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p.47 Before last inequality: "maximum wavelength to bandwidth" should be "minimum wavelength to bandwidth".

p.51 Line 5 from end: "minimum" should be "maximum".

p.52 Line 3 from end: " W/m^2 " should be " $W/m^2/Hz$ ".

p.107 Line 5 from end: "The Magellanic Clouds have" should be "The Small Magellanic Cloud has".

p.108 Line 14 from end: "[88]" should be "[85]".

p.108 Line 4 from end: "1.8 arcseconds" should be "0.18 arcseconds".

HARDWIRED DATA PROCESSING APPLIED TO
STELLAR SPECKLE INTERFEROMETRY.

By

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This thesis has been submitted to the University of London
for the degree of Doctor of Philosophy and to Imperial College
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For my parents and my dearest Sarah Jane
- who is always in my thoughts.

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Abstract.

The resolution limit of large ground based telescopes is not set by their optics, but by the "blurring" of the image due to atmospheric motions. Nevertheless such an image, which has a speckled appearance, contains diffraction limited information. Stellar speckle interferometry is a technique which overcomes the effects of atmospheric turbulence, and allows the fundamental resolution limit of a telescope to be achieved by the analysis of many short exposure images.

The first three chapters of this thesis give a general overview of astronomical interferometry, describe the theoretical basis of stellar speckle interferometry, and explain various approaches to the technological problems involved in recording astronomical speckle images. The remaining chapters detail the research involved in the application of a hardwired autocorrelator and a hardwired cross-correlator to the reduction of speckle data.

The ensemble average autocorrelation of a set of speckle images contains a broad component, or "seeing pedestal", due to the statistics of the seeing. A method of removing this atmospherically induced distortion has been proposed by Welter and Worden. It involves the subtraction of a suitably normalised average cross-correlation from the average autocorrelation. The development of a hardwired cross-correlator has allowed the Welter and Worden seeing correction to be performed on the autocorrelations produced by the existing hardwired autocorrelator.

A wide variety of astronomical objects has been observed and results are presented for binary stars, planetary nebulae and nebulae. The analysis of data undertaken as part of the Hipparcos astrometry programme suggests that 30% of astronomical sources are multiple at the diffraction limit of a 3.9 m. telescope. The diameters of Magellanic Cloud and Galactic planetary nebulae have also been derived and the mass of a Magellanic Cloud planetary nebula has been calculated.

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Chapter One.

CHAPTER ONE - INTRODUCTION.

1.1 The Bounds Upon Telescopes.

The atmosphere is not stationary: it is a turbulent medium with inherently unpredictable optical properties. This instability is of great importance to astronomers using large high quality optical telescopes, as it causes scintillation of the image and blurs the image of a point source into a "seeing" disc. The diameter of this disc varies from 0.5 arcseconds to 10 arcseconds depending upon the prevailing atmospheric conditions. Diffraction theory predicts that the fundamental resolution limit of a telescope is approximately equal to the ratio of the observational wavelength to the diameter of the aperture. Therefore, a seeing disc one arcsecond across imposes an optical resolution limit equivalent to that of a telescope with an aperture 12 cm. in diameter. This detrimental effect of atmospheric turbulence was recognised by Newton as early as 1704. In Opticks [1] he writes:

"If the Theory of making Telescopes could at length be fully brought into Practice, yet there would be certain Bounds beyond which Telescopes could not perform. For the Air through which we look upon the Stars, is in perpetual Tremor; as may be seen by the tremulous Motion of Shadows cast from high Towers, and by the twinkling of the fix'd Stars."

Newton concludes with a statement that predicted the need for site testing and the location of most modern optical observatories:

"The only Remedy is a most serene and quiet Air, such as may be found on the tops of the highest Mountains above the grosser Clouds."

The steady increase in the diameter of telescopes since the 17th. Century has not increased their practically attainable resolution, and to minimise the atmospherically induce blurring of images most modern telescopes are now situated at high altitude in climates that have still air. The ultimate example of this principle of "getting above the atmosphere" is the Hubble Space Telescope (HST). This is a

2.4 m. telescope which is to be placed in earth orbit. Freed from the distortion and absorption effects of the atmosphere, and with suitable instrumentation, the HST will have a resolution limit of 0.05 arcseconds and a magnitude limit of +28 [2]. However, there are observational techniques that allow ground based telescopes to attain their fundamental resolution limit and thus compete with the resolution of any space based instrument.

1.2 High Resolution Observing Techniques.

A few determined physicists have shown that by the use of special observing techniques it is possible to obtain information on scales smaller than the diameter of the seeing disc with ground based instruments. These observational methods fall into two basic categories: those which operate on a single telescope and give resolution down to the diffraction limit of the optics; and those which rely on the concepts of aperture synthesis to achieve baselines far greater than the diameter of any monolithic mirror.

The idea of utilising interferometry to overcome the effects of atmospheric seeing was first suggested by Fizeau [3] in 1868. He proposed to observe Young's fringes in stellar images, but it was not until 1890 that Michelson [4] fully developed this idea into a practical technique by the use of aperture masks. Later, Michelson and Pease [5] increased the baseline of the interferometer to 6 m. by constructing separate optics to be fitted to the mount of the Mount Wilson reflector. Over the next ten years, from careful fringe contrast measurements, the diameters of several bright giant stars were deduced and the separation of the binary Capella found to be 0.05 arcseconds. An attempt by Pease to further increase the baseline to 15 m. proved unsuccessful because of the difficulties in maintaining the stability of the two optical paths.

The next advance in stellar interferometry came with the invention of the intensity interferometer by Hanbury Brown and Twiss [6] in 1958. Such an instrument has been constructed at the Narrabri Observatory in Australia. It consists of two comparatively poorly figured 6.5 m. mirrors which can be moved on a circular track 300 m.

in diameter. The light gathered by each mirror is detected by photomultipliers and the amplitude fluctuations converted into electrical signals which are proportional to the light intensity. The two currents are amplified and multiplied before their correlation is measured at a central station. By measuring this correlation as a function of the mirror separation, stellar diameters can be derived.

Intensity interferometry has several advantages over amplitude interferometry; it is insensitive to atmospheric turbulence, the telescope figure and baseline instabilities. The coherence length must be less than $c/\Delta f$, where Δf is the electrical bandwidth of the amplifiers used. Typically $\Delta f = 10^8$ Hz. and, hence, the light paths must be maintained within 3 m. of each other. This is easily achieved. The major drawback of intensity interferometry is that the correlation depends on the energy received per unit spectral bandwidth. This means that to obtain an satisfactory signal to noise ratio long integrations must be made on bright stars. The magnitude limit is approximately +2, but the Narrabri interferometer has been used to measured the diameters of 32 bright stars.

As a result of the poor limiting magnitude of intensity interferometers interest in amplitude interferometry has undergone a renaissance since the early 1970s. One approach uses apertures of smaller diameter than the typical seeing cell size of 10 cm. These instruments are similar in construction to Michelson's original interferometer. Currie [7] describes an "amplitude" interferometer that combines the light from two apertures using a Köster prism. The resulting two output beams are detected and a digital correlator used to calculate the fringe visibility.

As the angular resolution of any interferometer is approximately λ/D , where D is the baseline separating the two apertures, an interferometer constructed by masking a single telescope will have a maximum resolution equal to the diffraction limit of the telescope optics. Therefore, a further extension of Michelson's interferometer uses two telescopes or siderostats separated by a long baseline. Labeyrie has been responsile for the development of such a long baseline stellar interferometer consisting of two 25 cm. telescopes

separated by a 12 m. baseline at the Centre d'Etudes et de Recherches Geodynamiques et Astronomiques (CERGA). Fringes have been observed during a study of the star Vega [8]. Further research at CERGA has led to the use of two mobile telescopes on a baseline variable from 6 to 67 m. This gives a resolution of 0.015 to 0.002 arcseconds [9]. In Australia Davis and Tango [10] have constructed an interferometer with two siderostats mounted on massive concrete supports separated by an 11.4 m. baseline. The limiting magnitude is estimated to be +8.5. Interference has been observed with starlight and observations on "standard" stars are planned. There is a proposal for a second generation instrument with a baseline range of 5 to 640 m. However, a major problem encountered in the use of such long baselines is maintaining instrumental stability. Active mechanisms are generally used to maintain the critical condition of temporal coherence at the focus where the beams from the separate optical components are combined. Also the use of apertures larger than the average seeing cell size means that the incoming wavefront will contain atmospherically induced distortions. These can be partially corrected by a control loop containing a wavefront sensor and active optics. The sensor detects the wavefront distortions and sends a signal through a servo system to move or deform the mirror in a manner which corrects for the observed distortion.

In principle it is possible to reconstruct the object under study by measuring the complex degree of coherence using a pair of apertures of varying separation. This is implicit in the van Cittert-Zernicke theorem which states that the complex degree of coherence is the normalised Fourier transform of the brightness distribution of the object. However, atmospheric turbulence or mechanical instabilities can produce random phase shifts in the two interfering beams. If the integration time is short enough to ensure that the fringe movement is negligible the true fringe contrast can be measured. However, the phase of the fringes will be random which prevents the unambiguous reconstruction of the object's brightness distribution. Also the limited coverage of the Fourier plane by two aperture interferometers constrains their performance to the estimation of a limited number of geometrical parameters, unless many consecutive observations are made with mobile apertures.

Some physicists and astronomers are now suggesting the application, at optical wavelengths, of the phase closure techniques routinely used in radio astronomy. There is, however, a very important difference between the proposed optical array telescopes and their existing radio counterparts. In radio astronomy it is possible to detect the relative phase of the signal received at each antenna by comparison with a very stable local oscillator or "clock", and store the result. The recorded signal can then be amplified before it is multiplied with the signals from the other antennae to produce the radio equivalent of an interference pattern. Phase closure algorithms are implemented to recover the phase of the complex degree of coherence and allow an image of the source to be made. This is the basis of radio heterodyne interferometry. Optical heterodyne interferometry is at present severely limited by the lack of suitable amplifiers and sufficiently stable lasers to act as local oscillators. With these considerations in mind consider an array of N telescopes. High angular resolution could be achieved and a large number of spatial frequencies simultaneously measured. The number of spatial frequencies depends on the redundancy of the array. If all beams are combined at one detector a completely redundant linear array of N telescopes covers $N-1$ spatial frequencies. For a totally non-redundant linear or two dimensional array $N(N-1)/2$ spatial frequencies are covered. However, in this non-redundant scheme the use of a single detector reduces the magnitude limit by $2.5\log(N/2)$ [11] and it is advisable to use a highly redundant array. Alternatively each pair of telescopes could be equipped with their own detector, however, this would limit the number of simultaneously observed frequency components to the order of N instead of N^2 as in radio astronomy.

Initial attempts at optical phase closure have been made by Baldwin [12]. He placed a mask containing between three and six holes (of similar diameter to the seeing cells) in the aperture of a single telescope. The sets of fringe patterns, produced by the interference of light from pairs of apertures, were then Fourier transformed to derive their amplitudes and phases. An example of an already existing phased array is the Multiple Mirror Telescope (MMT) [13]. This consists of six 1.8 m. mirrors arranged in a hexagonal configuration. All six mirrors may be co-phased by adjusting the optical path lengths

of the six beams to give an effective resolution diameter of 6.86 m. No active optics are available on the telescope to attempt to correct for the wavefront phase and tilt errors across the individual mirrors and speckle interferometry is used to overcome the effects of atmospheric turbulence.

1.3 Speckle Techniques.

The technique of stellar speckle interferometry, as proposed by Labeyrie in 1970 [14], is a generalised form of Fizeau interferometry which uses the full aperture of a telescope. Many short exposure images are recorded at the telescope focus. These images are then analysed to obtain diffraction limited information from their two dimensional visibility function (Chapter Two gives a detailed discussion of the theoretical basis of speckle interferometry). One very important property of the speckle interferometry transfer function is that it is insensitive to telescope aberrations. Thus a telescope of poor optical quality can achieve diffraction limited resolution if the atmospheric seeing is sufficiently poor [15].

The principle of detecting the "double speckles" present in the image of a binary star had already been used in a intuitive way by experienced binary star observers [16]. Such astronomers were able to recognise the binary character of the speckle pattern inside the wider modulation of the seeing disc. However, it was not possible to record the phenomenon until devices such as the Lallemand electronographic camera and the image intensifier were developed. In 1972 Gezari, Labeyrie and Stachnick [17] published the first speckle observations of several binary stars and the giant stars Betelgeuse and Mira Ceti. There have now been numerous observations of binary stars by McAlister and Morgan; McAlister has published a "Catalog of Interferometric Measurements of Binary Stars" and Morgan is eliminating binaries from the Hipparcos astrometry programme. In principle, the fringe visibility can be used to estimate the magnitude difference of a binary star, but this requires accurate calibration of the system. Strenuous efforts are now being made by several groups to compensate for the instrumental and atmospheric effects which can distort speckle data and prevent the derivation of photometric

information (see Chapter Eight).

Speckle observations are not now confined to bright binary stars. The development of photon counting detectors and the application of powerful computing techniques to data reduction have improved the magnitude limit to at least +16. The astronomical objects observed range from giant single stars to active galactic nuclei. The sources studied in association with this work include binary stars, bipolar nebulae, and planetary nebulae. The measurement of the diameter of several planetary nebulae has led to the derivation of nebulae masses. When this information is combined with spectral data certain conclusions can be drawn about the nature of the progenitor stars.

A recent extension to the technique of conventional speckle interferometry is Beckers' differential speckle interferometry. This combines high spatial and spectral resolution. Instead of computing the power spectrum of a single image, the cross spectrum is formed of two speckle images taken at slightly different wavelengths. The technique uses the fact that the spectrum of the light emitted from an object often varies spatially because of Doppler shifts, changes in chemical composition or Zeeman splitting. The differences in the relative positions of speckles at differing wavelengths can be accurately measured and differential speckle images of Betelgeuse have been reported by Hebden et al. [18]. Other areas of possible application include the study of spectroscopic binaries, stellar rotation and peculiar A type stars.

Speckle interferometry has also been extended into the near infra-red (2-5 μm .). At a wavelength of 0.5 μm . the diffraction limit of a 4 m. telescope is 0.03 arcseconds, approximately 30 times smaller than the seeing limit. However, at 5 μm . the diffraction limit is 0.3 arcseconds and the seeing limit is 0.6 arcseconds [15]. Therefore, the potential resolution of a telescope is only doubled by infra-red speckle interferometry. Nevertheless, there are a great many potentially resolvable infra-red objects and several groups are active in this area. Of course the technological problems associated with any infra-red observation must be overcome. The detectors employed are one dimensional and must be cooled to liquid nitrogen or liquid helium

temperatures. Careful calibration of the background emission from the sky and the telescope is also necessary. Despite these additional difficulties, which are not present in optical speckle interferometry, there are many published results. Some of the most important include McCarthy's measurement of the triple nature of MonR 2-IRS 3 [19], Chelli's reconstruction of η Carinae using the Knox-Thompson algorithm and various observations of Mira variables, the dust shells around Wolf-Rayet stars and Betelgeuse. With the imminent availability of two dimensional detectors speckle interferometry in the infra-red should become an increasingly important astronomical technique.

1.4 The Reduction of Speckle Data.

The reduction of speckle interferometry data is a statistical process involving the formation of ensemble averages. To achieve a satisfactory signal to noise ratio in a measurement, many individual short exposure images must be analysed. Therefore, a critical aspect of any speckle system is the speed at which the data can be processed.

The work presented here will describe the application of a hardwired autocorrelator and a hardwired cross-correlator to the processing of astronomical data. The data is recorded on video tape and the speed of these hardwired devices allows correlations to be calculated in real-time. A detailed description of the design and performance of the autocorrelator and the cross-correlator is given in Chapters Four and Five.

With the continued improvement in the speed of computing elements a new generation of devices to analyse speckle data can be seriously proposed. The development of array processors allows Fourier transforms to be performed quickly on large quantities of data while preserving the flexibility of a software system. However, the most exciting advance could be the introduction of parallel processing techniques to the problem of image reconstruction. With such an approach the now lengthy calculations involved in certain image reconstruction algorithms become far less time consuming. These ideas are developed in Chapter Eight.

1.5 Reconstruction of the Object Intensity.

Speckle interferometry as developed by Labeyrie, and as applied in most astronomical observations, is used to derive the object's energy spectrum, or equivalently its autocorrelation. However, it is impossible, in general, to reconstruct the object intensity from its energy spectrum alone. Even the intensity of unequal magnitude binary stars cannot be uniquely determined. To retrieve the object intensity the phase of the object's Fourier transform must be known. There are certain algorithms which use only the modulus of the object Fourier transform to attempt to reconstruct the object, but they assume that a unique solution exists and are not guaranteed to converge on the correct solution. Other methods use additional information contained in the speckle images to derive a unique solution. However, the practical difficulties of implementing such algorithms on astronomical data are not trivial. Brief descriptions of some reconstruction methods are given in Chapter Eight.

Chapter Two.

CHAPTER TWO - THE THEORY OF STELLAR SPECKLE INTERFEROMETRY.

2.1 Introduction.

The fundamental resolution limit of a telescope is set by the diffraction of light passing through the aperture. The resulting image of a point source has a bright central core of angular diameter $2.44 \lambda/D$ (where λ is wavelength and D the diameter of the aperture) surrounded by successively larger but fainter rings of light. This Fraunhofer diffraction pattern, known as the Airy function, constrains the maximum resolution, $\Delta\alpha$, to a value conventionally given by the Rayleigh Criterion: two point sources are said to be just resolved when the first dark ring of one image crosses the central maximum of the second image, as shown in Figure 2.1. Therefore:

$$\Delta\alpha = 1.22 \lambda/D. \quad \text{Eq. 2.1}$$

For example, at a wavelength of 500 nm. a telescope with a diameter of 4 m. is theoretically capable of resolving separations down to 0.03 arcseconds. Unfortunately, this limit is rarely reached by ground

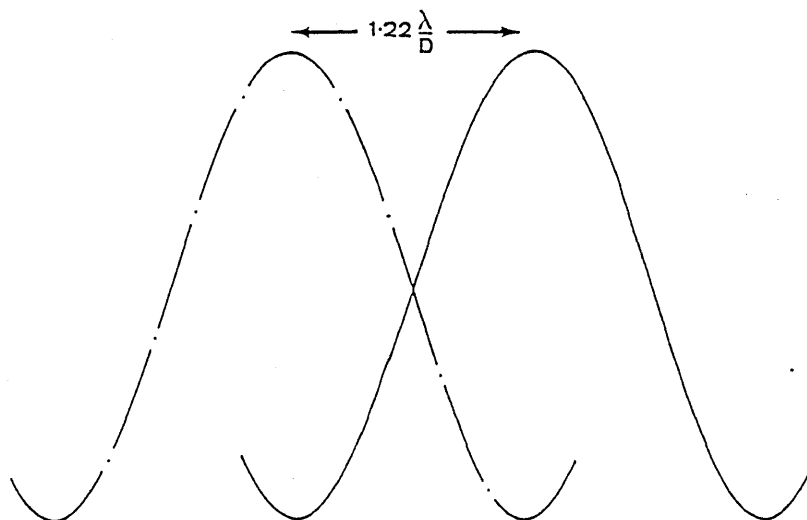


Figure 2.1 The Rayleigh Criterion for the Resolution of Two Point Sources.

based instruments because of atmospheric turbulence.

2.2 The Seeing Problem.

The Earth's atmosphere is a turbulent fluid containing many inhomogeneities of temperature, pressure and refractive index. The effect of these variations is to produce a random isotropic distribution of "seeing cells" throughout the atmosphere. These cells (which have a characteristic size of 10 cm. [20,21]) are not stationary of course, but continually change shape, merge with each other and drift across the sky. Therefore, any wavefront propagating through the atmosphere is subject to random fluctuations of phase. Figure 2.2 illustrates the effect of these phase fluctuations upon the image formed by a telescope. For an unperturbed plane wave the image is an Airy disc. However, in practice, the incoming wavefront is

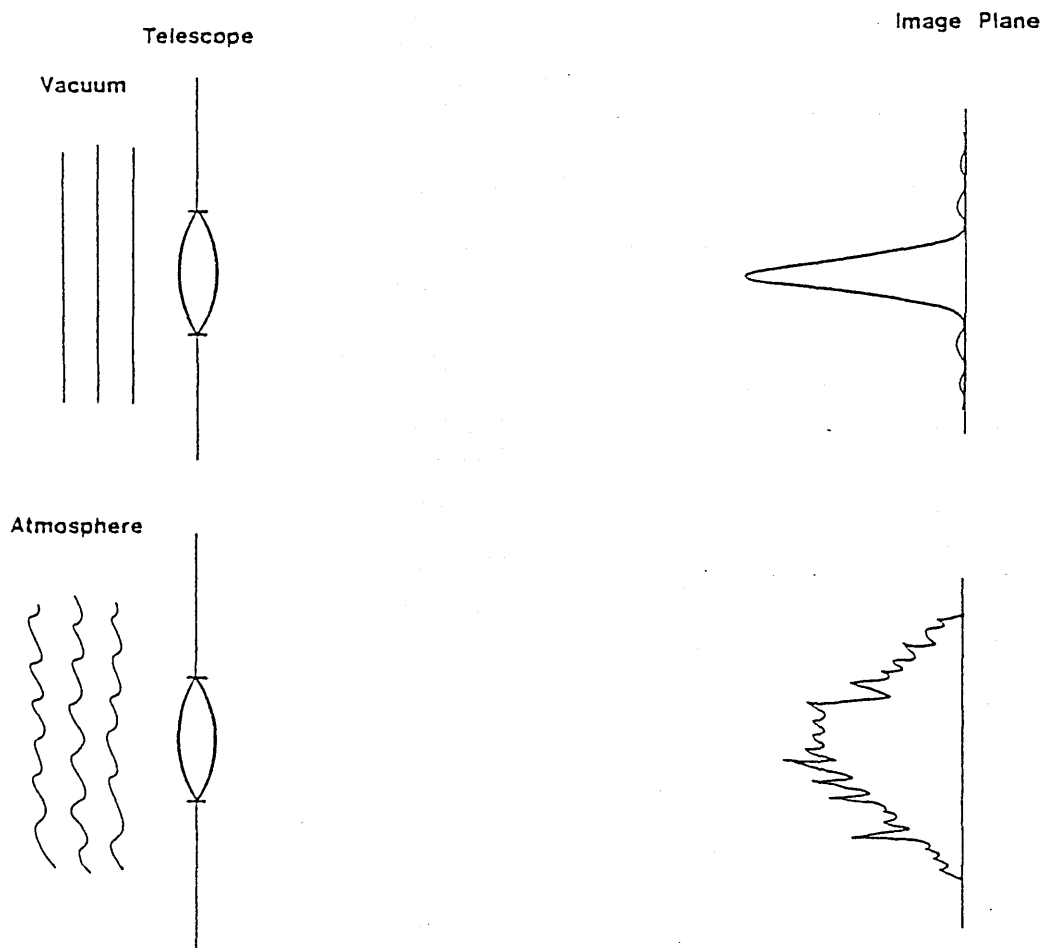


Figure 2.2 A Schematic Representation of the Effects of Atmospheric Turbulence.

distorted by the atmosphere. The aperture of the telescope contains many perturbing cells and at any instant a random assortment of these cells are in phase and interference will occur. As the in phase cells are randomly distributed across the aperture, the interference fringes, or speckles, have a size close to the diffraction limit of the telescope and are contained within a broad seeing disc. Further causes of atmospheric image degradation are the different perturbations imposed on wavefronts travelling through separate columns of the atmosphere. The maximum angular separation between two point sources for which the wavefront perturbations are highly correlated is known as the isoplanatic angle. Several measurements of this angle have been performed and results ranging from 2 to 10 arcseconds obtained [22,23]. This variation is probably due to the different seeing conditions encountered by the observers.

It is now useful to define two parameters: the decorrelation time T as the interval between statistically independent atmospheric states; and the speckle lifetime t as the period over which atmospheric changes are considered negligible. Of course no definitive values can be assigned to these time scales, but measurements performed by Scaddan and Walker [24], Dainty [25], and Ebersberger and Weigelt [22] suggest that typical values for T and t are 100 ms. and 10 ms. respectively. For exposure times longer than the decorrelation time the image produced by a telescope is made up of many speckle patterns randomly displaced by atmospheric turbulence. Because of this motion and integration time a "seeing disc" appears with an intense core which becomes gradually dimmer towards the edges. The seeing seldom allows the angular diameter of this disc to fall below one arcsecond, which is 30 times greater than the diffraction limit of a 4 m. telescope. Therefore, it is the prevailing atmospheric seeing which limits the practical resolution of an astronomical telescope.

2.3 Speckle Images.

For an exposure time comparable to the speckle lifetime, the image formed by a telescope will depend on the telescope's aperture function and one particular random distribution of atmospheric seeing cells; any atmospheric turbulence is effectively frozen. Therefore, the

intensity of each statistically independent quasi-monochromatic speckle image, $I(x,y)$, can be described by the following incoherent imaging equation:

$$I(x,y) = \iint_{-\infty}^{\infty} O(x',y') \cdot P(x - x', y - y') \cdot dx' dy'. \quad \text{Eq. 2.2}$$

This integral represents the convolution of the object intensity $O(x,y)$ with the instantaneous point spread function, $P(x,y)$, of the combined atmosphere and telescope system, therefore:

$$I(x,y) = O(x,y) \otimes P(x,y) \quad \text{Eq. 2.3}$$

where $\otimes$ denotes the convolution operator.

Consider the formation of an idealised speckle image of a point source. Let (u,v) be coordinates in the aperture plane, then the effective aperture function, $H(u,v)$, of the combined telescope and atmosphere system is modelled by a circular aperture of radius ρ containing stationary seeing cells of radius r_0 . As this function is rotationally symmetric [26]:

$$H(u,v) = H(w) \quad \text{where } w^2 = u^2 + v^2$$

$$\Rightarrow H(w) = \text{circ}(w/\rho) \cdot \text{III}(w/a) \otimes \text{circ}(w/r_0).$$

The first term is a circular aperture with unit transmission out to a radius of ρ and the second term is a circular Dirac comb of separation a which replicates the seeing cells over the full aperture. The instantaneous image intensity, $I(x,y)$, will also be rotationally symmetric and can, therefore, be derived from a one dimensional Hankel transform:

$$I(x,y) = I(r) \quad \text{where } r^2 = x^2 + y^2.$$

The Hankel Transform gives:

$$I(r) = \text{const.} \left| \int_0^{\infty} H(w) \cdot J_0(2\pi r w) \cdot w \cdot dw \right|^2,$$

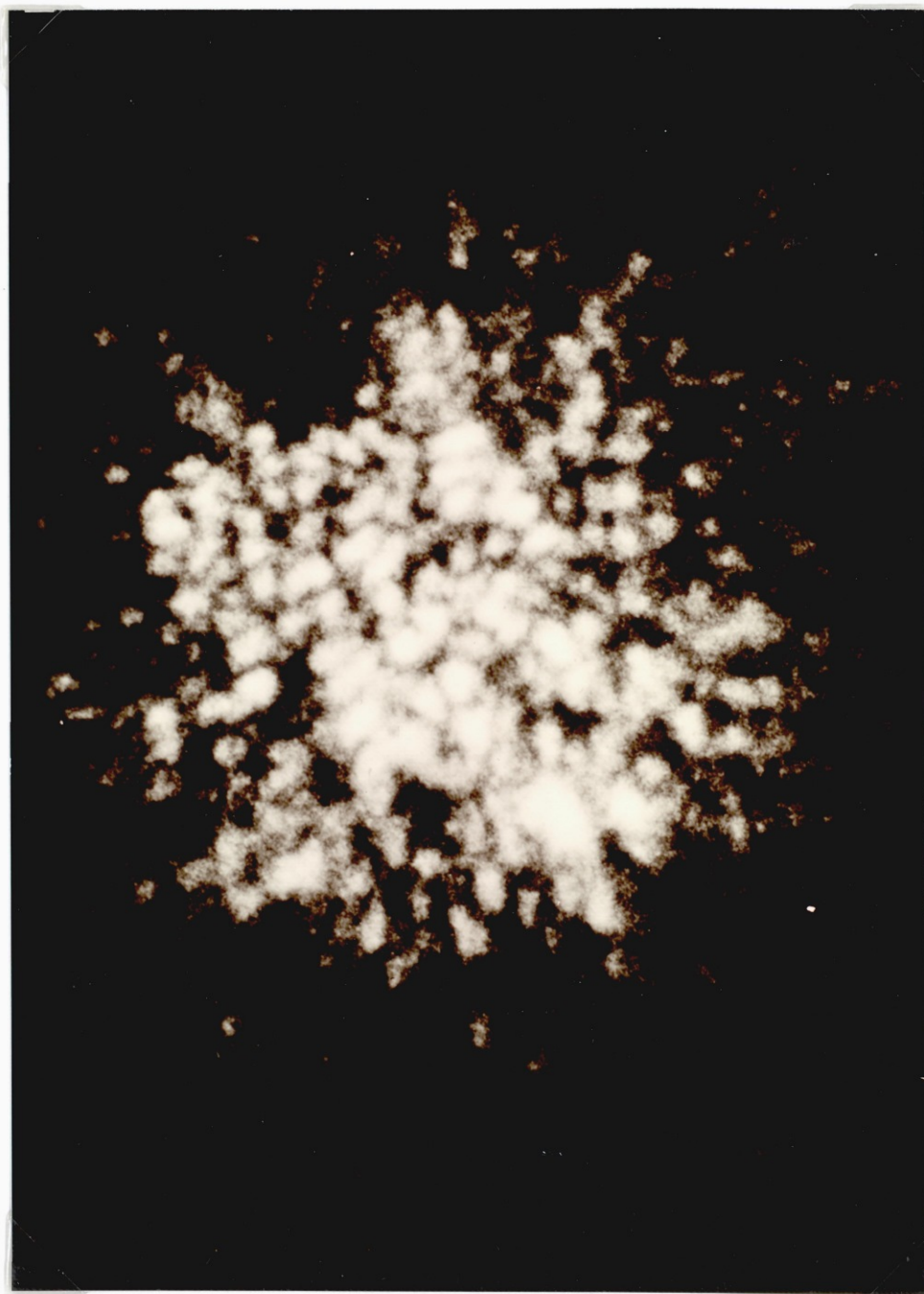


Plate 2.1 A Speckle Image of the Single Star α Centauri.

where $J_0(2\pi wr)$ is the zero order Bessel function of the first kind. As:

$$\int_0^x J_0(\zeta) \cdot \zeta \cdot d\zeta = x \cdot J_1(x)$$

where $J_1(x)$ is the first order Bessel function of the first kind,

$$\Rightarrow I(r) = \text{const.} [J_1(2\pi pr)/r]^2 \otimes \text{III}(ra) \cdot [J_1(2\pi r_0 r)/r]^2$$

$$= \text{CONST. jinc}^2(2\pi pr) \otimes \text{III}(ra) \cdot \text{jinc}^2(2\pi r_0 r).$$

This is the resultant intensity point spread function (PSF) of the telescope and atmosphere. Therefore, a point source $I(r)$ appears as a set of displaced diffraction limited images modulated by a broader Airy pattern. If $r_0 = 10$ cm. this broad pattern has a diameter of about 2.5 arcseconds. Actual speckle frames are not multiple aperture interferograms but Korrf et al. [27] have shown that such individual speckles have a characteristic size and shape similar to that of the Airy disc. A speckle image of a single star is shown in Plate 2.1.

The long exposure image is the ensemble average of these speckle exposures. Therefore, denoting an ensemble average by $\langle \rangle$, for M such exposures:

$$\begin{aligned} \langle I(x,y) \rangle &= 1/M \sum_m I_m(x,y) \\ &= O(x,y) \otimes \langle P(x,y) \rangle. \end{aligned}$$

It is the long exposure PSF, $\langle P(x,y) \rangle$, that determines the form of the seeing disc.

2.4 Data Reduction.

In stellar speckle interferometry a large number of short-exposure quasi-monochromatic images are analysed using Fourier techniques to extract diffraction limited information. The result is the average energy spectrum, or equivalently the average spatial autocorrelation,

of the object intensity.

Labeyrie [28] pioneered an optical processing method in which coherent light is passed through a speckle image recorded on cine film. The speckle image acts as a "grating" and the resulting diffraction pattern is the energy spectrum of the image intensity. This is formally defined as the squared modulus of the image's Fourier transform, so if lower case letters are used to indicate a Fourier transform and (u,v) are spatial frequency variables then from equation 2.3:

$$\begin{aligned} i(u,v) &= o(u,v) \cdot p(u,v) \\ \Rightarrow |i(u,v)|^2 &= |o(u,v)|^2 \cdot |p(u,v)|^2. \end{aligned}$$

By recording many such energy spectra on the same piece of photographic film the average energy or Wiener spectrum, $W(u,v)$, of the image is found:

$$\begin{aligned} W(u,v) &= \langle |i(u,v)|^2 \rangle \\ &= |o(u,v)|^2 \cdot \langle |p(u,v)|^2 \rangle. \end{aligned} \quad \text{Eq. 2.4}$$

In equation 2.4 $\langle |p(u,v)|^2 \rangle$ is the speckle transfer function. Korrf [20] and Dainty [15] have shown that this is greater than zero down to the diffraction limit of the telescope and, therefore, preserves high spatial frequency information. If N_s is the number of speckles per frame and $p_o(u,v)$ is the diffraction limited transfer function:

$$\langle |p(u,v)|^2 \rangle = |\langle p(u,v) \rangle|^2 + (1/N_s) \cdot p_o(u,v) \quad \text{Eq. 2.5}$$

As an example, the Wiener spectrum obtained from observations of a binary star appears as a set of fringes on a Gaussian-like background. The origin of this modulation is analagous to Young's slits in that pairs of corresponding speckles act as double slits and the background is a consequence of the long exposure transfer function. It is possible to estimate the speckle transfer function from observations of a nearby unresolved star under similar seeing conditions. In this case the object intensity is a Dirac delta function, $\delta(x,y)$:

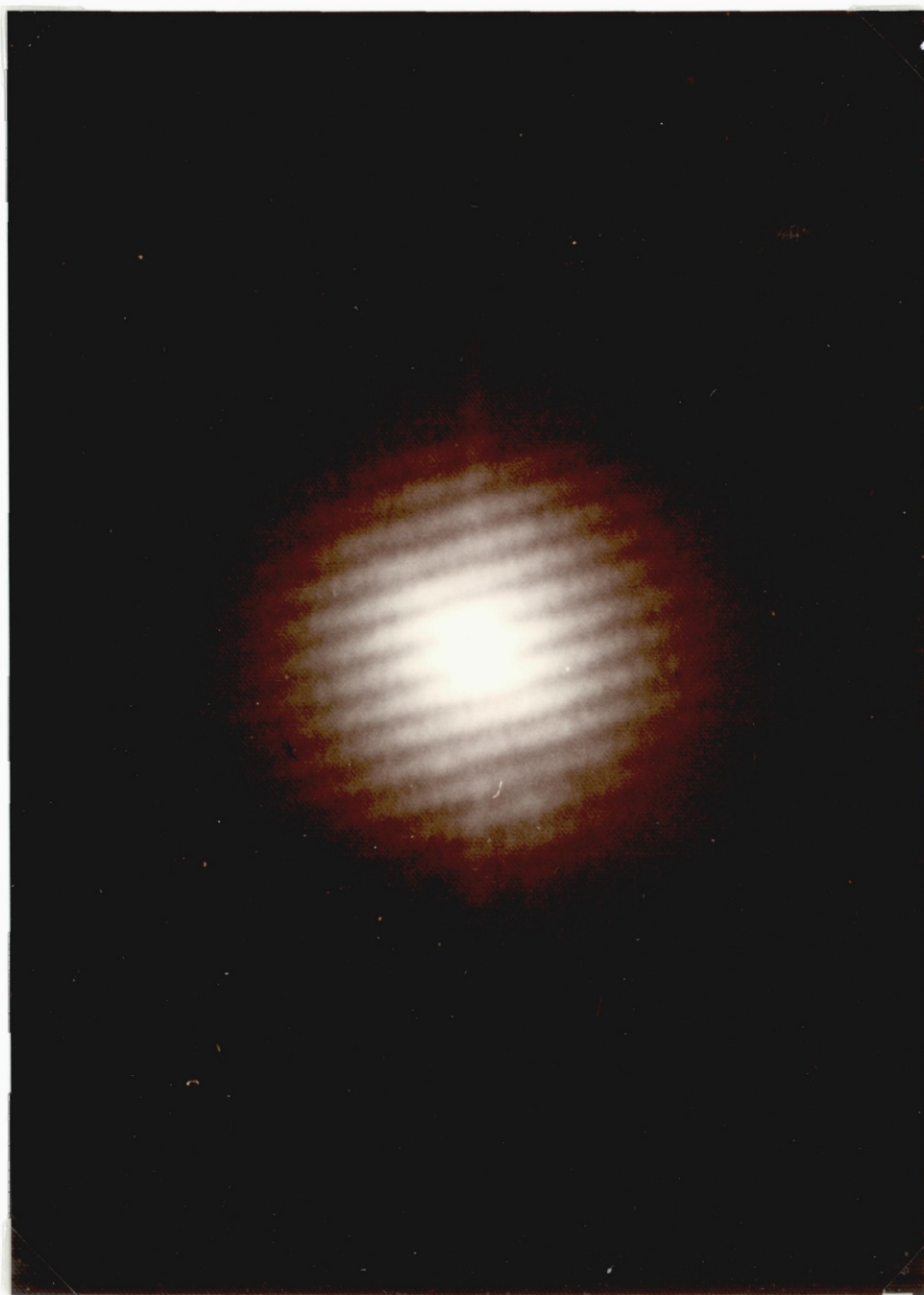


Plate 2.2. The Ensemble Averaged Energy Spectrum of the
Binary Star γ Lupi (separation = 0.56 arcsec. $m_V = 2.8$).

$$\begin{aligned}
I'(x,y) &= \delta(x,y) \otimes P(x,y) \\
\Rightarrow W'(u,v) &= \langle |i'(u,v)|^2 \rangle \\
&= \langle |p(u,v)|^2 \rangle.
\end{aligned}$$

Therefore, assuming the transfer function to remain unchanged between observations, from equation 2.4:

$$|o(u,v)|^2 = W(u,v)/W'(u,v).$$

The angular separation of the binary is inversely proportional to the fringe separation in this object energy spectrum and the position angle is given by the fringe orientation. The average energy spectrum of a binary star displaying such fringes is shown in Plate 2.2. However, under unstable seeing conditions the estimation of the transfer function can introduce considerable errors.

A method of estimating the object energy spectrum without the need to observe a point source has been described by Granrath [29]. For a set of transformed speckle frames the most intense fringe in a given spatial frequency range is normalised by the intensity of the D.C. component in the spectrum and then recorded. This process is repeated so that a "maximum magnitude" Fourier transform is built up. Such a transform directly estimates the combined object and telescope energy spectrum.

An alternative approach to speckle data reduction calculates the object's spatial autocorrelation. This is an even function which describes the distance over which $O(x,y)$ is coherent and is defined by:

$$\begin{aligned}
A(h,k) &= \iint_{-\infty}^{\infty} O^*(x,y) \cdot O(x+h,y+k) dx dy \\
&= O(x,y) * O(x,y),
\end{aligned} \tag{Eq. 2.6}$$

where $*$ denotes a complex conjugate and $*$ is the autocorrelation operator. The parameters of an astronomical object can be equivalently measured from an energy spectrum or an autocorrelation as these two functions are related by the Autocorrelation Theorem:

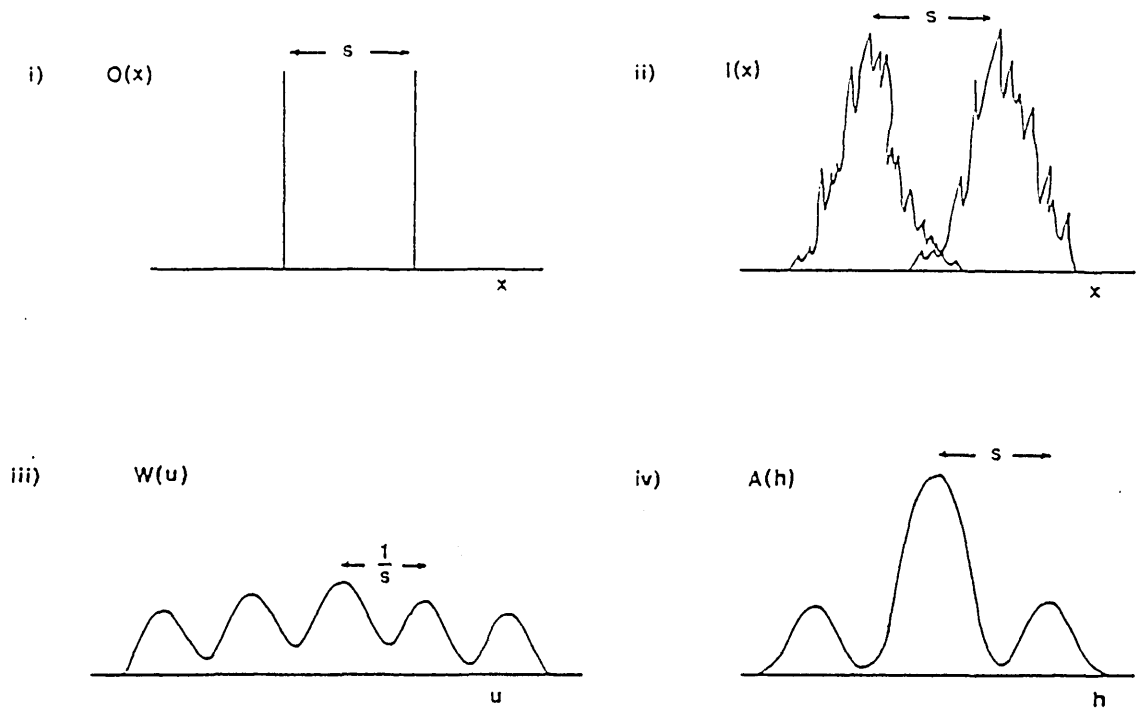


Figure 2.3 The Basic Processes Involved in Binary Star Speckle Interferometry.

- i) The Object Intensity. ii) The Instantaneous Image Intensity.
- iii) The Wiener Spectrum of the Image.
- iv) The Autocorrelation of the Image.

$$A(h,k) = \iint_{-\infty}^{\infty} |o(u,v)|^2 \cdot \exp(-2\pi i(uh + vk)) du dv.$$

Therefore, the autocorrelation and the energy spectrum form a Fourier transform pair. For a binary star the object autocorrelation contains three co-linear maxima. The distance between the central maximum and either one of the outer maxima is proportional to the binary separation and the position angle can be derived from the orientation of the three peaks. The basic steps and relationships involved in data reduction are summarised in Figure 2.3.

The optical processing method can be used to find an autocorrelation by simply forming the diffraction pattern of the energy spectrum. However, this approach requires a second, and consequently degrading, photographic exposure and development. Instead, the autocorrelation of

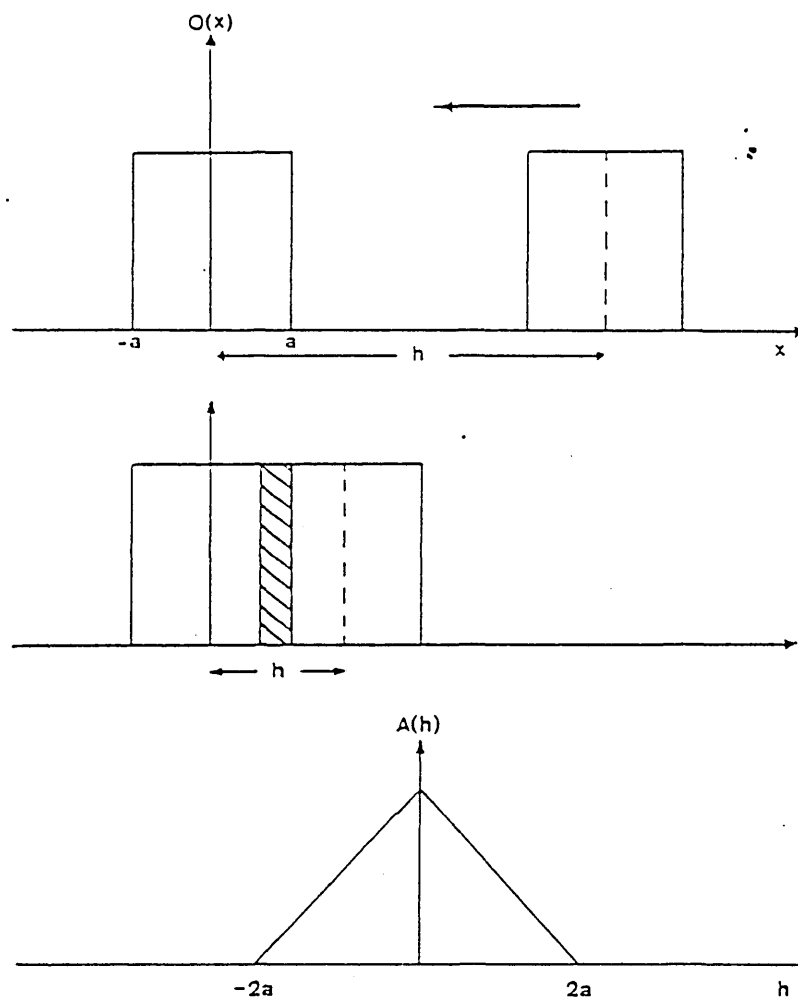


Figure 2.4 The Analogue Representation of Autocorrelation.

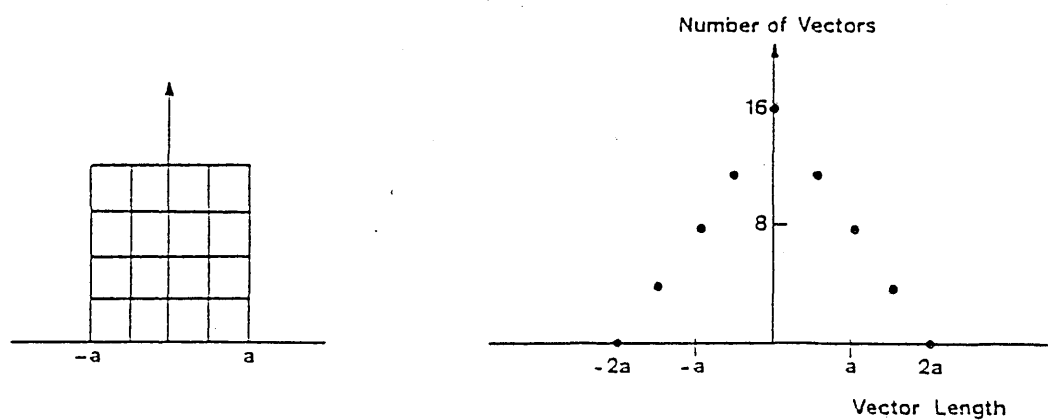


Figure 2.5 The Vector Autocorrelation of a "Top Hat" Function.

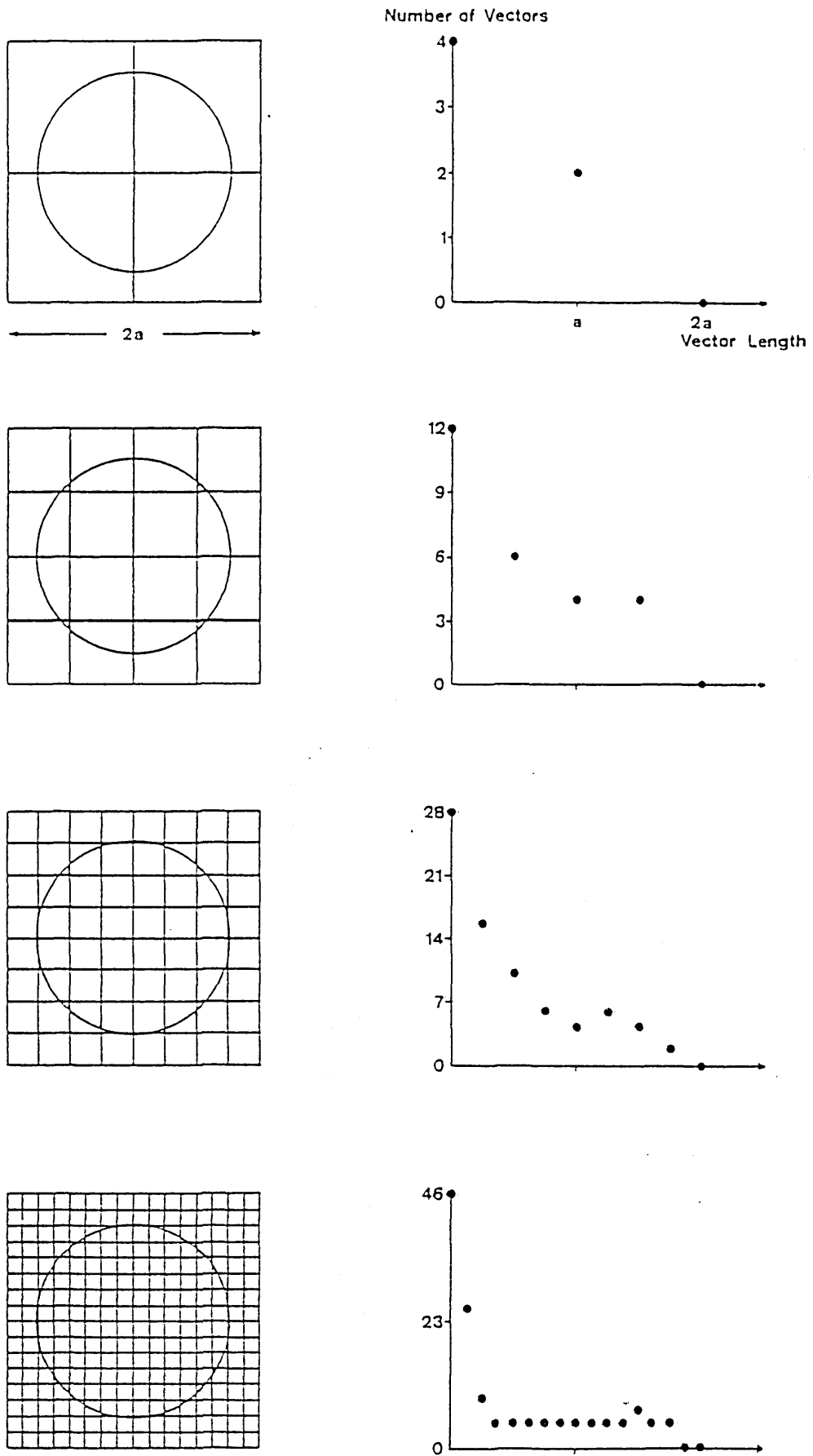


Figure 2.6 Vector Autocorrelations of an Annulus for Successively Smaller Pixel Sizes.

an image can be directly calculated by applying digital analysis techniques. Such an approach can be implemented in software or hardware and eliminates the non-linearities of photographic processing.

2.5 Digital Processing.

The "analogue" definition of autocorrelation, as given by equation 2.5, may be considered as the product of the overlapping areas between $O(x,y)$ and a displaced copy of $O(x,y)$ as a function of their relative displacement. Figure 2.4 illustrates the process of analogue autocorrelation, in one dimension, for a "top hat" function. As the function is moved through its replica the product of the overlapping areas increases linearly until it reaches a maximum where the origins of the functions coincide. Now consider an image having an origin at its top left-hand corner, x coordinates increasing towards the right, y coordinates increasing towards the bottom of the image and containing n photon events at coordinates (x_i, y_i) . The analogue autocorrelation is then equivalent to the distribution of vectors between any event and every other event, given by:

$$A(h,k) = \sum_i \sum_j (x_j - x_i, y_j - y_i).$$

The "vector" autocorrelation of the "top hat" function is shown in Figure 2.5. A coordinate system is imposed on the function and then the horizontal vectors between the centres of pixels are counted. There are 16 vectors of zero length, 12 of length 1 pixel, 8 of length 2 pixels, 4 of length 3 pixels and none of length 4 pixels. The envelope of this vector autocorrelation is identical to that of the analogue autocorrelation shown in Figure 2.4.

The most basic consideration in the formation of a vector autocorrelation is the size of the picture element, or pixel, between which vectors are to be measured. As an example consider the vector autocorrelations of an annulus calculated with various pixel sizes shown in Figure 2.6. Using only four pixels to cover the annulus, the distribution of vectors in the +x direction is: 4 of zero length; and two each of length +1. Therefore, the vector autocorrelation has a

triangular envelope. However, as the pixel size is reduced the envelope of the vector autocorrelation develops the features of the analogue autocorrelation of an annulus: a central maximum and local maxima in the region of $+2a$ and $-2a$. In the limit where the pixel size tends to zero the vector and analogue autocorrelations become equivalent.

In any practical speckle system the photon events recorded have some finite size and this must be considered when choosing the pixel size. If the pixel size is larger than a single photon event two spatially close events may be unresolved with a consequent loss in the number of vectors counted. Whereas, if the pixel size is less than the size of a photon event the autocorrelation will contain an extra component from vectors within a single photon event; this is obviously preferable as no information is lost.

2.6 The Vector Autocorrelation of Speckle Images.

A speckle image is made up of a random distribution of speckles, which in turn consists of one or more photon events. Therefore, vectors between speckles are randomly distributed and form the two dimensional autocorrelation of the long exposure seeing limited envelope. This "seeing pedestal" is analogous to the Gaussian-like background present in the image's energy spectrum. However, vectors between events within the same speckle produce a component in the autocorrelation which contains diffraction limited information.

Let us return to the example of a speckle image of an equal magnitude binary star with a separation smaller than the diameter of the seeing disc. The speckles will consist of two distinct speckle patterns as each contains a near-diffraction-limited image of the binary [27]. Therefore, vectors between pixels within the same speckle will have similar lengths and orientations. This produces an autocorrelation containing two peaks superimposed on the seeing pedestal, one due to positive vectors and the other due to negative vectors. An autocorrelation, as defined in equation 2.6, is an even function so that:

$$A(h,k) = A(-h,-k).$$

Therefore, because of this symmetry it is only necessary to count vectors in a single direction.

So far a method has been described for calculating the autocorrelation of a speckle image. However, to draw conclusions about the morphology of an astronomical object it is preferable to obtain the object autocorrelation. Forming the ensemble average autocorrelation of equation 2.3 gives:

$$\langle I(x,y) * I(x,y) \rangle = O(x,y) * O(x,y) + \langle P(x,y) * P(x,y) \rangle \quad \text{Eq. 2.7}$$

The average image autocorrelation is simply formed by adding the autocorrelations of many statistically independent speckle frames. But, to calculate the object autocorrelation requires the observation of a point source under similar seeing conditions and then a deconvolution operation. Fortunately equation 2.7 can be simplified and the need for a reference star observation eliminated.

