

DIE-LESS DRAWING

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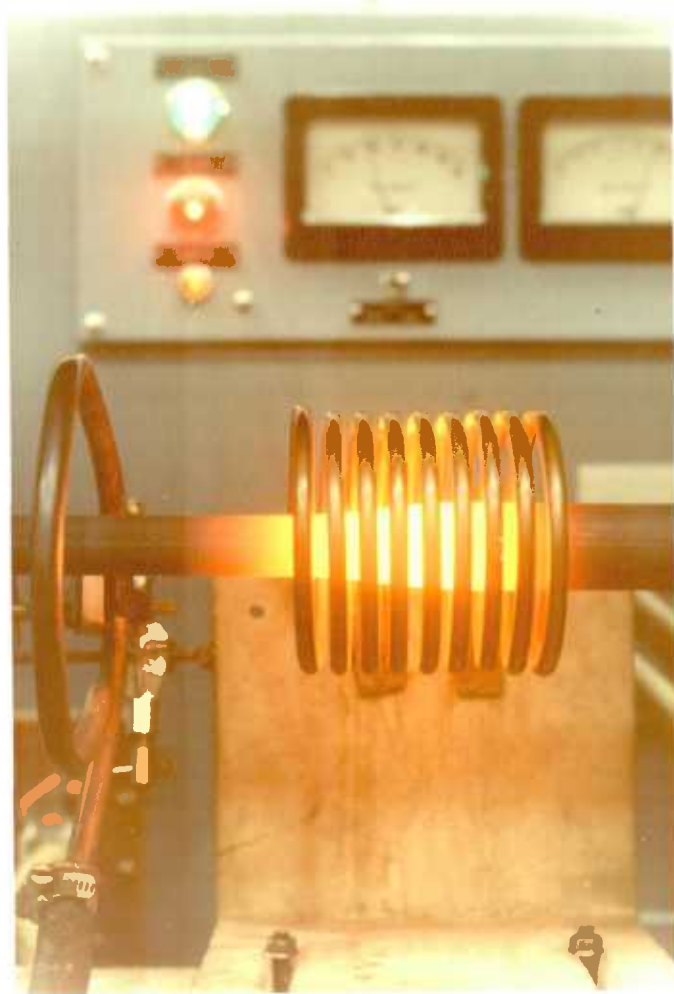
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SUMMARY

The new process of die-less drawing has been examined experimentally on a horizontal Buckton 100 ton testing machine. Six different materials were tested, mild steel, niobium mild steel, 18 - 8 stainless steel, two chrome steels, and a titanium alloy: success was achieved in all except the niobium mild steel.

Specimens of different cross-sections, ranging from 7.6 cm (3 in) diameter and 7.6 cm x 3.8 cm (3 in x 1½ in) rectangular to 2.5 cm (1 in) diameter and square, were drawn up to speeds of 7.6 cm/min (3 in/min). The maximum percentage reduction in area was 75% for steel and 84% for the titanium alloy.

Practical details are discussed at length and a brief outline of the metallurgical considerations is given.

Three different approaches have been made to establish an understanding of the process from a theoretical standpoint. The first is an analysis of the deformation zone considering plane sections remaining plane. The second analysis considers the bar to be in pure tension, allowing for the radial as well as the longitudinal temperature gradients. Strain hardening characteristics and strain rate sensitivity are considered, and the relative effects of each determined. The third technique developed is the finite element approach.

The finite element technique uses the similarity between the

elasticity and plasticity equations. Poisson's ratio is replaced by a value close to 0.5, and Young's modulus by a function of equivalent stress and temperature, and with displacement and strain becoming equivalent to velocity and strain rate respectively, an approximate solution is obtained. The program has been developed to modify the mesh which represents the deformation zone and give approximate flow lines.

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NOTATION

A	Cross-sectional area
A_I	Cross-sectional area of undrawn bar
A_F	Cross-sectional area of drawn bar
A_i	Cross-sectional area of i^{th} section
B	Constant
$[B]$	Strain displacement matrix
C	Distance moved by the draw head
D	Distance moved by the induction coil
$[D]$	Stress strain matrix
E	Young's modulus
F	Load on the specimen
$\{F\}$	Force matrix
K, K', K'', K_1, K_2	Constant
L	Load
M	Mass of draw head
Q	Activation energy
Q'	Quantity of heat
R	Gas constant
$R' = \frac{V_D}{V_C}$	Ratio of draw and coil velocities

T	Temperature
ΔT	Temperature difference
T_m	Melting point temperature
V	Velocity
V_C	Velocity of the induction coil
V_D	Velocity of the draw head
$V_C + V_D$	Velocity of the draw
c	Specific heat
e	Engineering strain
e	Superscript to denote single element
f, g, h	Function
$[k]$	Element stiffness matrix
k'	Conductivity
m	Strain rate sensitivity index
n, n'	Constant
r	Radius
t	Time
u	Specimen velocity
u', v'	Components of displacement
z	Direction of specimen axis
α, β, δ	Constant
δ	Displacement

ϵ	Natural strain
$\dot{\epsilon}$	Strain rate
ν	Poisson's ratio
ρ	Density
σ	Stress
$\bar{\sigma}$	Equivalent stress
σ'	Deviatoric stress
Δ	Area of element

CHAPTER ONE

INTRODUCTION

Holla, ye pampered Jades of Asia:
What, can ye draw but twenty miles a day?

Conquests of Tamburlaine, Part 1, 1.3980

Christopher Marlowe

1564 - 1593

1.1

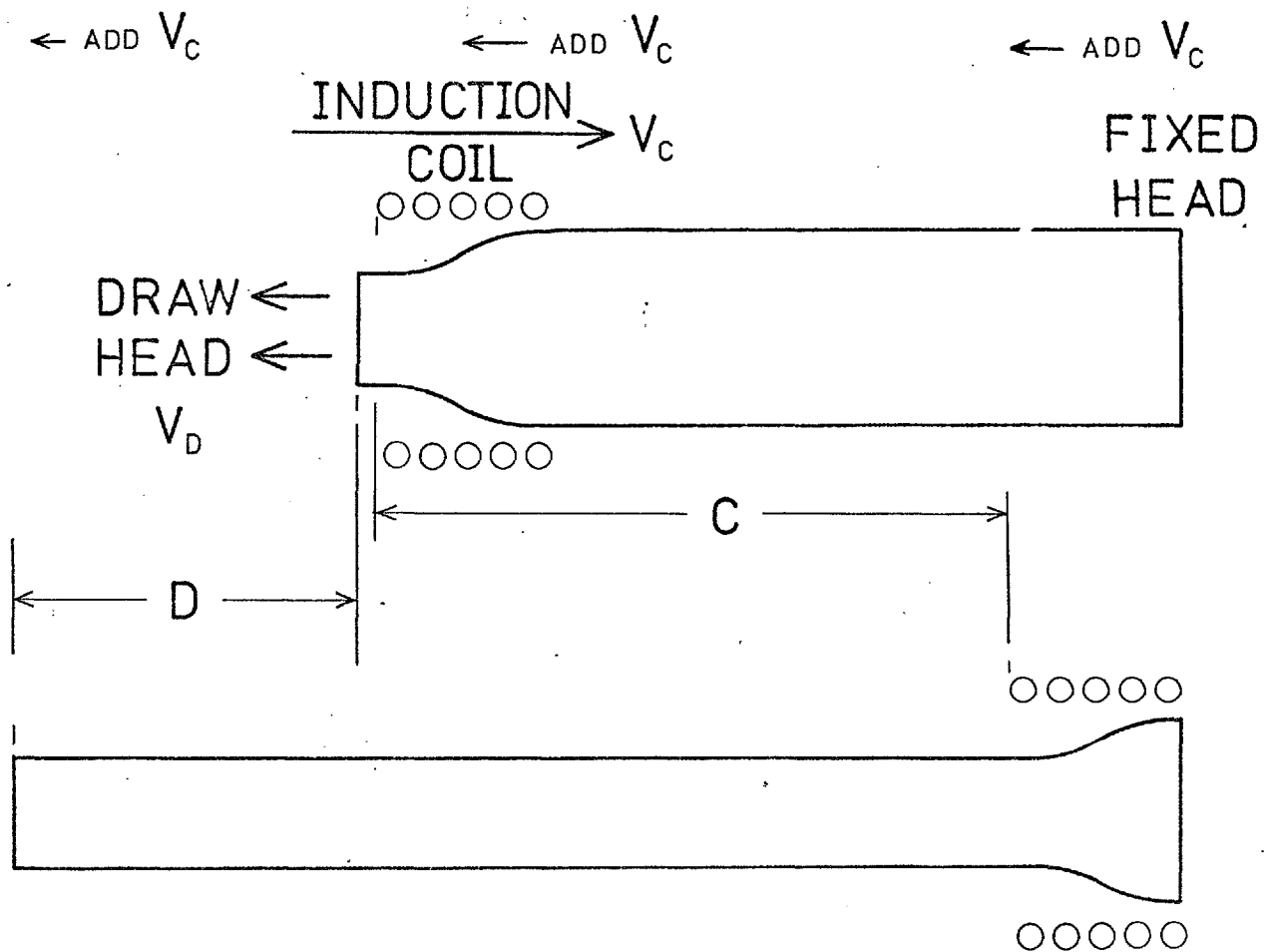
PREFACE

The viscous behaviour of alloys was reported as early as 1920 by Rosenhain, Haughton and Bingham (1) and Pearson (2) historic photograph of a tensile test-piece of extruded bismuth-tin eutectic alloy with an elongation of 1950 per cent is well known. Such extensive deformation is now called superplasticity.

The phenomenon of superplasticity, comprehensively reviewed by Johnson (3), is exhibited by many metals and alloys, both ferrous and non-ferrous, and is either a fine-grain-size or a transformation effect. In recent years a great amount of interest has developed in the phenomenon because of the possibility of commercial application. It is from this background that the idea of die-less drawing emerged with experimentation by Johnson (4) and Weiss and Kot (5).

Die-less drawing is a process where a longitudinal tensile force is applied to a bar a small length of which is heated. After a neck has formed the heat source is moved along the bar, the neck cooled and a reduced cross-section drawn from the bar. Only that part of the bar which is being deformed is heated. A schematic diagram of the process is shown in Figure 1.

It is easy to see die-less drawing purely as a superplastic process and superplasticity certainly contributes a great deal to a stable draw, but with a complex deforming zone of varying stress, strain,



By adding V_c in the direction shown above to the velocities of the draw head, induction coil, and fixed head, the coil may be regarded as stationary with material fed into the coil at a velocity V_c and the bar drawn from the coil with the velocity

$$V_c + V_D$$

FIGURE 1 Schematic diagram of die-less drawing

and strain rate fields and temperature gradients essential to stability, the process may be viable outside the confines of superplastic behaviour.

1.2

EQUIPMENT FOR DIE-LESS DRAWING

The equipment required for die-less drawing comprises a machine for applying a tensile force, some means whereby the deforming part of the specimen may be heated to the hot working range and a method of cooling the deformed bar. In order to control the process as precisely as possible heating is effected by an induction set which makes the design of the coil an added consideration.

Draw head speed is best controlled by a positive drive from a screw thread rotating at constant velocity and the induction coil must be designed to move in the opposite direction to the draw head. A range of speed ratios is needed to give a range of percentage reductions in area.

To draw mild steel or materials which rapidly form scale at the drawing temperature, an inert atmosphere or a method of controlling or removing the scale is essential. The problem was averted in the present work by concentrating on chromised mild steel, stainless steel and titanium which form a very thin oxide film at the working temperature and prevent scale formation.

A system of feed back control on the heat input and cooling device would be very useful but was not investigated in the present work.

It was however noted that stainless steel could be very readily stopped from necking by a slight increase in the pressure of the cooling air.

A final sizing using rolls or a die to give a small percentage reduction at the bottom of the die-less zone was suggested⁽⁶⁾ and considered to be useful equipment at the start of the experimental programme. The rolls or die would tend to close up cavities, precise sizing would be possible, and the rolls could be designed to extract heat effectively. This kind of equipment was not designed however, so that attention could be concentrated on the die-less drawing aspects of this metal working process.

1.2.1 Tensile Machine

All that is required for providing the load in a die-less drawing process is a machine which gives a steady pull over a long length. Ideally the pull should come from a screw thread to give a precise velocity, but a hydraulic machine in which oil is pumped into the cylinder at a constant rate gives a satisfactory draw.

Loads of the order of 40 kN (4 tons) are required to draw 5 cm (2 in) diameter bars of stainless steel, thus 10 cm (4 in) diameter bars will require about 160 kN (16 tons). One inch diameter titanium

requires only 6 kN (0.6 tons).

Many existing machines therefore can be used for die-less drawing and specialised design is unnecessary.

With the variations and oscillation of the diameter of the bars at the beginning of the draw, there is a variation in the load on a hydraulic machine. This results in accelerations and decelerations of the moving head. To keep these velocity changes to a minimum it is important to ensure that the moving parts of the machine are massive. This point is discussed later. Machines driven by a screw thread do not have this problem as the velocity can be kept constant throughout the draw.

The machine may be vertical or horizontal and perhaps the most important criterion in selecting a drawing machine is the length of the stroke.

1.2.2 Heating Techniques

Only that part of the bar which is being deformed is heated in the die-less drawing process, which makes induction heating by far the most convenient method. Other forms of heating may be used however and in an industrial process there is scope for a study of the various techniques.

The convenience of induction heating lies in the two principal advantages. Firstly high heating rates are available because the heat is generated in the body of the material. Secondly the power supplied is easily controlled.

The depth of induction heating is inversely proportional to the square root of the frequency, thus it is important to choose an induction set which matches the oscillator with the specimen diameter.

The optimum depth of heating, i.e. when the eddy currents are generated down to the centre line of the bar, expressed as diameters are plotted against frequency for steel specimens in Figure 2.

Specimens up to 30 cm (12 in) diameter may be heated at mains frequency.

1.2.3 Induction Coil Design

With specimen diameters chosen at about 5 cm (2 in) and larger, the frequency of the induction set was fixed at about 3 - 5 kHz. At these lower frequencies the eddy currents cannot be concentrated into particular regions. Radio frequencies permit the use of intensifiers which make high temperatures possible over a narrow band.

The important parameters at the lower frequencies are coil length, coil diameter and the number of turns. If there are insufficient turns (<6) it is not possible to load the coil as there is insufficient coupling between the coil and specimen.

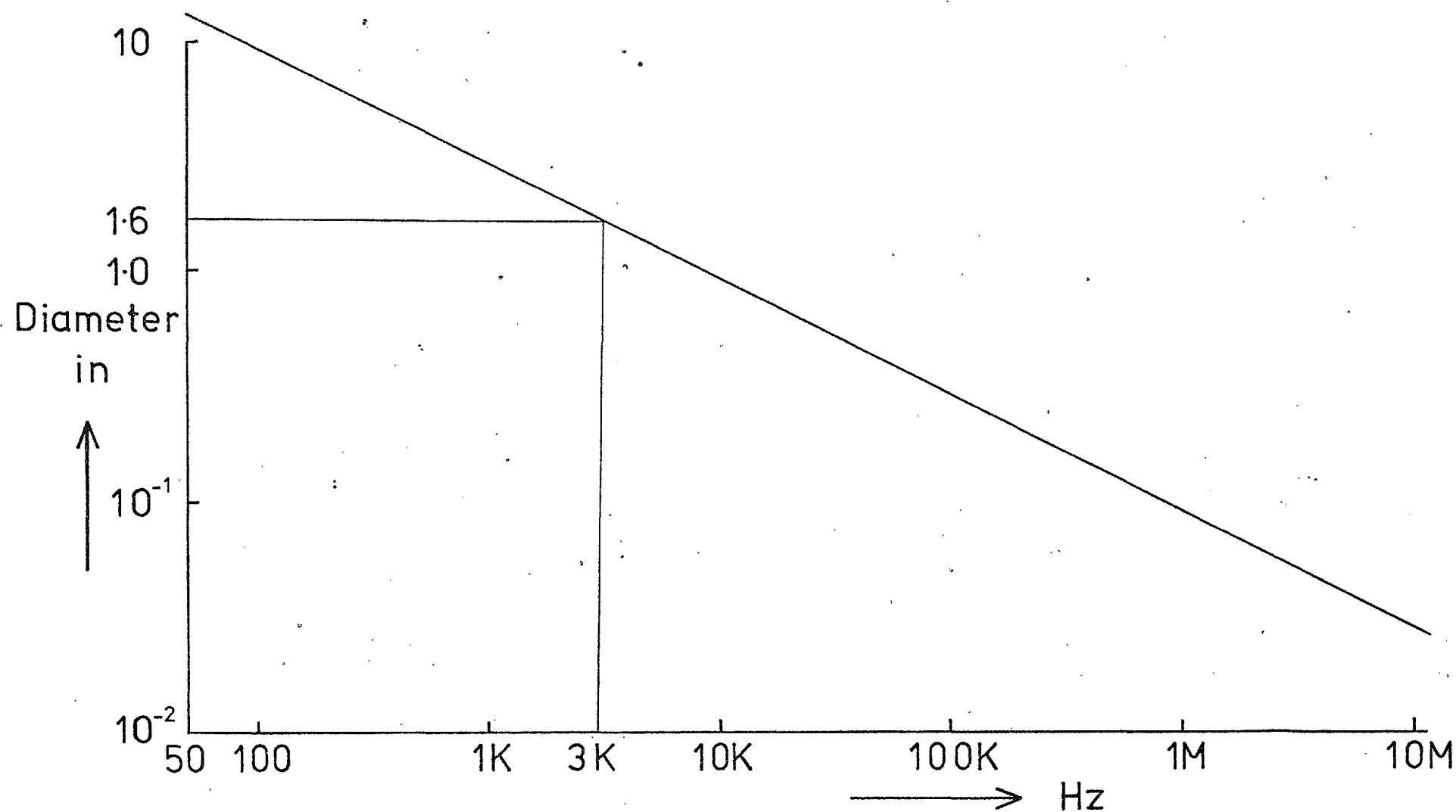


FIGURE 2 Optimum depth of heating expressed as diameter vs. frequency. Induction heating of steel.

1.3

REVIEW OF THE PROCESS1.3.1 Theoretical Approach

The analytical treatment of metal forming processes almost invariably deals with a contained plastic region where the boundary conditions play a vital role. Die-less drawing has no such constraints and a stable draw can only be arrived at by allowing the different parameters to take their "free" values with a fixed load as the only constant.

Johnson⁽³⁾ in his review of superplasticity points out that the limit of stable flow is marked by the onset of geometrical instabilities and goes on to discuss the whole question of stability. Rossard⁽⁷⁾, using a relationship, expressed as the product of a strain hardening function and a strain rate sensitivity function, of the form

$$\sigma = K \dot{\epsilon}^{n,m} \quad (1)$$

demonstrates both experimentally and theoretically the importance of the strain rate sensitivity index, m , a point corroborated by Backofen, Turner and Avery⁽⁸⁾ who neglect the strain hardening term and state "On a phenomenological basis, superplasticity can be rationalised in terms of a strain rate sensitivity". They set $K\dot{\epsilon}^n$ equal to a constant and obtain:

$$\sigma = K'\dot{\epsilon}^m \quad (2)$$

Since
$$\sigma = \frac{F}{A} \quad (3)$$

and
$$\dot{\epsilon} = \frac{1}{z} \frac{dz}{dt} = - \frac{1}{A} \frac{dA}{dt} \quad (4)$$

for an incompressible material.

Combining equations (2), (3) and (4)

$$\frac{dA}{dt} = - \left[\frac{F}{K'} \right]^{\frac{1}{m}} \frac{1}{A^{\frac{m-1}{m}}} \quad (5)$$

The implications of equation (5) are expressed in the graph, Figure 3, which shows $-\frac{dA}{dt}$ against A for different values of m . It is clear that when $m = 1$ the rate of change of area with time is independent of the area and the flow is Newtonian. For an m value of 0.2, an appropriate value for steel, the smaller the area becomes the more rapidly does the area reduce.

An analysis of the tensile test is given by Hart ⁽⁹⁾ in which he states his criterion for stability as follows: "If in the course of the deformation the magnitude of the cross-section difference does not increase, we shall call the deformation stable". Were this criterion to be used for die-less drawing, the process would not be feasible, but this work does not take into account temperature gradients, which play a major role in establishing a deforming zone which is stable.

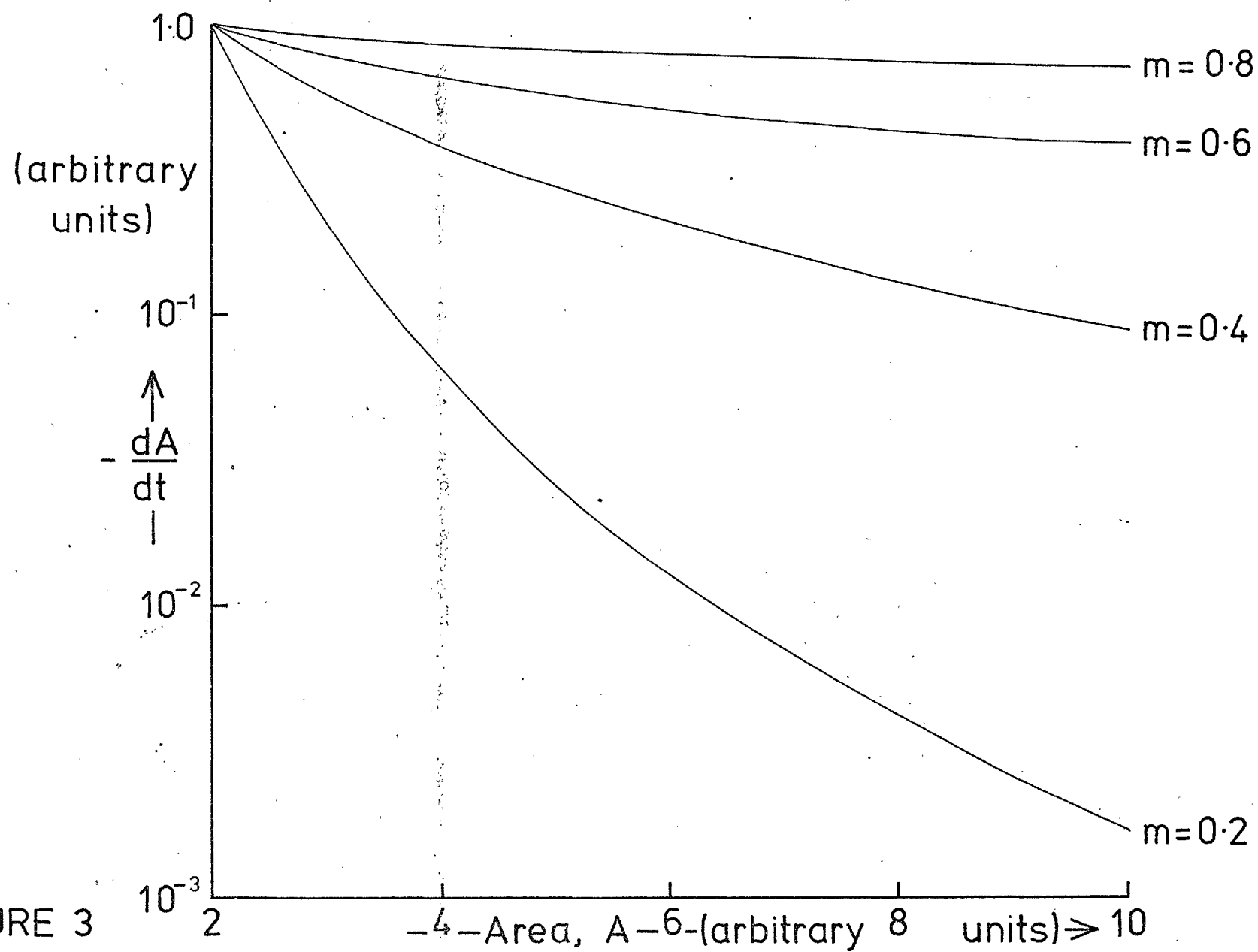


FIGURE 3

Above the homologous temperature, $0.5 T_m$, i.e. the temperature range usually associated with hot working, strain hardening is less pronounced and a proposal put forward by Sellars and Tegart⁽¹⁰⁾ relates the stress, strain rate, and temperature by the equation

$$\dot{\epsilon} = B(\sinh \alpha \sigma)^{n'} \exp \left[\frac{-Q}{RT} \right], \quad (6)$$

which is similar to the form used in describing secondary creep if the hyperbolic sine function is replaced. Expressed in multiaxial form the secondary creep equation is

$$\dot{\epsilon}_{ij} = K'' \bar{\sigma}^{n-1} \cdot \sigma'_{ij} \cdot \exp \left[\frac{-Q}{RT} \right] \quad (7)$$

These equations are used by Cornfield and Johnson⁽¹¹⁾ to make theoretical predictions of plastic flow in hot rolling for various temperature distributions in the rolled material. The constants in equation (7) were evaluated from Cook's data⁽¹²⁾ using the uniaxial form of the equation. Cornfield and Johnson are aware of the drawback of the assumption that strain rate is independent of strain. Their analysis uses the analogy between ϵ and $\dot{\epsilon}$ and between displacement and velocity in the elastic stress-strain equations for plane strain conditions, which may be written as follows

$$\begin{aligned} \epsilon_{xx} &= \frac{1 - \nu^2}{E} \left[\sigma_{xx} - \frac{\nu}{1 - \nu} \sigma_{yy} \right] = \frac{1}{E'} \left[\sigma_{xx} - \nu' \sigma_{yy} \right] \\ \epsilon_{yy} &= \frac{1}{E'} \left[\sigma_{yy} - \nu' \sigma_{xx} \right] \\ \epsilon_{xy} &= \frac{1 + \nu'}{E'} \gamma_{xy} \end{aligned} \quad (8)$$

where $E' = \frac{E}{1 - \nu^2}$ and $\nu' = \frac{\nu}{1 - \nu}$ are the effective Young's Modulus and Poisson's ratio respectively, for elastic plain strain conditions.

In general $0.25 < \nu < 0.5$, so that

$$\frac{16E}{15} < E' < \frac{4E}{3}, \text{ and}$$

$$\frac{1}{3} < \nu' < 1.$$

By analogy, equation (8) may be constrained to take the form of equation (7) as follows. Consider, for example, the xx component of equation (7)

$$\begin{aligned} \dot{\epsilon}_{xx} &= K'' \bar{\sigma}^{n-1} \exp \left[\frac{-Q}{RT} \right] \sigma'_{xx} \\ &= K'' \bar{\sigma}^{n-1} \exp \left[\frac{-Q}{RT} \right] \left[\sigma_{xx} - \frac{1}{3} (\sigma_{xx} + \sigma_{yy} + \sigma_{zz}) \right] \\ &= \frac{2K''}{3} \bar{\sigma}^{n-1} \exp \left[\frac{-Q}{RT} \right] \left[\sigma_{xx} - \frac{1}{2} (\sigma_{yy} + \sigma_{zz}) \right] \end{aligned}$$

The analogous elastic equation is

$$\epsilon_{xx} = \frac{1}{E} \left[\sigma_{xx} - \nu (\sigma_{yy} + \sigma_{zz}) \right]$$

so that we must replace ν by $\frac{1}{2}$ (or ν' by 1) and E by $\left\{ \frac{2K''}{3} \bar{\sigma}^{n-1} \exp \left[\frac{-Q}{RT} \right] \right\}^{-1}$

(or E' by $\frac{4E}{3}$, i.e. by $\left\{ \frac{1K''}{2} \bar{\sigma}^{n-1} \exp \left[\frac{-Q}{RT} \right] \right\}^{-1}$)

This is a very useful technique as it allows the problem to be solved using well developed finite element programs for elastic deformation.

A similar approach has been adopted by Sharman⁽¹³⁾ and applied to extrusion of steel. He uses three temperature profiles and calculates the results using the analogy between equations (7) and (8) described above but replacing $\dot{\sigma}^{n-1}$ with $(\sinh \alpha \dot{\sigma})^{n'}$. The hyperbolic sine function represents high temperature deformation more closely over a wider range of temperature and strain rate. His data is obtained from torsion tests where the flow stress becomes virtually independent of strain after a certain strain.

Equation (7), applied to uniaxial tension, becomes

$$\dot{\epsilon} = \frac{2}{3} K'' \sigma^n \exp \left[\frac{-Q}{RT} \right], \quad (9)$$

and is the starting point for an investigation of the growth of instabilities in the die-less drawing of rods and tubes by Cornfield⁽¹⁴⁾.

Writing equation (9) in the form of equation (2) we have

$$\sigma = K' \dot{\epsilon}^m = \left\{ \frac{1}{\frac{2}{3} K'' \exp \left[\frac{-Q}{RT} \right]} \right\}^{\frac{1}{n}} \dot{\epsilon}^{\frac{1}{n}}, \quad (10)$$

Thus

$$K' = \left\{ \frac{1}{\frac{2}{3} K'' \exp \left[\frac{-Q}{RT} \right]} \right\}^{\frac{1}{n}} \quad \text{and} \quad m = \frac{1}{n}.$$

From equation (5) we have

$$\frac{dA}{dt} = - \left[F \left\{ \frac{2}{3} K'' \exp \left[\frac{-Q}{RT} \right] \right\}^{\frac{1}{n}} \right]^n \cdot A^{1-n} \quad (11)$$

As A is a function of both z and t we have

$$\frac{dA}{dt} = \frac{\partial A}{\partial t} + u \frac{\partial A}{\partial z} = - \frac{2}{3} K'' \exp \left[\frac{-Q}{RT} \right] \cdot \frac{F^n}{A^{n-1}} \quad (12)$$

This equation, together with the equation of conservation of mass,

$$\frac{\partial A}{\partial t} + \frac{\partial (uA)}{\partial z} = 0, \quad (13)$$

and the heat conduction equation,

$$\frac{\partial T}{\partial t} + \frac{u \partial T}{\partial z} = \frac{D'}{A} \frac{\partial}{\partial z} \left(\frac{A \partial T}{\partial z} \right) + \frac{Q'(z)}{\rho c} \quad (14)$$

describe the die-less deformation. A steady state solution is assumed and small perturbations set up about this steady state.

Plane sections remaining plane are assumed to establish the equation

$$\frac{d\mathbf{x}}{dt} = \mathbf{P}\mathbf{x}$$

where $\mathbf{x}$ is a $2N$ vector made up of the temperatures and perturbations of area in the N sections, and $\mathbf{P}$ is a square matrix of order $2N$.

The eigen-values of the matrix were then investigated and if the real parts of all the eigen-values of $\mathbf{P}$ are negative, a stable solution exists. Cornfield gives his results in graphical form plotting the percentage reduction against the stress index n to give a line which

separates the stable and unstable zones.

Investigation of the die-less drawing process has been moving very slowly over the last few years: the subject was not even mentioned in the Review Course on Plasticity and Superplasticity in 1969⁽¹⁵⁾. The reason for this is probably its identification with superplasticity. Micrograin superplasticity occurs at strain rates of the order of 10^{-3} to 10^{-5} per second which would allow only very slow drawing speeds. Transformation superplasticity, the starting point for Weiss and Kot's⁽⁵⁾ work, involving the cycling of the specimen about a transformation temperature, is also slow, and Al-Naib and Duncan⁽¹⁶⁾ explicitly state that industrial utilisation would seem to be remote. There is, however, evidence already that die-less drawing can be satisfactorily established without superplastic behaviour, at least in the main body of the deforming material. As analyses are becoming more and more precise, especially with the powerful finite element techniques, theoretical predictions of complex deforming processes such as die-less drawing will be more easily made.

1.3.2 Experimental Approach

The foundations of any metal working process are laid by the experimental evaluation of the variables which control the material behaviour. A four-dimensional graph relating flow stress, strain, strain rate, and temperature is the requirement for investigating die-less drawing. Unfortunately the speeds used so far in the process give strain rates in the region 10^{-1} to 10^{-3} sec^{-1} , which are very slow compared with conventional forming methods such as rolling and extrusion and very fast when compared with creep. There is, therefore, a distinct lack of data, although the position is changing as many workers investigate superplastic phenomena.

Oelschlägel and Weiss⁽¹⁷⁾ have investigated the ferrite-austenite transformation in tension for steels and found a linear dependency between stress and strain rate. Similar conclusions were made by Ankara and West⁽¹⁸⁾ in the austenite-martensite transformation and Greenwood and Johnson⁽¹⁹⁾ working on uranium, zirconium, titanium, iron, iron + 0.39% carbon, and cobalt.

Weiss and Kot⁽⁵⁾, wishing to take advantage of the considerable increase in ductility during phase transformation, set up an experiment for die-less drawing. A lathe was modified to give a draw head driven by a spindle speed using a threaded rod. Steel specimens were heated using an induction coil, giving temperatures of 705°C and 815°C . The plain carbon steel, AISI 1018, was reduced

by up to 50% in area at draw head speeds in the range 1.2 cm/min (0.5 in/min) to 10.9 cm/min (4.3 in/min). Specimen diameters were 0.635 cm (0.250 in) diameter. Commercially pure titanium of the same diameter was also drawn at 1.2 cm/min by up to 50% reduction in area at temperatures of the order of 900°C, and Ti-6Al-4V, 0.95 cm (0.375 in) diameter, drawn at 2.5 cm/min (1.0 in/min) by up to 54.5% reduction in area at a temperature of 930°C. They concluded that wire drawing is feasible without a die for the steel, titanium, and Ti-6Al-4V and, noting that their calculations of strain rate for AISI 1018 are two or three orders of magnitude higher than the strain rates necessary to cause transformation plasticity, they consider the die-less process might be used on materials which do not undergo a transformation.

Johnson⁽⁴⁾ has carried out similar experiments on a range of materials using a modified lathe and concludes that the process has commercial possibilities.

Stabilized tensile testing (i.e. without local necking) has been investigated by Taraldsen⁽²⁰⁾. His approach is to apply a small stabilizing rolling pressure along the specimen continuously moving up and down the gauge length. The effect of the roll pressure is to elongate a small layer just underneath the rolls and to introduce a transverse pressure which makes the flow stress a combination of tensile and compressive stresses. Stabilization up to percentage reductions of about 75% were obtained quite easily with mild steel.

Roll pressures, measured in terms of the reduction in load on the specimen, were in the range 2% to 15% reduction and had little effect on Taraldsen's results; the smallest pressures being effective in preventing the formation of local necks.

The "superhigh plasticity" of commercial iron reported by Lozinsky and Simeonova⁽²¹⁾ led to a search for superplasticity in steels and attention was turned to micrograin superplasticity. Morrison⁽²²⁾ refined the grains of several steels by rolling in the lower austenite region for testing in the ferrite region, at temperatures in the neighbourhood of 700°C. Tensile testing was then carried out with each specimen subjected to eight rapid speed changes. A steady state, indicated by a constant flow stress, was reached at each speed ranging from 0.0125 cm/min (0.005 in/min) to 0.05 cm/min (0.02 in/min). Strain rate sensitivity indices of the order of 0.5 were obtained for several of the low alloy steels and over a limited range of strain and strain rate the relationship $\sigma = K \dot{\epsilon}^m$ could be applied.

Hayden and Brophy⁽²³⁾ carried out hot tensile tests of Ni-Cr-Fe alloys and found the fine-grained matrix structure exhibiting superplasticity and Tither⁽²⁴⁾ has obtained high values of the strain rate sensitivity index, ~ 0.5 for a niobium stabilized fine-grain structured mild steel using an Instron tensile testing machine. His testing temperature was 730°C and the strain rate in the region 10^{-5} to 10^{-4} sec⁻¹.

Backofen⁽⁸⁾, investigating superplasticity in an Al-Zn alloy, recognized the importance of a high m value in obtaining neck-free stable extension and also noted that m is a function of strain rate and temperature. In a later paper Lee and Backofen⁽²⁵⁾ dealing with isothermal tension tests on several titanium alloys and one of zirconium give results of the variations of m , plotting the strain rate sensitivity index against both temperature and strain rate. The work is illuminating, the data from one of their graphs being reproduced in Figure 4, and demonstrates in the clearest way that the commonly used relationship $\sigma = K\dot{\epsilon}^m$ is a considerable oversimplification of the behaviour of superplastic materials.

There is a natural tendency to believe that experimental data obtained from wire drawing could be used to develop an understanding of the die-less drawing process. This is unhappily not true for the following reasons. Wire drawing is a cold process; much higher speeds are encountered and the boundary conditions are very different. Wistreich's⁽²⁶⁾ investigation of the mechanics of wire drawing indicates that the mean die pressure is never likely to be much less than two-thirds of the mean yield stress, a fact that demonstrates that any comparison would be of little value.

The foregoing paragraphs naturally lead to the starting point from which an investigation into the die-less drawing process might begin; a beginning which must inevitably be experimental because of the complexities inherent in the process itself.

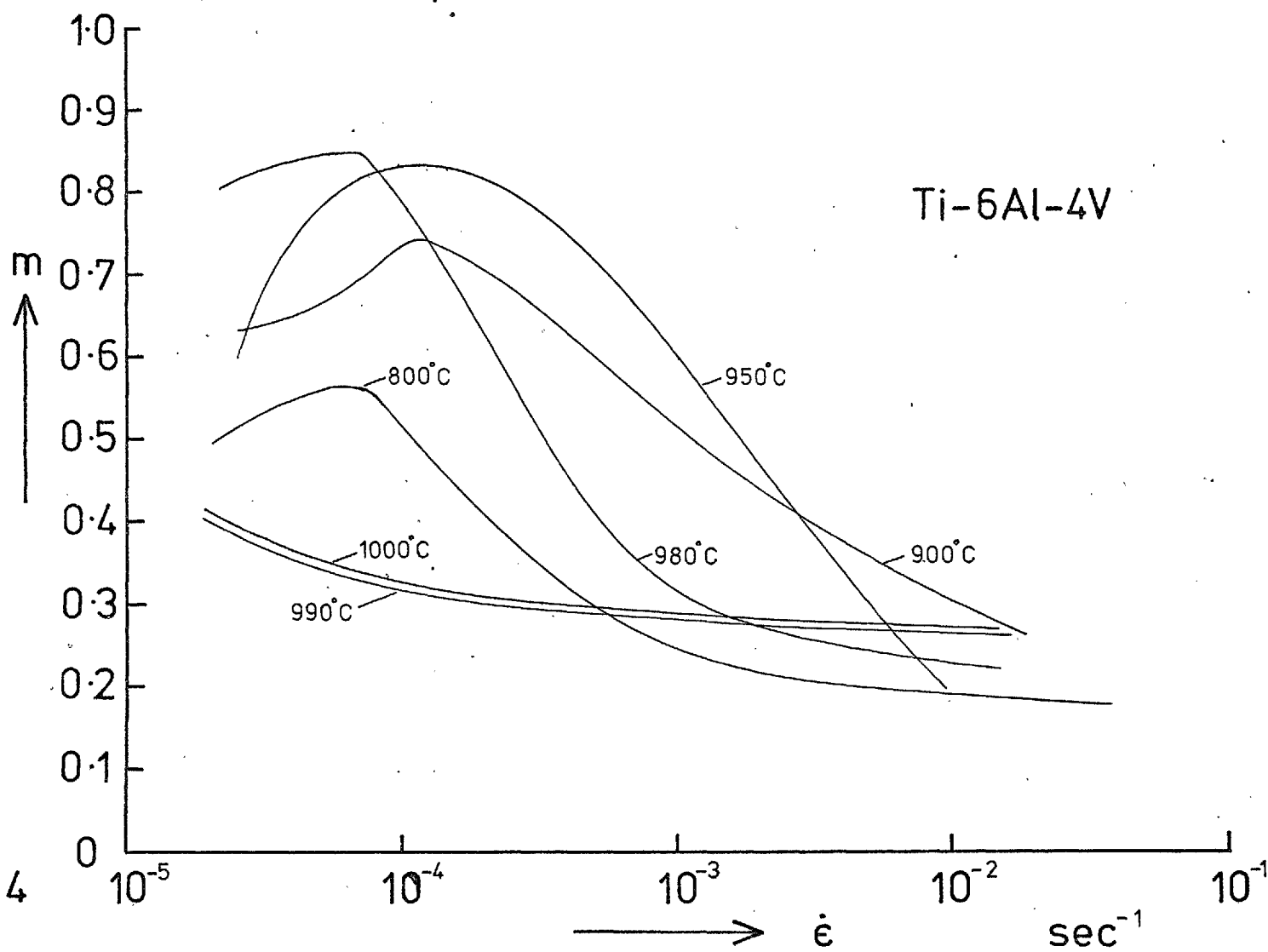


FIGURE 4

1.4

AIMS OF THE WORK

The main objective of the work is to establish a broad understanding of the process in order to make it possible for predictions to be made of how different metals behave during the complex forming process. The first stage was to show that a die-less drawing process is possible for steel bars of different cross-sections, up to diameters of 7.5 cm (3 in).

Having demonstrated the existence of the stability of the process experimentally, a complete theoretical analysis became the final aim, although it must be appreciated that real materials with widely differing strain rate sensitivities and strain hardening characteristics will be likely to present difficulties in the empirical approach.

Despite the slow speeds of the process at the present stage of its development the possibility of industrial application remains.

Variable cross-sections are unique to the process when it is compared with the use of normal dies, and the good die-less drawing properties of difficult-to-work materials like titanium may be exploited to the full; these are two aspects at least which make the final objective of industrial application attractive.

CHAPTER TWO

EXPERIMENTAL WORK

We first survey the plot, then draw the model;
And when we see the figure of the house,
Then must we rate the cost of the erection:
Which if we finds outweighs ability,
What do we then, but draw anew the model

King Henry IV Part II Act 1. Sc. 3

William Shakespeare

2.1

INTRODUCTION

The experimental programme was started with the intention of demonstrating the viability of the process for bars of the order of 5.0 cm (2 in) diameter and investigating the following parameters:

Stress	Strain rate sensitivity index
Strain	Material
Strain rate	Induction coil shape
Temperature	Temperature profile
	Specimen cross-section

During the testing it was found that other parameters played an important part in stable drawing: they are:-

Grain size	Position of zone in coil
Rate of heat extraction	Mass of moving head
Induction coil length	Heating technique

It is general practice to keep as many of the important parameters constant and to obtain results where only two parameters vary. This cannot be done with die-less drawing, as the process can only operate when stress, strain, strain rate, and temperature all vary to give a constant load.

With fifteen variables to consider a somewhat intuitive approach was adopted and whenever a simplification presented itself it was immediately

incorporated. For example, after the initial draws on mild steel, where the formation of scale caused difficulties with heat extraction, effort was concentrated on those materials which did not scale.

2.2

PROCEDURE

Testing each specimen was carried out as follows:

- (i) The bar was set up in the rig with the heating coil positioned a distance of 2.5 cm (1 in) from the start end. Reference to Figure 5 shows the position of each of the components.
- (ii) The setting of the gear box was adjusted to give the appropriate speed ratio, R' .
- (iii) Having balanced the Wheatstone bridge circuit of the load cell, the cell was calibrated using the steelyard set at a load close to the expected drawing load.
- (iv) The power to the induction coil was switched on and the operating current adjusted to give the chosen maximum temperature for the draw.
- (v) The chart recorder was switched on and the draw head set in motion by opening the hydraulic drive valve.
- (vi) When the neck represented about 30% reduction in area, the coil position was adjusted to place the developing deformation zone correctly.

