

Blast Resistance of Laminated Glass Facades

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Declaration of Originality

The work presented in this thesis was produced by me during the course of my PhD studies. Where contributions by others are used, they have been appropriately referenced in the text and in the bibliography.

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Abstract

The aim of this thesis is to improve the understanding of the behaviour of Polyvinyl Butyral (PVB) laminated annealed glass façade panels subjected to blast loading.

A full scale blast test was performed. During this, deflection and strain data were collected employing digital image correlation techniques (DIC). Local reaction forces were measured using several pairs of strain gauges on the support. The full field deflection and strain data obtained were in line with those observed in historical tests. The strain gauge data available showed that the reaction forces varied along the edge, with higher values being reached at the quarter length gauge locations. The results from this test and from other historical experiments were used to calculate the reaction forces along the entire perimeter of the glass pane. The results showed that the forces reach an early peak before the glass failure, and then rise gradually approaching a plateau at high central deflections.

To explain the specific form of this force time history, the detailed behaviour of the laminated material after the glass skins failed was studied. Existing experimental data was employed to fit a material model to the PVB material. Two Prony series models with different hyperelastic springs and a model employing a full finite deformation viscoelastic law derivation were employed. It was found that the finite deformation viscoelastic model could represent the material's behaviour more accurately and fully include its rate dependency.

One of the PVB models was employed to study the delamination between the glass and the membrane. Delamination energies were found for different speeds of deformation, and these parameters were employed to study the delamination of samples presenting different crack arrangements. The results showed that these had only a limited influence on the behaviour of the composite.

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Nomenclature

α_o	Ogden spring function constant
α_i	Hoo Fatt's hyperelastic springs constants
ε	Strain
$\dot{\varepsilon}$	Strain rate
$\dot{\varepsilon}_0$	Reference strain rate for Johnson-Cook
ε_1	Strain on top face of window support steel plate
ε_2	Strain on bottom face of window support steel plate
ε_a	Axial force strain of window support steel plate
ε_b	Bending strain of window support steel plate
ε_p	Plastic strain for Johnson-Cook
η	Damping function
θ	Angle of force F from the in plane position
λ	Material's stretch
$\dot{\lambda}$	Rate of stretch
λ_i	Inelastic stretch
λ_e	Elastic stretch
μ_o	Ogden spring function constant
μ_i	Hoo Fatt's and hyperelastic springs constants
ρ_r	Material's density
σ	Stress

$\sigma_o(\varepsilon)$	Hyperelastic stress function dependant on strain
τ_i	Prony series' individual relaxation time constants
ψ	Hyperelastic spring's work function
$\dot{\psi}$	Rate of work
ψ_{eq}	Equilibrium spring's work function
ψ_{oe}	Overstress spring's work function
A	Johnson-Cook constant
B	Johnson-Cook constant
$\mathbf{B}$	Left Cauchy-Green strain tensor
$\mathbf{B}_i$	Inelastic Left Cauchy-Green strain tensor
$\mathbf{B}_e$	Elastic Left Cauchy-Green strain tensor
C	Johnson-Cook constant
$C_1, C_2, \dots$	Reduced polynomial's hyperelastic springs constants
$C_{\eta 1}, C_{\eta 2}, \dots$	Damper's function constants
D	Horizontal distance between window's reaction centroid and strain gauges pair
d	Window support steel plate's thickness
d_0	Lever arm between inclined PVB membrane and centre of gauge's pair.
E	Young's modulus
F	Membrane force applied by PVB membrane on supports
$\mathbf{F}$	Deformation tensor
F_a	Axial force on support plate
$\mathbf{F}_e$	Elastic portion of deformation tensor

$\mathbf{F}_i$	Inelastic portion of deformation tensor
F_{ij}	Deformation tensor's component
I	Second moment of area
$\mathbf{I}$	Identity matrix
I_{1B} , I_{2B}	First and second invariants of the total Left Cauchy-Green strain tensor
I_{1B_e} , I_{2B_e}	First and second invariants of the elastic portion of the Left Cauchy-Green strain tensor
g_∞	Prony series' long term stiffness constant
g_i	Prony series' individual stiffness constants
$g(t)$	Time dependent Prony series
H	Vertical distance between window's membrane centroid and strain gauges pair
k	Stiffness parameter for Cohesive Zone Model
$\mathbf{L}$	Material's velocity gradient
M	Moment on support plate
N	Number of series' elements used
n	Johnson-Cook constant
P	Blast pressure
p	Hydrostatic pressure
P_0	Atmospheric pressure
P_r	Reflected pressure
P_s	Incident pressure
P_p	Peak blast overpressure

R	Stand off distance
S	Peak delamination stress for Cohesive Zone Model
S_n, S_s, S_t	Delamination stresses in the normal and the two shear directions
S_n^o, S_s^o, S_t^o	Peak delamination stresses in the normal and the two shear directions
$\mathbf{T}$	Cauchy true stress tensor
t	Time
$\mathbf{T}_{oe}$	Overstress spring Cauchy true stress tensor
t^p	Duration of the ositive phase of blast pressures
$\mathbf{X}$	Material's elements' original position coordinates' vector
$\mathbf{x}$	Material's element new coordinates' vector
X_j	Material's elements' original position coordinates' vector component
x_i	Material's element new coordinates' vector component
y	Maximum distance of the material's fibers from section's centroid
Z	Blast scaled distance
W	Equivalent TNT charge weight

Acronyms

CFD	Computational fluid dynamics
CZM	Cohesive zone modelling
DIC	Digital image correlation
FEA	Finite element analysis
FPS	Frames per second
LVDT	Linear variable differential transformer
PVB	Polyvinyl butyral
SDOF	Single degree of freedom
TNT	Trinitrotoluene

1. Introduction

In today's society the security of buildings needs to be the object of careful considerations due to the risk of both terrorist attacks and accidents. Large structures whose use causes a significant amount of people to congregate in their interior or vicinity, such as airports and office buildings, represent ideal targets for malicious attacks. Therefore the protection of these buildings is of high importance to guarantee the safety of bystanders during wilful or accidental explosions.

These structures are generally the object of careful architectural design, which often includes the use of large glazed facades for both aesthetic and natural lighting reasons. The failure of such glazing elements, and especially the glass shards produced in those situations, represent one of the main sources of injuries during blasts [1]. When the glazing elements break, glass parts can be propelled both inside and outside the building, injuring and potentially killing people in both areas. Additionally, once the building envelope is pierced, blast pressure waves will be able to penetrate the internal space. These will cause further injuries and, even where this is not the case, will cause significant damage to the internal fittings and equipment. This will add considerably to the overall cost and time required to restore the structure to its original function. Therefore, the design and detailing of the glazing elements is a very important part of the strategy to guarantee the resilience of buildings.

Several materials have been used for this purpose. Typical building glazing is built with monolithic glass panes. Either annealed glass or toughened glass can be used for these. Both of these materials will fail catastrophically once their limiting stress is reached. In the case of annealed glass, this maximum is determined by the presence of flaws in its matrix. Once the stress capacity is exceeded at one of these flaws locations, the cracks propagate quickly throughout the pane. Generally, relatively large shards with sharp edges are produced, their pattern depending on the exact stress and defects distribution. In the case of a blast, these fragments can be propelled in the building space, causing significant injury. Toughened glass panes, instead, underwent a tempering process to improve their capacity. A layer of material in a compression state is created on all surfaces, increasing the overall tensile stress limit of the structure. However, once this is exceeded, the whole panel fragments suddenly in small,

dull pieces. Whilst these in a normal application would not present the same level of risk as sharp shards, during a blast they can be propelled at high velocity, becoming a significant hazard. Therefore, both types of glass are not ideal for use in application where blast resistance is a priority. Figure 1-1 shows the effect of a blast on a monolithic glass façade, highlighting the amount of damage caused by in these situations. The picture was provided by David Smith at Arup Resilience Security and Risk.



Figure 1-1: Effect of an explosion on office buildings in London's Docklands area.

Laminated glass is a composite material which has been used for applications where resisting high levels of load is a key requirement. The material is composed of layers of glass, annealed or toughened, interposed with polymer membranes to which they are bonded. Several types of polymers are used for the internal membrane, the choice depending on the details of the glazing's application. For blast resistance the most common material used is Polyvinyl Butyral (PVB). This is bonded to the glass using a heating process at high pressure carried out in an autoclave. Whilst the designer is free to choose the thickness of the glass, PVB is produced in minimum thicknesses of 0.38 mm, and supplied only as multiples of this measure. A thickness of 1.52 mm is generally recommended for blast resistant designs [2].

During a blast, once their capacity is reached, the glass panes fragment. The splinters will however remain attached to the layers of PVB, which will continue to transfer the loads to the glazing's supports. The membrane will also prevent the pressure waves entering the building

space. Whilst the glazing units will need to be substituted after the event, this still represent a significantly more positive outcome than those typical of other kinds of glazing.

1.1. Project aims

The main challenge for designers is to decide on appropriate approaches to design all the elements of the glazing which will affect its performance. These include, between others, the thickness and number of the constituent layers and the restraint mechanism. The connection of the glazing panel to the supporting structure is especially important in this respect. In this project, some of these aspects have been considered, with a special attention to the reaction forces produced in a blast test. An understanding of their absolute level and of their evolution during a blast is essential to decide on specific laminate and connection designs.

The results of a new blast test have been examined to assess whether the reactions change along a window's perimeter. The test was performed on a standard size window using a realistic explosive quantity and stand-off distance. Optical measuring techniques and more traditional strain gauges were employed to collect the necessary data. The results from the blast test, together with data from blast tests performed by others [3], have been used to calculate reaction forces along the full length of the edges. The optically collected strain data were related to the gauge's information to produce a cracked laminate overall material property law, which was employed to estimate the reactions.

Following this, the characteristic of the cracked laminate material was studied to explain the observed features of the reaction forces. Firstly, the PVB material was characterised. Existing experimental data were used to fit material properties laws covering the high strain rate behaviour relevant to the situations of interest. Viscoelasticity was employed to achieve this. Several formulations were utilised, including Prony series and a full finite deformation non-linear viscoelastic model. The results of all the approaches were finally be compared.

Finally, the delamination of the glass fragments from the PVB membrane was considered. The PVB material properties previously derived were used in finite element analysis (FEA) models to determine both the physical properties of the process and the effect of different, realistic glass configurations on its macroscopic characteristics, specifically on the material's overall stresses. The aim of this work was to not only identify some likely levels of blast loading reaction forces, but also to clarify their time history behaviour and the likely effect of different glass fragmentation patterns.

2. Literature Review

Several aspects of the blast loading of windows are of interest for this research. These cover both the blast loading and the properties of the composite, in term of its constitutive materials, their interaction and the overall behaviour. Knowledge of these topics is necessary to then attempt to interpret and predict the performance of the system in a realistic situation.

2.1. Blast Pressure Theory

The pressures originating from explosions have been studied by several authors in the past, attempting to provide methods to describe both the general behaviour of the blast waves and how they interact with structures in their path. Some analytical solutions have been proposed for a selection of these issues, whilst empirical or numerical methods are commonly employed to describe the more complex cases. These will be examined below.

2.1.1. Blast wave characteristic and interactions with structures

The pressure's time history is initially characterised by a sharp rise of its level at the arrival of the shock wave. This is followed by an exponential decrease and eventually a negative phase, where suction is imposed on the supporting structure. Its general shape is shown in Figure 2-1. The area under the positive portion of the curve is the impulse which, together with the maximum pressure, represents a useful measure of the total energies imposed on structures.

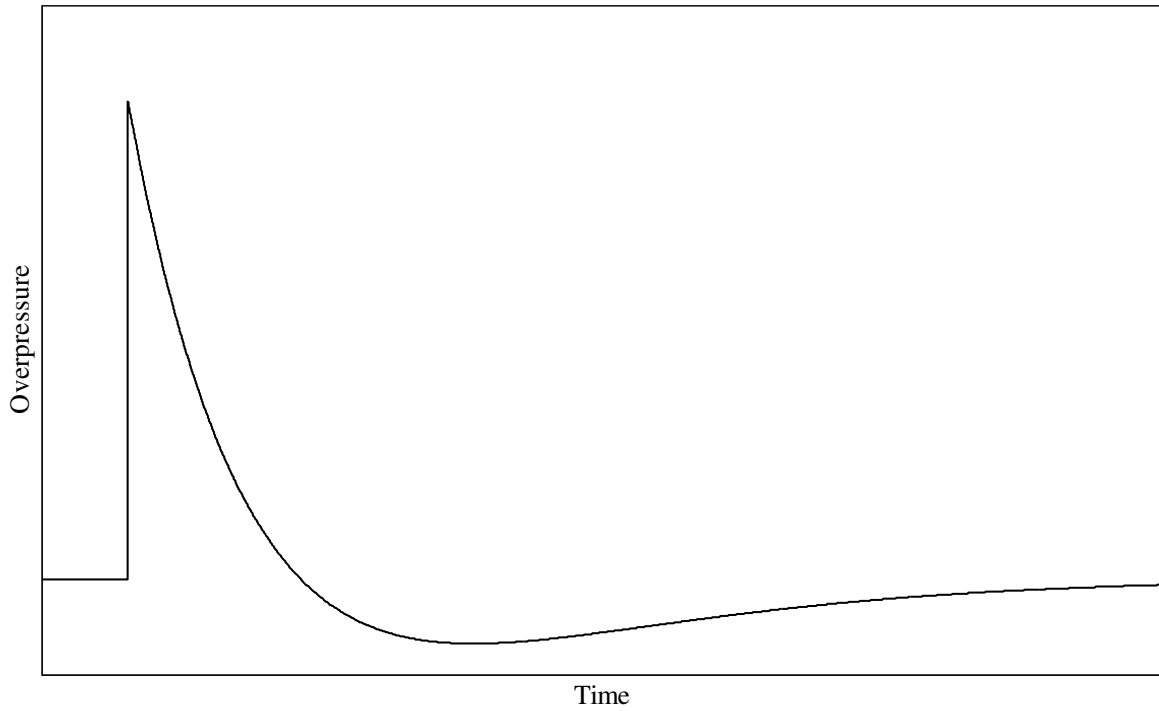


Figure 2-1: Typical shape of blast pressure wave time history.

The general time history of pressures can be described by the Friedlander equation [4]:

$$P = P_s e^{-\frac{t}{t^p}} \left(1 - \frac{t}{t^p} \right) \quad \text{Eq. 2-1}$$

Where P is the blast overpressure at time t , P_s is the peak overpressure and t^p is the positive phase duration. The equation is able to reproduce accurately the time history of the majority of the blast waves it is fitted to. The different pressures produced by distinct charges can be compared using scaling laws. One of these is described by Hopkinson [5] and Cranz et al. [6], and is given by:

$$Z = R / W^{1/3} \quad \text{Eq. 2-2}$$

Where Z is a scaled distance, R is the stand off distance of interest and W is the equivalent TNT charge mass. Blasts with the same scaled distance should in theory produce comparable effects. Should different types of explosives be used, their mass needs to be converted to an equivalent TNT one. This can be done using similitude parameters, which are also used to produce more complex predictions of blast effects. For example, Geers and Hunter [7]

present several similitude constants for different types of explosives to be used for blast wave predictions.

The interaction of blast waves and structures is also the object of several studies. The phenomenon of wave reflection is of special importance, as it can amplify substantially the loads imposed on targets when compared to the free field overpressures. Kingery and Coulter [8] studied the reflection of blast waves on finite targets. They performed 15 tests using eight targets at different distances to obtain data for different overpressure levels. After each test the targets were rotated by a set number of degrees to collect data about reflections at different angles. The collected data highlight the influence of the angle of incidence on the level of reflected pressures. Additionally, for 0° cases, the classic equation derived by Rankine and Hugoniot can be employed [9]:

$$P_r = 2P_s \left(\frac{7P_0 + 4P_s}{7P_0 + P_s} \right) \quad \text{Eq. 2-3}$$

where P_0 is the atmospheric pressure, P_s is the incident pressure and P_r is the reflected pressure. It should be noted that this is only valid for the limiting case of an infinite target, and hence cannot be used to convert the entirety of the blast pressure time history for a real structure. The initial peak though can be estimated, as size effects will be less significant at the early stages of loading.

2.1.2. Loading estimate methods

Whilst the methods presented above can provide guidance on the general behaviour of the blast waves, it is challenging to predict blast loading on specific target configurations analytically. Several empirical and FEA methods have been proposed to obviate this limit. Firstly, empirical solutions based on results from blast tests can be used [9]. These have been implemented in CONWEP [10], a software tool utilised to predict realistic loading considering explosive characteristics and target geometries. The issue with this method is that, where the analysed situation differs significantly from the tests used to derive the implemented functions, the results produced might not be realistic.

Alternatively, FEA software packages which can simulate the whole blast event exist. Air3D [11] is a relatively easy to use tool which implements computational fluid dynamics (CFD)

principles to simulate both the explosion event and the propagation of the pressure waves in potentially complex geometries of targets. The limitation of this tool is that fluid structures interactions are not included, and hence all the obstacles are treated as perfectly rigid. The only possibility offered consists of including a failure pressure for specific panels, which will then be suddenly removed if the results reach this limiting value. Other software, such as ANSYS AUTODYN, can couple the CFD analysis with a FEA model of the targets, including therefore the effects of structural movements on the pressure wave estimation. However, the use of such complex tools requires significant user skill and computer time, precluding their utilisation in at least some design situations.

2.2. Behaviour of Laminated Glass

Laminated glass is a composite material; therefore to explain its behaviour fully the material properties and interaction of the various constituent parts need to be considered carefully.

Whilst no Eurocode building standard is currently available to guide its design, the ASTM E1300 standard [12] can be used to design the glazing elements to resist wind and other horizontal forces. Blast loading can be included in the analysis using codes such as ASTM F2248 [13], which provide guidance on the derivation of equivalent 3 s loads to represent the blast pressures. These codes can therefore be used to achieve an approximate design for the panels. However, as the capacity design principles are based on a definition of failure coinciding with the cracking of the glass, the engineering solutions reached using them would likely be significantly oversized. More complex design methods, based on either single degree of freedom (SDOF) approaches or FEA analysis require the estimate of more precise parameters for the constituent materials and their interaction. The literature concerning these will be considered in turn.

2.2.1. PVB material model

The PVB material presents a non-linear, time dependant behaviour when subject to impulsive loads of the magnitudes of interest. Several studies have been performed in the past few years attempting to characterise it at a range of different strain rates. Some research concentrated on achieving estimates of the material modulus for use in design. Vallabhan et al. performed

shear tests on laminated glass samples and obtained estimated of the modulus at different strain levels [14]. Others, like Du Bois et al and Timmel et al, attempted to apply a hyperelastic law to simulate the material's characteristics. They employed experimental results to fit a hyperelastic model of the PVB material. Several different spring functions were considered, using a Mooney Rivlin representation for their final impact model [15], [16].

Several authors then attempted to fit viscoelasticity laws to model the time dependant part of the deformations. Xu and Li have used elastic models which were used for windscreen impact simulations [17]. Subsequently, they performed tests at strain rates of up to 4500 s^{-1} using a split Hopkinson bar apparatus to obtain stress strain curves. They then fitted a viscoelastic law to this, assuming a Mooney Rivlin hyperelastic spring to account for the non-linearity [18].

Iwasaki and Sato identified a shift in the PVB behaviour at high strain rates of 113 s^{-1} [19] and they included a viscoelastic material model fit for the lower strain rates cases (below 1 s^{-1}) [20]. Liu et al. also performed several tests at different strain rates, from $4 \times 10^{-4} \text{ s}^{-1}$ to 118 s^{-1} both in tension and in compression. Instead of trying to obtain a single material property law covering the full range of their tests, they the fitted the data with several viscoelastic material laws to cover the different rates and stress regimes [21].

In general, except for the work of Liu et al., these attempts did not include a model for the high strain rate behaviour of the material. This was generally included in research concerned with modelling the full composite using linear elastic – linear plastic laws with a strain rate dependency. For example, both Larcher [22] and Zhang et al. [23] applied this approach in their work. However, none of the proposed models can cover all ranges of strain rates. This could be important when considering blast loads, as it is likely that different areas of PVB will be deformed at different speeds.

2.2.2. Glass behaviour

The capacity of annealed glass is one of the parameters necessary to model the laminate's overall response. Griffith [24] suggested that material flaws have a strong influence on failures in brittle materials. These are present in significant quantities in annealed glass, their presence caused both by the production methods and by the subsequent use of the panels. A large study with a high number of glass samples was performed to estimate breaking strengths. A ring on ring test was used for these to avoid asymmetries in the stress

distribution in the samples [25]. This showed that samples failed between 30 and 120 MPa, with a 95% probability that the capacity would be greater than 45 MPa. This was then extrapolated to account for rate effects, and a failure stress of 80 MPa was calculated for impulsive loadings [3]. Recently, some authors started including more complex, strain rate dependent failure models in FEA models of laminated glass [26]. The Johnson Holmquist Ceramic constitutive model [27] was implemented in the software and used for this.

2.2.3. PVB – glass delamination

The failure of the bond between PVB and glass is of great importance to explain the overall behaviour of laminated panes. After the glass layers reach their capacity and fracture, the PVB membrane will tend to expand. The higher strains will occur at the crack locations, creating significant stress concentrations at the layer's connections in the vicinity of the edges of the glass fragments. Delamination is likely to take place at these positions and it is possible that this could absorb a significant part of the total energy imposed in the system.

Some studies have been performed to analyse this behaviour. Rahul-Kumar et al. [28] performed compressive shear tests at different rates to obtain estimates of the cohesive strength of the bond. The results show the significance of strain rate for the delamination characteristics, with different peak stresses being reached during the different tests. Muralidhar et al. [29] performed tensile tests of laminated glass specimens with a central pre crack. The force-displacement curves obtained reached a delamination plateau stress, which was maintained for much of the deformation. The author then used the results to determine delamination energies. The applied external work was calculated, and the strain energy of the PVB was deducted from this. It was then assumed that the remaining portion of the energy was absorbed by the failure progression. A hyperelastic material law was used for the PVB, ignoring strain rate effects. Butchart and Overend [30] used a similar approach. In her study a very similar experimental set up was used and two different delamination speeds considered, albeit both were low compared to those observed during blasts. A linear viscoelastic model was used for the membrane. The energy estimates were then produced with a closed form analytical method and compared. The results indicated rate dependency in the absorbed energies. However further investigations were deemed necessary to establish whether this was a real effect or an artefact of uncertainties in the material models used.

Iwasaki et al. [20] also employed a similar test set up to measure delamination energies. In this occasion, only one layer of glass was laminated to the PVB. Additionally, an initial area of PVB was already detached before the commencement of the loading phase. Higher speeds were achieved, up to 2.5 m/s. As mentioned above, a viscoelastic material law was used for the PVB. Whilst the method used for the calculation is not discussed in depth, energies were extracted, showing that a similar amount was absorbed at the speeds considered.

Hooper [3] also performed the same type of tests, applying though a greater range of speeds, up to 10 m/s. Additionally, cases with more complex crack patterns were considered to represent more closely the behaviour of a cracked laminate panel.

2.3. Overall response of laminated glass

Several studies, both experimental and analytical, have been conducted on the overall response of laminated glass to different loading situations, ranging from relatively slowly varying loads to impulsive impact and blast forces.

2.3.1. Low rate loading

The low rate behaviour of the material is of interest for the design of both glazing subject to regular wind loads and structural elements, such as beam and columns.

Norville et al. [31] and Behr et al. [32] performed studies on the strength of laminated glass, comparing it with annealed glass of similar thicknesses. They analysed static and transient wind loading, reaching the conclusion that the interlayer does seem to have a role in transferring shear forces, creating a composite action between the two glass plates, rather than simply acting as a spacer element.

Other quasi static studies were conducted in the context of the use of laminated glass for structural elements, such as beams or columns. As an example, Galuppi and Royer-Carafagni [33] concentrated on laminated glass beams. They formulated an analytical, time dependent solution based on linear glass properties and a linear viscoelastic PVB material law. They then compared the deflection results with a fully linear solution and with numerical FEA

results. The authors' conclusion was that the linear solution would significantly underestimate the deflections under highly impulsive loads, such as blast wave pressures.

2.3.2. Impact loading

Several studies were performed concerning impact loading of the material. This is an important situation for the use of this composite system in the automotive and aeronautical industries. Also, whilst some of the load characteristics are different from what seen during blasts, there are several similarities, such as the magnitude of the rate of loading and the expectation that the glass plies will fracture. Several impact tests have been performed on laboratory samples. Bennison et al. [34] conducted impact tests on circular plates of laminated glass. The data were compared with FEA simulations. The authors applied a linear elastic – viscoelastic material law to represent the PVB membrane and did not include a glass failure mechanism. They modelled instead the glass as pre fractured by reproducing the observed laboratory crack distribution in the model's initial geometry. The results were used to study the stress distribution through the glass thickness. The work was developed in a following paper by the same group [35], where force displacement graphs were produced and delamination was included in the finite element analysis. Pyttel et al. [36] and Du Bois et al. [15], [16] also performed impact tests and modelling. In this case the fracture of the glass was included through the use of an erosion mechanism, where the stiffness of glass elements was reduced to 0 when a limiting stress was reached. To achieve an improved agreement with empirical results, Pyttel et al. introduced an additional total strain energy factor was included to determine the sudden onset of cracking.

2.3.3. Blast loading studies

The blast loading case has been studied extensively, both with numerical and experimental studies. Some of these results have then been used to produce simplified design approaches which can practically be used in real projects.

Experimental studies

Several experimental programmes have been carried out to study the blast capacity of this material. Hooper et al. [37] and Stephens [38] performed blast tests measuring the deflections of laminated glass panes. Hooper's data is especially interesting as he used digital image correlation (DIC) techniques to obtain the full field 3D deflection and strain data, which will be used for further analysis. The same measurement technique was also employed by Kumar and Shukla [39]. This study used shock tube experiments on smaller samples comparing the performance of several different types of glazing materials. Wired glass, sandwich glass panels without interlayer membrane and laminated glass with the addition of protective films on both exterior surfaces were tested. DIC was employed to obtain full field deformation information and the tested samples were also analysed to compare their damage levels. The author concludes that laminated glass provided the best option to limit the effect of blast pressures, outperforming the other options on all the measured quantities.

Analytical solutions

Attempts have been made to analyse the observed responses using analytical solutions. Wei and Dharani [40] took such an approach and carried out studies using von Karman large deflection theories. They used this theory to estimate the deflection of non cracked glass panes and the probability of their failure. In another paper [41] the same authors combined these results with Griffith fracture energy balance theory to estimate the likely locations and density of cracks occurring at failure. Whilst this approach produced some realistic results for the pre-crack phase of deformation, the complexity of the post cracking behaviour caused this approach to not be as conclusive.

FEA studies

To overcome the issues with analytical solutions, FEA approaches were employed by several authors. Wei and Dharani [40] used FEA models to validate their analytical solutions. However their models were generally limited to the pre-crack phase. A possible technique to model the post glass cracking phase of the deformation consists in using a smeared material property for the composite, without considering its constitutive elements separately. Such an

approach was used by Hooper [3]. In his work he produced two separate models for the pre crack and post crack phases. He used both linear elastic and viscoelastic models for the PVB formulations before the glass failure. When a limiting stress was reached in the glass pane, the analysis was stopped and restarted with post-cracking material properties. The stiffness of the glass was reduced to 0 throughout and a plastic material model derived from laboratory experiments was applied to the PVB. The model deflection predictions are similar to the experimental data at early stages of the analysis, however the deflected shape and peak deflections diverge from the available data at later times in the analysis.

Recently more works have been produced using a single model attempting to accurately represent all phases of the system's behaviour. These models use a variety of 2D and 3D element formulations, including differing detail levels on material's properties and failure conditions. Larcher [22] performed a FEA analysis of this type. He used a layered model, employing an erosion technique to model failure. The author then applied a linear elastic PVB model for low strains. A limit was set, transforming the behaviour of this material to perfectly plastic beyond this point. The results of the model were then compared to experimental information and to the results of a smeared property model. The data showed that the authors' formulation matched experimental deflection to a greater extent than simpler models. Zhang et al. [23] also developed FEA models of windows subject to blast loading. In this case the aim was to produce pressure-impulse curves for use in design. In the study, dynamic properties were used for both the glass and PVB failure. Delamination was also included, albeit the authors lamented the lack of published dynamic properties for this element.

FEA of the system was also used to assess the individual contributions of its elements to energy absorption. Hidallana-Gamage et al. [42] produced such a study. In this work the authors used dynamic material models for the main components, including also the structural sealant. Failure parameters were also included for all individual components, however glass – PVB delamination was not included. The deflection, stresses time history and the energy absorption of each element were then compared, providing guidance on their relative importance. Amadio and Bedon [43] also performed numerical analyses of the blast events, similarly to the aforementioned authors. In this case though the focus was to verify whether viscoelastic absorption devices used as glazing support units could prevent some of the damage to the panels. The results showed that these could absorb significantly more energy than the standard options, improving the outcomes of the pressure loading.

SDOF approaches and design aids

The studies highlighted above attempted to consider several aspects of the composite's behaviour in detail. The results produced, though they agreed with experimental observations to a varying degree, seemed to generally be able to consider their key characteristics and could therefore be used for practical design work. However, their computational and expertise requirements are likely to be high and ill suited to the constraints of design offices. Historically, single degree of freedom (SDOF) simplifications have been used as a relatively accurate tool to produce designs within acceptable time scales. In this approach, the complex system of the laminated glass and its connections to the supports is approximated with an undamped spring model. Failure is considered to occur when a limiting deflection is reached. This limit is generally estimated from the results of blast trials performed in the past. The application of this approximation necessitates the introduction of mass and stiffness constants to produce estimates of the effective mass, stiffness and external force in the system. The values for these parameters have generally been calculated assuming large deflection bending to represent the deformation of the glass plate.

To calculate the support reactions, a key element of the design, a resistance function is assumed. This represents the internal force resisting the deflection. If the system behaved in a static manner, the function would be equal to the externally applied force which would cause the same magnitude of deflection [44]. Besides this limiting case, for simple system, such as a beam undergoing small amplitude vibrations, the resistance can also be calculated directly from structural principles. However, in the case of a blast loaded window's deflections, especially after cracking, obtaining such an estimate is not as straightforward. At present, a shape for the force function is assumed and calculations are carried out with reference to previous tests to verify its magnitude [1]. Factors can then be applied to this resistance and the externally applied force to produce estimates of the reactions on each of the four window sides [44].

This approach has been used by several authors in the past. Smith [1] suggests a linear resistance function's shape after the glass cracks. He then states that the curve presented has been used to produce estimates of the window capacity. Fischer and Häring [45] also conducted analysis of blast experimental data attempting to fit linear resistance functions. They then compared the deflections obtained from single degree of freedom models with

those obtained during the experiments. They conclude that a tri-linear response function is reasonable at representing the deflections observed during the tests.

The SDOF approach was used by Stewart and Netherton [46], [47]. They produced estimates of the risk probability of damage from explosions, including both parameters such as likelihood of blasts taking place in the target's vicinity and, using the SDOF approximation, the structural capacity of the glazing. Morrison [48] concentrated his work on improving the parameters estimation for SDOF methods. He studied the properties of the constitutive elements of the composite to produce closer approximations for use in industry.

The results from tests, these SDOF calculations and FEA models can be also employed to produce pressure-impulse diagrams with iso-damage curves, as done by Zhang et al. [23]. These can then be used to produce quick designs for specific windows. Their main limitation though is that the validity of the results will be restricted to the specific sizes considered when the diagrams were generated.

2.4. Restraint mechanism

One of the key aspects of window blast design is ensuring that the glass pane will be contained in the frame during the event. The restraint mechanisms employed for laminated glazing are important to guarantee this and need to be able to resist the reaction forces produced.

Structural silicone

One of the most common systems used for this purpose, especially with laminated glass, is structural silicone. The behaviour of this component has been the object of several studies. Ramesh et al. [49] studied the effect of aging and specimen size on the elastic properties of the silicone material, concluding that, whilst aging has a significant impact on the Young modulus of the material, the length of the specimen did not seem to. Meunier et al. [50] carried out tests on silicone material to characterise its properties. Tensile, pure shear, compression, plane strain compression and bulge tests were carried out, using digital image correlation techniques to obtain the full strain distribution across the samples. The authors then used the data to fit five hyperelastic material models, namely the Neo-Hookean,

Mooney, Gent, Haines and Wilson and Ogden models. These models were used to simulate a tensile experiment on a silicone membrane with several holes. They then compared the FEA results with experimental results for the membrane, finding that the models seemed to represent the material satisfactorily. Their material laws are therefore relevant for attempting to model the whole window system subject to blast.

Hooper [3] performed tensile experiments aimed at assessing the sealant's capacity under realistic blast loading. Structural silicone beads were applied to glass and aluminium substrates. The tests consisted of pulling the silicone at different angles using a high speed Instron servo hydraulic machine. This was thought to represent the load applied during a blast, as the window will deflect during the event and the force on the silicone will be applied at a varying angle. Hooper managed to use his results to derive a limiting capacity for the silicone being examined. This was then applied together with PVB strength data to obtain estimates of the maximum useful width of silicone to be used to fix blast resistant windows.

Glazing tape

Glazing tapes are increasingly being considered a possible substitute for silicone. New 3M [51] and Tremco [52] products are being marketed for blast resistant uses and are therefore attracting a rising amount of research to determine their properties. Most of the studies concentrates on low strain rates events, covering, for example, the effect of wind loading. Townsend et al. [53] produced a study of cumulative damage due to wind loading. The authors found that glazing tape designed according to industry standard for the peak 3 s gust will accumulate damage, however it will not generally be at risk of failure for the duration of its design life. The same authors [54], [55] also carried out work characterising the material properties of 3M VHB tape. They subjected the tapes to ramp to fail and creep tests. The tests were carried out at different temperatures and strain rates, the speed of the machine ranging from 5 mm/min to 500 mm/min. They produced a set of storage and loss moduli for the material and an estimate of the glass transition temperature of the material. The study though did not cover the higher strain rates commonly observed during blast events.

A 3M study conducted at the university of Waterloo [56] analysed VHB tape samples tested at higher strain rates. During the study both a high speed Instron and a Split Hopkinson bar

were used to produce the data, reaching rates of approximately 7000 s^{-1} . The produced diagrams could be used to evaluate the material properties at high strain rates.

2.5. Conclusion

The research described above shows that a significant amount of research has been performed in the past concerning the blast phenomena and the laminated materials of interest to this study.

The behaviour of the blast pressure waves has been studied with numerous field tests and analytical studies from an early age. Therefore several methods are available to predict the pressure intensities and the imposed loading on structures in their path. Whilst some of these are of complex application, relatively quick estimates can be obtained with empirical solutions and CFD applications.

The constituent materials of the composite were also considered by several authors. However, whilst simplified rate dependant solutions do exist to model their response to loading at rates above quasi static, there are few studies considering this aspect thoroughly. This is an important factor, especially for the PVB membrane, as it is likely that its behaviour will be complex, varying in the area of the glazing panels. Modelling it will therefore require formulations of wide applicability. There also seems to be a lack of delamination parameters derived for higher speed deformations, which might decrease the accuracy of the more complex simulations proposed by authors.

The overall blast event was the object of both practical tests and simulations. The experiments performed and reported provide insight on the deflections and strains which can be observed, together with information on likely failure mechanisms and limits. The results could though be analysed further and combined with the available and new knowledge relating to the material properties to provide more detailed information on the forces developed during the loading.

Experimental results are available to achieve this and it is proposed that this study will develop these points, using both blast test data and laboratory information to this effect. The material properties at high strain rates will be studied further and the results employed to interpret the results from realistic pressure waves loading.

3. Full Scale Blast Test

3.1. Introduction

The performance of laminated glazing subject to blast loads is affected by many aspects of the material behaviour and overall system design. To achieve a safe and efficient solution all the factors need to be considered to optimise the final product. However the influence of the properties of individual elements, such as material properties and restraint mechanisms, are often difficult to differentiate between and quantify. This can lead to significant uncertainties on how a specific design will perform during an explosion. The most conclusive method to assess the overall behaviour of a system is to carry out a full scale blast test on a complete assembly. Within this trial, much data on the performance of specific components can be obtained using relevant instrumentation. The information can then be used to glean more understanding on the general effects of blasts and the influence of specific component factors on the overall performance. In this chapter one such blast test, performed in May 2013, is described. Whilst several blast tests had been previously carried out by others, such as Hooper et al. [37], it was realised that additional information could be gathered with a test aimed at collecting extra data on the reaction forces. The results of this test will be described in this chapter, together with a novel method for analysing the data.

3.2. Method

The data collection and analysis method included both on site experimental techniques and post processing analytical techniques. All these were necessary to obtain and interpret the results of the experiment.

3.2.1. Experimental set up and data collection

The data collection methods used had to take into consideration both the blast set up and the exact data necessary to achieve the experimental objectives. The test geometry and the different data collection methods used will be described below.

Sample geometry and test pad disposition

The blast test was performed in a dedicated testing facility run by DNV GL Group at the RAF Spadeadam facility. A 100 kg TNT equivalent charge was used, the sample being located at a 23 m stand-off distance. The charge weight was dictated by the requirements of the other tests being performed at the same time. The stand-off distance was adjusted to maximise the displacement of the window whilst preventing catastrophic failure. To achieve this a SDOF model was employed to estimate the behaviour of the system before the test took place.

The test panel consisted of an approximately 1.5 x 1.2 m laminated glass window held in a steel frame. The laminate make up was two 3 mm thick annealed glass outer layers with a 1.52 mm PVB interlayer, producing a total thickness of 7.52 mm. It was sourced and installed in its frame by Structural Glazing Limited. A double sided silicone connection to the frame was provided as the restraint mechanism. Figure 3-1 shows a sketch of the support with the relevant dimensions. The test assembly was located in a steel cubicle provided by the test facility. The glazing was mounted into this with the longer sides horizontal. The window opening was in the centre of the front wall, 0.6 m above the ground. Figure 3-2 shows a sketch of the front of the cubicle with the relevant dimensions and Figure 3-3 shows the steel structure composing the front walls. The assembly ensured that the support conditions would be symmetric on the two vertical sides of the glazing, avoiding differences in the response at the two locations. The response on the horizontal sides would differ due to the effects of ground and the free edge.

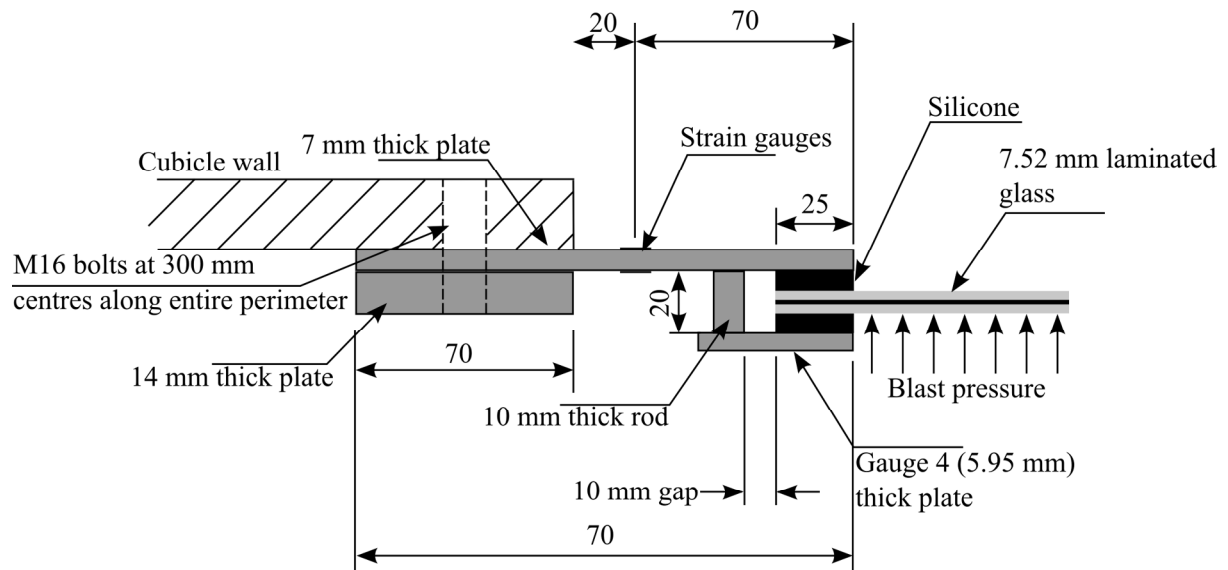


Figure 3-1: Section sketch of the glazing metal supports. All dimensions are in mm.

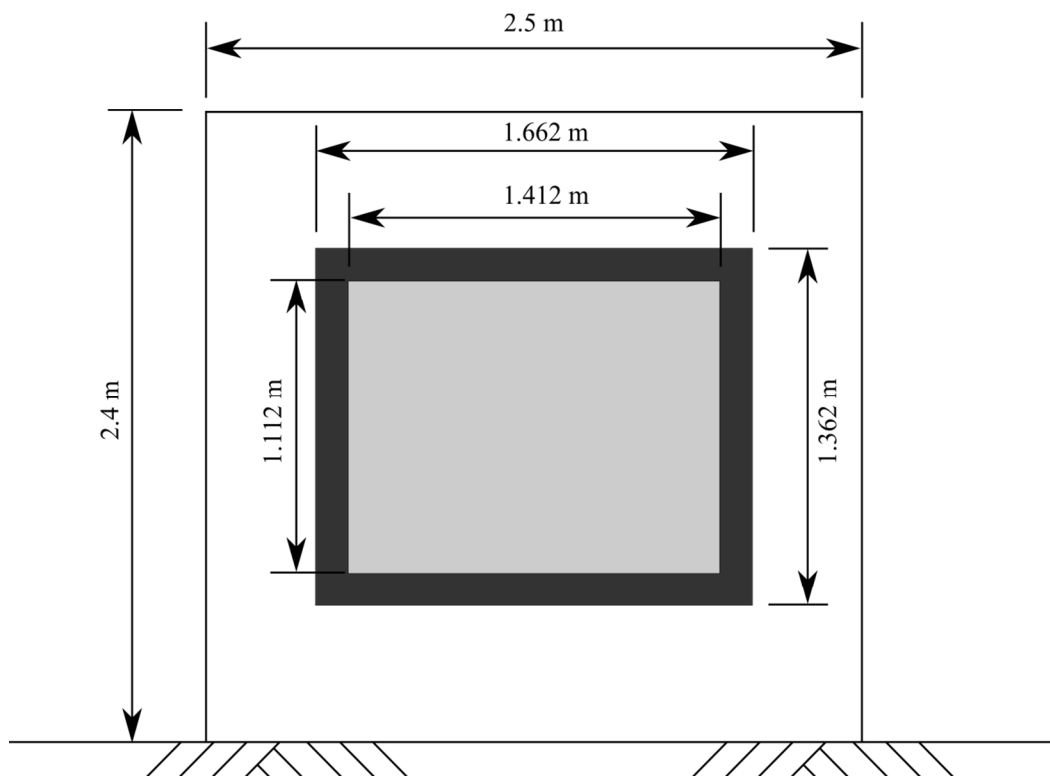


Figure 3-2: Cubicle elevation with dimensions.

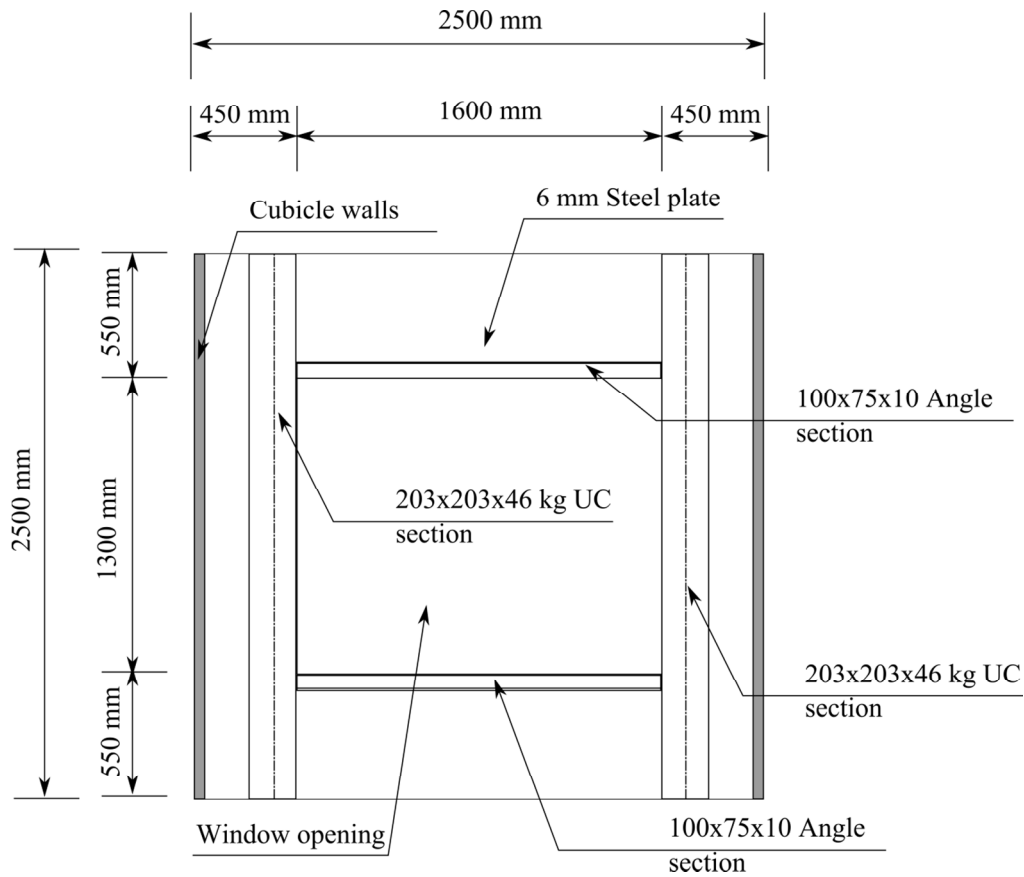


Figure 3-3: Steel structure of the front cubicle panel.

In this test, 100 kg of TNT equivalent explosive was employed. The liquid explosive was held in a spherical container to reduce irregularities in the blast pressure distribution due to non-symmetric explosive shapes. The explosion was initiated with a detonator placed in the middle of the charge. The explosive was positioned on polystyrene blocks 1 m high to minimise blast energy lost in ground shock and crater formation. Figure 3-4 shows a section of the test set up, showing both the charge and the cubicle containing the glazing sample. The test pad was shared with other cubicles at different stand offs. Their relative position were arranged to avoid the interactions of reflected waves.

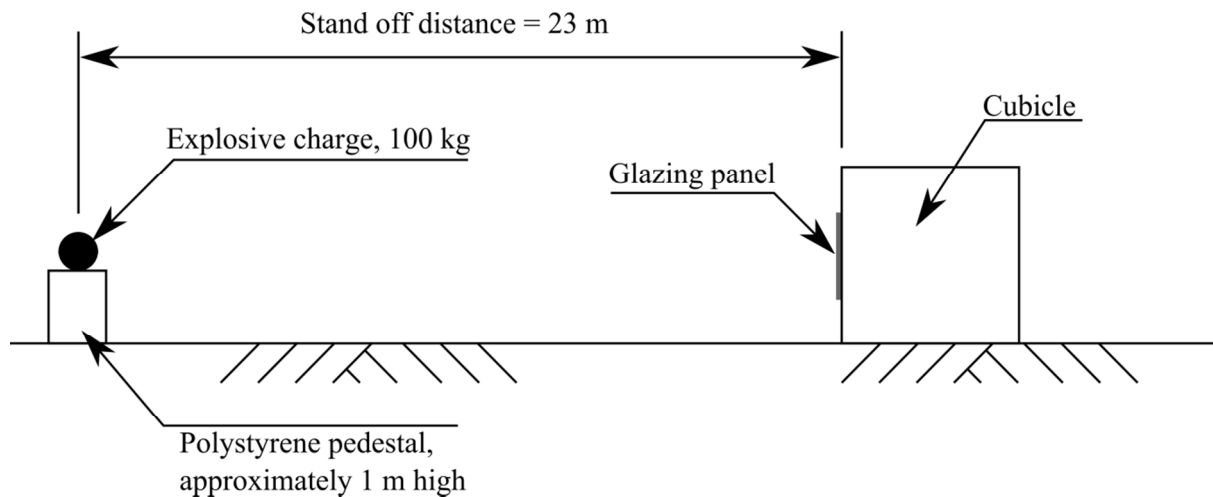


Figure 3-4: Section through the test pad, showing the charge and the cubicle containing the glazing sample.

The incident, side-on, pressures were collected employing a piezoelectric pressure gauge. PCB Piezotronics gauges, model 102-A06, were employed to collect the data. These were mounted on a steel disc aligned on the radius from the charge to avoid diffraction of the pressure wave around the sensors. The data was collected 1.4 m above the ground, 0.2 m higher than the centre line of the sample window. Figure 3-5 shows the disposition of the different elements on the test pad. The pressure gauge available for this analysis was located 25 m away from the charge, a distance 2 m greater than the tested panel, as data collection requirements for the other targets took precedence.

The test facility provided the recorded data already converted into pressures for use in this analysis.

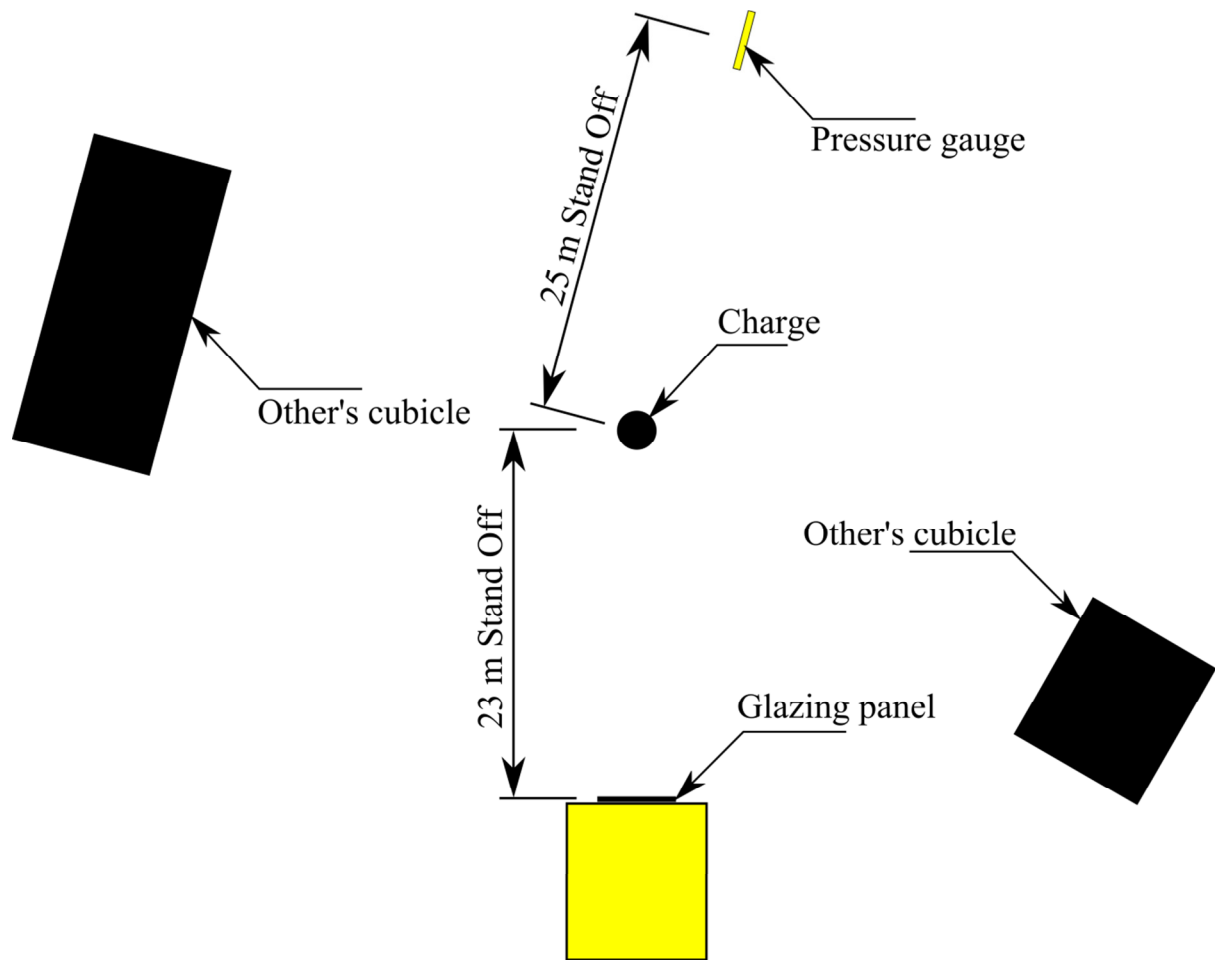


Figure 3-5: Plan view of the test pad, showing some of the other test cubicles present.

DIC data collection

Digital image correlation techniques (DIC) were employed to collect 3D deflection and strain data throughout the window surface. This measurement method requires two cameras orientated towards the sample at different angles taking pictures simultaneously throughout the test. The pairs of images are then used by a DIC software to track each dot of a speckle pattern on the sample and hence calculate the displacements and strains occurring across the area of interest in all three directions.

High speed cameras were employed to capture the transient deformations as these took place in a time period of only a few milliseconds during the blast. The instruments used were supplied by the test centre, consisting in a pair of Fastcam MC2 by Photron. This is a system of two satellite cameras controlled by a single separate processor unit. The equipment is capable of capturing grayscale images at a maximum resolution of 512 x 512 pixels at a rate

of 2000 frames per second (fps), or at a maximum rate of 10,000 fps when reducing the resolution to 512 x 96 pixels. In this test a resolution of 512 x 352 was used, as this provided enough field of view to cover the window area. The rate could then be raised to 3000 fps. The pixel size of the cameras is 10 μm , giving a total sensor size of 5.12 x 5.12 mm, of which 5.12 x 3.52 mm was used with the chosen resolution. Lenses of 8 mm focal length were used. The cameras were synchronised by the controlling processor unit. Two 1 kW halogen lamps lit the speckled surface. These were chosen since a high intensity of light was required to capture the images with short exposure times.

The two cameras were located on an extruded aluminium post on a heavy steel stand. As ground vibrations were expected to be transferred to the whole cubicle, the stand was isolated from the ground with rubber pads. Sand ballast was added around the supports to further minimise its movements. The cameras were positioned 2.5 m away from the window and 1.2 m apart. The height of the camera plane was 1.2 m to coincide with the centre line of the window pane. Figure 3-6 shows the layout. The angle subtended by the camera system was 27° as shown in the figure. This guaranteed that enough information could be obtained from the analysis of the separate pictures to calculate the out of plane displacement data [57].

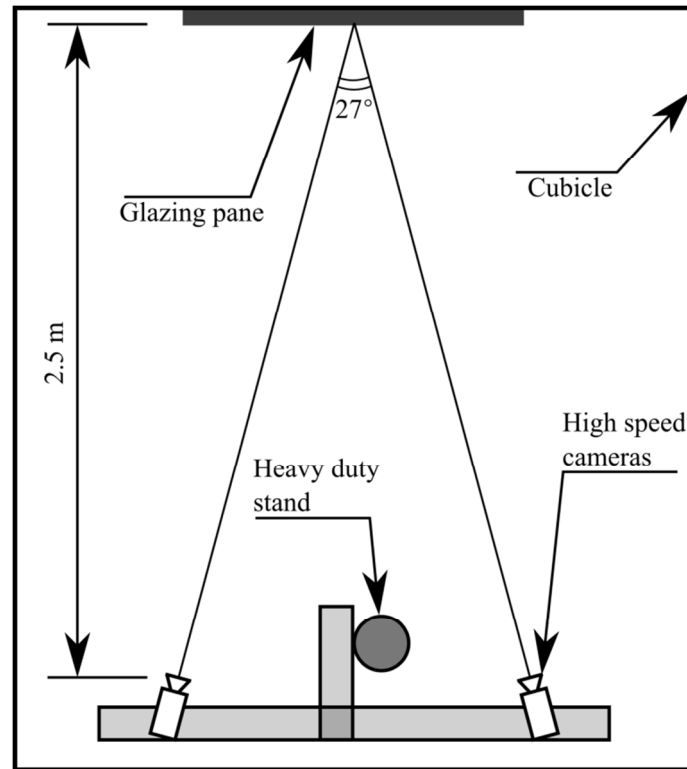


Figure 3-6: Plan view of the DIC set up inside the cubicle.

The software used calculated the movements of the sample by tracking each element of an irregular speckle pattern which was applied to the surface of the specimen. The size of each individual dot in this pattern needed to be sufficient for the algorithm to recognise it in the images. This minimum speckle area is generally considered to be 3 x 3 pixels. Therefore, the size of the window was divided by the resolution of the cameras to provide a minimum speckle size. In this case this was found to be approximately 9 mm, considering the window maximum dimension to be 1.5 m and the resolution to be 512 pixels. Before applying the speckling, the window was prepared by painting the internal surface with white matt paint and the outer surface with black paint. This was done to minimise the amount of light penetrating the window from the flash of the explosion, which could have overexposed the images. The black dots forming the speckle pattern was then applied randomly with matt paint. An approximate 50% ratio between black speckle area and white background was aimed at and a final cover of 32% was achieved. Figure 3-7 show an image of the final window surface.

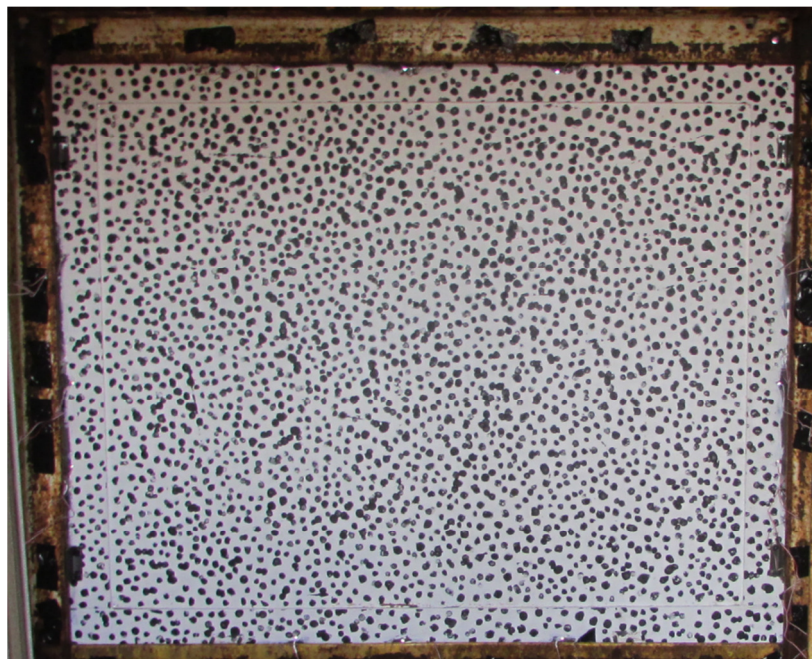


Figure 3-7: Speckle pattern on the glazing.

Before the test, pictures were taken of a standard calibration object. These pictures were to be used by the algorithm to locate the cameras and correct for picture distortions.

The DIC system was triggered by the test site staff before detonation to ensure that the whole pane excursion was captured. The pictures were then analysed to obtain deformation data using the procedure described below.

Strain gauges set up

Frame reaction data were collected using pairs of strain gauges attached to the window frame at several locations along each edge. The pairs were located on the supporting steel near the window restraint positions, as shown in Figure 3-1. Each gauge was located on one side of the plate. To obtain meaningful information it was necessary to align each gauge with minimal relative lateral displacement. A total of 24 strain gauges in 12 pairs were used. Three pairs were located at quarter points along each side. Figure 3-8 shows a sketch of the gauge locations along the perimeter.

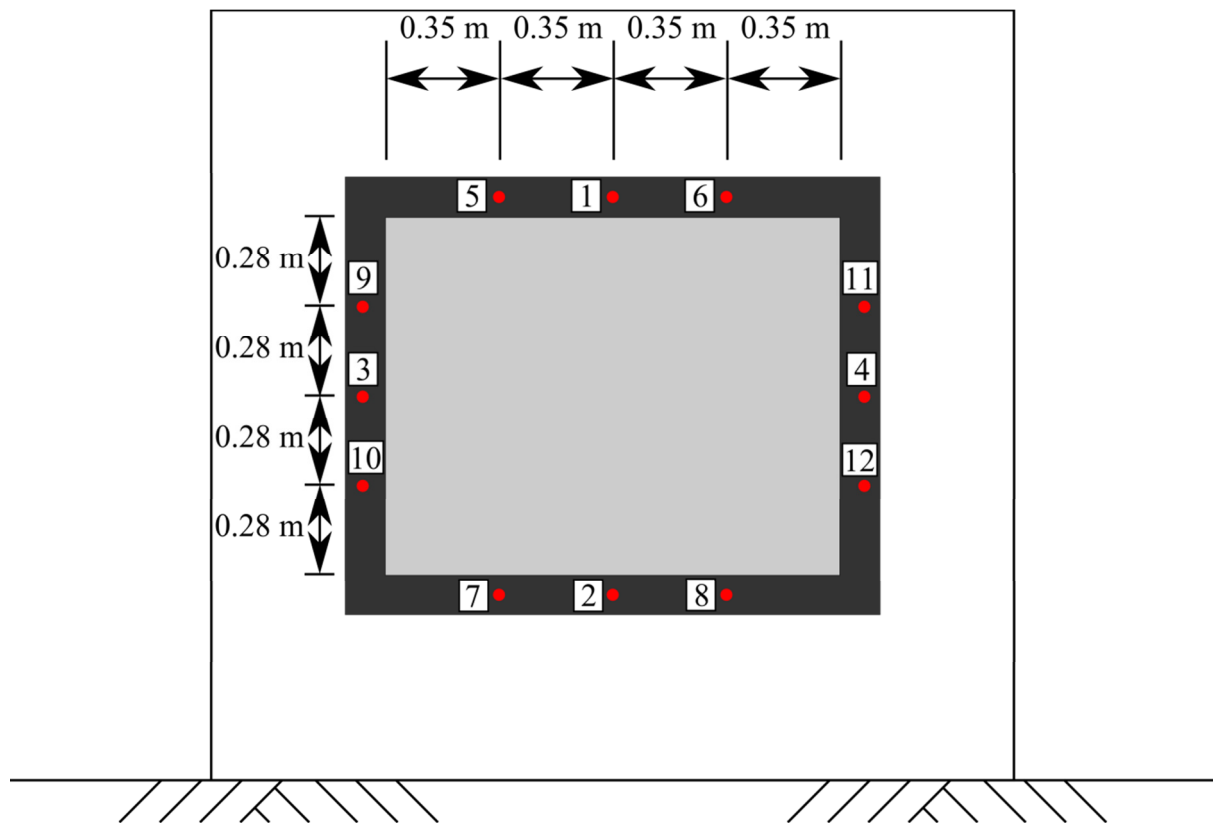


Figure 3-8: Cubicle front face elevation showing the location and numbering of the strain gauges used.

