

Nonlinear stability of prestressed stayed beam-columns

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Declaration

I confirm that this thesis is my own work and that any material from published or unpublished work from others is appropriately referenced.

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Abstract

Slender struts supported with prestressed cable-stays in conjunction with periodically-spaced crossarms distributed longitudinally, offer an attractive, innovative and pragmatic solution to reducing structural self-weight while maintaining buckling strength. The use of the crossarm–stay system generally results in a higher load-carrying capacity. Previous research has been primarily conducted for vertical stayed members, but there is a potential demand for very long struts to be used in the horizontal plane. Hence, the current thesis investigates the buckling and post-buckling behaviour of stayed beam-columns where the axial load and the self-weight act orthogonally to each other.

Mechanical modelling is employed to determine the zones of behaviour in the stayed beam-column. Two of the existing three zones for vertical stayed columns are each sub-divided into two further sub-zones owing to self-weight. It is found that there is considerable post-buckling reserve especially at relatively higher prestressing levels, and the arrangement where the upper crossarm is shorter than the lower crossarm has a beneficial effect in some cases.

A nonlinear analytical model for prestressed stayed beam-columns with doubly-symmetric and mono-symmetric configurations, based on the Rayleigh–Ritz method, is subsequently presented. It captures modal interactions for perfect geometries explicitly. It is also found that the varying prestressing levels and crossarm length ratios can lead to different buckling modes. From the obtained insight, finite element models are then employed to inves-

tigate the post-buckling behaviour including pre-cambering, imperfection and prestressing sensitivities. The actual optimum prestressing force is determined considering ultimate behaviour and efficiency indicators. It is found that mono-symmetric cases are more sensitive to the level of pre-cambering than doubly-symmetric cases, and for cases with upward pre-cambering, mono-symmetric arrangements perform better than their doubly-symmetric counterparts. Moreover, recommended upper and lower limits on prestressing levels are presented.

Finally, a set of design equations is proposed based on the findings, such that stayed beam-columns may be used safely in practice.

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Nomenclature

Coordinates, stress, strain and energy

ε_{Ttn}	Pre-existing strain within stay tn introduced by the initial prestressing force
Φ_P, Φ_W	Work done by the axial load and self-weight respectively
Φ	Total work done by external loads
$\Phi_{W,c}, \Phi_{W,a}$	Work done by the self-weight of the main member and crossarms respectively
η_{tn}	Slackening switch for stay tn
U_{au}, U_{al}	Bending energy of the upper and lower crossarms respectively
U_{ca}, U_{cb}	Axial and bending energy of the main member respectively
U_s	Total strain energy of all the stays
U_{stn}	Strain energy stored in stay tn
V	Total potential energy
x	Longitudinal direction along the main member
y	Transverse direction along the crossarms
z	Transverse direction out of plane

Structural geometries and material properties

L	Main member length
a_u, a_l	Lengths of the upper and lower crossrams respectively
α_u	Length ratio of the upper crossram a_u/L
α_l	Length ratio of the lower crossram a_l/L
α	Crossarm length ratio a_u/a_l
β_u	Angle between the main member and upper stays
β_l	Angle between the main member and lower stays
ϕ_s	Stay diameter
ϕ_{co}, ϕ_{ao}	Outside diameters of the main member and crossarms respectively
ϕ_{ci}, ϕ_{ai}	Inside diameters of the main member and crossarms respectively
l_{tn}	Original length of the stay tn
l'_{tn}	Current length of the stay tn after applying axially compressive load
$l_{w,tn}, l_{p,tn}$	Lengths of the stay tn after applying self-weight and prestress respectively
E_c	Young's Modulus of the main member
E_a	Young's Modulus of crossarms
E_s	Young's Modulus of stays
A_c	Cross-sectional area of the main member
A_a	Cross-sectional area of crossarms
A_s	Cross-sectional area of stays

I_c	Cross-sectional second moment of area of the main member
I_a	Cross-sectional second moment of area of crossarms
r_c	Cross-sectional radius of gyration for the main member
W_c	Section modulus of the main member
$K_{c,a}$	Axial stiffnesses of the main member
$K_{c,b}, K_{cb,half}$	Bending stiffnesses of total and half of the main member respectively
$K_{au,a}, K_{al,a}$	Axial stiffnesses of the upper and lower crossarms respectively
$K_{au,b}, K_{al,b}$	Bending stiffnesses of the upper and lower crossarms respectively
K_{su}, K_{sl}	Axial stiffnesses of the upper and lower stays respectively
ρ	Material density
$\bar{\lambda}$	Normalized slenderness of the main member
f_y	Yield stress of the main member
$\varepsilon_{s,y}$	Yield strain of stay material

Internal forces and external loads

$F_{u,f}, F_{l,f}$	Final axial forces within the upper and lower crossarms respectively at the instant of buckling
$F_{u,i}, F_{l,i}$	Initial axial forces within the upper and lower crossarms respectively after applying prestress
Q_a	Crossarm self-weight load
$P_{max,x}^C$	Maximum critical loads at the boundaries of zones
P_1^C, P_2^C	Buckling loads for Modes 1 and 2 respectively

$P_{\max}^C, P_{\min}^C$	Maximum and minimum critical buckling loads respectively in zones of behaviour
P_E	Euler buckling load of the main member
P_i, P_f	Internal axial forces within the main member after prestressing and at the instant of buckling respectively
P_y	Squash load
P_u	Ultimate load
q_w	Main member self-weight intensity
R_{au}, R_{al}	Total horizontal forces on the upper and lower crossarm tips from the upper and lower stay restraints respectively
$T_{l,R}$	Residual tensions within the lower stays at the instant of buckling
$T_{u,i-a}, T_{l,i-a}$	Applied prestressing forces within the upper and lower stays respectively to improve structural performance
$T_{u,f}, T_{l,f}$	Final prestressing forces within the upper and lower stays respectively at the instant of buckling
$T_{u,i}, T_{l,i}$	Achieved prestressing forces within the upper and lower stays respectively to improve structural performance
$T_{l,w-a}$	Applied prestressing force within the lower stays to counteract self-weight
$T_{l,w}$	Achieved prestressing force within the lower stays to counteract self-weight
T_0	Force within the lower stays after applying self-weight
$V_{u,f}, V_{l,f}$	Final horizontal forces on the upper and lower crossarm tips respectively at the instant of buckling

g	Gravitational acceleration
M	Bending moment at any cross-section
P	Externally-applied axial load
Q_1, Q_2	Vertical force on the main member ends resulting from buckling Modes 1 and 2 respectively
T_{mini}	Linear minimum prestressing forces
T_{opti}	Linear optimum prestressing forces

Kinematic quantities and degrees of freedom

Δ_{au}, Δ_{al}	End-shortening displacements of the upper and lower crossarms respectively from linear analysis
Δ_c	End-shortening displacement of the main member from linear analysis
δ_c	Normalized purely compressive end-shortening displacement of the main member
δ_m	Normalized vertical deflection at the midspan of the main member
$\Delta_{w,c}, \Delta_{p,c}$	End-shortening displacements of the main member after applying self-weight and prestress respectively
$\Delta_{p,au}, \Delta_{p,al}$	Axially compressive deformations of the upper and lower crossarm respectively from applying prestress
Δ_{su}, Δ_{sl}	Extensions of the upper and lower stays respectively from linear analysis
κ	Calibration factor to correct the crossarm shape functions
k_{au}, k_{al}	Circular frequencies for the deformed profile of the upper and lower crossarm shape functions respectively

$\mathcal{E}$	Main member end-shortening during applying axially compressive load
u_u, u_l	Generalized coordinates for the upper and lower crossarms respectively
θ_{cm}, θ_a	Rotations at midspan joint of the main member and crossarms respectively
$w_{au,b}, w_{al,b}$	Displacements of the upper and lower crossarm tips respectively owing to the crossarm bending
$w_{au,f}, w_{al,f}$	Total displacements of the upper and lower crossarm tips respectively
$w_{au,r}, w_{al,r}$	Rigid body displacements of the upper and lower crossarm tips respectively owing to the joint rotation
$w_{w,m}, w_{p,m}$	Midspan vertical displacements of the main member after applying self-weight and prestress respectively
w_m	Midspan vertical displacement of the main member during application of axially compressive load
$W_u(y), W_l(y)$	Shape functions of the upper and lower crossarms respectively
$W(x)$	Shape functions of the main member
u_i	Generalized coordinate for the main column
$W_i(x)$	Shape functions of the main member affine to Modes 1 and 2 and self-weight deformations respectively

Pre-cambering and imperfection

δ	Imperfection amplitude ratio
γ	Parameter to indicate the distribution of prestressing forces between the upper and lower stays
I_p	Pre-cambering index

$\mathcal{E}_0$	Main member end-shortening from imperfections compared to a perfect member
μ_1, μ_2	Scale factors introduced to represent the proportions of the initial imperfection shape corresponding to buckling Modes 1 and 2 respectively
w_{imp}	Initial imperfection shape for the main member

Structural efficiency

η, η_s	Structural efficiency indicators for the main member and stays respectively
$f_{y,\text{req}}, f_{s,\text{req}}$	Required resistances at the point of the maximum load for the main column and stays respectively

Design method

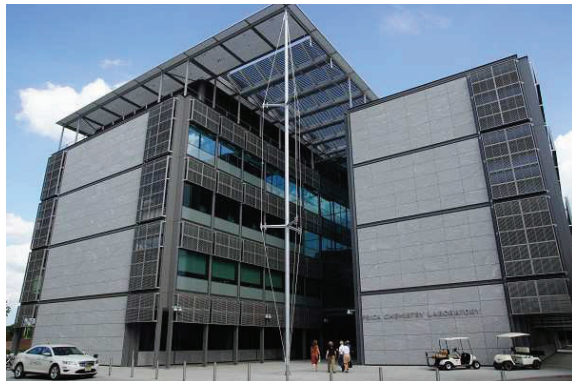
α_{EC3}	Imperfection factor referred to as α in BS EN 1993-1-1 (2005)
χ	Reduction factor for axial capacity of a structural member
C_{pr}	Prestress coefficient for stayed members
η_{EC3}	Generalized imperfection factor referred to as η in BS EN 1993-1-1 (2005)
η_{tot}	Total generalized imperfection factor for stayed members
η_{pr}	Prestress factor for stayed members
I_{cr}	Index to describe the close degree of the buckling loads of Modes 1 and 2
$P_{\text{u,design}}$	Design load-carrying capacity
χ_{int}	Reduction index for interactive buckling resistance relative to the pure Mode 2 buckling resistance

Chapter 1

Introduction

Slender struts supported with prestressed cable-stays in conjunction with periodically-spaced crossarms distributed longitudinally, offer an attractive, innovative and pragmatic solution to reducing structural self-weight while maintaining buckling strength. The use of the crossarm–stay system with pretensioning within the cable stays effectively provides intermediate supports that act to reduce the effective buckling wavelength which, in turn, results in a higher load-carrying capacity. Compared with conventional unstayed members of the same dimension, prestressed stayed members can achieve a higher load-carrying capacity while limiting self-weight. These members usually undertake structural loads as main columns for high-rise buildings and beams for long-span structures, as shown in Figure 1.1. Stays and multiple sets of crossarms within the stayed column system shown in Figure 1.1(a) allow the reduction of the cross-section area of the main column significantly, and the externally prestressing stays within the bridge in Figure 1.1(b) allow a relatively long span to be achieved.

Recently, huge hydraulic props have been adopted to support a basement excavation in London Paddington, which is shown in Figure 1.2. The props were made of thick tubular steel cross-sections and therefore had an intrinsically high self-weight. Potentially, if prestressed



(a) Frick Chemistry Laboratory, USA
(Princeton, 2010)



(b) San Miguelito creek footbridge, Mexico
(Madrazo-Aguirre *et al.*, 2015)

Figure 1.1: Applications of prestressed stayed structures: (a) column example; (b) bridge example.



(a) End part of huge hydraulic props



(b) Construction site with huge props

Figure 1.2: So-called ‘super struts’ used to support a basement excavation (London, 2012).

stayed beam-columns were to be employed as members to replace the huge hydraulic props, the size of these components could be decreased considerably. This could have some advantages in terms of reduced weight for lifting the props into place and increased space for working within the excavation. For example, for a specific stayed beam-column case defined as F3.7 in §3.4, its self-weight is 18.62 kg, and the achieved load-carrying capacity is 125.17 kN; however, for an unstayed beam-column with the same span, the minimum weight of steel to achieve the same load-carrying capacity is 21.69 kg. This implies that the steel consumption can be reduced by 14%. Considering engineering practice, the elements of the stayed members can be pre-fabricated, but the length of each piece should be limited to ensure transportation convenience. Hydraulic devices can be installed at the end of the main member, which can adjust the member length and axial force to support the excavation. The assembly process can be finished in situ, including welding members and prestressing stays. In addition, the risks associated with impact on the stays should be considered, such as prestressing loss, temperature effects and intrinsic imperfections of stays.

Research on the type of structural system shown in Figure 1.1(a) can be found initially from the 1960s (Chu & Berge, 1963; Mauch, 1967) with specific work on determining the buckling behaviour analytically (Smith *et al.*, 1975; Belenya, 1977) and the important relationship between the levels of pretensioning and the critical buckling load (Hafez *et al.*, 1979). The work presented in Smith *et al.* (1975) was verified in Temple (1977) and Temple *et al.* (1984), and imperfection sensitivity was investigated in Wong & Temple (1982). Experiments and numerical studies were conducted to validate the buckling behaviour (Jemah & Williams, 1990; De Araujo *et al.*, 2008; Osofero *et al.*, 2012, 2013). Subsequently, the relationship between prestressing levels and nonlinear post-buckling behaviour of stayed struts was determined numerically (Chan *et al.*, 2002) and then theoretically, for distinct post-buckling behaviour (Saito & Wadee, 2008, 2009a) and where interactive buckling occurs (Saito & Wadee, 2009b; Wadee *et al.*, 2013a; Yu & Wadee, 2017a). Design guidance for struts with one intermediate crossarm has also been presented (Wadee *et al.*, 2013b;

Tankova *et al.*, 2019). In addition, prestressed steel elements under tension and compression were studied in Gosaye *et al.* (2014, 2016), while, more recently, the stability and design of prestressed cold-formed steel members have also been investigated by Hadjipantelis *et al.* (2018, 2019a,b) and Wadee *et al.* (2020b).

Most of the aforementioned research has been conducted for vertical members where the direction of the self-weight and the axial load have been collinear. However, there is a potential demand for very long struts to be used in the horizontal plane, for example to support excavation works in the construction of large scale foundations, as represented in Figure 1.3. As can be seen in Figure 1.2, existing so-called ‘super struts’ are currently used; these are constructed from heavy tubular steel cross-sections such that the struts can withstand the earth pressures axially in conjunction with their considerable self-weight in bending. It has been demonstrated in the aforementioned research work that the most effective stayed columns can increase the load-carrying capacity of a conventional column by an order of magnitude. Hence, the current study considers the effect of placing stayed columns in the horizontal plane, thereby converting them into stayed beam-columns where the axial load and the self-weight act orthogonally to each other. This introduces new complexity to the mechanics of the system since the collinearity of the loading and self-weight no longer exists. The self-weight introduces a directionality that is similar to that found in cable-stayed bridges, where stays are regularly used to reduce the self-weight deflection of the deck, thereby increasing load-carrying capacity (Madrado-Aguirre *et al.*, 2015). Currently, there is a similar opportunity by reducing the length of the upper portion of the crossarm, as shown in Figure 1.3, to provide a net upwards force from the intermediate restraint. This enables the main beam-column member to be pre-cambered by a controlled extent to counteract the effect of the self-weight.

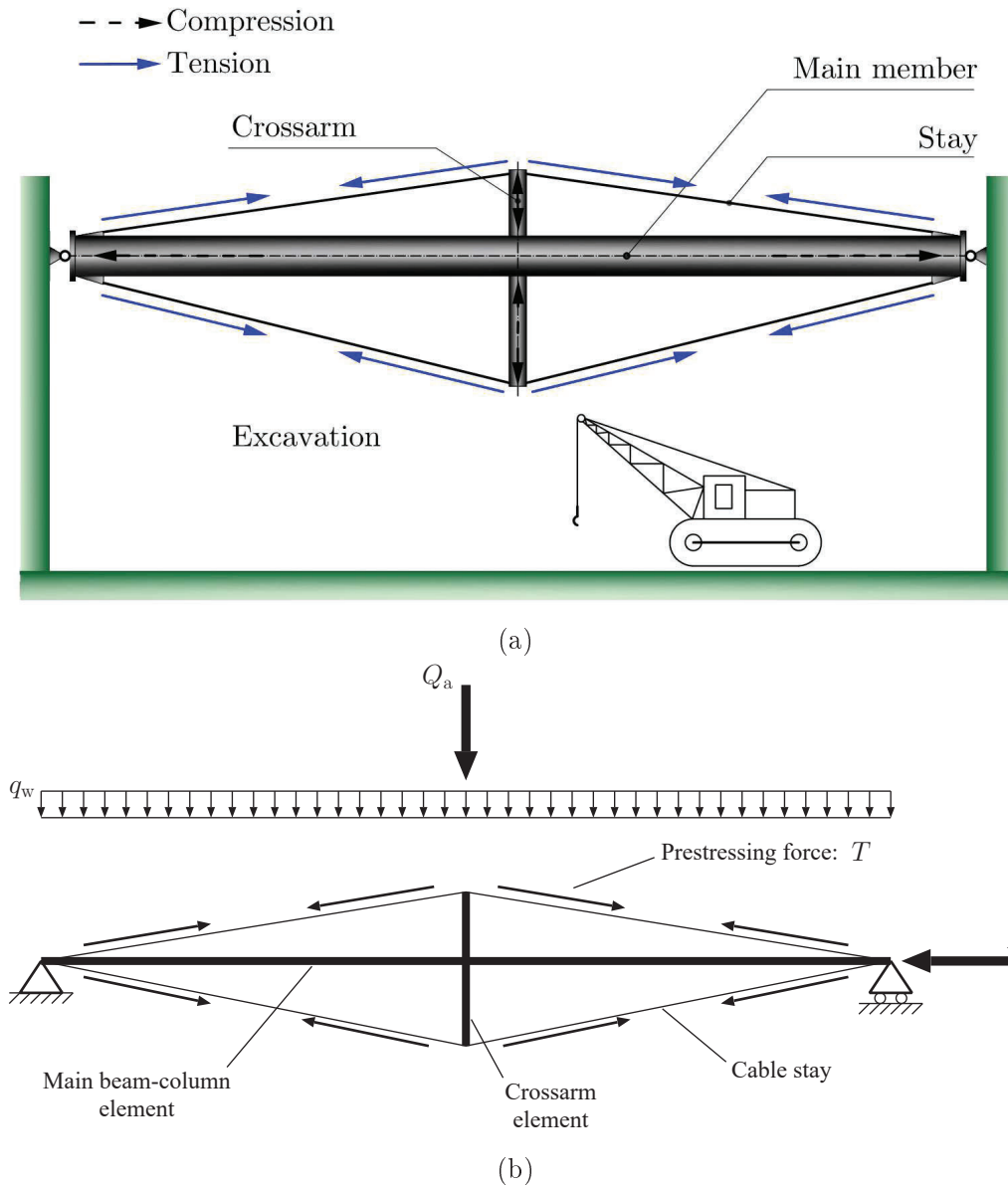


Figure 1.3: (a) Potential application of a prestressed stayed beam-column as an excavation prop. In (b), a prestressed stayed beam-column is depicted, under an axial load P and self-weight lateral loads Q_a from the crossarm and q_w from the beam-column element; cable stays are under an initial prestressing force T that put both the beam-column element and crossarms under axial compression before any external loading is introduced.

1.1 Research aims and objectives

The key aims and objectives of the present research project are:

1. To reveal the post-buckling behaviour and the ultimate load-carrying capacity, of doubly-symmetric and mono-symmetric prestressed stayed beam-columns under self-weight acting orthogonally to the axially compressive load.
2. To use the conclusions of objective 1 to determine the best configuration and true optimum prestressing level of the stayed beam-column to maximize efficiency.
3. To provide guidance for practical work based on the findings from the investigations in objectives 1 and 2.

In order to determine the critical buckling load and the theoretical optimum prestressing level as reference values for the basis of the establishment of future research and design rules, three principal zones of behaviour are identified with two of these each having two sub-zones that relate the critical buckling load to the initial prestressing force applied to the stayed cables. The purpose of determining these zones is to provide fixed points from which comparisons can be made.

A nonlinear analytical model for prestressed stayed beam-columns with doubly-symmetric and mono-symmetric configurations, based on the Rayleigh–Ritz method, is presented that captures modal interactions for perfect geometries explicitly for the first time. Numerical solutions of the analytically derived equilibrium equations are obtained from the continuation and bifurcation software AUTO-07P (Doedel & Oldeman, 2011). The analytical model is verified against a nonlinear finite element (FE) model developed within the commercial package ABAQUS (ABAQUS, 2016) quantitatively.

The interactive buckling behaviour revealed by the aforementioned analytical model is verified by the post-buckling behaviour of imperfect stayed beam-columns from FE modelling.

The true optimum prestressing force of the imperfect stayed column is determined from a numerical study on the ultimate behaviour while defining measures of structural efficiency of stayed beam-columns.

1.2 Basic stability concepts

1.2.1 Post-buckling response

Koiter (1945) proposed a general theory on the post-buckling behaviour of structures for the first time, and a nonlinear bifurcation theory for continuous elastic systems by minimizing the total potential energy of the structure was formulated. Thompson & Hunt (1973) investigated post-buckling problems by introducing a series of ‘generalized coordinates’ independently. In this theory, the post-buckling deformation and the potential energy of a structural system can be formulated by a number of discrete components of respective generalized coordinates u_n , which represents each component of the system. The total energy within the static system can be defined as the total potential energy function V using u_n , thus:

$$V = V(u_1, u_2, u_3, \dots, u_n, P), \quad (1.1)$$

where P is the external loading component, which comprises the axial load and self-weight in the present study.

Before the function V can be applied, two axioms relating total potential energy to equilibrium and its stability are introduced:

Axiom 1 A stationary value of the total potential energy with respect to the generalized coordinates is necessary and sufficient for the equilibrium of the system.

Axiom 2 A complete relative minimum of the total potential energy with respect to the

generalized coordinates is necessary and sufficient for the stability of an equilibrium state.

In the classic of a nonlinear elastic pin-ended strut, the post-buckling behaviour is well-known to be weakly stable (Thompson & Hunt, 1973), as represented in Figure 1.4. With-

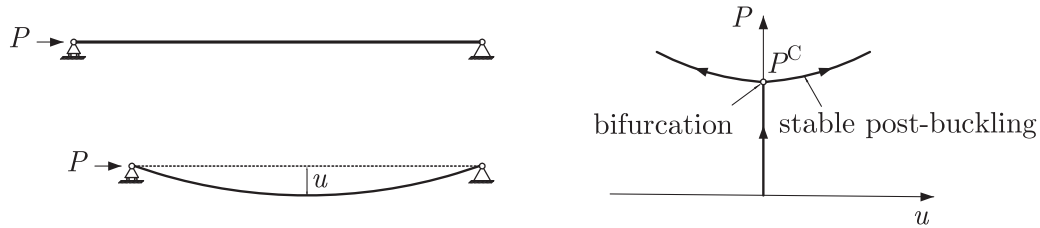


Figure 1.4: Post-buckling behaviour of a nonlinear elastic pin-ended strut.

out considering any initial geometric imperfections, the axially compressive load P would increase alongside the increasing magnitude of the generalized coordinate u within the elastic post-buckling range. In terms of a system that is simplified to a nonlinear pin-ended strut with a lateral spring restrained at the mid-span, the post-buckling behaviour has been determined to be unstable (Hunt *et al.*, 1993), as represented in Figure 1.5. The axially

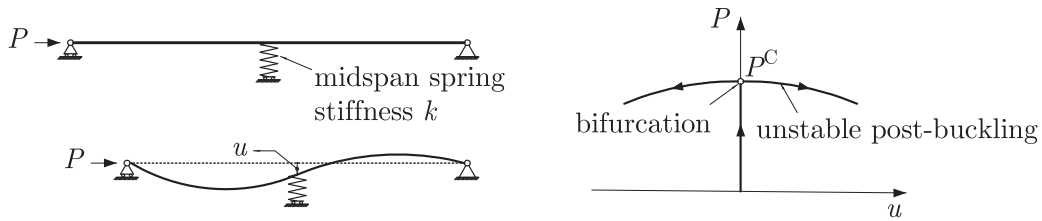


Figure 1.5: Post-buckling behaviour of a strut with a mid-span spring restraint.

compressive load P would decrease alongside the increasing magnitude of the generalized coordinate u within the elastic post-buckling range. However, for the prestressed stayed column system, the post-buckling is neither represented by the pure pin-ended strut nor the strut restrained with a mid-span spring. This is because the post-buckling behaviour is strongly linked to the effects of mode interaction, where a nonlinear mixture of the above cases affects the mechanical response, and stay slackening, where the sudden loss in stiffness changes the geometry and capacity in a discontinuous manner.

1.2.2 Interactive buckling

For systems with more than one degree of freedom, there exist several buckling modes, but the first mode is usually used as the dominant one in practical work. However, when the buckling loads of some modes are numerically close, the initially activated mode may be affected or even ‘jump’ to another, which leads to different post-buckling behaviour combined with more than one buckling modes: this is so-called ‘interactive buckling’, and this phenomenon may reduce the practical load-carrying capacity of the system.

A simple example from Augusti (1964), which can be applied to explain interactive buckling, is shown in Figure 1.6. The exact form of the total potential energy V is given by the

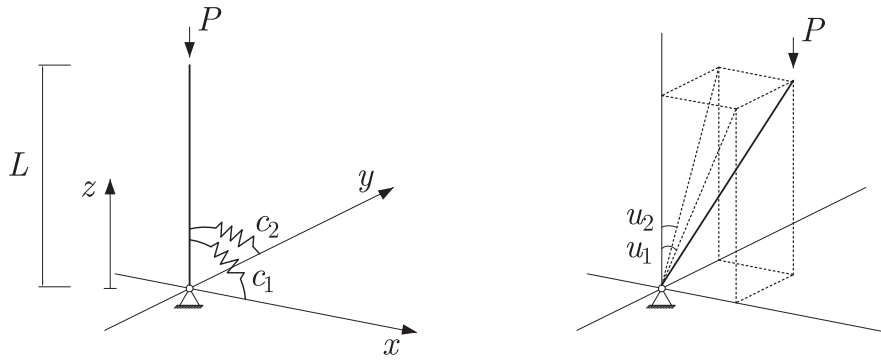


Figure 1.6: Model for interactive buckling from Augusti (1964); rigid link of length L is supported by rotational springs of stiffness c_1 and c_2 acting in different planes; u_1 and u_2 are the generalized coordinates representing planar rotations relative to the vertical axis.

expression as follows, where the first two terms represent the strain energy stored in the rotational springs and the third term represents the work done by the external load:

$$V = \frac{1}{2}c_1u_1^2 + \frac{1}{2}c_2u_2^2 - PL \left[1 - (1 - \sin^2 u_1 - \sin^2 u_2)^{1/2} \right]. \quad (1.2)$$

This can be expressed as a power series in u_1 and u_2 thus:

$$V = \frac{1}{2}c_1u_1^2 + \frac{1}{2}c_2u_2^2 - PL \left[\frac{1}{2}(u_1^2 + u_2^2) + \frac{1}{4}u_1^2u_2^2 - \frac{1}{24}(u_1^4 + u_2^4) \dots \right]. \quad (1.3)$$

From Axiom 1, the buckling loads of the first mode P_1^C and the second mode P_2^C can be determined respectively thus:

$$P_1^C = \frac{c_1}{L}, \quad P_2^C = \frac{c_2}{L}. \quad (1.4)$$

Defining the ratio of the rotational spring stiffnesses as $R = c_2/c_1$, if $R > 1$ then u_1 is an active coordinate and u_2 is a passive coordinate at the critical buckling load, since $P^C = P_1^C$. Obviously, if $R < 1$, the situation is reversed. However, if $R = 1$, then the two buckling loads are equal and both coordinates are active. Apart from the trivial fundamental paths in u_1 and u_2 respectively, the respective uncoupled post-buckling paths can be expressed thus:

$$P = \frac{c_1}{L} \left(\frac{1}{1 - u_1^2/6} \right), \quad P = \frac{c_2}{L} \left(\frac{1}{1 - u_2^2/6} \right). \quad (1.5)$$

In the uncoupled path analysis, the quartic cross-term present in Eq. (1.3) is not taken into account, but when the strut rotates in the active coordinate, the quartic cross-terms need to be considered to evaluate any mixing of the modes in the nonlinear range; this mixing is known as interactive buckling. Again from Axiom 1, two solutions for the buckling loads can be obtained, but now on the active uncoupled path, are thus:

$$P = \frac{c_1}{L}, \quad P = \frac{3c_1 + c_2}{4L}. \quad (1.6)$$

The first solution is the original critical buckling load, but the second solution represents a secondary bifurcation point S is triggered at a new load, where the post-buckling behaviour changes from a distinct modal behaviour to interactive buckling behaviour. Aside from the trivial fundamental path $u_1 = u_2 = 0$, the relationship between u_1 and u_2 by eliminating P is found to be thus:

$$u_2^2 - u_1^2 \left(\frac{3 + R}{1 + 3R} \right) = \frac{6(1 - R)}{1 + 3R}. \quad (1.7)$$

This describes a hyperbolic relationship (Supple, 1967), and it can be seen that the mode

interaction depends on R . Since the secondary bifurcation point S is triggered when $u_2 = 0$, a closed-form expression for u_1 at S can be obtained thus:

$$u_1^S = \pm \left[\frac{6(1-R)}{1+3R} \right]^{1/2}. \quad (1.8)$$

This quantity represents the deflected magnitude when the mode interaction is triggered. When the magnitude of u_1 at S , u_1^S , is very small, the interactive buckling would dominate the post-buckling behaviour. The complete post-buckling path for the case with $R = 1.05$ is shown in Figure 1.7, it is also worth noting that if $R = 1$, all bifurcation points (B_1 , B_2

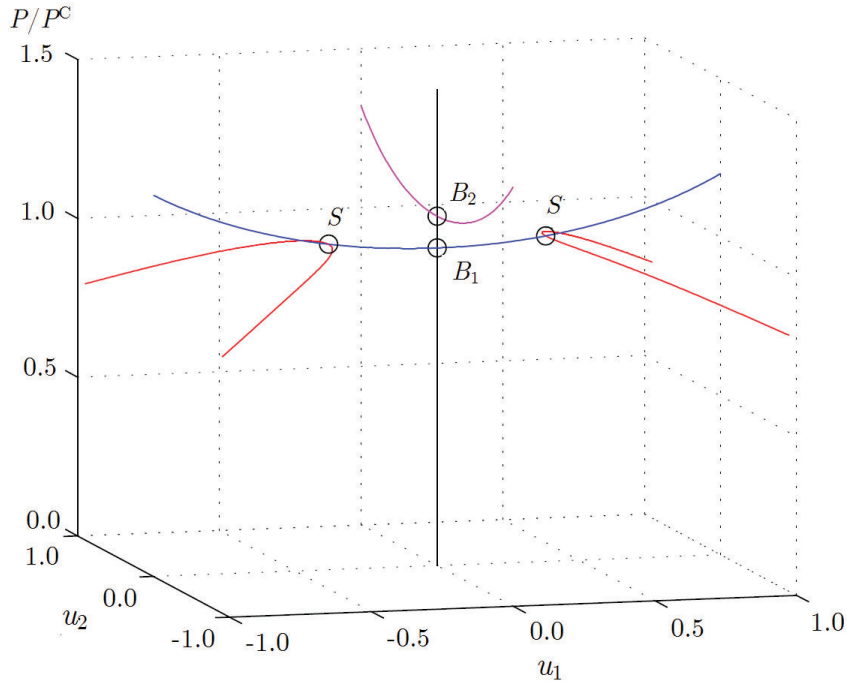


Figure 1.7: Complete post-buckling paths for the case with $R = 1.05$; B_1 and B_2 are the bifurcation points of the first and second modes respectively; S is a secondary bifurcation point. The interactive post-buckling path is unstable even though both distinct modes are stable.

and S) coalesce to one compound bifurcation point, which transforms to what appears to be a stable system to one that in reality is unstable, and is therefore highly sensitive to imperfections in the elastic range.

Interactive buckling can be observed in many structures, for example reticulated columns, sandwich struts, cylindrical shells, stiffened plates, in many different kinds of thin-walled structures and lattice materials (Hutchinson & Koiter, 1970; Thompson & Hunt, 1973; Hunt, 1986, 1989; Gioncu, 1994; Menken *et al.*, 1997; Hunt & Wadee, 1998; Wadee, 1999; Everall, 1999; Everall & Hunt, 2000; Olhoff & Seyranian, 2008; Becque & Rasmussen, 2009a,b; Yiatros & Wadee, 2011; Wadee & Gardner, 2012; Wadee & Farsi, 2014; Wadee & Bai, 2014; Martins *et al.*, 2015; Zschoernack *et al.*, 2016; Wadee *et al.*, 2020a). In particular, this phenomenon has been studied in the context of prestressed stayed members relatively recently (Saito & Wadee, 2009b; Wadee *et al.*, 2013a; Yu & Wadee, 2017a; Wu *et al.*, 2020).

1.3 Thesis outline

In Chapter 2, a review of the previous research on stayed columns is presented. The initial work on the critical buckling behaviour is introduced first, and further studies on the post-buckling behaviour, for example mode interaction, imperfection sensitivity and determining the true optimum prestressing level, are reviewed. Key recent work on the buckling and post-buckling behaviour of the single-bay and multi-bay doubly-symmetric prestressed stayed columns are reviewed in detail. In particular, the literature on the interactive buckling of stayed columns using analytical as well as numerical modelling and physical experiments are reviewed.

In Chapter 3, a mechanical model is employed to determine the relationship between prestressing forces within stays and critical buckling loads. As in the work by Hafez *et al.* (1979), a relationship between the initially applied prestressing force and the critical buckling load is established. However, instead of three clearly distinct zones of behaviour in Hafez *et al.* (1979), two of the three zones are each sub-divided into two further sub-zones of behaviour owing to the effect of the self-weight. The primary aim is to compare the ultimate loads of the studied stayed beam-columns, obtained numerically from ABAQUS,

with their critical buckling loads obtained from the currently presented analytical model in the different zones of behaviour. This is to study whether there is any significant post-buckling strength, and it is in fact determined that there is considerable post-buckling reserve especially at relatively higher prestressing levels. In addition, it is revealed that reducing the ratio of the upper to lower crossarm lengths from unity can have a beneficial effect, but an excessive reduction in the crossarm length ratio is also shown to promote highly unstable post-buckling behaviour. Finally, it is demonstrated that these conclusions determined from the current mechanical model, provide a good estimate for maximizing the load-carrying capacity. The study within that chapter has been published in Wu *et al.* (2019).

In Chapter 4, an analytical model is formulated based on small deflection assumptions, which allows the direct evaluation of the buckling loads for both doubly-symmetric and mono-symmetric arrangements under purely axial compression. The solutions of buckling loads are obtained by solving differential equations, and a simplified closed form equation is subsequently determined using regression techniques. These solutions show excellent agreement with the buckling eigenvalues from analysis within ABAQUS. This study aims to simplify the process for calculating the theoretical optimum prestressing force presented in Chapter 3, and enables the previously necessary numerical analysis to determine linear buckling loads to be omitted. Moreover, the obtained solutions for buckling loads are later used to develop design methods in Chapter 7.

In Chapter 5, a nonlinear analytical model is presented, which implements the Rayleigh-Ritz method to represent the geometrically nonlinear mechanical behaviour of the stayed beam-column as a multi-degree of freedom (MDOF) system. Bending of the main member and the crossarms is included, as is the explicit response of the prestressed cable stays. Beam-columns with doubly-symmetric and mono-symmetric crossarm arrangements are investigated in terms of their critical buckling and post-buckling behaviour. Numerical solutions of the analytically derived equilibrium equations are obtained from the numer-

ical continuation and bifurcation package AUTO-07P (Doedel & Oldeman, 2011). A sequence of instabilities that are intimately connected with stay slackening are found in the post-buckling range showing, quantitatively, how the post-buckling profile of the stayed beam-columns can be transformed from being predominantly antisymmetric to symmetric. Moreover, in terms of mono-symmetric stayed beam-columns, the effect from the varying prestressing levels leading to different buckling modes is investigated for the first time. The analytical model is verified against nonlinear FE modelling developed within ABAQUS, which has been shown in the literature to provide an excellent benchmark when compared to true physical experiments (Osofero *et al.*, 2012). The investigation presented in that chapter has been published in Wu *et al.* (2020).

In Chapter 6, numerical models are employed to study the structural response of imperfect stayed beam-columns. Both doubly-symmetric and mono-symmetric systems are examined. Although earlier chapters investigate the stability of this system, their focus was only on stayed beam-columns with ‘perfect’ geometries under a combination of axial compression and self-weight. The study in Chapter 6 investigates the structural performance of imperfect stayed beam-columns under axial compression in conjunction with their self-weight numerically. Different imperfection combinations are considered for interactive buckling, and the effects on the structural behaviour from varying prestressing and pre-cambering levels are investigated. The optimum prestressing force for ultimate strength, referred to as the true optimum prestressing force, is determined based on the ultimate load achieved and two bespoke structural efficiency indicators. The best structural arrangements are found to be strongly linked to the level of pre-cambering.

In Chapter 7, based on the obtained results from the studies of the buckling and post-buckling behaviour of horizontal stayed beam-columns, a preliminary study to develop design methods for this system is conducted. The methodology is based on the recent work presented by Tankova *et al.* (2019).

Chapter 8 summarizes the important results and conclusions of this project. Suggestions

for possible future work are also presented.

Chapter 2

Literature Review

Research on prestressed stayed systems can be found initially from the 1960s on determining the mechanical response analytically. The critical buckling load and optimum configurations were investigated firstly. With the development of finite element (FE) modelling and computing science, increasingly complex stayed systems have been investigated. The crossarm system of the stayed member has been extended from single-bay to multi-bay, including multiple spatial branches and those with additional stay systems. The nonlinear post-buckling behaviour of the imperfect cases has been studied, and the true optimum prestressing level based on the ultimate behaviour and material failure has been introduced. Recently, analytical and numerical models for the various types of stayed systems have been used to investigate interactive buckling behaviour, and physical experiments also have been conducted to validate the modelling. Therefore, in the present chapter, literature on these aforementioned topics is reviewed in detail.

2.1 Critical buckling loads

Chu & Berge (1963) developed an analytical model to determine the buckling load of prestressed stayed columns for the first time. The general equation was derived and solutions were obtained, and the results showed that the critical load of single-bay stayed columns could be four times the Euler buckling load (Timoshenko & Gere, 1961) of unstayed columns. The analytical solutions were verified against physical experiments, which showed excellent agreement. In order to optimize the configuration and obtain the maximum load-carrying capacity, optimization studies for the initial prestressing force and the member cross-section were conducted. It should be noted that the connections between the crossarms and the main column were simplified as pinned joints in their study, which was different from the practical work, as shown in Figure 2.1.

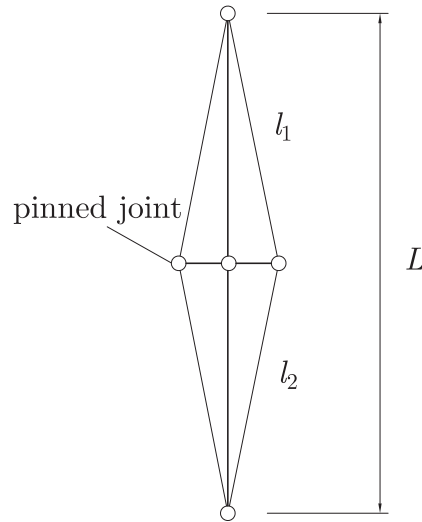


Figure 2.1: Pinned joint model for a prestressed stayed column by Chu & Berge (1963). The hollow circles represent the pinned joints.

Based on Chu & Berge (1963), Mauch (1967) introduced a structural index P/L^2 , where P is the buckling load and L is the length of the main column, to evaluate the efficiency of the structure. It was found that for a structure with a low structural index value the material consumption of the single-bay prestressed stayed column can be reduced by up to

50% compared to that of an unstayed column with the same buckling load and length.

Ellis (1971) at the Royal Military College of Canada conducted an undergraduate project for the design, building and testing of stayed columns. Different from the previous pinned joint model, the revised model adopted rigid joints between the crossarms and main column, and more complex stayed systems with multiple bays were introduced, as shown in Figure 2.2. This model showed that the buckling load of the stayed column could be a

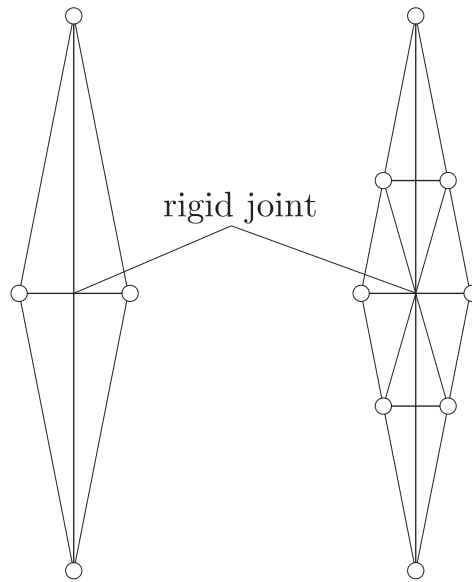


Figure 2.2: Rigid joint model for prestressed stayed column by Ellis (1971). Hollow circles represent pinned joints.

factor of seven times larger than the Euler buckling load of the unstayed column. Pearson (1971) continued this work and found that the critical buckling load of the stayed column was strongly linked to the crossarm length and the stay diameter for the first time.

Smith *et al.* (1975) developed the previous rigid joint model to take both symmetric and anti-symmetric buckling modes into account. The prestressing effect from the stays was simplified as spring supports, as shown in Figure 2.3. The maximum theoretical buckling load of the pin-ended single-bay stayed column was obtained, which was 8.18 times the Euler buckling load of the unstayed column. The buckling loads of these two modes could

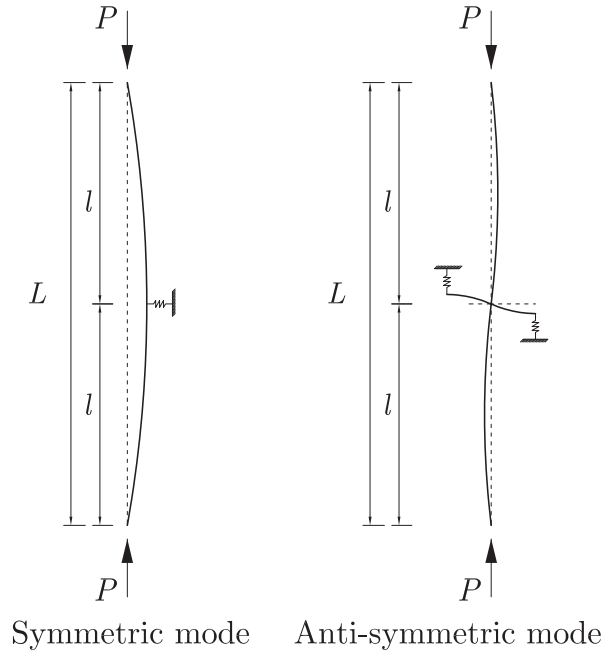


Figure 2.3: Simplified spring-support models of the symmetric and anti-symmetric buckling modes for the single-bay stayed column (Smith *et al.*, 1975).

be determined quantitatively by solving the governing differential equations that include the stiffnesses of the main column, crossarms and stays. This particular model will be developed further in Chapter 4 to make it applicable to both doubly-symmetric and mono-symmetric arrangements under purely axial compression. However, this analytical model only could be applied to the single-bay stayed column, and only two buckling shapes were considered, which implied that the simplified model was equivalent to a two degree of freedom system.

In order to overcome the aforementioned limitations in Smith *et al.* (1975), along with the development of computing science, Temple (1977) proposed a finite element (FE) method to determine the buckling load of multiple-crossarm stayed columns numerically. Since more degrees of freedom were introduced, this method led to more accurate solutions for buckling loads, and the results showed excellent agreement with theoretically exact solutions. It was also found that the buckling mode shape of the stayed column did not only comprise half-sine and full-sine waves. This research was the first to introduce FE

modelling to solve the buckling problems of stayed systems.

Belenya (1977) proposed an analytical model to investigate the critical buckling load of the single-bay stayed column. Ordinary differential equations were solved to obtain the solutions of buckling loads, and the results showed that the buckling load of the stayed column was significantly higher than that of unstayed columns, although the assumption of zero prestressing forces was adopted. The effects from this assumption were studied by comparing the results from physical experiments with and without prestressing forces, which showed that the column with zero prestressing forces possessed a 15% to 20% smaller buckling load as compared to the prestressed column.

Howson & Williams (1980) investigated the buckling load of ten types of stayed columns, a subset of which is illustrated in Figure 2.4. A self-developed plane frame computer

Crossarm number: $N =$

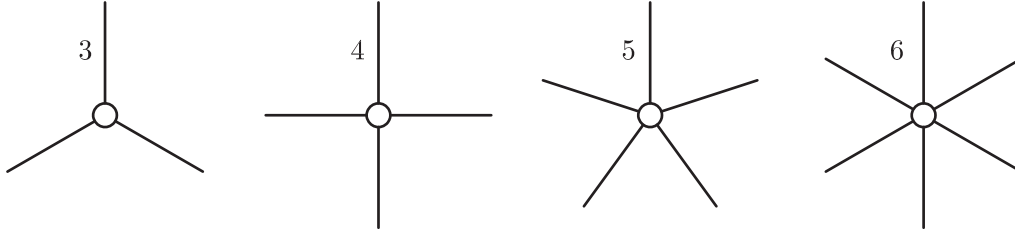


Figure 2.4: End view of the joint between the main member and crossarms, where $N = \{3, 4, 5, 6, \dots\}$ is the number of identical stay branch frames equally spaced round the thin cylindrical main member (Howson & Williams, 1980).

programme from Howson (1979) was applied. Minimum steel consumption was adopted as the optimization target to evaluate the efficiency of these structures. Further parametric studies for the varying crossarm lengths and joint numbers were conducted in Howson & Williams (1984). Based on steel consumption, optimum configurations were presented, and effects from the residual forces within the stays were studied.

2.2 Prestressing levels and initial imperfections

In the aforementioned literature, it was found that the critical buckling load of the stayed column was strongly linked to the prestressing levels. However, Hafez *et al.* (1979) presented an important relationship between the prestressing level and the critical buckling load for the first time. This relationship was represented by the zonal behaviour illustrated in Figure 2.5, and the optimum prestressing force, T_{opt} , of the single-bay stayed column

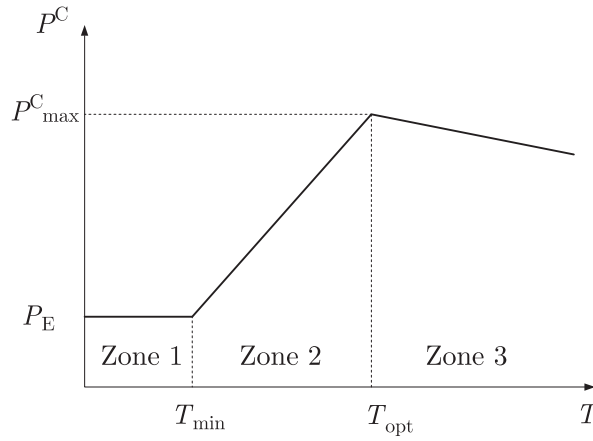


Figure 2.5: Zones of behaviour for prestressed stayed columns. The abscissa is the applied prestressing force within the cable stays T and the ordinate is the critical buckling load P^C . This relationship was first presented in Hafez *et al.* (1979).

was obtained analytically. The three zones of behaviour for the prestressed stayed column that were identified previously (Hafez *et al.*, 1979) were thus:

Zone 1 – defined where the initial prestressing force within the cable stays $T = [0, T_{\text{min}}]$, where under axial compression the prestressing force within the stays is released entirely before buckling occurs and the column buckles at the Euler load of the bare column, hence: $P^C = P_E$.

Zone 2 – defined where the initial prestressing force within the cable stays $T = [T_{\text{min}}, T_{\text{opt}}]$, where the column buckles above the Euler load since the stays are still under (a reduced) tension when the axial load is P_E . Subsequently, however, the column

buckles as soon as the cable tension is released completely, hence $P^C = [P_E, P_{\max}^C]$.

Zone 3 – defined where the initial prestressing force within the cable stays $T > T_{\text{opt}}$, where the column buckles while there is still tension within the cable stays. However, since increasing the prestress introduces more pre-compression in the main column, the critical load reduces as T is increased, hence: $P^C \leq P_{\max}^C$.

The optimum prestressing force of the stayed beam-column is investigated in Chapter 3 using similar methods. It should be noted that the critical buckling loads from the linear zonal behaviour in Hafez *et al.* (1979) can serve as reference values for the basis of the establishment of future design rules, but it does not reflect the nonlinear influences of geometric imperfections, residual stresses, second-order effects and post-buckling stability.

Wong & Temple (1982) investigated the effect of an initial out-of-straightness imperfection on the buckling load of the single-bay stayed column, using the nonlinear finite element (FE) modelling proposed in Temple (1977). The crossarm length and stay diameter were adopted as the varying parameters for the different configurations of stayed columns. The results showed that the buckling load of the stayed column was sensitive to imperfections. The difference between the buckling load from experiments and the imperfect FE models was less than 9%, while the difference between that from experiments and the previous perfect analytical models was up to 21%. Temple *et al.* (1984) studied this further to take varying sizes of the initial imperfections ($L/500$, $L/1000$ and $L/3000$) into account. For the case with the $L/1000$ imperfection, the effect on the buckling load considering varying prestressing levels was investigated. It was found that increasing the imperfection size would decrease the buckling load of the stayed column significantly; indeed, in cases with a prestressing force being close to $T_{\min}$, the buckling load was higher than the analytical solution found in Hafez *et al.* (1979). In terms of cases with a prestressing force close to T_{opt} , the buckling load did not reach the maximum value. These conclusions are sketched in Figure 2.6.

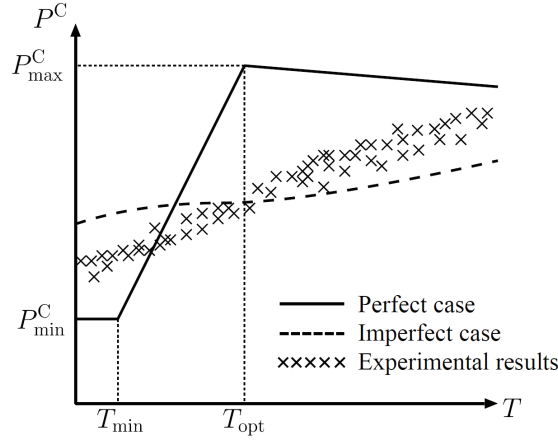


Figure 2.6: Qualitative comparisons of the zonal behaviour from the analytical solutions for perfect models (Hafez *et al.*, 1979), imperfect models and experiments (Temple *et al.*, 1984).

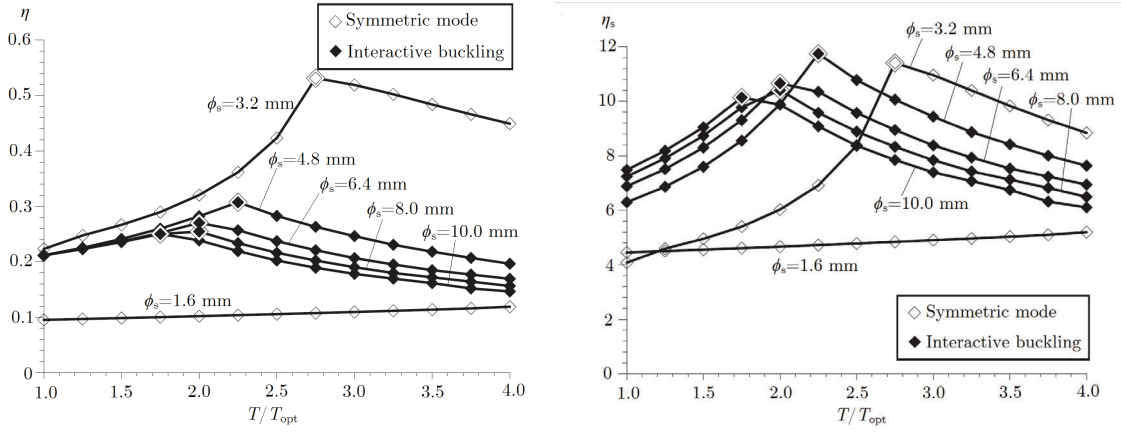
Chan *et al.* (2002) conducted parametric studies on the ultimate load of the imperfect single-bay stayed column using FE modelling, and the results showed excellent agreement with the results in Temple *et al.* (1984). Moreover, it was found that when the symmetric mode was the dominant buckling mode, increasing the crossarm length and stayed diameter would improve the buckling load considerably; when the anti-symmetric mode was the dominant buckling mode, this enhancing effect was not significant but increasing the crossarm stiffness was more efficient.

Owing to the linear elastic assumptions for studying the optimum prestressing force in the aforementioned literature, geometric nonlinearity is not taken into account, which therefore cannot represent the true behaviour. Saito & Wadee (2010) presented a study on the true optimum prestressing force of the imperfect single-bay stayed column that maximizes the load capacity. In order to evaluate the efficiency of the stayed column, two indicators, η and η_s were proposed, thus:

$$\eta = \frac{P_{\max}}{A_c f_{y,\text{req}}}, \quad \eta_s = \frac{P_{\max}}{A_s f_{s,\text{req}}} \frac{L/2}{l_s}, \quad (2.1)$$

where $P_{\max}$ was the maximum load-carrying capacity, and $f_{y,\text{req}}$ and $f_{s,\text{req}}$ were the required

resistances at the point of the maximum load for the main column and stays respectively; A_c and A_s were the cross-sectional areas of the main column and stays respectively; L and l_s were the lengths of the main column and stays respectively. These two indicators not only considered the load-carrying capacity $P_{\max}$, but also took the resistances of the main column and stays into account. These two indicators were adopted as the indices to investigate the true optimum prestressing force, and higher values of these indicators represented more efficient configurations, as shown in Figure 2.7. The symmetric and



(a) Efficiency indicator η for the main column

(b) Efficiency indicator η_s for the stays

Figure 2.7: Efficiency indicators versus prestressing levels in Saito & Wadee (2010) using the FE modelling; ϕ_s represents the stay diameter.

anti-symmetric buckling modes were defined as Modes 1 and 2 respectively, as shown in Figure 2.8. The true optimum prestressing force for the imperfect stayed column was determined quantitatively for the first time, and it was found that the optimum prestressing force proposed by Hafez *et al.* (1979) grossly underestimated the true optimum prestressing level after considering nonlinearity. In Chapter 6, these two efficiency indicators are revised and used to evaluate the structural efficiency of both the doubly-symmetric and mono-symmetric stayed beam-columns, which will allow the evaluation of the true optimum prestressing force for the presently investigated system.

From the aforementioned studies on the optimum prestressing force for the single-bay stayed columns, the theoretical and true optimum prestressing forces of the stayed column

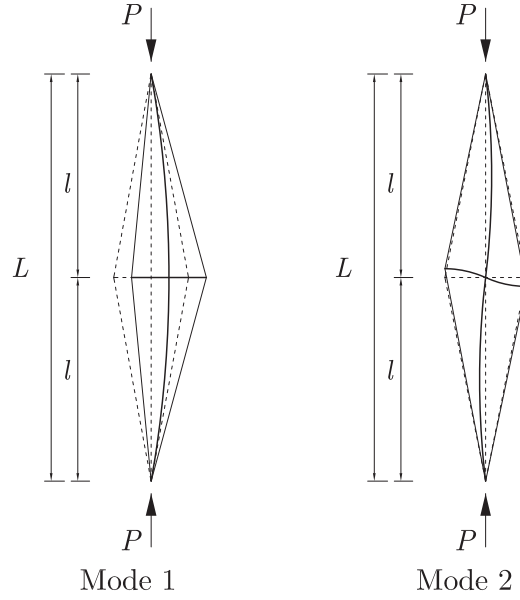


Figure 2.8: Buckling mode shapes of symmetric Mode 1 and anti-symmetric Mode 2 for the single-bay prestressed stayed column.

with three branch crossarms was studied in Li *et al.* (2016), where the cross-section of this system was illustrated by the case of $N = 3$ in Figure 2.4. Yu & Wadee (2017a) presented an analytical model for the stayed column with triple crossarms along the main column length, and the true optimum prestressing level of this system was investigated in Yu & Wadee (2017b). For the stayed columns with three branches or triple crossarms, it was shown that the true prestressing force was approximately $1.5T_{\text{opt}}-2.0T_{\text{opt}}$, which was significantly smaller than the optimum prestressing level for the most single-bay stayed columns (higher than $2.0T_{\text{opt}}$ as determined in Saito & Wadee (2010)). Lapira *et al.* (2017) conducted both analytical and numerical studies to present the theoretical and true optimum prestressing levels for multiple-crossarm prestressed stayed columns with additional stay systems, and it was also found that this type of the stayed column would obtain a higher load-carrying capacity with a higher than T_{opt} prestressing force.

2.3 Post-buckling behaviour

Following the previous literature on the buckling behaviour, Smith (1985) proposed an analytical model to study the load-carrying capacity, rather than the buckling load, of the single-bay prestressed stayed column, which was close to a true post-buckling study. The effect of residual forces within stays and geometric imperfections were taken into account.

Saito & Wadee (2008) presented a study on the post-buckling behaviour of the single-bay stayed column explicitly for the first time. The summation of a series of sine waves was adopted as the shape function of the main column, and a combination of sine, cosine and polynomial functions as the shape of the crossarm was determined by solving governing differential equations accounting for the wavelength of buckling crossarm profiles. Three types of Mode 2 buckling were found, as shown in Figure 2.9. The energy stored in

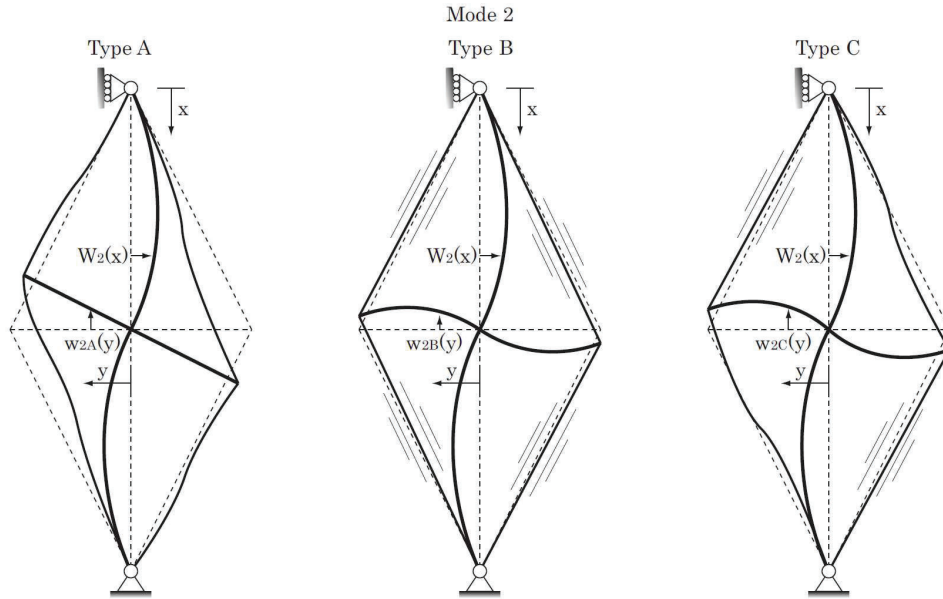


Figure 2.9: Three types of Mode 2 buckling in Saito & Wadee (2008). Type A: all of the stays are slack; Type B: all of the stays are active; Type C: only two of the four stays are active.

each component of the stayed column was calculated based on the Rayleigh–Ritz method, and the post-buckling equilibrium path was obtained by solving equilibrium equations

numerically using MAPLE (Bernardin *et al.*, 2011). It was found that the post-buckling behaviour of the single-bay stayed column was intimately related to the prestressing level. In Zone 1, a stable path would be observed, since the axial energy stored in the fundamental state could be absorbed by stays at the moment of buckling; in Zone 2, the anti-symmetric profile with a pair of slack stays led to stable or unstable post-buckling behaviour, *i.e.* Type B or C respectively, which was linked to prestressing levels; in Zone 3, the initial stage near to the critical load was in Type B with weakly stable behaviour, but the following stage was in Type C with unstable behaviour owing to the loss of stability from the stay slackening.

Saito & Wadee (2009a) studied further on the post-buckling behaviour of the single-bay stayed columns considering imperfections and material failure using both analytical and FE modelling. Parametric studies on prestressing levels and imperfection sizes were conducted. It was found that the true optimum prestressing force of the imperfect stayed column was higher than T_{opt} , *i.e.* within Zone 3 in Hafez *et al.* (1979). It was also found that stay fracture governed the failure when the prestressing force or imperfection size was relatively large, and the fracture failure usually occurred before reaching the ultimate load. Saito & Wadee (2010) conducted a more detailed study on the true optimum prestressing force considering the material resistance, which was mentioned in the discussion on Figure 2.7.

In order to propose a design method based on the aforementioned work, Wadee *et al.* (2013b) presented a general design procedure to determine the load-carrying capacity for the single-bay stayed column. A criterion to check structural design was introduced, thus:

$$\frac{\gamma_{\text{M1}} N_{\text{Ed}}}{N_{\text{max}}} \leq 1.0, \quad (2.2)$$

where N_{Ed} and N_{max} were the design load and the predicted ultimate load respectively, while γ_{M1} was a partial safety factor from Eurocode 3 for structural steel design (BS EN 1993-1-1, 2005). The design curve is shown in Figure 2.10.

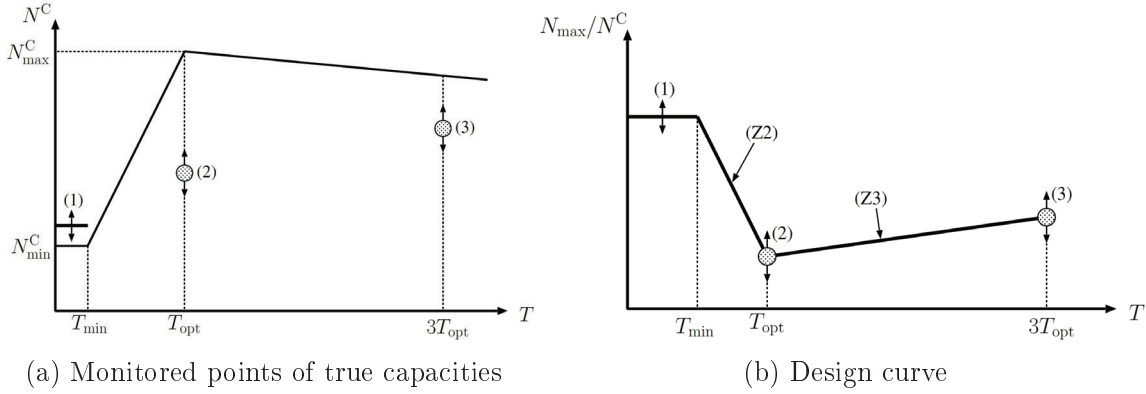


Figure 2.10: Design curve from Wadee *et al.* (2013b), and the prestressing level limit was $3T_{\text{opt}}$: (a) Zonal behaviour graph with monitored points showing the true load capacities for imperfect stayed columns. (b) Strength graph N_{max}/N^C versus prestressing levels corresponding to (a).