2.7 Cross-correlation.

The cross-correlation of two functions $f(x,y)$ and $g(x,y)$ measures their mutual spatial coherence and is given by:

$$\begin{aligned} C(h,k) &= \iint_{-\infty}^{\infty} f^*(x,y) \cdot g(x+h, y+k) dx dy \\ &= f(x,y) * g(x,y). \end{aligned} \quad \text{Eq. 2.8}$$

As with the autocorrelation function a vector cross-correlation can be defined. This is the distribution of vectors between points (x_i, y_i) in one function and all points (X_j, Y_j) in the other function. Thus:

$$C(h,k) = \sum_i \sum_j (X_j - x_i, Y_j - y_i)$$

Now consider two statistically independent speckle frames. The vector cross-correlation of these frames will only contain vectors between

spatially uncorrelated speckles. Therefore, if the speckles are contained in Gaussian envelopes, this cross-correlation will have a Gaussian envelope with a width greater than the widths of either of the individual envelopes [30]. In 1977 Welter and Worden suggested that the difference between the average autocorrelation and the average cross-correlation of statistically independent frames is "proportional to the autocorrelation of the image effectively uninfluenced by a turbulent atmosphere" [31]. Subsequently Bruck [32] and Walker [33] have shown that this statement is incorrect.

Consider the difference between the average autocorrelation and the average cross-correlation of a set of speckle images, $Q(h,k)$. For an object, $O(x,y)$, and a point spread function, $P(x,y)$, having stationary statistics the image, $I(x,y)$, is given by equation 2.3. If $I'(x,y)$ is an image statistically independent from $I(x,y)$:

$$Q(h,k) = \langle I(x,y) * I(x,y) \rangle - \langle I(x,y) * I'(x,y) \rangle \quad \text{Eq. 2.9}$$

If we now substitute equation 2.3 into equation 2.9 and for simplicity remove the coordinates:

$$Q = \langle (O \otimes P) * (O \otimes P) \rangle - \langle (O \otimes P) * (O \otimes P') \rangle \quad \text{Eq. 2.10}$$

Bruck [32] points out that the average cross-correlation is equal to the autocorrelation of the average, or long exposure, image. Therefore:

$$Q = \langle (O \otimes P) * (O \otimes P) \rangle - (\langle O \otimes P \rangle * \langle O \otimes P \rangle)$$

At this point it is convenient to take the Fourier transform of equation 2.10 and work in frequency space, therefore:

$$\begin{aligned} q &= \langle (o.p).(o.p)^* \rangle - (\langle o.p \rangle . \langle o.p \rangle^*) \\ &= (o.o^*) . [\langle p.p^* \rangle - (\langle p \rangle . \langle p \rangle^*)] \end{aligned} \quad \text{Eq. 2.11}$$

Now consider the speckle transfer function as given in equation 2.5:

$$\langle p.p^* \rangle = \langle p \rangle . \langle p \rangle^* + (1/N_s) . p_o$$

$$p_0 \approx \langle p \cdot p^* \rangle - (\langle p \rangle \cdot \langle p \rangle^*) \quad \text{Eq. 2.12}$$

Therefore, substituting Eq. 2.12 into Eq. 2.11:

$$q \approx (o \cdot o^*) \cdot p_0 \quad \text{Eq. 2.13}$$

Finally, taking the Fourier transform of equation 2.13 we obtain the result:

$$Q(h,k) \approx [O(x,y) * O(x,y)] \otimes P_0(x,y)$$

This result does not invalidate the Welter and Worden seeing correction, but does change its interpretation: the difference between the average autocorrelation and the average cross-correlation has been shown to be proportional to the diffraction limited image of the autocorrelation of the object intensity.

In implementing their seeing correction, Welter and Worden normalized the intensity of each frame so that the sum of the intensities was the same for any two frames [34]. By this operation they made the integral area of the autocorrelation equal to that of the cross-correlation. However, as the autocorrelation contains a central peak and may contain a speckle component absent from the cross-correlation, a simple differencing of the two over-corrects for the seeing pedestal. Several groups have noted this over-correction [35,36], and have proposed a modification to the original method in which the mean cross-correlation is normalised to the mean autocorrelation for separations outside the speckle component. Therefore:

$$Q(h,k) = \langle I(x,y) * I(x,y) \rangle - N \cdot \langle I(x,y) * I'(x,y) \rangle \quad \text{Eq. 2.14}$$

where N is some appropriate constant. Using Bruck's alternative formulation equation 2.14 becomes:

$$Q(h,k) = \langle I(x,y) * I(x,y) \rangle - N \cdot (\langle I(x,y) \rangle * \langle I(x,y) \rangle)$$

However, in the long exposure image, $\langle I(x,y) \rangle$, photon events may overlap and limit the validity of the vector autocorrelation. This

photon pile-up problem has been reported by Hege et al. [36]. In this case analogue processing is desirable. Other problems arise from the assumptions made in deriving $Q(h,k)$. If there are gross movements of the seeing disc, the image will be displaced from frame to frame. Therefore, the cross-correlation will contain some vectors at separations greater than the seeing disc diameter. These will broaden the cross-correlation relative to the autocorrelation. This problem can be eliminated by finding the centre of gravity of each frame and centring the images before processing.

A second limitation of the Worden seeing correction is its assumption that the complex amplitude of the wavefront in the telescope aperture obeys a Gaussian probability density and not the more accurate log normal model. The result of this is to invalidate the seeing correction for frequencies less than the seeing limit. However, if $D/r_0 > 10$ say, the frequencies below the seeing limit make up only a small fraction of the frequency plane. Therefore, for a telescope such as the Anglo-Australian Telescope (AAT), where $D/r_0 \approx 40$, the Worden seeing correction is satisfactory.

2.8 Signal to Noise Ratio.

The aim of speckle interferometry is to estimate some parameter of the observed object: a diameter, orientation or separation. The error in this estimate is determined by the fluctuations of the atmospheric seeing, the quantum nature of light and detector induced noise. A description of this error is given by the signal to noise ratio (SNR) defined as the ratio of the expected value, E , to the standard deviation of the estimate, $(\langle E^2 \rangle - \langle E \rangle^2)^{1/2}$:

$$SNR = E / (\langle E^2 \rangle - \langle E \rangle^2)^{1/2}$$

The first result is that for M statistically independent frames the SNR is simply given by:

$$SNR = SNR_1 \cdot M^{1/2}$$

where SNR_1 is the signal to noise ratio obtained from a single

frame. Therefore, speckle observations usually consist of a large number of individual frames. A further analysis of SNR can be made in either the spatial or frequency domains. I will concentrate on the results obtained by various authors for the SNR associated with the autocorrelation function of a set of images.

Dainty [37] and Walker [38] consider the autocorrelation of a photon counting observation of a binary star. The SNR is defined as the height of the binary peak above the local background divided by the probable error associated with the measurement. If Poisson statistics are assumed, this error is the square root of the value of the autocorrelation at the point considered. Walker's final expression for SNR is:

$$\text{SNR} = n \cdot (M \cdot A)^{\frac{1}{2}} \cdot f \cdot q' / 2A_s \cdot (1 + 2f + f^2) \quad \text{Eq. 2.15}$$

where n = number of photons per frame

M = number of frames

A = area of detector element

A_s = area of the seeing disc

$f = (2.5)^{-\Delta m}$

q' = mean square contrast of the speckle pattern.

Walker has derived a similar result for the SNR attainable when estimating the diameter of a giant star from the "seeing corrected" autocorrelation, $Q(h,k)$ [33]. The dependence on observational parameters is the same as in equation 2.15, but the f dependence is obviously absent and for a well resolved star the SNR is inversely proportional to the stellar diameter.

In speckle observations it is important to obtain the maximum SNR by choosing the optimum exposure parameters. The most easily varied observing conditions are the filter bandwidth, $\Delta\lambda$, and the exposure time, τ . Consider the effect of τ on SNR:

$$\begin{aligned} n &\propto \tau & \text{and} & & M &\propto \tau^{-1} \\ \Rightarrow \text{SNR} &\propto \tau^{\frac{1}{2}} \end{aligned}$$

However, the mean square speckle contrast is decreased as the exposure time is increased, and it is found that the maximum SNR is given when $\tau = 1.6t$ (t is the speckle lifetime). This condition corresponds to "partial freezing" of the speckle image.

The filter bandwidth affects two factors in equation 2.15: the number of photons per frame and the mean squared speckle contrast. In practice the bandwidth should be increased to maximise the number of photon events per frame up to the point where the SNR is compromised by the loss in speckle contrast. Walker [33] states that the SNR initially scales linearly with the number of photon events per frame and then tends asymptotically to a maximum value. He concludes that at low light levels speckle interferometry may be performed in white light.

The theory outlined in this chapter provides a starting point for the design of a practical speckle interferometry system. The interferometer must contain near diffraction limited magnification optics and a high gain sensor. Because of the turbulent nature of the atmosphere there is also a need for atmospheric dispersion correction, shuttering and wavelength filtering. These and other practical considerations are discussed more fully in Chapter Three.

Chapter Three.

CHAPTER THREE - IMPLEMENTING SPECKLE INTERFEROMETRY.

3.1 Introduction.

This chapter summarises the development of various speckle systems and highlights the technological difficulties of recording speckle images. The choice of observational parameters such as bandwidth and exposure time are also briefly discussed.

3.2 Speckle Interferometers.

An instrument used to obtain speckle images is referred to as either a "speckle camera" or a "speckle interferometer". Labeyrie [39] uses the term speckle interferometer and his convention is adopted in this text. Since Gezari et al's. first publication of astronomical speckle observations, at least thirteen such instruments have been constructed [39 to 49,35,17], but certain fundamental elements remain common to all:

- 1) a shutter to restrict the exposure times to a length comparable to the speckle lifetime;
- 2) an interference filter to define a spectral bandpass and maintain coherence;
- 3) some mechanism to correct for atmospheric dispersion;
- 4) a system of near-diffraction-limited optics (typically a microscope objective) to expand the image scale of the telescope;
- 5) an image intensifier and read out system.

3.3 Shuttering.

Speckle interferometry depends on the recording of many short exposures in which any atmospheric motion is "frozen". As mentioned in Chapter Two, this is achieved by constraining the exposure time, τ , to be comparable to the speckle lifetime. The exact exposure length adopted will also depend on the brightness of the object being observed and the signal to noise ratio required in the final results.

The effects of various exposure times on the speckle modulation

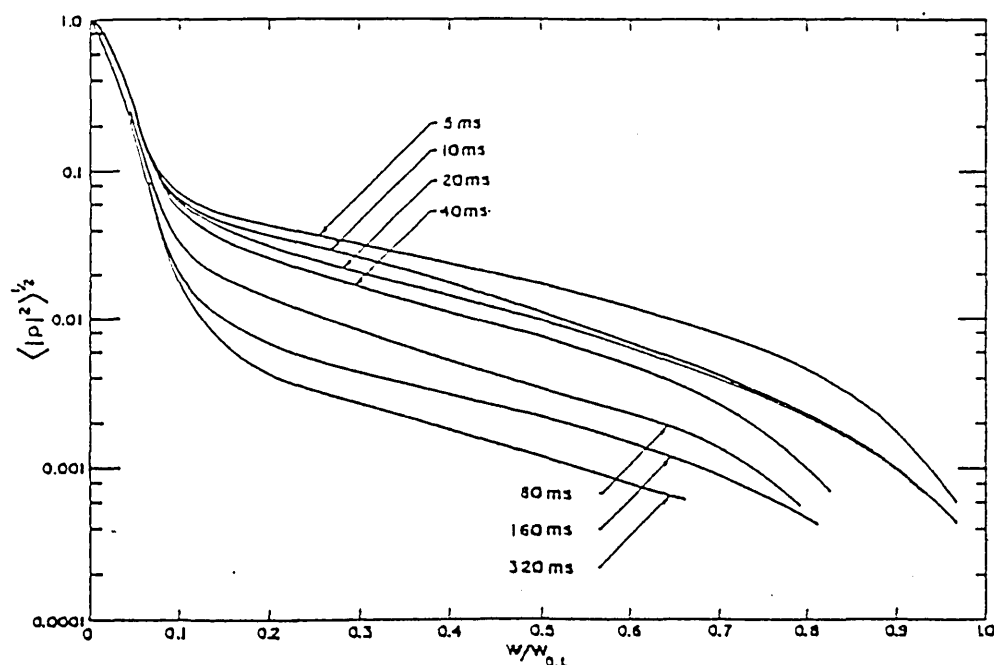


Figure 3.1. The Effect of Exposure Time on the Speckle MTF.

transfer function (MTF) are shown in Figure 3.1 [50]. The horizontal axis is the ratio of spatial frequency to the diffraction limited spatial frequency ($w/w_{D.L.}$), and the vertical axis gives the speckle MTF on a logarithmic scale. As the speckle lifetime increases the speckle MTF is lowered over the complete frequency range, although the MTF still contains frequencies above the seeing limit for lifetimes as long as 320 msec. Therefore, the transition between the speckle MTF and the long exposure MTF is gradual and experienced speckle observers often use exposure times greater than the speckle lifetime to maximise the signal to noise ratio (see section 2.8).

Because of the large variations in the speckle lifetime with weather conditions, geographical location and the time of observation, a speckle interferometer must be equipped with a controllable shutter. A synchronisation signal derived from the television or cine camera can be used to control an electro-mechanical shutter to provide exposure times from 1 to 100 msec. However, for most speckle systems an upper limit to the exposure time is set by some characteristic frame rate: for cine cameras this is 62 msec. and for television cameras this is 40 msec. in Britain and 33 msec. in the U.S.A.

A novel approach to shuttering in a speckle system has been employed

by H.A. McAlister in the Georgia State University Speckle Camera [48]. The instrument uses a dual microchannel plate intensifier and by gating the voltage on the photocathode the need for a mechanical shutter is eliminated. The gated "on times" can be varied from 1 to 30 msec.

3.4 Wavelength Filtering.

A quasi-monochromatic speckle image is formed with light of a bandwidth $\Delta\lambda$ centred at a wavelength λ . This finite bandwidth has two main effects: it allows chromatic dispersion; and causes a loss of speckle contrast as the atmospherically induced path differences approach the coherence length of the light.

Let us consider the effect of chromatic dispersion first. In a polychromatic speckle pattern, individual speckles will be radially "smeared" through an angle $\Delta\omega$ [51]. This is similar to the dispersive effect of a grating [15]. Hence, Figure 3.2 shows two points on a grating separated by a distance Δl . According to interference theory [52] a set of fringes (here representing the speckles) will be formed where:

$$\Delta l \cdot \sin\omega = n\lambda \quad n = 1, 2, 3, \dots$$

with $n = 1$ and assuming ω to be a small angle:

$$\Delta l \cdot \omega \approx \lambda. \quad \text{Eq. 3.1}$$

Differentiating equation 3.1 gives:

$$\Delta l \cdot \Delta\omega \approx \Delta\lambda. \quad \text{Eq. 3.2}$$

Substituting from equation 3.1 into equation 3.2 gives:

$$\frac{\lambda}{\Delta\lambda} \approx \frac{\omega}{\Delta\omega}.$$

Both Korff and Dainty suggest that at the edge of a seeing disc of

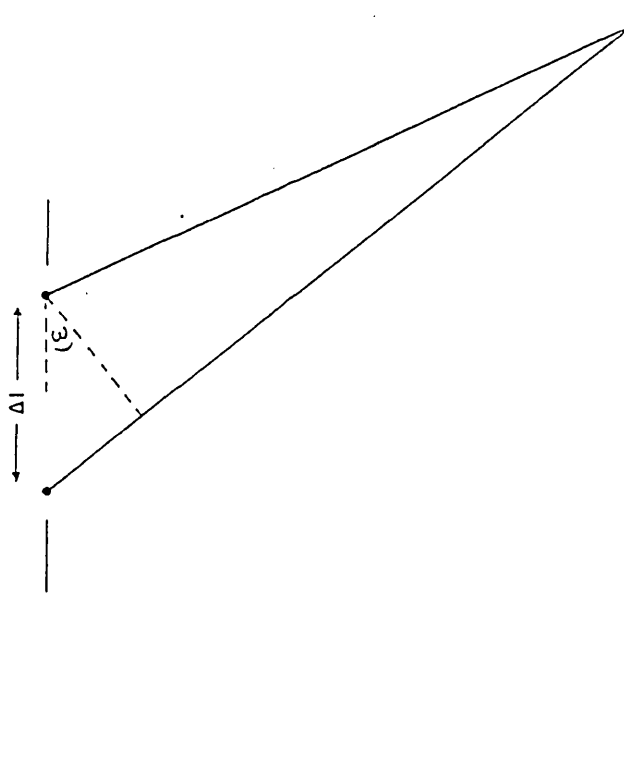


Figure 3.2 Points on a Grating.

angular diameter $\omega = \lambda/r_o$, the radial elongation of a speckle, $\Delta\omega$, should be constrained to be less than the speckle diameter $\omega_s = \lambda/D$. Hence:

$$\text{with } \omega_s/\Delta\omega > 1 \quad \text{and} \quad \frac{\omega_s}{\Delta\omega} = \frac{r_o \cdot \lambda}{D \cdot \Delta\lambda}$$

$$\Rightarrow \quad \frac{\lambda}{\Delta\lambda} > \frac{D}{r_o}$$

Therefore, for good observing conditions with $r_o = 10$ cm. and $D = 4$ m. the maximum wavelength to bandwidth ratio is:

$$\frac{\lambda}{\Delta\lambda} > 40.$$

With an interference filter centred at 500 nm. this analysis gives a maximum bandwidth of 13 nm. To allow wider bandwidth observations there have been attempts to design optics to correct for this dispersion [51,53].

The loss of contrast due to path length differences approaching the

coherence length of the light is a less serious problem. The coherence length ΔL is given by [52]:

$$\Delta L = \frac{\lambda^2}{\Delta\lambda}. \quad \text{Eq. 3.3}$$

From the Kolmogorov theory Roddier [54] calculates the standard deviation of the optical path length of light propagating through a turbulent atmosphere to be:

$$\sigma = \frac{2.62\lambda(D/r_0)^{5/6}}{2\pi}. \quad \text{Eq. 3.4}$$

Therefore, substituting for λ in equation 3.4 and if $\Delta L > \sigma$ then:

$$\frac{\lambda}{\Delta\lambda} > 0.42(D/r_0)^{5/6}.$$

With $r_0 = 10$ cm. and $D = 4$ m. then $\frac{\lambda}{\Delta\lambda} > 9$.

Dainty and Scaddan [55] have measured σ and obtained a value of approximately one wavelength. By analogy with the $\lambda/10$ figuring of conventional lenses this experimental result gives:

$$\frac{\lambda}{\Delta\lambda} > 10.$$

This result supports the derivation immediately above.

When making speckle observations of faint objects (say greater than magnitude +10 with a 4 m. telescope) a compromise is often made between the need for a narrow bandwidth to eliminate radial dispersion and the bandwidth required to maximize the number of photon events in each short exposure. An example of this is Hege's observation of the quasar PG1115+08 ($m_v = +16.2$) [56] in which he uses an effective bandwidth of 200 nm. centred at 600 nm.

3.5 Correction for Atmospheric Dispersion.

For all observations away from the zenith the atmosphere acts as a dispersive prism. The dependence of this dispersion on wavelength can be derived from Snell's Law. Figure 3.3 shows the refraction of a light ray at the boundary between a vacuum and the atmosphere. Light is incident at an angle ϕ and propagates from a region with refractive index $n = 1$ into a region where $n = n(\lambda)$.

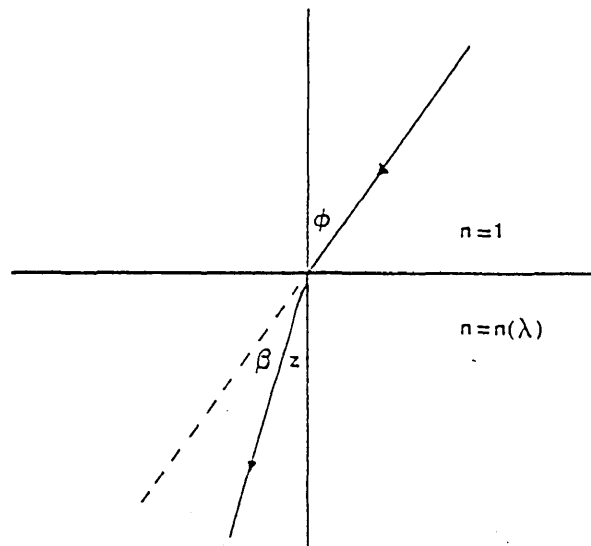


Figure 3.3 Refraction of Light by the Atmosphere.

From Snell's Law $\sin(\beta + z) = n(\lambda) \cdot \sin\beta$.

If β is small $\beta \approx \tan z \cdot (n(\lambda) - 1)$.

Therefore: $\frac{d\beta}{d\lambda} \approx \frac{dn(\lambda)}{d\lambda} \cdot \tan z$.

This chromatic dispersion is a serious problem because of its dependence on the zenith angle, z , and the high magnification used in speckle interferometry. Plate 3.3 shows how individual speckles are radially elongated. Therefore, most speckle systems contain some optics to correct for this chromatic aberration.

Two approaches to dispersion correction have been adopted. Labeyrie [39] and Arnold et al. [35] employ a system containing a figured diffraction grating which also defines the wavelength and bandwidth of the observations. By moving the sensor along the optical axis a position can be found where the displaced speckle patterns of

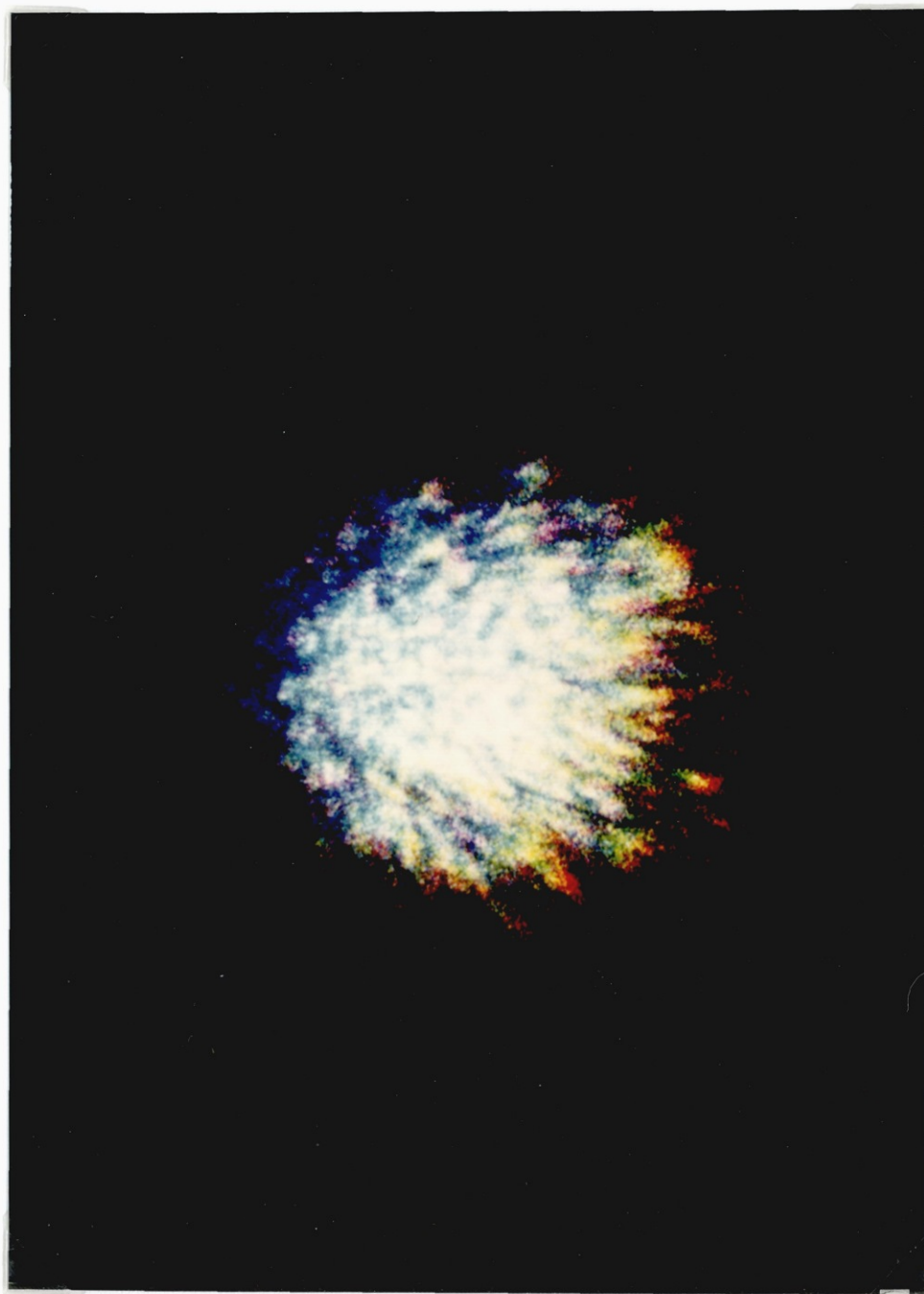


Plate 3.1. Radial Smearing of Speckles Due to
Atmospheric Dispersion.

different wavelengths overlap. At this point the atmospheric dispersion is corrected. However, a compromise must be made between the chosen bandwidth and the accuracy of the compensation.

A more widely adopted correction method uses a set of Risley prisms. Such a system contains two identical double prisms, each double prism consisting of two glass wedges of differing refractive index. The prism angles and refractive indices are chosen to compensate for atmospheric dispersion at some optimum wavelength and it is necessary to use different sets of prisms in the visible and the near ultra-violet. By rotating the double prisms with respect to each other the dispersive power can be varied to correct for dispersion as a function of hour angle and declination [57].

The need for atmospheric dispersion correction can be eliminated if the observational bandwidth is small. An example of this is Wood's use of a 2.4 nm. bandwidth filter in his observations of the OIII emission line of Magellanic Cloud planetary nebulae [58].

3.6 Magnification Optics.

The angular size of a single speckle in the image of a point source is λ/D . Therefore, for a telescope with a focal length F_1 the maximum spatial frequency present in the image (the diffraction limit) is:

$$\nu_0 = D/\lambda \cdot F_1.$$

Therefore, according to Shannon's sampling theorem, in order to recover the image it must be sampled at a frequency $\nu_s > 2\nu_0$. If we now consider a telescope with an F/15 focus and an observational wavelength of 500 nm. the sampling frequency $\nu_s > 270$ cycles/mm. This implies a minimum detector pixel size of 3.75 μm . However, for Charge Coupled Devices (CCDs) and television type photon counting detectors the pixel size is typically 30 μm . This problem is overcome by magnification optics which reduce the required sampling frequency.

For the Imperial College Interferometer [44] the available objectives range in focal length from 25 to 75 mm. and the image is focused onto the intensifier's photocathode by a relay lens with a focal length of 600 mm. If the image tube does not affect the image size, the magnification is given by the ratio of the focal lengths of the relay lens and the microscope objective. In fact the four stage EMI cascade tube used has a measured magnification of 0.96 at an operating voltage of 36 kV. [46]. Therefore, this system is capable of magnifications ranging from approximately 8 to 24. For a 50 mm. objective the magnification is 11.52 and $\nu_s = 23$ cycles/mm. This corresponds to a pixel size of 43 μm .

A secondary consideration to magnification in the design of a speckle interferometer is that of the field of view. In general the nature of speckle interferometry means that the observer is interested in objects whose size is close to that of the seeing limit. Therefore, the field of view, Γ , is normally a small multiple of λ/r_o . Assuming a sensor diameter, D_s , of 15 mm. (using only the central portion of the output phosphor of an EMI cascade tube), a magnification, M' , of 12 and a 3.8 m. F/15 telescope such as the AAT:

$$\Gamma = D_s/F_l.M' \approx 4.5 \text{ arcsecs.}$$

3.7 The Image Intensifier and Read Out System.

The need for image intensification is inherent in the technique of speckle interferometry: the short exposure times and narrow bandwidths limit the number of photons in each speckle image. For a main sequence star of magnitude m :

$$\log_{10} I = - (1.43 + 0.4m).$$

where I = photon flux per unit bandwidth at the earth (units of W/m^2). The number of photons in each speckle image, n , is then given by:

$$n = Q_e . I . D^2 . \pi . \lambda . \Delta\lambda . \tau / 4h.c,$$

where Q_e = overall detective quantum efficiency (DQE),
 D = telescope diameter,
 λ = wavelength of light,
 $\Delta\lambda$ = bandwidth,
 τ = exposure time,
 h = Planck's constant,
 c = velocity of light.

For typical values $m = 10$, $D = 4$ m., $\Delta\lambda = 20$ nm., $\lambda = 500$ nm., $\tau = 20$ ms., and $Q_e = 0.01$ there are about 500 photons in each speckle frame (Q_e contains contributions from the detector DQE (0.05) and from reflection and transmission losses in the telescope and interferometer). The magnitude limit of such observations can be easily estimated. In order to detect an average of at least two photons in each exposure the magnitude of the object observed must be greater than +16. Of course, this is not a definitive figure and fainter objects may be observed with better detectors or if conditions allow the extension of exposure time or bandwidth.

After Labeyrie's initial formulation the development of speckle interferometry has, to a large extent, been limited by the performance of the detector systems available. The first intensifier and read out combination used by Gezari [17] consisted of two optically coupled, electrostatically focused image tubes and a motorised film transport containing Kodak Tri-X film. Each stage of the intensifier provided a gain of about 10^3 , but the poor image quality of such tubes and the cumbersome photographic development imposed a magnitude limit of +9. The further development of sensor systems is summarised in the following table.

Name and Year	Intensifier and Read Out
Gezari 1972	Two ITT electrostatically focused tubes and Tri-X film.
Labeyrie 1974....	Silicon intensified target (SIT) vidicon.
Lynds 1974.....	RCA 3 stage magnetically focused tube and Tri-X film.
Beddoes 1976.....	EMI 4 stage magnetically focused tube and Tri-X cine film.
Schmidt 1976.....	Linear digicon detector.
Blazit 1977.....	Microchannel plate (MCP) intensifier and SIT vidicon.
Scaddan 1979.....	EMI 4 stage tube with a Plumbicon T.V. camera.
Arnold 1979.....	IPCS, 4 stage EMI tube with a Plumbicon T.V. camera.
Dudinov 1979.....	UM-92 3 stage magnetically focused tube and T-22 flim.
Hubbard 1979.....	Varo 4 stage intensifier and Plumbicon T.V. camera.
Weigelt 1981.....	EMI or Varo intensifier and photographic film.
McAlister 1982...	Dual MCP intensifier and CCD camera.
Papaliolios 1984.	MCP intensifier and photomultiplier tube system read out.

The sensors implemented by McAlister and Papaliolios deserve some detailed consideration as they both mark significant departures from the previously adopted technologies.

The McAlister system [48] (developed at Georgia State University) consists of an ITT 18 mm. diameter dual microchannel plate (MCP) intensifier fibre-optically coupled to a CCD camera. The MCP tube provides a luminous gain of 10^6 , which corresponds to an electron gain of approximately 10^4 [59]. This arrangement is very compact, rugged and has little distortion compared to cascade intensifiers. Another advantage of the MCP tube over the cascade intensifier is the shorter decay time of its output phosphor; an EMI four stage tube contains four phosphor screens so that the persistence of the output phosphor is determined by the convolution of four separate decay characteristics. When spectrally matched to an appropriate phosphor the CCD read out of the Georgia State Speckle Camera offers a high

responsive quantum efficiency (reaching 80% in the red). However, McAlister has compromised between spectral matching and resolution by employing a P-20 phosphor. This phosphor has a peak emission at approximately 555 nm., but a higher resolution than the "red phosphors" available. A CCD also offers a saturation limit of 10^5 counts per pixel, however, unlike Plumbicon or vidicon systems, it has some time independent read out noise. This Georgia State Speckle Camera is the logical application of current astronomical imaging techniques to speckle interferometry and has obviously evolved from existing speckle interferometers. In contrast the PAPA (precision analogue photon address) system, developed at Harvard University, adopts a novel approach to photon counting [49].

The optical arrangement of the PAPA detector is shown in Figure 3.4. The first stage is a high gain MCP image intensifier. Multiple images of the output phosphor are formed by the collimating lens and an array of 21 variable focal length lenses. Each image then falls onto a separate Gray coded mask and the light transmitted is registered by 21 photomultiplier tubes. One channel, containing no mask, determines whether a photon arrives and triggers the detection process. Other masks have successively finer opaque grating periods to define the various bits of the x or y position of a photon event. In this way a 19 bit Gray coded word from the photomultipliers can resolve the image into 512×512 pixels.

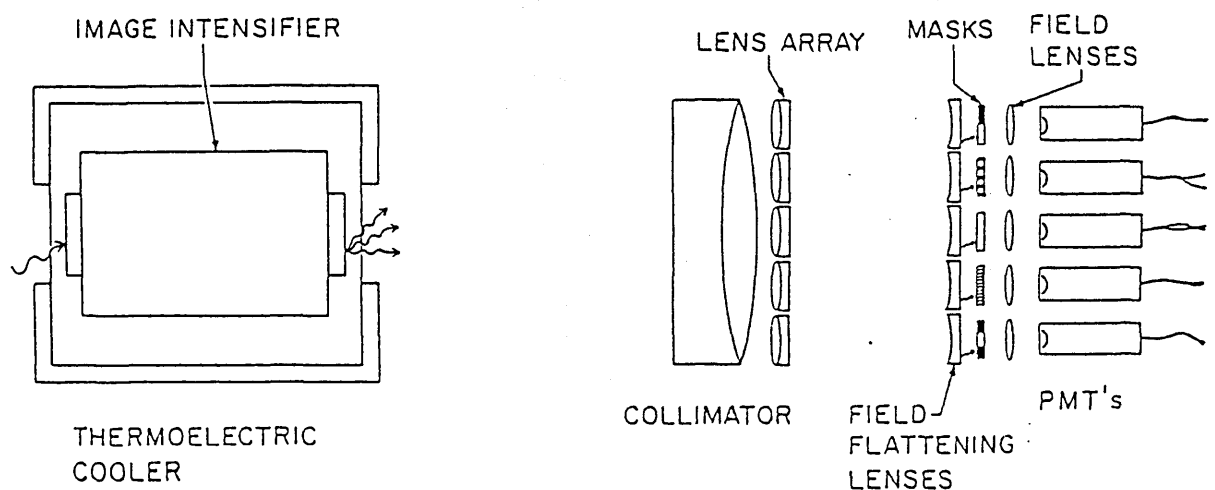


Figure 3.4. The PAPA Detector.

Gray coded rather than binary coded masks are used to minimize addressing errors when a photon event falls on the boundary of an opaque area and a transparent area. Gray code has the property that only one bit differs between adjacent pixels. The positions of the photon events are stored by integrating electronics and then encoded onto a video carrier to be recorded on video tape. A development of the PAPA detector is now commercially available with a time resolution of 200ns. and a spatial resolution of 512 X 512 pixels. Such characteristics make it ideally suited for speckle interferometry, however, every one of the 21 images must be correctly aligned with its corresponding mask which imposes stringent requirements on the mechanical stability of the optics.

3.8 The Imperial College Speckle Interferometer.

The Imperial College Speckle Interferometer, described by Scaddan et al. [44], possesses all the features required to record speckle images. It is a robust instrument which has been used on many telescopes: these include the Anglo-Australian Telescope (AAT); UKIRT; the 1.8 m. Perkins Telescope of the Lowell Observatory; the Isaac Newton Telescope (INT); the 74 inch telescope at Mount Stromlo; and the 1.9 m. telescope at the SAAO. A diagram of the interferometer is shown in Figure 3.5. Light from a telescope enters the interferometer

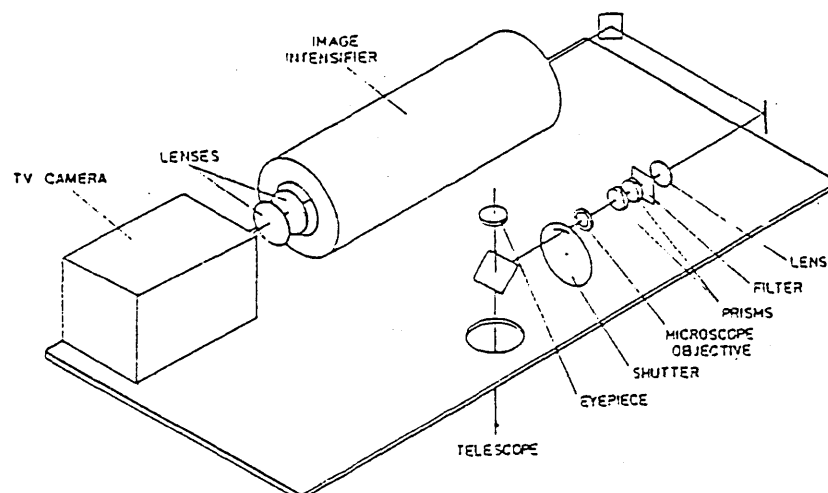


Figure 3.5 The Imperial College Speckle Interferometer.

through a hole in its baseplate and passes through a rotating sector shutter which determines the exposure time (typically 20 msec.). A microscope objective then magnifies the image. After passing through a set of prisms, to correct for atmospheric dispersion, and an optical filter, the light is focused onto the photocathode of an EMI four stage cascade tube. The intensified image is then recorded onto video tape from a Plumbicon television camera. The television picture shows an area of sky approximately four arcseconds square.

The results of observations made with the Imperial College Speckle Interferometer, and reduced using hardwired processing techniques developed at Imperial College, are given in Chapter Seven.

Chapter Four.

CHAPTER FOUR - DATA PROCESSING HARDWARE.

4.1 Approaches to Data Processing.

There are several approaches to the processing of speckle images. The coherent optical method (as described in Chapter Two) produces an ensemble average energy spectrum, however, the data analysis is slow and involves the non-linear process of photographic development which imposes a magnitude limit of approximately +8. To remove photographic processing most speckle systems now employ a T.V. camera and record the speckle images on video tape. Such an approach allows both analogue (intensity) and digital (photon counting) observations to be made. Data reduction then consists of digitising the speckle frames so that they can be analysed by a computing system.

The time taken to analyse speckle data is governed by the characteristic access time of the computing system and the algorithm chosen. If there is no need for real time data reduction a laboratory based mini-computer can be used. This "software" approach is flexible as various reduction algorithms can be adopted and compared. However, the digitisation of each frame with equipment such as the "Grinnell" digitiser and the following computer analysis can still be tedious. Hege et al. [60] note that the signal processing time can be 100 times longer than the actual observing time. This, coupled with the need to analyse a large number of frames to obtain a satisfactory signal to noise ratio, is a severe limitation on the approach. With the advent of fast array processors the Fourier transform of individual speckle frames can now be computed in real time, but before such computing elements were readily available various hardwired devices were constructed.

The autocorrelation method of analysis lends itself to real-time computation and is equivalent to the Fourier transform method. One bit hardwired vector autocorrelators have been described by Blazit et al. [61] and Hebden et al [62]. These devices work on the principle that the autocorrelation of a photon counting image is equal to the distribution of vectors between all possible pairs of photon events (see Chapter Two). However, when presented with a high light level

image (a classical intensity distribution), a one bit device can only assign the value of one or zero to a point in the image. Thus the autocorrelation of a non-photon counting record will be distorted by this "clipping". Information on separations and position angles is preserved in the "clipped" autocorrelation, but photometric information is lost. A four bit (16 levels) autocorrelator has been described by Vocak [63]. This device is able to process two 64×64 pixel pictures per second. This chapter briefly discusses the operation and specifications of the Imperial College Hardwired Autocorrelator and goes on to describe the development and use of a complementary device, the Imperial College Hardwired Cross-correlator.

4.2 The Imperial College Hardwired Autocorrelator.

The hardwired autocorrelator, which was designed and built by Mr. H.A. Vine (a Research Officer in the Imperial College Astrophysics Group), produces two dimensional autocorrelations of television images in real-time. The standard British television frame is made up of two interlaced fields, each containing 293 lines of image information. The field frequency is 50 Hz. and the autocorrelator processes alternate fields by examining the central 256 lines in a square 256×256 pixels. The coordinate system imposed on each television field is as shown in Figure 4.1. The origin is at the top left-hand corner. The x coordinates increase towards the right, up to 256, and the y

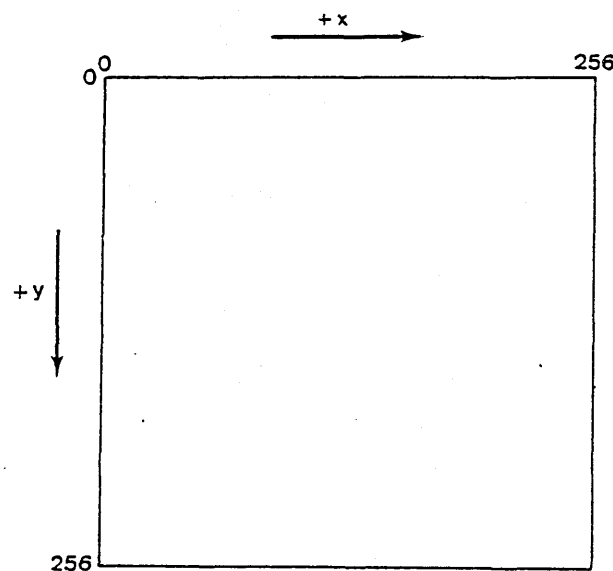


Figure 4.1 The Coordinate System Imposed on a Television Field.

coordinates increase towards the bottom of the field, again up to a maximum of 256. The processing of only odd or even fields relaxes the constraints on the operating speed of the autocorrelator, and although this effectively halves the resolution, little or no information is lost as each photon event spreads across at least two television lines. The autocorrelator contains two identical memories so that one field can be scanned, and the locations of the photon events recorded, while the coordinates stored from the previous field are processed. Therefore, the processing of a single field must be completed within 40 ms. If there are η occupied pixels in each field the number of vectors between all possible pairs of occupied pixels is, $\eta^2 - \eta$. However, due to the even nature of the autocorrelation function it is only necessary to count vectors in one direction. The autocorrelator also counts "zero length" vectors: the vector from an occupied pixel to itself. This allows the total number of occupied pixels to be read from the resulting autocorrelation. So, the number of vectors calculated for each field is $(\eta^2 + \eta)/2$. If $\eta^2 \gg \eta$ the number of vectors is approximately $\eta^2/2$. The vectors counted between three occupied pixels are shown in Figure 4.2. There are two vectors from pixel 1 to pixels 2 and 3, and there is one vector counted between pixel 2 and pixel 3. In addition to these vectors the three zero length vectors are also recorded giving a total of six vectors.

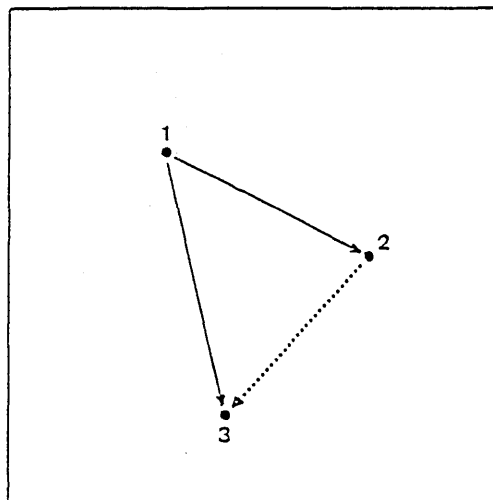


Figure 4.2 The Vectors Counted Between Three Occupied Pixels in a Single Field.

The vector difference between any two pixels is calculated by performing two eight bit subtractions, one each between the x and the y coordinates of the pixels, and then the result is stored in a 128 x 64 x 20 bit integrating memory. Only vectors with lengths less than or equal to 63 pixels are calculated. Therefore, if the time taken to calculate and store a single vector is T, for real-time operation:

$$(\eta^2/2).T < 40 \text{ ms.} \quad \text{Eq. 4.1}$$

The autocorrelator was conceived in the late 1970s and at that time the read-modify-write time of a integrating memory was approximately 300 ns. The Arithmetic Logic Unit (ALU) calculates the vector differences which are stored in a buffer memory while the integrating memory is accessed. Therefore the limiting factor on the operational speed of the autocorrelator is the read-modify-write time of the memory. If $T = 300 \text{ ns.}$ then from equation 4.1 we obtain $\eta = 516$, so for convenience the maximum number of occupied pixels per frame was chosen to be 512. The read-modify-write time of the latest memories is 80 ns. and an autocorrelator designed today would be correspondingly faster.

4.3 Features of the Autocorrelator's Operation.

Astronomical data can be reduced in realtime at the telescope or can be recorded on video tape (using a Sony AV-3620CE video tape recorder). In practice the data is always recorded for further reduction in the laboratory. The analogue signal from the television camera or video tape recorder undergoes one bit digitisation within the autocorrelator: a variable threshold level is applied to the video signal so that non-photon counting records are "clipped" and the positions of the speckles stored. There is also an upper threshold above which any signal is ignored, this discriminates against any ion events on the record. Two filters have been incorporated into the autocorrelator to eliminate sources of noise from the autocorrelation. The high frequency discriminator distinguishes between camera noise, which persists for less than the time taken to scan one pixel, and photon events, which typically cover several pixels, by discounting any signal that persists over

less than 80% of the pixel scan time. Tape drop-outs and damage to the video tape can produce bright lines across the television picture. The low frequency discriminator rejects any television line in which a signal persists over twenty two pixels.

As described in Section 4.2 a single British television frame consists of two interlaced fields. The television signal contains a series of synchronisation pulses on "odd" and "even" fields that are responsible for the accurate interlacing of the fields. Either odd or even fields can be autocorrelated by setting a switch on the front of the autocorrelator.

4.4 The Imperial College Hardwired Cross-correlator.

The cross-correlator was conceived as a complementary instrument for the autocorrelator. It is required to process television fields with the same constraints as outlined for the autocorrelator, but to derive vectors between fields separated by a predetermined time interval. This interval is adjustable from 2 to 998 field periods in steps of two field periods. Plate 4.1 shows the autocorrelator and cross-correlator.

At the start of a cross-correlation computation the field interval is selected by setting a thumbwheel switch. Autocorrelation and cross-correlation calculations can be carried out simultaneously. The autocorrelator examines the television signal at each pixel and if this lies above the adjustable threshold the coordinates of that pixel are stored as 2 x 8 bits in one of the two 512 x 16 bit memories. The recorded coordinates are then used to calculate the autocorrelation. These same coordinates are loaded into one of two 512 x 16 bit memories in the cross-correlator. If a field interval, F , is chosen the coordinates are stored for the first available field and for the F th. field. The cross-correlation is computed for the first and the F th. fields and then the coordinates from the $(F + F)$ th. field are written over those of the first field. The cross-correlation is performed for this new set of coordinates and so on.

Plate 4.1 The Autocorrelator and Cross-correlator.



4.5 The Cross-correlation Algorithm.

The vector cross-correlation of two images, $I(x,y)$ and $I'(x,y)$, is equivalent to the two dimensional histogram of vector differences between every event in one field and every other event in the second field. As the two fields will, in general, contain different distributions of photon events the cross-correlation of $I(x,y)$ with $I'(x,y)$ is not equal to the cross-correlation of $I'(x,y)$ with $I(x,y)$. Also the cross-correlation function is neither an odd nor even function. These properties can be summarised thus:

$$I(x,y) * I'(x,y) \neq I'(x,y) * I(x,y) \quad \text{Eq. 4.2}$$

and

$$I(x,y) * I'(x,y) = I'(-x,-y) * I(-x,-y). \quad \text{Eq. 4.3}$$

However, as the individual speckle images used to calculate the cross-correlation are statistically independent the ensemble average cross-correlation is equal to the autocorrelation of the long exposure image. Therefore:

$$\langle I(x,y) * I'(x,y) \rangle = \langle I(x,y) \rangle * \langle I(x,y) \rangle \quad \text{Eq. 4.4}$$

and as any autocorrelation is an even function then the ensemble average cross-correlation of statistically independent speckle images is also an even function. Because of this it is only necessary to store half of the cross-correlation. However, as the cross-correlation of $I(x,y)$ with $I'(x,y)$ is not generally symmetric ignoring one half completely would exclude some information and reduce the signal to noise of the average cross-correlation. To obtain this information use can be made of the property expressed in equation 4.3; the upper half of the cross-correlation of $I(x,y)$ with $I'(x,y)$ equals the lower half of the cross-correlation of $I'(x,y)$ with $I(x,y)$. The photon event coordinates of the first field are subtracted from those of the second field and the results stored in an incrementing memory. Then the photon event coordinates of the second field are subtracted from those of the first field and the corresponding memory locations incremented. In this way it is only necessary to store vector differences containing positive y coordinates.

There are several distinct reasons why this particular algorithm was chosen to calculate the cross-correlation; it is a similar approach as adopted to calculate the autocorrelation, thus reducing the design and hardware costs; and it avoids the problem of "photon pile up" that occurs when the cross-correlation is calculated by autocorrelating the long exposure image.

4.6 The Operation of the Cross-correlator.

The cross-correlator computes the cross-correlation of two fields F_1 and F_2 from the coordinates that it obtains from the autocorrelator. The coordinates of the i th. event in F_1 are subtracted from those of the j th. event in F_2 (where $j \geq i$) and a memory location which has an address given by the result of the subtraction is incremented. If the y subtraction is less than zero it is ignored and the $(j + 1)$ th. event in F_2 is considered. If the result of the y subtraction is greater than 63 it too is ignored and since all subsequent y differences will also exceed 63 the next value of i is considered. To avoid unnecessary calculations for each value of i the first value of j which gave y greater than or equal to zero is recorded; then for the subsequent i increment this j value is the first to be considered, since all previous y differences will be less than zero. When all the event coordinates of F_1 have been subtracted from those of F_2 , the coordinates of events in F_2 are subtracted from those of F_1 . In Figure 4.3 two fields have been superimposed to show the vectors that are calculated for $F_1 * F_2$ and for $F_2 * F_1$. After all such fields have been processed the number contained in the cross-correlation array (h,k) represents the number of times that a particular vector difference has been recorded. The extent of the cross-correlation array, and hence the extent of the autocorrelation array, is shown in Figure 4.4. To limit the number of operations and the amount of memory required, only vectors of lengths less than or equal to 63 are counted. Therefore, a "window" in the correlation is calculated; the h coordinate ranges from -64 to +63 and the k coordinate runs from 0 to +63.

The minimum time required to calculate the cross-correlation of two fields is 80 ms. as the smallest field separation is two. If the two

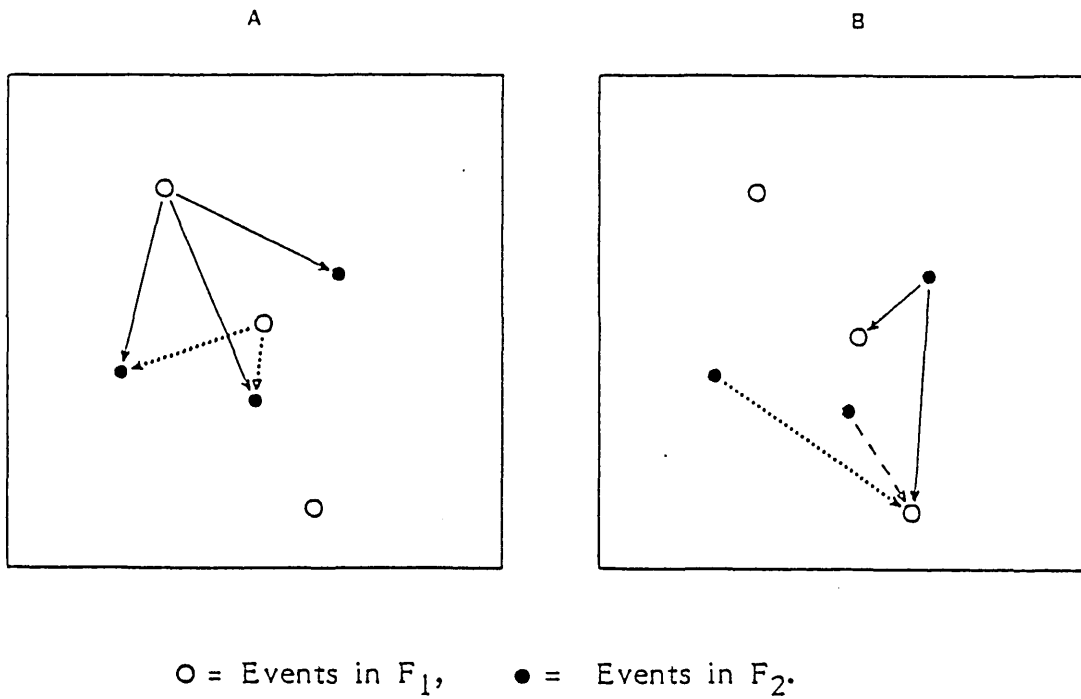


Figure 4.3 A) The Vectors Counted For $F_1 * F_2$,
 B) The Vectors Counted For $F_2 * F_1$.

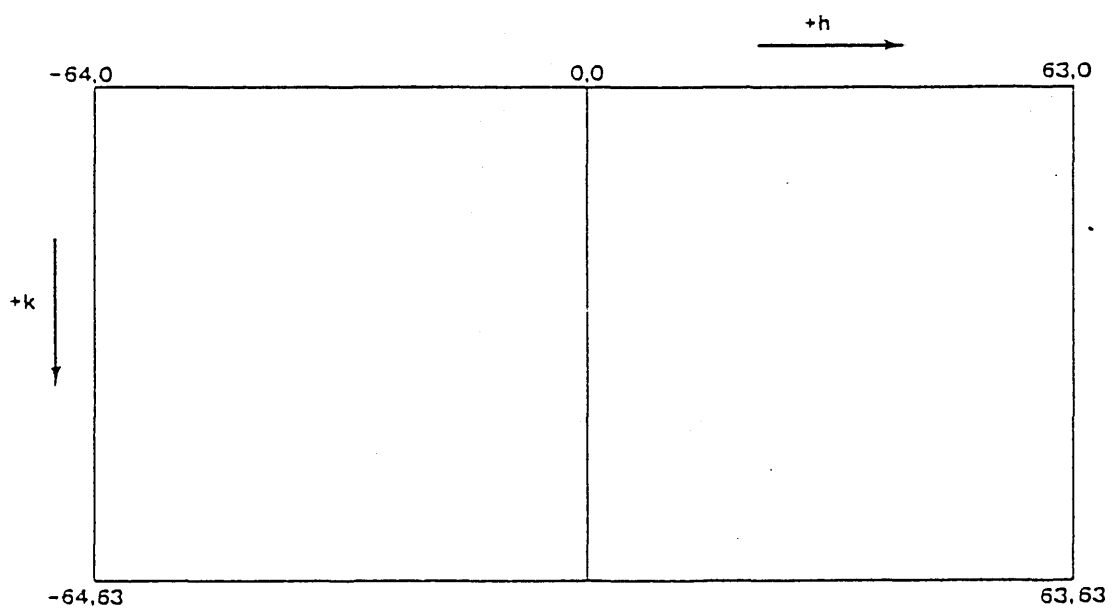


Figure 4.4 The Autocorrelation and Cross-correlation Array.

fields to be cross-correlated contain η and η' occupied pixels, then the total number of vectors to be calculated is $\eta.\eta'$. Also if the read-modify-write time of the cross-correlator is given by T then the following inequality must be satisfied:

$$(\eta.\eta'.T) < 80 \text{ ms.} \quad \text{Eq. 4.5}$$

As similar RAM chips are used in both the autocorrelator and cross-correlator the read-modify-write time of the two instruments is the same. Using current technology the operational speed of the cross-correlator could have been increased by at least a factor of three. However, to make the two instruments compatible the cross-correlator was designed and built to conform with the specifications of the autocorrelator. When equation 4.1 is compared with equation 4.5 it is seen that the maximum value of $\eta.\eta'$ is equal to the maximum value of η^2 . Therefore, as for the autocorrelator, the cross-correlator is capable of storing a maximum of 512 photon event coordinates from each field.

