached to the model.

By comparing the axial shortening that resulted from the two analyses with the experiments, it was found that the first set of conditions was closer to the test. Therefore detailed correlations between stresses and displacements were only carried out for the analysis with these boundary conditions. In addition these conditions are closer to what would be found in practice where the damaged area is part of a structure which provides additional longitudinal restraint.

4.4.4 Solution Time and Convergence

In analysing a shell under lateral load by the finite element method several factors contribute to long solution times. The need for an extensive model with areas of mesh refinement are major considerations in this respect.

The nature of the problem of lateral load application on a shell also creates convergence difficulties. If lateral load were applied directly to a shell as a point load some convergence problems would occur because an infinite stress is caused in the shell, resulting in a singularity. This is not found with the present analysis but the similar circumstances result in generally slow convergence.

The above factors contribute to the need for an extensive mesh representation and small loading steps to run the analysis successfully. For this reason, only one model has been analysed and correlated with the experiments. It is believed that this single analysis is representative of the response, since all the experiments have shown similar trends in relation to stresses and deformations due to denting.

4.5 CORRELATIONS BETWEEN NUMERICAL AND EXPERIMENTAL RESULTS

4.5.1 Load Dent Deflection Curves

Load versus central dent deflection for the two analyses performed are presented in Figure 4.3 together with the experimental curve. The analysis which included end rings gave a higher stiffness than the experiment, while the free end conditions produced a lower stiffness. As already discussed above, the presence of the end rings had a stiffening effect on the model by restricting any relative local axial movement in the area of the dent. The upper bound solution gave a load 34 percent higher than the test, while the lower bound produced a 26 percent lower load for a given displacement.

It can be seen that all three curves are very close, for displacements up to about 0.9mm which is the linear range of displacements. Between 0.9 and 1.6mm the two finite element curves have lower stiffness than the experimental curve. This is attributed to the fact that in the analyses all the load was applied through a single point until the first gap/contact element closed at $d=1.6\text{mm}$. As soon as the first gap closes a distinct change in the slope appears on the load deflection curve, followed by some softening until the next gap closes. This pattern is repeated between successive gaps. The overall effect on the load deflection curve, however, is very slight. This shows that the discretisation of contact assumed in the analysis is reasonable. The discrepancy in the loads before the first gap closed could be reduced by increasing the number of gap/contact elements in the area.

4.5.2 Overall Deflection Response

Deflections during denting are presented for four longitudinal sections in Figure 4.4. Both experimental and numerical results are shown at different levels of denting. These are found to be in close agreement. It can be seen that the adjacent ring stiffeners, designed according to the DnV Code [58], provided some resistance to lateral deformations but did not deform significantly. Radial deflections of the order of 30 percent of the maximum dent depth were recorded in the central section (D), for the adjacent ring. As a result, deflections spread along the whole length of the model. Outward deflections could be seen outside the dent on section (A). These are shown more clearly in Figure 4.5.

Three-dimensional plots of the overall deflected shapes obtained by the finite element analysis are shown in Figure 4.5. The deflection scale is exaggerated by 15 and 30 times for a better appreciation of the radial deflections. Large inward deflections can be seen in the dent area followed by outward deflections of the order of 12 percent

of the maximum dent depth. These outward deflections extend for about half the width of the dent itself. Further away circumferentially, inward deflections of the order of 5 percent occurred. These radial deflections die out at about 90 degrees from the dent centreline.

4.5.3 Stress Distributions at Peak Load.

In Figure 4.6 axial stresses around the midpanel line above the dent are plotted. Stresses on the inside and outside surfaces are plotted separately, rather than an average stress distribution, in order to demonstrate the high bending that occurred in areas adjacent to the dent.

Reasonable correlation can be seen between numerical and experimental results. Near the centre of the dent tensile stresses occur on both sides of the shell. These stresses were higher on the inside face corresponding to the inward deflections in the area. Outside the dent stresses became compressive with higher values close to yield on the inside surface. This correlates with the outward deflections seen in the area. Further away from the dent, stress values reduce to very small figures at around 90 degrees from the dent centre.

4.5.4 Stress Distributions in the Residual State

Figure 4.7 shows the distribution of axial stresses after the lateral load was removed. Some reduction occurred in the stresses which was higher on the inside surface and resulted in a slightly different distribution than at the peak. The experimental stresses measured near the centre of the dent were at yield at both faces and much higher than the numerical values. This discrepancy in the results could be attributed to the inability of the strain gauges to follow the unloading after they had been strained to very high levels. In this sense the numerical distribution in the residual state is likely to be more reliable than the experimental one.

The residual stresses are complex and it is therefore difficult to envisage what net effect these would have on the axial residual strength of the model. In addition they are found to vary significantly along the cylinder. There is, however, an area between 110 and 150 degrees, where compressive residual stresses occur at levels up to 30 percent of yield. These stresses may prove to be significant to the axial strength of the cylinder because they occur in the area where the onset of buckling was seen to occur in the tests.

4.6 FURTHER NUMERICAL RESULTS

Further results from the finite element analysis are plotted in Figures 4.8 and 4.9 showing the distribution of stresses at different levels along the cylinder. Inner and outer surface stresses are again plotted to show areas of bending. Average stresses are also plotted. At peak load (Figure 4.8) a complete reversal of stress can be seen between the dent line and the end rings. Stress distributions again correlate with the lateral displacement directions.

The compressive membrane stresses can again be observed outside the dent extending over all the sections. This is the result of the reaction to the pull-in generated on the end rings due to the membrane tension created by the dent.

As indicated previously, after load removal, high compressive residual stresses occur in the area between 110 and 150 degrees in all sections. The stresses are up to 40 percent of yield and may well have reduced the axial strength of the cylinder.

4.7 DISCUSSION AND CONCLUSIONS

A numerical model has been proposed for modelling the denting of a ring stiffened cylinder by a knife edge indenter. The model includes a gap/contact finite element, which has been developed to enable the gradual contact between the indenter and the shell to be represented.

The proposed numerical model has been used to analyse the denting of one of the experimental models (CY-8). Detailed experimental results correlate well with the analyses, thus validating the numerical model.

The analyses presented provide a full picture of the response of the stiffened cylinder subjected to lateral load, giving information on the deformations and stresses everywhere in the cylinder.

The predicted deflected shape takes the form of inward deflections in the circumferential area of the dent, which spread over the whole length of the model. These are followed by outward deflections outside the dent, extending for about half the width of the dent on either side. Upon unloading large residual deformations, of a similar form, remain in the cylinder. An important application of the proposed numerical model is to predict the deflected shape accurately for use as an initial state for subsequent numerical analysis, for assessing the residual strength of the damaged shell.

High levels of stresses develop during denting inside and around the dent area. Significant levels of residual stresses remain in the cylinder after unloading. For the geometry considered in this chapter, an area circumferentially outside the dent has been shown to develop compressive residual stresses up to 40 percent of yield. Considering that this is also the critical area under compressive loading, these stresses are expected to reduce the axial strength of the cylinder. Their effect on the strength is examined in Chapter 5 using the data on residual stresses generated here.

In conclusion, the proposed model predicts the response and extent of damage on a ring stiffened shell, for a given impact situation. Results of this analysis provide an initial set of data for residual strength assessment.

GAP/CONTACT ELEMENT CHARACTERISTICS

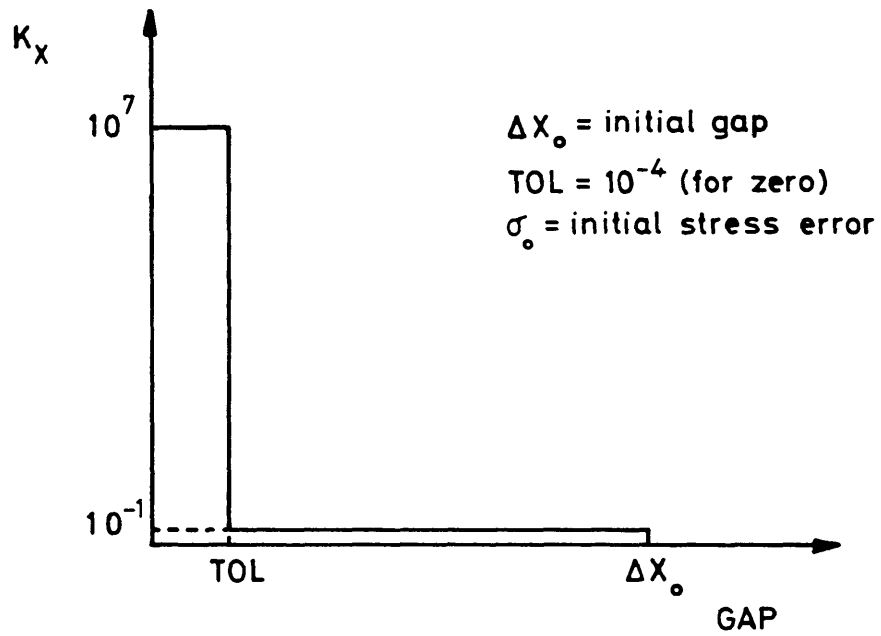
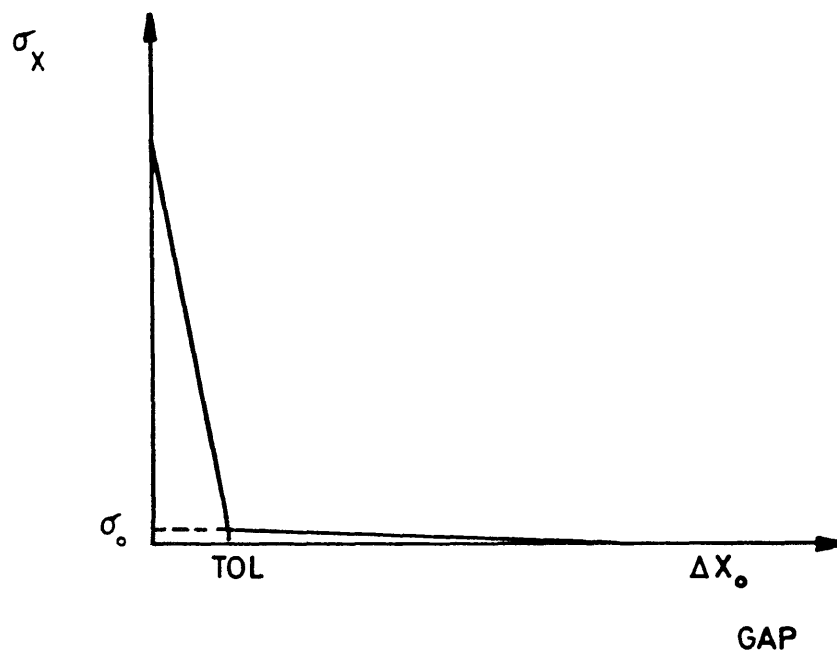
(a) Spring stiffness K_x Vs gap(b) Stress through spring σ_x Vs gap

Figure 4.1

FINITE ELEMENT MESH

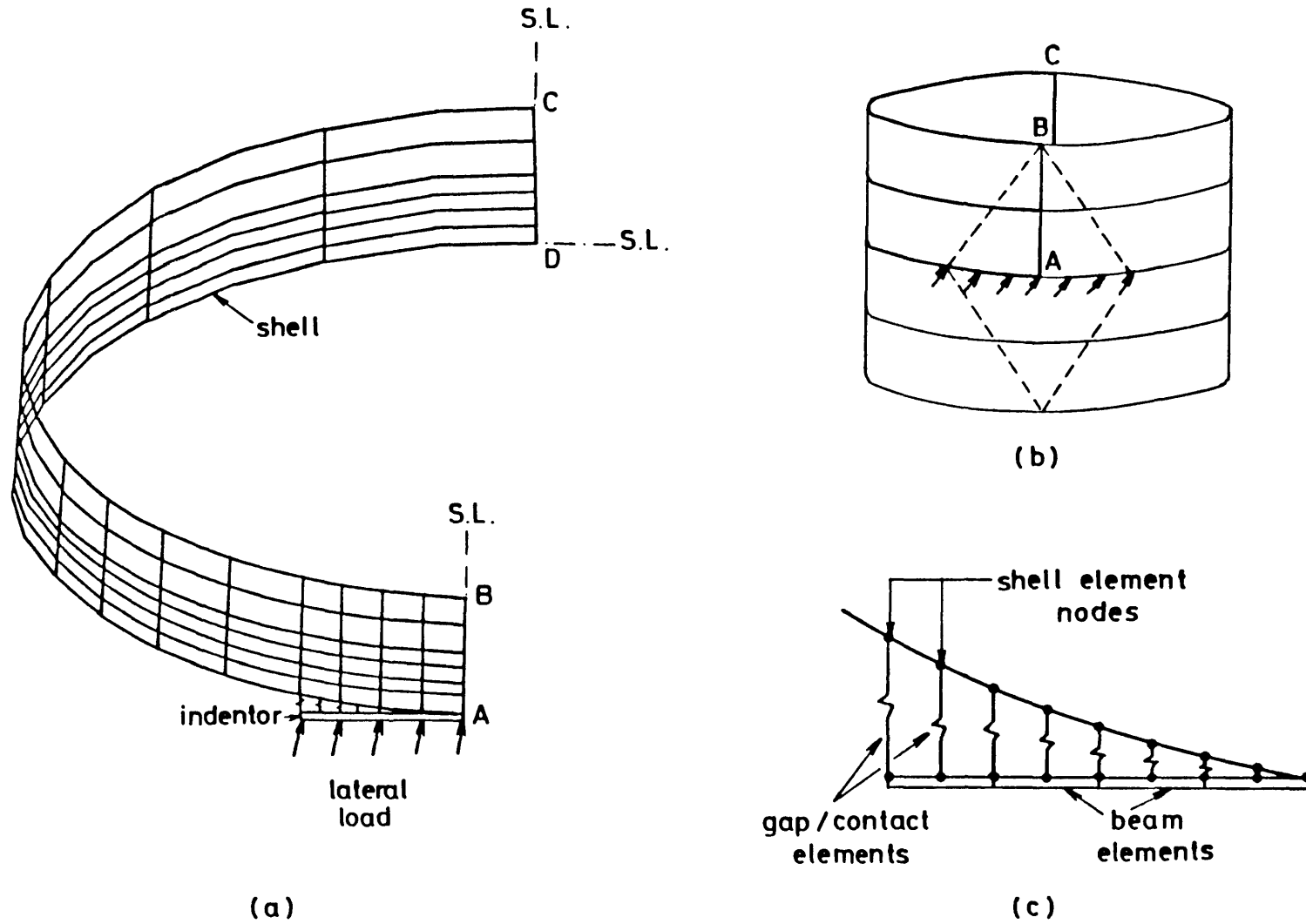


Figure 4.2

LOAD DENT DEFLECTION CURVES

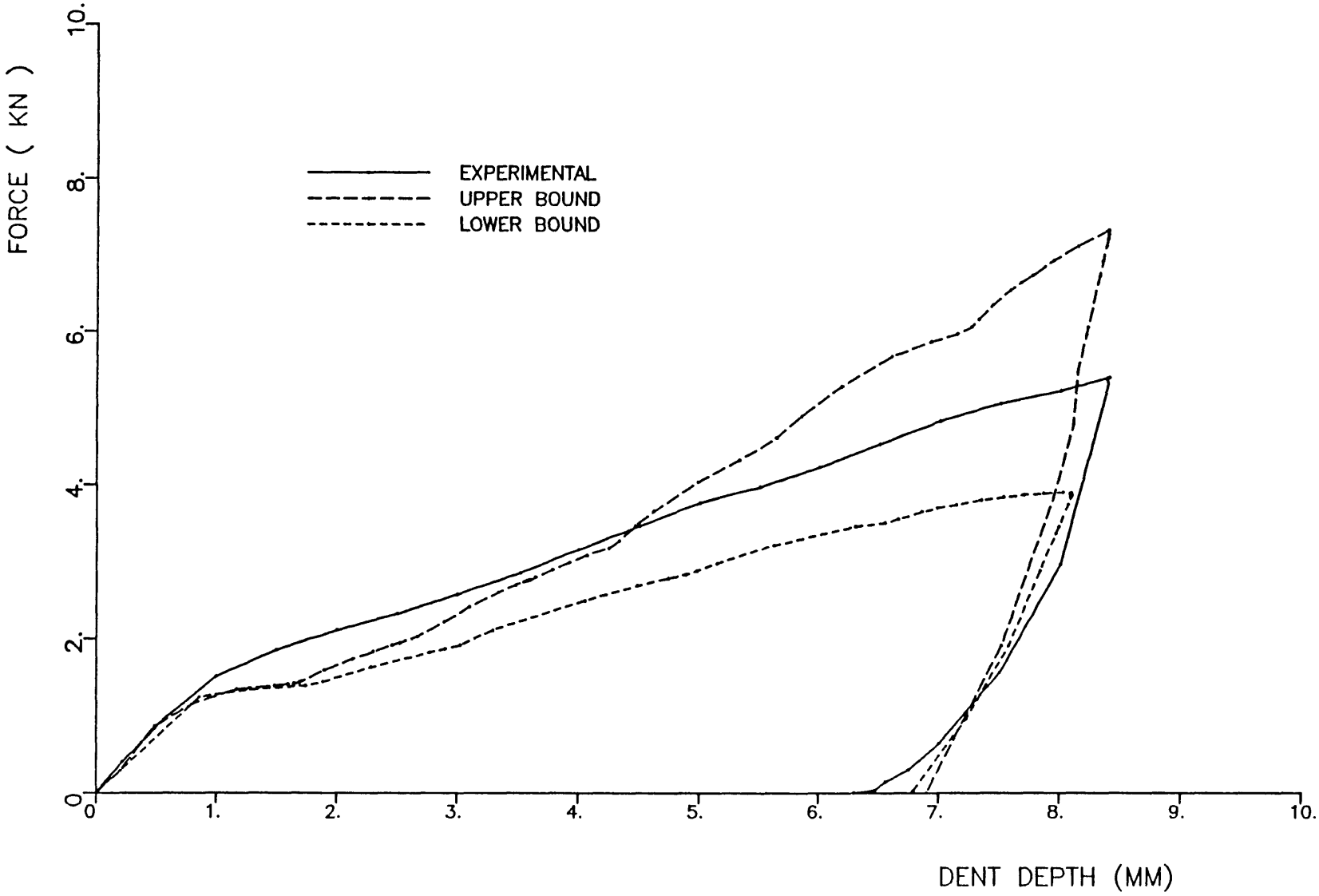


Figure 4.3

DENTING DEFLECTIONS - MODEL CY-8

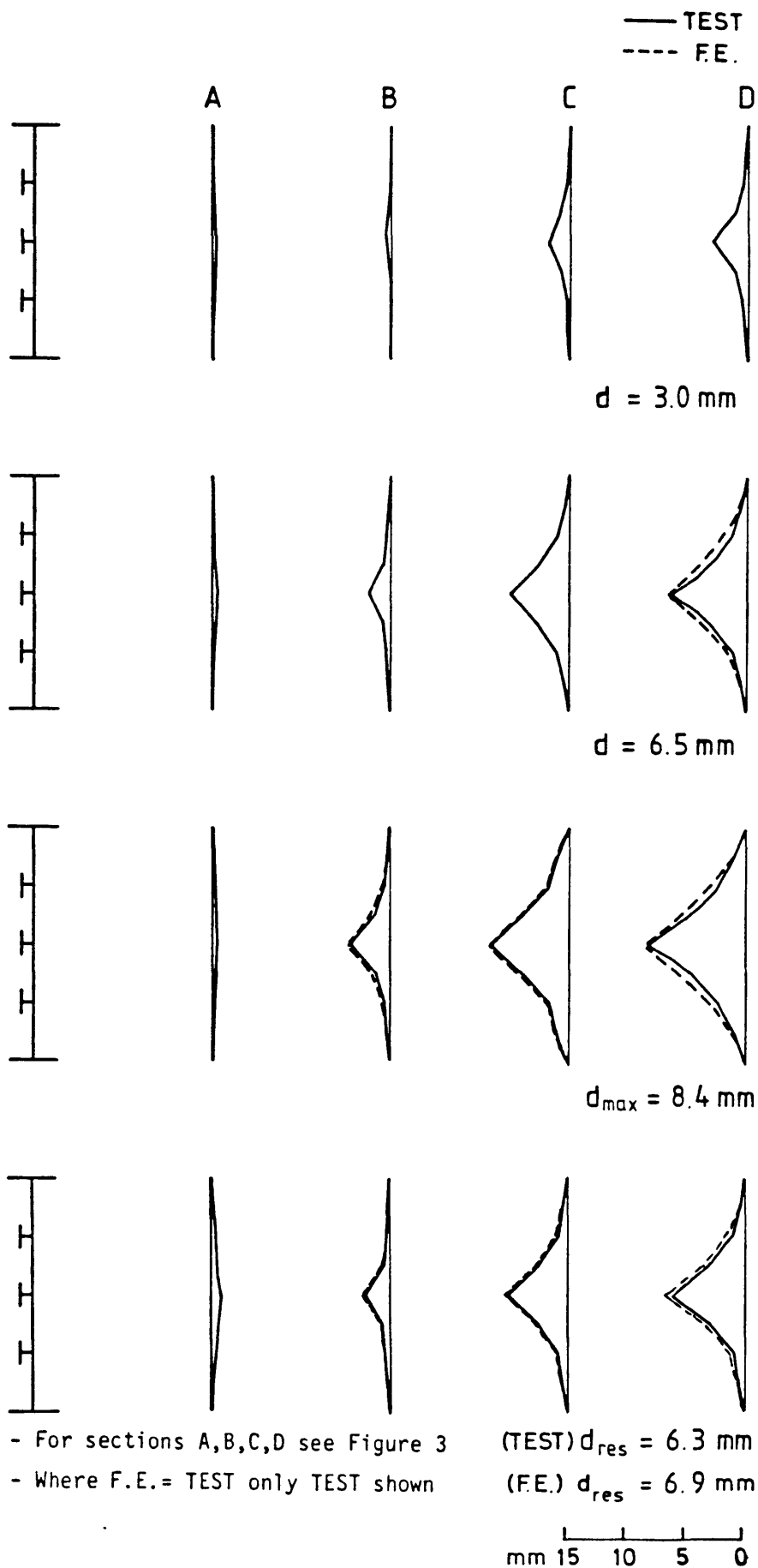
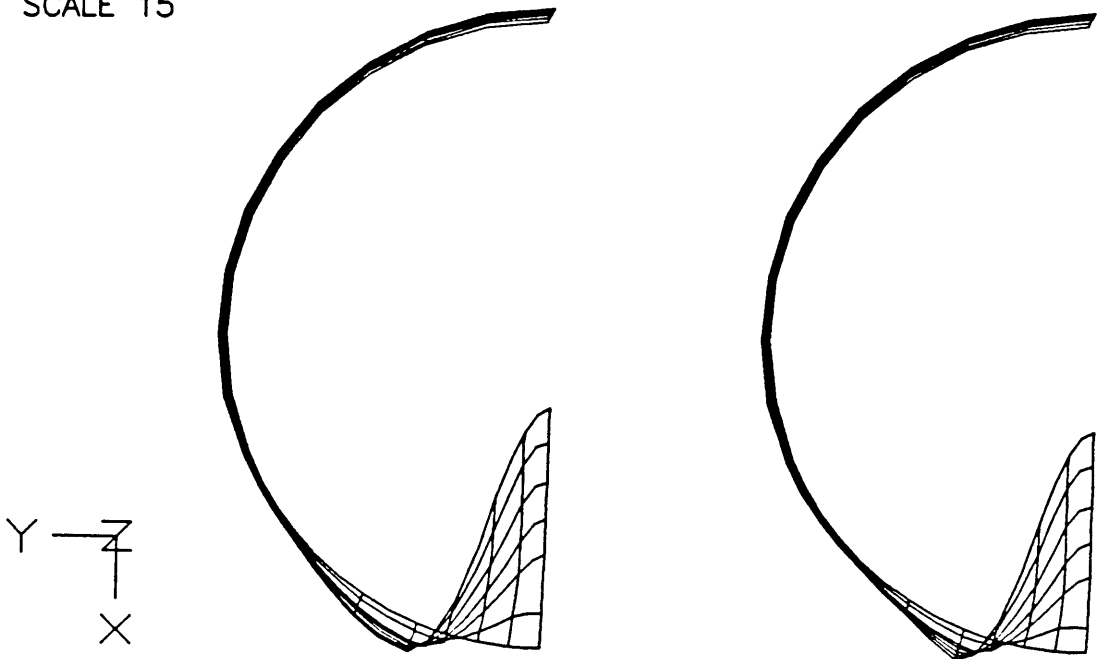


Figure 4.4

3-D PLOTS OF DEFLECTED SHAPES

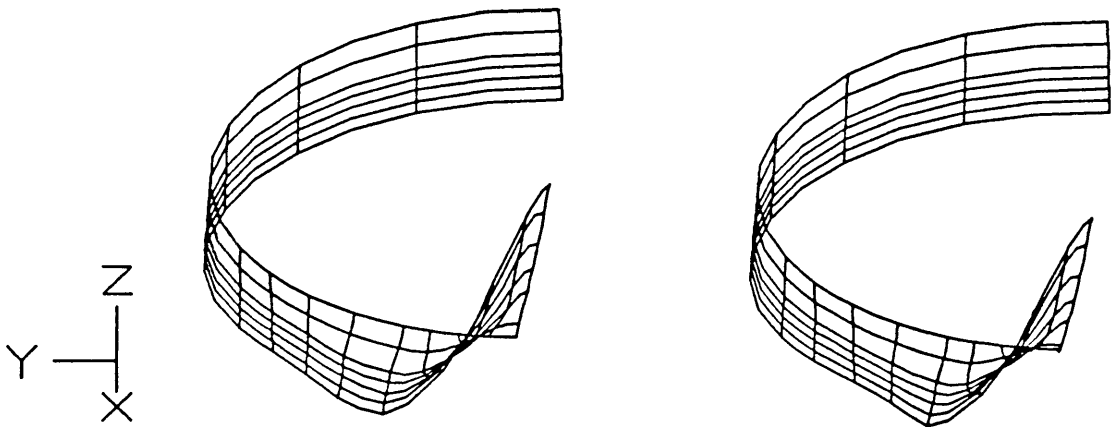
PLAN VIEW
SCALE 15



PEAK DEFORMATIONS

RESIDUAL DEFORMATIONS

PERSPECTIVE VIEW
SCALE 30



PEAK DEFORMATIONS

RESIDUAL DEFORMATIONS

Figure 4.5

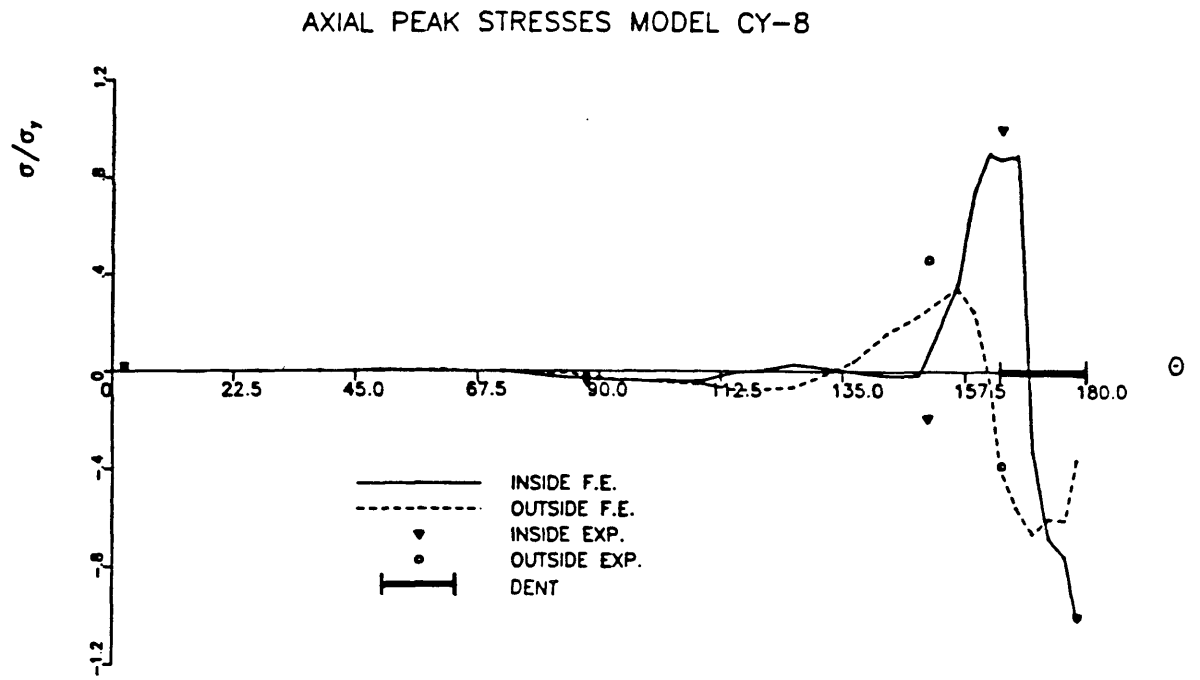


Figure 4.6

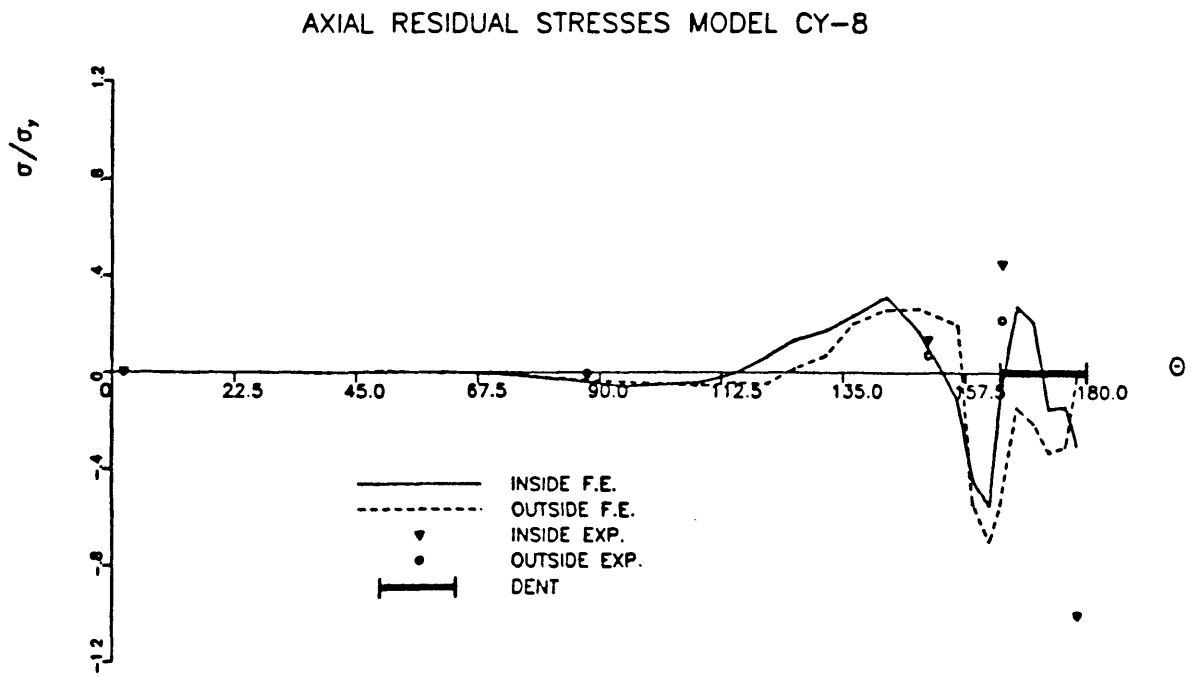
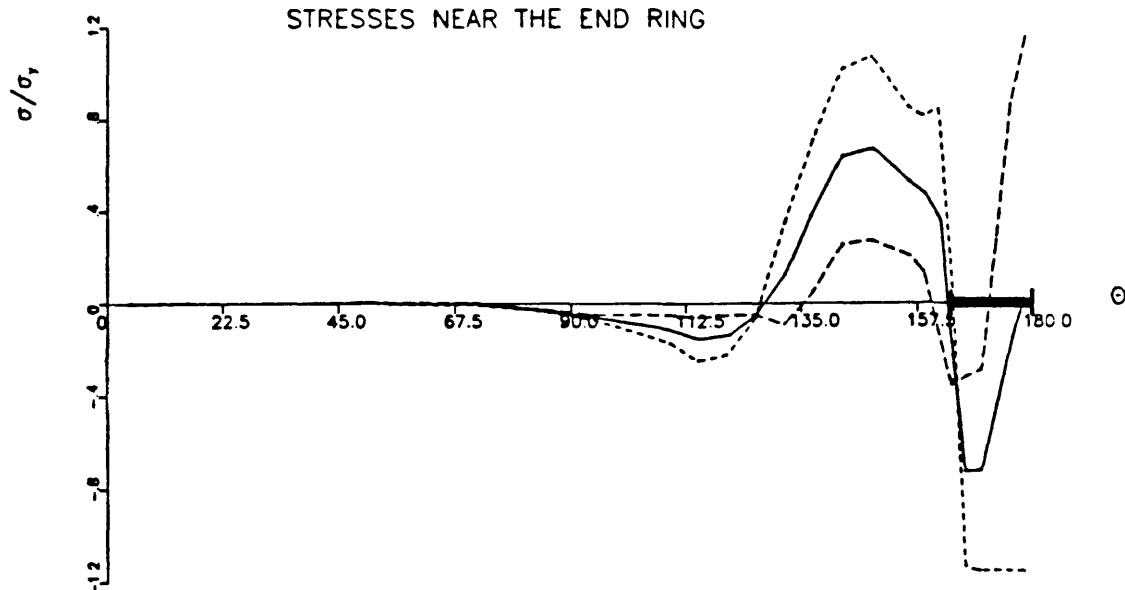
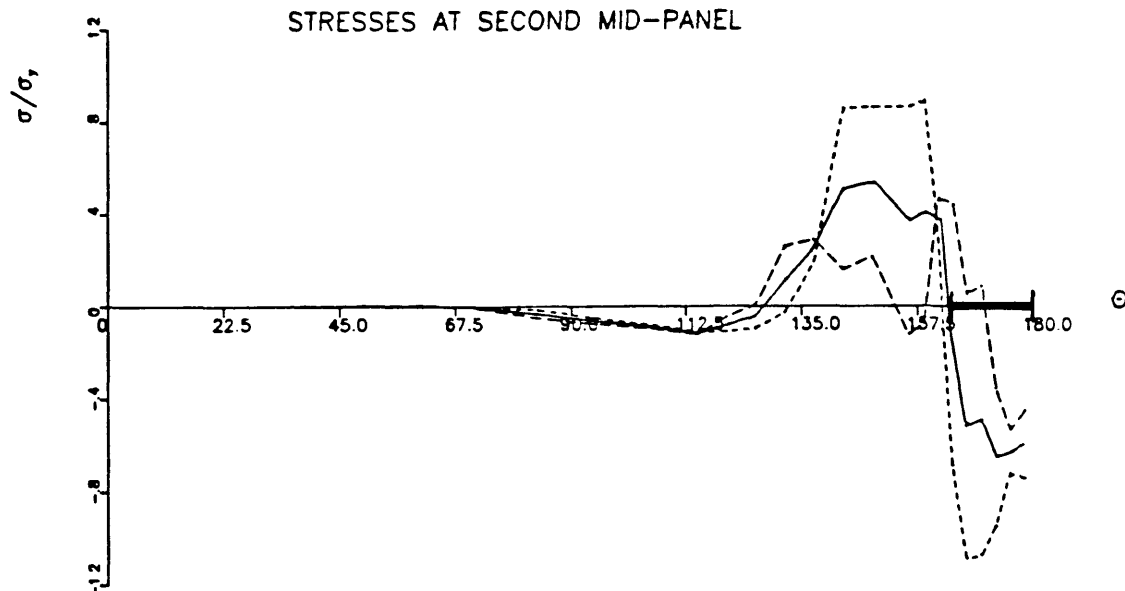


Figure 4.7

AXIAL PEAK STRESSES MODEL CY-8
FINITE ELEMENT DENTING TEST
STRESSES NEAR THE END RING



STRESSES AT SECOND MID-PANEL



STRESSES NEAR THE DENT LINE

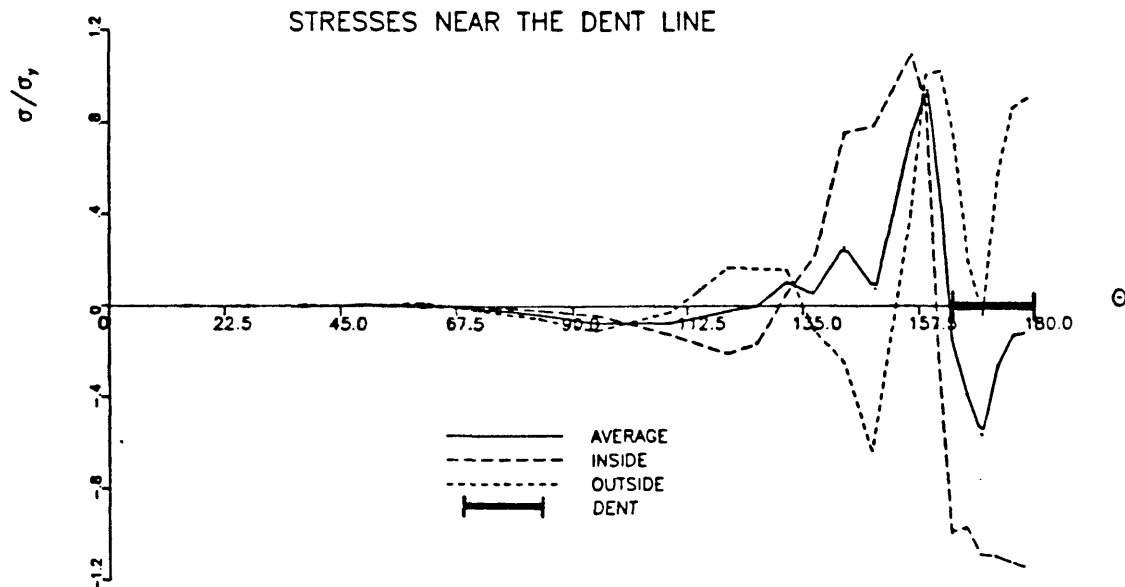


Figure 4.8

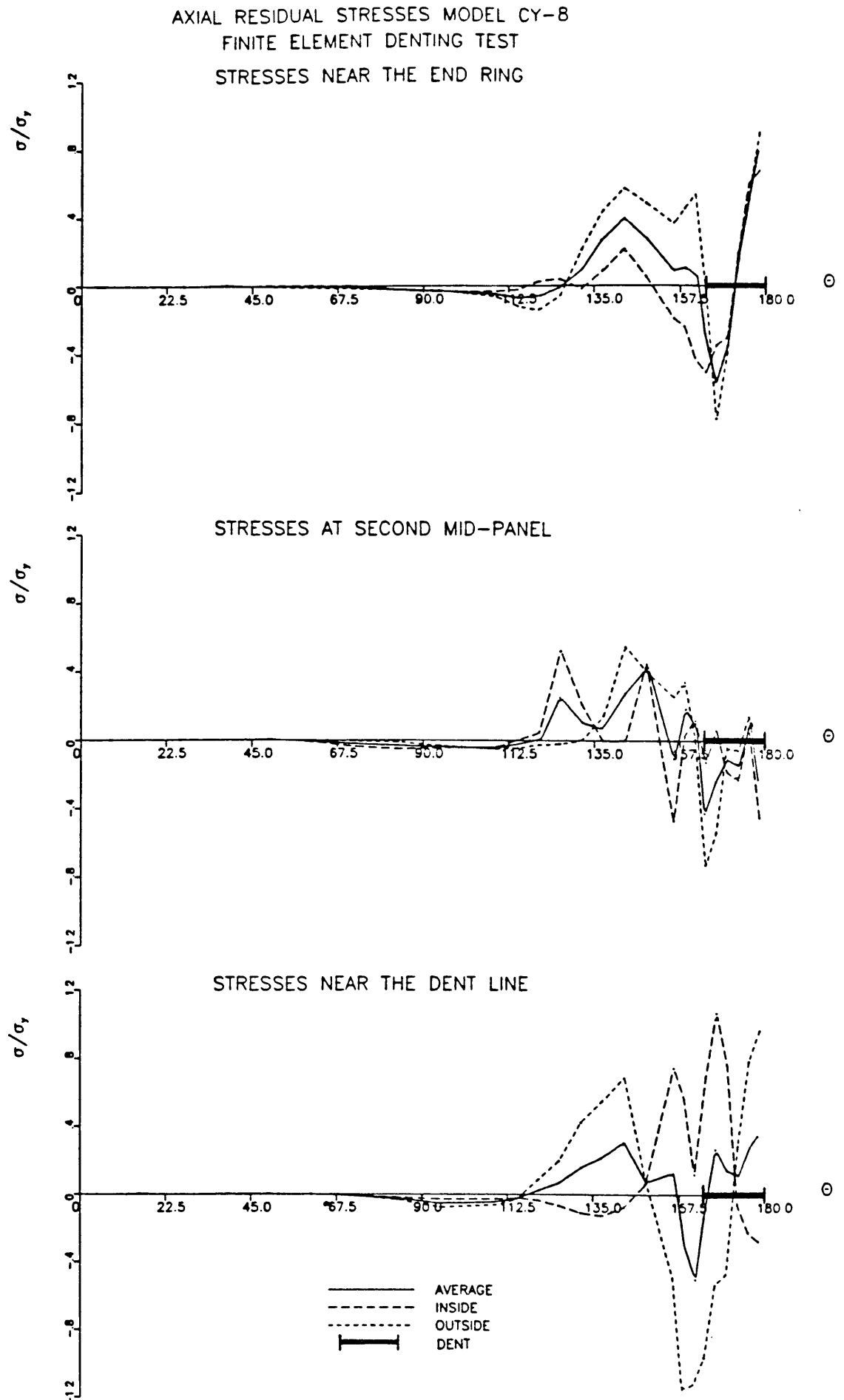


Figure 4.9

CHAPTER 5

NUMERICAL ANALYSES OF AXIAL COMPRESSION TESTS

5.1 INTRODUCTION

The aim of the work presented in this chapter was twofold. The primary objective was to develop a method capable of accurately analysing dented ring stiffened cylinders subjected to axial compressive loading. To this end, a non-linear finite element program was employed which could account for the effect of geometrical imperfections and residual stresses. The latter have been shown to have a significant effect on the strength of intact ring stiffened cylinders, as already discussed in Chapter 2. As the dent could be regarded as a large initial imperfection and is also associated with high residual stress levels, it would seem important to be able to account for these effects accurately.

The second objective of the work presented in this chapter was the following. It was important to be able to identify and quantify the strength reductions caused by the dent when the cylinders were subsequently loaded in axial compression. To this end the intact models also have to be analysed with the pre-dented imperfections so that a reliable equivalent datum could be established. By comparing directly the results of the two analyses the damage-induced effects could be understood and quantified.

The choice of a suitable model for the analysis of the dented cylinders is presented. This was based on a study of important modelling considerations, namely mesh sizes and boundary conditions. The numerical model finally chosen for representing the dented model was used for both sets of analyses so that all doubts about comparisons could be eliminated. Geometrical imperfections, measured before and after denting, were utilised to model the cylinders. Numerical results from the dented model analyses were correlated with the experimental results, thus validating the numerical

models [129].

The effect of boundary conditions is further examined by analysing all the models with three different sets of conditions.

The effect of residual stresses which had been disregarded in the rest of the analyses is examined by analysing one of the experimental models with and without the residual stresses predicted by the analysis described in the previous chapter.

Some useful observations are drawn by examining the detailed results and areas where this form of damage is likely to have a bigger effect are identified.

5.2 DEVELOPMENT OF FINITE ELEMENT MODEL REPRESENTATION

The finite element package FINAS was again employed to perform the correlations. The finite element modelling of both intact and damaged ring stiffened cylinders is discussed in this Section. Modelling the dented cylinder subjected to axial compression proved to be a very demanding exercise and much more involved than the intact model analysis. A study examining the effects of different modelling considerations was carried out before the final model choice was made. Of the above, loading and boundary conditions as well as the circumferential and longitudinal extent of the model, proved to be very significant when analysing the dented cylinder. The final choice was based on the particular requirements of the dented cylinder and the same model was adopted for the intact cylinder which was much less sensitive to the above considerations. Results of the above study and a full description of the model finally adopted are given in the next few Sections.

5.2.1 Circumferential and Longitudinal Extent of the Model

In choosing a suitable model the special needs arising from the particular case of the dented model were given due consideration. The first complication arose from the fact that the dented area could not be treated in isolation, because this would result in total loss of any possible stress redistribution capability towards the intact areas of the model. Hence there was a need to model the whole of the circumference of the model, which required an extensive mesh in this direction.

Initial trials with a full circumference model representing only the dented panel, resulted in very poor strength predictions mainly due to the strong interaction between the end conditions and the dented panel stress distribution. This highlighted the need to model the adjacent panels to avoid this effect. The finite element

model had to be extended to allow for this. This would have brought the number of elements required and the size of the problem close to practical and economic limits of large mesh computing.

With the above considerations in mind, two planes of symmetry were assumed, as shown in Figure 5.1a. These planes introduced circumferential and longitudinal symmetry about the centre of the dent. This enabled only a quarter of the cylinder to be analysed as shown in Figure 5.1b and lines AB, CD and AD were treated as symmetry lines.

5.2.2 Loading and Boundary Conditions

In applying the correct boundary conditions, difficulties arose, firstly from the fact that the exact loading and boundary conditions for the part analysed were unknown and secondly because the damaged configuration of the models proved to be particularly sensitive to this factor.

Two types of loading are usually available in finite element packages, namely stress and displacement. In the absence of any other data these would normally be applied uniformly on the ends of the cylinder. In the package used for these analyses only stress loading was available, so that was one clear option. Initial trials were performed implementing uniform stress with boundary restraints at the top line as shown in Figure 5.2. The radial deflections were restrained at the boundary as well as the longitudinal tangential rotation. This represented a clamped end condition. Results of these analyses have shown that this was a lower bound solution which underestimated the strength of the cylinder. It was therefore important to be able to apply different boundary conditions to model the tests.

A simple way was devised to produce uniform displacement at the top boundary of the model. This was achieved by introducing end rings of very high stiffness, at the top boundary of the model (e.g. E.R. in Figure 5.2). The same boundary restraints were applied here as in the uniform stress analyses. The presence of the end rings, however, forced the top boundary line to move as a rigid line in the axial direction, causing uniform axial shortening of the model when subjected to loading. An upper bound solution was obtained by assuming the latter conditions.

A modification was applied to the uniform displacement condition to obtain a closer representation of the experimental conditions. The new formulation allowed rotation of the end rings about the Y-axis, which was actually the case in the experiments where a ball seat was located between the cylinder and the cross beam. The top

boundary nodes were free in this direction (Figure 5.2) but the presence of the end rings restrained any local relative axial deflections. The two extra restraints introduced at nodes N_1 and N_2 , as shown in Figure 5.2, were required in this case to restrain the model globally in the X-direction. These conditions forced the top boundary line to deflect as a solid line while remaining free to rotate about the Y-axis. Results obtained from this analysis were generally in good agreement with the tests.

Through this study of different boundary conditions, it became clear that boundary choice has a pronounced effect on strength predictions for dented cylinders and it is essential that the true situation is represented accurately. For the purposes of the finite element correlations with the experiments the third set of boundary conditions, of uniform displacement with rotation, was employed. However, as practical conditions may relate to any of the three options and choice may be a limitation of the finite element package used, all three sets are further examined in Section 5.6.

5.2.3 Mesh Study

Three different meshes were examined with all three sets of boundary conditions. Mesh 1, which is shown in Figure 5.3a, extended from the dent line to the end of the adjacent panel. Meshes 2 and 3 included two and three adjacent panels respectively, as shown in Figures 5.3b and 5.3c.

Two models were analysed, CY-4 and CY-5, both in the damaged and intact state. Collapse strength predictions for both models are tabulated in Table 5.1 and load deflection curves of model CY-4 are plotted in Figures 5.4 and 5.5. It was clear from the results of collapse strength predictions that while the intact models were rather insensitive to mesh size and boundary conditions, the damaged models suffered large variations in strength.

The uniform stress predictions were the most sensitive to the change in the extent of the model. Collapse loads were found to increase with the larger models. This can be attributed to the relatively small stress redistribution allowed in Mesh 1. It was also clear from the results that as the number of panels increased the uniform stress results approached the uniform displacement predictions.

The uniform displacement condition was the least sensitive to model size because the load redistributed almost fully to the intact part of the shell in all cases and failure occurred when the average stress reached the collapse stress.

A larger variation in the strength predictions for the more slender model of the two (CY-5) can be observed. This reflects the higher sensitivity of slender shells to imperfections and modelling considerations. Through this mesh study it became apparent that there was little scope for assumptions and approximations regarding the longitudinal extent of the model. The full length should have been modelled for a good representation of the behaviour of the dented cylinders. This is contrary to intact model analysis, where a single panel analysis would be sufficient. The only case where only part of a dented cylinder may be analysed without great loss of accuracy is when uniform displacement boundary conditions apply.

5.2.4 Final Choice of the Finite Element Model

In the light of the study presented above Mesh 2 was chosen for the panel dented models of series 1 (Figure 5.3b). In this mesh, two shell elements were used longitudinally per panel. This was the minimum requirement for a one half wave deflected shape, which was expected for most of the geometries considered in this study. Four elements were used in the dented panel. This refinement was thought to be necessary to enable a better representation of this geometrically imperfect region, which suffered rapid variations in stress and deflections. This refinement also covered the more slender geometries which exhibited a lobular buckling tendency. Mesh refinement was also applied in the circumferential direction where thirteen elements were used in total for a half circumference. The elements had a smaller aspect ratio in the area of the dent as shown in Figure 5.3b.

Mesh 4 was used for analysing the ring dented models of series 2. This is a two-panel mesh shown in Figure 5.6. This mesh followed the same principles as mesh 2, i.e. the same symmetry lines through the dent and the same mesh refinement scheme. Models that included dents near the rings were analysed using a different mesh that included an additional panel below the dent line AD. This extension was required since the dent line was no longer a symmetry line.

The boundary conditions implemented at the loaded end of the finite element model were those of uniform displacement with rotation. Of the three sets of boundary conditions examined above, this set modelled the experiments most closely.

The finite element models described here were employed in analysing both the intact and dented configurations of all the experimental models.

5.3 INITIAL IMPERFECTIONS

In analysing the intact and dented models under axial compression loading it was important to model their geometry accurately. For this purpose the experimental initial imperfection measurements taken on the cylinders before and after denting were used. These readings, as explained previously in section 3.2.6, were analysed by a computer program using a least squares fit to determine deviations from the best fit cylinder. These deviations were then expressed as Fourier series coefficients. The latter were used in conjunction with the best fit radius to generate the nodal coordinates for the finite element models. Examples of imperfection states before and after denting are shown in Figures 3.26 and 3.27 and they refer to model CY-2.

5.4 AXIAL COMPRESSION ANALYSIS OF INTACT MODELS

Analysis of all the intact models was performed. These analyses provided a base strength for estimating the damage/induced strength reductions. Furthermore, information on the overall behaviour of the intact model was obtained which enabled a better understanding of the dent effect on the cylinder.

5.4.1 Collapse Load Predictions

Strength predictions of the intact models are shown in Table 5.2. The results show the normal strength-slenderness (R/t) reduction.

It is also worth noting that stocky models with nominally the same geometries, e.g. CY-4, 6 and 7, failed at load levels which were very close. This is not the case, however, for models with more slender geometries, like CY-1, 8 and 9. A larger variation can be seen in their strengths, which can be explained in terms of their higher imperfection sensitivity. In addition, the outward imperfection mode which was prominent in all the models coincided with the critical mode of the latter geometry, thus having a significant and variable effect on the strength, depending on the mode amplitude.

The load-axial deflection responses of all the models are plotted in Figures 5.7-5.15 alongside the numerical and experimental results for the dented shells. These are discussed in section 5.5.1.

5.4.2 Failure Modes

The failure modes predicted by the finite element analyses are discussed below.

Model CY-1 failed in an axisymmetric outward bulge. This model had a very narrow ring spacing of $\ell/R=0.15$ which made the relative expansion of the panel between rings the controlling feature. Figure 5.16a shows this mode developing uniformly in all three panels modelled.

Models CY-2 and CY-3 (Figures 5.17a and 5.18a) also exhibited collapse modes in the form of outward bulges. In these models the mode was not as well defined and uniform as in CY-1 and this was attributed to the wider ring spacing.

Analysis of model CY-4 predicted lobular deflections in the middle panel (Figure 5.19a) but a pronounced outward bulge was detected in the top panel, which possibly triggered collapse. In a cylinder of this geometry the critical mode is a lobular mode. In the finite element meshes used, however, the refinement in the top panel was not sufficient to accommodate a mode of this form, and hence an axisymmetric mode developed. In addition, the presence of the end ring and the clamped end restraints applied a restraining effect on the deflections. This induced high curvatures and triggered collapse. This end effect has been observed before in ring stiffened cylinder tests [71].

A lobular form of failure was predicted for the most slender cylinder of the series, model CY-5, which is shown in Figure 5.20a. In the top panel some outward deflections occurred but these were not as large as in model CY-4.

The collapse modes of models CY-6 and CY-7 are shown in Figures 5.21a and 5.22a. Both models show a lobular tendency combined with some outward bulging.

Axisymmetric outward bulging was predicted for the narrow ring spacing models CY-8 and CY-9. These modes of collapse are shown in Figures 5.23a and 5.24a. It is worth noting that in model CY-9 the lower panel deflected inwards in some areas. This may be attributed to the presence of some initial imperfections in this form which could also be responsible for the higher collapse load obtained for this model compared to model CY-8.

5.5 AXIAL COMPRESSION ANALYSIS OF DENTED CYLINDERS

Analyses of all the dented experimental models are presented below. Results of the analyses are correlated in detail with the experiments and compared with the corresponding intact model behaviour. The finite element model described in section 5.2.3 was employed in the analyses.