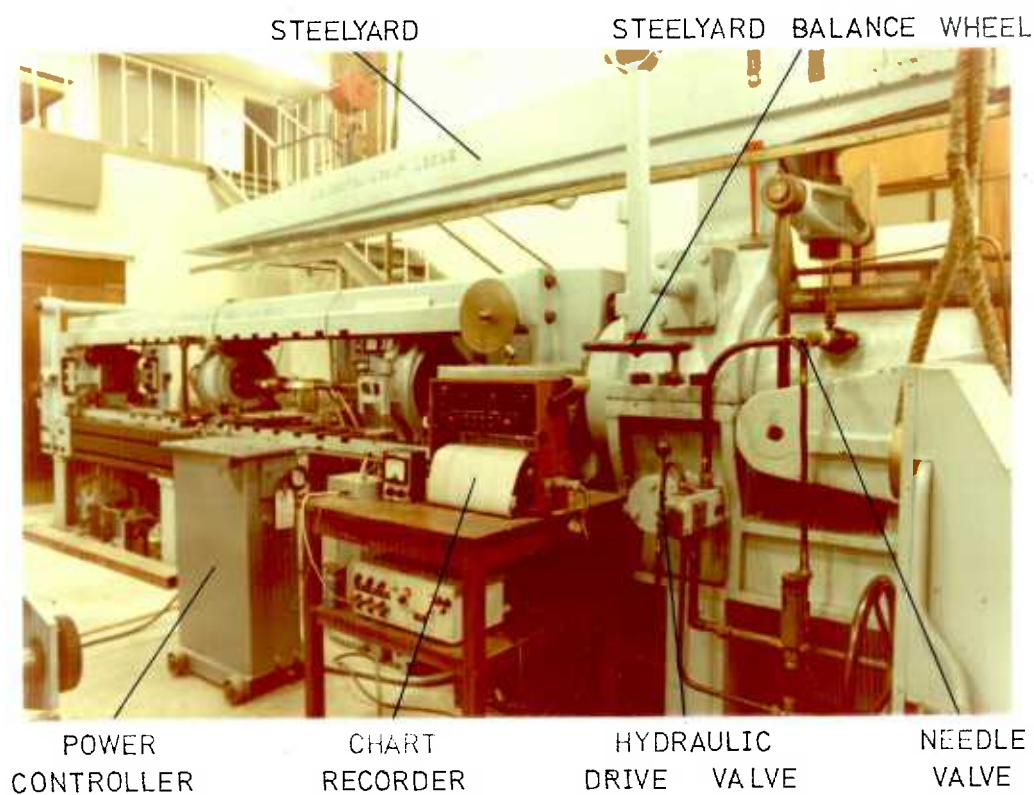
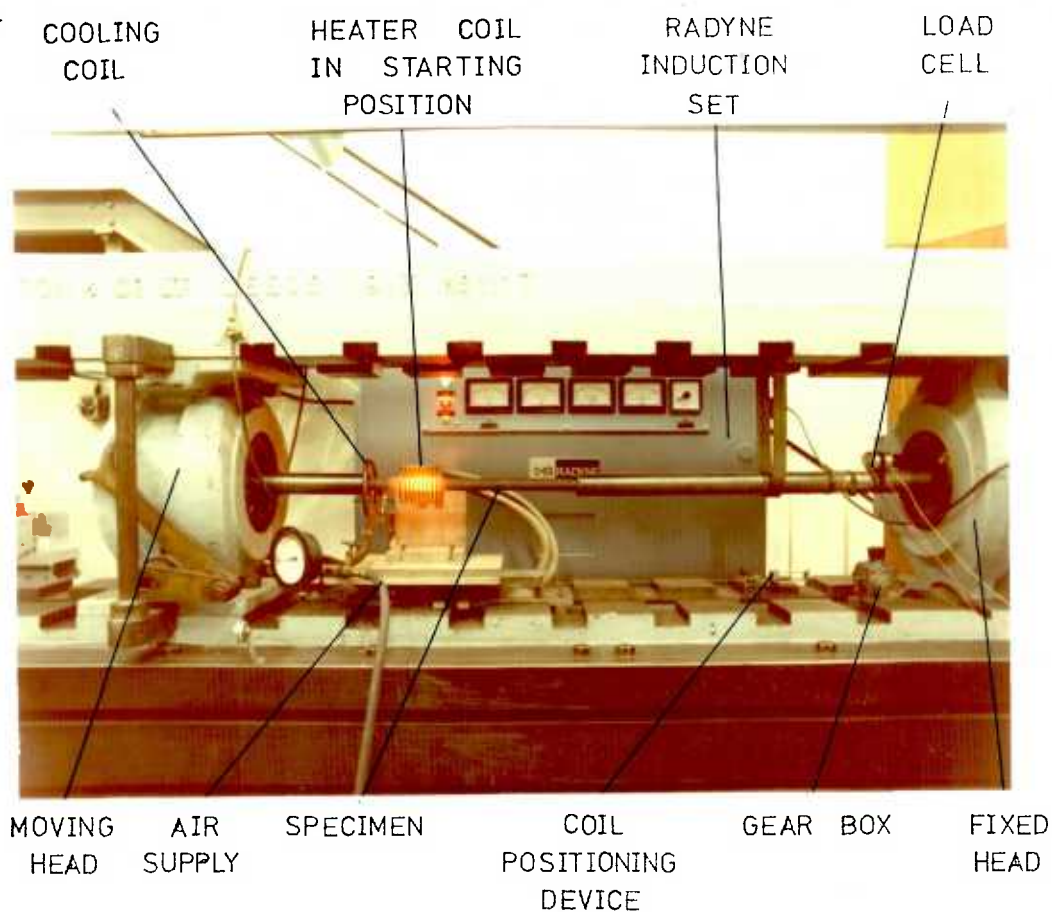


FIGURE 5 GENERAL ARRANGEMENT

This adjustment is unnecessary for superplastic materials like titanium.

(vii) Draw speed was set at the desired value by adjusting the needle valve in the by pass line of the hydraulic drive to the drawing head.

(viii) The air pressure supplying the cooling coil was adjusted.

N.B. Several starting procedures were tried such that adjustments (vi), (vii) and (viii) had to be made several times.

(ix) The successful draws were stopped and the load relieved immediately.

(x) After cooling, the diameters of the deformation zone were measured every 0.127 cm (0.05 in) or 0.254 cm (0.10 in) along the axis. A travelling microscope was used.

2.3

RELATED THEORY

Before proceeding with descriptions of the experimental work and the apparatus it is considered important to take up two theoretical points which are closely related to the experiments. The first deals with the view taken of the deforming zone and the second with a comparison of a hydraulic machine with a screw head type.

In the tests the undeformed bar is quite stationary and the heating coil moves along it in the opposite direction to the drawn material. It is convenient to view this by fixing the frame of reference so that it moves with the coil. To do this it is necessary to add a velocity of $-V_C$ to all parts of the system. This effectively fixes the coil in space so that material is apparently fed into the coil at a velocity V_C and drawn from the coil in the same direction at a velocity $V_C + V_D$ (see Figure 1). A simple relation exists between the velocity ratio, R' , and the percentage reduction in area.

$$\text{Volume of bar moving into coil} = A_I V_C,$$

$$\text{Volume of bar drawn from coil} = A_F (V_C + V_D)$$

$$\text{For no volume change } \frac{A_F}{A_I} = \frac{V_C}{V_C + V_D} = \frac{1}{1 + R'}$$

$$\text{Percentage reduction in area} = 100\left(1 - \frac{1}{1+R'}\right) = \frac{100 R'}{1 + R'}$$

Many of the experiments were carried out with $V_C = V_D$, i.e. $R' = 1$, giving a reduction in area of 50%.

Consider a thin section of the deforming zone under steady drawing conditions. After a time δt the section will be replaced by another section where it is important for equilibrium and stability that the stress, strain, strain rate and temperature should take the same values as those in the original section at the original time. If some perturbation allows the temperature to rise by a small amount in some section of the zone the flow stress will be reduced. The Buckton machine maintains constant load, F , however, as the load is a function of the oil pressure in the cylinder. We therefore have:

$$F = F_d + \delta F$$

where F_d is the load required for deformation and δF the excess load which will cause the moving parts to accelerate:

$$\delta F = \frac{d}{dt} (M V_D)$$

In the Buckton machine the moving head is massive ($\sim 5,000$ kg) keeping $\frac{dV_D}{dt}$ small. Nevertheless the acceleration changes the strain rate in the deforming zone.

Calculations based on observed results show that a change in force of 2 newtons (0.4 lbs) is sufficient to double the speed, V_D , from

2.46 cm/min (0.97 in/min) to 4.92 cm/min (1.94 in/min) in a second. Clearly this is very significant when the material being drawn is particularly strain rate sensitive.

It is found experimentally that the oscillations which occur at the start of a stable draw do allow the draw head velocity to change by a factor of the order of 2. The linking of the draw velocity and coil velocity, where any change in the draw velocity is mirrored in the coil velocity, counteracts these variations, but it does not eradicate them completely.

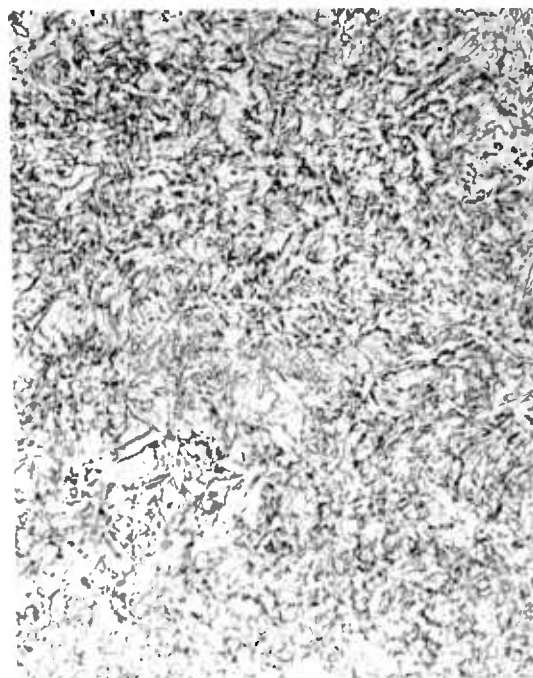
The original concept that the initial oscillations were only a function of the characteristics of the metal in the hot zone is apparently incorrect and the mass of the moving head plays a significant part in stability. The screw thread type of drawing machine does not have the problem discussed above, because it is velocity controlled rather than load controlled and extraneous accelerations do not occur.

2.4 METALLURGICAL CONSIDERATIONS

Owing to the complexities involved with a process in which fifteen variables play an essential part, it has not been possible to investigate the metallurgical considerations as fully as the subject warrants. An adumbration is included in the hope that this important area will shortly be given the close scrutiny it deserves. There is at this stage unhappily, a justification for not taking the metallurgical considerations ex cathedra because of the inherent complexities set out below.

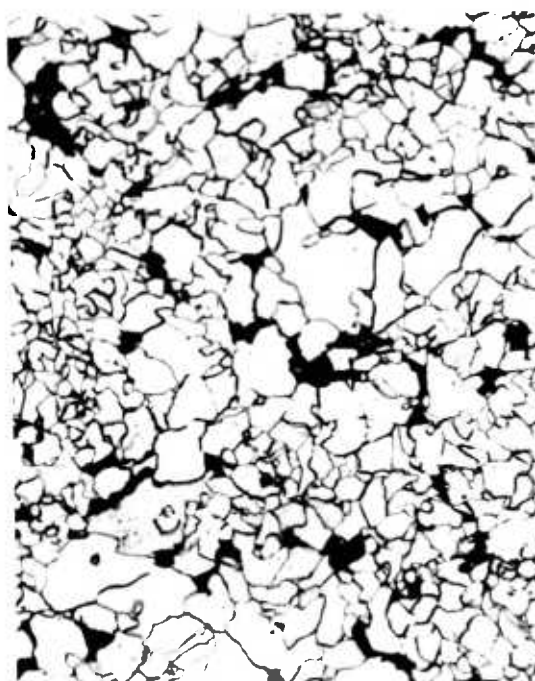
The properties of the drawn material are a function of grain size, temperature and temperature gradients, rate of heat extraction, temperature holding and therefore the speed of draw.

The importance of grain size in superplastic flow was recognised as early as 1934 by Pearson ⁽²⁾ and later by Backofen et alia ⁽⁸⁾. It was with this work in mind that a number of niobium stabilized fine-grain structured mild steel specimens were drawn. Despite repeated efforts varying both the temperature and the drawing speed, no stable draw was achieved. In each case after a promising initial necking phase, a pronounced localised neck formed which was usually followed by fracture. Care must be taken in drawing conclusions at this stage, because it may be possible to obtain a successful draw with this material by changing the temperature gradients and rate of heat extraction. Photographs of the micro-structure appear in Figure 6.



(a)

x500



(b)

x500

FIGURE 6 Micrographs of niobium stabilized fine-grain structured mild steel
(a) water quenched (b) air quenched

An example of the grain growth occurring in the heated zone of an unstabilized mild steel is shown in Figure 8.

The rate of heat extraction is clearly dependent on the speed of draw. Unfortunately the maximum speed of drawing is controlled by the heat input in the work reported here, and cooling techniques have been but thinly explored. It is perhaps not out of place to reiterate the suggestions that rollers be placed at the bottom of the deformation zone. The rollers in controlling the final dimensions could also be used to control the rate of cooling which might prove to be of metallurgical value.

There is evidence to suggest that cavities are not closed up by the die-less drawing process. A mild steel specimen was found with a flaw running along the whole of the length. After drawing it appeared to have reduced in cross-section in proportion with the reduction in area, i.e. the volume of the cavity appeared to remain constant. Since the flaw was not known to be of constant cross-section prior to deformation, the result is inconclusive. However a niobium mild steel bar drilled longitudinally before being drawn, does show that the volume of a large hole remains constant to a first approximation, see Figure 7.

Surface cavities tend to open up, a fact which is aggravated by the radial temperature gradients, which result in the surface of the deformation zone being cooler than the body of the material. This

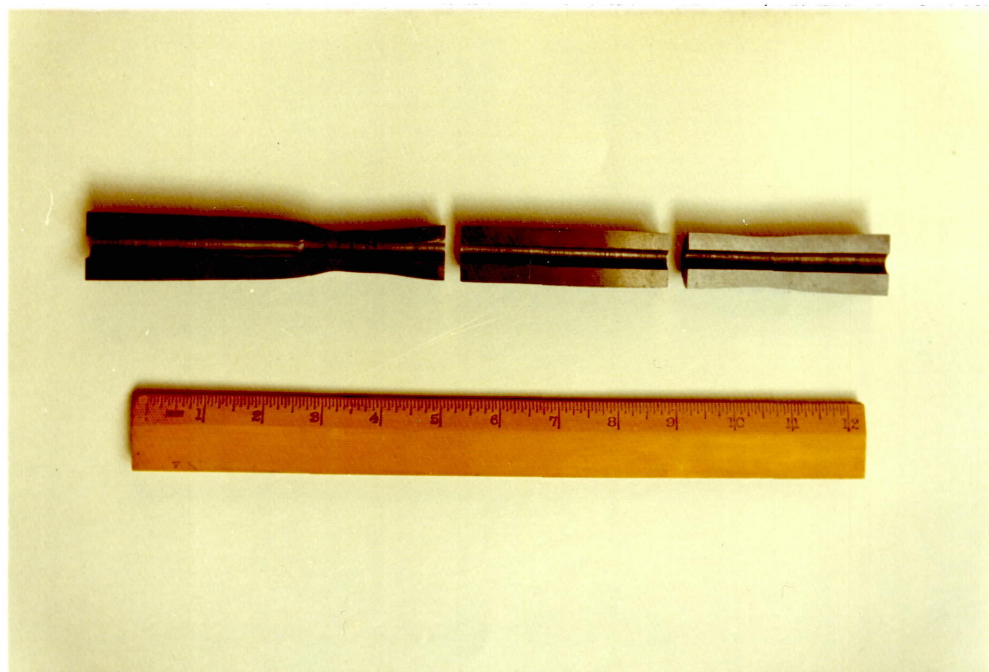


FIGURE 7 LONGITUDINAL SECTION OF NIOBIUM MILD STEEL

can be overcome to some extent by lagging the specimen with fibre glass tape.

Before proceeding to a consideration of the microstructures of a selection of drawn materials, it should be stressed that the final acceptance of a new process, from the metallurgical point of view, must come from the satisfactory performance of a component commercially produced by that new process. Rolls Royce (1971) Limited kindly allowed the author to report ⁽²⁷⁾ that the dieless drawing process has been established as a feasible process for titanium alloy tubes, drawn repeatedly at a 50% reduction in area from 1 in diameter tube to a final 0.015 in diameter. The Company is, in fact, in commercial production of these die-less drawn components.

Figure 8 shows the micrographs of a square mild steel specimen drawn unsuccessfully in an atmosphere of nitrogen. (a) shows the typical structure of ferrite and pearlite in undeformed material where the grain size is of the order of $28\text{ }\mu\text{m}$. A marked increase in grain size is shown in (b) where the percentage reduction in area is 50%. The larger grain size is the result of holding the temperature at $\sim 1050^{\circ}\text{C}$ as the bar is drawn slowly through the heated zone. A percentage reduction in area of approximately 90%, available because of the failure to establish a stable draw, enabled the heavily deformed material to be shown in (c). The photomicrographs in this and the ensuing figures were all taken with the direction of draw out of the page.

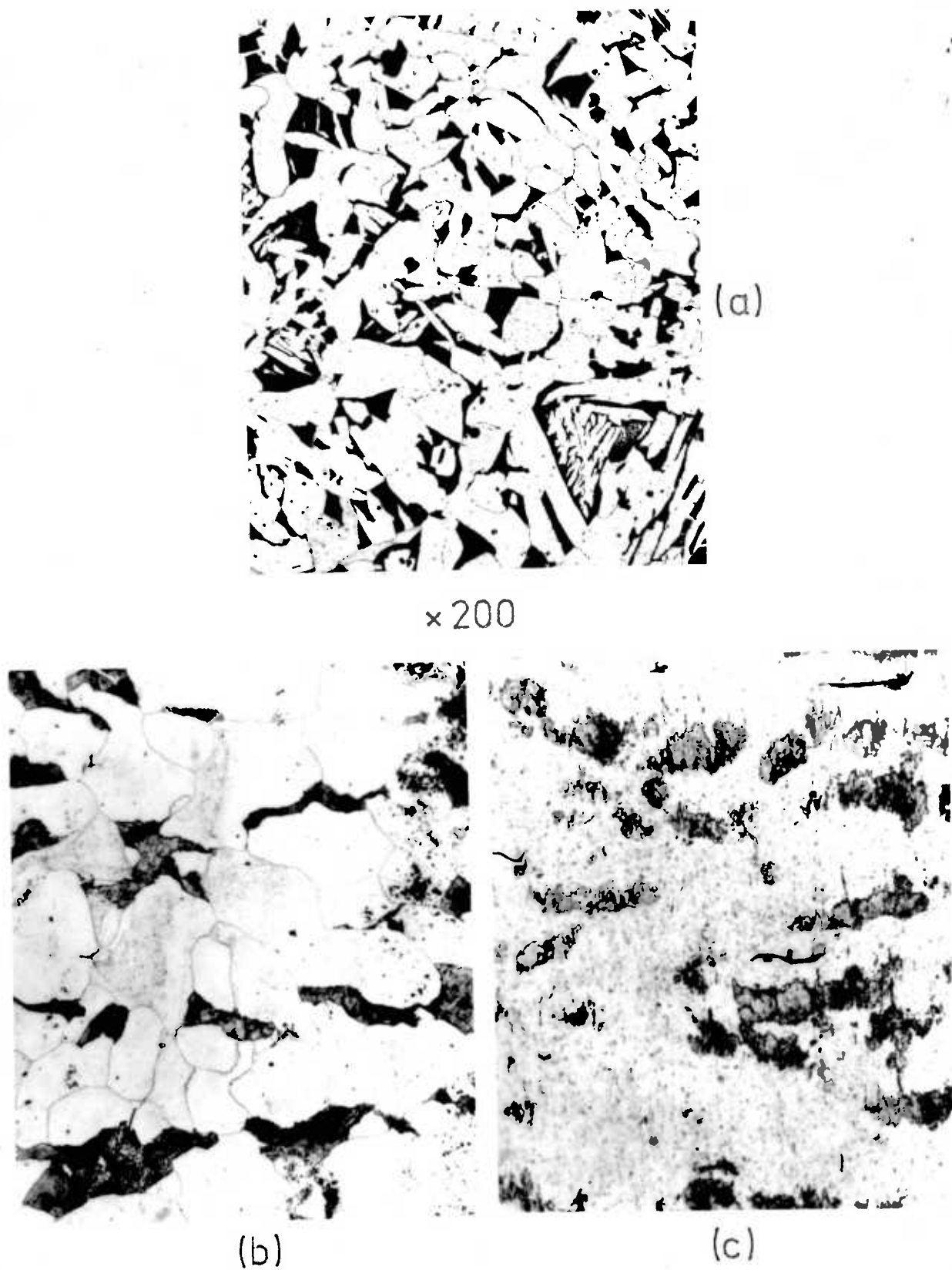
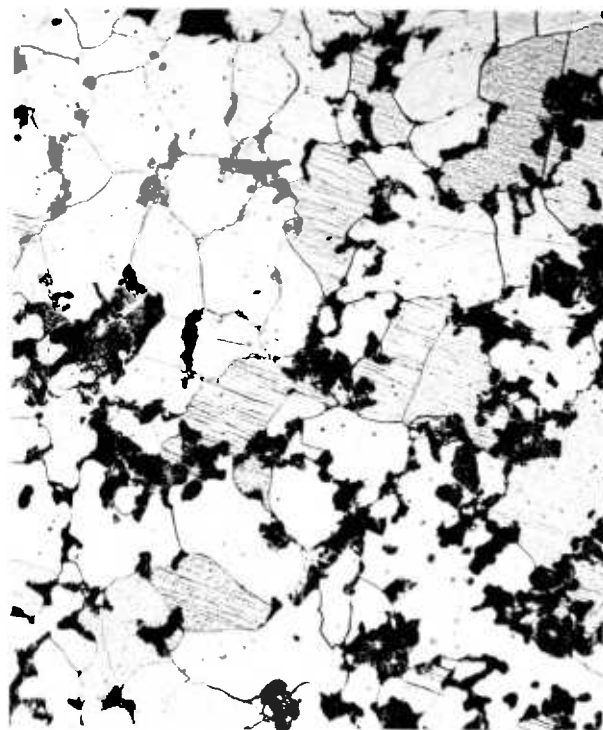


FIGURE 8 Micrographs of mild steel
unsuccessful draw
(a) undeformed
(b) 50% reduction (c) 90% reduction

A completely successful draw in a chromised mild steel square section is shown in Figure 9. There is no evidence of significant grain growth, but there is apparently, after the matrix recrystallization, cementite (Fe_3C) aligning along specific crystallographic planes to form a typical Widmanstätten structure. This results from a more rapid cooling as the temperature falls through the transformation temperature.

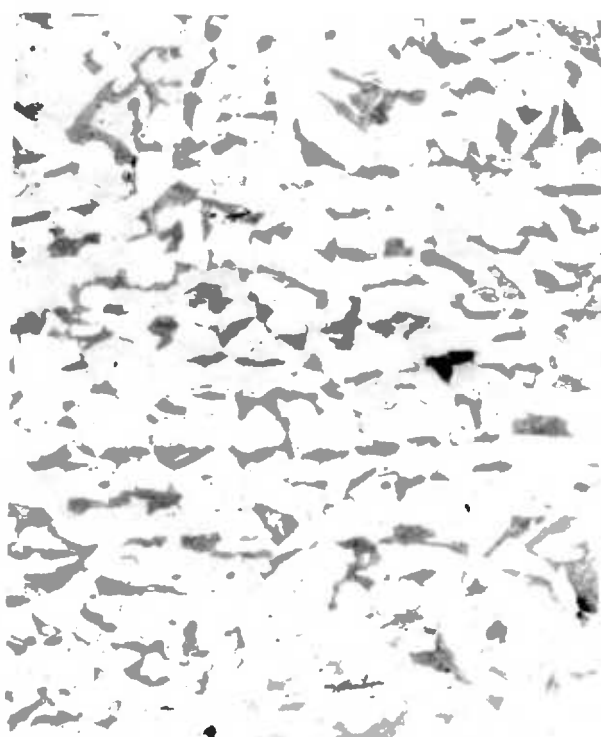
The micrographs of undeformed and deformed (50% reduction in area) rectangular mild steel bars are shown in Figure 10. Although a stable draw was established, the temperature gradients made the corners cooler than the main body and severe cracking and tears resulted. Again the typical ferrite and pearlite structure is apparent in the undeformed bar (a), and the pearlite of the deformed steel (b) tends to show a Widmanstätten structure presumably the result of an increased rate of cooling. It is clear also that with a stable draw, the material is not maintained at a high temperature long enough for any marked grain growth to occur.

The subject of Figure 11 is a 5 cm (2 in) diameter EN 58J stainless steel which gave a stable draw at 50% reduction in area but developed severe surface cracks despite the aluminium reflector set up with the intention of reducing the radial thermal gradients. (a) shows the undeformed material, an austenitic structure with marked twinning. The deformed material in (b) is severely worked and clear cracks easily discerned. Seemingly the mechanical twinning, which can result



(a)

×200



(b)

FIGURE 9 Micrographs of mild steel
successful draw
(a) undeformed
(b) 50% reduction

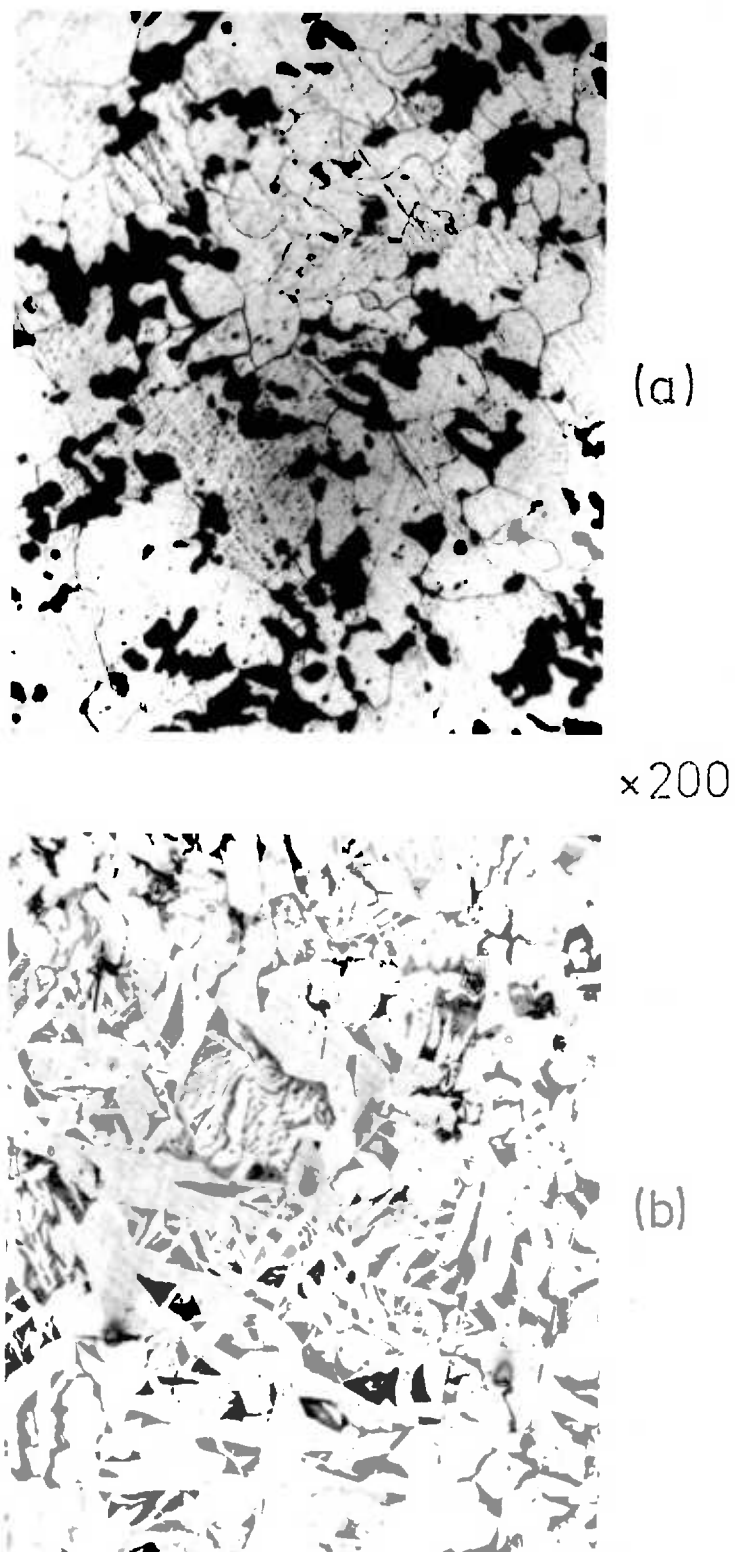


FIGURE 10 Micrographs of mild steel
stable draw (corner cracks)
(a) undeformed
(b) 50% reduction

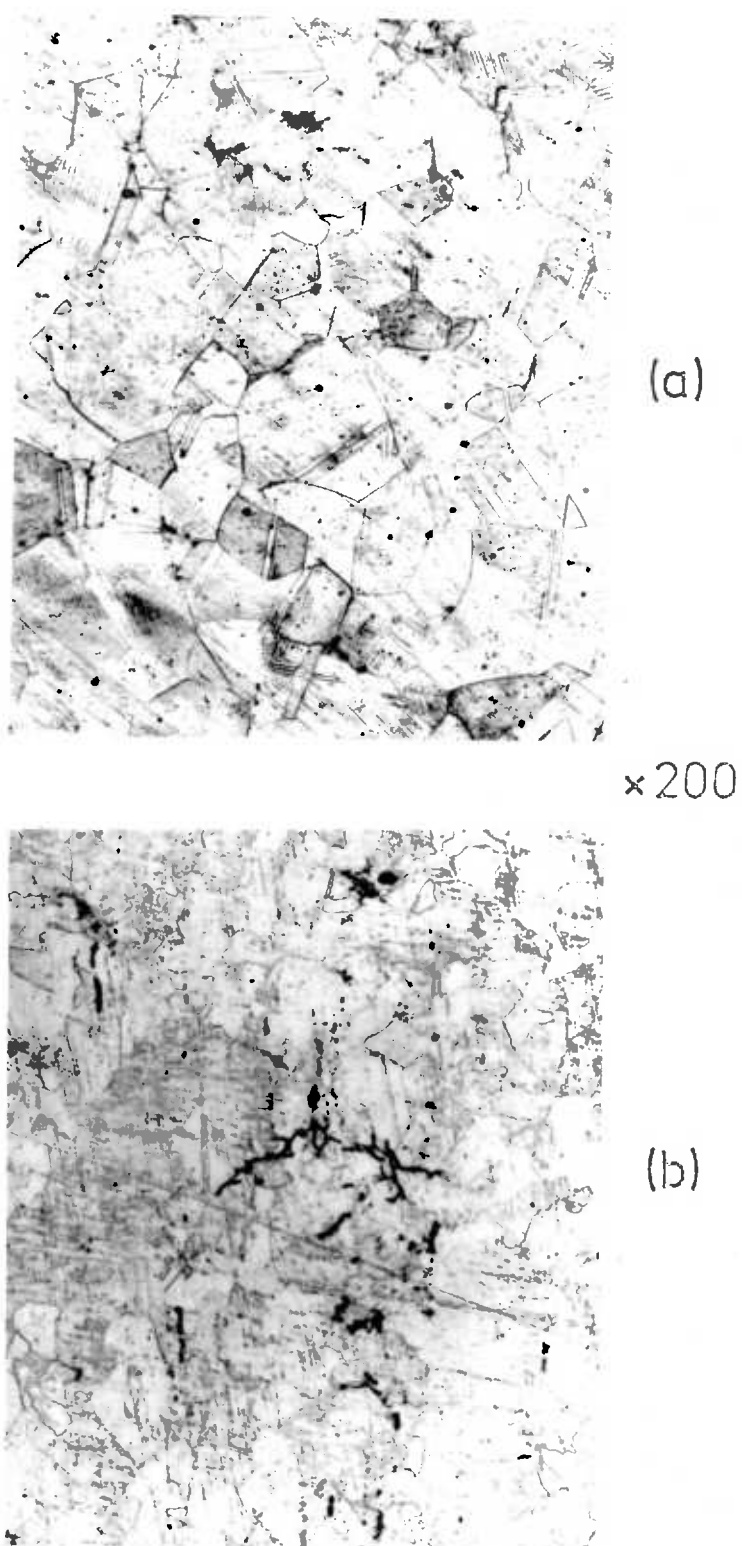


FIGURE 11 Micrographs of stainless steel
stable draw (surface cracks)
(a) undeformed (EN 58J)
(b) 50% reduction

in the material being easily deformed under different conditions, has probably allowed the heavy deformation to create cracks both internally and on the surface. The grain size of the specimen is $\sim 49 \mu\text{m}$.

The wholly austenitic structure of undeformed EN 58J stainless steel is shown in figure 12 (a). The specimen, 3.8 cm (1.5 in) diameter, was drawn very successfully and the surface of the drawn material was very good. Towards the end of the draw the percentage reduction in area was substantially increased to $\sim 80\%$ (The draw at this stage was unstable). A section of this heavily deformed steel was examined, see Figure 12 (b) and shows the σ -phase precipitation which reduces ductility. At this section of the drawn bar, surface cracks can be seen.

Figure 13 shows the undeformed and deformed material of the 2.5 cm (1.0 in) diameter titanium alloy bar at (a) and (b) respectively. The undeformed titanium has a uniform fine dispersion of the α (white) and β (black) phases. The deformed material does show a loss of grain structure with increased β precipitation in a general α matrix. The percentage reduction in area of the deformed material was $\sim 80\%$. To all appearances the surface of the drawn bar suggests a sound product, but at such a percentage reduction in area the limit of stable drawing has almost been reached.

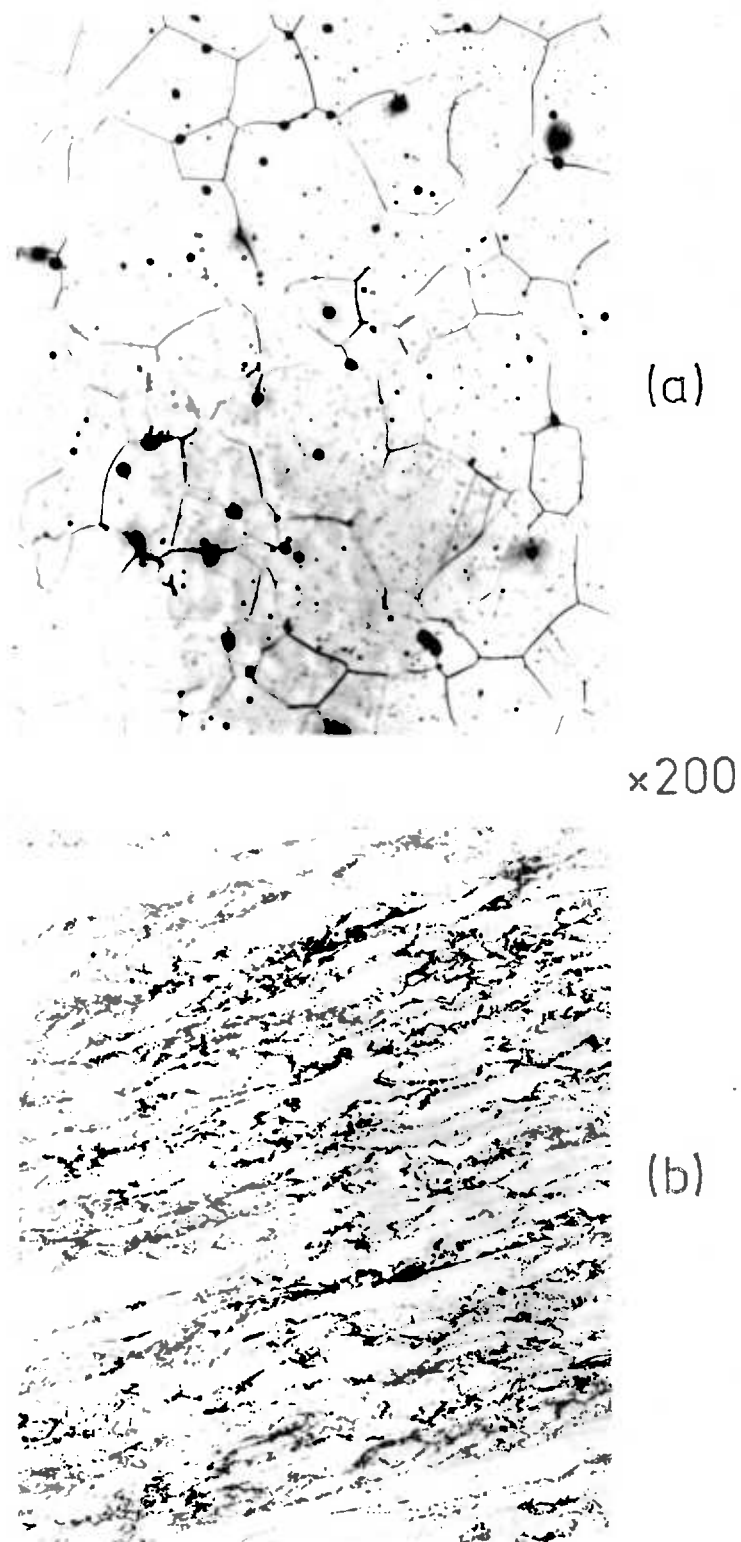


FIGURE 12 Micrographs of stainless steel
successful draw (EN 58J)
(a) undeformed
(b) 80% reduction

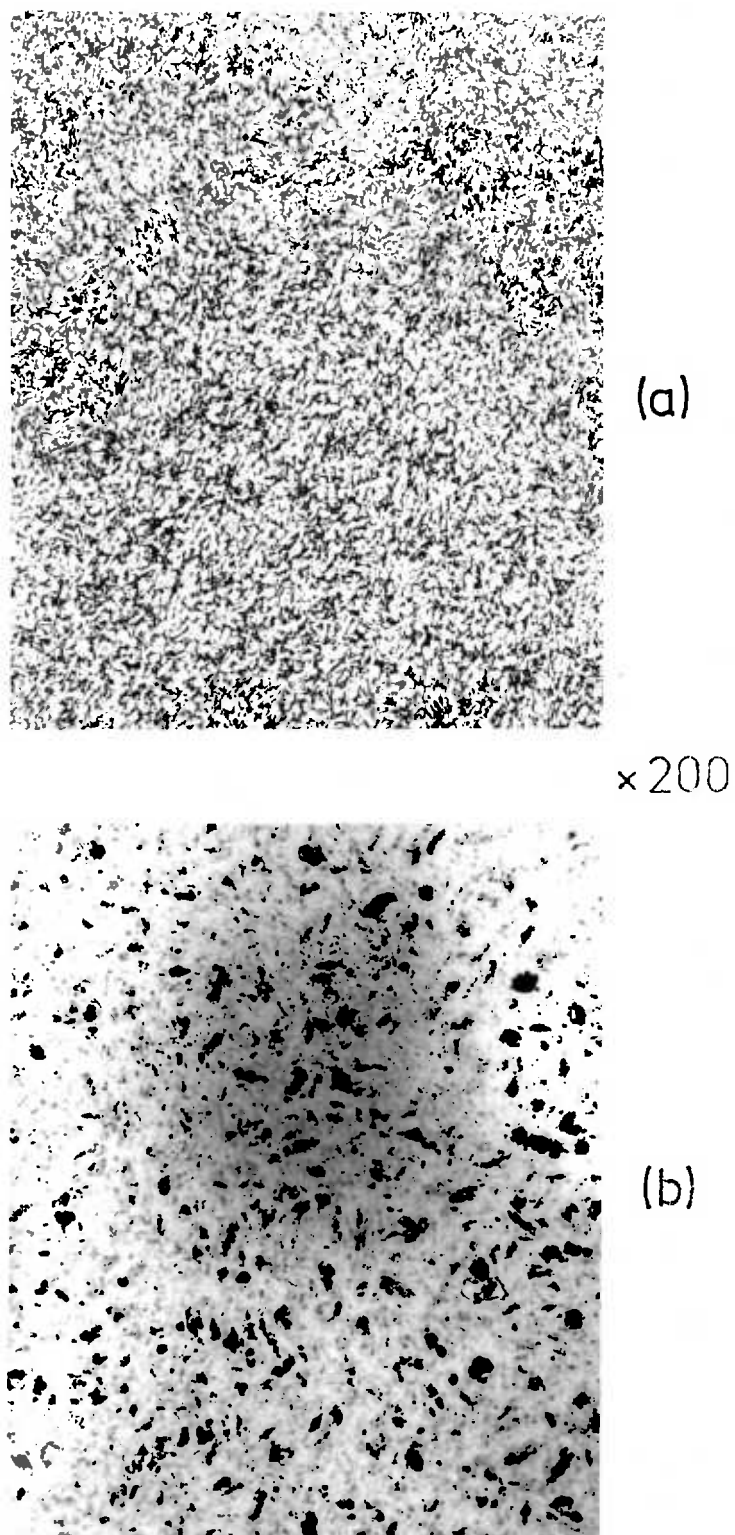


FIGURE 13 Micrographs of titanium alloy
successful draw (Ti-6Al-4V)
(a) undeformed
(b) 80% reduction

2.5

MATERIALS

The materials tested in the experimental programme were as follows:

- (i) Mild steel supplied by B.I.S.R.A.

7.6 cm (3 in) diameter

5.0 cm (2 in) diameter

7.6 cm x 3.8 cm (3 in x 1.5 in)

3.8 cm x 3.8 cm (1.5 in x 1.5 in)

- (ii) Niobium mild steel supplied by B.I.S.R.A.

0.07% C, 1.4% Mn, 0.03% Nb, Remainder Fe.

2.5 cm (1 in) diameter

Casts 2807 and 2808 were soaked at 1250°C for 1 hour then rolled under controlled conditions to a finishing temperature of 825°C and then air cooled⁽²⁴⁾.

Cast 2809 was finish rolled at 950°C and quenched directly into water⁽²⁴⁾.

- (iii) Stainless steel supplied by B.I.S.R.A.

EN 58J: 0.12% C, 0.20% Si, 2.00% Mn, 8.0/12.0% Ni,

17.0/20.0% Cr, 2.50/3.50% Mo, 0.04% S, 0.045% P,

Remainder Fe.

5.0 cm (2 in) diameter

3.8 cm (1.5 in) diameter

3.2 cm x 3.2 cm (1.25 x 1.25 in)

(iv) Chrome steel

0.19% C, 0.25% Si, 0.51% Mn, 3.20% Cr, 0.0055% S, 0.012% P,
Remainder Fe.

5.7 cm (2.25 in) diameter

(v) Chrome steel supplied by Kayser Ellison

KEA 162: 1.0% C, 0.65% Mn, 5.25% Cr, 1.0% Mo, 0.2% V,
Remainder Fe.

5.0 cm (2 in) diameter

3.8 cm (1.5 in) diameter

5.0 cm x 3.8 cm (2 in x 1.5 in)

(vi) Titanium alloy supplied by Rolls Royce (1971) Ltd.

6% Al, 4% V, Remainder Ti.

2.5 cm (1 in) diameter

2.6

EXPERIMENTAL APPARATUS2.6.1 Buckton 100 ton Testing Machine

The choice of equipment in research projects depends upon availability and adaptability. The large Buckton machine possesses both of these but after the first tests it was decided that very little adaptation was needed.

The machine is massively built on a horizontal bed 9 metres (28 feet) long, having a capacity for loads up to 1000 kN (100 tons). The general arrangement of the testing machine is shown in the photograph, Figure 14. The stroke is over 60 cm (2 feet) and originally the load was measured using the 6 m (20 feet) long steelyard having a 5 kN (10 cwt) balance.

Operation is hydraulic, driving a 36 cm (14 in) diameter piston which is attached to a sliding member the mass of which is roughly estimated to be 5,000 kg (weighing 50 kN or 5 tons). The moving head is keyed to the sliding member. Oil flow to the cylinder is adjusted by a control valve, and a needle valve placed in a by-pass line gives a fine control to the flow. Thus the load and speed of the draw are accurately controlled.