The cross-correlator contains a 50 Hz. clock which is synchronised to the television synchronisation signal from the autocorrelator at the selected field intervals. This clock solves the problem of loss of video synchronisation due to video tape drop outs or other interference occurring in the intervals between the reading of fields. If the synchronisation were lost then, of course, no computation would be performed for the fields concerned. The cross-correlation array is a 128 x 64 x 20 bit integrating memory consisting of forty 4K MOS static RAM chips (these are the pale chips in Plate 4.2) and a 20 bit, parallel loading, synchronous counter. The remaining cross-correlator digital circuitry is implemented by Schottky, Low Power Schottky and High Power CMOS integrated circuits.

Figure 4.5 shows a schematic diagram of the autocorrelator and cross-correlator. The separated television synchronisation signal and the digitised coordinates are fed from the autocorrelator to the cross-correlator. The arithmetic logic unit (ALU) then performs the appropriate subtractions. The comparator determines when the h and k limits are exceeded and increments the appropriate address counter.

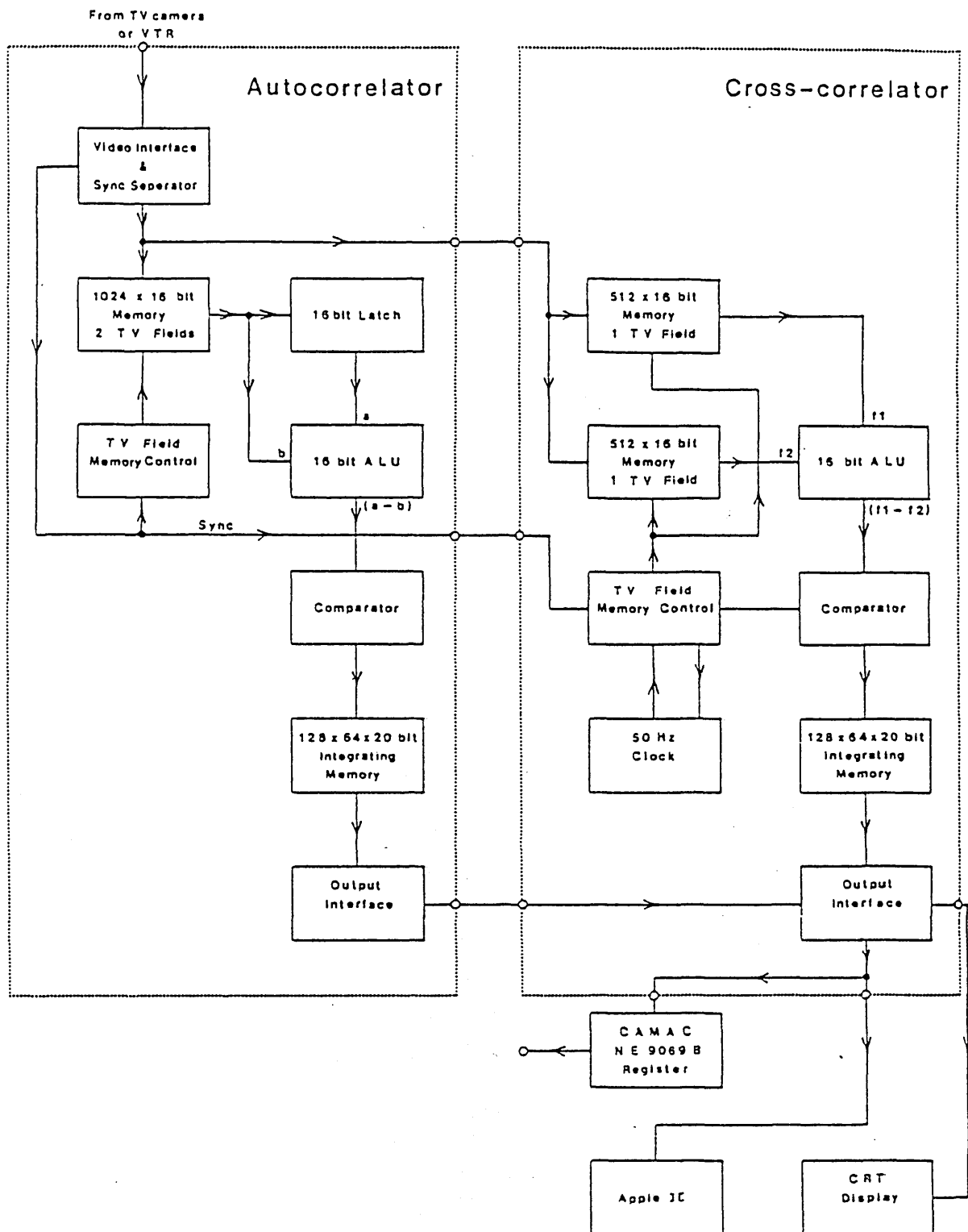


Figure 4.5 A Schematic Representation of the Autocorrelator and the Cross-correlator.

If the comparator determines that the result is within the 64×128 vector window the corresponding location in the memory is incremented.

One major deficiency of the present cross-correlator is that it is only able to store the photon event coordinates of two fields separated by some interval ranging from 1 to 496 frame periods. Therefore, the fields within this interval are not cross-correlated and as the field separation increases the fraction of the record processed decreases. To overcome this problem the same record is cross-correlated several times, each time starting at a different field. At present this is a rather "ad hoc" solution involving the identification of several fields at the beginning of the record and the manual starting of the cross-correlator. A far more elegant approach would involve the identification of individual fields by introducing an additional video signal. Then the cross-correlator could be modified so that operation would begin at some predetermined video field. The complete video record would then be cross-correlated after a number of passes equal to the field separation. The original cross-correlator design incorporated 25 separate 512×16 bit memories in which the photon event coordinates of 25 fields could be written in a cyclic manner. Thus while the 25th. field is stored the 24th. field would be cross-correlated with the first field. This option was rejected on the basis of cost and the extensive time required for construction, considering the limited man-power available. Also it did not possess a variable field interval so that the effects of various field separations on the cross-correlation could not be investigated.

Chapter Five.

CHAPTER FIVE - THE COMPUTATION OF AUTOCORRELATIONS AND CROSS-CORRELATIONS.

5.1 Operational Procedure.

Speckle images can be analysed at the telescope to produce autocorrelations and cross-correlations in real-time, but a video tape record of all observations is always made so that detailed data reduction can be undertaken in the laboratory. The video taped data is reduced simultaneously by the autocorrelator and by the cross-correlator. A particular record is generally analysed at various video levels with several combinations of discriminators. Plate 5.1 shows the video signal and the sampling level as displayed on an oscilloscope. The video signal is sampled above the horizontal line which defines the sampling threshold. At any time the operation of either correlator can be interrupted and the current results displayed on a storage oscilloscope, before data reduction is continued. The completed autocorrelation and cross-correlation are transferred to the memory of an Apple II micro-computer via two 6522 versatile interface adapters (VIA) for storage on floppy disc, normalisation, subtraction and display. The cross-correlation is normalised to autocorrelation at separations greater than the speckle component and then this normalised cross-correlation is subtracted from the autocorrelation by a short machine code routine. The result is proportional to the diffraction limited image of the object's autocorrelation.

5.2 The Effects of Sampling Level.

The sampling threshold imposed upon the video signal can be adjusted so that it is only detected above a certain voltage level. The majority of observational records contain discrete photon events (they are photon counting). Therefore, as the sampling threshold is decreased the number of detected events will increase in accordance with the Poissonian pulse height distribution of the photon events on the output phosphor of the image tube. Also because of the peaked intensity distribution within a single photon event the average number of pixels detected per event will increase. Therefore, the selection of the sampling threshold is a compromise between detecting

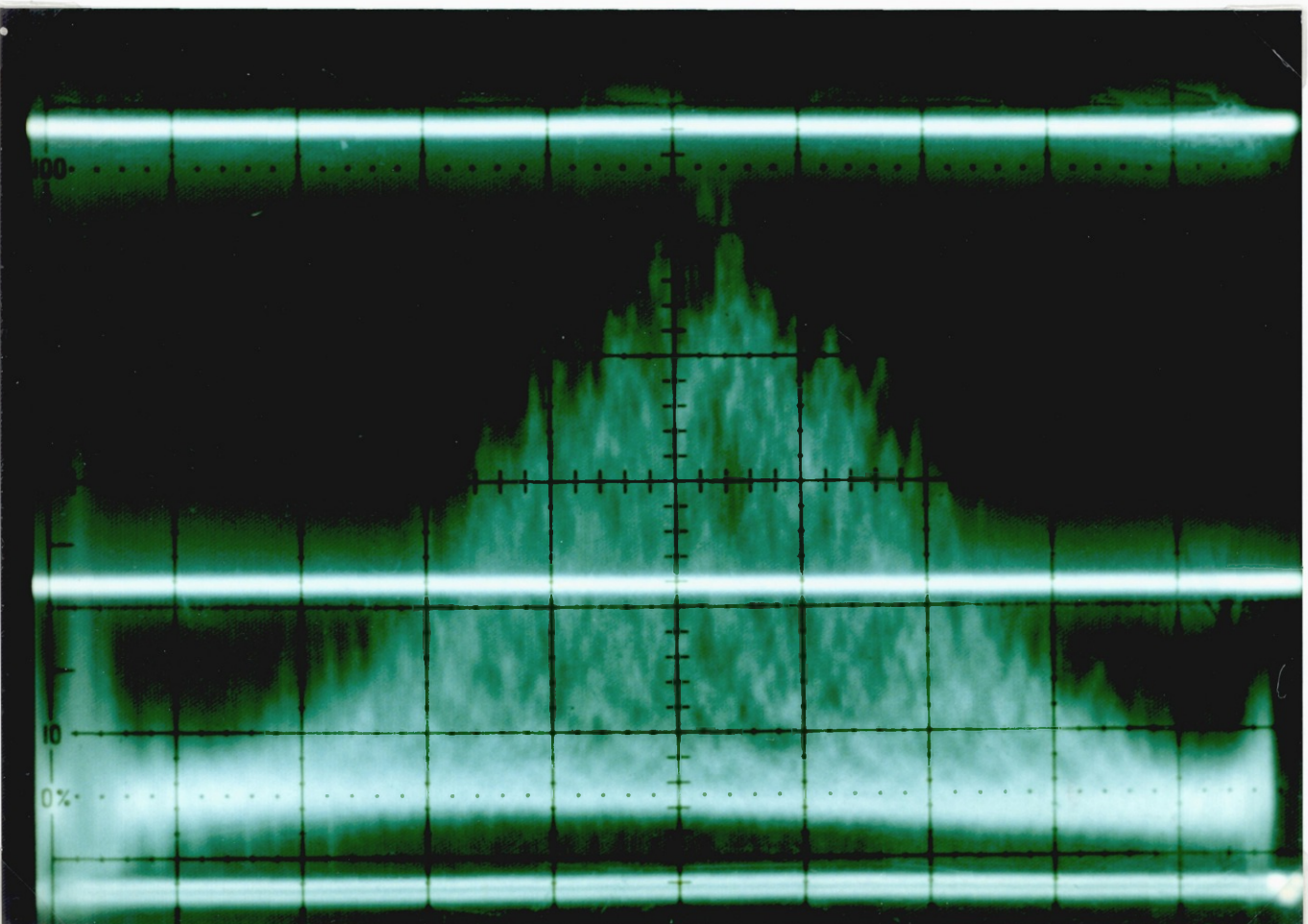


Plate 5.1 Video Signal and Video Levels
(The Two Bright Horizontal Lines Define the Sampling Window).

as many events as possible while avoiding the 512 pixel per frame saturation level. For high light level images each bright speckle is "clipped" by the imposition of a sampling threshold so that photometric data is lost.

The effects of two different sampling levels on the autocorrelation, the cross-correlation and the seeing corrected autocorrelation of a high light level record of a binary star are shown in Figures 5.1, 5.2, and 5.3. The solid curves are data derived at the lower sampling threshold. In Figure 5.1 the two autocorrelations have been normalised at the fifth pixel so that the relative number of vectors can be compared at various points in the profiles. Of course, the total number of vectors counted is greater for the lower sampling threshold. As the sampling level is decreased the number of detected pixels per speckle increases and more events are detected towards the periphery of the image. Therefore, the central component of the autocorrelation is broadened and the relative number of vectors from the 10th. to the 20th. pixels and from the 45th. to the 60th. pixels is increased. An important feature is that the position of the binary peak is independent of the sampling level.

In Figure 5.2 cross-correlations of the same image, taken at different sampling levels, are normalised at the fifth pixel. By lowering the sampling threshold the relative number of vectors counted increases at every point of the cross-correlation. However, this increase is most marked for separations greater than the 45th. pixel. This is because the intensity of the image falls off towards the edge of the seeing disc, and as the sampling level is lowered the relative increase in the number of central pixels detected is less than the relative increase in the number of peripheral pixels detected.

Figure 5.3 shows the differences between the autocorrelation and cross-correlation for the two sampling levels; both cross-correlations have been normalised to the appropriate autocorrelation at the 50th. pixel. The maxima of both seeing corrected autocorrelations are at the 35th. pixel, but the relative height of the maximum for the higher sampling threshold is greater. As the video level is increased, and

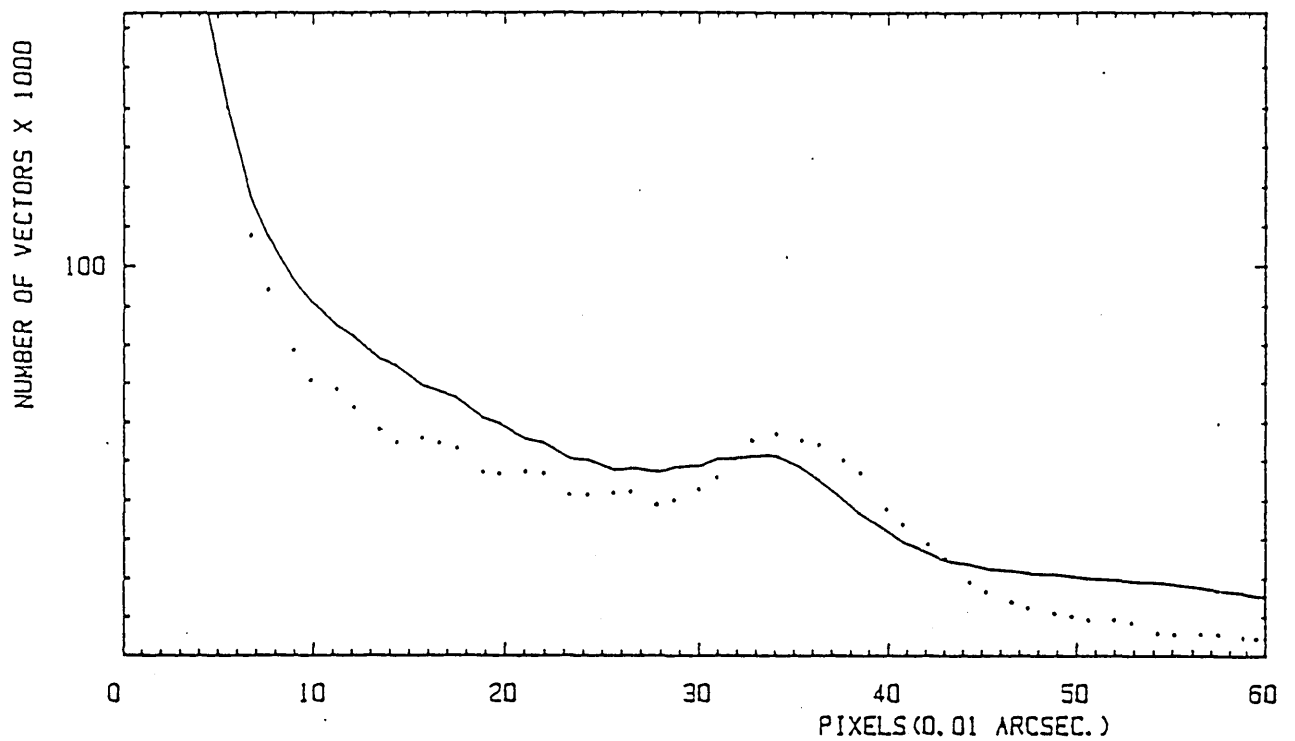


Figure 5.1 Autocorrelations at High and Low Sampling Levels.
 High Sampling Level, — Low Sampling Level.

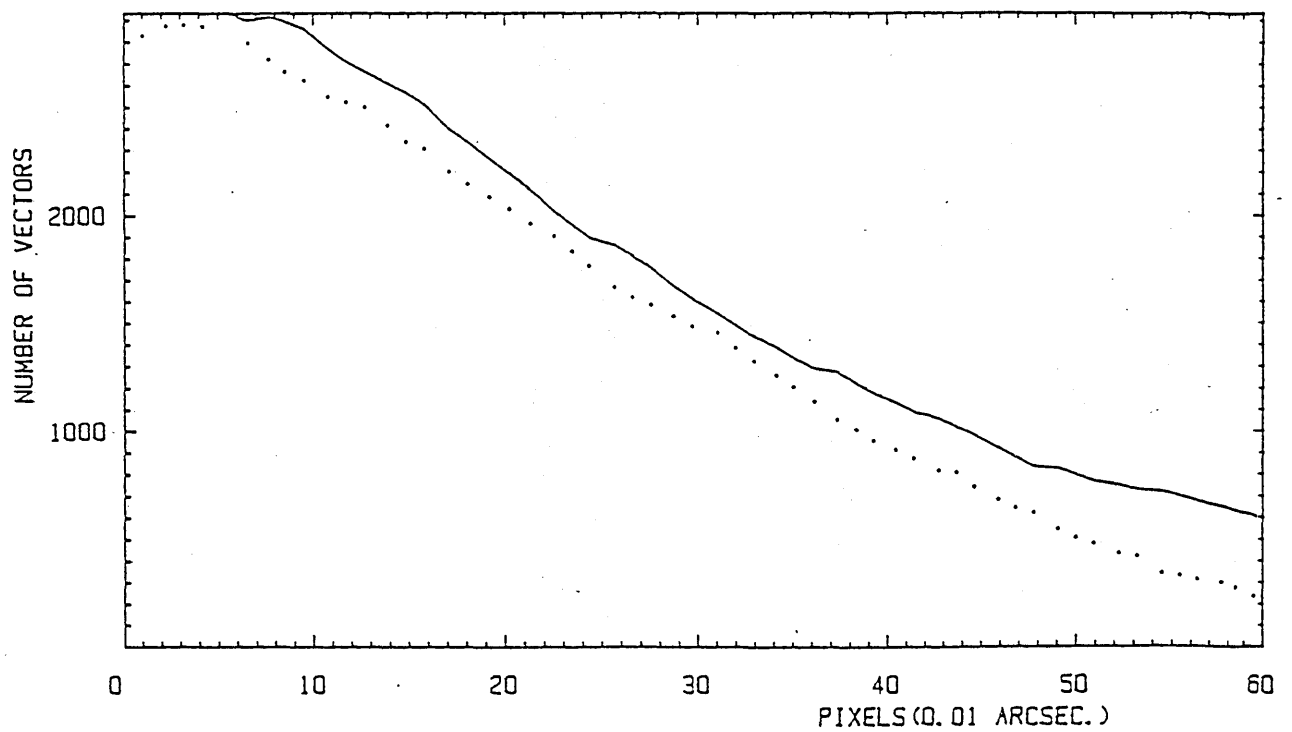


Figure 5.2 Cross-correlations at High and Low Sampling Levels.
 High Sampling Level, — Low Sampling Level.

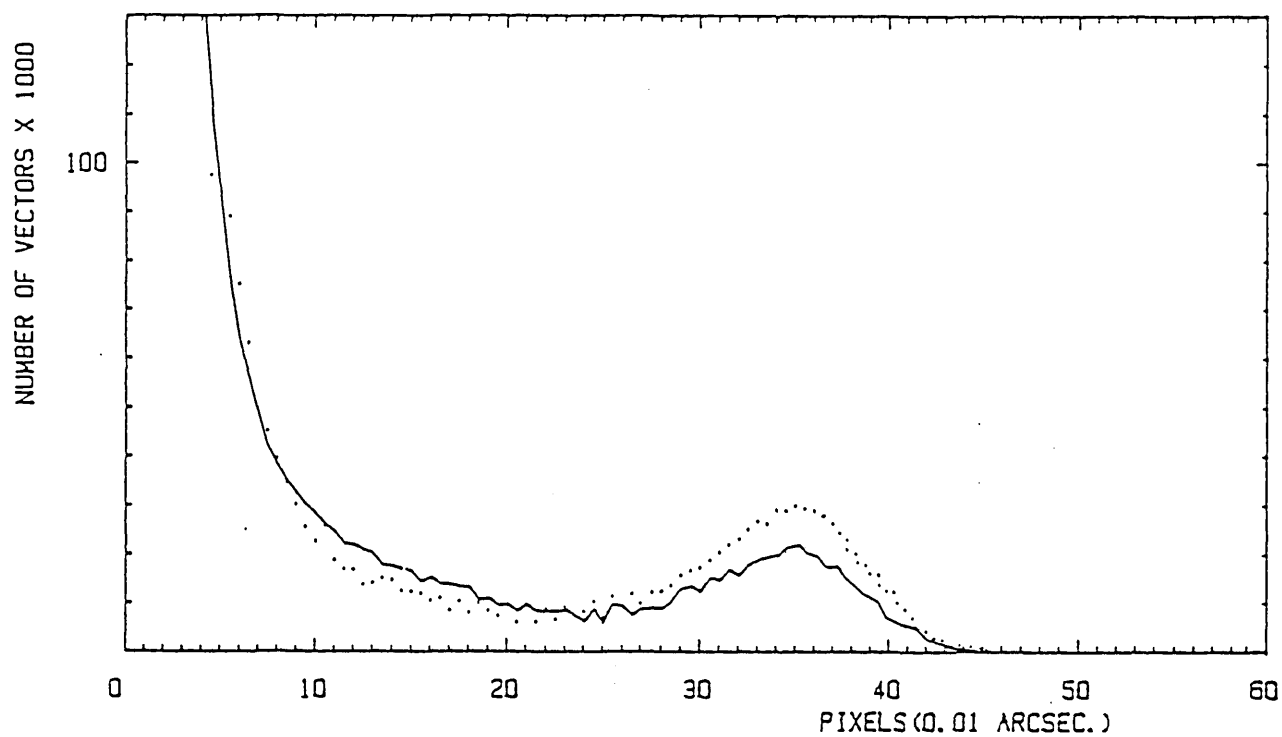


Figure 5.3 The Seeing Corrected Autocorrelation Calculated at Two Different Sampling Levels.

..... High Sampling Level, ——— Low Sampling level.

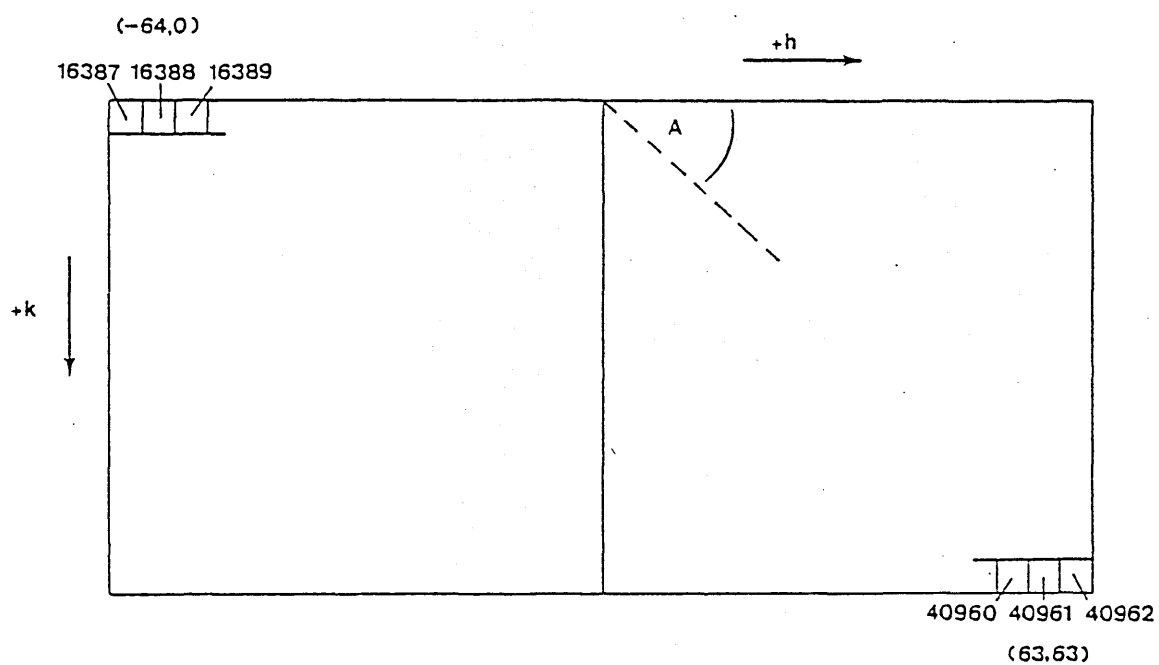


Figure 5.4 The Storage of Correlations in the Apple Memory.

the video signal is clipped at a lower voltage, the sampled image becomes wider. Therefore, the proportion of vectors contained in the binary peak falls. This demonstrates the difficulty in obtaining photometric information from an analogue image that is clipped at some level during processing. However, the determination of the binary separation and position angle is not affected by the sampling level.

5.3 Calibration Records.

During each observing run it is normal practice to record the motion of a star across the television screen as the telescope is tracked from east to west and then from north to south. These star trails indicate the orientation of the interferometer with respect to the sky. The cross-correlation of such an image will contain an elongation in the direction of the telescope's motion. Therefore, any small rotation of the image, due to misalignment of the optics or the magnetic and electric field of the image tube, will be apparent. Any rotation can then be taken into account when making measurements of position angles.

Records are also taken of the sky background and with the camera cap on. By autocorrelating these records it is possible to determine the sampling level at which the sky background and camera noise are detected.

For each object observed an observation of a nearby unresolved source is also made. Neutral density filters are often placed in the optical light path to attenuate the light from the reference star so that its image is of a similar intensity to that of the object. The data analysis is then performed on the object and the reference star. Our confidence in any results is considerably increased if after subtraction of the seeing components from the autocorrelations the reference star gives the result expected for a point source.

5.4 Data Analysis Software.

In order to manipulate and analyse the autocorrelations and cross-correlations derived by the data processing hardware a computer

program, APLOT5, has been written. Various options are displayed in a menu format. The LOAD option allows correlations to be transferred from the data processing hardware to the Apple microcomputer. A machine code routine configures the data direction registers of the VIA to act as either outputs or inputs and provides the handshaking necessary for data transfer. Each twenty bit number of an autocorrelation or cross-correlation is read and stored in three bytes of the Apple's random access memory. A correlation occupies 24 kbytes. of RAM starting from memory location 16387 (or \$4003 in hexadecimal) and finishing at location 40962 (\$A002). The data is read and stored in horizontal lines. Therefore, as shown in Figure 5.4, the contents of bytes 16387, 16388 and 16389 represent the value of a correlation at pixel (-64,0) and the value at (63,63) is stored in bytes 40960, 40961 and 40962. The number of vectors, N, present at a particular pixel is easily found by "PEEKING" the appropriate bytes and implementing a simple algorithm:

$$N = \text{PEEK}(\text{BYTE}) + 256 \times \text{PEEK}(\text{BYTE} + 1) + 256^2 \times \text{PEEK}(\text{BYTE} + 2)$$

When the complete correlation has been stored in the Apple a second machine code routine, SWAPMEM, is called which interchanges the contents of the Main memory from 16387 to 40962 with the contents of an auxiliary memory from 16387 to 40962. This auxiliary memory is held on a "Ramview" card and extends the memory of the Apple from 64 kbytes. to 128 kbytes. This allows two 24 kbyte. correlations to be simultaneously stored in the Apple. However, operations can only be performed on data stored in the main memory as this is where the program APLOT5 is held and run. As the SWAPMEM routine operates each time a correlation is loaded, the first correlation transferred to the Apple will finally be stored in the main memory and the second correlation to be loaded will be held in the auxiliary memory. Whether an autocorrelation or cross-correlation is to be transferred to the Apple is determined by setting a two way switch on the front panel of the autocorrelator. It is also possible to load correlations that have been previously stored on floppy disc. At any time the contents of these two memories may be interchanged via the program's menu.

The machine code routine SWAPMEM uses the inbuilt subroutine AUXMOVE to copy blocks of data from main memory to auxiliary memory or from auxiliary memory to main memory. Before calling this routine the data addresses must be put into byte pairs in page zero and the carry bit set to select the direction of the block move. To copy data from main to auxiliary memory the carry bit is set (SEC); to copy data from auxiliary to main memory the carry bit is cleared (CLC). When the SWAPMEM routine is called three bytes of auxiliary memory are copied into page three of main memory. Then the corresponding bytes from main memory over-write their counterparts in auxiliary memory. Finally the three bytes stored in page three are copied into the appropriate memory locations in main memory. This set of operations is performed for memory locations \$4003 to \$A000.

Via the PROFILE option the contents of the auxiliary and main memories may be interchanged and then the correlation present in the main memory displayed. Radial sections through the two dimensional correlation are defined by the correlation angle, A , measured from the horizontal axis in a clockwise direction. A single profile or a series of profiles separated by an angle increment greater than or equal to one may be displayed. The range of the horizontal axis can be varied within the limit of 0 to 63 pixels. The vertical axis is defined by adjustable lower and upper limits. Other programs allow particular profiles to be plotted by an Hewlett Packard 7470A graphics plotter and two dimensional contour plots to be printed on a dot matrix printer. The graphics plotter offers a resolution of forty points per millimeter and is generally used to produce the final hard copy of results for papers. The contour display represents each pixel as a black or white square. The colour of each pixel is dependent on the value of the correlation at that particular point. This method of display is similar to that of the storage oscilloscope, but with a higher resolution.

The MEAN option is often used in the analysis of objects that are assumed to be centro-symmetric. The mean profile of a set of profiles defined over any angle range within the 0 to 180° limits of the correlation angle is calculated. The value of the correlation at each pixel is stored in an array element defined by the distance of that

pixel from the origin to the nearest half pixel. Then an average profile is calculated by dividing the value stored in each array element by the number of increments to that array element. Therefore, a profile is plotted in half pixel steps that possesses a greater signal to noise ratio than any individual profile.

At the heart of the Welter and Worden seeing correction is the idea of normalising the average cross-correlation to the average autocorrelation of a speckle record. A cross-correlation stored in the main memory can be multiplied by any constant. The normalising constant is calculated as the ratio of the number of vectors present in the autocorrelation and cross-correlation for separations greater than the speckle component. For centro-symmetric objects it is also possible to normalise the azimuthally averaged cross-correlation to an azimuthally averaged autocorrelation. After normalisation the cross-correlation can be subtracted from the autocorrelation by choosing the SUBTRACT option. This subtracts the value at each pixel of the normalised cross-correlation from the value at the corresponding pixel of the autocorrelation. To perform this operation the autocorrelation must be held in the main memory and the cross-correlation in the auxiliary memory. After the subtraction is complete the seeing corrected autocorrelation is stored in the main memory and the autocorrelation is stored in the auxiliary memory. Unfortunately the present memory available does not allow three 24 kbyte. correlations to be simultaneously stored, so before the subtraction is performed the cross-correlation is stored on floppy disc using the SAVE option. Upto five correlations may be stored on a double density, single sided five inch floppy disc.

Chapter Six.

CHAPTER SIX - THE ANALYSIS OF AUTOCORRELATIONS AND CROSS-CORRELATIONS.

6.1 Introduction.

In Labeyrie's original formulation of speckle interferometry the ensemble average energy spectrum of a set of short exposure images is calculated. To recover the object's energy spectrum, $|o(u,v)|^2$, the average energy spectrum of the image, $\langle|i(u,v)|^2\rangle$, is divided by the speckle transfer function, $\langle|p(u,v)|^2\rangle$:

$$|o(u,v)|^2 = \langle|i(u,v)|^2\rangle / \langle|p(u,v)|^2\rangle.$$

In practice the speckle transfer function is estimated from observations of a reference star assuming similar seeing conditions. Unfortunately, as described in Chapter Two, the quality of the seeing depends upon several atmospheric parameters that vary with time. Therefore, under typically variable atmospheric conditions the application of the above equation can lead to considerable errors in the estimation of the object's energy spectrum. In the spatial domain the form of the autocorrelation function will depend on the seeing. In the case of poor seeing the seeing pedestal is broad and shallow while in good seeing it is steep and quickly falls to zero outside the speckle component.

Two separate approaches to seeing compensation have been suggested. The first involves measuring the seeing cell size, r_0 , and then calculating the speckle transfer function. Korff [20] has derived the speckle transfer function assuming that the complex amplitude of a wavefront propagating through the atmosphere can be described by log-normal statistics: the log of the amplitude and phase of the wave have Gaussian statistics. This is a more satisfactory model than the Gaussian model. Good agreement has been obtained between observed and theoretical transfer functions by Aime et al. [64] and by Chelli et al. [65], but the effects of finite bandwidth and exposure time may limit the use of this method.

Hebden [66] has investigated simple seeing component modelling in

the spatial domain. He starts from an expression for the atmospheric transfer function derived assuming atmospheric turbulence to possess a Kolmogorov spectrum:

$$p(u,v) = \exp[-3.44(\lambda \cdot w/r_0)^{5/3}], \quad \text{Eq. 6.1}$$

where $w^2 = u^2 + v^2$.

The long exposure image of a point source is given by the Fourier transform of $p(u,v)$. Both near-Gaussian (5/3 exponent) and Gaussian models have been fitted to the profiles of long exposure images by King [67], and as the autocorrelation of a Gaussian distribution is also Gaussian, one of the models adopted by Hebden for the seeing component in an autocorrelation has a Gaussian form. Another model adopted was the near-Gaussian distribution. This model stems directly from the form of equation 6.1 and Roddier's statement [54] that since the 5/3 exponent is almost Gaussian its Fourier transform will also be close to a Gaussian distribution. In approximating the speckle transfer function by these simple functions no attempt has been made to model the component proportional to the transfer function of the telescope. However, the seeing compensation should be adequate for all but the highest spatial frequencies. In practice, the validity of the seeing component modelling for the autocorrelation of a given object can be supported by comparison with the modelling of the seeing component of a reference star. Hebden finds that the near-Gaussian model fits the autocorrelation of a reference star better at intermediate vector lengths while the Gaussian model is preferable for separations greater than 0.5 arcseconds.

The second method of seeing compensation is immediately applicable in the spatial domain and involves the calculation of the ensemble average cross-correlation of many speckle images.

6.2 Application of the Welter and Worden Seeing Correction.

The Welter and Worden seeing correction consists of estimating the difference between the average autocorrelation and cross-correlation functions of a speckle record. The method has been successfully

implemented by Hege et al. [36], Worden and Stein [34], Arnold et al. [35], Wood [58] and Morgan et al. [68]. As previously shown in Chapter Two, assuming Gaussian statistics for any wavefront perturbations and negligible image movement between cross-correlated frames, the seeing compensation produces a function, $Q(h,k)$, that is proportional to the diffraction limited image of the object's autocorrelation. This section describes the seeing compensation as applied to the observation of a binary star.

The determination of binary star separations and position angles are perhaps the simplest and most widely published measurements made using stellar speckle interferometry. McAlister has reported over one thousand measurements and the groups of Morgan, Labeyrie and Weigelt have published several hundred observations of binary systems. Speckle interferometry is so successful in this area of observational astronomy for a variety of reasons: the measurements can be made rapidly on the brighter objects and yield an accuracy far in excess of visual observations. It is possible to determine separations to ± 1 pixel (typically this corresponds to ± 0.015 arcseconds for the Imperial College speckle system) and position angles to ± 2 degrees. In most cases it is possible to detect the characteristic secondary peak in the autocorrelation of a binary star without seeing compensation. However, to make an accurate determination of the separation and to have any hope of obtaining magnitude differences the effect of atmospheric seeing must be calibrated out of the autocorrelation.

Figure 6.1 shows the two dimensional autocorrelation of the binary star ADS1123 ($m = 6.7$, $\Delta m = 0.2$) obtained from 2500 television fields. The system was observed at the Coude focus of the 3.9 m. Anglo-Australian Telescope using a 50 mm. objective. This gives a pixel size of 0.01 arcseconds. The autocorrelation has a secondary peak superimposed on a seeing pedestal at an autocorrelation angle of 159° with a maximum value at a separation of 0.35 arcseconds. As explained in Chapter Two the secondary peak present in the autocorrelation of a binary star is composed of vectors between the two components present in each speckle. If the secondary peak is as large as that present in Figure 6.1 then identification is extremely

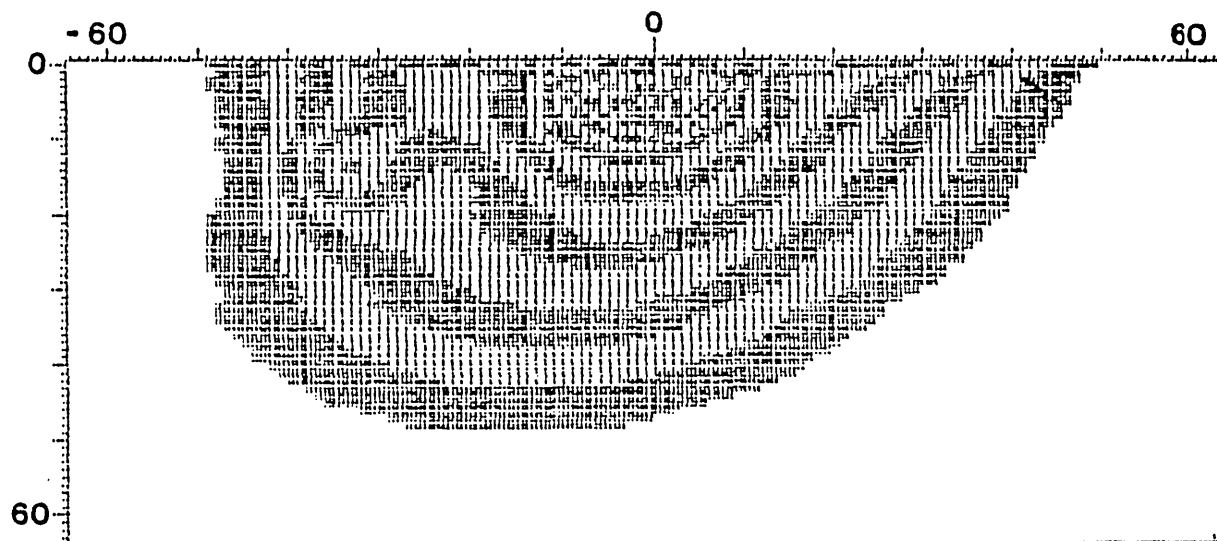


Figure 6.1 Autocorrelation of the Binary Star ADS1123.
Scale = 0.01 arcseconds/pixel.

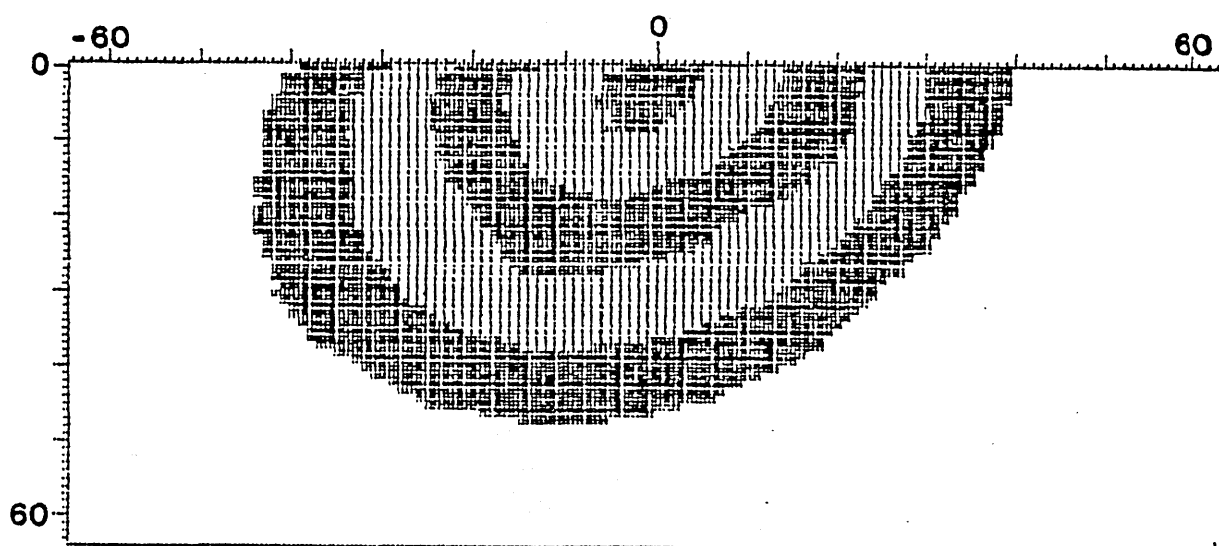


Figure 6.2 Cross-correlation of ADS1123 (Field Interval = 8).

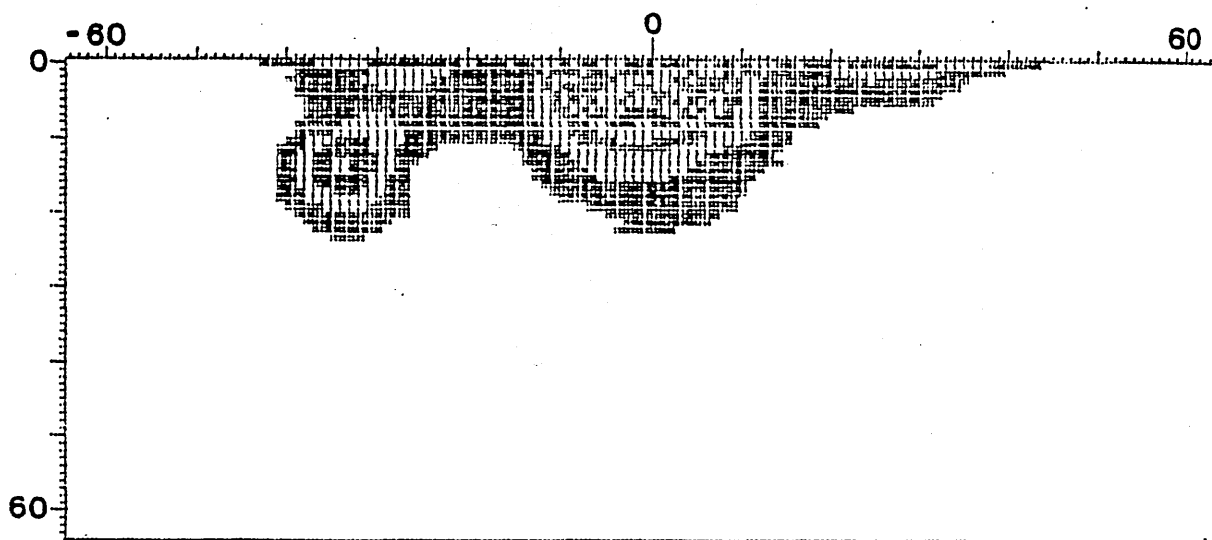


Figure 6.3 Seeing Corrected Autocorrelation of ADS1123.

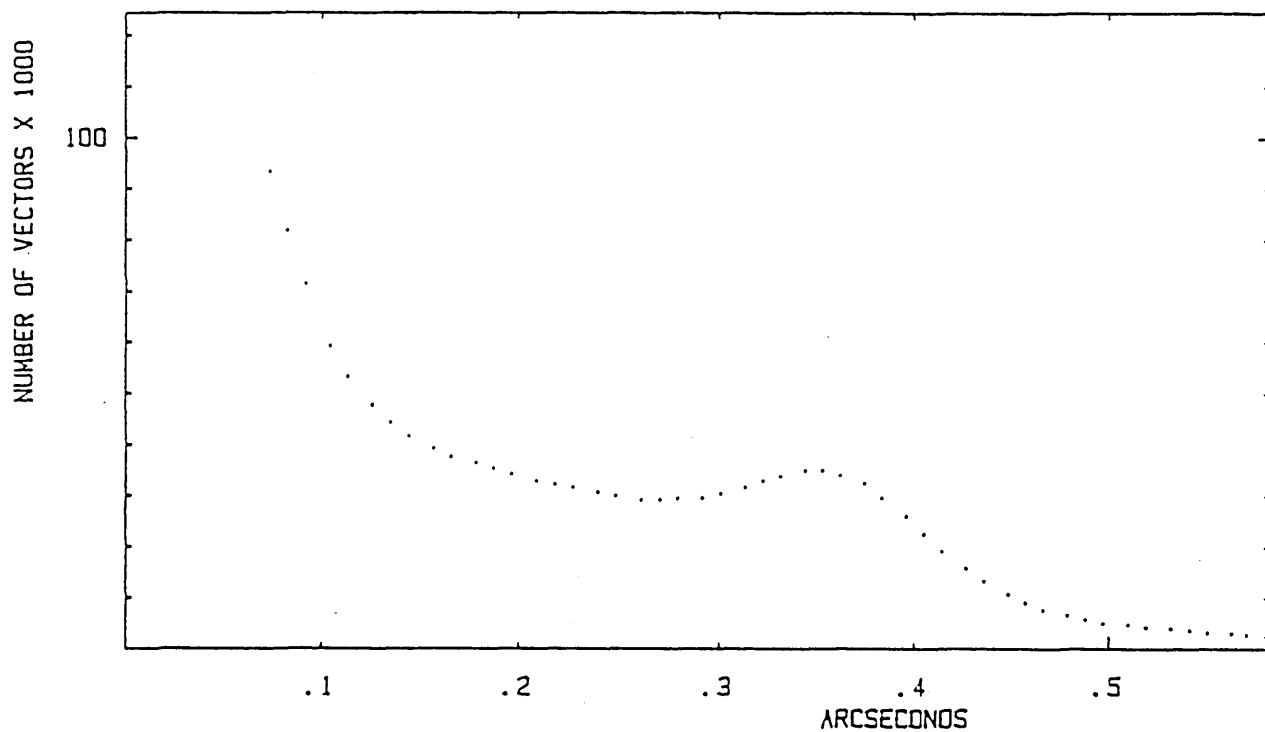


Figure 6.4 Radial Profile Through the Autocorrelation of ADS1123 at 159°.

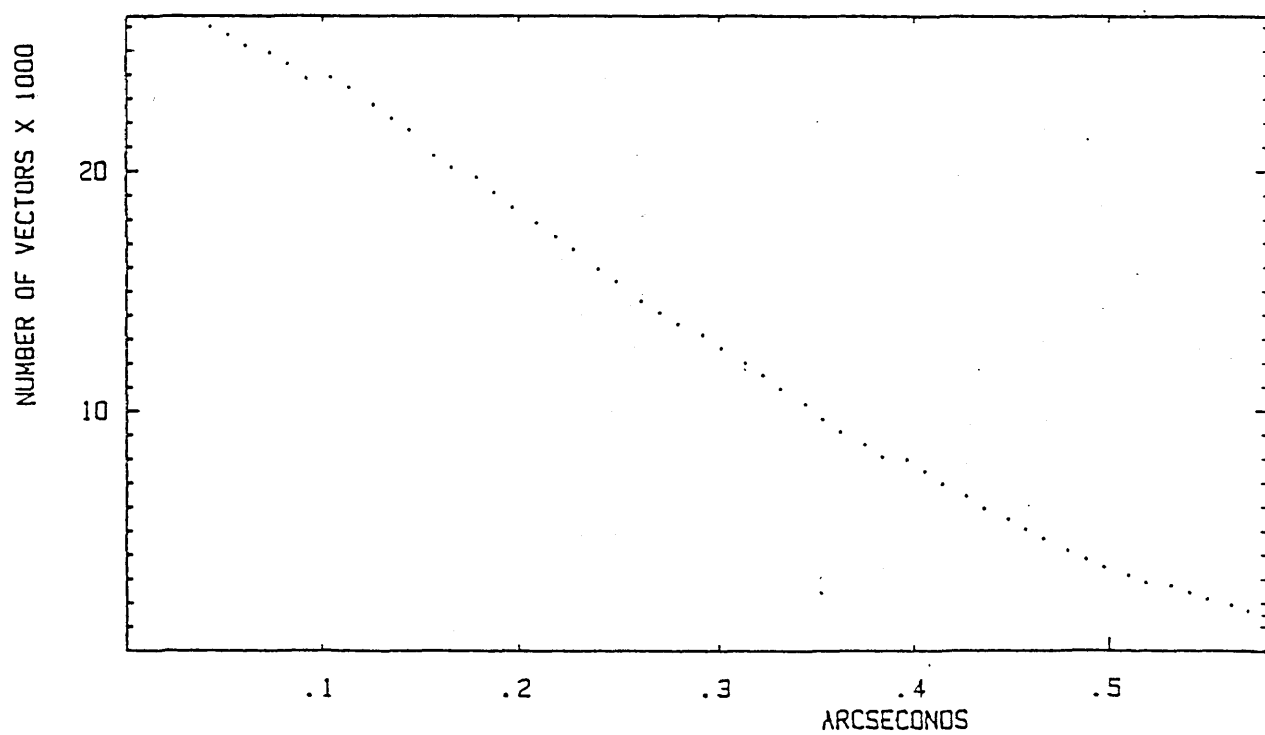


Figure 6.5 Radial Profile Through the Cross-correlation of ADS1123 at 159°.

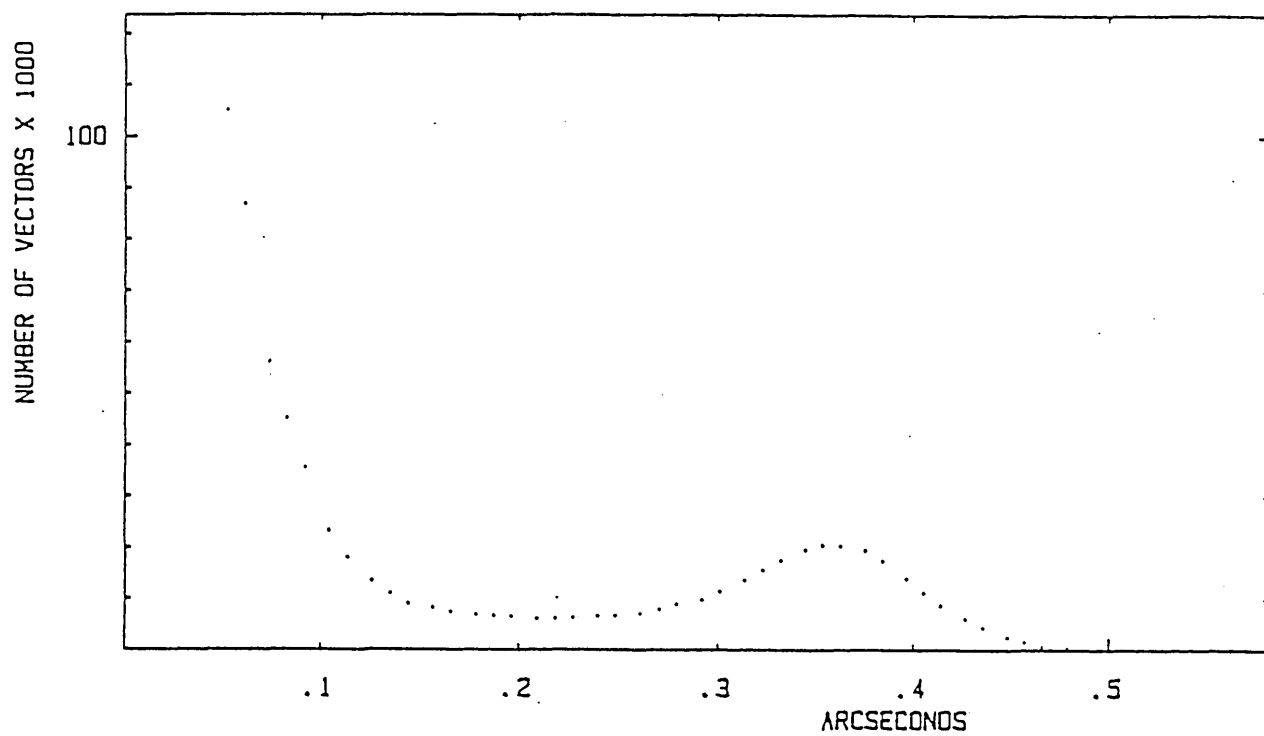


Figure 6.6 Radial Profile Through the Seeing Corrected Autocorrelation of ADS1123 at 159° .

