

DESIGN OF MULTICHANNEL NONRECURSIVE DIGITAL FILTERS
WITH APPLICATIONS TO SEISMIC REFLECTION DATA

by

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to my GRANDMOTHER with affection and gratitude

ABSTRACT

The basic principles of discrete multichannel least-squares stacking filters, designed in the time-space domain, are reviewed. An extensive study of these filters, with reference to seismic reflection data, is made with design examples and applications to synthetic and real seismograms. The emphasis is put on the filter parameters, namely the correlated noise and uncorrelated noise relative weightings and chatter. Multichannel filtering versus straight stacking studies are made with synthetic seismograms.

Multichannel least-squares filter theory is extended to the design of moveout filters for passing and/or rejecting events having exact hyperbolic moveouts within a time-gate. It is shown how to interpret the frequency-wave number responses of these filters, which are obtained after Fourier transformation of their time-distance domain coefficients. Design examples and their applications to synthetic and real seismograms are presented.

A frequency-wave number design technique is introduced mainly to overcome wave number aliasing, which is shown to be the main limitation of the least-squares method. This method, called the 'co-efficient mapping' technique, uses a reversible mapping formula with a sorting procedure for transforming a two-dimensional filter design problem into a one-dimensional frequency filter approximation. The desired frequency-wave number response is approximated in the one-dimensional frequency domain as a multiband frequency filter using an efficient Tchebycheff algorithm and mapped back to obtain the moveout filter. Design examples with the corresponding least-squares filters and their applications to synthetic and real reflection seismograms are given. The applications also include straight-stacked traces for respective comparisons.

The new mapping technique is extended to the design of circularly symmetric two-dimensional nonrecursive digital filters, which have applications not only in geophysics but also in other applied sciences, and illustrated with some design examples. A brief review of the two-dimensional filter theory is also presented.

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CHAPTER 1

INTRODUCTION

Data gathering in almost all branches of the Earth Sciences, as well as in other applied fields, is usually multidimensional. In exploration geophysics what is described as an anomaly in potential field data or an event in seismic data has some correlation from one recording point to another. Naturally, processing techniques applied to multidimensional data make use of these correlations. Perhaps the most outstanding example of all is a seismogram recorded for the detection and delineation of strata in the Earth's crust. A reflection seismogram contains so much information that the only events which are meaningful to the interpreter's eye are those which have some continuity across several recording positions.

With the vast expansion and development of digital recording and the computing industry, digital data processing of some kind is now accepted as routine before the final interpretation of the information received. Parallel with and contributing to this development is communication theory, which was well developed for continuous time and has been extended to the discrete variable. Also transform techniques, especially Fourier techniques, have been further developed for data analysing and processing (Cooley and Tukey, 1965; Cooley et al., 1967 and 1969).

Mathematical aspects of statistical communication were solved independently, around the same time, by Kolmogoroff (1939) and Wiener in the 1940's. The theory was developed for the continuous variable in one-dimensional space. It has been subsequently extended to n -dimensional filtering and prediction both in continuous and discrete form. The optimization criterion used in this unifying theory was that of least-squares. The theory was clarified and simplified by Levinson (1946 and 1947) before Wiener's own work was published (Wiener, 1949). This theory has been used for designing optimum systems for smoothing, prediction and filtering of messages received in such diverse fields of science as, for example, communication, meteorology, economics, sociology, biology, medicine, astronomy, photography and geophysics.

Since the initial work of Wadsworth et al. (1953), digital least-squares prediction and filtering techniques for geophysical applications have been developed by Robinson (1954 and 1957), Rice (1962), Robinson and Treitel (1967) and many others. The field of application was mainly seismic, and predominantly seismic reflection, data processing. Optimization procedures leading to a unique set of filter coefficients for a given input can be derived either in continuous (Wiener, 1949; Lee, 1960; Robinson, 1962) or discrete form (Robinson, 1967a and 1967b). Fortunately, sound mathematical theories developed for continuous variables giving physically nonrealizable operators which range from

minus infinity to plus infinity can be extended to discrete realizable forms through statistically reasonable working assumptions about the input data. In practice one deals mainly with stationary, or approximately stationary, stochastic (random) processes which are ergodic (Robinson, 1967a and 1967c; Bryan, 1967).

Among the many applications of digital filters speech communication, radar and sonar array processing, weather prediction, x-ray and general image enhancement, and seismic processing are the main categories. In the last decade one-dimensional digital frequency filter design methods have been developed using techniques other than least-squares. These methods are based on calculating the impulse response coefficients in order to approximate a desired ideal frequency response. Among other optimization techniques used, the min-max or Tchebycheff technique for designing one-dimensional frequency filters has been found to have many advantages over all other available methods both computationally and in approximating a given response. The design of two-dimensional operators using one-dimensional frequency filter design techniques has also been receiving attention in recent years, predominantly in electrical engineering literature.

In this study the design and application of so called moveout filters are the main objectives. Since the field of application is seismic reflection, a few important points about the statistical characteristics of a seismic record are in order. The predictive decomposition theorem of Wold

(1938) and the power spectrum factorisation theorem of Kolmogoroff (1939) are both valid for seismic reflection data. Each trace of a seismic record can be thought of as a superposition of wavelets (Ricker, 1953a and 1953b). It is the sum of both signal and noise components (Robinson, 1967c). In this study noise is considered to be the sum of two components; correlated noise and random noise. Signal and correlated noise correlate from trace to trace but they differ in moveout. The above stochastic model describing the input seismic traces has been used extensively in geophysical literature and it has been shown that it is a good working assumption in practice. This is explored further, both theoretically and practically, in the first part of this thesis by the design and application of least-squares moveout filters. In this section the problems and limitations of the least-squares moveout filter design using correlation functions is also explained. In the second part, a new method is introduced with the same objective. It employs a one-dimensional filter design algorithm to approximate a desired frequency-wave number response by means of a reversible mapping between the two domains. The one-dimensional algorithm used is the Tchebycheff (or min-max) approximation. A comparison of the least-squares and the new method is made, with examples. Finally, the extension of the latter technique to the design of two-dimensional circularly symmetric operators is presented, followed by the concluding remarks.

Selected examples of seismic reflection applications, using both synthetic and real data, given in this work help towards making a better assessment of optimum moveout filters. The synthetic applications are too voluminous to display; therefore, only characteristic combined examples are included and the general results obtained have been quoted. These examples include straight-stacked traces for comparisons with the multichannel results where appropriate.

CHAPTER 2

INTRODUCTION TO MULTICHANNEL LEAST-SQUARES

FILTER THEORY

In this chapter the basic design principles of discrete multichannel linear least-squares stacking operators are outlined. The use of multidimensional processing techniques is well justified by the fact that these techniques use the information which has some continuity from trace to trace on seismic reflection records. The least-squares techniques make use of all this information by means of the autocorrelation matrix of input traces. The autocorrelation matrix forms the main component of the normal equations, the solutions of which result in the multichannel filter coefficients.

2.1 PRINCIPLES OF DISCRETE MULTICHANNEL LINEAR LEAST-SQUARES STACKING OPERATORS

Multichannel operators designed in the least-squares sense for filtering of seismic data have been proposed by Burg (1964), Foster et al. (1964), Meyerhoff (1966) and Davis and Mercado (1968) in their continuous forms.

In its discrete form, multichannel linear least-squares filter theory is a generalization of the single channel theory. Single channel filter theory, based on Wiener-Kolmogoroff least-squares principles, has received

a great deal of attention in the exploration geophysics literature. Robinson (1967a) gives a tutorial explanation in his book on statistical communication and detection with relevant references. Optimum least-squares filter design input-output models for multichannel systems always have more than one input series and one or more output series. All inputs are filtered and then summed linearly to obtain a single output which has minimum error power (Figure 2.1.1). The error is the difference between a desired output or signal and actual final output (Robinson, 1963 and 1967b; Treitel, 1970).

A brief and simplified summary of multichannel move-out filters is outlined below. This treatment forms a basis for the theoretical and practical discussions presented in the next chapter. When discrete multiple series $\{x_{i,t}\}$ are the input to a digital linear time invariant filter with a set of coefficients $\{h_{i,j,k}\}$, the outputs will be

$$\begin{aligned}
 y_{j,t} &= \sum_{i=1}^N \sum_{k=0}^L x_{i,t} * h_{i,j,k} \\
 &= \sum_{i=1}^N \sum_{k=0}^L h_{i,j,k} x_{i,t-k}
 \end{aligned}
 \tag{2.1.1}$$

where $j=1,2,\dots,M$ and $t=0,1,\dots,T$. The lengths of the operators are $L+1$. M and N are the number of outputs and inputs, respectively. The lengths of the input series, $T+1$, are assumed to be the same and the output series will

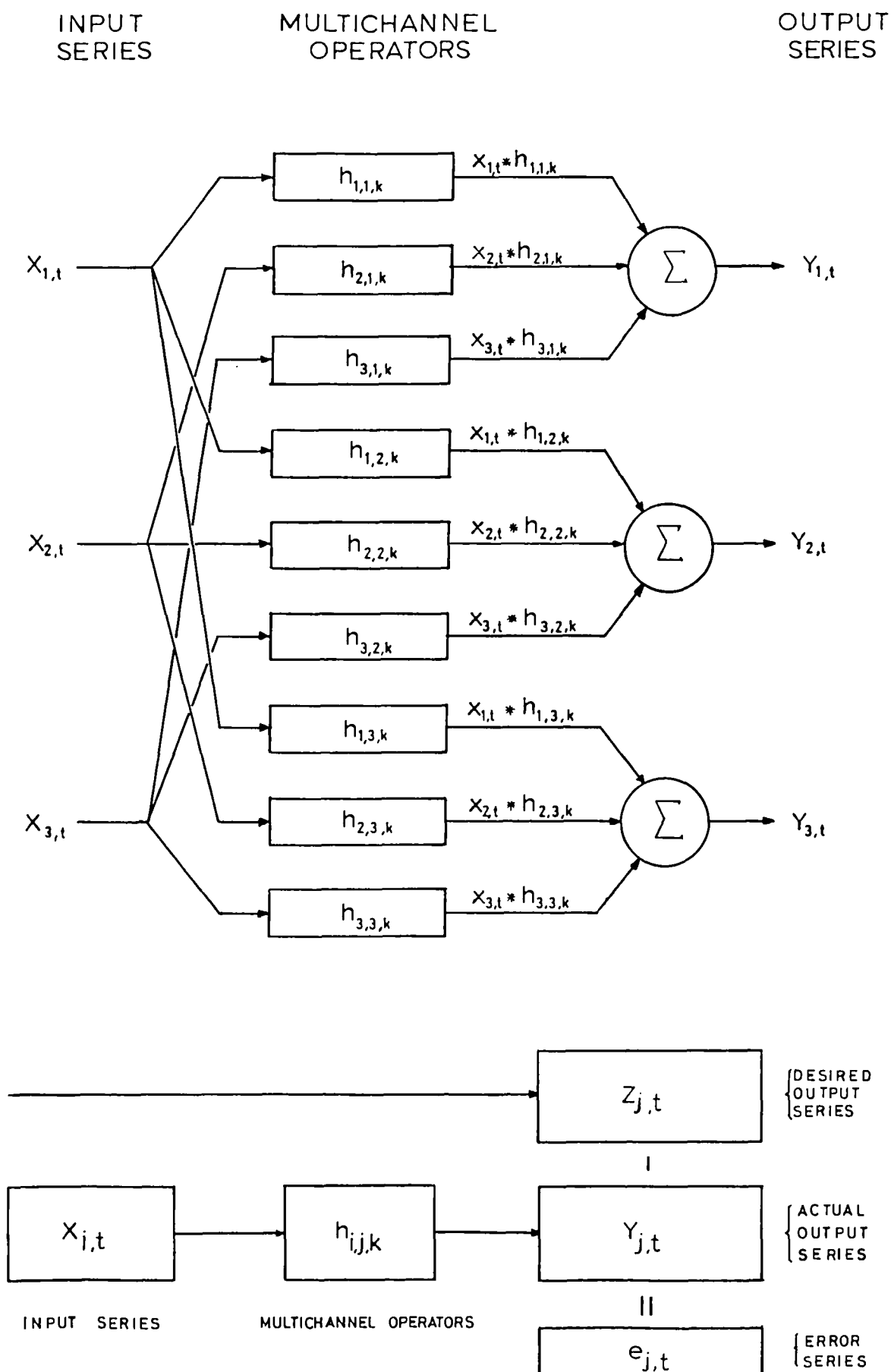


Figure 2.1.1 Block diagram of multichannel least-squares filtering input-output model (lower) and multichannel filter realization for 3 input 3 output case (upper).

have the length(s) $L+T+1$. Throughout this work t represents generally discrete time though sometimes continuous time (but only where stated). The symbol $*$ denotes convolution. By means of the z -transform Equation 2.1.1 can be written in a convenient matrix form where each entry becomes a matrix itself (Robinson and Treitel, 1964; Robinson 1966; Jury, 1967; Treitel, 1970). The multiply-sum operations become a matrix multiplication,

$$\begin{bmatrix} H_{11}(z) & H_{12}(z) & \dots & H_{1N}(z) \\ H_{21}(z) & H_{22}(z) & \dots & H_{2N}(z) \\ \dots & \dots & \dots & \dots \\ H_{M1}(z) & H_{M2}(z) & \dots & H_{MN}(z) \end{bmatrix} \begin{bmatrix} X_1(z) \\ X_2(z) \\ \vdots \\ X_N(z) \end{bmatrix} = \begin{bmatrix} Y_1(z) \\ Y_2(z) \\ \vdots \\ Y_M(z) \end{bmatrix} \quad 2.1.2$$

which can be shortened to the form $H(z) X(z) = Y(z)$ where $X(z)$, $Y(z)$ are the z -transforms of the multichannel input and output, respectively, and $H(z)$ is the z -transform of multichannel filter set. In the time domain the multichannel convolution in Equation 2.1.1 becomes

$$h_0 x_t + h_1 x_{t-1} + h_2 x_{t-2} + \dots + h_k x_{t-k} = y_t \quad 2.1.3$$

where each filter coefficient $h_0, h_1, \dots, h_k$ is an $M \times N$ matrix and x_t and y_t are $N \times (T+1)$ and $M \times (T+L+1)$ matrices, respectively.

There is an analogy with the single channel case.

This involves scalar operations, whereas multichannel processes

involve matrix operations. Operations involving matrix manipulations, namely matrix polynomials or polynomial matrices, are most conveniently given by Gantmacher (1959), Frazer et al. (1963) and are reviewed by Robinson (1967b) and Treitel (1970) for multichannel filtering purposes.

To find the set of coefficients $\{h_{i,j,k}\}$ in Equation 2.1.1, which makes the error or difference between the actual output y_t and some desired output z_t minimum, one solution is the least-squares approach originated by Wiener (1949). z_t , like y_t , is a $M \times (L+T+1)$ matrix. The optimization criterion is the mean-square-error between desired output and actual output. The operator $\{h_{i,j,k}\}$ is determined in such a way that the trace of the mean square-error matrix between z_t and y_t is minimized, i.e. the total mean-square-error is minimized. Mathematically,

$$I = \text{tr} E\{(z_t - y_t)(z_t - y_t)^T\} \quad 2.1.4$$

should be a minimum. In this study E denotes an averaging operator or expectation. The superscript T always denotes matrix transpose. It follows that, to be able to estimate the ensemble average given in Equation 2.1.4, all possible input series, x_t , and desired outputs, z_t , must be stationary and ergodic stochastic processes. Then ensemble averages can be estimated from the time averages. Wiener least-squares theory also assumes that $\{h_{i,j,k}\}$ is a linear time invariant operator.

The error between the desired output

$$z_t \equiv z(t) = \begin{bmatrix} z_1(t) \\ z_2(t) \\ \vdots \\ z_M(t) \end{bmatrix}$$

and the actual output

$$y_t \equiv y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ y_M(t) \end{bmatrix}$$

is

$$e_t = z_t - y_t = \begin{bmatrix} z_1(t) - y_1(t) \\ z_2(t) - y_2(t) \\ \dots\dots\dots \\ z_M(t) - y_M(t) \end{bmatrix} = \begin{bmatrix} e_1(t) \\ e_2(t) \\ \vdots \\ e_M(t) \end{bmatrix} \quad 2.1.5$$

which is an $M \times 1$ vector valued series. Sum of the mean squared errors, I , is the trace of the mean-square-error matrix defined by

$$\begin{bmatrix} E\{e_1^2(t)\} & E\{e_1(t)e_2(t)\} & \dots & E\{e_1(t)e_M(t)\} \\ E\{e_2(t)e_1(t)\} & E\{e_2^2(t)\} & \dots & E\{e_2(t)e_M(t)\} \\ \dots & \dots & \dots & \dots \\ E\{e_M(t)e_1(t)\} & E\{e_M(t)e_2(t)\} & \dots & E\{e_M^2(t)\} \end{bmatrix}$$

and

$$I = \text{tr } E\{e_t e_t^T\} = E\{e_1^2(t)\} + E\{e_2^2(t)\} + \dots + E\{e_M^2(t)\}$$

2.1.6

which is the total mean error power of the least-squares approximation.

From Equations 2.1.1 and 2.1.4

$$\begin{aligned} e_t &= z_t - y_t = z_t - \sum_{k=0}^L h_k x_{t-k} \\ e_t e_t^T &= z_t z_t^T - z_t \sum_{r=0}^L x_{t-r}^T h_r^T - \sum_{k=0}^L h_k x_{t-k} z_t^T \\ &\quad + \sum_{k=0}^L \sum_{r=0}^L h_k x_{t-k} x_{t-r}^T h_r^T \end{aligned} \quad 2.1.7.$$

Multichannel subscripts are omitted for convenience. The ensemble average of the above can be written as,

$$\begin{aligned} E\{e_t e_t^T\} &= E\{z_t z_t^T\} - \sum_{p=0}^L E\{z_t x_{t-p}^T\} h_p^T - \sum_{k=0}^L h_k E\{x_{t-k} z_t^T\} \\ &\quad + \sum_{k=0}^L \sum_{p=0}^L h_k E\{x_{t-k} x_{t-p}^T\} h_p^T \end{aligned} \quad 2.1.8$$

(Kolmogoroff, 1956). The lagged products and sums in the above equation are the correlation matrices involving inputs and desired output(s). These are, as defined, for stationary stochastic processes. The autocorrelation matrix of the input is given by

$$C_{xx}(p-k) = E\{x_{t-k}x_{t-p}^T\}$$

the crosscorrelation matrix between the desired output and input series is,

$$C_{zx}(p) = E\{z_t x_{t-p}^T\}$$

and the crosscorrelation matrix between input and desired output is,

$$C_{xz}(-k) = E\{x_{t-k} z_t^T\}$$

with the relation between the last two being

$$C_{zx}(k) = C_{xz}^T(-k) = E\{z_t x_{t-k}^T\}.$$

Hence Equation 2.1.8 becomes

$$\begin{aligned} E\{e_t e_t^T\} = E\{z_t z_t^T\} &- \sum_{p=0}^L C_{zx}(p) h_p^T - \sum_{k=0}^L h_k C_{xz}(-k) \\ &+ \sum_{k=0}^L \sum_{p=0}^L h_k C_{xx}(p-k) h_p \end{aligned} \quad 2.1.9.$$

processes. In the above equations $(h_0, h_1, \dots, h_L) \equiv h_{i,j,k}$ are the $M \times N$ matrices; k th filter coefficient between the i th input and j th output. The multichannel autocorrelation matrix $C_{xx}(\tau)$, $\tau = 0, \pm 1, \dots, \pm L$ and $C_{xx}(\tau) = C_{xx}(-\tau)$, is a block Toeplitz matrix, i.e. the $M \times N$ matrix elements along any block diagonal are the same. The crosscorrelation matrix $C_{zx}(\tau)$, $\tau = 0, 1, 2, \dots, L$, also has $M \times N$ matrix elements. As seen from Equations 2.1.10 and 2.1.11, the filter coefficients are calculated from autocorrelation and crosscorrelation functions of inputs and desired output(s) but they do not depend on these series directly. Therefore, input and desired output series must be stationary stochastic (random) processes and the set of operators obtained from the last equation are optimum in the least-squares sense for every input and desired output process having the same ensemble averages for their correlation functions.

Estimation of C_{xx} and C_{zx} is either done directly from input and desired output series (Blackman and Tukey, 1958; Jenkins and Watts, 1968) or from assumed suitable input and desired output models. Multichannel deconvolution which takes as desired output a spike at the zeroth sample leading to a crosscorrelation matrix which has spikes at zero positions has been given by Davies and Mercado (1968). The autocorrelation matrix is calculated from the input data in this case.

The solution of the normal equations in 2.1.10, leading to the matrix equation in 2.1.11, is done in a

recursive manner exploiting the block Töplitz character of the autocorrelation matrix (Wiggins and Robinson, 1965). This is an extension of the fast Levinson method from the single to multichannel case with added features to speed up computations and storage savings on a digital computer.

In practice, an input model of

$$x_t = s_t + r_t + n_t \quad 2.1.13$$

represents a general input process, where s_t , r_t and n_t are the multichannel N input series representing signal, correlated noise and uncorrelated or random noise, respectively. The desired output is taken as $z_t = s_t$. Putting Equation 2.1.13 in Equation 2.1.7

$$\begin{aligned} e_t &= s_t - \sum_{k=0}^L h_k (s_{t-k} + r_{t-k} + n_{t-k}) \\ &= s_t - \sum_{k=0}^L h_k s_{t-k} - \sum_{k=0}^L h_k r_{t-k} - \sum_{k=0}^L h_k n_{t-k} \end{aligned} \quad 2.1.14$$

is obtained. If the same steps for minimizing the mean-square-error, or the time average of the power in the error signal e_t , is followed, Equation 2.1.12 becomes

$$\sum_{k=0}^L h_k [C_{ss}(p-k) + C_{rr}(p-k) + C_{nn}(p-k)] = C_{ss}(p) \quad 2.1.15$$

where $p=0,1,\dots,L$, assuming that components of the input model are not correlated, i.e. they are statistically

independent. In other words,

$$C_{sr} = C_{rs} = C_{sn} = C_{ns} = C_{nr} = C_{rn} = 0 \quad 2.1.16$$

$$C_{x_i x_j}(\tau) = C_{s_i s_j}(\tau) + C_{r_i r_j}(\tau) + \delta_{i,j} C_{n_i n_j}(\tau) \quad 2.1.17$$

$$C_{z_i s_j}(\tau) = C_{s_i s_j}(\tau) \quad 2.1.18$$

where $i, j = 1, 2, \dots, N$ and $\tau = 0, 1, \dots, L$. $\delta_{i,j}$ is the Dirac delta pulse defined as $\delta_{i,j} = 0$ if $i \neq j$.

Sometimes it is necessary to emphasize the power of input signal or noise according to the problem at hand. This will be further explored in the next chapter with the design and application results of multichannel linear move-out filters. The power spectra of stationary stochastic ergodic processes are calculated from their Fourier transforms. If w_r and w_n are the correlated and uncorrelated noise power weighting factors relative to the signal power for all frequencies, then Equation 2.1.15 is modified to

$$\sum_{k=0}^L h_k [C_{ss}(p-k) + w_r C_{rr}(p-k) + w_n C_{nn}(p-k)] = C_{ss}(p) \quad 2.1.19$$

where $p = 0, 1, \dots, L$. It is also known that when a delay, t_d , is introduced into the desired output, the value of mean-square-error is reduced (Claerbout and Robinson, 1964; Wiggins, 1967). The value which makes the mean-square-

error a minimum is the optimum value for t_d . The desired output is now $z_t = s_{t-t_d}$ and the right hand side of Equation 2.1.15 becomes $C_{zs}(p) = C_{ss}(p-t_d)$. With the relative weights introduced into the same equation, it becomes

$$\sum_{k=0}^L h_k [C_{ss}(p-k) + w_r C_{rr}(p-k) + w_n C_{nn}(p-k)] = C_{ss}(p-t_d)$$

2.1.20

where $p = 0, 1, \dots, L$. These are the normal equations in their general form which will be used in this work for multichannel least-squares filter design. The computer program for the linear moveout stacking filters has been developed from that of Hubral (1970).

Operators calculated from Equation 2.1.20 are the approximations to the exact solutions. Exact solutions can only be obtained from infinitely long operators (physically unrealizable) with autocorrelations and cross-correlations calculated up to infinite lags.

2.2 AUTOCORRELATION AND CROSSCORRELATION MATRICES FOR GENERAL PURPOSE OPTIMUM LINEAR LEAST-SQUARES STACKING OPERATORS

In the last section it was shown that for the design of most general optimum linear multichannel filters in the least-squares sense all it is necessary to know are the autocorrelation matrix of the input processes and cross-

correlation matrix of the desired output and input processes. The stochastic model given in Equation 2.1.13 has been successfully used for designing optimum stacking operators for seismic data. The three-dimensional frequency and vector wave number least-squares filters for seismic arrays (Burg, 1964), ghost elimination filters (Schneider et al., 1964), suboptimum ghost and velocity filters (Foster et al., 1964), differential moveout filters for multiple attenuation (Schneider et al., 1965), optimum multichannel velocity pass and reject filters (Sengbush and Foster, 1968) and optimum stacking filters (Galbraith and Wiggins, 1968) fall in this category. The technique proposed by D'Hoeraene (1966) for the attenuation of multiples with differential moveout is similar to that of Schneider et al., (1965), except that in D'Hoeraene's method only the energy of multiples is minimized. The derivation and design of optimum least-squares multichannel filters has been extensively treated by Hubral (1970, 1972 and 1974). Cassano and Rocca (1973 and 1974) have solved Wiener-Hopf equations for multichannel moveout filters in frequency domain for the same stochastic model given by Galbraith and Wiggins. Olorunniwo (1973) used the same least-squares Wiener theory for designing multichannel velocity filters for bandvarying signals. The present study is the continuation of a research program at Imperial College into the theory and application of multichannel filters.

A stochastic model more suitable for seismic reflection traces proposed by Sengbush and Foster (1968) is

$$x_i(t) = s(t - \tau_i) + r(t - \tilde{\tau}_i) + n_i(t) \quad 2.2.1$$

where $i = 1, 2, \dots, N$. Here t , τ_i and $\tilde{\tau}_i$ are continuous variables. This makes it possible to represent moveouts which do not necessarily fall on sample points. τ_i and $\tilde{\tau}_i$ are independent random variables uniformly distributed over prescribed non-overlapping intervals. A general time window configuration for this model is given in Figure 2.2.1. The relative arrival times are random variables uniformly distributed between t_i and $t_i + \Delta t_i$, e.g. $t_{r-2} \leq a_{r-2} \leq t_{r-2} + \Delta t_{r-2}$. The choice of relative moveouts and positioning of the desired output trace (r in Figure 2.2.1) gives nonsymmetric, symmetric, centrosymmetric and trace (or channel) - symmetric windows. The latter three give multichannel filters with zero phase response.

The relative arrival times of signal or correlated noise for a reference channel r and i th channel can be

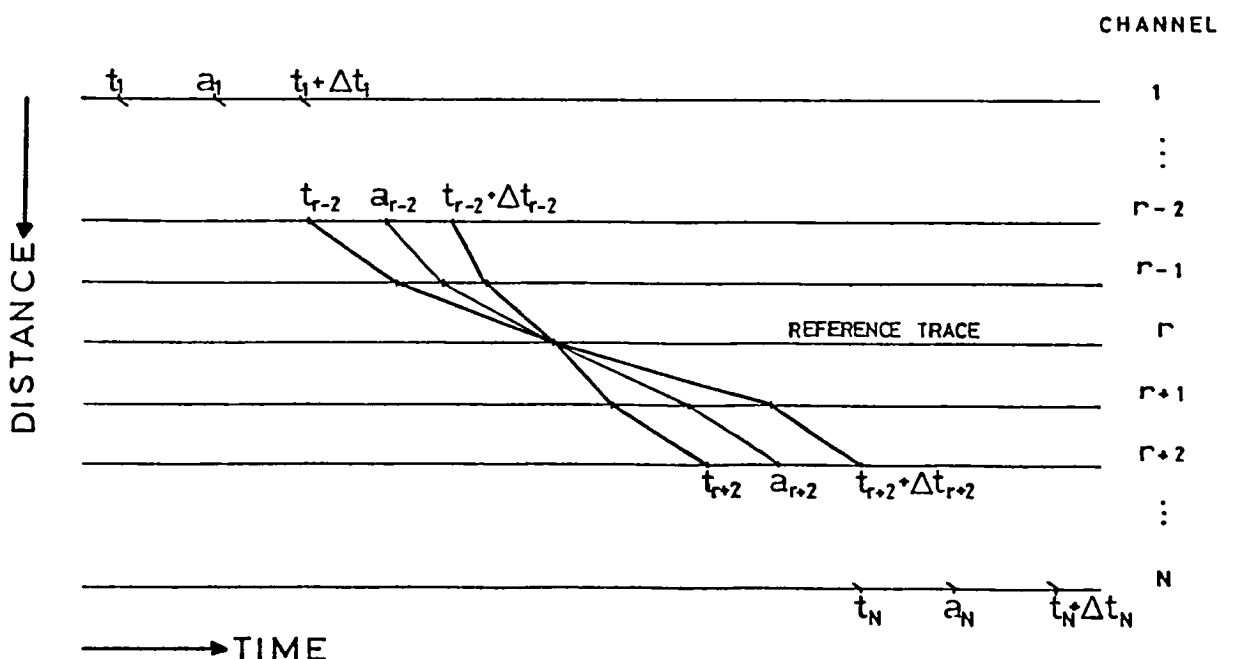


Figure 2.2.1 A general time window.

respectively defined as,

$$a_r = t_r + \frac{\Delta t_r}{2} + \eta + \gamma_r$$

$$a_i = t_i + \frac{\Delta t_i}{2} + \eta \frac{\Delta t_i}{\Delta t_r} + \gamma_i \quad 2.2.2$$

with $-\frac{\Delta t_r}{2} \leq \eta \leq \frac{\Delta t_r}{2}$, $-\frac{t_c}{2} \leq \gamma_i \leq \frac{t_c}{2}$ and $i = 1, 2, \dots, N$.

η and γ_i are the random variables with probability density functions:

$$p(\eta) = \frac{1}{\Delta t_r} \text{rec}\left(\frac{\eta}{\Delta t_r}\right)$$

$$p(\gamma_i) = \frac{1}{t_c} \text{rec}\left(\frac{\gamma_i}{t_c}\right), \quad i = 1, 2, \dots, N \quad 2.2.3.$$

Here t_c is the so called 'chatter' parameter introduced into the design to take into account signal and correlated noise trace independent random deviations from their moveout curves. The maximum value of these deviations is $\left|\frac{t_c}{2}\right|$. The parameter η forces the arrival times to fall on non-overlapping curves dividing the window widths, Δt_i , on the traces in the same ratio. The rectangular function $\text{rec}\left(\frac{t}{x}\right)$ is defined as,

$$\text{rec}\left(\frac{t}{x}\right) \equiv \begin{cases} 1 & \text{for } |t| < \frac{x}{2} \\ 0 & \text{for } |t| > \frac{x}{2} \end{cases} \quad 2.2.4$$

(Bracewell, 1965).

The autocorrelation matrix for each signal is

$$\begin{aligned}
C_{s_i s_j}(\tau) &= E\{s_i(t+\tau)s_j(t)\} \\
&= E\{s_i(t+\tau-a_i) s_j(t-a_j)\} \\
&= C_{s_i s_j}\left(\tau+t_j-t_i+\frac{1}{2}(\Delta t_j-\Delta t_i) + \frac{n}{\Delta t_r}(\Delta t_j-\Delta t_i) + \gamma_j-\gamma_i\right) \\
&= C_{s_i s_j}(\tau+t_{i,j}+\hat{n}+\gamma_j-\gamma_i)
\end{aligned} \tag{2.2.5}$$

$$\text{where } t_{i,j} = t_j - t_i + \frac{1}{2}(\Delta t_j - \Delta t_i)$$

$$\hat{n} = \left(\frac{n}{\Delta t_r}\right)(\Delta t_j - \Delta t_i) = \frac{n}{\Delta t_r} \tau_{i,j} \quad \text{and}$$

$$\tau_{i,j} = \Delta t_j - \Delta t_i \tag{2.2.6.}$$

Since $\hat{n}$ is dependent on the random variable n , its probability density function can be obtained from 2.2.6 and $p(n)$ as:

$$p(\hat{n}) = \frac{1}{\tau_{i,j}} \text{rec}\left(\frac{\hat{n}}{\tau_{i,j}}\right) \tag{2.2.7}$$

If the Fourier transform of both sides of Equation 2.2.5 is taken, it becomes

$$Q_{s_i s_j}(f) = Q_{ss}(f) e^{i2\pi t_{i,j}f} e^{i2\pi \hat{n}f} e^{i2\pi \gamma_j f} e^{-i2\pi \gamma_i f} \tag{2.2.8}$$

with the expectation

$$\begin{aligned}
E\{Q_{s_i s_j}(f)\} &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} Q_{ss}(f) e^{i2\pi t_{i,j}f} p(\hat{n}) e^{i2\pi \hat{n}f} p(\gamma_j) e^{i2\pi \gamma_j f} \\
&\quad p(\gamma_i) e^{i2\pi \gamma_i f} d\hat{n} d\gamma_j d\gamma_i \\
&= Q_{ss}(f) e^{i2\pi t_{i,j}f} \int_{-\infty}^{\infty} p(\hat{n}) e^{i2\pi \hat{n}f} d\hat{n} \int_{-\infty}^{\infty} p(\gamma_j) e^{i2\pi \gamma_j f} d\gamma_j \\
&\quad \int_{-\infty}^{\infty} p(\gamma_i) e^{-i2\pi \gamma_i f} d\gamma_i \\
&= Q_{ss}(f) e^{i2\pi t_{i,j}f} \text{sinc}(\tau_{i,j}f) \text{sinc}^2(t_c f)
\end{aligned}
\tag{2.2.9}$$

Note that throughout this thesis $i = \sqrt{-1}$ whenever it is the first character of an exponent. If it is assumed that signal has a flat response from zero to the Nyquist frequency, Equation 2.2.9 becomes

$$E\{Q_{s_i s_j}(f)\} = \text{rec}(f) e^{i2\pi t_{i,j}f} \text{sinc}(\tau_{i,j}f) \text{sinc}^2(t_c f) \tag{2.2.10}$$

where $\text{rec}(f)$ is defined in the same way as in Equation 2.2.4 above:

$$\text{rec}(f) \equiv \begin{cases} 1 & \text{for } |f| < 0.5 \\ 0 & \text{for } |f| > 0.5 \end{cases}$$

In the time domain Equation 2.2.10 becomes

$$E\{C_{s_i s_j}(\tau)\} = \int_{-0.5}^{0.5} \text{sinc}(\tau_{i,j}f) \text{sinc}^2(t_c f) \cos[2\pi f(t_{i,j} + \tau)] df \tag{2.2.11}$$

Under this broadband assumption, input trace autocorrelation functions become sinc functions:

$$E\{C_{s_i s_i}(\tau)\} = \text{sinc}(\tau) \quad 2.2.12.$$

With the broadband assumption, for each entry of correlated noise component in the input series the same equations as 2.2.10 and 2.2.11 are obtained, except this time $t_{i,j}$ and $\tau_{i,j}$ (Equation 2.2.6) are defined by correlated noise moveout window(s). That is

$$E\{C_{r_i r_j}(\tau)\} = \int_{-0.5}^{0.5} \text{sinc}(\tilde{\tau}_{i,j} f) \text{sinc}^2(t_c f) \cos[2\pi f(\tilde{t}_{i,j} + \tau)] df \quad 2.2.13.$$

For the random noise, the contribution to the autocorrelation matrix is

$$E\{C_{n_i n_j}(\tau)\} = \text{sinc}(\tau) \delta_{i,j}, \quad \text{where } \delta_{i,j} = 0 \quad \text{if } i \neq j, \quad 2.2.14.$$

The crosscorrelation between the desired output on the r th reference trace and the signal on the j th input trace can be written as

$$\begin{aligned} C_{s_r s_j}(\tau) &= E\{s_r(t + \tau - t_d) s(t - a_j)\} \\ &= C_{ss}(\tau + a_j - t_d) \\ &= C_{ss}\left(\tau + t_j + \frac{\Delta t_j}{2} + \eta \frac{\Delta t_j}{\Delta t_r} + \gamma_j - t_d\right) \\ &= C_{ss}(\tau + \tau_j + \tilde{\eta} + \gamma_j) \end{aligned} \quad 2.2.15$$

where t_d and a_j are as defined before and:

$$\tau_j = t_j + \frac{\Delta t_j}{2} - t_d$$

$$\tilde{n} = n \frac{\Delta t_j}{\Delta t_r}$$

$$p(\tilde{n}) = \frac{1}{\Delta t_j} \text{rec}\left(\frac{\tilde{n}}{\Delta t_j}\right) \quad 2.2.16$$

Repeating the same steps in Equations 2.2.8 and 2.2.9,

$$E\{Q_{s_r s_j}(f)\} = \text{rec}(f) e^{i2\pi\tau_j f} \text{sinc}(\Delta t_j f) \text{sinc}(t_c f) \quad 2.2.17$$

and

$$E\{C_{s_r s_j}(\tau)\} = \int_{-0.5}^{0.5} \text{sinc}(\Delta t_j f) \text{sinc}(t_c f) \cos[2\pi f(\tau_j + \tau)] df \quad 2.2.18$$

are obtained.

The integrals given in Equations 2.2.11 and 2.2.18 can be reduced to sine, cosine and integrals containing sinc functions which are calculated numerically.

If the input traces have variable gain with a random distribution it does not affect the crosscorrelation matrix and has the same effect of adding a white noise constant to the autocorrelation matrix (Baldwin, 1964; Galbraith and Wiggins, 1968).

A brief description of a general purpose moveout filter computer program is given in Appendix 1.

CHAPTER 3

APPLICATIONS OF OPTIMUM MULTICHANNEL LEAST-SQUARES (MLS) LINEAR MOVEOUT OPERATORS

Design characteristics of linear moveout multichannel least-squares (MLS) filters with their field and synthetic data applications are presented in this chapter. Emphasis will be put on the practical implications of the filter parameters.

In the past it has been shown that optimum multichannel filters, designed in the least-squares sense for seismic reflection data, have superior responses compared to conventional Common Depth Point (CDP) stacking. As a result of this, a better signal to noise (S/N) ratio is expected after optimum multichannel filtering. Applications to field and offshore data given in the recent literature (Cassano and Rocca, 1973 and 1974) indicate that the relative improvement brought about by multichannel filtering is small in some cases.

The multichannel processing of seismic data is expensive because either a large memory is devoted to storing the input data or data has to be transferred between memory and a peripheral storage device to carry out lengthy convolving and summing operations. With the fast expansion of land and especially offshore seismic exploration in more and more stratigraphically complex areas, there has been a big

demand for handling and processing large amounts of seismic reflection data. It is expected that specially designed computers and array processors used in industry will make multichannel filtering beneficial in conditions where stacking techniques are not adequate. Obviously, multichannel filters can always be applied to already CDP stacked data for further S/N improvements.

A thorough investigation of the published research shows that there are still many aspects of multichannel filtering which have not been solved. These problems can be categorized under three headings:

1. Matching the actual input data with the assumed model used in filter design.
2. Designing multichannel least-squares operators to approximate a desired response.
3. Undesired after effects of multichannel filtering e.g. spurious line-ups.

The theoretical arguments and practical results of this chapter is an attempt to show the advantages and limitations of optimum MLS filters. It is assumed that some sort of analysis is carried out before multichannel filter design to estimate relative moveouts for signal and correlated noise. This can be done either by simply measuring the moveouts from displayed records whenever it is possible or through more computer oriented methods such as dynamic correlation analysis techniques (Schneider and Backus, 1968), and velocity techniques (Cook and Taner, 1969; Robinson,

1969; Taner and Koehler, 1969). A more elaborate investigation for estimating frequency-wave number spectra of seismic traces is done by Hains (1973). In later sections it will be shown that an accurate velocity or moveout function is essential, but the knowledge of other statistics such as signal and noise power and chatter is not crucial for optimum multichannel least-squares operator design.

Throughout this thesis the parameters of the moveout filters designed are given on the page facing their frequency-wave number, or (f,k) , responses where appropriate. If the same filter is redesigned with only one varying parameter, this parameter is printed alongside the responses given. The synthetic reflection seismograms are presented in the same manner. All input parameters for generating the synthetic examples are given on the facing page when the data is first presented. These pages are self-contained.

3.1 ERROR IN THE OUTPUTS FROM LEAST-SQUARES MOVEOUT OPERATORS

In this section the error involved in approximating a desired response, or effectively a desired output, from an input model by multichannel least-squares (MLS) moveout filtering is investigated for different input models. The mean-square-errors involved in approximating some one-dimensional operators used in seismic data processing were given by Wiggins (1967) and Galbraith (1971).

The mean-square-error given in Equation 2.1.9 will reduce to

$$I_{\min} = \text{tr}C_{zz}(0) - \text{tr} \sum_{k=0}^L C_{zx}(k)h_k^T \quad 3.1.1$$

(Robinson, 1967b). In practice the normalized mean-square-error (NMSE) = $I_{\min}/\text{tr}C_{zz}(0)$, which may have values from zero to unity, is used as an indicator for the approximation. The NMSE decreases with increased filter length and/or input channels. It also varies with the delay introduced at the desired output. From the designer's point of view, for a fixed pass and reject window configuration, the smaller the error the better the approximation of the filter response which means improved desired output. In the remaining sections of this chapter it will be shown that this is not always the case. A design parameter such as uncorrelated noise weight, when increased in value, always increases the error but it is necessary for stability up to a certain value.

In the following examples, extracted from a large number of optimum MLS moveout filters designed and applied during this study, it will be shown how the NMSE varies with changing filter parameters. Maps, obtained by printing out NMSE for optimum length/optimum delay filters showed fairly complicated contouring with an increased number of prescribed moveout windows in the design. The optimum delay position for minimum error was generally around the half filter length except for very short operators. The

same mappings obtained for moveout filters designed to pass events with zero or small moveout and reject events with higher moveout showed that increasing chatter and uncorrelated noise weight always increase the NMSE.

In Figure 3.1.1A the NMSE versus delay curves of a simple moveout pass filter for different lengths are plotted. These filters have unit weight for their pass windows. The NMSE versus delay curves in Figure 3.1.1B are for the same filters with an added reject window. Note that the filter design parameters in this section are also given on the pages facing the corresponding figures. The immediate conclusion which can be generalized is that, although the added window increases the NMSE, the optimum delay positions do not change noticeably. Increasing the filter length always decreases the NMSE. Beyond a certain length, however, additional increases in length have very little effect on NMSE.

The NMSE versus filter length curves of 5×31 and 7×31 linear moveout filters are given in Figure 3.1.2. The curves A and B belong to the same filter with the reference output traces being $R=1.0$ and $R=3.0$, respectively. It is evident that the centre trace estimate is superior. This is verified by applying N input- N output filtering to synthetic data. In both cases increasing the filter length beyond about 12 does not reduce the NMSE substantially.

Comparing the curves C and D shows that although the moveout limits are the same for the corresponding

The design parameters of the filters, whose NMSE versus delay curves are given in Figure 3.1.1A, are:

Number of input channels,	$N = 6$
Length of filter,	$LF = 10, 20, 30, 40 \text{ and } 50$
Reference output trace,	$R = 3.5$
Pass moveout window (time unit/trace),	$0_s = 0.0 \text{ to } 0.75$
Uncorrelated noise weight,	$w_n = 1.0$
Chatter (time unit),	$t_c = 0.5$

The NMSE versus delay curves given in Figure 3.1.1B are for the same filters with the following added reject window:

Reject moveout window (time unit/trace),	$0_r = 1.5 \text{ to } 5.0$
Correlated noise or reject window weight,	$w_r = 11.0$

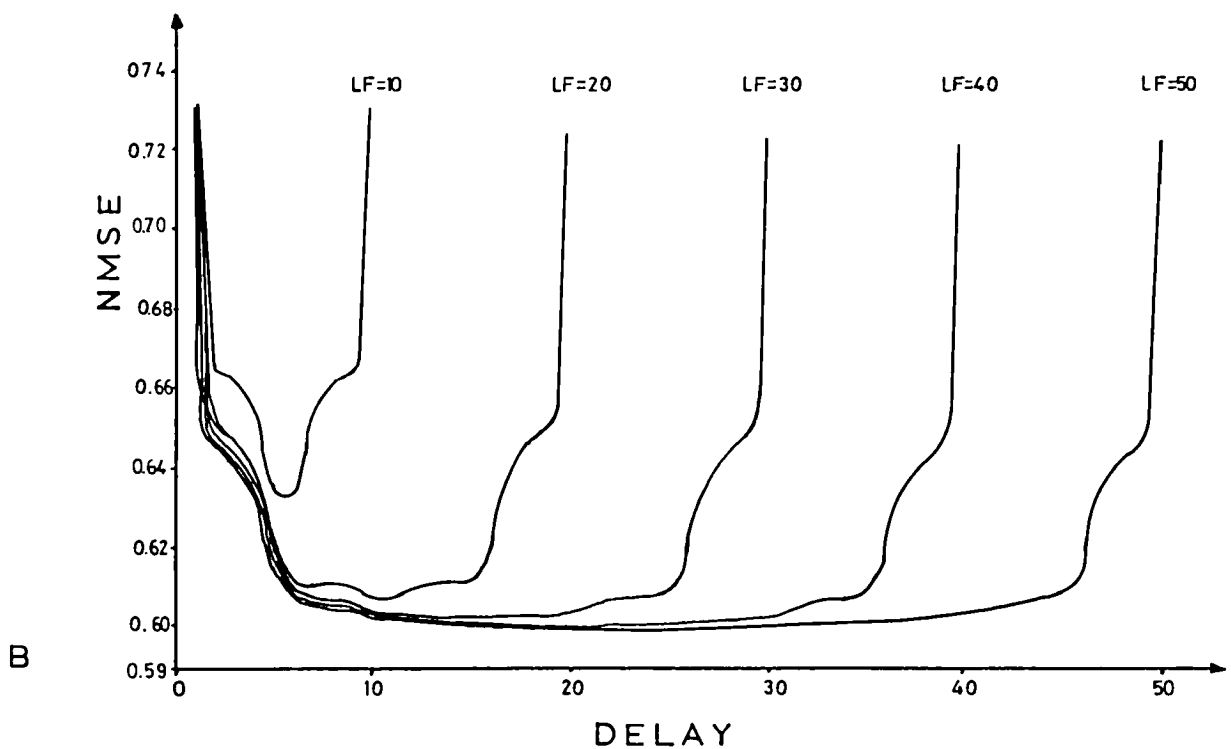
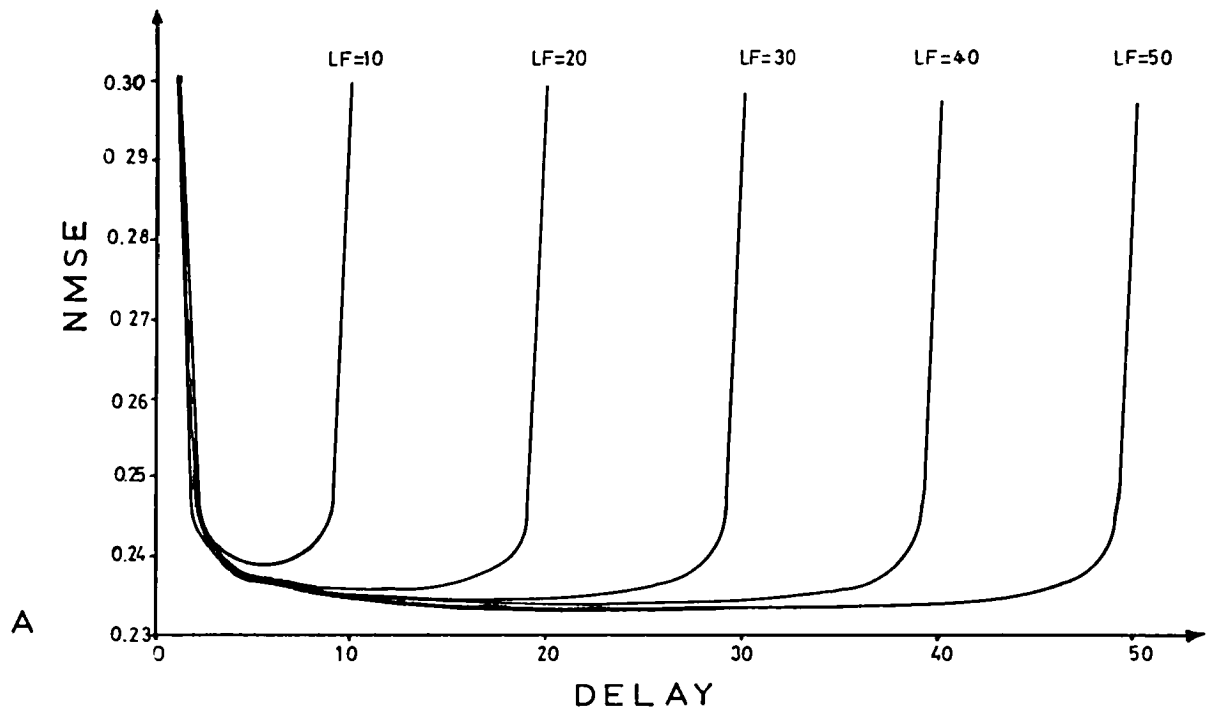


Figure 3.1.1 The normalized mean-square error (NMSE) versus delay curves: A) Simple pass moveout filters and B) Pass-reject moveout filters. The filter design parameters are given on the facing page.

filters, an added transition band in the reject region window makes the approximation less accurate. The curve E shows how reducing one of the reject region weights for the filter of Curve-D reduces the NMSE. One reason for this reduction is that aliasing of this reject region into the pass region is reduced. Secondly, the reject region with smaller weight is merged into the transition band to make approximation easier. Further decreases in correlated noise weight will lower the error more because the approximation problem reduces to a pass filter case, see Curve-F.

Relatively higher weighting of both correlated and uncorrelated noise is the same as effectively increasing the noise power assumed at the input. This naturally gives a larger noise level at the output which manifests itself with an increased error of approximation. This is more clearly seen if the error function is defined as

$$I_{w_r w_n} = \text{trE}\{s_{e_t} s_{e_t}^T\} + w_r \text{trE}\{r_{e_t} r_{e_t}^T\} + w_n \text{trE}\{n_{e_t} n_{e_t}^T\} \quad 3.1.2.$$

The normal equations obtained from this equation after minimization procedure will result in Equation 2.1.19 of the previous chapter, i.e. the same basic normal equations for designing the optimum MLS filters as presented in this study.

Curve-G in the same figure belongs to a filter with a similar moveout configuration to that of the filters of

The design parameters for the moveout filters, whose NMSE versus filter length curves are given in Figure 3.1.2, are:

Curve - A

$N = 5$, $LF = 31$, $R = 1.0$, $0_s = -1.0$ to 1.0 , $0_r = 1.0$ to 2.0 , $w_r = 1.0$, $w_n = 2.0$ and $t_c = 0.5$.

Curve - B

This curve is for the same filter as curve - A except the reference output trace is the centre trace, i.e. $R = 3.0$.

Curve - C

$N = 5$, $LF = 31$, $R = 3.0$, $0_s = -0.625$ to 0.625 , $0_r = 2.5$ to 5.0 , $w_r = 1.0$, $w_n = 1.0$ and $t_c = 0.0$.

Curve - D

$N = 5$, $LF = 31$, $R = 3.0$, $0_s = -0.625$ to 0.625 , $0_r^{(1)} = 2.5$ to 3.75 , $0_r^{(2)} = 4.125$ to 5.0 , $w_r^{(1)} = w_r^{(2)} = 1.0$, $w_n = 1.0$ and $t_c = 0.0$.

Curve - E

This curve is for the same filter as curve - D except $w_r^{(1)} = 0.1$.

Curve - F

This curve is for the same filter as curves - D and E except $w_r^{(1)} = 0.01$ and $w_r^{(2)} = 0.1$.

Curve - G

$N = 7$, $LF = 31$, $R = 4.0$, $0_s = 2.5$ to 3.375 , $0_r^{(1)} = -0.625$ to 0.625 , $0_r^{(2)} = 4.125$ to 5.0 , $w_1^{(1)} = w_1^{(2)} = 1.0$, $w_2 = 1.0$ and $t_c = 0.5$.

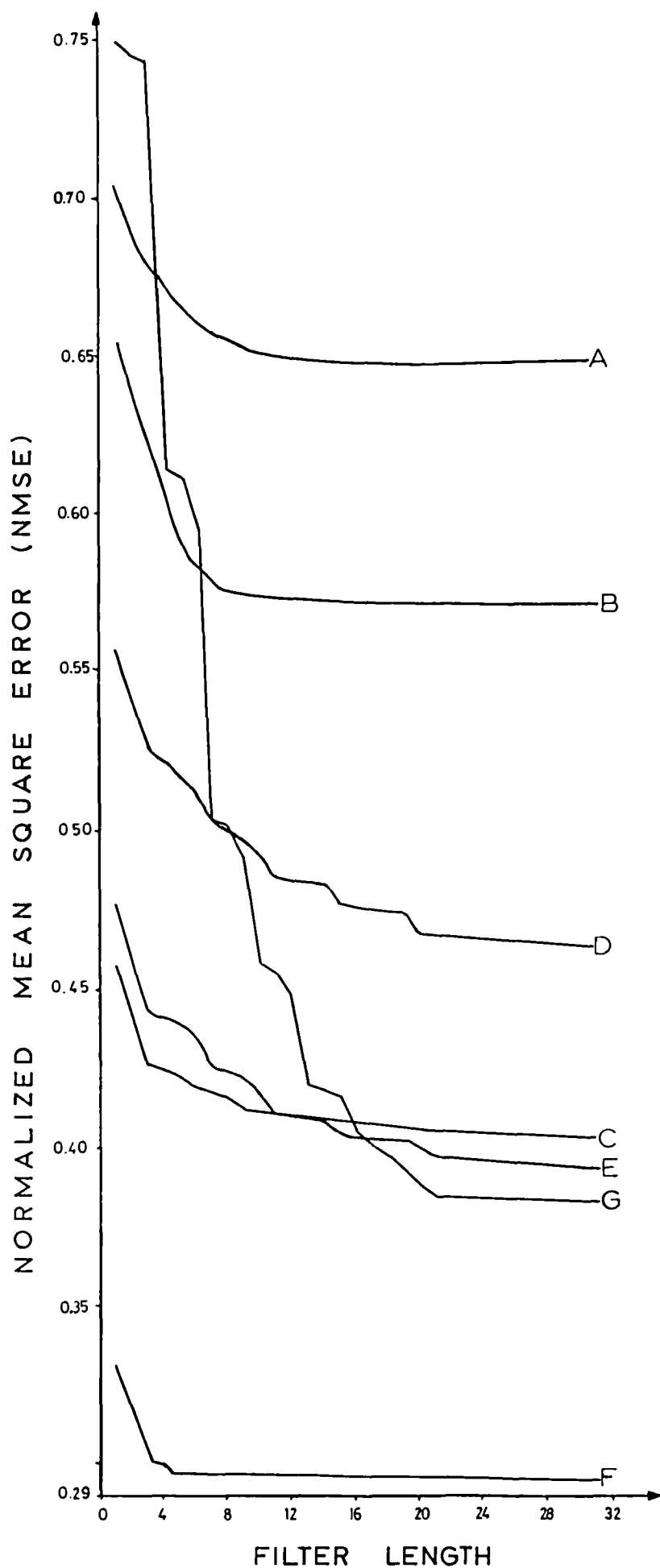


Figure 3.1.2 The normalized mean-square-error versus filter length curves for optimum least-squares moveout filters with various design parameters. The filter design parameters are given on the facing page.

Curves-E and F except the pass region is interchanged with one of the reject regions. The abrupt steplike changes in the NMSE versus filter length curve of this filter are due to the multiplicity of the prescribed moveout windows. A pass region in between reject regions makes approximation more difficult and the NMSE error is high. This is more pronounced at smaller filter lengths.

It is generally found that the NMSE versus filter length curve of a moveout filter is shaped by its moveout window configuration and window weightings. Although the relative error does not decrease substantially after certain lengths, long filters or filters with an increased number of input channels are required if a close approximation is desired near the origin of the (f,k) responses. For practical purposes the following approximate generalizations about the NMSE of the least-squares moveout filters can be made.

1. The optimum delay position for the desired output is equal to half of the filter length. For filters with an odd number of coefficients this value is almost always rounded to the nearest highest digit. For filters with a small number of coefficients the same rule applies with plus or minus one accuracy. When big moveouts, relative to the filter length, are involved, optimum delay position is less predictable. The relative values of NMSE for delay values around half the filter length, plus or minus one, differ by about 0.01 to 0.03 for short filters and 0.00001 or less

for filters having lengths greater than the biggest stepout time of any prescribed region, i.e. the NMSE changes slowly near mid-point delay.

2. The error decreases with increasing filter length or by increasing the number of input channels, and it is smallest for centre trace estimation. Filters with simple responses have smooth NMSE versus filter length curves which flatten out quickly after the length of the filter is long enough to cover the biggest stepout time amongst all of the prescribed windows. The same curves for filters with complex responses are steplike. They begin to die out after the filter length reaches to about twice the maximum stepout time. Relative weighting of prescribed moveout windows affects the shape of the NMSE versus filter lengths and the NMSE versus delay curves as well as the relative error.

3. Design results not presented here showed that increasing chatter and uncorrelated noise for stable filters increases the NMSE. This increase is relatively high if the windows with smaller moveouts are pass windows, which is almost always the case for seismic reflection applications. The reasons for these increases will become clearer in the later sections. Here it is sufficient to say that the added noise power in the input model naturally makes the output noise larger which means increased error in the approximation of the desired output. Increased chatter in the input model is equivalent to the increased

high frequency attenuation which results with a poor pass domain approximation at these frequencies.

3.2 PARAMETERS AFFECTING THE RESPONSE AND PERFORMANCE OF OPTIMUM MULTICHANNEL LEAST-SQUARES (MLS) STACKING OPERATORS

The practical response and performance of an optimum MLS moveout filter vary with changing correlated noise weight (w_r), uncorrelated noise weight (w_n) and chatter (t_c). In the remaining part of this chapter, the effects of varying the above parameters are investigated with special reference to the following,

1. Changes in the S/N ratio after multichannel filtering,
2. The efficiency of extracting signal events from severe overriding noise by stacking and moveout filtering,
3. The effect of moveout filtering on events falling into a pass moveout window,
4. Changes in the frequency content of the input data after moveout filtering,
5. The mixing effect of the multichannel operators.

In due course, it will be seen that the parameters w_n , w_r and t_c affect the filter response and performance sometimes in similar ways, i.e. they are interacting para-

meters. For this reason their effects on the filter responses and subsequent applications will be examined separately. It was found that tests with parameter combinations were less informative than the individual treatments presented.

Examples to illustrate the above points will be shown with synthetic and real data. Figure 3.2.1A shows a 24 trace, digitally recorded, seismic reflection split-spread record. Along with two other adjoining records from the same area the above data are used to examine the effect of varying the filter parameters w_r , w_n and t_c . Conclusions drawn from all the applications were the same and the results displayed here are found to be the most representative. These data have been moveout corrected, edited, 50 Hz notch filtered and deconvolved. The digitizing interval is 4 ms. The samples from 100 ms to 1200 ms are gated for processing. The number of input traces, N , for all the moveout filters applied was 5. The two beginning and two end traces of two sets of 12 trace data are estimated from adjoining traces, the rest are centre trace estimates with overlapping filtering. The linear moveout filters applied to this data are designed to pass events with zero or near zero moveouts (± 2.5 ms/trace or ± 0.625 time unit/trace) and reject the correlated noise events ('ground-roll' or shot noise) with about ± 10.0 to ± 20.0 ms/trace or ± 2.5 to ± 5.0 time unit/trace. The input for all the processed field data examples displayed in this

chapter is the data given in Figure 3.2.1A without further reference at figure captions.

Frequency spectrum analysis of the field data displayed in Figure 3.2.1A is given in Figure 3.2.2. The power spectra for signal and random noise are calculated using the multiple coherence algorithm of White (1973). The resultant power spectra plotted are the averages of the six estimates, each using four consecutive non-overlapping traces. Various low-pass, high-pass and bandpass frequency filtering of the same raw data have been made to compare with the moveout filtered results. The frequency filters which were designed in the Tchebycheff sense using the Parks and McClellan (1972) method, had lengths of 75 and approximated the pass and reject regions with about 20 dB deviation. Transition bands were always 4 Hz wide. Figure 3.2.1B and C show the best of the two bandpass filtered versions of the field data given in the same figure.

Applications of multichannel operators to real seismograms and their simulated versions showed that similar conclusions about the performance of MLS filters can be obtained from both. Therefore, synthetic models are powerful tools to simulate all possible signal and noise conditions that may be encountered in practice. A computer program using the same stochastic input model given in Equation 2.2.1 for generating synthetic seismic traces is briefly outlined in Appendix 2.

Although the superiority of multichannel filtering over straight stack has often been reported, the gain from

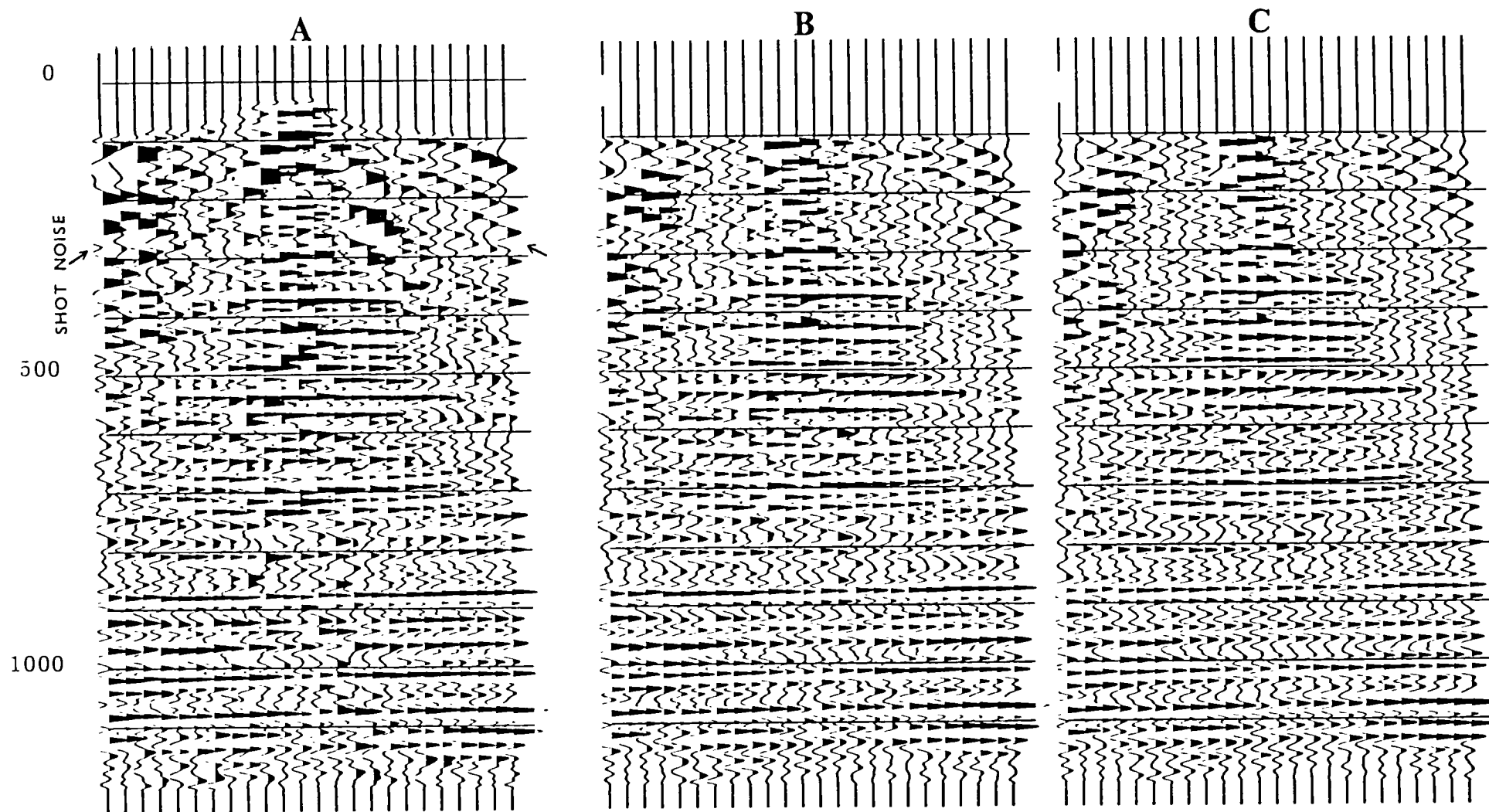


Figure 3.2.1 A) Raw field data used for the applications,
 B) 20-70 Hz bandpass and C) 25-65 Hz bandpass filtered versions
 of the same raw data.