Luo *et al.* (2015) proposed a new method for the static equilibrium form-finding analysis of a cable-strut system based on a nonlinear dynamic FE method. This method could be applied to study the structural stability within the construction progress, such as integral tow-lifting and tensioning process, which can ensure the safety for two practical projects of a rigid cable dome.

2.4 Interactive buckling

All the aforementioned literature investigates the buckling and post-buckling behaviour with a pure mode, such as pure Mode 1 or Mode 2, as presented in Figure 2.8. However, the interactive buckling mentioned in Chapter 1 sometimes governs the post-buckling behaviour, since the structural optimization by engineers can lead to buckling loads for different modes becoming similar.

Saito & Wadee (2009b) conducted a numerical study on the interactive buckling of the single-bay stayed column using FE modelling. Apart from the cases with the pure Mode 1 or 2 imperfection, complex cases with combined imperfections were investigated. The

combined imperfection cases would effectively make both the Modes 1 and 2 buckling be triggered simultaneously. They found that when the Mode 1 was critical, this mode also dominated the post-buckling behaviour; when Mode 2 was critical and the buckling loads of Modes 1 and 2 were numerically close, interactive buckling dominated the post-buckling behaviour, as shown in Figure 2.11.

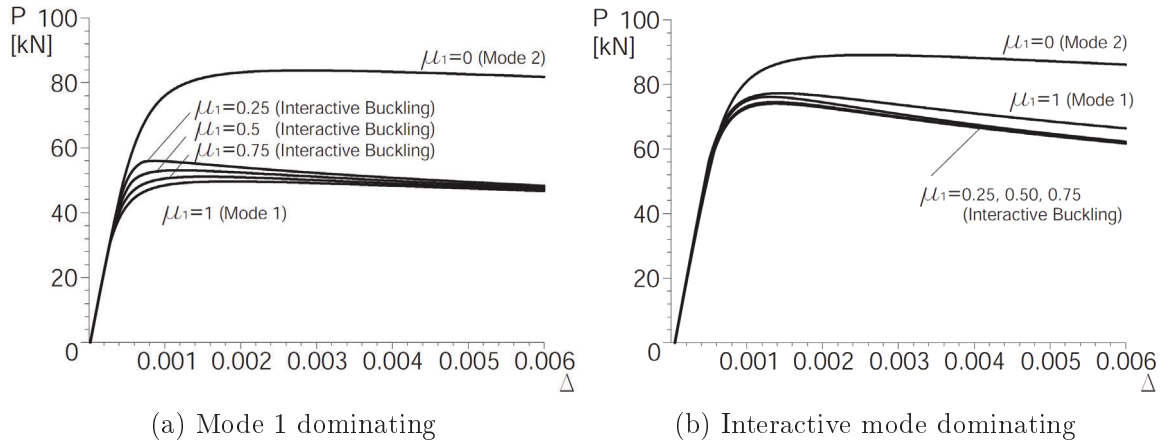


Figure 2.11: Equilibrium paths for the Mode 1, Mode 2 and interactive mode in Saito & Wadee (2009b); μ_1 is the scale factor for the Mode 1 imperfection, hence when $\mu_1 = 0$, the only imperfection present assumed the shape of pure Mode 2.

Wadee *et al.* (2013a) proposed a simplified link model to study the interactive buckling of the single-bay stayed column analytically, as illustrated in Figure 2.12. A stayed column was modelled by four rigid links with the three degrees of freedom, and the restrained effect from stays and crossarms was modelled by the longitudinal and rotational springs. The post-buckling behaviour was obtained by solving the nonlinear equilibrium equations. An interactive loop was observed in the equilibrium path only when Mode 2 was critical, as shown in Figure 2.13. These results provided theoretical evidence to explain the FE results in Saito & Wadee (2009b).

Yu & Wadee (2017a) presented a nonlinear analytical model for the prestressed stayed column with triple crossarms along the column length based on the Rayleigh–Ritz method. Mode interaction and the interactive loop were observed explicitly for the first time in the

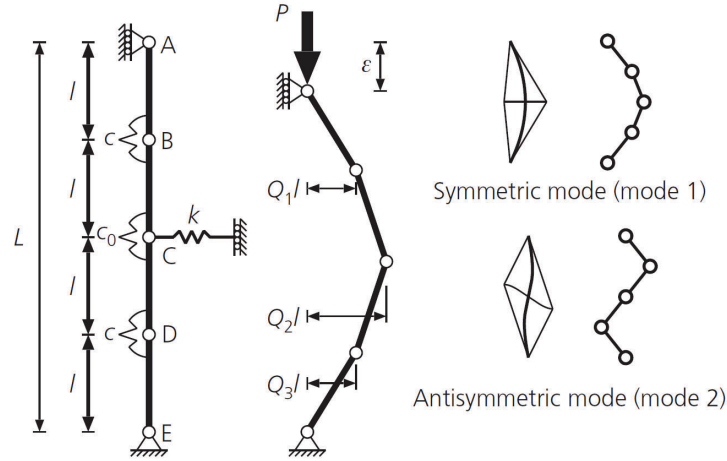


Figure 2.12: Model with rigid links and springs in Wadee *et al.* (2013a); Q_1 , Q_2 and Q_3 are the generalized coordinates describing the lateral pin displacements.

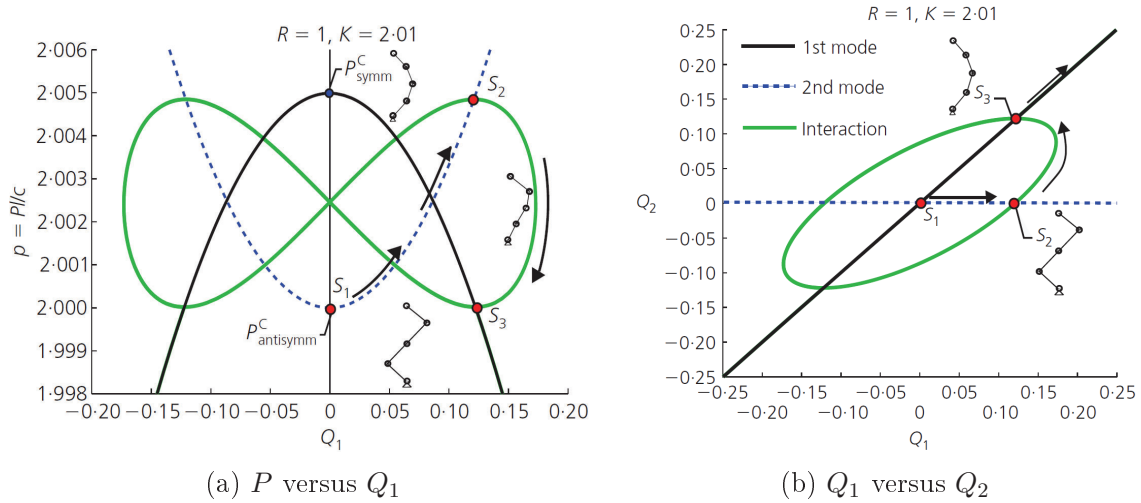


Figure 2.13: Interactive buckling shown in Wadee *et al.* (2013a); (a) Post-buckling response showing normalised load p plotted against Q_1 and (b) Q_2 plotted against Q_1 for the discrete strut model. Point S_1 is the critical bifurcation point, whereas points S_2 and S_3 are secondary bifurcation points on the post-buckling paths of the antisymmetric and symmetric modes respectively.

perfect model, where the bending of the main column and the crossarms were modelled explicitly. The unstable interactive buckling behaviour, *i.e.* ‘mode jumping’ and ‘snaking’, was triggered owing to stay slackening, as shown in Figure 2.14. These results showed that

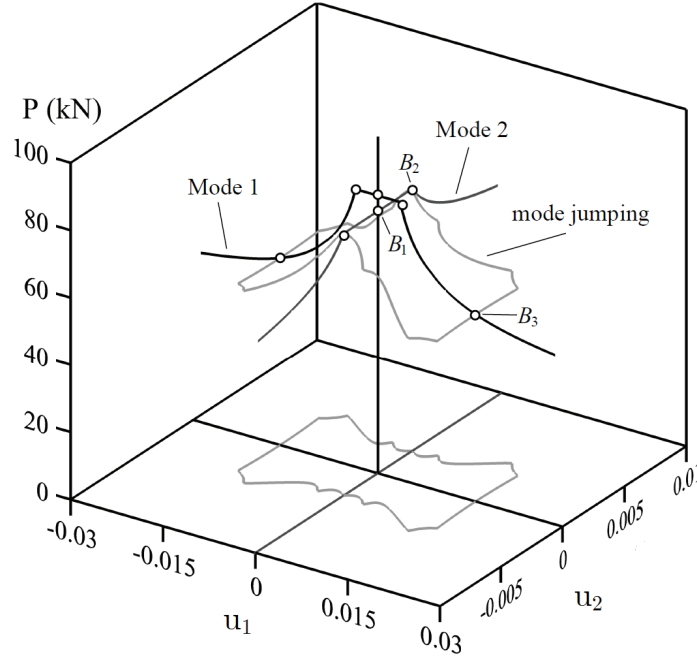


Figure 2.14: Full modal interaction: mode jumping from Mode 2 to Mode 1 as found by Yu & Wadee (2017a); P is the axial load, with u_1 and u_2 being the modal amplitudes.

mode interaction was common in stayed systems, and validated the conclusions of Saito & Wadee (2009b) and Wadee *et al.* (2013a).

Li *et al.* (2018) investigated the interactive buckling behaviour of the stayed column with pin-connected three branch crossarms, *i.e.* $N = 3$ in Figure 2.4, as shown in Figure 2.15. They found that for the cases with the pure Mode 2 imperfection, interactive buckling could not be triggered automatically when the crossarms were pin-connected to the main column, while interactive buckling could be triggered automatically when the crossarms were rigidly connected to the main column.

In addition, Pichal & Machacek (2017) investigated the critical buckling loads and post-buckling behaviour of stainless steel stayed columns with regards to 2D and 3D geometri-

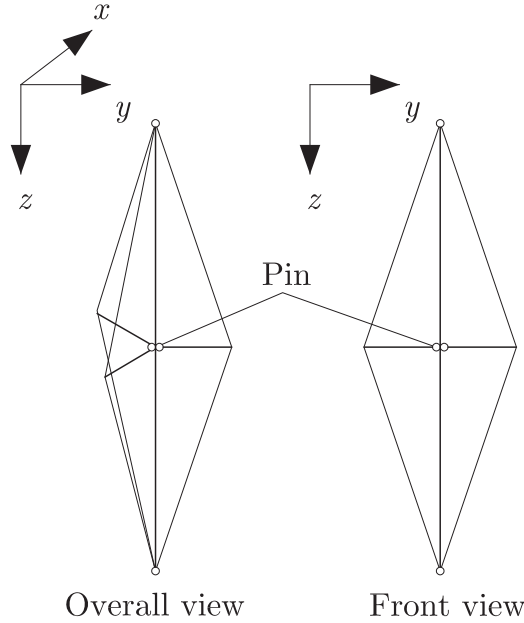


Figure 2.15: Stayed column with pin-connected three branch crossarms in Li *et al.* (2018).

cally and materially nonlinear analysis with imperfections using ANSYS software.

2.5 Experimental studies

Apart from the analytical and numerical studies in the aforementioned literature, physical experiments have been conducted since the 1990s to validate theoretical conclusions. Jemah & Williams (1990) tested the stayed column with bipods, as illustrated in Figure 2.16. Based on the varying N values shown in Figure 2.4 (Howson & Williams, 1980), both symmetric and anti-symmetric buckling modes were investigated. It was found that for the imperfect systems, the true optimum prestressing force was several times of the theoretical solution, and the buckling load was 10% below the theoretical solution. Van Steirteghem *et al.* (2005) conducted further experimental research stayed columns with bipods. They found that the stay diameter and crossarm length were strongly linked to the load-carrying capacity of stayed columns, and compared to the single-bay stayed column, the efficiency of the stayed column with bipods (defined as the minimum steel consumption for a given

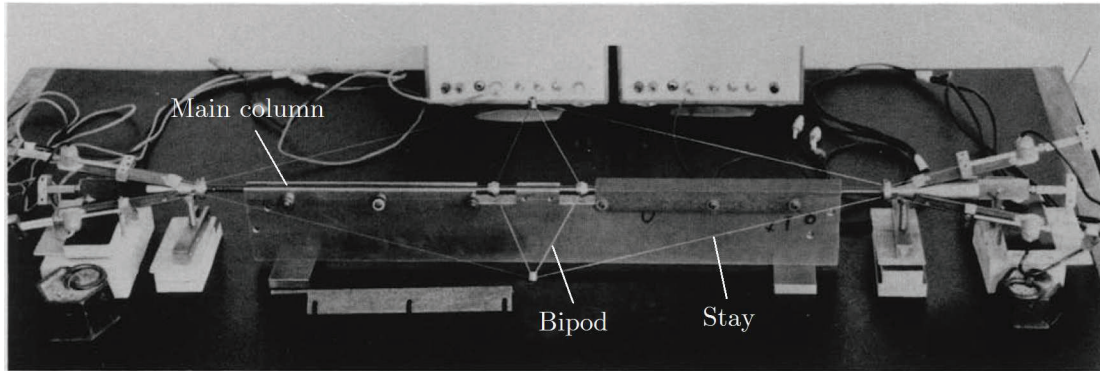


Figure 2.16: Experiments on prestressed stayed column with bipods in Jemah & Williams (1990).

load-carrying capacity) would increase by 20%.

De Araujo *et al.* (2006, 2008) conducted a series of full-scale experiments on 12 m single-bay prestressed stayed columns with 90 mm diameter stays, and the parametric studies on stay diameters, crossarm lengths, prestressing levels and imperfections were conducted using FE modelling. The results showed that the stays would increase the load-carrying capacity significantly. They also found that only investigating elastic behaviour was sufficient to analyse the ultimate behaviour of the stayed column owing to the high slenderness.

Osofero *et al.* (2012) conducted 18 full-scale experiments to investigate Mode 1 (symmetric), Mode 2 (anti-symmetric) and interactive buckling of the single-bay stayed column, as shown in Figure 2.17. The experimental results validated that when the Mode 2 buckling was critical and the buckling loads of Mode 1 and Mode 2 were numerically close, interactive buckling would be triggered (Wadee *et al.*, 2013a; Saito & Wadee, 2009b). This implied that the design method, which adopted the Mode 2 buckling load as the load-carrying capacity when the Mode 2 was critical, was unsafe. The numerical study using FE modelling following these experiments was presented in Osofero *et al.* (2013) and was used to develop the design method in Wadee *et al.* (2013b).

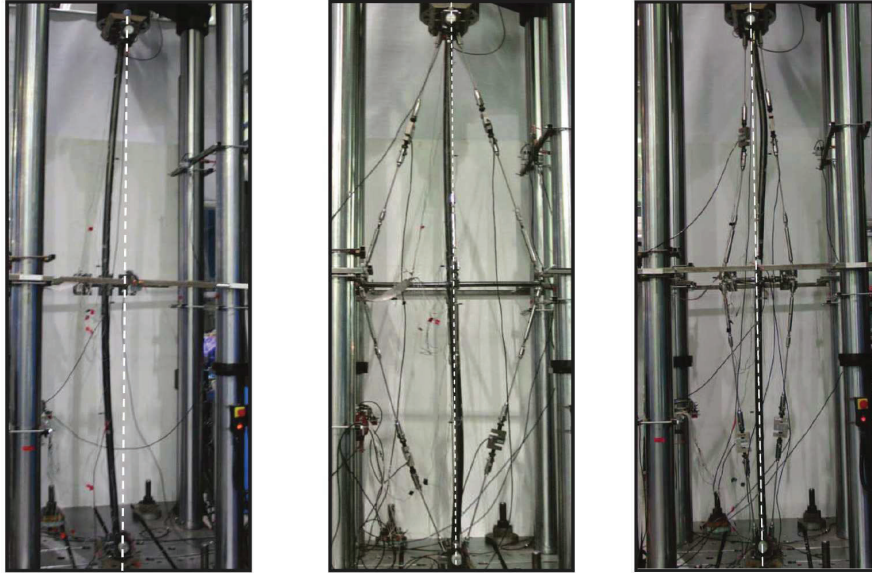


Figure 2.17: Buckling profiles of Mode 1, Mode 2 and interactive buckling from the experiments in Osofero *et al.* (2012).

Serra *et al.* (2015) also conducted full-scale tests on 12 m single-bay prestressed stayed columns. Apart from the aforementioned parameters, column cross-section areas and steel grades were adopted as the varying parameters, and high strength steel was included (Raoul & Günther, 2005). They found that the load-carrying capacity of the stayed column was strongly linked to the column cross-section area, but the high strength steel did not necessarily improve the performance of this system considerably owing to the high slenderness as mentioned in De Araujo *et al.* (2008). This was because the buckling load was still well below the material capacity even with the considerable increase due to the system of crossarms and stays.

2.6 Concluding remarks

This chapter summarized the existing literature on the buckling and post-buckling behaviour of the prestressed stayed column. The buckling load, interactive buckling, imperfection sensitivity and true optimum prestressing force have been investigated using

analytical as well as FE modelling and physical experiments. However, all of the aforementioned research work is conducted for vertical members where the directions of the self-weight and the axial load are collinear. Since there is a potential demand for very long struts to be used in the horizontal plane as shown in Figure 1.2, the present study is to investigate stayed beam-columns where the axially compressive load is in combination with self-weight. Therefore, in the subsequent chapters, the buckling and post-buckling behaviour of the single-bay prestressed stayed beam-column will be investigated in detail using both analytical and FE modelling techniques.

Chapter 3

Zones of behaviour

3.1 Introduction

From Hafez *et al.* (1979), a clear relationship between the initially applied prestressing force and the critical buckling load is established. More recently, the zones of behaviour of single-crossarm and multi-crossarm stayed columns have been investigated further adopting the linear eigenvalues from FE modelling (Saito & Wadee, 2009b; Yu & Wadee, 2017a; Lapira *et al.*, 2017). The aforementioned research is conducted for vertical doubly-symmetric members, where the direction of the self-weight and the axial load are collinear. However, as has been discussed, there is a potential demand for the mono-symmetric stayed beam-columns in horizontal plane. In the current chapter, mechanical modelling is employed to determine the relationships between the prestressing forces, the applied axial load and the residual forces within the main beam-column component and the stays. Three principal zones of behaviour are identified with two of these each having two sub-zones that relate the critical buckling load to the initial prestressing force applied to the stay cables. Moreover, the ultimate loads of the studied members, evaluated by conducting nonlinear FE analysis, are compared with their critical buckling loads obtained from the currently presented

analytical model.

3.2 General definitions and modelling

The numbering of the stays and the geometric parameters of the currently considered prestressed stayed beam-column are shown in Figure 3.1. The upper and lower crossarm

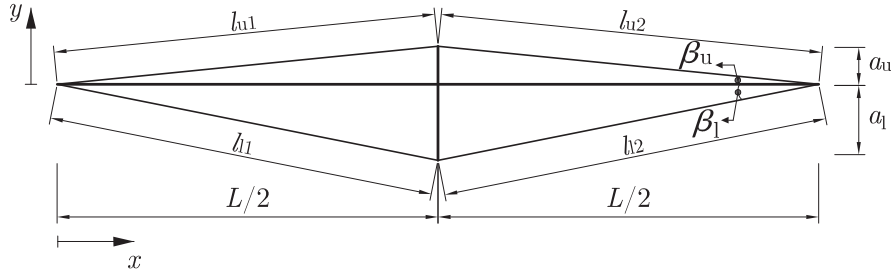


Figure 3.1: Stay numbering and geometric parameters of the prestressed stayed beam-column: L is the total length of the main beam-column element; a_u and a_l are the lengths of the upper and lower crossarms respectively; β_u and β_l are the angles between the main member with the upper and lower stays respectively; l_{tn} represents the original length of the stay tn , where $t = \{u, l\}$ corresponding to the upper and lower stays, and $n = \{1, 2\}$. Currently, $l_u = l_{u1} = l_{u2}$ and $l_l = l_{l1} = l_{l2}$.

ratios are defined respectively as follows:

$$\alpha_u = \frac{a_u}{L}, \quad \alpha_l = \frac{a_l}{L}, \quad (3.1)$$

and the crossarm length ratio α is introduced thus:

$$\alpha = \frac{\alpha_u}{\alpha_l} = \frac{a_u}{a_l}. \quad (3.2)$$

Based on the geometry shown in Figure 3.1, the length of each stay can be expressed thus:

$$\begin{aligned} l_u &= l_{u1} = l_{u2} = L\sqrt{\alpha_u^2 + 1/4}, \\ l_l &= l_{l1} = l_{l2} = L\sqrt{\alpha_l^2 + 1/4}. \end{aligned} \quad (3.3)$$

The main member element has a Young's modulus E_c , cross-sectional area A_c and second moment of area I_c about its axis of bending. Both upper and lower crossarms are assumed throughout the current work to have the same Young's modulus, cross-sectional area and second moment of area E_a , A_a and I_a respectively. Moreover, both upper and lower stays are also assumed throughout the current work to have the same Young's modulus and cross-sectional area E_s and A_s respectively. The axial stiffnesses of the main member $K_{c,a}$, upper crossarm $K_{au,a}$, lower crossarm $K_{al,a}$, each upper stay K_{su} and each lower stay K_{sl} are respectively defined as follows:

$$K_{c,a} = \frac{E_c A_c}{L}, \quad K_{au,a} = \frac{E_a A_a}{a_u}, \quad K_{al,a} = \frac{E_a A_a}{a_l}, \quad K_{su} = \frac{E_s A_s}{l_u}, \quad K_{sl} = \frac{E_s A_s}{l_l}. \quad (3.4)$$

In addition, the radius of gyration for the main member is defined thus:

$$r_c = \sqrt{\frac{I_c}{A_c}}. \quad (3.5)$$

As in previous work (Hafez *et al.*, 1979; Wadee *et al.*, 2013b; Osofero *et al.*, 2012), the connections between the crossarms and the main member element are assumed to be rigid, whereas the connections between the stay cables and the main member element and the crossarms are assumed to be pinned.

It should be noted that although the current work considers essentially a two-dimensional (2D) system, in reality, it is a three-dimensional (3D) system, the beam-column must be supported by three or more crossarm elements to be stable in the third dimension. An alternative strategy is to introduce lateral braces between two adjacent props to decrease the lateral effective length, to avoid buckling out of plane, and the problem would essentially return to 2D and the current model would be valid and circumvent the need to include imperfections out of the plane.

3.2.1 Loading steps and analytical model assumptions

The prestressed stayed beam-column studied herein is under bending, through the action of self-weight, in conjunction with axial compression that amplifies the bending displacement. This is naturally quite different from the previously studied prestressed stayed columns under purely axial compression (Hafez *et al.*, 1979; Saito & Wadee, 2008; Osofero *et al.*, 2012; Yu & Wadee, 2017a). Hence, it is necessary to define the adopted loading sequence, as given below and represented in Figure 3.2:

Step 1 With no pre-tension applied to all the stays, owing to its self-weight, the member assumes a curved profile similar to a plain beam.

Step 2 A specific prestressing force is introduced within the lower stays to counteract the self-weight effect and the midspan vertical displacement becomes zero relative to the supports.

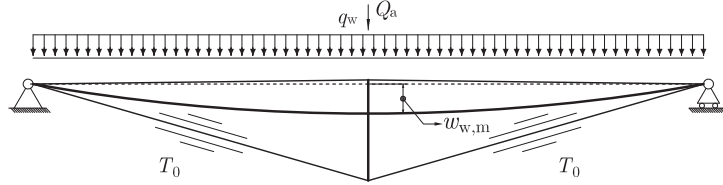
Step 3 Additional prestressing forces are applied within all the stays to improve the buckling performance of the system.

Step 4 Finally, an axially compressive load is applied until the system fails, usually by a combination of bending and buckling.

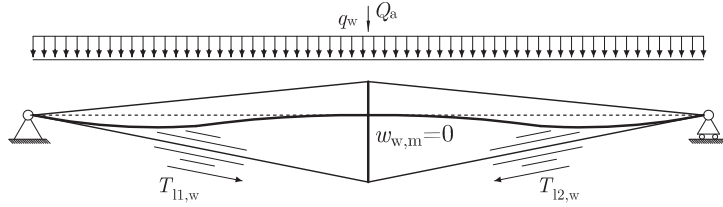
For modelling purposes, the following assumptions are made:

1. All the deflections are relatively small, although the effects of nonlinearity from combining bending and axial compression are included.
2. All materials are linearly elastic since the geometries are very slender and hence the strains are generally well below the yield strain.
3. The self-weight of all the stays is currently neglected, since the unit-length self-weight of the stays is less than 10% of the main member in most models.

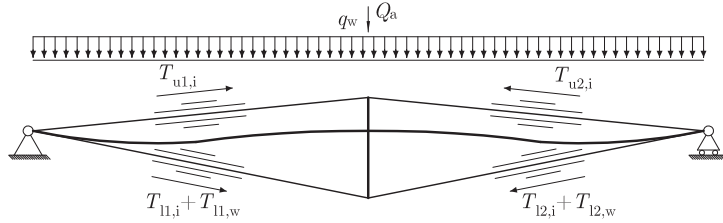
Loading step 1: Applying crossarm and main member self-weight Q_a and q_w



Loading step 2: Prestressing to counteract self-weight; increasing tension in the lower stays to $T_{l,w}$, $T_{l2,w}$ such that $w_{w,m}=0$



Loading step 3: Prestressing to improve buckling performance; increasing tension in all stays simultaneously to $T_{u1,i}$, $T_{u2,i}$, $T_{l1,i}$, $T_{l2,i}$ such that the axial forces within the crossarms are equal (to leading order)



Loading step 4: Applying compressive axial load P until system buckles

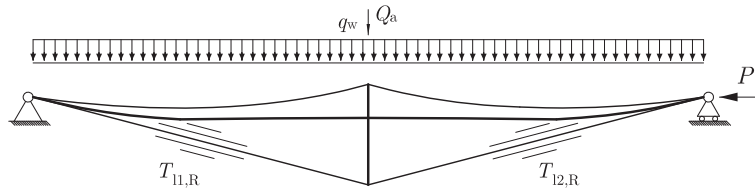


Figure 3.2: Loading steps for the prestressed stayed beam-column. In steps 1–3, all stays are assumed to remain straight; T_0 is the force within the lower stays in step 1; $w_{w,m}$ is the mid-span vertical displacement after applying self-weight and $T_{l,w} = T_{l1,w} = T_{l2,w}$ are the prestressing forces within the lower stays to counteract the midspan self-weight deflection; $T_{u,i} = T_{u1,i} = T_{u2,i}$ and $T_{l,i} = T_{l1,i} = T_{l2,i}$ are the prestressing forces applied within the upper and lower stays respectively to improve buckling performance; P is the compressive axial load; $T_{l,R} = T_{l1,R} = T_{l2,R}$ are the residual tensions within the lower stays when the system buckles.

4. When the self-weight is introduced in loading step 1, all the stays are assumed to be straight but without stress. Similarly, when applying the prestress within the lower stays to counteract the self-weight in loading step 2, the upper stays are straight but are assumed to have zero stress.
5. The main member is assumed to be perfectly horizontal when deducing the optimum prestressing forces and zones of behaviour. This is because after applying the self-weight and the prestressing force within the lower stays, the midspan vertical displacement of the main member returns to zero and the maximum vertical displacement of the main member remote from midspan is small.
6. When deducing the analytical expressions for the optimum prestressing forces and the boundaries demarcating the zones of behaviour, the prestressing force within the lower stays that counteracts the self-weight always exists. However, this force, alongside the self-weight, is no longer taken into account explicitly owing to the effect of them counteracting one another.

3.2.2 Finite element model

In the current study, finite element (FE) modelling is used to verify the theoretical results obtained. Following previous numerical studies of prestressed stayed columns using the commercial software ABAQUS, the Euler–Bernoulli beam element B23 is adopted to model the main horizontal member as well as the crossarms (Saito & Wadee, 2009b; Osofero *et al.*, 2013; Yu & Wadee, 2017a). Since all the stays are pinned to the ends of the main member and crossarms, these are modelled with the truss element T2D2 in conjunction with the ‘no compression’ option. The main member and crossarms are meshed as beam elements each with a length of 50 mm, and each stay is meshed as an individual truss element. Since the connection between the main member and crossarm is welded, it is assumed to be a fixed joint. Since the connections between member tips and stays can rotate without

constraints in the plane, they are assumed to be pinned. Buckling analysis is conducted to determine the linear eigenvalue and eigenmode profile. The obtained eigenvalue is applied to calculate $P_{FE,T=0}^C$ in Eq. (3.47), evaluate the linear optimum prestressing force, and the eigenmode profile is used to apply the imperfection profiles in nonlinear analysis. In Figure 3.2, General Static analysis based on the Newton–Raphson method is conducted in steps 1–3 to apply self-weight and prestressing forces, and then Riks analysis (Riks, 1979) is conducted in step 4 to obtain the nonlinear post-buckling behaviour.

3.3 Analytical modelling

The principal purpose of the current analytical work is to investigate the structural response in each of the loading steps described in Figure 3.2 based on the aforementioned assumptions by applying mechanical principles. Subsequently, the analytical expressions of the so-called optimum prestressing forces and how these demarcate the behaviour into separate zones are deduced.

3.3.1 Counteracting self-weight

The applied prestressing force within the lower stays at the beginning of loading step 2 to counteract the self-weight $T_{l,w-a}$, becomes $T_{l,w}$ at the end of loading step 2 owing to the loss in prestressing from the end-shortening displacement of the main element. In order to determine the expression for $T_{l,w-a}$, the structural response in loading steps 1 and 2 (see Figure 3.2) is studied.

In loading step 1, the stays and crossarms do not affect the structural response of the main member from the aforementioned assumptions. Hence, the main member can be considered to be a simply-supported beam under a vertical load with a distributed intensity q_w from the main member self-weight, alongside a midspan concentrated load Q_a from the crossarm

self-weight. Owing to linear theory, the effect of these two types of loads can be considered simply as their linear superposition, as shown in Figure 3.3.

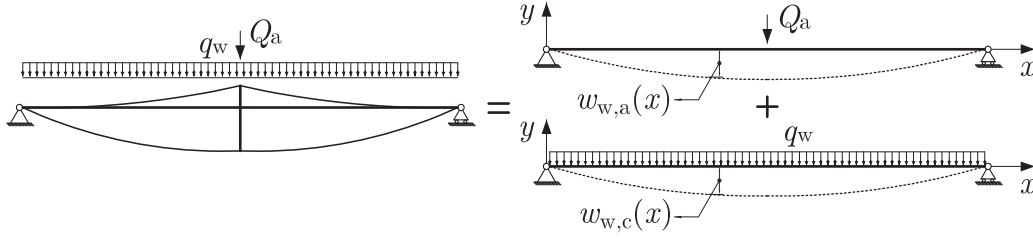


Figure 3.3: Simplifying the effect from the loads in loading step 1: $w_{w,a}(x)$ is the deflection function of a simply-supported beam under a midspan concentrated vertical load Q_a representing the self-weight of the crossarms; $w_{w,c}(x)$ is the deflection function of a simply-supported beam under a distributed vertical load q_w representing the self-weight from the main beam-column member.

Based on Euler–Bernoulli bending theory, $w_{w,a}(x)$ and $w_{w,c}(x)$ can be expressed thus, where $x = [0, L/2]$:

$$\begin{aligned} w_{w,a}(x) &= \frac{Q_a L^2 x}{48 E_c I_c} \left(3 - \frac{4x^2}{L^2} \right), \\ w_{w,c}(x) &= \frac{q_w L^3 x}{24 E_c I_c} \left(1 - \frac{2x^2}{L^2} + \frac{x^3}{L^3} \right). \end{aligned} \quad (3.6)$$

The total deflection function of the main member $w_w(x)$ is the superposition of $w_{w,a}(x)$ and $w_{w,c}(x)$, thus:

$$w_w(x) = w_{w,a}(x) + w_{w,c}(x). \quad (3.7)$$

The deformation of the system in loading step 1 is illustrated in Figure 3.4 and includes the end-shortening displacement $\Delta_{w,c}$, arising from w_w , a leading order approximation of which is given by the following expression (Thompson & Hunt, 1973):

$$\Delta_{w,c} = \frac{1}{2} \int_0^L w_w'^2(x) dx. \quad (3.8)$$

Hence, the midspan displacement $w_{w,m}$ can be expressed thus:

$$w_{w,m} = w_w(L/2) = \frac{Q_a L^3}{48 E_c I_c} + \frac{5 q_w L^4}{384 E_c I_c}. \quad (3.9)$$

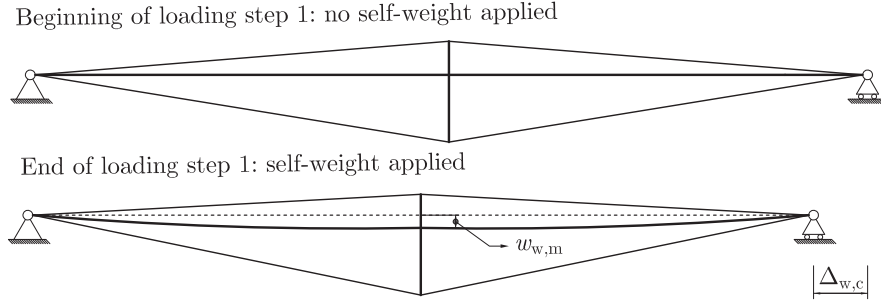


Figure 3.4: Deformation of the system under self-weight in loading step 1: $\Delta_{w,c}$ is the end-displacement and $w_{w,m}$ is the midspan vertical displacement of the main member.

If all the stays are straight but without stress, the length of stay n at the end of loading step 1, $l_{w,n}$, would be as follows:

$$\begin{aligned} l_{w,u} = l_{w,u1} = l_{w,u2} &= \sqrt{(a_u - w_{w,m})^2 + \left(\frac{L}{2} - \frac{\Delta_{w,c}}{2}\right)^2}, \\ l_{w,l} = l_{w,l1} = l_{w,l2} &= \sqrt{(a_l + w_{w,m})^2 + \left(\frac{L}{2} - \frac{\Delta_{w,c}}{2}\right)^2}. \end{aligned} \quad (3.10)$$

Equilibrium analysis at the end of loading step 2 is shown in Figure 3.5. The right-hand half of the main member is equivalent to a beam under a distributed load q with the left-hand support being clamped and the right-hand support being pinned with a roller allowing motion in x . Moreover, the member has reflective symmetry about the midspan of the main member. The vertical reaction force R_w is determined thus:

$$R_w = \frac{1}{2}(q_w L + Q_a). \quad (3.11)$$

The static equilibrium equations for the left-hand support are as follows:

$$\begin{aligned} R_w - T_{l,w} \sin \beta_1 - \frac{3}{16} q_w L &= 0, \\ T_{l,w} \cos \beta_1 - N_{ax} &= 0. \end{aligned} \quad (3.12)$$

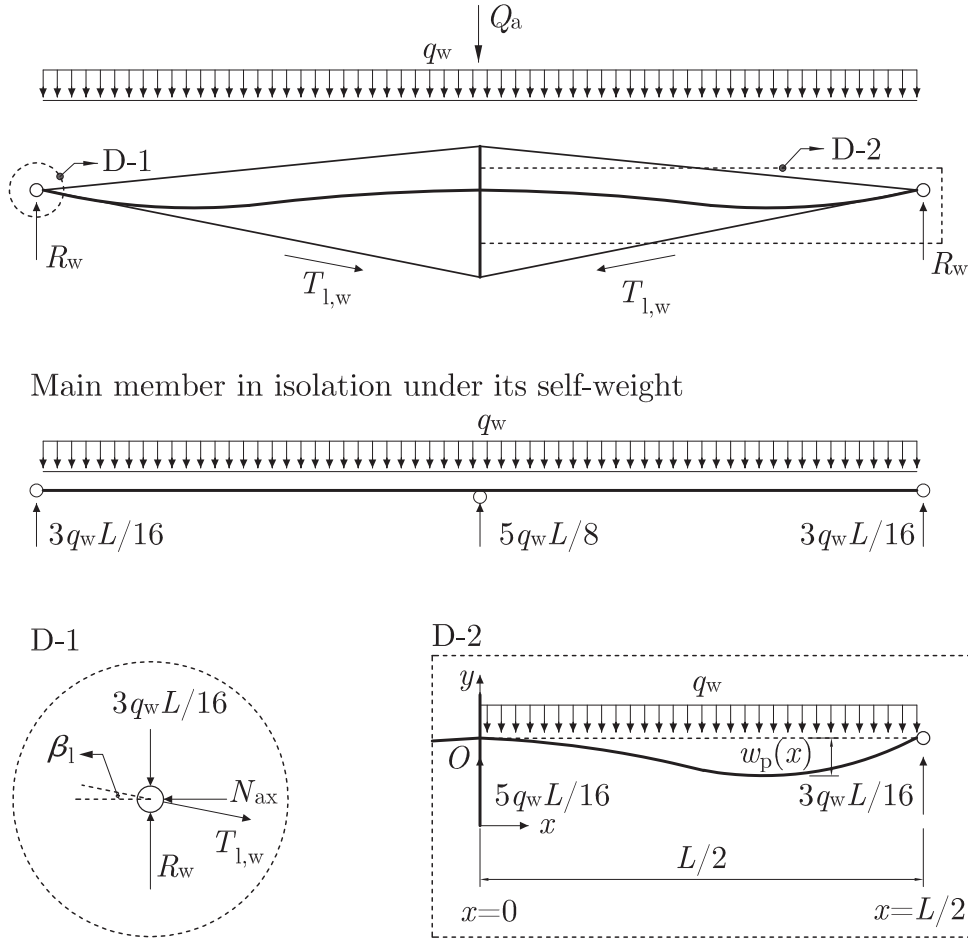


Figure 3.5: System at the end of loading step 2: the end reactions of main member are derived from a two-span continuous beam under a uniformly distributed load of intensity q_w ; ‘D-1’ shows the free-body diagram of the left-hand pinned support; ‘D-2’ details the simplified beam of the right-hand half of the main member; R_w is the vertical reaction force from the self-weight; N_{ax} is the axial force within the main member; $w_p(x)$ is the simplified beam deflection function for the right-hand half of the main member.

Substituting Eq. (3.11) into Eq. (3.12) and solving Eq. (3.12) gives:

$$N_{\text{ax}} = \frac{(5q_w L/8 + Q_a)}{2 \tan \beta_l}, \quad T_{l,w} = T_{l1,w} = T_{l2,w} = \frac{(5q_w L/8 + Q_a)}{2 \sin \beta_l}. \quad (3.13)$$

The deflection function of the simplified beam for the right-hand half of the main member, $w_p(x)$, can be again determined from Euler–Bernoulli bending theory thus:

$$w_p(x) = \frac{q_w L^2 x^2}{96 E_c I_c} \left(\frac{4x^2}{L^2} - \frac{5x}{L} + \frac{3}{2} \right). \quad (3.14)$$

The maximum vertical deflection $w_{w,c,\text{max}}$ is at $x = 0.290L$, which is:

$$w_{w,c,\text{max}} = \frac{q_w L^4}{2960 E_c I_c}. \quad (3.15)$$

In addition, the deformation after loading step 2 is illustrated in Figure 3.6. The dis-

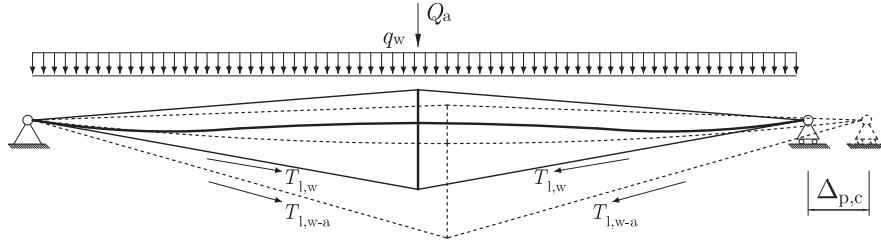


Figure 3.6: Deformation after loading step 2: $\Delta_{p,c}$ is the end-displacement of the main member.

placement $\Delta_{p,c}$ is given similarly to Eq. (3.8) but now including the axially compressive displacement, thus:

$$\Delta_{p,c} = \frac{N_{\text{ax}}}{K_{c,a}} + \frac{1}{2} \int_0^L w_p'^2(x) \, dx. \quad (3.16)$$

The axially compressive displacement of the lower crossarm $\Delta_{p,al}$ in loading step 2 can be determined from Hooke's law in one-dimension thus:

$$\Delta_{p,al} = \frac{2T_{l,w} \sin \beta_l}{K_{al,a}}. \quad (3.17)$$

If all the stays are straight but without stress from the aforementioned assumptions, the length of each stay would be as follows at the end of loading step 2:

$$\begin{aligned} l_{p,u} = l_{p,u1} = l_{p,u2} &= \sqrt{a_u^2 + \left(\frac{L}{2} - \frac{1}{2} \Delta_{p,c} \right)^2}, \\ l_{p,l} = l_{p,l1} = l_{p,l2} &= \sqrt{(a_l - \Delta_{p,a1})^2 + \left(\frac{L}{2} - \frac{1}{2} \Delta_{p,c} \right)^2}. \end{aligned} \quad (3.18)$$

Hence, the required lower stay prestressing force $T_{l,w-a}$ to achieve $T_{l,w}$ at the beginning of loading step 2, compensating for the end-shortening of the main member, can be expressed thus:

$$T_{l,w-a} = T_{l,w} + \frac{E_s A_s (l_{w,l} - l_{p,l})}{l_{w,l}}, \quad (3.19)$$

where $T_{l,w}$ was given in Eq. (3.13).

3.3.2 Linear optimum prestressing forces

In the previous sections, the prestressing forces within the lower stays that counteract the self-weight were investigated. However, to improve the buckling behaviour of the stayed beam-columns, additional prestressing forces would be required as in the case of conventional prestressed stayed columns. The processes for applying these additional prestressing forces and then the axially compressive force are in loading steps 3 and 4 respectively (see Figure 3.2). In the current section, the appropriate levels of the final additional prestressing forces within the upper and lower stays $T_{u,i}$ and $T_{l,i}$ in loading step 3, and the buckling behaviour in loading step 4 corresponding to specific prestressing levels are investigated, which leads to the solutions for the so-called linear optimum prestressing forces.

From the aforementioned assumptions, the respective prestressing forces within the upper

and lower stays T_u and T_l at the end of loading step 3 can be obtained thus:

$$T_u = T_{u,i}, \quad T_l = T_{l,i} + T_{l,w}, \quad (3.20)$$

where $T_{u,i}$ and $T_{l,i}$ are the additional prestressing forces within the upper and lower stays respectively to improve the buckling behaviour at the end of loading step 3. It should be noted that the following relationship needs to be satisfied to ensure that the internal axial forces within the upper and lower crossarms are equal to leading order (since this equation does not take the induced end-displacement into consideration):

$$T_{u,i} \sin \beta_u = T_{l,i} \sin \beta_l. \quad (3.21)$$

In order to reach a specific prestressing level accurately at the end of loading step 3, the loss of prestressing owing to the end-displacement of members during loading step 3 is taken into consideration, which is different from previous studies (Hafez *et al.*, 1979; Saito & Wadee, 2008). Hence, prestressing forces $T_{u,i-a}$ and $T_{l,i-a}$, are applied to the stays at the beginning of loading step 3 and they become $T_{u,i}$ and $T_{l,i}$ respectively at the end of loading step 3, which is illustrated in Figure 3.7. The forces $T_{u,i-a}$ and $T_{l,i-a}$ can be obtained in a

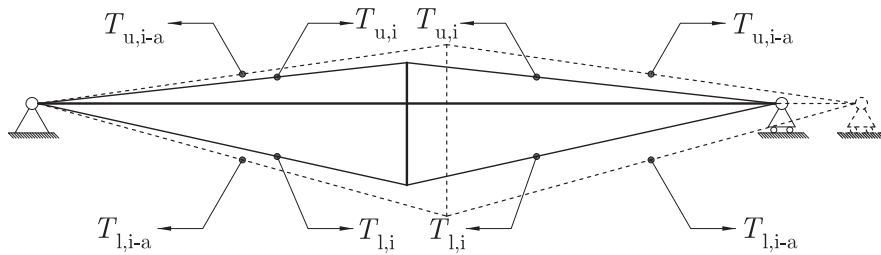


Figure 3.7: Initial and final prestressing forces ($T_{x,i-a}$, $T_{x,i}$) within loading step 3, where subscript ‘x’ represents either the upper (‘u’) or lower stays (‘l’).

similar way to Eq. (3.19) and expressed respectively thus:

$$\begin{aligned} T_{u,i-a} &= T_{u,i} + \left[\frac{(T_{u,i} \cos \beta_u + T_{l,i} \cos \beta_l) \cos \beta_u}{2K_{c,a}} + \frac{2T_{u,i} \sin \beta_u}{K_{au,a}} \right] \frac{E_s A_s}{l_{p,1}}, \\ T_{l,i-a} &= T_{l,i} + \left[\frac{(T_{u,i} \cos \beta_u + T_{l,i} \cos \beta_l) \cos \beta_l}{2K_{c,a}} + \frac{2T_{l,i} \sin \beta_l}{K_{al,a}} \right] \frac{E_s A_s}{l_{p,3}}. \end{aligned} \quad (3.22)$$

To formulate a revised general expression for the optimum prestressing forces similar to previous studies (Hafez *et al.*, 1979; Saito & Wadee, 2008), but that can be applied to both $\alpha = 1.0$ (*i.e.* equal crossarms) and $\alpha < 1.0$ (*i.e.* the upper crossarm being shorter than the lower crossarm) cases, the structural response in loading step 4 needs to be evaluated. The definitions of the forces in loading step 4 are illustrated in Figure 3.8, and the geometric

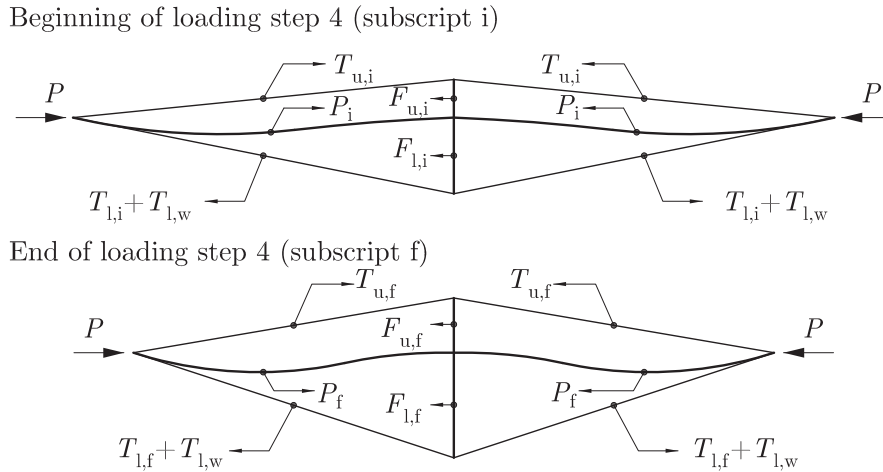


Figure 3.8: Axial force definitions within loading step 4: P is the externally-applied axial load; P_i and P_f are the internal axial forces within the main member; $F_{u,i}$ and $F_{u,f}$ are the internal axial forces within the upper crossarm; $F_{l,i}$ and $F_{l,f}$ are the internal axial forces within the lower crossarm; $T_{u,i}$ and $T_{u,f}$ are the applied prestressing forces within the upper stays; $T_{l,i}$ and $T_{l,f}$ are the prestressing forces within the lower stays applied in addition to $T_{l,w}$.

details for the deformation analysis in that loading step is shown in Figure 3.9.

From Figures 3.8–3.9, the process of deducing the optimum prestressing forces is now presented. The extensions of the upper stays Δ_{su} and the lower stays Δ_{sl} are related to

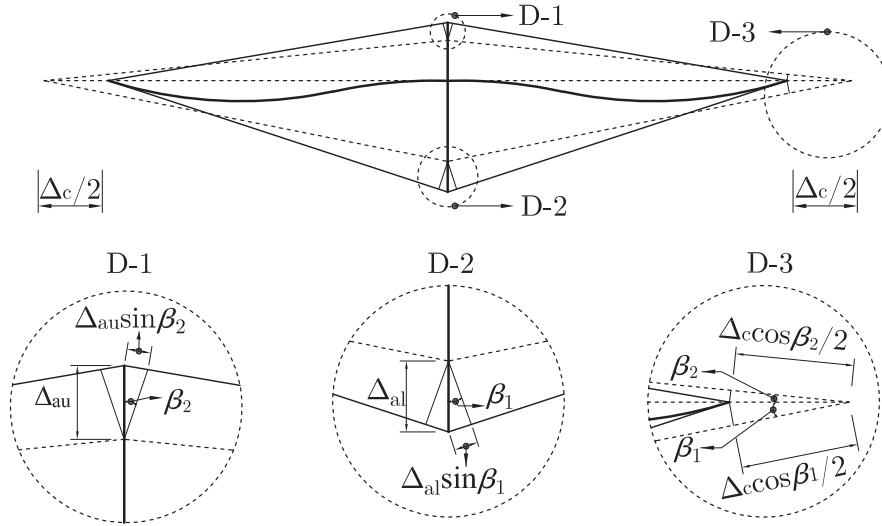


Figure 3.9: Geometric deflection analysis of prestressed stayed beam-columns in loading step 4: ‘D-1’ is the detailed figure for the deformation at the top of the upper crossarm; ‘D-2’ shows the bottom of the lower crossarm, and ‘D-3’ shows the right-hand pinned support of the main member; Δ_c is the total end-shortening displacement of the main member; Δ_{au} and Δ_{al} are the end-shortening displacements of the upper and lower crossarms respectively.

the change in the length of the main member and crossarms thus:

$$\Delta_{su} = \frac{\Delta_c \cos \beta_u}{2} - \Delta_{au} \sin \beta_u, \quad \Delta_{sl} = \frac{\Delta_c \cos \beta_l}{2} - \Delta_{al} \sin \beta_l. \quad (3.23)$$

The initial axial force in the main member P_i , induced by the initial prestressing forces within the stays, is:

$$P_i = T_{u,i} \cos \beta_u + T_{l,i} \cos \beta_l, \quad (3.24)$$

with the crossarm axial forces $F_{u,i}$ and $F_{l,i}$ being expressed respectively thus:

$$F_{u,i} = 2T_{u,i} \sin \beta_u, \quad F_{l,i} = 2T_{l,i} \sin \beta_l. \quad (3.25)$$

After applying the external axial load P , the final axial force in the main member P_f is:

$$P_f = T_{u,f} \cos \beta_u + T_{l,f} \cos \beta_l + P, \quad (3.26)$$

with the forces $F_{u,f}$ and $F_{l,f}$ being expressed respectively thus:

$$F_{u,f} = 2T_{u,f} \sin \beta_u, \quad F_{l,f} = 2T_{l,f} \sin \beta_l. \quad (3.27)$$

The main member axial displacement Δ_c can be expressed thus:

$$\Delta_c = \frac{P_f - P_i}{K_{c,a}}. \quad (3.28)$$

Substituting Eqs. (3.24) and (3.26) into Eq. (3.28) gives the following:

$$\Delta_c = \frac{P + (T_{u,f} - T_{u,i}) \cos \beta_u + (T_{l,f} - T_{l,i}) \cos \beta_l}{K_{c,a}}, \quad (3.29)$$

the upper and lower crossarm end-shortening displacements Δ_{au} and Δ_{al} can be expressed thus:

$$\Delta_{au} = \frac{F_{u,i} - F_{u,f}}{K_{au,a}}, \quad \Delta_{al} = \frac{F_{l,i} - F_{l,f}}{K_{al,a}}. \quad (3.30)$$

Substituting Eqs. (3.25) and (3.27) into Eq. (3.30) give the following:

$$\Delta_{au} = \frac{2(T_{u,i} - T_{u,f}) \sin \beta_u}{K_{au,a}}, \quad \Delta_{al} = \frac{2(T_{l,i} - T_{l,f}) \sin \beta_l}{K_{al,a}}. \quad (3.31)$$

The displacement of the upper stays Δ_{su} and the lower stays Δ_{sl} can be expressed thus:

$$\Delta_{su} = \frac{T_{u,i} - T_{u,f}}{K_{su}}, \quad \Delta_{sl} = \frac{T_{l,i} - T_{l,f}}{K_{sl}}, \quad (3.32)$$

where the stiffnesses of the upper and lower stays are $K_{su} = E_s A_s / l_u$ and $K_{sl} = E_s A_s / l_l$ respectively, with $l_u = l_{u1} = l_{u2}$ and $l_l = l_{l1} = l_{l2}$. Substituting Eqs. (3.29), (3.31) and (3.32) into Eq. (3.23) gives the following equations:

$$\begin{aligned} \frac{T_{u,i} - T_{u,f}}{K_{su}} = & \left[\frac{P + (T_{u,f} - T_{u,i}) \cos \beta_u + (T_{l,f} - T_{l,i}) \cos \beta_l}{2K_{c,a}} \right] \cos \beta_u \\ & - \frac{2(T_{u,i} - T_{u,f}) \sin^2 \beta_u}{K_{au,a}}, \end{aligned} \quad (3.33)$$

$$\frac{T_{l,i} - T_{l,f}}{K_{sl}} = \left[\frac{P + (T_{u,f} - T_{u,i}) \cos \beta_u + (T_{l,f} - T_{l,i}) \cos \beta_l}{2K_{c,a}} \right] \cos \beta_l - \frac{2(T_{l,i} - T_{l,f}) \sin^2 \beta_l}{K_{al,a}}. \quad (3.34)$$

Solving Eqs. (3.33) and (3.34) gives the following equations:

$$\frac{T_{u,i} - T_{u,f}}{P} = \frac{C_1}{C_3}, \quad \frac{T_{l,i} - T_{l,f}}{P} = \frac{C_2}{C_3}, \quad (3.35)$$

where:

$$\begin{aligned} C_1 &= K_{au,a} K_{su} (2K_{sl} \sin^2 \beta_l + K_{al,a}) \cos \beta_u, \\ C_2 &= K_{al,a} K_{sl} (2K_{su} \sin^2 \beta_u + K_{au,a}) \cos \beta_l, \\ C_3 &= 8K_{c,a} K_{sl} K_{su} \sin^2 \beta_l \sin^2 \beta_u + 2K_{al,a} K_{sl} K_{su} \cos^2 \beta_l \sin^2 \beta_u \\ &\quad + 2K_{au,a} K_{sl} K_{su} \cos^2 \beta_u \sin^2 \beta_l + 4K_{c,a} K_{al,a} K_{su} \sin^2 \beta_u + 4K_{c,a} K_{au,a} K_{sl} \sin^2 \beta_l \\ &\quad + K_{al,a} K_{au,a} K_{sl} \cos^2 \beta_l + K_{al,a} K_{au,a} K_{su} \cos^2 \beta_u + 2K_{c,a} K_{al,a} K_{au,a}. \end{aligned} \quad (3.36)$$

Defining new quantities $C_u = C_1/C_3$ and $C_l = C_2/C_3$, allow the following simplifications:

$$\frac{T_{u,i} - T_{u,f}}{P} = C_u, \quad \frac{T_{l,i} - T_{l,f}}{P} = C_l. \quad (3.37)$$

Combining Eqs. (3.26) and (3.29) gives:

$$\Delta_c = \frac{P_f - T_{u,i} \cos \beta_u - T_{l,i} \cos \beta_l}{K_{c,a}}. \quad (3.38)$$

Substituting Eqs. (3.31), (3.32) and (3.38) into Eq. (3.23) gives:

$$\frac{T_{l,i} - T_{l,f}}{K_{sl}} = \left(\frac{P_f - T_{u,i} \cos \beta_u - T_{l,i} \cos \beta_l}{2K_{c,a}} \right) \cos \beta_l - \frac{2(T_{l,i} - T_{l,f}) \sin^2 \beta_l}{K_{al,a}}, \quad (3.39)$$

hence:

$$T_{l,f} = - \frac{K_{sl} K_{al,a} [P_f - T_{u,i} \cos \beta_u - T_{l,i} \cos \beta_l] \cos \beta_l}{2K_{c,a} (2K_{sl} \sin^2 \beta_l + K_{al,a})} + T_{l,i}. \quad (3.40)$$

Substituting Eqs. (3.31), (3.32) and (3.38) into Eq. (3.23) gives:

$$\frac{T_{u,i} - T_{u,f}}{K_{su}} = \left(\frac{P_f - T_{u,i} \cos \beta_u - T_{l,i} \cos \beta_l}{2K_{c,a}} \right) \cos \beta_u - \frac{2(T_{u,i} - T_{u,f}) \sin^2 \beta_u}{K_{au,a}}, \quad (3.41)$$

hence:

$$T_{u,f} = T_{u,i} - \frac{K_{su}K_{au,a} (P_f - T_{u,i} \cos \beta_u - T_{l,i} \cos \beta_l) \cos \beta_u}{2K_{c,a} (2K_{su} \sin^2 \beta_u + K_{au,a})}. \quad (3.42)$$

Substituting Eq. (3.40), Eq. (3.42) into Eq. (3.26) after some manipulation gives:

$$\begin{aligned} P = P_f - & \left[T_{l,i} - \frac{K_{sl}K_{al,a} (P_f - T_{u,i} \cos \beta_u - T_{l,i} \cos \beta_l) \cos \beta_l}{2K_{c,a} (2K_{sl} \sin^2 \beta_l + K_{al,a})} \right] \cos \beta_l \\ & - \left[T_{u,i} - \frac{K_{su}K_{au,a} (P_f - T_{u,i} \cos \beta_u - T_{l,i} \cos \beta_l) \cos \beta_u}{2K_{c,a} (2K_{su} \sin^2 \beta_u + K_{au,a})} \right] \cos \beta_u. \end{aligned} \quad (3.43)$$

Defining a new quantity C_0 , thus:

$$C_0 = 1 + \frac{1}{2K_{c,a}} \left(\frac{K_{su}K_{au,a} \cos^2 \beta_u}{2K_{su} \sin^2 \beta_u + K_{au,a}} + \frac{K_{sl}K_{al,a} \cos^2 \beta_l}{2K_{sl} \sin^2 \beta_l + K_{al,a}} \right), \quad (3.44)$$

and substituting Eq. (3.44) into Eq. (3.43), the following expression is obtained:

$$P = (P_f - T_{u,i} \cos \beta_u - T_{l,i} \cos \beta_l) C_0. \quad (3.45)$$

From the relationships in Eq. (3.37), the optimum prestressing forces are determined based on the force within the lower stays being given by the condition $T_{l,f} = 0$, where the lower stay forces become $T_l = T_{l,w}$, or the upper stays become slack given by the condition $T_{u,f} = 0$ (since $T_u = 0$ also) when the system buckles, thus:

$$T_{opt1} = P_{\max}^C C_l, \quad T_{opt2} = P_{\max}^C C_u. \quad (3.46)$$

Hence, T_{opt1} is the optimum prestressing force based on the condition that $T_{l,f} = 0$ when the beam-column buckles; conversely, T_{opt2} is the optimum prestressing force based on the

condition that the upper stays become slack when the beam-column buckles. This leads to the evaluation of the maximum critical buckling load $P_{\max}^C$, thus:

$$P_{\max}^C = \frac{P_{\text{FE},T=0}^C}{C_0}, \quad (3.47)$$

where $P_{\text{FE},T=0}^C$ is evaluated using linear buckling analysis within FE models; in the current case this is obtained using the commercial code ABAQUS and is similar to the previously presented procedure (Saito & Wadee, 2009b).

From Eq. (3.21), when $T_{l,i} = T_{\text{opt}1}$, the optimum prestressing force within the upper stays is defined as $T_{\text{opt}12}$ and can be expressed thus:

$$T_{\text{opt}12} = \frac{T_{\text{opt}1} \sin \beta_l}{\sin \beta_u}. \quad (3.48)$$

Similarly when $T_{u,i} = T_{\text{opt}2}$, the optimum prestressing forces within the lower stays is defined as $T_{\text{opt}21}$ and can be expressed thus:

$$T_{\text{opt}21} = \frac{T_{\text{opt}2} \sin \beta_u}{\sin \beta_l}. \quad (3.49)$$

For the beam-column with double symmetry ($\alpha = 1.0$), $C_l = C_u$, and the expressions reduce to those in previous work (Hafez *et al.*, 1979; Saito & Wadee, 2008). When $\alpha < 1.0$, however, $C_l < C_u$ and the currently developed expressions need to be applied.

In addition, this solution can be adjusted to include the effect of support settlement and temperature changes in practice. In terms of the axially compressive force within the main member, it can be monitored and adjusted using an active force control device to maintain the required compressive load precisely, which can counteract the axial force change owing to these two effects (Nakata *et al.*, 2019); in terms of the stress within stays, the stress change from these two effects can be taken into account as part of an equivalent prestress,

thus:

$$T_{x,i} = T_{\text{equivalent}} + T_{\text{applied}}, \quad (3.50)$$

where $x = \{u, l\}$; $T_{\text{equivalent}}$ is the equivalent prestressing force accounting for effects of support settlement and temperature; T_{applied} is the externally applied prestressing force, and $T_{x,i}$ can be set to the desired level as described above.

