

Imperial College of Science, Technology and Medicine
Department of Electrical and Electronic Engineering

Carbon-Oriented Generation Planning: A Market-Driven Approach

Junkai Wang

Supervisor: Prof. Goran Strbac

Submitted in part fulfilment of the requirements for the degree of
Doctor of Philosophy in Electrical Engineering and
the Diploma of Imperial College, July 2024

Statement of Originality

In accordance with the requirements of the college, I affirm that this thesis and the research detailed within are my own work. This thesis and the work to which it refers are the results of my own efforts. Any ideas, data, images, or text borrowed from other sources have been declared in this thesis through proper citations.

This work has not previously been submitted to fulfil any degree requirement at this or any other institution. Where I have quoted from the work of others, the source is always given. I have also acknowledged all the main sources of help.

Junkai Wang

July 2024

‘All models are wrong, but some are useful.’

George E. P. Box

Copyright Declaration

The copyright for this thesis rests with the author. Unless otherwise indicated, its contents are licenced under a Creative Commons Attribution-NonCommercial 4.0 International Licence (CC BY-NC).

Under this licence, you may copy and redistribute the material in any medium or format. You may also create and distribute modified versions of the work. This is on the condition that: you credit the author and do not use it, or any derivative works, for a commercial purpose.

When reusing or sharing this work, ensure you make the licence terms clear to others by naming the licence and linking to the licence text. Where a work has been adapted, you should indicate that the work has been changed and describe those changes.

Please seek permission from the copyright holder for uses of this work that are not included in this licence or permitted under UK Copyright Law.

Junkai Wang

July 2024

Acknowledgements

First and foremost, I am deeply grateful to my supervisor, Professor Goran Strbac, for granting me the opportunity to engage in this fascinating research topic. His invaluable guidance, patience, and professional expertise greatly contributed to my research and academic growth during my doctoral program.

Special thanks are due to my colleagues from the Control and Power Research Group at Imperial College London: Dr. Dawei Qiu, Dr. Jiawei Wang, and Dr. Yujian Ye. Their academic support and technical insights were instrumental in shaping the direction and execution of my research. Beyond these, their encouragement and rigorous standards pushed me to refine my work to its best form. I appreciate Imperial College London for offering an exceptional research environment and allowing me to encounter these remarkable individuals.

I owe a profound debt of gratitude to my parents, Yongqiang Wang and Cuihong Wang. Their unwavering support, both emotionally and financially, has been the bedrock of my PhD journey. Their belief in my abilities and their endless unconditional love have been my constant source of strength and inspiration.

I would also like to extend my heartfelt thanks to my best friends: Jianli Gao, Jianhong Wang, Qian Chen, Guoxuan Cui, Yunhe Wei, Wangkun Xu, Junyu Mao, Yanshu Niu, Jian Wang, and Bo Yang. Their belief in me, their constant encouragement, and their presence through the highs and lows of this journey have been a source of immense comfort and motivation.

This journey would have been a lonely one without the support and camaraderie of these individuals. This PhD research could not have been accomplished without their bountiful support. I am sincerely thankful for having them in my life.

Abstract

Under the circumstances of the urgent global challenges posed by climate change, decarbonisation has become a critical priority. In this context, the GB government has set ambitious goals to decarbonise its power sector by 2035, aligning with the net-zero target by 2050. This thesis examines generation planning under stringent carbon emission restrictions from four distinct aspects: the value of market structure, the role of demand flexibility, the impact of energy storage, and the feasibility of achieving a fully decarbonised GB power system by 2035.

To meet these objectives, the proposed bi-level models encompass an upper-level problem, representing the decision-making process of the self-interested generation company, alongside two lower-level problems: the first lower-level problem represents the joint clearing process of the day-ahead energy market and the real-time balancing market to co-optimize the generation scheduling and reserve deployment; the second lower-level problem represents the clearing process of the green certificate market. Notably, the proposed models incorporate demand and wind power uncertainty through a series of chance constraints. Furthermore, the time-coupling demand flexibility and energy storage models are also integrated into the bi-level models.

Numerous case studies have provided valuable insights on generation planning under current carbon targets using real-world data from the 2021 GB power system and predicted data from the 2035 GB power system. Overall, the scope of the case studies is three-fold. Firstly, they quantitatively compare the role of vertically integrated monopoly utilities and deregulated energy markets. Secondly, they explore how demand flexibility and energy storage benefit market-driven generation planning by mitigating massive renewable uncertainty. Finally, they demonstrate that the high levels of demand flexibility, in conjunction with utility-scale energy storage, not only ensure the stable operation of a 100% renewable GB power system but also guarantee adequate economic benefits.

Contents

Acknowledgements	v
Abstract	vii
List of Tables	xvi
List of Figures	xix
List of Publications	xxi
Nomenclature	xxix
1 Introduction	1
1.1 Background	1
1.2 Research Motivation	2
1.3 Research Questions	3
1.4 Scope and Contributions	4
1.5 Thesis Structure	6
2 Market Models and Modelling Approaches	8
2.1 Literature Review	8
2.1.1 Generation Planning Model	8
2.1.2 Uncertainty Modelling	10
2.1.3 Demand Flexibility	11

2.1.4	Energy Storage	12
2.1.5	Low-Carbon Policy	13
2.2	Market Mechanisms	14
2.2.1	Energy Market	14
2.2.2	Balancing Market	15
2.2.3	Carbon Market	16
2.2.4	Green Certificate Market	16
2.3	Flexibility Models	17
2.3.1	Demand Flexibility	17
2.3.2	Energy Storage	17
2.4	Uncertainty Model	18
2.4.1	Chance Constraint	18
2.5	Solution Approaches	20
2.5.1	Centralised Approach	20
2.5.2	Game-Theoretic Approach	21
3	System-Driven Generation Planning	26
3.1	Introduction	26
3.2	Problem Statement	27
3.2.1	Structure of the Single-Level Optimisation Model	27
3.2.2	Modelling Assumptions	27
3.3	Mathematical Model	29
3.4	Solution	32
3.4.1	CCP Reformulation	32
3.4.2	Final LP	34
3.5	Case Studies	35
3.5.1	Data Descriptions and Implementation	35
3.5.2	System Outcomes and Business Cases	39

3.6	Conclusions	42
4	Market-Driven Generation Planning	44
4.1	Introduction	44
4.2	Problem Statement	45
4.2.1	Structure of the Bi-Level Optimisation Model	45
4.2.2	Modelling Assumptions	47
4.3	Mathematical Model	49
4.3.1	UL Problem: Investment Decisions of GenCo	49
4.3.2	LL1 Problem: Joint Clearing Process of DA Energy and RT Balancing Markets	51
4.3.3	LL2 Problem: Clearing Process of TGC Market	53
4.4	Solution	54
4.4.1	CCP Reformulation	55
4.4.2	MPEC Formulation	55
4.4.3	Linearization of MPEC	60
4.4.4	Final MIQP	68
4.5	Case Studies	69
4.5.1	Data Descriptions and Implementation	69
4.5.2	Market Outcomes and Business Cases	71
4.6	Conclusions	74
5	Impact of Demand Flexibility	76
5.1	Introduction	76
5.2	Problem Statement	77
5.2.1	Structure of the Bi-Level Optimisation Model	77
5.2.2	Modelling Assumptions	79
5.3	Mathematical Model	81
5.3.1	UL Problem: Investment Decisions of GenCo	81

5.3.2	LL1 Problem: Joint Clearing Process of DA Energy and RT Balancing Markets	83
5.3.3	LL2 Problem: Clearing Process of TGC Market	85
5.4	Solution	85
5.4.1	CCP Reformulation	86
5.4.2	MPEC Formulation	86
5.4.3	Linearization of MPEC	91
5.4.4	Final MIQP	98
5.5	Case Studies	100
5.5.1	Data Descriptions and Implementation	100
5.5.2	Impact of Demand Flexibility	101
5.6	Conclusions	110
6	Impact of Energy Storage	112
6.1	Introduction	112
6.2	Problem Statement	113
6.2.1	Structure of the Bi-Level Optimisation Model	113
6.2.2	Modelling Assumptions	115
6.3	Mathematical Model	117
6.3.1	UL Problem: Investment Decisions of GenCo	117
6.3.2	LL1 Problem: Joint Clearing Process of DA Energy and RT Balancing Markets	118
6.3.3	LL2 Problem: Clearing Process of TGC Market	121
6.4	Solution	121
6.4.1	CCP Reformulation	122
6.4.2	MPEC Formulation	122
6.4.3	Linearization of MPEC	127
6.4.4	Final MIQP	136

6.5	Case Studies	137
6.5.1	Data Descriptions and Implementation	137
6.5.2	Impact of Storage Capacity	139
6.6	Conclusions	146
7	Pathway towards Fully Decarbonised GB Power System by 2035	148
7.1	Introduction	148
7.2	Problem Statement	149
7.2.1	Structure of the Bi-Level Optimisation Model	149
7.2.2	Modelling Assumptions	151
7.3	Mathematical Model	153
7.3.1	UL Problem: Investment Decisions of GenCo	154
7.3.2	LL1 Problem: Joint Clearing Process of DA Energy and RT Balancing Markets	155
7.3.3	LL2 Problem: Clearing Process of TGC Market	157
7.4	Solution	157
7.4.1	CCP Reformulation	158
7.4.2	MPEC Formulation	159
7.4.3	Linearization of MPEC	164
7.4.4	Final MIQP	173
7.5	Case Studies	174
7.5.1	Data Descriptions and Implementation	174
7.5.2	Impact of Flexibility Resources	176
7.6	Conclusions	184
8	Conclusions and Future Work	186
8.1	Conclusions	186
8.2	Key Findings	187
8.3	Future Work	189

References	190
Appendix A: 2021 GB System Demand Data	200
Appendix B: GB Normalised Wind Power Output	201

List of Tables

3.1	Parameters of investment options.	36
3.2	Parameters of the GB power grid in 2021.	36
3.3	Investment decisions and business results for the system-driven scenario regarding the 2021 GB power system.	39
4.1	Parameters of investment options.	70
4.2	Parameters of the GB power grid in 2021.	70
4.3	Investment decisions and business results for two market design scenarios regarding the 2021 GB power system.	71
5.1	Parameters of investment options.	100
5.2	Parameters of the GB power grid in 2021.	100
5.3	Investment decisions and business results for three demand flexibility sce- narios regarding the 2021 GB power system.	102
6.1	Parameters of investment options.	137
6.2	Parameters of the GB power grid in 2021.	138
6.3	Parameters of the utility-scale ES in 2021.	138
6.4	Investment decisions and business results for three storage capacity sce- narios regarding the 2021 GB power system.	139
7.1	Parameters of investment options.	175
7.2	Parameters of the GB power grid in 2035.	175

7.3	Investment decisions and business results for three flexibility resource scenarios regarding the 2035 GB carbon target.	177
1	Hourly 2021 GB power system demand for the single representative day (in GW).	200
2	Hourly GB normalised wind power output for the single representative day (in %).	201

List of Figures

2.1	The deregulated markets from the perspective of GenCo.	15
2.2	Recasting the bi-level optimisation model into a MPEC.	22
3.1	Illustration of the two-stage single-level optimisation problem.	28
3.2	Data profiles of the 2021 GB power system.	38
3.3	Daily generation dispatches in the DA stage for the system-driven scenario regarding the 2021 GB power system.	40
3.4	Daily imbalances in the RT stage for the system-driven scenario regarding the 2021 GB power system.	41
3.5	Daily prices in the DA and RT stages for the system-driven scenario regarding the 2021 GB power system.	41
4.1	Illustration of the proposed two-stage bi-level optimisation model.	46
4.2	Daily generation dispatches in the DA energy market for the market-driven scenario regarding the 2021 GB power system.	72
4.3	Daily imbalances in the RT balancing market for the market-driven scenario regarding the 2021 GB power system.	72
4.4	Daily DA and RT market clearing prices for market-driven scenario in the 2021 GB power system.	73
5.1	Illustration of the proposed two-stage bi-level optimisation model.	78
5.2	Daily system demand for three demand flexibility scenarios regarding the 2021 GB power system.	102

5.3	Daily DA energy market clearing prices for three demand flexibility scenarios in the 2021 GB power system.	103
5.4	Daily RT balancing market clearing prices for three demand flexibility scenarios in the 2021 GB power system.	103
5.5	Daily generation dispatches in the DA energy market for the Z-DF case regarding the 2021 GB power system.	104
5.6	Daily imbalances in the RT balancing market for the Z-DF case regarding the 2021 GB power system.	104
5.7	Daily generation dispatches in the DA energy market for the L-DF case regarding the 2021 GB power system.	106
5.8	Daily imbalances in the RT balancing market for the L-DF case regarding the 2021 GB power system.	107
5.9	Daily generation dispatches in the DA energy market for the H-DF case regarding the 2021 GB power system.	109
5.10	Daily imbalances in the RT balancing market for the H-DF case regarding the 2021 GB power system.	109
6.1	Illustration of the proposed two-stage bi-level optimisation model.	114
6.2	Daily charging/discharging power output in the DA energy market for three storage capacity scenarios regarding the 2021 GB power system. . .	140
6.3	Daily charging/discharging power output in the RT balancing market for three storage capacity scenarios regarding the 2021 GB power system. . .	140
6.4	Daily DA energy market clearing prices for three storage capacity scenarios in the 2021 GB power system.	141
6.5	Daily RT balancing market clearing prices for three storage capacity scenarios in the 2021 GB power system.	141
6.6	Daily generation dispatches in the DA energy market for the L-SC case regarding the 2021 GB power system.	142
6.7	Daily imbalances in the RT balancing market for the L-SC case regarding the 2021 GB power system.	142

6.8	Daily generation dispatches in the DA energy market for the H-SC case regarding the 2021 GB power system.	145
6.9	Daily imbalances in the RT balancing market for the H-SC case regarding the 2021 GB power system.	145
7.1	Illustration of the proposed two-stage bi-level optimisation model.	150
7.2	Daily charging/discharging power output in the DA energy market for two flexibility resource scenarios regarding the 2035 GB carbon target.	177
7.3	Daily charging/discharging power output in the RT balancing market for two flexibility resource scenarios regarding the 2035 GB carbon target.	178
7.4	Daily system demand for two flexibility resource scenarios regarding the 2035 GB carbon target.	178
7.5	Daily generation dispatches in the DA energy market for the B-FR case regarding the 2035 GB carbon target.	179
7.6	Daily imbalances in the RT balancing market for the B-FR case regarding the 2035 GB carbon target.	179
7.7	Daily generation dispatches in the DA energy market for the H-FR case regarding the 2035 GB carbon target.	182
7.8	Daily imbalances in the RT balancing market for the H-FR case regarding the 2035 GB carbon target.	182

List of Publications

The results demonstrated in this thesis have been submitted for peer review. They have been showcased in the publications listed below.

Journal Papers

- **Junkai Wang**, Dawei Qiu, Goran Strbac, "Carbon-oriented Generation Planning: A Market-Driven Approach", *Applied Energy*, Submitted, March 2024.
- Dawei Qiu, Yi Wang, **Junkai Wang**, Chuanwen Jiang, Goran Strbac, "Personalized Retail Pricing Design for Smart Metering Consumers in Electricity Market", *Applied Energy*, Vol. 348, p. 121545, July 2023.

Conference Papers

- **Junkai Wang**, Dawei Qiu, Goran Strbac and Yujian Ye, "Market-Based Generation Planning with Carbon Target", *19th International Conference on the European Energy Market (EEM)*, Lappeenranta, Finland, 6-8 June 2023.
- Dawei Qiu, Jianhong Wang, **Junkai Wang** and Goran Strbac, "Multi-Agent Reinforcement Learning for Automated Peer-to-Peer Energy Trading in Double-Side Auction Market", *30th International Joint Conferences on Artificial Intelligence (IJCAI)*, Montreal, Canada, 19-27 August 2021.

Nomenclature

For ease of reference, a list of the notations utilised throughout this thesis is provided below. Additional symbols will be introduced and defined within the respective sections of the text.

List of Abbreviations

CCP	Chance-constrained programming
DA	Day-ahead
DC	Direct current
EPEC	Equilibrium problem with equilibrium constraints
ES	Energy storage
GB	Great Britain
GenCo	Generation company
KKT	Karush-Kuhn-Tucker
LL	Lower-level
LMP	Locational marginal price
LP	Linear programming
MIP	Mixed-integer programming
MIQP	Mixed-integer quadratic programming
MPEC	Mathematical programming with equilibrium constraints
QP	Quadratic programming
RES	Renewable energy source

RO	Robust optimisation
RPS	Renewable portfolio standard
RT	Real-time
SO	Stochastic optimisation
TGC	Tradable green certificate
TOU	Time-of-use
UC	Unit commitment
UL	Upper-level

Nomenclature of Chapter 2

A. Indices and Sets

$t \in T$	Index and set for time periods
-----------	--------------------------------

B. Parameters

L_t^{DA}	Baseline demand in the DA market at hour t (MW)
α	Demand flexibility limit
η^c	Charging efficiency of energy storage
η^d	Discharging efficiency of energy storage
s^{max}	Power capacity of energy storage (MW)
E^0	Initial energy level of energy storage (MWh)
E^{min}	Minimum energy level of energy storage (MWh)
E^{max}	Maximum energy level of energy storage (MWh)

C. Stochastic variables

L_t^R	Realisation of baseline demand at hour t (MW)
---------	---

D. Decision variables

$P_{t,o}^{DA}$	Power consumed by demand in the DA market at hour t and block o (MW)
$P_{t,o}^{dev}$	Deviated quantity of demand in the RT market at hour t and block o (MW)

$L_{t,o}^{SH}$	Demand shifting of demand at hour t and block o (MW)
$s_t^{c,DA}$	Charging power of energy storage in the DA market at hour t (MW)
$s_t^{d,DA}$	Discharging power of energy storage in the DA market at hour t (MW)
$s_t^{c,RT}$	Charging power of energy storage in the RT market at hour t (MW)
$s_t^{d,RT}$	Discharging power of energy storage in the RT market at hour t (MW)
E_t	Energy level of energy storage at hour t (MW)
λ_t^{DA}	DA market clearing price at hour t (£/MWh)
λ_t^{RT}	RT market clearing price at hour t (£/MWh)

Nomenclature of Chapter 3-7

A. Indices and sets

$d \in D$	Index and set for representative days
$t \in T$	Index and set for hours
$i \in I$	Index and set for generation technologies
$j \in J$	Index and set for demands
$e \in E$	Index for and set for energy storages
$b \in B$	Index for and set for production blocks of flexible generation technologies
$o \in O$	Index for and set for demand blocks
$n/m \in N$	Index and set for buses
$I^{FL} \subseteq I$	Subset of flexible generation technologies
$I^{MR} \subseteq I$	Subset of must-run generation technologies
$I^{RE} \subseteq I$	Subset of renewable generation technologies
Ψ_n	Set of generation units located at bus n
Λ_n	Set of buses connected to bus n
V^{LL1}	Set of variables of the LL1 problem
V^{LL2}	Set of variables of the LL2 problem

V^{MPEC}	Set of variables of the MPEC model
V^{MIQP}	Set of variables of the MIQP model
V^{BINARY}	Set of auxiliary binary variables
V^{SYS}	Set of variables of the system-driven generation planning model

B. Parameters

ω_d	Weighting factor of representative day d
ω_b^{FL}	Weighting factor of production block b
C_i^{IN}	Investment cost of generation technology i (£/MW/yr)
$C_{i,b}^{EN}$	Marginal energy cost of generation technology i at production block b (£/MWh)
$C_{i,b}^U$	Marginal upward reserve cost of generation technology i at block b (£/MWh)
$C_{i,b}^D$	Marginal downward reserve cost of generation technology i at block b (£/MWh)
$L_{j,d,t,o}^{DA}$	Baseline demand j in the DA market at day d , hour t and block o (MW)
λ^{CO_2}	Carbon price (£/tCO ₂)
η^{Grid}	System-wide carbon intensity (t/MWh)
$\eta_i^{CO_2}$	Carbon intensity of flexible generation technology i (t/MWh)
ϵ_i	Carbon allowance allocation ratio of renewable generation technology i
$k_{i,d,t}^{RE}$	Normalised output of renewable generation technology i at day d and hour t
α	Demand flexibility limit
b_{nm}	Susceptance of line $n - m$ (S)
C_{nm}^{max}	Transmission capacity of line $n - m$ (MW)
η_e^c	Charging efficiency of energy storage e
η_e^d	Discharging efficiency of energy storage e
s_e^{max}	Power capacity of energy storage e (MW)
E_e^0	Initial energy level of energy storage e (MWh)
E_e^{min}	Minimum energy level of energy storage e (MWh)

E_e^{max}	Maximum energy level of energy storage e (MWh)
$G_{i,b}^U$	Maximum upward reserve capacity of generation technology i at block b (MW)
$G_{i,b}^D$	Maximum downward reserve capacity of generation technology i at block b (MW)
l^{PGC}	Intercept of the inverse demand function of the green certificate market (£/MWh)
q^{PGC}	Slope of the inverse demand function of the green certificate market (£/MWh ²)
γ^{FGC}	Approximated price ratio of green certificate in the future green certificate market
γ^{RPS}	Renewable portfolio standard requirement ratio
θ^{PGC}	Green certificate price coefficient in the present green certificate market
λ_0^{RPS}	Penalty price for renewable portfolio standard deficiency (£/MWh)
λ^{FGC}	Approximated green certificate price in the future green certificate market (£/MWh)
$1 - \rho$	Confidence level of chance constraints

C. Stochastic variables

$L_{j,d,t}^R$	Realisation of baseline demand j at day d and hour t (MW)
$g_{i,d,t}^R$	Power output realisation of renewable technology i at day d and hour t (MW)

D. Decision variables

$G_{i,n}$	New capacity of generation technology i at bus n (MW)
$g_{i,d,t,n,b}^{DA}$	Power output of new capacity in the DA market using generation technology i at day d , hour t , bus n and block b (MW)
$g_{i,d,t,n,b}^U$	Upward reserve provided by new capacity in the RT market using generation technology i at day d , hour t , bus n and block b (MW)
$g_{i,d,t,n,b}^D$	Downward reserve provided by new capacity in the RT market using generation technology i at day d , hour t , bus n and block b (MW)

$g_{i,d,t,n}^{dev}$	Deviated power output of new capacity in the RT market using renewable generation technology i at day d , hour t and bus n (MW)
$P_{j,d,t,o}^{DA}$	Power consumed by demand j in the DA market at day d , hour t and block o (MW)
$P_{j,d,t,o}^{dev}$	Deviated quantity of demand j in the RT market at day d , hour t and block o (MW)
$L_{j,d,t,o}^{SH}$	Demand shifting of demand j at day d , hour t and block o (MW)
$s_{e,d,t}^{c,DA}$	Charging power of energy storage e in the DA market at day d and hour t (MW)
$s_{e,d,t}^{d,DA}$	Discharging power of energy storage e in the DA market at day d and hour t (MW)
$s_{e,d,t}^{c,RT}$	Charging power of energy storage e in the RT market at day d and hour t (MW)
$s_{e,d,t}^{d,RT}$	Discharging power of energy storage e in the RT market at day d and hour t (MW)
$E_{e,d,t}$	Energy level of energy storage e at day d and hour t (MW)
$\lambda_{d,t,n}^{DA}$	DA market clearing price at day d , hour t and bus n (£/MWh)
$\lambda_{d,t,n}^{RT}$	RT market clearing price at day d , hour t and bus n (£/MWh)
$\delta_{d,t,n}^{DA}$	Voltage angle in the DA market at day d , hour t and bus n (rad)
$\delta_{d,t}^{DA,ref}$	Voltage angle of reference bus in the DA market at day d and hour t (rad)
$\delta_{d,t,n}^{RT}$	Voltage angle in the RT market at day d , hour t and bus n (rad)
$\delta_{d,t}^{RT,ref}$	Voltage angle of reference bus in the RT market at day d and hour t (rad)
$Q_{i,d,t,n}^{CA}$	Carbon allowance allocated to renewable generation technology i at day d , hour t and bus n (t)
$Q_{i,d,t,n}^{emi}$	Carbon emissions from flexible generation technology i at day d , hour t and bus n (t)

$Q_{i,d}^{PGC}$	Green certificate quantity sold by renewable generation technology i at day d in the present green certificate market (MWh)
$Q_{i,d}^{FGC}$	Green certificate quantity sold by renewable generation technology i at day d in the future green certificate market (MWh)
$\lambda_{i,d}^{PGC}$	Green certificate price for renewable generation technology i at day d in the present green certificate market (MWh)

Chapter 1

Introduction

1.1 Background

In order to address the urgent threat of global warming and climate change, the necessity to mitigate carbon emissions is universally recognised. The power generation sector, a major contributor to global carbon emissions, is at a crucial historical turning point, stressing the importance of a carbon-oriented generation planning framework that prioritises the transformation from traditional fossil fuels to renewable energy sources (RESs). This transformation is already underway in several European countries, like Germany, Spain, and Denmark, showcasing substantial progress in renewable energy expansion, where the proportion of total electricity generation from RESs has reached 28.2%, 38.7%, and 48.5%, respectively [1].

Great Britain (GB) has emerged as a global leader in tackling climate change and has attained some remarkable achievements. Today, over 50% of the GB's electricity supply comes from RESs. Nevertheless, GB's power plants still emitted 53.7 million metric tonnes of carbon dioxide (CO₂) in 2022 [2]. Currently, GB's power system is undergoing a significant transition aimed at aligning with the 2050 Net Zero target by fully decarbonising the national power sector by 2035 [3]. This ambitious goal necessitates a comprehensive reform of the existing energy infrastructure to pave the way for deep decarbonisation, with a particular emphasis on increasing the deployment of RESs towards achieving a 100% renewable power system.

Despite GB's commitment to a zero-carbon future, several challenges are unavoidable [4]. First, a future powered entirely by renewables introduces significant uncertainty from a large variety of sources, notably the inherent intermittency and variability of renewable energy, as well as demand fluctuations. Second, although many studies show that the inte-

gration of a wide range of flexibility resources, like demand flexibility and energy storage (ES), is crucial for dealing with uncertainties and grid stability in a purely renewable system, the current energy market has not fully leveraged the potential of these resources. Third, while market design could offer a practical avenue to explore the role and value of flexibility resources, current studies often overlook system-wide carbon emission targets and the need for a cost-effective market mechanism that addresses energy price signal distortion and cost recovery in a completely decarbonised power grid. Aiming to handle these challenges, there are significant research spaces regarding the cost-effective electricity market design for promoting further integration of RESs.

1.2 Research Motivation

In the context of global commitments to reduce carbon emissions, there has been growing interest in the 100% renewable power system. However, the existing studies investigate this from the perspective of a central planner, validating the feasibility of a 100% renewable electricity system through a single-level optimisation model that incorporates mandatory environmental regulations. This approach fails to capture the interactions among stakeholders in the current deregulated market environment and neglects the economic interests of market participants. To overcome these shortcomings, it is essential to explore a carbon-oriented generation planning model that reflects the real-world electricity market environment through a market-based modelling approach, thereby exploring the path to a 100% renewable power system through economic incentives.

On the one hand, due to the massive RESs in a zero-carbon power grid, it is urgent to address the significant uncertainty they bring. It has been recognised that integrating a large amount of flexibility resources (e.g., demand flexibility, ES, electric vehicles, etc.) into the power grid with high renewable penetration is a promising strategy to effectively mitigate uncertainty, i.e., ES can accommodate excess wind power during windy periods and compensate for the energy shortage during peak-demand periods [5]. Additionally, flexibility resources play a crucial role in providing reserves in the balancing market, especially in a 100% renewable electricity system. However, the value of flexibility resources within a market-driven generation planning framework has not been fully explored yet.

On the other hand, due to the 'missing money' dilemma incurred by the increasing penetration of RESs, market-based renewable energy promotion policies have attracted researchers' attention, e.g., the cap-and-trade system, the tradable green certificate (TGC), and the feed-in premium [6]. Driven by market signals and incentives, these policies work by assigning monetary value to the environmental benefits of renewable energy, effectively making investments in renewable generation capacity more competitive compared

to conventional generation capacity [7]. Nevertheless, there are significant spaces for exploration regarding the potential impact of integrating these renewable energy promotion policies into a market-driven generation planning framework, along with the market outcomes and business cases.

1.3 Research Questions

This thesis focuses on designing cost-effective market mechanisms for generation planning, integrating more flexibility resources (e.g., demand flexibility and ES), and ultimately achieving a purely renewable electricity system. In this context, four sets of research questions are outlined as follows:

1. **Compared to the past vertically integrated power system, does the deregulated energy market still promote efficient operation and planning of the power system under today's carbon emission constraints?**

Although many studies have shown that the electricity system is more efficient in deregulated markets compared to vertically integrated utilities, these studies have mostly investigated from the view of economic benefits. Under today's strict environmental regulations, there is significant space to explore the role of market-driven generation planning in integrating large-scale renewable energy.

2. **Under GB's strict carbon emission restrictions, can demand flexibility in market-driven generation planning promote investment in renewable generation capacity while ensuring economic benefits?**

Previous works on demand flexibility have overlooked the time-coupling nature and emphasised the economic value. The role of time-coupling demand flexibility in market-driven generation planning under carbon emission restrictions has not been fully explored.

3. **Under GB's carbon targets, can ES in market-driven generation planning promote investment in renewable generation capacity? What is the impact of strict carbon targets on ES's energy arbitrage and provision of ancillary services?**

Many existing studies on ES focus on energy arbitrage. However, the role of ES in providing ancillary services in a market-driven generation planning model in the context of increasing renewable penetration has not been fully explored.

4. **By combining various flexibility resources (e.g., demand flexibility and ES), can the GB government's goal of achieving a fully decarbonised power system**

by 2035 be realised? Can this power system be a 100% renewable power system (excluding nuclear power)? Does the 'peak-flattening effect' of demand flexibility in the traditional energy structure still work in a 100% renewable energy system?

Under the GB government's commitment to a net-zero power sector, there is increasing interest in the 100% renewable energy system. However, there is a lack of research on attracting investment in renewable generation capacity by expanding the scale of flexibility resources to achieve a 100% renewable energy system. Moreover, the role and value of demand flexibility and ES in a purely renewable energy system have not been fully studied.

1.4 Scope and Contributions

The GB government's ambitious 2050 Net Zero target mandates a complete transition to a fully decarbonised power system by 2035. This rapid decarbonisation necessitates a cost-effective market mechanism that incentivizes generation planning aligned with GB's zero-carbon goals. However, existing research lacks a comprehensive study that simultaneously incorporates renewable energy promotion policies, demand flexibility, and utility-scale ES within a market-driven generation planning model to ensure the stable operation of a purely renewable electricity system. This thesis fills this knowledge gap by proposing a series of generation planning models to investigate the impacts of market structure, mandatory carbon emission limits, renewable energy promotion policies, demand flexibility, and utility-scale ES, respectively, and ultimately finding a feasible path towards a 100% renewable power system. This thesis offers six main contributions:

1. This thesis first proposes a system-driven single-level optimisation model to capture the characteristics of a traditional power system with a vertically integrated monopoly structure, serving as a benchmark to quantitatively analyse the impact of market structure.
2. This thesis then introduces a novel market-based two-stage bi-level optimisation framework to evaluate a 100% renewable electricity transformation plan for the GB power grid, aligning with the GB's 2035 target for a zero-carbon electricity system. The model employs a hierarchical structure, consisting of an upper-level (UL) problem and two lower-level (LL) problems. The UL problem represents the low-carbon investment strategies of the self-interested generation company (GenCo) in order to maximise its profit. The first LL problem represents the joint clearing process of

the day-ahead (DA) energy and real-time (RT) balancing markets for the target year, incorporating carbon emission limitations and the allocation of carbon allowances, aiming at the minimum system cost. Additionally, the models of time-coupling demand flexibility and ES are embedded into the first LL problem. The second LL problem represents the clearing process of the green certificate market, captured by an inverse demand function. The GenCo's decision-making process considers two sources of uncertainty: renewable energy and demand deviation. These uncertainties are modelled through a set of chance constraints.

3. The solution of the advanced two-stage bi-level optimisation model employs a single-level reduction approach. Initially, linearization methods are utilised to reformulate the chance constraints. Subsequently, this bi-level optimisation model is converted into a mathematical program with equilibrium constraints (MPEC) utilising the Karush-Kuhn-Tucker (KKT) conditions. Finally, leveraging the strong duality theorem and KKT complementarity conditions, the MPEC is further transformed into a mixed-integer quadratic programming (MIQP) problem, ensuring the efficient resolution of the global optimality via commercial branch-and-cut solvers.
4. The first set of case studies focuses on the quantitative comparison of investment decisions and business results derived from system-driven and market-driven generation planning models based on the data of the real-world 2021 GB electricity grid. The findings reveal that the system-driven model cannot reflect the current deregulated market environment, eventually facing the risk of a financial deficit in a centralised power system. On the contrary, a market-driven framework can ensure substantial profit and, meanwhile, foster investment in renewable energy.
5. The second set of case studies employs data from the 2021 GB power grid to investigate the role and value of demand flexibility and ES in promoting the integration of renewable generation capacity within a market-driven generation planning framework. The results demonstrate that demand flexibility and ES not only increase investment in renewable generation capacity but also enhance the GenCo's profitability.
6. The third set of case studies validates the feasibility of reaching an emission-free GB power system by 2035 through incorporating large-scale flexibility resources, adopting 2035 forecast data provided by the GB National Grid. The findings manifest that a combination of extensive demand flexibility and ES not only facilitates the stable operation of a 100% renewable power system while also securing adequate economic benefits. To our best knowledge, this is the first time a market-driven generation planning model has been used to investigate the energy structure

in the GB's 2035 fully decarbonised grid and factor in the role of flexibility resources in it.

Apart from the contributions mentioned above, this thesis offers valuable insights for multiple stakeholders in the power sector. Firstly, for GenCos, the investment decisions derived from our model can boost their profitability under the carbon emission restrictions imposed on the energy market. Secondly, market operators gain a comprehensive understanding of the generation resource mix in a decarbonised power grid and the importance of flexibility resources, guiding future power system reforms. Thirdly, policymakers can evaluate the impact of carbon emission limits and renewable energy promotion policies on economic benefits, helping to avoid market inefficiencies. Finally, providers of flexible resources can examine their market values and increase their incomes, thus accelerating the integration of flexible resources and improving power grid resilience.

1.5 Thesis Structure

The remainder of this thesis is organised as follows:

- **Chapter 2** first reviews the research related to generation planning from different perspectives. Then, we detail the concepts and mechanisms of various markets and the flexibility models employed in the following chapters. Finally, the solution approaches regarding the centralised approach and the game-theoretic approach are illustrated by the general optimisation models.
- **Chapter 3** proposes a single-level optimisation framework to capture generation planning with the carbon emission restrictions in a traditional power system regulated by vertically integrated utilities. The case studies use the data from the 2021 GB power system to investigate investment decisions and business results within a centralised framework. Furthermore, the results serve as a benchmark for analysing the impact of market structure.
- **Chapter 4** introduces a novel two-stage bi-level optimisation model, reflecting generation planning under carbon emission constraints in a deregulated energy market. This model is composed of an UL problem, representing a self-interested GenCo, and two LL problems: the first LL problem represents the clearing process of co-optimised DA and RT markets, and the second LL problem represents the clearing process of the green certificate market. The proposed model is solved by recasting it as a single-level MPEC through KKT optimality conditions. The case studies

quantitatively compare the economic benefits and the ability to integrate renewable energy between system-driven and market-driven generation planning models, adopting the data from the 2021 GB power system.

- **Chapter 5** focuses on the impact of demand flexibility on market-driven generation planning with carbon targets. The bi-level model employed in this chapter is in line with that present in Chapter 4, but enhanced with the incorporation of the time-coupling demand flexibility model into the first LL problem. Case studies are carried out based on the 2021 GB power system, revealing the dual role of demand flexibility in promoting investment in RESs and counteracting the uncertainty of renewable energy.
- **Chapter 6** explores the role of ES in market-driven generation planning with carbon emission limitations. The bi-level model proposed in this chapter is in line with that present in Chapter 4, but enhanced with the incorporation of the ES model into the first LL problem. The case studies utilise the data from the 2021 GB power system to demonstrate that ES can not only provide ancillary services for the growing scale of renewable energy but also enhance its profitability through energy arbitrage.
- **Chapter 7** investigates the path to achieve a 100% renewable GB power system by 2035 through integrating large-scale flexibility resources, i.e., demand flexibility and ES, into the power grid. The bi-level model utilised in this chapter is in line with that present in Chapter 4, but enhanced by the joint incorporation of the time-coupling demand flexibility model and the ES model into the first LL problem. The case studies are conducted using the predicted data of the 2035 GB power system, demonstrating that the GB's goal of completely decarbonising the whole power sector is feasible and that a 100% renewable power system can be further achieved through facilitating the expansion of flexibility resources.
- **Chapter 8** concludes this thesis by reviewing the key findings from case studies and identifying interesting topics for future research.
- **Appendix A and Appendix B** detail the data on 2021 GB system demand and GB normalised wind power output, respectively.

Chapter 2

Market Models and Modelling Approaches

2.1 Literature Review

The following subsections offer a comprehensive review of research works in the field of generation planning from different perspectives, i.e., electricity market modelling, uncertainty modelling, demand flexibility, ES, and environmental low-carbon policy.

2.1.1 Generation Planning Model

The past few years have witnessed a surge in research interest regarding generation planning problems. Numerous research studies have contributed to the development of generation planning modelling, which can be broadly categorised into two primary approaches: system-driven generation planning models and market-driven generation planning models.

System-driven generation planning models, also known as centralised generation planning models, typically employ single-level optimisation problems formulated from the perspective of the system operator, aiming to minimise the total costs. For instance, authors in [8] present a system-driven generation planning model to investigate the impact of applying convex relaxations to unit commitment (UC) constraints. Authors in [9] develop a system-driven generation planning framework, focusing on minimising the total present value of investment and operating costs while incorporating both emission allowances and renewable quota penalties. In [10], authors develop a system-driven generation planning model to explore the impact of demand elasticity and carbon policies. Authors in [11] incorporate carbon capture and storage systems into a system-driven multistage generation

planning model. Authors in [12] compare the efficiencies of investment decisions obtained from two formulations of the system-driven generation planning problem: energy-based and power-based formulations. In [13], authors address the coordinated planning problem of natural gas and power generation using a system-driven approach. Authors in [14] propose a system-driven generation planning model with the aim of reducing carbon emissions by 50% from the current level. Authors in [15] design a long-term system-driven generation planning model to integrate more RESs into Turkey's power system. Despite their prevalence, the system-driven generation planning models presented in the preceding studies exhibit inherent limitations in capturing the influence of market clearing mechanisms. Consequently, these models fall short of providing a comprehensive evaluation of the implicit financial viability associated with investment decisions made within a market-based framework.

Market-driven generation planning models, in contrast, leverage bi-level optimisation frameworks to capture the intricate interdependencies between the GenCo and the market operator. Typically, these frameworks comprise two types of coupled optimisation problems: the UL and the LL problem. The UL problem represents the objective of the GenCo, which is typically profit maximisation. Conversely, the LL problem depicts the market clearing process, aiming to maximise overall social welfare from the market operator's perspective. It is important to note that there can be one or multiple LL problems and UL problems, depending on the number of heterogeneous market participants. Several studies have employed the bi-level optimisation framework to address various generation planning challenges. Authors in [16] propose a novel bi-level optimisation model, addressing the generation planning challenge with a focus on the roles of demand flexibility and allocation mechanisms regarding reserve provision. Authors in [17] detail a stochastic bi-level optimisation model designed for wind power generation planning that leverages topology control methods for enhancing transmission capacity in a cost-effective manner. Furthermore, studies [18, 19] investigate the optimal investment decisions made by a producer in the jointly cleared electricity and renewable energy credit markets. The study [20] integrates wind power correlation uncertainties within a bi-level optimisation framework, aiming to assess optimal investment strategies for wind power producers within an electricity market environment. Authors in [21] introduce a bi-level model to quantify the effect of resource adequacy on generation planning, considering the simultaneous interaction between capacity, energy, and reserve markets. Authors in [22] offer a bi-level optimisation model investigating the strategic behaviours of a profit-seeking GenCo with an eye towards both future and spot markets. Authors in [23] investigate the investment behaviours of the examined producer to manipulate the market by strategic offering in a market-driven generation planning framework. Authors in [24]

propose a jointly market-driven planning model for wind power and transmission lines to co-optimize their investment decisions with the target of minimising total cost.

2.1.2 Uncertainty Modelling

The inherent uncertainty within power systems, particularly in the presence of high renewable energy penetration, presents a persistent challenge. This inherent uncertainty leads to inevitable mismatches between scheduling and real-time system states in electricity markets. To ensure that models accurately represent real-world market conditions and identify potential risks, incorporating uncertainty into power system modelling is crucial. Existing methods for modelling uncertainty can be broadly categorised into three main approaches: stochastic optimisation (SO), robust optimisation (RO), and chance-constrained programming (CCP) optimisation [25].

The first approach, SO, involves constructing a set of discrete future states, namely scenarios, that represent various possibilities. The model then evaluates these scenarios to identify the optimal strategy that performs well across this range of conditions [25]. This approach allows for the consideration of uncertainty in a comprehensive manner. For instance, in the context of market-based long-term generation investment planning, authors in [26] design a series of scenarios to capture the uncertainty associated with investments from rival GenCos. Authors in [27] employ a bi-level optimisation model incorporating scenario-based uncertainty from load and wind power to address the wind power generation planning problem from a profit-maximising wind power investor's perspective. Authors in [28] integrate a series of scenarios to model the renewable energy uncertainty in a generation planning framework, aiming to include 50% renewable power generation by 2030 in the GB power system. The second approach, RO, prioritises the performance of decisions under the worst-case scenario by using an uncertainty set that defines the possible range of uncertain parameters. Authors in [29] develop a multistage adaptive robust optimisation model for a generation planning problem, taking account of uncertainties introduced by large shares of RESs. Authors in [30] utilise a two-stage robust optimisation framework to address the generation planning problem under carbon growth constraints.

In contrast to SO and RO, CCP optimisation addresses uncertainty by directly incorporating it into chance constraints with predetermined confidence levels. This method allows for probabilistic modelling of uncertainties, providing a middle ground between the worst-case focus of RO and the scenario-based approach of SO. It aims to find solutions that are feasible under a certain probability, thereby reducing the conservatism of RO while also potentially lessening the computational burden associated with SO [31]. Several studies

have demonstrated the application of CCP optimisation in generation planning. The authors in [32] propose a novel long-term distributionally robust generation planning model that utilises CCP optimisation to factor in wind forecast error uncertainty by incorporating it into the chance constraints. Additionally, this model also considers UC characteristics and the risk of operational limit violations. The authors in [33] further explore CCP optimisation by incorporating it into a generation planning model with an eye to uncertainties arising from both demand growth and wind power forecast errors. The authors in [34] utilise CCP optimisation to build a generation planning model co-optimising wind farm capacity, ES capacity, and power transmission lines, explicitly considering wind power uncertainty.

2.1.3 Demand Flexibility

Demand flexibility, also known as flexible demand, plays a critical role in the electricity market. It allows for adjustments in power consumption on the demand side in response to power supply driven by economic incentives. This ability becomes increasingly important as the penetration of renewable energy increases. Demand flexibility offers solutions to mitigate the challenges associated with the inherent variability of RESs, thereby accelerating the transition towards a green and clean future. Several studies have explored the integration of demand flexibility into generation planning models. Authors in [35] utilise a piece-wise approximation of the load duration curve to model demand flexibility within a market-based generation planning framework. Similarly, authors in [36] explore the impact of demand-side response (a key component of demand flexibility) on integrated gas and electricity planning models. Focusing on the impact of price sensitivity on demand, authors in [37] present a stochastic generation planning model for quantitative analysis of demand flexibility, considering different levels of price elasticity of demand. The study [38] explores the economic benefits of demand flexibility based on time-of-use (TOU) pricing, a specific type of price-based demand response program, in the context of co-optimising generation and transmission planning. Authors in [39] assess the benefits of various flexibility mechanisms, including demand flexibility, to find the pathways to fully renewable systems.

However, the demand flexibility models employed in the above studies do not fully capture the time-coupling characteristic of demand flexibility. This key attribute refers to the reduction in demand during peak periods to offset the increase in demand during off-peak periods [40]. For example, in [41], the authors propose a centralised planning model including UC characteristics to examine the effect of time-coupling flexibility through smart charging and discharging operations of electric vehicles. In [42], a centralised

generation planning model is developed to investigate time-coupling demand flexibility facilitated by smart appliances in the context of TOU pricing.

2.1.4 Energy Storage

ES plays a vital role in the power system under high renewable energy penetration. This importance stems from their ability to address the challenges posed by the intermittent and variable nature of RESs. ES can mitigate the fluctuations from RESs through its charge-discharge cycles, improve the efficiency of renewable energy utilisation, and enhance grid stability and reliability. Consequently, ES is increasingly recognised as a promising solution to the issue of global warming. Within the deregulated electricity market environment, ES contributes to the power grid by integrating large-scale RESs primarily through two key applications: energy arbitrage and the provision of ancillary services [43].

The first application, energy arbitrage, involves exploiting price differentials in the electricity market. Recent literature emphasises significant progress in ES's energy arbitrage. Authors in [44] introduce a market-driven planning model for an ES entity to find the strategic investment decisions to maximise its profit through energy arbitrage, taking account of uncertainties arising from competing offering, bidding, and demand growth. In [45], two planning models are presented: one focusing on profitability constraints and the other on profit maximisation, both aimed at optimising investment decisions for arbitrage profits under the carbon emission limitations. The study [46] offers a comparative analysis of long-term storage arbitrage benefits derived from generation planning models that utilise various time series aggregation methods.

In terms of the second application, several studies have explored the role of ES in providing ancillary services. Authors in [47] focus on a quantitative analysis of the business results with storage sizing decisions derived from two ES modelling approaches: the discrete storage model and the continuous storage model. This analysis takes place within a generation planning framework that considers uncertainties introduced by RESs and balancing reserves provided by ES. Authors in [48] integrate ES into a centralised planning model for providing ancillary services, co-optimising distribution and generation, and accounting for the uncertainties related to demand, wind speed, and solar irradiation. Authors in [49] introduce a coordinated planning framework for generation and ES, aimed at optimising both the siting and sizing of ES systems alongside the primary frequency response services they offer.

2.1.5 Low-Carbon Policy

The power sector is a significant contributor to carbon dioxide (CO₂) emissions, a primary greenhouse gas responsible for global warming and climate change. This is largely due to the consumption of fossil fuels [50]. In response, a diverse range of low-carbon policies have been implemented to reduce carbon emissions and encourage the integration of large-scale renewable energy into the power grid. These policies can be categorised into two main types: carbon emission constraints and renewable energy promotion policies.

In terms of the first category, authors in [51] develop a stochastic generation planning model that incorporates a cumulative carbon emission limit, aiming to analyse multi-scale uncertainties using both scenario partitioning and scenario reduction methods. Authors in [52] introduce a generation planning model that accounts for carbon targets to examine the impact of operational flexibility in the context of stringent renewable portfolio standard (RPS) requirements. The studies [53, 54] explore the impact of carbon emission constraints on an integrated electricity and natural gas planning model, which includes electric vehicle fast-charging stations and addresses multi-scale uncertainties. Authors in [55] incorporate the carbon emission cost into a long-term generation planning model based on Portuguese and Spanish power systems. The carbon emission constraints employed in the aforementioned studies solely enforce a mandatory annual cap on system-wide carbon emissions. This approach fails to account for the economic implications for stakeholders in the power system within the current market environment.

The second category, renewable energy promotion policies, aims at fostering the growth and integration of RESs. These policies, often market-driven and paired with rigorous carbon targets, are established by governments and authoritative organisations to expand the scale of RESs in the energy structure. Authors in [56] propose and analyse the economic benefits of two incentive-based regulations: carbon tax and cap-and-trade scheme, using a generation planning model. Authors in [57] examine the role of four low-carbon policies—carbon tax, cap-and-trade scheme, RPS, and renewable production credit—in a generation planning framework from the perspective of environmental and economic benefits. Authors in [58] explore the combined effect of carbon tax and government subsidy on facilitating investment in renewable energy in a centralised generation planning model. In [59], authors incorporate the step-wise cap-and-trade scheme into a generation planning model, taking account of renewable energy uncertainty across multiple time scales.

2.2 Market Mechanisms

Historically, the GB's power sector, like many others around the world, operated under a vertically integrated monopoly structure. This model typically consists of a single entity encompassing the generation, transmission, and distribution of electricity [60]. While this approach offers advantages in terms of centralised planning and control, it also presents significant drawbacks over time. The lack of competition in the market hinders technological innovation and lowers efficiency, finally leading to high prices for consumers. Additionally, this monopoly structure makes it difficult to attract investment in new generation capacity and other infrastructure [60].

Recognising these shortcomings, the GB government initiated a series of reforms throughout the 1980s and 1990s, gradually transitioning the power sector towards a more competitive, market-based structure. This process, known as deregulation (or liberalisation), aims to introduce competition across the different stages of the power supply chain. Deregulation fosters competition among different GenCos, retailers, and other market participants, leading to potentially lower prices and a wider range of choices for consumers. Moreover, market competition incentivizes participants to improve technological innovation and efficiency, ultimately encouraging great investment and enhancing overall system reliability [61].

At the core of this thesis, today's electricity markets are subject to various strict environmental regulations, presenting new opportunities and challenges for GenCos, as illustrated in Figure 2.1. GenCos can participate in various markets that span different time scales, including the energy market, the balancing market, the carbon market, and the green certificate market. Traditionally, GenCos can engage in energy and reserve trading across different time horizons in the energy and balancing markets. The carbon market and green certificate market are established based on the vision of the current low-carbon transition, where GenCos trade carbon allowances and green certificates to reduce carbon emissions and expand the proportion of renewable energy. These markets are detailed in the following subsections.

2.2.1 Energy Market

The energy market is the central platform for buying and selling electricity for the following day, as well as determining electricity prices through the locational marginal pricing (LMP) scheme. In this thesis, the energy market is modelled as a pool-based, energy-only market with a day-ahead horizon, reflecting the physical realities of the GB power system. The market operator clears this energy market by matching the bids and

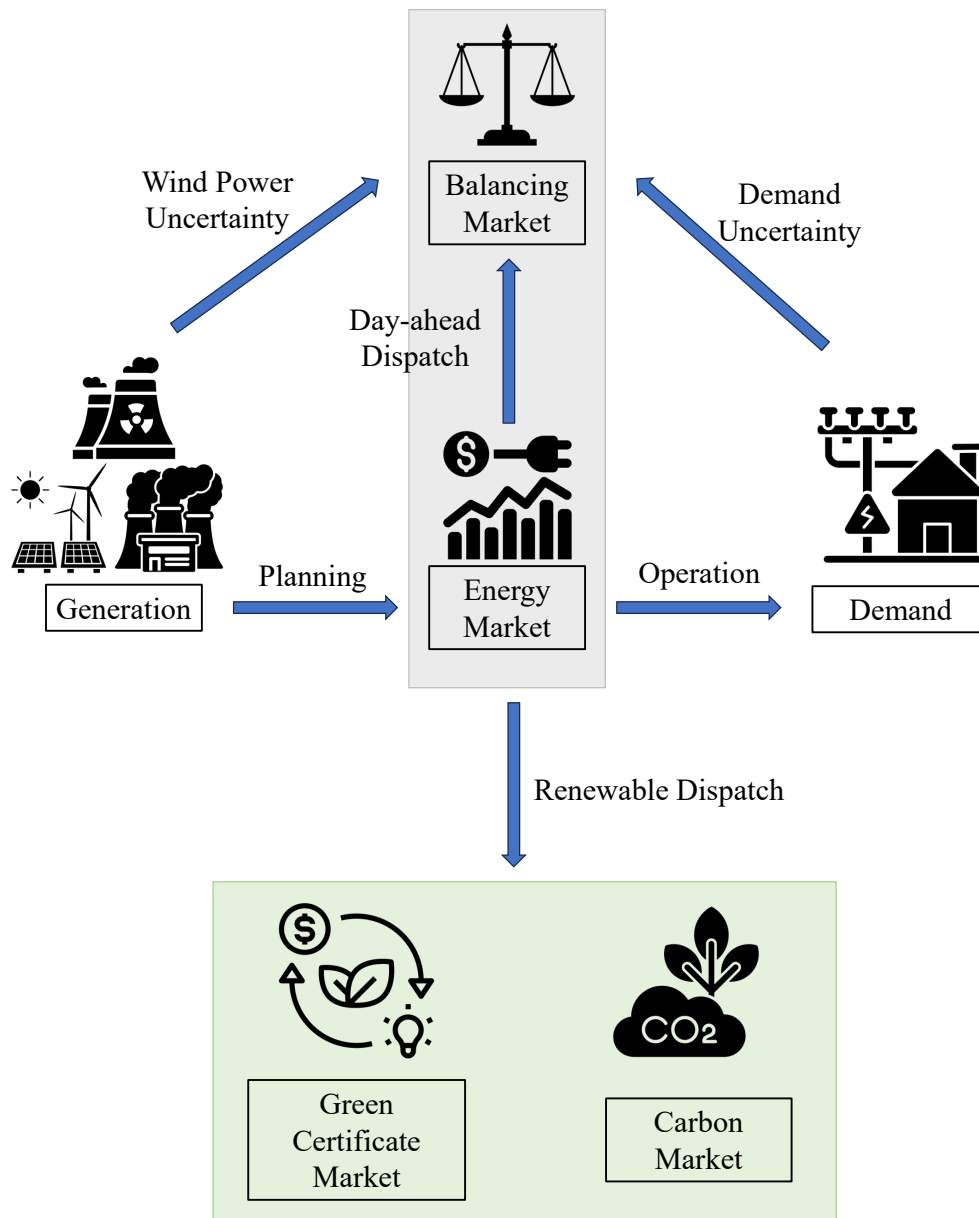


Figure 2.1: The deregulated markets from the perspective of GenCo.

offers based on the principle of merit order, which prioritises generation from the cheapest sources first, ensuring a cost-effective market clearing process.

2.2.2 Balancing Market

The balancing market plays a vital role in providing ancillary services to maintain grid stability by addressing mismatches between the forecasted and actual electricity supply and demand in real-time. In GB's power system, the National Grid Electricity System Operator (NG ESO) is responsible for this market [62]. In this thesis, the clearing of bal-

ancing markets is modelled by the co-optimisation framework for energy and balancing markets. The importance of the balancing market is increasing in the context of the GB's ambitious carbon goals, including achieving 'Net Zero' by 2050. In parallel, as the GB government continues to expand its renewable energy capacity and develop more sophisticated demand-side management technologies, the role of the balancing market becomes even more critical.

2.2.3 Carbon Market

The carbon market, particularly within the context of the GB's power system, represents a critical mechanism for driving the low-carbon transition. This market-based approach utilises economic incentives to reduce CO₂ and other greenhouse gas emissions. In the GB power system, the carbon market is achieved by the UK Emissions Trading Scheme (UK ETS), a cap-and-trade system that is operated by the GB government [63]. Specifically, the UK ETS will impose a cap on the total carbon emissions of the power sector on a yearly basis. This cap is gradually reduced over time, finally leading to a zero-carbon power system. The GenCos require allowances to emit carbon. These carbon allowances can be bought and sold on the carbon market. The prices are determined by the auctions [63]. In this thesis, the carbon emission cost is applied to the generators using fossil fuels. Furthermore, it is assumed that the free carbon allowances are allocated based on renewable generation dispatches.

2.2.4 Green Certificate Market

The transition towards a sustainable and low-carbon power system involves not only reducing carbon emissions but also encouraging the integration of renewable energy. While the carbon market addresses the former, the green certificate market plays a unique role in supporting the latter. GenCos can obtain the green certificates based on the green electricity they generate and then sell them on the green certificate market. Similar to the carbon market, the green certificate market offers a market-based approach to promoting investment in renewable energy capacity through an additional revenue stream. Additionally, it is a promising approach to dealing with the 'missing money' dilemma incurred by a 100% renewable energy system [64].

2.3 Flexibility Models

In this thesis, the models of two key flexibility resources, demand flexibility and ES, are incorporated into the generation planning framework to explore the pathway towards a 100% renewable power system. Thus, their models are introduced in the following subsections.

2.3.1 Demand Flexibility

A time-coupling demand flexibility model is employed across the DA and RT market horizons, accompanied by the uncertainty introduced by demand shifting, as expressed in (2.1)-(2.4). The variable L_t^{SH} represents the demand shifting with respect to the realisation of baseline demand L_t^R at period t . Constraint (2.1) expresses the DA demand consumption P_t^{DA} is limited by the predetermined ratio of demand flexibility α ($0 \leq \alpha \leq 1$), where subscript L_t^{DA} indicates baseline demand. Constraint (2.2) identifies the energy-neutral characteristic of the demand shifting, i.e., the total amount of demand reduction is equal to the total amount of demand increase on a daily basis. Chance constraint (2.3) represents that demand shifting is limited to the range of the predetermined proportion α regarding the realisation of baseline demand with a confidence level of $1 - \rho^L$. Chance constraint (2.4) expresses that the total demand across the DA and RT markets, $P_t^{DA} + P_t^{dev}$, follows the realisation of net baseline demand, $L_t^R + L_t^{SH}$, with a probability no less than $1 - \rho^L$.

$$(1 - \alpha) L_t^{DA} \leq P_t^{DA} \leq (1 + \alpha) L_t^{DA} \quad \forall t \quad (2.1)$$

$$\sum_t L_t^{SH} = 0 \quad (2.2)$$

$$\mathbb{P} \left[-\alpha L_t^R \leq L_t^{SH} \leq \alpha L_t^R \right] \geq 1 - \rho^L \quad \forall t \quad (2.3)$$

$$\mathbb{P} \left[P_t^{DA} + P_t^{dev} \geq L_t^R + L_t^{SH} \right] \geq 1 - \rho^L \quad \forall t \quad (2.4)$$

2.3.2 Energy Storage

A dynamic continuous model is employed in this study to capture the characteristics of ES across the DA and RT market horizons, as formulated in (2.5)-(2.11). Constraint (2.5) defines the ES's dynamic energy balance with charge-discharge cycles. Constraint (2.6) requires that the initial energy level of ES be identical to the energy level of the end period. Constraint (2.7) expresses the lower and upper bounds of the ES's energy level. Constraints (2.8)-(2.11) represent the power capacity for ES's charging and discharging

across the DA and RT markets.

$$E_t = E_{t-1} + \eta^c(s_t^{c,DA} + s_t^{c,RT}) - \frac{s_t^{d,DA} + s_t^{d,RT}}{\eta^d} \quad \forall t \quad (2.5)$$

$$E^0 = E_T \quad (2.6)$$

$$E^{min} \leq E_t \leq E^{max} \quad \forall t \quad (2.7)$$

$$0 \leq s_t^{c,DA} \leq s^{max} \quad \forall t \quad (2.8)$$

$$0 \leq s_t^{c,DA} + s_t^{c,RT} \leq s^{max} \quad \forall t \quad (2.9)$$

$$0 \leq s_t^{d,DA} \leq s^{max} \quad \forall t \quad (2.10)$$

$$0 \leq s_t^{d,DA} + s_t^{d,RT} \leq s^{max} \quad \forall t \quad (2.11)$$

The ES entity in this thesis is allowed to participate in the DA energy and RT balancing markets, thereby making profit by energy arbitrage utilising the price differentials across the DA and RT markets. The profit of ES entity can be formulated as follows:

$$Pro^{ES} = \sum_t (\lambda_t^{DA} s_t^{d,DA} + \lambda_t^{RT} s_t^{d,RT}) - \sum_t (\lambda_t^{DA} s_t^{c,DA} + \lambda_t^{RT} s_t^{c,RT}) \quad (2.12)$$

where λ_t^{DA} and λ_t^{RT} are clearing prices for DA and RT markets, respectively.