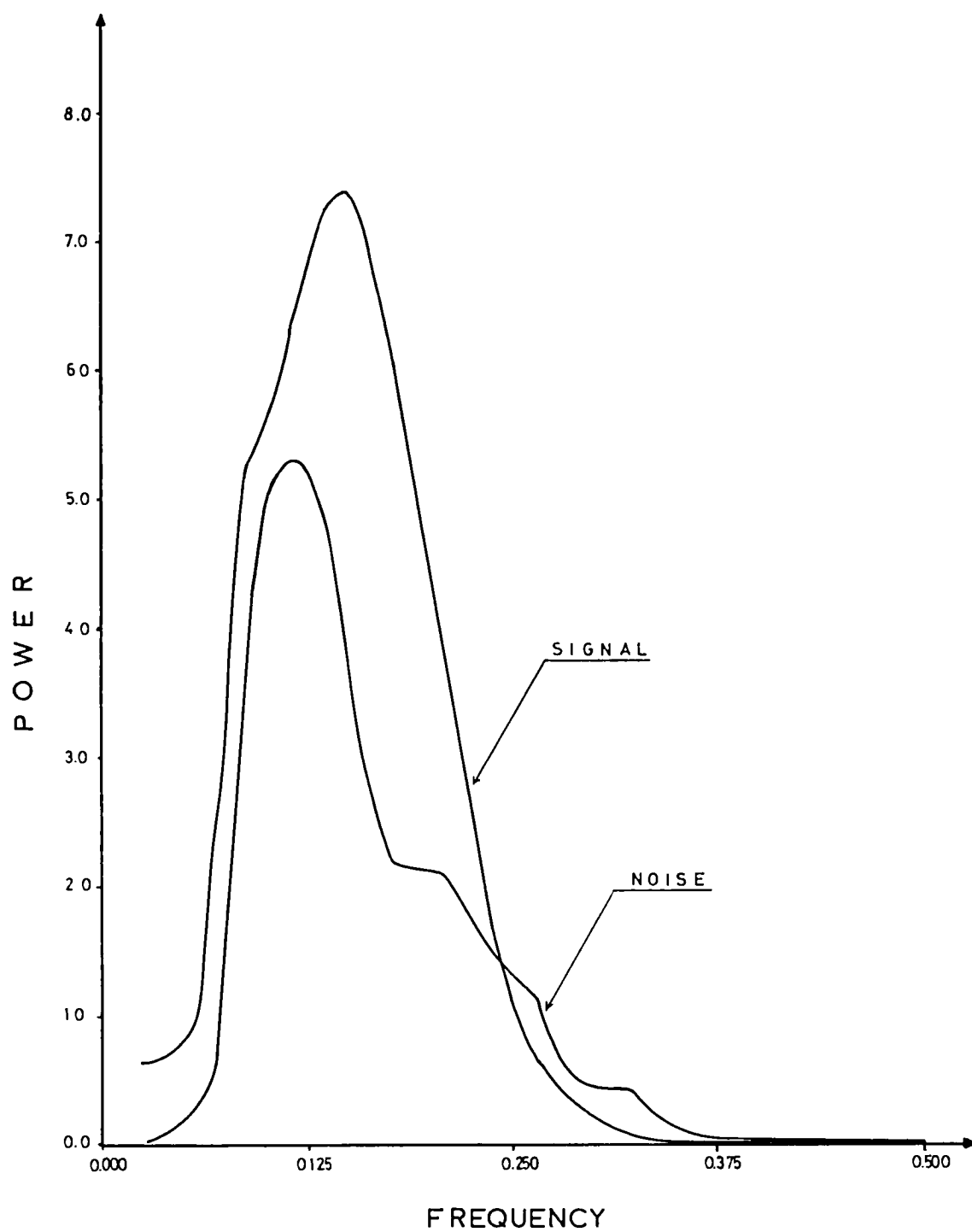


Figure 3.2.2 Power spectrum of the field data given in Figure 3.2.1A.

the more expensive multichannel processes under different signal and noise conditions is not clearly known. The horizontal or straight stacking technique has been described by Mayne (1962 and 1967), Marr and Zagst (1967) and others. A frequency domain semi-deterministic stacking technique suggested by Robinson (1968) is only feasible when there is a single source of correlated noise, i.e. all the correlated noise events to be attenuated have the same moveout function. The statistically optimum stacking or weighted-stack introduced by the same author, Robinson (1970), is only practical under certain signal-correlated noise conditions. It also requires the knowledge of S/N ratios and signal amplitudes for the traces stacked which, in practice, is not easily obtained. In the latter work it has been demonstrated that theoretically, conventional straight-stacking techniques are not optimal, but, in practice, little or nothing is gained from the more elaborate techniques mentioned above. In this study when comparative studies with the multichannel filter applications are presented only the conventional straight stacking technique is considered. By using broad band spikes Galbraith and Wiggins (1968) simulated seismic reflection traces and showed that correlated noise energy rejection is always one to two times greater for multichannel filters compared to straight stacking. They also showed that even for signal and noise events falling on nonlinear curves which are in the limits of the corresponding prescribed linear moveout window boundaries, MLS filters perform better than the straight stacking. Hubral (1970)

gave the same conclusions from the theoretical bases for certain moveout filters. Cassano and Rocca (1973 and 1974) published both design and real data application examples indicating the benefit of MLS filtering over straight stack. Simulated data applications show that the conclusions quoted above are very much dependent on signal-noise conditions of the input record and especially the amount of fluctuation (chatter) the signal arrivals show from the zero moveout. For further investigations, the straight stacks of all synthetic data inputs and their MLS filtered outputs will be presented whenever appropriate. The autocorrelations and adjacent trace crosscorrelations of input and output data as well as the autocorrelations of stacked traces are plotted to help to illustrate the improvements brought about by filtering and stacking at various stages of processing. The autocorrelations are normalized by their corresponding zero lag amplitudes, $C_{i,i}$, whereas the crosscorrelations are scaled by $1/[C_{i,i}(0)C_{i+1,i+1}(0)]^{\frac{1}{2}}$, $i=1,2,\dots,N-1$. Correlations peak at lags equal to the relative arrival time differences between signal and correlated noise events. Comparisons of the peaks corresponding to signal and correlated noise events for raw and processed data show the efficiency of processing in improving the signal to correlated noise ratio. Auto-correlations are also useful in judging the signal to uncorrelated noise ratio improvements since they should approach zero at lags other than zero if the traces had only

uncorrelated noise.

In all synthetic examples, signal and correlated noise wavelets were made to have the same frequency content with uncorrelated noise having a broader spectrum which covers all frequencies of signal and correlated noise. All wavelets are the impulse response of a Butterworth filter with 15-85 Hz passband for signal and correlated noise and 5-95 Hz for uncorrelated noise. The experiments not displayed, had 5-85 Hz passband for uncorrelated noise. Low-cut and high-cut slopes for band edges were 24 dB/octave. Amplitudes, polarities and arrival times of the spikes in prescribed time gates are random. Sometimes signal and correlated noise events were arranged to have certain arrival times and equal amplitudes for easy comparison.

All frequency-wave number, (f,k) , responses and their isometric plots presented in this study are contours of the multichannel filter amplitude spectrum $|H(f,k)|$ in decibels, i.e.

$$H_{\text{nor}}(f,k) = 20 \log_{10} \frac{|H(f,k)|}{|H_{\text{max}}(f,k)|} \quad 3.2.1$$

where $|H_{\text{max}}(f,k)|$ is the maximum amplitude in the $(0.0 \leq f \leq 0.5) - (0.0 \leq k \leq 1.0)$ range. Only the values between 0 to -30 dB are contoured. The -30 dB contour is drawn thicker to ease the interpretation of (f,k) plots.

The interpretations of the (f,k) responses for pass and reject windows bounded by linear moveouts are made using

$\tau = \frac{k}{f}$, where τ is the moveout per trace in time samples.

This is because a plane harmonic wave can be presented in the (f,k) space as a point since $v = \frac{f}{k}$, where $v = \frac{1}{\tau}$ is the apparent velocity and k the apparent wave number. Therefore, a linear moveout window boundary in the time-distance domain will correspond to the straight line $v = \frac{f}{k}$ passing through the origin in the corresponding (f,k) domain (Embree et al., 1963; Sengbush and Foster, 1968).

3.2.1 THE EFFECT OF CHATTER PARAMETER (t_c)

The stochastic input model for multichannel seismic traces given in Equation 2.2.1 is repeatedly reported and praised in the literature as a good working practical model. To simulate more realistic seismic traces a so called chatter (t_c) parameter is introduced to allow signal and correlated noise arrival times to differ from exact prescribed points. As a result of this, broadband spectra assumed in the design are multiplied by $\text{sinc}^2(t_c f)$ and $\text{sinc}(t_c f)$ functions (see Equations 2.2.10,11,17 and 18). Hubral (1970) pointed out that both functions are very similar for small values of t_c and $|f| < 0.5$. In fact, when uncorrelated noise weight (w_n) is zero, introducing chatter into filter design is equivalent to filtering the output with $B(f) = \text{rec}(f)\text{sinc}(t_c f)$. $B(f)$ is a high-cut filter for $t_c > 0$.

It is important to note that multiplying signal and correlated noise components of the autocorrelation matrix for input model traces by $\text{sinc}^2(t_c f)$ helps to stabilize this

matrix if it is singular. The role of the chatter parameter in moveout filter design becomes clearer when the autocorrelation matrix of the input model traces and cross-correlation matrix between the desired output and input traces are written in the time domain involving convolutions. Using Equation 2.2.15 with 2.2.10 and 2.2.17, the autocorrelation matrix can be written as,

$$E\{C_{x_i x_j}(\tau)\} = \frac{1}{t_c} \text{rec}\left(\frac{\tau}{t_c}\right) * \frac{1}{t_c} \text{rec}\left(\frac{\tau}{t_c}\right) * \left[\text{sinc}(\tau) * \frac{1}{|\tau_{i,j}|} \text{rec}\left(\frac{\tau + t_{i,j}}{|\tau_{i,j}|}\right) \right. \\ \left. + \text{sinc}(\tau) * \frac{1}{|\tilde{\tau}_{i,j}|} \text{rec}\left(\frac{\tau + \tilde{t}_{i,j}}{|\tilde{\tau}_{i,j}|}\right) \right] + \text{sinc}(\tau) \delta_{i,j}$$

signal entry

correlated noise entry

uncorrelated noise entry

3.2.1.1

and the crosscorrelation matrix

$$E\{C_{s_r s_j}(\tau)\} = \frac{1}{t_c} \text{rec}\left(\frac{\tau}{t_c}\right) * \text{sinc}(\tau) * \frac{1}{|\Delta t_j|} \text{rec}\left(\frac{\tau + \tau_j}{|\Delta t_j|}\right) \quad 3.2.1.2.$$

Equation 3.2.1.1 shows that autocorrelations obtained for all entries of signal and correlated noise are filtered by the triangle function

$$\Lambda(\tau t_c) = \frac{\pi}{t_c} \Lambda\left(\frac{\pi \tau}{t_c}\right) \quad 3.2.1.3.$$

It is clear from the latter three equations that broadband

signal and noise assumed for the input model, are tapered by sinc^2 and sinc functions before they are shifted and scaled for calculation of correlations. The signal and noise spectra of assumed input models entering into the autocorrelation matrix are more strongly frequency filtered than the signal in the crosscorrelation matrix. As a result of this, the original signal model input will differ from desired output with increasing chatter. The input correlated noise model will also be modified. It is expected that tapered models are more realistic than broadband assumption models because, in reality, no such wavelets with a flat spectrum are observed. Theory and practice show that seismic wavelets always have a tapered spectrum (Ricker, 1940, 1953a and 1953b; O'Doherty and Anstey, 1971). Intuitively, tapering the input model spectra as much as actual input spectra should improve the assumed model, but at high values of chatter, a desired output with higher frequency content would be required from a more tapered signal input.

The above arguments, combined with the fact that when one part of a multidimensional filter response is weighted relative to the other prescribed regions it is reflected on all other parts, show that the chatter parameter affects the whole frequency-wave number domain of a moveout filter. This works like a high-cut filter for large chatter values.

Normally, a chatter value incorporated into the input model of a moveout filter is unrealistic if it is greater

or equal to the smallest moveout which prescribes a pass or reject window. It will be demonstrated next that this is not necessarily the case. Even at very high chatter values, changes at the low frequencies of (f,k) response of moveout filters are marginal.

Optimum MLS moveout filters designed with uncorrelated noise weight and chatter parameters equal to zero are unstable and destroy the data when applied. A small amount of uncorrelated noise introduced into the design gives stable moveout filters. Now, it will be shown, by designing and applying moveout filters having zero uncorrelated noise for their input model, how increased chatter affects unstable operators. Figure 3.2.1.1 show the (f,k) responses of three of the linear moveout filters designed with $w_r=0.5$, $w_n=0.0$ and $t_c=0.25, 0.625, 1.0, 1.5, 3.0$ and 4.0 . Comparisons between these filters and the same filters designed with $w_n=0.03$ showed that a chatter value of about 1.0 to 1.5 is necessary to achieve the best possible pass and reject characteristics when $w_n=0.0$ (see Figures 3.2.1.5 and 3.2.1.6 for comparison). Chatter values about 1.0 and greater bring high frequency attenuation. At high chatter values the pass region has 6 to 18 dB attenuation where reject window approximation is as good as -30 dB or better except near the origin. Increasing chatter from very small values to about unity improves stability which provides better pass region approximation. In fact, the time domain coefficients show the stabilizing effect of increasing chatter

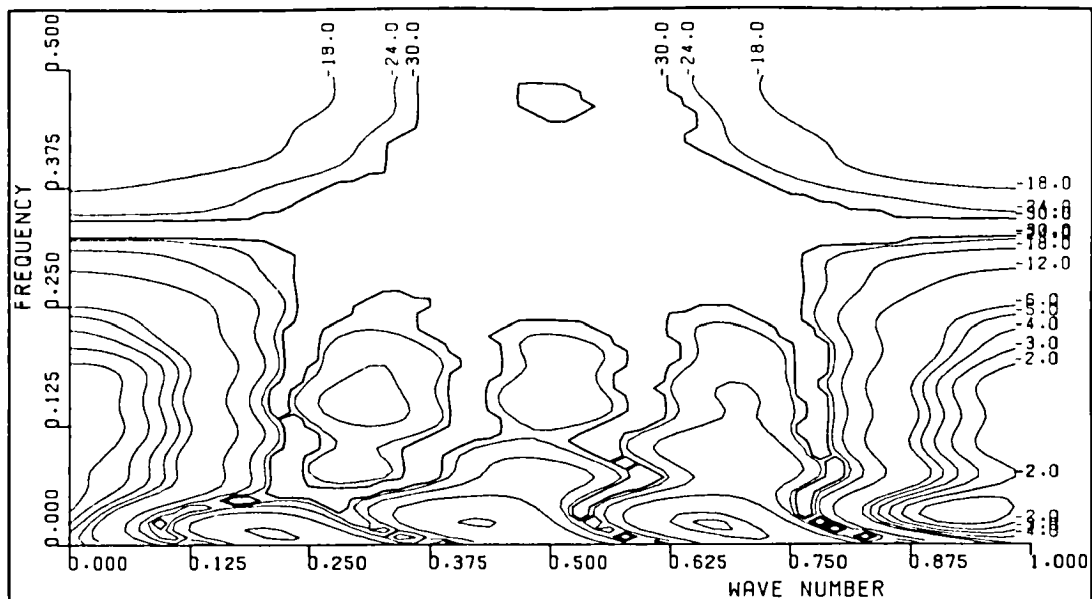
The common design parameters for all the optimum least-squares linear moveout filters in this chapter, which are applied to the raw data given in Figure 3.2.1A, i.e. all the filters whose (f,k) responses are presented:

Number of input channels,	$N = 5$
Length of filter,	$LF = 31$
Reference output trace,	$R = 3.0$
Delay index,	$t_d = 16$
Pass moveout window (time unit/trace),	$0_s = -0.625 \text{ to } 0.625$
Reject moveout window (time unit/trace),	$0_r = 2.5 \text{ to } 5.0$

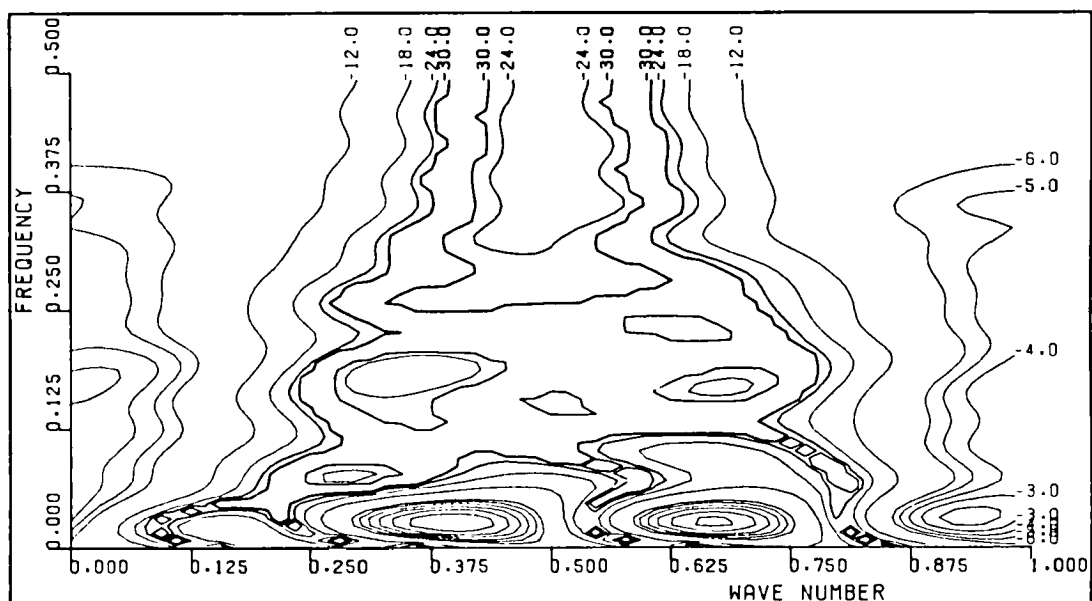
The additional design parameters for the varying chatter filters whose (f,k) responses are given on the facing page:

Correlated noise weight,	$w_r = 0.5$
Uncorrelated noise weight,	$w_n = 0.0$

TC=3.0



TC=1.0



TC=0.25

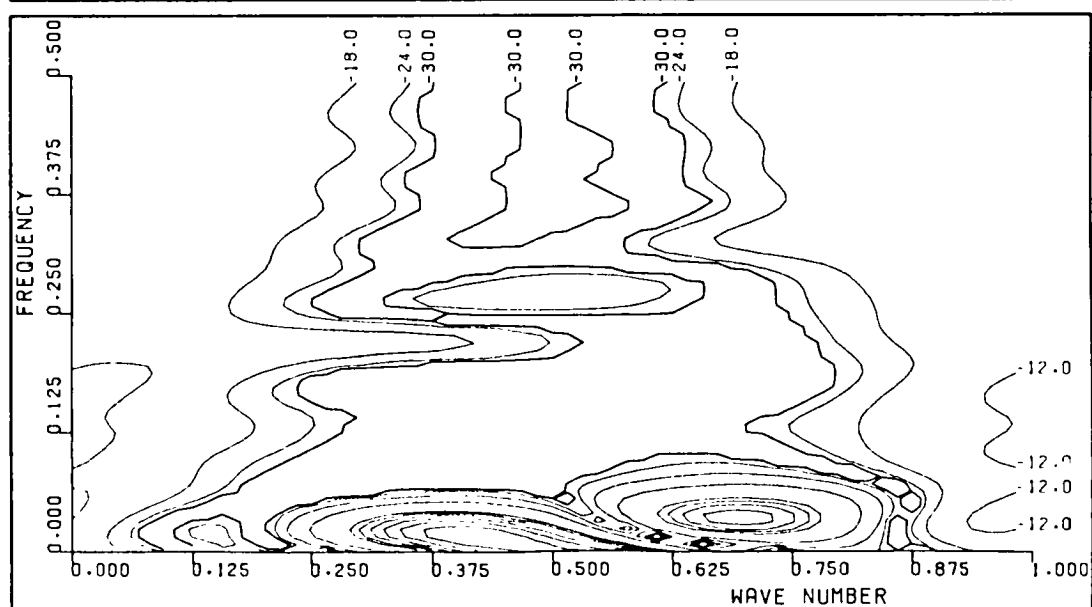


Figure 3.2.1.1 (f,k) responses of the MLS moveout filter designed with zero uncorrelated noise and varying chatter. See the facing page for the common filter parameters.

without further investigation being needed. If each filter of N channel filter set are normalized, stable filters have smaller coefficients towards both ends compared to the corresponding unstable filters.

The raw data given in Figure 3.2.1A have been processed by the above moveout filters designed with zero uncorrelated noise and varying chatter values as a practical example. The outputs of the filters, whose responses are given in Figure 3.2.1.1, are displayed in Figure 3.2.1.2. The outputs of the filters with very small chatter show low amplitude filter generated noise before the actual onset times of seismic arrivals. These features, indicative of the unstable filters applied, are clearly seen after the strong events at the beginning of the records. At the early part of the record the best signal to correlated noise ratio is achieved by the filter with $t_c=0.25$, see especially the outer traces for the event at about 375 ms. This is because the severe correlated noise at these traces is most efficiently removed by this filter. On the other hand, alignment and resolution of signal events after about 500 ms are improved with increasing chatter values up to about $t_c=1.0$ to 1.5. At relatively higher values, the resolution is greatly reduced due to high-frequency filtering. The signal to uncorrelated noise ratio is also reduced. In all cases, the correlated noise marked as shot noise at the beginning of the raw data is successfully removed.

As it will be shown in the next section, the stability problem of MLS moveout filters is solved by adding some

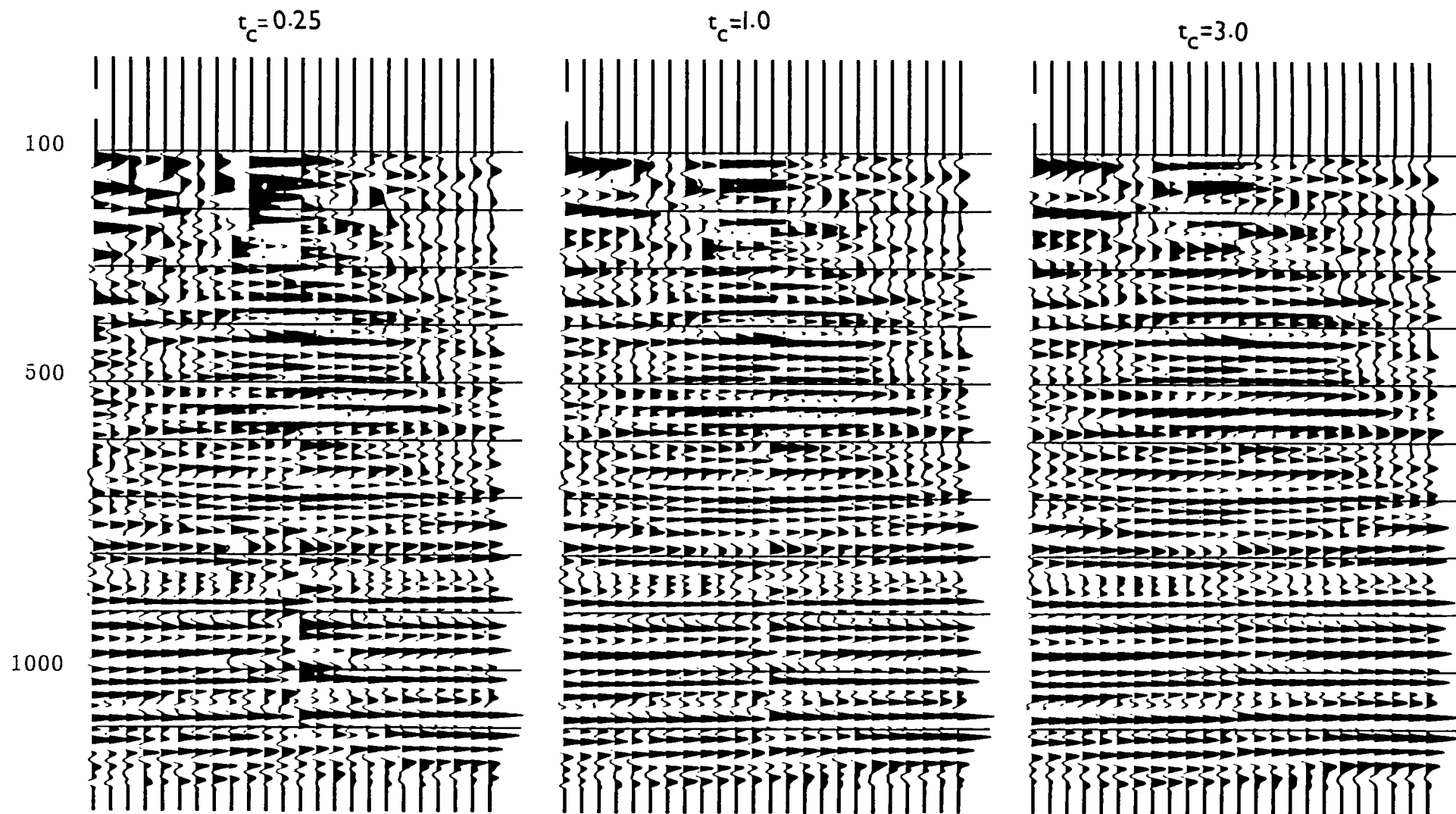


Figure 3.2.1.2 Outputs of the zero uncorrelated noise and varying chatter filters whose (f,k) responses are given in Figure 3.2.1.1.

random noise into the input model. In the remaining part of this section, the effect of chatter parameter on stable MLS moveout filters is examined. The synthetic examples with very low uncorrelated noise showed that, chatter values less than unity incorporated into the filter design do not make any visible changes at the output traces. Experiments have also shown that there is no gain in matching the chatter value used in designing the MLS moveout filter with the chatter value of the actual input. All synthetic data examples were in agreement with the theoretical arguments given previously in the sense that increased chatter is equivalent to increased high frequency attenuation and reduces the resolution of the data. At very high values of about greater than $t_c=3.0$ spurious low-frequency line-ups were observed due to high frequency-filtering.

The raw synthetic data given at the bottom of Figure 3.2.1.3 will be used to demonstrate the effect of chatter, as well as correlated and uncorrelated noise weights. This simulated data is processed by the moveout filter which has unit weights, $w_r=w_n=1.0$, and zero chatter for its input model. The output is displayed at the top of the same figure. The other common filter parameters are given on the page facing the raw and filtered data. The processed data show improved trace-to-trace alignment and S/N ratio. Note that the 'S/N ratio' will mean both signal to correlated and signal to uncorrelated noise ratio in this study. Similarity between the traces is also highly improved.

This can be most conveniently observed from the cross-correlation functions which are very similar to the auto-correlations for the filtered output. Therefore if after-stack reflection traces show a poor S/N ratio, it is possible to improve it by moveout filtering in cases when the signal is not severely corrupted by noise. The stack-after MLS filter trace is superior to the straight-stack trace for passing signal events, especially with high chatter. There is no improvement over straight-stack in rejecting correlated noise. Use of after-stack moveout filtering may give line-ups leading to false interpretations such as the correlated noise line-ups at 200.0 ms and 500 ms. It is also important to notice that the straight-stack trace shows the same undesirable results. This kind of false line-up generation, by moveout filtering and straight stacking, has been commonly observed for different synthetic signal and noise models. The features, discussed above, are common to all the synthetic examples mentioned in this chapter.

Figure 3.2.1.4 shows the results of processing the same raw data with the same common moveout filters using chatter values of $t_c=4.0$ ms and $t_c=12.0$ ms, i.e. one and three samples. The output of the filter with $t_c=2.5$ ms chatter showed no visible difference from the zero chatter filter output given in the previous figure. Even at values as high as $t_c=4.0$ ms the visible effect is marginal though high cut filtering is evident. As expected, the

Input parameters of the raw synthetic seismogram given in Figure 3.2.1.3:

Duration of all spikes (time gate in ms)	: 0 - 996
Sampling interval (ms)	: 4
Trace gains	: 1.0
Number of signal spikes on each trace	: 4
Number of correlated noise spikes on each trace	: 4
Correlated noise to signal ratio (over 0 - 996 ms)	: 1.0
Number of correlated noise spikes on each trace	: 100
Signal moveouts (ms/trace)	: 0.0, 0.0, 0.0 and 0.0
Correlated noise moveouts (ms/trace)	: 12.0, 7.5, 5.0 and 3.0
Arrival times for signal events (ms)	: 150, 325, 600 and 885
Arrival times for correlated noise events (ms)	: 0, 200, 500 and 700

Chatter values (in ms) for signal and correlated noise events added to the relative moveouts:

Trace	Spike	1	2	3	4
1		0.0	0.0	0.0	0.0
2		-2.0	-3.0	2.0	-1.0
3		1.0	1.0	1.0	0.0
4		-4.0	2.0	-1.0	1.0
5		2.0	0.0	-2.0	-1.0
6		3.0	-2.0	0.0	0.0

Common parameters for the MLS moveout filters applied to the above described raw data:

$N = M = 6$, $LF = 31$, $t_d = 16$, $0_s = -2.0$ to 2.0 ms/trace, $0_r = 3.0$ to 15.0 ms/trace.

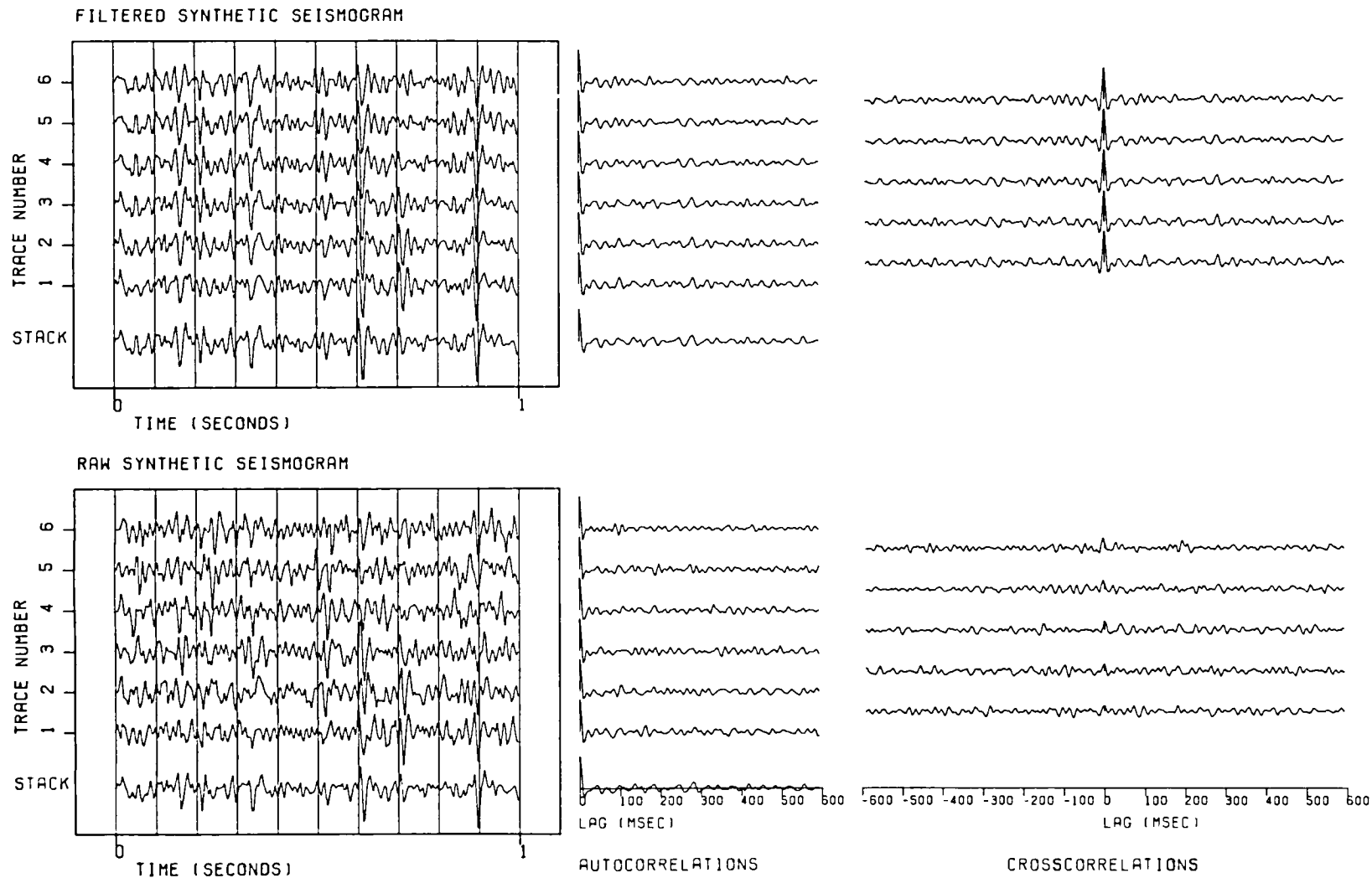


Figure 3.2.1.3 The raw synthetic data used for chatter and noise weight experiments (lower), and its filtered version by MLS filter designed with $t_c = 0.0$ and $w_r = w_n = 1.0$ (upper). See the facing page for the synthetic data and common filter design parameters.

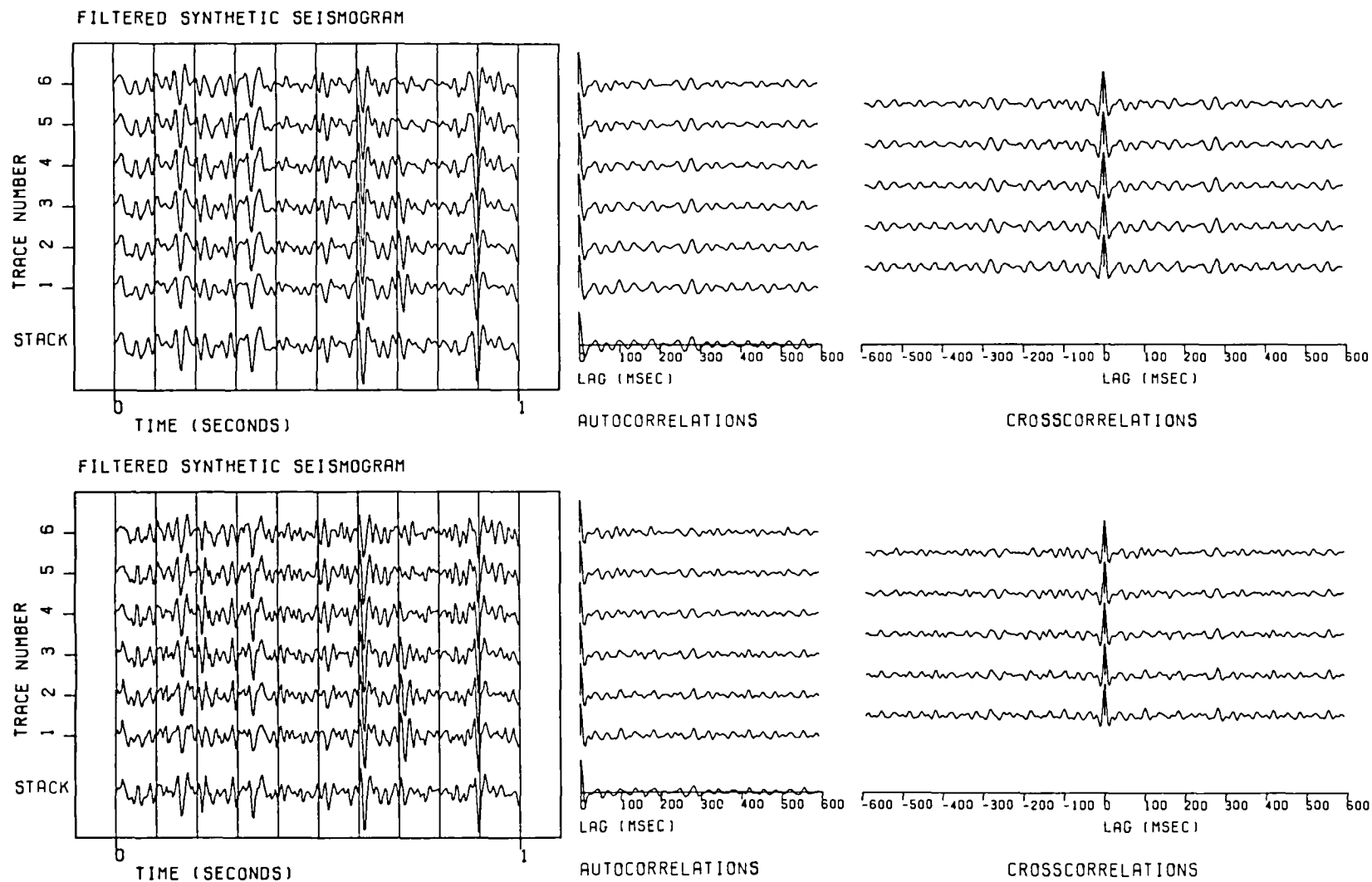


Figure 3.2.1.4 Effect of increasing chatter in the MLS moveout filter design. The output of the filter designed with $t_c = 4.0$ ms is shown at the bottom and that of $t_c = 12.0$ at the top. The raw data is given in Figure 3.2.1.3.

filter with 12.0 ms chatter does relatively more frequency filtering and reduces the resolution of data. Comparing the filtered traces, stacked traces, and correlations indicates that there is no gain in introducing chatter in the MLS moveout filter design in the case simulated. Further increase of chatter in the filter design results in low-frequency line-ups in the filtered output and signal distortion.

Three other sets of least-squares moveout filters with chatter values 0.0, 0.25, 0.625, 1.0, 1.5, 3.0 and 4.0 have been applied to the same reflection records, one of which is given in Figure 3.2.1A. The uncorrelated noise weights for these experiments were 0.03, 0.3 and 0.5. The results of the former two were indistinguishable by eye and the latter one showed slightly better trace-to-trace alignment and S/N ratio. Since chatter and uncorrelated noise weight have similar effects on moveout filter response the smallest of the above weights, i.e. $w_n=0.03$, which gives weakly stable filters, was chosen for the examples presented.

The desired (f,k) response of the linear moveout filters applied to field traces presented in this chapter is given in Figure 3.2.1.5 which also shows its least-squares approximation designed with $t_c=0.0$. The desired response includes expected extra reject regions (dashed lines) due to wave number aliasing. Figure 3.2.1.6 shows (f,k) responses of the same filter designed with $t_c=0.25$, 1.0 and 3.0. The only difference between these filters and corre-

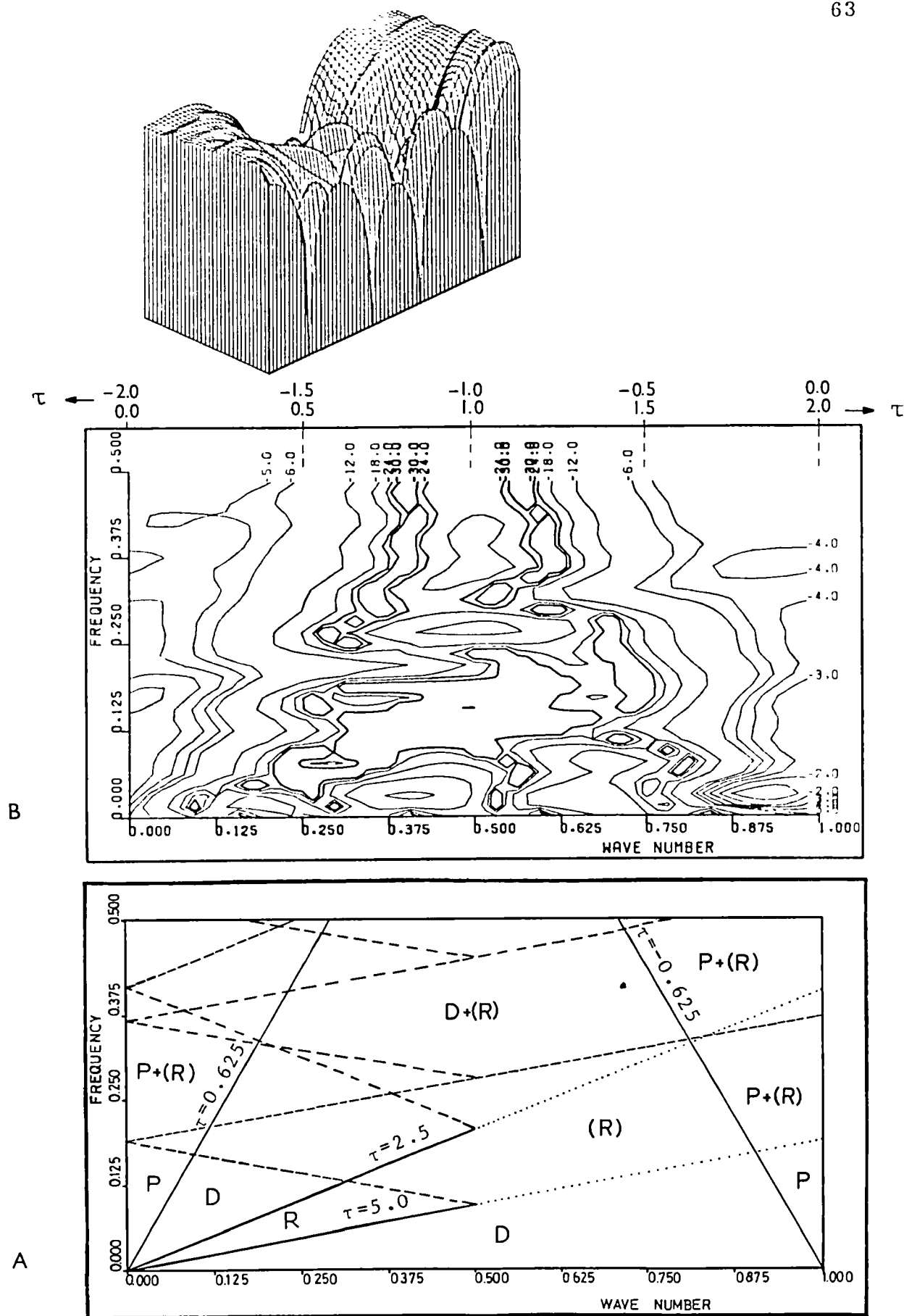
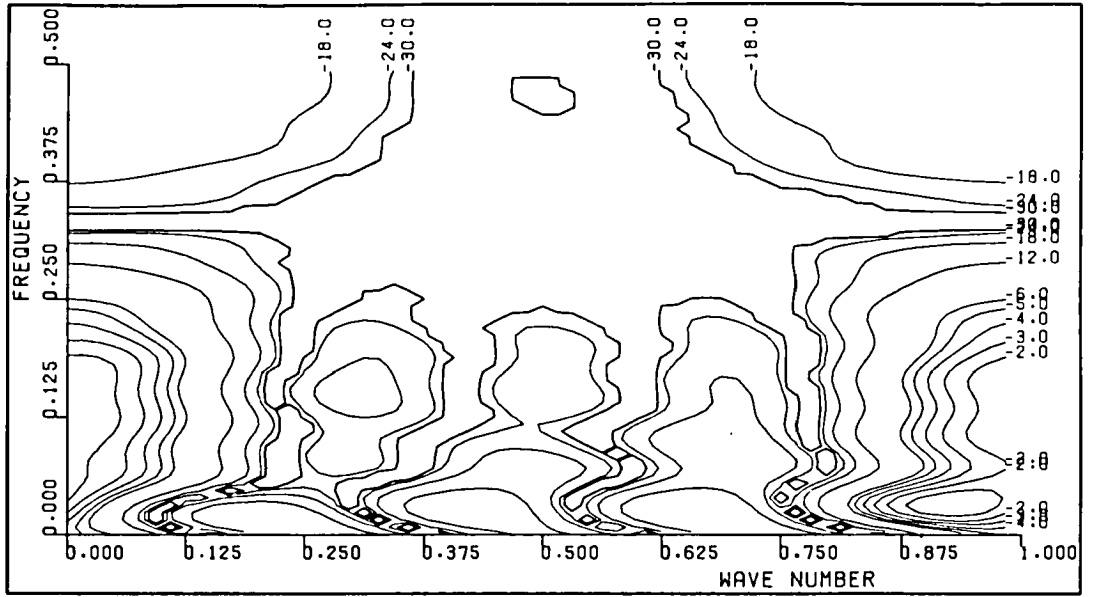
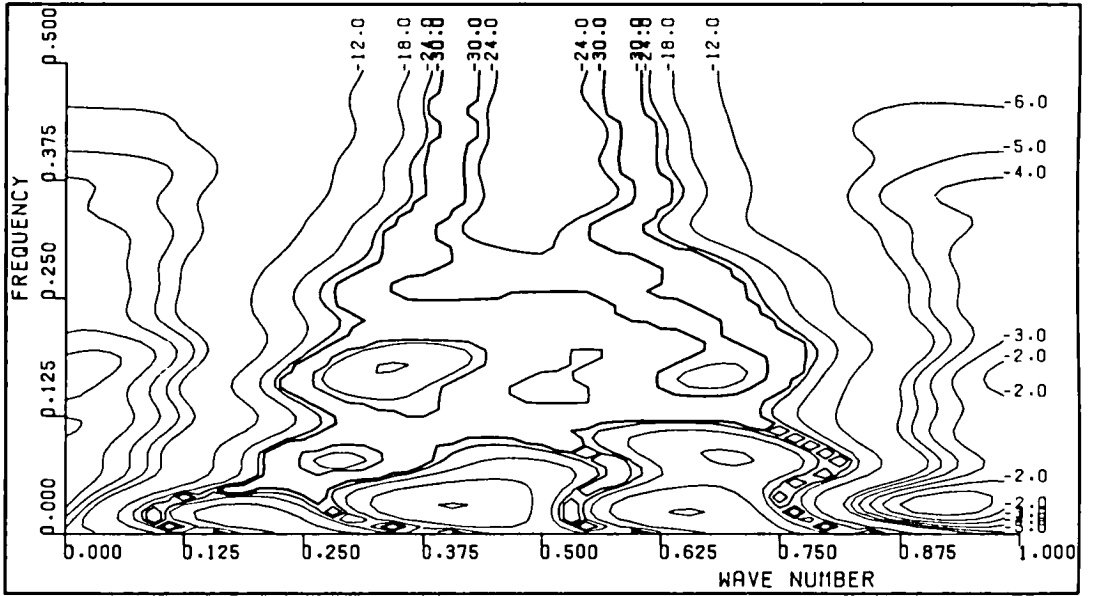


Figure 3.2.1.5 The desired (f,k) response of the linear moveout filter applied to real seismograms (A) and an actual response obtained with zero chatter in the input model (B). The isometric plot is viewed from the origin.

TC=3.0



TC=1.0



TC=0.25

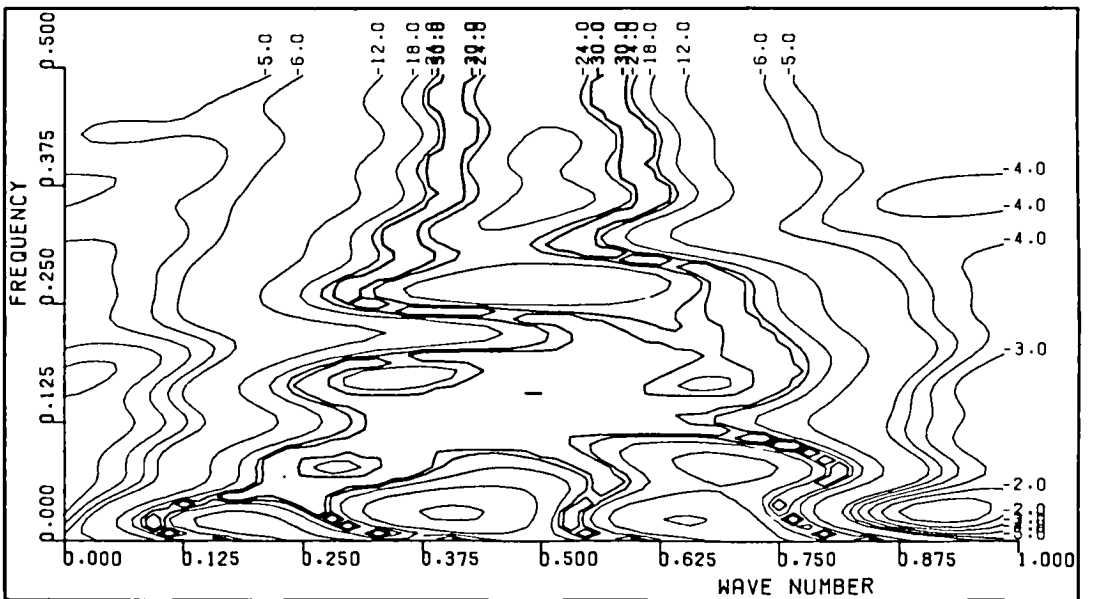


Figure 3.2.1.6 (f,k) responses for MLS moveout filter with varying chatter.

sponding filters whose responses were given in Figure 3.2.1.1 is the increase of uncorrelated noise weight from 0.0 to 0.03. The general conclusions from all design examples quoted and presented in this section may be summarized as follows.

1. Frequency-wave number responses of MLS filters designed with chatter values less than about 1.0, differ tremendously for filters designed from input models with zero uncorrelated noise and uncorrelated noise weight of about 0.03 to 0.5. This is because MLS moveout filters designed with zero uncorrelated noise and small chatter are not stable. Note that instability in this case refers to the singular auto-correlation matrix inherent in the normal equations.
2. The MLS moveout filters designed from input models with zero uncorrelated noise and about unit chatter and greater are very similar to the corresponding filters with uncorrelated noise high enough for stability, i.e. $w_n=0.03$ to 0.5.
3. The MLS moveout filters designed with chatter values much higher than $t_c=1.0$ behave like an all-reject filter at high frequencies. Frequency-wave number responses lose their fan-shaped pass-reject characteristics for linear moveout input models with small moveouts. The prescribed domains at low frequencies for all wave numbers are only marginally affected by the increasing chatter such as the reject region in the

given examples.

The outputs of the filters, whose (f,k) responses are given in Figure 3.2.1.6, for the raw data of Figure 3.2.1A are displayed in Figure 3.2.1.7. The output from the zero chatter filter whose response is given in Figure 3.2.1.5 shows the same visual characteristics as the filter designed with $t_c=0.25$ and is not presented. The other two sets of data processed by the same varying chatter moveout filters but uncorrelated noise weights 0.3 and 0.5 showed the same characteristics as the examples presented. The following are the main features;

1. There is very little variation in the output data when chatter values of up to unity are introduced into the input models of the MLS moveout filters.
2. When uncorrelated noise in the input data is very low ($w_n=0.03$) there is improvement in S/N ratio with increased chatter of up to about $t_c=1.5$. At higher chatter values than $t_c=1.5$, high frequency filtering with decreasing resolution becomes visible.
3. In practice, good results are obtained even for chatter values from $t_c=1.0$ to 1.5 which are much greater than those of the pass domain moveouts (± 0.625 time unit per trace) for the given examples. Generally, the effect of chatter is the same whatever the moveouts for the prescribed regions.
4. High-cut filtering due to increased chatter is useful

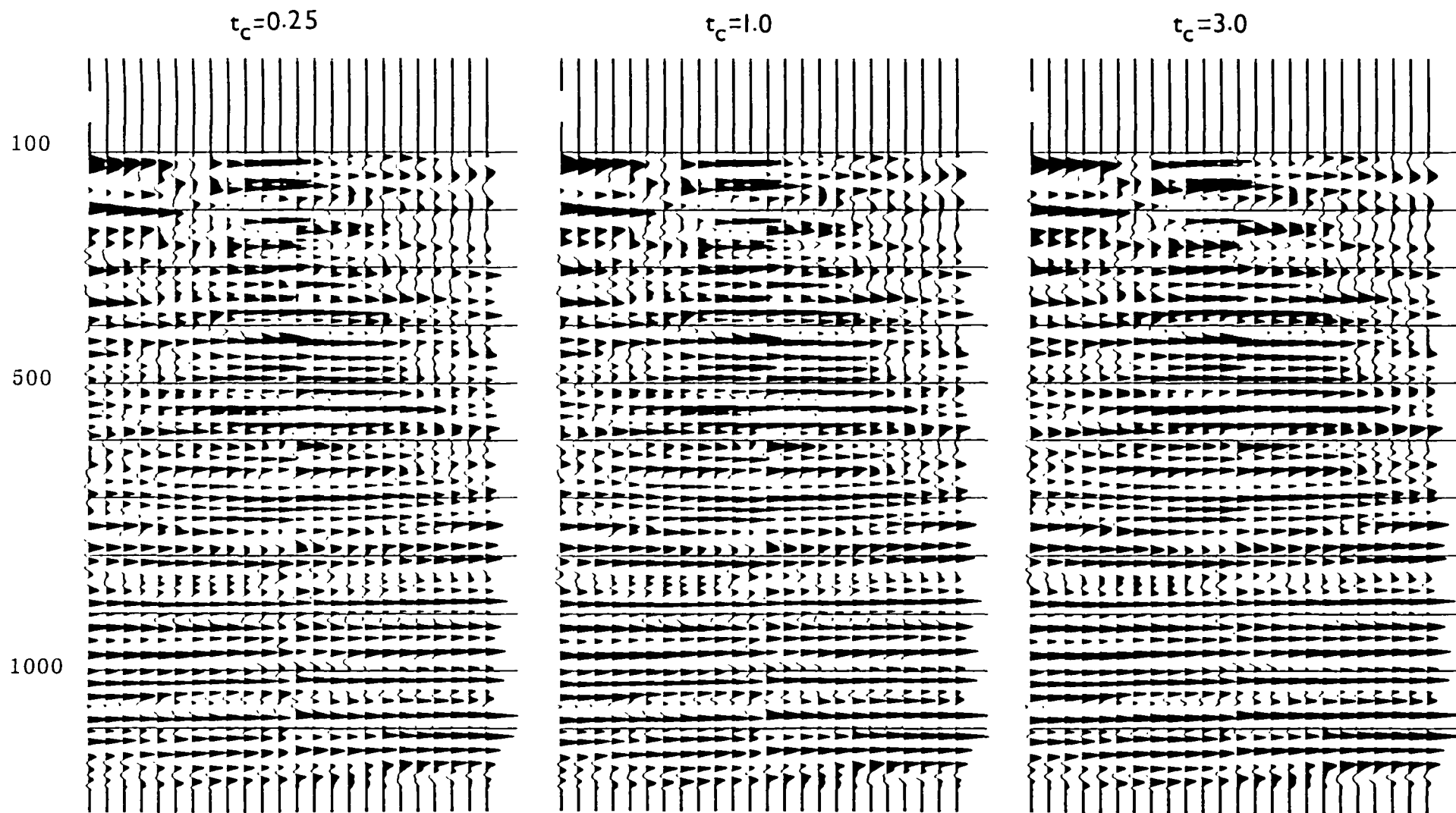


Figure 3.2.1.7 Outputs of the varying chatter filters whose (f,k) responses are given in Figure 3.2.1.6.

or at least harmless up to about $t_c=1.0$. This is because the raw data have the signal power contained at the normalized frequencies about 0.06 to 0.3 (see Figure 3.2.2). On the other hand, if moveout filters are designed to pass signal events which have power at higher frequencies only very small chatter should be included into the input model or, it should be omitted altogether.

5. Generally, it may be concluded that, although the chatter parameter incorporated into the design of MLS moveout filters makes the assumed input model more realistic, practically nothing is gained from it provided that enough uncorrelated noise is introduced for stability of the autocorrelation matrix and also no high frequency filtering is required.

3.2.2 THE EFFECT OF UNCORRELATED NOISE WEIGHTING FACTOR (w_n)

In the derivation of the normal equations for the optimum MLS moveout filters in Section 2.2, it was assumed that uncorrelated noise in the input model traces had a flat spectrum between zero and the Nyquist frequency and then zero power after that. Its contribution to the autocorrelation matrix of input traces was given in Equation 2.2.14. With the help of Equation 2.1.20, it may be written as:

$$\begin{aligned} E\{C_{n_i n_j}(0)\} &= w_n \text{sinc}(0) \delta_{i,j} \\ &= w_n \delta_{i,j} \end{aligned} \quad 3.2.2.1$$

where $\delta_{i,j}=0$ if $i \neq j$; and zero for all other lags. Therefore, the uncorrelated noise weight, w_n , introduced into moveout filter design is added to the main diagonal of the autocorrelation matrix.

When the noise in the input traces is completely random and signal to uncorrelated noise ratio is the same on all traces, straight stack gives the best possible noise suppression (Burg, 1964). Increasing the uncorrelated noise weight in MLS filter design improves signal to uncorrelated noise ratio with increased signal distortion (Hubral, 1970 and 1975). Hubral also showed that the random noise power at the output of a multichannel filter is equal to the power of the filter, i.e. sum of the squared time-space domain coefficients. This is true if the random noise on input traces is normalized to unity. Otherwise, noise power at the output of a filter depends on its input noise power. Hains (1973) demonstrated that, although multichannel filters are a powerful tool for improving signal to uncorrelated noise ratio, beyond about -12dB noise level on input traces no gain is obtained by their application. By calculating frequency-wave number diagrams of seismic reflection traces, Hains also showed that when the uncorrelated noise level reaches about -12dB, the detection of correlated events from these diagrams becomes difficult and eventually impossible. Another known aspect, which will be demonstrated later, is that increasing uncorrelated noise in the moveout filter input model brings suppression

to the high frequencies in the final design. It is worth mentioning that, usually the larger the reject window(s) the better the rejection of random noise becomes.

Intuitively, if the assumed input model is a very close approximation of the actual input traces, then any filter designed from this model works at its best. This leads to an impossible task of designing filters, passing and/or rejecting events correlated from trace-to-trace but having no appreciable power levels for their spectra in the frequency-wave number domain, i.e. the input model has a very high uncorrelated noise power relative to correlated event powers. Therefore, an input model which works better in practice, has to be chosen. Uncorrelated noise on field recorded traces may and will vary from one recording position to another. A moveout filter designed for a certain input model is applied to a large part, or all, of the data to be processed. For this reason, it is desirable that moveout filters are not too sensitive to the uncorrelated noise changes in the input traces. Although it is assumed that uncorrelated noise has a broadband spectrum, a sample of seismic input traces does not necessarily have a flat spectrum for the uncorrelated noise in the frequency-wave number domain. This is why correlated events at very low power cannot be detected from frequency-wave number diagrams of seismic input traces when the uncorrelated noise level is high.

As a result of the theory developed in the previous

chapter, an increase of relative random noise power in MLS filter design is equivalent to desiring increased rejection all over the (f,k) domain in the Nyquist range which also includes pass regions. From the designer's point of view, random noise is assumed in the input model but it is not included at the desired output; therefore, the higher the random noise power in the input model the more the MLS filter will be forced to see the input traces as random processes, i.e. tend to straight stack. Conventionally, it has been thought that mainly the high frequencies will be affected; this is a half truth.

The synthetic data applications which are not displayed here show the following generalizations;

1. Inputs with low or zero uncorrelated noise show good rejection of correlated and random noise when stacked or moveout filtered with operators having uncorrelated noise weights up to 2.0 are used. If there is no random noise in the input traces a value of $w_n=1.0$ to 2.0 is required in the filter design input model to avoid filter generated low amplitude noise before the onset times of signal events, that is, highly stable filters are required.
2. Synthetic traces having signal to uncorrelated noise ratios from $\frac{1}{6}$ to 1.0 processed with a moveout filter of unit weight, showed good recovery of signal events especially in the improvement of trace-to-trace alignments. Increasing uncorrelated noise weight

much higher than 2.0 for the same filter introduces spurious line-ups in the filtered output and reduces the resolution by broadening the signal events.

3. It is also observed that, an increase of uncorrelated noise weight in the filter design increases the mean-least-square-error which means inferior approximation of the prescribed pass and reject regions.

The common MLS moveout filter, used in the previous section for filtering of the synthetic traces presented, has been applied to the raw synthetic data of Figure 3.2.1.3, this time with $w_r=1.0$, $t_c=0.0$ and varying uncorrelated noise weight. The filtered output at the bottom of Figure 3.2.2.1 is the result of having $w_n=0.5$ and the top $w_n=5.0$. Compared with the filtered traces in Figure 3.2.1.3 which are the outputs of the same filter with $w_n=1.0$, changes are marginal. High frequency attenuation with increased uncorrelated noise weight is hardly noticeable. This is because the aliasing of the fan-shaped reject region into the high frequencies of the pass region is reduced with the increased uncorrelated noise weight. The experiments presented below will clarify this important aspect of uncorrelated noise in the MLS moveout filter design.