3.3.3 Zonal behaviour in doubly-symmetric stayed beam-columns

Zones of behaviour for vertical stayed columns under purely axial compression have been identified in previous studies (Hafez *et al.*, 1979; Saito & Wadee, 2008), as illustrated in Figure 2.5. In terms of the specific horizontal beam-column cases where $\alpha = 1.0$, the primary conclusions are similar to that for vertical stayed columns. The minimum prestressing force $T_{\min}$, which marks the boundary between Zones 1 and 2 (*i.e.* the prestressing level that ensures at least two stays are active at the point of buckling), can be determined from the relationships in Eq. (3.37). This is based on the force within the lower stays being given by the condition $T_{l,f} = 0$ (where $T_l = T_{l,w}$) or the upper stays become slack $T_{u,f} = 0$ when the system buckles thus:

$$T_{\min} = P_E C_l = P_E C_u, \quad (3.51)$$

where P_E is the Euler buckling load for a simply-supported strut with the cross-sectional properties of the main column element:

$$P_E = \frac{\pi^2 E_c I_c}{L^2}. \quad (3.52)$$

The prestressing forces marking the boundary between Zones 2 and 3 (*i.e.* the prestressing level that ensures all stays are active at the point of buckling), T_{opt1} , T_{opt2} , T_{opt12} and T_{opt21} can be calculated from Eqs. (3.46), (3.48) and (3.49) respectively. For this case,

$T_{\text{opt}} = T_{\text{opt1}} = T_{\text{opt12}} = T_{\text{opt2}} = T_{\text{opt21}}$. In Zone 1, where $T_{\text{u,i}} = T_{\text{l,i}} = [0, T_{\text{min}}]$, the critical buckling load P_{Z1}^C is a constant, thus:

$$P_{Z1}^C = P_E. \quad (3.53)$$

In Zone 2, where $T_{\text{u,i}} = T_{\text{l,i}} = [T_{\text{min}}, T_{\text{opt}}]$, $T_{\text{u,f}} = 0$ and $T_{\text{l,f}} = 0$ simultaneously when the system buckles, both the upper and lower stays provide stiffness until buckling. From Eq. (3.37), the critical buckling load P_{Z2}^C is given thus:

$$P_{Z2}^C = \frac{T_{\text{u,i}}}{C_u} = \frac{T_{\text{l,i}}}{C_l}. \quad (3.54)$$

In Zone 3, where $T_{\text{u,i}} = T_{\text{l,i}} > T_{\text{opt}}$, the critical buckling load P_{Z3}^C is given from Eq. (3.45), thus:

$$P_{Z3}^C = (P_{\text{max}}^C - T_{\text{u,i}} \cos \beta_u - T_{\text{l,i}} \cos \beta_l) C_0. \quad (3.55)$$

The critical buckling load P^C reaches its maximum P_{max}^C when $T_{\text{u,i}} = T_{\text{l,i}} = T_{\text{opt}}$; this is obtained from Eq. (3.47).

3.3.4 Zonal behaviour in mono-symmetric stayed beam-columns

In terms of the case where $\alpha < 1.0$, from the outlined assumptions, the reference minimum prestressing forces $T_{\text{min1,ref}}$ and $T_{\text{min2,ref}}$, can be determined from the relationships in Eq. (3.37), based on the condition where the upper stays become slack ($T_{\text{u,f}} = 0$), or the force within the lower stays is given by the condition $T_{\text{l,f}} = 0$, when the system buckles thus:

$$T_{\text{min1,ref}} = P_E C_l, \quad T_{\text{min2,ref}} = P_E C_u. \quad (3.56)$$

From Eq. (3.21), when $T_{l,i} = T_{\min 1, \text{ref}}$, the minimum prestressing force within the upper stays, defined as $T_{\min 12, \text{ref}}$, can be expressed thus:

$$T_{\min 12, \text{ref}} = \frac{T_{\min 1, \text{ref}} \sin \beta_l}{\sin \beta_u}. \quad (3.57)$$

Since $T_{\min 2, \text{ref}} < T_{\min 12, \text{ref}}$ when $\alpha < 1.0$, there must be a sub-zone within Zone 2 where there are residual forces within the upper stays and zero stress within the lower stays under the externally-applied axial load $P = P_E$. In this sub-zone, the lower stays would be inactive before the system buckles while the upper stays would be active until the system buckles. For this case, the expressions for C_l and C_u from Eq. (3.36), which are based on both the upper and lower stays being active, cannot be applied. Repeating the process in the previous sections gives revised expressions for parameters $C_{u, Z2}$ (without considering the lower stays and crossarm) and $C_{l, Z2}$ (without considering the upper stays and crossarm), thus:

$$\begin{aligned} C_{u, Z2} &= \frac{K_{au, a} K_{su} \cos \beta_u}{2K_{c, a} K_{au, a} + K_{au, a} K_{su} \cos^2 \beta_u + 4K_{c, a} K_{su} \sin^2 \beta_u}, \\ C_{l, Z2} &= \frac{K_{al, a} K_{sl} \cos \beta_l}{2K_{c, a} K_{al, a} + K_{al, a} K_{sl} \cos^2 \beta_l + 4K_{c, a} K_{sl} \sin^2 \beta_l}. \end{aligned} \quad (3.58)$$

The minimum prestressing forces $T_{\min 1}$ and $T_{\min 2}$, which marks the boundary between Zones 1 and 2, can be determined from the relationships in Eq. (3.37), based on the condition that the upper stays become slack ($T_{u, f} = 0$) or the force within the lower stays being given by the condition $T_{l, f} = 0$ when the system buckles, thus:

$$T_{\min 1} = P_E C_{l, Z2}, \quad T_{\min 2} = P_E C_{u, Z2}. \quad (3.59)$$

From Eq. (3.21), when $T_{l,i} = T_{\min 1}$, the minimum prestressing force within the upper stays, defined as $T_{\min 12}$, can be expressed thus:

$$T_{\min 12} = \frac{T_{\min 1} \sin \beta_l}{\sin \beta_u}. \quad (3.60)$$

Similarly, when $T_{u,i} = T_{\min 2}$, the minimum prestressing force within the lower stays, defined as $T_{\min 21}$, can be expressed thus:

$$T_{\min 21} = \frac{T_{\min 2} \sin \beta_u}{\sin \beta_l}. \quad (3.61)$$

With the prestressing levels within all the stays increasing, there is an additional sub-zone within Zone 2 where both the upper and lower stays are active until the system buckles, and the parameters obtained from the previous sections can be applied. The prestressing forces marking the boundary between Zones 2 and 3, $T_{\text{opt}1}$, $T_{\text{opt}2}$, $T_{\text{opt}12}$ and $T_{\text{opt}21}$ can be calculated from Eqs. (3.46), (3.48) and (3.49) respectively. In the current formulation, for most cases with $\alpha < 1.0$ (apart from $\alpha \rightarrow 0$), $T_{\min 21} < T_{\min 1} < T_{\text{opt}21} < T_{\text{opt}1}$ corresponding to $T_{\min 2} < T_{\min 12} < T_{\text{opt}2} < T_{\text{opt}12}$. The details of the responses within each zone can be described as follows:

Zone 1 – when $T_{l,i} = [0, T_{\min 21}]$, corresponding to $T_{u,i} = [0, T_{\min 2}]$, on applying P , the prestressing forces would disappear (excluding $T_{l,w}$ within the lower stays from the aforementioned assumptions) owing to the axial end-shortening displacement before Euler buckling occurs, and so the response replicates that of unstayed beam-columns. The expression for the buckling load P_{Z1}^C is thus:

$$P_{Z1}^C = P_E. \quad (3.62)$$

Zone 2 – when $T_{l,i} = [T_{\min 21}, T_{\text{opt}21}]$, corresponding to $T_{u,i} = [T_{\min 2}, T_{\text{opt}2}]$, on applying P , the prestressing forces within all the stays would disappear when the system buckles (excluding $T_{l,w}$ within the lower stays as described above), generally at a higher load compared to Zone 1. However, there are two different sub-zones within Zone 2, currently named ‘Zone 2-1’ and ‘Zone 2-2’ that are characterized from whether a residual force within the lower stays exists when the external load $P = P_E$, thus:

Zone 2-1 – when $T_{l,i} = [T_{\min 21}, T_{\min 1}]$, corresponding to $T_{u,i} = [T_{\min 2}, T_{\min 12}]$, and

$P = P_E$, there is a residual force within the upper stays (*i.e.* $T_{u,f} > 0$) while $T_{l,f} = 0$ within the lower stays. The restraining effect from the residual force within the upper stays only would increase P^C , where $P^C = [P_E, P_{\max,2-1}^C]$, with $P_{\max,2-1}^C$ being the maximum critical buckling load in this sub-zone. The expression for the buckling load P_{Z2-1}^C is:

$$P_{Z2-1}^C = \frac{T_{u,i}}{C_{u,Z2-1}}, \quad (3.63)$$

where $C_{u,Z2-1} = C_{u,Z2}$ in Eq. (3.58), and $P_{\max,2-1}^C = T_{\min12}/C_{u,Z2-1}$.

Zone 2-2 – when $T_{l,i} = [T_{\min1}, T_{\text{opt}21}]$, corresponding to $T_{u,i} = [T_{\min12}, T_{\text{opt}2}]$, and $P = P_E$, there is a residual force within both the upper and lower stays, *i.e.* $T_{u,f} > 0$ and $T_{l,f} > 0$. The restraining effect from the residual forces within both the upper and lower stays would increase P^C further, and $P^C = [P_{\max,2-1}^C, P_{\max,2-2}^C]$, where $P_{\max,2-2}^C$ is the maximum critical buckling load in this sub-zone (referred to previously as $P_{\max,1}^C$ in Wu *et al.* (2019)). The expression for the buckling load P_{Z2-2}^C is determined from linear interpolation thus:

$$P_{Z2-2}^C = P_{\max,2-1}^C + \left[\frac{P_{\max,2-2}^C - P_{\max,2-1}^C}{T_{\text{opt}2} - T_{\min12}} \right] (T_{u,i} - T_{\min12}), \quad (3.64)$$

where:

$$P_{\max,2-2}^C = P_{\max}^C = \frac{P_{\text{FE},T=0}^C}{C_0}. \quad (3.65)$$

Zone 3 – when $T_{l,i} > T_{\text{opt}21}$, corresponding to $T_{u,i} > T_{\text{opt}2}$, on applying P , the beam-column buckles while residual forces exist within all the stays. However, in a similar fashion to Zone 2, there are two different sub-zones within Zone 3, currently named ‘Zone 3-1’ and ‘Zone 3-2’ that are characterized by whether or not a residual force within the lower stays exists at the point of buckling, thus:

Zone 3-1 – when $T_{l,i} = [T_{\text{opt}21}, T_{\text{opt}1}]$, corresponding to $T_{u,i} = [T_{\text{opt}2}, T_{\text{opt}12}]$, on applying P , the force within the lower stays reduces to $T_{l,w}$, but there is a residual

force within the upper stays ($T_{u,f} > 0$) when the system buckles. The upper-stay residual force decreases P^C , and $P^C = [P_{\max,3-1}^C, P_{\max,2-2}^C]$, where $P_{\max,3-1}^C$ marks the minimum critical buckling load in this sub-zone (referred to previously as $P_{\max,2}^C$ in Wu *et al.* (2019)). The expression for the buckling load P_{Z3-1}^C is determined from linear interpolation thus:

$$P_{Z3-1}^C = P_{\max,2-2}^C + \left[\frac{P_{\max,3-1}^C - P_{\max,2-2}^C}{T_{\text{opt}12} - T_{\text{opt}2}} \right] (T_{u,i} - T_{\text{opt}2}), \quad (3.66)$$

where the minimum critical buckling load $P_{\max,3-1}^C$ in this sub-zone is given from Eq. (3.45), thus:

$$P_{\max,3-1}^C = (P_{\max}^C - T_{\text{opt}12} \cos \beta_u - T_{\text{opt}1} \cos \beta_l) C_0. \quad (3.67)$$

Zone 3-2 – when $T_{l,i} > T_{\text{opt}1}$, corresponding to $T_{u,i} > T_{\text{opt}12}$, on applying P , there are residual forces within the lower and upper stays when the system buckles, *i.e.* $T_{l,f} > 0$ and $T_{u,f} > 0$. Both the upper-stay and lower-stay residual forces reduce P^C , and $P^C < P_{\max,3-1}^C$. The expression for the buckling load P_{Z3-2}^C can be determined from Eq. (3.45), thus:

$$P_{Z3-2}^C = (P_{\max}^C - T_{u,i} \cos \beta_u - T_{l,i} \cos \beta_l) C_0. \quad (3.68)$$

The aforementioned zones of behaviour of prestressed stayed beam-columns with $\alpha < 1.0$ are illustrated in Figure 3.10.

3.4 Numerical verification

The principal purpose of the current section is to investigate the buckling and post-buckling behaviour of stayed beam-columns in the horizontal plane. It should be noted that in

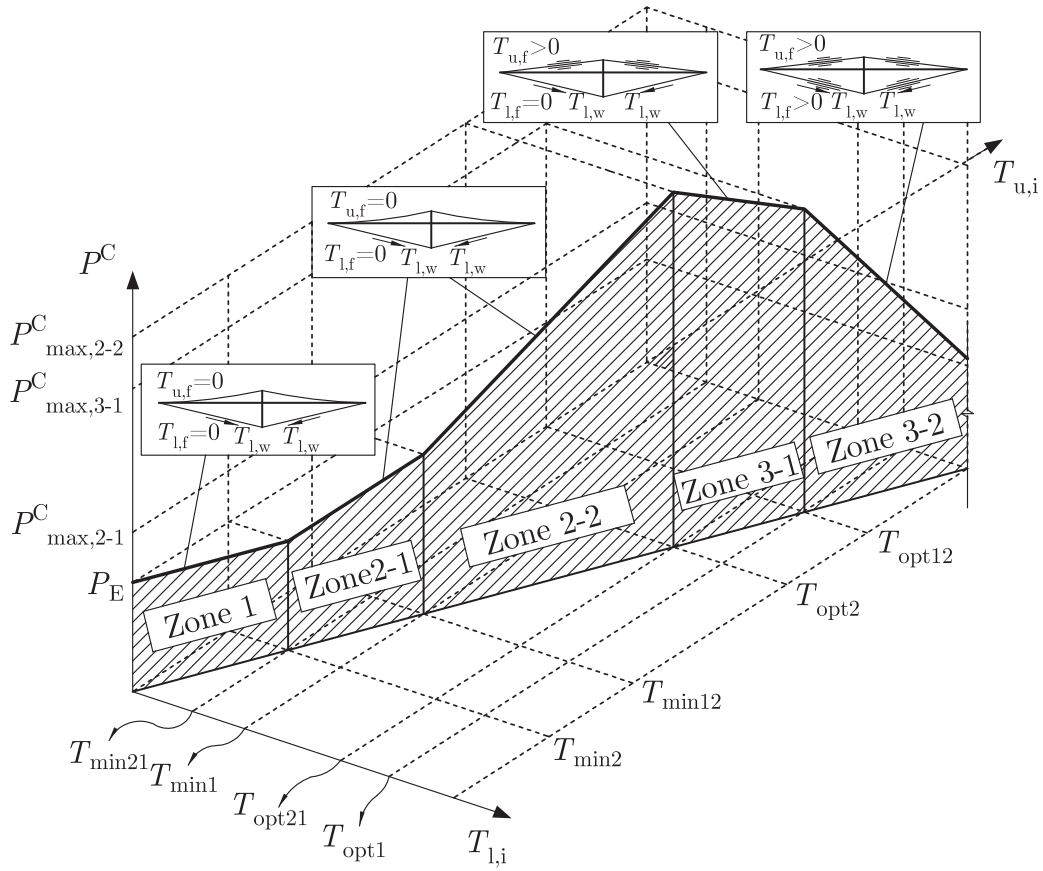


Figure 3.10: Zones of behaviour for prestressed stayed beam-columns with $\alpha < 1.0$: the graph shows the critical buckling load of the beam-column P^C plotted against different combinations of initial prestressing forces within the upper and lower stays ($T_{u,i}$ and $T_{l,i}$ respectively). Note that this is a line rather than a surface since the relationship between $T_{u,i}$ and $T_{l,i}$ is fixed through Eq. (3.21).

the current chapter initial geometric imperfections are not taken into consideration since the self-weight serves the purpose of perturbing the geometry to enable the nonlinear buckling problem to be solved. Firstly, using linear buckling analysis in ABAQUS, the optimum prestressing forces are obtained by substituting Eq. (3.47) into Eq. (3.46), and $T_{l,i} = T_{\text{opt}1}$ is selected as the benchmark prestressing force for the lower stays corresponding to $T_{u,i} = T_{\text{opt}12}$ for the upper stays; subsequently, the nonlinear mechanical behaviour is studied using Riks analysis.

Apart from the lower-crossarm length a_l , stay diameter ϕ_s and crossarm length ratio α , the material properties and structural dimensions of prestressed stayed beam-columns in the current study are fixed and are listed in Table 3.1. In terms of doubly-symmetric cases,

Table 3.1: Material properties and structural dimensions of the prestressed stayed beam-columns in the current study.

Length of main member	$L = 3050 \text{ mm}$
Main member Young's moduli	$E_c = 201 \text{ kN/mm}^2$
Crossarm Young's moduli	$E_a = 201 \text{ kN/mm}^2$
Stay Young's moduli	$E_s = 202 \text{ kN/mm}^2$
Outside diameter of main member	$\phi_{co} = 38.1 \text{ mm}$
Inside diameter of main member	$\phi_{ci} = 25.4 \text{ mm}$
Outside diameter of crossarm	$\phi_{ao} = 38.1 \text{ mm}$
Inside diameter of crossarm	$\phi_{ai} = 25.4 \text{ mm}$
Material density (steel)	$\rho = 7850 \text{ kg/m}^3$
Gravitational acceleration	$g = 9.81 \text{ m/s}^2$
Yield stress of main member	$f_y = 340 \text{ N/mm}^2$

12 configurations, as defined in Saito & Wadee (2009b), listed as F1–F6 and A1–A6 are studied. In terms of the mono-symmetric cases F3.1 to F3.9 and F4.1 to F4.9 (inclusive), the crossarm length ratio α is varied from 0.1 to 0.9 (inclusive), while the stay diameter and the lower-crossarm length remain the same as in cases F3 and F4 respectively. The detailed parameters of each studied case are listed in Tables 3.2, 3.3 and 3.4. Changing ϕ_s allows the stiffness of the cable to be varied; crossarm length ratio α_l , α_u and α were defined in Eqs. (3.1) and (3.2), and allows the angle of the stays relative to the main element to be varied, since $\beta_l = \arctan 2\alpha_l$ and $\beta_u = \arctan 2\alpha_u$.

Table 3.2: Detailed parameters of studied doubly-symmetric ($\alpha = 1.0$) cases, where $\alpha_1 = 0.1$ in cases F1–F6 and $\phi_s/(2r_c) = 0.21$ in cases A1–A6.

Case	F1	F2	F3	F4	F5	F6
ϕ_s (mm)	1.6	3.2	4.8	6.4	8.0	10.0
$\phi_s/(2r_c)$	0.07	0.14	0.21	0.28	0.35	0.44
Case	A1	A2	A3	A4	A5	A6
α_1	0.025	0.05	0.075	0.100	0.125	0.150

Table 3.3: Detailed parameters of studied mono-symmetric ($\alpha < 1.0$) cases, where $\alpha_1 = 0.1$ and $\phi_s/(2r_c) = 0.21$.

Case	F3.1	F3.2	F3.3	F3.4	F3.5	F3.6	F3.7	F3.8	F3.9
α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9

The normalized slenderness of the main member element $\bar{\lambda}$ is defined by the well known expression (Trahair *et al.*, 2008):

$$\bar{\lambda} = \sqrt{P_y/P_E}, \quad (3.69)$$

where P_y is the squash load, thus:

$$P_y = A_c f_y, \quad (3.70)$$

and P_E is the Euler buckling load of the main member element defined in Eq. (3.52). For the main member within the current study, $P_y = 214$ kN, $P_E = 17.7$ kN and $\bar{\lambda} = 3.477$. Note also that the main member radius of gyration from Eq. (3.5), $r_c = \sqrt{I_c/A_c} = 11.45$ mm.

Table 3.4: Detailed parameters of studied mono-symmetric ($\alpha < 1.0$) cases, where $\alpha_1 = 0.1$ and $\phi_s/(2r_c) = 0.28$.

Case	F4.1	F4.2	F4.3	F4.4	F4.5	F4.6	F4.7	F4.8	F4.9
α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9

3.4.1 Buckling analysis

Figure 3.11 shows the buckling loads for ‘Mode 1’ (symmetric eigenmode) and ‘Mode

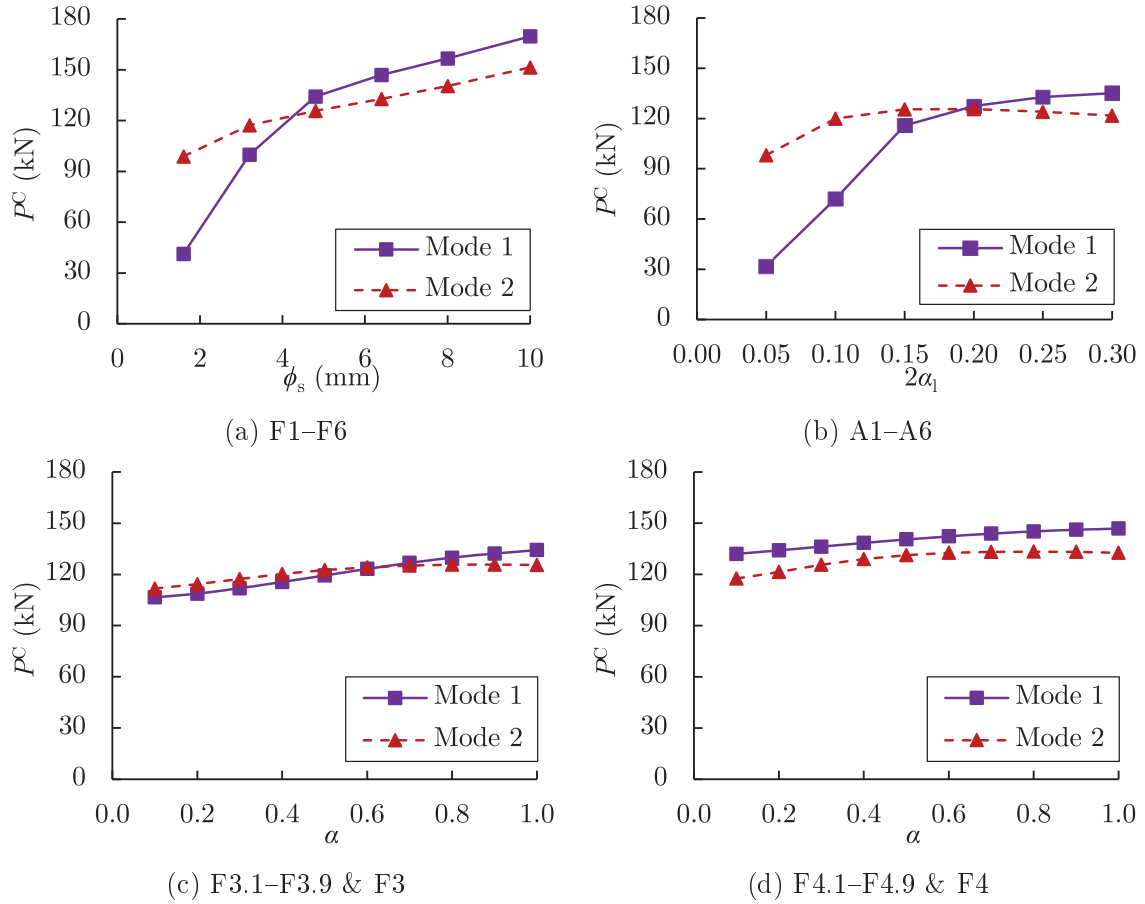


Figure 3.11: Linear buckling loads from ABAQUS linear buckling analysis: ‘Mode 1’ represents the buckling load of the eigenmode profile that has reflective symmetry about midspan; ‘Mode 2’ represents the buckling load of the eigenmode profile that has rotational symmetry about midspan.

2’ (antisymmetric eigenmode), which are illustrated in Figure 3.12, from linear buckling analysis within ABAQUS. From the expression of the optimum prestressing forces, the lower-stay optimum prestressing force T_{opt1} can be calculated. It should be noted that the linear buckling loads for Modes 1 and 2 are used as $P_{\text{FE},T=0}^C$ in Eq. (3.47). Based on Figures 3.11(a–b), in terms of the doubly-symmetric models F1–F6 and A1–A6, it is evident

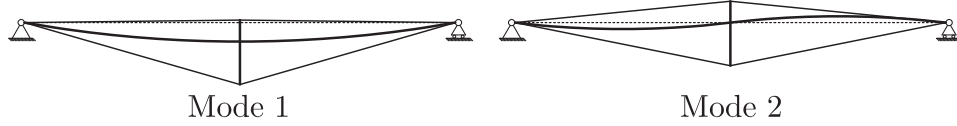


Figure 3.12: Illustration of buckling eigenmodes 1 (symmetric) and 2 (antisymmetric).

that the critical buckling mode changes from 1 to 2 with the stay diameter increasing, and the same trend is found when the crossarm length is increased. This conclusion is similar to that drawn in previous studies of stayed columns (Saito & Wadee, 2009b; Yu & Wadee, 2017a). The graphs in Figures 3.11(c–d) show that the critical buckling load for the mono-symmetric models is fundamentally independent of the crossarm ratio α , but the mode can switch from 1 to 2.

To provide reassurance of the adopted FE modelling described in §3.2.2 prior to the inclusion of self-weight and pre-cambering from mono-symmetric geometry, the result comparison between the adopted FE model in the present thesis and the FE model presented in Saito & Wadee (2009b) is shown in Table 3.5. It can be seen that the results from both

Table 3.5: Linear critical buckling load comparison between the adopted FE model and the FE model presented in Saito & Wadee (2009b); P_{present}^C is from the adopted FE model, and P_{Saito}^C is from Saito & Wadee (2009b).

Case	F1	F2	F3	F4	F5	F6
P_{present}^C (kN)	41.24	99.90	125.62	132.71	140.31	151.35
P_{Saito}^C (kN)	41.24	99.90	125.62	132.71	140.31	151.35
Case	A1	A2	A3	A4	A5	A6
P_{present}^C (kN)	31.93	72.04	115.85	125.62	124.00	121.81
P_{Saito}^C (kN)	31.93	72.04	115.85	125.62	124.00	121.81

methods are same, and the current FE modelling is confirmed to be valid such that further studies can be conducted on stayed beam-columns with self-weight and pre-cambering considered.

3.4.2 Zonal boundaries

Using the benchmark prestressing forces where $T_{l,i} = T_{\text{opt1}}$ and $T_{u,i} = T_{\text{opt12}}$, the buckling and post-buckling behaviour is investigated using Riks analysis within ABAQUS. It should be noted that the four loading steps are adopted, as described in Figure 3.2. In addition, the following initial prestressing force levels within the lower stays are considered:

$$T_{l,i}/T_{\text{opt1}} = \{0, 0.25, 0.50, 0.75, 1.00, 1.25, 1.50, 1.75, 2.00, 2.25, 2.50, 2.75, 3.00\} \quad (3.71)$$

and the corresponding upper-stay prestressing forces ($T_{u,i}$) can be obtained from Eq. (3.21). Based on this, the ultimate load of each model is investigated using nonlinear FE modelling. To verify the profile of the response zones identified in the previous section (Figure 3.10), both the analytical critical buckling loads P^C and the ultimate loads P_u from nonlinear FE analysis are presented in Figure 3.13. The methodology of using nonlinear FE analysis to verify the quality of the buckling load predictions has been used in earlier work (Lapira *et al.*, 2017). This is because in the current system, the self-weight introduces eccentricities that initiates bending within the system as soon as the external axial load is applied. Therefore, it is not possible to detect a buckling instability in the classical (linear) sense since the behaviour within the FE model is always nonlinear. Hence, an analysis of the post-buckling response is used to infer the quality of the analytical model predictions; this also has the advantage of determining whether there is any significant post-critical stiffness and strength that can be exploited.

In the doubly-symmetric cases presented, Figures 3.13(a–c) show that when the stay diameter ϕ_s is relatively large, a significantly higher load-carrying capacity can be obtained if the prestressing level within the lower stays is higher than T_{opt1} . For example, in terms of cases F3 and F5, the ultimate loads increase significantly when $T_{l,i}/T_{\text{opt1}}$ increases, while in case F1, the ultimate loads and the critical buckling loads basically coincide for $T_{l,i}/T_{\text{opt1}} \geq 1$. As can be seen, these trends are also observed in Figures 3.13(d–f) respectively where the

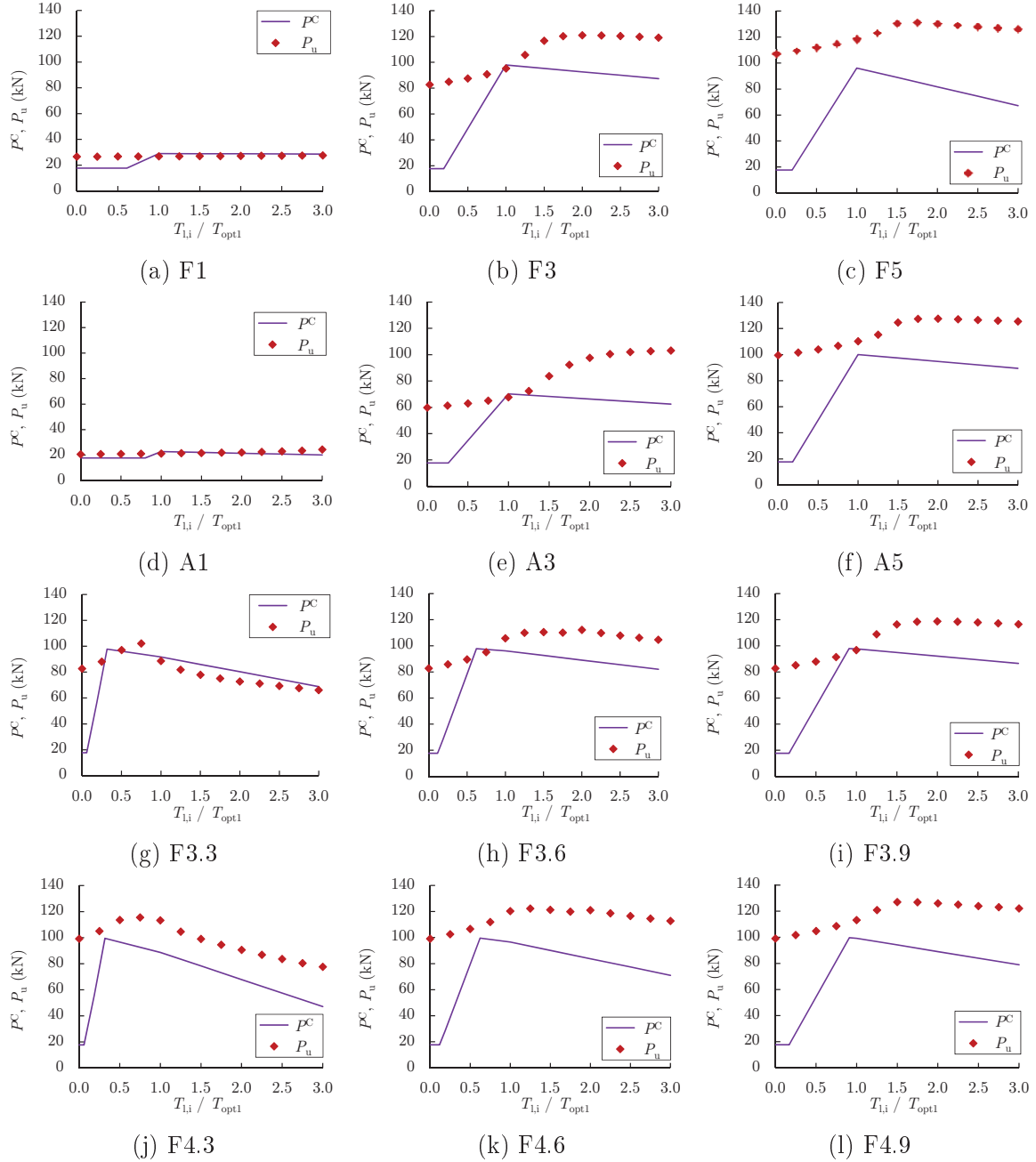


Figure 3.13: Plots of the analytically derived critical buckling loads P^C and numerically (FE) derived ultimate loads P_u versus the lower-stay relative prestressing level $T_{l,i}/T_{opt1}$.

crossarm lengths are progressively increased.

In the mono-symmetric cases presented, Figures 3.13(g–i) show that when α is small, higher prestressing levels may lead to a significant decline in the buckling load based on both the analytical curve and nonlinear FE results, *i.e.* the ultimate load of F3.3, with $\alpha = 0.3$, decreases obviously when $T_{l,i}/T_{opt1} > 0.75$ and essentially shadows the trend of the critical buckling load. However, in terms of cases F3.6 and F3.9 with a relatively larger value of α , the ultimate load becomes significantly higher than the critical buckling load where the lower-stay initial prestressing force $T_{l,i}/T_{opt1} > 1.0$ and a clear maximum can be observed. It can also be observed that similar trends appear in Figures 3.13(j–l), although the stay diameter and the critical buckling mode are different to those in Figures 3.13(g–i). Finally, as for the case of the prestressed stayed column under pure axial compression (Saito & Wadee, 2008, 2009a; Yu & Wadee, 2017a), it is significant that T_{opt1} , obtained from purely linear analysis, is not the true optimum prestressing force for the lower stays in the sense that it does not maximize the load-carrying capacity. Hence, introducing higher prestressing levels would be recommended in practice as determined in previous studies (Saito & Wadee, 2010; Wadee *et al.*, 2013b; Serra *et al.*, 2015; Yu & Wadee, 2017b; Lapira *et al.*, 2017); the true optimum prestressing levels are discussed in Chapter 6.

Note that the analytical results presented in Figure 3.13 are critical buckling loads P^C , while the numerical results are ultimate loads P_u . The critical buckling loads serve as reference values for the basis of the establishment of future design rules, while the difference between P^C and P_u reflects the combined influence of geometric imperfections, residual stresses, second-order effects and post-buckling stability. An enhanced analytical model that includes second-order effects is presented in Chapter 5.

3.4.3 Nonlinear equilibrium paths

To examine the nonlinear equilibrium behaviour of stayed beam-columns from various perspectives, the prestressing force within the lower stays $T_{l,i}$ is now fixed to $T_{\text{opt}1}$ and for the upper stays $T_{u,i} = T_{\text{opt}12}$. Figure 3.14 presents equilibrium diagrams of the externally-

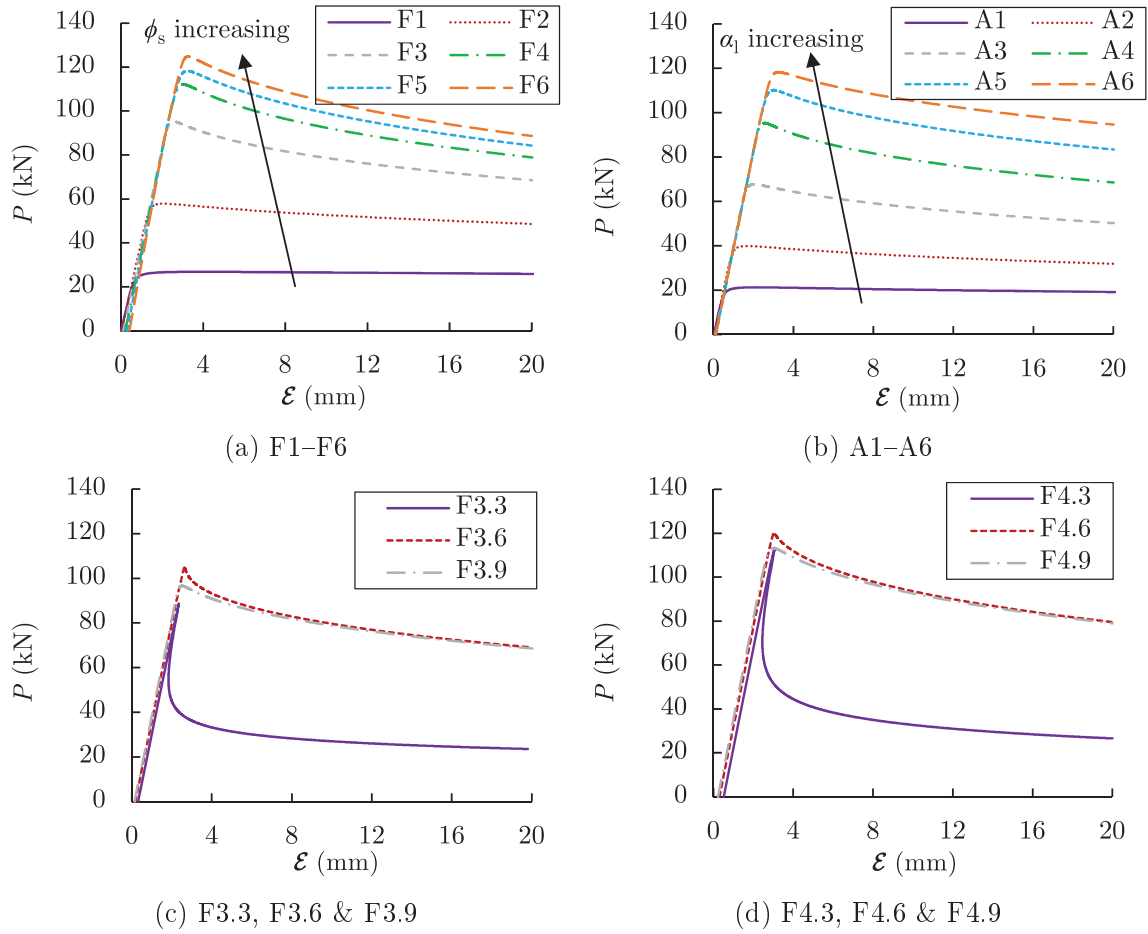


Figure 3.14: Externally-applied axial load P versus the end-shortening displacement ϵ of the main member when $T_{l,i}/T_{\text{opt}1} = 1.0$.

applied axial load P versus the corresponding end-shortening displacement ϵ to show the post-buckling behaviour of the beam-columns. For the doubly-symmetric cases, the graphs in Figures 3.14(a–b) indicate that increasing the stay diameter or the crossarm length both increase the ultimate load, but the post-buckling response is slightly more unstable when

these two parameters are larger. For the mono-symmetric cases, it can be observed in the graphs in Figures 3.14(c–d) that when α is small, such as in cases F3.3 and F4.3, the post-buckling response is hugely unstable when compared to models with a larger α (cases F3.6, F3.9, F4.6 and F4.9). Tables 3.6 and 3.7 present the corresponding numerical ultimate

Table 3.6: Ultimate loads of F1–F6 and A1–A6 when $T_{l,i}/T_{opt1} = 1.0$.

Case	F1	F2	F3	F4	F5	F6
P_u (kN)	27.5	59.2	96.7	112.7	118.6	124.8
Case	A1	A2	A3	A4	A5	A6
P_u (kN)	21.9	41.0	69.0	96.7	111.0	118.7

Table 3.7: Ultimate loads of cases F3.1–F3.9 and F4.1–F4.9 when $T_{l,i}/T_{opt1} = 1.0$.

Case	F3.1	F3.2	F3.3	F3.4	F3.5	F3.6	F3.7	F3.8	F3.9
P_u (kN)	34.2	60.6	85.8	106.0	107.3	107.7	105.3	101.1	98.3
Case	F4.1	F4.2	F4.3	F4.4	F4.5	F4.6	F4.7	F4.8	F4.9
P_u (kN)	42.0	84.0	112.2	117.4	120.7	121.2	117.9	115.4	113.9

loads. It is evident that the ultimate load increases with the stay diameter or crossarm length increasing since the effectiveness of the stays in resisting structural instability is also increased. It should also be noted that in the considered mono-symmetric cases, the ultimate load reaches the maximum value when $\alpha = 0.6$, *i.e.* in cases F3.6 and F4.6.

3.4.4 Sensitivity to prestress

Since it was revealed that the buckling and post-buckling behaviour of stayed members is strongly linked to the prestressing level in previous studies (Saito & Wadee, 2008, 2009b), the effect of an increase in the initial prestress is investigated currently using nonlinear FE analysis. Two different levels of prestressing forces, $T_{l,i}/T_{opt1} = \{1.0, 2.0\}$, are adopted. It should be noted that the corresponding upper-stay prestressing force satisfies the relationship in Eq. (3.21). The graphs in Figure 3.15 show comparisons of the curves of the

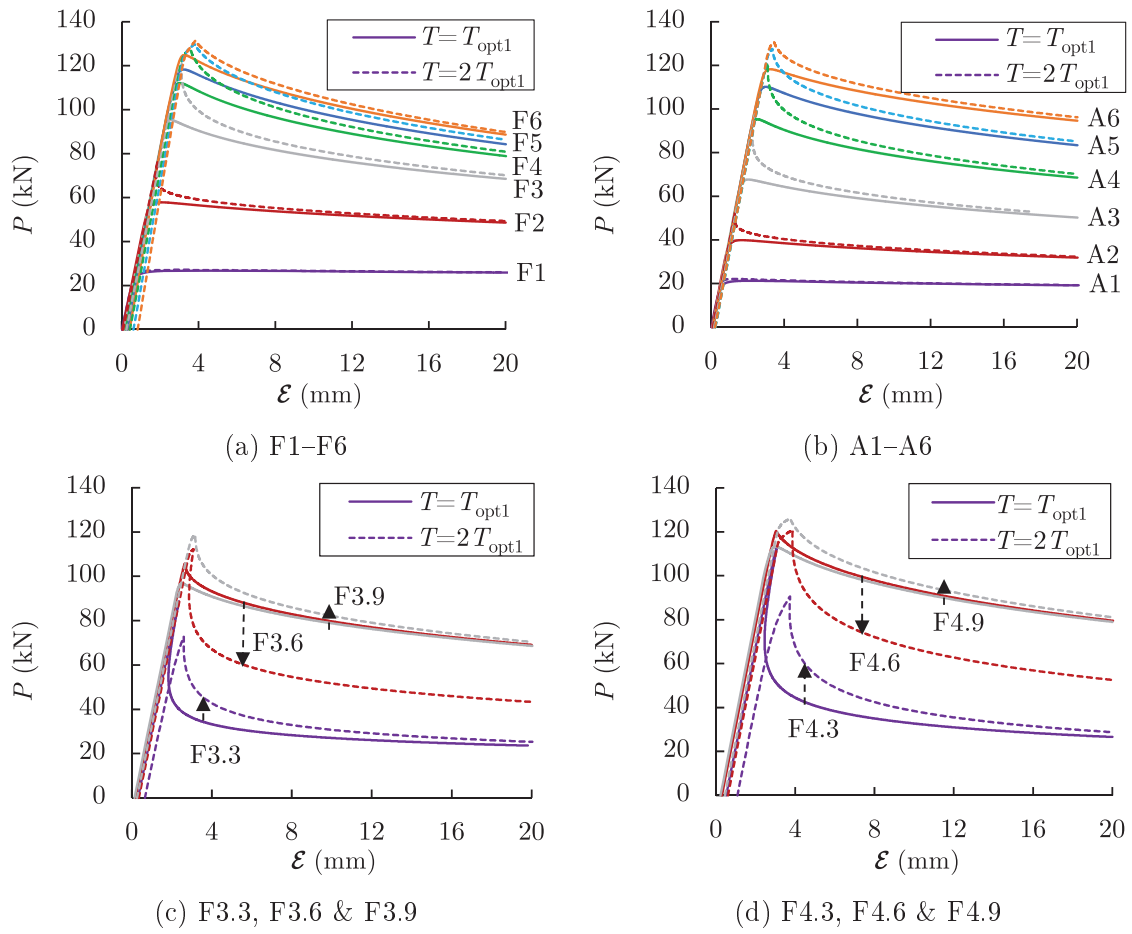


Figure 3.15: Comparisons of the externally-applied axial load P versus the end-shortening displacement $\mathcal{E}$ of the main member between $T_{l,i}/T_{opt1} = 1.0$ and 2.0 .

externally-applied axial load P versus the end-shortening displacement $\mathcal{E}$ of the main member between these two cases. It is observed that in terms of the doubly-symmetric models, F1–F6 and A1–A6, in Figures 3.15(a–b), the higher prestressing level increases the ultimate load, which can also be observed in Figure 3.13. However, the post-buckling response is relatively more unstable at $T_{l,i}/T_{opt1} = 2.0$ when compared to $T_{l,i}/T_{opt1} = 1.0$ in the initial post-buckling range but the residual capacity is generally similar, if slightly higher; these conclusions are similar to those in the previous studies on the purely axially-loaded cases (Saito & Wadee, 2008, 2009b).

In terms of the mono-symmetric models, F3.1–F3.9 and F4.1–F4.9, when α is small, such as in cases F3.3 and F4.3, the higher prestressing level reduces the ultimate load, but increases the post-buckling stability. When α is assigned an intermediate value, such as in cases F3.6 and F4.6, where $\alpha = 0.6$, the higher prestressing level increases the ultimate load, but decreases the post-buckling stability significantly. Finally, when α approaches unity, such as in cases F3.9 and F4.9, these behave similarly to the doubly-symmetric cases. A reason for these variations in response owing to the lower values of the mono-symmetry parameter α may be attributed to the more acute pre-cambering effect at higher prestressing levels.

3.4.5 Effect of pre-cambering

To study the post-buckling response further, the vertical displacement at midspan w_m and the forces within each stay T_n , where $n = \{l1, l2, u1, u2\}$ (see Figures 3.16 and 3.17) are investigated. However, at this point it is worth recalling that Equation (3.21), which relates the initial prestressing levels within the upper and lower stays, such that the restraining forces within both crossarms that inhibit the midspan displacement are equal, is only accurate to leading order. This is because that relationship deliberately neglects the induced end-displacement from the introduction of the prestressing forces through compressing and bending the main column element to ensure analytical solvability. Hence, for cases where

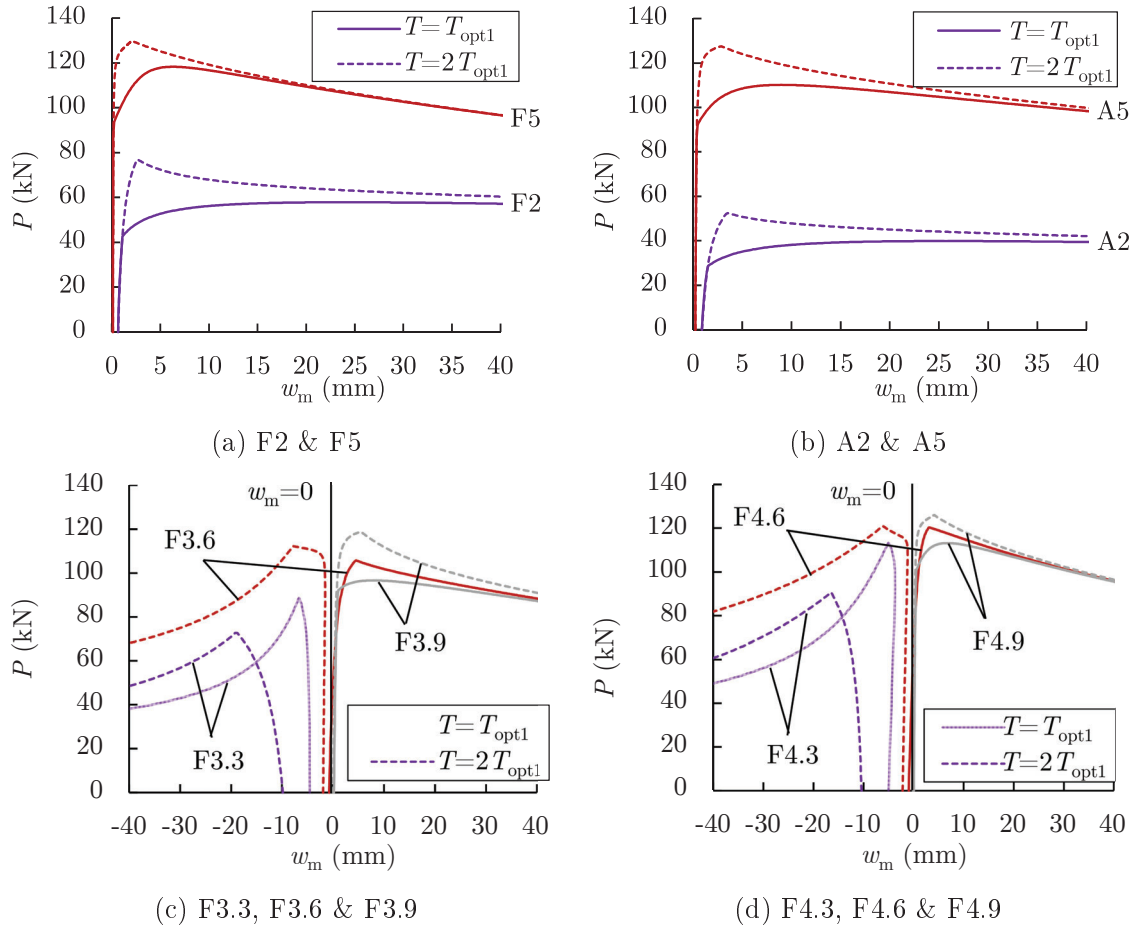


Figure 3.16: Comparisons of the externally-applied axial load P versus the vertical displacement at midspan w_m between $T_{l,i}/T_{opt1} = 1.0$ and 2.0 .

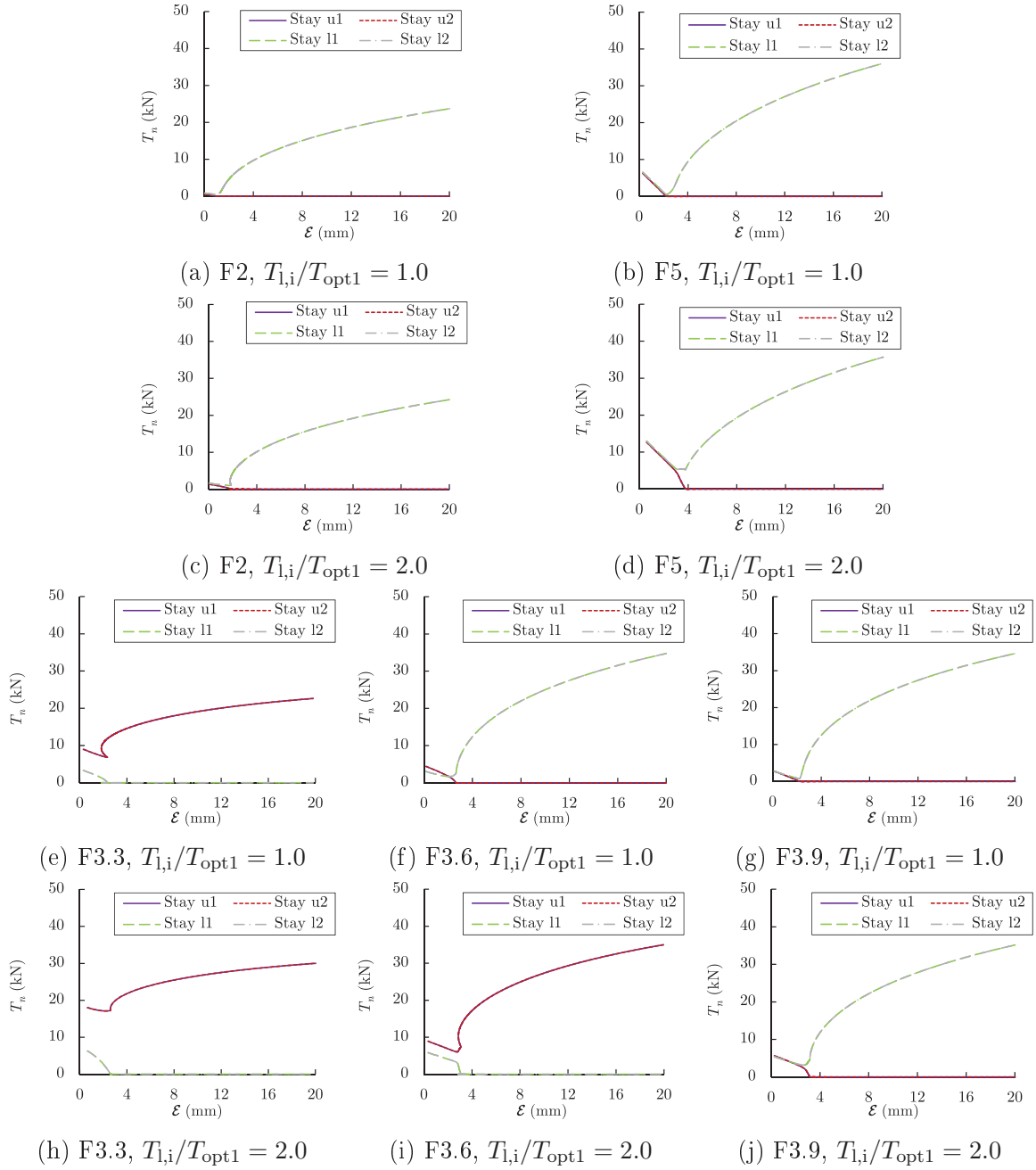


Figure 3.17: Forces within the individual upper and lower stays versus end-shortening displacement $\mathcal{E}$.

$\alpha < 1.0$ with higher prestressing levels, the initial lateral midspan displacement of the main column element is not necessarily zero. Therefore, there may be a significant pre-camber when the external axial load is introduced.

The graphs in Figure 3.16 compare the externally-applied axial load P versus the vertical displacement at midspan w_m between $T_{l,i}/T_{opt1} = 1.0$ and 2.0 . It should be noted that $w_m > 0$ represents a downward vertical displacement (*i.e.* the same direction as gravity). It is evident that in terms of the doubly-symmetric models presented in Figures 3.16(a–b), w_m is always positive for the downward displacement in both the $T_{l,i}/T_{opt1} = 1.0$ and 2.0 cases, which indicates that the pre-cambering effect is always smaller than the effect of the self-weight. This leads to a downward midspan displacement but higher prestressing levels increase the ultimate load.

However, the direction of w_m reverses in the case of the mono-symmetric models presented in the graphs in Figures 3.16(c–d). It can be seen that when α is small, such as in cases F3.3 and F4.3, $w_m < 0$ in both $T_{l,i}/T_{opt1} = 1.0$ and 2.0 cases, which shows that the pre-cambering effect is greater than the effect of the self-weight, leading to an upward midspan displacement. This is naturally associated with a potentially greater load-carrying capacity since the beam-column has to work against the gravitational force to deflect upwards. However, since $w_m < 0$ for both prestressing levels in these two cases, F3.3 and F4.3, the higher prestressing forces reduce the ultimate load because of the extra initial compression within the beam-column. For the cases where there is an intermediate level of α , such as in cases F3.6 and F4.6, $w_m > 0$ when $T_{l,i}/T_{opt1} = 1.0$ but $w_m < 0$ when $T_{l,i}/T_{opt1} = 2.0$. This demonstrates the cases where the beam-column switches from being self-weight dominant, when $T_{l,i}/T_{opt1} = 1.0$, to being pre-camber dominant, when $T_{l,i}/T_{opt1} = 2.0$. For these two cases, F3.6 and F4.6, the increase in prestressing level produces advantageous behaviour in terms of ultimate strength and initial post-buckling behaviour. Finally, when α approaches unity, *i.e.* the cases converge to the doubly-symmetric cases, such as F3.9 and F4.9, $w_m > 0$ in both prestressing levels and the behaviour is similar to the doubly-symmetric cases.

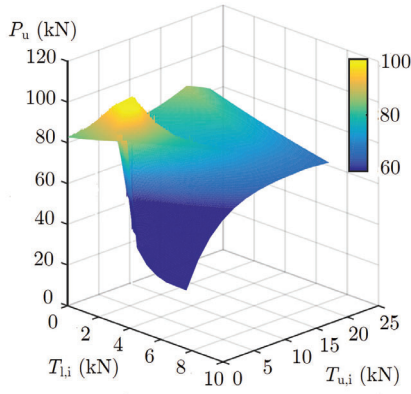
The graphs in Figure 3.17 confirm all the conclusions drawn from Figure 3.16 from the perspective of how the force changes within each stay as the end-shortening displacement is increased. It can be observed that the lower stays (l1 and l2) go slack finally in Figures 3.17(e), (h) and (i), which indicates that $w_m < 0$ and the pre-cambering effect dominates. The upper stays (u1 and u2) go slack in Figures 3.17(a), (b), (c), (d), (f), (g) and (j), which indicate that $w_m > 0$ and self-weight dominates. These results indicate that the pre-cambering effect from a small α or higher prestressing levels can affect the post-buckling response and the ultimate load significantly but there appears to be a combination of geometric and prestressing properties that maximize the effectiveness of the system. These matters will be revisited in Chapter 6.

3.4.6 Prestressing ratio of upper and lower stays

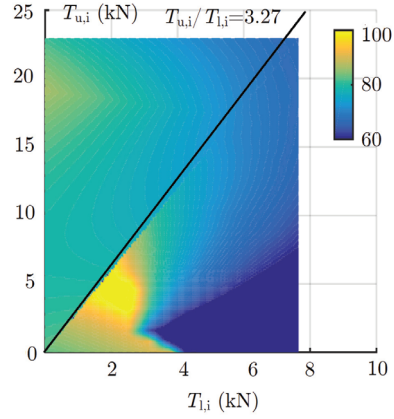
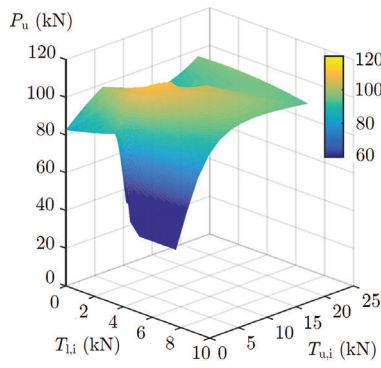
Hitherto, the ratio between the upper-stay and lower-stay prestressing forces is set from the geometric condition in Eq. (3.21). In order to verify that this ratio is approximately optimal, nonlinear FE analysis is conducted with cases F3.3, F3.5 and F3.7 being selected for this parametric study. In each model, the lower-stay prestressing force is assigned in turn from those defined in Eq. (3.71); once this is fixed, all of the following levels are studied for the upper-stay prestressing force in turn:

$$T_{u,i}/T_{l,i} = \{0, 0.25, 0.50, 0.75, 1.00, 1.25, 1.50, 1.75, 2.00, 2.25, 2.50, 2.75, 3.00\}. \quad (3.72)$$

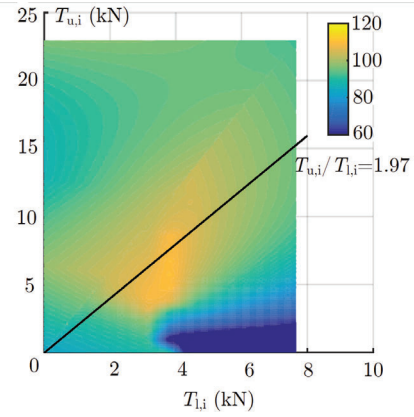
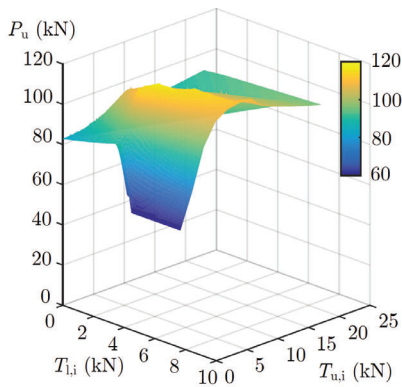
In total, there are 169 cases for each model, the results from which are compiled into the three-dimensional surfaces provided in Figures 3.18(a), (c) and (e) using linear interpolation. The graphs in Figures 3.18(b), (d) and (f) show planes of these surfaces in the plane of $T_{u,i}$ versus $T_{l,i}$. As can be seen, the larger ultimate loads P_u are numerically close to the line defined by the geometric condition $T_{u,i}/T_{l,i} = \sin \beta_l / \sin \beta_u$, which represents the prestressing ratio obtained from Eq. (3.21). The plots suggest that this particular pre-



(a) F3.3–3D surface

(b) F3.3–2D contour, $\sin \beta_l / \sin \beta_u = 3.27$ 

(c) F3.5–3D surface

(d) F3.5–2D contour, $\sin \beta_l / \sin \beta_u = 1.97$ 

(e) F3.7–3D surface

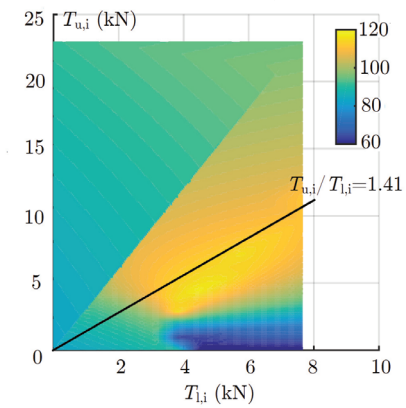
(f) F3.7–2D contour, $\sin \beta_l / \sin \beta_u = 1.41$

Figure 3.18: Effects from allowing different prestressing forces within the upper $T_{u,i}$ and lower stays $T_{l,i}$; the surfaces in (a), (c) and (e) are depicted as planes in (b), (d) and (f). The relationship from Eq. (3.21) is superimposed on the graphs in (b), (d) and (f).

stressing ratio provides a good initial estimate for maximizing the ultimate load; again, this is revisited in Chapter 6.

3.5 Concluding remarks

As far as the author is aware, this is the first study to investigate the buckling and post-buckling behaviour of prestressed stayed beam-columns. Although previous studies investigated the stability of prestressed stayed columns, their focus was only on doubly-symmetric stayed columns in the vertical plane where the applied force and the principal self-weight contributions were not only in the same direction but also collinear. Putting the stayed column within the horizontal plane changes this significantly and the flexural effect of the structural self-weight cannot be neglected. This opens up a whole new set of issues, including the possible desirability of making the structure mono-symmetric by adopting different crossarm lengths at midspan such that the prestressed system counteracts the effect of self-weight. It has been shown currently that the crossarm length ratio alongside the effect of self-weight have significant effects on the mechanical behaviour.

It has been found that the three distinct zones of behaviour that have been determined for purely axially-loaded prestressed stayed columns are significantly affected by the effect of self-weight. The principal effect is to sub-divide two of these zones into two further parts by taking the natural mono-symmetry of the geometry into account. The newly-derived zones for the beam-column have been verified by nonlinear FE results and, as has been determined for stayed columns, the true optimum prestressing level, in terms of maximizing the ultimate load-carrying capacity, is higher than the linearly-derived optimum level, which implies that the current theoretical study needs to be developed further to provide a fundamental understanding of the mechanical behaviour within the nonlinear range.

Apart from the well-known finding that increasing the stay diameter and crossarm lengths

can increase the ultimate load for doubly-symmetric stayed beam-columns, as is case for conventional stayed columns, the crossarm length ratio is also found to be a key parameter. It transpires that small crossarm length ratios should be avoided since this has been demonstrated to lead to extremely unstable post-buckling behaviour including snap-back responses. The current results demonstrate that intermediate crossarm length ratios, determined herein to be approximately 0.6, give the most favourable mechanical behaviour since they strike a balance between the effect of self-weight and the initial pre-cambering displacement. Finally, the prestressing force distribution between the upper and lower stays, determined from a simple force balancing equation, appears to provide a sensible estimate for the required combination to be adopted in stayed beam-columns to maximize the system effectiveness.