The fixed head is attached to two long 8.9 cm ($3\frac{1}{2}$ in) diameter bars. Connecting the fixed head with the stationary end of the specimen is a 5 cm (2 in) diameter bar. This is necessary to locate the specimen

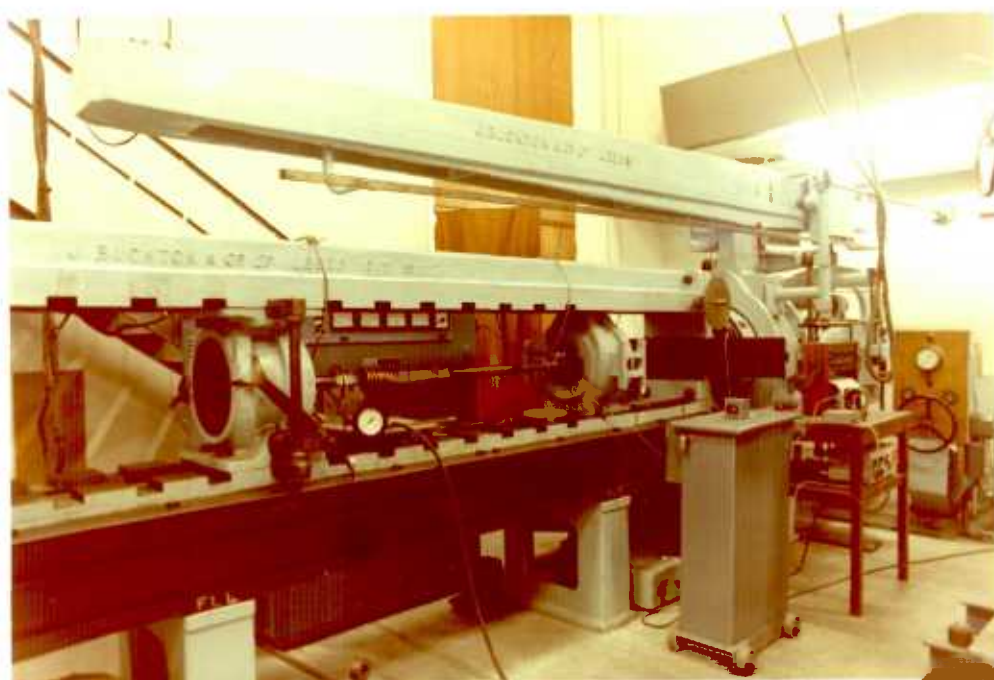


FIGURE 14 GENERAL ARRANGEMENT OF THE TESTING MACHINE

position within the range of the flexible leads from the Radyne set which supplies the induction heating coil and it also keeps the load cell at room temperature, by providing considerable length between the cell and the specimen.

2.6.2 Radyne Induction Set

The set, Model RD. 150, obtained from Radyne Limited, is a medium frequency valve generator operating between 2 and 5 kHz with a maximum continuously rated output of 15 kW. The unit takes a three phase supply up to 60 amp and a cooling water supply of 14-18 litres/min (3-4 galls/min). Its mass is 570 kg (weighing 1270 lb). A second floor standing unit, output power regulator Model RDOA 50 enables the power to be adjusted manually.

Heavy duty M.F. flexible leads, water cooled, are used to connect the M.F. output to the work coil. The two 120 cm (4 ft) leads are always held parallel to keep losses to a minimum.

Six working coils were used with eight to twelve turns ranging from 5 cm (2 in) to 10 cm (4 in) internal diameter. The first coil to be used was supplied by Radyne Limited and consists of twelve turns of square section copper tubing. The other coils were made from 0.95 cm (3/8 in) or 0.80 cm (5/16 in) diameter copper tubing. Loading the coils was never a problem but occasionally the loading was poor for the smaller specimens in the larger coils below the Curie temperature.

Table 1 shows the calculated time for the deformation zone to be heated by 1000°C as well as the time actually measured during the experiments.

The calculated time is worked out assuming that 100% of the energy is available for heating. Approximate efficiencies are also given. The values show that the efficiency falls as the specimen size reduces, but it is important to note that the table is very approximate because the values given are a function of coil coupling and material properties.

A cooling curve was plotted for a rectangular specimen from which it was found that heat losses from all sources, radiation, convection and conduction, amounted to about 4.2 kW or 28% of 15 kW. With non-circular stock there are always surface temperature gradients which together with radial gradients always cause difficulties. This point is taken up again in the section on heat transfer.

After some initial troubles with a faulty cut-out switch and some arcing in the generator cabinet, the heating equipment proved to be trouble free and a most effective heat source.

	STOCK SIZE				
	7.6 × 3.8 cm 3 × 1.5 in	7.6 cm 3 in dia	5 cm 2 in dia	3.8 × 3.8 cm 1.5 × 1.5 in	2.5 cm 1 in dia
TIME REQUIRED TO RAISE 1000°C 100% EFFICIENCY NO LOSSES (S)	80	130	60	40	15
ACTUAL TIME REQUIRED TO RAISE 1000°C (S)	360	600	200-300	265	150-200
EFFICIENCY	22%	22%	30 - 20%	15%	10-7.5%

The above table is approximate because the properties of each specimen change as the temperature rises, and the times are a function of coil coupling.

TABLE 1

2.6.3 Coil Carriage

The coil carriage is an arrangement to enable the heating coil and air cooling coil to move in the opposite direction to the draw head.

A horizontal plate is clamped to the fixed 8.9 cm ($3\frac{1}{2}$ in) diameter bars on which the fixed head of the Buckton is mounted. The plate is fitted with four cross pieces each end of which carries a ball bearing. The carriage runs smoothly on these bearings and is retained by two guides bearing on the underneath surfaces.

The sliding member of the Buckton is arranged to pull a wire wound round the pulley of a variable speed gear box. This is manufactured by Heynau Great Britain (H Drive Type 2W) and gives a velocity ratio, R , with a range of 0.33 to 3.0 with equal input and output pulley diameters. A wire wound round the output pulley in the opposite direction to the winding on the input is attached to the coil carriage. Changing the input and output pulley diameter ratio extends the range of the velocity ratio R .

The coil is mounted on the carriage using insulating board. The board also carries a cooling coil positioned about 7.5 cm (3 in) from the trailing end of the induction coil. Twelve hole 0.10 cm (0.040 in) diameter drilled on the inside of a single turn of copper tubing 10 cm (4 in) diameter allow air to blow on to the neck of the specimen.

2.6.4 Setting up the Draw

Figure 15 shows how the specimen is supported in the rig. The fixed head is extended using a long 5 cm (2 in) diameter bar to locate the specimen within the range of the Radyne flexible leads. A shorter 5 cm (2 in) diameter bar is attached to the moving head enabling the bar to be drawn to be secured at each end with 1 in B.S.W. studs.

Most of the specimens drawn were 46 cm (18 in) long, but different lengths can be accommodated by keying the moving head at different positions on the Buckton sliding member. The separation of the keying positions, seen in Figure 13, is 20 cm (8 in).

The starting position of the coil is indicated in Figure 15, and after the necking phase, the coil position can be adjusted by a screw thread wire shortening device in the wire connecting the draw head to the gear box.

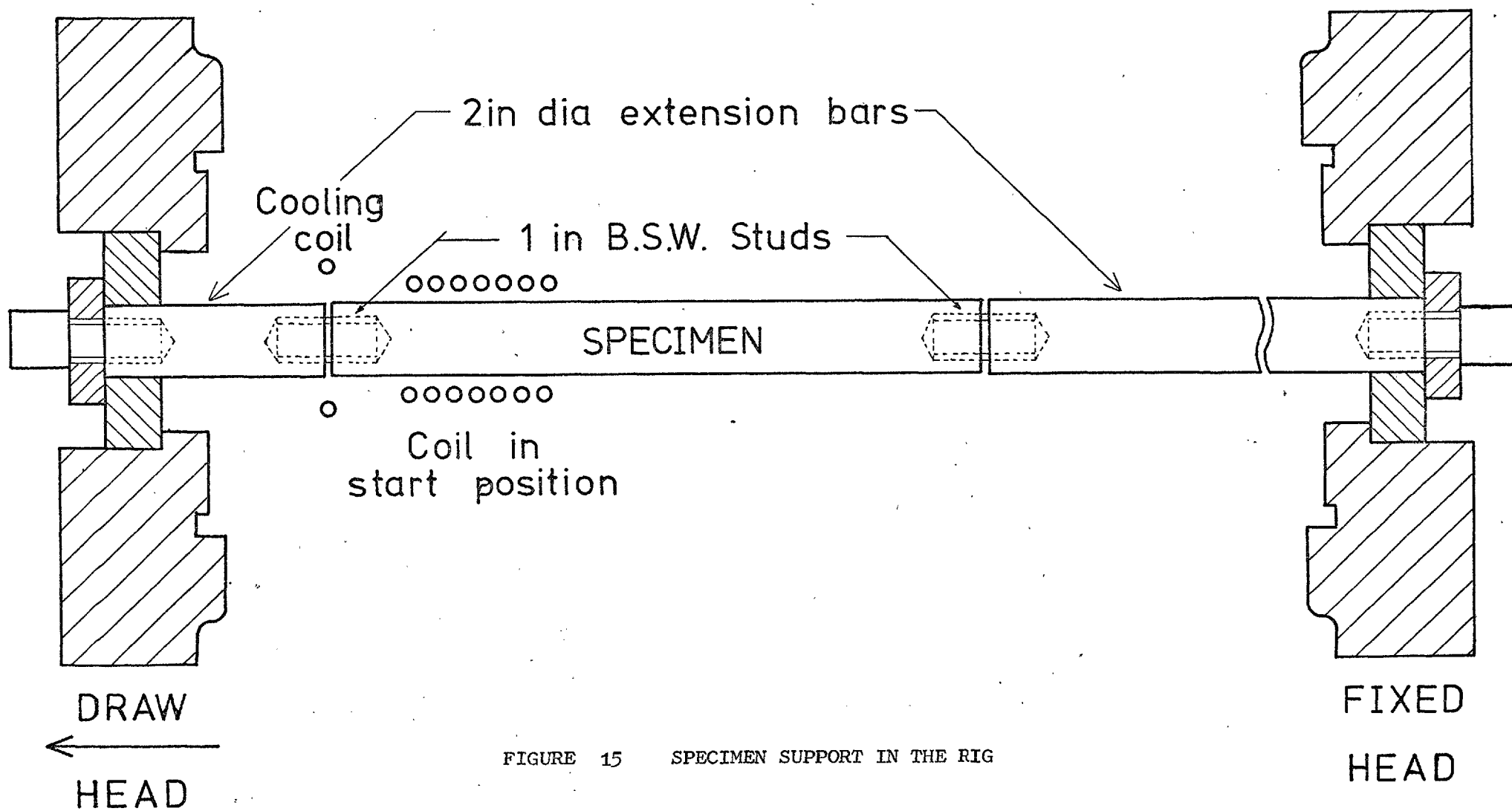


FIGURE 15 SPECIMEN SUPPORT IN THE RIG

2.6.5 Instrumentation

2.6.5.1. Load Measurement

A facility for measuring load was incorporated in the design of the Buckton testing machine, namely a steelyard of vast proportions. This was quite unsuitable for the present work, firstly because of the time taken to adjust the position of the 5 kN (10 cwt) slider and secondly because of the fluctuations in load resulting from the reaction of the steelyard on the oil pressure in the drive cylinder. The facility was however useful for calibration.

The load cell used consists of a Wheatstone Bridge arrangement, the wiring diagram of which appears in Figure 16. Four strain gauges arranged symmetrically around the 5 cm (2 in) extension bar are connected in series-parallel as shown, to form the active component of 100Ω . The dummy gauge as well as the two other gauges, which make up the bridge network, were fixed transversely on the bar within a centimetre or two of the active groups. Nevertheless, with the load cell situated at the remote end of the extension bar, i.e. 70 cm (27 in) from the heated work piece, the cell always operated at room temperature. A variable wire-wound 5Ω resistor provided the means for balancing the bridge.

Power is provided by a dry battery giving 4.5v d.c. The output is fed by a potential divider to an amplifier and micro-voltmeter and thence to a pen recorder.

FOUR STRAIN GAUGES
SYMMETRICALLY DISPOSED
ON EXTENSION BAR

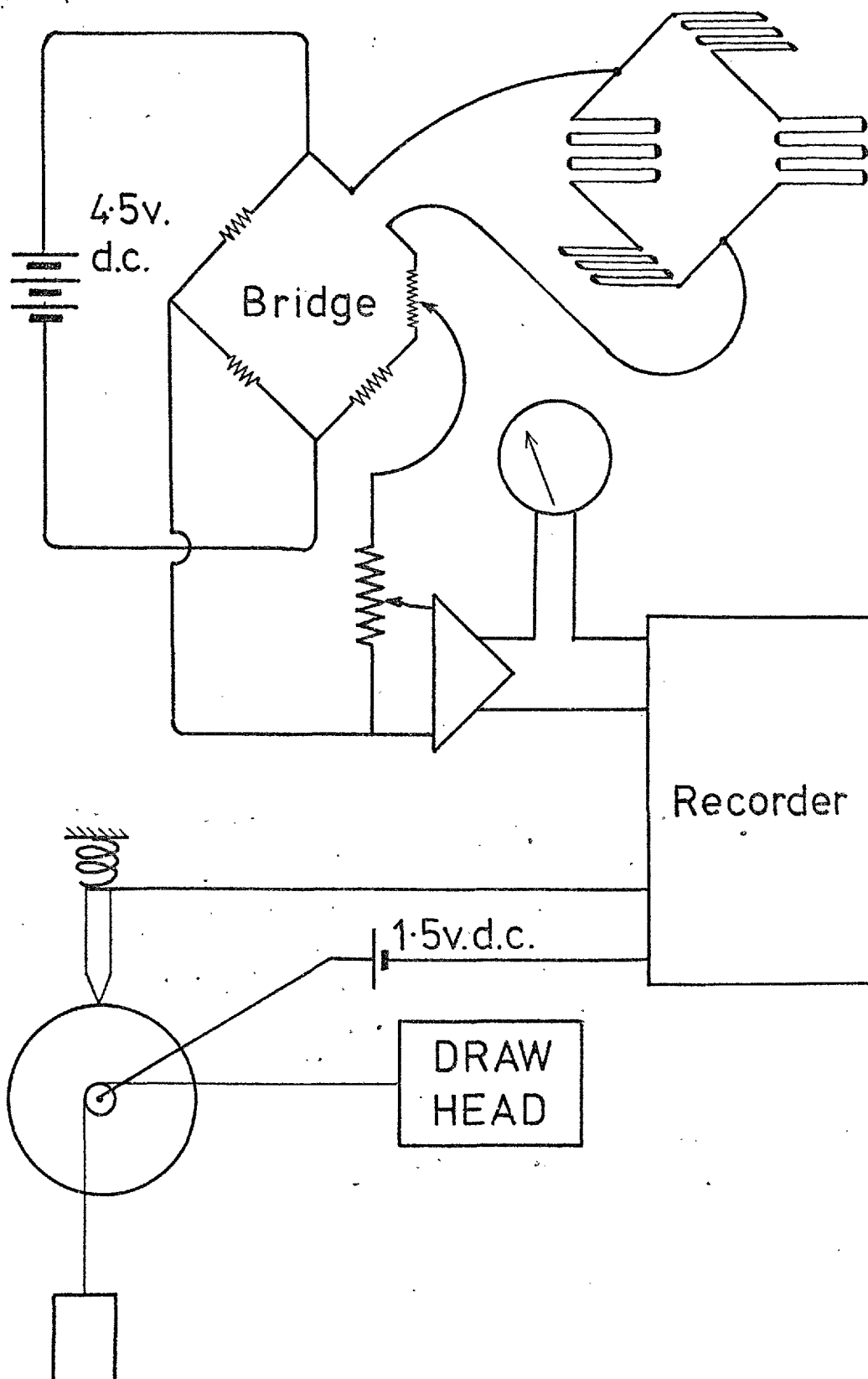


FIGURE 16 WIRING DIAGRAM OF THE INSTRUMENTATION

Before every test the cell was calibrated making selection of the range of load to be measured quite simple. The selection of the range is made by changing the range on the microvoltmeter.

2.6.5.2. Calibration

For every test the load cell was calibrated using the steelyard. The sliding weight is set at a value close to the expected steady load required for the pull. Tension is then established in the specimen which just balances this value. A potentiometer on the microvoltmeter is then adjusted for the appropriate range to give a convenient reading on the pen recorder chart. Typically a 5 cm amplitude for 10 kN (1 ton) is used for titanium and 5 cm amplitude for 50 kN (5 tons) in the case of 2 in diameter steel specimens.

2.6.5.3. Displacement Measurement

Two displacement transducers were incorporated in the rig. One is a simple visual device which is essential for observation at slow speeds and a second designed to give a displacement trace on the chart recorder, thus enabling velocities to be read.

The first consists of a series of hexagonal nuts (with each surface ground) bolted together and fixed on the flat surface of the sliding member of the tensile testing machine, see the photograph in Figure 17. The clock gauge is attached to the fixed head and is in contact with the upper surfaces of the nuts which always present a 30° slope to the

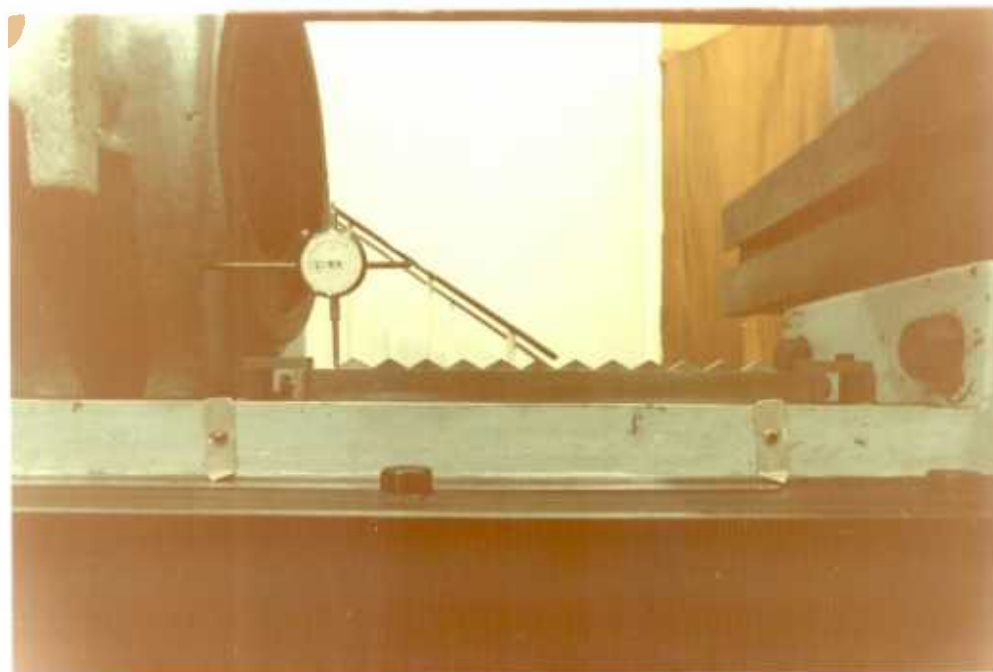


FIGURE 17 VISUAL DISPLACEMENT TRANSDUCER

clock contact. As the sliding member begins to move the clock records a rise or a fall, and velocities as slow as 0.05 cm/min (0.020 in/min) are readily observed.

The second displacement and velocity transducer is shown in the photograph of Figure 18. The disc has radial slots around the periphery at every 0.25 cm (0.100 in) and was turned so as to be truly round after filling the slots with araldite. The disc is mounted, again on the sliding member, using a shaft whose diameter is one tenth of that of the disc. A wire is wound round the shaft and attached to the fixed cylinder of the Buckton. The other end carries a weight to give steady rotation.

A spring loaded brush on the periphery makes and breaks contact as the sliding member moves thereby rotating the transducer disc, and by connecting through a 1.5v cell a square wave is recorded on the chart. The wiring diagram appears in Figure 16. With a 10:1 ratio of the disc to shaft diameter, the wavelength of the recorded square wave is 0.025 cm (0.010 in). Accurate positioning is always recorded, therefore, and by counting 20 to 50 waves on the chart an accurate velocity of the draw head is available at any stage during the draw. A simple calculation gives the velocity of the coil.

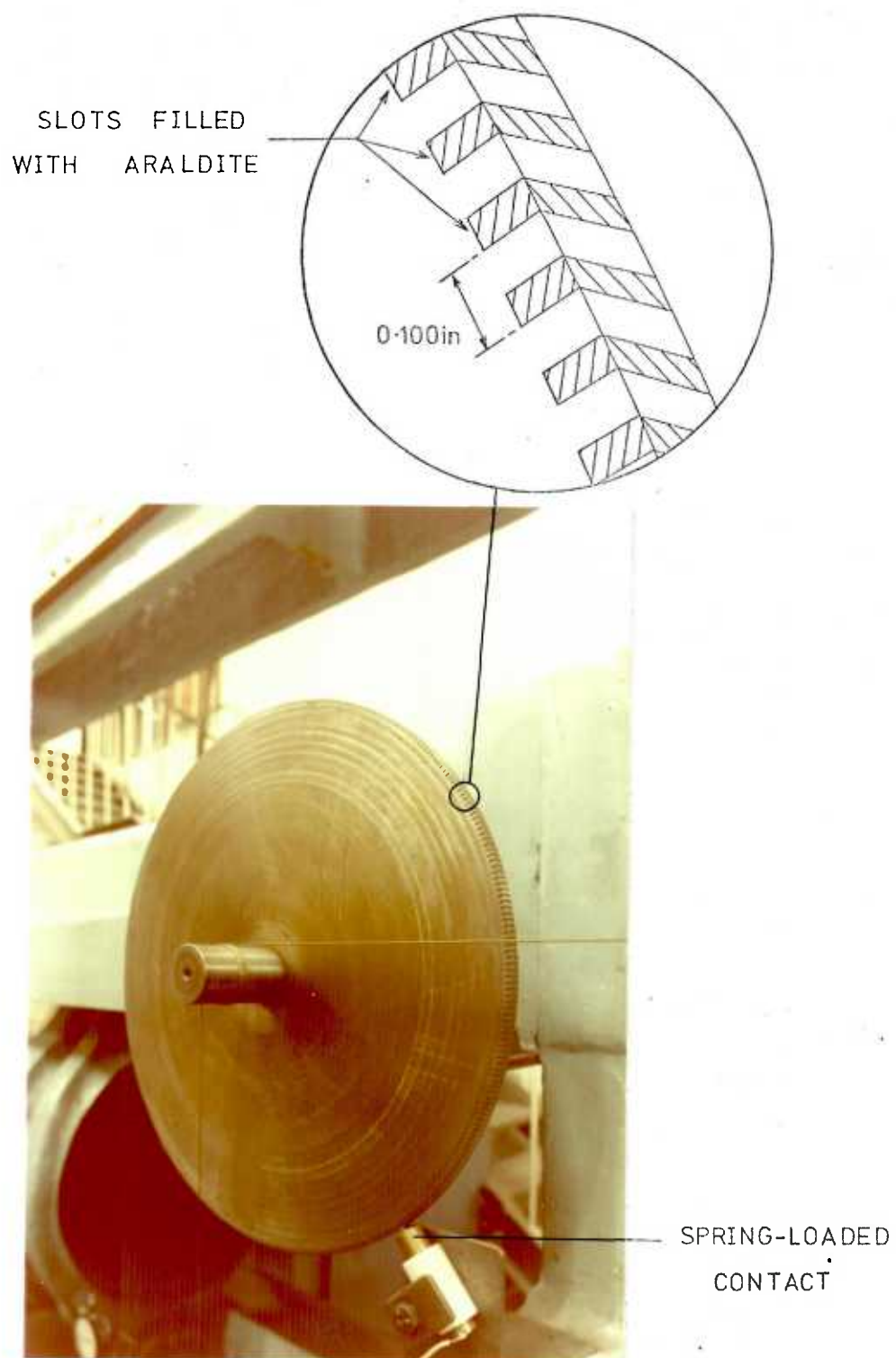


FIGURE 18 SQUARE WAVE DISPLACEMENT TRANSDUCER

2.6.5.4. Temperature Measurement

An optical pyrometer was used to measure temperature for all of the tests. Maximum temperatures are easily taken in this way.

Observation of the steady state over several minutes, with several pyrometer readings of the maximum temperature recorded, indicate an accuracy of better than $\pm 5^{\circ}\text{C}$.

2.7

EXPERIMENTS

2.7.1 Initial Draws

The first pull was a hot tensile test on mild steel of rectangular cross-section. Sections are shown in the photograph. Figure 19, Although this was not a draw, it has been included because it shows an interesting variation in the cross-section and also suggests that substantial reductions might be achieved.

The first set of drawn bars were all of mild steel with the coil speed equal to the drawn head speed to give a 50% reduction.

All the specimens were painted with an Acheson Colloids Dag material, Dag 1946, to stop scaling. The dag contains a glass powder and a bonding compound such that on heating, the glass powder melts and covers the metal with a protective coating of viscous liquid glass. Two other dag materials from Acheson Colloids, Dag 1871D and Dag 1981M, were tried but were ineffective from about 1000°C upwards where scaling



FIGURE 19 HOT TENSILE TEST CROSS-SECTIONS. MILD STEEL

is a severe problem.

Scale forms rapidly over 1000°C and does not allow heat to escape from the lower end of the deformation zone, by creating a kind of hot box (Figure 20).

Tests tried at the lower end of the hotworking range between 700°C and 1000°C resulted in the bar breaking at the narrow end of the deforming zone.

Nitrogen was used in some tests for cooling but this did not reduce the rate of scale formation to any great extent.

More than a dozen bars were drawn varying the drawing speed from 0.18 cm/min (0.07 in/min) to 2.0 cm/min (0.8 in/min) and the maximum temperature ranging from 800°C to 1100°C with no satisfactory result.

The first successful draw was carried out on a 5.7 cm ($2\frac{1}{4}$ in) diameter bar, thought at the time to be of mild steel. Later analysis showed it to be a 3.20% chrome steel. The draw head speed was 0.18 cm/min (0.07 in/min) and the temperature 1050°C . The R ratio was 0.428 giving a percentage reduction of 30%. Variations in the load were from 49 kN (4.9 tons) to 61 kN (6.1 tons). Two coats of the 1871D dag material had been applied and scaling was negligible, the chromic oxide providing an excellent keying material for the glass. It was later found that all the steels containing nickel and chromium were very resistant to scale formation without any protective layer.

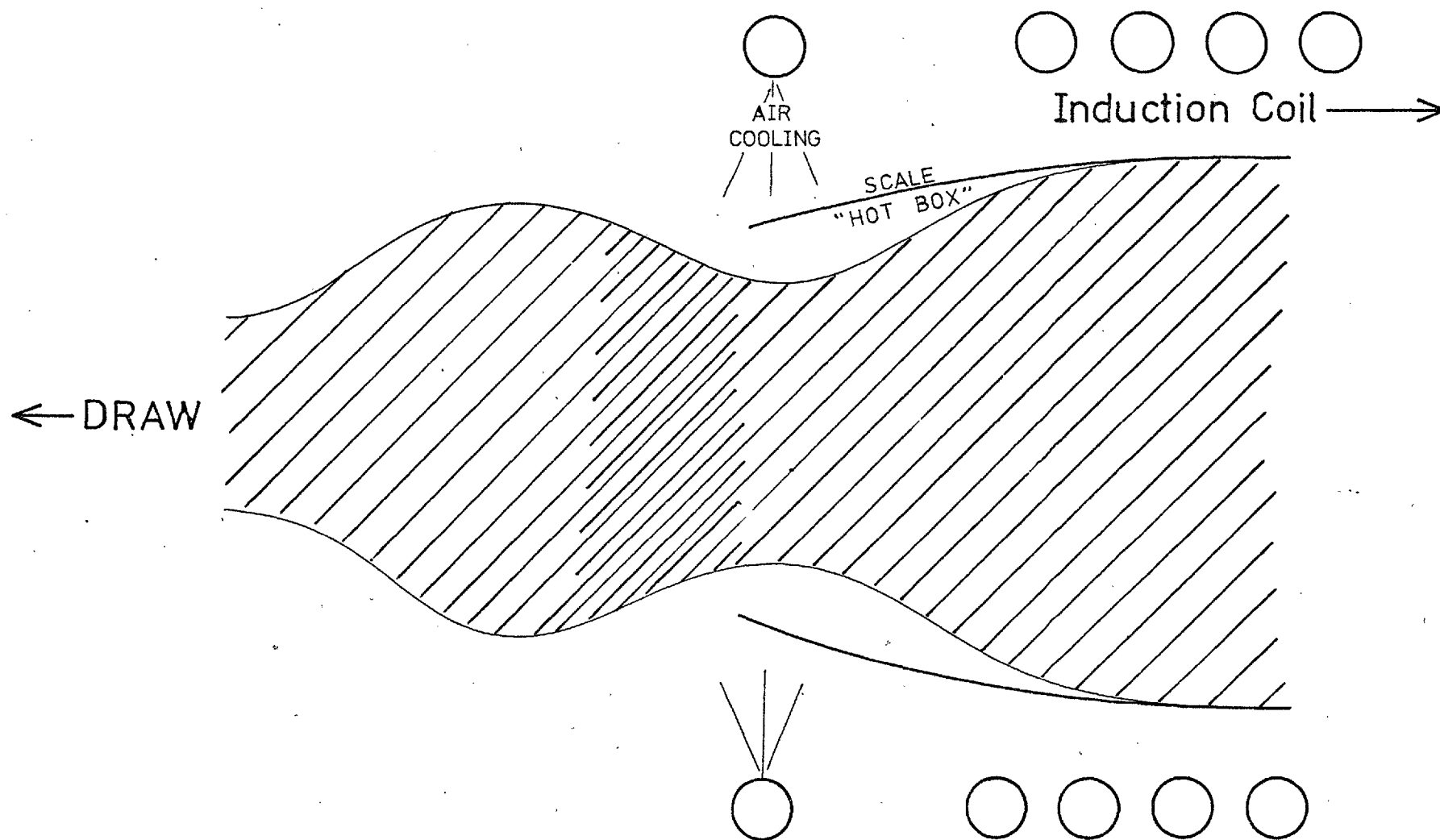


FIGURE 20 "HOT BOX" CREATED BY MILD STEEL DRAW

A second bar of the 5.7 cm ($2\frac{1}{4}$ in) diameter chrome steel was pulled successfully. Half way through the test the heat input was increased by about 25%. The draw continued successfully with the temperature rising from 1020°C to 1100°C, the speed from 0.2 cm/min (0.08 in/min) to 0.4 cm/min (0.16 in/min) with the load falling from about 65 kN (6.5 tons) to 45 kN (4.5 tons). Figure 21 indicates how the parameters varied.

The damped oscillation in the diameter of the drawn bar can also be seen in Figure 21. This is a clear indication of the stability of the deformation zone.

2.7.2 Techniques for Drawing Steel

The experimental problem for steels has two main aspects. The first concerns starting the process, which can prove difficult, whilst the second relates to the investigation of the drawing process itself, where the heat extraction is the major consideration.

Starting from a constant cross-section a neck is first produced which is inherently an unstable process. After the initial necking the position of the coil has to be adjusted to place the deformation zone centrally in the coil. The heat in the other half of the heated neck must be allowed to dissipate so that a temperature gradient is set up in the bar where the reduction is greatest. This must be done slowly to enable a long neck to form and approximate to the stable drawing configuration.

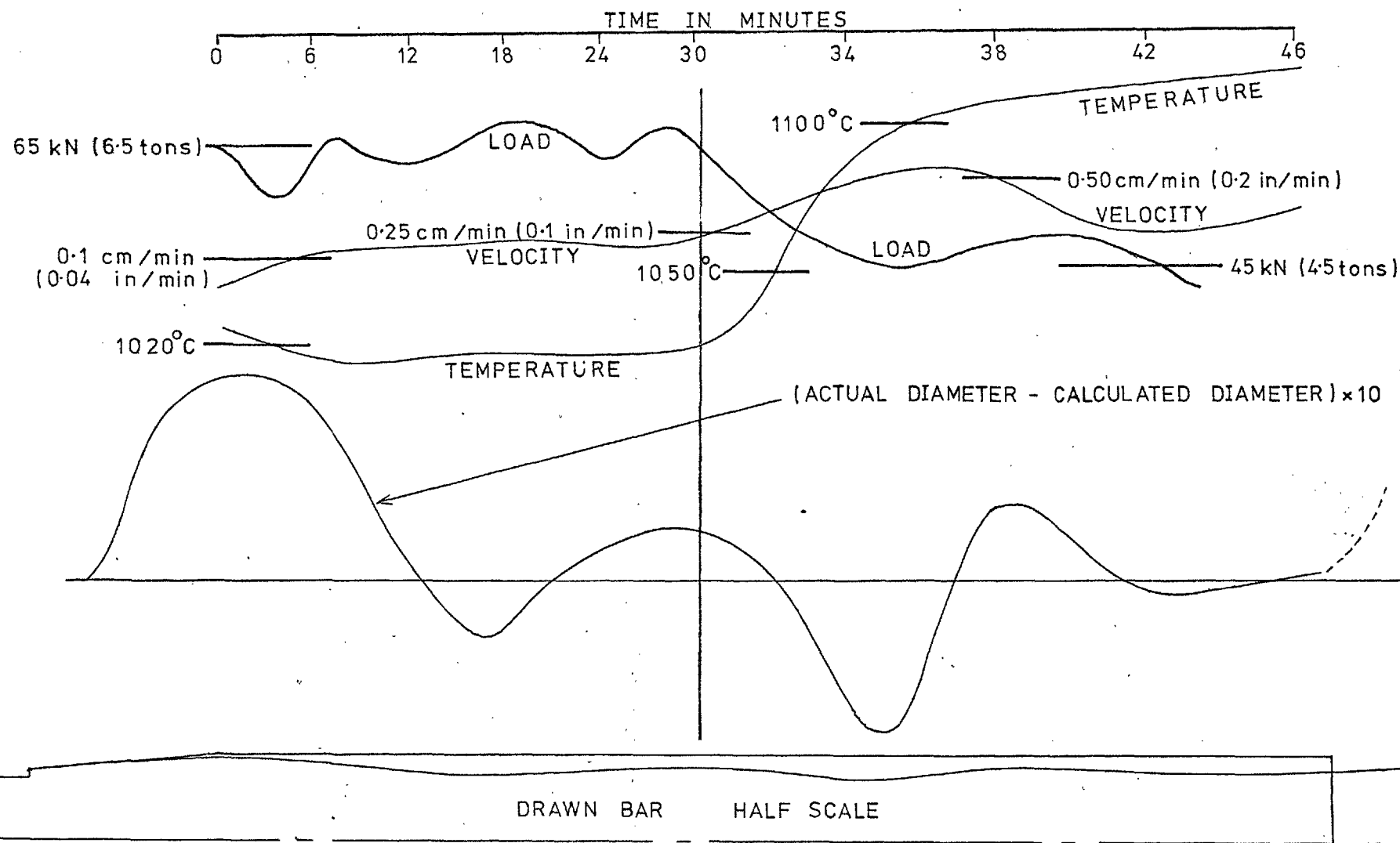


FIGURE 21 DIAGRAM SHOWING PARAMETER VARIATION. CHROME STEEL

Several bars were turned with an initial taper to assist the start but care must be taken not to overheat the taper bottom. Here the discontinuity is sufficient to lead to an early fracture.

Adjustment of the coil position was always found necessary whether the bar was tapered or not as the zone position in the coil is a function of the coil and moving head speeds and also of the amount of cooling. At slow speeds 0.2 cm/min (0.08 in/min) the bar begins to draw immediately the steel enters the coil whereas at about 2.5 cm/min (1.0 in/min) the deformation zone moves towards the trailing end of the coil.

With faster draws an attempt is made to gain speed rather quickly after the first elongation of the necked zone, because increases in the speed of the moving head always result in a reduced cross-section. If this cross-section is reduced below the value which follows from the ratio of the drawing speed to the coil speed, R_1 , an oscillation is set up. It is extremely difficult to start a steel draw without getting a varying diameter oscillating about the theoretical value. The oscillation is damped and disappears in about 2 or 3 cycles.

The drawing process itself is quite another problem. The stability of the draw is quite remarkable. Increases of heat input, load and speed of the moving head, although they lead to the oscillation (fortunately damped) of the drawn bar diameter, do not basically affect the draw. Breakdown of the process and fracture have been extremely rare after a stable draw has been established.

The major problem with drawing steel is heat extraction. This is closely related to scaling which, though no problem itself, leads to considerable variations in the rate of heat lost from the drawn bar.

Nitrogen cooling, rather than air, has already been referred to. This was followed by some experiments where the whole bar was contained in a pyrex tube. The experiments worked well and effectively stopped scaling, but the arrangement made the starting of the process (in particular the accurate coil adjustment) somewhat more difficult, and all the specimens broke during the necking stage. At one stage it was considered that a completely inert atmosphere should be contained within the press. This was dropped when easier techniques were found to be effective.

Owing to the scaling difficulties with mild steel, much of the experimental work was concentrated on EN 58, an austenitic 18-8 stainless steel. The material draws well but in rather a tight temperature band, i.e. $1020^{\circ} \pm 15^{\circ}\text{C}$. The steel draws outside this temperature range, but surface cracking develops. A useful feature of EN 58 is its sensitivity to variations in the cooling air, making far easier control of unwanted necks.

2.7.3 Techniques for Drawing Titanium

Whereas steel presents a double problem, that of starting as well as the draw itself, titanium has no starting problem and draws of up to 84% reduction in area are quite straightforward. The reason for this is simply expressed. If development of the initial neck is carried out slowly the material in the neck, subjected to increased strain rate, becomes strain rate hardened thus making an elongated neck easy to achieve. Then during the draw, the material moving towards the bottom end of the deformation zone is cooling down and slowing down, conditions which in titanium result in an ever-increasing strain rate sensitivity index, m . Thus the yield stress increases.

Reference to Figure 4 shows that the m value for titanium at 950°C increases from about 0.2 at a strain rate of 10^{-2} to over 0.8 at a strain rate of 10^{-4} .

2.7.4 Surface Protection

Several references have already been made to the problem of scaling, which causes difficulties in removing heat from the deformed bar.

Fortunately the two materials most closely examined, stainless steel and titanium, are readily heated to the upper band of the hot working range with minimal oxidation. Mild steel can easily be drawn, however, and the experiments involving surface protection are here summarised.

Three different glass dag materials were tried and are effective up to approximately 1000°C . Thicker glass coatings were attempted using dag and glass fibre wrapped around the specimen. It was always found that the glass film gave little protection over 1000°C due either to porosity or cracks.

Nitrogen in place of air used as the cooling gas does not reduce scaling. With air cooling the burning of the surface layer is sufficient to raise the temperature appreciably, resulting in melting during one test.

Nitrogen contained in a pyrex tube is effective, and chromising also works well. (A number of mild steel bars were subjected to a chromium salt atmosphere of about $1000^{\circ} - 1100^{\circ}\text{C}$ for 24 hours. A thin skin of chromium steel, 0.0025 cm (0.001 in) covers the whole bar. The technique was completely successful and the steel could be drawn up to 1100°C with or without the use of dag). The process of chromising, however, causes grain growth, due to a long high temperature soak.

Although the best means of over-coming the problem for mild steel is containing the whole experiment in an inert atmosphere, this is not a solution that recommends itself outside the laboratory.

2.7.5 Temperature Distribution

Temperature measurements were carried out for all the tests using an optical pyrometer. Accurate maximum surface temperatures were therefore

available but temperatures at each end of the hot zone could not be taken accurately owing to the substantial temperature gradients. Some approximate temperatures were observed and the results are shown in Figure 22.

As the heat generated within the specimen there is inevitably a radial gradient, i.e. the temperature rises as the depth increases. This was not investigated experimentally because in the steady state the problem is simply solved theoretically using the conduction equation. It was however noticed in surface cracks that the temperature inside was higher than that of the surface. This was corroborated by Proops⁽²⁸⁾ of Rolls Royce (1971) Limited who carried out some practical tests on inductively heated bars. He also found a cooler core which should be apparent, by inspection of the colour of the cross section, immediately after fracture (an all too common experience in the beginning). This cooler core was never observed in these experiments.

A series of tests was carried out using a polished aluminium tube approximately 30 cm (12 in) diameter and 60 cm (24 in) long. The tube was fixed to the coil carriage with the heated zone and coil centrally disposed. The object of the series was to reduce radiation losses thereby reducing the radial temperature gradient, at the surface, and in turn reduce the surface cooling. There was no evidence that this technique had any effect on the surface cracks. A possible explanation of this lies in the water cooled heating coil. The hot zone is contained completely within the coil, and radiation from the zone must be largely absorbed by the cold surface of the coil.

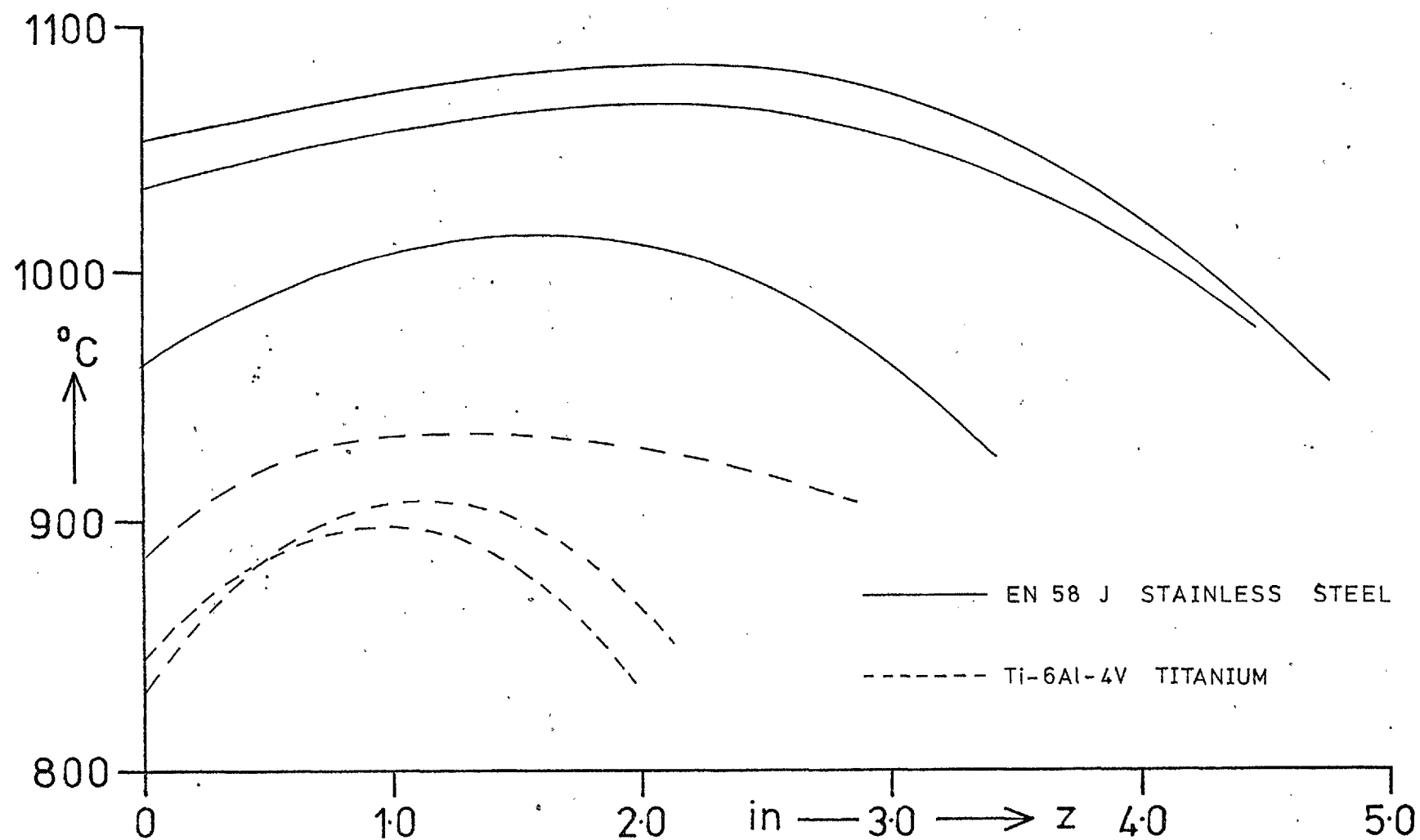


FIGURE 22 TEMPERATURE PROFILES

Surface cracking is particularly evident in stainless steel. The surface temperature gradient and cracks may be reduced by applying a glass dag and double wrapping the steel with glass fibre tape. There is nevertheless a narrow band of temperature near 1020°C where the stainless steel surface is good at a 50% reduction when the surface of the material is left unprotected.

Holes were drilled diametrically through several stainless steel specimens at intervals along the length. These were then filled with stainless steel pins 0.16 cm (0.062 in) diameter welded at the ends and filed to present a smooth surface. After the draw the deformation zone containing the pins was cut from the bar and slit longitudinally, see Figure 23. The flow of the material resulting from the temperature distribution shows that plane sections remain plane for the first half of the reducing cross-section.

The method is not wholly satisfactory as the structural change caused by the weld results in surface tearing and the flow in the narrow part of the deformation zone is not clear.

With temperature distribution a function of material, maximum temperature, coil diameter and length, drawing speed, and frequency there is scope here for more experimental work.