2.4 Uncertainty Model

In this thesis, all the proposed generation planning models take account of the uncertainties sourced from demand and renewable energy production, which are modelled through a series of chance constraints. Therefore, the mathematical formulations related to chance constraints are interpreted in this section.

2.4.1 Chance Constraint

Power systems are experiencing a growing integration of RESs, which are characterised by the uncertainties that arise from their volatility and intermittency. Traditional methods such as RO and SO have been prevalent in addressing these uncertainties. However, they exhibit certain limitations in the complex modelling of modern power systems.

The incorporation of RESs introduces multiple layers of uncertainty into power system operations. SO tackles these challenges by considering a range of possible scenarios. However, with the growing RESs, the need to account for a broader array of scenarios

escalates, thereby significantly increasing the computational burden. This high computational requirement for extensive scenarios will lead to a more time-consuming optimisation process and a heightened risk of non-convergence in the MIP gap [65]. In parallel, RO is designed to provide solutions that can withstand worst-case scenarios. While this approach ensures reliability, it tends to lead to solutions that are overly conservative [31]. In the context of extreme fluctuations incurred by large-scale renewable energy integration, utilising RO to cope with the absolute worst-case scenario will result in solutions that are not only excessively conservative but also economically inefficient, as they may require substantial overcapacity or stringent operational constraints.

Given these limitations, CCP has emerged as an attractive alternative [66]. CCP offers a more balanced approach to handling uncertainties in power systems with high levels of RES integration. Specifically, as illustrated by the constraints (3.21) and (3.22), the focus is on limiting decision variables that fulfil the chance constraints with a predetermined confidence level. CCP distinguishes itself from traditional RO and SO by striking a balance between these two approaches, as detailed in the [66]. On the one hand, in contrast to SO, CCP does not depend on a vast array of scenarios, which makes it computationally tractable. This aspect is particularly beneficial in power systems where the high penetration of RESs could otherwise necessitate a number of unwieldy scenarios in SO, leading to increased computational burdens. On the other hand, CCP transforms the rigid constraints of RO into probabilistic chance constraints. This approach ensures that system reliability and safety are maintained without the excessive conservatism characteristic of RO solutions.

The general chance-constrained optimisation problem can be expressed as follows:

$$\min_x c^T x \quad (2.13)$$

$$s.t. \quad \mathbb{P}_\xi(f(x, \xi) \leq 0) \geq 1 - \epsilon \quad (2.14)$$

where x represents decision variables and ξ is a random vector introducing uncertainties. The constraint (2.14) is a general chance constraint, which means that the inner function $f(x, \xi)$ should be no less than zero with a probability higher than $1-\epsilon$. It can be observed that the chance constraint (2.14) is nonlinear. Therefore, based on the techniques detailed in [67], it gives a special instance of the CCP problem that can be linearized, aligning with the mathematical form of chance-constrained uncertainties in our model. This special case of CCP can be detailed as follows:

$$\min_x c^T x \quad (2.15)$$

$$s.t. \quad \mathbb{P}(a^T x + b^T \xi + \xi^T D x \leq e) \geq 1 - \epsilon \quad (2.16)$$

where a , b , D , and e are the parameters with appropriate dimensions, and ξ represents the multivariate normal distribution with mean m and covariance Σ . As proved in [67], if ϵ is no more than 0.5, the special case (2.15)-(2.16) is equivalent to the following problem:

$$\min_x c^T x \quad (2.17)$$

$$s.t. \quad e - b^T m - (a + D^T m)^T x \geq \Phi^{-1}(1 - \epsilon) \sqrt{(b + D x)^T \Sigma (b + D x)} \quad (2.18)$$

where $\Phi^{-1}(\cdot)$ is the inverse cumulative distribution function of a standard normal distribution.

Given that the normal distributions adopted in this work are one-dimensional and they are not correlated with the decision variables (indicated by $D=0$), the equivalent constraint (2.18) can be further reduced to the following linear constraint:

$$e - b^T m - a^T x \geq \Phi^{-1}(1 - \epsilon) |b|s \quad (2.19)$$

where s is the standard deviation of the normal distribution.

2.5 Solution Approaches

In this thesis, two optimisation approaches are utilised: the centralised approach and the game-theoretic approach, corresponding to the two types of generation planning modelling summarised in the literature review, namely the system-driven model and the market-driven model. These two optimisation approaches are detailed in the following subsections.

2.5.1 Centralised Approach

The power system was once regulated under a vertically integrated structure, where the system operator handled generation, transmission, and distribution. In such a scenario, the centralised approach was used to model the whole system from the perspective of the system operator. Single-level optimisation serves as a fundamental centralised approach for addressing these system-driven generation planning problems. The general formulation is listed as (2.20)-(2.22).

$$\min_x f(x) \quad (2.20)$$

subject to:

$$g_j(x) \leq 0 \quad \forall j \quad (2.21)$$

$$h_k(x) = 0 \quad \forall k \quad (2.22)$$

The decision variables are denoted as x . Constraints (2.21)-(2.22) represent inequality and equality constraints, respectively. Since the single-level structure is perfectly compatible with the commercial solvers, this single-level optimisation problem can be solved efficiently.

2.5.2 Game-Theoretic Approach

The increasing penetration of RESs in the power system necessitates secure and cost-effective planning strategies within the complex market environment. In this context, bi-level optimisation has become a powerful game-theoretic approach for addressing the complexities of market-driven generation planning problems over the last two decades [68]. Unlike the traditional single-level optimisation framework employed in the centralised approach, bi-level optimisation acknowledges the hierarchical decision-making structure, reflecting the interactions among multi-level market participants in the real world. Generally, bi-level optimisation models the market-driven generation planning problems through two levels: an upper level for profit-seeking players (leader) and a lower level for energy market clearing (follower).

Typically, when the LL problem is convex, a bi-level optimisation problem can be transformed into a single-level problem, known as a MPEC, by replacing the LL problem with a set of equivalent conditions. As shown in Figure 2.2, this method of replacement can be divided into two types: the KKT optimality conditions and the primal-dual transformation. It is important to note that the bi-level model has a cyclic nature, thus requiring a fixed point for solution process. Typically, in generation planning problems, the optimal investment capacity in the LL problem is considered a fixed value (parameter).

MPEC Formulation through KKT Optimality Conditions

Regarding the first method, the KKT optimality conditions can be divided into three parts: equality conditions, stationarity conditions, and complementarity conditions. Let us illustrate these KKT conditions through a general bi-level optimisation problem, formulated in (2.23)-(2.28).

$$\min_{x, y_l} F(x, y_1^*, y_2^*, \dots, y_l^*) \quad (2.23)$$

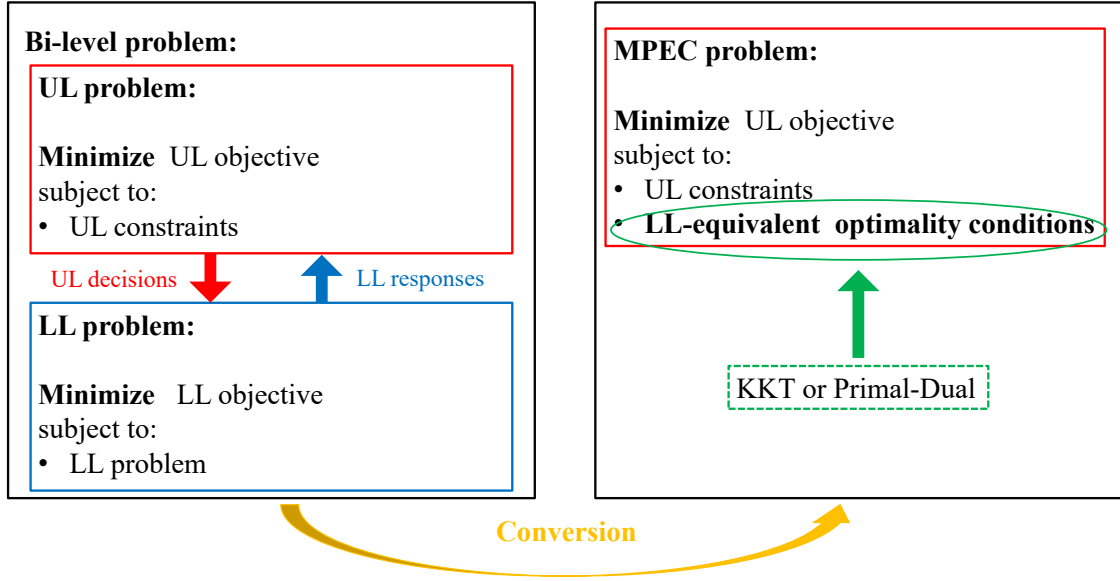


Figure 2.2: Recasting the bi-level optimisation model into a MPEC.

subject to:

$$G(x, y_1^*, y_2^*, \dots, y_l^*) \leq 0 \quad (2.24)$$

$$H(x, y_1^*, y_2^*, \dots, y_l^*) = 0 \quad (2.25)$$

$$y_l^* \in \arg \min_{y_l} \{f_l(x, y_l)\} \quad (2.26)$$

subject to:

$$g(x, y_l) \leq 0 \quad (2.27)$$

$$h(x, y_l) = 0 \quad \forall l \quad (2.28)$$

This bilevel optimisation problem (2.23)-(2.28) represents a single-leader multi-follower game, consisting of a UL problem (2.23)-(2.25) and N_l LL problems (2.26)-(2.28). It should be noted that the y_l^* indicates the optimal solution of the l -th LL problem. Additionally, although x represents the UL decision variables, they are treated as parameters from the view of the LL problems.

In detail, the KKT equality conditions consist of the equality constraints from the original problem. The KKT stationarity conditions are obtained by differentiating the Lagrangian equation with respect to the primal variables. The KKT complementarity conditions represent the complementary relationships between the inequality constraints of the original problem and the corresponding dual variables [69]. By replacing the LL problems (2.26)-(2.28) with their KKT conditions, the proposed bi-level optimisation problem

can be reduced to a single-level MPEC problem (2.29)-(2.36).

$$\min_{x, y_l, \mu_l, \lambda_l} F(x, y_l) \quad (2.29)$$

subject to:

UL constraints :

$$\{G(x, y_l) \leq 0 \quad (2.30)$$

$$H(x, y_l) = 0 \quad (2.31)$$

LL KKT equality conditions :

$$h(x, y_l) = 0 \quad (2.32)$$

LL KKT stationarity conditions :

$$\nabla_{y_l} f_l(x, y_l) + \mu_l \nabla_{y_l} g(x, y_l) + \lambda_l \nabla_{y_l} h(x, y_l) = 0 \quad (2.33)$$

$$\mu_l \geq 0 \quad (2.34)$$

$$\lambda_l \text{ is free} \quad (2.35)$$

LL KKT complementarity conditions :

$$\mu_l g(x, y_l) = 0 \quad \forall l \quad (2.36)$$

where μ_l and λ_l are dual variables corresponding to constraints (2.27) and (2.28), respectively. Note that the symbol ∇_{y_l} identifies the gradient with respect to variables y_l .

However, it should be noted that the KKT complementarity conditions (2.36) are still non-linear. Thus, to make the reduced problem compatible with branch-and-cut solvers, these non-linearities can be eliminated by the big-M method. Finally, the bi-level optimisation is converted into a mixed-integer programming (MIP) problem, which is formulated as follows:

$$\min_{x, y_l, \mu_l, \lambda_l, \psi_l} F(x, y_l) \quad (2.37)$$

subject to:

$$(2.30) - (2.35) \quad (2.38)$$

$$\{\mu_l \leq \psi_l M \quad (2.39)$$

$$-g(x, y_l) \leq (1 - \psi_l) M \quad (2.40)$$

$$\psi_l \in \{0, 1\} \quad \forall l \quad (2.41)$$

where ψ_l is the auxiliary binary variable and M is a large constant. It should be noted that

the large constants M involved in the respective constraints should be carefully selected in terms of the physical bounds of variables, since they determine the search space of the MIP problem and further computational efficiency is profoundly affected.

MPEC Formulation through Primal-Dual Transformation

In order to transform the bi-level optimisation problem into a single-level MPEC, the LL problems can be replaced by their equivalent primal and dual constraints, as well as strong duality equality constraints. Similarly, let us illustrate the process of primal-dual transformation using the following example of a bi-level optimisation problem (2.42)-(2.46):

$$\min_{x, y_l} F(x, y_1^*, y_2^*, \dots, y_l^*) \quad (2.42)$$

subject to:

$$G(x, y_1^*, y_2^*, \dots, y_l^*) \leq 0 \quad (2.43)$$

$$H(x, y_1^*, y_2^*, \dots, y_l^*) = 0 \quad (2.44)$$

$$y_l^* \in \arg \min_{y_l} \{c^T x + d(x)^T y_l\} \quad (2.45)$$

subject to:

$$Ax + B(x)y_l \leq b_1 : \mu_l \quad (2.46)$$

$$Cx + D(x)y_l = b_2 : \lambda_l \quad (2.47)$$

$$y_l \geq 0 : \gamma_l \quad \forall l \quad (2.48)$$

where μ_l , λ_l , and γ_l are dual variables corresponding to the LL constraints (2.46), (2.47), and (2.48), respectively. Compared with the bi-level optimisation present in the previous subsection, this illustrative example (2.42)-(2.48) includes N_l linear LL problems.

Since the LL problems can be seen as the linear x -parameterized linear problems, their dual problems are formulated as follows:

$$c^T x + \max_{\mu_l, \lambda_l, \gamma_l} \mu_l^T (b_1 - Ax) + \lambda_l^T (b_2 - Cx) \quad (2.49)$$

subject to:

$$B(x)^T \mu_l + D(x)^T \lambda_l = d(x) \quad (2.50)$$

$$\mu_l \geq 0 \quad (2.51)$$

$$\gamma_l \geq 0 \quad (2.52)$$

$$\lambda_l \text{ is free} \quad (2.53)$$

Due to the convexity of the LL problems, the strong duality theorem holds as follows:

$$d(x)^T y_l = \mu_l^T (b_1 - Ax) + \lambda_l^T (b_2 - Cx) \quad (2.54)$$

Finally, the bi-level optimisation problem is converted into a single-level optimisation problem by substituting the LL problems by their primal-dual equivalent conditions.

$$\min_{x, y_l, \mu_l, \lambda_l, \gamma_l} F(x, y_l) \quad (2.55)$$

subject to:

UL constraints :

$$\{G(x, y_l) \leq 0 \quad (2.56)$$

$$H(x, y_l) = 0 \quad (2.57)$$

LL primal constraints :

$$Ax + B(x)y_l \leq b_1 \quad (2.58)$$

$$Cx + D(x)y_l = b_2 \quad (2.59)$$

$$y_l \geq 0 \quad (2.60)$$

LL dual constraints :

$$B(x)^T \mu_l + D(x)^T \lambda_l = d(x) \quad (2.61)$$

$$\mu_l \geq 0 \quad (2.62)$$

$$\gamma_l \geq 0 \quad (2.63)$$

$$\lambda_l \text{ is free} \quad (2.64)$$

LL strong duality equality :

$$d(x)^T y_l = \mu_l^T (b_1 - Ax) + \lambda_l^T (b_2 - Cx) \} \quad \forall l \quad (2.65)$$

It should be noted that the KKT stationarity conditions of LL problems are equivalent to the equality constraints of the LL dual problems, and the KKT complementarity conditions of LL problems are consistent with the strong duality equality of primal-dual transformation [70].

Chapter 3

System-Driven Generation Planning

3.1 Introduction

The landscape of the GB power industry has been subject to significant evolution, shaped by the shift from the previous integrated operating monopoly structure to today's deregulated market structure. Historically, the GB's power system was characterised by an integrated operating monopoly structure, a model that centralised control over generation, transmission, and distribution within a single entity. This configuration, emblematic of many national power systems during the 20th century, was established on the rationale that electricity supply constituted a natural monopoly, best managed through a centralised approach to ensure its efficiency and reliability, thereby maximising social welfare [60].

In this context, this chapter proposes a system-driven generation planning framework that reflects the characteristics of generation planning in a traditional vertically integrated monopoly structure. It offers a holistic approach to generation capacity expansion with an overarching cost-minimising objective.

Despite the transition towards a deregulated market structure in the latter decades, the principles of system-driven generation planning retain their importance. Specifically, system-driven generation planning can serve as a benchmark, inspiring today's complex multi-layer market design. Moreover, in the context of the GB's commitment to achieve net-zero carbon emissions by 2050, the roles of various low-carbon policies, such as carbon allowance and green certificate, should also be incorporated into the system-driven model to uncover their implicit impacts on centralised generation planning, offering guidance for future integration into the market framework. Overall, in an era where the energy sector faces the dual challenges of designing cost-effective market mechanisms and navigating towards a low-carbon future, this system-driven generation planning model can provide invaluable insights for policymakers.

The system-driven generation planning model proposed in this chapter is established for quantitative analysis of the GB power grid regulated by an integrated monopoly. Additionally, the GB's carbon emission limits are incorporated into this model. A series of case studies is designed to evaluate its business results, utilising the data sourced from the 2021 GB power system. The findings clearly demonstrate that despite the system cost remaining low, the system-driven generation planning framework cannot recover the total cost and integrate more RESs.

3.2 Problem Statement

This section mathematically expands upon the decision-making process involved in the system-driven generation planning and outlines the key assumptions underpinning the modelling.

3.2.1 Structure of the Single-Level Optimisation Model

The system-driven model is presented within a two-stage, single-level LP optimisation framework, reflecting the vertically integrated structure of the electricity industry regulated by the centralised decision-maker, the system operator. This monopoly structure is visually represented in Figure 3.1, providing a clear depiction of the single-level modelling approach. Specifically, at the core of this single-level framework, the system operator pursues minimising total cost. In parallel, this cost-minimising pursuit is subject to a series of constraints stemming from the operation of the power system and environmental policies. The examined vertically integrated utility is characterised by a two-stage architecture that accommodates both the DA scheduling stage and the RT balancing stage, capturing the centralised system operator's perspective.

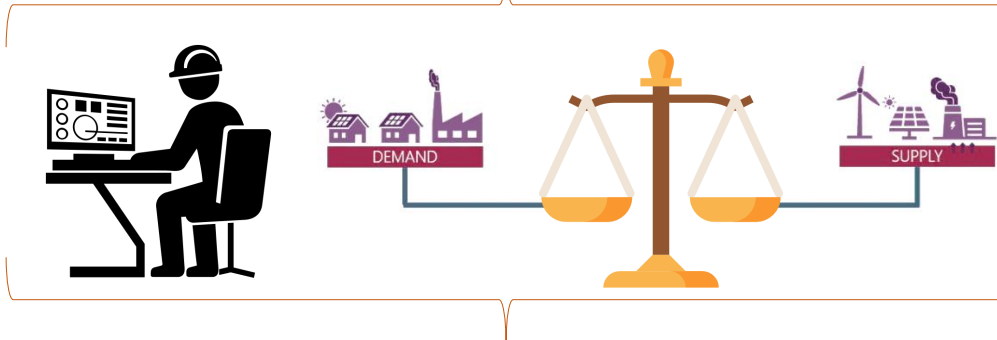
3.2.2 Modelling Assumptions

For comprehensive understanding, the major assumptions that frame the scope of the proposed system-driven generation planning are outlined below:

1. The proposed single-level optimisation model represents the system-driven single-stage generation planning faced by the system operator for the sake of minimising total cost. Specifically, power generation is under an integrated operating monopoly structure regulated by the system operator. The system operator is provided with various investment options for generation capacities utilising flexible, must-run, and

Investment decisions from the perspective of centralized system operator

Min System Cost + Investment Cost



subject to:

- Carbon allowance allocation constraints
- Demand-supply balance constraints in the DA and RT stages
- System-wide carbon emission regulatory constraints
- Generation operating constraints
- Upward and downward reserve constraints
- Transmission network constraints in the DA and RT stages
- Chance constraints for renewable energy uncertainty
- Chance constraints for demand uncertainty

Figure 3.1: Illustration of the two-stage single-level optimisation problem.

renewable technologies. Additionally, the flexible generation is divided into a series of production blocks to model its output characteristics. In parallel, demand blocks are also included in the proposed system-driven model.

2. To efficiently solve the model with uncertainties stemming from renewable power generation and demand, a set of chance constraints is utilised.
3. The primary focus of this work is to investigate the composition of system-level generation in future power networks, especially under the conditions of strict carbon emission regulations. Therefore, the existing generation capacity is ignored in the proposed system-driven model.
4. To maintain the convexity of the model, linear generation costs are implemented. Moreover, it is assumed that renewable generation technologies incur no cost.
5. In the system-driven model, it is assumed that the generation capacities of RESs and must-run technologies are utilised at their full capacities.

6. The network transmission is represented through a linear DC power flow approximation.
7. In response to decarbonisation, the proposed model integrates a comprehensive framework of carbon emission restrictions. On the one hand, the total flexible generation capacity within the system, which consumes fossil fuels, is constrained by an annual limit on carbon emissions. On the other hand, the flexible generation is charged for its carbon emissions. This framework is designed to guide the system towards a more sustainable and low-carbon future.

3.3 Mathematical Model

The system-driven generation planning is a single-level optimisation problem derived from the perspective of the system operator, and it can be formulated as follows:

$$\begin{aligned}
 \min_{V^{SYS}} \quad & \sum_d \omega_d \left[\underbrace{\sum_{i,t,n,b} C_{i,b}^{EN} g_{i,d,t,n,b}^{DA}}_{\text{DA energy cost}} + \underbrace{\sum_{i,t,n,b} (C_{i,b}^U g_{i,d,t,n,b}^U - C_{i,b}^D g_{i,d,t,n,b}^D)}_{\text{RT reserve cost}} \right. \\
 & \left. + \underbrace{\sum_{i \in I^{FL}, t, n} \lambda^{CO_2} Q_{i,d,t,n}^{emi}}_{\text{Carbon emission cost}} \right] + \underbrace{\sum_{i,n} C_i^{IN} G_{i,n}}_{\text{Investment cost}} \quad (3.1)
 \end{aligned}$$

where:

$$V^{SYS} = \{G_{i,n}, Q_{i,d,t,n}^{emi}, g_{i,d,t,n}^{DA}, g_{i,d,t,n}^U, g_{i,d,t,n}^D, P_{j,d,t,o}^{dev}, g_{i,d,t,n}^{dev}, \delta_{d,t,n}^{DA}, \delta_{d,t}^{DA,ref}, \delta_{d,t,n}^{RT}, \delta_{d,t}^{RT,ref}\} \quad (3.2)$$

subject to:

$$G_{i,n} \geq 0 \quad \forall i \in I, \forall n \quad (3.3)$$

$$G_{i,n,b} = \omega_b^{FL} G_{i,n} \quad \forall i \in I^{FL}, \forall n, \forall b \quad (3.4)$$

$$Q_{i,d,t,n}^{emi} = (g_{i,d,t,n}^{DA} + g_{i,d,t,n}^U - g_{i,d,t,n}^D) \eta_i^{CO_2} \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n \quad (3.5)$$

$$\sum_{i \in I^{FL}, t, n, b} \eta_i^{CO_2} (g_{i,d,t,n}^{DA} + g_{i,d,t,n}^U - g_{i,d,t,n}^D) \leq \eta^{Grid} \sum_{j,t,o} L_{j,d,t,o}^{DA} \quad \forall d \quad (3.6)$$

$$\sum_{j \in \Psi_n, o} L_{j,d,t,o}^{DA} + \sum_{m \in \Lambda_n} b_{nm} (\delta_{d,t,n}^{DA} - \delta_{d,t,m}^{DA}) - \sum_{i \in \Psi_n, b} g_{i,d,t,n,b}^{DA} = 0 \quad \forall d, \forall t, \forall n \quad (3.7)$$

$$\sum_{j \in \Psi_{n,o}} P_{j,d,t,o}^{dev} + \sum_{i \in I^{FL} \cap \Psi_{n,b}} (-g_{i,d,t,n,b}^U + g_{i,d,t,n,b}^D) + \sum_{i \in I^{RE} \cap \Psi_n} g_{i,d,t,n}^{dev} - \sum_{m \in \Lambda_n} b_{nm}(\delta_{d,t,n}^{DA} - \delta_{d,t,n}^{RT} - \delta_{d,t,m}^{DA} + \delta_{d,t,m}^{RT}) = 0 \quad \forall d, \forall t, \forall n \quad (3.8)$$

$$g_{i,d,t,n,b}^{DA} + g_{i,d,t,n,b}^U \leq G_{i,n,b} \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (3.9)$$

$$g_{i,d,t,n,b}^D \leq g_{i,d,t,n,b}^{DA} \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (3.10)$$

$$0 \leq g_{i,d,t,n,b}^U \leq G_{i,b}^U \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (3.11)$$

$$0 \leq g_{i,d,t,n,b}^D \leq G_{i,b}^D \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (3.12)$$

$$g_{i,d,t,n}^{DA} = G_{i,n} \quad \forall i \in I^{MR}, \forall d, \forall t, \forall n \quad (3.13)$$

$$g_{i,d,t,n}^{DA} = k_{i,d,t}^{RE} G_{i,n} \quad \forall i \in I^{RE}, \forall d, \forall t, \forall n \quad (3.14)$$

$$-C_{nm}^{max} \leq b_{nm}(\delta_{d,t,n}^{DA} - \delta_{d,t,m}^{DA}) \leq C_{nm}^{max} \quad \forall d, \forall t, \forall n, \forall m \in \Lambda_n \quad (3.15)$$

$$-\pi \leq \delta_{d,t,n}^{DA} \leq \pi \quad \forall d, \forall t, \forall n \quad (3.16)$$

$$\delta_{d,t}^{DA,ref} = 0 \quad \forall d, \forall t \quad (3.17)$$

$$-C_{nm}^{max} \leq b_{nm}(\delta_{d,t,n}^{RT} - \delta_{d,t,m}^{RT}) \leq C_{nm}^{max} \quad \forall d, \forall t, \forall n, \forall m \in \Lambda_n \quad (3.18)$$

$$-\pi \leq \delta_{d,t,n}^{RT} \leq \pi \quad \forall d, \forall t, \forall n \quad (3.19)$$

$$\delta_{d,t}^{RT,ref} = 0 \quad \forall d, \forall t \quad (3.20)$$

$$\mathbb{P} \left[g_{i,d,t,n}^{DA} - g_{i,d,t,n}^{dev} \leq g_{i,d,t,n}^R \right] \geq 1 - \rho^{RE} \quad \forall i \in I^{RE}, \forall d, \forall t, \forall n \quad (3.21)$$

$$\mathbb{P} \left[\sum_o L_{j,d,t,o}^{DA} + \sum_o P_{j,d,t,o}^{dev} \geq L_{j,d,t}^R \right] \geq 1 - \rho^L \quad \forall j, \forall d, \forall t \quad (3.22)$$

The objective function of the system-driven generation planning model is formulated in (3.1), aiming to minimise the overall system cost. Specifically, the overall system cost consists of four components: (i) the energy cost incurred by energy transactions in the DA stage; (ii) the reserve cost posed by reserve deployment in the RT stage; (iii) the carbon emission cost caused by flexible generation dispatches; and (iv) the investment cost for new generation capacity. The decision variables of the system-driven model include the total invested capacities across all generation technologies, carbon emission quantities, DA generation dispatches, RT upward and downward reserves, demand and renewable power deviations, DA voltage angles, and RT voltage angles, as outlined in the set (3.2). It is important to note that the subscript b in DA generation dispatches, used for compact notation, can be omitted when calculating dispatches for renewable and must-run

capacities.

The system-driven generation planning is bound by six sets of constraints, which are detailed as follows:

(i) Invested capacity constraints (3.3) - (3.4).

This set of constraints imposes restrictions on the new investments in generation capacities with various technologies. Constraint (3.3) ensures that the newly invested generation capacities by the GenCo are not negative, reflecting the physical reality that generation capacities cannot be less than zero. Constraint (3.4) determines the weighting factors of production blocks within flexible generation capacities.

(ii) Carbon-aware constraints (3.5) - (3.6).

This set of constraints imposes restrictions on carbon emissions. Constraint (3.5) quantifies the carbon emissions resulting from the use of fossil-fired flexible technologies. Constraint (3.6) limits system-wide carbon emissions from flexible generation capacities on a yearly basis.

(iii) Demand-supply balance constraints (3.7) - (3.8).

This set of constraints imposes the mandatory demand-supply balancing limits on the DA and RT stages for stable system operation. Constraint (3.7) ensures that demand and supply are balanced at each bus in the DA stage on an hourly basis. Constraint (3.8) expresses unexpected imbalances that arise from demand, and renewable power deviations are counteracted by the deployment of reserves in the RT stage.

(iv) Generation operating constraints (3.9) - (3.14).

This set of constraints regulates the operation of generation capacities using various technologies. Constraints (3.9) and (3.10) present the coupled limits of DA generation dispatches and RT reserve provisions of flexible technologies. Constraints (3.11) and (3.12) set the caps on the amount of upward reserve and downward reserve that can be deployed. Constraint (3.13) stipulates that the must-run generation capacities are required to operate at their full capacity during the DA stage. Constraint (3.14) incorporates normalised forecasting output parameters into the DA generation dispatches using renewable technologies.

(v) Network constraints (3.15) - (3.20).

This set of constraints models the power network topology and transmission across the DA stage and RT stage. Constraint (3.15) regulates the transmission capacities for linear DC power flow between two interconnected buses. The notation $m \in \Lambda_n$ is used to specify the buses m that are directly connected to a given bus n .

Constraint (3.16) imposes upper and lower limits on the voltage angles at each bus in the DA stage. Constraint (3.17) designates bus $n = 1$ as the slack bus in the DA stage. Accordingly, constraints (3.18) - (3.20) are analogous to those in the DA stage but are tailored for the RT stage.

(vi) Chance constraints for uncertainties (3.21) - (3.22).

This set of constraints relates to the uncertainties derived from demand and renewable power deviations. Chance constraint (3.21) accounts for the variability and unpredictability of wind generation, ensuring that the total wind power production is lower than its expected realisation with a specific confidence level of $1 - \rho^{RE}$. They mean that the probabilistic model regulates the actual wind power generation lower than its realisation with a certain degree of probability $1 - \rho^{RE}$. Chance constraint (3.22) ensures that the overall demand consumption is projected to be higher than the realisation of baseline demand, again with a defined confidence level of $1 - \rho^L$. Similarly, they mean the overall demand consumption is higher than the baseline demand realisation with a probability of $1 - \rho^L$.

3.4 Solution

This section details the solution approach for addressing the non-convex, system-driven generation planning model. The non-convexities incurred by the chance constraints are eliminated by the linearization techniques. After this, the proposed model can be transformed into a LP problem, which can be solved efficiently by commercial solvers.

3.4.1 CCP Reformulation

In our research, the chance constraints include the stochastic variables, denoted as g^R and L^R , for addressing the uncertainties related to renewable power and demand. According to the central limit theorem, these stochastic variables are modelled as following normal distributions:

$$g_{i,d,t,n}^R \sim \mathcal{N}(m_{i,d,t,n}^{RE}, (s_{i,d,t,n}^{RE})^2), \quad L_{j,d,t}^R \sim \mathcal{N}(m_{j,d,t}^L, (s_{j,d,t}^L)^2) \quad (3.23)$$

where parameter m is the mean of the corresponding normal distribution and parameter s is its standard deviation. It should be noted that the chance constraints (3.21) and (3.22) are non-convex. In order to make the proposed model solvable, the non-convexities in these chance constraints are required to be eliminated.

Based on the techniques detailed in [67], it gives a special instance of the CCP problem that can be linearized, aligning with the mathematical form of chance-constrained uncertainties in our model. This special case of CCP can be detailed as follows:

$$\min_x c^T x \quad (3.24)$$

$$s.t. \quad \mathbb{P}(a^T x + b^T \xi + \xi^T D x \leq e) \geq 1 - \epsilon \quad (3.25)$$

where a , b , D , and e are the parameters with appropriate dimensions, and ξ represents the multivariate normal distribution with mean m and covariance Σ . As proved in [67], if ϵ is no more than 0.5, the special case (3.24)-(3.25) is equivalent to the following problem:

$$\min_x c^T x \quad (3.26)$$

$$s.t. \quad e - b^T m - (a + D^T m)^T x \geq \Phi^{-1}(1 - \epsilon) \sqrt{(b + D x)^T \Sigma (b + D x)} \quad (3.27)$$

Given that the normal distributions adopted in this work are one-dimensional and they are not correlated with the decision variables (indicated by $D=0$), the equivalent constraint (3.27) can be further reduced to the following linear constraint:

$$e - b^T m - a^T x \geq \Phi^{-1}(1 - \epsilon) |b| s \quad (3.28)$$

where s is the standard deviation of the normal distribution.

To illustrate the linearization process, let us consider the example of wind power uncertainty represented by the chance constraint (3.21). In this example, the corresponding parameters are listed as follows:

$$\begin{aligned} e &= 0, a = [1, -1]^T, b = -1, \xi = g_{i,d,t,n}^R, m = m_{i,d,t,n}^{RE}, s = s_{i,d,t,n}^{RE}, \\ x &= [g_{i,d,t,n}^{DA}, g_{i,d,t,n}^{dev}]^T, \epsilon = \rho^{RE} \end{aligned} \quad (3.29)$$

Subsequently, the linear expression can be obtained by substituting the parameters (3.29) into the constraint (3.28):

$$m_{i,d,t,n}^{RE} - g_{i,d,t,n}^{DA} + g_{i,d,t,n}^{dev} \geq \Phi^{-1}(1 - \rho^{RE}) s_{i,d,t,n}^{RE} \quad \forall i \in I^{RE}, \forall d, \forall t, \forall n \quad (3.30)$$

Similarly, in terms of the chance constraint (3.22) representing the demand uncertainty,

the relevant parameters can be expressed as follows:

$$\begin{aligned} e &= \sum_o L_{j,d,t,o}^{DA}, a = -1, b = 1, \xi = L_{j,d,t}^R, m = m_{j,d,t}^L, s = s_{j,d,t}^L, \\ x &= \sum_o P_{j,d,t,o}^{dev}, \epsilon = \rho^L \end{aligned} \quad (3.31)$$

Finally, the chance constraint related to the demand-side uncertainty, (3.22), can be simplified into a linear constraint using the same methodology. The formulation is listed below:

$$-m_{j,d,t}^L + \sum_o P_{j,d,t,o}^{dev} + \sum_o L_{j,d,t,o}^{DA} \geq \Phi^{-1}(1 - \rho^L) s_{j,d,t}^L \quad \forall j, \forall d, \forall t \quad (3.32)$$

3.4.2 Final LP

Finally, the proposed system-driven model is converted into a single-level LP. The LP's linear objective function (3.33) and set of decision variables (3.34) concur with those in the original model. Furthermore, the LP's constraints are composed of the linear constraints in the original system-driven model (3.35) and the linearized chance constraints (3.36). The final LP is mathematically formulated as follows:

$$\begin{aligned} \min_{V^{SYS}} \sum_d \omega_d & \left[\sum_{i,t,n,b} C_{i,b}^{EN} g_{i,d,t,n,b}^{DA} + \sum_{i,t,n,b} (C_{i,b}^U g_{i,d,t,n,b}^U - C_{i,b}^D g_{i,d,t,n,b}^D) \right. \\ & \left. + \sum_{i \in I^{FL}, t, n} \lambda^{CO_2} Q_{i,d,t,n}^{emi} \right] + \sum_{i,n} C_i^{IN} G_{i,n} \end{aligned} \quad (3.33)$$

where:

$$V^{SYS} = \{ G_{i,n}, Q_{i,d,t,n}^{emi}, g_{i,d,t,n,b}^{DA}, g_{i,d,t,n,b}^U, g_{i,d,t,n,b}^D, P_{j,d,t,o}^{dev}, g_{i,d,t,n}^{dev}, \delta_{d,t,n}^{DA}, \delta_{d,t}^{DA,ref}, \delta_{d,t,n}^{RT}, \delta_{d,t}^{RT,ref} \} \quad (3.34)$$

subject to:

$$\text{Linear constraints of original system-driven model : (3.3) – (3.20)} \quad (3.35)$$

$$\text{Linearized chance constraints : (3.30) – (3.32)} \quad (3.36)$$

3.5 Case Studies

3.5.1 Data Descriptions and Implementation

The objective of case studies is to explore the impact of power generation under the supervision of a vertically integrated monopoly and subsequently serve as a benchmark to evaluate the value of the deregulated energy market discussed later. To this end, the case studies are carried out using the data from the GB power grid in 2021. All necessary data are presented in this subsection.

The case studies explore a range of investment options for the system operator, detailing the holistic technical data sourced from [31]. As described in Table 3.1, the technical specifications cover six distinct technologies for investment, including two variants of combined cycle gas turbines (CCGT1 and CCGT2), two variants of open cycle gas turbines (OCGT1 and OCGT2), one option for nuclear power, and one option for wind power. These technologies are differentiated based on their operational characteristics. Specifically, CCGT1, CCGT2, OCGT1, and OCGT2 are recognised as flexible generation technologies, nuclear power is categorised as a must-run generation source, and wind power is identified as a renewable power generation option. It is noteworthy that in terms of flexible generation technologies, the marginal cost for providing upward reserve is set at 1.1 times the marginal energy cost; conversely, the marginal cost for providing downward reserve is set at 0.9 times the marginal energy cost, indicating a discount for the ability to reduce output when demand falls below expectations. Furthermore, a comparison between the two pairs of technologies, CCGT1 versus CCGT2, and OCGT1 versus OCGT2, reveals a trade-off between cost and operational flexibility. CCGT1 and OCGT1 are characterised by lower investment and marginal energy costs, making them economically attractive options. However, these technologies have a relatively limited capability to provide reserves. On the other hand, CCGT2 and OCGT2, despite their higher investment and marginal energy costs, offer greater reserve provision capability.

As detailed in the fourth row of Table 3.1, the differentiation in environmental impact among the generation technologies is a key consideration for the system operator. CCGT technologies are noted for being more environmentally friendly compared to OCGT technologies due to lower carbon emissions. Furthermore, nuclear and wind power are recognised as sustainable energies with zero carbon emissions during operation.

The last five rows of Table 3.1 refer to the marginal energy costs associated with different generation blocks of flexible technologies (CCGT1, CCGT2, OCGT1, and OCGT2), with five production blocks designed to capture the input-output characteristics of these flexible technologies. The allocation of weights to these blocks is set as [0.1, 0.15, 0.2,

0.25, 0.3]. It should be clarified that although energy prices are not included in the proposed system-driven model, they can be determined by the marginal energy cost of marginal generation capacities. Accordingly, the same pricing mechanism is applied to derive the prices of both upward and downward reserves.

Table 3.1: Parameters of investment options.

Investment option		CCGT1	CCGT2	OCGT1	OCGT2	Nuclear	Wind
Type		Flexible	Flexible	Flexible	Flexible	Must-run	Renewable
Investment cost C^{IN} (£/MW/yr)		14413	17440	8610	10418	114849	35756
Carbon intensity η^{CO_2} (t/MWh)		0.39	0.39	0.65	0.65	0	0
Maximum upward reserve capacity G^U (MW)		4000	8000	4000	8000	-	-
Maximum downward reserve capacity G^D (MW)		4000	8000	4000	8000	-	-
Marginal energy cost C^{EN} (£/MWh)	Block 1	7.43	20.05	40.99	49.25	4.72	0
	Block 2	9.03	24.34	49.78	59.81		
	Block 3	10.62	28.64	58.56	70.36		
	Block 4	12.21	32.94	67.34	80.91		
	Block 5	13.81	37.23	76.13	91.47		

Table 3.2 displays crucial parameters for analysing the business cases derived from the system-driven model in the context of the 2021 GB power grid, focusing on the carbon intensity, carbon price, and system demand. These parameters are sourced from authoritative datasets: the system-wide carbon intensity is given by the National Grid Electricity System Operator (NG ESO) [71], the carbon price is collected from the UK Emissions Trading Scheme (UK ETS) [72], and the total system demand is provided by the National Grid Future Energy Scenarios (NG FES).

Table 3.2: Parameters of the GB power grid in 2021.

Year	2021
System-wide carbon intensity η^{Grid} (t/MWh)	183.11
Carbon price λ^{CO_2} (£/t)	71.54
Total system demand (TWh)	248.30

In order to serve as a benchmark, despite taking no account of the wind power deviation penalty, carbon allowance, or TGC transaction in the proposed system-driven model, their business results can be obtained using the two frameworks. Firstly, the wind power deviation penalty and carbon allowance revenue are formulated as below:

$$\text{Wind power deviation penalty} = \sum_d \omega_d \sum_{i \in I^{RE}, t, n} \lambda_{d,t,n}^{RT} g_{i,d,t,n}^{dev} \quad (3.37)$$

$$Q_{i,d,t,n}^{CA} = (g_{i,d,t,n}^{DA} - g_{i,d,t,n}^{dev}) \eta^{Grid} \epsilon_i \quad \forall i \in I^{RE}, \forall d, \forall t, \forall n \quad (3.38)$$

$$\text{Carbon allowance revenue} = \sum_d \omega_d \sum_{i \in I^{RE}, t, n} \lambda^{CO_2} Q_{i,d,t,n}^{CA} \quad (3.39)$$

Equation (3.38) defines the allocation of carbon allowances, aligning with environmental regulations. It should be noted that the values of DA wind generation dispatch $g_{i,d,t,n}^{DA}$, RT wind power deviation $g_{i,d,t,n}^{dev}$, and RT price $\lambda_{d,t,n}^{RT}$ are known since their optimal results are obtained by the proposed system-driven model. Thus, wind power deviation penalty and carbon allowance revenue can be calculated through (3.37) and (3.39), respectively. Moreover, it should be clarified that the wind power deviation penalty is derived from insufficient wind power generation. Conversely, if the actual wind power exceeds expectations, this term will be negative, representing a form of revenue.

Secondly, the TGC revenue is from the present TGC market and the future TGC market, which is derived through the following single-level optimisation framework:

$$\max_{V^{TGC}} \sum_d \omega_d \underbrace{\left[\sum_{i \in I^{RE}} (\lambda_{i,d}^{PGC} Q_{i,d}^{PGC} + \lambda^{FGC} Q_{i,d}^{FGC}) \right]}_{\text{Green certificate revenue}} \quad (3.40)$$

where:

$$V^{TGC} = \{Q_{i,d}^{PGC}, Q_{i,d}^{FGC}, \lambda_{i,d}^{PGC}\} \quad (3.41)$$

subject to:

$$Q_{i,d}^{PGC} + Q_{i,d}^{FGC} = \sum_{t,n} (g_{i,d,t,n}^{DA} - g_{i,d,t,n}^{dev}) \quad \forall i \in I^{RE}, \forall d \quad (3.42)$$

$$\lambda_{i,d}^{PGC} = l^{PGC} - q^{PGC} Q_{i,d}^{PGC} \quad \forall i \in I^{RE}, \forall d \quad (3.43)$$

$$\lambda^{FGC} = \gamma^{FGC} \lambda_0^{RPS} \quad (3.44)$$

The objective function (3.40) is to maximise the revenue from TGC transactions in the present TGC market (first term) and future TGC market (second term). It is necessary to explain that renewable generation dispatch and deviation have already been determined by solving the system-driven generation planning model. The related variables are collected in the set V^{TGC} , including TGC quantity trading in the present TGC market $Q_{i,d}^{PGC}$, TGC quantity trading in the future TGC market $Q_{i,d}^{FGC}$, and present TGC price $\lambda_{i,d}^{PGC}$. It should be noted that the estimated price for the future TGC market, λ^{FGC} , is known as a parameter according to (3.44). This set of TGC constraints (3.42)-(3.44) relates to the present TGC market and the future TGC market. Constraint (3.42) represents the total amount of TGCs related to wind generation dispatches in the DA stage and deviations in the RT stage. Constraint (3.43) expresses that the present TGC prices are captured by a

quantity-based inverse demand function. Constraint (3.44) approximates the future TGC price based on the penalty price of RPS deficiency.

It is observable that the bilinear term, $\lambda_{i,d}^{PGC} Q_{i,d}^{PGC}$, exists in objective function (3.40). To erase this bilinear term, it can be converted into a quadratic term by adopting constraints (3.43)-(3.44), which can be expressed as follows:

$$\begin{aligned} \sum_{i \in I^{RE}, d} (\lambda_{i,d}^{PGC} Q_{i,d}^{PGC} + \lambda^{FGC} Q_{i,d}^{FGC}) = \\ \sum_{i \in I^{RE}, d} [l^{PGC} Q_{i,d}^{PGC} - q^{PGC} (Q_{i,d}^{PGC})^2 + \lambda^{FGC} Q_{i,d}^{FGC}] \end{aligned} \quad (3.45)$$

The business cases related to wind power deviation, carbon allowance, and TGC can be derived from these two frameworks after the outcomes of the system-driven model are obtained. The comprehensive trading mechanisms for carbon allowance and TGC will be detailed later. Apart from the energy-related parameters, the additional parameters associated with carbon allowance and TGC are essential. As referenced from [73] and [64], the carbon allowance allocation ratio, ϵ_i , is set as 0.4; the RPS requirement ratio, γ^{RPS} , is set as 40%; the approximated price ratio in the future TGC market, γ^{FGC} , is set as 0.3; and the price coefficient in the present TGC market, θ^{PGC} , is also set as 0.3. It should be noted that the penalty price for RPS deficiency, λ_0^{RPS} , is assumed to be 1.1 times the carbon price.

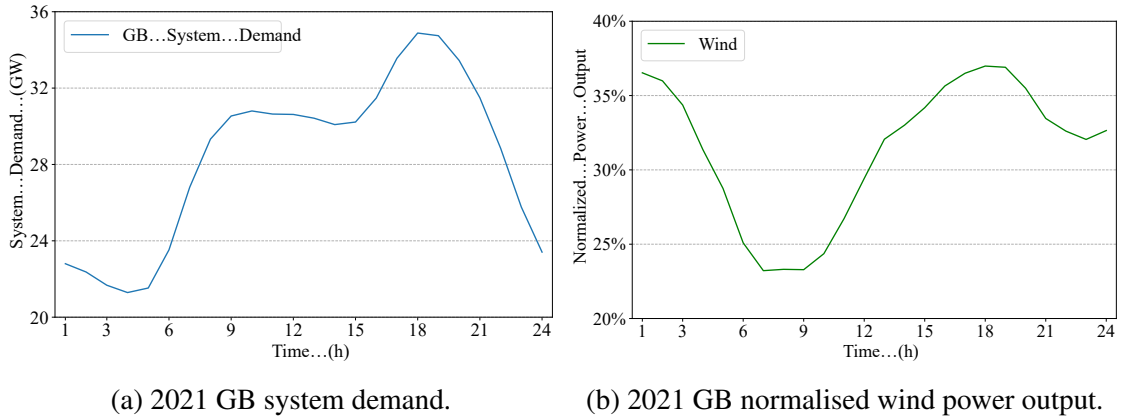


Figure 3.2: Data profiles of the 2021 GB power system.

The GB demand profile and the normalised power output of wind energy in 2021 are illustrated in Figure 3.2. The corresponding data is derived from the National Grid Electricity System Operator (NG ESO) and the research paper [16]. In line with the setting of production blocks, the demand is accurately modelled by the five demand blocks, incorporating weighting factors as [0.6, 0.1, 0.1, 0.1, 0.1]. By virtue of computational burden, the demand profile is simulated on a representative day with an hourly resolution

($T = \{1, 2, \dots, 24\}$), scaled across a target year ($\omega = 365$) to capture daily fluctuations in energy consumption and production patterns.

In order to enhance the realism of the experiments, chance constraints are employed in the proposed system-driven model to address the inherent uncertainties, which are categorised into two types: demand uncertainty and wind power production uncertainty. The related coefficients used in the chance constraints are given by [31]. Overall, the model sets confidence levels for all the chance constraints at 95% ($1 - \rho^L = 1 - \rho^{RE} = 95\%$), conveying the ρ -reliable threshold for both demand and wind power production uncertainty.

The linearized CCP formulations incorporate stochastic variables for wind power and demand realisations, which are modelled using two normal distributions, $\mathcal{N}(m^{RE}, (s^{RE})^2)$ for wind power and $\mathcal{N}(m^L, (s^L)^2)$ for demand. The means of these distributions (m^L and m^{RE}) are aligned with the baseline demand and DA power dispatch, respectively. Meanwhile, the standard deviations (s^L and s^{RE}), set at 1% of the baseline demand and 10% of the DA wind power dispatch, respectively, quantify the anticipated variability around these means.

The case studies are implemented in the developed MIQP model solved with the well-known solver, Gurobi 10.0, on a PC with a 6-core 3.50 GHz Intel(R) Xeon(R) E5-1650 processor and 64 GB of RAM [74]. All the cases are coded in Python 3.10, integrated with Pyomo 6.4.3 [75]. Due to the simplification of CCP, all cases are computationally tractable within a reasonable time.

3.5.2 System Outcomes and Business Cases

Table 3.3: Investment decisions and business results for the system-driven scenario regarding the 2021 GB power system.

CCGT1 (MW)	14,184	Investment cost (mil.£)	2,631
CCGT2 (MW)	-	DA energy cost (mil.£)	1,497
OCGT1 (MW)	-	DA energy revenue (mil.£)	2,970
OCGT2 (MW)	-	RT reserve cost (mil.£)	47
Nuclear (MW)	20,172	RT reserve revenue (mil.£)	72
Wind (MW)	3,562	Wind power deviation penalty (mil.£)	20
Carbon emission cost (mil.£)	1,902	Carbon allowance revenue (mil.£)	43
Green certificate revenue (mil.£)	552	Profit (mil.£)	-2,460

It should be clarified that the comprehensive investment decisions and business results derived from the system-driven scenario are listed in Table 3.3. Additionally, the DA generation dispatches and RT imbalances of the system-driven scenario are illustrated in Figures 3.3 and 3.4, respectively. The DA and RT price profiles are shown in Figure 3.5.

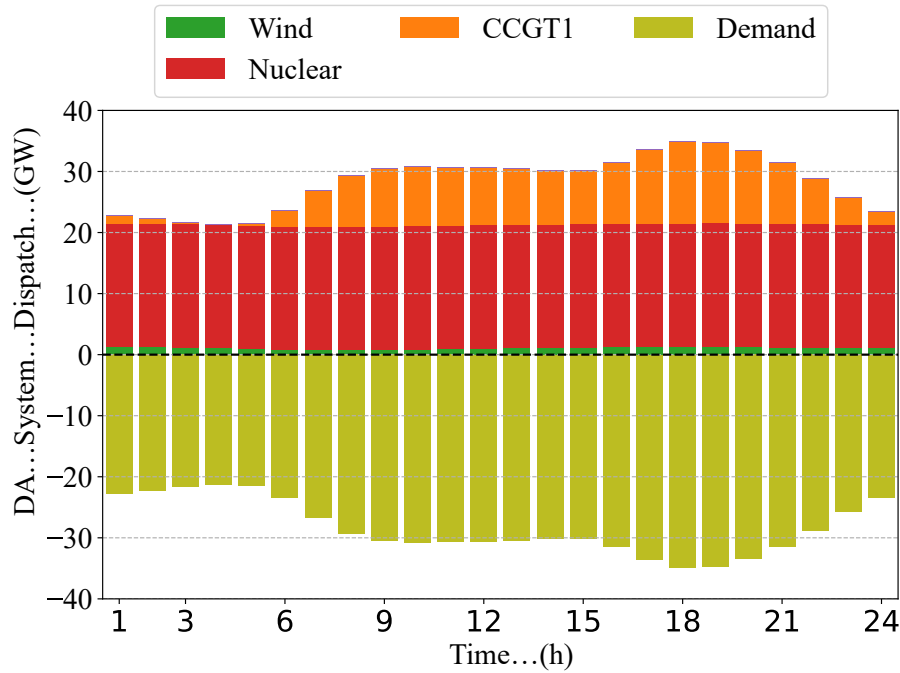


Figure 3.3: Daily generation dispatches in the DA stage for the system-driven scenario regarding the 2021 GB power system.

Analysis of Investment Decisions

The first to sixth row in the first column of Table 3.3 exhibits the optimal investment decisions derived from the system-driven model in the context of the 2021 GB power system. Under the rigid 2021 GB carbon limitations, emission-free generation technologies, i.e., nuclear and wind power, have become increasingly popular. Due to their 'must-run' characteristics in the DA stage, the total power generation from emission-free capacity is capped by the bottom level of baseline system demand, as illustrated in Figure 3.3. Additionally, it should be noted that the legends 'Demand' and 'Wind' in Figure 3.3 represent DA demand consumption and DA wind power dispatch, respectively, while in Figure 3.4, they depict the deviations in demand and wind power generation. However, it is observable that the investment in nuclear power is much greater than in wind power. This is because nuclear power has two main advantages in a system-driven scenario: on the one hand, despite the high investment cost of nuclear power, unlike the inherent intermittency and variability of wind power, nuclear does not introduce uncertainty that would increase the reserve requirement, thereby leading to increased system cost; on the other hand, compared with the high capacity factor of nuclear power, wind power generation is highly affected by the weather conditions, resulting in a relatively low capacity factor, ultimately making the efficiency of investing in wind power capacity quite low.

Flexible generation capacity plays two necessary roles: in the DA stage, since both

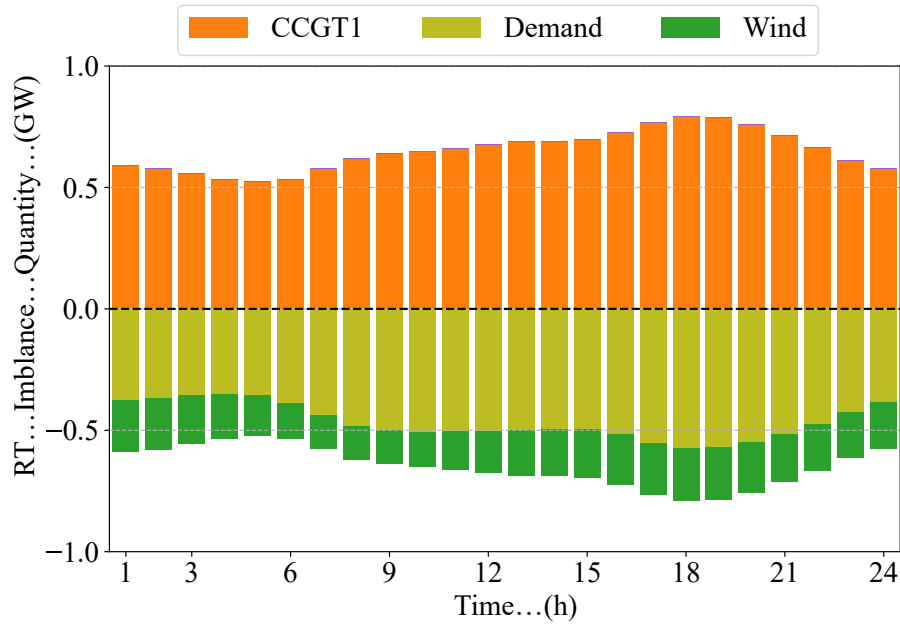


Figure 3.4: Daily imbalances in the RT stage for the system-driven scenario regarding the 2021 GB power system.

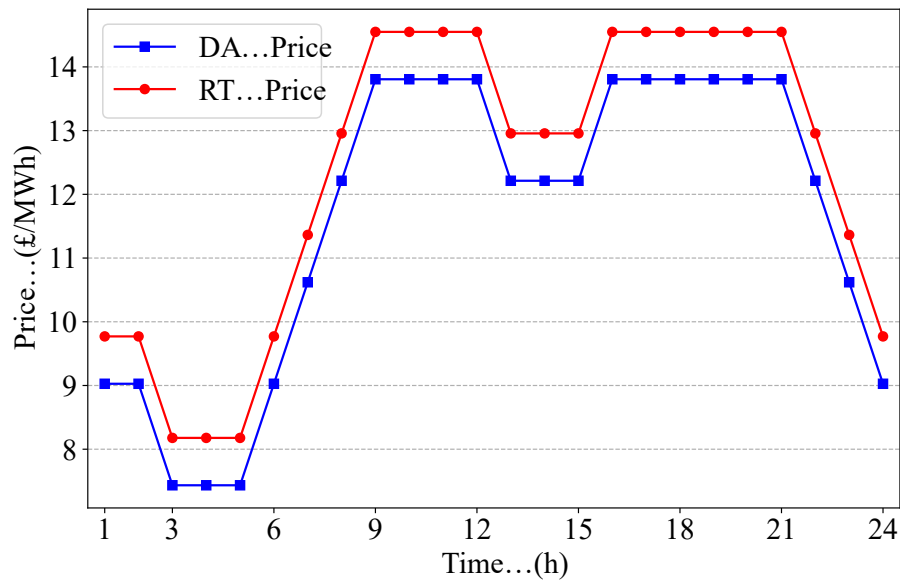


Figure 3.5: Daily prices in the DA and RT stages for the system-driven scenario regarding the 2021 GB power system.

wind and nuclear are inflexible, one of the primary roles of flexible generation capacity is to compensate for the peak-valley difference in system demand, as illustrated in Figure 3.3; then, in the RT stage, the necessary reserve needs to be provided by flexible generation to eliminate uncertainties from wind power and demand, as seen in Figure 3.4. It is evident that the flexible generation capacity in the optimal investment mix is entirely provided by CCGT1 technology. This phenomenon is due to the cost-effective advantage

of CCGT1 technology. Specifically, CCGT1 technology has three natural advantages compared to other flexible generation technologies: first, the low marginal energy cost characteristic of CCGT1 technology means that both its generation cost in the DA stage and reserve cost in the RT stage are low, aligning with the goal of minimising system cost; second, the investment cost of CCGT1 is quite low; and third, the carbon intensity of CCGT1 technology is the lowest among the available flexible generation technologies, facilitating compliance with the 2021 GB carbon limits and reducing carbon emission cost. Additionally, CCGT1 serves as the marginal unit in both the DA and RT stages; therefore, the clearing prices of the DA and RT stages are determined by the marginal cost of CCGT1, as illustrated in Figure 3.5.

Analysis of Business Cases

The business results of the system-driven case are fully displayed in the last two rows in the first column and the first to eighth row in the second column of Table 3.3. It is noteworthy that, since the system-driven scenario is committed to minimising the total system cost, all components of the system cost, including investment cost, DA energy cost, RT reserve cost, and carbon emission cost, are maintained at low levels. Simultaneously, the wind power deviation penalty remains low due to the low scale of integrated wind power. However, as the system-driven framework fails to capture the implicit economic benefits, all components of revenue, including DA energy revenue, RT reserve revenue, carbon allowance revenue, and TGC revenue, are also low, ultimately leading to a negative total profit for the GenCo, indicating a serious problem of cost recovery.