Notwithstanding the present findings, since the currently developed theoretical formulation is based on linear theory and it has been shown through nonlinear FE analysis that the behaviour is considerably more nuanced, further development is clearly required. Moreover, the current study is limited to perfect structures without taking imperfections into consideration. The present findings suggest that a nonlinear model to investigate imperfection sensitivity (Saito & Wadee, 2009a; Serra *et al.*, 2015) and the potential for interactive buckling, which are known issues in prestressed stayed columns (Saito & Wadee, 2009b; Yu & Wadee, 2017a), would lead to predicting and understanding the practical ranges for parameters and realistic arrangements. These matters are discussed in the subsequent chapters

Chapter 4

Explicit solutions for buckling loads

4.1 Introduction

In Chapter 3, linear buckling analysis needed to be conducted to obtain the buckling eigenvalues prior to evaluating the linear optimum prestressing force for stayed beam-columns, which is similar to the previous studies on vertical stayed columns (Saito & Wadee, 2009b; Yu & Wadee, 2017a; Lapira *et al.*, 2017). However, it is not convenient to introduce a process that depends on prior FE analysis into design guidances. The primary aim of the current chapter is to simplify the process of finding buckling loads to make it more applicable for engineering practice. Following similar existing work on the doubly-symmetric stayed columns in Smith *et al.* (1975), an analytical model, which can also be applied to mono-symmetric stayed members, is formulated. Theoretical solutions of buckling loads are obtained by solving differential equations, and closed form simplified solutions are formulated using regression techniques; the latter allowing explicit evaluation of the critical buckling load under purely axial compression. In addition, these solutions for the buckling loads are compared with the buckling eigenvalues from linear buckling analysis in ABAQUS and are found to compare very well. This enables the design guidance

developed in later chapters to be simplified.

4.2 Definitions and assumptions

Apart from the axial stiffness definitions from Eq. (3.4) and the assumptions adopted in Chapter 3, member bending stiffnesses are defined currently, and additional assumptions are made for formulating the mechanical models to investigate the buckling load. The bending stiffnesses of the main column member $K_{c,b}$, upper crossarm $K_{au,b}$ and lower crossarm $K_{al,b}$ can be defined as the following respectively:

$$K_{c,b} = \frac{E_c I_c}{L^3}, \quad K_{au,b} = \frac{E_a I_a}{a_u^3}, \quad K_{al,b} = \frac{E_a I_a}{a_l^3}. \quad (4.1)$$

In a similar method to the analytical study only for doubly-symmetric stayed columns in Smith *et al.* (1975), for a mono-symmetric stayed member, two qualitatively different buckling modes, Modes 1 and 2 illustrated in Figure 3.12, are also possible. The lowest buckling load between these two modes can be used to evaluate the linear optimum pre-stressing forces. Apart from the assumptions made in Chapter 3, the following are also made for the analytical model presently:

1. All the deflections are small, hence the model only considers critical buckling.
2. The axial deformation of the main column member and crossarms is neglected since it is small compared with the effect of stay deformation.
3. The connections between the crossarms and the main member are assumed to be perfectly rigid; the connection between the stays and the main member and between the stays and the crossarms are assumed to be ideal hinges.
4. There is no imperfection and deflection prior to buckling.

5. Buckling occurs in the plane of the main member in conjunction with the crossarms.
6. The prestressing level of the current study is within Zone 3, hence all stays are in tension at critical buckling.

4.3 Theoretical solutions

From all the aforementioned assumptions, the buckling load for each mode can be obtained using the beam differential equation. If second-order terms and the effect of the end-shortening displacement of the main member are neglected, the basic differential equation for beam bending is thus:

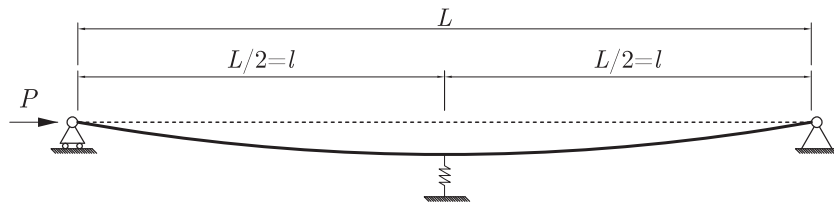
$$M = -E_c I_c \frac{d^2 w}{dx^2}, \quad (4.2)$$

where x and w are the main member axis and lateral deflection function respectively (see Figure 4.1); M is the bending moment at any cross-section. The theoretical buckling solution for each mode is formulated subsequently. Since the prestressing level for the present study is within Zone 3 as stated in the aforementioned assumptions, all the stays would be in tension before buckling.

4.3.1 Mode 1 buckling

The Mode 1 buckling shape of the stayed beam-column can be replaced by the model as shown in Figure 4.1. It comprises a simply-supported main member of length L , where the stays and crossarms are replaced by an elastic spring support that resists the vertical deflection of the main member. To establish the condition where the stayed beam-column becomes unstable, it is necessary to consider force equilibrium of the main member in its deflected shape using the free body diagram in Figure 4.1.

Model:



Free body diagram:

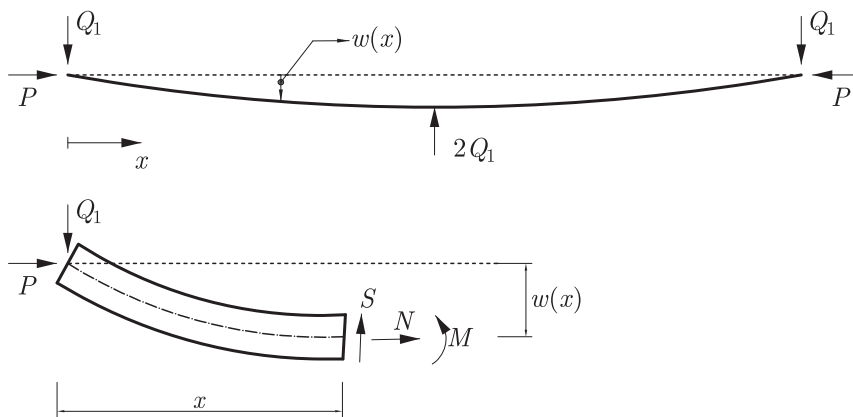


Figure 4.1: Model and free body diagram for Mode 1 buckling; Q_1 is the vertical force on the main member ends resulting from Mode 1 buckling; $2Q_1$ at the main member mid-span is the total vertical force from the stay restraints, currently modelled as a linear longitudinal spring; l is defined as the half-length the main member $L/2$; N , S and M are the cross-sectional axial force, shear force and moment respectively.

The bending moment at any cross-section M can be expressed thus:

$$M = Pw - Q_1x. \quad (4.3)$$

Substituting Eq. (4.3) into Eq. (4.2) gives:

$$E_c I_c \frac{d^2 w}{dx^2} + Pw = Q_1 x. \quad (4.4)$$

Dividing each term in Eq. (4.4) by $E_c I_c$ and using:

$$k = \sqrt{\frac{P}{E_c I_c}}, \quad (4.5)$$

then Eq. (4.4) can be rewritten as:

$$\frac{d^2 w}{dx^2} + k^2 w = \frac{Q_1}{E_c I_c} x, \quad (4.6)$$

which represents the governing differential equation for Mode 1 buckling. The general solution of Eq. (4.6) is:

$$w = A_1 \sin kx + A_2 \cos kx + \frac{Q_1}{P} x, \quad (4.7)$$

where A_1 and A_2 are constants that can be determined from the Mode 1 geometric boundary conditions, as shown in Figure 4.2. Three conditions are required to obtain the particular solution:

1. At $x = 0$, $w = 0$.
2. At $x = l$, $w = w_m$.
3. At $x = l$, $\frac{dw}{dx} = 0$.

The final condition is one of reflective symmetry that is specific to Mode 1. Applying these

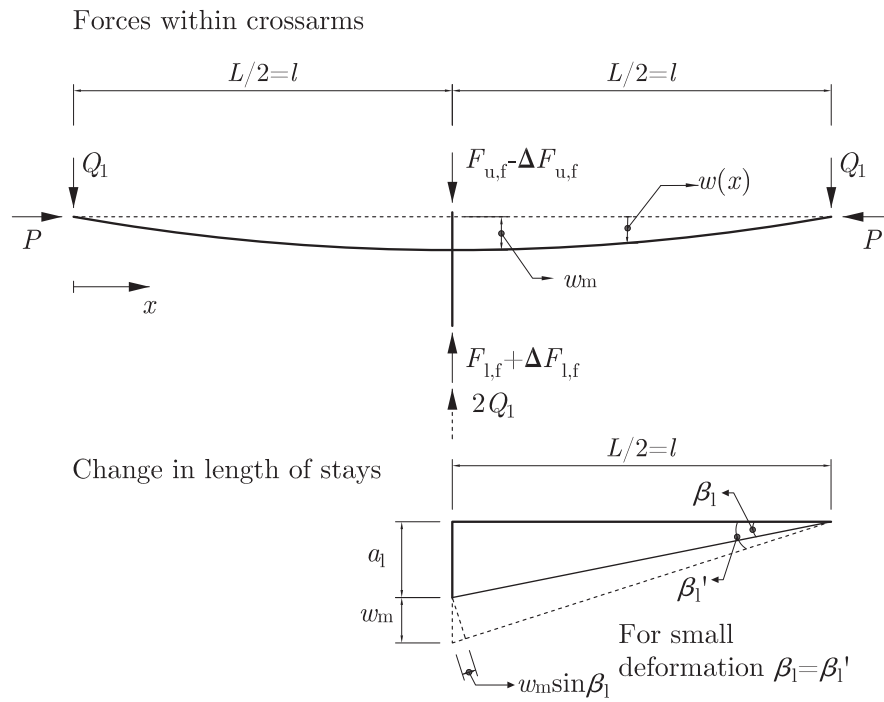


Figure 4.2: Forces and deformations resulting from Mode 1 buckling; w_m is the mid-span vertical displacement of the main member; $F_{u,f}$ and $F_{l,f}$ are the forces within the upper and lower crossarms respectively at the instant of buckling; $\Delta F_{u,f}$ and $\Delta F_{l,f}$ are the absolute changes of force within the upper and lower crossarms respectively resulting from the beam-column buckling.

boundary conditions leads to the following solution for the mid-span deflection $w = w_m$:

$$w_m = \frac{Q_1}{Pk}(kl - \tan kl). \quad (4.8)$$

To complete the buckling solutions, it is necessary to determine the relationship between Q_1 and w_m . This can be done by relating the forces to the deflections produced as a result of the beam-column buckling. The vertical equilibrium of forces acting on the main member at the crossarms (see Figure 4.2) requires that:

$$2Q_1 = (F_{l,f} + \Delta F_{l,f}) - (F_{u,f} - \Delta F_{u,f}). \quad (4.9)$$

From the relationship of the prestressing forces within the upper and lower stays presented in Chapter 3, the axial forces within the crossarms are equal, thus:

$$F_{u,f} = F_{l,f}, \quad (4.10)$$

which gives:

$$2Q_1 = \Delta F_{u,f} + \Delta F_{l,f} = 2\Delta T_{u,f} \sin \beta_u + 2\Delta T_{l,f} \sin \beta_l, \quad (4.11)$$

where $\Delta T_{u,f}$ and $\Delta T_{l,f}$ are the absolute changes of forces within the upper and lower stays respectively resulting from the beam-column buckling. Since the vertical displacement of the main member is assumed to be very small at buckling, the absolute changes in length of the upper and lower stays, $\Delta l_{u,m1}$ and $\Delta l_{l,m1}$, can be expressed respectively as shown in Figure 4.2 thus:

$$\Delta l_{u,m1} = w_m \sin \beta_u, \quad \Delta l_{l,m1} = w_m \sin \beta_l. \quad (4.12)$$

This leads to the expressions of the absolute changes of forces within the upper and lower

stays, $\Delta T_{u,f}$ and $\Delta T_{l,f}$ respectively, thus:

$$\begin{aligned}\Delta T_{u,f} &= K_{su} \Delta l_{u,m1} = K_{su} w_m \sin \beta_u, \\ \Delta T_{l,f} &= K_{sl} \Delta l_{l,m1} = K_{sl} w_m \sin \beta_l,\end{aligned}\tag{4.13}$$

where K_{su} and K_{sl} are the stiffness of upper and lower stays defined in Eq. (3.4). Substituting Eq. (4.13) into Eq. (4.11) gives:

$$Q_1 = K_{su} w_m \sin^2 \beta_u + K_{sl} w_m \sin^2 \beta_l.\tag{4.14}$$

Substituting Eq. (4.14) into Eq. (4.8) and rearranging gives:

$$\frac{(kl)^3}{kl - \tan kl} = D_1,\tag{4.15}$$

where the new quantity D_1 is:

$$D_1 = \frac{K_{su} \sin^2 \beta_u + K_{sl} \sin^2 \beta_l}{K_{cb,half}},\tag{4.16}$$

with the bending stiffness of half of the main member $K_{cb,half}$ being defined thus:

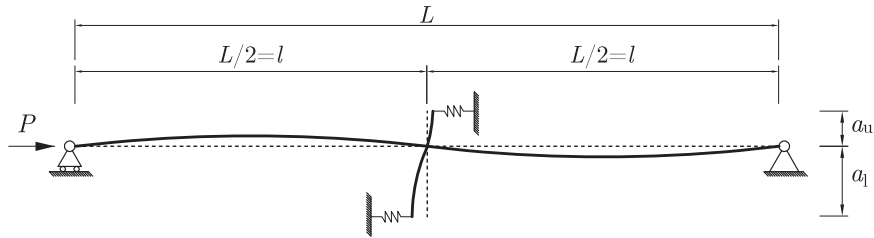
$$K_{cb,half} = \frac{E_c I_c}{l^3}.\tag{4.17}$$

4.3.2 Mode 2 buckling

The Mode 2 buckling shape of the stayed beam-column can be replaced by the model shown in Figure 4.3, where the restraint of the stays is replaced by an elastic spring support. A free body diagram of the forces for this model is also shown. In terms of this case, the bending moment at any cross-section can be expressed thus:

$$M = Pw - Q_2(l - x).\tag{4.18}$$

Model:



Free body diagram:

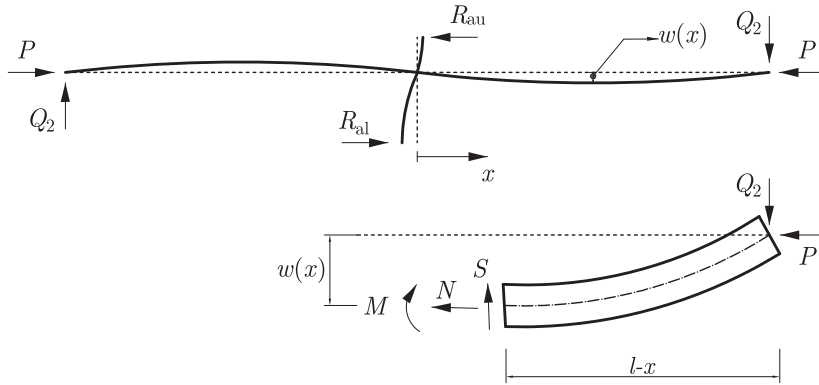


Figure 4.3: Model and free body diagram for Mode 2 buckling; Q_2 is the vertical force on the main member ends resulting from Mode 2 buckling; R_{au} and R_{al} are the total horizontal forces on the upper and lower crossarm tips from the upper and lower stay restraints respectively; N , S and M are the cross-sectional axial force, shear force and moment respectively.

Substituting Eq. (4.18) into Eq. (4.2) leads to:

$$\frac{d^2w}{dx^2} + k^2w = \frac{Q_2}{E_c I_c}(l - x), \quad (4.19)$$

which represents the governing differential equation for Mode 2 buckling. The general solution of Eq. (4.19) is:

$$w = A_3 \sin kx + A_4 \cos kx + \frac{Q_2}{P}(l - x), \quad (4.20)$$

where A_3 and A_4 are constants of integration that can be determined from the Mode 2 geometric boundary conditions, as shown in Figure 4.4; deflections $w_{au,r}$, $w_{au,b}$, $w_{au,f}$, and forces $V_{u,f}$ and $\Delta V_{u,f}$ are for the upper stays corresponding to deflections $w_{al,r}$, $w_{al,b}$, $w_{al,f}$, and forces $V_{l,f}$ and $\Delta V_{l,f}$ for the lower stays. Three conditions are required to obtain the particular solution:

1. At $x = 0$, $w = 0$.
2. At $x = l$, $w = 0$.
3. At $x = 0$, $\frac{dw}{dx} = \theta_{cm}$.

The final condition being the one for rotational symmetry that is specific to Mode 2. Applying these boundary conditions leads to the following equation:

$$\tan kl = \frac{kl}{1 + \frac{P\theta_{cm}}{Q_2}}. \quad (4.21)$$

To determine an appropriate solution for Mode 2 buckling, it is necessary to determine the relationship between θ_{cm} and Q_2 . This can be done by relating the forces to the deflections produced as a result of the main member buckling. From Figure 4.3, moment equilibrium acting on the free body of the main member in conjunction with the crossarms requires

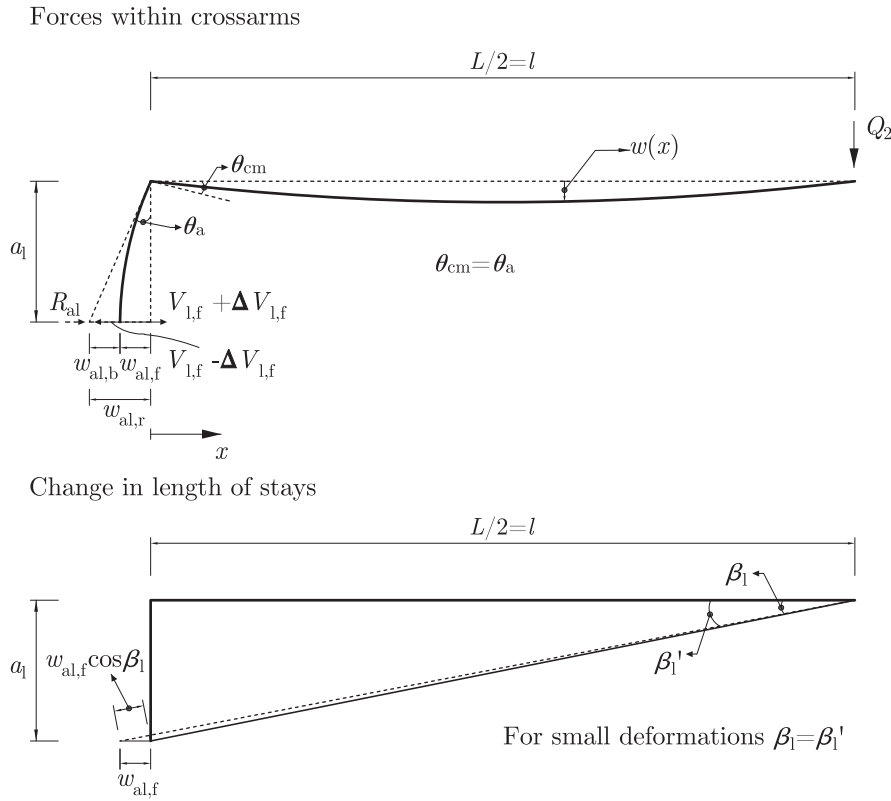


Figure 4.4: Forces, displacements and rotations resulting from Mode 2 buckling; θ_{cm} and θ_a are the rotational angles of the main member and the upper crossarm at their intersection point respectively; $w_{al,r}$ is the rigid body displacement of the lower crossarm tip owing to the joint rotation; $w_{al,b}$ is the displacement of the lower crossarm tip owing to the crossarm bending; $w_{al,f}$ is the final displacement of the lower crossarm tip; $V_{l,f}$ is the horizontal force on the lower crossarm tip at the instant of buckling; $\Delta V_{l,f}$ is the absolute change of the horizontal force on the lower crossarm tip resulting from buckling.

that the following expression is satisfied:

$$2Q_2l = R_{au}a_u + R_{al}a_l, \quad (4.22)$$

where $a_u = l \tan \beta_u$ and $a_l = l \tan \beta_l$ from the preceding geometric definitions. From the equilibrium of forces on the tip of the crossarm shown in Figure 4.4, the following equations are obtained:

$$R_{au} = 2\Delta V_{u,f}, \quad R_{al} = 2\Delta V_{l,f}, \quad (4.23)$$

where $\Delta V_{u,f}$ and $\Delta V_{l,f}$ are the absolute changes of the horizontal forces on the upper and lower crossarms respectively resulting from the buckling, given thus:

$$\Delta V_{u,f} = \Delta T_{u,f} \cos \beta_u, \quad \Delta V_{l,f} = \Delta T_{l,f} \cos \beta_l. \quad (4.24)$$

Since the lateral displacement of the main member is assumed to be very small at the point of instability, the absolute changes in length of the upper and lower stays, $\Delta l_{u,m2}$ and $\Delta l_{l,m2}$, can be expressed respectively, as shown in Figure 4.4, thus:

$$\Delta l_{u,m2} = w_{au,f} \cos \beta_u, \quad \Delta l_{l,m2} = w_{al,f} \cos \beta_l, \quad (4.25)$$

which lead to the expressions for the absolute changes of forces within the upper and lower stays, $\Delta T_{u,f}$ and $\Delta T_{l,f}$ respectively, thus:

$$\begin{aligned} \Delta T_{u,f} &= K_{su} \Delta l_{u,m2} = K_{su} w_{au,f} \cos \beta_u, \\ \Delta T_{l,f} &= K_{sl} \Delta l_{l,m2} = K_{sl} w_{al,f} \cos \beta_l. \end{aligned} \quad (4.26)$$

Substituting Eqs. (4.23)–(4.26) into Eq. (4.22) gives:

$$Q_2 = K_{su} w_{au,f} \cos \beta_u \sin \beta_u + K_{sl} w_{al,f} \cos \beta_l \sin \beta_l. \quad (4.27)$$

From the displacement definitions in Figure 4.4, the relationships among different displace-

ments of the upper and lower crossarm tips are as follows:

$$w_{\text{au},r} = w_{\text{au},f} + w_{\text{au},b}, \quad w_{\text{al},r} = w_{\text{al},f} + w_{\text{al},b}, \quad (4.28)$$

where the rigid body displacements of the upper and lower crossarm tips owing to the joint rotation, $w_{\text{au},r}$ and $w_{\text{al},r}$, are given respectively:

$$w_{\text{au},r} = \theta_a a_u, \quad w_{\text{al},r} = \theta_a a_l. \quad (4.29)$$

Based on Euler–Bernoulli bending theory, the bending displacements of the upper and lower crossarm tips, $w_{\text{au},b}$ and $w_{\text{al},b}$, are given thus:

$$w_{\text{au},b} = \frac{R_{\text{au}}}{3K_{\text{au},b}}, \quad w_{\text{al},b} = \frac{R_{\text{al}}}{3K_{\text{al},b}}. \quad (4.30)$$

Substituting Eqs. (4.23)–(4.26) into Eq. (4.30) gives:

$$w_{\text{au},b} = \frac{2K_{\text{su}}}{3K_{\text{au},b}} w_{\text{au},f} \cos^2 \beta_u, \quad w_{\text{al},b} = \frac{2K_{\text{sl}}}{3K_{\text{al},b}} w_{\text{al},f} \cos^2 \beta_l. \quad (4.31)$$

Substituting Eqs. (4.29) and (4.31) into Eq. (4.28) gives:

$$w_{\text{au},f} = \frac{a_u \theta_a}{1 + \frac{2K_{\text{su}}}{3K_{\text{au},b}} \cos^2 \beta_u}, \quad w_{\text{al},f} = \frac{a_l \theta_a}{1 + \frac{2K_{\text{sl}}}{3K_{\text{al},b}} \cos^2 \beta_l}. \quad (4.32)$$

Substituting Eq. (4.32) into Eq. (4.27) gives:

$$Q_2 = K_{\text{su}} \left[\frac{a_u \theta_a}{1 + \frac{2K_{\text{su}}}{3K_{\text{au},b}} \cos^2 \beta_u} \right] \cos \beta_u \sin \beta_u + K_{\text{sl}} \left[\frac{a_l \theta_a}{1 + \frac{2K_{\text{sl}}}{3K_{\text{al},b}} \cos^2 \beta_l} \right] \cos \beta_l \sin \beta_l. \quad (4.33)$$

From the aforementioned assumptions, $\theta_{\text{cm}} = \theta_a$ owing to the rigid joint at mid-span.

Substituting Eq. (4.33) into Eq. (4.21) gives:

$$\frac{(kl)^2 \tan kl}{kl - \tan kl} = D_2, \quad (4.34)$$

where the new quantity D_2 is:

$$D_2 = \frac{K_{su}}{K_{cb,half}} \left[\frac{\sin^2 \beta_u}{1 + \frac{2K_{su}}{3K_{au,b}} \cos^2 \beta_u} \right] + \frac{K_{sl}}{K_{cb,half}} \left[\frac{\sin^2 \beta_l}{1 + \frac{2K_{sl}}{3K_{al,b}} \cos^2 \beta_l} \right]. \quad (4.35)$$

4.4 Regression solutions

The buckling loads for Modes 1 and 2 can be obtained by solving Eqs. (4.15) and (4.34) respectively. However, these two equations are implicit and cannot be solved in closed form for kl and hence the load P . In order to simplify these transcendental equations to be able to give explicit solutions, regression is used to provide accurate approximations. Currently, $E_c = E_a = E_s$ and $I_c = I_a$ are assumed to simplify the study, although these assumptions are not strictly necessary.

The left-hand side of Eqs. (4.15) and (4.34) can be defined as two functions, $F_{1,theory}$ and $F_{2,theory}$ respectively, thus:

$$F_{1,theory} = \frac{(kl)^3}{kl - \tan kl}, \quad F_{2,theory} = \frac{(kl)^2 \tan kl}{kl - \tan kl}. \quad (4.36)$$

From Eqs. (4.15), (4.16), (4.34) and (4.35), the values of these two functions should be positive. Based on studying the numerical properties of these two functions, when $kl = [1.57, 4.49]$, $F_{1,theory}$ is positive, continuous and monotonically increasing; when $kl = [3.14, 4.49]$, $F_{2,theory}$ is positive, continuous and monotonically increasing. Both of these two functions approach positive infinity as $kl \rightarrow 4.49$.

To obtain an approximate, yet explicit and accurate expression for the buckling load, the required range of $F_{1,theory}$ and $F_{2,theory}$ needs to be investigated. From Eq. (3.21), $T_{l,i} \leq T_{u,i}$ can be obtained with $\beta_l \geq \beta_u$ and $\alpha \leq 1.0$, and:

$$T_{l,i} = T_{u,i} \frac{\sin \beta_u}{\sin \beta_l}. \quad (4.37)$$

From Eq. (4.37), the axial force within the main member arising from the stay stress can be expressed as:

$$T_{l,i} \cos \beta_l + T_{u,i} \cos \beta_u = \left(\frac{\sin \beta_u}{\sin \beta_l} \cos \beta_l + \cos \beta_u \right) T_{u,i} \leq \left(\frac{\sin \beta_u}{\sin \beta_l} \cos \beta_l + \cos \beta_u \right) f_y A_s, \quad (4.38)$$

where f_y is the yield stress of applied material, and the material currently is carbon steel. The relationship in Eq. (4.38) considers the load-carrying capacity of the main member from the perspective of a practical material strength. Considering the status where there is no external axial load, the stress within stays may lead to the buckling of the main member, and the following inequality needs to be satisfied, thus:

$$T_{l,i} \cos \beta_l + T_{u,i} \cos \beta_u \leq P^C, \quad (4.39)$$

where the critical load P^C can be determined based on Euler buckling, thus:

$$P^C = P_E = \frac{\pi^2 E_c I_c}{l_e^2}, \quad (4.40)$$

where the effective length $l_e = nl$, and n is the effective length factor for one-half of the main member. The relationship in Eq. (4.39) considers the load-carrying capacity of the main member from the perspective of elastic buckling. When the axial load applied to the main member $P = P^C$, substituting Eq. (4.40) into Eq. (4.5) gives:

$$kl = l \sqrt{\frac{P^C}{E_c I_c}} = \frac{\pi}{n}. \quad (4.41)$$

In order to maximize the material efficiency of both the stays and main member, from Eqs. (4.38) and (4.39), the following equation is obtained:

$$\frac{\pi^2 E_c I_c}{(nl)^2} = \left(\frac{\sin \beta_u}{\sin \beta_l} \cos \beta_l + \cos \beta_u \right) f_y A_s. \quad (4.42)$$

Replacing $f_y = E_s \varepsilon_{s,y}$ where $\varepsilon_{s,y}$ is the yield strain of the stay material and rearranging

Eq. (4.42) gives:

$$\frac{A_s l^2}{I_c} = \frac{1}{n^2 \varepsilon_{s,y}} \frac{\pi^2}{\frac{\sin \beta_u}{\sin \beta_l} \cos \beta_l + \cos \beta_u} = \frac{\Omega}{n^2 \varepsilon_{s,y}}, \quad (4.43)$$

where Ω is defined thus:

$$\Omega = \frac{\pi^2}{\frac{\sin \beta_u}{\sin \beta_l} \cos \beta_l + \cos \beta_u}. \quad (4.44)$$

4.4.1 Regression function: Mode 1

In terms of Mode 1 buckling, the range of kl depends on the boundary condition of half of the main member, as illustrated in Figure 4.5.

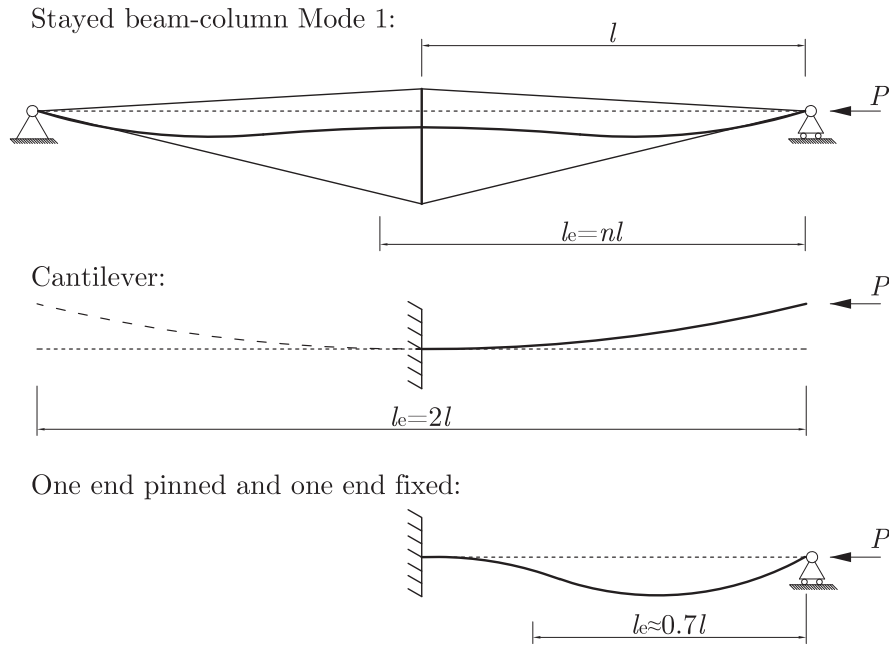


Figure 4.5: The range of the effective length of half of the main member for Mode 1 buckling; l_e is the effective length of each case.

From Figure 4.5, the effective length of half of the main member in the stayed beam-column is between that of the cantilever and that of the case where one end is pinned and the other is fixed. Hence, $n = [0.7, 2]$, and $kl = [\pi/2, \pi/0.7] \approx [1.571, 4.488]$, which approximates to the aforementioned range of $F_{1,\text{theory}}$ obtained numerically. Considering $E_c = E_a = E_s$,

substituting Eqs. (3.4) and (4.17) into Eq. (4.16) and rearranging gives:

$$D_1 = \frac{A_s l^2}{I_c} (\cos \beta_u \sin^2 \beta_u + \cos \beta_1 \sin^2 \beta_1). \quad (4.45)$$

Substituting Eq. (4.43) into (4.45) gives:

$$D_1 = \frac{\Omega}{n^2 \varepsilon_{s,y}} (\cos \beta_u \sin^2 \beta_u + \cos \beta_1 \sin^2 \beta_1). \quad (4.46)$$

From BS EN 1993-1-1 (2005), for the commonly used carbon steel, the yield stress $f_y = [215, 460]$ MPa, and Young's Modulus $E = 210$ GPa; hence, $\varepsilon_{s,y} = [0.10\%, 0.22\%]$. In addition, $\alpha_1 = [0.01, 0.15]$, $\alpha = [0.1, 1.0]$ and $n = (0.7, 2)$. Finally, $D_1 = [0.37, 1663.11]$ can be determined numerically, and $D_1 = [0, +\infty)$ corresponding to $kl = [\pi/2, \pi/0.7]$ can be adopted as the required range of $F_{1,\text{theory}}$ to determine $F_{1,\text{regression}}$. Applying regression techniques (Bethea, 1995), the following expressions are obtained:

$$F_{1,\text{regression}} = \begin{cases} -3.4206 + 7.0485 \tan(0.4573kl - 0.2671), & \text{if } kl \in (\pi/2, 2.42]; \\ 0.2259 + 11.2878 \tan(0.4753kl - 0.7939), & \text{if } kl \in [2.42, 4.18]; \\ 15.9457 + 2.3136 \tan(0.5139kl - 0.7384), & \text{if } kl \in [4.18, \pi/0.7). \end{cases} \quad (4.47)$$

The details for determining these expressions are presented in Appendix A.

4.4.2 Regression function: Mode 2

In terms of Mode 2 buckling, the range of kl also depends on the boundary condition half of the main member, as illustrated in Figure 4.6. In the case of Mode 2, $n = [0.7, 1]$, $kl = [\pi, \pi/0.7] \approx [3.142, 4.488]$, which approximates to the aforementioned range of $F_{2,\text{theory}}$ obtained numerically. Considering the assumptions where $E_c = E_a = E_s$, substituting Eqs.

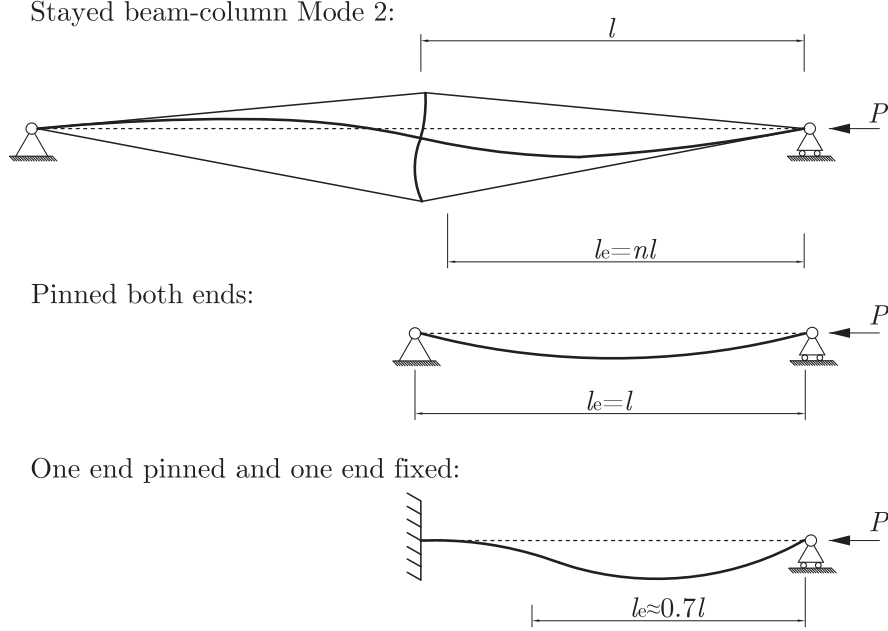


Figure 4.6: The range of the effective length of half of the main member for Mode 2 buckling; l_e is the effective length of each case.

(3.4) and (4.17) to Eq. (4.35), and rearranging gives:

$$D_2 = \frac{A_s l^2}{I_c} \left(\frac{\cos \beta_u \sin^2 \beta_u}{1 + \frac{2A_s l^2}{3I_a} \sin^3 \beta_u} + \frac{\cos \beta_l \sin^2 \beta_l}{1 + \frac{2A_s l^2}{3I_a} \sin^3 \beta_l} \right). \quad (4.48)$$

Considering $I_c = I_a$, substituting Eq. (4.43) to Eq. (4.48) gives:

$$D_2 = \frac{\Omega}{n^2 \varepsilon_{s,y}} \left(\frac{\cos \beta_u \sin^2 \beta_u}{1 + \frac{\Omega}{n^2 \varepsilon_{s,y}} \sin^3 \beta_u} + \frac{\cos \beta_l \sin^2 \beta_l}{1 + \frac{\Omega}{n^2 \varepsilon_{s,y}} \sin^3 \beta_l} \right). \quad (4.49)$$

Similarly, from BS EN 1993-1-1 (2005), $\varepsilon_{s,y} = [0.10\%, 0.22\%]$. Considering $\alpha_1 = [0.01, 0.15]$, $\alpha = [0.1, 1.0]$ and $n = [0.7, 1]$, $D_2 = [1.45, 23.29]$ can be determined numerically, and $D_2 = [1.45, 25.00]$ corresponding to $kl = [3.50, 4.33]$ can be adopted as the required range of $F_{2,\text{theory}}$ to determine $F_{2,\text{regression}}$. Applying regression techniques (Bethea, 1995), the following expression is obtained:

$$F_{2,\text{regression}} = -2.3799 + 3.2130 \tan(0.6999kl - 1.5753), \quad kl = [3.50, 4.33]. \quad (4.50)$$

The details for determining these expressions are also presented in Appendix A.

4.4.3 Error analysis

The comparison between the theoretical and regression functions is shown in Figure 4.7. It can be seen that the theory and regression functions both show excellent agreement for

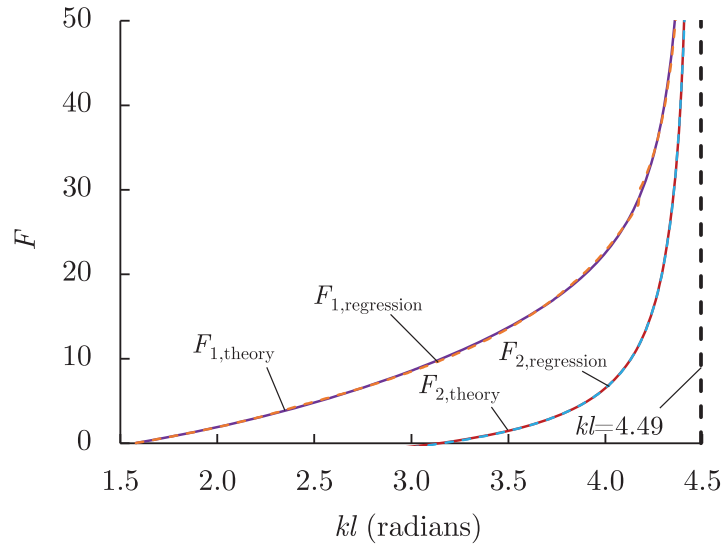


Figure 4.7: Comparison between the theory and regression functions for the buckling loads.

F_1 and F_2 . The distribution of the error between the theory and regression functions is shown in Figure 4.8. As can be seen, the absolute error for $F_{1,regression}$ is mostly less than 3%; the absolute error for $F_{2,regression}$ is mostly less than 0.4%. This level of accuracy is sufficient for engineering practice.

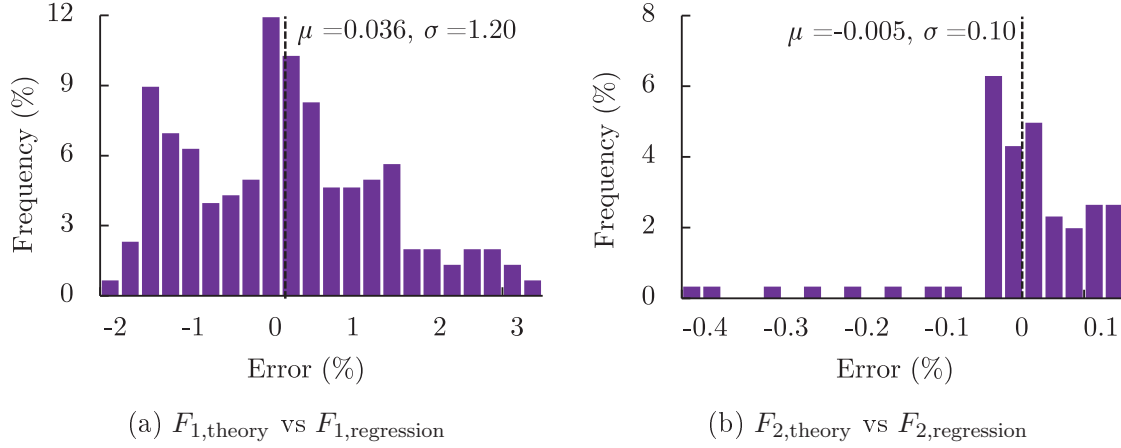


Figure 4.8: Distribution of the error between the theory and regression functions. ‘Frequency’ is the percentage of points where the error is within a specific range over all points; ‘Error’ is calculated by $100\% \times (F_{i,\text{regression}} - F_{i,\text{theory}})/F_{i,\text{theory}}$, where $i = \{1, 2\}$; μ and σ are the mean and standard deviation of the frequency respectively.

4.4.4 Buckling loads

Replacing $F_{1,\text{theory}}$ and $F_{2,\text{theory}}$ in Eqs. (4.15) and (4.34) with $F_{1,\text{regression}}$ and $F_{2,\text{regression}}$ in Eqs. (4.47) and (4.50) give:

$$D_i = F_{i,\text{regression}} = b_{i,1} + b_{i,2} \tan(b_{i,3}kl + b_{i,4}), \quad (4.51)$$

where $i = \{1, 2\}$ corresponds to Mode 1 or 2 respectively. Substituting Eq. (4.5) into Eq. (4.51) and rearranging gives the regression solution of the buckling load for Mode i , $P_{i,\text{regression}}^C$, thus:

$$P_{i,\text{regression}}^C = \frac{E_c I_c}{l^2} \left[\frac{\arctan\left(\frac{D_i - b_{i,1}}{b_{i,2}}\right) - b_{i,4}}{b_{i,3}} \right]^2, \quad (4.52)$$

where $b_{i,j}$ represents the parameters from the preceding regression study, and $i = \{1, 2\}$. In terms of Mode 1 buckling ($i = 1$), from Eq. (4.47), $b_{1,j}$, where $j = \{1, 2, 3, 4\}$, can be

determined thus:

$$\begin{cases} b_{1,1} = -3.4206, b_{1,2} = 7.0485, b_{1,3} = 0.4573, b_{1,4} = -0.2671, & \text{if } D_1 = (0, 4.42]; \\ b_{1,1} = 0.2259, b_{1,2} = 11.2878, b_{1,3} = 0.4753, b_{1,4} = -0.7939, & \text{if } D_1 = [4.42, 30.18]; \\ b_{1,1} = 15.9457, b_{1,2} = 2.3136, b_{1,3} = 0.5139, b_{1,4} = -0.7384, & \text{if } D_1 = [30.18, +\infty). \end{cases} \quad (4.53)$$

In terms of Mode 2 buckling ($i = 2$), from Eq. (4.50), $b_{2,j}$ can be determined thus:

$$b_{2,1} = -2.3799, b_{2,2} = 3.2130, b_{2,3} = 0.6999, b_{2,4} = -1.5753, \quad D_2 = [1.45, 25.00]. \quad (4.54)$$

4.5 FE verification

To verify the aforementioned theoretical and regression solutions of the buckling loads for Modes 1 and 2, linear buckling analysis within ABAQUS is conducted. The element type and size are the same as those for the numerical studies within Chapter 3. The aforementioned doubly-symmetric cases F1–F6, A1–A6 and mono-symmetric cases F3.1–F3.9 in Tables 3.2 and 3.3 are adopted as examples. Figures 4.9, 4.10 and 4.11 compare the buckling loads from the FE modelling, theoretical modelling and regression for the doubly-symmetric cases, *i.e.* F1–F6 and A1–A6, and mono-symmetric cases, *i.e.* F3.1–F3.9, respectively. They show excellent agreement for all these examples between the theoretical and regression solutions. Compared to the theoretical model, the stay and crossarm within the FE model both increase the restraint at the mid-span of the main member, which lead to different mid-span deflections and affect the buckling loads. This effect is seen to be most significant where the stay diameter is large.

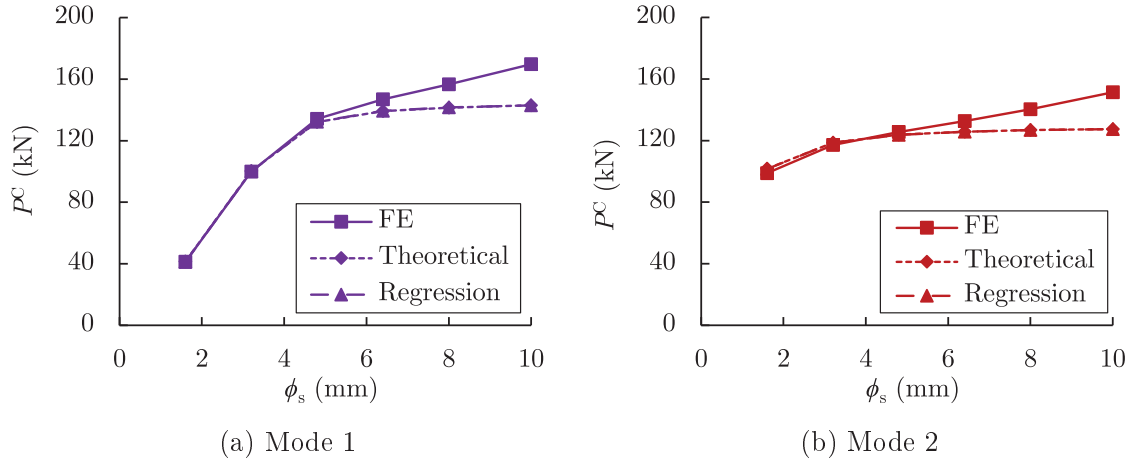


Figure 4.9: Buckling load comparison from FE modelling, theoretical modelling and regression expressions for cases F1–F6: P^C is the buckling load for Mode 1 or 2; ‘FE’ represents the results from the linear buckling analysis within ABAQUS; ‘Theoretical’ represents the results from the theoretical implicit equations; ‘Regression’ represents the results from the currently obtained explicit regression equations.

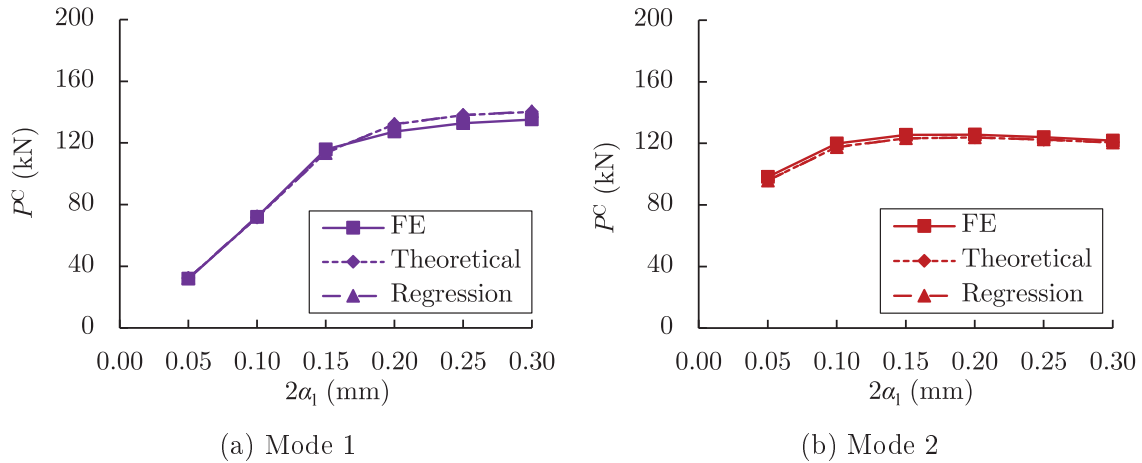


Figure 4.10: Buckling load comparison from FE modelling, theoretical modelling and regression for cases A1–A6. Axes and labels are identical to Figure 4.9.

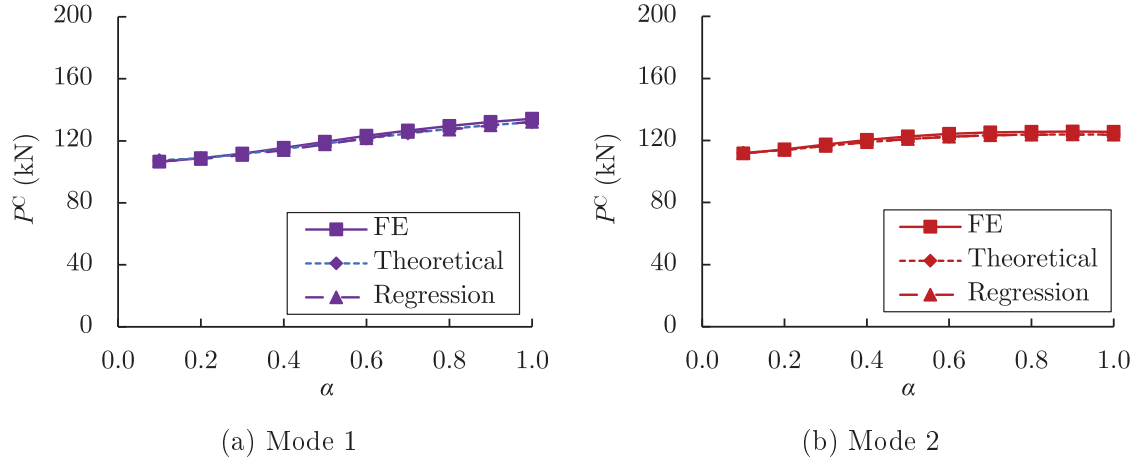


Figure 4.11: Buckling load comparison from FE modelling, theoretical modelling and regression for cases F3.1–F3.9. Axes and labels are identical to Figure 4.9.

4.6 Concluding remarks

The implicit theoretical and explicit regression solutions for buckling loads, which can be applied to both the doubly-symmetric and mono-symmetric stayed beam-columns are obtained; as far as the author is aware, the latter for the first time. The results show excellent agreement against each other as well as with the buckling eigenvalues from the linear buckling analysis using FE modelling. The regression solutions can be used to evaluate the theoretical optimum prestressing level that can simplify the preceding process of obtaining the linear optimum prestress obtained in Chapter 3 and make it more applicable for engineering practice.

Moreover, the theoretical and regression solutions presented currently are used to conduct the preliminary study of design methods for stayed beam-columns in Chapter 7.

Chapter 5

Analytical model for mode interaction

5.1 Introduction

In the previous chapters, the critical buckling behaviour of the stayed beam-column, *i.e.* the zonal behaviour and buckling loads, has been investigated using linear analytical modelling. From the existing studies (Saito & Wadee, 2009b; Wadee *et al.*, 2013a; Yu & Wadee, 2017a), it is well known that the theoretical compressive strength enhancements under certain configurations can only be obtained at the expense of triggering a sequence of destabilizing bifurcations. This can give rise to severely unstable interactive post-buckling behaviour including the so-called ‘mode jumping’ phenomenon. However, the aforementioned studies only focus on these phenomena in vertical doubly-symmetric stayed columns under purely axial compression. In the current chapter, following the analytical work based on the Rayleigh–Ritz method (Saito & Wadee, 2008; Yu & Wadee, 2017a), a nonlinear analytical model for horizontal stayed members with both the doubly-symmetric and mono-symmetric configurations under axial compression in conjunction with self-weight, is presented that captures modal interactions for perfect geometries explicitly. Moreover, the analytical model is verified using a nonlinear FE model formulated within ABAQUS and excellent

comparisons are obtained.

5.2 Nonlinear analytical modelling

The principal purpose of the current analytical work is to model the geometrically nonlinear mechanical behaviour of the stayed beam-column. Bending of the main member element and the crossarms is included, as is the explicit response of the prestressed cable stays. The Rayleigh–Ritz method is selected to form the basis of the model.

In the current analytical study, the numbering of the stays, the geometric parameters and crossarm ratios have been defined in Figure 3.1. The axial and bending stiffnesses of members within the currently considered prestressed stayed beam-column have been defined in Eqs. (3.4) and (4.1) respectively. The loading steps illustrated in Figure 3.2 are adopted, and the current Rayleigh–Ritz based analytical model is devised to study the structural response within loading step 4. The zonal behaviour and expressions for the benchmark prestressing forces for prestressed stayed beam-columns under self-weight and axial compression have also been obtained and presented in Chapter 3 (Wu *et al.*, 2019). The optimum prestressing force T_{opt1} within the lower stay corresponding to T_{opt12} within the upper stay is adopted as the benchmark prestressing force. It was indeed confirmed in Chapter 3 that the initial prestressing force should be considerably larger than the optimum prestressing level, *i.e.* within Zone 3, such that stays would not slacken before buckling and the ultimate load would be maximized. By assuming that the initial prestressing level places the beam-column within Zone 3, it allows the presented analytical model to be simplified somewhat.

The resulting nonlinear equilibrium equations can then be solved within the well-established numerical continuation and bifurcation software AUTO-07P (Doedel & Oldeman, 2011) to reveal the critical and post-buckling behaviour of the beam-columns, while retaining the

bifurcational structure of the mechanical system. This provides key information regarding the modal interactions and hence strong indications regarding any potential sensitivities to initial geometric imperfections.

5.2.1 Shape functions

The buckling profile of the main member element can be expressed as a combination of a symmetric shape W_1 for Mode 1, an antisymmetric shape W_2 for Mode 2 and a symmetric shape from the combination of self-weight and pre-cambering from the prestressing forces W_3 , thus:

$$\begin{aligned} W_1(x) &= \sum_{i=1}^m u_{2i-1} L \sin \frac{(2i-1)\pi x}{L}, \\ W_2(x) &= \sum_{i=1}^m u_{2i} L \sin \frac{2i\pi x}{L}, \\ W_3(x) &= u_{2m+1} L \frac{x(L-x)}{L^2} - 4u_{2m+1} L \frac{x^2(L-x)^2}{L^4}. \end{aligned} \tag{5.1}$$

In these expressions u_j are the generalized coordinates for the buckling mode shape contribution that contains j half-sine waves with L being the total length of the main column member; m is the total number of pairs of the sine functions. The adoption of a larger m value allows the expressions above to describe deformations that are closer to the actual profiles. However, of course, this results in more equations to be solved, thereby increasing the overall complexity of the model. The overall shape of the main member may be obtained by combining the two distinct buckling shapes that would break both the reflective and rotational symmetries if modal interactions are triggered. The deformations from the combination of self-weight and the pre-cambering effect from the prestressing forces are also included, thus:

$$W(x) = W_1(x) + W_2(x) + W_3(x). \tag{5.2}$$

The shape function is illustrated in Figure 5.1. The normalized vertical deflection at the

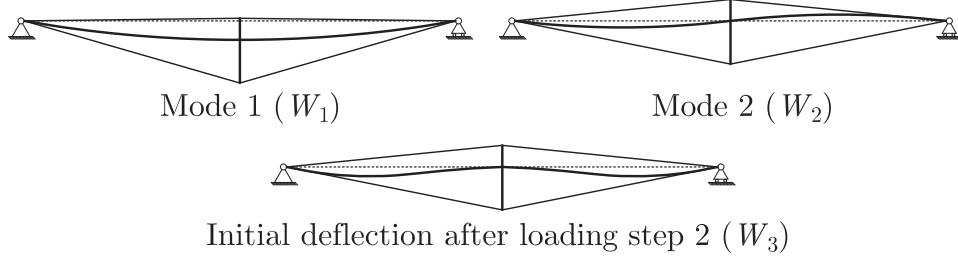


Figure 5.1: Deformed profile of each part of the shape function. ‘Mode 1’ is the buckling mode with reflective symmetry about midspan. ‘Mode 2’ is the buckling mode with rotational symmetry about midspan. When modal imperfections are introduced, these take the leading order expressions of Modes 1 and 2 (from W_1 and W_2 respectively) with specific scaling factors and a small amplitude.

midspan of the main member δ_m can be expressed thus:

$$\delta_m L = W(L/2), \quad (5.3)$$

while the rotation of the main member about the horizontal x axis at midspan θ_{cm} can be expressed to leading order thus:

$$\theta_{cm} = -W'(L/2), \quad (5.4)$$

where a prime denotes differentiation with respect to the longitudinal coordinate x . Since the joint between crossarms and the main member is rigid, the main member and crossarm rotations are assumed to be equal thus:

$$\theta_a = \theta_{cm}. \quad (5.5)$$

Presently, there is one set of crossarms in the considered prestressed stayed beam-columns. The shape function for an arbitrary crossarm, $W_t(y)$, where $t = \{u, l\}$ for the upper and lower crossarms respectively, is defined thus:

$$W_t(y) = H_{t0} + H_{t1}y + H_{t2} \sin k_{at}y + H_{t3} \cos k_{at}y, \quad (5.6)$$

where k_{at} is the ‘circular frequency’ for the horizontal deflection of a crossarm, as shown in Figure 5.2. The functional form of the shape function is derived from an analysis presented

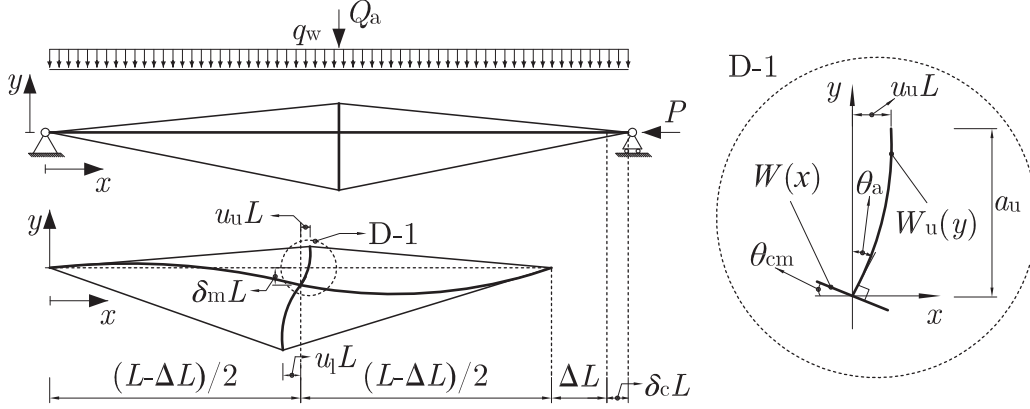


Figure 5.2: Deflected shape of the prestressed stayed beam-column: D-1 is a detailed figure for the shape-function definition of the upper crossarm; u_u and u_l are the generalized coordinates of the horizontal displacement of the upper and lower crossarm tips respectively; $W(x)$ and $W_u(y)$ are the shape functions of the main member and the upper crossarm respectively; θ_{cm} and θ_a are the respective angles at the intersection of the main member to the horizontal and the upper crossarm to the vertical; ΔL and $\delta_c L$ are the end-shortening displacements arising from the vertical deflection of the beam-column and from purely axial compression respectively.

in Saito & Wadee (2008) in which Euler–Bernoulli bending theory was applied to a beam-column representation of the upper and lower crossarms; the details of which are presented in Appendix B. The analysis shows that k_{au} and k_{al} can be expressed thus:

$$k_{au} = \frac{\kappa}{\sqrt{\alpha}} \sqrt{\frac{2T_{u,i}a_u}{E_a I_a l_u}}, \quad k_{al} = \kappa \sqrt{\frac{2T_{l,i}a_l}{E_a I_a l_l}}, \quad (5.7)$$

with κ being a calibration factor that is introduced to correct the crossarm shape since the main element and both crossarms bend simultaneously. The relationship between κ and α is calibrated numerically and the results are presented in §5.4.1.

Four boundary conditions are required for evaluating the precise shape function of this crossarm:

1. The lateral deflection at the joint is zero, hence $W_t(0) = 0$.

2. The horizontal deflection at the tip of the upper crossarm $W_t(a_t) = u_t L$.
3. The joint of the crossarm is rigidly connected to the main member, therefore the rotation of the crossarm at the connection is related to the rotation of the main member, hence $W'_t(0) = \theta_a = \theta_{cm}$, as discussed above.
4. The tip of the crossarm is pinned to the stays, hence there is no bending moment at the tip, therefore $W''_t(a_t) = 0$.

These conditions allow the shape function constants in Eq. (5.6) to be evaluated thus:

$$\begin{aligned}
 H_{t0} &= -\frac{u_t L - a_t \theta_a}{a_t k_{at} - \tan(k_{at} a_t)} \tan(k_{at} a_t), & H_{t1} &= \theta_a + \frac{k_{at}(u_t L - a_t \theta_a)}{a_t k_{at} - \tan(k_{at} a_t)}, \\
 H_{t2} &= -\frac{u_t L - a_t \theta_a}{a_t k_{at} - \tan(k_{at} a_t)}, & H_{t3} &= \frac{u_t L - a_t \theta_a}{a_t k_{at} - \tan(k_{at} a_t)} \tan(k_{at} a_t).
 \end{aligned} \tag{5.8}$$

5.2.2 Strain energy stored within stays

The initial lengths of the upper and lower stays are expressed in Eq. (3.3). When the external force P is applied, the beam-column is purely compressed. When the main member begins to deflect vertically by $W(x)$, the generalized deformed shape of the stayed beam-column is illustrated in Figure 5.2, and the expressions for the new lengths of each stay are thus:

$$\begin{aligned}
 l'_{u1} &= L \sqrt{(\Lambda + u_u)^2 + (\alpha_u - \delta_m)^2}, \\
 l'_{u2} &= L \sqrt{(\Lambda - u_u)^2 + (\alpha_u - \delta_m)^2}, \\
 l'_{l1} &= L \sqrt{(\Lambda - u_l)^2 + (\alpha_l + \delta_m)^2}, \\
 l'_{l2} &= L \sqrt{(\Lambda + u_l)^2 + (\alpha_l + \delta_m)^2},
 \end{aligned} \tag{5.9}$$

in which $\Lambda = (1 - \Delta)/2$. Apart from the strains generated by changing stay lengths, there is also a pre-existing strain introduced by the initial prestressing force that can be expressed thus:

$$\varepsilon_{Ttn} = \frac{T_{tn,i}}{E_s A_s}, \tag{5.10}$$

where $T_{tn,i}$ is the initial prestressing force applied within stay tn , where $t = \{u, l\}$ and $n = \{1, 2\}$. Therefore, the total strain in each stay can be expressed thus:

$$\varepsilon_{tn} = l'_{tn}/l_{tn} + \varepsilon_{Ttn} - 1, \quad (5.11)$$

where positive values for ε_{tn} imply tensile strain. The strain energy within stay tn , U_{stn} , can be expressed thus:

$$U_{stn} = \frac{1}{2} E_s A_s l_{tn} \varepsilon_{tn}^2, \quad (5.12)$$

where the non-trivial strain ε_{tn} must be positive. It may be observed that when the main member begins to deflect vertically, the strain energy within the stays on the convex side increases as the magnitude of the bending curvature in the main member element becomes larger, while the strain energy within the stays on the concave side decreases commensurately. Eventually, one or more stays on the concave side of the main member may go slack when the main member end-shortening displacement reaches certain values. Since the stays cannot resist compression or bending, these components of the strain energy would effectively vanish when the stay strains decrease to zero and below. To model this discontinuous behaviour in terms of the strain energy, a series of switches η_{tn} is introduced into the energy formulation, thus:

$$U_s = \frac{1}{2} E_s A_s \sum_{i=1}^8 \eta_{tn} l_{tn} \varepsilon_{tn}^2, \quad (5.13)$$

where:

$$\eta_{tn} = \begin{cases} 1 & \text{if } \varepsilon_{tn} > 0, \\ 0 & \text{if } \varepsilon_{tn} \leq 0. \end{cases} \quad (5.14)$$

Note that during the interactive buckling process, the stays on the concave side may become slack separately owing to the broken symmetry introduced by the nonlinear coupling of the symmetric Mode 1 and the antisymmetric Mode 2.