Model C2A-06-062LW-350 gauges produced by Vishay PG were used, with a resistance of 350 Ω . A 20 m long RJ45 cable was used to connect each gauge pair to the recording

equipment, controlled by the test site personnel. Prior to the positioning of the window frame each gauge pair was tested for failures and flatness. The recording equipment was triggered before the detonation as per the cameras. Data were collected at a frequency of 200 kHz.

3.2.2. Data analysis

The data collected during the test were analysed to produce the final results. The techniques used to analyse the pressure, optically collected deformation and the strain gauges data are presented here.

Pressure distribution

The side on pressure readings from the pressure gauge were converted into a reflected pressure using the method presented by Kingery and Coulter [8]. Equation 3-1 below was applied:

$$P_r = 2P_s \left(\frac{7P_0 + 4P_s}{7P_0 + P_s} \right) \quad \text{Eq. 3-1}$$

where P_0 is the atmospheric pressure, P_s is the incident pressure and P_r is the reflected pressure. This reflected value assumes an infinite surface with no pressure leakage around the edges and no enhancement due to ground proximity. Whilst these conditions were not respected at the cubicle location, it was decided that the errors due to the different standoff distance of the gauge were likely to be more significant for the peak pressure datum. The formula however was not used to obtain a full time history of reflected pressures, as the error for this information would have been very significant. For these reasons, a computational model was employed to estimate the pressures at 23 m stand-off. The software used was the Air3D package [58]. The 9th version of the program was used [11]. This package requires the input of blast parameters, namely the TNT equivalent explosive mass and the diameter of the charge. The layout of the test needs to be then modelled, specifying the position of the charge with respect to the ground level and the shape of any obstruction, in this case the test cubicle. The software can allow for a failure pressure to be specified for window elements, however this option was not employed in this case. The analysis is performed firstly in one dimension

until the blast wave reaches the ground, then in 2 dimensions until the wave reaches the target proximity and finally in 3 dimensions to compute the pressures on the area of interest. A half model of the cubicle was produced, setting a reflective symmetry boundary condition along the vertical window centre line. Output was requested at 50 mm intervals throughout the window area. Two models were created, one locating the cubicle at the same stand-off as the gauge, 25 m, and one with the correct 23 m distance. Additionally, an extra model was run without a cubicle to calculate the incident pressures at 25 m as this information could be directly compared with the experimental data.

The data obtained was analysed with Matlab to calculate the distribution of the pressures on the window area and estimates of the total pressure and impulse on the window. The 25 m peak pressure results could then be compared with the recorded pressures to evaluate eventual differences with the computational model.

DIC calibration

The DIC analysis package used was ARAMIS by GOM Optical Measuring Techniques. For the software to perform the calculations, it requires the input of calibration images. These are pictures of a proprietary object in which the software can recognise specific points. The algorithm can then compute the exact camera positions and account for picture deformations, such as lens barrelling. In this case however, due to problems with the cameras used, these pictures were not available. A new procedure had therefore to be applied to produce calibration parameters.

Whilst it is theoretically possible to produce a calibration using alternative algorithms without calibration objects [59], this technique was not implemented in the available software. It was decided that writing a proprietary algorithm would not be feasible, hence a method compatible with ARAMIS had to be found. Finally, it was decided to use virtual 3D images for the calibration. As a first step, the cameras and the vector of the direction they were aimed at had to be located with respect to the window. To do this, a 3D rendering software (PovRay) was employed. This software allows the use of script files to define images to be rendered. A generic file describing an image of a black rectangle of the same size as the window pane was prepared. Using Matlab, a series of 1296 rendering files were produced varying the camera position and its direction in a grid with steps of 40 mm in each direction. The x and y position

of the cameras and of the target point were being changed. An algorithm was then employed to compare the proportion of the lengths of the rectangle's sides to those measured in a pre-blast picture of the window. The image which presented the closest approximation to a geometrical similar shape to the window was then chosen. A further iteration with 1296 images was performed, this time cutting the steps to 8mm. The same analysis was done, and the position of the camera and its direction were assumed to be as indicated in the image generation file. This procedure was applied to both cameras. The camera positions found were then used to render images of the calibration object. A rectangular object with dimensions of 500 x 400 mm was modelled graphically and a series of 18 images representing different item rotations and positions was generated. The window frame was included in these for reference and was employed to cut the images to the size and proportion they would have had if they had been taken with the test cameras. Figure 3-9 shows an example of the sets of pictures used for this procedure. The images were then used in ARAMIS to produce a calibration. The software requires the input of the camera's sensor dimensions to produce the required data. As the produced calibration images were of a higher resolution, the pixel size used for the calibration was adjusted accordingly.

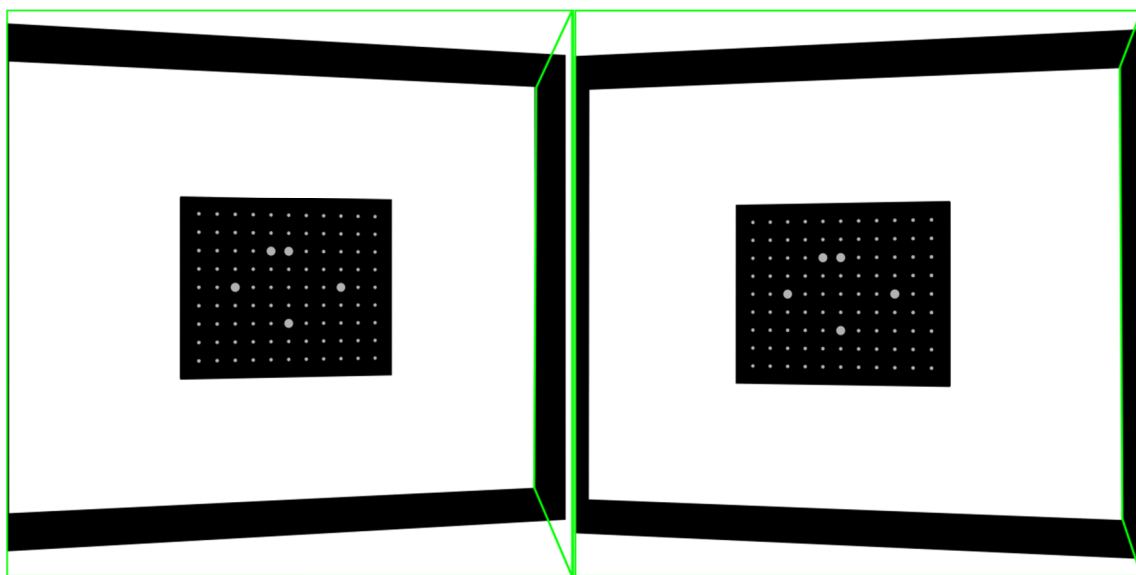


Figure 3-9: One of the sets of calibration pictures used for the procedure.

The software could then be used to analyse the pictures taken in the experiment. The only further difference from the standard procedure concerned the setting of analysis start points. These are employed by the software as reference locations on the sample's surface. Due to the non-standard calibration technique used, these had to be created manually by selecting the correct area of the panel in each picture at each time step.

A DIC calculation is performed based on facets, i.e. elements of the sample area which are tracked individually. In the case of this test, square facets of 10 x 10 pixels were employed.

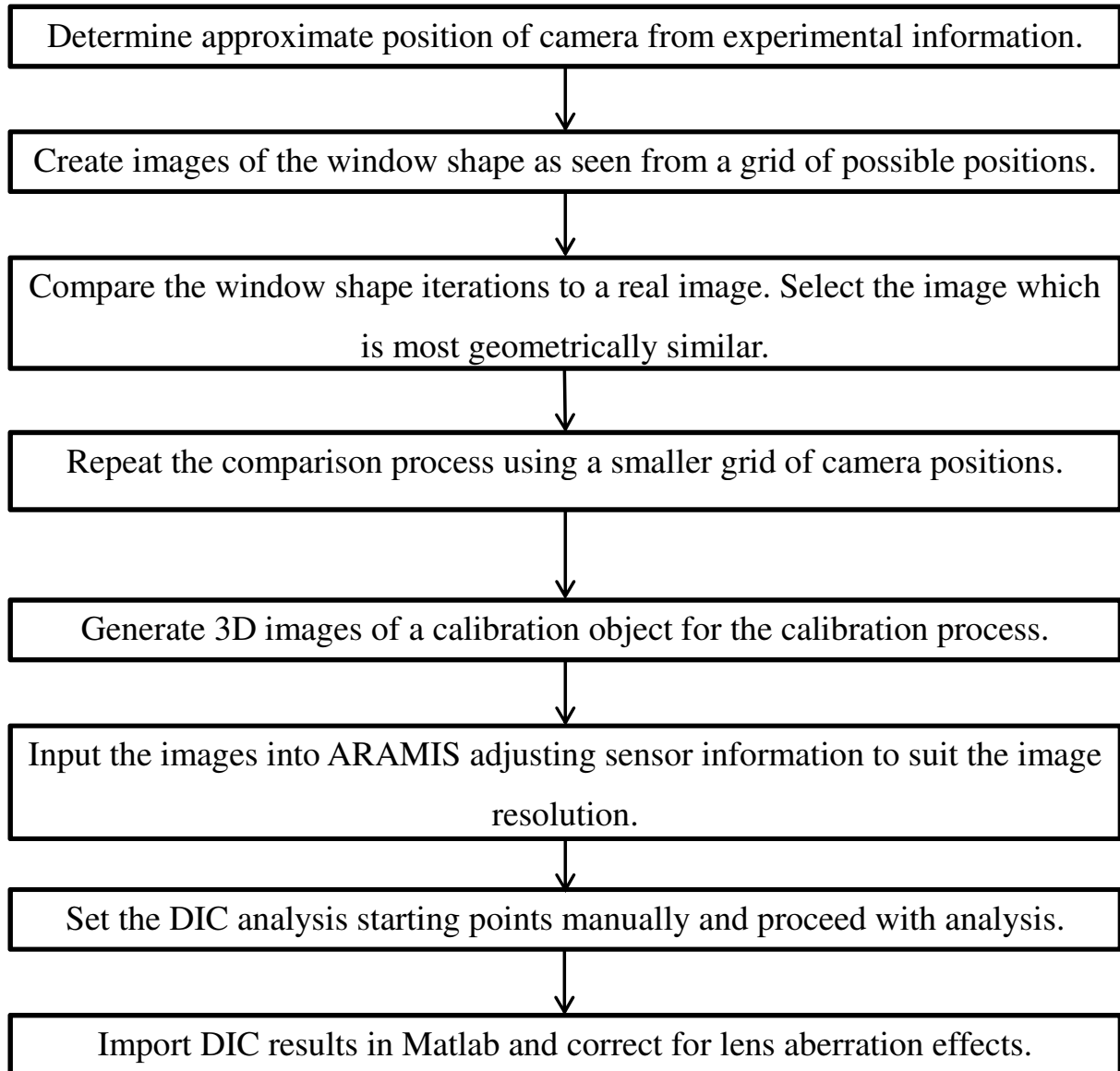


Figure 3-10: Flow chart summarising the calibration and DIC analysis process used for the experimental data.

The calibration technique used could not account for lens aberrations. Specifically, barrelling was very evident due to the very short focal length of the lenses used. The results produced therefore showed the surface of the specimen to be curved. This problem was corrected numerically during post processing. All the data were imported in Matlab. The curvature in the original coordinates was eliminated for further use of the data. The data points were also interpolated to a regular 20 mm grid. These were then used to produce sections and specific

plots to analyse the behaviour of the window at different time frames. A summary of the process described here is given in Figure 3-10 for clarity.

As this technique had not been used before, its results could not be assumed to be exact. To validate it, the method was applied to the third of Hooper et al.'s previous blast tests [37]. This experiment was chosen as it represented a typical test with respect to the window size and its calibration images allowed a calibration of higher precision when compared with other tests. As full calibration information was available, it was possible to directly compare the results of the technique with the data obtained through a normal calibration. The out of plane deflections, velocities and strains in the two main directions and the deflection angles at four locations were compared to produce error estimates. The deflection comparison was made between the DIC results obtained with the two techniques. The angles and velocities were also calculated from these results for this exercise. The strains were compared using both the alternative calibration DIC output and values recalculated using the out of plane deflection results, as explained below. The x and y direction strains were used.

DIC data analysis

The technique described above was applied to the available test images. Deflection data were obtained from the DIC analysis software and adjusted to correct for aberrations as discussed. It was decided to recalculate the strain data from the obtained movements instead of employing the data directly produced by the DIC algorithm. To do this, the deformed distance of nearby interpolated points was calculated using the DIC deflected coordinates information. This was then used to calculate the strain in the x and the y directions, as these would be more useful for later analysis.

The velocity was also calculated from the out of plane deflections. A centre point differentiation technique was used at each point of the grid, producing full field velocity information.

The displacement data were also used to calculate the springing angles at the location of the strain gauge pairs. The deflection profiles at the different positions along the edge were considered and angles were calculated using simple trigonometry.

Reaction force

The data from the strain gauges were analysed using a method similar to that described by Hooper et al. [37]. The strain gauges provide strain data on both side of the supporting steel plate, which will deform elastically under the reactions from the pane. These strains can therefore be easily converted into stresses, and these used to calculate the bending moments and axial forces acting on the angle plate. Figure 3-11 shows a diagram of the system with the main force and reactions acting on the steel plate, whilst Figure 3-12 shows a schematic of the main forces and plate strains.

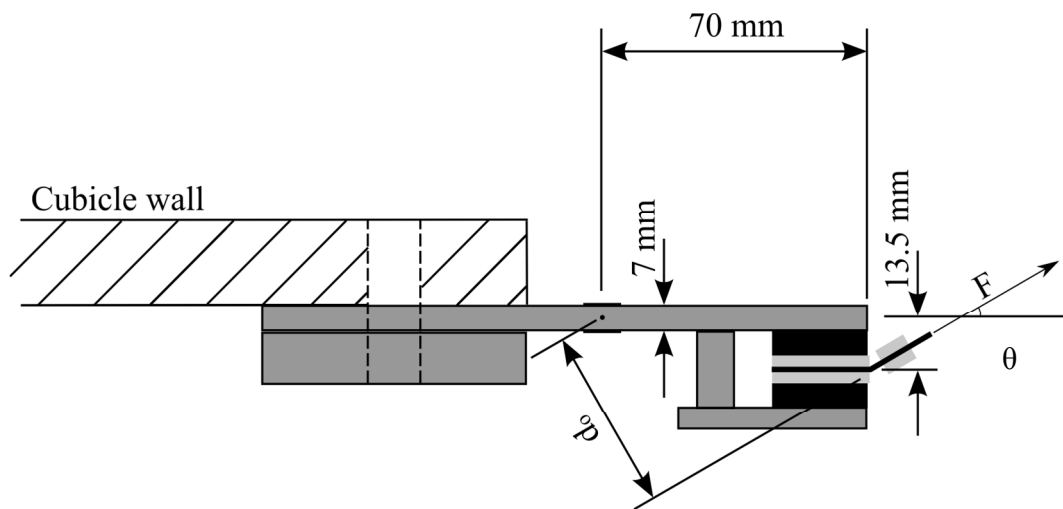


Figure 3-11: Sketch showing the forces acting on the strain gauges and the main dimensions used in the calculation.

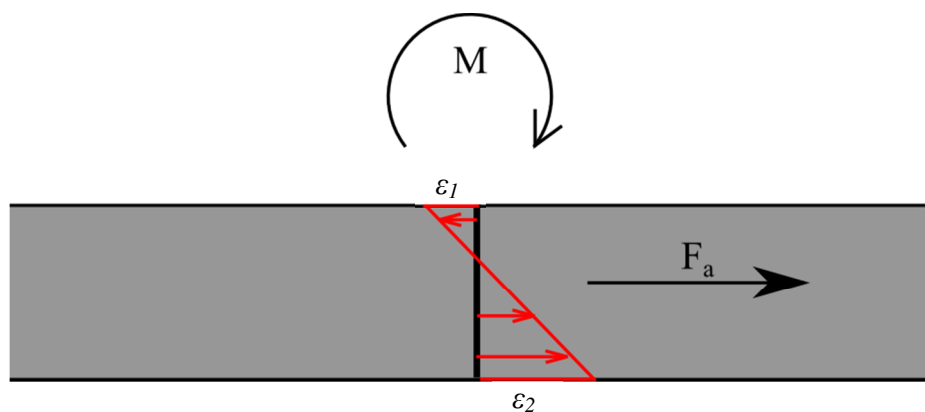


Figure 3-12: Strain distribution in the supporting plate's cross section.

The bending strain at the gauge location can be calculated from the recorded data as:

$$\varepsilon_b = \frac{\varepsilon_1 - \varepsilon_2}{2} \quad \text{Eq. 3-2}$$

Where ε_1 and ε_2 are the strains on opposite sides of the steel plate and will tend to be of opposite signs.

The strain due to the axial force is equal to:

$$\varepsilon_a = \frac{\varepsilon_1 + \varepsilon_2}{2} \quad \text{Eq. 3-3}$$

The bending moment M , which is measured in Newton-metres per unit width (Nm/m), applied to the plate is therefore equal to:

$$M (\text{Nm/ m}) = \frac{\varepsilon_b EI}{y} = \frac{\varepsilon_b E d^2}{6} = \frac{\varepsilon_b \times 2 \times 10^{11} \times 0.007^2}{6} \quad \text{Eq. 3-4}$$

Where I is the second moment of area, y is half the plate's thickness, equal to d , and E is the Young's modulus of steel. The axial force per meter width will be equal to:

$$F_a (\text{N/m}) = \varepsilon_a E d = \varepsilon_b \times 2 \times 10^{11} \times 0.007 \quad \text{Eq. 3-5}$$

These moments and forces will be caused by the window reactions. The bending moment at the strain gauges is:

$$M (\text{Nm/ m}) = F \times \cos(\theta) \times 0.0135 + F \times \sin(\theta) \times 0.070 \quad \text{Eq. 3-6}$$

And the axial force at the same positions is:

$$F_a (\text{N/m}) = F \times \cos(\theta) \quad \text{Eq. 3-7}$$

In theory therefore a system of equations could be set up using the four formulae listed above equating the forces measured by the gauges with the reaction forces. The reaction force and its angle θ could then be found. However, Hooper et al. found that even relatively small errors in the alignment of the gauges cause significant errors in this estimate. This was also found in this case, with the strain readings producing non realistic values for the reactions. The solution was to use the DIC data to estimate the angle at which the reaction force is applied. The force could be then calculated directly from either the bending moment or the

axial force information. The bending moment measurement was used, as Hooper had found the axial force strains to be the most affected by the alignment errors.

The reaction force therefore was given by:

$$F \times \cos(\theta) \times 0.0135 + F \times \sin(\theta) \times 0.07 = \frac{\epsilon_b \times 2 \times 10^{11} \times 0.007^2}{6} \quad \text{Eq. 3-8}$$

Which can be rearranged to give:

$$F(\text{N/m}) = \frac{\epsilon_b \times 2 \times 10^{11} \times 0.007^2}{6 \times (\cos(\theta) \times 0.0135 + \sin(\theta) \times 0.070)} \quad \text{Eq. 3-9}$$

The force so obtained will be in Newton per meter width. Once the forces were calculated at the locations of the gauge pairs using this formula, the results were presented and compared to examine the changes in reaction force along a window edge.

3.3. Results

The data collected during the test were analysed employing the techniques described above. The results are illustrated below.

3.3.1. Blast Pressures

The temperature on site at the time of the test was 18°C. The experimentally recorded peak pressure was converted to a reflected pressure with the formula stated above in paragraph 3.2.2. The peak pressure recorded by the site pressure gauge was 83.2 kPa and the positive phase lasted approximately 20 ms. The pressure wave reached the gauge 42.4 ms after detonation. The experimental pressure time history is shown in Figure 3-13 together with the Air3D CFD estimate. A comparison between the values obtained is shown in Table 3-1.

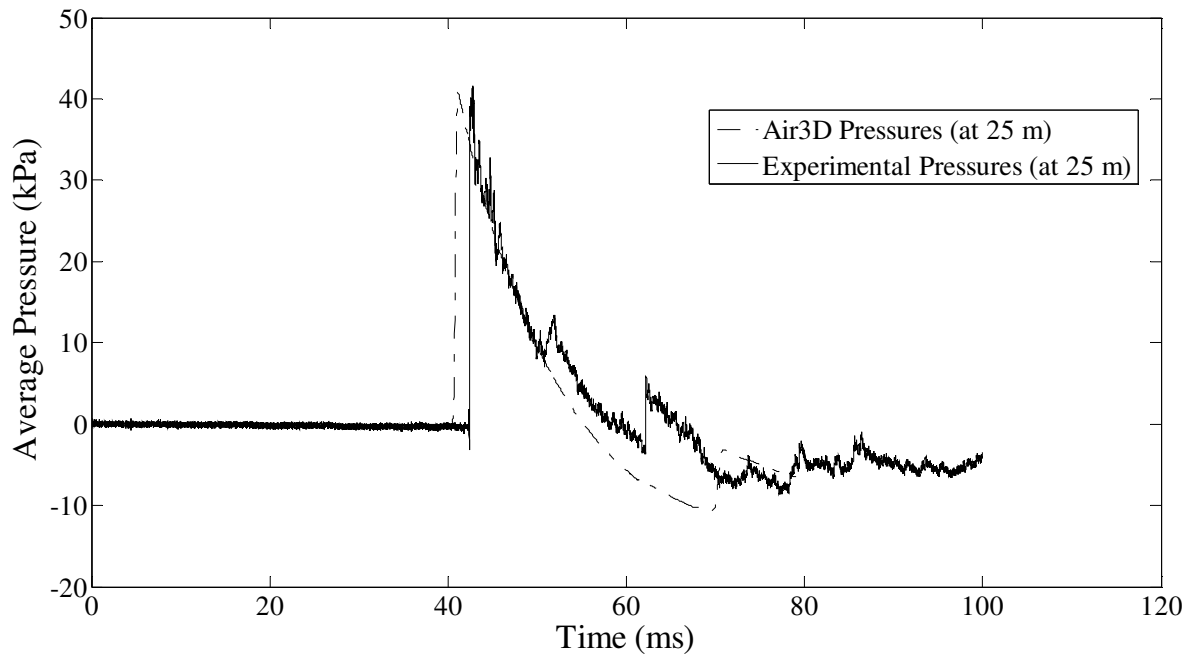


Figure 3-13: Pressure time history at 25 m stand-off. The experimental and CFD incident data are shown. The charge was set of at time 0 ms.

Table 3-1: Incident pressure results.

Case	Peak Incident Pressure (kPa)	Positive Phase Duration (ms)	Arrival Time (ms)	Incident Impulse (kPa ms)
Experimental	40.3	15.6	42.4	214.4
Air3D	41.6	14.8	40.5	242.4

The 25 m Air3D analysis with a cubicle produced a peak reflected pressure of 91.2 kPa, with a positive phase duration of 10 ms and an arrival time of 40.5 ms. The 23 m stand-off Air3D estimate is shown in Figure 3-14. The peak pressure in this case was 107.0 kPa at a time of arrival of 35.4 ms. Impulses were calculated for the computational data. The results for the reflected data are summarised in Table 3-2.

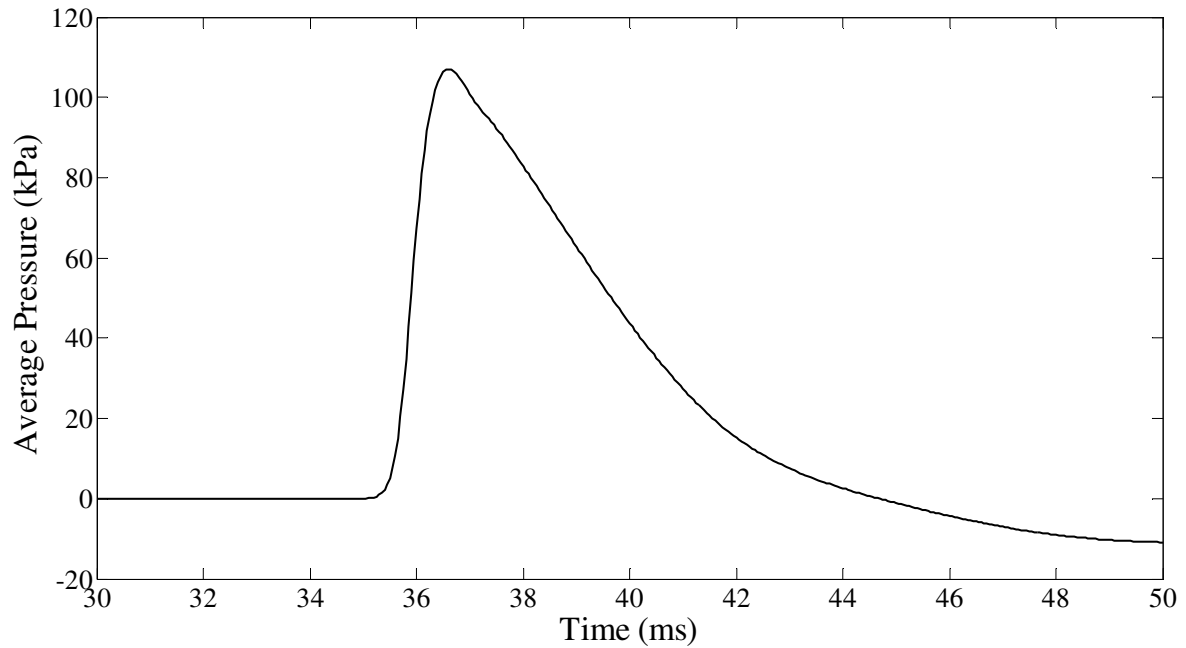


Figure 3-14: Pressure time history at 23 m stand off calculated with Air3D. The charge was set of at time 0 ms.

Table 3-2: Reflected pressure results.

Case	Peak Reflected Pressure (kPa)	Reflected Impulse (kPa ms)	Arrival Time (ms)
25 m Stand off experimental	93.8	N/A	42.4
25 m Stand off Air3D	91.2	359	40.5
23 m Stand off Air3D	107.0	406	35.4

The incident data shows a difference between the recorded peak pressures of 3.3%, whilst the impulse error is 13%. The reflected data presents similar errors. The calculated reflected peak pressure is 3% lower and the time of arrival 2 ms shorter. Some of the differences could be due to limitations in the CFD software. A mesh sensitivity study was run, and the mesh used (75 mm size) did not show significant differences when compared to a 50 mm mesh data, as shown in Figure 3-15. However, limitations in the modelling of the shock wave remain and are inherent in the software employed. Whilst these numerical imprecisions can explain some

of the minor difference in the blast peak and the more relevant difference in the arrival time, it is also possible that the limitations of the conversion equation used (Eq. 3-1) contributed to this difference. For example, ground effects are already significant in amplifying the pressure waves before this reached the target, potentially increasing its velocity.

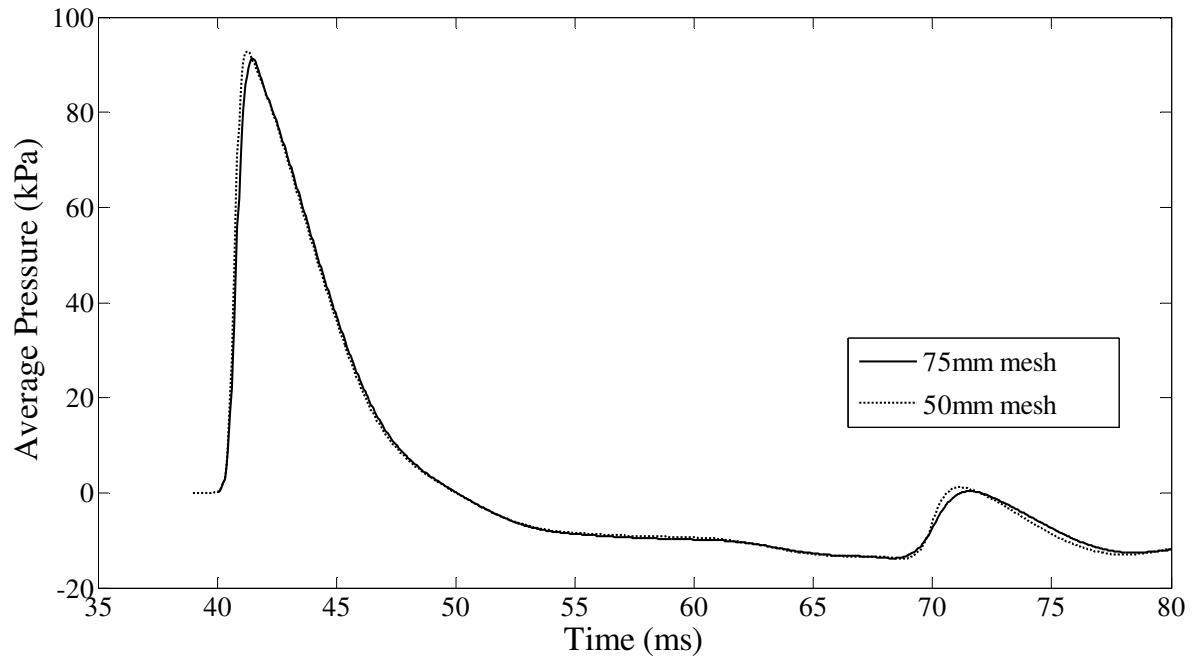


Figure 3-15: Comparison of Air3D results for two mesh sizes. The charge was set of at time 0 ms.

3.3.2. Window general observations

Unfortunately the outer glass ply fractured diagonally before the test when the pane was being mounted on the cubicle. The location of the crack is highlighted in Figure 3-16.



Figure 3-16: Existing diagonal crack location.

The glass plies fractured early during the blast, as expected. The paint adhered well to the surface, with only limited peeling. This prevented an accurate observation of the crack distribution throughout the pane. Where visible though, generally near the edges, it appeared that the glass fragments were small, making the glass panes completely opaque. Diagonal cracks appeared in both directions, with an increased density near the corners. The majority of the peeled paint is concentrated in these regions. Circumferential cracks are also evidenced by damage to the paint. These locations are shown in Figure 3-17.

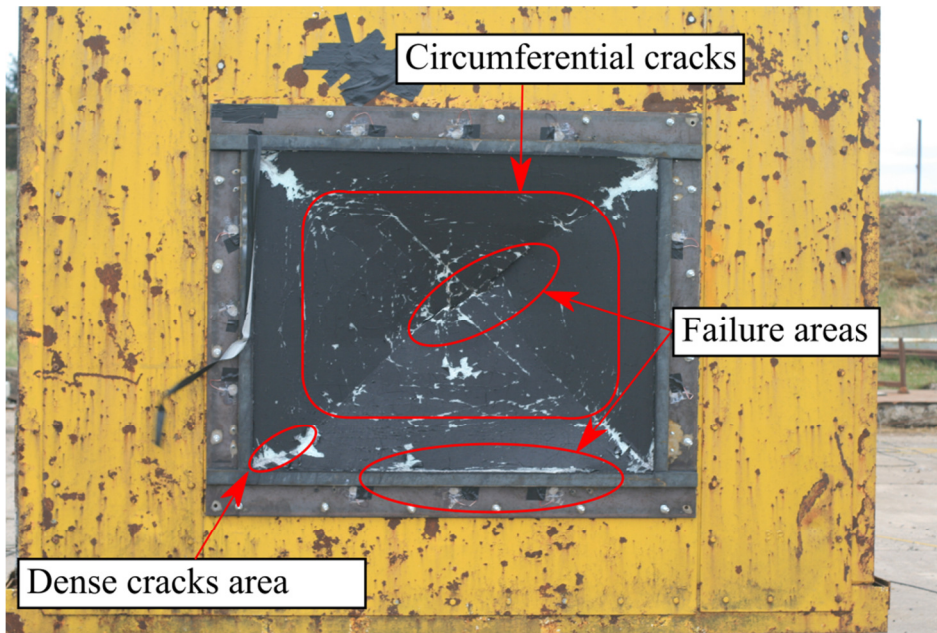


Figure 3-17: Front view of the tested panel, showing main areas of damage.

The PVB tore in two main locations, along one diagonal and along the bottom edge. The diagonal tear, shown in detail in Figure 3-18, took place in the central window area. This is most likely due to the pre-existing diagonal crack. The PVB failed towards the end of the inbound deflection, with an evident oscillation of the two sides in the outbound stroke.

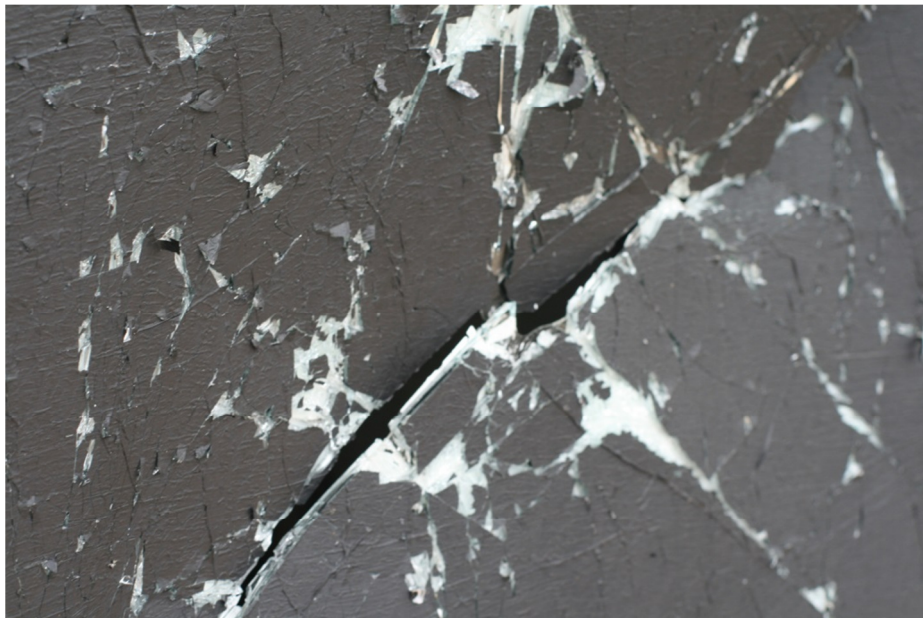


Figure 3-18: Detail of central diagonal failure.

Another tear was present on the bottom edge. This is similar to what observed by Hooper, who reported PVB failures occurring along the edges preferentially. Figure 3-19 shows a

detail of this failure. The PVB became torn near the window support at the edge of the silicone. Glass was still attached to the silicone connections. This, together with longitudinal fracture of the glass, delamination of PVB and silicone failure is one of the typical edge failure mechanisms observed in previous tests. The specific failure encountered here could have been favoured by the double sided restraints, which offered more resistance to the alternative mechanisms. Additionally, the sharper angle caused by the increased stiffness of the support system would have brought the PVB layer in closer contact with the sharp glass edges, causing it to be cut.

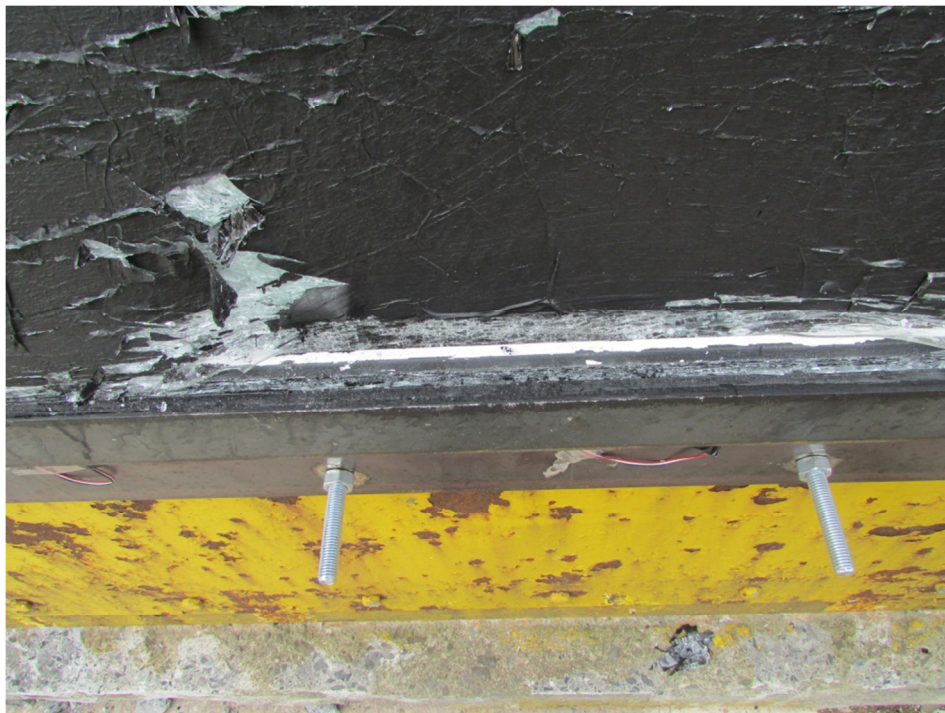


Figure 3-19: Detail of the pane failure at the bottom support.

3.3.3. Digital Image Correlation calibration comparison

The calibration technique was applied to a test whose real calibration data were available for comparison. Once the cameras were located with the technique explained above, the calibration pictures were generated.

The DIC analysis was performed using the generated calibration pictures and displacement and strain data. These were imported into Matlab, and interpolated to a regular grid for easy

comparison with the existing data. The initial coordinates were flattened out to account for barrelling distortions previously described.

Figure 3-20 shows a 3 dimensional plot of both sets of deflection data at 15 ms, whilst Figure 3-21 shows a comparison of these results, presenting both relative and absolute errors.

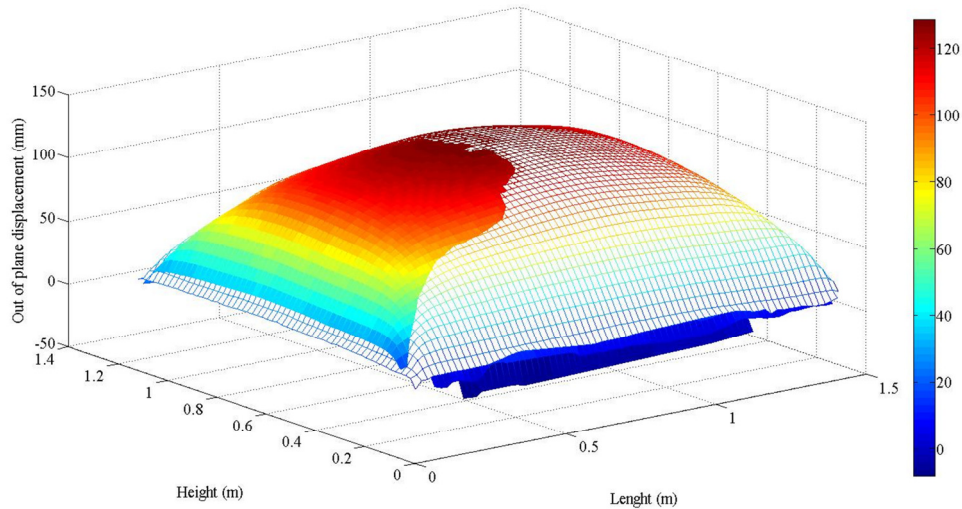


Figure 3-20: 3D plot of the deflection data. The shaded data set is the data obtained with the new calibration method, whilst the white facets curve shows the old calibration data.

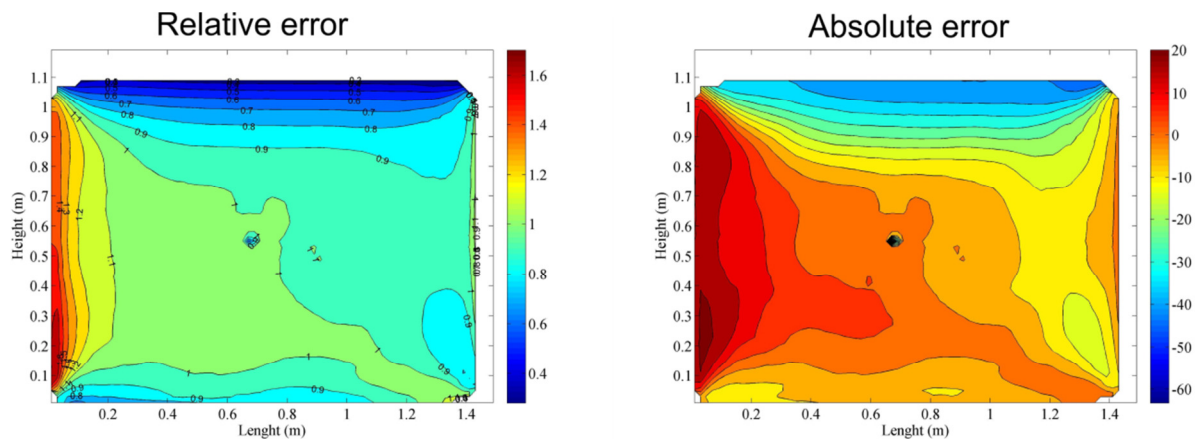


Figure 3-21: A plot of the relative and absolute deflection errors.

The deflections in most of the panel area are similar between the two sets, with errors within 20% of the fully calibrated DIC results. The bigger differences are observable along the edges. The 3D plot shows that the behaviour of the panel compares well between the two analyses, however there is some error in the starting position, which is different in the two data sets. This could justify the higher errors in these areas. A comparison of the velocities

obtained is shown in Figure 3-22. The situation is similar to that described for the deflections, with higher errors present along the edges. This could be expected, as this information was calculated from the out of plane displacement information.

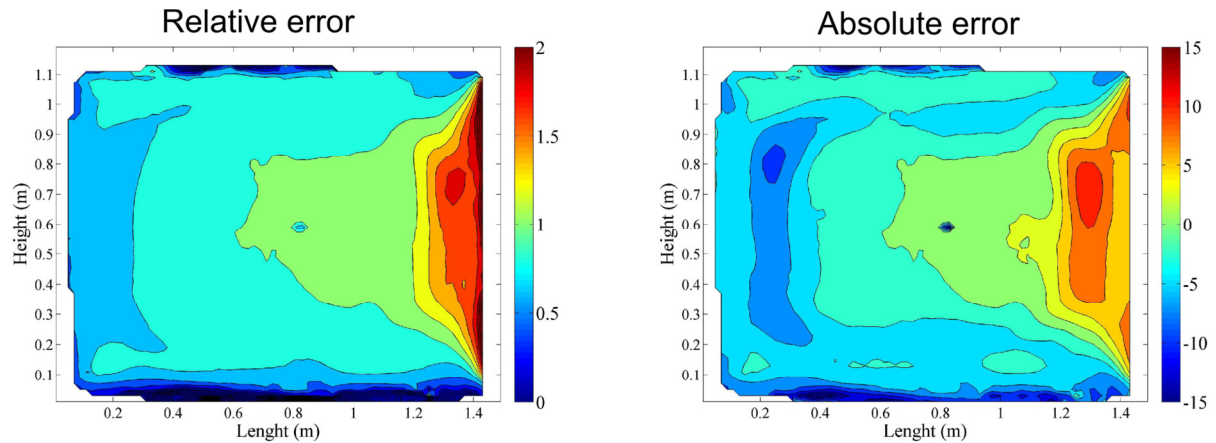


Figure 3-22: Relative and absolute differences in the velocity data obtained with the two calibrations.

The strain data are shown in Figure 3-23. As mentioned, two strain estimates are shown for the alternative calibration method, the DIC output and strains recalculated from the deflection information. The differences between the two calibrations' results are higher than for the deflection information. The general behaviour of the strains is similar, with the highest deformations in both directions concentrated along the window edges. However the magnitudes are different. For example, the strains in the x direction are 14% or 7% (depending on the calculation method) in the alternative calibration data, whilst the fully calibrated output shows around 3-4% strains. Therefore, whilst the order of magnitude of the strain values and information on their distribution of the pane can be obtained it is not possible to obtain results with greater accuracy. These uncertainties will need to be considered when utilising the data of the latest blast for further analyses in subsequent chapters.

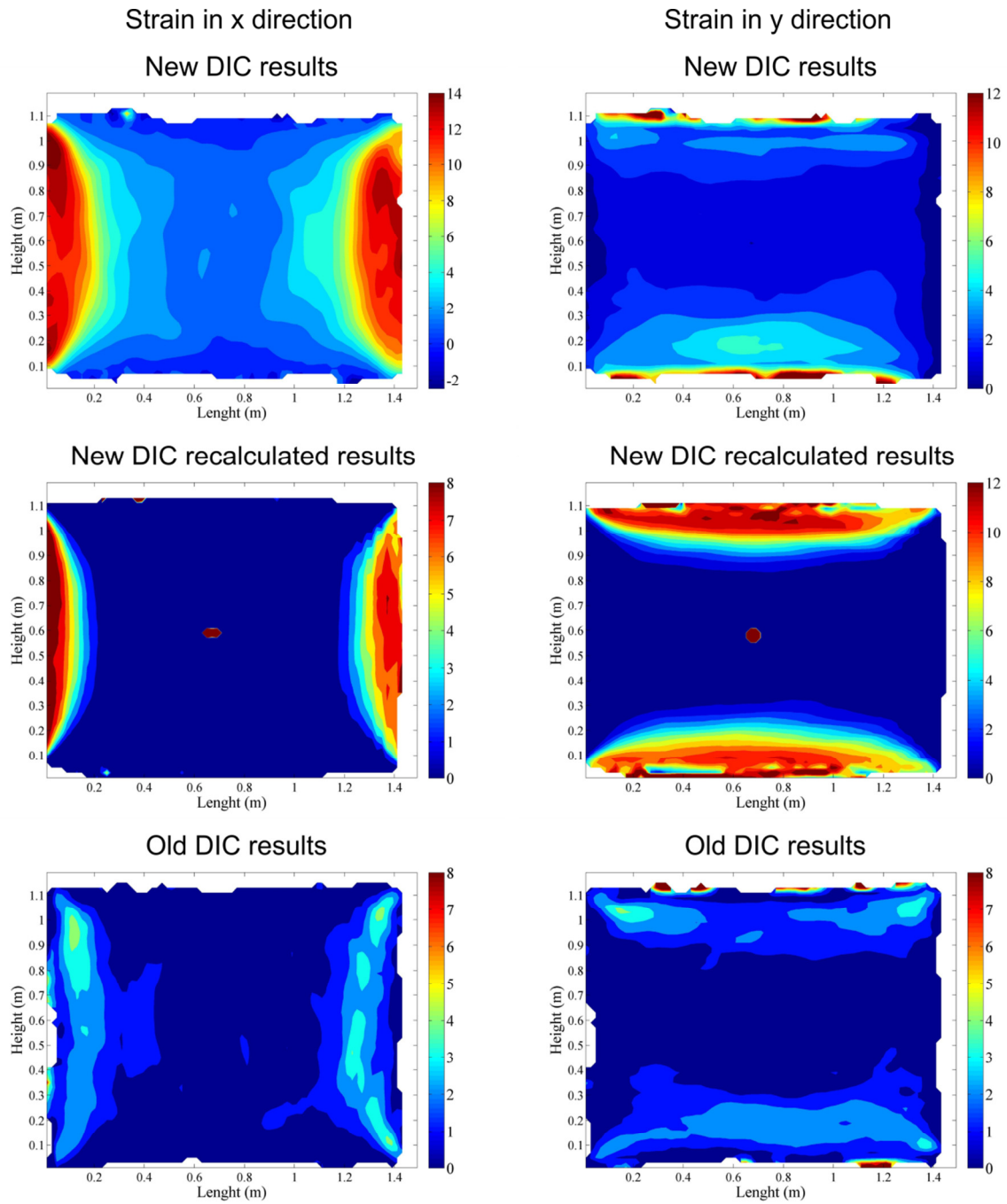


Figure 3-23: Comparison of strain data obtained with both calibrations and with two methods for the new calibration data. Both strains in the x and in the y direction are shown.

As the edge angles will be needed to calculate reaction forces, the likely uncertainty of these data was also estimated. The angles were calculated at the four central locations of each side. Figure 3-24 and Figure 3-25 show the results at the left hand and top window sides, with the relative errors shown in Figure 3-26. The maximum inaccuracy is 20% on the left hand side. This will be considered when assessing the results of further analyses.

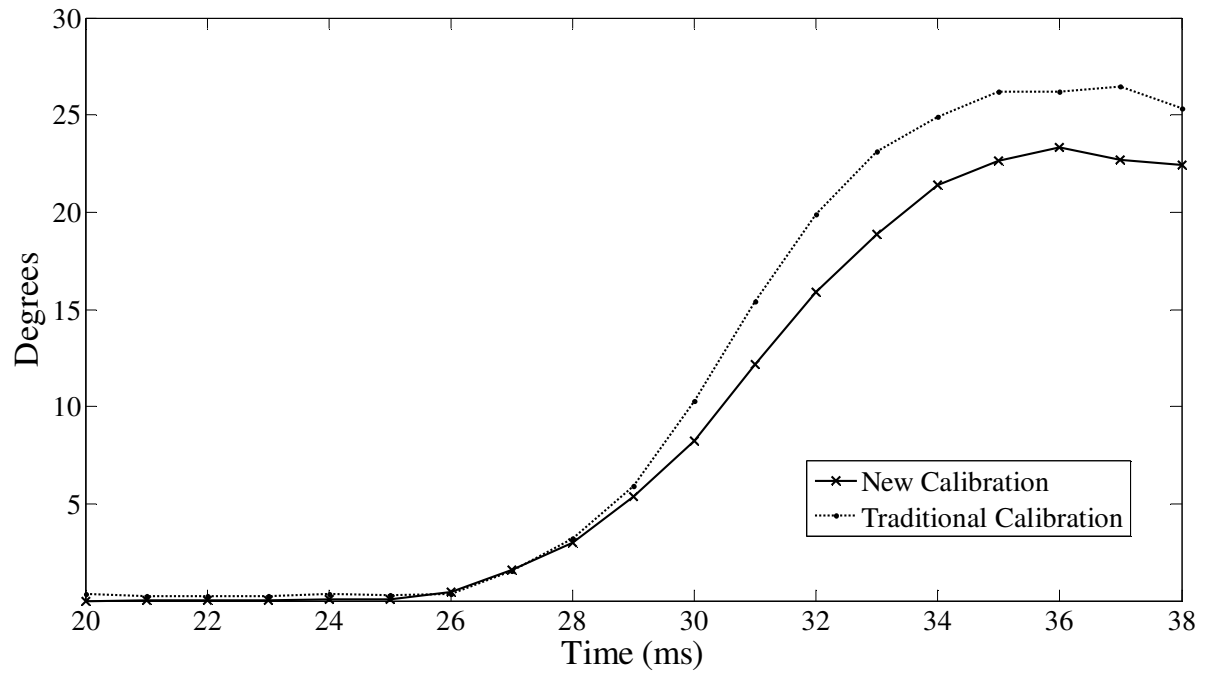


Figure 3-24: Comparison of angle measurements time history along the left hand vertical window side.

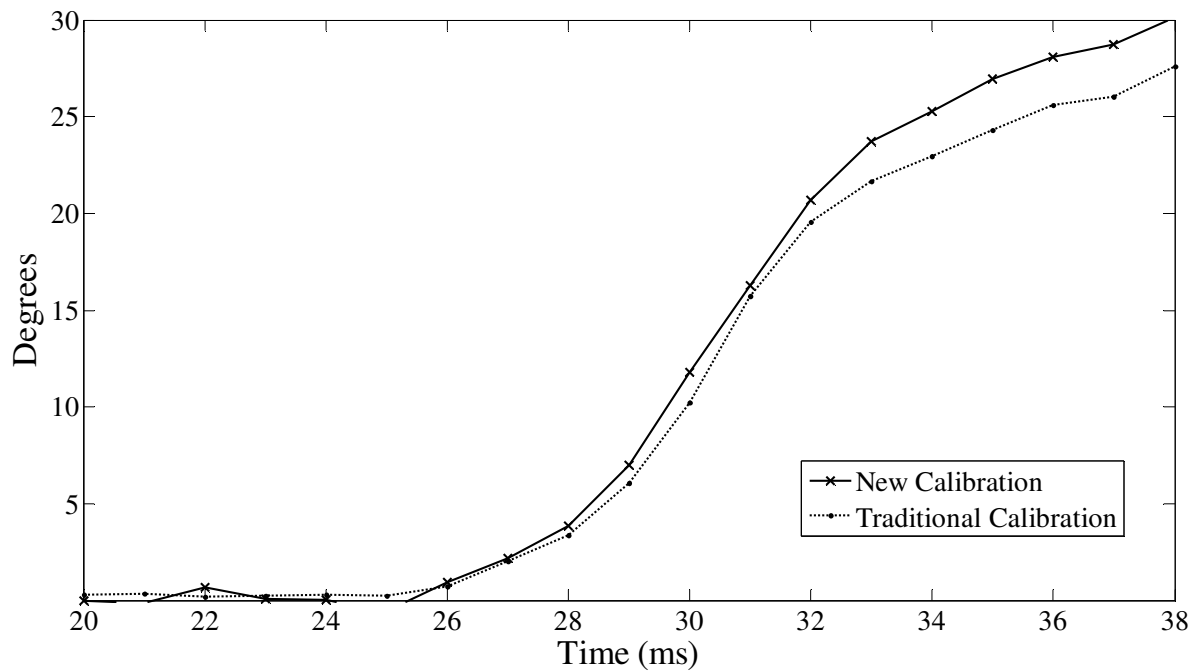


Figure 3-25: Comparison of angle measurements time history along the top window side.

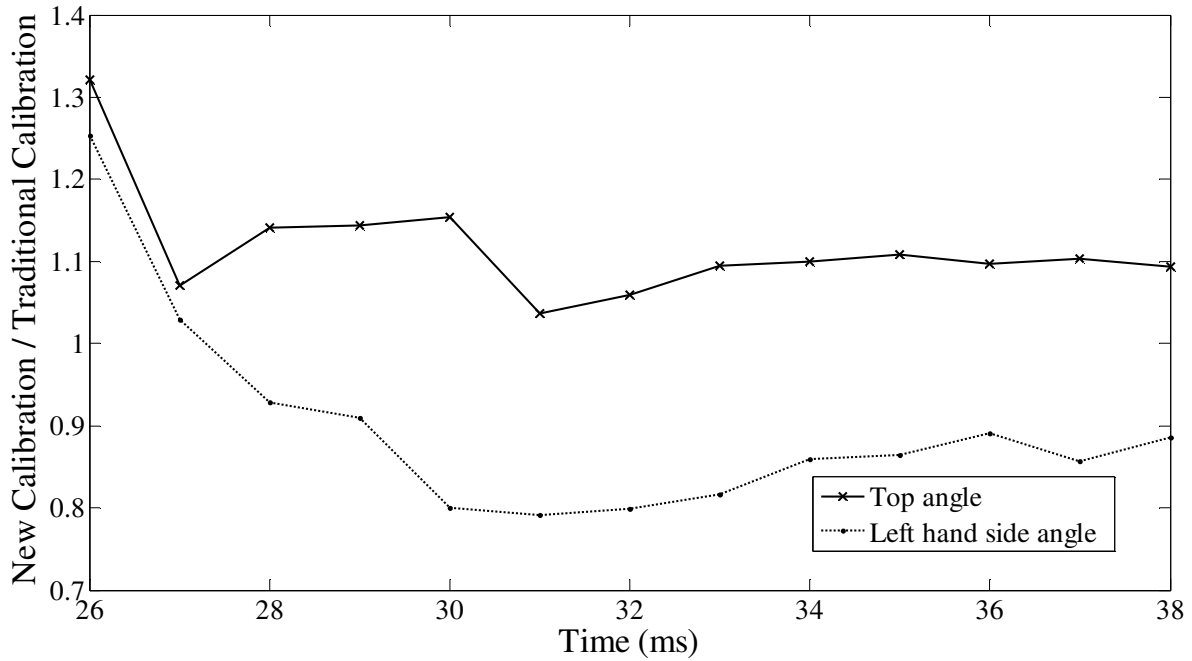


Figure 3-26: Relative error for the two data sets displayed above.

3.3.4. DIC Analysis Results

The high speed images from the test were processed using DIC software. The DIC algorithm could not produce results for the central area of the window when peak deflections were reached, as the central diagonal tear ruined the speckle pattern available at the latter stages of the deformation. The relevant information was obtained through interpolation of the available data. A total of 64 images was utilised for the analysis, producing data for a period of 21.3 ms. Figure 3-27 shows the out of plane deflection results. The low deflection area around the window edge represents results for the steel support, which was also covered with a speckle pattern to estimate its deflections. As can be seen in the series, these deflections are significantly lower than for the glass, peaking at 50 mm. The deflected shape of the window changes significantly during the event. The panel initially shows a relatively flat area in the middle. Most of the deformation at low deflections is concentrated around the edges, which could be an explanation for the higher cracking densities which are traditionally observed in these areas.

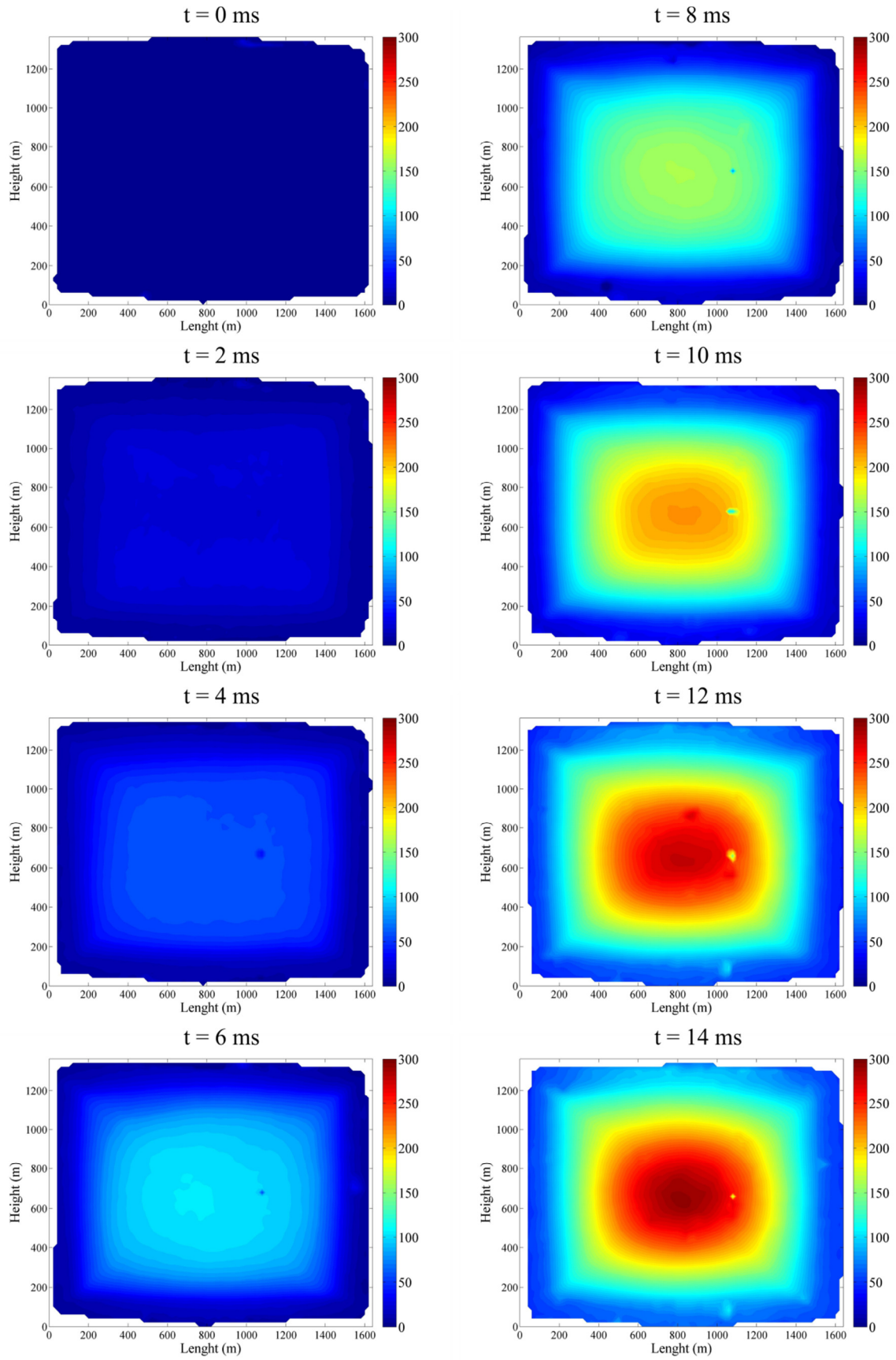


Figure 3-27: Out-of-plane deflections of the glass panel measured with the DIC method.
The deflection is in mm. Time 0 ms is at the blast wave arrival.