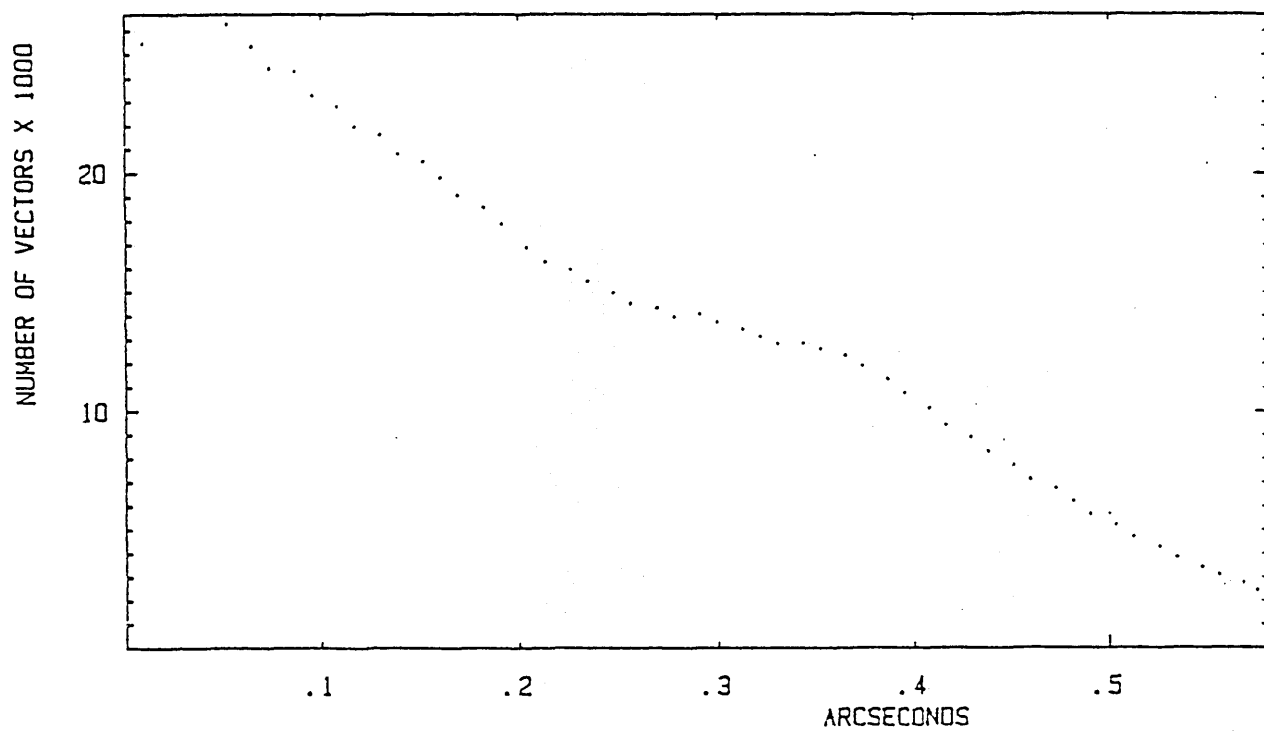


Figure 6.7 Radial Profile Through the Cross-correlation of ADS1123 at 159° (Field Interval = 2).

easy. Figure 6.4 shows a radial profile through the autocorrelation of ADS 1123 at an autocorrelation angle of 159° . For binary stars of lower visual magnitude the height of the peak above the local seeing background will be decreased as the number of vectors counted between the binary components becomes a smaller fraction of the number of vectors in the complete autocorrelation. Also the peak height will decrease as the magnitude difference of the binary increases. From practical experience binaries can be detected with magnitude differences of at least four. The secondary peak extends over several pixels because of the finite size of the binary components in each speckle. Therefore, even very small secondary peaks will produce a change in the gradient of a radial profile through the autocorrelation which extends over a number of consecutive pixels. This allows such peaks to be distinguished from random fluctuations in the data.

The position angle, P , of the binary is found by displaying many radial profiles through the autocorrelation. The angle of the profile that bisects the binary peak is termed the autocorrelation angle, A , and from this P can be easily calculated. The cross-correlation of a star trail record allows us to determine the orientation of the television image with respect to the sky and calculate any small rotational correction. When looking at the television image, if east is to the right-hand side, then from the convention for measuring position angle as shown in Figure 6.8:

$$P = A + 90^\circ + 180^\circ. \quad \text{Eq. 6.2}$$

The only method of removing the 180° ambiguity in the measurement of position angle, without phase retrieval, involves identifying the brightest component in individual speckles. This is possible for bright analogue records, but the method cannot be applied to photon counting records.

The seeing correction is achieved by the subtraction of a suitably normalised cross-correlation from the autocorrelation. The two correlations are derived with identical sampling levels and discriminator settings. One variable parameter is the interval between cross-correlated fields. Figure 6.2 shows the cross-correlation of

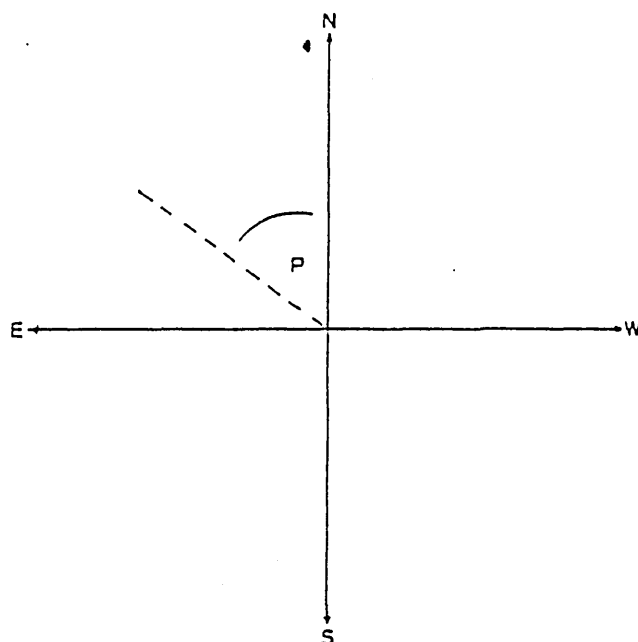


Figure 6.8 The Convention for Measuring Position Angle.

ADS 1123 calculated with a field interval of eight. At present no image centring is performed, therefore, the choice of field interval is a compromise between minimising any image motion between cross-correlated fields while maintaining statistical independence. It is hoped that in the future the cross-correlator will be modified to find the centre of gravity of each field and then add a constant "centring vector" to the position of the events in each field. The cross-correlation contains seeing information and is extended in the direction of the image's elongation, but the binary peak is absent. A radial profile through this function at an angle of 159° is shown in Figure 6.5. There is no photon spike present in the cross-correlation as vectors are measured between individual pixels in separate fields and not within photon events. To perform the seeing compensation the cross-correlation is normalised to the autocorrelation at large separations where no diffraction limited signal is expected and then subtracted from the autocorrelation. When observing centro-symmetric objects that are fainter than the example of the binary star given here, the mean profiles of the cross-correlation and autocorrelation are calculated to improve the signal to noise ratio. Then the seeing correction is performed by subtracting the normalised mean cross-correlation profile from the mean autocorrelation profile.

Figure 6.3 shows the seeing corrected autocorrelation. The binary peak is now more prominent, but the central photon spike is still present. It was found that for field intervals greater than 8 the application of the Welter and Worden method over-corrected the autocorrelation in the range 0° to 90° . This can be attributed to displacements of the image between cross-correlated fields on a scale comparable to the diameter of the seeing disc. A radial profile through the seeing corrected autocorrelation is shown in Figure 6.6. It is obvious that the contrast of the binary peak is increased and the position of its maximum is shifted from 0.35 to 0.36 arcseconds. Such a shift would be expected on subtracting a Gaussian seeing pedestal model.

For a binary star the signal, S , in the autocorrelation may be defined as the height of the binary peak above the local background, B [69]. Assuming Poisson statistics, the probable error in the measurement of the autocorrelation, N , is the square root of the sum of the signal and the background then:

$$\text{SNR} = S/(S + B)^{\frac{1}{2}}. \quad \text{Eq. 6.3}$$

Applying equation 6.3 to the autocorrelation of ADS1123 $S = 6150$ vectors $N = 188$ vectors and, therefore, $\text{SNR} = 33$; for the seeing corrected autocorrelation the normalising constant $M = 2$, $S = 14280$, $N = 219$ and $\text{SNR} = 65$. Therefore, by applying the cross-correlation seeing correction the SNR has been increased by a factor of order two.

Having derived the cross-correlation it is interesting to compare it with the Gaussian and near-Gaussian seeing pedestal models. In Figure 6.9 these two models have been fitted to a profile through the cross-correlation using a least squares algorithm. At high spatial frequencies the Gaussian model is more satisfactory than the near-Gaussian. The root mean squared (rms) deviation of the Gaussian model is 29 vectors and the rms deviation of the near-Gaussian model is 50 vectors.

The consequences of reducing the interval between cross-correlated fields is shown in Figure 6.7. For a field interval of two, the fields

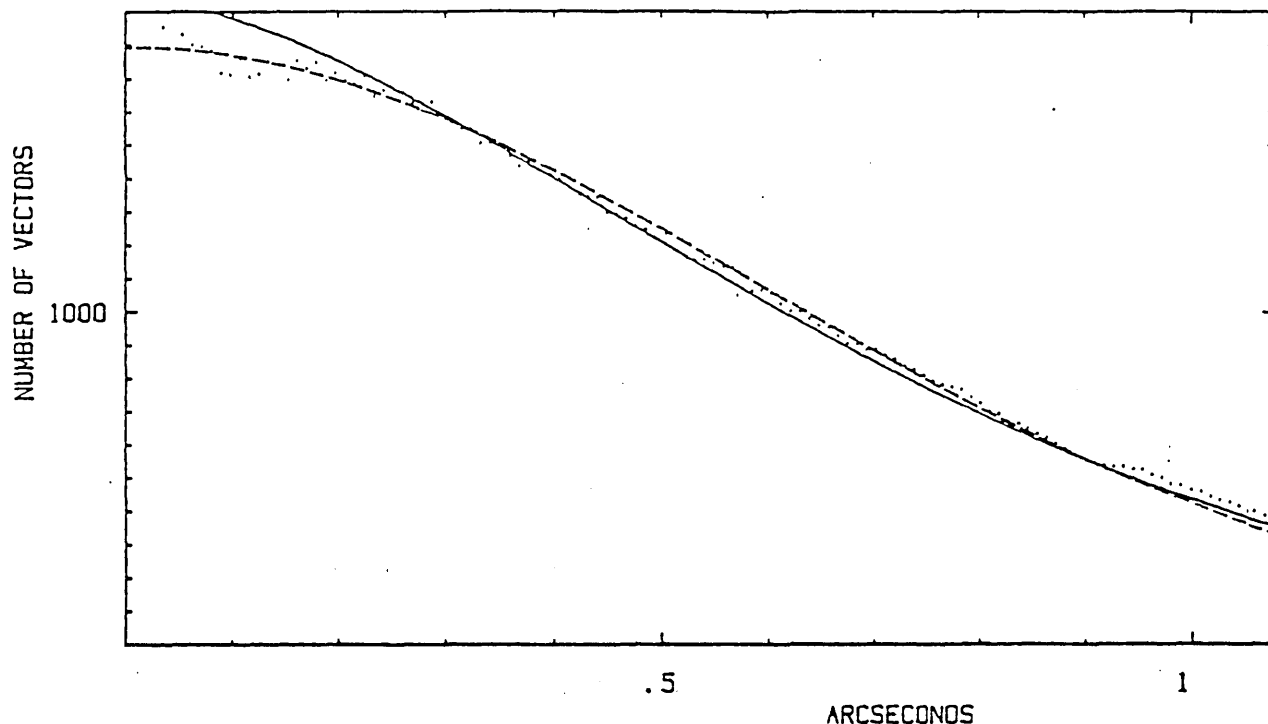


Figure 6.9 Near Gaussian (—) and Gaussian (-----) Seeing Pedestal Models Compared with the Cross-correlation (.....).

are no longer statistically independent and a "binary peak" is apparent in the cross-correlation. The time resolution of the cross-correlator is two television fields (40 msec.). As no peak is apparent in the cross-correlation calculated with a field interval of four, the speckle images decorrelate at some time greater than 40 msec. and less than 80 msec. This is entirely expected as for a frame of zero there would be total correlation and the cross-correlation would equal the autocorrelation.

The validity of the cross-correlation seeing correction is usually checked by performing the correction on an observation of a reference star. This is not strictly necessary, but if the result is consistent with that expected for a point source we can be confident of our results for extended objects. Results obtained with this method of seeing correction are presented in Chapter Seven.

6.3 The Photon Spike and Detector Response Function.

The photon events that appear on the output phosphor of the image intensifier tube are imaged onto a Plumbicon television camera so that they typically cover an area of several square pixels. Vectors counted

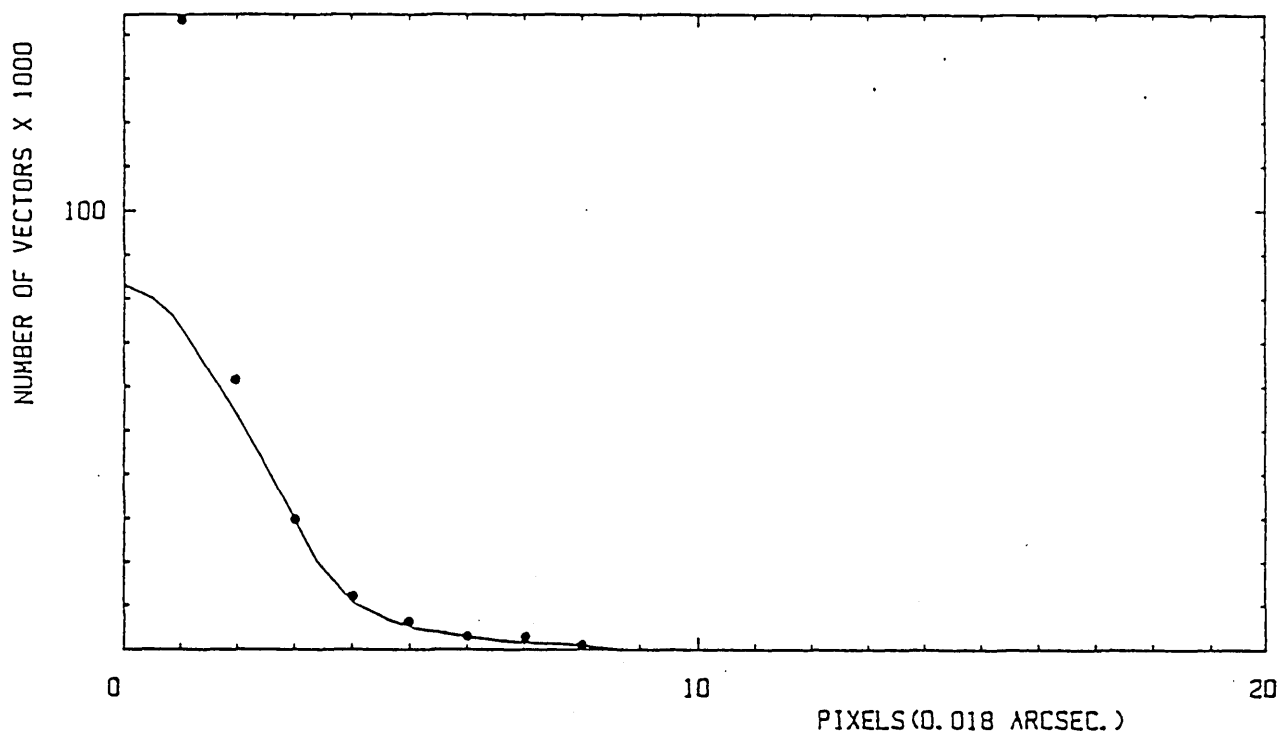


Figure 6.10 The Seeing Corrected Autocorrelation of a Point Source.

between pixels within a photon event contribute to a feature in the autocorrelation called the photon spike. Figure 6.10 shows the seeing corrected autocorrelation of a point source observed on the AAT. The photon spike extends out to the 3rd. pixel. However, the form of this seeing corrected autocorrelation is not that predicted by the analysis of Chapter Two, where the seeing corrected autocorrelation, $Q(h,k)$, is shown to be:

$$Q(h,k) \propto [O(x,y) * O(x,y)] \oplus P_0.$$

If the object is a point source $O(x,y) = \delta(x,y)$ and:

$$Q(h,k) \propto P_0.$$

That is, the seeing corrected autocorrelation of a point source is the Airy pattern of the telescope. So, the average autocorrelation of a point source will have the Airy pattern superimposed on a broad Gaussian-like background. These results are valid when the statistics of the seeing are assumed to be complex Gaussian. In physical terms this means that they apply in bad seeing, $D/r_0 > 10$. Goodman [70] and Lowenthal and Arsenault [71] have derived similar results for laser speckle patterns, and Bonneau et al. [72] have published

astronomical results in which the autocorrelation of a point source varies between an Airy pattern and the autocorrelation of an Airy pattern, depending on the quality of the seeing. For the AAT $D/r_0 \approx 40$ and, therefore, the data points in Figure 6.10 should show an Airy function. The discrepancy between the experimental observations and the theoretical predictions can be explained if the effect of the detector impulse function is considered.

For each photon event the number of pixels detected by the autocorrelator depends on the video sampling threshold and the size of the photon event on the output phosphor of the image tube. Therefore, the impulse response of the system, $S(x,y)$, may vary between photon events in the same image. However, a net impulse response, $\langle S(x,y) \rangle$, can be defined as the average of the individual response functions [73]. The measured image intensity, $I_s(x,y)$, is then given by:

$$I_s(x,y) = I(x,y) \otimes \langle S(x,y) \rangle \quad \text{Eq. 6.4}$$

The measured seeing corrected autocorrelation, $Q_s(h,k)$, is now given by:

$$Q_s = \langle I_s * I_s \rangle - \langle I_s * I_s' \rangle \quad \text{Eq. 6.5}$$

where the coordinates have been omitted for convenience. Substituting equation 6.4 into equation 6.5 gives:

$$\begin{aligned} Q_s &= [\langle S \rangle * \langle S \rangle] \otimes [\langle I * I \rangle - \langle I * I' \rangle] \\ &= [\langle S \rangle * \langle S \rangle] \otimes Q. \end{aligned}$$

The measured seeing corrected autocorrelation is the actual seeing corrected autocorrelation convolved with the autocorrelation of the impulse response. So, Figure 6.10 shows the autocorrelation of the impulse response, convolved with the Airy pattern. The solid curve in Figure 6.10 is the autocorrelation of a disc of radius 2 pixels convolved with the Airy pattern. This is a good fit to the experimental data outside the photon spike, which suggests that the impulse response of the system can be modelled by such a disc.

The impulse response of the system will blur the actual autocorrelation, but it will not change the position of any peaks in autocorrelation so the angular diameter of objects can still be determined. However, if accurate photometric data is to be derived the effects of the impulse response must be removed from any observation, and several approaches to correcting for the detector response function are discussed in Chapter Eight. At present no attempt is made to deconvolve the measured autocorrelations and the consequences for various results are considered in Chapter Seven.

6.4 The Modelling of Speckle Components.

Several image reconstruction algorithms exist which can be implemented on speckle data, but some can be susceptible to photon noise and they have, so far, produced no non-trivial astronomical results. Therefore, extensive research is underway in the field of image reconstruction techniques (see Chapter Eight). The successful application of these would allow both spatial and photometric information to be derived. However, the various algorithms for phase retrieval require extensive computational facilities and with a few simple assumptions a great deal of information can be derived from the seeing corrected autocorrelation. The first assumption is that the complex amplitude in the pupil of the telescope obeys complex Gaussian statistics. This is a satisfactory model under poor seeing conditions ie. when $D/r_0 > 10$. For a 4 m. telescope this inequality is normally satisfied and then the seeing corrected autocorrelation of a speckle record is proportional to the diffraction limited image of the autocorrelation of the object under study. Further if the object is assumed to be centro-symmetric it is possible to uniquely determine the object's intensity distribution.

A program named NEWMOD has been written in the FORTRAN programming language on a VAX 11/750 to calculate the two dimensional image of the autocorrelation of a given centro-symmetric object assuming an unaberrated telescope. This function is given by:

$$Q(h,k) = [O(x,y) * O(x,y)] \oplus P_o(x,y),$$

where $O(x,y)$ = object intensity distribution,
 $P_O(x,y)$ = point spread function of the telescope.

The point spread function of a diffraction limited telescope is known as the Airy pattern and is a function of the observational parameters given by:

$$U(\omega) = U.[2J_0(z)/z - 2\epsilon.J_0(\epsilon.z)/z]^2/(1 - \epsilon^2)^2,$$

where U = peak intensity,

ϵ = obscuration ratio of a circular aperture,

$z = \pi.\omega.D/\lambda$,

D = aperture of the telescope,

ω = sine of the angle subtended by a point in the image plane,

λ = wavelength,

J_0 = first order Bessel function.

The program prompts the entry of the observational parameters of wavelength, telescope aperture, obscuration ratio and pixel size. The value of the Airy pattern is calculated at every quarter pixel and stored in a two dimensional array. The Bessel functions are accessed from the extensive Numerical Algorithms (NAG) library available on the VAX. The one dimensional profile of the object to be autocorrelated can then be entered in quarter pixel steps in one of two ways. If the object under study is a planetary nebula an option can be chosen that calculates the radial brightness distribution according to the equations outlined by Harrington [74]. These equations and the modelling of planetary nebulae are discussed in Section 6.5. Any other object profile is entered from the keyboard. The one dimensional object profile is then expanded into a two dimensional centro-symmetric array assuming linearity between any two adjacent points in the original profile.

Autocorrelation can be considered as forming the product of the overlapping areas of a function and its replica as a function of the relative displacement of their origins. As the object array is centro-symmetric its autocorrelation will also be centro-symmetric and it is only necessary to perform any calculations in one dimension.

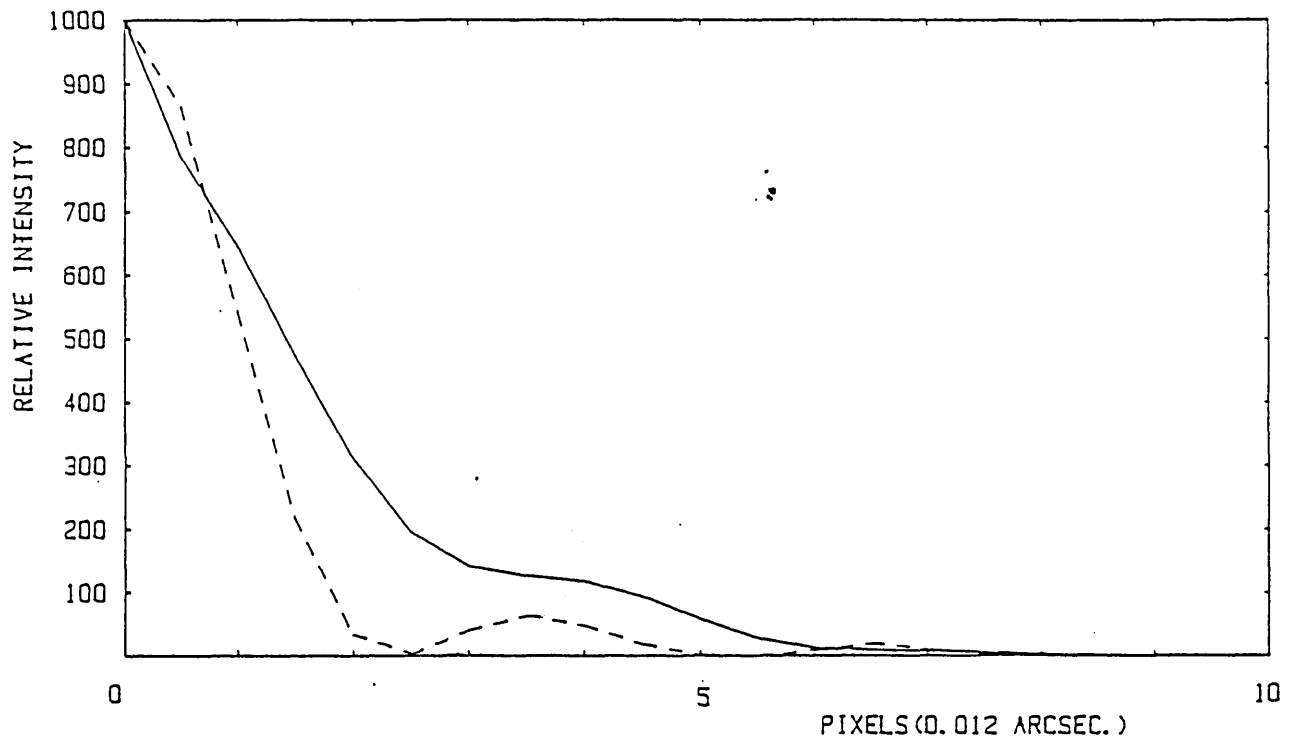


Figure 6.11 $Q(h,k)$ (----) and $Q'(h,k)$ (—) for $O(x,y) = \delta(x,y)$

The x axes of the object array and its replica are superimposed and the products of each pair of overlapping quarter pixels are summed as the origins of the two distributions are displaced in the x direction. This process continues until the two origins overlap and produces a one dimensional profile through the autocorrelation of the object array. This profile is then expanded into the complete two dimensional autocorrelation of the object.

The convolution of two functions is performed in a similar manner to autocorrelation. After reversing the signs of the coordinates of one function the convolution is found by calculating the overlapping areas of the two distributions as a function of their relative displacement. As the object array and the Airy pattern array are both symmetric reversing the signs of any coordinates has no effect and is therefore not required in this case. The x axes of the autocorrelation and the Airy pattern array are superimposed and the products of each pair of quarter pixels summed as one array is displaced with respect to the other. This produces a one dimensional profile through the diffraction limited image of the object's autocorrelation.

Let the image of an object's autocorrelation be denoted by $Q(h,k)$ and the autocorrelation of the image of an object be denoted by

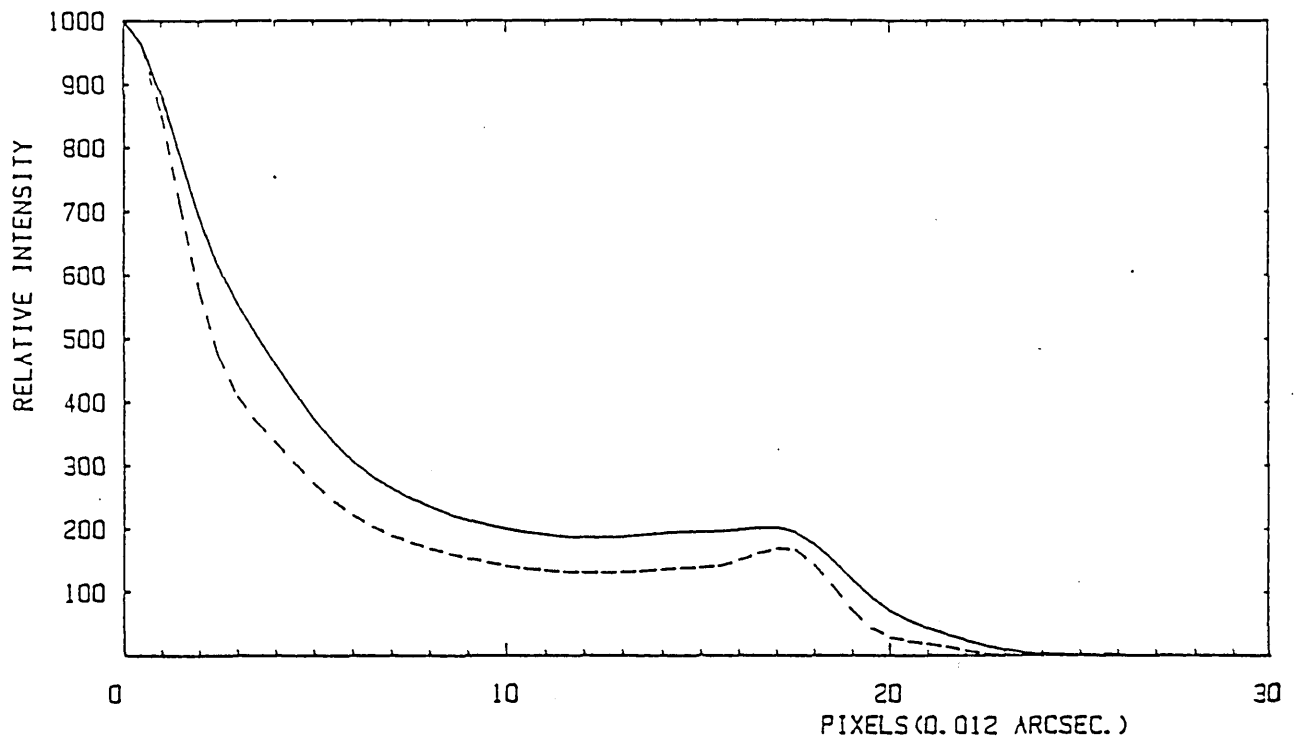


Figure 6.12 $Q(h,k)$ (-----) and $Q'(h,k)$ (—) for an Annulus of Inner Radius 8 Pixels and Outer Radius 9 Pixels.

$Q'(h,k)$. In Figure 6.11 the dashed line shows the function $Q(h,k)$ for a δ -function for AAT observations at 500 nm. and a pixel size of 0.012 arcseconds. The solid line indicates the function $Q'(h,k)$ for the same object, as would have been calculated in the original interpretation of the Welter and Worden seeing correction (see Section 2.7). The relative forms of these functions are easily understood when the mathematical expressions for $Q(h,k)$ and $Q'(h,k)$ are examined:

$$Q(h,k) = [O(x,y) * O(x,y)] \oplus P_o(x,y),$$

and

$$Q'(h,k) = [O(x,y) \oplus P_o(x,y)] * [O(x,y) \oplus P_o(x,y)].$$

Because of the two convolutions with an Airy pattern involved in calculating $Q'(h,k)$ the autocorrelation of the δ -function's image has a wider half-width than the image of the δ -function's autocorrelation.

Figure 6.12 shows profiles through the functions $Q(h,k)$ and $Q'(h,k)$ calculated for an annulus of inner radius 8 pixels and outer radius 9 pixels. The Airy pattern used is the same as in the calculation of the profiles in Figure 6.11. There are several important features to be

noted in Figure 6.12:

$$1) \quad \text{for } h = 16 \quad \frac{dQ(h,k)}{dh} = 0 \quad \text{and} \quad \frac{d^2Q(h,k)}{dh^2} < 0$$

and

$$\frac{dQ'(h,k)}{dh} = 0 \quad \text{and} \quad \frac{d^2Q'(h,k)}{dh^2} < 0$$

ie. the maxima in $Q(h,k)$ and $Q'(h,k)$ both occur at the 16th. pixel;

$$2) \quad \text{for } h = 16 \text{ and } h' = 12 \quad \frac{Q(h,k)}{Q(h',k)} > \frac{Q'(h,k)}{Q'(h',k)}$$

ie. the peak to background ratio is greater for $Q(h,k)$ than for $Q'(h,k)$;

$$3) \quad \text{at } h = 23 \quad Q(h,k) = 0 \quad \text{and at } h = 27 \quad Q'(h,k) = 0.$$

The similarities and differences between $Q(h,k)$ and $Q'(h,k)$ are important when modelling object distributions by comparison with seeing corrected speckle data. Any object distribution derived with the original interpretation of the Welter and Worden seeing correction will be slightly too narrow, but the position of any peaks in intensity will be correct.

6.5 The Derivation of Angular Diameters.

If the object under study can be considered to have a centro-symmetric intensity distribution the signal to noise ratio in the autocorrelation can be improved by taking an azimuthal average. The seeing correction is then made for both the object and reference star autocorrelations by subtracting a normalised azimuthally averaged cross-correlation. It is not strictly necessary to analyse data from a reference star, but it acts as a useful comparison with object data to show the difference between a point source speckle component and an extended source speckle component. Once the object's speckle component has been derived the theoretical speckle component produced by various models can be calculated with the NEWMOD program and compared with the

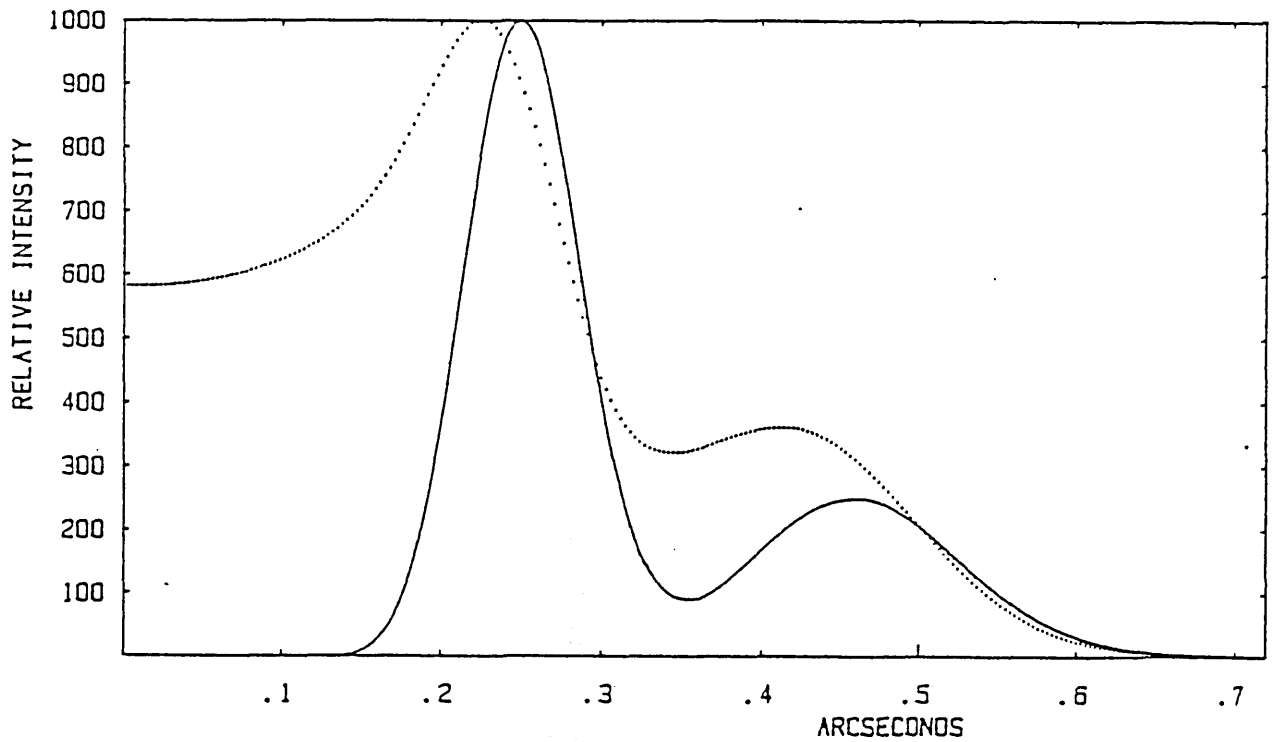


Figure 6.13 Radial Emissivity (—) and Radial Brightness (.....)
Models for a Planetary Nebula.

observed data. For objects such as asteroids, nebulae and resolvable stars, simple discs of uniform intensity have produced give good fits to the observed speckle components. The modelling procedure for planetary nebulae is somewhat more complicated.

Many planetary nebulae exhibit a high degree of circular symmetry and a ring structure. Therefore, it is convenient to model their radial emissivity and brightness distributions by functions similar to those proposed by Harrington and Feibelman [74]. The emissivity per unit volume is modelled by:

$$e(r) = \sum_{k=1}^K \sum_{n=0}^N a_{k,n} \cdot (r/H_k)^{2n} \cdot \exp[-(r/H_k)^2]$$

where r = radial distance from the centre,
 H = distance scale constant,
 a = constant.

Then the projected brightness distribution along the line of sight

whose line of closest approach to the centre is R is:

$$I(R) = (1/2\pi) \int_r^{\infty} e(r) \cdot r \cdot dr / (r^2 - R^2)^{\frac{1}{2}}$$

$$= \sum_{k=1}^K \sum_{n=0}^N b_{k,n} \cdot (R/H_k)^2 \cdot \exp[-(R/H_k)^2]$$

where $b_{k,n} = (H_k/4\pi^2) \sum_{l=n}^N l!! \cdot (1-n)!! \cdot 2^{n-l} \cdot a_{k,l} / (1-n)! \cdot n!$

Here $(0)!! = 1$ and $m!! = 1.3.5 \dots (2m-1)$ for $m \geq 1$

In Figure 6.8 the model emissivity, $e(r)$, and brightness, $I(R)$, distributions for planetary nebula are shown. These models were derived with $a_{1,12} = 1$, $a_{2,12} = 0.25$, $H_{1,12} = 0.072$ arcseconds and $H_{2,12} = 0.132$ arcseconds. Important features of these curves include the central infilling present in brightness and the reduction in the radius of the peaks when the emissivity is integrated to produce the brightness distribution. Results using such models for planetary nebulae are presented in Chapter Seven.

6.6 Sources of Error.

Any result obtained with speckle interferometry will be a estimate of some quantity such as a diameter or separation. As explained in Section 6.3 photon events typically cover several pixels and the measured autocorrelation is, in fact, the actual autocorrelation blurred by the impulse response of the system. Initial investigations suggest that this response function can be modelled by a disc of radius two pixels. Hence, the derived diameters of uniform brightness objects will be approximately two pixels too large. Arnold has previously noted this problem in his observations of Pluto [35]. In contrast, the positions of any peaks in the measured autocorrelation will be unaffected by a deconvolution with a uniform disc. Therefore, binary separations and the diameters of objects possessing peaks in intensity can be determined to ± 1 pixel.

The presence of ion events, camera noise and sky background photons on a record will, of course, introduce spurious signals. However, the use of discriminators and the careful choice of sampling level can exclude the effects of these noise sources from the autocorrelation. It is of great importance to obtain the maximum signal to noise ratio in the autocorrelation of observational records. Faint objects (fainter than magnitude 10 say) are usually observed for half an hour so that a large number of television fields can be analysed.

A considerable improvement in the accuracy of seeing compensation has been possible since the introduction of the cross-correlator. Before the adoption of the Welter and Worden seeing correction, variations in the quality of the seeing would produce a seeing pedestal with two distinct gradients. The resulting "kink" could be mistaken for an extended speckle component. The limitations of the present form of the Welter and Worden method have been discussed and the effect of image motion between cross-correlated frames noted. The application of the methods described in this chapter to astronomical data is described in Chapter Seven.

Chapter Seven.

CHAPTER SEVEN - ASTRONOMICAL OBSERVATIONS.

7.1 Introduction.

The Imperial College speckle system has been used to observe a wide range of astronomical objects. These include binary stars, nebulae, asteroids, active galaxies and galactic and extra-galactic planetary nebulae. The catholic nature of these observations demonstrates the improvements in magnitude limit and data reduction techniques since the first speckle observations of bright binary stars.

Several collaborative observing programmes have produced the results presented in this chapter: the search for structure in objects to be observed by the Hipparcos Astrometry Satellite is undertaken with Dr. Noel Argue of Cambridge University; the study of planetary nebulae is in collaboration with Dr. Mike Barlow of University College London; and the observations of NGC 2346 were undertaken in collaboration with Dr. John Meaburn of Manchester University.

7.2 Hipparcos Link Stars.

The Hipparcos Satellite of the European Space Agency will establish a stellar reference frame based on 100000 stars brighter than magnitude +13. Such a frame should be inertial to the best achievable accuracy. Therefore, it is hoped to assign coordinates to very distant quasi-motionless galaxies and quasars. However, too few extragalactic objects brighter than 13th. magnitude are known which can provide a direct link between the Hipparcos frame and an extragalactic frame. To overcome this problem it is proposed to observe a number of stars with both the Space Telescope and the Hipparcos Satellite.

A selection of 75 extragalactic radio sources will provide the basis of the inertial reference frame [75]. The Space Telescope's Fine Guidance System (FGS) has a magnitude limit of +17 and is capable of measuring the separation between an extragalactic source and a Hipparcos link star as long as this does not exceed 18 arcminutes (the field of the FGS). Therefore, each radio source must have an unresolved star brighter than 13th. magnitude within 18 arcminutes.

The FGS will be able to measure this separation to an accuracy of ± 0.002 arcseconds.

To achieve the required accuracy of measurement it is essential that none of the objects observed display structure above the Space Telescope's resolution limit of 0.05 arcseconds. Also there must be no companion present within the range 0.3 to 15 arcseconds. Such a companion would be present in the field of the Hipparcos Satellite and would perturb any measurement made unless it was at least 3.5 magnitudes fainter. An initial set of link stars has been chosen by examining sky survey plates, however these can only confirm that no companion exists down to the 2 arcsecond resolution of the plate. For separations between 0.05 and approximately 0.7 arcseconds it was decided to use speckle interferometry to eliminate any objects possessing multiple structure. The analysis of objects over the remaining separations between 0.7 and 2 arcseconds is the topic of an investigation by de Vegt at the Hamburger Sternwarte.

The observation of Hipparcos link stars by speckle interferometry has been undertaken by two groups: in the Northern Hemisphere McAlister is performing observations at the Lowell Observatory and for Southern Hemisphere objects Morgan's group, in collaboration with Dr. Noel Argue, has observed using the AAT and the 1.9 m. telescope at Mount Stromlo. The ability to reduce data in real-time has proved extremely useful during this programme and the observing run conducted in 1984 made particularly efficient use of two major telescopes. Initially as many link stars as possible were observed with the 1.9 m. telescope at Mount Stromlo. Immediately the autocorrelator allowed us to identify 16 of the objects to have multiple structure in the range 0.07 to 1.0 arcseconds. Ninety four stars were unresolved by the 1.9 m. telescope. Of these 94 stars, 29 were observed with the AAT over the range 0.03 to 0.7 arcseconds and 6 of them were resolved as multiple objects. On returning to Imperial College the entire data set was re-analysed by the author and the 123 stars were divided into three categories: those which showed no evidence of multiple structure; those which displayed some evidence of possible multiple structure; and those which showed strong evidence of multiple structure. A complete list of these results has been published by

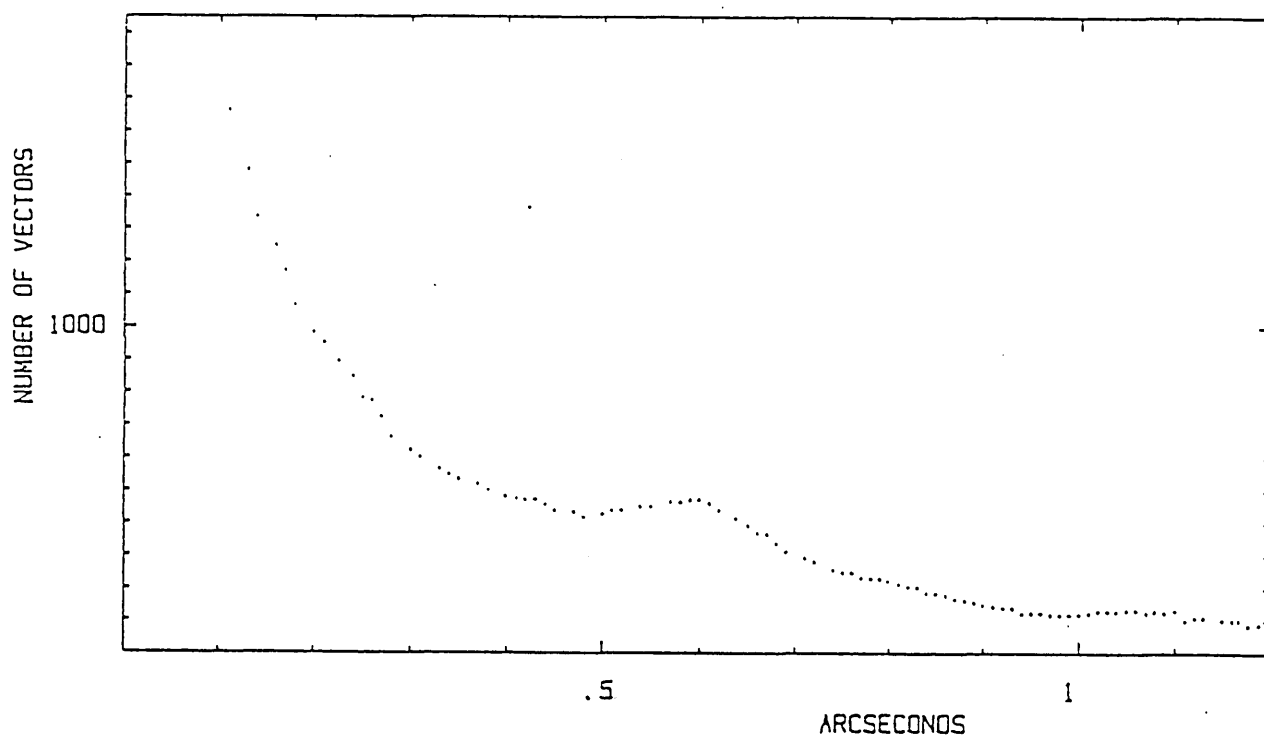


Figure 7.1 Radial Profile Through the Autocorrelation of AGK+0°2717 at an Autocorrelation Angle of 105° Showing a Binary Peak at 0.59 Arcseconds.

Argue et al. [76] and the paper appears in the Publications Section of this thesis. An example of the binary structure observed in a Hipparcos link star is given in Figure 7.1. During the data reduction no attempt was made to correct for the seeing pedestal as the main objective of the survey was merely to identify any objects with structure above 0.05 arcseconds.

To date 160 candidate stars have been observed in the Southern Hemisphere [76,77]. It is found that 30% of the stars chosen from sky survey plates as suitable link stars are resolved as multiple objects by the Imperial College speckle system used on the AAT. This severely limits the number of stars available to link the Hipparcos and extragalactic reference frames, and there may be long periods of time during which an object has no suitable guide star.

7.3 The Magellanic Cloud Planetary Nebula SMC N2.

A knowledge of the shell masses of planetary nebulae is of interest for three distinct reasons. Firstly to quantify the extent to which

low to intermediate mass stars contribute to the chemical enrichment of the interstellar medium. Secondly from the shell mass it is possible to derive the properties of the progenitor star. The masses of the central white dwarf stars in planetary nebulae can be determined by fitting model evolutionary paths through their position in the Hertzsprung-Russell (HR) diagram. Then the masses of these white dwarfs can be related to those of the progenitor stars on the asymptotic giant branch (AGB) of the HR diagram, and eventually the masses of the white dwarfs can be related to the original main sequence stellar masses. Finally a knowledge of planetary nebulae shell masses as a function of the central stellar masses will aid in determining the physical processes involved in the ejection mechanism.

In order to determine the shell mass of a planetary nebula the ion number density must be calculated as a function of the nebula's radius. However, to obtain the data required for this calculation a well resolved planetary nebula at a known distance must be observed. The planetary nebulae in our own galaxy are indeed easily resolved, but their distances are only known to an accuracy of approximately 50% [78] and it is necessary to look outside the Galaxy.

The Magellanic Clouds, which are visible in the Southern Hemisphere, are two dwarf irregular galaxies lying some 60 kiloparsecs (kpc) from our own galaxy. Their distances are known to an accuracy of 15% because each galaxy contains a number of independent distance calibrators [79]. At this point it is useful to introduce the concept of distance modulus. This is a measure of distance often used on galactic scales. The difference between the apparent magnitude, m , and the absolute magnitude, M , of an object is related to its distance from us in parsecs, d , by:

$$m - M = 5.(\log_{10}d - 1)$$

The Magellanic Clouds have an approximate distance modulus of 18.9 and the likely uncertainty in this is ± 0.3 . The remoteness of the Magellanic Clouds has limited the number of planetary nebulae that can be resolved by standard imaging techniques [80], and the majority of them are expected to have diameters of less than one arcsecond

[81,82]. The technique of speckle interferometry has been used to observe these planetary nebulae, and angular diameters and radial brightness distributions have been derived. In this section the results for SMC N2 are presented.

During November 1983 and October 1984 Dr. M. Barlow, of University College, and Mr. H. Vine, of Imperial College, conducted speckle observations of Magellanic Cloud planetary nebulae with the AAT. The observations were made through narrow band filters and the target planetary nebulae were chosen from those best observed at optical and ultraviolet wavelengths in previous surveys [81,82,83,84]. A second consideration in our choice of nebulae was that they be optically thin in the hydrogen Lyman continuum so that the ionised nebula mass is equal to the total shell mass.

The observations of SMC N2 were made through a 75 mm. objective lens and with the interferometer at the f/15 focus of the AAT. This gives a pixel size of 0.018 arcseconds and allows the detection of separations up to 1.1 arcseconds. The filter used had a 5 nm. bandpass centred on the OIII (500.7 nm.) emission line. This particular filter was chosen to suppress the stellar background; because its strength is typically ten times that of the H β line; and because the brightness distributions of most planetary nebulae in OIII line are very similar to those in the H β line [88]. SMC N2 was observed for 60 minutes and immediately afterwards an unresolved reference star, SAO 255683, was observed for 30 minutes. The azimuthally averaged autocorrelations of these objects are shown in Figures 7.2 and 7.3 respectively. The azimuthally averaged cross-correlations of these records are shown as the solid curves normalised to the corresponding autocorrelation at low spatial frequencies. The reference star autocorrelation in Figure 7.3 displays the smooth seeing pedestal of an unresolved object and the difference between the autocorrelation and the cross-correlation falls to zero at 1.8 arcseconds. This is consistent with a point source as shown in Section 6.3. The seeing corrected autocorrelation of SMC N2, displayed as the dashed line in Figure 7.2, possesses an extension out to 0.3 arcseconds.

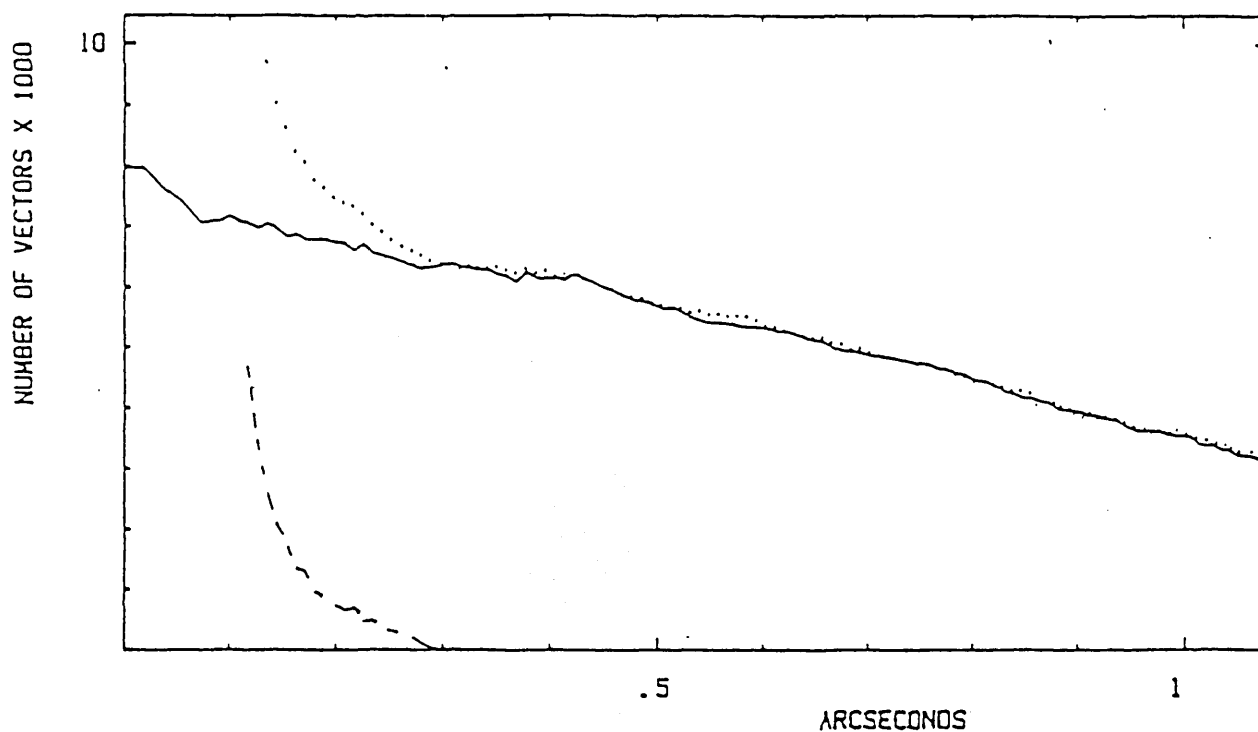


Figure 7.2 The Azimuthally Averaged Autocorrelation (.....), Cross-correlation (——) and Seeing Corrected Autocorrelation (----) of SMC N2.

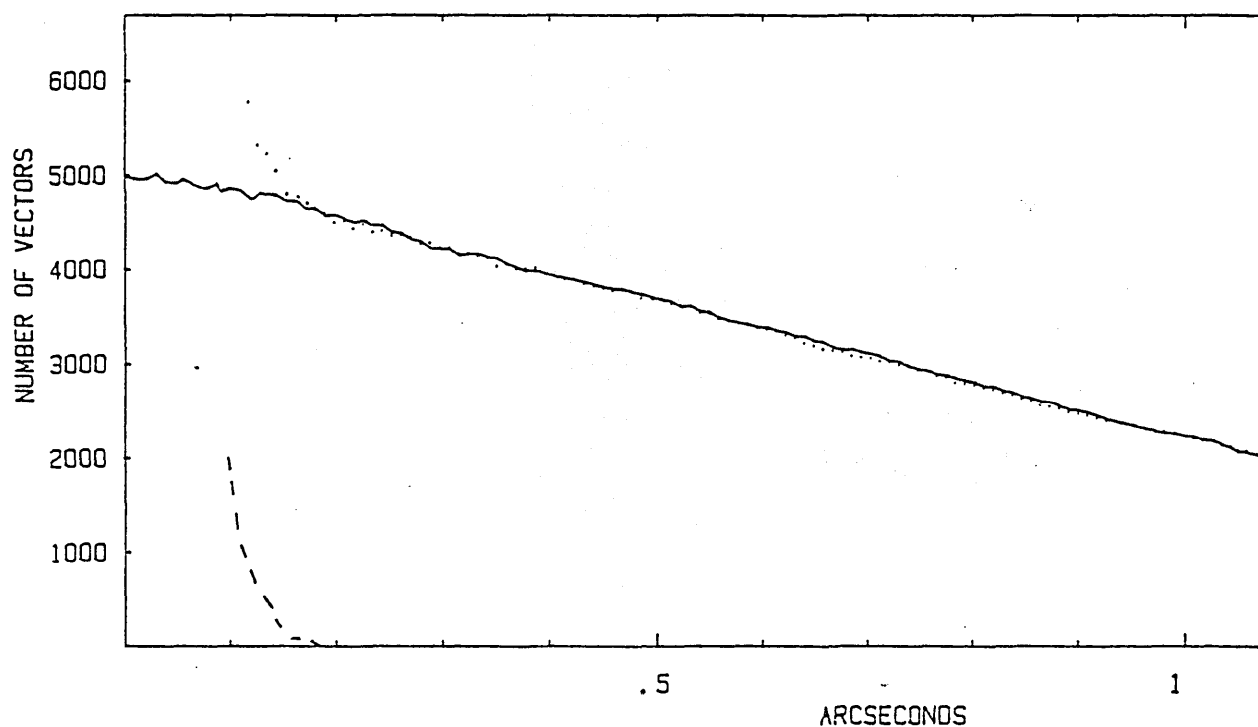


Figure 7.3 The Azimuthally Averaged Autocorrelation (.....), Cross-correlation (——) and Seeing Corrected Autocorrelation (----) of a Reference Star.