5.5.1 Load Deflection Responses

The collapse load predictions are shown in Table 5.2, together with the test results. It can be noted that the finite element predictions were, in general, higher than the corresponding experimental values, with the exception of one model, CY-2. The strength overestimate was of the order of 6 percent on average but in one case this was up to 19 percent. Several factors contributed to this loss of accuracy in the strength predictions and these are discussed below.

It has been shown that boundary conditions are extremely important in determining the response of damaged shells under axial compression. The idealised conditions assumed in the analyses, which included end rings of very high stiffness rigidly fixed at the ends of the model, may have introduced a restraining effect resulting in higher strength predictions.

The assumption of symmetry lines through the centre of the dent, which was necessary to reduce the problem to a manageable size, reduced the accuracy of the overall initial imperfection representation in the finite element model. Consequently this had some effect on the strength predictions.

The mesh refinement is another consideration which may have affected the results of the analyses. It is possible that for certain models the mesh was not fine enough to accommodate the critical mode of failure. This may have been the case for the slender model CY-5 for which the highest strength overestimate was obtained.

Finally, the fact that residual stresses were not included in the analyses may be partly responsible for the high strength predictions. As already shown in Chapter 4, the residual stress distribution is characterised by a significant level of compressive stress in the critical area outside the dent. Although the residual stresses predicted by the analysis were up to 40 percent of yield, their effect on the strength is not expected to be that high. This is because the effect of residual stresses is not directly additive to that of the large initial deformations which have already been accounted for in the analysis. The effect of residual stresses on the strength is specifically examined in

Section 5.7.

Load deflection plots for all the models are shown in Figures 5.7 – 5.15. These plots feature the intact and dented model analyses together with the experimental dented model curve. Reasonable correlation could be seen between the two curves referring to the dented model. In some cases (models CY-1, 3, 4 and 5) the initial slope of the experimental curve was different from the rest of the curve. The slope exhibited at higher loads was generally in good agreement with the finite element slope. This initial discrepancy in the slope of the experimental curve could be attributed to the initial "bedding in" of all the loaded faces in the experimental setup.

Comparisons between the two finite element curves show an initial loss of stiffness in the dented model which is introduced by the weak area of the dent. In addition, the slope of the dented model curve continues to drop slightly as the load increases. This response reflects the geometrical and material non-linearity introduced by the dent.

5.5.2 Stress Distributions

The axial stress distributions obtained by the finite element analyses show the same trends as those seen in the experimental results. Sample plots of four of the models analysed (models CY-2, CY-4, CY-5 and CY-6) are presented in Figures 5.24– 5.29, which show good agreement between experimental and numerical results. In these plots the axial stresses at peak load are presented.

In Figure 5.26 of model CY-4 for example very little stress can be seen in the dent area, accompanied by a high peak in stress near the edges of the dent. This very low axial stress through the dent is due to the high bending that takes place in this area. Figure 5.29, which presents the stresses on the outside and inside surfaces of the shell shows very clearly that high bending stresses up to yield develop in the dent area. The point at 160° where the inside and outside stresses have the same compressive magnitude corresponds to the peak observed in the axial stress plot (Figure 5.26). This point coincides with a straight generator, between inward and outward deflections. Away from this point the average axial stress reduces significantly. As shown in Figure 5.29, large bending stresses develop associated with the area of outward imperfection. Further away from the dent the mean stress increases to a second peak (at around 130°) which gradually reduces diametrically opposite the dent. Some inward bending can also be seen around part of the circumference (between 30° – 130°). This behaviour reflects the imperfection shape

as well as the overall load eccentricity introduced by the weak area of the dent.

Of the two areas of peak stresses observed in Figure 5.26 the first sharp peak (160°), although higher than the second one, does not appear to be responsible for failure. Instead failure is triggered when the second area of high stresses (at around 130°) reaches a level of stress close to the equivalent intact model failure stress. The exact level and location of these peak stresses appears to be an interaction between several effects such as stress capacity reduction in the imperfect area outside the dent. Furthermore, dent form interaction with critical failure modes and initial imperfections may also have an effect on the peak collapse stresses. The latter are examined later in this dissertation.

5.5.3 Failure Modes

The main features of the failure modes observed in the experiments were successfully reproduced by the numerical analyses. Dented models failed with the dent deflecting inwards and the adjacent area buckling outwards with the rest of the model assuming, in most cases, the expected collapse mode of the equivalent intact model. This can be appreciated by observing Figures 5.16–5.24 where three dimensional plots from the finite element analyses are presented with the corresponding intact modes. It should be remembered that the part of the cylinder shown in these figures represents the top left quarter of the dented cylinder with two assumed planes of symmetry.

In Figure 5.16b the failure mode of model CY-1 can be seen. The failure mode outside the dent coincides with the intact mode of failure (Figure 5.16a). The overall mode of collapse is in good agreement with the test observations, (Figure 3.52).

In model CY-2 where the dent depth was bigger, increased curvatures were observed inside and outside the dent (Figure 5.17b). As well as exaggerated inward and outward deflections in the immediate area of the dent there is a large area of inward deflection before the shell assumes an outward bulge mode. This mode agrees in general with the experimentally obtained collapse mode (Figure 3.56). In the latter, however, the outward bulge was continuous on one side of the dent and, in addition, some stiffener tripping occurred on the other side. This was not predicted in the analysis where stiffener imperfections were not included.

In model CY-3 (Figure 5.18b) a lobular form of buckling occurred. This agrees with the experimental mode of collapse (Figure 3.60). The intact model analysis, however, showed the model failing in an axisymmetric outward bulge (Figure 5.18a).

It has been suggested in the previous chapter that the large magnitude of the dent, which was non-sympathetic to the critical mode, prevented the latter from developing. Instead, a lobular mode was enhanced which was more compatible with the deflections of the dent and the adjacent areas.

A periodic mode of collapse can be seen in the centre panel of model CY-4 (Figure 5.19a) with buckles emanating from the dent region. The lobular form of failure predicted by the analyses agreed with the experiment, except that in the latter the buckles developed in the end panel (starting from a circumferential position corresponding to the critical area adjacent to the dent, Figure 3.64). In the finite element analysis the finer mesh in the centre panel may have allowed collapse to occur in this region with the area of coarser mesh preventing a closer correlation with the experiment.

Model CY-5, with the most slender geometry of all the models, failed in a periodic mode (Figure 5.20a). Buckles were again concentrated around the centre panel, starting from the dent. In the experiment, although a lobular mode was observed, this spread at an angle into the adjacent panels (Figure 3.68). A chain of buckles was observed on one side of the dent, starting in the adjacent panel and spreading towards the end panel. Failure was initiated at a circumferential location corresponding to the critical area adjacent to the dent, where higher stress concentrations occurred. In the finite element analysis the finer mesh in the centre panel and possibly the symmetry line assumed through the centre of the dent caused central panel failure as for model CY-4.

Figure 5.21b shows the failure mode for the ring dented model CY-6. The lobular mode of failure observed in the test (Figure 3.106) is also demonstrated by the analysis.

In Figure 5.22b model CY-7 is portrayed. This model was subjected to a dent near the ring. The dent line is marked in the figure and it can be observed that a series of small buckles developed involving the ring close to the dent and the adjacent panel. This agrees well with the experimental mode (Figure 3.110).

A good prediction was obtained for the failure mode of model CY-8 (Figure 5.22b). The dented ring deflected inward at the dent with an outward bulge developing outside it. In the rest of the circumference the panel above the dented ring failed in an axisymmetric outward bulge. A similar mode was obtained in the experiment, where on the one side of the dent the top panel failed and on the other side the panel below the dent failed (Figure 3.114).

Figure 3.24b shows the failure mode of the near ring dented model CY-9. The failure mode obtained by the analysis was very similar to the ring dented model CY-8, and agreed well with the experimental mode of collapse (Figure 3.118).

5.6 ANALYSES INCLUDING DIFFERENT BOUNDARY CONDITIONS

The effect of different boundary conditions is examined further by analysing all the experimental models assuming uniform stress and uniform displacement loading. Results of these analyses are compared with the previous analyses including uniform displacement with rotation and the effect of different boundary conditions on the failure mechanism and strength predictions is discussed below.

5.6.1 Axial Shortening

Under uniform stress, differential axial shortening occurs. In the dent area there are high longitudinal deflections which die out very quickly outside the dent and become uniform for the rest of the circumference (Figure 5.30a).

The uniform displacement condition prevents this relative movement along the boundary, which moves as a rigid body in the axial direction. Figure 5.30b shows this deflection.

The third boundary condition allows some rotation of the end ring to occur but in other respects is similar to the second condition. In Figure 5.30c the half cylinder deflections are plotted. The sinusoidal curve results from opening of the cylindrical form. In reality the top boundary line moves downwards and rotates as a solid body.

5.6.2 Collapse Load

The following general trend is observed in the results. The highest strength is obtained with uniform displacement while uniform stress produces the weakest result because of lack of redistribution potential. The third condition of line displacement with rotation gives an estimate between the other two but closer to the uniform displacement value (Table 5.3). The difference in the strength estimates is substantial. The three strength estimates for model CY-4 for example, are .47, .88 and .79 with a 46 percent difference between the two extreme values. It is interesting to compare corresponding values for the intact models. These are .96, .97 and .96 with a maximum difference of 1 percent. This highlights the increased sensitivity of the dented models to the boundary restraint. The explanation lies with

the degree of stress redistribution allowed in each case. A closer look at the stress distributions and failure modes shows how the model behaves in each case.

5.6.3 Stress Distribution and Failure Modes

A similar stress distribution occurs in the zone of the dent for all the boundary conditions. Figures 5.31 and 5.32 show the axial stress distributions for models CY-3 and CY-4.

An essentially uniform stress distribution could be seen in the intact area away from the dent when uniform stress was applied. The average axial stress around the intact part of the model at peak load was low and nowhere near the equivalent intact strength. In model CY-4 (Figure 5.32) for example, an average axial stress of .47 of yield was attained compared to the intact strength of .96. The latter suggests that little support was provided to the weak zone of the dent and the adjoining highly imperfect area. This was attributed to the very low stress redistribution possible in this case. The dent and adjacent area suffered severe bending which eventually lead to collapse of the model locally with the dent deflecting inwards and the adjacent zone buckling outwards. The failure mode did not extend to the intact area around the shell. An example from model CY-5 can be seen in Figure 5.33.

In the uniform displacement condition the stress away from the dent reached a uniformly high value before collapse occurred. Significant stress redistribution took place prior to collapse. Stress shed uniformly from the weak zones to the intact part of the model. Failure occurred only when the average stress in the intact part reached the equivalent intact model collapse stress. Figure 3.32 (model CY-4) supports this description of behaviour. The average value of stress in the intact part of the shell at peak load is in fact very close to .96 which is the equivalent intact model strength. The same is true for model CY-3 in Figure 5.31. It is worth noting here that in model CY-4 although the intact collapse stress was exceeded at the first sharp peak, which corresponded to the edge of the dent, this did not trigger collapse. Instead failure occurred when the rest of the circumference reached the undented failure stress indicating that the first local peak in stress is not responsible for collapse. The collapse mode associated with this analysis is an overall failure mode. This is a combination of the dent failing inwards, the adjacent area buckling outwards and the rest of the circumference failing in the intact model failure mode. A plot of model CY-5 in Figure 5.33 illustrates the latter.

When the third set of boundary conditions is applied a slope can be seen in the overall stress distribution. A higher value is observed near the dent which reduced

towards the other side (Figures 5.31 and 5.32). This distribution, as discussed above, is a reflection of the load eccentricity effect of the dent on the cylinder. The peak stress location which occurs after the first sharp peak outside the dent becomes the critical zone and failure occurs when this stress is close to the equivalent intact model collapse stress. Failure is in an overall mode as for the uniform displacement case (Figure 5.33).

5.6.4 Discussion of Results

The uniform stress condition, while it can give very satisfactory predictions for intact shells, has been shown to give very poor strength estimates for dented shells. The latter is attributed to the fact that this condition allows no stress redistribution and the model fails locally in the area of the dent.

The other two conditions are closer to what would be found in practice where the shells are part of a continuous structure which will provide additional axial restraint. Of these two conditions the uniform displacement provides more stress redistribution capability and therefore a lesser effect from the dent.

These observations should also apply to finite element analysis of other thin walled structures with local defects.

5.7 EFFECT OF RESIDUAL STRESSES ON DENTED CYLINDERS

A dented shell is characterised by both initial imperfections and residual stresses. So far only initial imperfections were included in the analyses. This was thought to be a reasonable approximation as the effect of the two on the strength is not directly additive [131,29], and the deformations alone would in general provide a sufficient representation of the response. In this section the implications of this assumption are examined by performing sample analyses with and without residual stresses.

Results from the numerical analyses of the denting process provide input data for these analyses. However, due to the upper and lower bound form of the denting analysis, results do not compare directly to the experimental model. Nevertheless, they do provide a feel for the relative effect that residual stresses have on the strength.

The residual state obtained from the upper bound denting analysis of model CY-8 provided the stress and deformation data required for this study. Two analyses have been performed with boundary conditions of uniform displacement with rotation. The

first of these analyses included only residual deformations, while the second one modelled the combined effect of residual deformations and stresses. A limitation of the program was that residual stresses could only be input as average stresses through the thickness of the shell which meant that the bending effect could not be modelled.

A difference of 4 percent of squash was attained between collapse strength predictions, with the initially stressed model exhibiting the lower strength. Both load-deflection paths showed rounded peaks (Figure 5.34). While the initial stiffness was similar the initially stressed shell suffered earlier loss of stiffness. After the first peak the slope of the latter approached the stress-free analysis path.

Average axial stress distributions around the model at peak loads are shown in Figure 5.35. The initial stress distribution is also shown. Three areas can be identified where the distribution has been affected by the inclusion of initial stresses. A reduction in stress could be seen in zone "A" due to the initial tensile stresses. In zone "B" an increase in the compressive stresses can be observed which results from the initial compressive stresses in the area. The overall distribution has a sharper slope (zone "C") in the initially-stressed analysis which reflects a higher load eccentricity caused by the combined effect of residual displacements and stresses. The most significant effect of the residual stresses on the collapse strength appeared to be the increase in compressive stresses in the critical area "B", which results in earlier yielding on the surface in compression. Hence, the earlier loss of stiffness and lower collapse load.

The 4 percent difference in the strength predictions obtained, is much smaller than the actual magnitude of the initial compressive stresses in the area. The limited information obtained from these analyses suggests that the net effect of the residual stresses on the strength may be small, but clearly more work is necessary to confirm this.

5.8 DAMAGE-INDUCED STRENGTH REDUCTIONS

Experimental collapse strengths of the dented models have been compared with the corresponding intact strengths which were provided by the finite element analysis. The strength reductions (Table 5.2) are expressed as a percentage of the intact strengths.

Figure 5.36, presents the strength reductions for the midpanel models of series 1. The strength reduction in the model with the narrow ring spacing, CY-1, is 10 percent higher than that of CY-2 ($R/t = 267$). The same trend is observed with

models CY-3 and CY-4. Again the model with the smaller ring spacing suffers a 3 percent larger strength knockdown, despite having a 20 percent smaller dent. This may suggest an increased sensitivity towards denting as the ring spacing reduces.

Another observation is that the failure modes are more concentrated around the dented panel in cylinders with low ℓ/R values. Models with widely spaced stiffeners failed in modes which spread more into the adjacent panels. This observation supports the above suggestion that models with small ℓ/R may be more sensitive to denting. It is possible that for a cylinder with low ℓ/R the dent area yields more in bending due to the presence of higher curvatures.

A second trend in the results can be identified by comparing models CY-4 and CY-5 (larger R/t). The latter suffers a bigger knockdown in strength even though it contains a smaller dent. This may again be attributed to the more contained form of dent, i.e. smaller deflections extending across the stiffener lines. This time this is due to the low lateral stiffness of the shell in relation to the rings. For shells with a wide ring spacing, like model CY-5, the dent is in sympathy with the lobular form of buckling expected of an intact cylinder of the same geometry. This means that the dent could have a more significant effect on the strength. In CY-4 the dent spreads more into adjacent panels and therefore the tendency of a lobular mode is reduced.

The above observations suggest that there may be an interaction between the ℓ/R and R/t parameters, with certain regions of high slenderness (R/t) and large values of ℓ/R causing more strength reduction due to the dent coinciding with the critical lobular form of failure. The latter is examined further in Chapter 6. The sensitivity of a shell like CY-5 to denting, if seen in association with its low lateral stiffness make it particularly vulnerable to damage.

Strength reductions obtained from the series 2 models are shown in Figure 5.37. Two panel dented models CY-1 and CY-4 of similar geometries are also included for comparison purposes. Among the more slender shells CY-8, CY-9 and CY-1, the ring dented model CY-8 showed a 2 percent bigger strength reduction. Model CY-1, although of similar geometry, included a much smaller dent, hence it is not directly comparable with the rest. The less slender shells, CY-6, CY-7 and CY-4, exhibited a 5 percent bigger reduction in strength from a near ring dent which is different from the trend observed above.

The failure modes associated with different dent locations were similar and corresponded essentially with panel buckling. Furthermore, shells with bigger dents

suffered more reduction in strength. Hence it would appear from these results that the effect of the dent location is not a significant factor in the determination of the residual strength of the shell.

5.9 DISCUSSION AND CONCLUSIONS

The finite element method has been applied in analysing the behaviour of ring stiffened cylinders including local damage in the form of a dent. The method produces good qualitative representation of the response. The accuracy of the strength predictions, however, was reduced due to several factors. Firstly, the sensitivity of the models to the assumed idealised boundary conditions affected the results. Secondly, limitations arising from the available computer power which necessitated the assumption of symmetry lines through the dent and prohibited extensive mesh refinement, also affected the strength predictions. Finally, the omission of residual stresses from the analyses had some effect on the results.

Numerical studies confirm the experimental findings regarding the main effect of the dent on the strength. The latter creates an area of weakness that results in an overall load eccentricity with high stresses in an area outside the dent. Analyses have shown that this becomes the critical area where collapse is triggered when the stress level is close to the equivalent intact model collapse stress. The exact level and location of these peak stresses appears to be affected by several factors, such as the stress capacity reduction in the imperfect area outside the dent and possibly by dent-form interactions with critical failure modes and initial imperfections.

The best representation of the experimental boundary conditions was achieved by applying uniform displacement with end rotation. Of the other two boundary conditions examined, uniform stress was found to be unsuitable for analysing dented shells as it does not allow any stress redistribution and it results in very low strength estimates. The other condition examined, that of uniform displacement, allows high stress redistribution towards the intact part of the shell and therefore results in much higher strength estimates. The latter condition and that of the uniform displacement with rotation are also closer to what would be found in practice where shells are continuous and provide axial restraint.

Sample analyses of one of the experimental models with and without residual stresses have shown a difference of only 4 percent of squash between the collapse strength predictions, with the initially stressed model exhibiting the lower value. This strength reduction is much lower than the magnitude of the compressive residual stresses present in the critical area outside the dent. The limited information obtained from

these analyses suggests that the net effect of the residual stresses on the strength may be small.

Certain trends have been identified in the dent-induced strength reductions. Models with smaller ring spacing suffered bigger strength reductions which was attributed to the higher curvatures associated with the dent formation resulting in higher yielding in the dent area. Larger strength reductions were also observed in cylinders with bigger R/t values, owing to the more localised form of the dent due to the low lateral stiffness of the panel in relation to the rings. For shells with wider ring spacing, for which a lobular buckling failure mode occurs, the dent may have a direct effect in triggering this mode prematurely. There are likely to be changes in behaviour as dent shape and buckling form interact.

It is also concluded that the location of the dent in relation to the rings does not have a significant effect on the strength or the failure mode of the cylinders.

The study presented in this chapter demonstrates the increasing complications arising from the analyses of dented shells as opposed to intact shells, using the nonlinear finite element method of analysis. Results of the analyses performed highlight some limitations of the method and provide guidance to its use in analysing dented shells.

Table 5.1: Mesh and Boundary Conditions Study

Model	Boundary Conditions	I/D	Mesh 1 σ_d/σ_y	Mesh 2 σ_d/σ_y	Mesh 3 σ_d/σ_y	Test σ_d/σ_y
CY-4	US	D	.43	.47	.53	.78
	UD	D	.88	.88	.87	
	UDR	D	.80	.79	.76	
	US	I	.96	.96	.96	
	UD	I	.96	.97	.96	
	UDR	I	.96	.96	.97	
CY-5	US	D	.39	.36	.48	.59
	UD	D	.80	.79	.75	
	UDR	D	.75	.70	.68	
	US	I	.84	.89	.84	
	UD	I	.89	.87	.85	
	UDR	I	.87	.88	.86	

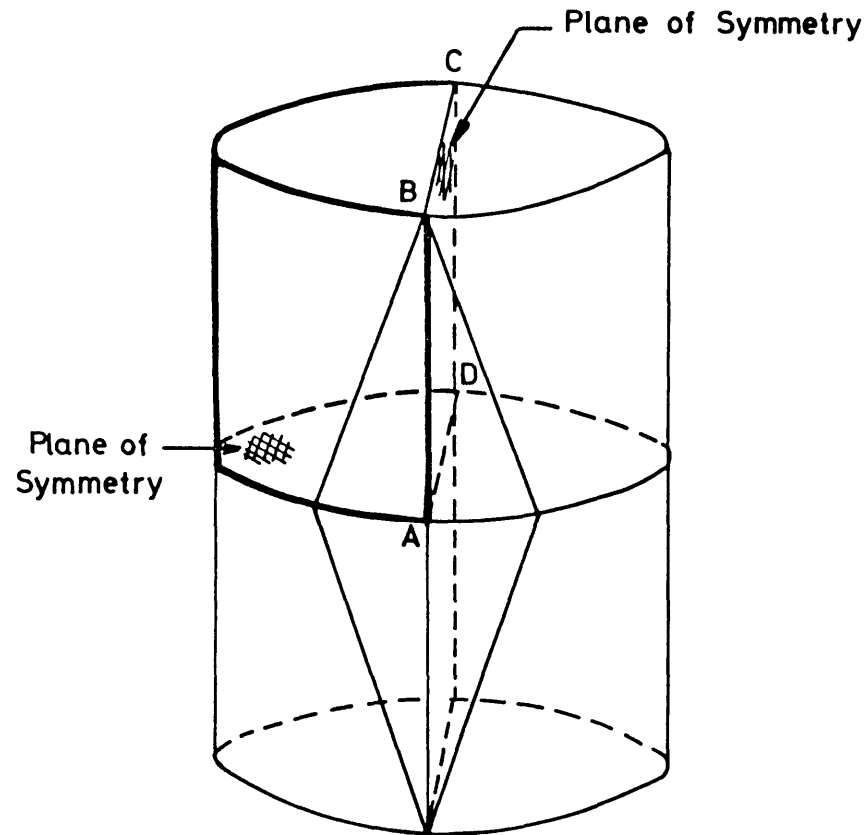
Table 5.2: Collapse Strengths and Damage Induced Strength Reductions

Model	F E σ_I/σ_y	F E σ_d/σ_y	Test σ_d/σ_y	Dent Depts d/D	% Strength Reduction
CY-1	.65	.58	.58	.0107	11
CY-2	.74	.62	.62	.0103	1
CY-3	.96	.81	.75	.0170	22
CY-4	.96	.79	.78	.0210	19
CY-5	.88	.70	.59	.0170	33
CY-6	.92	.77	.74	.0218	19
CY-7	.92	.73	.70	.0233	24
CY-8	.78	.59	.58	.0197	26
CY-9	.78	.68	.59	.0199	24

Table 5.3: Study of Boundary Conditions

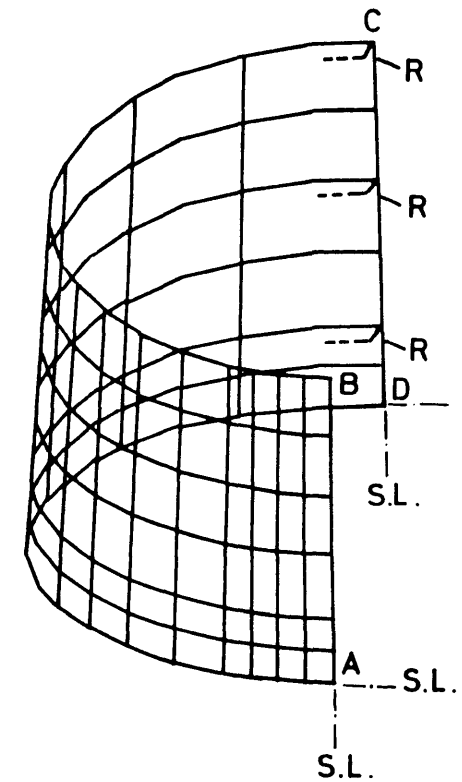
Model	Boundary Conditions	FE σ_i/σ_y	FE σ_d/σ_y	Test σ_d/σ_y
CY-1	US	.62	.51	.58
	UD	.64	.65	
	UDR	.65	.58	
CY-2	US	.76	.30	.73
	UD	.79	.78	
	UDR	.74	.62	
CY-3	US	.95	.49	.75
	UD	.97	.90	
	UDR	.96	.81	
CY-4	US	.96	.97	.78
	UD	.97	.88	
	UDR	.96	.79	
CY-5	US	.89	.36	.59
	UD	.87	.79	
	UDR	.88	.70	
CY-6	US	.91	.51	.74
	UD	.93	.86	
	UDR	.92	.77	
CY-7	US	.90	.47	.70
	UD	.92	.83	
	UDR	.92	.73	
CY-8	US	.68	.34	.58
	UD	.78	.63	
	UDR	.78	.59	
CY-9	US	.76	.40	.59
	UD	.84	.70	
	UDR	.88	.68	

FINITE ELEMENT MODEL FOR AXIAL COMPRESSION ANALYSIS



A = Centre of dent
 ABCD = Quarter analysed by F.E.

(a)



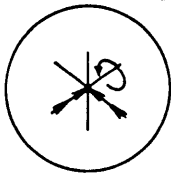
R = Ring
 S.L. = Symmetry Line

(b)

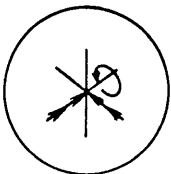
Figure 5.1

BOUNDARY CONDITIONS ON FINITE ELEMENT MODEL

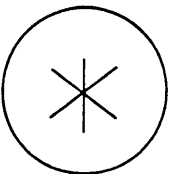
Uniform Stress
no end rings



Uniform Displacement
with end rings of high stiffness



Uniform Disp. + Rotation
with end rings of high stiffness



Key

S.L. = Symmetry Line
E.R. = End Rings
R = Ring

arrows represent restrained
degrees of freedom
along model boundaries

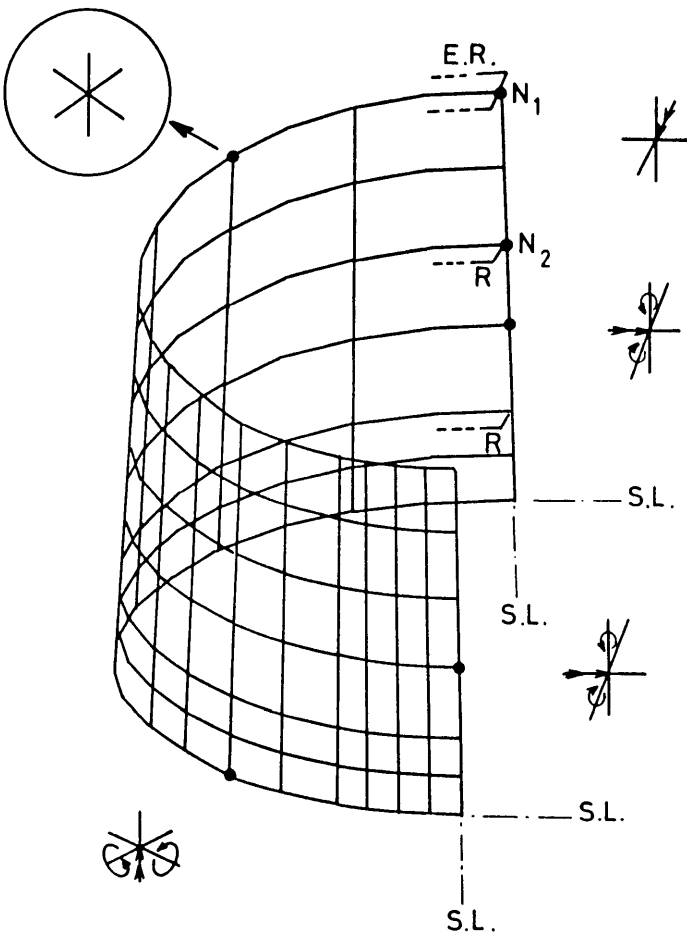


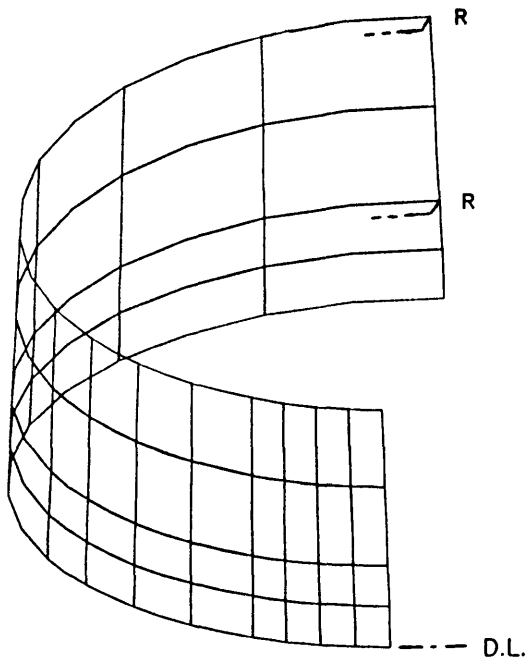
Figure 5.2

FINITE ELEMENT MESH STUDY

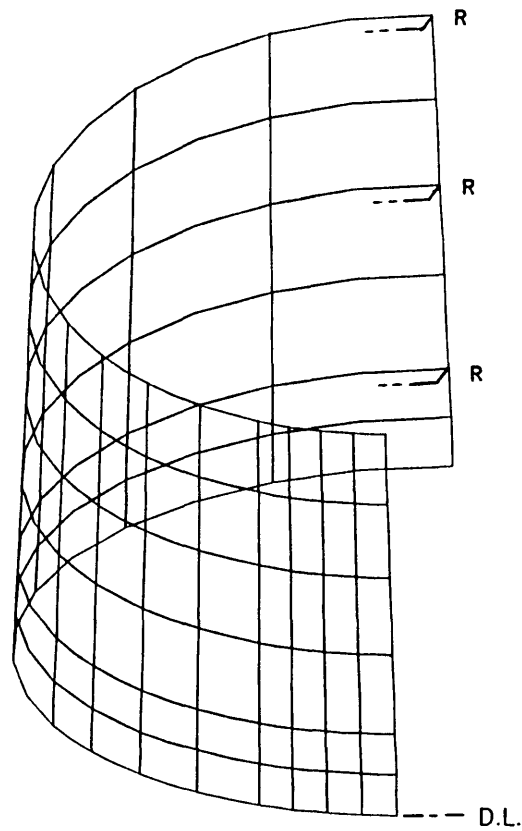
KEY

R = RING

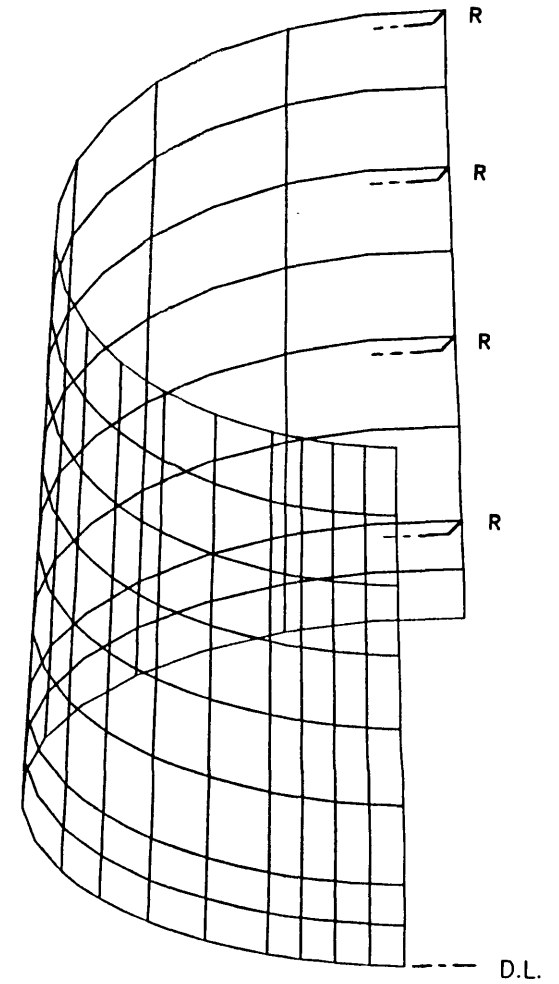
D.L. = DENT LINE



(a) MESH 1



(b) MESH 2



(c) MESH 3

Figure 5.3

LOAD DEFLECTION CURVES FOR DENTED CY-4

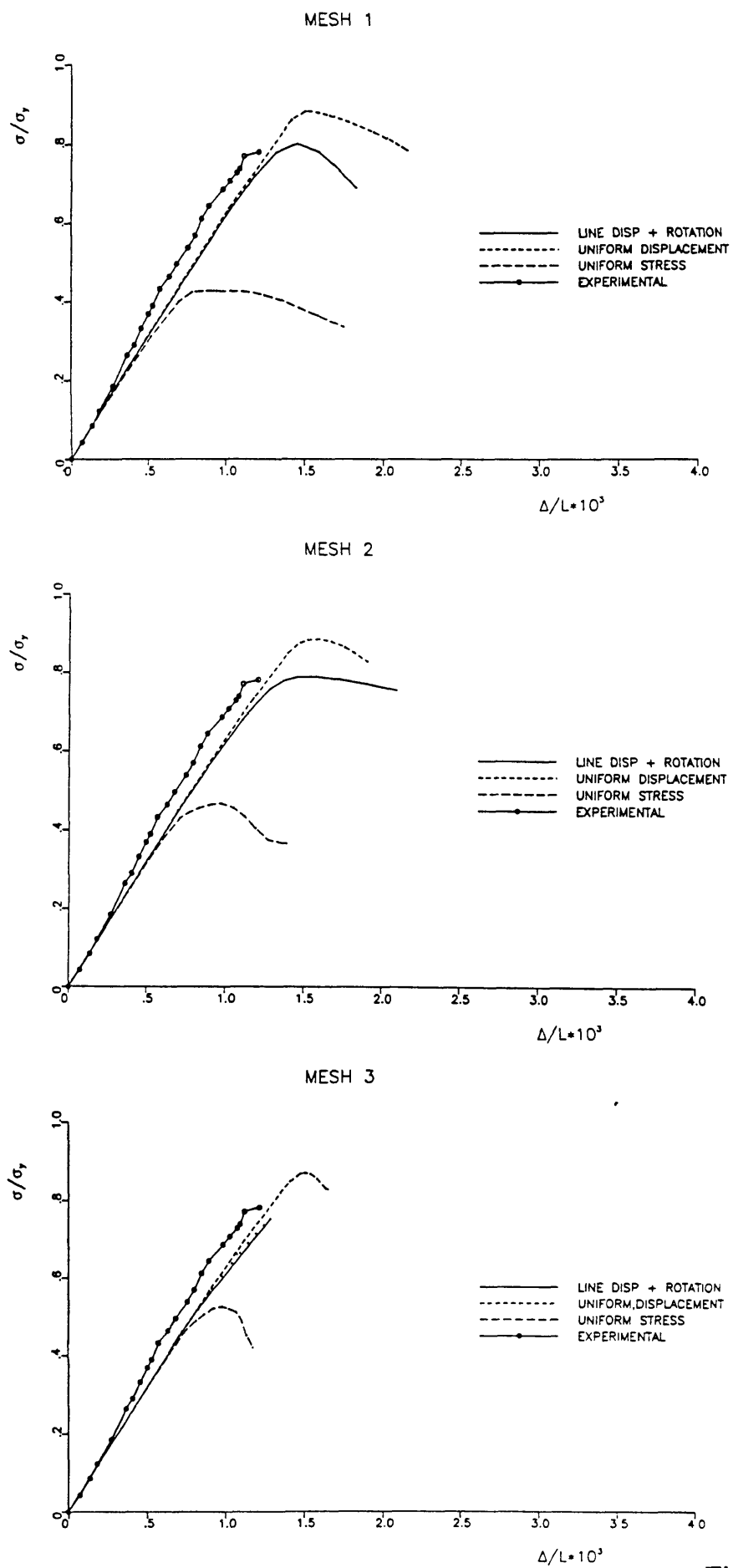
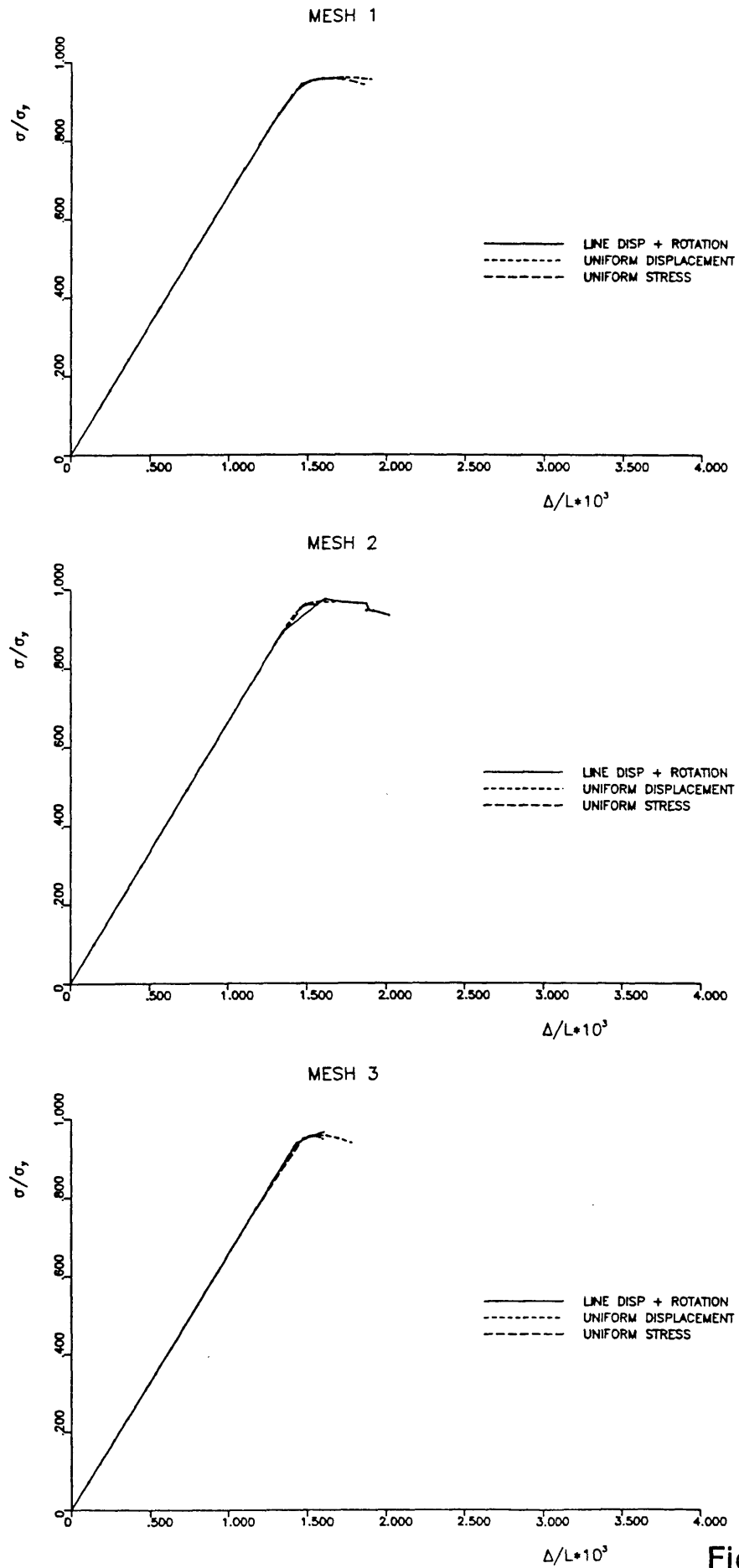