The MLS moveout filters designed with the varying uncorrelated noise weights $w_n=0.03, 0.07, 0.125, 0.2, 0.3, 0.35, 0.4, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 5.0, 10.0, 20.0$ and 40.0 have been applied to the same three records used for the chatter experiments in the previous section. These

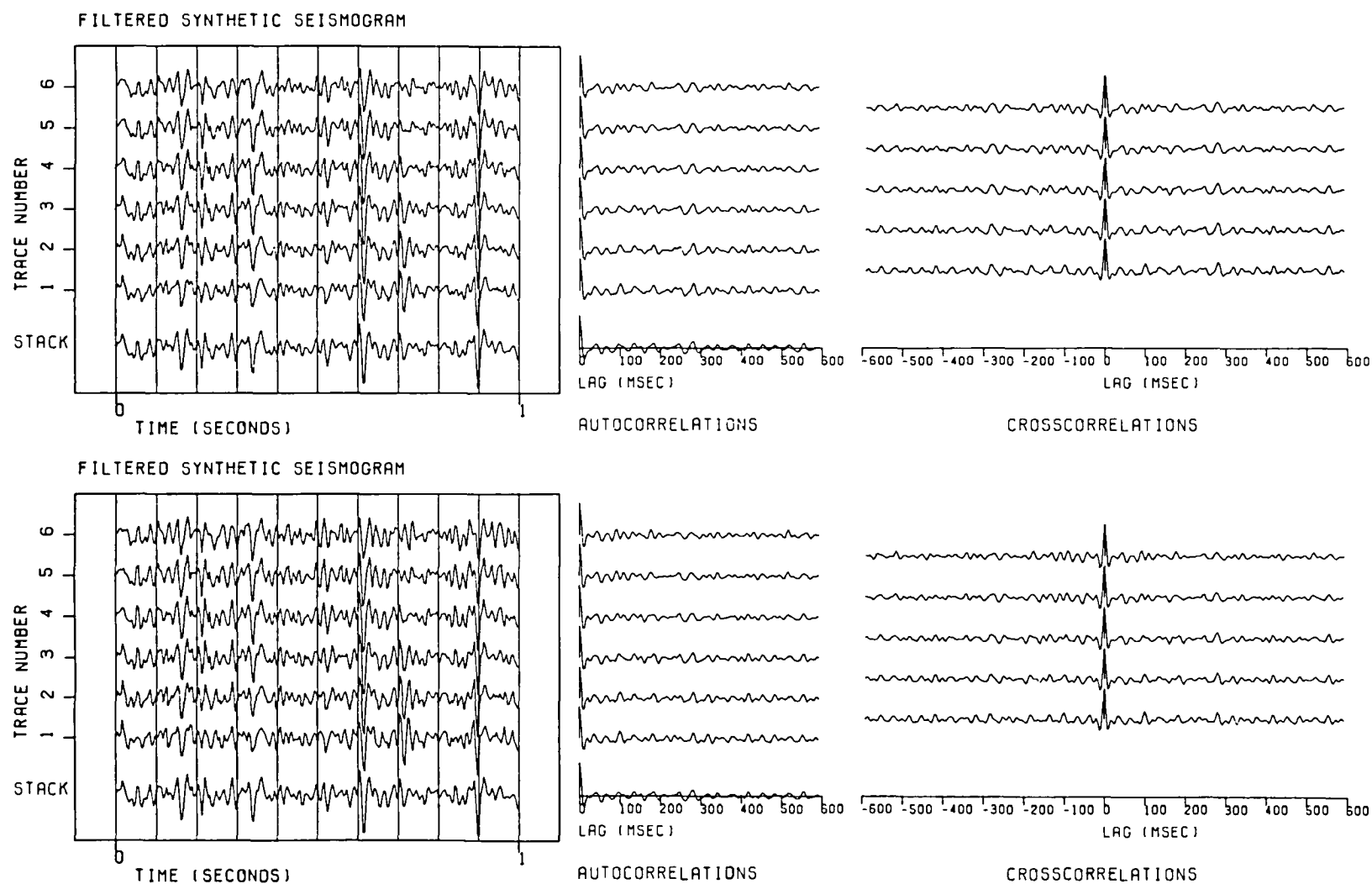
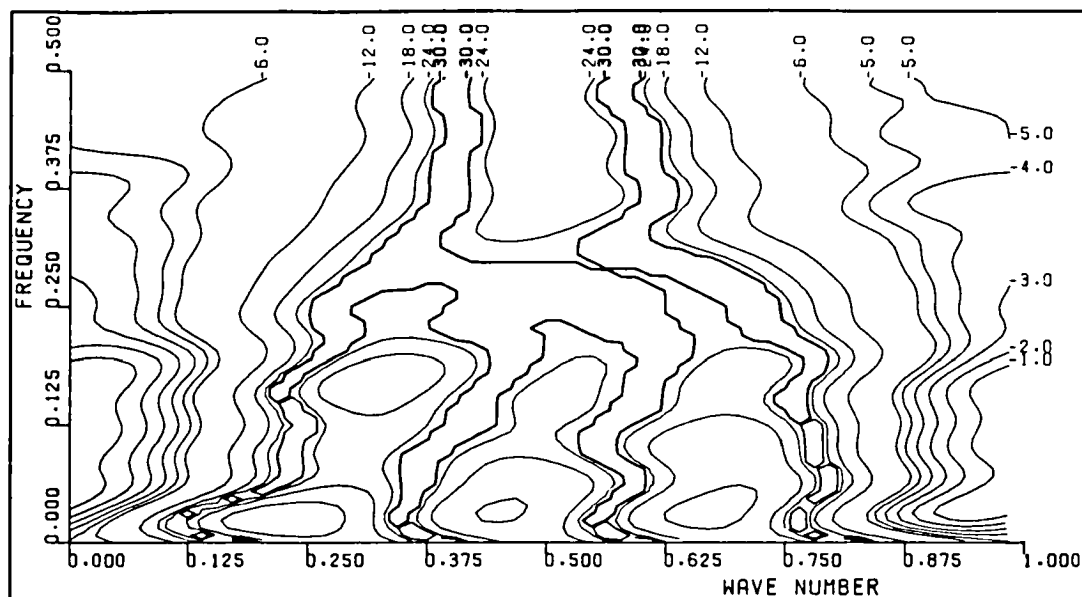


Figure 3.2.2.1 Effect of increasing uncorrelated noise in the MLS moveout filter design. The output of the filter designed with $w_n = 0.5$ is shown at the bottom and that of $w_n = 5.0$ at the top. The raw data is given in Figure 3.2.1.3.

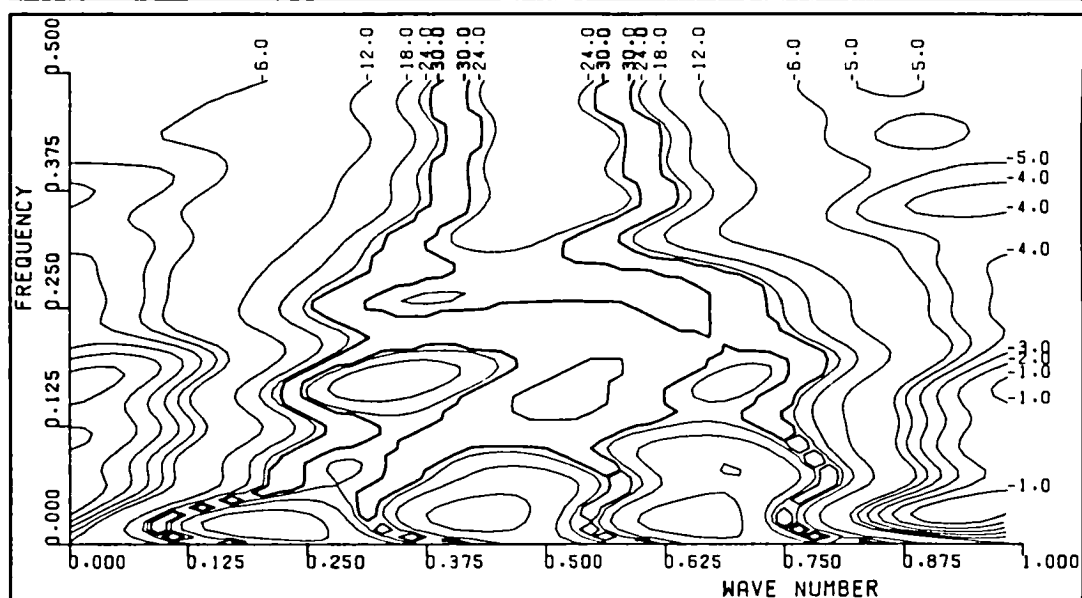
filters all have correlated noise weight $w_r=1.0$ and zero chatter in their input models. Figure 3.2.2.2 show the (f,k) responses of three of the above filters designed with $w_n=0.03, 0.3$ and 1.0 . Three other (f,k) responses for filters designed with uncorrelated noise weights greater than unity, i.e. $w_n=3.0, 10.0$ and 40.0 , are given in Figure 3.2.2.3. The general conclusions drawn from the above examples, which can be observed from the (f,k) responses presented, may be summarized as:

1. Increasing uncorrelated noise weight from small values to about unity improves pass and reject region approximations. This indicates that the operator is passing from an unstable state to stability, which was also observed with increased chatter, and can be deduced from the time-space domain coefficients.
2. If uncorrelated noise weights higher than unity are used in MLS moveout filter design, pass and reject region approximations deteriorate. Fan-shaped approximations of prescribed regions corresponding to linear moveout windows in the time-space domain are also lost. At weights greater than about $w_n=10.0$, the (f,k) responses become badly approximated two-dimensional low-pass filters. As a result of this, domains (the reject domain in the given examples) bounded with higher linear moveouts than the sampling interval do not alias or fold back onto domains with smaller moveouts.

WN=1.0



WN=0.3



WN=0.03

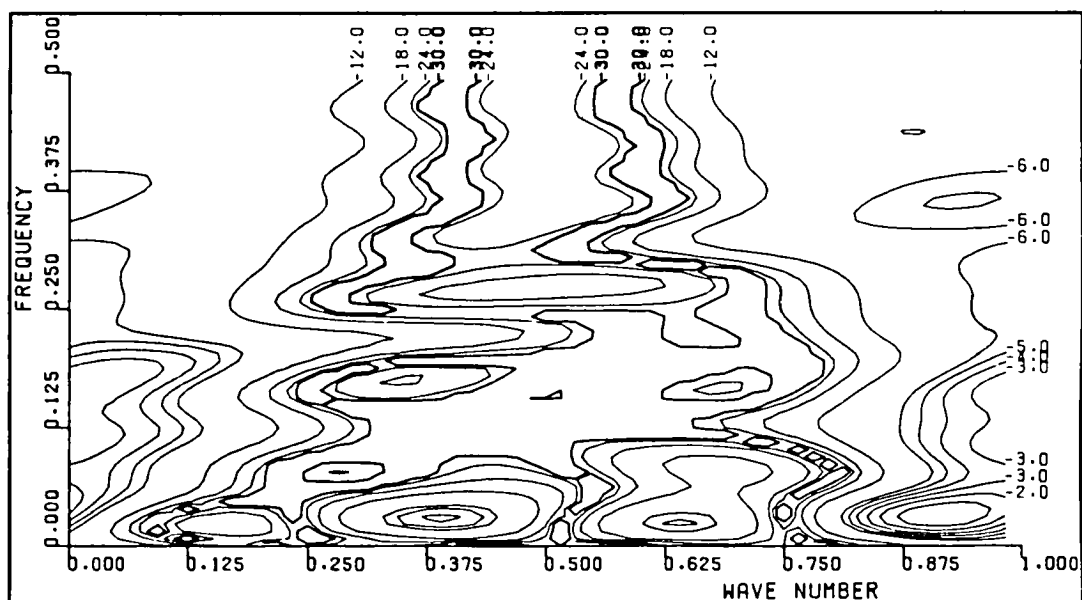


Figure 3.2.2.2 (f,k) responses for MLS moveout filter designed using input models with relatively small uncorrelated noise weights.

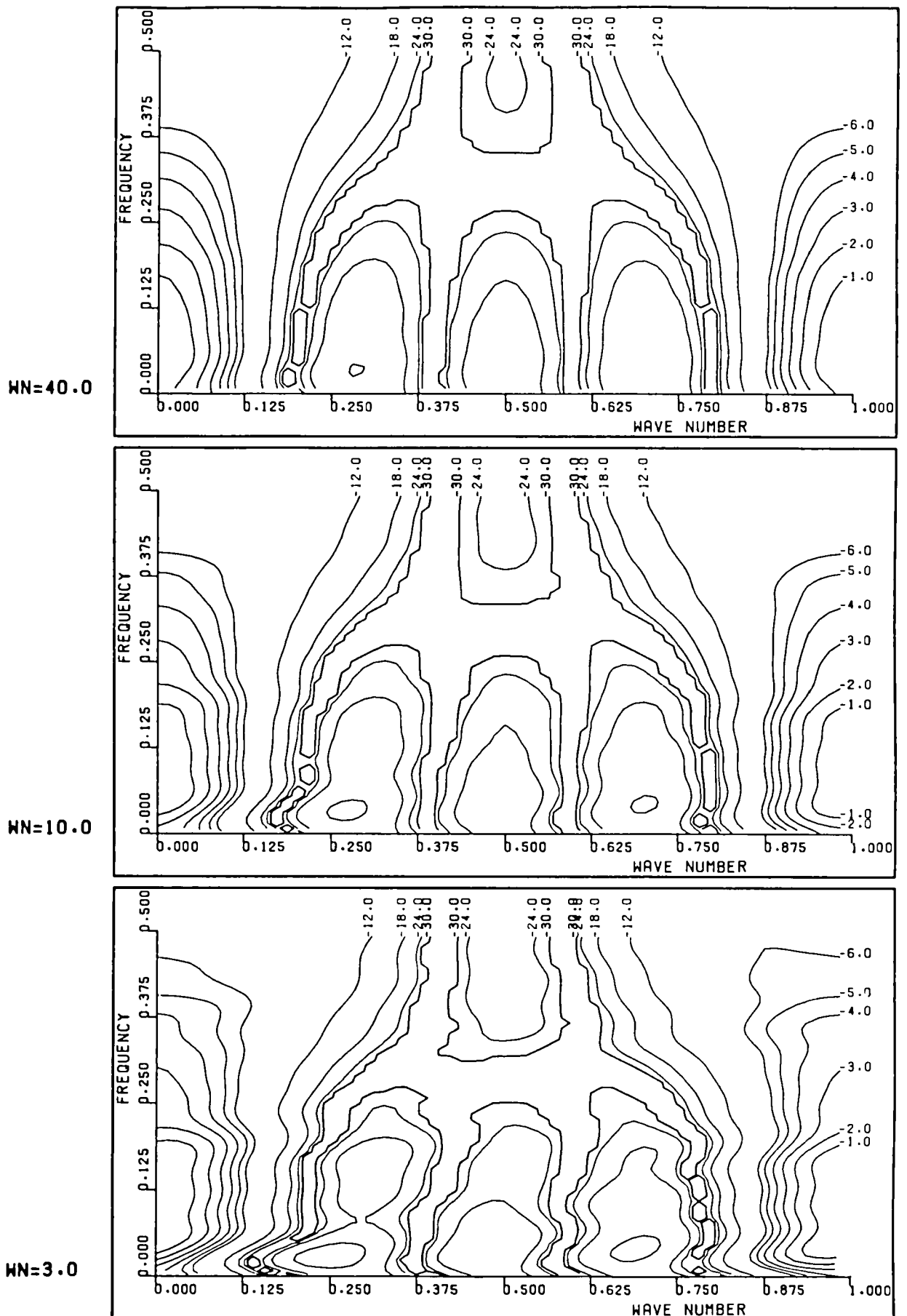


Figure 3.2.2.3 (f,k) responses for MLS moveout filter designed with uncorrelated noise weights higher than unity.

3. After a certain value of w_n , -6dB and -12dB contours become parallel to the frequency axis at frequencies less than about 0.375 and there is little change in the (f,k) response with further weight increase (Figure 3.2.2.3). Also the approximation of prescribed regions at low (f,k) values is very poor. This is very similar to the approximations obtained for the same pass and reject regions with the smaller number of coefficients.
4. With increased uncorrelated noise weight MLS centrosymmetric moveout filters tend to become symmetrical both in time-space and the corresponding frequency-wave number domain, i.e. they tend to straight-stack.

The processed data displayed in Figures 3.2.2.4 and 3.2.2.5 are the outputs of the filters whose responses are given in Figures 3.2.2.2 and 3.2.2.3, respectively. The input data are the same set of traces, given in Figure 3.2.1A, as were used in the previous section. The general results obtained are similar to those of varying chatter presented previously. Increasing the uncorrelated noise weight from very small values to about $w_n=3.0$ improved the S/N ratio leading to a better trace-to-trace alignment of horizontal or near horizontal events. Rejection of correlated and uncorrelated noise events in the first 100 to 500 ms of the raw data is good for all filtered outputs. At higher weights almost a total removal of noise with high frequency filtering of signal events is observed. The continuation of signal events, e.g. an event at about 475ms, has been brought out

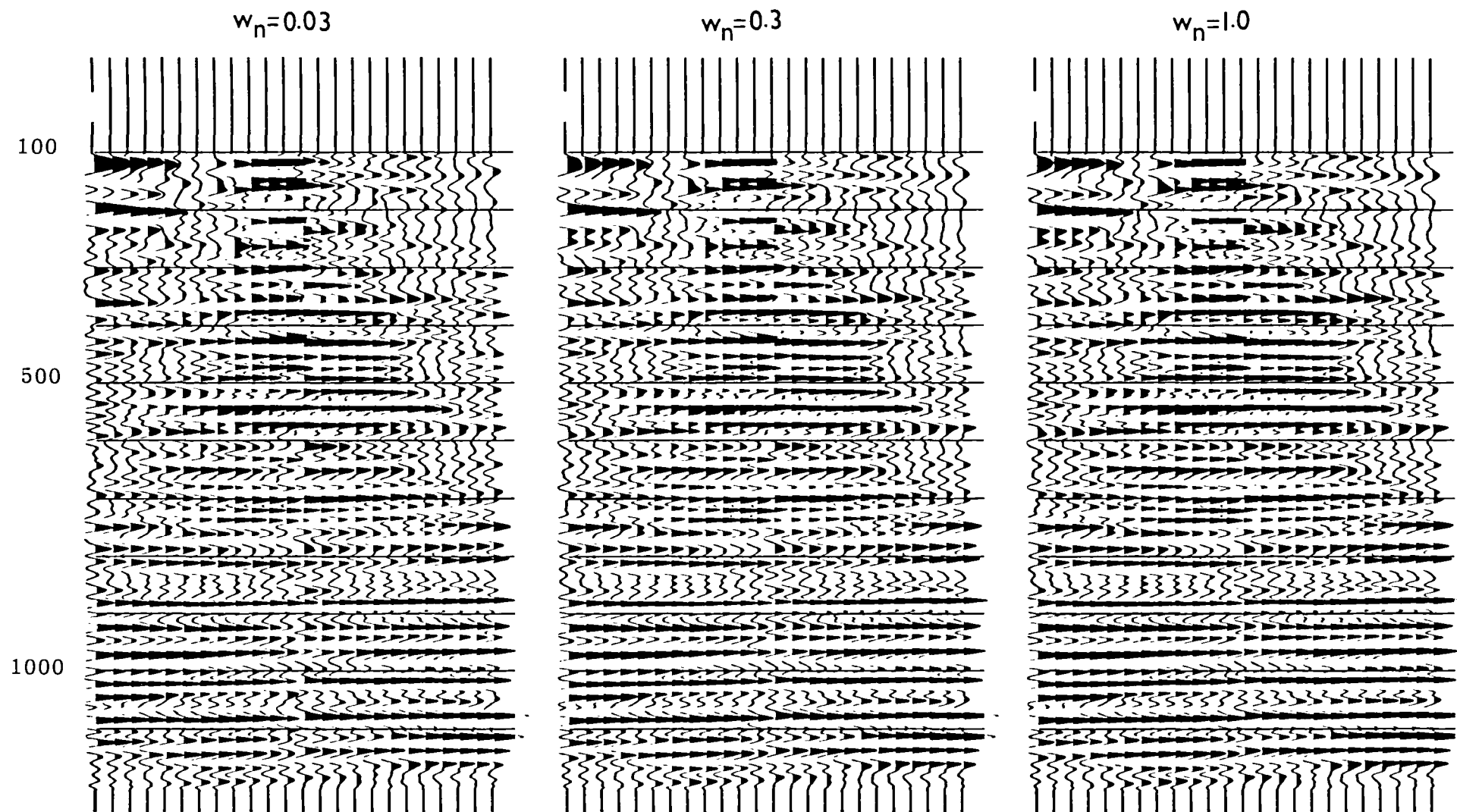


Figure 3.2.2.4 Outputs of the varying uncorrelated noise filters whose (f,k) responses are given in Figure 3.2.2.2.

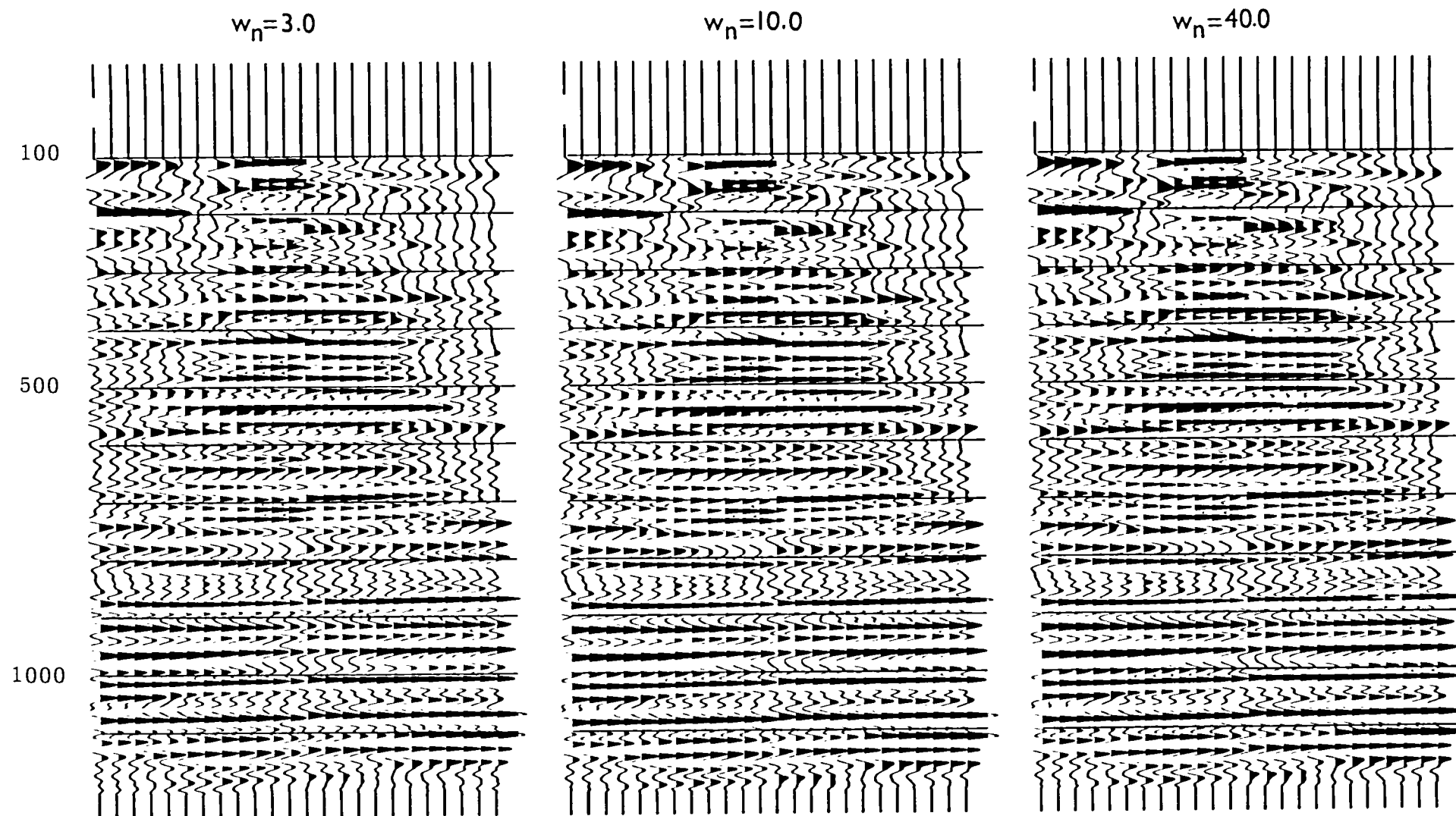


Figure 3.2.2.5 Outputs from the moveout filters designed with relatively higher uncorrelated noise weightings. The filter responses are given in Figure 3.2.2.3.

providing good trace-to-trace alignment across all traces. The undesirable low-frequency high-energy noise outburst line-ups from about 100 to 200 ms are typical for multi-channel filtered outputs and should not be mistaken as signal events. The events after 500 ms are strong arrivals showing very high S/N ratio. Trace-to-trace alignment as well as S/N ratio is greatly improved for this part, for the outputs of moveout filters with weights up to $w_n=3.0$, over the raw and frequency filtered data. At weights higher than about $w_n=5.0$ resolution decreases due to high frequency filtering.

It must be pointed out that increased uncorrelated noise in MLS moveout filter design does not necessarily give the results that are presented in this section. For example, if the pass and reject regions of the above common filter are interchanged, the disturbance caused by increased uncorrelated noise on the pass domain is relatively small. The triangular shape of the window is preserved at weights as high as $w_n=40.0$ and it aliases into the high frequencies sharply. Nevertheless, the results presented illustrate the most common case encountered in seismic reflection applications.

3.2.3 EFFECTS OF WEIGHTING CORRELATED EVENTS; ALIASING AND PRESCRIBED REGION APPROXIMATION

As was explained in Section 2.1, if desired, the least-squares approximation technique is suitable for changing

the relative power of a prescribed moveout window. Therefore, all pass and reject windows may be weighted to simulate an input model. It was also shown that changing the relative power of a window is equivalent to multiplying the corresponding correlation function with a suitable weight. In practice, relative weighting may be implemented in any one of the following three ways;

1. The multichannel filter designer estimates the powers contained in the pass and reject windows of input data and uses them to improve the agreement between the input model and the actual data.
2. Although the designer has some knowledge of power distributions in the prescribed windows, certain window(s) are purposely favoured to deal with a particular problem. This may be required, for example, when there is a need to improve very weak signal arrivals.
3. The weights of the prescribed regions are chosen in such a way that all regions are approximated equally well regardless of the actual input power distribution.

In due course, it will be seen that, if the least-squares technique given in the previous chapter is used for moveout filter design, the designer may have to favour a filter which is not necessarily based on the input model power distribution.

The behaviour of an MLS filter for linear moveouts

can be estimated for a given moveout using Parseval's Theorem (Hubral, 1970). Output energy at a constant moveout τ for filter series h_j in continuous frequency is

$$E(\tau) = \int_{-0.5}^{0.5} |H(f, \tau)|^2 df = 2 \int_0^{0.5} \left| \sum_{j=1}^N h_j e^{i2\pi j f \tau} \right|^2 df \quad 3.2.3.1$$

where h_j is an $N \times 1 \times (L+1)$ operator. In practice, as done throughout this study, the filter performance for correlated events is judged from their frequency-wave number responses. This is because performances estimated from $E(\tau)$ graphs will be less accurate than a complete representation such as the whole (f, k) diagram. Energy versus moveout curves for certain moveout filters calculated by Hubral showed that MLS filtering is always superior to straight stacking except at zero or near zero moveouts, where the difference between the two is marginal. It was also shown theoretically that increasing the relative correlated noise weight in the input model improves signal to correlated noise ratio.

The field recorded seismic reflection data are always frequency filtered before sampling in order to cut off frequencies above the Nyquist frequency (equal to the inverse of twice the sampling interval) so that aliasing in the time domain is avoided. In the spatial domain, sub-arrays at recording positions have to be designed to limit the data below the Nyquist wave number. The wavelengths in the recorded data are not usually known apriori and a trial and error process is required at the beginning of the

survey. Although there are many practical problems involved, choosing the distance between recording positions so as to give the upper Nyquist wave number limit (equal to the inverse of twice the trace spacing) which is greater than any wave number in the recorded data, is one way to avoid space aliasing (Hawes and Gerdes, 1974). Sub-geophone arrays used at recording positions discriminate against nonvertical travelling waves and help to give better recording of incoming signal energy over noise (Holzman, 1963; Krey and Toth, 1973; Newman and Mahoney, 1973). With improved field recording techniques long spread lengths are employed for better signal-multiple reflection moveout differentiation. Even the most sophisticated field recording techniques do not wave number filter all the unwanted correlated events and special processing techniques are used for further noise suppression. In this study, it has been assumed that a faithful, i.e. unaliased, recording of seismic reflection data has been achieved by means of appropriate frequency and wave number antialias filtering.

If multichannel filters for linear moveout windows are to be designed in the least-squares sense, aliasing in the wave number domain will pose problems when windows having moveouts per trace greater than the sampling interval used for digitizing the input processes are approximated. A one-dimensional operator designed for digital data sampled at a uniform interval ΔT cannot discriminate between the frequencies f and $f + \frac{m}{\Delta T}$ whenever m is an integer (Burg,

1964; Blackman and Tukey, 1958; Jenkins and Watts, 1968; Schwartz and Shaw, 1975). This can be generalized to multidimensional data and operators. If an n -dimensional digital operator is designed for equisampled data its frequency response, H , satisfies the following:

$$H(f_1 + \frac{m_1}{\Delta T_1}, f_2 + \frac{m_2}{\Delta T_2}, \dots, f_n + \frac{m_n}{\Delta T_n}) = H(f_1, f_2, \dots, f_n) \quad 3.2.3.2$$

whenever $m_1, m_2, \dots, m_n$ are integers. $\Delta T_1, \Delta T_2, \dots, \Delta T_n$ are the sampling intervals in n -dimensional space. Take an N input-one output moveout filter with impulse response $h_j(t, x)$. Its frequency-wave number response, $H(f, k)$, is periodic with $\frac{1}{\Delta T}$ and $\frac{1}{\Delta X}$ where ΔT and ΔX are the time and space domain uniform sampling intervals,

$$\begin{aligned} H(f, k) &= \sum_j h_j(t, x) \exp[-i2\pi(ft + kx)] \\ &= \sum_j h_j(p\Delta T, q\Delta X) \exp[-i2\pi(fp\Delta T + kq\Delta X)] \end{aligned} \quad 3.2.3.3$$

where p and q are integers. Take

$$\begin{aligned} H(f + \frac{m}{\Delta T}, k + \frac{n}{\Delta X}) &= \sum_j h_j(p\Delta T, q\Delta X) \exp\{-i2\pi[(f + \frac{m}{\Delta T})p\Delta T + (k + \frac{n}{\Delta X})q\Delta X]\} \\ &= \sum_j h_j(p\Delta T, q\Delta X) \exp[-i2\pi(fp\Delta T + kq\Delta X)] \end{aligned} \quad 3.2.3.4$$

therefore,

$$H(f + \frac{m}{\Delta T}, k + \frac{n}{\Delta X}) = H(f, k) \quad 3.2.3.5$$

whenever m, n, p and q are integers. If sampling intervals in both dimensions are taken as unity, the last equation becomes

$$H(f+m, k+n) = H(f, k) \quad 3.2.3.6.$$

As a result of uniform interval sampling, which is equivalent to multiplying continuous data with a Dirac comb, high frequency and wave number components are made to appear as erroneous low frequency and wave number components (Papoulis, 1962; Darby and Davies, 1967; Kanasewich, 1975). The least-squares technique under consideration assumes this aliased model as its input processes. The result is that, the operator is asked to treat all f and k values in the Nyquist range and their aliases alike. As was explained at the end of Section 3.2, a nondispersive wave with a linear moveout corresponds in (f, k) space to a straight line through the origin. Its slope is equal to the apparent velocity (v) of the nondispersive plane wave. When sampling intervals in time and space domains are taken as unity, spectral periods range from 0 to 1.0 or from $-\frac{1}{2}$ to $\frac{1}{2}$. Therefore, in the case of linear moveouts when the $v = \frac{f}{k}$ line intersects $k = 0.5$ line of (f, k) -space (when v is less than $|1.0|$) it folds back into the primary part of the (f, k) response, i.e. into $(0.0, 0.0) - (\mp 0.5, \mp 0.5)$. The desired response of the common filter, designed for the field data applications in this chapter, with aliasing of the reject

region into the pass and 'don't care' regions at high frequencies was given in Figure 3.2.1.5A. Simulated and field seismic reflection data applications showed that aliasing is a serious problem and limitation for optimum least-squares moveout filters. In the remaining part of this section the practical aspects of wave number aliasing will be the main discussion point.

Synthetic data applications of the least-squares moveout filters, using the input model given in Equation 2.2.1 with the added chatter to the signal and correlated noise arrivals, showed that signal to correlated noise ratio obtained from stack-after-moveout filtering trace is generally higher than straight-stacked trace for signal events with zero or near zero linear moveouts and varying linear moveout correlated noise arrivals. Moveout filters pass and reject all events falling into the corresponding window boundaries equally well. No gain in random noise reduction is obtained by moveout filtering over the straight-stack.

Series of synthetic data experiments have been made using an input model of only signal plus correlated noise. The number of input traces was 6. The correlated noise moveout window was varied from 1.0-2.0 to 22.0-23.0 samples per trace. Some chatter, varying from ± 0.25 to ± 0.5 sample, has been added to both signal and correlated noise arrivals. The pass moveout window covering ± 1.0 sample per trace linear moveouts was kept fixed. Beyond reject window boundaries of about 3.0 or 4.0 samples per trace,

w_r values had to be gradually reduced and w_n increased in the filter design input to obtain as high signal to correlated noise ratios as before. These changes in w_r and w_n have obviously reduced the aliasing of the reject regions into pass domains for the MLS moveout filters applied. Increased w_n made moveout filters more stable and low-amplitude filter generated noise before the onset times of signal events at filtered traces were removed. Signal to correlated noise ratios obtained from peak-to-peak amplitudes of straight-stacked and stack-after-moveout-filtered traces were about 12dB and 14dB, respectively, i.e. signal to correlated noise ratios of about 3.0-4.0 and 4.5-5.5, respectively. These were all for signal events with zero or near zero linear moveouts. The experiments also showed that multichannel filtering and straight stacking processes are not very sensitive to a small amount of chatter in the input traces. At a relatively large chatter values of about plus or minus one to two samples, multichannel filtering is always superior to straight stacking. This makes multichannel filtering highly preferable when the field recorded data has large residual statics which is often the case especially for land recorded data.

Applications of moveout filters to simulated data with different signal to correlated noise ratios showed that improvement brought about by filtering is not dependent on signal to correlated noise power ratio present in the input traces. Moveout filtering always makes a relative

improvement if the signal is not badly distorted by noise.

Increased w_r for better reject window approximation is always accompanied by degradation in the pass window approximation, even if there is no aliasing present. But if the signal and correlated noise windows are wide apart, relatively small weight increases for one window does not affect the other(s) considerably.

The raw synthetic data of Figure 3.2.1.3, which have been used to demonstrate the effect of chatter and uncorrelated noise, have been processed again as an illustrative example. The same common filter is applied to this data with varying w_r values. At the bottom of Figure 3.2.3.1 is shown the output of the filter with $w_r=0.3$ and at the top that of $w_r=15.0$. Two other trials not presented here were with weights $w_r=0.5$ and $w_r=2.5$. High-frequency filtering due to aliasing of the reject region with increased w_r is evident on signal and noise events. Although signal to uncorrelated noise ratio improves with increased correlated noise weight, uncorrelated noise line-ups created at about 50 ms and 940 ms are typical for MLS moveout filters with aliasing reject windows and high w_r . Note that signal to uncorrelated noise ratio for the signal event at 885 ms is reduced. This event has the lowest deviation from the zero moveout among the signal events, i.e. $t_c=\bar{t}\pm 0.25$ samples. Signal to correlated noise ratios for other signal events all increased with increasing w_r . Experience with simulated data examples showed that further correlated noise increases

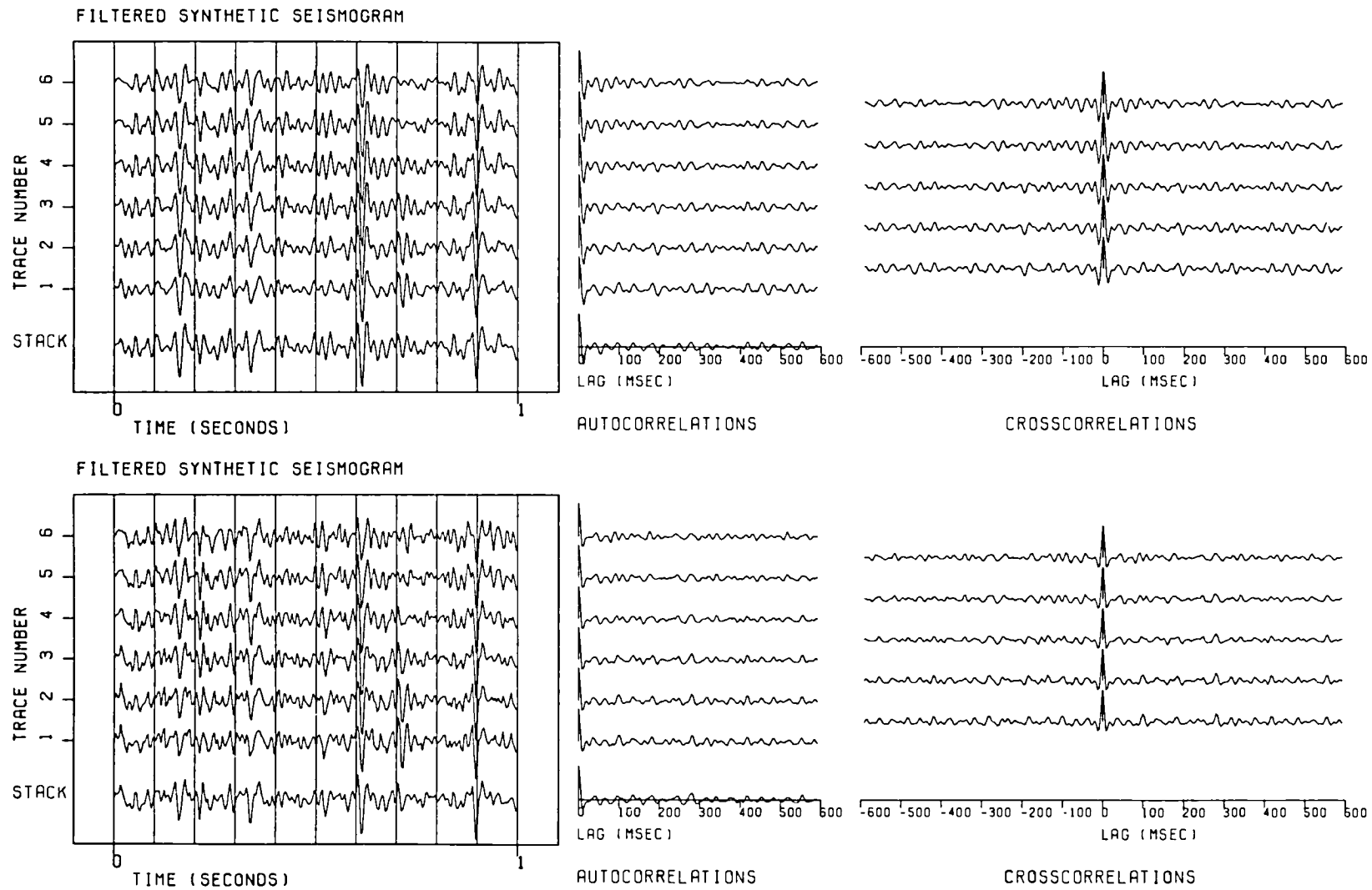


Figure 3.2.3.1 Effect of increasing correlated noise weight in the MLS moveout filter design. The output of the filter designed with $w_r = 0.3$ is given at the bottom and that of $w_r = 15.0$ at the top. The raw data is given in Figure 3.2.1.3.

create more and stronger uncorrelated noise line-ups when reject windows alias into the pass windows over a wide range of high frequencies. The resolution of data is also reduced.

All synthetic data experiments so far have been done for the N input-N output case for computational reasons. If centre trace estimate 'trace-overlapping' processing is employed more improvement over straight-stack should be obtained.

The (f,k) responses presented in the previous two sections showed that aliasing of the reject region into the high frequencies limited the approximation of the pass region to about -1 to -6dB at these frequencies. The same filter designed with varying correlated noise weights $w_r = 0.0, 0.1, 0.3, 0.5, 1.5, 2.5, 5.0, 7.5, 10.0$ and 15.0 to see the effect of prescribed region weighting, and especially the wave number aliasing. These filters have unit uncorrelated noise weight and zero chatter in their input models. Figures 3.2.3.2 and 3.2.3.3 show the (f,k) responses of these filters designed with weights $w_r = 0.0, 0.1, 0.5$ and $w_r = 2.5, 7.5, 15.0$, respectively. The response for the filter designed with unit weights and zero chatter was given in Figure 3.2.2.2. It is clear from the (f,k) responses shown that increasing the correlated noise weight from zero to unity changes the moveout filters from a pass-reject operator, similar to 'fan' or 'pie-slice' filters (Fail and Grau, 1963; Embree et al., 1963), to a pass-

reject operator with transition bands. At higher correlated noise weights the reject window is more clearly defined with an improved degree of approximation. And due to its aliasing into the high frequencies the pass window approximation decreases from about -1 to -3dB, to about -3 to -6dB. Note that the transition bands are not smooth either, and they also become reject regions with increased correlated noise weight. At $w_r=0.0$ the -1 to -3dB attenuation observed at the frequencies of about 0.325 to 0.5 for the pass region is due to the uncorrelated noise ($w_n=1.0$) introduced into the input model. At small frequency and wave numbers the selectivity of prescribed windows improves with increased w_r . At higher weights than $w_r=1.0$, due to the aliasing and pushing effect of the reject window the pass region is almost destroyed except for a very small area (see Figure 3.2.3.3). The bulging of the pass region into the 'don't care' region at very low frequencies, and wave numbers (about 0.75 to 1.0) is a common feature for all moveout filters with central symmetry, i.e. moveout filters with only one prescribed region for backwards sloping events (events with negative relative moveouts) close to zero moveout.

The above filters were applied to the same field data of previous sections to see the practical implications of correlated noise weighting. Most of the outputs are only marginally different for close w_r values. The outputs of the filters whose responses are given in Figures 3.2.3.2

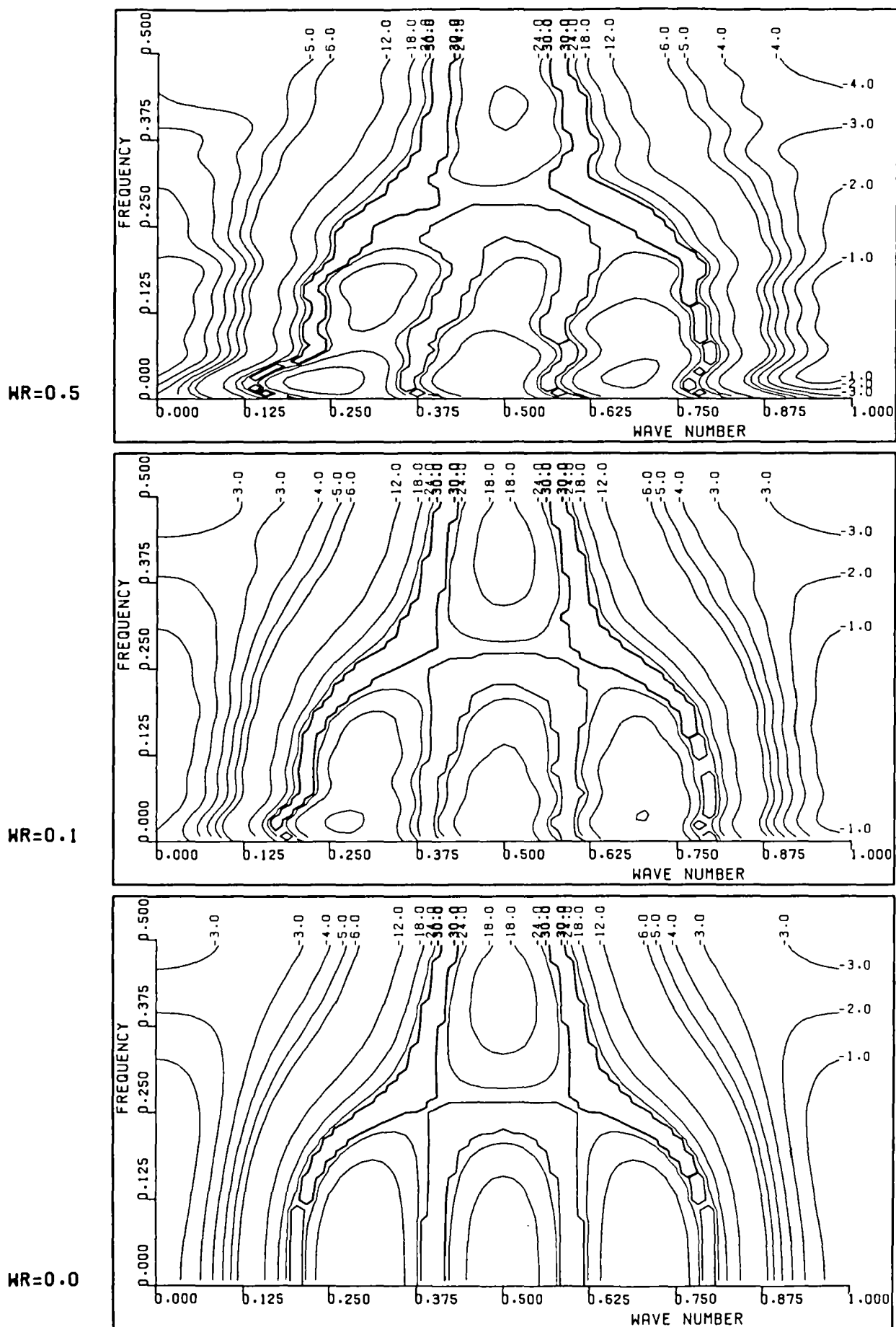


Figure 3.2.3.2 (f,k) responses for MLS linear moveout filter designed with correlated noise weights less than unity.

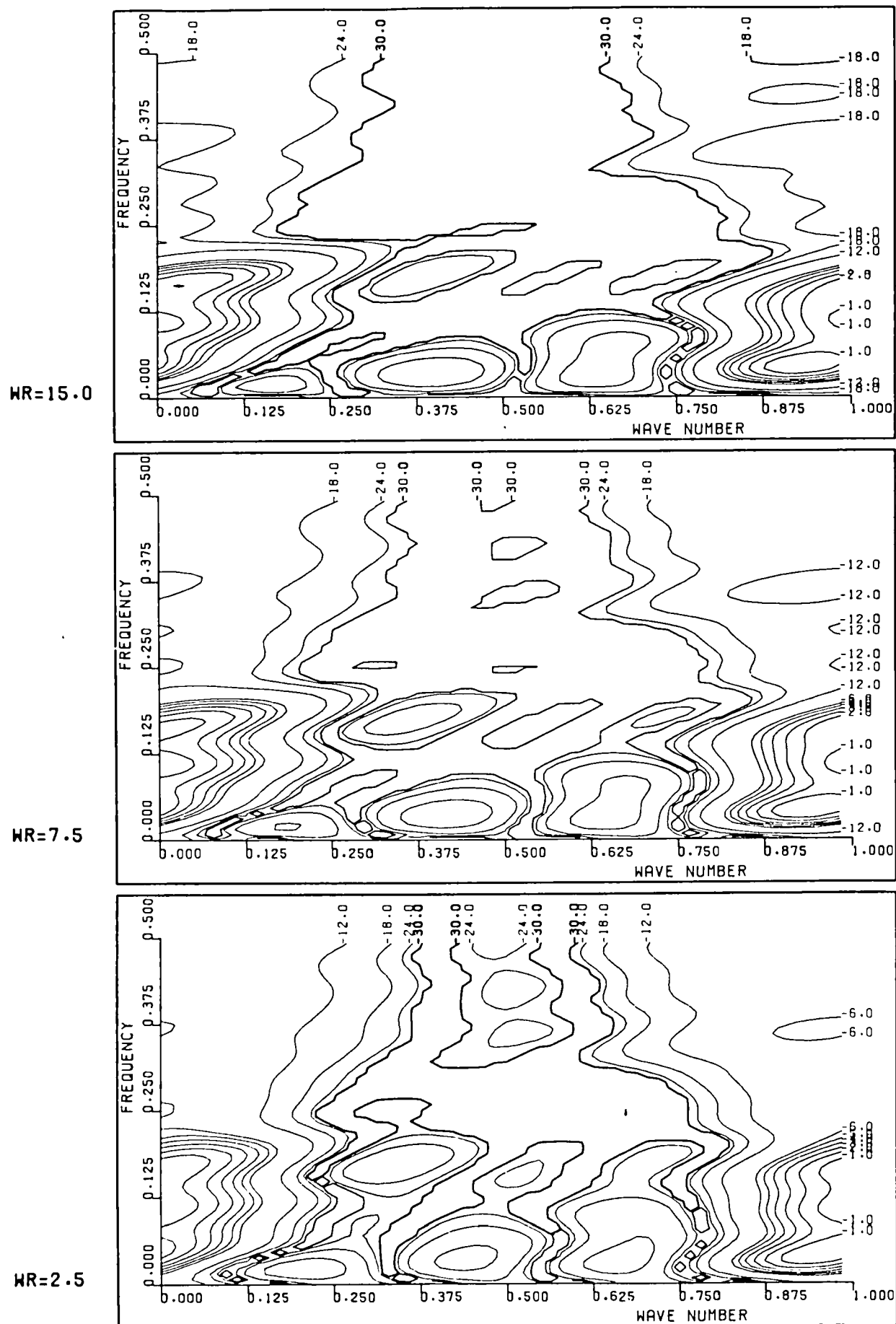


Figure 3.2.3.3 (f, k) responses of the MLS linear moveout filter designed with high correlated noise weights.

and 3.2.3.3 are given in Figures 3.2.3.4 and 3.2.3.5, respectively. The output of the unit weight ($w_r=w_n=1.0$) zero chatter filter was given in Figure 3.2.2.4. Rejection of the correlated noise at the beginning of the raw data is good for all values of w_r . Increasing w_r in the filter design improves trace-to-trace alignment and S/N ratio which can be observed from all events with moveouts falling into the pass region, e.g. events at about 375 ms, 500 ms and 1075 ms. Increases higher than about $w_r=1.0$, result in spurious line-ups, e.g. low-frequency high energy noise line-up at about 150 ms, and reduced resolution due to high-frequency filtering (Figure 3.2.3.5).

The linear moveout MLS filter design examples showed that changing the relative weighting of one prescribed region affects all of the (f,k) response. To reduce aliasing, either the aliased window relative weight is reduced or uncorrelated noise weight increased (see Figures 3.2.2.3 and 3.2.3.2). Either of the latter changes results in an inferior approximation to the desired response, especially at low and high frequencies. In practice, seismic reflection events have a power spectrum, peaking between frequencies of about 15 Hz to 75 Hz (Figure 3.2.2) corresponding to the frequencies 0.06 to 0.3 on the given (f,k) diagrams. This is why, although pass domains have been badly approximated or become reject domains at very low and high frequencies, the filtered outputs still preserve some of the signal power.

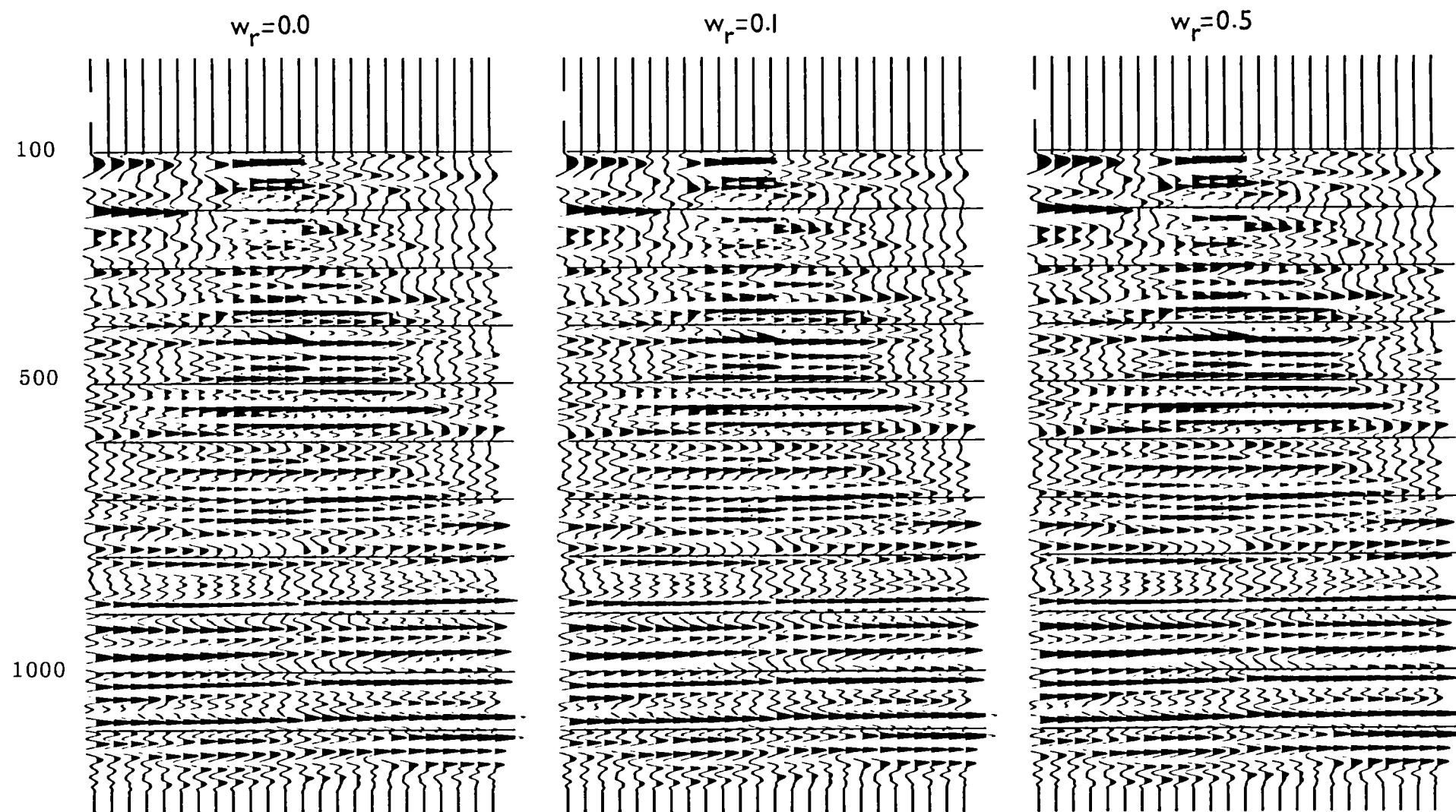


Figure 3.2.3.4 Outputs from the moveout filters designed with correlated noise weights less than unity. The filter responses are given in Figure 3.2.3.2.

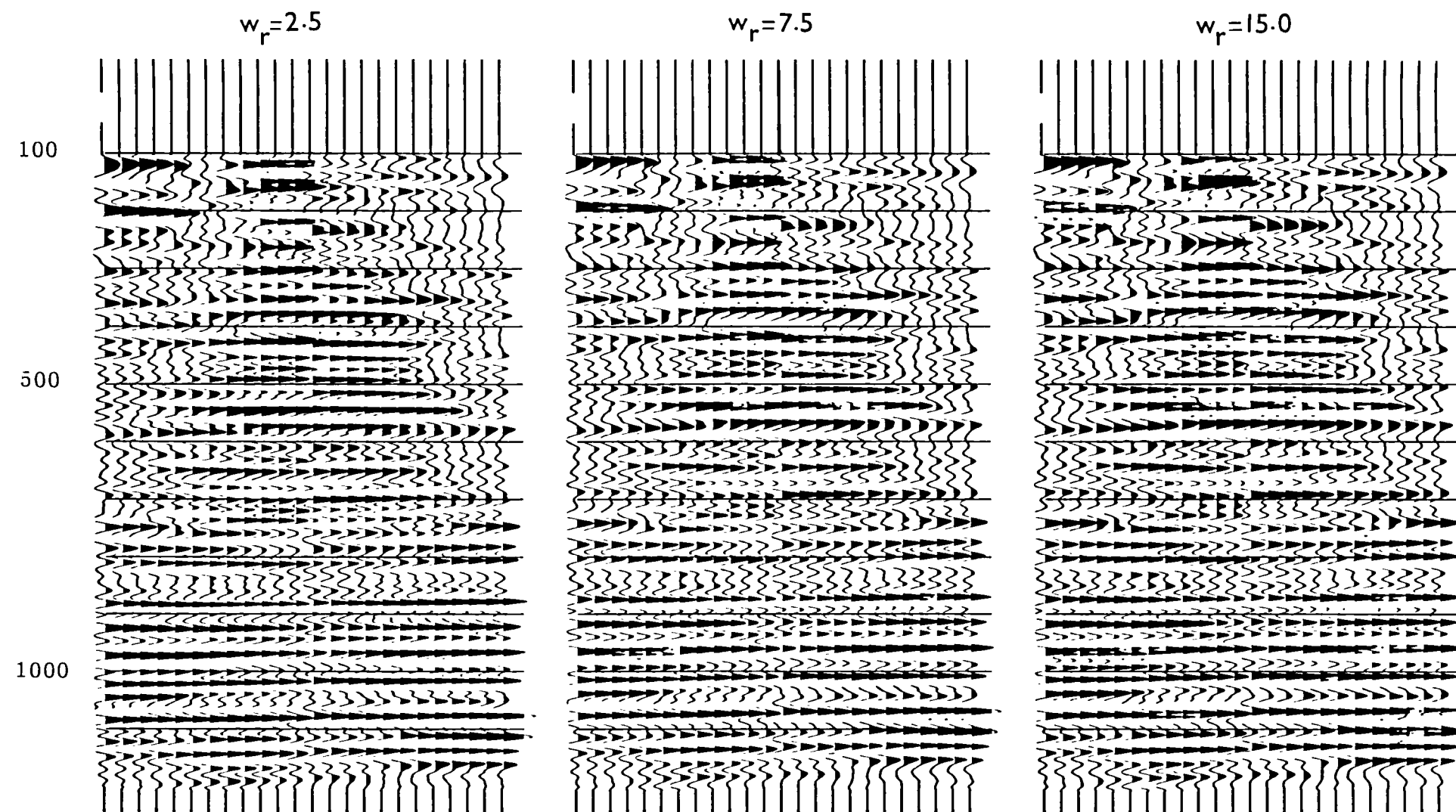


Figure 3.2.3.5 Outputs from the moveout filters with severely aliased reject regions. The filter responses are given in Figure 3.2.3.4.

As expected, frequency filtered versions of the same raw data compared with that of this section showed that increasing the aliasing reject region weight in moveout filtering is similar to band-pass filtering in one-dimension with narrower and narrower passbands. Power spectrum analysis of the moveout filtered data, i.e. the outputs of chatter, uncorrelated noise and correlated noise varying filters, verified the frequency filtering effects of these parameters. Comparing the optimum cases of moveout filtered outputs of this chapter and frequency filtered results (see Figure 3.2.1B and C), it is concluded that even powerful one-dimensional frequency filtering is inferior to multichannel processing in improving S/N ratio and trace-to-trace alignment. This is clearly seen especially from the early part of the processed data where S/N ratio is poor.

CHAPTER 4

MULTICHANNEL LEAST-SQUARES HYPERBOLIC MOVEOUT FILTERS

In this chapter multichannel input processes having correlated events falling on hyperbolae are treated. The aim is to design operators for filtering events with hyperbolic moveouts over a time gate rather than using fixed time hyperbolic moveout filters. Autocorrelation and crosscorrelation functions for hyperbolic moveout windows will be derived. Examples given also include input processes with hyperbolic plus linear moveout events.

4.1 AUTOCORRELATION AND CROSSCORRELATION MATRICES FOR MULTICHANNEL PROCESSES WITH HYPERBOLIC CORRELATED EVENTS

After necessary static corrections, a correlated seismic reflection event falling on a hyperbola will have the following arrival times if straight ray-path approximation is assumed.

$$a_i = \{a_0 + b[x_1 + (i-1)X]^2\}^{\frac{1}{2}} + \gamma_i, \quad i=1,2,\dots,N, \quad 4.1.1$$

where

a_0 : the square of two-way arrival time,

$b = \frac{1}{\bar{v}^2}$: where $\bar{v}$ is the r.m.s. seismic velocity,

x_1 : the distance between the source and the first recording

position,

- X : the distance between the recording positions,
 γ_i : the error in diverging from an exact hyperbola, i.e.
 so called chatter parameter as defined in Section
 2.2.

Contrary to the linear moveout case the relative moveouts calculated from the above equations are time and space dependent. For a given velocity the moveout curve is varied with the changing two-way time. A hyperbolic moveout filter may be designed for a fixed a_0 . The relative moveouts, necessary for the computation of autocorrelation and crosscorrelation matrices, for a given b and reference trace r are calculated from the relative arrival times given in Equation 4.1.1 (Curtis, 1971).

If input models have correlated events falling on hyperbolae and moveout filters designed for a time gate rather than a fixed time are desired, a more elaborate derivation of correlation functions is required. The possible values for a_0 and b can be defined by rectangular probability density functions

$$\text{pr}(a_0) = \frac{1}{A_0} \text{rec}\left(\frac{x-p}{A_0}\right)$$

and

4.1.2

$$\text{pr}(b) = \frac{1}{B} \text{rec}\left(\frac{x-s}{B}\right)$$

respectively. These distributions are shown in Figure

4.1.1. The probability density function for γ_i was given in Equation 2.2.3 as

$$\text{pr}(\gamma_i) = \frac{1}{t_c} \text{rec}\left(\frac{\gamma_i}{t_c}\right).$$

Following the same steps as in Section 2.2 the autocorrelation function entering into the normal equations for each signal entry is

$$\begin{aligned} C_{s_i s_j}(\tau) &= C_{ss}(\tau + a_j - a_i) \\ &= C_{ss}(\tau + \sqrt{a_0 + b u_2} - \sqrt{a_0 + b u_1} + \gamma_j - \gamma_i) \end{aligned} \quad 4.1.3$$

where $u_1 = [x_1 + (i-1)X]^2$, $u_2 = [x_1 + (j-1)X]^2$ and $i, j = 1, 2, \dots, N$.

In the frequency domain, by taking the Fourier transform of the above equation,

$$Q_{s_i s_j}(f) = Q_{ss}(f) e^{i2\pi f[(a_0 + b u_2)^{\frac{1}{2}} - (a_0 + b u_1)^{\frac{1}{2}}]} e^{i2\pi f \gamma_j} e^{-i2\pi f \gamma_i} \quad 4.1.4$$

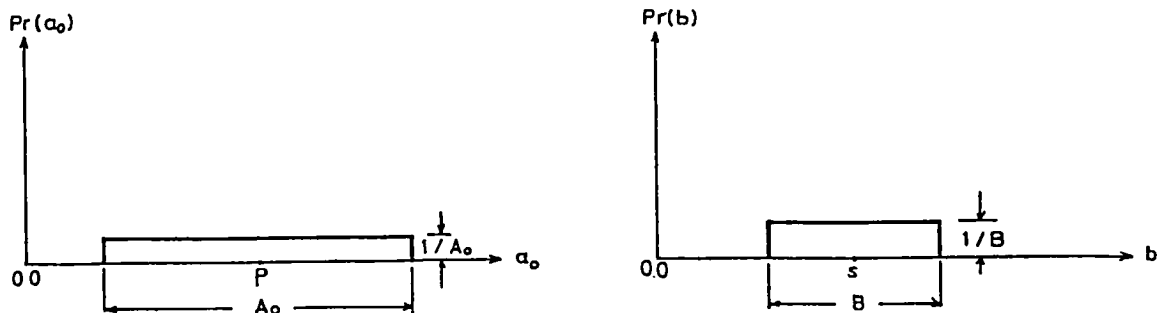


Figure 4.1.1 Probability density functions for two-way time (left) and apparent velocity (right).

is obtained. The last two equations correspond to Equations 2.2.5 and 2.2.8 which were derived for the linear moveout case. The expectation of the last equation is

$$E\{Q_{s_i s_j}(f)\} = Q_{ss}(f) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \text{pr}(a_0) \text{pr}(b) e^{i2\pi f[(a_0 + bu_2)^{\frac{1}{2}} - (a_0 + bu_1)^{\frac{1}{2}}]} da_0 db$$

$$\int_{-\infty}^{\infty} \text{pr}(\gamma_j) e^{i2\pi \gamma_j f} d\gamma_j \int_{-\infty}^{\infty} \text{pr}(\gamma_i) e^{-i2\pi \gamma_i f} d\gamma_i$$

4.1.5.