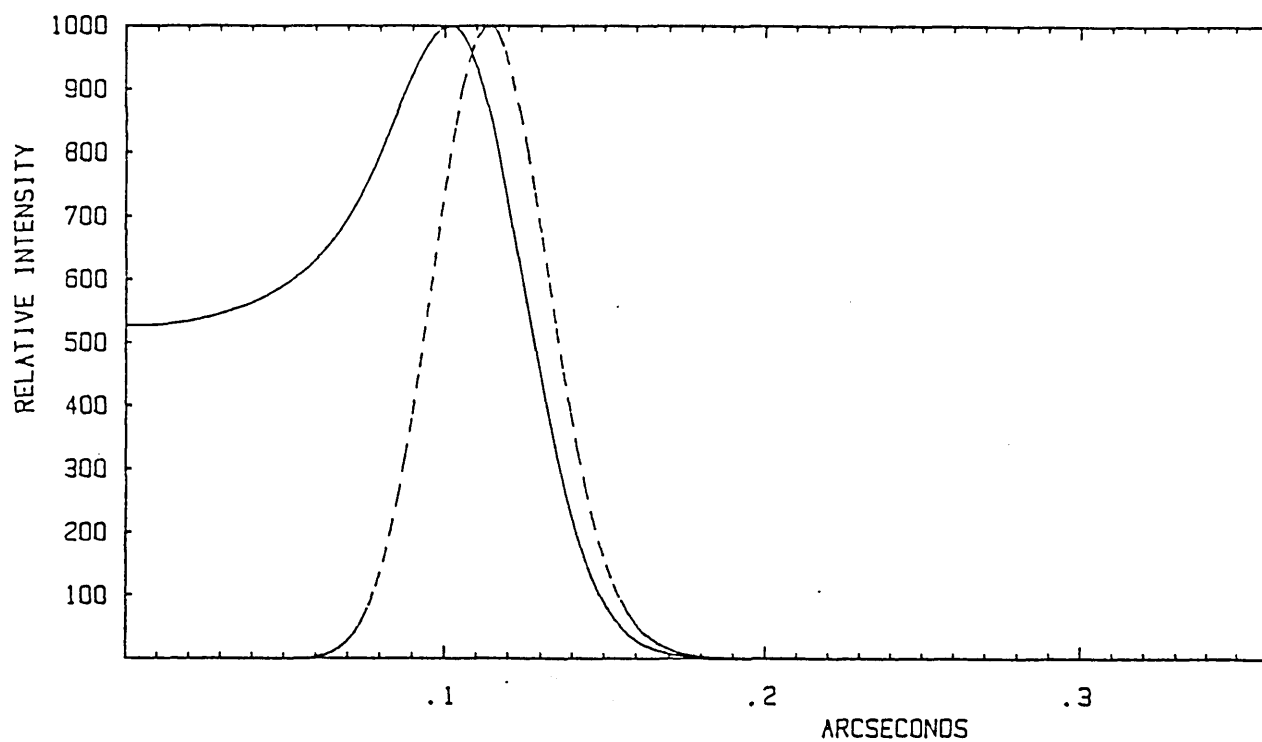


Figure 7.4 The Radial Emissivity (---) and
Brightness (—) Models for SMC N2.

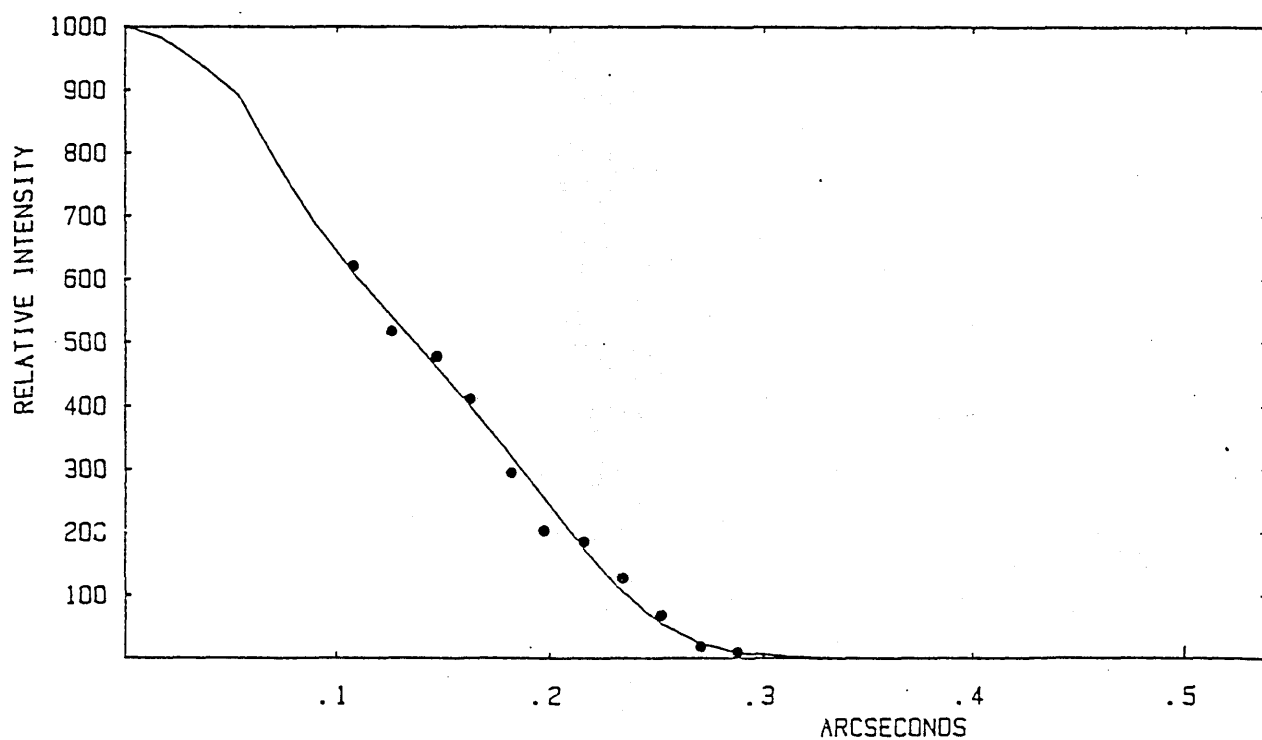


Figure 7.5 The Seeing Corrected Autocorrelation (• • •) of SMC N2
Normalised to the Diffraction Limited Image of the Brightness
Model's Autocorrelation (—).

With the implementation of the Welter and Worden seeing correction, and noting that the speckle component in Figure 7.2 lies well outside the photon spike, it is possible to model the brightness distribution of SMC N2 assuming azimuthal symmetry. This assumption can be made as no gross features occurred at specific angles within the 180° autocorrelation. To derive the planetary nebula's diameter and brightness distribution various models were autocorrelated. This autocorrelation was then convolved with the Airy pattern of the telescope and the resulting distribution was compared to the seeing corrected autocorrelation derived from our observations. Uniform or centrally peaked distributions yielded functions that did not resemble the experimental data. When these functions were normalised to the seeing corrected autocorrelation their intensity between 0.1 and 0.2 arcseconds was found to be too great. Therefore, as many galactic planetary nebulae exhibit a symmetrical ring or shell structure it was decided to experiment with models having such peaks in their brightness distributions. These models were derived from functions of the type described by Harrington and Fiebelman [74] (see Section 6.5).

One half of the radial brightness distribution of the best-fitting model is shown in Figure 7.4. This consists of an edge brightened disc; the peak intensity occurring at a radius of 0.1 arcseconds. The dashed curve in Figure 7.4 is the emissivity model which has been projected along the line of sight to produce the brightness distribution. The image of the brightness model's autocorrelation is shown in Figure 7.5 normalised to the observed seeing corrected autocorrelation. There is good agreement between this model and the seeing corrected autocorrelation down to 0.1 arcseconds. For separations less than 0.1 arcseconds the "photon spike" dominates the seeing corrected autocorrelation. As this "photon spike" contamination is not modelled the diffraction limited image of the brightness model's autocorrelation and the seeing corrected autocorrelation diverge. However, any deconvolution of the derived model with the impulse response of the speckle system (as described in Section 6.3) will not alter the position of the brightness peak.

7.4 The Mass of SMC N2.

The OIII brightness distribution of SMC N2 can be assumed to be the same as that for H β [85,86]. Then the total H β flux emitted by the nebula at a distance D is given by:

$$4\pi.D^2. I(H\beta) = \gamma.\alpha_{\text{eff}}(H\beta).h.\nu_{\beta} \cdot \int 4\pi.\epsilon.n_H^2(r).r^2.dr \quad \text{Eq. 7.1}$$

where $I(H\beta)$ = the de-reddened H β flux at the earth,
 $\alpha_{\text{eff}}(H\beta)$ = partial recombination coefficient of hydrogen,
 ϵ = the filling factor,
 $\gamma = (1 + n(\text{He}^+)/n(\text{H}^+) + 2n.(\text{He}^{2+})/n(\text{H}^+))$,
 $n()$ = the local density of the designated species,
 $n_H(r)$ = the density of hydrogen as a function of radius, r.

In a similar manner the emissivity distribution, $e(r)$, as a function of radius is:

$$e(r) = K.\gamma.\alpha_{\text{eff}}(H\beta).h.\nu_{\beta}.\epsilon.n_H^2(r) \quad \text{Eq. 7.2}$$

where K is some constant. Substituting from equation 7.2 into equation 7.1 leads to an equation for K:

$$K = (1 / I(H\beta).D^2) \cdot \int e(r).r^2.dr \quad \text{Eq. 7.3}$$

The integral in equation 7.3 can be numerically evaluated using the function derived for $e(r)$ which is shown as the dashed curve in Figure 7.4. Hence, K can be calculated and used to derive $n_H(r).\epsilon^{\frac{1}{2}}$ as:

$$n_H^2(r).\epsilon = e(r) / (K.\gamma.\alpha_{\text{eff}}(H\beta).h.\nu_{\beta}) \quad \text{Eq. 7.4}$$

In calculating $e(r)$ from the equations described in Section 6.4 the parameters used were: $k = 1$, $n = 10$, and $h = 0.036$ arcseconds. To allow the evaluation of $n_H(r).\epsilon^{\frac{1}{2}}$ an observed H β flux of $F(H\beta) = 1.7 \times 10^{-13}$ ergs/cm²/s [87,88] and a total reddening of 0.06 [89,90] were adopted. This gives a de-reddened H β flux of $I(H\beta) = 2.07 \times 10^{-13}$ ergs/cm²/s. The OIII electron temperature

used in calculating $\alpha_{\text{eff}}(\text{H}\beta)$ was taken as 13400 K. and the value of γ adopted was 1.13 [83]. Finally, the value of D adopted was 57.5 kpc. [91,92].

Having derived the radial density distribution of SMC N2, it is now possible to calculate the ionised nebula mass given by:

$$M(\text{PN}) = \mu \cdot m_{\text{H}} \cdot \epsilon^{\frac{1}{2}} \cdot \int n_{\text{H}}(r) \cdot \epsilon^{\frac{1}{2}} \cdot 4\pi \cdot r^2 \cdot dr, \quad \text{Eq. 7.5}$$

where m_{H} = mass of the hydrogen atom,
 $\mu = (1 + 4n_{\text{He}}/n_{\text{H}})$.

The integral in equation 7.5 has been numerically evaluated, and from the work of Aller [83] $\mu = 1.43$. This gives:

$$M(\text{PN}) = 0.107 \cdot \epsilon^{\frac{1}{2}} \cdot M_{\odot}.$$

To determine the nebula mass all that now remains is to derive a value for the filling factor. As the mean nebular electron density is a measure of the local electron density weighted by the local emissivity averaged over the entire nebula, it is possible to write:

$$\langle n_e \rangle = \int n_e^2 \cdot \epsilon \cdot n_i \cdot r^2 \cdot dr / \int n_e \cdot \epsilon \cdot n_i \cdot r^2 \cdot dr \quad \text{Eq. 7.6}$$

where n_i = local ion density,
 n_e = local electron density.

As $n_e = \gamma \cdot n_i$ and if we assume $n_i \propto n_{\text{H}}$, then equation 7.6 becomes:

$$\langle n_e \rangle \cdot \epsilon^{\frac{1}{2}} = \gamma \cdot \int n_{\text{H}}^3(r) \cdot \epsilon^{\frac{3}{2}} \cdot r^2 \cdot dr / \int n_{\text{H}}^2(r) \cdot \epsilon \cdot r^2 \cdot dr \quad \text{Eq. 7.7}$$

The numerical evaluation of equation 7.7 leads to a value for $\langle n_e \rangle \cdot \epsilon^{\frac{1}{2}}$ of $1.13 \times 10^4 / \text{cm}^3$. From observations of the electron density sensitive OII (372.6 nm., 372.9 nm.) doublet ratio $\langle n_e \rangle = 2850 / \text{cm}^3$. This gives:

$$\epsilon = 15.86.$$

This is clearly an unphysical result as the filling factor cannot exceed unity. There are two possible explanations for this; either the value of $\langle n_e \rangle$ is incorrect (this is unlikely as independent observers have obtained very similar values); or the assumption of a uniform filling factor over the nebula is too great a simplification of the actual situation. In the absence of a more accurate nebula model the value of $\epsilon = 0.7$ used by Wood [93] has been adopted. This gives a nebula mass of:

$$M(\text{PN}) = 0.09.M_{\odot}.$$

The derived nebula mass is surprisingly insensitive to the position and form of the emissivity in Figure 7.4. The nebula mass given by equation 7.5 is proportional to the integral of $n_H(r) \cdot \epsilon^{\frac{1}{2}} \cdot r^2 dr$ and to $\epsilon^{\frac{1}{2}}$. The value of $\epsilon^{\frac{1}{2}}$ is given by equation 7.7 which has a constant denominator for a fixed value of the H β flux, $I(\text{H}\beta)$ (see equation 7.1). Therefore, if we substitute for $\epsilon^{\frac{1}{2}}$ from equation 7.7 into equation 7.5, then substitute for $n_H(r) \cdot \epsilon^{\frac{1}{2}}$ from equation 7.4 and finally substitute for K from equation 7.3, the following result is obtained:

$$M(\text{PN}) \propto \frac{\int e^{\frac{3}{2}}(r) \cdot r^2 dr \cdot \int e^{\frac{1}{2}}(r) \cdot r^2 dr}{(\int e(r) \cdot r^2 dr)^{\frac{3}{2}} \cdot (\int e(r) \cdot r^2 dr)^{\frac{1}{2}}} \quad \text{Eq. 7.8}$$

For the Gaussian functions used to calculate the emissivity, $e(r)$, the ratios of the integrals in equation 7.8 remain constant if the half width of the derived emissivity is changed. For example, if the half width of the emissivity peak in Figure 7.4 is increased by 50%, the derived filling factor decreases from 15.86 to 12.93, but the derived nebula mass is unchanged. From equation 7.8 it can be seen that the nebula mass is also independent of the radius of the emissivity peak. Therefore, the ± 1 pixel accuracy to which the position of the peak can be found, and any deconvolution of the model with the impulse response of the system do not introduce an error in the derived nebula mass.

In conclusion it is the filling factor, ϵ , that is sensitive to any uncertainties in the modelling of the brightness distribution of

SMC N2. The value of ϵ derived from the best fitting model was greater than unity. Therefore, $\epsilon = 0.7$ was adopted as this value has been used by both Wood et al. [93] and Harrington and Fiebelman [74]. The most important source of error in the nebula mass estimate is the uncertainty with which the distance to the planetary nebula is known. If the distance modulus is taken as 18.8 ± 0.3 this leads to a error in the nebula mass of 30%. Therefore, $M(\text{PN}) = 0.09 \pm 0.027 M_{\odot}$.

The data for SMC N2 was originally analysed before the cross-correlator was in operation and the results of this analysis are presented in a paper in the Publications Section. The seeing pedestal was modelled by a smooth Gaussian function. The derived seeing corrected autocorrelation possessed several features absent from the seeing corrected autocorrelation calculated by the Welter and Worden method. These spurious features can be attributed to variations in the seeing which produced changes in the gradient of the autocorrelation. The cross-correlation also contains these features and, therefore, when the cross-correlation is subtracted from the autocorrelation they do not appear in the seeing corrected autocorrelation. This re-interpretation of the results for SMC N2 shows the value of the cross-correlator.

7.5 The Galactic Planetary Nebula VY 2-2.

To confirm the validity of the modelling procedure adopted in the analysis of the Magellanic Cloud Planetary nebulae it was decided to observe a compact galactic planetary nebula whose brightness distribution had been independently determined. The planetary nebula VY 2-2 is a suitable candidate as it has been observed by Seaquist and Davis [94] with the Very Large Array (VLA). The 15 GHz. map of VY 2-2 is shown in Figure 7.6. The dominant feature is the symmetric ring structure with a radius of 0.2 arcseconds. As the radio flux is due to hydrogen free-free emission the radio "brightness" should be the same as the H β brightness distribution.

VY 2-2 was observed with the AAT through the OIII filter and through a 1nm. bandpass H β filter. Figures 7.7 and 7.10 show the azimuthally averaged autocorrelations of observations made in the

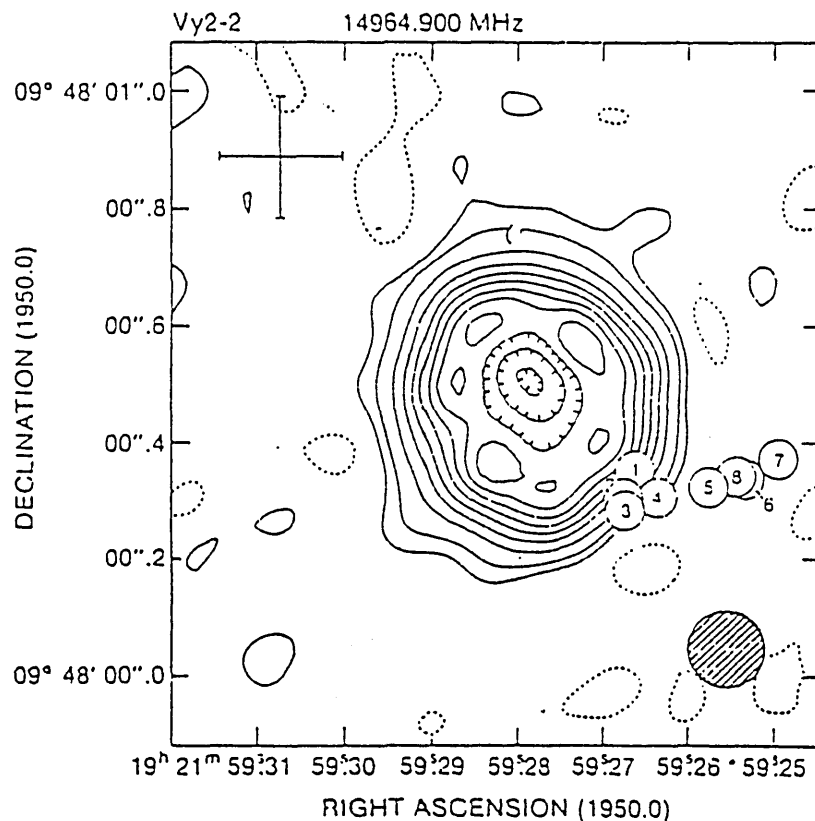


Figure 7.6 Seaquist and Davis' 15 GHz. VLA Map of VY 2-2.

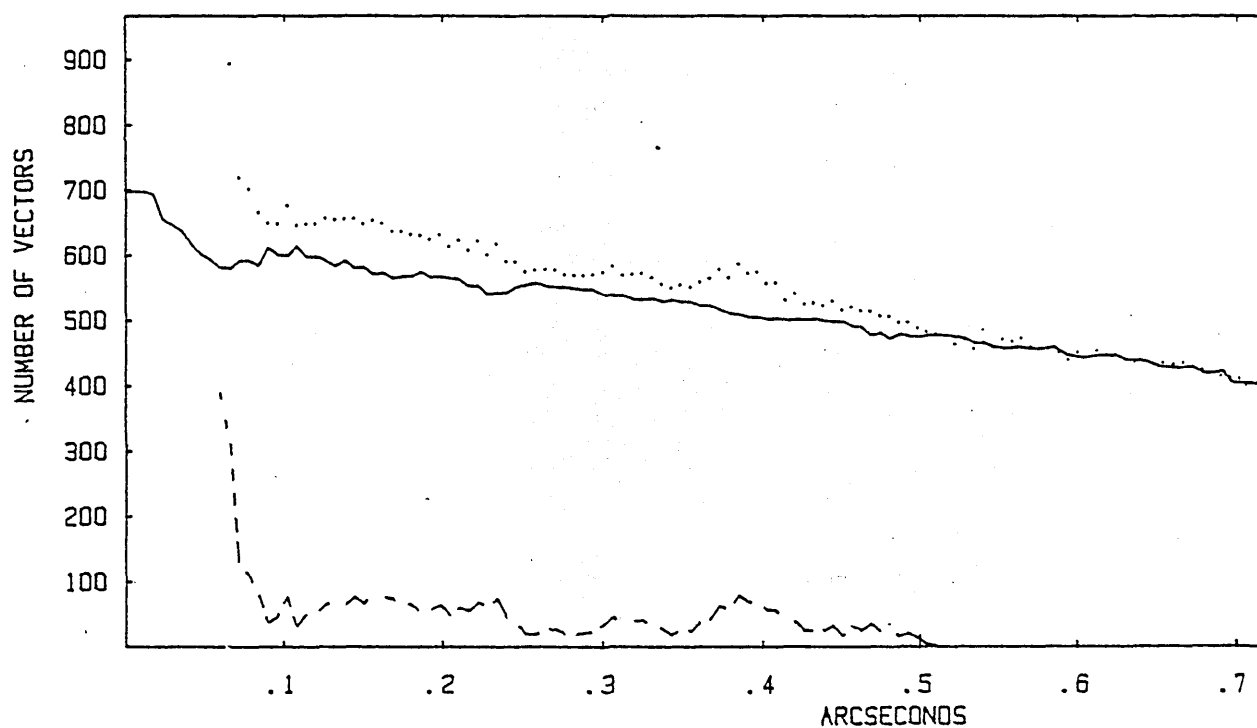


Figure 7.7 The Azimuthally Averaged Autocorrelation (.....), Cross-correlation (——) and Seeing Corrected Autocorrelation (---) of VY 2-2 in OIII.

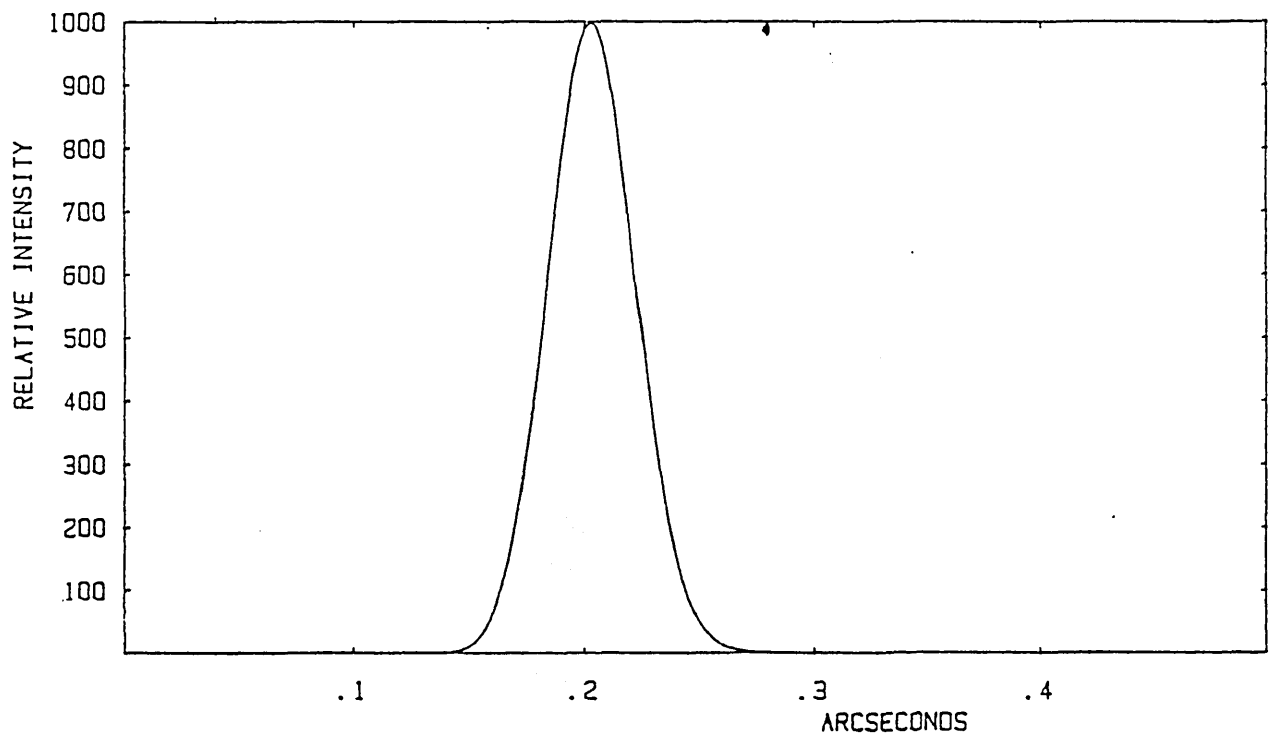


Figure 7.8 The Radial OIII Brightness Model of VY 2-2.

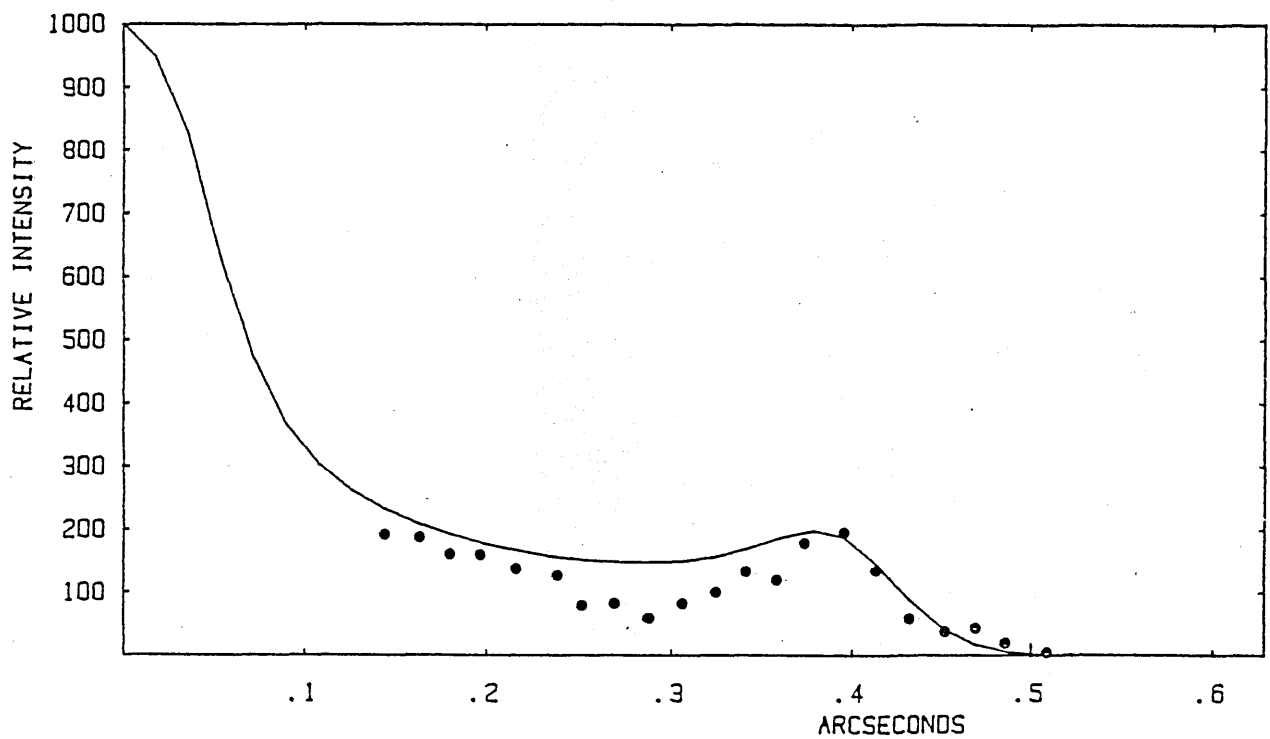


Figure 7.9 The Seeing Corrected Autocorrelation (• • •) of VY 2-2 in OIII Normalised to the Diffraction Limited Image of the Brightness Model's Autocorrelation (—).

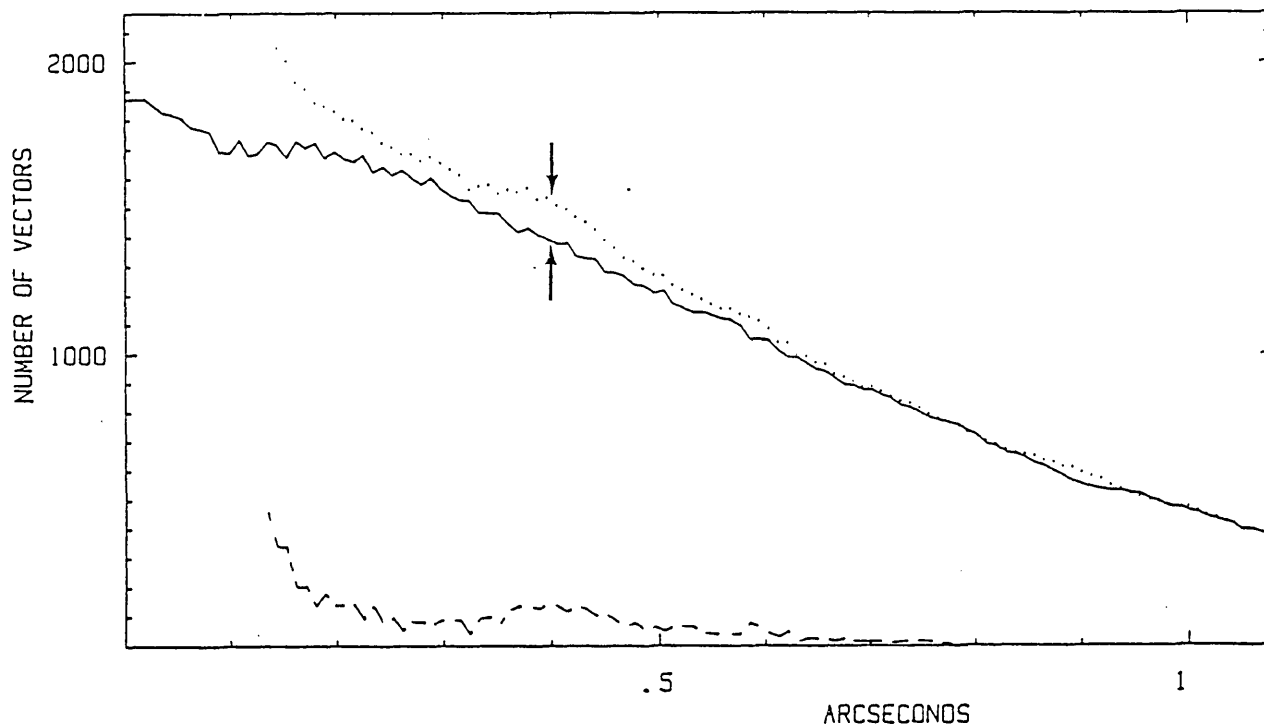


Figure 7.10 The Azimuthally Averaged Autocorrelation (.....), Cross-correlation (——) and Seeing Corrected Autocorrelation (----) of VY 2-2 in $H\beta$.

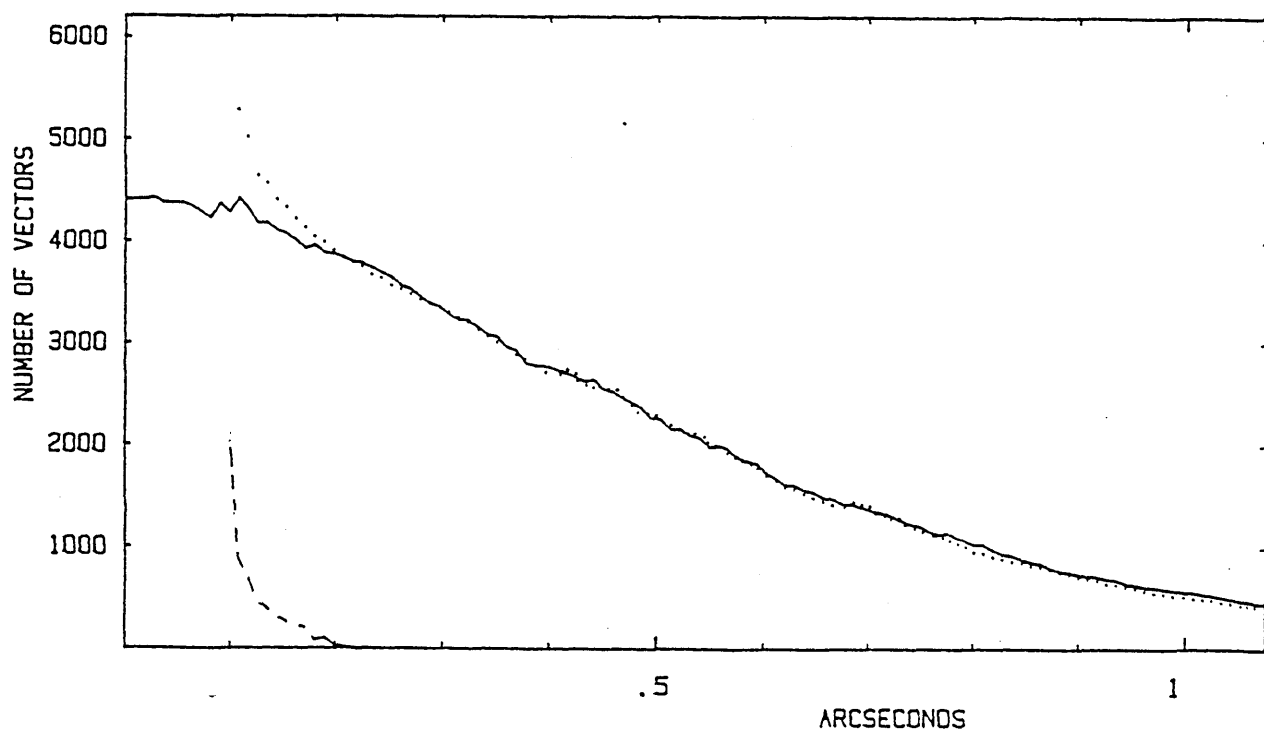


Figure 7.11 The Azimuthally Averaged Autocorrelation (.....), Cross-correlation (——) and Seeing Corrected Autocorrelation (----) of a Reference Star.

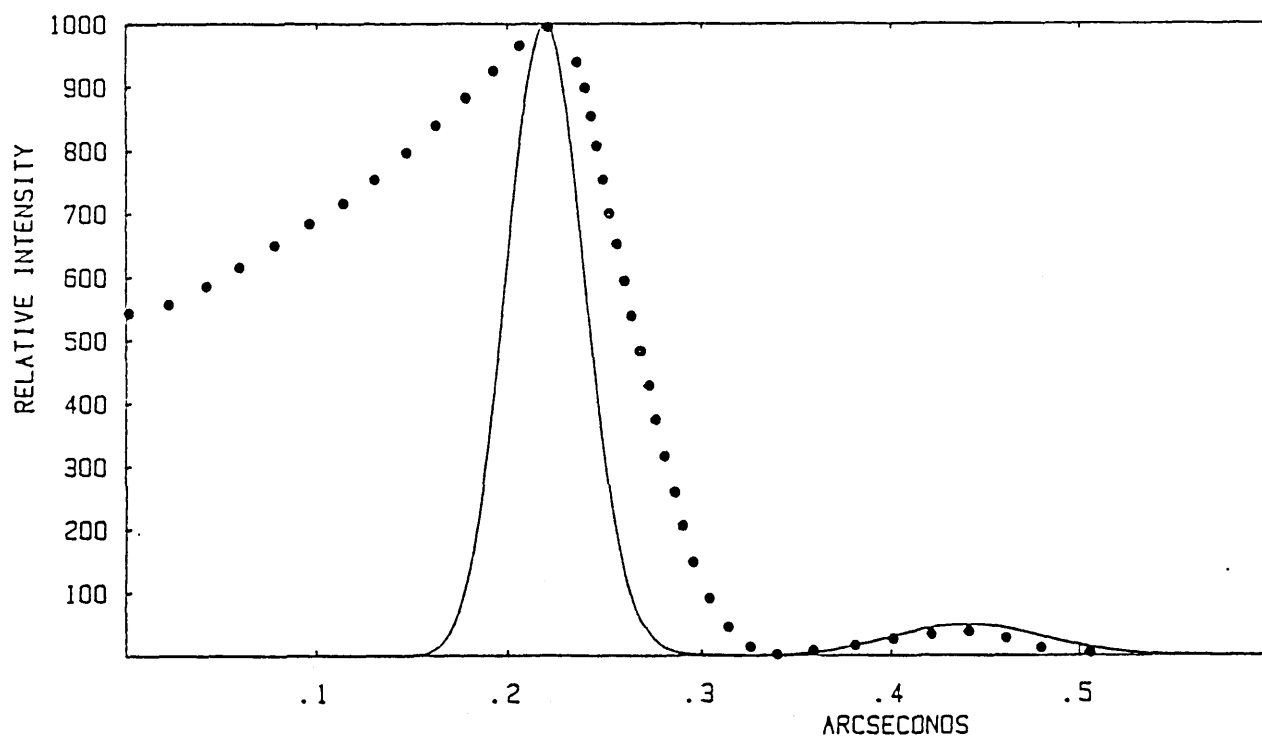


Figure 7.12 The Radial H β Brightness Model (—) of VY 2-2 and North South Profile Through Seaquist and Davis' 15 GHz. VLA Radio Map of VY 2-2 (• • •).

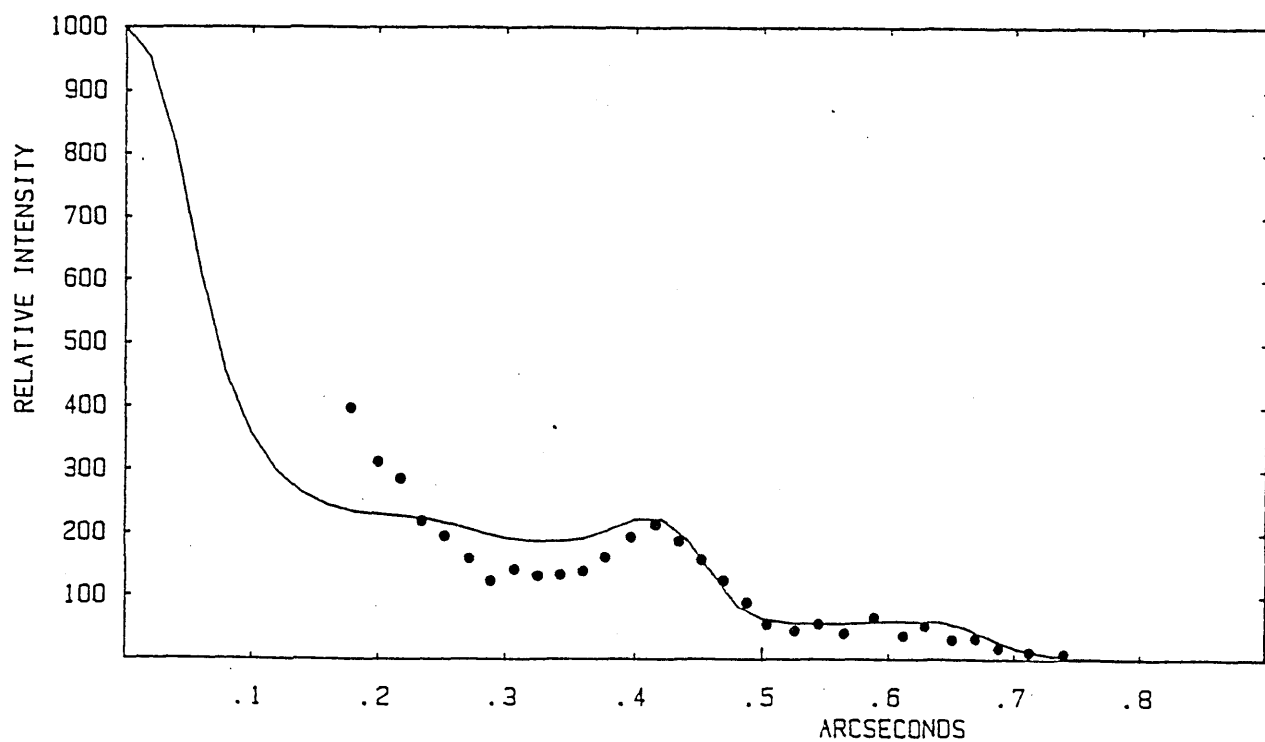


Figure 7.13 The Seeing Corrected Autocorrelation (• • •) of VY 2-2 in H β Normalised to the Diffraction Limited Image of the Brightness Model's Autocorrelation (—).

OIII and H β lines respectively. After subtraction of suitably normalised cross-correlations both seeing corrected autocorrelations show a feature at about 0.4 arcseconds. Figure 7.11 is the autocorrelation of a reference star, SAO 104799, and after subtraction of the cross-correlation the speckle component is consistent with that of a point source, falling to zero at 0.2 arcseconds. The brightness distributions constructed for VY 2-2 are shown in Figures 7.8 and 7.12. The diffraction limited images of the autocorrelations of these brightness distributions, normalised to the OIII and H β seeing corrected autocorrelations, are shown in Figures 7.9 and 7.13 respectively. The OIII distribution is a single azimuthally symmetric ring with a peak intensity at a radius of 0.205 arcseconds, while the H β data is best modelled by a ring which peaks at a radius of 0.22 arcseconds and a second ring of smaller amplitude at a radius of 0.43 arcseconds. The dotted curve in Figure 7.12 is a north-south profile through the 15 GHz. VLA radio map. There is good agreement between the position of the peaks in the model H β brightness distribution and those in the radio brightness distribution. However, the models derived from speckle observations do not contain the central infilling present in Sequist and Davis' result. A possible explanation for this is revealed in a study of deconvolution techniques by Cobb and McCarthy [95]. This shows that the CLEAN algorithm, used to restore VLA data, is not effective in resolving closely spaced delta functions as it produces significant infilling. However, it was found that the Maximum Entropy deconvolution algorithm is able to resolve such delta functions. Rodriguez [96] has used the Maximum Entropy algorithm to deconvolve recent VLA observations of VY 2-2. The resulting image is a ring of radius 0.2 arcseconds which is broken in three places and has no visible central infilling. There is also evidence of an extended gas cloud beyond the inner ring. This is in very good agreement with the H β brightness distribution derived from the speckle observations and confirms the validity of the data reduction methods. The signal to noise ratio of individual profiles through the autocorrelation of VY 2-2 is not sufficient to allow the detection of the breaks in the inner ring, but from the circular averages taken the dimensions of the planetary nebula have been determined.

7.6 The Planetary Nebula IC 4997.

IC 4997 is a young Galactic planetary nebula. It is of interest because it is at the beginning of the planetary nebula evolutionary sequence and its emission line intensities are variable [97]. The OIII/H γ ratio steadily declined from 1895 to 1962 [98]. A reversal of this trend began in 1962 and has continued to the present. The presence of the HeII emission line suggests a central star temperature of at least 65000 K. Electron densities have been given by Flower et al. [99] and updated by Feibelman [100]. An accurate determination of its angular diameter will allow an estimate of its linear diameter to be made and combining this result with the spectrophotometric observations allows the characteristics of the nebula and its central star to be derived.

Observations of IC 4997 were made on 13th. October 1984 using the AAT. A 75 mm. objective was used, the interferometer was placed at the f/15 cassegrain focus and the previously described H β filter was employed. Autocorrelation and cross-correlation of a 10 minute photon-counting observation produced the azimuthally averaged profiles shown in Figure 7.14. The seeing corrected autocorrelation is consistent with that of an unresolved source, falling to zero at 0.2 arcseconds. The seeing corrected autocorrelation of a reference star is shown in Figure 7.15 and this too falls to zero at 0.2 arcseconds. However, there is another possible model for the brightness distribution of the planetary nebula; a disc of uniform intensity with a diameter greater than the autocorrelator's field of view. In this case the speckle component occupies the whole autocorrelation field. Therefore, the point at which the speckle component departs from the seeing pedestal is not known and the Welter and Worden seeing correction cannot be correctly implemented. If the cross-correlation is normalised to the autocorrelation at low spatial frequencies no extension will be apparent in the seeing corrected autocorrelation. In fact the transfer function of a photon detector can be determined from observations of uniform low intensity light.

A 75 mm. microscope objective was used to observe IC 4997 and this gives an autocorrelation field of 1.1 arcseconds. Therefore, the two

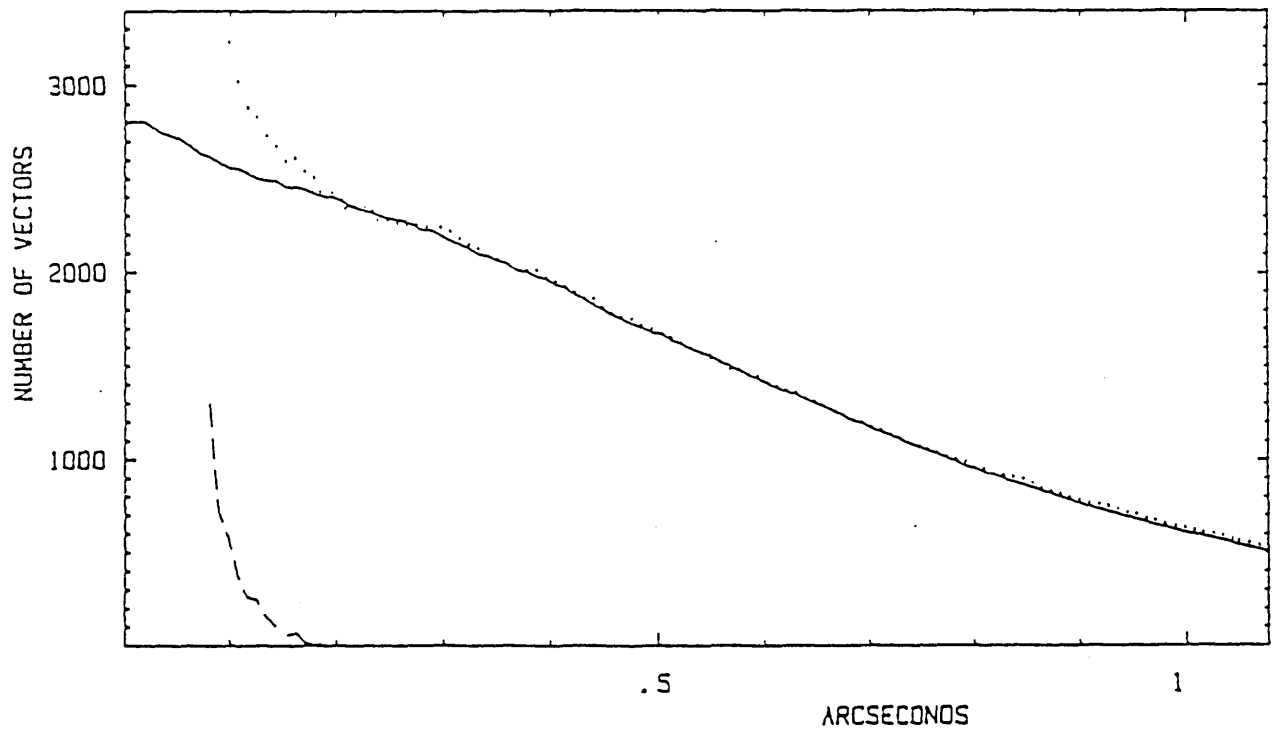


Figure 7.14 The Azimuthally Averaged Autocorrelation (.....), Cross-correlation (——) and Seeing Corrected Autocorrelation (---) of IC 4997.

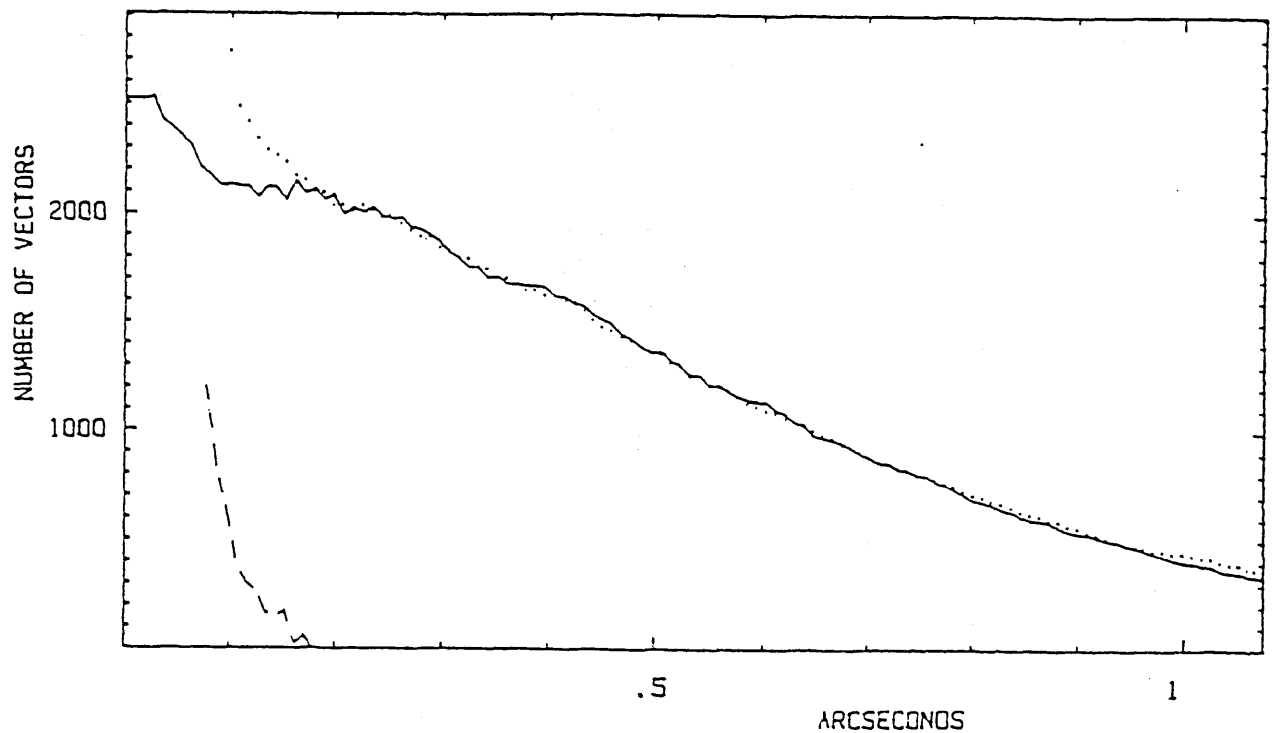


Figure 7.15 The Azimuthally Averaged Autocorrelation (.....), Cross-correlation (——) and Seeing Corrected Autocorrelation (---) of a Reference Star.

possible models for the planetary nebula are either an unresolved source or a disc of diameter greater than 1.1 arcseconds. In a survey of stellar planetary uniform nebulae [101], Kohoutek and Martin give the angular diameter of IC 4997 as 1.6 ± 0.4 arcseconds. This was determined from observations with a photoelectric area scanner on the 1.2 m. telescope at the Hamburg Observatory. This result supports the uniform disc model of IC 4997. An accurate determination of the angular diameter by speckle interferometry requires a larger autocorrelation field. To achieve this the focal length of the objective used can be increased or the observations made on a telescope with a larger plate scale.

7.7 The Bipolar Nebula NGC 2346.

NGC 2346 is a bipolar nebula with a central cool A5V star [102], but the surrounding nebula exhibits the typical high excitation emission lines of a planetary nebula [103]. This suggests the presence of a hot companion star, indeed, Mendez [104] has confirmed this binary nature by observing a 16 day period in the radial velocity of the stellar lines. From photometric observations Kohoutek [105] concludes that the stellar system is a close binary, but that the conditions for the eclipses are rapidly changing. However, the problem of how a small blue star could eclipse a larger A5V star still remained. Mendez et al. [106] has made further photometric observations which confirm the behaviour of the light curves and he suggests that this is consistent with an eclipse of the central binary system by a dust cloud of varying opacity.

To investigate the structure of the central source in NGC 2346 speckle interferometric observations were made on 11th. November 1981 using the AAT. An interference filter with a 55 nm. bandpass, centred on 500 nm., isolated a region of the spectrum dominated by the continuous light from the A5V star. Preliminary results have already been published [107] and the model derived for the central region of NGC 2346 has a central, bright, unresolved source surrounded by a uniform disc of light 0.54 arcseconds in diameter. The original speckle data has now been re-analysed with the autocorrelator and the

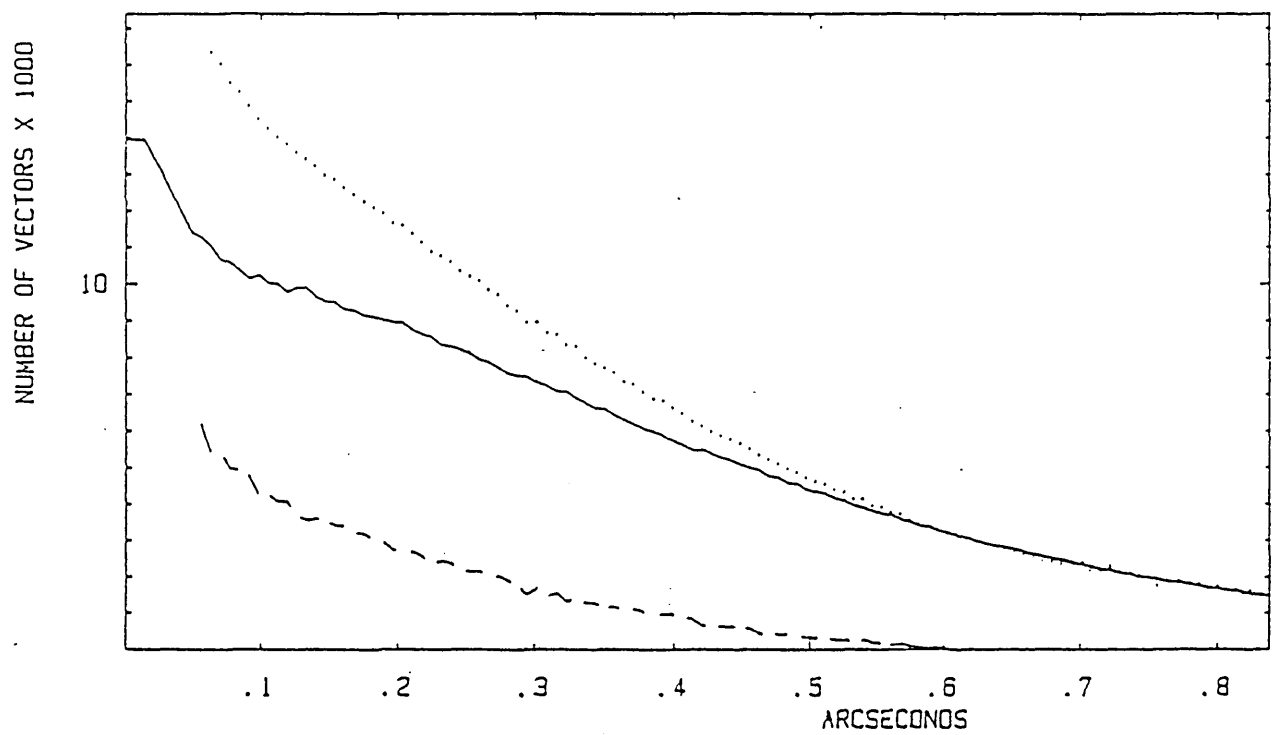


Figure 7.16 The Azimuthally Averaged Autocorrelation (.....), Cross-correlation (——) and Seeing Corrected Autocorrelation (----) of NGC 2346.