5.2.3 Strain energy from bending and axial compression

Strain energy from bending exists in the main member and the crossarm elements. With the introduction of the respective shape functions, the bending energy of the main member U_{cb} is given by the standard expression:

$$U_{cb} = \frac{1}{2} E_c I_c \int_0^L W''^2 dx. \quad (5.15)$$

The general expression for the bending energies stored within the upper and lower crossarms U_{at} , where $t = \{u, l\}$, is thus:

$$U_{at} = \frac{1}{2} E_a I_a \int_0^{a_t} W_t''^2 dy. \quad (5.16)$$

As shown in Figure 5.2, the main member also has a purely compressive end-shortening displacement $\delta_c L$ that is independent of $W(x)$. Hence, the contribution to the purely compressive strain energy U_{ca} is:

$$U_{ca} = \frac{1}{2} E_c A_c \int_0^L \delta_c^2 dx. \quad (5.17)$$

The crossarms, in turn, are assumed to be inextensional and so the model is only in fact valid for prestressed beam-columns with an initial prestressing force in Zone 3, where all stays are in tension at the point of initial instability. The purpose of this assumption is to simplify a highly complex system to one that can be modelled analytically. Moreover, previous work (Saito & Wadee, 2010) has demonstrated conclusively that the load-carrying capacity is maximized when the prestressing level is higher than the theoretical linear optimum prestressing force, so this assumption does not reduce the practical applicability of the model in the nonlinear range.

5.2.4 Work done by loads

The total work done by external loads Φ comprises two parts: the work done by the axial load Φ_P , and the work done by the self-weight Φ_W , thus:

$$\Phi = \Phi_P + \Phi_W. \quad (5.18)$$

The work done by the axial load Φ_P is:

$$\Phi_P = P\mathcal{E}, \quad (5.19)$$

where $\mathcal{E}$ is the main member end-shortening displacement. From Figure 5.2, $\mathcal{E}$ can be expressed thus:

$$\mathcal{E} = L(\delta_c + \Delta), \quad (5.20)$$

where ΔL is given by the well-known first-order expression for the inextensional end-shortening displacement of a strut (Thompson & Hunt, 1973) as follows:

$$\Delta L = \frac{1}{2} \int_0^L W'^2 \, dx, \quad (5.21)$$

and δ_c is simply thus:

$$\delta_c = \frac{P}{E_c A_c}. \quad (5.22)$$

The work done by the self-weight Φ_W comprises two parts: the work done by the self-weight of the main member $\Phi_{W,c}$ and the crossarms $\Phi_{W,a}$, thus:

$$\Phi_W = \Phi_{W,c} + \Phi_{W,a}. \quad (5.23)$$

The work done by the self-weight of the main member $\Phi_{W,c}$ is:

$$\Phi_{W,c} = \int_0^L q_w W(x) \, dx, \quad (5.24)$$

where the uniformly distributed load (UDL) intensity from the self-weight of the main member q_w is:

$$q_w = \rho A_c g, \quad (5.25)$$

with ρ being the density of the crossarm material and g is the acceleration due to gravity (9.81 m/s^2). The work done by the self-weight of the crossarms $\Phi_{W,a}$ can be expressed thus:

$$\Phi_{W,a} = Q_a \delta_m L = \rho g A_a (a_l + a_u) \delta_m L. \quad (5.26)$$

5.2.5 Total potential energy

The total potential energy of the system comprises the strain energy stored minus the work done by the external loads. The strain energy comprises the axial strain energy of the main member, the bending energies of the main member and the crossarms alongside the strain energy stored from stretching the stays. Therefore, the total potential energy V is obtained thus:

$$V = U_{ca} + U_{cb} + U_s + U_{au} + U_{al} - \Phi. \quad (5.27)$$

Invoking the stationary potential energy principle gives the following equilibrium equations:

$$\frac{\partial V}{\partial \delta_c} = 0, \quad \frac{\partial V}{\partial u_i} = 0, \quad \frac{\partial V}{\partial u_u} = 0, \quad \frac{\partial V}{\partial u_l} = 0, \quad (5.28)$$

where the subscript i ranges from 1 to $2m + 1$ (inclusive) and m was defined in Eq. (5.1). The first equation returns the pre-buckling path, thus:

$$\delta_c = \frac{P}{E_c A_c}. \quad (5.29)$$

The remaining expressions lead to a system of $2m + 3$ simultaneous nonlinear algebraic equations that are formulated within the symbolic computation package Maple (Maple, 2015) and are subsequently solved numerically using the continuation and bifurcation soft-

ware AUTO-07P (Doedel & Oldeman, 2011). This is a sophisticated and powerful package designed for solving nonlinear algebraic and ordinary differential equations. The principal feature of AUTO-07P is numerical continuation (Seydel, 1994), a technique that can solve nonlinear equations and trace evolving solution paths as parameters are varied while also pinpointing bifurcations and evaluating branching solution paths. The software was used successfully in Yu & Wadee (2017a) and is thus used in the present study.

5.3 FE modelling

In the current study, FE modelling is used to verify the results obtained from the analytical model. The element type and size are the same as those for the numerical studies within the previous chapters. In a similar fashion to the numerical study of interactive buckling within imperfect stayed columns (Saito & Wadee, 2009b), an initial imperfection shape w_{imp} for the main beam-column element is introduced, thus:

$$w_{\text{imp}} = \delta L \left[\mu_1 \sin \frac{\pi x}{L} + \mu_2 \sin \frac{2\pi x}{L} \right]. \quad (5.30)$$

In this expression, δ is the imperfection amplitude ratio, which in the current study is kept to be very small, whereas μ_1 and μ_2 are scale factors introduced to represent the proportions of the initial imperfection shape of the stayed beam-column that are affine to buckling Modes 1 and 2 of regular (unstayed) prismatic columns respectively – see Figure 5.1 and the leading terms of W_1 and W_2 presented in Eq. (5.1). The first order approximation of the end-shortening $\mathcal{E}_0$ is expressed as follows to ensure that imperfections of commensurate magnitude may be compared regardless of the detailed shape of the initial imperfections (Wadee, 2000; Wadee & Simões da Silva, 2005):

$$\mathcal{E}_0 = \int_0^L \frac{1}{2} w_{\text{imp}}'^2 \, dx. \quad (5.31)$$

Substituting Eq. (5.30) into (5.31) gives:

$$\mathcal{E}_0 = \frac{1}{4}\pi^2\delta^2L(\mu_1^2 + 4\mu_2^2). \quad (5.32)$$

Therefore, the relationship between μ_1 and μ_2 being:

$$\mu_1^2 + 4\mu_2^2 = 1, \quad (5.33)$$

which was determined from the basis that different initial imperfection shapes should have an equal initial end-shortening displacement such that a fair comparison can be made between imperfection shapes of different wavelengths. This relationship is used to introduce an initial imperfection to the stayed beam-column geometry to simulate either a pure mode or an interactive nonlinear buckling mode within FE modelling. Apart from the perfect case (without considering imperfections), other imperfection combination cases are listed in Table 5.1.

Table 5.1: Imperfection combination cases: cases 1 and 5 represent pure eigenmode buckling simulations for Modes 1 and 2 respectively; cases 2–4 represent those that are designed to trigger interactive buckling responses.

Case	1	2	3	4	5
μ_1	1.000	0.750	0.500	0.250	0.000
μ_2	0.000	0.331	0.433	0.484	0.500

Similar to the analytical modelling, the loading steps illustrated in Figure 3.2 are adopted, and the optimum prestressing force T_{opt1} within the lower stay corresponding to T_{opt12} within the upper stay is adopted as the benchmark prestressing force.

5.4 Results and discussion

The principal purpose of the current section is to investigate the nonlinear mechanical behaviour based on the currently described nonlinear analytical and FE models; the latter has been shown in the literature to provide an excellent benchmark when compared to actual physical experiments (Osofero *et al.*, 2012, 2013). For the examples in the current studies, the material properties and cross-section parameters described in Table 3.1 are adopted, and the aforementioned doubly-symmetric cases F1–F6, A1–A6 and mono-symmetric cases F3.1–F3.9 in Tables 3.2 and 3.3 are investigated. It has been demonstrated that for highly slender stayed column systems, purely elastic FE models provide an excellent prediction of the actual load-carrying capacity (De Araujo *et al.*, 2008) since the stresses within the system generally do not approach the material yield stress. Moreover, apart from within the prestressing sensitivity study, in all other models presented the stay prestressing forces are fixed to the following: $T_{l,i} = T_{\text{opt1}}$ and $T_{u,i} = T_{\text{opt12}}$.

From Figure 3.11, the maximum critical buckling loads P_{max}^C of ‘Mode 1’ (lowest symmetric eigenmode) and ‘Mode 2’ (lowest antisymmetric eigenmode) are obtained from linear buckling analysis within ABAQUS. From the expressions of the optimum prestressing forces in Wu *et al.* (2019), the lower-stay optimum prestressing force T_{opt1} can be calculated. As described in Chapter 3, the graphs in Figure 3.11(a–b) show that, in terms of the doubly-symmetric cases F1–F6 and A1–A6, the critical buckling mode changes from Mode 1 to 2 as the stay diameter is increased, and a similar trend is found when the crossarm length is increased. This conclusion is similar to the results from the previous studies of stayed columns (Saito & Wadee, 2009b). Figure 3.11(c) shows that in terms of the mono-symmetric cases F3.1–F3.9, the critical buckling loads are always numerically close, which is fundamentally independent of the crossarm ratio α . Since mode interaction is likely to dominate where the buckling loads of Modes 1 and 2 are numerically close, the results of buckling loads are used to guide the study of the analytical model that follows.

5.4.1 Calibration of κ and model validation

Before any meaningful results can be presented from the analytical model, the relationship between the calibration factor κ for the crossarm shape function and the ratio α is determined numerically. This is performed by determining the value of κ , which closely matches the ultimate load obtained from the FE model with the corresponding load from the analytical model, for the doubly-symmetric case F3 alongside the mono-symmetric cases F3.1–F3.9 (see Tables 3.2–3.3). The results from the current calibration process are formulated into a simple piecewise-linear equation relating κ to α , thus:

$$\kappa = \begin{cases} 1 & \text{where: } \alpha = [0, 0.8], \\ 2(1.3 - \alpha) & \text{where: } \alpha = [0.8, 1]. \end{cases} \quad (5.34)$$

This piecewise relationship demonstrates that the crossarm shape function circular frequencies, given in Eq. (5.7), only need to be adjusted when the crossarms are approximately of equal length (*i.e.* as α approaches unity). This is because the shape function given in Eq. (5.6) effectively assumes the considered crossarm to be a cantilever with an axial and a lateral load only arising from the stay attached to that crossarm tip, without reference to the *other* crossarm (alongside the loads on the other crossarm). Hence, when α reduces from unity, the effect of the other crossarm diminishes. Eq. (5.34) is implemented throughout presently in all the following results that are based on the nonlinear analytical model. Verification against a purely numerical FE model is presented in §5.4.4.

To provide reassurance of the adopted nonlinear analytical modelling prior to the inclusion of self-weight and pre-cambering from mono-symmetric geometry, the result comparison between the adopted analytical model and the analytical model presented in Saito & Wadee (2008) is shown in Table 5.2. It can be seen that the results from both methods are numerically close, and the current nonlinear analytical modelling is confirmed to be valid such that further studies may be conducted on stayed beam-columns with self-weight and

Table 5.2: Critical load comparison between the adopted analytical model and the analytical model presented in Saito & Wadee (2008); P_{present}^C is from the adopted analytical model, and P_{Saito}^C is from Saito & Wadee (2008).

Case	F1	F2	F3	F4	F5	F6
P_{present}^C (kN)	40.38	100.06	131.87	133.90	134.24	134.89
P_{Saito}^C (kN)	41.34	101.60	130.05	132.10	132.21	132.37
Case	A1	A2	A3	A4	A5	A6
P_{present}^C (kN)	23.99	48.70	73.76	103.69	125.43	133.13
P_{Saito}^C (kN)	24.07	50.03	73.87	102.30	124.67	131.08

pre-cambering considered.

5.4.2 Analytical solutions for equilibrium paths

The focus of the following two sub-sections concerns the post-buckling behaviour and the range of mechanical phenomena that are predicted by the analytical model. Five generalized coordinates are introduced to describe the main member shape to satisfy both the requirements of modelling accuracy and computational efficiency. The shape function therefore comprises two symmetric alongside two antisymmetric component waves and one for the self-weight deflection. The model has a total of 7 DOFs (5 DOFs for the main member deflection, with 1 DOF each for the upper and lower crossarm deflection amplitudes respectively) plus the purely compressive displacement component δ_c . The prestressing forces within the lower and upper stays, $T_{l,i} = T_{\text{opt1}}$ and $T_{u,i} = T_{\text{opt12}}$ respectively (Wu *et al.*, 2019), are fixed for the most part, but are also varied at specific points in the investigation.

Figure 5.3 presents equilibrium diagrams of the axially compressive load P versus the main member axial end-shortening displacement $\mathcal{E}$ for six separate cases. In each case, the load P initially increases linearly with $\mathcal{E}$ until buckling; geometrically nonlinear behaviour is then triggered with a sudden change in stiffness arising from the global instability. Indeed, the significant reduction in load-carrying capacity beyond the initial instability in most

of the cases presented, apart from cases A1 and F1, where Mode 1 is clearly critical, demonstrates the significant influence of nonlinearity. Moreover, the sharp corners in the equilibrium path beyond the initial change in slope, which lead to the significant reduction in load-carrying capacity, are fingerprints of stay slackening. This is an effect introduced by the instantaneous loss in structural stiffness when a cable-stay component goes from tension to compression (Saito & Wadee, 2008) as a result of the main beam-column element bending.

The graphs in Figure 5.3(a–c) show the equilibrium paths of P versus $\mathcal{E}$ with $\alpha_1 = 0.1$ and $\alpha = 1.0$ while ϕ_s is varied, whereas the graphs in Figure 5.3(d–f) show the equilibrium paths of P versus $\mathcal{E}$ with $\phi_s = 4.8$ mm and $\alpha = 1.0$ while α_1 is varied. It can be observed that apart from pure Mode 1 buckling, interactive buckling is triggered when the lowest buckling loads of Modes 1 and 2 are numerically close. Interactive buckling, in fact, dominates the behaviour in such cases, its path yielding the lowest ultimate load. However, in contrast with the existing studies on stayed columns in the vertical plane (Yu & Wadee, 2017a; Saito & Wadee, 2009b), pure Mode 2 buckling of the stayed beam-columns under self-weight cannot be triggered. The reason being that the self-weight is naturally symmetric, similar to the Mode 1 imperfection, which leads only to pure Mode 1 or interactive buckling being triggered in the post-buckling range.

Figure 5.4 shows the equilibrium paths of P versus $\mathcal{E}$ with $\alpha_1 = 0.1$ and $\phi_s = 4.8$ mm while α is varied. Similar nonlinear behaviour, as discussed above, can be observed after buckling with progressive loss in structural stiffness leading to ultimate collapse. It is evident that the ultimate loads ('ult') from the different equilibrium paths follow the trend: $P_{\text{UM1,ult}} < P_{\text{DI,ult}} < P_{\text{DM1,ult}}$, where the subscripts 'UM1', 'DM1' and 'DI' represent 'Upward Mode 1', 'Downward Mode 1' and 'Downward Interactive' buckling behaviour respectively. This indicates that the ultimate load of the mono-symmetric stayed beam-column is strongly linked to the perturbation direction, and that UM1 buckling is the weakest case. In particular, when α is small, such as 0.3, an example that is presented in

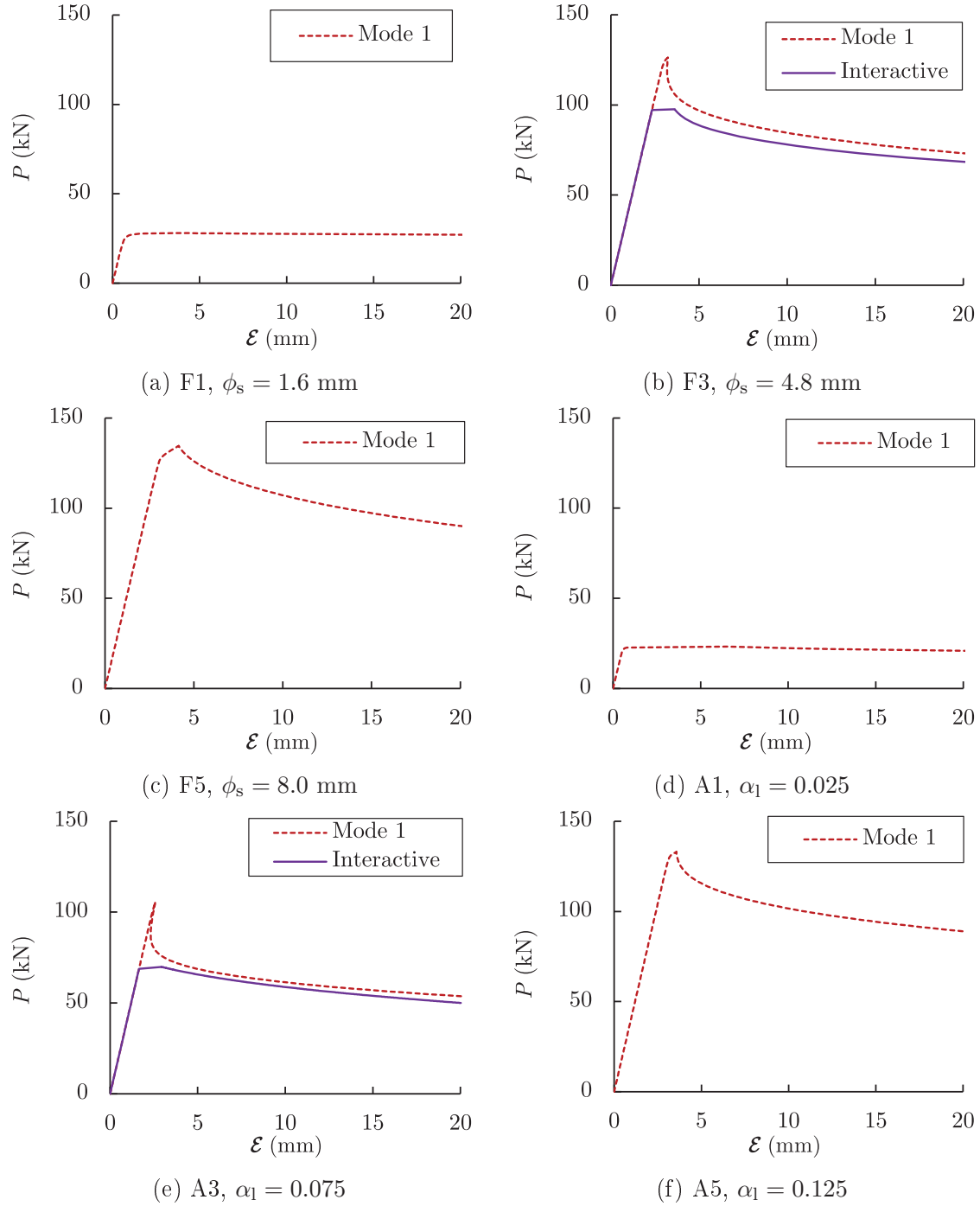


Figure 5.3: Equilibrium paths of the applied load P versus the main member end-shortening displacement ε for the doubly-symmetric cases. All results are obtained from the analytical model.

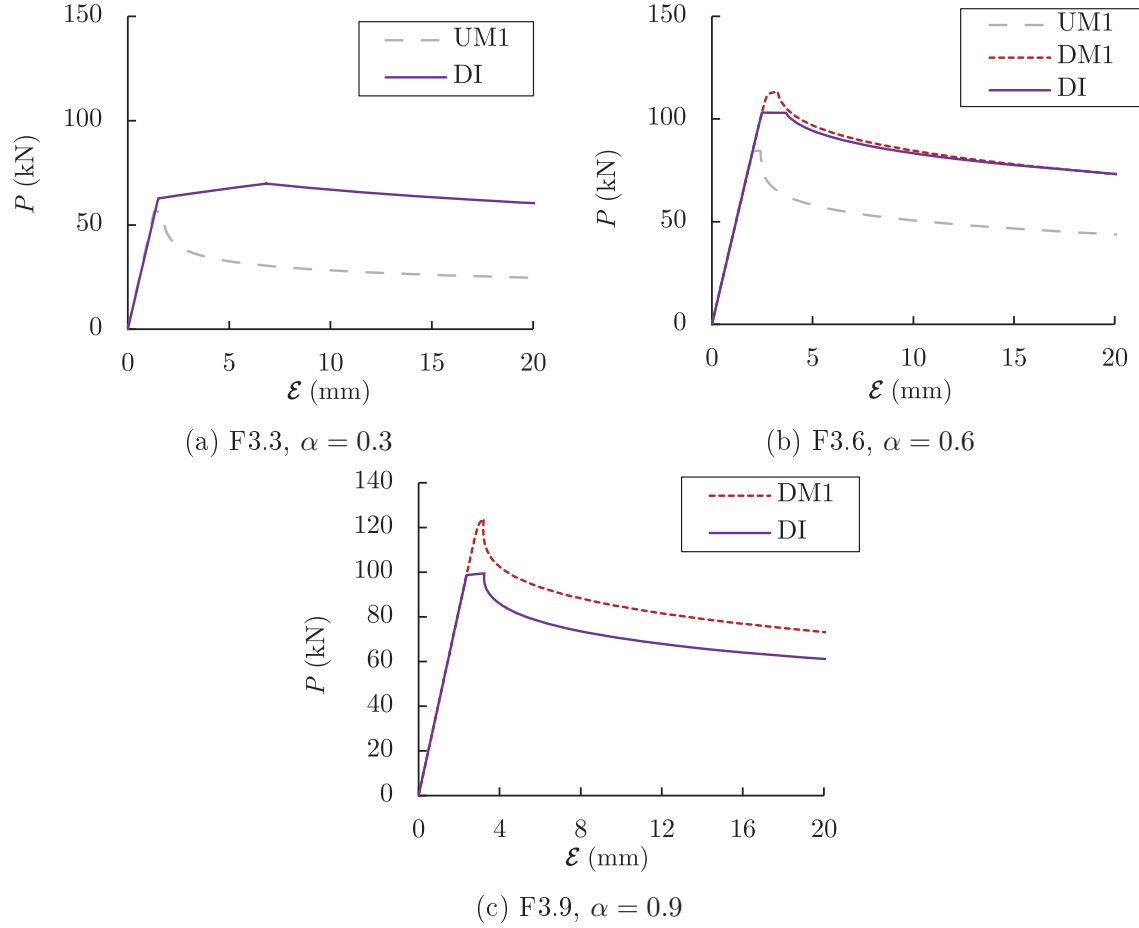


Figure 5.4: Equilibrium paths of P versus $\mathcal{E}$ with $\alpha_1 = 0.1$ and $\phi_s = 4.8$ mm while α is varied. All results are obtained from the analytical model. Nonlinear buckling responses: UM1: ‘Upward Mode 1’; DM1: ‘Downward Mode 1’; DI: ‘Downward interactive’ buckling.

Figure 5.4(a), the equilibrium path of DM1 cannot be triggered owing to the significant pre-cambering; when α is an intermediate value, such as 0.6, as shown in Figure 5.4(b), all three responses can be triggered. When α is close to 1.0, such as 0.9, which is presented in Figure 5.4(c), the equilibrium path of UM1 cannot be triggered owing to the only moderate pre-cambering, which is similar to the doubly-symmetric cases presented in Figure 5.3.

5.4.3 Analytical solutions for mode jumping

Figures 5.3 and 5.4 demonstrate that interactive buckling is observed when the buckling loads of Modes 1 and 2 are numerically close and is considered to be the most unstable scenario in those cases. The process for how these two modes interact is now discussed. Currently, u_1 and u_2 , generalized coordinates that were defined in Eq. (5.1), are adopted to present the respective contributions from Modes 1 (symmetric buckling) and 2 (antisymmetric buckling) of the main member. This is because, during the mechanical behaviour of interactive buckling, the breaking of symmetry is fundamental and the relationship between u_1 and u_2 shows its extent.

Figure 5.5 shows that in terms of the doubly-symmetric cases, such as F1–F6, initially when ϕ_s is small, such as in case F1, shown in Figure 5.5(a), the critical buckling load for Mode 2 is significantly larger than that for Mode 1 and only pure Mode 1 can be observed; the reverse is true for case F5 shown in Figure 5.5(b). An interaction loop within the equilibrium path can be observed such as in the intermediate case F3 presented in Figure 5.5(c). Figure 5.6 shows the equilibrium path of u_1 versus u_2 corresponding to Figure 5.5(c), where both exhibit discontinuities in slope that are signatures of the stays slackening sequentially on the concave side of the beam-columns, as shown in Figure 5.7. The strain within each stay ε_{tn} is monitored at different locations on the interaction loop corresponding to Figure 5.7, as listed in Table 5.3. Although pure Mode 2 is not observed, the interactive process properly begins when one of the upper stays goes slack after S_1 , which introduces an asymmetry beyond which both u_1 , as well as u_2 , grow. Observing Figures 5.5(c), 5.6 and 5.7, before reaching point B , self-weight is dominant, and there is a slight downward deflection shown since $u_1^B < 0$; between points B and S_1 , although the profile is technically mixed with Modes 1 and 2, Mode 2 begins dominating and leads to one pair of stays going slack; beyond point S_1 , one upper stay slackens, but the mode interaction is weak; beyond point S_2 , the other upper stay slackens, and the mode interaction intensifies; as point e is approached, the effect from self-weight begins to dominate with Mode 2 interacting with

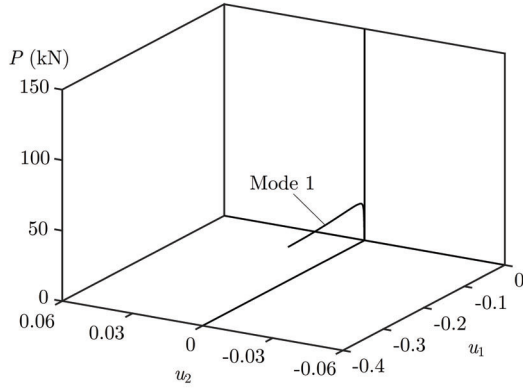
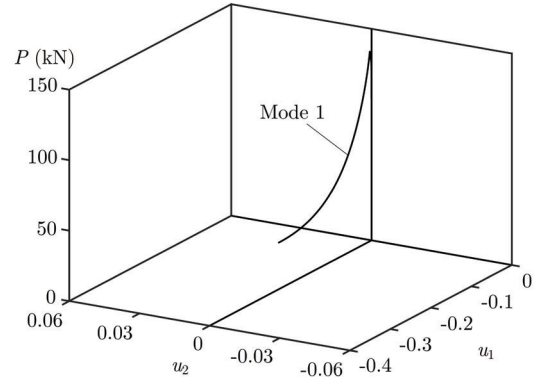
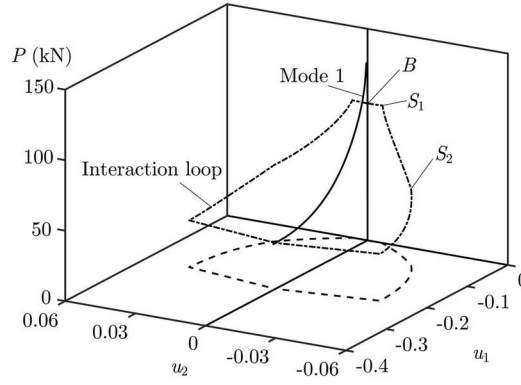
(a) F1, $P_1^C/P_2^C = 0.42$ (b) F5, $P_1^C/P_2^C = 1.12$ (c) F3, $P_1^C/P_2^C = 1.07$

Figure 5.5: Equilibrium paths of the axially compressive load P versus u_1 and u_2 with $\alpha_1 = 0.1$, $\alpha = 1.0$ and (a) $\phi_s = 1.6$ mm, (b) $\phi_s = 8.0$ mm, (c) $\phi_s = 4.8$ mm; B is a bifurcation point and points S_i mark where the stays slacken in sequence.

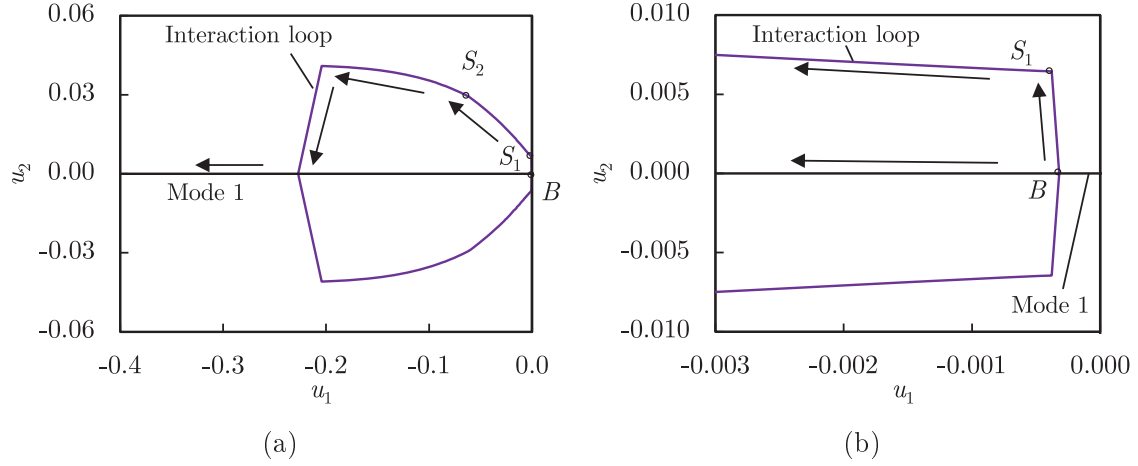


Figure 5.6: Equilibrium paths showing amplitudes of Mode 1 versus Mode 2 for case F3; (a) shows the large scale relationship; (b) shows details in the neighbourhood of the initial bifurcation point B .

Table 5.3: Strains within the four stays at the selected points for case F3. Note that the stay goes slack when the strain becomes negative. Points ‘a’–‘g’ are depicted in the graph presented in Figure 5.7. Although points ‘d’–‘g’ have very high tensile strains, this may be addressed in practice by using larger stay diameters. Nonlinear buckling descriptors: DM1: ‘Downward Mode 1’; M2d: ‘Mode 2 dominant’; DI: ‘Downward Interactive’ buckling.

Point	$\mathcal{E}$ (mm)	ε_{u1} (%)	ε_{u2} (%)	ε_{l1} (%)	ε_{l2} (%)	Description
<i>a</i>	2.127	0.062	0.062	0.078	0.078	DM1
<i>b</i>	3.622	0.083	slack	slack	0.086	M2d
<i>c</i>	9.339	0.090	slack	0.300	0.480	DI
<i>d</i>	147.7	slack	slack	0.930	1.500	DI
<i>e</i>	394.1	slack	slack	0.990	1.660	DI
<i>f</i>	442.5	slack	slack	2.040	2.040	DM1
<i>g</i>	442.9	slack	slack	2.050	2.050	DM1

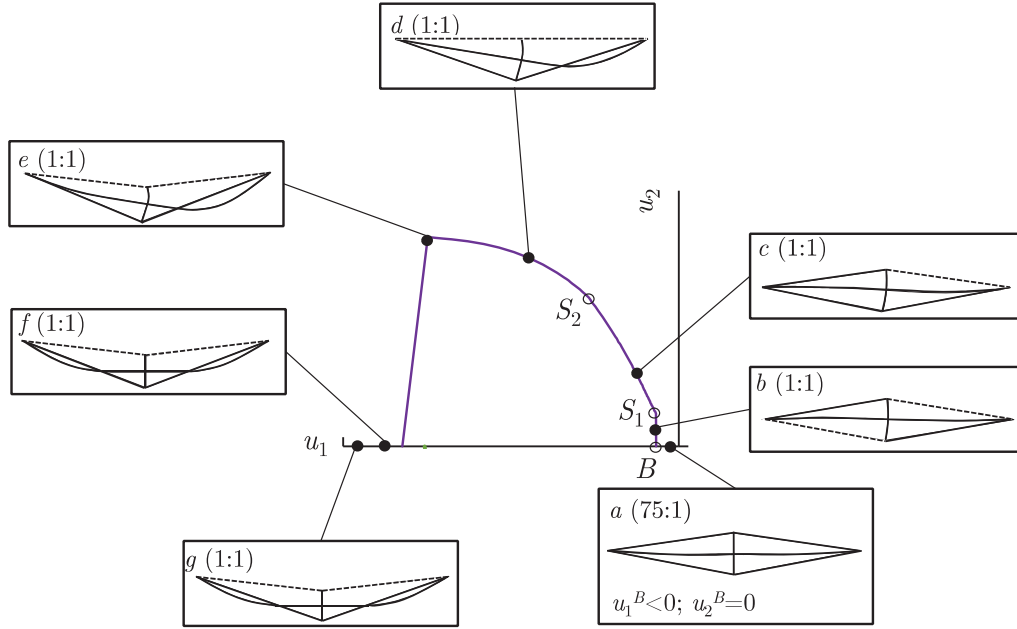


Figure 5.7: Detailed transformation of the post-buckling profile in the nonlinear mode jumping process for case F3. The solid circles pinpoint the combination of u_1 and u_2 for the corresponding deformation profile depicted; the hollow circles show the initial bifurcation point (B) and the subsequent points (S_i) that are signatures of the stays slackening where the slope is discontinuous; in the deformation profiles, the dashed lines represent slack stays and the ratio in the bracket is the amplification factor for visualization purposes.

Mode 1. However, beyond point e the profile jumps to pure Mode 1 at point f with the self-weight becoming totally dominant and reflective symmetry being restored. It should be stressed that only buckling in the downward deflection, *i.e.* where $u_1 < 0$, can be triggered owing to the self-weight. The complete interactive buckling process described presently is also known as ‘mode jumping’ (Everall & Hunt, 2000).

In terms of the mono-symmetric cases, such as F3.1–F3.9, along with the increase of α , the linear buckling loads of Modes 1 and 2 are always numerically close, as shown in Figure 3.11(c). The interaction loop can be observed in all 9 cases and three representative examples are shown in Figure 5.8; one each where Modes 1 and 2 are slightly more distinctly critical and the third where the critical buckling loads are within 1% of each other. From Figures 5.8(a) and (b), UM1 would be triggered potentially for cases F3.3 and F3.6

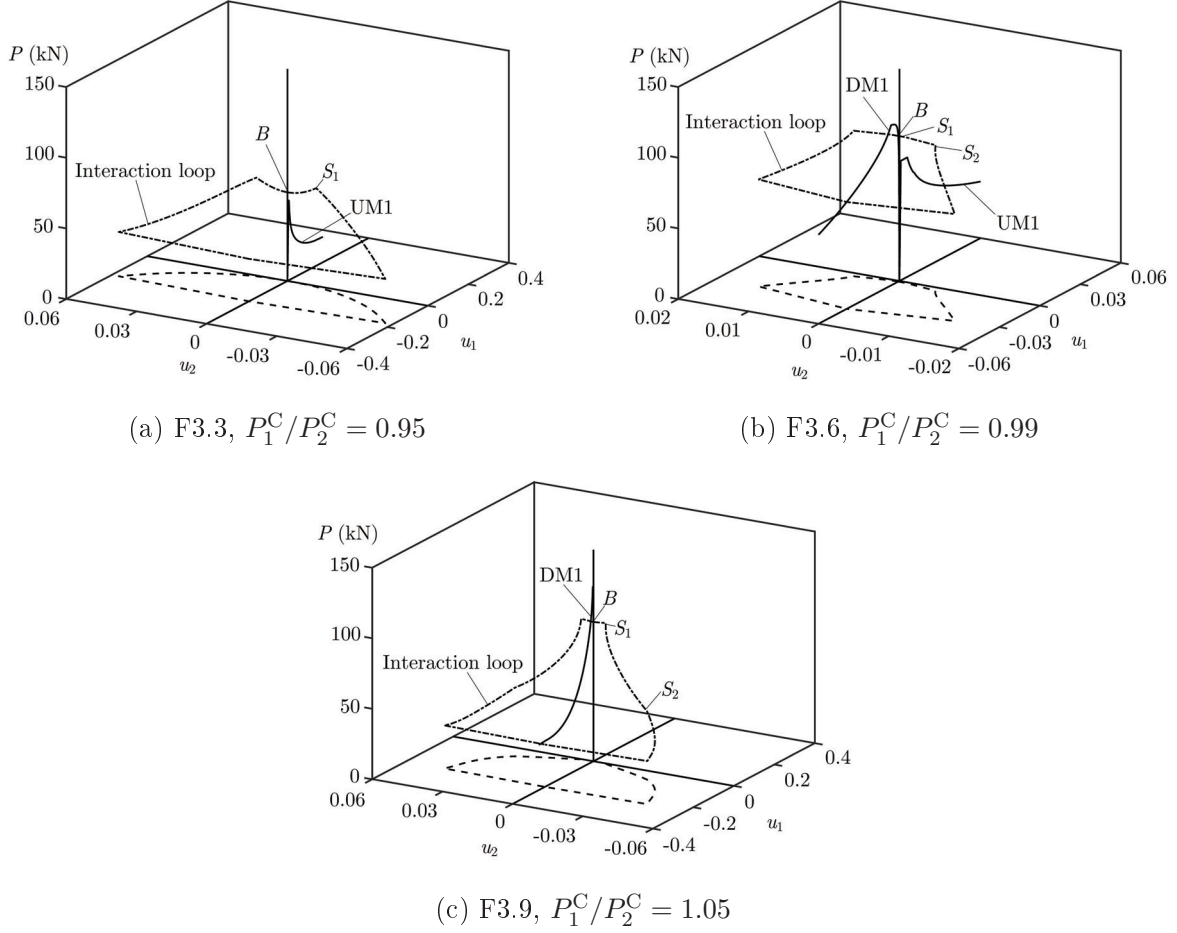


Figure 5.8: Equilibrium paths of P versus u_1 and u_2 with $\alpha_1 = 0.1$, $\phi_s = 4.8$ mm. Cases presented: (a) $\alpha = 0.3$, (b) $\alpha = 0.6$, (c) $\alpha = 0.9$. Axes and labels are the same as Figure 5.5.

owing to the weakest behaviour within this buckling mode; however, if there is a downward perturbation in these systems, DI and DM1 would be triggered since they have respectively the weakest responses for buckling with a downward deflection. It should be noted that for case F3.3, it is considerably more difficult to trigger DI owing to the strong pre-cambering. From Figure 5.8(c), only DI would be triggered for both upward and downward perturbations, since the pre-cambering is only moderate. Figure 5.9 shows the equilibrium path of u_1 versus u_2 for case F3.6 corresponding to Figure 5.8(b). There are also discontinuities in slope that are signatures of the stays slackening sequentially on the concave side of the buckling beam-columns, as shown in Figure 5.10. The strain within each stay ε_{tn} is moni-

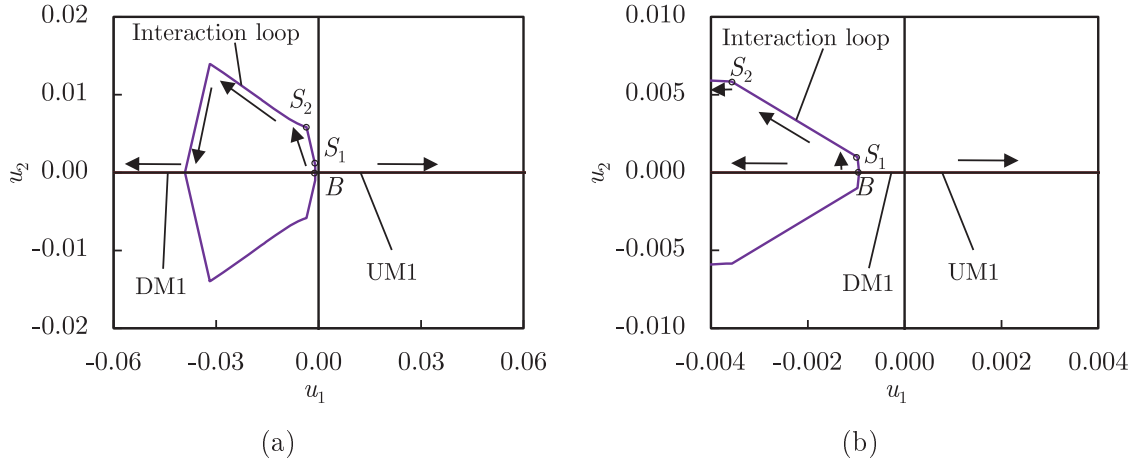


Figure 5.9: Equilibrium paths showing amplitudes of Mode 1 versus Mode 2 for case F3.6; (a) shows the large scale relationship; (b) shows details in the neighbourhood of the initial bifurcation point B .

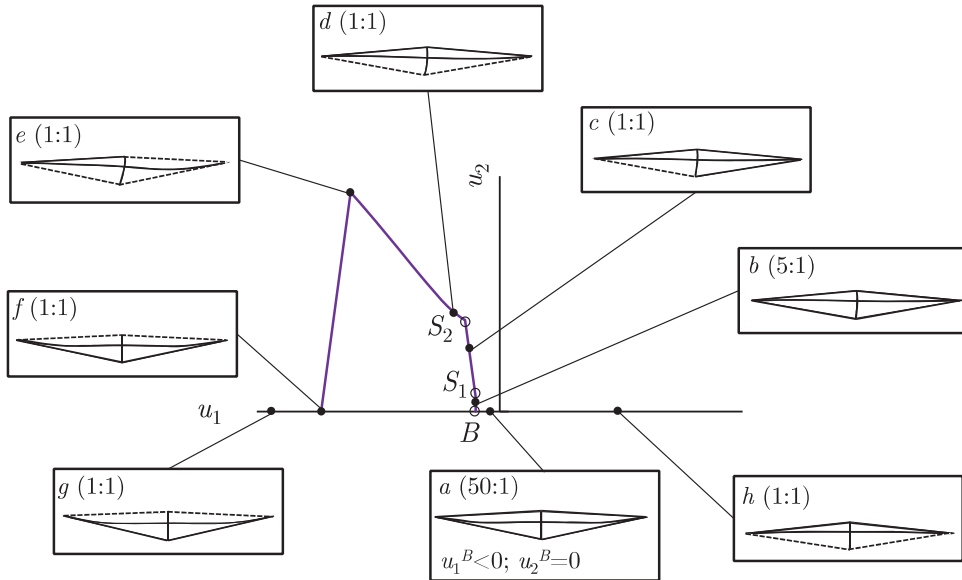


Figure 5.10: Detailed transformation of the post-buckling profile in the nonlinear mode jumping process for case F3.6. Axes and labels are the same as Figure 5.7.

tored at different locations on the interaction loop corresponding to Figure 5.10, as listed in Table 5.4. It can be seen that the ‘mode jumping’ phenomenon in the mono-symmetric

Table 5.4: Strains within the four stays at the selected points for case F3.6. Note that the stay goes slack when the strain is negative. Points ‘*a*’–‘*g*’ are depicted in the graph presented in Figure 5.10. Although points ‘*d*’–‘*g*’ have very high tensile strains, this may be addressed in practice by using larger stay diameters. Nonlinear buckling descriptors: UM1: ‘Upward Mode 1’; DM1: ‘Downward Mode 1’; M2d: ‘Mode 2 dominant’; DI: ‘Downward Interactive’ buckling.

Point	$\mathcal{E}$ (mm)	ε_{u1} (%)	ε_{u2} (%)	ε_{l1} (%)	ε_{l2} (%)	Description
<i>a</i>	2.372	0.130	0.130	0.050	0.050	DM1
<i>b</i>	2.512	0.140	0.120	0.030	0.050	M2d
<i>c</i>	3.677	0.210	0.070	slack	0.001	DI
<i>d</i>	4.860	0.440	0.020	slack	slack	DI
<i>e</i>	17.16	1.230	slack	slack	slack	DI
<i>f</i>	17.36	slack	slack	1.430	1.430	DM1
<i>g</i>	27.25	slack	slack	2.000	2.000	DM1
<i>h</i>	2.472	0.430	0.430	slack	slack	UM1

cases is similar to that observed in the doubly-symmetric cases. However, as opposed to the doubly-symmetric cases, there is a pre-cambering effect for the cases with $\alpha < 1.0$, which can lead to the UM1 buckling response being critical. In addition, this effect becomes stronger with α decreasing; Figure 5.8(a) shows that DM1 buckling cannot be triggered owing to the strong pre-cambering effect in case F3.3; Figure 5.8(b) shows that all three buckling responses can be triggered owing to the intermediate pre-cambering effect in case F3.6; Figure 5.8(c) shows that UM1 buckling cannot be triggered owing to the only slight pre-cambering effect in case F3.9.

5.4.4 FE verification of nonlinear behaviour

Since a range of nonlinear phenomena, in particular mode interaction, have been predicted in the results from the analytical model, the FE model is now used to verify those findings. The same structural geometries and material properties are used in the FE models to conduct both the linear and nonlinear analyses. Owing to the requirement of introducing

Table 5.5: Comparison between analytical and FE results to verify Eq. (5.34); $P_{u,AUTO}$ and $P_{u,FE}$ are the ultimate loads from analytical and FE modelling respectively.

Case	F3.5	F3.6	F3.7	F3.8	F3.9	F4.0
κ	1.0	1.0	1.0	1.0	0.8	0.6
$P_{u,AUTO}$ (kN)	108.24	103.11	101.55	88.93	99.48	97.60
$P_{u,FE}$ (kN)	103.89	102.11	98.77	84.48	94.98	93.86
$P_{u,AUTO}/P_{u,FE}$	1.04	1.00	1.03	1.05	1.05	1.04

imperfections as perturbations to obtain the near-perfect post-buckling response, an amplitude ratio of $\delta = 1/10000$ was adopted to ensure a fair comparison with the analytical model as far as possible. A mixture of the perturbations from the first two pure modes was used to trigger interactive buckling in the FE models. Case 1 in Table 5.1 was adopted as the imperfection combination to study pure Mode 1 buckling; case 3 in Table 5.1 was adopted as the imperfection combination to study interactive buckling. Initially though, to verify the piecewise-linear equation relating κ to α in Eq. (5.34), the ultimate loads obtained from the analytical and FE models are compared quantitatively, as shown in Table 5.5, and the difference between FE and analytical results is limited to 5%, which is a highly satisfactory result that justifies Eq. (5.34).

Figures 5.11 and 5.12 show the comparisons between the equilibrium paths of the load P versus the main member end-shortening displacement $\mathcal{E}$ for both $\alpha = 1.0$ and $\alpha < 1.0$ cases respectively. The graphs in Figures 5.11(b) and (d) as well as in Figures 5.12(c) and (d) show excellent agreement in the ultimate load and, perhaps more importantly, the far-field post-buckling response, which verifies the piecewise-linear equation relating κ to α in Eq. (5.34), where Mode 1 does not dominate the response. It is seen that the maximum load-carrying capacity and the nonlinear post-buckling response compare excellently for all cases between the analytical and FE models, which implies that the current analytical approach can capture the interactive buckling behaviour both qualitatively and quantitatively. In terms of the doubly-symmetric cases, since Mode 1 buckling is critical and dominates for cases F1 and A1, the equilibrium paths for Mode 1 buckling from both the FE and analytical models are compared in Figures 5.11(a) and (c). Since interactive

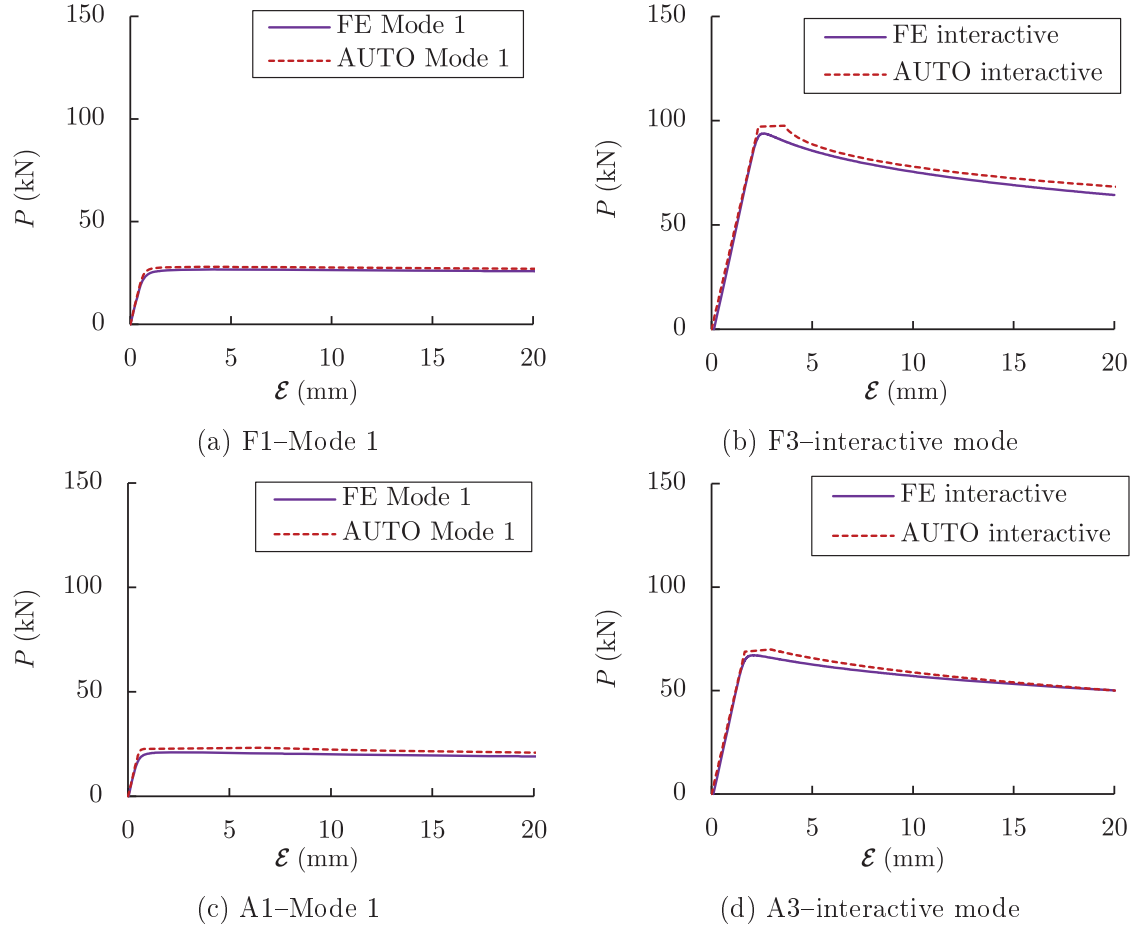


Figure 5.11: Comparisons between analytical and FE results of the equilibrium paths for doubly-symmetric cases ($\alpha = 1.0$).

buckling is effectively dominant for cases F3 and A3, the appropriate equilibrium paths from both the FE and analytical modelling are compared in Figures 5.11(b) and (d).

In terms of the mono-symmetric cases, since UM1 would be the observed behaviour for case F3.3 because of the strong pre-cambering, the equilibrium paths for this response from both the FE and analytical models are compared in Figure 5.12(a). For case F3.6, owing to the moderate pre-cambering, the possible responses DM1 and DI have similar quantitative behaviour; hence, both responses are presented in Figure 5.12(b) and (c). For case F3.9, owing to the relatively weak pre-cambering, which is quite similar to case F3 in the doubly-symmetric case presented in Figure 5.11(b), only the equilibrium paths from interactive

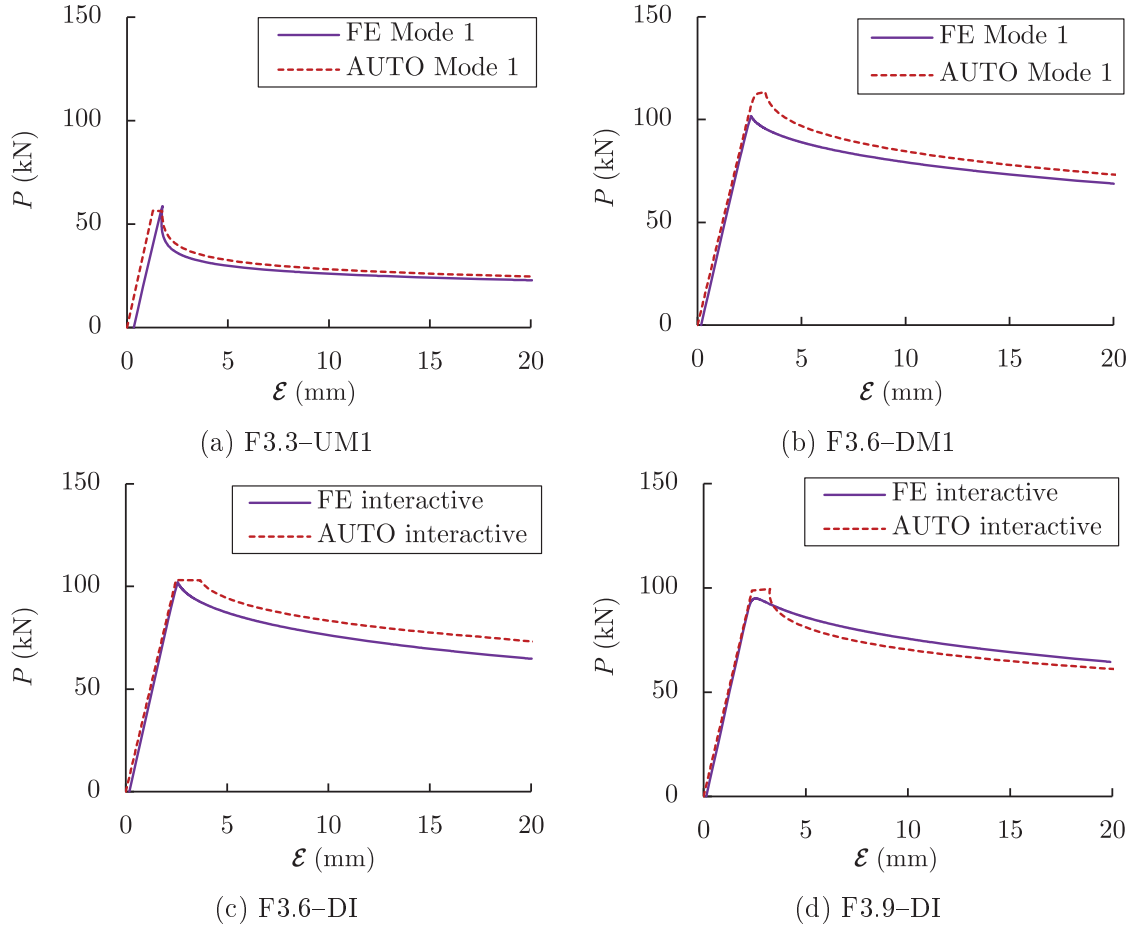


Figure 5.12: Comparisons between analytical and FE results of the equilibrium paths for mono-symmetric cases ($\alpha < 1.0$).

buckling are compared in Figure 5.12(d). All cases show excellent comparisons with only small differences between the FE and analytical models throughout the equilibrium paths. These differences are attributed to the various assumptions made within each modelling approach. In the analytical model, the main member is assumed to be a perfectly horizontal member after loading step 3. In contrast, within the FE model, there is a small deflection in the main beam-column element after applying the self-weight and prestressing force. This, alongside definitions of assumed crossarm shape functions, provides unavoidable variations currently and lead to the small differences that, nevertheless, do not significantly affect the overall results.

Figures 5.13 and 5.14 show the comparisons of the post-buckling profiles between the ana-

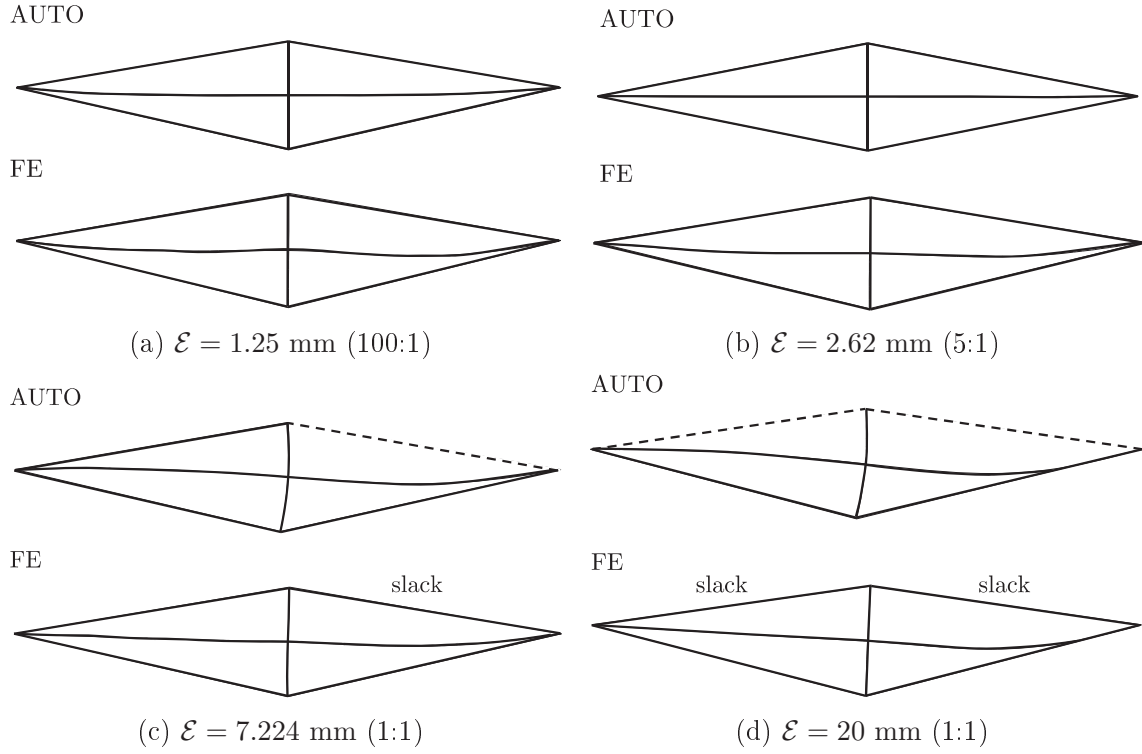


Figure 5.13: Comparisons between post-buckling profiles obtained from the analytical and the FE models for case F3. Dashed lines for the analytical model results (AUTO) represent slack stays.

lytical and FE models for specific end-shortening displacements $\mathcal{E}$. The comparisons show an excellent visual agreement in terms of buckling profiles between these two models even for very large displacements. This demonstrates that the analytical model established currently is quite capable of predicting the nonlinear behaviour accurately both qualitatively and quantitatively in the far-field post-buckling range with understandable divergence in prediction for very large deflections that are beyond the scope of the modelling assumptions made.

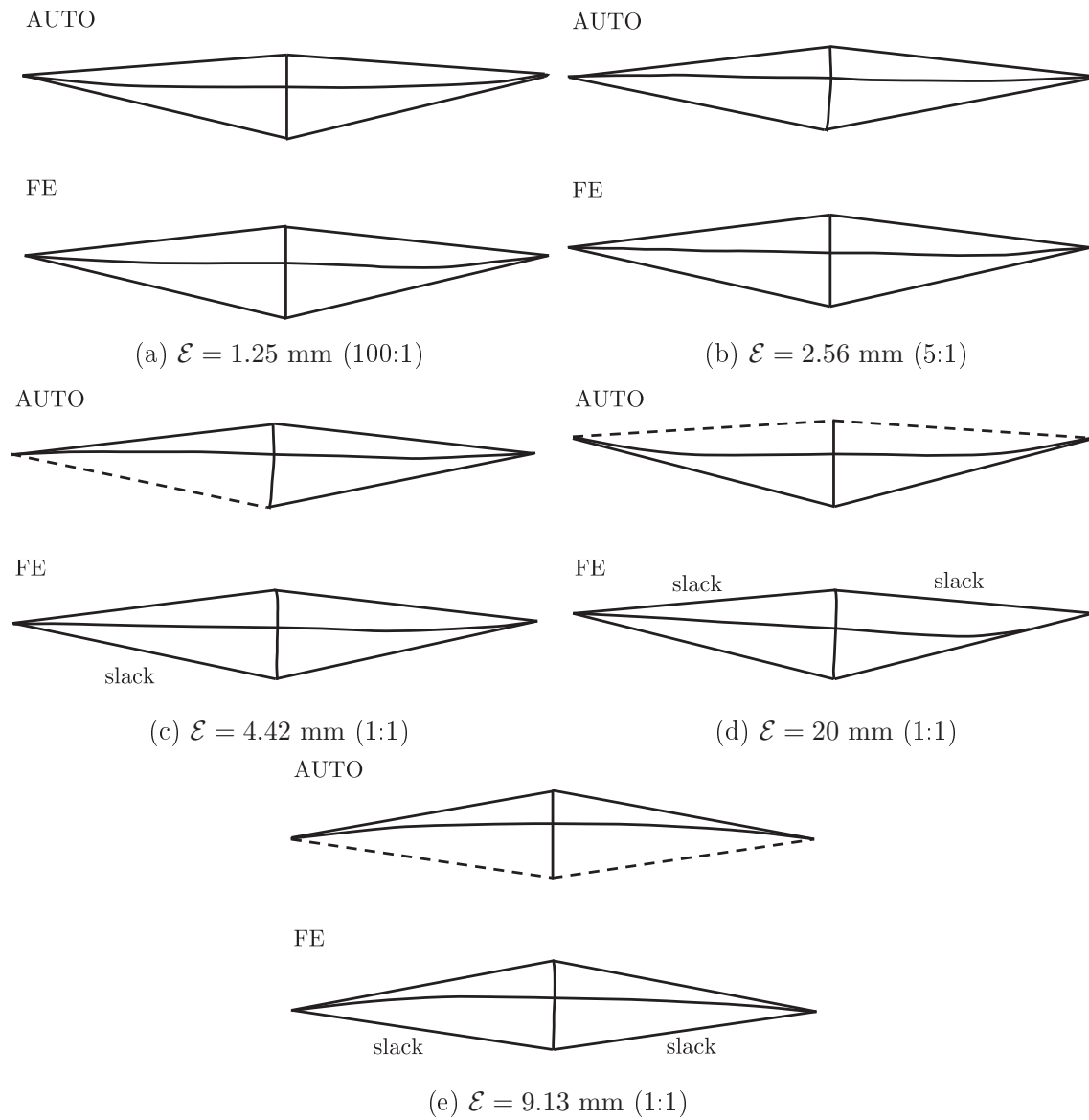


Figure 5.14: Comparisons between post-buckling profiles obtained from the analytical and the FE models for case F3.6. Dashed lines for the analytical model results (AUTO) represent slack stays. Note that in (e), upward Mode 1 is triggered unlike in the other cases (a–d).

