

RADIO EMISSION FROM EXTENSIVE AIR SHOWERS

A Thesis

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A B S T R A C T

A large part of this thesis describes theoretical investigations into the properties of extensive air showers with reference to radio emission. The first chapter indicates the possibility of radio emission helping to determine the mass of the primary cosmic ray particle, which is of considerable astrophysical interest. This is followed by a semi-quantitative discussion in which it is shown that it is of prime importance in radio emission calculations to establish the structure and development of the electron-photon cascades in the air shower.

A Monte Carlo model of an electron-photon cascade has been modified and used for this purpose. A detailed investigation into the properties of cascades in air has been necessary also because of serious differences compared with some of the earlier results. Differences of more than a factor of two in the lateral spread of electrons, for example, are believed to be resolved with the present model and an evaluation and comparison of results are detailed in Chapter 4.

The results of the radio emission calculations from individual cascades are used to determine estimates of radio field strengths for the whole air shower and this work is presented in Chapter 5.

In the last chapter, the experimental radio measurements carried out at the Haverah Park extensive air shower array are analysed and compared with theoretical predictions. From these results, it is concluded that the mechanism for radio emission is well established, but the technique is not sensitive enough to establish the mass of the primary particle with any degree of conviction.

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CHAPTER 1

INTRODUCTION1.1 Relativistic Particles in Space

The radiation which continually reaches the earth from outer space may be classified into two categories. The electromagnetic radiation covers a wide frequency spectrum and although much of this radiation is absorbed by the earth's atmosphere, visible light and radio frequencies are able to penetrate and can be detected at ground level. The second form of radiation consists of high energy atomic nuclei ($10^{10} - 10^{20}$ eV). The study of this radiation provides information about the sources and the intervening space through which it has passed. Cosmic ray physicists have been concerned with the detection of these particles with one or more of the following objectives in mind:-

- (1) To establish the origin of the primary cosmic radiation, i.e. whether they are of galactic or extragalactic origin.
- (2) To establish what sort of acceleration mechanism could be responsible for their very high energies.
- (3) To establish the primary composition of the radiation.

Closely related to the above objectives are the experimental measurements of the primary energy spectrum (Fig. 1.1), and the arrival directions of the cosmic rays. For energies below about 10^{13} eV, the mass composition is well established as being mostly protons, with only a small percentage of heavier nuclei (Ginzburg and Syrovatsky, 1964). However, the primary mass is not so easily established at higher energies and the work described in this thesis

INTEGRAL PRIMARY ENERGY SPECTRUM

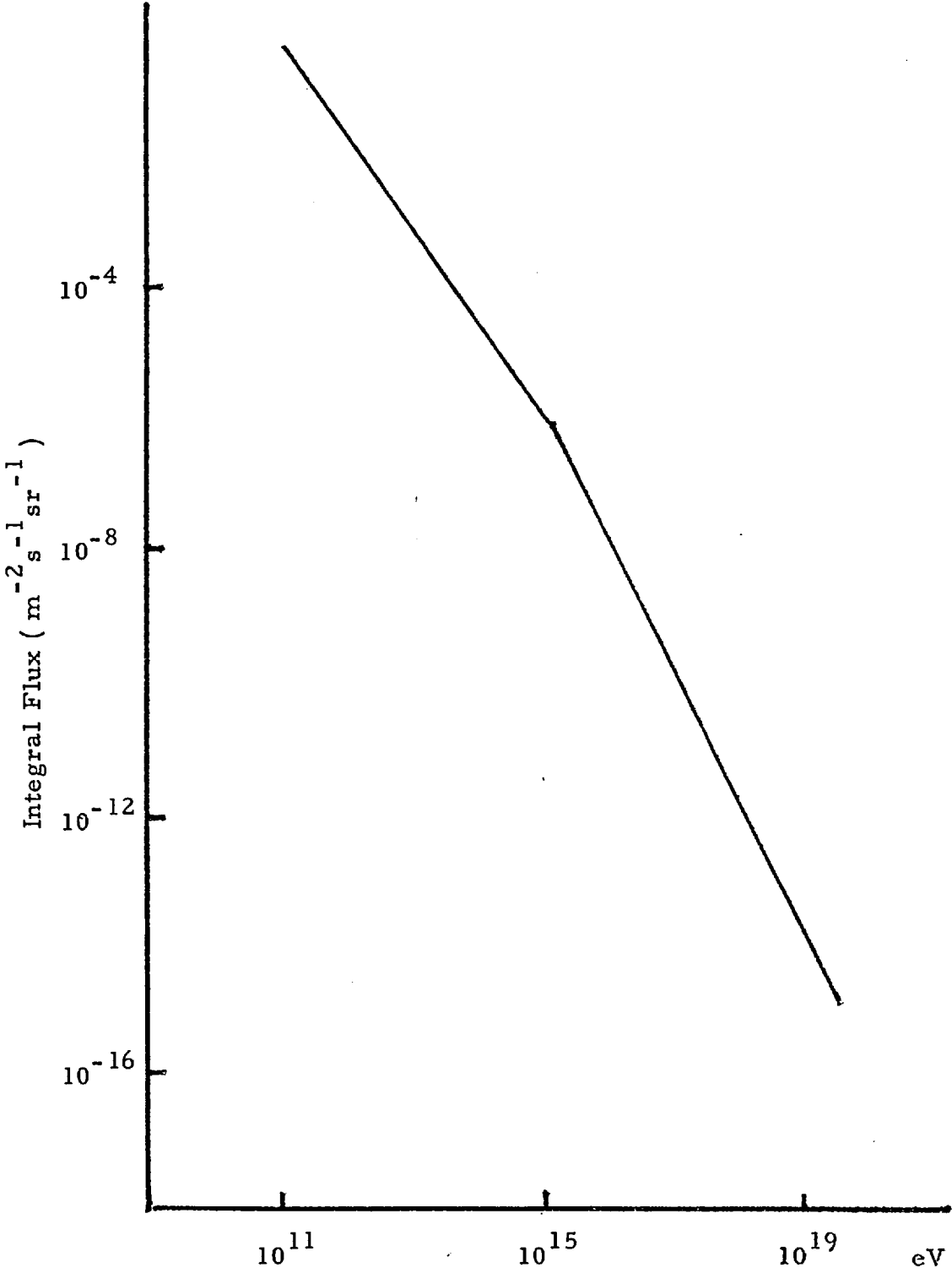


Fig. 1.1

represents part of the larger effort being devoted at the present time to finding out the mass spectrum of the high energy cosmic radiation.

1.2 Astrophysical Aspects of Cosmic Rays

Sixty years after the discovery of cosmic rays, the problem of their origin remains unresolved. The first experimental observation to be explained is the virtual absence of any detectable anisotropy for all energies from 10^{12} eV to above 10^{17} eV. One possibility is that the source distribution is isotropic, which would suggest that the primaries are of universal or extra-galactic origin. Alternatively, if one suggests a galactic origin, a confinement and storage mechanism by the galactic magnetic field must be considered.

The Larmor radius (R) for a particle of energy E eV in a field of B gauss is given by:-

$$R_{cm} = \frac{E_{ev}}{300B_{gauss}Z}$$

For a given energy, the radius of gyration is inversely proportional to the number of elementary charges, which in turn is related to the primary mass. It follows from this formula that a heavy 10^{17} eV primary particle could be contained within the galaxy, whereas a 10^{17} eV proton is almost certain to escape. High energy protons, if present in the galaxy, would therefore be extragalactic. This argument has been used to explain the 'kink' in the slope of the integral primary energy spectrum at around 10^{15} eV as shown in Fig. 1.1.

The determination of primary mass at these high energies is also of importance for another reason. In intergalactic space, high energy cosmic rays are liable to interact with the photons which constitute the 3^0K black-body radiation, losing energy in the

production of pions. These interactions should result in a high energy cut-off for cosmic ray protons above 10^{20} eV. However, there is no experimental evidence for such a cut-off in the energy spectrum. To overcome this puzzle, some theorists have suggested that high energy cosmic rays are all of galactic origin. Over galactic distances, the 3^0K interaction probability is small. The near isotropy of the radiation then requires the particles to be heavies. Extragalactic heavies would inevitably break up into separate nucleons by spallation. Protons from the local super cluster could survive the 3^0K absorption, so we have two competing theories: galactic heavies and super cluster protons. In a model in which the observed radiation comes from our own galaxy, magnetic field irregularities could partly explain the high degree of isotropy in the cosmic ray arrival directions. Fermi (1949) proposed a mechanism whereby the collision of fast-moving charged particles with moving magnetic fields not only results in a random scattering but also leads to a net gain in energy by the particles. However, such an acceleration mechanism is unsatisfactory for heavy nuclei.

Several possible sources have been suggested which could accelerate cosmic rays to high energies. A mathematical model of a supernova explosion has been investigated by Colgate et al. (1969). It is supposed that a very strong shockwave produced by gravitational collapse in the centre of a star can accelerate the ions in the outer layers. According to estimates by Ginzburg, supernovae and ordinary novae could be present in sufficient numbers within our own galaxy to account for all the observed energy spectrum. Burbidge and Hoyle (1964) on the other hand, have urged that cosmic rays are largely of extra-galactic origin and that accelerations may take place in the centres of strong extra-galactic radio sources. Arguments by Ginzburg and counter arguments by Burbidge have continued at successive international conferences.

Recent observations of cosmic rays with energies above 10^{18} eV have indicated that the radiation is not entirely isotropic. The

analysis carried out by Krasilnikov et al. (1974) suggests an arrival direction from a local cluster of galaxies. However, the same data analysed by Hillas and Ouldrige (1975) is claimed to indicate a galactic origin. It is clear that the experimental verification of the primary mass will provide an important clue for a more complete model of cosmic ray production.

1.3 The Detection of Extensive Air Showers

Low energy cosmic rays below 10^{13} eV have been studied directly by flying balloons which carry nuclear emulsions. The use of this technique is impracticable at higher energies due to the rapidly falling intensity of the radiation; the flux above 10^{16} eV is only one particle per (m^2sr) per year. However, cosmic rays of energy greater than 10^{14} eV can produce extensive air showers containing thousands of secondary particles. The atmosphere happens to be a very useful medium in which to study energetic primaries by observing the processes that take place. Its vertical thickness of 1000 g-cm^{-2} is sufficient to permit the highest energy air showers to reach their maximum development, where a count of the number of particles is closely related to the total energy. The low density of the atmosphere is responsible for the secondary particles being spread over a large area on the ground. Consequently, it is possible to study high energy primaries at ground level without using excessively large detectors. With an array of detectors, various parts of an extensive air shower may be sampled and some of the properties of the primary particle may be inferred. However, the indirect nature of the observations does result in a loss of some kinds of information. The current opinion is that the energy and arrival direction of the primary particle can be accurately determined, whereas other properties such as the primary mass remain uncertain.

Particle Detectors

In view of the considerable astrophysical interest in the high energy end of the cosmic ray spectrum, large particle-detecting

LOCATIONS OF HAVARAH PARK DETECTORS

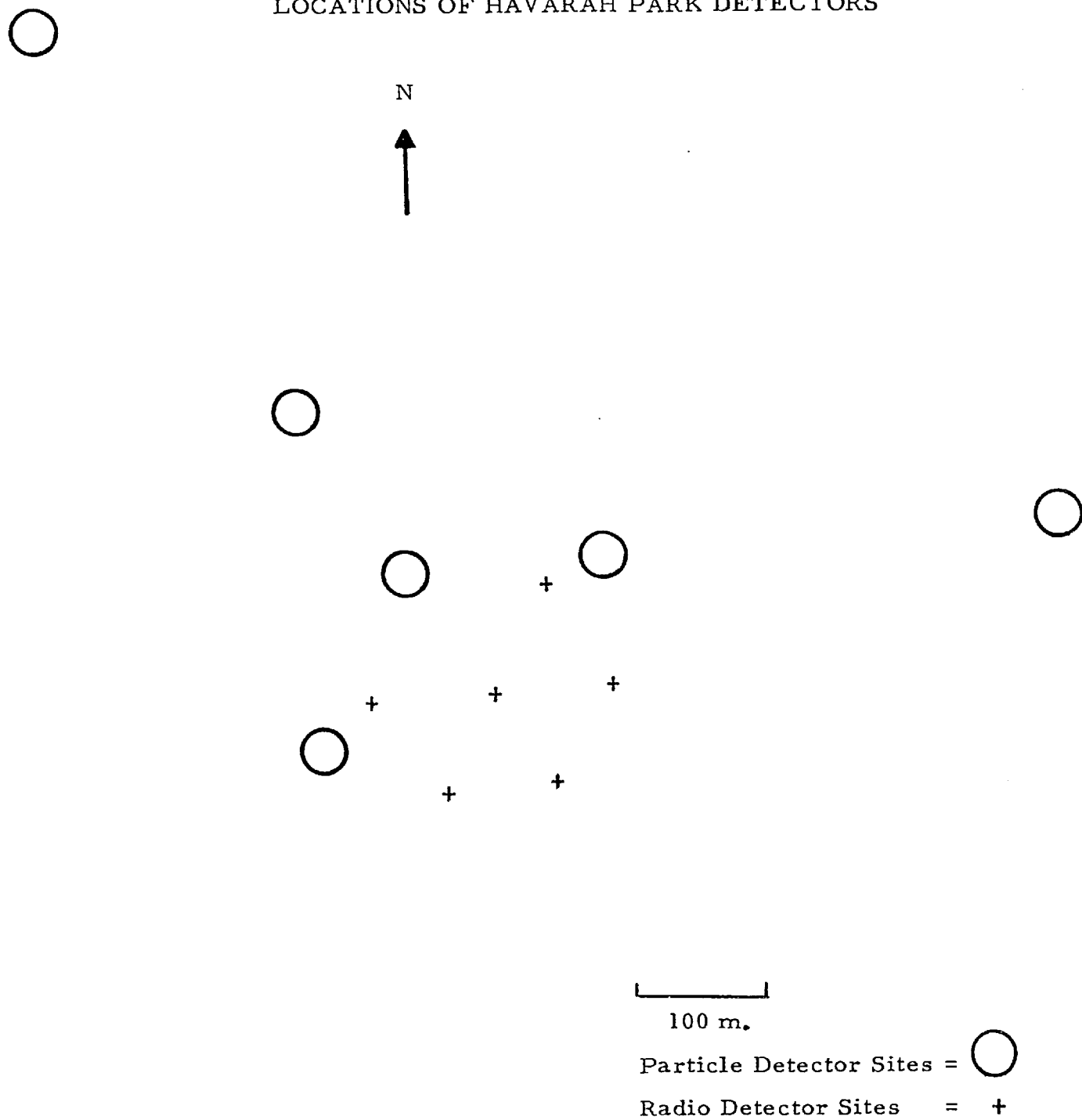


Fig. 1.2

arrays have been constructed: these include arrays at Haverah Park (Tennent, 1967), Sydney (Brownlee et al., 1967), Volcano Ranch (Linsley, 1963) and Yakutsk (Egorov et al., 1971). The detector normally consists of slabs of scintillator material, or a medium giving Cerenkov light such as water, with a surface area of a few m^2 . A photomultiplier is used to detect the light produced by the action of the secondary particles on the material at the arrival of an extensive air shower. Fig. 1.2 shows the locations of the detectors at the Haverah Park array. The effective collecting area is 0.6 km^2 ; a detailed description is given by Tennent (1967).

Atmospheric Cerenkov Detectors

The secondary particles of the extensive air shower generate Cerenkov light in the atmosphere which can then be detected using photomultipliers (Jelley, 1967). One advantage of atmospheric Cerenkov light detectors is that information relating to the particles is received from the whole shower. However, the main drawback is that the recording time is severely restricted because of sunlight, moonlight and clouds which must be absent during the observations.

Cerenkov light detectors are employed at Haverah Park as a subsidiary experiment (Smith and Turver, 1973) and at the Yakutsk air shower array (Egorov et al., 1971).

Radio Receivers

The charged particles in the air shower give rise to radio emission which can be detected using an antenna and a receiver tuned to a fixed frequency. Radio pulses coincident with air shower particles were first observed by Jelley (1965) using a receiver tuned to a frequency of 44 MHz. The main problem with radio measurements is that there is a high background noise from natural as well as man-made sources. As in the case of Cerenkov detectors, radio emission is also related to the distribution of the shower particles

in height. The recording time is unrestricted but only air showers $> 10^{17}$ eV in primary energy give radio pulses larger than the background noise.

Radio measurements have been carried out at Haverah Park to determine the mechanism responsible for radio emission and to provide supplementary information in conjunction with the particle arrays. The experimental details and the results are described in Chapter 6.

1.4 Fluctuations in Extensive Air Showers

The detailed structure of an extensive air shower is complicated (Chapter 2) but it is believed that the distribution in height of the secondary particles in the atmosphere is dependent on the primary mass. This arises from the differences in the interaction mean-free paths for protons and heavy nuclei. For primary protons with an interaction length of the order of 80 g-cm^{-2} , the first interaction in the atmosphere can fluctuate considerably in height and this to a large extent subjects the whole development of the air shower to similar displacements. On the other hand, heavy primary nuclei have a considerably shorter interaction length and this results in the shower development occurring much higher in the atmosphere (Fig. 1.3). Fluctuations for individual air showers of the same energy would also be greatly reduced for heavy primaries.

The possibility of detecting fluctuations in showers is therefore of considerable interest. However it requires a subtle analysis of the observations at ground level and so far the evidence is not wholly conclusive. Methods include fluctuations in particle lateral distributions and the relative proportions of muons to electrons in the shower at sea level (see next Chapter). Radio emission from the shower can also be used and is the subject of this thesis.

ELECTRON DEVELOPMENT CURVES FOR
 10^{17} eV EXTENSIVE AIR SHOWERS
(DIXON ET AL., 1974)

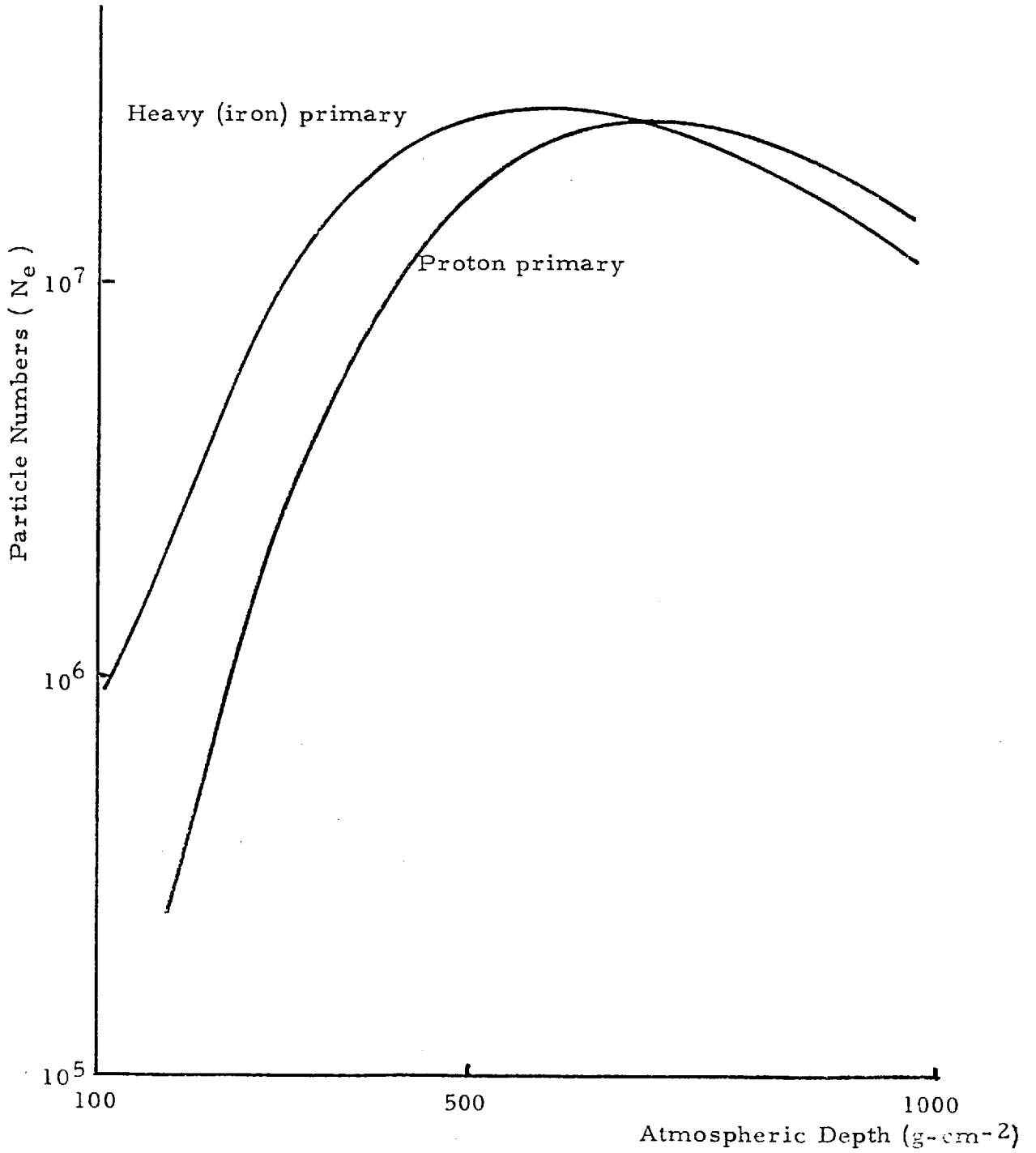


Fig. 1.3

1.5 Brief Summary of Radio Emission Theories

The question of radio emission theory was first considered by Jelley shortly after his discovery of atmospheric Cerenkov radiation. Relative to the amount of visible radiation produced, the conclusion was that radio emission would be far weaker. The reason for this is that at the lower radio frequencies, when the wavelength becomes longer than the distance between the individual particles in the shower, it becomes necessary to add their contributions coherently by amplitude rather than incoherently by intensity. This means that if one assumes that the numbers of positive and negative charges are equal, the total emission is reduced virtually to zero.

However, Askaryan (1965) was the first to point out that if the existence of a negative charge excess is recognised, phase coherence between different parts of the shower at longer wavelengths increases the possibility of emission.

Kahn and Lerch (1966) in a more detailed analysis, showed that even in the absence of charge excess, the separation of positive and negative charges in an air shower due to the geomagnetic field could give rise to substantial radio emission. Using a different approach, Colgate (1967) and later Allan (1969) applied more physical arguments to estimate the radio emission associated with the geomagnetic deflection.

A detailed investigation of the geomagnetic and charge excess contributions to radio emission will be the main subject of this thesis. This will be based on model simulations of extensive air showers in which the important processes for radio emission are considered.

1.6 The Lateral Spread of the Radio Emission

To measure the radio lateral distribution at Haverah Park, an

array of radio detectors is employed (Fig. 1.2). In this section, the lateral spread of the radiation about the shower axis is discussed in terms of a simple model, to give some indication of how it varies with frequency and with the height of maximum development.

The radiation intensity will not change appreciably from one point to another on the ground so long as most of the shower particles provide contributions that are in phase with one another. The intensity falls off in directions when the path differences become appreciable. Referring to Fig. 1.4, a line source of radiating particles of length L is considered at a height H above the ground. The situation is then analogous to the one of optical diffraction due to a single slit. In this case, a direction θ of the first diffraction minimum is specified by the condition that the extreme rays should have a path difference λ :-

$$L - L \cos \theta = \lambda$$

The corresponding radius on the ground R is given by $H \theta$ and solving for R gives the following:-

$$R \simeq \left(\frac{2\lambda}{L} \right)^{\frac{1}{2}} H$$

The crude model does give an order of magnitude estimate for the lateral spread which is proportional to $(\text{wavelength})^{\frac{1}{2}}$ or $(\text{frequency})^{-\frac{1}{2}}$. For example, a lateral spread of about 200 m is obtained for radiation at a frequency of 60 MHz (the receiver operating frequency used at Haverah Park) if one takes 5000 m as a reasonable value for both the height (H) and the effective length (L) of the shower.

It also follows from the above argument that the radio lateral spread is proportional to the height (H) of the shower particles above the ground. A shower developing higher in the atmosphere can be expected to produce a wider lateral spread of radio emission.

THE RADIAL SPREAD OF RADIO EMISSION

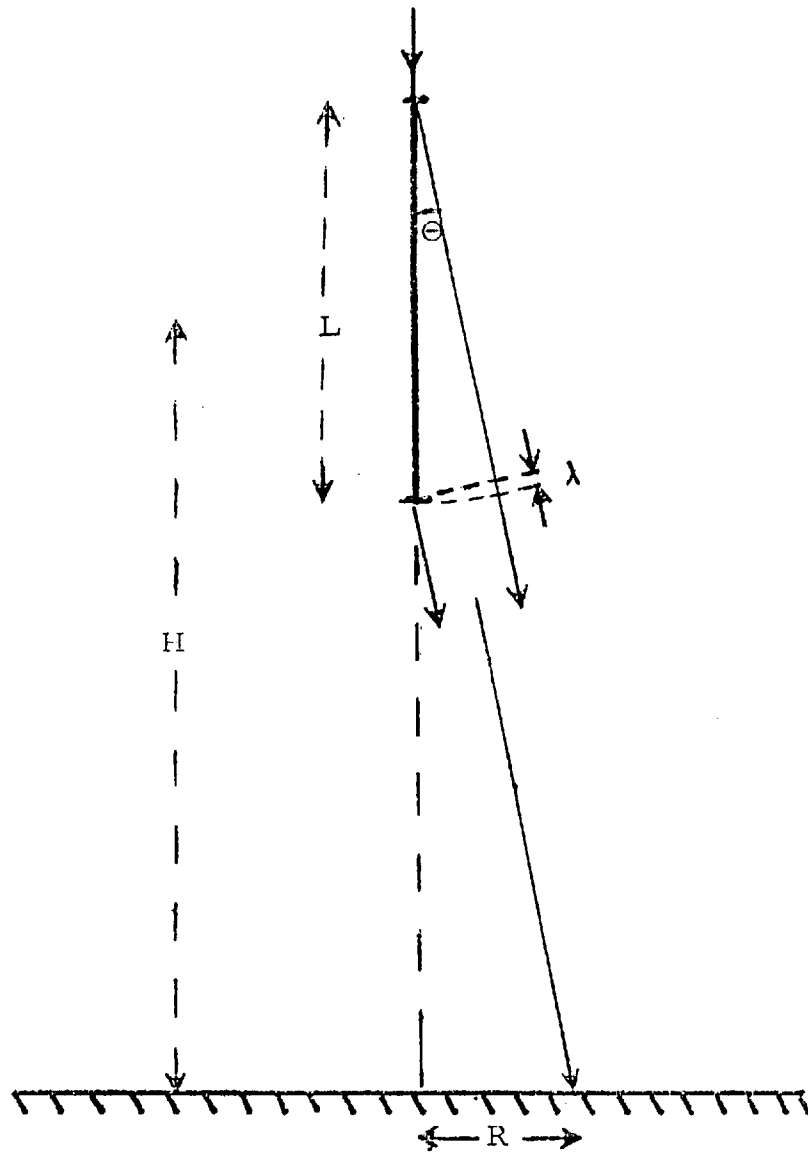


Fig. 1.4

RADIO LATERAL DISTRIBUTIONS AT 60 MHz

(ALLAN, 1971)

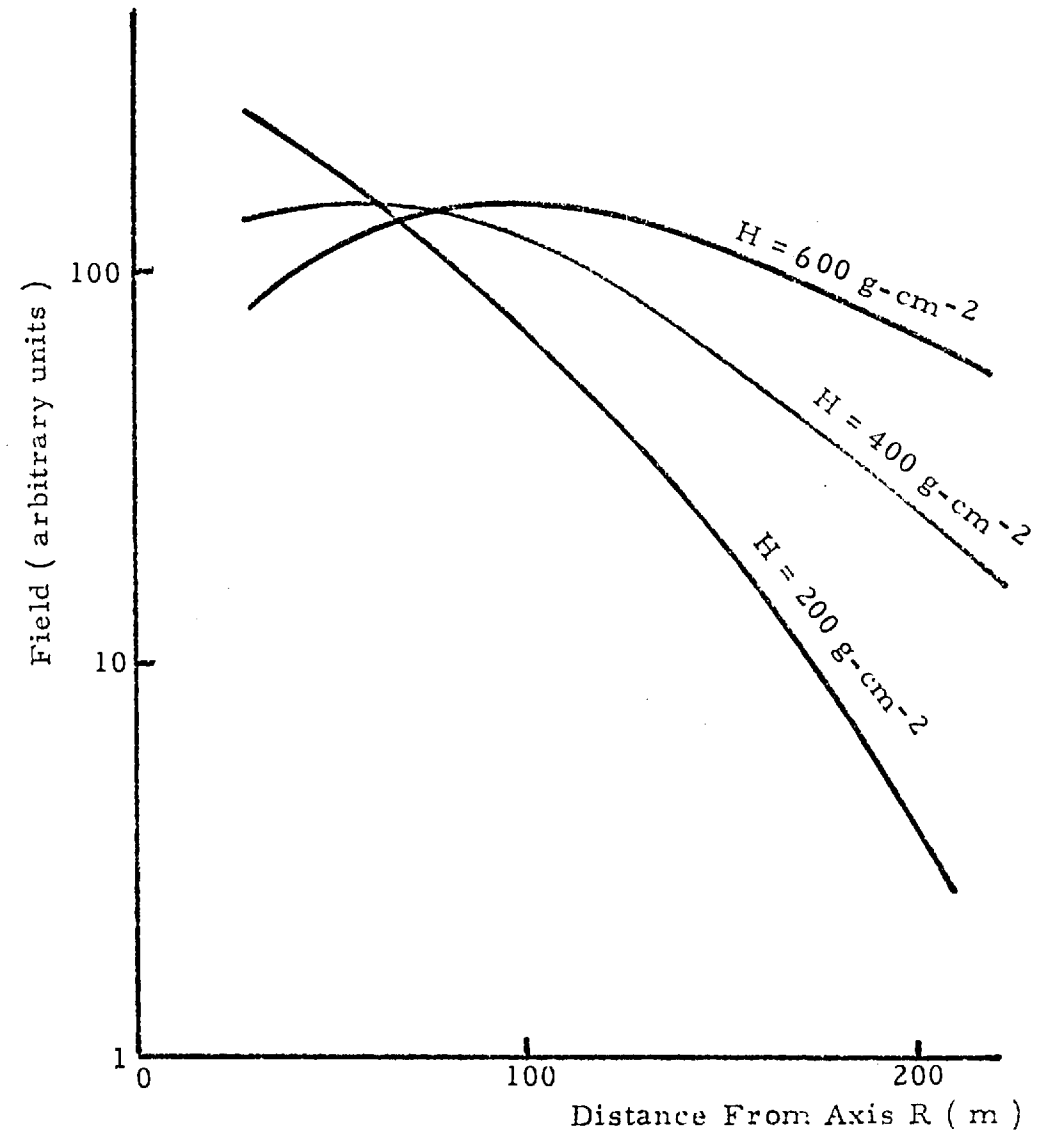


Fig. 1.5

The main interest in measuring radio lateral distributions arises from the possibility of being able to infer the height of maximum development of extensive air showers which could in turn provide some clue as to the primary mass.

A more quantitative analysis has been carried out by Allan (1971) to determine the expected dependence of the radio lateral distributions on fluctuations in extensive air showers. Important factors such as the refractive index of the atmosphere have been included and the lateral distributions at 60 MHz are shown in Fig. 1.5.

At around 100 m, the field strength is seen to be almost independent of fluctuations in the height of shower maximum (X_m) whereas at larger distances, the sensitivity to fluctuations is greater. In Chapter 6, the experimental radio data is analysed with the aim of establishing whether the shape of the radio lateral distributions shows any evidence of fluctuations in extensive air showers.

CHAPTER 2

THE STRUCTURE OF EXTENSIVE AIR SHOWERS

2.1 Introduction

In order to obtain theoretical estimates of the radio emission from, say, a 10^{17} eV extensive air shower, it is necessary to investigate the motions of charged particles in the shower. The particles in an extensive air shower can be divided into three principal components:-

- (1) The primary particle interacting with an atmospheric nucleus initiates a 'nuclear-active cascade' which consists mainly of charged and neutral pions and other particles that participate in strong nuclear interactions. This component constitutes the 'core' of the extensive air shower.
- (2) The secondary neutral pions (π^0) decay into photons which initiate electron-photon cascades and this is the 'soft component' of the air shower. The overlapping electron-photon cascades rapidly grow to become the most numerous particles in the shower, and it is these electrons and positrons that are responsible for the radio emission that is observed.
- (3) The secondary charged pions either interact as part of the nuclear-active cascade, or decay into muons. The muon component is a non-multiplying, weakly interacting group of particles which ultimately, at great depths underground, become the most numerous particles remaining in the shower.

In this Chapter, the electron-photon component is discussed in more detail, using semi-quantitative arguments, since it is the general features of this component that determines the nature of the radio-emission. It will be seen that for radio-emission purposes, the extensive air shower can be represented as a superposition of many electron-photon cascades in the atmosphere at various depths.

A three-dimensional Monte Carlo programme has been adapted to carry out studies of radio emission from an electron-photon cascade. A description of the programme with its modifications is presented in Chapter 3 and the results concerning important aspects of the structure of a cascade are discussed in Chapter 4. The radio emission calculations and results are detailed in Chapter 5.

2.2 The Nucleonic Component

The theoretical knowledge of the interactions of nuclear particles in this high energy region can only be based on the extrapolation of the experimental information from particle accelerator energies. The extrapolation is over some five orders of magnitude, and a wide choice of models can be offered for carrying it out (Gaisser 1973, Greider et al. 1973). It is largely a matter of adopting a model which is a best fit to E.A.S. experimental measurements. For example, the model adopted for high energy interactions should fit in with the characteristics of muons observed at large distances from the shower axis (section 2.3). Also, the attenuation rate of the shower (which is considerably slower than for electron-photon cascades) as predicted must be in agreement with measured values.

In this section, a simplified model is discussed to indicate the orders of magnitude involved. The assumptions are taken from Hillas (1965) and Orford and Turver (1968) but the numbers are adjusted in the interest of simple arithmetic.

The mean free path for interaction of an incoming primary proton with nucleons in the atmosphere is about 80 g cm^{-2} . The inelasticity K for the collision is about 0.5 which means that half of the proton energy goes into generating secondary pions at each collision. The number of pions so generated are assumed to be of the form:-

$$n = E^{1/2}/10^2 \quad \text{for } E < 10^{12} \text{ eV}$$

$$n = E^{1/2}/10^5 \quad \text{for } E > 10^{12} \text{ eV}$$

For a primary proton of say 10^{16} eV , about 12 proton-nucleon interactions can take place in the atmosphere, which is about 1000 g cm^{-2} in depth to ground level.

Pions are also strongly interacting particles with a mean free path of about 80 g cm^{-2} . At each interaction, a further generation of pions are produced as given by the expressions given above. As an example, with the launch of a 10^{16} eV primary proton it is now possible to estimate the number of particles that are present after three mean free paths. Assuming that the secondary particles share the available energy equally at each interaction, one obtains:-

$$1 \text{ proton of energy} \quad 1.25 \times 10^{15} \text{ eV}$$

$$1000 \text{ pions of energy} \quad 2.5 \times 10^{12} \text{ eV}$$

$$2 \times 10^4 \text{ pions of energy} \quad 1 \times 10^{11} \text{ eV}$$

$$2.5 \times 10^4 \text{ pions of energy} \quad 5 \times 10^{10} \text{ eV}$$

$$1 \times 10^5 \text{ pions of energy} \quad 1 \times 10^{10} \text{ eV}$$

The development of the cascade of secondary pions is modified by decay processes. The neutral pions (π^0) that are generated (about a third of the total number of pions) decay almost immediately

into two photons which then initiate electron-photon cascades. The π^0 particles never have a chance to undergo nuclear interactions. The charged pions may interact with another nucleus but can also decay into muons: this is characterised by a decay lifetime:-

$$T_{\text{decay}} = 3 \times 10^{-8} E / m_{\pi} c^2$$

where $m_{\pi} c^2$ is 1.5×10^8 eV. At an energy ~ 30 GeV, the mean free path for decay becomes equal to the mean free path for interaction (120 g cm^{-2} , which corresponds to about 2 km at the typical atmospheric density).

Below this energy, decay predominates over interaction and therefore after about three interaction lengths, the number of nuclear active secondaries has reached a maximum. The other particles are much more numerous. There are about 10 times as many muons, each carrying typically an energy of 10 GeV, and about 100 times as many electrons and positrons, each with an energy of 20 to 30 MeV.

2.3 The Muon Component

The muons are weakly interacting particles and once formed, they can reach sea level unimpeded. High energy muons rarely decay because the relativistic time dilation increases their lifetime. The attenuation length for a shower with respect to muon numbers (N_{μ}) is therefore long, and the fluctuations in N_{μ} are quite small. Measured values of N_{μ} are sometimes used, particularly in the U.S.S.R., as an index of primary energy (Allan 1971a).

The electron component in the shower is much more readily absorbed than the muons, consequently the relative proportion of muons and electrons (N_{μ}/N_e) in a shower changes with depth. An air

shower initiated by a heavy nucleus (which develops high in the atmosphere) will therefore contain a larger proportion of muons at sea level than a shower due to a primary proton. This suggests a way of finding the primary mass. However, although estimates of N_μ can be determined readily from muon density measurements with shielded particle detectors, the correlation between this parameter and primary cosmic ray parameters is dependent on the choice of model for high energy interactions and so the method does not give very definite answers. Detailed model calculations aiming at primary mass measurement have been carried out by the Durham Group (Dixon et al., 1974); they also indicate the sensitivity of the shower parameters to the model that is adopted.

Comparing the observed lateral spread of muons at sea level with predictions provides some check on the validity of the different models. Near the shower axis, the particles are predominantly electrons but with increasing radius, the muons grow comparatively more abundant. The large lateral spread of muons is related to their height of origin and the low energy typical of the charged pions when they decay. At these energies, the angles of emission are quite large and beyond 1000 metres from the shower core the muons are the dominant particles.

2.4 The Electron-photon Component

Each gamma photon from neutral pion decay will initiate an electron-photon cascade. The two processes responsible for the growth in the number of particles in such a cascade are bremsstrahlung and pair production. The mean free path for the two processes is about the same and equal to 1 radiation length or 40 g cm^{-2} in air. In a crude approximation, one can assume that the two secondary particles are produced with equal energy, and that ionisation loss can be neglected. In this case, the number of particles is doubled in about a radiation length and this process continues until the energy per particle is reduced to 80 MeV, which is the 'critical

energy'. The critical energy is defined as the energy at which the rate of loss by ionisation is equal to the rate of loss through radiation.

Below 80 MeV, ionisation loss cannot possibly be ignored. The ionisation loss in air is about 2 MeV per g cm^{-2} so that if one assumes that this is the only important process below critical energy, the cascade would come to an abrupt end at 40 g cm^{-2} after shower maximum. The shower attenuation will be more gradual than this if one allows for the photons, which will travel some distance before giving up their energy to electrons either through pair production or the Compton effect.

In this very simple model, the number of particles in the shower reaches a maximum given by $E_{\text{launch}}/E_{\text{critical}}$ at a depth given by:-

$$X_{\text{max}} = 40 n \text{ g cm}^{-2}$$

where n is the number of steps from launch energy to critical energy given by:-

$$2^n = E_{\text{launch}}/E_{\text{critical}}$$

Thus for a 10 GeV photon-initiated cascade, the number of particles at shower maximum is about 100 at a depth of about 250 g cm^{-2} from launch height. The number of particles is an overestimate largely as a result of neglecting ionisation loss above critical energy. More exact calculations are discussed later in this section.