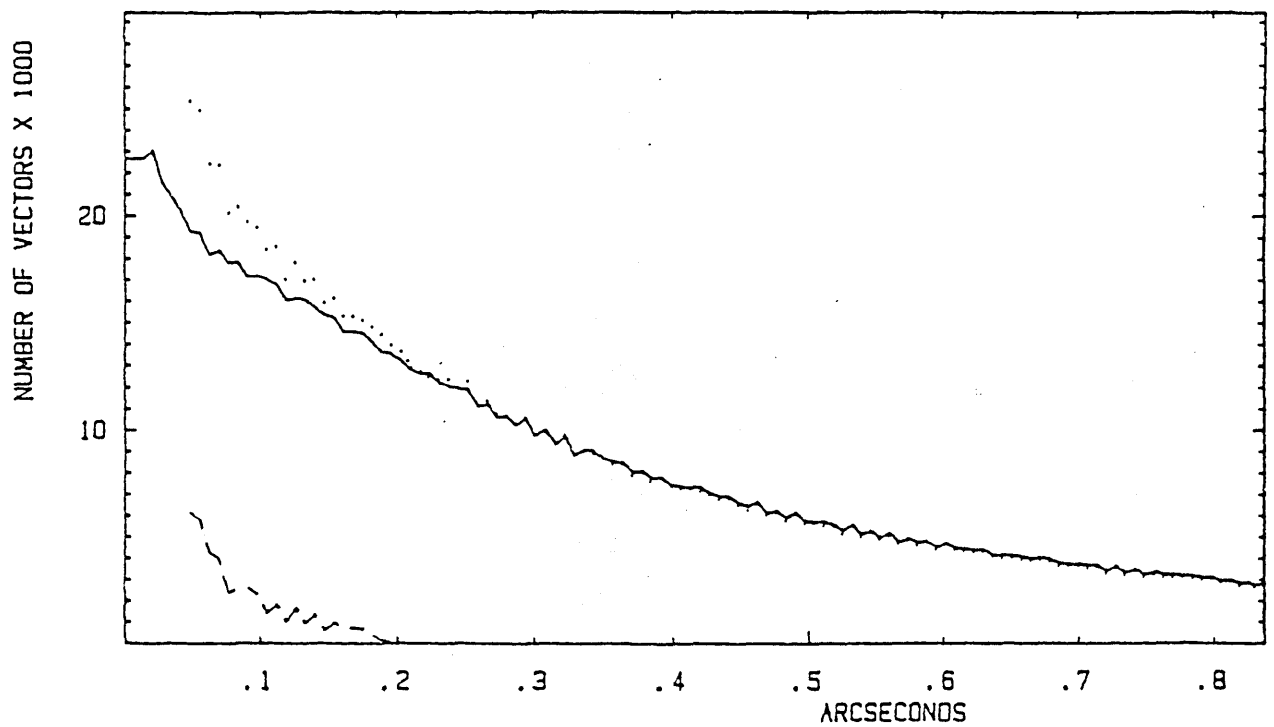


Figure 7.17 The Azimuthally Averaged Autocorrelation (.....), Cross-correlation (——) and Seeing Corrected Autocorrelation (----) of a Reference Star.

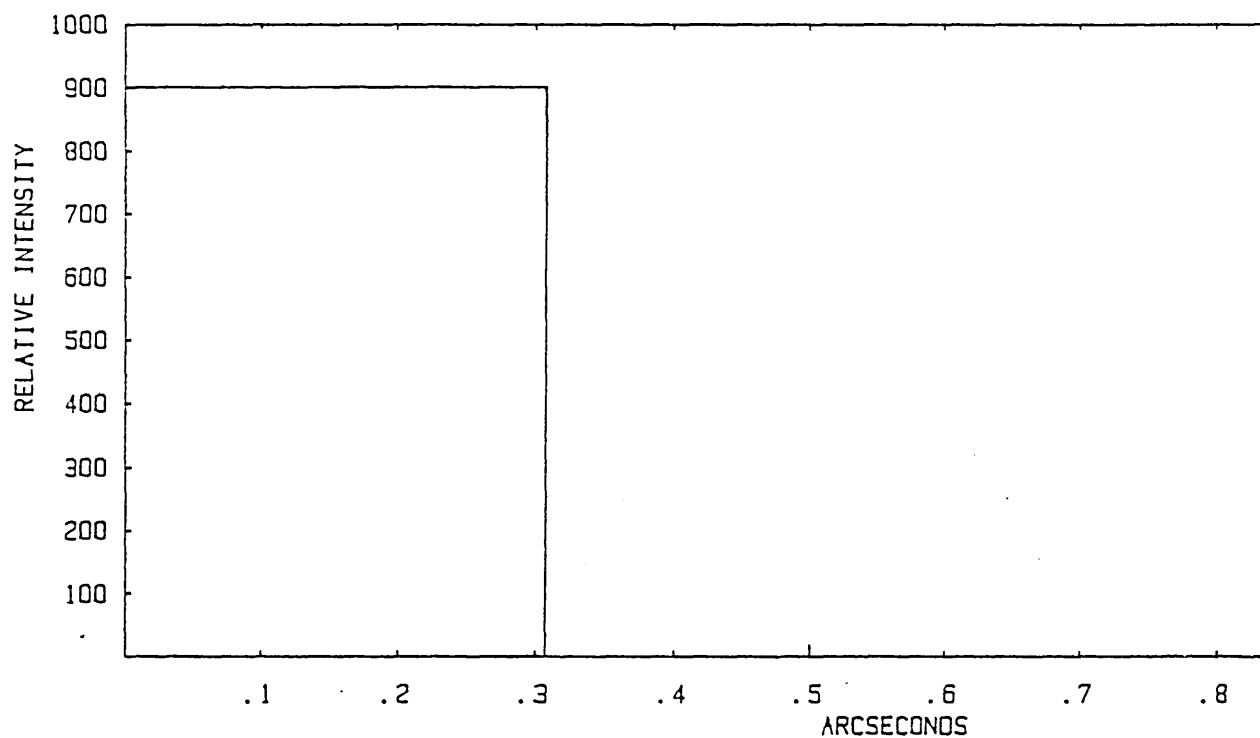


Figure 7.18 The Radial Brightness Model of NGC 2346.

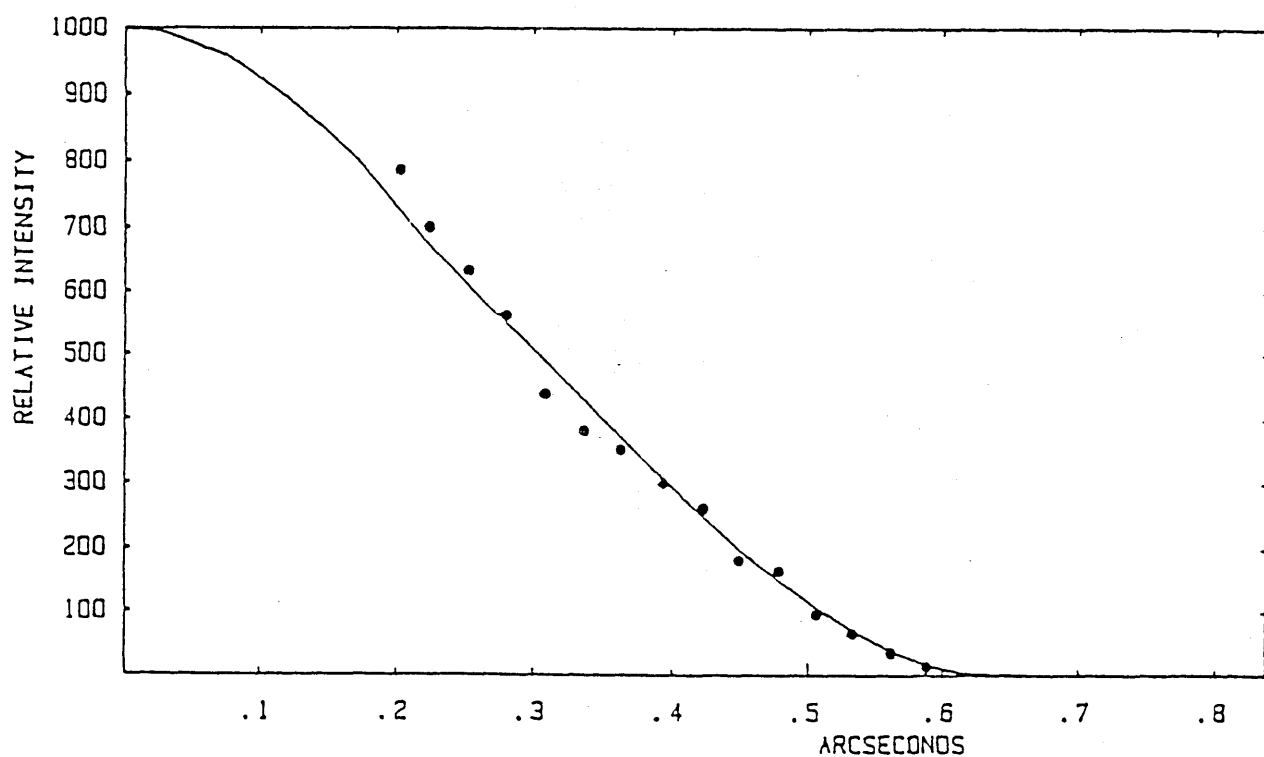


Figure 7.19 The Seeing Corrected Autocorrelation (• • •) of NGC 2346 Normalised to the Diffraction Limited Image of the Brightness Model's Autocorrelation (—).

cross-correlator to improve the accuracy of the seeing pedestal subtraction.

The azimuthally averaged autocorrelation and cross-correlation of NGC 2346 are shown in Figure 7.16 together with the seeing corrected autocorrelation. Figure 7.17 shows the same functions for a reference star. The speckle component present in Figure 7.16 extends out to 0.5 arcseconds and comparisons have been made between this profile and several model speckle components. The estimated brightness distribution shown in Figure 7.18 is a uniform disc 0.62 arcseconds in diameter. The speckle component calculated from this model is shown normalised to the seeing corrected autocorrelation in Figure 7.19. However, as discussed in Section 6.3 the derived brightness model should be deconvolved with the impulse response of the speckle system. Such a deconvolution will reduce the derived diameter by approximately the width of the impulse response. This impulse response has a diameter of four pixels, or 0.056 arcseconds. Therefore, the final estimate for the diameter of the central region of NGC 2346 is 0.564 ± 0.028 arcseconds.

The extended disc of light proposed is probably produced by the light of the central A5V star scattered by dust. Pottasch [108] gives the distance to NGC 2346 to be 640 to 900 pc. If adopt a value of 800 pc. then the diameter of the disc of dust will be approximately 451 AU. This is very much greater than the disc diameter of $60 R_{\odot}$ predicted by Mendez et al. [106] from occultation observations. However, such smaller dusty condensations could be contained within a larger dust cloud. The autocorrelation of NGC 2346 contains no evidence of the central binary, but if the binary separation is 0.03 AU (as suggested by Kohoutek [109]) at a distance of 800 pc. the orbit would subtend only 3×10^{-5} arcseconds. This separation is 1000 times smaller than the diffraction limit of the AAT. A disc of dust 0.564 arcseconds in diameter is consistent with the predictions of Mendez [110] that a rotating dust cloud of diameter less than 2 arcseconds must be present to explain the variable reddening of the central star.

Chapter Eight.

CHAPTER EIGHT - CONCLUSIONS AND FUTURE WORK.

8.1 Improvements to the Hardware.

Since the original design of the Imperial College Speckle Interferometer there have been several developments in the technology associated with photon counting systems. The high gain EMI image intensifiers used require a bulky solenoid for magnetic focussing and are no longer readily available. Alternative image intensifiers, which may be used in the future, contain microchannel plates (MCPs). Such intensifiers are compact, require no magnetic focussing, can be driven into a saturated state which has a peaked pulse height distribution and allow far more flexibility in the design of the speckle interferometer's optical layout. A high gain dual MCP intensifier with an active area of 40 mm. is currently being developed by the Photon Counting Group at Imperial College, the Royal Greenwich Observatory and Instrument Technology Limited. An 18 mm. diameter MCP intensifier is used in McAlister's speckle camera [48]. This instrument also employs microprocessor control of the dispersion correction prisms, the filter wheel and the video recorder used to record speckle images. The remote control achieved allows the prism settings to be continually adjusted to give the optimum dispersion correction.

As previously described in Chapter Three a novel photon counting camera has been developed by Papaliolios et al. [49]. This uses an MCP intensifier, but eliminates the constraints imposed by the recording of conventional television images. The device provides a time ordered list of photon positions at count rates up to 200 kHz. with 512 x 512 spatial resolution. The binary coded data is a convenient format for computer processing and the data rate is set by the number of photons detected, not the number of resolvable pixels. Additionally the arrival time of each photon can be specified to a precision of less than a microsecond.

The Sony reel to reel video recorders used in the Imperial College speckle system provide recordings of good quality, however, their performance has now been equalled, or superseded, by more modern

"U-matic" recorders. Such recorders use a convenient cassette format and have editing facilities that would be useful in removing damaged television fields from observational records.

The use of the cross-correlator has greatly enhanced the confidence with which we are able to correct for the effects of atmospheric seeing present in autocorrelations. However, there are several improvements that could be made to the cross-correlator. Each image should be centred before processing to remove any distortion from the resulting cross-correlation, and the ability to cross-correlate every pair of fields separated by a given interval would be very desirable. An interim solution to the second problem would involve the numbering of each field so that the cross-correlator could identify the fields processed. Then each time the record is cross-correlated a different set of fields could be analysed until every field were processed.

When autocorrelating a record it is important to not to sample any spurious signals. The various filters described in Chapter Four considerably reduce the effects of this noise and by examining the video signal on an oscilloscope it is possible to crudely judge the sampling level. We now propose to include in the system a real-time television display of the positions at which the autocorrelator detects a photon event. By comparing these television images with those analysed by the autocorrelator it will be possible to determine the level of sampled noise.

With the advent of new computer architectures it is now possible to envisage flexible, cost effective systems that will process speckle images in real-time. These new architectures use several individual processors in parallel. Future systems could contain a combination of dedicated signal processing elements (to perform correlations and Fourier transforms) and processors that could be programmed to implement various image reconstruction algorithms. Many computer manufacturers now market parallel processing systems and the application of this technology to the reduction of speckle data would be a significant advance.

8.2 Improvements to the Processing of Data.

The advantage of real-time operation offered by hardwired data processing is offset by the lack of flexibility inherent in such an approach to the analysis of speckle images. To allow sophisticated manipulations of the correlations produced by the autocorrelator and cross-correlator it is proposed to set up a data link to a VAX 11/750. The speed, memory capacity and software available on the VAX will allow us to perform data analysis in both the spatial and frequency domains and investigate photon spike removal and various image reconstruction algorithms (see Section 8.3).

It was mentioned in Chapter Five that photon events generally cover more than one pixel. This gives rise to an anomalous "photon spike" in the region of the origin. This photon spike limits the angular resolution of the system and makes the estimation of binary star magnitude differences impossible, as this requires the ratio of the secondary peak height to the height of the autocorrelation at the origin to be known. We intend to investigate methods of photon spike compensation.

In frequency space the removal of the photon spike corresponds to correcting for the detector transfer function and then subtracting the resulting noise bias [73]. It can be shown [111] that for an ideal detector (one that detects each photon event as a delta function) the measured ensemble average energy spectrum is given by:

$$\langle |i(u,v)|^2 \rangle_m = n^2 \cdot \langle |i(u,v)|^2 \rangle + n \quad \text{Eq. 8.1}$$

where n = average number of photons per frame,
 $|i(u,v)|^2$ = energy spectrum of the classical intensity,
normalised to unity at zero angular frequency.

It follows that the energy spectrum of the photon limited data is a biased estimate of the energy spectrum of the classical image. In this case the autocorrelation would contain a delta function at the origin with a height proportional to n . In the more realistic case of a detector with a transfer function, $|g(u,v)|^2$, not equal to unity

equation 8.1 becomes:

$$\langle |i(u,v)|^2 \rangle_m = [n^2 \cdot \langle |i(u,v)|^2 \rangle + n] \cdot |g(u,v)|^2 \quad \text{Eq. 8.2}$$

The detector transfer function can be determined in two ways. It can be directly measured, or it can be modelled. For a direct calibration, the detector is illuminated with uniform low intensity light. For these uncorrelated events equation 8.2 becomes:

$$\langle |i(u,v)|^2 \rangle_m = [n^2 + n] \cdot |g(u,v)|^2 \quad \text{Eq. 8.3}$$

The modelling approach requires a two dimensional least-squares fit to be made to the average energy spectrum at spatial frequencies above the telescope cut-off. The ensemble average energy spectrum is then divided at all frequencies by the derived detector transfer function. This operation makes the noise bias term constant over all spatial frequencies. After subtraction of the constant noise bias the detector transfer function and noise bias compensation are complete.

In the autocorrelation domain compensation for the detector transfer function causes the photon spike to collapse to a delta function at the origin. This delta function is removed by subtraction of the noise bias.

8.3 Solving the Phase Problem.

The energy spectrum, or equivalently the autocorrelation, of an object contains no obvious information about the phase of object's Fourier transform. Therefore, it is generally impossible to calculate the object intensity from it's energy spectrum alone and the reconstruction of the object intensity is often referred to as the "phase problem". The solution of the two dimensional phase problem is the subject of much current research and reviews are given by Dainty [15] and by Bates [112].

There are three basic approaches to the solution of the phase problem as it is encountered in speckle interferometry. The first is

the most simple and assumes some knowledge of the object. For example, if the object has a symmetric intensity its Fourier transform is purely real and from continuity arguments it can be deduced from the autocorrelation. This is the method used to deduce the object intensities of the planetary nebulae presented in this thesis. In the second approach it is assumed that the two dimensional phase problem "almost always" has a unique solution. If this is the case the object intensity can then be recovered from modulus information alone. There is much research into the uniqueness of solutions to the phase problem. Napier and Bates [113] have shown that a unique solution is more likely in the two dimensional case than in one dimension. However, the convergence of the various modulus only reconstruction algorithms is the subject of much debate. The third approach extracts additional information from speckle images to unambiguously reconstruct the object intensity. The following paragraphs discuss various image reconstruction algorithms.

Speckle holography is a reconstruction method that relies on the presence of a reference star in the same isoplanatic patch as the object under study. Then the autocorrelation of the object and the reference star contains a term equal to the object intensity. Weigelt [114] has applied this method to reconstruct multiple star systems, but the need for a reference star close to the object severely limits the application of speckle holography.

Fienup [115,116] has suggested various iterative algorithms for computing the object intensity from the modulus of its Fourier transform. The algorithms only differ in the way in which the next input estimate is derived from the preceding output estimate. An initial estimate of the object intensity is made and its Fourier transform calculated. Then the modulus of this transform is replaced by the known modulus and the result is inverse Fourier transformed to produce a new estimate of the object intensity. The modulus of this estimate is set to zero where it is known to be zero and where negative values occur. This output estimate provides the new input estimate for the next iteration. This algorithm can reconstruct simple non-symmetric objects, but it fails to converge to the correct solution for complicated objects [115]. Bates [117] has suggested a

phase closure algorithm that involves calculating the phase of each point in the Fourier transform of an object and setting the phase at the origin to zero. This can be used to find the initial object estimate for a Fienup solution or an improved version may be valuable on its own.

The ambiguity of the phase problem can be eliminated in a number of ways. Walker [118] suggests a method in which the speckle data is multiplied by a two dimensional exponential distribution. It can be shown that the energy spectra of the original speckle data and the exponentially modified speckle data contain sufficient information to give a unique solution to the phase problem. It then remains to find an iterative algorithm to uniquely reconstruct the object intensity. Walker [119] uses a variant of the Fienup algorithm which constrains the solution to be compatible with the energy spectra of the object and the exponentially modified object. There is only one solution that satisfies both sets of Fourier constraints and therefore a unique solution is guaranteed.

An intuitively simple approach to image reconstruction has been developed by Lynds, Harvey and Worden [120]. It supposes that each speckle is a distorted image of the object. To produce an "image" a few bright speckles are chosen from each frame and superimposed. Bates and Cady [121] have extended this approach and developed the "shift and add" technique. This involves shifting each image so that the brightest speckle is at the origin and then adding all such images. The process is performed for the object under study and a reference star. An image of the object is deconvolved using the reference star image and a standard deconvolution algorithm such as CLEAN. Recently Davey et al. [122] and Christou et al. [123] have developed further extensions to shift and add called "zero and add" and "weighted shift and add" respectively. Zero and add relies on the fact that speckle images may be characterised by maps of their zeros in the complex Fourier or z plane. The zeros are isolated when the zero maps of many speckle images are superimposed and the image is reconstructed by taking the inverse z transform. Computer simulations suggest that this imaging technique is insensitive to noise contamination. In weighted shift and add all the local maxima in a speckle image are

characterised by a weighted impulse function. The average cross-spectrum of the speckle image with the weighted impulse distribution is formed and then deconvolved by a division by the energy spectrum of the weighted impulse distribution. This deconvolution method avoids the problems caused by the zeros present in the energy spectrum of a reference star.

The Knox Thompson reconstruction algorithm [124] involves finding the average autocorrelation of the Fourier transform of speckle images. As the phase of the average autocorrelation of the transfer function's Fourier transform is approximately zero [15] it is possible to compute the phase differences between adjacent points in the spatial frequency domain. The phase of a point in the Fourier transform of the object intensity is then given by the sum of the phase differences from the origin to the point. Hence the modulus and phase can be estimated at each sampled point and the object reconstructed. Nisenson and Papaliolios [125] have investigated the performance of the Knox Thompson algorithm and it is found that it is sensitive to the presence of noise and to movement of the image within the field of view. Photon noise introduces a frequency dependent bias which must be removed to allow successful reconstruction.

The final reconstruction algorithm discussed here is the "speckle masking" technique developed by Weigelt and Wirtitzer [126,127]. Rather than calculating an autocorrelation (a correlation of order two) the method requires a third order triple-correlation to be found. The average triple-correlation of a set of speckle images is given by:

$$\langle T(\mathbf{x} + \mathbf{x}') \rangle = \langle [I(\mathbf{x}) \cdot I(\mathbf{x} + \mathbf{x}')] * I(\mathbf{x}) \rangle. \quad \text{Eq. 8.8}$$

Each speckle image is multiplied by a version of itself shifted by a vector $\mathbf{x}'$. The shift vector is chosen so that the resulting product mask contains a set of delta functions. For photon limited data a suitable product mask is produced for the separations between each photon event and every other photon event in the image. The product mask is then cross-correlated with the original speckle image to produce the four dimensional triple-correlation. The average

triple-correlation is Fourier transformed to produce the average bispectrum and after compensation for the speckle masking transfer function, the phase of the object's Fourier transform is calculated with a recursive algorithm. The phase is combined with the known modulus and the result inverse Fourier transformed to yield the diffraction limited object intensity. Such a triple-correlation method has several advantages over other reconstruction algorithms: no assumptions are made about the uniqueness of the solution; it can be readily applied to photon limited data [128], unlike shift and add methods which require bright speckles [121]; the bispectrum is highly redundant and therefore the phases can be calculated with a good signal to noise ratio [129]; and the triple-correlation is insensitive to noise [130]. All these advantages must be weighed against the large amount of data processing and memory required. A recent study of the requirements for a triple-correlator has been undertaken by Matcher at Imperial College [131]. He concludes that for a maximum vector shift of 64 pixels the triple-correlation requires an array with $(2 \times 64)^2$ elements. Assuming 2 bytes per element (this allows 65535 identical bivectors to be counted) a complete triple-correlation would occupy 512 Mbytes. The redundancy present in the bispectrum may allow this figure to be reduced. However, with the introduction of array processors and the falling cost of large quantities of memory the general application of this algorithm may soon be possible.

As yet image restoration techniques have only been applied to the simplest of astronomical objects. To turn diffraction limited imaging into a technique that is generally applicable the performances of the various algorithms available must be rigorously investigated and compared.

8.4 Observing Programmes.

There are several observing programmes being undertaken by the Speckle Interferometry Group at Imperial College. The observations of Magellanic Cloud planetary nebulae have been extended to a long term programme on the Anglo-Australian Telescope; 8 nights for speckle interferometry and 6 nights for spectroscopy. This is a collaborative programme with Dr. Mike Barlow and Dr. Robin Clegg of University

College London. Further observations of Hipparcos link stars may also be carried out in the near future.

A study of Northern Hemisphere novae and nova-like systems has been undertaken in conjunction with Dr. Mike Bode of Lancashire Polytechnic. Recent VLA and VLBI studies of such objects have shown the presence of high velocity jets and structure on scales of a few tenths of an arcsecond. Optical speckle observations were made in September 1986 using the 2.5 m. Isaac Newton Telescope and the results of these observations will be compared with existing radio data. The value of comparing optical and radio data has been demonstrated by the study of the galactic planetary nebula VY 2-2 presented in Chapter Seven.

8.5 Conclusions.

The analysis of speckle interferometry data is a statistical process which requires the reduction of many individual speckle images. Therefore, a highly efficient approach to data processing is required. The approach described in this work involves the application of real-time, hardwired correlators and it has produced many astronomical results (see Chapter Seven). However, with the increased computational speed offered by modern computers and array processors, a software based real-time speckle data reduction system can now be seriously considered.

To date most published stellar speckle interferometry results have been obtained from the analysis of autocorrelations or energy spectra. With the post processing of speckle images the technique's potential for diffraction limited imaging may soon be fully realised. When this is achieved the imaging resolution of optical telescopes will better that of dilute aperture radio telescopes, such as the VLA, and allow the direct comparison of high resolution radio and optical images.

Speckle interferometry implemented on a single telescope is an attractive technique because of its ability to yield diffraction limited information, its high magnitude limit and its good coverage of the Fourier plane. However, the diameter of a single mirror is

limited and multiple aperture instruments such as the National New Technology Telescope and the Very Large Telescope have been recently proposed as the next step in the evolution of the optical telescope. The feasibility of these designs has been proven by the construction of the MMT. Such telescope arrays provide large effective apertures and hence the potential for very high angular resolution. Hebden [18] has already used the MMT as a phased array and applied speckle techniques to overcome any aberrations and provide diffraction limited images.

Speckle interferometry is not limited to overcoming only atmospheric distortions. Weigelt [132] has proposed an Imaging Speckle Interferometer in Space (ISIS). The attractions of ISIS are its limiting magnitude of +24 and high resolution, 0.001 arcseconds for a 20 m. baseline. This multiple mirror instrument will produce speckle images because of deformations in the structure and other aberrations. An image reconstruction technique such as speckle masking could be applied to obtain diffraction limited images. However, the resolution of ISIS can be achieved and bettered by ground based telescope arrays (probably at considerably less cost), and the limiting magnitude of proposed terrestrial instruments, such as ESO's Very Large Telescope, is calculated to be +20 [133]. Therefore, either in space or on the ground speckle interferometry will continue to be a useful astronomical technique, and it will, no doubt, be applied whenever imaging through a turbulent medium is required.

Publications.

PUBLICATIONS.

Meaburn. J., Walsh. R., Morgan. B.L., Hebden. J.C., Vine. H., and Standley. C., Speckle observations of the central source in the bipolar nebula NGC 2346, Mon. Not. R. astr. Soc., 243, 35p., 1985.

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Speckle observations of the central source in the bipolar nebula NGC 2346

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Summary. The core of the bipolar nebula NGC 2346 has been observed with the Imperial College Speckle Interferometer combined with the 3.9-m Anglo-Australian telescope. A disc of light 0.54 arcsec in diameter surrounds an inner, bright, unresolved source. It is suggested that this extended region may be light from the central A5V star scattered off a cloud of dust ≈ 420 AU in diameter.

1 Introduction

NGC 2346 is a distinctly bipolar nebula which has a dominant cool A5V central star (Mendez 1978) but the radiatively ionized nebulosity exhibits a fairly high excitation emission-line spectrum (Fiebelman & Aller 1983). The presence of a hot companion is therefore suggested and the binary nature is confirmed by the 16-day period in the radial velocities of stellar lines which has been observed by Mendez & Niemela (1981). First evidence of an eclipse of the A5V star was found on 1981 November 29 by Kohoutek (1982) after which several deep, broad, minima were observed. (Kohoutek 1983) with a 17.2 ± 0.4 day period. Kohoutek concluded that the central stellar system is a semi-detached close binary having a semi-major axis of the relative orbit of ≈ 0.03 AU, and that the conditions for eclipses are changing rapidly. The problem of how a small blue star could eclipse a much larger A5V star remained unanswered. Mendez, Gathier & Niemela (1982) have confirmed this extraordinary behaviour in the light curves but suggest that these results are consistent with an eclipse of the binary system by a rotating dust cloud of varying opacity. The colour of the A5V star changes during an eclipse and there is possibly an anomalous value of the ratio of total to selective extinction (Kohoutek 1983).

Previously Wallerstein (1978) had suggested that the apparently binary companion of the M supergiant, VY Cma AB, could in fact be a consequence of a 'holey' rotating dust cloud. This was also one of the possibilities considered for the binary component of the central star in the

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bipolar Red Rectangle (Meaburn *et al.* 1983). From their observations of variable reddening Mendez *et al.* (1985) have even more recently suggested that an inhomogeneous dust cloud of ≤ 2 arcsec diameter exists around the central binary system of NGC 2346.

The aim of the present work was to detect any structure that may exist in the central source of NGC 2346 with angular dimensions between 0.03 and 0.9 arcsec.

2 Observations and results

For this purpose the speckle interferometer of Imperial College of Science and Technology was combined with the 3.9-m Anglo-Australian Telescope on 1981 November 11. The results were then analysed with the hard-wired autocorrelator. The technical details of these observations are exactly the same as for previous work on the Red Rectangle (Meaburn *et al.* 1983) and will not be repeated here.

An interference filter of 550 Å bandwidth, centred on 5000 Å, isolated a region of the spectrum. From a low-dispersion spectrum of the central star the emission-line contribution within this 550 Å bandwidth is only ≈ 8 per cent of the total flux in a 4×1.8 arcsec² aperture centred on the star. Therefore the speckle interferogram is expected to be dominated by the continuous light from the central A5V star.

Light was accepted in a 3-arcsec-square aperture, containing the central object, and the two-dimensional autocorrelations reveal any image structure with separations between 0.03 and 0.9 arcsec. No information is obtained for separations outside this range.

No secondary maximum in the autocorrelations which could have been interpreted as a binary component is detected at any position angle within this range of separation. However when the autocorrelations at all position angles are added together some interesting structure is revealed. This may be seen by comparing Fig. 1(a), which shows the integrated autocorrelation of the central source of NGC 2346, with Fig. 1(b) which shows the integrated autocorrelation of the reference star. First, it should be appreciated that the integrated autocorrelation of the reference star has all the characteristics of an unresolved source; the wing of the central photon spike merges with the Gaussian seeing pedestal at a separation of around 0.16 arcsec. In Fig. 1(a) the corresponding curve for NGC 2346 deviates significantly from this shape and shows a distinct 'kink' for a separation of 0.54 ± 0.02 arcsec which makes it impossible to fit a single convincing Gaussian to the seeing pedestal.

Model autocorrelations of a variety of sources, each with different structure can reproduce this behaviour. However, the simplest and most likely interpretation is that a central, bright, unresolved (< 0.03 arcsec) source is surrounded by a reasonably uniform disc of light ≈ 0.54 arcsec in diameter.

3 Conclusions

A disc of continuous light at a wavelength of 5000 Å could have two origins. It is possible that it could be a consequence of the atomic recombination continuum from an ionized gas. However this seems unlikely since Walsh (1983) did not find a corresponding intense unresolved central peak to the H α distribution as was seen in the comparable bipolar Mz-3 by Lopez & Meaburn (1983). More likely this extended disc is produced predominantly by the light of the central A5V star scattered by dust.

Pottasch (1983) gives a distance of between 640–900 pc to NGC 2346. Here we will adopt 800 pc in which case this dust disc will be ≈ 432 AU diameter for an angular diameter of 0.54 arcsec. This is massively larger than the $60 R_{\odot}$ predicted by Mendez *et al.* (1982) for the diameter of the dust concentration which would cause the occultations they observe. However

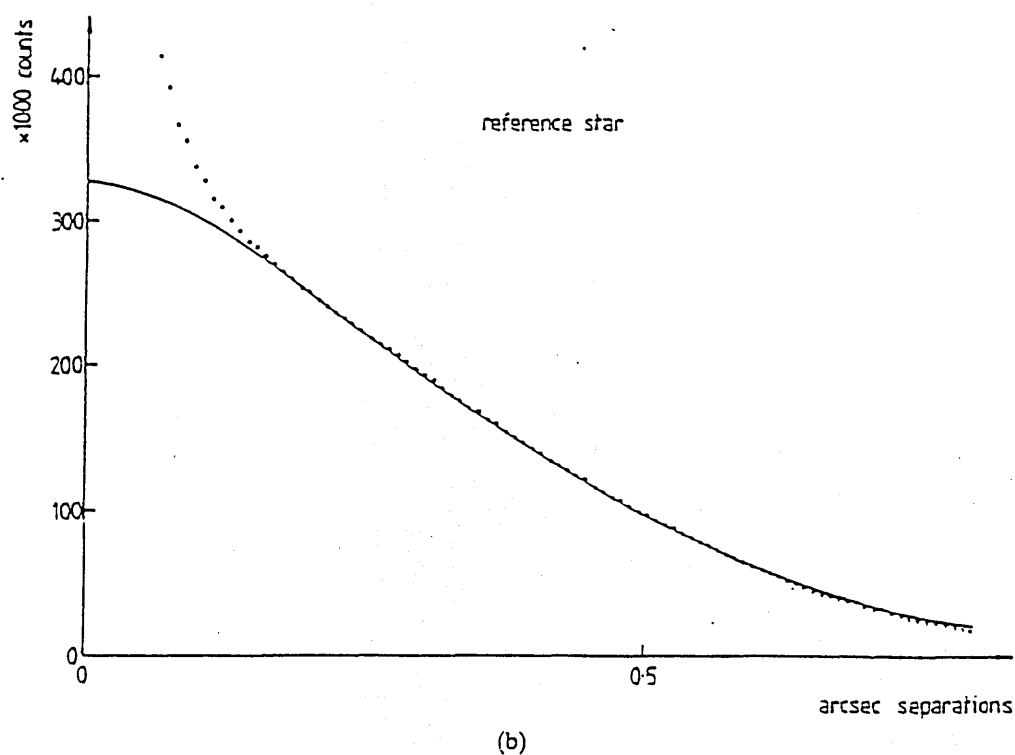
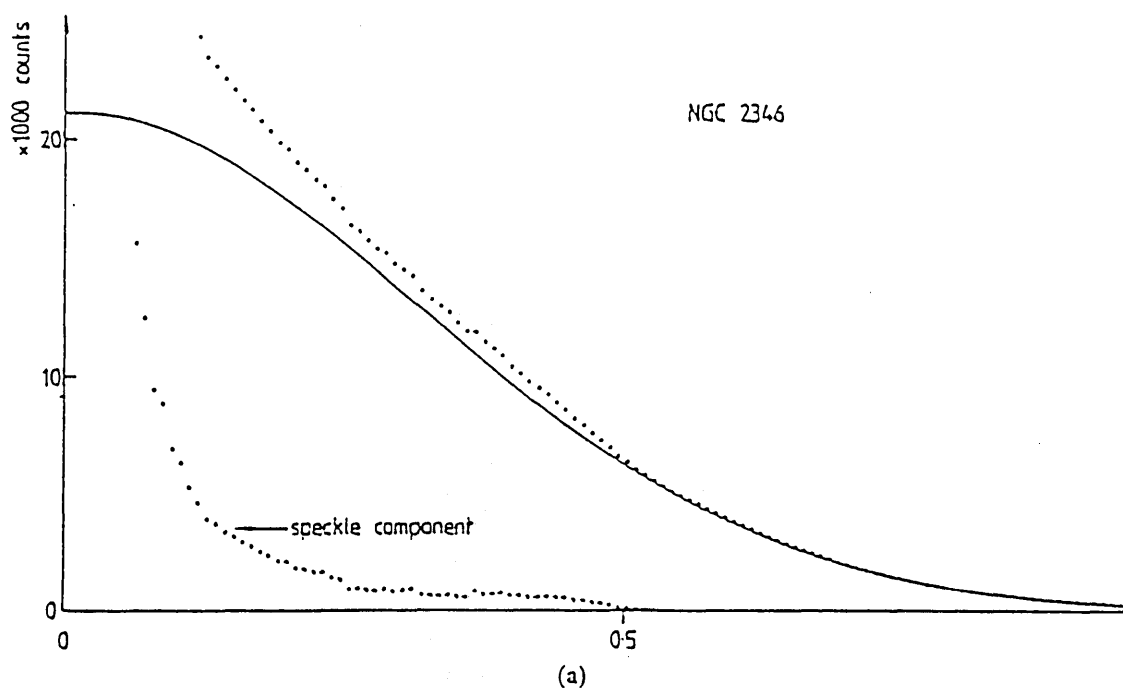


Figure 1. (a) The mean autocorrelation for all position angles for the core of NGC 2346. A distant kink in the curve at 0.54 arcsec prevents a single Gaussian (solid line) being fitted to the whole seeing pedestal. A residual speckle component is indicated. (b) As for Fig. 1(a) but for the reference star. The Gaussian seeing pedestal is shown as a solid line.

the presence of a dusty core to NGC 2346 which is suggested by the present observations strengthens the possibility that such small dusty condensations could exist within this larger volume. (Incidentally, if the binary system is separated by 0.03 AU as suggested by Kohoutek (1982) then, for a distance of 800 pc, the orbit would subtend only 3×10^{-5} arcsec which is 1000

times smaller than the resolution of the speckle interferometer.) This conclusion is remarkably consistent with the predictions of Mendez *et al.* (1984) that a rotating dust cloud of <2 arcsec must be present to explain their observations of the variable reddening of the central star.

Several further observations could be useful. For instance it would be interesting to speckle the core through a narrow-band filter centred on $H\alpha$ to completely eliminate the possibility that the disc is an ionized gas. Also, if the extended source is a stellar continuum scattered off dust, it should be enhanced in blue continuum light ($<4000 \text{ \AA}$) and far less intense in the red ($>6000 \text{ \AA}$). Speckle observations at these wavelengths are therefore desirable.

Longer integrations to achieve higher signal-to-noise ratios in the autocorrelation may also permit any change in shape of the extended cloud of dust, as a function of position angle, to be measured. Many bipolar nebulae have their central stars embedded in elongated discs of dense dust which could have been ejected from the central star in the early stages of the formation of a planetary nebula.

A hard-wired device is being constructed (Hebden, Morgan & Vine 1983) to cross-correlate sequential speckle frames (each $1/25$ s in duration). When this is working the seeing pedestal will be subtracted automatically from the final autocorrelation; consequently the shape of the component of the autocorrelation in Fig. 1 produced by the extended disc will be revealed more precisely.

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Speckle interferometry of *Hipparcos* link stars – II

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Summary. We extend our earlier list of stars tested by speckle interferometry for their suitability to act as links between *Hipparcos* and the *Space Telescope*. We used the Imperial College, London, speckle interferometer and autocorrelator, first on the 1.9-m telescope of the Mt Stromlo and Siding Spring Observatories, then, after discarding 16 stars showing evidence of binary nature, on the Anglo-Australian 3.9-m telescope. We give results for 123 stars. Combined with our 1984 list we estimate that 29 per cent of our original selection of ‘clean looking’ stars (on sky survey prints) are expected to be multiple at the resolution of the 3.9-m telescope, 0.03–0.7 arcsec.

1 Introduction

We present here a second list of stars which we have tested for their suitability as astrometric links between *Hipparcos* and the *Space Telescope*. There is a difference from the first list (Argue *et al.* 1984): here we initially tested as many stars as we could on the 1.9-m telescope of the Mt Stromlo and Siding Spring Observatories. A quick but, as it turned out, very reliable reduction of the video tapes enabled us to discard 16 of the objects as definitely multiple at the 0.07–1.0 arcsec level. The remaining objects could then be tested at the higher resolution (0.03–0.7 arcsec) attainable using the 3.9-m Anglo-Australian Telescope. In this way the efficiency of our use of the AAT was enhanced (although a few of the stars were reobserved as a check). Finally, in London, the entire video data from both telescopes were reduced or re-reduced.

2 Observations

At Mt Stromlo we had four consecutive nights commencing 1984 May 28/29. The weather was good enough for substantial progress only on the last two. The Imperial College speckle interferometer (Hebden, Morgan & Vine 1983) was mounted at the $f/18$ Cassegrain focus, and the

heavier parts of its equipment and the observers were on the movable platform while another observer selected the objects and identified them on the TV monitor in the comfort of the control room. The starlight was observed through filters of bandwidth 22 or 55 nm both centred at 500 nm. In most cases the speckle exposures were of duration 0.04 s, but this was reduced to 0.02 s or even 0.01 s as brightness or seeing demanded. The filters and exposure times are specified in the tables.

On the AAT we had three nights commencing 1984 June 15/16 with favourable weather only on the first. The interferometer was mounted at the $f/15$ R-C focus and operated there by the same two observers. The filters and exposure times were the same as on the 1.9-m telescope.

3 Results

Our results are given in Tables 1, 3 and 4, where the column UT specifies the time of start of the observation, 'Exp' the speckle exposure and FWHM the filter bandwidth.

4 Proportion of resolved stars

On the 1.9-m telescope we found 16 stars that had probably or possibly been resolved (Tables 3 and 4) and 94 unresolved. Of these 94, we tested 29 at the higher resolution of the 3.9-m telescope and found six of them to be resolved: therefore, if we had been able to test all 94, we would have expected 19 to have been resolved on the larger telescope, and we would have had to transfer (19-6) stars from Table 1 to Tables 3 or 4, so reducing Table 1 to 88 stars and increasing Tables 3 and 4 to a total of 35. Adding in the numbers from our previous paper, namely 27 unresolved by the 3.9-m telescope and 10 resolved, and allowing for the fact that SAO 231636 and CD-41°14398 are common to both papers, we expect 29 per cent of our candidate link stars, that

Table 1. Stars that showed no indication of multiplicity. The May measurements were made on the Mt Stromlo 1.9-m telescope, those of June on the Anglo-Australian 3.9-m telescope. The objects are listed in order of increasing RA. Equatorial coordinates for stars not identified in standard catalogues are given in Table 2.

Star	Date	UT (h m)	Exp (s)	FWHM (nm)	Scale (arcsec)
SAO 249926	84 May 31	08 25	0.02	55	0.1 -1.0
SAO 249941	84 May 31	08 34	0.02	55	0.1 -1.0
SAO 199266	84 May 30	08 11	0.04	55	0.1 -1.0
SAO 154848	84 May 31	08 56	0.04	55	0.1 -1.0
BD-13°2746	84 May 30	08 23	0.04	55	0.1 -1.0
SAO 179004	84 May 31	09 15	0.04	22	0.1 -1.0
SAO 179028	84 May 31	09 23	0.04	22	0.1 -1.0
SAO 179060	84 Jun 15	09 05	0.02	22	0.03-0.7
AGK+6°1340	84 Jun 15	09 15	0.04	22	0.03-0.7
SAO 222592	84 May 31	09 40	0.04	55	0.1 -1.0
BD-14°3331	84 Jun 15	09 29	0.04	55	0.03-0.7
BD-13°3381	84 Jun 15	09 22	0.04	22	0.03-0.7
AGK+0°1486	84 May 28	09 42	0.04	22	0.1 -1.0
SAO 180169	84 May 31	10 19	0.02	55	0.1 -1.0
SAO 180172	84 Jun 15	09 39	0.04	22	0.03-0.7
SAO 180174	84 May 30	10 06	0.04	22	0.1 -1.0
SAO 202795	84 Jun 15	09 44	0.01	22	0.03-0.7
CD-37°7473	84 May 31	10 44	0.04	22	0.1 -1.0
BD+0°2845	84 May 28	09 54	0.04	55	0.1 -1.0
AGK+0°1515	84 May 28	10 24	0.04	22	0.1 -1.0
1222+037(1)	84 Jun 15	09 59	0.04	55	0.03-0.7
AGK+2°1579	84 May 28	10 10	0.04	22	0.1 -1.0
CD-25°9428	84 May 31	10 53	0.04	55	0.1 -1.0
CD-24°10532	84 May 31	11 01	0.04	55	0.1 -1.0
SAO 181097	84 Jun 15	10 24	0.04	22	0.03-0.7

Table 1 - continued

Star	Date	UT (h m)	Exp (s)	FWHM (nm)	Scale (arcsec)
SAO 157525	84 Jun 15	10 31	0.01	22	0.03-0.7
1253-055(1)	84 May 31	11 19	0.04	55	0.1 -1.0
CD-31°9972	84 May 30	10 48	0.04	22	0.1 -1.0
SAO 203990	84 May 29	10 51	0.04	22	0.1 -1.0
BD-9°3615	84 May 30	11 09	0.04	55	0.1 -1.0
SAO 157711	84 Jun 15	10 48	0.04	22	0.03-0.7
CD-32°9235	84 Jun 15	11 31	0.04	55	0.03-0.7
CD-33°8944	84 May 30	11 13	0.04	55	0.1 -1.0
CD-43°6312	84 Jun 15	11 03	0.04	22	0.03-0.7
SAO 224510	84 Jun 15	11 24	0.04	55	0.03-0.7
SAO 158248	84 Jun 15	11 14	0.04	22	0.03-0.7
SAO 158625	84 May 29	11 53	0.04	22	0.1 -1.0
CD-37°9770	84 May 31	11 42	0.04	55	0.1 -1.0
CD-37°9771	84 Jun 15	11 40	0.04	55	0.03-0.7
BD-5°4027	84 Jun 15	11 52	0.04	55	0.03-0.7
SAO 140372	84 May 31	13 18	0.04	22	0.1 -1.0
CD-23°12184	84 May 31	13 58	0.04	55	0.1 -1.0
SAO 183344	84 Jun 16	12 15	0.04	55	0.03-0.7
CD-23°12192	84 May 31	14 09	0.04	55	0.1 -1.0
SAO 183357	84 Jun 16	12 08	0.04	22	0.03-0.7
SAO 183404	84 May 30	12 34	0.04	55	0.1 -1.0
SAO 183418	84 May 30	12 41	0.04	55	0.1 -1.0
SAO 121210	84 May 30	12 50	0.04	22	0.1 -1.0
SAO 121283	84 May 30	13 17	0.04	55	0.1 -1.0
SAO 257377	84 May 30	13 34	0.04	22	0.1 -1.0
1656-053(1)	84 May 31	14 20	0.04	55	0.1 -1.0
AGK+5°2203	84 Jun 16	13 19	0.04	55	0.03-0.7
1656-053(2)	84 May 31	14 33	0.04	55	0.1 -1.0
SAO 122400	84 May 30	14 03	0.04	55	0.1 -1.0
AGK+4°2133	84 May 31	14 47	0.04	55	0.1 -1.0
SAO 122404	84 May 30	14 15	0.02	55	0.1 -1.0
BD-13°4642	84 May 30	14 23	0.04	55	0.1 -1.0
BD-13°4650	84 May 31	15 04	0.04	55	0.1 -1.0
SAO 160626	84 May 30	14 30	0.04	55	0.1 -1.0
SAO 141833	84 May 30	15 01	0.02	55	0.1 -1.0
SAO 141841	84 May 30	14 48	0.04	55	0.1 -1.0
BD-3°4165	84 May 30	14 40	0.04	55	0.1 -1.0
SAO 185880	84 May 30	15 45	0.04	55	0.1 -1.0
SAO 103673	84 May 31	15 15	0.04	22	0.1 -1.0
SAO 103677	84 May 31	15 27	0.04	22	0.1 -1.0
AGK+10°2225	84 May 31	15 36	0.04	22	0.1 -1.0
SAO 187783	84 May 31	15 58	0.04	22	0.1 -1.0
SAO 187789	84 May 31	16 06	0.04	22	0.1 -1.0
SAO 187802	84 May 31	15 49	0.04	22	0.1 -1.0
SAO 187804	84 May 31	16 13	0.04	22	0.1 -1.0
SAO 188093	84 May 30	16 14	0.01	22	0.1 -1.0
SAO 254609	84 May 30	16 42	0.01	55	0.1 -1.0
SAO 211415	84 May 31	17 21	0.04	22	0.1 -1.0
CD-63°4536	84 May 30	16 56	0.04	55	0.1 -1.0
SAO 162858	84 May 30	17 20	0.04	55	0.1 -1.0
SAO 162862	84 May 31	17 47	0.01	22	0.1 -1.0
AGK+7°2758	84 May 31	18 05	0.04	22	0.1 -1.0
BD-16°5523	84 Jun 15	17 40	0.04	55	0.03-0.7
BD-7°5199	84 Jun 15	17 30	0.02	22	0.03-0.7
BD-17°6080	84 May 31	18 29	0.04	55	0.1 -1.0
BD-17°6085	84 Jun 15	17 53	0.04	22	0.03-0.7
SAO 163888	84 Jun 15	18 02	0.01	22	0.03-0.7
CD-41°14398	84 Jun 15	18 09	0.04	55	0.03-0.7
CD-41°14402	84 Jun 15	18 15	0.04	55	0.03-0.7
AGK+4°2915	84 Jun 15	18 26	0.01	22	0.03-0.7
AGK+4°2916	84 Jun 15	18 34	0.01	22	0.03-0.7
SAO 145482	84 May 30	18 58	0.04	55	0.1 -1.0
SAO 145490	84 Jun 15	18 41	0.04	22	0.03-0.7
BD-2°5568	84 May 31	18 58	0.04	55	0.1 -1.0
SAO 145504	84 Jun 15	18 46	0.04	22	0.03-0.7
AGK+0°2713	84 Jun 15	18 54	0.04	22	0.03-0.7
SAO 164768	84 May 31	19 23	0.01	22	0.1 -1.0
SAO 164774	84 May 30	19 29	0.04	22	0.1 -1.0
SAO 164854	84 Jun 15	19 22	0.01	22	0.03-0.7
SAO 146122	84 May 30	19 54	0.04	22	0.1 -1.0
SAO 231182	84 May 30	20 03	0.04	22	0.1 -1.0
CD-48°14514	84 Jun 15	19 51	0.04	55	0.03-0.7
SAO 231636	84 Jun 15	19 36	0.01	22	0.03-0.7
CD-48°10901	84 Jun 15	19 43	0.04	22	0.03-0.7
SAO 165730	84 Jun 15	19 59	0.02	22	0.03-0.7
SAO 191999	84 Jun 15	20 06	0.04	22	0.03-0.7

Table 2. Equatorial coordinates and approximate magnitudes for those stars in Table 1 that we failed to identify in standard catalogues. These coordinates were measured from sky survey prints taking as reference the AGK3 or, for fields south of -2° , the SAO positions for five or six nearby stars updated to the epoch of the survey print. The magnitudes were estimated from image diameters. The accuracy is about ± 1 arcsec for each coordinate and ± 0.5 for the magnitudes.

Star	RA (B1950.0) (h m s)	Dec ($^\circ$ ' ")	m
1222+037(1)	12 22 54.2	+03 50 03	11.5
1253-055(1)	12 53 01.1	-05 22 22	11.1
1656+053(1)	16 55 04.3	+05 21 03	11.1
1656+053(2)	16 56 11.2	+05 07 31	11.0

Table 3. Stars that showed some evidence of possible binary or multiple nature, listed in order of increasing RA. The May measurements were made with the Mt Stromlo 1.9-m telescope with scale 0.1–1.0 arcsec, the June with the Anglo-Australian 3.9-m telescope with scale 0.03–0.7 arcsec. The position angle (PA) is measured from north through east (with ambiguity 180°) at the epoch of date.

Star	Date	UT (h m)	Exp (s)	FWHM (nm)	Separation	PA ($^\circ$)
SAO 177329	84 Jun 15	08 55	0.01	22	0.46	225
SAO 118598	84 May 28	09 32	0.04	22		
SAO 202788	84 Jun 17	10 14	0.04	55		
AGK+0°1513	84 Jun 15	10 14	0.04	22	0.40	230
CD-31°9988	84 May 30	11 30	0.04	55	0.34	128
BD-8°3919	84 Jun 15	12 00	0.04	55		
AGK+2°1869	84 May 30	13 04	0.04	22	0.32	140
SAO 121281	84 Jun 16	14 44	0.04	22		
AGK+10°2219	84 May 30	15 56	0.04	55	0.44	151
BD-15°5413	84 May 30	17 28	0.02	55	0.71	185
AGK+8°2629	84 May 31	17 56	0.04	22	0.42	202
SAO 163176	84 May 30	17 36	0.02	55	0.36	94
BD-18°5570	84 May 30	17 44	0.02	55	0.38	159
SAO 144138	84 May 31	18 20	0.04	55	0.48	138

Table 4. Stars that showed stronger evidence for binary nature than those in Table 3.

Star	Date	UT (h m)	Exp (s)	FWHM (nm)	Separation (arcsec)	PA ($^\circ$)
CD-37°4708	84 May 31	08 44	0.02	55	0.46	165
AGK+1°1168	84 May 30	08 32	0.04	55	0.44	240
BD-5°3598	84 May 30	10 36	0.04	22	0.55	248
SAO 204292	84 Jun 15	10 55	0.04	22	0.38	116
SAO 103672	84 May 30	16 02	0.04	55	0.42	246
SAO 163324	84 Jun 15	17 47	0.04	55	0.16	174
AGK+0°2717	84 May 30	19 10	0.04	55	0.59	195
SAO-146083	84 May 31	19 57	0.04	22	0.63	219

had been selected on account of their clean appearance on the sky survey prints, in fact to be resolvable at the resolution of the 3.9-m telescope (0.03–0.7 arcsec). This result confirms the conclusion in our 1984 paper, that about 30 per cent are resolvable.