3.6 Conclusions

This chapter aims to explore the impact of the vertically integrated monopoly structure on generation planning with the current carbon emission limits. In parallel, it also serves as a benchmark for the market-driven framework discussed later. To this end, we first develop a chance-constrained single-level optimisation model to reflect the perspective of a centralised system operator, where the objective is to minimise the total cost and a series of constraints imposed by power system operation and environmental regulation. Afterward, the chance constraints employed to depict the uncertainties introduced by RESs and demand are converted into linear expressions by the linearization technique. Finally, this chance-constrained single-level optimisation problem is recast as a LP, which can be efficiently solved. The case studies have been carried out based on the 2021 GB power system, and the findings are detailed as follows:

1. The optimal investment mix derived from the proposed system-driven model indicates that wind power capacity accounts for 9%, suggesting that the wind energy integration capability of a vertically integrated electricity system is poor, conflicting with the current GB's vision to expand wind power scale.
2. The business results obtained from the proposed system-driven model indicate that total profit is negative (-£2,460 million). This implies that while a vertically integrated electricity system can ensure low costs, it fails to generate sufficient profit to recover costs, thereby losing economic vitality.

Chapter 4

Market-Driven Generation Planning

4.1 Introduction

The deregulation of the GB's power system, initiated in the 1980s and progressing through the 1990s, marked a shift towards a liberalised energy market, fundamentally transforming the landscape of generation planning. The GB's journey towards deregulation was characterised by the unbundling of generation, transmission, and distribution functions, along with the introduction of multi-scale market competitions, i.e., the wholesale energy market, retail market, local energy market, and so on [60]. This restructuring incorporates the impact of economic incentives into the investment decisions in power generation. The historic transition from a traditional vertically integrated monopoly to a competitive market environment introduced new complexities and set the stage for the emergence of market-driven generation planning.

In this context, market-driven generation planning becomes a crucial methodology that reflects the complexities of market dynamics in the expansion of generation capacity within a deregulated power system. This approach employs a bi-level optimisation framework, capturing the coupled relationships between the GenCos and market operator. The upper level of the framework focuses on the profit-maximising objectives of GenCos, strategizing investment decisions in response to market signals. Conversely, the lower level encompasses the perspectives of a cost-minimising market operator, ensuring the efficient allocation of resources and compliance with regulatory standards.

Nowadays, under the GB's ambitious zero-carbon targets, market-driven generation planning acquires enhanced significance. It becomes a powerful tool for factoring in the role and value of various low-carbon policies under the market framework aimed at steering generation planning towards a sustainable pathway. This planning approach particularly focuses on integrating renewable generation capacity and responding to market

signals for profitability.

The market-driven generation planning model proposed in this chapter is established for quantitatively analysing the impact of the GB's deregulated market environment. Additionally, multi-scale low-carbon policies, including carbon emission limits, carbon allowances, and green certificates, are incorporated into this model. A series of case studies are designed to evaluate market outcomes and business results, utilising the data sourced from the 2021 GB power system. The findings clearly demonstrate that the deregulated market framework not only sharply raises the total profit of the GenCo but also enhances the integration of renewable power capacity.

4.2 Problem Statement

This section generalises the decision-making process of the investigated market-driven generation planning and the main assumptions for its mathematical modelling.

4.2.1 Structure of the Bi-Level Optimisation Model

The market-driven model is presented as a two-stage bi-level optimisation framework, capturing the dynamic interplay between a profit-seeking GenCo at the UL problem and the intricate clearing processes of multiple markets at the two LL problems (LL1 and LL2). This structure is visually represented in Figure 4.1, providing a clear depiction of the comprehensive modelling approach. Specifically, at the core of this framework, the LL1 problem adopts a two-stage architecture that concurrently clears the DA energy market and the RT balancing market. Meanwhile, the LL2 problem employs the inverse demand function to capture the present TGC market's dynamics and estimate the price in the future TGC market, offering a strategic perspective on TGC trading.

The interdependence between the UL problem and the two LL problems is pronounced. The investment decisions made by GenCo at the UL significantly affect the market outcomes realised within the two LL problems. In turn, the business cases emerging from the LL1 problem, characterised by energy prices and generation dispatches in both the DA and RT markets, along with the TGC quantities and prices obtained in the LL2 problem, critically influence GenCo's profitability. This bidirectional feedback loop underscores the complex relationship between generation planning and market performance, highlighting the feature of capturing real-world market dynamics and GenCo's decision-making processes.

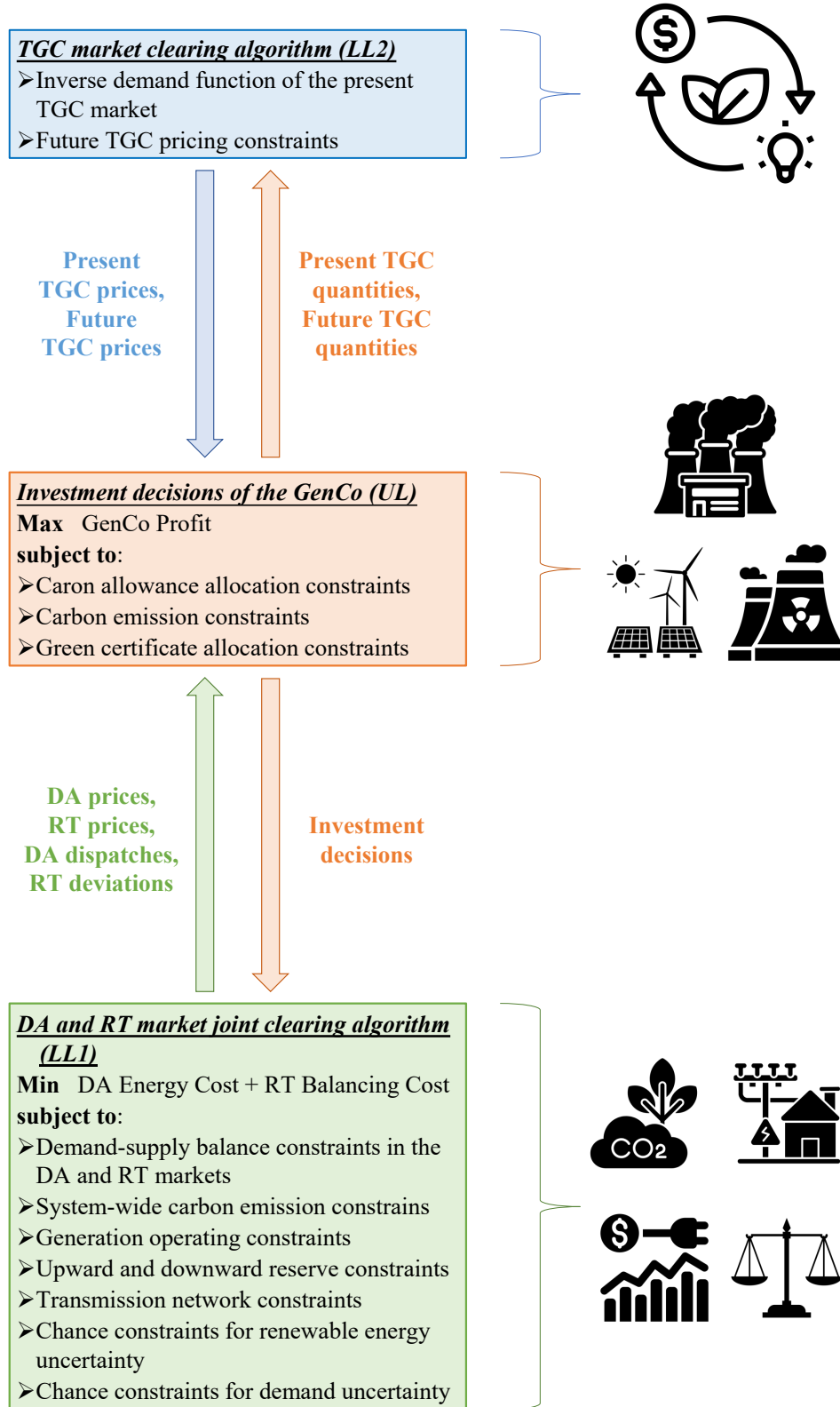


Figure 4.1: Illustration of the proposed two-stage bi-level optimisation model.

4.2.2 Modelling Assumptions

Aiming at clarity and completeness, the major assumptions of the proposed two-stage bi-level market-driven model are listed below:

1. The proposed two-stage bi-level optimisation model represents the market-driven generation planning faced by a self-interested GenCo seeking to maximise its long-term profit. The GenCo is allowed to invest in generation capacities utilising flexible, must-run, and renewable technologies. Furthermore, the flexible generation capacities are explicitly modelled by the production blocks to capture their output characteristics. Accordingly, demand blocks are also incorporated into the proposed model.
2. The examined GenCo is permitted to participate in four different markets. The two-stage energy markets adopted for this work include the DA energy market and the RT balancing market with an hourly time resolution. Furthermore, the proposed model takes into consideration the present TGC market and the future TGC market.
3. Based on the study in [76], its pricing scheme, namely LMP, is applied within the proposed two-stage energy market framework. This approach leverages the LMPs for determining the electricity prices at different locations in the network, which are calculated using the dual variables of the hourly demand-supply balance in both the DA and RT markets. The so-called LMP scheme reflects the marginal cost of supplying the next unit of electricity at a specific location, considering the effect of network transmission congestion, thereby facilitating efficient market operations and signaling appropriate economic incentives.
4. The model incorporates the pricing mechanism outlined in [64] for trading in the present and future TGC markets. Specifically, the prices in the present TGC market are determined using an inverse demand function. Furthermore, the future TGC price is estimated based on the penalty price associated with RPS deficiency.
5. Once the GenCo finalises its investment decisions, the sequential clearing process of the DA, RT, and TGC markets is arranged by the market operator. Specifically, the market operator first clears the DA energy market, taking into account the uncertainties incurred by demand and renewable generation as well as scheduling the generation dispatches. Subsequently, the depolyment of reserves is organised by market operator in the RT market to eliminate the unexpected uncertainties arising from wind power and demand-side deviations. Following this, the prices and quantities of TGC are determined based on the clearing results of the DA and RT

markets. By comparing the present TGC market clearing price with the estimated future TGC market price, the examined GenCo can choose either to trade its TGCs in the present TGC market or retain them for future trading, thus completing the transactions in the TGC market. Finally, the outcomes of these three markets are relayed to the examined GenCo, significantly influencing its profit-oriented investment decision-making process.

6. Although the DA and RT markets are cleared sequentially from the perspective of the examined GenCo, investment decisions are required to maximise the overall expected profit simultaneously [77]. Additionally, as highlighted in research works [78, 79], a co-optimised approach to managing energy and reserves significantly enhances the efficiency of generation dispatch compared to scenarios where energy and reserves are cleared separately. Therefore, this paper adopts a joint clearing model for the DA and RT markets. Furthermore, it is assumed that GenCo sells the TGCs in the present TGC market or withholds the TGCs for future trading after the outcomes of the energy markets are obtained. This assumption is logical since the allocation of TGCs becomes definitive after the results of renewable generation dispatches and deviations are available, which is more conducive to risk mitigation [64].
7. In order to facilitate the mathematical solution, the uncertainties sourced from RESs and demand are realised by a set of chance constraints.
8. As the primary objective of this paper is to explore the system-level generation mix of the future grid under stringent carbon emission regulations, the proposed generation planning model excludes the consideration of other GenCos.
9. In order to ensure the convex characteristics of the LL1 problem, linear generation costs are adopted in the proposed model. Additionally, it is assumed that renewable generation technologies are cost-free.
10. In the DA market, the generation capacities of renewable and must-run technologies are assumed to operate at their full capacity.
11. The network transmission is modelled based on a linear DC power flow approximation.
12. In the face of decarbonisation, a series of carbon emission restrictions and green energy incentives are imposed on the examined GenCo. Firstly, GenCo is limited by an annual cap on carbon emissions. Moreover, it is assumed that carbon emissions only depend on the generation of flexible technologies that consume fossil fuels.

Secondly, carbon emissions from the GenCo are subject to a carbon tax. Thirdly, carbon allowances are allocated to the GenCo based on its renewable energy generation, and the GenCo can profit by trading these carbon allowances. Fourthly, GenCo can earn TGCs through renewable power production and participate in the present and future TGC markets to gain profits. Finally, the RPS ratio is incorporated into the parameter setting of TGC markets. These environmental policies are set to work together towards a fully decarbonised power system.

4.3 Mathematical Model

This section details the specific formulations of the UL problem pertaining to GenCo and the two LL problems (LL1 and LL2) with respect to the multiple market clearing processes.

4.3.1 UL Problem: Investment Decisions of GenCo

The UL problem in the framework focuses on the decision-making process of the profit-driven GenCo, aiming to maximise its total profit through the optimal investment mix in light of the evolving market dynamics. It can be mathematically articulated through the following formulations:

$$\begin{aligned}
\max_{V^{UL}} \sum_d \omega_d & \left[\underbrace{\sum_{i \in \Psi_{n,t,n,b}} \lambda_{d,t,n}^{DA} g_{i,d,t,n,b}^{DA}}_{\text{DA energy revenue}} - \underbrace{\sum_{i,t,n,b} C_{i,b}^{EN} g_{i,d,t,n,b}^{DA}}_{\text{DA energy cost}} \right. \\
& + \underbrace{\sum_{i \in I^{FL} \cap \Psi_{n,t,n,b}} \lambda_{d,t,n}^{RT} (g_{i,d,t,n,b}^U - g_{i,d,t,n,b}^D)}_{\text{RT reserve revenue}} - \underbrace{\sum_{i \in I^{FL},t,n,b} (C_{i,b}^U g_{i,d,t,n,b}^U - C_{i,b}^D g_{i,d,t,n,b}^D)}_{\text{RT reserve cost}} \\
& - \underbrace{\sum_{i \in I^{RE} \cap \Psi_{n,t,n}} \lambda_{d,t,n}^{RT} g_{i,d,t,n}^{dev}}_{\text{Renewable deviation penalty}} - \underbrace{\sum_{i \in I^{FL},t,n} \lambda^{CO_2} Q_{i,d,t,n}^{emi}}_{\text{Carbon emission cost}} + \underbrace{\sum_{i \in I^{RE},t,n} \lambda^{CO_2} Q_{i,d,t,n}^{CA}}_{\text{Carbon allowance revenue}} \\
& + \underbrace{\sum_{i \in I^{RE}} (\lambda_{i,d}^{PGC} Q_{i,d}^{PGC} + \lambda^{FGC} Q_{i,d}^{FGC})}_{\text{Green certificate revenue}} \left. \right] - \underbrace{\sum_{i,n} C_i^{IN} G_{i,n}}_{\text{Investment cost}} \tag{4.1}
\end{aligned}$$

where:

$$V^{UL} = \{G_{i,n}, Q_{i,d,t,n}^{CA}, Q_{i,d,t,n}^{emi}, Q_{i,d}^{PGC}, Q_{i,d}^{FGC}\} \tag{4.2}$$

subject to:

$$G_{i,n} \geq 0 \quad \forall i \in I, \forall n \quad (4.3)$$

$$G_{i,n,b} = \omega_b^{FL} G_{i,n} \quad \forall i \in I^{FL}, \forall n, \forall b \quad (4.4)$$

$$Q_{i,d,t,n}^{CA} = (g_{i,d,t,n}^{DA} - g_{i,d,t,n}^{dev}) \eta^{Grid} \epsilon_i \quad \forall i \in I^{RE}, \forall d, \forall t, \forall n \quad (4.5)$$

$$Q_{i,d,t,n}^{emi} = (g_{i,d,t,n}^{DA} + g_{i,d,t,n}^U - g_{i,d,t,n}^D) \eta_i^{CO_2} \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n \quad (4.6)$$

$$Q_{i,d}^{PGC} + Q_{i,d}^{FGC} = \sum_{t,n} (g_{i,d,t,n}^{DA} - g_{i,d,t,n}^{dev}) \quad \forall i \in I^{RE}, \forall d \quad (4.7)$$

The objective function of the UL problem, defined as (4.1), is structured to maximise the total profit of the examined GenCo on a yearly basis. This comprehensive objective encompasses various aspects of GenCo's business cases through nine distinct elements: (i) energy revenue earned from invested generation capacities in DA market (first term); (ii) energy cost posed by generation in DA market (second term); (iii) reserve revenue earned from reserve provision of flexible generation capacities in RT market (third term); (iv) reserve cost posed by reserve provision in RT market (fourth term); (v) penalty incurred by renewable power deviations (fifth term); (vi) carbon emission cost incurred by flexible generation technologies (sixth term); (vii) revenue earned from trading carbon allowance (seventh term); (viii) revenue earned from TGC transaction (eighth term); and (ix) investment cost for procuring new generation capacities (last term). The decision variables in the UL problem are composed of the total invested capacities across all generation technologies, as well as the quantities of carbon allowances, emissions, and TGCs, as defined in (4.2). It should be clarified that the renewable deviation penalty captures the cost incurred by the unexpected curtailment of renewable power generation. Conversely, if the actual renewable power exceeds expectations, this term will be positive, implying a profit for overperformance. Note that although the DA generation dispatches are uniformly denoted with the subscript b , in practice, this subscript b can be omitted when calculating dispatches of renewable and must-run capacities. Additionally, the symbol $i/k/q/j/e \in \Psi_n$ indicates new-invested capacities using flexible/must-run/renewable generation technologies $i/k/q$, demand j , and ES unit e , are located at bus n .

The UL problem is bound by various constraints. Constraint (4.3) ensures that the new capacities invested by the GenCo are non-negative, reflecting the real-world physics. Constraint (4.4) details the weighting factors of the production blocks corresponding to flexible generation capacities. Constraint (4.5) presents the allocation of carbon allowances. Constraint (4.6) quantifies the carbon emissions produced by the operation of fossil-fired flexible generation technologies. Lastly, constraint (4.7) calculates the total quantities of TGCs that are generated based on renewable energy production in the DA market and its

deviation in the RT market.

4.3.2 LL1 Problem: Joint Clearing Process of DA Energy and RT Balancing Markets

From the perspective of the market operator, the two-stage electricity markets, namely the DA energy market and the RT balancing market, are jointly cleared in order to minimise the overall system cost, which can be mathematically expressed as follows:

$$\left\{ \min_{V^{LL}} \sum_{i,t,n,b} C_{i,b}^{EN} g_{i,d,t,n,b}^{DA} + \sum_{i,t,n,b} (C_{i,b}^U g_{i,d,t,n,b}^U - C_{i,b}^D g_{i,d,t,n,b}^D) \right. \quad (4.8)$$

where:

$$V^{LL1} = \left\{ g_{i,d,t,n,b}^{DA}, g_{i,d,t,n,b}^U, g_{i,d,t,n,b}^D, P_{j,d,t,o}^{dev}, g_{i,d,t,n}^{dev}, \delta_{d,t,n}^{DA}, \delta_{d,t}^{DA,ref}, \delta_{d,t,n}^{RT}, \delta_{d,t}^{RT,ref} \right\} \quad (4.9)$$

subject to:

$$\sum_{j \in \Psi_{n,o}} L_{j,d,t,o}^{DA} + \sum_{m \in \Lambda_n} b_{nm} (\delta_{d,t,n}^{DA} - \delta_{d,t,m}^{DA}) - \sum_{i \in \Psi_{n,b}} g_{i,d,t,n,b}^{DA} = 0 \quad : \lambda_{d,t,n}^{DA} \quad \forall t, \forall n \quad (4.10)$$

$$\begin{aligned} & \sum_{j \in \Psi_{n,o}} P_{j,d,t,o}^{dev} + \sum_{i \in I^{FL} \cap \Psi_{n,b}} (-g_{i,d,t,n,b}^U + g_{i,d,t,n,b}^D) + \sum_{i \in I^{RE} \cap \Psi_n} g_{i,d,t,n}^{dev} - \\ & \sum_{m \in \Lambda_n} b_{nm} (\delta_{d,t,n}^{DA} - \delta_{d,t,n}^{RT} - \delta_{d,t,m}^{DA} + \delta_{d,t,m}^{RT}) = 0 \quad : \lambda_{d,t,n}^{RT} \quad \forall t, \forall n \end{aligned} \quad (4.11)$$

$$\sum_{i \in I^{FL}, t, n, b} \eta_i^{CO_2} (g_{i,d,t,n,b}^{DA} + g_{i,d,t,n,b}^U - g_{i,d,t,n,b}^D) \leq \eta^{Grid} \sum_{j,t,o} L_{j,d,t,o}^{DA} \quad : \zeta_d \quad (4.12)$$

$$g_{i,d,t,n,b}^{DA} + g_{i,d,t,n,b}^U \leq G_{i,n,b} \quad : \mu_{i,d,t,n,b}^U \quad \forall i \in I^{FL}, \forall t, \forall n, \forall b \quad (4.13)$$

$$g_{i,d,t,n,b}^D \leq g_{i,d,t,n,b}^{DA} \quad : \mu_{i,d,t,n,b}^D \quad \forall i \in I^{FL}, \forall t, \forall n, \forall b \quad (4.14)$$

$$0 \leq g_{i,d,t,n,b}^U \leq G_{i,b}^U \quad : \phi_{i,d,t,n,b}^{U,-}, \phi_{i,d,t,n,b}^{U,+} \quad \forall i \in I^{FL}, \forall t, \forall n, \forall b \quad (4.15)$$

$$0 \leq g_{i,d,t,n,b}^D \leq G_{i,b}^D \quad : \phi_{i,d,t,n,b}^{D,-}, \phi_{i,d,t,n,b}^{D,+} \quad \forall i \in I^{FL}, \forall t, \forall n, \forall b \quad (4.16)$$

$$g_{i,d,t,n}^{DA} = G_{i,n} \quad : \gamma_{i,d,t,n} \quad \forall i \in I^{MR}, \forall t, \forall n \quad (4.17)$$

$$g_{i,d,t,n}^{DA} = k_{i,d,t}^{RE} G_{i,n} \quad : \theta_{i,d,t,n} \quad \forall i \in I^{RE}, \forall t, \forall n \quad (4.18)$$

$$-C_{nm}^{max} \leq b_{nm} (\delta_{d,t,n}^{DA} - \delta_{d,t,m}^{DA}) \leq C_{nm}^{max} \quad : \nu_{d,t,n,m}^{DA,-}, \nu_{d,t,n,m}^{DA,+} \quad \forall t, \forall n, \forall m \in \Lambda_n \quad (4.19)$$

$$-\pi \leq \delta_{d,t,n}^{DA} \leq \pi \quad : \xi_{d,t,n}^{DA,-}, \xi_{d,t,n}^{DA,+} \quad \forall t, \forall n \quad (4.20)$$

$$\delta_{d,t}^{DA,ref} = 0 \quad : \xi_{d,t}^{DA,ref} \quad \forall t \quad (4.21)$$

$$-C_{nm}^{max} \leq b_{nm}(\delta_{d,t,n}^{RT} - \delta_{d,t,m}^{RT}) \leq C_{nm}^{max} \quad : \nu_{d,t,n,m}^{RT,-}, \nu_{d,t,n,m}^{RT,+} \quad \forall t, \forall n, \forall m \in \Lambda_n \quad (4.22)$$

$$-\pi \leq \delta_{d,t,n}^{RT} \leq \pi \quad : \xi_{d,t,n}^{RT,-}, \xi_{d,t,n}^{RT,+} \quad \forall t, \forall n \quad (4.23)$$

$$\delta_{d,t}^{RT,ref} = 0 \quad : \xi_{d,t}^{RT,ref} \quad \forall t \quad (4.24)$$

$$\mathbb{P} \left[g_{i,d,t,n}^{DA} - g_{i,d,t,n}^{dev} \leq g_{i,d,t,n}^R \right] \geq 1 - \rho^{RE} \quad : \chi_{i,d,t,n}^{RE} \quad \forall i \in I^{RE}, \forall t, \forall n \quad (4.25)$$

$$\mathbb{P} \left[\sum_o L_{j,d,t,o}^{DA} + \sum_o P_{j,d,t,o}^{dev} \geq L_{j,d,t}^R \right] \geq 1 - \rho^L \quad : \chi_{j,d,t}^L \quad \forall j, \forall t \quad \forall d \quad (4.26)$$

The objective function of the LL1 problem, represented as (4.8), focuses on minimising two key components within the energy system: the total energy cost (first term) and the reserve cost (last two terms). This function reflects the overarching goal of reducing the overall system cost from the perspective of the market operator. The decision variables for the LL1 problem are grouped in the set (4.9), encompassing DA generation dispatches, RT upward and downward reserve provisions, demand and renewable power uncertainties, as well as DA and RT voltage angles.

The LL1 problem is subject to system balance constraints (4.10) - (4.11), system-wide carbon emission constraint (4.12), generation operating constraints (4.13) - (4.18), network constraints (4.19) - (4.24), and chance constraints (4.25) - (4.26), respectively. These constraints are detailed as follows:

(i) System balance constraints (4.10) - (4.11).

This set of constraints imposes demand-supply balance restrictions on the two-stage energy markets. Constraint (4.10) enforces the DA demand-supply balance at each bus, where the DA market clearing price is derived from the associated dual variable $\lambda_{d,t,n}^{DA}$. Similarly, constraint (4.11) guarantees the RT unexpected imbalance arises from the demand and renewable power deviations are counterbalanced by the reserve deployment, where the dual variable, $\lambda_{d,t,n}^{RT}$, constitutes the RT market clearing price.

(ii) System-wide carbon emission constraint (4.12).

This constraint limits the system-level carbon emissions derived from flexible generation capacities that consume carbon-intensive fossil fuels.

(iii) Generation operating constraints (4.13) - (4.18).

This set of constraints details the operational features of the generators using vari-

ous technologies across the DA energy market and the RT balancing market. Constraints (4.13) and (4.14) are instrumental in describing the coupled operational bounds for DA generation dispatches and RT reserves offered by flexible generation capacities. Constraints (4.15) and (4.16) define the upper bounds for the upward and downward reserve provisions, respectively. Constraint (4.17) mandates that generators using must-run technologies are operated at their full capacities in the DA market, while constraint (4.18) expresses that renewable generation capacities always operate at the maximum output normalised by their forecast parameters in the DA market.

(iv) Network constraints (4.19) - (4.24).

This set of constraints relates to the physical limitations of network transmission with DC power flow approximation across both DA and RT markets. Constraint (4.19) presents the transmission capacity limits of the linear DC power flow approximation between any two interconnected buses during the DA market. The notation $m \in \Lambda_n$ identifies the buses m that are directly connected to bus n . Constraint (4.20) sets the physical lower and upper bounds for the nodal voltage angles in the DA market. Constraint (4.21) designates bus $n = 1$ as the slack bus in the DA network model. The slack bus serves as a reference point for voltage angles. In parallel, constraints (4.22)-(4.24) extend the network transmission constraints into the RT market.

(v) Chance constraints for renewable energy and demand uncertainties (4.25) - (4.26).

This set of chance constraints manages the inherent uncertainties in wind power production and demand. Specifically, the chance constraint (4.25) is designed to express that the total wind power production does not exceed its actual realisation at the probability of $1 - \rho^{RE}$. Chance constraint (4.26) guarantees that the overall demand consumption is higher than its realisation with a probability of $1 - \rho^L$.

In the formulation of the LL1 problem, the dual variables associated with the specified constraints are meticulously recorded following the colon. It is important to emphasise that, from the market operator's viewpoint, the generation capacities resulting from investment decisions are treated as given parameters within the LL1 problem.

4.3.3 LL2 Problem: Clearing Process of TGC Market

The clearing process of the present TGC market and the forecast for the future TGC market can be formulated as follows:

$$V^{LL2} = \{\lambda_{i,d}^{PGC}, Q_{i,d}^{PGC}\} \quad (4.27)$$

$$\lambda_{i,d}^{PGC} = l^{PGC} - q^{PGC} Q_{i,d}^{PGC} \quad \forall i \in I^{RE}, \forall d \quad (4.28)$$

$$\lambda^{FGC} = \gamma^{FGC} \lambda_0^{RPS} \quad (4.29)$$

In the LL2 problem, the decision variables are encapsulated within the set (4.27), which notably includes the present TGC price and the TGC quantity trading in the present TGC market. It should be noted that the estimated future TGC price, λ^{FGC} , is known as a parameter since it is calculated by the other parameters. Additionally, readers might be confused that the TGC quantity trading in the present TGC market, $Q_{i,d}^{PGC}$, appears in both the variable sets V^{UL} and V^{LL2} . It is not surprising that the UL problem decides the total quantity of TGC quantity (e.g., $Q_{i,d}^{PGC} + Q_{i,d}^{FGC}$), while the LL2 problem determines the TGC quantity trading in the present TGC market, $Q_{i,d}^{PGC}$.

The present TGC market is structured based on a quantity-dependent inverse demand function. Under this framework, the present TGC price is determined by equation (4.28) that represents the linear correlation between the present price of TGC and its quantity. On the other hand, equation (4.29) articulates the estimation of future TGC price, which is related to the penalty price associated with RPS deficiency.

The determination of the coefficients is critical to the inverse demand function of the present TGC market, specifically l^{PGC} and q^{PGC} , which is also a fundamental aspect of accurately modelling the dynamics of the present TGC market. These coefficients are obtained from equations (4.30) and (4.31), respectively, capturing the relationship between the price and quantity of TGCs. The methodology employed to derive these coefficients is proved in [64].

$$l^{PGC} = \lambda_0^{RPS} \quad (4.30)$$

$$q^{PGC} = \frac{(1 - \theta^{PGC}) \lambda_0^{RPS}}{\gamma^{RPS} \sum_{j,d,t} L_{j,d,t}^{DA}} \quad (4.31)$$

4.4 Solution

The solution approach for the proposed two-stage bi-level optimisation model involves a series of steps, which are detailed in this section. Initially, the non-convex chance constraints present in the LL1 problem are linearized, as seen in Subsection 4.4.1. Following this linearization, the entire bi-level problem is transformed into a single-level MPEC, as

detailed in Subsection 4.4.2. This transformation leverages the KKT optimality conditions to replace the LL1 problem; simultaneously, it integrates the constraints from the LL2 problem with the UL problem. To further ensure the solvability, non-linear components identified within the MPEC problem are eliminated through linearization techniques, as explored in Subsection 4.4.3. Ultimately, in order to guarantee global optimality, the MPEC problem undergoes a final transformation into a computationally tractable MIQP, as described in Subsection 4.4.4.

4.4.1 CCP Reformulation

In this thesis, the chance constraints adopt the stochastic variables for tackling RES and demand uncertainties, g^R and L^R , following the normal distributions:

$$g_{i,d,t,n}^R \sim \mathcal{N}(m_{i,d,t,n}^{RE}, (s_{i,d,t,n}^{RE})^2), \quad L_{j,d,t}^R \sim \mathcal{N}(m_{j,d,t}^L, (s_{j,d,t}^L)^2) \quad (4.32)$$

where parameter m is the mean of the corresponding normal distribution and parameter s is its standard deviation. It should be noted that the chance constraints (4.25) and (4.26) are non-convex. In order to make the proposed model solvable, the non-convexities in these chance constraints are required to be eliminated. By employing the linearization process detailed in Subsection 3.4.1, chance constraints (4.25) and (4.26) present in the market-driven model are reformulated as follows:

$$m_{i,d,t,n}^{RE} - g_{i,d,t,n}^{DA} + g_{i,d,t,n}^{dev} \geq \Phi^{-1}(1 - \rho^{RE}) s_{i,d,t,n}^{RE} \quad : \chi_{i,d,t,n}^{RE} \quad \forall i \in I^{RE}, \forall d, \forall t, \forall n \quad (4.33)$$

$$-m_{j,d,t}^L + \sum_o P_{j,d,t,o}^{dev} + \sum_o L_{j,d,t,o}^{DA} \geq \Phi^{-1}(1 - \rho^L) s_{j,d,t}^L \quad : \chi_{j,d,t}^L \quad \forall j, \forall d, \forall t \quad (4.34)$$

4.4.2 MPEC Formulation

Since the continuity and convexity of the LL1 problem are maintained through linearized chance constraints (4.33)-(4.34), the LL1 problem can be replaced by a series of constraints obtained by its KKT optimality conditions. Moreover, the LL2 problem, characterised completely by constraints, is seamlessly combined with the UL problem. Consequently, the original two-stage bi-level optimisation problem (4.1)-(4.29) is converted into a single-level MPEC problem. This transformation is aimed at enhancing computational efficiency by avoiding the inherent challenges of bi-level optimisation problems.

KKT Conditions Associated with the LL1 Problem

In order to derive the KKT optimality conditions corresponding to the LL1 problem, its Lagrangian function $\mathcal{L}_{LL1}$ is essential, which is listed as follows:

$$\begin{aligned}
\mathcal{L}_{LL1} = & \sum_{i,d,t,n,b} C_{i,b}^{EN} g_{i,d,t,n,b}^{DA} + \sum_{i,d,t,n,b} (C_{i,b}^U g_{i,d,t,n,b}^U - C_{i,b}^D g_{i,d,t,n,b}^D) + \sum_{d,t,n} \lambda_{d,t,n}^{DA} \left[\sum_{j \in \Psi_{n,o}} L_{j,d,t,o}^{DA} \right. \\
& + \sum_{m \in \Lambda_n} b_{nm} (\delta_{d,t,n}^{DA} - \delta_{d,t,m}^{DA}) - \sum_{i \in \Psi_{n,b}} g_{i,d,t,n,b}^{DA} \left. + \sum_{d,t,n} \lambda_{d,t,n}^{RT} \left[\sum_{j \in \Psi_{n,o}} P_{j,d,t,o}^{dev} + \right. \right. \\
& \sum_{i \in I^{FL} \cap \Psi_{n,b}} (-g_{i,d,t,n,b}^U + g_{i,d,t,n,b}^D) + \sum_{i \in I^{RE} \cap \Psi_n} g_{i,d,t,n}^{dev} - \sum_{m \in \Lambda_n} b_{nm} (\delta_{d,t,n}^{DA} - \delta_{d,t,n}^{RT} - \\
& \delta_{d,t,m}^{DA} + \delta_{d,t,m}^{RT}) \left. \right] + \sum_d \zeta_d \left[\sum_{i \in I^{FL}, t, n, b} \eta_i^{CO_2} (g_{i,d,t,n,b}^{DA} + g_{i,d,t,n,b}^U - g_{i,d,t,n,b}^D) - \right. \\
& \eta^{Grid} \sum_{j,t,o} L_{j,d,t,o}^{DA} \left. \right] + \sum_{i \in I^{FL}, d, t, n, b} \mu_{i,d,t,n,b}^U (g_{i,d,t,n,b}^{DA} + g_{i,d,t,n,b}^U - G_{i,n,b}) + \\
& \sum_{i \in I^{FL}, d, t, n, b} \mu_{i,d,t,n,b}^D (g_{i,d,t,n,b}^D - g_{i,d,t,n,b}^{DA}) - \sum_{i \in I^{FL}, d, t, n, b} \phi_{i,d,t,n,b}^{U,-} g_{i,d,t,n,b}^U + \\
& \sum_{i \in I^{FL}, d, t, n, b} \phi_{i,d,t,n,b}^{U,+} (g_{i,d,t,n,b}^U - G_{i,b}^U) - \sum_{i \in I^{FL}, d, t, n, b} \phi_{i,d,t,n,b}^{D,-} g_{i,d,t,n,b}^D + \\
& \sum_{i \in I^{FL}, d, t, n, b} \phi_{i,d,t,n,b}^{D,+} (g_{i,d,t,n,b}^D - G_{i,b}^D) + \sum_{i \in I^{MR}, d, t, n} \gamma_{i,d,t,n} (g_{i,d,t,n}^{DA} - G_{i,n}) + \\
& \sum_{i \in I^{RE}, d, t, n} \theta_{i,d,t,n} (g_{i,d,t,n}^{DA} - k_{i,d,t}^{RE} G_{i,n}) + \sum_{d,t,n,m \in \Lambda_n} \nu_{d,t,n,m}^{DA,-} [-C_{nm}^{max} - b_{nm} (\delta_{d,t,n}^{DA} - \\
& \delta_{d,t,m}^{DA})] + \sum_{d,t,n,m \in \Lambda_n} \nu_{d,t,n,m}^{DA,+} [b_{nm} (\delta_{d,t,n}^{DA} - \delta_{d,t,m}^{DA}) - C_{nm}^{max}] + \\
& \sum_{d,t,n} \xi_{d,t,n}^{DA,-} (-\pi - \delta_{d,t,n}^{DA}) + \sum_{d,t,n} \xi_{d,t,n}^{DA,+} (\delta_{d,t,n}^{DA} - \pi) + \sum_{d,t,n=1} \xi_{d,t}^{DA,ref} \delta_{d,t}^{DA,ref} + \\
& \sum_{d,t,n,m \in \Lambda_n} \nu_{d,t,n,m}^{RT,-} [-C_{nm}^{max} - b_{nm} (\delta_{d,t,n}^{RT} - \delta_{d,t,m}^{RT})] + \sum_{d,t,n,m \in \Lambda_n} \nu_{d,t,n,m}^{RT,+} [b_{nm} (\delta_{d,t,n}^{RT} - \\
& \delta_{d,t,m}^{RT}) - C_{nm}^{max}] + \sum_{d,t,n} \xi_{d,t,n}^{RT,-} (-\pi - \delta_{d,t,n}^{RT}) + \sum_{d,t,n} \xi_{d,t,n}^{RT,+} (\delta_{d,t,n}^{RT} - \pi) + \\
& \sum_{d,t,n=1} \xi_{d,t}^{RT,ref} \delta_{d,t}^{RT,ref} + \sum_{i \in I^{RE}, d, t, n} \chi_{i,d,t,n}^{RE} [\Phi^{-1}(1 - \rho^{RE}) s_{i,d,t,n}^{RE} - m_{i,d,t,n}^{RE} + \\
& g_{i,d,t,n}^{DA} - g_{i,d,t,n}^{dev}] + \sum_{j,d,t} \chi_{j,d,t}^L [\Phi^{-1}(1 - \rho^L) s_{j,d,t}^L + m_{j,d,t}^L - \sum_o P_{j,d,t,o}^{dev} \\
& - \sum_o L_{j,d,t,o}^{DA} \left. \right] \tag{4.35}
\end{aligned}$$

According to the Lagrangian function (4.35), the comprehensive KKT optimality conditions are listed as below:

LL1 KKT equality conditions : (4.10) – (4.11), (4.17) – (4.18), (4.21), (4.24) (4.36)

LL1 KKT stationarity conditions :

$$\frac{\partial \mathcal{L}_{LL1}}{\partial g_{i,d,t,n,b}^{DA}} = C_{i,b}^{EN} - \lambda_{d,t,n}^{DA} + \eta_i^{CO_2} \zeta_d + \mu_{i,d,t,n,b}^U - \mu_{i,d,t,n,b}^D = 0 \quad \forall i \in I^{FL} \cap \Psi_n, \forall d, \forall t, \forall n, \forall b \quad (4.37)$$

$$\frac{\partial \mathcal{L}_{LL1}}{\partial g_{i,d,t,n}^{DA}} = C_i^{EN} - \lambda_{d,t,n}^{DA} + \gamma_{i,d,t,n} = 0 \quad \forall i \in I^{MR} \cap \Psi_n, \forall d, \forall t, \forall n \quad (4.38)$$

$$\frac{\partial \mathcal{L}_{LL1}}{\partial g_{i,d,t,n}^{DA}} = -\lambda_{d,t,n}^{DA} + \theta_{i,d,t,n} + \chi_{i,d,t,n}^{RE} = 0 \quad \forall i \in I^{RE} \cap \Psi_n, \forall d, \forall t, \forall n \quad (4.39)$$

$$\begin{aligned} \frac{\partial \mathcal{L}_{LL1}}{\partial g_{i,d,t,n,b}^U} = & C_{i,b}^U - \lambda_{d,t,n}^{RT} + \eta_i^{CO_2} \zeta_d + \mu_{i,d,t,n,b}^U - \phi_{i,d,t,n,b}^{U,-} + \phi_{i,d,t,n,b}^{U,+} = 0 \\ & \forall i \in I^{FL} \cap \Psi_n, \forall d, \forall t, \forall n, \forall b \end{aligned} \quad (4.40)$$

$$\begin{aligned} \frac{\partial \mathcal{L}_{LL1}}{\partial g_{i,d,t,n,b}^D} = & -C_{i,b}^D + \lambda_{d,t,n}^{RT} - \eta_i^{CO_2} \zeta_d + \mu_{i,d,t,n,b}^D - \phi_{i,d,t,n,b}^{D,-} + \phi_{i,d,t,n,b}^{D,+} = 0 \\ & \forall i \in I^{FL} \cap \Psi_n, \forall d, \forall t, \forall n, \forall b \end{aligned} \quad (4.41)$$

$$\frac{\partial \mathcal{L}_{LL1}}{\partial P_{j,d,t,o}^{dev}} = \lambda_{d,t,n}^{RT} - \chi_{j,d,t}^L = 0 \quad \forall j \in \Psi_n, \forall d, \forall t \quad (4.42)$$

$$\frac{\partial \mathcal{L}_{LL1}}{\partial g_{i,d,t,n}^{dev}} = \lambda_{d,t,n}^{RT} - \chi_{i,d,t,n}^{RE} = 0 \quad \forall i \in I^{RE} \cap \Psi_n, \forall d, \forall t, \forall n \quad (4.43)$$

$$\begin{aligned} \frac{\partial \mathcal{L}_{LL1}}{\partial \delta_{d,t,n}^{DA}} = & \sum_{m \in \Lambda_n} b_{nm} (\lambda_{d,t,n}^{DA} - \lambda_{d,t,m}^{DA}) - \sum_{m \in \Lambda_n} b_{nm} (\lambda_{d,t,n}^{RT} - \lambda_{d,t,m}^{RT}) + \\ & \sum_{m \in \Lambda_n} b_{nm} (\nu_{d,t,n,m}^{DA,+} - \nu_{d,t,m,n}^{DA,+}) - \sum_{m \in \Lambda_n} b_{nm} (\nu_{d,t,n,m}^{DA,-} - \nu_{d,t,m,n}^{DA,-}) + \xi_{d,t,n}^{DA,+} \\ & - \xi_{d,t,n}^{DA,-} + (\xi_{d,t}^{DA,ref})_{n=1} = 0 \quad \forall d, \forall t, \forall n \end{aligned} \quad (4.44)$$

$$\begin{aligned} \frac{\partial \mathcal{L}_{LL1}}{\partial \delta_{d,t,n}^{RT}} = & \sum_{m \in \Lambda_n} b_{nm} (\lambda_{d,t,n}^{RT} - \lambda_{d,t,m}^{RT}) + \sum_{m \in \Lambda_n} b_{nm} (\nu_{d,t,n,m}^{RT,+} - \nu_{d,t,m,n}^{RT,+}) - \\ & \sum_{m \in \Lambda_n} b_{nm} (\nu_{d,t,n,m}^{RT,-} - \nu_{d,t,m,n}^{RT,-}) + \xi_{d,t,n}^{RT,+} - \xi_{d,t,n}^{RT,-} + (\xi_{d,t}^{RT,ref})_{n=1} = 0 \quad \forall d, \forall t, \forall n \end{aligned} \quad (4.45)$$

LL1 KKT complementarity conditions :

$$0 \leq \left[\eta^{Grid} \sum_{j,t,o} L_{j,d,t,o}^{DA} - \sum_{i \in I^{FL}, t, n, b} \eta_i^{CO_2} (g_{i,d,t,n,b}^{DA} + g_{i,d,t,n,b}^U - g_{i,d,t,n,b}^D) \right] \perp \zeta_d \geq 0 \quad \forall d \quad (4.46)$$

$$0 \leq (G_{i,n,b} - g_{i,d,t,n,b}^{DA} - g_{i,d,t,n,b}^U) \perp \mu_{i,d,t,n,b}^U \geq 0 \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (4.47)$$

$$0 \leq (g_{i,d,t,n,b}^{DA} - g_{i,d,t,n,b}^D) \perp \mu_{i,d,t,n,b}^D \geq 0 \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (4.48)$$

$$0 \leq g_{i,d,t,n,b}^U \perp \phi_{i,d,t,n,b}^{U,-} \geq 0 \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (4.49)$$

$$0 \leq (G_{i,b}^U - g_{i,d,t,n,b}^U) \perp \phi_{i,d,t,n,b}^{U,+} \geq 0 \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (4.50)$$

$$0 \leq g_{i,d,t,n,b}^D \perp \phi_{i,d,t,n,b}^{D,-} \geq 0 \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (4.51)$$

$$0 \leq (G_{i,b}^D - g_{i,d,t,n,b}^D) \perp \phi_{i,d,t,n,b}^{D,+} \geq 0 \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (4.52)$$

$$0 \leq [C_{nm}^{max} + b_{nm}(\delta_{d,t,n}^{DA} - \delta_{d,t,m}^{DA})] \perp \nu_{d,t,n,m}^{DA,-} \geq 0 \quad \forall d, \forall t, \forall n, \forall m \in \Lambda_n \quad (4.53)$$

$$0 \leq [C_{nm}^{max} - b_{nm}(\delta_{d,t,n}^{DA} - \delta_{d,t,m}^{DA})] \perp \nu_{d,t,n,m}^{DA,+} \geq 0 \quad \forall d, \forall t, \forall n, \forall m \in \Lambda_n \quad (4.54)$$

$$0 \leq (\pi + \delta_{d,t,n}^{DA}) \perp \xi_{d,t,n}^{DA,-} \geq 0 \quad \forall d, \forall t, \forall n \quad (4.55)$$

$$0 \leq (\pi - \delta_{d,t,n}^{DA}) \perp \xi_{d,t,n}^{DA,+} \geq 0 \quad \forall d, \forall t, \forall n \quad (4.56)$$

$$0 \leq [C_{nm}^{max} + b_{nm}(\delta_{d,t,n}^{RT} - \delta_{d,t,m}^{RT})] \perp \nu_{d,t,n,m}^{RT,-} \geq 0 \quad \forall d, \forall t, \forall n, \forall m \in \Lambda_n \quad (4.57)$$

$$0 \leq [C_{nm}^{max} - b_{nm}(\delta_{d,t,n}^{RT} - \delta_{d,t,m}^{RT})] \perp \nu_{d,t,n,m}^{RT,+} \geq 0 \quad \forall d, \forall t, \forall n, \forall m \in \Lambda_n \quad (4.58)$$

$$0 \leq (\pi + \delta_{d,t,n}^{RT}) \perp \xi_{d,t,n}^{RT,-} \geq 0 \quad \forall d, \forall t, \forall n \quad (4.59)$$

$$0 \leq (\pi - \delta_{d,t,n}^{RT}) \perp \xi_{d,t,n}^{RT,+} \geq 0 \quad \forall d, \forall t, \forall n \quad (4.60)$$

$$0 \leq [m_{i,d,t,n}^{RE} - g_{i,d,t,n}^{DA} + g_{i,d,t,n}^{dev} - \Phi^{-1}(1 - \rho^{RE})s_{i,d,t,n}^{RE}] \perp \chi_{i,d,t,n}^{RE} \geq 0 \quad \forall i \in I^{RE}, \forall d, \forall t, \forall n \quad (4.61)$$

$$0 \leq [-m_{j,d,t}^L + \sum_o P_{j,d,t,o}^{dev} + \sum_o L_{j,d,t,o}^{DA} - \Phi^{-1}(1 - \rho^L)s_{j,d,t}^L] \perp \chi_{j,d,t}^L \geq 0 \quad \forall j, \forall d, \forall t \quad (4.62)$$

The KKT optimality conditions for the LL1 problem are a critical component in transforming the bi-level optimisation problem into a single-level MPEC. These conditions are segmented into three distinct parts, which are detailed as follows:

1. KKT equality conditions (4.36).

The first part of the KKT conditions encompasses the equality constraints that are directly extracted from the LL1 problem.

2. KKT stationarity conditions (4.37)-(4.45).

The second part involves the KKT stationarity conditions, which are derived from the first-order partial derivatives of the Lagrangian function of the LL1 problem $\mathcal{L}_{LL1}$ with respect to its primal variables. It is noteworthy that the order of taking the partial derivatives coincides with the sequence of LL1 decision variables listed

in the set V^{LL1} (4.9).

3. KKT complementarity conditions (4.46)-(4.62).

The third part consists of the KKT complementarity conditions, which describe the complementary slackness between the inequality constraints of the LL1 problem and their corresponding dual variables. The notation $a \perp b$ implies that either a is zero, or b is zero, or both are zero, indicating that for an inequality constraint, either the constraint is active (equality holds) and its dual variable is positive, or the constraint is inactive (inequality holds strictly) and the dual variable is zero.

Collectively, these KKT conditions provide a rigorous mathematical framework to embed the LL1 problem's optimality criteria into the MPEC problem.

Final MPEC Formulation

Since the LL2 problem consists solely of a series of constraints, it can be easily integrated into the MPEC. As a result, the MPEC formulation is mathematically given as below:

$$\begin{aligned}
 \max_{V^{MPEC}} \sum_d \omega_d \bigg[& \sum_{i \in \Psi_{n,t,n,b}} \lambda_{d,t,n}^{DA} g_{i,d,t,n,b}^{DA} - \sum_{i,t,n,b} C_{i,b}^{EN} g_{i,d,t,n,b}^{DA} + \\
 & \sum_{i \in I^{FL} \cap \Psi_{n,t,n,b}} \lambda_{d,t,n}^{RT} (g_{i,d,t,n,b}^U - g_{i,d,t,n,b}^D) - \sum_{i \in I^{FL},t,n,b} (C_{i,b}^U g_{i,d,t,n,b}^U - C_{i,b}^D g_{i,d,t,n,b}^D) \\
 & + \sum_{i \in I^{RE} \cap \Psi_{n,t,n}} \lambda_{d,t,n}^{RT} g_{i,d,t,n}^{dev} - \sum_{i \in I^{FL},t,n} \lambda^{CO_2} Q_{i,d,t,n}^{emi} + \sum_{i \in I^{RE},t,n} \lambda^{CO_2} Q_{i,d,t,n}^{CA} \\
 & + \sum_{i \in I^{RE}} (\lambda_{i,d}^{PGC} Q_{i,d}^{PGC} + \lambda^{FGC} Q_{i,d}^{FGC}) \bigg] - \sum_{i,n} C_i^{IN} G_{i,n} \quad (4.63)
 \end{aligned}$$

where:

$$V^{MPEC} = V^{UL} \cup V^{LL1} \cup V^{LL2} \cup V^{DUAL} \quad (4.64)$$

$$\begin{aligned}
 V^{DUAL} = \{ & \lambda_{d,t,n}^{DA}, \lambda_{d,t,n}^{RT}, \zeta_d, \mu_{i,d,t,n,b}^U, \mu_{i,d,t,n,b}^D, \phi_{i,d,t,n,b}^{U,-}, \phi_{i,d,t,n,b}^{U,+}, \phi_{i,d,t,n,b}^{D,-}, \phi_{i,d,t,n,b}^{D,+}, \gamma_{i,d,t,n}, \\
 & \theta_{i,d,t,n}, \nu_{d,t,n,m}^{DA,-}, \nu_{y,t,n,m}^{DA,+}, \xi_{d,t,n}^{DA,-}, \xi_{d,t,n}^{DA,+}, \xi_{d,t}^{DA,ref}, \nu_{d,t,n,m}^{RT,-}, \nu_{d,t,n,m}^{RT,+}, \xi_{d,t,n}^{RT,-}, \xi_{d,t,n}^{RT,+}, \\
 & \xi_{d,t}^{RT,ref}, \chi_{i,d,t,n}^{RE}, \chi_{j,d,t}^L \} \quad (4.65)
 \end{aligned}$$

subject to:

$$UL \text{ constraints : (4.3) – (4.7)} \quad (4.66)$$

$$\text{LL1 KKT equality conditions : (4.36)} \quad (4.67)$$

$$\text{LL1 KKT stationarity conditions : (4.37) – (4.45)} \quad (4.68)$$

$$\text{LL1 KKT complementarity conditions : (4.46) – (4.62)} \quad (4.69)$$

$$\text{LL2 constraints : (4.28) – (4.29)} \quad (4.70)$$

The objective function of the MPEC problem (4.63) harmonises with that of the UL problem (4.1). The set of decision variables in the MPEC problem (4.64) comprehensively includes the decision variables from the UL problem (4.2), both LL problems (4.9) and (4.27), as well as the dual variables emerging from the LL1 problem (4.65). The constraints of the MPEC problem encompass the original constraints from the UL problem (4.66), the KKT optimality conditions corresponding to the LL1 problem (4.67)-(4.69), and the constraints inherent in the LL2 problem (4.70).

4.4.3 Linearization of MPEC

To enhance the computational tractability of the proposed MPEC problem, the application of effective linearization techniques is crucial for transforming the inherently non-linear MPEC into a MIQP, which is well-suited for commercial branch-and-cut solvers, thereby guaranteeing global optimality. The non-linearities within the MPEC formulation arise from two sources:

1. The first source of non-linearity is found in the KKT complementarity conditions for the LL1 problem (4.69). These conditions include bilinear terms that are the products of primal variables and dual variables.
2. The second source of non-linearity stems from the calculation of GenCo's profit, represented by the MPEC's objective function (4.63). This includes bilinear terms formed by the products of energy prices and generation dispatches across the two-stage electricity markets and the products of TGC quantities and TGC prices.

The linearization of these non-linearities is meticulously addressed in this subsection, where different strategies and mathematical techniques are employed to reformulate these bilinear terms into linear equivalences.

Linearization of KKT Complementarity Conditions

The big-M approach, as outlined in [76], offers a linearization technique for handling the KKT complementarity conditions (4.69). This method effectively translates the KKT

complementarity conditions, which inherently include bilinear terms, into mixed-integer linear constraints that are solvable using MIP solvers.

The essential principle behind the big-M method involves the introduction of large positive constants (denoted as M^a and M^b) and auxiliary binary variables (denoted as β) to decompose the original complementarity condition $0 \leq a \perp b \leq 0$ into a set of linear conditions combined with binary variables. Specifically, a represents a primal variable in KKT conditions, and b represents the corresponding dual variable. The resulting equivalent mixed-integer linear conditions are formulated as follows: $\{a \geq 0, b \geq 0, a \leq \beta M^a, b \leq (1 - \beta)M^b, \beta \in \{0, 1\}\}$. This setup ensures that either a is zero or b is zero, guaranteeing the original complementary relationship but within a MIP framework that can be handled by commercial solvers.

The general form of linearized KKT complementarity conditions is formulated as below:

$$\mathcal{C}^{Comp} = \{0 \leq a^{Comp} \leq \beta M, 0 \leq b^{Comp} \leq (1 - \beta)M\} \quad (4.71)$$

By employing a series of auxiliary binary variables contained in set (4.72) in line with the general form, the comprehensive mixed-integer linearized equivalences for the KKT complementarity conditions (4.46)-(4.62) are listed as follows:

$$V^{BINARY} = \left\{ \beta_d^{CO_2}, \beta_{i,d,t,n,b}^U, \beta_{i,d,t,n,b}^D, \beta_{i,d,t,n,b}^{U,-}, \beta_{i,d,t,n,b}^{U,+}, \beta_{i,d,t,n,b}^{D,-}, \beta_{i,d,t,n,b}^{D,+}, \beta_{d,t,n,m}^{DA,net,-}, \right. \\ \left. \beta_{d,t,n,m}^{DA,net,+}, \beta_{d,t,n}^{DA,\pi,-}, \beta_{d,t,n}^{DA,\pi,+}, \beta_{d,t,n,m}^{RT,net,-}, \beta_{d,t,n,m}^{RT,net,+}, \beta_{d,t,n}^{RT,\pi,-}, \beta_{d,t,n}^{RT,\pi,+}, \right. \\ \left. \beta_{i,d,t,n}^{RE}, \beta_{j,d,t}^L \right\} \quad (4.72)$$

$$0 \leq \eta^{Grid} \sum_{j,t,o} L_{j,d,t,o}^{DA} - \sum_{i \in I^{FL}, t, n, b} \eta_i^{CO_2} (g_{i,d,t,n,b}^{DA} + g_{i,d,t,n,b}^U - g_{i,d,t,n,b}^D) \leq \beta_d^{CO_2} M \quad \forall d \quad (4.73)$$

$$0 \leq \zeta_d \leq (1 - \beta_d^{CO_2})M \quad \forall d \quad (4.74)$$

$$0 \leq G_{i,n,b} - g_{i,d,t,n,b}^{DA} - g_{i,d,t,n,b}^U \leq \beta_{i,d,t,n,b}^U M \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (4.75)$$

$$0 \leq \mu_{i,d,t,n,b}^U \leq (1 - \beta_{i,d,t,n,b}^U)M \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (4.76)$$

$$0 \leq g_{i,d,t,n,b}^{DA} - g_{i,d,t,n,b}^D \leq \beta_{i,d,t,n,b}^D M \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (4.77)$$

$$0 \leq \mu_{i,d,t,n,b}^D \leq (1 - \beta_{i,d,t,n,b}^D)M \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (4.78)$$

$$0 \leq g_{i,d,t,n,b}^U \leq \beta_{i,d,t,n,b}^{U,-} M \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (4.79)$$

$$0 \leq \phi_{i,d,t,n,b}^{U,-} \leq (1 - \beta_{i,d,t,n,b}^{U,-})M \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (4.80)$$

$$0 \leq G_{i,b}^U - g_{i,d,t,n,b}^U \leq \beta_{i,d,t,n,b}^{U,+} M \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (4.81)$$

$$0 \leq \phi_{i,d,t,n,b}^{U,+} \leq (1 - \beta_{i,d,t,n,b}^{U,+}) M \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (4.82)$$

$$0 \leq g_{i,d,t,n,b}^D \leq \beta_{i,d,t,n,b}^{D,-} M \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (4.83)$$

$$0 \leq \phi_{i,d,t,n,b}^{D,-} \leq (1 - \beta_{i,d,t,n,b}^{D,-}) M \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (4.84)$$

$$0 \leq G_{i,b}^D - g_{i,d,t,n,b}^D \leq \beta_{i,d,t,n,b}^{D,+} M \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (4.85)$$

$$0 \leq \phi_{i,d,t,n,b}^{D,+} \leq (1 - \beta_{i,d,t,n,b}^{D,+}) M \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (4.86)$$

$$0 \leq C_{nm}^{max} + b_{nm}(\delta_{d,t,n}^{DA} - \delta_{d,t,m}^{DA}) \leq \beta_{d,t,n,m}^{DA,net,-} M \quad \forall d, \forall t, \forall n, \forall m \in \Lambda_n \quad (4.87)$$

$$0 \leq \nu_{d,t,n,m}^{DA,-} \leq (1 - \beta_{d,t,n,m}^{DA,net,-}) M \quad \forall d, \forall t, \forall n, \forall m \in \Lambda_n \quad (4.88)$$

$$0 \leq C_{nm}^{max} - b_{nm}(\delta_{d,t,n}^{DA} - \delta_{d,t,m}^{DA}) \leq \beta_{d,t,n,m}^{DA,net,+} M \quad \forall d, \forall t, \forall n, \forall m \in \Lambda_n \quad (4.89)$$

$$0 \leq \nu_{d,t,n,m}^{DA,+} \leq (1 - \beta_{d,t,n,m}^{DA,net,+}) M \quad \forall d, \forall t, \forall n, \forall m \in \Lambda_n \quad (4.90)$$

$$0 \leq \pi + \delta_{d,t,n}^{DA} \leq \beta_{d,t,n}^{DA,\pi,-} M \quad \forall d, \forall t, \forall n \quad (4.91)$$

$$0 \leq \xi_{d,t,n}^{DA,-} \leq (1 - \beta_{d,t,n}^{DA,\pi,-}) M \quad \forall d, \forall t, \forall n \quad (4.92)$$

$$0 \leq \pi - \delta_{d,t,n}^{DA} \leq \beta_{d,t,n}^{DA,\pi,+} M \quad \forall d, \forall t, \forall n \quad (4.93)$$

$$0 \leq \xi_{d,t,n}^{DA,+} \leq (1 - \beta_{d,t,n}^{DA,\pi,+}) M \quad \forall d, \forall t, \forall n \quad (4.94)$$

$$0 \leq C_{nm}^{max} + b_{nm}(\delta_{d,t,n}^{RT} - \delta_{d,t,m}^{RT}) \leq \beta_{d,t,n,m}^{RT,net,-} M \quad \forall d, \forall t, \forall n, \forall m \in \Lambda_n \quad (4.95)$$

$$0 \leq \nu_{d,t,n,m}^{RT,-} \leq (1 - \beta_{d,t,n,m}^{RT,net,-}) M \quad \forall d, \forall t, \forall n, \forall m \in \Lambda_n \quad (4.96)$$

$$0 \leq C_{nm}^{max} - b_{nm}(\delta_{d,t,n}^{RT} - \delta_{d,t,m}^{RT}) \leq \beta_{d,t,n,m}^{RT,net,+} M \quad \forall d, \forall t, \forall n, \forall m \in \Lambda_n \quad (4.97)$$

$$0 \leq \nu_{d,t,n,m}^{RT,+} \leq (1 - \beta_{d,t,n,m}^{RT,net,+}) M \quad \forall d, \forall t, \forall n, \forall m \in \Lambda_n \quad (4.98)$$

$$0 \leq \pi + \delta_{d,t,n}^{RT} \leq \beta_{d,t,n}^{RT,\pi,-} M \quad \forall d, \forall t, \forall n \quad (4.99)$$

$$0 \leq \xi_{d,t,n}^{RT,-} \leq (1 - \beta_{d,t,n}^{RT,\pi,-}) M \quad \forall d, \forall t, \forall n \quad (4.100)$$

$$0 \leq \pi - \delta_{d,t,n}^{RT} \leq \beta_{d,t,n}^{RT,\pi,+} M \quad \forall d, \forall t, \forall n \quad (4.101)$$

$$0 \leq \xi_{d,t,n}^{RT,+} \leq (1 - \beta_{d,t,n}^{RT,\pi,+}) M \quad \forall d, \forall t, \forall n \quad (4.102)$$

$$0 \leq m_{i,d,t,n}^{RE} - g_{i,d,t,n}^{DA} + g_{i,d,t,n}^{dev} - \Phi^{-1}(1 - \rho^{RE}) s_{i,d,t,n}^{RE} \leq \beta_{i,d,t,n}^{RE} M \quad \forall i \in I^{RE}, \forall d, \forall t, \forall n \quad (4.103)$$

$$0 \leq \chi_{i,d,t,n}^{RE} \leq (1 - \beta_{i,d,t,n}^{RE}) M \quad \forall i \in I^{RE}, \forall d, \forall t, \forall n \quad (4.104)$$

$$0 \leq -m_{j,d,t}^L + \sum_o P_{j,d,t,o}^{dev} + \sum_o P_{j,d,t,o}^{DA} - \Phi^{-1}(1 - \rho^L) s_{j,d,t}^L \leq \beta_{j,d,t}^L M \quad \forall j, \forall d, \forall t \quad (4.105)$$

$$0 \leq \chi_{j,d,t}^L \leq (1 - \beta_{j,d,t}^L)M \quad \forall j, \forall d, \forall t \quad (4.106)$$

For notational simplicity, the unified symbol M has been used to denote the large constants introduced by the big-M method across various constraints listed above. However, it is crucial to acknowledge that in practical applications, the large constants (M) associated with different constraints should indeed be distinct to suit their physical bounds.