If it is assumed that $Q_{ss}(f) = \text{rec}(f)$ then the above equation becomes

$$E\{Q_{s_i s_j}(f)\} = \text{rec}(f) \text{sinc}^2(t_c f) \cdot I_1 \quad 4.1.6$$

where

$$I_1 = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \text{pr}(a_0) \text{pr}(b) e^{i2\pi f[(a_0 + bu_2)^{\frac{1}{2}} - (a_0 + bu_1)^{\frac{1}{2}}]} da_0 db \quad 4.1.7.$$

In the time domain Equation 4.1.6 is written as

$$E\{C_{s_i s_j}(\tau)\} = \int_{-\infty}^{\infty} \text{rec}(f) \text{sinc}^2(t_c f) I_1 e^{i2\pi f \tau} df \quad 4.1.8$$

and for the discrete case

$$E\{C_{s_i s_j}(\tau)\} = \int_{-0.5}^{0.5} \text{sinc}^2(t_c f) I_1 e^{i2\pi f \tau} df \quad 4.1.9$$

where I_1 is now

$$I_1 = \frac{1}{A_0 B} \int_{A_{01}}^{A_{02}} \int_{B_1}^{B_2} e^{i2\pi f [(a_0 + bu_2)^{\frac{1}{2}} - (a_0 + bu_1)^{\frac{1}{2}}]} da_0 db \quad 4.1.10$$

$$\begin{aligned} \text{with } A_{01} &= p - \frac{A_0}{2} \\ A_{02} &= p + \frac{A_0}{2} \\ B_1 &= s - \frac{B}{2} \\ B_2 &= s + \frac{B}{2} \end{aligned} \quad 4.1.11$$

as indicated in Figure 4.1.1. There is no direct analytical solution to the multiple integral in Equation 4.1.9. It may be expanded into a two-dimensional Taylor series and then solved analytically or calculated by numerical techniques, with a computer. The inclusion of the chatter parameter in the input model makes both analytical and numerical solutions over-cumbersome and expensive. In fact, it has been shown in the previous chapter that the chatter parameter may be omitted from MLS moveout filter design without any loss. Then Equation 4.1.9 becomes, in full,

$$E\{C_{s_i s_j}(\tau)\} = \frac{1}{A_0 B} \int_{-0.5}^{0.5} \int_{A_{01}}^{A_{02}} \int_{B_1}^{B_2} e^{i2\pi f [(a_0 + bu_2)^{\frac{1}{2}} - (a_0 + bu_1)^{\frac{1}{2}} + \tau]} da_0 db df \quad 4.1.12.$$

Integrating over f ,

$$E\{C_{s_i s_j}(\tau)\} = \frac{1}{A_0 B} \int_{A_{01}}^{A_{02}} \int_{B_1}^{B_2} \frac{\sin\{\pi[(a_0 + bu_2)^{\frac{1}{2}} - (a_0 + bu_1)^{\frac{1}{2}} + \tau]\}}{\pi[(a_0 + bu_2)^{\frac{1}{2}} - (a_0 + bu_1)^{\frac{1}{2}} + \tau]} da_0 db \quad 4.1.13.$$

In all the equations above $i, j = 1, 2, \dots, N$ and $\tau = 0, 1, \dots, L$ where N is the number of input channels and $L+1$ is the filter length.

The correlated noise entries of the autocorrelation matrix are derived in the same way, except for different B and s being used to describe the reject window(s) which do not overlap with the pass window(s) over the time gate $g_t = A_{01}$ to A_{02} .

The uncorrelated noise component is the same as that given in Equation 2.2.14.

The crosscorrelation matrix for hyperbolic moveout signal events can be calculated in a similar manner. The signal component on the j th input and r th desired output is

$$s_j(t) = s(t - a_j)$$

and

4.1.14

$$\begin{aligned} z_r(t) &= s_r(t) \\ &= s(t - a_r - t_d) \end{aligned}$$

respectively, where $a_r = \{a_0 + b[x_1 + (r-1)X]^2\}^{\frac{1}{2}}$ and

$$a_j = \{a_0 + b[x_1 + (j-1)X]^2\}^{\frac{1}{2}} + \gamma_j.$$

t_d is the delay allowed for the desired output signal and γ_j comes from the chatter parameter, as explained previously.

The crosscorrelations between the desired output and input processes are

$$\begin{aligned}
 C_{s_r s_j}(\tau) &= E\{s_r(t+\tau)s_j(t)\} \\
 &= E\{s(t+\tau-a_r-t_d)s(t-a_j)\} \\
 &= C_{ss}(\tau+a_j-a_r-t_d) \\
 &= C_{ss}(\tau+\sqrt{a_0+bu_2}-\sqrt{a_0+bR}-t_d) \quad 4.1.15
 \end{aligned}$$

where $u_2 = [x_1+(j-1)X]^2$, $R = [x_1+(r-1)X]^2$ and $j=1,2,\dots,N$.

If the same steps from Equation 4.1.4 to 4.1.11 are followed the expectation of the crosscorrelation function becomes

$$E\{C_{s_r s_j}(\tau)\} = \frac{1}{A_0 B} \int_{-0.5}^{0.5} \int_{A_{01}}^{A_{02}} \int_{B_1}^{B_2} e^{i2\pi f[(a_0+bu_2)^{\frac{1}{2}}-(a_0+bR)^{\frac{1}{2}}+\tau-t_d]} da_0 db df \quad 4.1.16.$$

Integrating over f ,

$$E\{C_{s_r s_j}(\tau)\} = \frac{1}{A_0 B} \int_{A_{01}}^{A_{02}} \int_{B_1}^{B_2} \frac{\sin\{\pi[(a_0+bu_2)^{\frac{1}{2}}-(a_0+bR)^{\frac{1}{2}}+\tau-t_d]\}}{\pi[(a_0+bu_2)^{\frac{1}{2}}-(a_0+bR)^{\frac{1}{2}}+\tau-t_d]} da_0 db \quad 4.1.17$$

where

$$j = 1, 2, \dots, N$$

$$\tau = 0, 1, 2, \dots, L.$$

All other variables are as defined before. Note that, as in the linear moveout case, the reference positions of output traces do not have to coincide with the positions of the input channels, and r does not have to be an integer.

The double integral in Equation 4.1.17 has the same form as the one in Equation 4.1.13 and cannot be solved using straightforward analytical methods. Similarly, it can also either be expanded into two-dimensional Taylor series then integrated or calculated numerically on a computer.

4.2 FOURIER DOMAIN INTERPRETATION OF CORRELATED EVENTS WITH NONLINEAR MOVEOUT FUNCTIONS

The first step for estimating the quality of a designed filter is its frequency response. In the case of least-squares moveout filters, the approximation problem is set up in the time-distance domain without requiring any knowledge of desired (f,k) response. For the linear moveout case, pass and reject window boundaries are straight lines because of the relation $v = \frac{f}{k}$ where $v = \frac{1}{\tau}$ is the velocity and τ is the moveout per trace. There is no simple way to find the (f,k) domain response of a set of arrivals falling on a nonlinear curve such as a hyperbola. Theoretically, a nonlinear moveout boundary can be thought of as an ensemble of the linear moveout boundaries. A correlated event with a nonlinear moveout function corresponds to an area on the (f,k) domain after Fourier transformation. The boundaries of this area are defined by the smallest and biggest relative moveouts of the nonlinear moveout curve. Assume an N channel input process having only one correlated event with the relative moveouts $\tau_i, i=1,2,\dots,N$, and no uncorrelated noise. Consider two channels such as r th and j th with spikes (or any other

events such as seismic wavelets) at relative moveouts τ_r and τ_j and assume that the remaining traces have zero power. If this data is Fourier transformed, the (f,k) diagram will show that all the power lineation falls on the straight line $v_{r,j} = \frac{f}{k}$ where $v_{r,j} = \frac{1}{\tau_{r,j}}$ and $\tau_{r,j} = \frac{\tau_j - \tau_r}{|j-r|}$. If all the combinations of two-trace pairs are considered in similar way, the smallest and biggest $\tau_{r,j}$ values will define approximate linear boundaries in the (f,k) plane. Outside these boundaries the correlated event with nonlinear moveout does not have any power. Since the distribution of relative moveouts between these two boundaries is not linear the power distribution is not uniform. In the case of functional moveout such as hyperbolic, the straight line boundaries on the (f,k) plane are obtained from the tangents having minimum and maximum slopes. In the discrete case this is also reduced to the above approximate solution.

To illustrate the above points, the (f,k) diagrams of simulated input processes have been computed with the same program which is used to compute the (f,k) responses of the multichannel filters presented in this thesis. All the cases tried were in agreement with the above arguments. Figure 4.2.1A shows the (f,k) diagram of a 7 trace synthetic seismogram having only one correlated event, and no uncorrelated noise. The seismic wavelet used in the simulation is the same one as the signal and correlated noise wavelet of synthetic data examples presented in this study. This event has exact hyperbolic moveout with the relative values 0.0,

1.65, 4.32, 7.93, 12.37, 17.53 and 23.29 samples. It corresponds to a reflection event at 81.25 samples (325 ms) two-way time and velocity 0.105 time unit per trace spacing (0.42 ms per trace spacing). The trace spacing and the source distance from the first recording position is taken as unity. To ease the comparison of (f,k) plots presented in this section with the (f,k) responses of the hyperbolic moveout filters given in the next section the positions of the input traces are reversed prior to Fourier transformation. The power spectrum of the input wavelet is plotted alongside of the (f,k) diagram to help the interpretation. The curvature of a hyperbolic moveout curve increases with the increased distance from the source. Therefore, following the previous arguments, the smallest moveout per trace is always defined by the two traces nearest to the source and the biggest by the furthest two traces. In the example given these values are 1.65 and 5.76 samples per trace. The corresponding $\tau_{\min} = 1.65$ and $\tau_{\max} = 5.76$ samples per trace lines superimposed on the (f,k) diagram shows that the highest power is contained between the fan-shaped region bounded by them and peaking at frequencies of about 0.08 where the input wavelet has its highest power. The highest power lineation lies on approximately the average relative moveout ($\tau_{av} = 3.71$) line. The split of power lineation around this average value line is typical because there are two main trends in a hyperbolic curve with low and high slopes.

Figure 4.2.1B shows the (f,k) diagram of the synthetic data with approximately the same moveout configuration as above but with unit spikes assigned at relative moveouts 0, 2, 4, 8, 12, 18 and 23 samples instead of seismic wavelets. The corresponding straight lines for the smallest relative moveout per trace, $\tau_{\min} = 2$, and the highest, $\tau_{\max} = 6$, with the average, $\tau_{av} = 4$, are superimposed on the same (f,k) diagram. The conclusions drawn are similar to the previous example. The highest power lineation starts from the origin because spikes have a flat spectrum for all frequencies. The high power lineations obtained at high frequencies beyond the $\tau_{\min} = 2$ line are due to aliasing of the power at low frequencies.

Synthetic data experiments with more than one non-linear correlated event showed complicated (f,k) diagrams. The maximum power is always contained between the smallest and biggest relative moveout limits. The (f,k) diagram given in Figure 4.2.1C is for the spike data of Figure 4.2.1B plus another event represented by unit spikes at relative moveouts 0, 1, 2, 4, 6, 9 and 12 samples. The latter corresponds to a reflection event at 150 samples (600 ms) two-way time with velocity 0.1125 time unit per trace spacing (0.45 ms per trace spacing). The trace spacing and source distance from the first recording position is again taken as unity. The actual relative moveouts are 0.0, 0.79, 2.09, 3.89, 6.18, 8.94 and 12.13 samples. The straight lines corresponding to the smallest, $\tau_{\min} = 1$, and the biggest,

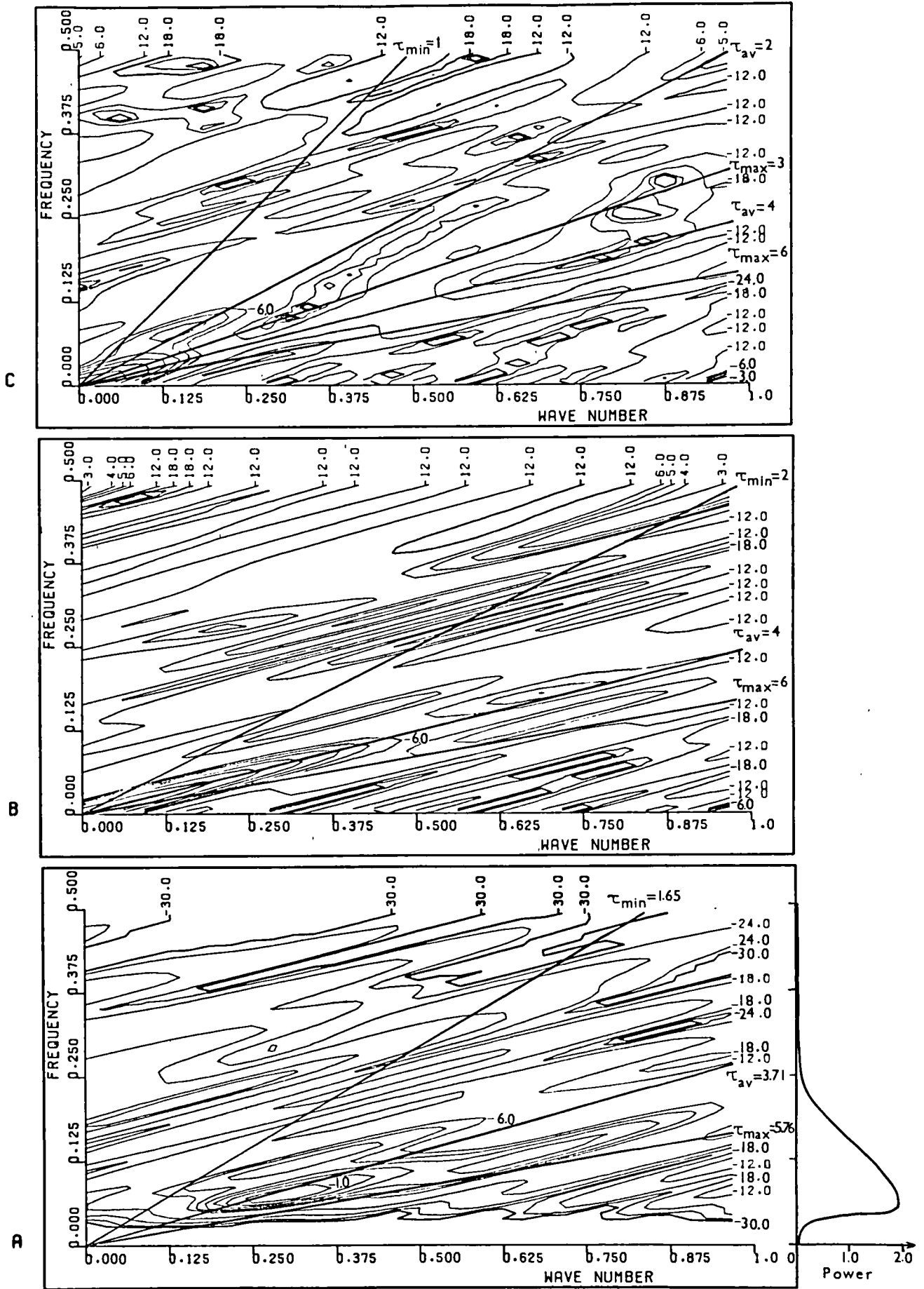


Figure 4.2.1 (f,k) plots of synthetic traces containing correlated events with hyperbolic moveouts.

$\tau_{\max} = 3$, relative moveouts per trace and their average, $\tau_{av} = 2$, are superimposed on the (f,k) diagram. The straight lines $\tau = 2, 4$ and 6 , correspond to the first event whose (f,k) diagram is given in Figure 4.2.1B. The highest power lineations at small frequency and wavenumbers are along the average relative moveout lines $\tau_{av} = 2$ and $\tau_{av} = 4$. At the other parts of the (f,k) plot, although there are lineations of power contours in accordance with the fan-shaped region defined by the smallest and biggest relative moveouts, a generalized interpretation is not possible especially at high frequencies.

Further investigations with different hyperbolic moveout events gave the same results as above. It has been found that the estimation of the moveout function from the (f,k) diagrams is not possible. The results obtained in this section are useful in estimating the effectiveness of the moveout filters, designed in the time domain, for passing and/or rejecting events falling on nonlinear moveout curves.

4.3 DESIGN AND APPLICATION OF LEAST-SQUARES OPERATORS WITH HYPERBOLIC MOVEOUT WINDOWS

The autocorrelation and crosscorrelation matrices of hyperbolic moveout filters (filters designed to pass and/or reject events falling on hyperbolic moveout curves) are computed by solving the double integrals in Equations 4.1.13 and 4.1.17. The analytical solutions using the two-dimensional Taylor expansion results in lengthy expressions

even when the expansion is limited to terms not greater than the second or third degree. The algorithm is inefficient because some of the common terms must either be calculated for each lag or kept on tape for future use which necessitates a great deal of tape reading and frequent rewinding operations. Numerical solution of the integrals on the computer was found to be more efficient than the Taylor expansion method in terms of both programming purposes and computer time. The integrands vary smoothly in both dimensions; therefore, a dense mesh for numerical integration is not necessary. The design experiments showed that a grid sampling of about 10 to 20 in velocity samples and 15 to 30 in time samples is usually adequate for approximate correlation computations. Obviously, these figures are functions of the velocities defining the window widths and the chosen time gate. Large gates and windows require a larger number of mesh points for accurate correlation computations. In practice, grids of excessive density are undesirable because of computational costs, and they may lead to roundoff errors depending on the computer used. Experience showed that, the use of a very dense grid does not necessarily improve the resultant filter response noticeably. Computational times for hyperbolic moveout filters, with adequate subdivisions for numerical integration, are not much higher than linear moveout cases.

Design experiments showed that unless enough uncorrelated noise is added to the input model, the resultant hyperbolic moveout filter is unstable. This immediately

shows up during computation by giving negative squared errors, which is also the case for hyperbolic moveout filters designed for a specific two-way time. The stable operators designed and applied in this chapter always had an uncorrelated noise weight greater than unity; and $w_n = 1.5$ to 3.0 has been found high enough in practice. It must be pointed out that, even for stable filters, although NMSE is not negative the NMSE versus delay curves are not well behaved as in the linear moveout case. This is probably because the numerically computed correlation functions are not accurate. The NMSE always decreases with increased delay up to the length of filter. Design and application experience showed that the approximate rules about the optimum delay position for linear moveout filters, given in Section 3.1, also work for hyperbolic moveout filters. Namely, the optimum delay position for the desired output is always about the half of the filter length.

Another important step before making any attempt to design hyperbolic moveout filters is the choice of the time gate. In practice, the width of the time gate is limited by the prescribed velocity windows for pass and reject regions. This is because for the same velocities a hyperbolic moveout boundary has a greater curvature at small times than at large times. Therefore care should be taken to ensure that the prescribed pass and reject moveout windows do not overlap over the entire time gate. It is expected that the designed filter will perform equally well for the whole time gate.

This is the case when the prescribed pass-reject regions are distinct. Time gates should be kept as small as possible because a filter designed for a narrower time gate performs better than the corresponding larger time gate filter. And this is true even if there are no over-lapping pass and reject regions in the latter case.

The (f,k) responses presented in this section are the Fourier transforms of the time-distance responses. The approximation accuracy for the prescribed hyperbolic windows is estimated by the approximate method given in Section 4.2; using the minimum, average and maximum moveouts per trace. The average moveout per trace is estimated from the relative moveout hyperbola prescribed by the 'average' velocity and two-way time corresponding to the mid-value of the time gate chosen. Let a hyperbolic moveout window be prescribed over velocity boundaries $v = v_{\min}$ to $v_{\max}$ and time gate $g_t = a_{\min}$ to $a_{\max}$. Then a useful criterion of assessment for the 'average' velocity is $v_{av} = \frac{v_{\min} + v_{\max}}{2}$ and the average two-way time is given by $a_{av} = \frac{a_{\min} + a_{\max}}{2}$. If the arrival times are $a_i, i=1,2,\dots,N$, obtained by substituting these values into Equation 4.1.1 and taking $\gamma_i = 0$, the average moveout is taken as $\tau_{av} = \frac{(a_2 - a_1) + (a_N - a_{N-1})}{2}$. The minimum moveout per trace is taken as $\tau_{\min} = a_2 - a_1$ where a_1 and a_2 are calculated using $a_{\max}$ and $v_{\max}$. The maximum moveout per trace is taken as $\tau_{\max} = a_N - a_{N-1}$ where a_{N-1} and a_N are calculated using $a_{\min}$ and $v_{\min}$. The corresponding $\tau_{\min}$, τ_{av} and $\tau_{\max}$ sample per trace lines are superimposed on the (f,k) responses presented.

Further checks with simulated data containing only signal and correlated noise events verified this approximate technique.

The first example is a hyperbolic pass-linear reject moveout filter designed for a relatively narrow time gate over smaller two-way times. The (f,k) response and its isometric plot for this filter is given in Figure 4.3.1. The pass domain around the average moveout per trace line for pass window, $(\tau_{av})_p$, is approximated with about 0 to -6dB. This is partly due to the multiple aliasing of the reject region into the pass region and is partly expected according to the results of Section 4.2. The reject band relative attenuation is about 18 to 30dB, except near the origin where the 'pass domain push' is evident. This kind of pushing is common to most of the moveout filters, designed in the least-squares sense, especially with pass regions prescribed for small velocities or high moveouts. In fact, the main reason is the sampling discrepancy between the frequency and wave number axes corresponding to the time and distance axes, respectively. In the above case there are 7 samples on the wavenumber axis from 0.0 to 1.0 and 13 samples on the frequency axis from 0.0 to 0.5, i.e. the filter has 7x25 coefficients. It can be reduced either by increasing the number of input traces, or increasing the relative weight of the disturbed window (the reject window in the above case) if there is no severe aliasing of the latter. The efficiency of this filter, over the full time and velocity ranges for which it is designed, is checked by

The design parameters for the filter whose (f,k) response and isometric plot are given in Figure 4.3.1:

Number of input channels,	$N = 7$
Length of filter,	$LF = 25$
Reference output trace,	$R = 4$
Delay index,	$t_d = 13$
Offset distance,	$x = 1.0$
Pass window velocities (trace/ms)	$v_p = 0.42 \text{ to } 0.45$
Reject window moveouts (ms/trace)	$O_r = 26.0 \text{ to } 32.0$
Correlated noise weight,	$w_r = 1.0$
Uncorrelated noise weight,	$w_n = 1.5$
Chatter	$t_c = 0.0$
Time gate (ms)	$g_t = 325 \text{ to } 600$
Subdivisions in time for numerical integration,	$d_t = 20$
Subdivisions in velocity for numerical integration,	$d_v = 10$

The trace spacing is taken as unit distance in all filter design and synthetic data examples presented in this chapter.

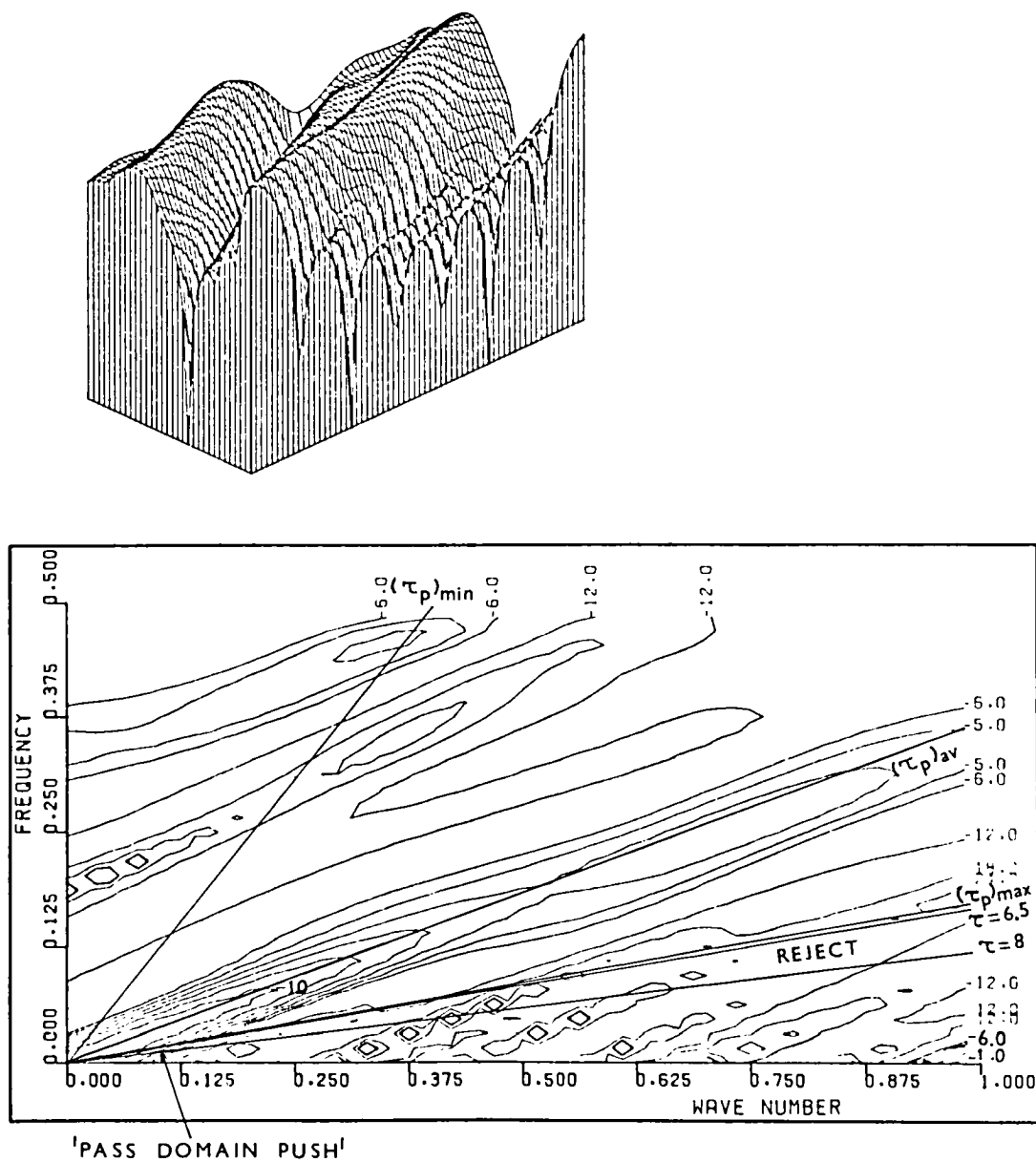


Figure 4.3.1 (f,k) response of a hyperbolic pass-linear reject moveout filter. The isometric plot is viewed from the origin. See the facing page for the design parameters.

processing the uncorrelated noise-free simulated seismic reflection traces given in Figure 4.3.2. The processed traces are given in the same figure. The performance of the filter for the signal events with moveout hyperbolae falling on or near the hyperbolic pass window boundaries is poorer as compared to the others, as expected. This feature of hyperbolic moveout filters, verified under different signal and correlated noise conditions, can be observed from the processed traces presented by comparing the events, for example, at 80, 260 and 900 ms representing the signal events falling on the upper boundary, mid-range and lower boundary of the pass window, respectively. The correlated noise suppression is about the same for all noise events. The signal to correlated noise ratio improvements (estimated from peak-to-peak amplitudes) are about 8 to 9dB.

The above raw data with 55 wavelets per trace of uncorrelated noise added are given in Figure 4.3.3. The signal to uncorrelated noise ratio over the duration of the record is 0.7. The output of the above filter for this data is given in the same figure. The general results obtained are similar to the previous uncorrelated noise-free case. The S/N ratio and trace-to-trace alignments have improved, especially for the signal events falling away from the hyperbolic pass moveout window boundaries, i.e. for the events at 260, 485 and 580 ms. These can also be observed from the plotted autocorrelation and crosscorrelation functions. The spurious line-up at about 50 ms is one of the characteristic features of the stacking operators on large noise pulse(s).

The input model parameters for the synthetic reflection seismogram of Figure 4.3.2:

Signal events:

Offset distance, $x = 1.0$

Two-way times (a_0 in ms)	Velocities (trace/ms)	Simulated onset times (ms)
325.0	0.435	80.0
325.0	0.45	325.0
325.0	0.42	420.0
462.5	0.42	260.0
462.5	0.435	485.0
462.5	0.45	580.0
600.0	0.42	645.0
600.0	0.435	740.0
600.0	0.45	900.0

Note that the simulated onset time means the arrival time assigned to an event in the given synthetic examples.

The correlated noise events have linear moveouts of 32, 29 and 26 ms/trace with the onset times 4.0, 165.0 and 810.0 ms, respectively.

The signal to correlated noise ratio over equivalent wavelets is unity for all the synthetic seismograms in this chapter. The sampling interval in time is 4.0 ms.

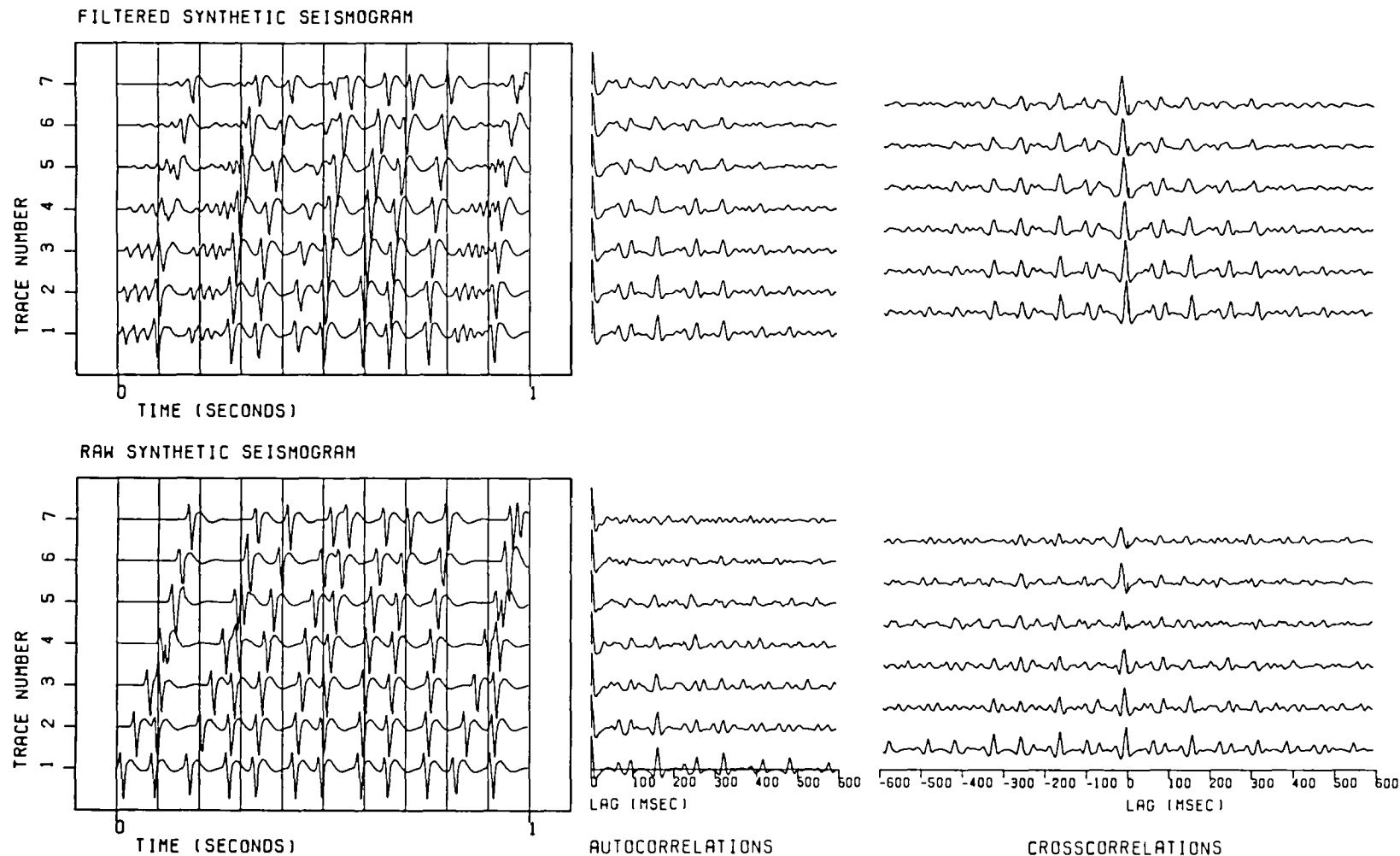


Figure 4.3.2 Raw synthetic reflection seismogram containing only correlated noise events with linear moveouts and signal events with hyperbolic moveouts (lower), and its processed version by the filter whose (f,k) response is given in Figure 4.3.1 (upper). The synthetic data parameters are given on the facing page.

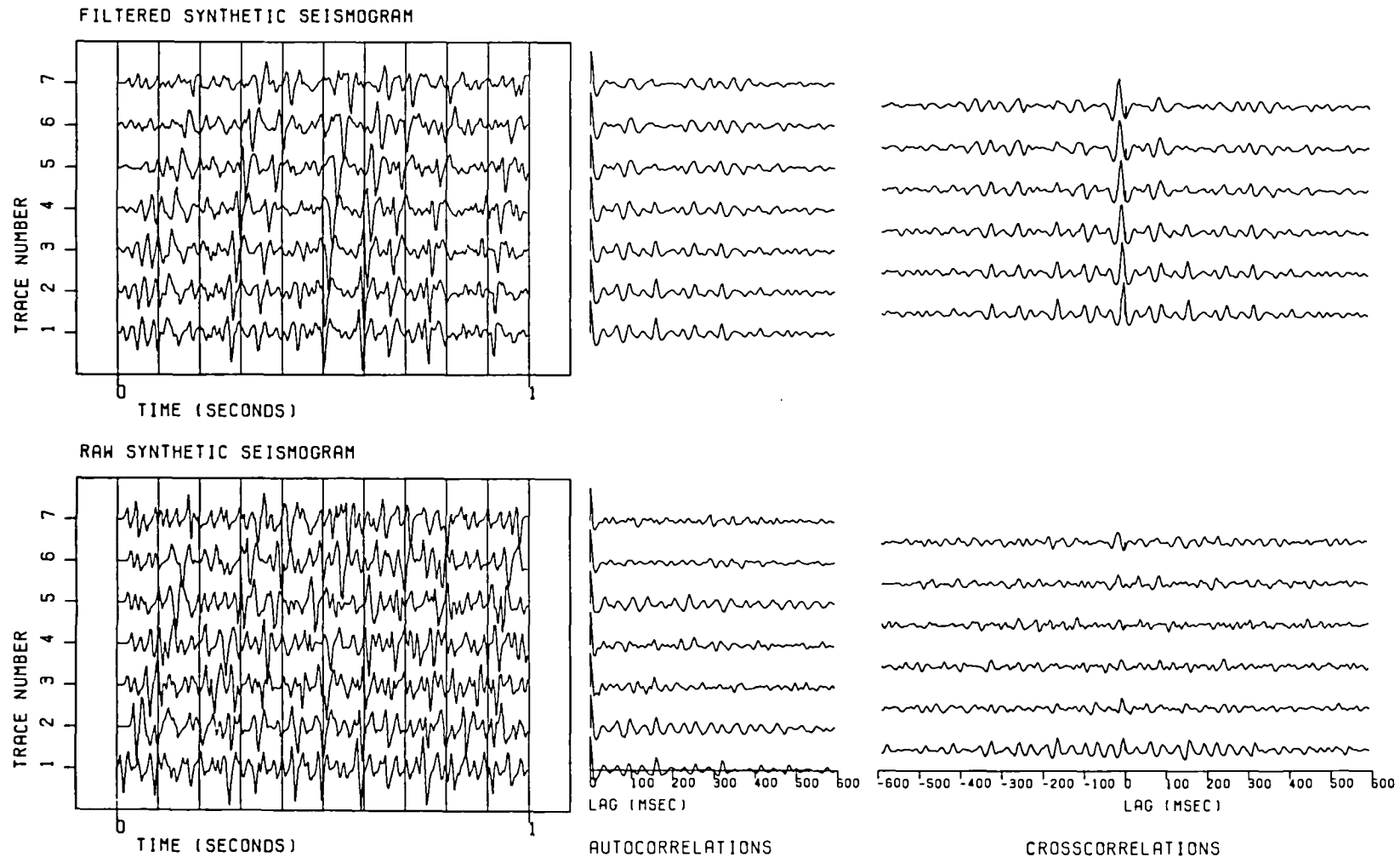


Figure 4.3.3 Raw synthetic seismogram of Figure 4.3.2 with 55 wavelets of random noise added to each trace (lower), and its processed version by the filter whose (f,k) response is given in Figure 4.3.1.

Equation 4.1.1 shows that a given velocity defines different arrival time hyperbolae for different two-way times. The hyperbolic moveout filters with the same prescribed velocity windows but different time gates, presented below, will also illustrate this point. Figure 4.3.4A and B show the (f,k) responses of a 7x25 hyperbolic pass-hyperbolic reject moveout filter designed for the time gates $g_t = 100$ to 500 ms and $g_t = 500$ to 900 ms, respectively. In both cases pass domains show about 0 to -6dB deviation around $(\tau_p)_{av}$ sample per trace lines. Note that there is pass and reject domain overlapping for the filter designed over smaller two-way times, i.e. over $g_t = 100$ to 500 ms. The reject domain attenuations are about 12 to 30dB. Unlike the pass domains, the reject domains do not have the best approximation area falling around the $(\tau_r)_{av}$ line.

The synthetic uncorrelated noise-free seismic reflection traces, containing signal and correlated noise events with moveout hyperbolae ranging over respective time and velocity gates of the above filters, have been processed to check the corresponding filter performances. Figure 4.3.5 shows the raw and processed data for the filter designed over $g_t = 100$ to 500 ms. The signal event at 75 ms represents an arrival with the smallest pass-velocity and lowest two-way time, i.e. the pass window boundary near to the reject window. This event is almost destroyed after filtering because the pass domain overlaps with the reject domain (Figure 4.3.4A).

The common design parameters for the hyperbolic pass-hyperbolic reject moveout filters whose (f,k) responses are given in Figure 4.3.4A and B:

Number of input channels,	$N = 7$
Length of filter,	$LF = 25$
Reference output trace,	$R = 4$
Delay index,	$t_d = 13$
Offset distance,	$x = 3.0$
Pass window velocities (trace/ms)	$v_p = 0.8 \text{ to } 1.0$
Reject window velocities (trace/ms)	$v_r = 0.28 \text{ to } 0.38$
Correlated noise weight,	$w_r = 1.0$
Uncorrelated noise weight,	$w_n = 2.0$
Chatter,	$t_c = 0.0$
Subdivisions in time for numerical integration,	$d_t = 25$
Subdivisions in velocity for numerical integration,	$d_v = 15$

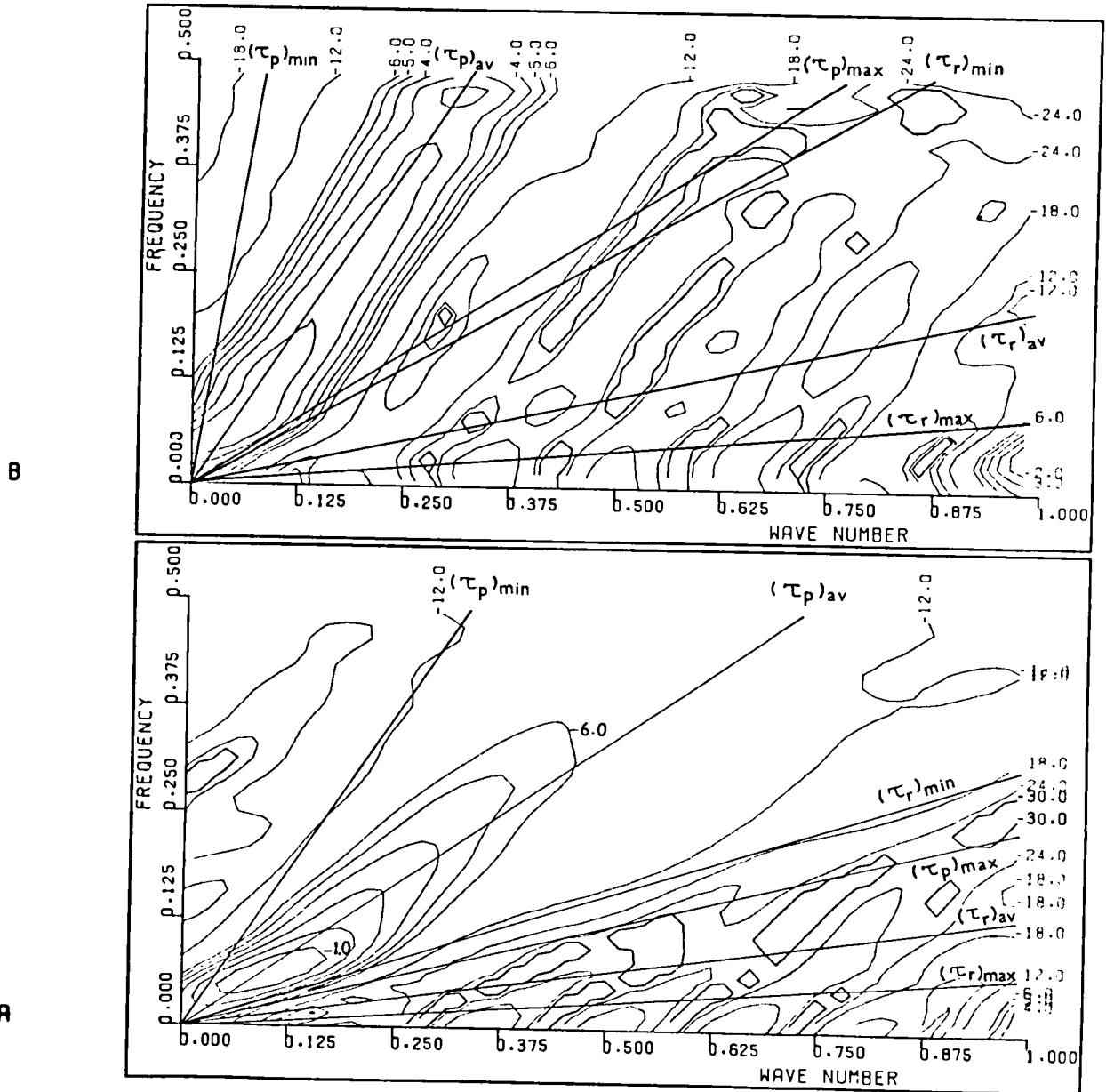
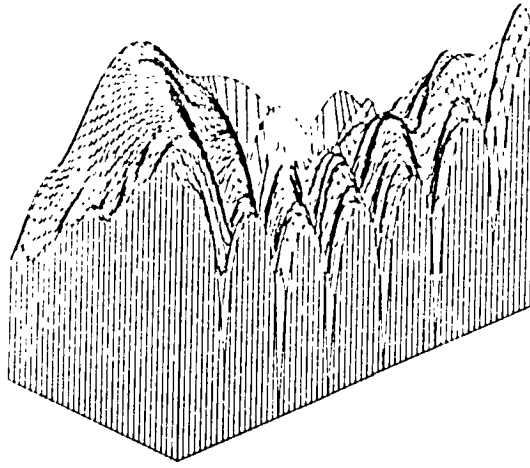


Figure 4.3.4 (f,k) response of 7x25 hyperbolic pass-hyperbolic reject moveout filter designed over the time gate $g_t = 100$ to 500 ms (A) and $g_t = 500$ to 900 ms (B). The isometric plot is viewed from the origin. See the facing page for the common design parameters.

The events having either mid-range two-way times or mid-range velocities or both are passed without distortion and the signal to correlated noise ratios are higher. The correlated noise events are attenuated at about the same degree of efficiency. In Figure 4.3.6 the input and output data for the filter designed over $g_t = 500$ to 900 ms are given. All the signal events are passed with very little or no distortion, as expected. The signal to correlated noise ratio is better than the previous corresponding application. Again, the ratio is higher for the signal events falling into the middle of the pass window, i.e. the events at 410,575 and 735 ms.

Sixty wavelets per trace of uncorrelated noise was added to the above raw synthetic data of Figures 4.3.5 and 4.3.6, and the new data were processed by the same corresponding filters mentioned above. The signal to uncorrelated noise ratio over the length of each trace is 0.75. The resultant raw and processed traces are given in Figures 4.3.7 and 4.3.8. The results concerning the signal and correlated noise events are the same as in the corresponding uncorrelated noise-free examples presented previously. The signal to uncorrelated noise ratios are improved after filtering in both cases, leading to better trace-to-trace alignments. The improvement brought about by the filter designed over the higher two-way times ($g_t = 500$ to 900 ms) are superior to the corresponding filter designed over the smaller two-way times ($g_t = 100$ to 500 ms).

The next example is an offshore data application.

The input model parameters for the synthetic reflection seismogram of Figure 4.3.5:

Offset distance, $x = 3.0$

Signal events:

Two-way times (a_0 , in ms)	Velocities (trace/ms)	Simulated onset times (ms)
100.0	0.8	75.0
200.0	0.9	240.0
300.0	0.8	410.0
300.0	0.9	575.0
300.0	1.0	735.0
500.0	1.0	910.0

Correlated noise events:

Two-way time (a_0 , in ms)	Velocities (trace/ms)	Simulated onset times (ms)
100.0	0.28	10.0
200.0	0.38	170.0
300.0	0.28	335.0
300.0	0.33	490.0
300.0	0.38	650.0
500.0	0.38	815.0

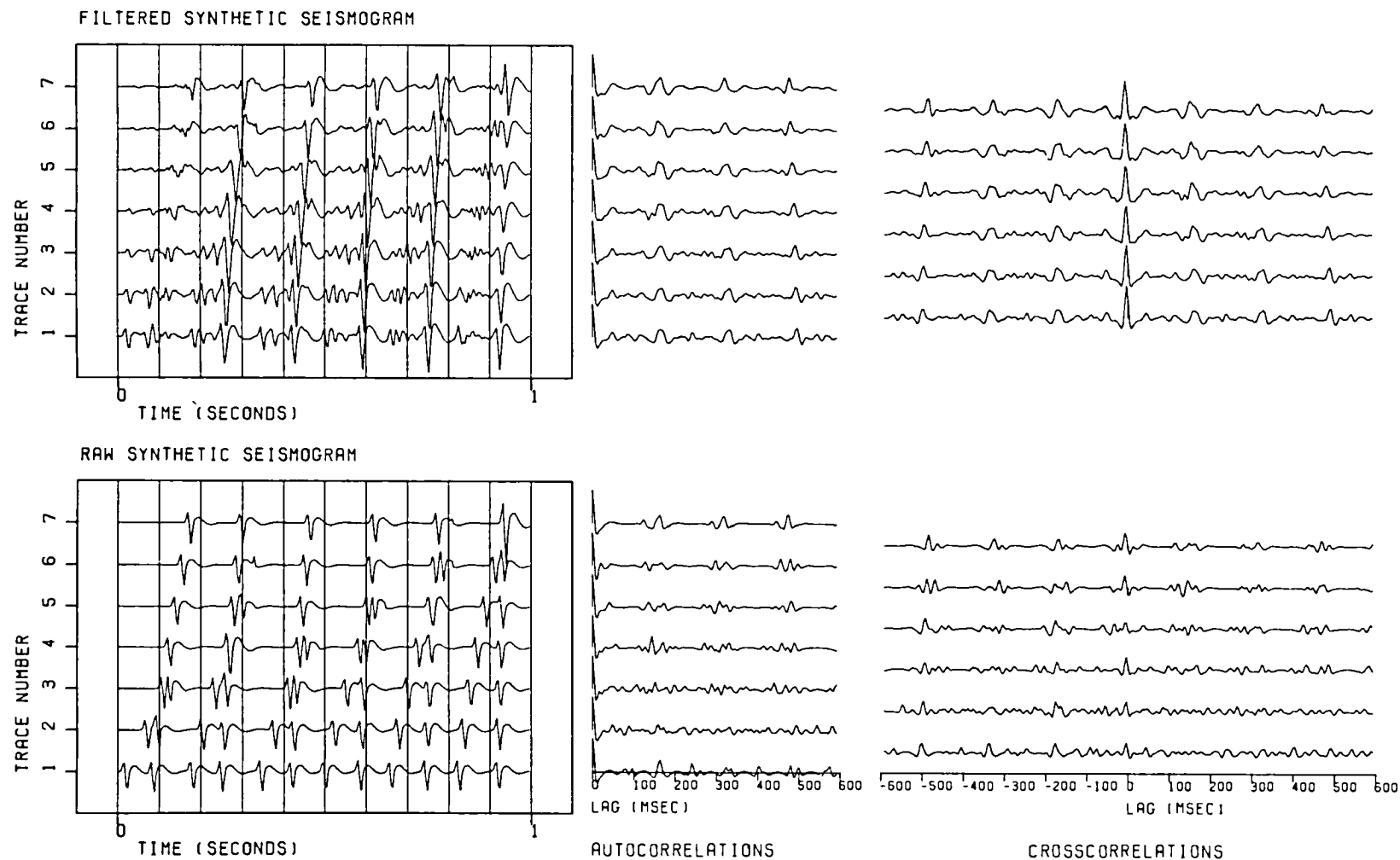


Figure 4.3.5 Raw synthetic seismogram containing signal and correlated noise events with hyperbolic moveouts (lower), and the output of filter designed over the time gate $g_t = 100$ to 500 ms for this data (upper). The synthetic data parameters are given on the facing page.

In common with all the other data from the same area, the record given in Figure 4.3.9A has a low-frequency correlated noise ('ground-roll' or shot noise) ranging across all the recording positions. The trace spacing for this data is 50 m and the offset distance (shot distance from the nearest recording position) is 222 m. The digitizing interval is 4 ms. The correlated noise event has approximate linear moveouts about 55 to 75 ms/trace and the hyperbolic events around 1200 ms have small curvatures. The above values enable the filter designer to employ relatively larger time gates for the filters applied, without the pass and reject regions overlapping. A number of 12-in/12-out filters have been applied to this data to remove the correlated noise event and pass all the events with hyperbolic moveouts in the prescribed time gates around 1200 ms two-way time. The outputs of the filters designed over the time gates $g_t = 800$ to 1500 ms and $g_t = 700$ to 1700 ms are given in Figures 4.3.9B and C, respectively. The common filter parameters are given on the page facing the same figure. The upper limit of the pass region velocity was maintained at a high value ($v = 2.0$ trace/ms) to allow hyperbolic events, arriving at two-way times greater than the upper limit of the corresponding time gates, to be passed. The processed data show that all hyperbolic events inside the prescribed time gates are passed with improved S/N ratio and trace-to-trace alignment. The correlated noise is successfully removed. The same results are valid for the events with

The input model parameters for the synthetic reflection seismogram of Figure 4.3.6:

Offset distance, $x = 3.0$

Signal events:

Two-way times (a_0 , in ms)	Velocities (trace/ms)	Simulated onset times (ms)
500.0	0.8	75.0
600.0	0.9	240.0
700.0	0.8	410.0
700.0	0.9	575.0
700.0	1.0	735.0
900.0	1.0	910.0

Correlated noise events:

Two-way times (a_0 , in ms)	Velocities (trace/ms)	Simulated onset times (ms)
500.0	0.28	10.0
600.0	0.38	170.0
700.0	0.28	335.0
700.0	0.33	490.0
700.0	0.38	650.0
900.0	0.38	815.0

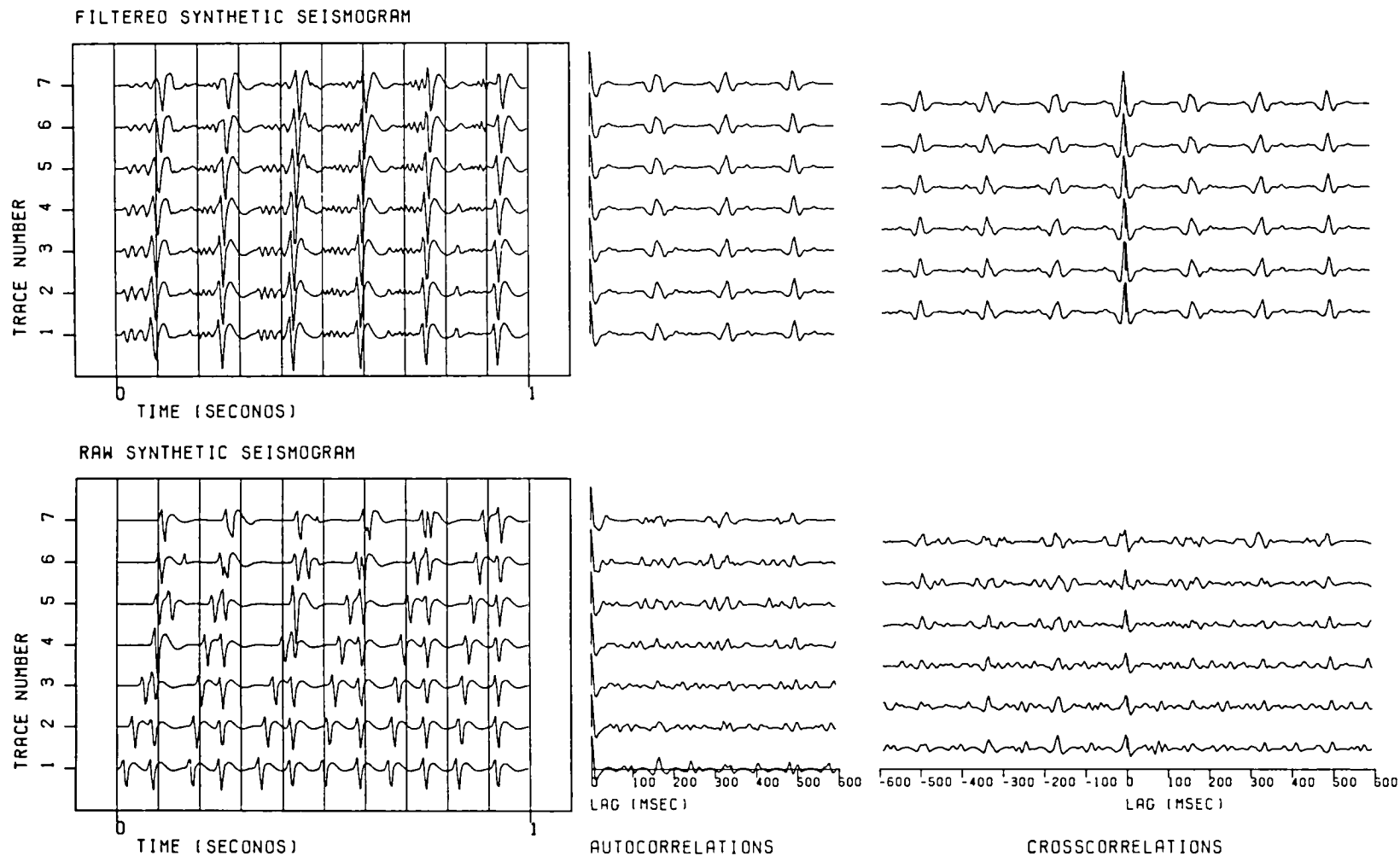


Figure 4.3.6 Raw synthetic seismogram containing signal and correlated noise events with hyperbolic moveouts (lower), and the output of filter designed over the time gate $g_t = 500$ to 900 ms for this data (upper). The synthetic data parameters are given on the facing page.

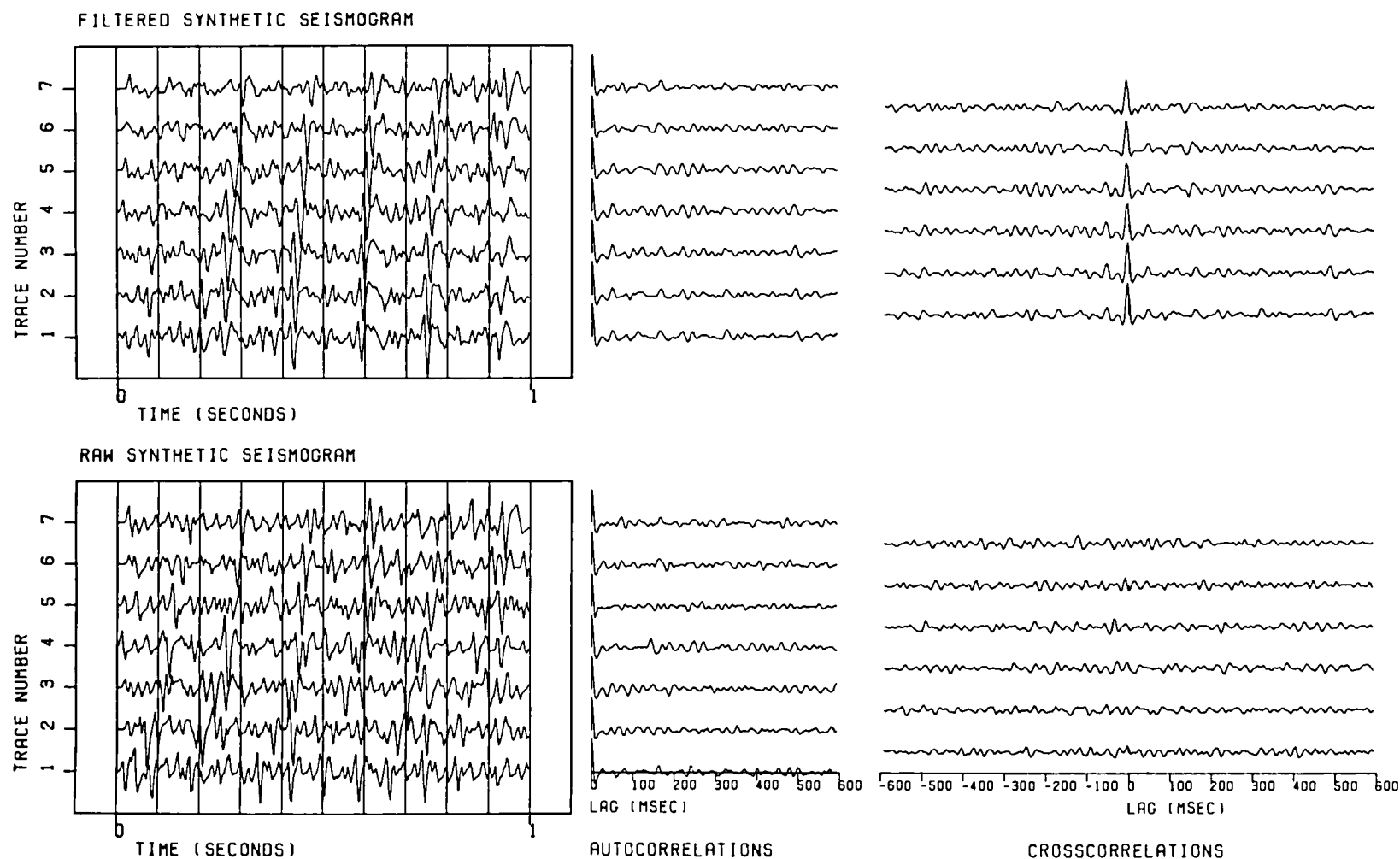


Figure 4.3.7 Raw synthetic seismogram of Figure 4.3.5 with 60 wavelets of random noise added to each trace (lower), and its processed version (upper) by the filter designed over the time gates $g_t = 100$ to 500 ms.

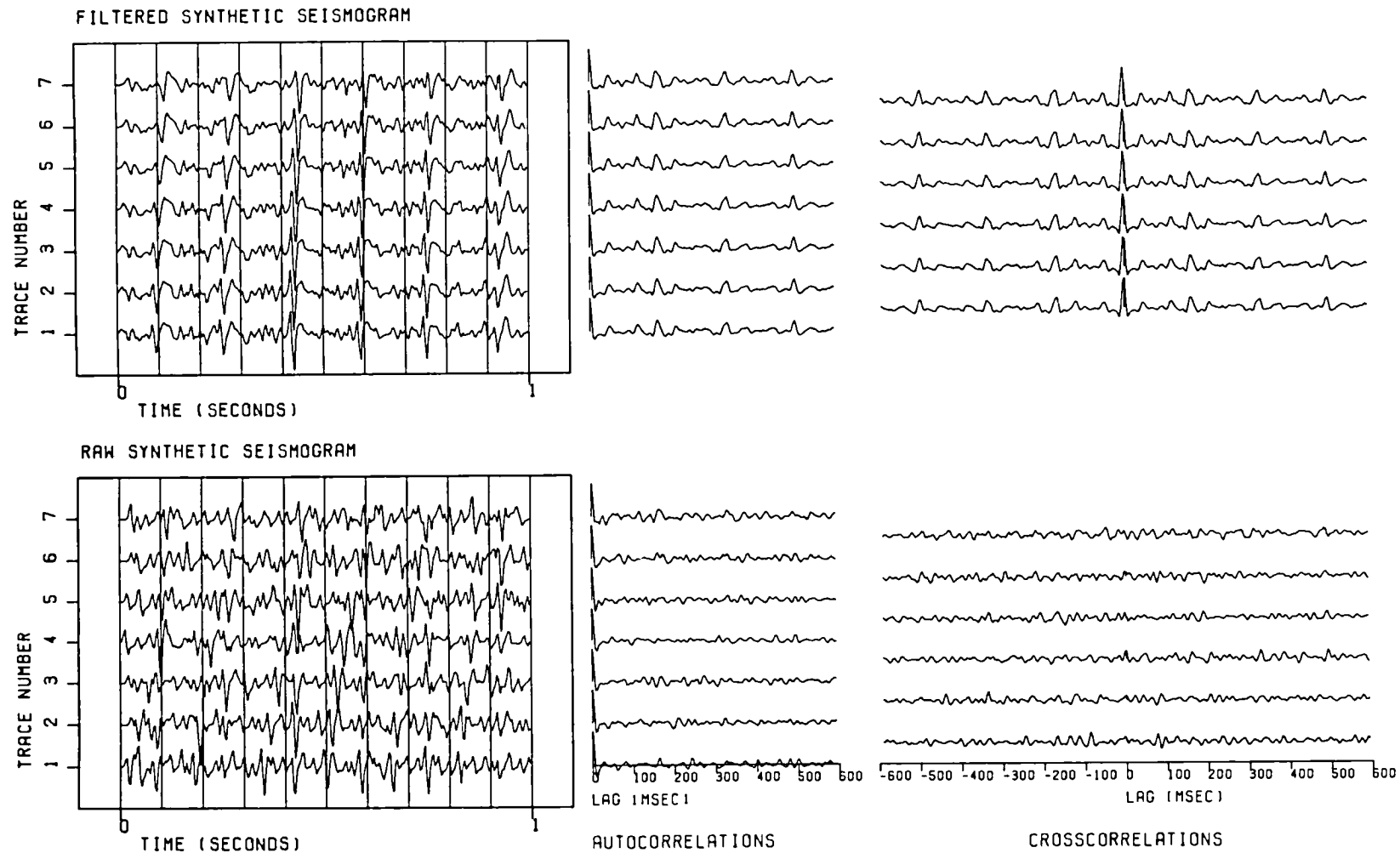


Figure 4.3.8 Raw synthetic seismogram of Figure 4.3.6 with 60 wavelets of random noise added to each trace (lower), and its processed version (upper) by the filter designed over the time gate $g_t=500$ to 900 ms.

hyperbolic moveouts arriving at two-way times greater than the upper limits of the prescribed time gates. There is no appreciable difference between the two outputs displayed. Similar applications showed that this is true even for the time gates as small as 500 ms ($g_t = 1050$ to 1550 ms).