2.5 The Distribution of Track Length in Electron-Photon Cascades

From the above model one may estimate the track length associated with particles above the critical energy. It is found to be 2^n radiation lengths or $40 E_{\text{launch}}/E_{\text{critical}} \text{ g cm}^{-2}$. The total track length below critical energy is also $40 \times 2^n \text{ g cm}^{-2}$, which is a direct consequence of

ionisation loss. However, ionisation loss takes place at all energies and so the true track length associated with the growth of the cascade is considerably less than that indicated by the above crude model. The number of particles at maximum is reduced by a factor of ~ 5 . A more careful treatment (such as in Approximation 'B' outlined below) shows that although the electron track length distribution shows a $1/E$ energy dependence well above critical energy as the simple model implies, this flattens out to give a total track length above zero energy given by $E_{\text{launch}}/2 \text{ g cm}^{-2}$. Some $3/4$ of the total track length is below E_{critical} and only $1/4$ is above E_{critical} . The median energy is about 30 MeV meaning that there are equal lengths of track above and below this energy. The value of the median energy is virtually independent of the launching energy of the cascade because of the negligibly small track length associated with the top end of the energy spectrum.

2.6 The Lateral Spread and Longitudinal Development of a Cascade

The three processes contributing to lateral spread in an electron-photon cascade are:-

- (1) Angles of emission of secondary particles at interaction point for the physical processes such as bremsstrahlung, pair production and Compton effect.
- (2) Multiple Coulomb scattering of electrons in air.
- (3) Deflections of the charged particles in the presence of the geomagnetic field.

The dominant mechanism contributing to the lateral spread in a cascade is multiple Coulomb scattering which increases with decrease in energy. Low energy charged particles therefore predominate further away from the cascade axis. The lateral spread will be a subject for detailed discussion in Chapter 4: the median

value at maximum development of the cascade turns out to be typically $1 - 2 \text{ g cm}^{-2}$ and independent of launching energy.

The longitudinal development of a cascade typically follows a smooth curve with a maximum at a depth which depends on the launching energy.

2.7 The Structure of the Shower Disc

A particle in a cascade that moves with velocity c along the shower axis marks the leading edge of the shower front. As a necessary corollary to the lateral spread in an air shower, most of the electrons lag behind the leading particle because of the effects of scattering. This not only results in the shower front being curved (Fig. 2.1) but also in the thickness of the disc to increase as the cascade develops in the atmosphere.

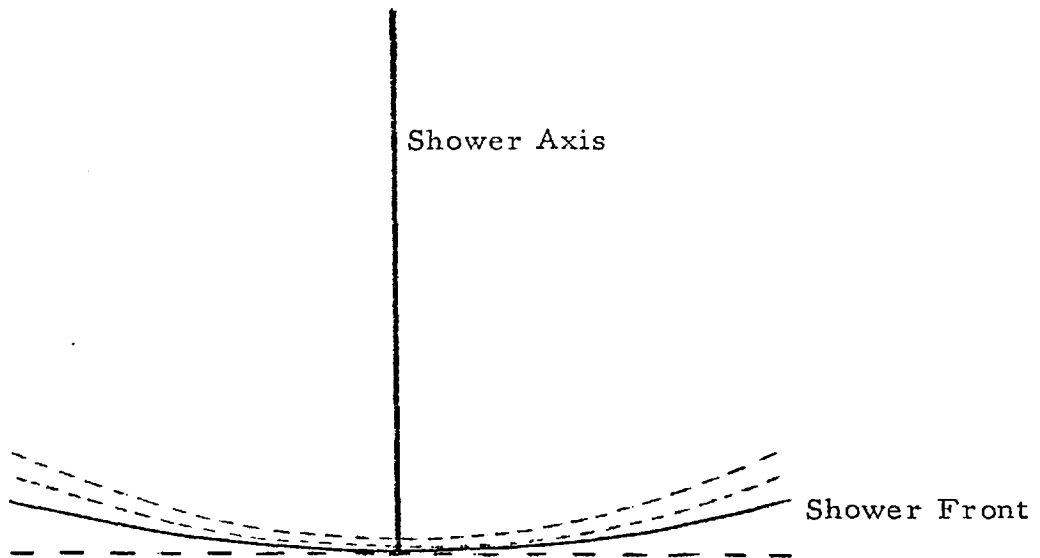


Fig. 2.1

Detailed calculations such as those from Monte Carlo simulations (Chapter 4) show that the thickness of a shower disc is typically less than a few metres. Greater lags are associated with the lower energy electrons since these in general are scattered more than high energy particles.

2.8 The Effect of the Geomagnetic Field

Positrons and electrons are deflected in opposite directions in the earth's magnetic field. The radius of curvature, r , in metres for the deflection is given by

$$r = 100 E \sin \alpha$$

where E is in MeV and α is the angle between the shower particle direction and the earth's magnetic field (taken to be 0.3 gauss). This deflection is small compared with multiple Coulomb scattering but it is important for radio emission because a net transverse electric dipole moment builds up as a result of the lateral displacements of electrons and positrons in opposite directions. In order to investigate this geomagnetic mechanism for radio emission, one must determine the increase in dipole moment as a function of time and this is again related to the structure of the shower disc as described above. This investigation has been carried out by Allan (1971), using analytical models for the shower structure. A similar method has been used by the author to calculate the radio emission, using a Monte Carlo model of electron-photon cascades (Chapter 5).

The total dipole moment that can be expected from an electron-photon cascade can be estimated by the following simple arguments. The radius of curvature of an electron track at an energy E MeV is approximately $300E$ metres for a vertical cascade, and the angular deflection $d\theta$ along a length dS of track is $dS/300E$. The residual range (through ionisation) for the particle may be written as

$(E/E_c)L$ metres where L is the radiation length. The eventual lateral displacement (dX) attributable to the angular deflection in dS is therefore $(dS/300E) \times (E/E_c)L$ metres. Since dS may be written as $(dE/E_c)L$, the lateral displacement is given by:-

$$dX = (1/300)(L^2/E_c^2) dE$$

and the total displacement, i.e. the total dipole moment is:-

$$X = (1/300)(L^2/E_c^2) E_p$$

For the radiation length (L) of 400 metres and the critical energy (E_c) of 80 MeV:-

$$X = (1/12)E_p \text{ metres, where } E_p \text{ is in MeV and the dipole moment is therefore } (1/12)E_p \text{ electron metres.}$$

The final dipole moment only depends on the primary energy of the cascade and it provides a check with more detailed calculations which are described in Chapter 3.

2.9 The Negative Charge Excess in a Cascade

It will appear that the number of electrons in a cascade exceeds the number of positrons by about 10%. This can be important in relation to radio emission and will be considered in detail in Chapter 5.

Three main processes contribute to the electron excess:-

- (1) The Compton process gives recoils of negative electrons only. The cross-section increases as the energy decreases, giving rise to an increasing electron excess at low energies.
- (2) Although in pair production, there is no charge excess to

begin with, the positrons can decay in flight, annihilating with an electron in the medium. Positrons have a probability of decaying in flight with a cross-section comparable to the Compton cross-section.

- (3) Any charged particle will undergo many collisions with atomic electrons, with mostly small energy transfers (primary ionisation). However, there is a finite probability of large energy transfers, in which case the ejected electron has sufficient energy to produce its own trail of ionisation called knock-on electrons or delta rays.

The relative contributions to charge excess by these three processes have been evaluated by Fujii and Nishimura (1969). The Compton process is shown to be the most important contribution to the charge excess in an electron-photon cascade.

2.10 Approximation 'B' Model of Cascades

So far, the properties of the cascades have been discussed using simple arguments. The need for more quantitative information on the structure of electron-photon cascades has resulted in the development of better models.

One such model involves the setting up of diffusion equations for the cascade process, taking into consideration the various production and absorption phenomena that take place in an infinitesimal layer in the medium. The method used to solve the diffusion equations is reviewed by Rossi (1965). To enable the mathematical equations to be solved, simplifying assumptions or approximations are made.

In Approximation 'A', only the pair production and bremsstrahlung processes are considered and, with these, the asymptotic formulae, i.e. the cross-sections corresponding to very high energy particles are

used for all energies. As one would expect, the results can only be accurate well above the critical energy.

In Approximation 'B', the ionisation loss of electrons is included and it is assumed to be continuous and energy-independent. Analytical solutions of the average behaviour of showers under this approximation are given by Snyder (1949) and other workers and are reviewed by Griesen (1956). The authors claim that both the electron longitudinal development and the energy spectrum should be correct to within 10% accuracy, in spite of the simplified assumptions. The effects of neglecting the Compton processes but using a constant pair production cross-section effectively introduce errors which tend to cancel each other in respect of track length because, in air, the sum of the Compton and pair cross-sections is nearly constant anyway. Since the ionisation loss predominates at low energies where the asymptotic bremsstrahlung cross-sections are inaccurate, this simplification is also justified.

The lateral spread of electrons has been extensively investigated by Kamata and Nishimura (1958) using the analytical approach under Approximation 'B', but including extra terms in their diffusion equations to represent the angular scattering of the electrons. The derived lateral distribution function for the electron-photon cascade is claimed to be accurate down to low energies and has been widely used in comparisons with data from extensive air shower experiments.

Greisen (1956) has fitted an empirical formula to the Nishimura-Kamata distributions:-

$$sf(x) = c(s)x^{s-1}(x+1)^{s-4.5}$$

where s is a parameter that enters into the solution of the diffusion equations and is known as the age parameter of the cascade. At the origin of the cascade $s = 0$; at shower maximum $s = 1$ and at the end of the cascade when the number of particles fall to unity,

$s = 2$. $c(s)$ is a normalisation constant with specific numerical values. For example, at $s = 1$, $c(s) = 5/4\pi = .398$; x is the radial distance in Molière units. The Molière unit (defined in Chapter 4) is about 80 metres in air at sea level, or one quarter of a radiation length.

2.11 Numerical Model of Cascades

With the aid of a computer it is possible to simulate an electron-photon cascade by following the tracks of individual **PARTICLES**. The various interaction cross-sections are sampled using Monte Carlo techniques and information on the various macroscopic properties of the cascade can be accumulated. The statistical accuracy of the results is, in principle, only dependent on the number of events that are sampled. By virtue of the 'dice rolling' technique that is employed, it is also possible to obtain information on fluctuations in cascade development. The approximations that are made and the physical processes that are neglected can be reduced to a degree that is only limited by the ingenuity of the programming technique employed and the computer time available. However, it is desirable to establish the physical processes that are important by making comparisons with less refined models. Moreover, even a relatively simple simulation programme tends to be lengthy and less amenable to checks for errors than analytical calculations. A Monte Carlo programme, based on the work of Butcher and Messel (1960) has been modified and adapted for the purposes of calculating the radio emission from individual electron-photon cascades in air. The cascade calculations are described and discussed in detail in the next two chapters.

2.12 Modelling the Whole Extensive Air Shower

Electrons are the most numerous particles in an extensive air shower and it is the motions of these particles that dominate in the

radio emission. It is therefore important to establish with reasonable precision the distribution in height of the electron-photon cascades. One method of doing this is to adopt a specific model for the nuclear interactions and so determine the longitudinal distribution of the π^0 particles that decay into (typically) 10 GeV photons. Alternatively, an empirical development curve for an extensive air shower may be used and individual 10 GeV electron-photon cascades may be folded to give the correct distribution in height. In the first instance, the cascades are supposed to be collinear and directed along the axis.

From the considerations in section 2.4, the choice of the primary photon energy (10 GeV in this case) is unimportant. However, the assumption that the cascades are collinear can prove unrealistic. In particular, if the lateral spread of a real extensive air shower turns out to be greater than the lateral spread of the model air shower, then one must assume that the angles the parent π^0 particles make with the shower axis are finite and important. Owing to the uncertainties that are inherent in the nuclear model that is adopted, this contribution to the lateral spread is difficult to evaluate theoretically. However, in Chapter 5, the importance of π^0 emission angles is considered by initially modelling an extensive air shower with zero π^0 angles and then comparing the lateral spreads as well as the predicted radio emission with experimental data.

CHAPTER 3

THE MONTE CARLO PROGRAMME3.1 Introduction

Two main approaches have been used in the solution of the problem concerning the development of the electron-photon cascade. One approach is the analytical method of Approximation 'B' described in the previous chapter. This necessarily requires various simplifications in order to reduce the mathematical expressions to manageable proportions. The other approach has been to computer-simulate the development of the electron-photon cascade by following the tracks of individual particles using Monte-Carlo techniques. The sampling technique of the various interaction processes was originally demonstrated by Butcher and Messel (1958, 1960). The general cross-sections for the basic interactions were used (Rossi, 1965) and, in principle, the complete cascade could be studied macroscopically from the launch of a primary electron or photon of any energy. The original Butcher and Messel programme is a one-dimensional calculation in the sense that electron scattering and lateral spread is not incorporated. However, the basic Monte Carlo programme has been extended to three dimensions by several workers (e.g. Baxter, 1967) to investigate various aspects of extensive air showers including the lateral spread. In addition to this, the effects of the geomagnetic field have been incorporated (Shutie, 1973) to calculate the radio emission from a cascade.

The present work on radio emission calculations was begun as an extension of the earlier work. However, certain features of that work proved to be in error and considerable corrections and modifications had to be made. The primary purpose of this thesis - the calculation of radio emission from simulated cascades - is

detailed in Chapter 5, but some more fundamental aspects of cascade structure are dealt with first in Chapter 4. In the present chapter, the objectives are:-

- (1) to outline the physical processes incorporated in the programme;
- (2) to describe its basic structure;
- (3) to summarise the results accumulated from cascade simulations and their significance, and
- (4) to summarise the modifications necessary to the original programme for a satisfactory cascade simulation.

3.2 Detailed Description of the Programme

In an electron-photon cascade, the initial energy of the primary photon is subdivided between more and more electrons and photons by bremsstrahlung and pair production. The behaviour of each of these particles is simulated in microscopic detail, one at a time, and if, at an interaction point, an additional particle is created, one of the two (the higher energy particle) must be stored with all its parameters. The particle parameters are defined as follows:-

- (1) Type: photon, electron or positron.
- (2) Space: coordinates (x y z) with respect to the coordinate frame of the cascade axis as in Fig. 3.1.
- (3) Time: defined as the intrinsic time lag of the particle in nanoseconds behind an imaginary particle moving with velocity c along the shower axis.

COORDINATE AXES OF PARTICLE AND SHOWER FRAME

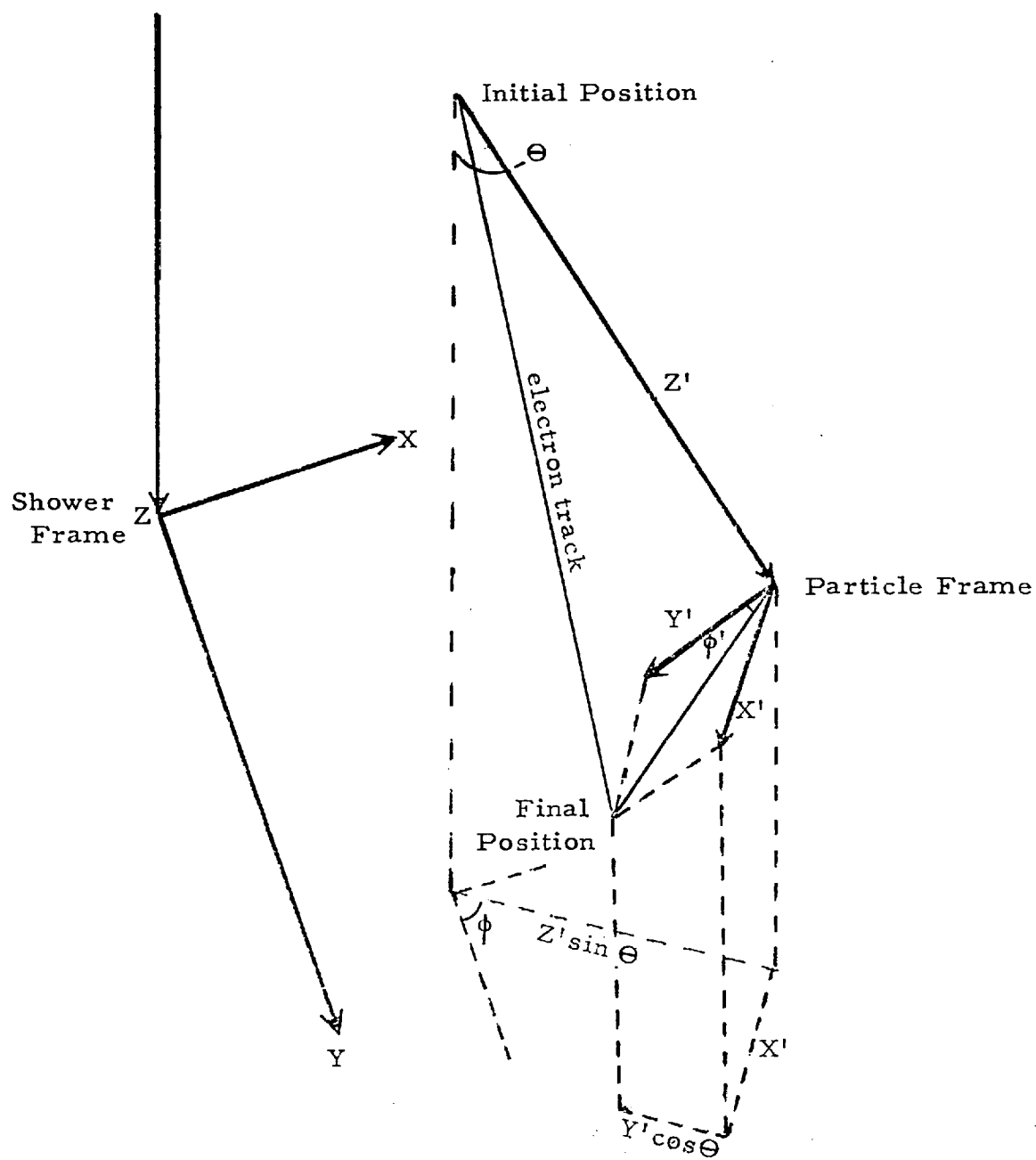


Fig. 3.1

- (4) Orientation: θ and ϕ define the direction of motion of the particle with respect to the cascade axis frame (Fig. 3.1).
- (5) Energy (E) in units of MeV.
- (6) A quantity (P) connected with the growth of the geomagnetic dipole moment.

When an interaction point is reached at which there are no further particles of that branch to be tracked, the programme returns to one of the stored particles. When all the particles have been tracked the single cascade simulation is complete. It is possible to run many cascades in order to gain information on fluctuations and to improve on statistics.

The cross sections for the physical processes are most conveniently defined by the corresponding interaction lengths in units of g cm^{-2} . All the cascades are simulated in air. Usually, lengths are given in g cm^{-2} ; for example, the atmospheric depth is given as the mass per unit area of atmosphere above sea level. When calculating radio emission, it is more appropriate to deal with geometrical distances in metres. The appropriate conversion between distances z (metres) and x (g cm^{-2}) is $x = x_0 e^{-z/z_0}$ where z_0 is the scale height (8000 metres), x_0 is 1032 g cm^{-2} at sea level, z is in metres above sea level and x is in g cm^{-2} measured from the top of the atmosphere. This is a sufficiently good approximation to a real atmosphere given by Tennent (1971).

However, the results presented in Chapter 4 are based on simulations carried out in a constant density atmosphere (10 metres per g cm^{-2}) to provide a more direct comparison with the calculations of other workers for air absorbers.

The programme is subdivided into two branches dealing with electrons and photons respectively.

The Photon Branch

Photons move in straight lines unless scattered (with change of energy) by an interaction; they are unaffected by the presence of the geomagnetic field. The distance a photon travels before interaction is determined using the probability function appropriate to its mean free path. To do this, a random number generator is used which gives a uniform distribution of random numbers from zero to unity. The product of the mean free path and the ^{minus} logarithm of the random number gives the distance through which the photon is to be moved. This technique is the essence of the Monte Carlo method when the mean free path is constant. For the true energy-dependent cross sections of pair production and the Compton process, effective simultaneously, a combination of procedures called the composition and the rejection method can be applied. These are described in detail by Messel and Crawford (1970). The method involves first moving the photon a distance appropriate to the shortest of the possible mean free paths, namely the asymptotic mean free path for pair production. A procedure is then followed to decide whether pair production, Compton effect or no interaction at all is to occur at this point. For a high energy photon, pair production is most likely to be accepted on the first consideration. For a low energy photon, if both the Compton and pair processes are rejected, the photon is moved an additional distance and the whole cycle repeated until one of the two processes is accepted. In this way, it is possible to decompose the energy-dependent cross sections into manageable steps.

The cross sections used are those given by Rossi (1965) and the simulated interaction results in the coordinates, energy and type of the secondary products to be defined. The orientation of the secondaries is related to conservation of momentum considerations which define the angles of emission of the electron pairs and Compton secondaries. A random azimuthal angle is selected and these angles are transformed to angles θ and ϕ which define the direction of motion with respect to the shower axis frame of reference.

The Electron Branch

Electrons do not retain either their energy or direction between interactions. The energy is continually degraded through ionisation, and multiple Coulomb scattering plays an important role in the lateral spread of electrons. Moreover, the geomagnetic field superimposes a mean curvature to the electron track. The treatment of electrons has therefore been divided into three main consecutive sections in the programme.

Bremsstrahlung and Ionisation Loss

The simulation of these two processes, as laid down by Butcher and Messel (1958, 1960), involves sampling a bremsstrahlung interaction path distance and then subtracting the ionisation energy loss for the distance moved. The ionisation loss energy dependence as used by Butcher and Messel, incorporates the relativistic rise at high energies (Fig. 4.3 in Chapter 4).

Electron Scattering

A Gaussian multiple scattering distribution as derived by Rossi (1965) is used. This approximation is valid for small angles and involves the sampling of independent Gaussian distributions of x and y displacements at the end of the electron track of the form:-

$$p(x) = A \exp (3x^2/\theta_s^2 z^3)$$

where z is the length of the track, x is the displacement in a direction perpendicular to z in the particle frame and θ_s^2 is the mean square scattering angle per g cm^{-2} of track length. The value of θ_s^2 is given by:-

$$\theta_s^2 = 10.43/E^2 \quad \text{for } E > 57 \text{ MeV}$$

$$\theta_s^2 = 1.16E^{-2} \ln (138.6E) \quad \text{for } E \leq 57 \text{ MeV}$$

CALCULATION OF MULTIPLE SCATTERING AND FORESHORTENING

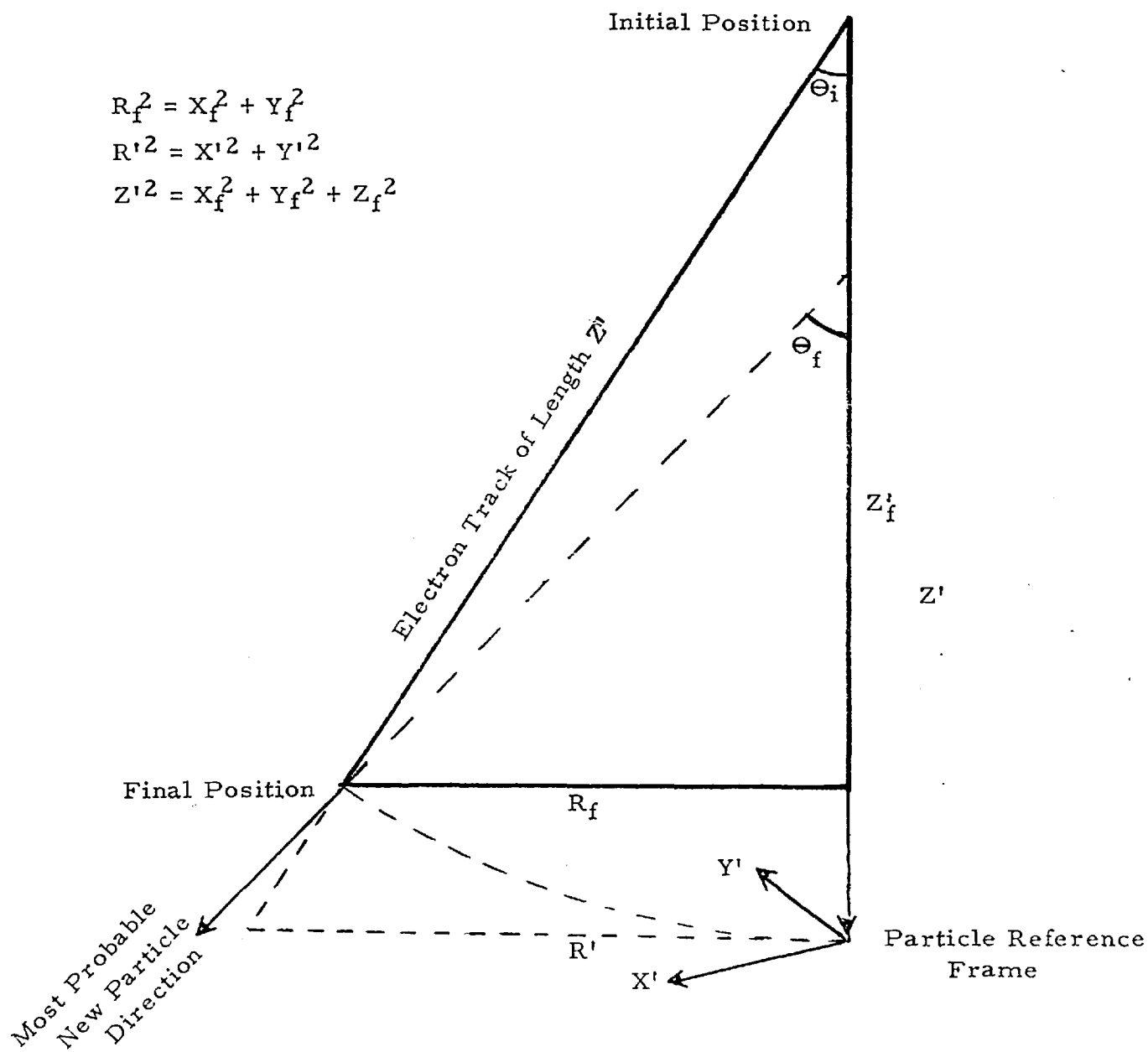


Fig. 3.2

The energy range $E \leq 57$ MeV is the region where the finite nuclear size modifies the range of impact parameters relevant in the scattering. The direction of motion of the electron after scattering is found by sampling a Gaussian angular distribution centred about a mean direction illustrated in Fig. 3.2. This direction is the line connecting the new particle position to a point two-thirds of the way back along the z axis in the particle frame, and is in accord with the Rossi angular distribution function (Rossi, 1965).

Single scattering, although not of significant importance, has been included and accounts for about 3% of all electron tracks. A comparison of the above treatment of electron scattering with other treatments (e.g. Messel and Crawford, 1970) will be given in Chapter 4. It will be shown that the angles involved are sufficiently small for the Gaussian approximation to be valid.

The scattering simulation just described leads to an increase in the electron path length which is inconsistent with the interaction cross section. To correct this effect, a track foreshortening procedure is adopted in the particle frame as shown in Fig. 3.2. Because of the importance that is attached to the treatment of electron scattering in the determination of the lateral spread of electrons, the following tests were carried out to justify the foreshortening procedure.

The section of the programme dealing with the scattering of electrons was used for electrons with a fixed initial energy, final energy and length of track (L). About 2000 such electrons were launched and a distribution of electrons as a function of lateral displacement (r/L) was constructed. The histograms of the distributions with and without foreshortening are illustrated in Fig. 3.3 for $\langle (r/L)^2 \rangle = 1$, which corresponds to comparatively large angle scatters associated with lower energy electrons. The solid lines in the diagram are the distributions obtained analytically for the stated $\langle (r/L)^2 \rangle$, without foreshortening. This is the Rayleigh

EFFECT OF FORESHORTENING ON LATERAL DISPLACEMENTS

r = Lateral Displacement
 L = Longitudinal Path Length

———— distributions without foreshortening for stated $\langle (r/L)^2 \rangle$
 ----- distributions after foreshortening
 histograms: distributions obtained from computer simulations

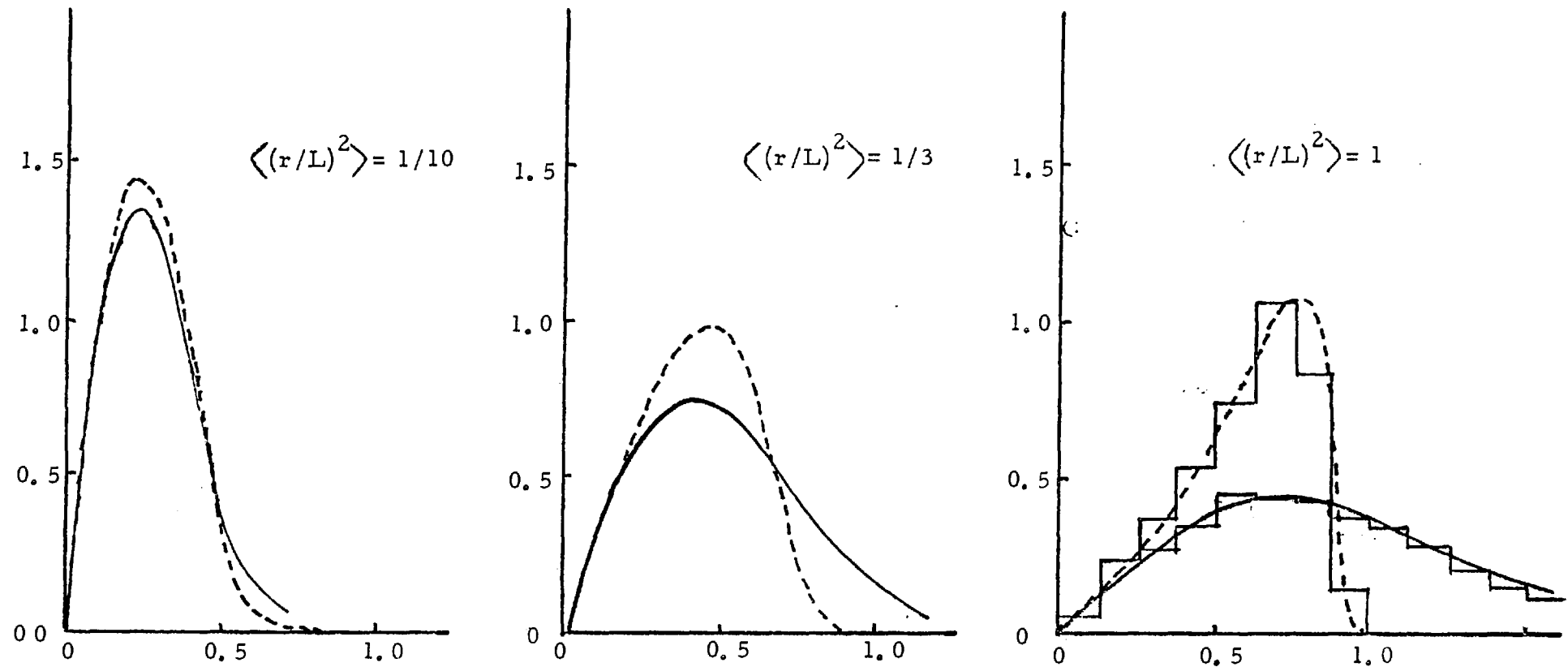


Fig. 3.3

DISTRIBUTION OF TRACK LENGTH AS A FUNCTION OF $\langle (r/L)^2 \rangle$

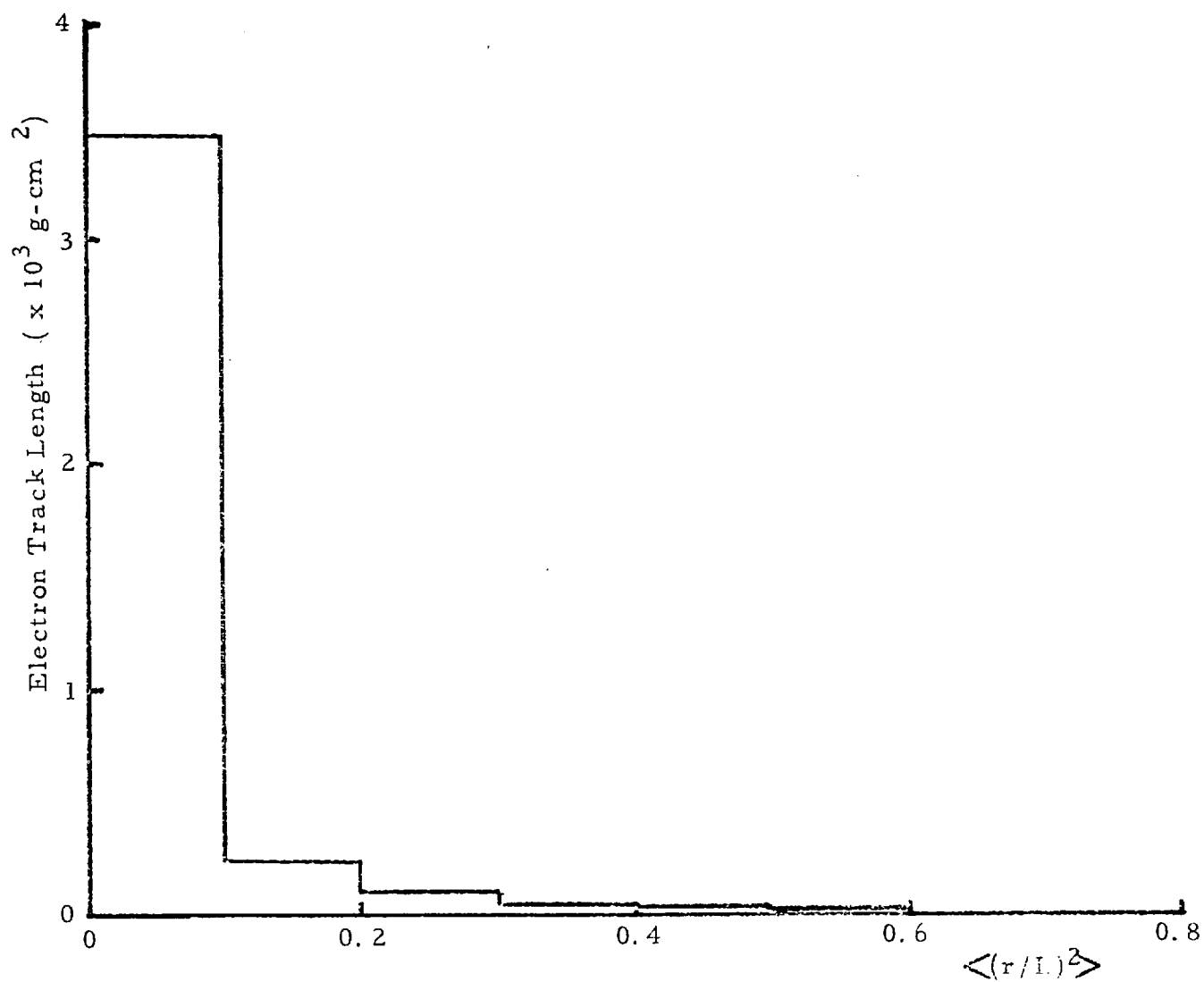


Fig. 3.4

distribution:-

$$f(r)dr = re^{-r^2}dr$$

For a total lateral displacement (r) where $r^2 = x^2 + y^2$, the Rayleigh distribution results from sampling two Gaussian distributions from the x and y displacements (Rossi, 1965). The dotted lines in the Figure represent the distributions after foreshortening and the degree to which foreshortening distorts the Rayleigh distribution is illustrated for three values of $\langle (r/L)^2 \rangle$.

A test simulation of the electron-photon cascade was carried out, in which the electron track length was determined as a function of $\langle (r/L)^2 \rangle$ as shown in Fig. 3.4. From this it was established that over 90% of the track length corresponded to $\langle (r/L)^2 \rangle$ less than 0.2, from which it was concluded that serious distortion occurs only with a very small fraction of the track length at the lowest energies.

Geomagnetic Deflections

In the absence of scattering, the path of an electron is a helix centred along the geomagnetic field lines. The effect of the geomagnetic field is only a small perturbation on the electron motion compared with multiple Coulomb scattering. It is, however, of major importance for radio emission and the treatment of this effect is discussed below.

For a particular electron track, the initial and final coordinates and the orientation due to scattering are first established. The additional influence of the geomagnetic field is then calculated separately to give an accurate estimate of the new final coordinates and orientation of the charged particle (Shutie, 1973).

CALCULATION OF MAGNETIC DEFLECTION

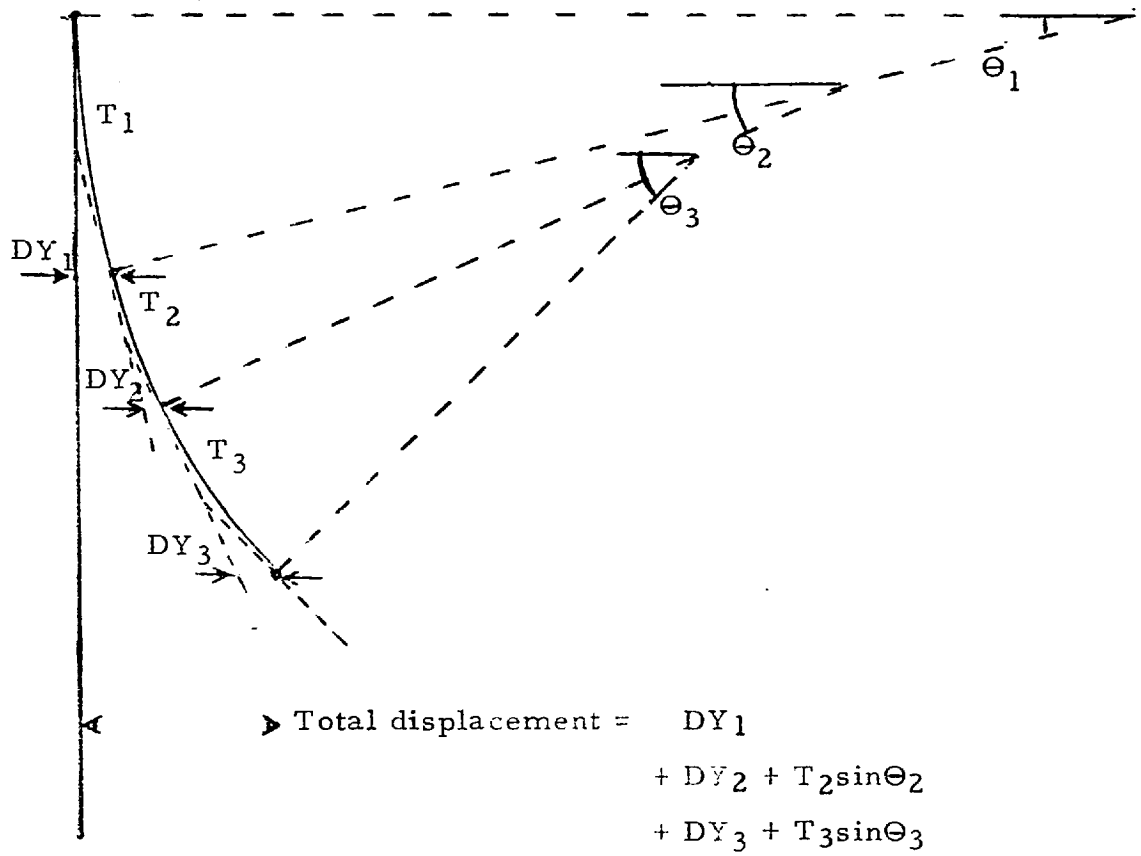


Fig. 3.5

It has already been stated that it is the manner in which the dipole moment rises with time that determines the radio-emission. Since electrons, ~~photons~~ and positrons are deflected in opposite directions by the geomagnetic field, it turns out that for a cascade launched vertically in the atmosphere, the dipole moment that is generated is predominantly in an East-West direction, along the 'y' coordinate of the shower axis. The summation of the total East-West deflections of all the charged particles as a function of time lag would yield the growth of the dipole moment in a straight-forward manner.