5.4.5 Prestressing sensitivity for equilibrium paths

The purpose of the following two sections is to conduct a parametric study on the effects of varying the initial prestressing levels. This factor can lead to different buckling and post-buckling behaviour of the stayed beam-columns. Both the aforementioned analytical and FE models are adopted in current section.

In order to study the sensitivity to prestress, the equilibrium responses of the stayed beam-columns with the initial prestressing level varying is investigated first. Figures 5.15 and

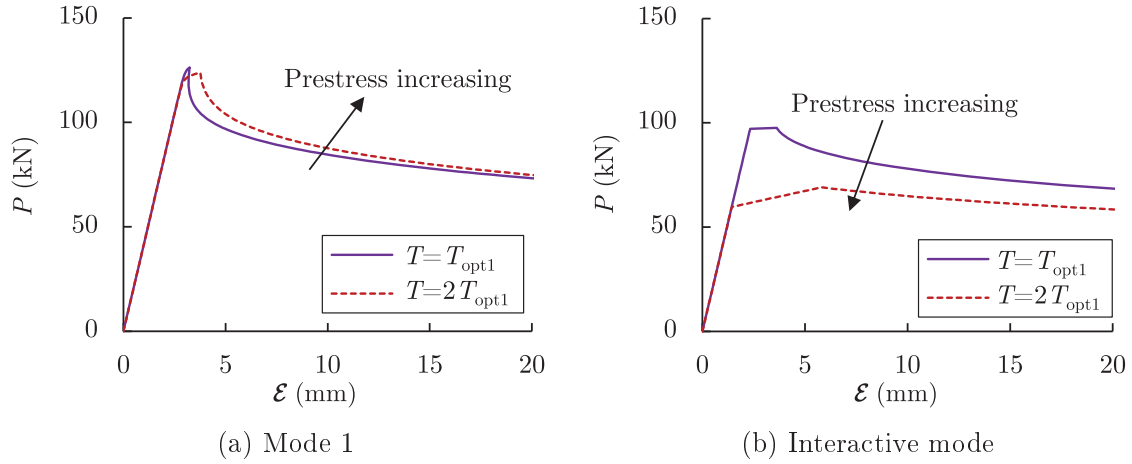


Figure 5.15: Equilibrium paths for case F3 from the present analytical model with increasing levels of initial prestress. Note the significantly increasing load plateau size in both graphs and the slightly higher residual load in (a), but not in (b), in the initial and advanced post-buckling stages respectively with increasing initial prestress.

5.16 show the equilibrium paths of the load P versus the main member end-shortening displacement $\mathcal{E}$ with the prestressing force varying for cases F3 and F3.6 respectively from the analytical model. In terms of the doubly-symmetric case F3, it can be seen that in both cases for Mode 1 and interactive buckling, increasing the prestressing force from T_{opt1} results in a decrease in the load-carrying capacity but makes the post-buckling behaviour more stable. This may imply that prestressed stayed beam-columns would benefit from the prestressing level being larger than T_{opt1} . The investigation of determining the true optimum prestressing force is presented in Chapter 6, but it is postulated that it would

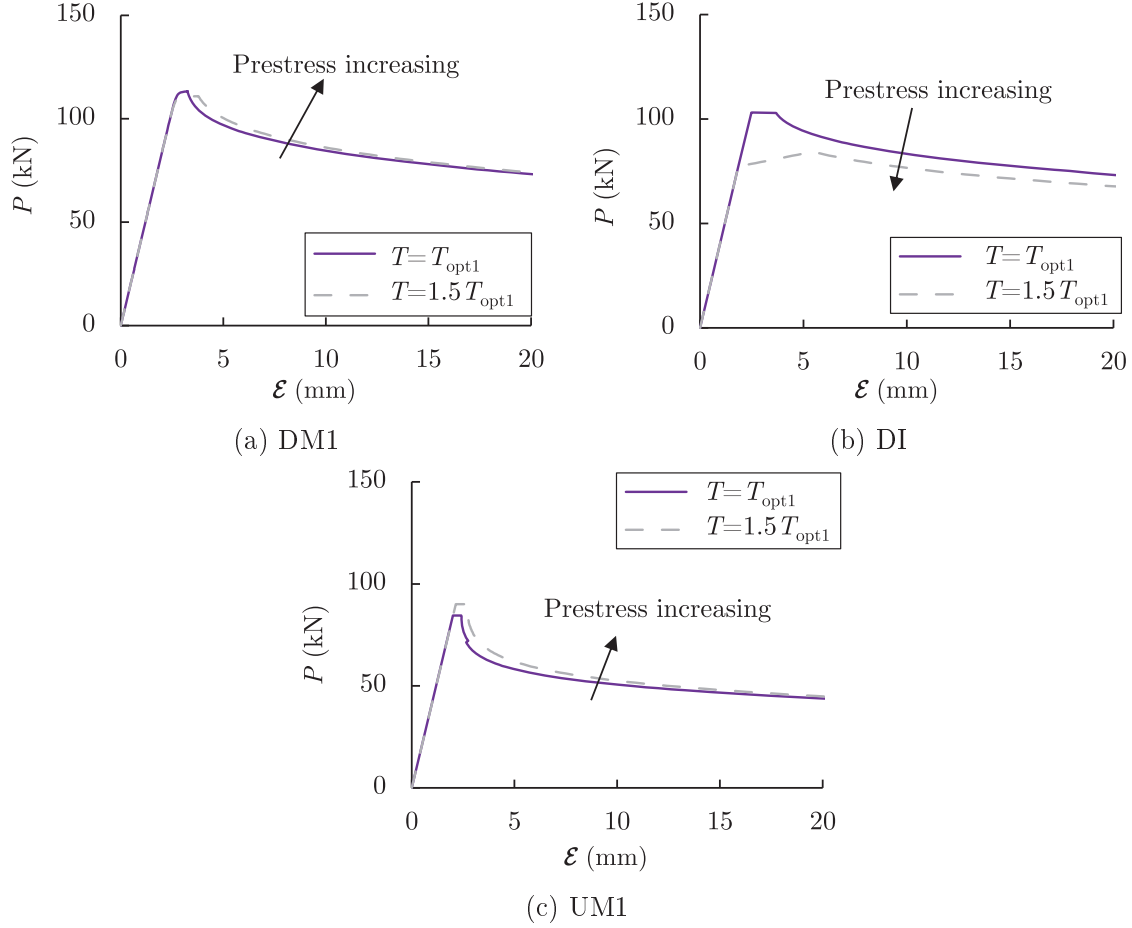


Figure 5.16: Equilibrium paths for case F3.6 from the present analytical model with increasing levels of initial prestressing force T . Note the significantly increasing load plateau size in all graphs and the slightly higher residual load in (a) and (c), but not in (b), in the initial and advanced post-buckling stages respectively with increasing T .

be significantly higher than T_{opt1} , as found for the vertical stayed column case (Saito & Wadee, 2010; Yu & Wadee, 2017b). It is worth recalling that the beam-columns with prestressing forces less than T_{opt1} would have significantly less load-carrying capacity as can be deduced from Figure 3.10. In terms of the mono-symmetric case F3.6 with the DM1 imperfection, the phenomenon and conclusions are similar to the doubly-symmetric cases; however, when the UM1 imperfection is critical, it can be seen that in the $1.5T_{\text{opt1}}$ case the load-carrying capacity significantly increases compared to the T_{opt1} case. Figure 5.17 shows the equilibrium paths of u_1 versus u_2 for case F3 with varying initial prestressing

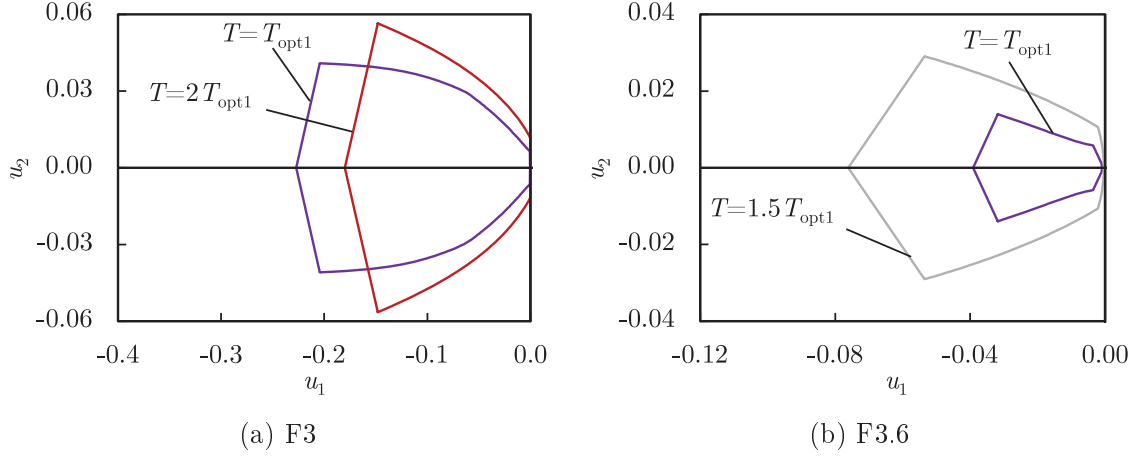


Figure 5.17: Equilibrium paths of u_1 versus u_2 with varying initial prestressing levels.

levels. Figure 5.17(a) shows that in terms of the doubly-symmetric case F3 with the prestressing level increasing, the interaction loop becomes larger, but u_1 becomes smaller. Figure 5.17(b) shows that in terms of the mono-symmetric case F3.6, the interaction loop becomes obviously larger in both u_1 and u_2 when the prestressing force increases from T_{opt1} to $1.5T_{\text{opt1}}$.

5.4.6 Prestressing sensitivity for nonlinear buckling response

As shown in Figure 5.17(b), the equilibrium paths for the mono-symmetric case F3.6 show a different phenomenon with the prestressing level increasing compared to the doubly-symmetric case F3. This may be owing to a qualitative change of buckling modes when the prestressing level increases. Figure 5.18 shows that the nonlinear buckling responses of the mono-symmetric cases F3.1–F3.9 are related to the combination of the crossarm ratio α and prestressing level. It can be seen that excessively high prestressing levels may reduce the pre-cambering effect derived from the length difference between the upper and lower crossarms. When α is smaller, *i.e.* $\alpha < 0.4$, the downward buckling mode still cannot be triggered with the relatively high prestressing levels, since the pre-cambering from the length difference between two crossarms is sufficiently strong to counteract any reduction

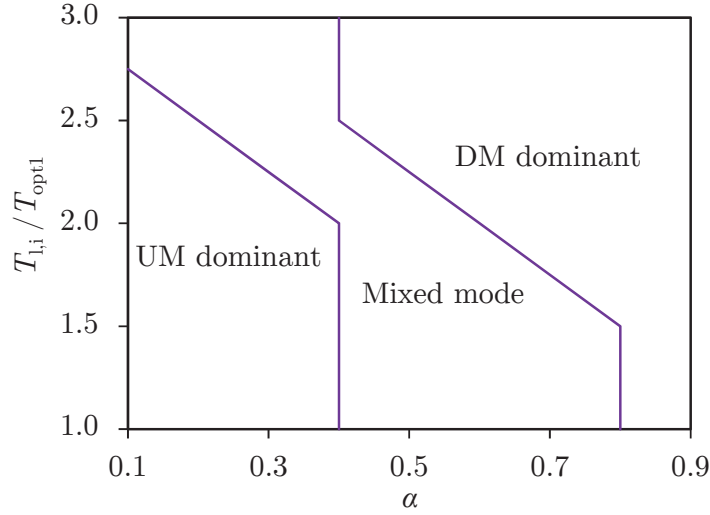


Figure 5.18: Parametric spaces for cases F3.1–F3.9 showing $T_{l,i}/T_{opt1}$ versus α showing regions where the effect from pre-cambering is varied: ‘UM dominant’ implies that DM1 cannot be triggered owing to strong pre-cambering; ‘DM dominant’ implies that UM1 cannot be triggered owing to insignificant pre-cambering; ‘Mixed mode’ implies that all the three nonlinear responses (UM1, DM1, DI) can be triggered owing to an intermediate level of pre-cambering.

from the prestressing; when α is larger, *i.e.* $\alpha > 0.8$, the upward buckling mode still cannot be triggered with the relatively low prestressing levels, since the pre-cambering effect from the length difference between the two crossarms is much less significant; when α has an intermediate value, all three responses can be triggered with relatively low prestressing levels, but the upward buckling mode cannot be triggered with high prestressing levels. Finally, it should be stressed that for larger α values, the required level of prestressing to eliminate the upward buckling mode is relatively smaller and for the doubly-symmetric beam-columns ($\alpha = 1.0$), the downward buckling response occurs regardless of the initial prestressing force. The relationship between pre-cambering and the level of prestress and mono-symmetry is developed further in the following chapter.

5.5 Concluding remarks

A nonlinear analytical model formulated with potential energy principles, in conjunction with the Rayleigh–Ritz method has been presented for prestressed stayed beam-columns and has been verified using the commercial FE package ABAQUS. Both linear and nonlinear analyses were conducted with parametric variations. The model results revealed, for the first time, the detailed process explaining how the two principal buckling modes interact when their corresponding buckling loads are in relatively close proximity. It was also clearly identified that mode interaction in prestressed stayed beam-columns is intrinsically linked to the stay slackening and crossarm bending phenomena, which were successfully modelled currently. The actual post-buckling response is usually dominated by nonlinear modal interactions that features mode jumping. Moreover, it has been found that, in terms of the mono-symmetric stayed beam-columns, varying the initial prestressing levels would naturally lead to different post-buckling behaviour.

The present results suggest that the true optimum prestress of stayed beam-columns is higher than the linearly obtained solution, as is the case in vertical prestressed stayed columns; this together with sensitivity to pre-cambering and initial imperfections is explored in the next chapter.

Chapter 6

Sensitivity and true optimum prestress

6.1 Introduction

In the previous chapters, the buckling and post-buckling behaviour, including zones of behaviour, buckling loads and mode interaction, were studied based on the analytical models validated by FE models. However, the focus of the previous chapters was only on stayed beam-columns with ‘perfect’ geometries. Following the numerical studies of imperfect vertical stayed columns on interactive buckling and true optimum prestressing forces (Saito & Wadee, 2009b, 2010; Yu & Wadee, 2017b), FE models are employed presently to study the structural response of imperfect stayed beam-columns. Both doubly-symmetric and mono-symmetric systems are examined. Different imperfection combinations are considered for interactive buckling, and the effects on the structural behaviour from varying prestressing and pre-cambering levels are investigated. The true optimum prestressing force for ultimate strength is determined based on the ultimate load achieved and two bespoke structural efficiency indicators. The structural performance is found to be strongly linked to the level of pre-cambering.

6.2 FE modelling

Similar to the previous chapters, for the examples in the current chapter, the material properties and cross-section parameters described in Table 3.1 are adopted, and the aforementioned doubly-symmetric cases F1–F6, A1–A6 and mono-symmetric cases F3.1–F3.9 in Tables 3.2 and 3.3 are investigated. The optimum prestressing force T_{opt1} within the lower stay corresponding to T_{opt12} within the upper stay is adopted as the benchmark prestressing force. All loading steps illustrated in Figure 3.2 are included in the current numerical modelling. The applied imperfection shape is defined from Eq. (5.30), and the imperfection combinations listed in Table 5.1 are adopted to trigger different buckling modes.

6.3 Prestress and pre-cambering

In Chapters 3 and 5, the mid-span displacement after loading step 3 described in Figure 3.2 was assumed to be zero by satisfying Eq. (3.21), which was derived on the assumption of linear behaviour. However, owing to the influence of geometric nonlinearity, namely the effect of the beam-column shortening in length due to introduction of prestress, an effect that is more pronounced for the mono-symmetric systems, the mid-span vertical displacement is in fact non-zero. This non-zero mid-span displacement essentially provides pre-cambering; the distribution of prestressing forces between upper and lower stays can be altered deliberately to achieve desired levels of pre-cambering (indicated by the mid-span displacement after applying the prestress in loading step 3, *i.e.* $w_{\text{p,m}}$) and affect the buckling and post-buckling behaviour in loading step 4. The principal purpose of the current section is to investigate the relationship between the distribution of the stay prestress and the level of pre-cambering. The FE models in the present section do not consider any initial geometric imperfections.

To study the effect on the pre-cambering from prestressing, a pre-cambering index I_{p} is

defined, thus:

$$I_p = 10000 \frac{w_{p,m}}{L}, \quad (6.1)$$

where $w_{p,m}$ is the mid-span displacement after introducing the prestress in loading step 3. The cases where $I_p = \{-5, -1, 0, 1, 5\}$ are adopted as the benchmark pre-cambering levels. Positive values of I_p and $w_{p,m}$ represent upward pre-cambering, while negative values correspond to downward pre-cambering, which is consistent with the direction of y axis and downward self-weight in FE modelling. The distribution of prestressing forces between the upper and lower stays is represented by a newly defined parameter γ , thus:

$$\gamma = \frac{T_{u,i} \sin \beta_u}{T_{l,i} \sin \beta_l}, \quad (6.2)$$

which is a ratio of the applied prestressing level to the first order prestressing level from Eq. (3.21). For the instance where $\gamma = 1$, Eqs. (3.21) and (6.2) are of course identical.

6.3.1 Doubly-symmetric cases

Doubly-symmetric cases ($\alpha = 1$) have upper and lower crossarms of identical length. Changes to the pre-cambering index I_p at $T_{l,i} = T_{opt1}$ and $T_{l,i} = 2T_{opt1}$ with increasing γ for cases F1 and A1, F2–F6, and A2–A6 are shown respectively in Figures 6.1, 6.2 and 6.3. It can be seen that the results in Figure 6.1 are significantly different from the results in Figures 6.2 and 6.3, implying that pre-cambering with $I_p \geq 0$, cannot be achieved by adjusting the prestressing distribution between the upper and lower stays while $T_{l,i} \leq 2T_{opt1}$. This is because the influence of the stays and crossarms are limited in these cases owing to the small stay diameter ϕ_s in case F1 and the small crossarm length a_l in case A1. In addition, considering the low load-carrying capacity of these two systems from the previous chapters, small stay diameters (*i.e.* $\phi_s/(2r_c) < 0.14$ corresponding to $\phi_s < 3.2$ mm currently) and crossarm lengths (*i.e.* $\alpha_l < 0.05$) should be avoided. However, for cases F2–F6 and A2–A6, a deliberate pre-cambering level, within the range of $I_p = [-5, 5]$, can

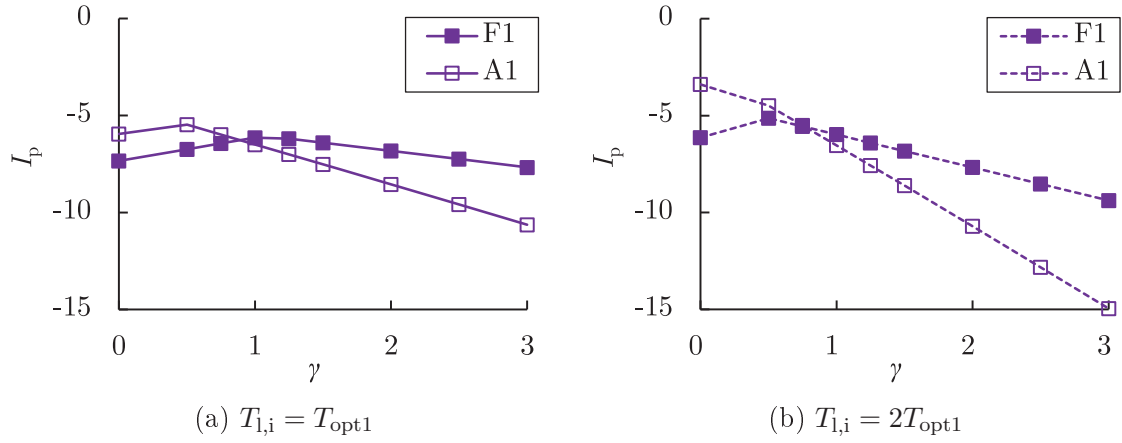
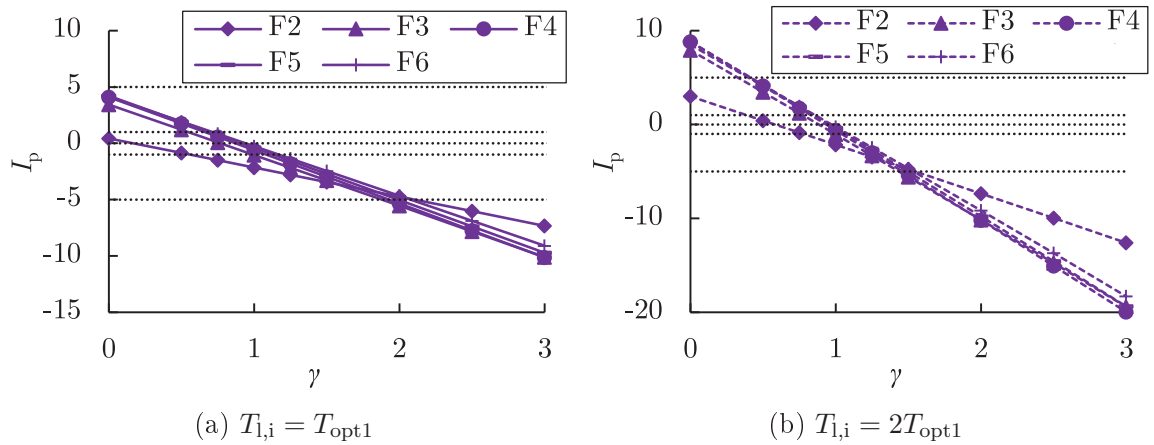


Figure 6.1: Pre-cambering index versus prestressing distribution for cases F1 and A1.

Figure 6.2: Pre-cambering index versus prestressing distribution for cases F2-F6. The dotted horizontal lines from bottom to top correspond to $I_p = \{-5, -1, 0, 1, 5\}$ respectively.

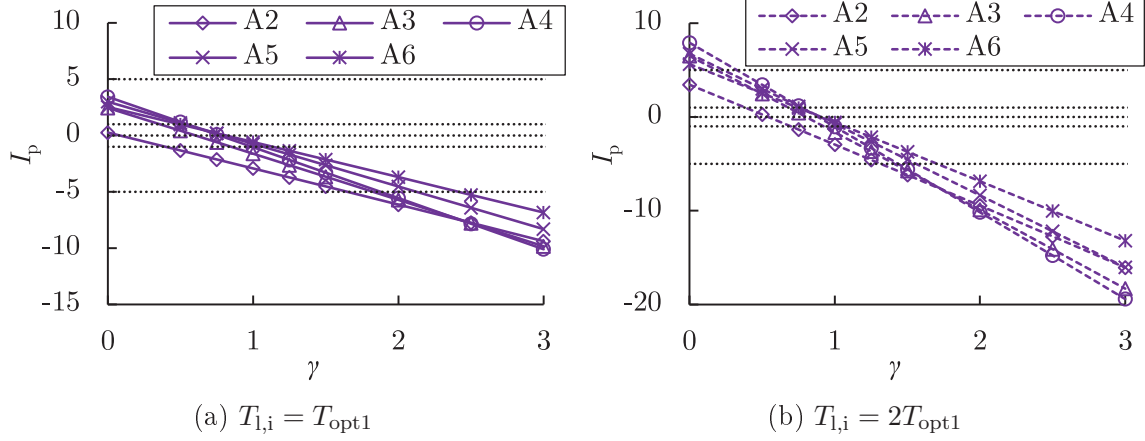


Figure 6.3: Pre-cambering index versus prestressing distribution for cases A2–A6. The dotted horizontal lines from bottom to top correspond to $I_p = \{-5, -1, 0, 1, 5\}$ respectively.

be achieved for most of these ten cases. The buckling and post-buckling behaviour of these ten cases will be investigated further in Section 6.4.

6.3.2 Mono-symmetric cases

Distinct from the doubly-symmetric cases, pre-cambering is more significant in mono-symmetric cases because the actual (nonlinear) behaviour deviates more significantly from Eq. (3.21) than in the doubly-symmetric cases, because when $\alpha < 1.0$ an upward force is introduced due to the end-shortening in the main member due to the prestressing.

Changes to the pre-cambering index I_p with increasing γ at $T_{l,i} = T_{opt1}$ for cases F3.1–F3.4, F3.5–F3.7, and F3.8, F3.9 and F3 are shown respectively in Figures 6.4, 6.5 and 6.6. From Figure 6.4, for cases F3.1–F3.4, it can be seen that the downward pre-cambering with $I_p \leq 0$ cannot be achieved by adjusting the prestressing distribution. Considering the unstable post-buckling behaviour of these four cases, as determined in the previous chapters, small crossarm length ratios (*i.e.* $\alpha < 0.5$) are best avoided. From Figures 6.5 and 6.6, for cases F3.5–F3.9 and F3, it can be seen that with increasing γ , I_p decreases consistently from positive to negative values. It implies that adjusting the distribution of prestress between

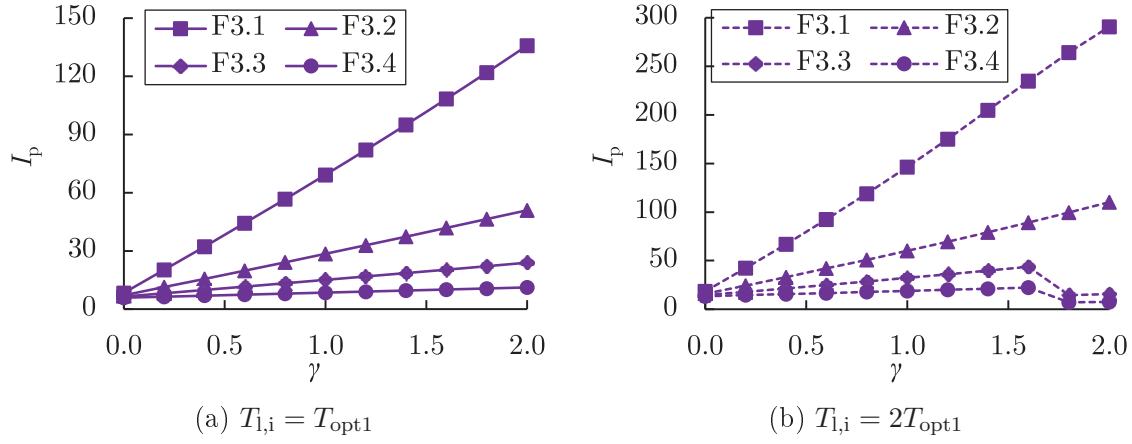
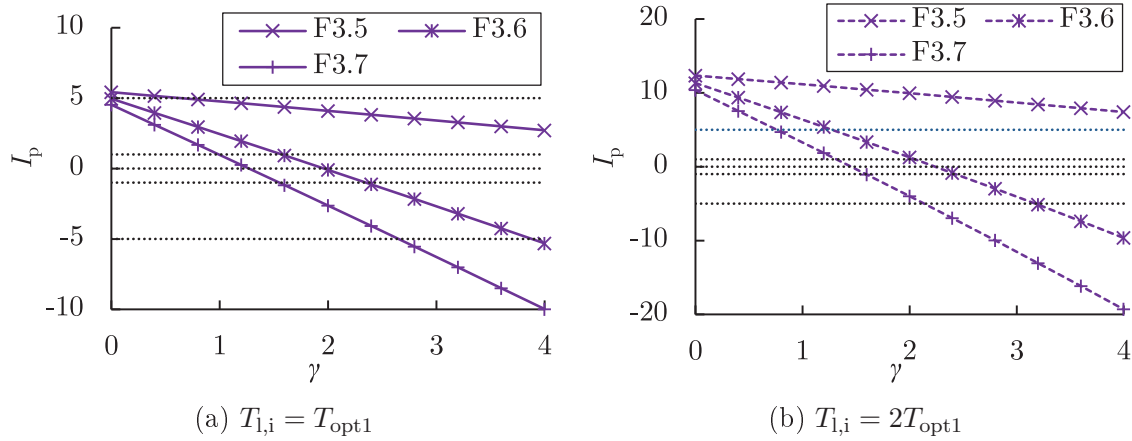


Figure 6.4: Pre-cambering index versus prestressing distribution for cases F3.1–F3.4.

Figure 6.5: Pre-cambering index versus prestressing distribution for cases F3.5–F3.7. The dotted horizontal lines from bottom to top correspond to $I_p = \{-5, -1, 0, 1, 5\}$ respectively. Note that case F3.5 only crosses $I_p = 5$ in (a).

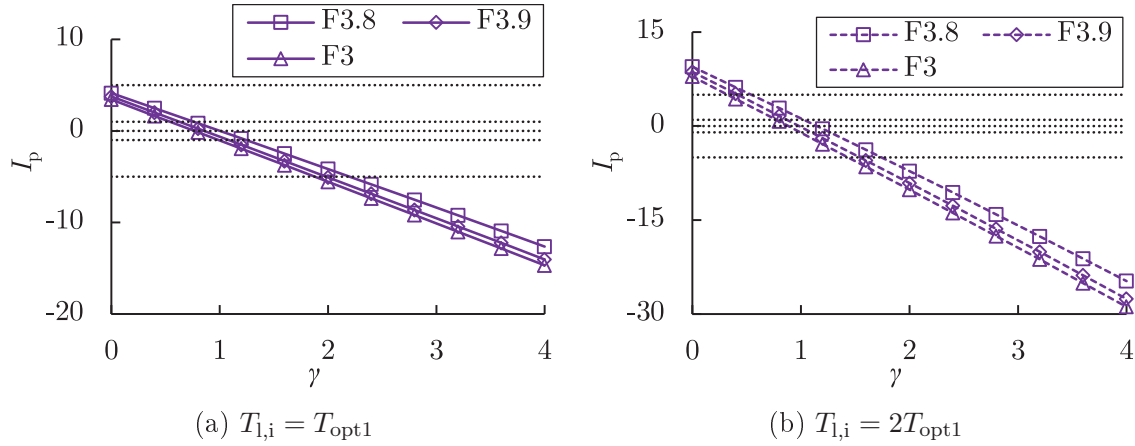


Figure 6.6: Pre-cambering index versus prestressing distribution for cases F3.8, F3.9 and F3. The dotted horizontal lines from bottom to top correspond to $I_p = \{-5, -1, 0, 1, 5\}$ respectively.

the upper and lower stays allows desired pre-cambering levels to be obtained, regardless of the initial member shapes with upward and downward pre-cambering. It is observed that for cases F3.8, F3.9 and F3, the gradients of their relationships are similar, while for cases F3.5–F3.7, the gradients of their relationships are significantly different.

Finally, it should be stressed that the analysis performed in this subsection purely shows the pre-cambering levels achieved after applying varying levels of prestress in loading step 3. In Section 6.4, the structural performance of stayed beam-columns with three specific pre-cambering levels ($I_p = \{-5, 0, 5\}$) is studied.

6.4 Pre-cambering, imperfection and prestressing sensitivities

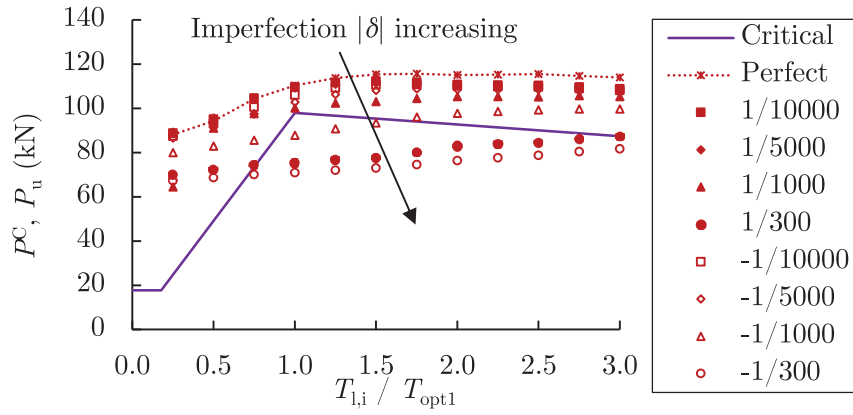
In the current section, the buckling and post-buckling behaviour of imperfect stayed beam-columns under axial compression in conjunction with self-weight are investigated using FE modelling. Following the advances in extending the zonal behaviour in Chapter 3 and

understanding the impact of mode interaction in Chapter 5 in perfect stayed beam-columns under the same loading case as studied herein, the nonlinear behaviour considering different pre-cambering levels, imperfections, and prestressing forces is now examined.

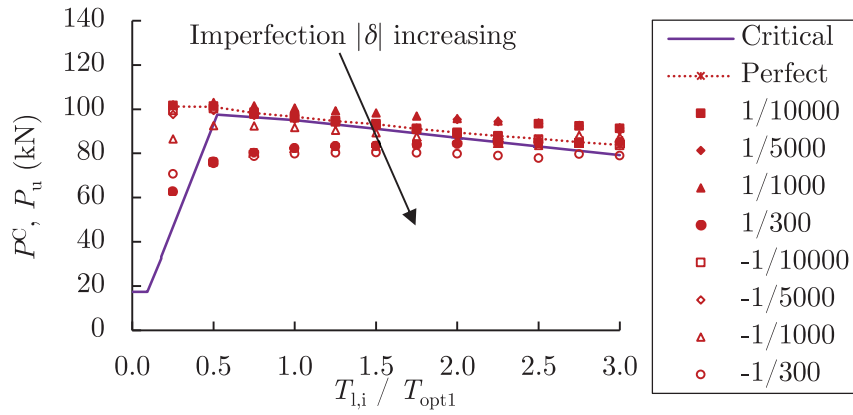
A geometric imperfection amplitude ratio δ of 1/1000 is adopted unless otherwise stated. The imperfection combinations in Table 5.1 are adopted to trigger the different buckling modes, including interactive buckling. In terms of the study on the sensitivity to varying imperfection amplitudes, combination 3 is adopted as the imperfection profile for cases F3 and F3.6. Owing to self-weight and pre-cambering from prestress, the principal deflection direction of the Mode 1 imperfection needs to be considered. A downward imperfection (*i.e.* the same as the self-weight direction) is amplified by the self-weight, and an upward imperfection (*i.e.* opposing the self-weight direction) is reduced by the self-weight. Although the displacement of the mid-span point is fixed after adjusting the prestressing distribution for a specific I_p value in step 3, the remainder of the member still deflects due to its self-weight. In terms of the studies on the sensitivity to varying imperfection combinations and prestressing levels, only the imperfections with the downward principal deflection (combined Mode 2 and downward Mode 1 imperfections) are adopted to trigger interactive buckling, since it was demonstrated in Chapter 5 that mode interaction only occurs in the downward buckling mode.

6.4.1 $I_p = -5$

Figure 6.7 shows the ultimate loads for cases F3 and F3.6 with varying imperfection amplitudes and the pre-cambering index $I_p = -5$. It can be seen that for both cases F3 and F3.6, the load-carrying capacity decreases with increasing imperfection amplitudes, but for case F3.6 the reduction is less significant than that for case F3. Compared to the imperfections with the upward principal deflection, the imperfections with the downward principal deflection lead to lower ultimate loads, as might be expected. In addition, for case F3.6, small imperfections in the upward principal deflection can in fact increase the



(a) F3



(b) F3.6

Figure 6.7: Critical buckling (solid line) and ultimate loads (all symbols) for cases F3 and F3.6 with a pre-cambering index $I_p = -5$ and varying imperfection amplitudes δ ; P^C is the critical buckling load from the analytically-defined zones in Chapter 3; P_u is the ultimate load from FE modelling.

ultimate load slightly.

Figure 6.8 shows the ultimate loads for the different imperfection combinations with $I_p =$

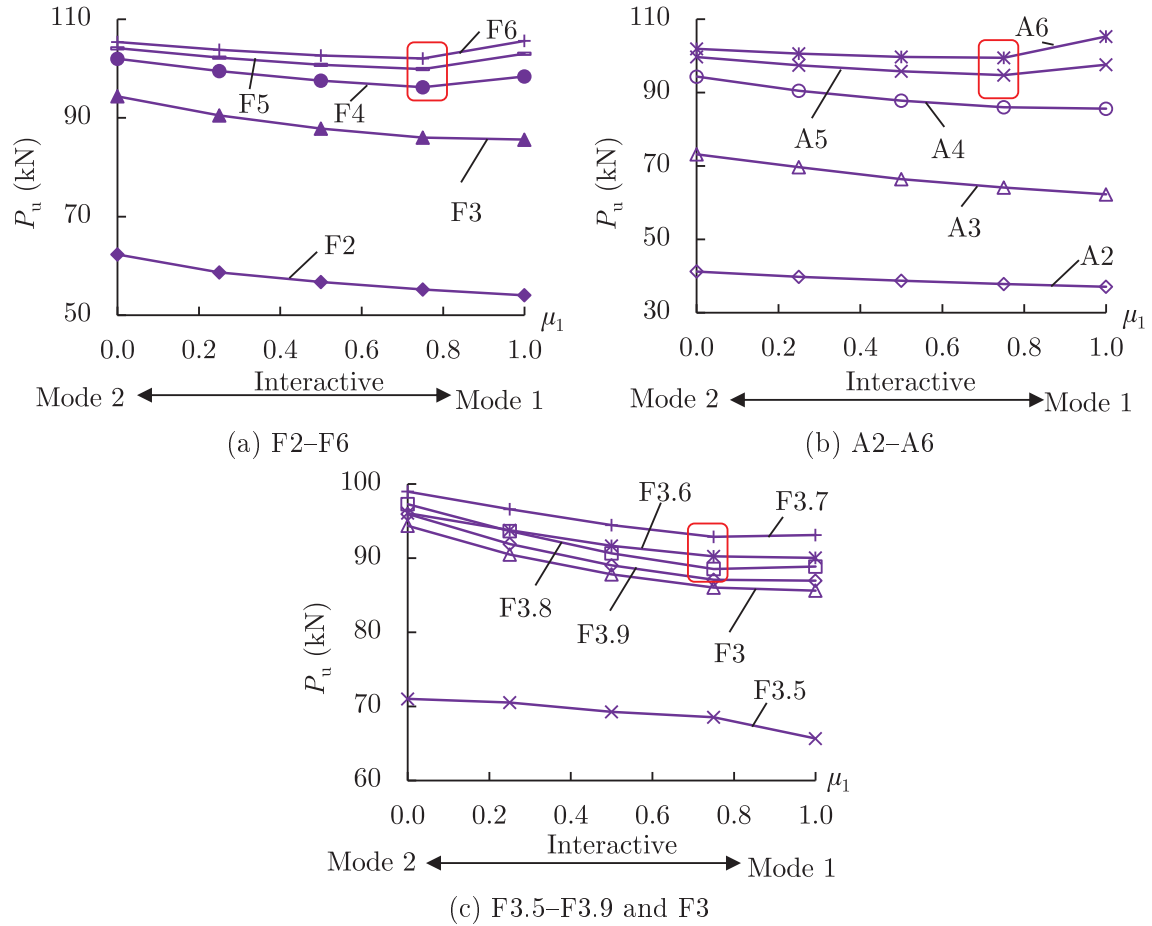


Figure 6.8: Sensitivity to the imperfection combinations represented by P_u versus μ_1 with $I_p = -5$ and (a) varying stay diameter ϕ_s , (b) varying lower crossarm length a_1 and (c) varying crossarm length ratio α . The encircled cases denote the minimum ultimate loads for the respective configurations.

-5 and a variation of the parameters ϕ_s , a_1 and α where $T_{l,i} = T_{opt1}$. For the doubly-symmetric cases, F2-F6 and A2-A6, as can be seen, when Mode 1 is critical, the lowest ultimate load is seen within the Mode 1 buckling case. However, when Mode 2 buckling is critical, the lowest ultimate load is always seen within the interactive buckling case, such as cases F4-F6 and A5-A6. For the mono-symmetric cases, interactive buckling becomes

dominant in cases F3.6–F3.8 that have intermediate values of the crossarm length ratio α . Moreover, for all cases, there are only small differences within the interactive buckling region regardless of the different values of μ_1 , *i.e.* different combinations of μ_1 and μ_2 . This result indicates that interactive buckling behaviour is not very sensitive to the combination of the combined imperfection profiles. These conclusions mirror those from the previous study on vertically stayed columns (Saito & Wadee, 2009b).

The ultimate loads for stayed beam-columns where $T_{l,i} = T_{\text{opt1}}$ and $T_{l,i} = 2T_{\text{opt1}}$ with $I_p = -5$ and varying values of ϕ_s , a_1 and α are shown in Figures 6.9, 6.10, and 6.11.

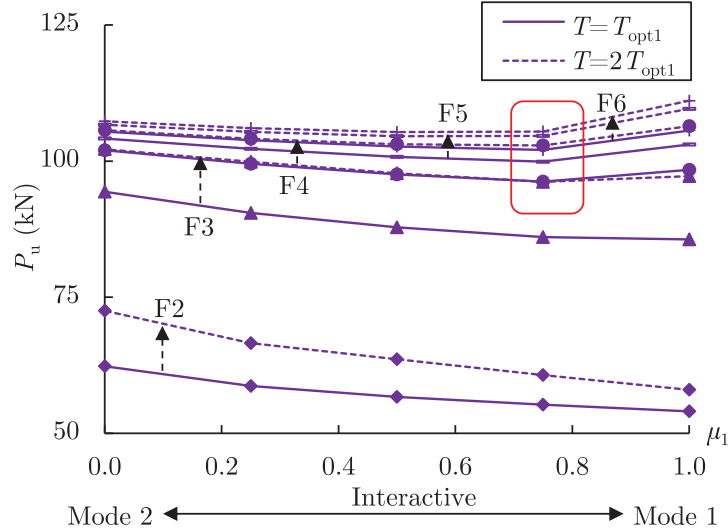


Figure 6.9: Sensitivity to the downward imperfection combinations represented by P_u versus μ_1 with $I_p = -5$ and varying stay diameter ϕ_s where $T_{l,i} = T_{\text{opt1}}$ and $T_{l,i} = 2T_{\text{opt1}}$. The arrows with the dashed lines denote an increase or decrease in the ultimate load from T_{opt1} to $2T_{\text{opt1}}$.

From Figures 6.9 and 6.10, for the doubly-symmetric cases, increasing the prestressing force results in a rise in the ultimate load for each imperfection combination. Although the maximum critical buckling load can be seen at $T_{l,i} = T_{\text{opt1}}$ from linear analysis in Chapter 3, T_{opt1} does not maximize the ultimate load, and introducing a prestressing force that is higher than this level is recommended in practice. From Figure 6.11, for the mono-symmetric cases F3.5–F3.7, increasing the prestressing level reduces the ultimate load,

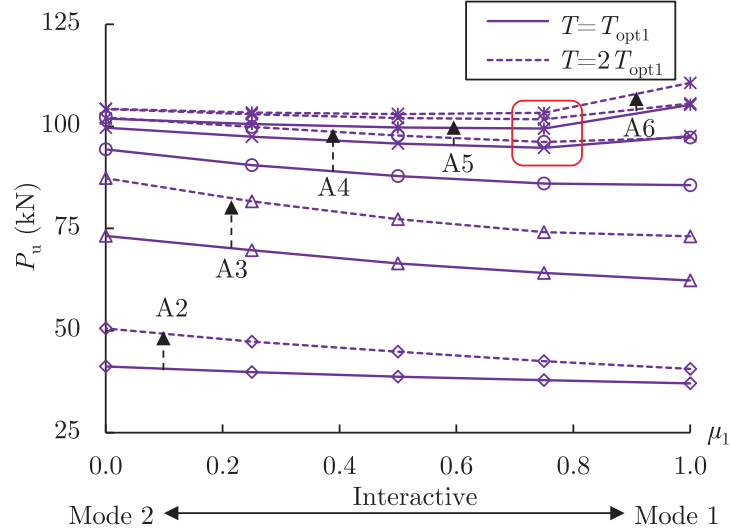


Figure 6.10: Sensitivity to the downward imperfection combinations represented by P_u versus μ_1 with $I_p = -5$ and varying lower crossarm length a_1 where $T_{l,i} = T_{opt1}$ and $T_{l,i} = 2T_{opt1}$. Axes and labels are the same as Figure 6.9.

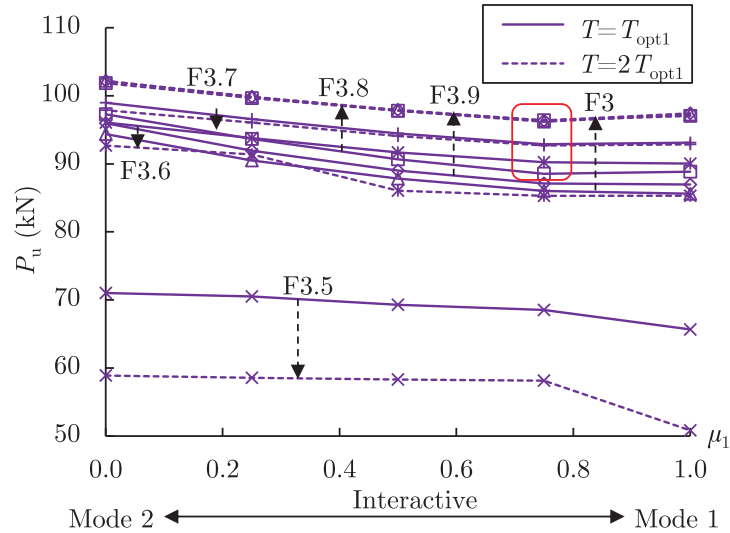


Figure 6.11: Sensitivity to the downward imperfection combinations represented by P_u versus μ_1 with $I_p = -5$ and varying crossarm length ratio α where $T_{l,i} = T_{opt1}$ and $T_{l,i} = 2T_{opt1}$. Axes and labels are the same as Figure 6.9.

while for other cases with larger α , *i.e.* cases F3.8, F3.9 and F3, the load-carrying capacity increases significantly from $T_{l,i} = T_{\text{opt1}}$ to $T_{l,i} = 2T_{\text{opt1}}$. In addition, for cases F3, A4, F3.9 and F3, at $T_{l,i} = T_{\text{opt1}}$, interactive buckling does not dominate the behaviour, while for $T_{l,i} = 2T_{\text{opt1}}$, interactive buckling is dominant.

6.4.2 $I_p = 0$

Figure 6.12 shows the ultimate loads for cases F3 and F3.6 with varying imperfection

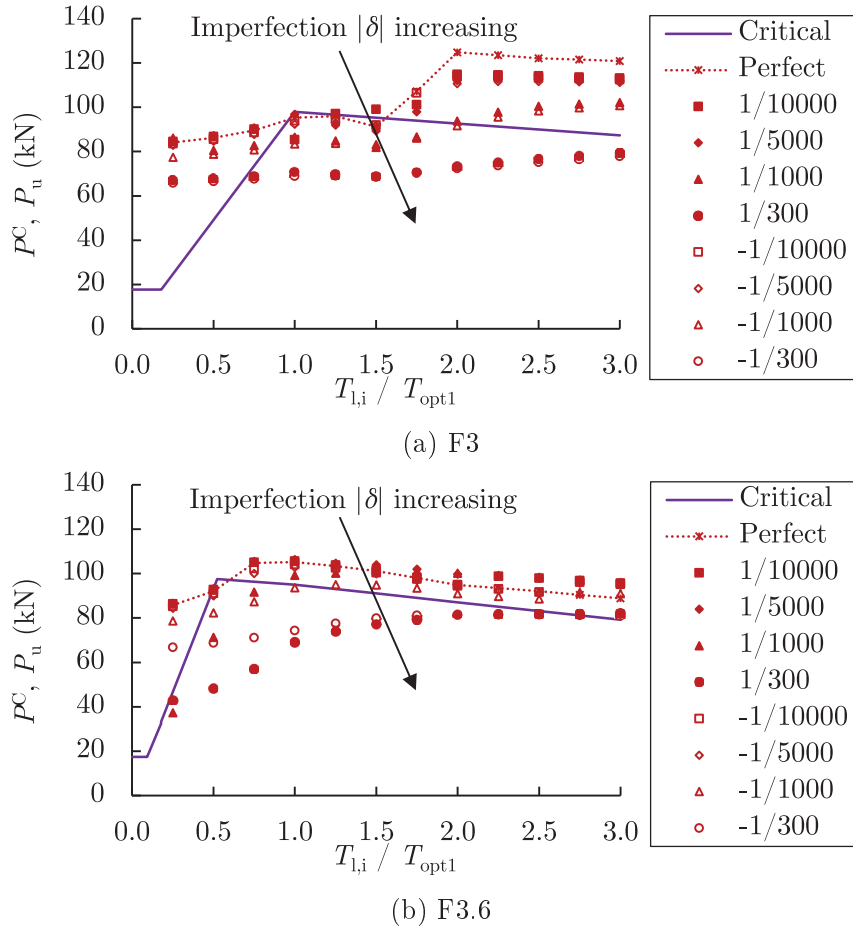


Figure 6.12: Critical buckling (solid line) and ultimate loads (all symbols) for cases F3 and F3.6 with $I_p = 0$ and varying δ . Axes and labels are the same as Figure 6.7.

amplitudes and the pre-cambering index $I_p = 0$. For case F3, the difference of ultimate

loads with imperfections in the different principal directions is small. However, for case F3.6, the reduction from the upward imperfections is more significant than that from the imperfections that are downwards.

Figure 6.13 shows the ultimate loads for the different imperfection combinations with $I_p = 0$

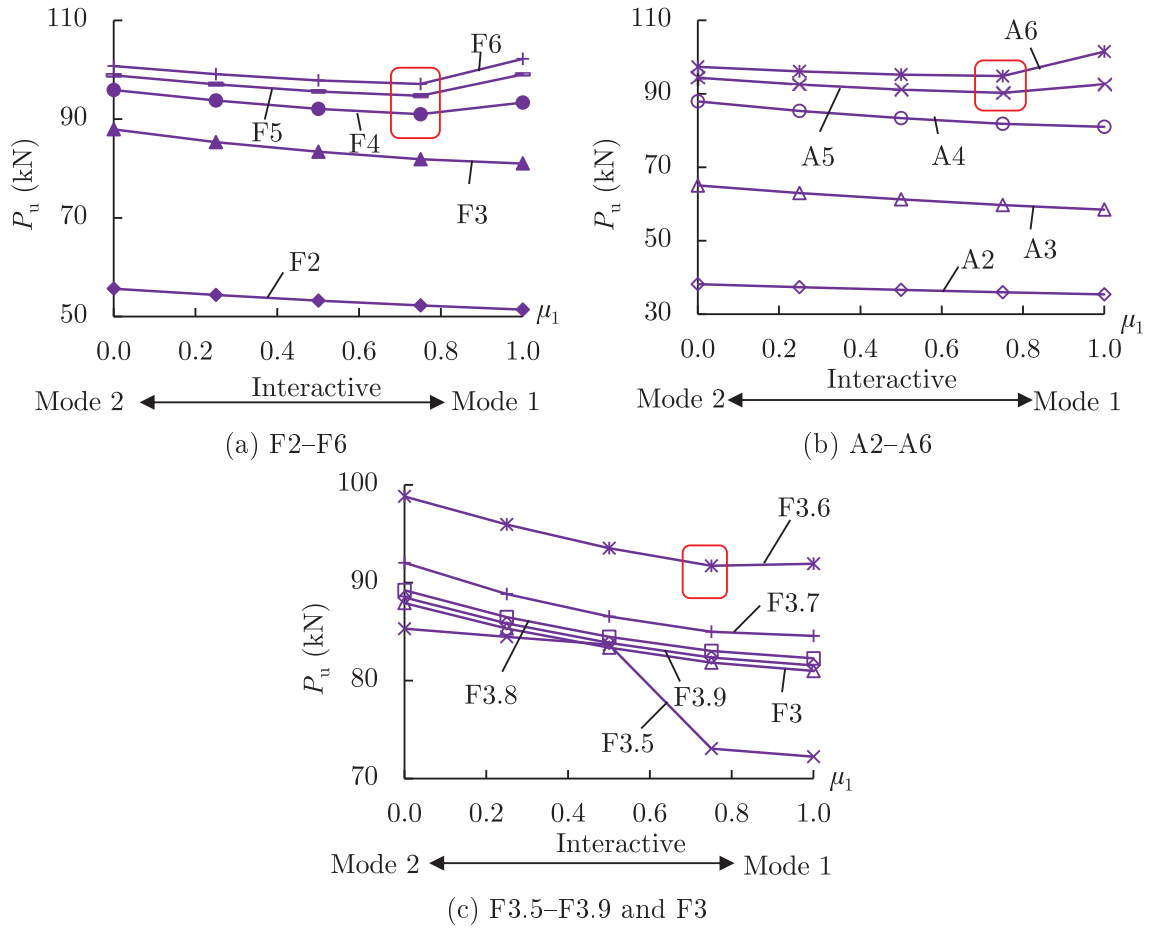


Figure 6.13: Sensitivity to the imperfection combinations represented by P_u versus μ_1 with $I_p = 0$ and (a) varying ϕ_s , (b) varying a_1 and (c) varying α .

and a variation of the parameters ϕ_s , a_1 and α where $T_{l,i} = T_{opt1}$. Comparing Figure 6.13 with 6.8, for the doubly-symmetric cases, the behaviour in these systems with this pre-cambering level is similar to that with $I_p = -5$, but the load-carrying capacity for the cases with $I_p = 0$ is lower than that with $I_p = -5$. The interactive buckling dominance does not change with the pre-cambering level varying. For the mono-symmetric cases with $I_p = 0$,

case F3.6 performs best, and the difference in load-carrying capacity between case F3.6 and the other cases is significant. For cases with $I_p = -5$, case F3.7 performs best, and the difference of the load-carrying capacity among cases F3.6–F3.9 and F3 is not significant.

The ultimate loads for stayed beam-columns where $T_{l,i} = T_{opt1}$ and $T_{l,i} = 2T_{opt1}$ with $I_p = 0$ and varying values of ϕ_s , a_1 and α are shown in Figures 6.14, 6.15, and 6.16. Comparing

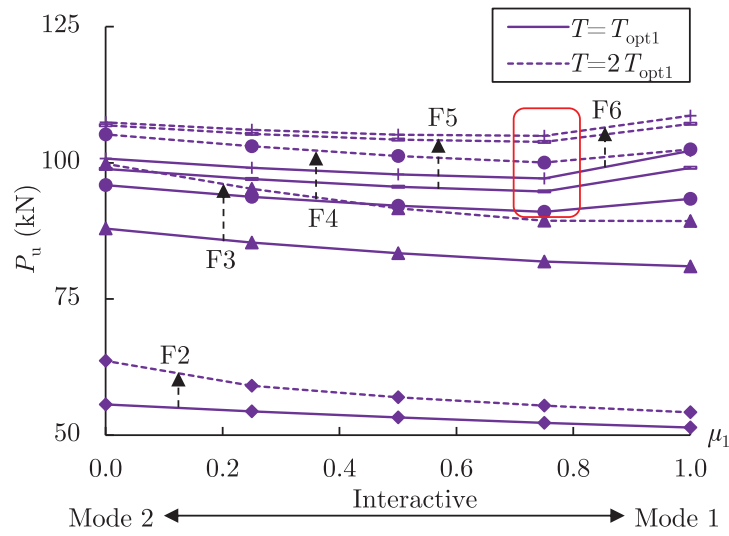


Figure 6.14: Sensitivity to the downward imperfection combinations represented by P_u versus μ_1 with $I_p = 0$ and varying ϕ_s where $T_{l,i} = T_{opt1}$ and $T_{l,i} = 2T_{opt1}$. Axes and labels are the same as Figure 6.9.

Figures 6.14 and 6.15 with Figures 6.9 and 6.10 respectively, for the doubly-symmetric cases, similar to the results where $T_{l,i} = T_{opt1}$, the behaviour in these systems with this pre-cambering level is similar to that with $I_p = -5$, but their load-carrying capacity with $I_p = 0$ is lower. Comparing Figure 6.16 with Figure 6.11, for case F3.7 where $I_p = 0$, the load-carrying capacity can be increased significantly by changing $T_{l,i} = T_{opt1}$ to $T_{l,i} = 2T_{opt1}$. However, when $I_p = -5$, increasing the prestressing level leads to a reduction of the load-carrying capacity. Moreover, when $T_{l,i} = 2T_{opt1}$, case F3.7 performs best as opposed to case F3.6 at $T_{l,i} = T_{opt1}$, and the ultimate loads of cases F3.6–F3.9 and F3 are numerically close.

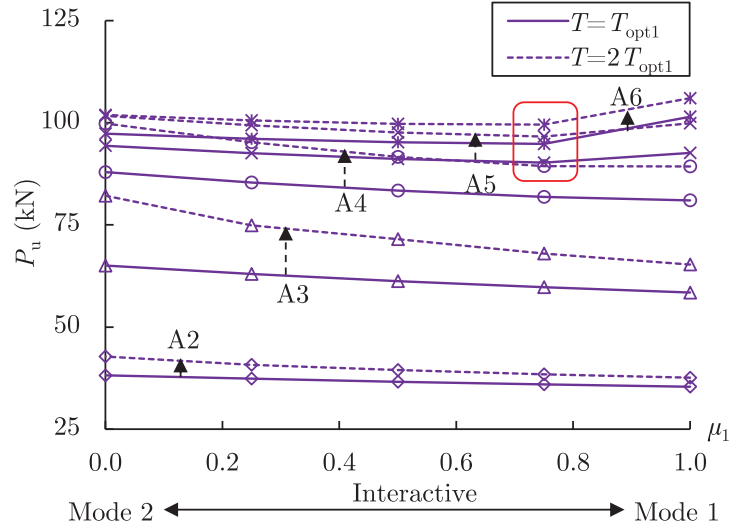


Figure 6.15: Sensitivity to the downward imperfection combinations represented by P_u versus μ_1 with $I_p = 0$ and varying a_1 where $T_{l,i} = T_{opt1}$ and $T_{l,i} = 2T_{opt1}$. Axes and labels are the same as Figure 6.9.

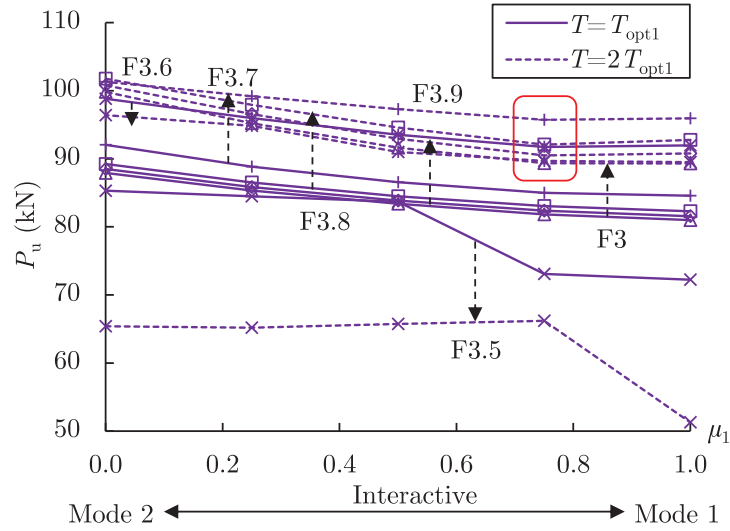


Figure 6.16: Sensitivity to the downward imperfection combinations represented by P_u versus μ_1 with $I_p = 0$ and varying α where $T_{l,i} = T_{opt1}$ and $T_{l,i} = 2T_{opt1}$. Axes and labels are the same as Figure 6.9.

6.4.3 $I_p = 5$

Figure 6.17 shows the ultimate loads for cases F3 and F3.6 with varying imperfection

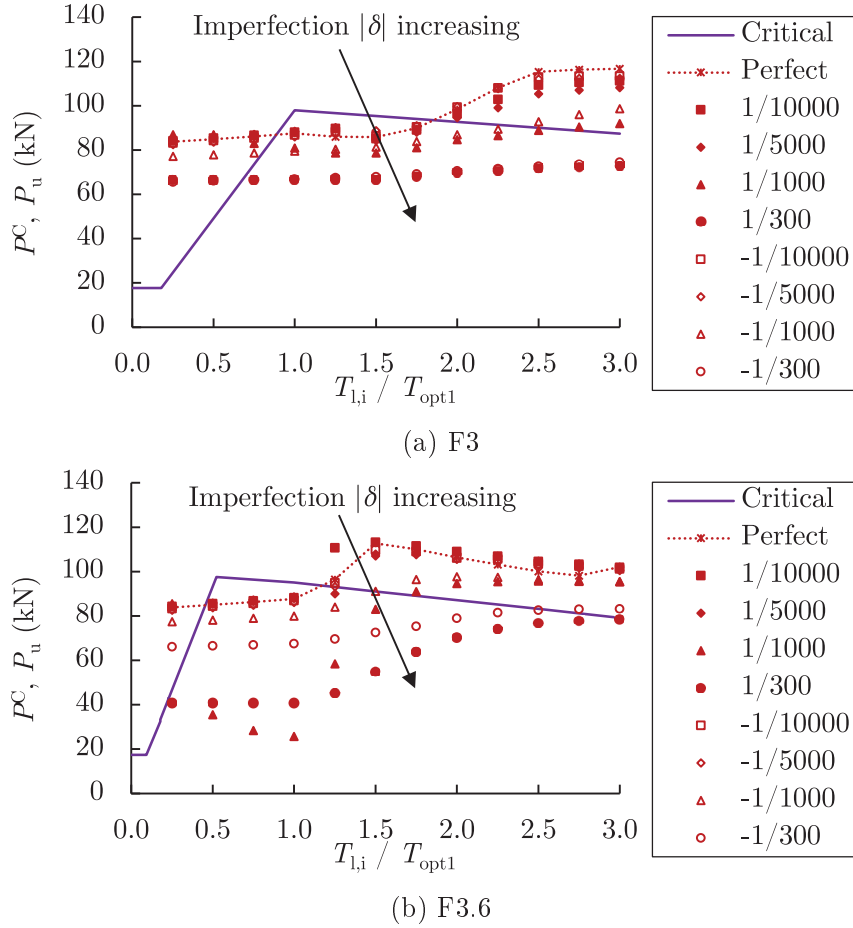


Figure 6.17: Critical buckling (solid line) and ultimate loads (all symbols) for cases F3 and F3.6 with $I_p = 5$ and varying δ . Axes and labels are the same as Figure 6.7.

amplitudes and pre-cambering index $I_p = 5$. For both cases F3 and F3.6, their ultimate loads with the upward imperfection are higher than those with the downward imperfection. For case F3.6, compared to the results with $I_p = -5$ and $I_p = 0$ in Figures 6.7 and 6.12, its load-carrying capacity with $I_p = 5$ is more sensitive in particular to upward imperfections. The reason for this phenomenon appears to be owing to the displacement from pre-cambering also being upward, *i.e.* in the same direction as the imperfection.