Further applications of hyperbolic moveout filters to simulated and field data showed that for good results very accurate velocity estimations must be made. Especially at low two-way times when a small change in velocity causes large changes in the corresponding moveout boundary. In practice, the results of this chapter may be used in removing anomalous events on reflection seismograms such as ghosts (Lindsey, 1960; Schneider et al., 1964), multiple reflections (Anstey and Newman, 1966; Watson, 1965) and water bottom reverberations (Backus, 1959). These events may have velocities that are very close to signal or primary reflection velocities. In these cases besides very accurate velocity estimation small time gates have to be chosen to avoid pass and reject windows overlapping.

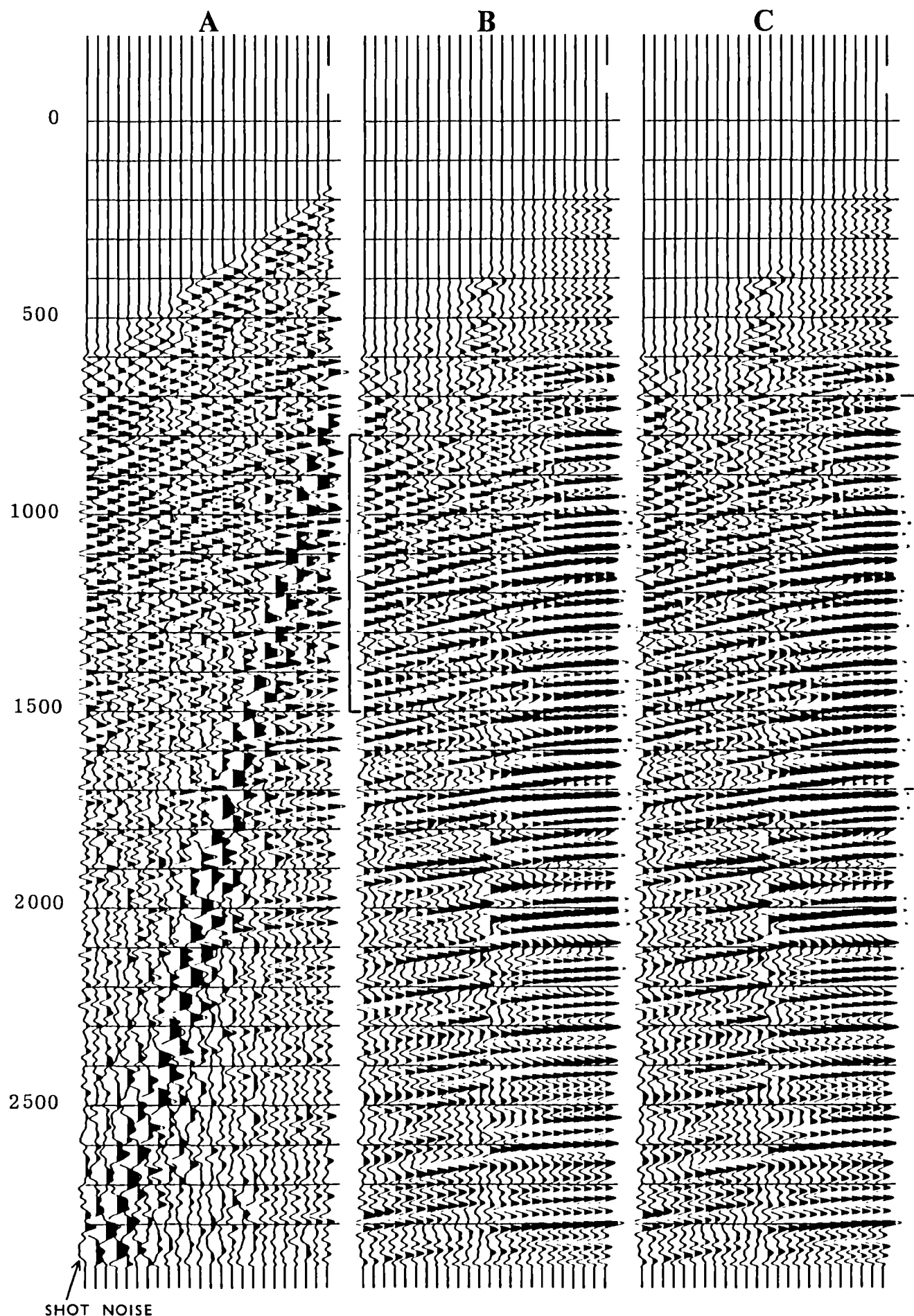


Figure 4.3.9 A) Offshore data with severe correlated noise (marked as shot noise), B) Output of 12x12x13 filter designed over time gate $g_t=800$ to 1500 ms and C) Output of the same filter designed over time gate $g_t=700$ to 1700 ms.

CHAPTER 5

MULTICHANNEL OPERATORS DESIGNED IN THE FREQUENCY DOMAIN BY TCHEBYCHEFF APPROXIMATION

In this chapter a new design technique for two-dimensional (2D) finite impulse response (FIR) operators is introduced with special reference to seismic reflection moveout filters. This technique is based on a one-to-one mapping method from multidimensional space to one-dimensional (1D) space and back, so that powerful 1D filter design techniques can be used for multidimensional approximation. Some design and application examples are given with the corresponding least-squares results for comparison.

The treatment of the least-squares moveout filters in the previous chapters with the already published work show that the wavenumber aliasing problem cannot be overcome adequately using the present techniques. A second undesirable limitation of least-squares techniques is the necessity of having long filter lengths when there are prescribed window(s) with large moveouts in the input model, however high the multiplicity of the input traces. On the other hand, if the desired response of a multi-dimensional operator is known it can be approximated in the frequency domain avoiding both of the above problems. This is because the response will be approximated in the Nyquist range which does not involve an aliased input model.

Secondly, increasing the number of coefficients in any one dimension improves the approximation to the desired response without requiring increase in any other dimension. The third natural reason for attempting to design moveout filters using other techniques is to approximate the desired response more accurately.

It is true that the input model for seismic reflection traces given in Equation 2.2.1 is very flexible in simulating any actual case. Therefore, any design technique using this model will also be a powerful tool in practice. But the design and application results of previous chapters showed that such a model inherits the above mentioned limitations. The aliasing windows bounded by high moveouts cannot be weighted highly, relative to the other windows bounded by lower moveouts. It has also been demonstrated in the previous chapters that the uncorrelated noise parameter is only necessary for the stability of the autocorrelation matrix, otherwise it has undesirable effects in approximating a desired response. The chatter parameter also has undesirable effects and can be omitted from the input model without any loss. Therefore, any design technique having an input model

$$x_i(t) = s(t-\tau_i) + r(t-\tilde{\tau}_i) \quad 5.1$$

will not be inferior to the one given in Equation 2.2.1 for practical purposes, provided that it leads to a stable moveout filter which approximates prescribed pass and reject regions well.

5.1 INTRODUCTION TO FINITE IMPULSE RESPONSE (FIR) DIGITAL FILTER DESIGN TECHNIQUES

Linear filtering can be implemented either numerically or optically. Instruments have been designed for analysis and filtering of seismic reflection data using coherent monochromatic light (Jakson, 1965; and Dobrin et al., 1965). The relevant theory explaining 2D systems is given by Papoulis (1968). It has been reported that optical techniques are limited to an accuracy of 3.0 to 5.0 percent while 0.01 percent is achieved by digital methods (Huang et al., 1971; and Fiasconaro, 1973). Optical processing is limited to the linear operations on images. An up to date coverage with an extensive literature survey is given by Lugt (1974).

The current approach to the design of digital filters is to calculate an impulse response which gives a desired frequency response or at least is a close approximation to it. There are two classes of digital filters defined by the length of their impulse responses. Digital filters with infinite impulse response (IIR), realised by factorising their z-transforms in 1D space, are also known as recursive filters. Difficulties involved with factorizing z-transforms in two-dimensions and relevant stability problems are discussed by Shanks et al. (1972) and Justice and Shanks (1973) and many others. The digital filters considered in this work have finite impulse responses (FIR). They are also called convolution filters or nonrecursive filters.

The design methods of 1D and 2D nonrecursive or FIR digital filters have been discussed extensively in the relevant literature (Rabiner et al., 1975; Mersereau and Dudgeon, 1975). These methods may be grouped into three general categories: 1. Windowing techniques, 2. Frequency sampling techniques and 3. Equiripple approximation techniques. The FIR or nonrecursive filters generally cannot realize an ideal response exactly because of their finite length. The above techniques are used to achieve the best possible approximate result.

The windowing technique uses suitable windows to truncate the impulse response of the ideal filter to avoid overshoots near discontinuities in the ideal frequency response, i.e. to avoid the Gibbs phenomenon (Kaiser, 1966). Helms (1968) showed that the Dolph-Tchebycheff window is optimum when controlling ringing in the frequency response as well as the overshooting at discontinuities. The problem of obtaining good windows in two-dimensions has been solved by Huang (1972) by extending 1D windows to 2D circular symmetric windows. The windowing method is based on calculating the discrete approximate Fourier series coefficients for a much denser grid than the actual number of filter coefficients under the window, which truncates the continuous desired response.

Frequency sampling techniques are implemented by sampling the continuous desired frequency response of the filter at a dense grid of about 16 to 20 times the filter length. The samples in the pass and reject domains are

set to one and zero respectively and the remaining samples in the 'don't care' regions or transition bands are changed to minimize the ripples in the pass and reject bands using an optimization method (Rabiner et al., 1971; and Hu and Rabiner, 1972). Another method presented by Gold and Jordan (1969) uses the direct approach of searching for 'optimum' values of the transition band to reduce the side lobes of the filter response. The filters obtained using the frequency sampling techniques are not the best approximations to the desired responses since only the transition band samples are permitted to vary.

The more recent digital filter design techniques employ Tchebycheff polynomials. If minimizing the maximum absolute value of the difference between the ideal filter response and approximate solution is chosen as the optimization criterion, then neither windowing nor frequency sampling techniques lead to an optimum set of coefficients. On the other hand, the best approximation to a continuous function over a closed interval, in 1D space, is unique with an error function of equiripple behaviour (Cheney, 1966). The FIR digital filter techniques use this fact to design linear and zero phase operators. Herrmann (1970) designed equiripple 1D FIR filters by solving a set of nonlinear equations which constrain the frequency response of the filter to differ from the desired response by a plus or minus constant value at points where the derivative of the frequency response is to be zero. This method is limited to FIR

filters with the number of coefficients less than about 40 samples for computational reasons. Hofstetter et al. (1971) designed linear phase FIR filters by constraining the maximum allowable errors in the pass and stop bands. In both methods the number of ripples in each band has to be chosen beforehand and they fail to approximate the band edges exactly. The 1D techniques mentioned so far have been most conveniently reviewed by Helms (1971) and Rabiner (1971). Among the several techniques presented in the recent literature; Rabiner (1972) demonstrated designing FIR digital filters with linear phase and arbitrary response using linear programming techniques, and Rabiner and Herrmann (1973) designed FIR linear phase filters with an even impulse response.

An efficient method, proposed by Parks and McClellan (1972), for designing optimum FIR linear phase filters make use of the Remes exchange algorithm (Remes, 1957). This method allows one to design very long multiband FIR filters. The response at the band edges are met exactly. The method is briefly outlined in Appendix 3. This algorithm has been used to design 1D FIR filters in this work for the purpose of approximating 2D frequency responses.

5.2 TWO-DIMENSIONAL DISCRETE OPERATORS

The z-transform of a 2D operator with the impulse response $h(n_1, n_2)$ is defined as

$$H(z_1, z_2) = \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} h(n_1, n_2) z_1^{-n_1} z_2^{-n_2} \quad 5.2.1.$$

For the 2D FIR systems the condition $-\infty < n_1, n_2 < \infty$ is always satisfied with

$$\sum_{n_1} \sum_{n_2} |h(n_1, n_2)| < \infty \quad 5.2.2$$

which secures stability.

The impulse response $h(n_1, n_2)$ and the frequency response $H(n_1, n_2)$ are related through the 2D Fourier transform with the change of variables $z_1 = \exp(i\omega_1)$ and $z_2 = \exp(i\omega_2)$ in Equation 5.2.1 as

$$H(e^{i\omega_1}, e^{i\omega_2}) = \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2-1} h(n_1, n_2) \exp[-i(\omega_1 n_1 + \omega_2 n_2)] \quad 5.2.3$$

where $h(n_1, n_2)$ is an $N_1 \times N_2$ operator. The discrete Fourier transform is calculated by evaluating Equation 5.2.3 at equally spaced discrete frequencies from zero to 2π or from $-\pi$ to π . Since the impulse response $h(n_1, n_2)$ has equispaced sampling in both dimensions, the frequency response $H(n_1, n_2)$ is periodic. If the sampling intervals are ΔT_1 and ΔT_2 then the periods will be $\frac{1}{\Delta T_1}$ and $\frac{1}{\Delta T_2}$; and Equation 5.2.3 may be written as

$$H(F_1, F_2) = \Delta T_1 \Delta T_2 \sum_{m_1=-M_1}^{M_1} \sum_{m_2=-M_2}^{M_2} h(m_1 \Delta T_1, m_2 \Delta T_2) \exp[-i2\pi(m_1 \Delta T_1 F_1 + m_2 \Delta T_2 F_2)] \quad 5.2.4$$

where $M_1 = \frac{N_1-1}{2}$ and $M_2 = \frac{N_2-1}{2}$. Taking ΔT_1 and ΔT_2 as unity,

the spectral periods in both dimensions become 1.0. With the minus sign adopted for the second dimension (spatial dimension in this case) in the exponent to represent a travelling wave along a line of sensors, the above equation becomes

$$H(F_1, F_2) = \sum_{m_1=-M_1}^{M_1} \sum_{m_2=-M_2}^{M_2} h(m_1, m_2) \exp[-i2\pi(m_1 F_1 - m_2 F_2)] \quad 5.2.5.$$

The choice of discrete frequencies

$$F_1 = \frac{k_1}{N_1}, \quad k_1 = 0, 1, 2, \dots, N_1 - 1$$

and

$$F_2 = \frac{k_2}{N_2}, \quad k_2 = 0, 1, 2, \dots, N_2 - 1$$

5.2.6

makes the last equation,

$$H(k_1, k_2) = \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2-1} h(n_1, n_2) \exp\left[-i2\pi\left(\frac{k_1 n_1}{N_1} - \frac{k_2 n_2}{N_2}\right)\right] \quad 5.2.7$$

with the inverse Fourier transform:

$$h(n_1, n_2) = \frac{1}{N_1 N_2} \sum_{k_1=0}^{N_1-1} \sum_{k_2=0}^{N_2-1} H(k_1, k_2) \exp\left[i2\pi\left(\frac{k_1 n_1}{N_1} - \frac{k_2 n_2}{N_2}\right)\right] \quad 5.2.8.$$

It is readily seen from the Equations 5.2.7 and 5.2.8 that the impulse response and the frequency response may be complex. On the other hand, the optimization techniques

such as linear programming for computing optimum filter coefficients require that both responses must be real. Fortunately, this problem is overcome if zero phase or linear phase filters are designed. Certain symmetries in the filter response such as

$$H(k_1, k_2) = H(-k_1, -k_2) \quad 5.2.9$$

and

$$\begin{aligned} h(n_1, n_2) &= h(N_1-1-n_1, n_2) \\ &= h(n_1, N_2-1-n_2) \end{aligned} \quad 5.2.10$$

make the impulse response and the frequency response pure real.

5.3 DESIGN OF OPTIMUM TWO-DIMENSIONAL DIGITAL FILTERS

As briefly mentioned in Section 5.1, there are 1D digital nonrecursive filter design techniques, using the second algorithm of Remes, for computing a set of coefficients which are optimum in the sense of Tchebycheff. In particular the method given by McClellan (1973) proved to be the most efficient both from the approximation point of view and computationally. The most logical approach to 2D filter design is to make use of powerful 1D techniques which produce optimal results, namely the use of linear programming and the direct extension of 1D Tchebycheff

technique to 2D field. The design techniques may be examined in two groups; first the direct approach, and second, mapping techniques. In the following sections these techniques will be discussed and a new mapping technique will be presented.

5.3.1 DIRECT APPROACH TO TWO-DIMENSIONAL FILTER DESIGN

Two-dimensional optimum FIR filters can be designed using linear programming techniques by letting all the filter coefficients vary. In practice, the linear phase and phase free filters have certain symmetries which give real coefficients. In this case, the number of independent coefficients computed are much less than the actual number of coefficients. With the increasing number of filter coefficients the computation time required grows even faster and the linear programming technique is limited to the maximum number of coefficients of 9×9 to 13×13 , according to the linear programming software package and the machine used for calculations (Hu and Rabiner, 1972; and Fiasconaro, 1973).

The 1D optimum filter design algorithm given in Appendix 3 enjoys the property that the functions used in the approximation satisfy the Haar condition which guarantees convergence. Multidimensional approximating functions do not have this property and unless special care is taken the algorithm may not converge due to the degeneracies. Kamp and Thiran (1975) introduced a perturbation technique to

obtain convergence and designed circularly symmetric 2D optimum Tchebycheff filters. The technique does not give a unique set of coefficients for all cases. The 2D filters designed using this technique provide closer approximations to the ideal filter responses than both the methods mentioned earlier (the windowing and frequency sampling techniques) and the maximally flat filters designed by the same authors (Kamp and Thiran, 1974). From here on, the optimum will be understood in the sense of Tchebycheff unless otherwise stated.

The error function in 2D Tchebycheff optimization does not show a simple alternation defined by the alternation theorem (see Appendix 3). The exchange algorithm becomes more complicated and time consuming, because there are no simple rules to find a set of points for the next iteration on the way to the final set of values for the optimum solution. Several points in both dimensions have to be exchanged at the same iteration level. The second complication in the extension of 1D Tchebycheff optimization to the 2D domain is that there are equiripple peaks as well as ridges in the prescribed regions of the 2D filter response which makes minimum and maximum detection in 2D space cumbersome and highly time consuming. A further time consuming aspect of 2D Tchebycheff filter design is the non-existence of Lagrange interpolation formula for the bidimensional reference function.

The above points show that the direct design of optimum multidimensional filters is computationally too costly, and furthermore it is not always possible to obtain

a unique set of coefficients for all desired responses required to be approximated. Almost all the techniques mentioned so far are directed towards the design of circularly symmetric digital filters (low-pass, bandpass, high-pass etc.) which have relatively simple responses. Figure 5.3.1.1 shows the Nyquist region of a circularly symmetric 2D filter (A) with the linear moveout filters having total symmetry (B) and central symmetry (C). The vertically hatched areas are the prescribed pass and/or reject regions. The horizontal hatching shows one of the possible areas of approximation which is sufficient for obtaining the whole response using symmetry. It is clear from the figure that, the number of coefficients used in the

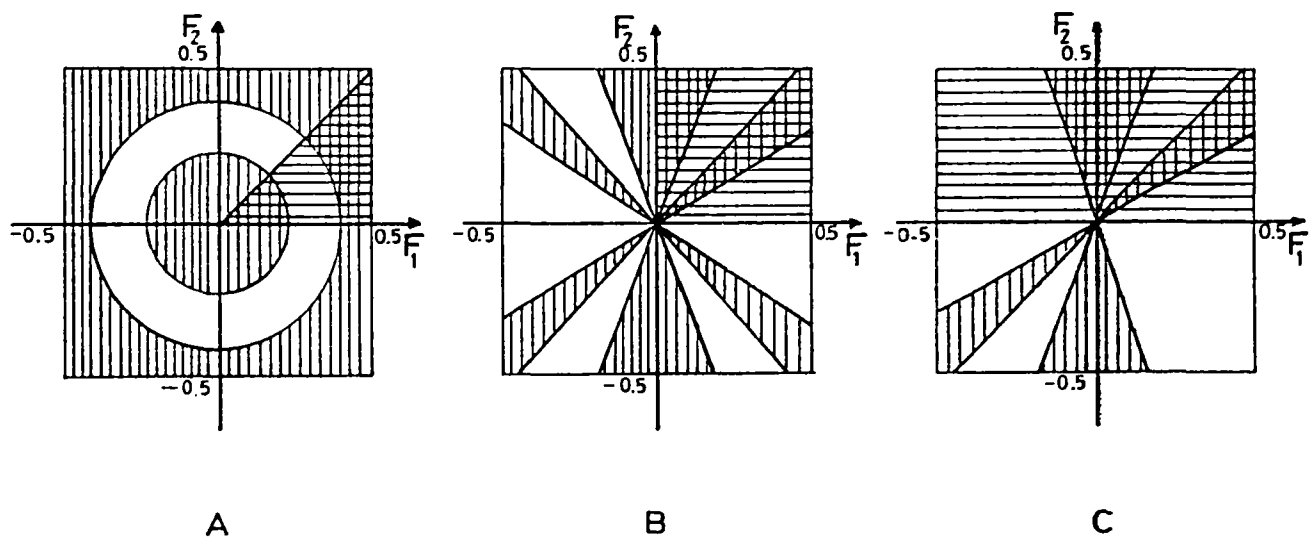


Figure 5.3.1.1 Nyquist regions of two-dimensional circularly symmetric and linear moveout operators. A) circularly symmetric, B) symmetric linear moveout and C) centrosymmetric linear moveout filter response.

approximation is doubled for the symmetric linear moveout filters and quadrupled in the case of centrosymmetric linear moveout filters leading to greater computational costs over circularly symmetric filters. Furthermore, the frequency response of a moveout filter is more complicated than that of circularly symmetric filters, especially around the origin, which will make the 2D search for extremes during the iteration very difficult. Consequently no attempt has been made in this study to design optimum moveout filters using the direct approaches mentioned. For further information on the theoretical and practical difficulties for designing 2D optimum filters, the reader is referred to Mersereau and Dudgeon (1975).

5.3.2 TWO-DIMENSIONAL FILTER DESIGN USING MAPPING TECHNIQUES

The theoretical, as well as practical (computational costs and computer memory) constraints imposed on the direct design of optimum multidimensional operators made it necessary to search for other means. McClellan and Parks (1972) extended their 1D min-max or Tchebycheff technique to the design of fan-filters using multistage mapping between 1D and 2D domains. McClellan (1973) applied the same method to the design of circularly symmetric filters. The pie-slice or fan-filters (Fail and Grau, 1963; Embree et al., 1963; Treitel et al., 1967) can only be implemented to pass or reject correlated events falling into a given moveout range and are inferior to the optimum least-squares filters discussed

in detail in the opening chapters of this study. More important, the line-ups of correlated noise, with moveouts close to the signal moveout range, will be more severe if the fan-filters are used for processing seismic reflection data. Furthermore, McClellan's method is constrained to the 2D filters with an equal number of coefficients in both dimensions which is not a desirable restriction especially for seismic applications.

Efforts to extend the above mentioned technique to the design of optimum linear moveout filters even with one pass and one reject region have been fruitless. This is because a line passing through the origin of the frequency response of a two-dimensional operator is not uniquely mapped to a region on the 1D frequency axis if McClellan's approach is used, i.e. it is not possible to map every fan-shaped region of 2D frequency domain onto separate nonoverlapping segments of the 1D counterpart.

Another attempt has been made to map the triangular and polygonal prescribed windows of 2D frequency-wave number response onto 1D frequency axis using conformal mapping techniques which were then found to be computationally impractical. First, simple linear transformations were sought then a more elaborate Schwarz-Cristoffel technique was tried. Available mapping functions which transfer two domains into each other (Nehari, 1952; and Moretti, 1964) involve solution of computationally costly numerical integrations.

The new mapping technique presented next, will overcome the problems mentioned above. The idea stems from the

fact that a multidimensional array stored by a computer is in a 1D form, i.e. stored columnwise. Therefore, if the subscripts of a multidimensional array are changed to map to a 1D array and an account of it is kept, then it is possible to map back by reversing the procedure. This method will be called the 'coefficient mapping' technique.

Amongst others, the simple transform

$$b(N_2 n_1 + n_2) = h(n_1, n_2) \quad 5.3.2.1$$

where $0 < n_1 < N_1 - 1$ and $0 < n_2 < N_2 - 1$, will map the 2D array $h(n_1, n_2)$ to the 1D array $b(\ell)$. If $h(n_1, n_2)$ is a finite sequence, $b(\ell)$ will be a finite sequence as well, and the mapping is reversible.

If the sequence $h(n_1, n_2)$ is the impulse response of a 2D operator, its frequency response $H(k_1, k_2)$ should be obtainable from the 1D sequence $b(N_2 n_1 + n_2)$ or from its Fourier transform. The z-transforms,

$$B(z) = \sum_{\ell=0}^{N_1 N_2 - 1} b(\ell) z^{-\ell} \quad 5.3.2.2$$

and

$$H(z_1, z_2) = \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2-1} h(n_1, n_2) z_1^{-n_1} z_2^{-n_2} \quad 5.3.2.3$$

of the two domains is mapped as

$$B(z) = \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2-1} h(n_1, n_2) z^{-(N_2 n_1 + n_2)} \quad 5.3.2.4$$

hence

$$B(z) = H(z^{N_2}, z) \quad 5.3.2.5.$$

If $b(N_2 n_1 + n_2)$ is introduced into Equation 5.3.2.4 instead of $h(n_1, n_2)$ with the F_1 and F_2 values given in Equation 5.2.6:

$$H(k_1, k_2) = \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2-1} b(N_2 n_1 + n_2) \exp \left[-i 2\pi \left(\frac{k_1 n_1}{N_1} - \frac{k_2 n_2}{N_2} \right) \right] \quad 5.3.2.6.$$

If the Fourier transform of $b(N_2 n_1 + n_2)$ is $B(\omega)$,

$$b(N_2 n_1 + n_2) = \frac{1}{2\pi} \int_{-\pi}^{\pi} B(\omega) \exp[i\omega(N_2 n_1 + n_2)] d\omega \quad 5.3.2.7$$

or in discrete form,

$$b(N_2 n_1 + n_2) = \frac{1}{N_1 N_2} \sum_{m=0}^{N_1 N_2} B(m) \exp \left[i \frac{2\pi}{N_1 N_2} m (N_2 n_1 + n_2) \right] \quad 5.3.2.8.$$

Substituting the above value of $b(N_2 n_1 + n_2)$ into Equation 5.3.2.6 gives

$$H(k_1, k_2) = \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2-1} \left\{ \frac{1}{N_1 N_2} \sum_{m=0}^{N_1 N_2} B(m) \exp \left[i \frac{2\pi}{N_1 N_2} m (N_2 n_1 + n_2) \right] \exp \left[-i 2\pi \left(\frac{k_1 n_1}{N_1} - \frac{k_2 n_2}{N_2} \right) \right] \right\} \quad 5.3.2.9.$$

By Fourier transforming the last equation, the impulse response $h(n_1, n_2)$ is obtained as,

$$h(n_1, n_2) = \frac{1}{N_1 N_2} \sum_{k_1=0}^{N_1-1} \sum_{k_2=0}^{N_2-1} H(k_1, k_2) \exp \left[i 2\pi \left(\frac{k_1 n_1}{N_1} - \frac{k_2 n_2}{N_2} \right) \right] \quad 5.3.2.10.$$

Therefore, once $b(N_2 n_1 + n_2)$ or $B(m)$ is approximated using a 1D FIR algorithm, 2D coefficients can be obtained by means of the last two equations.

The mapping technique independently developed above is similar to that already published work of Mersereau and Dudgeon (1974). The latter work refers to the function $B(z) = H(z^{N_2}, z)$ as a slice of $H(z_1, z_2)$ and $b(\ell)$ as a projection of $h(n_1, n_2)$. In the frequency domain, Equation 5.3.2.5 becomes

$$B(e^{i\omega}) = H(e^{iN_2\omega}, e^{i\omega}) \quad 5.3.2.11.$$

$H(e^{iN_2\omega}, e^{i\omega})$ is a doubly periodic function with period 2π in both dimensions. $B(e^{i\omega})$ is equivalent to $H(e^{iN_2\omega}, e^{i\omega})$ evaluated along a series of N_2 parallel lines in 2D frequency domain. In Figure 5.3.2.1 the 2D Fourier plane with superimposed parallel lines along which numerical values of $B(e^{i\omega})$ are equal to $H(e^{i\omega_1}, e^{i\omega_2})$, and a typical linear moveout filter frequency response to be approximated are shown. It is immediately clear from this figure that if the method of Mersereau and Dudgeon is used, the design of a linear moveout filter even with two prescribed regions amounts to designing a 1D filter with several pass and reject regions. Especially, the mapping of the area around the origin of 2D frequency response will result in too many very narrow pass

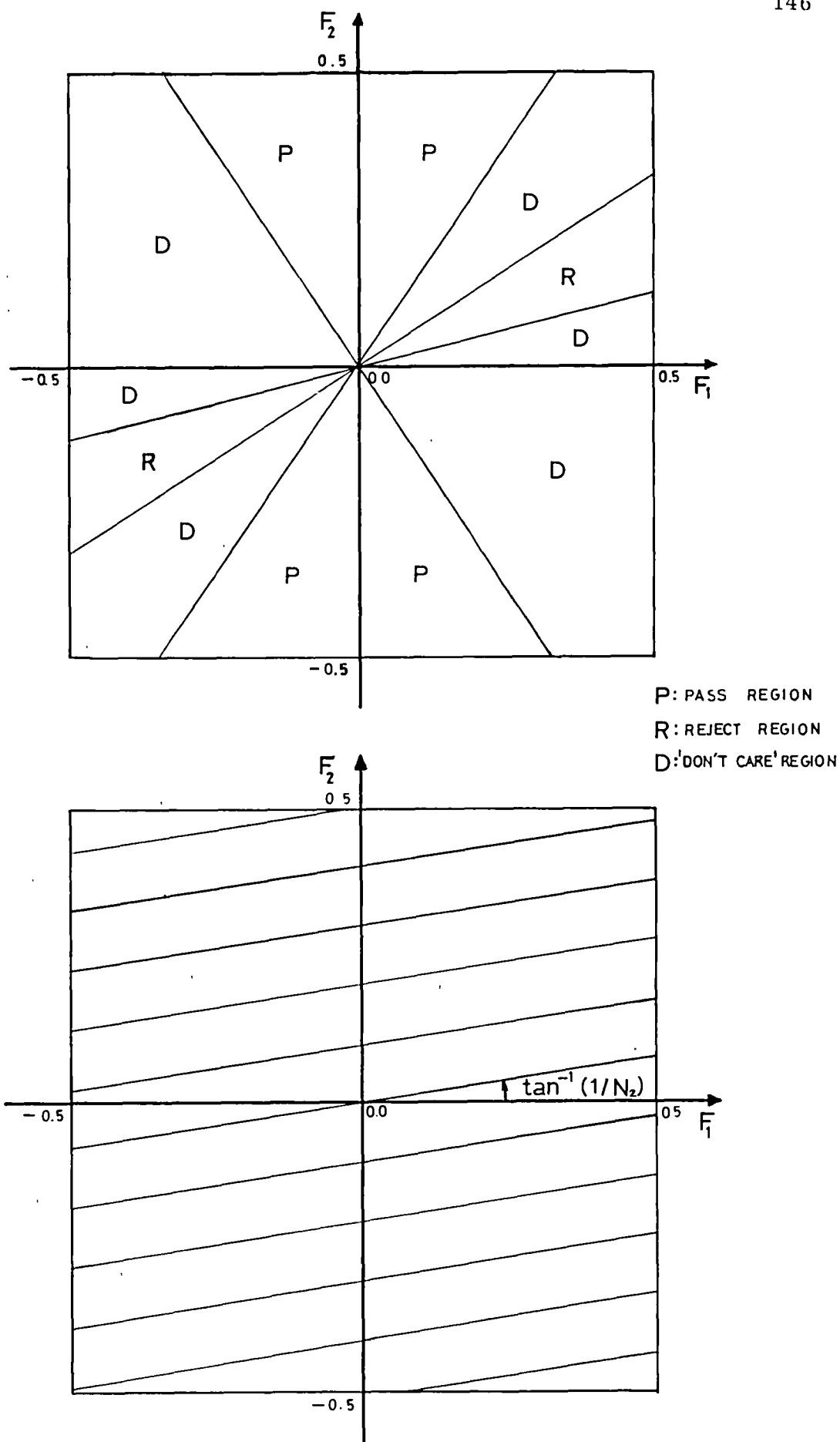


Figure 5.3.2.1 The 2D Fourier plane with superimposed parallel lines along which numerical values of $B(\exp i2\pi F)$ are equal to $H(\exp i2\pi F_1, \exp i2\pi F_2)$ (lower), and a typical linear moveout ideal filter response (upper).

and reject domains in the corresponding 1D frequency axis.

The new technique proposed has the advantage that there are an equal number of prescribed regions and transition regions in 1D frequency domain as there are in the corresponding 2D frequency domain. This is obtained by the sorting of 2D frequency domain coordinates for discrete coefficients so that the coefficients falling into the prescribed regions are brought together. A point in the 2D frequency or Fourier domain may be identified by the vector connecting the origin to this point. Let the length of the vector be denoted by R and the angle it makes with F_1 axis by A (Figure 5.3.2.2). From Figures 5.3.2.2 and 5.3.2.3, the coordinates of all the coefficients in 2D frequency domain are defined as the 2D functions,

$$R(k_1, k_2) = \left[\left(k_1 \frac{2\pi}{N_1-1} \right)^2 + \left(k_2 \frac{2\pi}{N_2-1} \right)^2 \right]^{\frac{1}{2}} \quad 5.3.2.12$$

and

$$A(k_1, k_2) = \tan^{-1} \left(\frac{k_2}{k_1} \frac{N_1-1}{N_2-1} \right) \quad 5.3.2.13$$

where $k_1 = 0, 1, \dots, N_1-1$, $k_2 = 0, 1, \dots, N_2-1$, and N_1 and N_2 are odd integers. With these new coordinates the angular frequency sample values in 2D Fourier domain are

$$\omega_1(k_1, k_2) = R(k_1, k_2) \cos A(k_1, k_2) \quad 5.3.2.14$$

$$\omega_2(k_1, k_2) = R(k_1, k_2) \sin A(k_1, k_2) \quad 5.3.2.15.$$

Putting the above sample values into Equation 5.2.3:

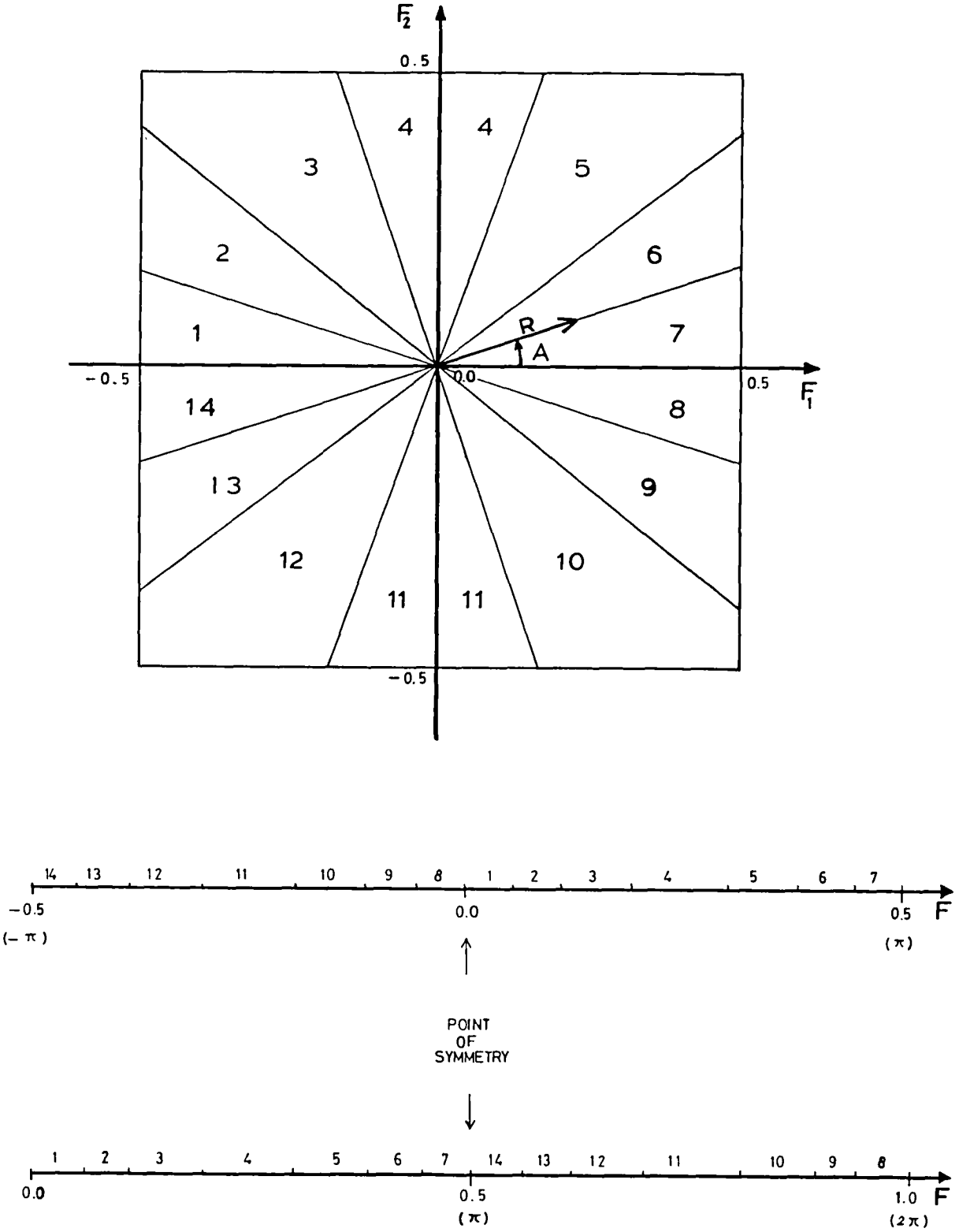


Figure 5.3.2.2 A linear moveout filter ideal response and its 1D frequency response mappings.

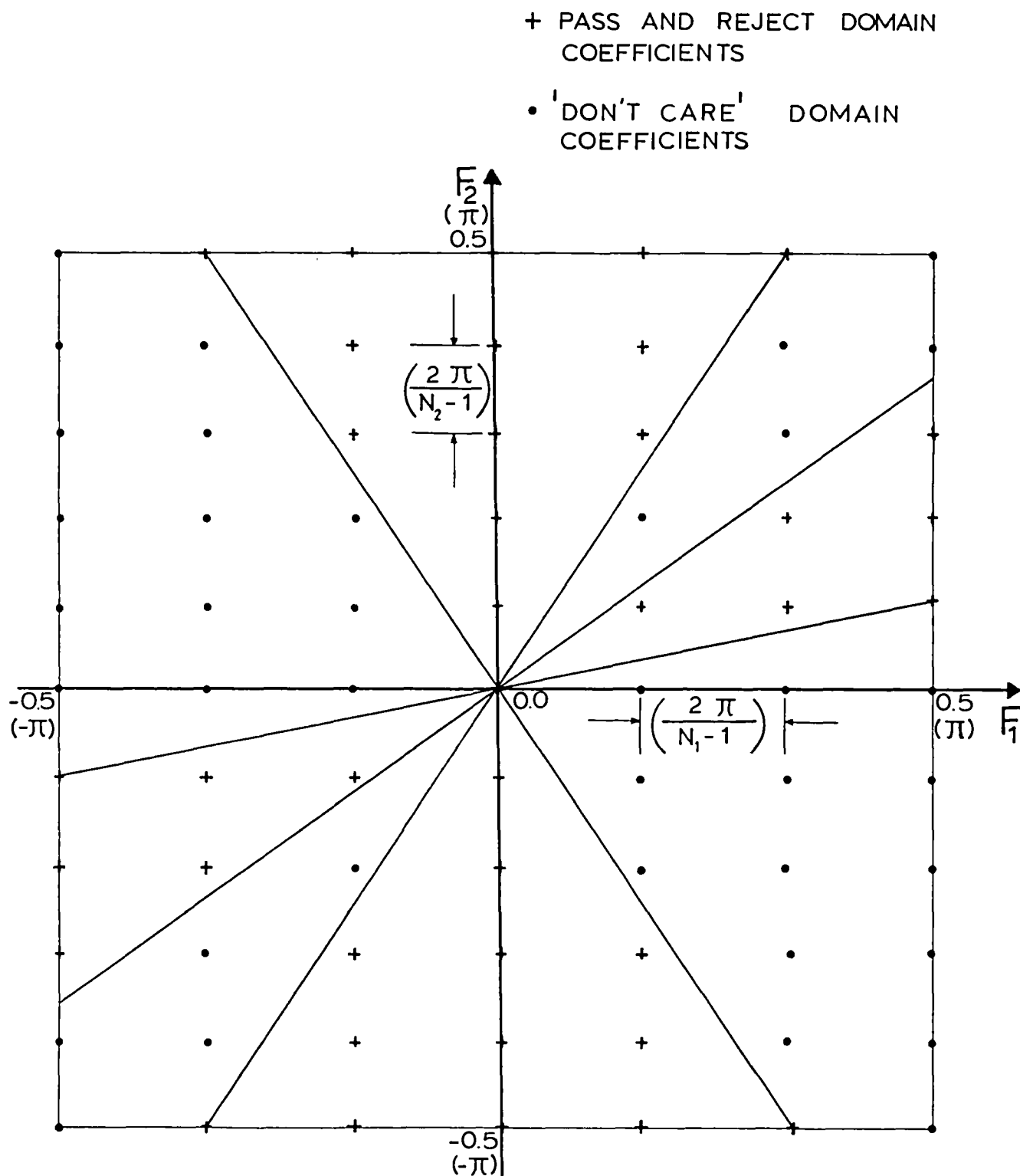


Figure 5.3.2.3 A typical linear moveout filter ideal response with 7x11 discrete Fourier transform sample locations (coefficients).

$$H(k_1, k_2) = H(R, A) = \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2-1} h(n_1, n_2) \exp \left\{ -i [n_1 R(k_1, k_2) \cos A(k_1, k_2)] \right\} \\ \exp \left\{ i [n_2 R(k_1, k_2) \sin A(k_1, k_2)] \right\}$$

5.3.2.16.

If $H(k_1, k_2)$ is written in terms of its corresponding 1D frequency response, as in Equation 5.3.2.9,

$$H(k_1, k_2) = \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2-1} \left\{ \frac{1}{N_1 N_2} \sum_{m=0}^{N_1 N_2} B(m) \exp \left[i \frac{2\pi}{N_1 N_2} m (N_2 n_1 + n_2) \right] \right\} D(n_1, n_2, R, A)$$

5.3.2.17

where

$$D(n_1, n_2, R, A) = \exp \left\{ -i [n_1 R(k_1, k_2) \cos A(k_1, k_2)] \right\} \\ \exp \left\{ i [n_2 R(k_1, k_2) \sin A(k_1, k_2)] \right\}$$

5.3.2.18.

Equation 5.3.2.9 and 5.3.2.16 show that if the order of the 2D frequency domain coefficients in the sequence $H(k_1, k_2)$ are changed with corresponding changes in $B(\ell)$, $R(k_1, k_2)$ and $A(k_1, k_2)$, these equations remain intact. In other words, if the order of coefficients in the 1D array $B(\ell)$ is changed, together with $R(k_1, k_2)$ and $A(k_1, k_2)$, the order of coefficients in the 2D array $H(k_1, k_2)$ is changed accordingly. If the values of the 2D sequences $R(k_1, k_2)$ and $A(k_1, k_2)$ are interchanged according to a criterion, $H(k_1, k_2)$ will change to another 2D sequence, $\tilde{H}(k_1, k_2)$. Equation 5.3.2.17 will become

$$\tilde{H}(k_1, k_2) = \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2-1} \left\{ \frac{1}{N_1 N_2} \sum_{m=0}^{N_1 N_2} \tilde{B}(m) \exp \left[i \frac{2\pi}{N_1 N_2} m (N_2 n_1 + n_2) \right] \right\} \tilde{D}(n_1, n_2, \tilde{R}, \tilde{A})$$

5.3.2.19

where $\tilde{D}(n_1, n_2, \tilde{R}, \tilde{A})$ is the same as $D(n_1, n_2, R, A)$ given in Equation 5.3.2.18 except $\tilde{R}(k_1, k_2)$ and $\tilde{A}(k_1, k_2)$ each have coefficients rearranged within them. In terms of set theory,

$$\begin{aligned} \{R(k_1, k_2)\} &= \{\tilde{R}(k_1, k_2)\} \\ \{A(k_1, k_2)\} &= \{\tilde{A}(k_1, k_2)\} \\ \{D(n_1, n_2, R, A)\} &= \{\tilde{D}(n_1, n_2, \tilde{R}, \tilde{A})\} \\ \{H(k_1, k_2)\} &= \{\tilde{H}(k_1, k_2)\} \end{aligned}$$

5.3.2.20

where the braces, $\{\cdot\}$, are used to describe a set.

Instead of mapping the impulse response coefficients, let the frequency domain coefficients be mapped in the same way as in Equation 5.3.2.1, namely

$$B(N_2 k_1 + k_2) = H(k_1, k_2)$$

5.3.2.21.

The 2D impulse response will be

$$h(n_1, n_2) = \frac{1}{N_1 N_2} \sum_{k_1=0}^{N_1-1} \sum_{k_2=0}^{N_2-1} B(N_2 k_1 + k_2) \exp \left[-i 2\pi \left(\frac{k_1 n_1}{N_1} - \frac{k_2 n_2}{N_2} \right) \right]$$

5.3.2.22.

It is readily seen that the last equation is similar to Equation 5.3.2.10, which can be written as:

$$h(n_1, n_2) = \frac{1}{N_1 N_2} \sum_{k_1=0}^{N_1-1} \left[\sum_{k_2=0}^{N_2-1} H(k_1, k_2) \exp \left(i 2\pi \frac{k_2 n_2}{N_2} \right) \right] \exp \left(-i 2\pi \frac{n_1 k_1}{N_1} \right)$$

5.3.2.23.

The inner summation is a 1D Fourier transform where k_1 is held fixed. Incidentally, a 2D discrete Fourier transform of a sequence can be performed in 1D domain by operating first, on rows, then columns or vice-versa.

Once the 2D desired response is prescribed, the mapping given in Equation 5.3.2.21 completely defines the corresponding 1D response. If the approximation is done in 1D domain $H(k_1, k_2)$ can be recovered from it by means of reverse mapping. The impulse response is obtained either from Equation 5.3.2.10 or 5.3.2.22. If the above straight 'coefficient mapping' is employed, the number of prescribed regions in 1D will be too high as it was in the case of mapping coefficients in the time domain. For the linear moveout filters Equation 5.3.2.13 may be used to order the coefficients according to the angle they make with the F_1 and F_2 axes of 2D Fourier domain (see Figures 5.3.2.3 and 5.3.2.4). The mapping of $A(k_1, k_2)$ values to a corresponding 1D domain is done in the same way as given in Equations 5.3.2.1 and 5.3.2.21, namely

$$\tilde{A}(N_2 k_1 + k_2) = A(k_1, k_2) \quad 5.3.2.24.$$

The following procedure has been programmed to calculate the impulse response of a linear moveout filter from its 1D projection.

1. Calculate $A(k_1, k_2)$ values for a half plane of (F_1, F_2) domain.
2. Map $A(k_1, k_2)$ into a 1D array $\tilde{A}(m)$ using Equation 5.3.2.24.

3. Put array $\tilde{A}(m) = \tilde{A}(N_2 k_1 + k_2)$ in decreasing order and save them in another 1D array $\hat{A}(m)$ while keeping a counting array of indices to be able to map $\hat{A}(m)$ back into $\tilde{A}(m)$.
4. Calculate the linear moveout boundaries in the same half plane of the (F_1, F_2) domain and repeat the above steps 2 and 3 to find cut-off angles ordered in the same sense as the frequency domain coordinates.
5. Compare moveout edge values calculated in step 4 above, with the ordered coordinates of discrete frequency domain coefficients calculated in step 3 bearing in mind that all points in array $\hat{A}(m)$ map to the 1D axis F where $0 \leq F \leq 0.5$.
6. Compute 1D frequency response using the edge values calculated in step 5 with a suitable 1D algorithm.
7. Re-order the 1D frequency response calculated above, using the array of indices calculated in step 3.
8. Map 1D frequency response into 2D frequency response using Equation 5.3.2.21. Calculate 2D impulse response, $h(n_1, n_2)$, if required. $H(n_1, n_2)$ can also be calculated from $B(N_2 k_1 + k_2)$ directly using Equation 5.3.2.22.

Figure 5.3.2.2 shows how a 2D frequency response is mapped to a 1D frequency axis if the above steps are followed. A brief description of the computer program for computing moveout filters using the above technique is given in Appendix 4.

5.4 PRACTICAL CONSIDERATIONS IN DESIGNING TWO-DIMENSIONAL FILTERS USING COEFFICIENT MAPPING TECHNIQUES

It was pointed out at the end of Section 5.2 that optimization techniques can only be used if the filter frequency response is real. Consequently, only linear phase and phase free digital FIR filters can be designed using these techniques. Let b_m , $m = 0, 1, \dots, M-1$, be the impulse response of a causal 1D FIR digital filter. If certain symmetries are imposed, the filter frequency response will have linear phase terms attached to its purely real coefficients. This is when

$$b_m = b_{M-1-m} \quad 5.4.1$$

the filter frequency response

$$B(\exp i 2 \pi F \Delta T) = \sum_{m=0}^{M-1} b_m \exp(-i 2 \pi F \Delta T m) \quad 5.4.2$$

will become, for the case when M is odd,

$$B(\exp i 2 \pi F \Delta T) = \exp[-i 2 \pi F \Delta T (\frac{M-1}{2})]$$

linear phase-term

$$\left\{ b_{(M-1)/2} + 2 \sum_{m=0}^{(M-3)/2} b_m \cos[2 \pi F \Delta T (\frac{M-1}{2} - m)] \right\}$$

purely real frequency domain coefficients

5.4.3.

When the sampling interval ΔT is taken as unity, the frequency

response is simplified to

$$\hat{B}(F) = \exp[-i2\pi F(M-1)] \sum_{m=0}^{M-1} d_m \cos(2\pi mF) \quad 5.4.4$$

where F is the normalized frequency variable and

$$d_{M-2-m} = 2b_m, \quad m = 0, 1, \dots, M-2 \text{ and } d_0 = b_{M-1} \quad 5.4.5.$$

The design problem in the frequency domain is to find either b_m or d_m . For the filters with an even number of coefficients similar results are obtained (Rabiner, 1972), see also Appendix 3.

In the same way if certain symmetries are maintained in the responses of 2D FIR filters, such as in Equations 5.2.9 and 5.2.10, the phase free and linear phase operators are obtained. If $h(n_1, n_2)$ is real, and symmetrical with respect to the origin, its spectrum $H(x_1, x_2)$ is real (Grau, 1966),

$$\begin{aligned} H(x_1, x_2) = & \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(n_1, n_2) \cos[2\pi(n_1 x_1 + n_2 x_2)] dn_1 dn_2 \\ & - i \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(n_1, n_2) \sin[2\pi(n_1 x_1 + n_2 x_2)] dn_1 dn_2 \end{aligned} \quad 5.4.6$$

where $i = \sqrt{-1}$.

If n_1 and n_2 are changed to $-n_1$ and $-n_2$, respectively, the quantity in the second double integral changes the sign, i.e. the second integral is zero. Therefore, for centrosymmetric

2D FIR filters the frequency response may be written as

$$H(k_1, k_2) = \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2-1} h(n_1, n_2) \cos \left[2\pi \left(\frac{k_1 n_1}{N_1} - \frac{k_2 n_2}{N_2} \right) \right] \quad 5.4.7$$

which is pure real. In the case of 2D FIR filters with an even number of coefficients in both dimensions, the frequency response is purely real, except with a linear phase term (Hu and Rabiner, 1972). With the normalized frequencies, the last equation becomes,

$$H(F_1, F_2) = \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2-1} h(n_1, n_2) \cos [2\pi(n_1 F_1 - n_2 F_2)] \quad 5.4.8$$

for $0 < F_1, F_2 \leq 1$, and

$$H(0, 0) = \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2-1} h(n_1, n_2) \quad 5.4.9.$$

All centrosymmetric linear moveout filters have pure real centrosymmetric frequency responses given in the last two equations.

Another practical problem arises when the 1D Tchebycheff algorithm of McClellan (1973) is used in conjunction with the coefficient mapping technique for designing 2D FIR filters with more than two prescribed regions. When the latter algorithm was used as part of the coefficient mapping technique to design linear moveout and circularly symmetric filters with greater than two prescribed regions, it was observed that the 1D frequency response approximation did

not always show monotonic behaviour in the transition or 'don't care' bands. Unless suitable weights are chosen for pass and reject bands, overshooting and undershooting near the band edge frequencies are unavoidable.

The problem of computing FIR digital filters with acceptable transition regions has been discussed by Rabiner et al. (1974) by designing multiband filters using McClellan et al. (1973) algorithm. In the present work the empirical equation of Herrmann et al. (1973) has been used for the calculation of optimum weights. If a multiband 1D FIR frequency response is treated as a sequence of high-pass or low-pass filters, the filter length (L) required to meet prescribed pass and reject region ripple sizes, can be obtained from the equation

$$L = \frac{D_{\infty}(\delta_1, \delta_2) - f(K)(F_s - F_p)^2}{(F_s - F_p)} + 1 \quad 5.4.10$$

where F_p and F_s are the passband and stopband cut-off frequencies for the low-pass filter treated in the sequence, respectively. δ_1 and δ_2 are the passband and stopband allowed ripples, respectively. The other parameters are defined as

$$K = \frac{\delta_1}{\delta_2},$$

$$D_{\infty} = [5.309 \times 10^{-3} (\log_{10} \delta_1)^2 + 7.114 \times 10^{-2} (\log_{10} \delta_1) - 0.4761] \log_{10} \delta_2 \\ - [2.66 \times 10^{-3} (\log_{10} \delta_1)^2 + 0.5941 (\log_{10} \delta_1) + 0.4778]$$

and

$$f(K) = 0.51244 \log_{10} K + 11.01217.$$

If $\delta_1 \leq 0.1$ and $\delta_2 \leq 0.1$, the above equations are valid within 1.3 percent relative error in L . Also it is assumed that the first band begins at the normalized frequency $F = 0.0$ and the last band ends at $F = 0.5$. Equation 5.4.10 shows that, if the multiband 1D filter response is divided into M low-pass filter sequences and (Rabiner et al., 1974),

$$L_1 \approx L_2 \approx \dots \approx L_M \quad 5.4.11$$

then acceptable transition bands are guaranteed without overshooting at the band edges. There are two ways of achieving the approximate equalities given in the last equation. First, by changing the widths of the transition bands, and second by changing the values of δ_1 and δ_2 relative to each other. The latter is equivalent to the changing the relative weights of the pass and reject regions, i.e. w_p and w_r .

In practice, F_p and F_s values are fixed by the corresponding 2D frequency domain cut-off edges (moveouts in seismic applications) and δ_1 and δ_2 are relatively changed to achieve the condition imposed in the Equation 5.4.11. Values of L and/or $F_s - F_p$ which are too big cause pass and reject domains ripples to be too small (smaller than about

10^{-8}). This makes the design algorithm lose accuracy depending on the computer used (computer word length dependent). Secondly, Equation 5.4.10 is not very accurate when ripples (δ_1 and δ_2) are much smaller than 10^{-6} . In these cases either the filter length is reduced or the transition widths, $F_s - F_p$, are made narrower. The foregoing remarks explain that there is a restriction on the length of the linear moveout FIR filter for a given moveout window configuration. This causes no problem in practical applications because it is always desirable to have short length filters which achieve sufficient accuracy in approximating a desired response.

The coefficient mapping technique algorithm can easily be adjusted to map certain coefficients of the 2D frequency response to a desired domain of 1D frequency axis. In the next section it will be shown that this is very useful especially in mapping the coefficient at the origin of the (f,k) response for filters with an odd number of coefficients.

5.5 DESIGNS AND APPLICATIONS OF THE TCHEBYCHEFF AND THE LEAST-SQUARES MOVEOUT FILTERS

In this section some design and application examples of linear moveout operators, designed using the least-squares technique and the coefficient mapping technique introduced in the previous section, are presented. The filters designed in the frequency domain via coefficient mapping will

be called Tchebycheff filters from here on because the 1D approximations of 2D responses are computed in min-max or Tchebycheff sense using the previously mentioned McClellan algorithm. This 1D algorithm assumes that the first band begins at $F = 0.0$ and the last band ends at $F = 0.5$. Due to this assumption, Tchebycheff linear moveout filters designed in this section have their reject window upper limit extended to infinity. The other 1D filter design techniques should be used if this is not suitable for the data to be processed. In seismic reflection applications, the rejection of all the noise energy, after certain moveouts, is desirable. In fact, the energy arriving with greater than about three to four samples per trace moveout transform into a very narrow triangular region of the (f,k) domain near to the wave number axis. If this area is left to be a 'don't care' band, there will still be about 6 to 12 dB or more deviation. This is always the case for the least-squares designs. In the given examples the reject window(s) for the least-squares filters will be kept as narrow as possible to avoid extra aliasing into the pass domain. Incidentally, this is a limitation of the least squares method. If passing or rejecting of events falling in large linear moveout windows having boundaries from very low to very high moveouts is required, aliasing will cause severe problems if the least-squares method is used for the filter design.

As explained in the previous section, if McClellan's algorithm is used for 1D filter design, too many coefficients

in the transition or 'don't care' bands require weights for the prescribed regions that are too small to avoid overshoots at the band edges. Consequently, even if only centrosymmetric linear moveout filters are required, the symmetric linear moveout filters are designed. In due course it will be seen that the filters designed with these added regions are not inferior to the centrosymmetric filters.

The first example is a 5×17 linear moveout filter. Figure 5.5.1A and B show the (f,k) responses of the centrosymmetric and symmetric least square designs, respectively. The centrosymmetric filter does not have the reject window for negative moveouts. The isometric plot given in this figure and the remaining examples always belongs to the top (f,k) response. The viewing angle for the isometric plots presented in this section is always from the origin of the (f,k) response.

It was pointed out at the end of the previous section that the coefficient mapping technique allows the designer to map the discrete Fourier coefficients of 2D filter response into a desired region of the 1D frequency axis. Table 5.5.1 shows the origin mappings for the Tchebycheff design examples of the same 5×17 symmetric filter. It includes pass and reject domain weightings for Tchebycheff as well as the above two corresponding least-squares designs. The pass and reject band weights are for the 1D frequency domain design corresponding to the 2D (f,k) response. The upper indices always increase from negative to positive moveout windows. In Table 5.5.1, $w_r^{(1)}$ denotes the weight

The moveout window configuration for 5x17 and 5x11 least-squares and Tchebycheff symmetric filters:

Pass moveout window (time unit/trace), $0_s = -0.625$ to 0.625
Reject moveout windows (time unit/trace), $0_r^{(1)} = -4.0$ to -2.5
 $0_r^{(2)} = 2.5$ to 4.0

The least-squares centrosymmetric moveout filters have only the second ($0_r^{(2)}$) reject region given above (Figures 5.5.1A and 5.5.4A).

The 5x17 least-squares moveout filters have the additional input parameters:

Delay index,	$t_d = 9$
Uncorrelated noise weight,	$w_n = 1.0$
Chatter,	$t_c = 0.5.$

All 5x17 and 5x11 filters are designed for the centre trace estimates, i.e. $R = 3.0$.