However, excessive computing time would be required to reduce the fluctuations due to electron scattering which superposes a large 'noise factor' on the dipole moment development curves. To eliminate the effect due to electron scattering, a procedure is applied which is illustrated in Fig. 3.5. The diagram illustrates the growth in the dipole moment as a result of geomagnetic deflections only; electron scattering and photon tracks are omitted for clarity. In each length of track T , the magnetic field produces a lateral displacement DY , corresponding (for a circular arc approximation) to an angular deflection $2DY/T$. The total angular deflection after n interactions is:-

$$\theta_n \approx \sum_{i=1}^n \frac{2(DY)_i}{T_i}$$

and the increment in magnetic displacement produced at the end of the n^{th} track is $\approx (\theta_{n-1}T_n) + DY_n$. In the programme $\sin \theta_i$ is determined at each electron interaction point and is referred to as the parameter (P) at the beginning of this section.

The next section describes how some electron tracks are themselves divided into a number of segments. In these cases, the same procedure described above is used to calculate the geomagnetic deflections at the end of each segment.

Segmentation of Electron Tracks

A procedure is applied whereby some electron tracks are split into a number of smaller segments, and the scattering and geomagnetic deflections are found separately for each segment of track. This procedure is applied for two main reasons.

The helical trajectory of an electron track due to the geomagnetic field is calculated on the assumption that the average energy determines the curvature at all points on the track. Dividing some of the tracks reduces the energy drop due to ionisation and renders the constant radius of curvature within each segment a closer approximation to the true spiralling motion of the electron. Secondly, the Rossi formulae used for scattering is only an approximation: it is correct only for small scattering angles. This implies that some inaccuracies could arise for some of the more oblique tracks which are typical of the lower energy electrons in the cascade. By dealing with shorter lengths of electron tracks, the scattering angles at each stage are reduced.

The obliquity (θ) and the length (L) of a track determines its geometrical lag ($\theta^2 L/2$). The magnitude of this lag is therefore used to determine the number of segments required for a given track.

The track is split into 2, 4 or 8 segments if the change in lag from the beginning to the end of the track exceeds 1, 3 and 7 nanoseconds respectively.

As an illustration of the relevance of segmenting electron tracks to multiple scattering and foreshortening, we consider low energy tracks for which ionisation (2MeV/g cm^{-2}) determines the range $Z(\text{g cm}^{-2})$. The value of $\langle\theta^2\rangle$ is $\langle\theta_s^2\rangle Z$, where $\langle\theta_s^2\rangle$ is the mean square scattering angle per g cm^{-2} . For an electron track with initial energy E_i and final energy E_f , $\langle\theta_s^2\rangle$ may be taken as $\sim 10/E_i E_f$ and therefore:-

$$\langle\theta^2\rangle \sim \frac{10}{E_i E_f} \cdot \frac{E_i - E_f}{2} \sim 5(1/E_f - 1/E_i)$$

For $E_f = 5$ MeV and a large value such as 80 MeV for E_i , the scattering angle is about a radian. However, by splitting an electron track into 8 steps of 10 MeV, the scattering angle still remains large for the last 15 - 5 MeV step but the range is only about 7 g cm^{-2} and the errors are confined to very low energy tracks.

Other Threshold Parameters

The programme only tracks the particles while their energy exceeds a specified cut-off value (e.g. 4 MeV). An electron, whose bremsstrahlung interaction point occurs below the cut-off energy, is tracked for that distance until the energy loss through ionisation drops to the cut-off energy.

Particles travelling backwards, i.e. those that have been scattered to angles greater than 90 degrees to the cascade axis, also cease to be tracked. This limitation is of no consequence from the macroscopic point of view of studying cascades in air. Moreover, the time delays associated with such low energy tracks are certainly too large to contribute to radio emission as is explained in Chapter 5.

Ground level is defined in the programme (in the shower frame of reference) as the depth below which particles are no longer tracked.

3.3 Detection of Programme Errors

In the course of a detailed study of the programme, it became clear that additional corrections to some of the earlier work were essential for satisfactory simulations of the cascade. These corrections are summarised below and tests on the various input cross sections are presented at the end of this Chapter.

A Deficit in Track Length and Total Dipole Moment

Below the critical energy, a deficit of track length by a factor of ~ 3 compared with estimates in Chapter 2 was found to be due to some of the electron tracks being discarded before the specified cut-off energy was reached. This was also partly responsible for a factor of 10 deficit in the geomagnetic dipole moment, i.e. about 40 electron-metres compared with the expected 800 electron-metres (section 2.7).

Cut-off Energies

Minor variations in terms of different cut-off energies (.6 - 10 MeV) defined in various parts of the programme have resulted in inaccurate cross sections at energies below 10 MeV. A generalised variable cut-off energy is now specified throughout the programme and all particles can be tracked until this cut-off energy is reached. The lowest cut-off energy that can be specified is set at 1 MeV.

Coordinate and Angular Transformations

The three coordinate frames of reference that are used in the programme are the particle frame, the cascade frame (Fig. 3.1) and the geomagnetic frame (used in the calculation of the helical trajectory of an electron due to the geomagnetic deflection). The coordinate and angular transformations have been checked 'in situ' by setting the scattering cross sections to zero and tracing a test particle that is launched at various angles to the cascade axis. The presentation of the original particle orientation between the successive interactions ensures that the transformations are correctly carried out. An error in the sign of one of the angular transformation equations was detected although this would have only affected some of the low energy oblique electron tracks.

Calculations of Time Lags

The 'intrinsic' time delay is defined as the lag (measured in nanoseconds) of the particle behind an imaginary particle moving with velocity c along the shower axis. This lag mainly arises from oblique electron tracks due to scattering (Fig. 3.6a). Another contribution could be included which allows for the fact that electron tracks should be curved between the scattering points as illustrated in Fig. 3.6b.

The following argument shows that this contribution is only a second order correction to the total time lag.

The additional delay (in units of $g \text{ cm}^{-2}$) is given by $L - D$ where $L = R\theta$ and $D = 2R \sin \theta/2$. Therefore

$$\begin{aligned} L - D &\approx R\theta - 2R[(\theta/2) - \frac{(\theta/2)^3}{6}] \\ &= R\theta^3/24 \\ &= L\theta^2/24 \end{aligned}$$

Since $\langle \theta^2 \rangle = \langle \theta_s^2 \rangle dL$ where $\langle \theta_s^2 \rangle$ is the mean square scattering angle per $g \text{ cm}^{-2}$, the lag may be written as:-

$$\begin{aligned} L - D &= \frac{\theta_s^2}{24} \int_0^L L dL \\ &= \theta_s^2 L^2 / 48 \text{ g cm}^{-2} \end{aligned}$$

The above expression may be compared with the contribution to the lag due to the obliquity of the electron track (Fig. 3.6a) which may be written as:-

$$L\theta_t^2/2$$

where θ_t is the mean angle of the track with the shower axis.

CALCULATION OF DELAY DUE TO OBLIQUITY
OF A PARTICLE TRACK

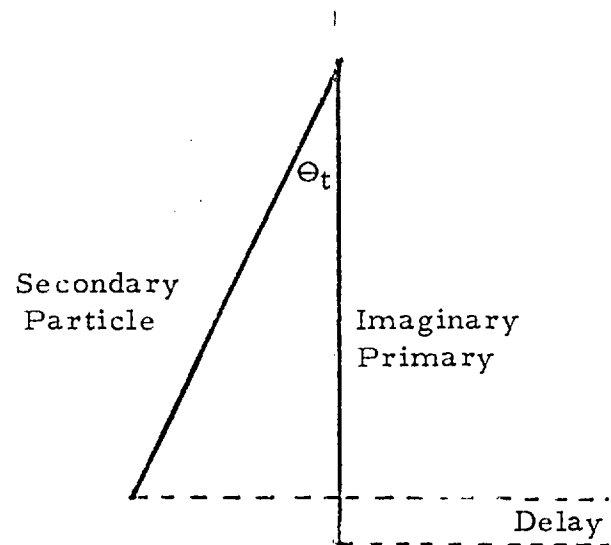


Fig. 3.6 a

CONTRIBUTION TO THE DELAY DUE TO
CURVATURE OF ELECTRON TRACKS

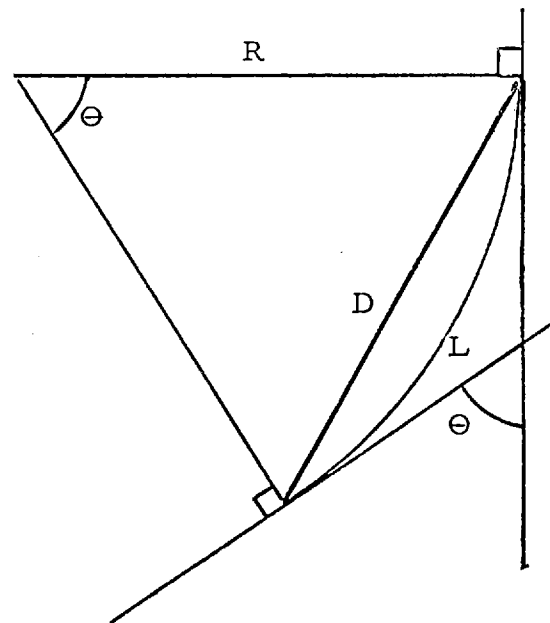


Fig. 3.6 b

Although θ_t would be $\sim \theta/2$ for a single arc, it will be substantially larger when one allows for the deflections in the previous history of the cascade. Estimating $\theta_t \sim \theta$, and substituting for θ_t^2 , the lag becomes $\theta_s^2 L^2/4$ and one finds that the additional delay due to curvature is only about 10%. This conclusion differs from that of Baxter (1967) who claimed that the curvature was important.

The 'L - D' contribution has been omitted from the present simulations and, in any case, the scattering angles are kept small by segmenting the electron tracks as described earlier.

An additional contribution to the geometrical lag arises from the deflection of electrons by the geomagnetic field. This contribution is incorporated in the programme.

The Pair Production Cross Sections

Baxter (1967) carried out investigations into the lateral spread of electrons as a function of depth. The Greisen expression (Chapter 2) for the lateral distribution function of Nishimura-Kamata was fitted to the data from the simulations and the corresponding age parameter (s) was ascribed to the simulated cascade as a function of depth. Baxter's results show a best fit of the lateral distribution for $s = 0.6$ at cascade maximum rather than 1.0. This was traced to an error in the decomposition of the pair production cross section in the programme which had the effect of ascribing a disproportionately high energy to one of the pair electrons. Correcting this resulted in an electron density independent of radius close to the axis at shower maximum. Such independence is to be expected on general grounds at shower maximum and agrees with the usual Nishimura-Kamata curves for $s = 1$.

3.4 Programme Output

In order to derive the various macroscopic properties of the simulated electron-photon cascades, it is necessary to introduce

various binning routines into the programme relating to the particle parameters. The choice of information to be binned should serve at least one of three purposes:-

- (a) To solve a specific problem relating to particular properties of electronic cascades such as the radio emission.
- (b) To provide a check against comparable data presented by other workers using different approaches to cascade studies.
- (c) To provide an internal self-consistency check on the programme.

The importance of (b) and (c) cannot be over-emphasised if one is to accept, with conviction, that the cascade is correctly simulated strictly according to the input assumptions. In this section, the main output information that is accumulated is summarised, but the presentation and discussion of the results is deferred to the next chapter.

Dumped Particles

These include particles below cut-off energy as well as backwards-going particles. Fourteen bins are arranged to accumulate the total number of such particles and their energies, as follows:-

- (a) Backward-going electrons.
- (b) Backward-going photons.
- (c) Electrons passing across the threshold energy for cut-off.
- (d) Electrons created below cut-off energy by bremsstrahlung.
- (e) Photons created below cut-off energy by bremsstrahlung.

- (f) Electrons created below cut-off energy by the pair or Compton process.
- (g) Compton photons created below cut-off energy.

Electron Energy Deposition

For each charged particle track, the energy deposited (through ionisation) is categorised according to:-

- (a) Location by depth, i.e. 10 depth bins in 103.2 g cm^{-2} intervals.
- (b) Location by radius, i.e. 4 radial bins with boundaries at 10, 30 and 100 metres away from cascade axis.
- (c) Energy range, i.e. 1 of 4 energy groupings of 0 - 10 MeV, 10 - 20 MeV, 20 - 80 MeV and above 80 MeV.
- (d) Type, i.e. electrons and positrons are grouped separately.

In addition, all the dumped particle energies are also deposited into this array according to the coordinates at the end of the particle tracks. Photons are labelled as negative electrons in this context. As an energy conservation check of prime importance, the total energy deposited into this array must equal the launch energy of the primary particle, minus the rest energy of the secondary electrons. This array gives an overall picture of the shower as well as some information on the ~~negat~~ive charge excess present.

Lateral and Longitudinal Electron Development

A two-dimensional array is employed with 40 depth boundaries separated in equal 25.8 g cm^{-2} intervals and 11 radial bins 5 metres wide. This array may be considered as equivalent to having 40 horizontal sheet detectors in the atmosphere, measuring the flux

of electrons passing through. Each 'sheet' is then divided into annular rings as a measure of lateral distribution to be plotted and compared with other cascade calculations. In addition, the mean and the r.m.s. lateral spread of electrons are computed at each of 40 depths as well as integrated over all depths.

The Track Length Distribution

The electron track length (in metres) is binned as a function of energy in 5 MeV steps at low energy (below the critical energy of 84 MeV). This is independent of three-dimensional scattering and provides an essential check with the relatively straight-forward one-dimensional aspects of the simulations. The computed track length is compared with track length distributions determined by other methods.

Photon Flux and Track Length

The photon flux is binned by counting at 10 depth partitions and within 4 energy ranges. This gives an indication of the development curves for the different energy ranges. The photon track length is also binned into a two-dimensional array of 4 energy groups and 10 radial bins of 10 metres width.

Time Structure of the Cascade

For each interaction point, the 'intrinsic lag' is computed and by sub-dividing the total particle track-length into bins according to the value of this lag a profile of the particle distribution across the thickness of the shower disc is obtained. One nanosecond time intervals are used and the time structure of the total track length describes disc properties averaged over the whole of the shower development.

Corresponding distributions are obtained for the charge excess and the geomagnetic dipole moment together with information on the way these vary with radial distance from the shower axis (Chapter 5).

Angular Distribution of Electrons and Photons

The electron track length for various energy ranges is binned as a function of angle to the cascade axis. In addition, the r.m.s. angular distribution of track length is binned as a function of energy for both electrons and photons. This enables some important comparisons of results to be made and also provides an additional check on the scattering cross-sections.

Arrived Particles

Electrons and photons reaching ground level are grouped separately into 4 energy ranges and 4 radial distances from the axis to give an overall ground level distribution for cascades from various heights in the atmosphere. This array is used later (Chapter 5) to determine the overall lateral distribution of particles at ground level for the whole simulated extensive air shower.

3.5 Checks Carried Out on the Input Cross-sections

In this chapter, various aspects of the programme have been described and the output information which relates to the structure of an electron-photon cascade has been listed. Before the output is examined in detail, it is necessary to check that the input data - the cross-sections - have been correctly fed into the programme.

The subroutines dealing with the pair production and Compton processes have been checked by launching a large number of photons (typically 2000) of fixed energy and finding their first interaction points. By repeating this for different photon energies, the variation of pair cross-sections and Compton cross-sections as a function of energy was found; it is plotted in Fig. 3.7. The distribution of energy between electrons and positrons is shown in Fig. 3.8 as a histogram and is found to be in accord with the equations given by Rossi (1965) for pair production. Similarly, the distribution of Compton recoil electrons is shown in Fig. 3.9

and agrees with distributions given by Evans (1965).

The subroutines dealing with the interactions of electrons have been tested in a similar way except that ionisation loss, which is expressed as a continuous function rather than a cross-section to be sampled, was 'turned off'. The distribution of the energy radiated in bremsstrahlung is shown in Fig. 3.10 to agree with the equations given by Rossi. Electron scattering has already been investigated in some detail in section 3.2 and is in accord with the scattering angles θ_s^2 plotted in Fig. 3.11 as a function of energy.

RELATIVE NUMBER OF COMPTON AND PAIR PRODUCTION
INTERACTIONS GENERATED IN THE COMPUTER
SIMULATIONS AS A FUNCTION OF PHOTON ENERGY

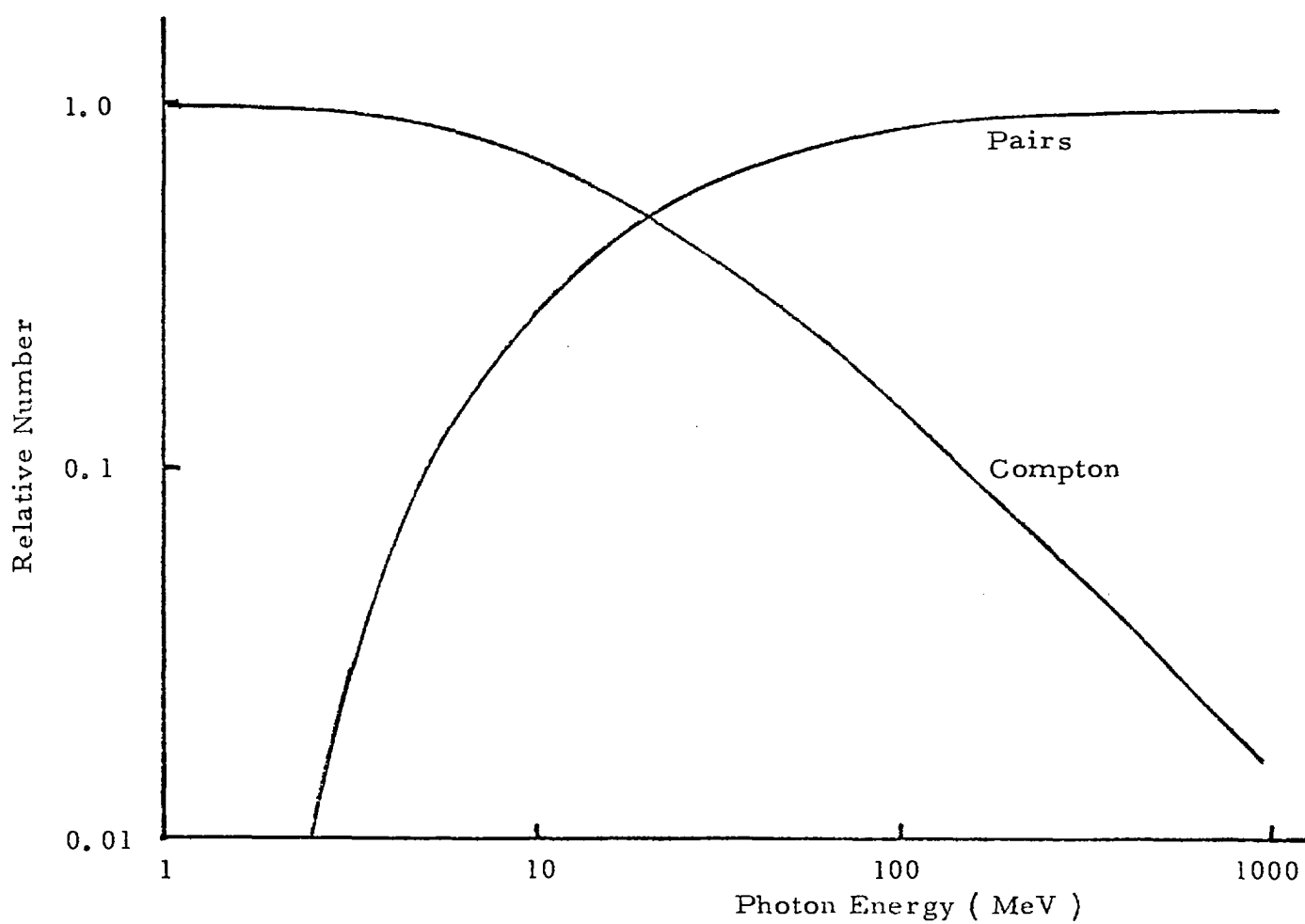


Fig. 3.7

DIFFERENTIAL PROBABILITY OF PAIR PRODUCTION
PER RADIATION LENGTH OF AIR FOR PHOTONS OF
ENERGY 10, 20, 100 AND 10^5 MeV

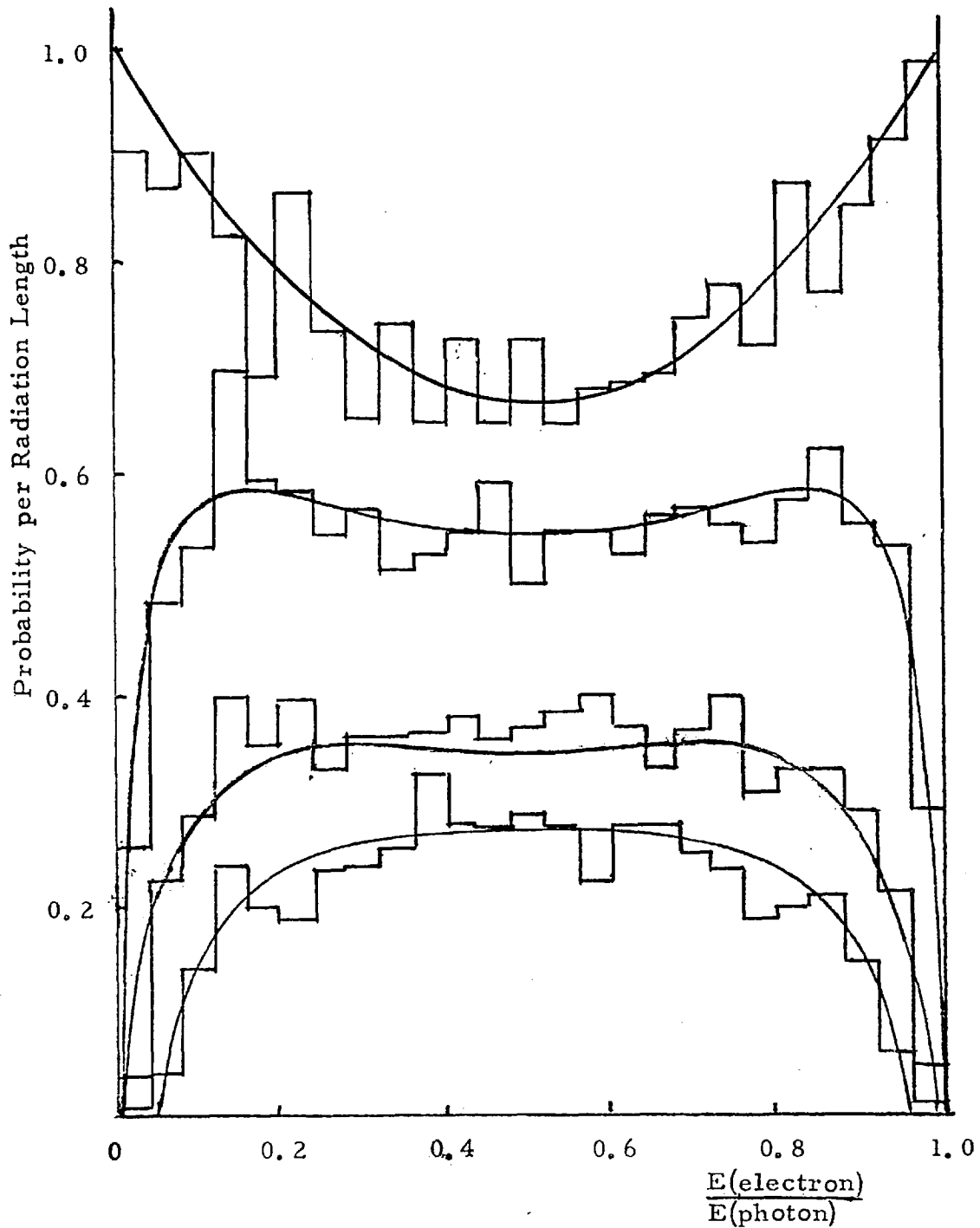


Fig. 3.8

DISTRIBUTION OF COMPTON RECOIL ELECTRONS

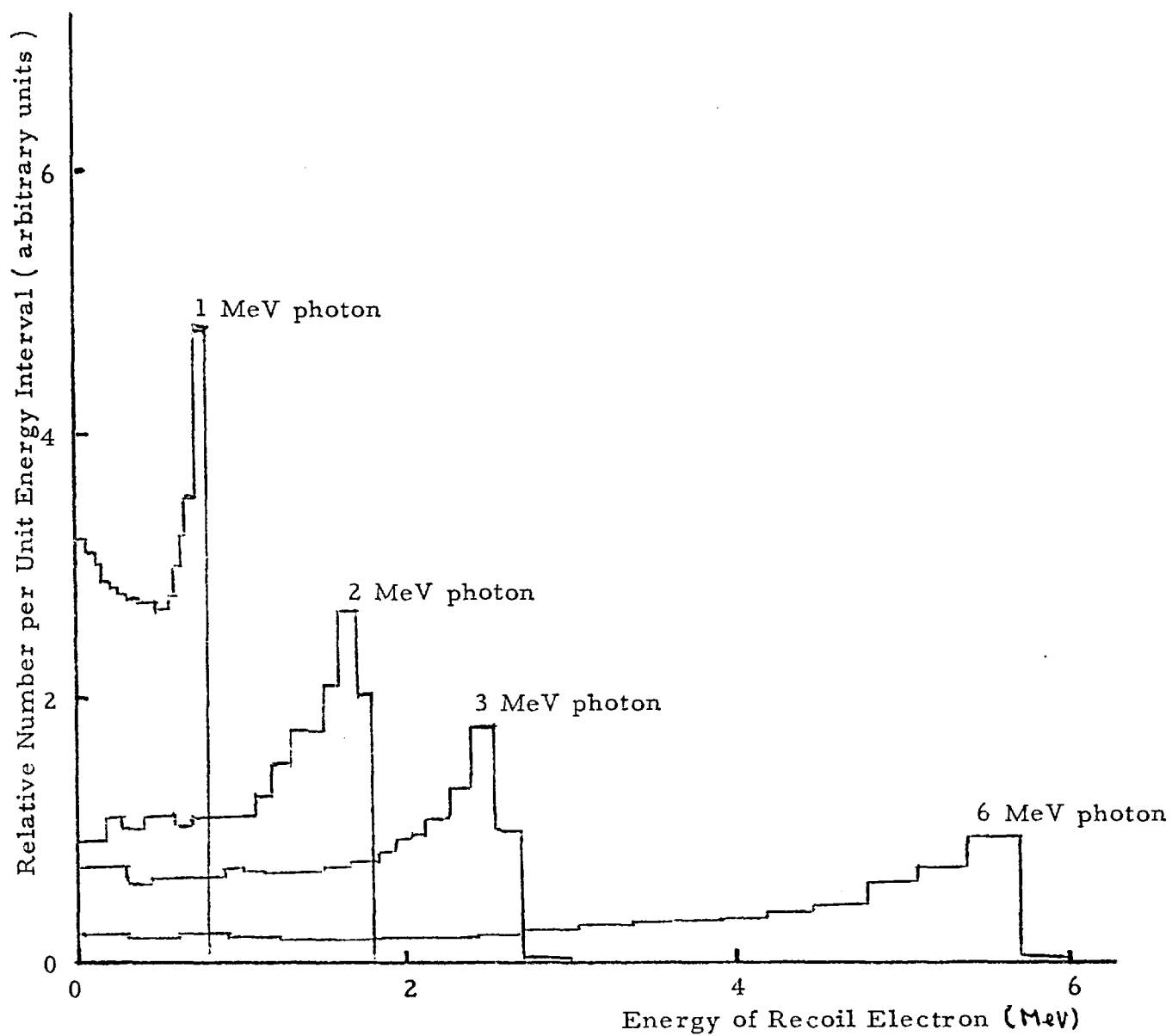


Fig. 3.9

DISTRIBUTION OF ENERGY RADIATED IN BREMSSTRAHLUNG
BETWEEN PHOTONS OF DIFFERENT QUANTUM ENERGIES

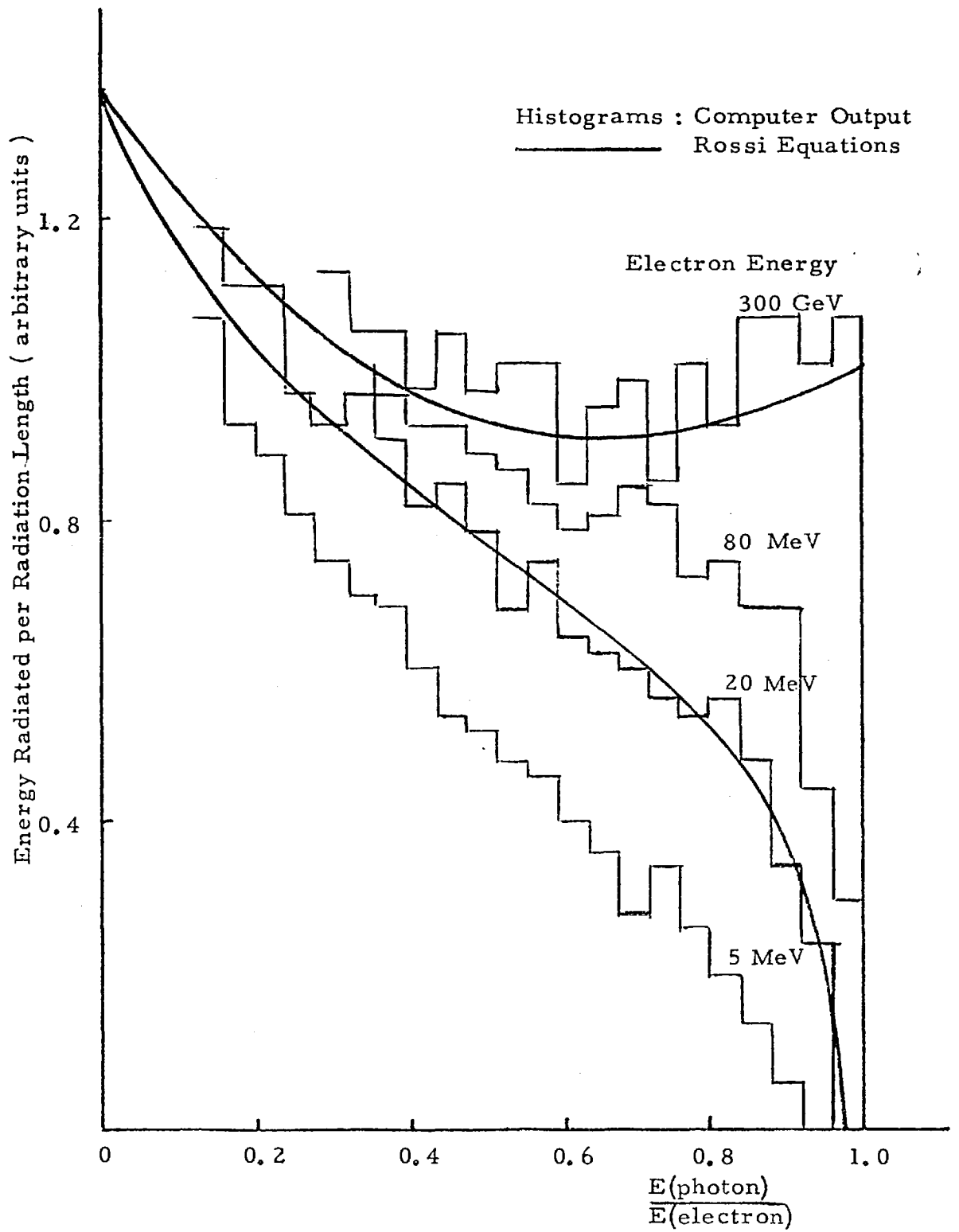


Fig. 3.10

VARIATION OF $E^2\langle\theta_s^2\rangle$ WITH ENERGY (ROSSI) WHERE $\langle\theta_s^2\rangle$ IS THE MEAN
SQUARE SCATTERING ANGLE PER G-CM⁻²

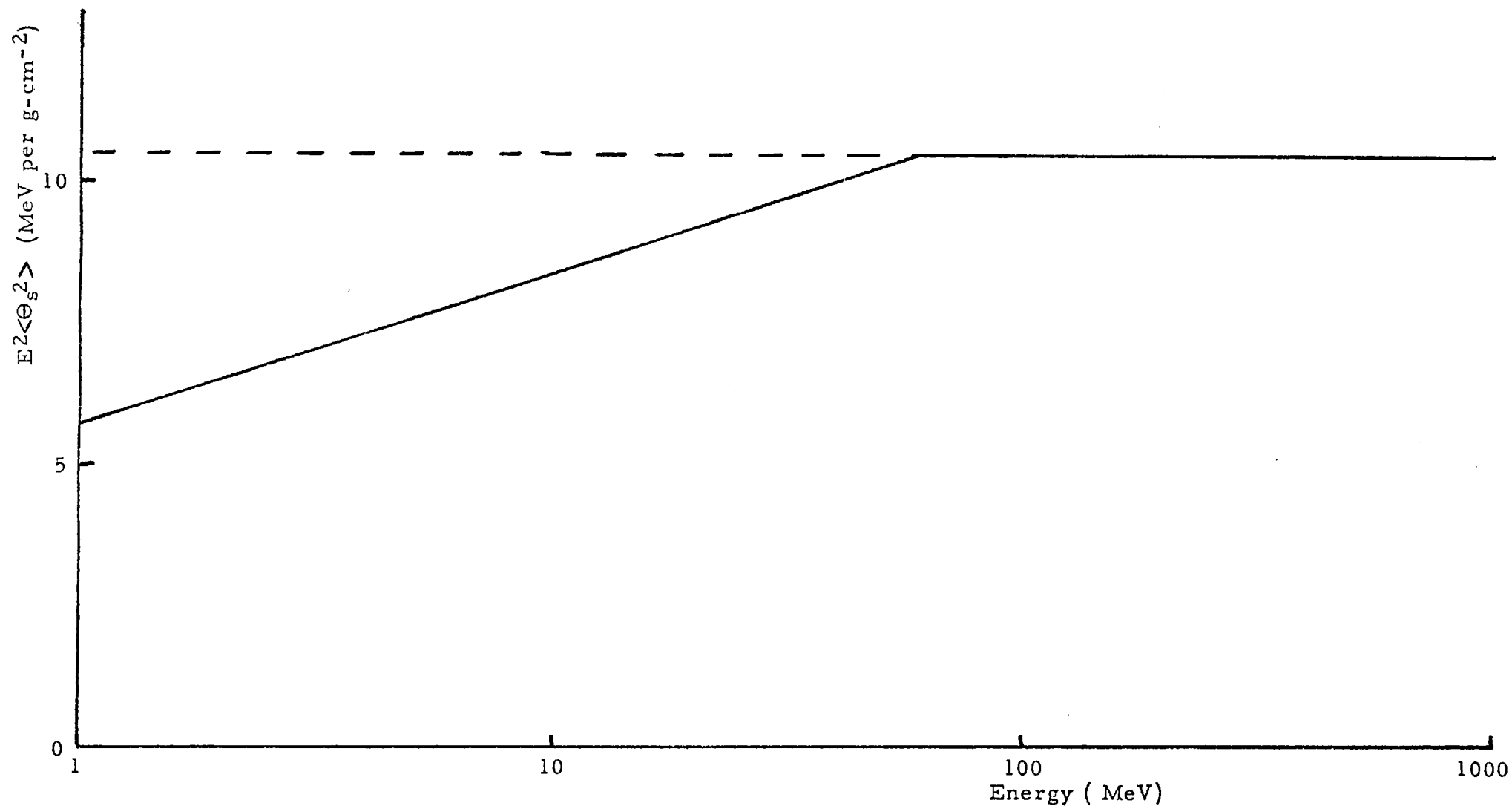


Fig. 3.11

CHAPTER 4

THE RESULTS FROM THE SIMULATIONS4.1 Introduction

In the preceding chapter, the Imperial College version of the Monte Carlo cascade programme has been described. In this chapter, some important results from the simulations are presented and compared with the results of other workers in the field. A constant density atmosphere is assumed, to facilitate these comparisons.

4.2 Choice of Cascade Parameters

Electron-photon cascades have been simulated for 1 GeV, 10 GeV and 100 GeV primary energies. The restriction to three primary energies is a consequence of limited computer time, particularly because a minimum number of separate runs is necessary to obtain reasonable statistics. For example, at shower maximum, there are ~ 15 particles in a 10 GeV cascade and ~ 150 particles in a 100 GeV cascade. For the same expenditure in computer time one can simulate ten times as many 10 GeV cascades as 100 GeV cascades and be much less subject to fluctuations occurring in the first few interactions. The 'track length' associated with particle energies above 1 GeV is about 2% to 3% of the total track length and so most of the results are not dependent on the launch energy. The launch of a 10 GeV primary photon has therefore been adopted as the 'standard' cascade.

The following is a summary of the simulations carried out:-

- (a) 'Standard' cascades with different cut-off energies imposed:
i.e., 1, 4, 10, 20, 80 MeV cut-offs.

- (b) 10 GeV cascades initiated by electrons.
- (c) Cascades for three different photon primary energies:
i.e. 1, 10 and 100 GeV.
- (d) 10 GeV photon initiated cascades simulated according to
the Approximation 'B' cross-sections.
- (e) Special cascade runs to evaluate the accuracy of our
results and the reliability of the programme: e.g. to
observe the effect of not segmenting the electron tracks,
etc.

What follows is a discussion of the various output tables
and graphs obtained.

4.3 Energy Deposition Tables

The energy deposited as a function of radius, depth and energy groupings is shown in Table 1 for electrons and positrons separately. This table gives a crude first view, showing most of the features of the cascade to be discussed. The integrals over all radii and depths are printed in the last row and column respectively. The last pair of tables for electrons and positrons show the lateral and depth dependence integrated over all energy. The energies of the dumped particles, i.e. all particles below the cut-off energy, as well as backwards scattered particles are also included in this table. It is therefore possible to check the overall energy balance of the simulated cascades, i.e. the total energy deposited must be equal to the launch energy of the primary particle, after allowing for the rest energies of the electrons below the cut-off energy.