Linearization of GenCo's Profit

To address the challenge of bilinear terms in GenCo's profit within the MPEC's objective function (4.63), the linearization technique outlined in [23] is employed. This approach leverages the strong duality theorem and the KKT optimality conditions to uncover implicit linear forms of these bilinear terms, thereby facilitating their incorporation into the model in a linearized structure.

Specifically, the bilinear terms ($\sum_{i \in I} \lambda_{d,t,n}^{DA} g_{i,d,t,n,b}^{DA}$, $\sum_{i \in I^{FL}} \lambda_{d,t,n}^{RT} g_{i,d,t,n,b}^U$, $\sum_{i \in I^{FL}} \lambda_{d,t,n}^{RT} g_{i,d,t,n,b}^D$, $\sum_{i \in I^{RE}} \lambda_{d,t,n}^{RT} g_{i,d,t,n}^{dev}$, and $\sum_{i \in I^{RE}} \lambda_{i,d}^{PGC} Q_{i,d}^{PGC}$) of the MPEC's objective function (4.63) need to be restructured as their linearized form by adopting the strong duality theorem and the KKT optimality conditions. The derivation process is detailed as follows.

First, in order to linearize the bilinear term $\sum_{i \in I} \lambda_{d,t,n}^{DA} g_{i,d,t,n,b}^{DA}$, the KKT stationarity condition (4.37) is rearranged as:

$$\lambda_{d,t,n}^{DA} = C_{i,b}^{EN} + \eta_i^{CO_2} \zeta_d + \mu_{i,d,t,n,b}^U - \mu_{i,d,t,n,b}^D \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (4.107)$$

Multiplying both sides by $g_{i,d,t,n,b}^{DA}$ ($\forall i \in I^{FL}$), it renders:

$$\begin{aligned} \lambda_{d,t,n}^{DA} g_{i,d,t,n,b}^{DA} &= C_{i,b}^{EN} g_{i,d,t,n,b}^{DA} + \eta_i^{CO_2} \zeta_d g_{i,d,t,n,b}^{DA} + \mu_{i,d,t,n,b}^U g_{i,d,t,n,b}^{DA} \\ &\quad - \mu_{i,d,t,n,b}^D g_{i,d,t,n,b}^{DA} \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \end{aligned} \quad (4.108)$$

Given that the term (4.108) exclusively accounts for the DA energy revenue derived from flexible generators, the same linearization procedure can be applied to address the revenue from must-run and renewable generators. Therefore, the KKT stationarity condition (4.38) is rewritten as:

$$\lambda_{d,t,n}^{DA} = C_i^{EN} + \gamma_{i,d,t,n} \quad \forall i \in I^{MR}, \forall d, \forall t, \forall n \quad (4.109)$$

Similarly, multiplying both sides by $g_{i,d,t,n}^{DA}$ ($\forall i \in I^{MR}$), it gives:

$$\lambda_{d,t,n}^{DA} g_{i,d,t,n}^{DA} = C_i^{EN} g_{i,d,t,n}^{DA} + \gamma_{i,d,t,n} g_{i,d,t,n}^{DA} \quad \forall i \in I^{MR}, \forall d, \forall t, \forall n \quad (4.110)$$

Subsequently, after the LL1 constraint (4.17) is plugged into (4.110), it renders:

$$\lambda_{d,t,n}^{DA} g_{i,d,t,n}^{DA} = C_i^{EN} g_{i,d,t,n}^{DA} + \gamma_{i,d,t,n} G_{i,n} \quad \forall i \in I^{MR}, \forall d, \forall t, \forall n \quad (4.111)$$

It should be emphasised that in terms of the realisation for renewable power generation, its mean is set as the DA renewable generation dispatch $g_{i,d,t,n}^{DA} (\forall i \in I^{RE})$, and its standard deviation is assumed to be proportional to the DA renewable generation dispatch. The specific setting will be detailed in the section on the case study. As a result, the chance constraint pertaining to renewable power production (4.33) should be rewritten as below:

$$g_{i,d,t,n}^{dev} \geq \Phi^{-1}(1 - \rho^{RE}) \Gamma g_{i,d,t,n}^{DA} \quad \forall i \in I^{RE}, \forall d, \forall t, \forall n \quad (4.112)$$

where Γ identifies the proportion of wind power deviation with respect to DA renewable generation dispatch.

Thus, the KKT stationarity condition (4.39) should be reformulated as follows:

$$-\lambda_{d,t,n}^{DA} + \theta_{i,d,t,n} + \Phi^{-1}(1 - \rho^{RE}) \Gamma \chi_{i,d,t,n}^{RE} = 0 \quad \forall i \in I^{RE}, \forall d, \forall t, \forall n \quad (4.113)$$

Then, rearranging the terms in the KKT stationarity condition (4.113) and multiplying both sides by $g_{i,d,t,n}^{DA} (i \in I^{RE})$, the energy revenue from renewable generation is recast as below:

$$\lambda_{d,t,n}^{DA} g_{i,d,t,n}^{DA} = \theta_{i,d,t,n} g_{i,d,t,n}^{DA} + \Phi^{-1}(1 - \rho^{RE}) \Gamma \chi_{i,d,t,n}^{RE} g_{i,d,t,n}^{DA} \quad \forall i \in I^{RE}, \forall d, \forall t, \forall n \quad (4.114)$$

By substituting the LL1 constraint (4.18) into the above expression (4.114), it further produces:

$$\lambda_{d,t,n}^{DA} g_{i,d,t,n}^{DA} = \theta_{i,d,t,n} k_{i,d,t}^{RE} G_{i,n} + \Phi^{-1}(1 - \rho^{RE}) \Gamma \chi_{i,d,t,n}^{RE} g_{i,d,t,n}^{DA} \quad \forall i \in I^{RE}, \forall d, \forall t, \forall n \quad (4.115)$$

In conclusion, the bilinear DA energy revenue, which is a critical component of the GenCo's profit, can be equivalently expressed by combining the expressions (4.108), (4.111), and (4.115) with respect to different generation technologies.

The next step is to deal with the bilinear terms related to the RT reserve revenues $\sum_{i \in I^{FL}} \lambda_{d,t,n}^{RT} g_{i,d,t,n}^U$ and $\sum_{i \in I^{FL}} \lambda_{d,t,n}^{RT} g_{i,d,t,n}^D$. To this end, we first rearrange the terms in the KKT stationarity conditions (4.40) and (4.41), and then multiplying both sides by

$g_{i,d,t,n,b}^U$ and $g_{i,d,t,n,b}^D$, respectively, it renders:

$$\begin{aligned} \lambda_{d,t,n}^{RT} g_{i,d,t,n,b}^U = & C_{i,b}^U g_{i,d,t,n,b}^U + \eta_i^{CO_2} \zeta_d g_{i,d,t,n,b}^U + \mu_{i,d,t,n,b}^U g_{i,d,t,n,b}^U - \phi_{i,d,t,n,b}^{U,-} g_{i,d,t,n,b}^U + \\ & \phi_{i,d,t,n,b}^{U,+} g_{i,d,t,n,b}^U \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \end{aligned} \quad (4.116)$$

$$\begin{aligned} \lambda_{d,t,n}^{RT} g_{i,d,t,n,b}^D = & C_{i,b}^D g_{i,d,t,n,b}^D + \eta_i^{CO_2} \zeta_d g_{i,d,t,n,b}^D - \mu_{i,d,t,n,b}^D g_{i,d,t,n,b}^D + \phi_{i,d,t,n,b}^{D,-} g_{i,d,t,n,b}^D - \\ & \phi_{i,d,t,n,b}^{D,+} g_{i,d,t,n,b}^D \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \end{aligned} \quad (4.117)$$

Furthermore, by employing the KKT complementarity conditions (4.49) - (4.52), we obtain the following expressions:

$$g_{i,d,t,n,b}^U \phi_{i,d,t,n,b}^{U,-} = 0 \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (4.118)$$

$$G_{i,b}^U \phi_{i,d,t,n,b}^{U,+} = g_{i,d,t,n,b}^U \phi_{i,d,t,n,b}^{U,+} \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (4.119)$$

$$g_{i,d,t,n,b}^D \phi_{i,d,t,n,b}^{D,-} = 0 \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (4.120)$$

$$G_{i,b}^D \phi_{i,d,t,n,b}^{D,+} = g_{i,d,t,n,b}^D \phi_{i,d,t,n,b}^{D,+} \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (4.121)$$

By substituting KKT complementarity conditions (4.118)-(4.121) into the equations for RT upward and downward reserve revenues, denoted as (4.116) and (4.117) respectively, it gives:

$$\begin{aligned} \lambda_{d,t,n}^{RT} g_{i,d,t,n,b}^U = & C_{i,b}^U g_{i,d,t,n,b}^U + \eta_i^{CO_2} \zeta_d g_{i,d,t,n,b}^U + \mu_{i,d,t,n,b}^U g_{i,d,t,n,b}^U + \\ & \phi_{i,d,t,n,b}^{U,+} G_{i,b}^U \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \end{aligned} \quad (4.122)$$

$$\begin{aligned} \lambda_{d,t,n}^{RT} g_{i,d,t,n,b}^D = & C_{i,b}^D g_{i,d,t,n,b}^D + \eta_i^{CO_2} \zeta_d g_{i,d,t,n,b}^D - \mu_{i,d,t,n,b}^D g_{i,d,t,n,b}^D - \\ & \phi_{i,d,t,n,b}^{D,+} G_{i,b}^D \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \end{aligned} \quad (4.123)$$

After this, the bilinear term of the renewable deviation penalty $\sum_{i \in I^{RE}} \lambda_{d,t,n}^{RT} g_{i,d,t,n}^{dev}$ should be addressed. To this end, we employ the KKT complementarity condition (4.61) related to the uncertainty of renewable power production and reformulate it as follows:

$$[-g_{i,d,t,n}^{dev} - \Phi^{-1}(1 - \rho^{RE})\Gamma g_{i,d,t,n}^{DA}] \chi_{i,d,t,n}^{RE} = 0 \quad \forall i \in I^{RE}, \forall d, \forall t, \forall n \quad (4.124)$$

By combining the KKT stationarity condition (4.43) with the complementary expres-

sion (4.124), the bilinear term $\lambda_{d,t,n}^{RT} g_{i,d,t,n}^{dev}$ is recast as follows:

$$\lambda_{d,t,n}^{RT} g_{i,d,t,n}^{dev} = \chi_{i,d,t,n}^{RE} g_{i,d,t,n}^{dev} = -\Phi^{-1}(1 - \rho^{RE}) \Gamma g_{i,d,t,n}^{DA} \chi_{i,d,t,n}^{RE} \quad \forall i \in I^{RE}, \forall d, \forall t, \forall n \quad (4.125)$$

Collectively, an alternative representation for the first four bilinear terms in the MPEC's objective function (the left side of the following equation) can be derived by integrating the expressions of (4.108), (4.111), (4.115), (4.122), (4.123), (4.125), and summing over their respective indices, which is formulated as follows:

$$\begin{aligned} & \sum_{i \in I, d, t, n, b} \lambda_{d,t,n}^{DA} g_{i,d,t,n,b}^{DA} + \sum_{i \in I^{FL}, d, t, n, b} \lambda_{d,t,n}^{RT} (g_{i,d,t,n,b}^U - g_{i,d,t,n,b}^D) - \sum_{i \in I^{RE}, d, t, n} \lambda_{d,t,n}^{RT} g_{i,d,t,n}^{dev} = \\ & \sum_{i \in I, d, t, n, b} C_{i,b}^{EN} g_{i,d,t,n,b}^{DA} + \sum_{i \in I^{FL}, d, t, n, b} C_{i,b}^U g_{i,d,t,n,b}^U - \sum_{i \in I^{FL}, d, t, n, b} C_{i,b}^D g_{i,d,t,n,b}^D + \\ & \sum_{i \in I^{FL}, d, t, n, b} \eta_i^{CO_2} \zeta_d (g_{i,d,t,n,b}^{DA} + g_{i,d,t,n,b}^U - g_{i,d,t,n,b}^D) + \sum_{i \in I^{FL}, d, t, n, b} \phi_{i,d,t,n,b}^{U,+} G_{i,b}^U + \\ & \sum_{i \in I^{FL}, d, t, n, b} \phi_{i,d,t,n,b}^{D,+} G_{i,b}^D + \sum_{i \in I^{FL}, d, t, n, b} \mu_{i,d,t,n,b}^D (g_{i,d,t,n,b}^D - g_{i,d,t,n,b}^{DA}) + \\ & \sum_{i \in I^{FL}, d, t, n, b} \mu_{i,d,t,n,b}^U (g_{i,d,t,n,b}^{DA} + g_{i,d,t,n,b}^U) + \sum_{i \in I^{MR}, d, t, n} \gamma_{i,d,t,n} G_{i,n} + \sum_{i \in I^{RE}, d, t, n} \theta_{i,d,t,n} k_{i,d,t}^{RE} G_{i,n} \end{aligned} \quad (4.126)$$

It is observable that some nonlinear terms on the right side of equation (4.126) can be eliminated through the application of the KKT complementarity conditions (4.46), (4.47) and (4.48), which are expressed as below:

$$\zeta_d \eta^{Grid} \sum_{j,t,o} L_{j,d,t,o}^{DA} = \zeta_d \sum_{i \in I^{FL}, t, n, b} \eta_i^{CO_2} (g_{i,d,t,n,b}^{DA} + g_{i,d,t,n,b}^U - g_{i,d,t,n,b}^D) \quad \forall d \quad (4.127)$$

$$\mu_{i,d,t,n,b}^U G_{i,n,b} = \mu_{i,d,t,n,b}^U (g_{i,d,t,n,b}^{DA} + g_{i,d,t,n,b}^U) \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (4.128)$$

$$\mu_{i,d,t,n,b}^D (g_{i,d,t,n,b}^{DA} - g_{i,d,t,n,b}^D) = 0 \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (4.129)$$

By substituting the above KKT complementarity conditions (4.127)-(4.129) into the equation (4.126), the representation for the first four bilinear terms in the MPEC's objective function can be partially linearized, which is given as follows:

$$\begin{aligned} & \sum_{i \in I, d, t, n, b} \lambda_{d,t,n}^{DA} g_{i,d,t,n,b}^{DA} + \sum_{i \in I^{FL}, d, t, n, b} \lambda_{d,t,n}^{RT} (g_{i,d,t,n,b}^U - g_{i,d,t,n,b}^D) - \sum_{i \in I^{RE}, d, t, n} \lambda_{d,t,n}^{RT} g_{i,d,t,n}^{dev} = \\ & \sum_{i \in I, d, t, n, b} C_{i,b}^{EN} g_{i,d,t,n,b}^{DA} + \sum_{i \in I^{FL}, d, t, n, b} C_{i,b}^U g_{i,d,t,n,b}^U - \sum_{i \in I^{FL}, d, t, n, b} C_{i,b}^D g_{i,d,t,n,b}^D + \end{aligned}$$

$$\begin{aligned}
& \sum_{j,d,t,o} \eta^{Grid} \zeta_d L_{j,d,t,o}^{DA} + \sum_{i \in I^{FL},d,t,n,b} \phi_{i,d,t,n,b}^{U,+} G_{i,b}^U + \sum_{i \in I^{FL},d,t,n,b} \phi_{i,d,t,n,b}^{D,+} G_{i,b}^D + \\
& \sum_{i \in I^{FL},d,t,n,b} \mu_{i,d,t,n,b}^U G_{i,n,b} + \sum_{i \in I^{MR},d,t,n} \gamma_{i,d,t,n} G_{i,n} + \sum_{i \in I^{RE},d,t,n} \theta_{i,d,t,n} k_{i,d,t}^{RE} G_{i,n} \quad (4.130)
\end{aligned}$$

However, it is noticeable that the presence of the nonlinearities posed by the last three bilinear terms of equation (4.130) needs to be further addressed. In order to remove these non-linearities, the strong duality theorem of the LL1 problem is utilised, which is expressed as follows:

$$\begin{aligned}
& \sum_{i,d,t,n,b} C_{i,b}^{EN} g_{i,d,t,n,b}^{DA} + \sum_{i,d,t,n,b} (C_{i,b}^U g_{i,d,t,n,b}^U - C_{i,b}^D g_{i,d,t,n,b}^D) = \sum_{j,d,t,o} \lambda_{j,d,t}^{DA} L_{j,d,t,o}^{DA} \\
& - \sum_{j,d,t,o} \eta^{Grid} \zeta_d L_{j,d,t,o}^{DA} - \sum_{i \in I^{FL},d,t,n,b} \mu_{i,d,t,n,b}^U G_{i,n,b} - \sum_{i \in I^{FL},d,t,n,b} \phi_{i,d,t,n,b}^{U,+} G_{i,b}^U \\
& - \sum_{i \in I^{FL},d,t,n,b} \phi_{i,d,t,n,b}^{D,+} G_{i,b}^D - \sum_{i \in I^{MR},d,t,n} \gamma_{i,d,t,n} G_{i,n} - \sum_{i \in I^{RE},d,t,n} \theta_{i,d,t,n} k_{i,d,t}^{RE} G_{i,n} \\
& - \sum_{d,t,n,m \in \Lambda_n} C_{nm}^{max} (\nu_{d,t,n,m}^{DA,-} + \nu_{d,t,n,m}^{DA,+}) - \sum_{d,t,n} \pi(\xi_{d,t,n}^{DA,-} + \xi_{d,t,n}^{DA,+}) \\
& - \sum_{d,t,n,m \in \Lambda_n} C_{nm}^{max} (\nu_{d,t,n,m}^{RT,-} + \nu_{d,t,n,m}^{RT,+}) - \sum_{d,t,n} \pi(\xi_{d,t,n}^{RT,-} + \xi_{d,t,n}^{RT,+}) \\
& + \sum_{j,d,t} [\Phi^{-1}(1 - \rho^L) s_{j,d,t}^L + m_{j,d,t}^L] \chi_{j,d,t}^L \quad (4.131)
\end{aligned}$$

It is observed that the last three nonlinear terms in equation (4.130) are also present within the strong duality equality (4.131). Given that the remaining terms in (4.131) are linear, this property facilitates the complete linearization of equation (4.130) by incorporating (4.131) into it. Hence, the linear representation for the first four bilinear terms in the MPEC's objective function is formulated as follows:

$$\begin{aligned}
& \sum_{i \in I,d,t,n,b} \lambda_{d,t,n}^{DA} g_{i,d,t,n,b}^{DA} + \sum_{i \in I^{FL},d,t,n,b} \lambda_{d,t,n}^{RT} (g_{i,d,t,n,b}^U - g_{i,d,t,n,b}^D) - \sum_{i \in I^{RE},d,t,n} \lambda_{d,t,n}^{RT} g_{i,d,t,n}^{dev} = \\
& \sum_{j,d,t,o} \lambda_{j,d,t}^{DA} L_{j,d,t,o}^{DA} - \sum_{d,t,n,m \in \Lambda_n} C_{nm}^{max} (\nu_{d,t,n,m}^{DA,-} + \nu_{d,t,n,m}^{DA,+}) - \sum_{d,t,n} \pi(\xi_{d,t,n}^{DA,-} + \xi_{d,t,n}^{DA,+}) \\
& - \sum_{d,t,n,m \in \Lambda_n} C_{nm}^{max} (\nu_{d,t,n,m}^{RT,-} + \nu_{d,t,n,m}^{RT,+}) - \sum_{d,t,n} \pi(\xi_{d,t,n}^{RT,-} + \xi_{d,t,n}^{RT,+}) \\
& + \sum_{j,d,t} [\Phi^{-1}(1 - \rho^L) s_{j,d,t}^L + m_{j,d,t}^L] \chi_{j,d,t}^L \quad (4.132)
\end{aligned}$$

The final step is to handle the last bilinear term related to the TGC revenue $\sum_{i \in I^{RE}} \lambda_{i,d}^{PGC} Q_{i,d}^{PGC}$. It can be converted into a quadratic term by adopting LL2 constraint (4.28),

which is presented as follows:

$$\sum_{i \in I^{RE}, d} \lambda_{i,d}^{PGC} Q_{i,d}^{PGC} = \sum_{i \in I^{RE}, d} [l^{PGC} Q_{i,d}^{PGC} - q^{PGC} (Q_{i,d}^{PGC})^2] \quad (4.133)$$

Ultimately, the substitution of expressions from (4.132) and (4.133) into the bilinear MPEC's objective function (4.63) leads to a comprehensive linearization of the GenCo's profit representation, which is reformed as follows:

$$\begin{aligned} & \sum_d \omega_d \left\{ - \sum_{i,t,n,b} C_{i,b}^{EN} g_{i,d,t,n,b}^{DA} - \sum_{i,t,n,b} (C_{i,b}^U g_{i,d,t,n,b}^U - C_{i,b}^D g_{i,d,t,n,b}^D) + \sum_{j,t,o} \lambda_{j,d,t}^{DA} L_{j,d,t,o}^{DA} \right. \\ & - \sum_{t,n,m \in \Lambda_n} C_{nm}^{max} (\nu_{d,t,n,m}^{DA,-} + \nu_{d,t,n,m}^{DA,+}) - \sum_{t,n} \pi (\xi_{d,t,n}^{DA,-} + \xi_{d,t,n}^{DA,+}) \\ & - \sum_{t,n,m \in \Lambda_n} C_{nm}^{max} (\nu_{d,t,n,m}^{RT,-} + \nu_{d,t,n,m}^{RT,+}) - \sum_{t,n} \pi (\xi_{d,t,n}^{RT,-} + \xi_{d,t,n}^{RT,+}) \\ & + \sum_{j,t} [\Phi^{-1}(1 - \rho^L) s_{j,d,t}^L + m_{j,d,t}^L] \chi_{j,d,t}^L - \sum_{i \in I^{FL}, t,n} \lambda^{CO_2} Q_{i,d,t,n}^{emi} + \sum_{i \in I^{RE}, t,n} \lambda^{CO_2} Q_{i,d,t,n}^{CA} \\ & \left. + \sum_{i \in I^{RE}} [l^{PGC} Q_{i,d}^{PGC} - q^{PGC} (Q_{i,d}^{PGC})^2 + \lambda^{FGC} Q_{i,d}^{FGC}] \right\} - \sum_{i,n} C_i^{IN} G_{i,n} \quad (4.134) \end{aligned}$$

As a result of this linearization process, the MPEC problem is simplified and reformulated into a MIQP, and its quadratic objective function is presented in (4.134).

4.4.4 Final MIQP

Through the process of linearization, the MPEC is ultimately transformed into a MIQP that can be solved efficiently. The MIQP's objective function (4.135) represents the GenCo's profit maximisation strategy within a linearized form that retains the essence of the original problem while being computationally solvable. The variable set for the MIQP (4.136) is the integration of the variable set of the MPEC problem (4.64) and the auxiliary binary variable set (4.72) introduced in the linearized LL1 KKT complementarity conditions. The constraints of the MIQP are comprised of three key components: i) original constraints from the UL problem (4.137), ii) linearized KKT optimality conditions of the LL1 problem (4.138), and iii) constraints from the LL2 problem (4.139). The final MIQP is provided as below:

$$\begin{aligned} \min_{V^{MIQP}} & \sum_d \omega_d \left\{ - \sum_{i,t,n,b} C_{i,b}^{EN} g_{i,d,t,n,b}^{DA} - \sum_{i,t,n,b} (C_{i,b}^U g_{i,d,t,n,b}^U - C_{i,b}^D g_{i,d,t,n,b}^D) + \sum_{j,t,o} \lambda_{j,d,t}^{DA} L_{j,d,t,o}^{DA} \right. \\ & - \sum_{t,n,m \in \Lambda_n} C_{nm}^{max} (\nu_{d,t,n,m}^{DA,-} + \nu_{d,t,n,m}^{DA,+}) - \sum_{t,n} \pi (\xi_{d,t,n}^{DA,-} + \xi_{d,t,n}^{DA,+}) \end{aligned}$$

$$\begin{aligned}
& - \sum_{t,n,m \in \Lambda_n} C_{nm}^{max} (\nu_{d,t,n,m}^{RT,-} + \nu_{d,t,n,m}^{RT,+}) - \sum_{t,n} \pi (\xi_{d,t,n}^{RT,-} + \xi_{d,t,n}^{RT,+}) \\
& + \sum_{j,t} [\Phi^{-1}(1 - \rho^L) s_{j,d,t}^L + m_{j,d,t}^L] \chi_{j,d,t}^L - \sum_{i \in I^{FL},t,n} \lambda^{CO_2} Q_{i,d,t,n}^{emi} \\
& + \sum_{i \in I^{RE},t,n} \lambda^{CO_2} Q_{i,d,t,n}^{CA} + \sum_{i \in I^{RE}} [I^{PGC} Q_{i,d}^{PGC} - q^{PGC} (Q_{i,d}^{PGC})^2 + \lambda^{FGC} Q_{i,d}^{FGC}] \Big\} \\
& - \sum_{i,n} C_i^{IN} G_{i,n}
\end{aligned} \tag{4.135}$$

where:

$$V^{MIQP} = V^{MPEC} \cup V^{BINARY} \tag{4.136}$$

subject to:

$$\text{UL constraints : (4.3) – (4.7)} \tag{4.137}$$

LL1 KKT conditions :

$$(4.10) – (4.11), (4.17) – (4.18), (4.21), (4.24), (4.37) – (4.45), (4.73) – (4.106) \tag{4.138}$$

$$\text{LL2 constraints : (4.28) – (4.29)} \tag{4.139}$$

4.5 Case Studies

4.5.1 Data Descriptions and Implementation

The case studies aim to investigate the impact of the deregulated market environment on the GenCo's optimal investment decisions and the corresponding business cases. To this end, the data from the GB power grid in 2021 is utilised to verify the proposed market-driven generation planning model. All necessary data are detailed below.

Table 4.1 presents comprehensive data about investment options for the GenCo. It is noteworthy that in terms of flexible generation technologies, the marginal cost for providing upward reserve is set at 1.1 times the marginal energy cost; conversely, the marginal cost for providing downward reserve is set at 0.9 times the marginal energy cost.

Table 4.2 displays crucial parameters for the 2021 GB power system. Additionally, the allocation of weights to these blocks is set as [0.1, 0.15, 0.2, 0.25, 0.3], and the demand is also accurately modelled by the five demand blocks, incorporating weighting factors as [0.6, 0.1, 0.1, 0.1, 0.1]. By virtue of computational burden, the demand profile is simulated on a representative day with an hourly resolution ($T = \{1, 2, \dots, 24\}$), scaled

Table 4.1: Parameters of investment options.

Investment option		CCGT1	CCGT2	OCGT1	OCGT2	Nuclear	Wind
Type		Flexible	Flexible	Flexible	Flexible	Must-run	Renewable
Investment cost C^{IN} (£/MW/yr)		14413	17440	8610	10418	114849	35756
Carbon intensity η^{CO_2} (t/MWh)		0.39	0.39	0.65	0.65	0	0
Maximum upward reserve capacity G^U (MW)		4000	8000	4000	8000	-	-
Maximum downward reserve capacity G^D (MW)		4000	8000	4000	8000	-	-
Marginal energy cost C^{EN} (£/MWh)	Block 1	7.43	20.05	40.99	49.25	4.72	0
	Block 2	9.03	24.34	49.78	59.81		
	Block 3	10.62	28.64	58.56	70.36		
	Block 4	12.21	32.94	67.34	80.91		
	Block 5	13.81	37.23	76.13	91.47		

across a target year ($\omega = 365$) to capture daily fluctuations in energy consumption and production patterns.

Table 4.2: Parameters of the GB power grid in 2021.

Year	2021
System-wide carbon intensity η^{Grid} (t/MWh)	183.11
Carbon price λ^{CO_2} (£/t)	71.54
Total system demand (TWh)	248.30

The additional parameters associated with carbon allowance and TGC are essential. Specifically, the carbon allowance allocation ratio, ϵ_i , is set as 0.4; the RPS requirement ratio, γ^{RPS} , is set as 40%; the approximated price ratio in the future TGC market, γ^{FGC} , is set as 0.3; and the price coefficient in the present TGC market, θ^{PGC} , is also set as 0.3. It should be noted that the penalty price for RPS deficiency, λ_0^{RPS} , is assumed to be 1.1 times the carbon price.

The related coefficients used in the chance constraints should be given. Overall, the model sets confidence levels for all the chance constraints at 95% ($1 - \rho^L = 1 - \rho^{RE} = 95\%$), conveying the ρ -reliable threshold for both demand and wind power production uncertainty. The linearized CCP formulations incorporate stochastic variables for wind power and demand realisations, which are modelled using two normal distributions, $\mathcal{N}(m^{RE}, (s^{RE})^2)$ for wind power and $\mathcal{N}(m^L, (s^L)^2)$ for demand. The means of these distributions (m^L and m^{RE}) are aligned with the baseline demand and DA power dispatch, respectively. Meanwhile, the standard deviations (s^L and s^{RE}), set at 1% of the baseline demand and 10% of the DA wind power dispatch, respectively, quantify the anticipated variability around these means.

In order to evaluate the value of the market-driven model, the system-driven model is employed as a benchmark. Therefore, two market design cases are established for the

quantitative analysis of their business cases:

1. **System-driven case:** The investment decisions and business results are obtained through the system-driven generation planning model, aligning with the description in Chapter 3. This case constitutes the benchmark.
2. **Market-driven case:** The investment decisions and business results are derived from the market-driven generation planning model.

4.5.2 Market Outcomes and Business Cases

Table 4.3: Investment decisions and business results for two market design scenarios regarding the 2021 GB power system.

	System-driven case	Market-driven case
CCGT1 (MW)	14,184	1,331
CCGT2 (MW)	-	-
OCGT1 (MW)	-	-
OCGT2 (MW)	-	13,050
Total flexible capacity (MW)	14,184	14,381
Nuclear (MW)	20,172	15,749
Wind (MW)	3,562	17,243
Investment cost (mil.£)	2,631	2,494
DA energy cost (mil.£)	1,497	5,055
DA energy revenue (mil.£)	2,970	19,835
RT reserve cost (mil.£)	47	177
RT reserve revenue (mil.£)	72	982
Wind power deviation penalty (mil.£)	20	637
Carbon allowance revenue (mil.£)	43	209
Green certificate revenue (mil.£)	552	2,044
Carbon emission cost (mil.£)	1,902	3,269
GenCo Profit (mil.£)	-2,460	11,438

Observably, the comprehensive investment decisions and business results regarding two market design scenarios are listed in Table 4.3. Furthermore, the DA generation dispatches and RT imbalances of the market-driven scenario are illustrated in Figures 4.2 and 4.3, respectively. The DA and RT price profiles for the market-driven scenario are shown in Figure 4.4.

Analysis of Investment Decisions

The optimal investment decisions derived from the market-driven model are presented in the third column of Table 4.3, from rows two to eight. Similar to the system-driven

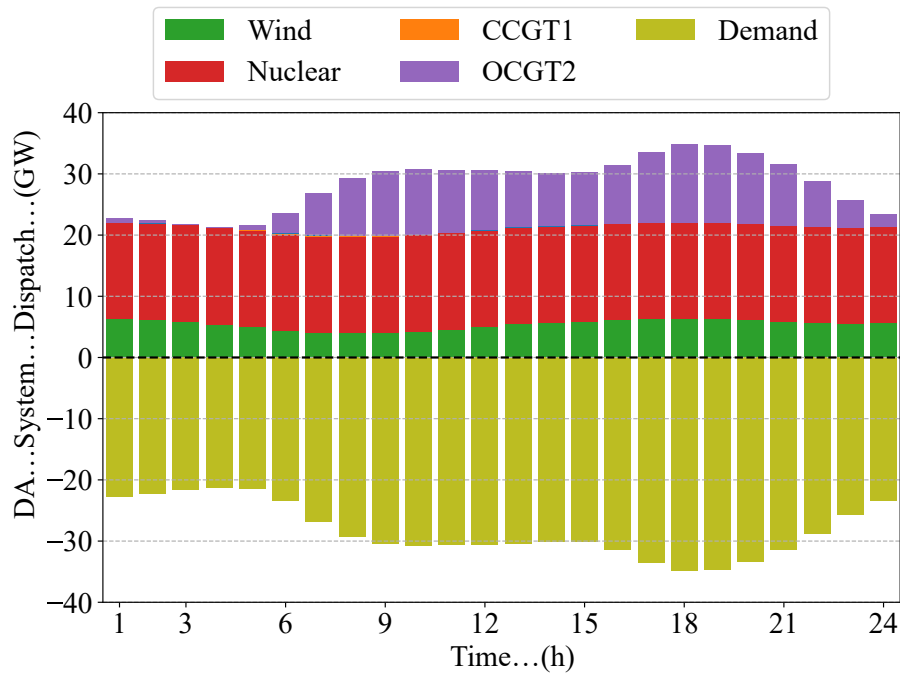


Figure 4.2: Daily generation dispatches in the DA energy market for the market-driven scenario regarding the 2021 GB power system.

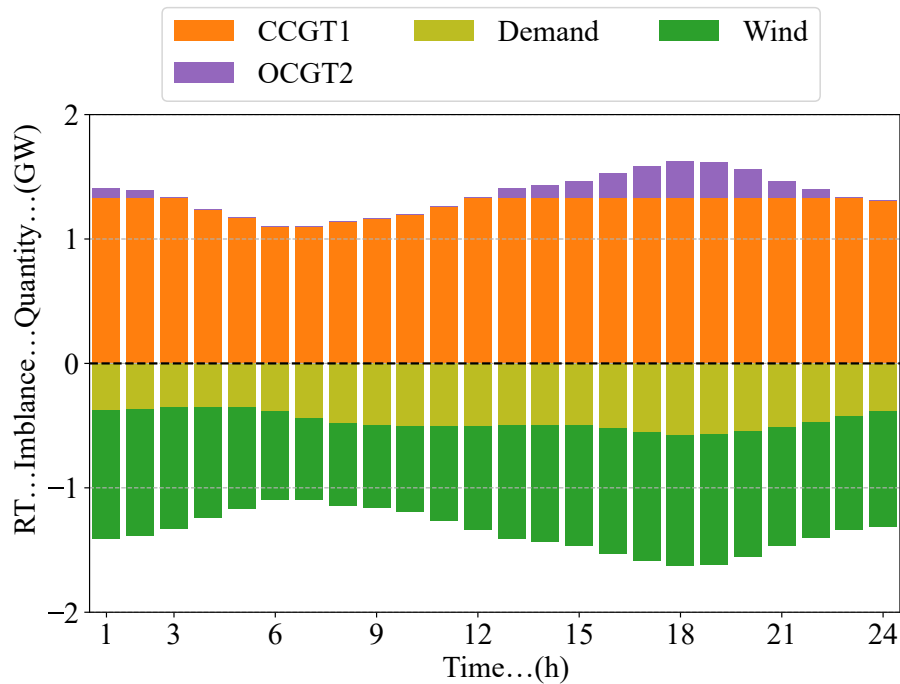


Figure 4.3: Daily imbalances in the RT balancing market for the market-driven scenario regarding the 2021 GB power system.

case, due to the 2021 GB carbon emission restrictions, zero-emission generation technologies, namely nuclear and wind power, remain highly popular. Moreover, their total

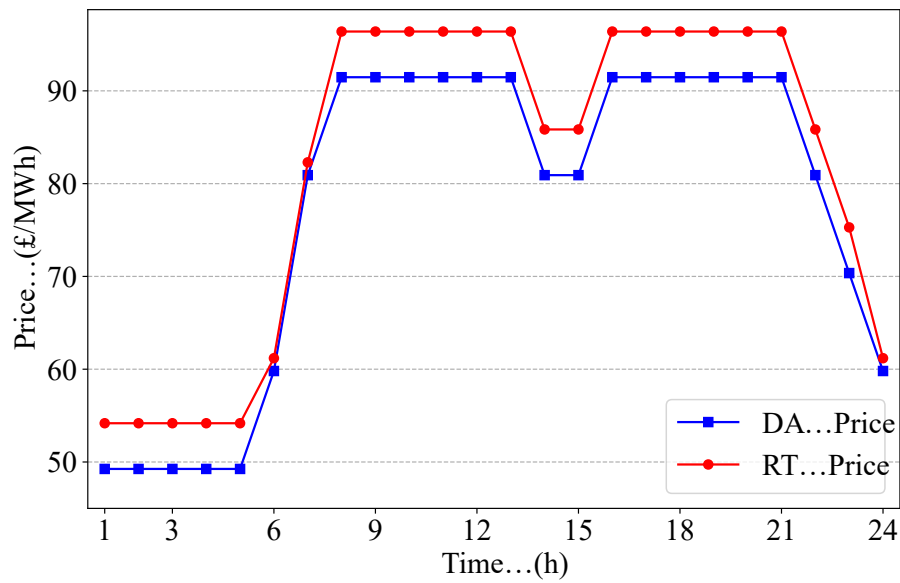


Figure 4.4: Daily DA and RT market clearing prices for market-driven scenario in the 2021 GB power system.

generation capacity is still constrained by the minimum system demand level due to their 'must-run' operational characteristics, as illustrated in Figure 4.2. However, a difference is observed in the market-driven case, where the investment in wind power capacity significantly exceeds that in nuclear power capacity. This is because, compared to the system-driven model, the market-driven model captures the implicit economic value of wind power, thereby integrating more wind power capacity. Specifically, the uncertainty caused by wind power leads to an increased reserve requirement, subsequently raising the overall system cost, which contradicts the objective pursued in the system-driven case. Conversely, in the market-driven case, the increased reserve requirement introduced by wind power has the potential to increase revenue from reserve provision, thereby enhancing the GenCo's profit.

The total flexible generation capacity invested in the market-driven case is slightly higher than in the system-driven case. This is because, in addition to compensating for the peak-valley difference of system demand in the DA market, flexible generation capacity is also used to meet the increased reserve need in the RT market, caused by the integration of large-scale wind generation capacity, as shown in Figure 4.3. Unlike the system-driven case, in the market-driven case, the flexible generation capacity comprises a large proportion of OCGT2 capacity (91%) and a small proportion of CCGT1 capacity (9%). This is due to OCGT2's high marginal energy cost, which can raise clearing prices in both the DA and RT markets, thereby significantly increasing profitability. Specifically, a large high-marginal-cost OCGT2 capacity is strategically scheduled in the DA market to fill the peak-valley gap in system demand, followed by deployment as a provider of

marginal reserve capacity in the RT market, thus raising clearing prices in both the DA and RT markets, as seen in Figure 4.4. Furthermore, in order to reduce the reserve cost, the cheap CCGT1 capacity is employed in the RT market to provide reserve.

Analysis of Business Cases

The business results of the market-driven case are fully displayed in the third column of Table 4.3, from rows nine to eighteen. Compared to the system-driven case, investment costs in the market-driven case are reduced due to the substitution of a large amount of expensive nuclear power capacity with wind power capacity. In the DA market, despite an increase in energy costs, energy revenue surges, leading to a significant increase in profit sourced from the DA market. A similar trend is observed in the RT market. This is because, under a market-based framework, the clearing prices in both DA and RT markets are strategically increased to align with the objective of maximising profit. A substantial capacity of high-polluting OCGT2 is invested in to raise market clearing prices; thus, carbon emission costs surge but comply with the 2021 GB carbon emission limits. As the market-driven case integrates more wind energy, the wind power deviation penalty significantly rises; concurrently, carbon allowance revenue and TGC revenue also increase. Overall, since the market-driven case aims at maximising profits, although many components of the system cost, such as DA energy cost, RT reserve cost, wind power deviation penalty, and carbon emission cost, are notably increased, the revenue from various sources, including DA energy revenue, RT reserve revenue, carbon allowance revenue, and TGC revenue, surges even more. Consequently, GenCo earns a substantial profit, thereby avoiding the risk of cost recovery present in the system-driven case.

4.6 Conclusions

This chapter focuses on the role of the deregulated energy market in generation planning under carbon emission restrictions. To this end, a two-stage bi-level optimisation model is developed to quantitatively analyse the investment decisions and business results of the GenCo. It is composed of the UL problem, which identifies the self-interested GenCo, and two LL problems. The first LL (LL1) problem represents the joint clearing process of the DA and RT markets, accompanied by the chance-constrained uncertainty and environmental regulation derived from the GB's carbon targets. The second LL (LL2) problem represents the clearing process of the TGC market. After the linearization of chance constraints, the proposed model is first converted into the MPEC problem by KKT conditions and then solved by recasting it as a MIQP with the strong duality theorem. The

proposed model is not only a powerful tool for investigating the investment strategy of the GenCo, but also a valuable reference for market design from the perspective of policy-makers. The case studies have been conducted in the context of the 2021 GB power system, and the findings are detailed as follows:

1. Compared with the system-driven model, the optimal investment mix obtained from the proposed market-driven model shows an increase in the share of wind power capacity from 9% to 28%. This increase indicates that the deregulated energy market significantly promotes wind energy integration, aligning with the GB government's plan to expand the proportion of renewable energy in the grid.
2. There is a significant difference in the investment preferences for flexible generation capacity between the system-driven and market-driven models. In the optimal investment decisions of the system-driven model, flexible generation capacity entirely comes from the low-cost CCGT1 technology. In contrast, in the market-driven model, high-cost OCGT2 generation capacity accounts for 91% of the total flexible generation capacity. This is because, in the market-driven model, high-marginal-cost OCGT2 capacity is strategically deployed in both DA and RT markets as marginal capacity, ensuring high clearing prices in the DA and RT markets.
3. Compared to the negative profit in the system-driven model, the profit obtained from the market-driven model increases to £11,438 million, ensuring sufficient profitability for GenCo and thus avoiding the risk of cost non-recovery.

Chapter 5

Impact of Demand Flexibility

5.1 Introduction

Demand flexibility has become an important concept in the modern power system, especially within the context of the deregulated energy market. Demand flexibility refers to the capacity of demands to adjust their consumption patterns in response to external signals, such as price changes and economic incentives [70]. This adjustment can range from reducing overall consumption during peak demand periods to shifting demand-side electricity use to periods with abundant renewable energy. In the deregulated market, demand flexibility emerges as a vital element in reducing the need for expensive peaking power plants and flattening the demand profile, thereby facilitating a more cost-effective and resilient generation landscape to maintain demand-supply balancing. In practice, demand flexibility in the GB power system is facilitated through the Demand Flexibility Service (DFS), operated by NG ESO from 2022, to manage near 350 MW of flexible demand [62].

The model of time-coupling demand flexibility proposed in this chapter is embedded in the LL1 problem, participating in the joint clearing process of DA energy and RT balancing markets. Specifically, it spans the time scales of both the DA and RT markets. It possesses the capability to modify the demand pattern in the DA market by a predetermined proportion, thereby allowing for adjustments in anticipation of market dynamics. Additionally, it influences demand deviations in the RT market, providing a mechanism for strategic utilisation of the discrepancies between forecasted and actual demand. Furthermore, this time-coupling demand flexibility embodies the operational characteristic of maintaining the net-zero energy shift during a representative day.

The market-driven generation planning model proposed in this chapter focuses on factoring in the impact of demand flexibility in a deregulated energy market under the strin-

gent carbon emission limits imposed by the GB's net-zero target. To this end, a series of case studies with different levels of demand flexibility are designed. The findings clearly demonstrate that demand flexibility plays a dual role in the context of market-driven generation planning. Firstly, it enhances GenCo's profitability. By allowing for more strategic and flexible electricity use to respond to market signals, demand flexibility can significantly improve the economic performance of GenCo. Secondly, demand flexibility facilitates the integration of higher levels of renewable power capacity into the system. By smoothing out the variability in demand to match the intermittent supply from RESs, demand flexibility supports a higher penetration of renewables, contributing to the decarbonisation of the power sector.

5.2 Problem Statement

This section outlines the decision-making framework for the market-driven generation planning, taking into account demand flexibility, and details the foundational assumptions for its modelling.

5.2.1 Structure of the Bi-Level Optimisation Model

This chapter introduces a market-driven generation planning model using a two-stage, bi-level optimisation framework to determine the role of demand flexibility in the current market environment regulated by the carbon emission constraints. In brief, the proposed bi-level optimisation framework captures the coupled relationship between GenCo and market operators: the UL problem represents GenCo pursuing profit maximisation, the first LL (LL1) problem represents the joint clearing process of the DA and RT markets, and the second LL (LL2) problem represents the clearing of the green certificate market. This hierarchical structure is shown in Figure 5.1. Different from previously proposed market models, the LL1 problem of the model introduced in this chapter accommodates time-coupling demand flexibility. Specifically, this demand flexibility spans the horizons of the DA and RT markets and is capable of altering the demand pattern in the DA market by a fixed proportion, as well as influencing the demand deviations in the RT market. Furthermore, this time-coupling demand flexibility is energy-natural in a typical day.

The interdependence between the UL problem and the two LL problems is pronounced. The investment decisions made by GenCo at the UL significantly affect the market outcomes realised within the two LL problems. In turn, the business cases emerging from the LL1 problem, characterised by energy prices and generation dispatches in both the DA and RT markets, along with the TGC quantities and prices obtained in the LL2 prob-

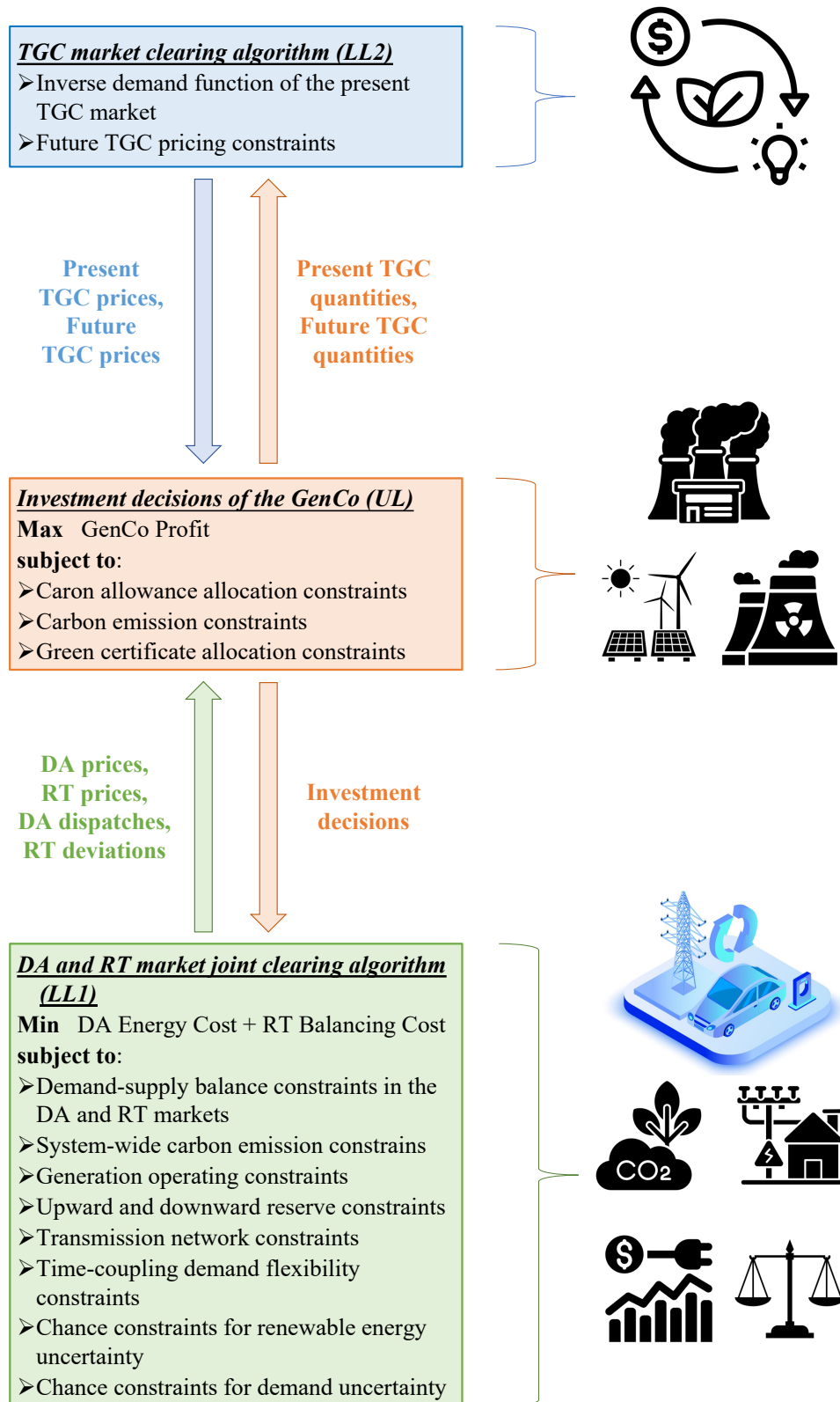


Figure 5.1: Illustration of the proposed two-stage bi-level optimisation model.

lem, critically influence GenCo's profitability. This bidirectional feedback loop underscores the complex relationship between generation planning and market performance, highlighting the feature of capturing real-world market dynamics and GenCo's decision-making processes.

5.2.2 Modelling Assumptions

For clarity and completeness, the key assumptions underpinning the proposed two-stage bi-level optimisation model are listed below:

1. The proposed two-stage bi-level optimisation model represents the market-driven generation planning faced by a self-interested GenCo seeking to maximise its long-term profit. The GenCo is allowed to invest in generation capacities utilising flexible, must-run, and renewable technologies. Furthermore, the flexible generation capacities are explicitly modelled by the production blocks to capture their output characteristics. Accordingly, demand blocks are also incorporated into the proposed model.
2. The examined GenCo is permitted to participate in four different markets. The two-stage energy markets adopted for this work include the DA energy market and the RT balancing market with an hourly time resolution. Furthermore, the proposed model takes into consideration the present TGC market and the future TGC market.
3. Based on the study in [76], its pricing scheme, namely LMP, is applied within the proposed two-stage energy market framework. This approach leverages the LMPs for determining the electricity prices at different locations in the network, which are calculated using the dual variables of the hourly demand-supply balance in both the DA and RT markets. The so-called LMP scheme reflects the marginal cost of supplying the next unit of electricity at a specific location, considering the effect of network transmission congestion, thereby facilitating efficient market operations and signaling appropriate economic incentives.
4. The model incorporates the pricing mechanism outlined in [64] for trading in the present and future TGC markets. Specifically, the prices in the present TGC market are determined using an inverse demand function. Furthermore, the future TGC price is estimated based on the penalty price associated with RPS deficiency.
5. Once the GenCo finalises its investment decisions, the sequential clearing process of the DA, RT, and TGC markets is arranged by the market operator. Specifically,

the market operator first clears the DA energy market, taking into account the uncertainties incurred by demand and renewable generation as well as scheduling the generation dispatches. Subsequently, the depolyment of reserves is organised by market operator in the RT market to eliminate the unexpected uncertainties arising from wind power and demand-side deviations. Following this, the prices and quantities of TGC are determined based on the clearing results of the DA and RT markets. By comparing the present TGC market clearing price with the estimated future TGC market price, the examined GenCo can choose either to trade its TGCs in the present TGC market or retain them for future trading, thus completing the transactions in the TGC market. Finally, the outcomes of these three markets are relayed to the examined GenCo, significantly influencing its profit-oriented investment decision-making process.

6. Although the DA and RT markets are cleared sequentially from the perspective of the examined GenCo, investment decisions are required to maximise the overall expected profit simultaneously [77]. Additionally, as highlighted in research works [78, 79], a co-optimised approach to managing energy and reserves significantly enhances the efficiency of generation dispatch compared to scenarios where energy and reserves are cleared separately. Therefore, this paper adopts a joint clearing model for the DA and RT markets. Furthermore, it is assumed that GenCo sells the TGCs in the present TGC market or withholds the TGCs for future trading after the outcomes of the energy markets are obtained. This assumption is logical since the allocation of TGCs becomes definitive after the results of renewable generation dispatches and deviations are available, which is more conducive to risk mitigation [64].
7. In order to facilitate the mathematical solution, the uncertainties sourced from RESs and demand are realised by a set of chance constraints.
8. As the primary objective of this paper is to explore the system-level generation mix of the future grid under stringent carbon emission regulations, the proposed generation planning model excludes the consideration of other GenCos.
9. In order to ensure the convex characteristics of the LL1 problem, linear generation costs are adopted in the proposed model. Additionally, it is assumed that renewable generation technologies are cost-free.
10. In the DA market, the generation capacities of renewable and must-run technologies are assumed to operate at their full capacity.

11. The network transmission is modelled based on a linear DC power flow approximation.
12. In the face of decarbonisation, a series of carbon emission restrictions and green energy incentives are imposed on the examined GenCo. Firstly, GenCo is limited by an annual cap on carbon emissions. Moreover, it is assumed that carbon emissions only depend on the generation of flexible technologies that consume fossil fuels. Secondly, carbon emissions from the GenCo are subject to a carbon tax. Thirdly, carbon allowances are allocated to the GenCo based on its renewable energy generation, and the GenCo can profit by trading these carbon allowances. Fourthly, GenCo can earn TGCs through renewable power production and participate in the present and future TGC markets to gain profits. Finally, the RPS ratio is incorporated into the parameter setting of TGC markets. These environmental policies are set to work together towards a fully decarbonised power system.
13. Time-coupling demand flexibility is implemented through a generic and technology-agnostic model. The demand flexibility model permits adjustments in demand, either decreasing or increasing, within a specified proportion for each time interval, while maintaining a net-zero demand-side energy shift throughout the market duration [80]. Essentially, the sum of demand reductions matches the sum of demand increases, ensuring energy neutrality over the market horizon. It is assumed that demand flexibility does not compromise the quality or satisfaction of consumer experiences.

5.3 Mathematical Model

This section elaborates on the specific formulations of the UL problem related to the GenCo and the two LL problems (LL1 and LL2), which correspond to various market clearing processes.