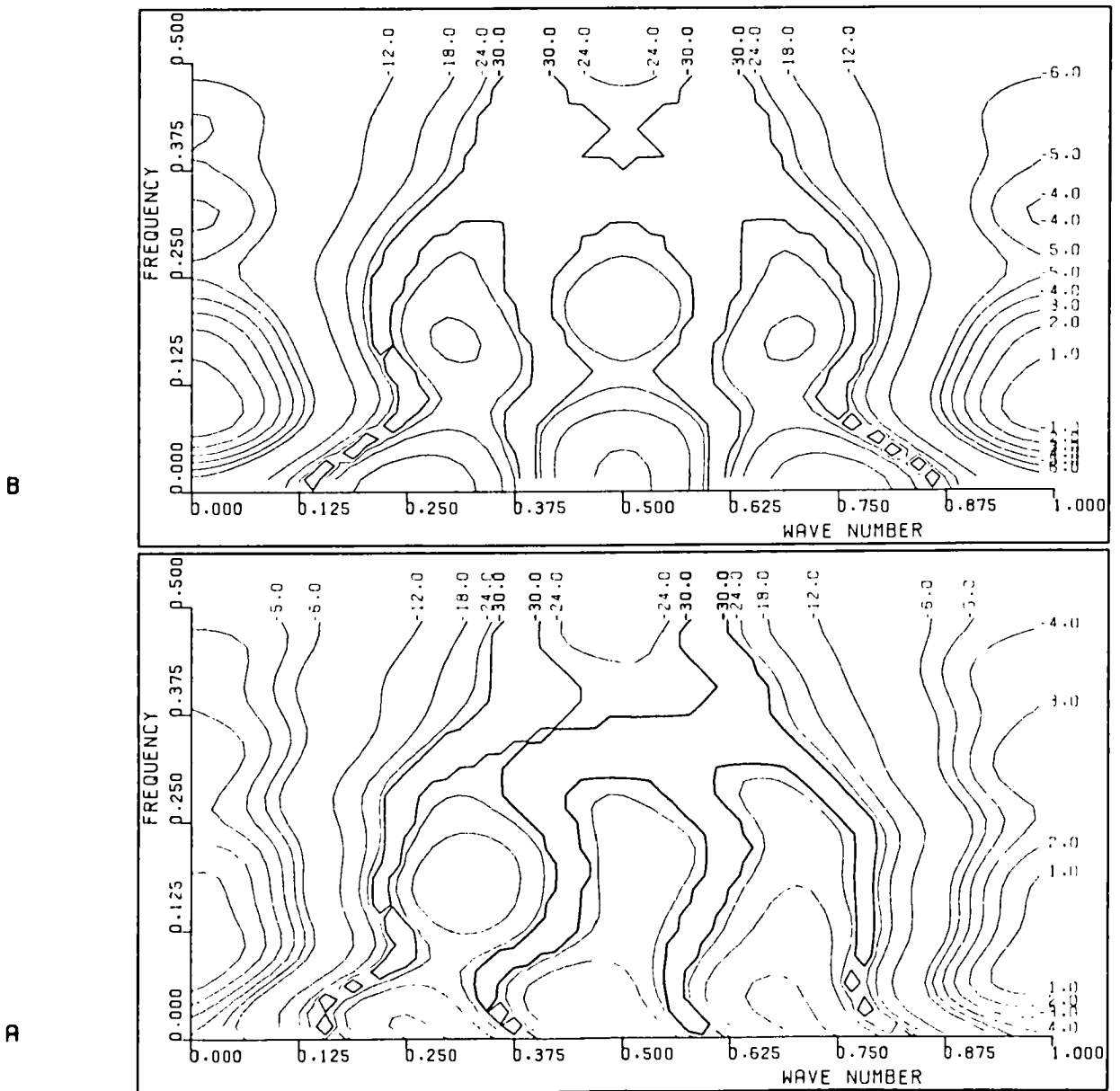
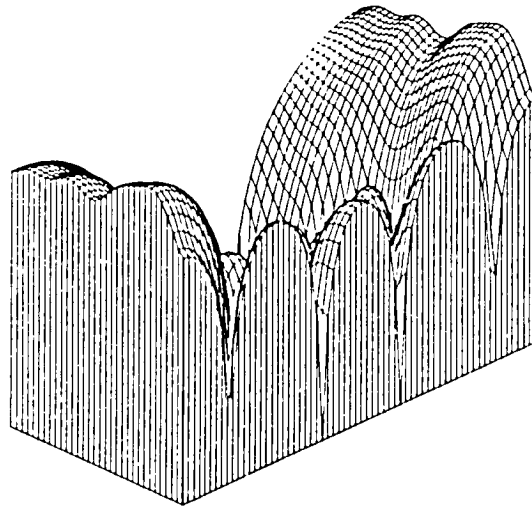
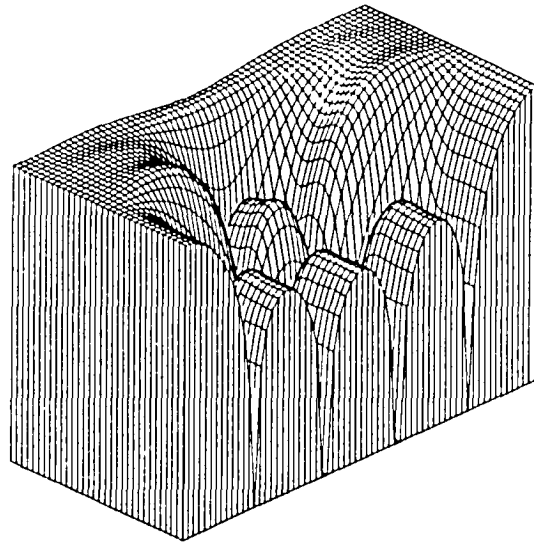
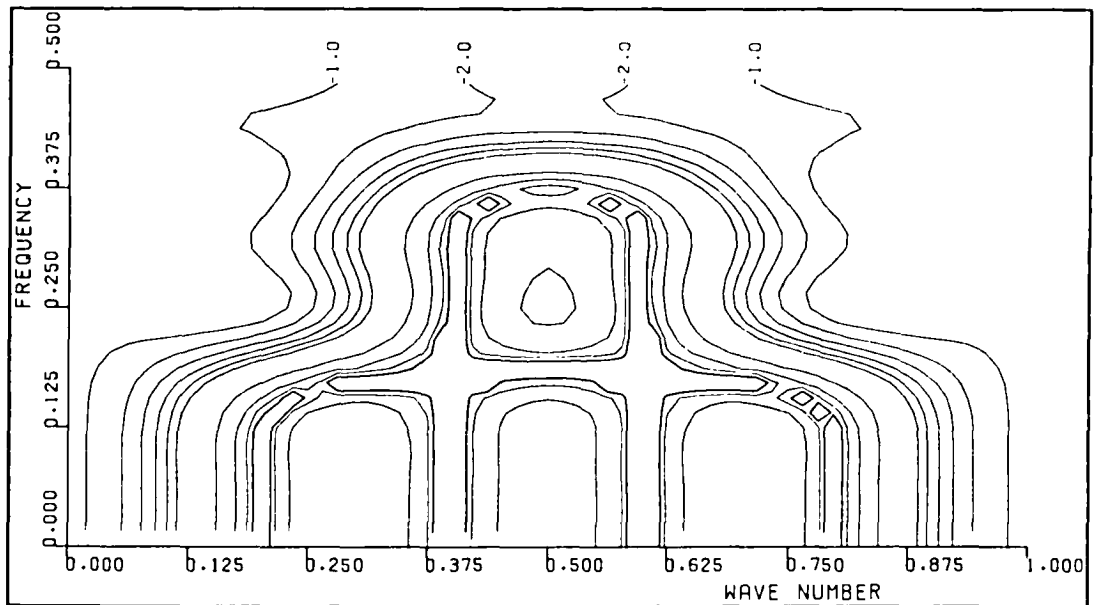


Figure 5.5.1 A) (f,k) response of 5×17 centrosymmetric least-squares filter (Filter 5.5.1) and B) (f,k) response and its isometric plot for the corresponding symmetric least-squares filter (Filter 5.5.2). See the facing page for the common design parameters.



B



A

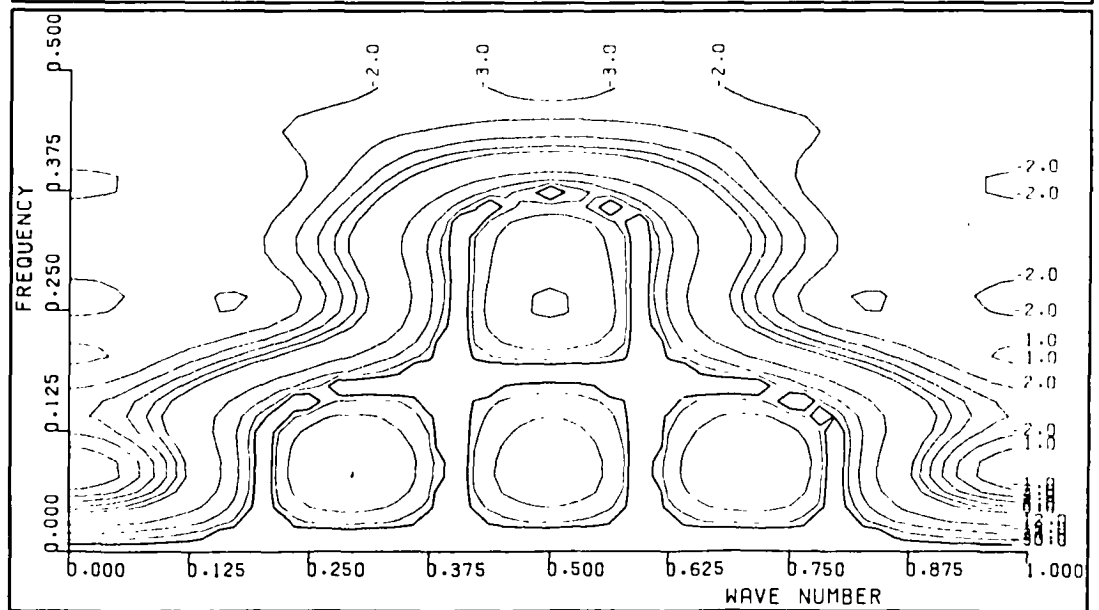
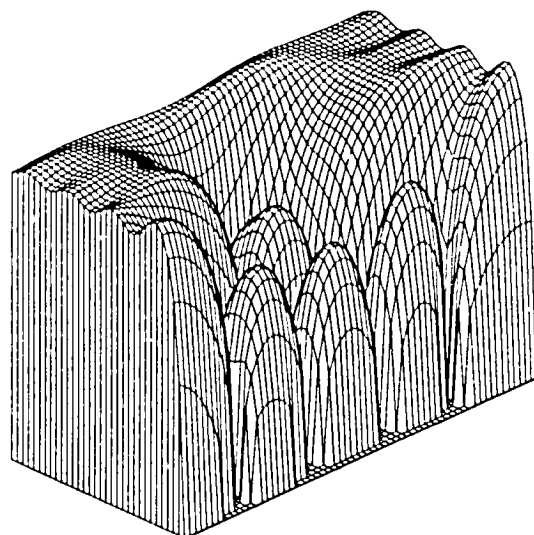
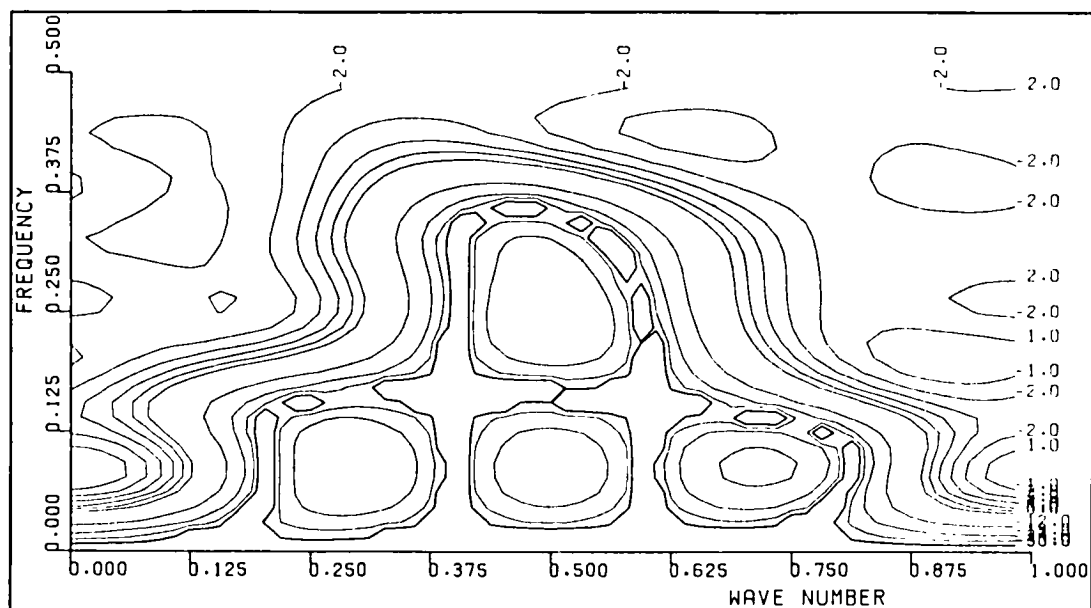


Figure 5.5.2 A) (f,k) response of 5x17 Tchebycheff filter with 'reject-origin' (Filter 5.5.3), and B) (f,k) response and its isometric plot for the same filter with 'pass-origin' (Filter 5.5.4).



B



A

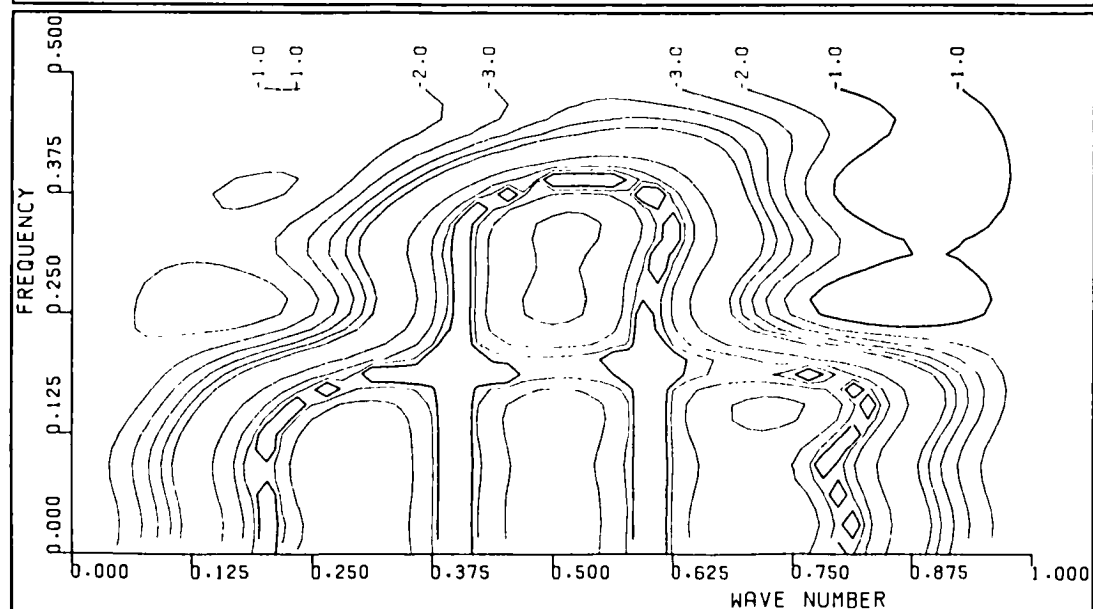


Figure 5.5.3 A) (f,k) response of Filter 5.5.5 and B) (f,k) response and its isometric plot of Filter 5.5.6.

Table 5.5.1 The additional design parameters for 5x17 linear moveout filters.

Filter No.	Figure No.	Pass Domain weight (w_p)	Reject Domain Weights		Origin Mapping Moveout (Sample/trace)
			$w_r^{(1)}$	$w_r^{(2)}$	
5.5.1	5.5.1A	1.0	0.0	1.0	Least-squares
5.5.2	5.5.1B	1.0	1.0	1.0	designs
5.5.3	5.5.2A	0.47829690E-08	0.48351437E-08	0.48351437E-08	125.0
5.5.4	5.5.2B	"	"	"	0.0
5.5.5	5.5.3A	0.47829690E-08	0.15534707E-09	0.48351437E-08	-1.0
5.5.6	5.5.3B	0.15009445E-08	0.28762352E-08	0.93772342E-08	-1.75

for the reject window $0_r^{(1)} = -4.0$ to -2.5 and $w_r^{(2)}$ to $0_r^{(2)} = 2.5$ to 4.0 . Figures 5.5.2A and B show the (f,k) responses of the 'reject-origin' and 'pass-origin' Tchebycheff filters, respectively, i.e. the Tchebycheff filters (Filter 5.5.3 and 5.5.4) with the origin coefficients are mapped to the pass and reject domains of the corresponding 1D frequency axis. In Figures 5.5.3A and B the same Tchebycheff filter responses with origins mapped into the 'don't care' regions are given (Filters 5.5.5 and 5.5.6). That is, the origin coefficient of the 2D frequency response is mapped into the 1D frequency axis like a coefficient falling onto -1.0 sample/trace and -1.75 sample/trace moveout lines, respectively.

The second example is a 5x11 linear moveout filter with the same moveout window configuration as the first example.

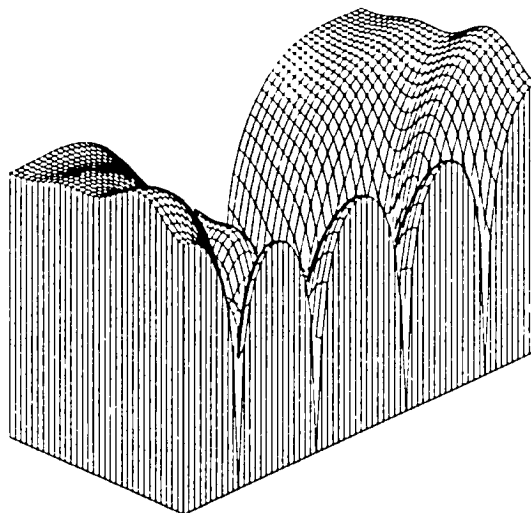
The differing parameters for the least-squares designs are $t_d = 6$, $w_n = 0.5$ and $t_c = 0.0$. The remaining parameters for the least-squares and Tchebycheff filters are given in Table 5.5.2. Figure 5.5.4A and B show the (f,k) responses

Table 5.5.2 The additional design parameters for 5x11 linear moveout filters.

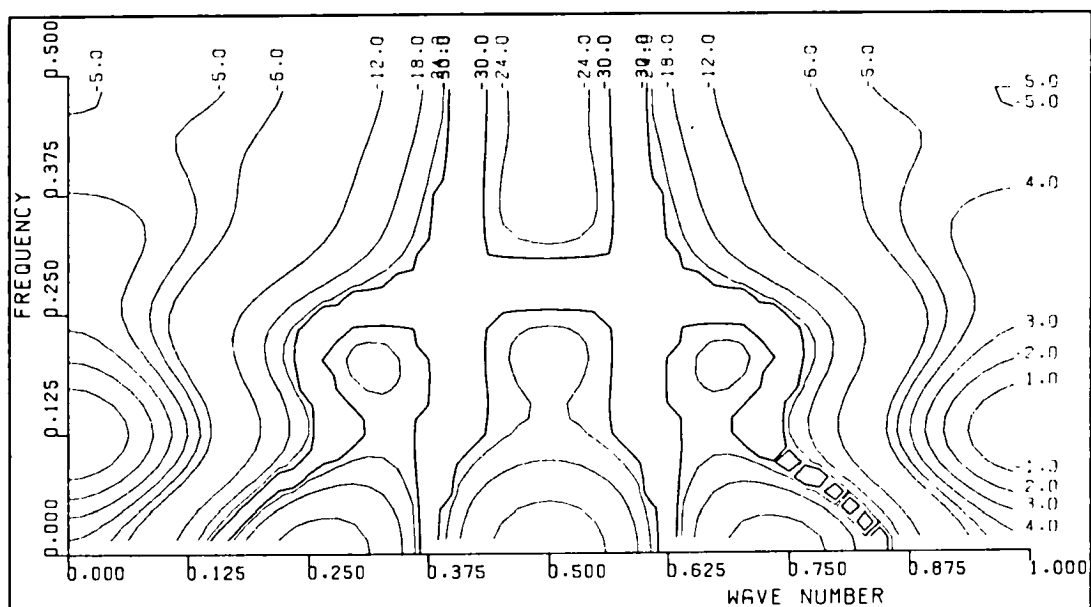
Filter No.	Figure No.	Pass Domain Weight (w_p)	Reject Domain Weights		Origin Mapping Moveout (sample/trace)
			$w_r^{(1)}$	$w_r^{(2)}$	
5.5.7	5.5.4A	1.0	0.0	0.5	Least-squares designs
5.5.8	5.5.4B	1.0	0.5	0.5	
5.5.9	5.5.5A	0.1E-04	0.57905E-05	0.57905E-05	125.0
5.5.10	5.5.5B	0.1E-05	0.46441E-04	0.46441E-04	0.0
5.5.11	5.5.6A	0.1E-06	0.30169E-03	0.30169E-03	0.0
5.5.12	5.5.6B	0.1E-08	0.94930E-02	0.94930E-02	0.0

of the least-squares centrosymmetric and symmetric filters, respectively. The (f,k) responses of the 'reject-origin' and the 'pass-origin' Tchebycheff designs are given in Figure 5.5.5A and B, respectively, and the 'pass-origin' Tchebycheff filters with varying region weightings are given in Figure 5.5.6A and B.

The general conclusions obtained from the comparisons of the least-squares and Tchebycheff moveout filter design examples given above may be summarized as:



B



A

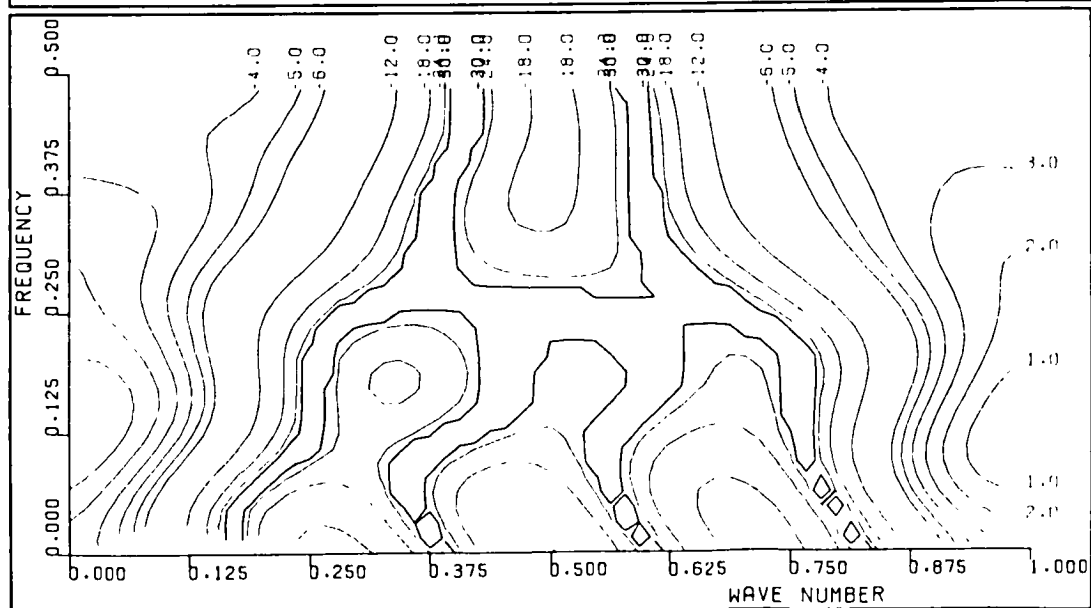
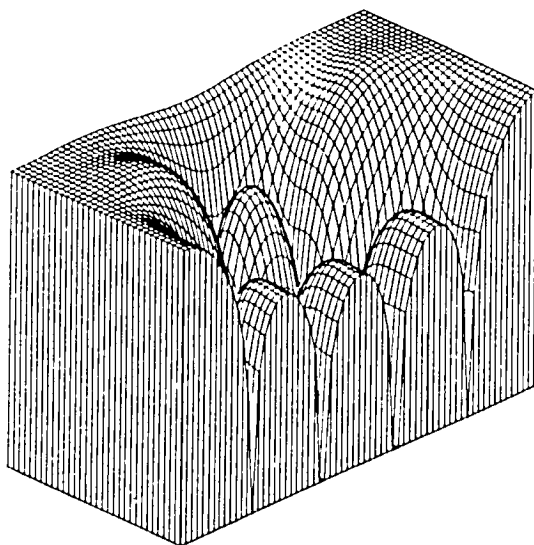
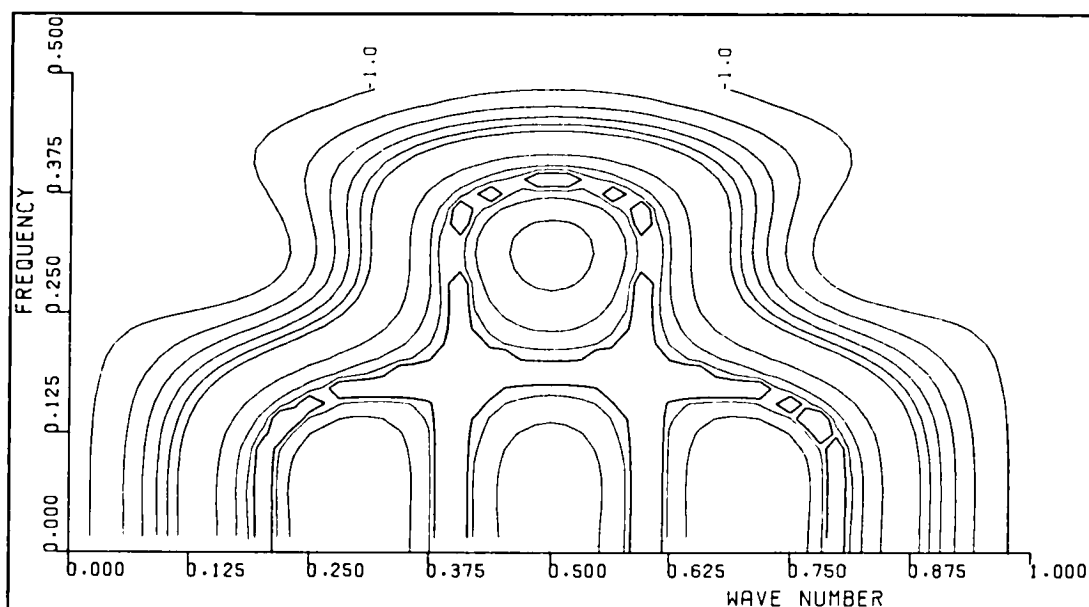


Figure 5.5.4 A) (f,k) responses of 5×11 centrosymmetric least-squares filter (Filter 5.5.7) and B) (f,k) response and its isometric plot for the corresponding symmetric least-squares filter (Filter 5.5.8).



B



A

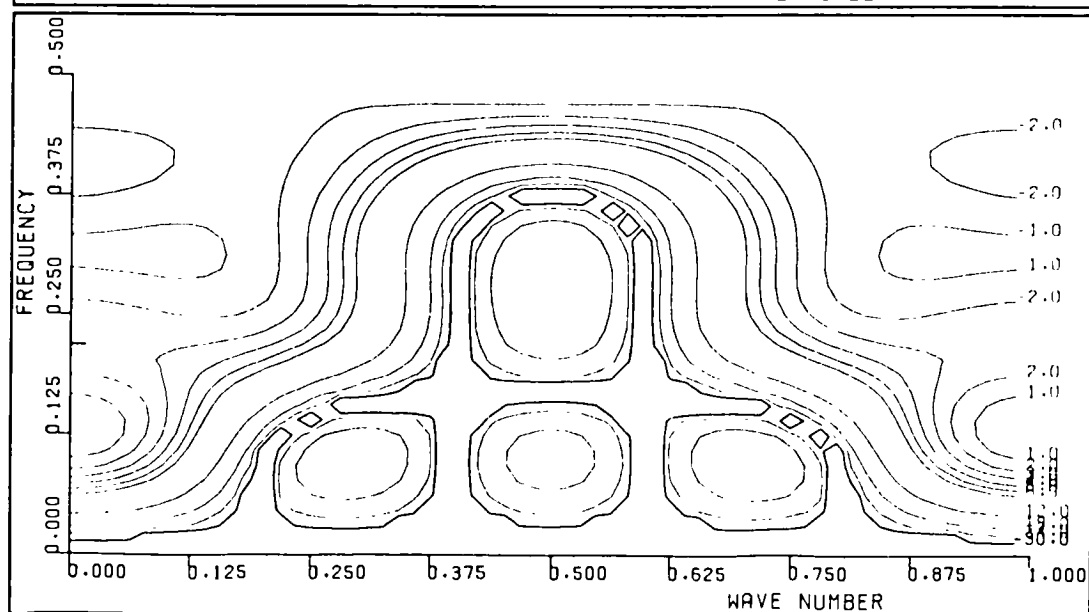
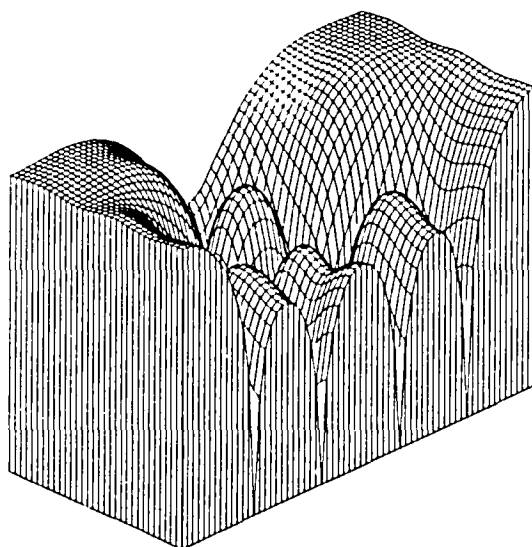
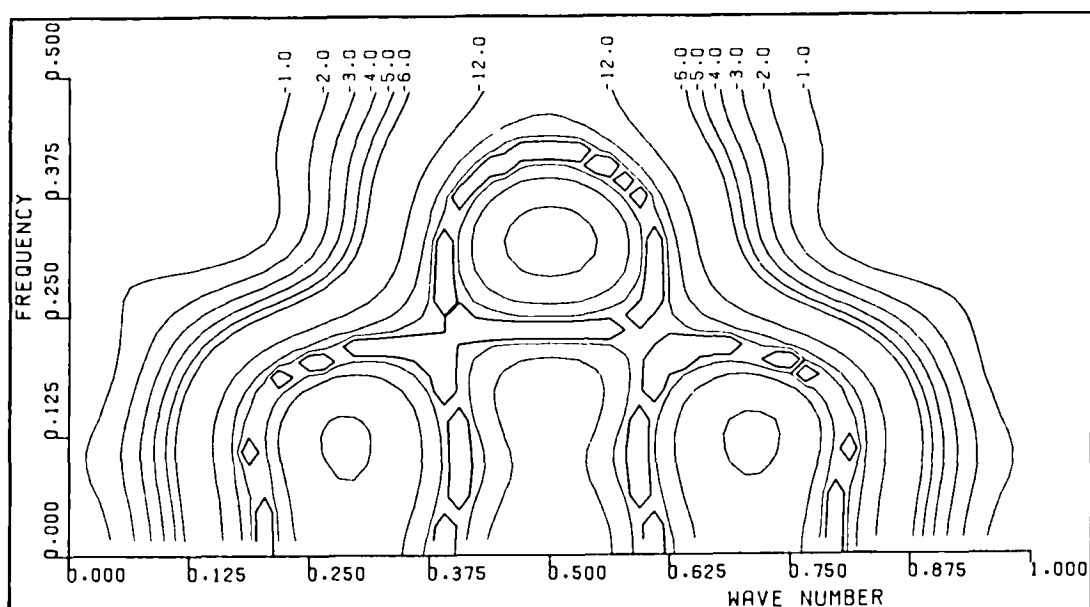


Figure 5.5.5 A) (f,k) response of 5x11 'reject-origin' Tchebycheff filter (Filter 5.5.9) and B) (f,k) response and its isometric plot for the same filter with 'pass-origin' (Filter 5.5.10).



B



A

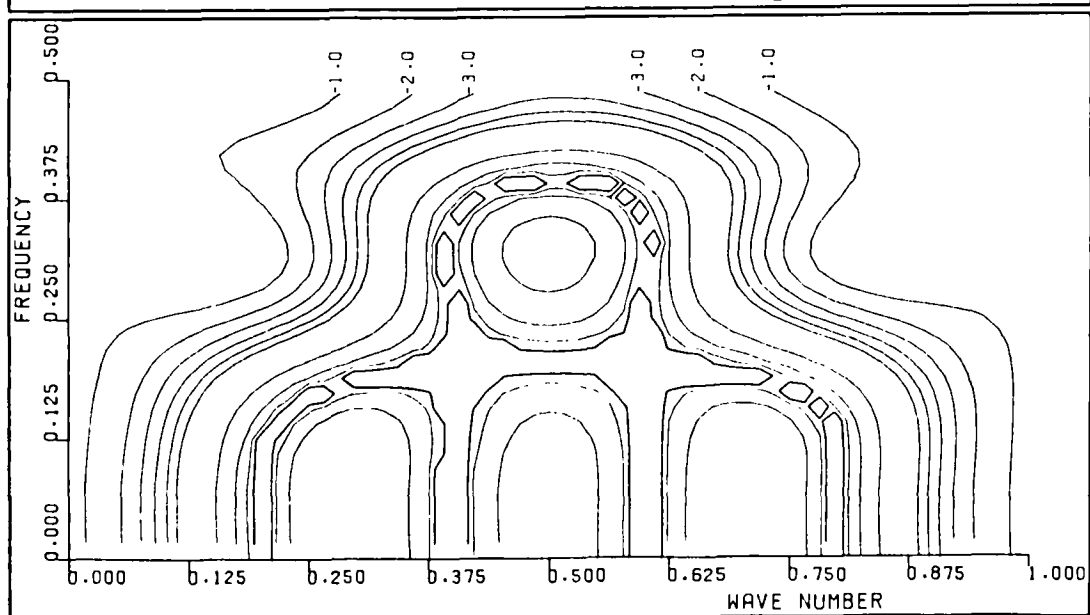


Figure 5.5.6 A) (f,k) response of 5x11 'pass-origin' Tchebycheff filter (Filter 5.5.11) with $(w_r/w_p) \approx 3017$ and B) (f,k) response and its isometric plot for the same filter (Filter 5.5.12) with $(w_r/w_p) \approx 9493 \times 10^3$.

1. Tchebycheff moveout filters with 'pass-origins' have (f,k) responses (Figures 5.5.2B, 5.5.5B, 5.5.6A and 5.5.6B) similar to the least-squares moveout filters (Figures 5.5.1A,B and 5.5.4A,B) at low frequencies and wave numbers. They approximate the remaining part of the pass window almost perfectly with about 0 to 1dB deviation compared to about 1 to 6dB deviation of the corresponding least-squares designs. The reject domain approximations are marginally better for the least-squares filters for these examples.
2. The 'reject-origin' Tchebycheff moveout filters always introduce about 1 to 30dB low-cut filtering at the very low frequencies of the pass window (Figures 5.5.2A and 5.5.5A). This reject area is pushed towards the origin of the (f,k) response with increasing filter length. The rest of the pass window is approximated with about 1 to 2dB deviation. The reject domain approximation is better than the corresponding 'pass-origin' and the least-squares designs.
3. Tchebycheff moveout filters with origins mapped into the transition or 'don't care' regions are not symmetric (Figure 5.5.3A and B). Depending on the mapping moveout for the origin coefficient, the desired response is approximated to a degree between the corresponding 'pass-origin' and 'reject-origin' designs.
4. The transition bands of the above moveout filters with a low order of coefficients are not smooth in the corresponding fan-shaped regions. Tchebycheff moveout

filters have smoother transition bands than the least-squares designs.

5. Deviations from the desired response show ripples similar to 1D designs for Tchebycheff moveout filters. These ripples, at regions with large moveouts and narrow windows, are too big and consequently poor approximations to the desired responses are obtained (the reject windows in the above given examples).
6. The aliasing of the linear moveout windows with large moveouts, when the least-squares technique is used, is avoided by employing the new (f,k) domain design technique.
7. The new technique can easily be manipulated to reject low frequencies, if desired. This is done by mapping the origin coefficient of 2D (f,k) response into a reject region or into a transition band near to a reject region of the corresponding 1D frequency response. The low-cut frequency can be changed by changing the filter length and/or mapping moveout for the origin coefficient.

The above 5×17 and 5×11 moveout filters applied to synthetic seismograms with varied noise conditions. Figure 5.5.7 show a 5 trace synthetic reflection seismogram and the outputs of 5×17 filters (Filters 5.5.1 to 5.5.6) for this data. In Figure 5.5.8 the same raw data and outputs of 5×11 filters (Filters 5.5.7 to 5.5.12) are given. All the filtered traces of this section are the centre trace estimates.

The input parameters for the raw synthetic seismogram given in Figure 5.5.7:

```

Duration of all spikes (time gate in ms)      : 0.0 - 996.0
Sampling interval (ms)                        : 4.0
Trace gains                                  : 1.0
Number of signal spikes on each trace         : 4
Number of correlated noise spikes on each trace : 4
Correlated noise to signal power ratio        : 1.0
Number of uncorrelated noise spikes on each trace: 75
Uncorrelated noise to signal power ratio      : 2.0
Signal moveouts (ms/trace)                   : 0.0, 0.0, 0.0, 0.0
Correlated noise moveouts (ms/trace)          : 20.0, 15.0, 10.0, 15.0
Arrival times for signal events (ms)          : 200.0, 450.0, 625.0,
                                              900.0
Arrival times for correlated noise events (ms): 0.0, 325.0, 525.0,
                                              750.0

```

The chatter values (in ms) added to signal spike arrival times:

	Spike	1	2	3	4
Trace					
1		0.0	0.0	4.0	8.0
2		0.0	-2.0	0.0	0.0
3		0.0	1.0	-2.0	-6.0
4		0.0	-1.0	1.0	5.0
5		0.0	2.0	-3.0	-7.0

The fourth correlated noise event (at 750.0 ms) has the same chatter values, added to its relative moveouts, as the fourth spike of the signal event. The correlated noise to signal power ratio given above is for the whole trace lengths and over the duration (38 samples) of the equivalent wavelets. The uncorrelated noise to signal power ratio is over the trace lengths.

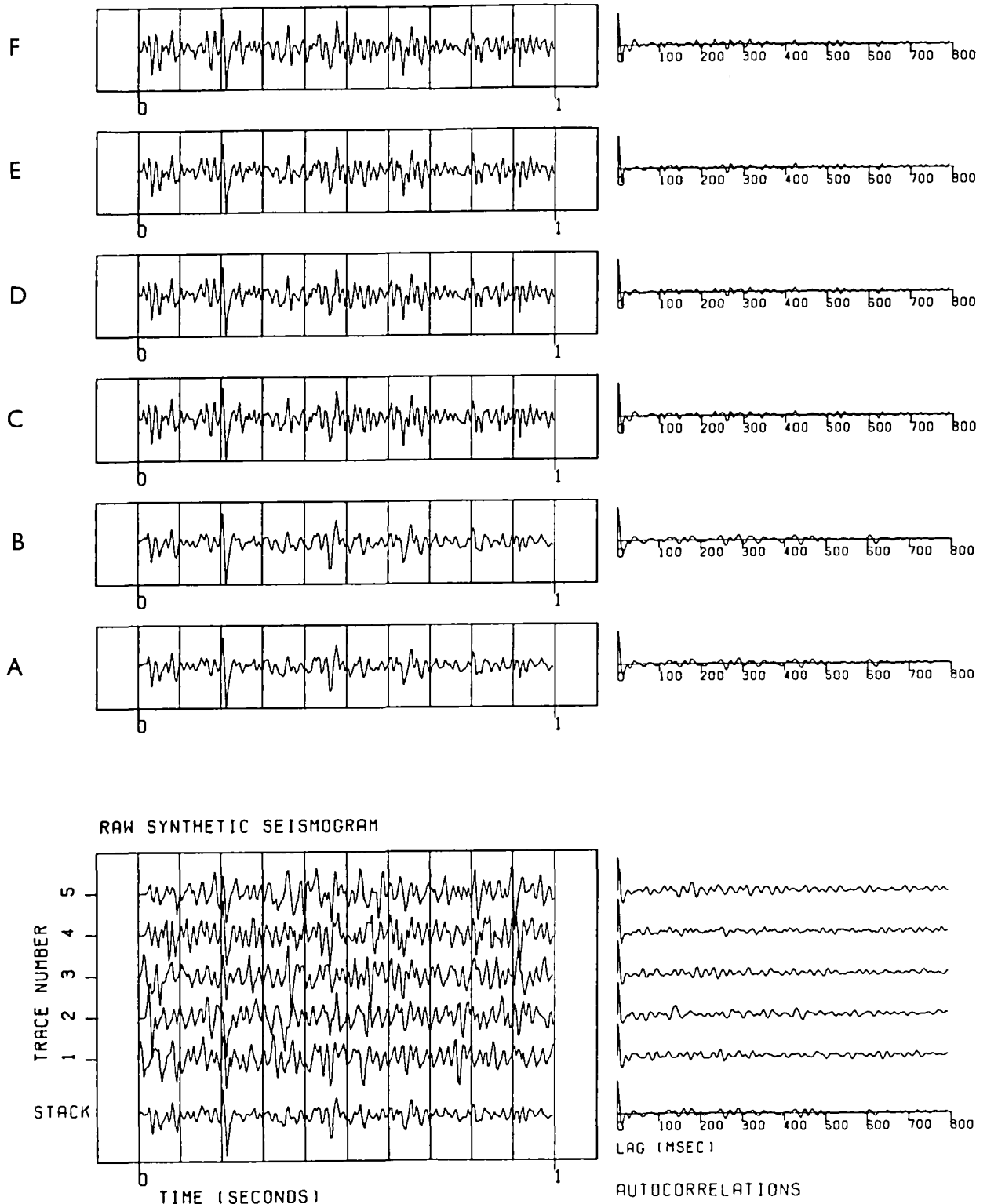


Figure 5.5.7 Applications of 5x17 moveout filters to synthetic data. The centre trace estimates are: A) Least-squares symmetric, B) Least-squares centrosymmetric, C) Tchebycheff 'pass-origin', D) Tchebycheff 'reject-origin', E) Tchebycheff with origin mapped to -1 sample/trace moveout and F) Tchebycheff origin mapped to -1.75 samples/trace moveout.

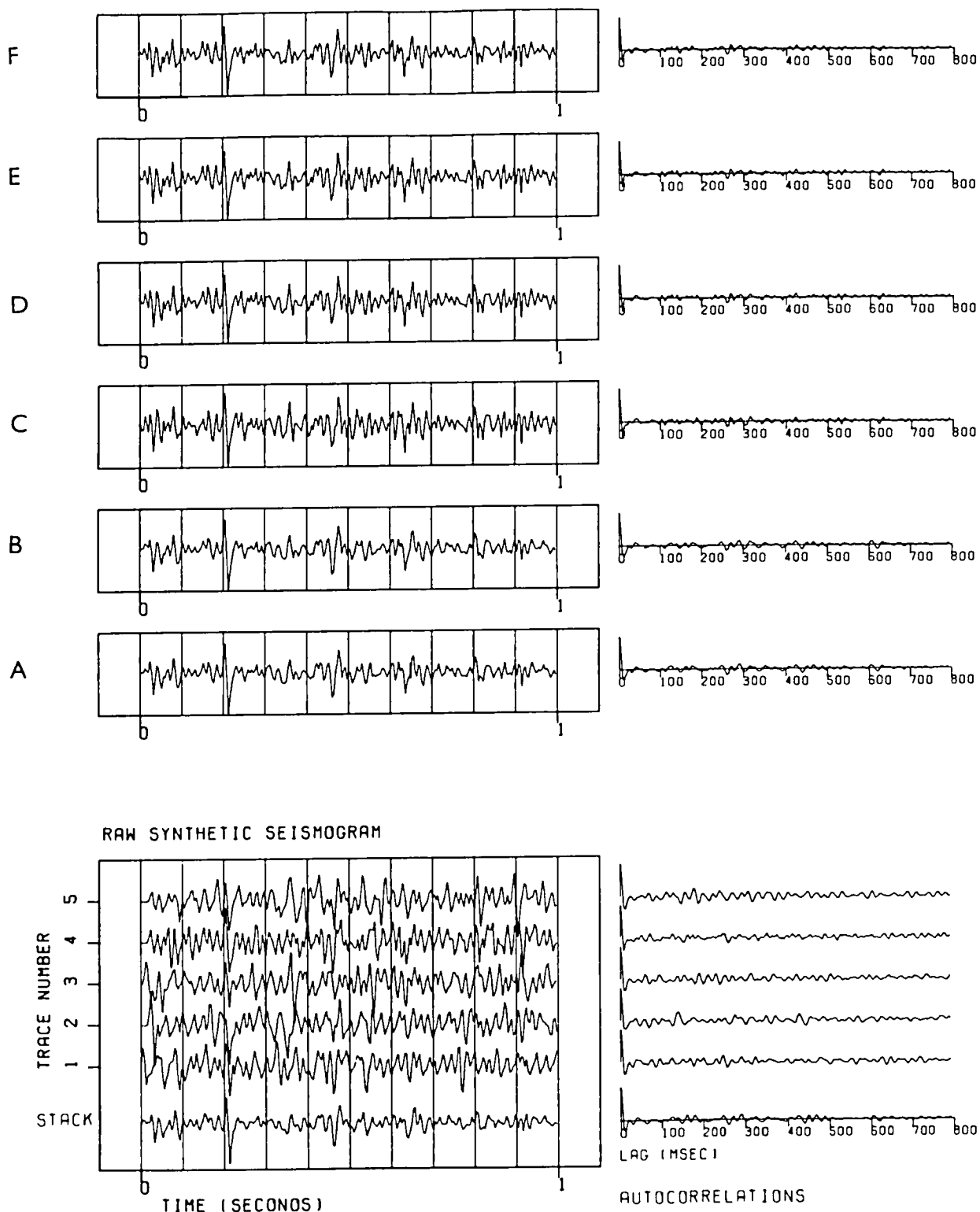


Figure 5.5.8 Applications of 5x11 moveout filters to synthetic data. The centre trace estimates are: A) Least-squares symmetric, B) Least-squares centrosymmetric, C) Tchebycheff 'pass-origin', D) Tchebycheff 'reject-origin', E) Tchebycheff 'pass-origin' with $(w_r/w_p) \approx 3017$, F) Tchebycheff 'pass-origin' with $(w_r/w_p) \approx 9493 \times 10^3$.

In all synthetic data applications the processed traces A,B and C,D,E,F are the outputs of the least-squares and Tchebycheff moveout filters, respectively. Figure 5.5.9 shows another synthetic seismogram with different signal and correlated noise event distributions. The outputs of 5x17 filters (Filters 5.5.1 to 5.5.6) for this data are given in the same figure. In Figure 5.5.10, the same raw data and the outputs of 5x11 filters (Filters 5.5.7 to 5.5.12) are presented. The two synthetic data applications illustrated are the combined examples extracted from a large number of simulated and processed data. The conclusions drawn from these experiments, which have important implications in seismic reflection data processing applications, are summarized below.

1. The events with zero moveout per trace are passed with comparable results by the Tchebycheff and the least-squares filters as well as the straight stacking process (the signal event at 200 ms in Figures 5.5.7 and 5.5.8). Tchebycheff filters recover events corrupted by noise better than the other two processes, e.g. the signal event at 200 ms in Figures 5.5.9 and 5.5.10.
2. Tchebycheff moveout filters are always superior to the least-squares filters and straight stacking in recovering the signal events with relatively big chatter (the signal events at 900 ms in Figures 5.5.7 and 5.5.8, and at 625 ms in Figures 5.5.9 and 5.5.10). This property of Tchebycheff moveout filters is very important in

The raw synthetic reflection seismogram in Figure 5.5.9 on the facing page has the same input parameters as the one given in Figure 5.5.7 with the changes:

Number of signal spikes on each trace	:	3
Number of correlated noise spikes on each trace:		3
Arrival times for signal events (ms)	:	200.0, 450.0, 625.0
Arrival times for correlated noise events (ms)	:	150.0, 325.0, 525.0
Correlated noise moveouts (ms/trace)	:	20.0, 15.0, 10.0

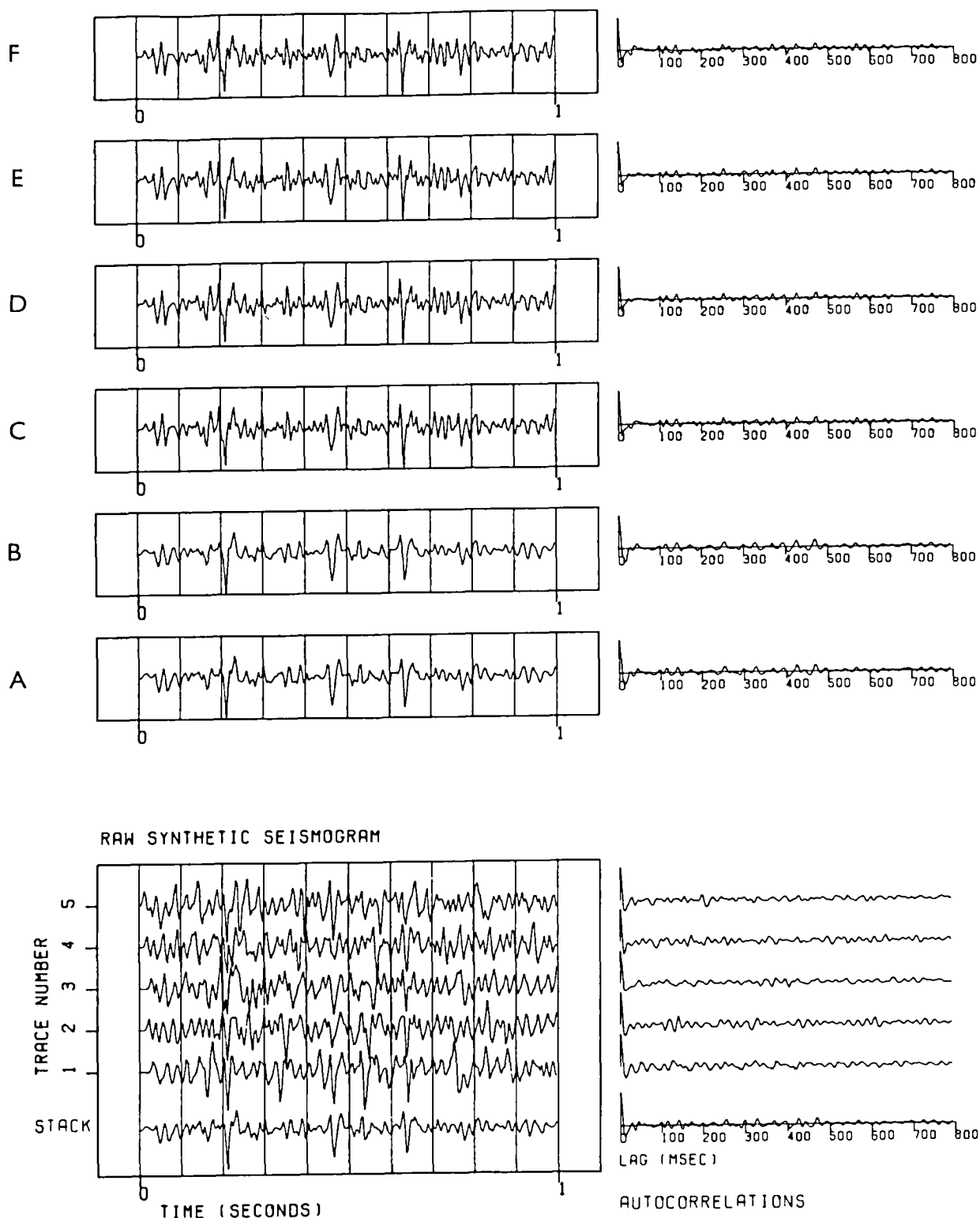


Figure 5.5.9 Applications of 5x17 moveout filters to synthetic data. The centre trace estimates are: A) Least-squares symmetric, B) Least-squares centro-symmetric, C) Tchebycheff 'pass-origin', D) Tchebycheff 'reject-origin', E) Tchebycheff with origin mapped to -1 sample/trace moveout and F) Tchebycheff with origin mapped to -1.75 sample/trace moveout.

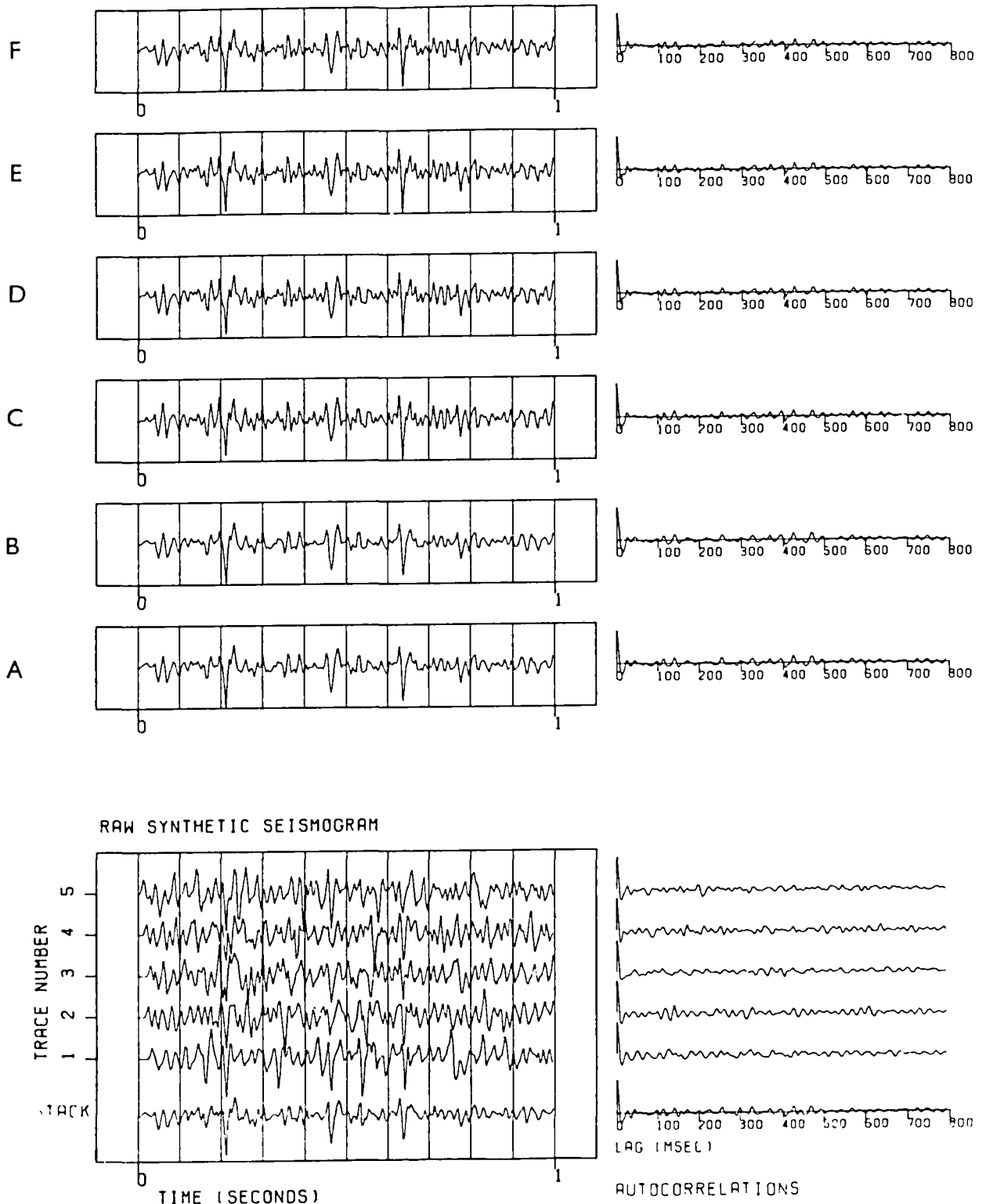


Figure 5.5.10 Applications of 5x11 moveout filters to synthetic data. The centre trace estimates are: A) Least-squares symmetric, B) Least-squares centrosymmetric, C) Tchebycheff 'pass-origin', D) Tchebycheff 'reject-origin', E) Tchebycheff 'pass-origin' with $(w_r/w_p) \approx 3017$ and F) Tchebycheff 'pass-origin' with $(w_r/w_p) \approx 9493 \times 10^3$.

practice, because the real seismic reflection events have always some jitter left after corrections, especially in the case of land data.

3. Straight stacking achieves the best random noise attenuation. Tchebycheff filtered outputs are noisier than the outputs of the least-squares filters.
4. Signal to correlated noise ratios for the Tchebycheff filtered traces are relatively higher, though not substantially, for filters with higher relative reject domain weights.
5. The design and application of 5x17 and 5x11 Tchebycheff moveout filters show that there is no gain with increased filter length for the pass and reject moveout configuration case presented.
6. The outputs of Tchebycheff filters with different origin mappings differ marginally because signal and correlated noise events have relatively very little power between the frequencies 0.0 to 0.05 where the origin mappings have most effect. For the same reason uncorrelated noise is also marginally affected. Improvements in the reject domain approximation by origin mappings other than into pass domain is paid by degradation in pass domain approximation. Therefore, signal to correlated noise ratio changes are relatively small.

The remaining examples of this section are on real seismograms. The three records processed in Chapter 3 have, once more, been used to compare the coefficient mapping and

the least-squares moveout filters. The raw data given in Figure 5.5.11A is found to be the most representative. In this figure the same record is plotted from trace 3 to 22 for easy comparisons with the centre trace estimate processed results. The samples from 100 ms to 1200 ms are gated for filtering. The processed data presented are the outputs of the latter 5x11 filters, i.e. Filters 5.5.7,8,9, 10,11 and 12 of Table 5.5.2. The outputs of the centrosymmetric least-squares filter (Filter 5.5.7), given in Figure 5.5.11C, and that of symmetric least-squares filter (Filter 5.5.8) in Figure 5.5.12A show no visible difference. The 'pass-origin' Tchebycheff filter (Filter 5.5.10) output for the same data is given in Figure 5.5.11D. In Figure 5.5.12B,C and D the outputs of the 'reject-origin' Tchebycheff filter (Filter 5.5.9) and the 'pass-origin' Tchebycheff filters (Filters 5.5.11 and 5.5.12) with higher reject window weights are presented. The filtered records show that all the least-squares and Tchebycheff filters applied, are equally efficient in removing the correlated noise at the beginning of the raw data. The Tchebycheff filter outputs have a higher S/N ratio compared to the least-squares filter outputs. Consequently, the Tchebycheff filtered outputs show small but significantly improved trace-to-trace continuation. This is equally true for all the Tchebycheff filter outputs shown.

The last example is an offshore data application. All the records from this area have the same correlated noise events, the so called 'ground-roll' or shot noise, with high moveouts persisting across all traces. Figure 5.5.17A

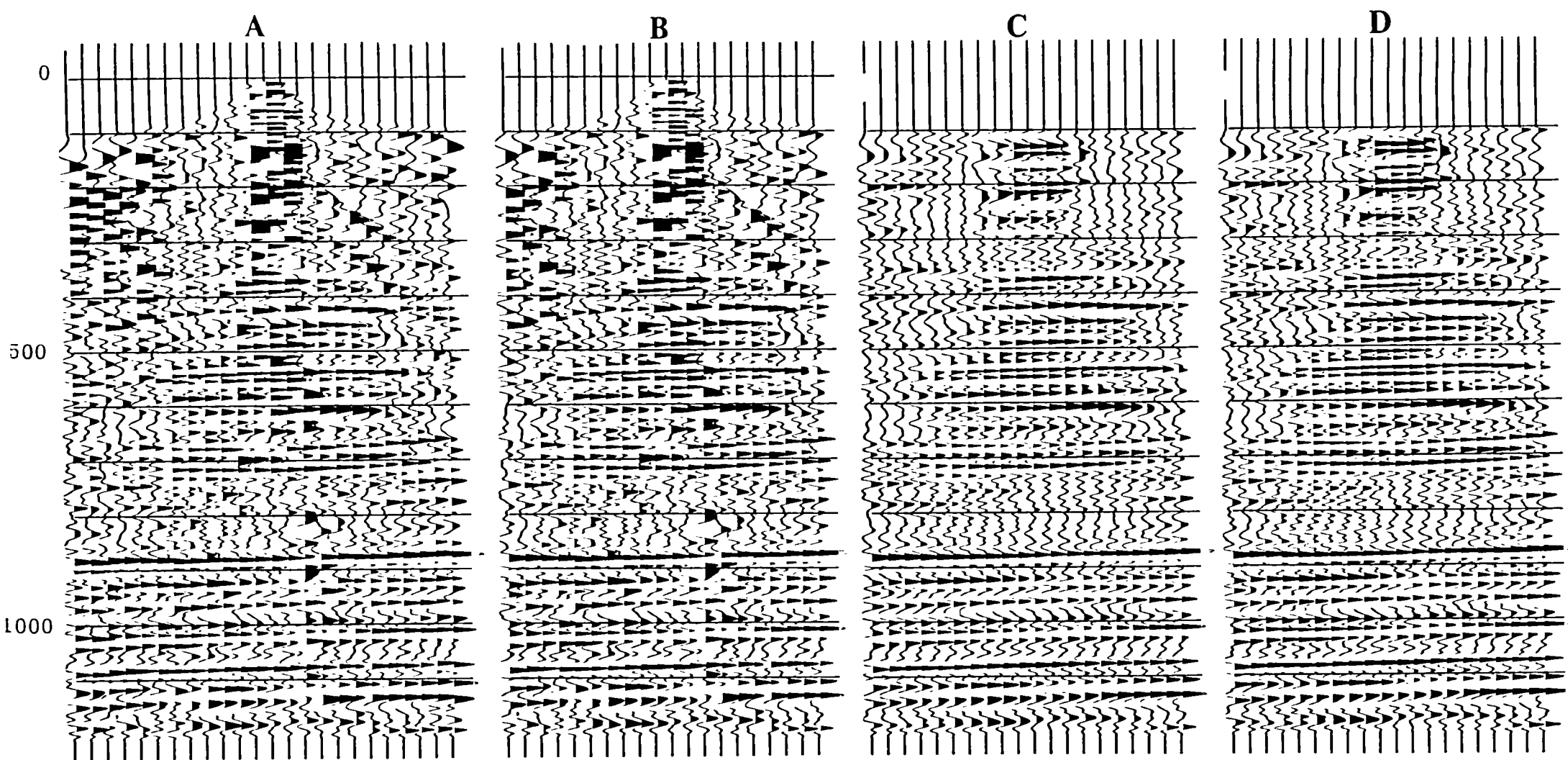


Figure 5.5.11 A) Raw field data, B) The same raw data plotted from trace 3 to 22, C) Output of 5x11 centrosymmetric least-squares filter and D) Output of 5x11 'pass-origin' Tchebycheff filter.

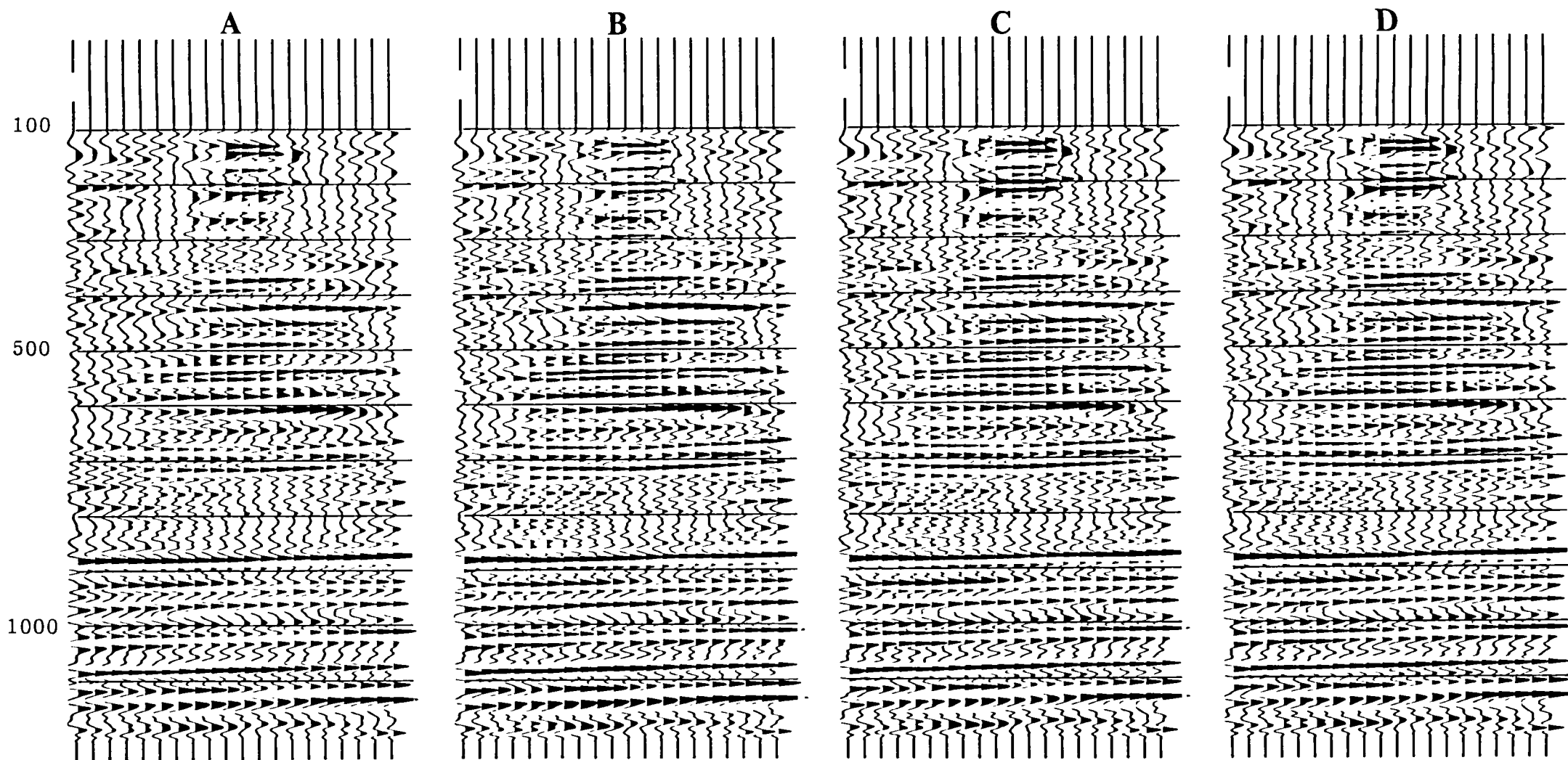


Figure 5.5.12 Outputs of 5x11 moveout filters for the raw data of Figure 5.5.11. A) Least-squares symmetric, B) Tchebycheff 'reject-origin', C) Tchebycheff 'pass-origin' with $(w_r/w_p) \approx 3017$ (Filter 5.5.11) and D) Tchebycheff 'pass-origin' with $(w_r/w_p) \approx 9493 \times 10^3$ (Filter 5.5.12).

shows one of the records without normal moveout correction. In this figure the same record plotted from the trace 4 to 21 to ease the comparisons with the corresponding centre trace estimate processed results. The sampling rate is 4.0 ms. The moveout filters applied to this data are linear pass-linear reject operators. The pass window boundaries are estimated using the approximate method described in Section 4.2. Since linear moveout filters are designed to pass events with hyperbolic moveouts, this will also serve as an example for filtering events whose moveouts fall into the boundaries of a moveout window which does not have the same moveout function. Table 5.5.3 shows some of the design parameters for 7x29 least-squares and Tchebycheff filters whose (f,k) responses and applications to the above offshore data are compared. In Figure 5.5.13 the (f,k) responses of centrosymmetric least-squares filters for varying correlated noise weights are given (Filters 5.5.13,14 and 15). Figure 5.5.14 shows the (f,k) responses for corresponding symmetric least-squares filters (Filters 5.5.16,17 and 18). With the increasing correlated noise weight, the improvement of the reject window approximation is evident from the given responses. This improvement is at the expense of the degradation of the pass window approximation mainly due to the strong multiple aliasing of the reject window.