The following conclusions may be deduced with regard to the overall structure of a typical electron-photon cascade (Table 1):-

TABLE OF ENERGY DEPOSITED IN MeV FOR A 10 GeV CASCADE

No. of cascades =, 500

Cut-off energy = 4 MeV

Depth bins in 103.2 g-cm⁻² intervals

Energy range 0-10 MeV

ELECTRONS						POSITRONS					
Radial bins †						Radial bins †					
	1	2	3	4	sum	1	2	3	4	sum	
1 *	56	10	2	0	68	25	6	1	0	32	
2 *	284	168	82	12	545	132	87	40	3	262	
3 *	195	259	214	61	729	93	128	98	17	336	
4 *	66	134	170	84	454	31	66	76	24	198	
5 *	14	41	80	61	197	6	20	35	19	81	
6 *	3	11	26	29	69	1	5	12	10	28	
7 *	0	3	7	12	22	0	1	3	4	9	
8 *	0	0	2	4	7	0	0	1	1	3	
9 *	0	0	0	1	1	0	0	0	0	0	
10 *	0	0	0	1	1	0	0	0	0	0	
sum *	618	627	583	266	2093	289	314	267	80	950	

Energy range 10-20 MeV

	1	2	3	4	sum	1	2	3	4	sum	
1 *	13	4	1	0	18	9	3	1	0	13	
2 *	80	68	30	2	180	57	49	20	1	127	
3 *	62	102	77	14	254	44	71	51	6	171	
4 *	22	54	62	20	158	15	37	39	10	101	
5 *	5	17	31	16	69	3	12	18	7	41	
6 *	1	4	10	8	24	0	3	6	4	14	
7 *	0	1	3	3	7	0	1	2	2	4	
8 *	0	0	1	1	2	0	0	0	1	1	
9 *	0	0	0	0	1	0	0	0	0	0	
10 *	0	0	0	0	0	0	0	0	0	0	
sum *	183	250	215	66	713	128	175	138	31	472	

Energy range 20-80 MeV

	1	2	3	4	sum	1	2	3	4	sum	
1 *	38	7	1	0	45	32	5	1	0	39	
2 *	249	134	37	1	422	211	112	28	1	353	
3 *	182	214	108	10	514	151	177	83	6	416	
4 *	62	111	96	16	286	53	90	68	11	222	
5 *	14	36	48	15	113	12	26	34	8	80	
6 *	3	9	15	8	35	2	7	12	5	26	
7 *	1	1	4	4	10	0	2	3	2	7	
8 *	0	1	1	1	3	0	1	1	1	2	
9 *	0	0	0	0	1	0	0	0	0	0	
10 *	0	0	0	0	0	0	0	0	0	0	
sum *	548	512	311	57	1428	462	420	230	33	1145	

Energy range 80-10⁴ MeV

ELECTRONS

Radial bins †

		1	2	3	4	sum
Depth bins	1 *	177	2	0	0	179
	2 *	583	81	8	0	672
	3 *	316	152	34	1	503
	4 *	97	76	27	2	202
	5 *	17	23	13	2	54
	6 *	5	5	5	1	16
	7 *	0	1	1	0	3
	8 *	0	0	0	0	0
	9 *	0	0	0	0	0
	10 *	0	0	0	0	0
sum *	1195	340	88	6	1630	

POSITRONS

Radial bins [†]

1	2	3	4	sum
168	2	0	0	170
534	73	6	0	613
289	134	25	0	448
88	63	23	1	174
13	17	15	1	46
4	6	5	1	15
0	1	1	0	2
0	0	0	0	1
0	0	0	0	0
0	0	0	0	0
1096	296	75	4	1470

Energy range 0-10⁴ MeV

		1	2	3	4	sum			1	2	3	4	sum
Depth bins	1 *	283	23	4	0	310			235	16	3	0	254
	2 *	1196	451	156	16	1819			934	322	95	5	1356
	3 *	755	727	433	86	2000			577	509	256	30	1372
	4 *	246	376	356	123	1100			186	256	206	45	694
	5 *	50	117	172	94	433			35	75	102	36	247
	6 *	12	29	56	47	144			8	22	35	19	84
	7 *	2	6	15	19	41			1	5	8	9	23
	8 *	0	1	4	7	12			0	1	3	3	7
	9 *	0	0	0	2	3			0	0	0	1	1
	10 *	0	0	0	1	1			0	0	0	0	0
sum *	2544	1729	1196	395	5864			1975	1206	709	147	4037	

† Radial bin no.

1

2

3

4

Radial distance ranges	0-10m	10-30m	30-100m	100-oom
------------------------	-------	--------	---------	---------

(Energy deposited is rounded to nearest MeV)

TABLE 1

- (a) The longitudinal development of the cascade due to a 10 GeV primary photon is a smooth rise to a maximum at a depth of about 200 g cm^{-2} from the launching point followed by an almost exponential decay towards the 'tail' of the cascade.
- (b) The lateral spread increases with depth and the cascade has a median width integrated over all depths of less than 1.5 g cm^{-2} .
- (c) The lower energy electrons show a wider lateral spread.
- (d) About a third of the total energy is deposited below 10 MeV and a third of the total energy is deposited above the critical energy ($\sim 80 \text{ MeV}$).
- (e) The negative charge excess is predominantly associated with low energy particles.
- (f) The charge excess increases with radial distance from the core.

The basic structure of the cascade is in accord with the qualitative features of the cascade discussed in Chapter 2.

4.4 The Track Length Distribution and the Energy Spectrum

Much of the track length is associated with low energy particles and it is clear that if a median energy with respect to track length is to be determined, one must ascribe a length of track to the particles below cut-off energy (the dumped particles). Where an energy-independent ionisation loss cross-section is adopted, this presents no problem, since in this case a track length can be assigned proportional to the energy deposited. Both the calculations carried out by Richards - Nordheim (1948) and by Nishimura - Kamata (Nishimura, 1967) are for a constant ionisation

INTEGRAL TRACK LENGTH DISTRIBUTIONS FOR ELECTRONS AND PHOTONS

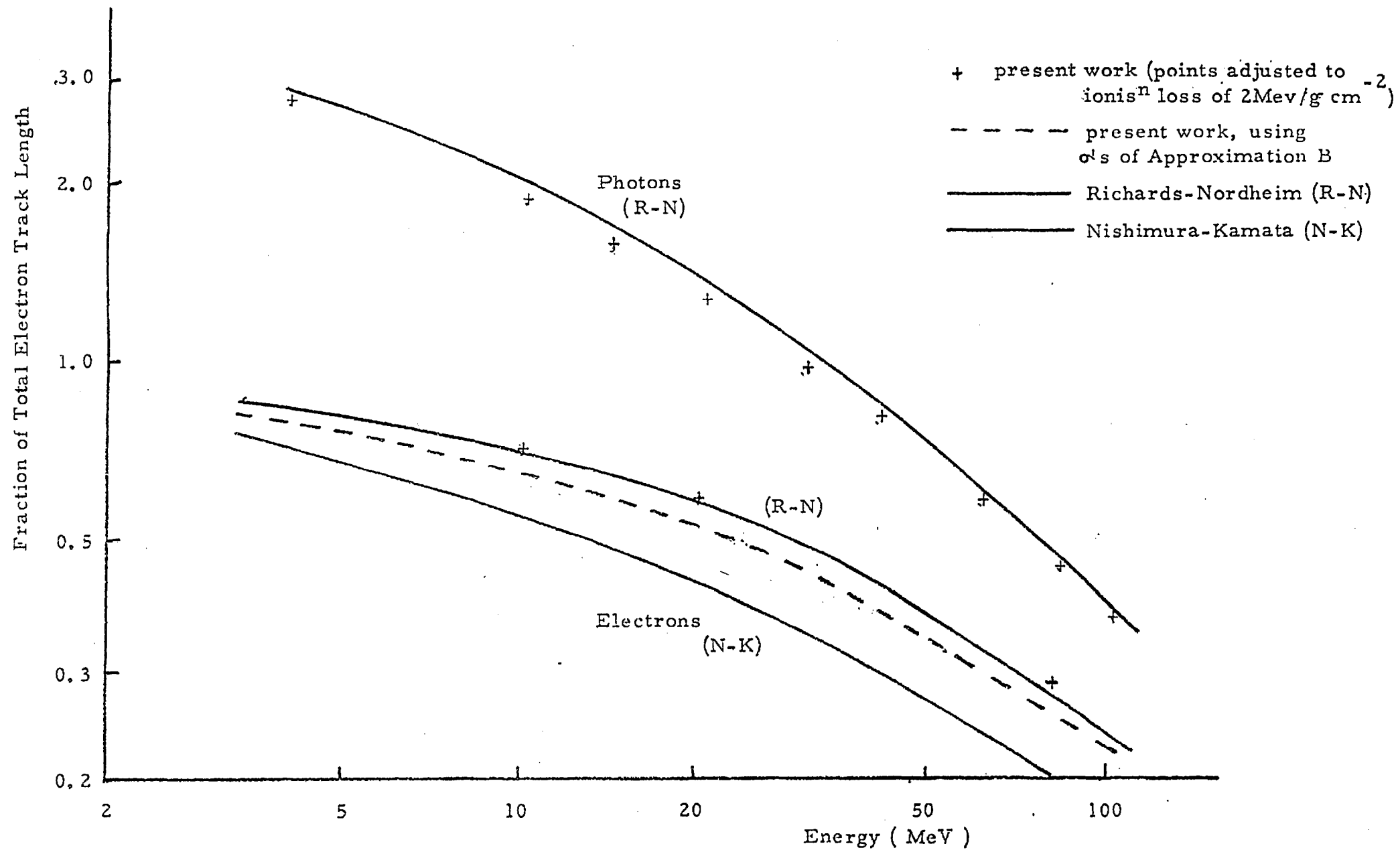


Fig. 4.1

INTEGRAL ENERGY SPECTRUM OF ELECTRONS

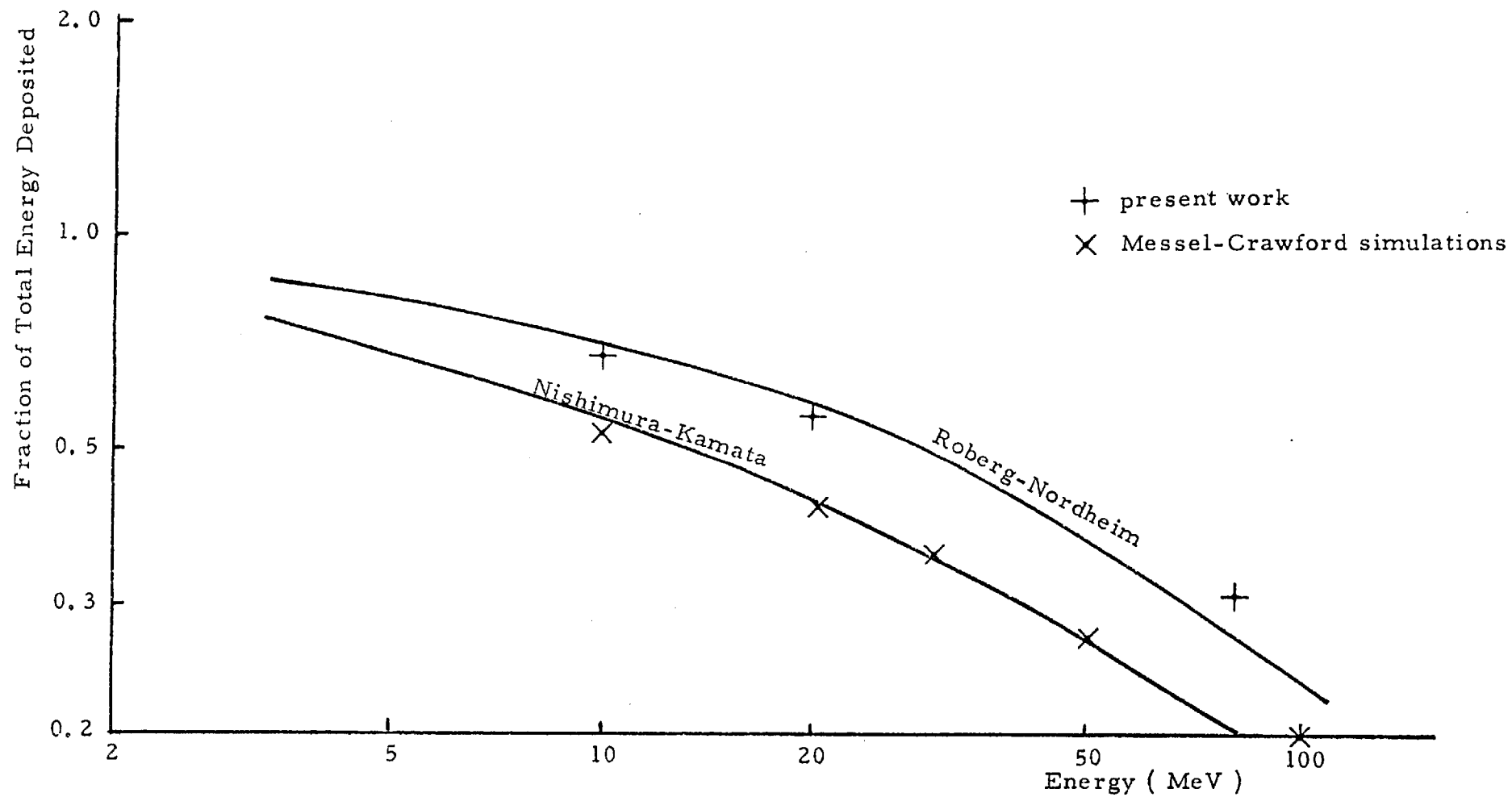


Fig. 4.2

A COMPARISON OF THE ADOPTED ENERGY DEPENDENCE OF IONISATION LOSS IN CASCADE CALCULATIONS

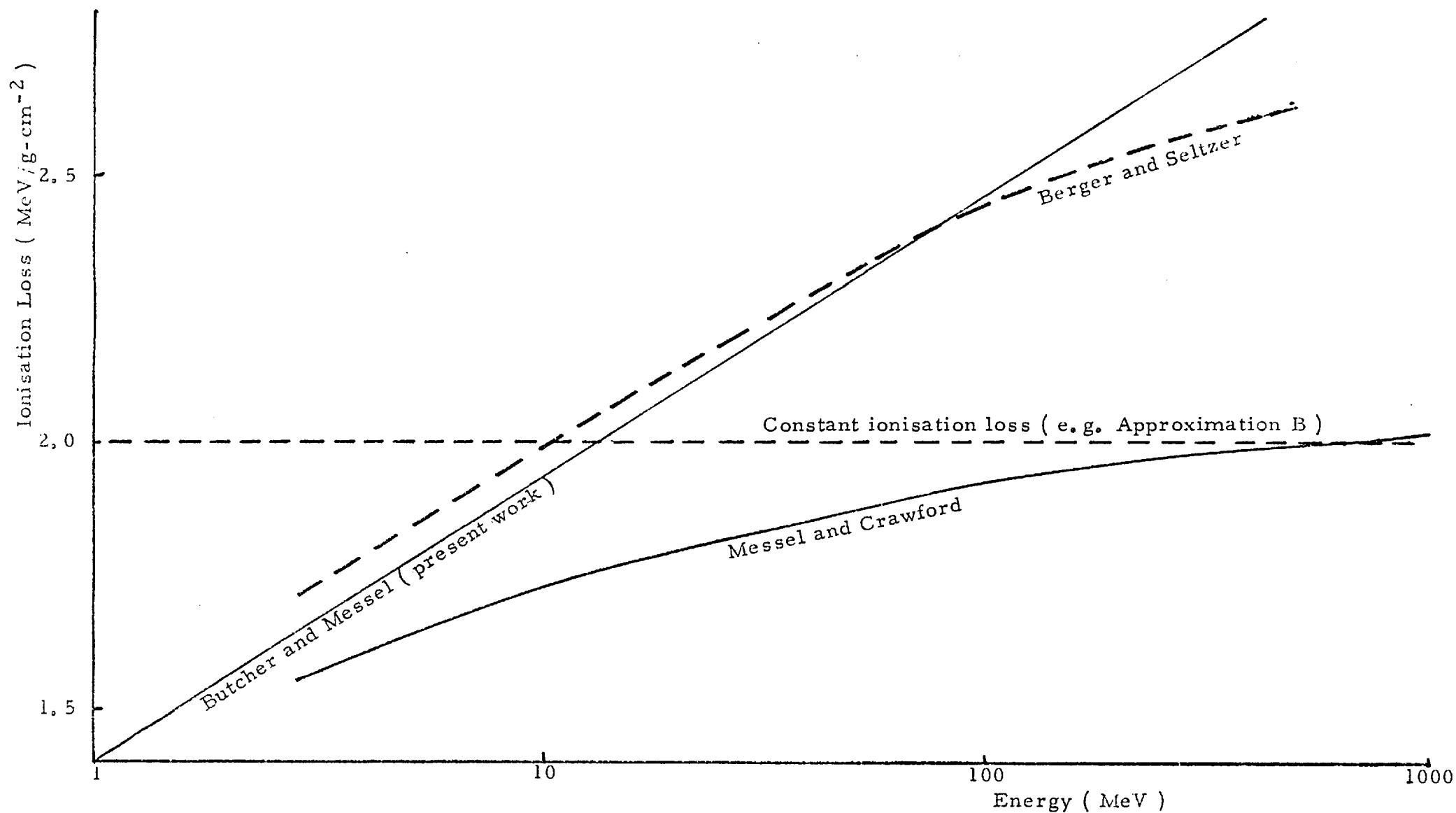


Fig. 4.3

loss of 2 MeV/g cm^{-2} . Richards and Nordheim carried out numerical calculations, using energy-independent cross-sections for the interactions above the critical energy (84 MeV), and an approximation to the true energy dependence at low energies. The Nishimura-Kamata results are based on Approximation 'B', in which the Compton process is ignored and other processes are given their asymptotic cross-sections at all energies.

In Fig. 4.1 we show the integral track length distribution for electrons and photons, on the assumption of constant ionisation loss. This distribution can be obtained despite the energy cut-off because the energy 'dumped' below the cut-off is known. The median energy of this distribution from our simulations is to be compared with the median energies from the other calculations mentioned above.

The value for our median electron energy is 30 MeV and the track length distributions for electrons and photons agree well with the Richards - Nordheim results. However, the Nishimura - Kamata results give a median energy of 12 MeV, i.e. a substantially greater length of track at low energies. As a quantitative check as to whether the change in the cross-sections could account for this large difference, we carried out some simulations under Approximation 'B' also. Although we obtain a somewhat smaller median energy of 21 MeV, it is still substantially greater than the 12 MeV given by the Nishimura - Kamata distribution.

Fig. 4.2 shows the integral energy spectra for electrons, i.e. the energy deposited as ionisation by particles with energies greater than E . This is plotted from our simulations using the energy-dependent ionisation loss and is directly based on Table 1. The energy spectra deduced from the simulations of Messel-Crawford (described in section 4.7) and the original Butcher-Messel results are also shown. The distribution curves for the Richards - Nordheim and Nishimura - Kamata remain unaltered whereas the curves

from the Monte Carlo calculations show large differences in the energy spectrum. However, in this case, the differences can be ascribed to the differences in the ionisation loss dependence on energy that is adopted, as shown in Fig. 4.3.

It is clear, however, that above the critical energy, ionisation loss is of little importance and the differences in the energy spectra at high energies do not reflect differences in the main physical processes of bremsstrahlung and pair production.

4.5 The Longitudinal Development Curves

The longitudinal development curves are plotted for different cut-off energies, for three different primary energies, and for simulations under the Rossi cross-sections and in Approximation 'B'.

The areas under the longitudinal development curves for different cut-off energies should be consistent with the track length distribution of Fig. 4.1. The curves for 1 and 4 MeV cut-offs (Fig. 4.4) appear to coincide, apparently implying that there is no track length associated with the energy interval 1 - 4 MeV. However, the proportion of particles in this energy range is about 12%. Moreover, it will be seen that about half of these are scattered backwards and are therefore omitted from the plot. The remaining 6% are also mostly very inclined tracks which would reduce the probability of their crossing a depth bin boundary because of the factor $\cos \theta$, where θ is the inclination of the track to the shower axis. The angular distribution of electrons is discussed in detail in the next section.

The comparison of electron initiated showers with photon initiated showers (Fig. 4.6) indicates that although the development curves are similar in shape (as one would expect), they are separated in depth by approximately one radiation length, which corresponds to the primary photon interaction distance before pair production.

LONGITUDINAL DEVELOPMENT CURVES AND LATERAL
DISTRIBUTIONS OF ELECTRONS FOR DIFFERENT CUT-OFF
ENERGIES

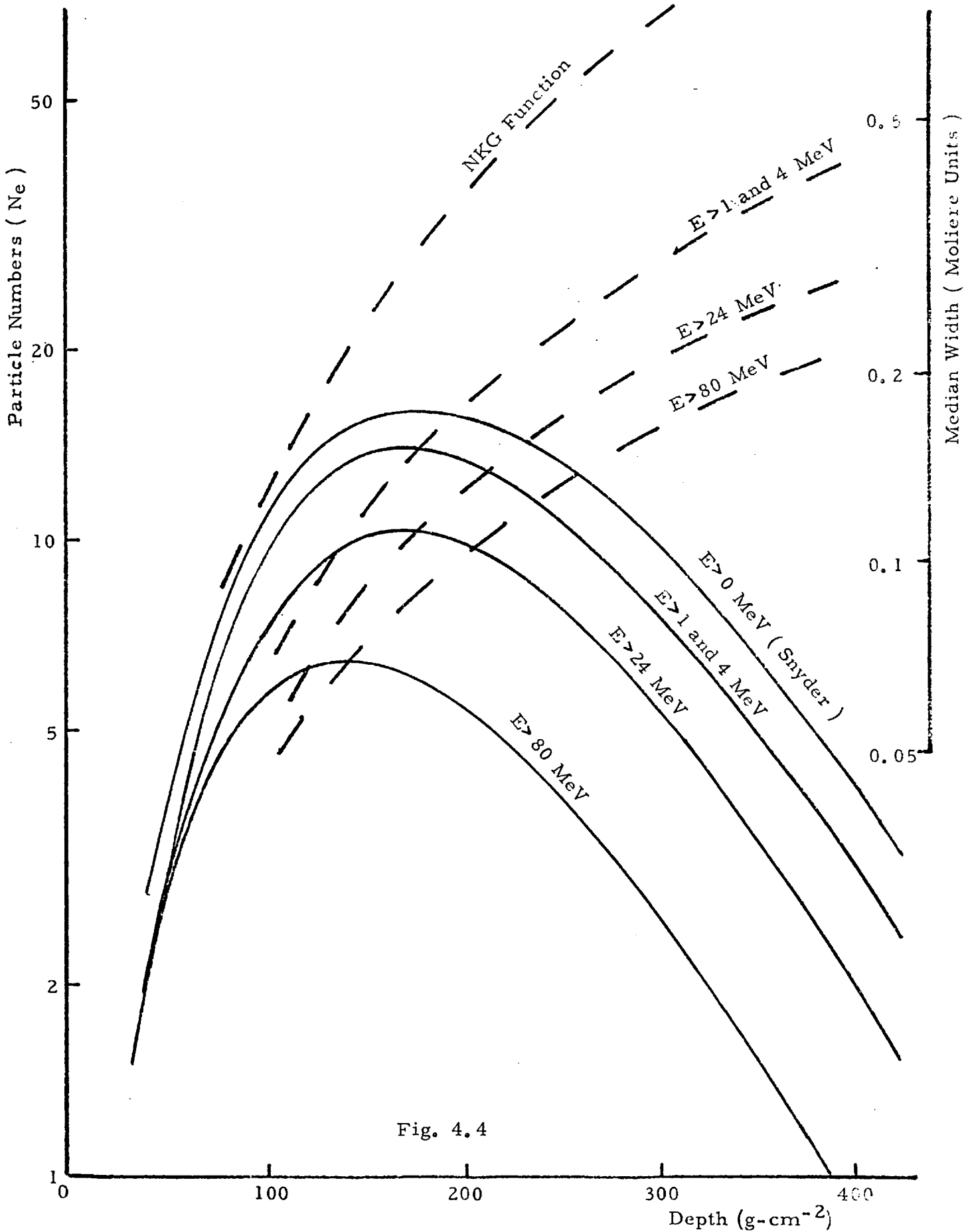


Fig. 4.4

ELECTRON DEVELOPMENT CURVES FOR SIMULATED
CASCADES OF DIFFERENT PRIMARY ENERGIES

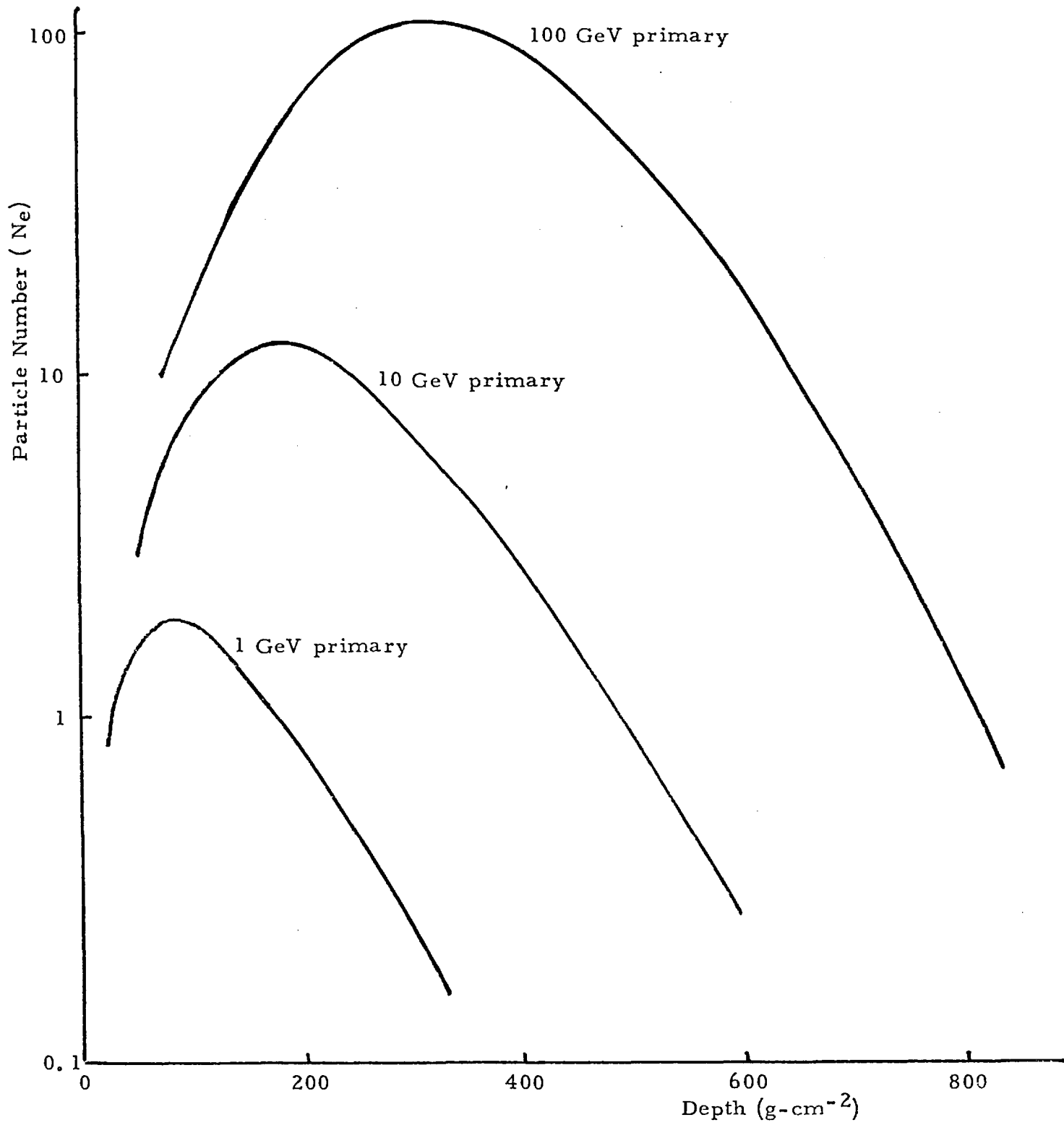


Fig. 4.5

ELECTRON DEVELOPMENT CURVES FOR
PHOTON AND ELECTRON INITIATED 10 GeV
CASCADES

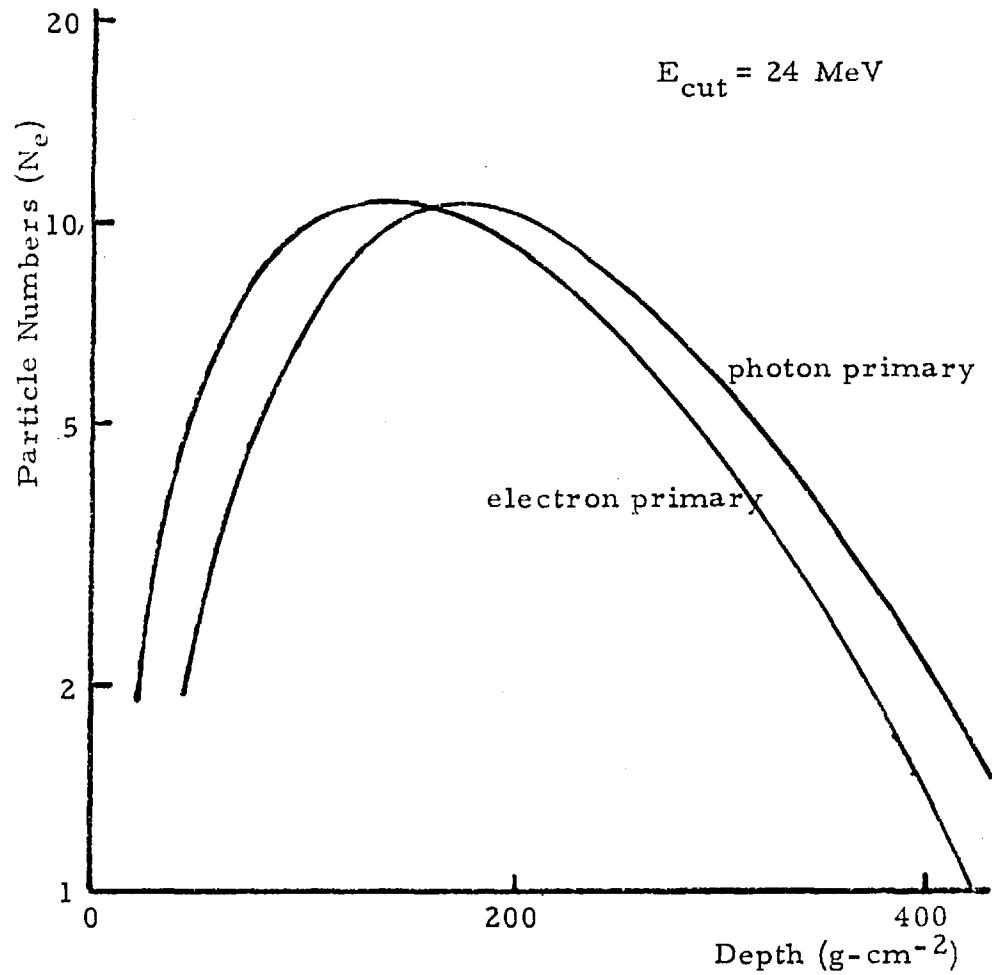


Fig. 4.6

ELECTRON DEVELOPMENT CURVES FOR
10 GeV CASCADES UNDER APPROXIMATION B

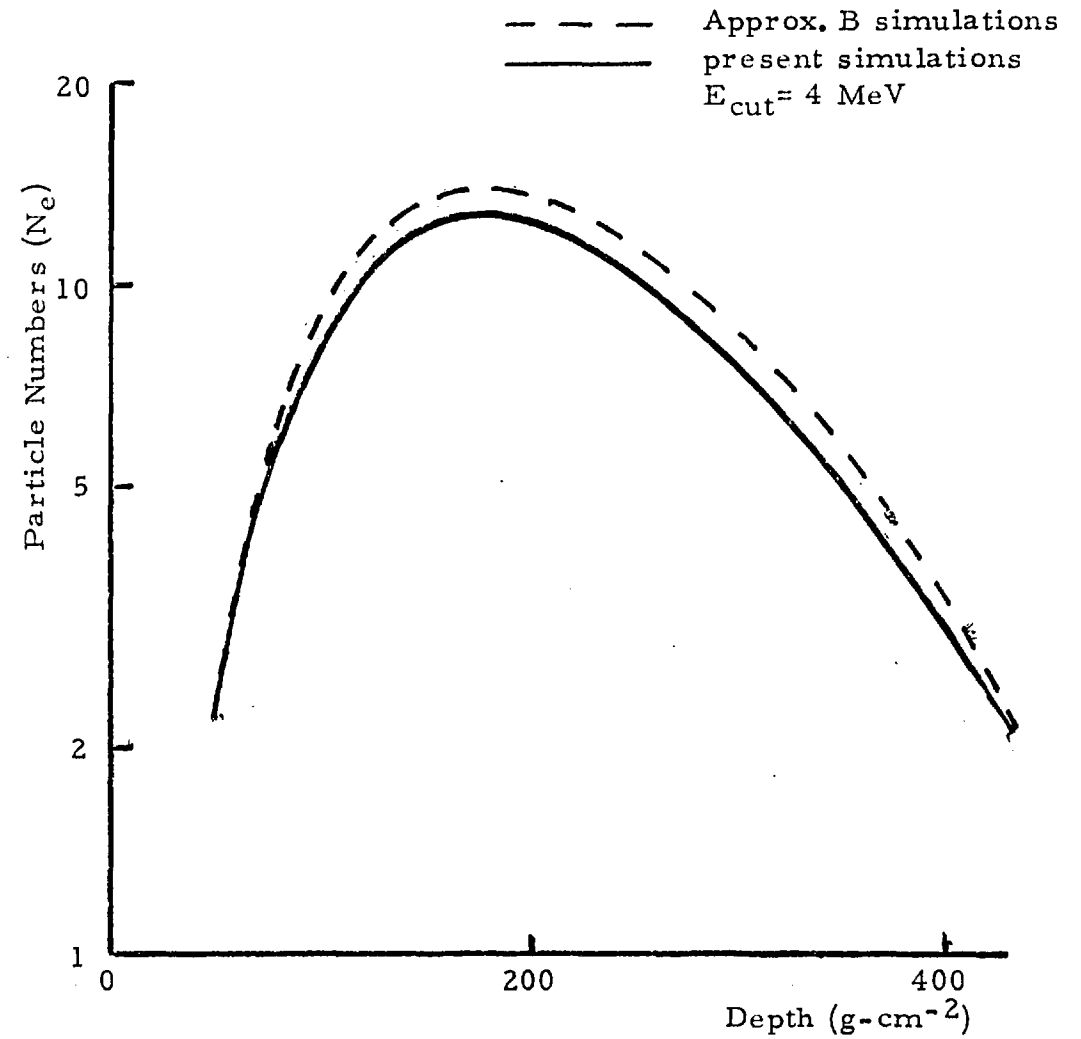


Fig. 4.7

The comparisons of development curves for 1, 10 and 100 GeV primaries (Fig. 4.5) are also in accord with expectations, although fluctuations in the rise to maximum of the 100 GeV development curve would be expected to be considerable.

4.6 The Angular Distribution of Electrons and Photons

Fig. 4.8 shows the angular distribution of electron track length for different energy ranges from our simulations. A negligible proportion of electrons above 5 MeV are scattered through more than 90° , but below about 1 MeV the angular distribution could well be isotropic. Since back-scattered particles are not tracked in the simulations, the lowest useful cut-off energy suitable for simulations is about 4 MeV.

Fig. 4.9 compares the rms scattering angles of the electron and photon track length, with those of Roberg and Nordheim (1949). For electrons, the results are virtually indistinguishable. The Roberg-Nordheim photon angles do show a bulge near the critical energy where their estimate is about 30% greater than the photon angles from our simulations.