Figure 6.18 shows the ultimate loads for the different imperfection combinations with

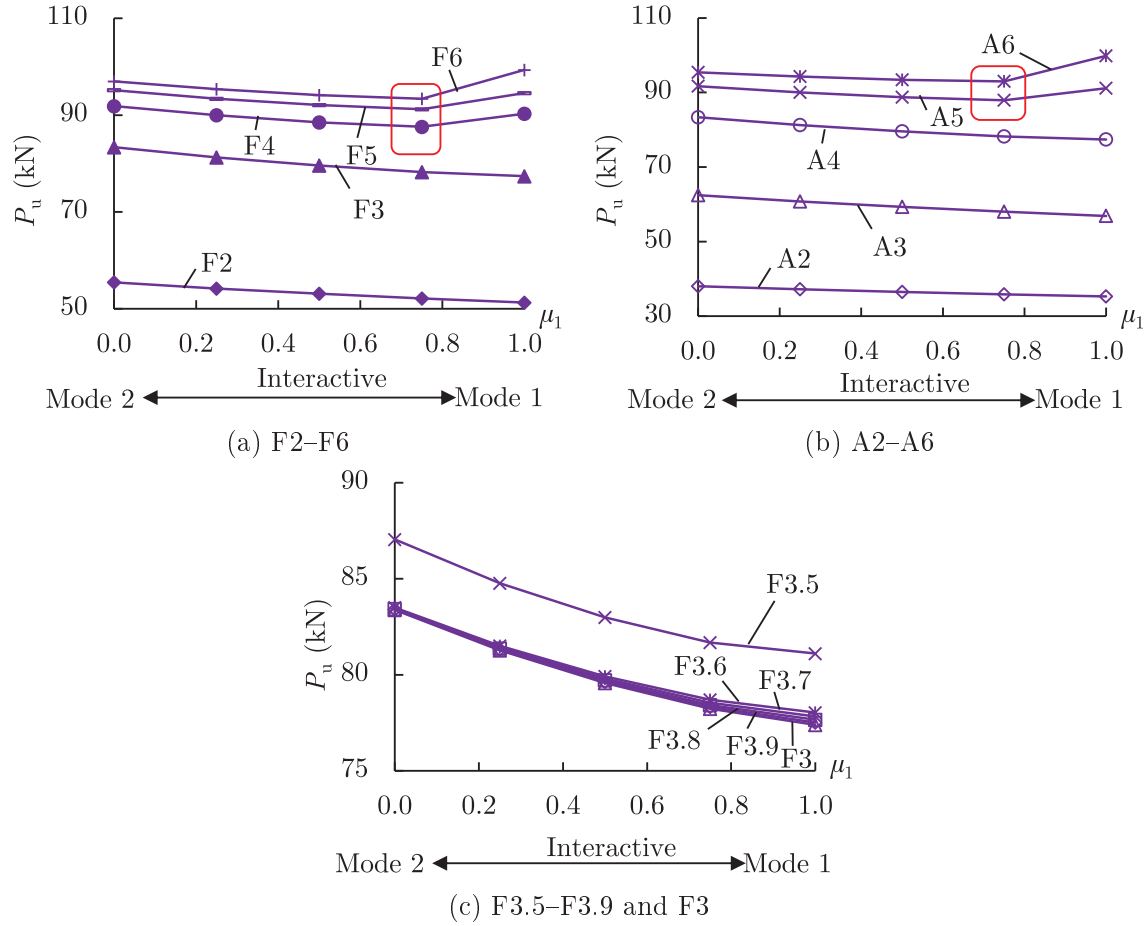


Figure 6.18: Sensitivity to the imperfection combinations represented by P_u versus μ_1 with $I_p = 5$ and (a) varying ϕ_s , (b) varying a_1 and (c) varying α .

$I_p = 5$ and a variation of the parameters ϕ_s , a_1 and α where $T_{l,i} = T_{opt1}$. Comparing Figures 6.18 with 6.13, for the doubly-symmetric cases, the load-carrying capacity of these systems with $I_p = 5$ is smaller than that with $I_p = 0$, but the trends are essentially the same as those with $I_p = 0$ and $I_p = -5$. For the mono-symmetric cases, with increasing α , the ultimate loads decrease, and case F3.5 performs significantly better than F3.6–F3.9 and F3. Moreover, there is no interactive buckling dominance observed.

The ultimate loads for stayed beam-columns where $T_{l,i} = T_{opt1}$ and $T_{l,i} = 2T_{opt1}$ with $I_p = 5$ and varying values of ϕ_s , a_1 and α are shown in Figures 6.19, 6.20, and 6.21. Compared

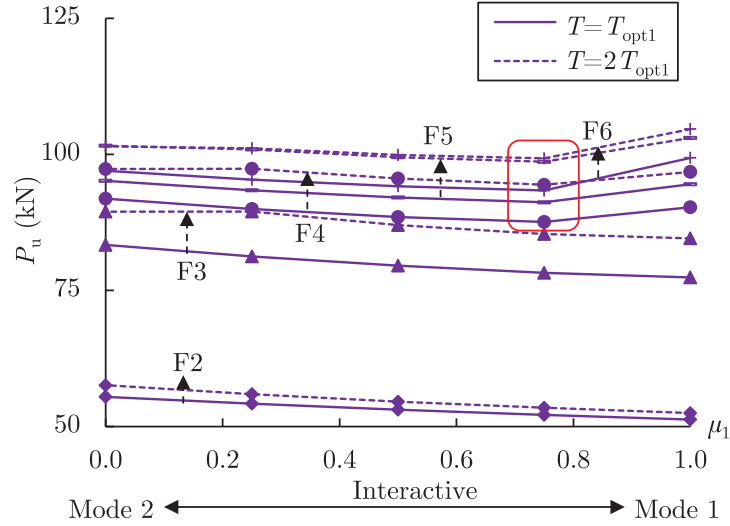


Figure 6.19: Sensitivity to the downward imperfection combinations represented by P_u versus μ_1 with $I_p = 5$ and varying ϕ_s where $T_{l,i} = T_{opt1}$ and $T_{l,i} = 2T_{opt1}$. Axes and labels are the same as Figure 6.9.

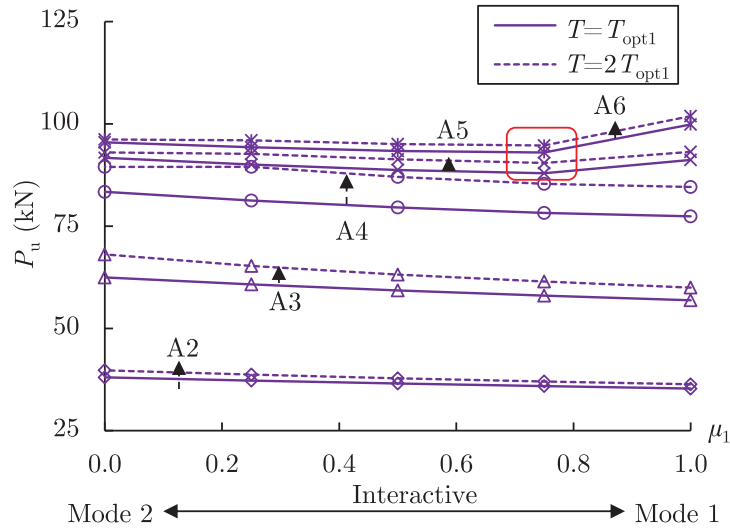


Figure 6.20: Sensitivity to the downward imperfection combinations represented by P_u versus μ_1 with $I_p = 5$ and varying a_1 where $T_{l,i} = T_{opt1}$ and $T_{l,i} = 2T_{opt1}$. Axes and labels are the same as Figure 6.9.

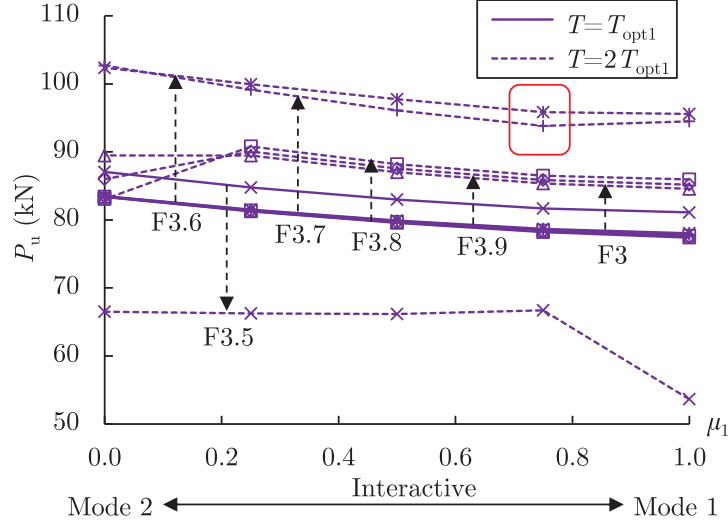


Figure 6.21: Sensitivity to the downward imperfection combinations represented by P_u versus μ_1 with $I_p = 5$ and varying α where $T_{l,i} = T_{opt1}$ and $T_{l,i} = 2T_{opt1}$. Axes and labels are the same as Figure 6.9.

with the results from these systems with $I_p = -5$ and $I_p = 0$, for the doubly-symmetric cases, similar to the results where $T_{l,i} = T_{opt1}$, the load-carrying capacity of these systems with $I_p = 5$ is smaller than that with $I_p = 0$, but the pattern is essentially the same as that with both $I_p = 0$ and $I_p = -5$. Comparing Figure 6.21 with Figure 6.16, for the mono-symmetric cases where $I_p = 5$, increasing the prestressing level reduces the load-carrying capacity of case F3.5, but for $I_p = 0$ the reduction effect was not only in case F3.5 but also in case F3.6; for $I_p = -5$, cases F3.5–F3.7 show this trend. Moreover, when $T_{l,i} = 2T_{opt1}$, for cases F3.6 and F3.7, interactive buckling becomes dominant, and the load-carrying capacity of case F3.7 is found to be the highest.

6.4.4 Summary

For the doubly-symmetric cases, the ultimate load, buckling mode and interactive buckling dominance are not sensitive to varying pre-cambering levels, although the load-carrying capacity decreases slightly with changing the pre-cambering level I_p from -5 to 5 . The ul-

ultimate loads of the doubly-symmetric cases can be increased by increasing the prestressing levels. However, for the mono-symmetric cases, the buckling and post-buckling behaviour are very sensitive to the level of pre-cambering. For $I_p = -5$ and $I_p = 0$, the cases with intermediate crossarm length ratios (*i.e.* cases F3.6 and F3.7) have relatively high load-carrying capacities, but the cases with either small or large crossarm length ratios (*i.e.* cases F3.5 or F3 respectively) have relatively low load-carrying capacities, which implies that the intermediate crossarm length ratios are more suitable for practice; the lowest ultimate loads usually occur in the systems with combined imperfections (*i.e.* imperfection combinations 2–4 defined in Table 5.1), which demonstrates the dominance of interactive buckling. For $I_p = 5$, reducing the crossarm length ratio increases the load-carrying capacity and interactive buckling does not dominate.

6.5 True optimum prestressing force

In the current section, the true optimum prestressing force is investigated based on the preceding studies on imperfection and prestressing sensitivities. The imperfection amplitude ratio $\delta = 1/1000$ is adopted, and the imperfection combinations with both the upward and downward principal deflections are taken into account, where the combination leading to the lowest buckling load is adopted in the FE models, as shown in Table 6.1. In terms of the doubly-symmetric systems, for cases F2, A2 and A3, combination 1 is adopted owing to dominant Mode 1 buckling; for cases F3–F6 and A4–A6, combination 3 is adopted as the representative shape to trigger interactive buckling that generally leads to the lowest ultimate loads. In terms of the mono-symmetric cases F3.5–F3.9, combination 1 from Table 5.1 is adopted for the upward imperfection, since when $\alpha < 1$, Mode 1 buckling with an upward deflection leads to a relatively lower load-carrying capacity from the mode interaction study in Chapter 5; combination 3 is adopted as the representative shape to trigger interactive buckling which generally leads to the lowest ultimate loads in downward buckling modes based on the findings from the imperfection sensitivity studies in the current

Table 6.1: Imperfection combinations and directions for the study of true optimum prestressing forces. Based on the definitions in Table 5.1, ‘U1’ and ‘D1’ represent the upward and downward imperfection with combination 1 respectively, and ‘U3’ and ‘D3’ represent the upward and downward imperfection with combination 3 respectively.

Case	F2	F3	F4	F5	F6
Downward	D1	D3	D3	D3	D3
Upward	U1	U3	U3	U3	U3
Case	A2	A3	A4	A5	A6
Downward	D1	D1	D3	D3	D3
Upward	U1	U1	U3	U3	U3
Case	F3.5	F3.6	F3.7	F3.8	F3.9
Downward	D3	D3	D3	D3	D3
Upward	U1	U1	U1	U1	U1

chapter and from Chapter 5.

To maximize the ultimate load, it has been demonstrated that increasing the initial prestressing level beyond the linearly obtained optimum in Chapter 3 is a viable strategy. Apart from determining the true optimum prestressing force based on the prestressing force leading to the maximum ultimate load, structural efficiency is also taken into account, since high prestressing levels usually increase the load-carrying capacity but may reduce the structural efficiency (Saito & Wadee, 2010), particularly when larger cables are required to introduce higher prestressing forces. In Saito & Wadee (2010), the true optimum prestressing forces for imperfect single-bay stayed columns were presented. In order to evaluate the efficiency of the stayed column, two indicators, η and η_s were proposed, thus:

$$\eta = \frac{P_u}{A_c f_{y, \text{req}}}, \quad \eta_s = \frac{P_u}{A_s f_{s, \text{req}}} \frac{L/2}{l_s}, \quad (6.3)$$

where P_u is the ultimate load, while $f_{y, \text{req}}$ and $f_{s, \text{req}}$ are the required strengths when $P = P_u$ for the main column and stays respectively; A_c and A_s are the cross-sectional areas of the main column and stays respectively; L and l_s are the lengths of the main column and stays respectively. These two indicators not only consider the load-carrying capacity P_u , but also take the material strength of the main column and stays into account. Higher values

of these indicators η and η_s imply that the constituent materials are being utilized to a fuller extent and hence represent more efficient prestressing levels and configurations. For the specific case when $\eta = 1.0$, elastic buckling analysis is sufficient and it is unnecessary to consider the resistance of cross-sections. The true optimum prestressing level can be determined by comparing the values of η and η_s , which provides suggestions for optimum structural configurations and prestressing levels. Since the stay lengths and prestressing forces are not equal in mono-symmetric stayed beam-columns in Chapter 3, the structural efficiency indicator for the stays η_s is adjusted slightly with a new definition, thus:

$$\eta_s = \min(\eta_{s,u1}, \eta_{s,u2}, \eta_{s,l1}, \eta_{s,l2}), \quad (6.4)$$

where:

$$\eta_{s,i} = \frac{P_u}{A_s f_{s,req,i}} \frac{L/2}{l_i}, \quad (6.5)$$

and $i = \{u1, u2, l1, l2\}$ corresponding to each stay, as illustrated in Figure 3.1. However, the definition of the structural efficiency indicator for the main member η is unchanged from the doubly-symmetric stayed beam-column definition.

6.5.1 $I_p = -5$

Figures 6.22, 6.23 and 6.24 show P_u , η and η_s for varying prestressing levels with the downward pre-cambering level $I_p = -5$ for cases F2–F6, A2–A6, and F3.5–F3.9 and F3 respectively. It can be seen that many high values of the efficiency indicators occur when $T_{l,i}/T_{opt1} \leq 0.5$, but these prestressing levels are not recommended owing to the intrinsically low load-carrying capacity at such prestressing levels, which provides requirements of very small column and stay sizes. From Figures 6.22 and 6.23, for the doubly-symmetric cases at $T_{l,i}/T_{opt1} = 3$, the ultimate loads of most cases do not reach the maximum values, although the gradients of their curves of P_u versus $T_{l,i}/T_{opt1}$ approach zero, but if the aforementioned high values of the efficiency indicators within $T_{l,i}/T_{opt1} \leq 0.5$ are discarded,

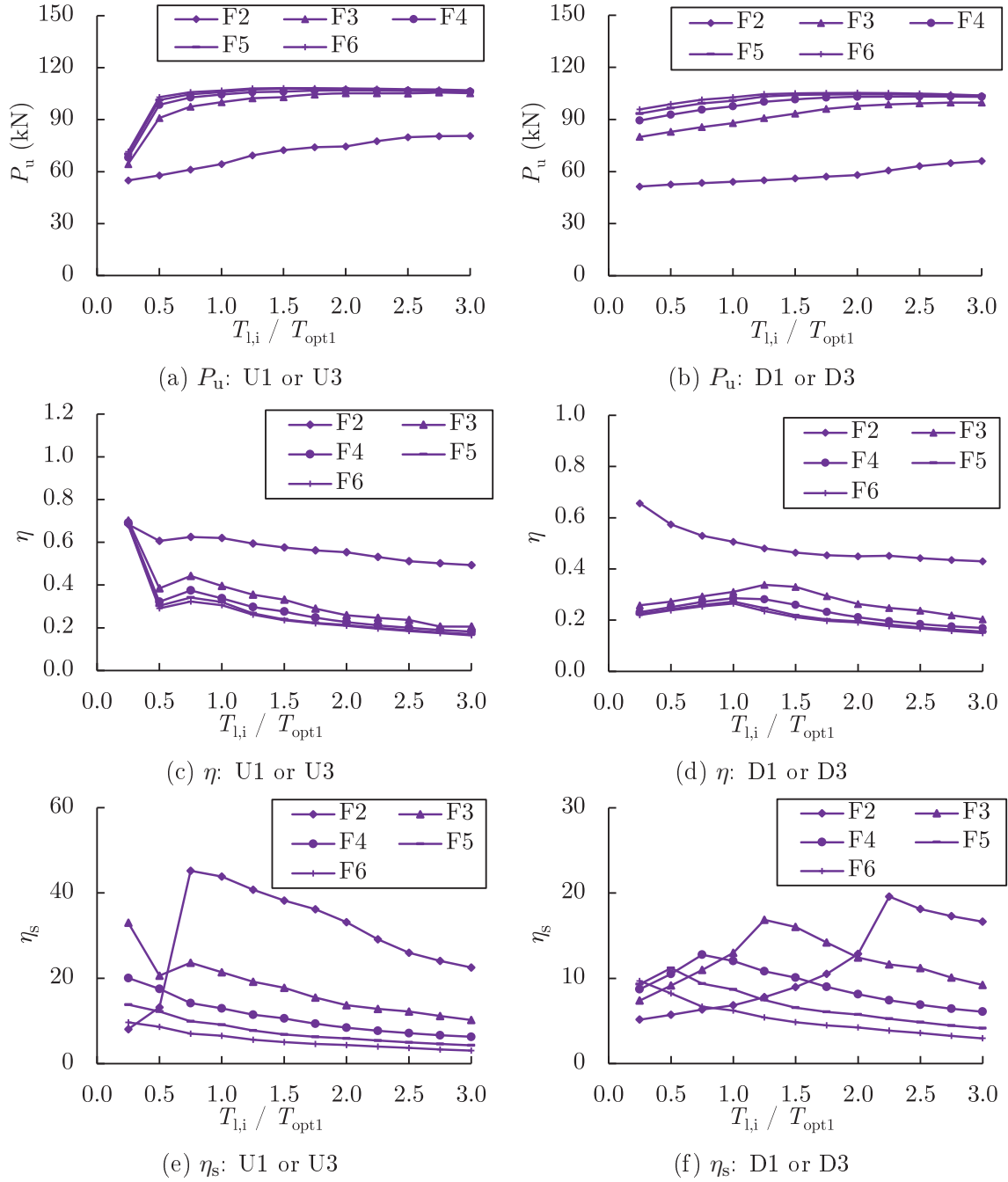


Figure 6.22: Efficiency indicators to determine the true optimum prestressing force for cases F2–F6 with $I_p = -5$.

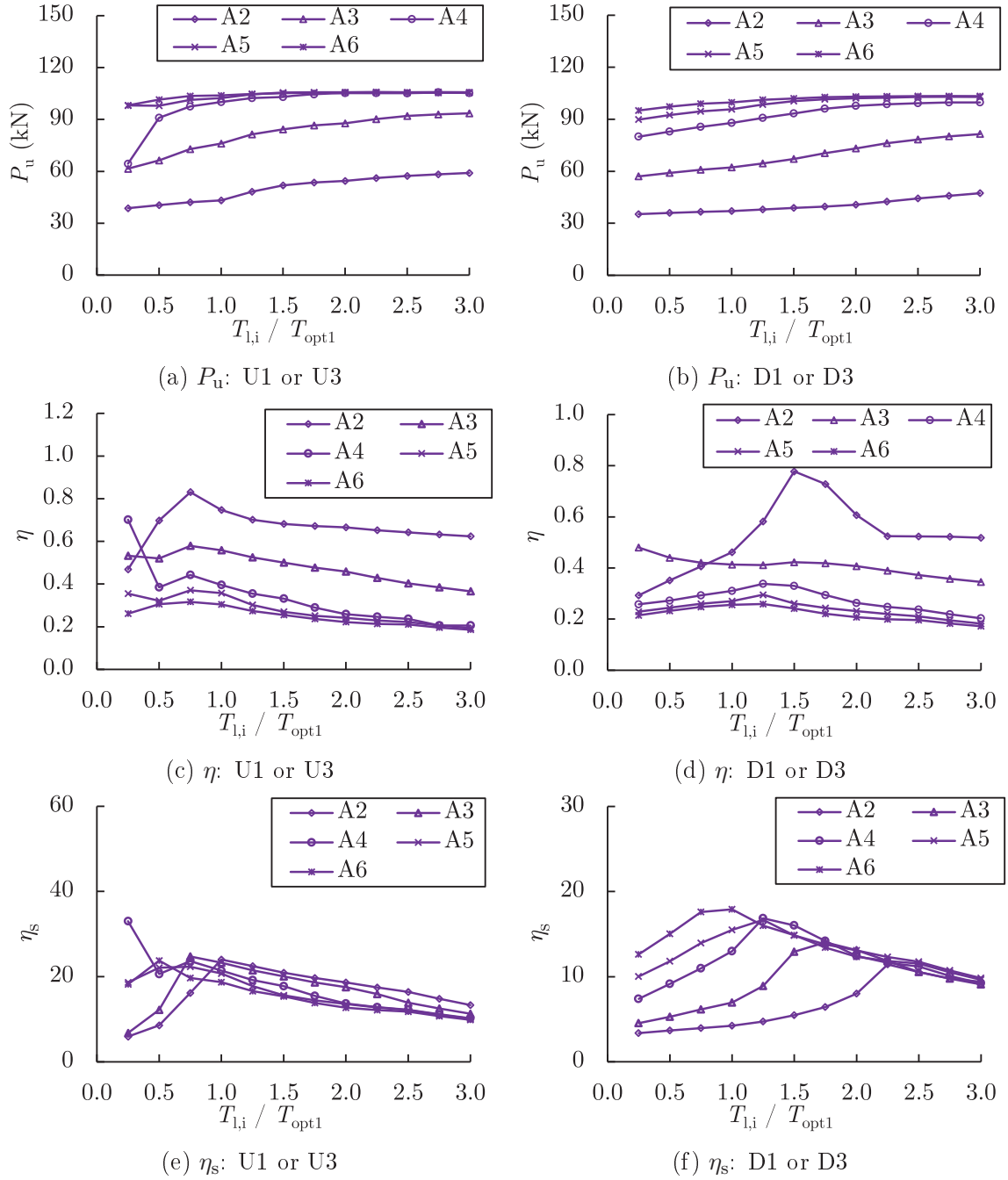


Figure 6.23: Efficiency indicators to determine the true optimum prestressing force for cases A2–A6 with $I_p = -5$.

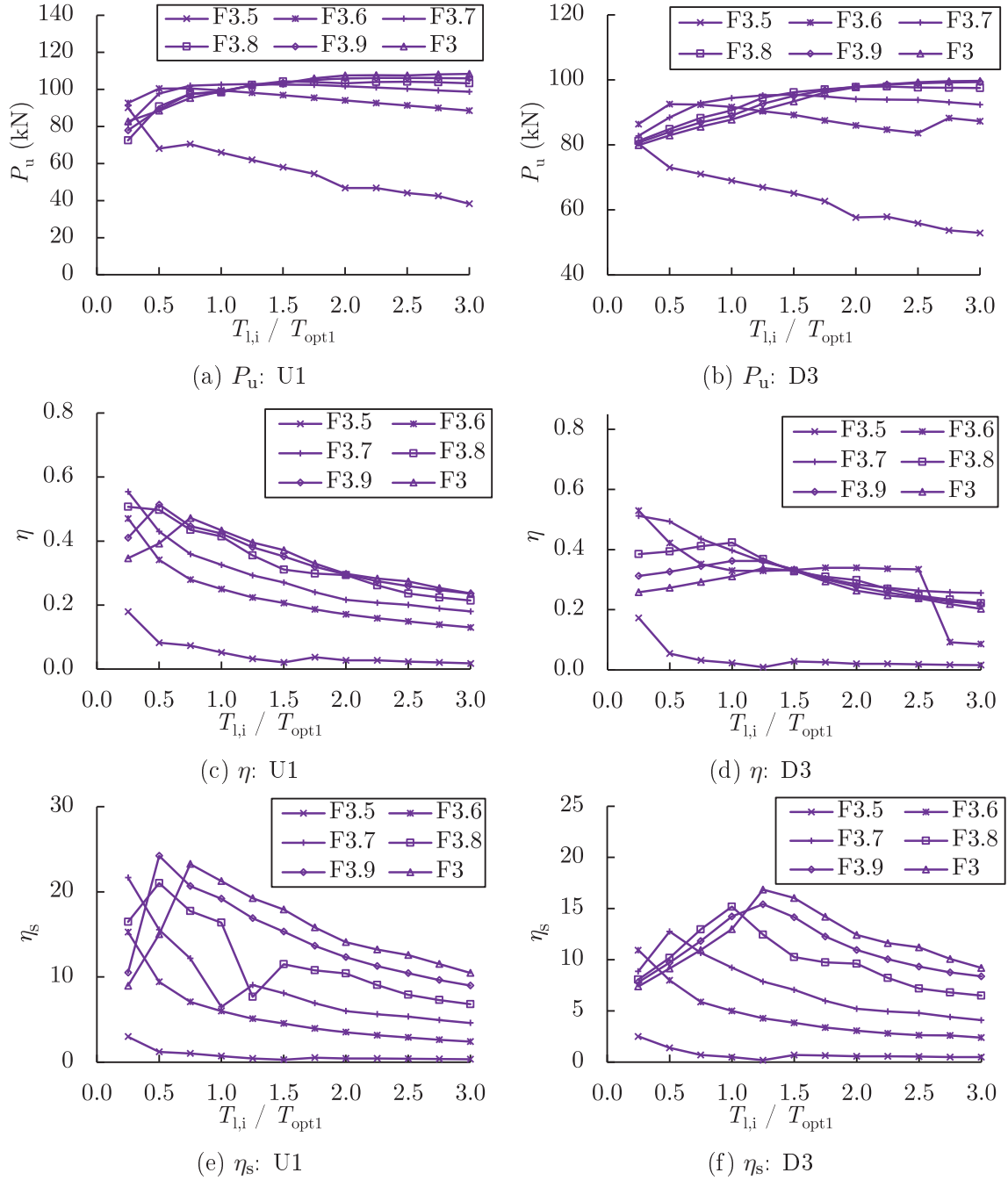


Figure 6.24: Efficiency indicators to determine the true optimum prestressing force for cases F3.5–F3.9 and F3 with $I_p = -5$.

the structural efficiency indicators in fact peak within the range $T_{l,i}/T_{opt1} = [0.5, 3]$. From Figure 6.24, in contrast with the results of the doubly-symmetric cases, the ultimate loads of cases F3.5–F3.8 reach their maximum values before approaching $T_{l,i}/T_{opt1} = 3$. Comparing Figure 6.24 with Figures 6.22 and 6.23, considering both load-carrying capacity and structural efficiency, for $I_p = -5$, most doubly-symmetric cases perform better than the mono-symmetric cases.

6.5.2 $I_p = 0$

Figures 6.25 and 6.26 show P_u , η and η_s for varying prestressing levels with the zero pre-cambering level $I_p = 0$ for cases A2–A6, and F3.5–F3.9 and F3 respectively. For cases F2–F6, similar trends with cases A2–A6, presented in Figure 6.25, are observed. From Figure 6.25, for the doubly-symmetric cases with upward imperfections, when $T_{l,i}/T_{opt1} \leq 1$, the efficiency indicators reduce to a local minimum as $T_{l,i}$ is increased. Subsequently, this is followed by a local maximum in the efficiency indicators as $T_{l,i}$ is further increased. From Figure 6.26, for the mono-symmetric cases with upward imperfections, the extremely high values of efficiency indicators occur when $T_{l,i}/T_{opt1} \leq 0.5$ due to the small ultimate loads at such prestressing levels; hence these are again discarded. Comparing Figure 6.26 with Figure 6.25, considering both load-carrying capacity and structural efficiency, for the systems with $I_p = 0$ and upward imperfections, the doubly-symmetric cases perform better than the mono-symmetric cases, but for the systems with $I_p = 0$ and downward imperfections, the reverse is found. For all cases, comparing the results from the systems with $I_p = 0$ with those from $I_p = -5$, it is apparent that for $I_p = 0$, the required prestressing levels that maximize the values of the structural efficiency indicators are generally higher than that for the case where $I_p = -5$.

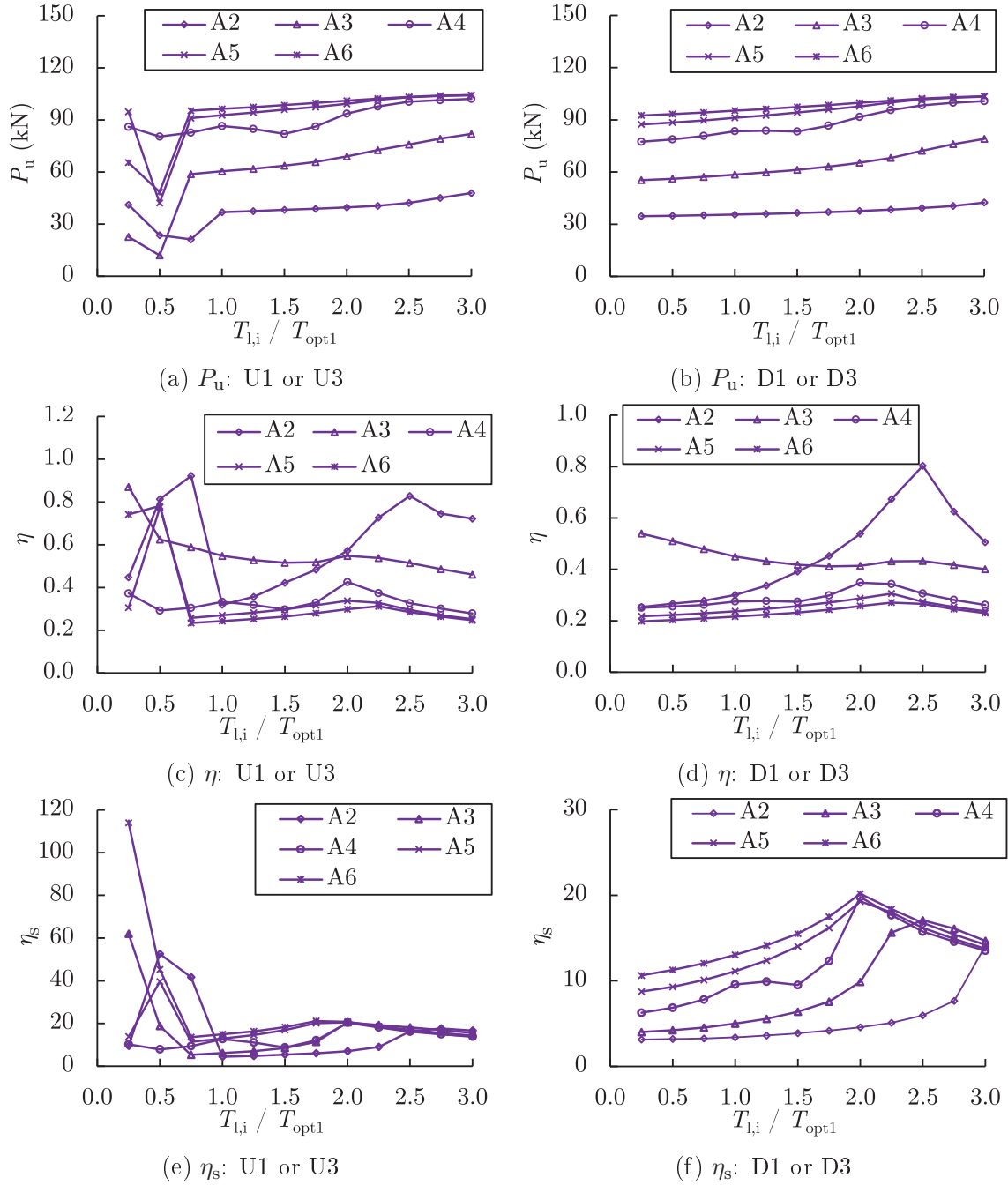


Figure 6.25: Efficiency indicators to determine the true optimum prestressing force for cases A2–A6 with $I_p = 0$.

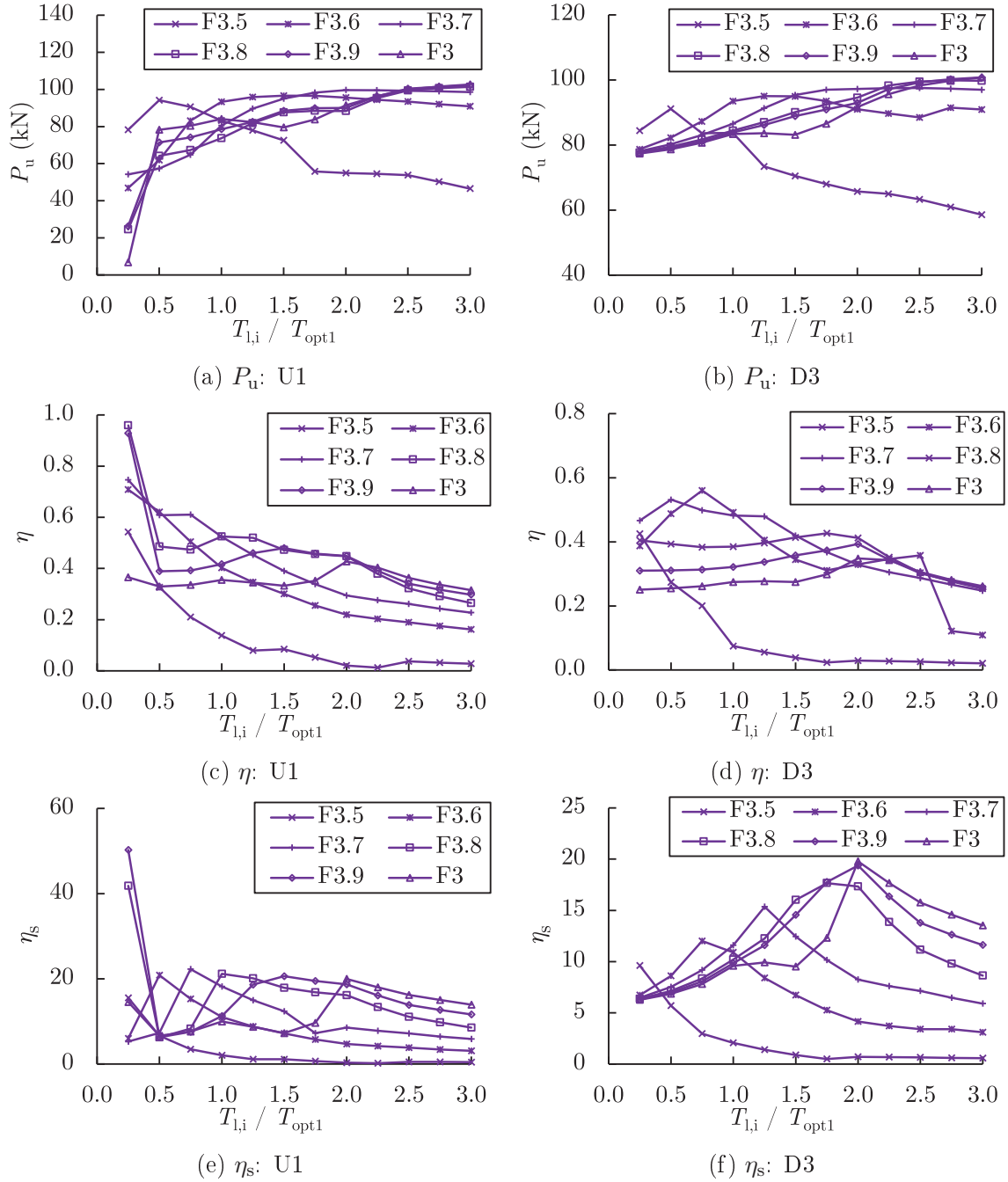


Figure 6.26: Efficiency indicators to determine the true optimum prestressing force for cases F3.5–F3.9 and F3 with $I_p = 0$.

6.5.3 $I_p = 5$

Figure 6.27 shows P_u , η and η_s plotted against varying prestressing levels with upward pre-cambering $I_p = 5$ for mono-symmetric cases F3.5–F3.9 and F3. For doubly-symmetric cases F2–F6 and A2–A6, similar trends with cases A2–A6 in Figure 6.25 are observed, but for $I_p = 5$, the reducing section on the curves of ultimate loads versus increasing prestressing levels and the increasing section on the efficiency indicator curves versus prestressing levels when $T_{l,i}/T_{opt1} \leq 1$ are more pronounced than when $I_p = 0$. Comparing Figure 6.27 with Figure 6.26, for the mono-symmetric cases with upward imperfections, for $I_p = 5$ the extremely high values of efficiency indicators when $T_{l,i}/T_{opt1} \leq 0.5$ are more pronounced than that for $I_p = 0$. For $I_p = 5$, the mono-symmetric cases perform better than doubly-symmetric cases in terms of both load-carrying capacity and structural efficiency, because the values of the efficiency indicators reach the peak values within $T_{l,i}/T_{opt1} = [1, 3]$, and higher structural efficiency can be achieved compared to the doubly-symmetric cases in the same geometric and prestressing arrangements. For all cases, changing I_p from 0 to 5, the required prestressing levels to maximize the structural efficiency indicators are increased. The reason for this trend is postulated to be that the upward pre-cambering reduces the effect from self-weight, and for $I_p = 5$ structural efficiency can become higher with much higher prestressing levels than those for $I_p = 0$. Note also that the maximum values of η_s are higher when $I_p = 5$.

6.5.4 Summary

Although most of the ultimate loads do not reach maximum values within the range $T_{l,i}/T_{opt1} = [1, 3]$, the required prestressing forces to maximize structural efficiency are within the range of $T_{l,i}/T_{opt1} = [1, 3]$. Considering both load-carrying capacity and structural efficiency, the recommended configurations of stayed beam-columns for deliberate pre-cambering levels are shown in Table 6.2. Moreover, when $T_{l,i}/T_{opt1} < 1$, although high

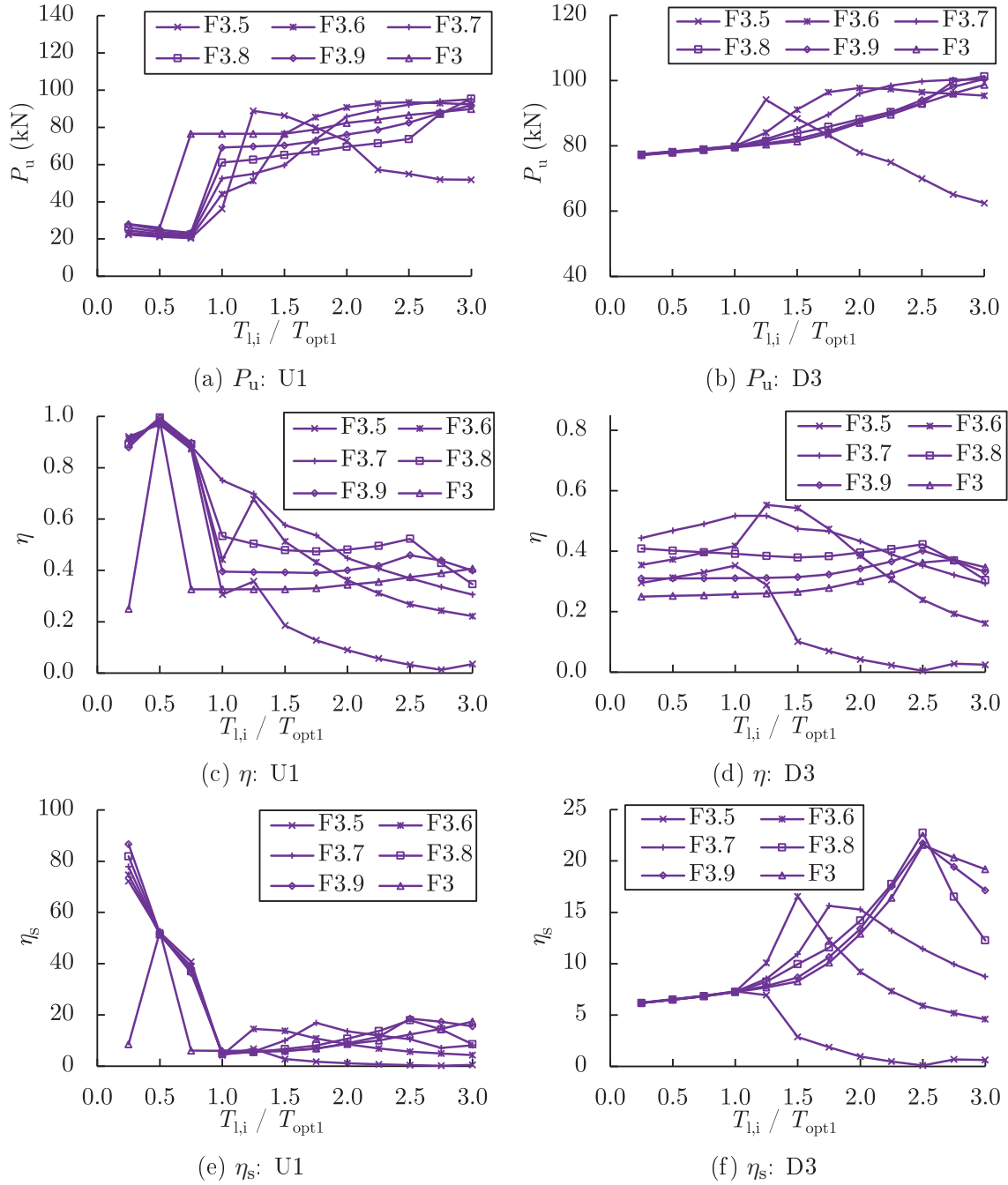


Figure 6.27: Efficiency indicators to determine the true optimum prestressing force for cases F3.5–F3.9 and F3 with $I_p = 5$.

Table 6.2: Recommended configurations for deliberate pre-cambering levels. ‘D’ and ‘U’ represent downward and upward imperfections respectively. Ticks (✓) and crosses (✗) signify configurations that are recommended and not recommended respectively.

I_p	-5	0	5
Mono-symmetric	✗	✓(D)	✓
Doubly-symmetric	✓	✓(U)	✗

values of η and η_s are observed, the ultimate loads at such prestressing levels are very small; conversely, when $T_{l,i}/T_{opt1} > 3$, although the load-carrying capacity is high, the structural efficiency at such prestressing levels is relatively low. Hence, $T_{l,i}/T_{opt1} < 1$ and $T_{l,i}/T_{opt1} > 3$ should be avoided for the configurations presented. In addition, $\alpha < 0.5$ is not recommended due to the low structural efficiency and issues with highly imperfection sensitive behaviour promoted by strongly unstable post-buckling characteristics.

6.6 Concluding remarks

Following the findings from previous chapters that interactive buckling would affect the structural performance of perfect stayed beam-columns under the combined action of axial compression and self-weight acting orthogonally, the structural performance of imperfect stayed beam-columns has been studied further using FE modelling. Firstly, the relationship between the induced pre-cambering level and the distribution of prestressing between the upper and lower stays were investigated. Then, under different pre-defined levels of pre-cambering, the ultimate behaviour considering the effects from the varying prestressing levels alongside varying principal imperfection directions, their amplitudes and shapes were investigated. Through applying different imperfection combinations, it transpired that interactive buckling can reduce the ultimate load. Moreover, the true optimum prestressing force for stayed beam-columns was studied based on the ultimate load and efficiency indicators.

From the study of the prestress and pre-cambering, excessively small crossarm lengths (*i.e.*

$\alpha_1 < 0.05$), stay diameters (*i.e.* $\phi_s/(2r_c) < 0.14$) and crossarm length ratios (*i.e.* $\alpha < 0.5$) should be avoided, since the desired pre-cambering level cannot be achieved by adjusting the prestressing distribution between the upper and lower stays, and the ultimate loads are small in conjunction with highly unstable post-buckling behaviour identified in the previous chapters. The crossarm length ratio $\alpha = 0.5$ is not recommended either, but this is primarily owing to an intrinsically low structural efficiency. In terms of the buckling and post-buckling behaviour, for doubly-symmetric cases, the structural performance is not particularly sensitive to the pre-cambering level, although the load-carrying capacity decreases slightly by changing the pre-cambering index I_p from -5 to 5 . However, for mono-symmetric cases, interactive buckling dominance, imperfection and prestressing sensitivities, and optimum crossarm length ratios are strongly linked to the pre-cambering levels. In terms of the true optimum prestressing force, although most of the ultimate loads do not reach maximum values within $T_{l,i}/T_{opt1} = [1, 3]$, the structural efficiencies are indeed maximized within this range. With changing I_p from -5 to 5 , the required prestressing forces to obtain the peak values of the structural efficiency indicators increase. Therefore, $T_{l,i}/T_{opt1} = [1, 3]$ is recommended for the cases presented, since this range achieved a good balance of load-carrying capacity and structural efficiency. Moreover, doubly-symmetric cases perform best for zero pre-cambering, while mono-symmetric cases perform best for upward pre-cambering.

Since the performance of the stayed beam-columns under axial compression in conjunction with self-weight has been studied in some depth, a preliminary study to develop a design method for stayed beam-columns is conducted in the next chapter.

Chapter 7

Preliminary study of design methods

7.1 Introduction

In the previous chapters, the buckling and post-buckling behaviour of perfect and imperfect stayed beam-columns are investigated in some depth. Although there exist design methods for stayed members (Wadee *et al.*, 2013b; Tankova *et al.*, 2019), they do not consider mono-symmetric geometry and pre-cambering from prestress. The proposed design approach is based preliminarily on analytical methods, can be adapted to varying pre-cambering levels and allows for the reduction in load-carrying capacity from interactive buckling found in Chapters 5 and 6. This method is calibrated by using the FE results in Chapter 6. A step-by-step procedure for the design method is summarized in a flowchart, and a design example is given to illustrate the application of the method.

7.2 Regression solutions for γ

Following the pre-cambering study in Chapter 6, a simple method to evaluate γ , defined in Eq. (6.2), is useful for design practice. The aim of this section is to investigate this using Nonlinear Least Squares (NLS) regression (Bethea, 1995), which is also mentioned in Appendix A.

From Figures 6.2, 6.3, 6.5 and 6.6, it can be seen that when $\alpha_1 \geq 0.05$, $\phi_s/(2r_c) \geq 0.14$ and $\alpha \geq 0.5$, a linear function can be adopted to describe the relationship between I_p and γ , thus:

$$I_p = b - k\gamma, \quad (7.1)$$

where b and k are the intercept and gradient of the linear function respectively. Considering the properties of the graphs of I_p versus γ , the following expressions can be obtained using NLS, thus:

$$\begin{aligned} b &= 0.455 \frac{T_{l,i}}{T_{opt1}} \left[\ln \left(5.661 \frac{\phi_s}{2r_c} + 7.889\alpha_1 - 1.577 \right) + 8.655 \right] \ln(5.335(1 - \alpha) + e), \\ k &= 1.441 \frac{T_{l,i}}{T_{opt1}} \left[1.358 \ln \left(\frac{\phi_s}{2r_c} + 1 \right) - 0.438 \ln(\alpha_1 + 1) + 1.408 \right]^2 \ln(-3.052(1 - \alpha) + e), \end{aligned} \quad (7.2)$$

where the mathematical constant $e \approx 2.718$. The comparison between regression and FE results is shown in Figure 7.1.

7.3 Buckling mode evaluation

In the previous chapters, the buckling modes, including the pure and interactive modes are investigated analytically and numerically. The principal aim of this section is to propose a simple quantitative method to evaluate the buckling mode of a stayed beam-column, and to determine the reduction on the load-carrying capacity from interactive buckling. From

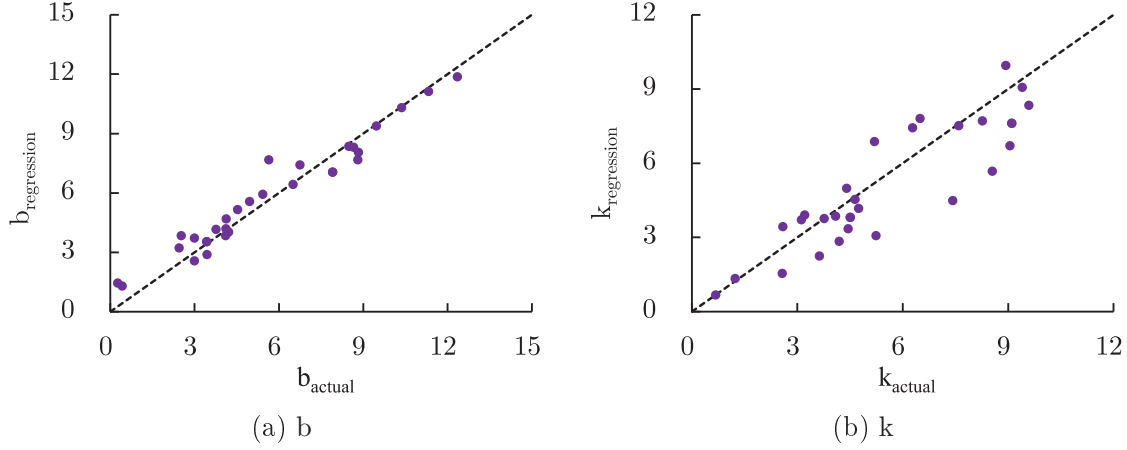


Figure 7.1: Comparison between regression and FE results; $b_{\text{regression}}$ and $k_{\text{regression}}$ are from regression, and b_{actual} and k_{actual} are from FE results; the dashed lines represent $b_{\text{regression}} = b_{\text{actual}}$ and $k_{\text{regression}} = k_{\text{actual}}$.

Chapter 6, it has been found that doubly-symmetric cases perform best with zero pre-cambering, and mono-symmetric cases perform best with upward pre-cambering. Hence, the results of doubly-symmetric cases with $I_p = 0$ and mono-symmetric cases with $I_p = 5$ are adopted to investigate the design method. In addition, similar to Chapter 6, only the results of cases F2–F6, A2–A6 and F3.5–F3.9 are taken into account. Since interactive buckling for mono-symmetric cases is only observed in downward buckling from Chapter 5, this section only investigates the dominant mode in downward buckling. However, both the upward Mode 1 (UM1) and dominant downward buckling mode, *i.e.* downward Mode 1 (DM1), downward Mode 2 (DM2) or downward interactive buckling (DI), should be considered to check the load-carrying capacity in design practice.

7.3.1 Reduction index for interactive buckling

Interactive buckling has been investigated analytically and numerically in Chapters 5 and 6 respectively, and it can reduce the load-carrying capacity. In terms of doubly-symmetric cases with $I_p = 0$, it has been found that, when Mode 1 is critical, this mode also dominates the post-buckling behaviour; when Mode 2 is critical and the buckling loads of Modes 1

and 2 are numerically close, interactive buckling dominates the post-buckling behaviour. This conclusion is similar to that drawn in previous studies of vertical doubly-symmetric stayed columns (Saito & Wadee, 2009b; Yu & Wadee, 2017a). In terms of mono-symmetric cases with $I_p = 5$, it has been found that, when $\alpha = [0.5, 1.0]$, the interactive buckling dominance of the mono-symmetric cases with $\alpha < 1$ is similar to that of their corresponding doubly-symmetric case with $\alpha = 1$, for example that of cases F3.5–F3.9 is similar to that of case F3. In addition, Figure 5.18 has shown that the dominant buckling mode of the mono-symmetric cases is strongly linked to the combination of the crossarm ratio α and prestressing level.

To express the close degree of Modes 1 and 2 buckling loads quantitatively, a new index is defined as follows:

$$I_{cr} = 100 \frac{(P_1^C - P_2^C)}{P_2^C}, \quad (7.3)$$

where P_1^C and P_2^C are the critical buckling loads of Modes 1 and 2 (illustrated in Figure 3.12) respectively, which can be obtained from Eqs. (4.52)–(4.54). Moreover, to take the reduction on the load-carrying capacity from interactive buckling into account quantitatively, an additional interactive buckling reduction factor χ_{int} that defines the interactive buckling resistance relative to the pure Mode 2 buckling resistance is defined as follows:

$$\chi_{int} = \frac{P_{u,int}}{P_{u,2}}, \quad (7.4)$$

where $P_{u,int}$ is the load-carrying capacity (represented by the ultimate load) for interactive buckling, and $P_{u,2}$ is the load-carrying capacity for pure Mode 2 buckling. In order to investigate χ_{int} numerically, this reduction factor can be determined by:

$$\chi_{int} = \frac{\min\{P_{u,c2}, P_{u,c3}, P_{u,c4}\}}{P_{u,c5}}, \quad (7.5)$$

where $P_{u,ci}$ represents the ultimate load of models with imperfection combination i defined in Table 5.1.

Therefore, when $I_{cr} < 0$, pure Mode 1 buckling is dominant; when $I_{cr} \geq 0$ and $\chi_{int} < 1$, interactive buckling is dominant; when $I_{cr} \geq 0$ and $\chi_{int} = 1$, pure Mode 2 buckling is dominant.

7.3.2 Regression expression

In terms of doubly-symmetric cases with $I_p = 0$, from Figures 3.11, 6.14 and 6.15, the obtained values of I_{cr} and χ_{int} for cases F2–F6 and A2–A6 are listed in Table 7.1. Based

Table 7.1: Values of I_{cr} and χ_{int} for cases F2–F6 and A2–A6. From Eq. (7.3), positive I_{cr} indicates that Mode 2 is critical, while negative I_{cr} indicates that Mode 1 is critical. Note that χ_{int} values corresponding to negative I_{cr} values are disregarded, since interactive buckling is not dominant when Mode 1 is critical.

Case	F2	F3	F4	F5	F6
I_{cr}	-14.69	6.86	10.66	11.66	12.10
$\chi_{int} (T_{l,i} = T_{opt1})$	0.94	0.93	0.95	0.96	0.96
$\chi_{int} (T_{l,i} = 2T_{opt1})$	0.87	0.90	0.95	0.97	0.98
Case	A2	A3	A4	A5	A6
I_{cr}	-39.93	-7.69	6.86	7.15	10.94
$\chi_{int} (T_{l,i} = T_{opt1})$	0.94	0.92	0.93	0.96	0.97
$\chi_{int} (T_{l,i} = 2T_{opt1})$	0.90	0.83	0.90	0.95	0.98

on the I_{cr} , χ_{int} and $T_{l,i}/T_{opt1}$ values shown in Table 7.1, a regression function can be obtained using linear regression (Bethea, 1995) based on the Ordinary Least Squares (OLS) method, thus:

$$\chi_{int} = \begin{cases} 0.9205 + 0.0022I_{cr} \frac{T_{l,i}}{T_{opt1}} & \text{where: } I_{cr} \frac{T_{l,i}}{T_{opt1}} = [0, 36.14), \\ 1 & \text{where: } I_{cr} \frac{T_{l,i}}{T_{opt1}} = [36.14, +\infty), \end{cases} \quad (7.6)$$

where $T_{l,i}/T_{opt1} = [1, 3]$ from the study of the true optimum prestressing forces in Chapter 6. A comparison between the sample results from Table 7.1 and the regression values from Eq. (7.6) is shown in Figure 7.2. From Eqs. (7.4) and (7.6), the ultimate load considering the interactive buckling reduction effect can be calculated based on the ultimate load for pure Mode 2 buckling.

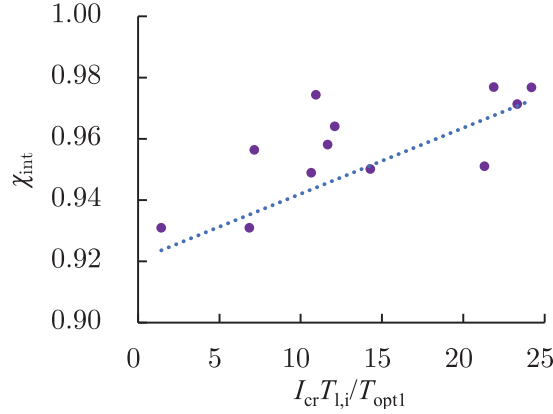


Figure 7.2: Relationship between χ_{int} and $I_{\text{cr}}T_{l,i}/T_{\text{opt1}}$ for the sample results from Table 7.1; the dashed line is the regression function from Eq. (7.6).

In terms of the mono-symmetric systems with $I_p = 5$, from Figures 3.11 and 6.21, the obtained values of I_{cr} and χ_{int} for cases F3.5–F3.9 are listed in Table 7.2. In contrast with

Table 7.2: I_{cr} values for cases F3.5–F3.9 and F3.

Case	F3.5	F3.6	F3.7	F3.8	F3.9	F3
I_{cr}	-2.61	-0.80	1.21	3.23	5.13	6.86
$\chi_{\text{int}} (T_{l,i} = T_{\text{opt1}})$	0.94	0.94	0.94	0.94	0.94	0.93
$\chi_{\text{int}} (T_{l,i} = 2T_{\text{opt1}})$	0.99	0.94	0.91	1.00	1.00	0.90

the doubly-symmetric cases, the interactive buckling dominance of the mono-symmetric cases, with $\alpha < 1$, depends primarily on whether the corresponding doubly-symmetric case, with $\alpha = 1$, is dominated by interactive buckling, regardless of the critical buckling mode. From Figure 5.18, for the studied mono-symmetric cases with $\alpha \geq 0.5$, ‘UM dominant’ can be ignored since interactive buckling does not dominate when the mode is upward. Therefore, when interactive buckling dominance is observed in a doubly-symmetric case (for example, case F3), it also can be observed in the corresponding mono-symmetric cases with $\alpha \geq 0.5$ from this doubly-symmetric case (for example, cases F3.5–F3.9 from case F3). From Table 7.2, it can be seen that the χ_{int} value for case F3 is smaller than all the χ_{int} values for cases F3.5–F3.9, so the χ_{int} value of case F3 can be applied to cases F3.5–F3.9 conservatively.

7.4 Flexural buckling of simply supported columns

Before investigating the stayed member, the design method for unstayed columns is reviewed herein. The vertical displacements of perfect and imperfect simply supported compressed horizontal members with length L are illustrated in Figure 7.3. The differential

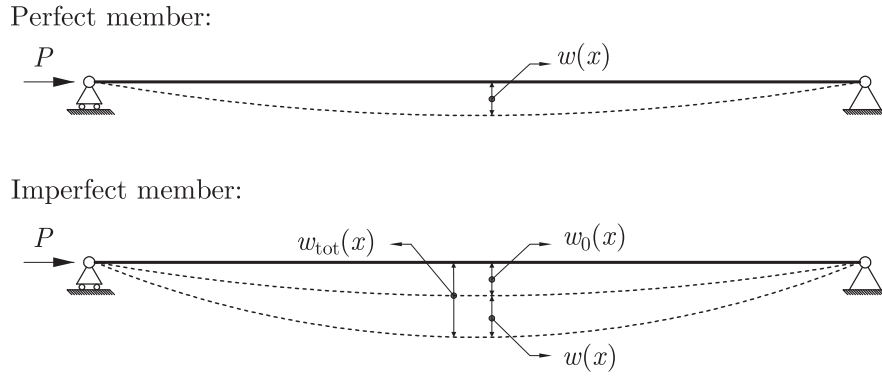


Figure 7.3: Vertical displacements of perfect and imperfect simply supported compressed horizontal members with length L . $w(x)$ and $w_0(x)$ are the vertical displacements from axial compression and initial deflection respectively, and $w_{\text{tot}}(x) = w(x) + w_0(x)$ is the total vertical displacement of the imperfect member. For unstayed members, $w_0(x)$ only comprises out-of-straightness imperfections referred to as w_{imp} in the study of stayed members.

equation describing the deflection of the perfect member is:

$$E_c I_c \frac{d^2 w(x)}{dx^2} + P w(x) = 0, \quad (7.7)$$

where application of the boundary conditions $w(0) = w(L) = 0$ yields the critical load $P^C = \pi^2 E_c I_c / L^2$. Similarly, the differential equation describing the deflection of the imperfect member is:

$$E_c I_c \frac{d^2 w(x)}{dx^2} + P w(x) = -P w_0(x). \quad (7.8)$$

The amplification relationship can thus be established as following:

$$\bar{w}_{\text{tot}} = \bar{w} + \bar{w}_0 = \frac{1}{1 - P/P^C} \bar{w}_0, \quad (7.9)$$

where each function with an overline represents its absolute amplitude. The design equation is established on the basis of a first yield criterion for the imperfect column. It accounts for the axial load and the second-order bending moment that arises from amplification of the initial imperfection:

$$\frac{P}{A_c f_y} + \frac{M^{\text{II}}}{W_c f_y} \leq 1.0 \rightarrow \frac{P}{A_c f_y} + \frac{P w_{\text{tot}}}{W_c f_y} \leq 1.0, \quad (7.10)$$

where W_c is the section modulus of the column, and M^{II} is the second-order bending moment. Substituting Eq. (7.9) into Eq. (7.10) leads to:

$$\frac{P}{A_c f_y} + \frac{P}{W_c f_y} \frac{1}{(1 - P/PC)} \bar{w}_0 \leq 1.0. \quad (7.11)$$

Considering the well-known normalized slenderness of the main member element $\bar{\lambda}$:

$$\bar{\lambda} = \sqrt{\frac{A_c f_y}{PC}}, \quad (7.12)$$

and the buckling reduction factor of the column χ :

$$\chi = \frac{P}{A_c f_y}, \quad (7.13)$$

the limiting condition of Eq. (7.11) can be expressed as:

$$\begin{aligned} \chi + \frac{\chi}{(1 - \chi \bar{\lambda}^2)} \frac{A_c \bar{w}_0}{W_c} &= 1.0 \rightarrow \chi + \frac{\chi}{(1 - \chi \bar{\lambda}^2)} \eta_{\text{EC3}} = 1.0 \\ \rightarrow \chi + \frac{\chi}{(1 - \chi \bar{\lambda}^2)} \alpha_{\text{EC3}} (\bar{\lambda} - 0.2) &= 1.0, \end{aligned} \quad (7.14)$$

where η_{EC3} is the generalized imperfection factor (referred to as η in BS EN 1993-1-1 (2005)) thus:

$$\eta_{\text{EC3}} = \frac{A_c \bar{w}_0}{W_c} = \frac{A_c \bar{w}_{\text{imp}}}{W_c} = \alpha_{\text{EC3}} (\bar{\lambda} - 0.2), \quad (7.15)$$

where α_{EC3} is the imperfection factor (referred to as α defined in BS EN 1993-1-1 (2005)).