To visualise this effect more accurately, Figure 3-28 shows a plot of the curvature of the window and the out of plane deflection 4 ms after the arrival of the blast wave. These were calculated along a horizontal cut taken at the centre of the window. The plot shows clearly the flat central area both the curvature and in the deflections. Later, as the deflections grows, the curved area increases, moving towards the centre of the pane. Ultimately the whole window assumes a deformed shape similar to what could be expected for static or harmonic deflections. The deflection of the mid point of the glass panel peaked at 297 mm. Figure 3-29 shows a time history of this datum.

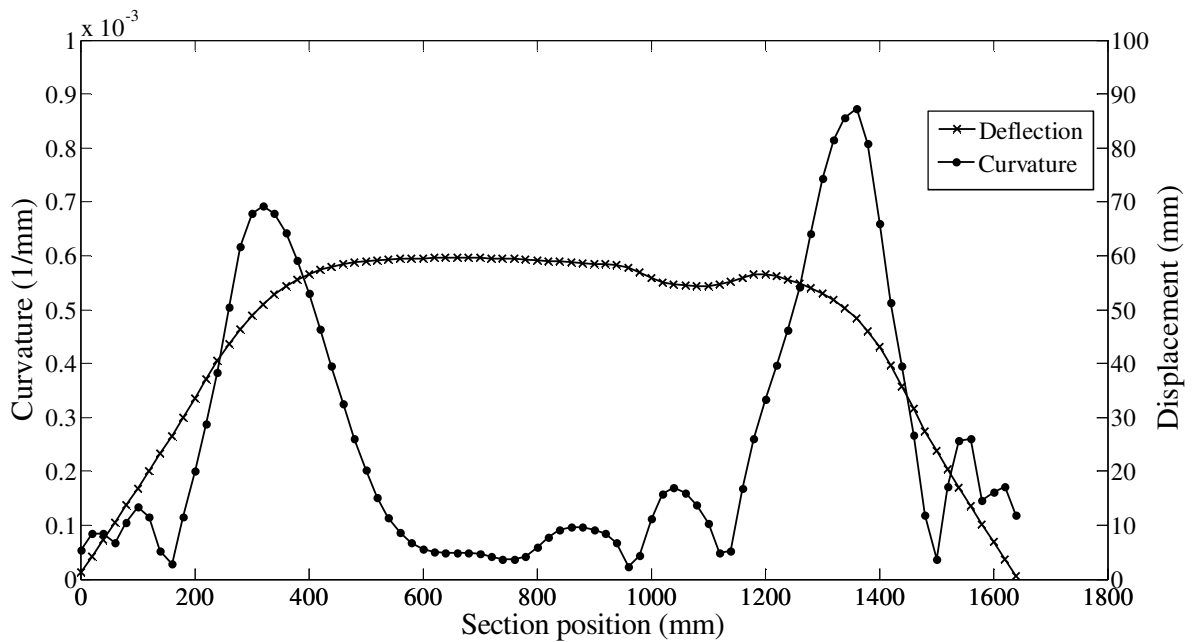


Figure 3-28: A plot of the curvature and deflections along a horizontal cut taken across the centre of the window.

Figure 3-30 shows a plot of the central velocity, whilst Figure 3-31 shows the calculated normal velocity across the area of the pane. As expected from the deflection results and literature analyses [37], the area of high velocity decreases as the deflection progresses. The maximum velocity recorded was 34 m/s.

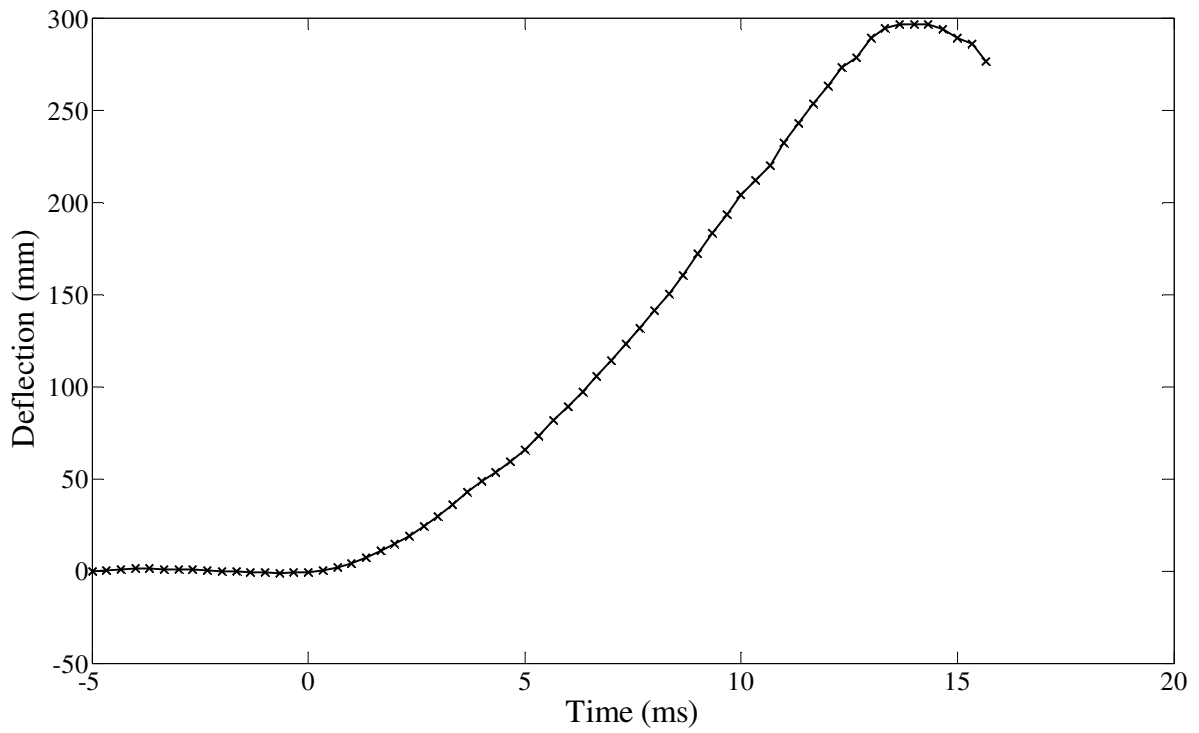


Figure 3-29: Central deflection time history from DIC data. Time 0 ms is at the blast wave arrival.

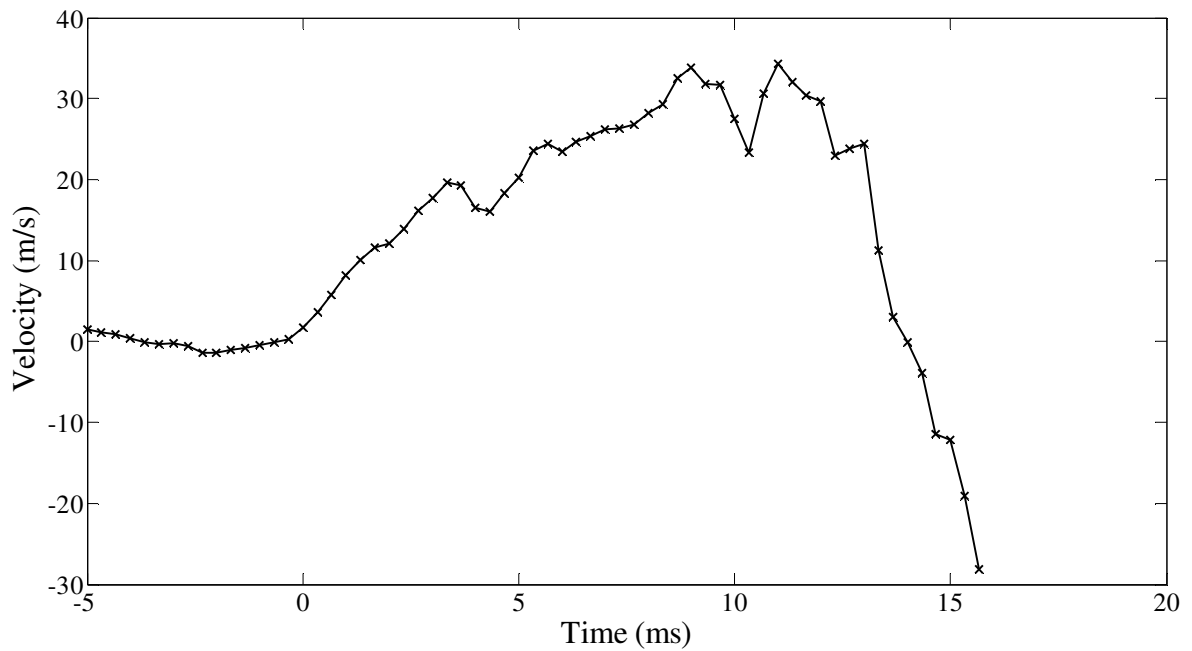


Figure 3-30: Central velocity time history from DIC data. Time 0 ms is at the blast wave arrival.

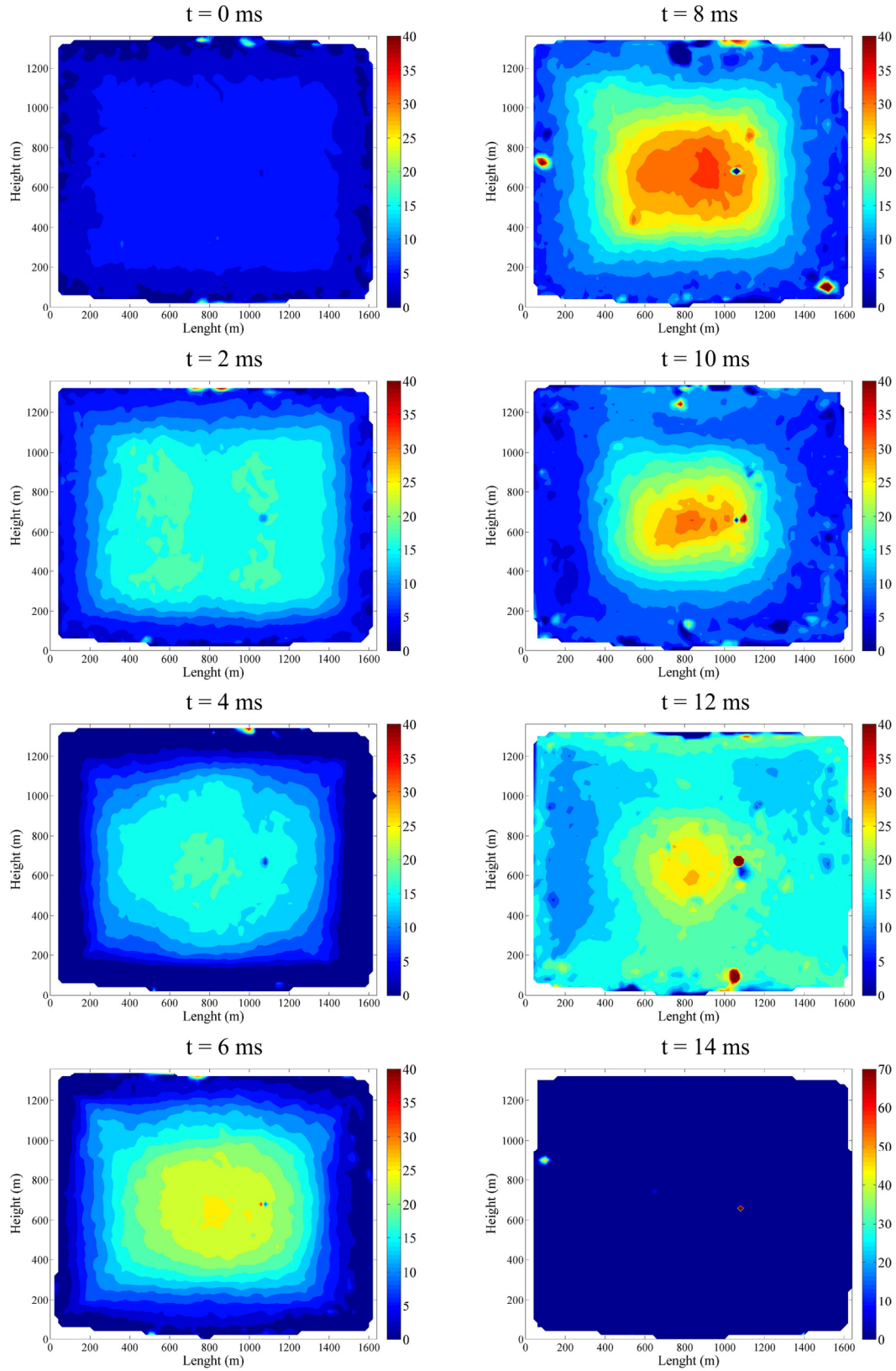


Figure 3-31: Out of plane velocities measured with the DIC method. The velocity is given in m/s. Time 0 ms is at the blast wave arrival.

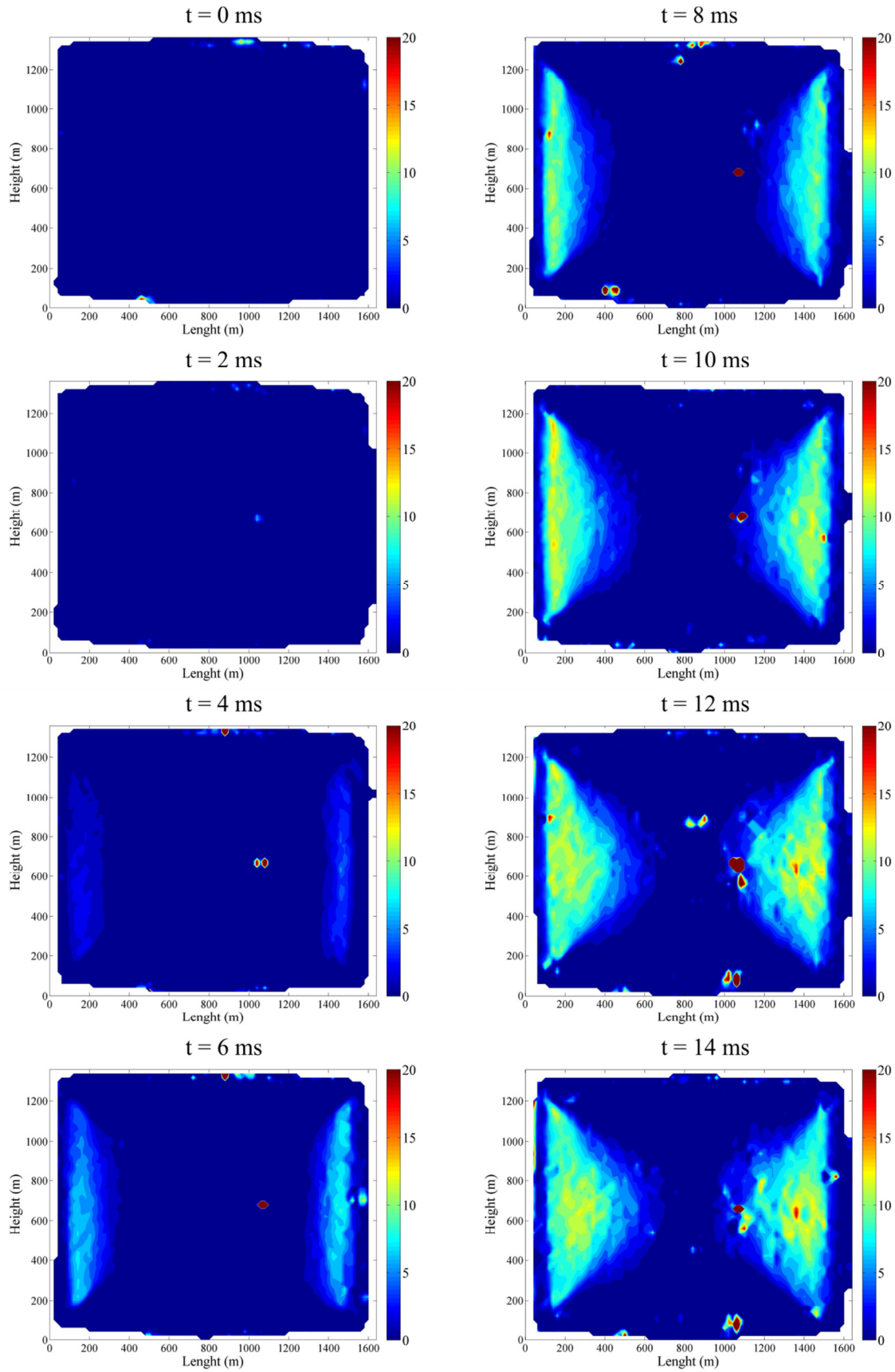


Figure 3-32: Strain in x direction calculated from the DIC deflection data. The strain is given as a percentage. Time 0 ms is at the blast wave arrival.

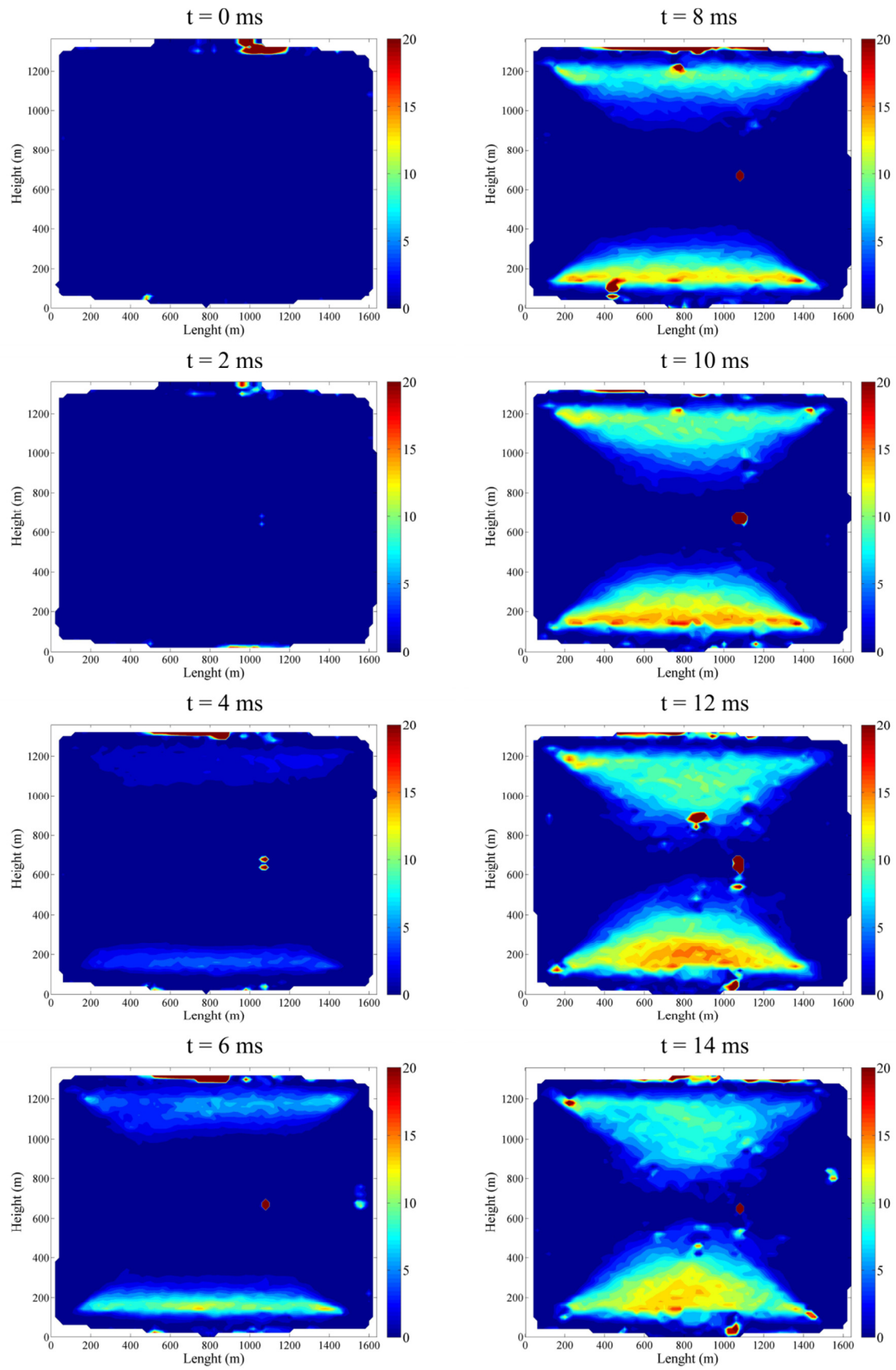


Figure 3-33: Strain in y direction calculated from the DIC deflection data. The strain is given as a percentage. Time 0 ms is at the blast wave arrival.

Figure 3-32 shows DIC results for the strain in the horizontal direction, whilst Figure 3-33 shows the vertical strain. In both cases, larger strains seemed to be concentrated along the edges. This is consistent with the observed deformed shape and curvatures described above. As can be seen, the vertical strains were larger. The average values peaked at 13%, with only limited locations displaying higher values. The edge horizontal strains showed an average of 12% strain. However it should be considered that, as observed in the comparison described in paragraph 3.3.3, the calibration method errors were bigger for this type of results. Therefore care should be taken when interpreting these exact values. Unfortunately the resolution of the picture and focus was not sufficient in this experiment to observe isolated crack locations through the strain data.

3.3.5. Reaction data

Several gauges failed during the blast test. This was due to both to atmospheric conditions during the night the window spent on site and to some cables being damaged during the positioning of the sample in the cubicle. These issues severely curtailed the amount of information available to assess the variation of reaction forces along the window edges. However, all the pairs along the top edge of the window and two on the left hand edge survived. Figure 3-35 shows the location of the failed and working gauges.

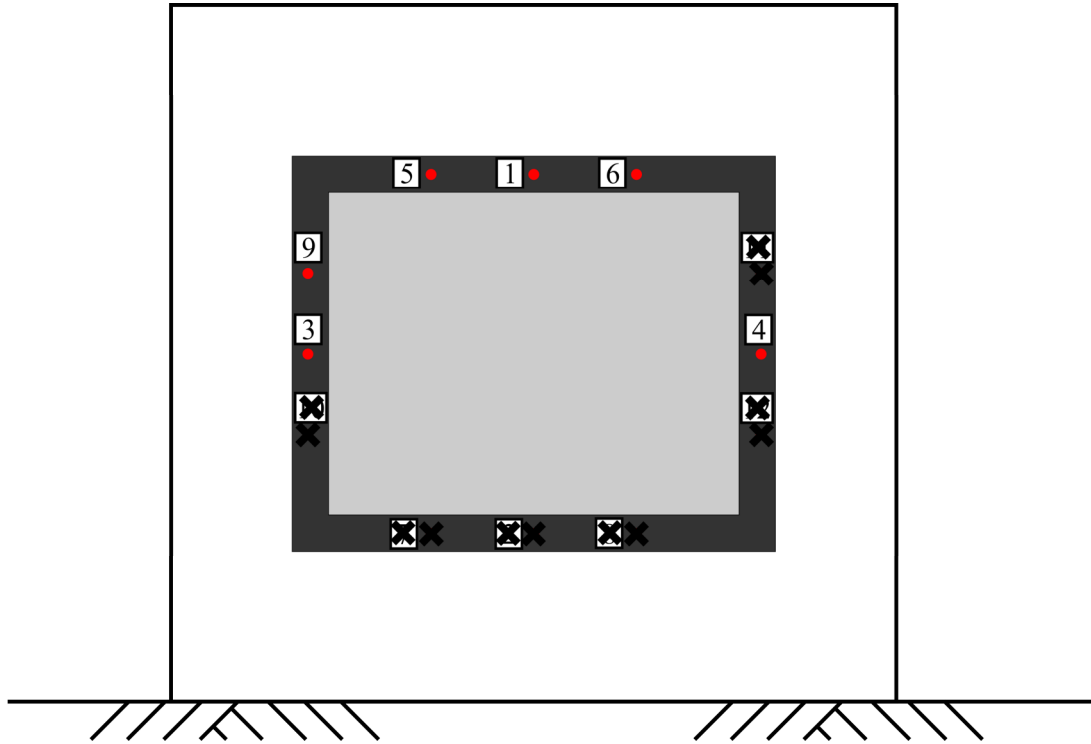


Figure 3-34: Cubicle elevation showing location of broken gauges.

The deflection angle was calculated using the DIC data. Errors within 20% are expected, in line with the comparison results illustrated above in section 3.3.3. Typical angles observed ranged between 20 and 30 degrees at the various gauge positions. The formulae described above were applied to find the reaction forces. Figure 3-35 to Figure 3-37 show some of the results, specifically those collected along the top window edge. It can be seen that the noise present in the data is high, especially during the advanced phases of the deformation. Whilst this prevents accurate reaction measurements, the relative magnitude of the forces can still be assessed. In general, all the reaction forces exhibited a plateau at high deflections, indicative of a maximum force being reached.

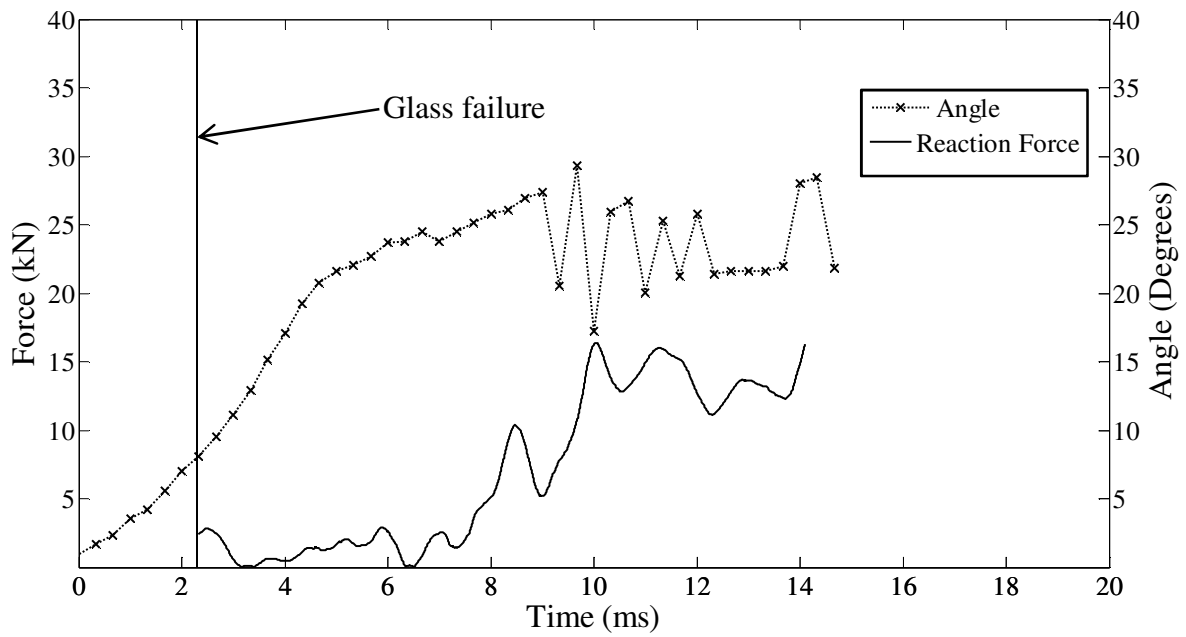


Figure 3-35: Angle and reaction force at gauge location 1 as shown in Figure 3-35. Time 0 ms is at the blast wave arrival.

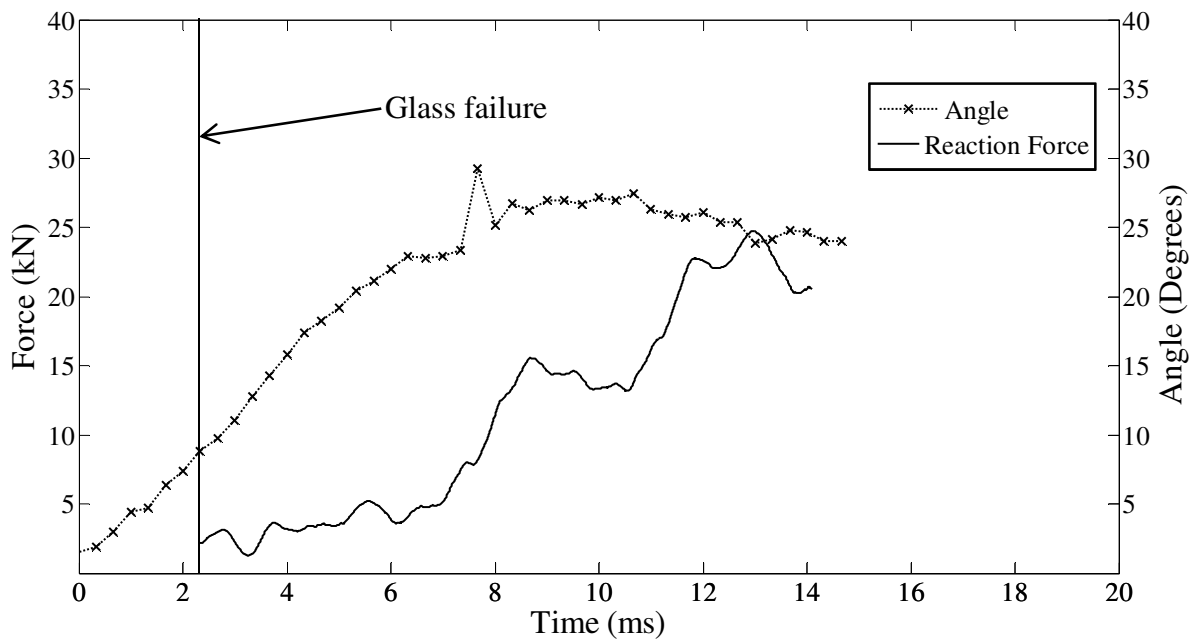


Figure 3-36: Angle and reaction force at gauge location 5 as shown in Figure 3-35. Time 0 ms is at the blast wave arrival.

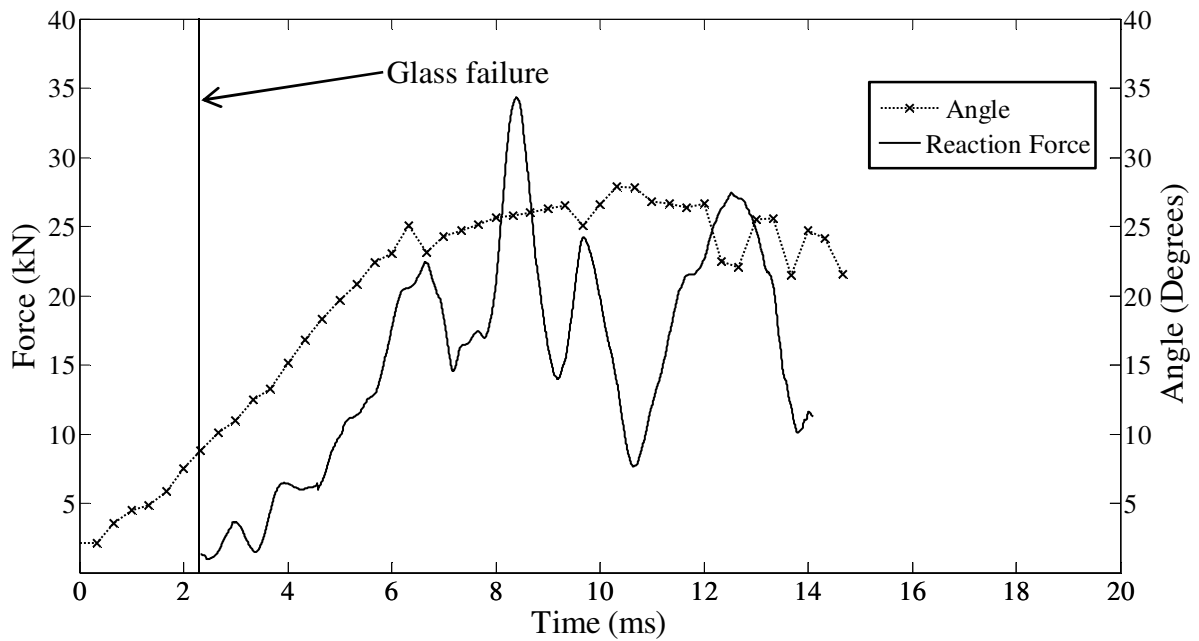


Figure 3-37: Angle and reaction force at gauge location 6 as shown in Figure 3-35. Time 0 ms is at the blast wave arrival.

Figure 3-38 shows the three forces calculated along the top edge. The plateau of the central force is lower than that measured at the quarter points. This is related to the shape of the deflection observed in previous tests [37], which would justify higher forces away from the centre of the panel. This effect will be studied in more detail. Figure 3-39 instead shows the two forces calculated along the left hand edge. The variation between these is smaller, indicating a more constant loading on the shorter edge.

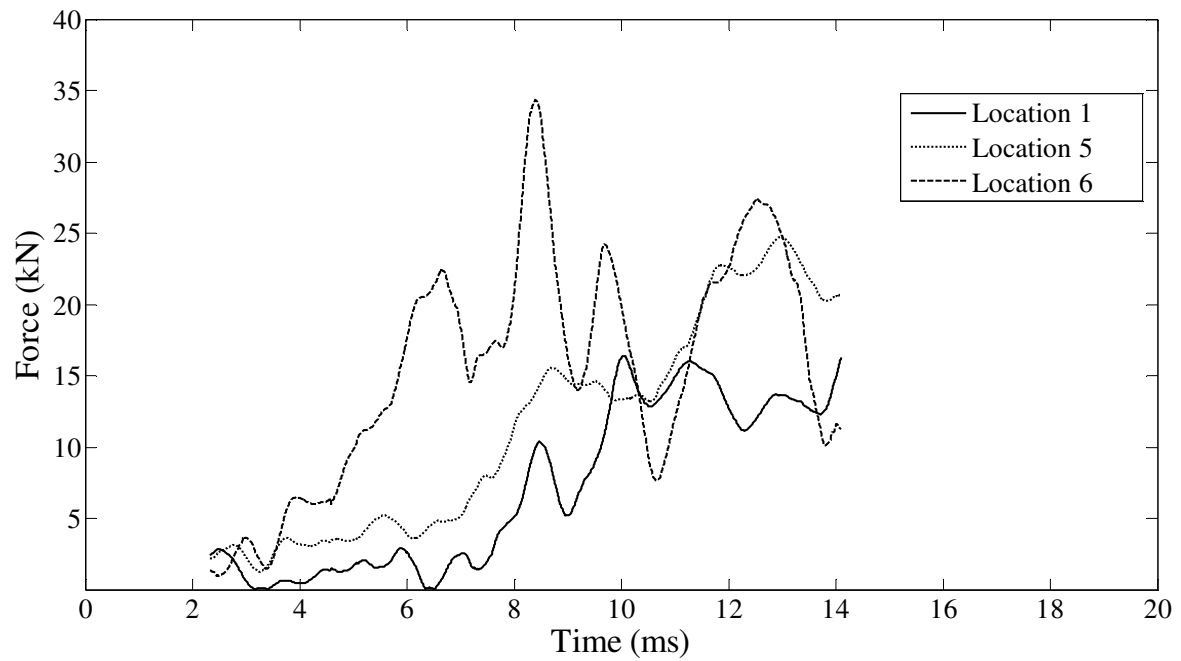


Figure 3-38: Reaction force data obtained on the top edge of the window. The gauge locations are as indicated in Figure 3-35. Time 0 ms is at the blast wave arrival.

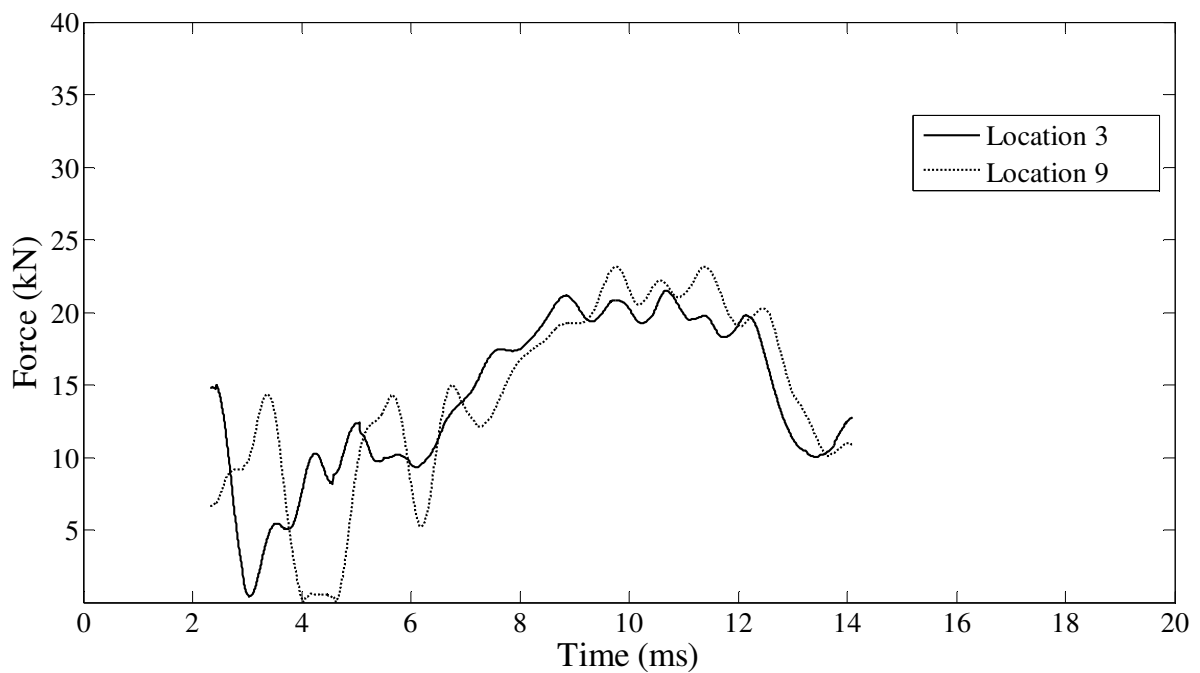


Figure 3-39: Reaction force data obtained on the left hand side edge of the window. The gauge locations are as indicated in Figure 3-35. Time 0 ms is at the blast wave arrival.

3.4. Discussion

The test pane failed both in the centre and along its bottom edge. Edge failure is typical, as that is the area where high curvature and tension forces will cause a greater amount of damage. Central tearing in the PVB is less common. In this case it was probably caused by the pre-existing crack which crossed the window diagonally. This would have weakened the local area and a tear initiated in this location at approximately the time when maximum deflections, and hence the maximum curvature in the centre, was reached.

The edge failure was due to a tear in the PVB. In the past, other failure mechanisms have been observed, including glass or silicone failures and debonding of the PVB membrane from the glass plies. However, in this test the double sided silicone joints strengthened the edge connection capacity, therefore causing a PVB tear to become more likely as a failure mechanism.

The DIC data were obtained through a non standard calibration procedure due to the failure of obtaining a calibration using the available images. When the results obtained were compared with fully calibrated results from a previous test, it could be seen that the deflection data, especially in the internal area of the window, were accurate, with errors well within 20%. These inaccuracies though increased towards the edges, highlighting uncertainties in the deflection in these areas. The velocity results showed a similar trend. However, the calculated angles showed a maximum error of about 15% at higher deflections. This, together with the 3D plots of deflection shown in Figure 3-20, might indicate that the calculated deflected shape is accurate, displaying however errors on the exact window dimensions' estimate.

The calculated strains showed a much larger error, with the estimated data in some cases being in cases twice the magnitude of the calibrated data. This is most likely due to the compounding of smaller deflection errors when the strains are calculated, together with the fact that the higher strains are recorded along the window edges where the deflection uncertainties are higher. Therefore, care must be taken when using these data for further analyses, especially if very accurate strain values are required. It is though useful to note that the trend of the data is realistic, showing similar patterns near the window edges. This will

help when using these data to calculate and compare material properties later, as the level of errors will be similar throughout the area of interest.

The deformed shape-time history shown in Figure 3-29 corresponds to those observed by Hooper in previous tests. The deformation is initially concentrated along the window edge, with a largely flat central section. As time progresses, the deformation fronts moves towards the centre, until a fully curved surface is reached. As Hooper explained, this phenomenon is influenced by the deformation speed and wave speed in the panel. Interestingly, the initial diagonal crack in the window did not affect significantly the deflection time history. This is probably due to the crack being on one side only of the window pane. Therefore, a hinge was not formed until after the glass failure. Additionally, the crack was not located in areas of high deformation, hence it can be assumed that it would not affect the overall blast capacity of the pane significantly until ultimate failure was reached.

Maximum strain locations were related to the deflected shape. Whilst, as discussed, the exact magnitude of the strains are uncertain due to the calibration technique used, the greater deformations seem to be consistently located near the window edges where the highest curvatures were also observed. This is consistent with the observed deflected shape and had been seen in previous tests as well.

As the pressure gauges were at a greater distance from the charge than the test window, it was necessary to utilise Air3D to produce loading pressure estimates for further analysis. Some differences between the experimental and computational data were noted. When the CFD analysis was run for a stand-off of 25 m, the same as that of the pressure gauges, its arrival time and peak pressures were different from the recorded ones. As mentioned, this is most likely due to limitation in the analysis software as well as in the conversion formula used to obtain experimental reflected pressures. However, as no experimental data was available at the correct distance, the Air3D data was employed for further stages of the analysis.

The reaction forces at the locations of the strain gauges were found to be similar to results previously reported by Hooper [3]. The springing angles of 20° to 30° are similar to literature data, as are the reaction magnitudes of 20-30 kN/m. However, a high level of noise was present in the data at higher deflections. Additionally, as discussed by Hooper, the reaction force calculation is sensitive to misplacement of the gauges and changes in the supports' geometry due to the blast loads effects. Also, in this case, the angles calculated from DIC included a relevant error, estimated to be up to 20%. The force calculation though proved to

be less sensitive to this uncertainty, producing an error of $\pm 15\%$ for angles of relevant magnitudes. Whilst these problems need to be considered, the data is still useful for comparison purposes. As shown in Figure 3-38, the reaction forces are higher at the quarter positions of the span. This relates well to deflected shapes described in literature, which included the presence of the two deflection peaks at some distance away from the centre of the pane. Whilst this was not detected clearly here, the lower camera resolution and non-standard calibration might have caused this effect to be hidden. The more constant force along the vertical edge also can be interpreted as evidence for this phenomenon, as in this direction the peaks would be aligned along the window centreline, removing higher order effects and producing a more uniform distribution of reaction forces.

3.5. Conclusion

A blast test was performed to assess the behaviour of a laminated glass panel under blast loading. The distribution of reaction forces along each edge was required; therefore several strain gauge pairs were applied along each edge to measure this datum at several locations. The panel was loaded with an estimated pressure of 107.0 kPa and an impulse of 405.7 kPa ms. Due to problems with the calibration images taken during the test set up, an alternative calibration method was used. Its implications were assessed employing historical data from a previous blast test. It was concluded that the deflection data obtained is accurate to within 20% of values measured using a calibration pictures in most of the window area, however the strain information is not reliable beyond order of magnitude comparisons.

The failure mechanisms observed were similar to those seen in previous tests. However, as the support condition was changed from single sided silicone to double sided, tearing of the PVB was encouraged rather than other possible mechanisms observed by Hooper. An existing glass crack crossing the window diagonally also affected the failure, initiating a PVB tear in the central area of the window.

The deflected shape and maximum deflection of the window were also comparable with other historical results. A maximum displacement of 297 mm and a maximum velocity of 34 m/s were measured. This indicates that the double sided joint and the existing crack did not affect these parameters significantly before failure.

One of the main objectives of this test was to measure reaction forces at several points on the window's perimeter. These were calculated at the strain gauge pairs' locations. Although many of these sets had failed due to damage either before or during the test, a full set of readings were available from three gauge pairs along the top horizontal edge and from two gauge pairs' along the left hand edge. Significant noise precludes a detailed analysis of the absolute magnitudes, however it could be seen that the forces along the longer edge were not constant, having higher values at the quarter points locations than at the centre. The values were more uniform along the shorter edge. This can be related to previously noted deflected shapes. These showed more than one peak, caused by higher order modes of vibration and interaction of stress waves. Generally, forces of 20-30 kN/m were recorded, which is comparable with literature data [3]. The reaction force distribution will be studied in more detail in Chapter 4, where the full reaction forces at pre and post cracking will be estimated.

4. Blast Reaction Forces

4.1. Introduction

An important consideration in designing a blast resistant glazing system is the connection between panes and its supports. During a blast, significant reaction forces are applied to the restraint mechanism, both out of the window plane and in the window plane, especially after the glass cracks. Calculating these forces is therefore a very important step in improving and optimising the design of blast resilient windows. Whilst in previous work forces at local points were found directly from strain gauge readings, ideally the whole reaction along each edge is needed to inform future design.

The objective of this part of the work was to produce such an estimate using available blast experimental data. Information from both the test described in Chapter 3 and from previous tests performed by Hooper et al. [37] have been employed. The aim was to calculate the overall reaction forces both prior to and after cracking of the glass pane. Different techniques have been used for the two situations, as in them the panel behaves in significantly different manners. Results from different tests data sets have then been compared and their implication for the window design and the behaviour of the laminated material considered.

4.2. Method

In addition to the blast test described in the previous chapter, Hooper et al. [37] performed several blast tests on laminated glass. His 2nd, 3rd and 4th tests are employed in this study. The specimens and techniques used for these were similar to those described in the previous chapter. The test panes consisted of two 3 mm thick plies of annealed glass with a 1.52 mm PVB interlayer. The explosive charges used ranged from 30 kg to 100 kg and the target was located between 8 m and 30 m away from the explosives. Data collection was achieved employing the digital image correlation techniques (DIC) to provide 3D full field information on deflections and strains in all directions. Two Photron S3 cameras were used to take images of the window at 1 ms intervals. In Hooper et al.'s tests, two window panes were tested side by side in the same cubicle, whilst in the further test a single window was placed

in a smaller cubicle. Therefore, slightly different boundary conditions applied on the two vertical sides of the windows, with the central edge showing an increased flexibility in Hooper et al.'s tests due to the cubicle construction. Figure 4-1 shows the typical DIC set up used in the historical tests. Differently from what described in Chapter 3, the cameras were placed vertically.

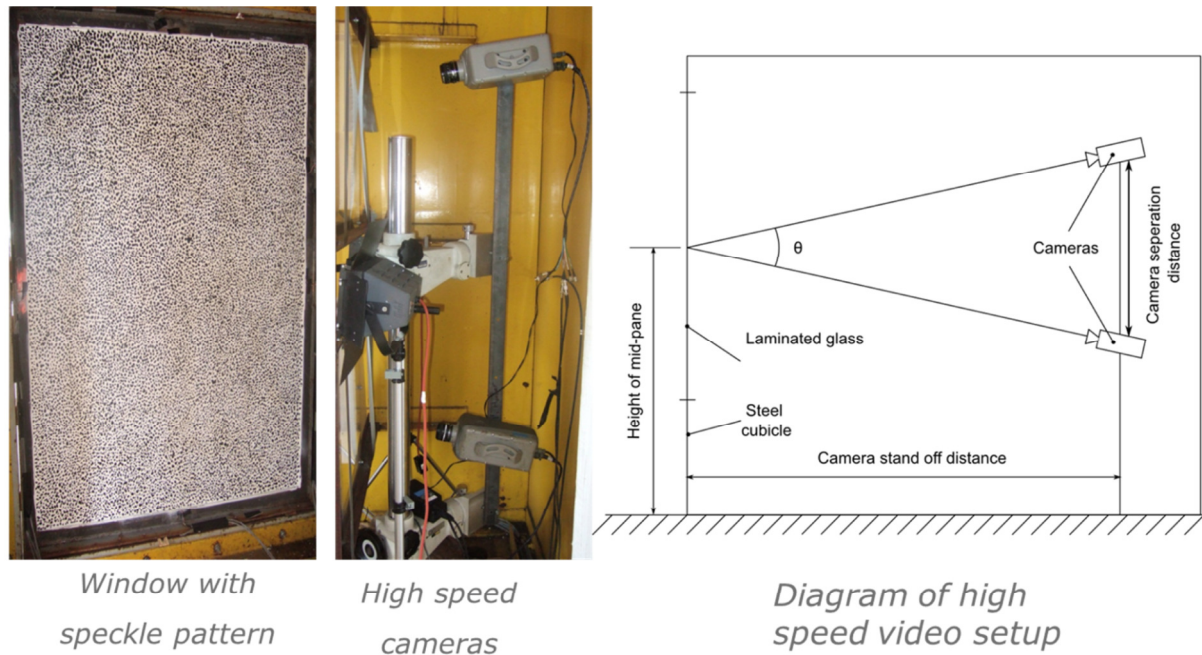


Figure 4-1: DIC set up used in Hooper et al.'s blast tests [37].

All the tests included at least four pairs of strain gauges located at the centre of each window side on the supporting steel angles of the frame. These were used to measure the local reaction force. The disposition of the gauges was similar to that used in the most recent test, shown in Figure 3-1. Therefore the only difference in this respect between Hooper's tests and the most recent one was the number of gauge pairs, four in Hooper's tests instead of the 12 used in the latest experiment reported in Chapter 3. The general characteristics of the blasts considered is summarised in Table 4-1 below. For tests 1 and 4 the pressure and impulse data were obtained from CFD simulations using Air3D, whilst experimental values were available for the other two cases.

For all the four tests considered the available 3D deflection and strain data were interpolated at regular grid points. As described in the previous chapter, it was found that as the deflections increased the DIC technique proved unable to provide data for a growing band of

facets along the edges. An extrapolation technique was therefore used to obtain the out of plane deflections. This involved firstly fitting a 4th order polynomial surface to the available interpolated information. Afterwards, the experimental data were substituted by the fitted results either where the deflected values were not found or where they presented unrealistically high positive or negative data points along the edges.

Table 4-1: Summary of main blast parameters of the tests considered in this work.

Test	Author	Charge Weight (kg)	Stand-off (m)	Peak Reflected Pressure (kPa)	Impulse (kPa x ms)
1	Hooper	15	13	31.2	284
2	Hooper	30	16	132	413
3	Hooper	30	14	152	461
4	Del Linz	100	23	107	406

Whilst it would have been ideal to employ the same technique to the strain data, their more noisy nature prevented an acceptable fit to be achieved. It was therefore considered that extrapolated points would not be realistic.

All the blast test data obtained was employed to calculate reactions of both the pre-cracked and the post-cracked pane. Due to the different behaviour of the glass in the two situations, separate approaches were employed.

4.2.1. Pre-cracked reactions

In the pre-cracked phase, the applied blast loading will generate in the panes a combination of out of plane bending and in plane membrane actions. Soon after the blast wave reaches the target, as the deflections will be small, membrane forces will be negligible in accordance with traditional small deflection theory. However, the window can deflect significantly compared with its thickness before cracking takes place. For example, in the tests considered here, the central deflection generally reached up to 40 mm before the glass shattered. In these later stages therefore the small deflection theory assumptions would not be valid and membrane forces will be significant.

If the strains on both window faces were available, the bending and direct stresses could have been separated and accounted for. However, the DIC data were only collected for one face, preventing this analysis to be performed directly. Finite element modelling was therefore used to estimate the proportion of surface stresses attributable to bending and to membrane actions at each point in the window. It was decided that, whilst the FEA values of stress and strain might not accurately reflect real deformations taking place during a test, it would be acceptable to use only the proportion between the different actions. This consideration was also strengthened by the fact that no other kind of direct measurements could be used. A pre-cracked FEA model similar to that developed by Hooper was employed. However, in this situation the simulation was developed further by including additional elements. These included the steel support structure and a layer of silicone between the glass and the supports. The model was developed using ABAQUS 6.10. The laminated glass was represented with 2D shell stress elements using reduced integration (S4R). The section was defined as a layered composite, using the real thicknesses of the 3 layers (3 mm for the glass and 1.52 mm for the PVB). The elements were of square shape, with a side length of 2 mm at the silicone interface and 4 mm elsewhere. A mesh size sensitivity analysis was run and elements of this dimension were found to be small enough to avoid size effects. Steel support angles with the same dimensions as in the experiment were added. Again, 2D reduced integration shell elements were employed for these. The element dimension varied between 2 mm in the areas of contact with the silicone and 4 mm in other areas. In this case a uniform material section was used, setting the thickness to the real steel dimensions. The layer of silicone was included using 3 dimensional stress elements with reduced integration (C3D8R). This was considered necessary due to the high through thickness deformation expected in this material. Cubic elements were used, with a side length of 2 mm. The overall silicone thickness assumed was 6 mm, with a bite depth of 20 mm. Three elements were therefore used across the 6 mm thickness to achieve the necessary quality of through thickness results. Tie conditions were set between the different parts, without any allowance for sliding. A similar model was used for test 4. A different geometry was used to reflect the actual dimensions of the glass support as well as of the double sided silicone. Additionally, approximately 4 mm elements were generally used in the steel and glass parts, whilst 4 x 2 x 2 mm elements were applied to the silicone. The dimensions were changed only in the contact areas to ensure that the mesh at the interfaces between the parts corresponded exactly. Specifically, the element dimensions in the glass and the steel were reduced to 4 x 2 mm.

The glass, PVB and steel were all given linear elastic material properties. Both the steel and the glass will behave linearly in the range of deformation of interest. Whilst the PVB would be likely to show some non linearity, its much lower stiffness implies that its contribution to load resistance can be considered negligible before the glass layers fail. Hence, for simplicity, it was also considered acceptable to employ a linear model for this element as well. Table 4-2 summarises the constants used for these three materials. The silicone, due to the expected large strains acting on it, was instead represented using a hyperelastic material model.

Table 4-2: Summary of linear elastic material properties applied in the FEA models.

Material	Material Density (kg/m ³)	Young's modulus (GPa)	Poisson's ratio
Glass	2530	72	0.22
PVB	1100	0.5	0.485
Steel	7800	200	0.3

A Mooney-Rivlin model was used. This is a hyperelastic model, able to represent the non-linear deformations of rubber materials. Its work function is given by [60]:

$$W = C_{mr1} (I_1 - 3) + C_{mr2} (I_2 - 3) \quad \text{Eq. 4-1}$$

where W is the work done by the deformation, I_1 and I_2 are the invariants of the deformation tensor and C_{mr1} and C_{mr2} are materials fitting constants determined with experimental data. The details of hyperelastic formulations will be described in more detail in Chapter 5. The constants used were derived by Meunier et al. [50], with values of $C_{mr1} = 0.14$ MPa and $C_{mr2} = 0.023$ MPa. Figure 4-2 and Figure 4-3 show a view of the overall geometry and a detail of the mesh in the area of the connection between the different parts.

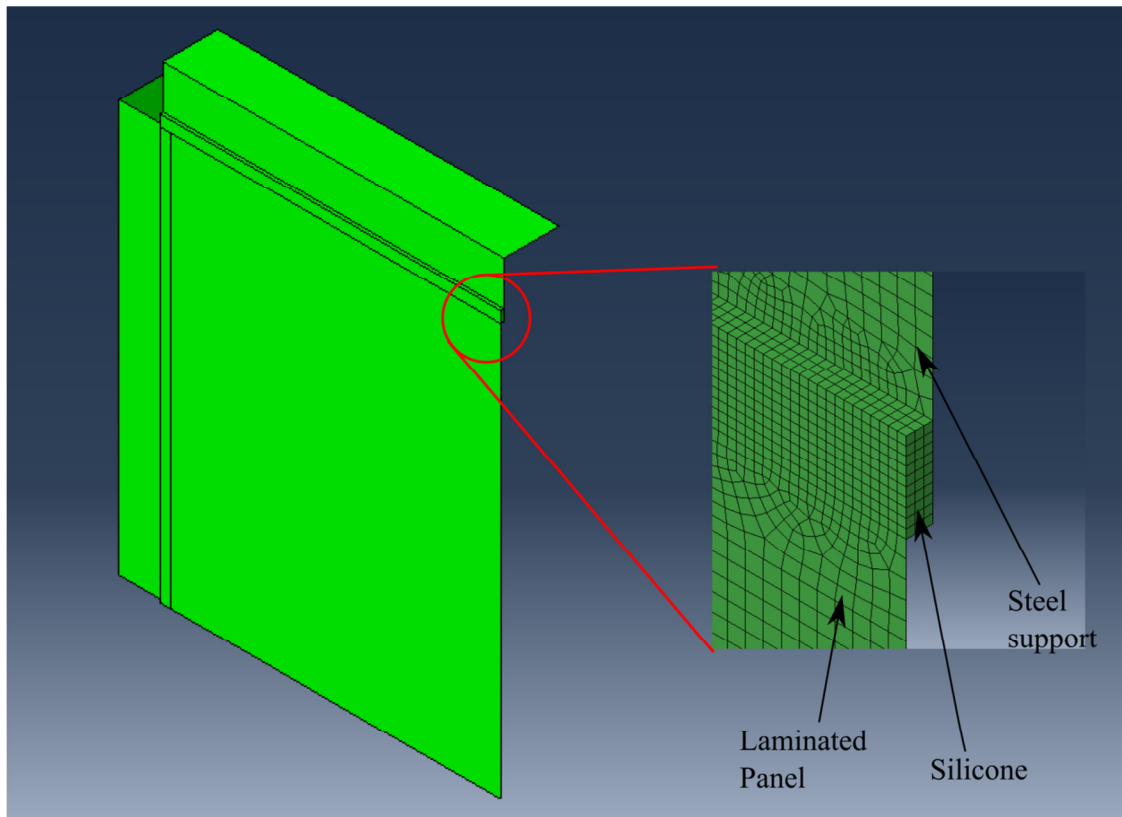


Figure 4-2: An image of the FEA model geometry employed for tests 1 to 3. The detail shows the mesh in the single sided joint area.

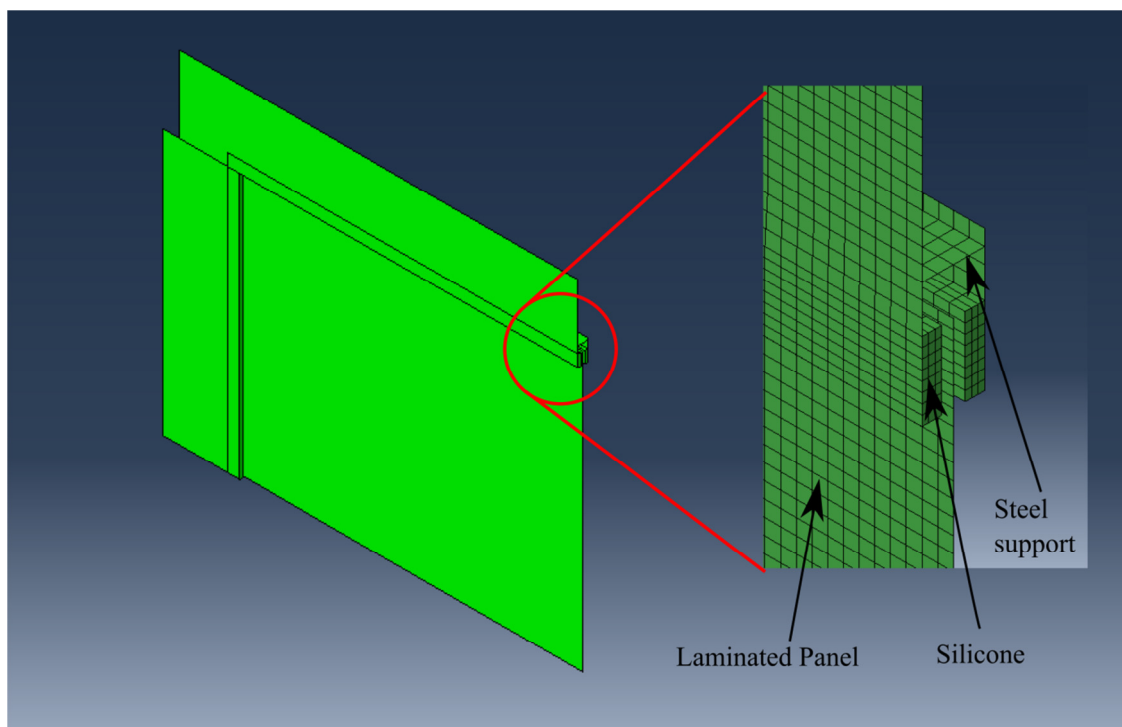


Figure 4-3: An image of the FEA model geometry employed for test 4. The detail shows the mesh in the double sided joint area.

The load was applied as a uniformly distributed pressure over the whole glass pane. Its pressure amplitude time history was estimated from computation fluid dynamics (CFD) models of the experiments. The Air3D [58] analysis code was used for this. This produced pressure estimates at 0.0001 s intervals at points on a 50 x 50 mm grid on a quarter of the window. It was observed that the pressure on the glass varied only by 10% over the whole area. Considering this and the fact that the FEA model would be used only to find the ratio of bending and axial stresses, it was decided that an average pressure would be appropriate for this application. This was therefore calculated from the Air3D output and inputted in the FEA model.

The model was set to run until a limiting stress of 80 MPa, was reached anywhere in the glass elements. The strain data were then extracted and the proportion of bending and axial strains was calculated. These proportions were then applied to the experimental DIC strain data. As a checking procedure, the central deflection was also obtained and compared with the recorded experimental data.