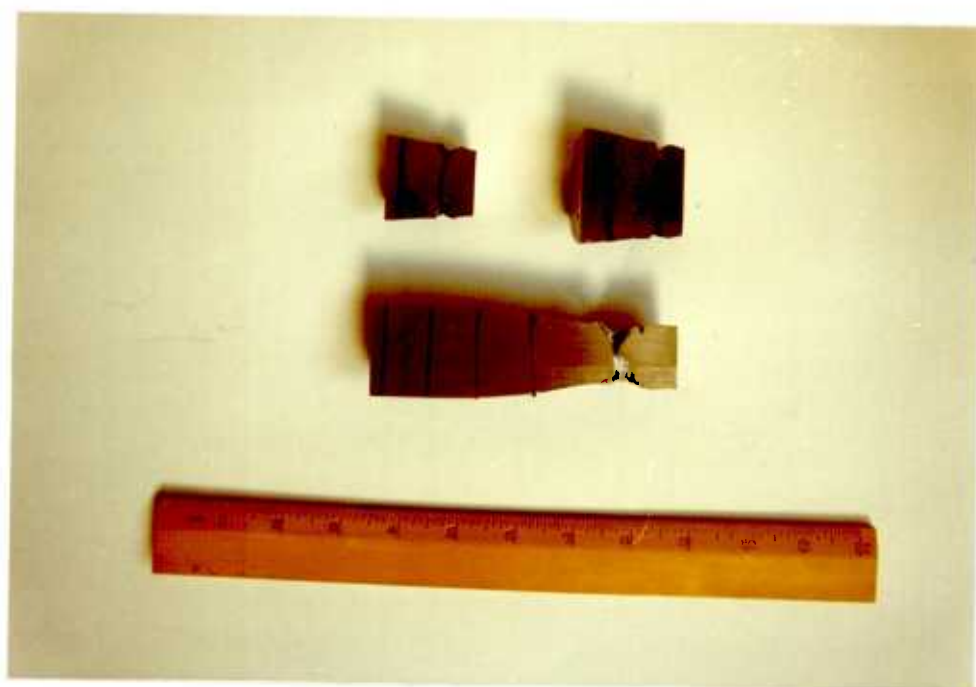


FIGURE 23 LONGITUDINAL SECTIONS OF DEFORMATION ZONES
FITTED WITH PINS. STAINLESS STEEL

2.7.6 Heat Extraction

Induction heating is an ideal method of heating the deformation zone. It is found also that at the slow speeds of draw which are associated with the phenomenon of superplasticity, heat extraction is straightforward: Natural radiation and convection with conduction along the bar occasionally supplemented by some forced convection from the cooling coil is quite adequate to set up an effective temperature gradient.

For many of the tests the cooling coil was not used, and in those runs where cooling was used the pressure of air inside the cooling coil varied from less than 1 p.s.i., except in a few of the fastest runs where pressures up to 50 p.s.i. were necessary.

Slight increases in the cooling air were sufficient to stop the necking resulting from an increase in the drawing speed. A feed back control system controlling the cooling from, say, a diameter sensor would improve the experimentation. The time to devise such a system debarred its use.

In the latter stages of the experiments, runs were carried out at the fastest rate made possible by the power of the induction set.

Unfortunately the diameter cannot be reduced much below 2.5 cm (1 in) as the efficiency and coupling becomes poor. The cooling coil was able to cope with the heat extraction without any difficulty with drawing speeds of 7.5 cm/min (3 in/min) for both titanium and steel at 67% reduction in area.

It is the author's strong opinion that speeds higher, perhaps by a factor of 10, would be possible on the assumption that all the necessary heat could be extracted at the end of the deformation zone. Water cooling has been used by Johnson⁽⁴⁾ and a most interesting series of experiments could be tried using a 150 kVA induction set. The order of magnitude of the drawing speed might then be 75 cm/min (30 in/min).

2.7.7 Variable Cross-sections

A number of tests were carried out using a variable R. The speed ratio gear box was replaced by a pulley and profiled board. The moving head was attached to the pulley and rotated the profile at a constant velocity. The coil carriage was attached to a wire round the profile which at the beginning has a radius of twice that of the pulley. The radius gradually reduces to half that of the pulley. The shape, R' values and expected percentage reductions are shown in Figure 24. An example of the tapered specimens obtained by this technique is shown in the photograph Figure 30.

In some experiments the coil speed was changed instantaneously. A satisfactory result was obtained when the coil speed was increased. Reducing the bar diameter by a decrease of the coil speed could not be carried out effectively with the gear box, owing to its inherent design.

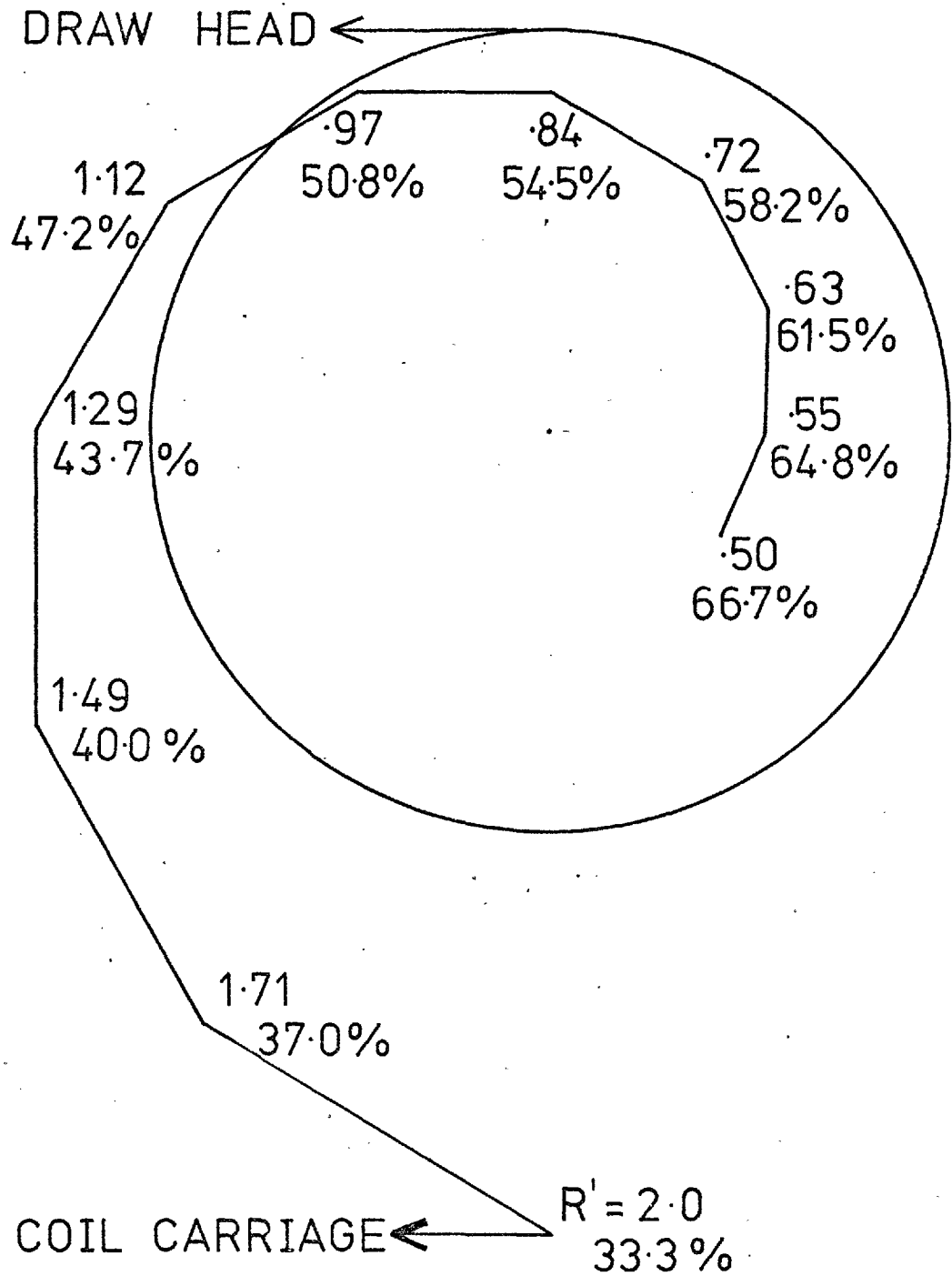


FIGURE 24 PROFILE FOR TAPERING

Drawing and upsetting at stages along the bar is a possibility and may find industrial application. No experiments were attempted in this investigation.

2.7.8 Drawing Speeds

When the heating equipment was being investigated at the start of the programme, frequency was considered to be the important factor in its selection, power output being largely neglected and the highest output chosen purely on economic grounds.

The Radyne induction set with a power output of 15 kW raises the mass 5.0 cm (2 in) diameter stainless steel 2.5 cm (1 in) long through 1000°C in 16 seconds (neglecting all losses). Hence the maximum possible speed for the 5 cm (2 in) steel is 9.8 cm/min (3.85 in/min) and if losses are considered the maximum realistic speed is of the order 2 - 3 cm/min (~ 1 in/min). Reducing the specimen diameter unfortunately also reduces the efficiency of the heating system and it was found experimentally that the maximum realistic speed for drawing 2.5 cm (1 in) diameter stainless steel is 7.6 cm/min (3 in/min). This limitation, that the maximum speeds obtained are controlled by the heat input rather than the material properties in the deformation zone, is the reason for the already stated opinion that higher drawing speeds are possible.

Whilst examining variations in speeds of draw, it was observed that at slow speed the material begins to deform immediately on entry into

the heating coil. As the velocity increases the deformation zone moves further into the coil where the coil characteristic result in a temperature rise for a constant setting of the Radyne power controller. With further increase of velocity the deforming zone moves to the trailing end of the heating coil where the temperature falls, and at the maximum realistic speed, discussed above, the end of the hot zone is drawn outside the coil.

Oscillations in the draw, which always occur initially in steels, are accompanied by oscillations of the zone position relative to the coil. A valuable contribution could be made by investigating the points discussed above more fully, and it is suggested that a screw thread type of drawing machine would be an advantage, thereby avoiding the oscillations resulting from the machine characteristics which are discussed in section 2.3.

2.8

PARAMETER EVALUATION2.8.1 Procedure

The list of fifteen variables given in Section 2.1 must be examined more closely before proceeding to the experimental results.

Mass of moving head Fixed by the choice of equipment at the start of the project.

Heating Technique - Fixed by the choice of equipment at the start of the project

Induction coil shape - Fixed by the choice of simple, circular, easy to manufacture coils.

Induction coil length - Considered to be fixed to give a heated zone of about two diameters. Since the heated length is a function of the speed of draw, the material properties, and specimen diameter, the heated zone was sometimes three or even four times the diameter.

Specimen cross-section - After the problems of corner cracking in square and rectangular specimens, the result of cooler corners, it was decided to concentrate on round specimens.

Position of heated zone in coil - Considered to be fixed in the centre of the coil. Oscillation of the relative transverse position and the position being a function of drawing speed were noted and are discussed

in Section 2.7.8.

Material, Grain size, and Strain rate sensitivity index - A full investigation of all the materials originally chosen was not possible because the experimental programme would have been too long, and too many specimens would have been required. The grain size effect has not been examined and the strain rate sensitivity index in many cases is unknown at the working temperatures used in the experiments.

Temperature profile and rate of heat extraction - The process depends very largely on obtaining the correct temperature profile and this is inevitably a function of the rate at which the heat is extracted. In each experiment the air pressure was adjusted (sometimes natural convection sufficed) to give a stable draw, but no measurements were made to establish exactly what the rate of heat extraction was.

Stress, strain, strain rate, and temperature - These are the four variables which must vary throughout the heated zone to set up the stable draw, and are discussed in the theoretical sections.

2.8.2 Experimental Measurements and Results

The following measurements were taken during every test: -

- 1 Time taken from switching on Radyne set to switching on chart recorder.

- 2 The maximum temperature, taken at the beginning and subsequently read at intervals until the end of the test.
- 3 The medium frequency voltage, grid current, and power voltage and current on the Radyne set.
- 4 Load, displacement and velocity, recorded continuously from the load cell and the square wave output from the transducer.

On the occasions when the stable draw was established, 3 above was repeated and the temperatures of the top and bottom of the deforming zone sometimes recorded. "Stable draw" in this context means simply that the bar is being drawn at a constant cross-section. It is reasonable to suppose that to achieve this a true steady state has been determined and that all the oscillations in the different parameters have disappeared. This true steady state is not always obtained, as the following example shows:

2.5 cm (1 in) diameter titanium

Over 51 cm drawn bar at 1.478 ± 0.038 cm diameter

(20 in drawn bar at 0.582 ± 0.015 in diameter)

Percentage reduction in area is $66\% \pm 1.7\%$

Load varies from 0.02 kN to 0.056 kN

(0.2 tons to 0.56 tons)

Velocity varies from 3.66 cm/min to 5.72 cm/min

(1.44 in/min to 2.25 in/min)

Lateral position of the deformation zone in the coil

varying appreciably. Temperature constant at 1010°C .

The above example, which leads to the results and discussion, shows that the process can operate in an apparently satisfactory way with the drawn diameter only varying by $\pm 2\frac{1}{2}\%$, when the load and velocity vary by as much as $\pm 47\%$ and $\pm 22\%$ respectively.

A selection of the specimens is photographed in Figures 25 to 30. Figure 25 (a) shows four 7.6 cm (3 in) diameter mild steel bars which were unsuccessfully drawn at the beginning of the programme. They are representative of a number of bars which failed to draw a constant cross-section, in the temperature range 1000°C to 1200°C , mainly the result of bad scaling as discussed in Section 2.7.1.

Figure 25 (b) shows three 7.6 cm x 3.8 cm (3 in x 1.5 in) mild steel specimens, two of which were chromised. The central bar shows a break at a very modest reduction in area, a typical result when attempting to draw at the lower end of the hot working range. Corner cracking is very apparent in the other two specimens. The section on the left of the top bar was removed for metallurgical examination.

Corner cracking does not occur in the three 3.8 cm x 3.8 cm (1.5 in x 1.5 in) mild steel specimens shown in Figure 26 (a). The bottom bar is chromised and apparently starts well, but a stable draw could not be established at this stage of the experimental work.

A selection of 2.5 cm (1 in) diameter niobium mild steel specimens appear in Figure 26 (b). Despite repeated efforts, this fine grained material, which could be drawn down to very large reduction in area



(a)



(b)

FIGURE 25 (a) 7.6 cm (3 in) DIAMETER MILD STEEL SPECIMENS
(b) 7.6 cm x 3.8 cm (3 x 1½ in) MILD STEEL SPECIMENS



(a)



(b)

FIGURE 26 (a) 3.8 cm x 3.8 cm ($1\frac{1}{2}$ x $1\frac{1}{2}$ in) MILD STEEL SPECIMENS
(b) 2.5 cm (1 in) DIAMETER NIOBIUM MILD STEEL SPECIMENS

(on the right of the bottom bar) always failed to give a stable draw. Severe necking (on the left of the bottom bar and on the right of the second bar drawn) could not be overcome.

The three 5 cm (2 in) diameter and one 3.8 cm ($1\frac{1}{2}$ in) diameter EN 58 J stainless steel specimens, Figure 27 (a), show the difficulty of establishing a stable draw, the bad surface cracks and the good surface obtainable in the 3.8 cm diameter bar. This bar was drawn with two coats of dag and was wrapped with fibre glass tape during the experiment.

The top 5 cm (2 in) diameter chrome steel specimens shown in Figure 27 (b) are good examples of stable draws. The surface is good and the damped oscillation is clearly seen. Both were reduced by 50% in area. The third bar on the bottom was one of the first attempts to draw a taper. Oscillations of the diameter are evident, but the draw is apparently stable when the reduction in area is 66% on the right of the bar.

Figure 28 shows some good results with chromised mild steel specimens. All the surfaces are good, but oscillations in the 7.6 cm (3 in) diameter bars are not fully controlled.

Some more good results obtained with 3.8 cm ($1\frac{1}{2}$ in) diameter EN 58 J stainless steel specimens appear in Figure 29. The top specimen in (a) has a good finish and the last 15 cm (6 in) of the bar, giving a percentage reduction in area of slightly more than 50%, is 2.654 cm



(a)



(b)

FIGURE 27 (a) 5 cm (2 in) AND 3.8 cm ($1\frac{1}{2}$ in) DIAMETER EN 58 J

STAINLESS STEEL SPECIMENS.

(b) 5 cm (2 in) DIAMETER CHROME STEEL SPECIMENS



(a)



(b)

FIGURE 28 (a) 3.8 x 3.8 cm ($1\frac{1}{2}$ x $1\frac{1}{2}$ in) CHROMISED MILD STEEL SPECIMENS
(b) 7.6 cm (3 in) DIAMETER CHROMISED MILD STEEL SPECIMENS



(a)



(b)

FIGURE 29 (a) 3.8 cm ($1\frac{1}{2}$ in) DIAMETER EN 58 J STAINLESS STEEL SPECIMENS
(b) 3.8 cm ($1\frac{1}{2}$ in) DIAMETER EN 58 J STAINLESS STEEL SPECIMENS

(1.045 in) $\pm$ 0.020 cm (0.008 in). The middle bar of (a) drawn between 1010°C and 1025°C gives a good surface and is reduced by more than 75% in area. The end of the bar was removed for metallurgical examination. The third bar in (a) has been included because the neck on the right of the bar represents a percentage reduction in area of 88%. However, no stable draw was ever achieved at this percentage reduction in area.

Other examples are shown in Figure 29 (b), the lower two bars showing results from two draws on each specimen at a percentage reduction in area of 50%, giving an overall reduction of 75%. Surface cracks appear on a helical line at this reduction. The severe oscillation, still damped, was aggravated by the length of the heated zone. On the second draw, the specimen diameter was such that approximately four diameters was the length of the heated zone, making a stable configuration of the deforming metal more difficult to achieve.

Some tapered specimens are shown in Figure 30 (a). The chrome steel specimen starts at 33% and tapers to 66% reduction in area. The tear on the right of the bar is the result of a step increase in the diameter which allowed a crack to propagate from the effective 'notch' produced half way down the deformation zone. The middle and lower titanium bars are tapered from 33% to 66% and 50% to 75% reduction in area respectively.

Some examples of the 2.5 cm (1 in) diameter titanium specimens appear in the photograph of Figure 30 (b).



(a)



(b)

FIGURE 30 (a) (i) 5.7 cm ($2\frac{1}{4}$ in) CHROME STEEL TAPERED SPECIMEN
(ii) 2.5 cm (1 in) DIAMETER TITANIUM TAPERED SPECIMEN
(iii) 2.5 cm (1 in) DIAMETER TITANIUM TAPERED SPECIMEN
(b) 2.5 cm (1 in) DIAMETER TITANIUM SPECIMENS

2.8.3 Discussion of the Results

The broad understanding of the die-less drawing process which was the initial aim, has been taken a long way with the experimental programme. A summary of the conclusions which can be drawn is as follows: -

- (1) Die-less drawing is possible with mild steel, chrome steels, EN 58 J stainless steel and titanium.
- (2) Percentage reductions in area up to 75% for steel and 84% for titanium are possible.
- (3) Round and square cross-sections may be drawn.
- (4) Speeds, up to the highest velocities possible for the maximum rate at which heat can be applied, of the order of 7.6 cm/min (3 in/min) give satisfactory stable draws.
- (5) Good surface finishes can be obtained on the drawn bars.
- (6) Although the specimens photographed in Figure 25 (a) represent unsatisfactory draws, we can nevertheless draw the conclusion that non-constant cross-section drawing under controlled conditions is possible.

The techniques for drawing steel and titanium have been developed and the photographs of the previous section suggest that there are possibilities for industrial application. All the experimental programme was carried out on the Buckton testing machine which is basically too powerful a machine especially for the titanium pulls, and the hydraulic control causes problems which would not present themselves with a screw thread drawing device. Some suggestions for developing the experimental work using such a device are included in Section 4.5.

CHAPTER THREE

THEORETICAL WORK

With no more said, they leapt from their
chariots, shook hands and pledged each other.
But Zeus the Son of Cronos must have robbed
Glaucus of his wits, for he exchanged with
Diomedes golden armour for bronze, a hundred
oxen's worth for the value of nine.

The Iliad Book VI

Homer

3.1 Introduction

The problem of analysing the process of die-less drawing has been approached in three different ways. The first is to make the assumption that plane sections remain plane and examine the varying parameters along the deformation zone. The assumption enables the relative effects of strain hardening, strain rate sensitivity, change of diameter or percentage reduction in area, and stress-temperature effects to be found.

The second approach is to examine the deformation zone as a series of concentric tubes each one of which is in simple tension. This type of analysis cannot be applied to large percentage reductions in area, but because of its simplicity, approximate solutions may be computed very rapidly.

The third method is the powerful finite element technique which is just reaching the stage where large deformation plastic flow can be analysed. A wholly satisfactory analysis has, however, not yet been found and further development work is necessary as discussed in Section 4.5.

Although not included in the present theoretical work, a combination of the simple tension and finite element approaches may prove to be valuable. The rapid computation of the simple tension method could be used to start the finite element procedure, thereby reducing the number of iterations required in inverting large matrices which result in lengthy and costly programs for fine mesh work.

3.2

THE THEORETICAL APPROACH3.2.1 General Assumptions

The heat distribution in the deforming specimen (discussed in Section 3.3) is critical in establishing a stable draw. For the plane sections work, it is assumed that throughout each section the temperature is constant, i.e. the radial temperature gradient is zero. For the rest of the analytical study, the heat is considered to be generated uniformly within the deformation zone and the temperature distribution determined from the conduction equation. It is further assumed that a convection term resulting from the material flow is zero.

In plane sections, the profile of the deforming zone is assumed, accurately measured from the zone. One can assume a measured temperature profile, but because the experimental temperatures are less accurately measured along the length of the zone the former assumption was adopted.

The simple tension solution uses the assumption that the specimen may be broken down into a series of short concentric tubes which have no interacting forces apart from the axial load, and deform according to the uniaxial form of the secondary creep equation and the condition of constant volume. Although at first sight this somewhat sweeping assumption appears unacceptable, the speed with which the solution is computed and the results obtained suggest that the method may form a satisfactory theoretical basis from which to develop.

The finite element technique assumes a linear relationship between the radial and axial velocities with the r and z coordinates, that the material is isotropic and free from initial strain, and that the approximation in computing the volume integral is sufficiently close. This approximation involves the assumption that:

$$\bar{r} = \left(\frac{r_i + r_j + r_p}{3} \right) \quad \text{and} \quad \bar{z} = \left(\frac{z_i + z_j + z_p}{3} \right)$$

in calculating the mean value of the $[B]$ strain rate-velocity matrix. The properties of the material, including the average temperature taken from the nodal temperatures, in each of the triangular ring elements are assumed to be constant.

3.2.2 Theoretical Models

The ideal theoretical model is one which enables the temperature distribution to be analysed in terms of the heat input, the conduction, convection and radiation, and the convection resulting from the material flow. Having obtained the distribution of temperature, the model should enable the profile of the bar and deformation zone, the stress distribution and the flow lines to be evaluated given the material properties.

Unfortunately the material properties relating the stress, strain rate and temperature are not generally available and in the present work little attention has been paid to the strain hardening characteristics.

Effect of strain hardening is reduced at elevated temperatures and the experiments of Cook⁽¹²⁾, giving true stress-strain curves for steel in compression, show an appreciable reduction of the effect as the temperature rises from 900°C to 1100°C. Nevertheless the strain hardening characteristics are still marked up to natural strains of 0.2 or 0.3.

Strain rate sensitivity is not only a much bigger effect but is also playing a significant role at the narrow end of the zone where there is no strain hardening. It is the narrow part of the zone which is critical when considering stability, and the neglect of strain hardening is quite justified here. Thus the models presented in the ensuing sections do go a fair way towards a broad understanding of the process.

3.2.3 Stress-Strain-Strain Rate Temperature Relationships

After the analysis which shows the relative effects of strain hardening, strain rate sensitivity and temperature (described in Section 3.4.2.), a considerable effort was put into considerations of the equations of state. The intention was to find an empirical form which would describe the behaviour of materials with both strain hardening characteristics and strain rate sensitivity.

Very often the stress strain relationships at constant temperature are well expressed by:

$$\sigma = \sigma_0 (1 + K\epsilon)^n$$

or

$$\sigma = K' \sigma_0 \epsilon^n$$

The secondary creep equation:

$$\dot{\epsilon} = \frac{2}{3} K'' \sigma^n \exp \left[\frac{-Q}{RT} \right] \quad (9)$$

has been used extensively (10), (11), (13), (14), (29) to account for a temperature dependence, but the activation energy is generally found to be a function of temperature and the stress function replaced by a hyperbolic sine function sometimes gives a better representation.

Using the data from Cook⁽¹²⁾ and Woolman and Mottram⁽³⁰⁾ for stainless steel and Lee and Backofen⁽²⁵⁾ for titanium, curve fitting was attempted using equation of the form:

$$\sigma = K_1 \dot{\epsilon}^f(T, \epsilon)$$

Very little progress was made.

The closest approach to an empirical expression which fits the available data for stainless steel is:

$$\sigma = K_2 \left\{ \frac{T^* - T}{\alpha + 2/\epsilon} \right\}^n \dot{\epsilon}^m \quad (16)$$

where K_2 , α , T^* , n and the strain rate sensitivity index, m , are all constant. The expression was found by noting the near parabolic form of the stress temperature curves at different strains. There is a reasonable fit at 900°C and 1000°C between strains of 0.05 and 0.7, but the strain hardening is over estimated between 0.05 and 0.10.

A value of 0.2 was taken for the strain rate sensitivity index, m , as an upper bound. Cook's data at strain rates from 1.5 to 100 sec^{-1} gives a range of m values from 0.07 to 0.11 between 900°C and 1100°C rising to 0.146 at 1200°C.

Near parabolic curves are obtained when stress is plotted against the log of strain rate for titanium. These curves plotted for a number of temperatures led to the expression:

$$\sigma = \frac{\{T_i^* - \gamma \log \dot{\epsilon} - T\}^2}{\beta (-\log \dot{\epsilon})} + 8 \dot{\epsilon}^{0.301} \quad (17)$$

where T_i^* , β and γ are constants.

Both equations (16) and (17) proved to be clumsy when used and it was decided that the secondary creep equation (9) should be used to develop both the simple tension and finite element approach.

3.2.4. Stability

After the clear demonstration of stability in the experimental work, it was hoped that an analytical study, following the start made by Cornfield⁽¹⁴⁾ (see Section 1.3.1.) would be possible. Owing to the time spent on the finite element work, however, this work was not developed.

There is nevertheless in the Section on plane sections a necessary though not sufficient condition that the flow is stable, i.e. a condition

of equilibrium. The work on a simple tension solution also implies stability, since a constant cross-section is "drawn" in the solution of the computer program.

Stability in the process is more complex when the load is applied hydraulically, as the speed varies and sets up oscillations and varying strain rates. An analysis of the stabilities of both the hydraulic and screw thread (constant velocity) method of dieless drawing might prove to be most interesting.

3.3

HEAT TRANSFER

Much has been made of the superplastic effect where a hardening occurs even though the strain rate decreases towards the narrow end of the deformation zone, due to an increase in the strain rate sensitivity index. A similar hardening occurs if a temperature gradient can be set up in the zone, a hardening which is applicable to all materials. Heat is applied only in that region where the metal is deforming and therefore there is a natural gradient at each end of the deforming zone exactly where we need it. Thus a complete analysis of heat transfer must be developed before accurate predictions of die-less drawing behaviour can be made. A very approximate solution is presented in the following sections as a full analysis is a large problem in itself.

3.3.1 Heat Balance

A steady state is required for a stable draw where the heat generated in the hot zone is equal to the heat lost by radiation, conduction, and convection, both natural and forced.

The quantity of heat generated in the zone is most easily found by plotting a cooling curve immediately after the power input is switched off. Values obtained show that for 5 cm (2 in) diameter stainless steel specimens approximately 25% of the heat input is generated in the zone, falling to about 7½% for 2.5 cm (1 in) diameter material. This compares favourably with the efficiencies quoted in Table 1, and the figure of 25% efficiency is taken for the heat balance which follows.

A rough heat balance was calculated for an EN 58 J stainless steel specimen using Stefan's fourth power law, setting Stefan's constant at 5.75×10^{-5} erg. cm.⁻² °C⁻⁴ s.⁻¹ and the emissivity at 0.5, for the radiation. The natural convection term for a horizontal cylinder in air was taken as:

$$1.0 \times 10^{-4} \left(\frac{\Delta T}{d} \right)^{\frac{1}{4}} \text{ cal. s.}^{-1} \text{ cm.}^{-2} \text{ °C}^{-1},$$

and the uniaxial form of the conduction equation

$$\frac{dQ'}{dt} = -k'. A. \frac{dT}{dz} \text{ assumed.}$$

The calculation shows that the percentage heat losses are as follows:

37½%	radiated
10 %	convected
35 %	conducted at entry
17½%	conducted at exit

3.3.2 Temperature Distribution

With the thermal conductivity, k' , effectively constant over the small temperature range in the deforming metal and the quantity of heat, Q' , generated in the specimen per unit volume assumed constant, only one unknown scalar quantity, T , remains. The steady state heat conduction equation:

$$\frac{\partial}{\partial r} (k'r \frac{\partial T}{\partial r}) + \frac{\partial}{\partial z} (k'r \frac{\partial T}{\partial z}) + rQ' = 0 \quad (18)$$

together with the boundary conditions defines the problem, and a finite element analysis is possible. Zienkiewicz⁽³¹⁾ describes how the analysis is developed, where, with the aid of the calculus of variation, a volume integral is minimized and a "stiffness" matrix set up. The mesh which is used is shown in Figure 31. Experimental surface temperatures give the boundary conditions and Q' is assessed at 25% of the energy input per unit volume. An example of the temperature contours obtained is shown in Figure 32.

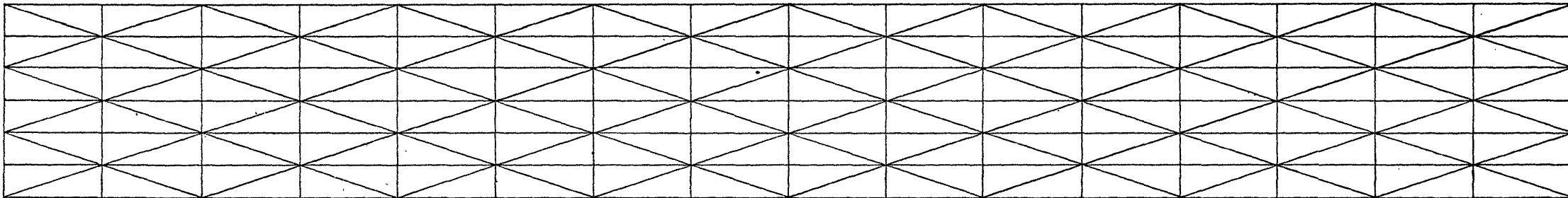


FIGURE 31 MESH USED FOR THE FINITE ELEMENT ANALYSIS OF TEMPERATURE DISTRIBUTION AND SOLUTION IN SIMPLE TENSION

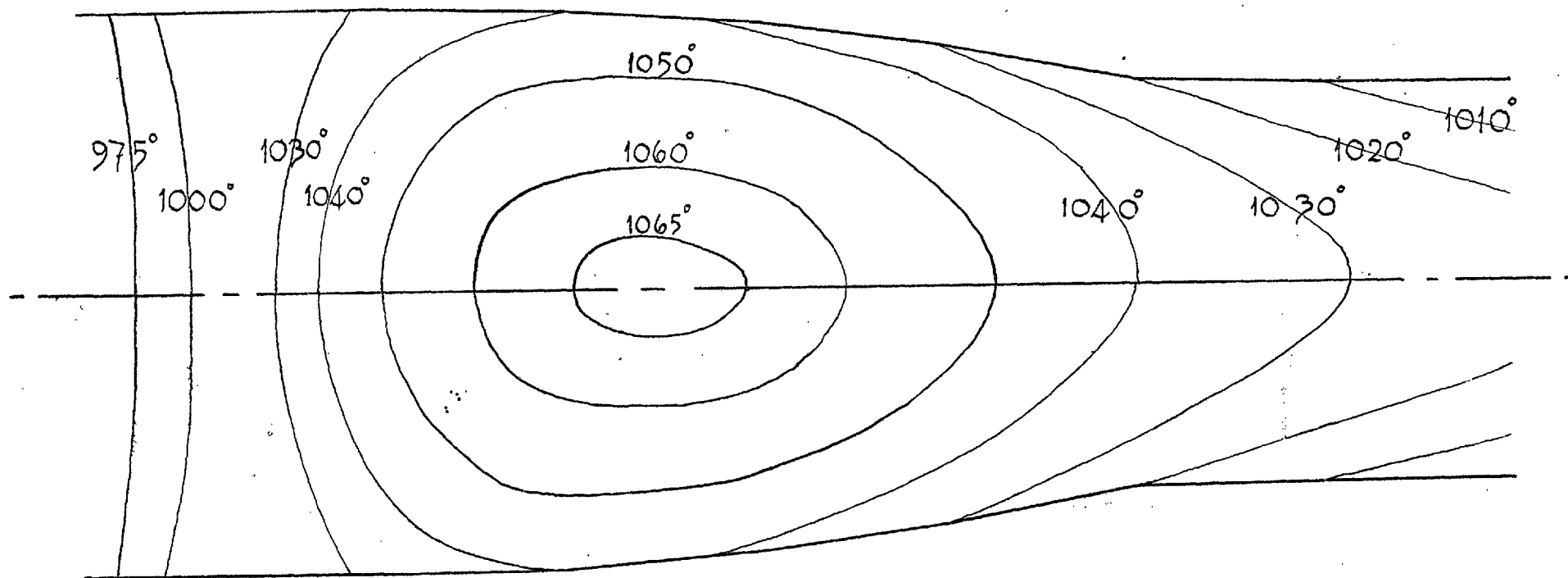


FIGURE 32 TEMPERATURE CONTOURS IN STAINLESS STEEL SPECIMENS

The above analysis does not take into account the convection term resulting from the material flow, which will tend to push the contours of Figure 32 to the right. The justification of neglect can be found in evaluating the Peclet number. This number, which may be regarded as the ratio between the convection and conduction effects, is defined by

$$Pe = \frac{\rho H c_p V}{k'}$$

where H is a linear dimension which we may take here as the diameter of the bar. Putting in the values for stainless steel we obtain

$$Pe = 68.8 V \text{ } ^\circ\text{C. cm.}^{-1}\text{s.}$$

A velocity of 0.87 cm/min (0.34 in/min) gives a Peclet number of 1, and a velocity of 7.6 cm/min (3 in/min) gives $Pe = 8.7$. Both these numbers are in the region where the convection term is only just becoming significant, but it should be noted that for speeds higher than 7.6 cm/min (3 in/min) the convection term due to flow should be included.

The temperature distribution, calculated from the experimental surface temperatures and the finite element technique, forms the basis for the simple tension and finite element solution of the drawing problem.

In the former, the temperature distribution calculation is integral with the program, whereas in the latter the temperature distribution is calculated separately and fed into the finite element program as data.

3.4

THE THEORETICAL SOLUTION3.4.1 Boundary Conditions

The boundary conditions specified for the temperature distribution work are:

1. The surface temperatures are designated over the whole length and for the nodal points at the ends. These are experimental values and for the ends a zero radial temperature gradient is assumed initially.
2. The isothermals are normal to the centre line.

Modifications of the end nodal point temperatures were possible when the radial gradients close to the ends were computed, enabling a more accurate distribution to be found.

The mesh generation for all the analytical work was for a constant cross-section representation (More precisely the profiling of the mesh used at the start of the analytical work was found to be unnecessary). Although the boundary conditions were applicable to a profiled specimen, the iterative techniques were such that the initial errors were corrected in the final iteration where the mesh is profiled.

The nodal points of entry were associated with the initial stress and velocity for the simple tension analysis. The stress and strain rate were set at zero. A limiting maximum velocity was specified to stop the hotter central regions from excessive "deformation".

The initial and final velocities are also set for the finite element analysis. The nodal points at entry were constrained to move only in the z direction, as were all the nodal points on the centre line. One, two, or three nodal points on the surface at entry were similarly constrained but the remaining nodal points were free to take up positions in a profiled mesh. The above constraint represents zero radial velocity.

3.4.2. Plane Sections

The simplifying assumptions that plane sections remain plane and that the temperature of a thin section is constant, enable some progress to be made in understanding the process.

Accurate measurement of the deformation zone gives the cross-sectional areas and because the material is considered to be incompressible,

$$\frac{A_I}{A_i} = 1 + \frac{\delta z}{z}$$

At each section we can therefore obtain the engineering strain $e_z = \frac{\delta z}{z}$, from which we calculate the natural strain:

$$\epsilon_z = \ln(1 + e_z)$$

With material fed into the deformation zone at an axial velocity, V_c , the axial velocity of each section is determined from

$$V_i = \frac{A_I}{A_i} \cdot V_c$$

The slope of the velocity curve along the z axis gives the axial strain rate:

$$\dot{\epsilon}_z = \frac{V_i - V_{i-1}}{\delta z}$$

Since the load, L , is constant the stress at each section is

$$\sigma_i = \frac{L}{A_i}$$

A simple computer program has been developed to obtain tables of the parameters for each specimen from the measured diameters of the deformation zone.

Gunasekera⁽³²⁾ has taken the program a step further, for, since σ_i and ϵ_i are evaluated, a point can be obtained on the stress-strain-temperature curves which gives a temperature by straightforward linear interpolation.

The dynamic yield stress can be expressed as the product of a strain hardening function and a strain rate sensitivity function:

$$\sigma = \sigma_0 g(\epsilon) f(\dot{\epsilon})$$

If we now take temperature into account by making σ_0 a function of temperature only we obtain

$$\sigma = h(T)g(\epsilon) f(\dot{\epsilon})$$

then

$$L = A\sigma = A h(T) g(\epsilon) f(\dot{\epsilon}) \quad (19)$$

This may be written in differential form

$$\frac{dL}{L} = \frac{dA}{A} + \frac{dh}{h} + \frac{dg}{g} + \frac{df}{f} \quad (20)$$

If we write finite differences in place of the differentials and consider that each function varies with respect to one variable only, we may write:

$$\frac{\delta L}{L} = \frac{\delta A}{A} + \frac{\frac{\partial h}{\partial T} \delta T}{h} + \frac{\frac{\partial g}{\partial \epsilon} \delta \epsilon}{g} + \frac{\frac{\partial f}{\partial \dot{\epsilon}} \delta \dot{\epsilon}}{f} \quad (21)$$

A necessary but not sufficient condition for stability is given by $\frac{\delta L}{L} = 0$.

The sign value of the first term is negative, as it represents the effect of the reducing area, whereas the second term, that of stress-temperature, begins negative as the temperature increases, becomes zero at the maximum temperature, and then becomes positive as the temperature reduces as one approaches the drawn bar. The sign values of the latter two terms, strain hardening and strain rate effect, are both positive.

All the finite differences are known from the experimental data, i.e. δA , δT , $\delta \epsilon$, and $\delta \dot{\epsilon}$. The strain hardening function can be taken as:

$$g = (1 + B\epsilon)^n$$

or taken from the experimental data of Cook⁽¹²⁾.

The rate of change of stress with temperature is constant for stainless steel, the only material for which we have useful data. Thus three of the four terms can be evaluated and the strain rate sensitivity term set at the values which make the summation of the four terms zero, i.e. $\frac{\delta L}{L} = 0$. When each term is plotted against z , an indication of the relative effects of reduction, strain hardening, temperature, and strain rate sensitivity is obtained.

Figure 33 shows these relative effects where the "strain rate sensitivity" effect is assumed to be the dotted curve. The dotted curve in fact represents values such that $\frac{\delta L}{L} = 0$ is satisfied at every point along the zone.

Table 2 shows the result taken from the print-out of the program used to calculate the varying parameters along the deformation zone. The values are typical of a successful draw with 5 cm (2 in) diameter En 58 J stainless steel reduced by 53%. The temperature is calculated using the interpolation technique. Table 3 gives the results of a successful draw with the titanium alloy giving a percentage reduction in area of 83%.

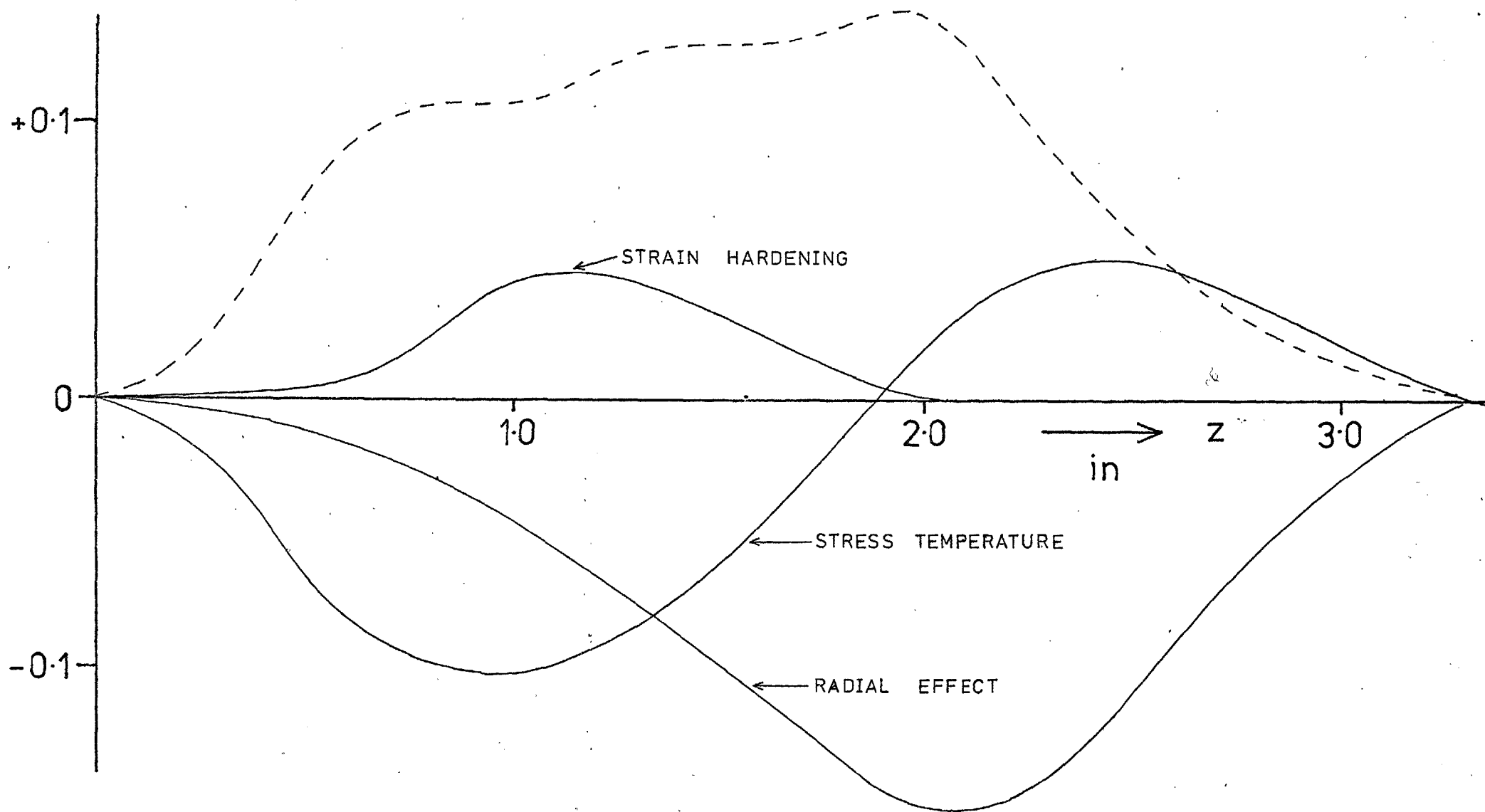


FIGURE 33 RELATIVE EFFECTS OF REDUCING AREA, STRESS-TEMPERATURE DEPENDENCE, AND STRAIN HARDENING

TABLE 2.

Results from the print-out of the program used
to calculate stress, strain, strain rate and
temperature in the deformation zone.

Plane sections remaining plane.

EN 58 J stainless steel.