This bulge can be explained in terms of their assumption of constant (asymptotic) values for the cross-sections down to the critical energy, followed by an abrupt change to more accurate cross-sections below E_{critical} . In particular, the Roberg-Nordheim asymptotic approximation for bremsstrahlung cross-sections just above critical energy, gives too high a probability that an electron of energy E (see Fig. 4.10) will radiate photons of energy close to E . If the angles of emission of these photons are determined by the parent electron angle, then it is clear that these angles are ascribed to photons with overestimated energies and this appears to be the main cause of the bulge. As a further check on the internal consistency of our programme, it is possible to calculate the photon angles graphically, from the differential

DISTRIBUTION OF SCATTERING ANGLES AT DIFFERENT
ELECTRON ENERGIES AVERAGED OVER THE CASCADE

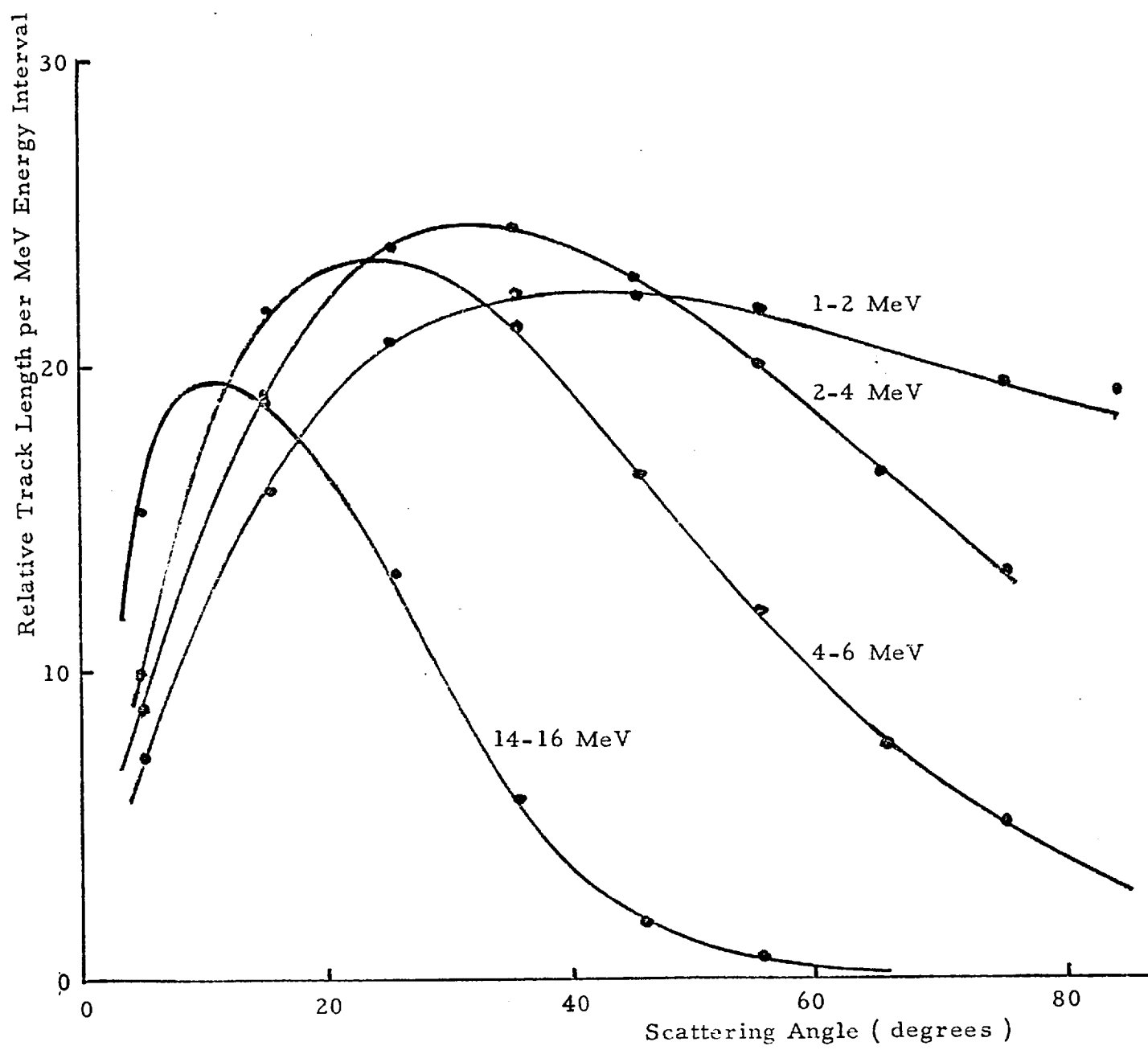


Fig. 4.8

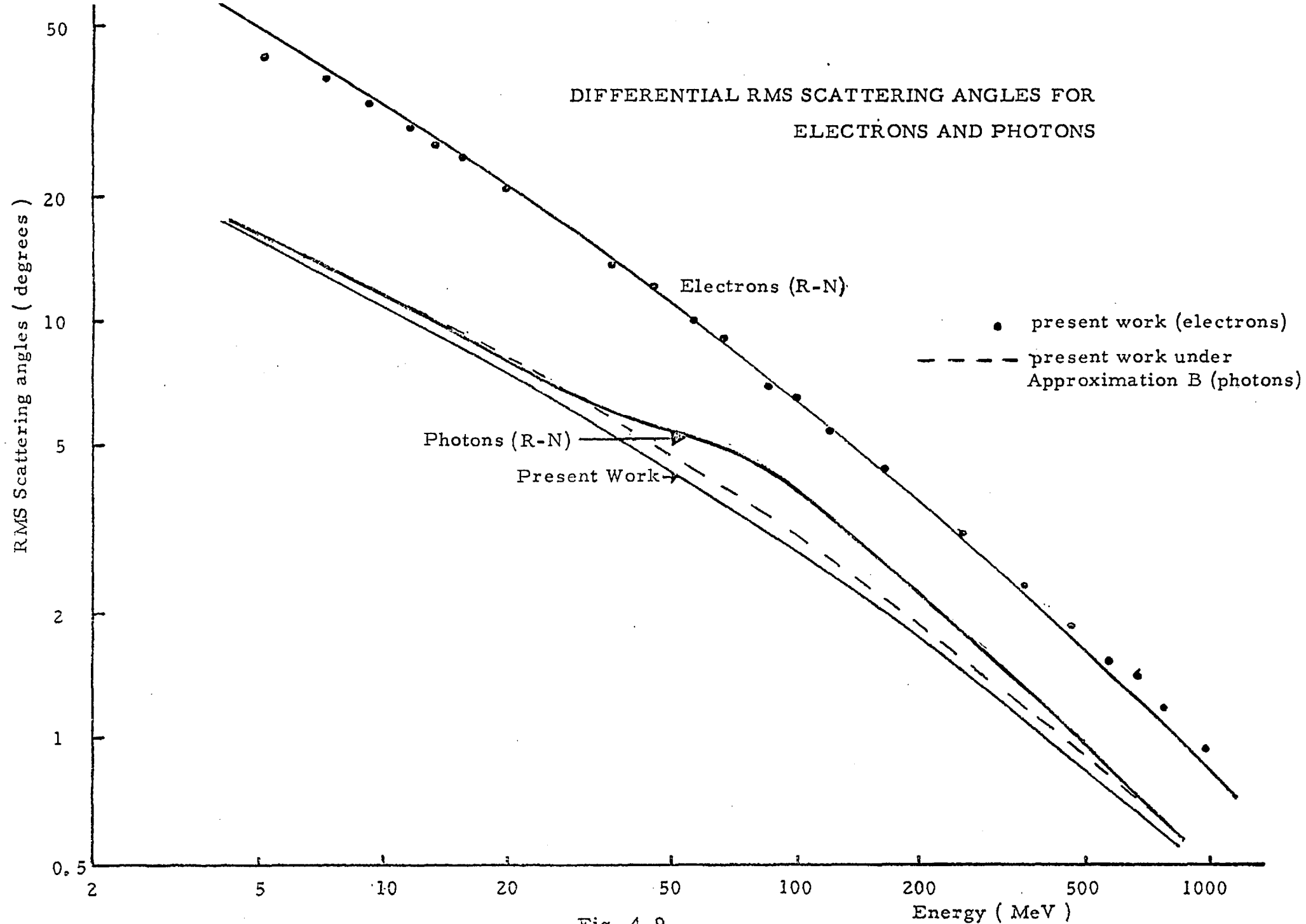


Fig. 4.9

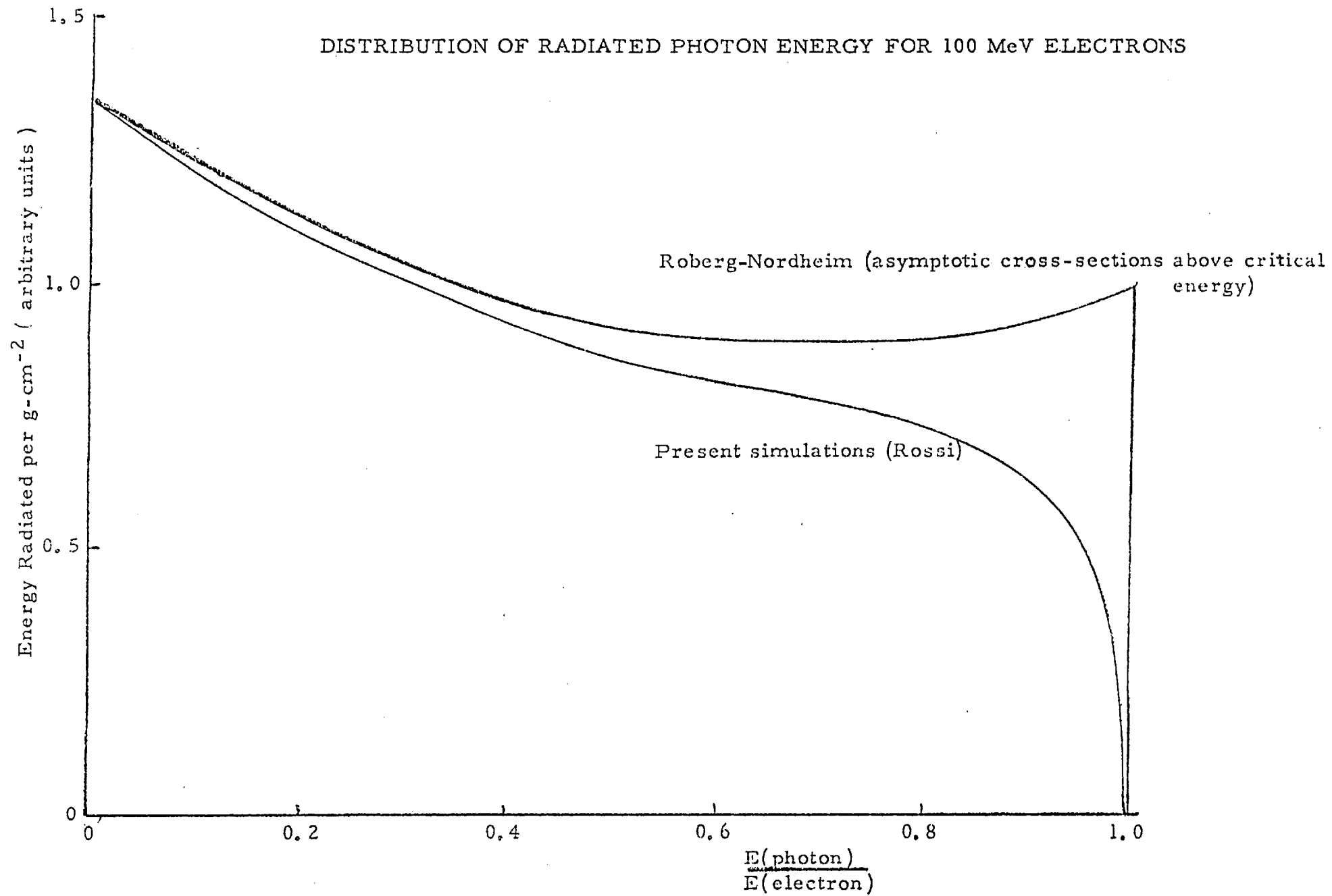


Fig. 4.10

bremsstrahlung cross-sections given by Rossi (1965) and using the track length distribution and scattering angles of electrons given in Fig. 4.1 and Fig. 4.9. Allan (private communication) has shown the photon angles deduced are in agreement with the computer simulations.

4.7 The Lateral Distribution

The lateral distribution of the electrons constitutes an important topic for discussion, partly because this significantly influences the radio emission (discussed in Chapter 5) and also because large differences are apparent when comparing the lateral spreads as derived by several other workers.

In order to be able to make as many direct comparisons of lateral spreads as possible, the median, mean and r.m.s. values of lateral width were computed as a function of depth (Fig. 4.11). The median, mean and r.m.s. values of lateral spread, integrated over all depths, were also computed for both electrons and photons.

Roberg and Nordheim calculate the particle r.m.s. lateral displacements integrated over the whole shower:-

	Electrons $E_{\text{cut}} = 4\text{MeV}$	Electrons $E_{\text{cut}} = 80\text{MeV}$	Photons $E_{\text{cut}} = 4\text{MeV}$	Photons $E_{\text{cut}} = 80\text{MeV}$
Present simulations	0.60	0.18	1.2	0.28
Roberg-Nordheim	0.80	0.29	1.6	0.39

The numbers are in Molière units where 1 Molière unit is $\sim \frac{1}{4}$ radiation length $\sim 10 \text{ g cm}^{-2}$ in air (Nishimura, 1967). The lateral spreads of both electrons and photons are substantially wider than our values and this may be attributed to the fact that photon track lengths are longer than the electron tracks so that the wider photon

CHARACTERISTICS OF LATERAL DISTRIBUTION FUNCTIONS

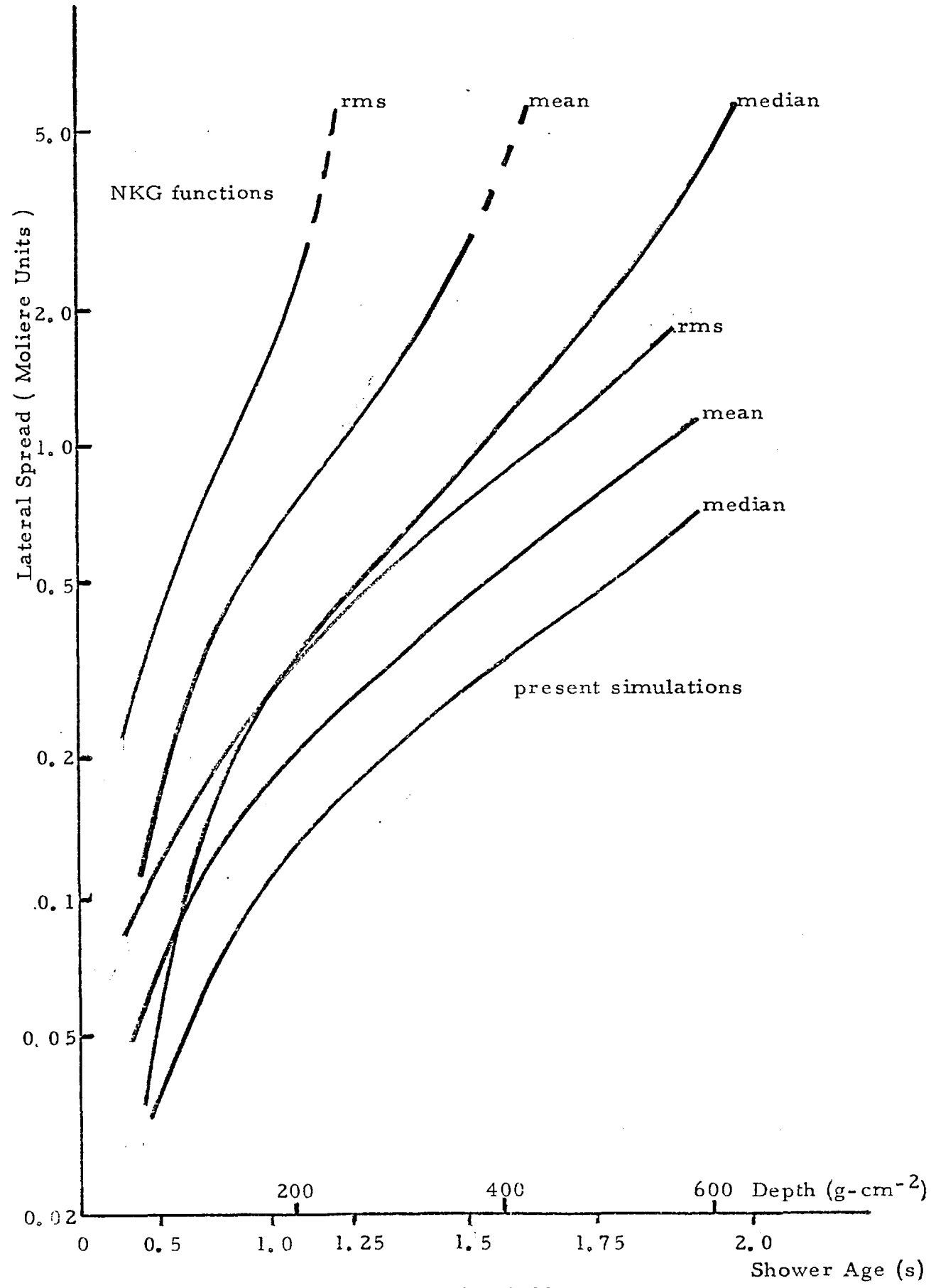


Fig. 4.11

scattering angles around 84 MeV lead to a considerable radial displacement of the subsequent electron. After taking into account the difference in the input assumptions between the Roberg-Nordheim calculations and our simulations, the two are in quite acceptable agreement.

The Nishimura-Kamata analytical calculations under Approximation 'B' (Kamata and Nishimura, 1958) give median and r.m.s. lateral displacements of electrons at shower maximum ($s = 1$):-

	Nishimura-Kamata -Private Communication $E_{\text{cut}} = 4 \text{ MeV}$	Present Simulations (500 Cascades- $E_{\text{cut}} = 4 \text{ MeV}$)	Present Simulations under Approximation 'B' (80 Cascades- $E_{\text{cut}} = 4 \text{ MeV}$)
median	0.27	0.12	0.12
r.m.s.	0.71	0.30	0.26

TABLE 2

It is clear that the discrepancy between the Nishimura-Kamata results and our simulations cannot be due to the differences in input cross-sections, but must be related to their derivation of a substantially lower median electron energy as shown in Fig. 4.2.

Greisen (1956) has fitted an empirical formula to the Nishimura-Kamata distributions (given in Chapter 2) for electrons above 0 MeV. The median, mean and r.m.s. lateral spread as a function of depth is plotted for this NKG function in Fig. 4.11. This gives a median value of 0.32 Molière units at shower maximum. The curves of Fig. 4.11 also illustrate the merits of expressing the lateral spread in terms of the median width rather than quoting the r.m.s. width. In the latter case, a disproportionate weighting is given to the few particles at large distances from the shower axis. Quoting the median width gives the radius of a circle which encloses one half of all the particles in a plane normal to the shower axis. The values of the r.m.s. lateral spread are considerably more sensitive to fluctuations

in the number of particles well away from the axis in the case of the computer simulations. The r.m.s. values from the NKG function actually diverge at a distance corresponding to $s = 1.25$ beyond shower maximum. It is also seen that the lateral spread averaged over the whole shower is somewhat greater than the lateral spread at shower maximum. This is as expected since the shower becomes wider as it develops and there is more track length associated with $s > 1$ than with $s < 1$.

Messel and Crawford (1970) have carried out extensive simulation studies of electron-photon cascades in different absorbers including air. Refinements in their programme include a more accurate treatment of electron scattering, involving much smaller electron step lengths. Other refinements include taking into account the photo-electric effect for low energy photons and positron annihilation. The following comparison of lateral spreads at shower maximum shows that their values are even wider than the Nishimura-Kamata values by a factor of ~ 2 :-

	Present simulations ($E_{\text{cut}} = 4 \text{ MeV}$)	Messel & Crawford ($E_{\text{cut}} = 1 \text{ MeV}$)	Messel & Crawford ($E_{\text{cut}} = 5 \text{ MeV}$)
median	0.12	0.60	0.60
mean	0.20	0.84	0.80
r.m.s.	0.30	1.21	1.10

TABLE 3

Since the publication of their results, Crawford (private communication) has drawn attention to an error in their programme, which effectively made their Compton cross-section zero for their simulations in air absorbers. To investigate the suggestion that this could explain their very wide lateral spreads, a simulation has been carried out with our programme with this error artificially introduced. The effect has been to increase the lateral spread only by about 30% and therefore the discrepancy with the Messel-Crawford simulations remains an open one.

LATERAL DISTRIBUTIONS OF ELECTRONS FOR SIMULATED
CASCADES OF DIFFERENT PRIMARY ENERGIES

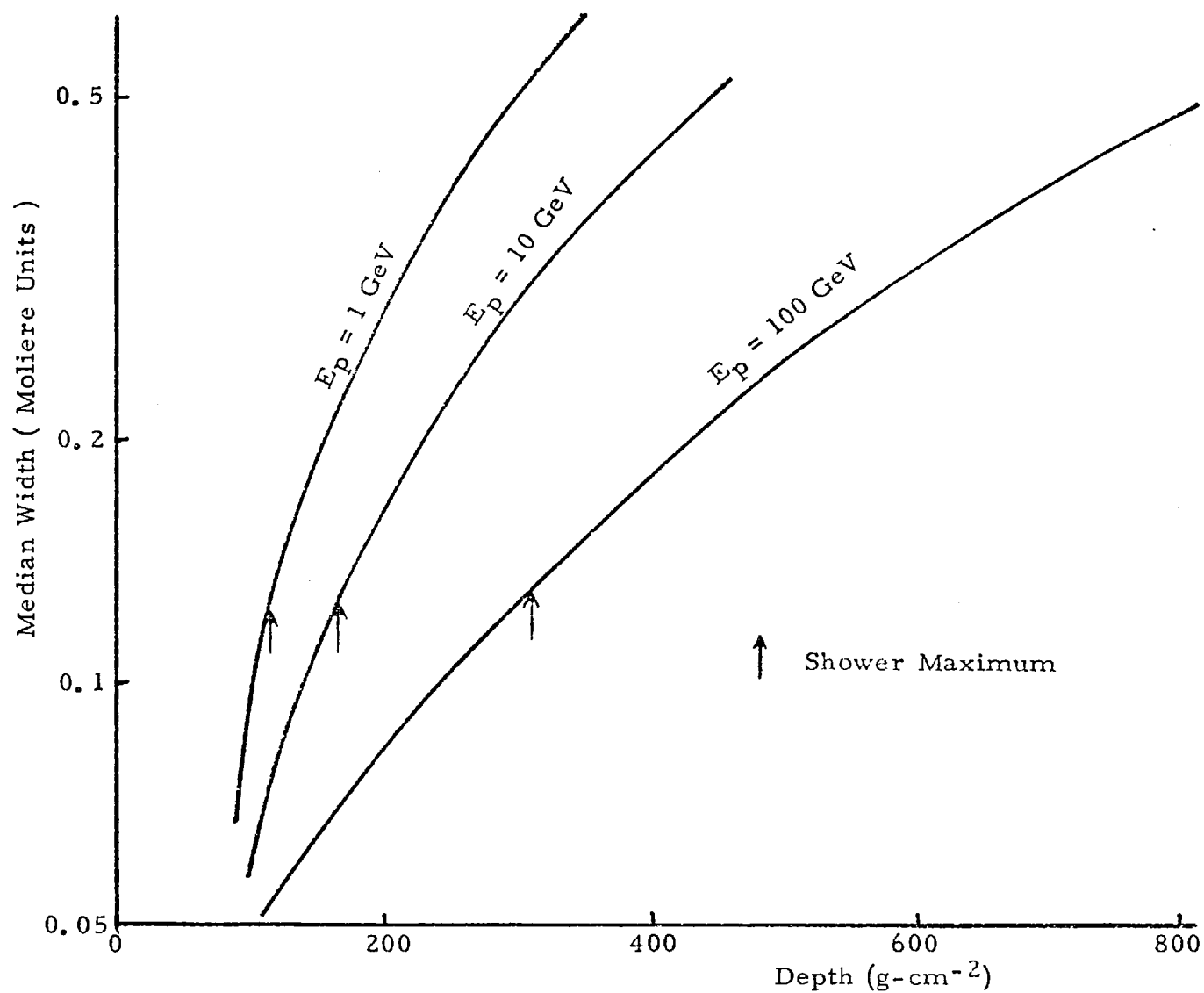


Fig. 4.12

By courtesy of Dr. R. J. Hynds, simulations have been carried out at Imperial College for one hundred 1 GeV cascades, using the electron-photon transport programme developed by Berger and Seltzer (1968) and made available by the U.S. National Bureau of Standards. The same ^{CROSS-}~~cross-~~sections for the pair production, Compton effects and bremsstrahlung processes have been used as in our Monte Carlo programme. These cross-sections and details of the absorber material are defined by the programme user and are part of the numerical input data which must be provided prior to the simulations (Berger and Seltzer, 1964). The quantity derived was the radial distribution of energy deposition in a thin disc perpendicular to the shower axis near shower maximum. The median width from this simulation for electrons above 4 MeV is about 0.1 to 0.2 Molière units. This compares well with our median width of 0.15 Molière units.

It will be noticed that the value for the lateral spread of energy deposition is somewhat wider than the corresponding values for the lateral spread of electron flux (Table 3). This arises from the fact that the electron tracks further away from the axis are more oblique and this results in a reduced "flux counting rate", whereas the energy deposited remains unchanged.

The median width at shower maximum is found to be virtually independent of the launch energy of the cascades. Simulations have been carried out for 1, 10, and 100 GeV primary photons (Fig. 4.12). The shapes of the lateral distributions are also indistinguishable.

4.8 A Comparison with Experimental Data

Experimental data from Stanford is available on the development of cascades in water. Crannell et al. (1969) have studied the distribution of ionisation round the axis of a 1 GeV electron beam when it is directed into a tank of water. The measurements give the distribution of energy deposited as a function of depth and

radius and this is a quantity very similar to the distribution of charged particle track. Cascade properties in air and water should be very similar and a direct comparison of the median radial distance for energy deposition can be made. The widths are given in Molière units.

	At Shower Maximum	Averaged over all Depths
Present simulations	0.15	0.20
Crannell experimental values for water	0.17	0.24

TABLE 4

The experimental widths are considerably narrower than those of Nishimura-Kamata, although they are somewhat wider than the values from our simulations. However, many arguments (discussed by Yuda, 1969) can be made to show that instrumental effects would tend to lead to an over-estimate in the measured lateral spread, whereas it would be very difficult to reconcile experimental values which were narrower than theoretical values. However, this is what we must suppose if the Nishimura-Kamata values are to be accepted.

4.9 An Evaluation of Programme Refinements

The various precautions taken in the development of the simulation programme and described in Chapter 3 increase one's confidence that the results obtained from it should be reliable. As a final check, tests were carried out in which some refinements were removed from the programme and the resultant effects on shower width and other properties examined. The most important single test was on the importance of the step procedure.

For simulations of cascades in which none of the electron tracks were segmented, no significant changes have been observed in the lateral distributions. Therefore, the segmenting of electron tracks is considered as not crucial for the treatment of electron scattering and it only makes a difference of less than about 10% in the radio calculations in the next chapter.

The importance of the large angle single Coulomb scatters, which occurs for about 3% of the electron interactions, has been evaluated by simulating cascades in which no single scatters are allowed. As expected, this made no significant changes to the median lateral spreads.

Other refinements temporarily removed to test their importance include the angles of emission of pair electrons, and the angles of emission in other processes. None of these play a significant role in determining the lateral spread of the shower, as is demonstrated by the close similarity of the results from simulations under Approximation 'B' where none of them are considered.

CHAPTER 5

RADIO EMISSION CALCULATIONS FROM MONTE CARLO SIMULATIONS

5.1 Introduction

In the previous chapter, the structure of an electron-photon cascade has been discussed in detail and the results from the Monte Carlo simulations and the calculations of other workers have been compared and evaluated. This chapter is primarily concerned with the calculation of the radio emission from an air shower as seen at an antenna at ground level at various distances from the shower axis. An isothermal atmosphere with a density varying exponentially with height is used as an adequate representation of a real atmosphere and is incorporated in the programme.

A folding procedure is used to simulate an extensive air shower from the individual electron-photon cascades launched at various heights in the atmosphere. This procedure and the results obtained from it are detailed in this chapter.

5.2 The Feynman Formulation of Electromagnetic Theory

In Feynman's treatment of electromagnetism (1963), a direct link is established between the motion of a charge (q) and the field (E) as seen by an observer at any point in space:-

$$E = \frac{-q}{4\pi\epsilon_0} \left[\frac{e_r}{r^2} + \frac{r}{c} \frac{d}{dt} \left(\frac{e_r}{r^2} \right) + \frac{1}{c^2} \frac{d^2 e_r}{dt^2} \right] \text{ Vm}^{-1}$$

where r is the distance from the charge to the observer and e_r is the unit vector in the direction of the charge motion as seen by the observer at an apparent time t . By apparent time is meant the

time at which a light beam carrying information about the particle position would reach the observer.

Feynman shows that this expression is directly equivalent to the Maxwell formulation of electromagnetism. For charged particles moving towards the observer at velocities $\approx c$, as in the electron-photon cascade, the third term in the expression is the dominant one, since the apparent acceleration becomes large because of time compression. Using this term only, the basic expression can be written in terms of the apparent angular acceleration as seen by an observer at an apparent time (t):-

$$E(t) = \frac{q}{4\pi\epsilon_0 c^2} \frac{d^2\theta}{dt^2}$$

The direction of the observed electric field is along the perpendicular to the line of sight and in the plane of the observed charged particle motion. θ is angular displacement of a charged particle as seen by the observer.

5.3 The Radio Emission from an Electron-photon Cascade

The method used here for calculating the radio emission is, in principle, the same as that used by Allan (1971b). In this method, one sums the angular deflections (θ) of the separate particles to obtain the total angular polarisation ($n\theta$) and one finds how $n\theta$ varies with time as seen at an antenna at a fixed distance from the shower axis. The electron-photon cascade simulation gives us this angular polarisation as a sum of the angular displacements of the charged particle tracks in 1 nanosecond time intervals, for six radial distances from the shower core. Ideally, in order to derive the frequency spectra in the region of 60 MHz, narrower time bins would have been preferred. However, besides computer core limitations, the necessary reduction in the statistical fluctuations in the structure of the

simulated cascades would entail an excessive computer running time.

The 'apparent' time delay used in the calculation is made up of three parts:-

- (1) The 'intrinsic' time lag which is the time lag behind the shower front, as described in Chapter 4.
- (2) The geometrical time lag appropriate to the difference in path lengths to the antenna for different particles in the cascade.
- (3) The additional delay due to the atmospheric refractive index. As shown in Fig. 5.1, for a particle at position B, the additional delay as observed at A due to the refractive index may be written as:-

$$\begin{aligned} dt &= \int_A^B \frac{\mu dx}{c} - \int_A^B \frac{dx}{c} \\ &= \int_A^B (\mu - 1) \frac{dx}{c} \end{aligned}$$

For an isothermal atmosphere with a scale height of 8000 metres, $(\mu - 1)$ may be written as:-

$$\mu - 1 = (\mu_0 - 1) e^{-z/8000}$$

where μ_0 is the refractive index at sea level and $(\mu_0 - 1) = 2.7 \times 10^{-4}$. By geometry:- $dx = dz \left(\frac{D}{z_0} \right)$ and the delay term may be written as:-

$$dt = \frac{\mu_0 - 1}{c} \frac{D}{z_0} \int_0^{z_0} e^{-z/8000} dz \text{ which reduces to}$$

CALCULATION OF DELAY DUE TO REFRACTIVE INDEX

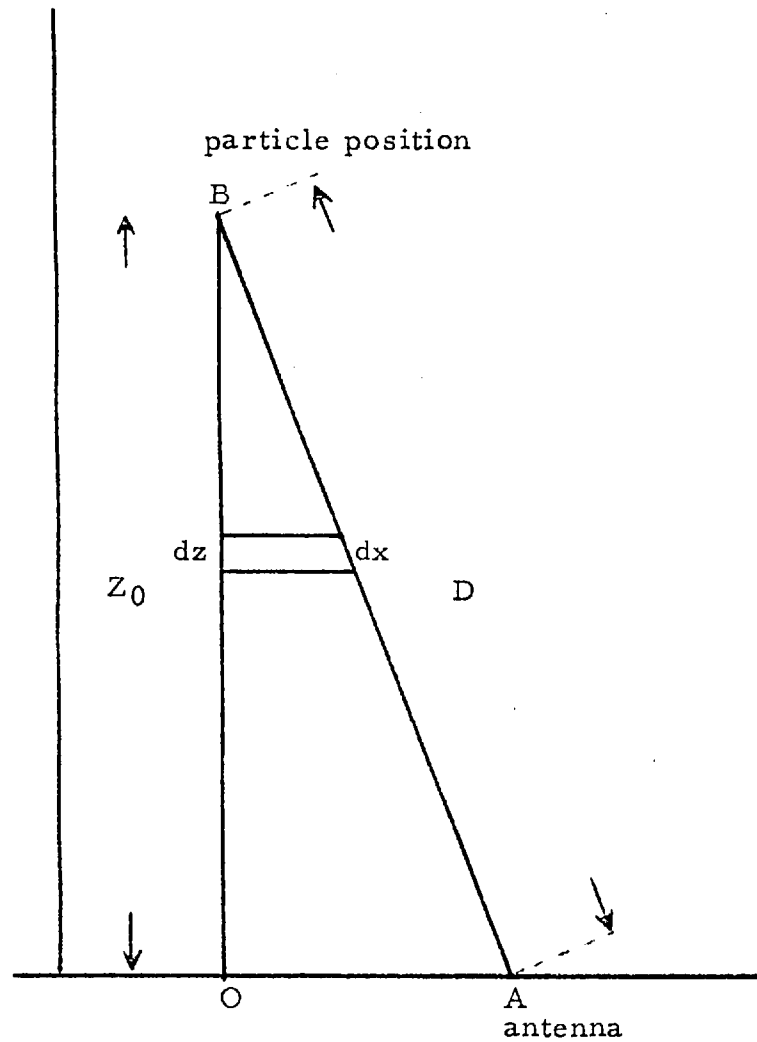


Fig. 5.1

ANGULAR POLARISATION FOR SINGLE ELECTRON TRACKS

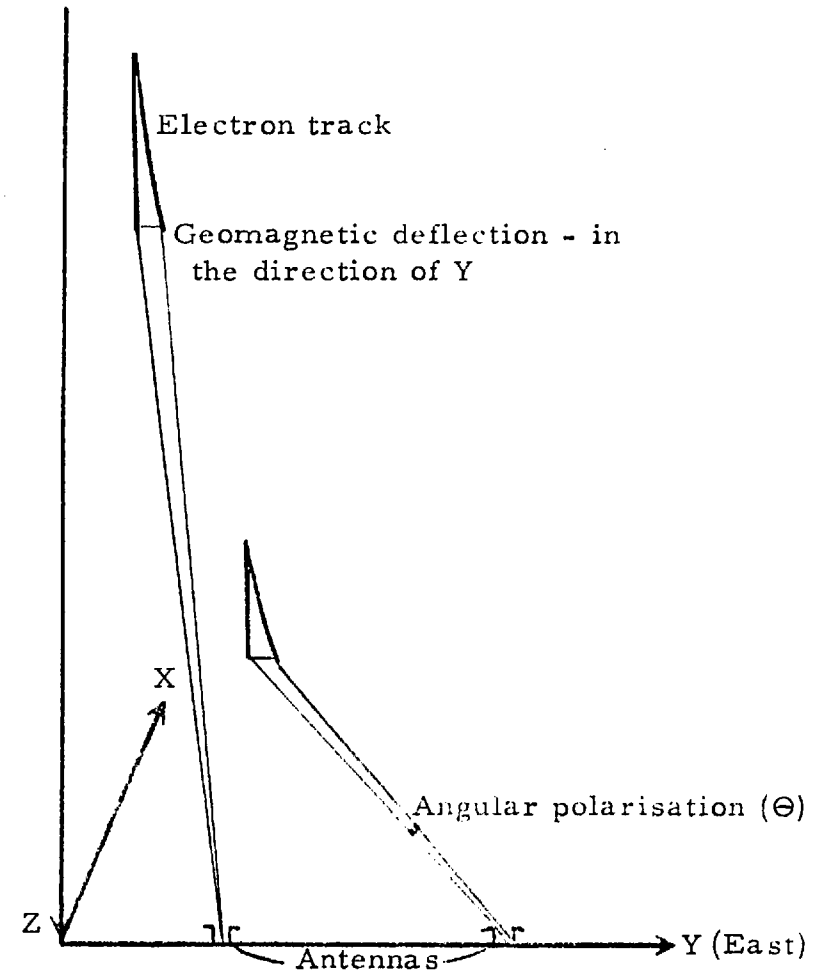


Fig. 5.2

$$dt = \frac{7.2 \times D}{z_0} (1 - e^{-z_0/8000}) \text{ nanoseconds,}$$

where z_0 and D are in metres.

This delay term contributes negatively to the total delay and can have a major effect when geometrical delays and multiple scattering are less pronounced.

The field strength $E(t)$ at an antenna is obtained straightforwardly as the second derivative of the polarisation according to the Feynman expression. However, the direction of the electric field will vary according to the positions of the charged particle tracks in the cascade relative to the antenna and also depending on the height of launch of the cascade (Fig. 5.2). It is therefore required that the angular polarisation as seen at each antenna is resolved into three mutually perpendicular directions at each antenna and is stored as three separate ' $\theta - t$ ' loci.

For the geomagnetic mechanism and for a vertical shower, the direction of the electric field is predominantly in an east-west direction perpendicular to the geomagnetic field lines.

For the charge excess mechanism, the electric field is radially polarised from the shower axis and therefore for the antenna positions illustrated, its direction is also predominantly on the plane of the E - W axis and the vertical.

On the other hand, the angular polarisation resolved in the x-direction is negligible for either of the two mechanisms and therefore has not been recorded in the programme. Similarly, the angular polarisation resolved in the z (vertical) direction for the geomagnetic mechanism can also be neglected. Thus, for a group of simulations at a specific height, the following three curves as a function of time in nanoseconds are obtained for each antenna:-

- (a) The angular polarisation resolved in the E - W direction for the geomagnetic mechanism, $\theta_y(\text{Geom.})$.
- (b) The angular polarisation resolved in the E - W direction for the charge excess mechanism, $\theta_y(\text{excess})$.
- (c) The angular polarisation resolved in the vertical direction for the charge excess mechanism, $\theta_z(\text{excess})$.

5.4 The $(n\theta - t)$ Curves for an Electron-photon Cascade

In this section, we begin with the simplifying assumptions that there is no particle scattering and that the refractive index of air is $= 1$. In such circumstances, an antenna located at ground level on the cascade axis will 'see' simultaneously all interactions that take place in the cascade. This is because the particles travel at the same speed as the light which carries the information to the antenna. If we now allow for multiple Coulomb scattering, the lower energy cascade particles lag behind those of higher energy and (as seen from the antenna) their interactions will take place at a later apparent time. The lag for a particle of energy E is proportional to $1/E^2$. Deflection by the earth's magnetic field causes an electric dipole moment to be associated with these particles. The deflection is proportional to $1/E$ and the particle numbers are also assumed to be proportional to $1/E$. It follows that particles of energy (E) provide a dipole moment that is proportional to the time delay and that the antenna sees an angular polarisation ($n\theta$) increasing linearly with time (t). Ultimately, at low values of particle energy, the approximations used in the above argument break down and as $E \rightarrow 0$, the dipole moment approaches a limiting value (see page 27).

For an antenna away from the cascade axis, additional geometrical delays are introduced which are different for particles at different heights above the ground. The effect is to impose a curvature on the $(n\theta - t)$ curve as shown in Fig. 5.3a.

TIME DEPENDENCE OF THE ANGULAR POLARISATION AS SEEN
FROM AN ANTENNA AT 50m AND 300m FROM THE AXIS OF A

10 GeV CASCADE

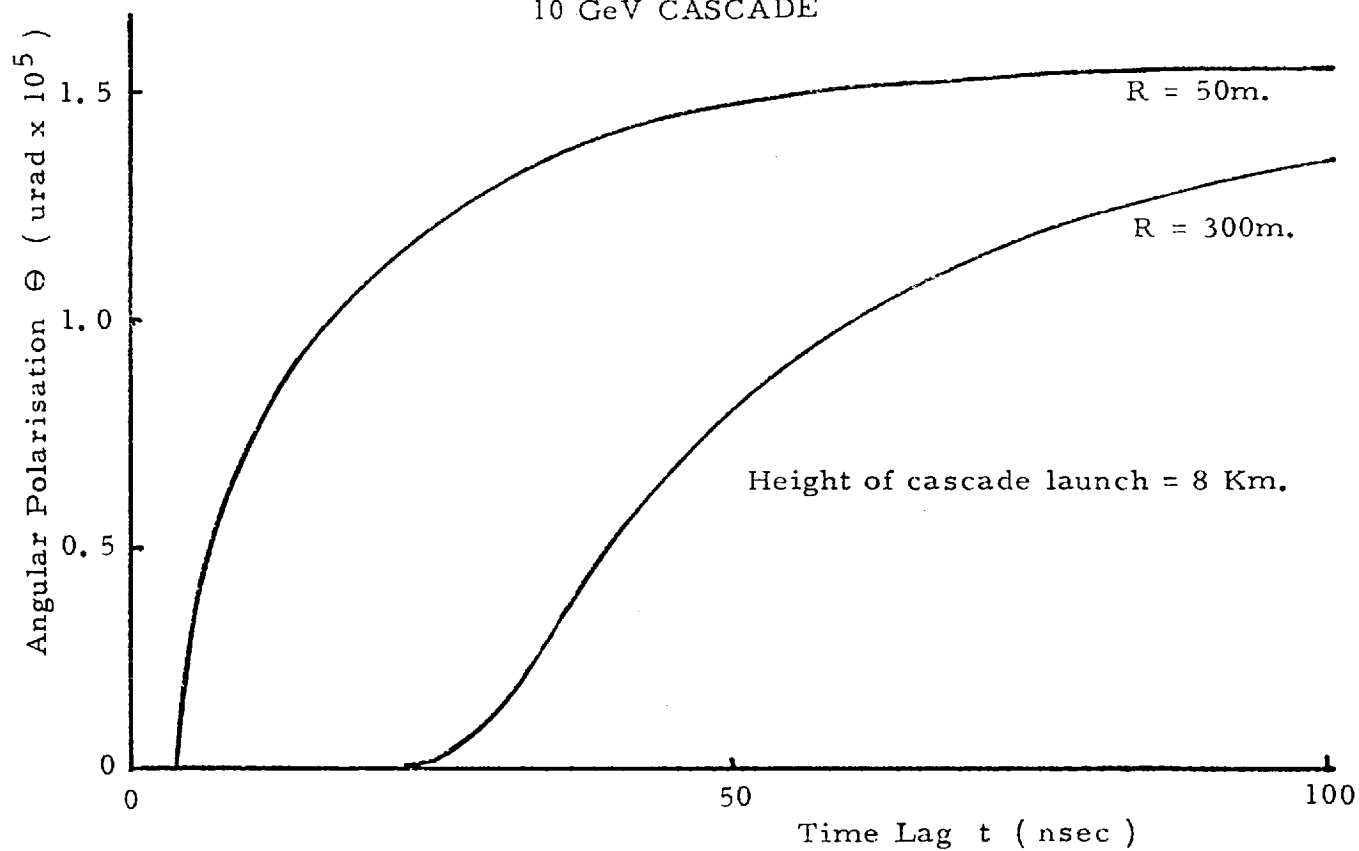


Fig. 5.3 a

RADIO PULSE WAVEFORM

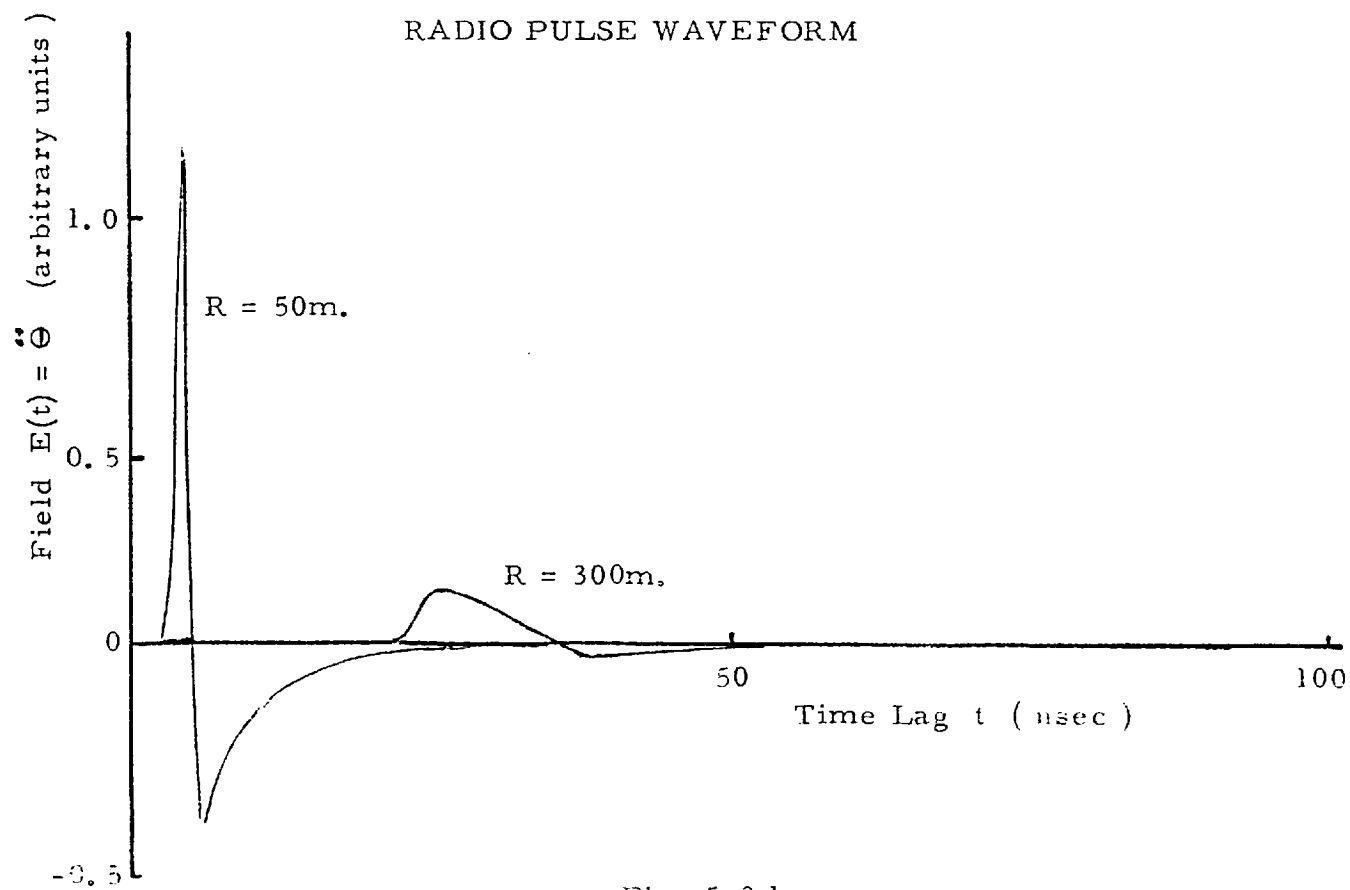


Fig. 5.3 b

The atmospheric refractive index, discussed in section 5.3, helps to counteract the effects of geometrical delays and in a practical air shower situation, the $(n\theta - t)$ curve is most nearly a straight line for an antenna position ~ 100 m from the cascade axis.