In Figure 5.5.15 and 5.5.16 the (f,k) responses of the same symmetric Tchebycheff filters are presented. Figure 5.5.15 shows the 'pass-origin' filters (Filters 5.5.19,20 and 21),

Table 5.5.3 Some of the design parameters for 7x29 least-squares and Tchebycheff linear moveout filters.

Filter No.	Figure No.	Pass Domain Weight (w_p)	Reject Domain Weights		Origin Mapping Moveout (sample/trace)
			$w_r^{(1)}$	$w_r^{(2)}$	
5.5.13	5.5.13	1.0	0.01	0.0	Least-squares designs
5.5.14	"	1.0	0.3	0.0	
5.5.15	"	1.0	1.0	0.0	
5.5.16	5.5.14	1.0	0.01	0.01	
5.5.17	"	1.0	0.3	0.3	
5.5.18	"	1.0	1.0	1.0	
5.5.19	5.5.15	0.1E-02	0.18628918E-05	0.18628918E-05	0.0
5.5.20	"	0.1E-03	0.25760472E-04	0.25760472E-04	0.0
5.5.21	"	0.1E-05	0.17373645E-02	0.17373645E-02	0.0
5.5.22	5.5.16	0.1E-02	0.18628918E-05	0.18628918E-05	125.0
5.5.23	"	0.1E-03	0.25760472E-04	0.25760472E-04	125.0
5.5.24	"	0.1E-05	0.17373645E-02	0.17373645E-02	125.0

and Figure 5.5.16 the 'reject-origin' filters (Filters 5.5.22, 23 and 24). The varying reject region weight to pass region weight ratios (w_r/w_p) or ripple ratios are printed alongside the corresponding responses. The actual weights are given in Table 5.5.3. Note that Tchebycheff filters are designed to reject all events having moveouts with an absolute value greater than 13.75 sample per trace (55.0 ms per trace). The characteristics observed from these (f,k) responses are

The common design parameters for 7x29 symmetric least-squares and Tchebycheff moveout filters:

Number of input channels,	$N = 7$
Length of filter,	$LF = 29$
Reference output trace,	$R = 4.0$
Delay index (least squares),	$t_d = 15$
Pass moveout window (time unit/trace),	$0_s = -4.5 \text{ to } 4.5$
Reject moveout windows (time unit/trace),	$0_r^{(1)} = -18.75 \text{ to } -13.75$ $0_r^{(2)} = 13.75 \text{ to } 18.75$
Uncorrelated noise weight (least-squares),	$w_n = 0.3$
Chatter (least-squares),	$t_c = 0.5$

The centrosymmetric 7x29 least-squares filters whose responses are given on the facing page have the moveout configuration:

Pass moveout window (time unit/trace),	$0_s = 0.0 \text{ to } 4.5$
Reject moveout window (time unit/trace),	$0_r = 13.75 \text{ to } 18.75$

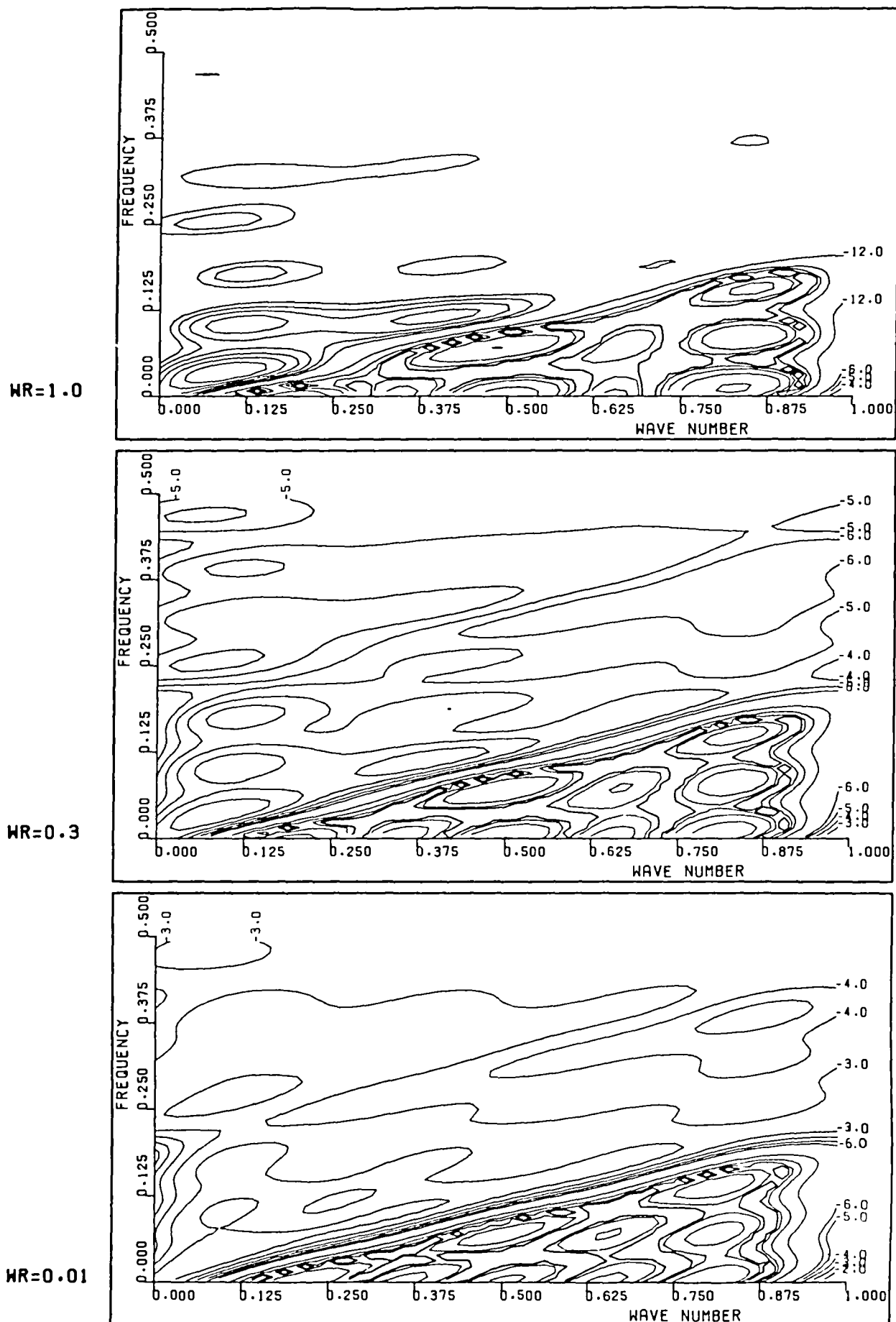


Figure 5.5.13 (f,k) responses of 7x29 centro symmetric least-squares moveout filters ($\theta_s = 0.0$ to 4.5 and $\theta_r = 13.75$ to 18.75). See facing page for the common design parameters.

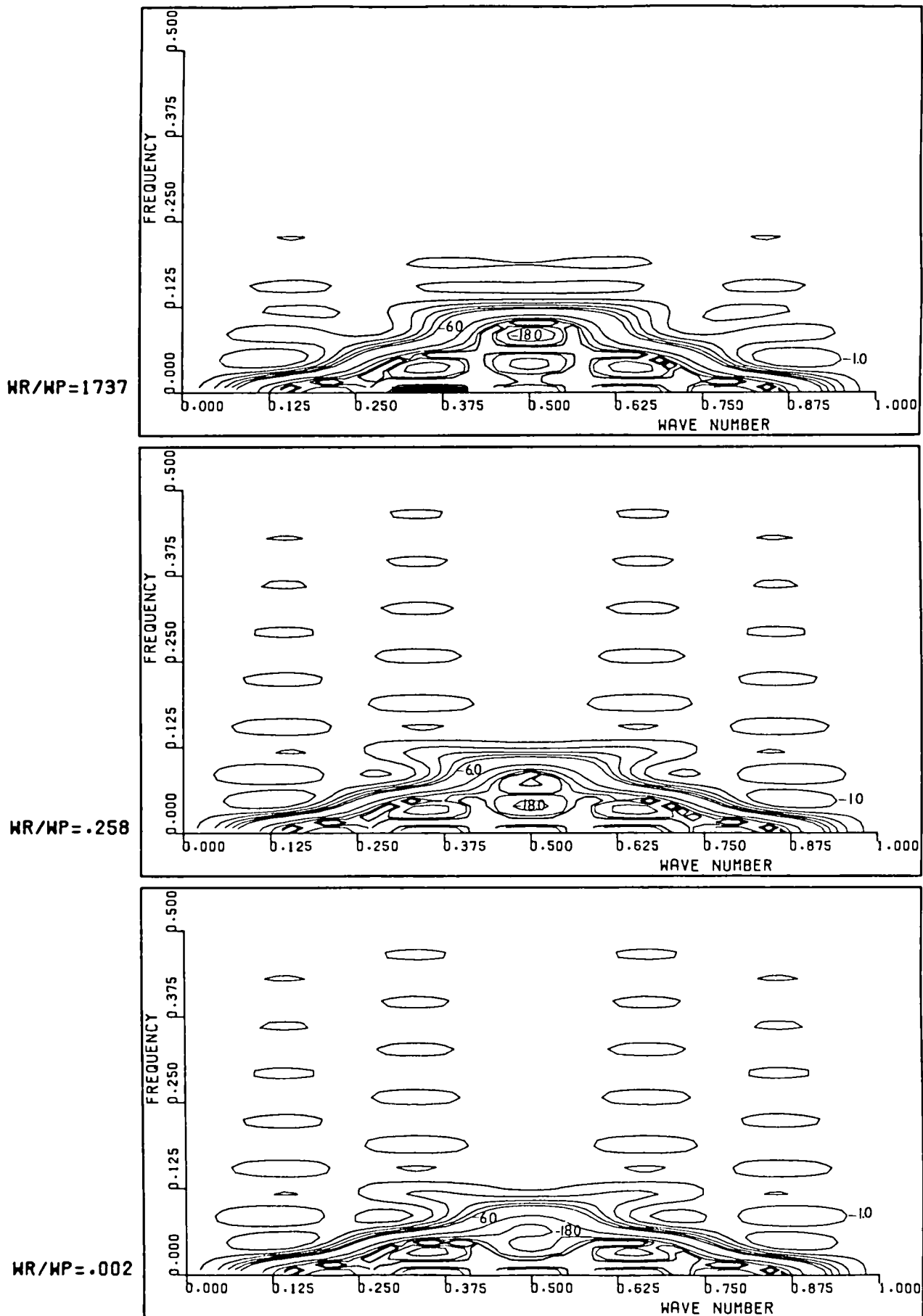
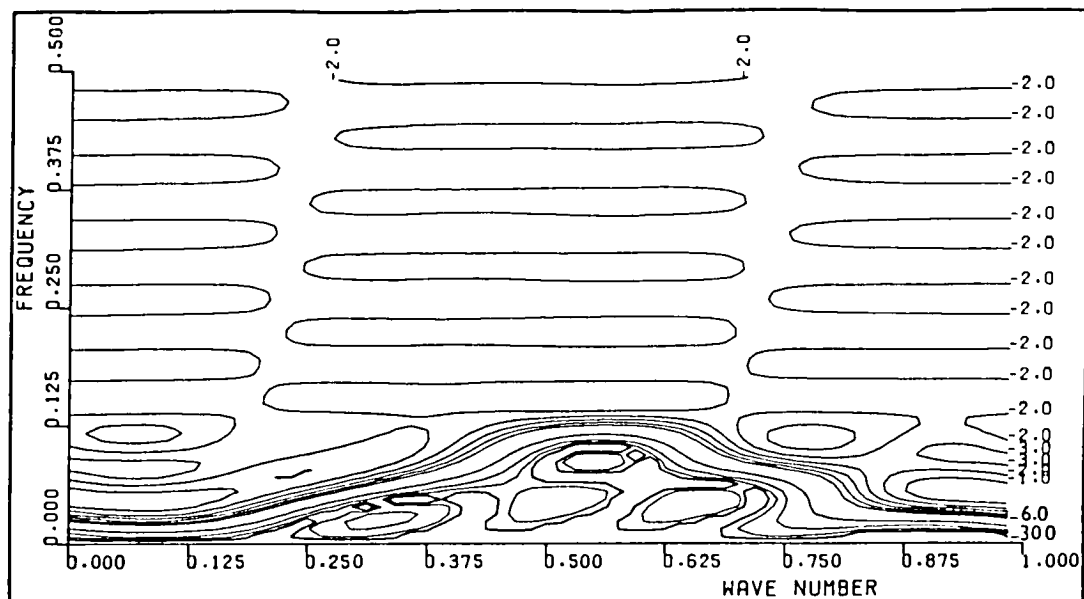
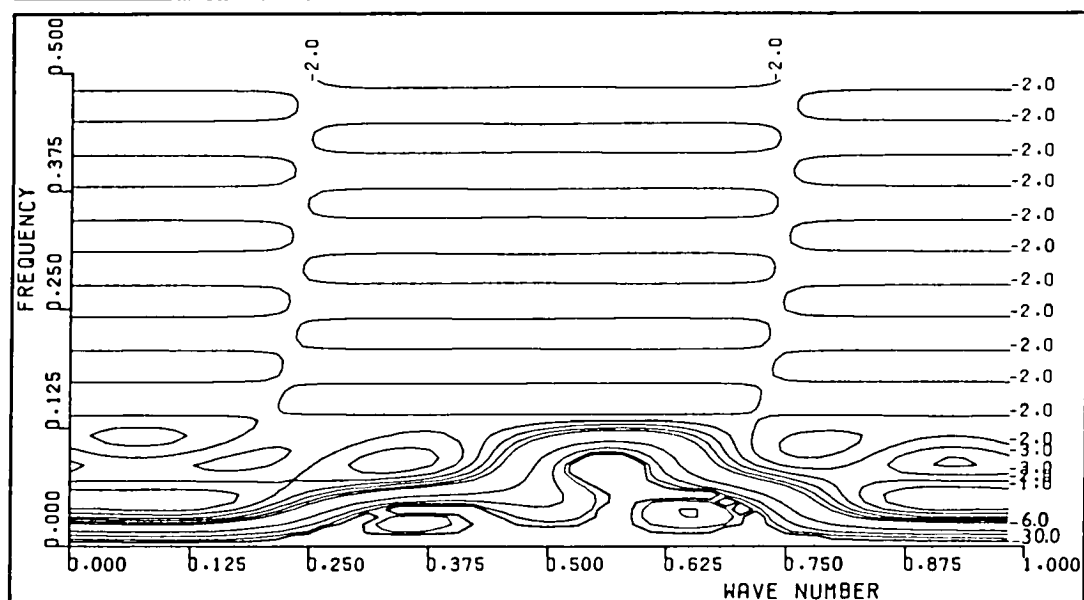


Figure 5.5.15 (f,k) responses of 7x29 'pass-origin' Tchebycheff moveout filters.

HR/HP=1737



HR/HP=.258



HR/HP=.002

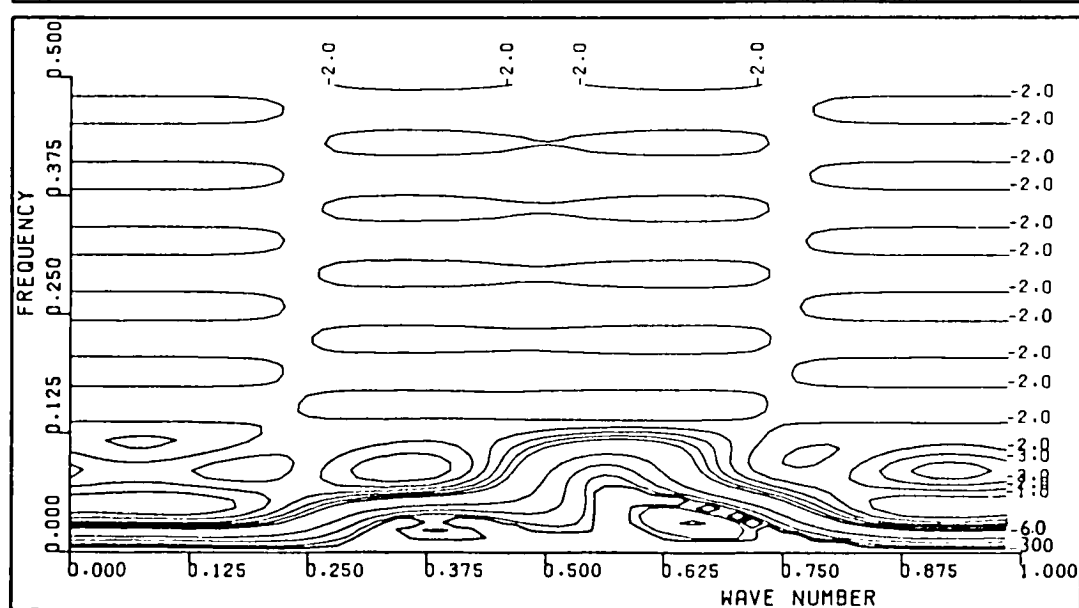


Figure 5.5.16 (f,k) responses of 7x29 'reject-origin' Tchebycheff moveout filters.

similar to those of the previous Tchebycheff filters with a low number of coefficients. Only big changes (about 10^3) of relative weighting of the prescribed regions affect the (f,k) responses considerably. On the other hand, the effect of the origin mapping is substantial, especially at the low frequency-wave number region. The pass domains of the (f,k) responses show the typical equiripple approximation characteristics of the 1D Tchebycheff or min-max design.

The responses presented in the latter four figures give similar conclusions to the previous examples: Tchebycheff moveout filters approximate pass windows consistently better than the least-squares moveout filters whatever the origin mapping for the former. The reject domains are approximated marginally better by the least-squares filters.

The outputs of the least-squares designs, Filters 5.5.13,14 and 15, for the raw data of Figure 5.5.17 are given in Figure 5.5.18A,B and C respectively. The outputs of the 'pass-origin' Tchebycheff designs, Filters 5.5.19,20 and 21, are presented in Figure 5.5.19A,B and C, respectively. And the outputs of the 'reject-origin' filters, Filters 5.5.22, 23 and 24, for the same raw data are displayed in Figure 5.5.20A,B and C, respectively. The processed data of Figure 5.5.18, compared with the raw data given in Figure 5.5.17B show that the least-squares moveout filters with varying correlated noise weights are equally efficient in removing the strong correlated noise event marked as shot noise.

The same processed data have very low S/N ratios which increase, though not substantially, with decreasing correlated noise weight. This is due to the reduced aliasing effect of the reject window into the pass window. The outputs of the Tchebycheff filters (Figures 5.5.19 and 5.5.20) do not differ significantly from one another. The correlated noise attenuation is not as good as it was as in the case of the least-squares outputs. The Tchebycheff filtered outputs show better trace-to-trace continuation even at times where there is high correlated noise arrivals overriding the events falling into the pass domains of the filters applied, e.g. events at about 1000 ms, 1300 ms and 1680 ms. The main reason for the remaining extra correlated noise on the Tchebycheff filter outputs, relative to the least-squares filter outputs, is the wave number aliasing during the recording. All the energy falling into the pass window of a Tchebycheff filter, including the aliased energy, is passed with about the same degree of efficiency. Consequently, even the most powerful multichannel techniques, that is, the techniques providing moveout filters with 'good' pass and reject window approximations, can only be effective in practice if the wave number aliasing is avoided during the field recording.

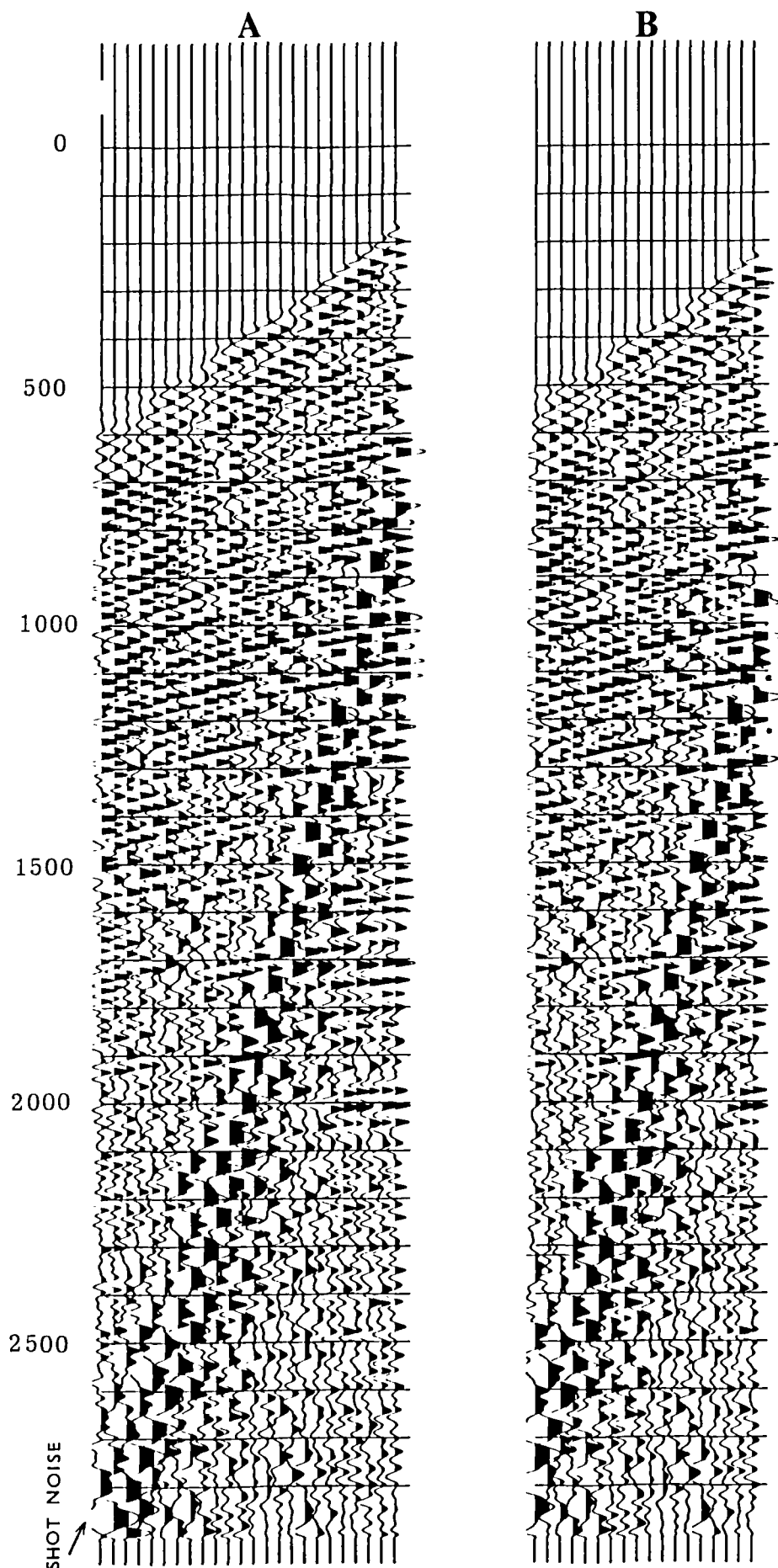


Figure 5.5.17 A) Offshore data with strong correlated noise (marked as shot-noise) and B) The same data plotted from trace 4 to 21.

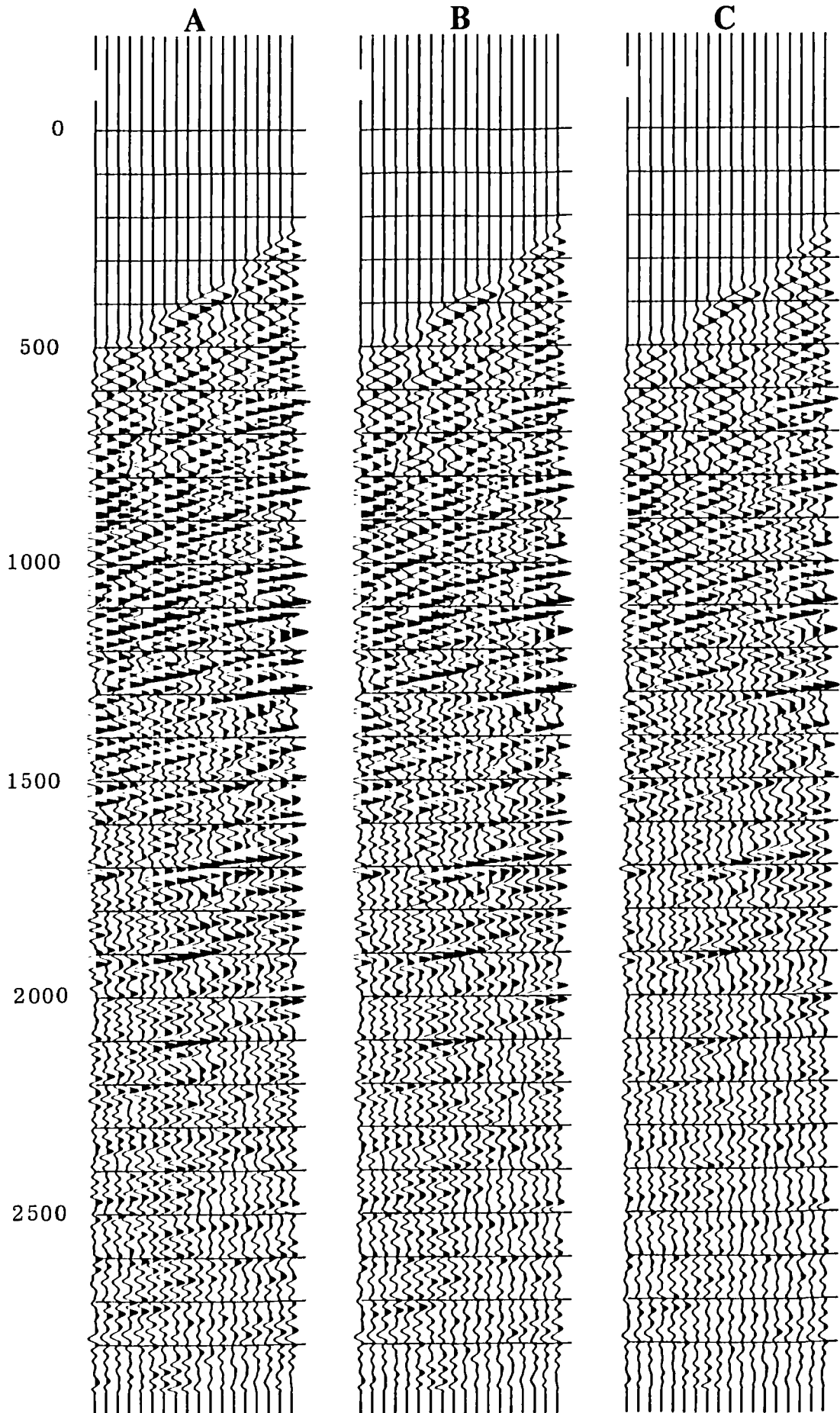


Figure 5.5.18 Outputs of 7x29 least-squares moveout filter designed with increasing correlated noise weights. The raw data are given in Figure 5.5.17.

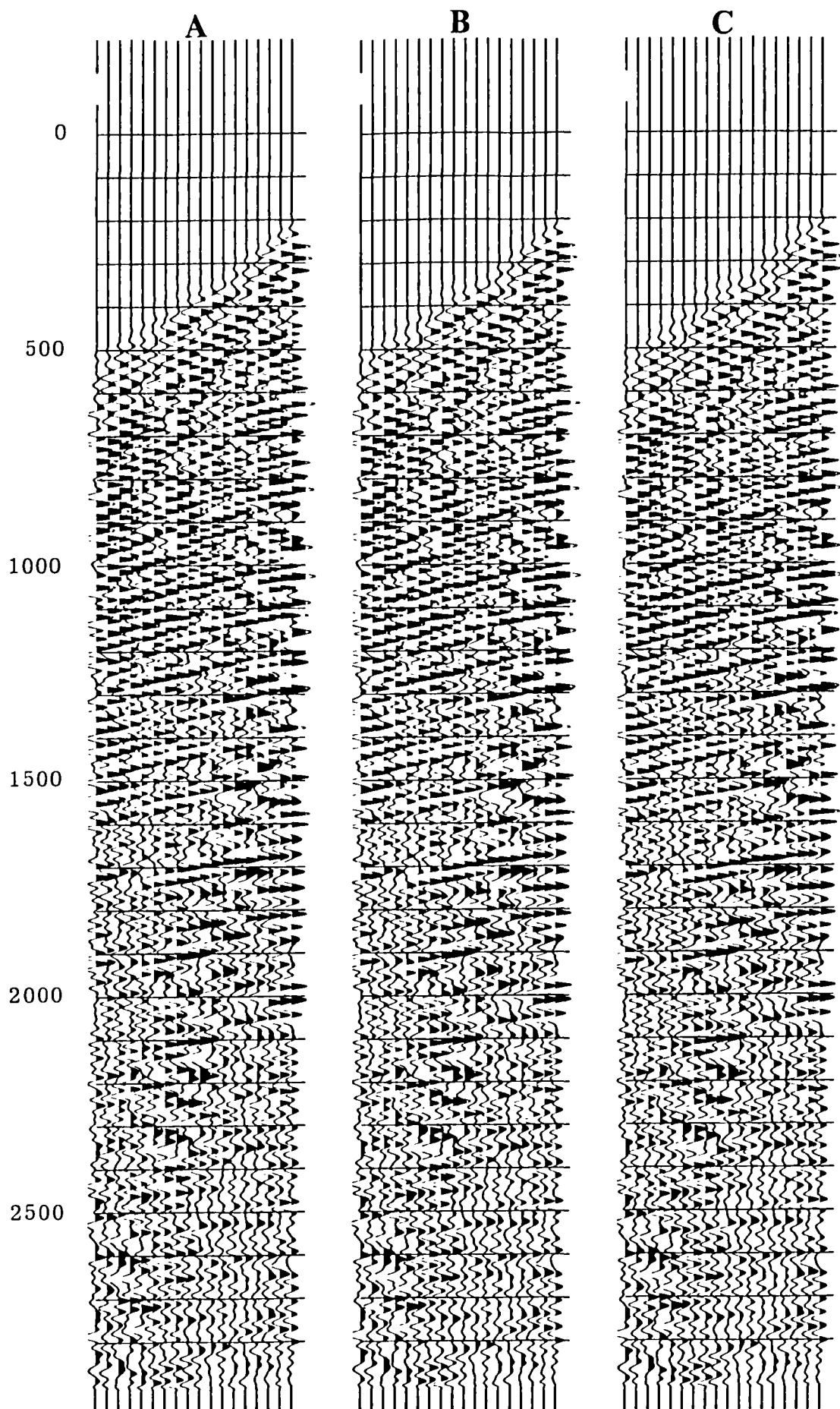


Figure 5.5.19 Outputs of 7x29 'pass-origin' Tchebycheff move-out filter designed with different w_r/w_p ratios. The raw data are given in Figure 5.5.17.

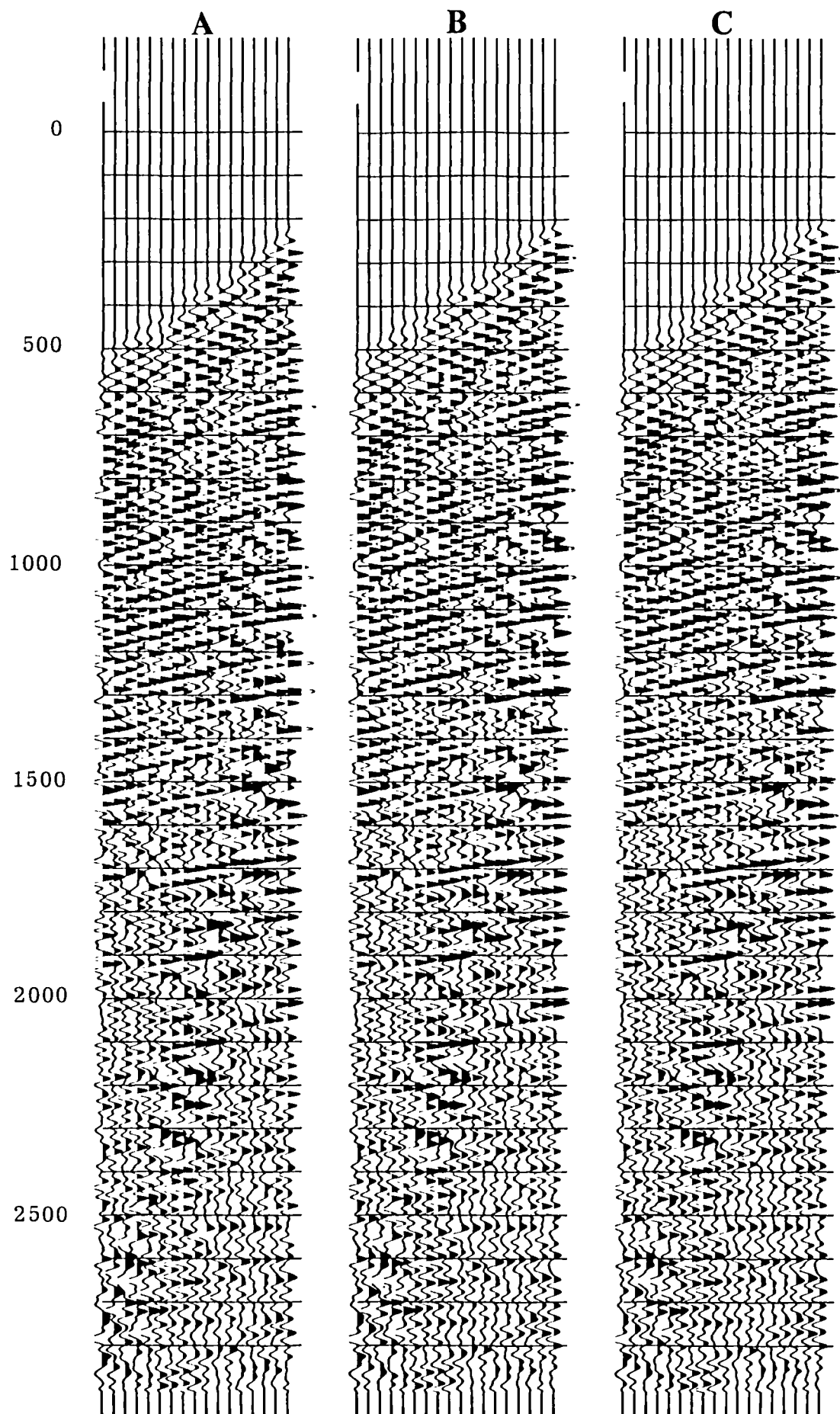


Figure 5.5.20 Outputs of 7x29 'reject-origin' Tchebycheff moveout filter designed with different w_r/w_p ratios. The raw data are given in Figure 5.5.17.

CHAPTER 6

TWO-DIMENSIONAL (2D) CIRCULARLY SYMMETRIC OPERATORS

In this chapter, the design techniques for two-dimensional (2D) filters are discussed. First an overview of previous approaches is given. Then the coefficient mapping technique, introduced in Chapter 5, is extended to the design of circularly symmetric nonrecursive filters. Finally some design examples of common filters are presented.

6.1 INTRODUCTION TO THE DESIGN TECHNIQUES FOR TWO-DIMENSIONAL CIRCULARLY SYMMETRIC NONRECURSIVE FILTERS

At the beginning of the previous chapter the designs of 2D linear shift invariant (LSI) operators were briefly reviewed. Most of the published design methods are for operators having circular symmetry in the Fourier domain. Hu and Rabiner (1972) and Fiasconaro (1973) used linear programming techniques to design 2D LSI filters. Kamp and Thiran (1974 and 1975) introduced techniques for designing maximally flat and Tchebycheff circularly symmetric 2D LSI filters. All these methods make a direct search in the 2D frequency or Fourier plane for an optimum set of coefficients and are limited to low order sets because of computational costs and/or storage problems on digital computers. It has been reported that the largest optimal filter designed using any of the above mentioned methods was 17×17 with an octagonal symmetry.

The second approach for designing 2D linear phase filters is McClellan's (1973) mapping method. This algorithm maps a 1D frequency response $B(\exp i\omega)$ into a 2D response $H(\exp i\omega_1, \exp i\omega_2)$ using the equation

$$\cos \omega = A \cos \omega_1 + B \cos \omega_2 + C \cos \omega_1 \cos \omega_2 + D \quad 6.1.1.$$

The method has been used to design circularly symmetric low-pass filters and 'pie-slice' filters (McClellan and Parks, 1972). Another method recently introduced, is Mersereau and Dudgeon's (1974 and 1975) projection technique. As outlined in the previous chapter, the ideal response is first approximated in the 1D frequency domain then it is projected to the 2D Fourier plane. The 2D response approximated in this way does not have all the symmetries of the ideal response. This method, like the others mentioned above, is not suitable for multichannel filters having complicated responses.

Two-dimensional discrete processing techniques were used in geophysics, for the enhancement of potential field data, long before fast digital computers were in operation. These techniques are the second derivative and the upward and downward-continuation methods. It has been shown that the second derivative methods such as Henderson and Zietz (1949), Elkins (1951) and Rosenbach (1953) are equivalent to a filtering operation with a response

function

$$H(\omega_1, \omega_2) = 4\pi^2(\omega_1^2 + \omega_2^2) \quad 6.1.2$$

(Swartz, 1954). Dean (1958) showed that the upward and downward-continuation operations on potential field data are equivalent to filtering with the following response functions

$$H_{up}(\omega_1, \omega_2) = \exp[-h(\omega_1^2 + \omega_2^2)^{\frac{1}{2}}] \quad 6.1.3$$

and

$$H_{down}(\omega_1, \omega_2) = \exp[h(\omega_1^2 + \omega_2^2)^{\frac{1}{2}}] \quad 6.1.4$$

respectively, where h is the distance between the gravitational source and the plane on which the continuation of its potential field is estimated. Meskó (1965 and 1966) compared the second-derivative and the continuation methods in terms of filter theory. Darby and Davies (1967) computed Henderson-Zietz, Elkins and Rosenbach second derivative responses using Meskó's results. According to their results only Rosenbach's method shows good circular symmetry like the theoretical second derivative response given in Equation 6.1.2, and they are all poor approximations to the theoretical response. They also designed 2D filters using circular ramp functions and inverse Fourier transforms after some smoothing of the desired response. These filters showed

good circular symmetry and were better than Rosenbach's second derivative operator in approximating the theoretical second derivative response. A 2D wave number domain design introduced by Wiggins (1966) is suitable for all desired response approximations including circularly symmetric filters. This method is based on computing a set of coefficients by minimizing the error between a desired and actual response. Zurflueh (1967) presented design and application examples of linear wavelength filters based on Darby and Davies (1967) and Dean (1958) design techniques. A review of the 2D frequency analysis and filters for potential field data is given by Fuller (1967) including strike sensitive operators which are similar to 'pie-slice' or fan-filters. Optimum second-derivative and downward continuation filters are designed and applied by Clarke (1969) using Wiener or the least-squares filter theory. The direct design technique of Lavin and Devane (1970) gives low-pass filters with good circular symmetry. This technique provides a means of error estimation in the approximating of a given response. Battacharyya (1972) designed 2D low-pass filters for potential field applications by computing the coefficients up to the cut-off wave number while having the coefficients falling outside the pass region (coefficients after the cut-off wave number) specified according to a suitable fall-off rate. The unknown coefficients are computed analytically from the circularly symmetric wave number response. Most of the potential field oriented 2D filter design techniques

mentioned above do not give a set of optimum coefficients. For optimum results, a desired response is specified and all the filter coefficients are varied to obtain the best approximation to the ideal response, using a suitable error criterion. The basic principles of 2D digital filtering are given by Clement (1973) where 2D frequency aliasing, 2D z-transform and 2D discrete convolution are explained. Appell (1974) analysed linear filters designed in the least-squares sense for separating regional and local gravity anomalies. The input model and the desired output was chosen as the gravity field of a buried point mass. He also designed minmax filters by making all the errors in the output equal. It was found that the latter filter did not produce any spurious (fake) anomalies where the optimum least-squares filter produce localized false concentrations. The transition width of the optimum least-squares filters were narrower than the minmax filter. The minmax filter has a more evenly distributed error curve. An intermediate filter compromising on the error and transition width of the above two designs was also presented.

6.2 DESIGN OF TWO-DIMENSIONAL CIRCULARLY SYMMETRIC FILTERS USING THE COEFFICIENT MAPPING METHOD

The coefficient mapping technique presented in the previous chapter, Section 5.3.2, can easily be extended to the design of any multichannel filter with prescribable frequency domain responses. In this chapter the argument

is confined to the design of the most common type filters, namely, circularly symmetric 2D FIR filters.

It was shown that the Fourier domain coefficients of a 2D filter can be mapped onto a 1D frequency axis using Equation 5.3.2.21, namely,

$$B(N_2 k_1 + k_2) = H(k_1, k_2) \quad 6.2.1.$$

The distance of every coefficient from the origin of the 2D frequency domain, for an odd number of coefficients in both dimensions, was given in Equation 5.3.2.13 as

$$R(k_1, k_2) = \left[\left(k_1 \frac{2\pi}{N_1 - 1} \right)^2 + \left(k_2 \frac{2\pi}{N_2 - 1} \right)^2 \right]^{\frac{1}{2}} \quad 6.2.2.$$

If the direct mapping given in Equation 6.2.1 is used to estimate the cut-off frequencies on the 1D frequency axis, there will be too many pass and reject regions, as it was in the case of moveout filters. On the other hand, $R(k_1, k_2)$ values given in Equation 6.2.2 are most conveniently used for the ordering of the 2D frequency domain coefficients falling into pass, reject and transition bands. The eight steps for 1D-2D frequency domain coefficient mapping and design, given at the end of Section 5.3.2, are the basis of the circularly symmetric filter design method using the coefficient mapping technique. The only difference is that, instead of the angles $A(k_1, k_2)$ the radii $R(k_1, k_2)$ values are used to implement the sorting of the coefficients in 1D frequency domain. This is because when the coefficients

are ordered in this way, there will be as many prescribed and 'don't care' domains in the 1D frequency domain as there would be in the corresponding 2D domain. Consequently, the calculation of the cut-off frequencies in the corresponding 1D frequency axis is straightforward.

To reduce the problem to a 1D FIR linear phase filter design, the coefficients of the upper half-plane of the 2D Fourier domain are mapped to the 1D frequency axis, $0.0 \leq F \leq 0.5$, after the sorting procedure. The upper half-plane includes coefficients falling on half of the first principle axis, i.e. $-0.5 \leq F_1 \leq 0$ or $0 \leq F_1 \leq 0.5$. Figure 6.2.1 shows the locations of the frequency samples for a 15×15 grid. After the mapping and sorting, 1D desired response cut-off frequencies will be defined in a form sketched at the bottom of the same figure. The lower half-plane coefficients and coefficients falling on one half of F_1 axis are estimated from the symmetry.

It was pointed out in the previous chapter that the coefficient mapping technique can be formulated in different forms. The one adopted so far is most suitable for the digital filter design problems. According to the nature of the desired 2D response, the same convention can be used but this time mapping is done rowwise, as

$$B(k_1 + N_1 k_2) = H(k_1, k_2) \quad 6.2.3.$$

Depending on the distribution of the transition zone coefficients in the 2D frequency plane, the rowwise mapping

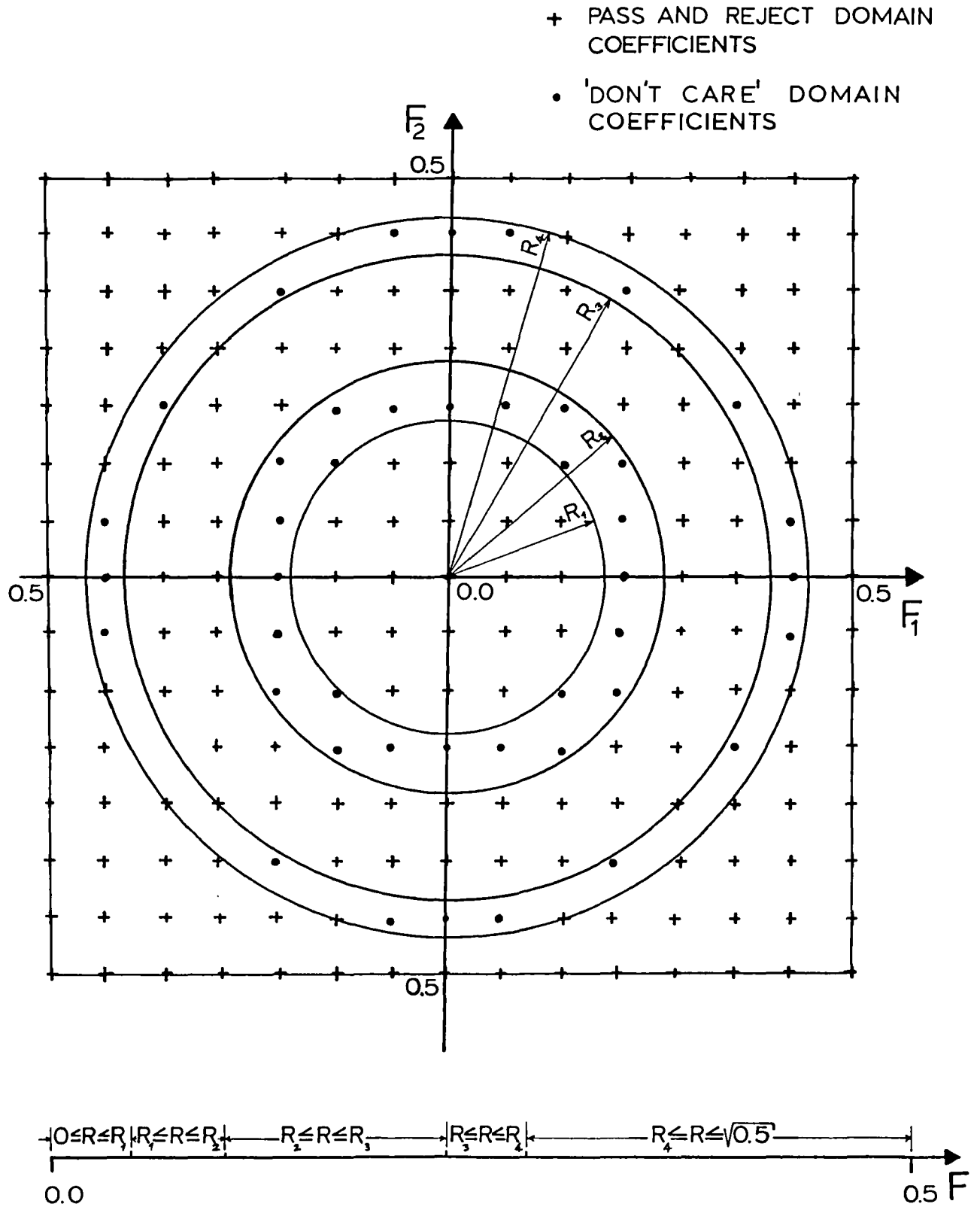


Figure 6.2.1 The locations of the frequency domain samples for a 15x15 2D filter with a superimposed circularly symmetric desired response (upper), and the desired response of the same 2D filter as mapped into the 1D frequency axis (lower).

should be used to improve the symmetry required for the desired response.

The computer program for computing 2D circularly symmetric filters using the above technique is similar to that of linear moveout filter program outlined in Appendix 4. The only difference is that, in this case the radii of 2D discrete Fourier coefficients are used for sorting of the coefficients as explained above.

6.3 DESIGN EXAMPLES OF TWO-DIMENSIONAL CIRCULARLY SYMMETRIC FILTERS

This section contains some examples of circularly symmetric filters designed using the coefficient mapping algorithm presented in the previous section. Experience with 2D filter designs of this kind, using the new technique, showed that, to obtain good symmetries the choice of the number of coefficients in both dimensions is important. Therefore, before any computational attempt it is advisable to make a sketch of the desired response with the 2D discrete Fourier coefficients superimposed. Another good reason for making this desired response sketch is to make sure that more than one coefficient falls into each transition or 'don't care' band for an adequate approximation of the corresponding 1D frequency response.

The first example is an 11x11 low-pass filter with the following parameters:

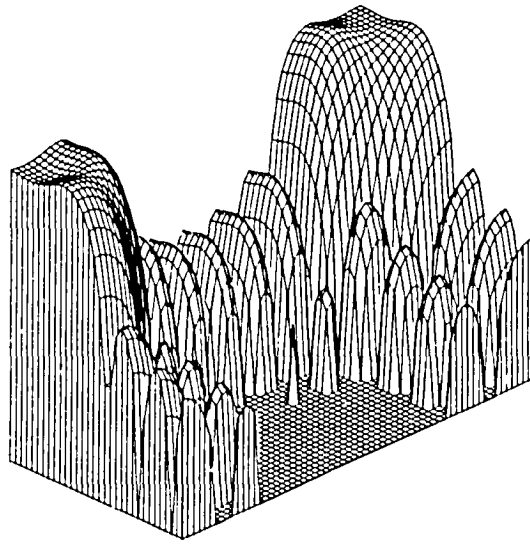
$$\begin{aligned}
 F_p &= 0.0 \text{ to } 0.2 & \text{with } w_p &= 10.0 \\
 F_r &= 0.3 \text{ to } 0.5 & \text{with } w_r &= 1.0 & \text{and} \\
 0.2 &< F_d < 0.3
 \end{aligned}$$

where

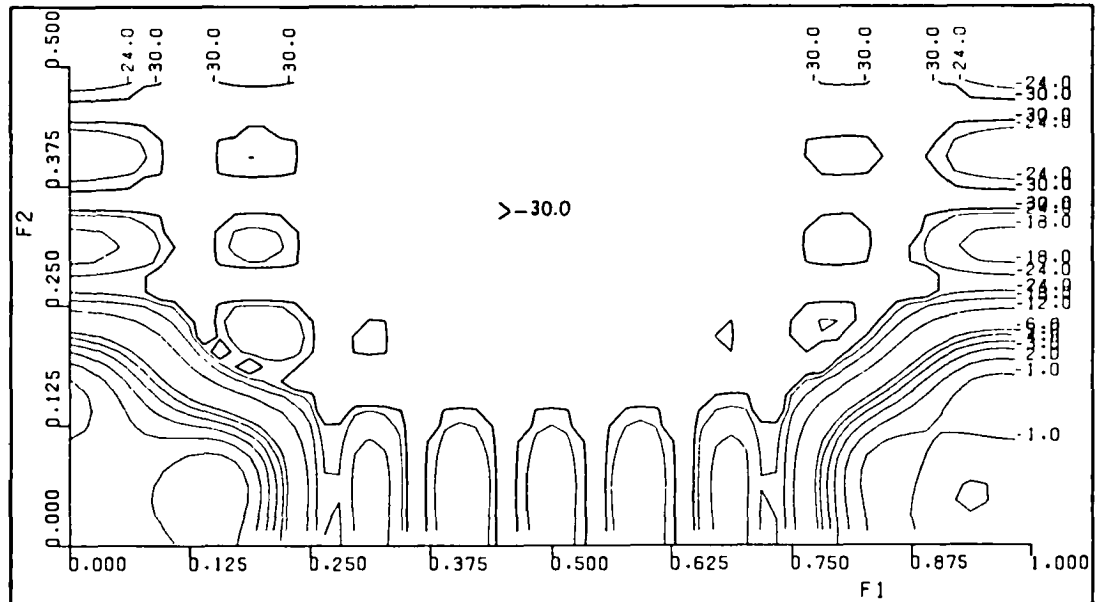
F_p : Normalized band edge frequencies for pass band,
 F_r : Normalized band edge frequencies for reject band,
 F_d : Normalized band edge frequencies for transition band,
 w_p : Weight for pass domain,
 w_r : Weight for reject domain.

Figure 6.3.1A shows the frequency response of this filter designed via columnwise mapping, i.e. using Equation 6.2.1. The same filter is redesigned this time employing the row-wise mapping Equation 6.2.3. The frequency response for the latter design is given in the same figure above (Figure 6.3.1B) with its isometric plot. It is evident from the responses presented that a good approximation to the desired response is obtained using either mapping convention. A closer examination shows that the rowwise mapping improves the overall approximation, pass and reject domains alike. The pass domain approximation is nearly as good as -1 dB. The reject domain attenuation is about 30 dB except at small (F_1, F_2) where it is about 18 to 30 dB. The maximum attenuations achieved by the columnwise and rowwise mapping filters are 94.72 and 106.20 dB, respectively.

The second example is another low-pass filter with 9x9 coefficients and the following input parameters:



B



A

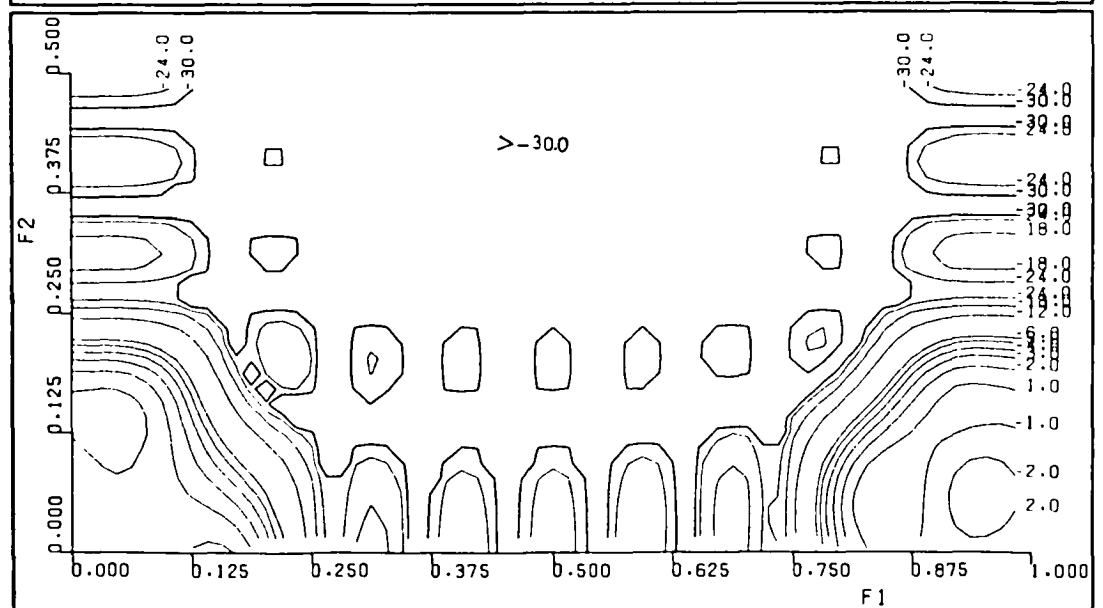
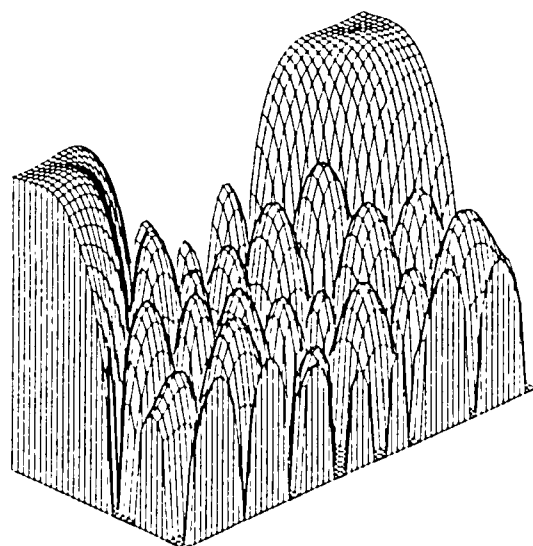
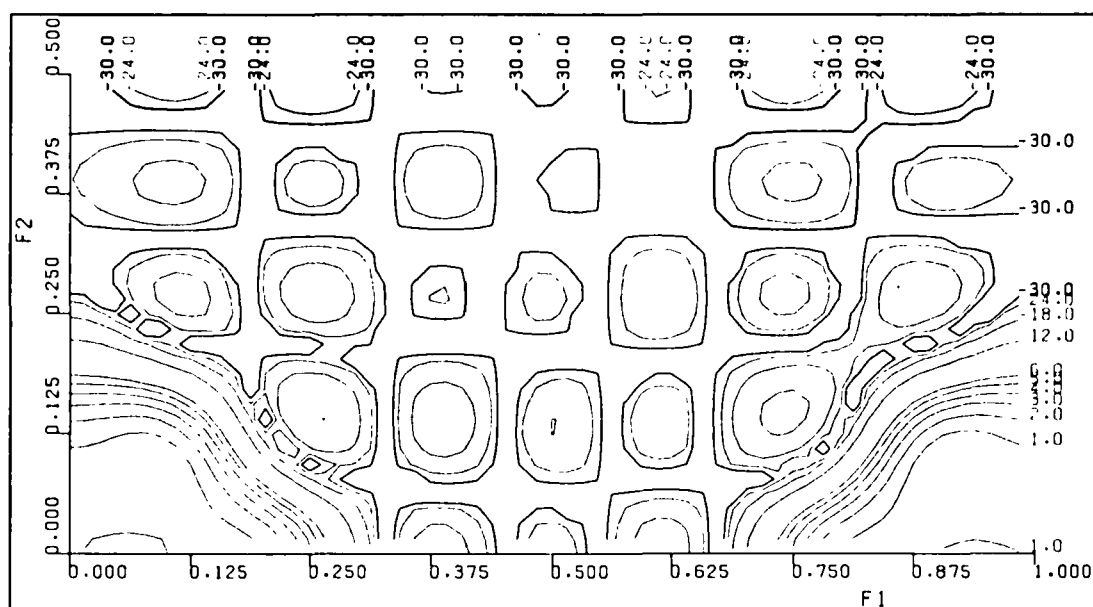


Figure 6.3.1 Frequency response of 11x11 low-pass filter with band edge frequencies: 0.0,0.2,0.3,0.5. A) Designed via columnwise mapping, B) Designed via rowwise mapping. The isometric plot is viewed from (1.0,0.5).



B



A

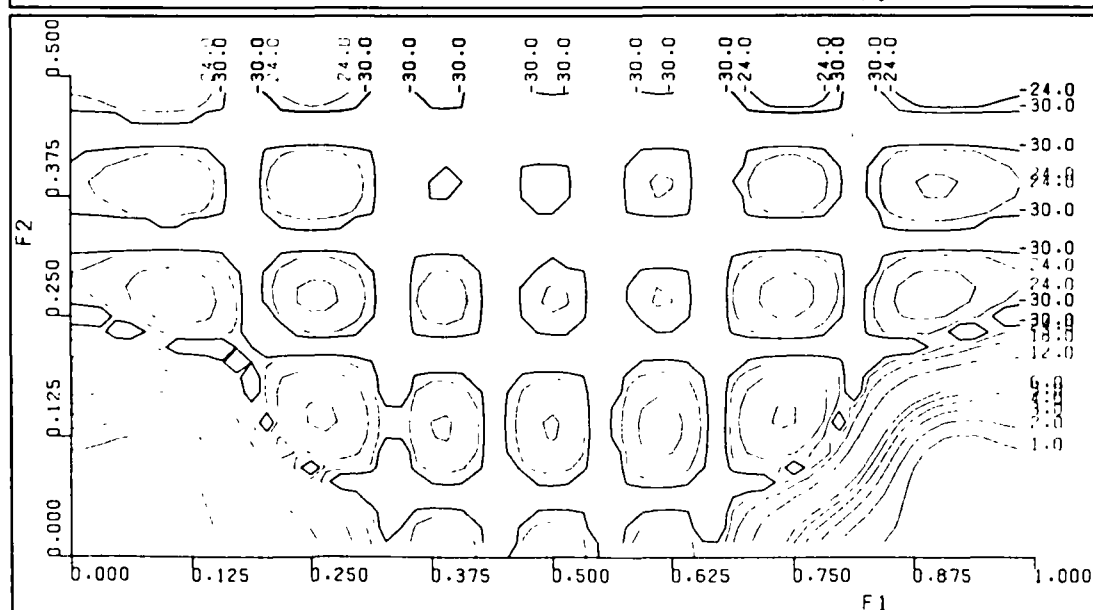


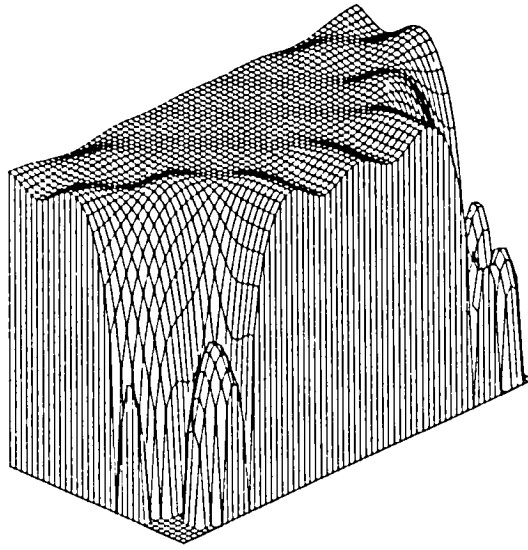
Figure 6.3.2 Frequency response of 9x9 low-pass filter with band edge frequencies: 0.0,0.2,0.2625,0.5 A) $w_p = w_r = 1$, B) $(w_r/w_p) = 0.1$. The isometric plot is viewed from (1.0,0.5).

$$\begin{aligned}
 F_p &= 0.0 \text{ to } 0.2 && \text{with } w_p = 1.0 \\
 F_r &= 0.2625 \text{ to } 0.5 && \text{with } w_r = 1.0 \quad \text{and} \\
 0.2 &< F_d < 0.2625.
 \end{aligned}$$

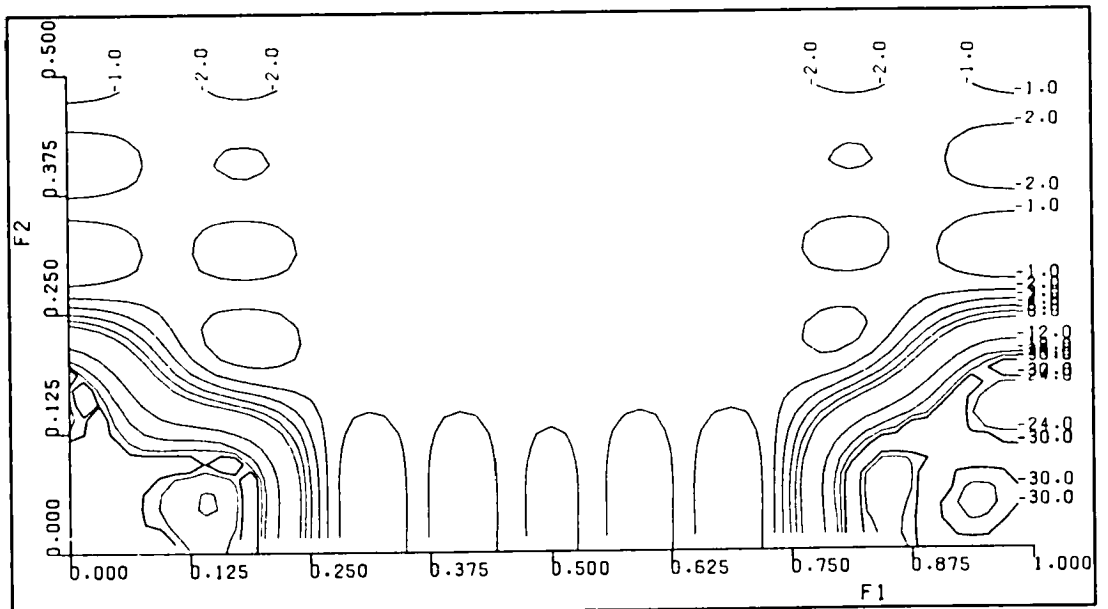
The above filter and the remaining examples of this section are all designed using rowwise mapping. The frequency response of this filter is given in Figure 6.3.2A. The latter filter is redesigned this time with $w_p = 10.0$ and the frequency response for the new filter is given in Figure 6.3.2B with its isometric plot. Comparison of the latter two responses, computed for the same cut-off frequencies, indicates that pass band approximation is improved at the expense of reject domain degradation even with a small increase in the relative weighting of the pass domain of the 2D low-pass filter. The pass band is approximated with about 1 dB deviation except at band edges where it is about 2 dB to 6 dB. The reject band attenuation is about 12 to 30 dB near the pass band and better than about 24 dB is achieved at higher frequencies. The maximum attenuations obtained by these filters designed with $w_p = w_r = 1.0$ and $w_p = 10.0, w_r = 1.0$ are 88.46 dB and 103.52 dB, respectively.