5.3.1 UL Problem: Investment Decisions of GenCo

The UL problem within this framework captures the decision-making process of the profit-driven GenCo, pursuing to maximise its total profit by determining the optimal investment mix, taking into account the multi-scale market dynamics. This UL problem

is mathematically formulated as follows:

$$\begin{aligned}
\max_{V^{UL}} \sum_d \omega_d & \left[\underbrace{\sum_{i \in \Psi_{n,t,n,b}} \lambda_{d,t,n}^{DA} g_{i,d,t,n,b}^{DA}}_{\text{DA energy revenue}} - \underbrace{\sum_{i,t,n,b} C_{i,b}^{EN} g_{i,d,t,n,b}^{DA}}_{\text{DA energy cost}} \right. \\
& + \underbrace{\sum_{i \in I^{FL} \cap \Psi_{n,t,n,b}} \lambda_{d,t,n}^{RT} (g_{i,d,t,n,b}^U - g_{i,d,t,n,b}^D)}_{\text{RT reserve revenue}} - \underbrace{\sum_{i \in I^{FL},t,n,b} (C_{i,b}^U g_{i,d,t,n,b}^U - C_{i,b}^D g_{i,d,t,n,b}^D)}_{\text{RT reserve cost}} \\
& - \underbrace{\sum_{i \in I^{RE} \cap \Psi_{n,t,n}} \lambda_{d,t,n}^{RT} g_{i,d,t,n}^{dev}}_{\text{Renewable deviation penalty}} - \underbrace{\sum_{i \in I^{FL},t,n} \lambda^{CO_2} Q_{i,d,t,n}^{emi}}_{\text{Carbon emission cost}} + \underbrace{\sum_{i \in I^{RE},t,n} \lambda^{CO_2} Q_{i,d,t,n}^{CA}}_{\text{Carbon allowance revenue}} \\
& + \underbrace{\sum_{i \in I^{RE}} (\lambda_{i,d}^{PGC} Q_{i,d}^{PGC} + \lambda^{FGC} Q_{i,d}^{FGC})}_{\text{Green certificate revenue}} \left. \right] - \underbrace{\sum_{i,n} C_i^{IN} G_{i,n}}_{\text{Investment cost}} \quad (5.1)
\end{aligned}$$

where:

$$V^{UL} = \{G_{i,n}, Q_{i,d,t,n}^{CA}, Q_{i,d,t,n}^{emi}, Q_{i,d}^{PGC}, Q_{i,d}^{FGC}\} \quad (5.2)$$

subject to:

$$G_{i,n} \geq 0 \quad \forall i \in I, \forall n \quad (5.3)$$

$$G_{i,n,b} = \omega_b^{FL} G_{i,n} \quad \forall i \in I^{FL}, \forall n, \forall b \quad (5.4)$$

$$Q_{i,d,t,n}^{CA} = (g_{i,d,t,n}^{DA} - g_{i,d,t,n}^{dev}) \eta^{Grid} \epsilon_i \quad \forall i \in I^{RE}, \forall d, \forall t, \forall n \quad (5.5)$$

$$Q_{i,d,t,n}^{emi} = (g_{i,d,t,n}^{DA} + g_{i,d,t,n}^U - g_{i,d,t,n}^D) \eta_i^{CO_2} \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n \quad (5.6)$$

$$Q_{i,d}^{PGC} + Q_{i,d}^{FGC} = \sum_{t,n} (g_{i,d,t,n}^{DA} - g_{i,d,t,n}^{dev}) \quad \forall i \in I^{RE}, \forall d \quad (5.7)$$

The objective function of the UL problem is to maximise the total profit of the examined GenCo, consisting of nine components, as defined in (5.1). Constraint (5.3) ensures that the new capacities invested by the GenCo are non-negative. Constraint (5.4) details the weighting factors of the production blocks corresponding to flexible generation capacities. Constraint (5.5) presents the allocation of carbon allowances. Constraint (5.6) quantifies the carbon emissions produced by the operation of fossil-fired flexible generation technologies. Lastly, constraint (5.7) calculates the total quantities of TGCs that are generated based on renewable energy production in the DA market and its deviation in the RT market.

5.3.2 LL1 Problem: Joint Clearing Process of DA Energy and RT Balancing Markets

From the viewpoint of the market operator, the LL1 problem represents that the joint clearing of the two-stage electricity markets, specifically the DA energy market and the RT balancing market, is undertaken with the objective of minimising the overall system cost. It needs to be emphasised that the focus of this chapter, demand flexibility, is accurately incorporated into the LL1 constraints. The formulation of this LL1 problem can be expressed as follows:

$$\left\{ \min_{V^{LL}} \sum_{i,t,n,b} C_{i,b}^{EN} g_{i,d,t,n,b}^{DA} + \sum_{i,t,n,b} (C_{i,b}^U g_{i,d,t,n,b}^U - C_{i,b}^D g_{i,d,t,n,b}^D) \right. \quad (5.8)$$

where:

$$V^{LL1} = \{ P_{j,d,t,o}^{DA}, g_{i,d,t,n,b}^{DA}, g_{i,d,t,n,b}^U, g_{i,d,t,n,b}^D, P_{j,d,t,o}^{dev}, g_{i,d,t,n}^{dev}, \delta_{d,t,n}^{DA}, \delta_{d,t}^{DA,ref}, \delta_{d,t,n}^{RT}, \delta_{d,t}^{RT,ref}, L_{j,d,t,o}^{SH} \} \quad (5.9)$$

subject to:

$$\sum_{j \in \Psi_{n,o}} P_{j,d,t,o}^{DA} + \sum_{m \in \Lambda_n} b_{nm} (\delta_{d,t,n}^{DA} - \delta_{d,t,m}^{DA}) - \sum_{i \in \Psi_{n,b}} g_{i,d,t,n,b}^{DA} = 0 \quad : \lambda_{d,t,n}^{DA} \quad \forall t, \forall n \quad (5.10)$$

$$\begin{aligned} & \sum_{j \in \Psi_{n,o}} P_{j,d,t,o}^{dev} + \sum_{i \in I^{FL} \cap \Psi_{n,b}} (-g_{i,d,t,n,b}^U + g_{i,d,t,n,b}^D) + \sum_{i \in I^{RE} \cap \Psi_n} g_{i,d,t,n}^{dev} - \\ & \sum_{m \in \Lambda_n} b_{nm} (\delta_{d,t,n}^{DA} - \delta_{d,t,n}^{RT} - \delta_{d,t,m}^{DA} + \delta_{d,t,m}^{RT}) = 0 \quad : \lambda_{d,t,n}^{RT} \quad \forall t, \forall n \end{aligned} \quad (5.11)$$

$$\sum_{i \in I^{FL}, t, n, b} \eta_i^{CO_2} (g_{i,d,t,n,b}^{DA} + g_{i,d,t,n,b}^U - g_{i,d,t,n,b}^D) \leq \eta^{Grid} \sum_{j,t,o} L_{j,d,t,o}^{DA} \quad : \zeta_d \quad (5.12)$$

$$g_{i,d,t,n,b}^{DA} + g_{i,d,t,n,b}^U \leq G_{i,n,b} \quad : \mu_{i,d,t,n,b}^U \quad \forall i \in I^{FL}, \forall t, \forall n, \forall b \quad (5.13)$$

$$g_{i,d,t,n,b}^D \leq g_{i,d,t,n,b}^{DA} \quad : \mu_{i,d,t,n,b}^D \quad \forall i \in I^{FL}, \forall t, \forall n, \forall b \quad (5.14)$$

$$0 \leq g_{i,d,t,n,b}^U \leq G_{i,b}^U \quad : \phi_{i,d,t,n,b}^{U,-}, \phi_{i,d,t,n,b}^{U,+} \quad \forall i \in I^{FL}, \forall t, \forall n, \forall b \quad (5.15)$$

$$0 \leq g_{i,d,t,n,b}^D \leq G_{i,b}^D \quad : \phi_{i,d,t,n,b}^{D,-}, \phi_{i,d,t,n,b}^{D,+} \quad \forall i \in I^{FL}, \forall t, \forall n, \forall b \quad (5.16)$$

$$g_{i,d,t,n}^{DA} = G_{i,n} \quad : \gamma_{i,d,t,n} \quad \forall i \in I^{MR}, \forall t, \forall n \quad (5.17)$$

$$g_{i,d,t,n}^{DA} = k_{i,d,t}^{RE} G_{i,n} \quad : \theta_{i,d,t,n} \quad \forall i \in I^{RE}, \forall t, \forall n \quad (5.18)$$

$$-C_{nm}^{max} \leq b_{nm}(\delta_{d,t,n}^{DA} - \delta_{d,t,m}^{DA}) \leq C_{nm}^{max} : \nu_{d,t,n,m}^{DA,-}, \nu_{d,t,n,m}^{DA,+} \quad \forall t, \forall n, \forall m \in \Lambda_n \quad (5.19)$$

$$-\pi \leq \delta_{d,t,n}^{DA} \leq \pi : \xi_{d,t,n}^{DA,-}, \xi_{d,t,n}^{DA,+} \quad \forall t, \forall n \quad (5.20)$$

$$\delta_{d,t}^{DA,ref} = 0 : \xi_{d,t}^{DA,ref} \quad \forall t \quad (5.21)$$

$$-C_{nm}^{max} \leq b_{nm}(\delta_{d,t,n}^{RT} - \delta_{d,t,m}^{RT}) \leq C_{nm}^{max} : \nu_{d,t,n,m}^{RT,-}, \nu_{d,t,n,m}^{RT,+} \quad \forall t, \forall n, \forall m \in \Lambda_n \quad (5.22)$$

$$-\pi \leq \delta_{d,t,n}^{RT} \leq \pi : \xi_{d,t,n}^{RT,-}, \xi_{d,t,n}^{RT,+} \quad \forall t, \forall n \quad (5.23)$$

$$\delta_{d,t}^{RT,ref} = 0 : \xi_{d,t}^{RT,ref} \quad \forall t \quad (5.24)$$

$$(1 - \alpha) L_{j,d,t,o}^{DA} \leq P_{j,d,t,o}^{DA} \leq (1 + \alpha) L_{j,d,t,o}^{DA} : \psi_{j,d,t,o}^{-}, \psi_{j,d,t,o}^{+} \quad \forall j, \forall t, \forall o \quad (5.25)$$

$$\sum_{t,o} L_{j,d,t,o}^{SH} = 0 : \sigma_{j,d} \quad \forall j \quad (5.26)$$

$$\mathbb{P} \left[-\alpha L_{j,d,t}^R \leq \sum_o L_{j,d,t,o}^{SH} \leq \alpha L_{j,d,t}^R \right] \geq 1 - \rho^L : \chi_{j,d,t}^{SH,-}, \chi_{j,d,t}^{SH,+} \quad \forall j, \forall t \quad (5.27)$$

$$\mathbb{P} \left[g_{i,d,t,n}^{DA} - g_{i,d,t,n}^{dev} \leq g_{i,d,t,n}^R \right] \geq 1 - \rho^{RE} : \chi_{i,d,t,n}^{RE} \quad \forall i \in I^{RE}, \forall t, \forall n \quad (5.28)$$

$$\mathbb{P} \left[\sum_o P_{j,d,t,o}^{DA} + \sum_o P_{j,d,t,o}^{dev} \geq L_{j,d,t}^R + \sum_o L_{j,d,t,o}^{SH} \right] \geq 1 - \rho^L : \chi_{j,d,t}^L \quad \forall j, \forall t \quad \forall d \quad (5.29)$$

The LL1 problem's objective function is to minimise the overall system cost, including the energy cost (first term) and the reserve cost (last two terms). The LL1 problem is subject to various sets of constraints: system balance constraints (5.10)-(5.11), system-wide carbon emission constraint (5.12), generation operating constraints (5.13)-(5.18), network constraints (5.19)-(5.24), demand flexibility constraints (5.25)-(5.26), and chance constraints (5.27)-(5.29).

Due to the incorporation of flexible demand, some new features should be clarified. Specifically, the decision variable set V^{LL1} includes the new variables $P_{j,d,t,o}^{DA}$ and $L_{j,d,t,o}^{SH}$, which are employed to model flexible demand and demand shifting, respectively. Additionally, the demand consumption in the DA demand-supply constraint (5.10) is replaced by the variable $P_{j,d,t,o}^{DA}$, instead of parametric baseline demand $L_{j,d,t,o}^{DA}$. Furthermore, the constraints (5.25)-(5.27) are proposed to accurately reflect the impact of demand flexibility. Constraint (5.25) indicates that the DA demand consumption, relative to the baseline demand for each hour, is confined within a predetermined ratio α ($0 \leq \alpha \leq 1$). Constraint (5.26) ensures that the time-coupling demand shifting results in energy neutrality over the course of a single representative day. Chance constraint (5.27) conveys that the hourly shifting of baseline demand is confined to a specified proportion, α , of the baseline

demand realisation, with a probability exceeding $1 - \rho^L$.

5.3.3 LL2 Problem: Clearing Process of TGC Market

The clearing process for the present TGC market and the projections for the future TGC market are formulated as follows:

$$V^{LL2} = \{\lambda_{i,d}^{PGC}, Q_{i,d}^{PGC}\} \quad (5.30)$$

$$\lambda_{i,d}^{PGC} = l^{PGC} - q^{PGC} Q_{i,d}^{PGC} \quad \forall i \in I^{RE}, \forall d \quad (5.31)$$

$$\lambda^{FGC} = \gamma^{FGC} \lambda_0^{RPS} \quad (5.32)$$

The present TGC market is built on a quantity-dependent inverse demand function. Under this framework, the present TGC price is determined by equation (5.31) that represents the linear correlation between the present price of TGC and its quantity. On the other hand, equation (5.32) indicates the estimation of the future TGC price. The necessary coefficients are obtained from equations (5.33) and (5.34), as proved in [64].

$$l^{PGC} = \lambda_0^{RPS} \quad (5.33)$$

$$q^{PGC} = \frac{(1 - \theta^{PGC}) \lambda_0^{RPS}}{\gamma^{RPS} \sum_{j,d,t} L_{j,d,t}^{DA}} \quad (5.34)$$

5.4 Solution

The methodology for solving the proposed two-stage bi-level optimisation model unfolds in a structured sequence of steps, detailed in this section. Initially, the challenge of non-convex chance constraints in the LL1 problem is addressed by applying linearization techniques, a process described in Subsection 5.4.1. After this, the model undergoes a transformation into a single-level MPEC and its comprehensive procedure is detailed in Subsection 5.4.2. This transformation employs the KKT optimality conditions to effectively substitute the LL1 problem, while concurrently incorporating the constraints from the LL2 problem into the UL problem. To enhance the computational tractability, the next step involves tackling the non-linear components appearing in the MPEC formulation, as discussed in Subsection 5.4.3. Finally, the model is converted into a MIQP problem, as depicted in Subsection 5.4.4.

5.4.1 CCP Reformulation

In this chapter, the chance constraints employ stochastic variables to address uncertainties associated with RESs and demand, denoted as g^R and L^R respectively. These stochastic variables are assumed to follow normal distributions, which are formulated as follows:

$$g_{i,d,t,n}^R \sim \mathcal{N}(m_{i,d,t,n}^{RE}, (s_{i,d,t,n}^{RE})^2), \quad L_{j,d,t}^R \sim \mathcal{N}(m_{j,d,t}^L, (s_{j,d,t}^L)^2) \quad (5.35)$$

where parameter m is the mean of the normal distribution and parameter s is the standard deviation of the normal distribution. It should be noted that the chance constraints (5.27), (5.28), and (5.29) are non-convex. In order to make the proposed model solvable, the non-convexities in these chance constraints are required to be eliminated. By employing the linearization process detailed in Subsection 3.4.1, the chance constraints (5.27)-(5.29) of the LL1 problem can be reformulated as follows:

$$\alpha m_{j,d,t}^L + \sum_o L_{j,d,t,o}^{SH} \geq \Phi^{-1}(1 - \rho^L) \alpha s_{j,d,t}^L \quad : \chi_{j,d,t}^{SH,-} \quad \forall j, \forall d, \forall t \quad (5.36)$$

$$\alpha m_{j,d,t}^L - \sum_o L_{j,d,t,o}^{SH} \geq \Phi^{-1}(1 - \rho^L) \alpha s_{j,d,t}^L \quad : \chi_{j,d,t}^{SH,+} \quad \forall j, \forall d, \forall t \quad (5.37)$$

$$m_{i,d,t,n}^{RE} - g_{i,d,t,n}^{DA} + g_{i,d,t,n}^{dev} \geq \Phi^{-1}(1 - \rho^{RE}) s_{i,d,t,n}^{RE} \quad : \chi_{i,d,t,n}^{RE} \quad \forall i \in I^{RE}, \forall d, \forall t, \forall n \quad (5.38)$$

$$-m_{j,d,t}^L - \sum_o L_{j,d,t,o}^{SH} + \sum_o P_{j,d,t,o}^{dev} + \sum_o P_{j,d,t,o}^{DA} \geq \Phi^{-1}(1 - \rho^L) s_{j,d,t}^L \quad : \chi_{j,d,t}^L \quad \forall j, \forall d, \forall t \quad (5.39)$$

Note that $\Phi^{-1}(\cdot)$ is the inverse cumulative distribution function of a standard normal distribution.

5.4.2 MPEC Formulation

Given that the continuity and convexity of the LL1 problem are preserved via the linearized chance constraints (5.36)-(5.39), it becomes feasible to substitute the LL1 problem with a set of constraints derived from its KKT optimality conditions. Furthermore, the LL2 problem, defined entirely by its constraints, is efficiently integrated into the UL problem. As a result, the two-stage bi-level optimisation framework (5.1)-(5.32), undergoes a transformation into a single-level MPEC problem.

KKT Conditions Associated with the LL1 Problem

To derive the KKT optimality conditions corresponding to the LL1 problem, it is crucial to construct its Lagrangian function, denoted as $\mathcal{L}_{LL1}$. This function integrates the

objective of the LL1 problem with its constraints, weighted by their respective Lagrange multipliers, and is formulated as follows:

$$\begin{aligned}
\mathcal{L}_{LL1} = & \sum_{i,d,t,n,b} C_{i,b}^{EN} g_{i,d,t,n,b}^{DA} + \sum_{i,d,t,n,b} (C_{i,b}^U g_{i,d,t,n,b}^U - C_{i,b}^D g_{i,d,t,n,b}^D) + \sum_{d,t,n} \lambda_{d,t,n}^{DA} \left[\sum_{j \in \Psi_{n,o}} P_{j,d,t,o}^{DA} \right. \\
& + \sum_{m \in \Lambda_n} b_{nm} (\delta_{d,t,n}^{DA} - \delta_{d,t,m}^{DA}) - \sum_{i \in \Psi_{n,b}} g_{i,d,t,n,b}^{DA} \left. \right] + \sum_{d,t,n} \lambda_{d,t,n}^{RT} \left[\sum_{j \in \Psi_{n,o}} P_{j,d,t,o}^{dev} + \right. \\
& \sum_{i \in I^{FL} \cap \Psi_{n,b}} (-g_{i,d,t,n,b}^U + g_{i,d,t,n,b}^D) + \sum_{i \in I^{RE} \cap \Psi_n} g_{i,d,t,n}^{dev} - \sum_{m \in \Lambda_n} b_{nm} (\delta_{d,t,n}^{DA} - \delta_{d,t,n}^{RT} - \\
& \delta_{d,t,m}^{DA} + \delta_{d,t,m}^{RT}) \left. \right] + \sum_d \zeta_d \left[\sum_{i \in I^{FL},t,n,b} \eta_i^{CO_2} (g_{i,d,t,n,b}^{DA} + g_{i,d,t,n,b}^U - g_{i,d,t,n,b}^D) - \right. \\
& \eta^{Grid} \sum_{j,t,o} L_{j,d,t,o}^{DA} \left. \right] + \sum_{i \in I^{FL},d,t,n,b} \mu_{i,d,t,n,b}^U (g_{i,d,t,n,b}^{DA} + g_{i,d,t,n,b}^U - G_{i,n,b}) + \\
& \sum_{i \in I^{FL},d,t,n,b} \mu_{i,d,t,n,b}^D (g_{i,d,t,n,b}^D - g_{i,d,t,n,b}^{DA}) - \sum_{i \in I^{FL},d,t,n,b} \phi_{i,d,t,n,b}^{U,-} g_{i,d,t,n,b}^U + \\
& \sum_{i \in I^{FL},d,t,n,b} \phi_{i,d,t,n,b}^{U,+} (g_{i,d,t,n,b}^U - G_{i,b}^U) - \sum_{i \in I^{FL},d,t,n,b} \phi_{i,d,t,n,b}^{D,-} g_{i,d,t,n,b}^D + \\
& \sum_{i \in I^{FL},d,t,n,b} \phi_{i,d,t,n,b}^{D,+} (g_{i,d,t,n,b}^D - G_{i,b}^D) + \sum_{i \in I^{MR},d,t,n} \gamma_{i,d,t,n} (g_{i,d,t,n}^{DA} - G_{i,n}) + \\
& \sum_{i \in I^{RE},d,t,n} \theta_{i,d,t,n} (g_{i,d,t,n}^{DA} - k_{i,d,t}^{RE} G_{i,n}) + \sum_{d,t,n,m \in \Lambda_n} \nu_{d,t,n,m}^{DA,-} [-C_{nm}^{max} - b_{nm} (\delta_{d,t,n}^{DA} - \\
& \delta_{d,t,m}^{DA})] + \sum_{d,t,n,m \in \Lambda_n} \nu_{d,t,n,m}^{DA,+} [b_{nm} (\delta_{d,t,n}^{DA} - \delta_{d,t,m}^{DA}) - C_{nm}^{max}] + \\
& \sum_{d,t,n} \xi_{d,t,n}^{DA,-} (-\pi - \delta_{d,t,n}^{DA}) + \sum_{d,t,n} \xi_{d,t,n}^{DA,+} (\delta_{d,t,n}^{DA} - \pi) + \sum_{d,t,n=1} \xi_{d,t}^{DA,ref} \delta_{d,t}^{DA,ref} + \\
& \sum_{d,t,n,m \in \Lambda_n} \nu_{d,t,n,m}^{RT,-} [-C_{nm}^{max} - b_{nm} (\delta_{d,t,n}^{RT} - \delta_{d,t,m}^{RT})] + \sum_{d,t,n,m \in \Lambda_n} \nu_{d,t,n,m}^{RT,+} [b_{nm} (\delta_{d,t,n}^{RT} - \\
& \delta_{d,t,m}^{RT}) - C_{nm}^{max}] + \sum_{d,t,n} \xi_{d,t,n}^{RT,-} (-\pi - \delta_{d,t,n}^{RT}) + \sum_{d,t,n} \xi_{d,t,n}^{RT,+} (\delta_{d,t,n}^{RT} - \pi) + \\
& \sum_{d,t,n=1} \xi_{d,t}^{RT,ref} \delta_{d,t}^{RT,ref} + \sum_{j,d,t,o} \psi_{j,d,t,o}^- [(1 - \alpha) L_{j,d,t,o}^{DA} - P_{j,d,t,o}^{DA}] + \\
& \sum_{j,d,t,o} \psi_{j,d,t,o}^+ [P_{j,d,t,o}^{DA} - (1 + \alpha) L_{j,d,t,o}^{DA}] + \sum_{j,d} \sigma_{j,d} \sum_{t,o} L_{j,d,t,o}^{SH} + \\
& \sum_{j,d,t} \chi_{j,d,t}^{SH,-} [\Phi^{-1} (1 - \rho^L) \alpha s_{j,d,t}^L - \alpha m_{j,d,t}^L - \sum_o L_{j,d,t,o}^{SH}] + \\
& \sum_{j,d,t} \chi_{j,d,t}^{SH,+} [\Phi^{-1} (1 - \rho^L) \alpha s_{j,d,t}^L - \alpha m_{j,d,t}^L + \sum_o L_{j,d,t,o}^{SH}] + \\
& \sum_{i \in I^{RE},d,t,n} \chi_{i,d,t,n}^{RE} [\Phi^{-1} (1 - \rho^{RE}) s_{i,d,t,n}^{RE} - m_{i,d,t,n}^{RE} + g_{i,d,t,n}^{DA} - g_{i,d,t,n}^{dev}] +
\end{aligned}$$

$$\sum_{j,d,t} \chi_{j,d,t}^L [\Phi^{-1}(1 - \rho^L) s_{j,d,t}^L + m_{j,d,t}^L + \sum_o L_{j,d,t,o}^{SH} - \sum_o P_{j,d,t,o}^{dev} - \sum_o P_{j,d,t,o}^{DA}] \quad (5.40)$$

Based on the Lagrangian function defined in (5.40), the comprehensive KKT optimality conditions for the LL1 problem are detailed as follows:

$$\text{KKT equality conditions : (5.10) – (5.11), (5.17) – (5.18), (5.21), (5.24), (5.26) } \quad (5.41)$$

LL1 KKT stationarity conditions :

$$\frac{\partial \mathcal{L}_{LL1}}{\partial P_{j,d,t,o}^{DA}} = \lambda_{d,t,n}^{DA} + \psi_{j,d,t,o}^+ - \psi_{j,d,t,o}^- - \chi_{j,d,t}^L = 0 \quad \forall j \in \Psi_n, \forall d, \forall t, \forall n, \forall o \quad (5.42)$$

$$\frac{\partial \mathcal{L}_{LL1}}{\partial g_{i,d,t,n,b}^{DA}} = C_{i,b}^{EN} - \lambda_{d,t,n}^{DA} + \eta_i^{CO_2} \zeta_d + \mu_{i,d,t,n,b}^U - \mu_{i,d,t,n,b}^D = 0 \quad \forall i \in I^{FL} \cap \Psi_n, \forall d, \forall t, \forall n, \forall b \quad (5.43)$$

$$\frac{\partial \mathcal{L}_{LL1}}{\partial g_{i,d,t,n}^{DA}} = C_i^{EN} - \lambda_{d,t,n}^{DA} + \gamma_{i,d,t,n} = 0 \quad \forall i \in I^{MR} \cap \Psi_n, \forall d, \forall t, \forall n \quad (5.44)$$

$$\frac{\partial \mathcal{L}_{LL1}}{\partial g_{i,d,t,n}^{DA}} = -\lambda_{d,t,n}^{DA} + \theta_{i,d,t,n} + \chi_{i,d,t,n}^{RE} = 0 \quad \forall i \in I^{RE} \cap \Psi_n, \forall d, \forall t, \forall n \quad (5.45)$$

$$\begin{aligned} \frac{\partial \mathcal{L}_{LL1}}{\partial g_{i,d,t,n,b}^U} &= C_{i,b}^U - \lambda_{d,t,n}^{RT} + \eta_i^{CO_2} \zeta_d + \mu_{i,d,t,n,b}^U - \phi_{i,d,t,n,b}^{U,-} + \phi_{i,d,t,n,b}^{U,+} = 0 \\ &\forall i \in I^{FL} \cap \Psi_n, \forall d, \forall t, \forall n, \forall b \end{aligned} \quad (5.46)$$

$$\begin{aligned} \frac{\partial \mathcal{L}_{LL1}}{\partial g_{i,d,t,n,b}^D} &= -C_{i,b}^D + \lambda_{d,t,n}^{RT} - \eta_i^{CO_2} \zeta_d + \mu_{i,d,t,n,b}^D - \phi_{i,d,t,n,b}^{D,-} + \phi_{i,d,t,n,b}^{D,+} = 0 \\ &\forall i \in I^{FL} \cap \Psi_n, \forall d, \forall t, \forall n, \forall b \end{aligned} \quad (5.47)$$

$$\frac{\partial \mathcal{L}_{LL1}}{\partial P_{j,d,t,o}^{dev}} = \lambda_{d,t,n}^{RT} - \chi_{j,d,t}^L = 0 \quad \forall j \in \Psi_n, \forall d, \forall t \quad (5.48)$$

$$\frac{\partial \mathcal{L}_{LL1}}{\partial g_{i,d,t,n}^{dev}} = \lambda_{d,t,n}^{RT} - \chi_{i,d,t,n}^{RE} = 0 \quad \forall i \in I^{RE} \cap \Psi_n, \forall d, \forall t, \forall n \quad (5.49)$$

$$\begin{aligned} \frac{\partial \mathcal{L}_{LL1}}{\partial \delta_{d,t,n}^{DA}} &= \sum_{m \in \Lambda_n} b_{nm} (\lambda_{d,t,n}^{DA} - \lambda_{d,t,m}^{DA}) - \sum_{m \in \Lambda_n} b_{nm} (\lambda_{d,t,n}^{RT} - \lambda_{d,t,m}^{RT}) + \\ &\sum_{m \in \Lambda_n} b_{nm} (\nu_{d,t,n,m}^{DA,+} - \nu_{d,t,m,n}^{DA,+}) - \sum_{m \in \Lambda_n} b_{nm} (\nu_{d,t,n,m}^{DA,-} - \nu_{d,t,m,n}^{DA,-}) + \xi_{d,t,n}^{DA,+} \\ &- \xi_{d,t,n}^{DA,-} + (\xi_{d,t}^{DA,ref})_{n=1} = 0 \quad \forall d, \forall t, \forall n \end{aligned} \quad (5.50)$$

$$\begin{aligned} \frac{\partial \mathcal{L}_{LL1}}{\partial \delta_{d,t,n}^{RT}} = & \sum_{m \in \Lambda_n} b_{nm} (\lambda_{d,t,n}^{RT} - \lambda_{d,t,m}^{RT}) + \sum_{m \in \Lambda_n} b_{nm} (\nu_{d,t,n,m}^{RT,+} - \nu_{d,t,m,n}^{RT,+}) - \\ & \sum_{m \in \Lambda_n} b_{nm} (\nu_{d,t,n,m}^{RT,-} - \nu_{d,t,m,n}^{RT,-}) + \xi_{d,t,n}^{RT,+} - \xi_{d,t,n}^{RT,-} + (\xi_{d,t}^{RT,ref})_{n=1} = 0 \quad \forall d, \forall t, \forall n \end{aligned} \quad (5.51)$$

$$\frac{\partial \mathcal{L}_{LL1}}{\partial L_{j,d,t,o}^{SH}} = \sigma_{j,d} + \chi_{j,d,t}^{SH,+} - \chi_{j,d,t}^{SH,-} + \chi_{j,d,t}^L = 0 \quad \forall j \in \Psi_n, \forall d, \forall t, \forall n \quad (5.52)$$

LL1 KKT complementarity conditions :

$$0 \leq [\eta^{Grid} \sum_{j,t,o} L_{j,d,t,o}^{DA} - \sum_{i \in I^{FL},t,n,b} \eta_i^{CO_2} (g_{i,d,t,n,b}^{DA} + g_{i,d,t,n,b}^U - g_{i,d,t,n,b}^D)] \perp \zeta_d \geq 0 \quad \forall d \quad (5.53)$$

$$0 \leq (G_{i,n,b} - g_{i,d,t,n,b}^{DA} - g_{i,d,t,n,b}^U) \perp \mu_{i,d,t,n,b}^U \geq 0 \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (5.54)$$

$$0 \leq (g_{i,d,t,n,b}^{DA} - g_{i,d,t,n,b}^D) \perp \mu_{i,d,t,n,b}^D \geq 0 \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (5.55)$$

$$0 \leq g_{i,d,t,n,b}^U \perp \phi_{i,d,t,n,b}^{U,-} \geq 0 \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (5.56)$$

$$0 \leq (G_{i,b}^U - g_{i,d,t,n,b}^U) \perp \phi_{i,d,t,n,b}^{U,+} \geq 0 \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (5.57)$$

$$0 \leq g_{i,d,t,n,b}^D \perp \phi_{i,d,t,n,b}^{D,-} \geq 0 \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (5.58)$$

$$0 \leq (G_{i,b}^D - g_{i,d,t,n,b}^D) \perp \phi_{i,d,t,n,b}^{D,+} \geq 0 \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (5.59)$$

$$0 \leq [C_{nm}^{max} + b_{nm} (\delta_{d,t,n}^{DA} - \delta_{d,t,m}^{DA})] \perp \nu_{d,t,n,m}^{DA,-} \geq 0 \quad \forall d, \forall t, \forall n, \forall m \in \Lambda_n \quad (5.60)$$

$$0 \leq [C_{nm}^{max} - b_{nm} (\delta_{d,t,n}^{DA} - \delta_{d,t,m}^{DA})] \perp \nu_{d,t,n,m}^{DA,+} \geq 0 \quad \forall d, \forall t, \forall n, \forall m \in \Lambda_n \quad (5.61)$$

$$0 \leq (\pi + \delta_{d,t,n}^{DA}) \perp \xi_{d,t,n}^{DA,-} \geq 0 \quad \forall d, \forall t, \forall n \quad (5.62)$$

$$0 \leq (\pi - \delta_{d,t,n}^{DA}) \perp \xi_{d,t,n}^{DA,+} \geq 0 \quad \forall d, \forall t, \forall n \quad (5.63)$$

$$0 \leq [C_{nm}^{max} + b_{nm} (\delta_{d,t,n}^{RT} - \delta_{d,t,m}^{RT})] \perp \nu_{d,t,n,m}^{RT,-} \geq 0 \quad \forall d, \forall t, \forall n, \forall m \in \Lambda_n \quad (5.64)$$

$$0 \leq [C_{nm}^{max} - b_{nm} (\delta_{d,t,n}^{RT} - \delta_{d,t,m}^{RT})] \perp \nu_{d,t,n,m}^{RT,+} \geq 0 \quad \forall d, \forall t, \forall n, \forall m \in \Lambda_n \quad (5.65)$$

$$0 \leq (\pi + \delta_{d,t,n}^{RT}) \perp \xi_{d,t,n}^{RT,-} \geq 0 \quad \forall d, \forall t, \forall n \quad (5.66)$$

$$0 \leq (\pi - \delta_{d,t,n}^{RT}) \perp \xi_{d,t,n}^{RT,+} \geq 0 \quad \forall d, \forall t, \forall n \quad (5.67)$$

$$0 \leq [P_{j,d,t,o}^{DA} - (1 - \alpha) L_{j,d,t,o}^{DA}] \perp \psi_{j,d,t,o}^- \geq 0 \quad \forall j, \forall d, \forall t, \forall o \quad (5.68)$$

$$0 \leq [(1 + \alpha) L_{j,d,t,o}^{DA} - P_{j,d,t,o}^{DA}] \perp \psi_{j,d,t,o}^+ \geq 0 \quad \forall j, \forall d, \forall t, \forall o \quad (5.69)$$

$$0 \leq [\alpha m_{j,d,t}^L + \sum_o L_{j,d,t,o}^{SH} - \Phi^{-1}(1 - \rho^L) \alpha s_{j,d,t}^L] \perp \chi_{j,d,t}^{SH,-} \geq 0 \quad \forall j, \forall d, \forall t \quad (5.70)$$

$$0 \leq [\alpha m_{j,d,t}^L - \sum_o L_{j,d,t,o}^{SH} - \Phi^{-1}(1 - \rho^L) \alpha s_{j,d,t}^L] \perp \chi_{j,d,t}^{SH,+} \geq 0 \quad \forall j, \forall d, \forall t \quad (5.71)$$

$$0 \leq [m_{i,d,t,n}^{RE} - g_{i,d,t,n}^{DA} + g_{i,d,t,n}^{dev} - \Phi^{-1}(1 - \rho^{RE}) s_{i,d,t,n}^{RE}] \perp \chi_{i,d,t,n}^{RE} \geq 0 \quad \forall i \in I^{RE}, \forall d, \forall t, \forall n \quad (5.72)$$

$$0 \leq [-m_{j,d,t}^L - \sum_o L_{j,d,t,o}^{SH} + \sum_o P_{j,d,t,o}^{dev} + \sum_o P_{j,d,t,o}^{DA} - \Phi^{-1}(1 - \rho^L) s_{j,d,t}^L] \perp \chi_{j,d,t}^L \geq 0 \quad \forall j, \forall d, \forall t \quad (5.73)$$

The KKT optimality conditions for the LL1 problem play a significant role in converting the bi-level optimisation model into a single-level MPEC. These conditions are divided into three main categories: i) KKT equality conditions (5.41), ii) KKT stationarity conditions (5.42)-(5.52), and iii) KKT complementarity conditions (5.53)-(5.73).

Final MPEC Formulation

Given that the LL2 problem is exclusively composed of a set of constraints, it seamlessly integrates into the MPEC. Consequently, the MPEC formulation is mathematically presented as follows:

$$\begin{aligned} \max_{V^{MPEC}} \sum_d \omega_d \left[\sum_{i \in \Psi_{n,t,n,b}} \lambda_{d,t,n}^{DA} g_{i,d,t,n,b}^{DA} - \sum_{i,t,n,b} C_{i,b}^{EN} g_{i,d,t,n,b}^{DA} + \right. \\ \sum_{i \in I^{FL} \cap \Psi_{n,t,n,b}} \lambda_{d,t,n}^{RT} (g_{i,d,t,n,b}^U - g_{i,d,t,n,b}^D) - \sum_{i \in I^{FL},t,n,b} (C_{i,b}^U g_{i,d,t,n,b}^U - C_{i,b}^D g_{i,d,t,n,b}^D) \\ + \sum_{i \in I^{RE} \cap \Psi_{n,t,n}} \lambda_{d,t,n}^{RT} g_{i,d,t,n}^{dev} - \sum_{i \in I^{FL},t,n} \lambda^{CO_2} Q_{i,d,t,n}^{emi} + \sum_{i \in I^{RE},t,n} \lambda^{CO_2} Q_{i,d,t,n}^{CA} \\ \left. + \sum_{i \in I^{RE}} (\lambda_{i,d}^{PGC} Q_{i,d}^{PGC} + \lambda^{FGC} Q_{i,d}^{FGC}) \right] - \sum_{i,n} C_i^{IN} G_{i,n} \quad (5.74) \end{aligned}$$

where:

$$V^{MPEC} = V^{UL} \cup V^{LL1} \cup V^{LL2} \cup V^{DUAL} \quad (5.75)$$

$$\begin{aligned} V^{DUAL} = \{ \lambda_{d,t,n}^{DA}, \lambda_{d,t,n}^{RT}, \zeta_d, \mu_{i,d,t,n,b}^U, \mu_{i,d,t,n,b}^D, \phi_{i,d,t,n,b}^{U,-}, \phi_{i,d,t,n,b}^{U,+}, \phi_{i,d,t,n,b}^{D,-}, \phi_{i,d,t,n,b}^{D,+}, \gamma_{i,d,t,n}, \\ \theta_{i,d,t,n}, \nu_{d,t,n,m}^{DA,-}, \nu_{y,t,n,m}^{DA,+}, \xi_{d,t,n}^{DA,-}, \xi_{d,t,n}^{DA,+}, \xi_{d,t}^{DA,ref}, \nu_{d,t,n,m}^{RT,-}, \nu_{d,t,n,m}^{RT,+}, \xi_{d,t,n}^{RT,-}, \xi_{d,t,n}^{RT,+}, \\ \xi_{d,t}^{RT,ref}, \psi_{j,d,t,o}^-, \psi_{j,d,t,o}^+, \sigma_{j,d}, \chi_{j,d,t}^{SH,-}, \chi_{j,d,t}^{SH,+}, \chi_{j,d,t}^{RE}, \chi_{j,d,t}^L \} \quad (5.76) \end{aligned}$$

subject to:

$$\text{UL constraints : (5.3) – (5.7)} \quad (5.77)$$

$$\text{LL1 KKT equality conditions : (5.41)} \quad (5.78)$$

$$\text{LL1 KKT stationarity conditions : (5.42) – (5.52)} \quad (5.79)$$

$$\text{LL1 KKT complementarity conditions : (5.53) – (5.73)} \quad (5.80)$$

$$\text{LL2 constraints : (5.31) – (5.32)} \quad (5.81)$$

5.4.3 Linearization of MPEC

To improve the computational tractability of the proposed MPEC problem, effective linearization techniques are necessary to convert the inherently non-linear MPEC into a MIQP, which is compatible with commercial branch-and-cut solvers. The non-linear elements within the MPEC formulation primarily originate from two sources: the KKT complementarity conditions associated with the LL1 problem (5.80), and the bilinear terms present in the GenCo's profit calculation, as outlined in the MPEC's objective function (5.74). The process of linearizing these non-linearities is detailed in this subsection.

Linearization of KKT Complementarity Conditions

The big-M method is adopted to decompose the KKT complementarity condition into a set of linear conditions mixed with binary variables. Specifically, a represents a primal variable in KKT conditions, and b represents the corresponding dual variable. The resulting equivalent mixed-integer linear conditions are formulated as follows: $\{a \geq 0, b \geq 0, a \leq \beta M^a, b \leq (1 - \beta)M^b, \beta \in \{0, 1\}\}$. Note that M^a and M^b identify the large positive constants. By following this linearization process, the KKT complementarity conditions associated with the LL1 problem (5.80) can be reformed into a series of disjunctive constraints, which are expressed as follows:

$$V^{BINARY} = \left\{ \beta_d^{CO_2}, \beta_{i,d,t,n,b}^U, \beta_{i,d,t,n,b}^D, \beta_{i,d,t,n,b}^{U,-}, \beta_{i,d,t,n,b}^{U,+}, \beta_{i,d,t,n,b}^{D,-}, \beta_{i,d,t,n,b}^{D,+}, \beta_{d,t,n,m}^{DA,net,-}, \right. \\ \left. \beta_{d,t,n,m}^{DA,net,+}, \beta_{d,t,n}^{DA,\pi,-}, \beta_{d,t,n}^{DA,\pi,+}, \beta_{d,t,n,m}^{RT,net,-}, \beta_{d,t,n,m}^{RT,net,+}, \beta_{d,t,n}^{RT,\pi,-}, \beta_{d,t,n}^{RT,\pi,+}, \right. \\ \left. \beta_{j,d,t,o}^{PDA,-}, \beta_{j,d,t,o}^{PDA,+}, \beta_{j,d,t}^{SH,-}, \beta_{j,d,t}^{SH,+}, \beta_{i,d,t,n}^{RE}, \beta_{j,d,t}^L \right\} \quad (5.82)$$

$$0 \leq \eta^{Grid} \sum_{j,t,o} L_{j,d,t,o}^{DA} - \sum_{i \in I^{FL,t,n,b}} \eta_i^{CO_2} (g_{i,d,t,n,b}^{DA} + g_{i,d,t,n,b}^U - g_{i,d,t,n,b}^D) \leq \beta_d^{CO_2} M \quad \forall d \quad (5.83)$$

$$0 \leq \zeta_d \leq (1 - \beta_d^{CO_2}) M \quad \forall d \quad (5.84)$$

$$0 \leq G_{i,n,b} - g_{i,d,t,n,b}^{DA} - g_{i,d,t,n,b}^U \leq \beta_{i,d,t,n,b}^U M \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (5.85)$$

$$0 \leq \mu_{i,d,t,n,b}^U \leq (1 - \beta_{i,d,t,n,b}^U)M \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (5.86)$$

$$0 \leq g_{i,d,t,n,b}^{DA} - g_{i,d,t,n,b}^D \leq \beta_{i,d,t,n,b}^D M \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (5.87)$$

$$0 \leq \mu_{i,d,t,n,b}^D \leq (1 - \beta_{i,d,t,n,b}^D)M \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (5.88)$$

$$0 \leq g_{i,d,t,n,b}^U \leq \beta_{i,d,t,n,b}^{U,-} M \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (5.89)$$

$$0 \leq \phi_{i,d,t,n,b}^{U,-} \leq (1 - \beta_{i,d,t,n,b}^{U,-})M \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (5.90)$$

$$0 \leq G_{i,b}^U - g_{i,d,t,n,b}^U \leq \beta_{i,d,t,n,b}^{U,+} M \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (5.91)$$

$$0 \leq \phi_{i,d,t,n,b}^{U,+} \leq (1 - \beta_{i,d,t,n,b}^{U,+})M \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (5.92)$$

$$0 \leq g_{i,d,t,n,b}^D \leq \beta_{i,d,t,n,b}^{D,-} M \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (5.93)$$

$$0 \leq \phi_{i,d,t,n,b}^{D,-} \leq (1 - \beta_{i,d,t,n,b}^{D,-})M \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (5.94)$$

$$0 \leq G_{i,b}^D - g_{i,d,t,n,b}^D \leq \beta_{i,d,t,n,b}^{D,+} M \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (5.95)$$

$$0 \leq \phi_{i,d,t,n,b}^{D,+} \leq (1 - \beta_{i,d,t,n,b}^{D,+})M \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (5.96)$$

$$0 \leq C_{nm}^{max} + b_{nm}(\delta_{d,t,n}^{DA} - \delta_{d,t,m}^{DA}) \leq \beta_{d,t,n,m}^{DA,net,-} M \quad \forall d, \forall t, \forall n, \forall m \in \Lambda_n \quad (5.97)$$

$$0 \leq \nu_{d,t,n,m}^{DA,-} \leq (1 - \beta_{d,t,n,m}^{DA,net,-})M \quad \forall d, \forall t, \forall n, \forall m \in \Lambda_n \quad (5.98)$$

$$0 \leq C_{nm}^{max} - b_{nm}(\delta_{d,t,n}^{DA} - \delta_{d,t,m}^{DA}) \leq \beta_{d,t,n,m}^{DA,net,+} M \quad \forall d, \forall t, \forall n, \forall m \in \Lambda_n \quad (5.99)$$

$$0 \leq \nu_{d,t,n,m}^{DA,+} \leq (1 - \beta_{d,t,n,m}^{DA,net,+})M \quad \forall d, \forall t, \forall n, \forall m \in \Lambda_n \quad (5.100)$$

$$0 \leq \pi + \delta_{d,t,n}^{DA} \leq \beta_{d,t,n}^{DA,\pi,-} M \quad \forall d, \forall t, \forall n \quad (5.101)$$

$$0 \leq \xi_{d,t,n}^{DA,-} \leq (1 - \beta_{d,t,n}^{DA,\pi,-})M \quad \forall d, \forall t, \forall n \quad (5.102)$$

$$0 \leq \pi - \delta_{d,t,n}^{DA} \leq \beta_{d,t,n}^{DA,\pi,+} M \quad \forall d, \forall t, \forall n \quad (5.103)$$

$$0 \leq \xi_{d,t,n}^{DA,+} \leq (1 - \beta_{d,t,n}^{DA,\pi,+})M \quad \forall d, \forall t, \forall n \quad (5.104)$$

$$0 \leq C_{nm}^{max} + b_{nm}(\delta_{d,t,n}^{RT} - \delta_{d,t,m}^{RT}) \leq \beta_{d,t,n,m}^{RT,net,-} M \quad \forall d, \forall t, \forall n, \forall m \in \Lambda_n \quad (5.105)$$

$$0 \leq \nu_{d,t,n,m}^{RT,-} \leq (1 - \beta_{d,t,n,m}^{RT,net,-})M \quad \forall d, \forall t, \forall n, \forall m \in \Lambda_n \quad (5.106)$$

$$0 \leq C_{nm}^{max} - b_{nm}(\delta_{d,t,n}^{RT} - \delta_{d,t,m}^{RT}) \leq \beta_{d,t,n,m}^{RT,net,+} M \quad \forall d, \forall t, \forall n, \forall m \in \Lambda_n \quad (5.107)$$

$$0 \leq \nu_{d,t,n,m}^{RT,+} \leq (1 - \beta_{d,t,n,m}^{RT,net,+})M \quad \forall d, \forall t, \forall n, \forall m \in \Lambda_n \quad (5.108)$$

$$0 \leq \pi + \delta_{d,t,n}^{RT} \leq \beta_{d,t,n}^{RT,\pi,-} M \quad \forall d, \forall t, \forall n \quad (5.109)$$

$$0 \leq \xi_{d,t,n}^{RT,-} \leq (1 - \beta_{d,t,n}^{RT,\pi,-})M \quad \forall d, \forall t, \forall n \quad (5.110)$$

$$0 \leq \pi - \delta_{d,t,n}^{RT} \leq \beta_{d,t,n}^{RT,\pi,+} M \quad \forall d, \forall t, \forall n \quad (5.111)$$

$$0 \leq \xi_{d,t,n}^{RT,+} \leq (1 - \beta_{d,t,n}^{RT,\pi,+})M \quad \forall d, \forall t, \forall n \quad (5.112)$$

$$0 \leq P_{j,d,t,o}^{DA} - (1 - \alpha)L_{j,d,t,o}^{DA} \leq \beta_{j,d,t,o}^{PDA,-}M \quad \forall j, \forall d, \forall t, \forall o \quad (5.113)$$

$$0 \leq \psi_{j,d,t,o}^{-} \leq (1 - \beta_{j,d,t,o}^{PDA,-})M \quad \forall j, \forall d, \forall t, \forall o \quad (5.114)$$

$$0 \leq (1 + \alpha)L_{j,d,t,o}^{DA} - P_{j,d,t,o}^{DA} \leq \beta_{j,d,t,o}^{PDA,+}M \quad \forall j, \forall d, \forall t, \forall o \quad (5.115)$$

$$0 \leq \psi_{j,d,t,o}^{+} \leq (1 - \beta_{j,d,t,o}^{PDA,+})M \quad \forall j, \forall d, \forall t, \forall o \quad (5.116)$$

$$0 \leq \alpha m_{j,d,t}^L + \sum_o L_{j,d,t,o}^{SH} - \Phi^{-1}(1 - \rho^L)\alpha s_{j,d,t}^L \leq \beta_{j,d,t}^{SH,-}M \quad \forall j, \forall d, \forall t \quad (5.117)$$

$$0 \leq \chi_{j,d,t}^{SH,-} \leq (1 - \beta_{j,d,t}^{SH,-})M \quad \forall j, \forall d, \forall t \quad (5.118)$$

$$0 \leq \alpha m_{j,d,t}^L - \sum_o L_{j,d,t,o}^{SH} - \Phi^{-1}(1 - \rho^L)\alpha s_{j,d,t}^L \leq \beta_{j,d,t}^{SH,+}M \quad \forall j, \forall d, \forall t \quad (5.119)$$

$$0 \leq \chi_{j,d,t}^{SH,+} \leq (1 - \beta_{j,d,t}^{SH,+})M \quad \forall j, \forall d, \forall t \quad (5.120)$$

$$0 \leq m_{i,d,t,n}^{RE} - g_{i,d,t,n}^{DA} + g_{i,d,t,n}^{dev} - \Phi^{-1}(1 - \rho^{RE})s_{i,d,t,n}^{RE} \leq \beta_{i,d,t,n}^{RE}M \quad \forall i \in I^{RE}, \forall d, \forall t, \forall n \quad (5.121)$$

$$0 \leq \chi_{i,d,t,n}^{RE} \leq (1 - \beta_{i,d,t,n}^{RE})M \quad \forall i \in I^{RE}, \forall d, \forall t, \forall n \quad (5.122)$$

$$0 \leq -m_{j,d,t}^L - \sum_o L_{j,d,t,o}^{SH} + \sum_o P_{j,d,t,o}^{dev} + \sum_o P_{j,d,t,o}^{DA} - \Phi^{-1}(1 - \rho^L)s_{j,d,t}^L \leq \beta_{j,d,t}^L M \quad \forall j, \forall d, \forall t \quad (5.123)$$

$$0 \leq \chi_{j,d,t}^L \leq (1 - \beta_{j,d,t}^L)M \quad \forall j, \forall d, \forall t \quad (5.124)$$

Linearization of GenCo's Profit

In order to deal with the challenge of bilinear terms in GenCo's profit within the MPEC's objective function (5.74), the strong duality theorem and the KKT optimality conditions are leveraged to uncover implicit linear forms of these bilinear terms, thereby facilitating their incorporation into the model in a linearized structure.

Specifically, the bilinear terms ($\sum_{i \in I} \lambda_{d,t,n}^{DA} g_{i,d,t,n,b}^{DA}$, $\sum_{i \in I^{FL}} \lambda_{d,t,n}^{RT} g_{i,d,t,n,b}^U$, $\sum_{i \in I^{FL}} \lambda_{d,t,n}^{RT} g_{i,d,t,n,b}^D$, $\sum_{i \in I^{RE}} \lambda_{d,t,n}^{RT} g_{i,d,t,n}^{dev}$, and $\sum_{i \in I^{RE}} \lambda_{i,d}^{PGC} Q_{i,d}^{PGC}$) of the MPEC's objective function (5.74) need to be restructured as their linearized form by adopting the strong duality theorem and the KKT optimality conditions. The derivation process is detailed as follows.

First, in order to linearize the bilinear term $\sum_{i \in I} \lambda_{d,t,n}^{DA} g_{i,d,t,n,b}^{DA}$, the KKT stationarity

condition (5.43) is rearranged as:

$$\lambda_{d,t,n}^{DA} = C_{i,b}^{EN} + \eta_i^{CO_2} \zeta_d + \mu_{i,d,t,n,b}^U - \mu_{i,d,t,n,b}^D \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (5.125)$$

Multiplying both sides by $g_{i,d,t,n,b}^{DA}$ ($\forall i \in I^{FL}$), it renders:

$$\begin{aligned} \lambda_{d,t,n}^{DA} g_{i,d,t,n,b}^{DA} = & C_{i,b}^{EN} g_{i,d,t,n,b}^{DA} + \eta_i^{CO_2} \zeta_d g_{i,d,t,n,b}^{DA} + \mu_{i,d,t,n,b}^U g_{i,d,t,n,b}^{DA} \\ & - \mu_{i,d,t,n,b}^D g_{i,d,t,n,b}^{DA} \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \end{aligned} \quad (5.126)$$

Given that the term (5.126) exclusively accounts for the DA energy revenue derived from flexible generators, the same linearization procedure can be applied to address the revenue from must-run and renewable generators. Therefore, the KKT stationarity condition (5.44) is rewritten as:

$$\lambda_{d,t,n}^{DA} = C_i^{EN} + \gamma_{i,d,t,n} \quad \forall i \in I^{MR}, \forall d, \forall t, \forall n \quad (5.127)$$

Similarly, multiplying both sides by $g_{i,d,t,n}^{DA}$ ($\forall i \in I^{MR}$), it gives:

$$\lambda_{d,t,n}^{DA} g_{i,d,t,n}^{DA} = C_i^{EN} g_{i,d,t,n}^{DA} + \gamma_{i,d,t,n} g_{i,d,t,n}^{DA} \quad \forall i \in I^{MR}, \forall d, \forall t, \forall n \quad (5.128)$$

Subsequently, after the LL1 constraint (5.17) is plugged into (5.128), it renders:

$$\lambda_{d,t,n}^{DA} g_{i,d,t,n}^{DA} = C_i^{EN} g_{i,d,t,n}^{DA} + \gamma_{i,d,t,n} G_{i,n} \quad \forall i \in I^{MR}, \forall d, \forall t, \forall n \quad (5.129)$$

It should be emphasised that in terms of the realisation for renewable power generation, its mean is set as the DA renewable generation dispatch $g_{i,d,t,n}^{DA}$ ($\forall i \in I^{RE}$), and its standard deviation is assumed to be proportional to the DA renewable generation dispatch. The specific setting will be detailed in the section on the case study. As a result, the chance constraint pertaining to renewable power production (5.38) should be rewritten as below:

$$g_{i,d,t,n}^{dev} \geq \Phi^{-1}(1 - \rho^{RE}) \Gamma g_{i,d,t,n}^{DA} \quad \forall i \in I^{RE}, \forall d, \forall t, \forall n \quad (5.130)$$

where Γ identifies the proportion of wind power deviation with respect to DA renewable generation dispatch.

Thus, the KKT stationarity condition (5.45) should be reformulated as follows:

$$-\lambda_{d,t,n}^{DA} + \theta_{i,d,t,n} + \Phi^{-1}(1 - \rho^{RE}) \Gamma \chi_{i,d,t,n}^{RE} = 0 \quad \forall i \in I^{RE}, \forall d, \forall t, \forall n \quad (5.131)$$

Then, rearranging the terms in the KKT stationarity condition (5.131) and multiplying

both sides by $g_{i,d,t,n}^{DA}$ ($i \in I^{RE}$), the energy revenue from renewable generation is recast as below:

$$\lambda_{d,t,n}^{DA} g_{i,d,t,n}^{DA} = \theta_{i,d,t,n} g_{i,d,t,n}^{DA} + \Phi^{-1}(1 - \rho^{RE}) \Gamma \chi_{i,d,t,n}^{RE} g_{i,d,t,n}^{DA} \quad \forall i \in I^{RE}, \forall d, \forall t, \forall n \quad (5.132)$$

By substituting the LL1 constraint (5.18) into the above expression (5.132), it further produces:

$$\lambda_{d,t,n}^{DA} g_{i,d,t,n}^{DA} = \theta_{i,d,t,n} k_{i,d,t}^{RE} G_{i,n} + \Phi^{-1}(1 - \rho^{RE}) \Gamma \chi_{i,d,t,n}^{RE} g_{i,d,t,n}^{DA} \quad \forall i \in I^{RE}, \forall d, \forall t, \forall n \quad (5.133)$$

In conclusion, the bilinear DA energy revenue, which is a critical component of the GenCo's profit, can be equivalently expressed by combining the expressions (5.126), (5.129), and (5.133) with respect to different generation technologies.