Typical curves are shown in Fig. 5.3a. The corresponding field strengths as a function of time $E(t)$ are found according to Feynman from the second derivative of these curves - Fig. 5.3b. However, the quantity of physical interest is not the pulse waveform but its frequency components, since only a narrow part of the whole frequency spectrum can be observed free from interference by such things as TV transmissions and broadcasting.

The frequency spectrum of the pulse is related closely to the pulse duration. The Fourier transform gives for the frequency component E_ν of a pulse waveform $E(t)$:

$$E_\nu = \int_{-\infty}^{\infty} E(t) e^{-2\pi i \nu t} dt$$

where

$$E(t) = \int_{-\infty}^{\infty} E_\nu e^{2\pi i \nu t} d\nu$$

When the $(n\theta - t)$ curve is a straight line, rising linearly from zero, $E(t)$ is a delta function and E_ν is independent of frequency and proportional to the slope of the $(n\theta - t)$ curve. A rounding of the $(n\theta - t)$ curve leads to a spectrum which is flat over a considerable frequency range but falls to zero both at $\nu = 0$ and $\nu = \infty$. The upper limit corresponds to a frequency $\sim (1/2\pi\tau)$ where τ is the pulse duration, i.e. the time it takes for the $(n\theta)$ curve to build up to its maximum slope. The lower frequency limit is similarly $\sim (1/2\pi T)$ where T gives the time constant for the approach of the $(n\theta)$ curve to its final value. It is easily shown from the formulae above that $E_\nu = 0$ at zero frequency, since $e^{2\pi i \nu t}$ is then $= 1$ and:-

$$E_v \propto \int_{-\infty}^{\infty} E(t) dt \sim \int_{-\infty}^{\infty} \frac{d^2\theta}{dt^2} dt \sim \dot{\theta}_{\text{final}} - \dot{\theta}_{\text{initial}} = 0.$$

5.5 The Folding of Electron-photon Cascades

As outlined in Chapter 2, we choose to treat an extensive air shower as a superposition of many 10 GeV electron-photon cascades launched from various heights along the shower axis. The number to be launched at each height is chosen to fit the overall shape of the shower longitudinal electron development curve. The question arises however whether one then obtains an overall shower lateral spread compatible with experimental measurements. The present simulations give a lateral spread for individual electron-photon cascades that is somewhat narrower than previous estimates and narrower also than the electron spread in real air showers. To match the model to a real shower it would be necessary to allow for the obliquity of the separate cascades with respect to the shower axis which results from the transverse momentum of the parent π^0 particles in the nuclear active cascade. Estimates of angular distribution of pions can be made from the energy spectrum of pions. This has been attempted by several workers (Turver et al., 1973) using various nuclear model simulations but these are largely speculative. For this reason and also because of computer time limitations, only parallel cascades launched along the shower axis are considered in the computation. The effect of the angles of emission of pions is discussed later in this section.

For the present calculations on the radio emission from an extensive air shower, the following procedure has been adopted:-

- (1) A suitable cut-off energy was first selected for the 10 GeV electron-photon cascades. Although very low energy electrons are expected to be greatly delayed and therefore relatively unimportant for radio emission in the MHz frequency region, it still remains true that the lower the energy, the greater the

deflection by the geomagnetic field. Also the electron charge excess increases with decreasing energy. Trials with different cut-offs show that about 8 MeV is the maximum cut-off energy that can be selected without significantly affecting the important parts of the angular polarisation curves.

- (2) A suitable number of cascades (e.g. 10 cascades) are launched at each of 40 heights, 25.8 g cm^{-2} apart in a standard isothermal atmosphere of scale height 8000 metres. The angular polarisation curves for each of these heights are stored on magnetic tape. In addition, the lateral distributions of electrons and photons at ground level are calculated and stored for each height of launch.
- (3) A separate programme is used to fold the electron-photon cascade development curves into a suitable development curve for an extensive air shower. The exact slope of the development curve (Fig. 5.4) is not crucial but is based on model calculations by a group at Durham University (Dixon et al., 1974). The programme yields the required number of cascades to be launched at each of the 40 heights to obtain the desired electron development for the whole shower. The angular polarisation curves are scaled according to these launching numbers and are then integrated over all depths to obtain the overall $(n\theta - t)$ curves for the six radial distances (antennas) from the shower axis. These curves are shown in Fig. 5.5 for a 10^{17} eV shower with the longitudinal development of an average proton-initiated shower. The corresponding lateral distributions of electrons and photons at ground level for the folded cascades are also calculated and shown in Table 5.

5.6 The $(n\theta - t)$ Curves for an Extensive Air Shower

In this section, it will be shown that the field strength

ELECTRON DEVELOPMENT CURVES FOR PROTON
INITIATED 10^{17} eV AIR SHOWERS

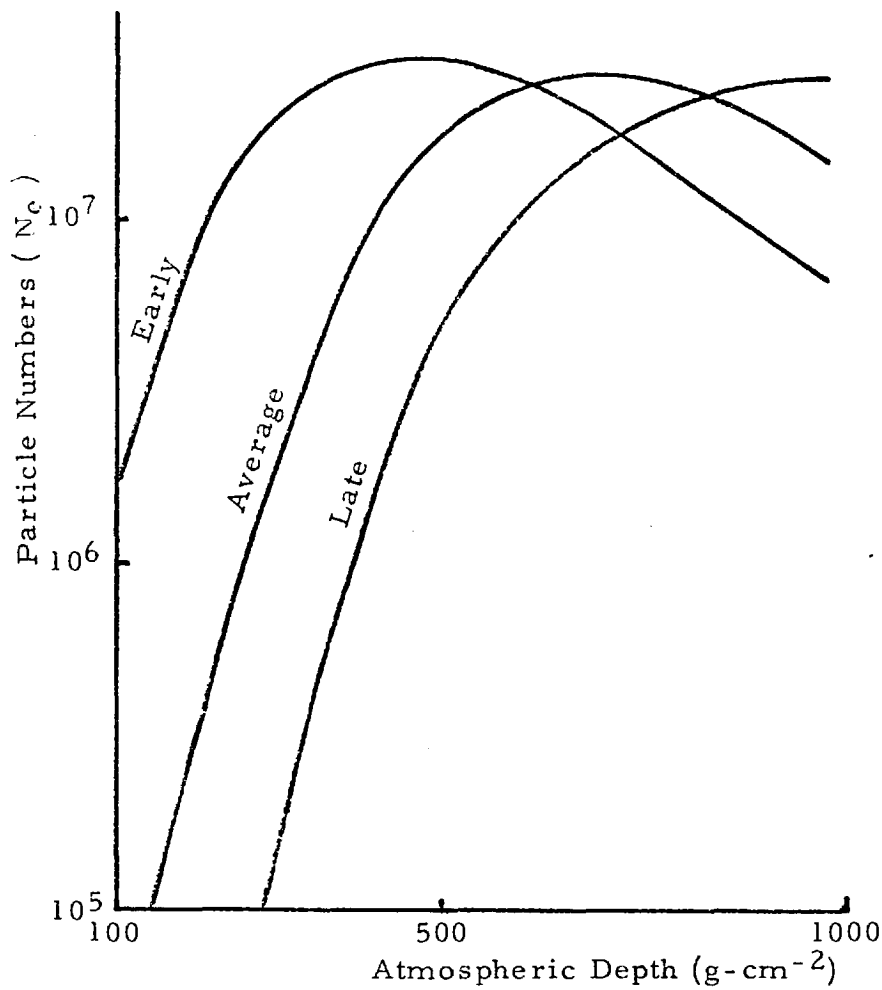
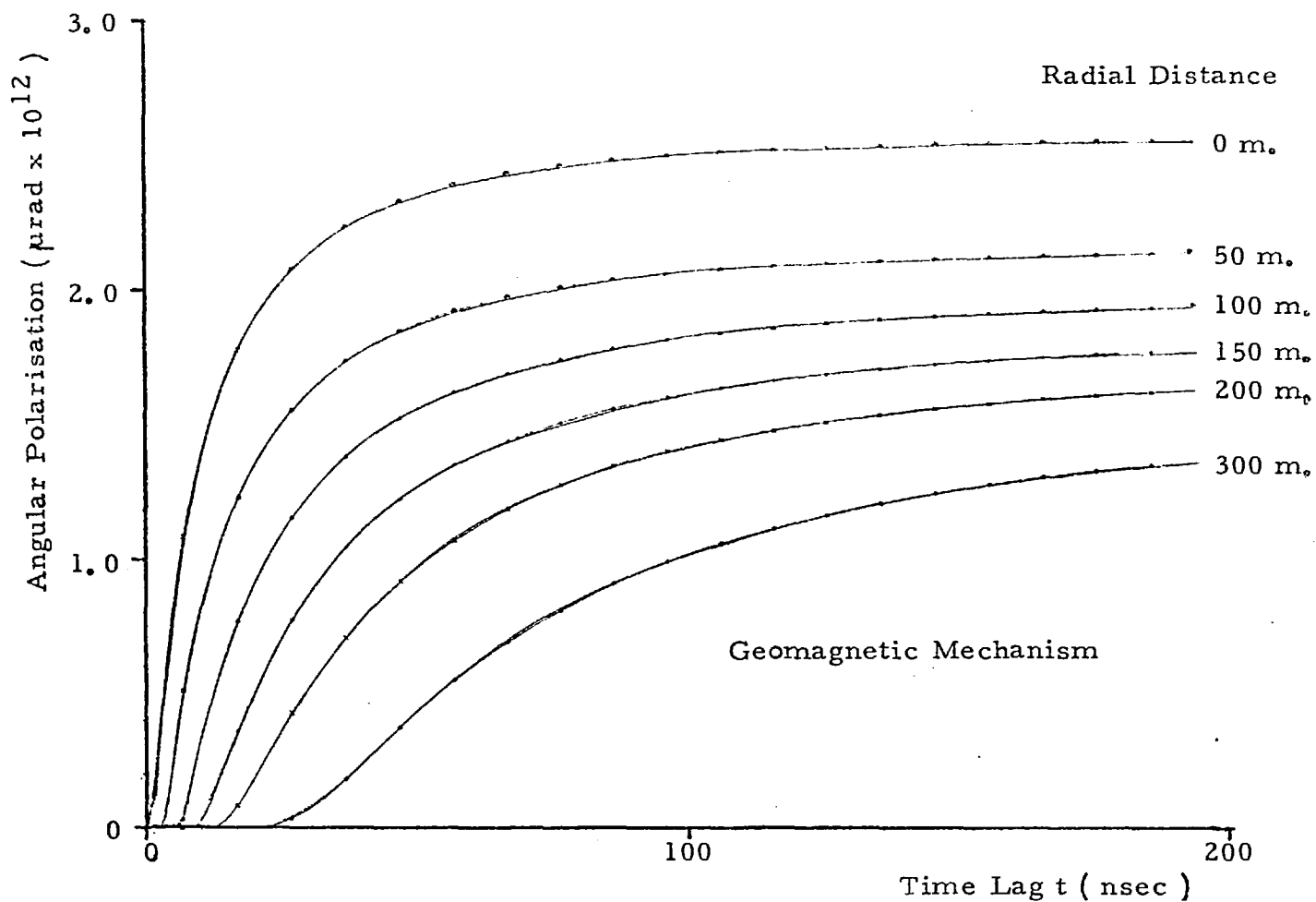


Fig. 5.4

LATERAL DISTRIBUTIONS AT GROUND LEVEL FOR THE
SIMULATED AIR SHOWER

0-10m	0 -30m	0-100m	0-250m	0-∞m	Radial distances
8.7×10^6	1.6×10^7	2.2×10^7	2.3×10^7	2.3×10^7	No. of electrons
1.1×10^7	2.9×10^7	6.4×10^7	9.4×10^7	1.2×10^8	No. of photons

TABLE 5



ANGULAR POLARISATION CURVES FOR A SIMULATED 10^{17} eV
EXTENSIVE AIR SHOWER

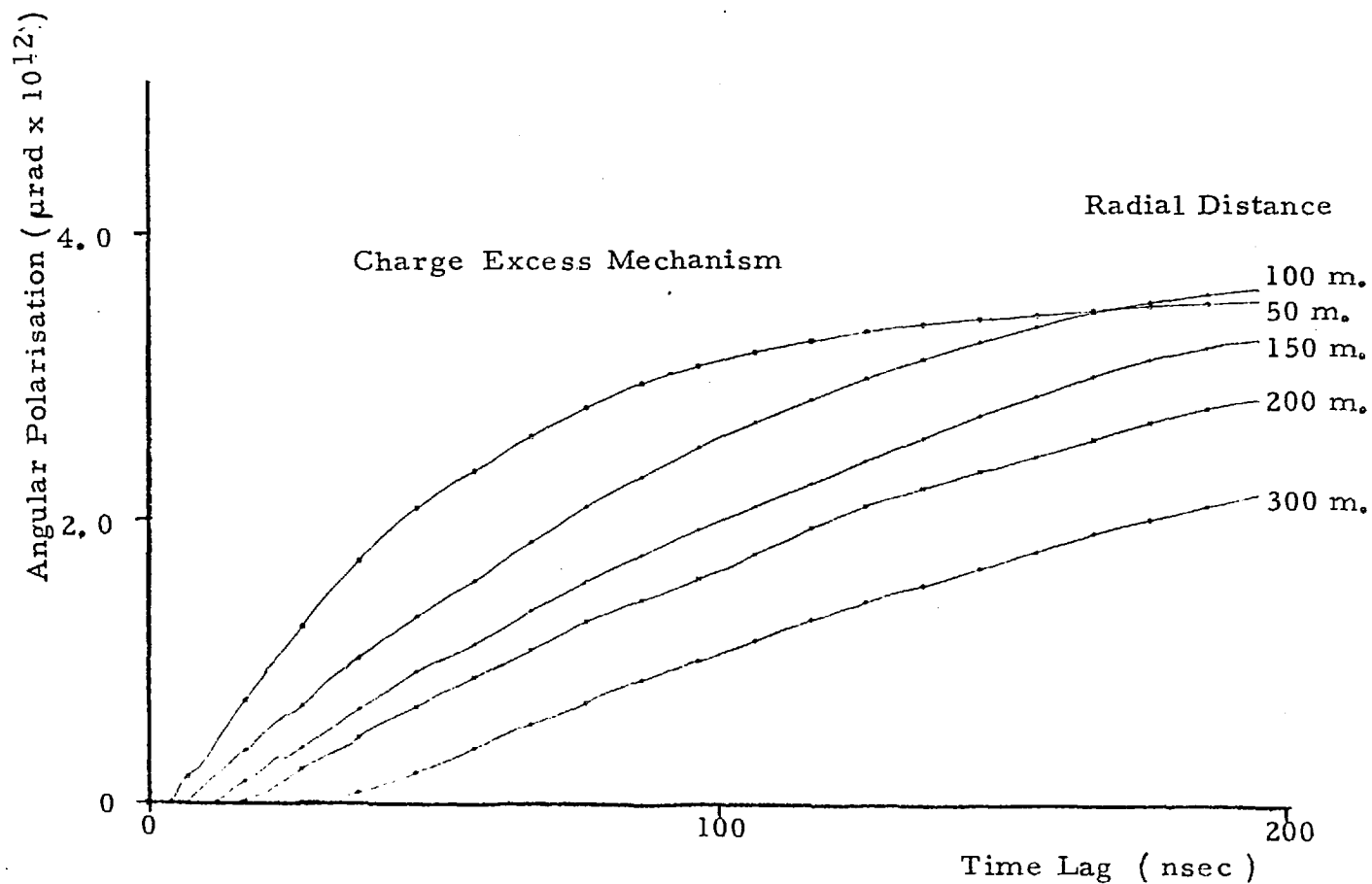


Fig. 5.5

DISTRIBUTION OF THE DELAY IN THE ARRIVAL OF THE RADIO SIGNAL
FROM SIMULATED CASCADES AT VARIOUS DEPTHS IN THE ATMOSPHERE

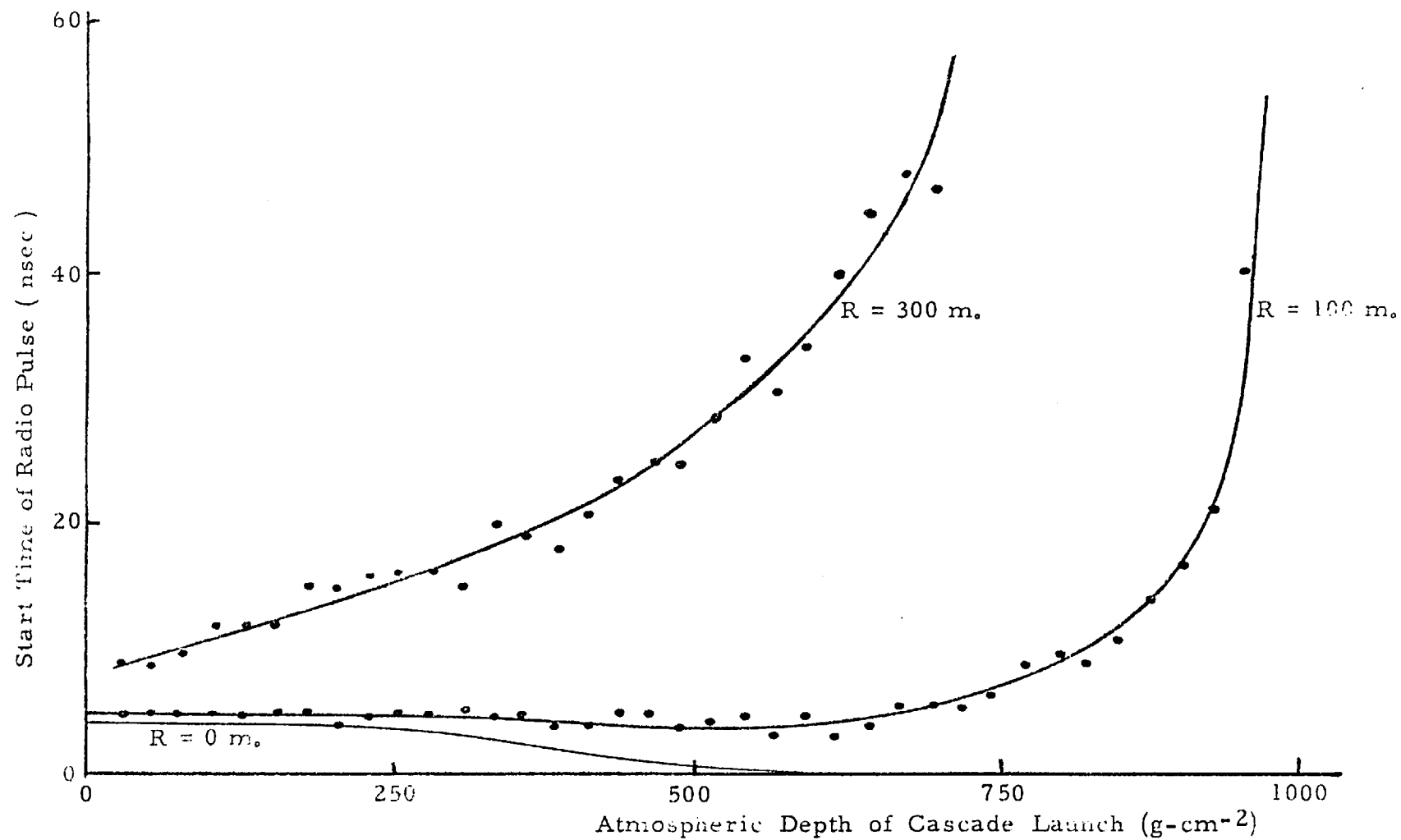


Fig. 5.6

prediction for antennas nearer to the shower axis is in the first instance an over-estimate. This is partly due to (a) digitising errors in the first few crucial nanoseconds in the angular polarisation curves for antennas close to the shower axis and (b) having so far assumed that the launching of the component cascades is along the shower axis.

Fig. 5.5 shows that the rise in the angular polarisation appears to be very abrupt for antennas less than 100 metres from the axis (almost a delta function in the radio pulse amplitude). However, the true 'start times' (i.e. the times when the antennas first receive information from a charged particle in the simulated air shower), do in fact occur somewhat sooner than depicted from the graphs, especially in the case of the angular polarisation curves for the charge excess mechanism. It seems that the sampling fluctuations near the start time when θ is small (Fig. 5.6) are biased towards greater time delay which results in a somewhat sharper rise in angular polarisation than would actually be the case for the shower. In order to overcome the problem of sampling fluctuations and the limits set by 1 nanosecond digits, it was decided to fit a suitable analytical expression for $\theta(t)$ to the $(n\theta - t)$ curves. The analytical expression used (originally suggested by Allan) consists of three exponential terms of the form:-

$$\theta = \theta_{\max} (1 - ae^{-t/T} + be^{-mt/T} - ce^{-nt/T})$$

where the three time constants T , T/m and T/n characterise three distinct parts of the radio pulse waveform (Fig. 5.7). The coefficients a , b and c are determined in terms of the time constants by the requirement that θ and its first and second derivatives must be zero at the origin, or the 'start time' when t is zero in the expression. It is also assumed that the 'overswing time' T is longer than the fall time T/m which in turn is longer than the characteristic rise time of the pulse T/n . Figures 5.7 and 5.8 show how different chosen values of m and n

ANALYTICAL ANGULAR POLARISATION FUNCTIONS

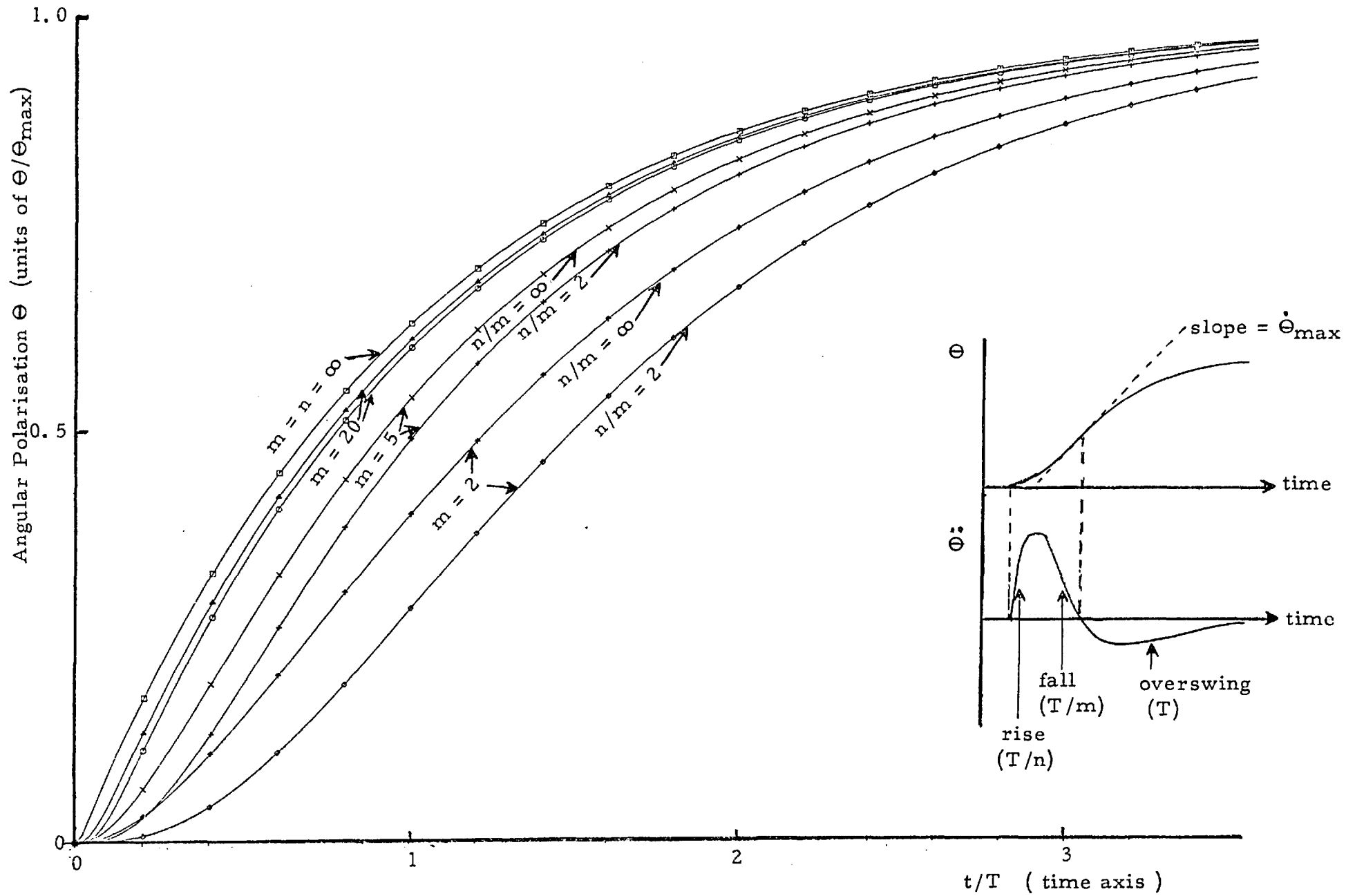


Fig. 5.7

FREQUENCY SPECTRA FOR THE ANALYTICAL POLARISATION CURVES

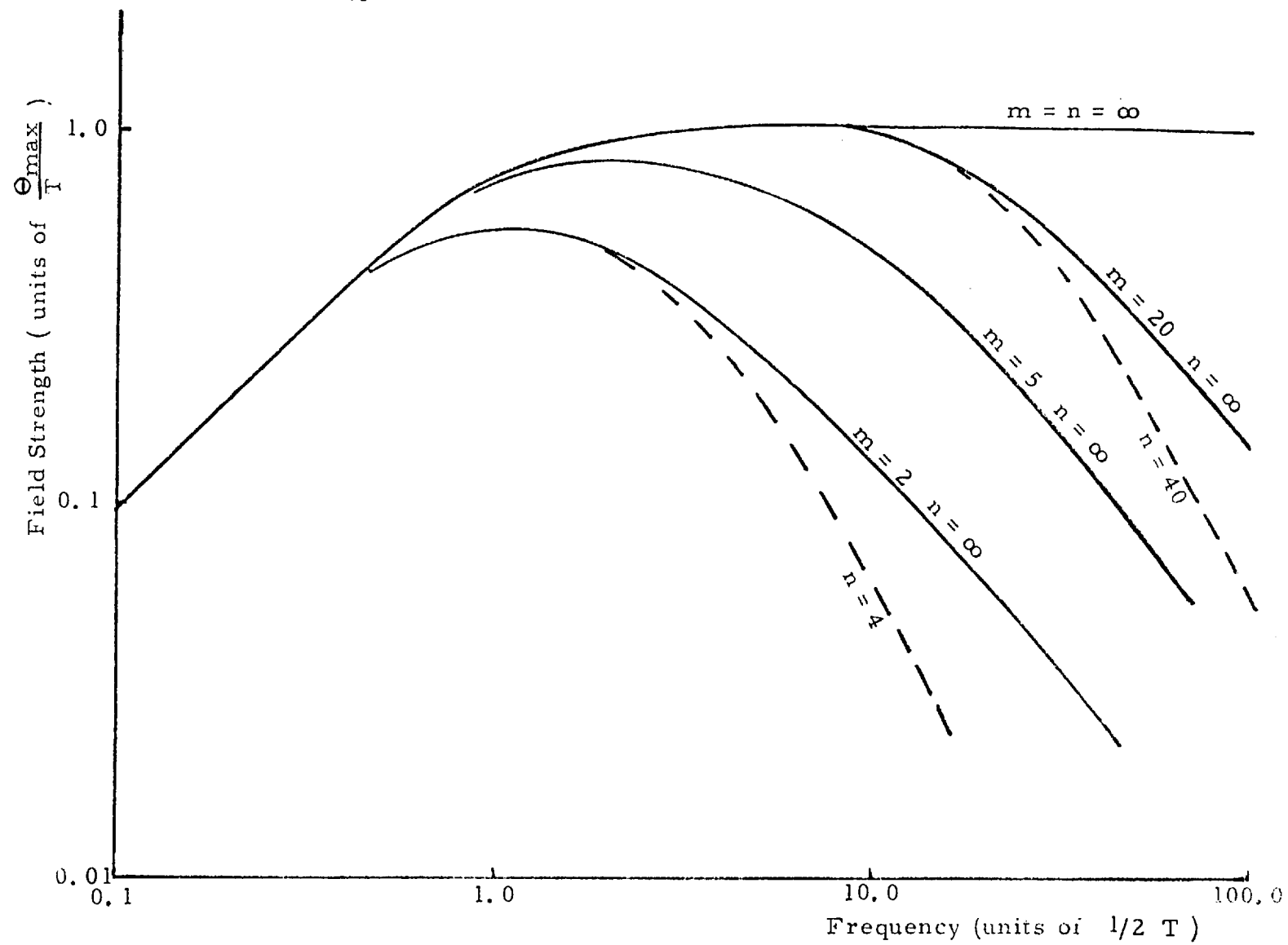


Fig. 5.8

affect the shapes of the angular polarisation curves and their frequency spectra. The overswing time (T) can be determined quite accurately and quite a reasonable estimate may be obtained for T/n by curve fitting near the kink and using the knowledge of a well-defined start time. However, T/n is not possible to estimate with any degree of accuracy from the graphs.

The frequency spectrum of the radio pulse falls off at high and at low frequencies. The range of frequencies over which the spectrum remains substantially flat extends over a range between $n/2\pi T$ and $m/2\pi T$ Hz (where T is in seconds). The amplitude in this 'mid band' region is proportional to $\theta_{\max}/T$, which relates closely to the maximum slope of the $(n\theta - t)$ curves at the point of inflection. Table 6 gives values of field strengths for the two mechanisms of radiation in the 'mid band' frequency regions, for different distances from the shower axis.

		Field Strength	Frequency Range
		$\mu\text{V m}^{-1} \text{ MHz}^{-1}$	MHz
100m from axis	G-M	2.8	8 - 50
	C-E	1.2	3 - 25
300m from axis	G-M	0.7	3 - 12
	C-E	0.5	1.5 - 6

TABLE 6

Experimental measurements are centred around 60 MHz which is somewhat above the mid-frequency band, on the falling part of the spectrum. The spectrum falls at a rate depending on the rise times of the pulse waveforms T/n , which cannot be assessed with any great accuracy. However, one can expect on physical grounds that the rise

time is neither zero nor is it longer than the fall time T/m . The field strength at 60 MHz from these considerations, is lower than in the mid-frequency bands. For the geomagnetic mechanism, the 100 m values will be reduced by factors ~ 2 and the 300 m values by factors ~ 5 . The charge excess values can be expected to be reduced by correspondingly larger factors to give about 10% to 15% of the geomagnetic field strengths at 60 MHz.

The experimental values at 60 MHz are somewhat lower than those suggested by theory. A field strength of about $0.2 \mu V$ is observed at 100 m from the axis of a vertical 10^{17} eV shower (Chapter 6). However, several reasons can account for this difference. The lateral scatter in the launching of the component cascades has been ignored as well as the time delays that could be introduced by obliquity of the particles in the nuclear active cascade. These particles remain close to the shower axis and any lateral spread takes place as a result of the transverse momentum (P_t) acquired at the moment of formation and is typically 0.5 GeV/c. The angular deflection of the secondary nucleons and pions (θ) is given by P_t/P_{11} where P_{11} is the longitudinal momentum. As an illustration, the angular deflection of a pion with a longitudinal momentum of 10 GeV/c would be 5×10^{-2} radians. The corresponding geometrical lag may be taken as $\frac{1}{2}L\theta^2$ where L is the interaction length of the pion which may be typically 1000 metres. The additional lag involved in the launching of some of the component cascades is over a metre or several nanoseconds and this could result in an appreciable increase in the time constant T/m and therefore a reduced value for the field strength at 60 MHz.

It has already been stated that the median lateral spread of electrons at ground level is narrower for the simulated air shower compared with experimental estimates by a factor of ~ 2 . Making some allowance for the obliquity of the nuclear-active particles is therefore not incompatible with lateral spread obtained from the simulations.

5.7 The Structure of the Shower Disc

As a by-product of the work described in this chapter, information has been obtained on how the shower particles are distributed across the shower disc and the way this varies with radial distance from the shower axis. Corresponding distributions as a function of 'intrinsic' lag have also been obtained for the geomagnetic and charge excess dipole moments (see section 3.4). These distributions were obtained from simulations of 10 GeV electron-photon cascades in a constant density atmosphere, 10 m being equivalent to 1 g cm^{-2} . All these distributions are characterised by an abrupt rise at the leading edge of the disc followed by a decay in an approximately exponential manner towards the trailing edge of the disc. The disc properties are averaged over the whole of the shower development. The half width of each distribution increases with the radial distance from the cascade axis as shown in Fig. 5.9. The median radial widths are given as 12 m for particle numbers, 21 m for the geomagnetic dipole moment and 25 m for the charge excess dipole moment. The half widths at these median radii are 0.5 m, 2.8 m and 1.8 m respectively.

The energy dependence of charge excess is shown in Fig. 5.10 and is compared with calculations made by Fujii and Nishimura (1969). In both curves, only the Compton contribution to the charge excess is included. Our simulations show a somewhat greater charge excess at low energies. Fujii and Nishimura evaluated the contributions of other processes (see Chapter 2) to the charge excess; they found that the total value is not very different, though still slightly less than that obtained in the present simulations.