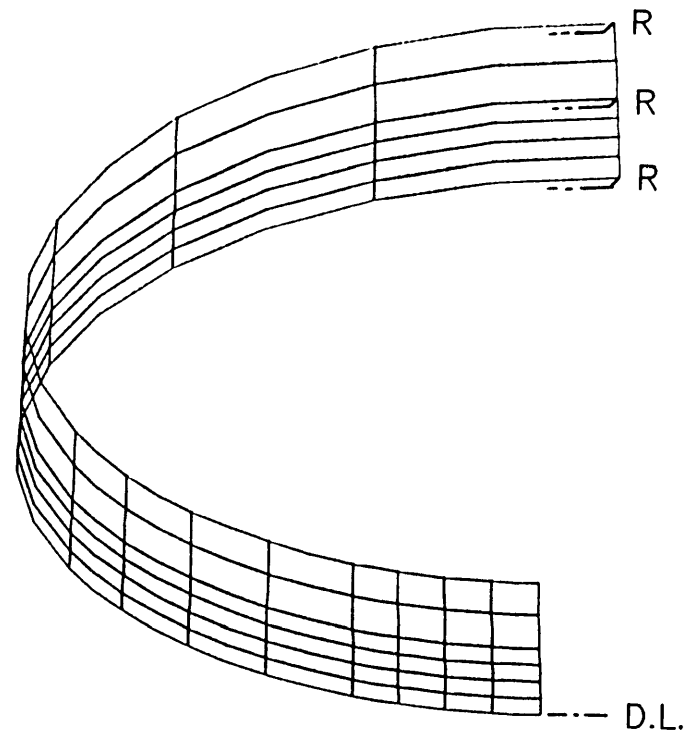
Figure 5.4

LOAD DEFLECTION CURVES FOR INTACT CY-4



FINITE ELEMENT MESH FOR RING DENTED MODELS

MESH 4



KEY

R = RING

D.L. = DENT LINE

Figure 5.6

AXIAL LOAD DEFLECTION CURVES CY-1
AXIAL COMPRESSION TEST

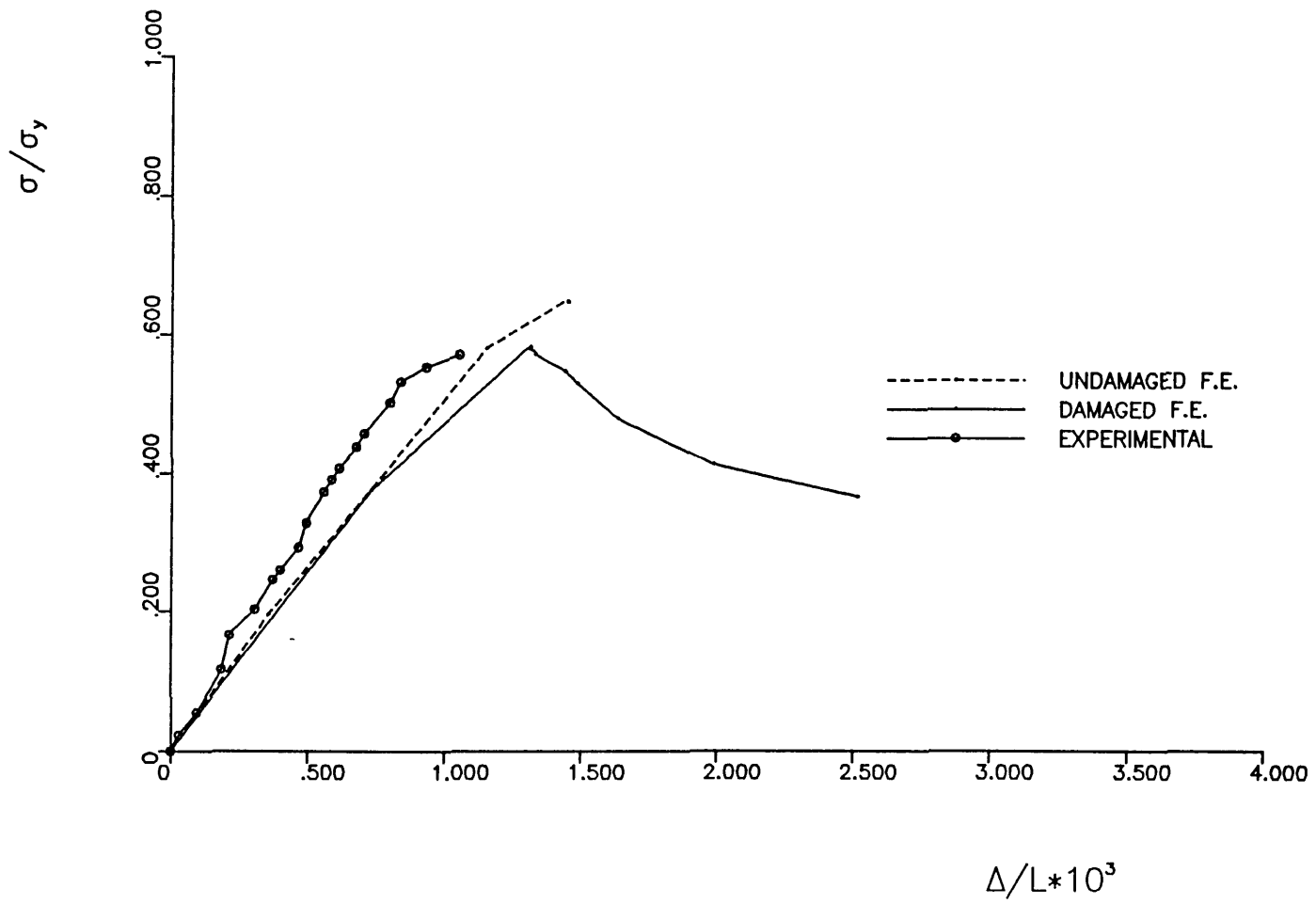


Figure 5.7

AXIAL LOAD DEFLECTION CURVES CY-2
AXIAL COMPRESSION TEST

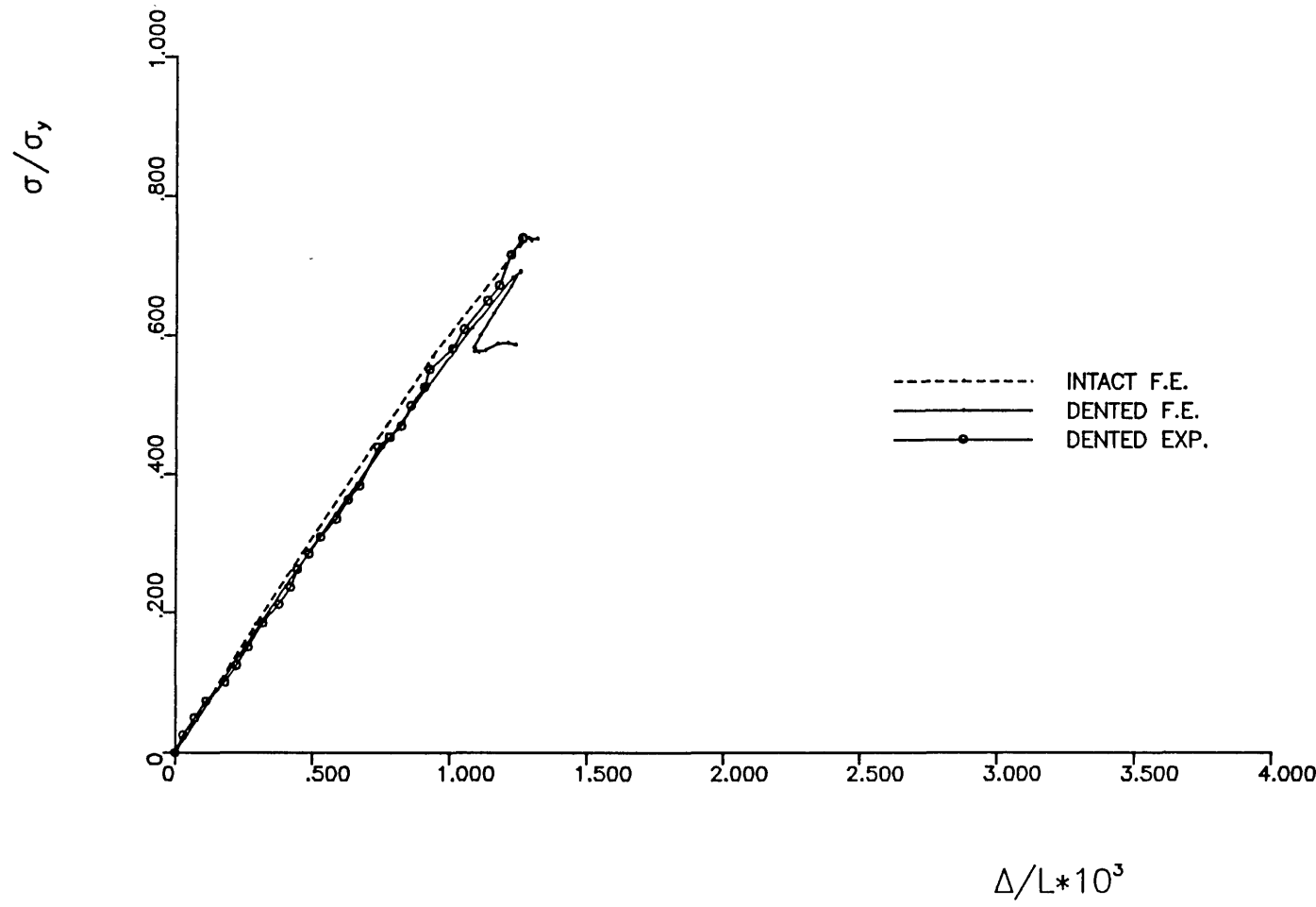


Figure 5.8

AXIAL LOAD DEFLECTION CURVES CY-3

AXIAL COMPRESSION TEST

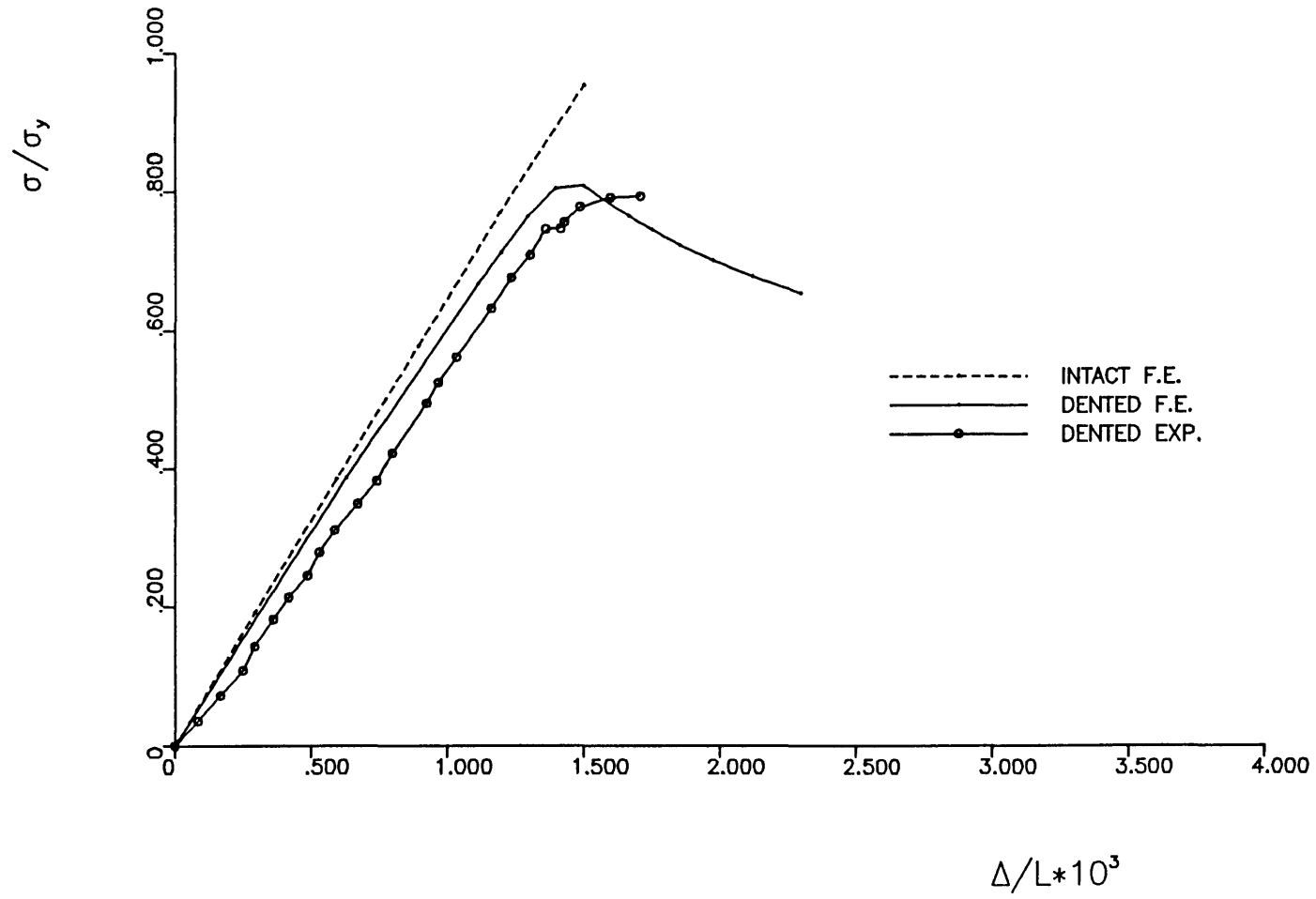


Figure 5.9

AXIAL LOAD DEFLECTION CURVES CY-4
AXIAL COMPRESSION TEST

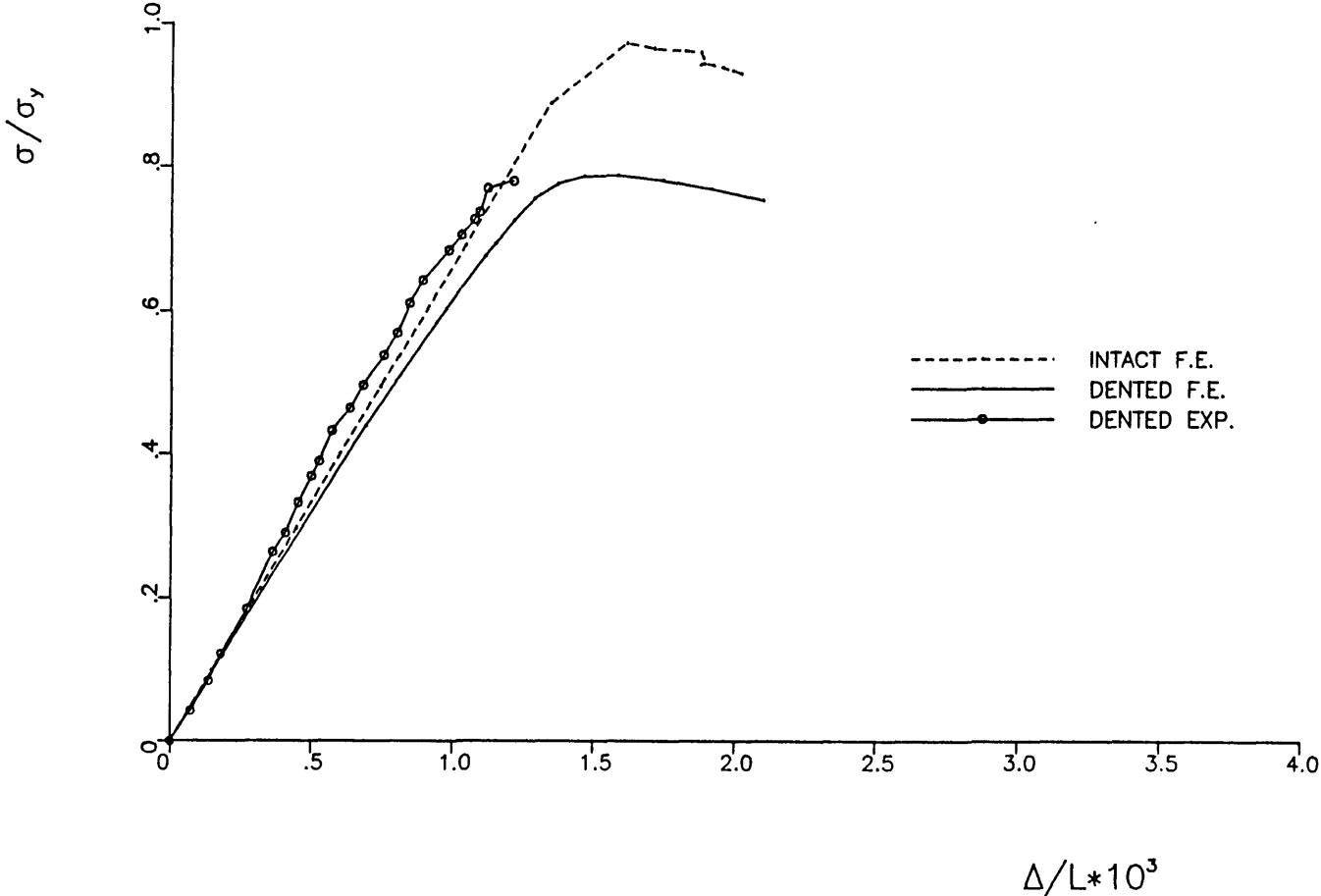


Figure 5.10

AXIAL LOAD DEFLECTION CURVES CY-5
AXIAL COMPRESSION TEST

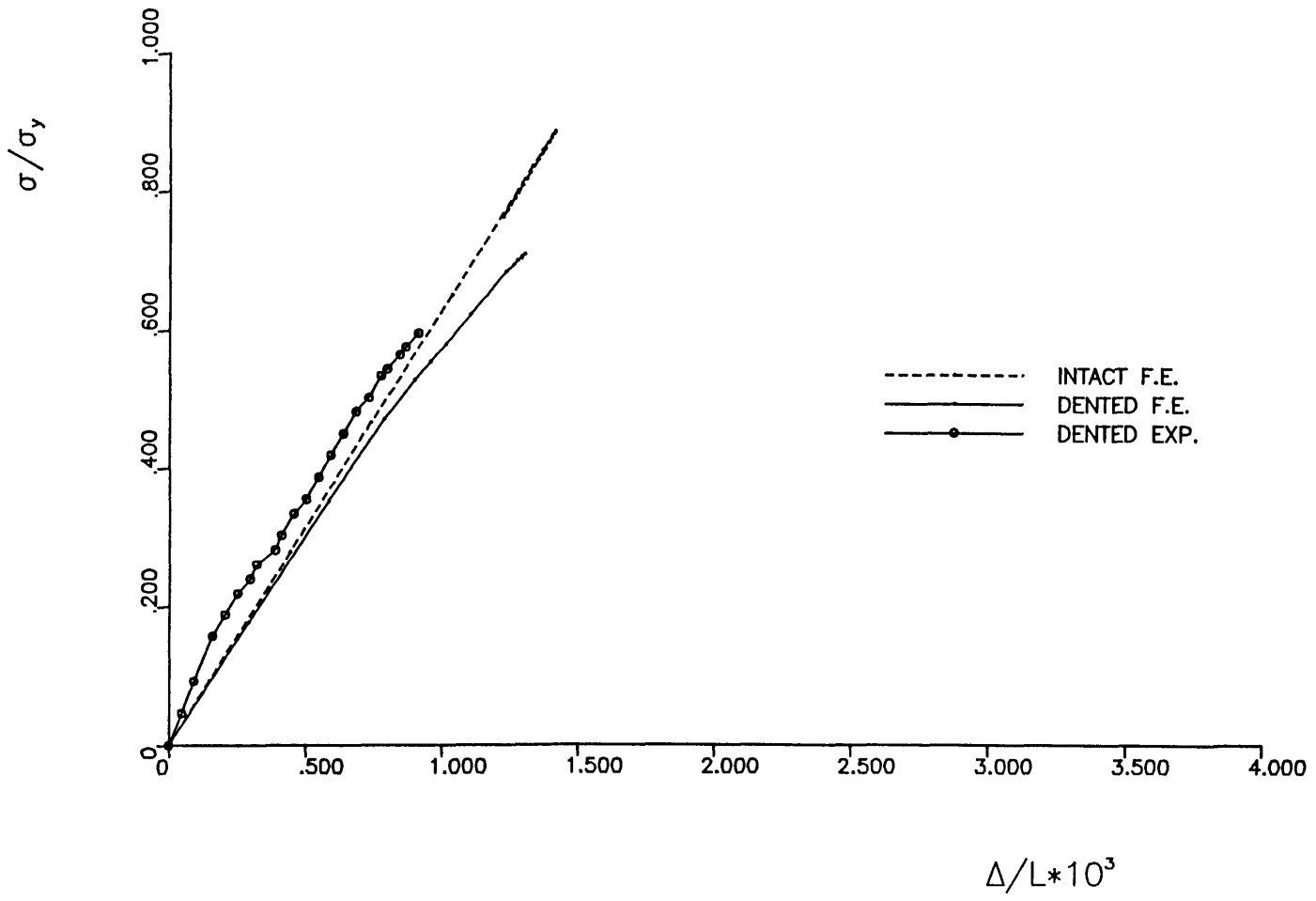


Figure 5.11

AXIAL LOAD DEFLECTION CURVES CY-6
AXIAL COMPRESSION TEST

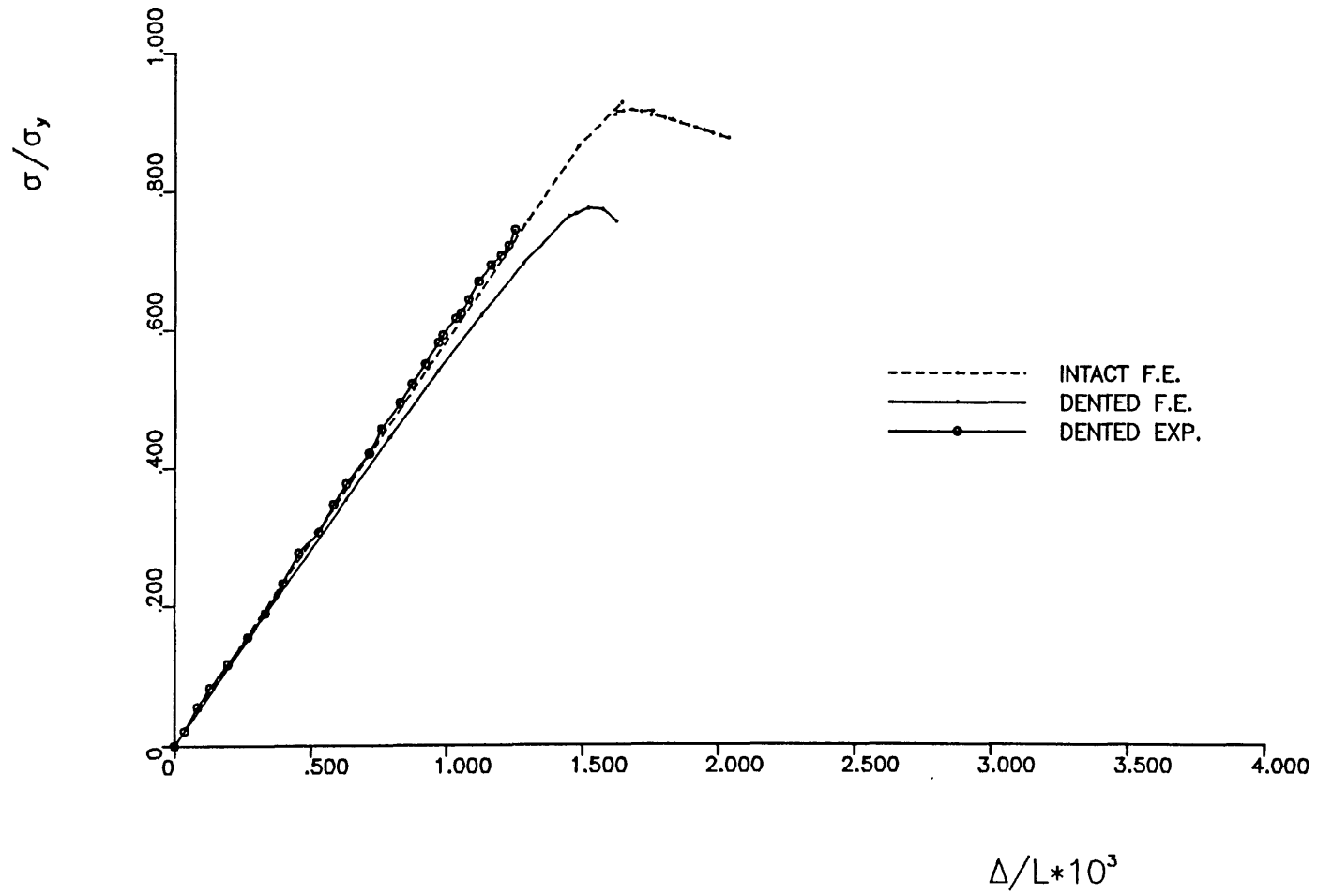


Figure 5.12

AXIAL LOAD DEFLECTION CURVES CY-7

AXIAL COMPRESSION TEST

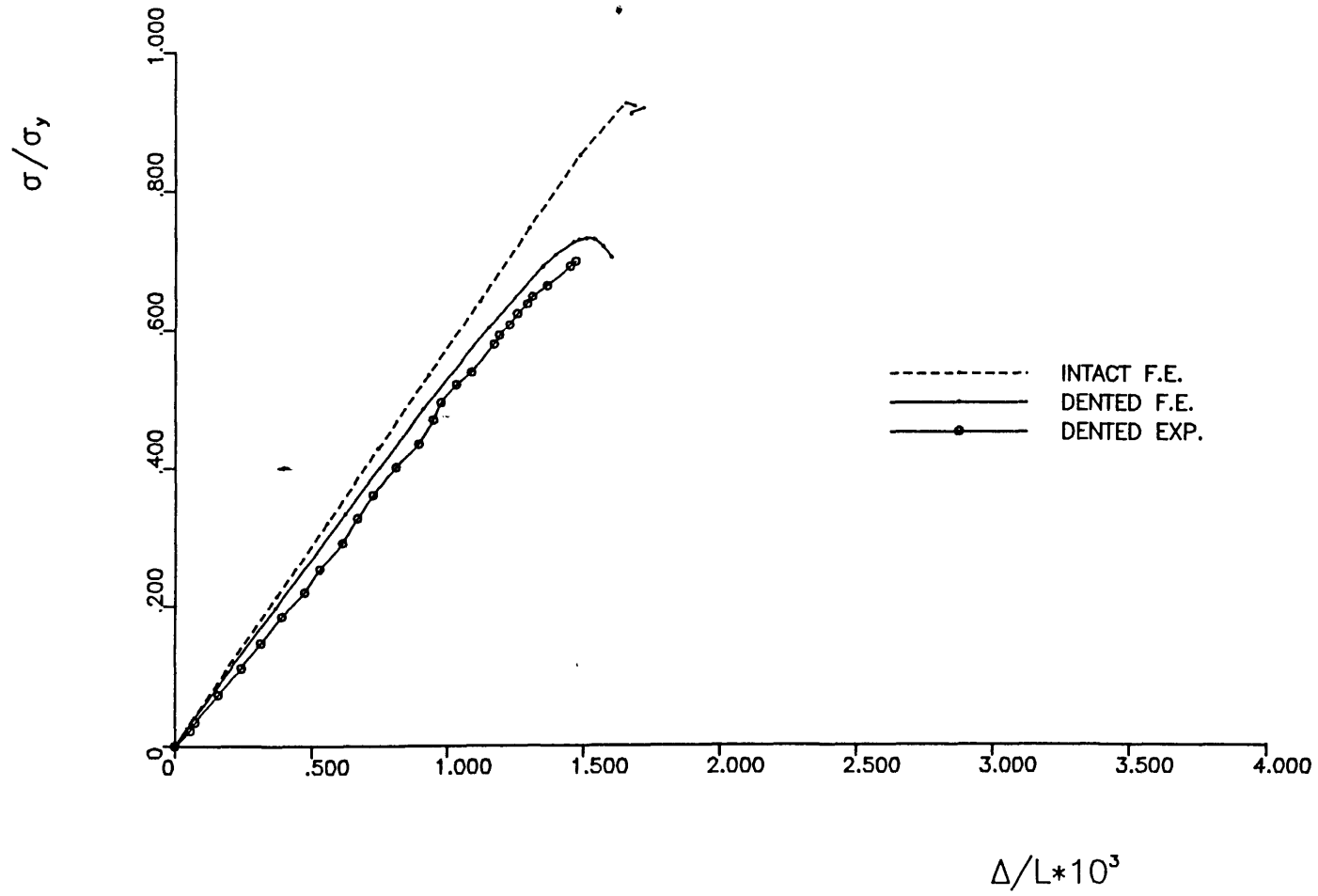


Figure 5.13

AXIAL LOAD DEFLECTION CURVES CY-8
AXIAL COMPRESSION TEST

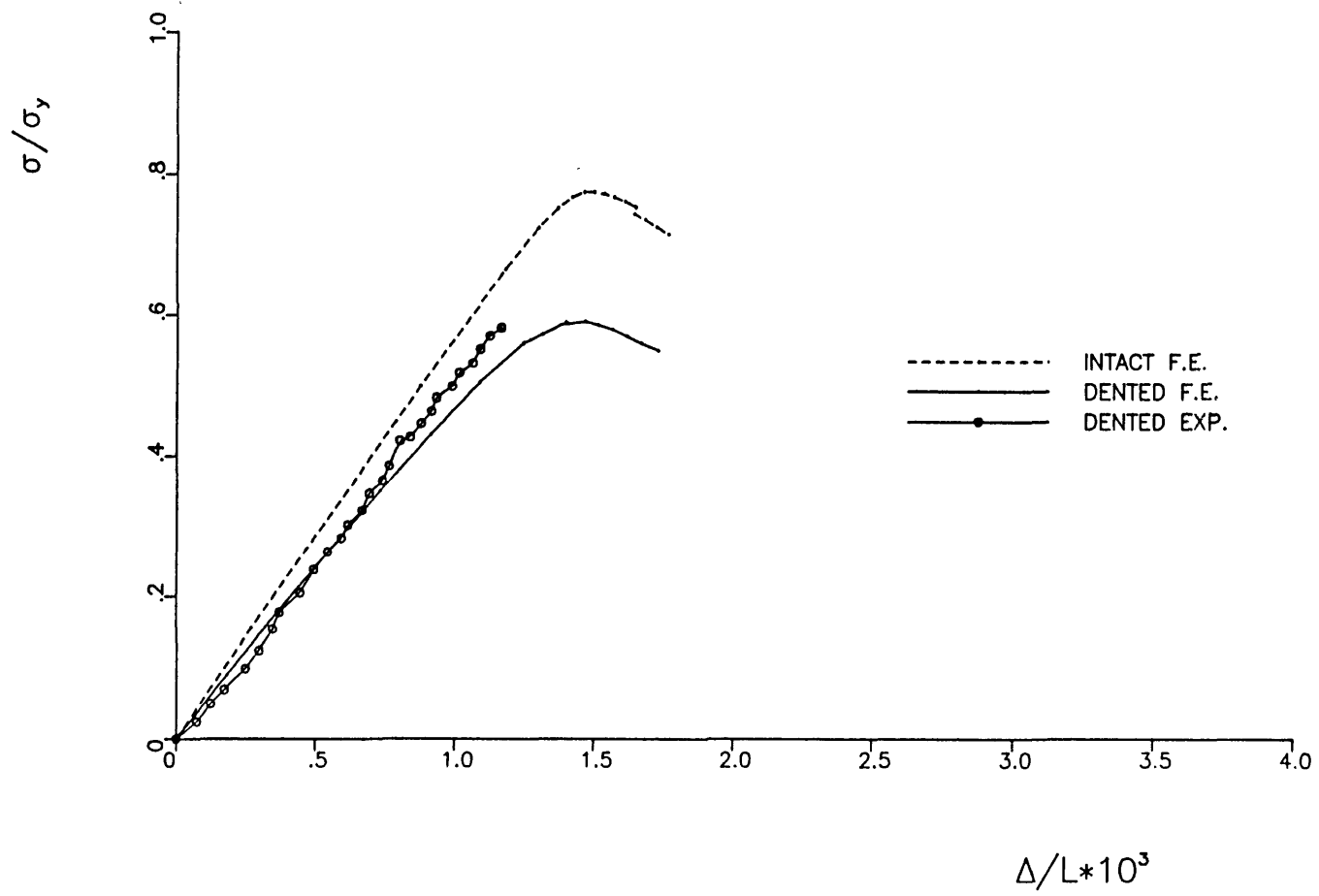


Figure 5.14

AXIAL LOAD DEFLECTION CURVES CY-9

AXIAL COMPRESSION TEST

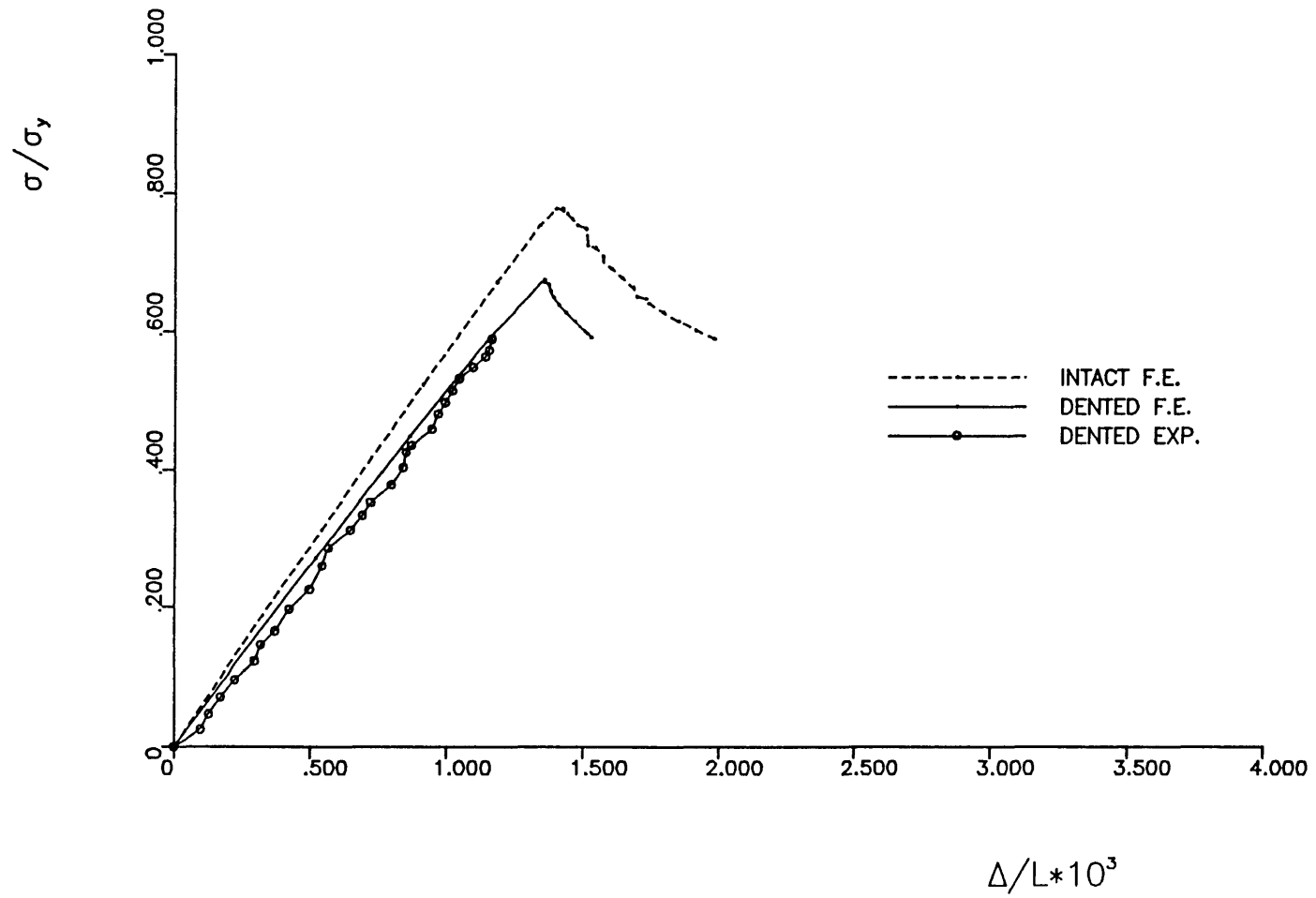
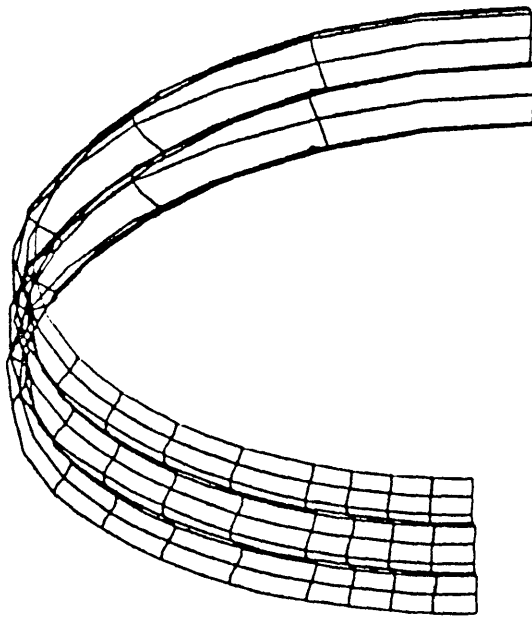


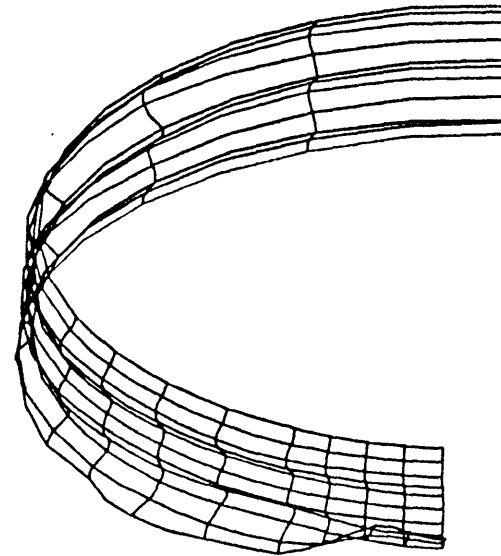
Figure 5.15

FINITE ELEMENT PREDICTED COLLAPSE MODES

MODEL CY-1



(a) INTACT

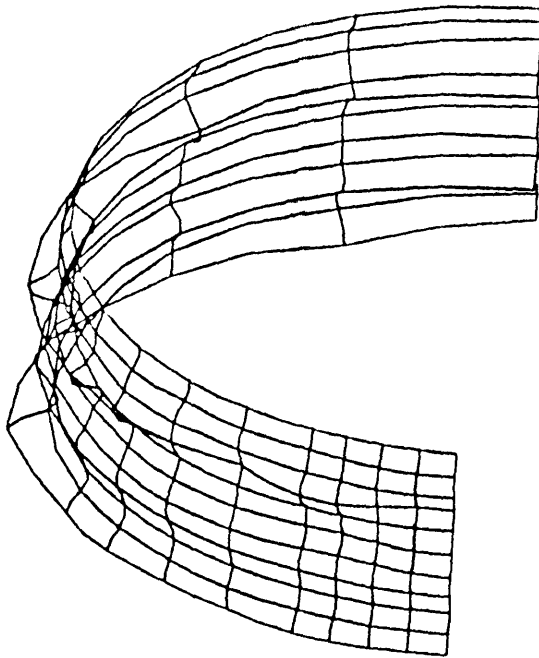


(b) DENTED

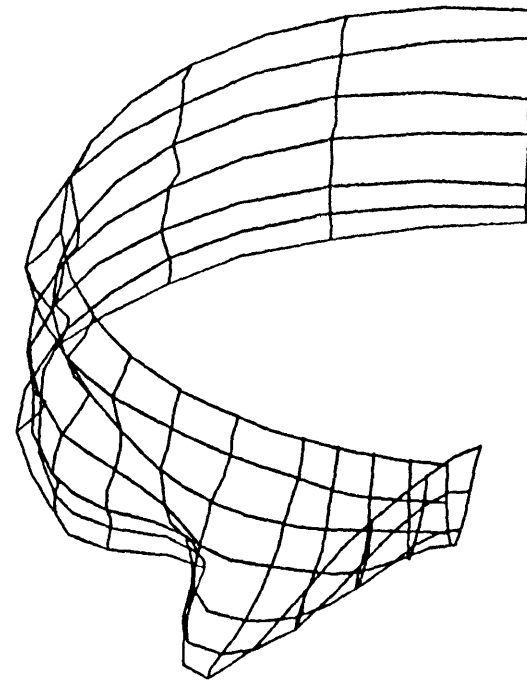
Figure 5.16

FINITE ELEMENT PREDICTED COLLAPSE MODES

MODEL CY-2



(a) INTACT

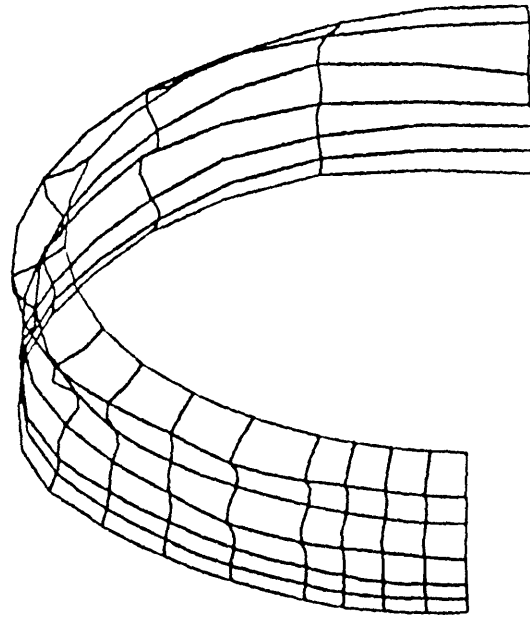


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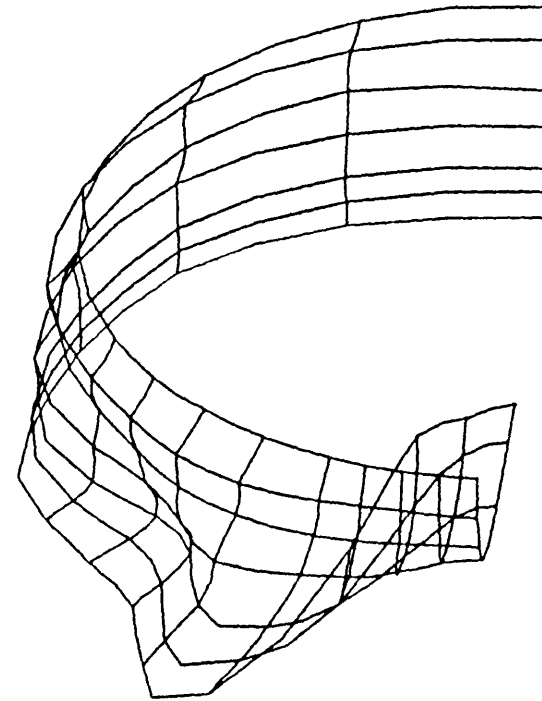
Figure 5.17

FINITE ELEMENT PREDICTED COLLAPSE MODES

MODEL CY-3



(a) INTACT

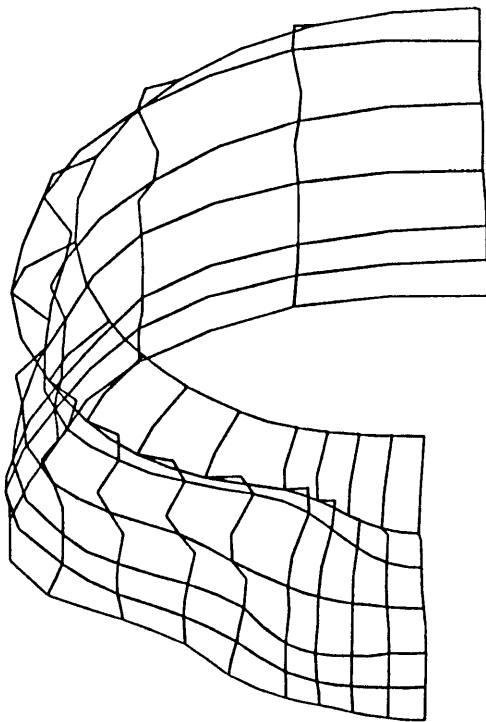


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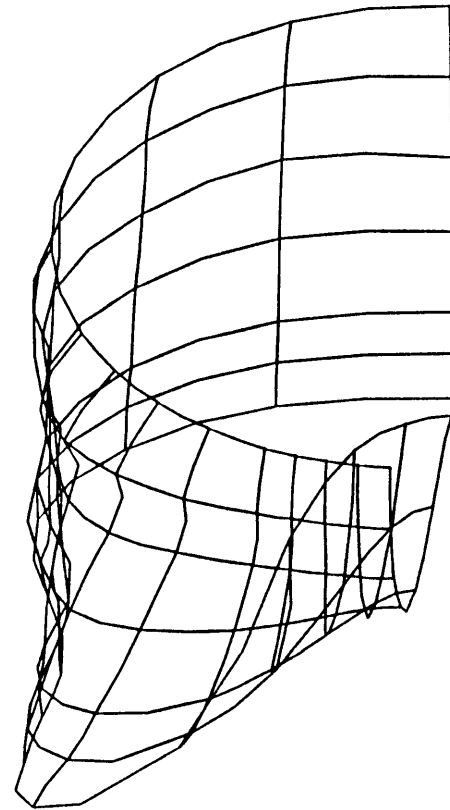
Figure 5.18

FINITE ELEMENT PREDICTED COLLAPSE MODES

MODEL CY-4



(a) INTACT

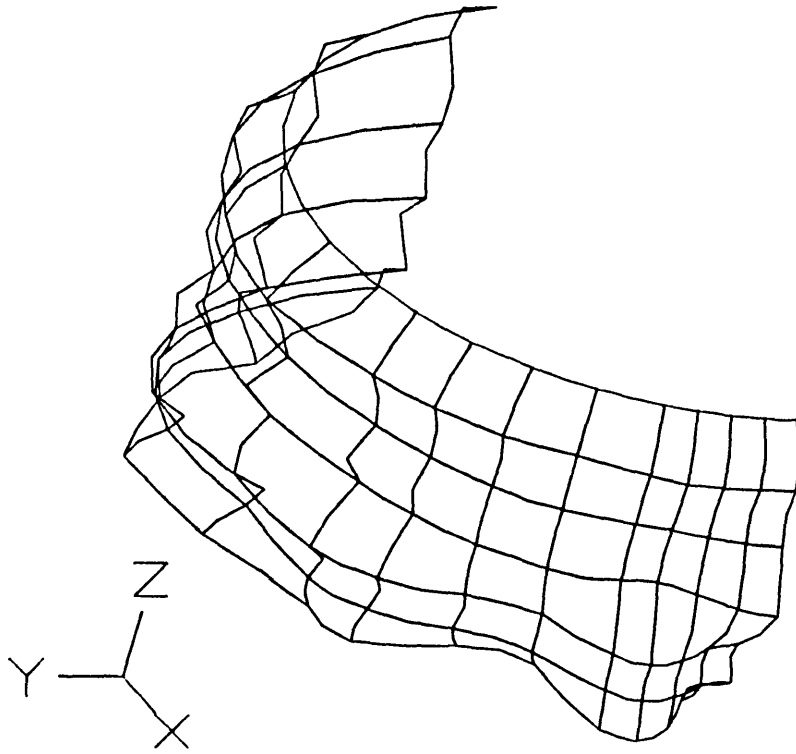


(b) DENTED

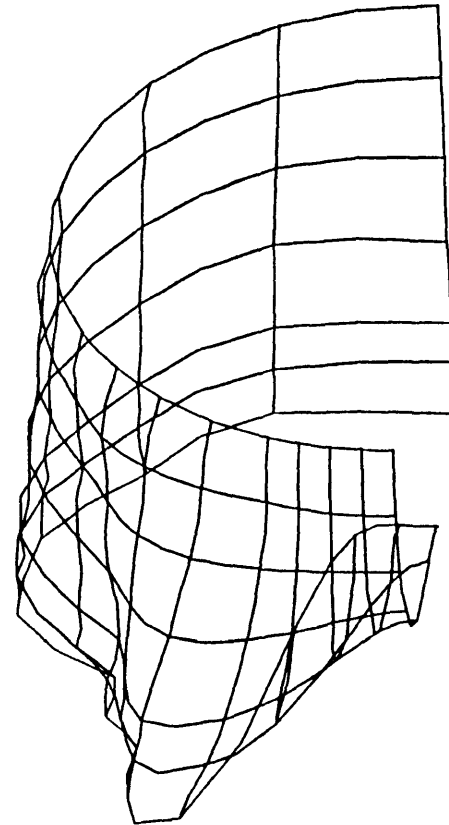
Figure 5.19

FINITE ELEMENT PREDICTED COLLAPSE MODES

MODEL CY-5



(a) INTACT

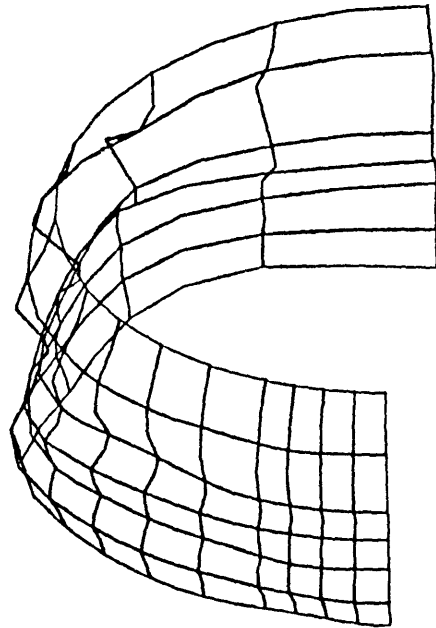


(b) DENTED

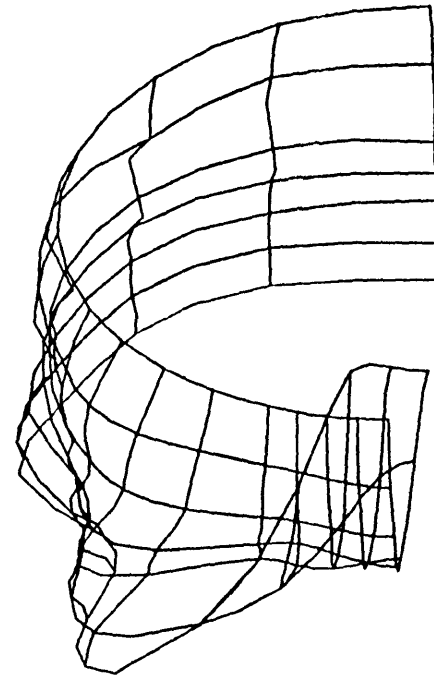
Figure 5.20

FINITE ELEMENT PREDICTED COLLAPSE MODES

MODEL CY-6



(a) INTACT



(b) DENTED

Figure 5.21

FINITE ELEMENT PREDICTED COLLAPSE MODES

MODEL CY-7

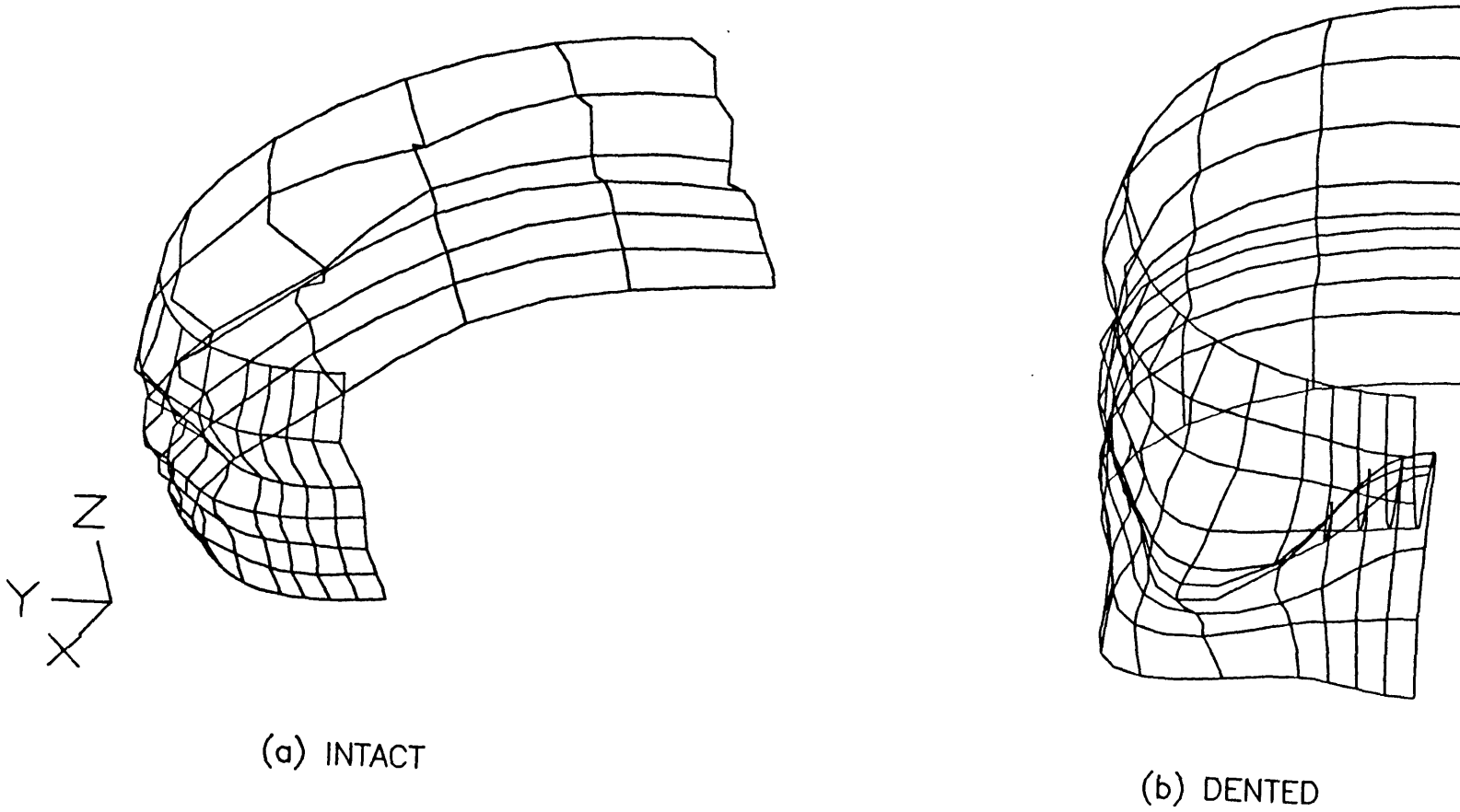
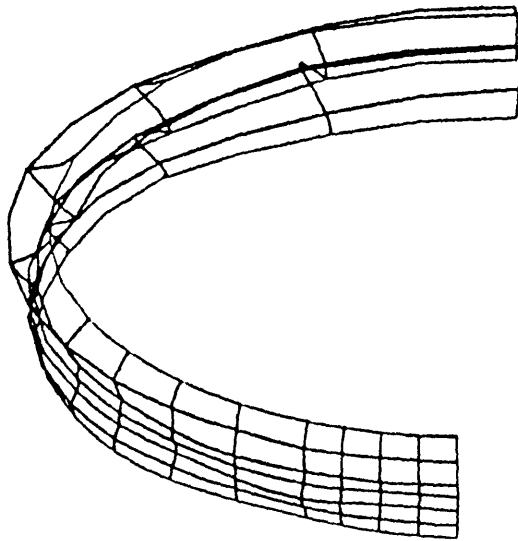