Once the bending strains were found, the bending stresses and moments were calculated along sections at 20 mm intervals in both directions. The shear force could then be found by numerically differentiating the moments with respect of the distance along the window's span. It was assumed that the out of plane reaction due to the bending moment at the edges would be equal to the magnitude of the shear force. As the strain data were found to be quite noisy, they were filtered to obtain a smoother output. It was assumed that the glass could not resist stresses higher than 80 MPa. Therefore, if a stress higher than this was found in the data, the stress was reduced to 80 MPa. Fourier transforms were then employed to eliminate some of the noise, eliminating higher order frequencies from the data set. Additionally, an average of the last three points of the bending moment diagram slope was employed when calculating the reaction force. The results at each facet were then numerically integrated to calculate the total reaction force along the edges of the window.

The membrane strains were converted to stresses using the same linear elastic model for the glass which was used in the FEA model. Again, Fourier transforms were employed to reduce the noise in the data and an average of the last three data points near each edge was taken to obtain edge stress values. To calculate the total membrane force at each point, the stress was multiplied by the glass plies thickness, as it was considered that the modulus of the PVB was much smaller and would provide only a very small contribution to the overall result. This contribution would therefore be negligible, especially considering the amount of noise, and

hence uncertainty, present in the data. The calculated force would be acting in the direction of the plane of the glass. To estimate the in-plane and out-of-plane components, the glass angle at each time step was calculated using the out of plane deflection data at each facet. The membrane reaction was therefore split and the out of plane component could be added to the bending reaction calculated previously.

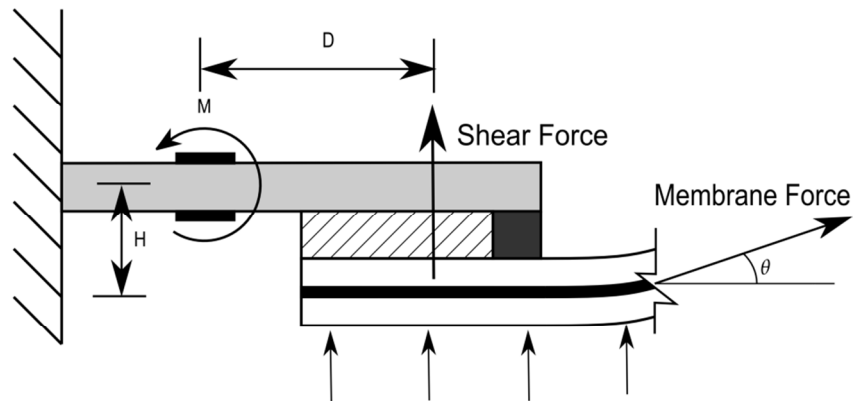


Figure 4-4: Representations of the reactions assumed in the pre glass cracking force calculation.

As a checking procedure, the moment at the strain gauges location was calculated using the recorded membrane and bending force. Figure 4-4 below shows a schematic of the forces assumed for this calculation. The shear force and out of plane membrane force were multiplied by D , whilst the in plane component of the membrane force was multiplied by H . The dimensions used for the four cases considered are listed in Table 4-3.

Table 4-3: Dimensions used for reaction calculations.

Test	D (mm)	H (mm)
1 to 3	31	9.75
4	57.5	13.5

4.2.2. Post-cracked reactions

After glass cracking takes place, it is assumed that no bending stiffness will be left in the laminated glazing. Therefore, the blast forces will be resisted entirely through membrane

action of the PVB. Hooper et al. [37] developed a method to calculate the reaction force at the gauge location, as described in detail in the previous chapter. Briefly, this employs the data from the strain gauges pairs and the glazing angle calculated with the out-of-plane deflection DIC data. Whilst in theory data from the gauges would provide sufficient information for a determinate solution of the axial and bending forces, in practice the system's sensitivity to misalignment of the gauges proved too high to reliably estimate both the bending moment and the axial force acting using solely their data. Hooper et al. showed that errors have a more significant effect on the axial force component estimate than on the bending moment. Using the DIC records to provide the angle at which the reaction force is applied allowed the membrane force in the pane to be calculated by considering only the equation for the bending moment component.

To calculate reaction forces at points along the edge away from the location of the strain gauges, a material model linking the recorded DIC strains and stresses in the laminate was required. In this research such a material model is fitted at each strain gauge pair location and then applied along the edge on which the pair is located.

To obtain a stress strain curve for the cracked laminate, an analysis similar to Hooper et al.'s was performed at the gauges locations. The resulting force, divided by the PVB thickness, provided the stresses in the glass at each time point. The DIC strains at the locations nearest to the gauges were then extracted, averaging the results over an area of 3 x 3 facets to reduce the effects of noise. This enabled a stress strain curve to be plotted for each edge, and a chosen material model to be fitted using a least squares difference approach. The calculated strain was also differentiated with respect to time to provide strain rate estimates for use in the fit. In test 4, this analysis could not be performed on one window edge due to the failure of the pair of strain gauges. The data obtained on the top edge was applied to both locations to obtain useful results.

As the stress-strain data showed that the stress reached a plateau after an initial stiff linear increase, a plastic material model was used to represent the composite material. It is realised that this model does not reflect a real physical characteristic of the PVB material, which in itself does not show plastic behaviour in the same way as metals do. However, it was considered that for the purpose of this analysis such a model would be useful for capturing the shape of the observed data. Similarly to the method applied by Hooper et al. for their FEA models [37], a Johnson-Cook [61] model was used, whose constitutive equation is:

$$\sigma = \left(A + B \varepsilon_p^n \right) \left(1 + C \ln \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right) \quad \text{Eq. 4-2}$$

Where A , B , C and n are constants to be determined, ε_p is the plastic strain, $\dot{\varepsilon}$ is the strain rate and $\dot{\varepsilon}_0$ is a reference strain rate. In this case a reference rate of 1 s^{-1} was assumed.

This model provided enough flexibility to account for rate effects and was at the same time straightforward to apply. Since, as mentioned above, plasticity models are only used to represent the observed shape of the data, it was decided that it would not be appropriate to employ more sophisticated, physically based models in this occasion.

Once the model was calibrated, the stress levels at each point could easily be found, allowing the calculation of the membrane force throughout the perimeter. Again, the glass springing angle was calculated at each point along the edge to split the total reaction force in the glass into out of plane and in plane reactions in the frame.

4.3. Results

4.3.1. DIC Results - Extrapolation of deflection data

The original out of plane deflection data were extrapolated to cover areas where the DIC analysis did not produce results, as described above. Figure 4-5 below shows a plot of the original data and a plot of the interpolated deflections for one of the blasts. Sets of data obtained in this manner were employed for further steps of the analysis.

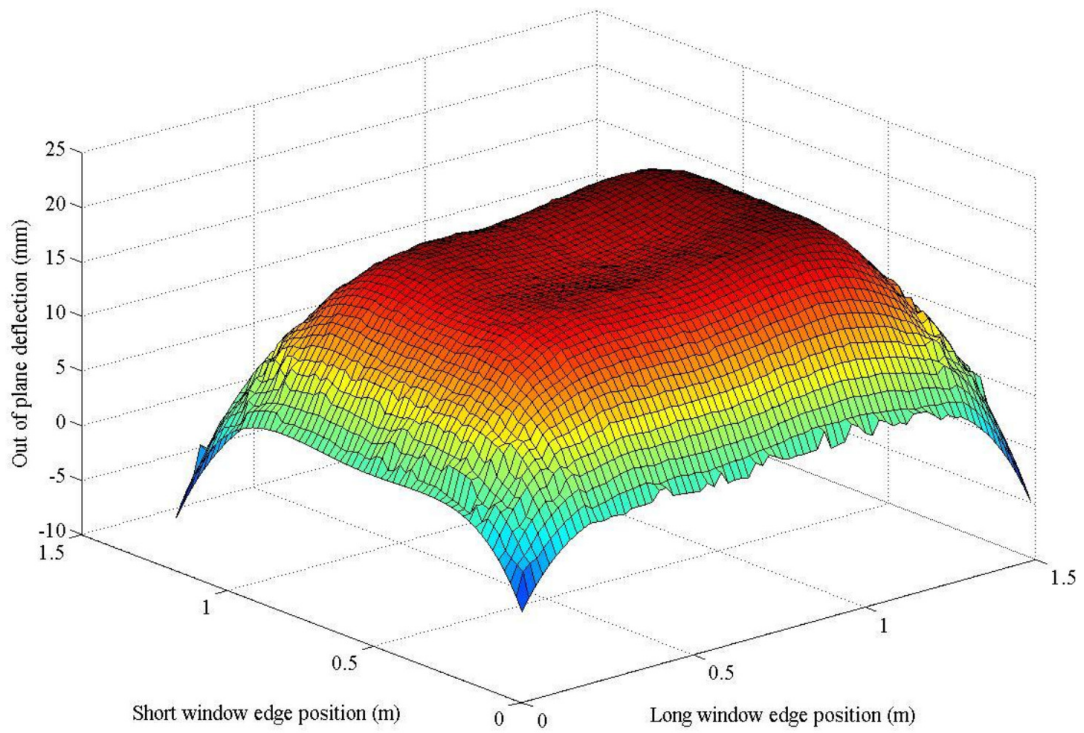
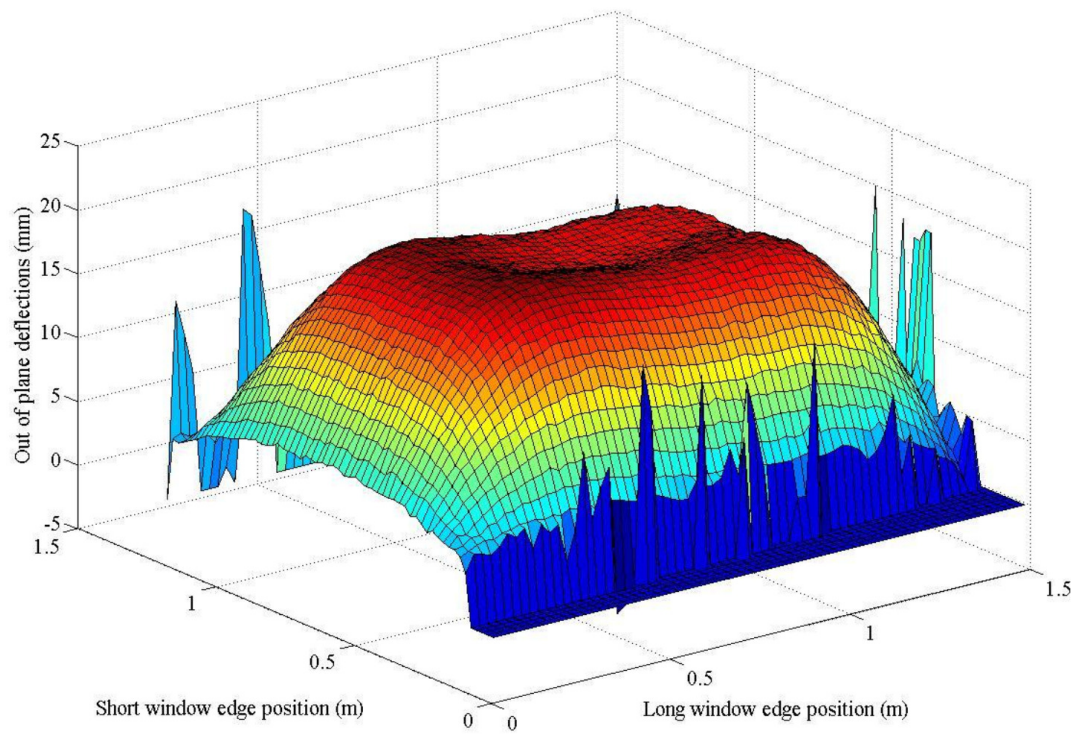


Figure 4-5: Original out of plane deflection data (top) and the same data with missing and unrealistic points substituted (bottom).

4.3.2. Pre-cracked reactions

As discussed above, the proportions of surface strains due to out of plane bending and in plane membrane forces were calculated using a FEA model of the glass pre-cracking phase. Figure 4-6 and Figure 4-7 show sample plots of the central deflection time history obtained from the FEA model and the DIC data for Tests 2 and 4. In general, the FEA deflections are lower than the recorded ones, however the trends are similar. The results for the latest test show a larger discrepancy, although the trend seems to follow the experimental data. In Hooper's test data shown in Figure 4-6, the central deflection reached at the glass cracking condition was 22.2 mm, which is compatible with recorded test values, in which failure took place at 12 mm to 35 mm as shown in Figure 4-18 to Figure 4-21 below. This comparison of the measured and estimated failure deflections produces the same result for all of Hooper's cases, whilst the latest blast test shows a larger discrepancy. This will be discussed below.

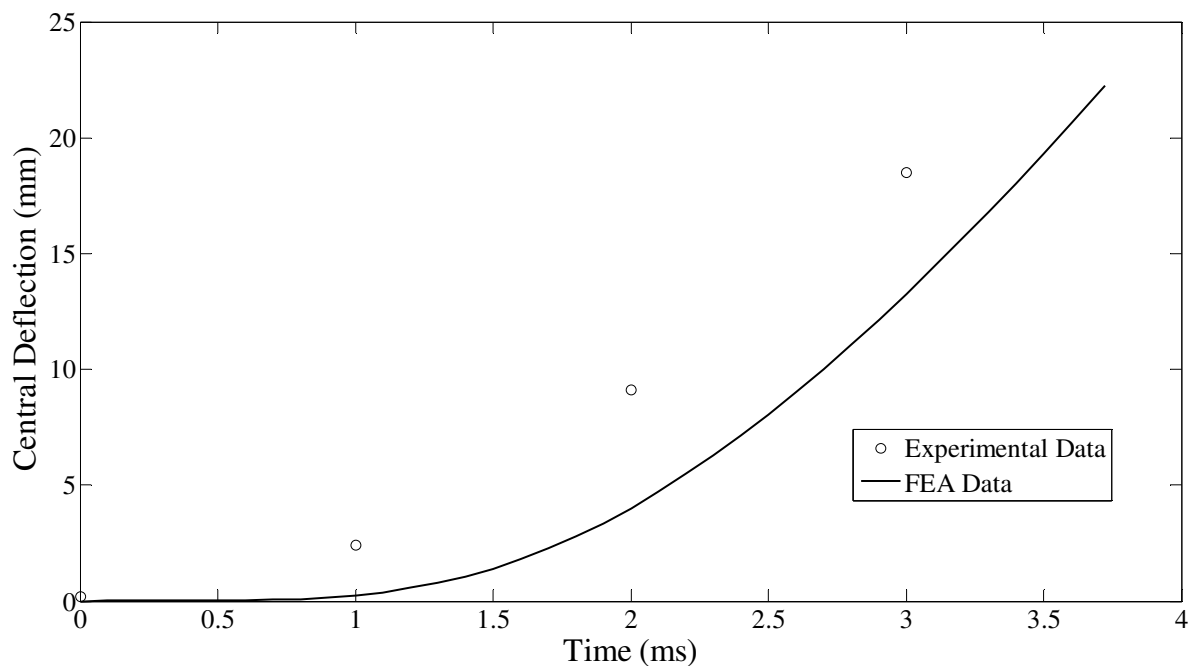


Figure 4-6: Comparison of central deflection obtained through DIC and FEA modelling for Test 2. Time = 0 ms is the blast wave arrival time.

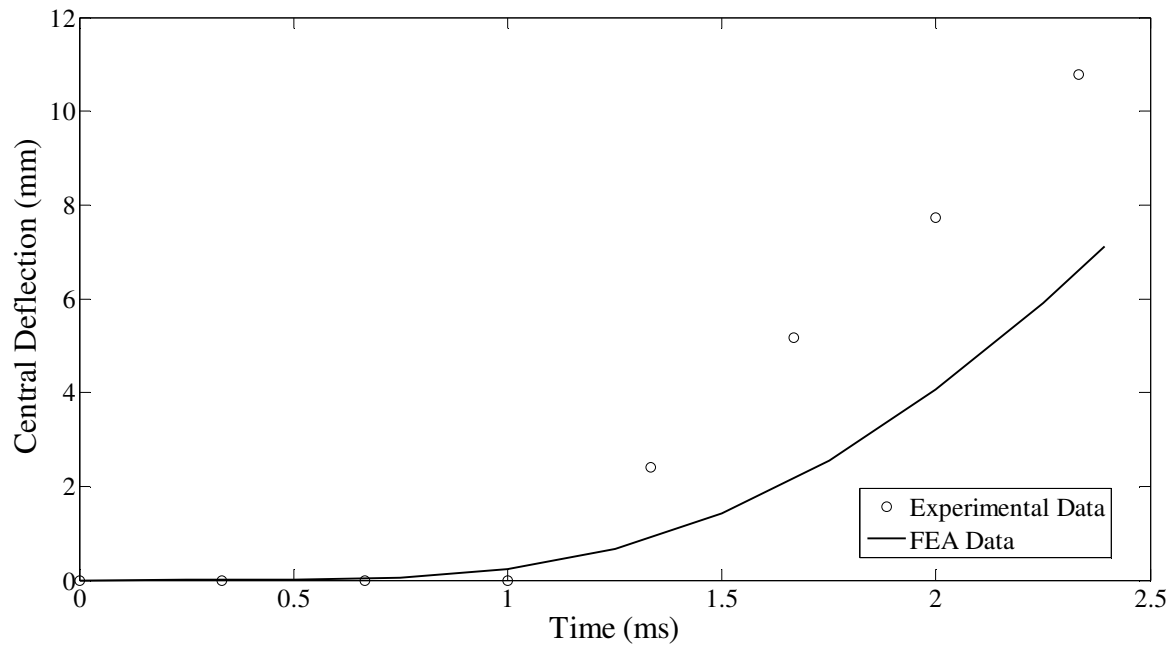


Figure 4-7: Comparison of central deflection obtained through DIC and FEA modelling for Test 4. Time = 0 ms is the blast wave arrival time.

Figure 4-8 presents the proportion of stresses due to bending moments and membrane forces found through the FEA model for Test 2 as an example. The proportion of bending strain was found to be higher towards the edges of the panel, whilst it decreased to almost zero in the central area. The membrane proportion shows the opposite behaviour, rising significantly in the centre of the window.

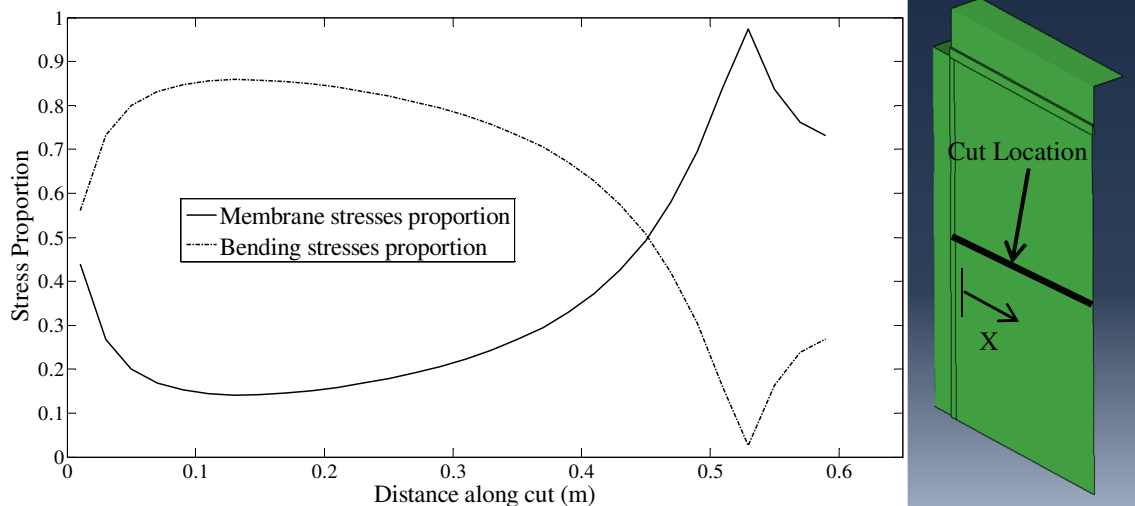


Figure 4-8: Proportion of the stresses due to bending moments and membrane forces found with the FEA model for Test 2.

These proportions were applied to the recorded DIC strains, and the strain along each cut in both directions considered in turn. As described above, the data so obtained were filtered to improve the shear force estimate. Figure 4-9 shows a typical plot of the bending stresses across a cut, showing the original data and the filtered result.

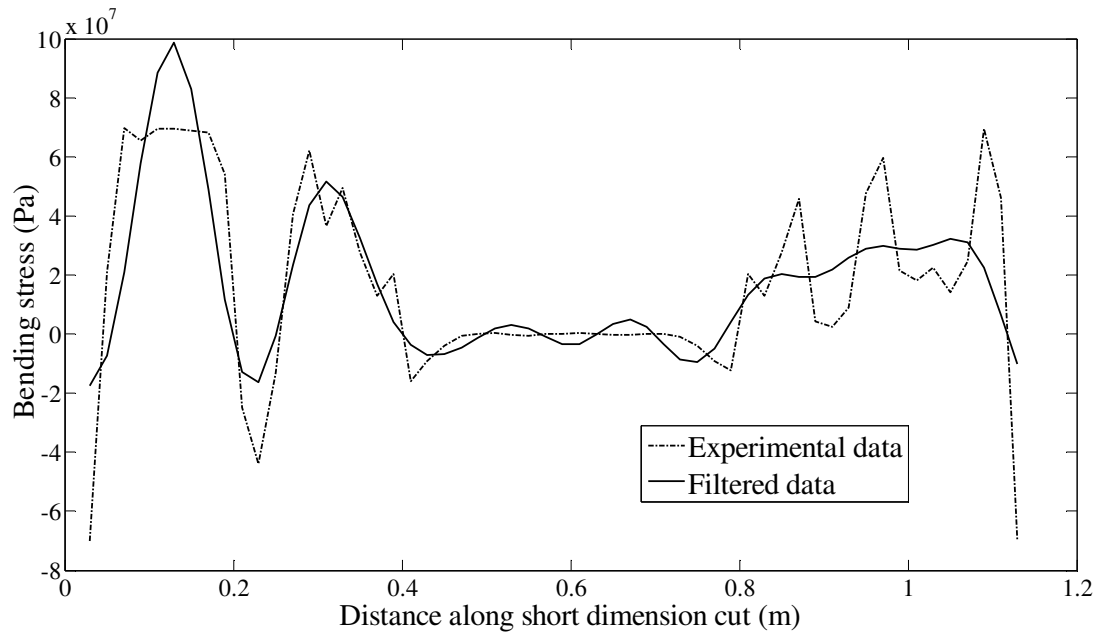


Figure 4-9: Samples of bending stress data. Both the original DIC data and the filtered curve are shown.

The plot shows that the maximum bending strains are reached near the supports, tapering to the lowest values towards the centre of the panel. This behaviour is typical and can be seen throughout the specimens, although it becomes more pronounced as the time increases. The membrane stresses were calculated using the same filtering techniques.

The overall out-of-plane reactions calculated along one of the edges are presented in Figure 4-10. It can be seen that the membrane force component is significantly lower, even though it increases as the loading progresses.

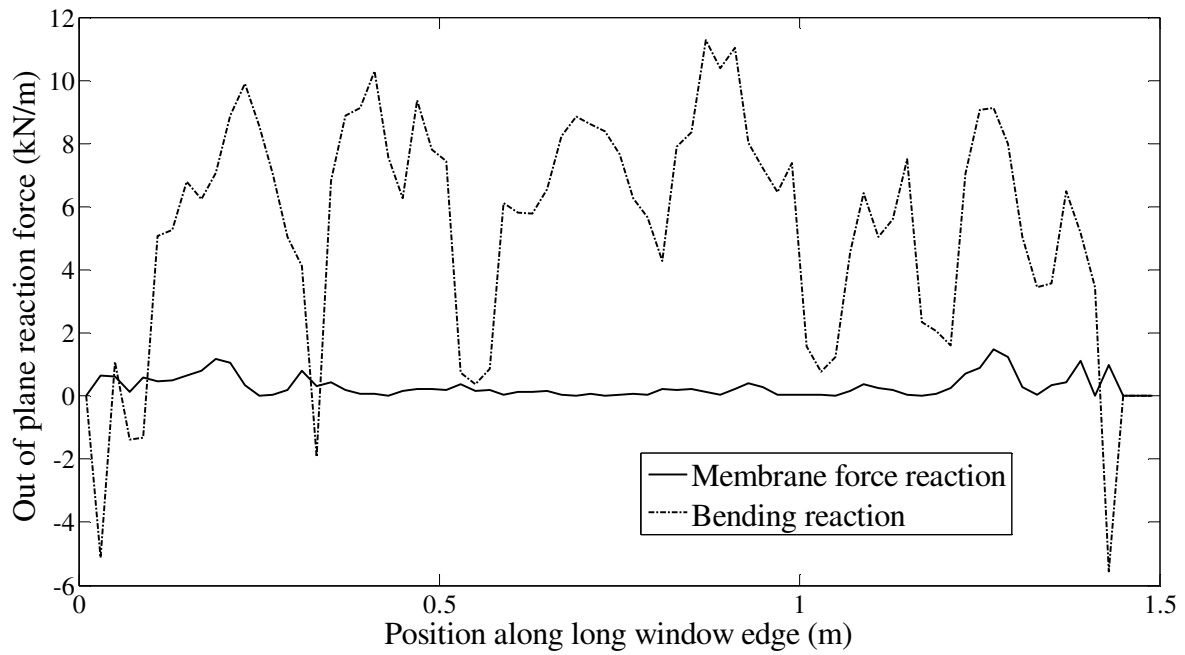


Figure 4-10: Out-of-plane reaction force along one edge. The bending and membrane components are shown separately.

As a checking procedure, the in-plane and out-of-plane reactions were employed to back calculate the bending moments at the strain gauge locations. Figure 4-11 and Figure 4-12 show examples of the results obtained, including both the moment calculated from gauge data and from the analytical results. The two sample graphs show that the quality of the results is variable, with the fit showing a good agreement in some locations, whilst it differs significantly from the measured reactions at others.

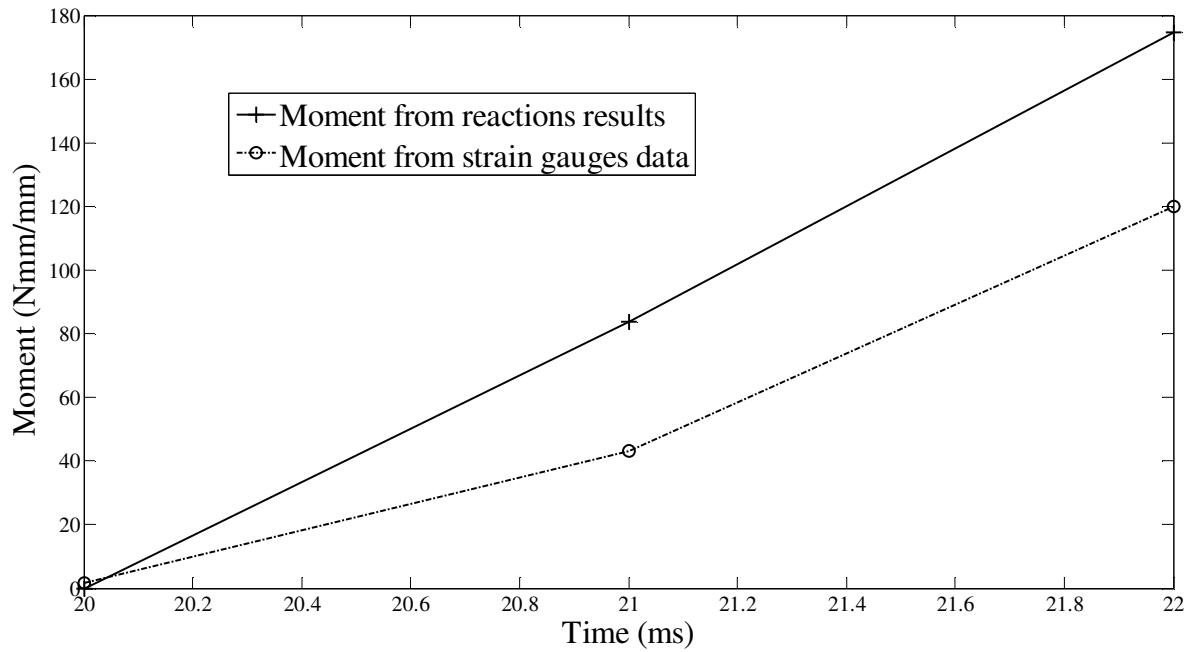


Figure 4-11: Bending moment at the gauges location on the bottom side of the panel in Test 1 calculated from the gauges results and from the analysis results.

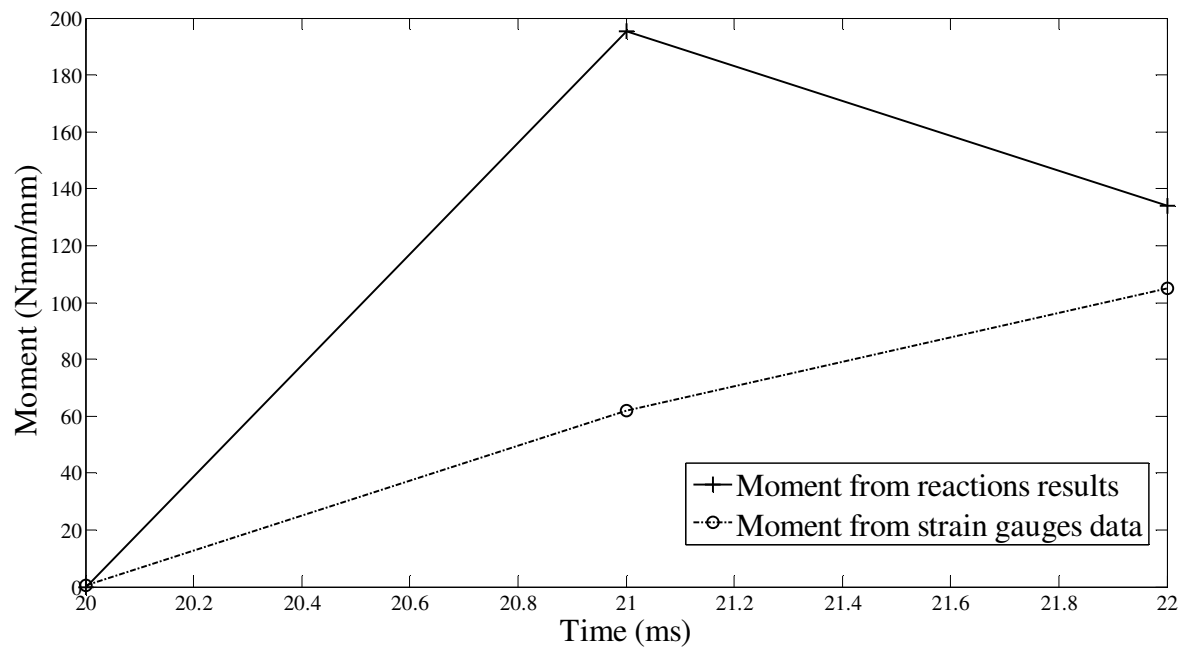


Figure 4-12: Bending moment at the gauges location on the top side of the panel in Test 1 calculated from the gauges results and from the analysis results.

The overall reaction results will be shown below, together with the post crack reaction results.

4.3.3. Post crack reactions

The material model was fitted to stress strain curves produced as detailed above. A typical stress strain curve together with the data fit is shown below in Figure 4-13.

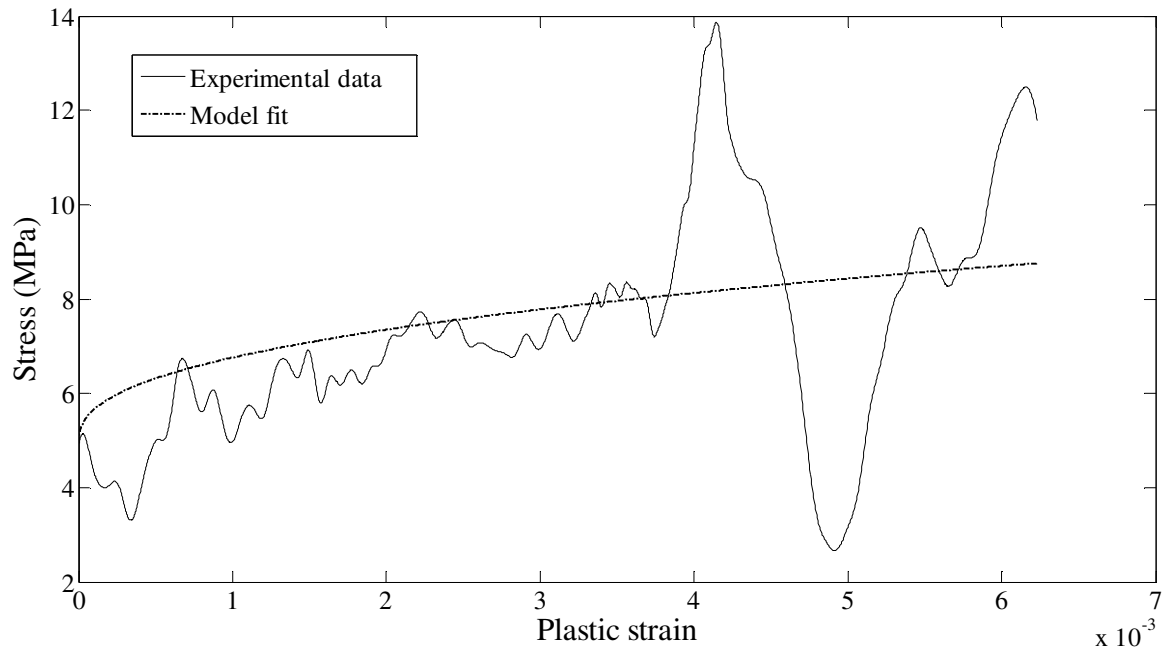


Figure 4-13: Stress strain curves for the estimate of cracked glass material model.

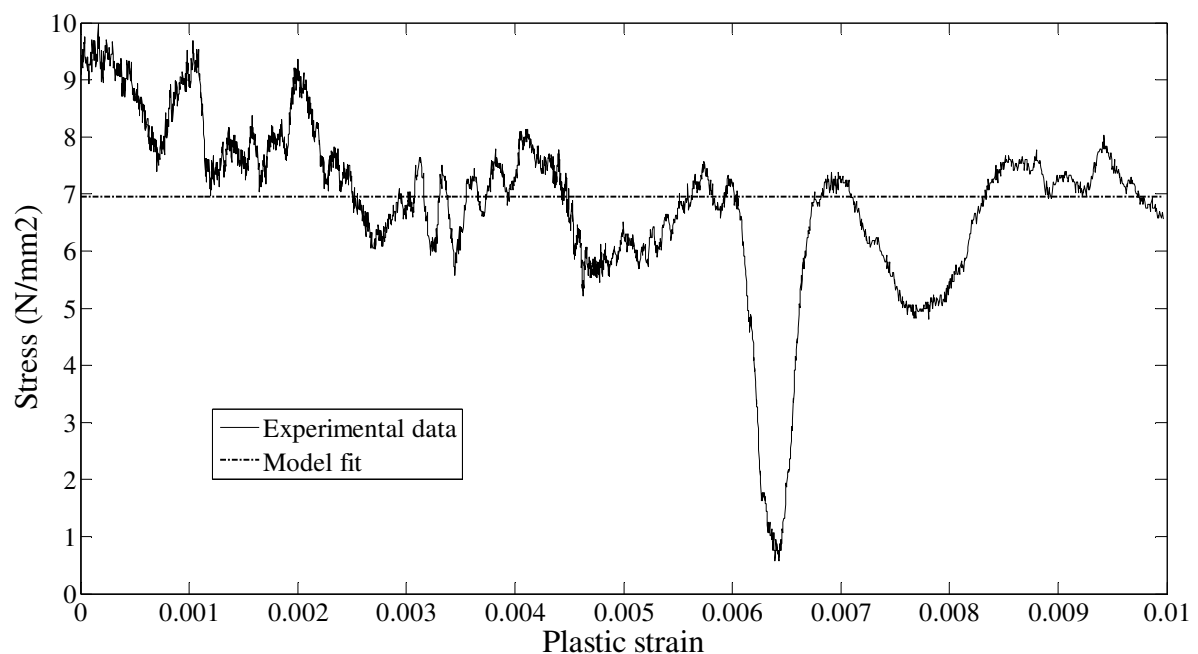


Figure 4-14: Stress strain curve and model fit for Test 1 the top of the window frame.

As can be seen, the data were noisy. Additionally, a few data sets did not display the typical expected behaviour, as can be seen in Figure 4-14. As a result of this, in some instances the best fit found for the material model was a straight line. Table 4-4 below shows the Johnson-Cook material model coefficients obtained for Test 2 as a typical example. Figure 4-17 shows a legend for the location of the sides. In general, it can be observed that the rate dependency was not strong, probably due to the lack of data at well defined strain rates. Also, in some cases a straight line provided the best fit for the available data, as is indicated by the constant n being equal to 0.

Table 4-4: Johnson Cook fit coefficients for Test 2. The results for each window side are shown. Refer to Figure 4-17 for a legend of the sides' locations.

Side	A (MPa)	B (MPa)	C	n
Top	9.38	2.32	0.0	0
Bottom	7.37	6.01	0.0	0.11
Wall	7.01	0.84	0.0	0
Central	3.6	7.76	0.11	0.05

These models were employed on their respective sides to calculate the reactions at all points. Figure 4-15 shows a plot of the reactions along an edge of one of the windows at two time steps. The reactions seem to reach two maxima near the ends of the window side.

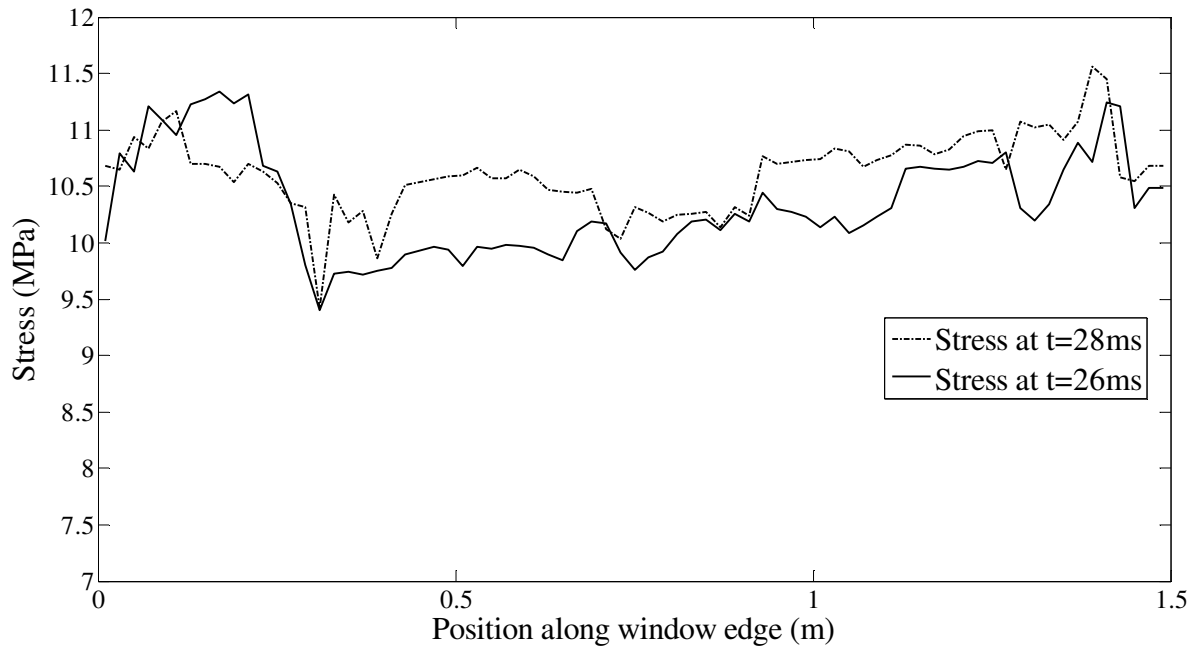


Figure 4-15: PVB reaction stress along the bottom edge of Test 1 at time steps $t = 26$ ms and $t = 28$ ms.

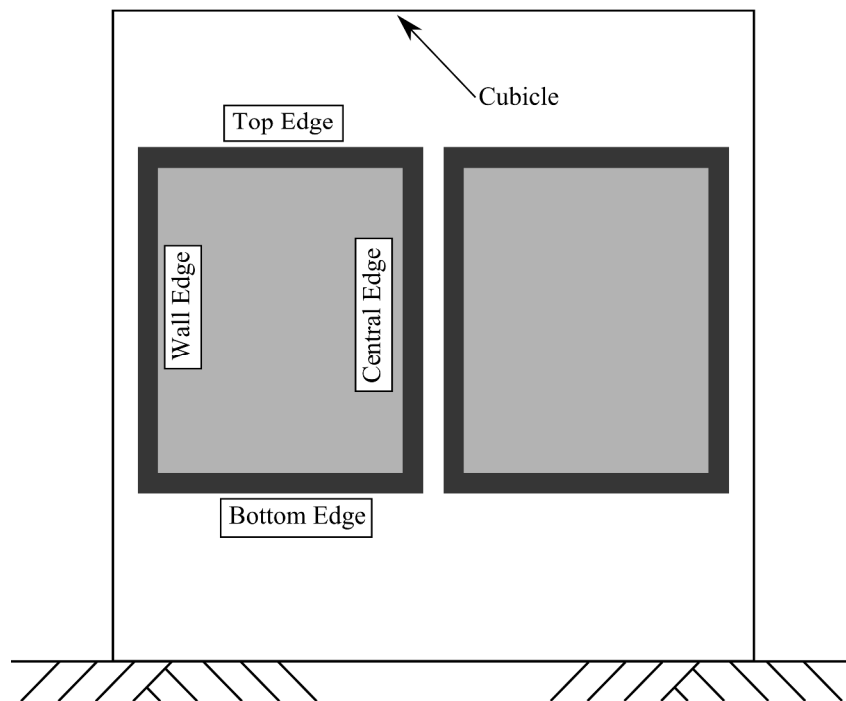


Figure 4-16: Legend of edge locations for reaction data plots.

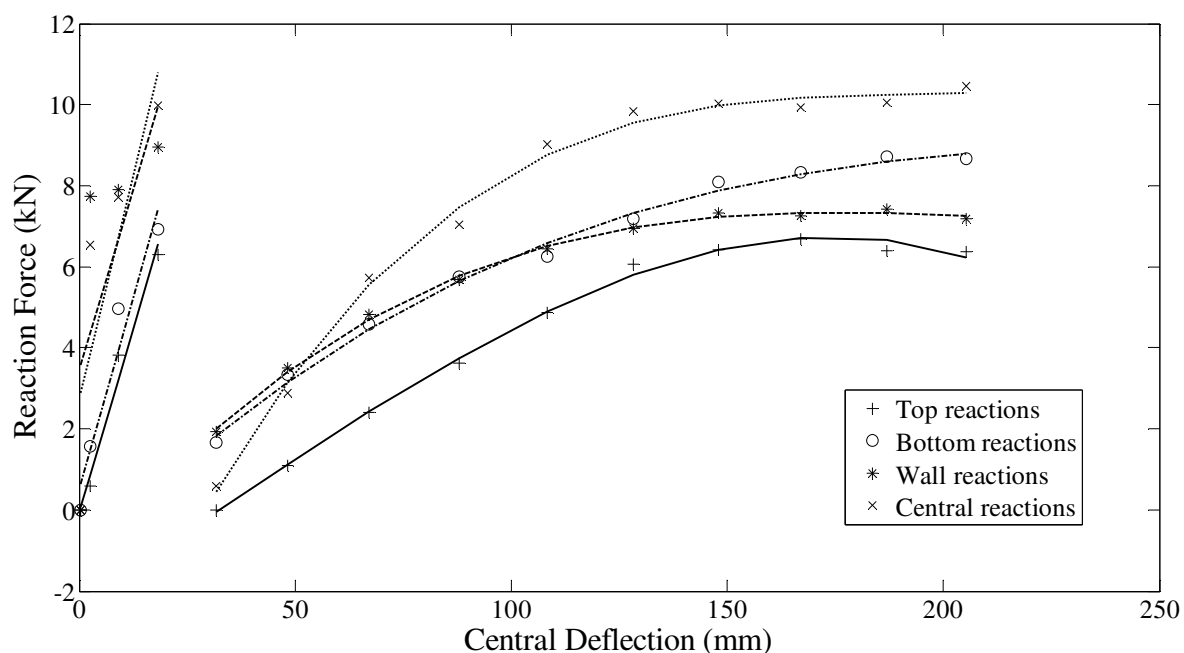


Figure 4-17: Out of plane reactions on individual panel sides for Test 3. The edge locations are shown in Figure 4-16.

Figure 4-17 shows the out of plane reactions on each side of the glazing panel during Test 2. The legend of the edge locations is shown in Figure 4-16. It can be seen that the forces reach a first peak during the pre-cracked phase, up to a central displacement of approximately 30 mm. After dropping they increase again, reaching a plateau level at higher displacements. The reactions of the wall and central sides reach higher values as these sides were significantly longer. The total out of plane reaction forces are shown in Figure 4-18 to Figure 4-21. As seen for the individual sides, in all four cases the edge reactions reduce significantly when the glass shatters. The pre-cracked reactions in tests 1 to 3 peaked between 35 and 50 kN. This could be due to the maximum capacity of the window before the failure of the glass is reached. In test 4 this peak was slightly higher at approximately 55 kN, possibly due to differences in the test set up. The post-cracked reaction magnitude instead varied between the tests, ranging between 21 kN in the test 4 and almost 55 kN in test 3. In all cases the reaction force approached a plateau level. This was similar to the observed behaviour of the laminated glass material. The results for test 4 showed a significantly greater level of noise, due to the different recording equipment used. The general shape of the results and magnitude of the loads was however similar to what was measured in the previous tests.

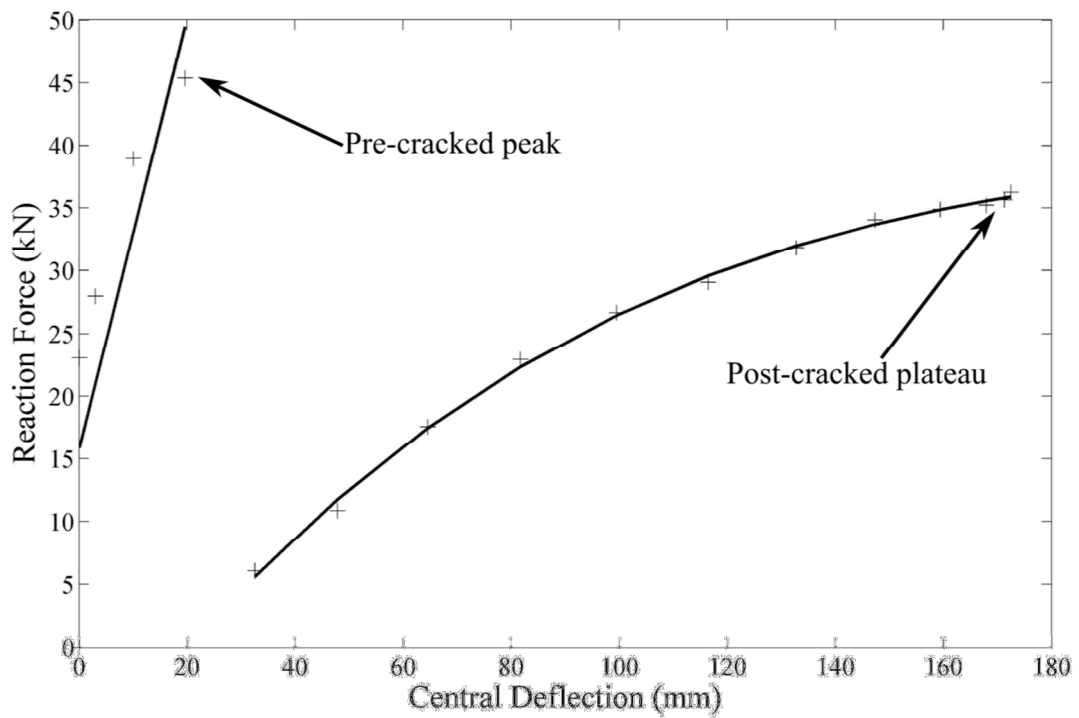


Figure 4-18: Total out of plane reactions forces for Test 1. The pre crack peak at low deflection is apparent.

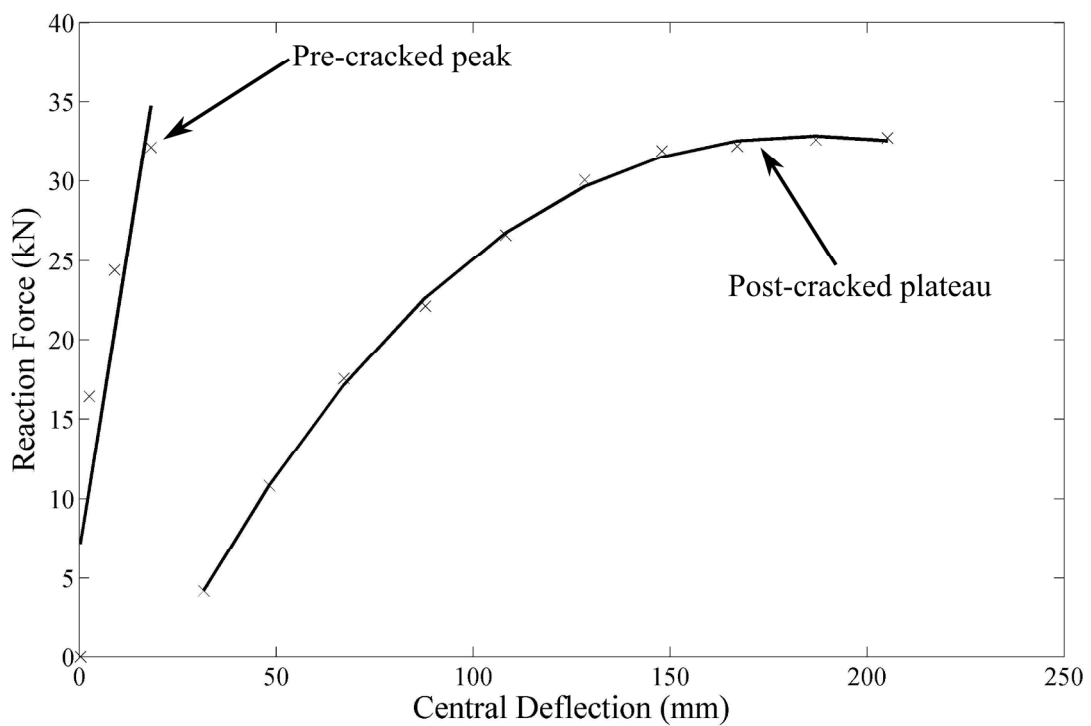


Figure 4-19: Total out of plane reactions forces for Test 2. The pre crack peak at low deflection is apparent.

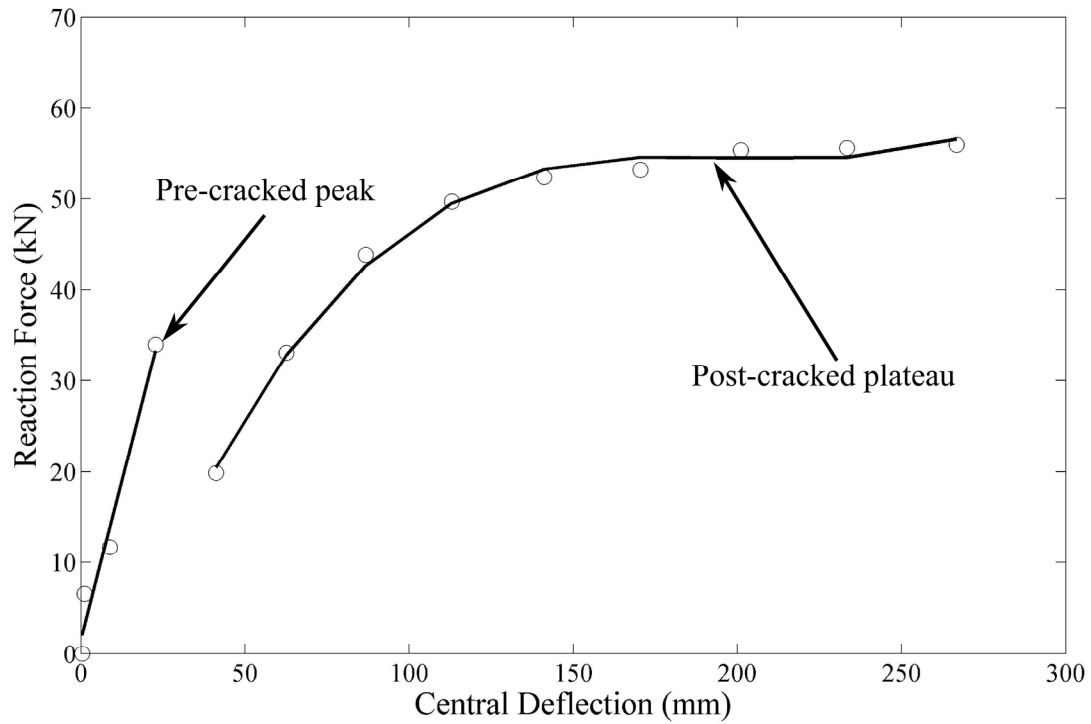


Figure 4-20: Total out of plane reactions forces for Test 3. The pre crack peak at low deflection is apparent.

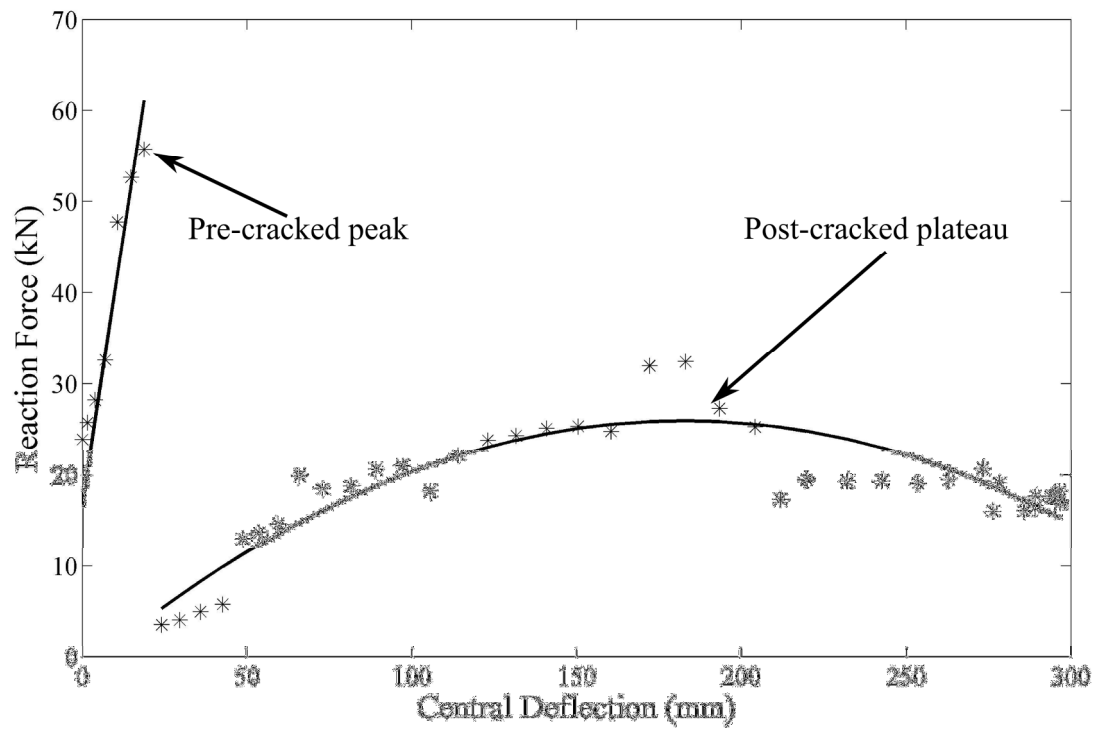


Figure 4-21: Total out of plane reactions forces for Test 4. The pre crack peak at low deflection is apparent.

The in plane forces were also considered. For these, pre-cracked reactions have been ignored. They would be dependant only on the small membrane forces present, and noise would have prevented achieving a realistic estimate. The results showed a somewhat different behaviour. A plateau level was reached as in the out of plane case, however the rise time was very short and the forces were significantly more consistent as the deflection increased.

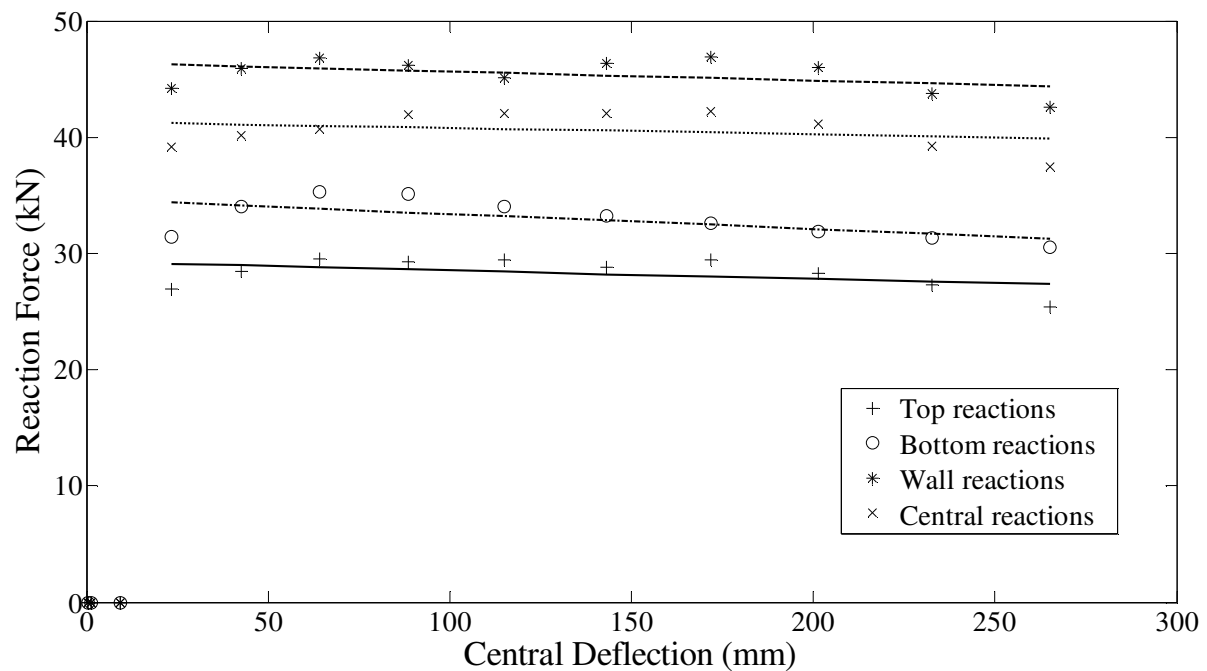


Figure 4-22: Total in plane edge reaction force along each edge for Test 3. See Figure 4-16 for the location of the edges.

Figure 4-22 shows a typical split between the reactions on the four window sides during Test 3. It can be observed that the shorter sides, the top and the bottom, exhibited a lower total reaction force. The highest reaction force acted on the cubicle's wall side, due to its higher stiffness. This distribution, whilst likely, is not always realised, as other factors, such as the blast pressure distributions can affect the reaction forces. As an example, the results shown in Figure 4-17 display a different distribution of forces.

Figure 4-23 to Figure 4-26 present the total in plane force results for all the blasts considered. As mentioned, the rise time of the reactions was lower than the time difference between the images, 0.3 ms in the latest test, after the failure of the glass for most of the blasts. Test 2, in Figure 4-24, showed reactions which increased as the deflection increases. However, this gradual increase was still small compared with the initial magnitude. The out-of-plane pre-

cracked peak forces and plateau levels and in plane plateau levels are summarised in Table 4-5.

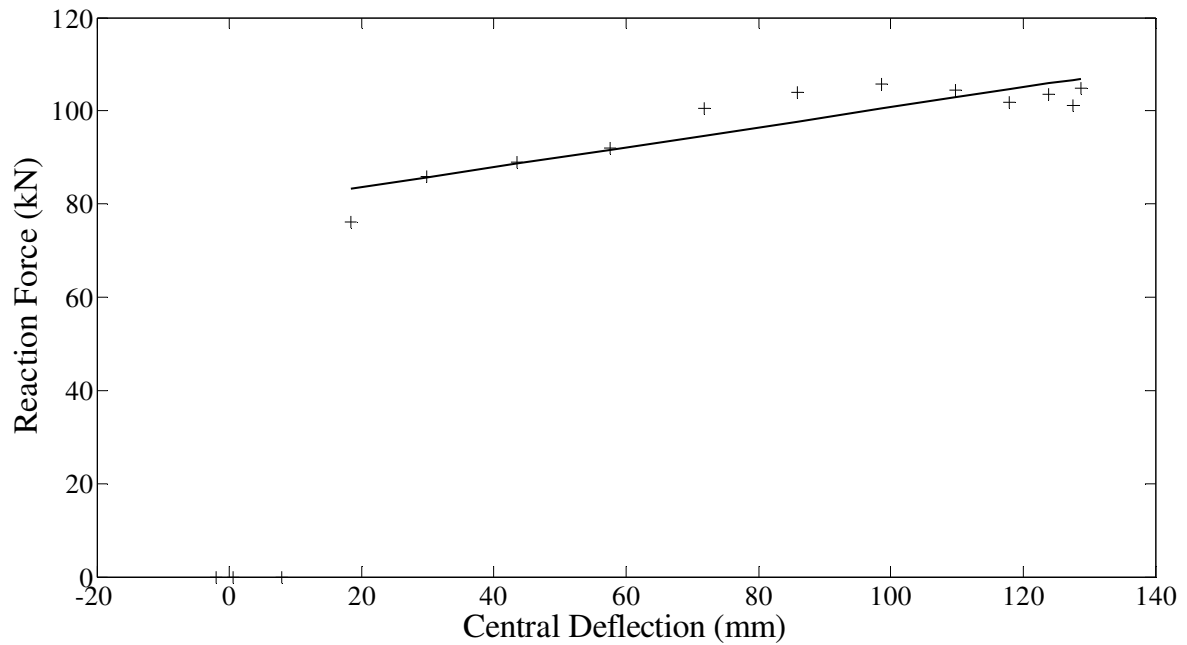


Figure 4-23: Total in plane reactions for Test 1.