DIAMETER	LENGTH	STRESS	STRAIN	NATSTRAIN	ST. RATE	VELOC	REDUCT	TEMP
INS.	INS.	TNS/IN2			/SECX	INS/MIN		C
2.000000	0.000000	2.037185	0.000000	0.000000	0.000000	.220000	0.00	1033.0
1.998000	.100000	2.041265	.002003	.002001	.007344	.220441	.20	1034.3
1.993000	.200000	2.051521	.007037	.007012	.018458	.221548	.70	1037.2
1.989000	.300000	2.059780	.011091	.011030	.014866	.222440	1.10	1039.3
1.988000	.400000	2.061853	.012109	.012036	.003731	.222664	1.20	1039.7
1.980000	.500000	2.078548	.020304	.020101	.030049	.224467	1.99	1043.3
1.973000	.600000	2.093323	.027557	.027184	.026593	.226062	2.68	1046.0
1.961000	.700000	2.119021	.040171	.039385	.046253	.228838	3.86	1050.0
1.951000	.800000	2.140799	.050861	.049610	.039198	.231190	4.84	1052.8
1.932000	.900000	2.183113	.071632	.069183	.076159	.235759	6.68	1057.3
1.919000	1.000000	2.212792	.086201	.082686	.053418	.238964	7.94	1059.9
1.898000	1.100000	2.262028	.110370	.104693	.088620	.244281	9.94	1063.3
1.880000	1.200000	2.305551	.131734	.123751	.078335	.248981	11.64	1065.8
1.860000	1.300000	2.355399	.156203	.145141	.089720	.254365	13.51	1068.1
1.839000	1.400000	2.409500	.182760	.167850	.097375	.260207	15.45	1070.1
1.809000	1.500000	2.490080	.222314	.200746	.145033	.268909	18.19	1072.4
1.784000	1.600000	2.560358	.256812	.228578	.126492	.276499	20.43	1073.8
1.756000	1.700000	2.642661	.297212	.260217	.148134	.285387	22.91	1075.0
1.726000	1.800000	2.735325	.342698	.294681	.166783	.295394	25.52	1075.7
1.690000	1.900000	2.853100	.400511	.336837	.211981	.308112	28.60	1075.9
1.655000	2.000000	2.975051	.460374	.378692	.219496	.321282	31.52	1075.5
1.620000	2.100000	3.104992	.524158	.421442	.233876	.335315	34.39	1074.4
1.581000	2.200000	3.260068	.600281	.470179	.279118	.352062	37.51	1072.4
1.546000	2.300000	3.409349	.673559	.514952	.268686	.368183	40.25	1069.8
1.510000	2.400000	3.573852	.754309	.562075	.296084	.385948	43.00	1066.4
1.492000	2.500000	3.660605	.796894	.586059	.156143	.395317	44.35	1064.3
1.455000	2.600000	3.849147	.889444	.636283	.339351	.415678	47.07	1059.3
1.435000	2.700000	3.957188	.942478	.663965	.194460	.427345	48.52	1056.2
1.416000	2.800000	4.064096	.994957	.690622	.192421	.438890	49.87	1052.8
1.395000	2.900000	4.187377	1.055472	.720506	.221889	.452204	51.35	1048.8
1.382000	3.000000	4.266526	1.094324	.739231	.142458	.460751	52.25	1046.1
1.381000	3.100000	4.272707	1.097358	.740679	.011125	.461419	52.32	1045.9
1.376000	3.200000	4.303815	1.112628	.747933	.055990	.464778	52.67	1044.8
1.370000	3.300000	4.341595	1.131174	.756673	.067999	.468858	53.08	1043.4

DIAMETER	LENGTH	STRESS	STRAIN	NATSTRAIN	ST. RATE	VELOC	REDUCT	TEMP
CM.	CM.	MN/M2			/SECK	CM/S		C
5.080000	0.000000	31.462805	0.000000	0.000000	0.000000	.009313	0.00	1033.0
5.074920	.254000	31.525825	.002003	.002001	.007344	.009332	.20	1034.3
5.062220	.508000	31.684206	.007037	.007012	.018458	.009379	.70	1037.2
5.052060	.762000	31.811772	.011091	.011030	.014856	.009417	1.10	1039.3
5.049520	1.016000	31.843784	.012109	.012036	.003731	.009426	1.20	1039.7
5.029200	1.270000	32.101628	.020304	.020101	.030049	.009502	1.99	1043.3
5.011420	1.524000	32.329818	.027557	.027184	.026593	.009570	2.68	1046.0
4.980940	1.778000	32.726702	.040171	.039385	.046253	.009687	3.86	1050.0
4.955540	2.032000	33.063048	.050861	.049610	.039198	.009787	4.84	1052.8
4.907280	2.286000	33.716555	.071632	.069183	.076159	.009980	6.68	1057.3
4.874260	2.540000	34.174918	.086201	.082686	.053418	.010116	7.94	1059.9
4.820920	2.794000	34.935343	.110370	.104693	.088620	.010341	9.94	1063.3
4.775200	3.048000	35.607521	.131734	.123751	.078335	.010540	11.64	1065.8
4.724400	3.302000	36.377391	.156203	.145141	.089720	.010768	13.51	1068.1
4.671060	3.556000	37.212939	.182760	.167850	.097375	.011015	15.45	1070.1
4.594860	3.810000	38.457434	.222314	.200746	.145033	.011384	18.19	1072.4
4.531360	4.064000	39.542829	.256812	.228578	.126492	.011705	20.43	1073.8
4.460240	4.318000	40.813929	.297212	.260217	.148134	.012081	22.91	1075.0
4.384040	4.572000	42.245052	.342698	.294681	.166783	.012505	25.52	1075.7
4.292600	4.826000	44.064011	.400511	.336837	.211981	.013043	28.60	1075.9
4.203700	5.080000	45.947452	.460374	.378692	.219496	.013601	31.52	1075.5
4.114800	5.334000	47.954283	.524158	.421442	.233876	.014195	34.39	1074.4
4.015740	5.588000	50.349330	.600281	.470179	.279118	.014904	37.51	1072.4
3.926840	5.842000	52.654859	.673559	.514952	.268686	.015586	40.25	1069.8
3.835400	6.096000	55.195483	.754309	.562075	.296084	.016338	43.00	1066.4
3.789680	6.350000	56.535311	.796894	.586059	.156143	.016735	44.35	1064.3
3.695700	6.604000	59.447206	.889444	.636283	.339351	.017597	47.07	1059.3
3.644900	6.858000	61.115818	.942478	.663965	.194460	.018091	48.52	1056.2
3.596640	7.112000	62.766935	.994957	.690622	.192421	.018580	49.87	1052.8
3.543300	7.366000	64.670917	1.055472	.720506	.221889	.019143	51.35	1048.8
3.510280	7.620000	65.893313	1.094324	.739231	.142458	.019505	52.25	1046.1
3.507740	7.874000	65.988776	1.097358	.740679	.011125	.019533	52.32	1045.9
3.495040	8.128000	66.469217	1.112628	.747933	.055990	.019676	52.67	1044.8
3.479800	8.382000	67.052704	1.131174	.756673	.067999	.019848	53.08	1043.4

TABLE 3.

Results from the print-out of the program used
to calculate stress, strain, strain rate and
temperature in the deformation zone.

Plane sections remaining plane.

Titanium alloy, Ti-6Al-4V.

DIAMETER	LENGTH	STRESS	STRAIN	NATSTRAIN	ST. RATE	VELOC	REDUCT	TEMP
INS.	INS.	TNS/IN2			/SEC	INS/MIN		C
1.003000	0.000000	1.252979	0.000000	0.000000	0.000000	0.164100000	0.00	658.4
1.003000	0.050000	1.252979	0.000000	0.000000	0.000000	0.164100000	0.00	658.4
1.003000	0.100000	1.252979	0.000000	0.000000	0.000000	0.164100000	0.00	658.4
1.001000	0.150000	1.257991	0.004000	0.003992	0.000219	0.164756399	0.40	886.3
1.000000	0.200000	1.260508	0.006009	0.005991	0.000110	0.165086077	0.60	874.7
0.998000	0.250000	1.265565	0.010045	0.009995	0.000221	0.165748408	0.99	886.2
0.995000	0.300000	1.273208	0.016145	0.016016	0.000334	0.166749402	1.59	892.7
0.991000	0.350000	1.283507	0.024365	0.024073	0.000450	0.168098229	2.38	897.2
0.985000	0.400000	1.299192	0.036882	0.036218	0.000685	0.170152364	3.56	910.0
0.971000	0.450000	1.336926	0.066998	0.064849	0.001647	0.175094292	6.28	941.4
0.955000	0.500000	1.382098	0.103050	0.098079	0.001972	0.181010473	9.34	945.7
0.937000	0.550000	1.435709	0.145837	0.136135	0.002340	0.188031783	12.73	949.5
0.909000	0.600000	1.525520	0.217514	0.196811	0.003921	0.199794110	17.87	970.6
0.881000	0.650000	1.624029	0.296134	0.259386	0.004301	0.212695661	22.85	970.5
0.847000	0.700000	1.757029	0.402281	0.338100	0.005806	0.230114310	28.69	978.0
0.817000	0.750000	1.888433	0.507154	0.410223	0.005737	0.247324041	33.65	972.9
0.783000	0.800000	2.055995	0.640886	0.495236	0.007315	0.269269351	39.06	977.5
0.747000	0.850000	2.258939	0.802854	0.589371	0.008860	0.295848413	44.53	979.3
0.707000	0.900000	2.521778	1.012626	0.699440	0.011474	0.330271896	50.31	982.7
0.665000	0.950000	2.850378	1.274880	0.821927	0.014345	0.373307879	56.04	984.0
0.630000	1.000000	3.175884	1.534666	0.930062	0.014210	0.415938717	60.55	977.3
0.586000	1.050000	3.670713	1.929589	1.074862	0.021602	0.480745486	65.87	984.7
0.554000	1.100000	4.107014	2.277799	1.187172	0.019047	0.537886838	69.49	973.0
0.524000	1.150000	4.590744	2.663864	1.298518	0.021118	0.601240010	72.71	970.0
0.491000	1.200000	5.228567	3.172909	1.428613	0.027845	0.684774316	76.04	973.4
0.473000	1.250000	5.634085	3.496552	1.503311	0.017703	0.737884123	77.76	943.9
0.454000	1.300000	6.115528	3.880790	1.585307	0.021018	0.801937709	79.51	946.5
0.444000	1.350000	6.394105	4.103122	1.629852	0.012162	0.837422271	80.40	905.3
0.430000	1.400000	6.817243	4.440827	1.693931	0.018473	0.892839789	81.62	925.9
0.429000	1.450000	6.849062	4.466222	1.698588	0.001389	0.897007063	81.71	809.3
0.426000	1.500000	6.945867	4.543482	1.712623	0.004226	0.909685451	81.96	852.2
0.422000	1.550000	7.078167	4.649070	1.731491	0.005776	0.927012404	82.30	862.7
0.422000	1.600000	7.078167	4.649070	1.731491	0.000000	0.927012404	82.30	401.0
0.421000	1.650000	7.111832	4.675938	1.736236	0.001476	0.931421493	82.38	806.6
0.419000	1.700000	7.179887	4.730253	1.745760	0.002971	0.940334567	82.55	834.5
0.419000	1.750000	7.179887	4.730253	1.745760	0.000000	0.940334567	82.55	399.0
0.418000	1.800000	7.214282	4.757704	1.750539	0.001502	0.944839157	82.63	805.6

DIAMETER	LENGTH	STRESS	STRAIN	NATSTRAIN	ST. RATE	VELOC	REDUCT	TEMP
CM.	CM.	MN/M2			/SEC	CM/S		C
2.547620	0.000000	19.351329	0.000000	0.000000	0.000000	0.006946900	0.00	658.4
2.547620	.127000	19.351329	0.000000	0.000000	0.000000	0.006946900	0.00	658.4
2.547620	.254000	19.351329	0.000000	0.000000	0.000000	0.006946900	0.00	658.4
2.542540	.381000	19.428734	.004000	.003992	.000219	0.006974688	.40	886.3
2.540000	.508000	19.467611	.006009	.005991	.000110	0.006988644	.60	874.7
2.534920	.635000	19.545715	.010045	.009995	.000221	0.007016683	.99	886.2
2.527300	.762000	19.663757	.016145	.016016	.000334	0.007059058	1.59	892.7
2.517140	.889000	19.822816	.024365	.024073	.000450	0.007116158	2.38	897.2
2.501900	1.016000	20.065047	.036382	.036218	.000685	0.007203117	3.56	910.0
2.466340	1.143000	20.647819	.066998	.064849	.001647	0.007412325	6.28	941.4
2.425700	1.270000	21.345479	.103050	.098079	.001972	0.007662777	9.34	945.7
2.379980	1.397000	22.173460	.145837	.136135	.002340	0.007960012	12.73	949.5
2.308860	1.524000	23.560521	.217514	.196811	.003921	0.008457951	17.87	970.6
2.237740	1.651000	25.081923	.296134	.259386	.004301	0.009004116	22.85	970.5
2.151380	1.778000	27.136000	.402281	.338100	.005806	0.009741506	28.69	978.0
2.075180	1.905000	29.165440	.507154	.410223	.005737	0.010470051	33.65	972.9
1.988820	2.032000	31.753319	.640886	.495236	.007315	0.011399069	39.06	977.5
1.897380	2.159000	34.887628	.802854	.589371	.008860	0.012524249	44.53	979.3
1.795780	2.286000	38.946983	1.012626	.699440	.011474	0.013981510	50.31	982.7
1.689100	2.413000	44.021959	1.274880	.821927	.014345	0.015803367	56.04	984.0
1.600200	2.540000	49.049158	1.534666	.930062	.014210	0.017608072	60.55	977.3
1.488440	2.667000	56.691431	1.929589	1.074862	.021602	0.021351559	65.87	984.7
1.407160	2.794000	63.429768	2.277739	1.187172	.019047	0.022770543	69.49	973.0
1.330960	2.921000	70.900627	2.663864	1.298518	.021118	0.025452494	72.71	970.0
1.247140	3.048000	80.751327	3.172909	1.428613	.027845	0.028988779	76.04	973.4
1.201420	3.175000	87.014248	3.496552	1.503311	.017703	0.031237095	77.76	943.9
1.153160	3.302000	94.449779	3.880790	1.585307	.021018	0.033906363	79.51	946.5
1.127760	3.429000	98.752185	4.103122	1.629852	.012162	0.035450876	80.40	905.3
1.092200	3.556000	105.287240	4.440827	1.693931	.018473	0.037796884	81.62	925.9
1.089660	3.683000	105.778662	4.466222	1.698588	.001389	0.037973299	81.71	809.3
1.082040	3.810000	107.273748	4.543482	1.712623	.004226	0.038510017	81.96	852.2
1.071880	3.937000	109.317012	4.649070	1.731491	.005776	0.039243525	82.30	862.7
1.071880	4.064000	109.317012	4.649070	1.731491	0.000000	0.039243525	82.30	401.0
1.069340	4.191000	109.836949	4.675938	1.736236	.001470	0.039430177	82.38	806.6
1.064260	4.318000	110.888014	4.730253	1.745760	.002971	0.039807497	82.55	834.5
1.064260	4.445000	110.888014	4.730253	1.745760	0.000000	0.039807497	82.55	399.0
1.061720	4.572000	111.419214	4.757704	1.750539	.001502	0.039998191	82.63	805.6

Temperature plots which show the surface temperature along the length of the deformation zone are shown in Figures 34 and 35, calculated using the interpolation technique. The flatter curves for stainless steel were successful draws, and the steep fall towards the drawn end of the zone indicates the "necking" of an unstable draw. All the titanium curves are from stable draws. The temperatures compare favourably with the observed temperatures plotted in Figure 22.

Typical experimental profiles of the deforming zone, the starting point for the plane sections work, are shown in Figures 39, 40 and 41, and strain rate variations in Figures 42 and 43.

3.4.3 Solution in Simple Tension

At the stage when the finite element method appeared to be intractable, it was decided to develop a solution where the rod is considered to be an assemblage of thin tubes.

The mesh, see Figure 31, is generated in terms of the number of nodal z and nodal r points for an axisymmetrical constant cross-section bar. The equivalent stiffness matrix is set up and the temperature distribution found, as in Section 3.3.2, using the conduction equation:

$$\frac{\partial}{\partial r} \left(k' r \frac{\partial T}{\partial r} \right) + \frac{\partial}{\partial z} \left(k' r \frac{\partial T}{\partial z} \right) + \dot{r} Q' = 0.$$

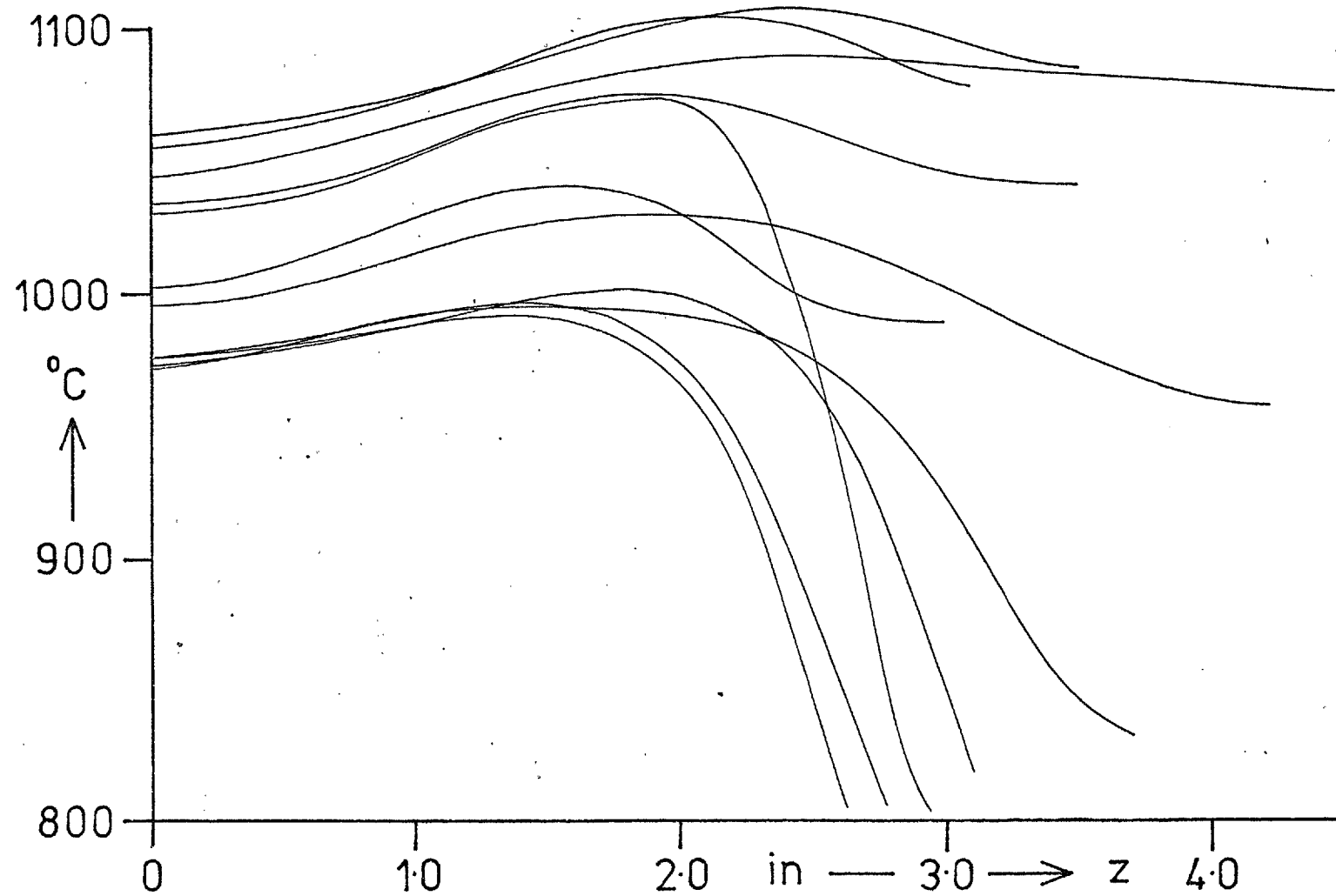


FIGURE 34 TEMPERATURE VS. Z CALCULATED USING PLANE SECTION APPROACH. STAINLESS STEEL

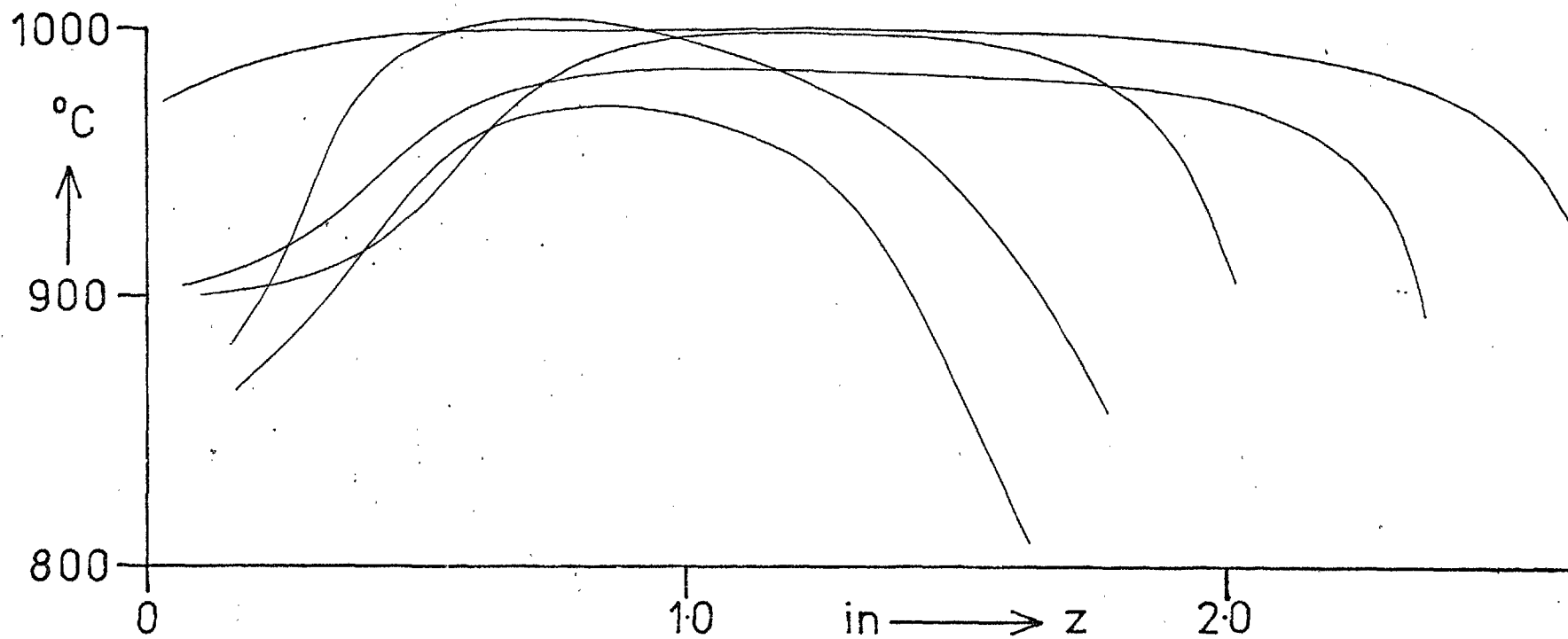


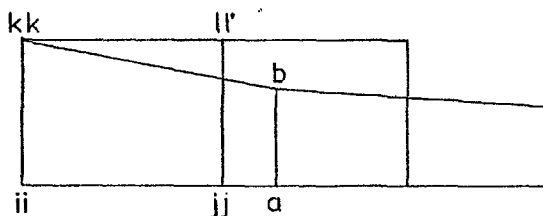
FIGURE 35 TEMPERATURE VS. Z CALCULATED USING PLANE SECTION APPROACH T1-6A1-4V

The same mesh is used for "plastically deforming" the bar, but instead of using triangular ring elements rectangular ring elements are employed. Initialization is carried out setting a constant stress (equivalent to a constant load), the velocity V_c at every nodal point, and the average absolute temperature is calculated for each rectangular element. Strains and strain rates are effectively set at zero.

The elemental tubes are now considered to be subjected to a pure tension and plastically deformed according to the equation of state

$$\dot{\epsilon} = K \sigma^n \exp \left[\frac{-Q}{RT} \right]$$

(cf. equation 9), in the following way.



The element stress is taken at point ii and the strain rate calculated using the average absolute temperature and the known constants.

From the known strain rate, referenced to point ii and assumed constant throughout the element, the velocity of the point jj and ll is found relative to point ii and kkc respectively. Hence we may calculate new z coordinates for the points jj and ll. The condition of constant volume enables the new r coordinate of points jj and ll to be found, and the engineering strain can be calculated. The new

coordinates of jj and ll are shown above in the points a and b respectively, and these become the ii and kk for the next element.

The stress for the next element is calculated from:

$$\sigma_{jj} = \frac{r_{kk}^2 - r_{ii}^2}{r_{ll}^2 - r_{jj}^2} \cdot \sigma_{ii}$$

and the computer program continues for each element in the tube and then proceeds with the remaining elements in precisely the same way.

The mesh is now advanced a step into the "heated zone". Several elements of "undeformed bar" are added to simulate the motion into the coil and an equal number of elements removed from the "drawn bar". The new nodal points are initialized and a new temperature distribution found. The temperature distribution on the second iteration is appreciably modified because, instead of "seeing" a constant cross-section as in the first iteration, the coordinates of the mesh are appreciably changed. The program now recalculates a new set of coordinates, stresses, strains, and strain rates.

It was found that after only five iterations a "stable draw" is established and very little computer time is needed to obtain the profile of the bar.

Some considerable time was spent using this program together with some of the relationships which have been discussed in Section 3.2.3. No significant progress was made and only the results from the creep

equation (9) are included.

A useful feature of the program is that a limiting velocity can be set so that the hotter elements, contained by cooler material, do not accelerate out of the centre of the bar.

A plot of the stabilized mesh coordinates is shown in Figure 36, using a typical temperature profile obtained when drawing En 58 J stainless steel. The initial stress value, coil velocity and limiting or final velocity were taken from drawing experiments where the percentage reduction in area was 50% and the constants from Sellars and Tegart⁽¹⁰⁾. An experimental profile is also shown and makes a fair comparison.

The strain rates determined from the pure tension program are shown in Figure 42. The curve obtained from an experimental deformation zone assuming plane sections remain plane (see Section 3.4.2.) is included in the figure.

3.4.4. Finite Element Method

The finite element work began by applying experimentally determined temperature distributions to the finite element displacement method developed by Gunasekera⁽³³⁾ and Haines⁽³⁴⁾. Small incremental displacement steps were applied to the mesh representing an undeformed bar. Once again the "heated portion" was fixed and the output or "drawn" displacements set at twice the input displacement for a percentage reduction in area of 50%. It was hoped that the solution

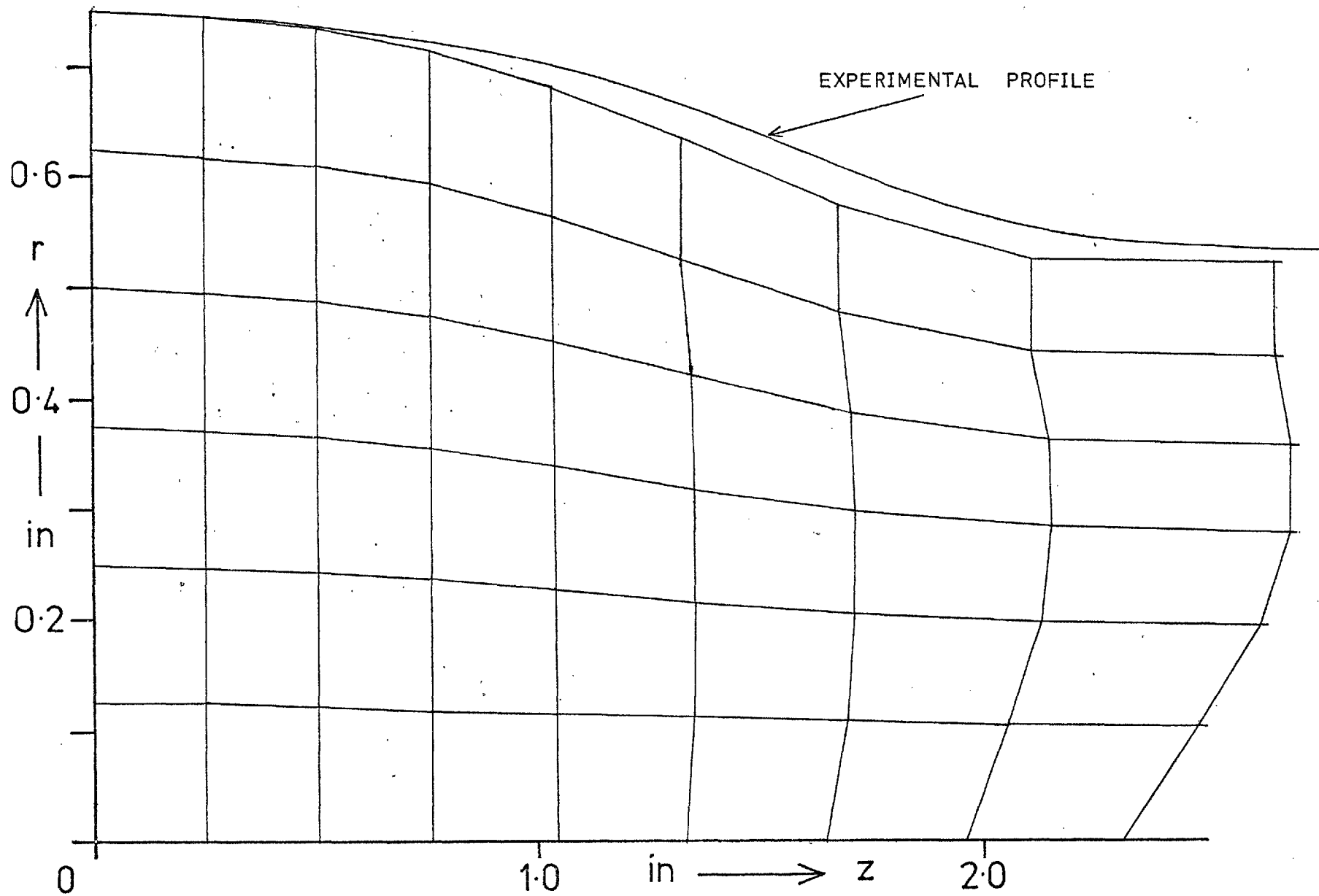


FIGURE 36 MESH DEFORMATION OBTAINED FROM SIMPLE TENSION ANALYSIS

to the transient stage would approach the steady state solution, but little progress was made and because of the excessive computer time requirements the approach was dropped.

The main effort into a finite element analysis of the process developed from the techniques discussed in Section 1.3.1. using the displacement method of the elastic stress-strain program.

The strain-displacement relationship is derived from a linear representation of the r and z displacements in terms of the coordinates.

$$\{e\} = [B]\{\delta\}^e$$

where

$$\{e\} = \begin{Bmatrix} e_z \\ e_r \\ e_\theta \\ e_{rz} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial v'}{\partial z} \\ \frac{\partial u'}{\partial r} \\ r/u' \\ \frac{\partial u'}{\partial z} + \frac{\partial v'}{\partial r} \end{Bmatrix} \quad (22)$$

$$[B] = [B_i, B_j, B_p]$$

where

$$[B_i] = \frac{1}{2\Delta} \begin{bmatrix} 0 & c_i \\ b_i & 0 \\ a_i/r + b_i + c_i z/r & 0 \\ c_i & b_i \end{bmatrix}$$

$$a_i = r_j z_p - r_p z_j$$

$$b_i = z_j - z_p$$

$$c_i = r_p - r_j$$

Equation(22) may be differentiated with respect to time to give

$$\{\dot{\epsilon}\} = [B] \{\dot{\delta}\}^e \quad (23)$$

where the matrix $[B]$, composed of elements related only to the r and z coordinates of the mesh, remains unchanged.

The elasticity matrix $[D]$ which relates the stresses and strains is derived from the elastic equations for an isotropic material:-

$$\begin{aligned} \epsilon_z &= \frac{1}{E} \left\{ \sigma_z - \nu(\sigma_r + \sigma_\theta) \right\} \\ \epsilon_r &= \frac{1}{E} \left\{ \sigma_r - \nu(\sigma_\theta + \sigma_z) \right\} \\ \epsilon_\theta &= \frac{1}{E} \left\{ \sigma_\theta - \nu(\sigma_z + \sigma_r) \right\} \\ \epsilon_{rz} &= \frac{2(1 + \nu)}{E} \cdot \tau_{rz} \end{aligned} \quad (24)$$

We have, on solving for the stresses

$$\{\sigma\} = \begin{Bmatrix} \sigma_z \\ \sigma_r \\ \sigma_\theta \\ \tau_{rz} \end{Bmatrix} = [D] \{\epsilon\} \quad (25)$$

$$\text{where } D = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1 & \frac{\nu}{1-\nu} & \frac{\nu}{1-\nu} & 0 \\ \frac{\nu}{1-\nu} & 1 & \frac{\nu}{1-\nu} & 0 \\ \frac{\nu}{1-\nu} & \frac{\nu}{1-\nu} & 1 & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2(1-\nu)} \end{bmatrix} \quad (26)$$

Recalling the secondary creep equation (7) we can write the axial component of the strain rate

$$\begin{aligned} \dot{\epsilon}_z &= K'' \bar{\sigma}^{n-1} \exp \left[\frac{-Q}{RT} \right] \sigma_z' \\ &= \frac{2}{3} K'' \bar{\sigma}^{n-1} \exp \left[\frac{-Q}{RT} \right] \left[\sigma_z - \frac{1}{2} (\sigma_r + \sigma_\theta) \right] \end{aligned} \quad (27)$$

and once again we can see the similarity between equations (24) and (27). We must replace ν by $\frac{1}{2}$ and E by $\left\{ \frac{2}{3} K'' \bar{\sigma}^{n-1} \exp \left[\frac{-Q}{RT} \right] \right\}^{-1}$ to make the equations (24) and (27) identical if the strains are replaced by strain rates and equation (25) above becomes:

$$\{\sigma\} = [D] \{\dot{\epsilon}\} \quad (28)$$

where the stress-strain matrix, $[D]$, becomes a stress-strain rate matrix the components of which are identical, i.e. equation (26) defining $[D]$ above is unchanged.

The element stiffness is now found, using an approximation to simplify the computation of the volume integral which must be taken over the complete ring of material in the axisymmetrical analysis.

$$[k] = 2\pi [B]^T [D] [B] \bar{r} \Delta \quad (29)$$

where

$$\bar{r} = \frac{(r_i + r_j + r_p)}{3}$$

and the $[B]$ matrix is evaluated using the coordinates of the centroid.

The mathematical model which reduces the elements to a set of stiffnesses, solves the elastic problem by setting the forces equal to the stiffnesses multiplied by the displacements,

$$\{F\}^e = \Delta \cdot 2\pi \bar{r} [B]^T [D] [B] \{\delta\}^e \quad (30)$$

When applied to elastic problems only very small displacements of the mesh are encountered, but noting that the strain-displacement relationship can be differentiated to give a strain rate - velocity relation, equation 23, it would appear possible to rewrite equation (30) above relating forces to velocities:

$$\{F\}^e = \Delta \cdot 2\pi \bar{r} [B]^T [D] [B] \{\dot{\delta}\}^e \quad (31)$$

The only difference is that strains are replaced by strain rates and displacements are replaced by velocities.

Application to plasticity problems with large displacements appears straightforward but Poisson's ratio, ν , for an incompressible material is 0.5 and the $[D]$ matrix becomes singular because of the $1 - 2\nu$ term in the denominator of equation (26). Hermann⁽³⁵⁾ resolves the problem using a variational theorem which is valid for both

compressible and incompressible materials. He formulates elastic field equations by utilizing displacements and a function of the mean pressure, but the computation time is appreciably increased by the necessity of solving for the mean pressure function in each element. By putting a value close but not equal to 0.5, the components of the $[D]$ matrix can be evaluated and some progress can nevertheless be made.

The mesh which is generated represents a constant cross-section specimen and the program is run to obtain a set of axial and radial velocities at each nodal point. Since the direction of material flow is defined by these velocities, the data may be used to modify the coordinates of the mesh in the following way. In figure 37 the arrowed lines represent the directions of the material flow which we express as a slope, S .

$$S = \frac{V_r}{V_z}$$

The intermediate nodal point, I , has a slope, S_I , represented by the dotted line IA . IB is the slope, S_J , of the adjacent nodal point, and the new radial coordinate of nodal point, J , is modified to a position midway between points A and B .

$$r_J = r_I + \frac{(S_J + S_I) \cdot Z_n}{2} \quad (32)$$

A single modification of the mesh causes appreciable "plastic deformation" and if the mesh is refined (Figure 37 represents a very course mesh)

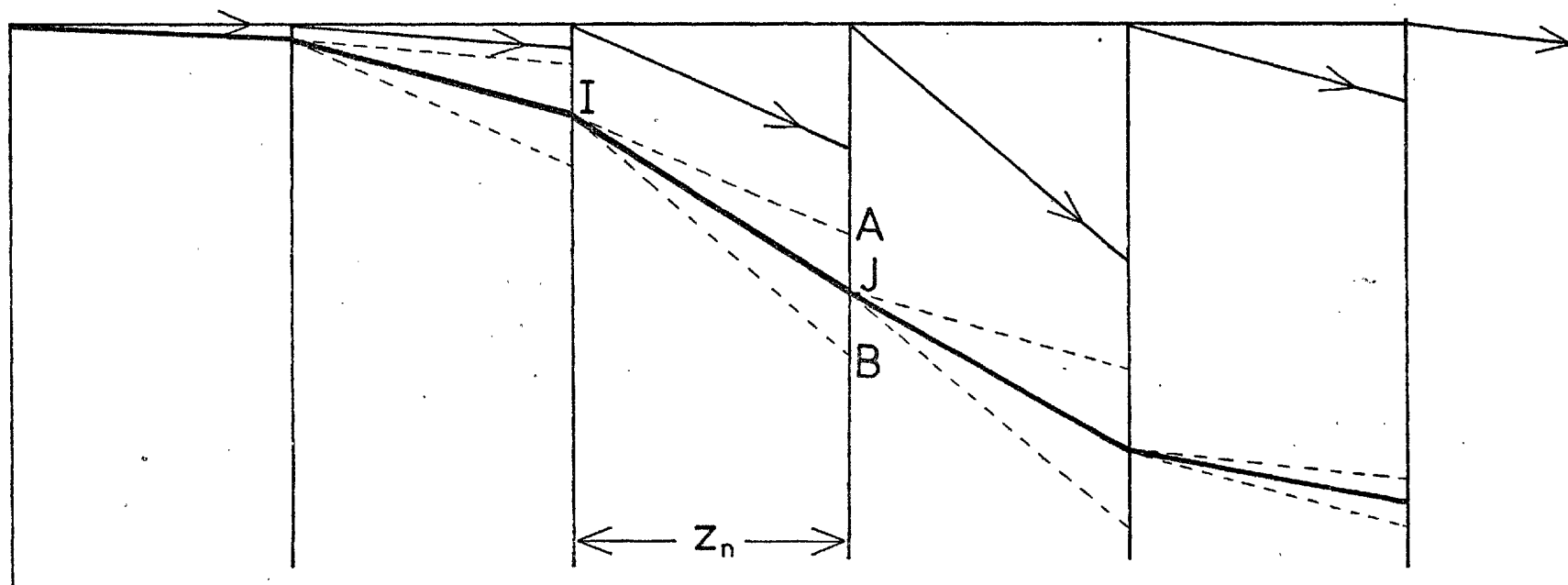


FIGURE 37 MODIFICATION OF COORDINATES USING VELOCITY SLOPES

the profile of the bar and flow lines in the material are found in very few iterations. With the fine mesh used in the present work it was necessary to use an under-relaxation factor of 10 for several iterations to avoid crossed flow lines. The factor was removed after ten iterations.

Having modified the coordinates of the mesh, the program proceeds to iterate on the stresses using the equation of state,

$$\bar{\sigma} = K' \bar{\epsilon}^{1/n} \exp\left[\frac{Q}{nRT}\right] \quad (33)$$

The absolute temperatures at each nodal point were calculated using the finite element technique separately and read in as data. New velocities were calculated again and the program continued to give sufficient iterations for close tolerances.

Summarizing the program we have: -

1. Initialize and generate the constant cross-section mesh. Read in the temperatures.
2. Evaluate the strain rate-displacement matrix, $[B]$.
3. Calculate the areas of the elements
4. Evaluate the stress matrix, $[D]$, and $[B]^T [D] [B]$.
5. Initialize the loads

6. Invert the stiffness matrix
7. Set the boundary conditions, initial and final velocities and zero radial velocities at entry and along the centre line
8. Iterate to give axial and radial velocities at each nodal point. Between 200 and 1500 iterations are required.
9. Modify the coordinates to fit the slopes determined from the r and z velocity ratios.
10. Compute the stresses and strain rates.
11. Iterate equivalent stresses. (~ 50 iterations)
12. Return to 3.
13. Calculate the forces
14. Print the solution.