The third example is an 11x11 high-pass filter designed with the same parameters as the first example except in this case the pass and reject regions are interchanged:

$$\begin{aligned}
 F_r &= 0.0 \text{ to } 0.2 && \text{with } w_r = 1.0 \\
 F_p &= 0.3 \text{ to } 0.5 && \text{with } w_p = 1.0 \quad \text{and} \\
 0.2 &< F_d < 0.3.
 \end{aligned}$$



B



A

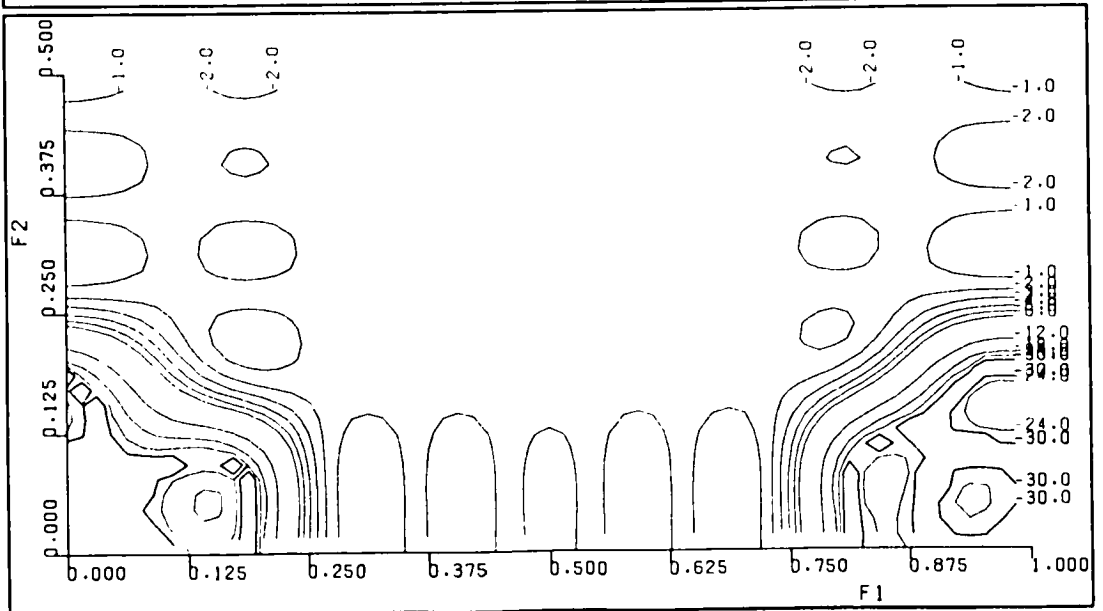


Figure 6.3.3 Frequency response of 11x11 high-pass filter with band edge frequencies: 0.0,0.2,0.3,0.5. A) $w_p = w_r = 1$, B) $(w_r/w_p) = 0.1$. The isometric plot is viewed from (0.0,0.0).

The frequency responses of the above filter and the same filter designed with $w_p = 10.0$, $w_r = 1.0$ are given in Figure 6.3.3. The general features of the transition band and divergence from the circular symmetry are the same as in the corresponding low-pass filter (Figure 6.3.1B). The pass domain approximation is as good as -1 to -2 dB at small frequencies whereas better than -1 dB approximation is achieved at high frequencies. Relatively small increase of the pass domain weight made only small changes. The reject domain attenuations are about 24 to 30 dB or higher. The maximum attenuations achieved are about 110.08 dB and 91.98 dB corresponding to these filters with $w_p = 1.0$ and $w_p = 10.0$, respectively.

The fourth example is a 7x11 bandpass filter. The design parameters for this filter are:

$$F_p = 0.225 \text{ to } 0.375 \quad \text{with } w_p = 0.1\text{E-}01$$

$$F_r^{(1)} = 0.0 \text{ to } 0.175 \quad F_r^{(2)} = 0.425 \text{ to } 0.5 \quad \text{with } w_r^{(1)} = w_r^{(2)} = 0.15867981\text{E-}03$$

and

$$0.175 < F_d^{(1)} < 0.225, \quad 0.375 < F_d^{(2)} < 0.425.$$

The frequency response of this filter is given in Figure 6.3.4 with its isometric plots. The approximation of the pass and reject bands is not as good as it was in the case of the low pass filters. This is usually the case for the bandpass filters which need higher order coefficients to approximate the desired response reasonably well. The maximum attenuation achieved by this filter is 92.01 dB.

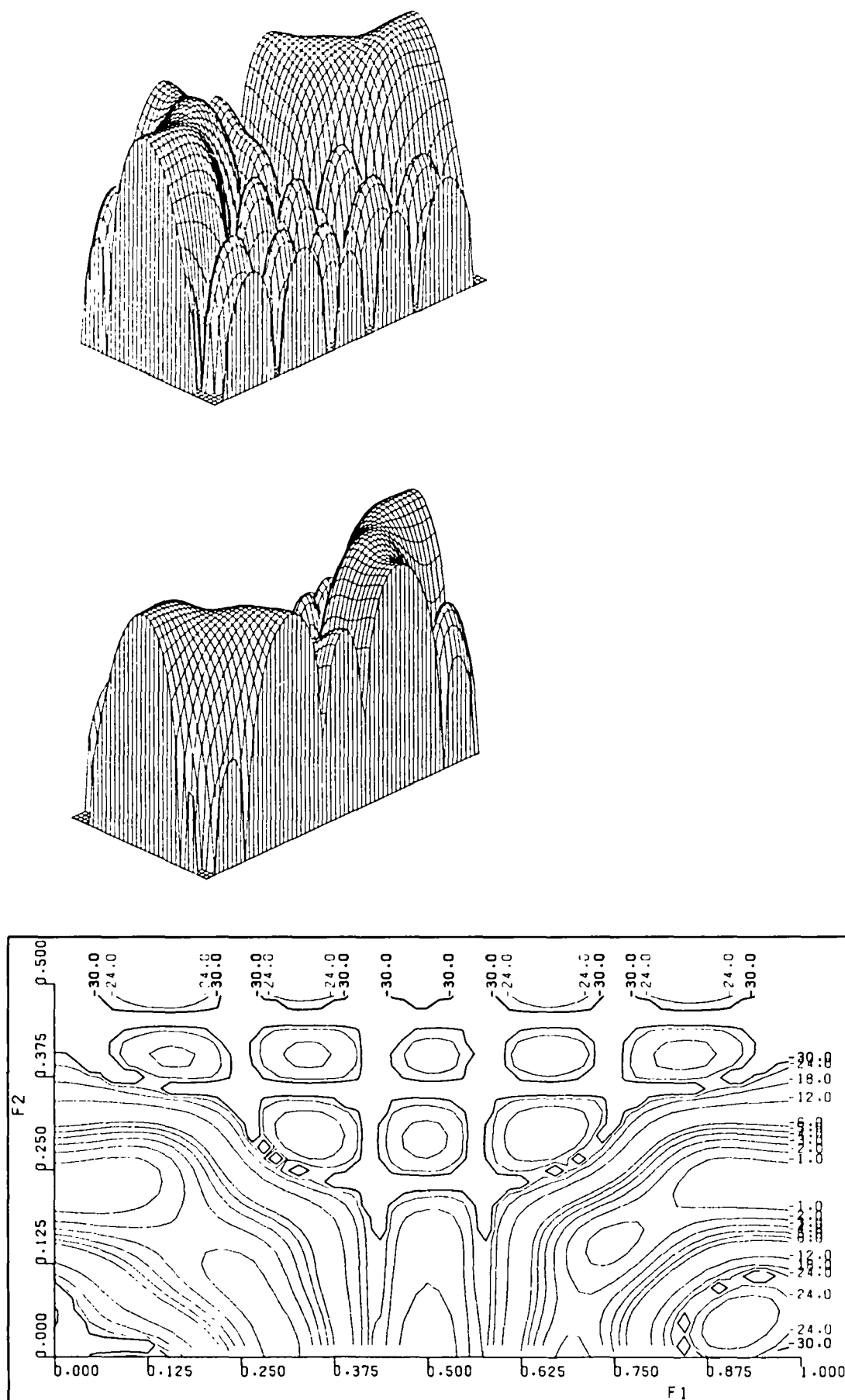


Figure 6.3.4 Frequency response of 7x11 bandpass filter with band edge frequencies: 0.0,0.175,0.225,0.375,0.425,0.5. The lower isometric plot is viewed from (0.0,0.0) and the upper from (1.0,0.5).

The fifth circularly symmetric filter design example is also a bandpass filter with 15x15 coefficients. This filter has very narrow transition bands. The design parameters are:

$$F_p = 0.10278 \text{ to } 0.23333 \quad \text{with } w_p = 0.28242918\text{E-}03$$

$$F_r^{(1)} = 0.0 \text{ to } 0.066667, F_r^{(2)} = 0.26389 \text{ to } 0.5$$

$$\text{with } w_r^{(1)} = w_r^{(2)} = 0.27483211\text{E-}03$$

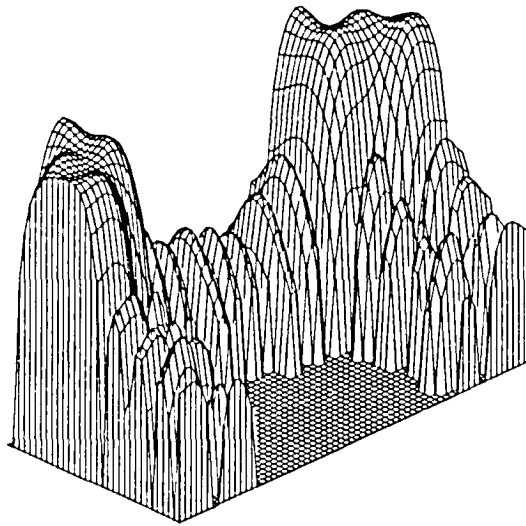
and

$$0.066667 < F_d^{(1)} < 0.10278, 0.23333 < F_d^{(2)} < 0.26389.$$

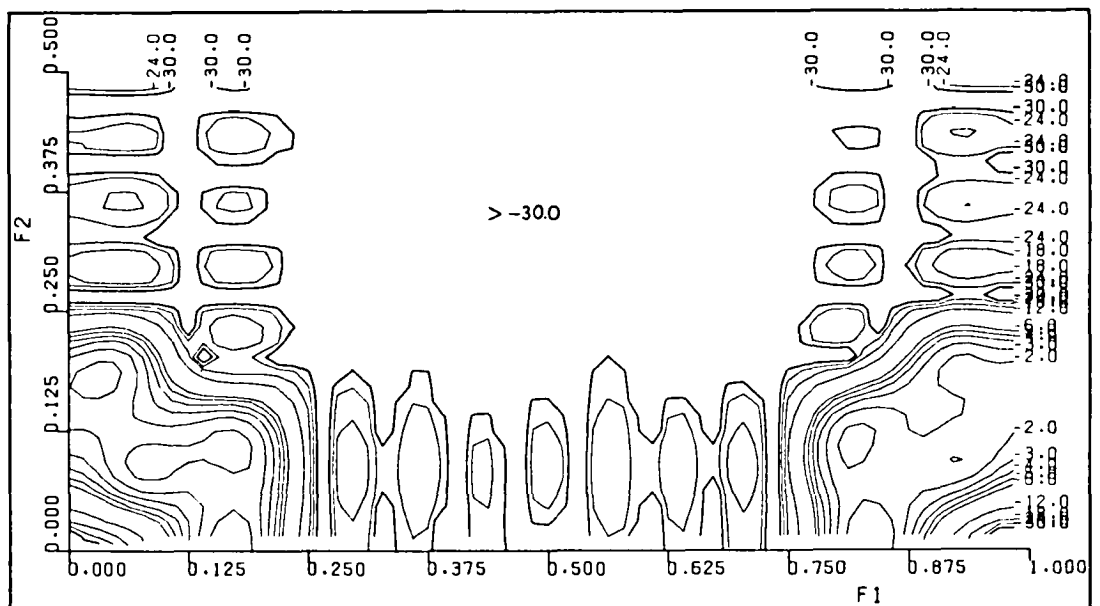
Figure 5.3.5A shows the frequency response of the above filter. The same filter is redesigned this time with the weights:

$$w_p = 0.1\text{E-}01 \text{ and } w_r^{(1)} = w_r^{(2)} = 0.35497699\text{E-}05.$$

The frequency response of the latter filter is given in Figure 6.3.5B with its isometric plot. These responses show typical characteristics of 2D bandpass filters designed using the coefficient mapping technique. The desired circularly symmetric responses are not approximated as good as in the case of simpler responses such as low-pass filters. The increased pass band relative weight did not make substantial changes. The pass band approximation is as good as -1 to -2 dB except at band edges where a -3 to -6 dB approximation is achieved. The reject band attenuation is about -24 to -30 dB near the origin and a better than a -30 dB attenuation is obtained at regions away from the frequency axes. The maximum attenuation obtained by the latter filter



B



A

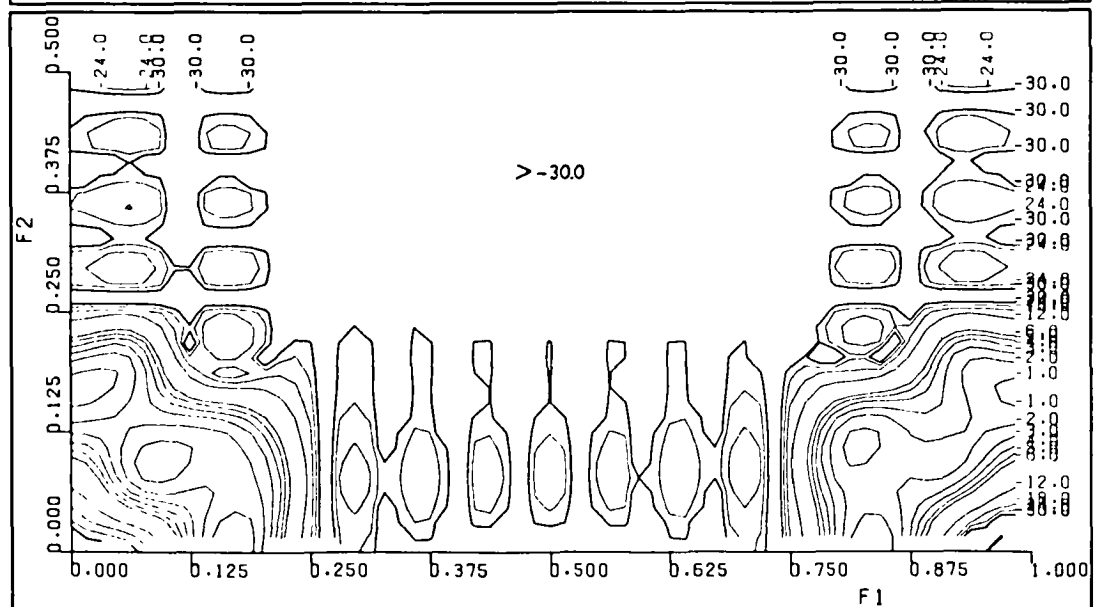


Figure 6.3.5 Frequency response of 15x15 bandpass filter with band edge frequencies: 0.0, 0.066667, 0.102780, 0.233333, 0.26389, 0.5. A) $(w_r/w_p) \approx 1$, B) $(w_r/w_p) \approx 2818$. The isometric plot is viewed from (1.0, 0.5).

is 90.77 dB while the previous design with almost equal pass and reject domain weights achieved 87.01 dB.

The sixth and last example is a 15x15 band reject filter designed with the same cut-off frequencies as the above filter whose frequency response is given in Figure 6.3.5A, except that the pass and reject domains are interchanged. That is:

$$F_p^{(1)}=0.0 \text{ to } 0.0066667 \quad F_p^{(2)}=0.26389 \text{ to } 0.5 \quad \text{with } w_p^{(1)}=w_p^{(2)}=0.27483211\text{E-}03$$

$$F_r=0.10278 \text{ to } 0.23333 \quad \text{with } w_r=0.28242918\text{E-}03$$

and

$$0.066667 < F_d^{(1)} < 0.10278, \quad 0.23333 < F_d^{(2)} < 0.26389.$$

The frequency response is given in Figure 6.3.6A. Figure 6.3.6B shows the frequency response and its isometric plot for the same band reject filter designed using the weights:

$$w_p^{(1)}=w_p^{(2)}=0.35497699\text{E-}05 \quad \text{and } w_r=0.1\text{E-}01.$$

The frequency response of the filter with almost equal weights (Figure 6.3.6A) has a better symmetry than that of the higher reject region weight (Figure 6.3.6B). There is slight improvement in the reject domain approximation with increased weight. The pass domain approximations are about -2 to -5 dB at the low frequencies (the first pass band) and -1 to -3 dB at the high frequencies (the second pass band). The reject domain attenuation is about -24 to -30 dB and higher. The filter with approximately equal weights achieved a maximum attenuation of 61.95 dB, and that of the higher reject weight 70.53 dB.

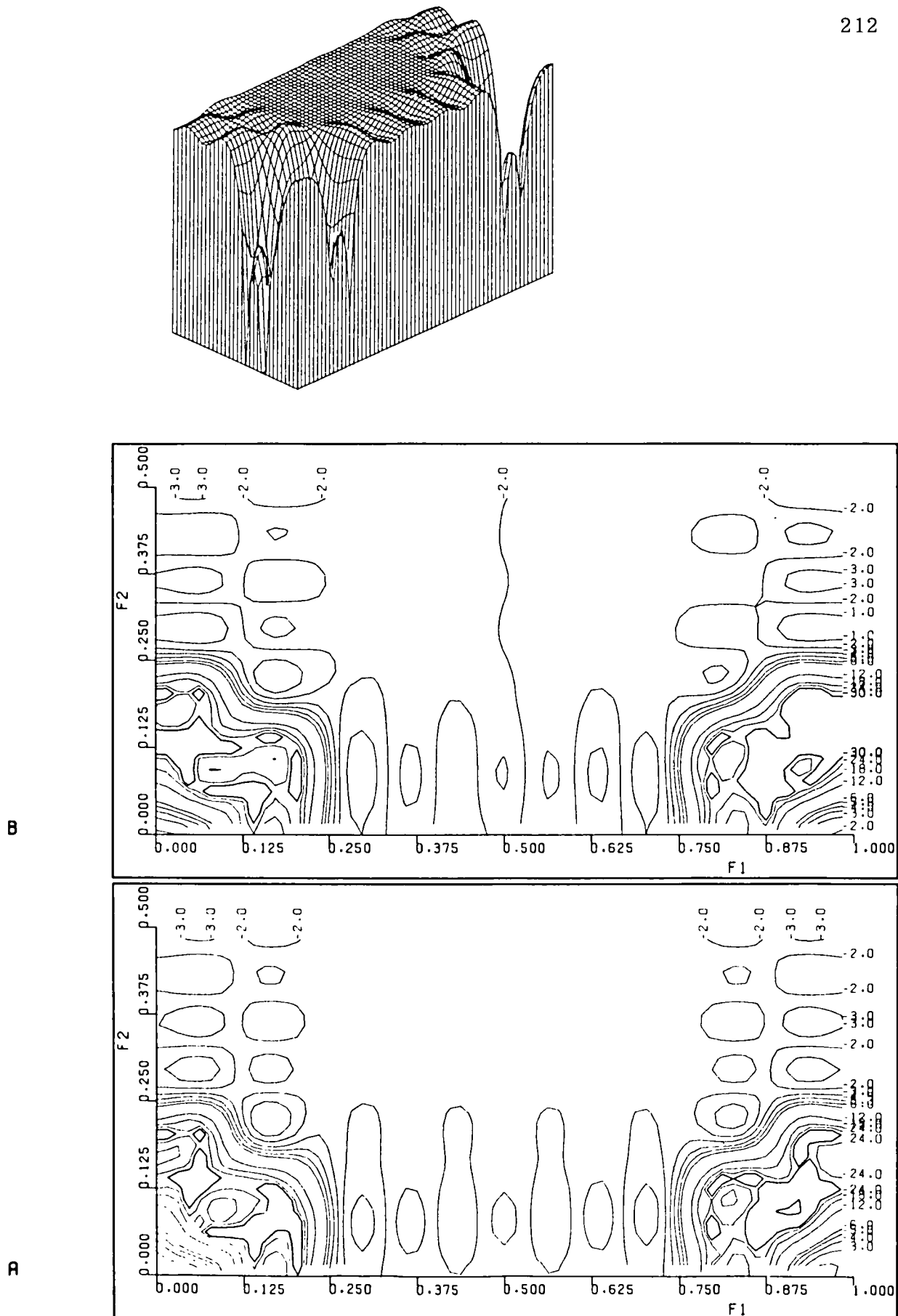


Figure 6.3.6 Frequency response of 15x15 band-reject filter with band edge frequencies: 0.0,0.066667,0.10278,0.233333,0.26389,0.5. A) $(w_r/w_p) \approx 1.0$, B) $(w_r/w_p) \approx 0.35E-4$. The isometric plot is viewed from (0.0,0.0).

CHAPTER 7

CONCLUDING REMARKS

The material presented in the various sections of this thesis is usually conclusive. Consequently, some of the results given here are the summary of points already covered in detail.

The optimum multichannel least-squares (MLS) moveout filter design technique allows the designer to emphasize the signal or noise power in the input processes. In the final derivations only the autocorrelation matrix of the input processes and the crosscorrelation matrix between the desired output and the input processes are required to compute the filter coefficients, which are obtained using suitable input and desired output models. The input model also incorporates a chatter parameter to account for divergencies of signal and correlated noise arrivals from their prescribed moveout curves. However, the actual results obtained from the use of this conventional input model describing the seismic reflection input traces are not always in full agreement with the theory. Some additional aspects of optimum MLS moveout filters, which have gained emphasis and clarity in this study, are summarized below.

The normalized mean-square-error (NMSE) at the outputs of optimum MLS moveout filters, which always increases with increased correlated noise, uncorrelated noise and chatter in the filter design input model, is dependent on the pass-

reject window configuration and the corresponding window weightings. The NMSE versus filter length curves indicate that for filters with simple responses a filter length of about as long as the biggest stepout time in the moveout window configuration is required, whilst twice this length is necessary for filters with complicated responses.

Including chatter in the input model of optimum MLS moveout filters is equivalent to high-cut frequency filtering. It also helps to stabilize the autocorrelation matrix of the model input processes. In practice there is no gain from the inclusion of this parameter, contrary to its theoretical significance in making the input models more realistic, even if the chatter values in the input traces and in the filter design are matched. Since the high frequency filtering, if needed, and stability of the autocorrelation matrix can be obtained more conveniently by other means, the chatter parameter should be omitted from the input model of optimum MLS moveout filters.

The autocorrelation matrix of input model traces is always singular if the input has zero chatter and no uncorrelated noise. In the case of linear moveout filters a relatively small amount of uncorrelated noise ($w_n = 0.3$ to 0.5) is enough to stabilize the autocorrelation matrix, which has very little undesirable effect on the (f,k) responses such as the high frequency filtering. Linear moveout filters for seismic reflection applications become badly approximated low-pass filters when very high uncorrelated noise power is

added to the input models. There is no gain in uncorrelated noise rejection if the relative uncorrelated noise power in the filter design is matched to, or even higher than, the corresponding relative power in the input traces. For linear moveout filters, uncorrelated noise relative weightings of about $w_n = 0.3$ to 1.0 give the best results both in design and application.

Wave number aliasing is a serious limitation of optimum MLS moveout filters especially in the case of linear moveout filters. Even if wave number aliasing is avoided during the recording, it is inherited in the MLS moveout filters unless aliasing regions relative weightings are kept very small and this results in very poor approximation of these regions. In seismic reflection applications aliasing becomes a severe problem only when reject domains alias into the frequencies where signal power is contained. If the aliasing of the correlated noise is not avoided during the recording, increased correlated noise weighting in the filter design increases the rejection of aliased noise as well as uncorrelated noise and signal power at high frequencies. This improves the alignment of signal arrivals but causes signal distortion and reduces the resolution of the data. Spurious line-ups of low frequency noise is created with increased aliased reject region weightings. Increased aliased reject region weighting is similar to frequency filtering in one-dimension with increasingly narrower pass bands. There is no extra improvement in the signal to correlated noise ratio if the relative correlated noise

power of the filter input model is matched to the corresponding power distribution of the input data. In seismic reflection applications, if there is no disturbance due to wave number aliasing, the reject domain relative weightings of higher than unity should be used.

Optimum MLS filters can be designed for processing data containing signal and correlated noise events falling on hyperbolic moveout curves. If these filters are designed over a time gate rather than for a fixed two-way time the autocorrelation and crosscorrelation functions entering the normal equations are reduced to two-fold integrals in time and velocity which cannot be solved analytically. Numerical solutions on computer, at comparable costs to the linear moveout cases, are feasible. The design algorithm also includes filters with pass and/or reject regions having different moveout functions, such as linear and hyperbolic. Uncorrelated noise relative weightings higher than unity ($w_n = 1.5$ to 3.0), are required to obtain stable hyperbolic moveout filters. These filters are very effective over the entire time gate for which they are designed if their pass and reject domains do not overlap. An accurate apparent velocity estimation is essential to obtain good results in practical applications. The approximate method presented enables the designer to judge the effectiveness of hyperbolic moveout filters, designed in the time-distance domain, after Fourier transforming into the (f,k) domain.

A greater insight into the design of moveout filters is gained when the approximation problem is considered in

the (f,k) domain. This approach also provides a bridge between the filter design techniques considered in geophysical and electrical engineering literature. The moveout filters designed in the (f,k) domain do not involve aliased regions because the approximation is implemented in the Nyquist range.

Linear moveout filters can be designed in the (f,k) domain as 2D filters using the input model $x_i(t) = s(t-\tau) + r(t-\tilde{\tau})$ which contains signal (s) and correlated noise (r). The Tchebycheff (or min-max) technique giving optimum filters in 1D (McClellan, 1973), which has many advantages over other available techniques, cannot be extended to 2D directly. The available indirect design techniques using mapping methods between 1D and 2D domains are not suitable for moveout filters with fan-shaped pass, reject and transition bands. A new method called the 'coefficient mapping' technique, which uses a reversible mapping formula to map the (f,k) domain discrete coefficients and 1D frequency domain coefficients into each other, is introduced to approximate linear moveout filters using 1D techniques. Direct mapping results in too many pass, reject and transition bands in the 1D frequency axis. A sorting criterion using the angles of the spherical coordinates of the (f,k) domain coefficients is suitable for bringing together the coefficients falling into the same prescribed bands. Consequently, each prescribed band in the (f,k) response corresponds to an equivalent band in the 1D frequency axis $0 \leq F \leq 0.5$. Linear spectral window boundaries in the (f,k) domain are mapped uniquely to the 1D

frequency axis to define the band edges for 1D design. The desired response, approximated in 1D domain as a frequency filter, is mapped back into the (f,k) domain after re-sorting. This technique has the flexibility of mapping the origin coefficient of (f,k) domain into any region of 1D frequency axis. Only pass and reject domain mappings give symmetrical moveout filters. The (f,k) responses of linear moveout filters designed, using the coefficient mapping technique with the McClellan (1973) 1D Tchebycheff algorithm, show ridges and troughs similar to the 1D Tchebycheff design ripples. Design results indicate that the reject domain weights should be kept as high as possible because they are poorly approximated compared to the pass domains.

The coefficient mapping technique approximates the pass domains of moveout filters consistently better than corresponding optimum least-squares filters. The reject domain approximations are marginally better for least-square filters in some cases. The transition or 'don't care' regions of moveout filters are selective. Moveout filters designed using the coefficient mapping technique have smoother transition bands compared to the corresponding optimum MLS moveout filters.

Performances of multichannel filters and straight stacking are dependent on the input traces. Stack-after-multichannel filtering traces do not have much higher signal to correlated noise ratios compared to straight-stacked traces when signal arrivals have about maximum ± 0.5 samples chatter. Multichannel filtering and straight stacking processings are not very sensitive to a small amount of chatter in the input traces. At relatively high chatter values of about

± 0.5 to ± 2 samples, multichannel filtering is always superior to straight stacking. In fact, if the chatter in the input traces is higher than about ± 1 sample, straight-stacked and multichannel filtered traces preserve only a distorted signal. Moveout filters designed in the (f,k) domain using the coefficient mapping technique are superior to the optimum MLS moveout filters and straight stacking in recovering the signal events with relatively high chatter. They also recover signal events corrupted by noise better than the other two techniques. All three techniques pass zero moveout signal events with comparable results. Straight stacking achieves the best random noise rejection.

A serious problem of multichannel techniques is the spurious line-ups created due to the mixing effect of these operators. They are: 1. High energy noise outburst line-ups and 2. Line-ups of correlated noise with relative moveouts close to the pass window boundary. The straight-stacked traces also show similar features. If moveout filters are applied to already CDP stacked data for further S/N ratio and trace-to-trace alignment improvements, care should be taken to avoid misinterpretation of these fake line-ups.

The coefficient mapping technique introduced in this study may be extended to the approximation of any desired response provided that a suitable sorting of the coefficients is made in the case of complicated desired responses. Some common circularly symmetric filters designed using this technique approximates pass domains with about -1 to -2dB deviations or better. Reject domain relative attenuations are about

24 to 30 dB near the transition bands and frequency axes, and usually better than about 30 dB at the other regions of the 2D frequency responses. Filters with very narrow transition bands are successfully approximated provided that filter lengths are high. The frequency responses show ridges and troughs similar to those of 1D Tchebycheff frequency responses. It is not possible to obtain exact circular symmetry using the new technique at present. This requires further consideration.

There are two aspects of the coefficient mapping algorithm which have not been discussed. One is the question of uniqueness of the best approximation. A unique set of coefficients is obtained for any multichannel filter using the new technique provided that the 1D algorithm that is incorporated gives a unique set of coefficients such as the Tchebycheff algorithm used in this study. The second question is the unsolved theoretical issue of the best approximation relevant to the design examples presented in this study. Although the frequency responses of 2D filters designed in this study show an equiripple behaviour similar to the 1D Tchebycheff frequency responses, there is no theoretical or practical criterion with which to judge the efficiency of these filters with reference to the best approximation in the Tchebycheff sense.

The final remarks to conclude this work concern future multichannel filter design techniques as well as the multichannel input. To obtain successful results from the multichannel techniques, the wave number aliasing should be avoided

during the recording of seismic reflection data. The optimum MLS moveout filters treated extensively in this thesis and elsewhere do not require further substantial studies. Theoretical and practical material already provided is adequate to approach the practical cases that are encountered in seismic reflection applications.

In the design of multichannel optimum Tchebycheff filters many problems remain. A sound theoretical development into the possibility of multidimensional Tchebycheff systems is required. In the meantime, (f,k) domain design techniques for moveout filters and other multidimensional desired response approximations have to be confined to the suboptimum Tchebycheff methods like the direct design and mapping techniques, or other optimization techniques using linear programming approaches.

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APPENDIX 1

A GENERAL PURPOSE LEAST-SQUARES MOVEOUT FILTER

PROGRAM - MOVFLT

1. PURPOSE OF PROGRAM

The purpose of this program is to compute the coefficients of a moveout filter designed in the least-squares sense.

2. PROGRAM DESCRIPTION

Program MOVFLT computes the optimum multichannel filter coefficients in the least-squares sense designed to pass and reject correlated events having different moveouts. First the autocorrelation and crosscorrelation matrices are calculated then the normal equations are solved in the time domain. This program can determine the coefficients of multichannel filters designed to pass and/or reject events with (a) linear moveout, (b) non-functional moveout, (c) hyperbolic moveout - for fixed time and (d) hyperbolic moveout over a time gate. In the last two cases computation of the correlation functions are only approximations. Filter coefficients are calculated according to the modes chosen for the filter length, delay and NMSE. It calculates (a) fixed length and fixed delay filters, (b) fixed length and optimum delay filters, (c) optimum length and fixed delay filters, (d) optimum filter length and optimum delay

filters, (e) fixed length and variable delay filters for a set NMSE base, (f) variable length and fixed delay filters for a set NMSE base, (g) variable length and variable delay filters with a set NMSE base.

3. THE SUBROUTINES AND FUNCTIONS

READ2 : Reads in the data for filter specifications and input-output modes.

WRITE2 : Writes out the input data.

TEST2 : Tests the input data against the array dimensions and stops the program in case of incompatibility.

MULFIL : The main subroutine for controlling the generation of multichannel filter coefficients according to the input modes.

TWOINT : Computes the value of a two-fold integral giving the two-dimensional function and integral limits.

ZERO : Stores the floating-point number 0.0 in each location of a one dimensional array (Robinson, 1967b).

AUTOMX : Computes a component of the autocorrelation matrix.

MOVWN1 : Determines the moveout window configuration for a reference point from a given configuration.

MOVWN2 : Determines the moveout window configuration for linear moveout case for a given reference point.

MOVWN3 : Determines the moveout window configuration for hyperbolic moveout case for a given reference point, given the parameters for the hyperbolic

moveout function. This is the case for fixed-time.

- CALCA : Computes the expected value of an element of the input autocorrelation matrix for a broadband spectrum.
- CROSMX : Computes the crosscorrelation matrix between the desired output and input signal from assumed models.
- CALCB : Computes the expected value of an element of the crosscorrelation matrix for a broadband spectrum input-output model.
- | | | |
|-------|---|--|
| CALC1 | } | These subroutines are called by CALCA and CALCB to compute the expected values of the correlation matrices for signal and correlated noise components according to the relative moveouts and chatter values. |
| CALC2 | | |
| CALC3 | | |
| CALC4 | | |
| CALC5 | | |
| ONE | } | Called by CALC1, CALC2 and CALC3. |
| TWO | | |
- SINCIN : Returns the value of the integral $\int_{-\infty}^T \frac{\text{SIN}(T)}{T} dT$.
- SSIMEQ : Solves a set of simultaneous equations $(A)(B) = C$ for A.
- MOVE : Shifts the elements of a one-dimensional array to another one-dimensional array (Robinson, 1967b).
- INVERT : Inverts a symmetric matrix.
- EROCAL : Computes the NMSE.
- SOLVE : Computes the multichannel filter coefficients for a filter of length LF given that the calculations for length LF-1 is done, N.B. $LF \geq 2$.

- GRAPH : Prints a graph of one-dimensional array whose coefficients lie between 0 and 1 as a function of element position.
- MINERR : Finds the minimum value of a two-dimensional array and its positions in two-dimensions assuming all values of the array are non-negative.
- TRANMX : Transposes the two-dimensional error matrix.
- CONTUR : Prints a map of a two-dimensional normalized array. The values are rounded to the nearest integer after multiplying by 10.
- PUTOT2 : Outputs the multichannel filter coefficients according to the mode specified.

The hierarchy of the above SUBROUTINES and FUNCTIONS is given in Figure A1.1.

MOVFLT

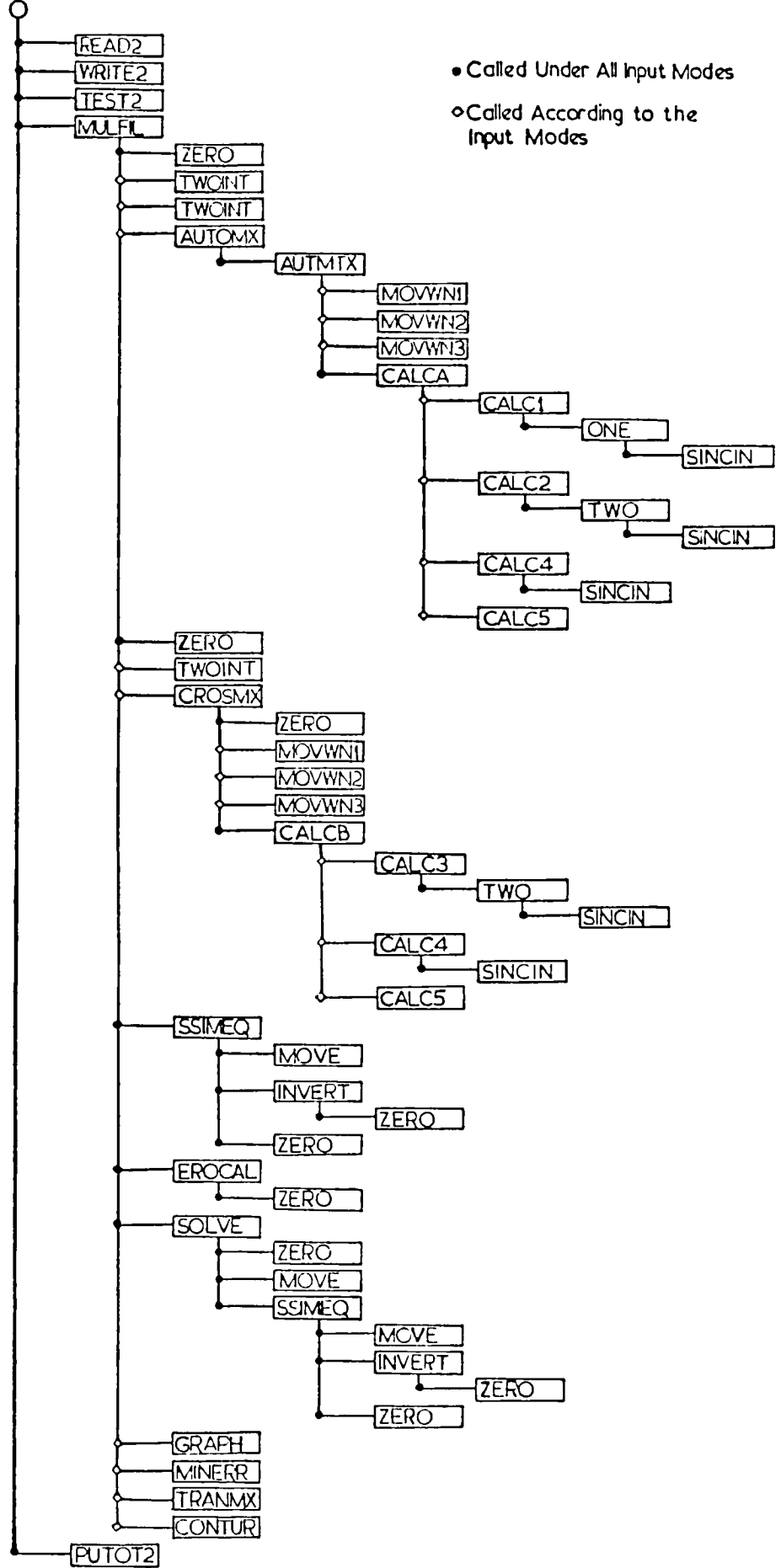


Figure A1.1 Hiearch of subroutines and functions for program MOVFLT.

APPENDIX 2

OUTLINE OF SYNTHETIC SEISMOGRAM GENERATION

PROGRAM - SNSEIS

1. THE INPUT MODEL

This program generates a synthetic seismogram according to the stochastic model

$$x_i(t) = (s(t-\tau_{i,k}) + \sqrt{w_r} r(t-\tilde{\tau}_{i,\ell}) + \sqrt{w_n} n_i(t))g_i$$

where $x_i(t)$: i th trace

$s(t)$: signal wavelet

$r(t)$: correlated noise wavelet

$n_i(t)$: uncorrelated noise wavelet on the i th trace

$\tau_{i,k}$: signal moveout on the i th trace for the k th
signal spike

$\tilde{\tau}_{i,\ell}$: correlated noise moveout on the i th trace for
the ℓ th correlated noise spike

w_r : correlated noise to signal energy ratio

w_n : uncorrelated noise to signal energy ratio

g_i : gain on the i th trace.

2. PROGRAM DESCRIPTION

Program begins with reading of the modes of generation of the synthetic seismogram. The input data is read and printed out then tested to see that the array dimensions are

compatible. All signal and noise spike series are random in amplitude and arrival time and are produced using a uniform probability distribution random number generator. The spikes are not restricted to arrive at sample points. Signal and noise wavelets are the impulse responses of Butterworth filters for specific cut-off frequencies and cut-off rates. A signal trace is constructed by interpolating the signal wavelets, shifting them to their corresponding arrival times and resampling them. Then the trace is normalized to its rms energy. The same convention is used for the correlated noise trace and multiplied by $\sqrt{w_r}$. An uncorrelated noise trace is generated using the same interpolating, shifting and resampling procedure and multiplied by $\sqrt{w_n}$. Signal and correlated noise traces are shifted according to their relative moveouts and summed. The N-trace seismogram is completed by repeating the last step and adding different uncorrelated noise traces at each step. A gain is applied to each trace. A straight stack of input traces is computed if desired. Autocorrelations and adjacent trace crosscorrelations of input traces and autocorrelation of the stacked trace are computed. A suitable plot of input traces their stack and correlation functions is made when desired.

3. SUBROUTINES AND FUNCTIONS

READ1 : Reads data for input-output modes and synthetic seismogram specifications.

WRITE1 : Writes out the input data.

TEST1 : Tests the input data against the maximum array dimensions if exceeded stops execution.

SEKSHN : The main subroutine which controls the generation of synthetic seismogram.

TRACE : Generates a trace by convolving a wavelet with random spike series.

WAVGEN : Generates a wavelet which is the impulse response of a Butterworth filter by making use of the subroutine FILTER.

FILTER : Calculates the approximate impulse response of a Butterworth filter from a specified frequency response by making use of three other subroutines; GEOP, CONVOL and SCALL.

SPIKER : Computes arrival times and amplitudes of signal and noise spikes for each call. These are random numbers stored in two different arrays.

RANDOM : Generates n random numbers drawn from a population of zero mean and a standard deviation 1.0.

BUILD : Performs the continuous time equivalent of the convolution of a random series with a wavelet.

SHIFT : Time shifts a wavelet according to its arrival time and replaces it into another array.

NORMAL : Normalizes an array by dividing each element by the rms energy of the array.

MUVOUT : Controls the moveout shifting of signal and correlated noise wavelets. Moveouts may be randomly distributed between two specified limits, deterministic or prescribed in advance (read in from cards).

- GAINER : Applies gains to a set of input traces. Gains may be deterministic, random between two limits or predefined (read in from cards).
- FMOVE : Computes a moveout value for a particular trace according to the moveout function defined e.g. linear or hyperbolic. This is supplied by the user.
- FGAIN : Computes or determines a gain for the *i*th input trace according to the user's specification.
- STACK : Stacks *N* traces (horizontal or straight-stack).
- PUTOT1 : Outputs the synthetic seismogram.
- PLTGRM : Plots a synthetic seismogram by making use of the subroutine PLOTTR.
- PLOTTR : Plots a synthetic seismogram. The straight-stack and its autocorrelation are plotted. The autocorrelations of input series and their adjacent trace crosscorrelations are computed and plotted.
- AXES : Draws and labels an axis.
- MACRO : Computes the autocorrelations and adjacent trace crosscorrelations of a multichannel input series (modified from Robinson, 1967b).
- CROSS : Computes the crosscorrelation of two vectors (Robinson, 1967b).
- DOT : Computes the dot product of two vectors (Robinson, 1967b).

The hierarchy of the above SUBROUTINES and FUNCTIONS is given in Figure A2.1.

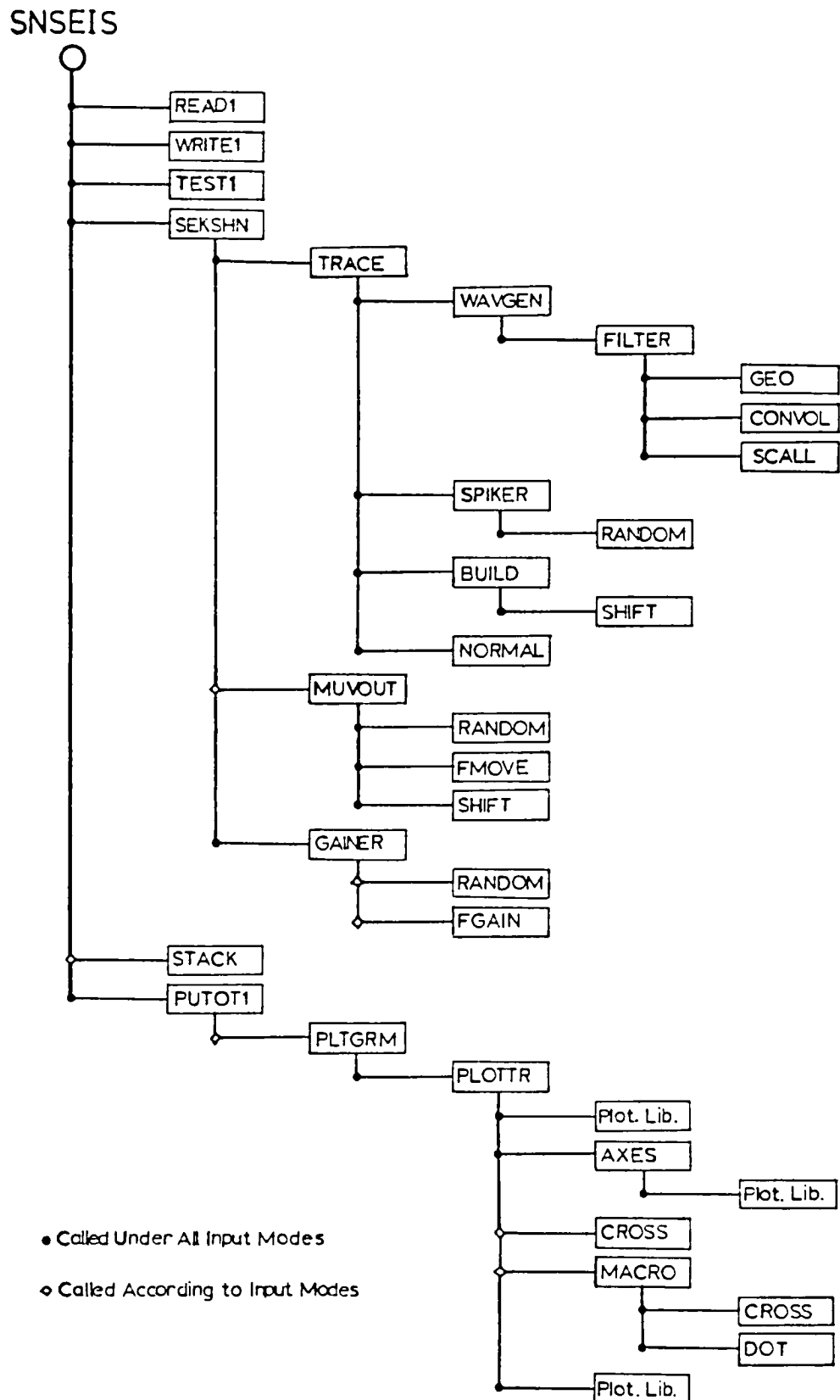


Figure A2.1 Hierarchy of subroutines and functions for program SNSEIS.

APPENDIX 3

TCHEBYCHEFF APPROXIMATION FOR 1D FINITE IMPULSE RESPONSE (FIR) FILTERS WITH LINEAR PHASE

The technique outlined here is the same as given by McClellan (1973) and McClellan et al. (1973). The frequency response of a FIR digital filter of length N and impulse response $\{h(k)\}$ is

$$H(F) = \sum_{k=0}^{N-1} h(k) \exp(-i2\pi kF) \quad A3.1$$

where $F = \frac{\omega}{2\pi}$. For linear phase filters it can be written as

$$H(F) = G(F) \exp[i(p+2\pi qF)] \quad A3.2$$

where $G(F)$ is pure real. $G(F)$ can be used to approximate the desired magnitude response. The term $\exp[i(p+2\pi qF)]$ contributes only to the phase response. The function $G(F)$ is different for odd and even N , and different symmetries of the impulse response. Note that to obtain linear phase filters certain symmetries in the impulse response have to be maintained. These are, the positive symmetry $h(k) = h(N-1-k)$ and negative symmetry $h(k) = -h(N-1-k)$. Therefore $G(F)$ can be expressed in one of the following forms;

1. Positive symmetry, odd length

$$G(F) = \sum_{k=0}^M a(k) \cos(2\pi kF) \quad A3.3$$

where $M = \frac{N-1}{2}$, $a(0) = h(M)$ and $a(k) = 2h(M-k)$ for $k = 1, 2, \dots, M$.

2. Positive symmetry, even length

$$G(F) = \sum_{k=1}^M b(k) \cos[2\pi(k-\frac{1}{2})F] \quad A3.4$$

where $M = \frac{N}{2}$ and $b(k) = 2h(M-k)$ for $k = 1, 2, \dots, M$.

3. Negative symmetry, odd length

$$G(F) = \sum_{k=1}^M c(k) \sin 2\pi kF \quad A3.5$$

where $M = \frac{N-1}{2}$, $c(k) = 2h(M-k)$ for $k = 1, 2, \dots, M$ and $h(M) = 0$.

4. Negative symmetry, even length

$$G(F) = \sum_{k=1}^M d(k) \sin[2\pi(k-\frac{1}{2})F] \quad A3.6$$

where $M = \frac{N}{2}$ and $d(k) = 2h(M-k)$ for $k = 1, 2, \dots, M$.

The approximation problem in the Tchebycheff sense is to choose the coefficients $G(F)$ to minimize

$$\|E(F)\| = \max_F W(F) |D(F) - G(F)| \quad A3.7$$

where $D(F)$: the desired magnitude response,

$W(F)$: a positive weight function

both being continuous on the compact subset $F \in [0, 0.5]$.

$G(F)$ can be written as

$$G(F) = Q(F)P(F) \quad A3.8$$

and the approximation problem set up in Equation A3.7 becomes:

Choose the coefficients $P(F)$ to minimize

$$\|E(F)\| = \max_{\hat{F}} \hat{W}(F) |\hat{D}(F) - P(F)| \quad A3.9$$

where $\hat{F} = F - \{\text{end points where } Q(F) = 0\}$,

$$\hat{W}(F) = Q(F)W(F)$$

and

$$\hat{D}(F) = \frac{D(F)}{Q(F)}.$$

$P(F)$ is a linear combination of cosine functions. The approximation problem for the above four cases can be carried out with one algorithm which can only do cosine approximation. The following theorems show that the approximation is unique and optimum in the sense of Tchebycheff. The proofs for these theorems can be found in the books on approximation theory such as Cheney (1966). The approximation problem formulated in A3.9 has unique solution on compact set $\hat{F}$ (Remes, 1957). The necessary and sufficient conditions

for the best approximation is guaranteed by the alternation theorem which assumes that $P(F)$ satisfies the Haar condition: The system of continuous functions $\{g_1, g_2, \dots, g_n\}$ is said to satisfy the Haar condition if every nontrivial linear combination of the polynomial $\sum_{i=0}^{M-1} c_i g_i$ has more than $M-1$ roots. Expressed otherwise, the set of continuous functions $g_1, g_2, \dots, g_M$ satisfies the Haar condition if each determinant

$$D[x_1, x_2, \dots, x_M] = \begin{vmatrix} g_1(x_1) & \dots & g_M(x_1) \\ \dots & \dots & \dots \\ g_1(x_M) & \dots & g_M(x_M) \end{vmatrix} \quad A3.10$$

made up from M distinct points $x_1, x_2, \dots, x_M$ is nonzero. A set of functions which satisfies the Haar condition is sometimes called a Tchebycheff system.

Haar's Unicity Theorem: The best approximation to a continuous function f by a generalized polynomial $\sum_{i=0}^M c_i g_i$ is unique for all choices of f if $\{g_1, g_2, \dots, g_n\}$ satisfies the Haar condition.

Alternation Theorem: If $P(F)$ is a linear combination of M cosine functions, then a necessary and sufficient condition that $P(F)$ be the unique best weighted Tchebycheff approximation to a continuous function $\hat{D}(F)$ on the set $\hat{F}$ is that the weighted error function $E(F) = \hat{W}(F)[\hat{D}(F) - P(F)]$ exhibits in F at least $M+1$ alternations. That is,

$$E(F_i) = -E(F_{i+1}) \text{ and } |E(F_i)| = \max_F |E(F)|, i = 1, 2, \dots, M.$$

The unique best approximation of 1D frequency response is obtained due to the fact that the set of functions $\{\cos 2\pi kF\}$, $k = 0, 1, \dots, M$, satisfies the Haar condition.

The weighted error function $E(F)$ exhibits at least $M+1$ alternations (or extremal frequencies) including the frequencies $F=0$ and $F=0.5$. The approximation algorithm begins with an initial guess of $M+1$ points and exchanges points until it obtains $M+1$ points of alternation. Therefore, at the i th iteration step the error function is forced to have magnitude $|\rho|$ with alternating signs. The error function $E(F)$ is evaluated at about a grid of $20M$ equally spaced points. $M+1$ extremal frequencies requires the solution of $M+1$ equations at each iteration. The analytical computation has been found more efficient than solving the matrix obtained from

$$W(F_k)[G(F_k) - D(F_k)] = -(-1)^k \rho \quad A3.11$$

$k = 0, 1, \dots, M$, for ρ and the filter coefficients. ρ is calculated analytically then $G(F)$ is interpolated on the $F_0, F_1, \dots, F_M$ using the Lagrange's interpolation formula. Finally, the error function $D(F)$ is calculated from $E(F) = W(F)[D(F) - G(F)]$ on F . When $|E(F)| \leq |\rho|$ the best approximation is achieved. If not, a new set of $M+1$ extremal points are chosen using Remes exchange method: The

new frequencies are chosen in such a way that $|\rho|$ will be increased at the next iteration. This suggests the choice of the local extremums of the error function $E(F)$ and $|E(F)| \geq |\rho|$. Ultimately, $|\rho|$ is forced to converge to its upper bound and the filter coefficients at this stage of the iteration are the optimum set of coefficients.

APPENDIX 4

TWO-DIMENSIONAL (2D) FILTER DESIGN PROGRAM USING MAPPING TECHNIQUES - ONETWO

1. PURPOSE OF PROGRAM

The purpose of this program is to compute the coefficients of a N input one output moveout filter designed to pass and reject events falling into linearly bounded time windows.

2. PROGRAM DESCRIPTION

Program ONETWO calculates the impulse response of a multichannel input one output linear moveout filter designed in the frequency domain. The 2D desired response with fan shaped pass and reject regions is mapped to 1D frequency axis using the mapping formula $F(LF \times N_1 + N_2) = FF(N_1, N_2)$ where $N_1 = 1, 2, \dots, N$ and $N_2 = 1, 2, \dots, LF$. The 1D approximation is done in Tchebycheff sense using McClellan's (1973) method and mapped back to 2D response by reversing the above mapping. The distribution of 2D frequency domain coefficients is arranged so that there are as many prescribed and 'don't care' regions in 1D response as in 2D. The pass and reject domain weights for 1D approximation are usually calculated in advance for practical reasons, using the Equation 5.4.10 given in Section 5.4.

3. THE SUBROUTINES AND FUNCTIONS

- ONETWO : This is the main program for reading and writing out the modes and 2D moveout filter specifications and controlling the computation of 2D impulse response. It does the necessary mappings and finally the calculation of 2D filter coefficients from the approximated 1D response. The 2D filter coefficients are outputted according to the modes defined.
- ORDFIX : Puts angular spherical coordinates of the upper half plane 2D frequency domain coefficients, which are mapped into a 1D array, into decreasing order. Defines an array of indices which relates ordered and original arrays.
- MAXSN : Finds the maximum element of a 1D array (Robinson, 1967b).
- COSTR : Computes the cosine transform (Robinson, 1967b).
- ONEFIL : Computes the finite impulse response (FIR) of 1D linear phase filters using the Remes exchange algorithm. Outputs the input parameters for 1D filter and filter coefficients (modified from McClellan, 1973).
- ERROR : Prints the error message and stops the program if the filter length read in is less than 3 or exceeds the maximum length specified for dimensioning the related arrays.
- EFF : Returns the desired magnitude response of a 1D filter as a function of frequency.

WATE : Returns the weight function of a 1D filter as
a function of frequency.

REMEZ : Impliments the Remes exchange algorithm for
the weighted Tchebycheff approximation of a
continuous function with a sum of cosines.

GEE : Evaluates the 1D frequency response using the
Lagrange interpolation formula.

D : Returns the Lagrange interpolation coefficients.

OUCH : Prints the message in case the 1D algorithm
diverges.

The hierarchy of the above SUBROUTINES and FUNCTIONS
is given in Figure A4.1. The block diagram of the program
ONETWO is given in Figure A4.2.

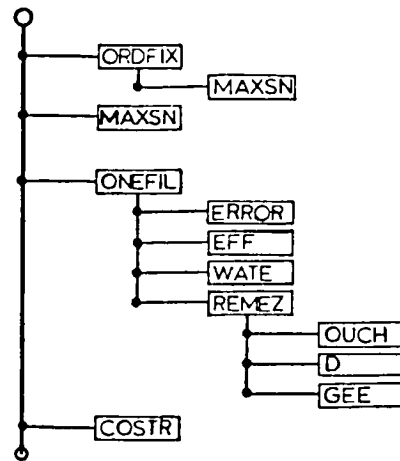


Figure A4.1 Hierarchy of subroutines and functions for program ONETWO.

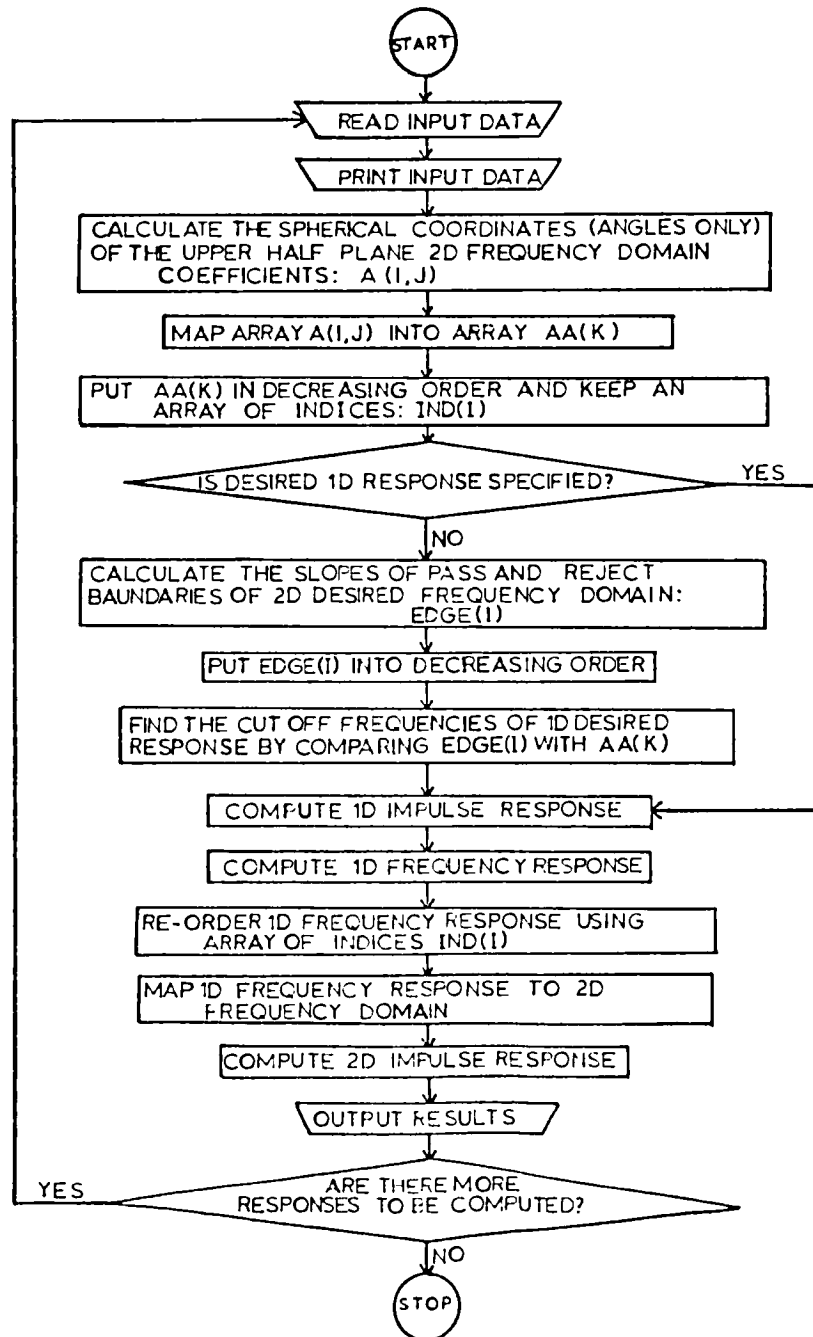


Figure A4.2 Overall flowchart for program ONETWO.