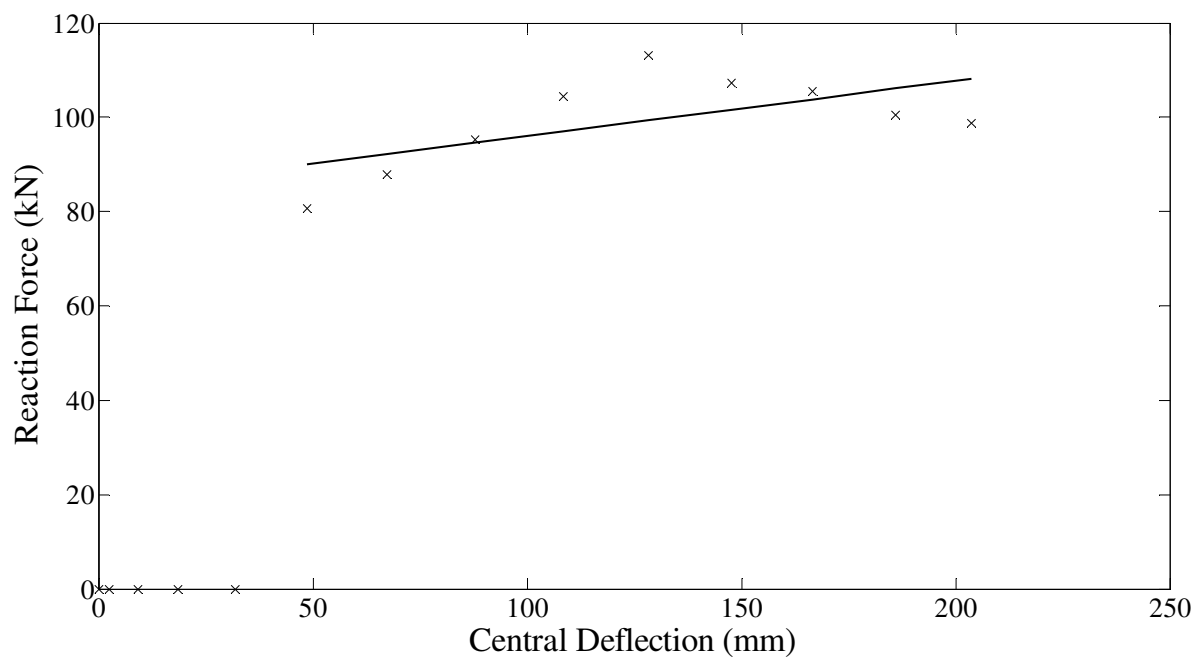


Figure 4-24: Total in plane reactions for Test 2.

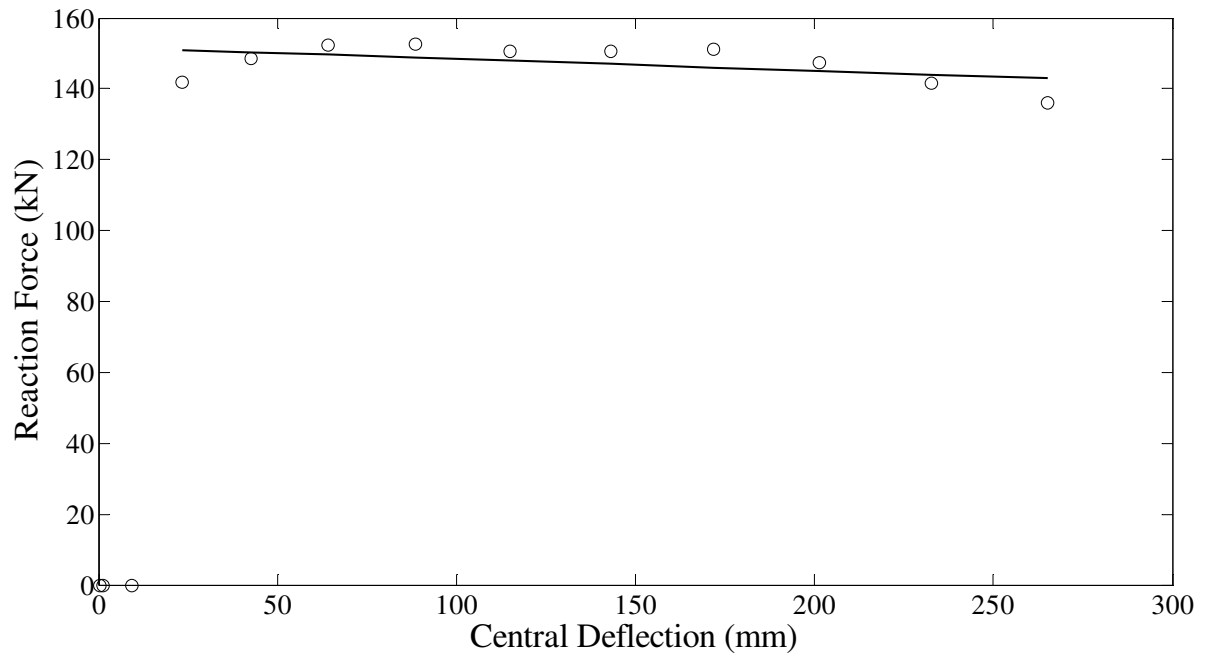


Figure 4-25: Total in plane reactions for Test 3.

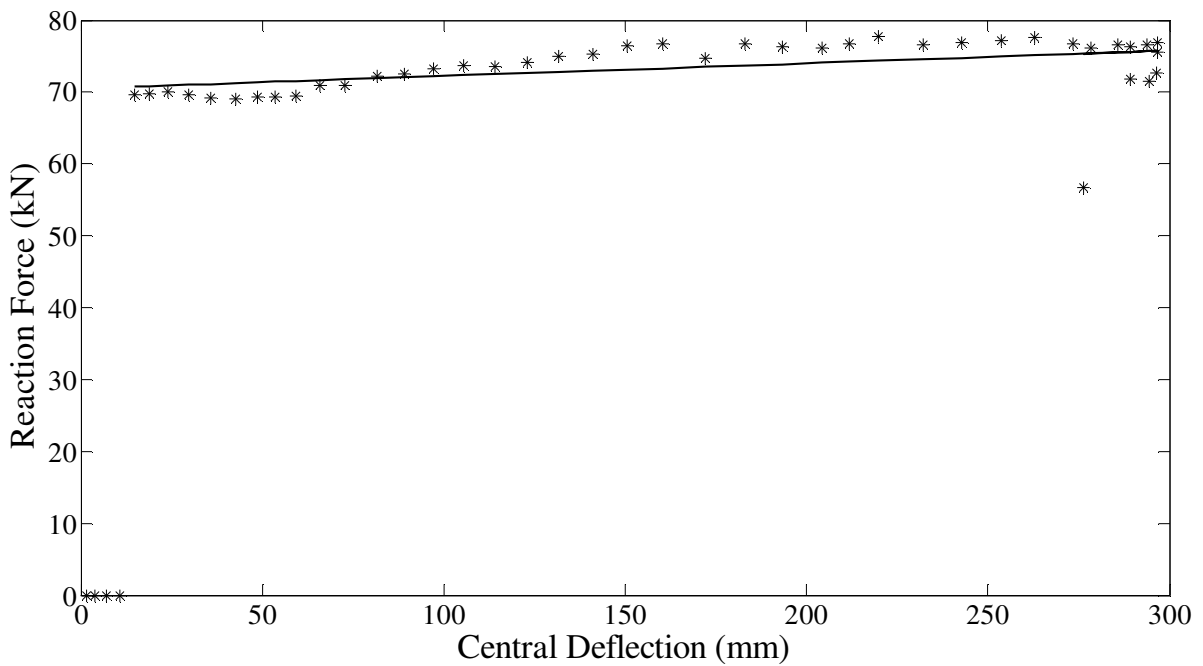


Figure 4-26: Total in plane reactions for Test 4.

Table 4-5: Summary of the pre-cracked out-of-plane peaks and out-of-plane and in-plane plateaux for the blasts considered.

Test	Out of plane pre crack reactions peak (kN)	Out of plane post crack plateau (kN)	In plane post crack plateau (kN)
1	39	35.2	90.8
2	32	31.7	78.3
3	34	54.5	90.4
4	56	21.0	73.6

4.4. Discussion

The analysis described above relies on several assumptions. However, it is possible to assess its reliability considering some of the results shown.

When considering the pre-cracked results, it was noted that the proportions of bending and axial force plots shown above correlate with the deflected shape observed during the experiments. The window exhibits most deformation near the edges, being relatively flat in the central area, especially at earlier time points. This would indicate that most of the bending moment would also be concentrated along the edges, which is confirmed in Figure 4-8. Figure 4-9 again confirms this result, showing far higher stresses near the edges. This is not just an effect of applying the FEA proportions to the raw data. Figure 4-27 below shows a total strain plot in the same location and at the same time step used for Figure 4-9, where again the increases in strains at the edges can be seen.

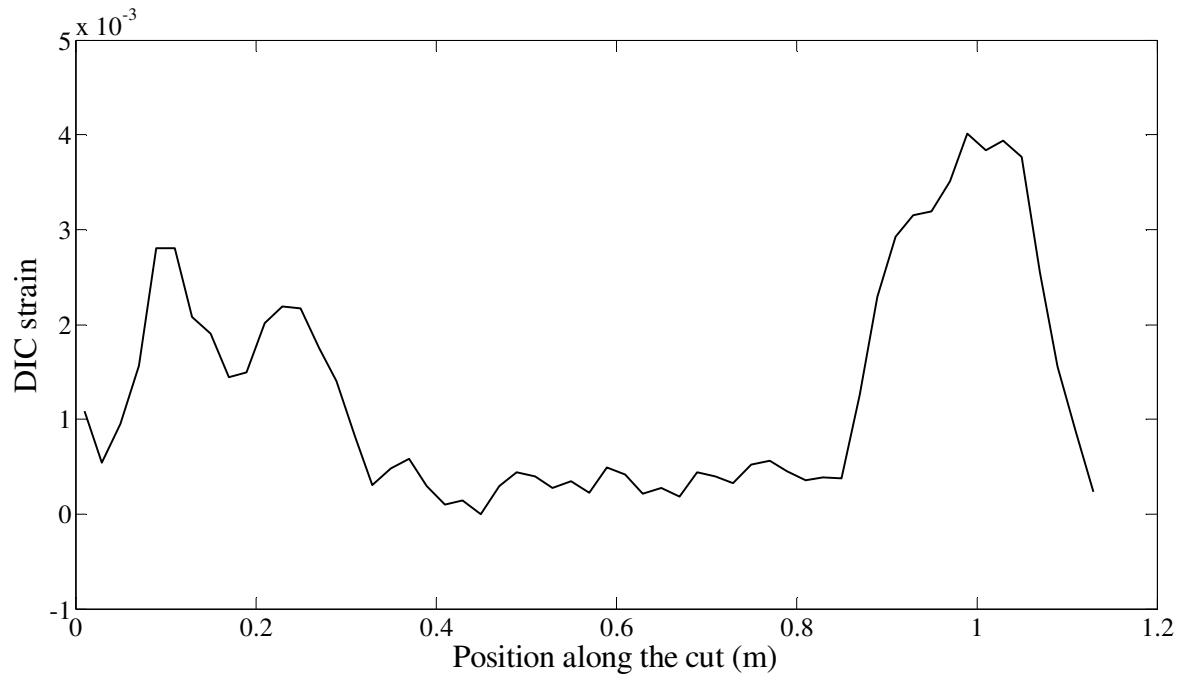


Figure 4-27: DIC strains along a cut in test 2.

The noise in the data in this phase however is fairly high. The error in the DIC strain measurements is in the order of 100 microstrains, which is a significant percentage of the strains observed before the failure of the glass. The filtering shown in Figure 4-9 goes towards reducing this effect, however it cannot eliminate it completely at all points. Given the vast quantity of data to be processed, an automated system had to be employed. The filtering and the averaging over the last few data points does seem to reduce the variability of the data along each edge, as can be seen in Figure 4-10, however this cannot be completely eliminated. It is most likely that the total reaction forces produced will be of the right magnitude; however it can be difficult to decide whether each individual datum can be considered accurate. As an example, the curves shown in Figure 4-11 and Figure 4-12 show that some calculated reactions do follow a similar trend to the strain gauges data, however others depart from it, sometimes significantly.

The issue of small strains recorded was less significant for the post-cracked analysis. In this case however one of the major issues was the material model employed to calculate the reactions. Hooper [37] employed the Johnson-Cook model for his FEA model. Whilst this does not represent the theoretical physical behaviour of the material, it does show a similar trend as seen in the recorded data. It is hypothesised that the plateaux in the cracked laminated glass stresses are caused by the delamination of the glass fragments from the PVB membrane. This complex behaviour is of course very different from plasticity in metals,

however the final stress strain curves seem to be similar, hence justifying the adoption of the model equation.

The plateau stresses calculated here are between 10-15 MPa for a strain rate of 3 s^{-1} . This is similar to the results obtained through Hooper et al.'s laboratory tests, giving credence to the results obtained above. The laboratory results were not directly employed as the material properties might be heavily dependent on crack spacing and other cracked laminate characteristics. The fit therefore had to be performed again for the available blast data. In test 4 no data from strain gauges on the bottom of the pane were available and results from another side were used. More importantly, the material model fits for this test cannot be compared directly to the results obtained for other cases. As explained in Chapter 3, the DIC calibration technique used does not guarantee the accuracy of the strain data. However, the distribution of the strains was found to be more reliable. Therefore, in this case it was decided that the reactions could be calculated, provided that the Johnson-Cook fit was performed with the same DIC strains which would be used for the reaction calculation. As the reactions obtained proved to be of the same general magnitude and presented the same time history behaviour seen in tests 1 to 3, it is likely that the method managed to produce reliable estimates.

The analysis performed here produced an estimate of the reactions along each edge. These could then be compared with the results obtained in the previous chapter. In this case as well, as shown in Figure 4-15, two peaks are present away from the side's central point. This ties in well with previous results, including those shown in Chapter 3.

The pre-cracked peak reactions show some consistency between the tests analysed. This is due to the inherent capacity of the glass plies. Supporting this is the fact that the reaction peak for Test 4 is higher than for Tests 1 to 3. In this case the window size was slightly reduced to allow the use of a larger frame, which included double sided rebates. Smaller pane dimensions imply a shorter span in all directions, which will cause a reduction of the bending moments generated by the applied loading. Therefore, the glass layers could resist higher pressures before the bending and membrane force stresses caused its fracture.

The post-cracked reactions clearly approach a plateau level at high deflections. This behaviour is similar to what was observed when considering the cracked glass stress strain behaviour, linking these two aspects. In further chapters the behaviour of the cracked

laminates will be studied to justify this behaviour and assess the effect of parameters such as the density of cracks on the plateau forces observed.

The in-plane reactions showed a different behaviour, increasing very quickly to their plateau levels. Their increased steadiness compared to the out-of-plane reactions was probably an effect of the geometric variations of the glass angles θ . Whilst the out-of-plane reactions are proportional to $\sin \theta$, the in-plane ones are proportional to $\cos \theta$. This tends to vary much more slowly at the angles recorded, which were in the region of 30° , as shown in the previous chapter. This would cause the increase in stability of the recorded forces.

4.5. Conclusion

In this chapter the available experimental data was employed to calculate reaction forces along the edges of the glazing panels. Pre-cracked reactions were estimated using a finite element model to split bending moments and membrane forces strains from the total deformations measured with DIC. These components were then calculated, and reaction forces found. The post-cracked reactions were found using the DIC strains, assuming that only membrane forces would be acting in this phase. A material model for the cracked laminate was fitted using the strain gauge data and DIC strains in the region at the centre of each panel. This was then used to convert DIC strains along all the edges into stresses.

The out-of-plane reactions show a distinct early peak before glass failure. In the first three tests this seems to be of a similar magnitude, whilst in the fourth it is higher. The size of this force is related to the geometric properties of the panel and hence to the maximum forces which can be resisted by the glass before shattering. Hence, as the panel used for Test 4 was smaller, higher pressures could be applied before the bending moment stresses reached the material's capacity.

Both in-plane and out-of-plane post-cracked reactions reached a plateau as the central deflections increased. This is compatible with the observed behaviour of the cracked laminate composite material, which shows a distinct plateau in its stress strain graph. One possible cause for this is the delamination between the PVB interlayer and the outer glass plies, which will tend to detach as the strains increase. This aspect of the material will be examined in the next two chapters in an attempt to explain the observed behaviour. The PVB material properties will be studied first, as they will influence the delamination behaviour. Afterwards, the delamination will be investigated and its relation to the reaction results presented.

5. PVB Material Properties

5.1. Introduction

Previous chapters analysed the behaviour of laminated glass plates subjected to real blast loads. The observed reaction forces followed a specific pattern, reaching a plateau which was subsequently maintained until either the window deflection started reducing or the failure of the laminated plate was reached. Explaining the presence of such a plateau requires a deeper understanding of the constituent materials of the cracked laminated plate and of their relation. Material models are therefore required to explain the behaviour of the composite at different strain rates. The PVB membrane was the specific focus of this chapter. Several constitutive equations have been fitted to PVB data, attempting to explain its behaviour over a range of strain rates.

Figure 5-1 shows the typical chemical structure of Polyvinyl Butyral. In common with other polymer materials, it exhibits a high degree of non-linearity in its strain response, which complicates its modelling. Additionally, it presents a high strain rate dependency, showing a significant shift in its behaviour at higher strain rates compared to lower rate deformations. To build a complete model of the composite material it is important to be able to represent the PVB behaviour at all rates of interest. Specifically, Hooper et al. [37] measured strain rates as high as 30 s^{-1} during his blast tests, hence a model being able to account for the PVB behaviour from quasi static levels to this rate would provide a strong basis for future finite element analyses.

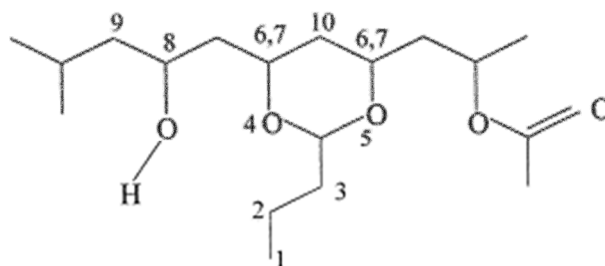


Figure 5-1: Typical chemical structure of the PVB polymer chain [62].

In this study, experimental data from uniaxial tests at different strain rates were used to model the behaviour at the rates of interest. Both the hyperelastic characteristics of the material and its time dependence were taken into account using visco-hyperelastic formulations. Two

general approaches were attempted. Firstly, Prony series were combined with two different hyperelastic models. Both a reduced polynomial model and a Hoo Fatt model as proposed by Hoo Fatt and Ouyang [63] were used. Following this, a different approach based on a full mathematical solution of finite deformation viscoelasticity was applied, as proposed and developed by several authors including Huber and Tsakmakis [64] and Hoo Fatt and Ouyang [63]. The results from the various approaches were then compared considering their fit to the experimental results and their ease of application.

5.2. Methods

5.2.1. Experimental data

Tensile tests at different strain rates were performed on the same PVB material to obtain the data necessary for the model calibration. The material tested in all cases was Saflex PVB produced by Solutia Inc. with product number RB-41. The equipment used was different for the high and the low strain rate experiments, as no single machine could produce the range of desired speeds.

All the tests at a rate above 0.2 s^{-1} and some of the tests at rates below this were performed by Hooper et al. [65]. For all these experiments the samples were cut in a dog bone shape. The height of the samples was 75 mm, with a 20 mm gauge length. The gauge length width was 4 mm and the material thickness was 0.76 mm throughout. The tests were performed on a servo-hydraulic Instron tensile testing machine, employing a loss motion device to ensure a constant strain rate at the higher speeds. Lightweight titanium alloy grips were used to minimise inertial effects due to their mass. Strain data were collected optically using a high speed camera. An image analysis was performed on each picture to obtain the deformations. Force data were collected using a piezoelectric load cell for speeds above 0.1 ms^{-1} and a strain gauge load cell for lower speeds. Tests were run at speeds of 0.005 ms^{-1} , 0.1 ms^{-1} , 0.32 ms^{-1} , 1 ms^{-1} , 2 ms^{-1} , 5 ms^{-1} and 10 ms^{-1} , providing results for average rates from 0.2 s^{-1} to 400 s^{-1} .

Lower rate tests were performed by Wang. The specimens in this case were cut following the dimensions specified in British Standard 37-2005 [66]. The size of the specimens is detailed in Figure 5-2. Wang employed a low load and low speed single column 1kN Zwick tensile testing machine. The strain data were collected optically in a manner similar to that used by

Hooper, with the difference that high speed cameras were not needed in this case. A Matlab routine was employed to identify black lines in the images recorded during the tests. These lines had been drawn at regular intervals on the samples. The distance between these could then be used to calculate the strain at each time step. The inbuilt force sensor of the machine was used to capture loading data. Tests were run at strain rates of 0.01 s^{-1} , 0.02 s^{-1} , 0.1 s^{-1} and 0.2 s^{-1} .

Data from the two sets of experiments have been employed to produce material models for the PVB. As both sets of experiments produced data for the rate of 0.2 s^{-1} , these curves will be compared.

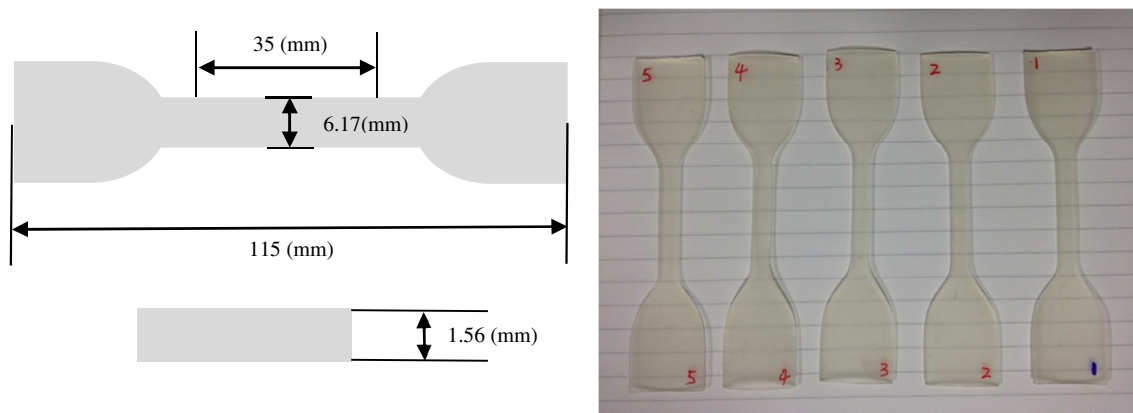


Figure 5-2: Dimensions of Dumb-bell PVB specimen.

5.2.2. Experimental Results

The experimental results are presented here before the material models will be described in detail.

The data is shown as true stress versus stretch ratio λ , which is equal to the length of the sample at time t (l_t) over the original length (l_0) :

$$\lambda(t) = \frac{l_t}{l_0} \quad \text{Eq. 5-1}$$

The results of the low rate tests are shown in Figure 5-3. As expected the curves present nonlinear behaviour together with significant strain rate sensitivity. At these rates however, the shape of the curves does not change significantly, mostly showing an increase in the overall stiffness as the deformation speed increases.

The tests performed at strain rates above and including 2 s^{-1} are shown in Figure 5-4. The grips in the tests at a rate of 200 s^{-1} appear to have slipped at a stretch of roughly 2.5. The material non linearity is still very evident, as is the rate sensitivity. Additionally the shape of the curves at the higher strain rates, starting from a rate of 8 s^{-1} , changes significantly, exhibiting a much higher stiffness at small strains. This is consistent with past observations by Iwasaki and Sato [20]. The effect becomes marked for rates of 20 s^{-1} and above. Ideally a PVB material model used for finite element application should be able to represent this switch in behaviour to ensure that all situations can be modelled accurately.

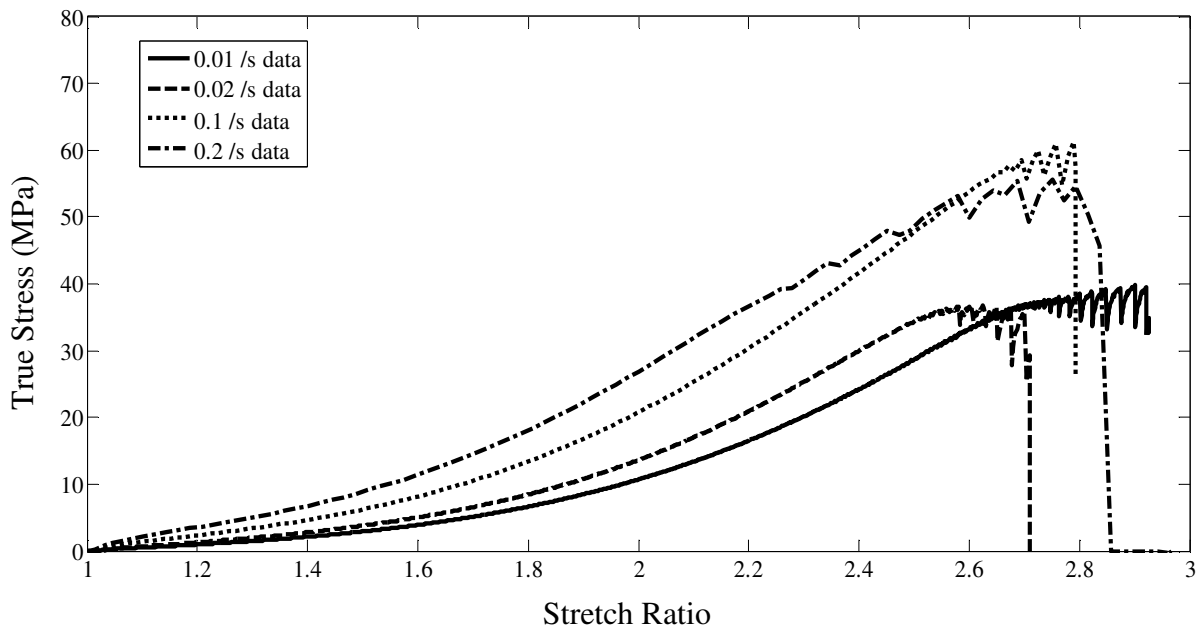


Figure 5-3: Low rate true Stress - stretch curves for PVB under uniaxial tensile loading.
Only Wang's data for the 0.2 s^{-1} case are presented for clarity.

The failure point of the material did not change significantly with the rate, the final stretch varying between ≈ 2.7 and ≈ 3.2 depending on the specific test.

Figure 5-5 shows a plot of the tests at 0.2 s^{-1} performed by Hooper et al. and Wang. The curves are similar. Whilst there is a difference at stretches above 1.5, their divergence is limited and will not affect the accuracy of the material model significantly. Therefore the data from the two sets of experiments were used directly without applying corrections for shape effects.

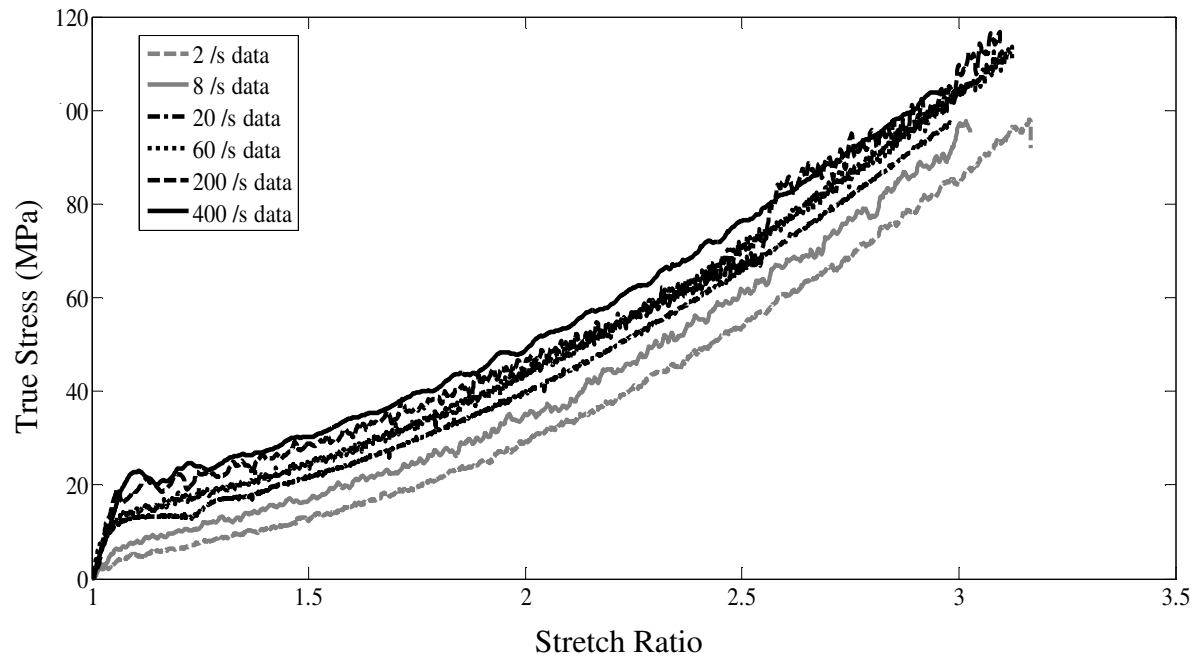


Figure 5-4: High rate true stress – stretch curves for PVB under uniaxial tensile loading.

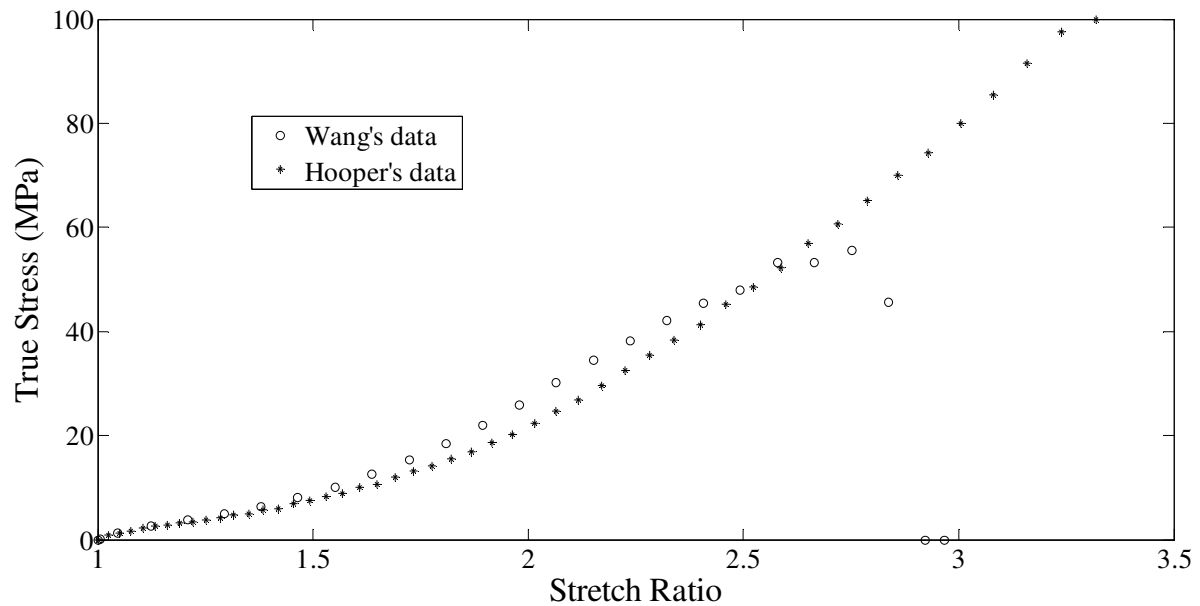


Figure 5-5: A comparison of the results from Hooper [65] and Wang at a strain rate of 0.2 s^{-1} .

5.2.3. Material model fit

The experimental results presented above highlight the high degree of rate dependence of the PVB material. Material models need to take this into account and ideally include a mechanism to represent this change in behaviour. Hooper et al. [65] argued that a viscoelastic model employing a Prony series would not be able to cover all the strain rates of interest. Specifically, it is challenging to ensure that a single Prony series model will guarantee the changes in the small strain behaviour observed in the experiments. This issue could be overcome by using different models for different rate conditions. As an example, the models proposed by Liu et al. [21] represented different strain rate ranges and loading type and hence ensure all the material behaviour was covered.

However, work by several authors, for example that by Hoo Fatt and Ouyang [63] shows that models which exhibit the change in behaviour can be derived using finite deformation viscoelasticity laws. Through these, different hyperelastic models can be combined, ensuring that the whole range of deformation speeds is covered. The technique has been described thoroughly by Huber and Tsakmakis [64] and has been used by Hoo Fatt and Ouyang [63], Arruda and Boyce [67], Amin et al. [68], [69] and others.

Both the Prony series and the finite deformation viscoelasticity approaches have been used in this study. As mentioned above, two Prony series models have been developed using different hyperelastic models, specifically a Hoo Fatt model and a reduced polynomial model. As it was not possible to model the entire PVB behaviour with a single set of parameters, two parameters fits have been performed for each hyperelastic formulation, one for the lower strain rate and one for the higher strain rate data sets. In this case, the “low rate” model covers tests up to and including an average rate of 8 s^{-1} , whilst the “high rate” models cover rates higher than this. The rigorous finite deformation viscoelasticity derivation has then been employed to formulate a single model covering the entire range of rates. This required the use of different hyperelastic springs and a nonlinear viscosity function. The different material models are described in more detail below.

Basic Assumptions and definitions

When deriving the material models described above, certain assumptions have been made with regards to the PVB behaviour. The main assumption is that the material has been

considered incompressible. This implies that no change of volume takes place during the tests.

The coordinates of a point in the material before deformations take place ($t = 0$) are defined by the vector $\mathbf{X}$. If this element of material is moved by the deformation, its new coordinates will be given by a new vector, $\mathbf{x}$. The deformation tensor $\mathbf{F}$ is given by $F_{ij} = \frac{\partial x_i}{\partial X_j}$. The product of the diagonal members of the deformation tensor should be 1 for an incompressible material. Therefore, if F_{11} is equal to the stretch ratio (λ), F_{22} and F_{33} will be equal to $\frac{1}{\sqrt{\lambda}}$.

For uniaxial tension, $\mathbf{F}$ will therefore be equal to:

$$\mathbf{F} = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \frac{1}{\sqrt{\lambda}} & 0 \\ 0 & 0 & \frac{1}{\sqrt{\lambda}} \end{bmatrix} \quad \text{Eq. 5-2}$$

The left Cauchy-Green strain tensor $\mathbf{B}$ is given by $\mathbf{B} = \mathbf{F}\mathbf{F}^T$. Therefore, in this case:

$$\mathbf{B} = \begin{bmatrix} \lambda^2 & 0 & 0 \\ 0 & \frac{1}{\lambda} & 0 \\ 0 & 0 & \frac{1}{\lambda} \end{bmatrix} \quad \text{Eq. 5-3}$$

The two invariants of $\mathbf{B}$ are defined by $I_1 = \text{tr}(\mathbf{B}) = \lambda^2 + 2\lambda^{-1}$ and $I_2 = \frac{1}{2}\{(\text{tr}(\mathbf{B}))^2 - \text{tr}(\mathbf{B}\mathbf{B})\} = \lambda^{-2} + 2\lambda$.

These quantities, especially $\mathbf{B}$ and its invariants, will be used below to derive all the material models.

Prony series derivation

These viscoelastic material models include a non-linear hyperelastic function $\sigma_0(\epsilon)$, dependent on strain, combined with a Prony series, $g(t)$, to include the time dependency.

The overall material model for a step strain relaxation test is shown in Eq. 5-4:

$$\sigma = \sigma_0(\epsilon) \times g(t) \quad \text{Eq. 5-4}$$

The hyperelastic function has been derived from the basic equation [69]:

$$\mathbf{T} = -p\mathbf{I} + 2\frac{\partial\psi}{\partial I_1}\mathbf{B} - 2\frac{\partial\psi}{\partial I_2}\mathbf{B}^{-1} \quad \text{Eq. 5-5}$$

where $\mathbf{T}$ is the Cauchy true stress tensor. The function implies that a function representing the work of the spring (ψ) needs to be differentiated with respect to the first two strain invariants of the strain tensor $\mathbf{B}$. Lamber-Diani and Rey [70] argued that to determine the portion of the equation relating to I_2 , a biaxial test would be required. As this was not available here, work functions related solely to I_1 were chosen for this analysis. The specific hyperelastic functions will be derived below.

The time dependent function $g(t)$ was assumed to take the form of a Prony series:

$$g(t) = g_\infty + \sum_1^N g_i \exp\left(-\frac{t}{\tau_i}\right) \quad \text{Eq. 5-6}$$

The model parameter fit was performed assuming six terms of this series, using either the τ_i terms derived by Hooper or others at more regular logarithmic intervals to facilitate the fitting process. As stated above, Eq. 5-4 is valid for stress relaxation test data. As only data from ramp tests were available for the model calibrations, the related convolution integral would have needed to be solved. As a solution of this would not be straightforward to obtain in this case, the method described by Goh et al. [71] was employed to perform the material model fit. A square minimisation technique was applied to determine the coefficients. The Solver function of Microsoft Excel 2012 was used to solve the optimisation problem.

Hoo Fatt formulation

Hoo Fatt and Ouyang [63] used a model similar to an Ogden spring when deriving a material law for a polymer material. The rationale for this formulation was to be able to represent the sharp change in stiffness which took place at small strains. As the behaviour of the material under consideration here is similar, the same hyperelastic law has been employed. However, in this case the model was expanded into a summation in a similar manner to the classic Ogden model. This allowed the sharp stiffness change to be reproduced with a single hyperelastic spring in the Prony series approach. The work function used is:

$$\psi = \sum_i \mu_i (I_1 - 3)^{\alpha_i} \quad \text{Eq. 5-7}$$

Two terms were used. If this function is substituted into Eq. 5-5, the hyperelastic strain is given by:

$$\mathbf{T} = -p\mathbf{I} + 2\alpha_1\mu_1(I_1 - 3)^{(\alpha_1-1)}\mathbf{B} + 2\alpha_2\mu_2(I_1 - 3)^{(\alpha_2-1)}\mathbf{B} \quad \text{Eq. 5-8}$$

The term p in the equation above can be found considering the boundary conditions $\sigma_{22} = \sigma_{33} = 0$. Substituting into Eq. 5-8, a formula for p is derived:

$$p = 2\alpha_1\mu_1(I_1 - 3)^{(\alpha_1-1)}\frac{1}{\lambda} + 2\alpha_2\mu_2(I_1 - 3)^{(\alpha_2-1)}\frac{1}{\lambda} \quad \text{Eq. 5-9}$$

and therefore the final hyperelastic expression for uniaxial tension is given by:

$$\sigma_{11} = 2\left(\lambda^2 - \frac{1}{\lambda}\right)\left(\alpha_1\mu_1(I_1 - 3)^{(\alpha_1-1)} + \alpha_2\mu_2(I_1 - 3)^{(\alpha_2-1)}\right) \quad \text{Eq. 5-10}$$

Reduced Polynomial formulation

Various authors in the past have used a polynomial formulation to represent the PVB material, e.g. [29]. As discussed above, a model independent of I_2 was necessary in this case. Therefore a reduced polynomial formulation was used in this work. Whilst this model does not guarantee an equally accurate representation of the change in stiffness at higher rates, it was considered that this procedure would guarantee the straightforward application of the model in finite element software for further analysis of the composite material.

The work function in this case is given by:

$$\psi = \sum_i C_i (I_1 - 3)^i \quad \text{Eq. 5-11}$$

A third order model was employed. Using the function above with Eq. 5-5 the stress tensor is given by:

$$\mathbf{T} = -p\mathbf{I} + 2\left(C_1 + 2C_2(\mathbf{I}_1 - 3) + 3C_3(\mathbf{I}_1 - 3)^2\right)\mathbf{B} \quad \text{Eq. 5-12}$$

The term p can be found using the same method described above, giving:

$$p = 2\left(C_1 + 2C_2(\mathbf{I}_1 - 3) + 3C_3(\mathbf{I}_1 - 3)^2\right)\frac{1}{\lambda} \quad \text{Eq. 5-13}$$

Inserting this in Eq. 5-12 and deriving an equation for σ_{11} gives:

$$\sigma_{11} = 2\left(\lambda^2 - \frac{1}{\lambda}\right)\left(C_1 + 2C_2(\mathbf{I}_1 - 3) + 3C_3(\mathbf{I}_1 - 3)^2\right) \quad \text{Eq. 5-14}$$

5.2.4. Finite deformation viscoelasticity

Model Derivation

The finite deformation viscoelasticity models are derived by solving the mathematical equations representing a “springs and dampers” system, similar to those assumed in traditional viscoelasticity theory. However, in this case all small strain assumptions are avoided to produce a solution valid for any level of deformation. For the situation of interest here a system of two nonlinear springs and a damper is assumed, as shown in Figure 5-6. This is the same system as model A described by Huber and Tsakmakys [64], and was also used by several other authors previously mentioned [63], [68].

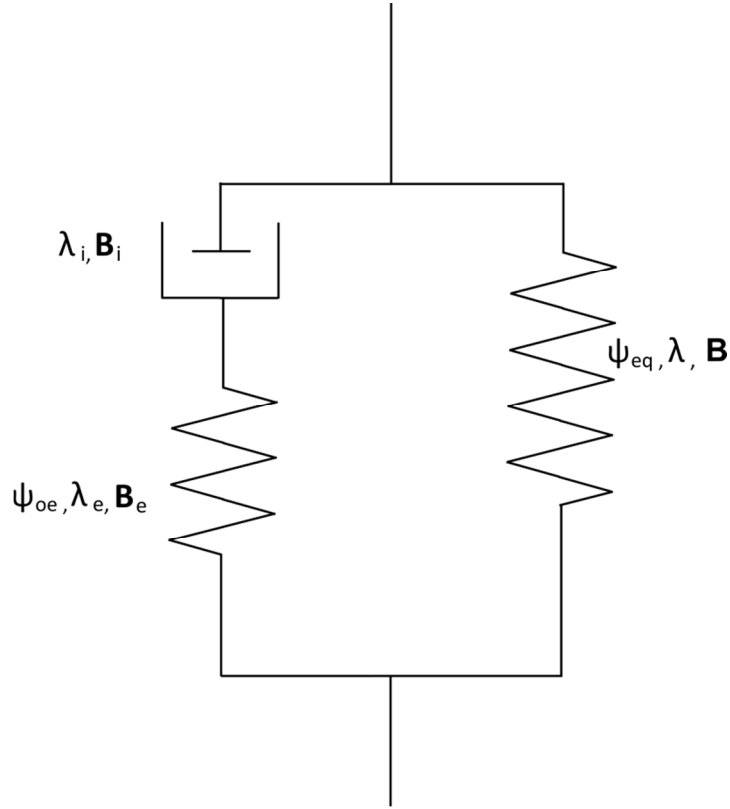


Figure 5-6: A summary of the material model assumed to model PVB.

The total observed stress will be equal to the sum of the stresses affecting each spring. The single spring will be referred to as the equilibrium spring (“eq” in symbols), whilst the spring in series with the damper will be referred to as the overstress spring (“oe” in symbols). The deformation of the equilibrium spring and the summation of that of the damper and overstress spring series will be the same and equal to the overall measured deformation. Therefore, as shown in Figure 5-6, the equilibrium deformation of the spring will be equal to $\mathbf{B}$. The deformation of the overstress spring is instead dependant on the viscous behaviour of the damper, introducing a time dependency in the model.

Huber and Tsakmakys [64] and Hoo Fatt and Ouyang [63] provide a detailed derivation of the general equations governing this model. The first step of the method, introduced by Lubliner [72] in the context of finite deformation plasticity, is to decompose the overall system deformation into an equilibrium and an instantaneous part, $\mathbf{F} = \mathbf{F}_e \mathbf{F}_i$. $\mathbf{F}_i$ represents the deformation state which would be reached if the load was instantaneously removed from the deformed sample. It could therefore be associated with the deformation of the damper. $\mathbf{F}_e$ is instead assumed to be the deformation of the overstress spring, hence the strain responsible for the stress acting in the damper and spring series.

Additionally, it is assumed that the system will obey the thermodynamic condition [69]:

$$-\rho_r \dot{\psi} + \mathbf{S}_E \cdot \mathbf{L} \geq 0 \quad \text{Eq. 5-15}$$

where $\mathbf{S}_E$ is Cauchy (true) stress and $\mathbf{L}$ is the velocity gradient, $\mathbf{L} = \dot{\mathbf{F}}\mathbf{F}^{-1}$, measuring the rate of change of the velocity in the volume of the material at a given moment in time.

Huber and Tsakmakys show the derivation of a system of equations from these assumptions, and conclude with a material model given by:

$$\mathbf{T} = -p\mathbf{I} + 2 \frac{\partial \psi_{eq}}{\partial I_{1B}} \mathbf{B} - 2 \frac{\partial \psi_{eq}}{\partial I_{2B}} \mathbf{B}^{-1} + 2 \frac{\partial \psi_{oe}}{\partial I_{1B_e}} \mathbf{B}_e - 2 \frac{\partial \psi_{oe}}{\partial I_{2B_e}} \mathbf{B}_e^{-1} \quad \text{Eq. 5-16}$$

$$\dot{\mathbf{B}}_e = \mathbf{L}\mathbf{B}_e + \mathbf{B}_e \mathbf{L}^T - \frac{2}{\eta} \mathbf{B}_e \mathbf{T}_{oe} \quad \text{Eq. 5-17}$$

Where $\mathbf{T}_{oe} = 2 \frac{\partial \psi_{oe}}{\partial I_{1B_e}} \mathbf{B}_e - 2 \frac{\partial \psi_{oe}}{\partial I_{2B_e}} \mathbf{B}_e^{-1}$, $\mathbf{B}_e$ is the left Cauchy-Green strain tensor for the

overstress spring and η represents the viscosity function of the damper. Spring functions similar to those derived above for the Prony series models can then be substituted into the equations. This derivation method has been used here, assuming a one term Ogden [73] function for the equilibrium spring and a three terms Hoo Fatt's spring function for the overstress response. Substituting these springs into the system above, the following equations are obtained for uniaxial tension:

$$\sigma_{11} = \mu_o \lambda^{\alpha_o} - \mu_o \lambda^{-\frac{1}{2}\alpha_o} + 2 \left(\lambda^2 - \frac{1}{\lambda} \right) \times \quad \text{Eq. 5-18}$$

$$\times \left(\alpha_1 \mu_1 (I_1 - 3)^{(\alpha_1 - 1)} + \alpha_2 \mu_2 (I_1 - 3)^{(\alpha_2 - 1)} + \alpha_3 \mu_3 (I_1 - 3)^{(\alpha_3 - 1)} \right)$$

$$\dot{\mathbf{B}}_e = \mathbf{L}\mathbf{B}_e + \mathbf{B}_e \mathbf{L}^T - \frac{2}{\eta} \mathbf{B}_e 2 \left(\lambda^2 - \frac{1}{\lambda} \right) \times \quad \text{Eq. 5-19}$$

$$\times \left(\alpha_1 \mu_1 (I_1 - 3)^{(\alpha_1 - 1)} + \alpha_2 \mu_2 (I_1 - 3)^{(\alpha_2 - 1)} + \alpha_3 \mu_3 (I_1 - 3)^{(\alpha_3 - 1)} \right)$$

The constants of these equations were fitted using the method explained below. Additional detail of the derivation is provided in Appendix A.

Data fit method

Hoo Fatt and Ouyang presented a method to fit the model constants for this kind of mathematical representation. In her paper, the constants for the two springs were obtained first. At low, quasi static, rates it is assumed that the damper will offer no resistance to the deformation. Therefore, the overstress spring will not be stretched, giving $\mathbf{F}_i = \mathbf{F}$ and $\mathbf{F}_e = \mathbf{I}$. The measured stress will hence be caused by the deformation of the equilibrium spring. Therefore, data sets in this range of rates can be used to fit the parameters of the equilibrium spring directly. As the two lowest strain rates data available were similar (see Figure 5-3), it was assumed that the quasi static condition was being approached. The Ogden model spring could then be fitted with the data from the lowest rate experiment (0.01 s^{-1}).

Again, the data for the higher rates, at 200 s^{-1} and 400 s^{-1} , showed less variation than data at lower speeds. It was therefore assumed that these sets would approximate an instantaneous response of the material. In this condition, it can be assumed that the damper will not move, and hence that the overstress spring deformation will be equal to the total deformation of the material, specifically $\mathbf{F}_i = \mathbf{I}$ and $\mathbf{F}_e = \mathbf{F}$. As the total measured stress is equal to the sum of the stresses acting on the two springs and the equilibrium spring stress can be calculated using the material constants obtained in the previous (quasi static) step, the stress acting on the overstress spring could be found. This allowed a stress strain curve to be derived for the overstress spring, whose material constants could then be fitted. This procedure was followed using the 400 s^{-1} data.

Figure 5-7 shows the two extreme stress-stretch curves together with the fitted models.

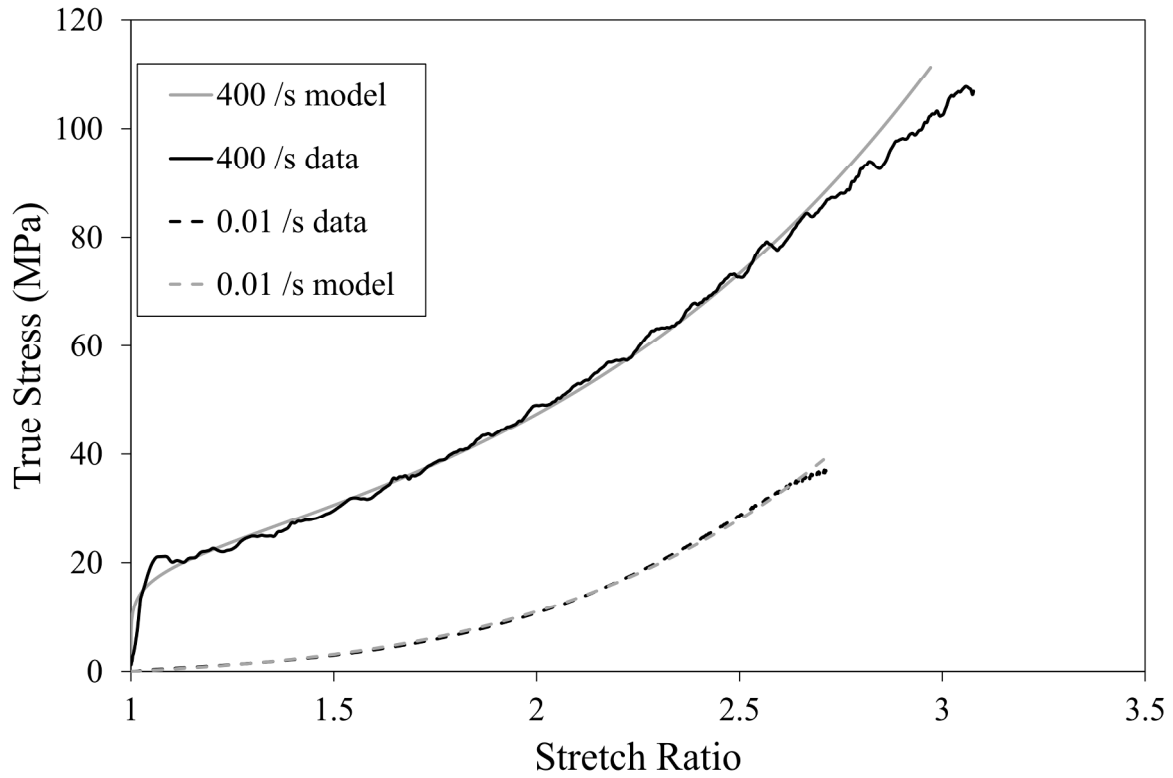


Figure 5-7: The lowest and highest available strain rate data were used to fit the two spring models. The figure shows the raw data and the model fits.

Once the parameters of the springs had been fitted, the viscous function had to be considered. The data showed that a constant would not be sufficient to model the different material behaviour. A function as used by Hoo Fatt and Ouyang was assumed and the parameters were fitted to the data. To achieve this, the method described in the same paper was followed. This assumed a matrix $\mathbf{F}_e$ of the form:

$$\mathbf{F}_e = \begin{pmatrix} \lambda_e & 0 & 0 \\ 0 & \frac{1}{\sqrt{\lambda_e}} & 0 \\ 0 & 0 & \frac{1}{\sqrt{\lambda_e}} \end{pmatrix} \quad \text{Eq. 5-20}$$

Therefore $\mathbf{B}_e$ is given by:

$$\mathbf{B}_e = \begin{pmatrix} \lambda_e^2 & 0 & 0 \\ 0 & \frac{1}{\lambda_e} & 0 \\ 0 & 0 & \frac{1}{\lambda_e} \end{pmatrix} \quad \text{Eq. 5-21}$$

Using this assumption, λ and λ_e could be substituted in the overstress strain rate differential equation, which becomes:

$$\dot{\lambda}_e = \frac{\dot{\lambda}\lambda_e}{\lambda} - \frac{2\mathbf{T}_{oe}}{3\eta} \lambda_e \quad \text{Eq. 5-22}$$

Therefore, $\eta = \frac{2\lambda\lambda_e\mathbf{T}_{oe}}{3(\dot{\lambda}\lambda_e - \dot{\lambda}_e\lambda)}$. The values of η found in this way were fitted to the proposed equation:

$$\eta = \left\{ C_{\eta 1} \left(1 - \exp \left(C_{\eta 2} (I_{1B} - 3) \right) \right) + C_{\eta 3} \right\} \left(C_{\eta 4} I_{1B_e}^3 + C_{\eta 5} I_{1B_e}^2 + C_{\eta 6} I_{1B_e} + C_{\eta 7} \right) \quad \text{Eq. 5-23}$$

This function was substituted into the system of equations of the model. Values for η and the required deformation invariants were obtained from the available data. $\mathbf{T}_{oe}$ was calculated by subtracting the equilibrium spring stress from the experimentally measured stress. This data was used to calculate λ_e from the known overstress spring equation. As this equation is nonlinear and difficult to invert, the stretch was found using a numerical solver in Matlab [74]. The overstress rate was then found through numerical differentiation. A polynomial was fitted to the λ_e data to achieve this, since otherwise the noise present in the experimental curve would have prevented realistic estimates being obtained. The values of η obtained were then used to fit the constants required.

Once the parameters were obtained, the resulting system of equations was solved using Matlab. An inbuilt numerical solver was employed for the differential equation.

5.3. Results

5.3.1. Prony series - Hoo Fatt spring

Figure 5-8, Figure 5-9 and Figure 5-10 show the experimental data and the fitted material models for the low strain rate (up to 8 s^{-1}) cases. The plots show that this function does not represent the material behaviour at the lowest strain rates accurately (Figure 5-8). The fit improves for the curves in Figure 5-9. However, the quality decreases again for the two higher rates (Figure 5-10), especially for the 8 s^{-1} data set, where the initial behaviour is not captured by the model. Whilst the shapes of the stress-stretch curves change slightly throughout the range of rates, it is at 8 s^{-1} that the stress curve starts to show the increase in initial stiffness that cannot be modelled by the basic hyperelastic spring. A plot of the relative error of these fits will be shown later.

Figure 5-11 and Figure 5-12 show the data for the high strain rate (from 20 s^{-1}) cases. The model captures the initial stiffness quite accurately. Additionally, the model follows the experimental results at higher stretches more accurately than the low rate cases. The fit again seems to be of lower quality for the highest rate cases, where the initial stiffness does not match the experimental stiffness. However, the model again follows the experimental data closely after this stage. Figure 5-20, Figure 5-21 and Figure 5-22 show plots of the relative errors for selected strain rates and will be discussed below. Table 5-1 and Table 5-2 summarise the parameters which were used in both the low and high rate cases.

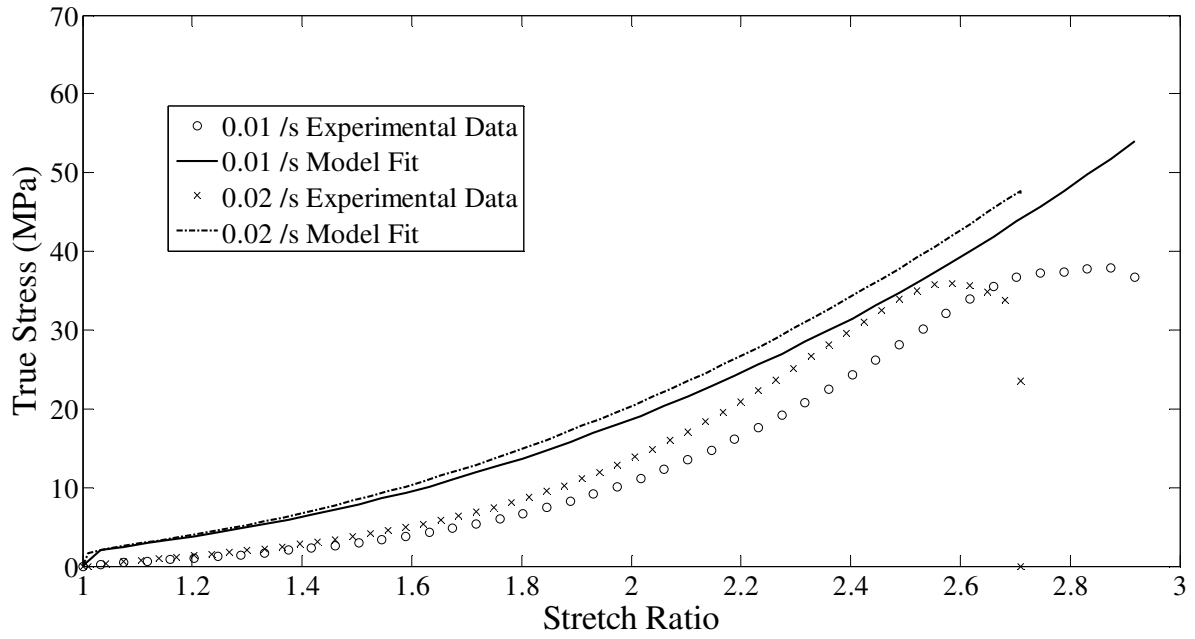


Figure 5-8: Low strain rate Prony series with Hoo Fatt spring fit for the tests at rates 0.01 s^{-1} and 0.02 s^{-1} .

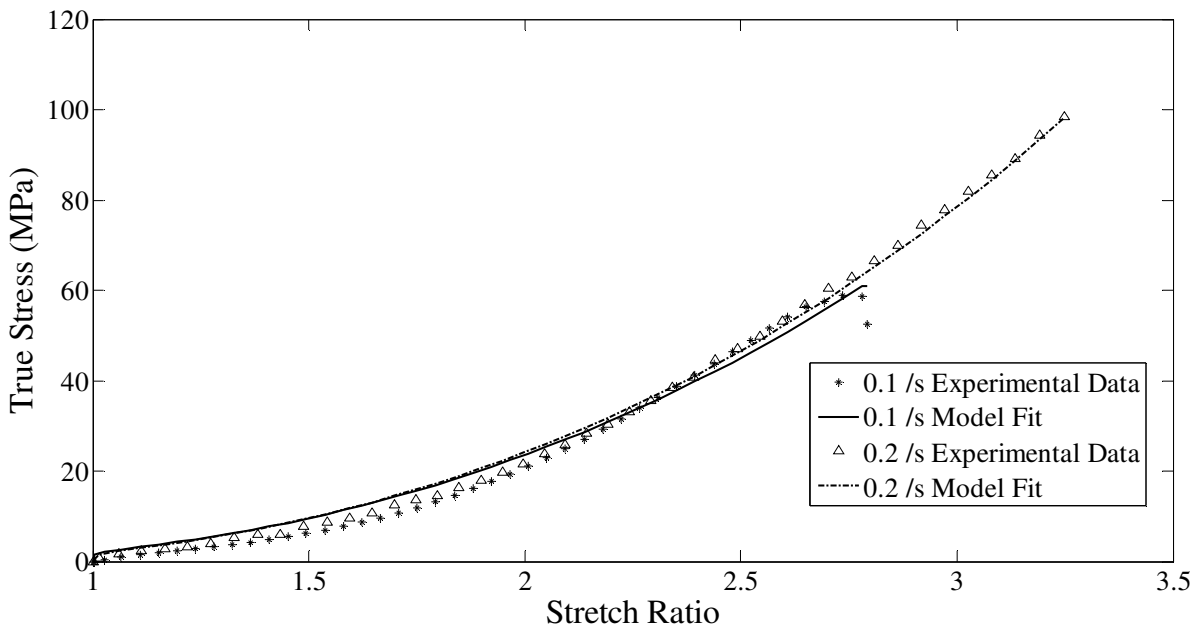


Figure 5-9: Low strain rate Prony series with Hoo Fatt spring fit for the tests at rates 0.1 s^{-1} and 0.2 s^{-1} . Hooper's data are shown for the 0.2 s^{-1} case.