Work was carried out using two meshes, one, the same as used for the simple tension analysis shown in Figure 31, and a finer mesh shown in Figure 38. Deformation zone profiles for stainless steel are shown in Figure 39, 40, and 41. The somewhat unsatisfactory strain rate variations, shown in Figure 42, show that the finite element approach requires further work before the analysis can be used with

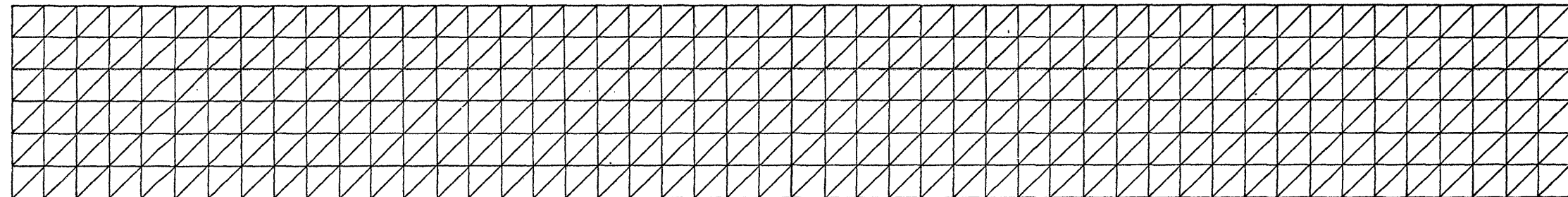


FIGURE 38 FINE MESH USED FOR FINITE ELEMENT ANALYSIS (576 ELEMENTS)

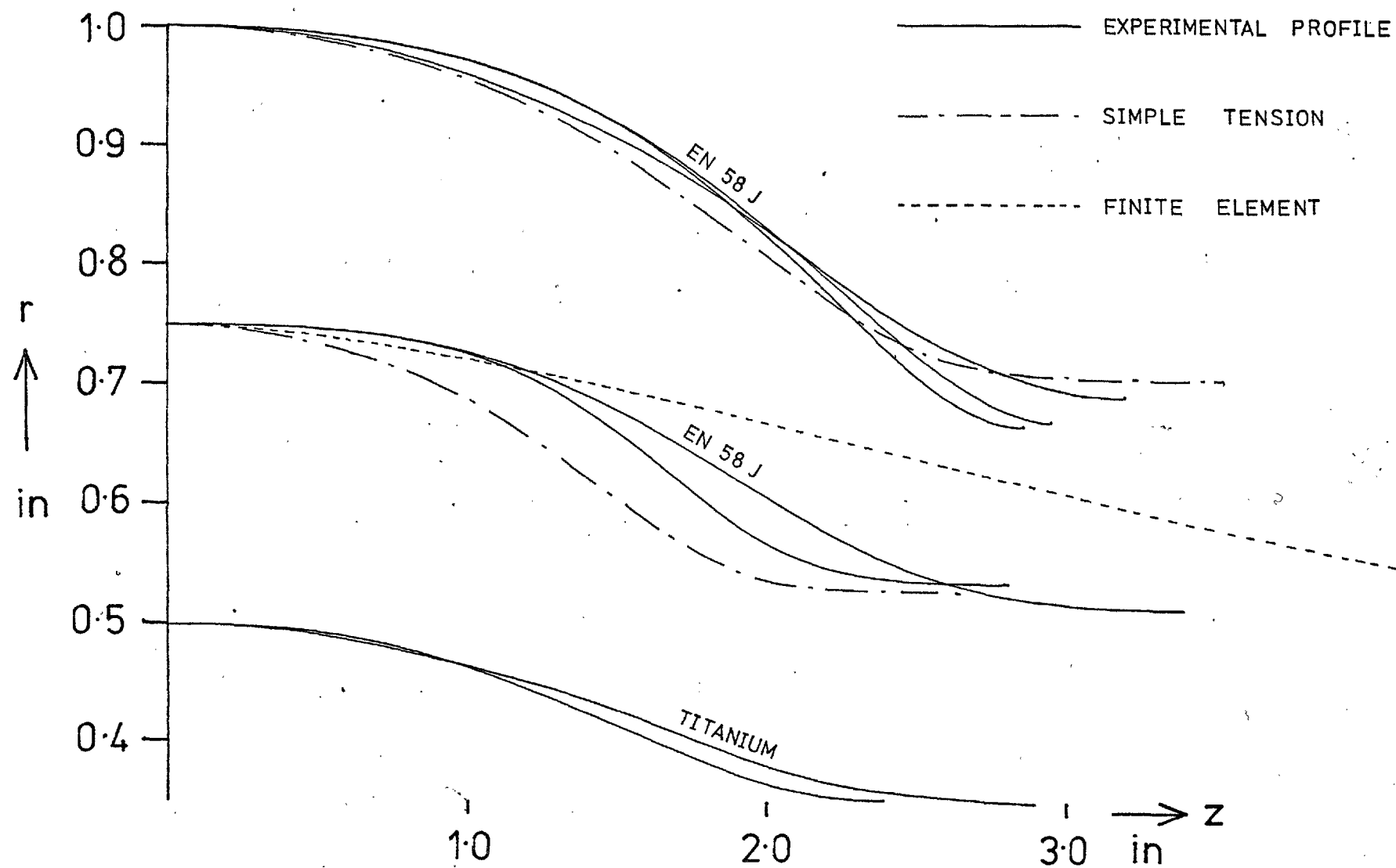


FIGURE 39 DEFORMATION ZONE PROFILES FOR EN 58 J STAINLESS STEEL AND Ti-6Al-4V TITANIUM 50% REDUCTION IN AREA

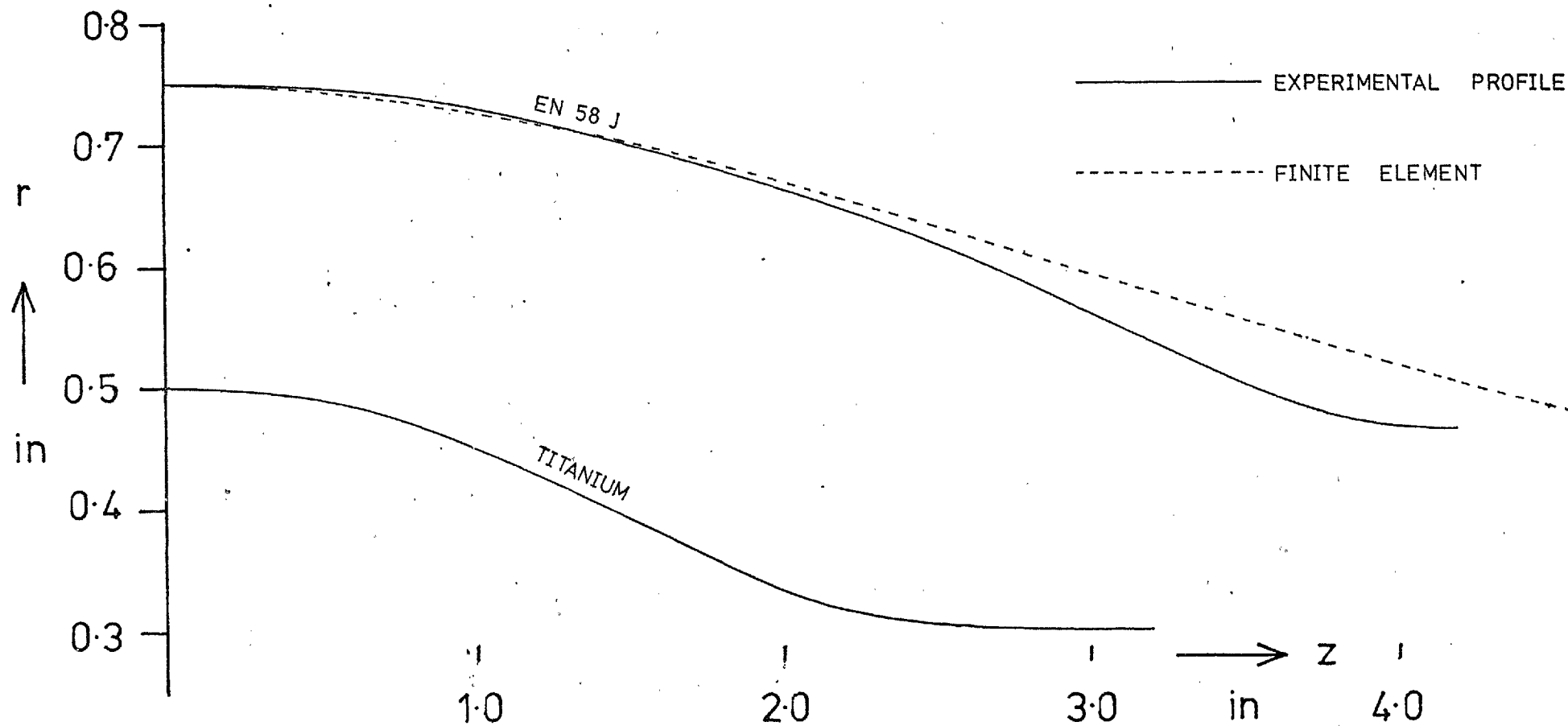


FIGURE 40 DEFORMATION ZONE PROFILES FOR EN 58 J STAINLESS STEEL AND Ti-6Al-4V TITANIUM 66% REDUCTION IN AREA

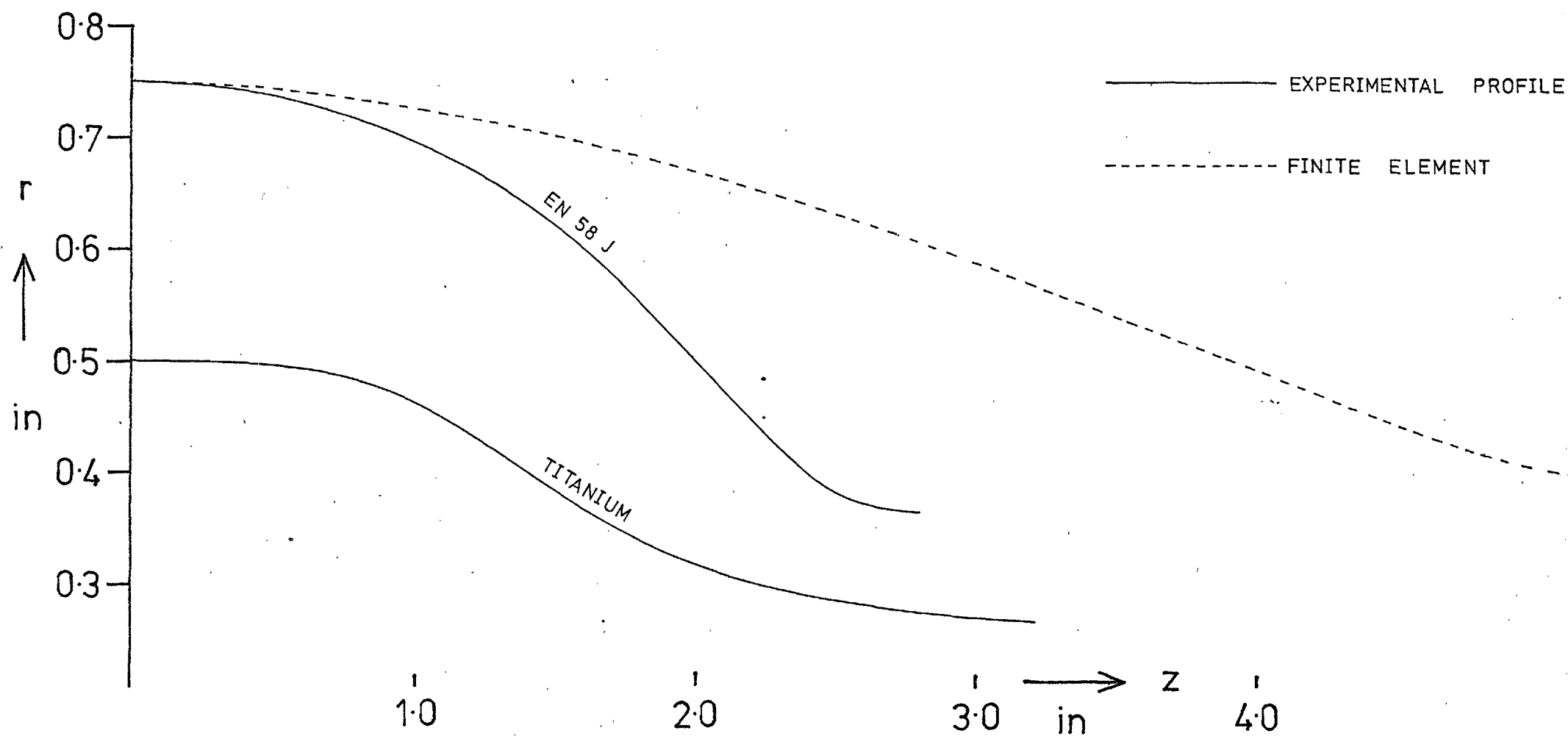


FIGURE 41 DEFORMATION ZONE PROFILES FOR EN 58 J STAINLESS STEEL AND Ti-6Al-4V TITANIUM 75% REDUCTION IN AREA

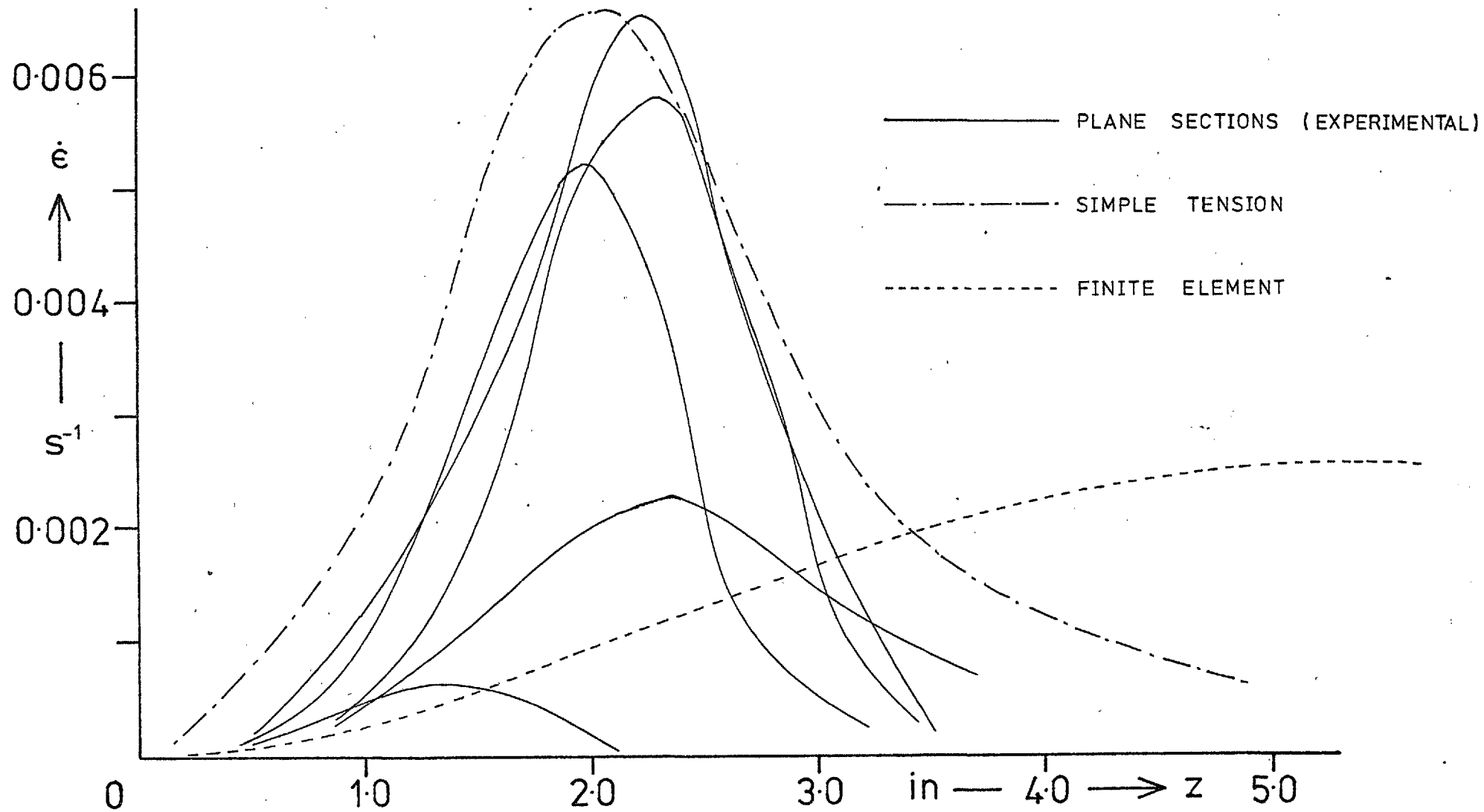


FIGURE 42 STRAIN RATE VARIATIONS ALONG THE DEFORMATION ZONE EN 58 J STAINLESS STEEL

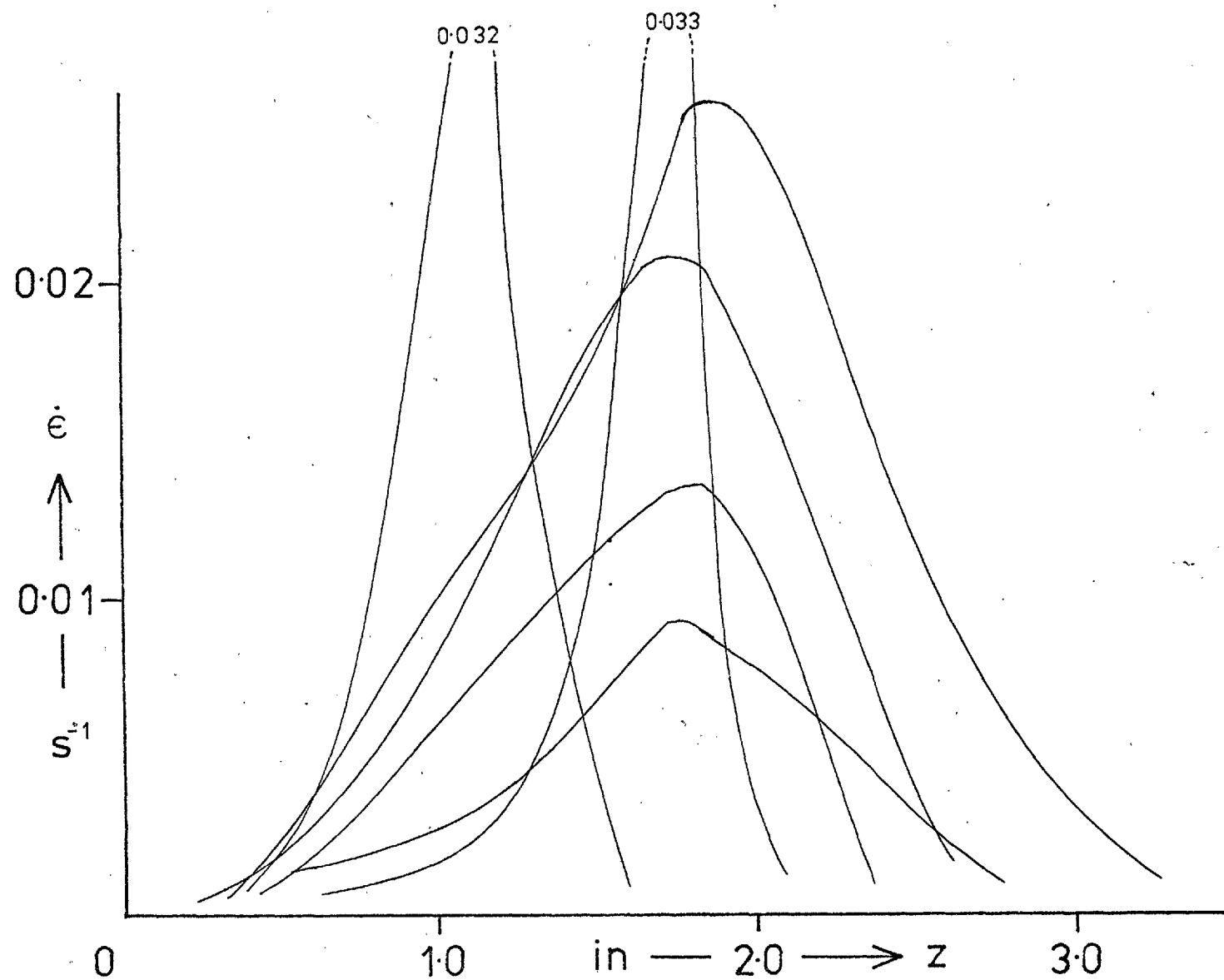


FIGURE 43 STRAIN RATE VARIATIONS ALONG THE DEFORMATION ZONE Ti-6Al-4V TITANIUM

confidence. This point is taken up in the discussion.

3.4.5 Stress Distribution

Throughout all the work on finite element analysis, oscillations of the elemental stresses were very apparent particularly when Poisson's ratio approached 0.5. A series of computer runs was carried out varying ν from 0.48 to 0.499999 and an "optimum" value found, $\nu = 0.4975$, where the profile of the deforming zone was closest to the experimental shape. At this "optimum" value the oscillations, negligible at $\nu = 0.48$ and severe at $\nu = 0.4975$, were quite marked and an averaging method was devised to give the nodal stress values. This effectively removed the oscillations.

When the mesh was refined the profile obtained was a little further from the experimental curve and it was clear that the chosen finite element approach was not converging towards the experimental solution. The point is discussed more fully in the next chapter. It was nevertheless decided to present the stress distributions obtained in the analysis as they may prove useful for future comparisons.

Figures 44 and 45 show the stress contours for a 50% reduction in area of stainless steel with meshes of 192 and 576 elements respectively. The first half inch of the figures in these and the subsequent figures are distorted by the boundary conditions. Some oscillations are apparent in the finer mesh hoop and shear stresses. Axial stress contours (in tension) show similar form in both meshes, but the increase

in stress as one proceeds from the centre line towards the outside of the bar is reduced in the fine mesh. Radial, hoop, and shear stress show a marked change.

Axial and shear stresses show similar forms in Figure 46 and 47 which represent a 66% reduction in area. The radial and hoop stresses are quite different however, and oscillations in the finer mesh are present again.

Comparison of the stresses, which are shown in Figures 48 and 49 for a 75% reduction in area, demonstrate similar forms in all four cases.

3.5

THEORETICAL AIMS

3.5.1. Data Analysis

A feature of the whole project has been the lack of data available, particularly in the range of strain rates associated with the process. The process, however, gives a deformation zone which can be accurately measured, and rather than starting from a constant cross-section mesh, the accurately measured zone can be taken as the starting point. This approach, the other way round from the analysis previously described in the present work, will enable data to be obtained for the drawn material.

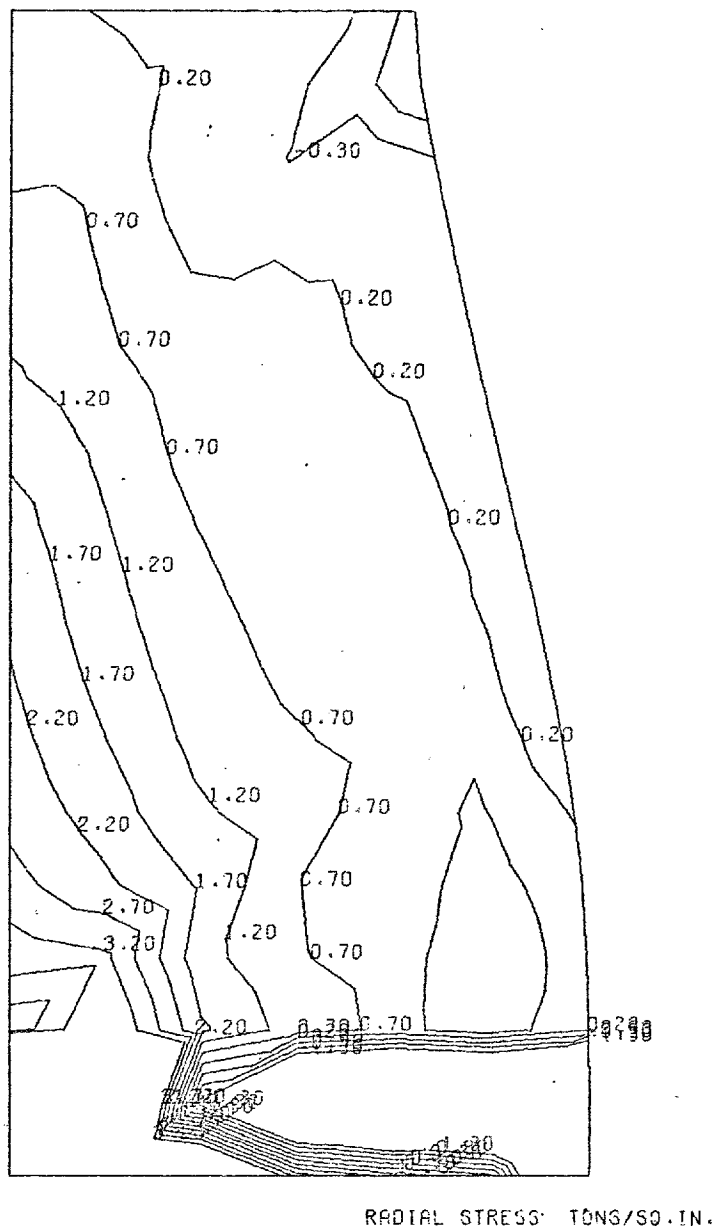


FIGURE 44 STRESS CONTOURS FROM THE FINITE
ELEMENT ANALYSIS FOR STAINLESS
STEEL AT 50% REDUCTION IN AREA
(192 ELEMENTS)

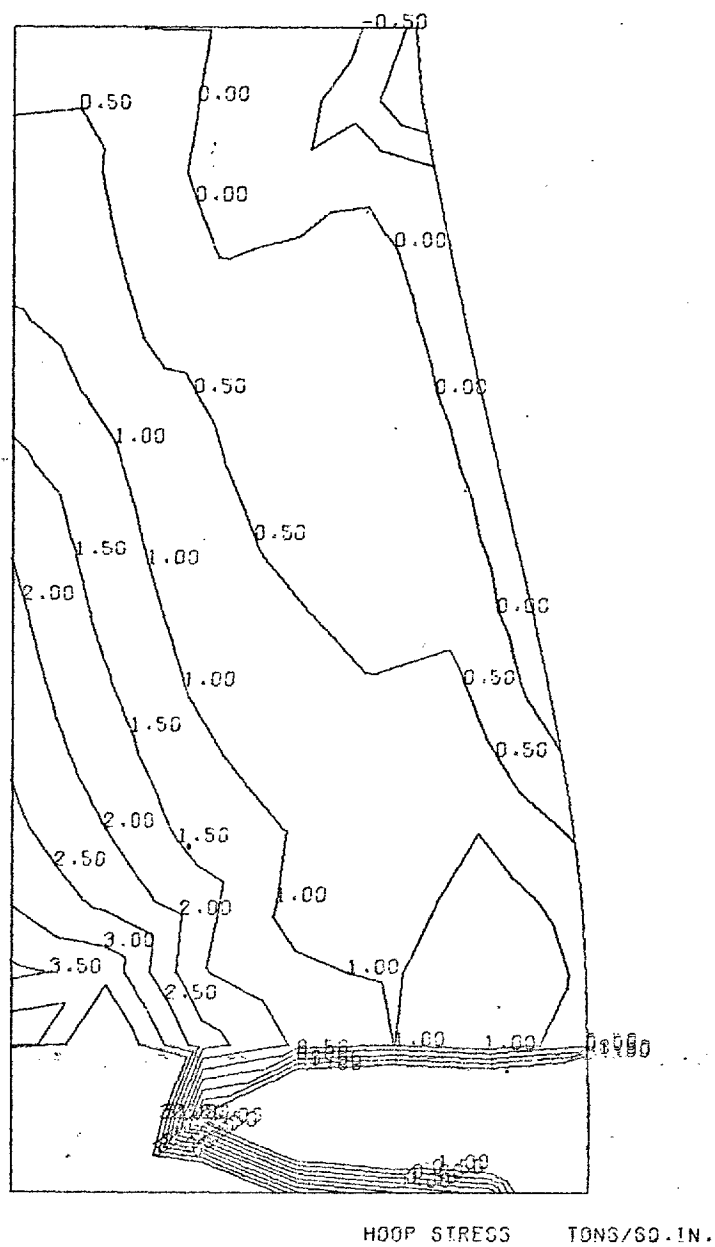


FIGURE 44 STRESS CONTOURS FROM THE FINITE
ELEMENT ANALYSIS FOR STAINLESS
STEEL AT 50% REDUCTION IN AREA
(192 ELEMENTS)

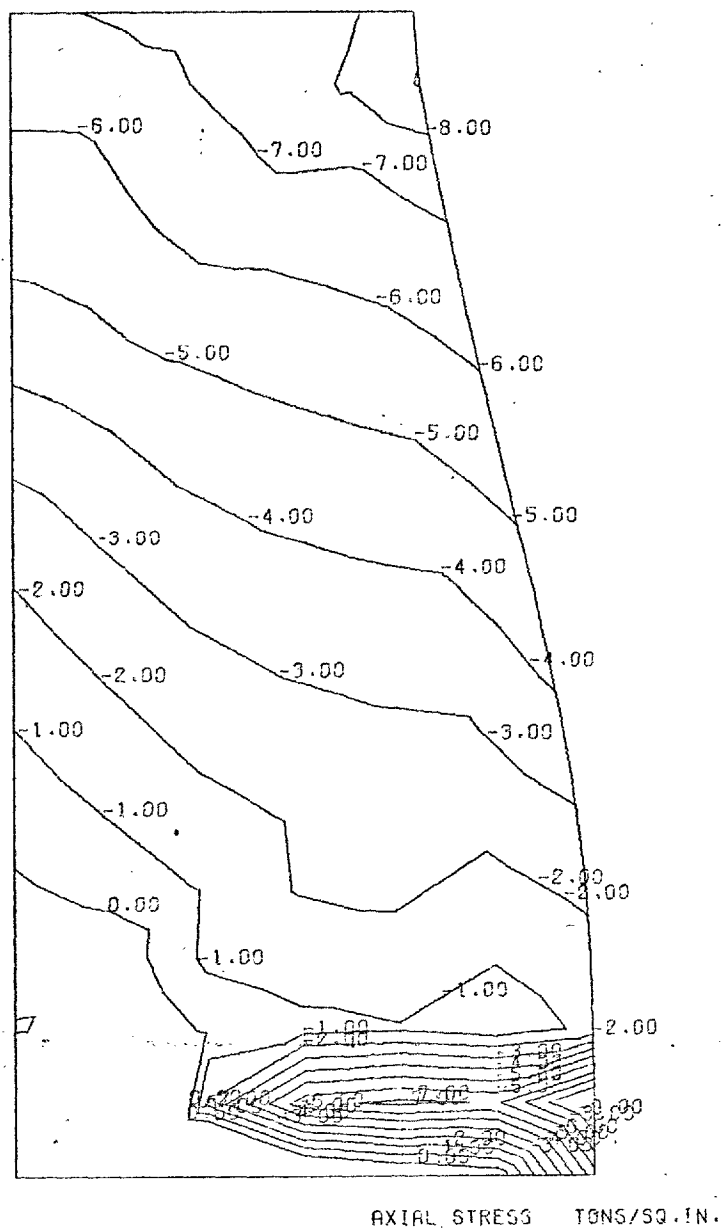


FIGURE 44 STRESS CONTOURS FROM THE FINITE
ELEMENT ANALYSIS FOR STAINLESS
STEEL AT 50% REDUCTION IN AREA
(192 ELEMENTS)

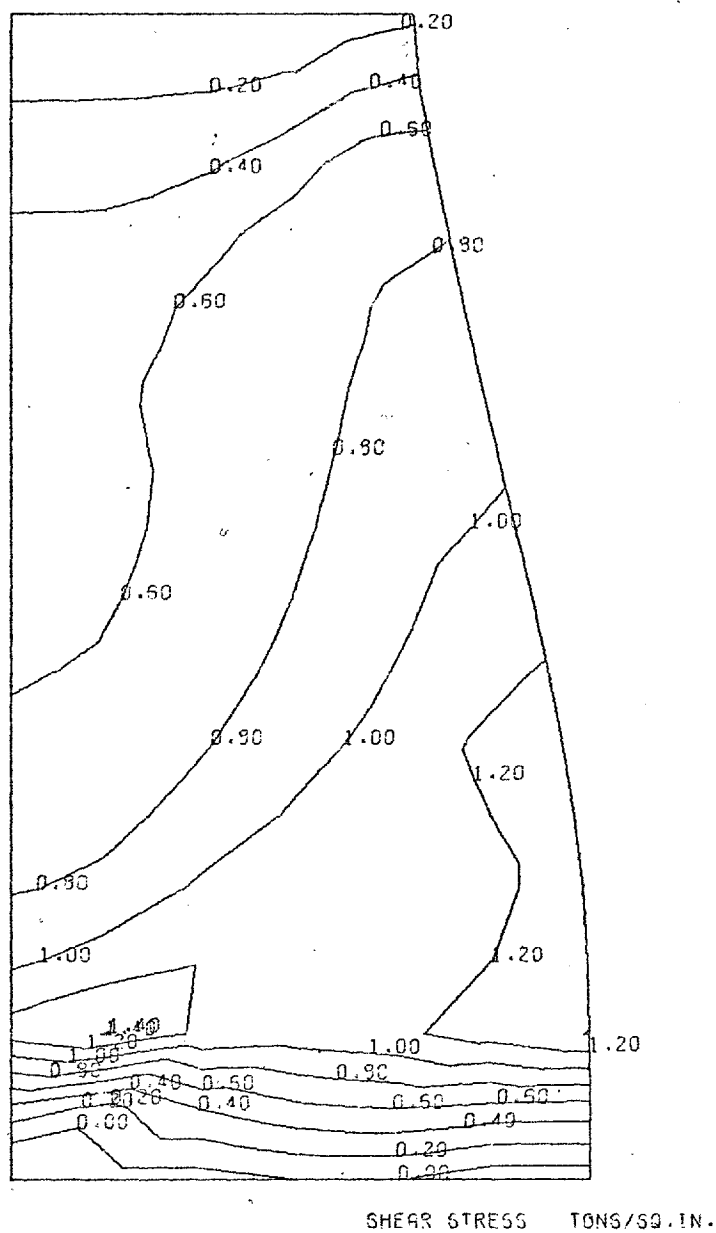
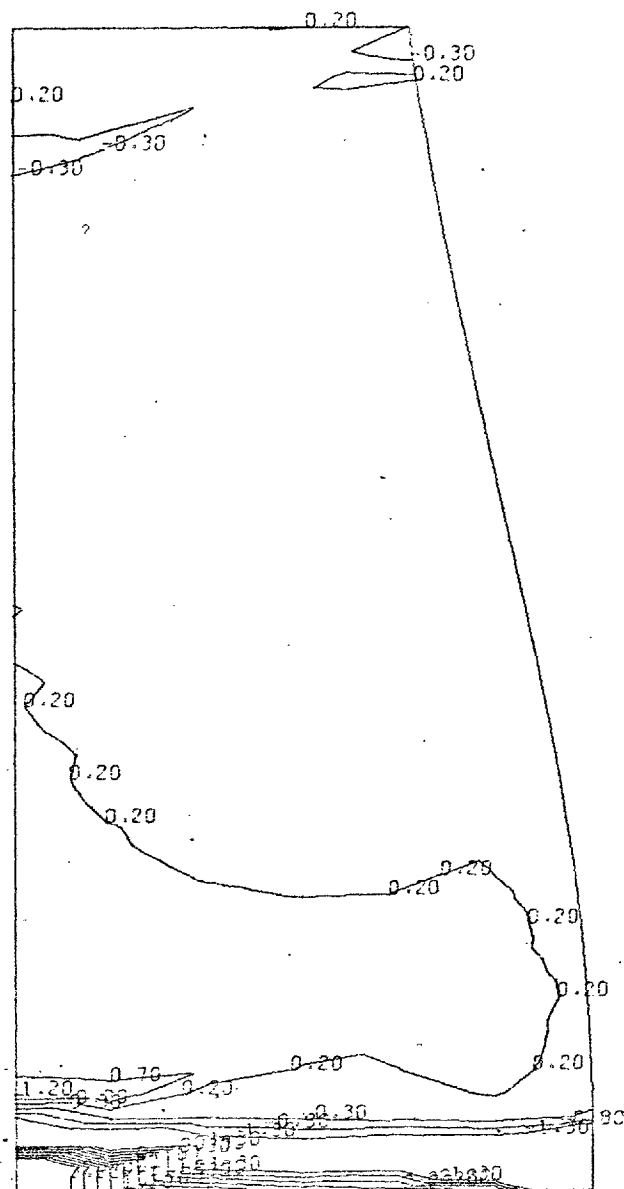


FIGURE 44 STRESS CONTOURS FROM THE FINITE
ELEMENT ANALYSIS FOR STAINLESS
STEEL AT 50% REDUCTION IN AREA
(192 ELEMENTS)



RADIAL STRESS TNS/SQ. IN.

FIGURE 45 STRESS CONTOURS FROM THE FINITE
ELEMENT ANALYSIS FOR STAINLESS
STEEL AT 50% REDUCTION IN AREA
(576 ELEMENTS)

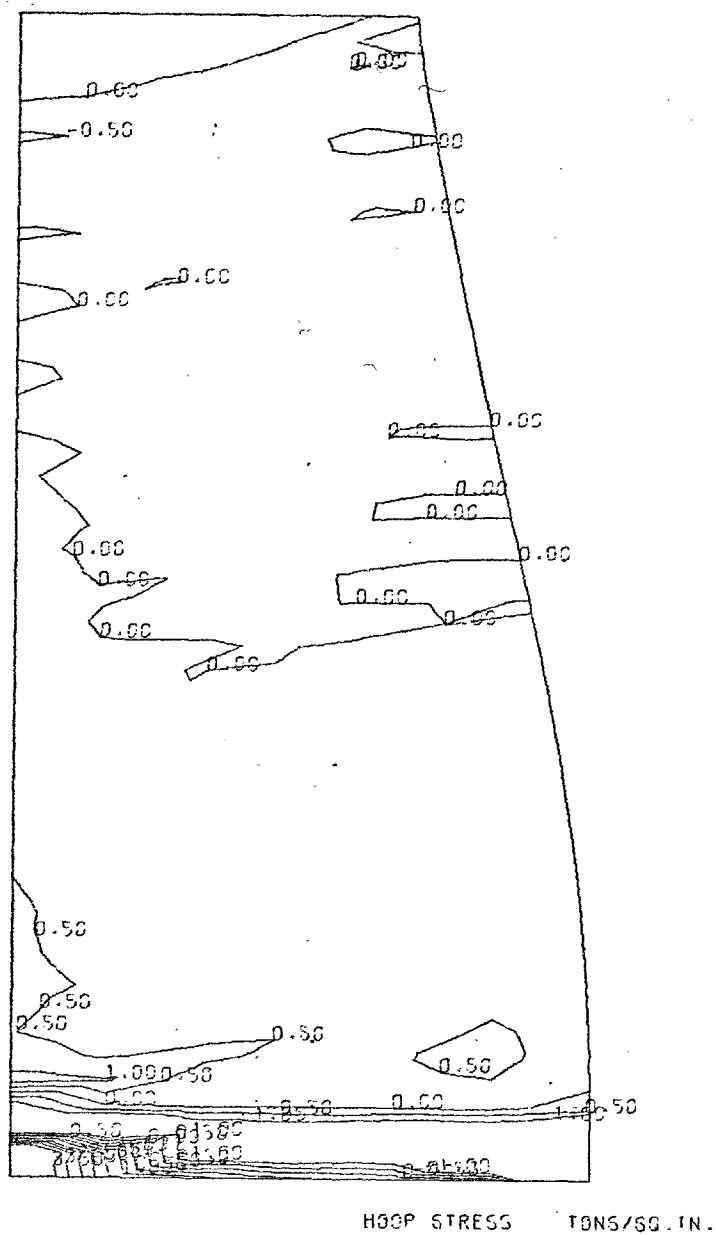


FIGURE 45 STRESS CONTOURS FROM THE FINITE
ELEMENT ANALYSIS FOR STAINLESS
STEEL AT 50% REDUCTION IN AREA
(576 ELEMENTS)

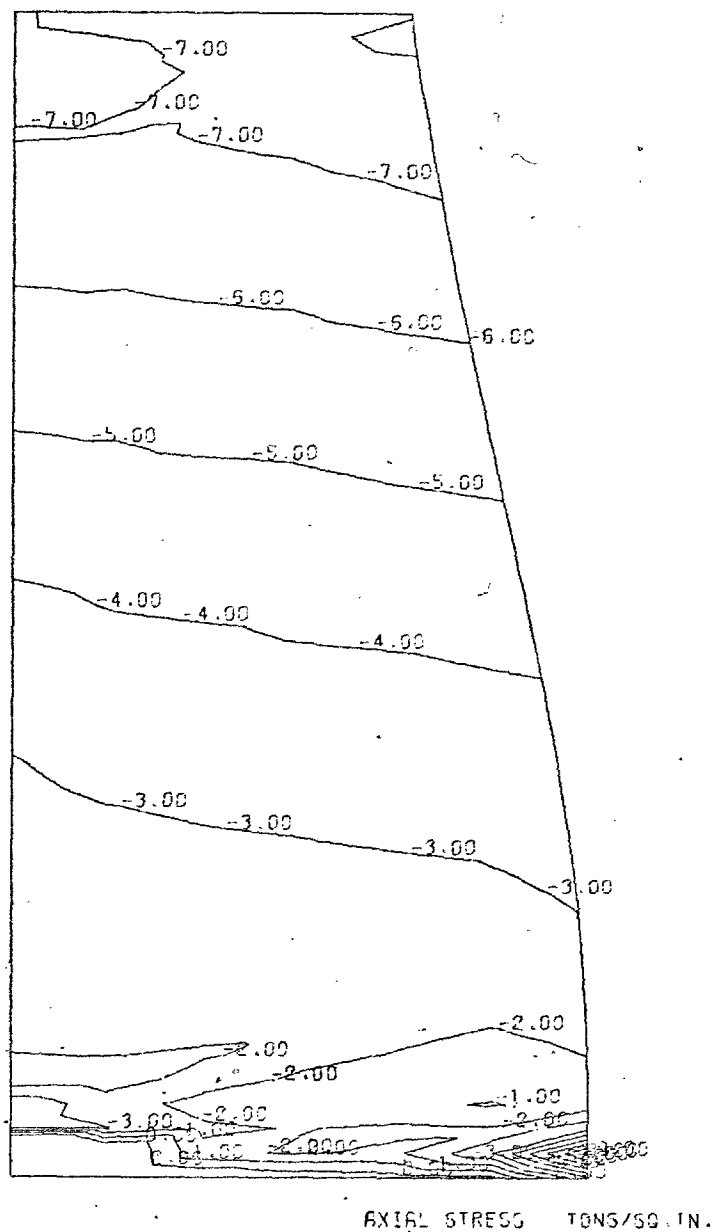


FIGURE 45 STRESS CONTOURS FROM THE FINITE
ELEMENT ANALYSIS FOR STAINLESS
STEEL AT 50% REDUCTION IN AREA
(576 ELEMENTS)

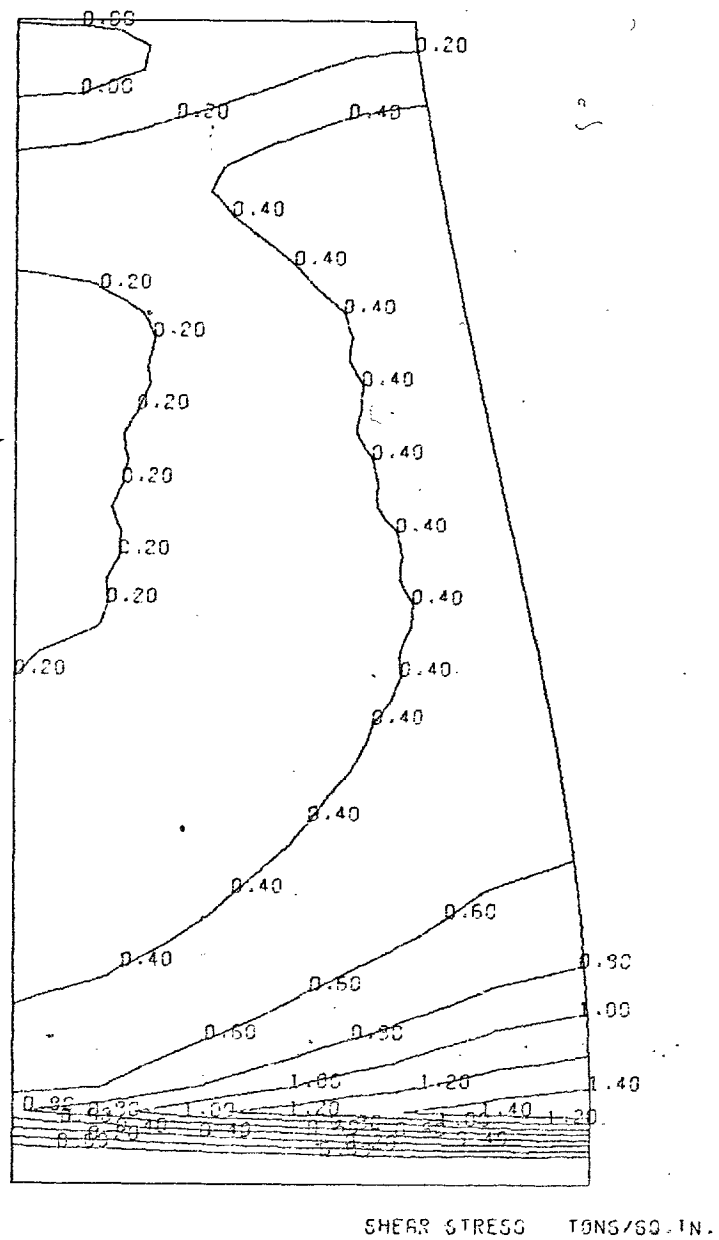


FIGURE 45 STRESS CONTOURS FROM THE FINITE
ELEMENT ANALYSIS FOR STAINLESS
STEEL AT 50% REDUCTION IN AREA
(576 ELEMENTS)

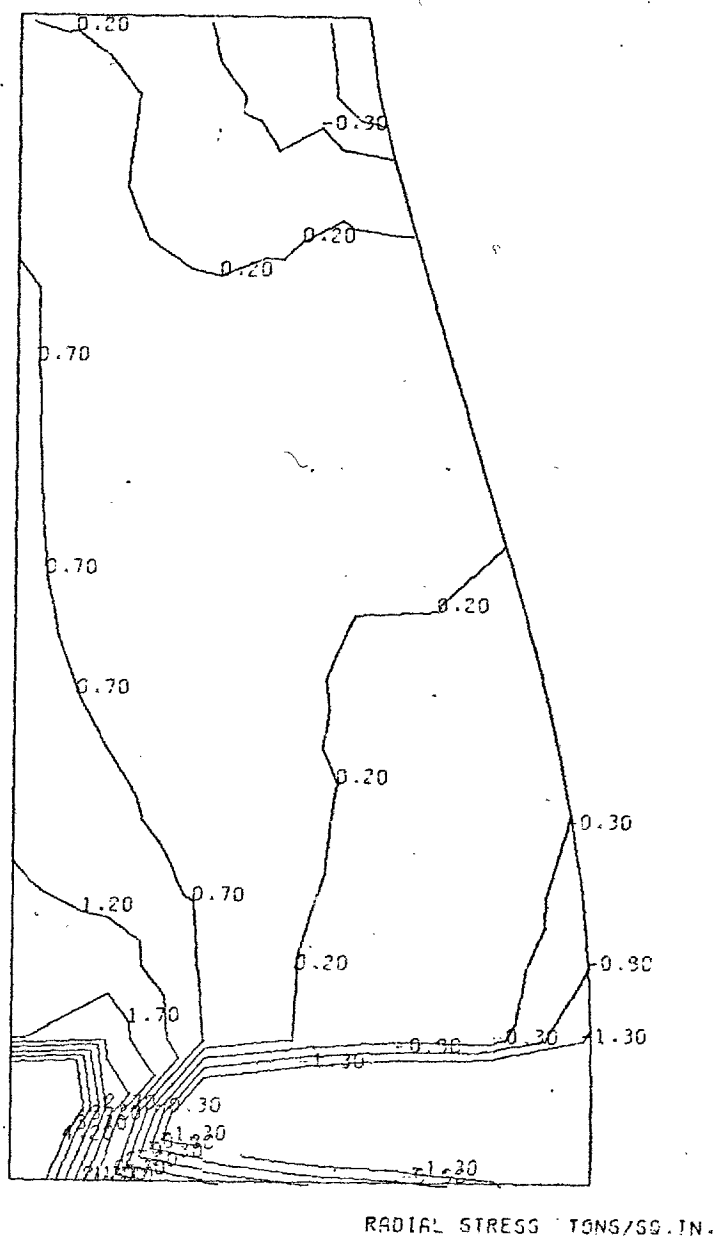


FIGURE 46 STRESS CONTOURS FROM THE FINITE
ELEMENT ANALYSIS FOR STAINLESS
STEEL AT 66% REDUCTION IN AREA
(192 ELEMENTS)