The next step is to deal with the bilinear terms related to the RT reserve revenues $\sum_{i \in I^{FL}} \lambda_{d,t,n}^{RT} g_{i,d,t,n,b}^U$ and $\sum_{i \in I^{FL}} \lambda_{d,t,n}^{RT} g_{i,d,t,n,b}^D$. To this end, we first rearrange the terms in the KKT stationarity conditions (5.46) and (5.47), and then multiplying both sides by $g_{i,d,t,n,b}^U$ and $g_{i,d,t,n,b}^D$, respectively, it renders:

$$\begin{aligned} \lambda_{d,t,n}^{RT} g_{i,d,t,n,b}^U = & C_{i,b}^U g_{i,d,t,n,b}^U + \eta_i^{CO_2} \zeta_d g_{i,d,t,n,b}^U + \mu_{i,d,t,n,b}^U g_{i,d,t,n,b}^U - \phi_{i,d,t,n,b}^{U,-} g_{i,d,t,n,b}^U + \\ & \phi_{i,d,t,n,b}^{U,+} g_{i,d,t,n,b}^U \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \end{aligned} \quad (5.134)$$

$$\begin{aligned} \lambda_{d,t,n}^{RT} g_{i,d,t,n,b}^D = & C_{i,b}^D g_{i,d,t,n,b}^D + \eta_i^{CO_2} \zeta_d g_{i,d,t,n,b}^D - \mu_{i,d,t,n,b}^D g_{i,d,t,n,b}^D + \phi_{i,d,t,n,b}^{D,-} g_{i,d,t,n,b}^D - \\ & \phi_{i,d,t,n,b}^{D,+} g_{i,d,t,n,b}^D \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \end{aligned} \quad (5.135)$$

Furthermore, by employing the KKT complementarity conditions (5.56) - (5.59), we obtain the following expressions:

$$g_{i,d,t,n,b}^U \phi_{i,d,t,n,b}^{U,-} = 0 \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (5.136)$$

$$G_{i,b}^U \phi_{i,d,t,n,b}^{U,+} = g_{i,d,t,n,b}^U \phi_{i,d,t,n,b}^{U,+} \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (5.137)$$

$$g_{i,d,t,n,b}^D \phi_{i,d,t,n,b}^{D,-} = 0 \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (5.138)$$

$$G_{i,b}^D \phi_{i,d,t,n,b}^{D,+} = g_{i,d,t,n,b}^D \phi_{i,d,t,n,b}^{D,+} \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (5.139)$$

By substituting KKT complementarity conditions (5.136)-(5.139) into the equations for RT upward and downward reserve revenues, denoted as (5.134) and (5.135) respec-

tively, it gives:

$$\lambda_{d,t,n}^{RT} g_{i,d,t,n,b}^U = C_{i,b}^U g_{i,d,t,n,b}^U + \eta_i^{CO_2} \zeta_d g_{i,d,t,n,b}^U + \mu_{i,d,t,n,b}^U g_{i,d,t,n,b}^U + \phi_{i,d,t,n,b}^{U,+} G_{i,b}^U \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (5.140)$$

$$\lambda_{d,t,n}^{RT} g_{i,d,t,n,b}^D = C_{i,b}^D g_{i,d,t,n,b}^D + \eta_i^{CO_2} \zeta_d g_{i,d,t,n,b}^D - \mu_{i,d,t,n,b}^D g_{i,d,t,n,b}^D - \phi_{i,d,t,n,b}^{D,+} G_{i,b}^D \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (5.141)$$

After this, the bilinear term of the renewable deviation penalty $\sum_{i \in I^{RE}} \lambda_{d,t,n}^{RT} g_{i,d,t,n}^{dev}$ should be addressed. To this end, we employ the KKT complementarity condition (5.72) related to the uncertainty of renewable power production and reformulate it as follows:

$$[-g_{i,d,t,n}^{dev} - \Phi^{-1}(1 - \rho^{RE})\Gamma g_{i,d,t,n}^{DA}] \chi_{i,d,t,n}^{RE} = 0 \quad \forall i \in I^{RE}, \forall d, \forall t, \forall n \quad (5.142)$$

By combining the KKT stationarity condition (5.49) with the complementary expression (5.142), the bilinear term $\lambda_{d,t,n}^{RT} g_{i,d,t,n}^{dev}$ is recast as follows:

$$\lambda_{d,t,n}^{RT} g_{i,d,t,n}^{dev} = \chi_{i,d,t,n}^{RE} g_{i,d,t,n}^{dev} = -\Phi^{-1}(1 - \rho^{RE})\Gamma g_{i,d,t,n}^{DA} \chi_{i,d,t,n}^{RE} \quad \forall i \in I^{RE}, \forall d, \forall t, \forall n \quad (5.143)$$

Collectively, an alternative representation for the first four bilinear terms in the MPEC's objective function (the left side of the following equation) can be derived by integrating the expressions of (5.126), (5.129), (5.133), (5.140), (5.141), (5.143), and summing over their respective indices, which is formulated as follows:

$$\begin{aligned} & \sum_{i \in I, d, t, n, b} \lambda_{d,t,n}^{DA} g_{i,d,t,n,b}^{DA} + \sum_{i \in I^{FL}, d, t, n, b} \lambda_{d,t,n}^{RT} (g_{i,d,t,n,b}^U - g_{i,d,t,n,b}^D) - \sum_{i \in I^{RE}, d, t, n} \lambda_{d,t,n}^{RT} g_{i,d,t,n}^{dev} = \\ & \sum_{i \in I, d, t, n, b} C_{i,b}^{EN} g_{i,d,t,n,b}^{DA} + \sum_{i \in I^{FL}, d, t, n, b} C_{i,b}^U g_{i,d,t,n,b}^U - \sum_{i \in I^{FL}, d, t, n, b} C_{i,b}^D g_{i,d,t,n,b}^D + \\ & \sum_{i \in I^{FL}, d, t, n, b} \eta_i^{CO_2} \zeta_d (g_{i,d,t,n,b}^{DA} + g_{i,d,t,n,b}^U - g_{i,d,t,n,b}^D) + \sum_{i \in I^{FL}, d, t, n, b} \phi_{i,d,t,n,b}^{U,+} G_{i,b}^U + \\ & \sum_{i \in I^{FL}, d, t, n, b} \phi_{i,d,t,n,b}^{D,+} G_{i,b}^D + \sum_{i \in I^{FL}, d, t, n, b} \mu_{i,d,t,n,b}^D (g_{i,d,t,n,b}^D - g_{i,d,t,n,b}^{DA}) + \\ & \sum_{i \in I^{FL}, d, t, n, b} \mu_{i,d,t,n,b}^U (g_{i,d,t,n,b}^{DA} + g_{i,d,t,n,b}^U) + \sum_{i \in I^{MR}, d, t, n} \gamma_{i,d,t,n} G_{i,n} + \sum_{i \in I^{RE}, d, t, n} \theta_{i,d,t,n} k_{i,d,t}^{RE} G_{i,n} \end{aligned} \quad (5.144)$$

It is observable that some nonlinear terms on the right side of equation (5.144) can be eliminated through the application of the KKT complementarity conditions (5.53), (5.54)

and (5.55), which are expressed as below:

$$\zeta_d \eta^{Grid} \sum_{j,t,o} L_{j,d,t,o}^{DA} = \zeta_d \sum_{i \in I^{FL}, t, n, b} \eta_i^{CO_2} (g_{i,d,t,n,b}^{DA} + g_{i,d,t,n,b}^U - g_{i,d,t,n,b}^D) \quad \forall d \quad (5.145)$$

$$\mu_{i,d,t,n,b}^U G_{i,n,b} = \mu_{i,d,t,n,b}^U (g_{i,d,t,n,b}^{DA} + g_{i,d,t,n,b}^U) \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (5.146)$$

$$\mu_{i,d,t,n,b}^D (g_{i,d,t,n,b}^{DA} - g_{i,d,t,n,b}^D) = 0 \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (5.147)$$

By substituting the above KKT complementarity conditions (5.145)-(5.147) into the equation (5.144), the representation for the first four bilinear terms in the MPEC's objective function can be partially linearized, which is given as follows:

$$\begin{aligned} & \sum_{i \in I, d, t, n, b} \lambda_{d,t,n}^{DA} g_{i,d,t,n,b}^{DA} + \sum_{i \in I^{FL}, d, t, n, b} \lambda_{d,t,n}^{RT} (g_{i,d,t,n,b}^U - g_{i,d,t,n,b}^D) - \sum_{i \in I^{RE}, d, t, n} \lambda_{d,t,n}^{RT} g_{i,d,t,n}^{dev} = \\ & \sum_{i \in I, d, t, n, b} C_{i,b}^{EN} g_{i,d,t,n,b}^{DA} + \sum_{i \in I^{FL}, d, t, n, b} C_{i,b}^U g_{i,d,t,n,b}^U - \sum_{i \in I^{FL}, d, t, n, b} C_{i,b}^D g_{i,d,t,n,b}^D + \\ & \sum_{j,d,t,o} \eta^{Grid} \zeta_d L_{j,d,t,o}^{DA} + \sum_{i \in I^{FL}, d, t, n, b} \phi_{i,d,t,n,b}^{U,+} G_{i,b}^U + \sum_{i \in I^{FL}, d, t, n, b} \phi_{i,d,t,n,b}^{D,+} G_{i,b}^D + \\ & \sum_{i \in I^{FL}, d, t, n, b} \mu_{i,d,t,n,b}^U G_{i,n,b} + \sum_{i \in I^{MR}, d, t, n} \gamma_{i,d,t,n} G_{i,n} + \sum_{i \in I^{RE}, d, t, n} \theta_{i,d,t,n} k_{i,d,t}^{RE} G_{i,n} \end{aligned} \quad (5.148)$$

However, it is noticeable that the presence of the nonlinearities posed by the last three bilinear terms of equation (5.148) needs to be further addressed. In order to remove these non-linearities, the strong duality theorem of the LL1 problem incorporating demand flexibility constraints is utilised, which is expressed as follows:

$$\begin{aligned} & \sum_{i,d,t,n,b} C_{i,b}^{EN} g_{i,d,t,n,b}^{DA} + \sum_{i,d,t,n,b} (C_{i,b}^U g_{i,d,t,n,b}^U - C_{i,b}^D g_{i,d,t,n,b}^D) = - \sum_{j,d,t,o} \eta^{Grid} \zeta_d L_{j,d,t,o}^{DA} \\ & - \sum_{i \in I^{FL}, d, t, n, b} \mu_{i,d,t,n,b}^U G_{i,n,b} - \sum_{i \in I^{FL}, d, t, n, b} \phi_{i,d,t,n,b}^{U,+} G_{i,b}^U - \sum_{i \in I^{FL}, d, t, n, b} \phi_{i,d,t,n,b}^{D,+} G_{i,b}^D \\ & - \sum_{i \in I^{MR}, d, t, n} \gamma_{i,d,t,n} G_{i,n} - \sum_{i \in I^{RE}, d, t, n} \theta_{i,d,t,n} k_{i,d,t}^{RE} G_{i,n} - \sum_{d,t,n,m \in \Lambda_n} C_{nm}^{max} (\nu_{d,t,n,m}^{DA,-} + \nu_{d,t,n,m}^{DA,+}) \\ & - \sum_{d,t,n} \pi (\xi_{d,t,n}^{DA,-} + \xi_{d,t,n}^{DA,+}) - \sum_{d,t,n,m \in \Lambda_n} C_{nm}^{max} (\nu_{d,t,n,m}^{RT,-} + \nu_{d,t,n,m}^{RT,+}) - \sum_{d,t,n} \pi (\xi_{d,t,n}^{RT,-} + \xi_{d,t,n}^{RT,+}) \\ & - \sum_{j,d,t,o} (1 + \alpha) L_{j,d,t,o}^{DA} \psi_{j,d,t,o}^+ + \sum_{j,d,t,o} (1 - \alpha) L_{j,d,t,o}^{DA} \psi_{j,d,t,o}^- + \sum_{j,d,t} \alpha [\Phi^{-1} (1 - \rho^L) s_{j,d,t}^L - \\ & m_{j,d,t}^L] (\chi_{j,d,t}^{SH,-} + \chi_{j,d,t}^{SH,+}) + \sum_{j,d,t} [\Phi^{-1} (1 - \rho^L) s_{j,d,t}^L + m_{j,d,t}^L] \chi_{j,d,t}^L \end{aligned} \quad (5.149)$$

By employing the strong duality equation (5.149), the bilinear function (left side of

the following expression) can be linearized as below:

$$\begin{aligned}
& \sum_{i \in I, d, t, n, b} \lambda_{d, t, n}^{DA} g_{i, d, t, n, b}^{DA} + \sum_{i \in I^{FL}, d, t, n, b} \lambda_{d, t, n}^{RT} (g_{i, d, t, n, b}^U - g_{i, d, t, n, b}^D) - \sum_{i \in I^{RE}, d, t, n} \lambda_{d, t, n}^{RT} g_{i, d, t, n}^{dev} = \\
& - \sum_{d, t, n, m \in \Lambda_n} C_{nm}^{max} (\nu_{d, t, n, m}^{DA, -} + \nu_{d, t, n, m}^{DA, +}) - \sum_{d, t, n, m \in \Lambda_n} C_{nm}^{max} (\nu_{d, t, n, m}^{RT, -} + \nu_{d, t, n, m}^{RT, +}) \\
& - \sum_{d, t, n} \pi(\xi_{d, t, n}^{DA, -} + \xi_{d, t, n}^{DA, +}) - \sum_{d, t, n} \pi(\xi_{d, t, n}^{RT, -} + \xi_{d, t, n}^{RT, +}) - \sum_{j, d, t, o} (1 + \alpha) L_{j, d, t, o}^{DA} \psi_{j, d, t, o}^+ \\
& + \sum_{j, d, t, o} (1 - \alpha) L_{j, d, t, o}^{DA} \psi_{j, d, t, o}^- + \sum_{j, d, t} \alpha [\Phi^{-1}(1 - \rho^L) s_{j, d, t}^L - m_{j, d, t}^L] (\chi_{j, d, t}^{SH, -} + \chi_{j, d, t}^{SH, +}) \\
& + \sum_{j, d, t} [\Phi^{-1}(1 - \rho^L) s_{j, d, t}^L + m_{j, d, t}^L] \chi_{j, d, t}^L
\end{aligned} \tag{5.150}$$

The final step is to handle the last bilinear term related to the TGC revenue $\sum_{i \in I^{RE}} \lambda_{i, d}^{PGC} Q_{i, d}^{PGC}$. It can be converted into a quadratic term by adopting LL2 constraint (5.31), which is presented as follows:

$$\sum_{i \in I^{RE}, d} \lambda_{i, d}^{PGC} Q_{i, d}^{PGC} = \sum_{i \in I^{RE}, d} [l^{PGC} Q_{i, d}^{PGC} - q^{PGC} (Q_{i, d}^{PGC})^2] \tag{5.151}$$

Finally, by the substitution of expressions from (5.150) and (5.151) into the bilinear MPEC's objective function (5.74), the GenCo's profit is reduced to a linearized form:

$$\begin{aligned}
& \sum_d \omega_d \left\{ - \sum_{i, t, n, b} C_{i, b}^{EN} g_{i, d, t, n, b}^{DA} - \sum_{i, t, n, b} (C_{i, b}^U g_{i, d, t, n, b}^U - C_{i, b}^D g_{i, d, t, n, b}^D) + \sum_{j, t, o} \lambda_{j, d, t}^{DA} L_{j, d, t, o}^{DA} \right. \\
& - \sum_{t, n, m \in \Lambda_n} C_{nm}^{max} (\nu_{d, t, n, m}^{DA, -} + \nu_{d, t, n, m}^{DA, +}) - \sum_{t, n, m \in \Lambda_n} C_{nm}^{max} (\nu_{d, t, n, m}^{RT, -} + \nu_{d, t, n, m}^{RT, +}) \\
& - \sum_{t, n} \pi(\xi_{d, t, n}^{DA, -} + \xi_{d, t, n}^{DA, +}) - \sum_{t, n} \pi(\xi_{d, t, n}^{RT, -} + \xi_{d, t, n}^{RT, +}) - \sum_{j, t, o} (1 + \alpha) L_{j, d, t, o}^{DA} \psi_{j, d, t, o}^+ \\
& + \sum_{j, t, o} (1 - \alpha) L_{j, d, t, o}^{DA} \psi_{j, d, t, o}^- + \sum_{j, t} \alpha [\Phi^{-1}(1 - \rho^L) s_{j, d, t}^L - m_{j, d, t}^L] (\chi_{j, d, t}^{SH, -} + \chi_{j, d, t}^{SH, +}) \\
& + \sum_{j, t} [\Phi^{-1}(1 - \rho^L) s_{j, d, t}^L + m_{j, d, t}^L] \chi_{j, d, t}^L - \sum_{i \in I^{FL}, t, n} \lambda^{CO_2} Q_{i, d, t, n}^{emi} + \sum_{i \in I^{RE}, t, n} \lambda^{CO_2} Q_{i, d, t, n}^{CA} \\
& \left. + \sum_{i \in I^{RE}} [l^{PGC} Q_{i, d}^{PGC} - q^{PGC} (Q_{i, d}^{PGC})^2 + \lambda^{FGC} Q_{i, d}^{FGC}] \right\} - \sum_{i, n} C_i^{IN} G_{i, n}
\end{aligned} \tag{5.152}$$

5.4.4 Final MIQP

Through the linearization process, the MPEC is successfully converted into a MIQP. This transformation enables efficient resolution of the optimisation problem. The MIQP's objective function, (5.153), preserves the GenCo's profit maximisation strategy. The vari-

able set for the MIQP, denoted as (5.154), merges the variable set from the MPEC problem (5.75), with the set of auxiliary binary variables (5.82), which were introduced during the linearization of the LL1 KKT complementarity conditions. The constraints of the MIQP are organised into three categories: i) original constraints from the UL problem (5.155), ii) linearized KKT optimality conditions of the LL1 problem (5.156), and iii) constraints from the LL2 problem (5.157). The final MIQP is formulated as follows:

$$\begin{aligned}
\min_{V^{MIQP}} \sum_d \omega_d \Big\{ & - \sum_{i,t,n,b} C_{i,b}^{EN} g_{i,d,t,n,b}^{DA} - \sum_{i,t,n,b} (C_{i,b}^U g_{i,d,t,n,b}^U - C_{i,b}^D g_{i,d,t,n,b}^D) + \sum_{j,t,o} \lambda_{j,d,t}^{DA} L_{j,d,t,o}^{DA} \\
& - \sum_{t,n,m \in \Lambda_n} C_{nm}^{max} (\nu_{d,t,n,m}^{DA,-} + \nu_{d,t,n,m}^{DA,+}) - \sum_{t,n,m \in \Lambda_n} C_{nm}^{max} (\nu_{d,t,n,m}^{RT,-} + \nu_{d,t,n,m}^{RT,+}) \\
& - \sum_{t,n} \pi (\xi_{d,t,n}^{DA,-} + \xi_{d,t,n}^{DA,+}) - \sum_{t,n} \pi (\xi_{d,t,n}^{RT,-} + \xi_{d,t,n}^{RT,+}) - \sum_{j,t,o} (1 + \alpha) L_{j,d,t,o}^{DA} \psi_{j,d,t,o}^+ \\
& + \sum_{j,t,o} (1 - \alpha) L_{j,d,t,o}^{DA} \psi_{j,d,t,o}^- + \sum_{j,t} \alpha [\Phi^{-1}(1 - \rho^L) s_{j,d,t}^L - m_{j,d,t}^L] (\chi_{j,d,t}^{SH,-} + \chi_{j,d,t}^{SH,+}) \\
& + \sum_{j,t} [\Phi^{-1}(1 - \rho^L) s_{j,d,t}^L + m_{j,d,t}^L] \chi_{j,d,t}^L - \sum_{i \in I^{FL}, t, n} \lambda^{CO_2} Q_{i,d,t,n}^{emi} \\
& + \sum_{i \in I^{RE}, t, n} \lambda^{CO_2} Q_{i,d,t,n}^{CA} + \sum_{i \in I^{RE}} [l^{PGC} Q_{i,d}^{PGC} - q^{PGC} (Q_{i,d}^{PGC})^2 + \lambda^{FGC} Q_{i,d}^{FGC}] \Big\} \\
& - \sum_{i,n} C_i^{IN} G_{i,n}
\end{aligned} \tag{5.153}$$

where:

$$V^{MIQP} = V^{MPEC} \cup V^{BINARY} \tag{5.154}$$

subject to:

$$\text{UL constraints : (5.3) – (5.7)} \tag{5.155}$$

LL1 KKT conditions :

$$(5.10) – (5.11), (5.17) – (5.18), (5.21), (5.24), (5.26), (5.42) – (5.52), (5.83) – (5.124) \tag{5.156}$$

$$\text{LL2 constraints : (5.31) – (5.32)} \tag{5.157}$$

5.5 Case Studies

5.5.1 Data Descriptions and Implementation

This set of experiments is organised in the 2021 GB deregulated market environment. Therefore, the data from the 2021 GB power system is detailed below.

Table 5.1 presents comprehensive data about investment options for the GenCo. It is noteworthy that in terms of flexible generation technologies, the marginal cost for providing upward reserve is set at 1.1 times the marginal energy cost; conversely, the marginal cost for providing downward reserve is set at 0.9 times the marginal energy cost.

Table 5.1: Parameters of investment options.

Investment option		CCGT1	CCGT2	OCGT1	OCGT2	Nuclear	Wind
Type		Flexible	Flexible	Flexible	Flexible	Must-run	Renewable
Investment cost C^{IN} (£/MW/yr)		14413	17440	8610	10418	114849	35756
Carbon intensity η^{CO_2} (t/MWh)		0.39	0.39	0.65	0.65	0	0
Maximum upward reserve capacity G^U (MW)		4000	8000	4000	8000	-	-
Maximum downward reserve capacity G^D (MW)		4000	8000	4000	8000	-	-
Marginal energy cost C^{EN} (£/MWh)	Block 1	7.43	20.05	40.99	49.25	4.72	0
	Block 2	9.03	24.34	49.78	59.81		
	Block 3	10.62	28.64	58.56	70.36		
	Block 4	12.21	32.94	67.34	80.91		
	Block 5	13.81	37.23	76.13	91.47		

Table 5.2 displays crucial parameters for the 2021 GB power system. Additionally, the allocation of weights to these blocks is set as [0.1, 0.15, 0.2, 0.25, 0.3], and the demand is also accurately modelled by the five demand blocks, incorporating weighting factors as [0.6, 0.1, 0.1, 0.1, 0.1]. By virtue of computational burden, the demand profile is simulated on a representative day with an hourly resolution ($T = \{1, 2, \dots, 24\}$), scaled across a target year ($\omega = 365$) to capture daily fluctuations in energy consumption and production patterns.

Table 5.2: Parameters of the GB power grid in 2021.

Year	2021
System-wide carbon intensity η^{Grid} (t/MWh)	183.11
Carbon price λ^{CO_2} (£/t)	71.54
Total system demand (TWh)	248.30

The additional parameters associated with carbon allowance and TGC are essential. Specifically, the carbon allowance allocation ratio, ϵ_i , is set as 0.4; the RPS requirement ratio, γ^{RPS} , is set as 40%; the approximated price ratio in the future TGC market, γ^{FGC} ,

is set as 0.3; and the price coefficient in the present TGC market, θ^{PGC} , is also set as 0.3. It should be noted that the penalty price for RPS deficiency, λ_0^{RPS} , is assumed to be 1.1 times the carbon price.

The related coefficients used in the chance constraints should be given. Overall, the model sets confidence levels for all the chance constraints at 95% ($1 - \rho^L = 1 - \rho^{RE} = 95\%$), conveying the ρ -reliable threshold for both demand and wind power production uncertainty. The linearized CCP formulations incorporate stochastic variables for wind power and demand realisations, which are modelled using two normal distributions, $\mathcal{N}(m^{RE}, (s^{RE})^2)$ for wind power and $\mathcal{N}(m^L, (s^L)^2)$ for demand. The means of these distributions (m^L and m^{RE}) are aligned with the baseline demand and DA power dispatch, respectively. Meanwhile, the standard deviations (s^L and s^{RE}), set at 1% of the baseline demand and 10% of the DA wind power dispatch, respectively, quantify the anticipated variability around these means.

This section is dedicated to exploring the impact of different levels of demand flexibility ($\alpha = 0\%, 5\%, 15\%$) on integrating renewable energy expansion within the 2021 GB electricity system. It also aims to assess the market clearing outcomes and the business cases under these varying degrees of demand flexibility, contributing to the acceleration of GB's transition towards a sustainable and clean power system. To align with this overarching scope, three distinct cases related to demand flexibility are designed as follows:

1. **Zero demand flexibility (Z-DF) case:** The demand side exhibits no flexibility, quantitatively represented by $\alpha = 0\%$. This case constitutes the benchmark for comparison.
2. **Low demand flexibility (L-DF) case:** The demand side manifests the low flexibility of $\alpha = 5\%$. This value is derived from the real-world data of the 2021 GB power system provided by NG FES [81].
3. **High demand flexibility (H-DF) case:** The demand side manifests the high flexibility of $\alpha = 15\%$.

5.5.2 Impact of Demand Flexibility

Notably, the comprehensive investment decisions and business results corresponding to the three demand flexibility scenarios are listed in Table 5.3. The system demand profiles with different levels of demand flexibility in the three scenarios are exhibited in Figure 5.2. Additionally, the clearing prices of the three demand flexibility scenarios across the DA and RT markets are shown in Figures 5.3 and 5.4, respectively.

Table 5.3: Investment decisions and business results for three demand flexibility scenarios regarding the 2021 GB power system.

	Z-DF case	L-DF case	H-DF case
CCGT1 (MW)	1,331	2,347	3,424
CCGT2 (MW)	-	-	-
OCGT1 (MW)	-	-	3,356
OCGT2 (MW)	13,050	9,868	3,877
Total flexible capacity (MW)	14,381	12,215	10,656
Nuclear (MW)	15,749	10,547	5,654
Wind (MW)	17,243	35,534	52,556
Investment cost (mil.£)	2,494	2,441	2,384
DA energy cost (mil.£)	5,055	4,771	4,027
DA energy revenue (mil.£)	19,835	21,451	22,578
RT reserve cost (mil.£)	177	108	112
RT reserve revenue (mil.£)	982	627	599
Wind power deviation penalty (mil.£)	637	1,313	1,822
Carbon allowance revenue (mil.£)	209	430	636
Green certificate revenue (mil.£)	2,044	2,997	3,837
Carbon emission cost (mil.£)	3,269	3,269	3,269
GenCo Profit (mil.£)	11,438	13,603	16,036

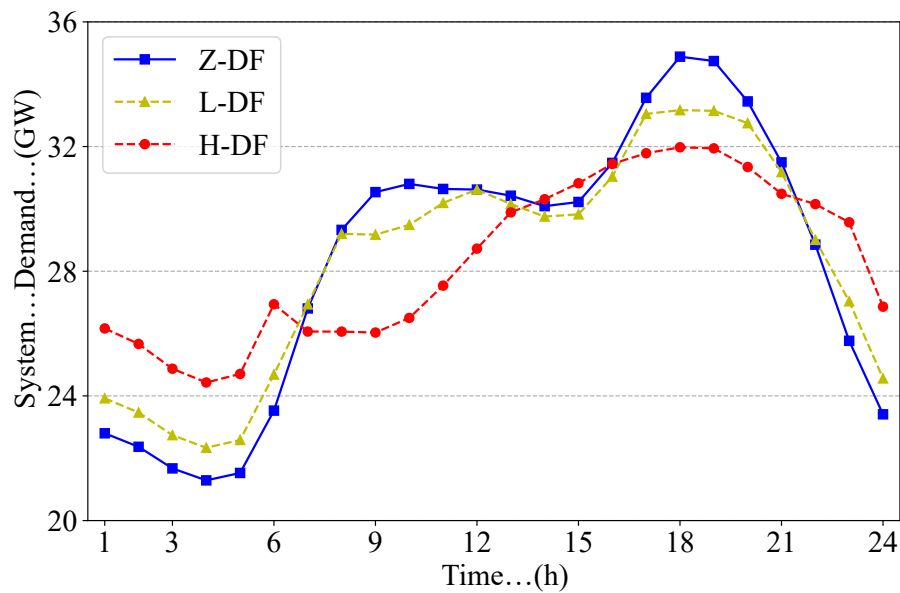


Figure 5.2: Daily system demand for three demand flexibility scenarios regarding the 2021 GB power system.

Readers may have noticed that the setup for the Z-DF case aligns with the market-driven case presented in Chapter 4. However, their emphases are different: the market-driven case focuses on market design, whereas the Z-DF case in this section concentrates on exploring the impact of demand flexibility on market-based generation planning. Read-

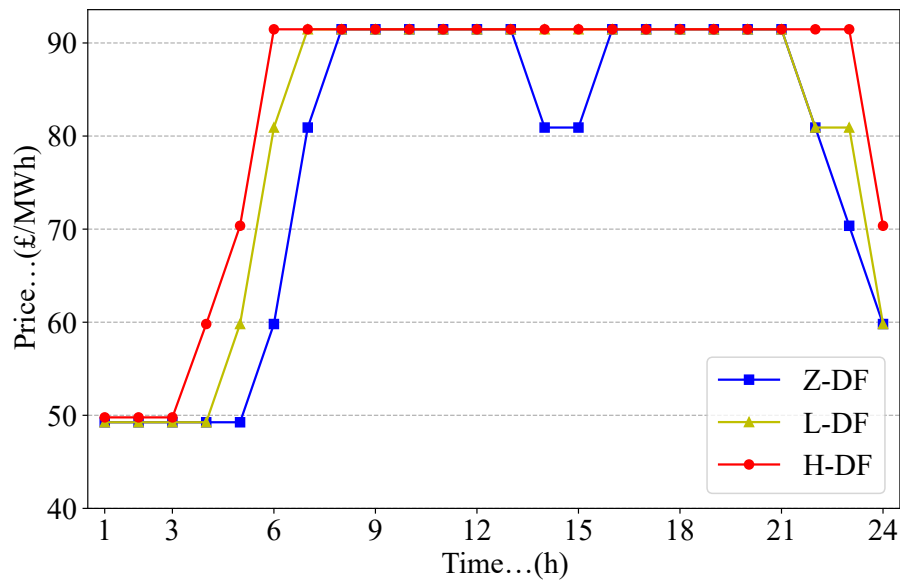


Figure 5.3: Daily DA energy market clearing prices for three demand flexibility scenarios in the 2021 GB power system.

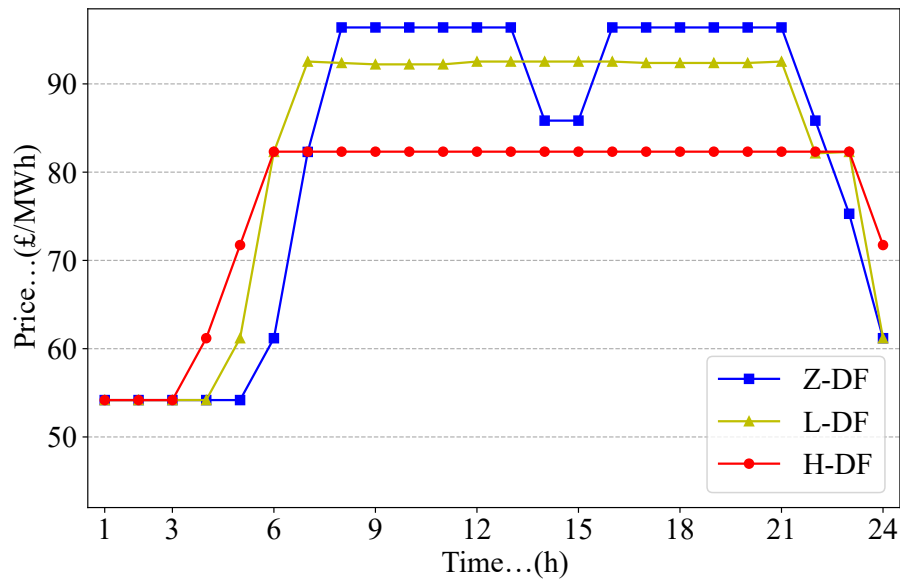


Figure 5.4: Daily RT balancing market clearing prices for three demand flexibility scenarios in the 2021 GB power system.

ers interested in the analysis of market design are referred to Subsection 4.5.2.

Z-DF Case Analysis

As outlined in the second to eighth row in the second column of Table 5.3, the optimal investment strategy still places significant emphasis on nuclear and flexible generation capacities, with wind power holding a lesser portion of the total generation mix. This

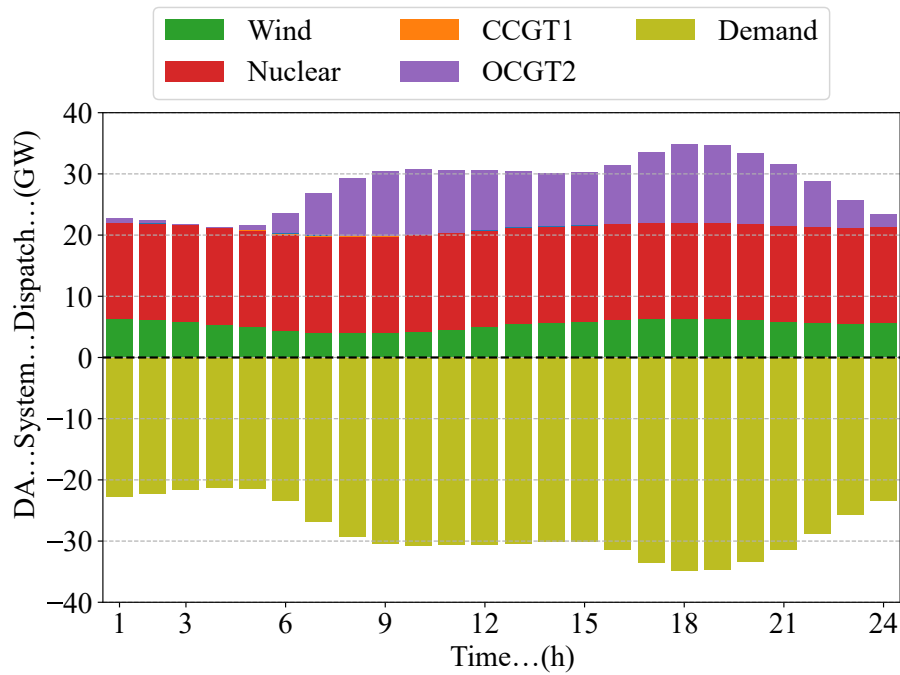


Figure 5.5: Daily generation dispatches in the DA energy market for the Z-DF case regarding the 2021 GB power system.

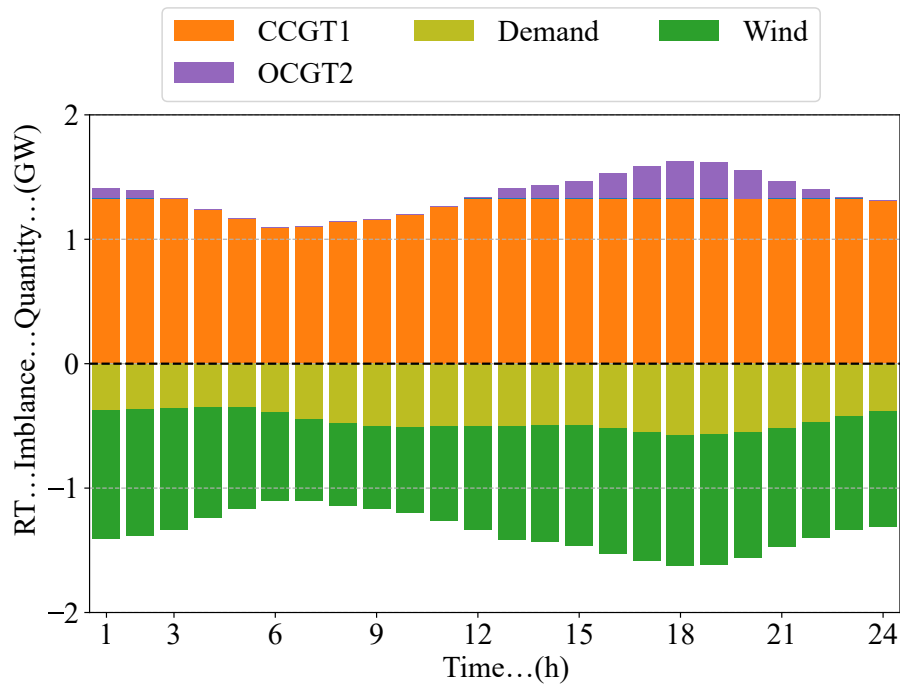


Figure 5.6: Daily imbalances in the RT balancing market for the Z-DF case regarding the 2021 GB power system.

preference stems from several key factors in the absence of demand flexibility, including:

1. Despite the benefits of zero energy costs and zero emissions, wind power requires

a relatively high investment cost, second only to nuclear power. This economic barrier makes wind power a less attractive investment option in the Z-DF case.

2. Wind power production is highly dependent on weather conditions, leading to potential mismatches with demand patterns. As depicted in Figure 5.5, an example of this is observed in the DA generation dispatch for the Z-DF case during hour 9, where there is a significant mismatch between peak demand periods and low wind power generation, attributed to a low normalised wind power output factor at the time.
3. The inherent variability and unpredictability of wind power necessitate higher reserve capacity in the RT balancing market, as shown in Figure 5.6. This requirement often leads to increased utilisation of flexible generation capacity, which may rely on fossil fuels, thereby raising pollution levels.

In the Z-DF case, under the 2021 carbon emission constraints for GB, the investment strategy limits the inclusion of high-pollution flexible generation capacity. On the other hand, nuclear power, despite its high costs and lack of operational flexibility, remains a competitive choice due to its emission-free operation and the absence of generation deviations. Furthermore, compared with other flexible generation technologies, optimal investment decisions reveal a reliance on flexible generation capacity with higher operational cost, i.e., OCGT2, making up 91% of the total flexible generation capacity. This trend is directly linked to the completely inflexible demand pattern in the power system. In this context, the flexible generation capacity fulfils three key roles:

1. In the DA energy market, it compensates for the fluctuations between demand peaks and valleys, ensuring the operational stability of power system.
2. In the RT balancing market, it offers the necessary deployment of reserves to manage the uncertainties introduced by demand and wind generation.
3. It plays a crucial role in the process of joint DA and RT market clearing, contributing to sustaining high market clearing prices across DA and RT markets, which in turn secures the profitability of the GenCo.

The demand profile in the Z-DF case, depicted in Figure 5.2, illustrates pronounced peaks and significant variations between these peaks and valleys due to the absence of demand flexibility. Typically, the extensive capacity of OCGT2 is deployed to bridge these variations in the DA market, whereas a smaller proportion of CCGT1 capacity is utilised primarily for reserve provision in the RT market, as shown in Figures 5.5 and

5.6, respectively. This strategic dispatch not only maintains operational stability but also maximises economic benefits. Specifically, in an inflexible demand scenario, heavy reliance on OCGT2 during the DA period ensures that the marginal capacity is frequently served by OCGT2 units, thereby rising DA market clearing prices over long durations, as observed in Figure 5.3. As a result of low deviations from wind power generation and demand, low-cost generation dispatches from OCGT1 units are sufficient for covering reserve requirements, while high RT market prices are attained through the strategic utilisation of flexible capacity coupling across DA and RT markets, as highlighted in Figure 5.4.

L-DF Case Analysis

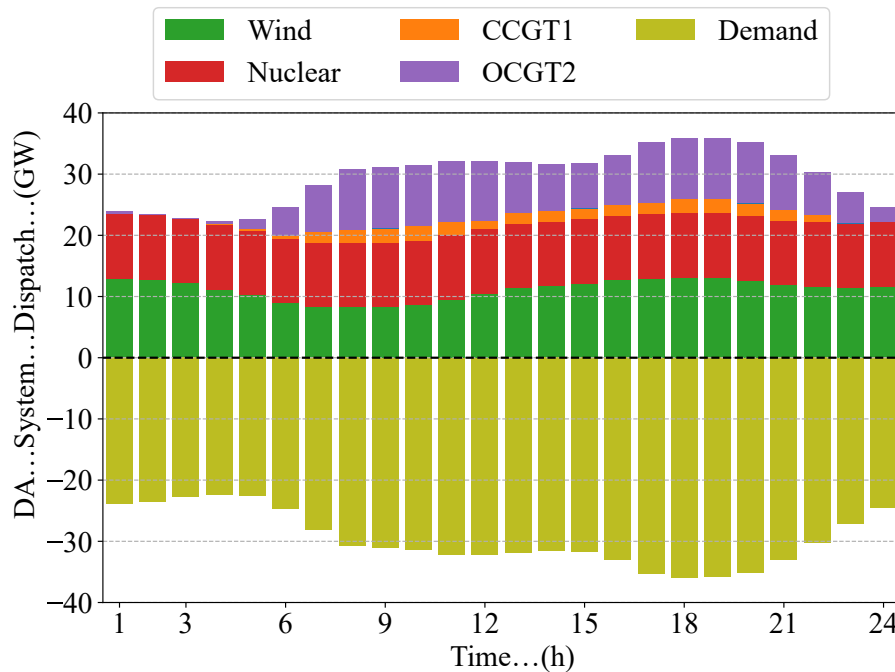


Figure 5.7: Daily generation dispatches in the DA energy market for the L-DF case regarding the 2021 GB power system.

The second to eighth row in the third column of Table 5.3 illustrates that with the introduction of demand flexibility in the L-DF case, there is a remarkable shift in the optimal investment mix towards increased wind capacity, alongside a reduction in nuclear capacity and a decrease in flexible generation capacity. This shift reveals the effect of demand flexibility on enhancing the competitiveness of wind energy, thereby motivating greater investment in wind power. Meanwhile, the increased competitiveness of wind power, due to demand flexibility, renders nuclear power, a high-investment-cost option, less attractive. In detail, this phenomenon is primarily attributed to the impact of demand flexibil-

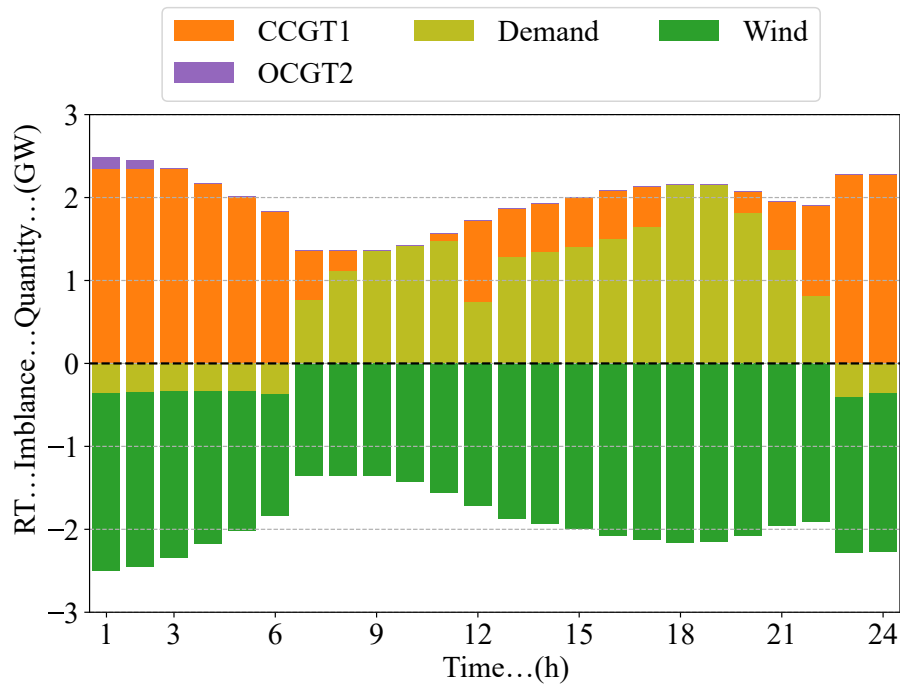


Figure 5.8: Daily imbalances in the RT balancing market for the L-DF case regarding the 2021 GB power system.

ity in the RT market, where increased demand flexibility expands the demand deviation, which can be used to counteract the uncertainty associated with wind power generation, as depicted in Figure 5.8.

The incorporation of demand flexibility offers substantial economic and environmental advantages. Economically, compared with the Z-DF case, utilising demand flexibility as a mechanism to counteract wind power uncertainty presents a cost-effective alternative to the conventional approach of relying on carbon-intensive flexible generation for reserve provision. Environmentally, replacing the need for reserves from high-emission flexible generation with demand deviation to balance wind power uncertainty directly contributes to reducing carbon emissions. This approach is especially critical in achieving the 2021 GB carbon emission targets, highlighting the dual benefits of demand flexibility in promoting a more economic and sustainable energy landscape.

In a power system including demand flexibility, a notable 'peak-flattening effect' on the demand profile is observed, as depicted in Figure 5.2. This effect arises from demand shifting, which effectively reduces peak demand and increases off-peak demand in adjacent time periods, thereby maintaining daily energy neutrality. Essentially, demand flexibility acts to smooth the demand profile, mitigating the difference between peak and off-peak demands and further reducing reliance on flexible generation capacity. In the context of the L-DF case, this smoothing of the demand profile leads to optimal invest-

ment decisions favouring a lower capacity of flexible generation technologies. Moreover, the smooth demand profile means that only a minimal amount of expensive flexible capacity is required to serve as the marginal capacity, which in turn maintains DA market clearing prices and boosts revenue for the self-interested GenCo, as shown in Figures 5.7 and 5.3. This effect of demand flexibility results in a decreased share of high-marginal-cost flexible generation (e.g., OCGT2), making up 81% of the total flexible capacity, a reduction from 91% seen in the Z-DF case.

Contrary to the trends in the DA market, the peak values of RT clearing prices in the L-DF case are found to be lower than those in the Z-DF case, as shown in Figure 5.4. This decrease is attributed to the utilisation of demand deviation as a cost-free balancing resource, effectively decreasing the marginal cost associated with reserve provision. Notably, the market clearing price profiles for both DA and RT markets exhibit a smoother pattern due to the peak-flattening effect, as illustrated in Figures 5.3 and 5.4.

The ninth to eighteenth row in the third column of Table 5.3 provides a detailed overview of the business outcomes in the L-DF case. Notably, the shift from Z-DF to L-DF does not manifest in significant changes in overall investment costs. This is because the reduction in costly nuclear capacity investment, as demand flexibility increases, is offset by a corresponding rise in wind capacity investment. In the DA energy market, compared with the Z-DF case, the enhancement in cost-free wind power generation facilitated by demand flexibility leads to a reduction in energy costs. Moreover, the consequent increase in market clearing prices substantially boosts energy revenue. In the RT balancing market, demand deviation acts as a zero-cost resource to counterbalance the uncertainty introduced by wind power, effectively diminishing the requirement for reserve provision. However, as previously discussed, RT market clearing prices experienced a decline. This results in a simultaneous decrease in both the cost and revenue associated with reserves. Additionally, the wind power deviation penalty grows with the increase in demand flexibility due to the heightened uncertainty from the significant growth in wind power capacity. Interestingly, carbon emission costs remain constant across all cases, as carbon emissions in each case reach the 2021 GB emission limit. The income derived from renewable promotion policies, including carbon allowance and TGC revenues, exhibits a marked increase in the L-DF case, boosted by the substantial growth in wind generation capacity. In conclusion, the expansion of demand flexibility in the L-DF case, despite leading to a higher wind power deviation penalty and reduced RT market revenue, brings about a significant uplift in DA energy profit and income from renewable promotion policies. This collectively boosts the overall profit for the examined GenCo.

H-DF Case Analysis

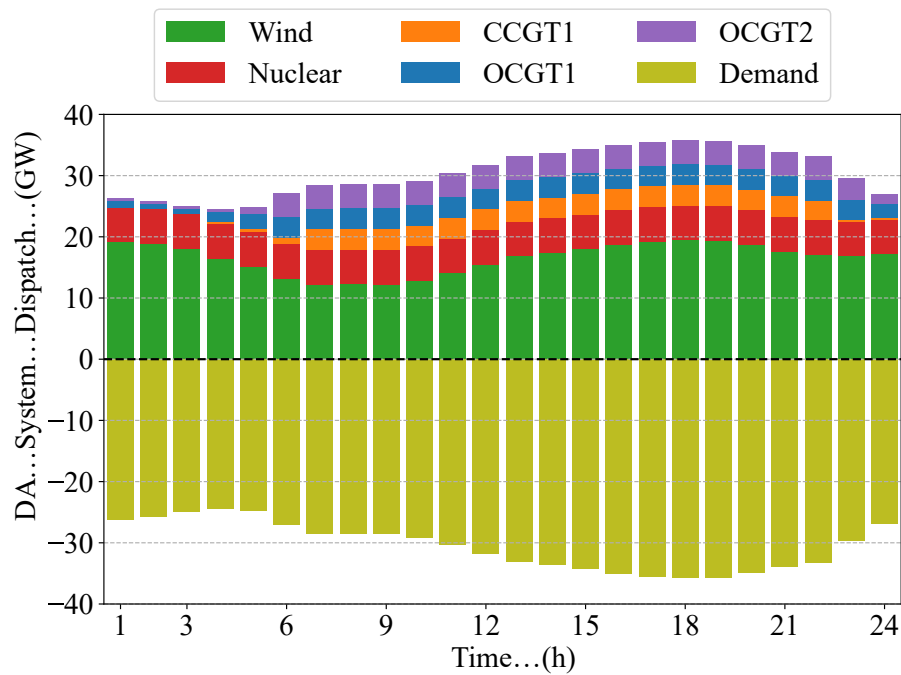


Figure 5.9: Daily generation dispatches in the DA energy market for the H-DF case regarding the 2021 GB power system.

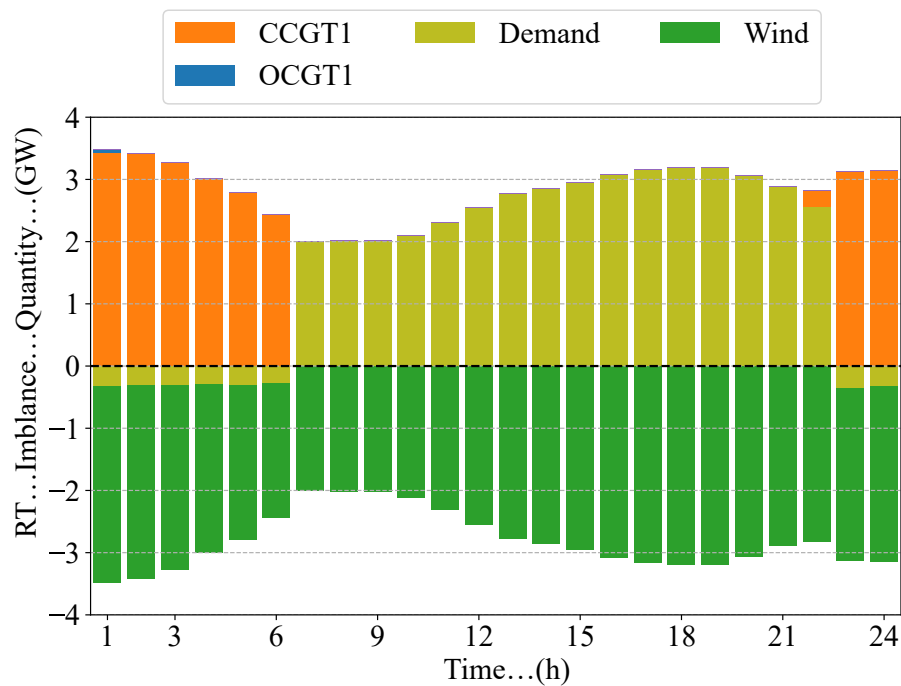


Figure 5.10: Daily imbalances in the RT balancing market for the H-DF case regarding the 2021 GB power system.

As demand flexibility is further increased in the H-DF case, there is a notable expansion in wind power capacity alongside a reduction in both nuclear and flexible generation capacities, as shown in the second to eighth row in the fourth column of Table 5.3. Moreover, the share of high-marginal-cost flexible generation (e.g., OCGT2) within the total flexible capacity significantly decreases to 54% in the H-DF case from 81% in the L-DF case. These investment trends observed in the L-DF case become even more remarkable in the H-DF case, coinciding with the analysis discussed in the L-DF case. In line with the L-DF case, wind power, as depicted in Figure 5.9, plays a dominant role in the DA generation dispatches. Figure 5.10 illustrates that, compared to the L-DF scenario, an abundance of demand flexibility is included in the H-DF case, enabling demand deviation to act as balancing resources during the hours 7-22. This enhanced flexibility leads to a smoother demand profile, as seen in Figure 5.2, facilitating the maintenance of DA market clearing prices at peak values for longer time periods, as shown in Figure 5.3. Consistent with the analysis in the L-DF case, peak prices in the RT market experience a further reduction, as demonstrated in Figure 5.4.

Interestingly, despite the availability of free balancing resources during hours 7-22 in the RT market, RT clearing prices do not drop to zero. This phenomenon occurs because, during these hours, the marginal reserve capacity continues to be provided by flexible technology (e.g., OCGT1). Specifically, the interrelationship of flexible generation capacity across both DA and RT markets implies that the utilisation of flexible generation to its near-maximum capacity in the DA market can still influence RT market clearing prices, even in the absence of flexible generation dispatch in the RT market.

The ninth to eighteenth row in the fourth column of Table 5.3 details the business results for the H-DF case. According to the analysis of the L-DF case, the increase in demand flexibility leads to similar trends across various components, with the exception of RT reserve costs. In the H-DF case, despite the majority of wind power deviation being neutralised by demand deviation, there remain time periods where reserve is supplied by flexible generation. Additionally, the marked increase in wind power capacity necessitates a higher reserve capacity, resulting in a rise in RT reserve costs compared to the L-DF case. Nevertheless, this rise in RT reserve cost is relatively modest compared with the substantial economic benefits incentivized by abundant demand flexibility in the H-DF case. Consequently, the GenCo's profit further grows in the H-DF case.

5.6 Conclusions

This chapter explores how demand flexibility influences market-driven generation planning under carbon emission restrictions. For this purpose, a two-stage bi-level optimisa-

tion model is formulated to conduct a quantitative assessment of GenCo's investment decisions and market outcomes under different levels of demand flexibility. The model consists of an UL problem, which characterises the profit-maximising GenCo, and two LL problems. The first LL (LL1) problem represents the joint clearing of the DA and RT markets, factoring in chance-constrained uncertainty and environmental regulations stemming from GB's carbon targets. Moreover, the time-coupling demand flexibility model is incorporated into the LL1 problem. The second LL (LL2) problem represents the clearing process for the TGC market. Following the linearization of the chance constraints, the model is transformed into a MPEC through the KKT conditions, subsequently being solved by recasting it as a MIQP utilising the strong duality theorem. The developed model serves as a tool for exploring GenCo's investment strategies in a system including flexible demand and offers insights into market design for policymakers. The case studies have been carried out in the context of the 2021 GB power system, and the findings are detailed as follows:

1. Compared to the scenario without demand flexibility, when demand flexibility is 5%, wind capacity expands by 106%, and with 15% flexibility, the expansion reaches 205%. This implies that a more flexible system can accommodate a larger share of wind power capacity, since demand flexibility can be strategically used to offset wind power uncertainties.
2. Compared to the scenario without demand flexibility, with 5% flexibility, profits increase by 19%, and with 15% flexibility, profits increase by 40%, indicating that demand flexibility can create a more efficient market environment to benefit the GenCo. Specifically, the demand flexibility brings about the so-called 'peak-flattening effect' in the DA market, which smooths the demand profile, thereby enabling the expensive generators to easily serve as the marginal capacities. Therefore, DA market clearing prices boost in many periods.

Chapter 6

Impact of Energy Storage

6.1 Introduction

ES stands as a vital innovation within the modern power system, particularly within the GB deregulated market. At its essence, ES involves the capture of energy produced at one time for use at a later time, a mechanism that is increasingly important in the context of the GB's plan for expanding the scale of ES solutions [82]. In the deregulated energy market with large-scale RESs, ES is a crucial flexibility resource for mitigating the intermittency of RESs and enhancing the overall reliability of the power system. In practice, the GB has recognised the critical role of ES in its transition to a low-carbon electricity system and has set ambitious plans to expand its scale. For example, the GB government funded £30 million to develop new ES technologies [83]. Additionally, the GB government has set an ambitious plan to achieve 30 GW of capacity by 2030, particularly with the utility-scale battery ES [84].

The ES model proposed in this chapter is incorporated into the LL1 problem, which indicates the joint clearing process for DA and RT markets. In detail, this dynamic ES model is designed to span both DA and RT market horizons, offering the capability of charging and discharging to provide operational flexibility. This function is essential for maintaining stability in a system with significant renewable energy penetration. By allowing for the storage of excess renewable energy during periods of low demand and its release for reserve deployment across the DA and RT markets, ES enhances the reliability of the power grid. Additionally, the ES entity can earn profit through energy arbitrage—buying electricity when prices are low and selling it when prices are high.

Under the GB's ambitious goal of a fully decarbonised power system, the market-driven generation planning model proposed in this chapter is employed to explore the impact of ES. To this end, the case studies accommodating different sizes of ES are car-

ried out based on the 2021 GB power system. Their results clearly demonstrate that ES has a triple function in the current energy market environment. Firstly, it enhances the integration of a higher capacity of renewable power into the system. Secondly, it increases GenCo's profitability. Thirdly, the profit of the ES entity increases through energy arbitrage, improving its economic viability.

6.2 Problem Statement

This section outlines the decision-making framework for market-driven generation planning, taking into account ES, and details the foundational assumptions for its modelling.

6.2.1 Structure of the Bi-Level Optimisation Model

This chapter presents a market-driven generation planning model utilising a two-stage, bi-level optimisation framework to explore the impact of ES in the current deregulated market environment, subject to carbon emission regulations. In brief, this bi-level optimisation framework reflects the interconnected dynamics between GenCo and market operators: the UL problem represents GenCo's profit-maximising pursuit; the first LL (LL1) problem indicates the joint clearing of the DA and RT markets; and the second LL (LL2) problem indicates the clearing of the green certificate market. This hierarchical structure is illustrated in Figure 6.1. Different from previously proposed market models, the market-driven generation planning model introduced in this chapter accommodates an ES model within its LL1 problem. Specifically, this ES model spans the DA and RT market horizons and is capable of charging and discharging in both DA and RT markets to provide operational flexibility for ensuring grid stability. Generally, the final energy level of the ES model is set to be the same as its initial energy level.

The interdependence between the UL problem and the two LL problems is pronounced. The investment decisions made by GenCo at the UL significantly affect the market outcomes realised within the two LL problems. In turn, the business cases emerging from the LL1 problem, characterised by energy prices and generation dispatches in both the DA and RT markets, along with the TGC quantities and prices obtained in the LL2 problem, critically influence GenCo's profitability. This bidirectional feedback loop underscores the complex relationship between generation planning and market performance, highlighting the feature of capturing real-world market dynamics and GenCo's decision-making processes.

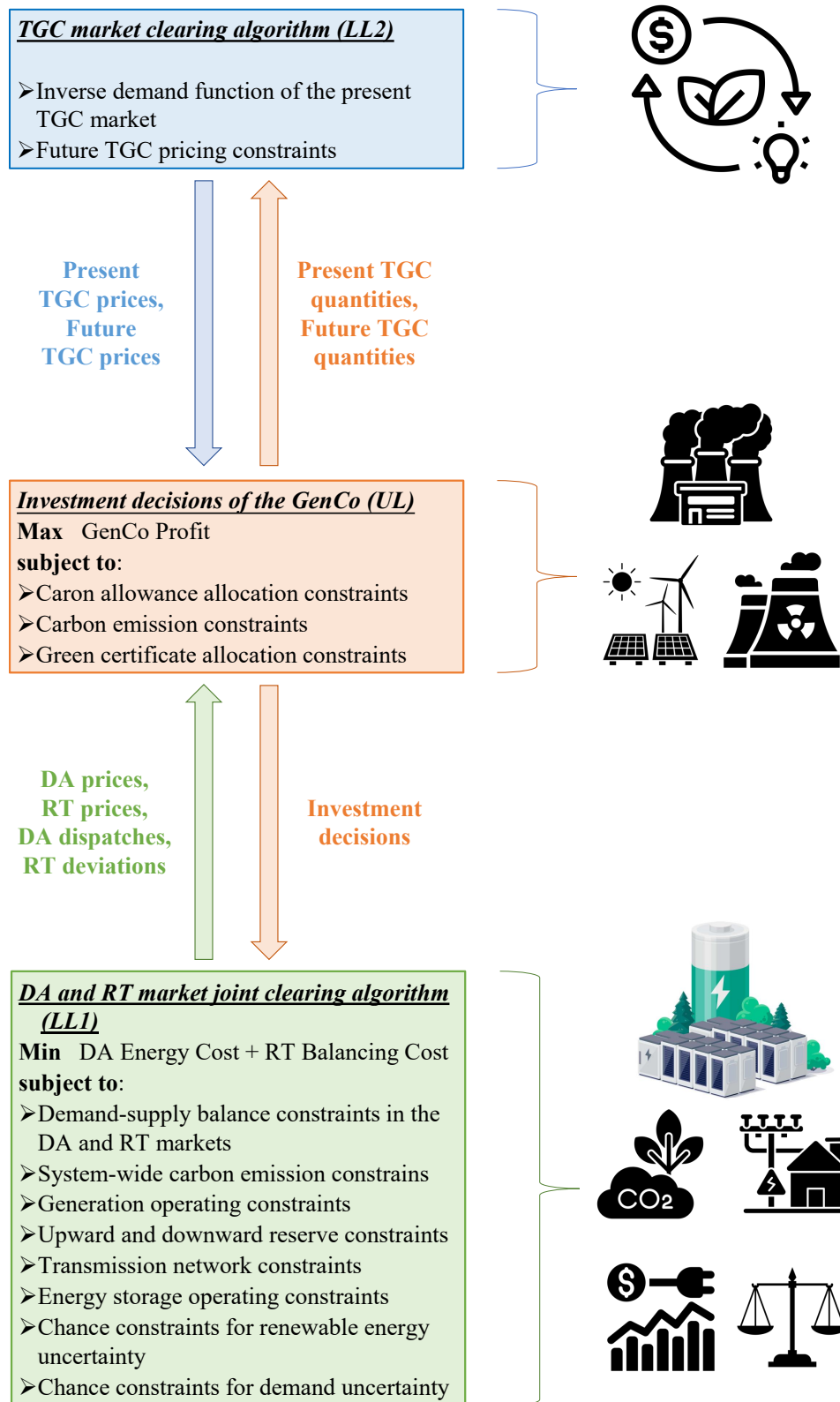


Figure 6.1: Illustration of the proposed two-stage bi-level optimisation model.