Acknowledgments

We are grateful to the Director of the Mt Stromlo and Siding Spring Observatories, Professor D. S. Matthewson, for time on the Mt Stromlo telescope, and to PATT for allocating three nights

on the AAT. The skill of our night assistants, Mr David J. Manuel at Mt Stromlo and Mr Frank F. Freeman at Siding Spring, in rapidly acquiring objects scattered widely over the sky ensured a high observing efficiency. Professor Raynor L. Duncombe and Dr Paul D. Hemenway of the University of Texas at Austin had cooperated in drawing up the observing list. Costs were covered by a travel grant from the SERC which is gratefully acknowledged. CS is grateful to the SERC for his Research Studentship, and to Imperial Chemical Industries PLC for his Educational Scholarship. HV is grateful to the SERC for support as Assistant Research Officer.

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The President. We now come to the programme as advertised on that purple thing [laughter]. Dr. Barlow of University College, London, is going to tell us about speckle interferometry of planetary nebulae.

Dr. M. J. Barlow. One can list at least three important reasons for determining the masses of shells of planetary nebulae. The first reason is to be able to quantify the recycling and enrichment of the interstellar medium by the low-to-intermediate mass stars which are responsible for planetary nebulae. The second is to be able to link the masses of the resulting white dwarfs to the masses of the progenitor red-giant stars, such as carbon stars and Mira variables, and eventually, right back to the masses of the original main-sequence stars. The third reason is to be able to diagnose the planetary-nebula ejection mechanism itself. The work I shall describe was carried out in collaboration with the speckle interferometry group of B. Morgan, C. Standiey and H. Vine at Imperial College, London, to determine the masses of a significant sample of planetary nebulae in the Magellanic Clouds. To derive the mass of a planetary shell, one needs to determine the number-density distribution of hydrogen as a function of radius. This requires that the nebula be well-resolved and that its distance be accurately known. Planetary nebulae in our Galaxy satisfy the first requirement but unfortunately their distances are usually uncertain by at least 50 per cent, since no good-quality intrinsic distance calibration exists. This distance uncertainty translates into a factor of three uncertainty in the nebular masses. By contrast, a 15 per cent uncertainty in the distances of Magellanic Cloud nebulae, whose distances are independently known, translates into a much smaller uncertainty of 40 per cent in the derived nebular masses. However, since good angular-diameter estimates are required and since almost all Magellanic Cloud nebulae have angular diameters of less than one arcsecond, little use has been made of them thus far. Although observations of Magellanic Cloud nebulae constitute high priority objectives for

two of the core teams of the Hubble Space Telescope, we have been able to demonstrate that the use of standard speckle interferometric techniques at the AAT allows us to derive angular diameters, brightness distributions and masses for these objects. We have imaged the nebula with a 50 ångström wide [O III] λ 5007 ångström filter, using typical exposure times of 1500 seconds and frame-rates of 25 hertz. The [O III] line, as the dominant coolant, is by far the strongest line emitted by most nebulae, for which its intensity distribution closely matches that of the hydrogen emission line. We have chosen to observe nebulae which independent diagnostics indicate are optically thin in the hydrogen Lyman continuum, so that the ionized mass should represent the entire mass of each nebula. The autocorrelation diagram for the nebula SMC N2 clearly shows three peaks, at 0.16, 0.44 and 0.60 arcseconds. We have been able to match this with a model consisting of a spherical double-shell nebula with an inner-ring radius of 0.22 arcseconds and an outer-ring radius of 0.38 arcseconds. We have modelled the intensity distribution as a function of radius by a double-Gaussian, and the intensity is proportional to the product of the proton and electron number-density distributions, times the nebular filling factor (the fraction of nebular volume occupied by gas). Since we know the absolute $H\beta$ flux emitted by the nebula, we can solve to obtain the run of electron density times the root of the filling factor. In order to derive the filling factor, we have obtained AAT service observations of the [O II] doublet ratio, which is proportional to the local electron density times the local emissivity, averaged over the entire nebula. The observed doublet ratio yields a mean electron density of 2850 per cubic centimetre for SMC N2, whereas the fit to our speckle observations yields 1360 per cubic centimetre times the root of the filling factor for this quantity. Therefore we find the filling factor to be 0.23 and the ionized mass of the nebula to be 0.27 solar masses. In order to confirm our technique we have also obtained speckle interferograms of the compact Galactic nebula VY 2-2, which VLA radio observations have shown to consist of a single ring of diameter 0.4 arcseconds. Our speckle observations, in both [O III] and $H\beta$, have yielded a single autocorrelation peak at 0.4 arcseconds, in agreement with a model fitted to the VLA data. We have used the Manchester échelle spectrograph on the AAT to derive an expansion velocity of 17 kilometres per second for SMC N2, by observations of the [O III] λ 5007 ångström linewidth. In conjunction with the outer nebular radius of 0.105 parsecs yielded by the speckle observations, this implies an age of 6000 years for SMC N2. We have recently obtained speckle observations of 20 more Magellanic Cloud nebulae at the AAT, which is the only telescope available to UK astronomers which has the requisite light-collecting power, angular resolution, and ability to look at the Magellanic Clouds. From our analysis of these observations it is hoped to derive the mass of the ejected nebula as a function of central-star mass and thereby as a function of the mass of the progenitor red giant star.

The President. You assumed in modelling the linewidths in the spectra that the nebula was of uniform surface brightness. Couldn't you have used the structure inferred from the speckle interferometry?

Dr. Barlow. We did in fact use the speckle results, but since the nebula was entirely within the slit of the spectrograph, the exact brightness profile turns out not to matter.

Dr. R. C. Smith. How many planetary nebulae in the Magellanic Clouds are accessible by this technique?

Dr. Barlow. We have observed about 20 so far. The total luminosity range is about a factor of 200—we can certainly observe the first 2½ to 3 magnitudes of this luminosity range.

Speckle Interferometry at Imperial College

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Abstract

The design and performance of the Imperial College hard-wired cross-correlator is described. Its use in conjunction with the Imperial College hard-wired autocorrelator is detailed and illustrated by a comparison of data reduced with and without cross-correlation.

Introduction

The Imperial College speckle interferometer and hard-wired autocorrelator (previously described by Hebden et al.¹) are routinely used to study binary stars, active galaxies and planetary nebulae. The ensemble average vector autocorrelation function of a set of short exposure images is calculated in real time. For an image having an origin in its bottom right-hand corner and containing m photon events at coordinates (x_i, y_i) , this vector autocorrelation, $A_1(h, k)$, is given by the distribution of vectors between each event and every other event, hence:

$$A_1(h, k) = \sum_{i=1}^m \sum_{j=1}^m (x_i - x_j, y_i - y_j). \quad (1)$$

The addition of a hard-wired cross-correlator has allowed us to implement the Welter and Worden method² for calculating the seeing corrected autocorrelation function $A_0(h, k)$.

The seeing corrected autocorrelation

The cross-correlation of two functions $f(x, y)$ and $g(x, y)$ measures their mutual spatial coherence and is given by:

$$C(h, k) = \iint f^*(x, y) \cdot g(x + h, y + k) dx dy. \quad (2)$$

For two images comprising individual photon events with coordinates (x_i, y_i) and (x_j, y_j) this "analogue" cross-correlation is equivalent to the distribution of vectors between every event in one image and every other event in the second image. Thus:

$$C(h, k) = \sum_{i=1}^m \sum_{j=1}^m (x_i - x_j, y_i - y_j). \quad (3)$$

The vector cross-correlation of two statistically independent frames will only contain vectors between spatially uncorrelated speckles. Therefore, if the speckles are contained in a Gaussian envelope their cross-correlation will also have a Gaussian envelope with a width greater than either of the individual widths.

In 1977 Welter and Worden proposed that the difference between the average autocorrelation and the average cross-correlation of statistically independent frames was proportional to the autocorrelation of the object intensity. Therefore, denoting an ensemble average by $\langle \rangle$:

$$A_0(h, k) = \langle A_1(h, k) \rangle - \langle C(h, k) \rangle. \quad (4)$$

In implementing this seeing correction, Welter and Worden normalized the intensity of each frame so that the sum of the intensities was the same for any two frames.³ By this operation they made the integral area of the autocorrelation equal to that of the cross-correlation. However, as the autocorrelation contains a central peak and may contain a speckle component absent from the cross-correlation, a simple differencing of the two over-corrects for the seeing pedestal. Several groups have noted this over-correction^{4,5} and have proposed a modification to the original method in which the average cross-correlation is normalized to the average autocorrelation for separations outside the speckle component:

$$A_0(h,k) = \langle A_1(h,k) \rangle - M \cdot \langle C(h,k) \rangle \quad (5)$$

where M is some appropriate constant. An alternative formulation of equation 5 has been given by Bruck and Sodin.⁶ They point out that for statistically independent speckle images the average cross-correlation is equal to the autocorrelation of the long exposure image, however, in such an image photon events may overlap and limit the validity of the vector autocorrelation. This photon "pile up" problem has been reported by Hege.⁷ A second problem arises from gross movements of the seeing disc. If this is displaced from frame to frame, the average cross-correlation will contain some vectors at separations greater than the seeing disc diameter. These will broaden the cross-correlation relative to the autocorrelation. This problem can be eliminated by either centring each frame before processing or by choosing the temporal separation between processed frames so that any motion is minimised, while maintaining statistical independence.

The hard-wired cross-correlator

Design requirements

The standard British T.V. system uses pictures consisting of two interlaced fields, each containing 293 lines of image information. The field frequency is 50 Hz. and the autocorrelator processes alternate fields by examining the central 256 lines in a square of 256 x 256 pixels. A maximum of 512 occupied pixels per field is allowed and the vectors within a 64 x 128 pixel window are recorded.

The cross-correlator was conceived as a complementary instrument for the autocorrelator. It is required to process T.V. fields with the same constraints as outlined for the autocorrelator, but to derive vectors between fields separated by a predetermined time interval. This interval is adjustable from 2 to 998 field periods.

Performance

At the start of a cross-correlation computation the field interval is selected by setting a thumbwheel switch. Autocorrelation and cross-correlation calculations can be carried out simultaneously. The autocorrelator examines the T.V. signal at each pixel and if this lies above an adjustable detection threshold the coordinates of that pixel are stored as 2 x 8 bits in one of two 512 x 16 bit memories. If signals are detected in 512 pixels subsequent signals within the field are ignored. The recorded coordinates are then used to calculate the autocorrelation.

These same coordinates are loaded into one of two 512 x 16 bit memories in the cross-correlator. If a field interval, f , is chosen, the coordinates are stored for the first available field and for the f th. field. The cross-correlation is computed for the first and for the f th. fields and then the coordinates from the $(i+f)$ th. field are written over those of the first field. The cross-correlation is performed for this new set of coordinates and so on.

The cross-correlator computes the cross-correlation of two fields F_1 and F_2 . The coordinates of the i th. event in F_1 are subtracted from those of the j th. event in F_2 (where $j \in i$) and a memory location which has an address given by the result of the subtraction is incremented. If the y subtraction is < 0 , it is ignored and the $(j+1)$ th. event in F_2 considered. If the result of the y subtraction is > 63 it too is ignored and since all subsequent y differences will also exceed 63 the next value of i is considered. To avoid unnecessary calculations for each value of i the first value of j which gave $y \geq 0$ is recorded; then for the subsequent i increment this j value is the first to be considered, since all previous y differences will be < 0 . When all the event coordinates of F_1 have been subtracted from those of F_2 the coordinates of events in F_2 are subtracted from those in F_1 .

The number contained in the cross-correlation array pixel (h,k) represents the number of times that particular vector difference has been recorded.

The cross-correlator contains a 50 Hz. clock which is synchronised to the T.V. synchronisation signal from the autocorrelator at the selected field intervals. This clock solves the problem of loss of video synchronisation due to video tape drop outs or other interference occurring in the intervals between the reading of fields. If synchronisation were lost then, of course, no computation would be performed for those fields concerned.

The cross-correlation array is a 128 x 64 x 20 bit integrating memory consisting of forty 4K MOS static RAM chips and a 20 bit, parallel loading, synchronous counter. The

remaining cross-correlator digital circuitry is implemented by Schottky, Low Power Schottky and High Power CMOS integrated circuits.

Figure 1 shows a schematic diagram of the cross-correlator. The separated T.V. synchronisation signal and the digitised coordinates are fed from the autocorrelator to the cross-correlator. The arithmetic logic unit (ALU) then performs the appropriate subtractions. The comparator determines when the h and k limits are exceeded and increments the appropriate address counter. If the comparator determines that the result is within the 64×128 vector window the corresponding location in the integrating memory is incremented.

Operational procedure

Video taped data is reduced simultaneously by the autocorrelator and cross-correlator. At any time the operation of either instrument can be interrupted and the current results displayed on a storage oscilloscope, before data reduction is continued.

Autocorrelations and cross-correlations are transferred to an Apple microcomputer via two 6522 versatile interface adapters (VIA) for storage on floppy disc, normalization, subtraction and display. The cross-correlation is normalized to the autocorrelation at separations greater than the speckle component and then this normalized cross-correlation is subtracted from the autocorrelation by a short machine code routine. The result is the seeing corrected autocorrelation function.

Results

Examples of the autocorrelation, cross-correlation and seeing corrected autocorrelation for the binary star ADS 1123 ($m = 6.7$, $\Delta m = 0.2$) are shown in Figures 2-4. The system was observed on the 3.9m. Anglo-Australian Telescope. Figure 2 is the autocorrelation obtained from 2500 T.V. fields and Figure 3 is the cross-correlation derived, with an eight frame separation, from the same record. The autocorrelation has a binary peak superimposed on a "seeing pedestal" at an autocorrelation angle of 159 degrees with a maximum value at a separation of 0.35 arcseconds (see Figure 5). The cross-correlator contains seeing information and is extended in the direction of the image's elongation, but the binary peak is absent (see Figure 6). The seeing corrected autocorrelation as given by equation 5 is shown in Figure 4, and Figure 7 is a radial profile through this function at 159 degrees. The seeing correction enhances the contrast of the binary peak and shifts the position of its maximum to 0.36 arcseconds. Such a shift would be expected on subtracting a Gaussian seeing pedestal model.

It was found that for frame intervals > 8 applying equation 5 over-corrected the autocorrelation in the range of autocorrelation angles 0 to 90 degrees. We attribute this to gross image movement between T.V. frames; at the present we do not perform any image centring before processing. Also for a separation of two frames the statistical independence of the frames is predictably lost and a "binary peak" is apparent in the cross-correlation (see Figure 8).

Signal to noise ratio (SNR)

For a binary star the signal, S, in the autocorrelation may be defined as the height of the binary peak above the local background.⁷ Assuming the probable error in the measurement of the autocorrelation, N, to be the square root of the sum of the signal and the background then:

$$SNR = \frac{S}{\sqrt{(S + B)}} \quad (6)$$

Applying equation 6 to the autocorrelation of ADS 1123 $S = 6150$ vectors, $N = 138$ vectors and, therefore, $SNR = 33$; for the seeing corrected autocorrelation the normalizing constant, $M = 2$, $S = 14280$, $N = 219$ and $SNR = 65$. Therefore, in this example, by applying the cross-correlation seeing correction the SNR has been increased by a factor of order two.

Future work

The cross-correlator will be used in conjunction with the autocorrelator to analyse recent speckle observations of planetary nebulae in the Magellanic Clouds and other records of binary stars, active galaxies, asteroids and nebulae. The statistics of atmospheric turbulence will be studied and speckle lifetimes derived.

Acknowledgements

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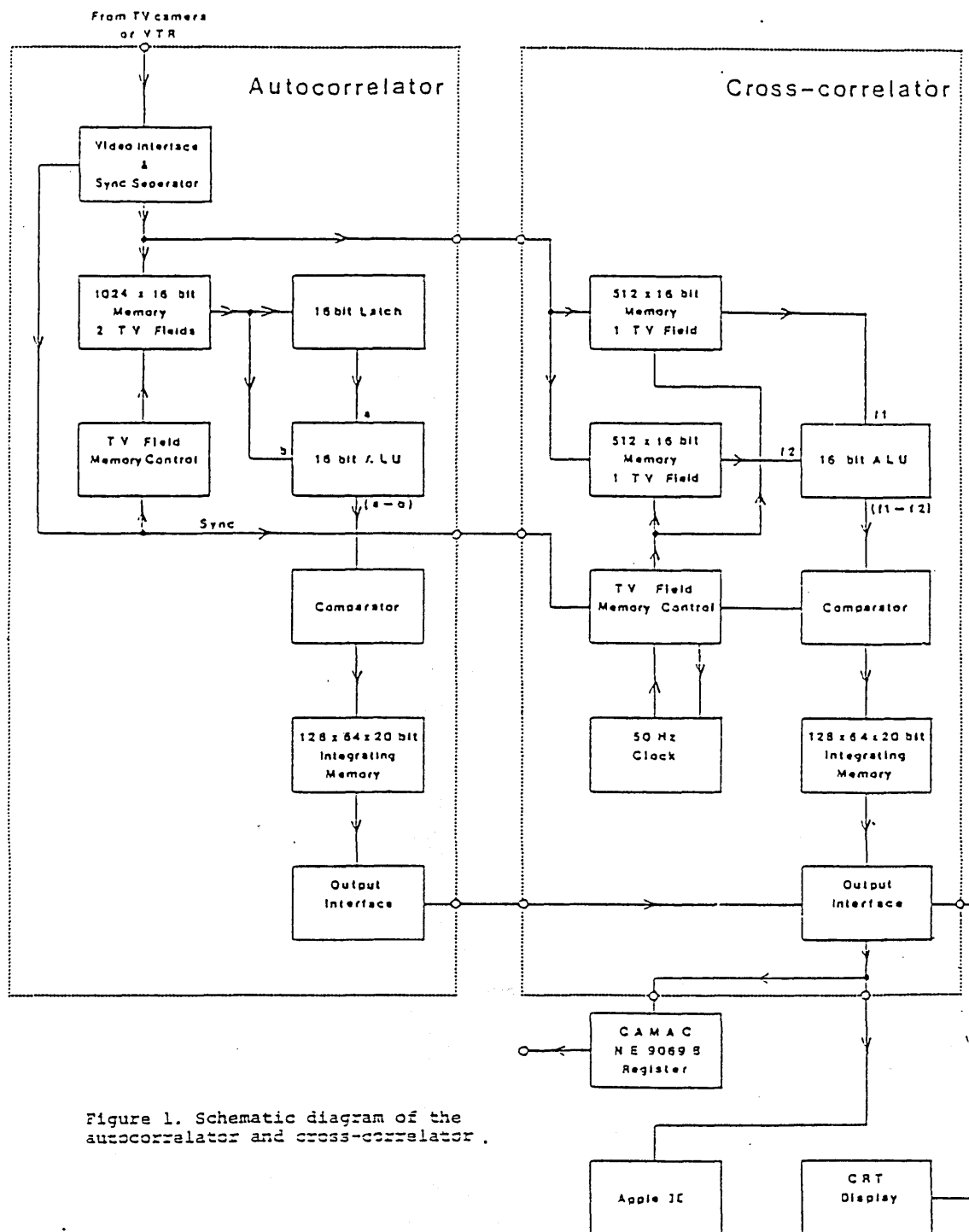


Figure 1. Schematic diagram of the autocorrelator and cross-correlator.

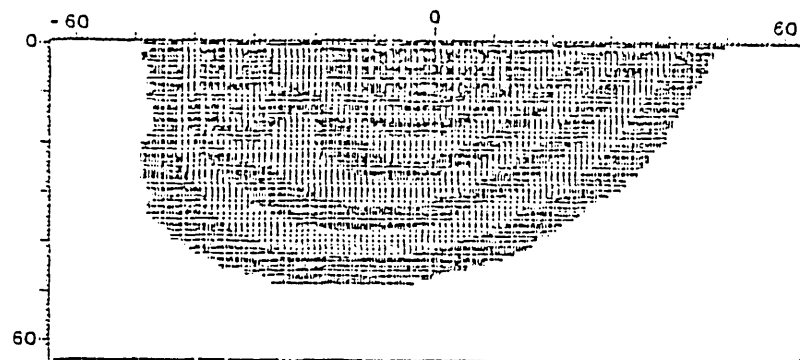


Figure 2. Autocorrelation of the binary star ADS 1123 ($m = 6.7$ $\Delta m = 0.2$)
Scale = 0.01 arcseconds/pixel.

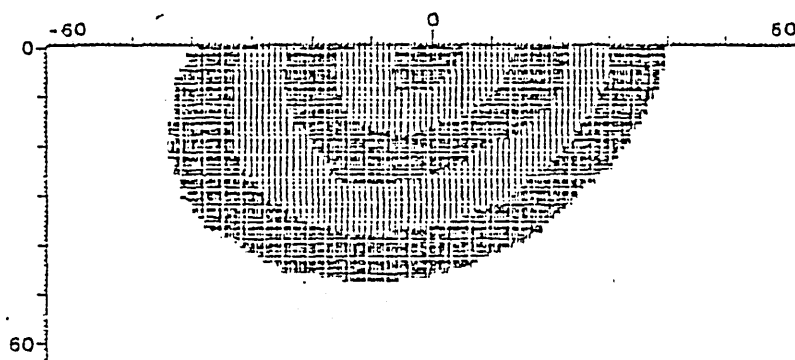


Figure 3. Cross-correlation of ADS 1123 (frame interval of eight frames).

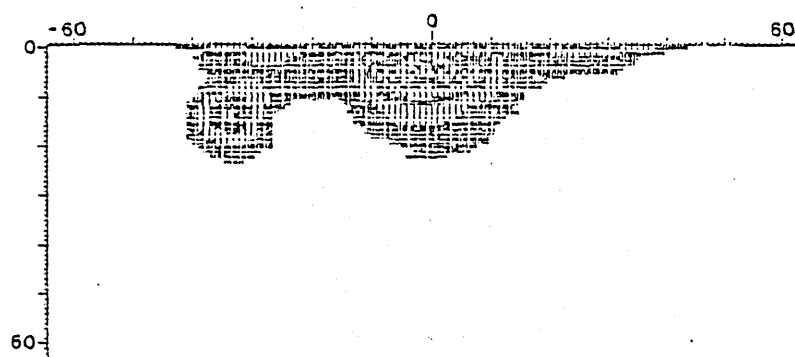


Figure 4. Seeing corrected autocorrelation of ADS 1123.

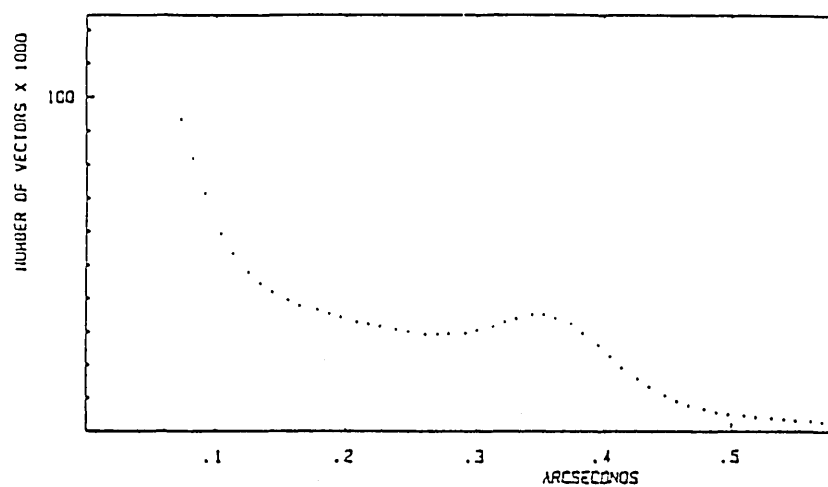


Figure 5. Radial profile through the autocorrelation of ADS 1123 at 159° .

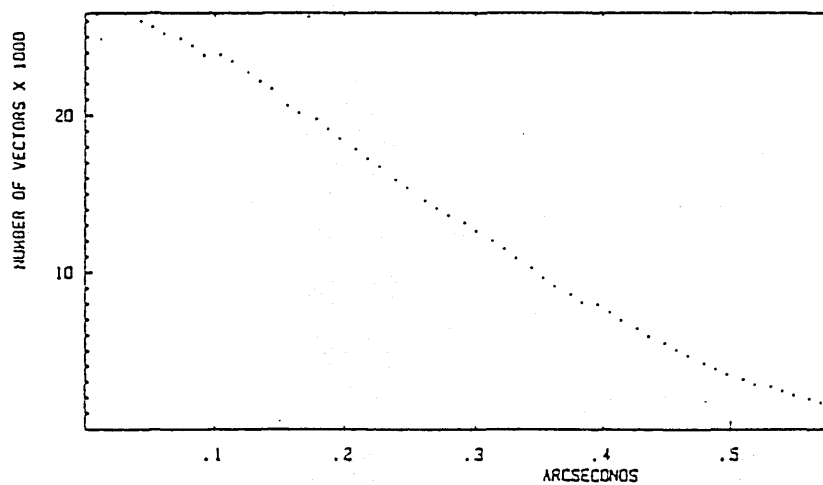


Figure 6. Radial profile through the cross-correlation of ADS 1123 at 159° (frame interval of eight frames).

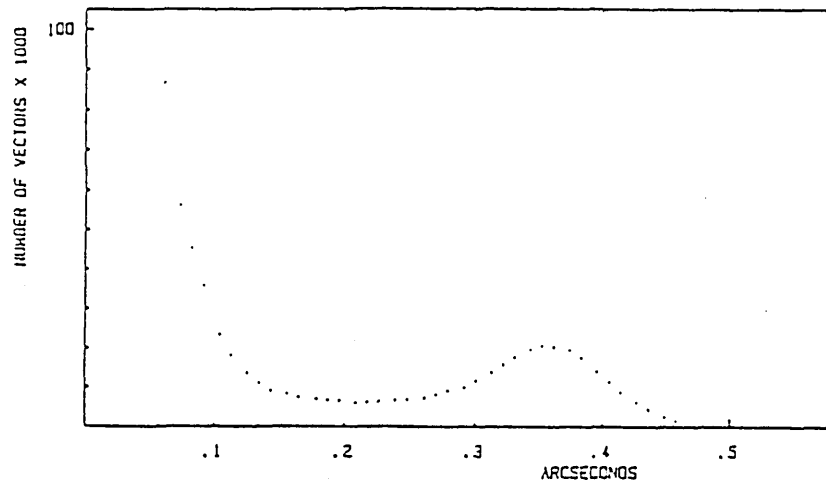


Figure 7. Radial profile through the seeing corrected autocorrelation of ADS 1123 at 1590.

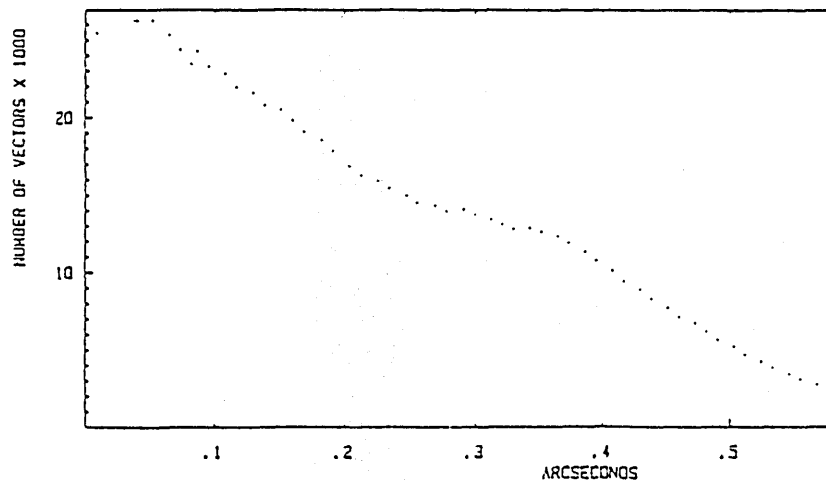


Figure 8. Radial profile through the cross-correlation of ADS 1123 at 1590 (frame interval of two frames).

The determination of the mass of a Magellanic Cloud planetary nebula by speckle interferometry

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Summary. We have resolved a Magellanic Cloud planetary nebula by speckle interferometry for the first time. Our observations of SMC N2 show it to have a double-ring structure, the rings having angular radii of 0.22 and 0.38 arcsec, corresponding to absolute radii of 0.06 and 0.10 pc. Our speckle observations of the galactic planetary nebula VY 2–2 show a ring of angular diameter 0.4 arcsec, in agreement with previous VLA radio data. We have determined the radial hydrogen density profile for SMC N2 and have derived masses of $0.09 M_{\odot}$ and $0.27 M_{\odot}$ for the inner and outer shells respectively, so that the total mass of this optically thin nebula is $0.36 M_{\odot}$. The nebular expansion velocity of SMC N2 has been derived from a high-resolution spectrum of the [O III] 5007 Å line and expansion ages of between 5000 and 12 000 yr have been derived for the shells. A reanalysis of the available nebular data leads us to derive a central star effective temperature of 110 000 K and a luminosity of $4340 L_{\odot}$, corresponding to a stellar mass of $0.59 M_{\odot}$, from a comparison with published evolutionary tracks. The stellar evolutionary age since leaving the AGB is about 8000 yr, consistent with the nebular expansion age.

1 Introduction

The masses of planetary nebula shells are of interest for at least three distinct reasons. (i) A knowledge of the shell masses can allow a quantification of the contribution which is made to the recycling and chemical enrichment of the interstellar medium by the low-to intermediate-mass stars which are responsible for planetary nebulae (PN). (ii) The central stars of PN evolve to become white dwarfs. Since the masses of central stars can be determined by fitting theoretical evolutionary tracks through their observed L , T_{eff} positions in the H–R diagram, the determination of nebular masses will allow the masses of white dwarfs to be linked to those of the progenitor asymptotic giant branch (AGB) stars. With a subsequent understanding of the mass-loss processes which occur prior to the planetary nebula formation stage of AGB stars, the

masses of white dwarfs may eventually be linked back to the original main-sequence stellar masses. (iii) The physical processes responsible for planetary nebula formation are of interest in their own right. A determination of the masses of planetary nebula shells as a function of stellar core mass may help to diagnose the PN ejection mechanism itself.

In order to derive the mass of a planetary nebula, one needs to determine the ion number density as a function of absolute radius. This requires that the nebula be well resolved and that its distance be accurately known. The first requirement is satisfied by PN in our own Galaxy but unfortunately their distances are usually uncertain by at least 50 per cent (see Gathier *et al.* 1983) because no good intrinsic calibrator exists. To illustrate the effect of this degree of distance uncertainty on mass determinations, consider a PN at distance D , with absolute radius R , mean ion density n , and a filling factor ϵ , where ϵ is the fraction of the nebular volume which is occupied by ions of mean density n . The mass of the PN, $M(\text{PN})$, is proportional to $\epsilon n \theta^3 D^3$, where $\theta = R/D$ is the nebular angular radius. The observed $\text{H}\beta$ flux, by recombination theory, is proportional to $\epsilon n^2 R^3/D^2$, or $\epsilon n^2 \theta^3 D$. For a nebula with a measured angular radius and $\text{H}\beta$ flux we have $n \propto (\epsilon D)^{-1/2}$, so that we obtain $M(\text{PN}) \propto \epsilon^{1/2} D^{5/2}$. If the value of n is known independently, then, since $\epsilon \propto 1/n^2 D$, we obtain $M(\text{PN}) \propto D^2/n$. Thus a 50 per cent uncertainty in D translates into a factor of 2.3 uncertainty in $M(\text{PN})$.

Gathier *et al.* (1983) have carried out a radio interferometric study of PN in the direction of the central bulge region of our Galaxy and have derived the nebular masses from their measured radio fluxes and diameters, under the assumption that they had filling factors of unity and that they were close to the Galactic Centre, for which a distance of 9 kpc was adopted. However, the PN towards the Galactic Centre are heavily obscured and so their properties are difficult to study at other wavelengths.

The PN in the Magellanic Clouds are very lightly obscured, and so their spectroscopic properties are relatively easy to study. Their distances are known relatively accurately because of the existence of a sufficient number of independent distance calibrators in each galaxy. A likely distance modulus uncertainty of 0.3 mag translates into an uncertainty of only 30 per cent in the derived nebular masses. However, good angular diameter estimates are required. Standard imaging techniques have only managed to resolve about half a dozen relatively old and faint PN in the Magellanic Clouds (Jacoby 1980), and the majority of the currently known PN are expected to have angular diameters of <1 arcsec (see Webster 1969, 1976). Although spaceborne imaging will undoubtedly be of benefit for the study of Magellanic Cloud PN, we will show that the use of standard speckle interferometric techniques at the 3.9-m Anglo-Australian Telescope has allowed us to derive angular diameters, radial brightness distributions and nebular masses for these objects. We present here our first speckle observations to resolve a Magellanic Cloud planetary nebula, SMCN2, obtained on 1983 November 18 when weather conditions only permitted observations of this PN. A second observing run in 1984 October yielded speckle interferometric data on 20 more Magellanic Cloud PN and those data will be described in a subsequent publication. As a check on our techniques, we also obtained speckle observations in 1984 October of the bright compact galactic PN Vy2-2, whose subarcsec structure had been previously determined at VLA radio wavelengths (Sequist & Davis 1983). Our observations and analysis of Vy2-2 are included in Section 3.

2 Observations

The Imperial College speckle interferometer was operated in photon counting mode at the $f/15$ focus of the 3.9-m Anglo-Australian Telescope. The results were analysed with a hard-wired autocorrelator. A detailed description of these instruments is given by Hebdén, Morgan & Vine (1983). A 75-mm objective was used for the observations of SMCN2, yielding a 4.6-arcsec field of

view and a pixel size of 0.018 arcsec. With this plate-scale, the maximum observable structural separation corresponds to about 1.1 arcsec, while the limit to the smallest measurable structural separation is determined by the width of the central photon spike in the autocorrelation plot; this spike corresponds to vectors between pixels within the same photon event. At the plate-scale used here, the base radius of the photon spike was about 0.14 arcsec.

We imaged through a 50 Å bandpass filter centred on the [O III] 5007 Å line. This filter was chosen because: (i) the narrow filter strongly suppresses the dense Magellanic Cloud stellar background and background photons from the moonlit sky. (ii) The [O III] 5007 Å line is usually the strongest line emitted by PN. Its strength is typically 10 times that of H β , allowing nebulae to be speckled which are too faint for H β observations. (iii) Ionization structure models of PN predict (e.g. figs 2.5 and 2.6 of Osterbrock 1974), and images of galactic PN confirm (e.g. Atherton *et al.* 1978), that the brightness distribution of a PN in the [O III] line is very similar to that in the H β line, for central star temperatures in excess of about 40 000 K. This encompasses the majority of PN. Nebulae with central star temperatures below this value can be easily recognized by their low-excitation spectra. Confirming this, our speckle data for the galactic PN Vy 2-2 (Section 3) show near agreement between the [O III] and H β structures.

In the selection of PN, it is important to choose objects which are likely to be optically thin in the hydrogen Lyman continuum, in order that the ionized nebular mass be equal to the total mass of the ejected shell. The best indicator of this is if the stellar He II Zanstra temperature is significantly larger than the H I Zanstra temperature but, in the absence of data on the central star, other selection criteria can be utilized, such as the presence of relatively weak lines from low ion stages. In general we preferred the lower density ($n_e < 5000 \text{ cm}^{-3}$) and/or the high-excitation PN, since by evolutionary arguments these are more likely to be optically thin. We chose to observe SMC N2 (Henize 1956) because of the very high luminosity, $3 \times 10^4 L_\odot$, that had been derived for its central star by Stecher *et al.* (1982). However, as will be seen in Section 6 and in the Appendix, our reanalysis of the available data leads us to derive a much lower stellar luminosity.

SMC N2 was imaged in the [O III] line for 60 min at a frame-rate of 25 Hz. Two-dimensional spatial autocorrelations over all azimuthal angles were produced between all pixel events on each frame and the results were summed to yield the dotted plot in Fig. 1(a). This plot can be seen to exhibit significant structure, most noticeably three broad peaks, corresponding to vectors between points separated by 12 pixels (0.22 arcsec), 23 pixels (0.41 arcsec) and 32 pixels (0.59 arcsec). A nearby unresolved reference star, SAO 255683, was also observed for 30 min immediately after the SMC N2 observation, through an additional neutral density filter, and the autocorrelation plot in Fig. 1(b) was produced. In addition to the central photon spike, the autocorrelation plot for SAO 255683 exhibits the normal extended seeing pedestal, which can be approximated by a near-Gaussian fit [i.e. $\exp-(x/a)^{5/3}$, see Roddier 1981]. The slope of the outer seeing pedestal of SMC N2 in Fig. 1(a) is not the same as that of the reference star in Fig. 1(b), presumably due to changes in the seeing during the period of 2 hr over which both observations were made. A number of near-Gaussian curves [e.g. the solid line in Fig. 1(a)] were normalized to the outer part of the SMC N2 autocorrelation and then subtracted to yield the speckle component. While the resulting speckle components [e.g. the dashed curve in Fig. 1(a)] thus showed different absolute amplitudes at the peaks, the positions of the peaks were of course invariant and the relative amplitudes of the peaks (which along with the positions are modelled in Section 3) varied much more slowly than their absolute levels.

In order to investigate the significance of the peaks and troughs present in the SMC N2 autocorrelation shown in Fig. 1(a), we also fitted a near-Gaussian to the dotted points and then compared the positive and negative excursions from this near-Gaussian, at separations larger than 0.2 arcsec, with the deviations expected on the basis of counting statistics alone. The ratio of the observed to expected deviations was found to range up to 25 for SMC N2, whereas for the case

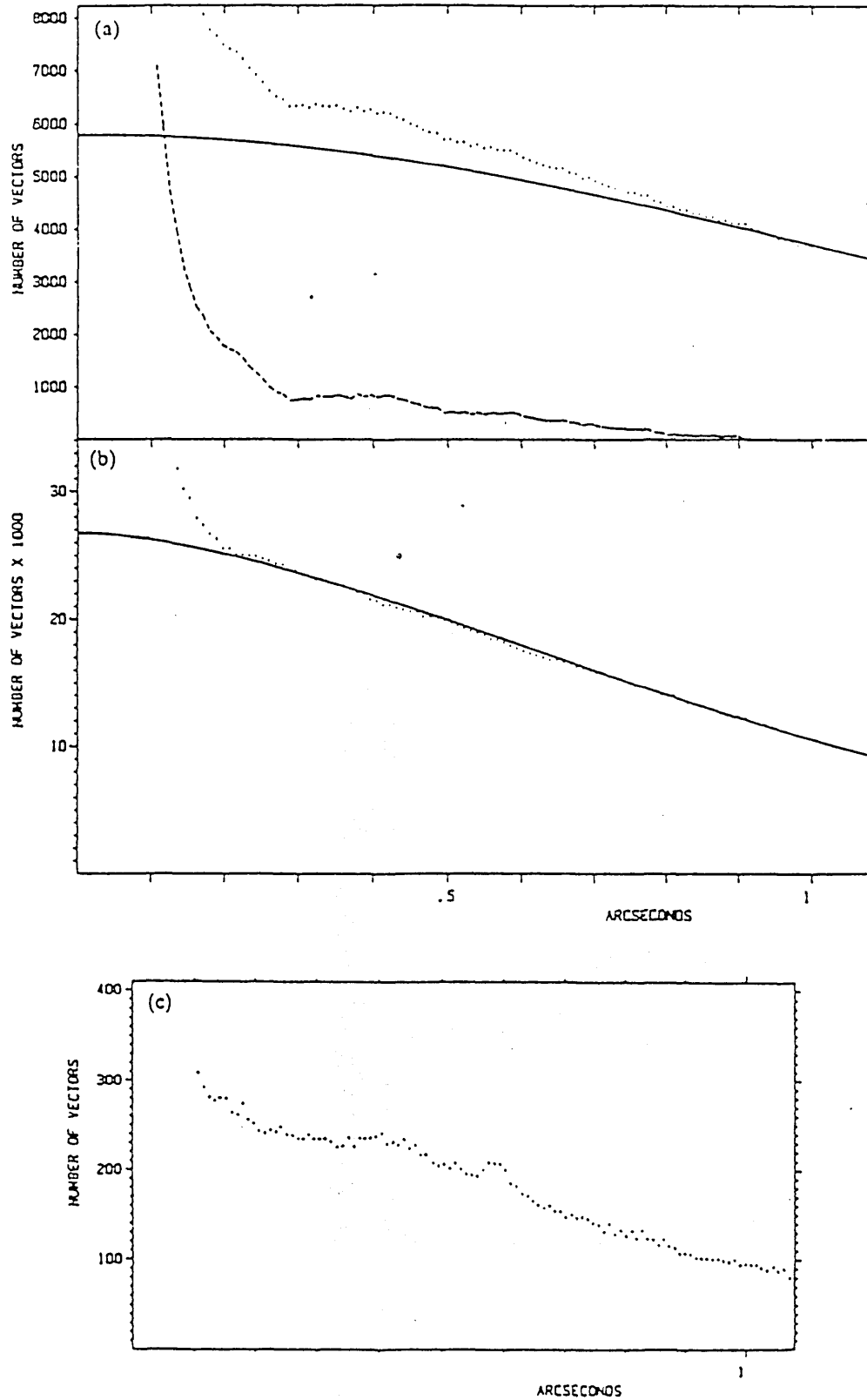


Figure 1. (a) and (b) The dotted plot in (a) (upper) shows the mean autocorrelation of a 60 min observation, made in 1983 November, of SMCN2 in the [O III] line. The dotted plot in (b) (lower) shows the mean autocorrelation of a 30 min observation, made through a neutral density filter and [O III] filter, of the nearby unresolved reference star SAO 255683. The solid curve in (b) is a near-Gaussian fit (see text) to the seeing pedestal of the SAO 255683 autocorrelation plot. The solid curve in (a), which is also a near-Gaussian fit to the seeing pedestal, has been normalized at 0.9 and 1.01 arcsec to the SMCN2 autocorrelation and subtracted to yield the speckle component, shown as a dashed curve in (a). (c) The mean autocorrelation of a 6 min observation of SMCN2 in the [O III] line, made in 1984 October.

of the near-Gaussian fitted to the reference star autocorrelation in Fig. 1(b), the ratio of observed to expected deviations did not exceed 10.

During our 1984 October AAT observing run, SMCN2 was reobserved with the same instrumental set-up as described above but, due to a recording equipment failure, only 6 min of useful data were obtained. Fig. 1(c) shows the resulting autocorrelation. Although the overall signal is much lower than for the 1983 data shown in Fig. 1(a), the peaks at 0.41 and 0.59 arcsec in the 1983 data can be clearly seen in the 1984 data as well.

3 Autocorrelation models

The presence of more than one peak in the autocorrelation function for SMCN2 suggests the presence of more than one peak in its brightness distribution. In order to model the intrinsic brightness distribution, we assumed azimuthal symmetry for SMCN2. The total number of counts which we obtained prevents us making detailed statements about the degree of symmetry.

Our approach was to autocorrelate a range of azimuthally symmetric input-model angular brightness distributions over all azimuthal angles and to compare the resulting plots with the speckle component in Fig. 1(a). Uniform, edge-brightened, or centrally peaked discs were found not to yield autocorrelation functions which resembled it in any way. Since many galactic PN exhibit ring structure, we therefore experimented with models having such peaks in their radial brightness distributions. We found it convenient to construct the distributions using functions of the type described by Harrington & Feibelman (1983). The angular intensity distribution of the best-fitting azimuthally symmetric model is shown in Fig. 2(a) and its autocorrelation is shown in Fig. 2(b), along with the observed speckle component. The model has a double-ring structure, with peaks at angular distances of 0.22 and 0.38 arcsec from the centre. The positions of the peaks in the autocorrelation plot are sensitive to the separation of the peaks in the radial-intensity model, allowing the positions of the latter to be determined to ± 1 pixel (± 0.018 arcsec). The feature at 0.22 arcsec in the autocorrelation plot corresponds to vectors between points on the inner ring and points on the outer ring at the same side of the nebula. The feature at 0.41 arcsec corresponds to vectors between points on the inner ring which are at opposite sides of the nebula, while the feature at 0.59 arcsec corresponds to vectors between points on the inner ring and points on the outer ring which are at opposite sides of the nebula. The position of the first autocorrelation peak is redundantly specified by the positions of the second and third peaks and its presence provides a valuable confirmation of the validity of the model. A fourth peak is expected to be present at about 0.76 arcsec, corresponding to vectors between points on the outer ring at opposite sides of the nebula, but our signal is too low at that separation to easily reveal it.

The relative strengths of the two peaks in the model intensity distribution determines the contrast of the features in the autocorrelation. A decrease in the relative intensity of the outer ring leads to a reduction in the contrast of the autocorrelation features at 0.22 and 0.59 arcsec, while an increase in the relative intensity of the outer ring causes them to become increasingly prominent relative to the feature at 0.41 arcsec. The closest fit to the observed contrast of the autocorrelation features was obtained with a double-ring model with an intensity ratio of 3:1 at the peaks [Fig. 2(a)].

The formulation of Harrington & Feibelman (1983) predicts a certain degree of central infilling for the projected intensity distribution of any spherically symmetric shell-like PN, due to the projection of emission from the outer regions on to the centre. However, we are able to set an upper limit to the brightness of any central infilling in SMCN2 of less than 20 per cent of the intensity at the peak of the inner ring, since any central infilling will contribute vectors which will tend to wash out the contrast of the features which correspond to vectors between the intensity peaks. In this respect, SMCN2 appears to resemble galactic PN such as the Ring Nebula NGC

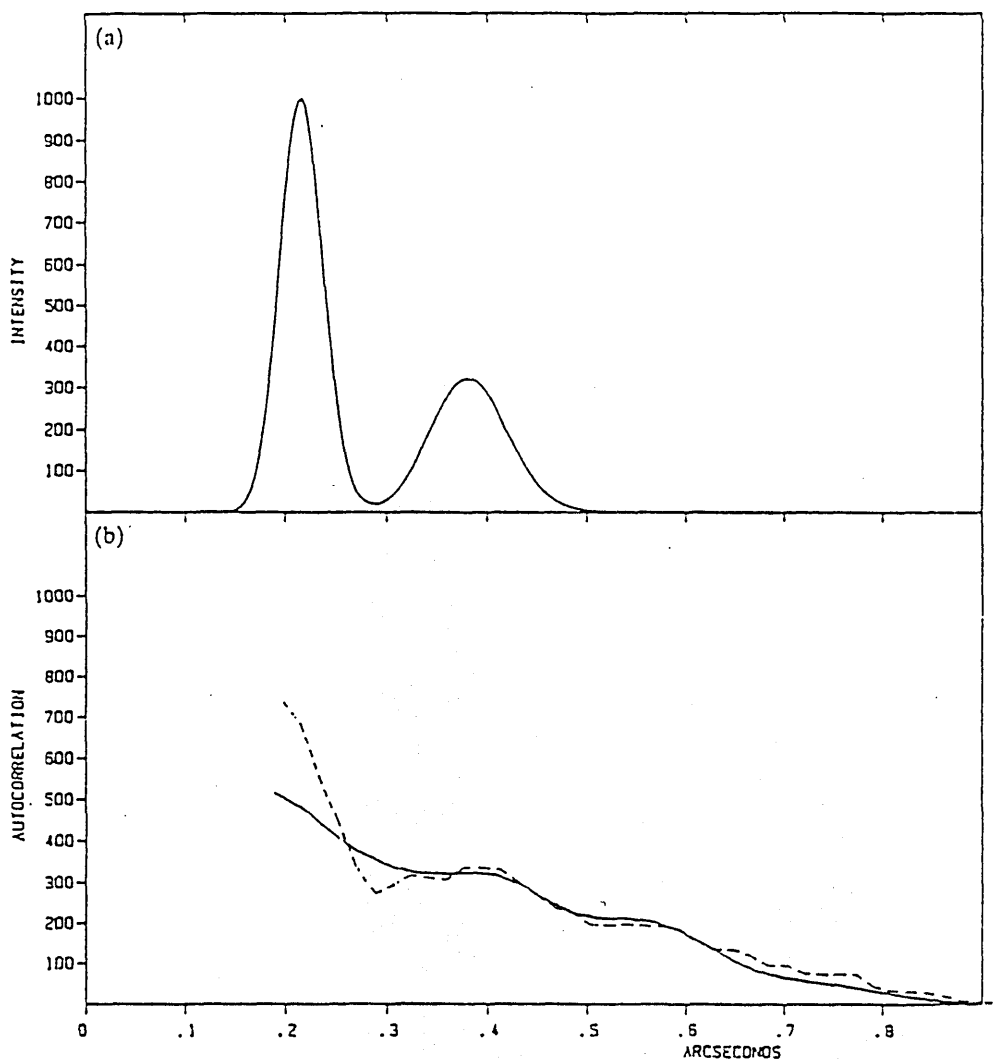


Figure 2. (a) (upper) shows the best-fitting model angular intensity distribution for SMCN2 in the $[O\text{III}]$ line, the autocorrelation of which yields the solid curve in (b) (lower). For comparison, the speckle component of SMCN2 [dashed plot in (b)] has been normalized to the model autocorrelation at a separation of 0.36 arcsec. The observed speckle component and the model autocorrelation are not plotted for separations less than 0.2 arcsec, because of the presence of the central photon spike in the former.

6720 (Atherton *et al.* 1978), which also exhibit a very low degree of central infilling. A toroidal structure, rather than a spherical shell, seen in projection, as assumed here, might provide an explanation for this behaviour. In converting our derived azimuthally symmetric angular intensity distribution into a spherically symmetric radial-density distribution (Section 4), we have assumed that Fig. 2(a) gives a good fit to the spherically symmetric radial emissivity distribution of the nebula as well as to the projected angular brightness distribution. Since the predicted differences between these two functions occur mainly in the central region (see fig. 5 of Harrington & Feibelman 1983), where little mass is contained, this seems a reasonable assumption.

In order to further confirm the validity of our modelling procedure, it appeared useful to us to obtain speckle observations of a compact PN whose subarcsec brightness distribution had been independently determined by other means. Very few such objects exist, but fortunately the VLA radio study of the galactic PN Vy 2-2, by Seaquist & Davis (1983), showed it to be a suitable comparison object. Their 15-GHz radio map shows Vy 2-2 to be dominated by a symmetric

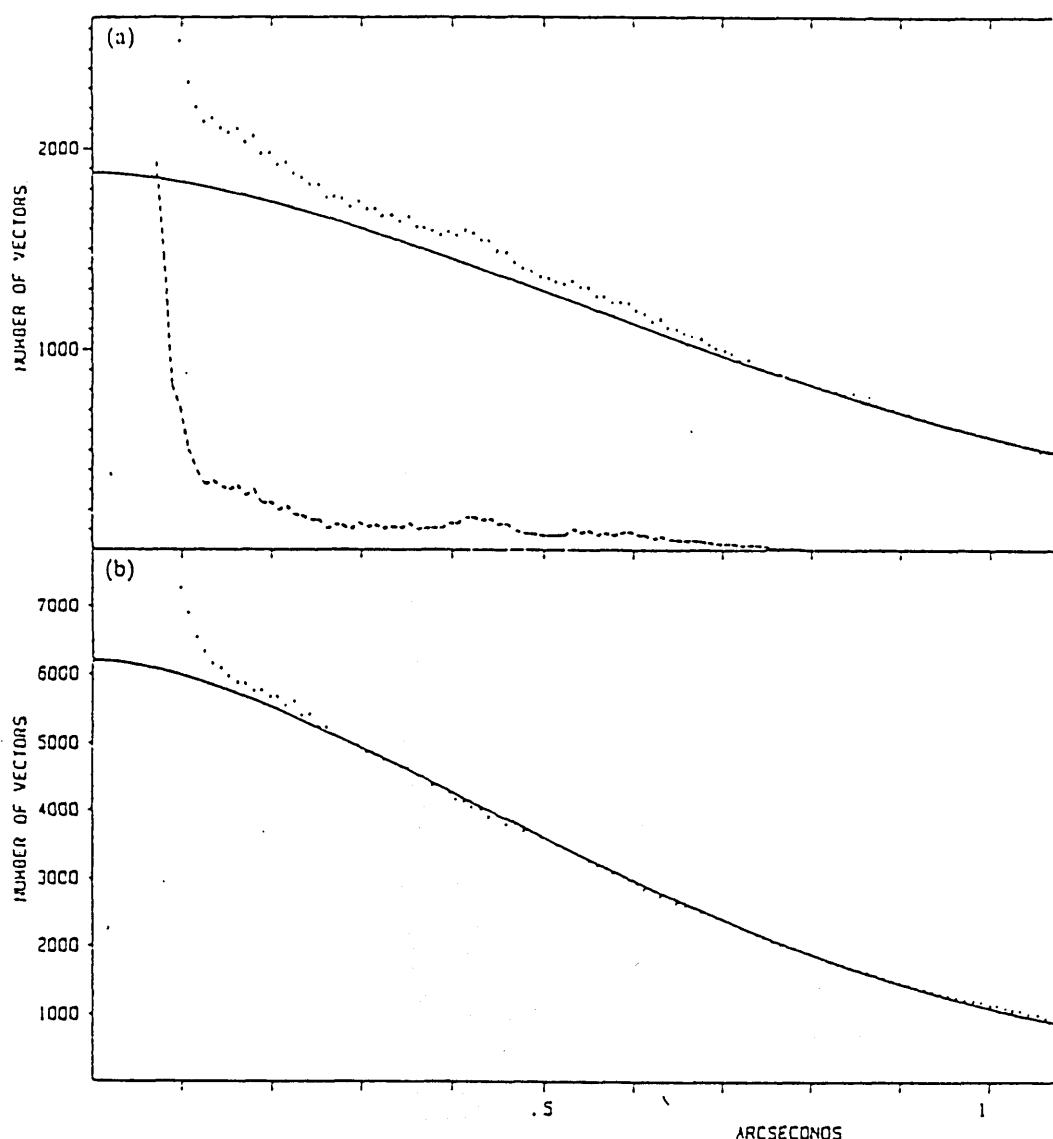


Figure 3. As for Fig. 1, but for an 11 min $H\beta$ observation of Vy 2-2, shown as the dotted plot in (a) (upper); and for a 6 min $H\beta$ observation of the nearby unresolved reference star SAO 104799, shown in (b) (lower). The fit to the seeing pedestal in (a) (solid curve) has been normalized to the mean autocorrelation of the observations at 0.79 and 1.00 arcsec and subtracted to yield the speckle component [dashed plot in (a)].

single-ring structure, with the peak of its intensity distribution at an angular radius of about 0.2 arcsec. Such a structure should give rise to a speckle interferometric autocorrelation plot with a single peak at about 0.4 arcsec, corresponding to vectors between points on the ring at opposite sides of the nebula. Since the VLA radio flux is due to hydrogen free-free emission, the $H\beta$ brightness profile should be the same as that found in the radio, provided no differential reddening exists across the face of the nebula. Following our arguments in Section 2, the [O III] brightness distribution should also be similar to that of $H\beta$. The blackbody hydrogen Zanstra temperature found by Shaw & Kaler (1985) for this optically thick PN was 41 000 K, putting it just above the minimum stellar effective temperature required for the [O III] brightness distribution to extend out to nearly the same radius as that of $H\beta$ (Section 2).