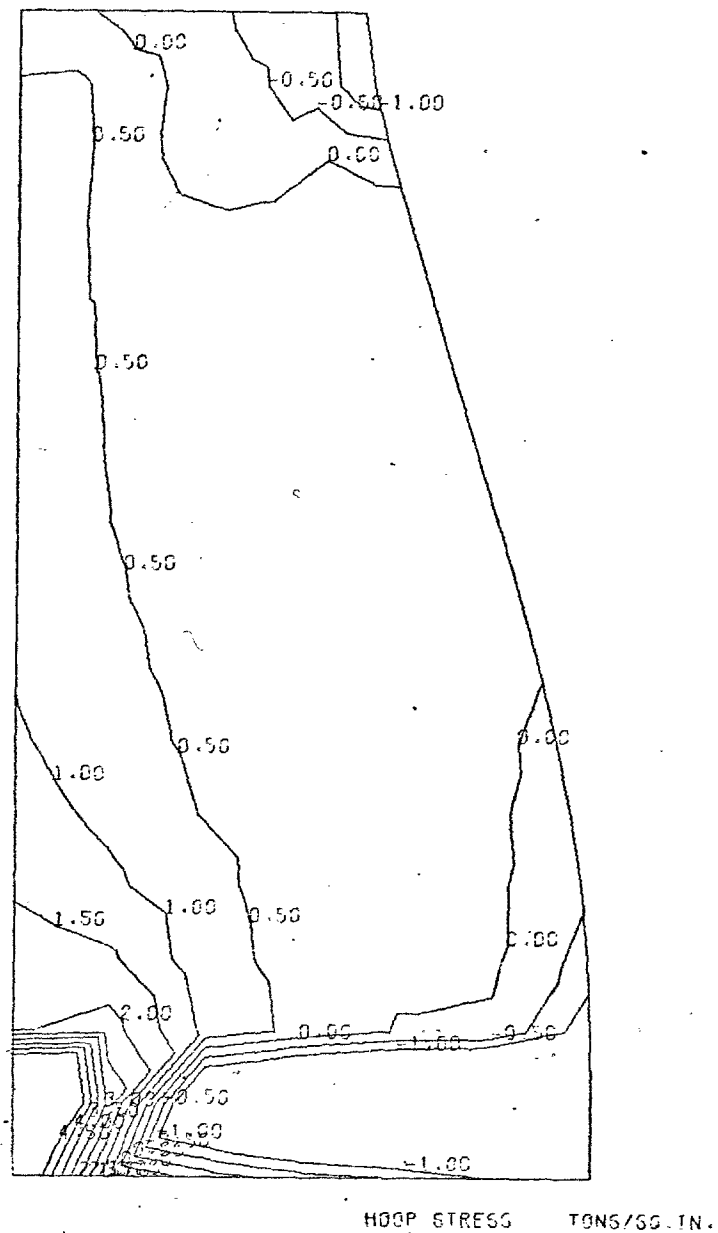


FIGURE 46 STRESS CONTOURS FROM THE FINITE
ELEMENT ANALYSIS FOR STAINLESS
STEEL AT 66% REDUCTION IN AREA
(192 ELEMENTS)

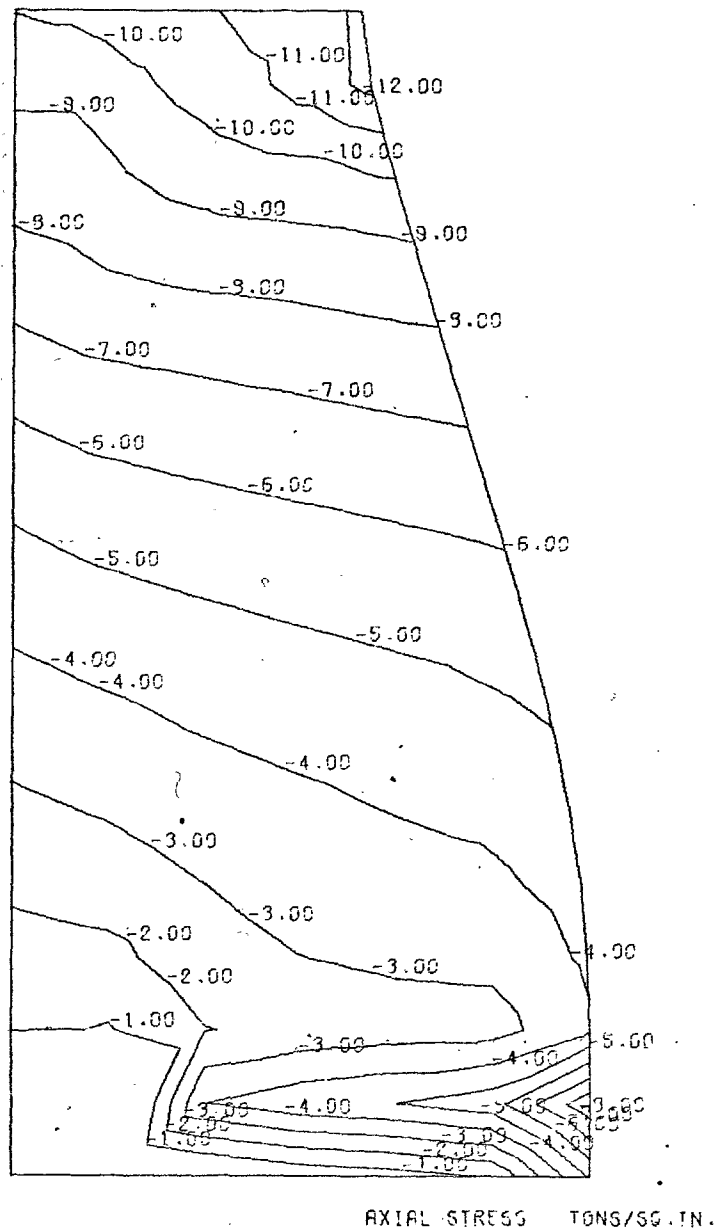


FIGURE 46 STRESS CONTOURS FROM THE FINITE
ELEMENT ANALYSIS FOR STAINLESS
STEEL AT 66% REDUCTION IN AREA
(192 ELEMENTS)

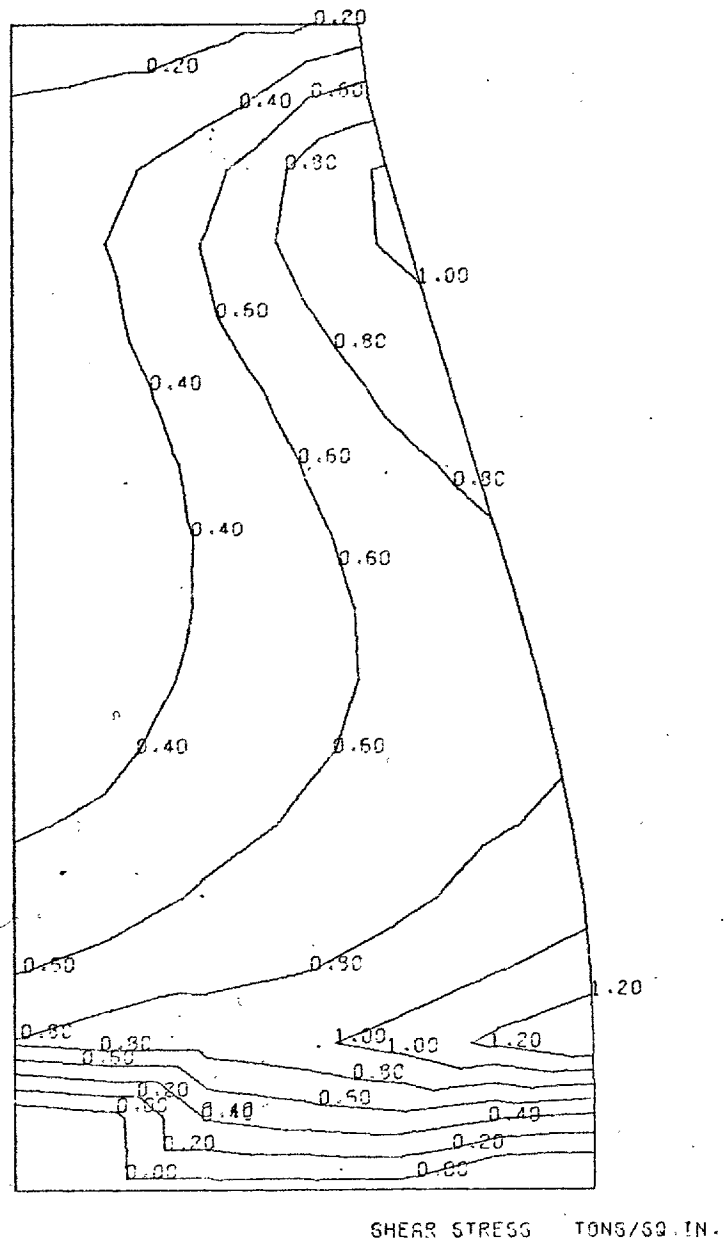


FIGURE 46 STRESS CONTOURS FROM THE FINITE
ELEMENT ANALYSIS FOR STAINLESS
STEEL AT 66% REDUCTION IN AREA
(192 ELEMENTS)

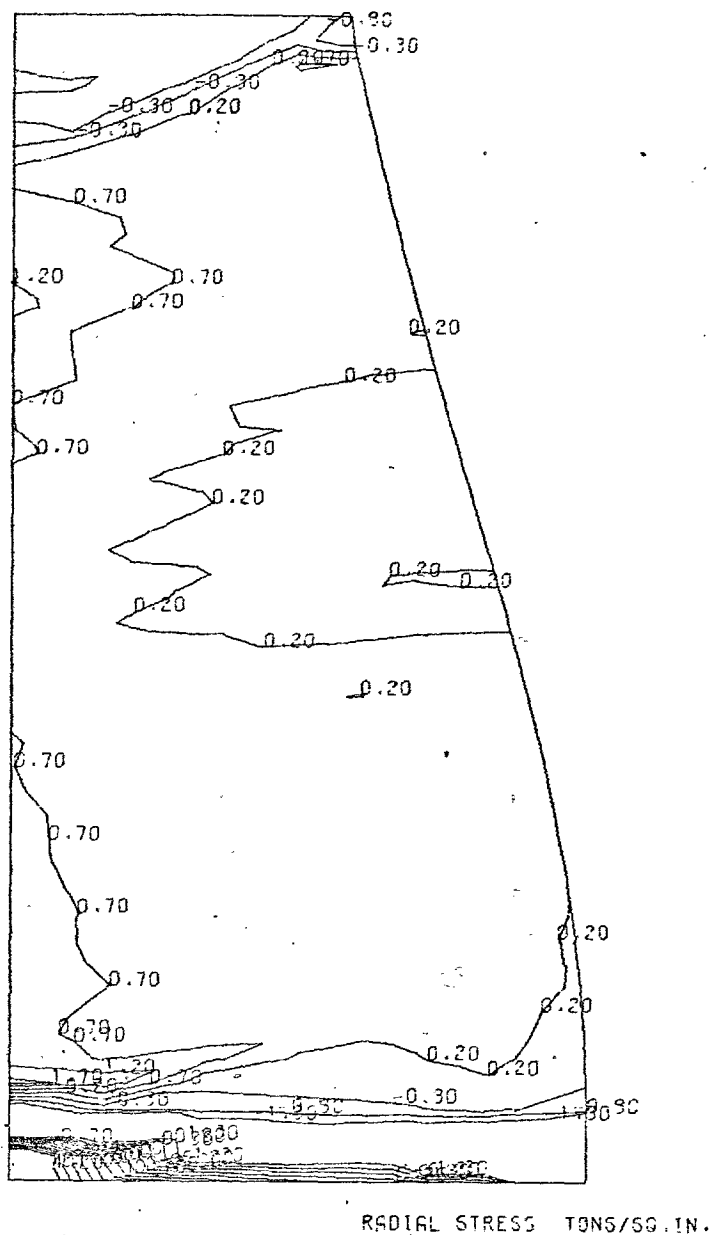


FIGURE 47 STRESS CONTOURS FROM THE FINITE
ELEMENT ANALYSIS FOR STAINLESS
STEEL AT 66% REDUCTION IN AREA
(576 ELEMENTS)

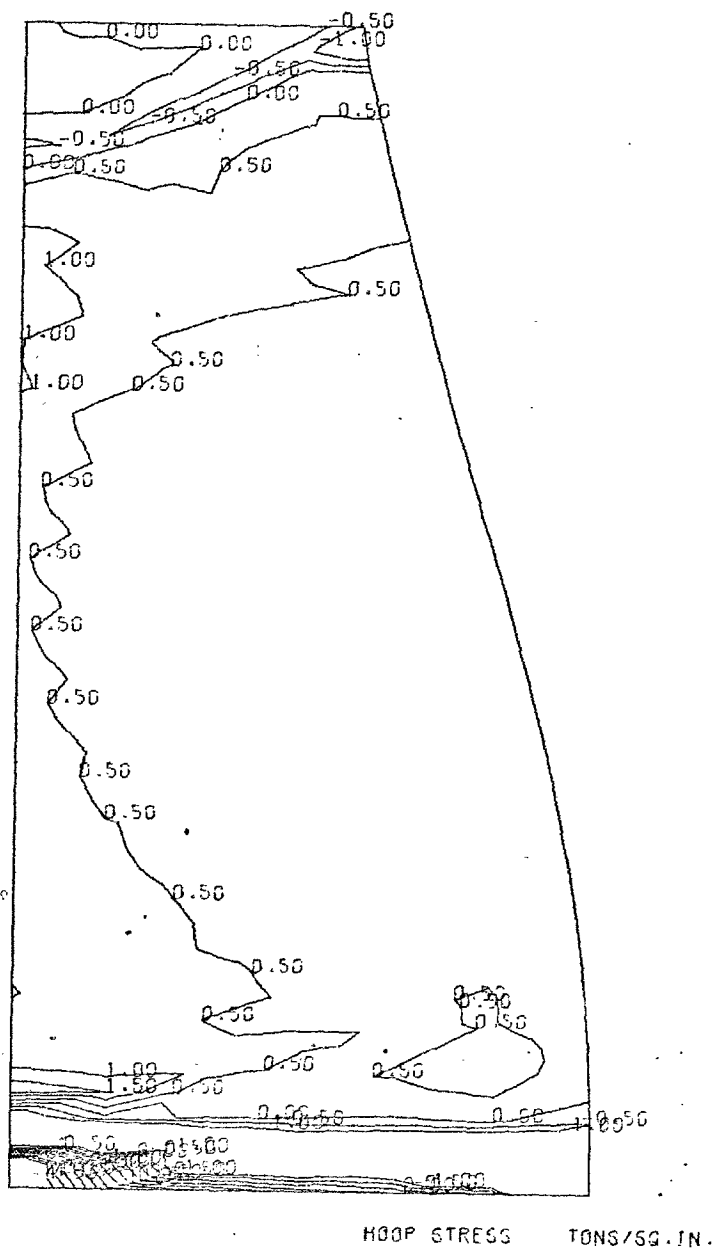


FIGURE 47 STRESS CONTOURS FROM THE FINITE
ELEMENT ANALYSIS FOR STAINLESS
STEEL AT 66% REDUCTION IN AREA
(576 ELEMENTS)

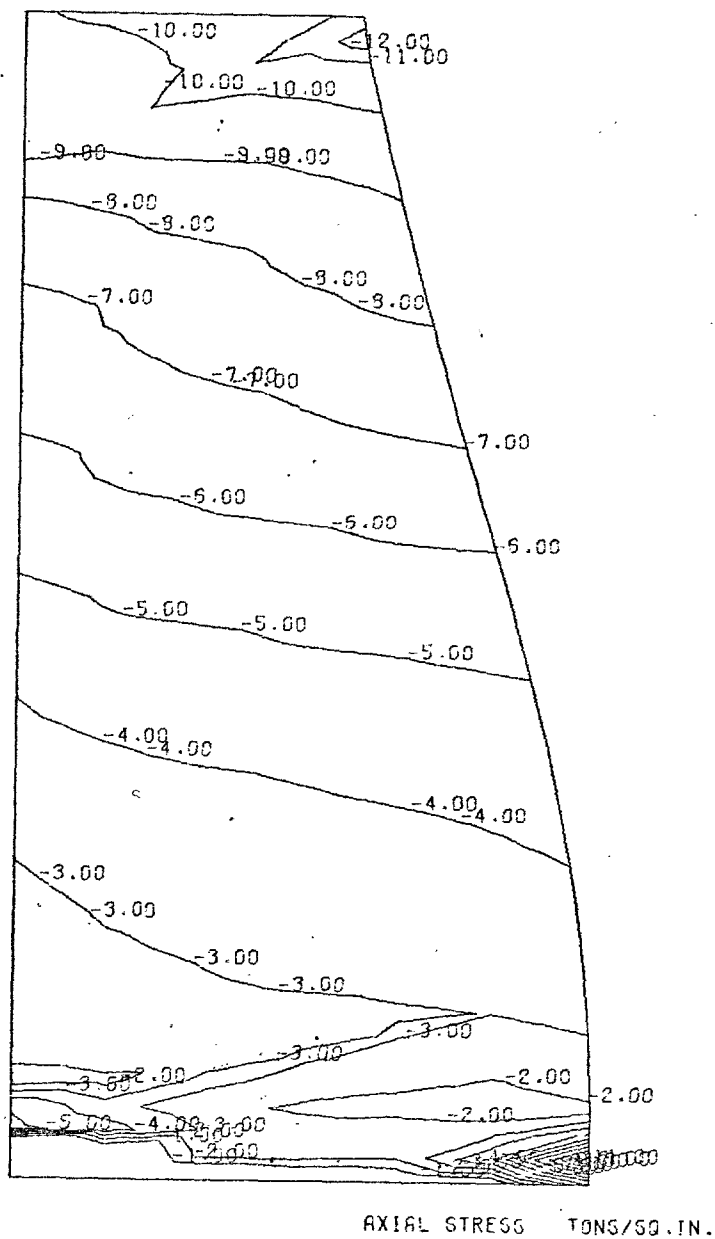


FIGURE 47 STRESS CONTOURS FROM THE FINITE
ELEMENT ANALYSIS FOR STAINLESS
STEEL AT 66% REDUCTION IN AREA
(576 ELEMENTS)

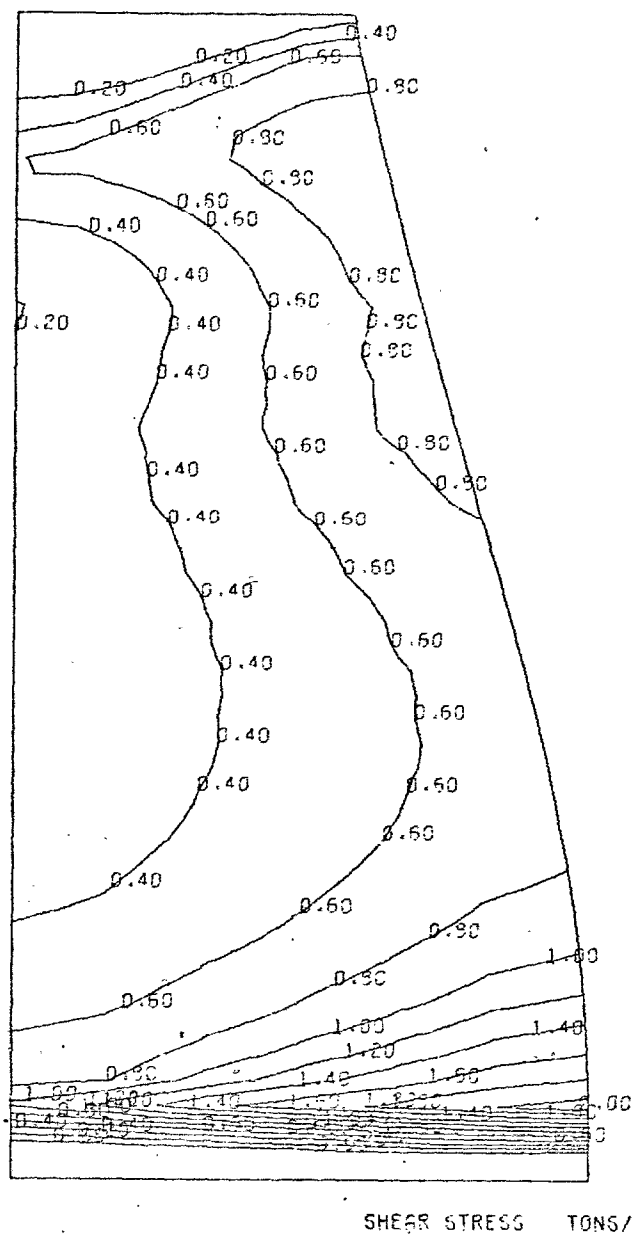


FIGURE 47 STRESS CONTOURS FROM THE FINITE
ELEMENT ANALYSIS FOR STAINLESS
STEEL AT 66% REDUCTION IN AREA
(576 ELEMENTS)

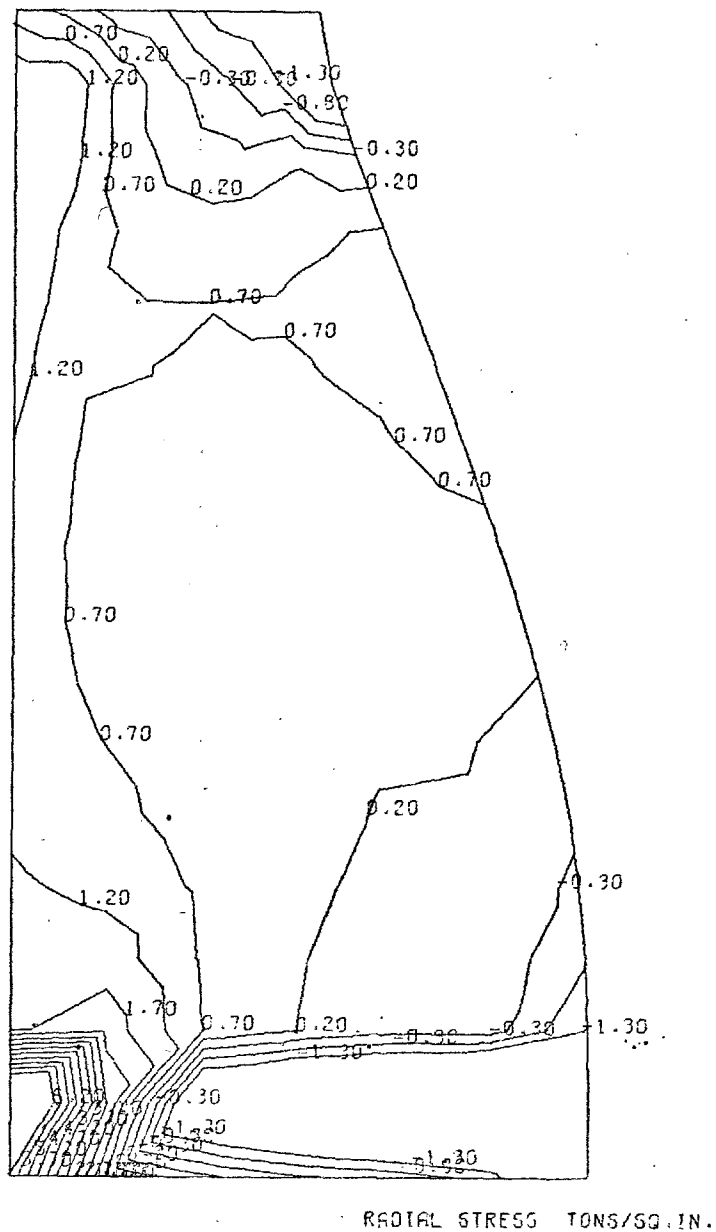


FIGURE 48 STRESS CONTOURS FROM THE FINITE
ELEMENT ANALYSIS FOR STAINLESS
STEEL AT 75% REDUCTION IN AREA
(192 ELEMENTS)

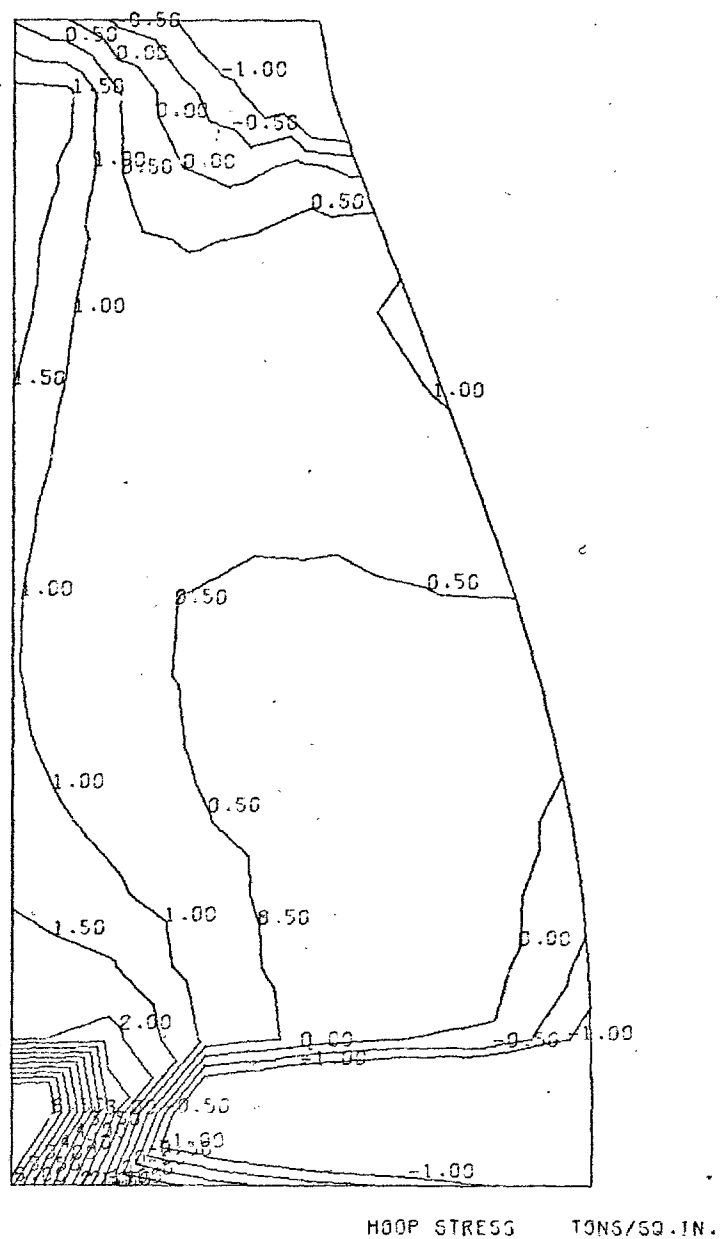


FIGURE 48 STRESS CONTOURS FROM THE FINITE
ELEMENT ANALYSIS FOR STAINLESS
STEEL AT 75% REDUCTION IN AREA
(192 ELEMENTS)

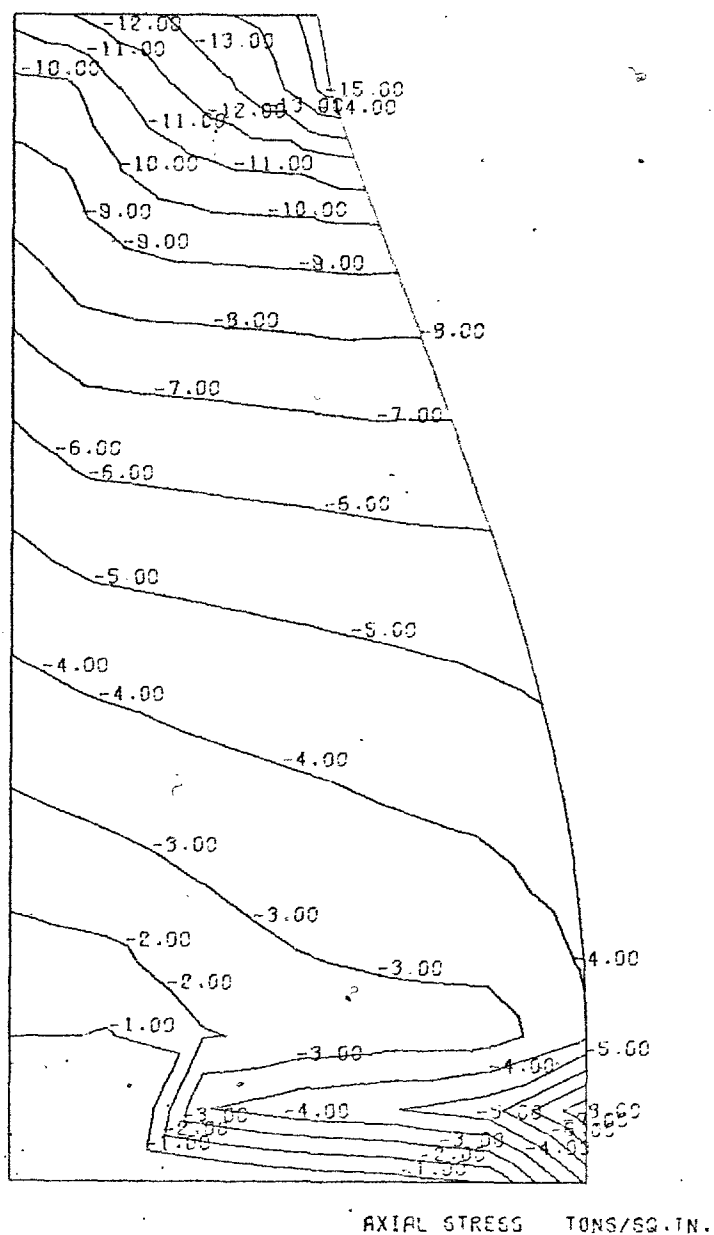


FIGURE 48 STRESS CONTOURS FROM THE FINITE
ELEMENT ANALYSIS FOR STAINLESS
STEEL AT 75% REDUCTION IN AREA
(192 ELEMENTS)

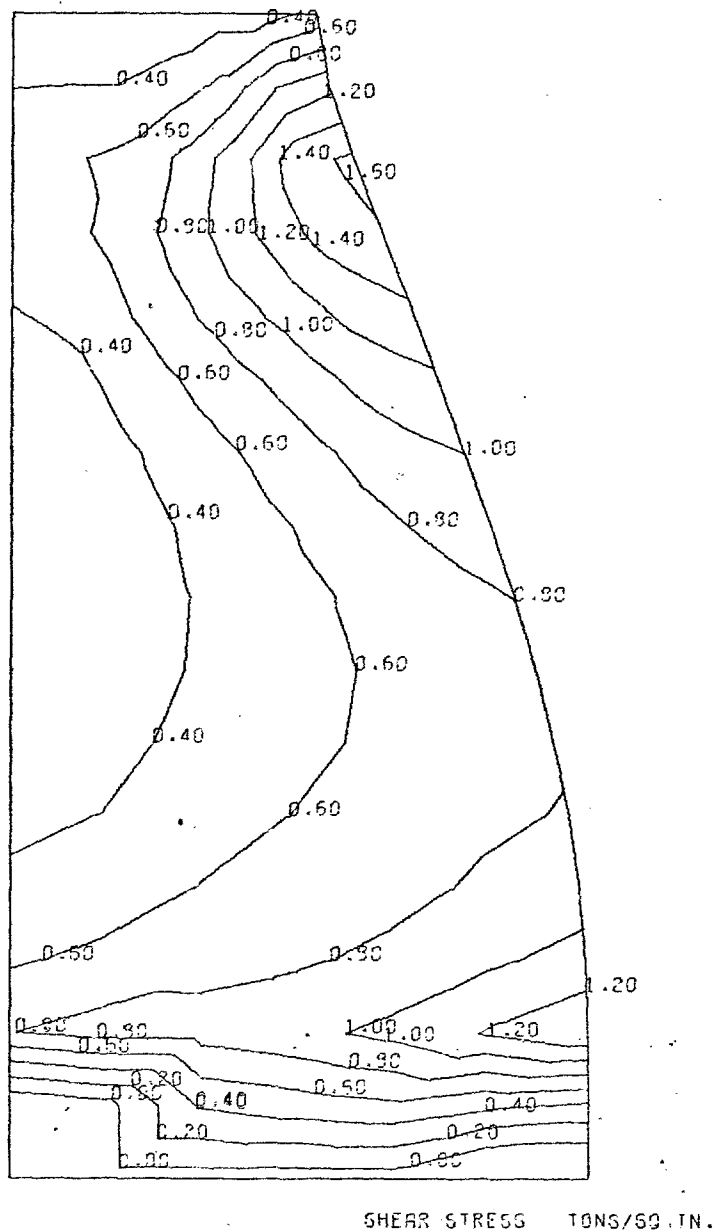


FIGURE 48 STRESS CONTOURS FROM THE FINITE
ELEMENT ANALYSIS FOR STAINLESS
STEEL AT 75% REDUCTION IN AREA
(192 ELEMENTS)

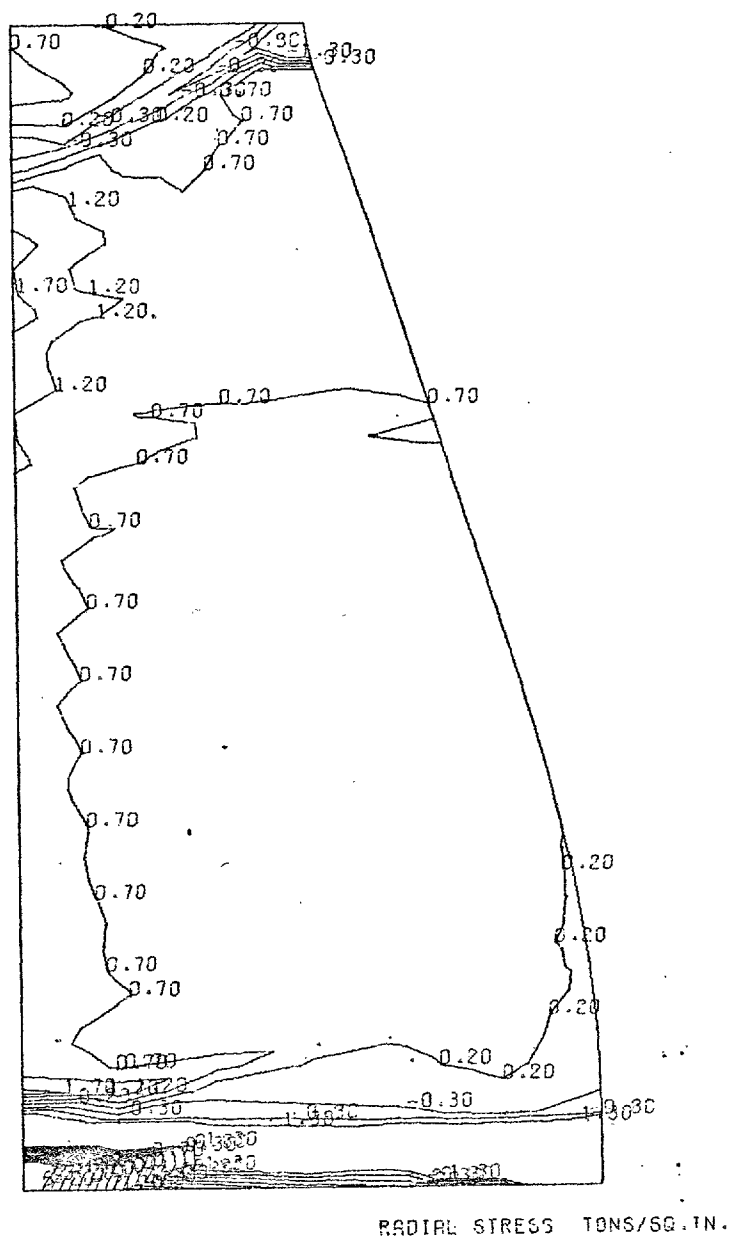


FIGURE 49 STRESS CONTOURS FROM THE FINITE
ELEMENT ANALYSIS FOR STAINLESS
STEEL AT 75% REDUCTION IN AREA
(576 ELEMENTS)

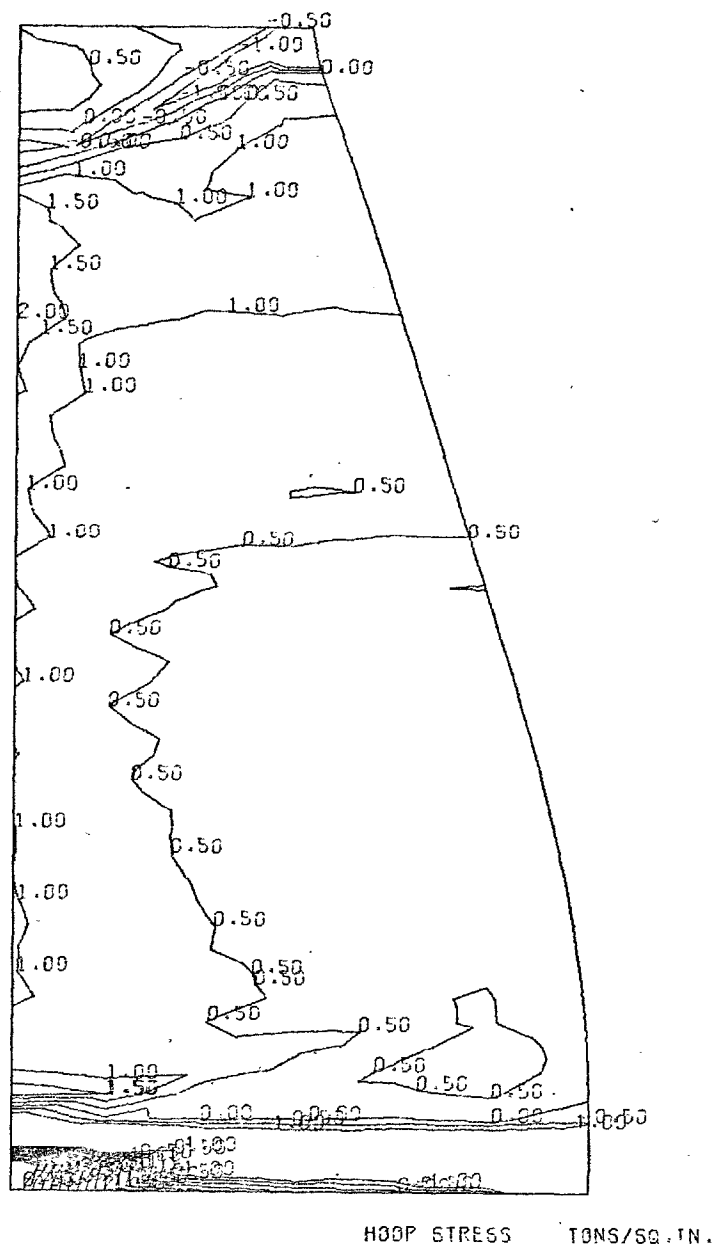


FIGURE 49 STRESS CONTOURS FROM THE FINITE
ELEMENT ANALYSIS FOR STAINLESS
STEEL AT 75% REDUCTION IN AREA
(576 ELEMENTS)

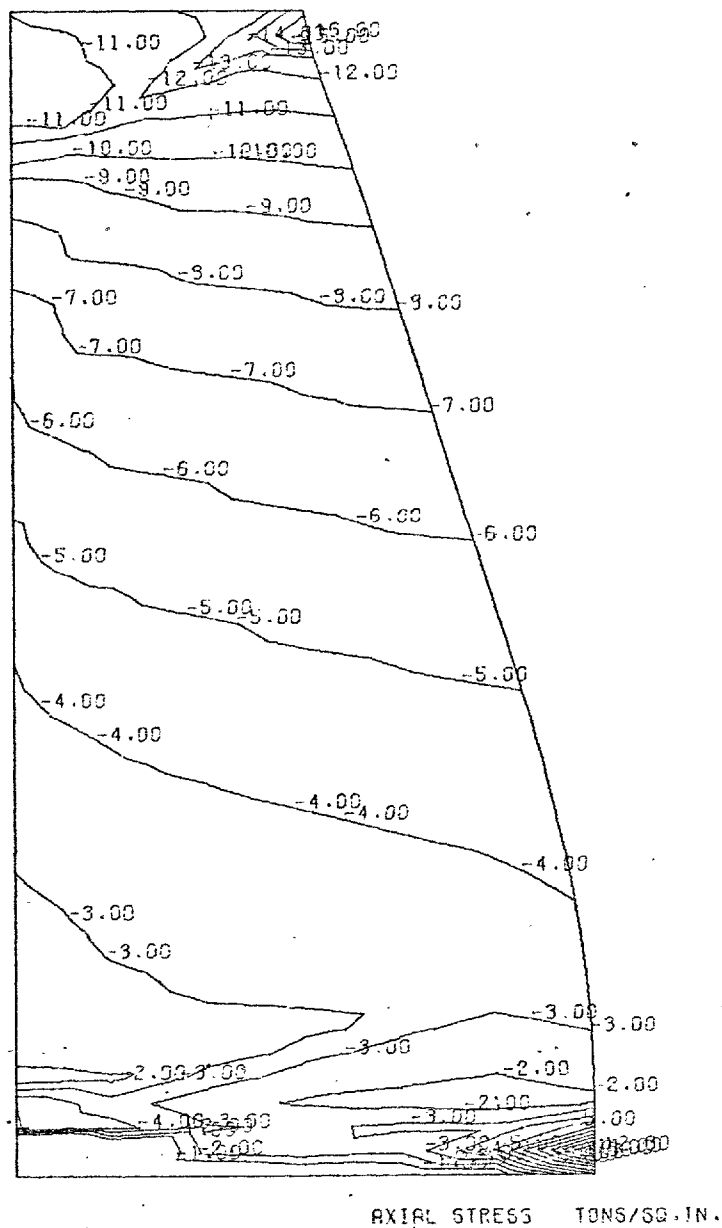
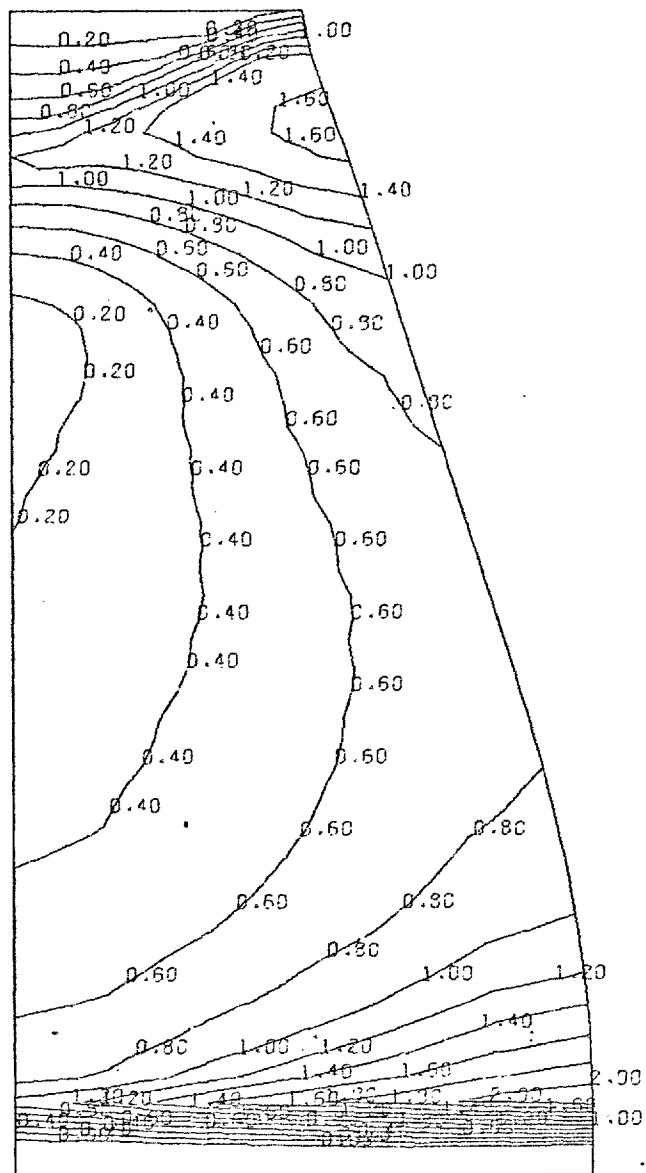


FIGURE 49 STRESS CONTOURS FROM THE FINITE
ELEMENT ANALYSIS FOR STAINLESS
STEEL AT 75% REDUCTION IN AREA
(576 ELEMENTS)



SHEAR STRESS TONS/SQ. IN.