6.2.2 Modelling Assumptions

For clarity and completeness, the foundational assumptions supporting the proposed two-stage bi-level optimisation model are detailed below:

1. The proposed two-stage bi-level optimisation model represents the market-driven generation planning faced by a self-interested GenCo seeking to maximise its long-term profit. The GenCo is allowed to invest in generation capacities utilising flexible, must-run, and renewable technologies. Furthermore, the flexible generation capacities are explicitly modelled by the production blocks to capture their output characteristics. Accordingly, demand blocks are also incorporated into the proposed model.
2. The examined GenCo is permitted to participate in four different markets. The two-stage energy markets adopted for this work include the DA energy market and the RT balancing market with an hourly time resolution. Furthermore, the proposed model takes into consideration the present TGC market and the future TGC market.
3. Based on the study in [76], its pricing scheme, namely LMP, is applied within the proposed two-stage energy market framework. This approach leverages the LMPs for determining the electricity prices at different locations in the network, which are calculated using the dual variables of the hourly demand-supply balance in both the DA and RT markets. The so-called LMP scheme reflects the marginal cost of supplying the next unit of electricity at a specific location, considering the effect of network transmission congestion, thereby facilitating efficient market operations and signaling appropriate economic incentives.
4. The model incorporates the pricing mechanism outlined in [64] for trading in the present and future TGC markets. Specifically, the prices in the present TGC market are determined using an inverse demand function. Furthermore, the future TGC price is estimated based on the penalty price associated with RPS deficiency.
5. Once the GenCo finalises its investment decisions, the sequential clearing process of the DA, RT, and TGC markets is arranged by the market operator. Specifically, the market operator first clears the DA energy market, taking into account the uncertainties incurred by demand and renewable generation as well as scheduling the generation dispatches. Subsequently, the depolyment of reserves is organised by market operator in the RT market to eliminate the unexpected uncertainties arising from wind power and demand-side deviations. Following this, the prices and quantities of TGC are determined based on the clearing results of the DA and RT

markets. By comparing the present TGC market clearing price with the estimated future TGC market price, the examined GenCo can choose either to trade its TGCs in the present TGC market or retain them for future trading, thus completing the transactions in the TGC market. Finally, the outcomes of these three markets are relayed to the examined GenCo, significantly influencing its profit-oriented investment decision-making process.

6. Although the DA and RT markets are cleared sequentially from the perspective of the examined GenCo, investment decisions are required to maximise the overall expected profit simultaneously [77]. Additionally, as highlighted in research works [78, 79], a co-optimised approach to managing energy and reserves significantly enhances the efficiency of generation dispatch compared to scenarios where energy and reserves are cleared separately. Therefore, this paper adopts a joint clearing model for the DA and RT markets. Furthermore, it is assumed that GenCo sells the TGCs in the present TGC market or withholds the TGCs for future trading after the outcomes of the energy markets are obtained. This assumption is logical since the allocation of TGCs becomes definitive after the results of renewable generation dispatches and deviations are available, which is more conducive to risk mitigation [64].
7. In order to facilitate the mathematical solution, the uncertainties sourced from RESs and demand are realised by a set of chance constraints.
8. As the primary objective of this paper is to explore the system-level generation mix of the future grid under stringent carbon emission regulations, the proposed generation planning model excludes the consideration of other GenCos.
9. In order to ensure the convex characteristics of the LL1 problem, linear generation costs are adopted in the proposed model. Additionally, it is assumed that renewable generation technologies are cost-free.
10. In the DA market, the generation capacities of renewable and must-run technologies are assumed to operate at their full capacity.
11. The network transmission is modelled based on a linear DC power flow approximation.
12. In the face of decarbonisation, a series of carbon emission restrictions and green energy incentives are imposed on the examined GenCo. Firstly, GenCo is limited by an annual cap on carbon emissions. Moreover, it is assumed that carbon emissions only depend on the generation of flexible technologies that consume fossil fuels.

Secondly, carbon emissions from the GenCo are subject to a carbon tax. Thirdly, carbon allowances are allocated to the GenCo based on its renewable energy generation, and the GenCo can profit by trading these carbon allowances. Fourthly, GenCo can earn TGCs through renewable power production and participate in the present and future TGC markets to gain profits. Finally, the RPS ratio is incorporated into the parameter setting of TGC markets. These environmental policies are set to work together towards a fully decarbonised power system.

13. As the share of renewables in the energy mix grows, utility-scale ES is installed in the considered power grid. The ES entities can participate in both DA and RT markets to balance power supply and demand consumption under high renewable production [85].
14. To maintain the convexity of the LL1 problem, followed by the previous work [47], the continuous storage model is utilised to describe the dynamic behaviour of the utility-scale ES during the market horizon.
15. In line with the concept of non-merchant storage [86], although the ES of the proposed model can make a profit by energy arbitrage, its primary goal is to provide the auxiliary service, i.e., reserve deployment, to maintain the operation stability of the power system under high penetration of renewable power production.

6.3 Mathematical Model

This section details the formulation of the UL problem associated with the GenCo and the two LL problems, LL1 and LL2, each corresponding to different market clearing processes.

6.3.1 UL Problem: Investment Decisions of GenCo

Within this framework, the UL problem represents the decision-making process of the profit-driven GenCo, which seeks to maximise its total profit. The mathematical formulation of this UL problem is presented as follows:

$$\max_{V^{UL}} \sum_d \omega_d \left[\underbrace{\sum_{i \in \Psi_{n,t,n,b}} \lambda_{d,t,n}^{DA} g_{i,d,t,n,b}^{DA}}_{\text{DA energy revenue}} - \underbrace{\sum_{i,t,n,b} C_{i,b}^{EN} g_{i,d,t,n,b}^{DA}}_{\text{DA energy cost}} \right]$$

$$\begin{aligned}
& + \underbrace{\sum_{i \in I^{FL} \cap \Psi_{n,t,n,b}} \lambda_{d,t,n}^{RT} (g_{i,d,t,n,b}^U - g_{i,d,t,n,b}^D)}_{\text{RT reserve revenue}} - \underbrace{\sum_{i \in I^{FL,t,n,b}} (C_{i,b}^U g_{i,d,t,n,b}^U - C_{i,b}^D g_{i,d,t,n,b}^D)}_{\text{RT reserve cost}} \\
& - \underbrace{\sum_{i \in I^{RE} \cap \Psi_{n,t,n}} \lambda_{d,t,n}^{RT} g_{i,d,t,n}^{dev}}_{\text{Renewable deviation penalty}} - \underbrace{\sum_{i \in I^{FL,t,n}} \lambda^{CO_2} Q_{i,d,t,n}^{emi}}_{\text{Carbon emission cost}} + \underbrace{\sum_{i \in I^{RE,t,n}} \lambda^{CO_2} Q_{i,d,t,n}^{CA}}_{\text{Carbon allowance revenue}} \\
& + \underbrace{\sum_{i \in I^{RE}} (\lambda_{i,d}^{PGC} Q_{i,d}^{PGC} + \lambda^{FGC} Q_{i,d}^{FGC})}_{\text{Green certificate revenue}} \Big] - \underbrace{\sum_{i,n} C_i^{IN} G_{i,n}}_{\text{Investment cost}} \tag{6.1}
\end{aligned}$$

where:

$$V^{UL} = \{G_{i,n}, Q_{i,d,t,n}^{CA}, Q_{i,d,t,n}^{emi}, Q_{i,d}^{PGC}, Q_{i,d}^{FGC}\} \tag{6.2}$$

subject to:

$$G_{i,n} \geq 0 \quad \forall i \in I, \forall n \tag{6.3}$$

$$G_{i,n,b} = \omega_b^{FL} G_{i,n} \quad \forall i \in I^{FL}, \forall n, \forall b \tag{6.4}$$

$$Q_{i,d,t,n}^{CA} = (g_{i,d,t,n}^{DA} - g_{i,d,t,n}^{dev}) \eta^{Grid} \epsilon_i \quad \forall i \in I^{RE}, \forall d, \forall t, \forall n \tag{6.5}$$

$$Q_{i,d,t,n}^{emi} = (g_{i,d,t,n}^{DA} + g_{i,d,t,n}^U - g_{i,d,t,n}^D) \eta_i^{CO_2} \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n \tag{6.6}$$

$$Q_{i,d}^{PGC} + Q_{i,d}^{FGC} = \sum_{t,n} (g_{i,d,t,n}^{DA} - g_{i,d,t,n}^{dev}) \quad \forall i \in I^{RE}, \forall d \tag{6.7}$$

The objective function of the UL problem is to maximise the total profit of the examined GenCo, consisting of nine components, as defined in (6.1). Constraint (6.3) ensures that the new capacities invested by the GenCo are non-negative. Constraint (6.4) details the weighting factors of the production blocks corresponding to flexible generation capacities. Constraint (6.5) presents the allocation of carbon allowances. Constraint (6.6) quantifies the carbon emissions produced by the operation of fossil-fired flexible generation technologies. Lastly, constraint (6.7) calculates the total quantities of TGCs that are generated based on renewable energy production in the DA market and its deviation in the RT market.

6.3.2 LL1 Problem: Joint Clearing Process of DA Energy and RT Balancing Markets

The LL1 problem represents the joint clearing process of the two-stage energy markets, i.e., the DA energy market and the RT balancing market, with the aim of minimising the overall system cost from the perspective of the market operator. It is noteworthy that in this chapter, the ES is incorporated into the constraints of the LL1 problem. The

mathematical expression of the LL1 problem is detailed as follows:

$$\left\{ \min_{V^{LL}} \sum_{i,t,n,b} C_{i,b}^{EN} g_{i,d,t,n,b}^{DA} + \sum_{i,t,n,b} (C_{i,b}^U g_{i,d,t,n,b}^U - C_{i,b}^D g_{i,d,t,n,b}^D) \right. \quad (6.8)$$

where:

$$V^{LL1} = \{g_{i,d,t,n,b}^{DA}, g_{i,d,t,n,b}^U, g_{i,d,t,n,b}^D, P_{j,d,t,o}^{dev}, g_{i,d,t,n}^{dev}, E_{e,d,t}, s_{e,d,t}^{c,DA}, s_{e,d,t}^{d,DA}, s_{e,d,t}^{c,RT}, s_{e,d,t}^{d,RT}, \delta_{d,t,n}^{DA}, \delta_{d,t}^{DA,ref}, \delta_{d,t,n}^{RT}, \delta_{d,t}^{RT,ref}\} \quad (6.9)$$

subject to:

$$\begin{aligned} & \sum_{j \in \Psi_{n,o}} L_{j,d,t,o}^{DA} + \sum_{e \in \Psi_n} s_{e,d,t}^{c,DA} + \sum_{m \in \Lambda_n} b_{nm}(\delta_{d,t,n}^{DA} - \delta_{d,t,m}^{DA}) - \sum_{i \in \Psi_{n,b}} g_{i,d,t,n,b}^{DA} - \\ & \sum_{e \in \Psi_n} s_{e,d,t}^{d,DA} = 0 \quad : \lambda_{d,t,n}^{DA} \quad \forall t, \forall n \end{aligned} \quad (6.10)$$

$$\begin{aligned} & \sum_{j \in \Psi_{n,o}} P_{j,d,t,o}^{dev} + \sum_{i \in I^{FL} \cap \Psi_{n,b}} (-g_{i,d,t,n,b}^U + g_{i,d,t,n,b}^D) + \sum_{e \in \Psi_n} (s_{e,d,t}^{c,RT} - s_{e,d,t}^{d,RT}) + \\ & \sum_{i \in I^{RE} \cap \Psi_n} g_{i,d,t,n}^{dev} - \sum_{m \in \Lambda_n} b_{nm}(\delta_{d,t,n}^{DA} - \delta_{d,t,n}^{RT} - \delta_{d,t,m}^{DA} + \delta_{d,t,m}^{RT}) = 0 \quad : \lambda_{d,t,n}^{RT} \quad \forall t, \forall n \end{aligned} \quad (6.11)$$

$$E_{e,d,t} = E_{e,d,t-1} + \eta_e^c(s_{e,d,t}^{c,DA} + s_{e,d,t}^{c,RT}) - \frac{s_{e,d,t}^{d,DA} + s_{e,d,t}^{d,RT}}{\eta_e^d} : \tau_{e,d,t}^{ES} \quad \forall e, \forall d, \forall t \geq 2 \quad (6.12)$$

$$E_{e,d,1} = E_e^0 + \eta_e^c(s_{e,d,1}^{c,DA} + s_{e,d,1}^{c,RT}) - \frac{s_{e,d,1}^{d,DA} + s_{e,d,1}^{d,RT}}{\eta_e^d} : \tau_{e,d}^{ES,1} \quad \forall e, \forall d \quad (6.13)$$

$$E_e^0 = E_{e,d,T} : \tau_{e,d}^{ES,T} \quad \forall e, \forall d \quad (6.14)$$

$$E_e^{min} \leq E_{e,d,t} \leq E_e^{max} : \tau_{e,d,t}^{ES,-}, \tau_{e,d,t}^{ES,+} \quad \forall e, \forall d, \forall t \quad (6.15)$$

$$0 \leq s_{e,d,t}^{c,DA} \leq s_e^{max} : \tau_{e,d,t}^{c,DA,-}, \tau_{e,d,t}^{c,DA,+} \quad \forall e, \forall d, \forall t \quad (6.16)$$

$$0 \leq s_{e,d,t}^{c,DA} + s_{e,d,t}^{c,RT} \leq s_e^{max} : \tau_{e,d,t}^{c,RT,-}, \tau_{e,d,t}^{c,RT,+} \quad \forall e, \forall d, \forall t \quad (6.17)$$

$$0 \leq s_{e,d,t}^{d,DA} \leq s_e^{max} : \tau_{e,d,t}^{d,DA,-}, \tau_{e,d,t}^{d,DA,+} \quad \forall e, \forall d, \forall t \quad (6.18)$$

$$0 \leq s_{e,d,t}^{d,DA} + s_{e,d,t}^{d,RT} \leq s_e^{max} : \tau_{e,d,t}^{d,RT,-}, \tau_{e,d,t}^{d,RT,+} \quad \forall e, \forall d, \forall t \quad (6.19)$$

$$\sum_{i \in I^{FL}, t, n, b} \eta_i^{CO_2} (g_{i,d,t,n,b}^{DA} + g_{i,d,t,n,b}^U - g_{i,d,t,n,b}^D) \leq \eta^{Grid} \sum_{j,t,o} L_{j,d,t,o}^{DA} : \zeta_d \quad (6.20)$$

$$g_{i,d,t,n,b}^{DA} + g_{i,d,t,n,b}^U \leq G_{i,n,b} : \mu_{i,d,t,n,b}^U \quad \forall i \in I^{FL}, \forall t, \forall n, \forall b \quad (6.21)$$

$$g_{i,d,t,n,b}^D \leq g_{i,d,t,n,b}^{DA} : \mu_{i,d,t,n,b}^D \quad \forall i \in I^{FL}, \forall t, \forall n, \forall b \quad (6.22)$$

$$0 \leq g_{i,d,t,n,b}^U \leq G_{i,b}^U : \phi_{i,d,t,n,b}^{U,-}, \phi_{i,d,t,n,b}^{U,+} \quad \forall i \in I^{FL}, \forall t, \forall n, \forall b \quad (6.23)$$

$$0 \leq g_{i,d,t,n,b}^D \leq G_{i,b}^D : \phi_{i,d,t,n,b}^{D,-}, \phi_{i,d,t,n,b}^{D,+} \quad \forall i \in I^{FL}, \forall t, \forall n, \forall b \quad (6.24)$$

$$g_{i,d,t,n}^{DA} = G_{i,n} : \gamma_{i,d,t,n} \quad \forall i \in I^{MR}, \forall t, \forall n \quad (6.25)$$

$$g_{i,d,t,n}^{DA} = k_{i,d,t}^{RE} G_{i,n} : \theta_{i,d,t,n} \quad \forall i \in I^{RE}, \forall t, \forall n \quad (6.26)$$

$$-C_{nm}^{max} \leq b_{nm}(\delta_{d,t,n}^{DA} - \delta_{d,t,m}^{DA}) \leq C_{nm}^{max} : \nu_{d,t,n,m}^{DA,-}, \nu_{d,t,n,m}^{DA,+} \quad \forall t, \forall n, \forall m \in \Lambda_n \quad (6.27)$$

$$-\pi \leq \delta_{d,t,n}^{DA} \leq \pi : \xi_{d,t,n}^{DA,-}, \xi_{d,t,n}^{DA,+} \quad \forall t, \forall n \quad (6.28)$$

$$\delta_{d,t}^{DA,ref} = 0 : \xi_{d,t}^{DA,ref} \quad \forall t \quad (6.29)$$

$$-C_{nm}^{max} \leq b_{nm}(\delta_{d,t,n}^{RT} - \delta_{d,t,m}^{RT}) \leq C_{nm}^{max} : \nu_{d,t,n,m}^{RT,-}, \nu_{d,t,n,m}^{RT,+} \quad \forall t, \forall n, \forall m \in \Lambda_n \quad (6.30)$$

$$-\pi \leq \delta_{d,t,n}^{RT} \leq \pi : \xi_{d,t,n}^{RT,-}, \xi_{d,t,n}^{RT,+} \quad \forall t, \forall n \quad (6.31)$$

$$\delta_{d,t}^{RT,ref} = 0 : \xi_{d,t}^{RT,ref} \quad \forall t \quad (6.32)$$

$$\mathbb{P} \left[g_{i,d,t,n}^{DA} - g_{i,d,t,n}^{dev} \leq g_{i,d,t,n}^R \right] \geq 1 - \rho^{RE} : \chi_{i,d,t,n}^{RE} \quad \forall i \in I^{RE}, \forall t, \forall n \quad (6.33)$$

$$\mathbb{P} \left[\sum_o L_{j,d,t,o}^{DA} + \sum_o P_{j,d,t,o}^{dev} \geq L_{j,d,t}^R \right] \geq 1 - \rho^L : \chi_{j,d,t}^L \quad \forall j, \forall t \quad \forall d \quad (6.34)$$

The LL1 problem's objective function is to minimise the overall system cost, including the energy cost (first term) and the reserve cost (last two terms). The LL1 problem is subject to various sets of constraints: system balance constraints (6.10)-(6.11), ES constraints (6.12)-(6.19), system-wide carbon emission constraint (6.20), generation operating constraints (6.21)-(6.26), network constraints (6.27)-(6.32), and chance constraints (6.33)-(6.34).

Due to the introduction of ES, some new components should be clarified. First, the decision variable set V^{LL1} is expanded to include new variables $E_{e,d,t}$, $s_{e,d,t}^{c,DA}$, $s_{e,d,t}^{d,DA}$, $s_{e,d,t}^{c,RT}$, and $s_{e,d,t}^{d,RT}$, designed to represent ES energy level, ES DA charging power, ES DA discharging power, ES RT charging power, and ES RT discharging power, respectively. Second, charging and discharging power dispatches are incorporated into the balance constraints (6.10)-(6.11) of DA and RT markets, respectively. Additionally, the constraints (6.12)-(6.19) are employed to capture the dynamic behaviours of ES across the two-stage energy markets. Constraints (6.12) and (6.13) enforce the time-linking energy balance of the ES unit. Constraint (6.14) ensures that each ES unit maintains energy neutrality throughout the market period of one typical day. Constraint (6.15) defines the upper and lower limits

for the ES energy level. Constraints (6.16)-(6.19) specify the maximum power capacities for the charging and discharging operations of ES units during the DA and RT market horizons, respectively.

6.3.3 LL2 Problem: Clearing Process of TGC Market

The process for clearing the present TGC market, along with the prediction for the TGC price in the future TGC market, is structured as follows:

$$V^{LL2} = \{\lambda_{i,d}^{PGC}, Q_{i,d}^{PGC}\} \quad (6.35)$$

$$\lambda_{i,d}^{PGC} = l^{PGC} - q^{PGC} Q_{i,d}^{PGC} \quad \forall i \in I^{RE}, \forall d \quad (6.36)$$

$$\lambda^{FGC} = \gamma^{FGC} \lambda_0^{RPS} \quad (6.37)$$

The present TGC market is built on a quantity-dependent inverse demand function. Under this framework, the present TGC price is determined by equation (6.36) that represents the linear correlation between the present price of TGC and its quantity. On the other hand, equation (6.37) indicates the estimation of the future TGC price. The necessary coefficients are obtained from equations (6.38) and (6.39), as proved in [64].

$$l^{PGC} = \lambda_0^{RPS} \quad (6.38)$$

$$q^{PGC} = \frac{(1 - \theta^{PGC}) \lambda_0^{RPS}}{\gamma^{RPS} \sum_{j,d,t} L_{j,d,t}^{DA}} \quad (6.39)$$

6.4 Solution

The solution approach for the proposed two-stage bi-level optimisation model is organised into a series of steps, as outlined in this section. The initial step addresses the non-convex chance constraints present in the LL1 problem by employing linearization techniques, as described in Subsection 6.4.1. Following this linearization, the model is restructured into a single-level MPEC, with the detailed transformation process provided in Subsection 6.4.2. This critical step utilises the KKT optimality conditions to replace the LL1 problem and simultaneously integrates the LL2 problem's constraints into the UL problem.

To further ensure the model's computational feasibility, subsequent efforts focus on addressing and linearizing the non-linear elements identified within the MPEC formulation, as detailed in Subsection 6.4.3. Finally, the MPEC model is transformed into a

MIQP problem, as illustrated in Subsection 6.4.4.

6.4.1 CCP Reformulation

In this chapter, the chance constraints utilise stochastic variables to depict uncertainties related to RESs and demand, represented by g^R and L^R , respectively. It is assumed that these stochastic variables adhere to normal distributions. The formulation of these normal distributions is as follows:

$$g_{i,d,t,n}^R \sim \mathcal{N}(m_{i,d,t,n}^{RE}, (s_{i,d,t,n}^{RE})^2), \quad L_{j,d,t}^R \sim \mathcal{N}(m_{j,d,t}^L, (s_{j,d,t}^L)^2) \quad (6.40)$$

where parameter m is the mean of the normal distribution and parameter s is the standard deviation of the normal distribution. It should be noted that the chance constraints (6.33) and (6.34) are non-convex. In order to make the proposed model solvable, the non-convexities in these chance constraints are required to be eliminated. By following the linearization process detailed in Subsection 3.4.1, the chance constraints (6.33)-(6.34) are reformulated as follows:

$$m_{i,d,t,n}^{RE} - g_{i,d,t,n}^{DA} + g_{i,d,t,n}^{dev} \geq \Phi^{-1}(1 - \rho^{RE}) s_{i,d,t,n}^{RE} \quad : \chi_{i,d,t,n}^{RE} \quad \forall i \in I^{RE}, \forall d, \forall t, \forall n \quad (6.41)$$

$$-m_{j,d,t}^L + \sum_o P_{j,d,t,o}^{dev} + \sum_o L_{j,d,t,o}^{DA} \geq \Phi^{-1}(1 - \rho^L) s_{j,d,t}^L \quad : \chi_{j,d,t}^L \quad \forall j, \forall d, \forall t \quad (6.42)$$

Note that $\Phi^{-1}(\cdot)$ is the inverse cumulative distribution function of a standard normal distribution.

6.4.2 MPEC Formulation

After linearizing the LL1 problem, it becomes practicable to replace the LL1 problem with a collection of its KKT optimality conditions. Additionally, since the LL2 problem is entirely characterised by constraints, it can be seamlessly incorporated into the UL problem. Consequently, this enables the proposed two-stage bi-level optimisation problem (6.1)-(6.37) to be transformed into a single-level MPEC.

KKT Conditions Associated with the LL1 Problem

In order to derive the KKT optimality conditions of the LL1 problem, it is necessary to construct its Lagrangian function $\mathcal{L}_{LL1}$, which combines the LL1 objective function and

constraints weighted by their respective dual variables. It is formulated as follows:

$$\begin{aligned}
\mathcal{L}_{LL1} = & \sum_{i,d,t,n,b} C_{i,b}^{EN} g_{i,d,t,n,b}^{DA} + \sum_{i,d,t,n,b} (C_{i,b}^U g_{i,d,t,n,b}^U - C_{i,b}^D g_{i,d,t,n,b}^D) + \sum_{d,t,n} \lambda_{d,t,n}^{DA} \left[\sum_{j \in \Psi_{n,o}} L_{j,d,t,o}^{DA} \right. \\
& + \sum_{e \in \Psi_n} s_{e,d,t}^{c,DA} + \sum_{m \in \Lambda_n} b_{nm} (\delta_{d,t,n}^{DA} - \delta_{d,t,m}^{DA}) - \sum_{i \in \Psi_{n,b}} g_{i,d,t,n,b}^{DA} - \sum_{e \in \Psi_n} s_{e,d,t}^{d,DA} \Big] + \\
& \sum_{d,t,n} \lambda_{d,t,n}^{RT} \left[\sum_{j \in \Psi_{n,o}} P_{j,d,t,o}^{dev} + \sum_{i \in I^{FL} \cap \Psi_{n,b}} (-g_{i,d,t,n,b}^U + g_{i,d,t,n,b}^D) + \sum_{e \in \Psi_n} (s_{e,d,t}^{c,RT} - s_{e,d,t}^{d,RT}) \right. \\
& + \sum_{i \in I^{RE} \cap \Psi_n} g_{i,d,t,n}^{dev} - \sum_{m \in \Lambda_n} b_{nm} (\delta_{d,t,n}^{DA} - \delta_{d,t,n}^{RT} - \delta_{d,t,m}^{DA} + \delta_{d,t,m}^{RT}) \Big] \\
& + \sum_{e,d,t \geq 2} \tau_{e,d,t}^{ES} [E_{e,d,t} - E_{e,d,t-1} - \eta_e^c (s_{e,d,t}^{c,DA} + s_{e,d,t}^{c,RT}) + \frac{s_{e,d,t}^{d,DA} + s_{e,d,t}^{d,RT}}{\eta_e^d}] \\
& + \sum_{e,d} \tau_{e,d}^{ES,1} [E_{e,d,1} - E_e^0 - \eta_e^c (s_{e,d,1}^{c,DA} + s_{e,d,1}^{c,RT}) + \frac{s_{e,d,1}^{d,DA} + s_{e,d,1}^{d,RT}}{\eta_e^d}] + \\
& \sum_{e,d} \tau_{e,d}^{ES,T} (E_e^0 - E_{e,d,T}) + \sum_{e,d,t} \tau_{e,d,t}^{ES,-} (E_e^{min} - E_{e,d,t}) + \sum_{e,d,t} \tau_{e,d,t}^{ES,+} (E_{e,d,t} - E_e^{max}) \\
& - \sum_{e,d,t} \tau_{e,d,t}^{c,DA,-} s_{e,d,t}^{c,DA} + \sum_{e,d,t} \tau_{e,d,t}^{c,DA,+} (s_{e,d,t}^{c,DA} - s_e^{max}) - \sum_{e,d,t} \tau_{e,d,t}^{c,RT,-} (s_{e,d,t}^{c,DA} + s_{e,d,t}^{c,RT}) + \\
& \sum_{e,d,t} \tau_{e,d,t}^{c,RT,+} (s_{e,d,t}^{c,DA} + s_{e,d,t}^{c,RT} - s_e^{max}) - \sum_{e,d,t} \tau_{e,d,t}^{d,DA,-} s_{e,d,t}^{d,DA} + \sum_{e,d,t} \tau_{e,d,t}^{d,DA,+} (s_{e,d,t}^{d,DA} - \\
& s_e^{max}) - \sum_{e,d,t} \tau_{e,d,t}^{d,RT,-} (s_{e,d,t}^{d,DA} + s_{e,d,t}^{d,RT}) + \sum_{e,d,t} \tau_{e,d,t}^{d,RT,+} (s_{e,d,t}^{d,DA} + s_{e,d,t}^{d,RT} - s_e^{max}) \\
& + \sum_d \zeta_d \left[\sum_{i \in I^{FL},t,n,b} \eta_i^{CO_2} (g_{i,d,t,n,b}^{DA} + g_{i,d,t,n,b}^U - g_{i,d,t,n,b}^D) - \eta^{Grid} \sum_{j,t,o} L_{j,d,t,o}^{DA} \right] \\
& + \sum_{i \in I^{FL},d,t,n,b} \mu_{i,d,t,n,b}^U (g_{i,d,t,n,b}^{DA} + g_{i,d,t,n,b}^U - G_{i,n,b}) + \\
& \sum_{i \in I^{FL},d,t,n,b} \mu_{i,d,t,n,b}^D (g_{i,d,t,n,b}^D - g_{i,d,t,n,b}^{DA}) - \sum_{i \in I^{FL},d,t,n,b} \phi_{i,d,t,n,b}^{U,-} g_{i,d,t,n,b}^U + \\
& \sum_{i \in I^{FL},d,t,n,b} \phi_{i,d,t,n,b}^{U,+} (g_{i,d,t,n,b}^U - G_{i,b}) - \sum_{i \in I^{FL},d,t,n,b} \phi_{i,d,t,n,b}^{D,-} g_{i,d,t,n,b}^D + \\
& \sum_{i \in I^{FL},d,t,n,b} \phi_{i,d,t,n,b}^{D,+} (g_{i,d,t,n,b}^D - G_{i,b}) + \sum_{i \in I^{MR},d,t,n} \gamma_{i,d,t,n} (g_{i,d,t,n}^{DA} - G_{i,n}) + \\
& \sum_{i \in I^{RE},d,t,n} \theta_{i,d,t,n} (g_{i,d,t,n}^{DA} - k_{i,d,t}^{RE} G_{i,n}) + \sum_{d,t,n,m \in \Lambda_n} \nu_{d,t,n,m}^{DA,-} [-C_{nm}^{max} - b_{nm} (\delta_{d,t,n}^{DA} - \\
& \delta_{d,t,m}^{DA})] + \sum_{d,t,n,m \in \Lambda_n} \nu_{d,t,n,m}^{DA,+} [b_{nm} (\delta_{d,t,n}^{DA} - \delta_{d,t,m}^{DA}) - C_{nm}^{max}] + \\
& \sum_{d,t,n} \xi_{d,t,n}^{DA,-} (-\pi - \delta_{d,t,n}^{DA}) + \sum_{d,t,n} \xi_{d,t,n}^{DA,+} (\delta_{d,t,n}^{DA} - \pi) + \sum_{d,t,n=1} \xi_{d,t}^{DA,ref} \delta_{d,t}^{DA,ref} +
\end{aligned}$$

$$\begin{aligned}
& \sum_{d,t,n,m \in \Lambda_n} \nu_{d,t,n,m}^{RT,-} [-C_{nm}^{max} - b_{nm}(\delta_{d,t,n}^{RT} - \delta_{d,t,m}^{RT})] + \sum_{d,t,n,m \in \Lambda_n} \nu_{d,t,n,m}^{RT,+} [b_{nm}(\delta_{d,t,n}^{RT} \\
& - \delta_{d,t,m}^{RT}) - C_{nm}^{max}] + \sum_{d,t,n} \xi_{d,t,n}^{RT,-} (-\pi - \delta_{d,t,n}^{RT}) + \sum_{d,t,n} \xi_{d,t,n}^{RT,+} (\delta_{d,t,n}^{RT} - \pi) + \\
& \sum_{d,t,n=1} \xi_{d,t}^{RT,ref} \delta_{d,t}^{RT,ref} + \sum_{i \in I^{RE}, d,t,n} \chi_{i,d,t,n}^{RE} [\Phi^{-1}(1 - \rho^{RE}) s_{i,d,t,n}^{RE} - m_{i,d,t,n}^{RE} + \\
& g_{i,d,t,n}^{DA} - g_{i,d,t,n}^{dev}] + \sum_{j,d,t} \chi_{j,d,t}^L [\Phi^{-1}(1 - \rho^L) s_{j,d,t}^L + m_{j,d,t}^L - \sum_o P_{j,d,t,o}^{dev} \\
& - \sum_o L_{j,d,t,o}^{DA}] \tag{6.43}
\end{aligned}$$

Based on the Lagrangian function (6.43), the KKT optimality conditions are detailed as follows:

$$\text{KKT equality conditions : (6.10) – (6.14), (6.25) – (6.26), (6.29), (6.32)} \tag{6.44}$$

LL1 KKT stationarity conditions :

$$\frac{\partial \mathcal{L}_{LL1}}{\partial g_{i,d,t,n,b}^{DA}} = C_{i,b}^{EN} - \lambda_{d,t,n}^{DA} + \eta_i^{CO_2} \zeta_d + \mu_{i,d,t,n,b}^U - \mu_{i,d,t,n,b}^D = 0 \quad \forall i \in I^{FL} \cap \Psi_n, \forall d, \forall t, \forall n, \forall b \tag{6.45}$$

$$\frac{\partial \mathcal{L}_{LL1}}{\partial g_{i,d,t,n}^{DA}} = C_i^{EN} - \lambda_{d,t,n}^{DA} + \gamma_{i,d,t,n} = 0 \quad \forall i \in I^{MR} \cap \Psi_n, \forall d, \forall t, \forall n \tag{6.46}$$

$$\frac{\partial \mathcal{L}_{LL1}}{\partial g_{i,d,t,n}^{DA}} = -\lambda_{d,t,n}^{DA} + \theta_{i,d,t,n} + \chi_{i,d,t,n}^{RE} = 0 \quad \forall i \in I^{RE} \cap \Psi_n, \forall d, \forall t, \forall n \tag{6.47}$$

$$\begin{aligned}
\frac{\partial \mathcal{L}_{LL1}}{\partial g_{i,d,t,n,b}^U} &= C_{i,b}^U - \lambda_{d,t,n}^{RT} + \eta_i^{CO_2} \zeta_d + \mu_{i,d,t,n,b}^U - \phi_{i,d,t,n,b}^{U,-} + \phi_{i,d,t,n,b}^{U,+} = 0 \\
&\forall i \in I^{FL} \cap \Psi_n, \forall d, \forall t, \forall n, \forall b \tag{6.48}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \mathcal{L}_{LL1}}{\partial g_{i,d,t,n,b}^D} &= -C_{i,b}^D + \lambda_{d,t,n}^{RT} - \eta_i^{CO_2} \zeta_d + \mu_{i,d,t,n,b}^D - \phi_{i,d,t,n,b}^{D,-} + \phi_{i,d,t,n,b}^{D,+} = 0 \\
&\forall i \in I^{FL} \cap \Psi_n, \forall d, \forall t, \forall n, \forall b \tag{6.49}
\end{aligned}$$

$$\frac{\partial \mathcal{L}_{LL1}}{\partial P_{j,d,t,o}^{dev}} = \lambda_{d,t,n}^{RT} - \chi_{j,d,t}^L = 0 \quad \forall j \in \Psi_n, \forall d, \forall t \tag{6.50}$$

$$\frac{\partial \mathcal{L}_{LL1}}{\partial g_{i,d,t,n}^{dev}} = \lambda_{d,t,n}^{RT} - \chi_{i,d,t,n}^{RE} = 0 \quad \forall i \in I^{RE} \cap \Psi_n, \forall d, \forall t, \forall n \tag{6.51}$$

$$\frac{\partial \mathcal{L}_{LL1}}{\partial E_{e,d,t}} = \tau_{e,d,t}^{ES} - \tau_{e,d,t+1}^{ES} + \tau_{e,d,t}^{ES,+} - \tau_{e,d,t}^{ES,-} = 0 \quad \forall e, \forall d, \forall t \leq T-1 \tag{6.52}$$

$$\frac{\partial \mathcal{L}_{LL1}}{\partial E_{e,d,T}} = \tau_{e,d,T}^{ES} - \tau_{e,d}^{ES,T} + \tau_{e,d,T}^{ES,+} - \tau_{e,d,T}^{ES,-} = 0 \quad \forall e, \forall d \tag{6.53}$$

$$\frac{\partial \mathcal{L}_{LL1}}{\partial s_{e,d,t}^{c,DA}} = \lambda_{d,t,n}^{DA} - \eta_e^c \tau_{e,d,t}^{ES} + \tau_{e,d,t}^{c,DA,+} - \tau_{e,d,t}^{c,DA,-} + \tau_{e,d,t}^{c,RT,+} - \tau_{e,d,t}^{c,RT,-} = 0 \quad \forall e, \forall d, \forall t \quad (6.54)$$

$$\frac{\partial \mathcal{L}_{LL1}}{\partial s_{e,d,t}^{d,DA}} = -\lambda_{d,t,n}^{DA} + \frac{\tau_{e,d,t}^{ES}}{\eta_e^d} + \tau_{e,d,t}^{d,DA,+} - \tau_{e,d,t}^{d,DA,-} + \tau_{e,d,t}^{d,RT,+} - \tau_{e,d,t}^{d,RT,-} = 0 \quad \forall e, \forall d, \forall t \quad (6.55)$$

$$\frac{\partial \mathcal{L}_{LL1}}{\partial s_{e,d,t}^{c,RT}} = \lambda_{d,t,n}^{RT} - \eta_e^c \tau_{e,d,t}^{ES} + \tau_{e,d,t}^{c,RT,+} - \tau_{e,d,t}^{c,RT,-} = 0 \quad \forall e, \forall d, \forall t \quad (6.56)$$

$$\frac{\partial \mathcal{L}_{LL1}}{\partial s_{e,d,t}^{d,RT}} = -\lambda_{d,t,n}^{RT} + \frac{\tau_{e,d,t}^{ES}}{\eta_e^d} + \tau_{e,d,t}^{d,RT,+} - \tau_{e,d,t}^{d,RT,-} = 0 \quad \forall e, \forall d, \forall t \quad (6.57)$$

$$\begin{aligned} \frac{\partial \mathcal{L}_{LL1}}{\partial \delta_{d,t,n}^{DA}} = & \sum_{m \in \Lambda_n} b_{nm} (\lambda_{d,t,n}^{DA} - \lambda_{d,t,m}^{DA}) - \sum_{m \in \Lambda_n} b_{nm} (\lambda_{d,t,n}^{RT} - \lambda_{d,t,m}^{RT}) + \\ & \sum_{m \in \Lambda_n} b_{nm} (\nu_{d,t,n,m}^{DA,+} - \nu_{d,t,m,n}^{DA,+}) - \sum_{m \in \Lambda_n} b_{nm} (\nu_{d,t,n,m}^{DA,-} - \nu_{d,t,m,n}^{DA,-}) + \xi_{d,t,n}^{DA,+} \\ & - \xi_{d,t,n}^{DA,-} + (\xi_{d,t}^{DA,ref})_{n=1} = 0 \quad \forall d, \forall t, \forall n \end{aligned} \quad (6.58)$$

$$\begin{aligned} \frac{\partial \mathcal{L}_{LL1}}{\partial \delta_{d,t,n}^{RT}} = & \sum_{m \in \Lambda_n} b_{nm} (\lambda_{d,t,n}^{RT} - \lambda_{d,t,m}^{RT}) + \sum_{m \in \Lambda_n} b_{nm} (\nu_{d,t,n,m}^{RT,+} - \nu_{d,t,m,n}^{RT,+}) - \\ & \sum_{m \in \Lambda_n} b_{nm} (\nu_{d,t,n,m}^{RT,-} - \nu_{d,t,m,n}^{RT,-}) + \xi_{d,t,n}^{RT,+} - \xi_{d,t,n}^{RT,-} + (\xi_{d,t}^{RT,ref})_{n=1} = 0 \quad \forall d, \forall t, \forall n \end{aligned} \quad (6.59)$$

LL1 KKT complementarity conditions :

$$0 \leq (E_{e,d,t} - E_e^{min}) \perp \tau_{e,d,t}^{ES,-} \geq 0 \quad \forall e, \forall d, \forall t \quad (6.60)$$

$$0 \leq (E_e^{max} - E_{e,d,t}) \perp \tau_{e,d,t}^{ES,+} \geq 0 \quad \forall e, \forall d, \forall t \quad (6.61)$$

$$0 \leq s_{e,d,t}^{c,DA} \perp \tau_{e,d,t}^{c,DA,-} \geq 0 \quad \forall e, \forall d, \forall t \quad (6.62)$$

$$0 \leq (s_e^{max} - s_{e,d,t}^{c,DA}) \perp \tau_{e,d,t}^{c,DA,+} \geq 0 \quad \forall e, \forall d, \forall t \quad (6.63)$$

$$0 \leq (s_{e,d,t}^{c,DA} + s_{e,d,t}^{c,RT}) \perp \tau_{e,d,t}^{c,RT,-} \geq 0 \quad \forall e, \forall d, \forall t \quad (6.64)$$

$$0 \leq (s_e^{max} - s_{e,d,t}^{c,DA} - s_{e,d,t}^{c,RT}) \perp \tau_{e,d,t}^{c,RT,+} \geq 0 \quad \forall e, \forall d, \forall t \quad (6.65)$$

$$0 \leq s_{e,d,t}^{d,DA} \perp \tau_{e,d,t}^{d,DA,-} \geq 0 \quad \forall e, \forall d, \forall t \quad (6.66)$$

$$0 \leq (s_e^{max} - s_{e,d,t}^{d,DA}) \perp \tau_{e,d,t}^{d,DA,+} \geq 0 \quad \forall e, \forall d, \forall t \quad (6.67)$$

$$0 \leq (s_{e,d,t}^{d,DA} + s_{e,d,t}^{d,RT}) \perp \tau_{e,d,t}^{d,RT,-} \geq 0 \quad \forall e, \forall d, \forall t \quad (6.68)$$

$$0 \leq (s_e^{max} - s_{e,d,t}^{d,DA} - s_{e,d,t}^{d,RT}) \perp \tau_{e,d,t}^{d,RT,+} \geq 0 \quad \forall e, \forall d, \forall t \quad (6.69)$$

$$0 \leq [\eta^{Grid} \sum_{j,t,o} L_{j,d,t,o}^{DA} - \sum_{i \in I^{FL},t,n,b} \eta_i^{CO_2} (g_{i,d,t,n,b}^{DA} + g_{i,d,t,n,b}^U - g_{i,d,t,n,b}^D)] \perp \zeta_d \geq 0 \quad \forall d \quad (6.70)$$

$$0 \leq (G_{i,n,b} - g_{i,d,t,n,b}^{DA} - g_{i,d,t,n,b}^U) \perp \mu_{i,d,t,n,b}^U \geq 0 \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (6.71)$$

$$0 \leq (g_{i,d,t,n,b}^{DA} - g_{i,d,t,n,b}^D) \perp \mu_{i,d,t,n,b}^D \geq 0 \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (6.72)$$

$$0 \leq g_{i,d,t,n,b}^U \perp \phi_{i,d,t,n,b}^{U,-} \geq 0 \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (6.73)$$

$$0 \leq (G_{i,b}^U - g_{i,d,t,n,b}^U) \perp \phi_{i,d,t,n,b}^{U,+} \geq 0 \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (6.74)$$

$$0 \leq g_{i,d,t,n,b}^D \perp \phi_{i,d,t,n,b}^{D,-} \geq 0 \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (6.75)$$

$$0 \leq (G_{i,b}^D - g_{i,d,t,n,b}^D) \perp \phi_{i,d,t,n,b}^{D,+} \geq 0 \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (6.76)$$

$$0 \leq [C_{nm}^{max} + b_{nm}(\delta_{d,t,n}^{DA} - \delta_{d,t,m}^{DA})] \perp \nu_{d,t,n,m}^{DA,-} \geq 0 \quad \forall d, \forall t, \forall n, \forall m \in \Lambda_n \quad (6.77)$$

$$0 \leq [C_{nm}^{max} - b_{nm}(\delta_{d,t,n}^{DA} - \delta_{d,t,m}^{DA})] \perp \nu_{d,t,n,m}^{DA,+} \geq 0 \quad \forall d, \forall t, \forall n, \forall m \in \Lambda_n \quad (6.78)$$

$$0 \leq (\pi + \delta_{d,t,n}^{DA}) \perp \xi_{d,t,n}^{DA,-} \geq 0 \quad \forall d, \forall t, \forall n \quad (6.79)$$

$$0 \leq (\pi - \delta_{d,t,n}^{DA}) \perp \xi_{d,t,n}^{DA,+} \geq 0 \quad \forall d, \forall t, \forall n \quad (6.80)$$

$$0 \leq [C_{nm}^{max} + b_{nm}(\delta_{d,t,n}^{RT} - \delta_{d,t,m}^{RT})] \perp \nu_{d,t,n,m}^{RT,-} \geq 0 \quad \forall d, \forall t, \forall n, \forall m \in \Lambda_n \quad (6.81)$$

$$0 \leq [C_{nm}^{max} - b_{nm}(\delta_{d,t,n}^{RT} - \delta_{d,t,m}^{RT})] \perp \nu_{d,t,n,m}^{RT,+} \geq 0 \quad \forall d, \forall t, \forall n, \forall m \in \Lambda_n \quad (6.82)$$

$$0 \leq (\pi + \delta_{d,t,n}^{RT}) \perp \xi_{d,t,n}^{RT,-} \geq 0 \quad \forall d, \forall t, \forall n \quad (6.83)$$

$$0 \leq (\pi - \delta_{d,t,n}^{RT}) \perp \xi_{d,t,n}^{RT,+} \geq 0 \quad \forall d, \forall t, \forall n \quad (6.84)$$

$$0 \leq [m_{i,d,t,n}^{RE} - g_{i,d,t,n}^{DA} + g_{i,d,t,n}^{dev} - \Phi^{-1}(1 - \rho^{RE})s_{i,d,t,n}^{RE}] \perp \chi_{i,d,t,n}^{RE} \geq 0 \quad \forall i \in I^{RE}, \forall d, \forall t, \forall n \quad (6.85)$$

$$0 \leq [-m_{j,d,t}^L + \sum_o P_{j,d,t,o}^{dev} + \sum_o L_{j,d,t,o}^{DA} - \Phi^{-1}(1 - \rho^L)s_{j,d,t}^L] \perp \chi_{j,d,t}^L \geq 0 \quad \forall j, \forall d, \forall t \quad (6.86)$$

As a crucial step for transitioning the bi-level optimisation model to a single-level MPEC, the KKT optimality conditions for the LL1 problem are categorised into three groups: i) KKT equality conditions (6.44), ii) KKT stationarity conditions (6.45)-(6.59), and iii) KKT complementarity conditions (6.60)-(6.86).

Final MPEC Formulation

Since the LL2 problem consists solely of constraints, it can be directly integrated into the MPEC. As a result, the formulation of the MPEC is depicted as follows:

$$\begin{aligned}
\max_{V^{MPEC}} \sum_d \omega_d \bigg[& \sum_{i \in \Psi_{n,t,n,b}} \lambda_{d,t,n}^{DA} g_{i,d,t,n,b}^{DA} - \sum_{i,t,n,b} C_{i,b}^{EN} g_{i,d,t,n,b}^{DA} + \\
& \sum_{i \in I^{FL} \cap \Psi_{n,t,n,b}} \lambda_{d,t,n}^{RT} (g_{i,d,t,n,b}^U - g_{i,d,t,n,b}^D) - \sum_{i \in I^{FL,t,n,b}} (C_{i,b}^U g_{i,d,t,n,b}^U - C_{i,b}^D g_{i,d,t,n,b}^D) \\
& + \sum_{i \in I^{RE} \cap \Psi_{n,t,n}} \lambda_{d,t,n}^{RT} g_{i,d,t,n}^{dev} - \sum_{i \in I^{FL,t,n}} \lambda^{CO_2} Q_{i,d,t,n}^{emi} + \sum_{i \in I^{RE,t,n}} \lambda^{CO_2} Q_{i,d,t,n}^{CA} \\
& + \sum_{i \in I^{RE}} (\lambda_{i,d}^{PGC} Q_{i,d}^{PGC} + \lambda^{FGC} Q_{i,d}^{FGC}) \bigg] - \sum_{i,n} C_i^{IN} G_{i,n} \tag{6.87}
\end{aligned}$$

where:

$$V^{MPEC} = V^{UL} \cup V^{LL1} \cup V^{LL2} \cup V^{DUAL} \tag{6.88}$$

$$\begin{aligned}
V^{DUAL} = \{ & \lambda_{d,t,n}^{DA}, \lambda_{d,t,n}^{RT}, \tau_{e,d,t}^{ES}, \tau_{e,d}^{ES,1}, \tau_{e,d}^{ES,T}, \tau_{e,d,t}^{ES,-}, \tau_{e,d,t}^{ES,+}, \tau_{e,d,t}^{c,DA,-}, \tau_{e,d,t}^{c,DA,+}, \tau_{e,d,t}^{c,RT,-}, \\
& \tau_{e,d,t}^{c,RT,+}, \tau_{e,d,t}^{d,DA,-}, \tau_{e,d,t}^{d,DA,+}, \tau_{e,d,t}^{d,RT,-}, \tau_{e,d,t}^{d,RT,+}, \zeta_d, \mu_{i,d,t,n,b}^U, \mu_{i,d,t,n,b}^D, \phi_{i,d,t,n,b}^{U,-}, \\
& \phi_{i,d,t,n,b}^{U,+}, \phi_{i,d,t,n,b}^{D,-}, \phi_{i,d,t,n,b}^{D,+}, \gamma_{i,d,t,n}, \theta_{i,d,t,n}, \nu_{d,t,n,m}^{DA,-}, \nu_{y,t,n,m}^{DA,+}, \xi_{d,t,n}^{DA,-}, \xi_{d,t,n}^{DA,+}, \\
& \xi_{d,t}^{DA,ref}, \nu_{d,t,n,m}^{RT,-}, \nu_{d,t,n,m}^{RT,+}, \xi_{d,t,n}^{RT,-}, \xi_{d,t,n}^{RT,+}, \xi_{d,t}^{RT,ref}, \chi_{i,d,t,n}^{RE}, \chi_{j,d,t}^L \} \tag{6.89}
\end{aligned}$$

subject to:

$$\text{UL constraints : (6.3) – (6.7)} \tag{6.90}$$

$$\text{LL1 KKT equality conditions : (6.44)} \tag{6.91}$$

$$\text{LL1 KKT stationarity conditions : (6.45) – (6.59)} \tag{6.92}$$

$$\text{LL1 KKT complementarity conditions : (6.60) – (6.86)} \tag{6.93}$$

$$\text{LL2 constraints : (6.36) – (6.37)} \tag{6.94}$$

6.4.3 Linearization of MPEC

In order to convert the nonlinear MPEC into a computationally tractable MIQP, this subsection details the process of linearizing the MPEC. The nonlinear elements of the MPEC primarily stem from two key sources: the KKT complementarity conditions related to the LL1 problem (6.93), and the bilinear terms embedded in the GenCo's profit, formulated by the MPEC's objective function (6.87).

Linearization of KKT Complementarity Conditions

The big-M method is adopted to decompose the KKT complementarity condition into a set of linear conditions mixed with binary variables. Specifically, a represents a primal variable in KKT conditions, and b represents the corresponding dual variable. The resulting equivalent mixed-integer linear conditions are formulated as follows: $\{a \geq 0, b \geq 0, a \leq \beta M^a, b \leq (1-\beta)M^b, \beta \in \{0, 1\}\}$. Note that M^a and M^b identify the large positive constants. Based on this linearization procedure, the KKT complementarity conditions linked to the LL1 problem (6.93) are transformed into a set of disjunctive constraints. These disjunctive constraints are formulated as follows:

$$V^{BINARY} = \left\{ \begin{aligned} &\beta_{e,d,t}^{E,-}, \beta_{e,d,t}^{E,+}, \beta_{e,d,t}^{c,DA,-}, \beta_{e,d,t}^{c,DA,+}, \beta_{e,d,t}^{c,RT,-}, \beta_{e,d,t}^{c,RT,+}, \beta_{e,d,t}^{d,DA,-}, \beta_{e,d,t}^{d,DA,+}, \beta_{e,d,t}^{d,RT,-}, \\ &\beta_{e,d,t}^{d,RT,+}, \beta_{d,t}^{CO_2}, \beta_{i,d,t,n,b}^U, \beta_{i,d,t,n,b}^D, \beta_{i,d,t,n,b}^{U,-}, \beta_{i,d,t,n,b}^{U,+}, \beta_{i,d,t,n,b}^{D,-}, \beta_{i,d,t,n,b}^{D,+}, \\ &\beta_{d,t,n,m}^{DA,net,-}, \beta_{d,t,n,m}^{DA,net,+}, \beta_{d,t,n}^{DA,\pi,-}, \beta_{d,t,n}^{DA,\pi,+}, \beta_{d,t,n,m}^{RT,net,-}, \beta_{d,t,n,m}^{RT,net,+}, \beta_{d,t,n}^{RT,\pi,-}, \\ &\beta_{d,t,n}^{RT,\pi,+}, \beta_{i,d,t,n}^{RE}, \beta_{j,d,t}^L \end{aligned} \right\} \quad (6.95)$$

$$0 \leq E_{e,d,t} - E_e^{min} \leq \beta_{e,d,t}^{E,-} M \quad \forall e, \forall d, \forall t \quad (6.96)$$

$$0 \leq \tau_{e,d,t}^{ES,-} \leq (1 - \beta_{e,d,t}^{E,-}) M \quad \forall e, \forall d, \forall t \quad (6.97)$$

$$0 \leq E_e^{max} - E_{e,d,t} \leq \beta_{e,d,t}^{E,+} M \quad \forall e, \forall d, \forall t \quad (6.98)$$

$$0 \leq \tau_{e,d,t}^{ES,+} \leq (1 - \beta_{e,d,t}^{E,+}) M \quad \forall e, \forall d, \forall t \quad (6.99)$$

$$0 \leq s_{e,d,t}^{c,DA} \leq \beta_{e,d,t}^{c,DA,-} M \quad \forall e, \forall d, \forall t \quad (6.100)$$

$$0 \leq \tau_{e,d,t}^{c,DA,-} \leq (1 - \beta_{e,d,t}^{c,DA,-}) M \quad \forall e, \forall d, \forall t \quad (6.101)$$

$$0 \leq s_e^{max} - s_{e,d,t}^{c,DA} \leq \beta_{e,d,t}^{c,DA,+} M \quad \forall e, \forall d, \forall t \quad (6.102)$$

$$0 \leq \tau_{e,d,t}^{c,DA,+} \leq (1 - \beta_{e,d,t}^{c,DA,+}) M \quad \forall e, \forall d, \forall t \quad (6.103)$$

$$0 \leq s_{e,d,t}^{c,DA} + s_{e,d,t}^{c,RT} \leq \beta_{e,d,t}^{c,RT,-} M \quad \forall e, \forall d, \forall t \quad (6.104)$$

$$0 \leq \tau_{e,d,t}^{c,RT,-} \leq (1 - \beta_{e,d,t}^{c,RT,-}) M \quad \forall e, \forall d, \forall t \quad (6.105)$$

$$0 \leq s_e^{max} - s_{e,d,t}^{c,DA} - s_{e,d,t}^{c,RT} \leq \beta_{e,d,t}^{c,RT,+} M \quad \forall e, \forall d, \forall t \quad (6.106)$$

$$0 \leq \tau_{e,d,t}^{c,RT,+} \leq (1 - \beta_{e,d,t}^{c,RT,+}) M \quad \forall e, \forall d, \forall t \quad (6.107)$$

$$0 \leq s_{e,d,t}^{d,DA} \leq \beta_{e,d,t}^{d,DA,-} M \quad \forall e, \forall d, \forall t \quad (6.108)$$

$$0 \leq \tau_{e,d,t}^{d,DA,-} \leq (1 - \beta_{e,d,t}^{d,DA,-}) M \quad \forall e, \forall d, \forall t \quad (6.109)$$

$$0 \leq s_e^{max} - s_{e,d,t}^{d,DA} \leq \beta_{e,d,t}^{d,DA,+} M \quad \forall e, \forall d, \forall t \quad (6.110)$$

$$0 \leq \tau_{e,d,t}^{d,DA,+} \leq (1 - \beta_{e,d,t}^{d,DA,+}) M \quad \forall e, \forall d, \forall t \quad (6.111)$$

$$0 \leq s_{e,d,t}^{d,DA} + s_{e,d,t}^{d,RT} \leq \beta_{e,d,t}^{d,RT,-} M \quad \forall e, \forall d, \forall t \quad (6.112)$$

$$0 \leq \tau_{e,d,t}^{d,RT,-} \leq (1 - \beta_{e,d,t}^{d,RT,-}) M \quad \forall e, \forall d, \forall t \quad (6.113)$$

$$0 \leq s_e^{max} - s_{e,d,t}^{d,DA} - s_{e,d,t}^{d,RT} \leq \beta_{e,d,t}^{d,RT,+} M \quad \forall e, \forall d, \forall t \quad (6.114)$$

$$0 \leq \tau_{e,d,t}^{d,RT,+} \leq (1 - \beta_{e,d,t}^{d,RT,+}) M \quad \forall e, \forall d, \forall t \quad (6.115)$$

$$0 \leq \eta^{Grid} \sum_{j,t,o} L_{j,d,t,o}^{DA} - \sum_{i \in I^{FL}, t, n, b} \eta_i^{CO_2} (g_{i,d,t,n,b}^{DA} + g_{i,d,t,n,b}^U - g_{i,d,t,n,b}^D) \leq \beta_d^{CO_2} M \quad \forall d \quad (6.116)$$

$$0 \leq \zeta_d \leq (1 - \beta_d^{CO_2}) M \quad \forall d \quad (6.117)$$

$$0 \leq G_{i,n,b} - g_{i,d,t,n,b}^{DA} - g_{i,d,t,n,b}^U \leq \beta_{i,d,t,n,b}^U M \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (6.118)$$

$$0 \leq \mu_{i,d,t,n,b}^U \leq (1 - \beta_{i,d,t,n,b}^U) M \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (6.119)$$

$$0 \leq g_{i,d,t,n,b}^{DA} - g_{i,d,t,n,b}^D \leq \beta_{i,d,t,n,b}^D M \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (6.120)$$

$$0 \leq \mu_{i,d,t,n,b}^D \leq (1 - \beta_{i,d,t,n,b}^D) M \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (6.121)$$

$$0 \leq g_{i,d,t,n,b}^U \leq \beta_{i,d,t,n,b}^{U,-} M \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (6.122)$$

$$0 \leq \phi_{i,d,t,n,b}^{U,-} \leq (1 - \beta_{i,d,t,n,b}^{U,-}) M \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (6.123)$$

$$0 \leq G_{i,b}^U - g_{i,d,t,n,b}^U \leq \beta_{i,d,t,n,b}^{U,+} M \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (6.124)$$

$$0 \leq \phi_{i,d,t,n,b}^{U,+} \leq (1 - \beta_{i,d,t,n,b}^{U,+}) M \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (6.125)$$

$$0 \leq g_{i,d,t,n,b}^D \leq \beta_{i,d,t,n,b}^{D,-} M \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (6.126)$$

$$0 \leq \phi_{i,d,t,n,b}^{D,-} \leq (1 - \beta_{i,d,t,n,b}^{D,-}) M \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (6.127)$$

$$0 \leq G_{i,b}^D - g_{i,d,t,n,b}^D \leq \beta_{i,d,t,n,b}^{D,+} M \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (6.128)$$

$$0 \leq \phi_{i,d,t,n,b}^{D,+} \leq (1 - \beta_{i,d,t,n,b}^{D,+}) M \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (6.129)$$

$$0 \leq C_{nm}^{max} + b_{nm} (\delta_{d,t,n}^{DA} - \delta_{d,t,m}^{DA}) \leq \beta_{d,t,n,m}^{DA,net,-} M \quad \forall d, \forall t, \forall n, \forall m \in \Lambda_n \quad (6.130)$$

$$0 \leq \nu_{d,t,n,m}^{DA,-} \leq (1 - \beta_{d,t,n,m}^{DA,net,-}) M \quad \forall d, \forall t, \forall n, \forall m \in \Lambda_n \quad (6.131)$$

$$0 \leq C_{nm}^{max} - b_{nm} (\delta_{d,t,n}^{DA} - \delta_{d,t,m}^{DA}) \leq \beta_{d,t,n,m}^{DA,net,+} M \quad \forall d, \forall t, \forall n, \forall m \in \Lambda_n \quad (6.132)$$

$$0 \leq \nu_{d,t,n,m}^{DA,+} \leq (1 - \beta_{d,t,n,m}^{DA,net,+}) M \quad \forall d, \forall t, \forall n, \forall m \in \Lambda_n \quad (6.133)$$

$$0 \leq \pi + \delta_{d,t,n}^{DA} \leq \beta_{d,t,n}^{DA,\pi,-} M \quad \forall d, \forall t, \forall n \quad (6.134)$$

$$0 \leq \xi_{d,t,n}^{DA,-} \leq (1 - \beta_{d,t,n}^{DA,\pi,-})M \quad \forall d, \forall t, \forall n \quad (6.135)$$

$$0 \leq \pi - \delta_{d,t,n}^{DA} \leq \beta_{d,t,n}^{DA,\pi,+}M \quad \forall d, \forall t, \forall n \quad (6.136)$$

$$0 \leq \xi_{d,t,n}^{DA,+} \leq (1 - \beta_{d,t,n}^{DA,\pi,+})M \quad \forall d, \forall t, \forall n \quad (6.137)$$

$$0 \leq C_{nm}^{max} + b_{nm}(\delta_{d,t,n}^{RT} - \delta_{d,t,m}^{RT}) \leq \beta_{d,t,n,m}^{RT,net,-}M \quad \forall d, \forall t, \forall n, \forall m \in \Lambda_n \quad (6.138)$$

$$0 \leq \nu_{d,t,n,m}^{RT,-} \leq (1 - \beta_{d,t,n,m}^{RT,net,-})M \quad \forall d, \forall t, \forall n, \forall m \in \Lambda_n \quad (6.139)$$

$$0 \leq C_{nm}^{max} - b_{nm}(\delta_{d,t,n}^{RT} - \delta_{d,t,m}^{RT}) \leq \beta_{d,t,n,m}^{RT,net,+}M \quad \forall d, \forall t, \forall n, \forall m \in \Lambda_n \quad (6.140)$$

$$0 \leq \nu_{d,t,n,m}^{RT,+} \leq (1 - \beta_{d,t,n,m}^{RT,net,+})M \quad \forall d, \forall t, \forall n, \forall m \in \Lambda_n \quad (6.141)$$

$$0 \leq \pi + \delta_{d,t,n}^{RT} \leq \beta_{d,t,n}^{RT,\pi,-}M \quad \forall d, \forall t, \forall n \quad (6.142)$$

$$0 \leq \xi_{d,t,n}^{RT,-} \leq (1 - \beta_{d,t,n}^{RT,\pi,-})M \quad \forall d, \forall t, \forall n \quad (6.143)$$

$$0 \leq \pi - \delta_{d,t,n}^{RT} \leq \beta_{d,t,n}^{RT,\pi,+}M \quad \forall d, \forall t, \forall n \quad (6.144)$$

$$0 \leq \xi_{d,t,n}^{RT,+} \leq (1 - \beta_{d,t,n}^{RT,\pi,+})M \quad \forall d, \forall t, \forall n \quad (6.145)$$

$$0 \leq m_{i,d,t,n}^{RE} - g_{i,d,t,n}^{DA} + g_{i,d,t,n}^{dev} - \Phi^{-1}(1 - \rho^{RE})s_{i,d,t,n}^{RE} \leq \beta_{i,d,t,n}^{RE}M \quad \forall i \in I^{RE}, \forall d, \forall t, \forall n \quad (6.146)$$

$$0 \leq \chi_{i,d,t,n}^{RE} \leq (1 - \beta_{i,d,t,n}^{RE})M \quad \forall i \in I^{RE}, \forall d, \forall t, \forall n \quad (6.147)$$

$$0 \leq -m_{j,d,t}^L + \sum_o P_{j,d,t,o}^{dev} + \sum_o P_{j,d,t,o}^{DA} - \Phi^{-1}(1 - \rho^L)s_{j,d,t}^L \leq \beta_{j,d,t}^L M \quad \forall j, \forall d, \forall t \quad (6.148)$$

$$0 \leq \chi_{j,d,t}^L \leq (1 - \beta_{j,d,t}^L)M \quad \forall j, \forall d, \forall t \quad (6.149)$$

Linearization of GenCo's Profit

In order to deal with the challenge of bilinear terms in GenCo's profit within the MPEC's objective function (6.87), the strong duality theorem and the KKT optimality conditions are leveraged to uncover implicit linear forms of these bilinear terms, thereby facilitating their incorporation into the model in a linearized structure.