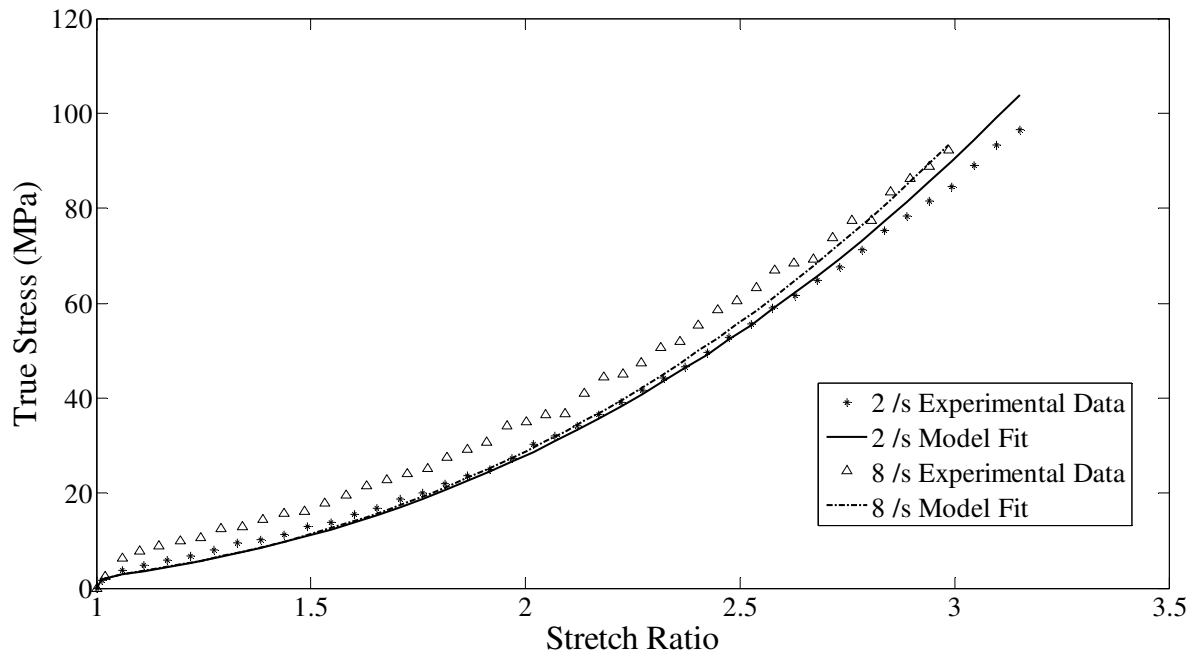


Figure 5-10: Low strain rate Prony series with Hoo Fatt spring fit for the tests at rates 2 s^{-1} and 8 s^{-1} .

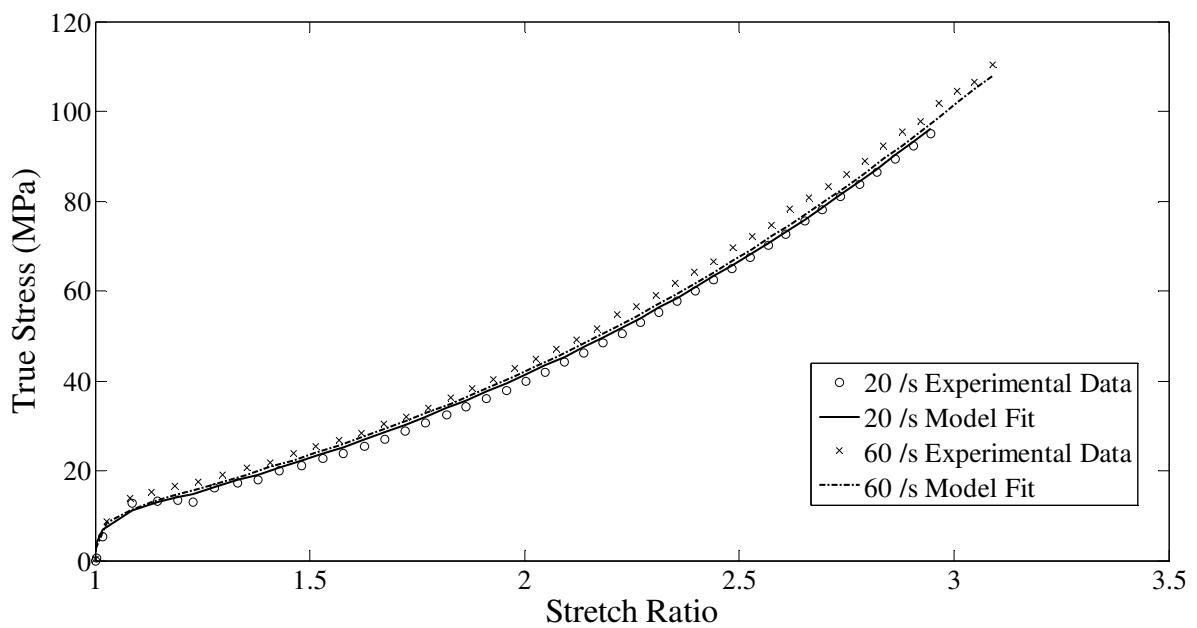


Figure 5-11: High strain rate Prony series with Hoo Fatt spring fit for the tests at rates 20 s^{-1} and 60 s^{-1} .

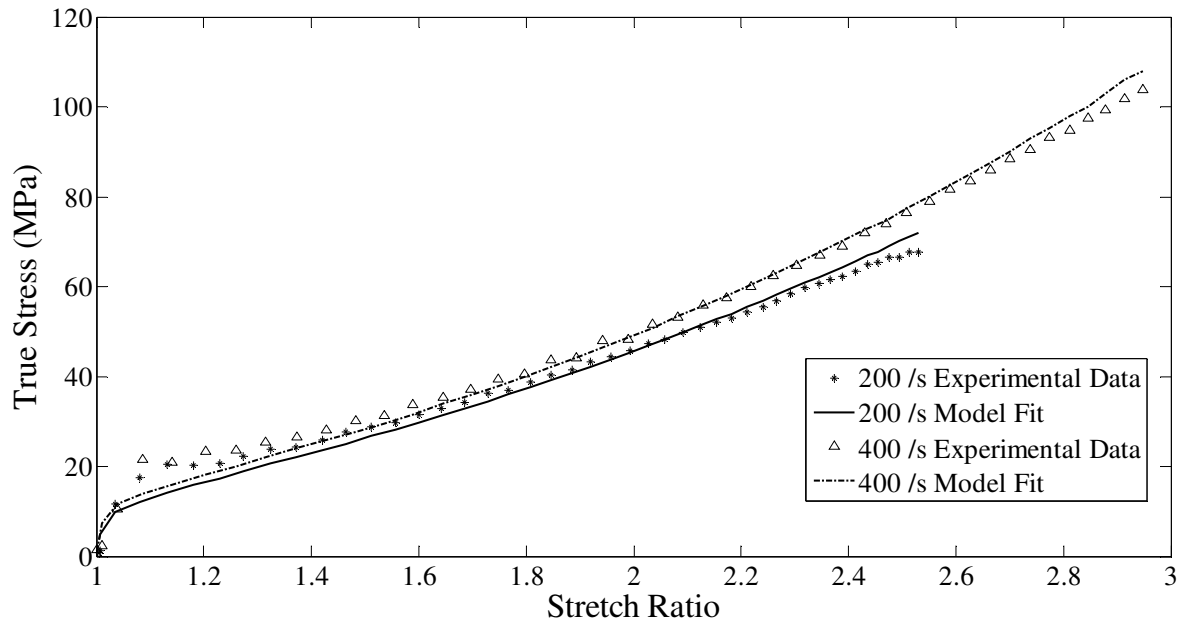


Figure 5-12: High strain rate Prony series with Hoo Fatt spring fit for the tests at rates 200 s^{-1} and 400 s^{-1} .

Table 5-1: Material parameters for the Hoo Fatt's spring.

Parameter	Low rate (up to 8 s^{-1})	High rate (above to 20 s^{-1})
μ_1 (MPa)	4.37	13.6
α_1	1.38	1.37
μ_2 (MPa)	5.68	80.2
α_2	0.618	0.645

Table 5-2: Prony series parameters for the Hoo Fatt's spring viscoelastic model.

Parameter	Low rate (up to 8 s ⁻¹)	High rate (above to 20 s ⁻¹)
τ_1 (s)	2.45×10^{-5}	2.45×10^{-5}
g_1	0.1	0.759
τ_2 (s)	2.21×10^{-3}	2.21×10^{-3}
g_2	0.1004	0.0624
τ_3 (s)	4.98×10^{-2}	4.98×10^{-2}
g_3	0	0
τ_4 (s)	0.624	0.624
g_4	0.139	0
τ_5 (s)	24.9	24.9
g_5	0.213	0
τ_6 (s)	1000	1000
g_6	0.224	0
g_{inf}	0.224	0.178

5.3.2. Prony series – Reduced polynomial spring

The fitting of the reduced polynomial spring model was again performed for the low strain rate and the high strain rate cases separately. The low strain rate (up to 8 s⁻¹) results are shown in Figure 5-13, Figure 5-14 and Figure 5-15. The results are similar to those for the Hoo Fatt spring model. The model follows the experimental data for the 0.1 s⁻¹ and 0.2 s⁻¹ data sets in Figure 5-14. However, the slower rates (Figure 5-13) and the higher rates (Figure

5-15) do not show a good agreement. The lower rates stiffness is over estimated, producing higher stresses than those produced experimentally. The 8 s^{-1} case again shows that the model could not capture the initial stiffness, producing lower stresses than the experiment.

Table 5-3: Material parameters for the reduced polynomial hyperelastic spring.

Parameter	Low rate (up to 8 s^{-1})	High rate (above to 20 s^{-1})
C1 (MPa)	40.7	311
C2 (MPa)	75.7	0.0112
C3 (MPa)	0	1.3

Table 5-4: Prony series parameters for the reduced polynomial viscoelastic model.

Parameter	Low rate (up to 8 s^{-1})	High rate (above to 20 s^{-1})
$\tau_1 \text{ (s)}$	1×10^{-4}	1×10^{-6}
g_1	0	0
$\tau_2 \text{ (s)}$	1×10^{-3}	1×10^{-5}
g_2	0.898	0.948
$\tau_3 \text{ (s)}$	1×10^{-2}	1×10^{-3}
g_3	0	0.0151
$\tau_4 \text{ (s)}$	0.1	0.1
g_4	0	0.00112
$\tau_5 \text{ (s)}$	1	10
g_5	0	0
$\tau_6 \text{ (s)}$	100	1000
g_6	0.102	0
g_{inf}	0	0.0354

The results of the fits for the higher strain rates (above 20 s^{-1}) are shown in Figure 5-16 and Figure 5-17. In this case the model follows the experimental results fairly closely once the material softens, though without modelling the initial deformation accurately. This is probably due to limitations in the spring model used, specifically that the mathematical formulation of the reduced polynomial model could not capture the change in stiffness without employing a much greater number of constants. Table 5-3 and Table 5-4 show the parameters which were used for these fits.

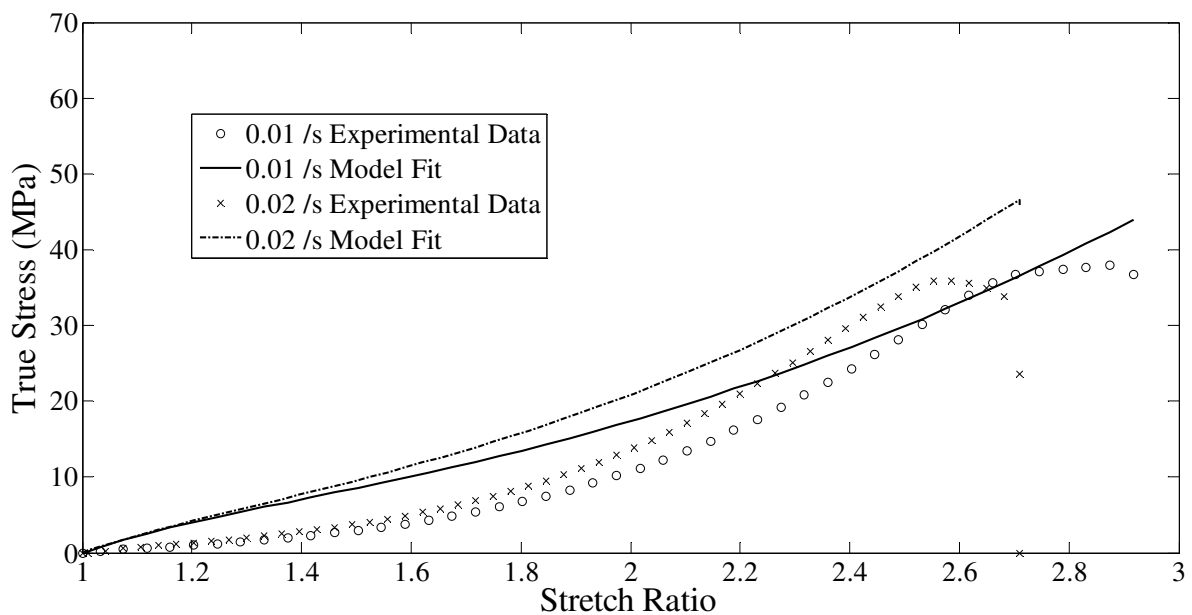


Figure 5-13: Low strain rate Prony series with the reduced polynomial spring fit for the tests at rates 0.01 s^{-1} and 0.02 s^{-1} .

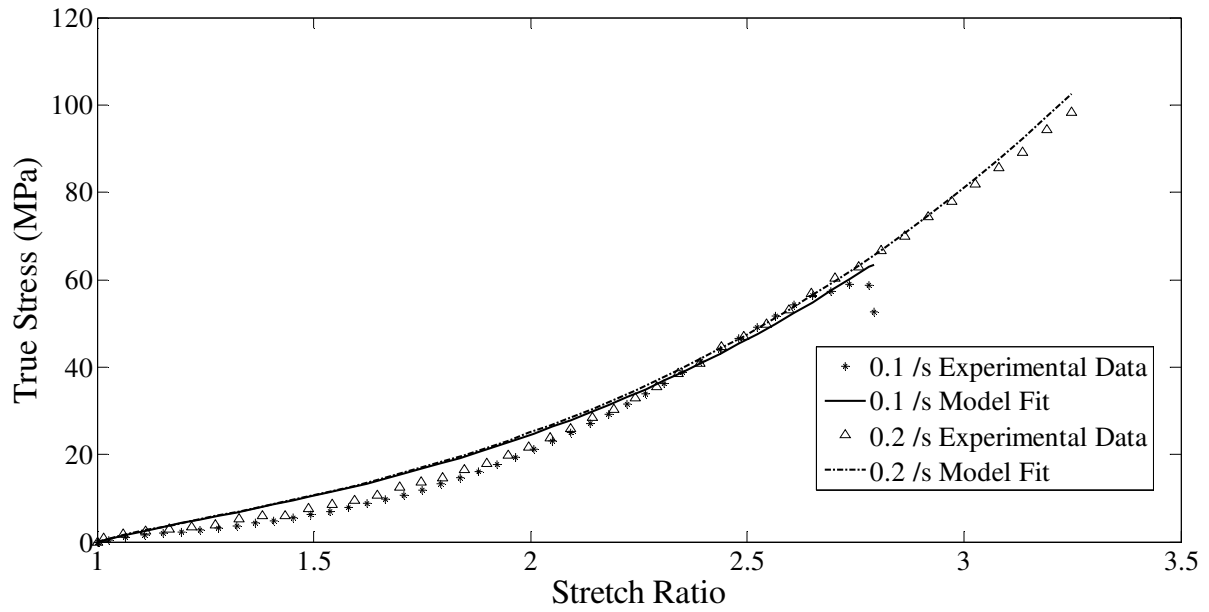


Figure 5-14: Low strain rate Prony series with the reduced polynomial spring fit for the tests at rates 0.1 s^{-1} and 0.2 s^{-1} . Hooper's data are shown for the 0.2 s^{-1} case.

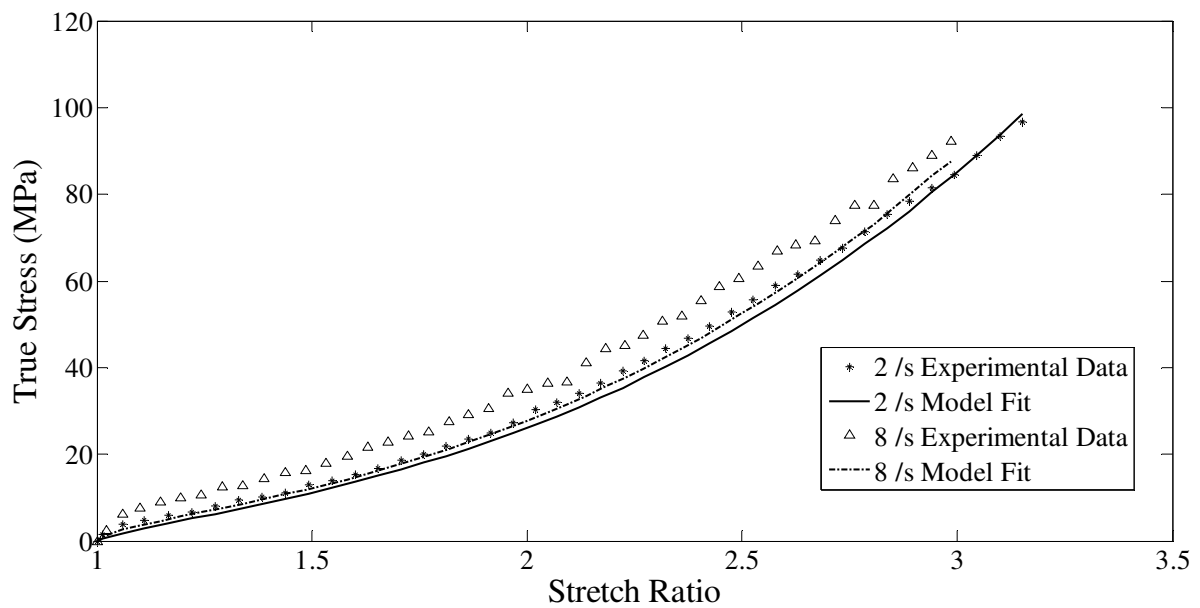


Figure 5-15: Low strain rate Prony series with the reduced polynomial spring fit for the tests at rates 2 s^{-1} and 8 s^{-1} .

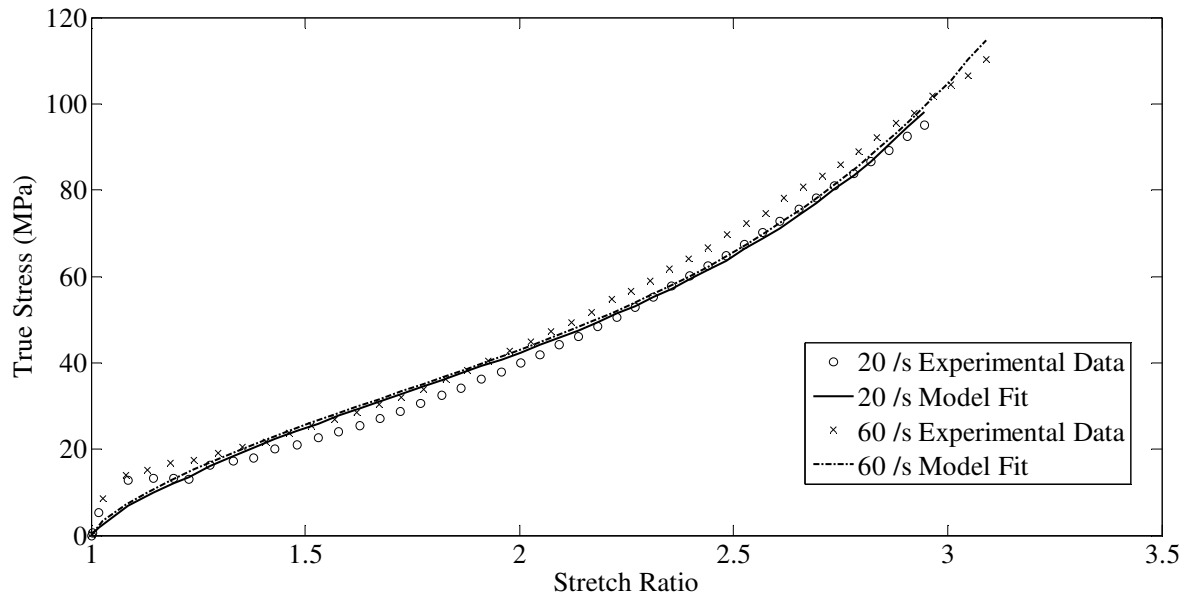


Figure 5-16: High strain rate Prony series with the reduced polynomial spring fit for the tests at rates 20 s^{-1} and 60 s^{-1} .

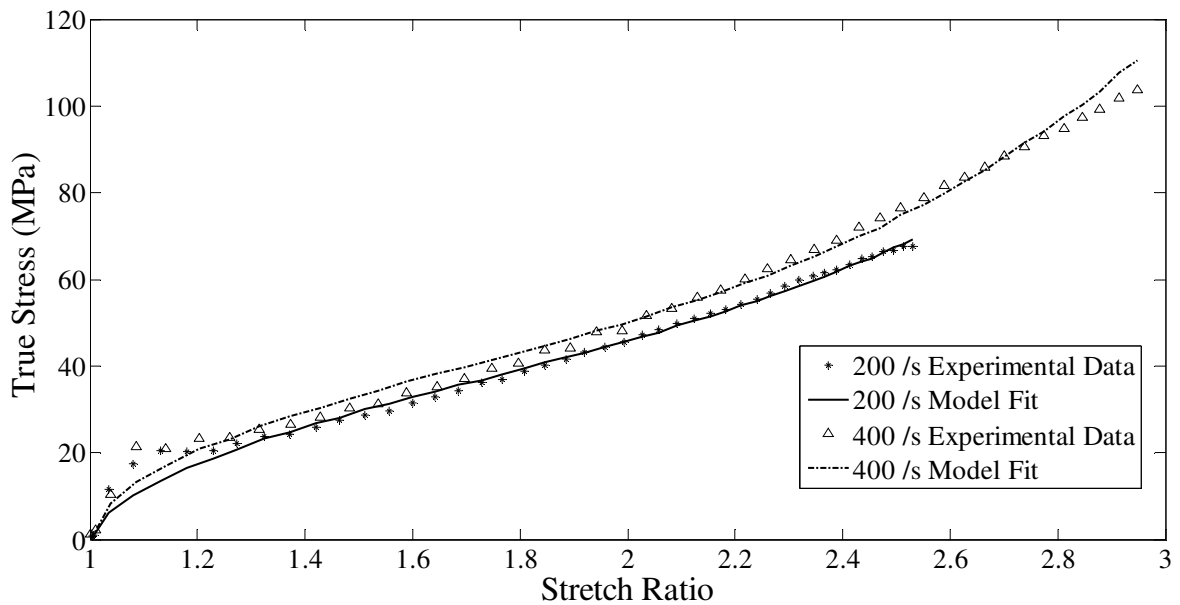


Figure 5-17: High strain rate Prony series with the reduced polynomial spring fit for the tests at rates 200 s^{-1} and 400 s^{-1} .

5.3.3. Finite deformation viscoelasticity

Figure 5-18 and Figure 5-19 show selected plots of the experimental data with their respective model fits obtained using finite deformation viscoelasticity. The quality of the fit is

again lower for the 8 s^{-1} . However, in this case the model overestimates the initial stiffness rather than underestimating it as in the Prony series results. The lower rate cases show more accurate results than for the Prony series fittings, especially in the 0.02 s^{-1} case. At the lower rates the model shows some noise at low stretches, up to about $\lambda = 1.1$. This is due to the difficulty of reaching a stable numerical solution for the nonlinear differential equation in these cases. Also, the model does not capture the sudden jump in the 200 s^{-1} curve above $\lambda = 2.5$. However, it is thought that this was due to grip slippage during the test. The model is able to switch between the high rate behaviour and the low rate behaviour, as can be seen by the difference between the 200 s^{-1} and the 0.02 s^{-1} curves. Table 5-5 and Table 5-6 shows the parameters which were fitted for the two springs.

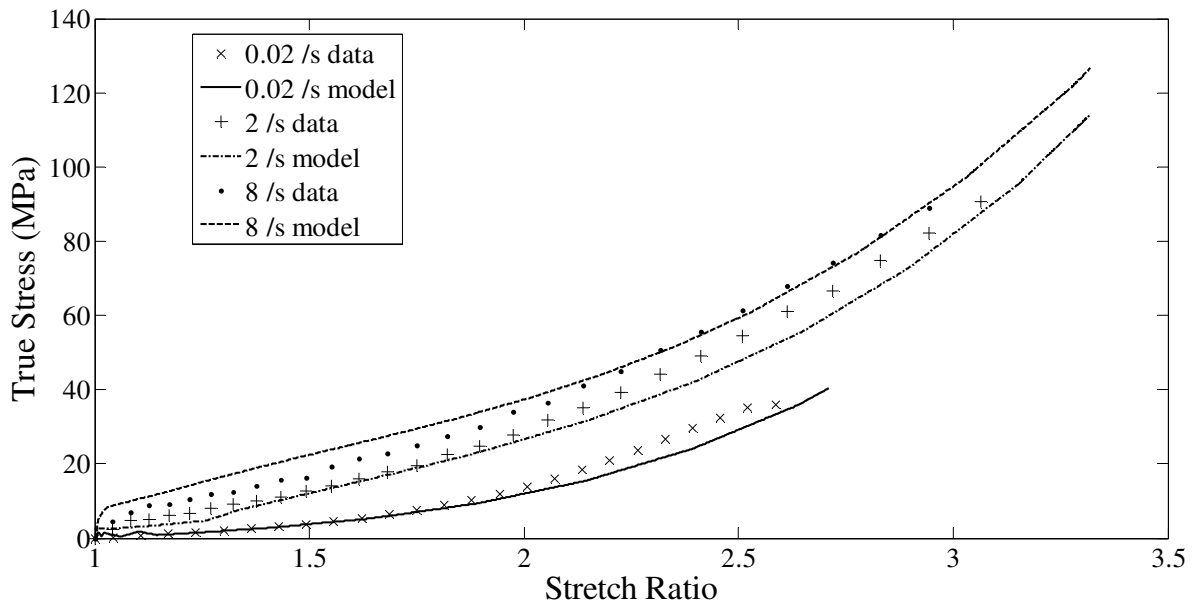


Figure 5-18: Finite deformation viscoelasticity material model fit for the rates 0.02 s^{-1} , 2 s^{-1} and 8 s^{-1} .

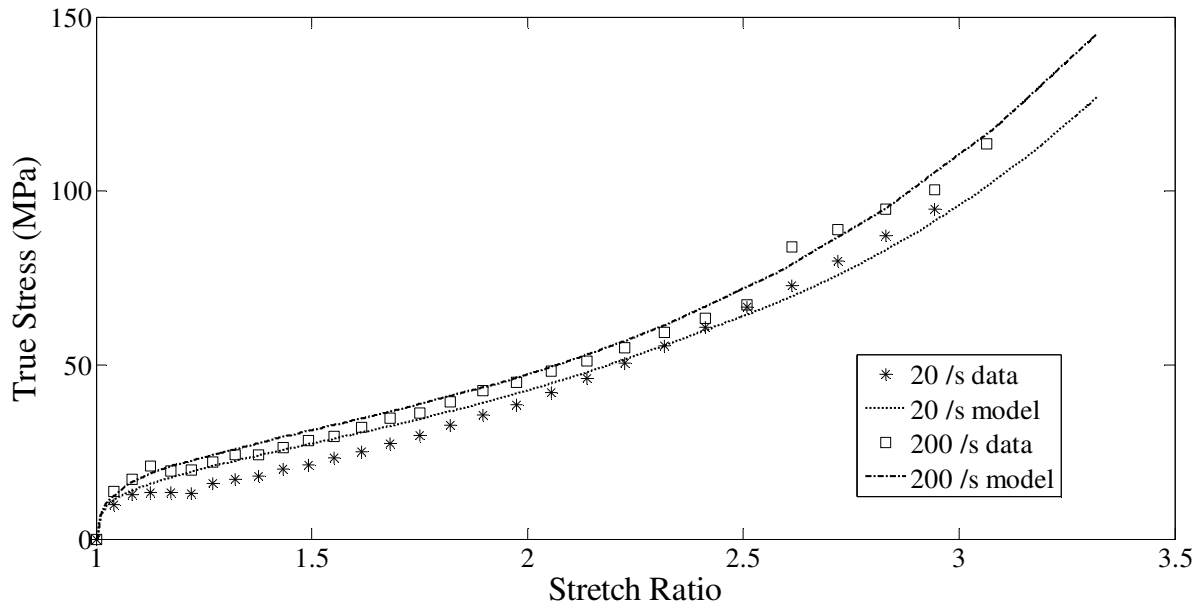


Figure 5-19: Finite deformation viscoelasticity material model fit for the rates 20 s^{-1} and 200 s^{-1} .

Table 5-5: Hyperelastic spring parameters for finite deformation viscoelasticity.

μ_0 (MPa)	α_0	μ_1 (MPa)	α_1	μ_2 (MPa)	α_2	μ_3 (MPa)	α_3
0.632	4.141	117.373	0.746	16.035	0.806	-121.468	0.768

Table 5-6: Damping function parameters for finite deformation viscoelasticity.

$C\eta_1$ (MPa s)	$C\eta_2$	$C\eta_3$ (MPa s)	$C\eta_4$	$C\eta_5$	$C\eta_6$	$C\eta_7$
1.131	-0.248	0.026	-1.527	6.626	4.876	-5.893

5.4. Discussion

All models can capture aspects of the PVB behaviour at the various strain rates, though showing different limitations. The Prony series models for low rates struggle to represent the data at both the bottom and the top end of their rate range. They produce stresses greater than

measured for the slower tests. The 8 s^{-1} curve fit instead does not show the initial stiffness, which starts being evident at this stage. The high rate (above 20 s^{-1}) fits show a good agreement beyond the initial stiffness region. However, both models struggle to account for the 200 s^{-1} and 400 s^{-1} cases. This error is much smaller in the Hoo Fatt spring model, which shows the correct behaviour, albeit without reaching the correct stress, whilst the reduced polynomial model struggles to show the initial stiffness in any of the high rate cases. Employing more terms might have improved this. However, the results might then show significant oscillation, precluding their use in finite element models. After the initial stiffness phase though the high rate models produce a better fit than the low rate models. The greater accuracy of the high strain rate fits might be due to the fact that the shape of the stress stretch curves for these cases shows relatively less variation, allowing a single model to capture their behaviour. It could therefore be argued that at least a third model could be fitted for intermediate rates, including the 8 s^{-1} rate. More experiments would be needed at speeds above and below this rate to achieve this.

The rigorous finite deformation viscoelasticity is able to fit most of the data sets considered. Again, the fit quality is lower at lower stretches, however it tends to improve significantly as the deformation increases. Additionally, the model can represent the higher stiffness region at high strain rate employing just one set of constants, thereby opening the possibility of using a single model to represent the PVB material for a range of situations. This however, comes at the cost of greater model complexity, both in terms of the model derivation and of the procedure needed to fit the material constants.

Figure 5-20, Figure 5-21 and Figure 5-22 show plots of the relative errors for strain rates of 0.02 s^{-1} , 8 s^{-1} and 200 s^{-1} . These data were calculated by dividing experimental stresses by modelled stresses at each point. It can be seen that the relative error for the finite deformation model, shown in black lines for each rate, is generally lower, i.e. the curves are closer to 1. The greater errors of all the models tend to be at very low strains. This is most likely due to the difficulty of mathematically representing the sharp change in stiffness observed in the available data. Also, confirming the observation made before, the Prony series models show greater relative errors for the lower strain rate cases, again especially at low strains. The finite deformation model performs better in this region, even though it does show high relative errors at very low stretches. The quality of the fits discussed above is therefore highlighted in these results. However, it should also be considered that a relatively small absolute error will translate to a more significant relative error if the stress magnitude is smaller, as is the case at

stretch ratios approaching 1. This should also be borne in mind when considering the seemingly constantly improving fit quality as the stretches increase. In fact, in this case the larger absolute value of the stress will tend to minimise relative errors. Whilst the limitations of this error estimate need to be considered, the plots shown below are still a useful tool to compare the different models at different strain rates, showing in general that more accurate results are produced by the finite deformation viscoelasticity model.

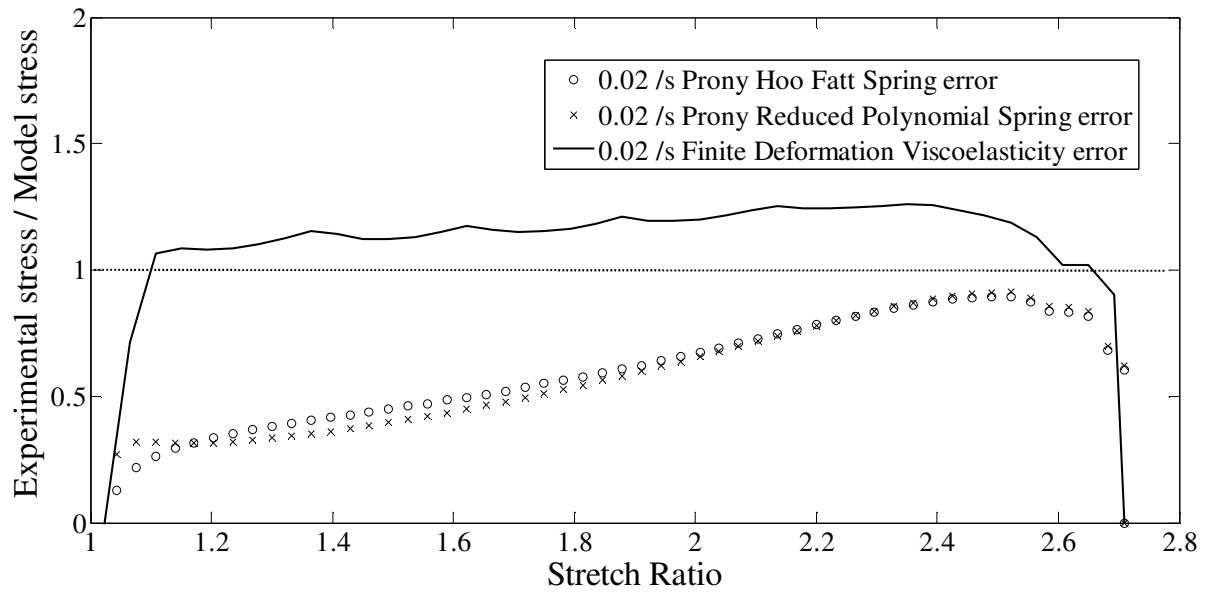


Figure 5-20: Relative fit error for the 0.02 s⁻¹ strain rate case.

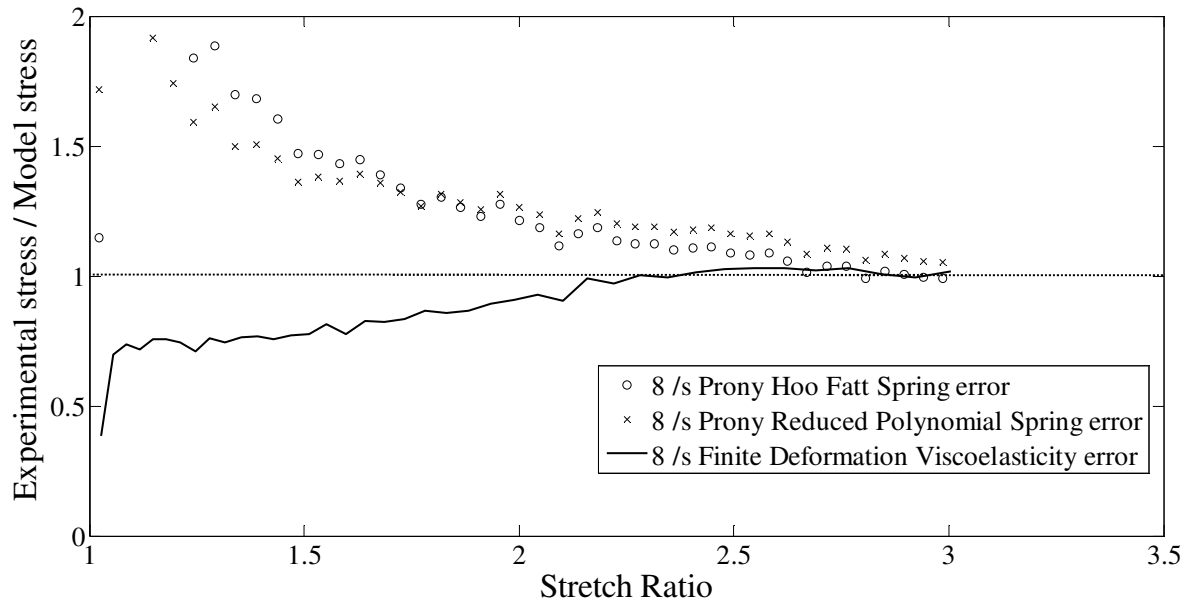


Figure 5-21: Relative fit error for the 8 s⁻¹ strain rate case.

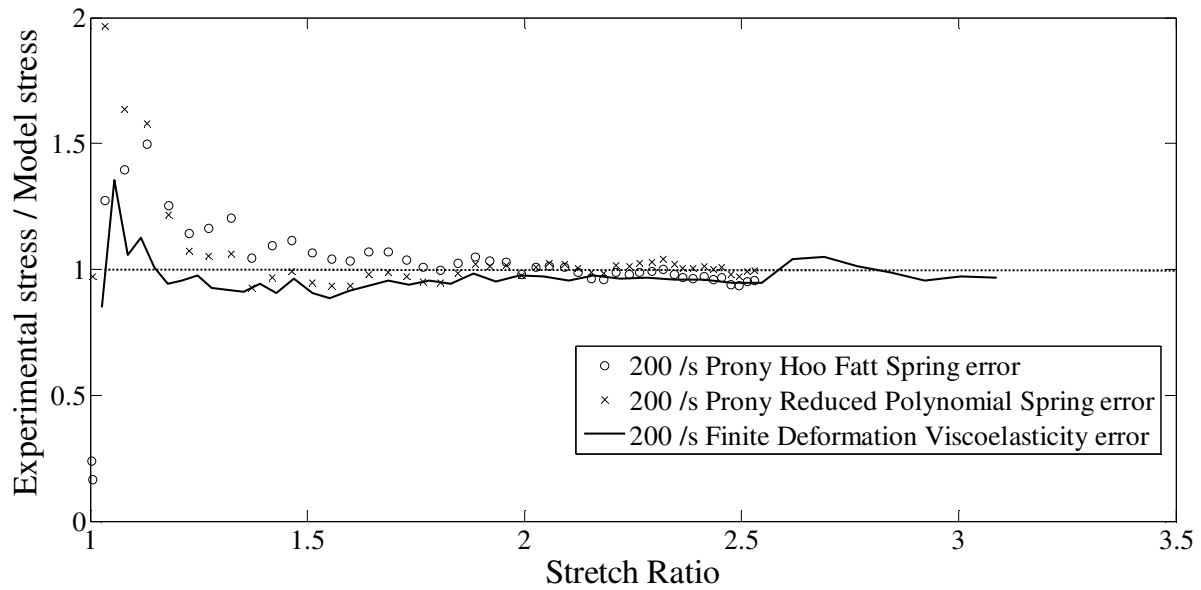


Figure 5-22: Relative fit error for the 200 s^{-1} strain rate case.

5.5. Conclusion

The PVB tensile experimental data demonstrated the high level of non-linearity and strain rate dependency exhibited by this material. Further analysis of the overall laminated glass composite material will require reliable material models for this important component, which will need to be modelled to account for various loading condition and strain rates. Two methods have been used to produce three material models which account for this shift in behaviour. In the first approach, two hyperelastic functions were used to produce four separate Prony series viscoelastic models. These were fitted to the high rate experimental data, including 20 s^{-1} and above, and to the low rate experimental data, including 8 s^{-1} and below.

The Hoo Fatt hyperelastic spring model produced more accurate results for the high rate fit, even though the quality deteriorated somewhat at higher rates. The low rate fit did not manage to capture the change in behaviour of the material in the range considered. This caused it not to represent closely the small strain behaviour of the fastest case, as the experimental curve started to show an initial higher stiffness. Additionally the behaviour of the lower rate data sets was not captured accurately. The same limitations were observed in the reduced polynomial model, which produced results of similar quality. This representation modelled more accurately the material behaviour at intermediate stretches whilst it produced worse results at lower deformations. However, the results followed a similar pattern to that

observed previously. These results suggest that, ideally, a third Prony series model might be needed for intermediate strain rates, thereby avoiding the need to produce a single model which has to account for these changes in behaviour. Both Prony series models performed significantly better at rates above 20 s^{-1} , especially at larger stretch levels. The Hoo Fatt spring performed better at lower stretches, as it was better able to represent the rapid change in stiffness. The reduced polynomial model could not manage this, showing higher errors at this stage.

To overcome the Prony series limitations, a finite viscoelasticity model was also derived following the method reported by Huber and Tsakmakys [64]. This was shown to account for the entire range of rates considered, generally with fit qualities which at least matched the results of the Prony series. However, both the model derivation and fitting procedure are significantly more complex. Additionally, the model in the form presented requires a large number of constants to be determined (eight for the springs and seven for the damper). Also, as the model is not currently included in commercially available finite element software, a user material subroutine would need to be developed in order to apply it.

If the phenomena of interest can be limited to a specific small range of strain rates, it would probably be advantageous to employ one of the Prony series viscoelastic models, as these require fewer constants to be included and they are commonly supported by FE software. However, if the rates of interest are not known or are spread over a larger range, the finite viscoelasticity model might well prove advantageous. In this case the large number of constants to be fitted would still be smaller than the total number of parameters for both the Prony series models and the overall accuracy would be improved.

In the next chapter the delamination of glass layers from the PVB membrane was studied employing FEA models. To achieve this, the PVB membrane was modelled employing the reduced polynomial Prony series model. Whilst the finite viscoelasticity model could provide a better approximation of the behaviour, it was decided that the complications relating to its implementation would make this not feasible for this project. The chosen model is instead already implemented in ABAQUS, making its use straightforward.

6. Glass – PVB Delamination

6.1. Introduction

The cracked laminate behaviour will be affected by two main elements, the PVB membrane material properties and the behaviour of its interface with the glass fragments. In the previous chapter the properties of the membrane were studied in detail. Some of the results obtained will now be used to consider the delamination behaviour of the composite material.

During a blast, the pressure wave causes the glass panes to fragment at relatively low displacements. As the PVB membrane is subsequently stretched, the glass fragments will tend to detach from its surface through a delamination process. This absorbs considerable energy, and tends to cause a plateau in the forces recorded as a steady-state equilibrium is reached and the delamination progresses at a constant rate. As a first step to estimate the delamination effect on a realistic window system, the basic energy absorption information needs to be determined. Hooper [3] performed several tensile tests at different speeds on laminated glass samples with varying numbers of cracks. In the present study some of these tests are employed to calculate delamination energies using one of the PVB models derived in the previous chapter. Previous authors, for example Muralidhar et al. [29], developed analytical solutions to calculate the dissipated bond energy directly from measured the external work. However, the PVB model used in this work has a more complex algebraic formulation, which prevented the derivation of a closed form solution for the delamination energy. Therefore, FEA models were used to fit the delamination energy parameters instead of explicit analytical solutions. Both the plateau stresses obtained experimentally and the speed of the delamination visible in the available high speed pictures were used to compare the experimental and modelling results.

Once the energy values had been determined, a parametric study was performed on the effect on the plateau stresses of different crack configurations. Whilst in a real blast situation the cracks are concentrated in several typical areas, as described in Chapter 3, their exact density and orientation varies. Additionally, some cracks might not be aligned in the two panes, introducing significant complexities in the material's behaviour. The effect of these factors therefore needs to be considered to inform possible designs. Experimental data were available

for 20 mm and 10 mm crack spacing. This information was extended to 5 mm distances through FEA analysis to produce estimates of wider application. Additionally, diagonal cracks and cracks in different positions on the two sides of the PVB have been explored to assess their effect on the overall behaviour.

6.2. Method

The delamination energies and effect of crack distribution were the subject of separate investigations. Additionally, the delamination speed, one of the parameters used to verify the accuracy of the results, was extracted from the experimental images. The procedure used for each of these steps is described below.

6.2.1. Experimental data

The tests were performed by Hooper [3] and consisted in laboratory tensile experiments on pre-cracked laminated glass rectangular samples, of size 150 x 60 mm. The glass layers were 3 mm thick. Whilst the PVB membrane thickness was varied, only the results from samples with a 1.52 mm thick interlayer were used in this analysis. The cracks were generated by scoring the glass with a dedicated tool and applying a sharp shock to propagate full depth cracks.

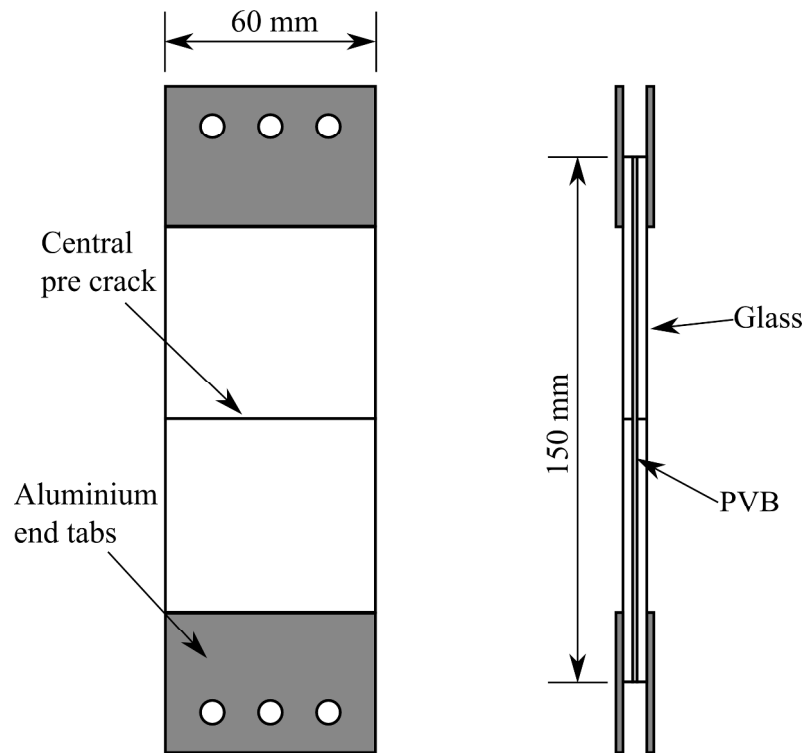


Figure 6-1: Diagram of a typical single crack experimental sample.

Specimens were prepared with three crack patterns: a single crack at mid length and multiple cracks at 20 mm and 10 mm spacing. Aluminium tabs were bonded to each end of the specimen to connect them to the actuator arm of the testing machine. The tests were performed using an Instron high speed servo-hydraulic testing machine at several speeds. The 0.1 m/s, 1 m/s and 10 m/s results are analysed here. Figure 6-1 shows a diagram of a single crack specimen. The loads were measured using a piezoelectric load cell and the displacements were recorded by an LVDT sensor measuring the position of the actuator. Photoelasticity techniques were used to highlight the position of the delamination front either side of the cracks as the test progressed. A Photron FASTCAM SA1 high speed camera was used to record colour pictures at different frame rates depending on the speed of the test.

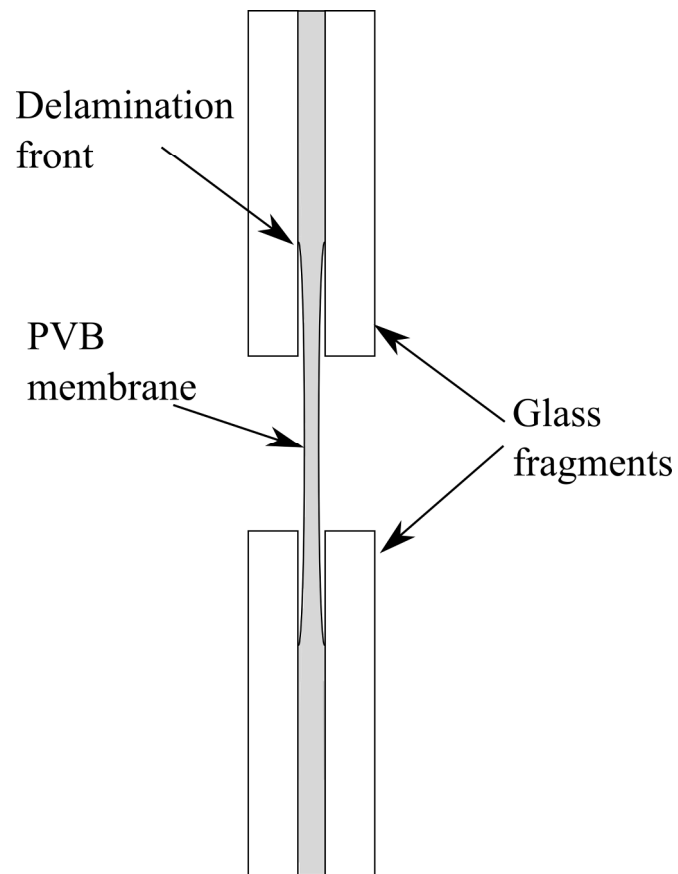


Figure 6-2: Section showing the expected PVB behaviour during the tests.

Figure 6-2 shows the expected behaviour of the PVB membrane during the test. Delamination is expected to take place on both sides and both above and below the crack locations. Figure 6-3 shows a typical test image. A delamination front and a glass edge are identified in the figure.

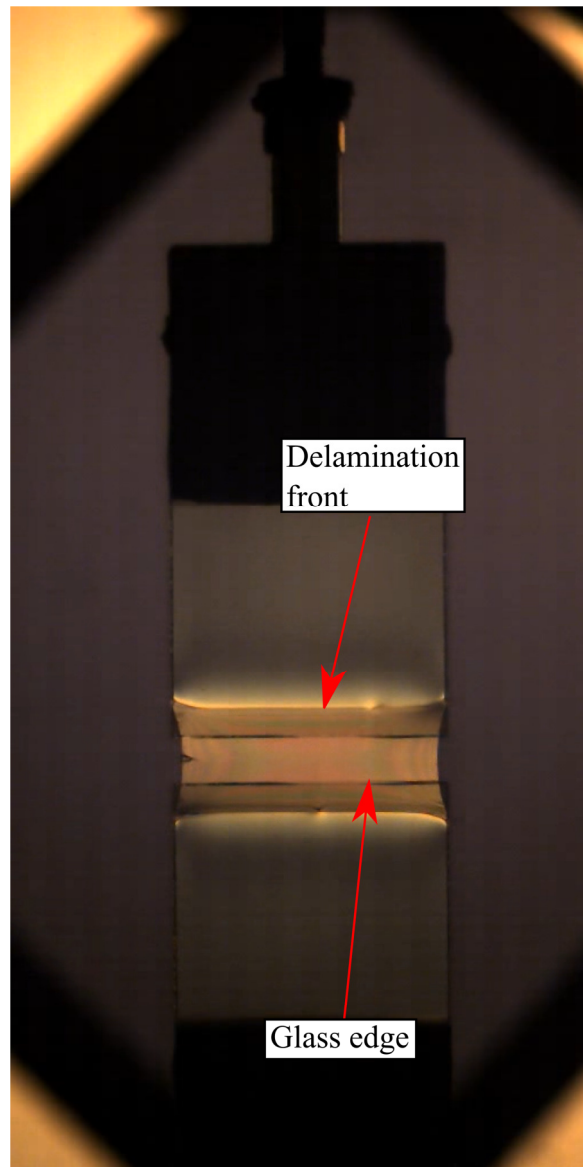


Figure 6-3: Image of a single crack sample, obtained using a high speed camera.

The recorded force and positions of the delamination fronts were employed in the present study to assess the delamination stress plateaux and the propagation speeds of the failure events. Figure 6-4 shows an example plot of the experimental data. The load had been converted to stress dividing the force data by the unloaded PVB cross-sectional area. The plateau stress values employed in later analyses are given in Table 6-1.

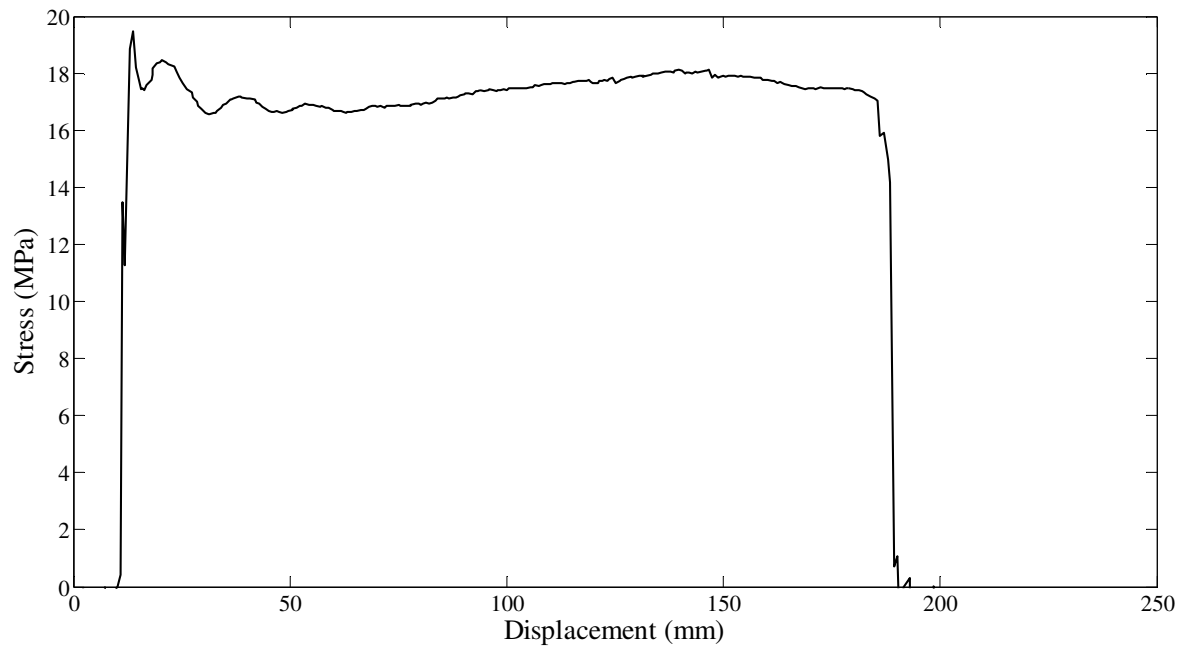


Figure 6-4: Experimental stress-displacement data for a 1 m/s loading speed on a 10 mm crack spacing specimen.

Table 6-1: Experimental plateau stresses used in the analysis [3].

Crack Configuration	Test Speed (m/s)	Repetitions	Plateau Stress (MPa)
Single Crack	0.1	1	13.2
Single Crack	1	1	21.3
Single Crack	10	1	26.3
10 mm Spacing	1	1	17.5
20 mm Spacing	1	1	16.5

Whilst the force readings were already available, the speed of the delamination propagation was calculated for use in this study. The approximate positions of the four main elements, the top and bottom delamination fronts and the glass edges, were determined in the images. Their distance could then be found at several time steps and the delamination velocity calculated.

Two methods were employed for this, depending on the test information available. Where the images were clear enough, measurements could be performed with an automatic procedure using Matlab. An algorithm was prepared to locate areas where the gradient of the intensity of the colour was highest. The software then assumed that these areas indicated the location of either a delamination front or of a glass edge and recorded its position. The distance between the uppermost two locations and the lower two was then calculated, thus providing the length of the delaminated area in the picture. The procedure was repeated on all the available images and the speed of delamination was obtained. As a checking procedure, the same algorithm modified the original pictures, including in them a horizontal white line at each of the four locations together with a vertical line to indicate the section along which the analysis was performed. A sample of these modified pictures is shown in Figure 6-5 below.

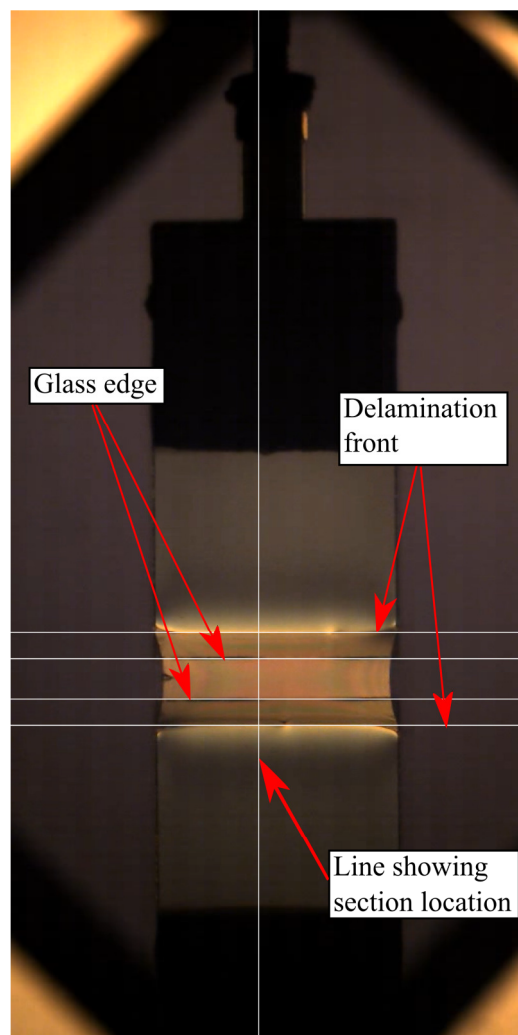


Figure 6-5: Modified experimental picture with white lines inserted at delamination and glass edges locations.

Where the images were complicated by the presence of unwanted glass fragments or in the case of “multiple cracks” specimens when more delamination fronts were present, the automated algorithm did not produce acceptable results. In these situations some of the pictures, taken at regular intervals, were modified by hand. White horizontal lines one pixel wide were included at the relevant locations. The delaminated areas and speeds were then determined from these. Four pictures were employed for each test.

6.2.2. Delamination energies

The procedure used to determine the PVB to glass delamination energies from these tests is similar to that applied to peel tests. It involves calculating the external work done from the force displacement information, and then deducting the dissipated energy components, in this case solely the internal strain energy of the PVB material. The remaining energy can be assumed to be absorbed by the delamination process.

In previous work using similar test set ups ([29], [30], [20]), the delamination energies of PVB-to-glass interfaces was determined using results from predominantly lower speed tests, below and assuming different PVB material properties. Muralidhar et al. derived closed form equations considering several hyperelastic material formulations. A polynomial formulation was chosen to calculate the delamination energy in the experimental cases considered. Butchart and Overend instead employed a linear-elastic viscoelastic model. Iwasaki did include some higher strain rate tests. He then used an elastic PVB model to calculate the delamination energies. In the present work a more realistic material model, including both non-linear hyperelasticity and viscoelastic effects has been formulated. However, the complexity of the material model prevented a closed form analytical solution for the delamination energy from being formulated. Instead, a finite element model employing cohesive elements was used.

FEA Modelling

The analysis was performed with ABAQUS release 6.10. Three main elements were included in all the models employed: (i) the internal PVB membrane, (ii) the glass fragments and (iii) two layers of cohesive elements connecting the glass and the PVB parts.

The geometry of the models was slightly modified from the experimental geometry due to numerical issues. A 5 mm long PVB part was included on the top and bottom of the main body of the materials. The overall dimension was kept constant at 150 mm. This part was added to dampen vibrations, which would otherwise have prevented significant information from being extracted from the FEA data. However, as potentially this addition could have significantly modified the results, such as the plateau stresses, the data obtained from one model without these end parts were compared with those obtained when they were included. All other material and boundary condition parameters were kept constant. The results of this comparison are shown below. Additionally, a half model with a boundary condition on the vertical symmetry plane was employed to increase the computational efficiency of the calculations.

Once a basic model geometry was fixed, it was used for all the speeds and material properties considered. This final geometry consisted of a model of 150 x 30 mm overall dimensions. The glass layers were 3 mm thick and of 140 mm overall height. They were split into separate parts at mid-length to simulate the pre-existing crack. The PVB interlayer was modelled as 1.52 mm thick. Separate PVB components were used for the top and bottom composite areas. The cohesive zone was represented with two 0.01 mm thick element layers added to each face of the membrane. Tie conditions were set between these and the glass faces touching them. The PVB parts were also connected with a tie condition. Only the PVB material elements were included in the constraint, excluding the cohesive layers. This was done to avoid numerical issues which emerged when the cohesive elements were included in the tie. Figure 6-6 shows the geometry of the models with the main dimensions.