FIGURE 49 STRESS CONTOURS FROM THE FINITE
ELEMENT ANALYSIS FOR STAINLESS
STEEL AT 75% REDUCTION IN AREA
(576 ELEMENTS)

More work is required on equations of state, but it is worth noting that the free forming which occurs in die-less drawing offers scope for data analysis on available machines which other techniques do not offer.

3.5.2. Predictions of Drawing Behaviour

Predictions of the drawing characteristics of some materials can already be made to some extent. However, the full analysis comprising the heat transfer and a developed finite element approach has not yet been completed.

It is possible that the full analytical study can be effected following the lines taken in this chapter but using Hermann's⁽³⁵⁾ variational theorem, but the approach would prove extremely expensive in computer time. An alternative approach is formulating the problem using a stream function and the velocity components as the basic unknowns in the finite element analysis. This is discussed in Section 4.3.3.

It is confidently expected that a full analytical treatment will shortly be possible. Only very little adaptation will be necessary to extend the analysis to problems such as rolling, extrusion or any large deformation metal forming process.

CHAPTER FOUR

DISCUSSION

Even so we in like manner, as soon as
we were born, began to draw to our end.

Wisdom of Solomon iii v. 13

4.1

INTRODUCTION

The first stage of the objectives set out in Section 1.4, that of showing that a die-less drawing process is possible for steel bars, has been successfully completed.

Difficulties which were anticipated regarding the properties of real materials have been augmented by the difficulties of obtaining physical convergence with the finite element method. Nevertheless the methods already described and discussed in the ensuing pages are considered adequate to make predictions sufficiently accurate for industrial use.

Some of the conclusions were only made after the considerations which are described in the Appendix.

4.2

EXPERIMENTAL METHOD

The photographs, Figures 25 to 30, give a representation of the 160 drawn specimens which were tested during the experimental programme. Approximately one third of the total number of bars tested were stable draws and sufficient data has been extracted to make subsequent drawing experiments with titanium, EN 58 J stainless steel, and mild steel very nearly 100%.

The titanium alloy Ti-6Al-4V presents no difficulties and it would appear that many superplastic alloys behave in a similar way.

Johnson⁽³⁶⁾ gives a list of many metals and alloys which exhibit the phenomenon, having strain rate sensitivity indices greater than 0.5.

Success with the titanium alloy is all the more remarkable because loads less than 10kN (1 ton) were obtained from a 1000kN (100 ton) testing machine; loads at which the machine is least likely to give a steady pull.

There is little doubt that the successful draws with stainless steel and chromised mild steel have nothing to do with superplastic phenomena. Calculations from the available data⁽¹²⁾ give m values less than 0.15, and another mechanism to explain the equilibrium of the process must be found. Hardening due to the temperature gradient offers a satisfactory explanation, since there is no evidence of strain hardening. In point of fact Cook's data show strain softening at the strains encountered at the thinner section of the deformation zone. Stainless steel shows sensitivity to a slight increase or decrease in the rate of heat extraction, and Figure 33 shows the stress temperature effect to be most important at the drawn end of the zone. Here the experimentally determined axial temperature gradient is of the order of $20^{\circ}\text{C}/\text{cm}$ ($50^{\circ}\text{C}/\text{in}$), Figure 22, which may be compared with $12^{\circ}\text{C}/\text{cm}$ ($30^{\circ}\text{C}/\text{in}$) computed by the plane sections approach, Figure 34. The unstable draws give computed axial temperature gradients of $120^{\circ}\text{C}/\text{cm}$ ($300^{\circ}\text{C}/\text{in}$) in Figure 34, gradients which could not be established in the experiments and which account for the instability.

A similar argument is not applicable to the titanium alloy. Firstly there was no example of instability and secondly the very rapid increase in the m value as the strain rate decreases (see Figure 4) causes the necessary hardening at the thinnest sections of the zone.

Chromising was a satisfactory method of stopping scale formation, thus enabling a steady rate of cooling to establish the stable drawing of mild steel. Unfortunately it is an expensive process and allows grain growth to occur with deleterious effects on the material drawing properties. Berkatekt manufactured by Berk Limited, and a new lubricant⁽³⁷⁾ is reported to be effective when used on mild steel at temperatures in excess of 1000°C , but the simplest method would be to find some method of extracting the heat at a constant rate. This might be a simple scraping device which removes the scale at the cooling sections thus allowing a constant rate of heat loss.

Although the experimental work on temperature distribution was sufficient to achieve the success reported, there is a good deal that could be done to develop the experimental method. For example, a secondary heat source warming the skin of the material, reflectors including aluminizing the inside of the heating coil, or lagging the bar with glass fibre tape are possible methods of minimising the radial temperature gradients, which can produce surface cracks.

4.3

THEORETICAL METHOD

4.3.1 Plane Sections

There is experimental and some theoretical work which shows that plane sections remain plane over the first half of the deformation zone at least. Figure 23 shows the experimental results when pins were fitted in the specimen and Figure 36 gives the resultant mesh deformation from simple tension analysis. Both show that plane sections remain substantially plane for much of the deformation zone. It is reasonable therefore, to use the method of plane sections to give an analysis of the varying parameters along the z axis of the heated zone.

The favourable comparison between the computed temperature profiles (Figure 34) and experimental profiles is further evidence confirming plane sections. The higher computed temperatures are to be expected since they represent values determined from a constant temperature section, i.e. zero radial gradients, whereas the experimental temperatures are taken at the surface.

It is interesting to speculate on the value of this straightforward approach in determining deformation shapes. The evidence of the present work is that it is certainly adequate when the percentage reduction in area does not exceed 50%; more accurate temperature profiles will be needed to check its validity at greater reduction ratios.

4.3.2 Simple Tension

The advantage of using this simple approach lies in the speed of computation. Approximately 10 seconds of computer time are required to obtain a solution compared with over 1000 seconds for the finite element program (refined mesh of 576 elements) and about 400 seconds for the program using 192 elements.

The computed diameters are within 10% of those obtained experimentally, and the strain rate variation shown in Figure 42 compares favourably with the variations computed using the plane sections approach together with the experimental profiles.

The mesh deformation obtained from an originally constant cross-section 'bar' (Figure 36), also supports the assumption of plane sections remaining plane. It is not recommended, however, that the method should be used for large percentage reductions in area.

4.3.3 Finite Elements

Prior to the conclusions which are drawn from the Appendix, it was found that the predictions of the profile of the deforming bar which emerged from the analytical study using finite elements (Figures 39 to 42) did not give a very satisfactory fit to the experimental profiles. This is most clearly shown in the strain rate curve of Figure 42, where the characteristic peak predicted by the other two theoretical approaches is absent.

The following points were given careful consideration in an attempt to explain the result:

1. Oscillating stresses.
2. Approach to singularity of the stiffness matrix.
3. Iterative techniques.
4. Modification of the mesh to obtain flow lines.

It was decided that the latter two points were not affecting the result. Iterating on the stresses gave the same result as iterating on the strain rates and only the characteristics of convergence were changed. The differences in the results from the two mesh sizes were quite small with the refined mesh giving a profile made up of straight lines which never deviate from the curved line by more than 0.0013 cm (0.0005 in). The oscillating stresses and approach to singularity appeared to be the most likely cause of the poor result. Boundary conditions were changed to constrain the flow at entry and reduce the oscillations. On finding this to be of little use, it was concluded that the oscillating stresses were caused by the approach to singularity and time prevented further work on this investigation.

In the light of the Appendix, the principal cause of the poor fit was found to be the inaccurate temperature data. On correcting these data, a much better fit is evident as shown in Figure 50.

Alexander and Zienkiewicz⁽³⁸⁾ have suggested that the errors and failure to obtain physical convergence are due to the triangular element formulation, and that isoparametric elements might prove to be better. There was insufficient time to investigate this.

4.4

CONCLUSIONS

The conclusions which may be drawn from the experimental programme have already been listed on page 104. Another important conclusion may be drawn from the fact that the draws were carried out on the hydraulic Buckton machine where full velocity control is not possible. This disadvantage is turned to an advantage when it is appreciated that this underlines the remarkable stability of the process.

The analytical study enables the following conclusions to be made:

1. Plane sections do remain plane over a substantial part of the deformation zone at a percentage reduction in area of 50% or less. It is therefore possible to evaluate the parameters along the deformation zone, and from a known profile the axial temperature distribution can be found. Similarly, from an accurate temperature distribution, predictions can be made of the shape taken up by the deforming bar.
2. From the relative effects of strain hardening, strain rate sensitivity, and the stress-temperature behaviour, strain hardening is found to play a minor role at the start of the deformation, thereafter having no effect.
3. The effect of strain rate sensitivity plays an important part in drawing titanium and other superplastic materials; but for steels the major effect is the hardening due to the temperature gradient.

4. Die-less drawing is possible without superplasticity.
5. An indication of the flow characteristics is possible by considering the bar to be in pure tension.
6. Despite the problems which have not entirely been solved with the finite element method (see Appendix) predictions may be made which approximate to the true solution.

Although a true physical convergence has not been obtained with the finite element analysis, the predicted diameters are within 5% of the experimentally determined values, compared with 10% from the pure tension analysis.

Changing Poisson's ratio from 0.49 to 0.495 alters the predicted diameters by less than 1% in both programs of 192 elements and 576 elements. Refinement of the mesh causes changes of up to 3% in the diameters. The loads calculated from the finite element program give values in tension within approximately 3%.

Here, the facts speak for themselves and show that the finite element results give a much better result than was first realised. A much more optimistic picture is apparent when the points discussed in the Appendix are taken into account. We can conclude, therefore, that the results are very satisfactory in predicting the profiles, and that the errors of a few per cent are caused by the approach to singularity in the stiffness matrix.

4.5

SUGGESTIONS FOR FUTURE WORK

The list of suggestions has become a formidable one and is the result of the complexities of the process. The success of the experimental work was not matched by the theory, principally because of the failure to obtain the physical convergence using finite elements, and development of this analysis should take first place, under the two following headings:

1. The elastic analogy approach is considered to be basically sound and with the modification of the mesh using velocity slopes a full solution should emerge. Herrmann's⁽³⁵⁾ contribution to avoid the matrix singularity coupled with an initial simple tension solution to reduce computer time, is one possible approach: another is to use isoparametric elements or to develop a stream function formulation.⁽³⁹⁻⁴²⁾
2. A complete analysis of the heat transfer problem taking into account draw speed remains to be perfected,

The most important aspect of the experimental work which requires a fuller study is the metallurgical work which should be the main thread for the following experimental suggestions:

1. Large diameter, 5 cm (2 in), tests using a screw thread (constant velocity) machine.
2. Experimental determination of the temperature distribution (a function of material, maximum temperature, coil diameter and length, draw speed, and frequency).

3. Develop a feed back control system to control the rate of cooling.
4. Investigate tapered specimens and non-constant cross-sections, rectangular and triangular stock, and tubes.
5. Combine die-less drawing with final shaping rollers.
6. Collect data using finely controlled die-less drawing experiments.

An interesting academic study would be to compare the stability of hydraulically drawn bars with those drawn at true constant velocity.

APPENDIX

NOTES ON THE FINITE ELEMENT ANALYSIS

The results which appear in Figures 39 to 42 show that the finite element work gives a poor representation of the die-less drawing process: the progress that can be made by putting a value close but not equal to 0.5 for Poisson's ratio would seem to be minimal. Furthermore doubts raised on examination of Herrmann's paper⁽³⁵⁾ become more explicit in that of Pin Tong⁽³⁹⁾ who states that "the conventional assumed stress hybrid formulation^(44,45) may lead to serious errors for nearly incompressible materials, because the influence matrix becomes nearly singular."

It was felt at the time of writing the discussion that the problem had to some extent been overcome by averaging the stresses, but as the profile of the deformation was affected by varying Poisson's ratio and by refining the mesh, there was little satisfaction in the result. Indeed the poorer result from a refined mesh shows that the numerical convergence does not represent a physical convergence. It was concluded that the poor solution was wholly caused by the close approach to singularity in the stiffness matrix, although this was questioned by Fenner⁽⁴⁶⁾ and Alexander and Zienkiewicz.⁽³⁸⁾

Fenner⁽⁴⁶⁾ suggested that the poor profile could be the result of the convection term being neglected (see Section 3.3.2). To investigate

* Note. "Convection" here refers to the heat transport due to the motion of the body.

this the program was run again with $\nu = 0.4975$, but with a temperature distribution appreciably modified by setting the radial gradient at zero along the whole length of the deformation zone. The profile was modified to such a small extent, that it can confidently be stated that the convection term resulting from the material flow has a negligible effect.

While investigating Herrmann's⁽³⁵⁾ approach, that of reformulating elastic field equations that are valid for all admissible values of Poisson's ratio, the idea of a device for using an exact effective value of 0.5 for ν for the rectangular ring element was conceived. Poisson's ratio is set at $0.5 + \delta\nu$ for one of the triangular ring elements of an adjacent pair, and at $0.5 - \delta\nu$ for the other.

The justification for using such a device is two-fold. Firstly, for the whole of the zone a precise value of 0.5 for ν is used, and secondly for each triangular element, values of ν which are sufficiently far from 0.5 enable the computation to avoid the close approach to singularity of the stiffness matrix. Moreover each rectangular element is made up of a triangular half where the volume of the material increases on deformation due to using an effective Poisson's ratio of $0.5 - \delta\nu$ and a half where the volume decreases because of using an effective Poisson's ratio of $0.5 + \delta\nu$.

To examine the device more closely, we may write down the axial strain rates of adjacent elements as follows:

$$\dot{\epsilon}_{z1} = \frac{2}{3} K'' \sigma^{-n-1} \exp \left[\frac{-Q}{RT} \right] \left[\sigma_{z1} - \frac{1}{2} (\sigma_{r1} + \sigma_{\theta 1}) + \delta \nu (\sigma_{r1} + \sigma_{\theta 1}) \right]$$

$$\dot{\epsilon}_{z2} = \frac{2}{3} K'' \sigma^{-n-1} \exp \left[\frac{-Q}{RT} \right] \left[\sigma_{z2} - \frac{1}{2} (\sigma_{r2} + \sigma_{\theta 2}) - \delta \nu (\sigma_{r2} + \sigma_{\theta 2}) \right]$$

where the subscripts 1 and 2 denote adjacent elements. If we assume the properties of the rectangular element are unchanged throughout the region such that

$$\sigma_{z1} = \sigma_{z2}, \quad \sigma_{r1} = \sigma_{r2} \quad \text{etc.},$$

it is immediately apparent that the arithmetic mean of $\dot{\epsilon}_{z1}$ and $\dot{\epsilon}_{z2}$ satisfies equation 27 where Poisson's ratio is precisely 0.5.

On a phenomenological basis it is difficult to accept the paradox of considering a rectangular ring element of constant properties made up of two triangular ring elements with quite different Poisson's ratios, but the mesh is rigid, each nodal point being precisely fixed in space for each calculation of the nodal velocities. The "deformation" takes place in the program by a geometrical modification of the mesh by the slopes technique, quite independently of the velocity and stress computation.

Some effort was put into the investigation of the device described above, but once again the numerical convergence, which presented difficulties at first, did not give the necessary physical convergence. The reason for the failure to obtain the physical convergence has not been fully understood and time has precluded further study.

During this stage of the analysis, it became evident that the temper-

ature profile had not been satisfactorily matched with the experimental results shown in Figure 22, as the position of the experimentally determined temperature at the thinnest point of the deformation zone did not coincide with the position determined by the computation. The temperature distribution was therefore computed again and repunched for feeding as data into the program.

Figure 50 shows the much improved deformation zone profiles which were obtained with the changed temperature distribution. The two curves shown, where $\nu = 0.49$, are for the two meshes used, and apparently the refined mesh gives a better result, giving a close fit to one of the two experimental curves which represent the approximate limits of a stable configuration in stainless steel. It must be stated however, that as Poisson's ratio approaches 0.5 the refined mesh gives a poorer fit to the experimental curves: this was noted at $\nu = 0.4975$, the value which appeared earlier to be an optimum. Figure 50 has been included mainly to demonstrate that the poor solution shown in Figure 39 was principally the fault of the temperature distribution. As a considerable improvement in the fit is evident, the stress contours have been included and are shown in Figures 51 and 52: those of the finer mesh (576 elements) in the latter are likely to be more realistic than those shown in Figures 44 and 45. Figure 52 shows a compressed region in the radial stress diagram near to the axis as well as the tension on the surface as one would expect. The shear stress shows a maximum on the surface at the maximum slope of the deformation zone. Once again the boundary conditions distort the contours at the ends of the diagrams.

It was decided to put the above ideas and work done in an appendant form because the work was done after the thesis had been virtually completed. We can conclude that an approximate solution to the problems of large-deformation plastic flow may be obtained using values of Poisson's ratio close to 0.5, but more analytical work must be done to obtain the true physical convergence before any real confidence can be placed in the method.

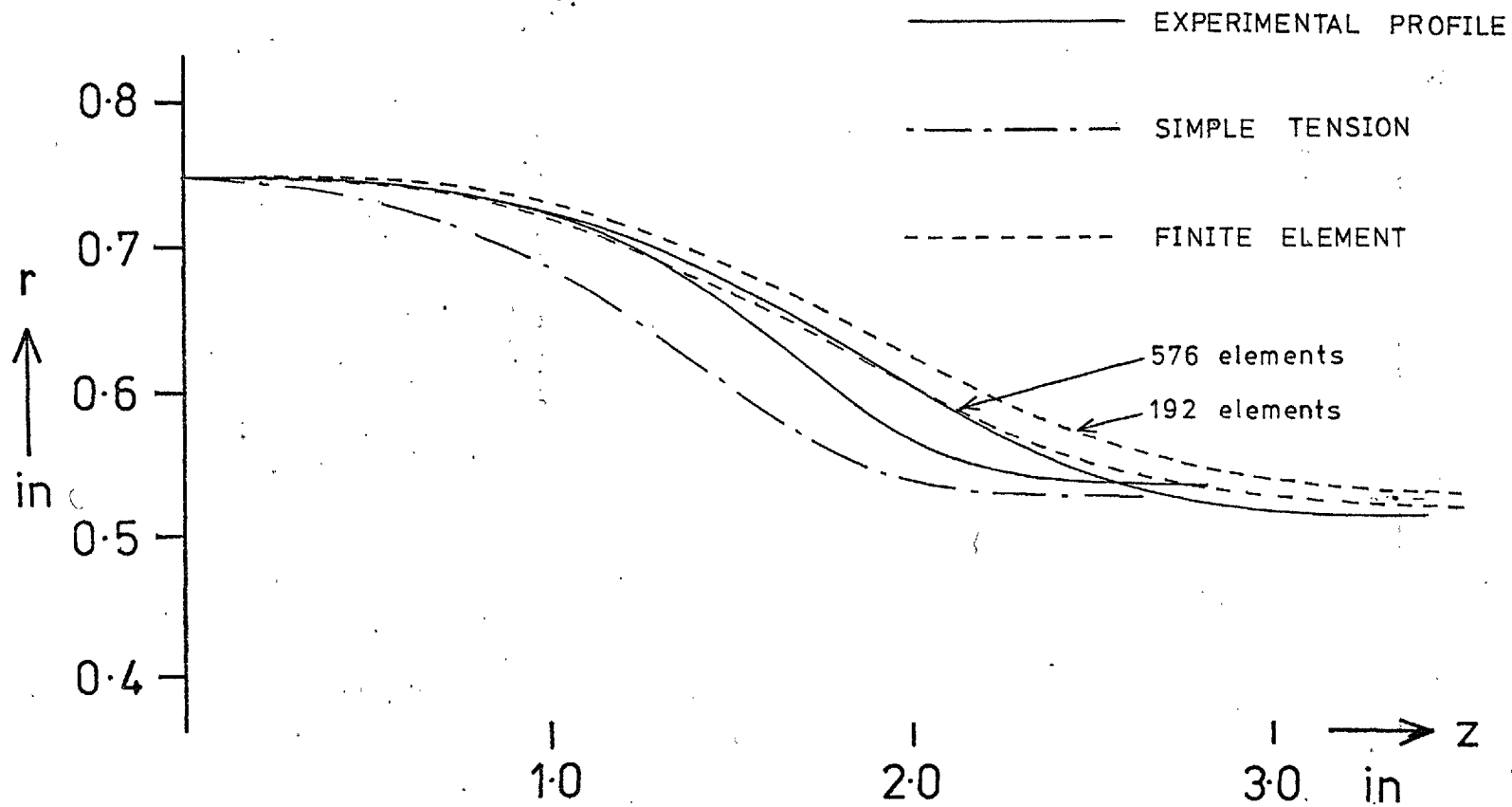
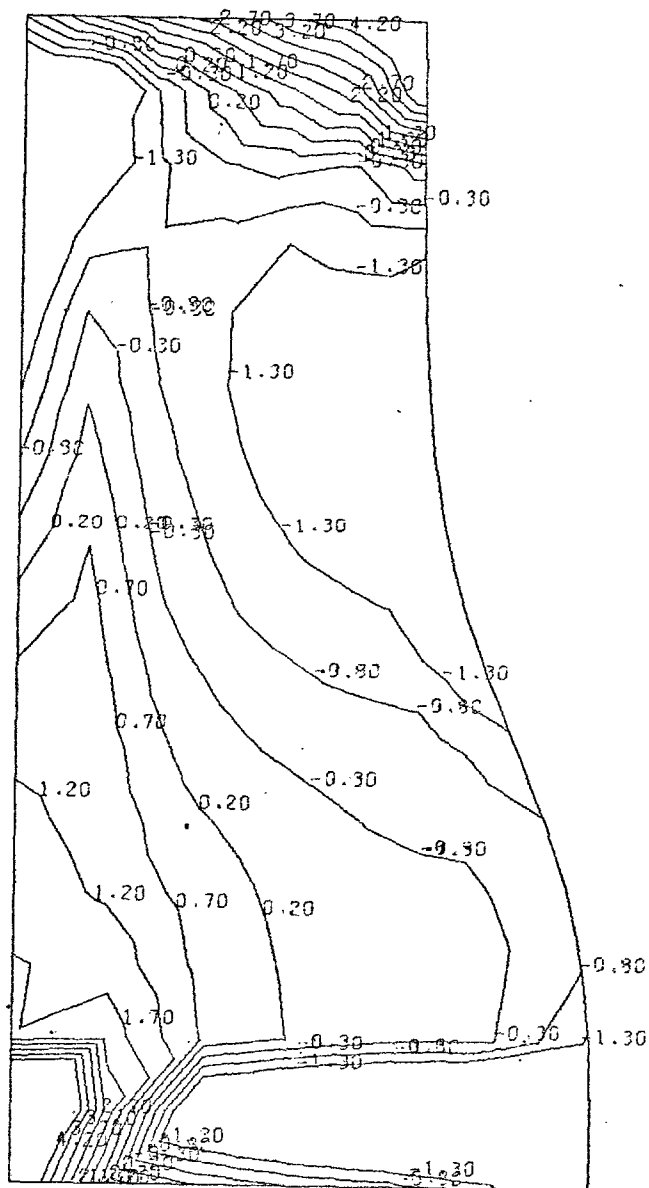


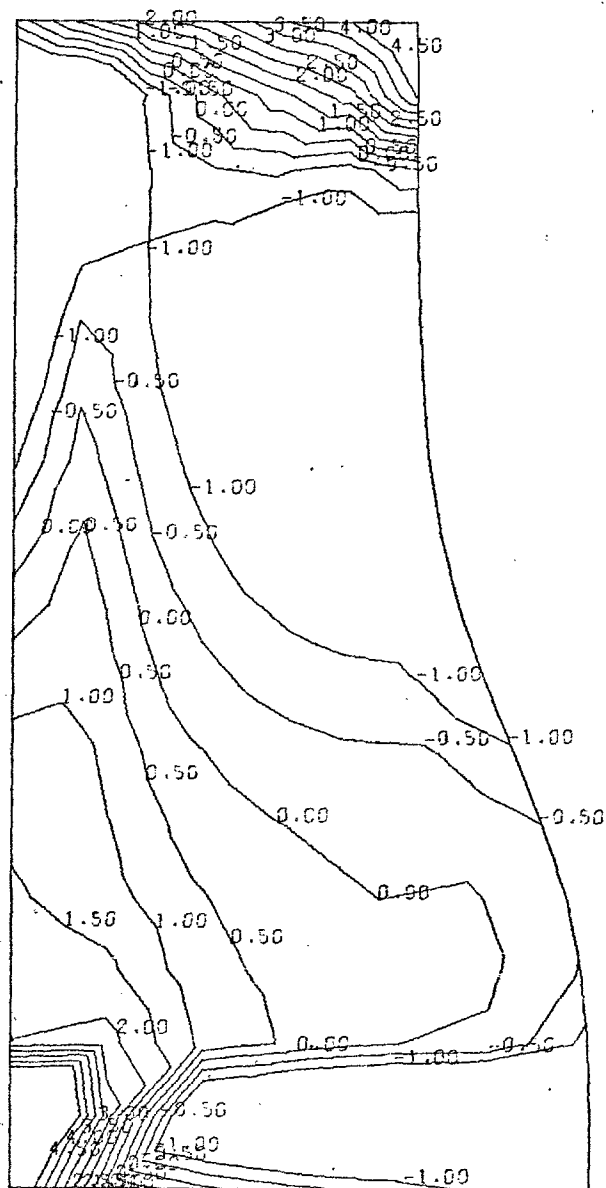
FIGURE 50 DEFORMATION ZONE PROFILES FOR EN 58 J STAINLESS STEEL (COMPARE FIGURE 39)



RADIAL STRESS TONS/SQ. IN.

MODIFIED

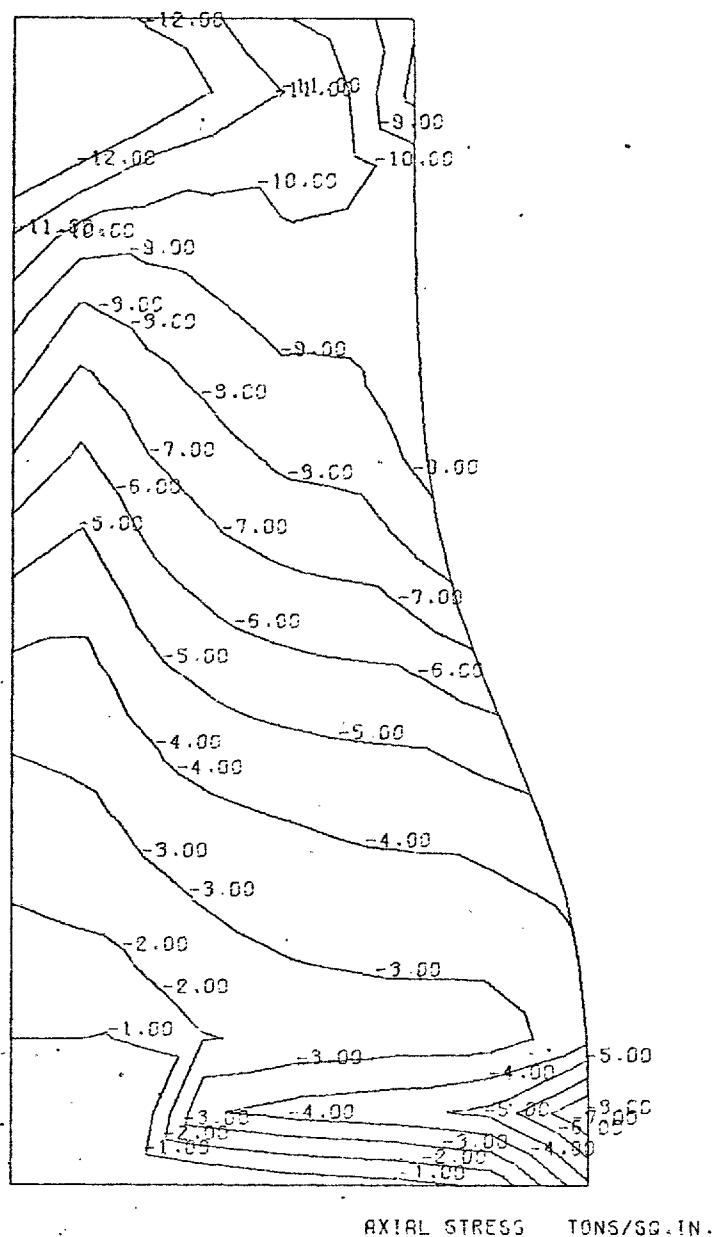
FIGURE 51 STRESS CONTOURS FROM THE FINITE
ELEMENT ANALYSIS FOR STAINLESS
STEEL AT 50% REDUCTION IN AREA
(192 ELEMENTS)



HGGP STRESS TONS/SQ. IN.

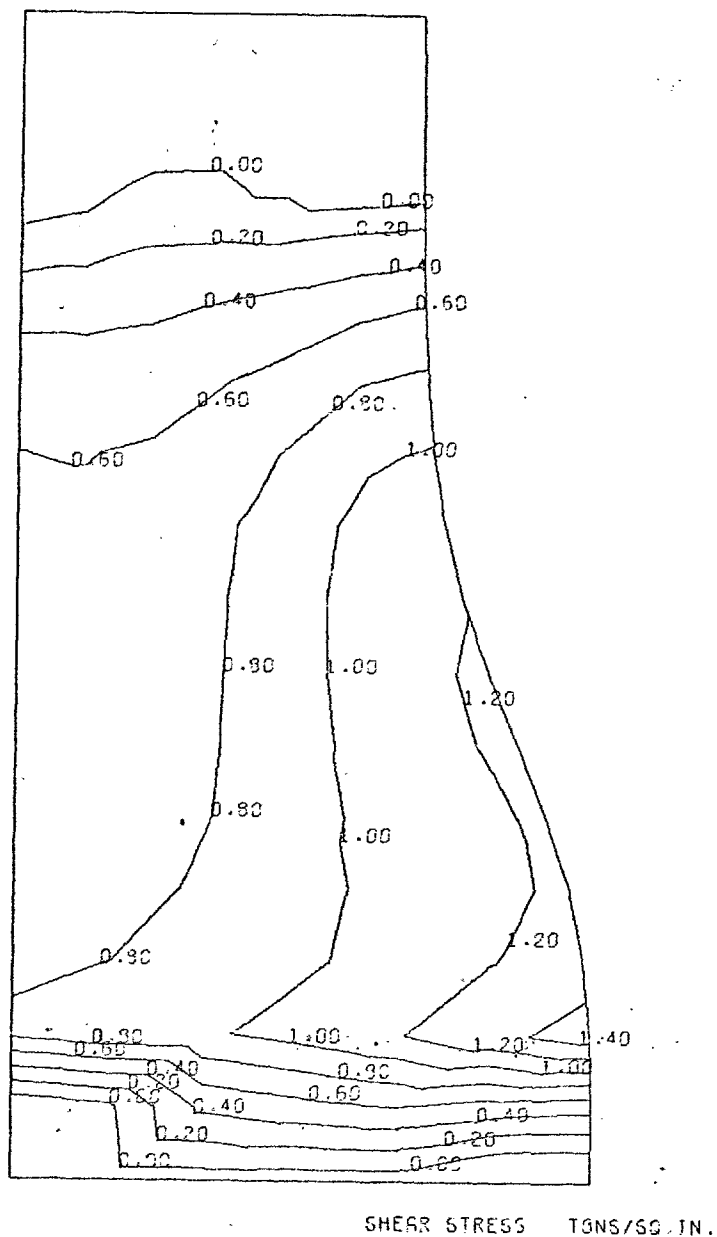
MODIFIED

FIGURE 51 STRESS CONTOURS FROM THE FINITE
ELEMENT ANALYSIS FOR STAINLESS
STEEL AT 50% REDUCTION IN AREA
(192 ELEMENTS)



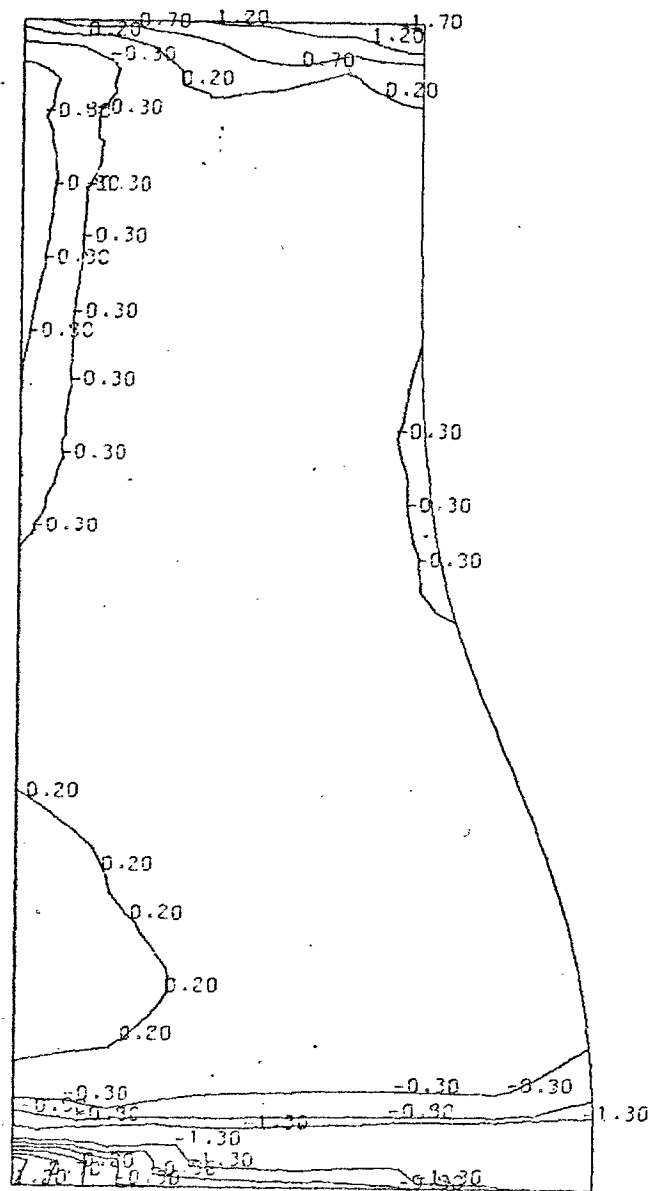
MODIFIED

FIGURE 51 STRESS CONTOURS FROM THE FINITE
ELEMENT ANALYSIS FOR STAINLESS
STEEL AT 50% REDUCTION IN AREA
(192 ELEMENTS)



MODIFIED

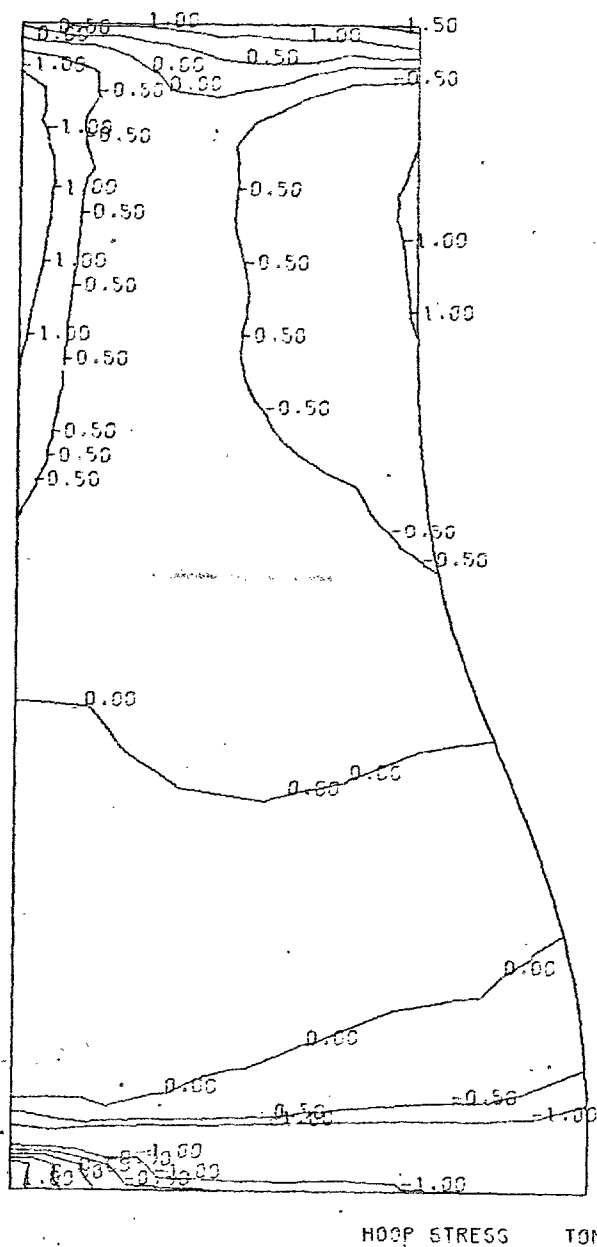
FIGURE 51 STRESS CONTOURS FROM THE FINITE
ELEMENT ANALYSIS FOR STAINLESS
STEEL AT 50% REDUCTION IN AREA
(192 ELEMENTS)



RADIAL STRESS TONS/SG. IN.

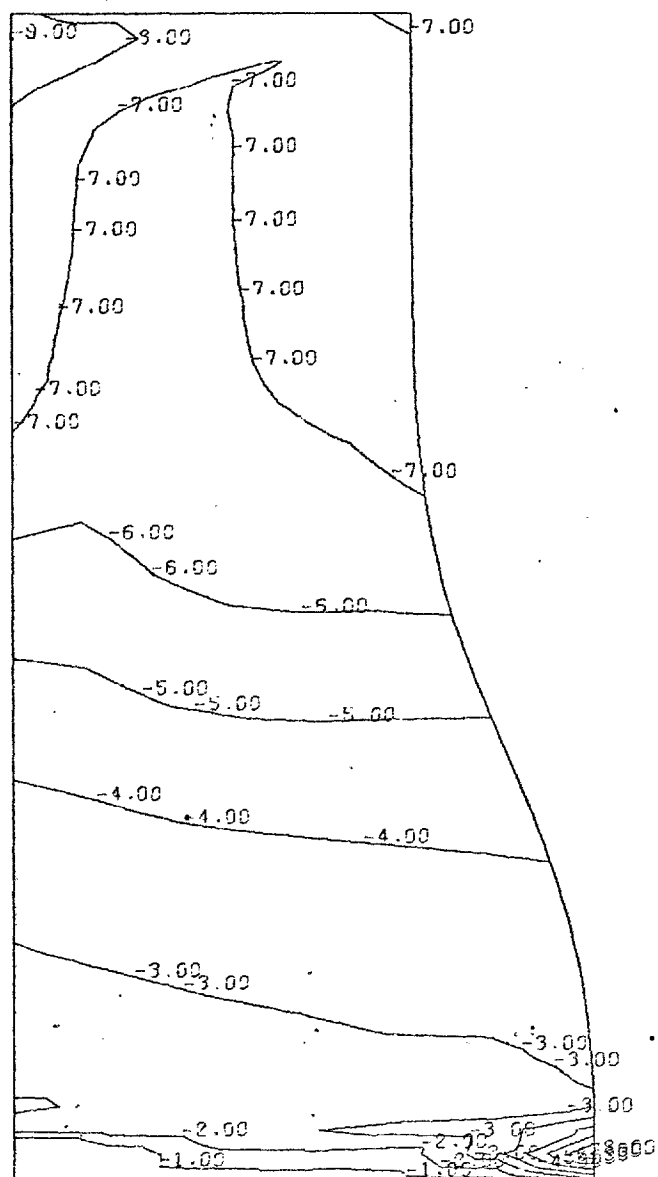
MODIFIED

FIGURE 52 STRESS CONTOURS FROM THE FINITE
ELEMENT ANALYSIS FOR STAINLESS
STEEL AT 50% REDUCTION IN AREA
(576 ELEMENTS)



MODIFIED

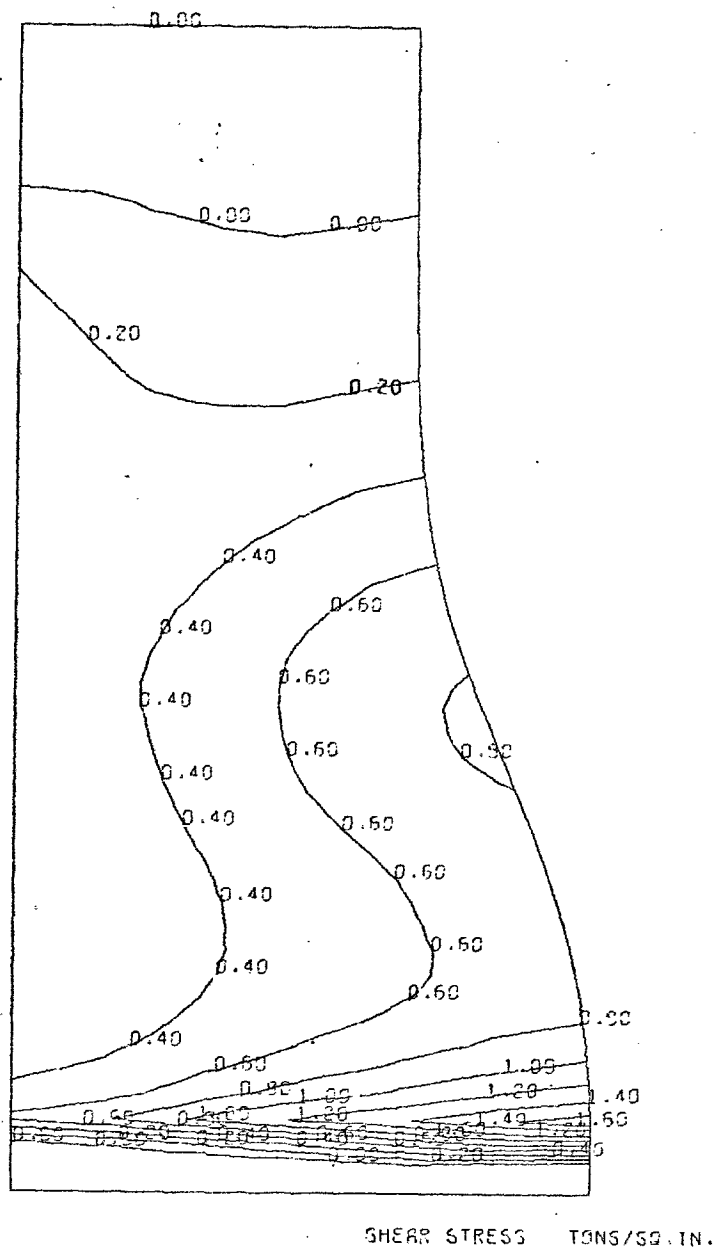
FIGURE 52 STRESS CONTOURS FROM THE FINITE
ELEMENT ANALYSIS FOR STAINLESS
STEEL AT 50% REDUCTION IN AREA
(576 ELEMENTS)



AXIAL STRESS TONS/50.IN.

MODIFIED

FIGURE 52 STRESS CONTOURS FROM THE FINITE
ELEMENT ANALYSIS FOR STAINLESS
STEEL AT 50% REDUCTION IN AREA
(576 ELEMENTS)



MODIFIED

FIGURE 52 STRESS CONTOURS FROM THE FINITE
ELEMENT ANALYSIS FOR STAINLESS
STEEL AT 50% REDUCTION IN AREA
(576 ELEMENTS)

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