DISTRIBUTION OF HALF-WIDTHS ACROSS THE SHOWER DISC

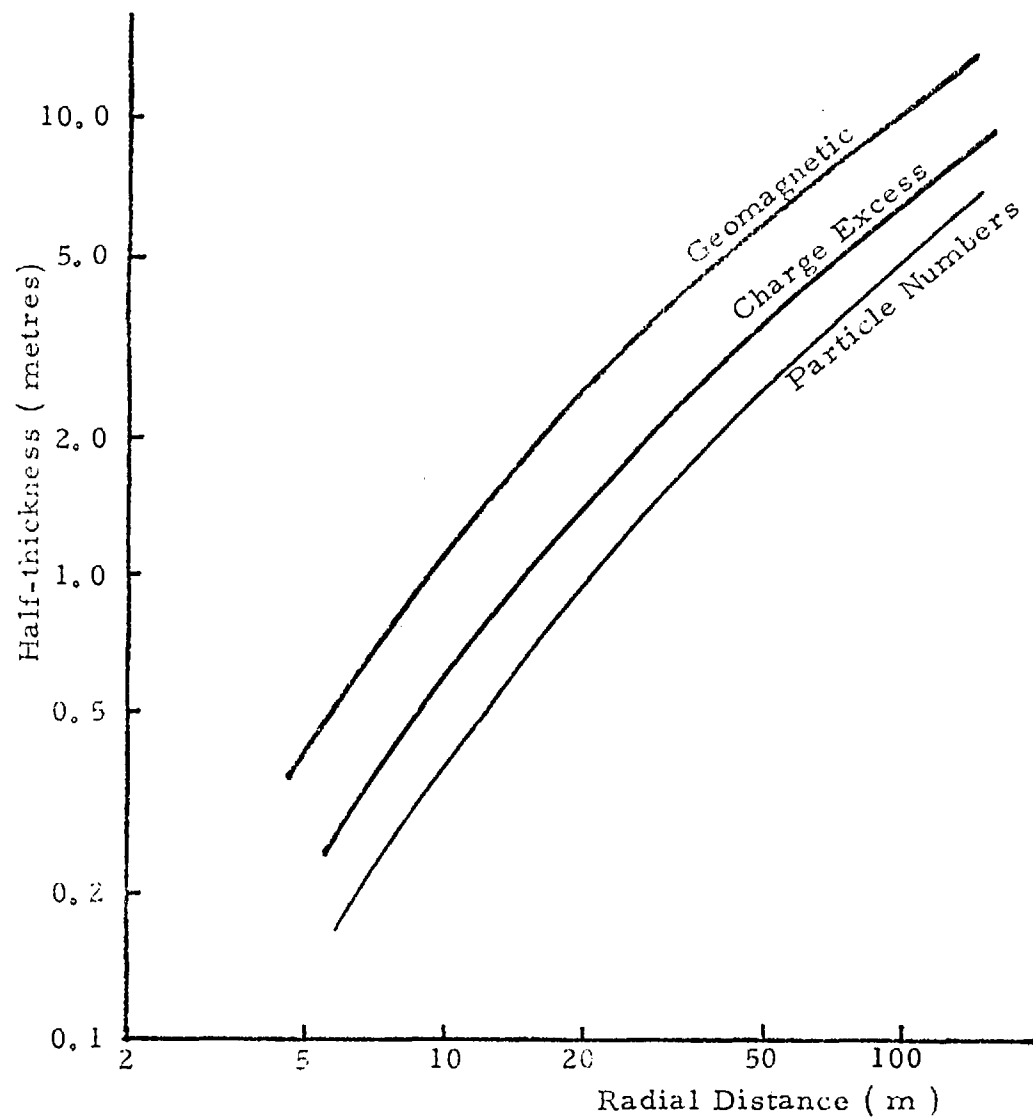


Fig. 5.9

THE ENERGY DEPENDENCE OF CHARGE EXCESS

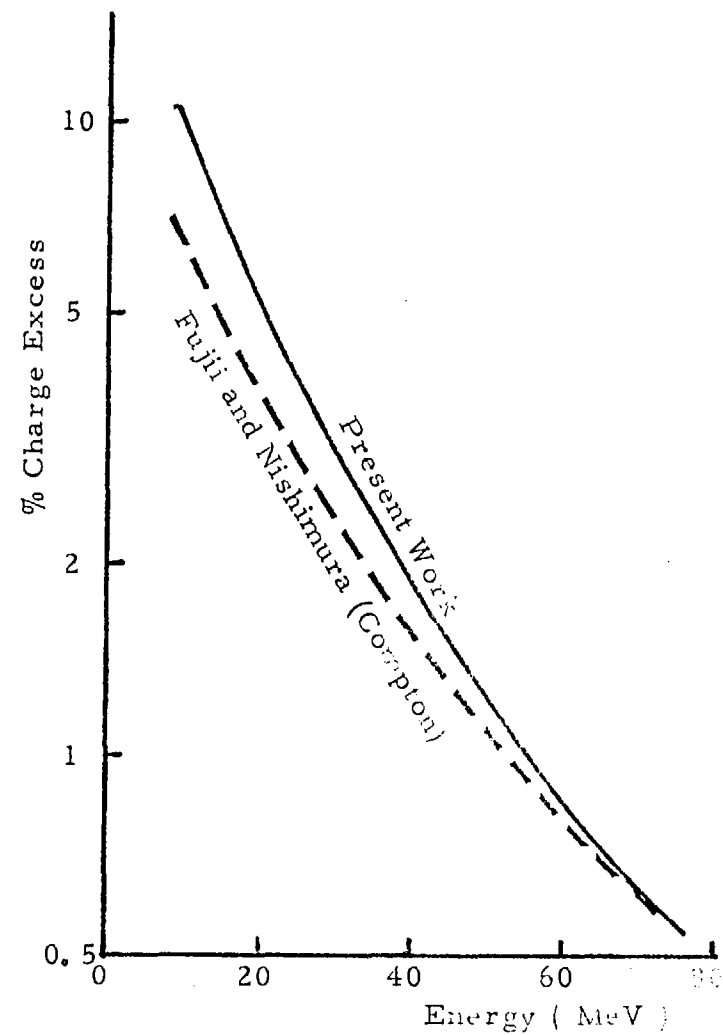


Fig. 5.10

CHAPTER 6

THE ANALYSIS OF THE HAVERAH PARK EXPERIMENTAL RADIO DATA6.1 Introduction

In this chapter, an outline of the current radio experiment is presented, together with an analysis of the radio data obtained over the last three years of almost continuous (24 hr/day) operation. The two main objectives in the radio experiment have been:

- (a) To determine the dependence of the radio field strength on the perpendicular distance from the shower axis and to investigate the sensitivity of the shape of this radio lateral distribution to the height of maximum development of the shower. The extent to which this height is found to fluctuate would be a clue to the nature of the primary particles initiating the showers.
- (b) To study quantitatively the degree to which the geomagnetic radiation mechanism dominates over other mechanisms such as charge-excess by polarisation measurements.

Radio field strengths have been measured and analysed for over 1000 showers up to the present time. Whereas it has been the prime objective to associate any fluctuations in the radio lateral distributions with fluctuations in shower development, it has turned out that the study of fluctuations is very close to the limits of accuracy attainable. As the data over the past three years has accumulated, it has become clear that although remarkably consistent field strength measurements have been attained, compatible with theoretical predictions, the radio technique cannot be used to

identify fluctuations in extensive air showers with any degree of conviction.

6.2 The Experimental Radio Array

Antennae and Operating Frequencies

In an ideal case, to give a faithful reproduction of a radio pulse as a function of time, one could desire to employ a very wide band antenna and receiver with a frequency response ranging from DC to well above 100 MHz. However, in practice it is necessary to restrict the bandwidth of the receiver and also to select a particular operating frequency that is free from man-made radio interference as well as atmospheric effects. Fig. 6.1 shows that intrinsic receiver noise is dominant at frequencies much above 60 MHz and ionospheric noise presents a problem at frequencies well below 60 MHz.

From May 1972 onwards, a single operating frequency of around 60 MHz has been employed for all antennae. Pilot experiments using other frequencies have been carried out (Prah, 1971) in order to ascertain the operating frequency with the best signal to noise performance. Powerful TV transmissions near 60 MHz necessitate the careful design of filters to avoid trouble from pick-up in the wings of the receiver pass-band. Bandwidths of about 6 MHz have been employed, with supplementary filters to eliminate the TV frequencies. Five antenna sites have had receiving systems in operation from May 1972 to May 1973. In May 1973, the spacing of the antenna sites was increased somewhat and a sixth antenna site was installed (Fig. 6.2). The new arrangement, which is considered to be near optimum for the receiving system as a whole, has been in continuous operation until the present day.

Each antenna site is supplied with two receiving channels, and each channel is matched to a pair of parallel folded half-wave dipoles which are half a wavelength apart and a quarter wavelength

COMPARISON OF NOISE LEVELS IN A 1 KHz BANDWIDTH
(DAVID AND VOGEL, 1969)

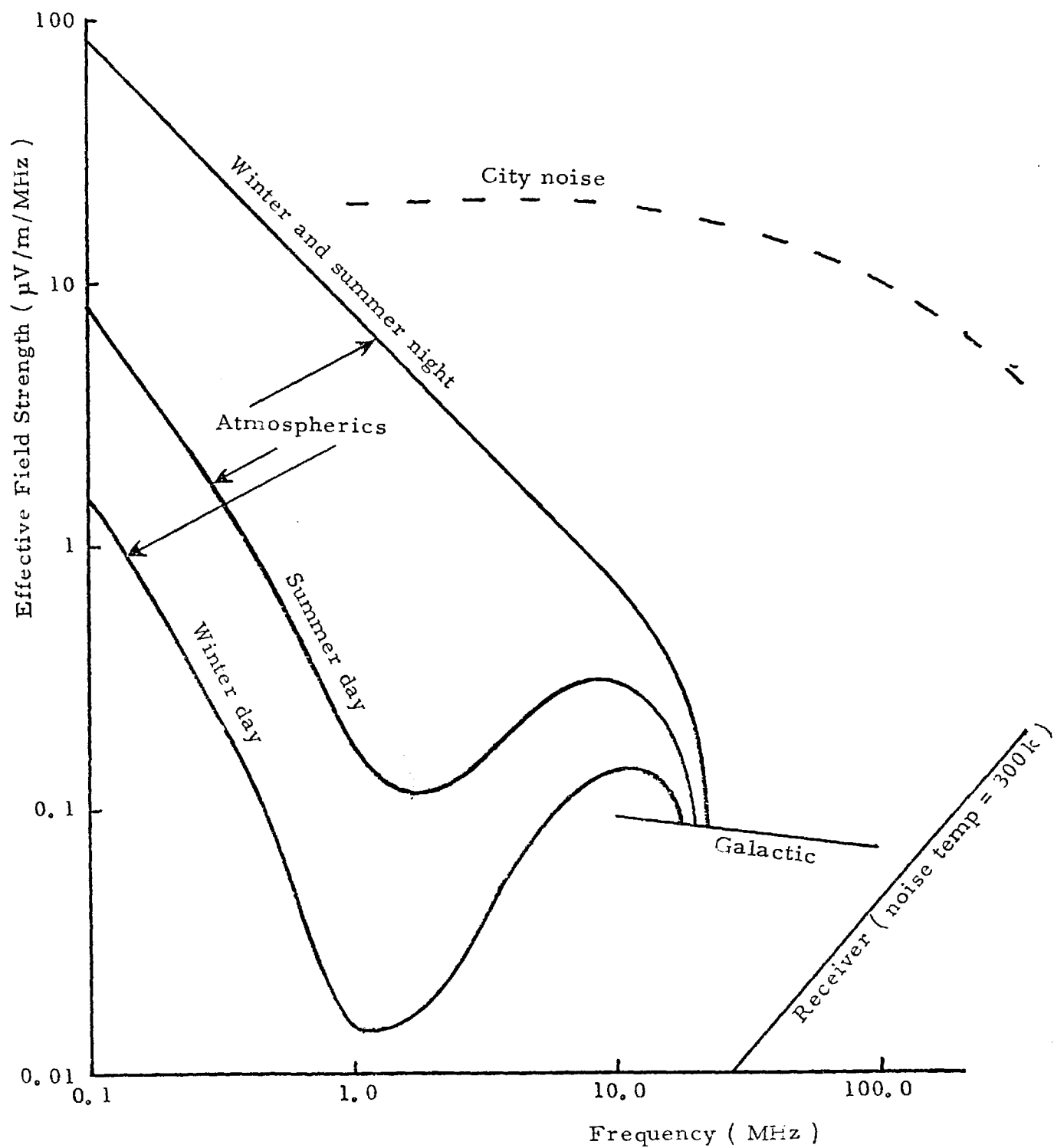
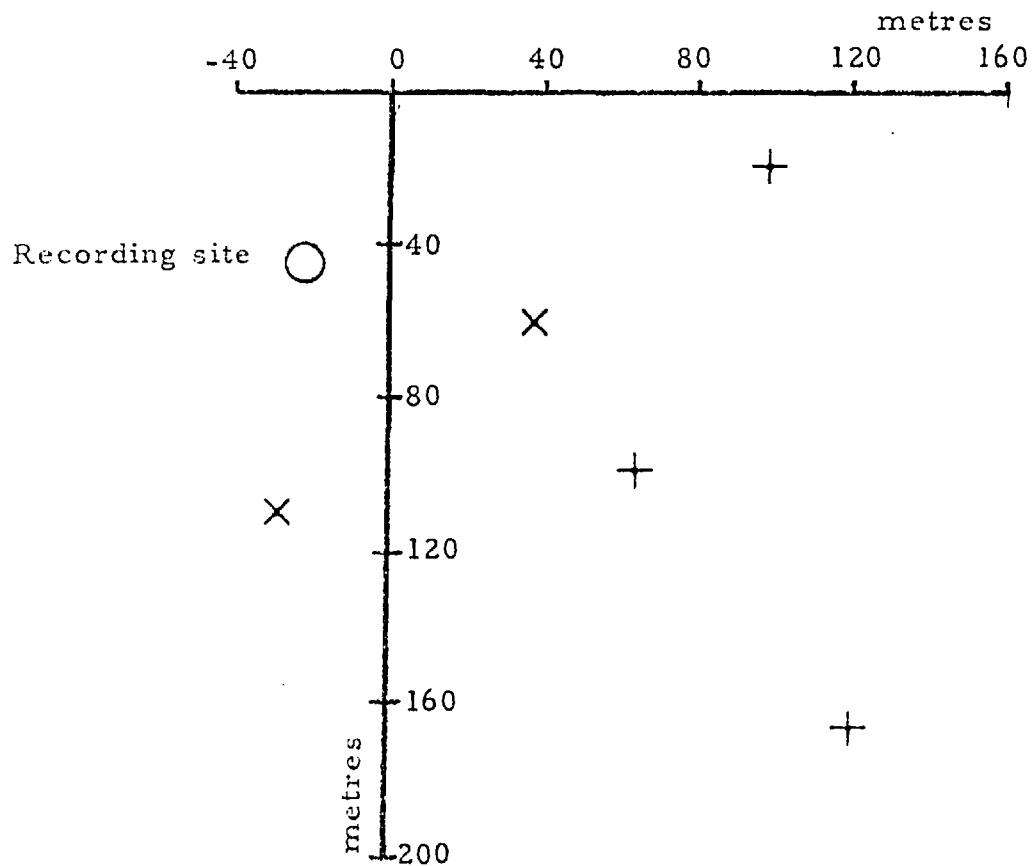


Fig. 6.1

LOCATIONS OF THE 60 MHz RADIO ANTENNAS

1972 TO MAY 1973



FROM MAY 1973 TO APRIL 1975

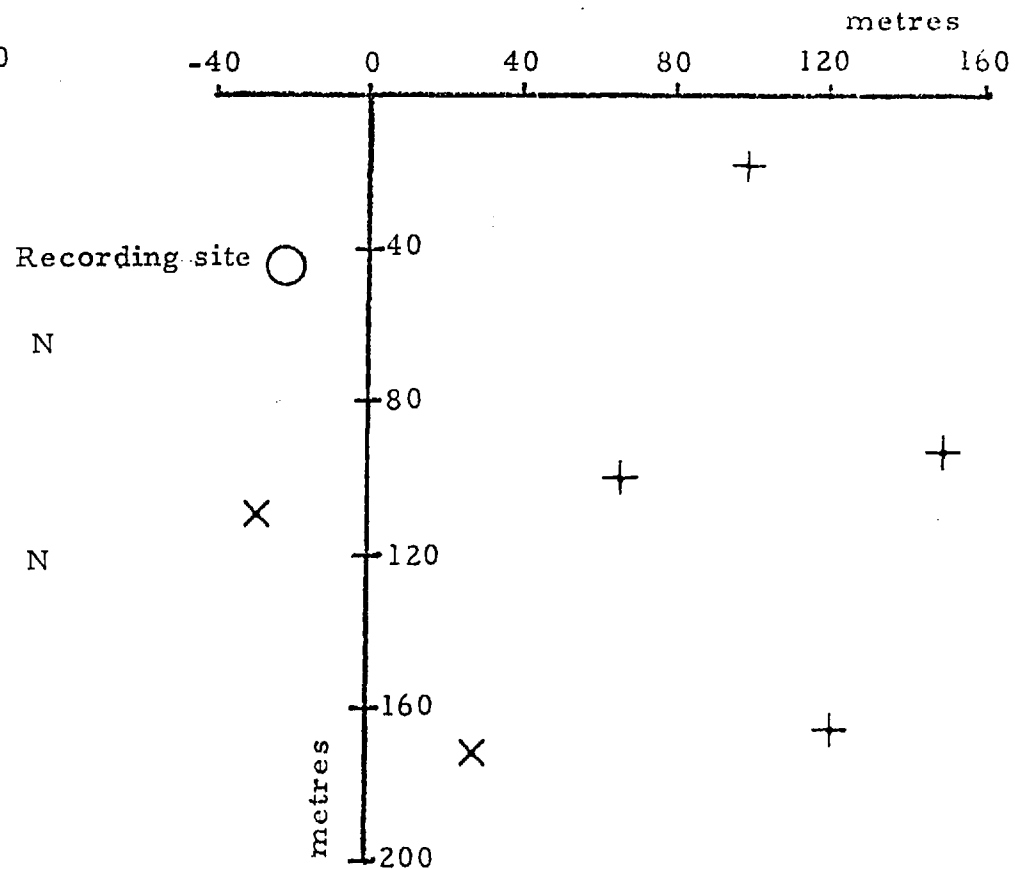


Fig. 6.2

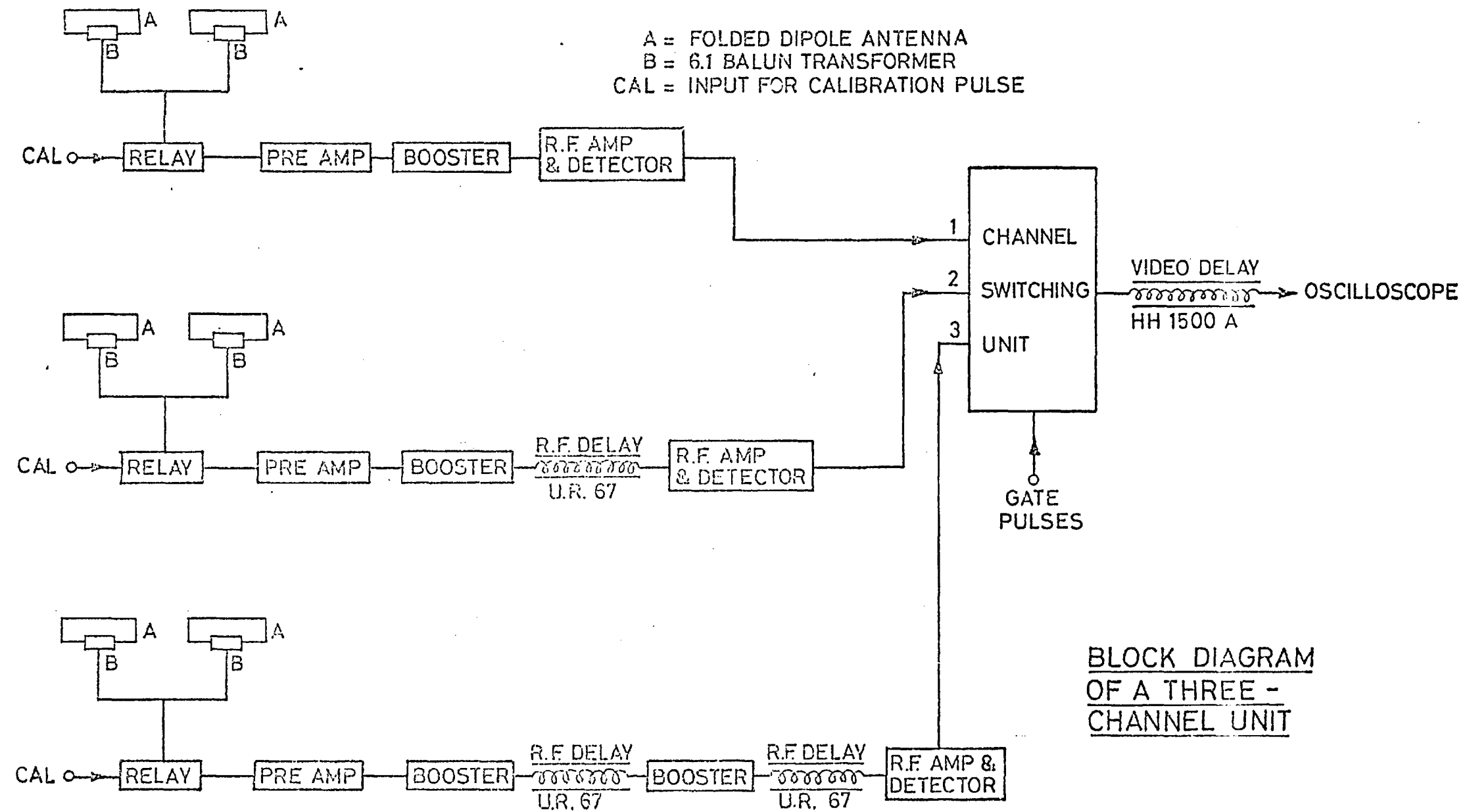
above the conducting ground surface. The two pairs of antennae are arranged to form the sides of a square (a "boxing ring" arrangement). The two mutually perpendicular horizontal components of the radiated field that are measured can be used to obtain the total field strength as well as polarisation information. The polar response of the antenna pair is a single lobe with a broad maximum towards the zenith and a null towards the horizon (Neat, 1968). It is believed that this smooth variation in antenna gain can be relied upon for all shower zenith angles not exceeding much above 45° .

The Receiver, Trigger and Display System

The signal from an antenna pair is passed through a pre-amplifier located at the antenna site and then through cables and booster amplifiers to a central recording hut, where the main amplification of the signal takes place. After a linear detecting stage, the signals are displayed on the time base of a high speed oscilloscope and photographed. Suitable lengths of delay cables are incorporated so that the radio pulses from three channels arrive in a consecutive time sequence to enable them all to be displayed on single time trace. The use of two double beam oscilloscopes and cameras allows the maximum of 12 traces to be photographed on two frames of 35 mm film. A multiplex unit is used to switch each set of three radio channels onto one time base in order to maintain the original signal to noise performance. Fig. 6.3 shows a flow diagram for a set of three channels.

The entire sequence of operations involved in the recording process is initiated by a trigger pulse from the Haverah Park particle array, which is received when this array records the presence of an air shower whose energy is usually above 10^{17} eV. A master clock is used to give a 6-digit readout, which is recorded on the film with the radio traces to identify the shower event in terms of the time of arrival of the shower.

A = FOLDED DIPOLE ANTENNA
 B = 6:1 BALUN TRANSFORMER
 CAL = INPUT FOR CALIBRATION PULSE



BLOCK DIAGRAM
OF A THREE -
CHANNEL UNIT

Fig. 6.3

System Calibrations

From May 1973, a system has been installed whereby after every radio event a calibration pulse in the form of a short burst from a 60 MHz oscillator is applied to the receiver inputs at each antenna site. The outputs are photographed a few seconds after the radio traces and they are used in the calculation of the field strengths, to correct for short term variations in gain. This variation can be as high as 30% during the summer when greater temperature variations between night and day can occur.

In addition to this 'on the spot' calibration, the operational routine includes gain checks on alternate days when reloading the films and a full calibration check on gain, linearity and bandwidth every two months (Shutie, 1973). The rigorous procedure adopted has resulted in very consistent field strength measurements and all experimental data from May 1973 onwards can be considered as having reached near the experimental limits of accuracy that is attainable for the system. The error in field strength measurements due to gain and bandwidth uncertainties is to within 10%.

6.3 Radio Data Reduction

The radio data for each shower event is recorded on two frames on each of two rolls of 35 mm film. Six radio traces are recorded on the first frame, followed by six calibration pulses and a message register number to identify the shower event (Fig. 6.4). The films, together with all the receiver calibration data are sent to Imperial College for film reading and data analysis.

The Particle Array Analysis

The analysis of the radio data relies heavily on the information derived about each shower from the 150 m and 500 m

TYPICAL TRACES RECORDED ON 6 CHANNELS FOR AN
EXTENSIVE AIR SHOWER

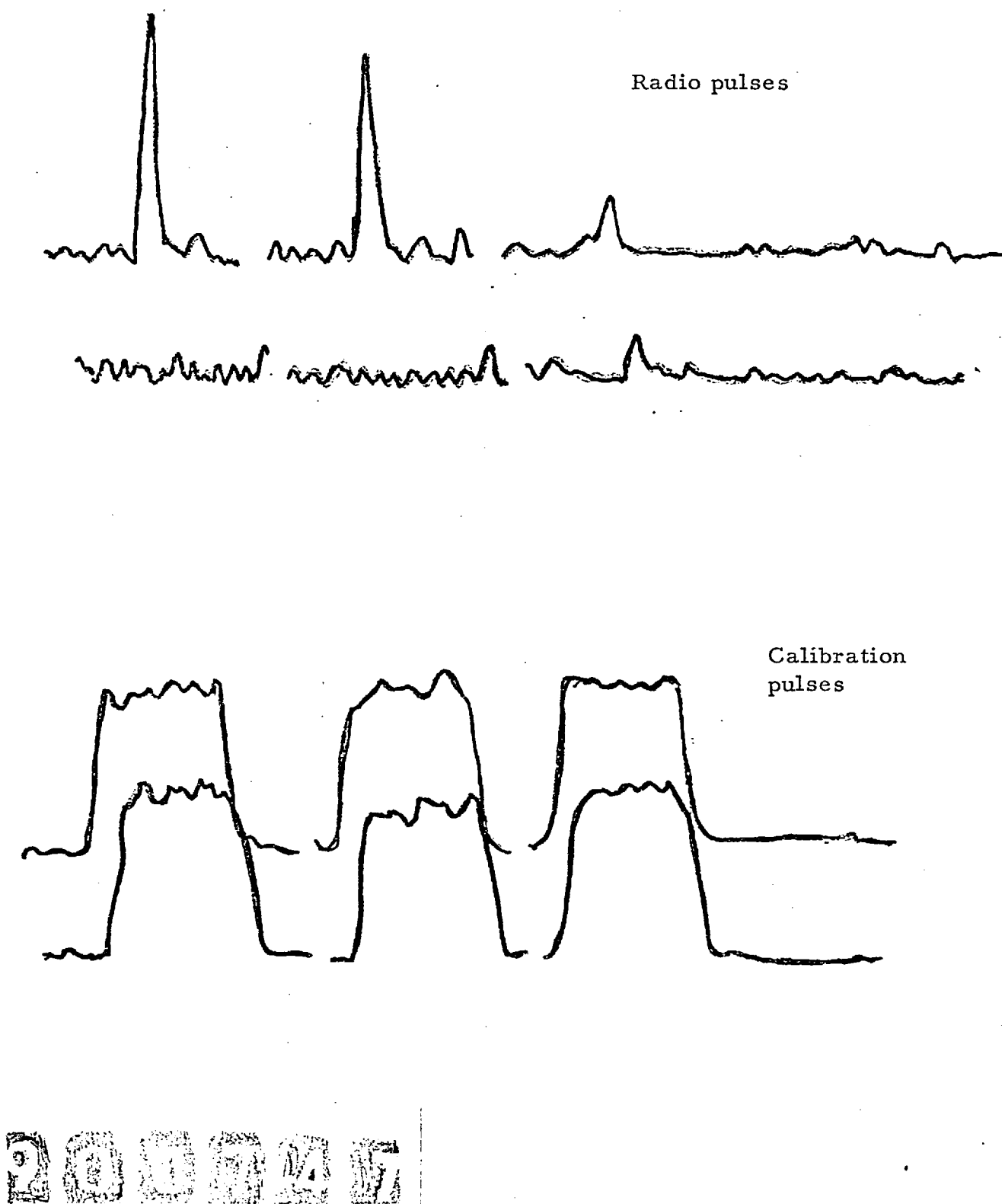


Fig. 6.4

particle arrays at Haverah Park (Fig. 1.2). This array has a collecting area of 0.6 km^2 and a threshold detection energy of about $5 \times 10^{16} \text{ eV}$ (Tennent, 1967). From the particle data analyses carried out by the Leeds University group, estimates of the following parameters are provided and used in the radio data analyses.

- (a) The message register number to identify the shower events.
- (b) The primary energy E_p which is estimated from particle density measurements at a distance of 500 m (ρ_{500}) from the shower axis (Hillas, 1971).
- (c) The zenith and azimuthal angles (θ and ϕ), which are estimated from shower timing measurements at the particle detectors (Hollows, 1968).
- (d) The shower core location (x and y) which is estimated by fitting an empirical lateral distribution function to the particle density measurements (Allan 1960, Evans 1971).
- (e) A χ^2 parameter which is used as a measure of internal consistency of the particle data.

Only those showers for which χ^2 is less than 2 are considered as acceptable in the radio data analysis. For such showers, the core location is believed to be accurate to within $\pm 20 \text{ m}$ and the accuracy in axis orientation to be within 3° . The estimate of the primary energy is believed to be within 25%.

The collecting area for the radio array is typically less than 0.2 km^2 and the arrival directions are restricted by the fact that the shower axis must be inclined at an appreciable angle to the geomagnetic field lines for radio detection from the geomagnetic mechanism. About 1 in 8 shower events detected by the particle

array give detectable radio pulses. However, strict selection criteria are applied when film reading and analysing the radio data and the collection rate are reduced to about 1 in 20 or about 1 or 2 'good' radio events per day.

Film Measurements

The three measurements that are taken (Fig. 6.4) for each channel are (a) pulse height of largest pulse visible on trace; (b) the peak noise level and (c) calibration pulse height. The following criteria are applied in the selection of genuine radio events:-

- (1) The measured pulse must be at least twice the height of the largest noise pulse on one or more channels.
- (2) The pulse must arrive in the expected 'time window' on the trace (about 0.5 microseconds wide).
- (3) The base lines of the traces must be level; any other shape implies the presence of TV carrier wave interference on the traces. However, the use of bandpass filters with very steep slopes from May 1973 has virtually eliminated this problem.

The calibration data, together with the data from the particle array are used in the computation of the field strength components for the pulses on each channel. The perpendicular distances from the antenna site to the shower axis is calculated to give the radio lateral distributions for the analysed showers. The results of this analysis as well as polarisation checks to establish the radiation mechanism are described in the rest of this chapter.

6.4 Polarisation Measurements

For the radio pulses measured at each antenna site, a polarisation check is carried out by comparing the measured polarisation with a theoretical estimate based on the orientation of the shower axis with respect to the magnetic field lines. The polarisation is expressed theoretically as a ratio for the mutually perpendicular horizontal components which correspond to the antenna positions. Fig. 6.5 shows the variation of these ratios with the zenith angle (θ) and the azimuthal angle (ϕ) as measured from the geomagnetic north. The maximum and minimum experimental ratios are computed by taking

$$\text{Maximum Ratio} = \frac{S_1 + N_1}{S_2 - N_2} \text{ and Minimum Ratio} = \frac{S_1 - N_1}{S_2 + N_2}$$

where S and N represent the pulse height and peak noise level respectively for the horizontal components 1 and 2. The pulse height is taken to be the peak noise level if no pulse is seen. If the corresponding theoretical prediction is outside these limits, the polarisations are initially labelled as 'inconsistent' with the geomagnetic mechanism.

Straightforward application of this ratio test is generally consistent with a geomagnetic origin of the radiation but about 1 in every 10 comparisons are 'inconsistent' at first sight. Before trying to ascribe these cases to other mechanisms of radiation, the following additional checks were carried out:-

- (1) From Fig. 6.5 it can be seen that for certain combinations of θ and ϕ the predicted ratios are very poorly defined so that even very small errors in the estimates of axis orientation can give rise to misleading predictions. This consideration reduces the number of measurements incompatible with geomagnetic predictions by a factor of more than 2.

PREDICTED RATIOS OF THE HORIZONTAL FIELD COMPONENTS FOR THE GEOMAGNETIC
POLARISATION OF RADIATION AS A FUNCTION OF ZENITH AND AZIMUTHAL ANGLES

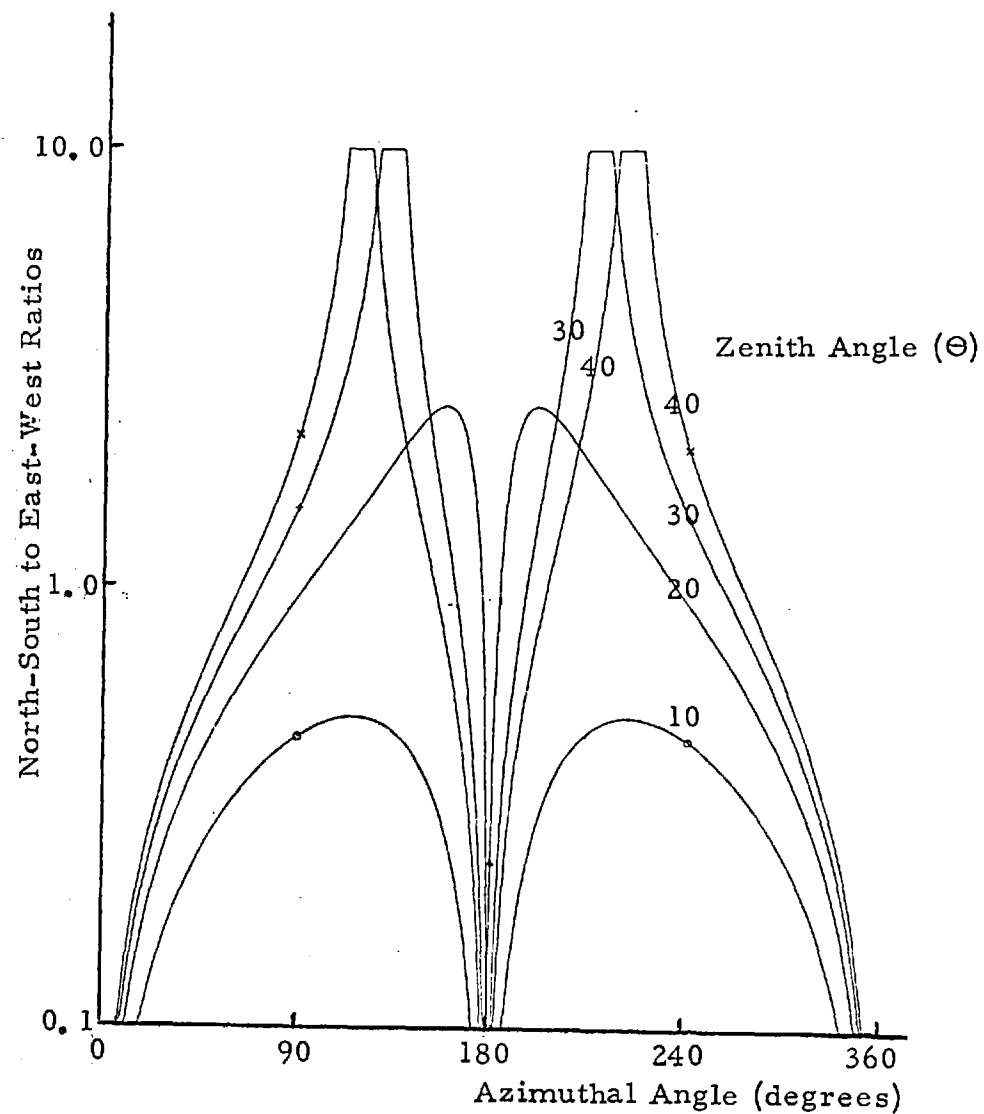
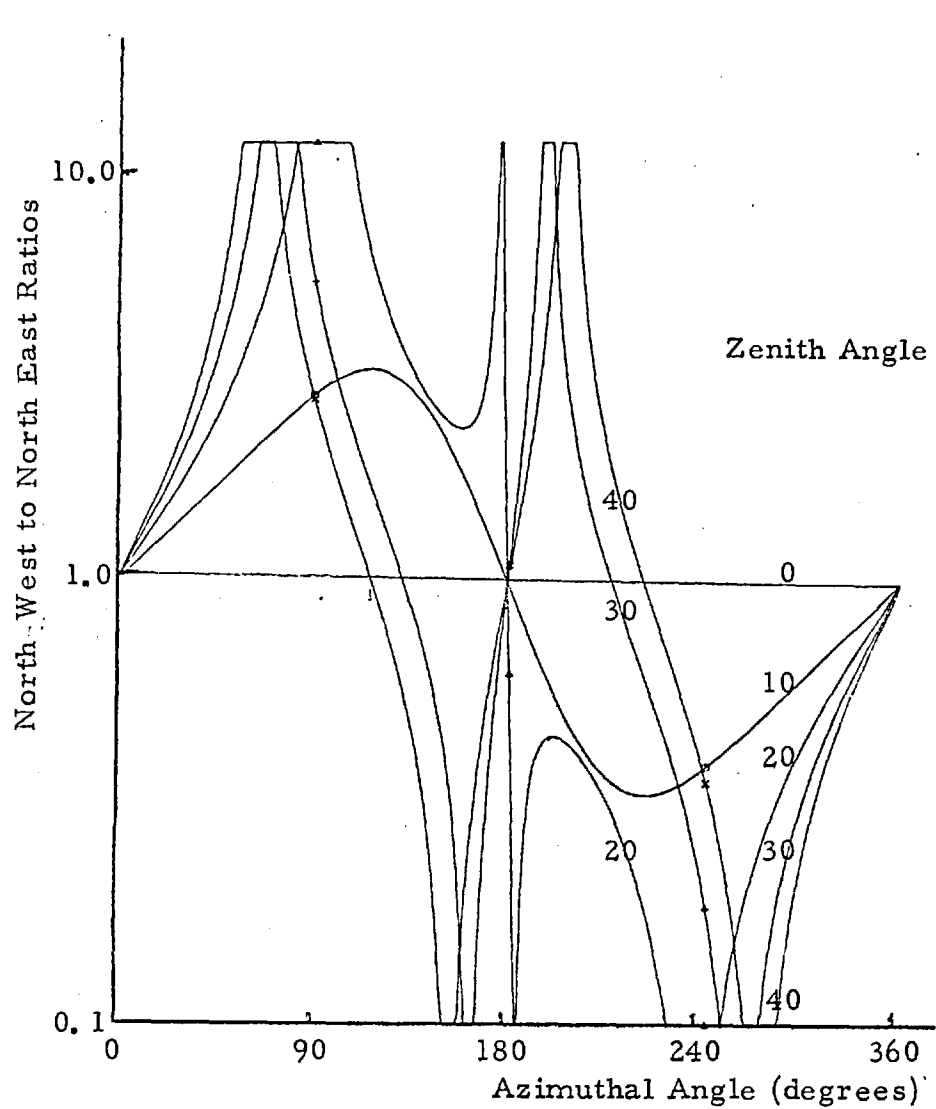


Fig. 6.5

- (2) The remaining polarisation anomalies were then checked again from the film to ensure that the channels have not been saturated.

Of the 1000 showers that have been analysed, only a dozen then remained with polarisations quite incompatible with the geomagnetic predictions. Out of the 12 showers, ten of these also had anomalously high field strengths associated with them and they are discussed in section 6.6.

The geomagnetic radiation mechanism is further confirmed by the lack of any radio pulses from showers where axes are nearly parallel to the magnetic field lines. Most of the radio pulses were recorded for showers coming from the geomagnetic north for which the Lorentz force ($eV \times B$) is greatest when the geomagnetic field is perpendicular to the shower axis.

Prior to the above analysis, an investigation was carried out to determine the degree to which all the polarisation data was consistent with theoretical polarisations obtained for a combination of geomagnetic and charge excess components. The number of 'inconsistencies' was determined for various percentages of charge excess components and the analysis showed that a 10% charge excess was compatible with the data although no significant improvement in the correlation was obtained. On the other hand, a charge excess component in excess of 20% showed a steep rise in the number of 'inconsistencies' with the experimental polarisation data.

6.5 Radio Lateral Distributions

The total field strength in a plane perpendicular to the shower axis is computed from the measured horizontal components (Shutie, 1973). Having confirmed the dominance of the geomagnetic radiation mechanism, we normalise the field strengths with respect

to the angle (α) between the shower axis and the geomagnetic field and also with respect to primary energy. The normalised field strength

$$E_v^N = \frac{E_v}{E_p \sin \alpha}$$

is the field strength that would have been seen for a shower in the same geometrical configuration with respect to the antenna if the energy had been 10^{17} eV and the geomagnetic field had been perpendicular to the shower axis. The parameter that remains which can affect the lateral distribution is the geometrical distance of the maximum development of the shower from the ground level antennae. Generally, increasing the zenith angle of a shower moves its maximum further from the antenna and the radio lateral distribution should be somewhat flatter. The shift in to the maximum in g cm^{-2} is approximately proportional to $\sec \theta$ where θ is the zenith angle. The experimental lateral distributions are therefore grouped into 5 zenith angle ranges up to a maximum of 50° . These intervals correspond to increments of 100 g cm^{-2} in atmospheric thickness; results from three of these are plotted on Fig. 6.6. All the graphs show the same qualitative features: a dense accumulation of data up to 150 m from the shower axis and a severe lack of data in the region beyond 200 m, where simple theory (Chapter 1) indicates that the field strengths should be sensitive to fluctuations in the shower development. The spread in values of field strengths near 100 m can be explained wholly in terms of experimental error and any fluctuations can be considered as too small to be measurable. The field strength at around 100 m is also independent of zenith angle at a value of $\sim 0.6 \text{ } \mu\text{V/m/MHz}$.