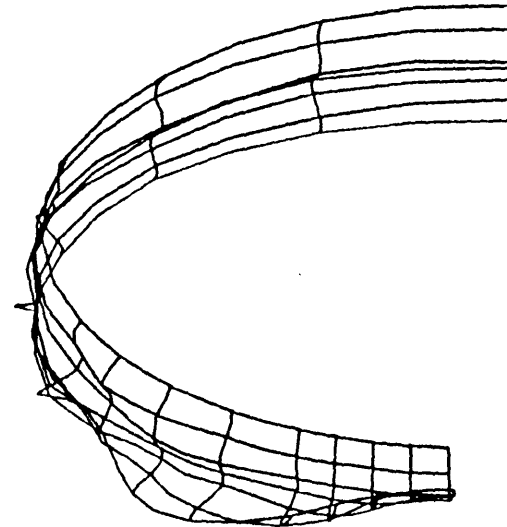
Figure 5.22

FINITE ELEMENT PREDICTED COLLAPSE MODES

MODEL CY-8



(a) INTACT

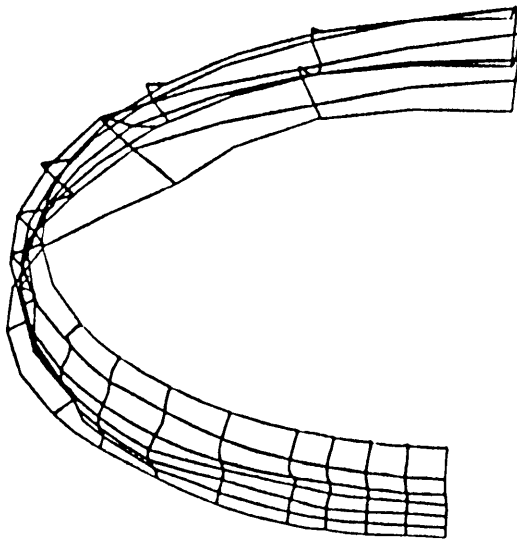


(b) DENTED

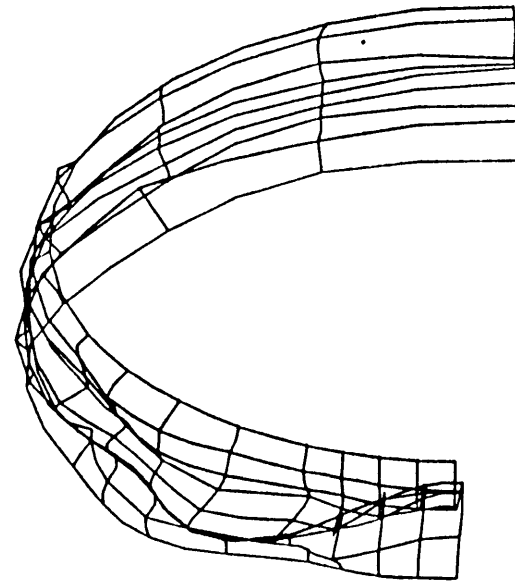
Figure 5.23

FINITE ELEMENT PREDICTED COLLAPSE MODES

MODEL CY-9



(a) INTACT



(b) DENTED

Figure 5.24

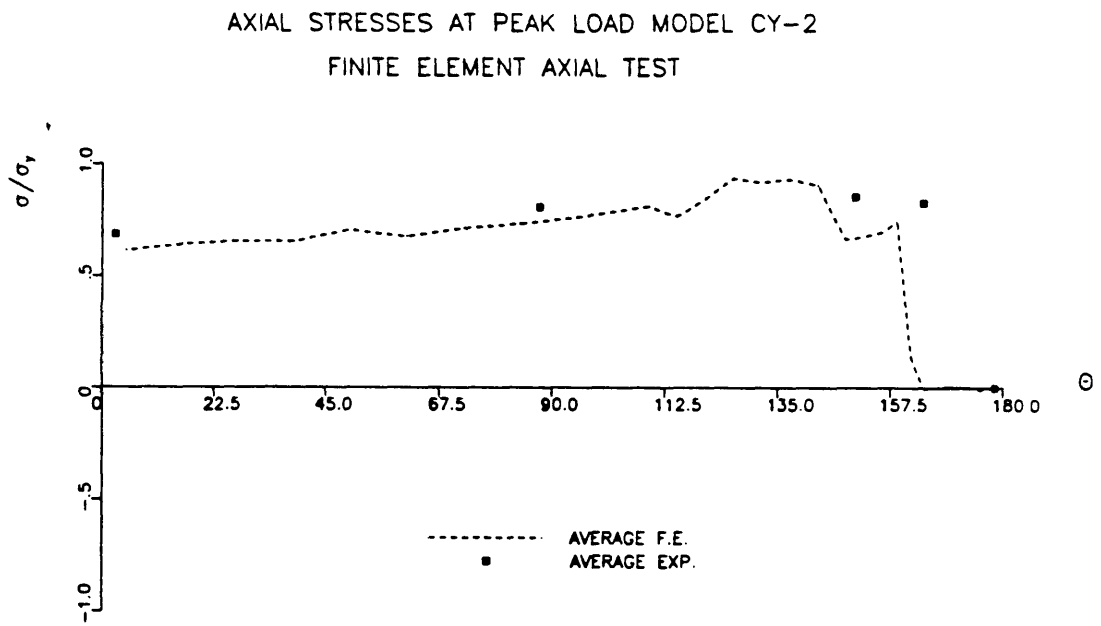


Figure 5.25

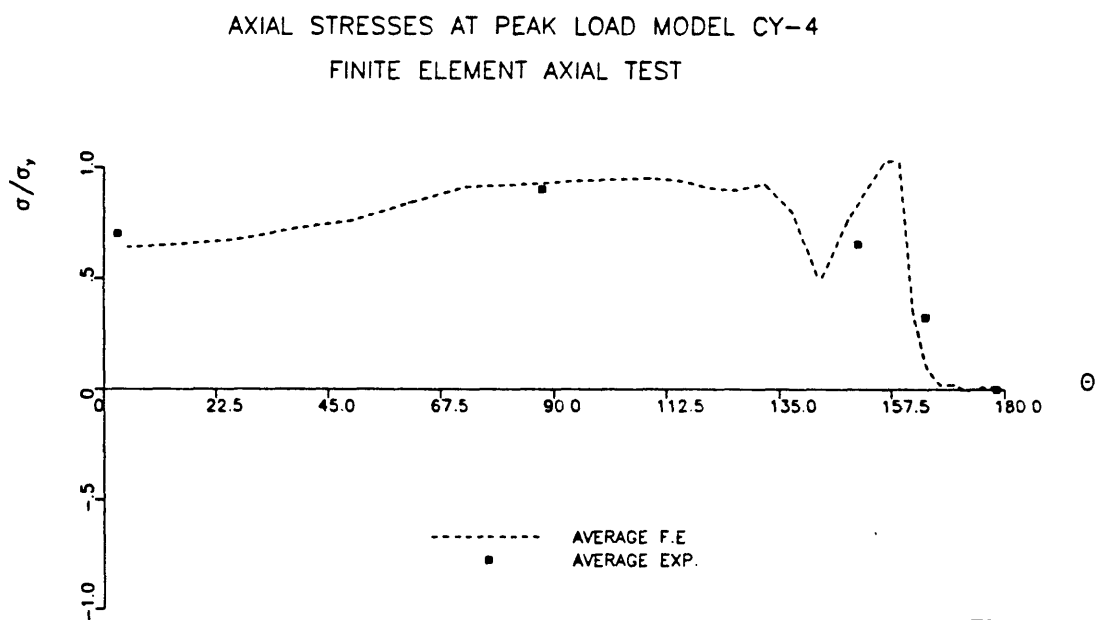


Figure 5.26

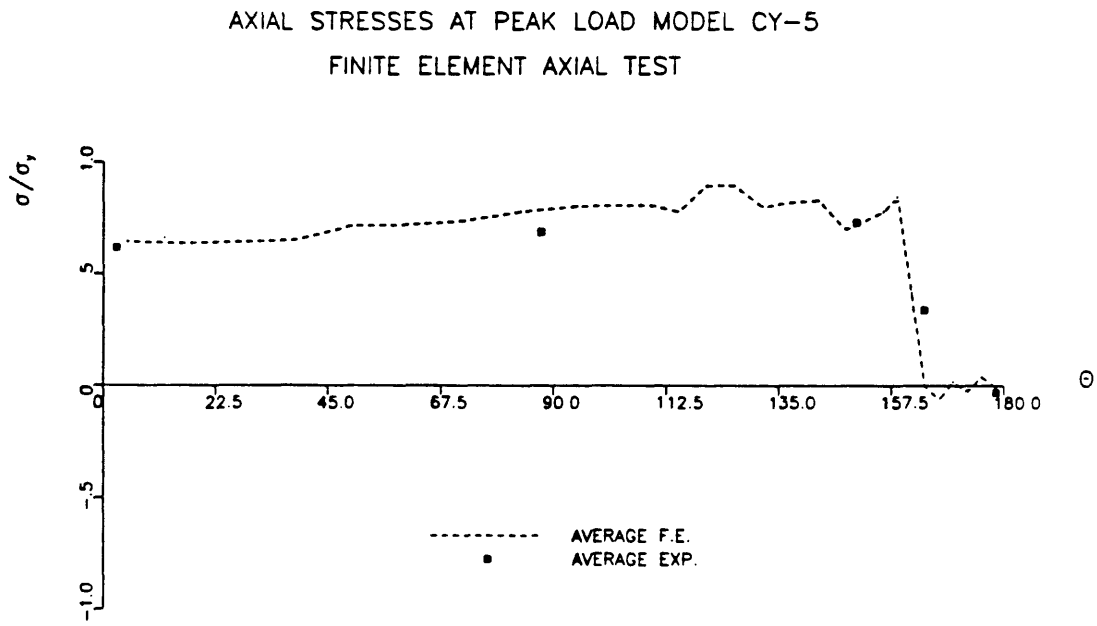


Figure 5.27

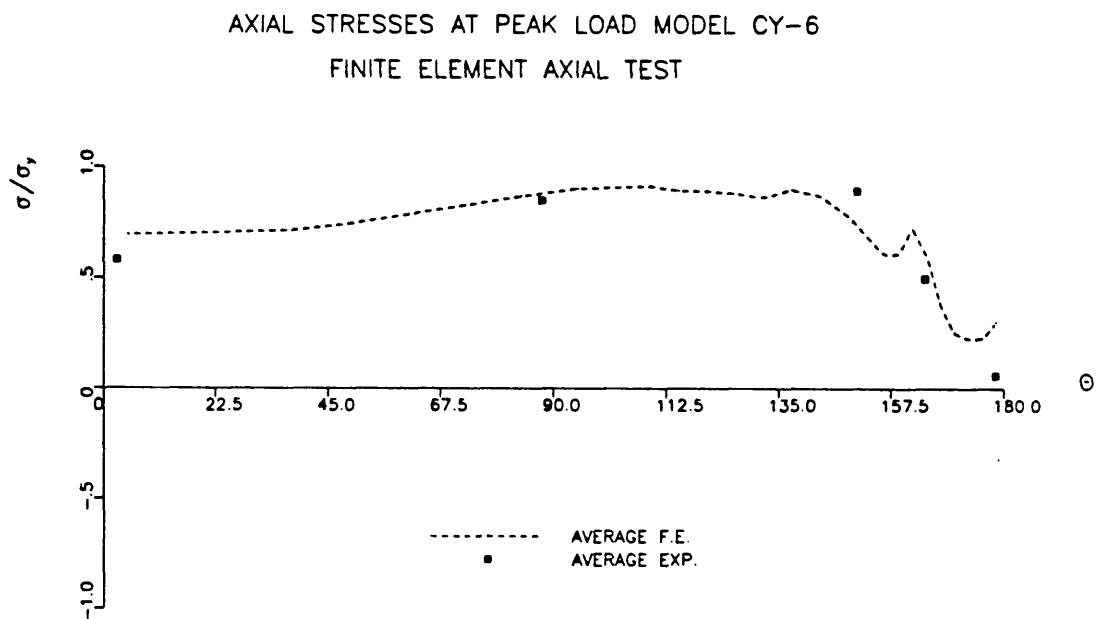


Figure 5.28

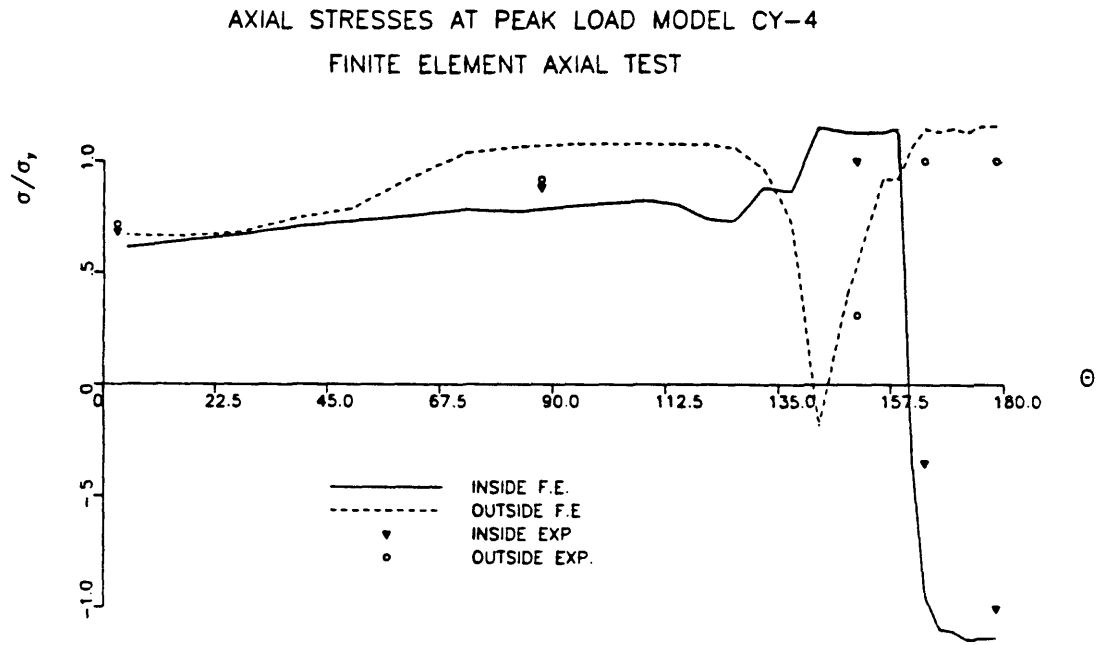


Figure 5.29

AXIAL SHORTENING FOR DIFFERENT BOUNDARY CONDITIONS

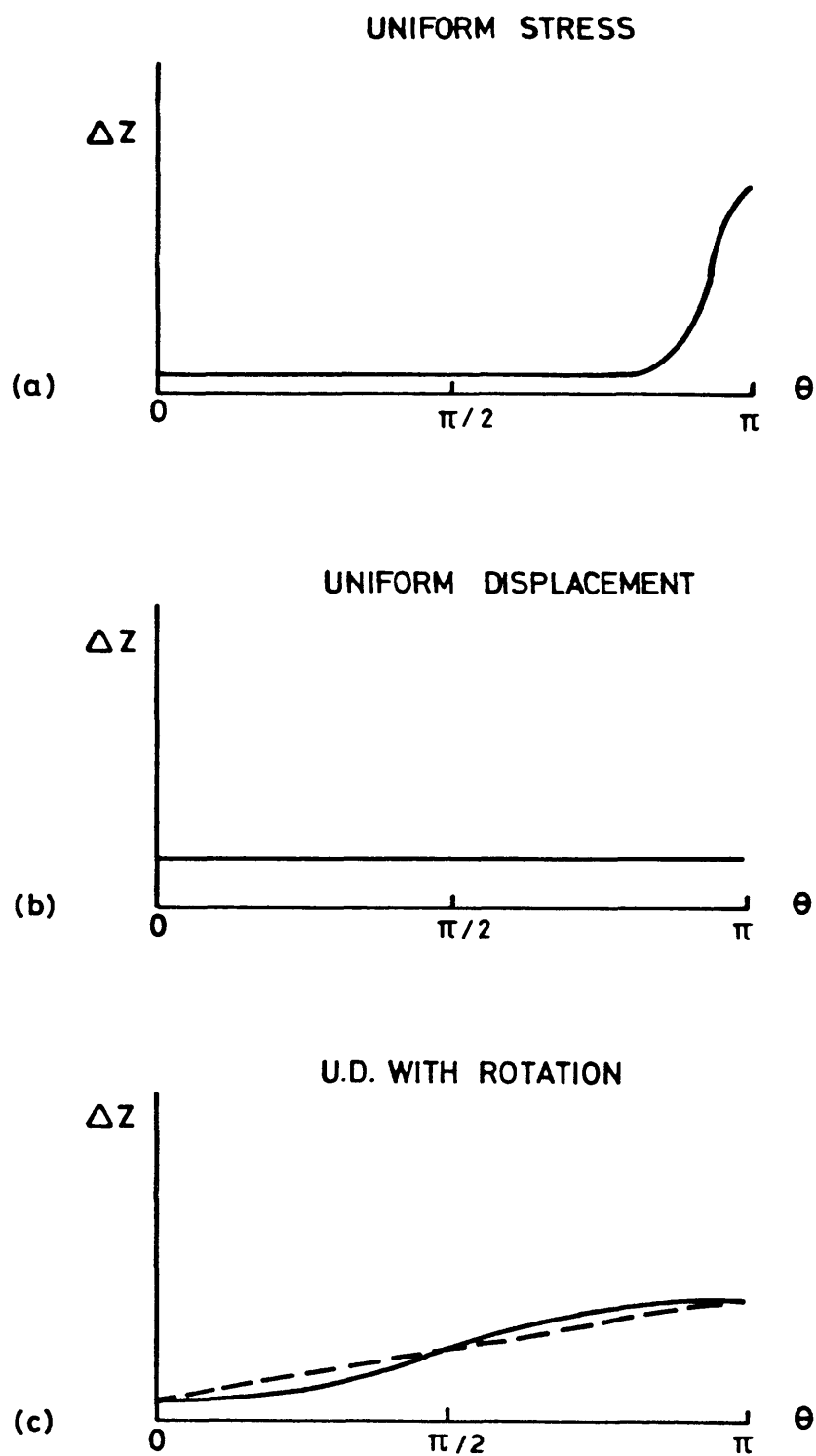


Figure 5.30

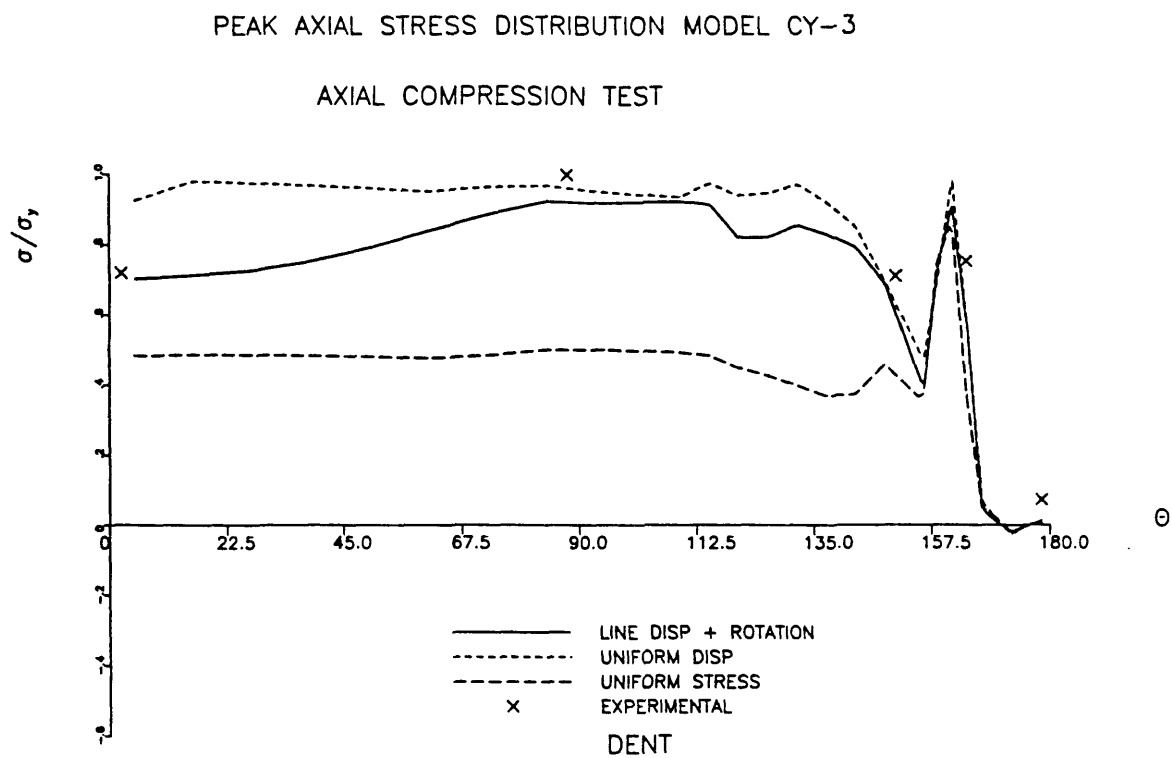


Figure 5.31

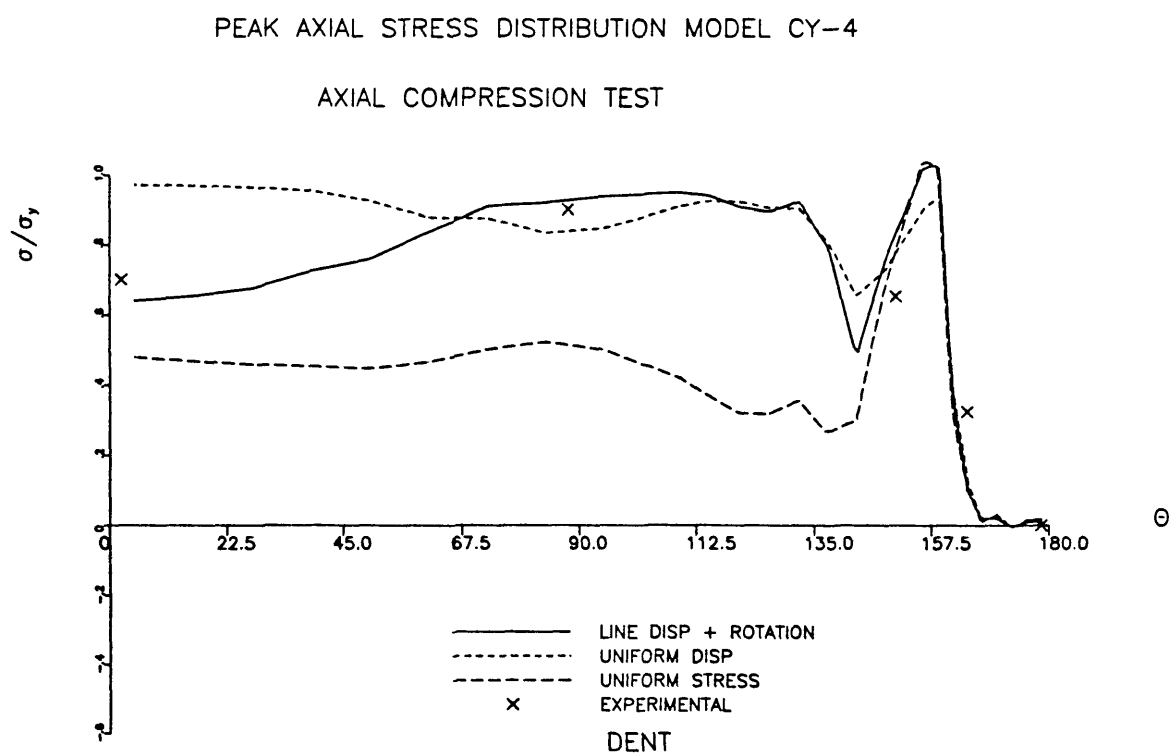
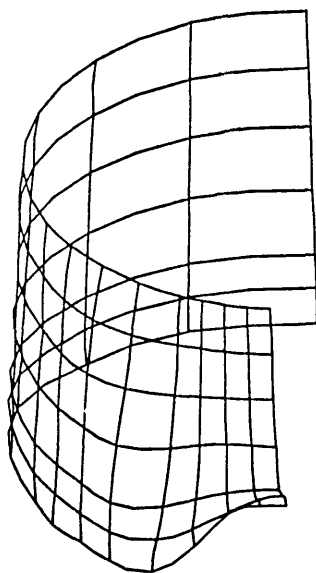
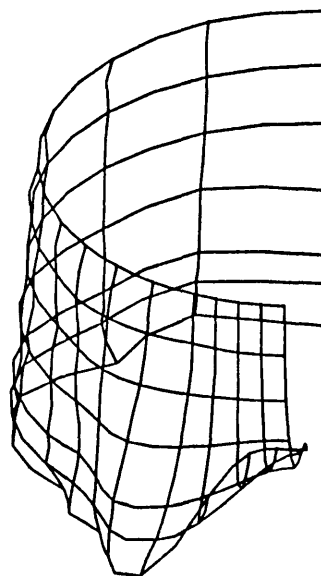


Figure 5.32

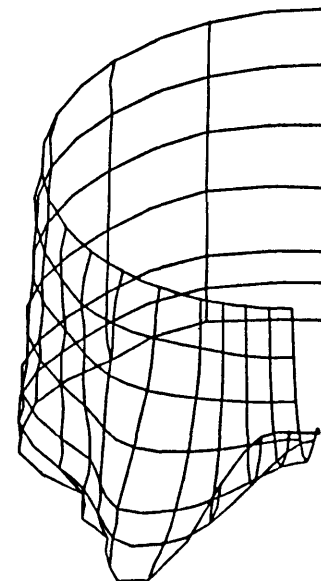
FAILURE MODES FOR DIFFERENT BOUNDARY CONDITIONS
MODEL CY-5



UNIFORM STRESS



UNIFORM DISPLACEMENT



U.D. WITH ROTATION

Figure 5.33

LOAD AXIAL DEFLECTION CURVES FOR DENTED SHELL
 ($R/t=267$, $l/R=.15$, $t=.6\text{mm}$, $d=7.7\text{mm}$)

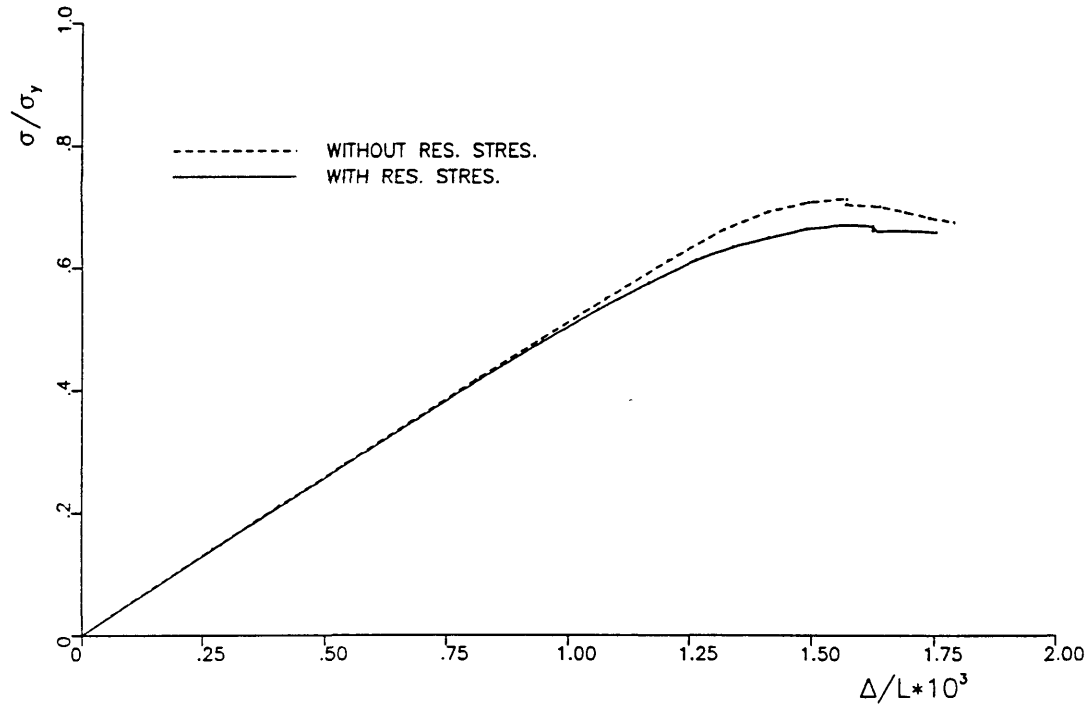


Figure 5.34

AXIAL STRESSES AT PEAK LOAD
 STRESSES NEAR THE DENT LINE

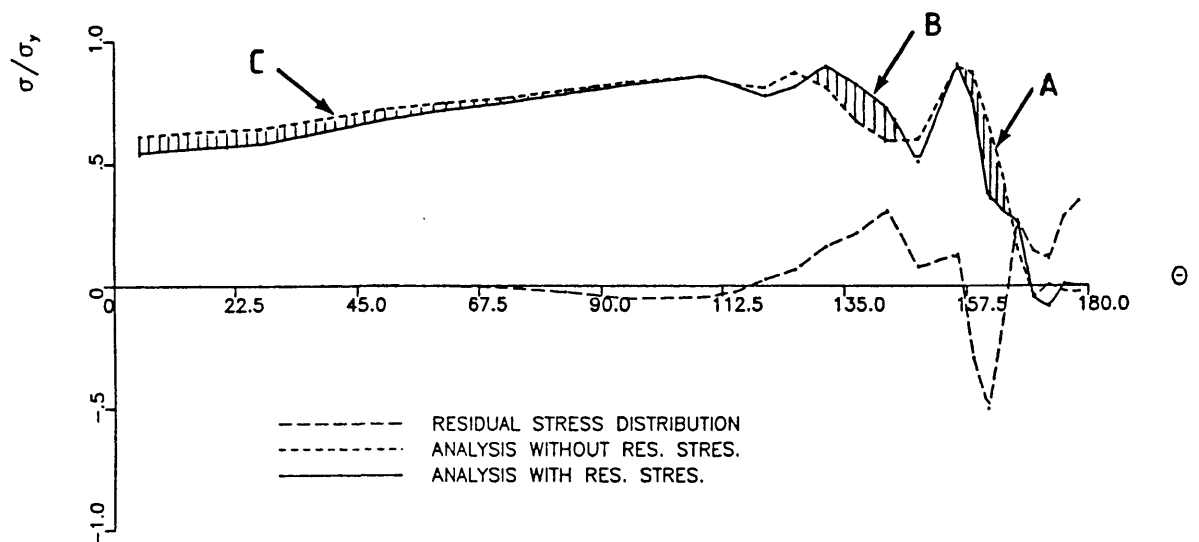


Figure 5.35

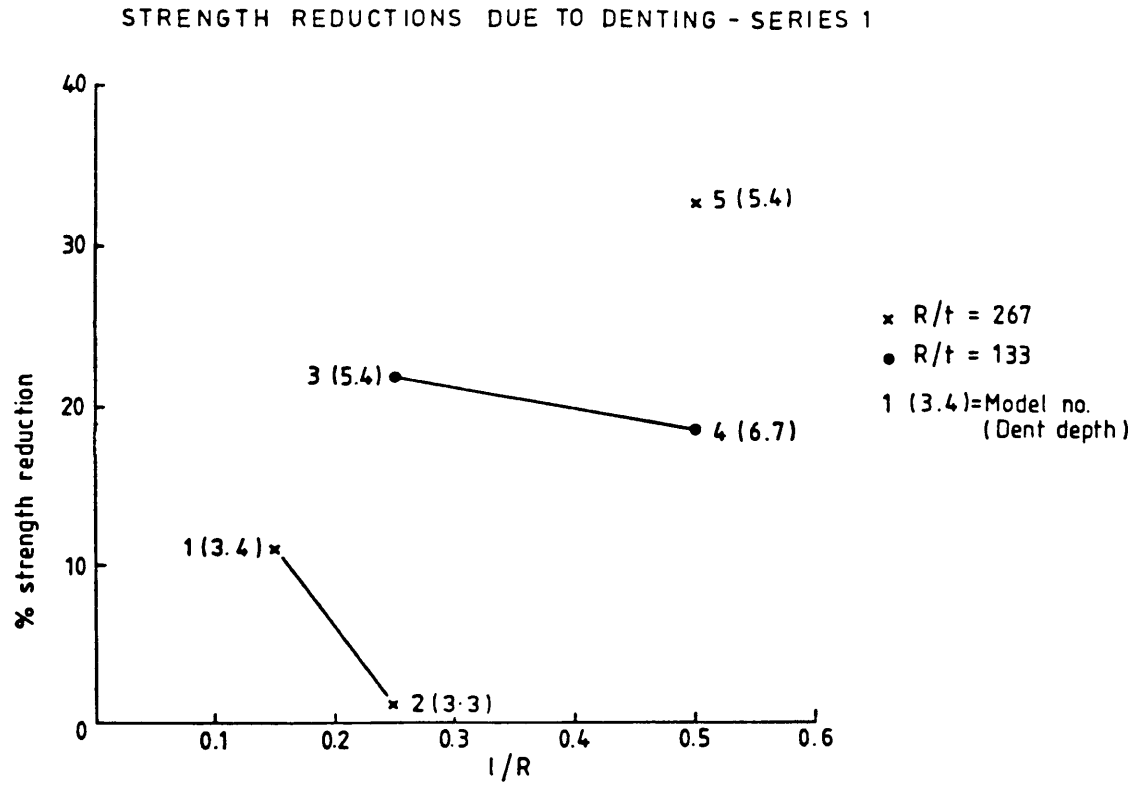


Figure 5.36

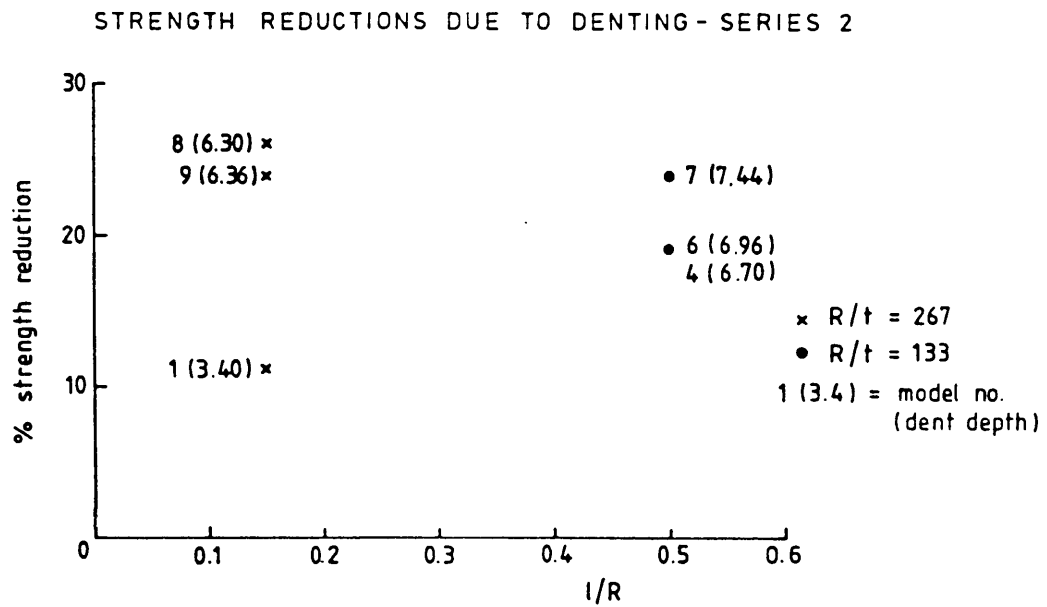


Figure 5.37

CHAPTER 6

THE EFFECT OF DIFFERENT PARAMETERS ON THE STRENGTH OF SLENDER RING STIFFENED CYLINDERS

6.1 INTRODUCTION

A systematic study of the effect of denting on shells of high slenderness outside the scope of the testing is presented. Given their low lateral stiffness and general sensitivity to imperfections it seemed important to examine their behaviour further. In addition, experimental correlations pointed towards an increase in sensitivity to denting for such geometries.

The numerical models developed in Chapters 4 and 5 are employed in this study. Despite some limitations of these models their use could provide useful results on the relative behaviour and trends.

The following aspects of behaviour are examined:

- The effect of the dent and possible interaction with the critical modes of collapse are examined by analysing initially perfect shells.
- The effect of the initial imperfections present in the shell before denting and the way they interact with the dent itself is investigated by considering initial imperfections with different forms.
- By analysing different levels of denting, the rate of strength deterioration with increasing dent depth is assessed.
- The effect of the outward imperfections associated with the dent which are present in the critical area circumferentially outside it is also examined.
- Results of the numerical study are compared with a simple analytical procedure.

6.2 STUDY ON SLENDER GEOMETRIES

6.2.1 Geometries Analysed

This study is not intended to cover the whole spectrum of offshore geometries, but to complement the work covered by the experimental programme and correlation exercises. Analyses performed concentrate on more slender geometries.

For the purposes of this study only a limited number of analyses have been performed. A value of 0.3 was assumed for the ring spacing parameter (ℓ/R) and two values of (R/t) of 300 and 500 were examined.

Five panel shells were analysed, stiffened by T-ring stiffeners. These have been designed according to the DnV specifications [59] and details of their cross-sections are shown in Table 6.1.

6.2.2 Dent Levels

Different levels of denting have been analysed. Dents were in the range of 0 to 3 percent of the diameter.

Results of the experiments have shown that strength reductions of significant magnitude occur for this range of dents. The energy input associated with such dents relates to the operational impact defined by the DnV specifications, [21,22].

A further consideration is the dent wavelength which compares with the elastic critical wavelength of the shells. More significant strength reductions are therefore likely to occur. The outward deflections adjacent to the dent have been shown to increase in magnitude as the dent depth increases. The effect of these may be more marked for larger dent sizes.

6.2.3 Initial Imperfections

The choice of a single imperfection mode that would be representative of practical offshore shells is an impossible task as is an examination of all possible modes. A number of cases have been examined, which include the initially perfect geometry. This was expected to isolate the dent effect from any interaction with the shell imperfections.

Two cases with different imperfection modes have also been chosen. Firstly, an

axisymmetric mode was chosen as imperfections in the axial direction have been found to be the most significant in determining the strength of ring-stiffened cylinders subjected to axial loading. Furthermore, the axisymmetric mode consisting of a single outward bulge represents the most dominant mode found in ring stiffened shells with narrow ring spacings. Stiffener welding causes a barrelling effect in the panels during manufacture. This has been observed by previous researchers [69,71], and in all the experimental models tested in this study.

Previous research [131] has suggested that the magnitude of a particular longitudinal mode is inversely proportional to the number of half-wavelengths per panel. More recent research by Agelidis [69] and Tsang [71] who compared imperfections in experimental models found reasonable agreement with this rule. They also showed that the first few modes are the most significant and can adequately describe the behaviour of a cylinder.

Following these findings, only 1, 3 and 5 half wavelength modes have been superimposed to produce the axisymmetric imperfections (Table 6.2). The tolerance specified by the DnV Rules [59] has been adopted to represent the maximum magnitude of the combined imperfections.

A periodic mode has been chosen for the third set of initial imperfections. This choice is aimed at examining the possible interaction between the initial imperfections and a sympathetic dent form. A mode with ten full waves around the circumference was superimposed over the previous axisymmetric mode (Table 6.4). The DnV tolerance was again taken for the maximum imperfection.

6.2.4 Models Employed in the Analyses

The numerical model described in Chapter 4 was used to damage the intact shells. Each configuration was subjected to mid-panel dents of different levels and subsequently loaded under axial compression.

Comparisons made between the deflections during loading and those after unloading show a very similar pattern (Figure 4.5). Because of this the dented shapes for the axial compression analyses were extracted from the loading path. This reduced the number of analyses significantly since only one was required for each intact configuration which could be used for the range of dent depths.

The finite element model described in section 5.2.4 (Figure 5.3b) was adopted for the axial loading analyses. The boundary conditions implemented at the top boundary

were those of uniform displacement with rotation. Residual stresses were neglected in these analyses.

6.3 RESULTS OF ANALYSES - SHELLS WITH R/t OF 300

6.3.1 Dent Deflections

One of the dented cylinders is shown in Figure 6.1 with an exaggeration factor of 20 applied to the displacements. This shell includes one of the largest dents examined with d/D of .0274. As seen in Figure 6.2, a similar pattern of deflections develops for all dent depths. The deflections shown in these figures correspond to the initially perfect shell. These were found to be very similar to the initially imperfect shells, and similar to previous correlations.

The magnitude of the outward deflections is of the order of 15 to 25 percent of the dent depth, for the dent levels considered. These deflections may, however, be of importance in the determination of the residual strength.

The wavelength of these imperfections and the dent are tabulated in Table 6.3, non-dimensionalised with respect to the shell circumference. The wavelength of the outward deflections is about three times bigger than the dent itself for the smallest dent (Figure 6.3). As the dent increases, however, the wavelength of the outward deflections reduces and becomes smaller than the dent wavelength. At the point where the two curves cross, d/D of .0181, the two wavelengths are equal. This means that a cylinder which is dented at this level would have three half-waves of equal wavelength. This was later shown to be of significance to the knock-down effect on strength.

Inward deflections of the order of 2 percent of the dent depth occur further away from his dent (Figure 6.4). These spread around the cylinder circumference and are of much longer wavelengths.

6.3.2 Initially Perfect Shells

6.3.2.1 Load Deflection Paths

Analyses of seven different dent levels were performed for this configuration, ranging between 0 and 27.4 percent of the diameter (Table 6.4). Load-axial deflection paths for each of the dented models are presented in Figures 6.4-6.9 together with the intact cylinder response.

The perfect cylinder exhibited a linear response up to very close to the peak. At that point the stiffness dropped significantly and the load deflection path formed a rounded peak.

The dented models exhibited an initially linear response which showed more loss of stiffness as the load increased. The initial stiffness exhibited by these cylinders was, as expected, lower than that of the intact shell.

The non-linearity in response was highest in cylinders which included large dents and was due to both geometric and material non-linearity.

One of the load deflection curves corresponding to a dent of $d/D = .0274$ (Figure 6.9), showed a peak at around .63 of yield, followed by a small drop in the load. After this the cylinder carried more load before collapse finally occurred at .70 of yield. There is an indication that a small buckle developed locally in the dent area causing this premature unloading.

6.3.2.2 Stress Distribution and Failure Modes

Peak load axial stress distributions for two of the models are shown in Figures 6.10 and 6.11. Both figures indicate the following. High bending inside the dent area produces yield on both surfaces. The average axial stress carried by the dent area was very close to zero. At the corner of the dent both surfaces reached yield in compression. A number of stress reversals indicate outward and inward deflections. This indicates a periodic mode of deflection which lead to a similar mode of collapse. No bending could be seen in the rest of the circumference. Compressive stresses developed there which reduced in magnitude away from the dent, reflecting the load eccentricity effect.

Failure modes for two of the models are shown in Figure 6.12. The intact cylinder failed in an axisymmetric outward bulge in the end panel. The dented shells exhibited a lobular form of failure in the dented panel combined with an axisymmetric outward bulge in the end panel. A closer look at the failure mode and the development of stresses during the loading process gives a clear indication as to how these failures occur.

The intact model suffers axisymmetric outward deflections in the prebuckling state. In the end panel, because of the restraining effect of the end rings high plasticity was introduced. Failure was initiated at that location (Figure 6.12). This behaviour has been observed previously in intact stiffened shell experiments [71,76] and also in

some of the dented models of this thesis.

Deflections exhibited by the dented shells were of a lobular form in the dented panel, as indicated above, but in addition the end panel deflected outwards as above for the intact configuration. The presence of the dent enhanced the lobular form of deflections which spread to the adjacent panel. The effect of the end rings, however, still controlled the behaviour of the last panel. Yielding began to develop in the dent area in the early stages of loading and this spread around the dent as the load increased. The load at which this yielding was initiated varied between .609 and .437 of the squash load of the cylinder. Yield occurred earlier as the dent size increased.

Yielding developed at a much higher load in the last panel in a region circumferentially well outside the line of the dent (Figure 6.13). This was a common feature of all the shells of this group and is due to the fact that this area corresponds to the most highly stressed area around the cylinder. The combination of the high axial stress and ring restraint causes yield. Failure followed shortly after the onset of plasticity in the top panel. The dented shell carried slightly more load after yield than the intact equivalent. The shell which contained the biggest dent failed at the onset of the plasticity. This is possibly because in this shell the centre panel failed before the end panel.

6.3.2.3 Residual Strength

The axial residual strengths are tabulated in Table 6.4 and plotted in Figure 6.14 versus the dent depth parameter d/D . A significant drop in strength of 24 percent was obtained for the smallest dent with d/D of .0027. The rate of strength reduction, however, reduced considerably as the dent depth increased. No sudden changes in the response have been observed.

Collapse strength predictions from a simple analytical procedure based on an effective section approach [12] are plotted alongside all the numerical results (Figure 6.14) for comparison purposes. Bigger strength reductions were predicted by the finite element analysis in the area of small dents. A difference of up to 9 percent is found in strength predictions for a dent of $d/D = .0027$.

If the dent is regarded as a half-wave imperfection, it is interesting to relate its wavelength to that of the elastic critical mode of collapse. Table 6.2 shows the number of full waves per circumference corresponding to each dent compared with the critical number of waves. This table shows that the smallest dent ($d/D = .0027$)

which produced the most significant strength reduction has a wavelength which corresponds to 15 full waves. This is equal to the elastic critical number of waves. The second biggest dent ($d/D = .0052$) corresponds to 11 full waves which is close to the critical number of waves and again for this shell the finite element strength prediction is lower.

This may suggest that for certain geometries and dent depths the dent may interact with the possible modes of collapse and where the dent wavelength is compatible with the critical mode, it acts as a sympathetic imperfection. As a result the dent has a bigger relative effect on the strength.

6.3.3 Shells with Axisymmetric Initial Imperfections

6.3.3.1 Load Deflection Paths

Six different levels of denting were applied. Load versus axial deflections are plotted in Figures 6.15–6.20. The intact cylinder exhibited a gradual drop in stiffness as the load increased. This was caused by geometric non-linearity due to the initial imperfections. Some plasticity also developed around the centre panel, reducing the stiffness of the shell, at around .568 of yield. It can be seen that the stiffness reduced significantly after this point.

The dented shells also exhibited a nonlinear load–deflection path. The initial stiffness dropped with increasing dent depth (Figure 6.15). The shells which included larger dents had more rounded peaks. The nonlinearity in the initial part of the curve was again due to geometrical effects, a result of the large initial imperfections associated with the dent and the initial axisymmetric imperfections in the cylinder. At between .259 and .302 of squash plasticity began to develop near the dent. This spread part-way round the circumference of the middle panel and for one of the shells (Figure 6.24) it occurred to a lesser extent in the adjacent panel.

6.3.3.2 Stress Distributions and Failure Modes

Stress distributions at peak load for two of the dented cylinders ($d/D = .0147$ and .228) are shown in Figures 6.21 and 6.22. When compared with the dented but otherwise perfect models, more bending is apparent around the rest of the circumference outside the dent area. The average axial stresses after the first peak dropped again outside the dent and then the distribution evened out in the rest of the shell. Failure was due to plasticity in the critical area. Yielding began to develop in the dent area early during the loading at around .3 of squash and this spread around the

whole of the circumference and to a lesser extent to the adjacent panels, (Figure 6.24).

An outward bulge failure developed outside the dent area with its maximum value adjacent to the dent. The failure plots show that the adjacent panels also deflect in the outward bulge mode. Failure, however, was triggered in the centre panel where the outward deflections associated with the dent increased the radial deflections and triggered collapse in that panel.

The axisymmetric form of the initial imperfections determined the prebuckling deformations and the failure mode of the cylinder. The lobular form of the dent and its associated imperfections, although of a larger magnitude, did not trigger failure in a periodic mode.

6.3.3.3 Residual Strength

Collapse stress predictions are plotted in Figure 6.25 alongside the simple analytical predictions [12]. In this plot the strength values are non-dimensionalised with respect to the perfect cylinder strength. A third curve is also plotted showing the simple analytical strength predictions relating to the effect of dents on an initially perfect shell for comparison purposes.

A smooth strength reduction can be observed with increasing dent depth, unlike the initially perfect shell (Figure 6.14). This is similar to the trend observed in the strength degradation curve predicted by the simple analytical method, except that the finite element predictions were higher by about 15 percent.

The presence of axisymmetric initial imperfections appears to determine the prebuckling deformations and the failure mode of the model. The significance of this is that the dent, for the range considered here, did not significantly affect the overall form of collapse and effectively only acted as a weak zone reducing the effective section of the cylinder. The outward deflections associated with the dent which increased in magnitude with loading did not appear to have any added effect on the strength deterioration.

6.3.4 Shells with Periodic Initial Imperfections

6.3.4.1 Load Deflection Paths

Load deflection curves for the levels of denting analysed are plotted alongside the

intact equivalent shell response. A nonlinear response can be seen in all the load deflection paths (Figures 6.25–6.29). The initial stiffness again reduced with increasing dent depth. The initial part of the curves reflected the geometric nonlinearity caused by the presence of the initial imperfections and the dent imperfections in the case of the dented shells. This was accompanied by some material nonlinearity. The intact shell started to yield at around .534 of squash. Some yielding at a lower load of .37 in the more severely dented model was apparent.

Shells with the smaller dents (Figures 6.26 and 6.27) exhibited sudden unloading, while for larger dents the stiffness reduced gradually upon approaching collapse. This indicated that significant plasticity developed before failure.

6.3.4.2 Stress Distribution and Failure Modes

Stress distributions at peak load show that deflections around the cylinder are of a lobular nature (Figures 6.30 and 6.31). High bending stresses can be seen around the circumference. Higher stresses reaching yield develop inside the dent.

It is interesting to note that the bending stresses at around θ of 120 were higher than those at θ of 150 which are closer to the dent. This effect is explained by looking at the failure mode plots in Figure 6.32. By comparing the deflections in the intact shell with those for d/D of .0089 it is clear that the dent coincides with the point of maximum outward initial imperfections. Hence dent deflections were out of phase with the initial imperfection form. This reverses the deflections in elements 2 to 4 from inward to outward and increases the inward deflections further away. At this latter location high bending occurs which is reflected in the stress distribution plot.

The intact shell fails in a lobular mode comprising twenty half-waves around the circumference. This is the mode of the initial imperfection. Very little yielding develops before collapse.

All the dented cylinders fail with the dent deflecting inwards and the rest of the cylinder failing in a lobular form. The shells that included the smaller dents, although they exhibited some surface yielding, failed mainly elastically in a lobular mode extending across all the panels. This elastic form of failure is reflected in the sudden drop in the load deflection path.

The two shells, which included the larger dents failed in a lobular mode but in this

case the failure mode was concentrated around the centre panel (Figure 6.31, $d/D = .0230$). Failure involved significant plasticity (Figure 6.33) in the area of the large deflections outside the dent. This is reflected in the well rounded peaks of the load deflection path (Figure 6.29).

6.3.4.3 Residual Strength

The strength degradation curve is different from the smooth curve predicted by the simple analysis (Figure 6.34). A slightly wavy response was obtained from the finite element analyses with a very mild average downward slope. In fact some increase in strength could be observed for the first and third dented shells while for the second one hardly any reduction could be seen in the strength. Although these results seem surprising the following explanation is proposed for them. As the dent is similar in form to the periodic initial imperfection mode some interaction may have occurred. As discussed in the previous section, for this case the presence of the dent locally reduced the magnitude of the imperfections in the critical area outside the dent and to some extent cancelled the mode present. This resulted in an increase in the critical stress in the area. For the smaller dents this effect was bigger than the weakness introduced by the dent itself, hence resulting in an overall increase in strength.

The significance of this is that the dent form can interact with the initial imperfection mode to either increase or decrease the strength depending on the magnitude, direction and relative wavelengths. Judging from the magnitude of the increase in strength obtained here, however, the interaction effect is small.

It is possible that when the initial imperfections and the dent are in phase a bigger strength reduction may occur due to the two effects being additive. The latter, however, has not been observed in the present analyses and is yet to be confirmed.

6.4 RESULTS OF ANALYSES – SHELLS WITH R/t OF 500

6.4.1 Dent Deflections

Figures 6.35–6.37 present results on the deflections due to denting. The overall response to denting (Figure 6.36) is very similar to that observed in the previous geometry (Figure 6.2) and so is the relation between the inward and outward deflections of the dent for different levels of denting (Figures 6.37 and 6.3). This is perhaps a little surprising since more localised deflections would be expected to occur in the more slender of the two geometries. It appears, however, that for the range

of very high slenderness examined herein there is little dependence of the deflection on the slenderness parameter.

6.4.2 Initially Perfect Shells

The perfect intact shell failed in a purely elastic manner which is reflected in the load deflection path (Figures 6.38–6.45). The finite element strength prediction was .682 of yield which is very close to the elastic critical strength, .678 of yield, as predicted by classical elastic theory.

An increasing drop in stiffness was again observed as the dent increased in depth. All the dented models except for one (shell with d/D of .0104) failed prior to any yielding.

The two larger dents exhibited a first peak with a sudden unloading after which the cylinder could re-load. Axial stress distributions again reflect the lobular mode of deflections induced by the dent imperfections (Figures 6.46 and 6.47). All models failed in a lobular form of failure (Figure 6.48) which was triggered in an area well outside the dent (θ of 145 and 137) where the axial stress reached the equivalent intact strength. As observed in the experiments and in previous analyses, higher stresses that reached yield occurred at the edge of the dent, but these did not trigger collapse.

Residual strengths are plotted in Figure 6.49. Again the strength degradation curve has a wavy form, unlike the simple analysis results. Two areas can be identified in which the dent causes bigger strength reductions than the simple analysis predicts. Firstly as in the case of the shells with R/t of 300, the smaller dents were found to reduce the strength significantly. Comparisons between the dent wavelength and the elastic critical wavelengths show that these dents are closer to the critical value. The critical lobular form of failure comprises 18 full waves around the circumference. The first two dents corresponded to 16 and 11 full waves. This reinforces the belief that when the dent is close to the critical wavelength it may result in a greater loss of strength than the simple analytical method would predict by acting as a sympathetic imperfection.

A second range of dents that appear to cause a relatively large effect on the strength is that between d/D of .190 and .0267. The wavelengths of these dents were well away from the critical wavelength being around three times the latter. There is another point which characterises the two smaller dents (d/D of .0190 and 0.230) that could explain the large drop in strength. Comparisons between the dent inward and

outward deflections (Figure 6.37) have shown that around this level of denting the two wavelengths are equal. In other words these shells contained three half-waves of the same wavelength forming a well defined periodic initial imperfection. This may be the reason for the early failure of these cylinders.

It would be interesting to see what the effect could be when the wavelength of the dent inward and outward imperfections are equal and also coincide with the critical wavelength. The latter is a potentially critical combination.

6.4.3 Shells with Axisymmetric Initial Imperfections

All shells exhibited nonlinear response due to both geometric effects and material yielding (Figures 6.50–6.57). The shape of the load deflection paths was very similar for all the models, which indicates that no change occurred in the response and failure mechanism as the dent level increased. Another feature of the load–deflection paths was that they all converged to the same curve in the post-buckling range. The latter was also seen in the previous set of shells with axisymmetric imperfections. The significance of this is that the dent only affects the pre-buckling path and collapse load but not the post-buckling path.

Stress distributions (Figures 6.58 and 6.59) reflect the high bending that develops around the cylinder due to the axisymmetric form of initial imperfections. Failure modes shown in Figure 6.60 show the centre panel failing in a mode determined by the shape of the initial imperfections. Even in the case of these very slender shells the lobular form of the dent and its associated imperfections do not overcome the tendency to fail in the form of the dominant general imperfection mode.

As seen in the previous less slender group of shells, the residual strengths predicted by the analyses for a smooth curve which is similar in form to the simple analytical curve (Figure 6.61). Finite element predictions are again higher and this may be attributed to the non-sympathetic form of the dent to the axisymmetric tendency of the initial imperfections.

These strength predictions are all clearly lower than those induced by similar dents in initially perfect shells. It appears, however, that while general initial imperfections reduce the base strength of the shell they mitigate to some extent the interaction effect of the dent by predetermining the collapse. It would be interesting to examine cases where imperfections are of a more general form and not of a dominant mode. In such a case dent–form interaction may still be possible.

Another conclusion that could be drawn from the results of both sets of geometries is that larger dents which are accompanied by larger outward imperfections in the critical area do not have any increasingly worsening effect on the strength.

6.4.4 Shells with Periodic Initial Imperfections

Load deflection responses for these shells are shown in Figures 6.62–6.66. A highly nonlinear response can be seen, again due to the presence of initial imperfections and the dent. Stress distributions (Figure 6.67 and 6.68) show the lobular mode of deflections spreading well outside the dent.

The intact shell failed elastically in the mode of the initial imperfections. All the dented shells failed in a lobular mode (Figure 6.69). The intact areas well outside the dent assumed the initial imperfection mode. Failure was triggered in the critical area outside the dent. In the first three dented shells failure occurred before the onset of plasticity when the stress in the critical area was close to the intact equivalent collapse stress. In the case of the two bigger dents, yielding in the critical area developed before collapse occurred.

The three bigger dents caused an increase in strength of up to 12 percent (Figure 6.70). Surprising as it may seem, this was also observed in the previous geometry, but to a lesser extent. This was attributed to a possible interaction between the dent form and periodic mode of imperfections which seems to have a bigger effect in the more slender geometry.

As for the previous geometry, only negative interaction was observed, which resulted in increases in the collapse loads mainly because the dent form was out of phase with the imperfection mode.

6.5 DISCUSSION AND CONCLUSIONS

The biggest relative effect on the strength has been induced on the perfect shells. The latter exhibited wavy strength degradation curves indicating certain areas of higher sensitivity to the dent form. Firstly, for smaller dents the wavelength of which was close to the critical elastic wavelength, higher relative strength reductions occurred. This suggests that the dent may interact with the critical mode of collapse and act as a sympathetic imperfection. This phenomenon, however, is only possible if there is no other preferred mode of collapse due to a dominant imperfection mode.

In the case of the more slender shells examined above, another area has been identified where higher strength reductions may occur. This has been observed in the range of dents for which the wavelengths of the inward and outward deflections are the same. Such a dent comprises three continuous half-waves of the same wavelength, thus forming a well defined initial imperfection that can trigger collapse in a similar mode. Perhaps a more critical combination would occur when the wavelengths of inward and outward deflections are the same and equal to the elastic critical wavelength. More work is needed to confirm this effect.

When initial imperfections were included in the shells two distinct types of behaviour were observed corresponding to the two sets of imperfections examined.

Where the dent was completely non-sympathetic to the initial axisymmetric imperfection mode no interaction effects were observed. The dent only acted as an area of "weakness" and the initial imperfections determined the failure mode. A smooth strength reduction was observed with increasing dent depth. Collapse loads were higher than simple analytical predictions.

When a lobular mode of initial imperfections was included a difference in behaviour was observed. The dent being similar in form with the imperfections, interacted with the imperfections in the critical area, thus affecting the strength. A variable effect on the strength was obtained depending on the magnitude and relative wavelengths of the dent and the general imperfections. For the analyses performed the interaction effect resulted in an increase in strength by reducing the imperfections in the critical area and increasing the collapse stress there. In theory it is possible that the opposite may occur, i.e. that the dent form will be completely in phase with the imperfection mode, causing a further reduction on the strength. Such response, however, has not been observed in the analyses and is yet to be confirmed.

To summarise the effect of the initial imperfections, it would be true to say that while these reduce the base strength of the cylinder they appear to mitigate the effect of the dent by precluding the latter from triggering a critical collapse mode.

In practice the response will vary between the three cases considered above for the initial state of the shell. Of these only the initially perfect shells showed areas where the dent may have an added reduction effect by acting as a sympathetic imperfection. Results generally show a small increase in sensitivity to denting as the slenderness increases.

Comparisons of the results and the simple analytical predictions have shown the latter to provide a lower bound for most of the cases analysed. Even in the cases where bigger relative strength reductions were predicted by the finite element analyses, a 10 percent added reduction on the simple predictions would be sufficient to provide safe results.

Within the limits of available computer power which is responsible for some of the limitations of the numerical models used, these results have been obtained. There would, however, be scope for confirming these findings using more refined models when this is possible. Furthermore, more work would be necessary to confirm the range of geometries in which the above conclusions remain valid.