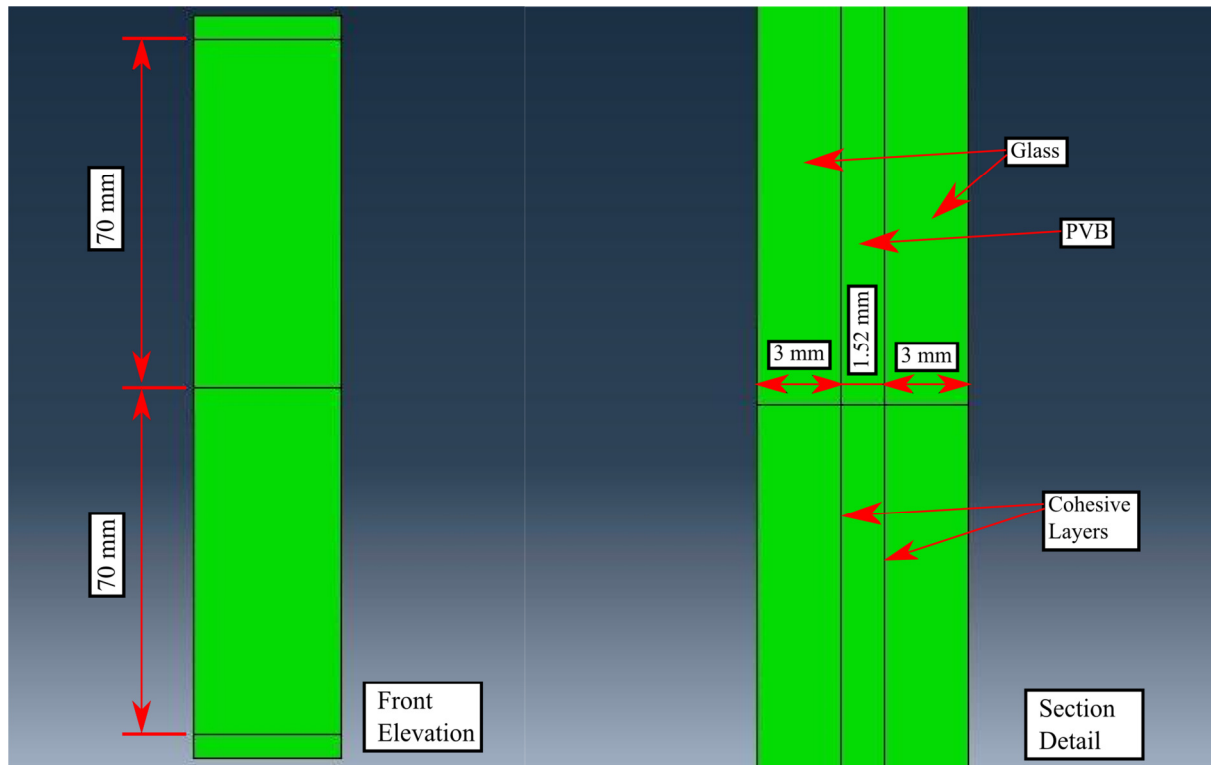


Figure 6-6: Main dimensions and features of the single crack delamination models.

The glass parts were meshed with 1 mm cubic stress elements (C4S4). The PVB membrane was meshed with the same type of elements. However, their thickness was reduced to 0.76 mm to ensure at least two elements were present in the thickness direction of the layer. It was considered that, for this analysis, through thickness effects would not be dominant and hence the use of two elements would be acceptable in this direction.

The cohesive elements were also set to have 1 mm sides. However, their thickness was 0.01 mm. They were generated from the PVB mesh, and hence were a feature of the same parts of the model. The typical mesh at one of the corners of the model is shown in Figure 6-7.

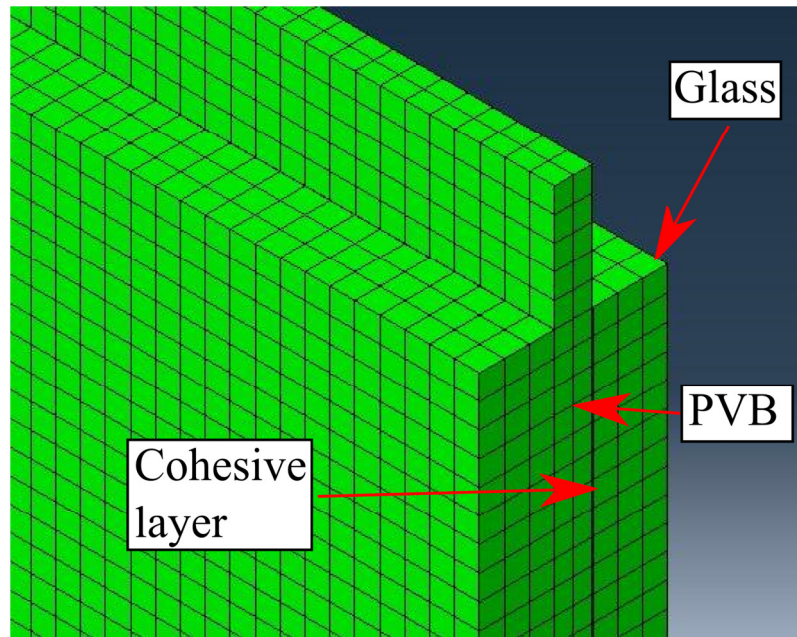


Figure 6-7: Detail of the mesh used in the model. The view shows a corner of the model.

The properties of the main components are summarised in Table 6-2. Glass was represented using linear elastic material properties, as the stresses it would be subject to would generally not be enough to justify non-linear approaches. Its Young's modulus was 70 GPa, with a density of 2500 kg/m^3 and a Poisson's ratio of 0.33. The PVB was modelled with the reduced polynomial viscoelastic models derived in the previous chapter. As it had been decided that the details of the material model employed for this material would affect the energy parameters, each FEA model was run with both the low rate and the high rate coefficients derived. The cohesive elements were modelled using a cohesive zone modelling (CZM) technique. This, as described by Chandra et al. [75], requires the user to specify of the behaviour of the material before a peak stress is reached, the peak stress itself and a degradation law which governs the loss of capacity past the peak. A bilinear formulation was employed, assuming that the behaviour both before and after the peak capacity could be modelled with linear laws. Figure 6-8 shows a stress-displacement plot of such a law. The failure during the tests will take place through a combination of mode I and mode II failure deformations. Muralidhar et al. [29] state that their results indicate that for this situation mode-mixity can be ignored. This approach was implemented here defining the same parameters for the failure modes. A quadratic damage initiation model available in the software represented by Eq. 6-1 was used to define the onset of failure [76].

$$\left\{ \frac{S_n}{S_n^o} \right\}^2 + \left\{ \frac{S_s}{S_s^o} \right\}^2 + \left\{ \frac{S_t}{S_t^o} \right\}^2 \quad \text{Eq. 6-1}$$

where S_n , S_s and S_t are the stresses in the normal and the two shear directions and S_n^o , S_s^o and S_t^o are the capacities for the same modes.

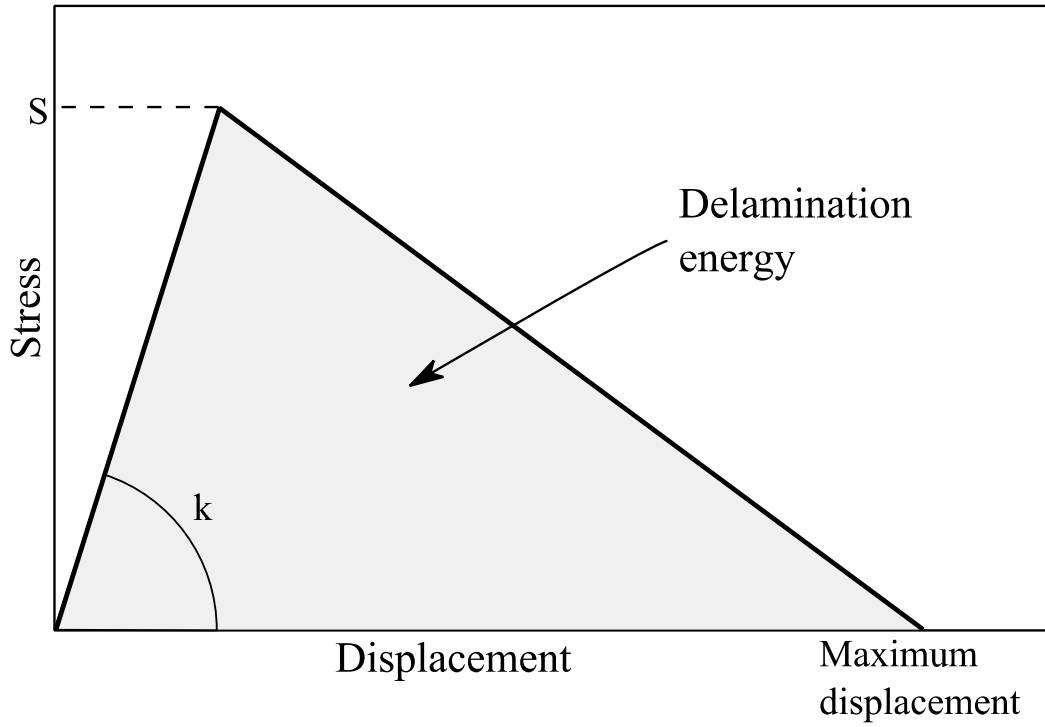


Figure 6-8: Cohesive zone model stress-displacement law used in this study.

A traction elasticity formulation was used for the pre-peak phase. The traction stiffness (k) parameters were determined assuming the cohesive material would behave in a similar way to the PVB bulk material. Employing the Young's modulus derived by Hooper [3], a stiffness parameter was calculated assuming a mesh size of 1 mm square and of 0.01 mm thickness. Therefore, the tensile stiffness of the cohesive elements was assumed to be equal to 5300 GN/m. A shear modulus was estimated from this assuming isotropic behaviour and a Poisson's ratio of 0.5. The onset of damage therefore took place in tension at displacements of 0.002 mm for the 10 m/s case, representing a strain of 0.2 considering the element thickness. Typical ultimate failure displacements were instead in the range of 0.35 mm to 0.7 mm. As no direct measurements were available for these quantities, a sensitivity analysis was performed. The assumed value was considered to be the minimum for the stiffness. The model was also analysed with stiffness values of 10^5 GN/m and 10^6 GN/m. No higher value

was employed due to convergence issues of the FEA model. The results obtained with higher k values were then compared with those obtained with the original stiffness.

Table 6-2: Summary of the material models employed in FEA models.

Material	Model type	Summary of parameters
Glass	Linear elastic	$E = 70 \text{ GPa}$, $\rho = 2500 \text{ kg/m}^3$, $\nu = 0.33$
PVB	Viscoelastic – Prony series with a reduced polynomial spring	See Table 5-3 and Table 5-4 for fit parameters
Cohesive elements	Traction separation	$k = 53 \text{ MN/m}$

A quadratic stress traction separation failure condition was applied to govern the degradation of the bond. The peak stress (S) was estimated from information provided by Rahul-Kumar et al. [28]. In his work the bond capacity between PVB and glass was measured using compressive shear tests at different strain rates. The peak stresses from his tests were employed to determine the failure stress parameter. Firstly, his results were extracted and plotted from data available in the paper. These data, plotted with a logarithmic horizontal axis, are shown in Figure 6-9. An exponential law was then fitted to the available points and used to extrapolate the values to higher rates relevant to those used in this study. The fit to the data is given by:

$$S = 0.782 \ln(\dot{\epsilon}) + 8.22 \quad \text{Eq. 6-2}$$

where S is the peak stress and $\dot{\epsilon}$ is the strain rate.

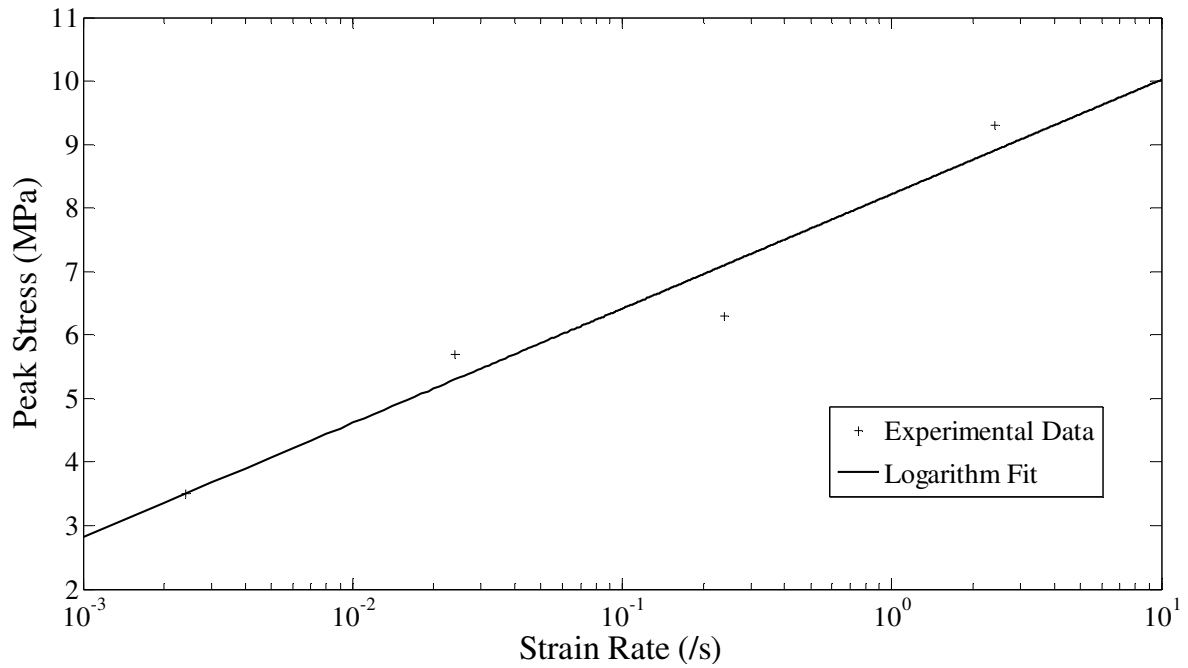


Figure 6-9: Peak stress against the strain rate from Jagota [77].

As extracting strain rate information directly from the experimental results proved unfeasible, an approximate value was determined by dividing the overall displacement speed by the sample length. A sensitivity analysis was then performed to check the effect of the peak stress. Several 10 m/s speed models were run with the low and high rate material properties. In each case, the peak stress was varied by 25% both above and below the value found with Rahul-Kumar et al.'s data. The total cohesive energy was firstly kept constant when changing the peak stress. Where differences in the plateau results were observed, a further model was run refitting the delamination energy. The difference in the energies could then be assessed. The final cohesive peak stresses used are shown in Table 6-3 for the three rates modelled.

Table 6-3: Peak stresses used in the models.

Speed (m/s)	Peak stress (MPa)
0.1	7.68
1	9.48
10	11.28

The last parameter of the cohesive zone modelling, representing the final displacement at failure, was used to fit the experimental data.

A symmetry boundary condition was set on one vertical side. A pinned condition was set at the bottom PVB surface, whilst a velocity condition was set on the top surface. The location of these is shown in Figure 6-10.

The 0.1 m/s model was run with artificially increased mass for some elements. This was done to increase the stable analysis increment to a level that avoided using an excessive number of steps to cover the required analysis time. A minimum step time of 2.5×10^{-7} s was enforced through an ABAQUS option. As this can affect the quality of the model results a model was run without the condition for comparison. The full analysis time could not be achieved without this modification, as the maximum advised number of steps, two millions [76], would have been reached. A 0.08 s period, instead of the full 0.5 s period, was run and the results up to that point were compared.

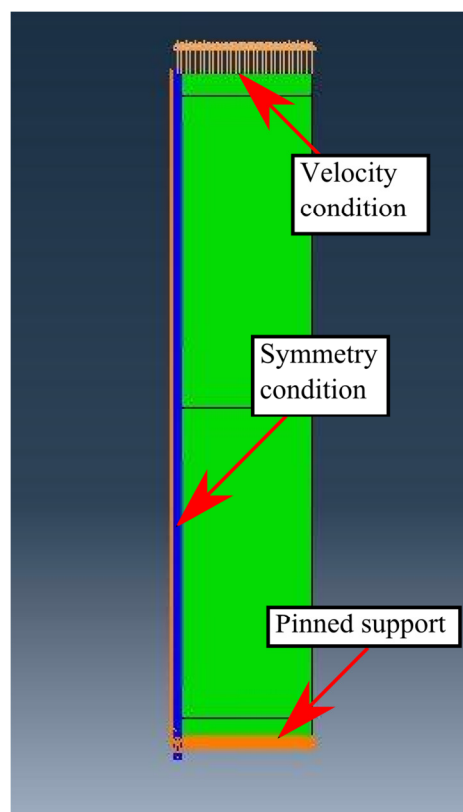


Figure 6-10: Boundary conditions applied in the model.

The models were run to fit the experimental data for the 0.1, 1 and 10 m/s tests. Therefore six models were run in total, two at each speed. All the models were run for a total deformation of 50 mm, therefore analysis times of 0.5 s, 0.05 s and 0.005 s were set respectively. Once the stress plateau levels seen in the experimental results were obtained, the delamination energy was calculated from the peak stress and failure displacement parameters used in the cohesive formulation. The delamination speed was also extracted for comparison with experimental results. Element failure data at four time steps were used for this.

The main steps of this analysis are summarised in Figure 6-11 below.

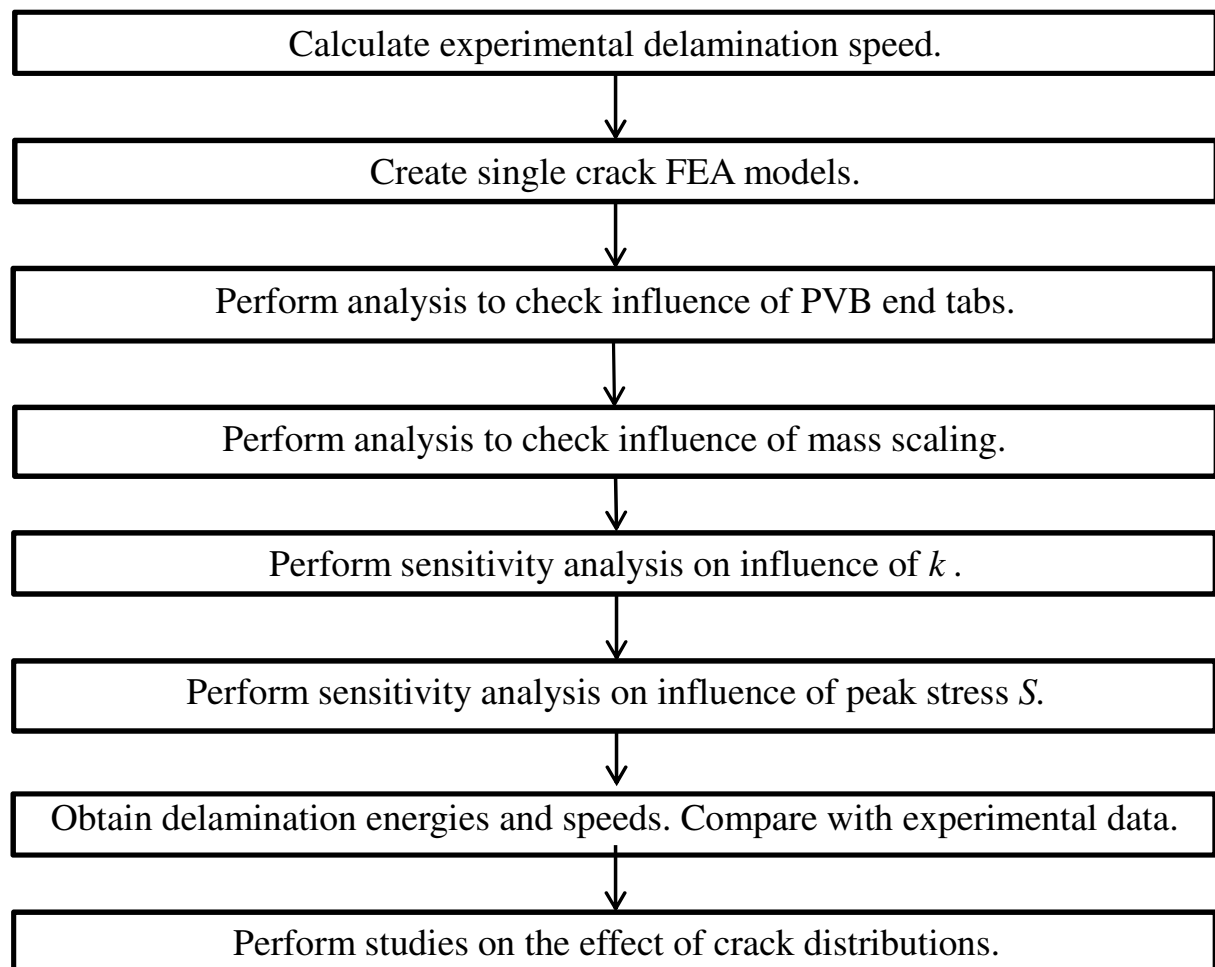


Figure 6-11: Steps undertaken to perform the delamination energy analysis.

6.2.3. Parametric studies

Crack spacing

The influence of crack density was considered an important parameter which might affect the overall material behaviour. Its effect was therefore studied with several FEA models. Crack spacings of 20 mm, 10 mm, and 5 mm were considered. The two lower crack densities had already been the subject of laboratory tests, to which the numerical results were compared.

The models employed were very similar to those used for the delamination energy studies. Separate glass and PVB parts were used for each section between cracks. These parts were then connected to each other in the same manner described above in section 6.2.2. The same mesh types and sizes were used. The only exception was the 5 mm crack spacing model, where the mesh dimension was reduced to 0.5 x 0.5 mm along the main surfaces. However, the through thickness dimension was kept constant at 1 mm to reduce the computational time. The same material properties summarised in Table 6-2 were used for the main components, applying the appropriate delamination failure constants determined in the previous study. Specifically, as 1 m/s models were run, the constants pertaining to this speed were employed. Figure 6-12 shows a typical model with 20 mm crack spacing.

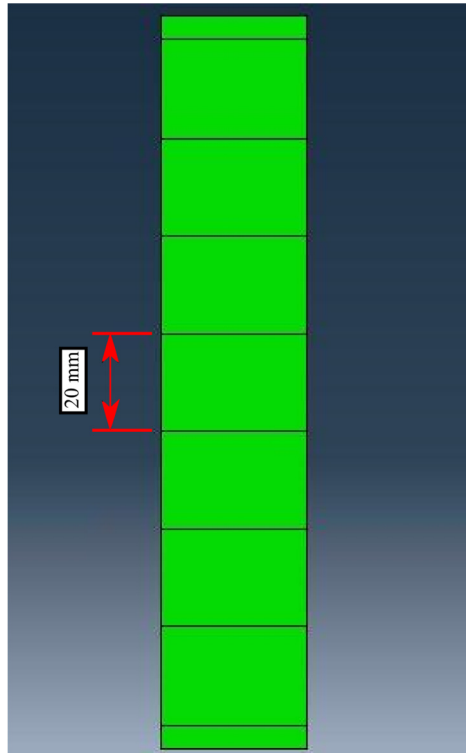


Figure 6-12: Front view of a multiple cracks, 20 mm spacing model.

All the models were run at a displacement rate of 1 m/s, and both PVB material models were used. Therefore, a total of six models were run for this section. Again, the plateau stresses and delamination speeds were extracted for evaluation. For the 10 mm and 20 mm cases these were also compared with experimental data. However, this was not available for the remaining crack spacing of 5 mm.

Crack inclination

Models were also run to assess the effect of the inclination of the cracks on the material properties. A 20 mm crack spacing were used for all cases. Three models were run with different inclinations. In one model, one side of the cracks was shifted by 20 mm, producing an angle of 18.43° . In a second model a shift of 40 mm was used, creating an angle of 33.69° . A third model was run using a 4.76° angle; however, in this case the direction of the inclination was alternated creating non parallel crack positions. Figure 6-13 shows the three models with the cracks locations and main dimensions. The three models were run using the same material properties as before, including both PVB models. Therefore six models were run in total. The entire model width was included without symmetry conditions due to the

non-symmetric crack patterns. Elements of the same type as stated above were employed; however, the size was changed to account for the inclination. 1 mm sides were used as a general guideline, and the size was modified as needed by the mesh generation algorithm to fit the shape of the part. All the models were run at 1 m/s.

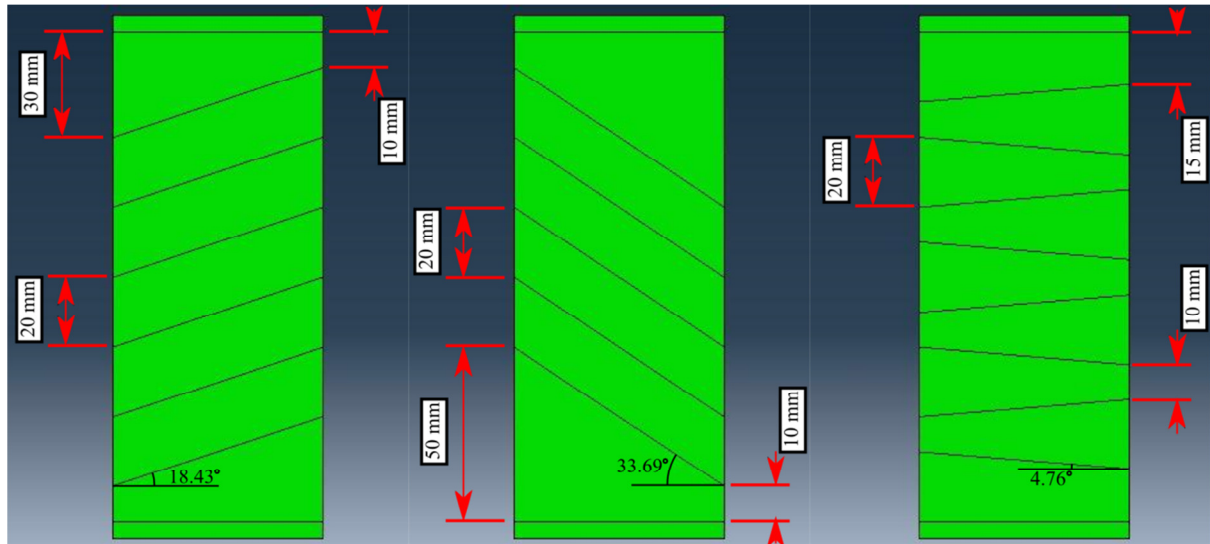


Figure 6-13: Front view with dimensions of the three inclined cracks models.

The plateau stresses were extracted for comparison between the models.

Staggered cracks

The effect of cracks not being aligned on the two sides of the PVB was considered next. A 20 mm crack spacing was again used and two cases were considered, one where the crack lines were displaced by 10 mm, and one where they were displaced by 5 mm. Figure 6-14 presents the main model dimensions and components. The model with 10 mm misalignment is shown. The only difference in the 5 mm misalignment case is the length of the top and bottom glass parts.

To avoid numerical instabilities, the geometry of the parts of the model had to be modified for this task. The PVB membrane was divided in two separate layers along the centre of the thickness. The two halves were then constrained to each other with a tie condition. The PVB parts on the two sides were staggered to follow the glass sections. This ensured that each glass fragment was connected to just one PVB part. Additionally, to ensure that the failure would occur at the staggered crack locations rather than at the top or bottom of the specimen,

the first and last glass fragments on each side were tied without a failure condition. The same material properties and mesh sizes were used as described above. Again, the plateau stresses were extracted for evaluation. The results of the previously described 20 mm spacing model without staggering were also employed for comparison.

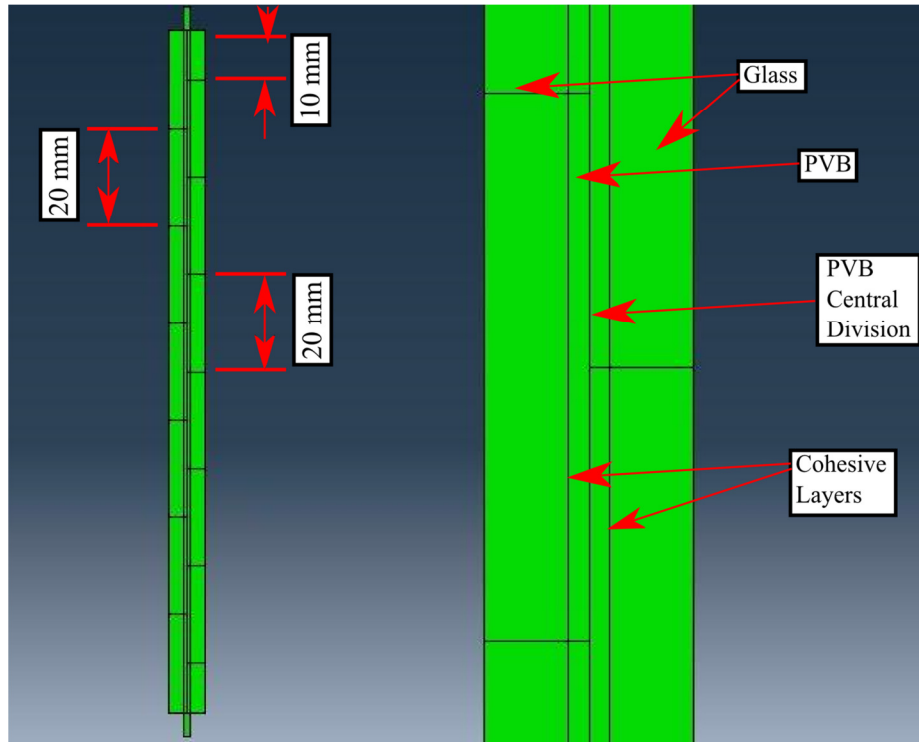


Figure 6-14: Main dimensions and components of the staggered cracks models.

6.3. Results

The results for the models described above will be considered here in turn. The delamination speeds extracted from the experimental pictures will be described first, as these will be used to assess the FEA results.

6.3.1. Experimental data analysis

The pictures available were analysed as described above. The automated algorithm was employed for the 0.1 m/s experimental images to identify the delamination fronts and glass edges in every picture. This allowed the progress of the phenomenon to be monitored throughout the deformation to estimate its general behaviour.

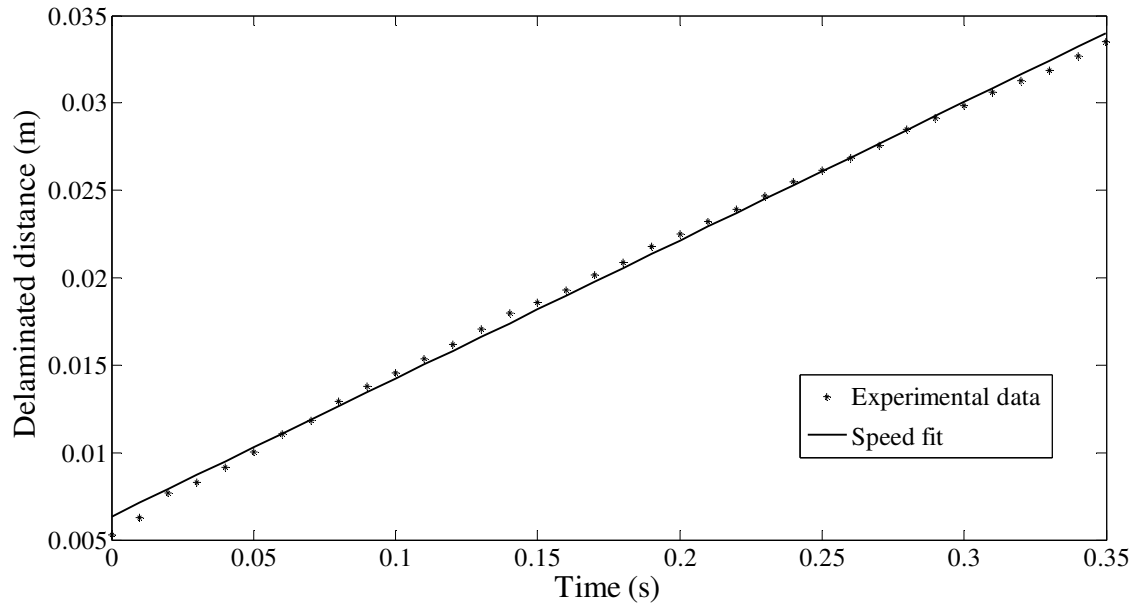


Figure 6-15: Delamination progress time history for 0.1 m/s test.

The delaminated distance data are shown in Figure 6-15, together with a plot produced using the estimated overall speed. As can be seen, the speed varies moderately during the course of the experiment. However, as the speed variations were small compared to the uncertainties and would be difficult to quantify in cases where only a limited number of pictures were analysed, it was considered that a constant value would represent an acceptable approximation.

Whilst manually preparing the images for other experimental speeds, it was noticed that frequently the exact location of the key positions was difficult to estimate exactly. Often an area of roughly three pixels could have been used for each point. Therefore, an error of ± 1 pixel for the position of each element was accounted for. In single cracks four such elements were identified, two delamination fronts and two glass edges, giving a total error range of 4 pixels. In a multiple crack specimen this had to be multiplied by the total number of cracks, five cracks in 20 mm spacing specimens and ten in 10 mm spacing specimens. Considering this and the total delaminated distance to be measured, the overall error was estimated to be 5% in single crack and 20% in multiple cracks specimens. This was added to the results. The data obtained are shown below in Table 6-4 with the estimated errors.

Table 6-4: Experimental delamination speeds for different crack configurations and displacement speed.

Crack Configuration	Speed Test (m/s)	Delamination speed (m/s)
Single Crack	0.1	0.079±0.004
Single Crack	1	1.40±0.07
Single Crack	10	9.05±0.50
10 mm Spacing	1	0.80±0.16
20 mm Spacing	1	0.95±0.20

The results show that the delamination speed is related to the deformation speed. Generally it seems to be lower than the test velocity, except for the 1 m/s test with a single crack where the results indicate a higher speed. The data also show a decrease in the delamination speed as the number of cracks increase, with the 10 mm spacing test producing the lowest delamination speed for the 1 m/s set of experiments. However, it should be noted that the multiple cracks results do overlap when the uncertainties are considered, hence this result might not be significant. This information was used to compare the model results presented below.

6.3.2. Single crack modelling

The models described above were run to fit the delamination energy parameters. As discussed, the values of the cohesive elements stiffness, peak stress, the effects of mass scaling and the presence of PVB end tabs were considered in order to measure their significance on the model results.

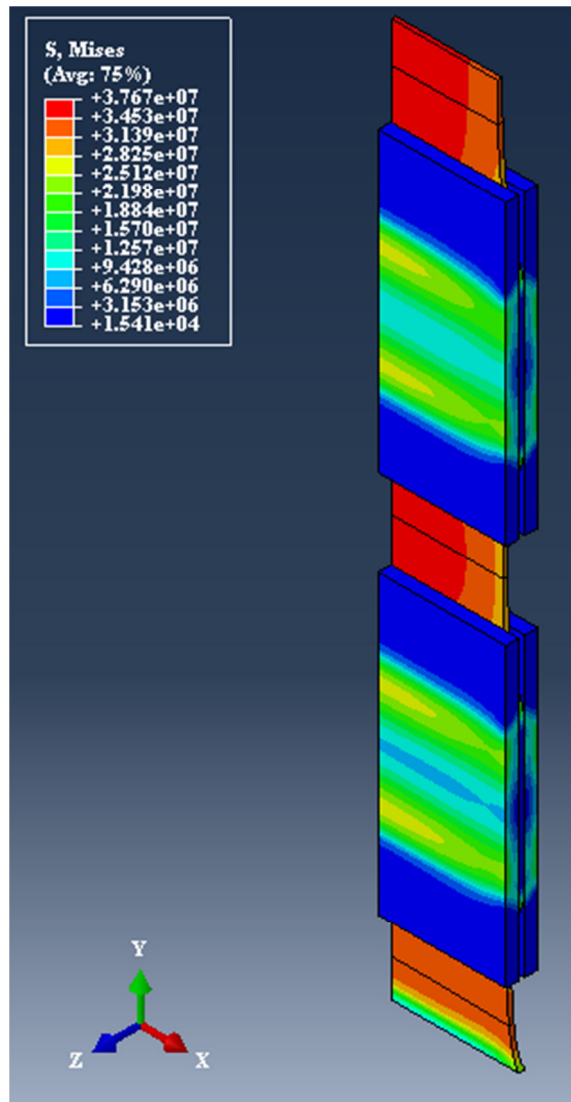


Figure 6-16: Typical stress distribution in a single crack sample during deformation.
The stresses are given in Pa.

Figure 6-16 shows typical stress contour plot for a single crack specimen. The stress variation in the glass parts between the laminated and delaminated areas is visible, as well as the deformation of the detached PVB membrane. The delamination progresses from the top and the bottom of the specimen as well as from the central crack due to the presence of the PVB end tabs.

The effect of the PVB end tabs was considered first. Figure 6-17 shows a plot of the PVB reaction stresses versus overall displacement for a 1 m/s model with high rate material properties both with and without free PVB ends. The model without the end parts presented greater oscillations in the early part of the deformation. However, the steady-state plateau reached is very similar, giving confidence in the validity of the results of the models used.

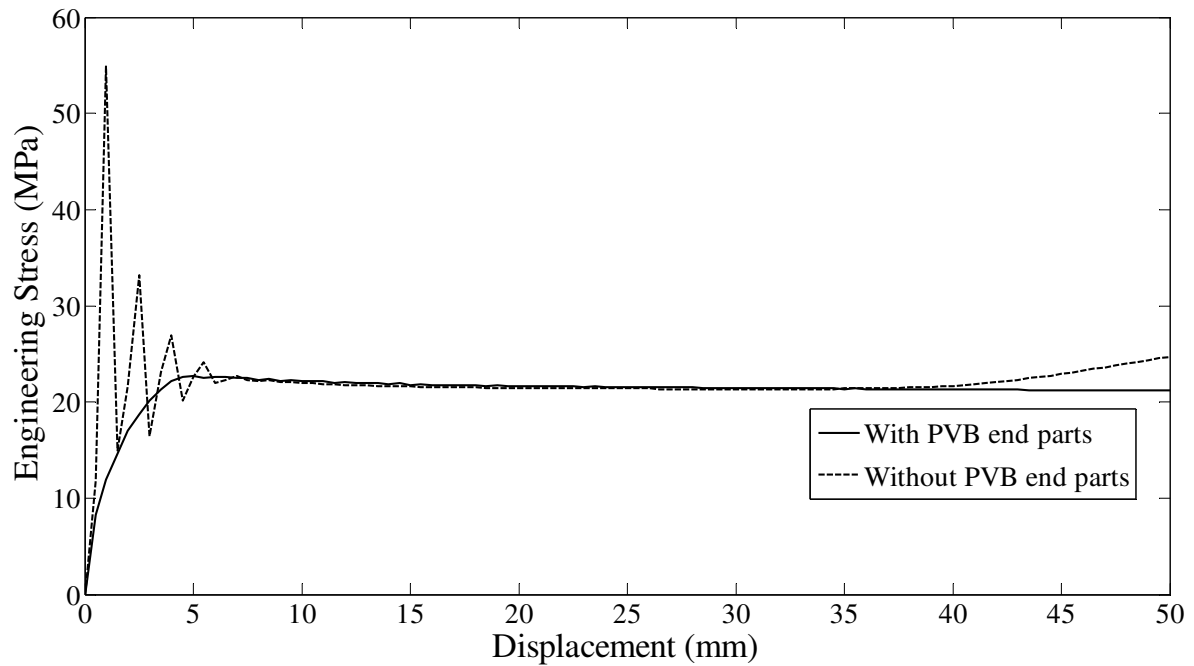


Figure 6-17: Comparison of the models with and without PVB ends.

The effects of mass scaling were considered next. The analysis was run on a 0.1 m/s model, as this was the case when mass scaling was used to limit the total number of analysis steps. The 80 ms model was run with the high strain rate PVB material properties. The increase in mass was equal to a factor of 73, however the ratio between kinetic energy to internal energy remained below 0.1%. This indicated a quasi static condition which minimised the importance of the extra mass [78]. The stress-displacement results are shown in Figure 6-18 together with data from the same model run for the full 0.5 s period with mass scaling. It can be seen that, whilst the initial peak is marginally lower without the scaling, the curves mostly superimpose. Mass scaling was therefore used in these cases.

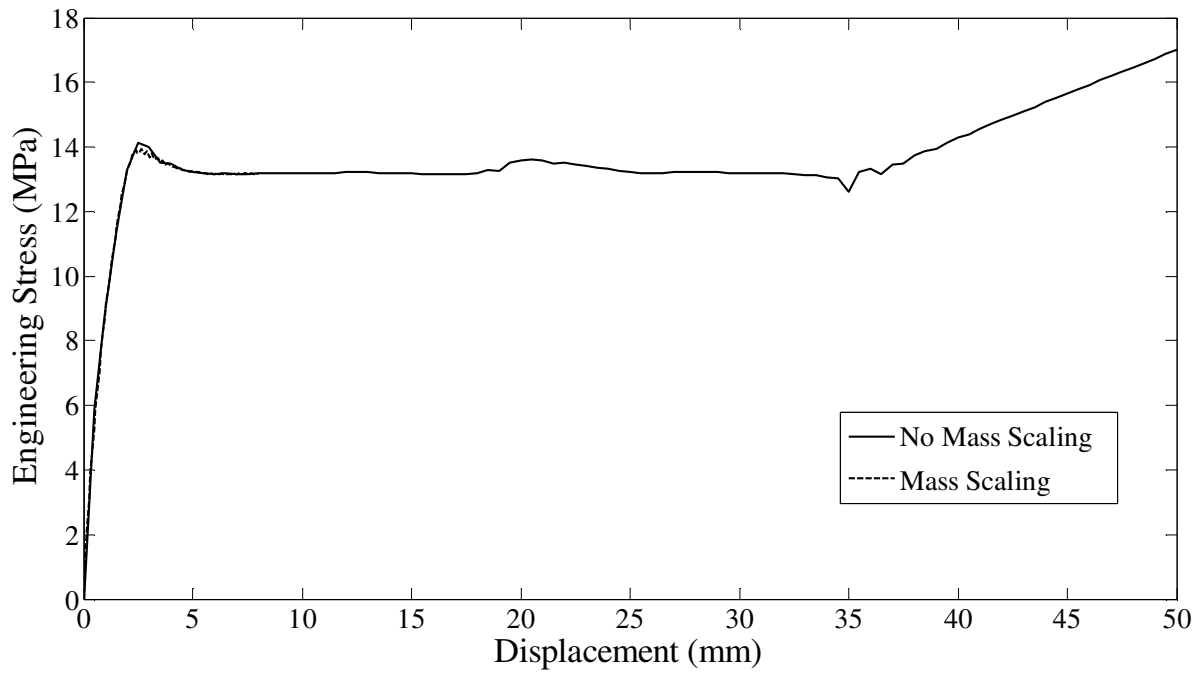


Figure 6-18: Stress-Displacement plot of the 0.1 m/s model with high rate material properties results when run with and without mass scaling.

The effect of varying the initial stiffness (k) of the cohesive elements was considered next. Three models were run, with k varying between 5.2×10^7 N/m and 1×10^9 N/m. The low rate material properties and a test speed of 10 m/s were used. The results are plotted in Figure 6-19. Again, no significant difference between the plateau results can be seen, with the three curves being in good agreement.

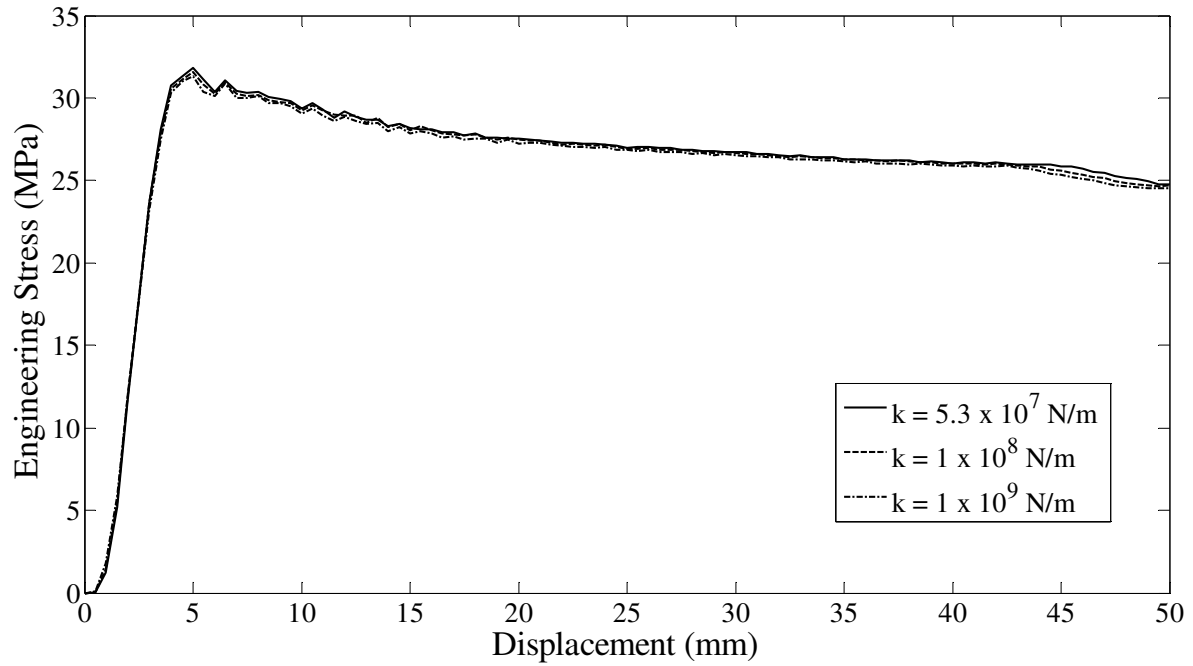


Figure 6-19: Stress-Displacement plot of the 10 m/s model results when run with different values of cohesive elements stiffness k .

The peak delamination stress (S) was then varied by 25% above and below the assumed value. The study was performed on both material model types to highlight potential differences in their behaviour. In all cases the plateau levels did change by a relatively small amount. As an example, the stresses of the low rate model changed by 2.7% when the peak stress was increased by 25%. The effect of this on the estimated energy was calculated, and the possible uncertainty estimated at 7%.

**Table 6-5: Results of models where the delamination peak stress was varied by 25%.
The original cases are included for comparison.**

Material model	Peak Stress (MPa)	Energy (J)	Modelling plateau stress (MPa)	Experimental plateau stress (MPa)
Low rate	11.28 *	1630	26.3	26.3
Low rate	14.1	1630	27.0	26.3
Low rate	14.1	1550	26.3	26.3
Low rate	9.02	1630	25.6	26.3
Low rate	9.02	1740	26.3	26.3
High Rate	11.28 *	2765	26.3	26.3
High Rate	14.1	2765	26.7	26.3
High Rate	14.1	2670	26.3	26.3
High Rate	9.02	2765	26.0	26.3
High Rate	9.02	2890	26.3	26.3

* The original cases are included for comparison

Having completed this parametric study, the energy parameters were then fitted to the experimental data. Figure 6-20 shows a typical graph of results, in this case for the 0.1 m/s model with low rate material properties.

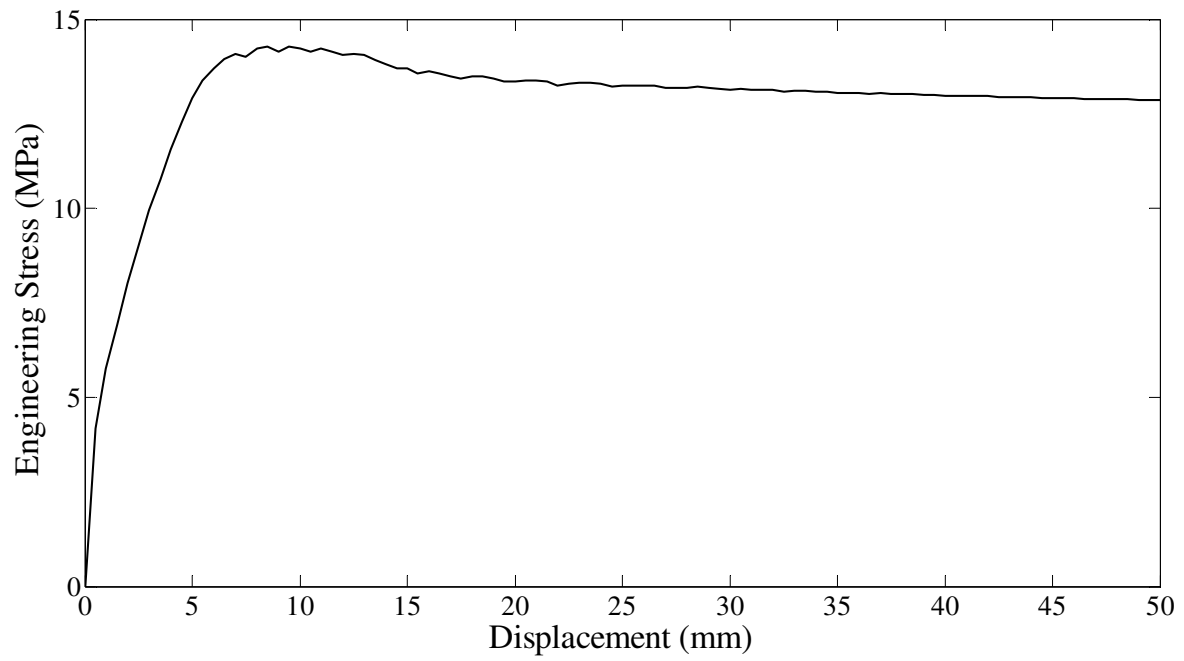


Figure 6-20: Stress-displacement results of the model run at 0.1 m/s with low rate material properties.

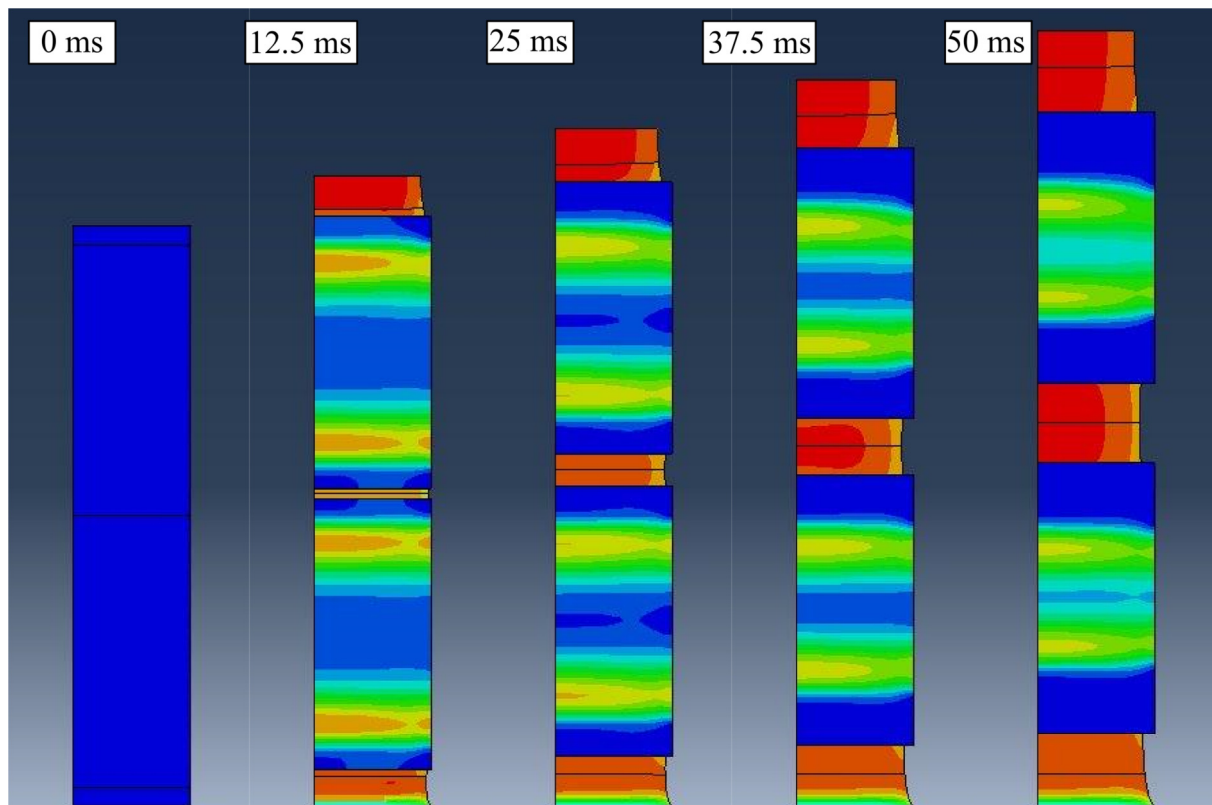


Figure 6-21: Plots of the stress results for a 1 m/s model at several time points.

The delamination front could be seen moving along the PVB layer as the analysis progressed. Figure 6-21 shows the stress distribution in the glass and PVB through a selection of frames obtained from the model. The stress distribution in the glass parts indicates the position of the delamination fronts, with high level of stress indicative of an area where the bond is still effective. The detail of a delamination location is shown in Figure 6-22. This presents an elevation of a glass fragment, indicating the position of the failure fronts. The PVB stresses increase rapidly beyond the failures, as the membrane has to resist the loading in the detached area.

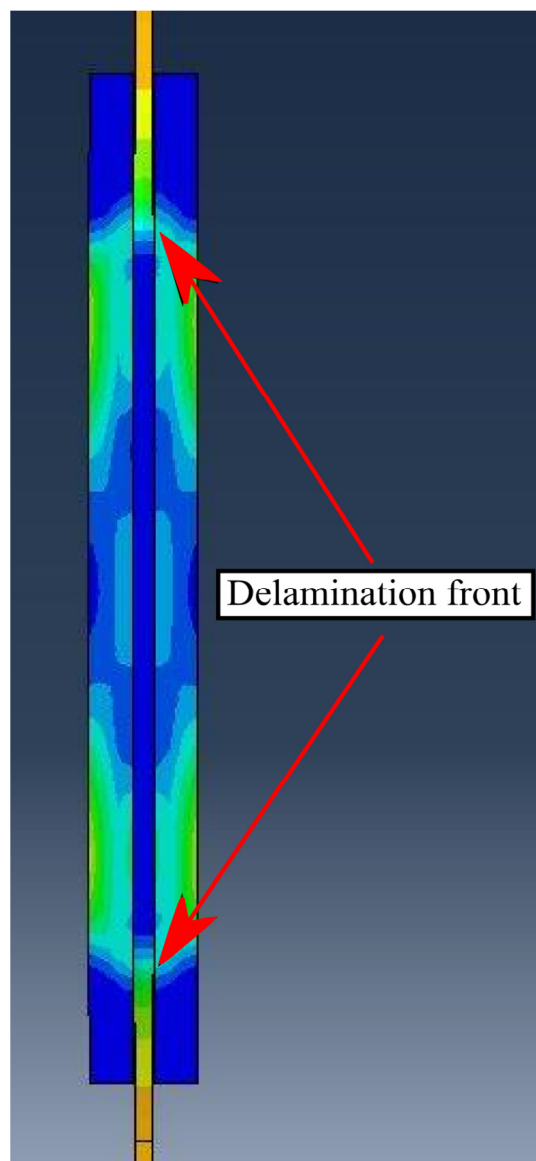


Figure 6-22: A detail of the side elevation of the model taken at 25 ms.

The results of the energy fit are shown in Table 6-6 below. The delamination energies varied between 3230 J and 1170 J depending on the deformation speed and the PVB material properties used. It is significantly that the delamination speeds are often different from the experimentally measured ones. The largest difference is shown by the 0.1 m/s model with high rate material properties, which has a 390% error. In general the 0.1 m/s case shows the worst agreement, with a 120% error in the low rate properties case as well. The 1 m/s model has the lowest errors, especially for the low rate material properties. In most cases the speed of the model exceeds the experimental one, signifying an excessive stiffness in the material models employed. Possible explanations for these results will be discussed in section 6.4.

Table 6-6: Summary of energy and delamination speed results for single crack models.

Deformation Speed (m/s)	Material Model	Model Plateau Stress (MPa)	Energy (J)	Model Delamination Speed (m/s)	Experimental Delamination Speed (m/s)	% Difference
0.1	Low rate	13.2	2690	0.17	0.079	120
0.1	High Rate	13.2	1170	0.39	0.079	390
1	Low rate	21.3	3225	1.12	1.43	28
1	High Rate	21.3	2890	2.24	1.43	57
10	Low rate	26.3	1630	14.67	9.05	62
10	High Rate	26.3	2765	18.13	9.05	100

The energies found here were used in the following parametric studies. As it proved difficult to choose the correct material model for the 1 m/s case, both were used for subsequent analyses.

6.3.3. Effect of crack spacing study

The models with different crack spacings were run as described, utilising the delamination parameters obtained with the single crack models. All the models were run at 1 m/s speed, using both the low rate and high rate PVB material properties. In all cases the delamination spread both above and below each crack location and at the top and bottom edges of the model. Figure 6-23 shows the stress distribution in the glass and PVB of a 20 mm crack spacing model at five time steps. In the image, the stress variations near the cracks at the top and bottom edges of the glass pieces highlight the presence of delamination fronts.

It can be seen that the area of glass subject to stress decreases as the delaminated areas spread away from the initial positions.

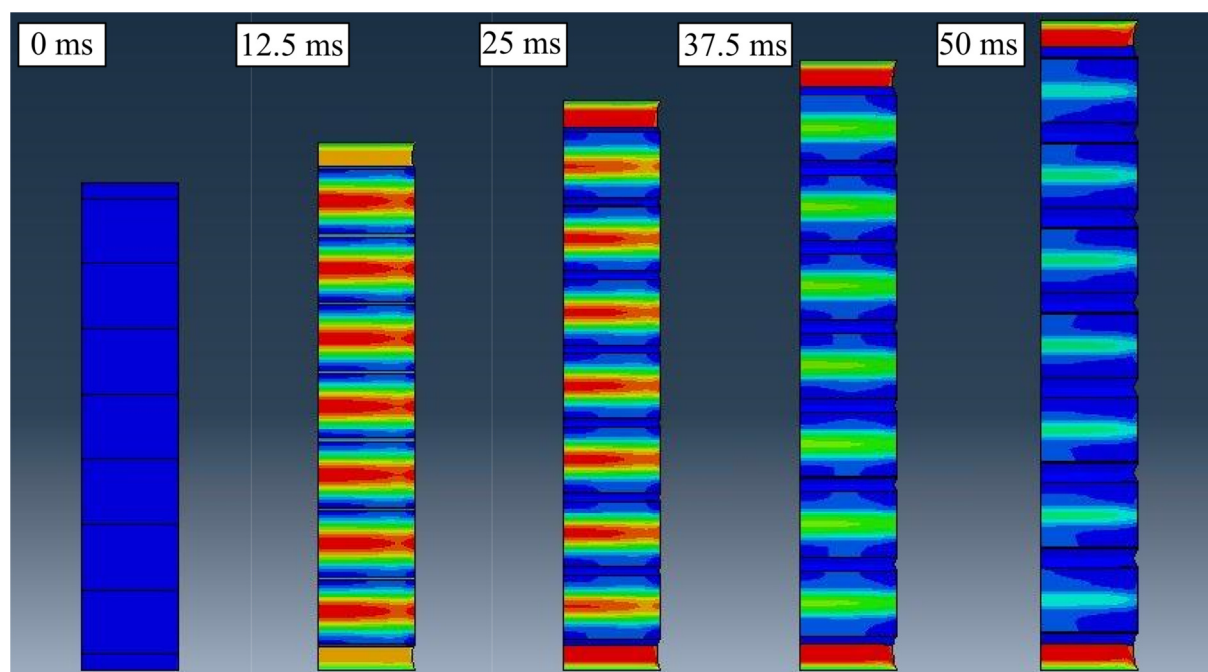


Figure 6-23: Plots of the stresses of a 20mm crack spacing model at several time points.

Both the peak stresses and the delamination speeds were extracted from the models. The results are shown in Table 6-7 below, together with a comparison with the experimental data where this is available. The plateau stresses show a smaller error when compared with the laboratory data. The largest difference is 30%, with the smallest being 10%. This indicates that the models can predict this parameter within acceptable error. The delamination speeds show similar differences when compared with experimental results to those highlighted above for the single crack cases.

Table 6-7: Results of the models with different crack spacings. The experimental results are included where relevant for comparison.

Crack Spacing (mm)	Material Model	Model Plateau Stress (MPa)	Experimental Plateau Stress (MPa)	% Difference	Model Delamination Speed (m/s)	Experimental Delamination Speed (m/s)	% Difference
20	Low rate	18.5	16.5	15	1.2	0.95±0.2	30
20	High Rate	21.1	16.5	30	1.7	0.95±0.2	80
10	Low rate	16.1	17.5	10	1	0.80±0.16	25
10	High Rate	21.7	17.5	25	1.1	0.80±0.16	40
5	Low rate	15.4	N/A	N/A	0.95	N/A	N/A
5	High Rate	21.2	N/A	N/A	1.1	N/A	N/A

The experimental results are included where relevant for comparison.

The effect of crack density on the stress results was small, with the biggest difference being 20% between the models using the low rate material model. This highlights the relative insensitivity of the phenomenon to the density of initial failures.

6.3.4. Inclined Cracks study

The effects of crack inclination were considered next. The models described above were run using the parameters found in previous parts of the study. Figure 6-24 shows stress distributions at different time steps in one of the models. Again, the spread of the delamination fronts can be deduced from the stress distribution in the glass, with the variations near the cracks and at the top and bottom edges indicating the presence of delamination. The failures seem to spread away from the cracks, with the failure front mostly parallel to the crack direction.