We observed Vy 2-2 with the AAT on 1984 October 12 and 13, imaging it through the [O III] filter and through a 10 Å bandpass $H\beta$ filter. Two different objectives were used to obtain two independent plate-scales. The dotted curve in Fig. 3(a) shows the autocorrelation of an 11-min

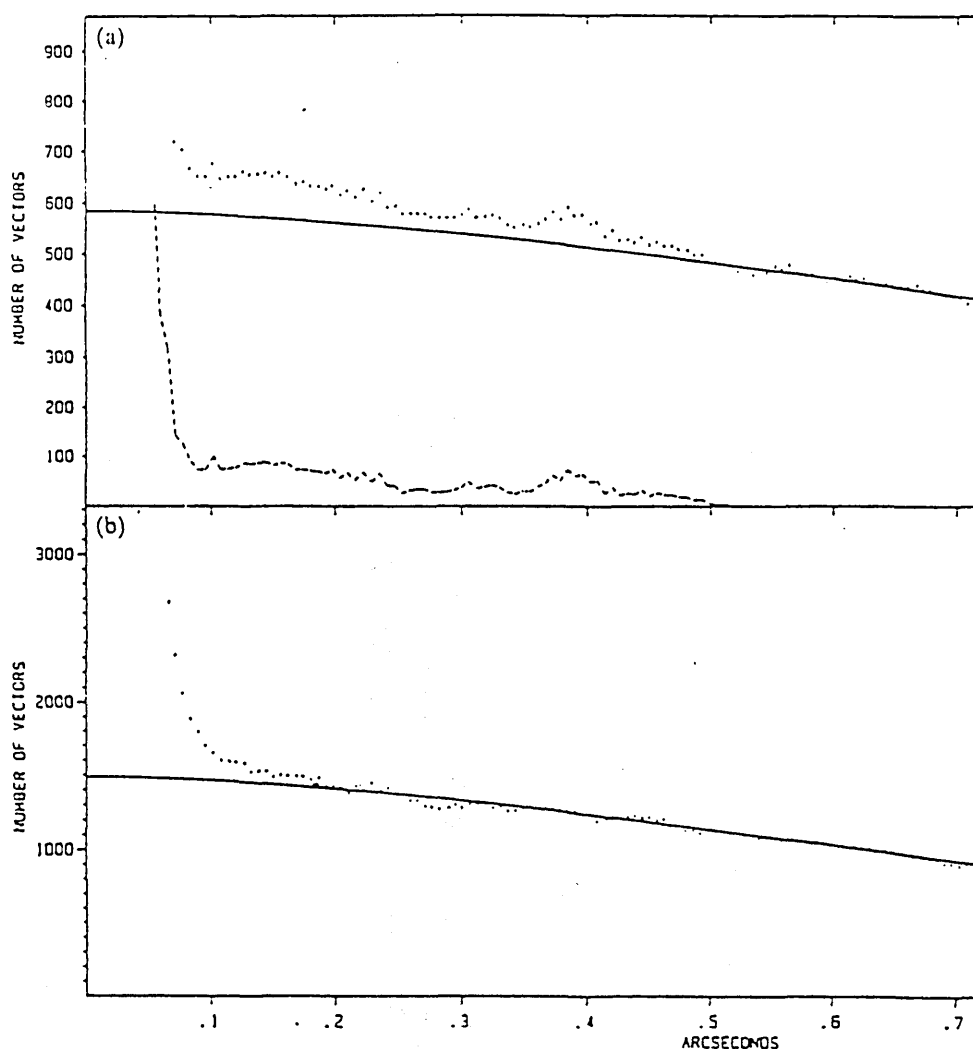


Figure 4. As for Fig. 1, but for a 6.5 min [O III] observation of Vy 2-2, shown in (a) (upper); and a 6 min [O III] observation of SAO 104799, made through a neutral density filter and shown in (b) (lower). The fit to the seeing pedestal in (a) (solid curve) has been normalized to the mean autocorrelation of the observations at 0.53 and 0.68 arcsec and subtracted to yield the speckle component [dashed plot in (a)]. These observations were made using a 50-mm objective, whereas all other observations were made using a 75-mm objective.

observation in the $H\beta$ line using a 75-mm objective, from which we have subtracted a fit to the seeing pedestal (solid line) to yield the speckle component (dashed line). The fit to the seeing pedestal was derived from observations, made immediately afterwards, of the nearby unrevolved reference star SAO 104799 [Fig. 3(b)]. The dotted plot in Fig. 4(a) shows the autocorrelation of a 6.5-min observation of Vy 2-2 in the [O III] line using a 50-mm objective, from which we have similarly subtracted a fit to the seeing pedestal derived from the adjacent observations of SAO 104799 which are shown in Fig. 4(b). The speckle components in Figs 3(a) and 4(b) both show a feature at about 0.4 arcsec, in agreement with expectations, with the [O III] feature at a slightly smaller separation than the $H\beta$ feature.

Model-brightness distributions have been constructed to fit the $H\beta$ and [O III] autocorrelations and are shown in Figs 5 and 6, respectively. The [O III] speckle component can be fitted by the autocorrelation of a single azimuthally symmetric ring peaking at a radius of 0.20 arcsec, while the radius at which the model $H\beta$ brightness distribution peaks is 0.22 arcsec. The degree of central

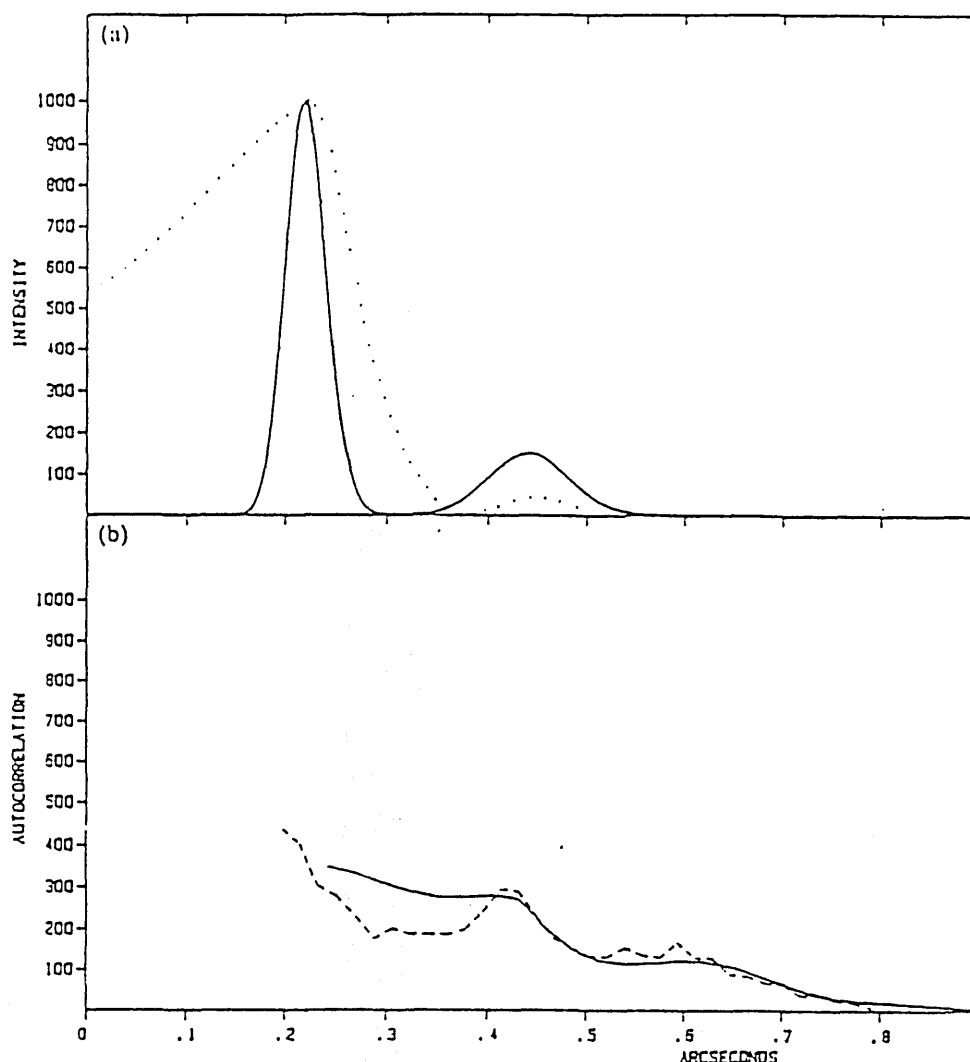


Figure 5. The solid curve in (a) (upper) is the best-fitting model angular intensity distribution for the $H\beta$ observations of Vy 2-2. Its autocorrelation is shown by the solid curve in (b), where it is compared with the speckle component of the mean autocorrelation of the Vy 2-2 observations, shown by the dashed plot. The dotted curve in (a) shows the free-free angular brightness distribution which Seaquist & Davis (1983) derived from their 15-GHz VLA radio observations of Vy 2-2.

infilling in $H\beta$ and $[O III]$ is constrained by our data to be less than about 10 per cent of the peak brightness – this is significantly lower than implied by the 15-GHz radio map in fig. 3 of Seaquist & Davis (1983): the dotted curve in Fig. 5(a) shows the north-south radial brightness distribution found by the VLA. Cobb & McCarthy (1986), in a comparative study of deconvolution techniques for infrared speckle interferometry, have found that the 'CLEAN' algorithm, used for restoring VLA data, is not good for resolving closely spaced delta functions, since it tends to produce significant infilling. In other respects our model $H\beta$ brightness distribution shows encouraging agreement with the VLA data. For example, it was found that a weak feature is present in our $H\beta$ autocorrelation plot at a separation of about 0.6 arcsec and required a weak secondary peak in the model brightness distribution at a radius of about 0.4 arcsec [Fig. 5(a), solid curve]. This weak secondary peak is also present in the VLA cross-cut [Fig. 5(a), dotted curve], although at a lower amplitude.

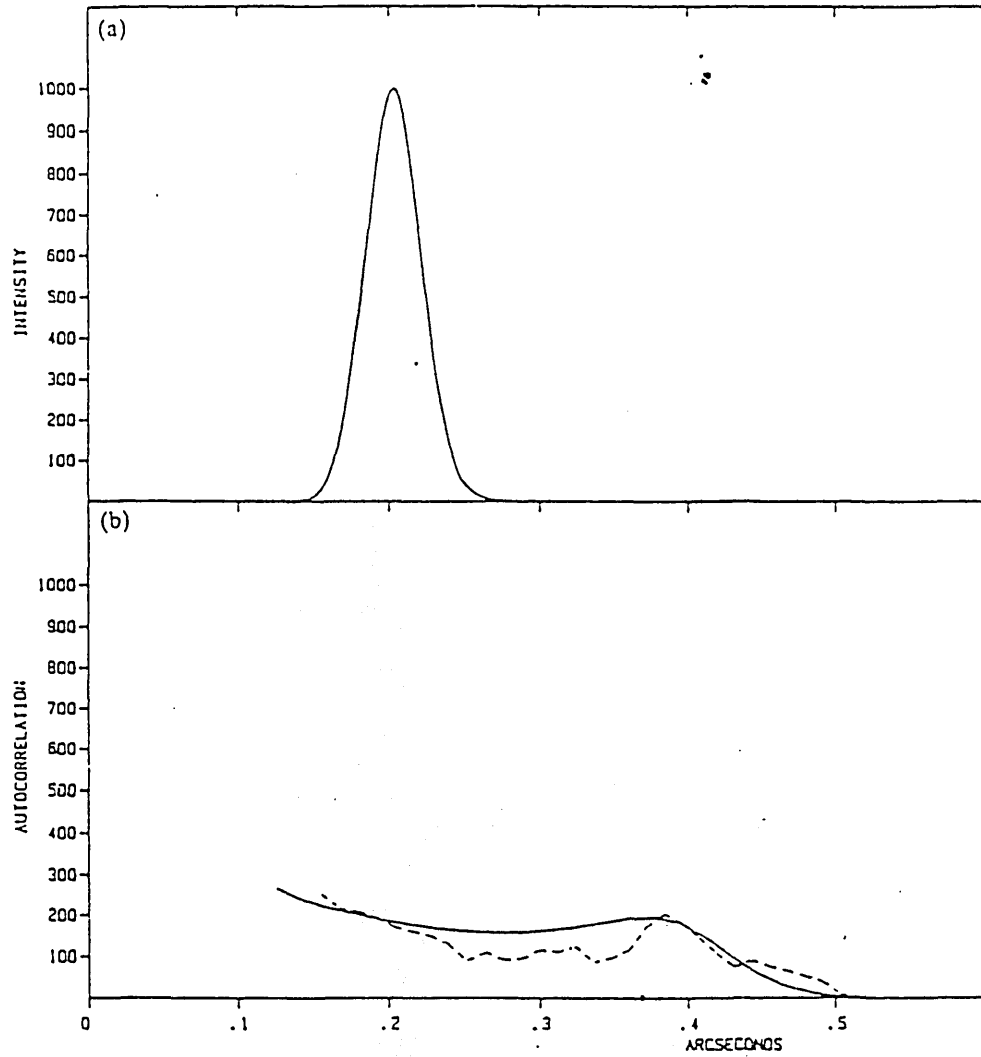


Figure 6. As for Fig. 5, but for the [O III] observations of VY 2-2.

4 The mass of SMC N2

We shall assume that the [O III] brightness profile for SMC N2 [Fig. 2(b)] is the same as that for H β . Our nebular ionization structure model for SMC N2 (Appendix) shows this to be a good approximation. The volume emissivities of both [O III] and H β are proportional to $\epsilon n_e n_i$, where n_e is the local electron density, n_i is the local ion density and ϵ is the filling factor. The total H β flux emitted by a nebula at distance D is given by

$$4\pi D^2 I(\text{H}\beta) = \gamma \alpha_{\text{eff}}(\text{H}\beta) h\nu_\beta \int \epsilon n_{\text{H}}^2(r) 4\pi r^2 dr \quad (1)$$

where $I(\text{H}\beta)$ is the dereddened H β flux observed at Earth, $\alpha_{\text{eff}}(\text{H}\beta)$ is the partial recombination coefficient of hydrogen for the emission of H β photons of energy $h\nu_\beta$ (from Brocklehurst 1971), and $n_e = \gamma n(\text{H}^+)$ where $\gamma = [1 + n(\text{He}^+)/n(\text{H}^+) + 2n(\text{He}^{2+})/n(\text{H}^+)]$. Similarly, $I(r)$, the line emissivity distribution as a function of radius r , is given by

$$I(r) = K \gamma \alpha_{\text{eff}}(\text{H}\beta) h\nu_\beta \epsilon n_{\text{H}}^2(r) \quad (2)$$

where K is a constant. Substituting from (2) into (1) gives

$$K = [I(\text{H}\beta)D^2]^{-1} \int I(r)r^2 dr. \quad (3)$$

Using the function which generated $I(r)$ in Fig. 2(a), we can numerically evaluate $\sum I(r)r^2 \Delta r$ and thereby determine the value of K using equation (3). We can then derive $n_{\text{H}}(r)\sqrt{\epsilon}$ since, by (2), we have

$$n_{\text{H}}(r)\sqrt{\epsilon} = \left[\frac{I(r)}{K\gamma\alpha_{\text{eff}}(\text{H}\beta)h\nu_{\text{H}}} \right]^{1/2}. \quad (4)$$

In Fig. 7 we plot $n_{\text{H}}\sqrt{\epsilon}$ as a function of both the angular radius θ and the absolute radius r . For the numerical evaluation of $n_{\text{H}}\sqrt{\epsilon}$ from $I(\text{H}\beta)$, we have used the functions defined in the appendix of Harrington & Feibelman (1983). The coefficients, defined there, which yield the functions shown in Figs 2(a) and 7 are $k=2$, $n=25$, $h_1=0.043$, $h_2=0.076$, $a'_{1,25}=1.0$, $a'_{2,25}=0.32$, all other $a'_{k,n}=0$, and $N_0=3.3 \times 10^{-9} \text{ cm}^{-3}$. For the evaluation of $n_{\text{H}}\sqrt{\epsilon}$ we adopted an observed $F(\text{H}\beta)=1.70 \times 10^{-13} \text{ erg cm}^{-2} \text{ s}^{-1}$, from the fluxes measured by Osmer (1976) and Webster (1983). We adopted a reddening of $E(B-V)=0.06$, corresponding to $E(B-V)=0.02$ by galactic foreground reddening (McNamara & Feltz 1980) plus $E(B-V)=0.04$ from dust within the SMC (Caldwell & Coulson 1985). The dereddened $\text{H}\beta$ flux is thus $I(\text{H}\beta)=2.07 \times 10^{-13} \text{ erg cm}^{-2} \text{ s}^{-1}$. We used $T_e=13\,400 \text{ K}$, the $[\text{O III}]$ electron temperature from the work of Aller *et al.* (1981), for the evaluation of $\alpha_{\text{eff}}(\text{H}\beta)$. From their results we also obtained $\gamma=1.13$. Finally, we adopted a distance of 57.5 kpc for the SMC, corresponding to a distance modulus of 18.8 (Rich, Da Costa & Mould 1984; Reid & Strugnell 1986).

The mass of the ionized nebula is given by

$$\begin{aligned} M(\text{PN}) &= \mu m_{\text{H}} \int \epsilon n_{\text{H}}(r) 4\pi r^2 dr \\ &= 4\pi \mu m_{\text{H}} \sqrt{\epsilon} \int n_{\text{H}}(r) \sqrt{\epsilon} r^2 dr, \end{aligned} \quad (5)$$

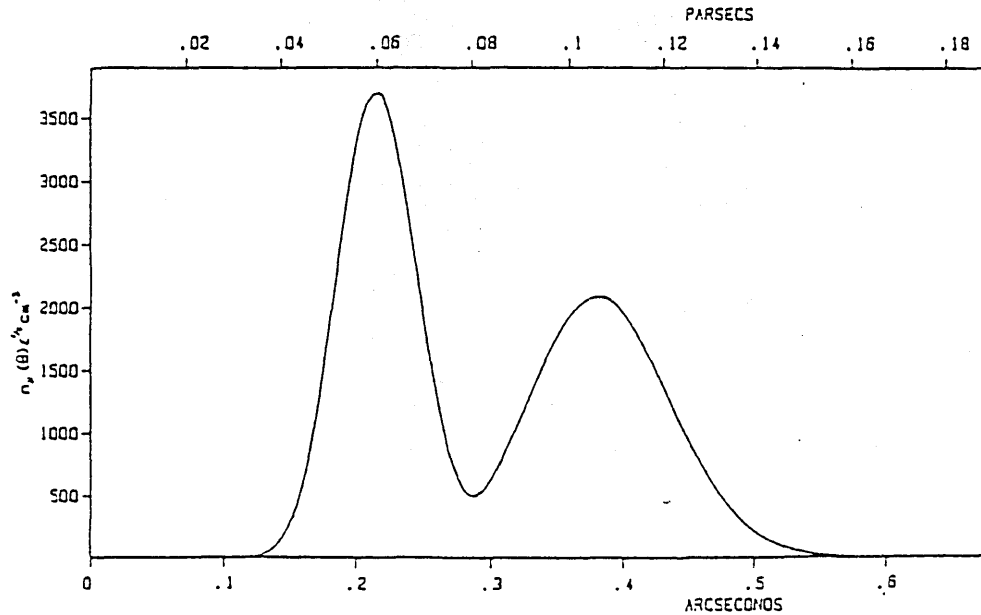


Figure 7. A plot of the derived density distribution of SMCN2, $n_{\text{H}}\epsilon^{1/2}$ (see text), as a function of angular radius (lower abscissa) and absolute radius (upper abscissa).

where m_{H} is the mass of a hydrogen atom and $\mu = [1 + 4n(\text{He})/n(\text{H})]$. The results of Aller *et al.* (1981) yield $\mu = 1.43$ for SMCN2. We have numerically evaluated the integral in (5) to obtain

$$M(\text{PN}) = 0.53 \sqrt[3]{\epsilon} M_{\odot}. \quad (6)$$

We find that 76 per cent of the mass (versus 64 per cent of the emission) is located in the outer shell.

In order to determine the nebular mass, it now remains to derive the value of the filling factor for SMCN2. To do this we have made use of observations of the electron-density-sensitive [O II] 3726, 3729 Å doublet ratio. The mean nebular electron density $\langle n_e \rangle$ yielded by this ratio is a measure of the local electron density in the nebula averaged over the entire nebula. In a manner similar to equation (3) of Harrington & Feibelman (1983), we can write

$$\langle n_e \rangle = \frac{\int n_e \epsilon n_{\text{H}} r^2 dr}{\int \epsilon n_{\text{H}} r^2 dr} \quad (7)$$

This can be written as

$$\langle n_e \rangle \sqrt[3]{\epsilon} = \gamma \frac{\int n_{\text{H}}^2 \epsilon r^2 dr}{\int n_{\text{H}} \epsilon^{1/2} r^2 dr} \quad (8)$$

if we assume that the filling factor is the same throughout the nebula and that O^+ has the same radial distribution in the nebula as H^+ . Since we know $n_{\text{H}} \sqrt[3]{\epsilon}$ as a function of r (Fig. 7), we can evaluate the integral in (8). We obtain $\langle n_e \rangle \sqrt[3]{\epsilon} = 1970 \text{ cm}^{-3}$. The value of $\langle n_e \rangle$ for SMCN2 was derived from the [O II] doublet ratio which was measured as part of PATT service observations at the AAT on 1985 January 4 of a number of Magellanic Cloud PN (Barlow & Walsh, in preparation). The 3726.0/3728.7 line intensity ratio was found to be 1.69, in agreement with the value of 1.7 measured by Webster (1976). A ratio of 1.69, in conjunction with an electron temperature of 13 400 K, gives $\langle n_e \rangle = 2850 \text{ cm}^{-3}$, using the collision strengths of Pradhan (1976) and the transition probabilities of Zeippen (1982). Thus we obtain $\epsilon = 0.48$ and $M(\text{PN}) = 0.37 M_{\odot}$, when assuming that O^+ has the same distribution as H^+ . A better estimate of ϵ can be obtained from an ionization structure model constructed to reproduce the observed absolute emission-line fluxes from SMCN2. Such a model is described in the Appendix, where it is shown that the observed [O II] 3726/3729 ratio of 1.69 implies $\epsilon = 0.45$.

We therefore have for SMCN2

$$\epsilon = 0.45$$

$$M(\text{PN}) = 0.36 M_{\odot}. \quad (9)$$

The inner nebular shell contains $0.09 M_{\odot}$, while the outer shell contains $0.27 M_{\odot}$.

We now consider how sensitive the derived nebular mass is to uncertainties in the input parameters and assumptions.

- (i) Our speckle observations are fitted by a double-ring structure. We have made the standard

assumption that these rings correspond to spherical shells seen in projection on the sky. We may also consider the case where the observed rings are instead due to tori whose planes happen to lie perpendicular to our line-of-sight. Such structures might arise, for example, as a result of equatorial rather than spherical ejection by the central star. If ω is the solid angle occupied by the tori, as seen from the central star, then we may repeat the previous analysis by substituting ω for 4π on the right-hand side of equation (1). Equation (2) is unchanged, while the right-hand side of equation (3) is multiplied by $\omega/4\pi$. Since the other parameters are unchanged, the value of K derived from equation (3) will be a factor of $\omega/4\pi$ smaller than for the spherical case and $n_{\text{H}}(r)\sqrt{\epsilon}$, given by equation (4), will be a factor of $(4\pi/\omega)^{1/2}$ larger. Since its numerator is fixed by the $\text{H}\beta$ flux, equation (8) implies that $\langle n_e \rangle \sqrt{\epsilon}$ changes by the inverse of the product of $\omega/4\pi$ and $n_{\text{H}}(r)\sqrt{\epsilon}$, i.e. $\langle n_e \rangle \sqrt{\epsilon}$ is proportional to $(4\pi/\omega)^{1/2}$. Since $\langle n_e \rangle$ is fixed by the $[\text{O II}]$ observations, this implies that the value of $n_{\text{H}}(r)$ is the same for both the spherical shell and the torus cases, and that only ϵ changes, being proportional to $4\pi/\omega$. Since the filling factor ϵ cannot exceed unity and since the spherical case gave $\epsilon=0.45$, we see that the smallest covered fraction of the sky that is allowed for the torus case is equal to 0.45. The mass of the PN for the torus case is given by equation (5) with a reduction in the solid angle from 4π to ω , which is exactly balanced by the increase in ϵ by $4\pi/\omega$. The derived nebular mass is therefore the same for both geometries.

(ii) The quantities best determined by the speckle observations are the angular radii of the radial intensity peaks shown in Fig. 2(a). With these radii fixed, we have investigated the effects of varying the widths and relative intensities of the two peaks. The nebular mass given by equation (5) is proportional to the product of $\sqrt{\epsilon}$ and the integral of $\sqrt{\epsilon} n_{\text{H}}(r) r^2 dr$. The former quantity is given by equation (8) which, for a fixed $\text{H}\beta$ flux, has a fixed numerator, from equation (1). We can therefore substitute for $\sqrt{\epsilon}$ from equation (8) into equation (5), to see that $M(\text{PN})$ is invariant for a fixed value of $\langle n_e \rangle$. For example, when the FWHM's of the radial intensity peaks in Fig. 2(a) are both increased by 25 per cent, the autocorrelation that results is smoother and gives a significantly poorer fit to the observed speckle component in Fig. 2(b), while the derived filling factor decreases from 0.45 to 0.36, and the total nebular mass remains unchanged. If, on the other hand, the ratio of the areas of the two radial intensity peaks is changed from the value of 2.2 shown in Fig. 2(a) to 1.0 (i.e. identical peaks but unchanged positions), the autocorrelation that results gives a significantly poorer fit to the speckle component in Fig. 2(b), the derived filling factor increases to 0.64, but the derived nebular mass is again unchanged.

(iii) The derived nebular mass is also insensitive to the radii of the intensity peaks. From equations (3) and (4), changing the radii by a factor of y leads to a change $y^{-3/2}$ in $n_{\text{H}}\sqrt{\epsilon}$ at the new radius yr , relative to the previous value of $n_{\text{H}}\sqrt{\epsilon}$ at radius r . The integral of $n_{\text{H}}\epsilon^{1/2} r^2 dr$ therefore changes by a factor of $y^3/y^{3/2}$. Since the numerator of equation (8) is unchanged, the value of $\sqrt{\epsilon}$ changes by $y^{-3/2}$. Because the terms outside and inside the integral in equation (5) for the nebular mass change by factors of $y^{-3/2}$ and $y^{3/2}$, the nebular mass remains the same. This result holds even if the radii of the two peaks are changed by different factors. However, the radii of the shells cannot be reduced by more than 30 per cent or else the derived filling factor will exceed unity. Our observations in fact constrain the radii of the peaks of the shells to ± 6 per cent.

(iv) Finally, the formal uncertainty of ± 0.08 , from counting statistics, for our $[\text{O II}]$ doublet ratio translates into an uncertainty of ± 15 per cent in the derived nebula mass.

We conclude that it is primarily the filling factor ϵ which is sensitive to the uncertainties in our fitting procedures, our best estimate for ϵ being 0.45. The total nebular mass, on the other hand, is surprisingly insensitive to these uncertainties. The uncertainty in the distance modulus of the SMC is probably the most important source of error in the mass estimate. As discussed in Section 1, a distance modulus uncertainty of ± 0.3 mag translates into a mass uncertainty of ± 30 per cent, giving us $M(\text{PN}) = 0.36 \pm 0.12 M_{\odot}$ for SMC N2.

5 The expansion age of SMC N2

Having determined an absolute radius for SMC N2, we are in a position to derive an expansion age for the nebula from its measured expansion velocity. The spectrum illustrated in Fig. 8 is the profile of the [O III] 5007 Å line emitted by SMC N2. It was obtained for us by Dr J. Meaburn using the University of Manchester Echelle Spectrometer (Meaburn *et al.* 1984) on the AAT. The projected slit width was 1 arcsec and the exposure time was 350 s. The instrumental resolution, as determined from the FWHM of comparison arc lines, was 12.9 km s^{-1} . The observed profile of the [O III] line in Fig. 8 is due to a convolution of the intrinsic line profile with the instrumental profile, so the FWHM of 36.9 km s^{-1} found for the observed profile using a single Gaussian fit (dashed curve in Fig. 8) implies an intrinsic line profile FWHM of 34.6 km s^{-1} after deconvolution. We may attribute the observed profile to a superposition of the effects of the expansion of the nebula along with the thermal line broadening. The latter depends on the mass of the ion and for $T_e(\text{O III}) = 13\,400 \text{ K}$ the predicted thermal line broadening corresponds to a Gaussian with a FWHM of 6.1 km s^{-1} . After deconvolution we thus obtain a FWHM of 34.1 km s^{-1} which can be attributed to the effects of nebular expansion alone. Robinson *et al.* (1982) have shown that a spherically symmetric nebula with an expansion velocity proportional to radius (as seems to be implied by data on other PN, see below) will yield a line profile whose FWHM is nearly equal to twice the nebular expansion velocity at the radius of peak emission, if the nebula is observed with an aperture larger than the nebular diameter. On this basis we derive an expansion velocity of 17 km s^{-1} which, with the outer shell radius of 0.105 pc derived in Section 4, implies an expansion age of 6000 yr for SMC N2, provided the velocity has been constant with time.

Since our speckle observations have shown that SMC N2 possesses two shells, we have also fitted the observed [O III] profile with two Gaussians, whose relative intensities, central positions and FWHM's were allowed to vary until the residuals between the observed and computed profiles were minimized (to do this we used the line fitting program ELF, written by Dr P. J. Storey, within the STARLINK package DIPSO of Howarth & Maslen 1984). The best fit was obtained with components which were offset by 4 km s^{-1} and whose FWHM's were 29.6 and

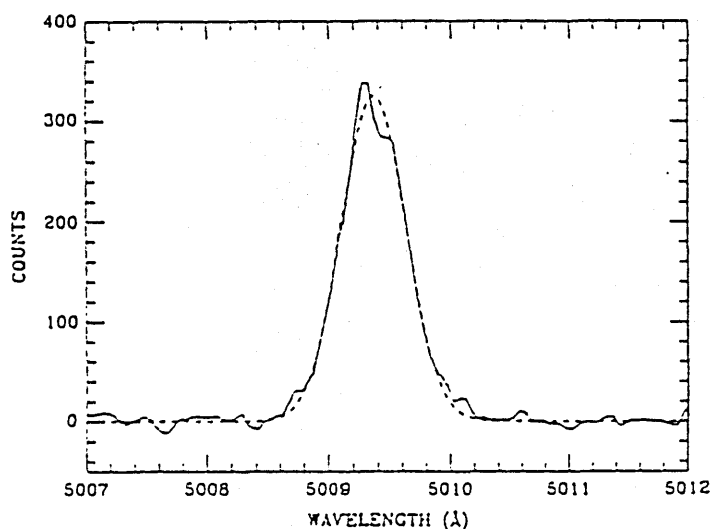


Figure 8. The profile of the [O III] 5007 Å line emitted by SMC N2, obtained with the University of Manchester Echelle Spectrometer on 1983 November 17 (see text). The dashed curve shows a single-Gaussian fit to the observed profile and has a FWHM of 36.9 km s^{-1} . The centroid of the Gaussian fit yields a heliocentric radial velocity of $+164.8 \text{ km s}^{-1}$ for SMC N2.

44.1 km s⁻¹, respectively contributing 42 and 58 per cent of the total emission. After correction for the effects of instrumental and thermal line broadening, these widths translate into expansion velocities of 13 and 20.8 km s⁻¹ for the two components. Since our speckle results (Section 4) gave similar relative intensities for the inner and outer nebular shells (36 and 64 per cent), we respectively identify the lower and higher expansion velocities with the inner and outer shells. This identification is supported by the fact that the ratio of the expansion velocities of the two components is nearly equal to the ratio of the radii of the two shells – velocity mappings of many galactic PN have shown that the expansion velocity within a nebula usually increases linearly with angular radius (Osterbrock 1974, p. 156).

Observational studies (e.g. Smith 1971; Robinson, Reay & Atherton 1982) have also shown that for a sample of galactic PN with some assumed common distance scale, the measured expansion velocity for each PN appears to be proportional to its absolute outer nebular radius. We can derive expansion ages for the two shells of SMC N2 on the basis of this model if we assume that each shell was initially ejected with the same constant expansion velocity, u , and thereafter attained a velocity at radius r given by $v(r) = u + k(r - R)$, where k is a constant (different for each shell) and R is the radius at which acceleration first occurred [$v(r) = u$ for $r \leq R$]. If we adopt $u = 10$ km s⁻¹, representative of AGB wind expansion velocities (Knapp & Morris 1985), we derive expansion ages of 5000 and 7000 yr for the inner and outer shells, respectively, if we assume that both shells started accelerating at $r = R = 0$. If we assume that both shells started accelerating at the time that the second shell was ejected then we derive expansion ages of 5000 and 8000 yr for the inner and outer shells. If we adopt an initial velocity of $u = 5$ km s⁻¹, which is at the lower end of measured AGB wind expansion velocities, then the assumption of acceleration starting at $r = 0$ leads to ages of 7000 and 9000 yr for the inner and outer shells, while the assumption that the acceleration commenced when the second shell was ejected leads to ages of 7000 and 12 000 yr for the two shells. Finally, the assumption that the outer shell at 0.105 pc has always had its current expansion velocity of 20.8 km s⁻¹ leads to a lower limit of 5000 yr for the expansion age of the nebula, while constant velocity expansion of the 0.06 pc radius inner shell at its current velocity of 13 km s⁻¹ would imply a minimum expansion age of 4500 yr.

6 Discussion

SMC N2 is one of three Magellanic Cloud PN for which Stecher *et al.* (1982) derived stellar luminosities, from *IUE* data, which were close to the theoretical maximum for core masses below the Chandrasekhar limit. For SMC N2, the central star mass of $0.9 M_{\odot}$, which they obtained from their derived luminosity of $33\,000 L_{\odot}$, is so high that the time-scale for a decline of the initial stellar luminosity by a factor of 10 should only be a few hundred years ($T \propto M^{-9.6}$, Iben & Renzini 1983). Such a time-scale is incompatible with the nebular expansion age of greater than 5000 yr which is implied by our data. Tylenda (1984) has also argued, on independent grounds, that the stellar luminosities and masses were overestimated by Stecher *et al.*

In the Appendix we describe how we have analysed the available data on SMC N2 in order to derive the nebular ionization structure and the central star parameters. We used the Stoy energy balance method, and the stellar He II Lyman continuum photon flux implied by the nebular He II 4686 Å recombination line flux, in order to define the range of allowable central star temperatures and luminosities, and then constructed nebular ionization structure models using the code described by Harrington *et al.* (1982), so as to find the best-fitting case. We found that no stellar blackbody model could satisfy the observations. The best-fitting model was produced by a central star with an NLTE energy distribution corresponding to $T_{\text{eff}} = 110\,000$ K and $\log g = 5.75$, and a luminosity of $4340 L_{\odot}$. Neglecting distance modulus uncertainties, the uncertainty in the derived luminosity is $\pm 400 L_{\odot}$. Our derived luminosity of $4340 L_{\odot}$ is a factor of 7.6 smaller than

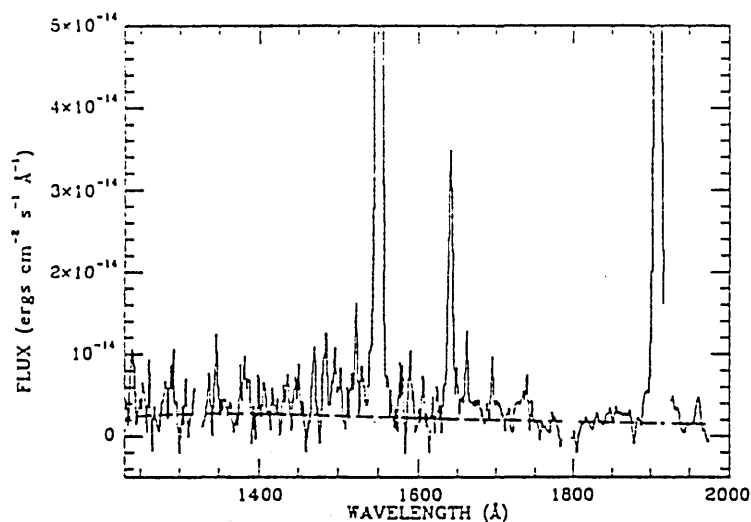


Figure 9. The observed *IUE* low-resolution spectrum of SMCN2, formed by merging the spectra from SWP 20454 and SWP 20527. Gaps in the spectrum indicate the positions of rescans. The dashed curve shows the predicted reddened continuum, formed by adding the theoretical nebular continuum to the continuum of the NLTE stellar model which provides the best fit to the nebular observations (see Appendix).

that derived by Stecher *et al.*, of which only a factor of 1.3 can be attributed to the slightly smaller distance modulus adopted here. We attribute most of the discrepancy to the fact that Stecher *et al.*'s luminosity was based on a He II Zanstra temperature determination at 1640 Å. As is apparent from Fig. 9, the continuum signal-to-noise level in the *IUE* SWP spectra of SMCN2 is sufficiently low that very large uncertainties can arise from this method.

The central star mass can be derived by plotting its luminosity and temperature on an H-R diagram and interpolating between the central star evolutionary tracks which have been published for a range of core masses. Interpolation between the hydrogen-burning tracks calculated by Schönberner (1981, 1983) yielded a central star mass of $0.593 M_{\odot}$ and an evolutionary age since T_{eff} was equal to 5000 K of 8100 yr. A comparison with the hydrogen-burning tracks calculated by Wood & Faulkner (1984) yielded a central star mass of $0.60 M_{\odot}$ and an evolutionary age since T_{eff} was equal to 10 000 K of about 12 000 yr. The alternative Wood & Faulkner case of PN ejection during a helium shell flash, followed by evolution while helium-burning, gives a central star mass of $0.64\text{--}0.65 M_{\odot}$ and an evolutionary age of only 3000 yr. The L , T_{eff} position of SMCN2 implies that it has reached the high-temperature edge of its evolutionary track and begun to slowly decline in luminosity, so that the Paczynski (1971) relation for steady hydrogen-shell burning cannot be used to derive the core mass. The luminosity just after leaving the AGB would have been $M_{\text{bol}} = -4.65$ for a $0.593 M_{\odot}$ core, according to the Schönberner hydrogen-burning evolutionary tracks, and $M_{\text{bol}} = -5.2$ according to the Wood & Faulkner helium-burning tracks. The hydrogen-burning central star evolutionary age of 8000–12 000 yr since leaving the AGB region is compatible with the nebular expansion ages derived in Section 5, whereas the helium-burning evolutionary age of 3000 yr is uncomfortably short compared to the minimum nebular expansion age of 5000 yr estimated in Section 5.

Our derived mass of $0.59 M_{\odot}$ and radius of $0.181 R_{\odot}$ for the central star of SMCN2 imply an effective gravity corresponding to $\log g = 5.7$, consistent with the central star surface gravity of $\log g = 5.75$ used for our nebular ionization structure modelling (Appendix). The C/O abundance ratio found from our nebular ionization structure modelling was 4.5 ± 1 (Maran *et al.* 1982 found C/O=2). Maran *et al.* concluded that the AGB progenitor was a Carbon star, but the luminosity

of $M_{\text{bol}} = -6.6$ which Stecher *et al.* (1982) derived for the central star was considerably brighter than the highest luminosity found for Magellanic Cloud Carbon stars: $M_{\text{bol}} = -6$ (e.g. Cohen *et al.* 1981; Wood, Bessell & Fox 1983; Iben 1984). The initial luminosity of $M_{\text{bol}} = -4.7$ to -5.2 derived here for SMCN2 instead puts it squarely within the $M_{\text{bol}} = -4$ to -6 range found for Magellanic Cloud Carbon stars. The derived luminosity is insensitive to the assumed radial structure of the nebula, although both the present luminosity, and that of Stecher *et al.* (1982), are based on the assumption of a spherically symmetric nebula. In the event of a toroidal geometry, our analysis in Section 4 showed that the smallest fraction of the sky that could be occupied by the nebula, as seen from the star, is 0.45. The maximum stellar luminosity for the case of toroidal geometry is therefore $4340/0.45 = 9640 L_{\odot}$ ($M_{\text{bol}} = -5.2$), corresponding to a core mass of $0.67 M_{\odot}$ for hydrogen-burning and an evolutionary age of 2000 yr. Since this is significantly shorter than the minimum nebular expansion age and since the increase in stellar luminosity would lead to a higher degree of ionization in the nebula than observed, we prefer the results of the spherically symmetric analysis.

An AGB progenitor star mass of about $1 M_{\odot}$ is implied by the hydrogen-burning stellar core mass of $0.59 M_{\odot}$ and the nebular shell mass of $0.36 M_{\odot}$ which we have derived. The shell mass is significantly larger than the $0.11 M_{\odot}$ predicted for a $0.59 M_{\odot}$ core by Wood *et al.* (1983) from their derivation of AGB star pulsational masses assuming first overtone pulsation and core masses assuming hydrogen-shell burning. Their analysis did predict a shell mass of $0.36 M_{\odot}$ for a hydrogen-shell burning core mass at the tip of the AGB of $0.64 M_{\odot}$, which, if the star had since switched to helium-shell burning, would be consistent with our nebular mass of $0.36 M_{\odot}$ and our core mass of $0.64 M_{\odot}$ derived for the case of helium-burning. However, as discussed above, the helium-burning stellar evolutionary age of 3000 yr is rather short compared to the minimum nebular expansion age of 5000 yr or the most likely expansion age of 8000 yr. Another possibility comes from the alternative analysis by Wood *et al.* of their AGB star data for the case of fundamental mode pulsation (and hydrogen-shell burning as before). This yielded a total pulsational mass of just under $1.0 M_{\odot}$ for a core mass of $0.60 M_{\odot}$, i.e. a shell mass of about $0.4 M_{\odot}$, which appears to be consistent with our results. Their analysis predicts that a small change in core mass should be accompanied by a large change in shell mass, so it will be of interest to see if this is confirmed by the extension of the type of analysis presented here to a larger sample of planetary nebulae.

Acknowledgments

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Appendix: An ionization structure model for SMC N2

In order to determine more accurately the filling factor in SMC N2, and thus the nebular mass, and secondly to determine the central star parameters, we have produced a spherically symmetric ionization structure model for the nebula using the code written by Dr J. P. Harrington (see Harrington *et al.* 1982, and Clegg *et al.* 1986 for a description) and implemented on the UCL STARLINK node by Dr R. E. S. Clegg.

Column 3 of Table A1 lists the relative intensities of the emission lines which have been observed from SMCN2, on a scale where $H\beta=100$. The fluxes of the lines between 1200 and 2000 Å were obtained from the merged *IUE* spectrum resulting from SWP 20454 (100 min) and SWP 20527 (90 min). These images, from the programme of Maran *et al.*, were obtained from the *IUE* data-bank. The fluxes of the lines between 3000 and 4000 Å were obtained from the AAT service spectrum of Barlow & Walsh, described in Section 4, while the fluxes of the lines longwards of 4000 Å are from Aller *et al.* (1981). The [Ne v] 3426 Å line is blended with the O III 3429 Å Bowen line, with about one half of the total flux estimated to be due to [Ne v]. As discussed in Section 4, we have adopted a total extinction of $E(B-V)=0.06$ to SMCN2, with $E(B-V)=0.02$ due to foreground galactic extinction and $E(B-V)=0.04$ due to extinction within

Table A1. Observed, dereddened and model fluxes for SMCN2.

Ion	$\lambda(\text{\AA})$	Observed	Dered.	Model ($\epsilon=0.45$)
OIV	1401	<40	<58	3.2
CIV	1549	929	1293	989
HeII	1640	139.4	189.4	198.0
CIII]	1908	753	993	1290
[NeV], OIII	3426, 29	7.2	7.6 (NeV 3.8)	2.4 (NeV)
[OII]	3726	17.5	18.7	18.9
[OII]	3729	10.4	11.1	11.2
[NeIII]	3868	70.5	74.4	73.1
[NeIII]	3967	20.1	21.0	22.6
[OIII]	4363	12.8	13.1	13.1
HeI	4471	3.7	3.8	4.0
HeII	4686	28.4	28.6	28.3
H β	4861	100	100	100
[OIII]	4959	297	296	297
[OIII]	5007	(855)	(853)	854
HeI	5876	11.5	11.1	10.5
[SIII]	6312	0.8	0.8	0.8
H α	6563	310	292	288
[NII]	6584	8.8	8.3	8.3
[SII]	6717, 31	4.0	3.8	1.1
[OII]	7325	5.3	4.9	2.6

Nebular abundances relative to hydrogen (by number):

He : 0.109
 C : 6.75×10^{-4}
 N : 2.00×10^{-5}
 O : 1.48×10^{-4}
 Ne : 2.60×10^{-5}
 S : 1.90×10^{-6}

the SMC. Column 4 of Table A1 lists the dereddened relative line intensities, where we have assumed that the fit to the LMC reddening law presented by Howarth (1983) provides a reasonable approximation to the less well determined SMC extinction law. Inspection of Table A1 shows that with this dereddening the relative intensities of He II $\lambda\lambda$ 1640 and 4686 are consistent with the theoretical ratio (Column 5).

The starting values of the elemental abundances used in the models were derived from an empirical analysis of the dereddened line intensities. For this we adopted $T_e = 13\,400$ K, the [O III] electron temperature, and $n_e = 2850$ cm⁻³, the [O II] electron density (Section 4) along with the collision strengths tabulated by Mendoza (1983) and the empirical ionization correction factors listed by Barker (1983). We obtained the following abundances (by number relative to hydrogen): He:0.109; C: 6.5×10^{-4} ; N: 2.7×10^{-5} ; O: 1.75×10^{-4} ; Ne: 2.6×10^{-5} ; and S: 2.0×10^{-6} .

In order to determine the parameters of the central star, we used the Stoy energy-balance method as developed by Preite-Martinez & Pottasch (1983). The available dereddened line intensities give $\rho = 36.0$, where ρ is the ratio of the flux in collisionally excited lines to the flux in the H β recombination line. To this, we added a contribution of $\rho = 5.7$ from unobserved collisionally excited lines (e.g. C III 977 Å, He I 10830 Å, and the infrared fine structure lines), which we derived by normalizing the predicted relative intensities of observed and unobserved lines from the same ion to the fluxes of the observed lines. For the case of a central star with a blackbody energy distribution, the final value of $\rho = 41.7$ implies an effective temperature of 128 000 K. Case II of Preite-Martinez & Pottasch (1983) was assumed, corresponding to a nebula which is optically thick in the He II Lyman continuum and optically thin in the hydrogen Lyman continuum (these assumptions are supported by the observed He⁺/He²⁺ ratio of 3.4 and were confirmed by the ionization structure modelling). The stellar radius is given by the requirement that the star emit the number of He II Lyman continuum photons implied by the nebular He II 4686 Å flux. For a distance of 57.5 kpc, the dereddened He II 4686 Å flux of 5.92×10^{-14} erg cm⁻² s⁻¹ yields $4\pi R_*^2 N(\text{He}^+) = 2.64 \times 10^{46}$ s⁻¹, the total number of photons emitted by the star shortwards of 228 Å. A blackbody temperature of 128 000 K implies a stellar radius of $0.112 R_\odot$ and a luminosity of $3030 L_\odot$.

The nebular radial density distribution that was used in the ionization structure modelling is that shown in Fig. 7, with the filling factor ϵ as a free parameter to be derived. It was found that a blackbody model was unable to maintain the observed degree of ionization in the nebula, for any value of the filling factor. The ratio of the predicted line fluxes from low-ionization stages to those from higher ionization stages was much larger than observed. By lowering the blackbody stellar temperature, the overall stellar luminosity required to provide the He II ionizing flux could be raised, thereby increasing the degree of ionization in the nebula, but such models then produced values for the electron temperature and ρ which were too low.

We therefore turned to NLTE stellar energy distributions to power the nebula. Because of the energy redistribution around the He II edge at 228 Å, NLTE atmospheres can produce more nebular heating (higher ρ) than blackbodies of the same temperature, since relatively more stellar photons are emitted just longwards of 228 Å, producing significant heating after ionizing hydrogen, and fewer photons are available for ionizing He⁺. In order to supply the same number of He II Lyman continuum photons as a blackbody giving the same value of ρ , an NLTE model atmosphere with a He II absorption edge must be more luminous, and so a higher degree of ionization in the nebula can be expected.

A grid of Auer & Mihaias (1972) hydrogen and helium blanketed NLTE model-atmosphere energy distributions, calculated on the University of London CRAY-1S, have been analysed (Clegg, work to be published) in order to obtain the appropriate values of χ , the energy gain coefficients defined by Preite-Martinez & Pottasch (1983); along with values for $N(\text{He}^+)$ and

$N(\text{H})$, the fluxes of stellar photons emitted per unit surface area shortwards of 228 and 912 Å, respectively. Grids of increasing T_{eff} were calculated for several values of $\log g$, all models having $n(\text{He})/n(\text{H})=0.1$. It was found that the observed ρ of 41.7 required an NLTE effective temperature in the region of 110 000 K, the exact value being a function of surface gravity.

In order to derive the appropriate gravity for the central star of SMCN2, we have made use of the following technique, which relies on the known distance of the nebula. The stellar radius and effective temperature which are required at a given $\log g$ in order to give the correct He II Lyman continuum flux, determine the stellar luminosity. The luminosity and the effective temperature can be used to derive the stellar mass, by comparison with theoretical evolutionary tracks such as those of Schönberner (1981, 1983). The surface gravity implied by this mass and by the stellar radius can then be checked for consistency with the surface gravity of the model atmosphere. As an example, an NLTE model atmosphere with $T_{\text{eff}}=110\,000$ K and $\log g=6.5$ would require a stellar radius of $0.28 R_{\odot}$ in order to maintain the nebular He II ionization in SMCN2. The resulting luminosity of $10^4 L_{\odot}$ and the effective temperature imply a stellar mass of $0.675 M_{\odot}$. This mass and the stellar radius of $0.28 R_{\odot}$ give $\log g=5.4$, inconsistent with the $\log g=6.5$ of the model atmosphere. As the surface gravity of a model atmosphere is lowered and the He II 228 Å absorption jump decreases, the stellar radius required for the necessary He II ionizing flux also decreases. When $T_{\text{eff}}=110\,000$ K and $\log g=5.7$, the required stellar radius is $0.176 R_{\odot}$, yielding a luminosity of $4095 L_{\odot}$, a stellar mass of $0.59 M_{\odot}$ and $\log g=5.73$, which is consistent with the surface gravity of the model atmosphere. In order to test the validity of this technique, we have calculated a grid of nebular ionization structure models for SMCN2 using central stars with NLTE effective temperatures ranging from 100 000 to 130 000 K and $\log g$'s ranging from 5.2 to 6.7. The model atmosphere which produced the best fit to the observed $T_e(\text{O III})$ and ρ , and to the observed level of ionization in the nebula, had $T_{\text{eff}}=110\,000$ K and $\log g=5.75$. The required stellar radius was $0.181 R_{\odot}$ and so the stellar luminosity was $4340 L_{\odot}$. The effective temperature and the luminosity give a stellar mass of $0.593 M_{\odot}$, upon comparison with the Schönberner evolutionary tracks, and the radius and mass give $\log g=5.69$, consistent with the surface gravity of the model atmosphere.

This model atmosphere energy distribution and luminosity was used for our nebular ionization structure modelling, with the elemental abundances and the filling factor as adjustable parameters. As mentioned earlier, the radial hydrogen density distribution that was used is that shown in Fig. 7. Our philosophy for judging the goodness of fit of a model to the observations was to try to match, (i) the fluxes of the electron-temperature-sensitive [O III] $\lambda\lambda 4363, 4959, 5007$; (ii) the fluxes of the electron-density-sensitive [O II] 3726 and 3729 Å lines; and (iii) the total flux in the strong coolant lines of C IV 1549 Å and C III] 1908 Å. The final values of the O and C abundances were determined by (i) and (ii), respectively, while the final N, Ne and S abundances were those that gave the best fits to the fluxes in [N II] $\lambda 6584$, [Ne III] $\lambda\lambda 3868, 3967$ and [S III] $\lambda 6312$, respectively. The final abundances are listed at the foot of Table 1 and are close to the starting values obtained from the empirical abundance analysis.

It was found that although the overall flux in the [O II] $\lambda\lambda 3726, 3729$ lines was sensitive to the stellar luminosity, the 3726/3729 flux ratio was sensitive only to the filling factor ϵ . The observed 3726/3729 ratio was matched by $\epsilon=0.45$. Column 5 of Table A1 lists the predicted relative fluxes from our final best-fit model with $\epsilon=0.45$. The main discrepancy with the observations exhibited by the final model is that the predicted fluxes in C IV $\lambda 1549$ and C III] $\lambda 1908$ are respectively 30 per cent too low and 30 per cent too high, although their combined cooling flux is equal to that observed. Models with the inner nebular hole (Fig. 7) filled to a maximum density of 40 per cent of that at the peak of the inner shell, as allowed by the speckle observations, produced no significant changes in the predicted fluxes since very little mass is contained in the central regions. A nebular model with $\epsilon=1.0$ produced the correct ratio of C IV $\lambda 1549$ to C III] $\lambda 1908$, since

the lower mean density implied by a higher filling factor gave a higher degree of ionization in the nebula. However, the 3726/3729 ratio of 1.43 predicted by the $\epsilon=1.0$ model was inconsistent with the observed ratio of 1.69, while the overall [O II] 3726, 3729 Å flux was a factor of 2 too low.

The nebular mass corresponding to the best-fit model with $\epsilon=0.45$ is $0.36 M_{\odot}$ and the nebula is optically thin in the hydrogen Lyman continuum. The unreddened apparent central star magnitude at V (5500 Å) is predicted to be 21.4, while the unreddened stellar flux at 1308 Å is predicted to be $2.25 \times 10^{-15} \text{ erg cm}^{-2} \text{ s}^{-1} \text{ Å}^{-1}$. In Fig. 9 we have plotted the *observed* 1200–2000 Å spectrum of SMCN2, which results from merging SWP 20454 and SWP 20527. The spectra were extracted using the STARLINK package IUE DR, written by J. R. Giddings. Also plotted in Fig. 9 is the continuum which results from reddening the sum of the theoretical nebular continuum and the model central star continuum. The galactic and SMC extinctions and reddening laws which were used are those described at the beginning of the Appendix. It can be seen that there is satisfactory agreement between the predicted and observed continuum levels.

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