Table 6.1: Geometrical Parameters

R/t	ℓ/R	R	t	H_f	t_f	H_w	t_w
300	0.3	300	1	6	1.2	4	0.5
500	0.3	500	1	10	2	6	1

Table 6.2: Imperfection Modes

Imperfection Modes	n	m
Perfect	0	0
Axisymmetric	1, 3, 5	0
Periodic	1, 3, 5	10

Table 6.3: Deflections due to Denting
($R/t = 300$, $\ell/R = 0.3$)

d/D	$\theta_d/2\pi$	$\delta_{do}/2\pi$
.0027	.033	.111
.0052	.046	.108
.0102	.064	.106
.0147	.078	.092
.0201	.091	.083
.0228	.096	.081
.0274	.106	.081

Table 6.4: Analyses of Initially Perfect Shells
 ($R/t=300$, $\ell/R=0.3$, $n=0$, $m=0$)

d/D	σ_d/σ_y	σ_d/σ_{ip}	n_{cr}	n_d	n_{do}
0	.898	1	15	-	-
.0027	.758	.844	15	15.0	4.5
.0052	.755	.840	15	10.9	4.6
.0102	.727	.810	15	7.8	4.7
.0147	.726	.808	15	6.5	5.4
.0228	.696	.775	15	5.2	6.2
.0274	.681	.758	15	4.7	6.2

Table 6.5: Analyses of Shells with Axisymmetric Initial Imperfections ($R/t=300$, $\ell/R=0.3$, $n=0$, $m=1,3,5$)

d/D	σ_d/σ_y	σ_d/σ_i	σ_d/σ_{ip}	n_{cr}	n_d	n_{do}
0	.612	1	.682	15	-	-
.0052	.605	.988	.674	15	10.9	4.6
.0102	.567	.926	.631	15	7.8	4.7
.0147	.561	.917	.625	15	6.5	5.4
.0228	.504	.824	.561	15	5.2	6.2
.0274	.493	.806	.549	15	4.7	6.2

Table 6.6: Analyses of Shells with Periodic Initial Imperfections ($R/t=300$, $\ell/R=0.3$, $n=10$, $m=1,3,5$)

d/D	σ_d/σ_y	σ_d/σ_i	σ_d/σ_{ip}	n_{cr}	n_d	n_{do}
0	.633	1	.705	15	-	-
.0046	.655	1.063	.730	15	11.6	4.5
.0089	.627	1.018	.700	15	8.3	4.6
.0130	.635	1.031	.707	15	6.9	5.1
.0230	.587	.953	.654	15	5.2	6.1

Table 6.7: Deflections due to Denting
($R/t = 500$, $\ell/R = 0.3$)

d/D	$\theta_d/2\pi$	$\theta_{do}/2\pi$
.0024	.031	.111
.0051	.046	.106
.0104	.065	.108
.0151	.078	.094
.0190	.088	.089
.0203	.091	.089
.0267	.104	.089
.0306	.112	.089

Table 6.8: Analyses of Initially Perfect Shells
($R/t=500$, $\ell/R=0.3$, $n=0$, $m=0$)

d/D	σ_d/σ_y	σ_d/σ_i	n_{cr}	n_d	n_{do}
0	.68	1	18	-	-
.0024	.383	.857	18	16.0	6.5
.0051	.556	.818	18	11.0	4.7
.0104	.562	.826	18	7.7	4.6
.0151	.551	.810	18	6.4	5.3
.0190	.452	.665	18	5.7	5.6
.0203	.447	.657	18	5.5	5.6
.0267	.478	.703	18	4.8	5.6
.0306	.301	.737	18	4.5	5.6

Table 6.9: Analyses of Shells with Axisymmetric Initial Imperfections ($R/t=500$, $\ell/R=0.3$, $n=0$, $m=1,3,5$)

d/D	σ_d/σ_y	σ_d/σ_i	σ_d/σ_{ip}	n_{cr}	n_d	n_{do}
0	.478	1	.703	18	-	-
.0051	.466	.975	.685	18	11.0	4.7
.0104	.432	.904	.635	18	7.7	4.6
.0151	.445	.931	.654	18	6.4	5.3
.0190	.412	.862	.606	18	5.7	5.6
.0203	.409	.856	.601	18	5.5	5.6
.0267	.412	.862	.606	18	4.8	5.6
.305	.409	.856	.601	18	4.5	5.6

Table 6.10: Analyses of Shells with Periodic Initial Imperfections ($R/t=500$, $\ell/R=0.3$, $n=10$, $m=1,3,5$)

d/D	σ_d/σ_y	σ_d/σ_i	σ_d/σ_{ip}	n_{cr}	n_d	n_{do}
0	.438	1	.644	18	-	-
.0033	.405	.925	.600	18	13.7	4.5
.0067	.401	.916	.590	18	9.6	4.6
.0091	.440	1.005	.650	18	8.2	4.6
.0139	.490	1.119	.720	18	6.6	5.0
.0220	.470	1.073	.690	18	5.3	5.5

DEFLECTED SHAPE OF DENTED CYLINDER ($R/t=300$, $l/R=0.3$, $n=0$, $m=0$, $d/D=.0274$)

DISPLACEMENT FACTOR = 20

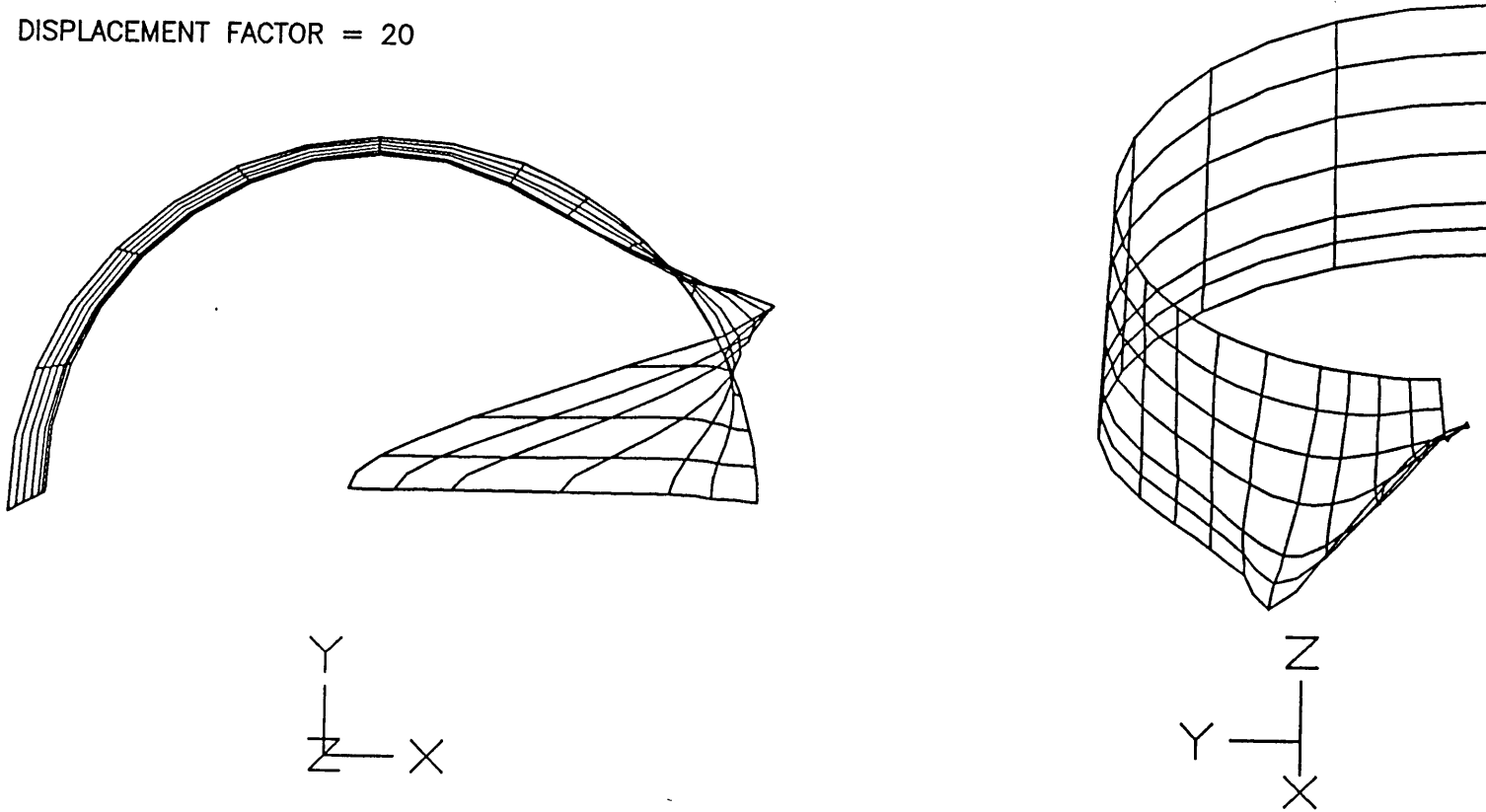
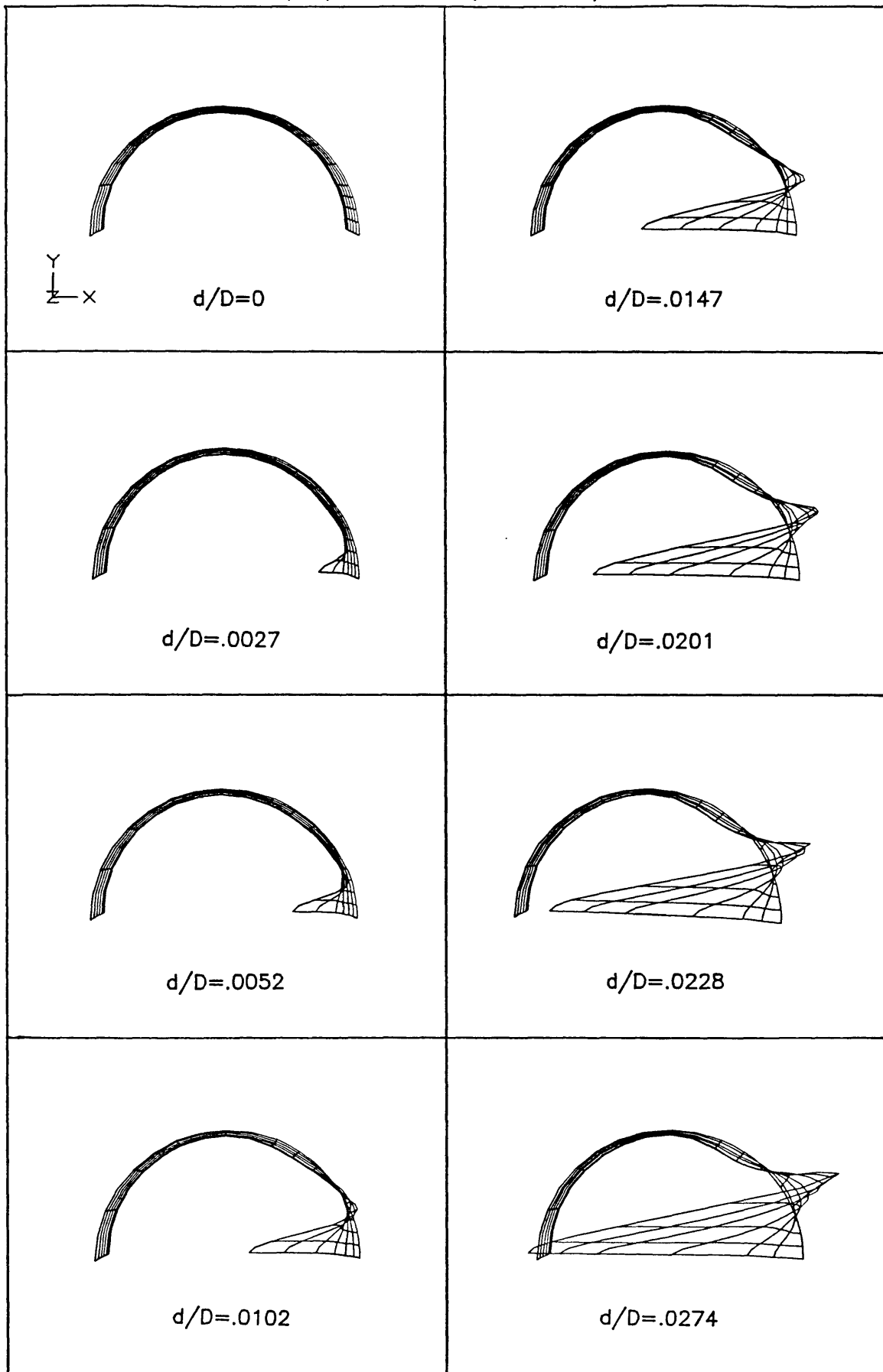


Figure 6.1

DEFLECTIONS FOR DIFFERENT LEVELS OF DENTING
 ($R/t=300$, $l/R=0.3$)



DISPLACEMENT FACTOR = 40

Figure 6.2

RELATIVE WAVELENGTHS OF DENT DEFLECTIONS
 ($R/t=300$, $l/R=0.3$)

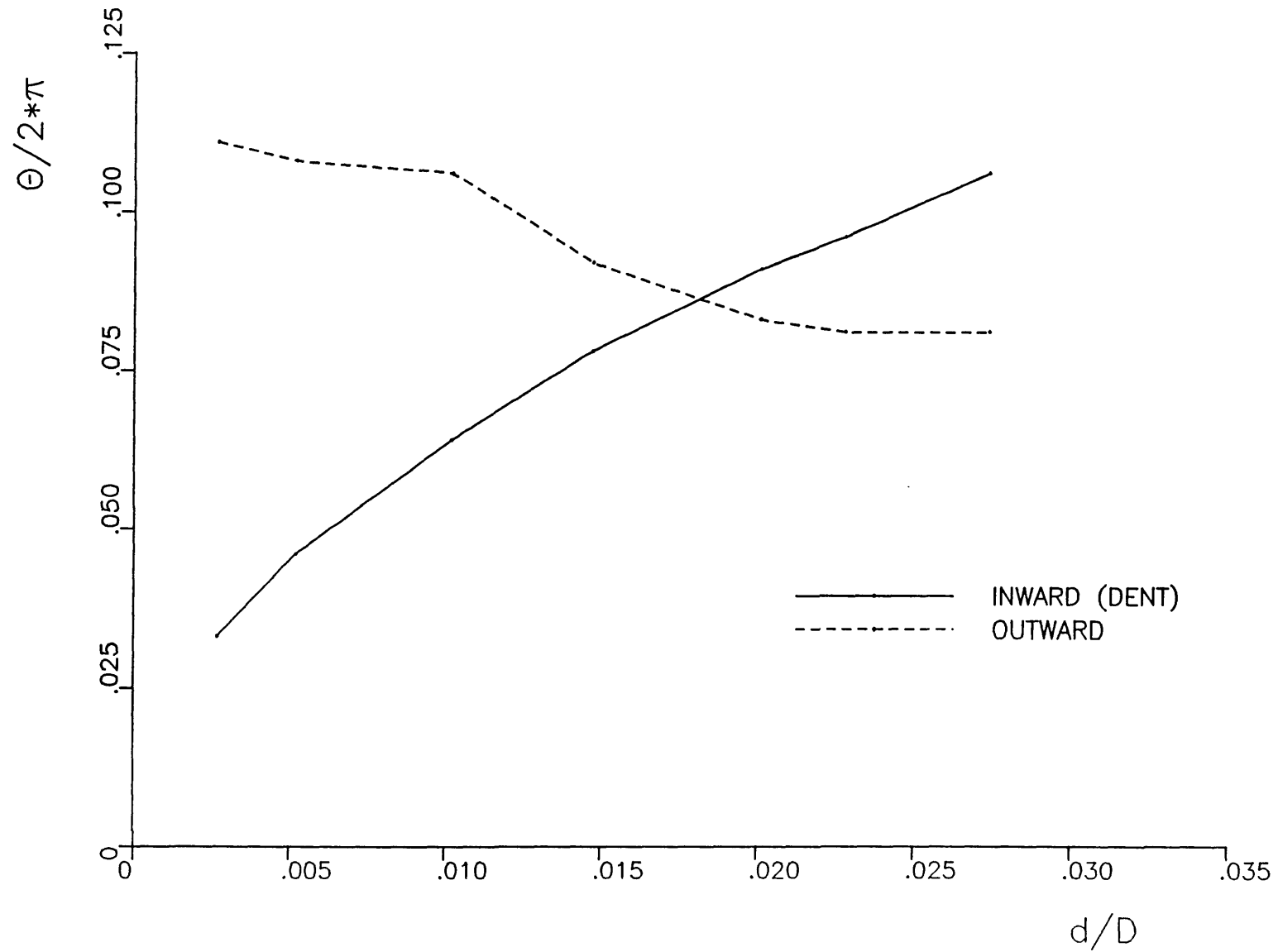


Figure 6.3

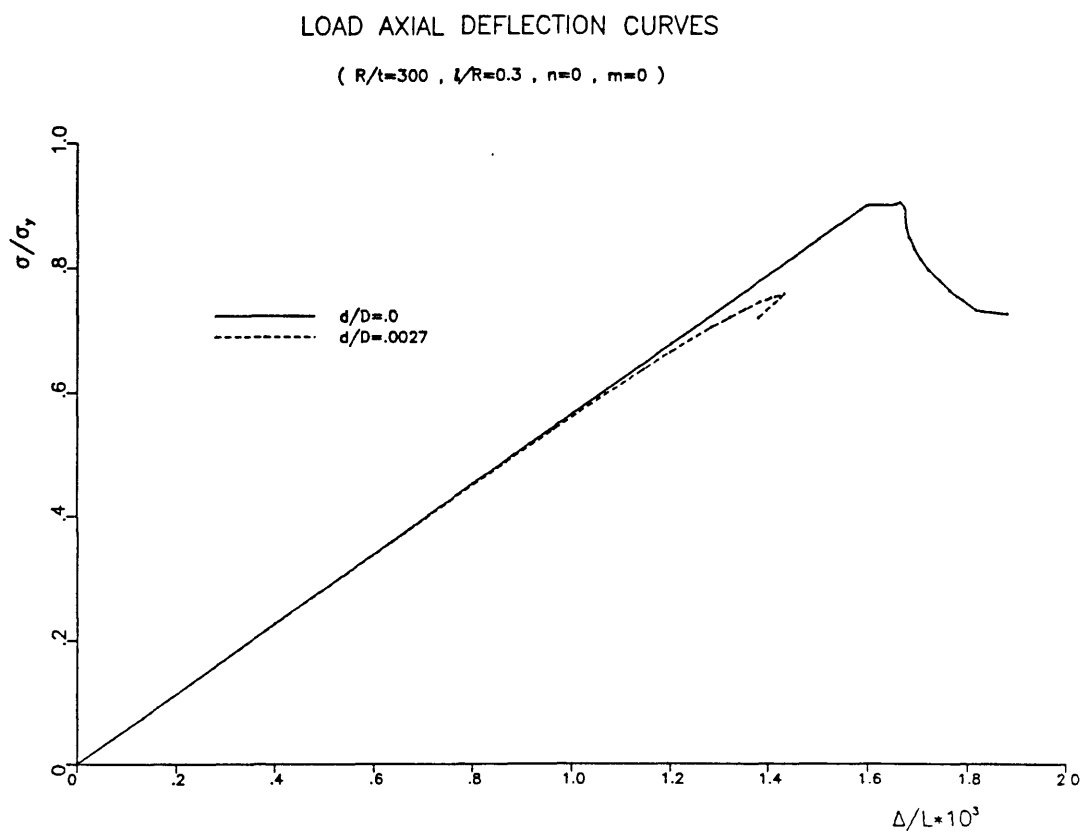


Figure 6.4

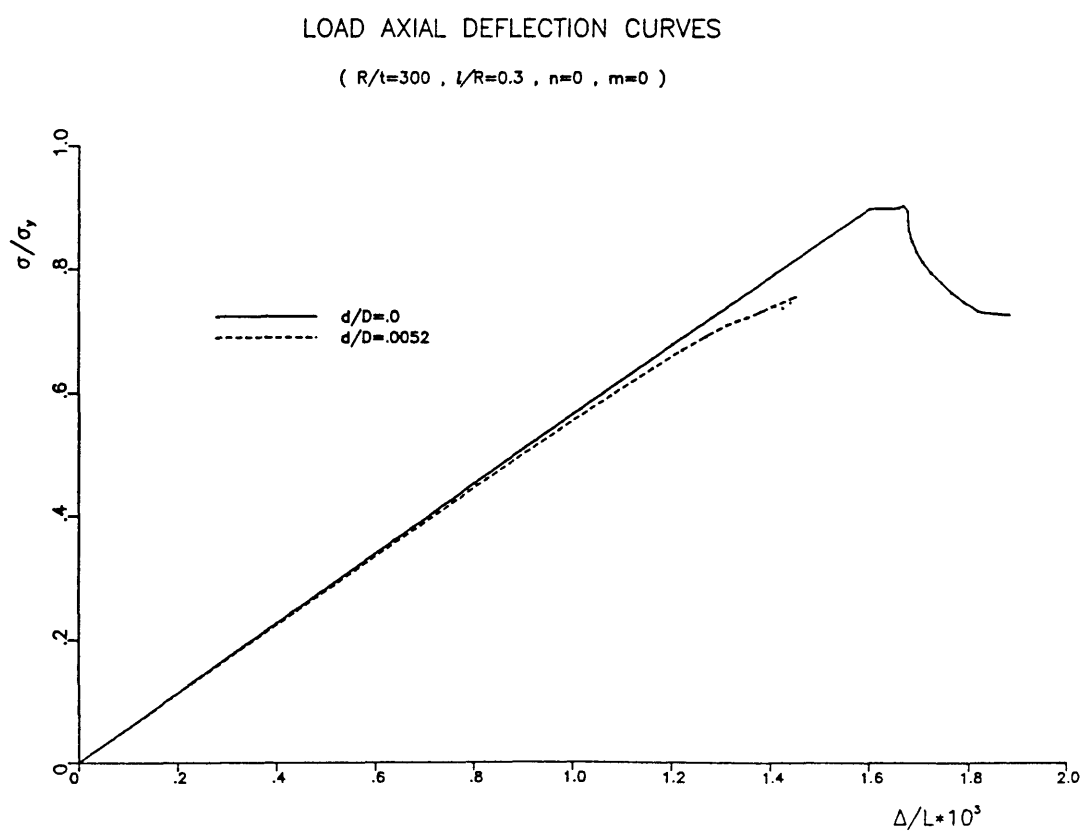


Figure 6.5

LOAD AXIAL DEFLECTION CURVES

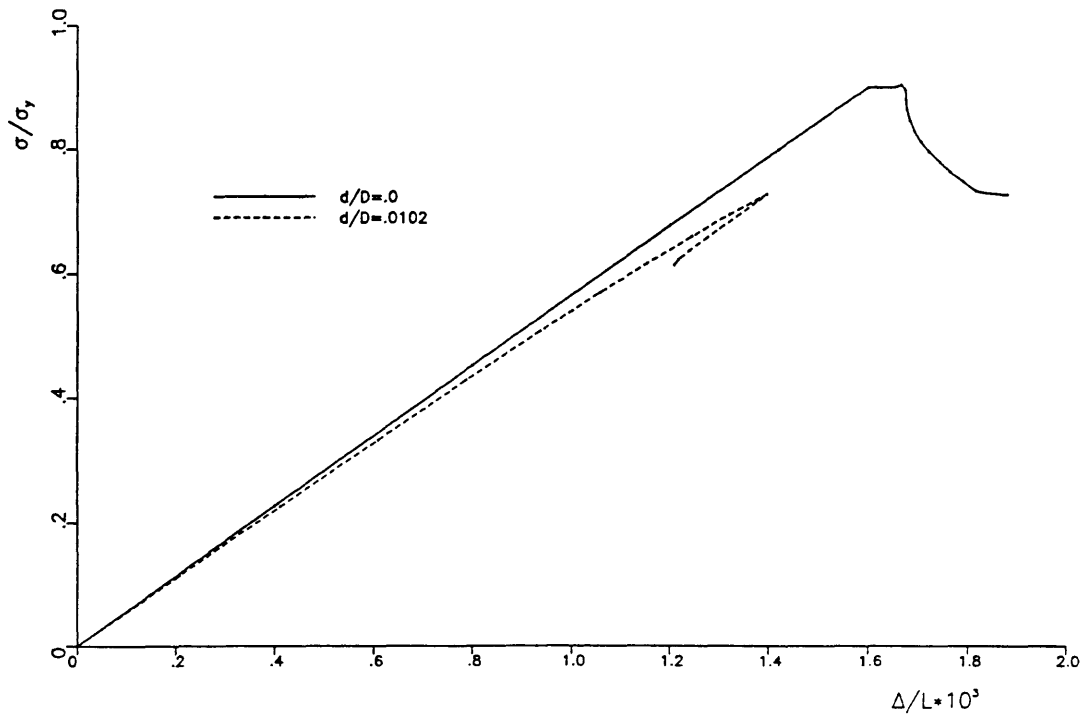
($R/t=300$, $l/R=0.3$, $n=0$, $m=0$)

Figure 6.6

LOAD AXIAL DEFLECTION CURVES

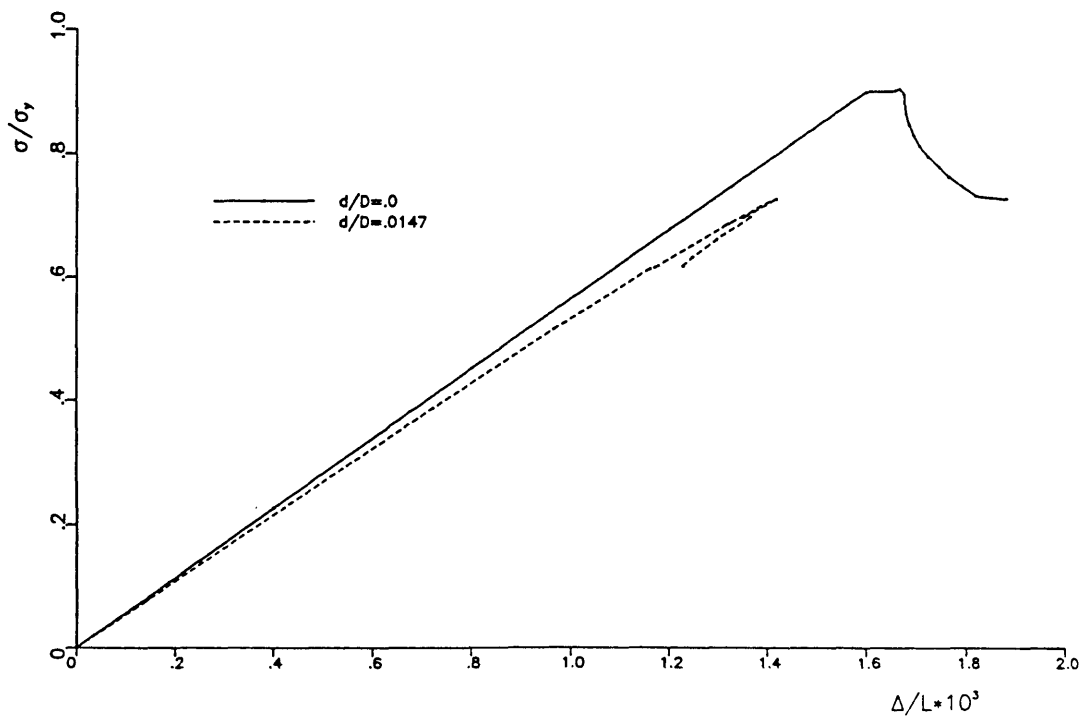
($R/t=300$, $l/R=0.3$, $n=0$, $m=0$)

Figure 6.7

LOAD AXIAL DEFLECTION CURVES

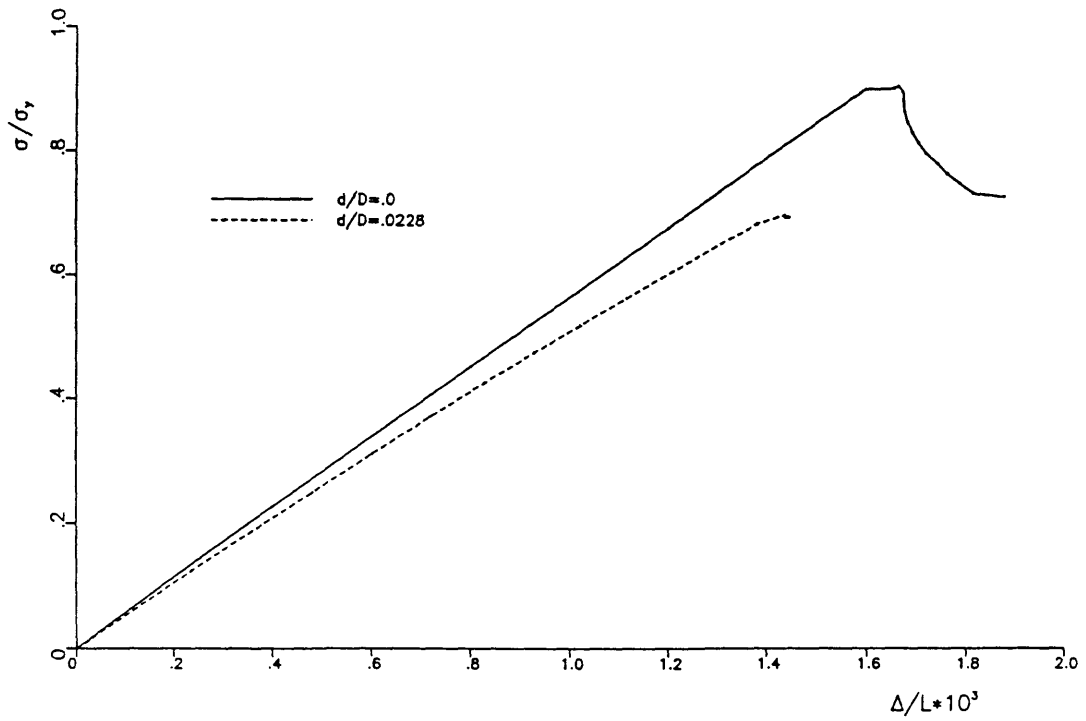
($R/t=300$, $l/R=0.3$, $n=0$, $m=0$)

Figure 6.8

LOAD AXIAL DEFLECTION CURVES

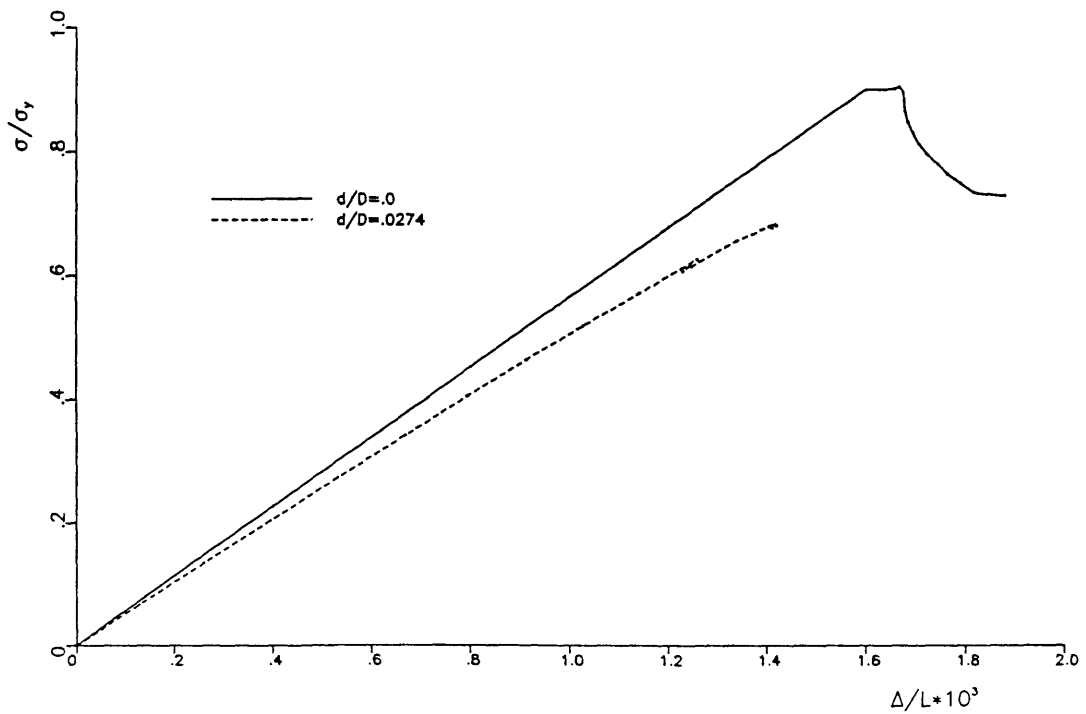
($R/t=300$, $l/R=0.3$, $n=0$, $m=0$)

Figure 6.9

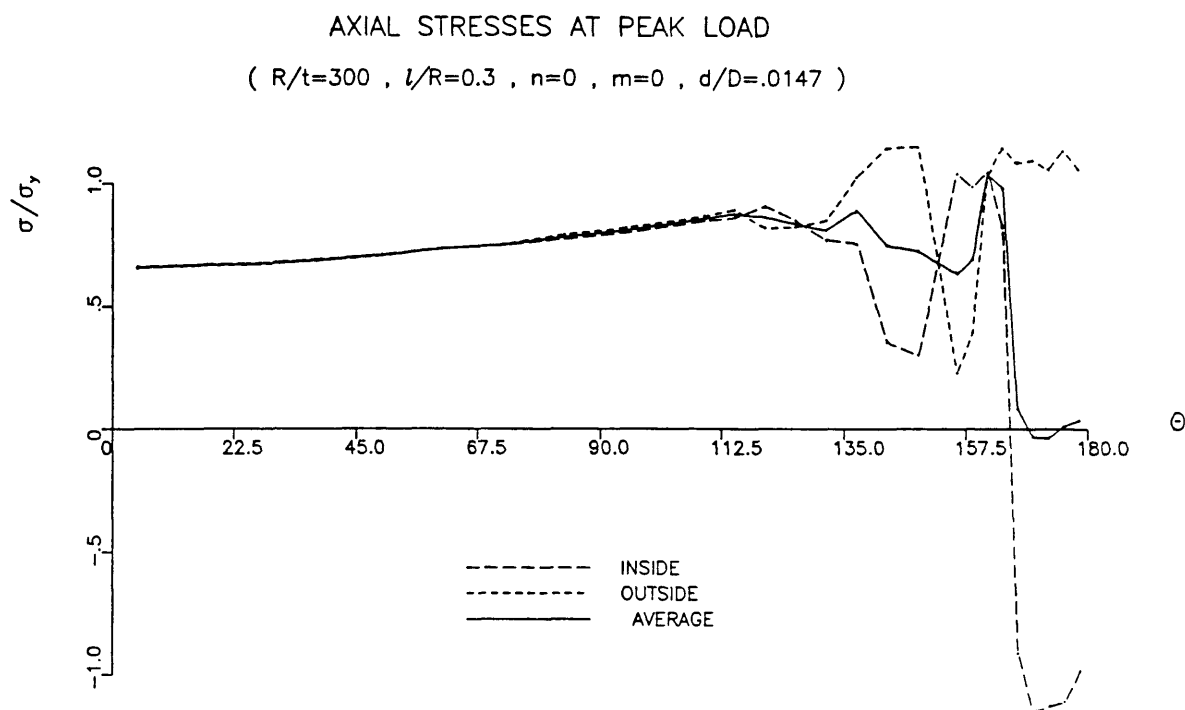


Figure 6.10

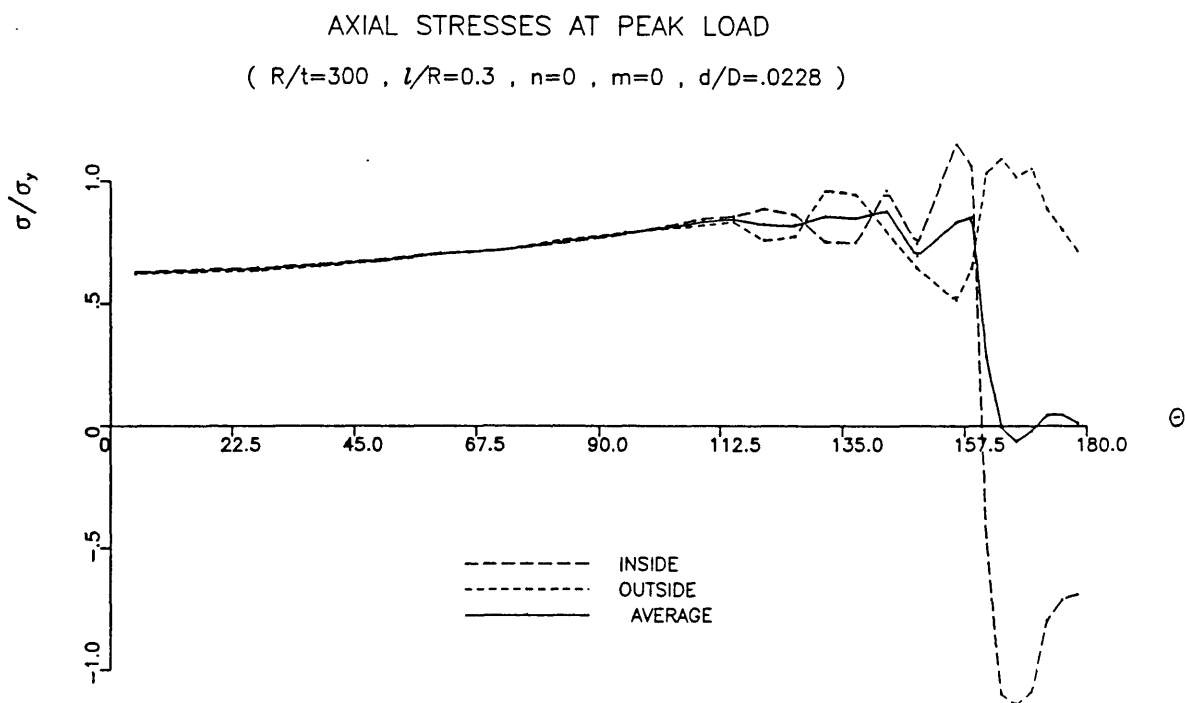


Figure 6.11

FAILURE MODES FOR DIFFERENT LEVELS OF DENTING
($R/t=300$, $l/R=0.3$, $n=0$, $m=0$)

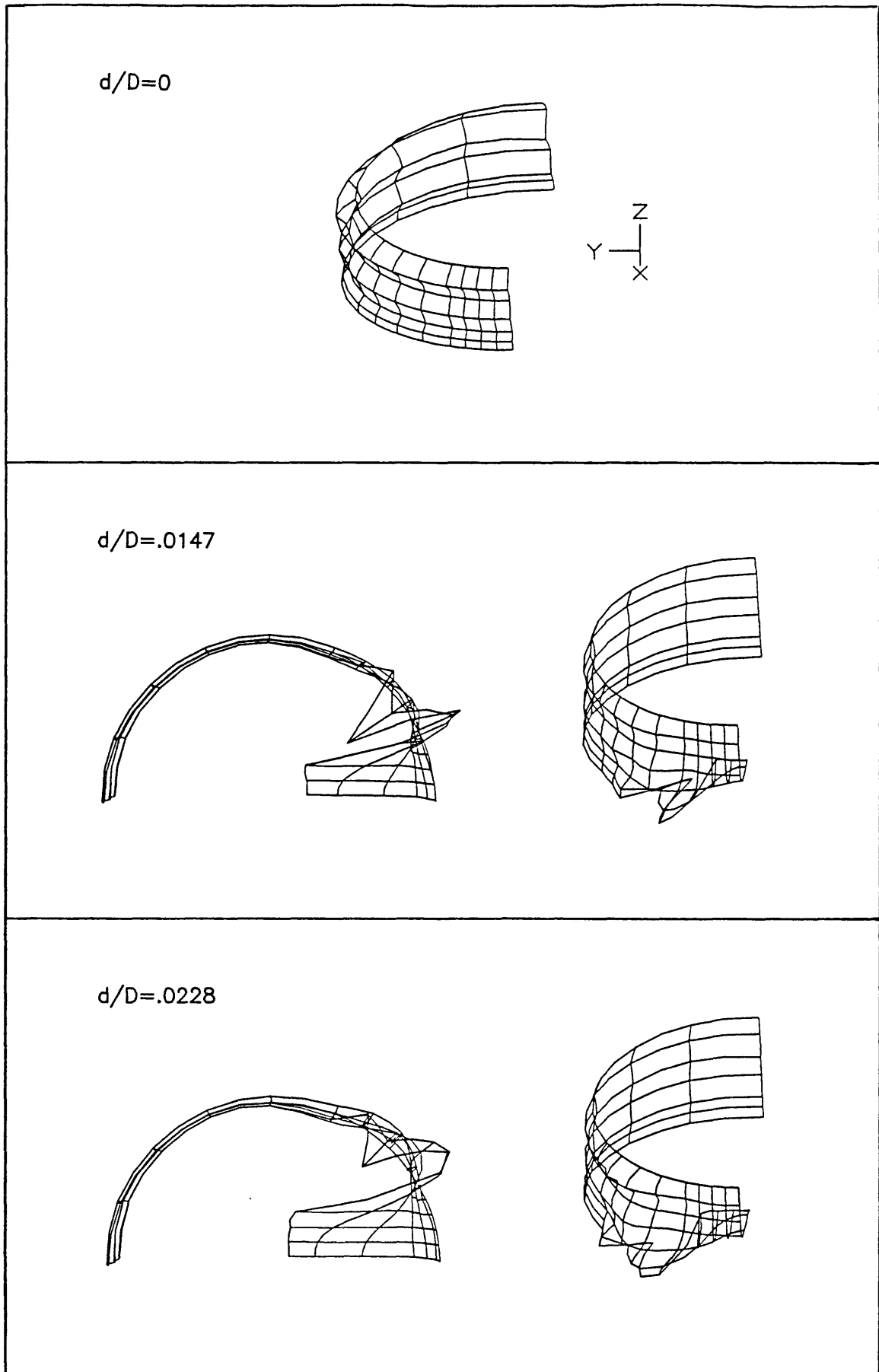
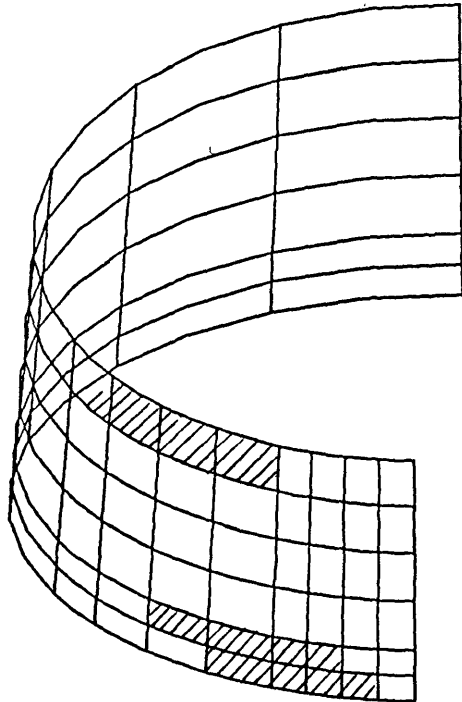


Figure 6.12

PLASTICITY PATTERNS FROM AXIAL COMPRESSION ANALYSIS
($R/t=300$, $l/R=0.3$, $n=0$, $m=0$)

$$d/D=.0147$$

PEAK LOAD



MAXIMUM LOAD

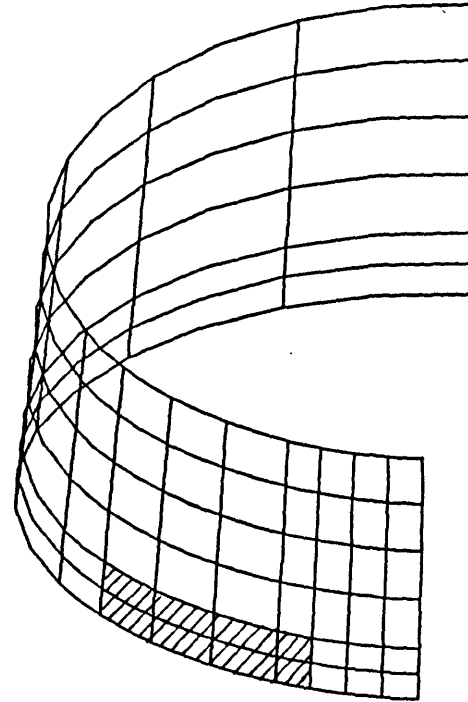


Figure 6.13

AXIAL RESIDUAL STRENGTH ($R/t=300$, $l/R=0.3$, $n=0$, $m=0$)

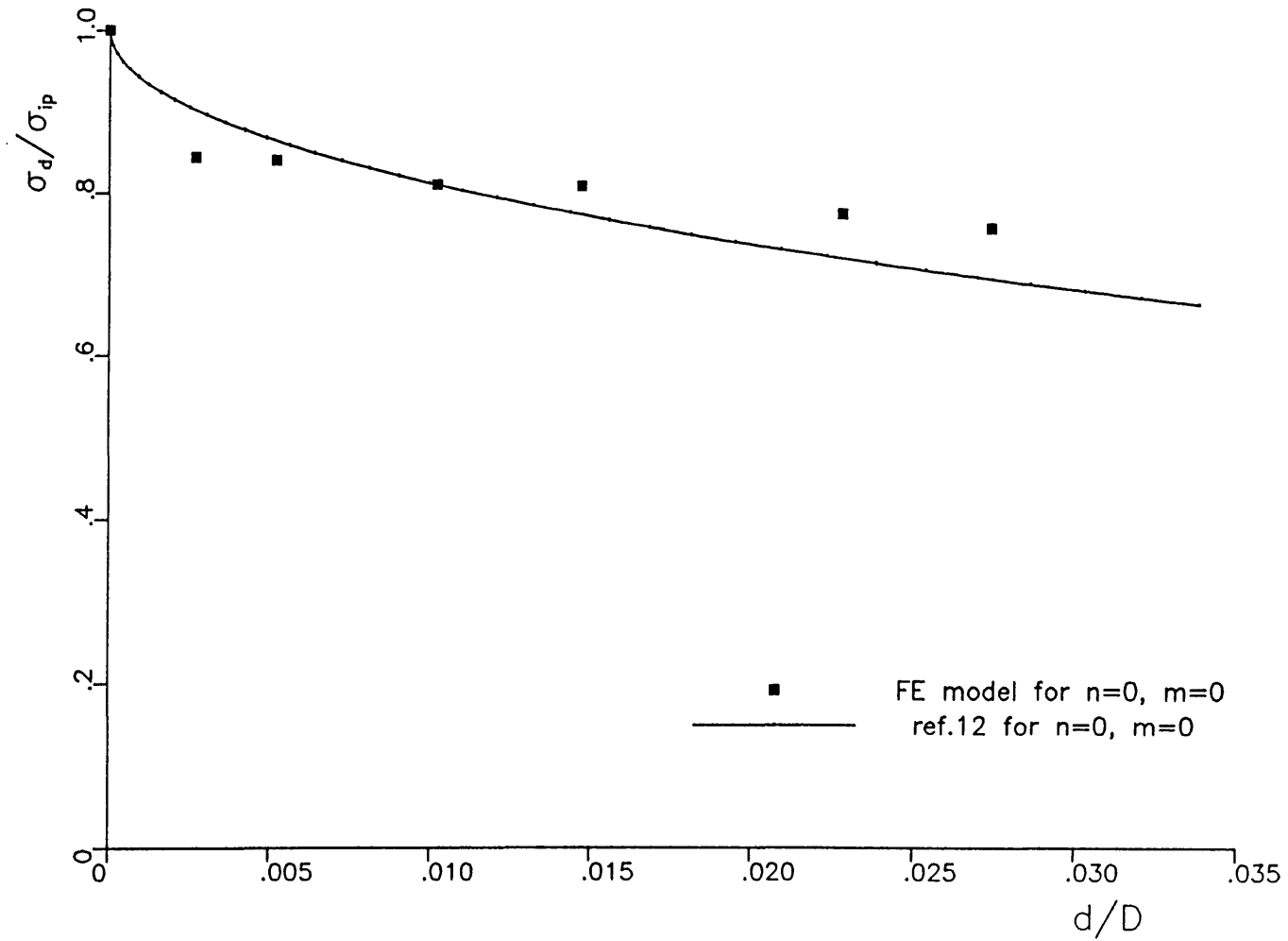


Figure 6.14

LOAD AXIAL DEFLECTION CURVES FOR DIFFERENT LEVELS OF DENTING

($R/t=300$, $l/R=0.3$, $n=0$, $m=1,3,5$)

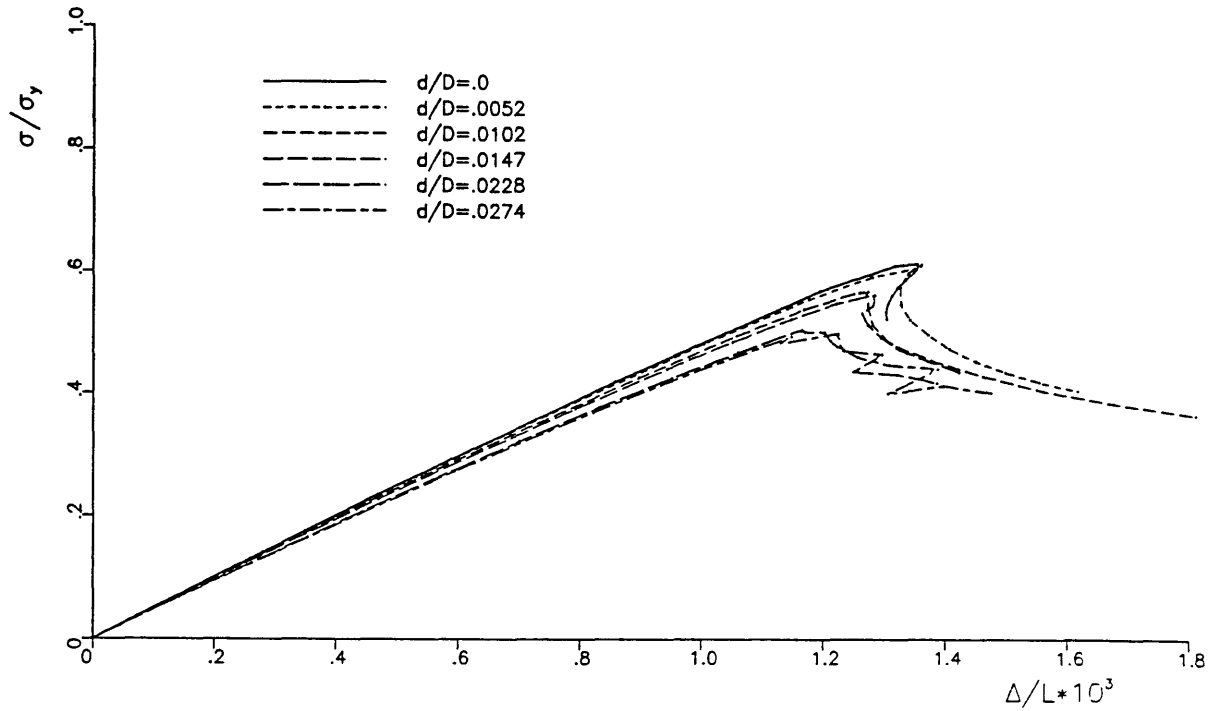


Figure 6.15

LOAD AXIAL DEFLECTION CURVES

($R/t=300$, $l/R=0.3$, $n=0$, $m=1,3,5$)

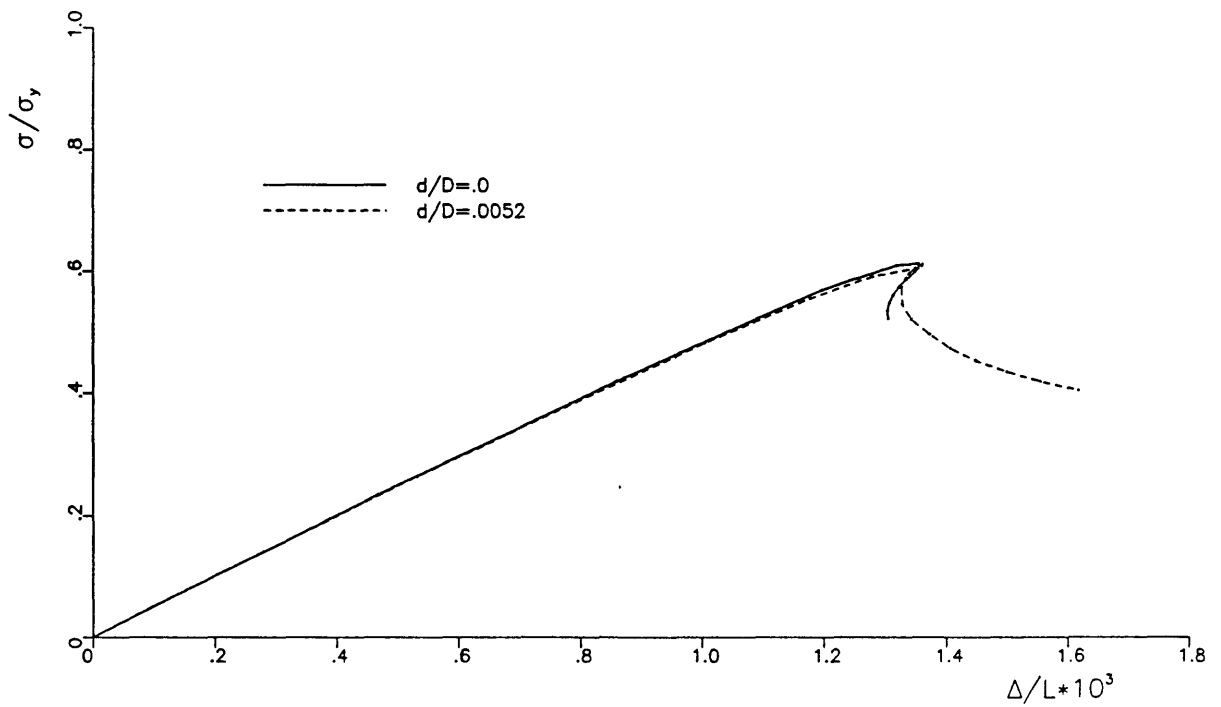


Figure 6.16

LOAD AXIAL DEFLECTION CURVES

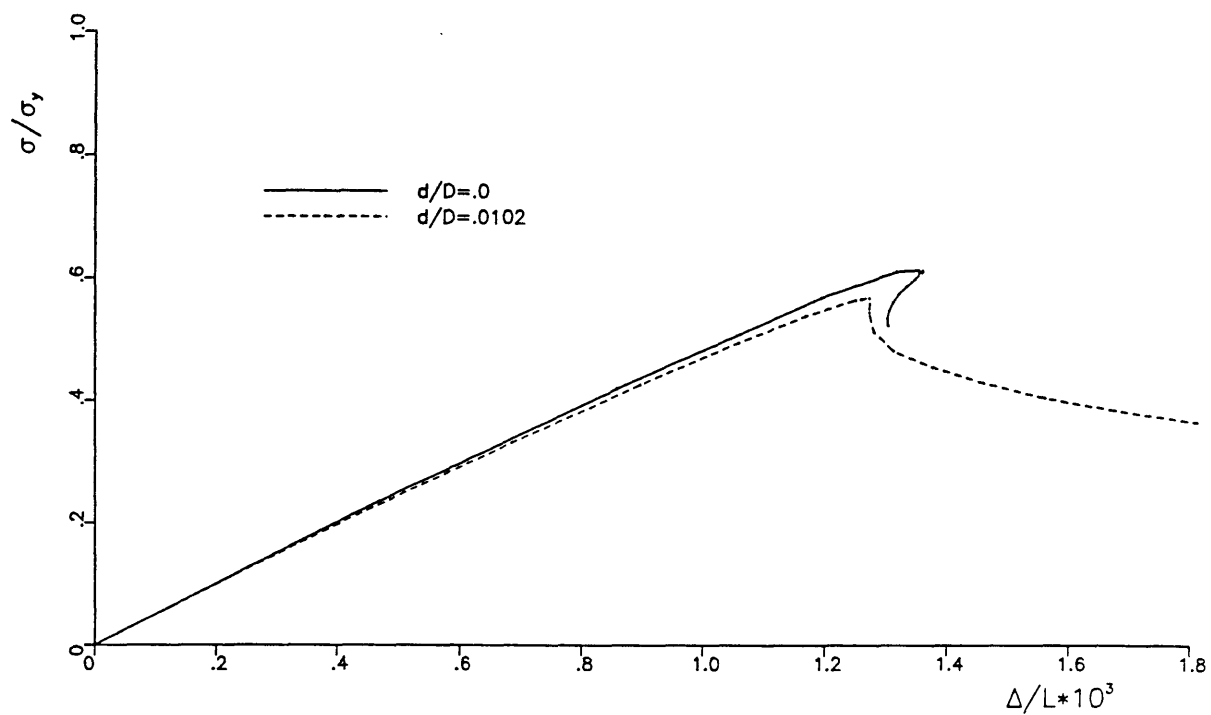
($R/t=300$, $l/R=0.3$, $n=0$, $m=1,3,5$)

Figure 6.17

LOAD AXIAL DEFLECTION CURVES

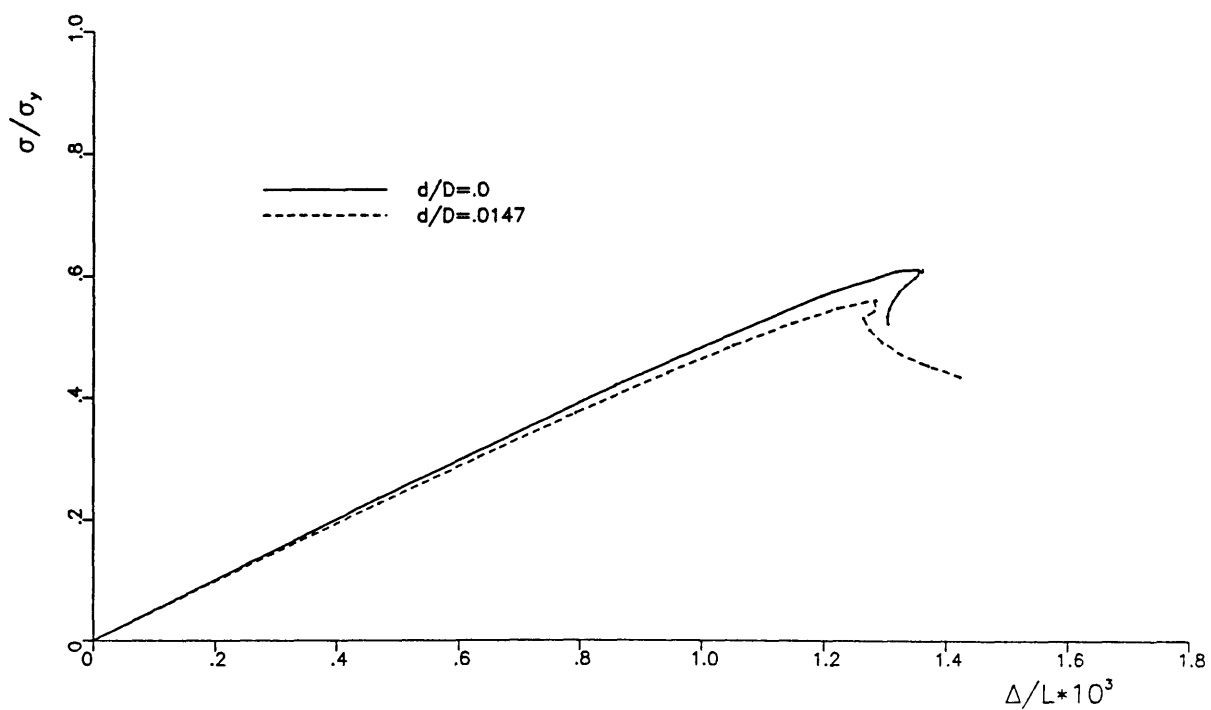
($R/t=300$, $l/R=0.3$, $n=0$, $m=1,3,5$)