It remains to be seen whether the fluctuations in the field strength at around 200m are partly attributable to fluctuations in the shower development. The field strengths do show an increase with zenith angle at these distances which is in qualitative agreement with theoretical expectations. In order to improve on

RADIO LATERAL DISTRIBUTIONS FOR ZENITH ANGLES 0 TO 24 DEGREES

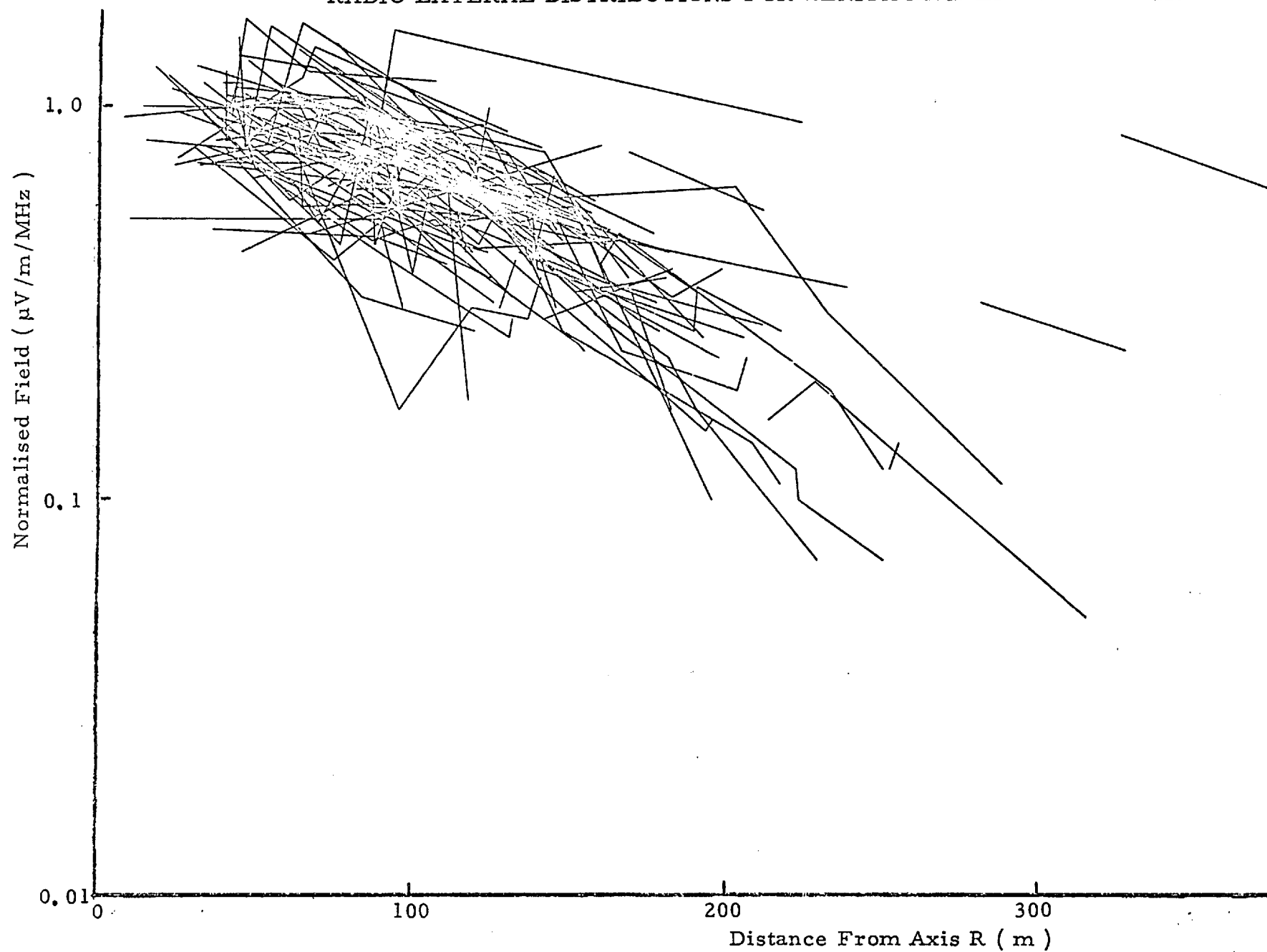


Fig. 6.6 a

RADIO LATERAL DISTRIBUTIONS FOR ZENITH ANGLES 24 TO 34 DEGREES

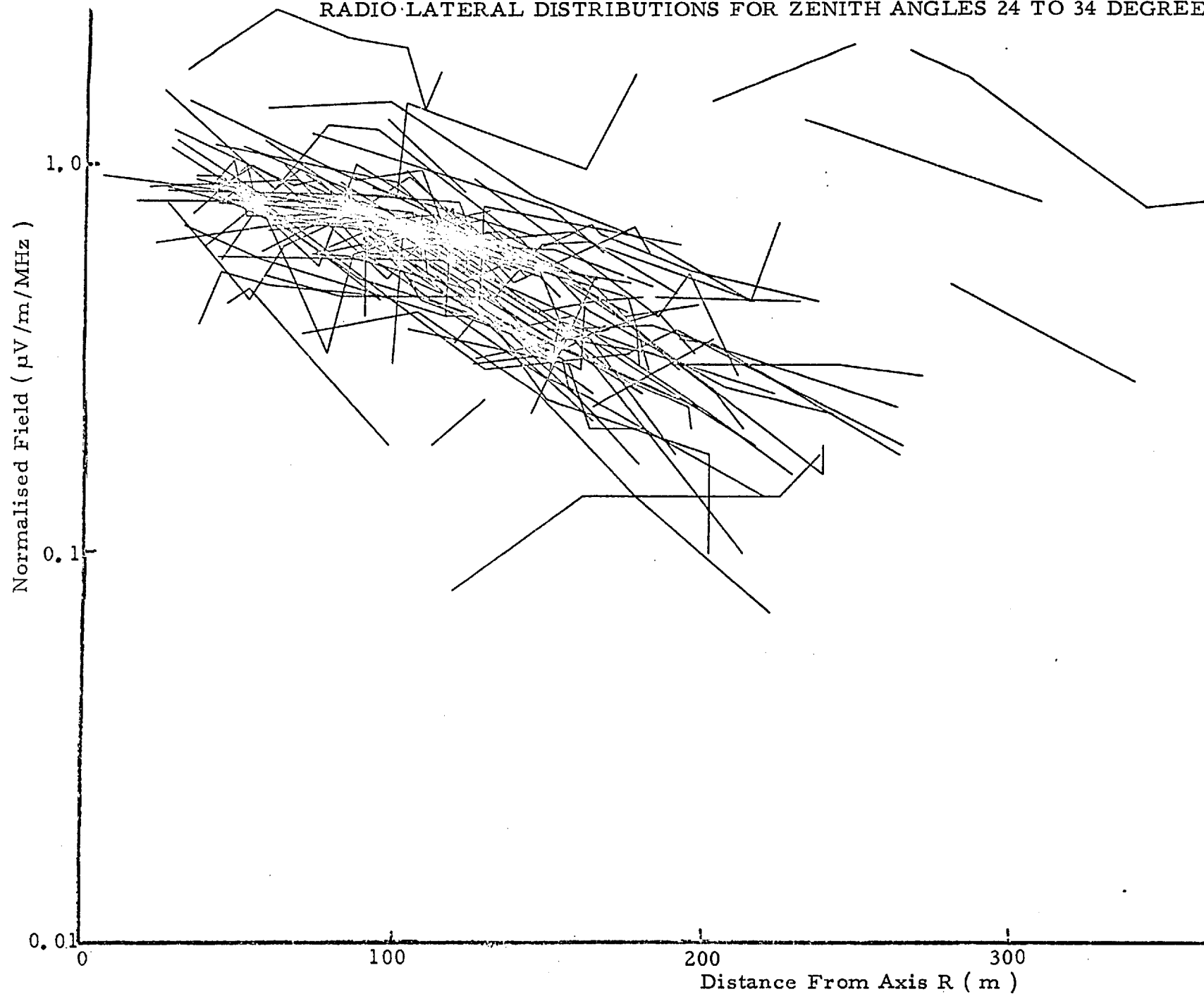


Fig. 6.6 b

RADIO LATERAL DISTRIBUTIONS FOR ZENITH ANGLES 34 TO 40 DEGREES

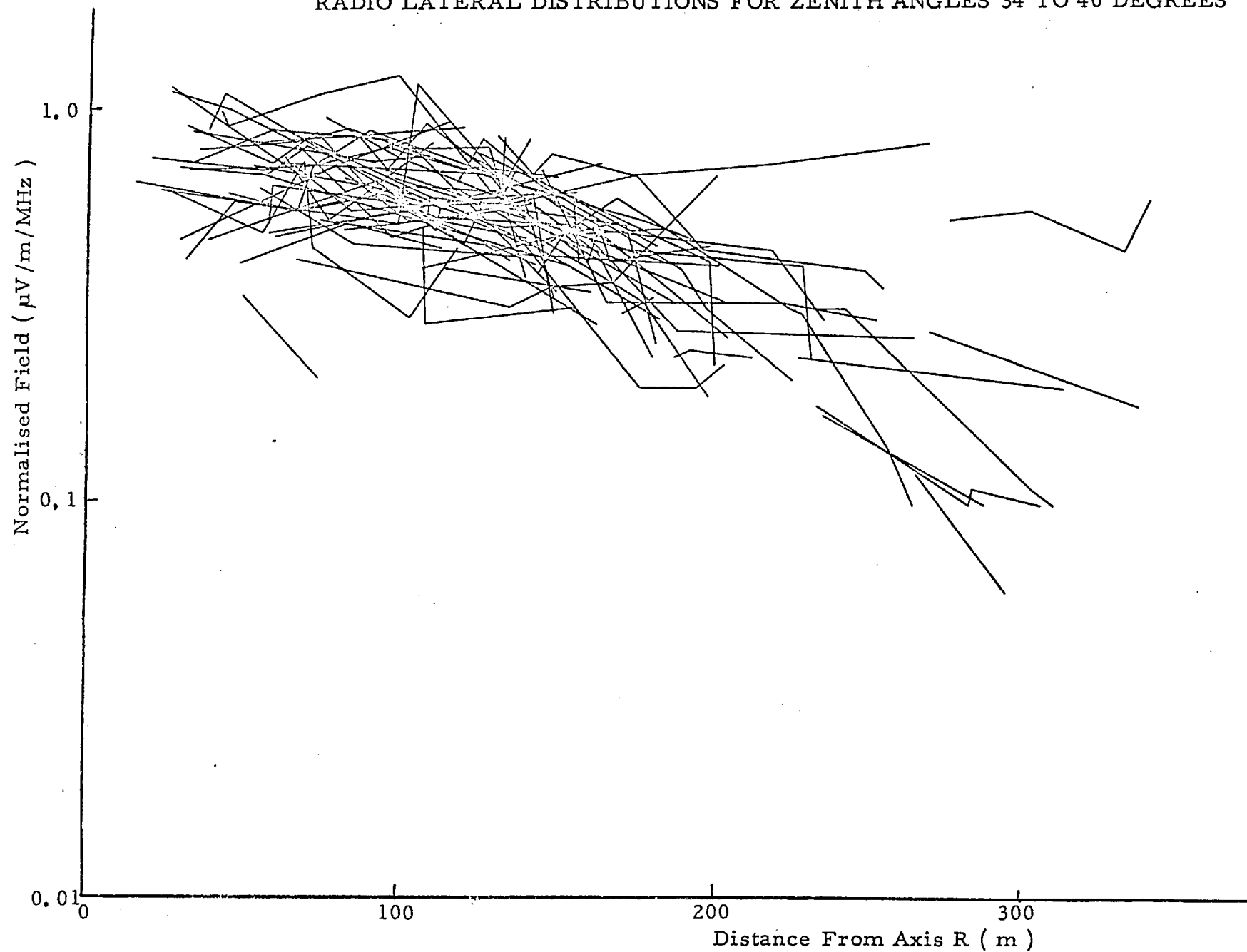


Fig. 6.6 c

statistics at the larger radial distances, the following procedure has been adopted to convert all the measured field strengths for inclined showers to values appropriate for "equivalent vertical showers". A common lateral distribution curve has been fitted for each of the 5 zenith angle ranges. The 'anomalous' showers have been excluded in the curve fitting and are considered later. The fitting has been done by dividing the experimental lateral distributions into 20 m radial bins and a harmonic mean of the normalised field strengths has been calculated. From the 'best fit' curves, the empirical correction factors for this zenith angle effect are obtained. The smoothed out version for the corrections is shown in Fig. 6.7. Each radio data point is then divided by the appropriate correction factor to give a "vertically normalised" radio field strength. Fig. 6.8 shows the results for four radial intervals as a scatter diagram of field against primary energy E_p .

6.6 Discussion of Results

Sensitivity to Fluctuations

A sensitivity to fluctuations at the larger radial distances may be claimed only if the scatter in field strength is greater than that accounted for by experimental error. Inspection of Fig. 6.8 may indeed suggest a slight increase in scatter for the larger radial distances, but any increase of so small a magnitude may easily be accounted for by an increase in experimental error due to smaller pulse height measurements. A least squares straight line fit to the scatter diagrams is represented by Fig. 6.9, which shows the scatter in the radio lateral distribution at 5×10^{17} eV. No significant increase in the spread of normalised field strengths is evident at around 230 m compared with 100 m. This confirms that the scatter due to experimental error exceeds any scatter due to fluctuations.

CORRECTION FACTORS TO NORMALISE INCLINED SHOWERS TO EQUIVALENT VERTICAL SHOWERS

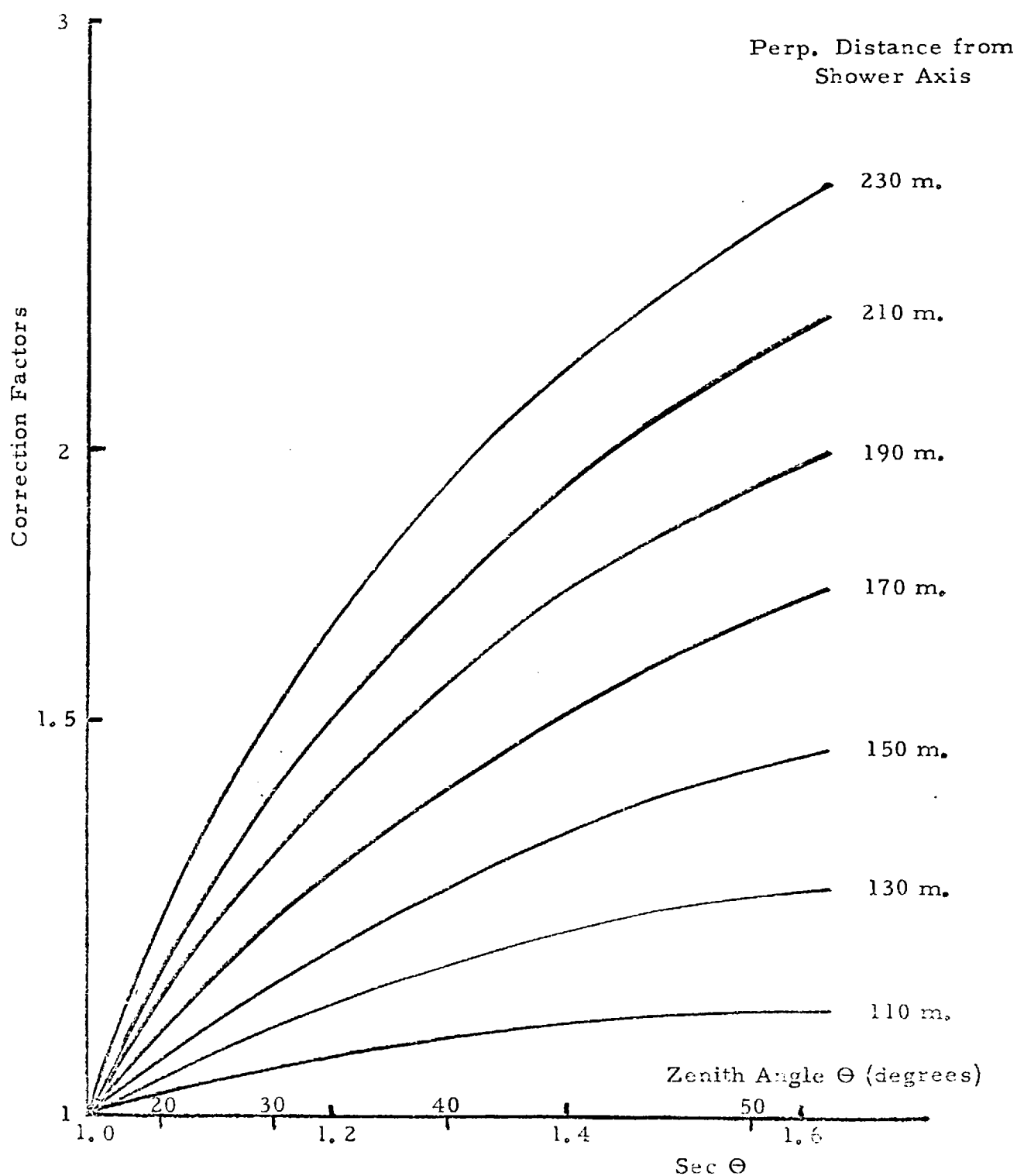


Fig. 6.7

FIELD STRENGTHS NORMALISED TO EQUIVALENT VERTICAL SHOWERS AS A FUNCTION OF ENERGY
AT 4 DISTANCES FROM SHOWER AXES (R)

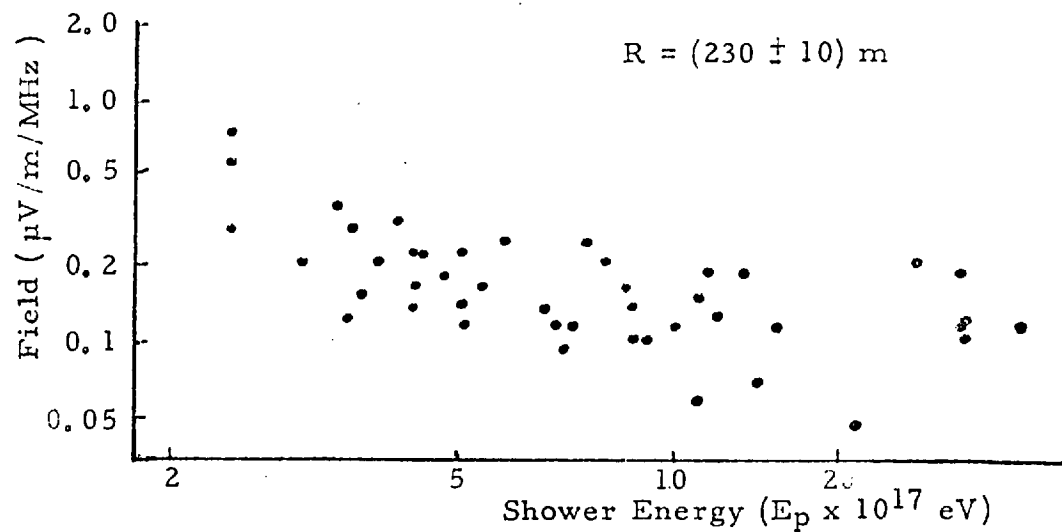
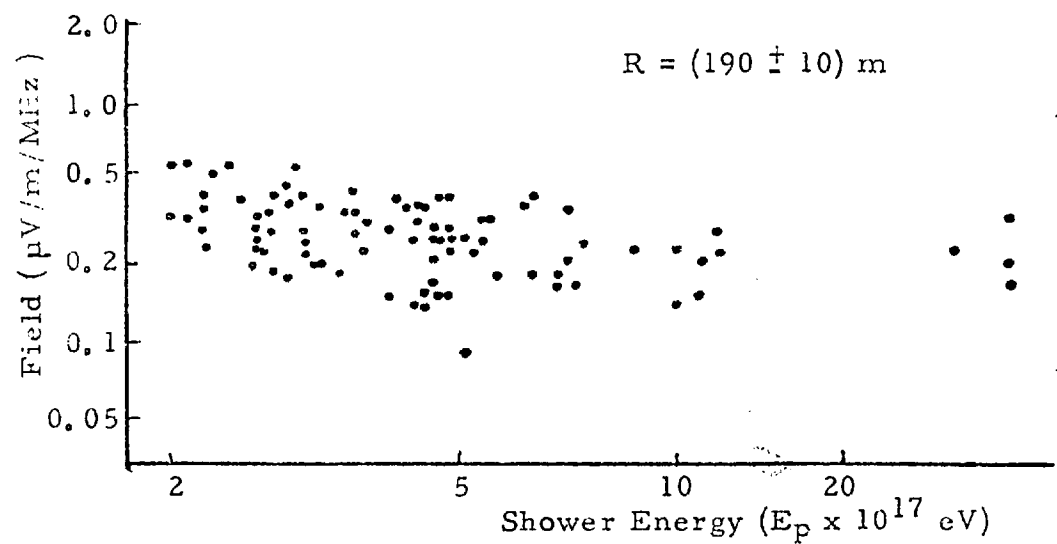
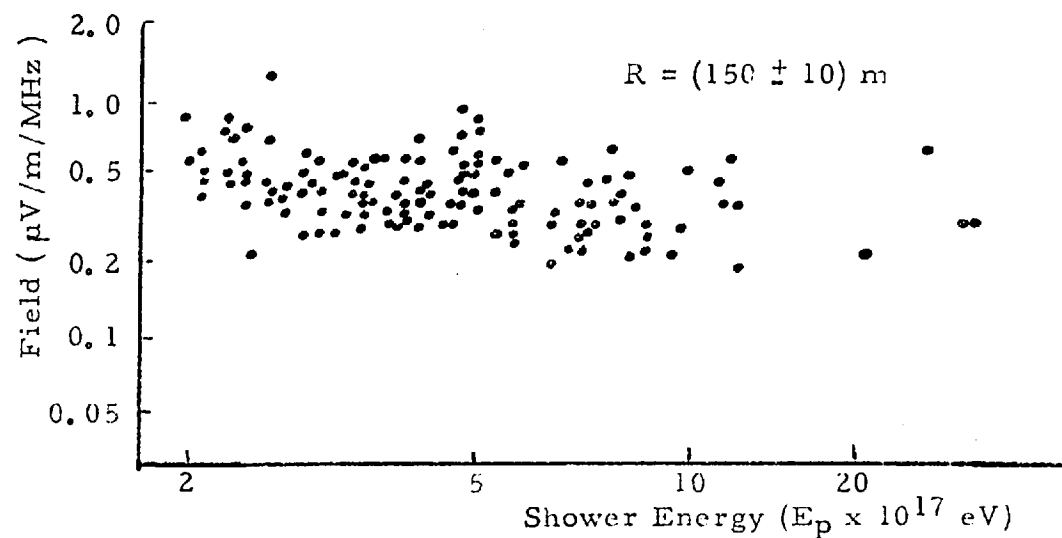
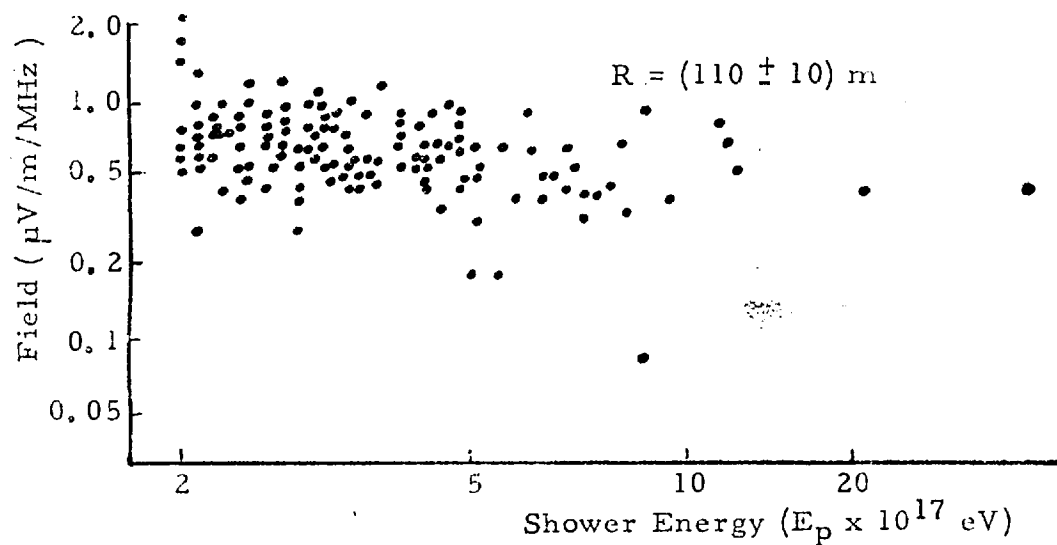


Fig. 6.8

THE SCATTER IN NORMALISED FIELDS AT A
SHOWER ENERGY OF 5×10^{17} eV AS A FUNCTION
OF R

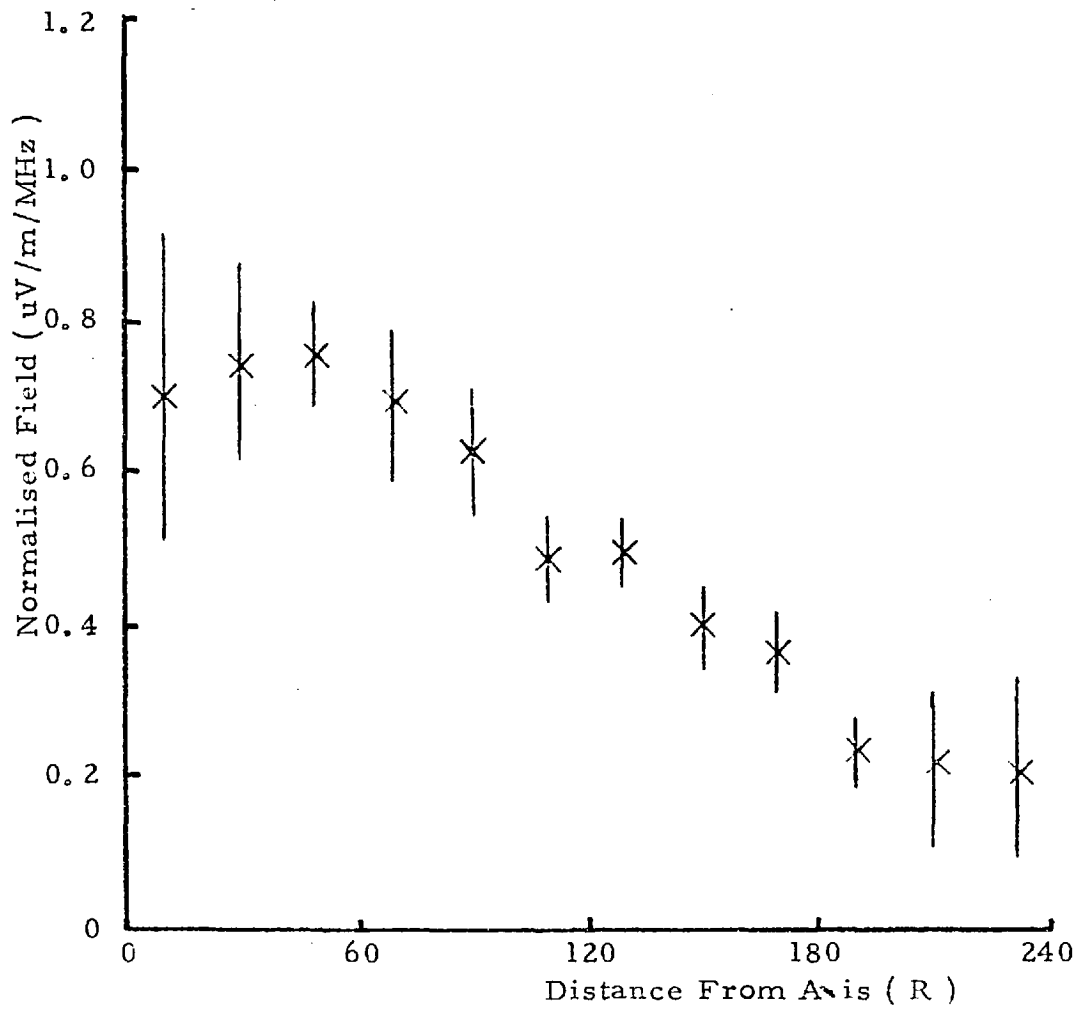


Fig. 6.9

SLOPES OF THE FIELD AGAINST SHOWER
ENERGY DISTRIBUTIONS AS A FUNCTION OF R

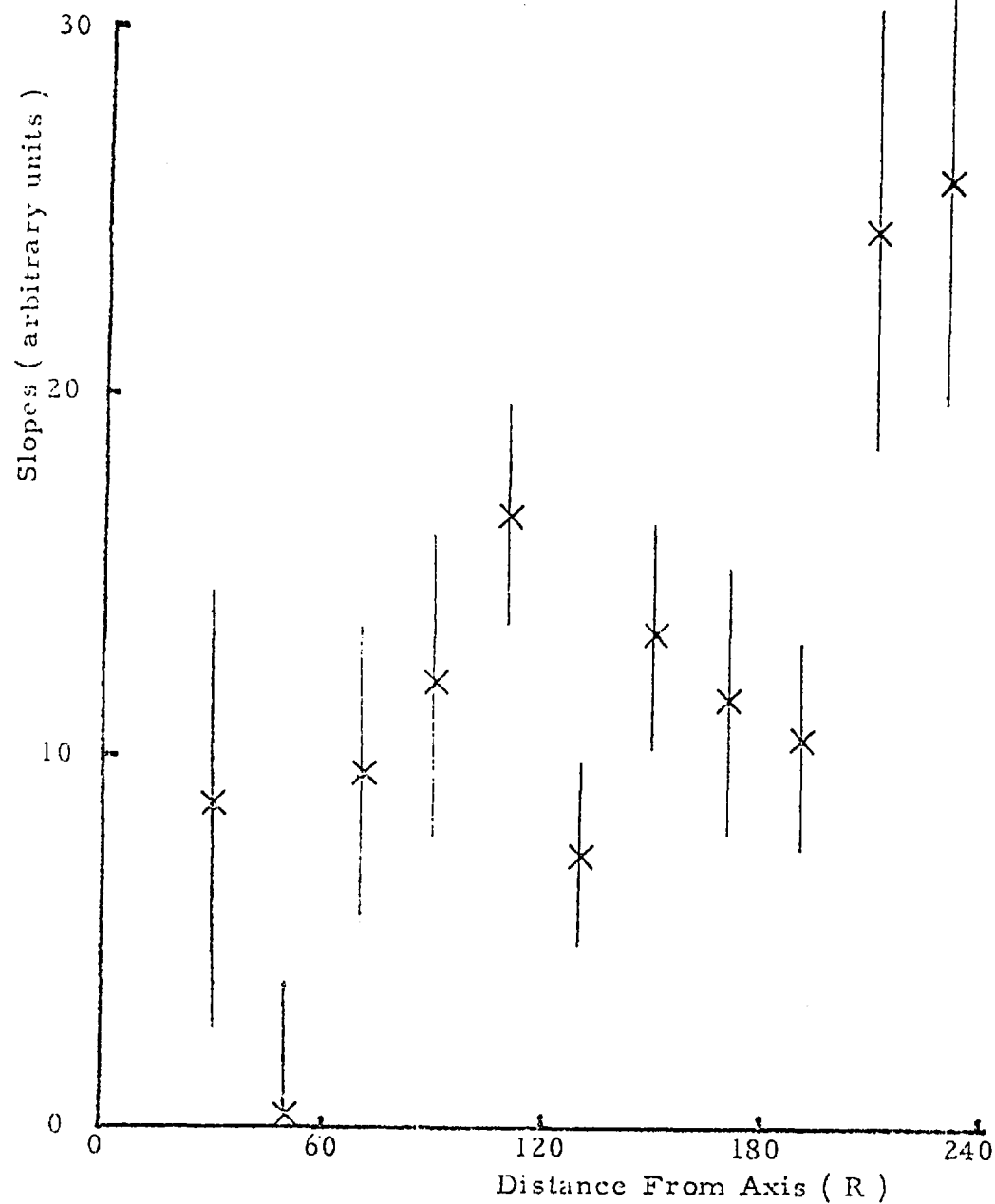


Fig. 6.10

MEAN VALUES OF THE FIELD AT 3 SHOWER
ENERGIES AS A FUNCTION OF R

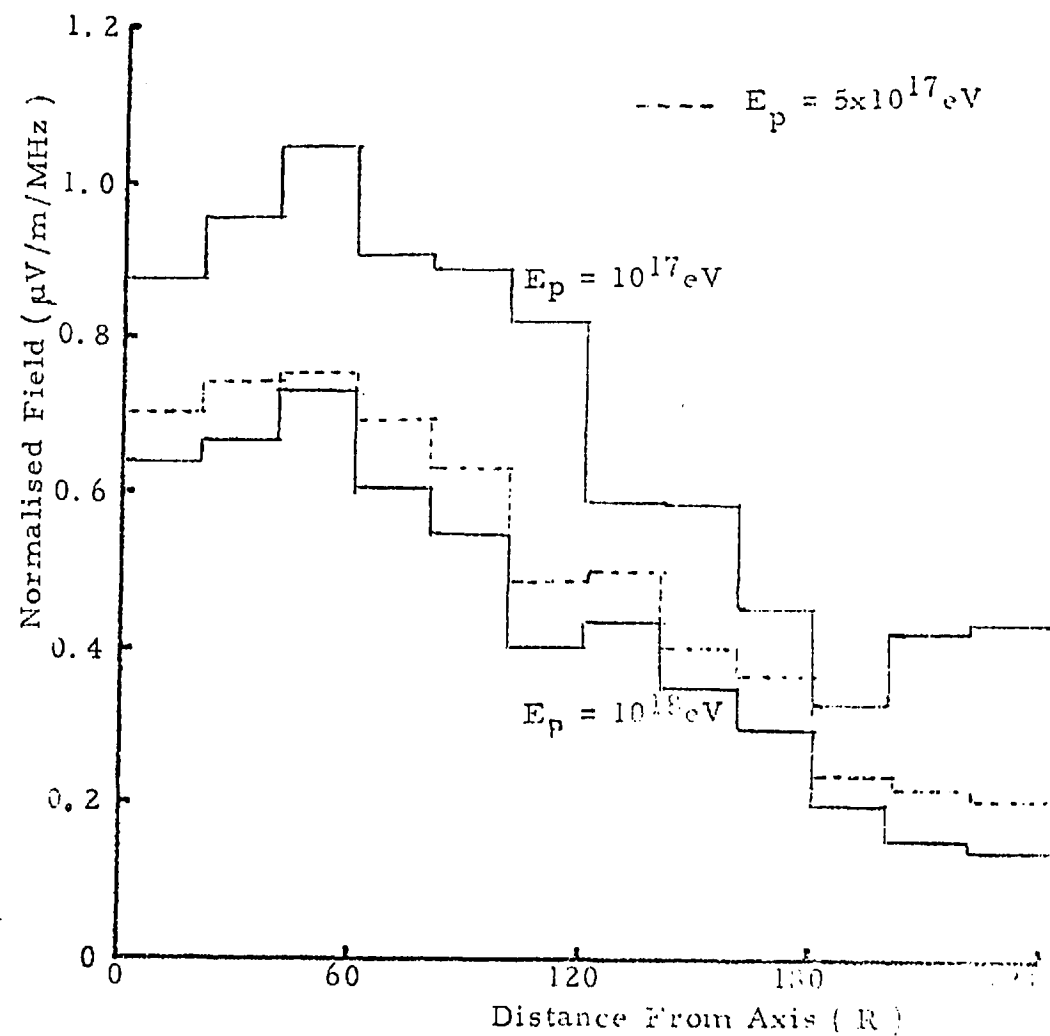


Fig. 6.11

The Variation of the Normalised Field Strength with E_p

If it were true that changes in the primary energy made no significant change in the height of shower maximum, then the normalised field strength would remain constant with primary energy. However, simulations of EAS development (Dixon et al., 1973) suggest that the height of maximum should decrease slightly with increasing primary energy by approximately 60 g cm^{-2} per decade on average. This would result in a decrease in the normalised field strength with increasing primary energy for the larger radial distances from the antenna. Such an effect can be seen in Fig. 6.8. The normalised field strength at 230 m from the axis can be seen to become less as the energy increases whereas that at 110 m is more nearly constant. Fig. 6.10 is an attempt to quantify the effect. It shows the slopes (obtained by least squares fits) of the field strengths versus primary energy. The finite value obtained for the slope at $\sim 100 \text{ m}$ is contrary to the expectation. Examination of Fig. 6.11 shows that low energy showers (10^{17} eV) give anomalously high normalised field strengths. The tendency for the measured normalised field strength to be overestimated at low energies may be because the values being measured are smaller. The lower the signal to noise ratio, the greater is the possibility of overestimating the field strength.

An alternative hypothesis is that the above effect arises because the values of E_p are underestimated for the smaller showers which are nearer to the threshold energy for particle array detection. In either case, it appears unwise to attempt a quantitative estimate of the shift in the shower maximum with energy.

The Anomalous Showers

A total of 10 showers (out of 1000) gave normalised field strengths which were larger than expected by a factor of ~ 5 . On the lateral distribution plots, they appear to form a completely

separate sub-set away from the main distribution of showers. Such an abnormally 'flat' distribution could not be obtained from the geomagnetic mechanism of radio emission even if one postulated that the height of maximum were near the top of the atmosphere.

The predicted polarisations for these showers are neither correlated with the geomagnetic mechanism nor with the charge excess mechanism of radiation, although the radio pulses appear to be very well defined on the film records. Reference to meteorological records show that six of these showers occurred within some hours of times when thundery weather conditions were reported within 50 miles of Haverah Park. One may speculate that very high local electric fields could be responsible for producing charge separation among the shower particles (Charman, 1968) in excess of that produced by geomagnetic charge separation. However, the polarisation of the measured field strengths does not show a clear correlation with this mechanism of radiation. A cross-check with the particle data measurements and analysis shows nothing unusual or inconsistent in either the timing measurements or the particle density measurements for these showers.

A Final Search for Fluctuations

The anomalous showers so far discussed are anomalous in giving too large a field strength. A search was also carried out for showers which would normally have given observable field strengths but did not do so. The absence of a radio pulse where one is expected could result from an extensive air shower whose height of maximum fluctuated very much closer to the ground. In this instance, an upper limit on the radio field strength could be determined from the noise level readings. From the Haverah Park particle data analysis, a list of all the showers with energies above 5×10^{17} eV which fell within the collecting area of the radio array was compiled. However, it was found that nearly all the

showers in this list corresponded to showers already analysed by radio. The remainder of the showers had no radio records either because the noise levels were abnormally high or because the radio receivers were inoperative at the time.

6.7 Conclusions

A large part of the work described has been devoted to establishing some of the important features of electron-photon cascades which are relevant in determining the radio field strengths from extensive air showers. The theoretical calculations are found to be compatible with experimental measurements after making allowances for some of the less certain properties of the nuclear active component in the air showers such as the angular spread of the pions.

The geomagnetic mechanism for radio emission should be the dominant process according to the calculations, and this is confirmed by the analysis of the polarisation data from the Haverah Park radio array. The very small scatter in the normalised field strengths at distances near 100 metres from the shower axis as a result of good signal to noise ratios also gives supporting evidence that the estimates of the primary energies from the Haverah Park particle data are consistent to within about 30%.

However, the presence of noise from external sources means that field strengths at larger distances from the shower axis cannot be measured with a sufficient degree of precision to achieve the main objective of solving the problem of primary mass. For this reason, it has been decided to cease carrying out radio measurements and the Haverah Park radio array was closed down in April 1975. It remains to be seen whether the other techniques, such as the atmospheric Cerenkov light detection or a subtle analysis of the particle data can give more definite answers on the mass of the primary particles.

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