7.5 Flexural buckling of simply supported stayed beam-columns

The principal aim of this section is to derive buckling curves applicable to both doubly-symmetric and mono-symmetric stayed beam-columns. Based on the preceding procedure for the flexural buckling of simply supported columns, the design method for doubly-symmetric stayed columns presented in Tankova *et al.* (2019) is revised further to take mono-symmetric geometry and pre-cambering into account. From the preceding study of the buckling mode evaluation, since the ultimate load for interactive buckling can be derived from that for pure Mode 2 buckling through Eq. (7.4), only pure buckling modes are investigated in this section.

7.5.1 Basic methodology

Considering the axial force within the main member from the stays and the applied axial load, Eq. (7.9) can be transformed to:

$$w_{\text{tot}} = w + w_0, \quad \bar{w}_{\text{tot}} = \bar{w} + \bar{w}_0 = \frac{1}{1 - P_f/P_f^C} \bar{w}_0, \quad (7.16)$$

where P_f is the final internal force within the main member at failure defined in Figure 3.8, and P_f^C is the critical load for the main member. P_f can be derived from Eq. (3.45) thus:

$$P_f = \frac{P}{C_0} + T_{u,i} \cos \beta_u + T_{l,i} \cos \beta_l, \quad (7.17)$$

where P is applied axial load, and C_0 can be obtained from Eq. (3.44). P_f^C can be derived from Eq. (3.45) thus:

$$P_f^C = \frac{P_n^C}{C_0} + T_{u,i} \cos \beta_u + T_{l,i} \cos \beta_l, \quad (7.18)$$

where P_n^C is the critical load of the stayed beam-column for Mode n and $n = \{1, 2\}$ which can be obtained from Eqs. (4.52)–(4.54). It should be noted that w_0 in Eq. (7.16) comprises not only out-of-straightness imperfections (w_{imp}) but also the deflection from any deliberate pre-cambering level (w_p), as discussed in Chapter 6, thus:

$$w_0 = w_{\text{imp}} + w_p. \quad (7.19)$$

The imperfection direction and shape $w_{\text{imp}} = w_{\text{imp},m}$ for different buckling modes, *i.e.* $m = \{\text{dm1}, \text{dm2}, \text{um1}\}$ corresponding to DM1, DM2 and UM1 respectively, can be expressed, from Eq. (5.30), as:

$$w_{\text{imp}} = \begin{cases} w_{\text{imp},\text{dm1}} = -\bar{w}_{\text{imp},\text{dm1}} \sin \frac{\pi x}{L} & \text{for DM1,} \\ w_{\text{imp},\text{dm2}} = \bar{w}_{\text{imp},\text{dm2}} \sin \frac{2\pi x}{L} & \text{for DM2,} \\ w_{\text{imp},\text{um1}} = \bar{w}_{\text{imp},\text{um1}} \sin \frac{\pi x}{L} & \text{for UM1.} \end{cases} \quad (7.20)$$

From W_3 in Eq. (5.1), the deflection from prestress considering a deliberate pre-cambering level I_p can be expressed approximately as:

$$w_p(x) = -16 \left[w_{\text{wc},\text{max}} \frac{x(L-x)}{L^2} - 4w_{\text{wc},\text{max}} \frac{x^2(L-x)^2}{L^4} \right] + w_{p,m} \sin \frac{\pi x}{L}, \quad (7.21)$$

where $w_{\text{wc},\text{max}}$ can be calculated from Eq. (3.15) for the deflection amplitude after step 2 in Figure 3.2; $w_{p,m}$ corresponds to a deliberate pre-cambering level I_p in Eq. (6.1). The coefficient ‘ -16 ’ in Eq. (7.21) is determined by calibration such that the first part of this equation is equal to the minimum value of $-w_{\text{wc},\text{max}}$. Substituting Eq. (7.21) into Eq. (7.19) gives:

$$\begin{aligned} w_0 &= -16 \left[w_{\text{wc},\text{max}} \frac{x(L-x)}{L^2} - 4w_{\text{wc},\text{max}} \frac{x^2(L-x)^2}{L^4} \right] + w_{p,m} \sin \frac{\pi x}{L} + w_{\text{imp}} \\ &\approx w_{p,m} \sin \frac{\pi x}{L} + w_{\text{imp}}, \end{aligned} \quad (7.22)$$

since in this thesis $w_{wc,max} = 2.5 \times 10^{-5}L$, which is significantly smaller than $w_{p,m}$ and w_{imp} .

In Tankova *et al.* (2019), the slenderness $\bar{\lambda}$ of stayed members is given by:

$$\bar{\lambda} = \sqrt{\frac{A_c f_y}{P_f^C}}, \quad (7.23)$$

and the buckling reduction factor for axial compression χ is:

$$\chi = \frac{P_f}{A_c f_y}. \quad (7.24)$$

Moreover, the design equation for stayed members can be expressed as:

$$\chi + \frac{\chi}{(1 - \chi \bar{\lambda}^2)} \eta_{tot} = 1.0, \quad (7.25)$$

where η_{tot} is the total generalized imperfection factor, and its expression is based on η_{EC3} from Eq. (7.15) for unstayed members; η_{tot} has different expressions for different buckling modes. From Tankova *et al.* (2019), χ can be given by:

$$\chi = \frac{1}{\Phi + \sqrt{\Phi^2 - \bar{\lambda}^2}}, \quad (7.26)$$

where Φ is:

$$\Phi = 0.5 \left(1 + \eta_{tot} + \bar{\lambda}^2 \right). \quad (7.27)$$

Eqs. (7.26) and (7.27) constitute the proposed model for computing the buckling resistance of prestressed stayed beam-columns. However, it requires calibration of the imperfection η_{tot} , which is unknown at this point. It will be calibrated on the basis of the numerical results from previous chapters. From Eqs. (7.17) and (7.24), if Mode 1 is critical, the design

load-carrying capacity $P_{u,\text{design}}$ is given by:

$$P_{u,\text{design}} = C_0(\chi A_c f_y - T_{u,i} \cos \beta_u - T_{l,i} \cos \beta_l); \quad (7.28)$$

if Mode 2 is critical, from Eq. (7.4), Eq. (7.28) should be revised to:

$$P_{u,\text{design}} = \chi_{\text{int}} C_0(\chi A_c f_y - T_{u,i} \cos \beta_u - T_{l,i} \cos \beta_l). \quad (7.29)$$

It should be noted that self-weight is not considered, because it is counteracted by $T_{l,w}$ in loading step 2 from Figure 3.2, and the small deflection at the end of loading step 2 is ignored from Eq. (7.22).

7.5.2 Mode 1 buckling

From the analytical and numerical studies on the interactive buckling in Chapters 5 and 6, DM1 is the first symmetric buckling mode for doubly-symmetric cases, while UM1 is the first symmetric buckling mode for mono-symmetric cases. The deflection and force analysis of the stayed beam-column for DM1 and UM1 are illustrated in Figures 7.4 and 7.5 respectively.

From Figures 7.4 and 7.5, the first yield criterion of the imperfect stayed beam-column for Mode 1 buckling can be expressed by:

$$\frac{P_f}{A_c f_y} + \frac{M^{\text{II}}}{W_c f_y} - \frac{LQ_1}{2W_c f_y} = 1.0, \quad (7.30)$$

where the additional term beyond Eq. (7.10) is for the restraint introduced by the stays. Q_1 can be derived from Eq. (4.14), thus:

$$Q_1 = (K_{\text{su}} \sin^2 \beta_u + K_{\text{sl}} \sin^2 \beta_l)w \left(\frac{L}{2} \right) = (K_{\text{su}} \sin^2 \beta_u + K_{\text{sl}} \sin^2 \beta_l)\bar{w}. \quad (7.31)$$

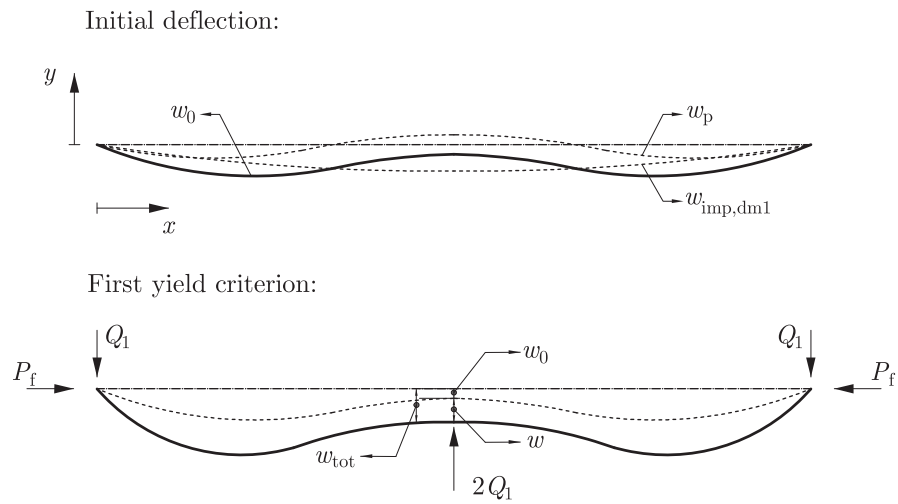


Figure 7.4: Initial deflection and first yield criterion for DM1. Q_1 is the vertical force at the end of the main member.

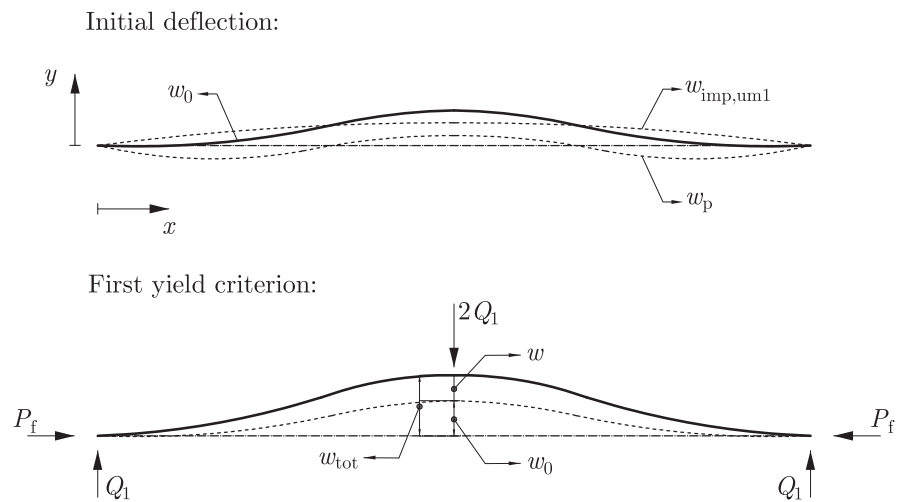


Figure 7.5: Initial deflection and first yield criterion for UM1. Q_1 is the vertical force at the end of the main member.

From Eq. (7.16), $\bar{w}$ can be expressed as:

$$\bar{w} = \frac{P_f/P_f^C}{1 - P_f/P_f^C} \bar{w}_0. \quad (7.32)$$

Substituting Eqs. (7.31) and (7.32) into Eq. (7.30) and rearranging gives:

$$\frac{P_f}{A_c f_y} + \frac{P_f}{W_c f_y} \frac{1}{(1 - P_f/P_f^C)} \bar{w}_0 - \frac{(K_{su} \sin^2 \beta_u + K_{sl} \sin^2 \beta_l) L}{2 W_c f_y} \frac{P_f/P_f^C}{(1 - P_f/P_f^C)} \bar{w}_0 = 1.0. \quad (7.33)$$

Considering Eqs. (7.15), (7.23) and (7.24), Eq. (7.33) can be transformed to the following expression:

$$\chi + \frac{\chi}{(1 - \chi \bar{\lambda}^2)} \eta_{EC3} \frac{\bar{w}_0}{\bar{w}_{imp}} \left[1 - \frac{(K_{su} \sin^2 \beta_u + K_{sl} \sin^2 \beta_l) L}{2 P_f^C} \right] = 1.0. \quad (7.34)$$

Comparing Eqs. (7.34) with (7.25), η_{tot} for Mode 1 buckling is:

$$\eta_{tot} = \eta_{EC3} \frac{\bar{w}_0}{\bar{w}_{imp}} \left[1 - \frac{(K_{su} \sin^2 \beta_u + K_{sl} \sin^2 \beta_l) L}{2 P_f^C} \right]. \quad (7.35)$$

In terms of DM1 in doubly-symmetric cases with zero pre-cambering, *i.e.* $I_p = 0$, $w_{p,m} = 0$ and $w_{imp} = w_{imp,dm1}$, from Eqs. (7.20) and (7.22), the following equations are obtained:

$$w_0 \approx -\bar{w}_{imp,dm1} \sin \frac{\pi x}{L}, \quad \bar{w}_0 \approx \bar{w}_{imp,dm1}. \quad (7.36)$$

In terms of UM1 in mono-symmetric cases with upward pre-cambering, *i.e.* $I_p > 0$, $w_{p,m} > 0$ and $w_{imp} = w_{imp,um1}$, from Eqs. (7.20) and (7.22), the following equations are obtained:

$$w_0 \approx (w_{p,m} + \bar{w}_{imp,um1}) \sin \frac{\pi x}{L}, \quad \bar{w}_0 \approx w_{p,m} + \bar{w}_{imp,um1}. \quad (7.37)$$

7.5.3 Mode 2 buckling

From the analytical and numerical investigations into interactive buckling in Chapters 5 and 6, pure Mode 2 is not observed in the studied models owing to self-weight, but interactive buckling is triggered in downward mode regardless of whether the configuration is doubly-symmetric or mono-symmetric. Hence, DM2 is investigated as the basis to obtain the load-carrying capacity considering interactive buckling. The deflection and force analysis of the stayed beam-column for DM2 are illustrated in Figure 7.6.

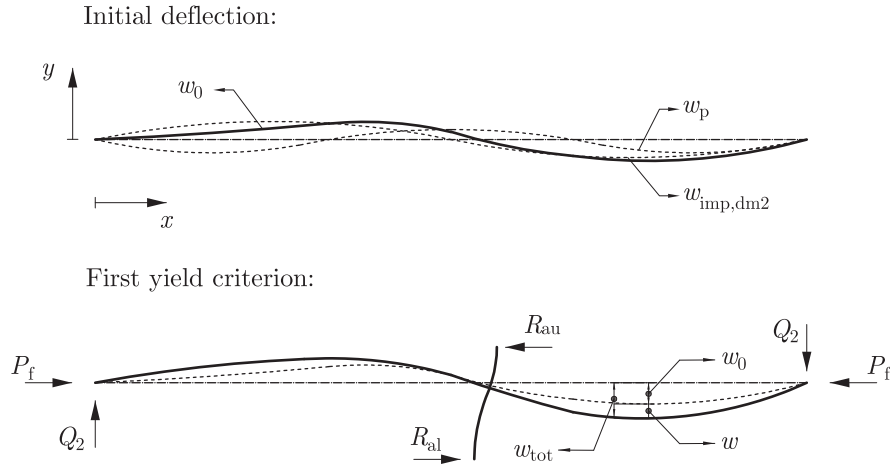


Figure 7.6: Initial deflection and first yield criterion for DM1. Q_2 is the vertical force at the end of the main member.

From Eqs. (7.16) and (7.19), the first order derivation of w_{tot} is:

$$w'_{\text{tot}} = w' + w'_0, \quad w'_0 = w'_{\text{imp,dm2}} + w'_p. \quad (7.38)$$

From Eq. (7.20), the following equation can be obtained:

$$w'_{\text{imp,dm2}} = \bar{w}_{\text{imp,dm2}} \frac{2\pi}{L} \cos \frac{2\pi x}{L}. \quad (7.39)$$

From Eq. (7.21), the following equation can be obtained:

$$w'_p = -16 \frac{w_{wc, \max}}{L} \left[1 - 10 \left(\frac{x}{L} \right) + 24 \left(\frac{x}{L} \right)^2 - 16 \left(\frac{x}{L} \right)^3 \right] + w_{p, m} \frac{\pi}{L} \cos \frac{\pi x}{L}. \quad (7.40)$$

Substituting Eqs. (7.39) and (7.40) into Eq. (7.38) gives:

$$\begin{aligned} w'_0 &= -16 \frac{w_{wc, \max}}{L} \left[1 - 10 \left(\frac{x}{L} \right) + 24 \left(\frac{x}{L} \right)^2 - 16 \left(\frac{x}{L} \right)^3 \right] + w_{p, m} \frac{\pi}{L} \cos \frac{\pi x}{L} + \bar{w}_{\text{imp, dm2}} \frac{2\pi}{L} \cos \frac{2\pi x}{L} \\ &\approx w_{p, m} \frac{\pi}{L} \cos \frac{\pi x}{L} + \bar{w}_{\text{imp, dm2}} \frac{2\pi}{L} \cos \frac{2\pi x}{L}, \end{aligned} \quad (7.41)$$

and:

$$w'_0 \left(\frac{L}{2} \right) \approx w'_{\text{imp, dm2}} \left(\frac{L}{2} \right) + w'_p \left(\frac{L}{2} \right) = -\bar{w}_{\text{imp, dm2}} \frac{2\pi}{L}. \quad (7.42)$$

Considering Eqs. (7.16) and (7.38), the total rotational displacement of the mid-span joint $w'_{\text{tot}}(L/2)$ can be determined by:

$$w'_{\text{tot}} \left(\frac{L}{2} \right) = w' \left(\frac{L}{2} \right) + w'_0 \left(\frac{L}{2} \right) = \frac{1}{1 - P_f/P_f^C} w'_0 \left(\frac{L}{2} \right). \quad (7.43)$$

Substituting Eq. (7.42) into Eq. (7.43) gives:

$$\begin{aligned} w'_{\text{tot}} \left(\frac{L}{2} \right) &= \frac{1}{1 - P_f/P_f^C} \left(-\bar{w}_{\text{imp, dm2}} \frac{2\pi}{L} \right), \\ w' \left(\frac{L}{2} \right) &= \frac{P_f/P_f^C}{1 - P_f/P_f^C} \left(-\bar{w}_{\text{imp, dm2}} \frac{2\pi}{L} \right). \end{aligned} \quad (7.44)$$

From Figure 7.6, the first yield criterion of the imperfect stayed beam-column for Mode 2 buckling can be expressed by:

$$\frac{P_f}{A_c f_y} + \frac{M^{\text{II}}}{W_c f_y} - \frac{L Q_2}{4 W_c f_y} = 1.0, \quad (7.45)$$

where Q_2 can be derived from Eq. (4.33), thus:

$$Q_2 = \left[\frac{K_{su}a_u \cos \beta_u \sin \beta_u}{1 + \frac{2K_{su}}{3K_{au,b}} \cos^2 \beta_u} + \frac{K_{sl}a_l \cos \beta_l \sin \beta_l}{1 + \frac{2K_{sl}}{3K_{al,b}} \cos^2 \beta_l} \right] \theta_a, \quad (7.46)$$

where $\theta_a = |w'(L/2)|$ is the absolute rotational angle of the crossarms at the mid-span joint illustrated in Figure 4.4. Considering Eqs. (7.15), (7.23) and (7.24), substituting Eqs. (7.44) and (7.46) into Eq. (7.45) and rearranging gives:

$$\chi + \frac{\chi}{(1 - \chi\bar{\lambda}^2)} \eta_{EC3} \left[\frac{\bar{w}_0}{\bar{w}_{imp}} - \frac{\pi}{2P_f^C} \left(\frac{K_{su}a_u \cos \beta_u \sin \beta_u}{1 + \frac{2K_{su}}{3K_{au,b}} \cos^2 \beta_u} + \frac{K_{sl}a_l \cos \beta_l \sin \beta_l}{1 + \frac{2K_{sl}}{3K_{al,b}} \cos^2 \beta_l} \right) \right] = 1.0. \quad (7.47)$$

Comparing Eqs. (7.47) with (7.25), η_{tot} for Mode 2 buckling is:

$$\eta_{tot} = \eta_{EC3} \left[\frac{\bar{w}_0}{\bar{w}_{imp}} - \frac{\pi}{2P_f^C} \left(\frac{K_{su}a_u \cos \beta_u \sin \beta_u}{1 + \frac{2K_{su}}{3K_{au,b}} \cos^2 \beta_u} + \frac{K_{sl}a_l \cos \beta_l \sin \beta_l}{1 + \frac{2K_{sl}}{3K_{al,b}} \cos^2 \beta_l} \right) \right]. \quad (7.48)$$

In terms of DM2 in doubly-symmetric cases with zero pre-cambering, *i.e.* $I_p = 0$, $w_{p,m} = 0$ and $w_{imp} = w_{imp,dm2}$, from Eqs. (7.20) and (7.22), the following equations are obtained:

$$w_0 \approx \bar{w}_{imp,dm2} \sin \frac{2\pi x}{L}, \quad \bar{w}_0 \approx \bar{w}_{imp,dm2}. \quad (7.49)$$

In terms of DM2 in mono-symmetric cases with upward pre-cambering, *i.e.* $I_p > 0$, $w_{p,m} > 0$ and $w_{imp} = w_{imp,dm2}$, from Eqs. (7.20) and (7.22), the following equations are obtained:

$$w_0 \approx w_{p,m} \sin \frac{\pi x}{L} + \bar{w}_{imp,dm2} \sin \frac{2\pi x}{L}, \quad (7.50)$$

and $\bar{w}_0$ needs to be determined numerically.

7.5.4 Calibration and validation

Based on the analytical expressions given in the preceding studies, the imperfection η_{tot} is calibrated against the numerical results. From Tankova *et al.* (2019), η_{tot} can be split into three terms, thus:

$$\eta_{\text{tot}} = \eta_{\text{pr}} |C_{\text{pr}}| \eta_{\text{EC3}}, \quad (7.51)$$

where η_{EC3} can be determined from Eqs. (7.15) (BS EN 1993-1-1, 2005). From Eqs. (7.35) and (7.48), C_{pr} can be expressed as:

$$C_{\text{pr}} = \begin{cases} \frac{\bar{w}_0}{\bar{w}_{\text{imp}}} \left[1 - \frac{(K_{\text{su}} \sin^2 \beta_{\text{u}} + K_{\text{sl}} \sin^2 \beta_{\text{l}}) L}{2P_{\text{f}}^{\text{C}}} \right] & \text{for Mode 1 buckling,} \\ \left[\frac{\bar{w}_0}{\bar{w}_{\text{imp}}} - \frac{\pi}{2P_{\text{f}}^{\text{C}}} \left(\frac{K_{\text{su}} a_{\text{u}} \cos \beta_{\text{u}} \sin \beta_{\text{u}}}{1 + \frac{2K_{\text{su}}}{3K_{\text{au,b}}} \cos^2 \beta_{\text{u}}} + \frac{K_{\text{sl}} a_{\text{l}} \cos \beta_{\text{l}} \sin \beta_{\text{l}}}{1 + \frac{2K_{\text{sl}}}{3K_{\text{al,b}}} \cos^2 \beta_{\text{l}}} \right) \right] & \text{for Mode 2 buckling.} \end{cases} \quad (7.52)$$

The reason for adopting absolute values of C_{pr} in Eq. (7.51) is that the different imperfection profiles, Mode 2 buckling shapes and levels and distributions of prestress within the stays may lead to the different directions of cross-sectional moment within the main member, which may give negative values of C_{pr} indicating that the moment direction is opposite to that assumed. In Eq. (7.51) only η_{pr} is unknown, and substituting Eq. (7.51) into Eq. (7.25) and rearranging gives:

$$\eta_{\text{pr}} = \frac{(1 - \chi)(1 - \chi \bar{\lambda}^2)}{\chi |C_{\text{pr}}| \eta_{\text{EC3}}}, \quad (7.53)$$

which can be used to calibrate η_{pr} . From the FE results in Chapter 6, when $\alpha_1 \geq 0.05$, $\phi_{\text{s}}/(2r_{\text{c}}) \geq 0.14$, $\alpha \geq 0.5$ and $T_{\text{l,i}}/T_{\text{opt1}} = [1, 3]$, η_{pr} can be determined by the following regression expression thus:

$$\eta_{\text{pr}} = \frac{p_1 X + p_2}{X^2 + p_3 X + p_4} \Theta, \quad (7.54)$$

where:

$$\begin{cases} p_1 = 12.127, p_2 = -5.056, p_3 = -0.745, p_4 = 0.106 & \text{for Mode 1 buckling,} \\ p_1 = 25.846, p_2 = -35.681, p_3 = -0.673, p_4 = -1.409 & \text{for Mode 2 buckling;} \end{cases} \quad (7.55)$$

X is:

$$X = \begin{cases} 100(T_{l,i} \sin \beta_l + T_{u,i} \sin \beta_u) \alpha_1 \frac{\phi_s}{2r_c} & \text{for Mode 1 buckling,} \\ \sqrt{100(T_{l,i} \sin \beta_l + T_{u,i} \sin \beta_u) \alpha_1 \frac{\phi_s}{2r_c}} & \text{for Mode 2 buckling;} \end{cases} \quad (7.56)$$

Θ is:

$$\Theta = \begin{cases} \frac{(T_{l,i}/T_{\text{opt1}})^2}{-23.015\alpha^2 + 36.860\alpha - 12.576} & \text{for Mode 1 buckling,} \\ (T_{l,i}/T_{\text{opt1}})^2(0.128\alpha^2 - 0.107\alpha + 0.031) & \text{for Mode 2 buckling.} \end{cases} \quad (7.57)$$

To validate the proposed design method, comparisons between the design load-carrying capacity and the ultimate load from FE modelling are shown in Figure 7.7. It can be seen

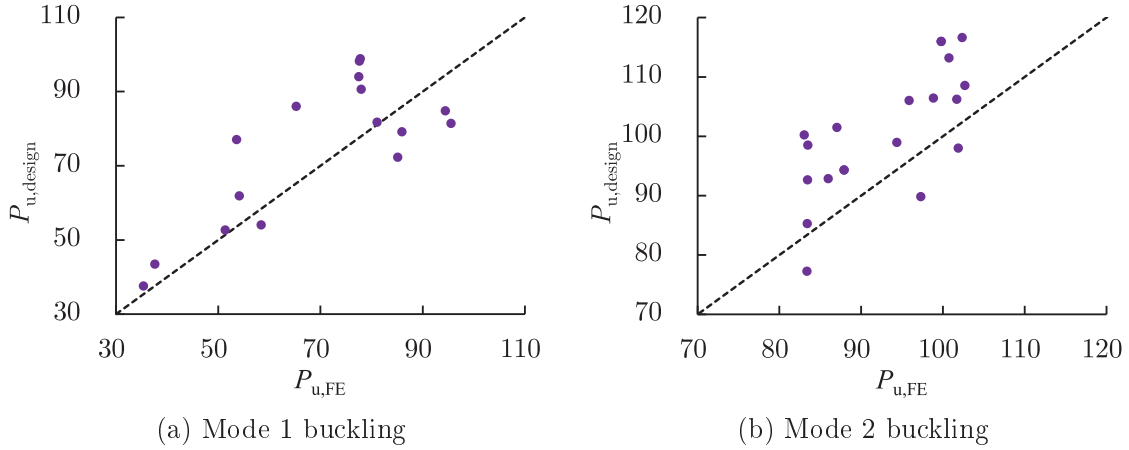


Figure 7.7: Comparisons between the load-carrying capacity from the proposed design method and the ultimate load from FE modelling; $P_{u,\text{design}}$ and $P_{u,\text{FE}}$ are the design load-carrying capacity and the ultimate load from FE results in Chapter 6 respectively; the dashed lines represent $P_{u,\text{design}} = P_{u,\text{FE}}$.

that the basic trend is well predicted, although this design method needs further refinement and assessment; this, together with reliability analysis, are recommended for future studies.

To compare the proposed design method with Tankova *et al.* (2019), the designed load-carrying capacity for Mode 1 buckling from both is listed in Table 7.3. Since Tankova *et al.*

Table 7.3: Comparison between the proposed and Tankova *et al.* (2019) design methods. $P_{u,Tankova}$ is the designed load-carrying capacity from the design method in Tankova *et al.* (2019).

Case	F2	A2	A3	F2	A2	A3
$T_{l,i}/T_{opt1}$	1	1	1	2	2	2
$P_{u,FE}$ (kN)	51.40	35.38	58.45	54.19	37.61	65.29
$P_{u,design}$ (kN)	52.67	37.51	54.00	61.84	43.41	86.03
$P_{u,Tankova}$ (kN)	74.55	72.14	69.78	83.86	68.61	83.13

(2019) only presented the design method for Mode 1 buckling of doubly-symmetric stayed columns, only cases F2, A2 and A3 can be included in this comparison due to the Mode 1 buckling dominance and doubly-symmetric geometry. It can be seen that the proposed method provides a more accurate evaluation for the load-carrying capacity of stayed beam-columns. The reason for this may be that the proposed methodology takes the self-weight and pre-cambering into account.

7.6 Design procedure

Based on the studies described above, the following design procedure is proposed, as described below and represented in Figure 7.8:

Step 1 Give initial dimension parameters for a stayed beam-column. Considering that the load-carrying capacity of a stayed member is typically 4 to 8 times that of the unstayed member with the same configuration, the dimension of the main member can be considering Euler buckling. In addition, from the studies in Chapter 6, it is recommended that $\alpha_1 \geq 0.05$, $\phi_s/(2r_c) \geq 0.14$ and $\alpha \geq 0.5$.

Step 2 Calculate P_1^C and P_2^C for Modes 1 and 2 buckling respectively from Eqs. (4.52)–(4.54).

Step 3 Calculate T_{opt1} and $T_{\text{l,w-a}}$ from Chapter 3 based on P_1^C and P_2^C from the regression solutions, and give an appropriate prestressing level. From Chapter 6, it is recommended that $T_{\text{l,i}}/T_{\text{opt1}} = [1, 3]$.

Step 4 Evaluate buckling mode. If Mode 2 is critical, calculate χ_{int} from Eq. (7.6).

Step 5 Give an appropriate pre-cambering level I_p , and calculate γ from Eqs. (7.1) and (7.2). From Chapter 6, in terms of doubly-symmetric cases, $I_p = 0$; in terms of mono-symmetric cases, $I_p > 0$.

Step 6 Calculate P_f^C from Eq. (7.18).

Step 7 Calculate w_0 and $\bar{w}_0/\bar{w}_{\text{imp}}$ from Eqs. (7.36)–(7.37) and (7.49)–(7.50) corresponding to the different critical modes.

Step 8 Calculate $\bar{\lambda}$ from Eq. (7.23).

Step 9 Calculate η_{EC3} , C_{pr} and η_{pr} from Eqs. (7.15), (7.52) and (7.54)–(7.57) respectively.

Step 10 Calculate η_{tot} , Φ and χ from Eqs. (7.51), (7.27) and (7.26) respectively.

Step 11 Calculate $P_{\text{u,design}}$ from Eqs. (7.28) and (7.29).

Step 12 Check if $P_{\text{u,design}}$ provides the required load-carrying capacity.

For doubly-symmetric members, it is only necessary to check the load-carrying capacity for the dominant downward buckling mode (*i.e.* DM1, DM2 or DI); however, for mono-symmetric members, in addition to the dominant downward buckling, it is also necessary to check the load-carrying capacity for upward buckling (*i.e.* UM1).

7.7 Design example

In this section, an example is presented in order to illustrate the application of the proposed design method. Mono-symmetric F3.8 is adopted as the example, and the material proper-

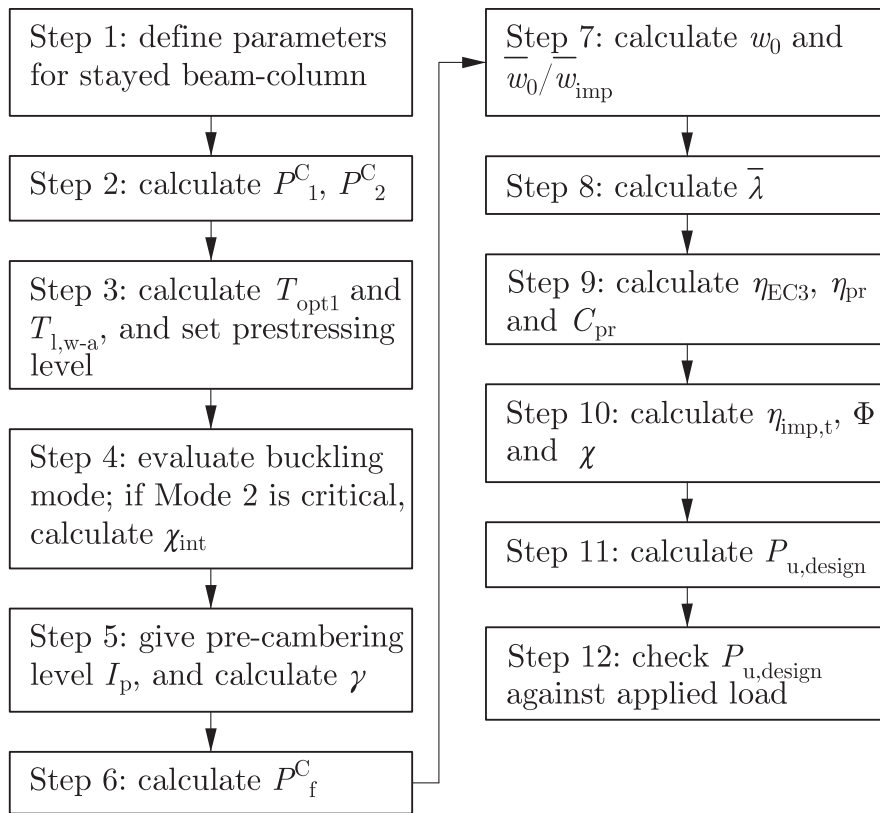


Figure 7.8: Design procedure for stayed beam-columns.

ties and cross-section parameters are described in Table 3.1, and the geometry parameters are listed in 3.3.

To determine critical buckling loads P_1^C and P_2^C , D_1 and D_2 are obtained from Eqs. (4.16) and (4.35) respectively thus:

$$\begin{aligned} D_1 &= \frac{K_{su} \sin^2 \beta_u + K_{sl} \sin^2 \beta_l}{K_{cb, half}} = 31.78, \\ D_2 &= \frac{K_{su}}{K_{cb, half}} \left[\frac{\sin^2 \beta_u}{1 + \frac{2K_{su}}{3K_{au, b}} \cos^2 \beta_u} \right] + \frac{K_{sl}}{K_{cb, half}} \left[\frac{\sin^2 \beta_l}{1 + \frac{2K_{sl}}{3K_{al, b}} \cos^2 \beta_l} \right] = 10.76. \end{aligned} \quad (7.58)$$

From Eqs. (4.52)–(4.54), P_1^C and P_2^C can be determined, thus:

$$\begin{aligned} P_1^C &= P_{1, regression}^C = \frac{E_c I_c}{l^2} \left[\frac{\arctan \left(\frac{D_1 - 15.9457}{2.3136} \right) + 0.7384}{0.5139} \right]^2 = 127.21 \text{ kN}, \\ P_2^C &= P_{2, regression}^C = \frac{E_c I_c}{l^2} \left[\frac{\arctan \left(\frac{D_2 + 2.3799}{3.2130} \right) + 1.5753}{0.6999} \right]^2 = 123.70 \text{ kN}. \end{aligned} \quad (7.59)$$

From Eqs. (3.46) and (3.47), the linear optimum prestressing force for the lower stays is:

$$T_{opt1} = P_{max}^C C_1 = \frac{\min\{P_1^C, P_2^C\}}{C_0} C_1 = 3.07 \text{ kN}. \quad (7.60)$$

From Eq. (3.19), the required applied prestressing force within the lower stays to counteract self-weight is:

$$T_{l, w-a} = T_{l, w} + \frac{E_s A_s (l_{w, l} - l_{p, l})}{l_{w, l}} = 2.26 \text{ kN}. \quad (7.61)$$

Considering both structural load-carrying capacity and efficiency, $T_{l, i} = 2T_{opt1} = 6.14 \text{ kN}$ is adopted.

Based on evaluating the buckling mode for mono-symmetric cases, since the character of case F3.8 is derived from case F3, the critical downward buckling mode of case F3.8 is the same as that of case F3, and χ_{int} of case F3 can be applied to case F3.8 safely. Similar to the calculation shown in Eq. (7.59), the critical buckling loads of case F3 for Modes 1 and

2 are $P_1^C = 132.27$ kN and $P_2^C = 123.74$ kN respectively, and Mode 2 is critical. From Eq. (7.3), the value of I_{cr} for case F3 is:

$$I_{cr} = 100 \frac{P_1^C - P_2^C}{P_2^C} = 6.86 > 0. \quad (7.62)$$

From Eq. (7.6), χ_{int} for case F3 where $T_{l,i}/T_{opt1} = 2$ is:

$$\chi_{int} = 0.9205 + 0.0022 I_{cr} \frac{T_{l,i}}{T_{opt1}} = 0.95 < 1, \quad (7.63)$$

and interactive buckling is dominant. Hence, for case F3.8 where $T_{l,i}/T_{opt1} = 2$, Mode 2 is critical. Moreover, DI is dominant in downward buckling, and $\chi_{int} = 0.95$. Since case F3.8 is mono-symmetric, both UM1 and DI (based on DM2) need to be checked to determine the load-carrying capacity.

Setting the pre-cambering level $I_p = 5$, from Eqs. (7.1) and (7.2), the relationship between I_p and γ can be obtained, thus:

$$I_p = 9.37 - 5.67\gamma, \quad (7.64)$$

and $\gamma = 0.77$. From Eq. (6.2), the prestressing force within the upper stays is:

$$T_{u,i} = \gamma T_{l,i} \frac{\sin \beta_l}{\sin \beta_u} = 5.87 \text{ kN}. \quad (7.65)$$

From Eq. (7.18), P_f^C is:

$$P_f^C = \begin{cases} \frac{P_1^C}{C_0} + T_{u,i} \cos \beta_u + T_{l,i} \cos \beta_l = 132.42 \text{ kN} & \text{for UM1,} \\ \frac{P_2^C}{C_0} + T_{u,i} \cos \beta_u + T_{l,i} \cos \beta_l = 129.10 \text{ kN} & \text{for DM2.} \end{cases} \quad (7.66)$$

From Eqs. (7.37) and (7.50), corresponding to UM1 and DM2 respectively, $\bar{w}_0/\bar{w}_{imp}$ can

be determined, thus:

$$\frac{\bar{w}_0}{\bar{w}_{\text{imp}}} = \begin{cases} \frac{w_{\text{p,m}} + \bar{w}_{\text{imp,um1}}}{\bar{w}_{\text{imp,um1}}} = \frac{1.525+3.05}{3.05} = 1.50 & \text{for UM1,} \\ \frac{\bar{w}_0}{\bar{w}_{\text{imp,dm2}}} = \frac{2.684}{1.525} = 1.76 & \text{for DM2,} \end{cases} \quad (7.67)$$

where $\bar{w}_0 = 2.684$ mm for DM2 is obtained numerically. From Eq. (7.23), $\bar{\lambda}$ can be determined, thus:

$$\bar{\lambda} = \sqrt{\frac{A_c f_y}{P_f^C}} = \begin{cases} 1.275 & \text{for UM1,} \\ 1.291 & \text{for DM2.} \end{cases} \quad (7.68)$$

From Eq. (7.15), η_{EC3} can be obtained from BS EN 1993-1-1 (2005), thus:

$$\eta_{\text{EC3}} = \alpha_{\text{EC3}}(\bar{\lambda} - 0.2) = \begin{cases} 0.21(1.275 - 0.2) = 0.226 & \text{for UM1,} \\ 0.21(1.291 - 0.2) = 0.229 & \text{for DM2,} \end{cases} \quad (7.69)$$

where $\alpha_{\text{EC3}} = 0.21$. From Eq. (7.52), C_{pr} can be obtained, thus:

$$C_{\text{pr}} = \begin{cases} \frac{\bar{w}_0}{\bar{w}_{\text{imp}}} \left[1 - \frac{(K_{\text{su}} \sin^2 \beta_u + K_{\text{sl}} \sin^2 \beta_l)L}{2P_f^C} \right] = -1.082 & \text{for UM1,} \\ \left[\frac{\bar{w}_0}{\bar{w}_{\text{imp}}} - \frac{\pi}{2P_f^C} \left(\frac{K_{\text{su}} a_u \cos \beta_u \sin \beta_u}{1 + \frac{2K_{\text{su}}}{3K_{\text{au,b}}} \cos^2 \beta_u} + \frac{K_{\text{sl}} a_l \cos \beta_l \sin \beta_l}{1 + \frac{2K_{\text{sl}}}{3K_{\text{al,b}}} \cos^2 \beta_l} \right) \right] = 0.821 & \text{for DM2.} \end{cases} \quad (7.70)$$

From Eq. (7.56), X in Eq. (7.54) can be obtained thus:

$$X = \begin{cases} 100(T_{\text{l,i}} \sin \beta_l + T_{\text{u,i}} \sin \beta_u) \alpha_1 \frac{\phi_s}{2r_c} = 3.53 & \text{for UM1,} \\ \sqrt{100(T_{\text{l,i}} \sin \beta_l + T_{\text{u,i}} \sin \beta_u) \alpha_1 \frac{\phi_s}{2r_c}} = 1.88 & \text{for DM2.} \end{cases} \quad (7.71)$$

From Eq. (7.57), Θ in Eq. (7.54) can be obtained thus:

$$\Theta = \begin{cases} \frac{(T_{\text{l,i}}/T_{\text{opt1}})^2}{-23.015\alpha^2 + 36.860\alpha - 12.576} = 1.833 & \text{for UM1,} \\ (T_{\text{l,i}}/T_{\text{opt1}})^2 (0.128\alpha^2 - 0.107\alpha + 0.031) = 0.109 & \text{for DM2.} \end{cases} \quad (7.72)$$

Substituting Eqs. (7.55), (7.71) and (7.72) into Eq. (7.54) gives:

$$\eta_{\text{pr}} = \begin{cases} \frac{12.127X-5.056}{X^2-0.745X+0.106} \times 2.053 = 6.963 & \text{for UM1,} \\ \frac{25.846X-35.681}{X^2-0.673X-1.409} \times 0.075 = 1.636 & \text{for DM2.} \end{cases} \quad (7.73)$$

From Eq. (7.51), η_{tot} can be obtained thus:

$$\eta_{\text{tot}} = \eta_{\text{pr}} |C_{\text{pr}}| \eta_{\text{EC3}} = \begin{cases} 6.963 \times 1.082 \times 0.226 = 1.701 & \text{for UM1,} \\ 1.636 \times 0.821 \times 0.229 = 0.308 & \text{for DM2.} \end{cases} \quad (7.74)$$

From Eq. (7.27), Φ is:

$$\Phi = 0.5 \left(1 + \eta_{\text{tot}} + \bar{\lambda}^2 \right) = \begin{cases} 0.5 (1 + 1.701 + 1.275^2) = 2.164 & \text{for UM1,} \\ 0.5 (1 + 0.308 + 1.291^2) = 1.488 & \text{for DM2.} \end{cases} \quad (7.75)$$

From Eq. (7.26), χ is:

$$\chi = \frac{1}{\Phi + \sqrt{\Phi^2 - \bar{\lambda}^2}} = \begin{cases} \frac{1}{2.164 + \sqrt{2.164^2 - 1.275^2}} = 0.256 & \text{for UM1,} \\ \frac{1}{1.488 + \sqrt{1.488^2 - 1.291^2}} = 0.449 & \text{for DM2.} \end{cases} \quad (7.76)$$

Finally, from Eq. (7.28) and (7.29), the design load-carrying capacity is:

$$P_{\text{u,design}} = \begin{cases} C_0(\chi A_c f_y - T_{\text{u,i}} \cos \beta_{\text{u}} - T_{\text{l,i}} \cos \beta_{\text{l}}) = 58.05 \text{ kN} & \text{for UM1,} \\ \chi_{\text{int}} C_0(\chi A_c f_y - T_{\text{u,i}} \cos \beta_{\text{u}} - T_{\text{l,i}} \cos \beta_{\text{l}}) = 96.89 \text{ kN} & \text{for DI.} \end{cases} \quad (7.77)$$

In Chapter 6, from the results of case F3.8 where $T_{\text{l,i}}/T_{\text{opt1}} = 2$ using FE modelling, the ultimate loads for UM1 and DI are 69.64 kN and 88.21 kN respectively, and the errors are within 20%. For UM1, the load-carrying capacity is underestimated, but for DI, this design method overestimates the load-carrying capacity. This implies that there are reliability

risks induced by this methodology, which should be addressed with further research.

7.8 Concluding remarks

A preliminary methodology for the design of both doubly-symmetric and mono-symmetric stayed beam-columns is proposed in this chapter. This method is based on analytical methods similar to BS EN 1993-1-1 (2005) and Tankova *et al.* (2019) for flexural buckling, and it can be adapted to varying pre-cambering levels and allows for the reduction in load-carrying capacity from interactive buckling. It can be applied using the step-by-step procedure illustrated in Figure 7.8. In addition, a design example is given to demonstrate the application of the method.

The calibration of this method against FE results is presented. It can be seen that the basic trend is well predicted, although the calibration are based on a limited number of FE results. Therefore, the concept and methodology from the proposed design method can be developed further based on more experimental and numerical results, including varying steel types and grades, main member lengths and cross-section types. Moreover, the variables, parameters and relationships in this method should be refined further to capture the complex structural performance to avoid the risk of overestimating the load-carrying capacity, which would make the proposed design concept much more applicable to engineering practice.

Chapter 8

Conclusions

Prestressed stayed beam-columns satisfy a potential demand for very long struts to be used in the horizontal plane, for example to support excavation works in the construction of large scale foundations. The conventional beam-columns, so-called ‘super struts’, which are currently used, are constructed from heavy tubular steel cross-sections, while the stayed beam-columns can be a more effective structure to achieve a long span. The buckling and post-buckling behaviour of stayed columns has been investigated fully (Saito & Wadee, 2008, 2009b; Wadee *et al.*, 2013a; Yu & Wadee, 2017a). However, most of the existing studies are focused on the vertical doubly-symmetric members where the direction of the self-weight and the axial load are collinear. The current study considered the effect of placing stayed columns in the horizontal plane, thereby converting them into stayed beam-columns where the axial load and the self-weight act orthogonally to each other, and both doubly-symmetric and mono-symmetric geometries were taken into account.

8.1 Summary of findings

It was shown in Chapter 3 that the crossarm length ratio alongside the effect of self-weight have significant effects on the mechanical behaviour. The study determined that the existing three distinct zones of behaviour for purely axially-loaded prestressed stayed columns sub-divide two of these zones into two further parts by taking the natural mono-symmetry of the geometry into account. The newly-derived analytical zones for the beam-column were verified by nonlinear FE modelling and, as has been determined for stayed columns, the true optimum prestressing level, in terms of maximizing the ultimate load-carrying capacity, is higher than the linearly-derived optimum level. Moreover, the crossarm length ratio was also found to be a key parameter. The results showed that small crossarm length ratios should be avoided since this was demonstrated to lead to extremely unstable post-buckling behaviour including snap-back responses. The intermediate crossarm length ratios, determined herein to be approximately 0.6, give the most favourable mechanical behaviour since it strikes a balance between the effect of self-weight and the initial pre-cambering displacement. Finally, the prestressing force distribution between the upper and lower stays, determined from a simple force balancing relationship provides a sensible estimate for the combination needed to be adopted in stayed beam-columns to maximize the system effectiveness.

Following the study of the zonal behaviour and the benchmark prestress, it was determined that linear buckling analysis through FE modelling is necessary to obtain the buckling eigenvalues prior to evaluating the linear optimum prestressing force. To simplify this process and make it more applicable for engineering practice, a linear analytical model was formulated in Chapter 4, which in combination with regression analysis allows the explicit evaluation of the buckling loads for both the doubly-symmetric and mono-symmetric arrangements under purely axial compression. Their results showed excellent agreement with linear buckling analysis through FE modelling.

Based on the aforementioned linear optimum prestressing force, a nonlinear analytical model formulated with potential energy principles, in conjunction with the Rayleigh–Ritz method was presented for prestressed stayed beam-columns and was verified using the commercial FE package ABAQUS in Chapter 5. The analytical results revealed that the detailed process explaining how the two principal buckling modes interact nonlinearly when their corresponding buckling loads are in relatively close proximity. It was also clearly identified that mode interaction in prestressed stayed beam-columns is intrinsically linked to the stay slackening and crossarm bending phenomena, which were also successfully modelled. The actual post-buckling response is usually dominated by nonlinear modal interactions that features mode jumping, and the different values of the crossarm length ratio lead to different dominant mechanical responses owing to the buckling directions. Moreover, it was found that, for the mono-symmetric stayed beam-columns, varying initial prestressing levels would naturally lead to different post-buckling behaviour. The results implied again that the true optimum prestressing force for stayed beam-columns is higher than the linearly obtained solution.

FE modelling was employed to investigate the structural performance of imperfect stayed beam-columns further in Chapter 6. Through applying different imperfection combinations, it transpired that interactive buckling can reduce the ultimate load, which was revealed analytically based on the Rayleigh–Ritz method in Chapter 5. From the study of the prestress and pre-cambering, excessively small crossarm lengths, stay diameters and crossarm length ratios should be avoided, since the desired pre-cambering level cannot be achieved by adjusting the prestressing distribution between the upper and lower stays, and the buckling loads are small in conjunction with highly unstable post-buckling behaviour that was found in Chapters 3 and 5. In terms of the buckling and post-buckling behaviour, mono-symmetric cases are more sensitive to the pre-cambering level. For mono-symmetric cases, interactive buckling dominance, imperfection and prestressing sensitivities, and optimum crossarm length ratios are strongly linked to the pre-cambering levels. Considering both load-carrying capacity and structural efficiency, doubly-symmetric cases perform

best with zero pre-cambering, but mono-symmetric cases perform best with upward pre-cambering. In terms of the true optimum prestressing levels, they are recommended to be significantly above the linearly obtained optimum to maximize structural efficiency.

Finally, based on the results in Chapter 5 and 6, a preliminary attempt at formulating a design method for horizontal stayed beam-columns was presented in Chapter 7. The analytical solutions from Chapters 3 and 4 were adopted to evaluate the critical load, mode and linear optimum prestressing force. Moreover, a set of design equations was proposed based on the findings. The proposed design approach can be adapted to varying pre-cambering levels and allows for the reduction in load-carrying capacity from interactive buckling. It was shown that this method can predict the basic trend of the load-carrying capacity well, although admittedly it needs further refinement and assessment.

To conclude, the buckling and post-buckling behaviour of the prestressed stayed beam-columns was investigated analytically and numerically, considering both the linear and nonlinear responses. Results from the FE and the analytical models show excellent comparisons both qualitatively and quantitatively. Similar to the phenomena determined in studies on vertical stayed columns, interactive buckling is found to be important in horizontal stayed beam-columns. Moreover, the crossarm length ratio for the mono-symmetric system can affect the buckling direction and buckling mode significantly owing to the level of pre-cambering. Finally, the true optimum prestressing level was discussed for different structural configurations based on efficiency indicators, and a basic design curve has been proposed.

8.2 Suggestions for future work

In the present thesis, the single-bay prestressed stayed beam-column was investigated. It has been sufficient to study the fundamentals of the buckling and post-buckling behaviour,

including zones of behaviour, buckling loads, modal interaction and pre-cambering. This, in turn, would enable guidance for designers to be established in future such that the system may be safely and efficiently used in practice. Following the preliminary study of the design method herein, a more substantial study of the stayed beam-column design needs to be conducted in the future. The concept and methodology from the proposed design method can be developed further based on more experimental and numerical results, including varying steel types and grades, main member lengths and cross-section types.

The practical struts usually work in combined loading cases, including external uniformly distributed load and end moments apart from purely self-weight, and the externally applied bending loads inevitably lead to more complex structural performance. The effects from different loading cases, imperfection directions and pre-cambering levels would introduce more complexity. This study would provide guidance to the design of the stayed beam-columns under combined loads, and be extended to the configuration optimization corresponding to different loading cases, for example optimizing the crossarm length ratio for struts under eccentric compression.

In addition, although the current study considers essentially a two-dimensional (2D) system, in reality, it is a three-dimensional (3D) system, the beam-column must be supported by three or more crossarm elements to be stable in the third dimension. An alternative strategy, which has been mentioned herein, is to introduce lateral braces between two adjacent props to decrease the lateral effective length, to avoid buckling out of plane. In more general cases, the out-of-plane buckling would be considered. In cases such as these, it is recommended to develop a three-dimensional model that takes the deflections about both axes into account. It would be worth investigating this to determine whether there is interaction between the buckling modes in two perpendicular axes.

It should be noted that although the FE modelling adopted in the presented thesis has been validated by the previous experiments of stayed columns and the nonlinear behaviour of stayed beam-columns has been investigated both analytically and numerically in detail,

experiments on stayed beam-columns would still be valuable in future. This can validate the captured complex phenomena and conclusions that have been made presently.

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Appendix A

Regression for buckling loads

In Chapter 4, Nonlinear Least Squares (NLS) (Bethea, 1995) is adopted to obtain the simplified solutions of the buckling loads. The details of the method are presented currently. The regression model can be written in general as:

$$Y_t = h(\beta; x_t) + u_t, \quad (\text{A.1})$$

where Y_t , x_t and u_t are the dependent variable, a vector of independent variables and an error term respectively; β is the unknown parameter within $h(\cdot)$ which is a nonlinear scalar function obtained from regression. The NLS is defined by:

$$\hat{\beta}_{\text{NLS}} = \operatorname{argmin}_{\tilde{\beta} \in B} \sum_{t=1}^T (y_t - h(\tilde{\beta}; x_t))^2, \quad (\text{A.2})$$

where $\hat{\beta}_{\text{NLS}}$ and $\tilde{\beta}$ are the evaluated value and trial value for β in Eq. (A.1); B is the set of β to conduct optimization. The sum of the error squared between the true values and fitted values (represented as R^2) is adopted as the cost function in this optimization.

From Eq. (4.36) and Figure 4.7, it can be seen that $F_{1,\text{theory}}$ and $F_{2,\text{theory}}$ approaches

infinity at $kl = \pi/0.7$. Moreover, since explicit simplified solutions are required such that the solutions can be obtained without requiring iterations, only simple functions are taken into account. Considering the aforementioned two properties for $F_{1,\text{theory}}$ and $F_{2,\text{theory}}$, the results based on $F_{i,\text{regression}} = a+be^{cx+d}$ and $F_{i,\text{regression}} = a+b \tan(cx+d)$ are compared, and the latter is adopted finally owing to the excellent fit that is found. All these computations were conducted within Matlab (MathWorks, 2018).

In terms of $F_{1,\text{regression}}$ for $F_{1,\text{theory}}$, a piecewise function comprising three functions is considered to reduce the error to be smaller than 3%. After trials, 2-stage regression can provide the best fitting results. At the first stage, the first and second intervals are thought of as a single interval, and the boundary point between this interval and the third one is determined using optimization to minimize R^2 in all the data. At the second stage, the boundary point between the first and the second interval is determined using the same methodology, but R^2 is calculated based on the data in the first and second intervals. The optimization result is shown in Eq. (4.47); the interval limits of kl of 4.18 and 2.42 are determined from the first and second stage optimizations respectively. In terms of $F_{2,\text{regression}}$ for $F_{2,\text{theory}}$, one function is sufficiently accurate, and the result is shown in Eq. (4.50). The error analysis is shown in Figures 4.7 and 4.8.

Appendix B

Crossarm shape function coefficients

In Chapter 5, the shape functions of the crossarms are derived from an analysis presented in Saito & Wadee (2008) which applied Euler–Bernoulli bending theory to a stayed beam-column. The details of that are presented currently.

Similar to Saito & Wadee (2008), three types of buckled states considering stay slackening are distinguished:

Type A : all of the stays are slack.

Type B : all of the stays are active.

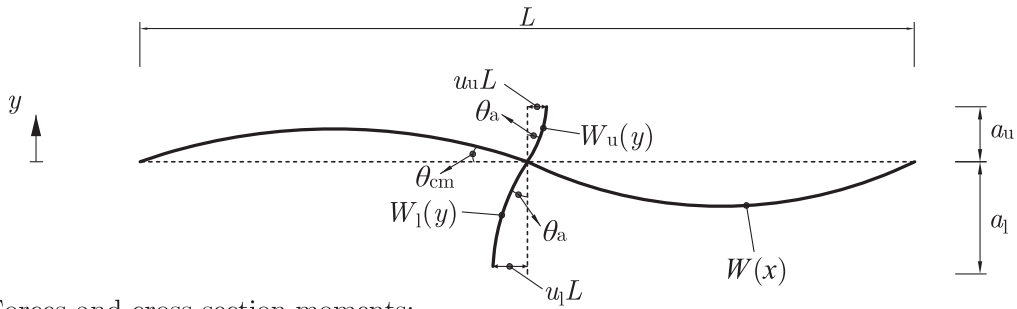
Type C : two stays are active.

Figure B.1 shows the displacements, deformations, forces and moments for crossarms. The bending moment for the upper crossarm cross-section $M_{\text{au}X}$ is given by:

$$M_{\text{au}X} = -F_{\text{au}X}[u_{\text{u}}L - W_{\text{u}}(y)] + R_{\text{au}X}(a_{\text{u}} - y). \quad (\text{B.1})$$

Discarding the higher-order terms and the effect of the end-shortening of the crossarm, the

Displacements and deformations:



Forces and cross-section moments:

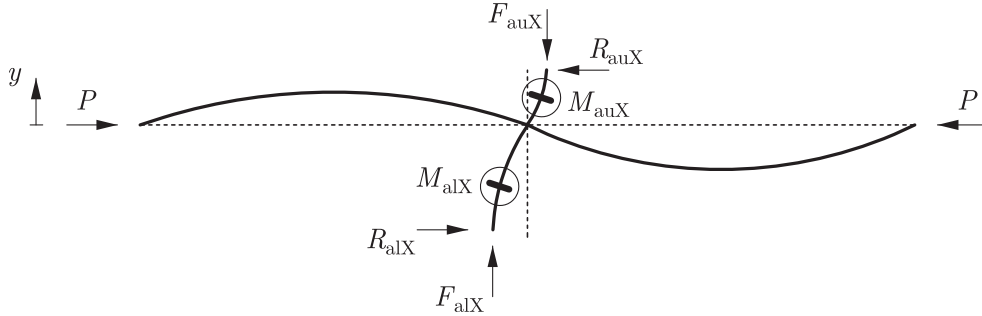


Figure B.1: Displacement, deformation, force and moment analysis for the upper and lower crossarms; $W(x)$ is the shape function of main member defined in Eq. (5.2), and $W_t(y)$, where $t = \{u, l\}$, is the shape function for an arbitrary crossarm defined in Eq. (5.6). The main member and crossarm rotations are equal as shown in Eq. (5.5); u_u and u_l are the generalized coordinates of the horizontal displacement of the upper and lower crossarm tips respectively; R_{atX} , F_{atX} and M_{atX} where $t = \{u, l\}$ are the horizontal force at an arbitrary crossarm tip, axial force within an arbitrary crossarm, and moment at an arbitrary crossarm cross-section. Note that the subscript X represents the buckling type classification which can be either B or C.

basic differential equation for the bending of the crossarm takes the form:

$$M_{\text{au}X} = -E_a I_a W_u''(y). \quad (\text{B.2})$$

Substituting Eq. (B.2) into (B.1) leads to:

$$W_u''(y) + k_{\text{au}X}^2 W_u(y) = -\frac{R_{\text{au}X}}{E_a I_a} (a_u - y) + u_u L k_{\text{au}X}^2, \quad (\text{B.3})$$

where:

$$k_{\text{au}X} = \sqrt{\frac{F_{\text{au}X}}{E_a I_a}}. \quad (\text{B.4})$$

The general solution of Eq. (B.3) is:

$$W_u(y) = H_{\text{au}X} \sin k_{\text{au}X} y + K_{\text{au}X} \cos k_{\text{au}X} y - \frac{R_{\text{au}X}}{k_{\text{au}X}^2 E_a I_a} (a_u - y) + u_u L. \quad (\text{B.5})$$

To obtain an approximate shape function with fixed coefficients for each item in Eq. (B.5), the state at the end of loading step 3 (*i.e.* after applying prestressing forces to stays see Figure 3.2) is adopted to determine the forces within the system. At this state, $F_{\text{au}X}$ is:

$$F_{\text{au}X} = 2T_{\text{u},i} \sin \beta_u = 2T_{\text{u},i} \frac{a_u}{l_u}, \quad (\text{B.6})$$

and substituting Eq. (B.6) into (B.4) leads to:

$$k_{\text{au}X} = \sqrt{\frac{2T_{\text{u},i} a_u}{E_a I_a l_u}}. \quad (\text{B.7})$$

Since $k_{\text{au}X}$ is the circular frequency for the deformed profile of the upper crossarm, it is strongly linked to the effect from the lower crossarm. When α reduces from unity, the effect of the lower crossarm diminishes, which leads to the reduction on the constraint at the mid-span joint. This reduction would lead to more waves in the buckling profile

indicated by the larger circular frequency $k_{\text{au}X}$. Hence, $k_{\text{au}X}$ can be expressed as:

$$k_{\text{au}} = \frac{\kappa}{\sqrt{\alpha}} \sqrt{\frac{2T_{\text{u,i}}a_{\text{u}}}{E_{\text{a}}I_{\text{a}}l_{\text{u}}}}, \quad (\text{B.8})$$

where κ is a calibration factor introduced to correct the crossarm shape to account for the ‘support’ having an additional crossarm on the other side. Although at the end of step 3, $R_{\text{au}X}$ in Eq. (B.5) is zero, the item related to y is also necessary to take the stress difference between the left-hand upper stay and right-hand upper stay into account in step 4. Therefore, the shape function of the upper crossarm can be expressed as:

$$W_{\text{u}}(y) = H_{\text{u}0} + H_{\text{u}1}y + H_{\text{u}2} \sin k_{\text{au}}y + H_{\text{u}3} \cos k_{\text{au}}y. \quad (\text{B.9})$$

Similarly, the shape function of the lower crossarm also can be obtained. The expressions of shape functions for both crossarms are shown in Eqs. (5.6) and (5.7). Moreover, considering the boundary conditions of crossarms, the coefficients for them are obtained in Eq. (5.8), and κ is determined based on numerical calibration that is presented in §5.4.1.