Figure 6.18

LOAD AXIAL DEFLECTION CURVES

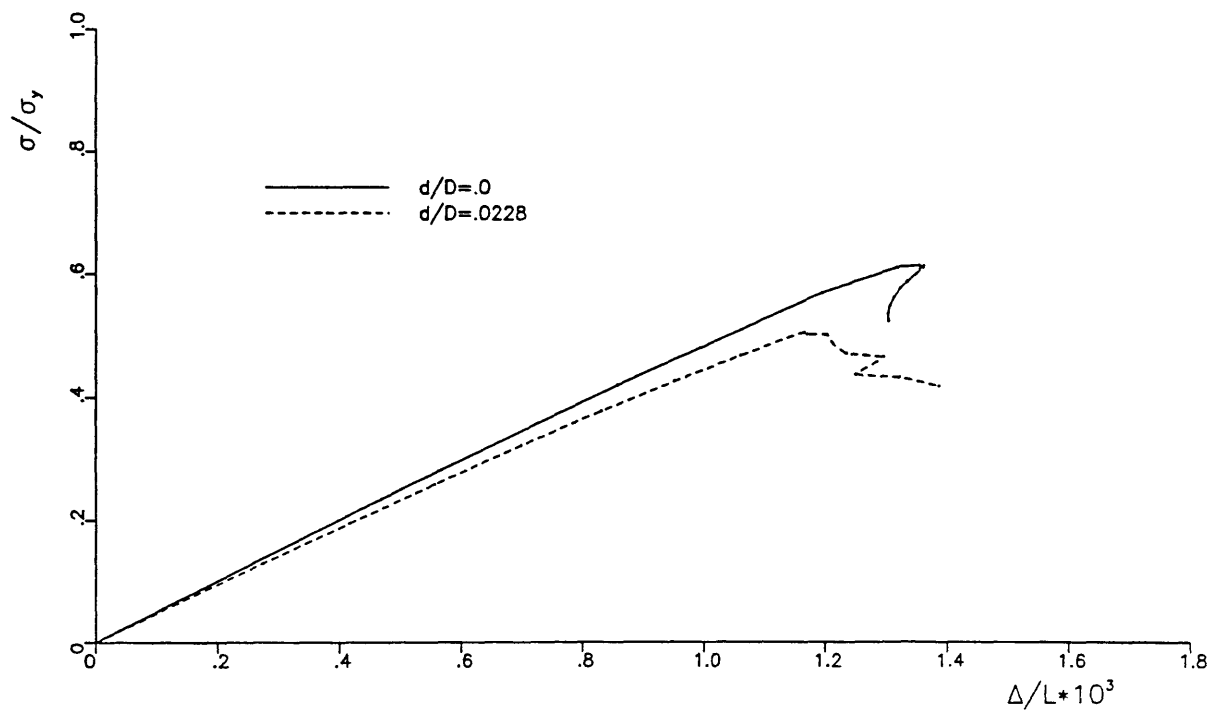
($R/t=300$, $l/R=0.3$, $n=0$, $m=1,3,5$)

Figure 6.19

LOAD AXIAL DEFLECTION CURVES

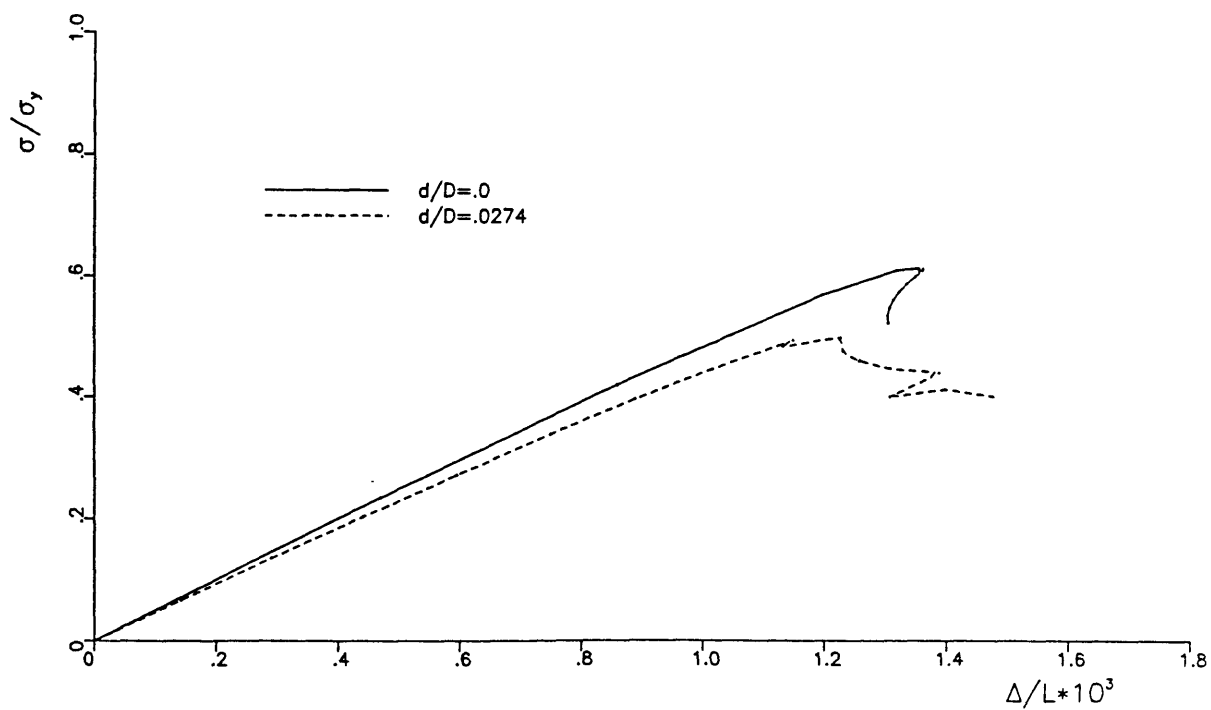
($R/t=300$, $l/R=0.3$, $n=0$, $m=1,3,5$)

Figure 6.20

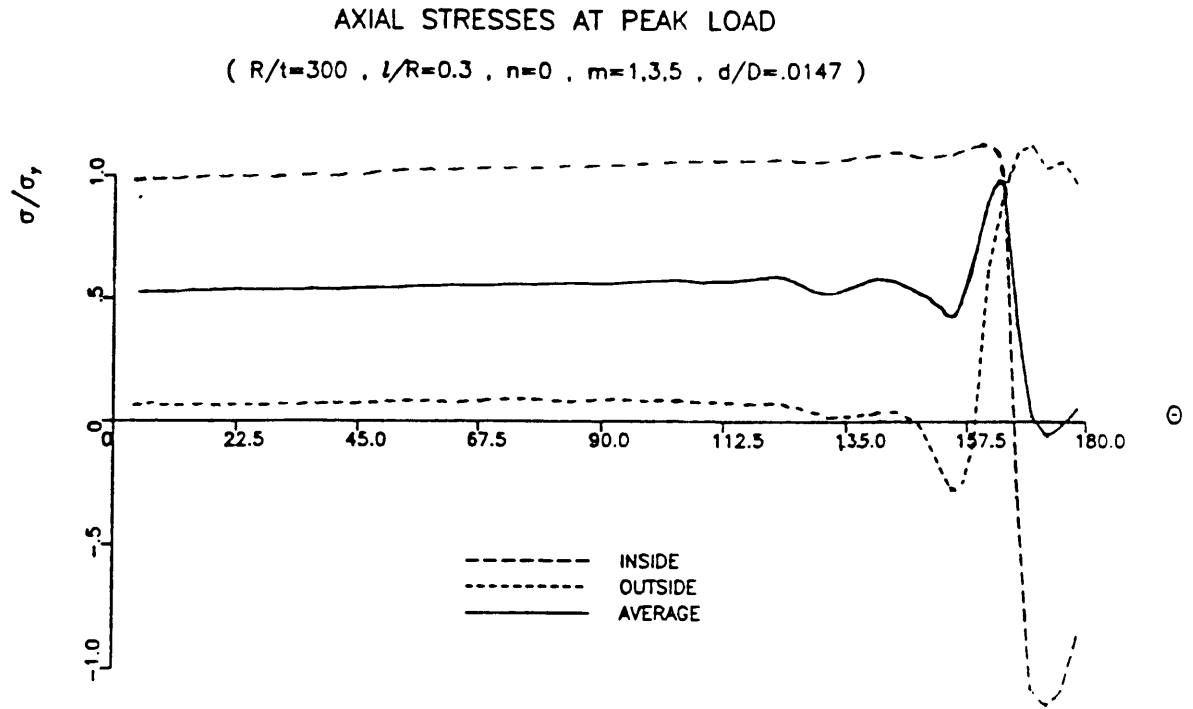


Figure 6.21

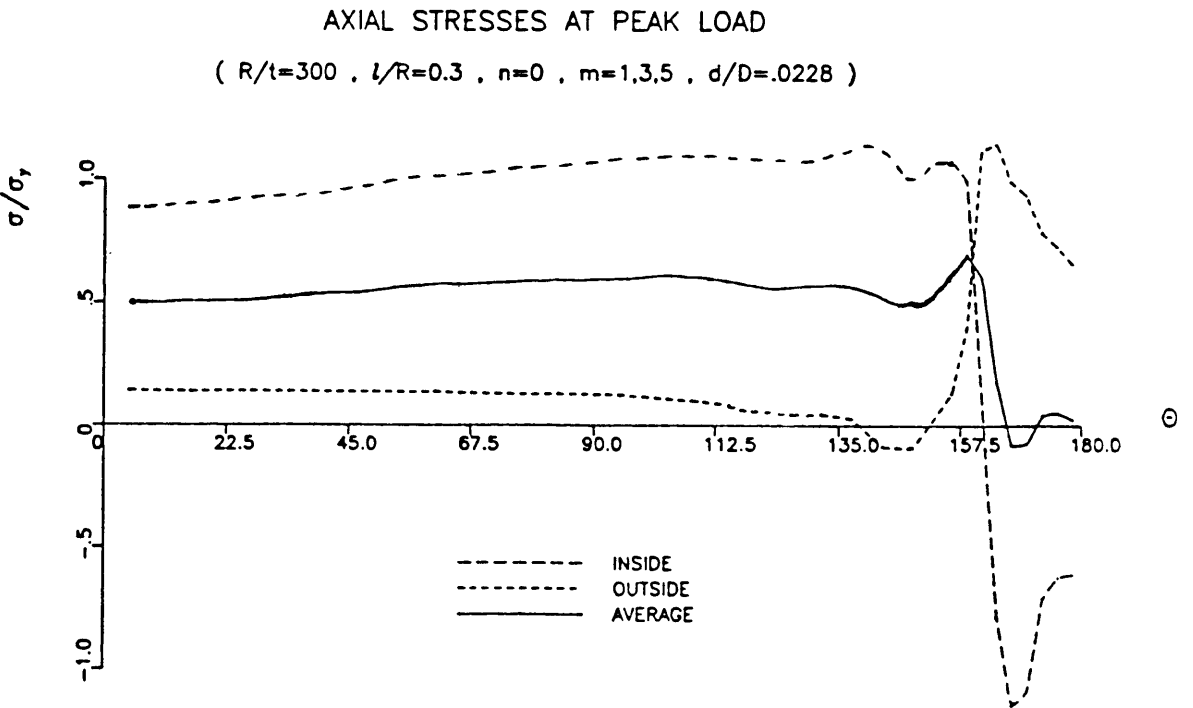


Figure 6.22

FAILURE MODES FOR DIFFERENT LEVELS OF DENTING
($R/t=300$, $l/R=0.3$, $n=0$, $m=1,3,5$)

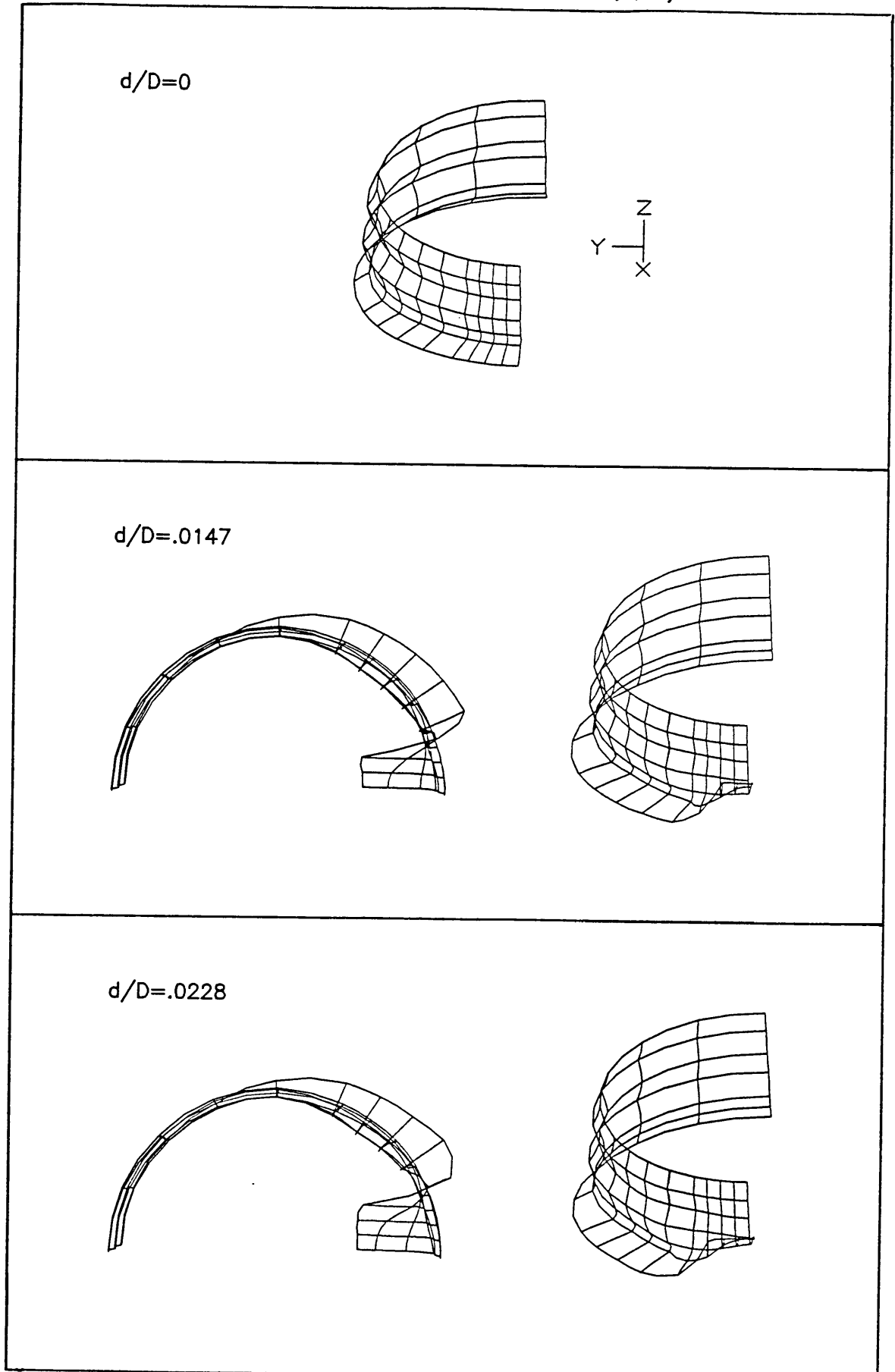
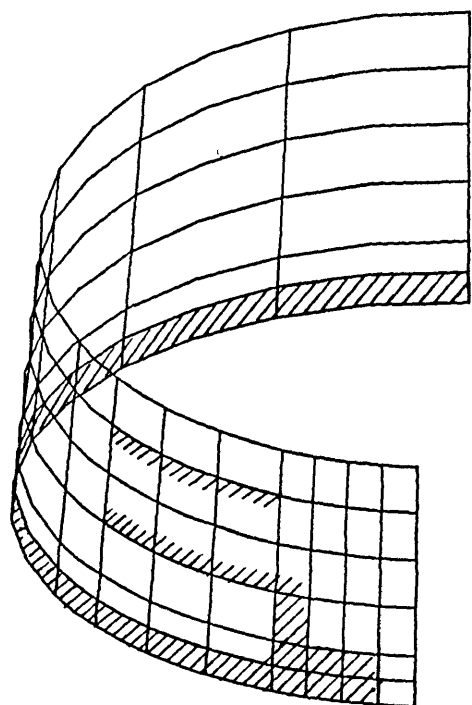


Figure 6.23

PLASTICITY PATTERNS FROM AXIAL COMPRESSION ANALYSIS
 ($R/t=300$, $l/R=0.3$, $n=0$, $m=1,3,5$)

$$d/D=.0147$$

PEAK LOAD



MAXIMUM LOAD

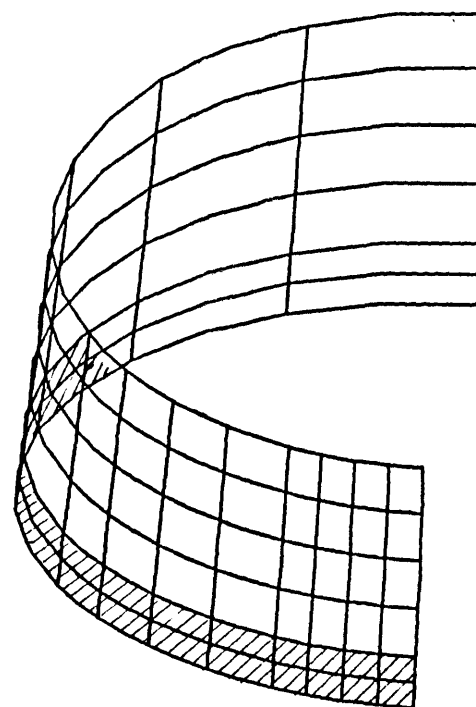


Figure 6.24

AXIAL RESIDUAL STRENGTH ($R/t=300$, $l/R=0.3$, $n=0$, $m=1,3,5$)

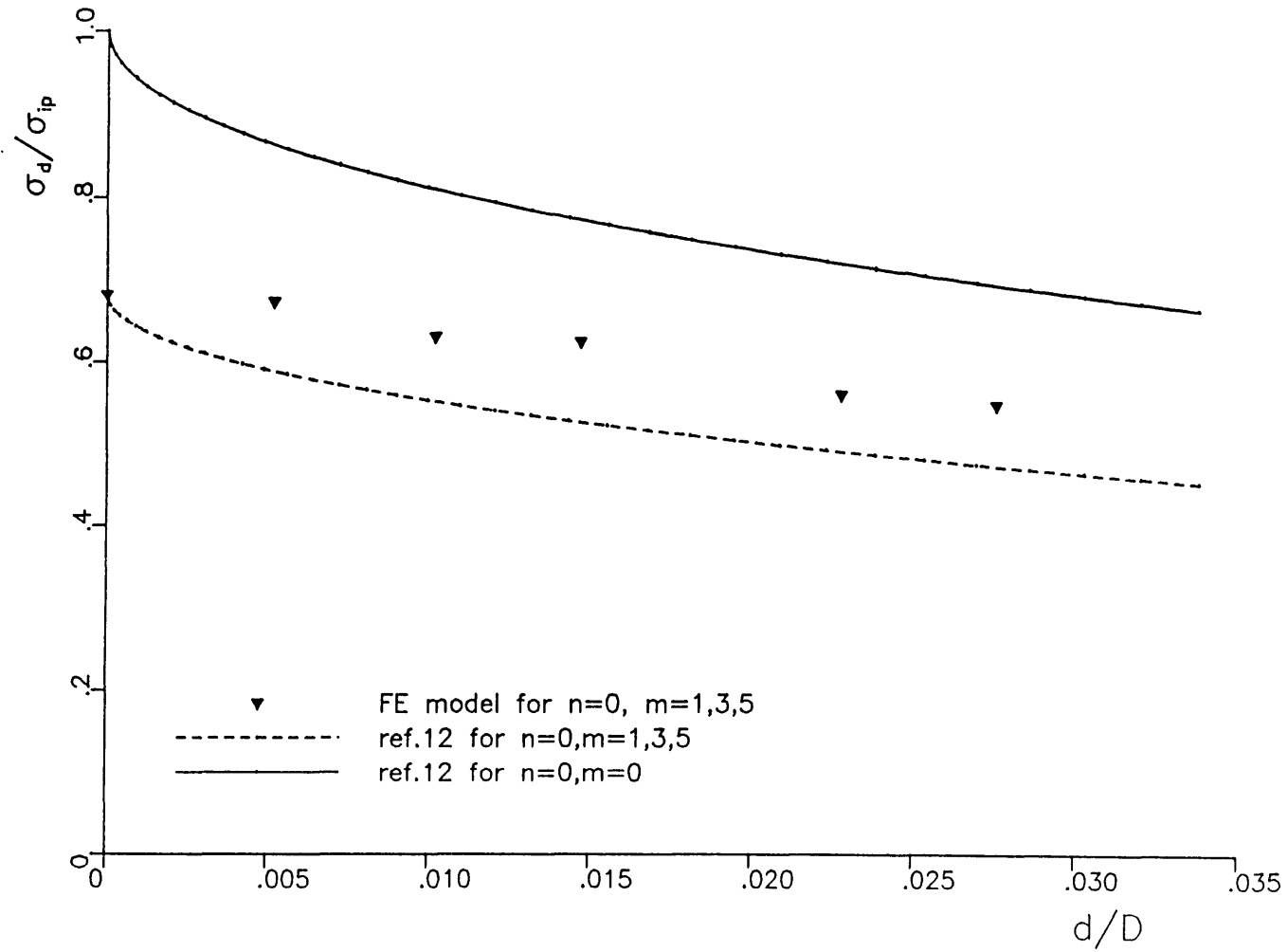


Figure 6.25

LOAD AXIAL DEFLECTION CURVES

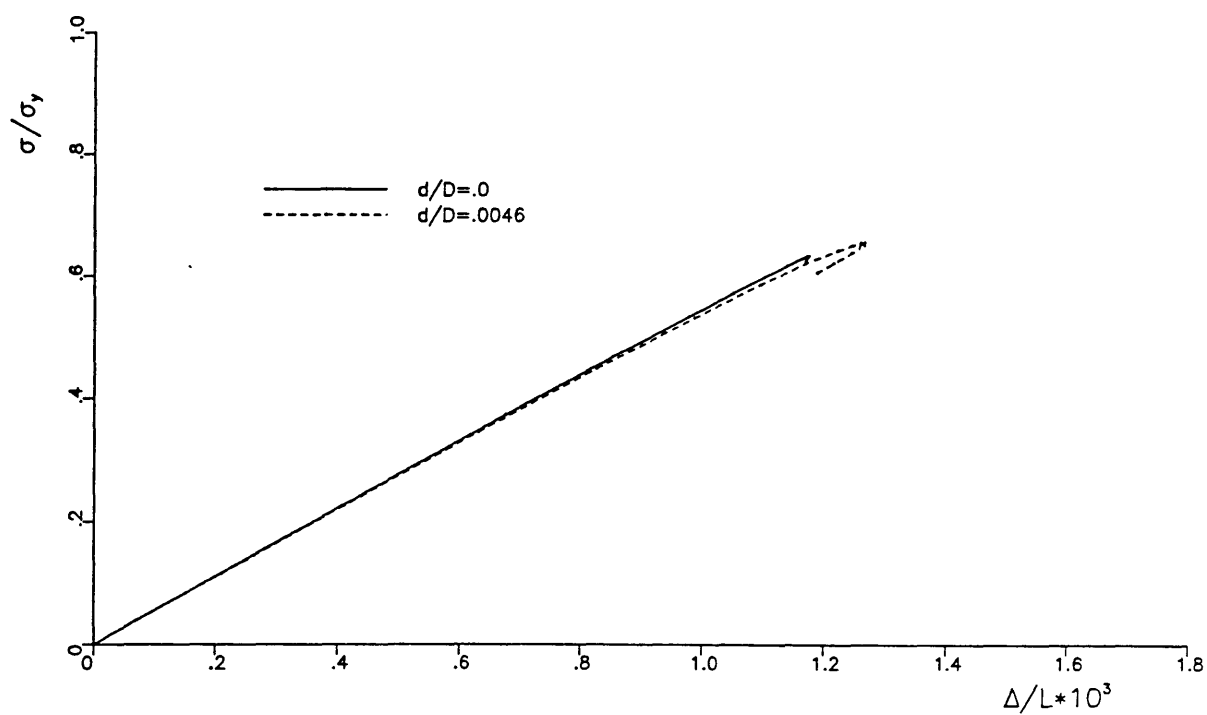
($R/t=300$, $l/R=0.3$, $n=10$, $m=1,3,5$)

Figure 6.26

LOAD AXIAL DEFLECTION CURVES

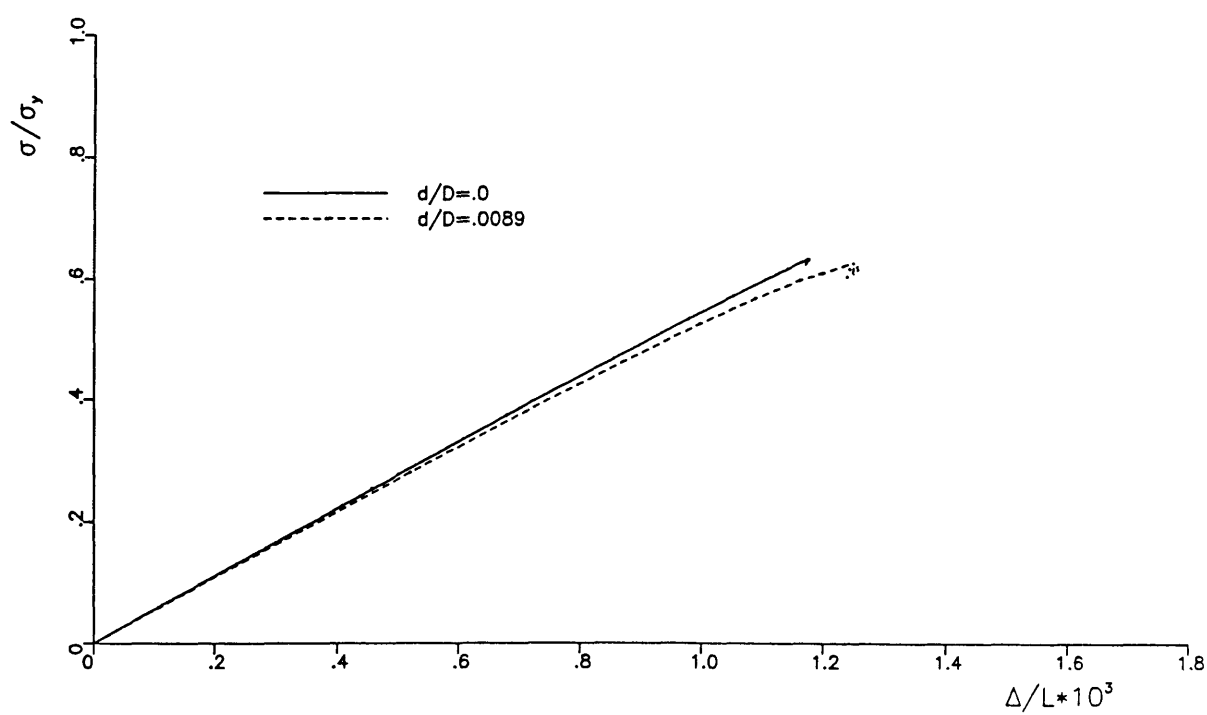
($R/t=300$, $l/R=0.3$, $n=10$, $m=1,3,5$)

Figure 6.27

LOAD AXIAL DEFLECTION CURVES

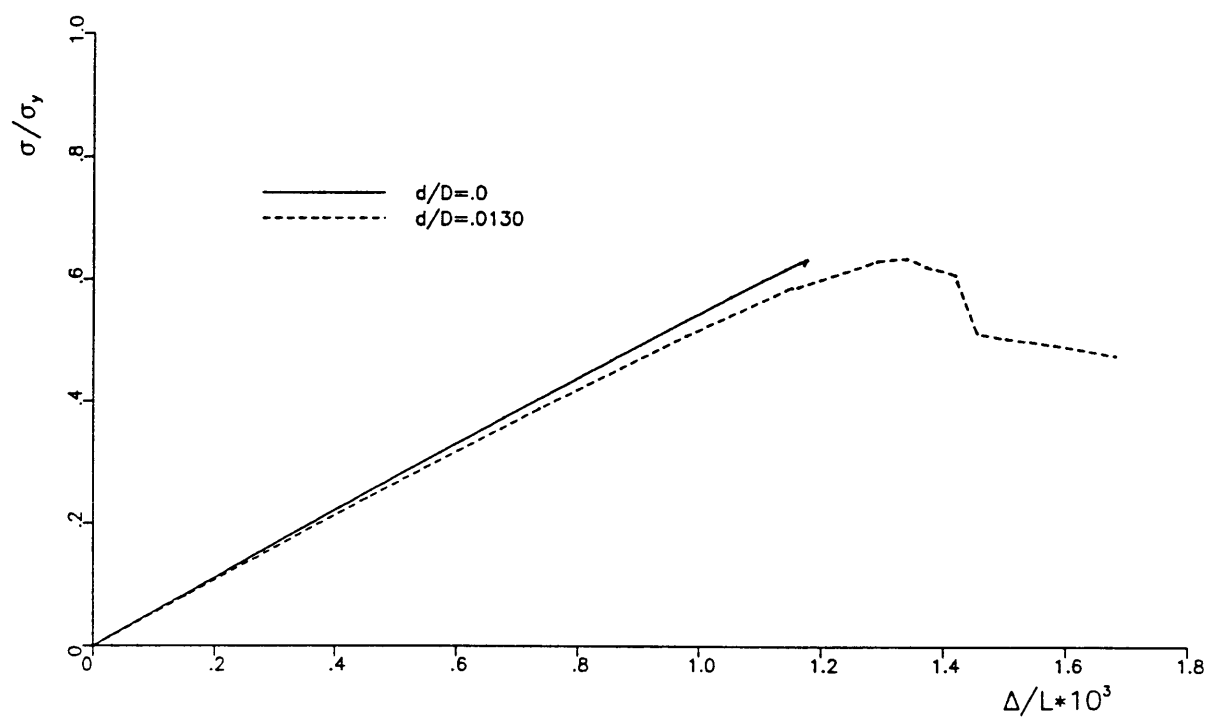
($R/t=300$, $l/R=0.3$, $n=10$, $m=1,3,5$)

Figure 6.28

LOAD AXIAL DEFLECTION CURVES

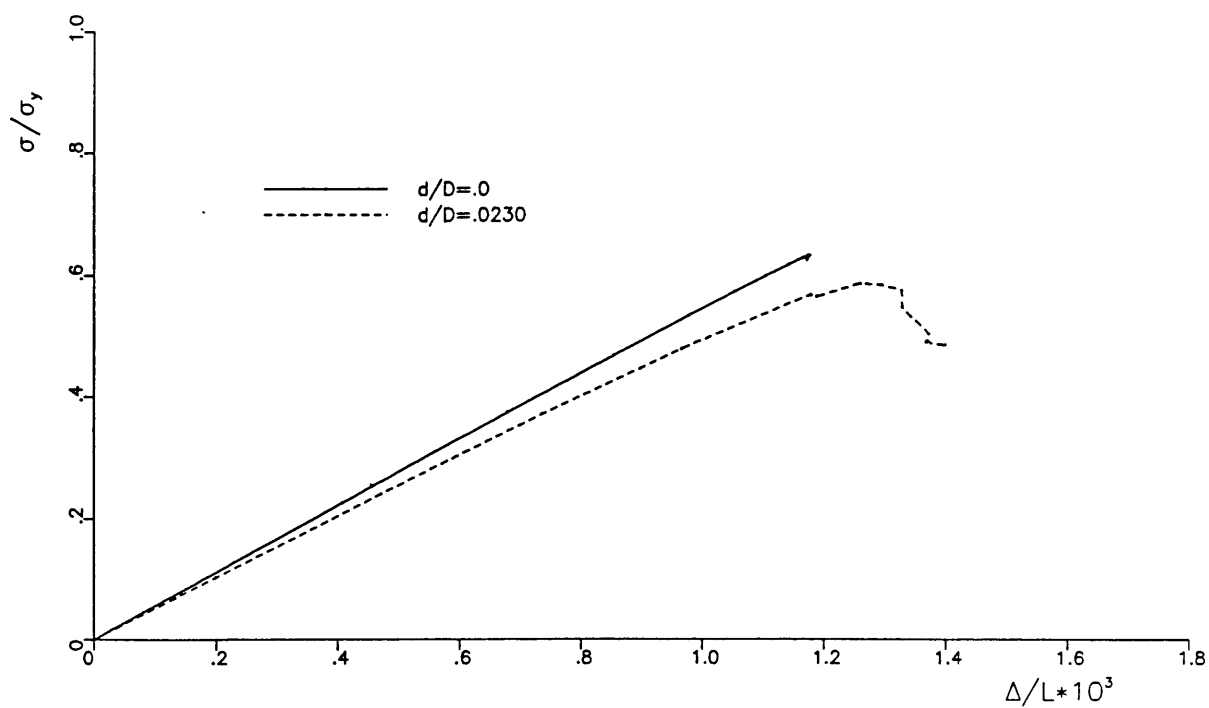
($R/t=300$, $l/R=0.3$, $n=10$, $m=1,3,5$)

Figure 6.29

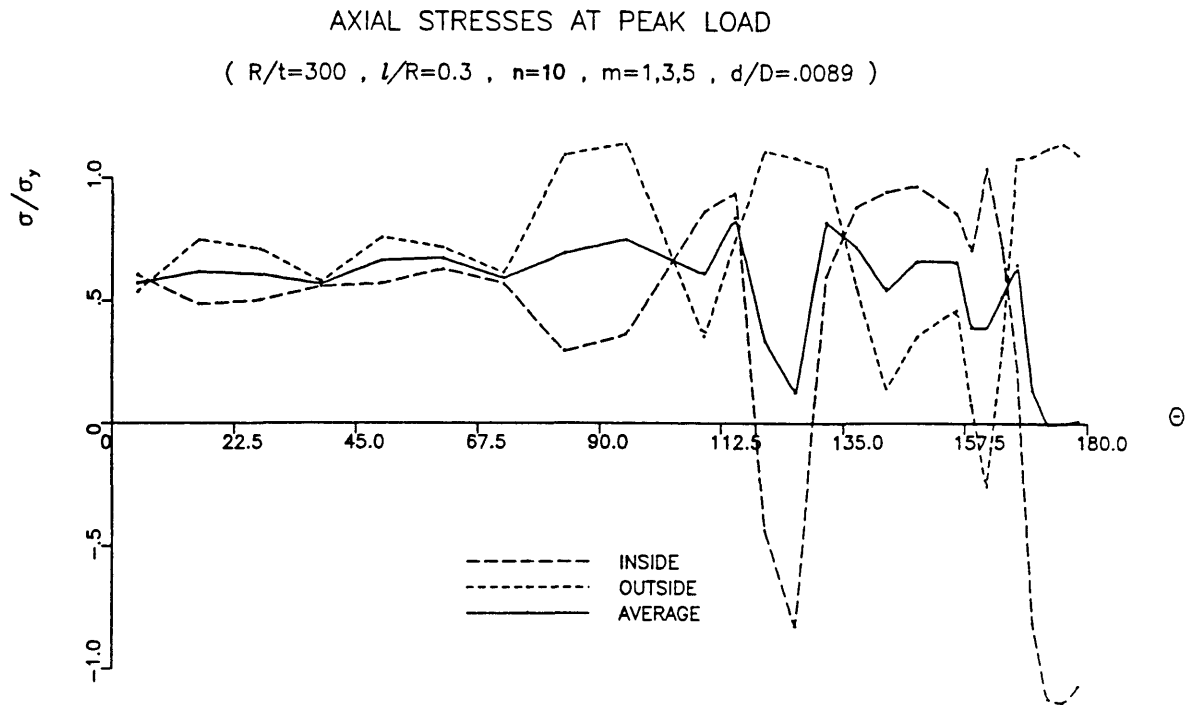


Figure 6.30

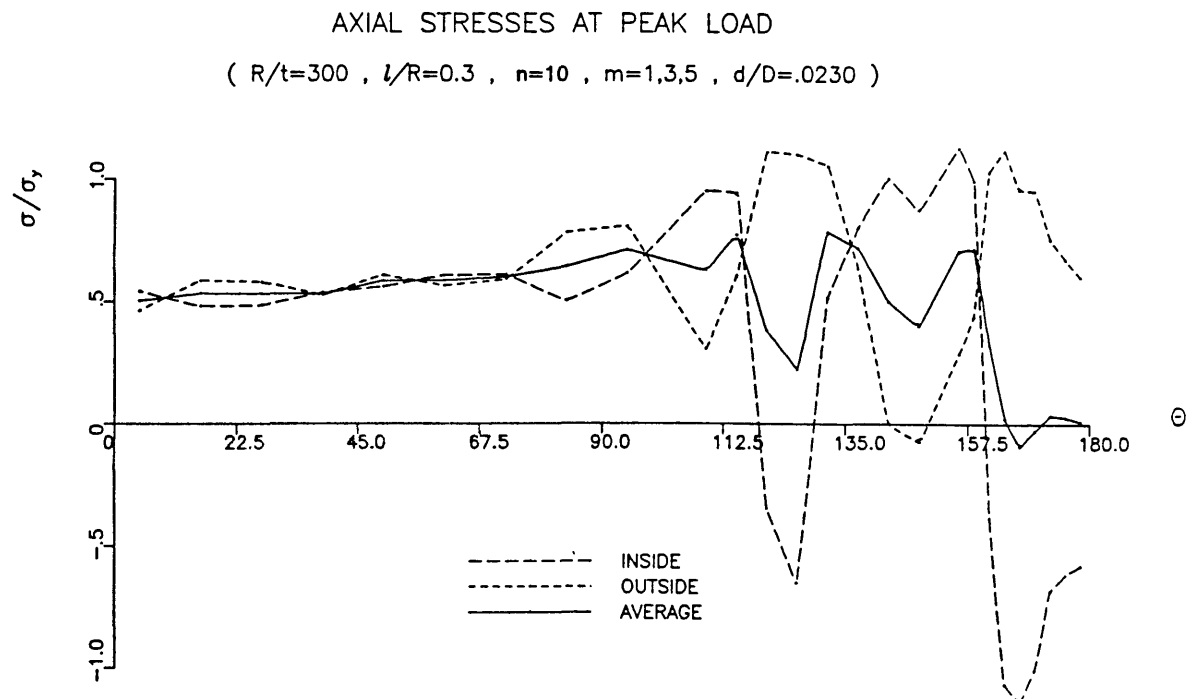


Figure 6.31

FAILURE MODES FOR DIFFERENT LEVELS OF DENTING

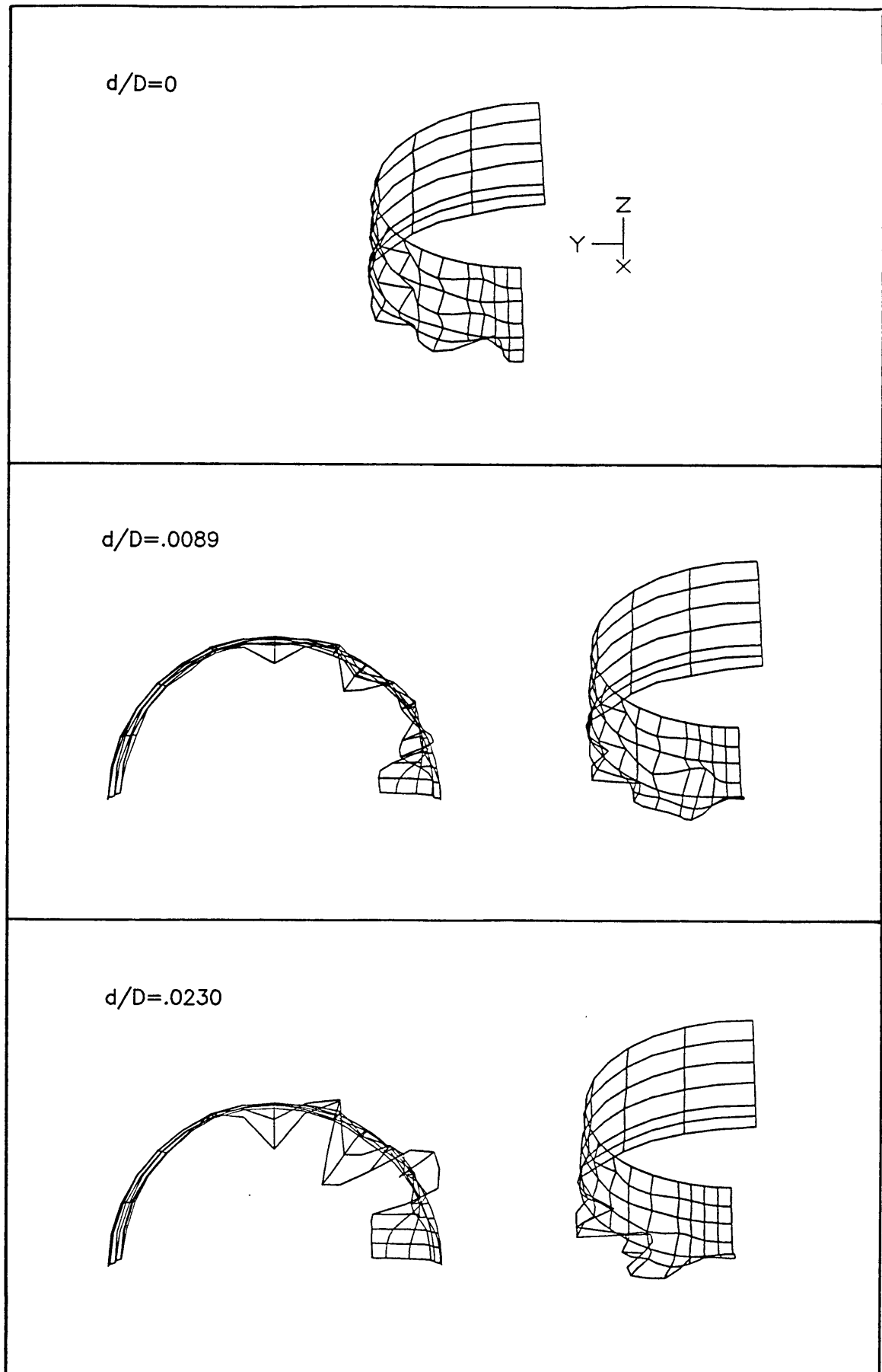
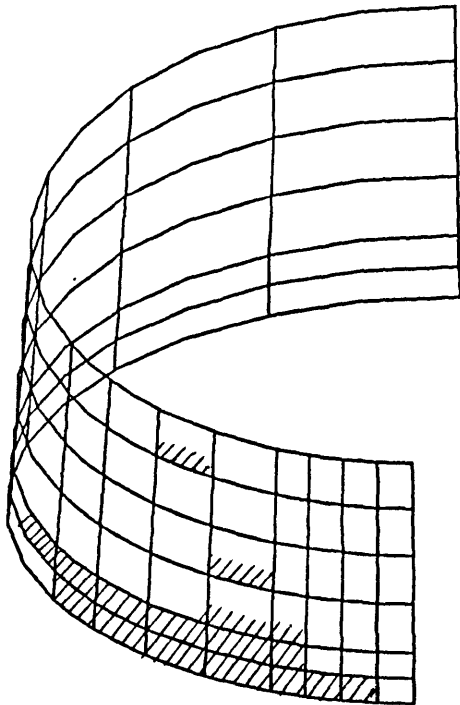
 $(R/t=300 , l/R=0.3 , n=10 , m=1,3,5)$ 

Figure 6.32

PLASTICITY PATTERNS FROM AXIAL COMPRESSION ANALYSIS
($R/t=300$, $l/R=0.3$, $n=10$, $m=1,3,5$)

$d/D=.0230$

PEAK LOAD



MAXIMUM LOAD

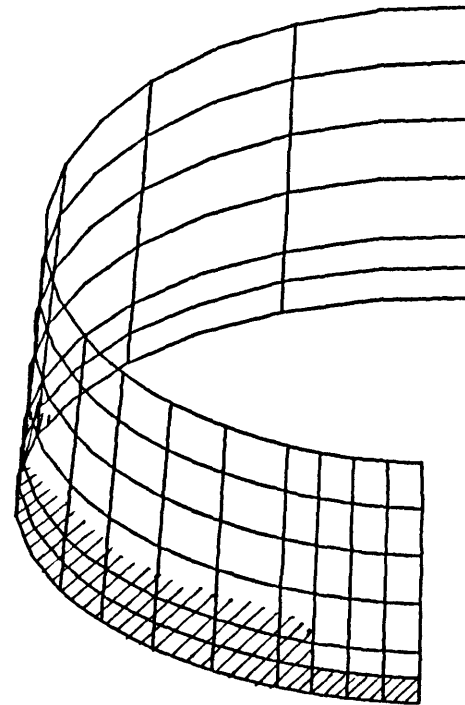


Figure 6.33

AXIAL RESIDUAL STRENGTH ($R/t=300$, $l/R=0.3$, $n=10$, $m=1,3,5$)

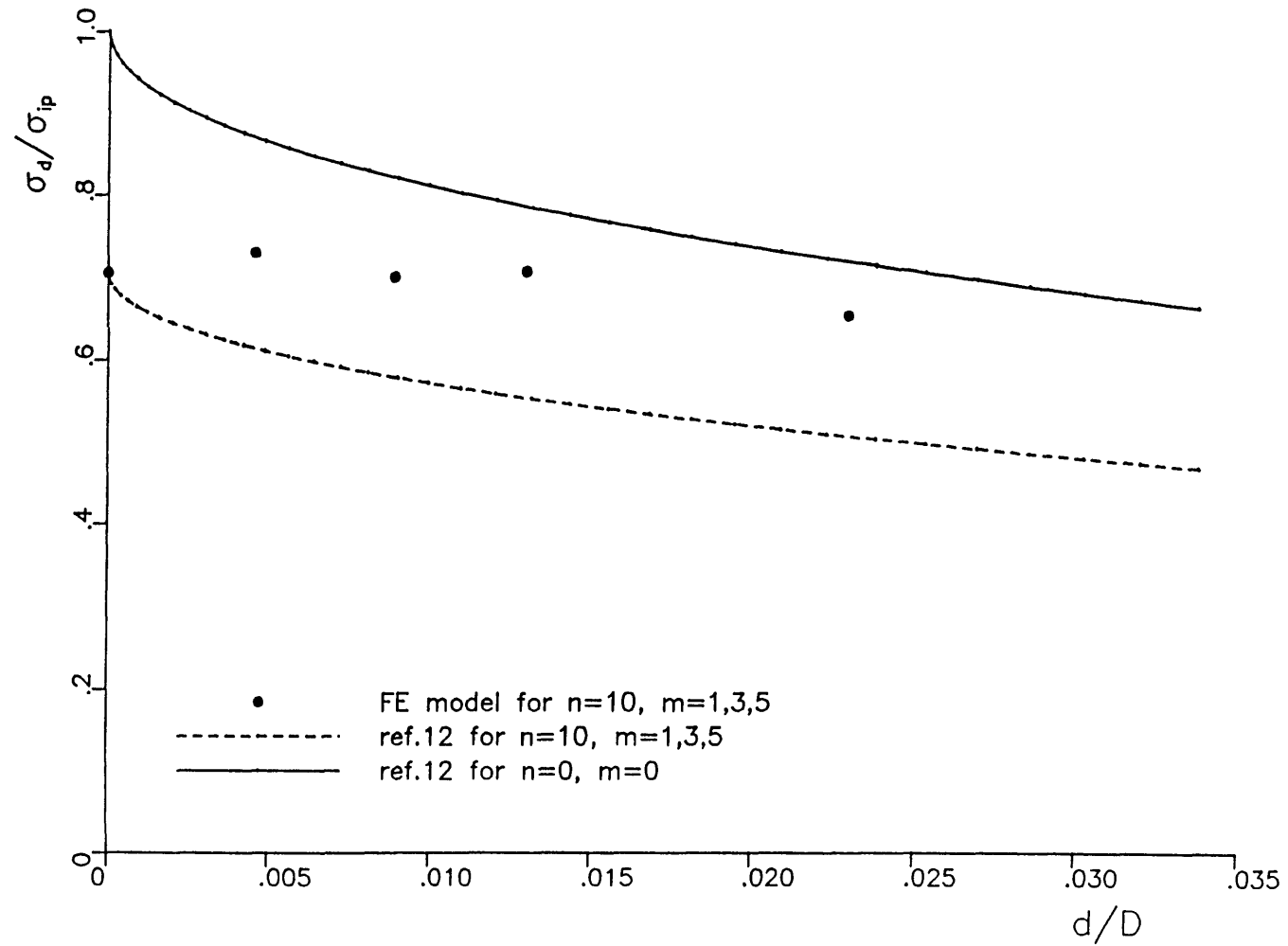


Figure 6.34

DEFLECTED SHAPE OF DENTED CYLINDER ($R/t=500$, $l/R=0.3$, $n=0$, $m=0$, $d/D=.0306$)

DISPLACEMENT FACTOR = 20

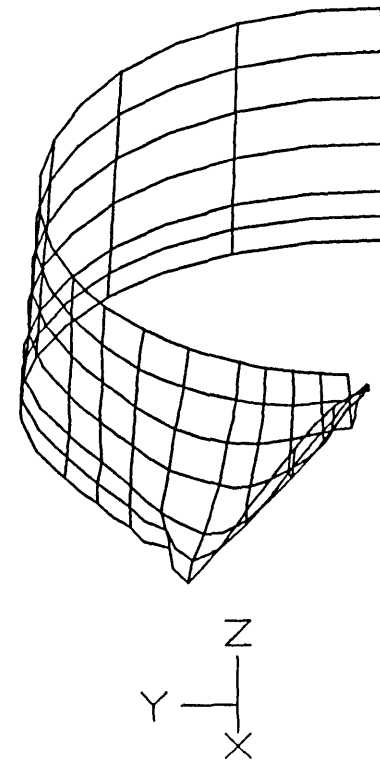
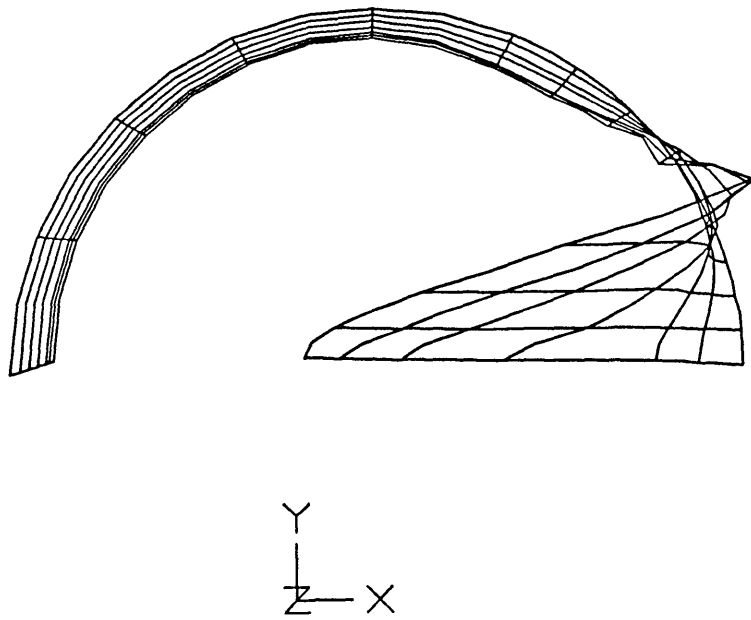
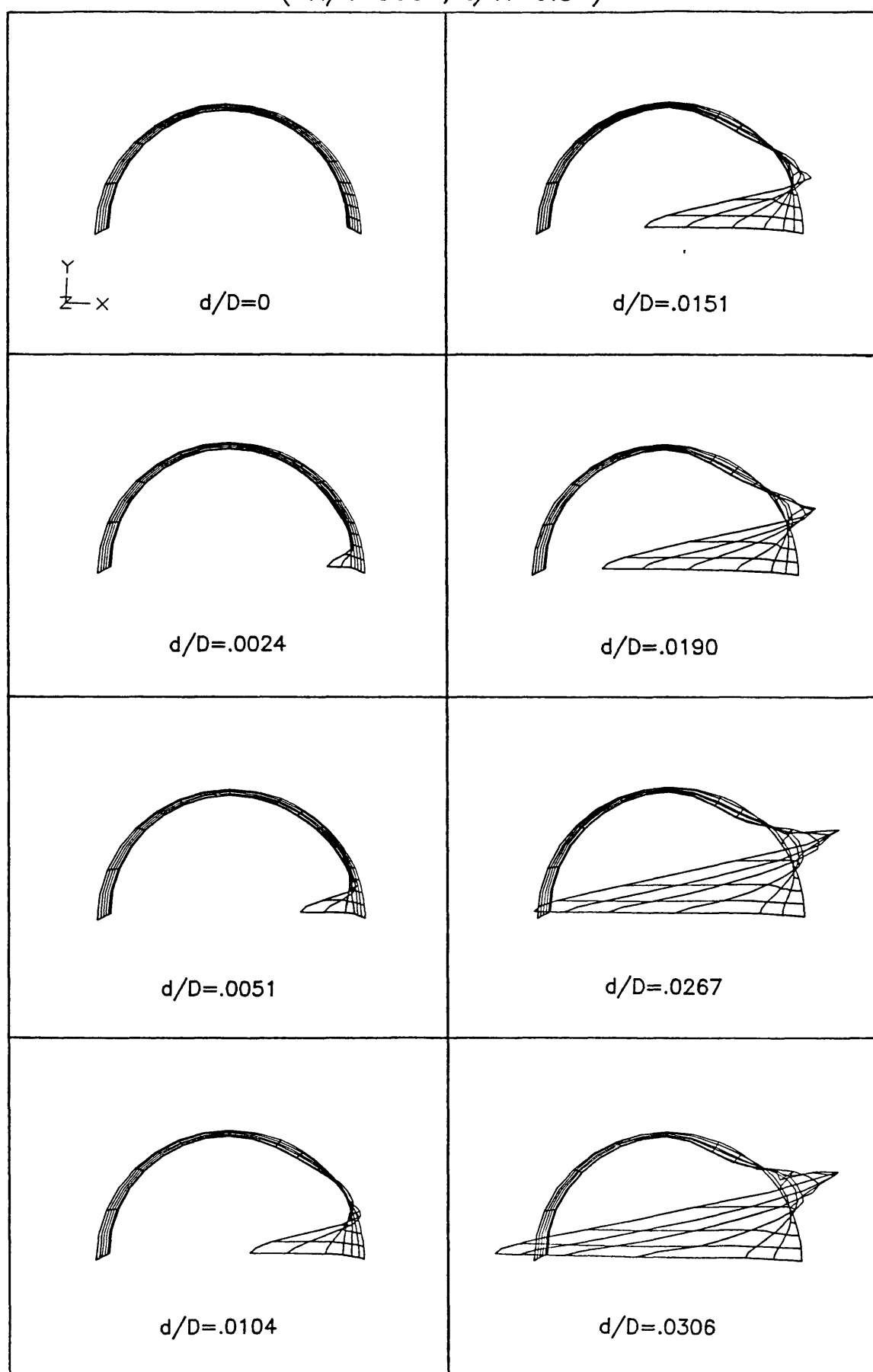