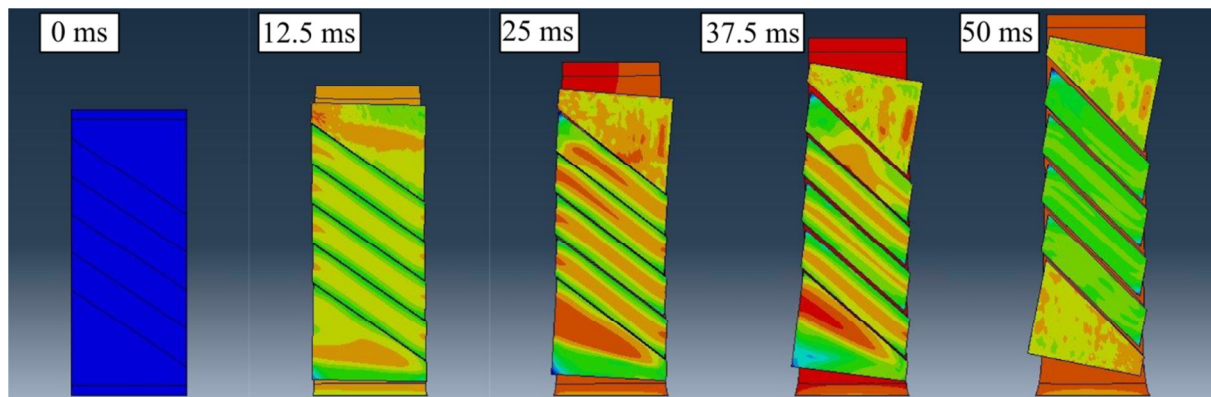


Figure 6-24: Stress plots at several time points during the analysis of a 33.7° inclination model.

The plateau stress results were collected from each of the models and are presented in Table 6-8. Again, their variation is small, with a maximum difference of 5% for the low rate models. Additionally, the model plateau stresses are similar to those observed in the cases with no crack inclination, supporting the view that inclination does not have a major effect on the cracked material behaviour.

Table 6-8: Results of the models with different crack inclination.

Crack Configuration	Material Model	Model Plateau Stress (MPa)
20 mm spacing, inclined at 18.4°	Low rate	16.7
20 mm spacing, inclined at 18.4°	High Rate	20.9
20 mm spacing, inclined at 33.7°	Low rate	17.6
20 mm spacing, inclined at 33.7°	High Rate	20.9
20 mm spacing, inclined at 4.8°, alternating direction	Low rate	16.4
20 mm spacing, inclined at 4.8°, alternating direction	High Rate	21.5

6.3.5. Staggered crack study

The two “staggered cracks” models were then considered, similarly to the cases described above. Three dimensional views of the stress distribution are shown in Figure 6-25. In this case the position of delamination fronts is less clear when considering only the glass stresses, probably due to the more complex situation created by the asymmetric geometry. However, the PVB deformed shape shows that two failure fronts move through the sample, starting from the top and the bottom of the model.

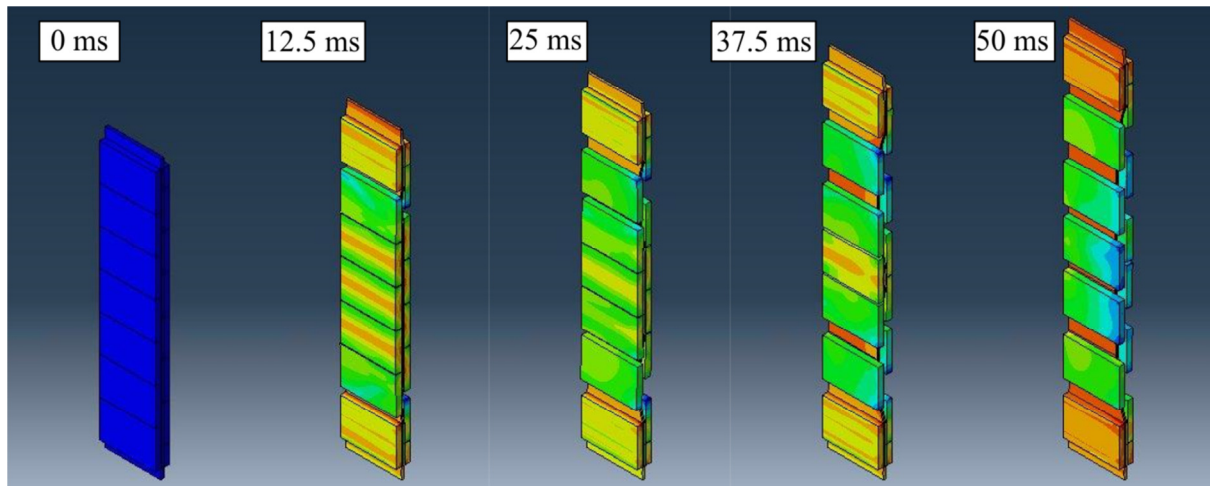


Figure 6-25: Plots of the stress results of a 10 mm stagger model at several time points.

This can be also seen in Figure 6-26, where a detail of the model is shown. In these frames, the delamination progresses downward, crossing a crack location in its travel. This behaviour can be related to the overall stress data. Figure 6-27 shows the stress displacement information for one of the 10 mm misalignment models. The stress reaches a peak at a displacement of about 15 mm, before dropping to a plateau level which is maintained until the end of the simulation. This indicates that a higher level of work needs to be done on the material before a stable delamination process is initiated. However, after this point, the fronts will move through the sample in a manner similar to that observed for the aligned cracks models.

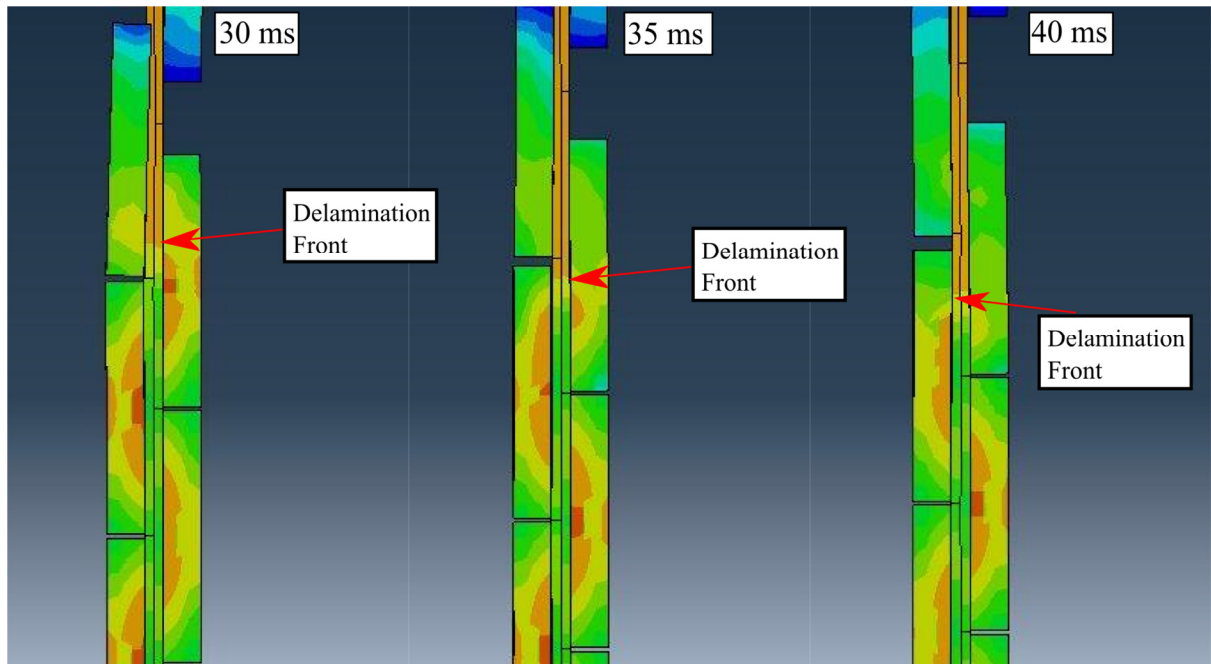


Figure 6-26: Section view of stress contours taken at different time points highlighting the movement of the delamination front.

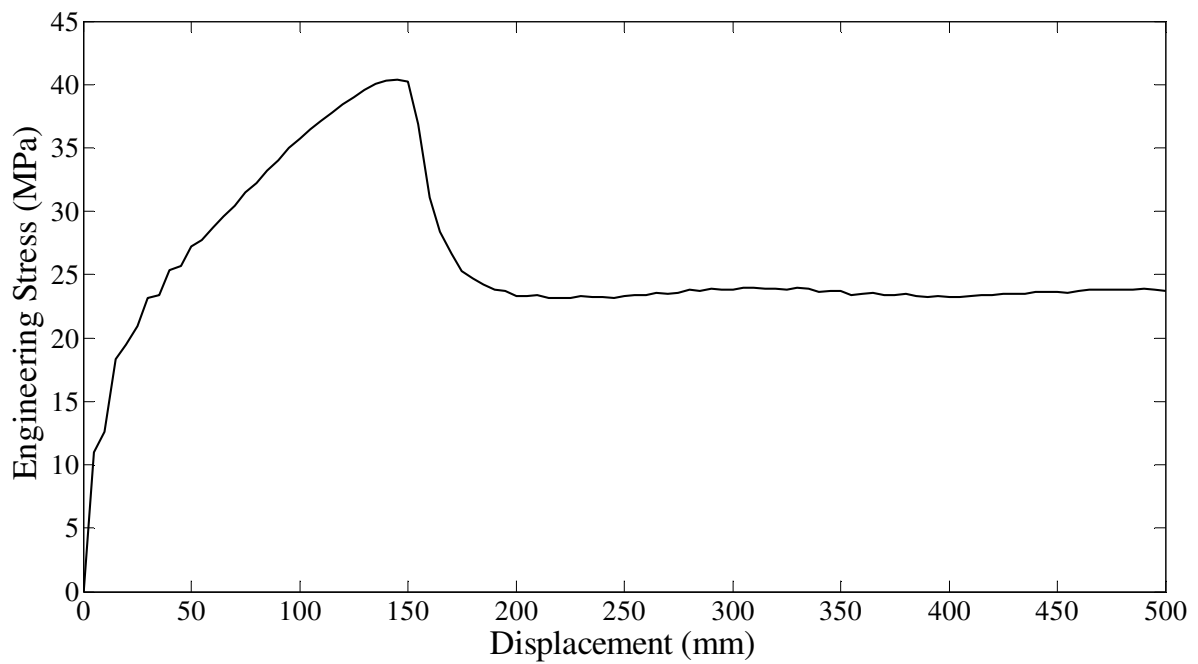


Figure 6-27: Stress-displacement plot for the FEA model with cracks 10mm out of alignment.

Table 6-9: Results of models with not aligned cracks on the two laminate faces.

Crack Configuration	Material Model	Model Plateau Stress (MPa)
20 mm spacing, 10 mm out of phase	Low rate	23.5
20 mm spacing, 10 mm out of phase	High Rate	22.0
20 mm spacing, 5 mm out of phase	Low rate	18.6
20 mm spacing, 5 mm out of phase	High Rate	21.6

The plateau stress information for all the models are shown in Table 6-9. The high rate PVB models again show very similar data, which is also analogous to that seen in previous cases. The low rate model with 5 mm stagger also conforms to this observation. The 10 mm misalignment, low rate PVB model instead shows a higher plateau stress, similar to that observed for a single crack.

6.4. Discussion

The model runs presented above underline several aspects of both the delamination energies fitting process and of the behaviour of the cracked composite materials with different failure configurations. These will be considered in turn.

6.4.1. Delamination energies

Two values of delamination energy were obtained for each deformation speed, producing energies ranging from 1170 J and 3225 J. The exact magnitudes proved to be very dependent on the material model utilised. This could be expected, as the strain energy in the material is one of the variables deducted from the total external work to determine the delamination

energy. The effect shown here though is large, with the two energies being often different by a factor of two. It is therefore very important to use a realistic material model for all these studies. This is complicated in the situations which were considered here as the strain rates were consistently high and varied significantly between the different cases. This implied that a simple quasi static approximation would not be appropriate and the more complex models derived in the previous chapter had to be used. The delamination speeds could provide a measure of the accuracy of the material models used in these cases. When these were compared to the experimental data, large difference could be seen between the different options. The worst fit was obtained in the 0.1 m/s case, with both models showing too high delamination progression. A stiffer model will produce higher delamination speed, as more PVB will have to be freed to stretch to allow for the imposed overall movement. Therefore it can be argued that for the slowest test both material models proved too rigid. This can also be related to the material model fit results shown in the Chapter 5, where the lower rate model showed a higher stiffness than seen in the experimental curves.

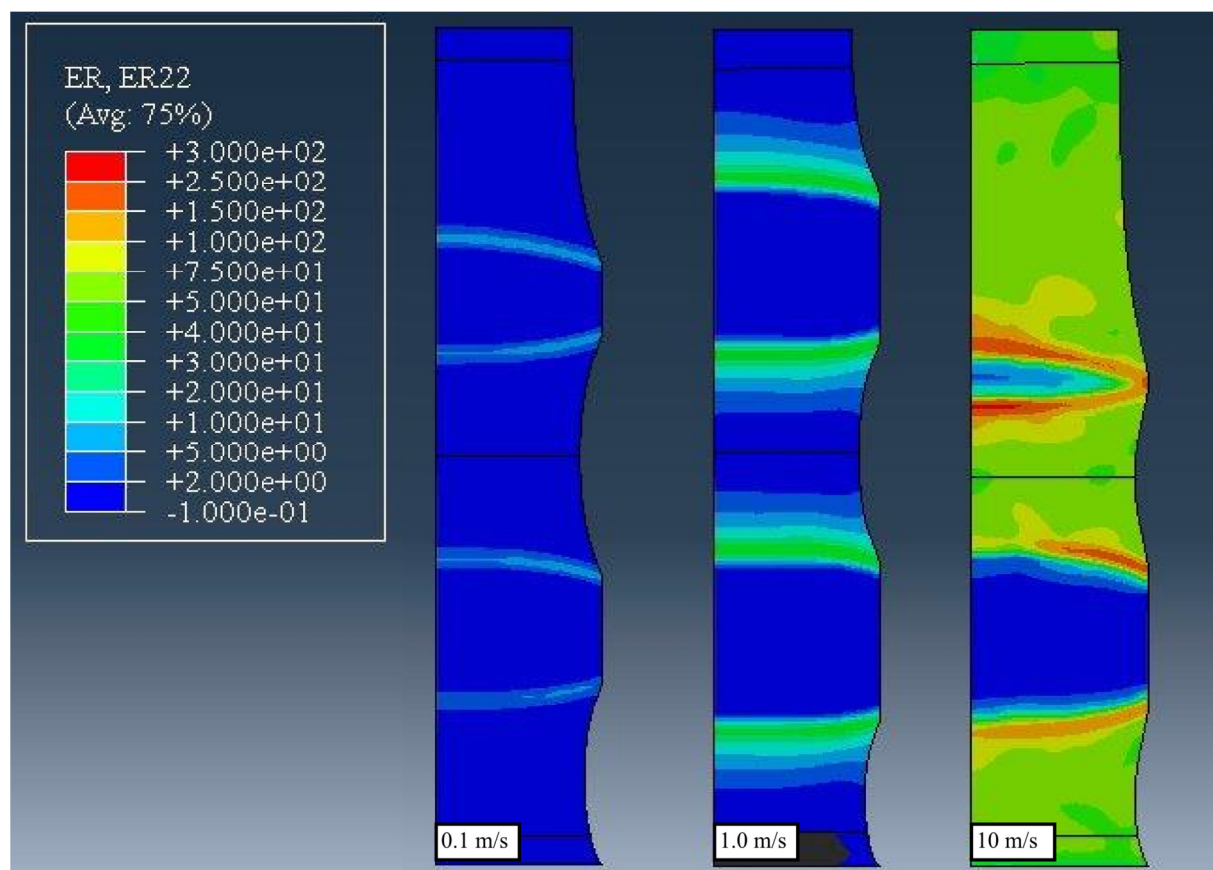


Figure 6-28: PVB strain rate in single crack models.

Additionally, the strain rate will vary significantly along the length of the delaminating sample. Figure 6-28 shows a plot of the strain rate distribution in three single crack models run at different speeds. Whilst the rate in the 0.1 m/s model peaks at 7.5 /s and was therefore in the range used for the derivation of the low rate model, in the other models these maxima are significantly higher, with rates of 42 /s for 1 m/s deformation speed and 280 /s for 10 m/s. Additionally, the deformation speed drops considerably away from the failure front. This is especially important in the 1 m/s case, where the rate is below 3 /s in delaminated areas. This implies that, ideally, a single material model will be required to model both ranges of rates to obtain better results. Otherwise, as seen in this study, it will be challenging to model the phenomenon accurately.

The peak rates observed lend credibility to the peak delamination stresses used. Whilst they are significantly higher than those used to estimate the employed stresses, alternative values derived using these higher rates are within the 25% variations considered in the sensitivity analysis. For example, for the 1 m/s case the estimated rate was 6.66 /s, whilst the measured rate was 42 /s. These would produce peak stresses of 9.48 MPa and 11.14 MPa respectively, a difference of 18%. Additionally, the uncertainties present due to the difference between the material models are much larger than those seen with the peak stresses variations, diminishing the importance of this parameter at this stage.

Given these uncertainties, it is difficult to assess the trend of the energy in relation with the deformation speed. The data though does seem to show that its order of magnitude will not change in the range considered here.

Whilst the values obtained cannot be trusted absolutely, it was considered that this did not preclude performing the subsequent parametric study on the effects of different crack configurations, as the data produced would still be useful for comparison purposes.

6.4.2. Parametric studies

The results for different crack configurations have been described and compared to the available experimental data above. These were all run at the same deformation speed of 1 m/s to allow for easy comparison of the results.

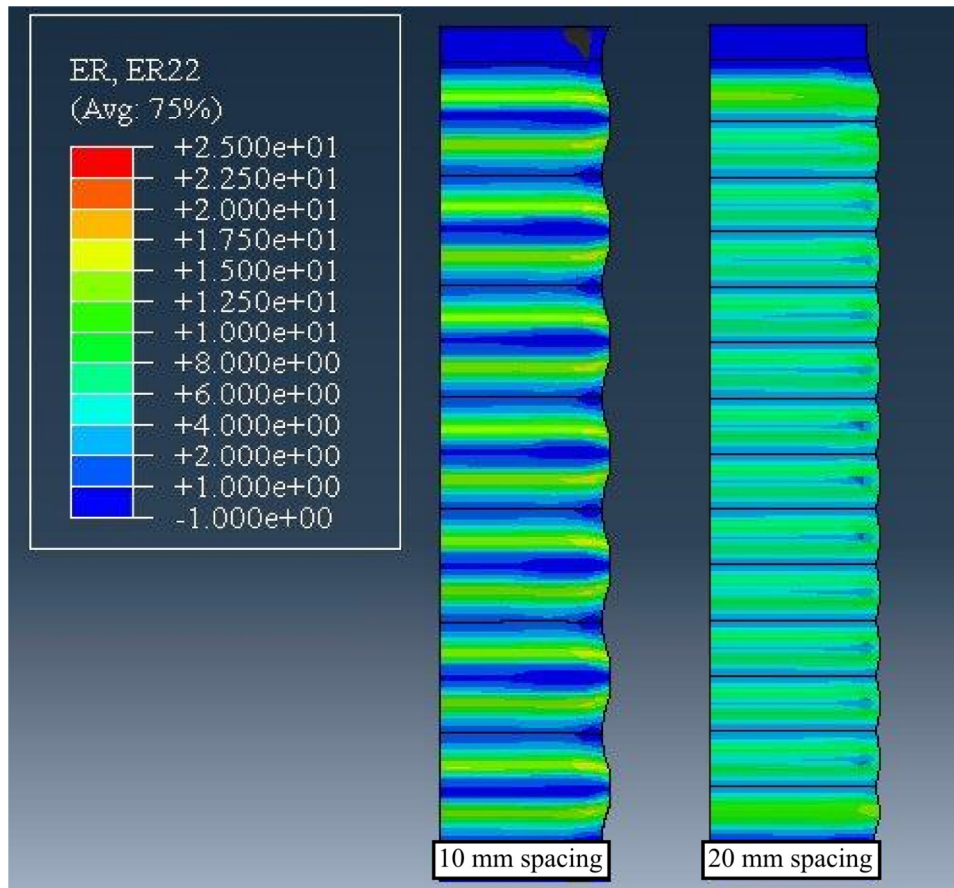


Figure 6-29: PVB strain rate in two multiple crack models.

In the cases with 10 mm and 20 mm horizontal cracks spacings, the delamination speeds were compared to experimental data. It was evident that neither material model captures the full behaviour of the material. Again, the need for a unique material model covering both ranges was highlighted, as shown by the strain rate data shown in Figure 6-29. The plateau levels obtained through modelling were much more similar to the experimental data available than the delamination speeds, with the largest error being 30%. The plateau levels shown by the various models were also similar, with only a few exceptions. Figure 6-30 shows a plot of the plateau stress levels for different crack densities (calculated in terms of crack / mm) and Figure 6-31 shows a plot of the same values, this time compared to the crack angles. In the case of the high strain rate models the data appear to lie around a line of constant stress. In the case of lower rate models there is a higher variation, especially between the single crack specimen and the multiple crack specimens. However, when the multiple crack cases are compared the differences were not high, with a maximum of 20%, and the plot suggests a trend of diminishing variations should the density be increased further. These results are in agreement with the observations by Hooper [3], as his results showed only minor differences

between the different crack spacings and also between samples with regularly spaced cracks and a randomly cracked samples.

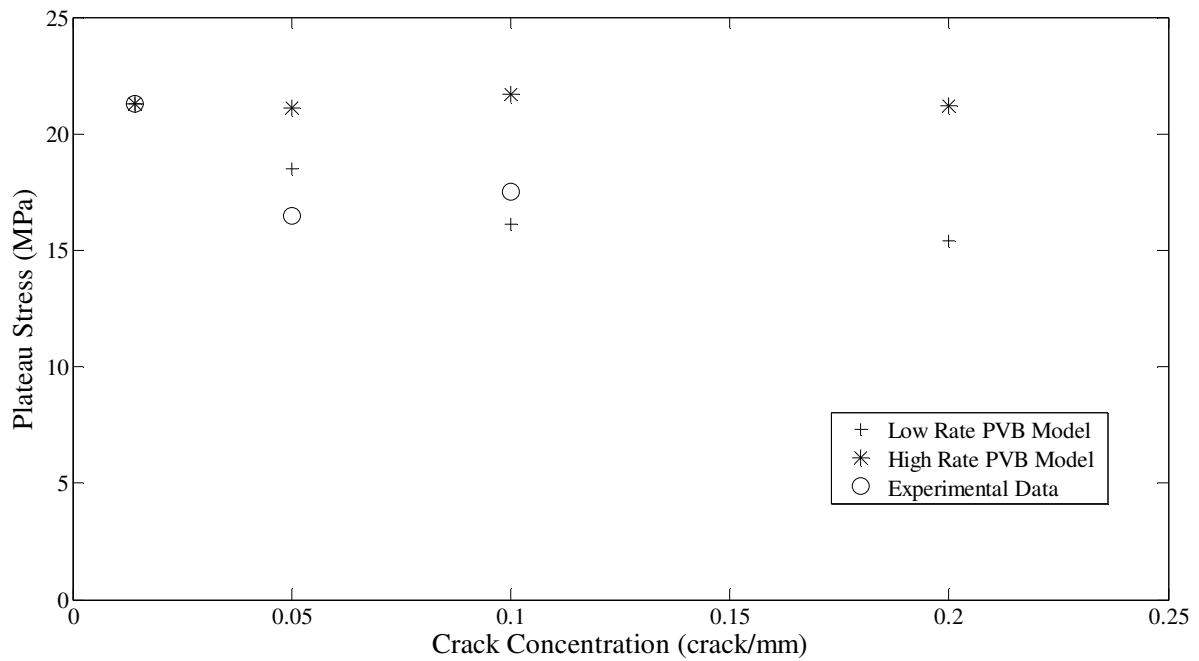


Figure 6-30: Plot of the plateau stresses versus the crack concentration.

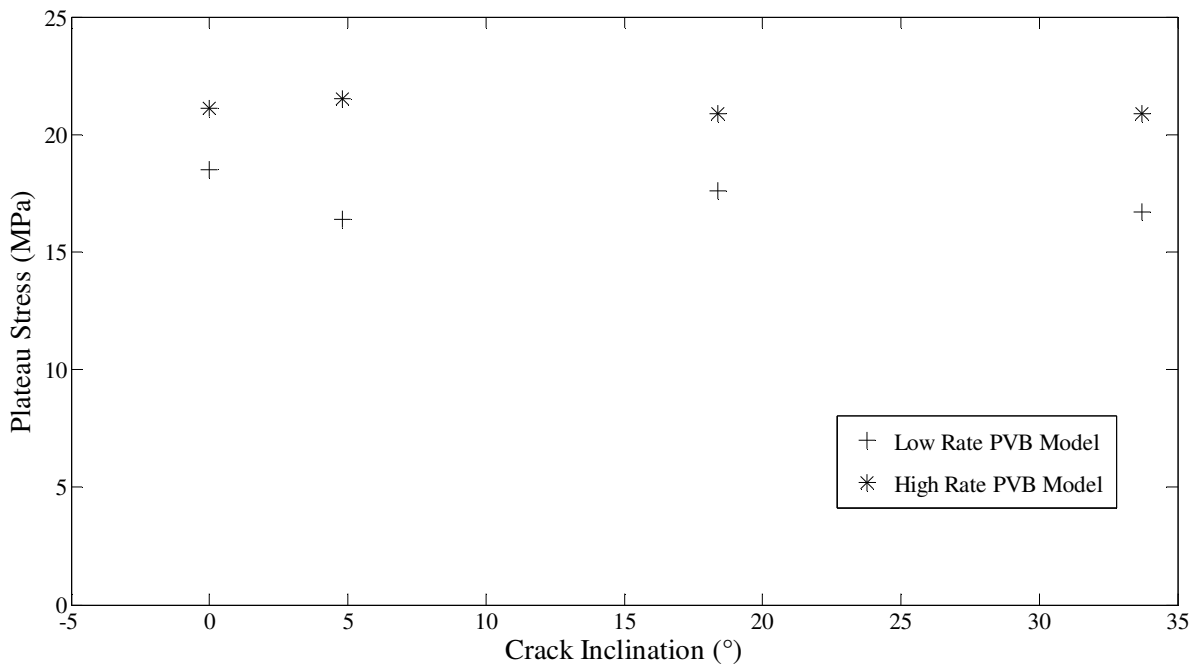


Figure 6-31: Plot of the plateau stresses versus the inclination of the cracks.

The “staggered cracks” cases also show similar plateau stresses, except for the lower rate model, which shows a discrepancy for the 10 mm misalignment case. This is shown in Figure

6-32. However, as could be seen in Figure 6-27 above, a higher peak stress was reached before the forces stabilise. This indicates that where there are no areas with aligned cracks, a higher force is required to initiate delamination. Once the process is started though, the failure travels through the sample progressively, without changing its behaviour significantly from previous cases. This is also shown in Figure 6-26, where the front is seen travelling downwards through a glass crack location instead of propagating in both directions from there.

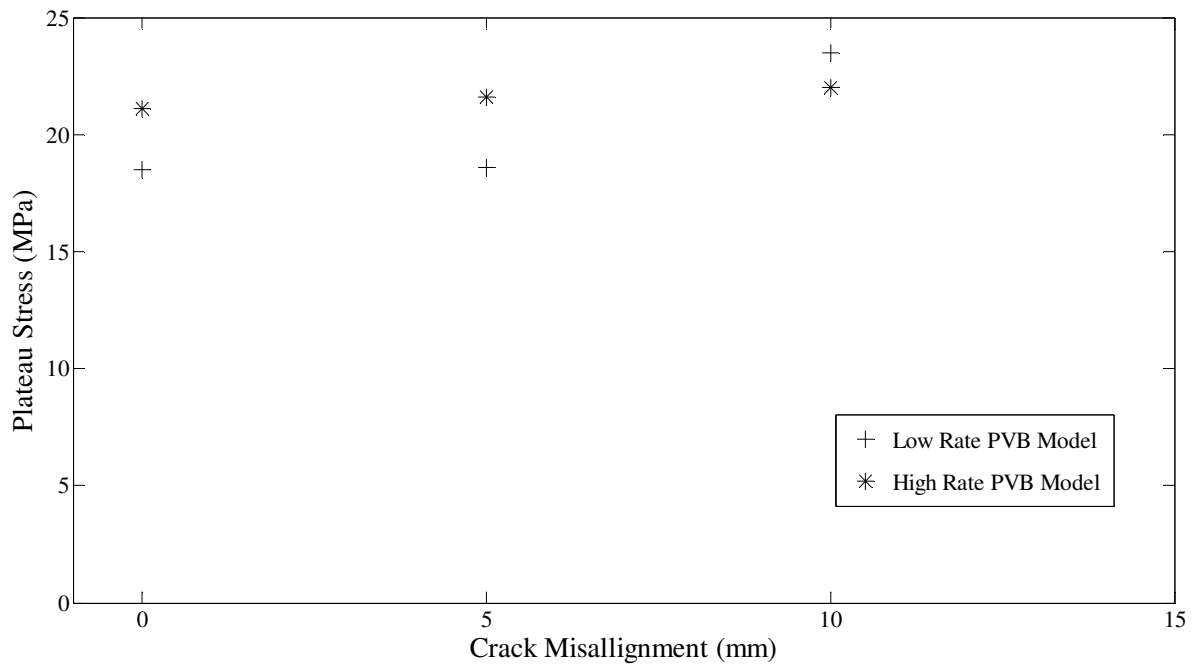


Figure 6-32: Plot of the plateau stresses versus the amount of crack misalignment.

This can also explain the difference in the results of the lower rate PVB model. In the 5 mm stagger configuration, whilst a higher activation stress is required, several delamination fronts initiate in the small areas between cracks, as can be seen in Figure 6-33. After the delamination begins, the system behaves similarly to the multiple cracks specimens. In contrast, with 10 mm misalignment delamination fronts originate at only two locations, as could be seen in Figure 6-25. This causes the model to behave similarly to the single crack specimens. This did not affect the plateau stress of the higher rate PVB models, as little change was caused by the crack density parameter. However, the lower rate PVB model is affected more significantly, causing the jump in plateau stresses.

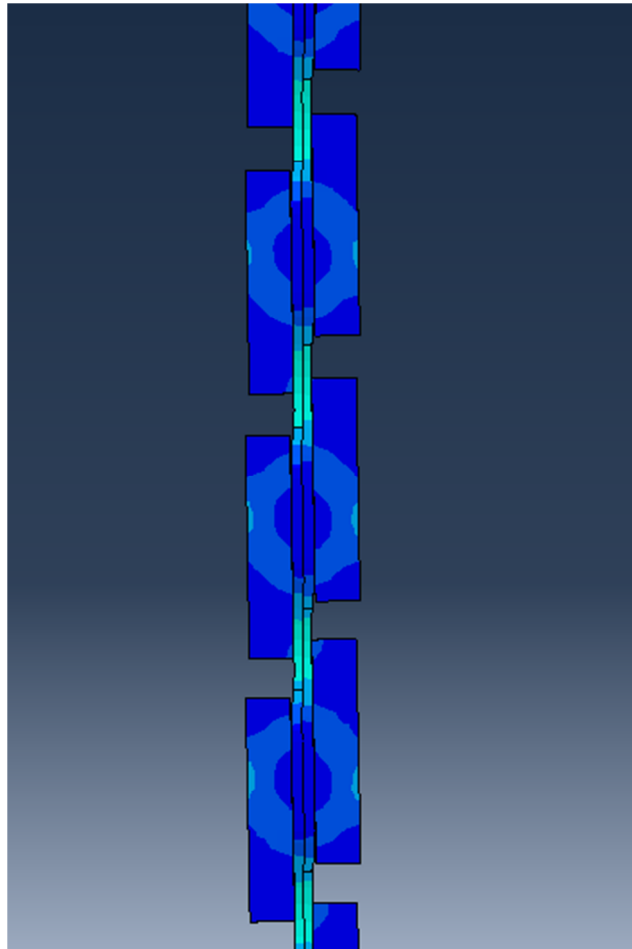


Figure 6-33: Model detail showing stresses in the glass and PVB for a low rate PVB material model with 5 mm crack misalignment.

These are important considerations to decide the relevance of this case to engineering design. The results imply that, unless only a small minority of cracks are aligned and the crack spacing is quite large, the delamination will simply originate at the aligned locations and travel through the rest of the glass without changing the overall behaviour significantly. Whilst it is inevitable that, under blast, many cracks in the window will not be aligned, many of them will be. Furthermore, the crack density is likely to be higher than the one used here, causing the absolute amount of misalignment between failure lines to be small, therefore making this parameter unlikely to affect the forces of the system.

6.5. Conclusion

The available experimental data was utilised to obtain values for the delamination energies and to investigate the effect of different crack patterns which are likely to occur under blast

loading. The effects of different materials properties were considered, and their influence estimated using a second experimental measure, the speed of delamination growth.

The results showed that the magnitudes of the energies of delamination are likely to be in the order a few thousand Joules for the specimens tested, with recorded values between 1170 J and 3225 J. Additionally, their variation in the speed range of interest is unlikely to exceed this level of difference.

However, especially in the case of slower deformation speeds, neither of the material models managed to capture the entire process accurately, therefore reducing the reliability of the energy values obtained. In cases where the results were in closer agreement, as in the 1 m/s tests, it was still challenging to decide which of the two models represented the phenomenon more accurately. Therefore, for further use in this study, both models were used for each case analysed.

Whilst the imprecisions in the material models used, especially for the low rate model, contributed to this error, the strain rate plots presented above in paragraphs 6.4.1 and 6.4.2 also highlight the great variability of this parameter in the areas of interest. Therefore, a single material model to represent both strain rate ranges is needed to obtain more accurate energy measurements. One such model was derived in Chapter 5, not only covering the full range of required speeds, but also achieving a lower level of errors at low rates. Consequently, including this material law in the FEA software and using it to repeat the energy fitting would constitute an important follow up to this work and would be very likely to produce significantly more accurate results.

The two energy parameters obtained for the 1 m/s case were used for the parametric study on crack configurations. The high rate results for all cases show very constant results, indicating a lack of sensitivity of the plateau stresses to all the variables considered. The low rate PVB model results showed a larger variation, with the plateau stress decreasing as the crack concentration increased. This variation was still only 20% when comparing realistic crack spacings of 20 mm and 5 mm. The results were much closer to each other when considering inclined cracks. One of the staggered crack results, with a 10 mm misalignment, presents the only significant change, bringing the results in line with those of the single crack specimens, as explained in section 6.4.2.

These results might indicate that a softer material model will be more influenced by the rate changes which are caused by having a different number of cracks and hence delamination zones. This effect though decreased as the crack spacing approaches 5 mm, and it is likely to be less important with further reductions of this distance. This is also borne out by Hooper's tests [3], where only small differences were observed between the different crack spacings, even when the results obtained with a randomly finely cracked specimen were taken into consideration.

The results suggest that the crack configuration is unlikely to have an important effect on the overall composite behaviour, especially with the small crack spacings which are typically observed in tested panels. Designers therefore should be able to rely on standard results in this respect. However, the delamination energies and plateau stresses are affected by the speed of the overall deformation, which will be dependent on the exact blast loading. This could explain the different reaction levels observed in Chapter 4. Once more precise estimates are obtained by using an improved material model, the data could be used to produce a more accurate overall smeared material property law similar to that used in Chapter 4, without the disadvantage of high noise levels being present in the forces and strains recorded during blast tests.

7. Conclusion

In this project, the response of laminated glazing panels to blast loading was studied, paying particular attention to the reaction forces which were applied to the supporting structure. The variation of the loading in the panels' geometry and its time history were analysed, as well as the properties of the constituent elements of the cracked laminate and their relationship.

To achieve this, a blast test was performed and its results investigated to obtain new reaction distribution information. The information was then combined with existing data to produce a full reaction estimate, both in terms of its geometrical distribution and its evolution in time. The properties of the cracked laminate were then studied. Firstly, the behaviour of the PVB membrane across a range of strain rates was characterised. Then the delamination of the glass fragments from the membrane was considered, producing both energy absorption estimates and an assessment of the effect of different crack distributions.

7.1. Blast Test Results and Analysis

A blast test was performed on a laminated glass pane using a 100 kg TNT equivalent explosive charge at a 23 m stand-off distance. Data were collected using optical techniques (DIC), strain gauges and a pressure gauge. The information was then analysed to provide information on both the general behaviour of the glass plate and the distribution of reaction forces along its edge. A novel calibration technique was employed to extract the full field displacement and strain information of the glass pane. This allowed for results to be obtained from the experimental information in the absence of calibration images, albeit at the cost of higher uncertainties.

The results obtained showed that the pane movement and strains presented a similar behaviour to those observed in literature tests. Maximum deflections of 297 mm and velocities of 34 m/s were recorded. Whilst the uncertainties in the strain measurements are higher and prevent an accurate assessment of their magnitudes, their geometrical distribution confirmed previous observations, with higher strains present near the perimeter of the window.

Although several of the strain gauges used failed during the test, enough data was obtained to reach some conclusions on the variation of reaction forces along two of the edges. The

recorded loads were in the range of 20 – 30 kN/m and did not increase further once this level was reached. Whilst significant noise was present, the data also showed a variation of loading along the longer glazing sides, with higher forces observed away from the central position. On the shorter sides instead the forces seemed more constant.

The information obtained was then combined with historical blast test information to calculate the overall reaction forces acting on the supports. These were found using conjunctly both the strain gauge information and the DIC strain output. The post-cracked reaction information from the gauges at the supports was used to calculate the stresses acting on the membrane, and this was combined with the DIC strains to produce stress-strain curves for the cracked material. A plastic material property was fit to these and applied throughout the panel's perimeter to calculate the post-cracked reactions.

A different method was applied for pre-cracked reactions. The DIC output was employed, together with a FEA analysis, to estimate the bending moments acting in the window. Their variation were then employed to calculate shear forces and hence reaction forces.

Both the overall time history of the reactions and their distribution along the window edges were considered. The distribution confirmed the observations from the data of the blast test. The forces varied in magnitude along the longer edges, presenting two separate peaks at some distance from the centre.

The time history of the reaction showed an initial peak before the window shattered, dropping close to zero immediately after. The magnitude of these first maxima ranged between 32 and 56 kN depending on the blast and window construction characteristics. Afterwards, the force rose more gradually and approached a plateau. This ranged between 21 kN and 55 kN for out-of-plane forces and between 74 kN and 91 kN for in-plane reactions.

As the forces did not increase for the latter part of the deformation, it was decided that a mechanism other than simply increasing the strain energy had to contribute to the absorption of the blast pressures. The delamination of glass fragments from the central membrane could significantly contribute to the energy absorption and therefore was studied next.

7.2. PVB Characterisation

To be able to assess the delamination phenomenon discussed above, it was considered important to use a realistic material property law for the PVB membrane. This needed to be able to represent the behaviour of this layer at the strain rates of interest, which were likely to be significantly higher than the quasi static level.

To achieve this, existing experimental data from uniaxial tensile tests at different strain rates were employed to fit viscoelastic material property laws. Prony series with different hyperelastic springs were considered, and two main models were fitted. One of these employed a reduced polynomial hyperelastic formulation, whilst the other employed a spring function proposed by Hoo Fatt and Ouyang [63]. Each of these was split into low rate (below and including 8 s^{-1}) and high rate (from 20 s^{-1}) models, separately covering a different range of strain rates. The results for both spring types showed a closer agreement to experimental data for the higher rate cases, whilst the stiffness tended to be overestimated for lower rate data.

Afterwards, a single material law derived with non linear finite deformation viscoelasticity was employed. Two different non linear springs were combined in parallel, with a non linear damper located in series with one of the springs. This method produced a single material model which could account for the observed behaviour at all rates. Additionally, the errors at low strain rates were significantly smaller for this approach than those seen in the Prony series models. However, these improvements came at a cost of an increase in the model's mathematical complexity and a greater number of fitting constants if compared to each individual Prony series model. Additionally, this model is not included as a matter of course in standard FEA software, requiring therefore the development of user material routines for its use. It was decided that for the following delamination study the reduced polynomial models would be used for its ease of application to the FEA solver used, ABAQUS.

7.3. Delamination Modelling

The data from existing laboratory tensile tests of glass specimens with pre cracks were available for this analysis. The experiments consisting in single crack specimens loaded at different speeds were used for the first phase of this study. The geometry of the sample and loading conditions were reproduced using FEA models in ABAQUS. These were then

employed to estimate the delamination energy parameters using both the low rate and the high rate material properties laws derived previously. To achieve this, one of the constants determining the cohesive zone model (CZM) parameters were varied to match the FEA stress plateau levels to the recorded ones. The absorbed energy was then calculated finding the area underneath the CZM stress-deformation graph. The results showed that the exact material definition employed affected the delamination energies significantly. Therefore a second parameter, the speed of delamination, was used to assess which PVB material model produced the better representation of the real system for each deformation speed considered. Whilst this showed that one of them, the low strain rate model, produced a better approximation of the experimental results, neither captured the behaviour exactly. This is likely due to both the models' intrinsic limitations, for example the excessive stiffness of the low rate law mentioned above, and by the complexity of the PVB strain rate distribution throughout the samples. However, all the estimated delamination energies varied between 1000 J and 3500 J, giving confidence that the final values would be of a similar magnitude.

The parameters found in this manner were then used to study the effect of different crack distributions in the samples. Several densities, inclinations and misalignments were considered to assess their impact on plateau delamination stresses. Two experimental cases were available for comparison, with 10 mm and 20 mm distance between the cracks.

When considering the crack densities and inclinations, the results indicated that these factors only have a limited impact on the final stresses. Whilst, again, the comparison with the available experimental results showed significant differences in the delamination speed, the observed stress plateaux all showed an error of 30% or less. For the low strain models this was limited further to a maximum 15% difference. These also showed the largest difference between the crack patterns considered, which was though contained to within 20% of the absolute plateau levels for realistic crack concentrations. The high rate PVB models instead showed more constant reactions throughout, with differences of less than 1 MPa on absolute magnitudes of between 21 and 22 MPa.

The possibility of crack misalignments was also considered. This more complex situation caused a larger deviation from previously observed plateaux levels. This was again more evident in the low strain rate models. The difference could be explained by the different delamination propagation conditions created in the cases studied. The largest misalignment

considered caused the delamination to spread in a manner similar to that seen with single cracks, hence causing the low rate PVB stresses to be higher.

Whilst the exact results obtained cannot be considered definitive due to the issues with PVB material property laws highlighted above, the models indicate that in general realistic crack distributions on their own will not affect the delamination stresses importantly. Additionally, the presence of plateau stresses is strong evidence for this to be one of the main causes of the time history behaviour of the reactions observed in the blast test data.

The lack of importance of cracks distribution highlighted here leaves the differences in the strain rates as the main element to differentiate between the reaction forces seen in separate blasts. In this respect the results presented, if refined using an improved material law, could be used to produce more accurate smeared material properties for the composite material after the glass failure. This would provide a powerful tool for practical design of façade elements.

7.4. Further Work

The results obtained with this research could be expanded in several ways with further studies. Some further work ideas will be discussed here.

7.4.1. Blast testing

In Chapter 3 it was attempted to analyse the variation of reaction forces along the glazing's edges through experimental data. As discussed, a significant proportion of the strain gauges used for this failed. Therefore the same study could be repeated to collect more information.

In the same test, a pre crack introduced into the sample accidentally did not seem to affect the results. Further tests could be performed to explore whether this was due to specific factors connected with the test or whether it is a common feature of the panes' behaviour. There is some uncertainty on whether the pre-cracked phase contributes significantly to the overall response of the material, as the energy absorbed in this initial situation is relatively small when compared with the total amount. A test with a pre cracked specimen would therefore yield valuable data in this respect.

In the same context, the opposite situation could be also tested by employing glazing panels composed of significantly thicker glass plies or of an increased number of layers. In this case the maximum pre-cracked forces would increase, possibly increasing the contribution of this portion of the behaviour.

7.4.2. Bending capacity of cracked laminated glass

The blasts tests with more glass layers would also introduce new complexities in the form of a possible residual bending resistance of the cracked material. This effect could be studied and quantified through laboratory experiments. Some unpublished preliminary laboratory tests were performed to measure its contribution on thin laminates with two glass layers. The results showed some potential minor contributions from bending at low deflections. It is likely that additional layers would be constrained by PVB membranes to a much higher extent, intensifying this effect and potentially making it an important factor in the behaviour of the glazing panes. Both low and high rate tests could be performed, attempting to quantify this contribution and helping to explain potential blast test measurements. DIC techniques could be employed to obtain strain data throughout the sample thickness, providing definitive evidence of the importance of the various mechanisms. The implementation of this technique could though be complicated by the cracks, which would introduce areas of low quality in the speckle pattern.

7.4.3. PVB material properties

In the present study, the effect of strain rates on the PVB membranes was analysed. A finite viscoelasticity material law was derived which provided predictions of the behaviour of the material in the cases considered. This material model could be included into FEA software for further use in simulations of more complex cases.

The effect of temperature variations was not considered at this stage. It is likely that, in real usage, the glazing elements will be at a range of temperatures when hit by a blast pressure wave. Considering normal ambient conditions, realistic temperatures in the UK could vary between 0°C and 30°C. Additionally, it is possible that the blast itself will cause a raise in the temperature. This could take place either through the thermal radiation emitted or through

indirect heating due to the high strain energies imparted by the pressure waves. As the glass transition temperature of the membrane is approximately 20°C [3], the difference in response based on this parameter could prove very significant. Laboratory tests at a range of temperature and strain rates could be performed. The results could then be used to expand the material models derived here.

7.4.4. Glass – PVB delamination

Once the available PVB material law is implemented in a FEA software, the delamination study presented here could be repeated to obtain more precise values of delamination energies. This would avoid the need to choose an appropriate material representation by estimating the strain rates likely to occur during the event of interest. Additional laboratory tests could also be performed on some of the crack patterns considered numerically to further verify the model.

Both these steps would increase the confidence in the modelling results of the delamination energies and the plateau stresses for different crack patterns. The numerical data could then be used to produce more complex FEA models of the whole window system. Additionally, the collected information could be employed to determine smeared material parameters for use in simple analyses run for practical design purposes.

7.4.5. Alternative interlayer materials

A laboratory and numerical analysis project similar to that performed on PVB could be carried out on alternative interlayers materials. Whilst PVB is the more common option for blast resistance, project specific factors and requirements could lead designers to different choices, such as the DuPont SentryGlass system [79]. Both the material properties and the delamination energies of these alternative options could be assessed using similar methods to those presented in this work.

7.4.6. Alternative holding mechanisms

The results of this study were based on results obtained on silicone bonded windows. Alternative options could be considered. Glazing tapes, for example, are proposed by

manufacturers as an alternative to the use of silicone joints [51], [52]. Laboratory tests similar to those done by Hooper on the silicone [3] could be carried out to test their capacities. DIC could be employed as a data collection device, providing therefore at the same time a realistic strain distribution throughout the deformation process. Blast tests could then be performed to measure the overall pane behaviour in a realistic situation.

7.5. Conclusion

In this research several aspects of the behaviour of laminated glass when subjected to blast loading have been examined. The data produced regarding the reactions forces, PVB material models and delamination phenomena will help to improve the design of glazing to resist this type of loads. It will be possible therefore to optimise the designs further than achieved in current practice, producing safer and more cost effective solutions. The possibilities for further studies highlighted in section 7.4 could lead to further improvements in the understanding of these structural elements and hence would again help to produce improved designs.

Additionally, several elements developed here could be incorporated in the design for alternative loading cases. Impact loading of laminated glass is another area of significant interest due to its applications in the automotive and aviation industries. Elements such as the PVB high rate behaviour and delamination will be relevant to such applications and it is hoped that the results obtained with this project will be helpful to practitioners in these areas as well.

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Appendix A – Finite Deformation Viscoelasticity Law Derivation

Nomenclature

$\dot{\mathbf{B}}_e$	Elastic deformation strain rate
$\mathbf{D}$	Symmetric part of overall deformation velocity gradient
$\mathbf{D}_i$	Symmetric part of inelastic deformation velocity gradient
$I_{3\mathbf{B}}$	Third invariant of the total Left Cauchy-Green strain tensor
$I_{3\mathbf{B}_e}$	Third invariant of the elastic Left Cauchy-Green strain tensor
$\mathbf{S}$	Weighted Cauchy stress tensor
$\mathbf{S}_e$	Deviatoric portion of weighted Cauchy total stress tensor
$\mathbf{S}_E^{eq}$	Deviatoric portion of weighted Cauchy equilibrium spring stress tensor
$\mathbf{S}_E^{oe}$	Deviatoric portion of weighted Cauchy overstress spring stress tensor
$\mathbf{W}$	Antisymmetric part of overall deformation velocity gradient
$\mathbf{W}_i$	Antisymmetric part of inelastic deformation velocity gradient

Derivation

The method developed by Amin [69] is used in the following derivation. The final result though are equivalent to those obtain by other authors. As discussed above in Chapter 5, the first step of the derivation of finite deformation viscoelasticity laws consists in multiplicatively decompose the deformation matrix $\mathbf{F}$ in its inelastic and elastic parts, $\mathbf{F}_e$ and $\mathbf{F}_i$.

As mentioned above, the left Cauchy Green strain tensor $\mathbf{B}$ is given by:

$$\mathbf{B} = \mathbf{F}\mathbf{F}^T \quad \text{Eq. A-1}$$

And therefore:

$$\mathbf{B}_e = \mathbf{F}_e \mathbf{F}_e^T \quad \text{Eq. A-2}$$

And

$$\mathbf{B}_i = \mathbf{F}_i \mathbf{F}_i^T \quad \text{Eq. A-3}$$

The velocity gradient defined in Chapter 5 ($\mathbf{L} = \dot{\mathbf{F}}\mathbf{F}^{-1}$) can be split in its symmetric and anti-symmetric parts:

$$\mathbf{L} = \dot{\mathbf{F}}\mathbf{F}^{-1} = \mathbf{D} + \mathbf{W} \quad \text{Eq. A-4}$$

Where $\mathbf{D} = \frac{1}{2}(\mathbf{L} + \mathbf{L}^T)$ and $\mathbf{W} = \frac{1}{2}(\mathbf{L} - \mathbf{L}^T)$. The elastic and inelastic velocity gradients and their components are formulated in the same manner.

The strain time rate of the elastic deformation is given by:

$$\dot{\mathbf{B}}_e = \dot{\mathbf{F}}_e \mathbf{F}_e^T + \mathbf{F}_e \dot{\mathbf{F}}_e^T \quad \text{Eq. A-5}$$

Where $\mathbf{F}_e$ can be expressed as $\mathbf{F}_e = \mathbf{F}\mathbf{F}_i^{-1}$ and therefore $\dot{\mathbf{F}}_e = \frac{d}{dt}(\mathbf{F}\mathbf{F}_i^{-1}) = \mathbf{L}\mathbf{F}_e - \mathbf{F}_e \mathbf{L}_i$. Inserting this result in Eq. A-5 and with some manipulation, the inelastic velocity gradient is defined by:

$$\dot{\mathbf{B}}_e = -2\mathbf{F}_e \mathbf{D}_i \mathbf{F}_e^T + \mathbf{B}_e \mathbf{L}_i^T + \mathbf{L} \mathbf{B}_e \quad \text{Eq. A-6}$$

As the material is assumed to be incompressible, the determinant of the deformation matrix and of its components is equal to 1. The weighted Cauchy stress is given by $\mathbf{S} = (\det \mathbf{F}) \mathbf{T}$ and therefore in this case is equal to the Cauchy stress $\mathbf{T}$. the hydrostatic stress component is included in the stress tensor, giving:

$$\mathbf{S} = -p\mathbf{I} + \mathbf{S}_E \quad \text{Eq. A-7}$$

The $\mathbf{S}_E$ component of the stress can then be split in the equilibrium and overstress components, giving:

$$\mathbf{S}_E = \mathbf{S}_E^{eq} + \mathbf{S}_E^{oe} \quad \text{Eq. A-8}$$

The Clausius Duhem inequality [80] is a thermodynamic inequality which is commonly used to define the stress components. It takes the form:

$$-\rho_r \dot{\psi} + \mathbf{S}_E \cdot \mathbf{L} \geq 0 \quad \text{Eq. A-9}$$

The rate of energy portion of the equation, $-\rho_r \dot{\psi}$, can be split additively in the equilibrium and overstress portions. This will give:

$$\rho_r \psi = \psi_{eq}(I_B, I_{2B}) + \psi_{oe}(I_{B_e}, I_{2B_e}) \quad \text{Eq. A-10}$$

And

$$\rho_r \dot{\psi} = \frac{\partial \psi_{eq}}{\partial I_B} \dot{I}_B + \frac{\partial \psi_{eq}}{\partial I_{2B}} \dot{I}_{2B} + \frac{\partial \psi_{oe}}{\partial I_{B_e}} \dot{I}_{B_e} + \frac{\partial \psi_{oe}}{\partial I_{2B_e}} \dot{I}_{2B_e} \quad \text{Eq. A-11}$$

Where $\dot{I}_B = \mathbf{I} \cdot \dot{\mathbf{B}}$, $\dot{I}_{2B} = (I_B \mathbf{I} - \mathbf{B}) \cdot \dot{\mathbf{B}}$ and similar equations are used for $\dot{I}_{B_e}$ and $\dot{I}_{2B_e}$.

The strain tensors $\mathbf{B}$ and $\mathbf{B}_e$ can be expressed in terms of their invariants. As an example, $\mathbf{B}$ is given by $\mathbf{B} = I_B \mathbf{I} - I_{2B} \mathbf{B}^{-1} + I_{3B} \mathbf{B}^{-2}$. The equation for $\mathbf{B}_e$ is similar. I_{3B} is the third invariant of the strain tensor, and it is given by:

$$I_{3B} = \det(\mathbf{B}) \quad \text{Eq. A-12}$$

Therefore, for an incompressible material, I_{3B} will be equal to 1.

Substituting these expressions in Eq. A-11, the following is obtained:

$$\begin{aligned} \rho_r \dot{\psi} = & \left\{ \frac{\partial \psi_{eq}}{\partial I_B} \mathbf{I} + \frac{\partial \psi_{eq}}{\partial I_{2B}} (I_{2B} \mathbf{B}^{-1} + \mathbf{B}^{-2}) \right\} \cdot \dot{\mathbf{B}} + \\ & + \left\{ \frac{\partial \psi_{oe}}{\partial I_{B_e}} \mathbf{I} + \frac{\partial \psi_{oe}}{\partial I_{2B_e}} (I_{2B_e} \mathbf{B}_e^{-1} + \mathbf{B}_e^{-2}) \right\} \cdot \dot{\mathbf{B}}_e \end{aligned} \quad \text{Eq. A-13}$$

The second part of the inequality is considered next.

The velocity gradient $\mathbf{L}$ can be expressed as:

$$\mathbf{L} = \dot{\mathbf{F}}_e \mathbf{F}_e^{-1} + \mathbf{F}_e \dot{\mathbf{F}}_i \mathbf{F}_i^{-1} \mathbf{F}_e^{-1} = \mathbf{L}_e + \mathbf{F}_e \mathbf{L}_i \mathbf{F}_e^{-1} \quad \text{Eq. A-14}$$

Using this, the stress power can be split to highlight the contributions of the elastic and inelastic deformations:

$$\mathbf{S}_E \cdot \mathbf{L} = \mathbf{S}_E^{eq} \cdot \mathbf{L} + \mathbf{S}_E^{oe} \cdot \mathbf{L}_e + (\mathbf{F}_e^{-1} \mathbf{S}_E^{oe} \mathbf{F}_e) \cdot \mathbf{L}_i^T \quad \text{Eq. A-15}$$

Considering that $\mathbf{S}_E^{eq}$ is an isotropic function of $\mathbf{B}$, giving $\mathbf{B}^{-1} \mathbf{S}_E^{eq} \mathbf{B} = \mathbf{S}_E^{eq}$ and with some algebraic manipulation, the following result can be obtained:

$$\mathbf{S}_E \cdot \mathbf{L} = \frac{1}{2} (\mathbf{B}^{-1} \mathbf{S}_E^{eq}) \cdot \dot{\mathbf{B}} + \frac{1}{2} (\mathbf{B}_e^{-1} \mathbf{S}_E^{oe}) \cdot \dot{\mathbf{B}}_e + (\mathbf{F}_e^{-1} \mathbf{S}_E^{oe} \mathbf{F}_e) \cdot \mathbf{D}_i \quad \text{Eq. A-16}$$

The results shown in Eq. A-13 and Eq. A-16 can be inserted in the original inequality, giving

$$\begin{aligned} & \left\{ \frac{1}{2} (\mathbf{B}^{-1} \mathbf{S}_E^{eq}) - \frac{\partial \psi_{eq}}{\partial I_{\mathbf{B}}} \mathbf{I} + \frac{\partial \psi_{eq}}{\partial I_{2\mathbf{B}}} (I_{2\mathbf{B}} \mathbf{B}^{-1} + \mathbf{B}^{-2}) \right\} \cdot \dot{\mathbf{B}} + (\mathbf{F}_e^{-1} \mathbf{S}_E^{oe} \mathbf{F}_e) \cdot \mathbf{D}_i \\ & + \left\{ \frac{1}{2} (\mathbf{B}_e^{-1} \mathbf{S}_E^{oe}) - \frac{\partial \psi_{oe}}{\partial I_{\mathbf{B}_e}} \mathbf{I} + \frac{\partial \psi_{oe}}{\partial I_{2\mathbf{B}_e}} (I_{2\mathbf{B}_e} \mathbf{B}_e^{-1} + \mathbf{B}_e^{-2}) \right\} \cdot \dot{\mathbf{B}}_e \geq 0 \end{aligned} \quad \text{Eq. A-17}$$

One way to satisfy this relationship is to set the terms in brackets multiplied by the strain rate tensors to 0 and then to ensure the $(\mathbf{F}_e^{-1} \mathbf{S}_E^{oe} \mathbf{F}_e) \cdot \mathbf{D}_i$ term is greater than 0. $\mathbf{S}_E^{eq}$ and $\mathbf{S}_E^{oe}$ can then be determined:

$$\mathbf{S}_E^{eq} = 2 \frac{\partial \psi_{eq}}{\partial I_{\mathbf{B}}} \mathbf{B} + 2 \frac{\partial \psi_{eq}}{\partial I_{2\mathbf{B}}} (I_{2\mathbf{B}} \mathbf{I} + \mathbf{B}^{-1}) \quad \text{Eq. A-18}$$

$$\mathbf{S}_E^{oe} = 2 \frac{\partial \psi_{oe}}{\partial I_{\mathbf{B}_e}} \mathbf{B}_e + 2 \frac{\partial \psi_{oe}}{\partial I_{2\mathbf{B}_e}} (I_{2\mathbf{B}_e} \mathbf{I} + \mathbf{B}_e^{-1}) \quad \text{Eq. A-19}$$

As the hydrostatic portion of the stress ($-p\mathbf{I}$) is undetermined and will need to be defined with respect of the boundary conditions, the terms which are proportional to $\mathbf{I}$ can be included in its function and eliminated from the equations above, giving:

$$\mathbf{S}_E^{eq} = 2 \frac{\partial \psi_{eq}}{\partial I_B} \mathbf{B} + 2 \frac{\partial \psi_{eq}}{\partial I_{2B}} \mathbf{B}^{-1} \quad \text{Eq. A-20}$$

$$\mathbf{S}_E^{oe} = 2 \frac{\partial \psi_{oe}}{\partial I_{B_e}} \mathbf{B}_e + 2 \frac{\partial \psi_{oe}}{\partial I_{2B_e}} \mathbf{B}_e^{-1} \quad \text{Eq. A-21}$$

These functions, considering the equivalence of $\mathbf{T}$ and $\mathbf{S}$ for incompressible materials, give the solution presented in Eq. 5-16:

$$\mathbf{T} = -p\mathbf{I} + 2 \frac{\partial \psi_{eq}}{\partial I_{1B}} \mathbf{B} - 2 \frac{\partial \psi_{eq}}{\partial I_{2B}} \mathbf{B}^{-1} + 2 \frac{\partial \psi_{oe}}{\partial I_{1B_e}} \mathbf{B}_e - 2 \frac{\partial \psi_{oe}}{\partial I_{2B_e}} \mathbf{B}_e^{-1} \quad \text{Eq. A-22}$$

To satisfy the second element of the inequality, it is sufficient to set $\mathbf{D}_i$ to be positive. A function which has been used for this is:

$$\mathbf{F}_e \mathbf{D}_i \mathbf{F}_e^T = \frac{1}{\eta} \mathbf{B}_e \mathbf{S}_E^{oe} \quad \text{Eq. A-23}$$

Where η is a function representing the viscosity and is always above 0. This solution can be combined with the relationship given in Eq. A-6, giving $\dot{\mathbf{B}}_e = \mathbf{B}_e \mathbf{L}^T + \mathbf{L} \mathbf{B}_e - \frac{2}{\eta} \mathbf{B}_e \mathbf{S}_E^{oe}$, and if the relationship between $\mathbf{T}$ and $\mathbf{S}$ is applied again, the final formula used in Chapter 5 is obtained:

$$\dot{\mathbf{B}}_e = \mathbf{B}_e \mathbf{L}^T + \mathbf{L} \mathbf{B}_e - \frac{2}{\eta} \mathbf{B}_e \mathbf{T}_{oe} \quad \text{Eq. A-24}$$