Figure 6.35

DEFLECTIONS FOR DIFFERENT LEVELS OF DENTING
($R/t=500$, $l/R=0.3$)



DISPLACEMENT FACTOR = 40

Figure 6.36

RELATIVE WAVELENGTHS OF DENT DEFLECTIONS
($R/t=500$, $l/R=0.3$)

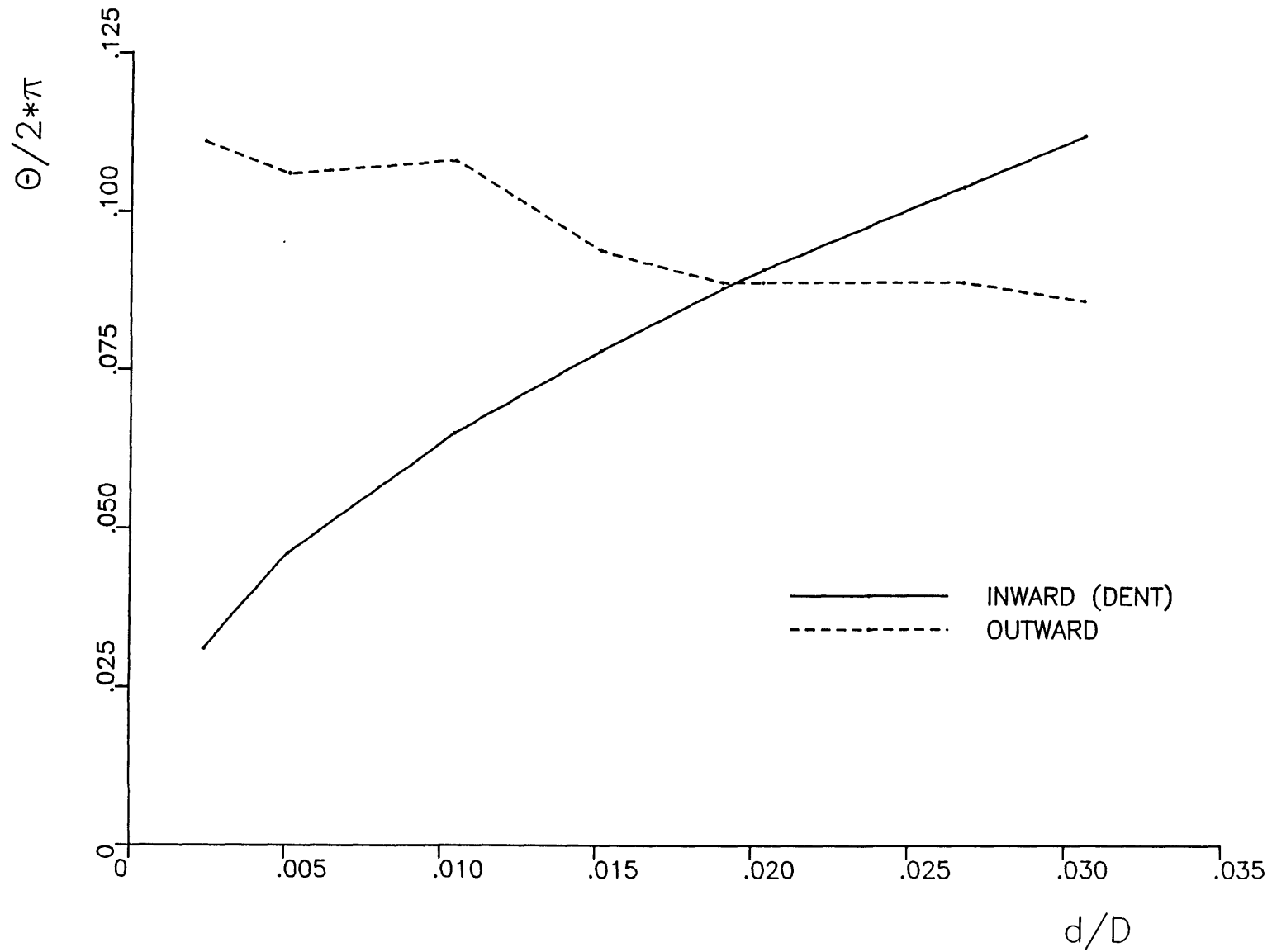


Figure 6.37

LOAD AXIAL DEFLECTION CURVES

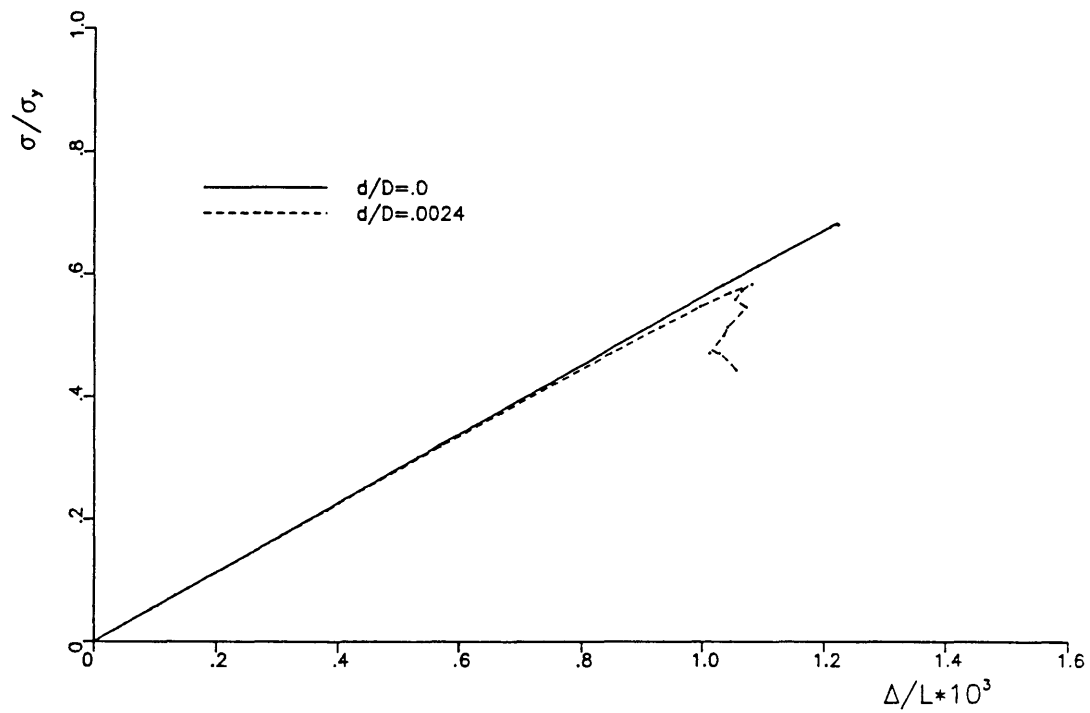
($R/t=500$, $l/R=0.3$, $n=0$, $m=0$)

Figure 6.38

LOAD AXIAL DEFLECTION CURVES

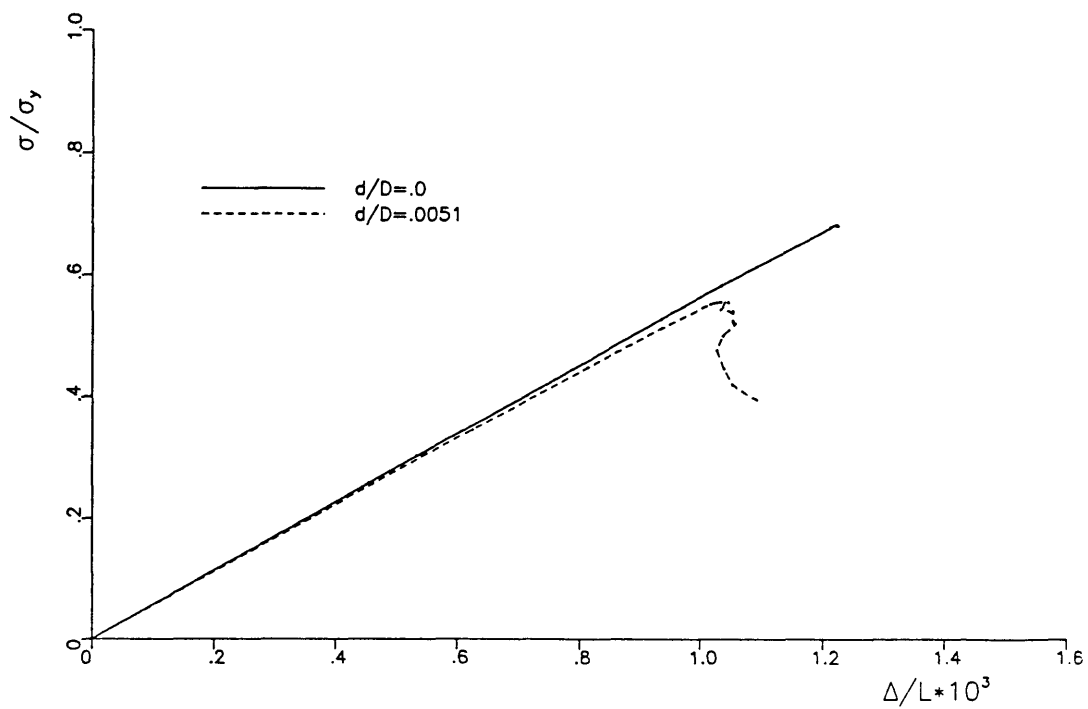
($R/t=500$, $l/R=0.3$, $n=0$, $m=0$)

Figure 6.39

LOAD AXIAL DEFLECTION CURVES

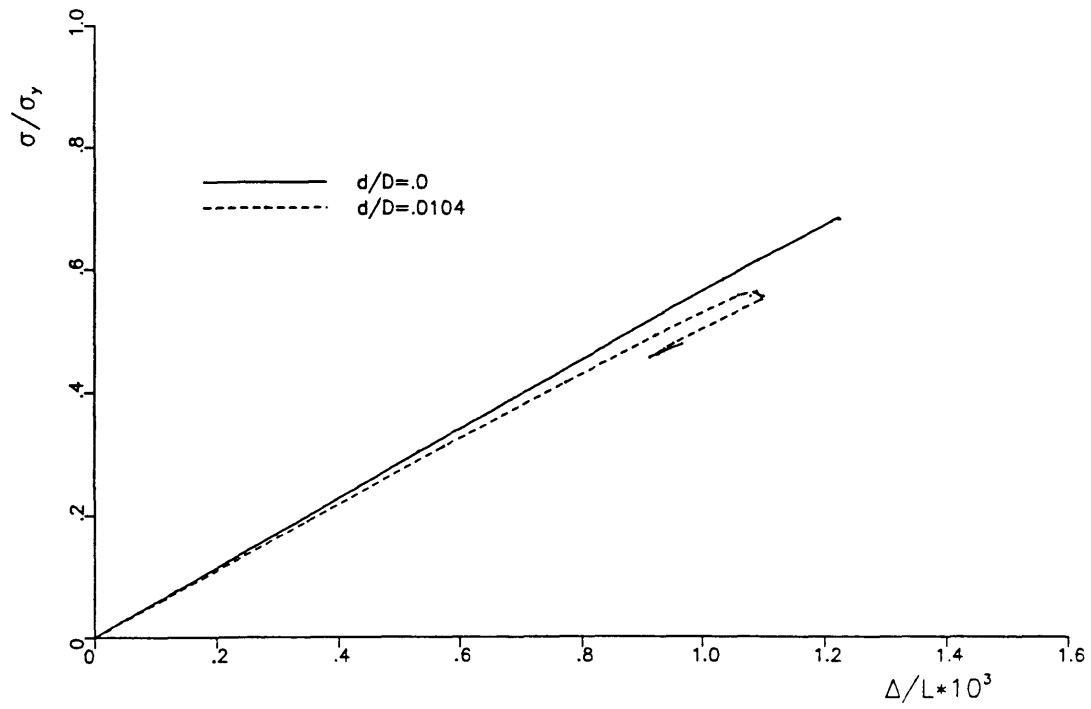
($R/t=500$, $L/R=0.3$, $n=0$, $m=0$)

Figure 6.40

LOAD AXIAL DEFLECTION CURVES

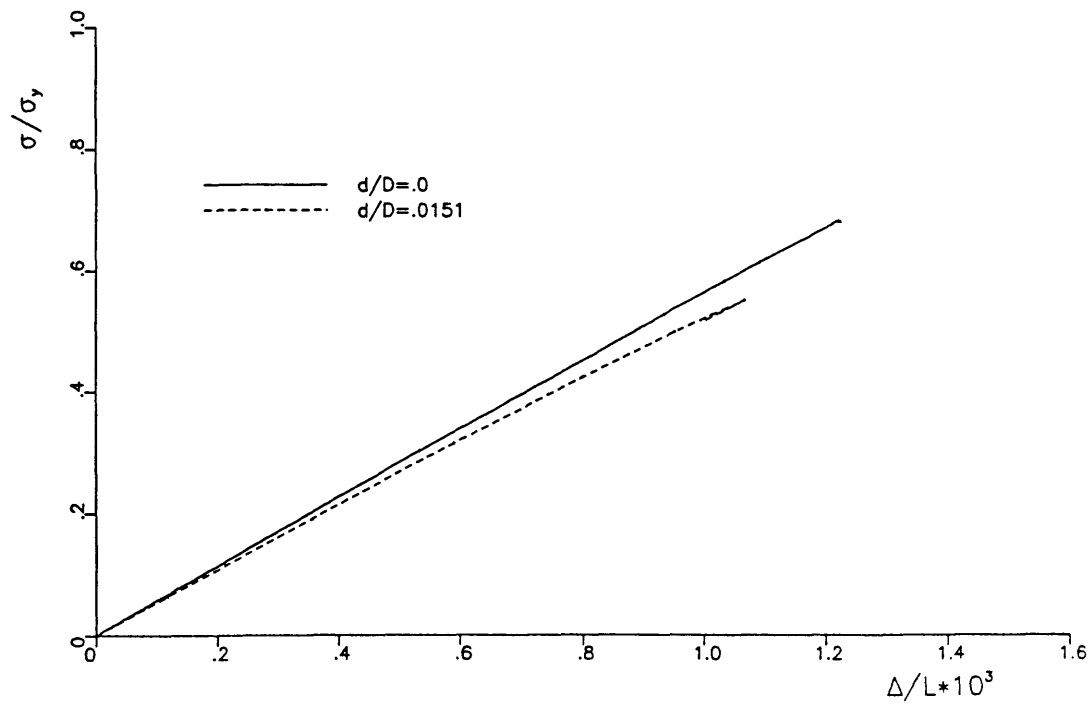
($R/t=500$, $L/R=0.3$, $n=0$, $m=0$)

Figure 6.41

LOAD AXIAL DEFLECTION CURVES

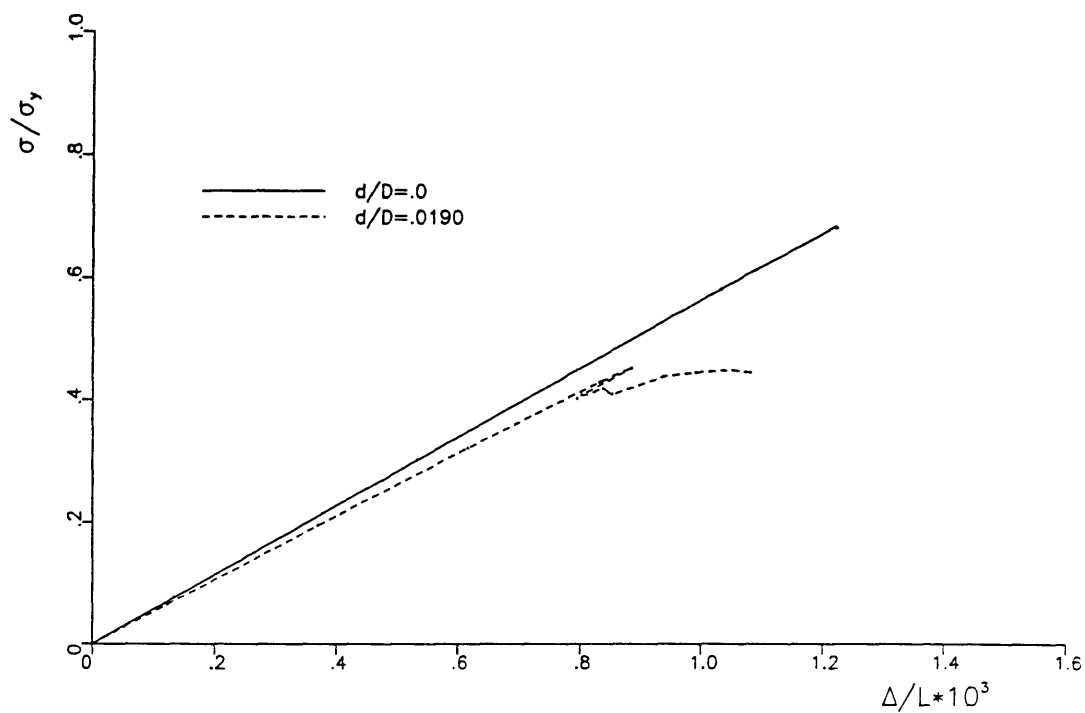
($R/t=500$, $l/R=0.3$, $n=0$, $m=0$)

Figure 6.42

LOAD AXIAL DEFLECTION CURVES

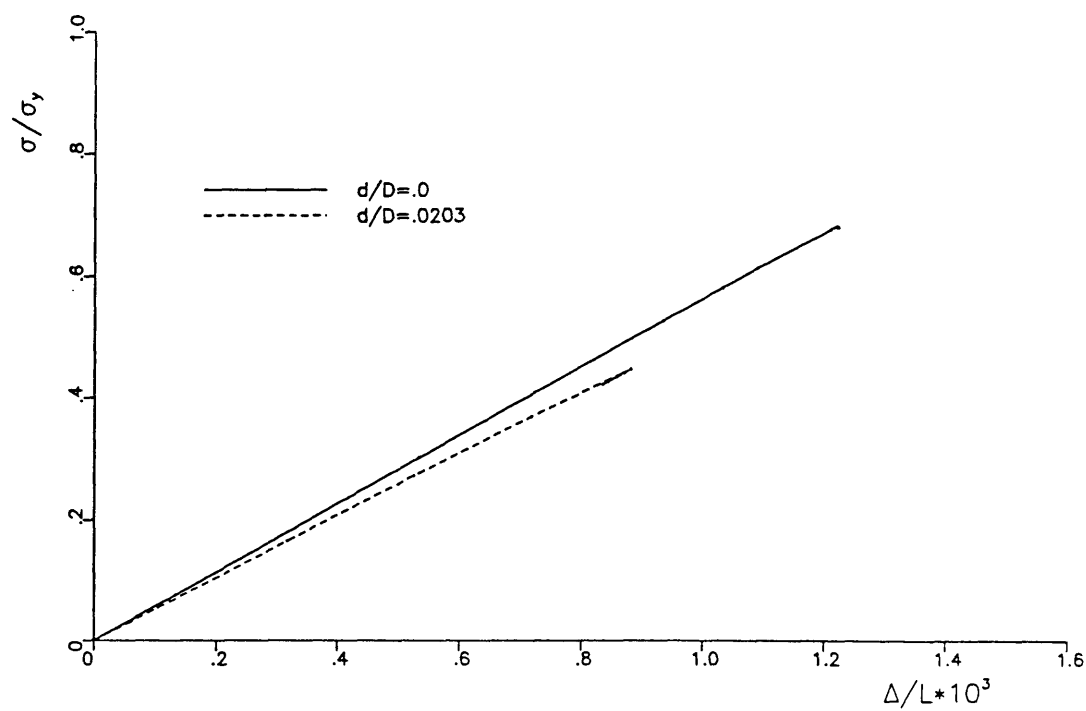
($R/t=500$, $l/R=0.3$, $n=0$, $m=0$)

Figure 6.43

LOAD AXIAL DEFLECTION CURVES

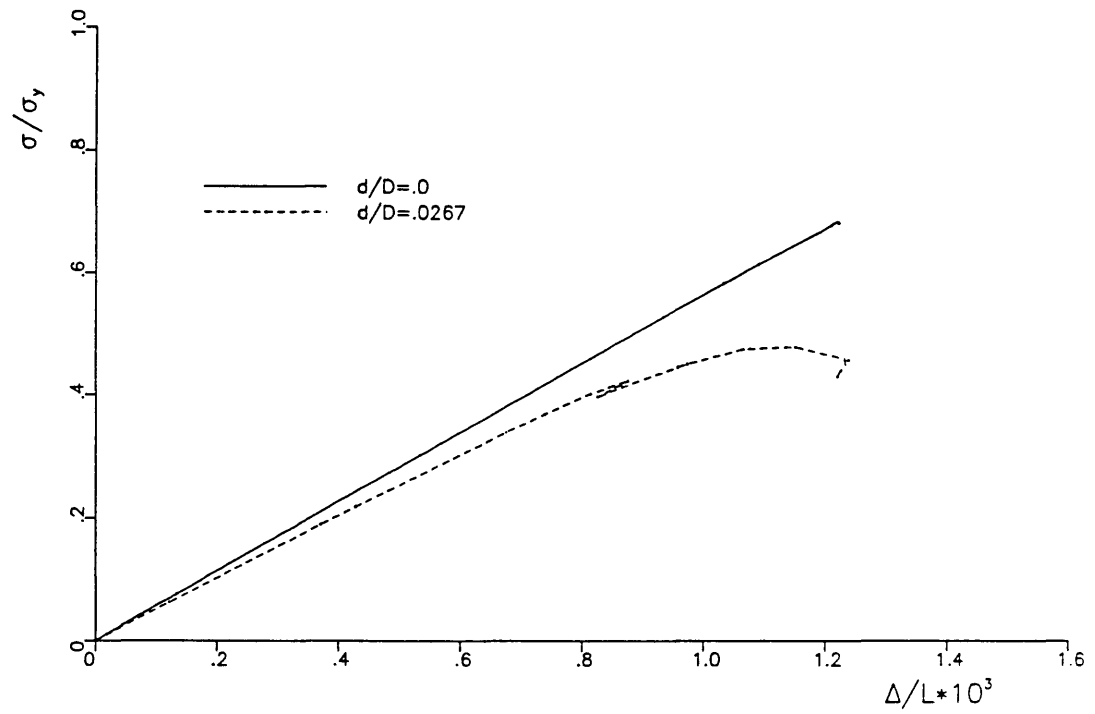
($R/t=500$, $l/R=0.3$, $n=0$, $m=0$)

Figure 6.44

LOAD AXIAL DEFLECTION CURVES

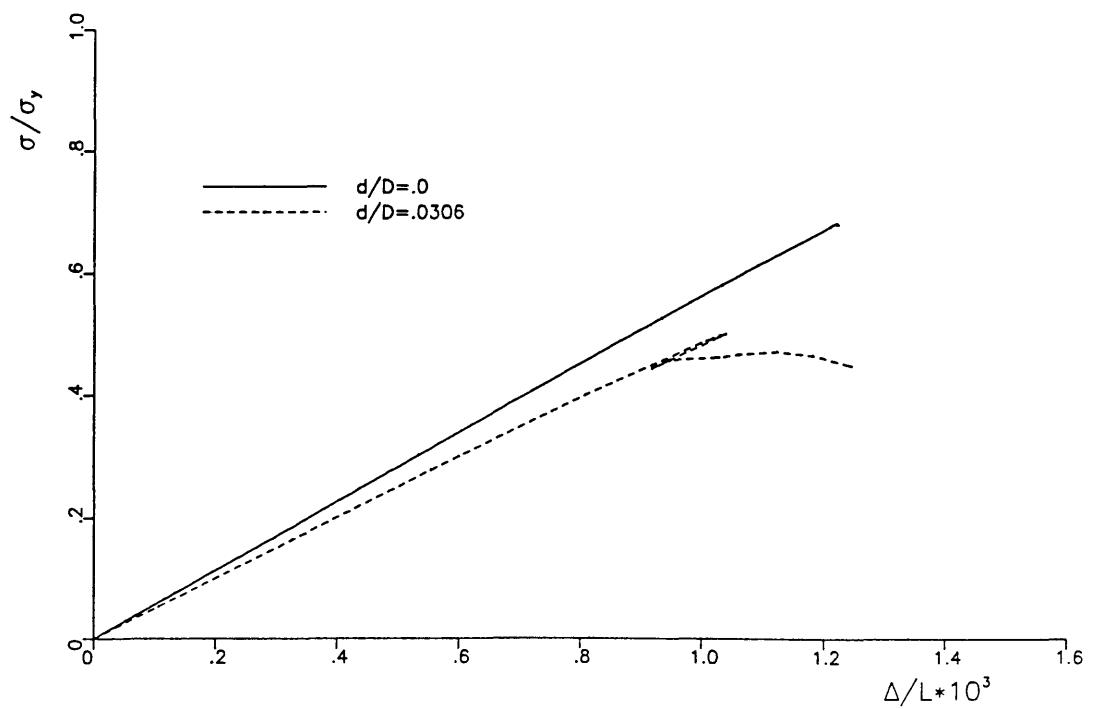
($R/t=500$, $l/R=0.3$, $n=0$, $m=0$)

Figure 6.45

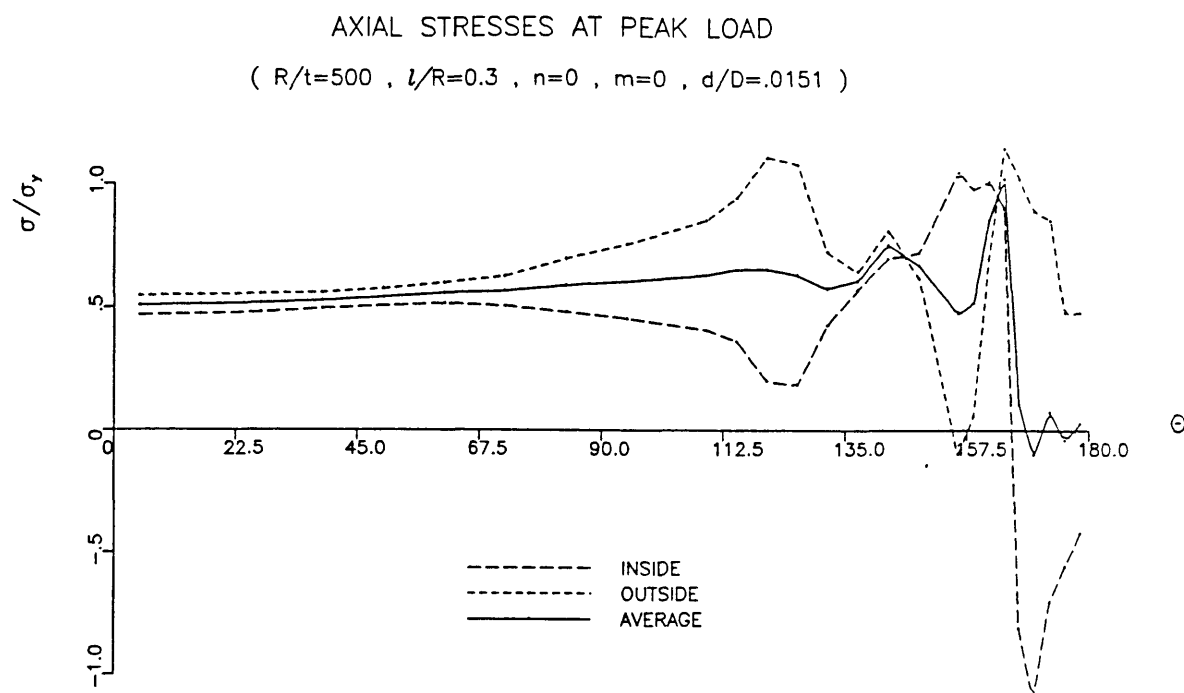


Figure 6.46

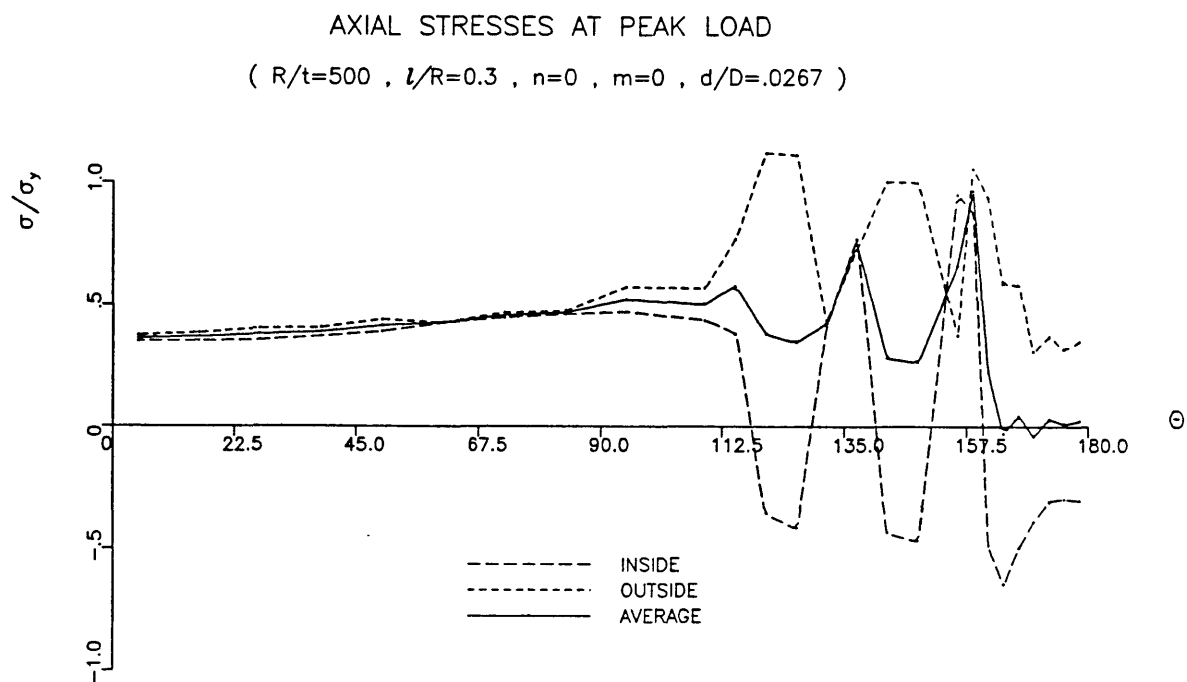


Figure 6.47

FAILURE MODES FOR DIFFERENT LEVELS OF DENTING
($R/t=500$, $l/R=0.3$, $n=0$, $m=0$)

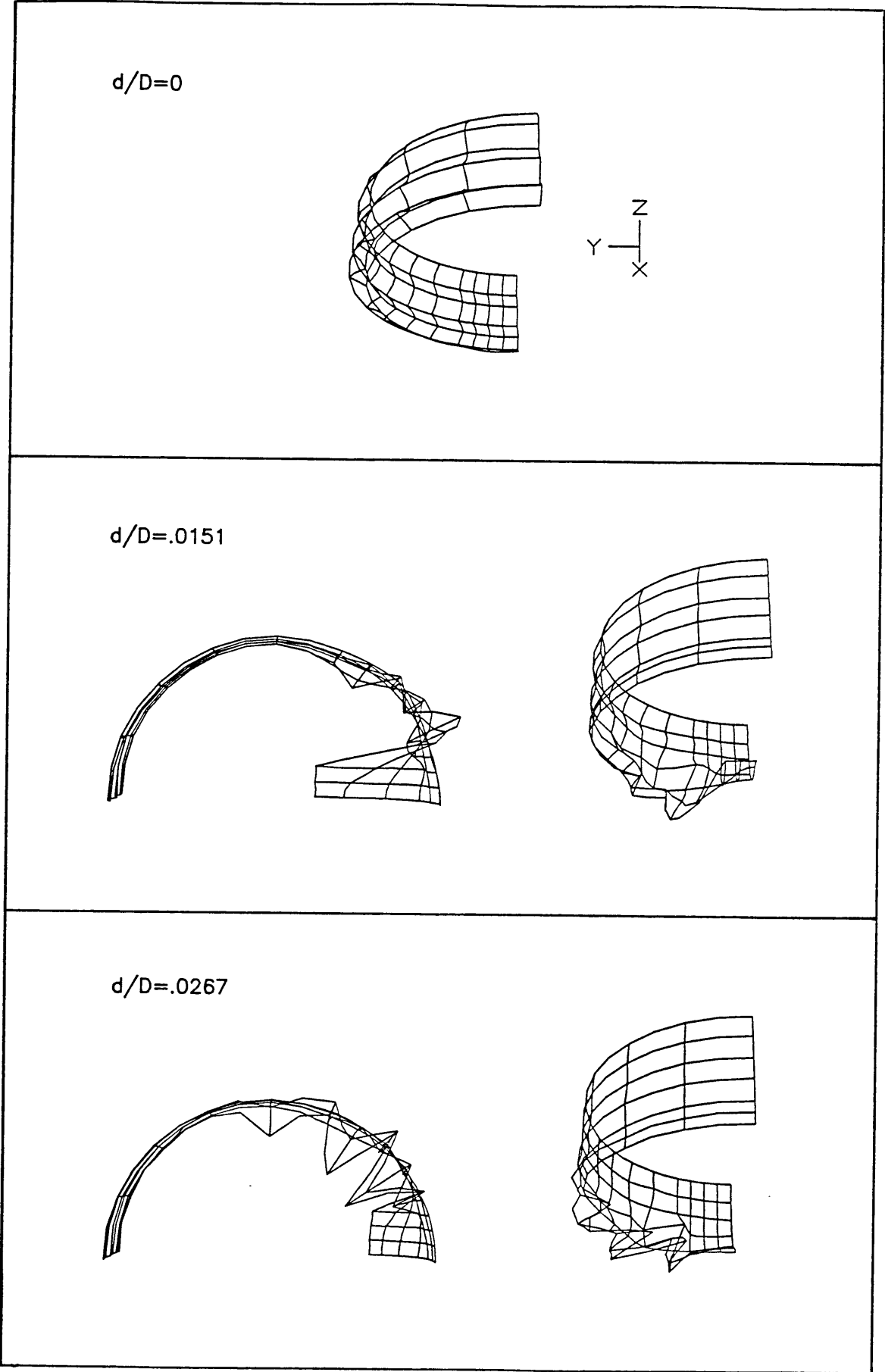


Figure 6.48

AXIAL RESIDUAL STRENGTH ($R/t=500$, $l/R=0.3$, $n=0$, $m=0$)

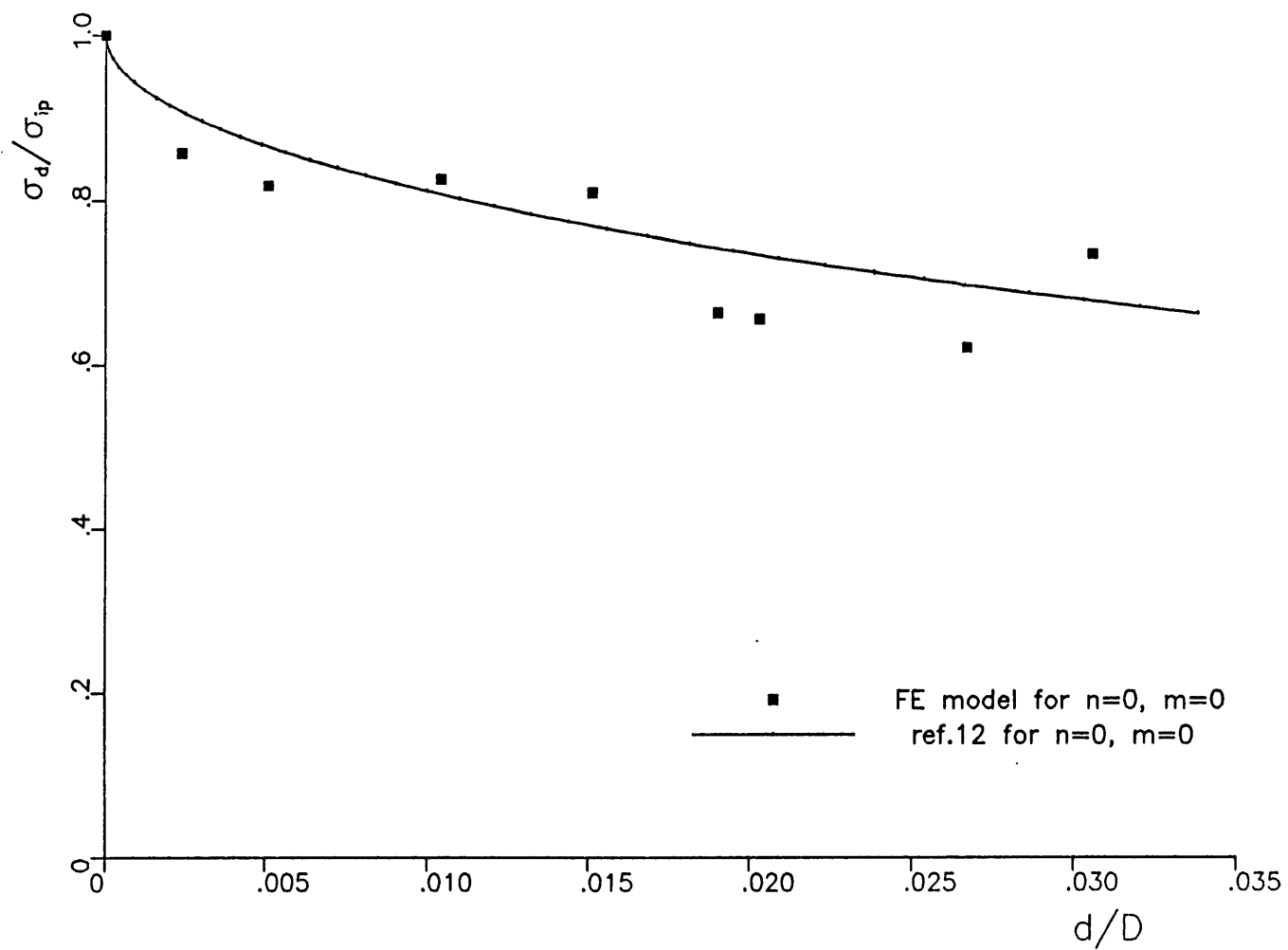


Figure 6.49

LOAD AXIAL DEFLECTION CURVES FOR DIFFERENT LEVELS OF DENTING

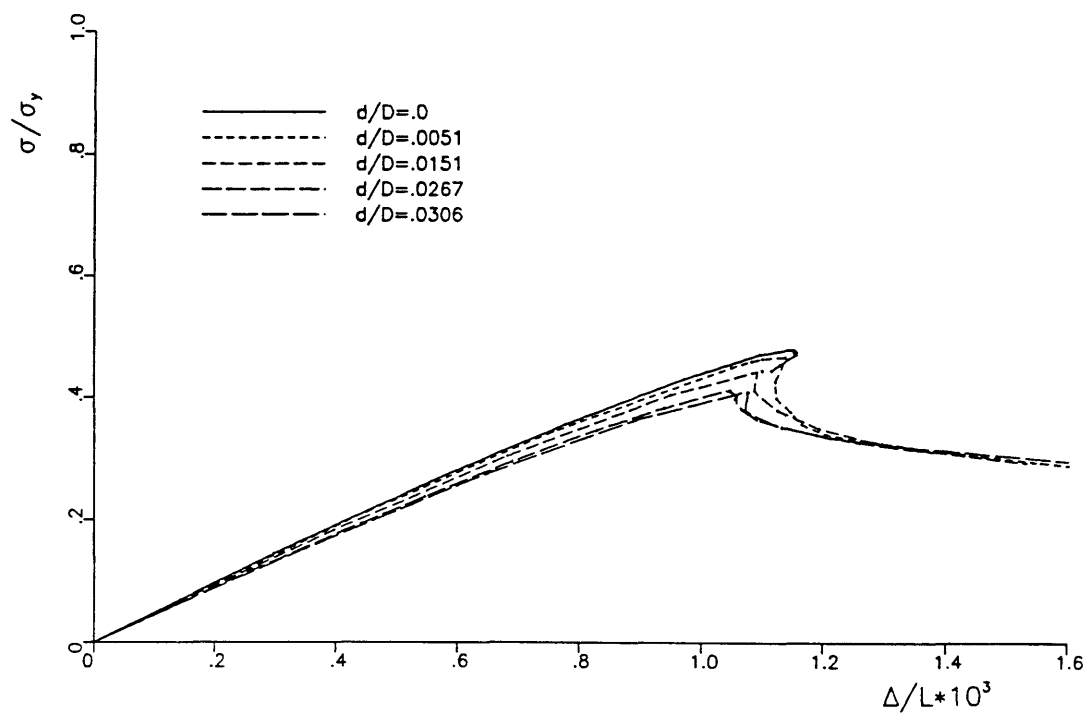
($R/t=500$, $l/R=0.3$, $n=0$, $m=1,3,5$)

Figure 6.50

LOAD AXIAL DEFLECTION CURVES

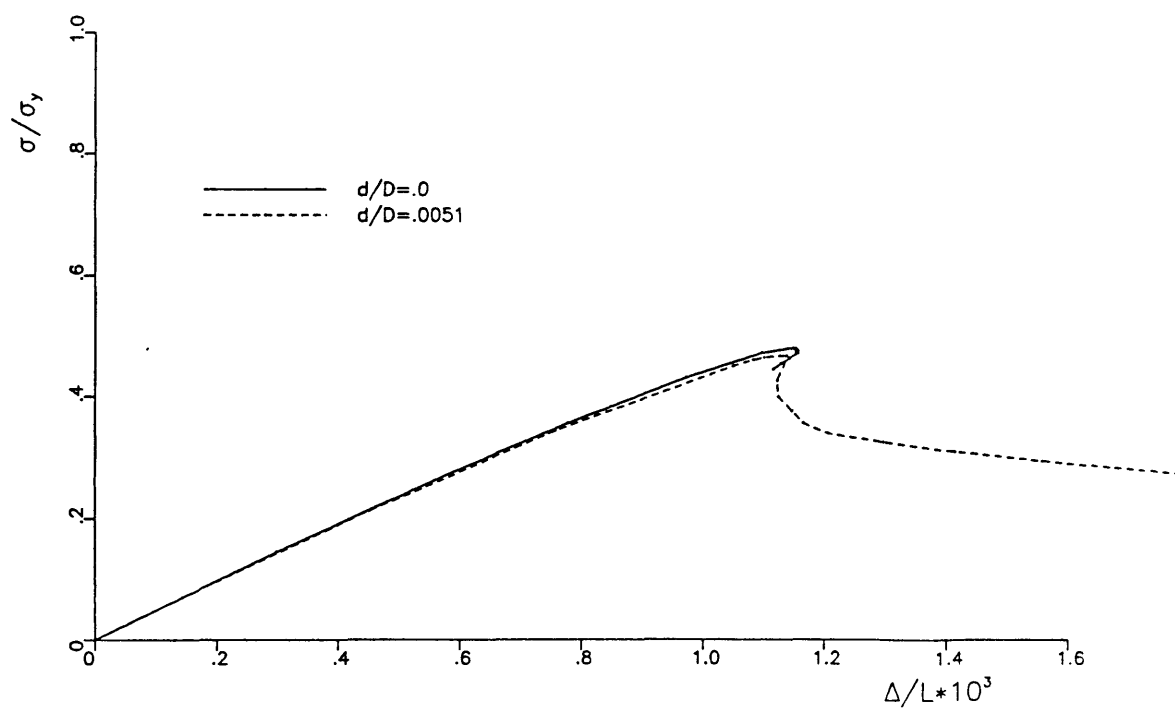
($R/t=500$, $l/R=0.3$, $n=0$, $m=1,3,5$)

Figure 6.51

LOAD AXIAL DEFLECTION CURVES

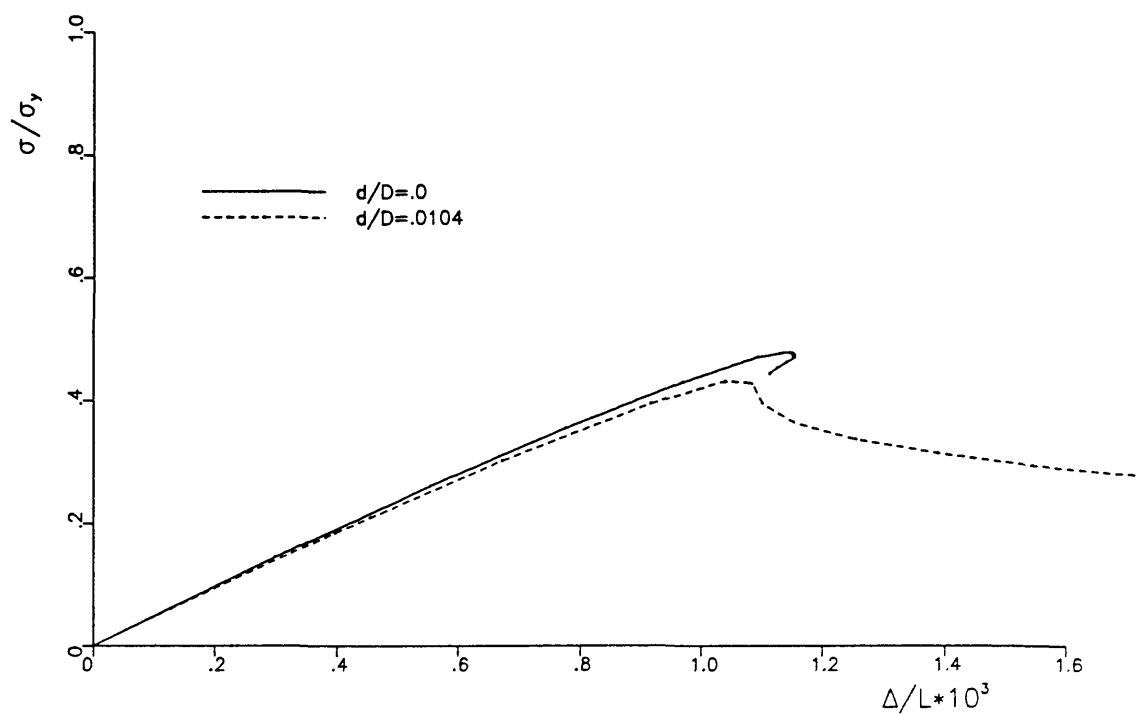
($R/t=500$, $l/R=0.3$, $n=0$, $m=1,3,5$)

Figure 6.52

LOAD AXIAL DEFLECTION CURVES

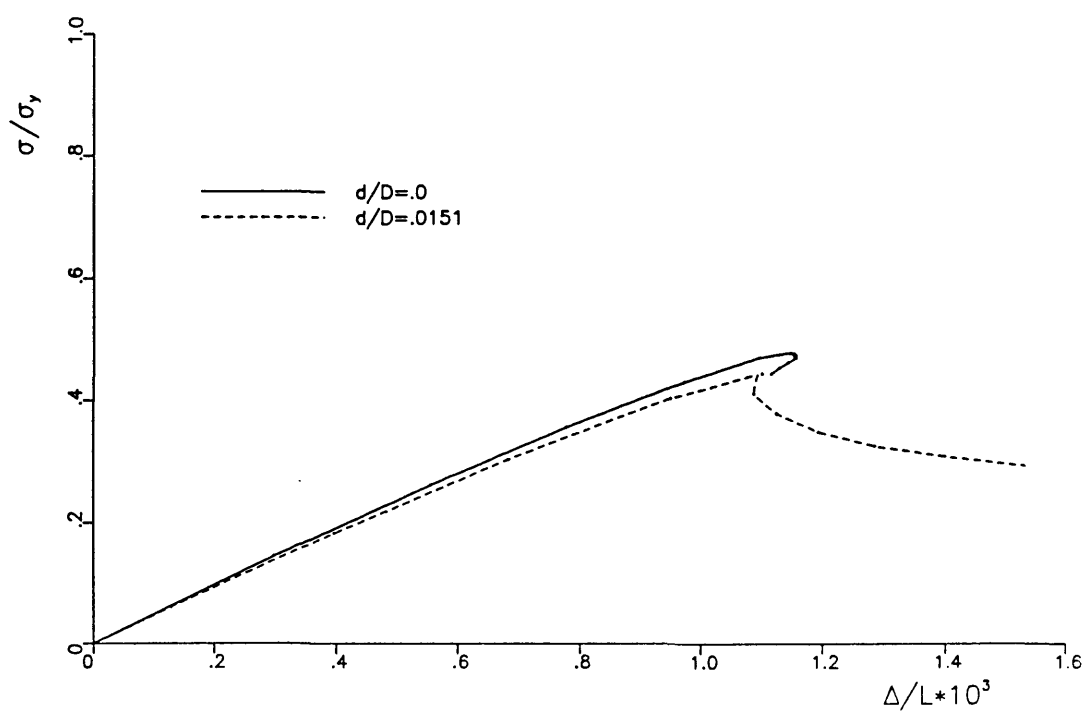
($R/t=500$, $l/R=0.3$, $n=0$, $m=1,3,5$)

Figure 6.53

LOAD AXIAL DEFLECTION CURVES

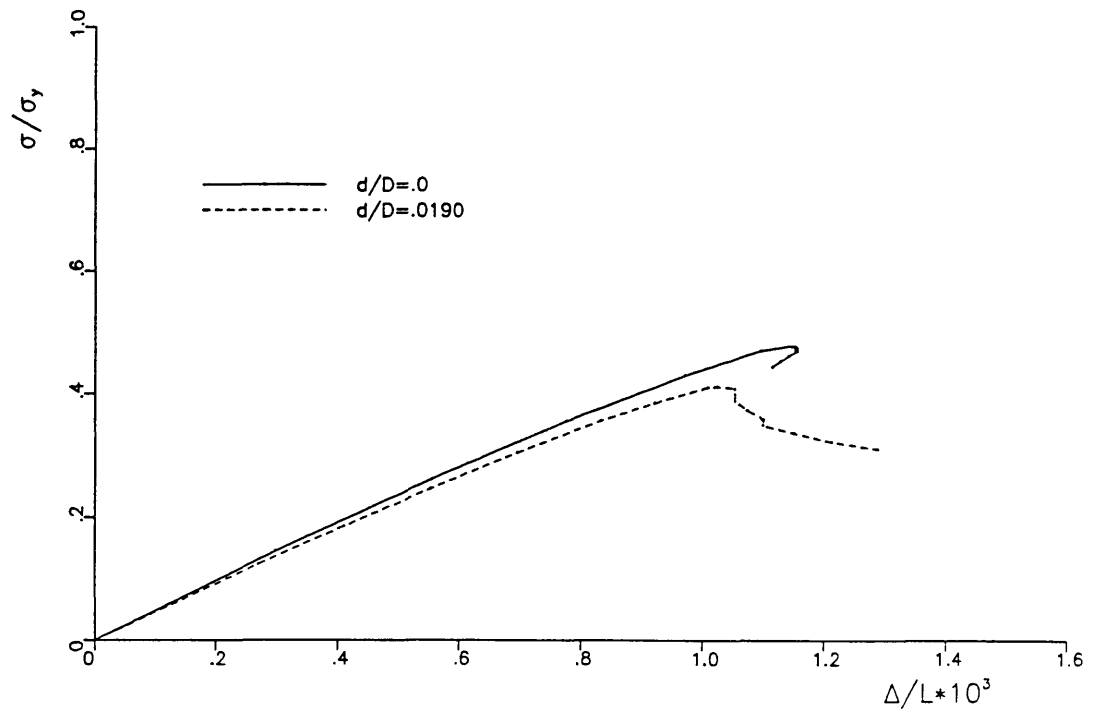
($R/t=500$, $l/R=0.3$, $n=0$, $m=1,3,5$)

Figure 6.54

LOAD AXIAL DEFLECTION CURVES

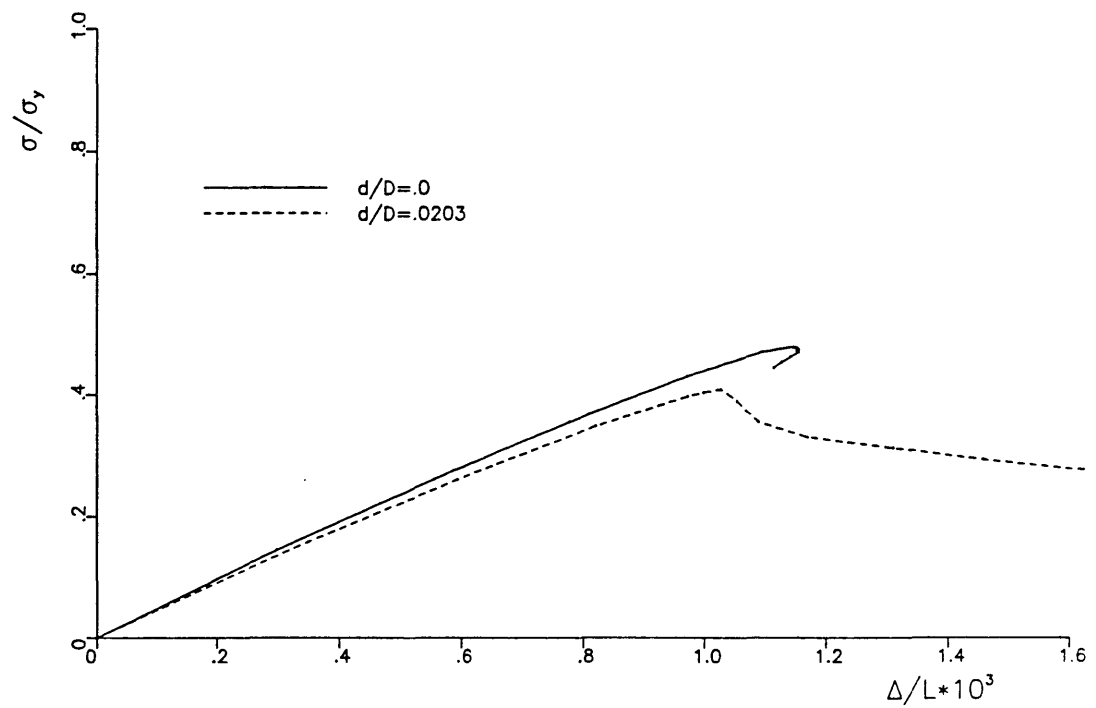
($R/t=500$, $l/R=0.3$, $n=0$, $m=1,3,5$)

Figure 6.55

LOAD AXIAL DEFLECTION CURVES

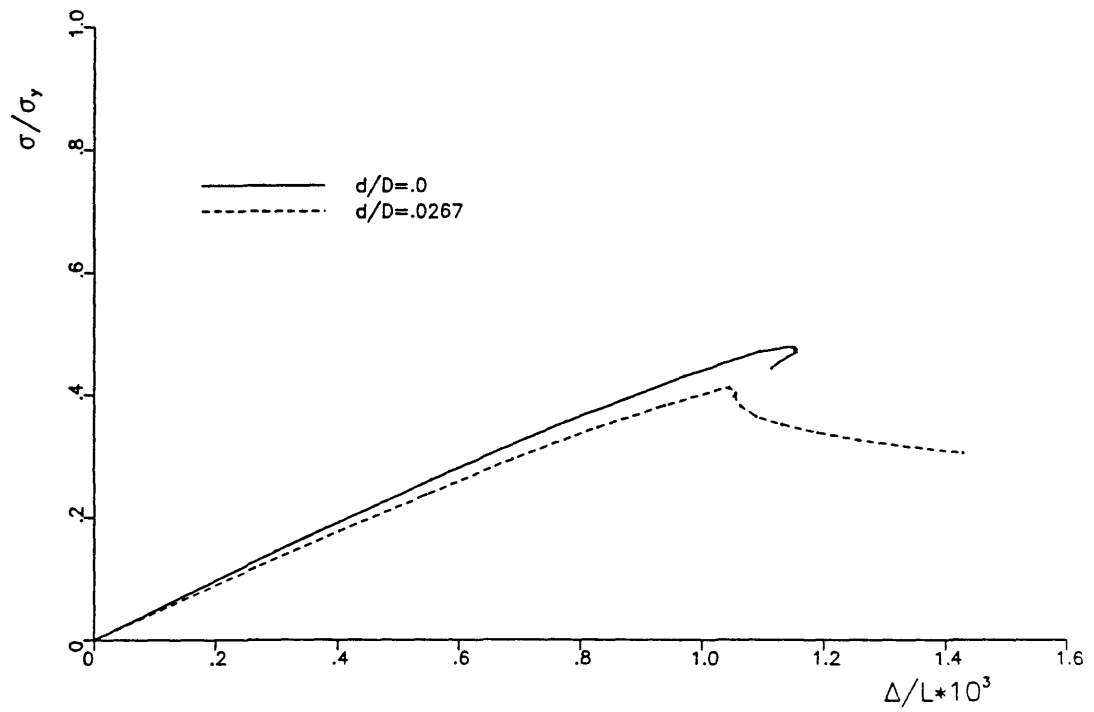
($R/t=500$, $l/R=0.3$, $n=0$, $m=1,3,5$)

Figure 6.56

LOAD AXIAL DEFLECTION CURVES

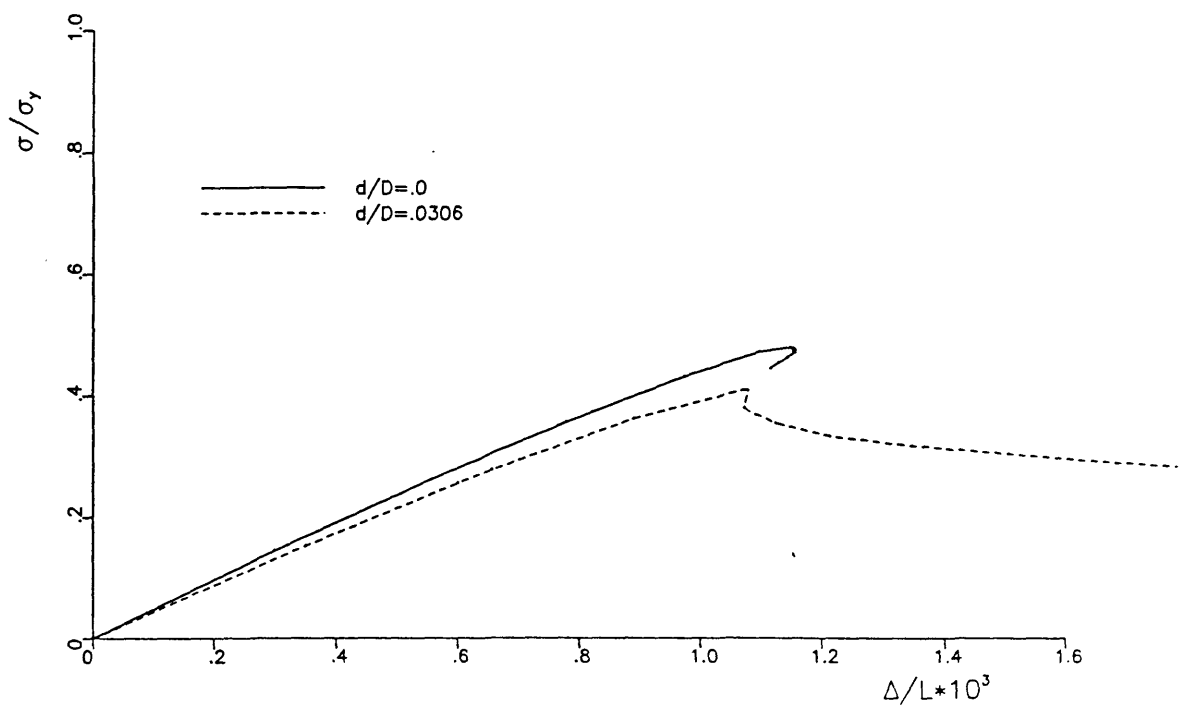
($R/t=500$, $l/R=0.3$, $n=0$, $m=1,3,5$)

Figure 6.57

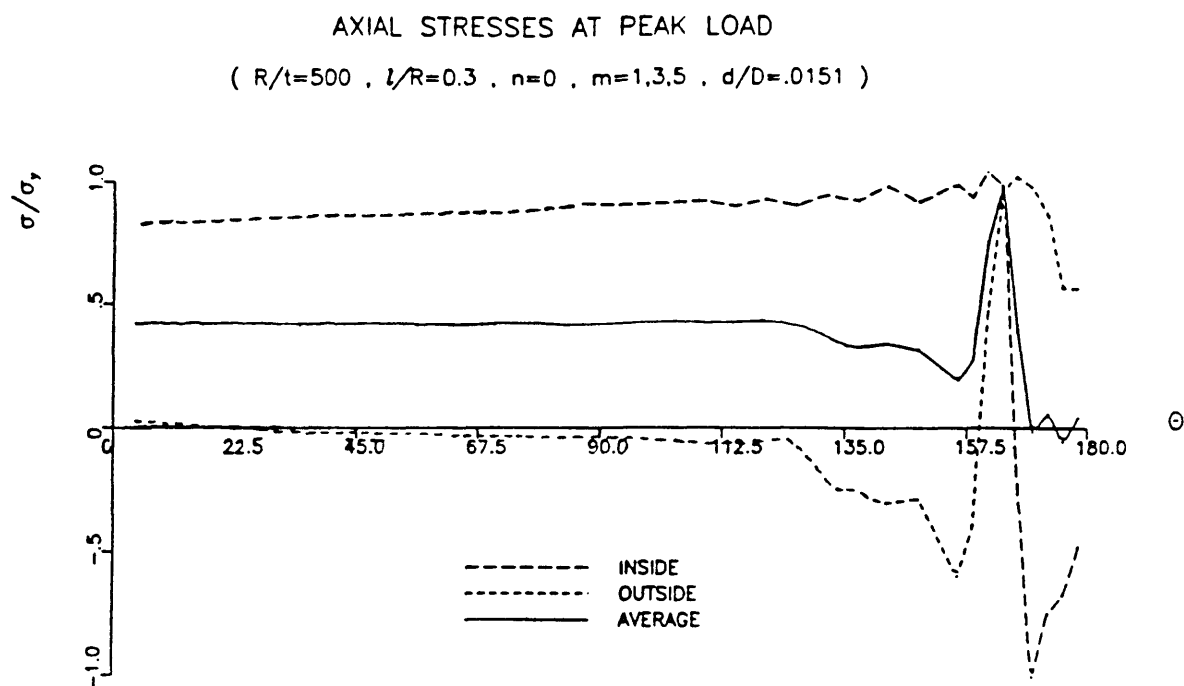


Figure 6.58

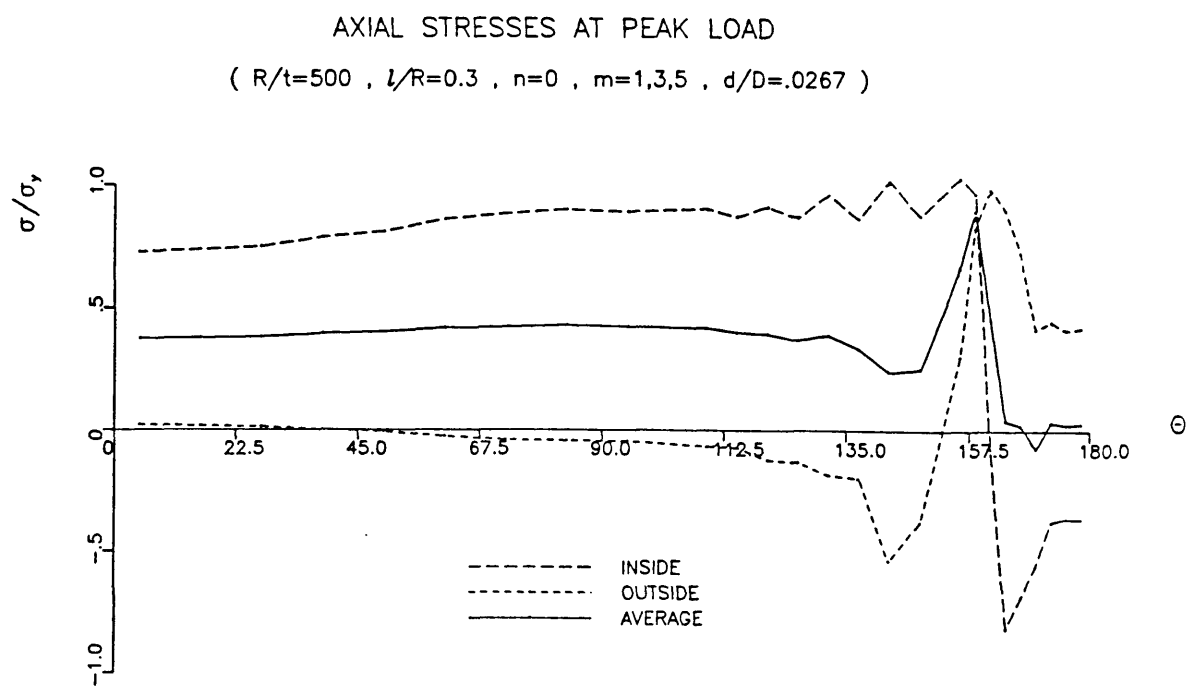


Figure 6.59

FAILURE MODES FOR DIFFERENT LEVELS OF DENTING

($R/t=500$, $l/R=0.3$, $n=0$, $m=1,3,5$)

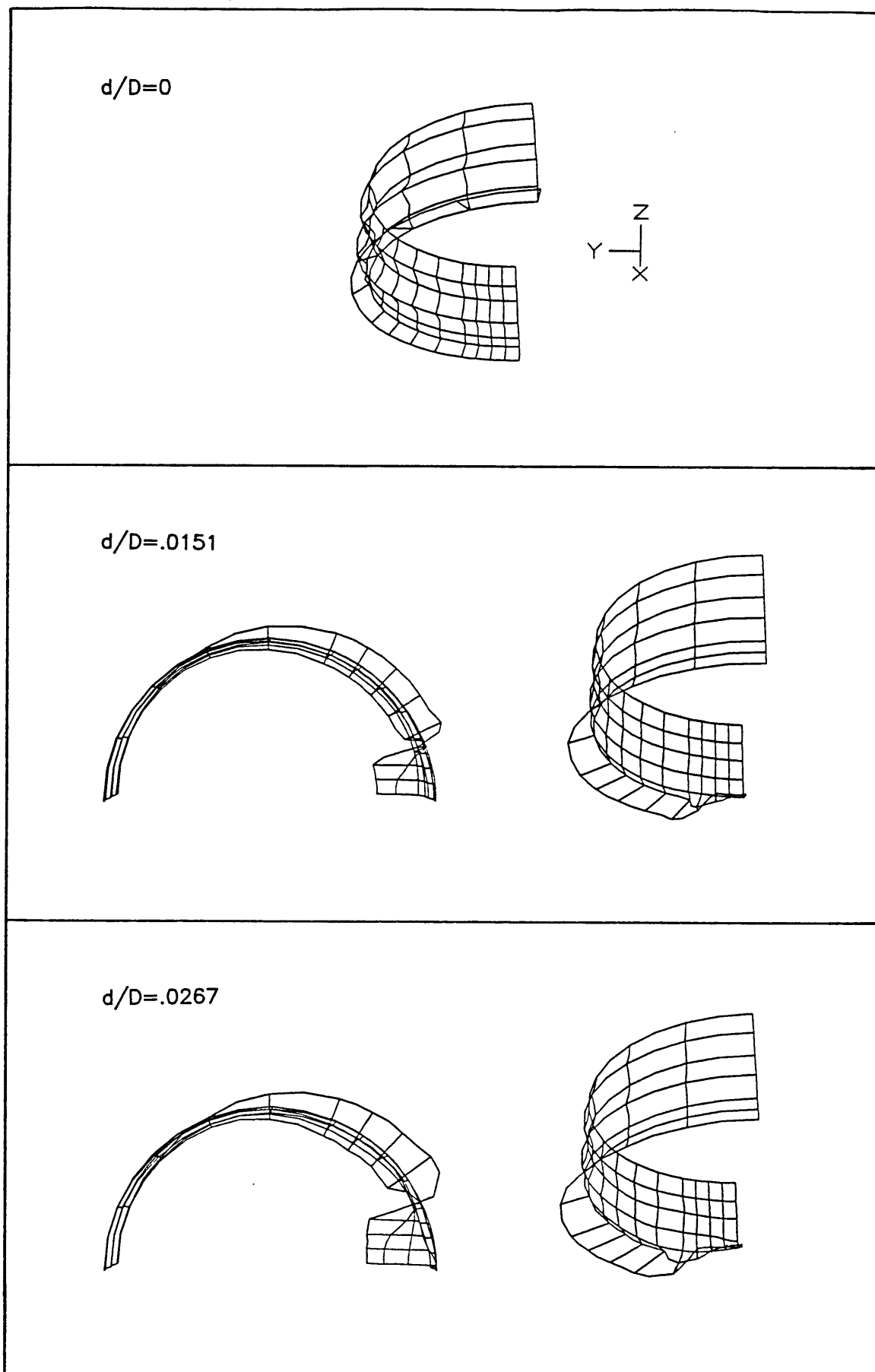


Figure 6.60

AXIAL RESIDUAL STRENGTH ($R/t=500$, $l/R=0.3$, $n=0$, $m=1,3,5$)

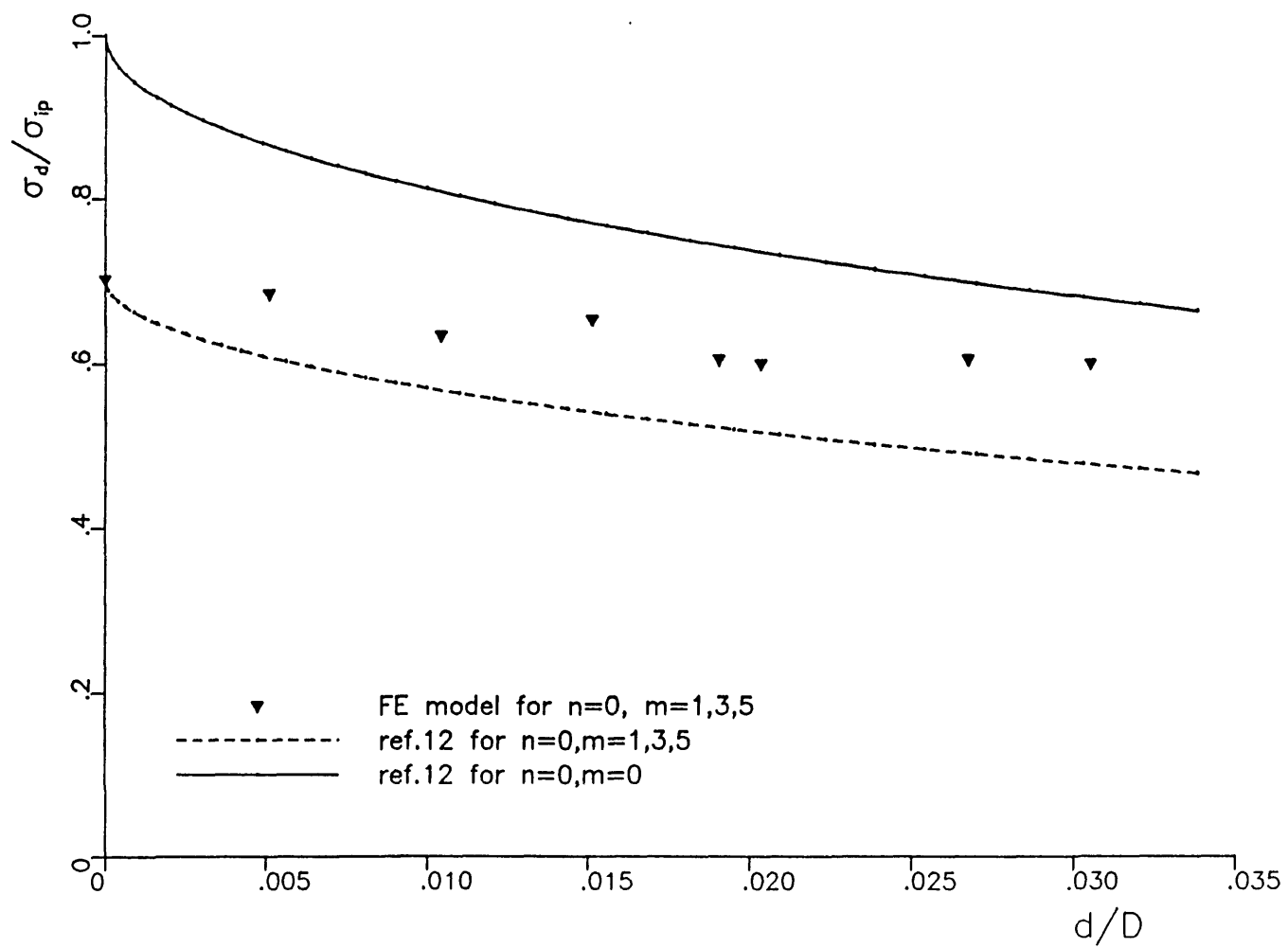


Figure 6.61

LOAD AXIAL DEFLECTION CURVES

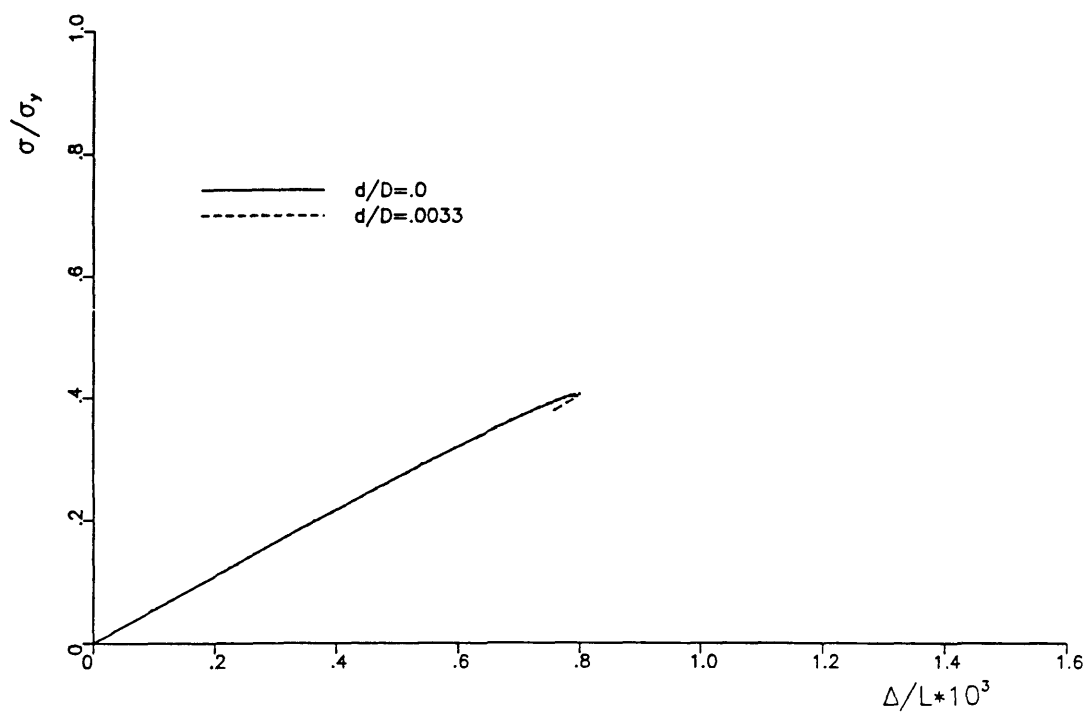
($R/t=500$, $l/R=0.3$, $n=10$, $m=1,3,5$)

Figure 6.62

LOAD AXIAL DEFLECTION CURVES

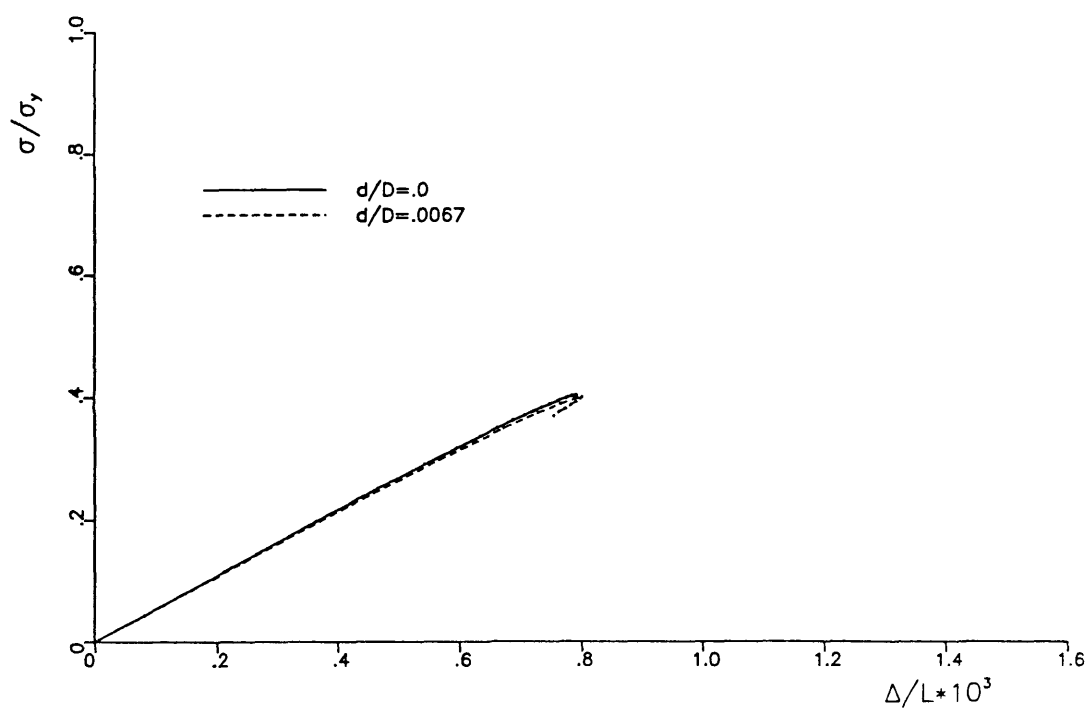
($R/t=500$, $l/R=0.3$, $n=10$, $m=1,3,5$)

Figure 6.63

LOAD AXIAL DEFLECTION CURVES

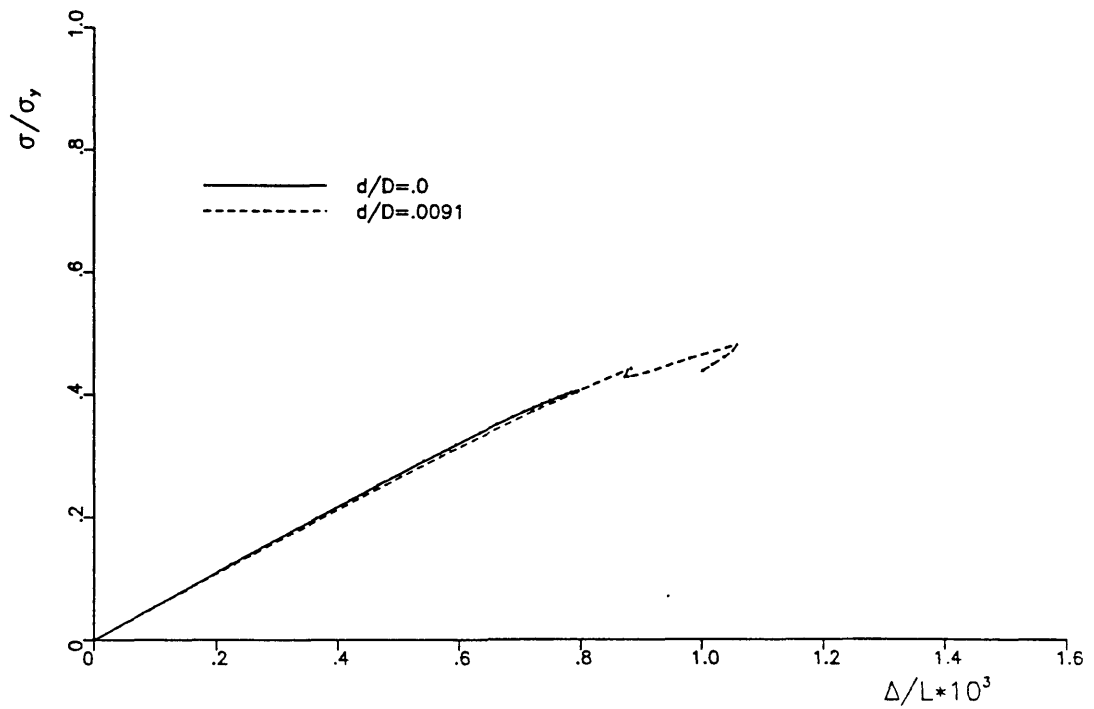
($R/t=500$, $l/R=0.3$, $n=10$, $m=1,3,5$)

Figure 6.64

LOAD AXIAL DEFLECTION CURVES

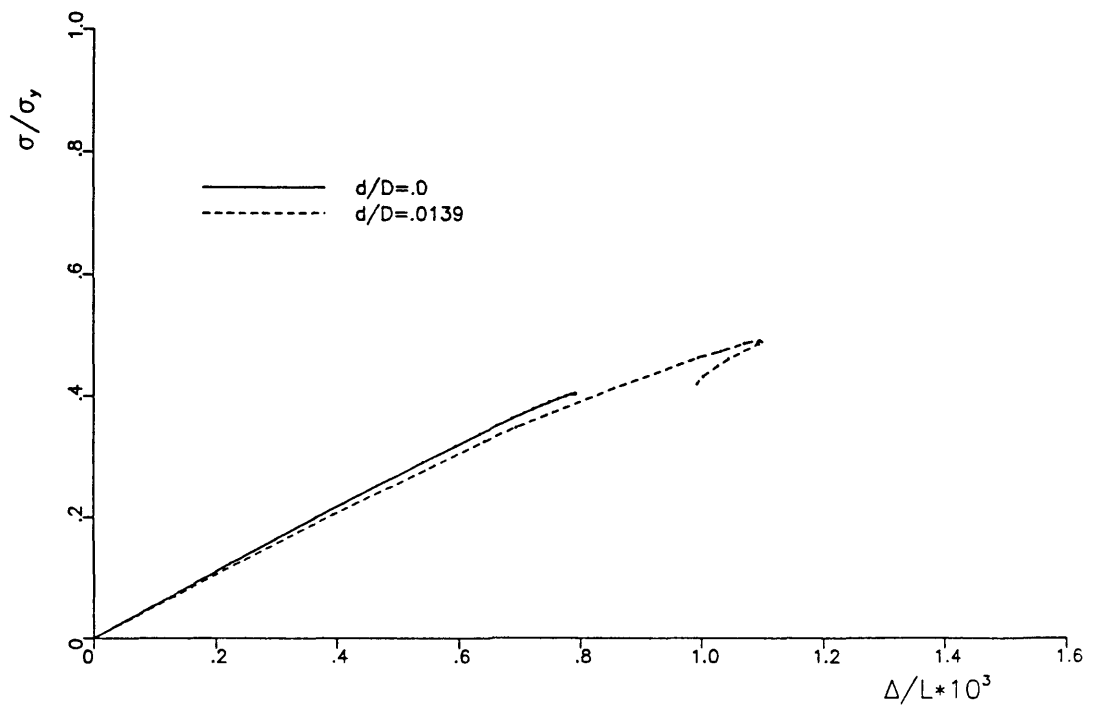
($R/t=500$, $l/R=0.3$, $n=10$, $m=1,3,5$)

Figure 6.65

LOAD AXIAL DEFLECTION CURVES

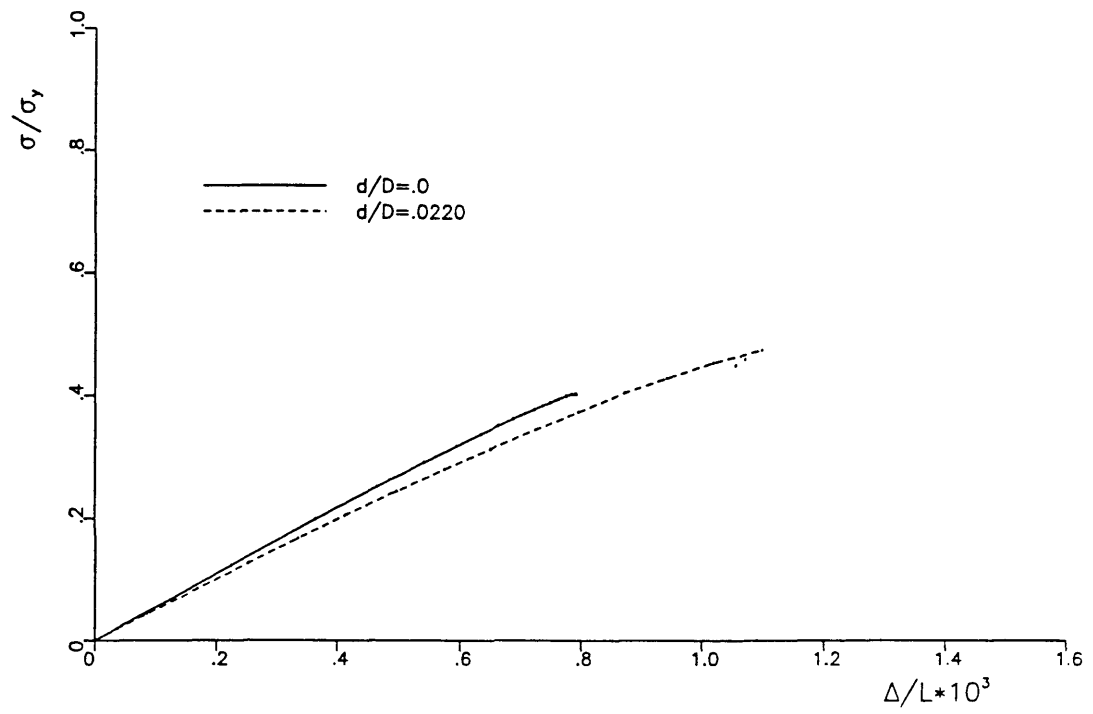
($R/t=500$, $l/R=0.3$, $n=10$, $m=1,3,5$)

Figure 6.66

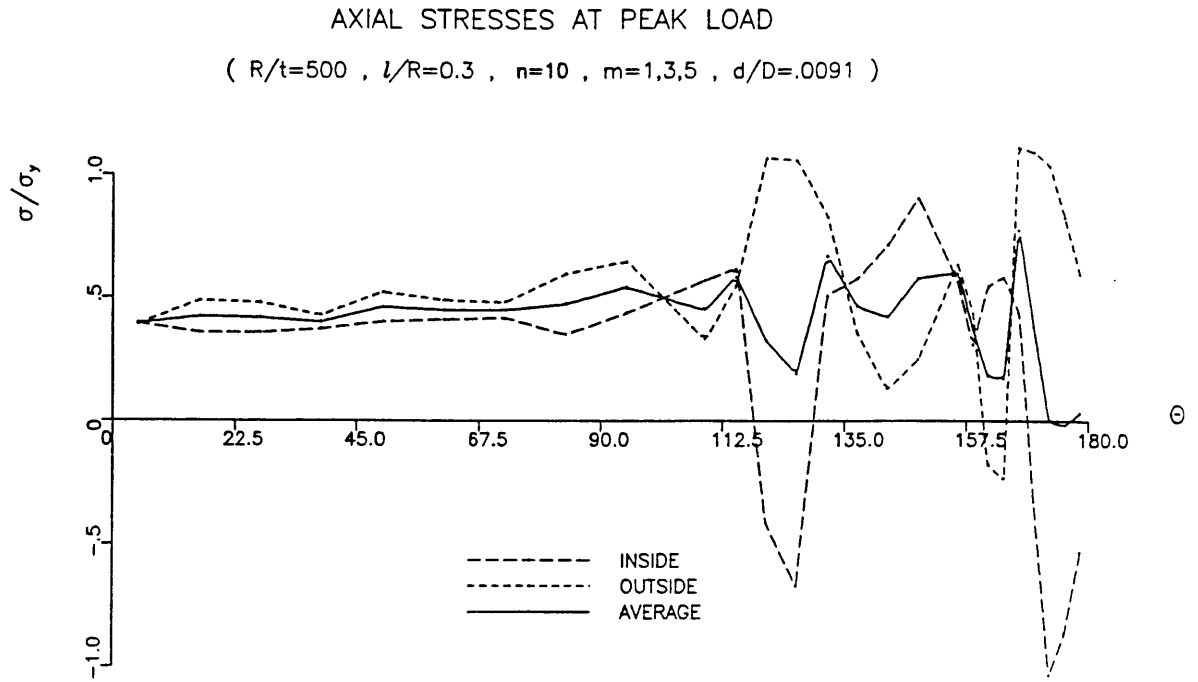


Figure 6.67

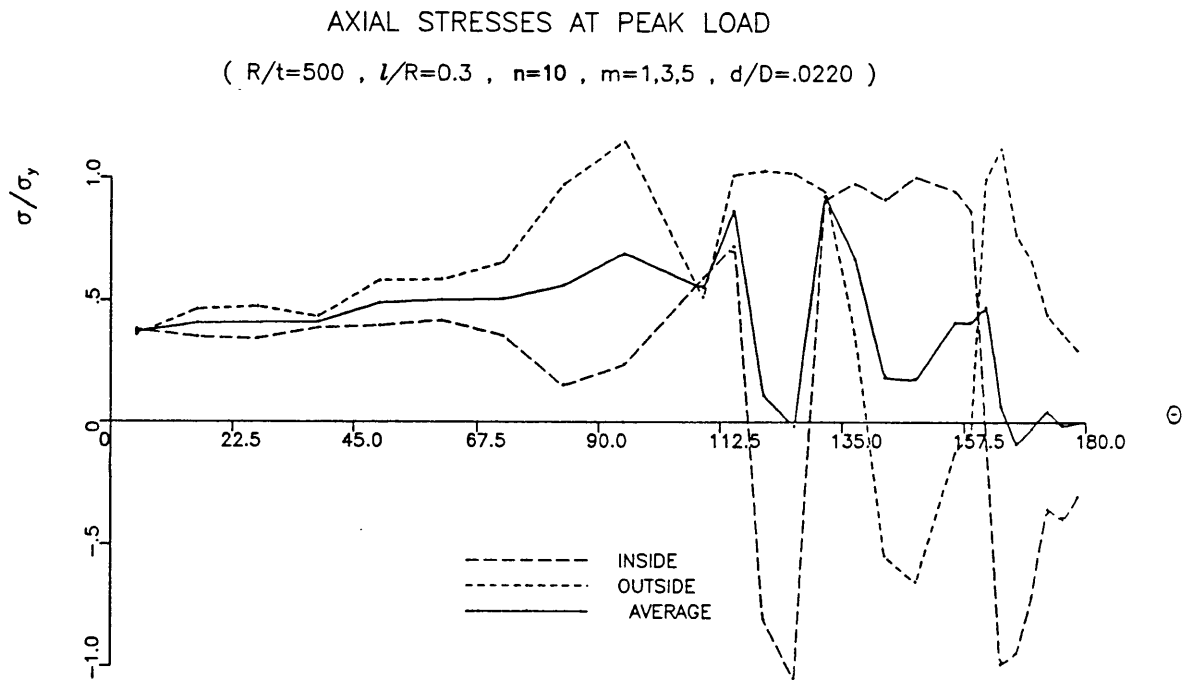


Figure 6.68

FAILURE MODES FOR DIFFERENT LEVELS OF DENTING

($R/t=500$, $L/R=0.3$, $n=10$, $m=1,3,5$)

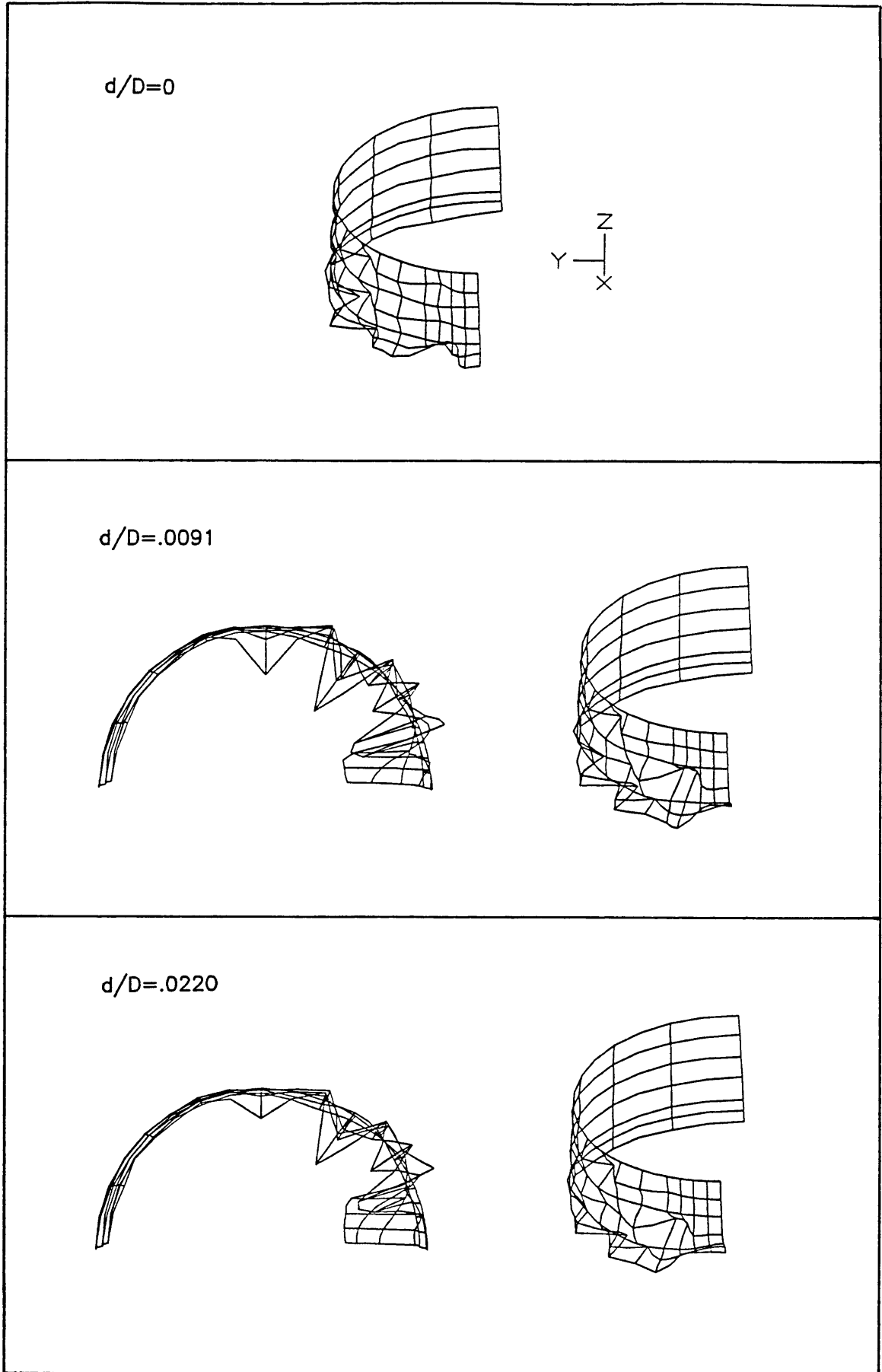


Figure 6.69

AXIAL RESIDUAL STRENGTH ($R/t=500$, $l/R=0.3$, $n=10$, $m=1,3,5$)

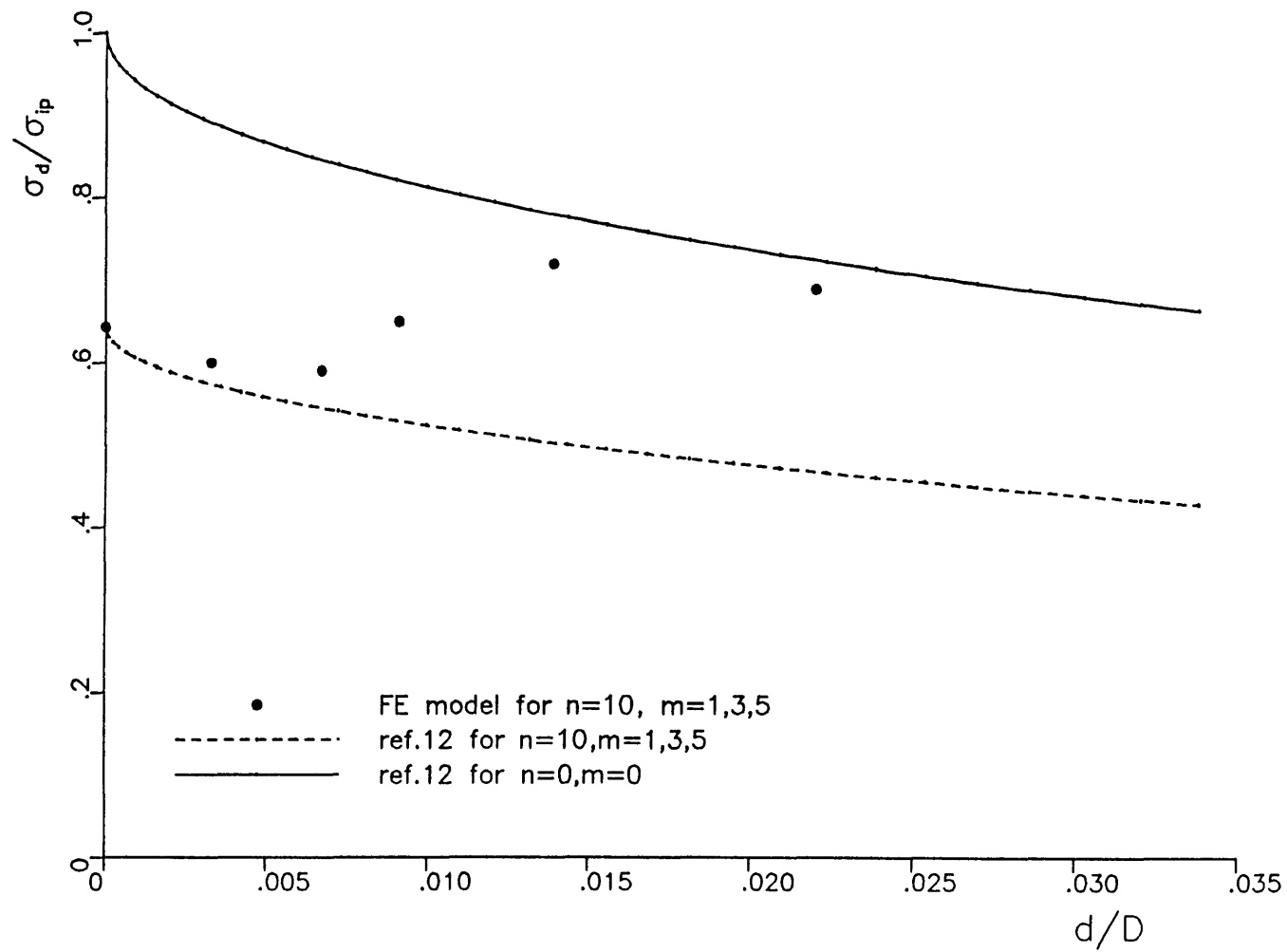


Figure 6.70

CHAPTER 7

CONCLUSIONS

7.1 GENERAL CONCLUSIONS

The problem of ship impact damage to the ring stiffened legs of offshore platforms has been the subject of this dissertation.

Appropriate numerical models for damage assessment have been developed using the nonlinear finite element method of analysis. Experimental work provided validation of these numerical models. Their sensitivity to different modelling considerations has been examined and some limitations of the finite element method of analysis for the present application have been identified.

The combined experimental and numerical studies performed produced an extensive data covering a wide range of practical geometries. Results of these studies provide guidance for damage assessment of real impact situations.

7.1.1 Experiments

Experimental techniques have been applied successfully to provide data on the response of ring stiffened shells subjected to denting and subsequent axial compressive loading. Results have been used to validate the numerical models.

Shells subjected to midpanel denting, as expected, exhibited lower lateral stiffness as the slenderness and ring spacing increased. These parameters were less important when the dents were applied to the rings. In the latter case the bending stiffness of the rings themselves mainly determined the response.

Stiffeners designed according to DnV specifications for axial compressive loading were shown to deflect readily under lateral load and deformations spread through the stiffeners over the whole length of the model. The main inward deflections of the dent were accompanied by smaller outward deflections immediately outside it and even smaller inward deflections further away around the circumference.

In the axial compression tests the slender shells showed a sudden failure, while the stockier ones suffered a small but significant reduction in stiffness prior to collapse. This behaviour is in line with equivalent intact shell behaviour.

The presence of the dent acts mainly as an area of "weakness" which results in stress redistribution around the model leading to highly stressed areas outside the dent. Collapse is triggered in these highly stressed areas when the stress level is close to the equivalent intact model collapse stress. Failure mode takes the form of inward buckling inside the dent and outward buckling adjacent to it. The rest of the circumference generally fails in the critical mode of the equivalent intact shell.

The location of the dent in relation to the rings does not appear to have any significant effect on the strength. Even in the cases of ring dents including large initial rotations of the stiffener no sign of stiffener distress was observed.

7.1.2 Numerical Studies

The non-linear finite element method of analysis has been employed in developing a model for the denting process. This includes a new gap-contact element which models the gradual contact between the indenter and the shell. The analysis predicts the response of the shell to denting both during loading and unloading and provides a full picture of the residual stresses and deformations remaining in the shell after the load is removed. Results of the analysis were correlated with experimental data and gave good agreement. Analyses performed demonstrated the importance of the axial boundary condition at the ends of the shell in determining the lateral stiffness. The predicted residual state of deformations and stresses comprises an initial state for residual strength assessment.

The finite element method was also employed in developing a model for the axial compression analysis of the dented shell. An appropriate modelling technique has been proposed after examining the effects of different modelling considerations. All the experimental models were analysed and results were in reasonable agreement with the experiments. Analyses produced a good qualitative picture of the response to axial loading.

The accuracy of the strength predictions, however, was affected by various factors such as the sensitivity of the model to the assumed idealised boundary conditions. The latter were shown to be of great significance in defining the degree of axial stress redistribution possible and consequently the collapse stress of the shell. Another factor which affected the accuracy of the strength predictions was the computer power available which limited the mesh refinement possible and also necessitated the introduction of symmetry lines to reduce the problem at hand. In addition, the omission of residual stresses from the analyses affected the results. All the above had some effect on the numerical predictions which were generally higher than the experimental ones.

A study on the effect of different boundary conditions has shown that uniform stress, while it provides a good strength estimate for an intact shell, is inappropriate for the analysis of dented shells. This is due to the complete loss of stress redistribution capability towards the intact part of the shell, which results in early failure. The condition of uniform displacement with rotation was the most representative of the test conditions. Uniform displacement only, which gave the highest residual strength predictions, could also be appropriate in certain situations in practice.

One of the experimental models was analysed including the residual stresses as predicted from the denting analysis. By comparing with the analysis including residual deformations only, the net effect of the residual stresses could be assessed. The latter were found to reduce the strength further by only a small percentage despite the fact that they reach a significant compressive level of stress in the critical area outside the dent. This may suggest that the deformations alone would be sufficient to give a reasonable representation of the response, but more work is needed to confirm this.

Analyses of the intact configurations provided a base strength and detailed information on the response for comparison purposes and enabled an estimation of the strength loss induced by the dent to be made. Examination of the dent induced strength reductions has shown that the dent location in relation to the rings has little effect on the mechanism of failure and strength reduction. Some increase in loss of strength was exhibited by shells of narrow ring spacing but the highest relative strength reduction of up to 33 percent occurred in the most slender shell with the highest R/t and ℓ/R . This indicated that in shells of high slenderness which have a tendency to fail in a lobular mode, the dent may have an added effect on the strength by acting as a sympathetic imperfection to the critical mode of failure.

The latter effect was examined further in a systematic numerical study of shells of high slenderness outside the scope of the experiments. Analyses of both initially perfect and imperfect shells were performed. The biggest relative effect on the strength was obtained in the initially perfect shells. A wavy strength degradation curve was obtained indicating areas where higher relative strength reductions occur, mainly where the dent wavelength was of the order of the critical wavelength. This suggests that the dent may interact with the critical failure modes and reduce the strength further by acting as a sympathetic imperfection.

The latter did not, however, occur in the initially imperfect shells where the presence of the well-defined initial imperfections dominated the response. Initial imperfections were shown to reduce the base strength of the cylinder but to mitigate the effect of the dent by predetermining the failure mode and precluding the dent from triggering a critical collapse mode.

In practice where a general form of initial imperfections would be present, the behaviour would vary between the two extreme cases of zero or well-defined imperfection modes examined above.

Comparisons with a simple effective section approach [12] have shown the latter to give lower bound strength predictions for most of the cases analysed. Even in the cases where bigger strength reductions were predicted by the finite element analyses, the difference was within 10 percent. This suggests that the analysis may be employed in this range of slender geometries with a 10 percent added reduction on the strength prediction sufficient to provide safe results.

The study of slender shells concludes that the net effect on the strength resulting from dent-form interactions with critical failure modes and initial imperfections is small. Consequently, little increase in sensitivity to denting may be expected as the slenderness increases. However, more work is needed to confirm the range of geometries over which this conclusion remains valid.

7.2 RECOMMENDATIONS FOR FURTHER WORK

An immediate extension of the scope of this dissertation is the employment of the numerical models developed in a wider examination of parameters such as dent depth (d/D), slenderness (R/t), ring spacing (ℓ/R) and initial imperfection form. Another important extension would be to examine different loading combinations including axial pressure, bending and torsional loads. Work done on shells subjected to pressure

loading in particular [50] indicated an increased sensitivity to denting which shows that there is scope for more work in this direction. Both numerical and experimental work would be useful.

Results of this study would provide the basis for further development of simple analytical procedures to account for the effects of all the above parameters. The procedure should be in a simple form suitable for designers faced with quick assessment of a collision situation.

Another aspect that deserves further attention is the effect of different forms of damage such as indentation shape or dent orientation to the cylinder axis. Experiments could be performed and the numerical models extended to model these different forms of damage.

Dynamic tests should enable an assessment to be made of strain-rate effects on the response to lateral load and on the residual strength. Similarly the models could be extended to enable dynamic analysis to be performed. Such analyses would show any necessary corrections that should be applied to the static predictions.

It would also be of interest to examine the performance of the ring stiffened legs under impact when the prestressing effect of the end load is included during the impact.

Finally it would be of interest to examine the subsequent response of dented shells to cyclic loading, representing the wave-action which the damaged leg would have to withstand after collision. Such an investigation would establish the fatigue characteristics of the damaged member. It is quite possible that high local curvatures around the dent could give problems in fatigue.

Results from each one of the above, should be implemented in a unified simple design approach for damage assessment, with the ultimate objective of establishing simple, realistic and safe guidance.

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