Specifically, the bilinear terms ($\sum_{i \in I} \lambda_{d,t,n}^{DA} g_{i,d,t,n,b}^{DA}$, $\sum_{i \in I^{FL}} \lambda_{d,t,n}^{RT} g_{i,d,t,n,b}^U$, $\sum_{i \in I^{FL}} \lambda_{d,t,n}^{RT} g_{i,d,t,n,b}^D$, $\sum_{i \in I^{RE}} \lambda_{d,t,n}^{RT} g_{i,d,t,n}^{dev}$, and $\sum_{i \in I^{RE}} \lambda_{i,d}^{PGC} Q_{i,d}^{PGC}$) of the MPEC's objective function (6.87) need to be restructured as their linearized form by adopting the strong duality theorem and the KKT optimality conditions. The derivation process is detailed as follows.

First, in order to linearize the bilinear term $\sum_{i \in I} \lambda_{d,t,n}^{DA} g_{i,d,t,n,b}^{DA}$, the KKT stationarity

condition (6.45) is rearranged as:

$$\lambda_{d,t,n}^{DA} = C_{i,b}^{EN} + \eta_i^{CO_2} \zeta_d + \mu_{i,d,t,n,b}^U - \mu_{i,d,t,n,b}^D \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (6.150)$$

Multiplying both sides by $g_{i,d,t,n,b}^{DA}$ ($\forall i \in I^{FL}$), it renders:

$$\begin{aligned} \lambda_{d,t,n}^{DA} g_{i,d,t,n,b}^{DA} = & C_{i,b}^{EN} g_{i,d,t,n,b}^{DA} + \eta_i^{CO_2} \zeta_d g_{i,d,t,n,b}^{DA} + \mu_{i,d,t,n,b}^U g_{i,d,t,n,b}^{DA} \\ & - \mu_{i,d,t,n,b}^D g_{i,d,t,n,b}^{DA} \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \end{aligned} \quad (6.151)$$

Given that the term (6.151) exclusively accounts for the DA energy revenue derived from flexible generators, the same linearization procedure can be applied to address the revenue from must-run and renewable generators. Therefore, the KKT stationarity condition (6.46) is rewritten as:

$$\lambda_{d,t,n}^{DA} = C_i^{EN} + \gamma_{i,d,t,n} \quad \forall i \in I^{MR}, \forall d, \forall t, \forall n \quad (6.152)$$

Similarly, multiplying both sides by $g_{i,d,t,n}^{DA}$ ($\forall i \in I^{MR}$), it gives:

$$\lambda_{d,t,n}^{DA} g_{i,d,t,n}^{DA} = C_i^{EN} g_{i,d,t,n}^{DA} + \gamma_{i,d,t,n} g_{i,d,t,n}^{DA} \quad \forall i \in I^{MR}, \forall d, \forall t, \forall n \quad (6.153)$$

Subsequently, after the LL1 constraint (6.25) is plugged into (6.153), it renders:

$$\lambda_{d,t,n}^{DA} g_{i,d,t,n}^{DA} = C_i^{EN} g_{i,d,t,n}^{DA} + \gamma_{i,d,t,n} G_{i,n} \quad \forall i \in I^{MR}, \forall d, \forall t, \forall n \quad (6.154)$$

It should be emphasised that in terms of the realisation for renewable power generation, its mean is set as the DA renewable generation dispatch $g_{i,d,t,n}^{DA}$ ($\forall i \in I^{RE}$), and its standard deviation is assumed to be proportional to the DA renewable generation dispatch. The specific setting will be detailed in the section on the case study. As a result, the chance constraint pertaining to renewable power production (6.41) should be rewritten as below:

$$g_{i,d,t,n}^{dev} \geq \Phi^{-1}(1 - \rho^{RE}) \Gamma g_{i,d,t,n}^{DA} \quad \forall i \in I^{RE}, \forall d, \forall t, \forall n \quad (6.155)$$

where Γ identifies the proportion of wind power deviation with respect to DA renewable generation dispatch.

Thus, the KKT stationarity condition (6.47) should be reformulated as follows:

$$-\lambda_{d,t,n}^{DA} + \theta_{i,d,t,n} + \Phi^{-1}(1 - \rho^{RE}) \Gamma \chi_{i,d,t,n}^{RE} = 0 \quad \forall i \in I^{RE}, \forall d, \forall t, \forall n \quad (6.156)$$

Then, rearranging the terms in the KKT stationarity condition (6.156) and multiplying

both sides by $g_{i,d,t,n}^{DA}$ ($i \in I^{RE}$), the energy revenue from renewable generation is recast as below:

$$\lambda_{d,t,n}^{DA} g_{i,d,t,n}^{DA} = \theta_{i,d,t,n} g_{i,d,t,n}^{DA} + \Phi^{-1}(1 - \rho^{RE}) \Gamma \chi_{i,d,t,n}^{RE} g_{i,d,t,n}^{DA} \quad \forall i \in I^{RE}, \forall d, \forall t, \forall n \quad (6.157)$$

By substituting the LL1 constraint (6.26) into the above expression (6.157), it further produces:

$$\lambda_{d,t,n}^{DA} g_{i,d,t,n}^{DA} = \theta_{i,d,t,n} k_{i,d,t}^{RE} G_{i,n} + \Phi^{-1}(1 - \rho^{RE}) \Gamma \chi_{i,d,t,n}^{RE} g_{i,d,t,n}^{DA} \quad \forall i \in I^{RE}, \forall d, \forall t, \forall n \quad (6.158)$$

In conclusion, the bilinear DA energy revenue, which is a critical component of the GenCo's profit, can be equivalently expressed by combining the expressions (6.151), (6.154), and (6.158) with respect to different generation technologies.

The next step is to deal with the bilinear terms related to the RT reserve revenues $\sum_{i \in I^{FL}} \lambda_{d,t,n}^{RT} g_{i,d,t,n,b}^U$ and $\sum_{i \in I^{FL}} \lambda_{d,t,n}^{RT} g_{i,d,t,n,b}^D$. To this end, we first rearrange the terms in the KKT stationarity conditions (6.48) and (6.49), and then multiplying both sides by $g_{i,d,t,n,b}^U$ and $g_{i,d,t,n,b}^D$, respectively, it renders:

$$\begin{aligned} \lambda_{d,t,n}^{RT} g_{i,d,t,n,b}^U = & C_{i,b}^U g_{i,d,t,n,b}^U + \eta_i^{CO_2} \zeta_d g_{i,d,t,n,b}^U + \mu_{i,d,t,n,b}^U g_{i,d,t,n,b}^U - \phi_{i,d,t,n,b}^{U,-} g_{i,d,t,n,b}^U + \\ & \phi_{i,d,t,n,b}^{U,+} g_{i,d,t,n,b}^U \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \end{aligned} \quad (6.159)$$

$$\begin{aligned} \lambda_{d,t,n}^{RT} g_{i,d,t,n,b}^D = & C_{i,b}^D g_{i,d,t,n,b}^D + \eta_i^{CO_2} \zeta_d g_{i,d,t,n,b}^D - \mu_{i,d,t,n,b}^D g_{i,d,t,n,b}^D + \phi_{i,d,t,n,b}^{D,-} g_{i,d,t,n,b}^D - \\ & \phi_{i,d,t,n,b}^{D,+} g_{i,d,t,n,b}^D \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \end{aligned} \quad (6.160)$$

Furthermore, by employing the KKT complementarity conditions (6.73) - (6.76), we obtain the following expressions:

$$g_{i,d,t,n,b}^U \phi_{i,d,t,n,b}^{U,-} = 0 \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (6.161)$$

$$G_{i,b}^U \phi_{i,d,t,n,b}^{U,+} = g_{i,d,t,n,b}^U \phi_{i,d,t,n,b}^{U,+} \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (6.162)$$

$$g_{i,d,t,n,b}^D \phi_{i,d,t,n,b}^{D,-} = 0 \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (6.163)$$

$$G_{i,b}^D \phi_{i,d,t,n,b}^{D,+} = g_{i,d,t,n,b}^D \phi_{i,d,t,n,b}^{D,+} \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (6.164)$$

By substituting KKT complementarity conditions (6.161)-(6.164) into the equations for RT upward and downward reserve revenues, denoted as (6.159) and (6.160) respec-

tively, it gives:

$$\lambda_{d,t,n}^{RT} g_{i,d,t,n,b}^U = C_{i,b}^U g_{i,d,t,n,b}^U + \eta_i^{CO_2} \zeta_d g_{i,d,t,n,b}^U + \mu_{i,d,t,n,b}^U g_{i,d,t,n,b}^U + \phi_{i,d,t,n,b}^{U,+} G_{i,b}^U \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (6.165)$$

$$\lambda_{d,t,n}^{RT} g_{i,d,t,n,b}^D = C_{i,b}^D g_{i,d,t,n,b}^D + \eta_i^{CO_2} \zeta_d g_{i,d,t,n,b}^D - \mu_{i,d,t,n,b}^D g_{i,d,t,n,b}^D - \phi_{i,d,t,n,b}^{D,+} G_{i,b}^D \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (6.166)$$

After this, the bilinear term of the renewable deviation penalty $\sum_{i \in I^{RE}} \lambda_{d,t,n}^{RT} g_{i,d,t,n}^{dev}$ should be addressed. To this end, we employ the KKT complementarity condition (6.85) related to the uncertainty of renewable power production and reformulate it as follows:

$$[-g_{i,d,t,n}^{dev} - \Phi^{-1}(1 - \rho^{RE})\Gamma g_{i,d,t,n}^{DA}] \chi_{i,d,t,n}^{RE} = 0 \quad \forall i \in I^{RE}, \forall d, \forall t, \forall n \quad (6.167)$$

By combining the KKT stationarity condition (6.51) with the complementary expression (6.167), the bilinear term $\lambda_{d,t,n}^{RT} g_{i,d,t,n}^{dev}$ is recast as follows:

$$\lambda_{d,t,n}^{RT} g_{i,d,t,n}^{dev} = \chi_{i,d,t,n}^{RE} g_{i,d,t,n}^{dev} = -\Phi^{-1}(1 - \rho^{RE})\Gamma g_{i,d,t,n}^{DA} \chi_{i,d,t,n}^{RE} \quad \forall i \in I^{RE}, \forall d, \forall t, \forall n \quad (6.168)$$

Collectively, an alternative representation for the first four bilinear terms in the MPEC's objective function (the left side of the following equation) can be derived by integrating the expressions of (6.151), (6.154), (6.158), (6.165), (6.166), (6.168), and summing over their respective indices, which is formulated as follows:

$$\begin{aligned} & \sum_{i \in I, d, t, n, b} \lambda_{d,t,n}^{DA} g_{i,d,t,n,b}^{DA} + \sum_{i \in I^{FL}, d, t, n, b} \lambda_{d,t,n}^{RT} (g_{i,d,t,n,b}^U - g_{i,d,t,n,b}^D) - \sum_{i \in I^{RE}, d, t, n} \lambda_{d,t,n}^{RT} g_{i,d,t,n}^{dev} = \\ & \sum_{i \in I, d, t, n, b} C_{i,b}^{EN} g_{i,d,t,n,b}^{DA} + \sum_{i \in I^{FL}, d, t, n, b} C_{i,b}^U g_{i,d,t,n,b}^U - \sum_{i \in I^{FL}, d, t, n, b} C_{i,b}^D g_{i,d,t,n,b}^D + \\ & \sum_{i \in I^{FL}, d, t, n, b} \eta_i^{CO_2} \zeta_d (g_{i,d,t,n,b}^{DA} + g_{i,d,t,n,b}^U - g_{i,d,t,n,b}^D) + \sum_{i \in I^{FL}, d, t, n, b} \phi_{i,d,t,n,b}^{U,+} G_{i,b}^U + \\ & \sum_{i \in I^{FL}, d, t, n, b} \phi_{i,d,t,n,b}^{D,+} G_{i,b}^D + \sum_{i \in I^{FL}, d, t, n, b} \mu_{i,d,t,n,b}^D (g_{i,d,t,n,b}^D - g_{i,d,t,n,b}^{DA}) + \\ & \sum_{i \in I^{FL}, d, t, n, b} \mu_{i,d,t,n,b}^U (g_{i,d,t,n,b}^{DA} + g_{i,d,t,n,b}^U) + \sum_{i \in I^{MR}, d, t, n} \gamma_{i,d,t,n} G_{i,n} + \sum_{i \in I^{RE}, d, t, n} \theta_{i,d,t,n} k_{i,d,t}^{RE} G_{i,n} \end{aligned} \quad (6.169)$$

It is observable that some nonlinear terms on the right side of equation (6.169) can be eliminated through the application of the KKT complementarity conditions (6.70), (6.71)

and (6.72), which are expressed as below:

$$\zeta_d \eta^{Grid} \sum_{j,t,o} L_{j,d,t,o}^{DA} = \zeta_d \sum_{i \in I^{FL}, t, n, b} \eta_i^{CO_2} (g_{i,d,t,n,b}^{DA} + g_{i,d,t,n,b}^U - g_{i,d,t,n,b}^D) \quad \forall d \quad (6.170)$$

$$\mu_{i,d,t,n,b}^U G_{i,n,b} = \mu_{i,d,t,n,b}^U (g_{i,d,t,n,b}^{DA} + g_{i,d,t,n,b}^U) \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (6.171)$$

$$\mu_{i,d,t,n,b}^D (g_{i,d,t,n,b}^{DA} - g_{i,d,t,n,b}^D) = 0 \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (6.172)$$

By substituting the above KKT complementarity conditions (6.170)-(6.172) into the equation (6.169), the representation for the first four bilinear terms in the MPEC's objective function can be partially linearized, which is given as follows:

$$\begin{aligned} & \sum_{i \in I, d, t, n, b} \lambda_{d,t,n}^{DA} g_{i,d,t,n,b}^{DA} + \sum_{i \in I^{FL}, d, t, n, b} \lambda_{d,t,n}^{RT} (g_{i,d,t,n,b}^U - g_{i,d,t,n,b}^D) - \sum_{i \in I^{RE}, d, t, n} \lambda_{d,t,n}^{RT} g_{i,d,t,n}^{dev} = \\ & \sum_{i \in I, d, t, n, b} C_{i,b}^{EN} g_{i,d,t,n,b}^{DA} + \sum_{i \in I^{FL}, d, t, n, b} C_{i,b}^U g_{i,d,t,n,b}^U - \sum_{i \in I^{FL}, d, t, n, b} C_{i,b}^D g_{i,d,t,n,b}^D + \\ & \sum_{j,d,t,o} \eta^{Grid} \zeta_d L_{j,d,t,o}^{DA} + \sum_{i \in I^{FL}, d, t, n, b} \phi_{i,d,t,n,b}^{U,+} G_{i,b}^U + \sum_{i \in I^{FL}, d, t, n, b} \phi_{i,d,t,n,b}^{D,+} G_{i,b}^D + \\ & \sum_{i \in I^{FL}, d, t, n, b} \mu_{i,d,t,n,b}^U G_{i,n,b} + \sum_{i \in I^{MR}, d, t, n} \gamma_{i,d,t,n} G_{i,n} + \sum_{i \in I^{RE}, d, t, n} \theta_{i,d,t,n} k_{i,d,t}^{RE} G_{i,n} \end{aligned} \quad (6.173)$$

Nonetheless, it is noticeable that the presence of the nonlinearities posed by the last three bilinear terms of equation (6.173) needs to be further addressed. In order to remove these non-linearities, the strong duality theorem of the LL1 problem incorporating the ES model is utilised, which is expressed as follows:

$$\begin{aligned} & \sum_{i,d,t,n,b} C_{i,b}^{EN} g_{i,d,t,n,b}^{DA} + \sum_{i,d,t,n,b} (C_{i,b}^U g_{i,d,t,n,b}^U - C_{i,b}^D g_{i,d,t,n,b}^D) = - \sum_{e,d} E_e^0 (\tau_{e,d}^{ES,1} - \tau_{e,d}^{ES,T}) \\ & - \sum_{e,d,t} E_e^{max} \tau_{e,d,t}^{ES,+} + \sum_{e,d,t} E_e^{min} \tau_{e,d,t}^{ES,-} - \sum_{e,d,t} s_e^{max} \tau_{e,d,t}^{c,DA,+} - \sum_{e,d,t} s_e^{max} \tau_{e,d,t}^{c,RT,+} - \\ & \sum_{e,d,t} s_e^{max} \tau_{e,d,t}^{d,DA,+} - \sum_{e,d,t} s_e^{max} \tau_{e,d,t}^{d,RT,+} + \sum_{j,d,t,o} \lambda_{j,d,t}^{DA} L_{j,d,t,o}^{DA} - \sum_{j,d,t,o} \eta^{Grid} \zeta_d L_{j,d,t,o}^{DA} - \\ & \sum_{i \in I^{FL}, d, t, n, b} \mu_{i,d,t,n,b}^U G_{i,n,b} - \sum_{i \in I^{FL}, d, t, n, b} \phi_{i,d,t,n,b}^{U,+} G_{i,b}^U - \sum_{i \in I^{FL}, d, t, n, b} \phi_{i,d,t,n,b}^{D,+} G_{i,b}^D \\ & - \sum_{i \in I^{MR}, d, t, n} \gamma_{i,d,t,n} G_{i,n} - \sum_{i \in I^{RE}, d, t, n} \theta_{i,d,t,n} k_{i,d,t}^{RE} G_{i,n} \\ & - \sum_{d,t,n,m \in \Lambda_n} C_{nm}^{max} (\nu_{d,t,n,m}^{DA,-} + \nu_{d,t,n,m}^{DA,+}) - \sum_{d,t,n} \pi (\xi_{d,t,n}^{DA,-} + \xi_{d,t,n}^{DA,+}) \\ & - \sum_{d,t,n,m \in \Lambda_n} C_{nm}^{max} (\nu_{d,t,n,m}^{RT,-} + \nu_{d,t,n,m}^{RT,+}) - \sum_{d,t,n} \pi (\xi_{d,t,n}^{RT,-} + \xi_{d,t,n}^{RT,+}) \end{aligned}$$

$$+ \sum_{j,d,t} [\Phi^{-1}(1 - \rho^L) s_{j,d,t}^L + m_{j,d,t}^L] \chi_{j,d,t}^L \quad (6.174)$$

Utilising the strong duality equation (6.174), the bilinear function (left side of equation (6.175)) can be linearized as follows:

$$\begin{aligned} & \sum_{i \in I, d, t, n, b} \lambda_{d,t,n}^{DA} g_{i,d,t,n,b}^{DA} + \sum_{i \in I^{FL}, d, t, n, b} \lambda_{d,t,n}^{RT} (g_{i,d,t,n,b}^U - g_{i,d,t,n,b}^D) - \sum_{i \in I^{RE}, d, t, n} \lambda_{d,t,n}^{RT} g_{i,d,t,n}^{dev} = \\ & - \sum_{e,d} E_e^0 (\tau_{e,d}^{ES,1} - \tau_{e,d}^{ES,T}) - \sum_{e,d,t} E_e^{max} \tau_{e,d,t}^{ES,+} + \sum_{e,d,t} E_e^{min} \tau_{e,d,t}^{ES,-} - \sum_{e,d,t} s_e^{max} \tau_{e,d,t}^{c,DA,+} \\ & - \sum_{e,d,t} s_e^{max} \tau_{e,d,t}^{c,RT,+} - \sum_{e,d,t} s_e^{max} \tau_{e,d,t}^{d,DA,+} - \sum_{e,d,t} s_e^{max} \tau_{e,d,t}^{d,RT,+} + \sum_{j,d,t,o} \lambda_{j,d,t}^{DA} L_{j,d,t,o}^{DA} \\ & - \sum_{d,t,n,m \in \Lambda_n} C_{nm}^{max} (\nu_{d,t,n,m}^{DA,-} + \nu_{d,t,n,m}^{DA,+}) - \sum_{d,t,n} \pi (\xi_{d,t,n}^{DA,-} + \xi_{d,t,n}^{DA,+}) \\ & - \sum_{d,t,n,m \in \Lambda_n} C_{nm}^{max} (\nu_{d,t,n,m}^{RT,-} + \nu_{d,t,n,m}^{RT,+}) - \sum_{d,t,n} \pi (\xi_{d,t,n}^{RT,-} + \xi_{d,t,n}^{RT,+}) \\ & + \sum_{j,d,t} [\Phi^{-1}(1 - \rho^L) s_{j,d,t}^L + m_{j,d,t}^L] \chi_{j,d,t}^L \end{aligned} \quad (6.175)$$

The final step is to handle the last bilinear term related to the TGC revenue $\sum_{i \in I^{RE}} \lambda_{i,d}^{PGC} Q_{i,d}^{PGC}$. It can be converted into a quadratic term by adopting LL2 constraint (6.36), which is presented as follows:

$$\sum_{i \in I^{RE}, d} \lambda_{i,d}^{PGC} Q_{i,d}^{PGC} = \sum_{i \in I^{RE}, d} [l^{PGC} Q_{i,d}^{PGC} - q^{PGC} (Q_{i,d}^{PGC})^2] \quad (6.176)$$

Finally, by the substitution of expressions from (6.175) and (6.176) into the bilinear MPEC's objective function (6.87), the GenCo's profit is simplified to a linearized form:

$$\begin{aligned} & \sum_d \omega_d \left\{ - \sum_{i,t,n,b} C_{i,b}^{EN} g_{i,d,t,n,b}^{DA} - \sum_{i,t,n,b} (C_{i,b}^U g_{i,d,t,n,b}^U - C_{i,b}^D g_{i,d,t,n,b}^D) - \sum_e E_e^0 (\tau_{e,d}^{ES,1} - \tau_{e,d}^{ES,T}) \right. \\ & - \sum_{e,t} E_e^{max} \tau_{e,d,t}^{ES,+} + \sum_{e,t} E_e^{min} \tau_{e,d,t}^{ES,-} - \sum_{e,t} s_e^{max} \tau_{e,d,t}^{c,DA,+} - \sum_{e,t} s_e^{max} \tau_{e,d,t}^{c,RT,+} - \\ & \sum_{e,t} s_e^{max} \tau_{e,d,t}^{d,DA,+} - \sum_{e,t} s_e^{max} \tau_{e,d,t}^{d,RT,+} + \sum_{j,t,o} \lambda_{j,d,t}^{DA} L_{j,d,t,o}^{DA} \\ & - \sum_{t,n,m \in \Lambda_n} C_{nm}^{max} (\nu_{d,t,n,m}^{DA,-} + \nu_{d,t,n,m}^{DA,+}) - \sum_{t,n} \pi (\xi_{d,t,n}^{DA,-} + \xi_{d,t,n}^{DA,+}) \\ & - \sum_{t,n,m \in \Lambda_n} C_{nm}^{max} (\nu_{d,t,n,m}^{RT,-} + \nu_{d,t,n,m}^{RT,+}) - \sum_{t,n} \pi (\xi_{d,t,n}^{RT,-} + \xi_{d,t,n}^{RT,+}) \\ & \left. + \sum_{j,t} [\Phi^{-1}(1 - \rho^L) s_{j,d,t}^L + m_{j,d,t}^L] \chi_{j,d,t}^L - \sum_{i \in I^{FL}, t, n} \lambda^{CO_2} Q_{i,d,t,n}^{emi} + \sum_{i \in I^{RE}, t, n} \lambda^{CO_2} Q_{i,d,t,n}^{CA} \right\} \end{aligned}$$

$$+ \sum_{i \in I^{RE}} \left[l^{PGC} Q_{i,d}^{PGC} - q^{PGC} (Q_{i,d}^{PGC})^2 + \lambda^{FGC} Q_{i,d}^{FGC} \right] \Big\} - \sum_{i,n} C_i^{IN} G_{i,n} \quad (6.177)$$

6.4.4 Final MIQP

By applying linearization techniques, the MPEC is finally transformed into a MIQP model. The MIQP's objective function, (6.178), continues to embody the GenCo's profit, but within a linearized framework. The decision variable set for the MIQP (6.179) blends the variable set of the original MPEC model (6.88) with an additional set of auxiliary binary variables (6.95). The MIQP's constraints are made of three principal categories: i) the original UL problem constraints (6.180), ii) the linearized KKT optimality conditions for the LL1 problem (6.181), and iii) the constraints from the LL2 problem (6.182). Thus, the comprehensive framework for the final MIQP formulation is provided as follows:

$$\begin{aligned} \min_{V^{MIQP}} \sum_d \omega_d \Big\{ & - \sum_{i,t,n,b} C_{i,b}^{EN} g_{i,d,t,n,b}^{DA} - \sum_{i,t,n,b} (C_{i,b}^U g_{i,d,t,n,b}^U - C_{i,b}^D g_{i,d,t,n,b}^D) - \\ & \sum_e E_e^0 (\tau_{e,d}^{ES,1} - \tau_{e,d}^{ES,T}) - \sum_{e,t} E_e^{max} \tau_{e,d,t}^{ES,+} + \sum_{e,t} E_e^{min} \tau_{e,d,t}^{ES,-} - \sum_{e,t} s_e^{max} \tau_{e,d,t}^{c,DA,+} \\ & - \sum_{e,t} s_e^{max} \tau_{e,d,t}^{c,RT,+} - \sum_{e,t} s_e^{max} \tau_{e,d,t}^{d,DA,+} - \sum_{e,t} s_e^{max} \tau_{e,d,t}^{d,RT,+} + \sum_{j,t,o} \lambda_{j,d,t}^{DA} L_{j,d,t,o}^{DA} \\ & - \sum_{t,n,m \in \Lambda_n} C_{nm}^{max} (\nu_{d,t,n,m}^{DA,-} + \nu_{d,t,n,m}^{DA,+}) - \sum_{t,n} \pi (\xi_{d,t,n}^{DA,-} + \xi_{d,t,n}^{DA,+}) \\ & - \sum_{t,n,m \in \Lambda_n} C_{nm}^{max} (\nu_{d,t,n,m}^{RT,-} + \nu_{d,t,n,m}^{RT,+}) - \sum_{t,n} \pi (\xi_{d,t,n}^{RT,-} + \xi_{d,t,n}^{RT,+}) \\ & + \sum_{j,t} [\Phi^{-1}(1 - \rho^L) s_{j,d,t}^L + m_{j,d,t}^L] \chi_{j,d,t}^L - \sum_{i \in I^{FL},t,n} \lambda^{CO_2} Q_{i,d,t,n}^{emi} + \\ & \sum_{i \in I^{RE},t,n} \lambda^{CO_2} Q_{i,d,t,n}^{CA} + \sum_{i \in I^{RE}} [l^{PGC} Q_{i,d}^{PGC} - q^{PGC} (Q_{i,d}^{PGC})^2 + \lambda^{FGC} Q_{i,d}^{FGC}] \Big\} \\ & - \sum_{i,n} C_i^{IN} G_{i,n} \end{aligned} \quad (6.178)$$

where:

$$V^{MIQP} = V^{MPEC} \cup V^{BINARY} \quad (6.179)$$

subject to:

$$\text{UL constraints : (6.3) – (6.7)} \quad (6.180)$$

LL1 KKT conditions :

$$(6.10) - (6.14), (6.25) - (6.26), (6.29), (6.32), (6.45) - (6.59), (6.96) - (6.149) \quad (6.181)$$

$$\text{LL2 constraints : } (6.36) - (6.37) \quad (6.182)$$

6.5 Case Studies

6.5.1 Data Descriptions and Implementation

This section conducts a sensitivity analysis of the role of ES within the 2021 GB deregulated energy market. Therefore, the real data for the 2021 GB power system is provided below.

Table 6.1 presents comprehensive data about investment options for the GenCo. It is noteworthy that in terms of flexible generation technologies, the marginal cost for providing upward reserve is set at 1.1 times the marginal energy cost; conversely, the marginal cost for providing downward reserve is set at 0.9 times the marginal energy cost.

Table 6.1: Parameters of investment options.

Investment option		CCGT1	CCGT2	OCGT1	OCGT2	Nuclear	Wind
Type		Flexible	Flexible	Flexible	Flexible	Must-run	Renewable
Investment cost C^{IN} (£/MW/yr)		14413	17440	8610	10418	114849	35756
Carbon intensity η^{CO_2} (t/MWh)		0.39	0.39	0.65	0.65	0	0
Maximum upward reserve capacity G^U (MW)		4000	8000	4000	8000	-	-
Maximum downward reserve capacity G^D (MW)		4000	8000	4000	8000	-	-
Marginal energy cost C^{EN} (£/MWh)	Block 1	7.43	20.05	40.99	49.25	4.72	0
	Block 2	9.03	24.34	49.78	59.81		
	Block 3	10.62	28.64	58.56	70.36		
	Block 4	12.21	32.94	67.34	80.91		
	Block 5	13.81	37.23	76.13	91.47		

Table 6.2 displays crucial parameters for the 2021 GB power system. Additionally, the allocation of weights to these blocks is set as [0.1, 0.15, 0.2, 0.25, 0.3], and the demand is also accurately modelled by the five demand blocks, incorporating weighting factors as [0.6, 0.1, 0.1, 0.1, 0.1]. By virtue of computational burden, the demand profile is simulated on a representative day with an hourly resolution ($T = \{1, 2, \dots, 24\}$), scaled across a target year ($\omega = 365$) to capture daily fluctuations in energy consumption and production patterns.

The additional parameters associated with carbon allowance and TGC are essential. Specifically, the carbon allowance allocation ratio, ϵ_i , is set as 0.4; the RPS requirement ratio, γ^{RPS} , is set as 40%; the approximated price ratio in the future TGC market, γ^{FGC} , is set as 0.3; and the price coefficient in the present TGC market, θ^{PGC} , is also set as 0.3.

Table 6.2: Parameters of the GB power grid in 2021.

Year	2021
System-wide carbon intensity η^{Grid} (t/MWh)	183.11
Carbon price λ^{CO_2} (£/t)	71.54
Total system demand (TWh)	248.30

It should be noted that the penalty price for RPS deficiency, λ_0^{RPS} , is assumed to be 1.1 times the carbon price.

The related coefficients used in the chance constraints should be given. Overall, the model sets confidence levels for all the chance constraints at 95% ($1 - \rho^L = 1 - \rho^{RE} = 95\%$), conveying the ρ -reliable threshold for both demand and wind power production uncertainty. The linearized CCP formulations incorporate stochastic variables for wind power and demand realisations, which are modelled using two normal distributions, $\mathcal{N}(m^{RE}, (s^{RE})^2)$ for wind power and $\mathcal{N}(m^L, (s^L)^2)$ for demand. The means of these distributions (m^L and m^{RE}) are aligned with the baseline demand and DA power dispatch, respectively. Meanwhile, the standard deviations (s^L and s^{RE}), set at 1% of the baseline demand and 10% of the DA wind power dispatch, respectively, quantify the anticipated variability around these means.

Additionally, the data related to the utility-scale ES is detailed in Table 6.3.

Table 6.3: Parameters of the utility-scale ES in 2021.

Year	2021
Energy capacity E^{max} (MWh)	2034.91
Power capacity s^{max} (MW)	1871.16
Minimum energy level E^{min} (MWh)	$0.2E^{max}$
Initial energy level E^0 (MWh)	$0.2E^{max}$
Charging efficiency η^c (%)	90
Discharging efficiency η^d (%)	90

The case studies focus on investigating the influence of distinct sizes of utility-scale ES on the integration of wind power within optimal investment strategies and their business results. To this end, three cases have been designed to assess the impact of varying levels of storage capacity on investment decisions within the 2021 GB power system, which are outlined as follows:

1. **Zero storage capacity (Z-SC) case:** This case is built on a 2021 GB power grid devoid of the participation of ES, serving as the foundational benchmark for comparison.
2. **Low storage capacity (L-SC) case:** The utility-scale ES in the 2021 GB power

grid has an energy capacity of 2034.91 MWh and a power capacity of 1871.16 MW, reflecting the real specifications provided by the NG FES.

3. **High storage capacity (H-SC) case:** The utility-scale ES in the 2021 GB power grid is expanded to 4069.82 MWh for energy capacity and 3742.32 MW for power capacity, based on twice the values provided by the NG FES.

6.5.2 Impact of Storage Capacity

Table 6.4: Investment decisions and business results for three storage capacity scenarios regarding the 2021 GB power system.

	Z-SC case	L-SC case	H-SC case
CCGT1 (MW)	1,331	1,829	2,000
CCGT2 (MW)	-	-	-
OCGT1 (MW)	-	4,012	3,748
OCGT2 (MW)	13,050	7,307	6,788
Total flexible capacity (MW)	14,381	13,148	12,536
Nuclear (MW)	15,749	9,080	6,639
Wind (MW)	17,243	42,093	53,512
Investment cost (mil.£)	2,494	2,474	2,540
DA energy cost (mil.£)	5,055	4,201	3,618
DA energy revenue (mil.£)	19,835	18,388	17,959
RT reserve cost (mil.£)	177	469	610
RT reserve revenue (mil.£)	982	1,645	1,919
Wind power deviation penalty (mil.£)	637	1,397	1,763
Carbon allowance revenue (mil.£)	209	510	648
Green certificate revenue (mil.£)	2,044	3,321	3,884
Carbon emission cost (mil.£)	3,269	3,269	3,269
GenCo Profit (mil.£)	11,438	12,054	12,610
DA ES revenue (mil.£)	0	-32	-77
RT ES revenue (mil.£)	0	72	162
Total ES revenue (mil.£)	0	40	84

The comprehensive investment decisions and business results corresponding to the three storage capacity scenarios are detailed in Table 6.4. Additionally, the charging and discharging power outputs for the three storage capacity scenarios across the DA and RT markets are illustrated in Figures 6.2 and 6.3, respectively. The clearing prices of the three storage capacity scenarios across the DA and RT markets are shown in Figures 5.3 and 5.4, respectively.

It is important to note that the Z-SC case, which served as a benchmark, is actually consistent with the market-driven case in Chapter 4 and the Z-DF case in Chapter 5,

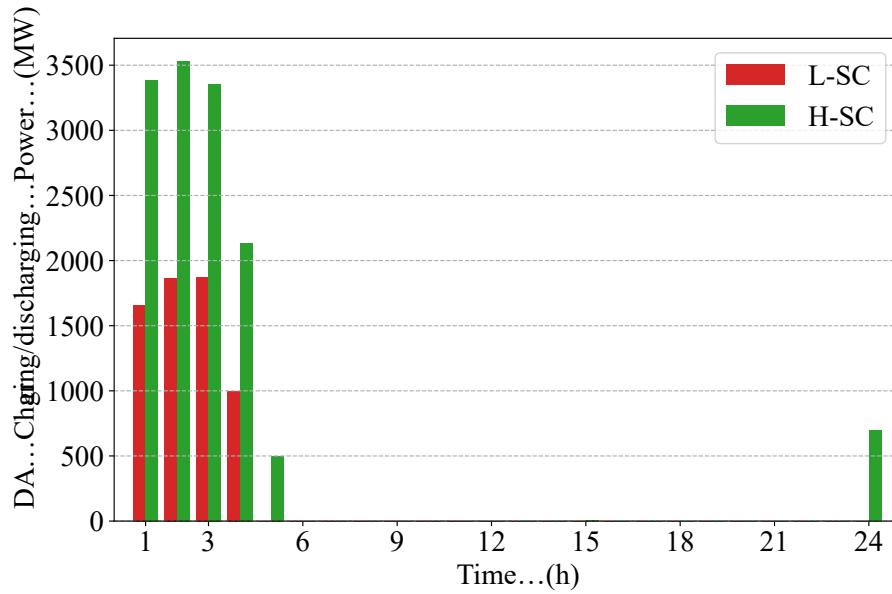


Figure 6.2: Daily charging/discharging power output in the DA energy market for three storage capacity scenarios regarding the 2021 GB power system.

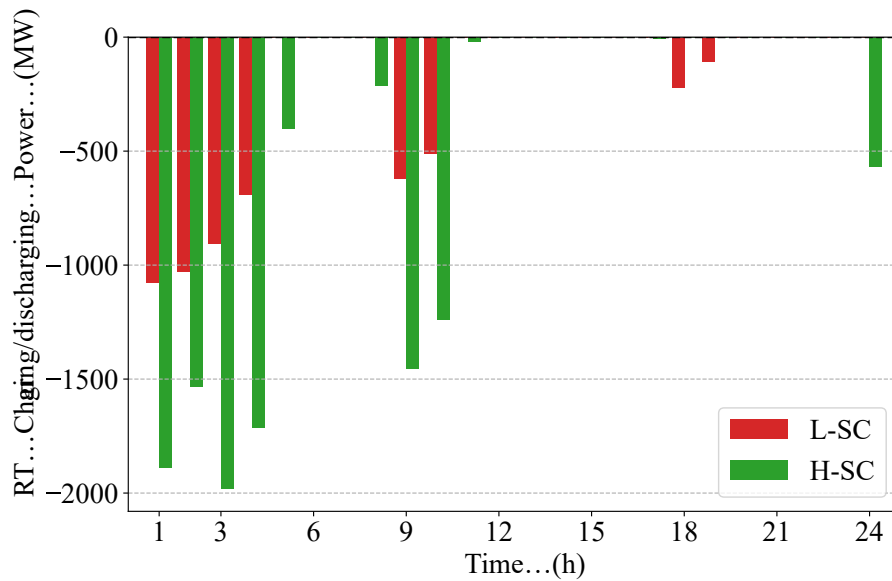


Figure 6.3: Daily charging/discharging power output in the RT balancing market for three storage capacity scenarios regarding the 2021 GB power system.

which are analysed from the perspective of the role of market design and the impact of demand flexibility, respectively. In particular, as the case design is based on another flexibility resource, ES, the analysis of the Z-SC case will be similar to that of the Z-DF case. Therefore, in order to save space, readers can refer to the market-driven case analysis in Section 4.5.2 and the Z-DF case analysis in Section 5.5.2 for a comprehensive evaluation of the Z-SC case.

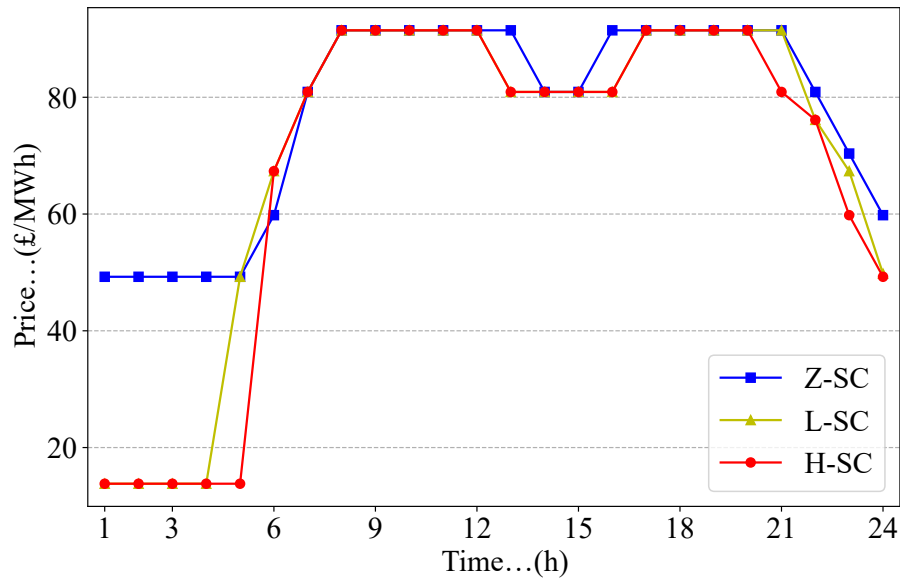


Figure 6.4: Daily DA energy market clearing prices for three storage capacity scenarios in the 2021 GB power system.

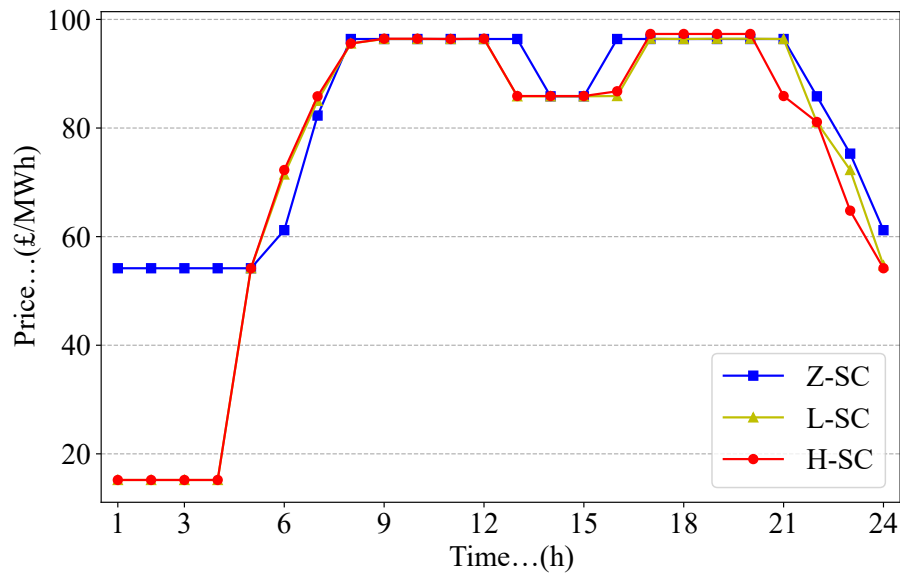


Figure 6.5: Daily RT balancing market clearing prices for three storage capacity scenarios in the 2021 GB power system.

L-SC Case Analysis

The integration of utility-scale ES into the 2021 GB power grid brings about a transformation in the optimal investment decisions, markedly reducing the capacities using nuclear and flexible generation technologies while substantially increasing wind power capacity, as described in the second to eighth row in the third column of Table 6.4. This transformation is due to the ES's role as a flexibility resource capable of managing the

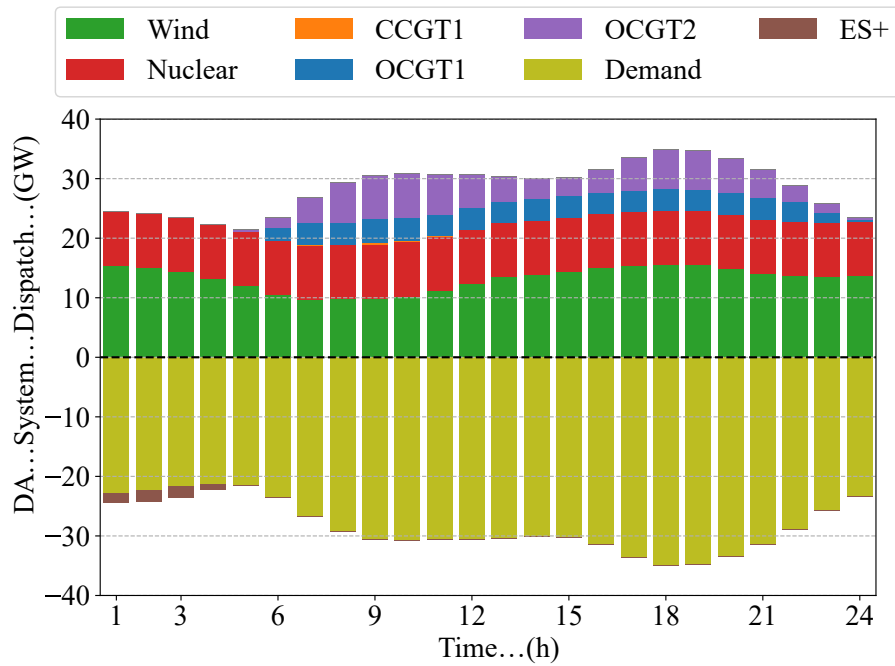


Figure 6.6: Daily generation dispatches in the DA energy market for the L-SC case regarding the 2021 GB power system.

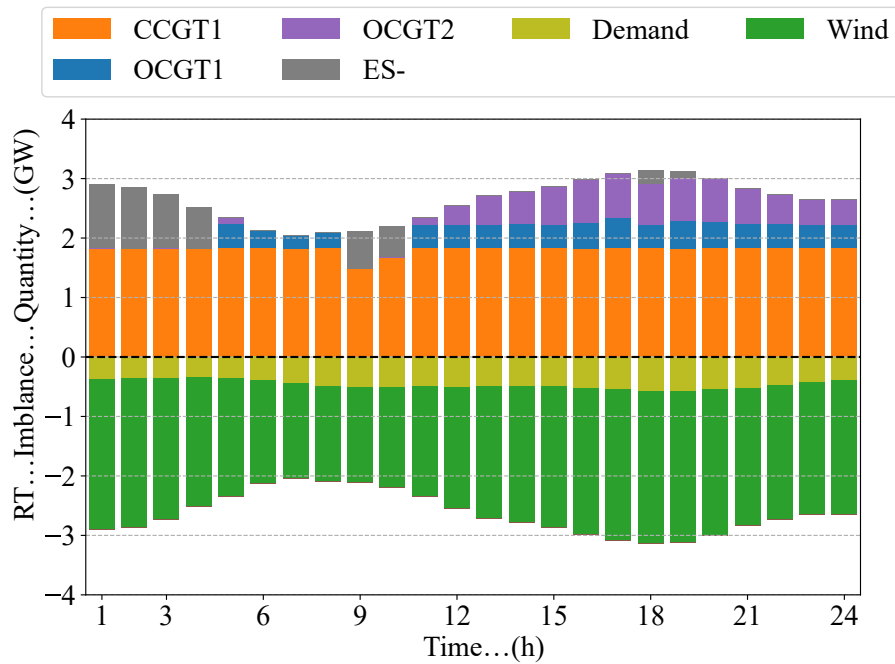


Figure 6.7: Daily imbalances in the RT balancing market for the L-SC case regarding the 2021 GB power system.

significant uncertainties associated with large-scale wind power generation. Specifically, ES stores the excess wind power generation during periods of low demand in the DA market, facilitating the stable operation of a power grid with high wind penetration, as shown

in Figure 6.6. Subsequently, in the RT market, the stored energy can be discharged to meet reserve requirements, as seen in Figure 6.7. This operational flexibility provided by the ES not only offers a cost-effective solution to reserve provision but also contributes to carbon emission reductions by utilising stored clean energy to fulfil reserve requirements. It should be noted that the legends 'ES+' and 'ES-' respectively indicate ES charging and discharging. Moreover, the charging and discharging cycles of the ES across the DA and RT markets are illustrated in Figures 6.2 and 6.3. Notably, positive values indicate ES charging, while negative values indicate ES discharging.

Despite the similarities between the roles of ES and demand flexibility in enhancing grid stability and efficiency, the integration of ES does not lead to a smoothing of the DA and RT market clearing price profiles. This phenomenon can be attributed to two primary factors:

1. The primary objective of the ES in this model is to offer ancillary services rather than perform energy arbitrage, which limits its impact on prices across the DA and RT markets.
2. The scale of ES capacity relative to system demand is significantly smaller than that of a 5% demand flexibility in the L-DF case, making it insufficient to produce significant fluctuations to smooth the price profiles across the whole market horizon.

Therefore, as shown in Figures 6.4 and 6.5, the DA and RT market clearing price profiles are broadly consistent with the inflexible system demand profile (see the blue line in Figure 5.2).

The observed dips in DA and RT prices during the early hours reflect the influence of ES operations on market outcomes. Specifically, in the DA market, the huge gap between the abundant wind power and low demand during hours 1–4 allows ES to charge with the surplus wind energy, leading to the cheap CCGT1 capacity providing the marginal reserve capacity. In the RT market, extensive ES discharging meets the high reserve requirements during windy hours 1–4, once again positioning cheap CCGT1 as the marginal reserve capacity provider.

The ninth to twenty-first row in the third column of Table 6.4 accommodates the comprehensive business results for the L-SC case. Despite the reduction of investments in expensive nuclear power, there is a steep increase in wind generation capacity, resulting in a negligible change in investment cost. In the DA market, both energy costs and revenue experience a decline. The decrease in energy costs is a direct consequence of replacing nuclear and flexible generation with zero-cost wind power. However, this transition also leads to a drop in energy revenue since excess wind power generation decreases market

prices during hours 1–4. Conversely, in the RT market, both reserve costs and revenue exhibit an increase. Although the ES is employed to deploy reserves, its limited scale proves inadequate for fully mitigating the uncertainties introduced by large-scale wind power. This inadequacy results in higher reserve costs due to the need for additional reserve capacity to manage wind power uncertainty. Nonetheless, the substantial rise in reserve requirements, compared to the Z-SC case, also leads to increased reserve revenue. The extensive integration of wind power, despite increasing wind power uncertainty and the associated wind power deviation penalty, also brings significant policy-driven benefits. Notably, revenues from carbon allowance and TGC have seen a dramatic increase, reflecting the environmental and regulatory incentives designed to promote renewable energy integration. It is noteworthy that the carbon emission costs remain consistent across all storage capacity cases, given that total emissions hit the cap set by the GB government for 2021. Generally, despite a slight dip in the prime revenue source, DA energy revenue, the revenue from RT reserve, TGC, and carbon allowance all increased substantially, cumulatively resulting in a modest increase in the GenCo's total profit. Furthermore, the analysis extends beyond the GenCo's perspective to include the business outcomes for ES entities. As detailed in the last three rows in the third column of Table 6.4, despite not being profit-driven, ES entities manage to secure profits through participation in the DA and RT markets to provide reserves.

H-SC Case Analysis

With the expansion of ES capacity in the H-SC case, the optimal investment decisions in the 2021 GB power system exhibit a further shift towards wind generation capacity, alongside a corresponding decrease in both nuclear and flexible generation capacities, following the trends observed in the L-SC case and amplifying them, as detailed in the second to eighth row in the fourth column of Table 6.4.

In line with the L-SC case, the expanded ES in the H-SC case still plays a crucial role in managing the grid's stability and efficiency by leveraging excess wind energy. Specifically, ES is charged with excess wind power generation during windy periods, hours 1–5, in the DA market; subsequently, it discharges to meet varying reserve requirements during hours 1–5 and 9–10 in the RT market, as shown in Figures 6.8 and 6.9. It needs to be clarified that the logic behind these two periods of reserve requirements is different. In detail, the discharging during hours 1–5 addresses the increased reserve requirement incurred by massive wind power generation, while the need for discharging during hours 9–10 arises due to the substantial utilisation of flexible capacity in the DA market, leaving ES to fulfil the reserve requirements.

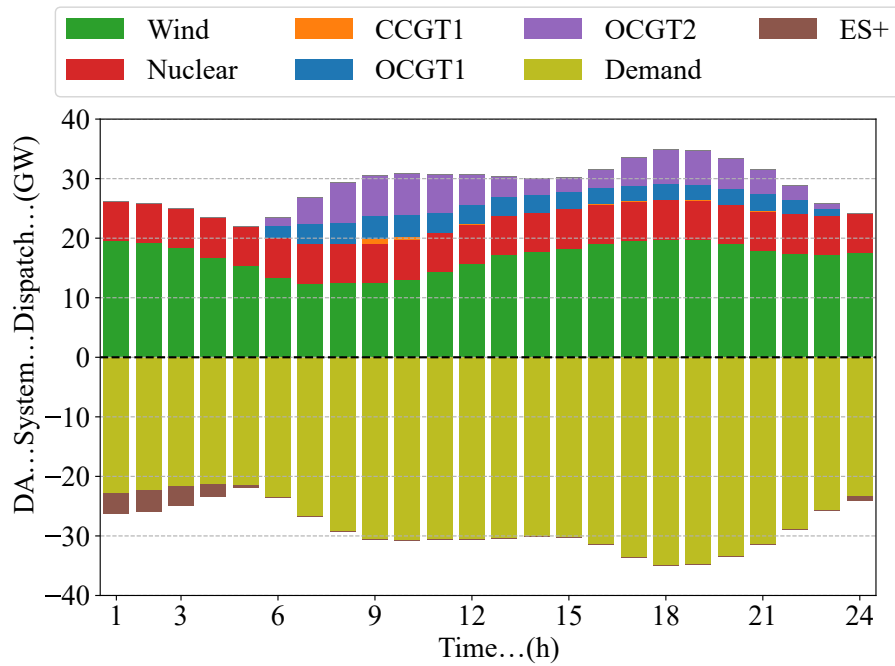


Figure 6.8: Daily generation dispatches in the DA energy market for the H-SC case regarding the 2021 GB power system.

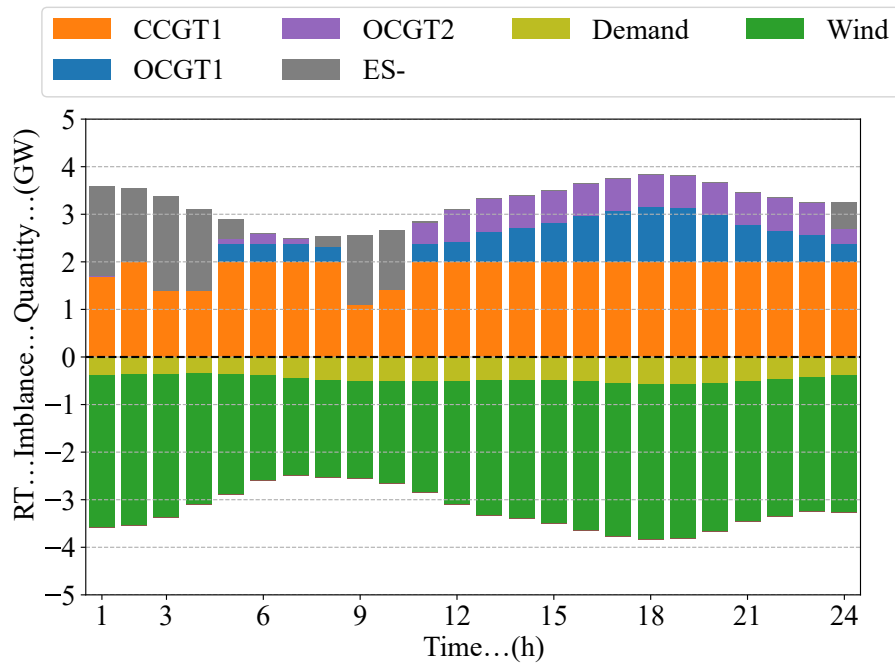


Figure 6.9: Daily imbalances in the RT balancing market for the H-SC case regarding the 2021 GB power system.

Furthermore, as seen in Figures 6.2, 6.3, 6.4, and 6.5, unlike the price drop during hours 1–5, despite the considerable discharging from ES in the RT market during hours 9–10, it does not result in a decrease in market clearing prices. This phenomenon can

be attributed to the coupling operational dynamics of the flexible generation capacity across the DA and RT markets. During the windy hours 1–5, the abundant wind power generation in the DA market allows for the deployment of low-cost CCGT1 technology as the marginal reserve provider. However, during the low-wind hours 9–10, since the cheap flexible generation capacity has already been heavily deployed in the DA market, it is necessary to utilise the expensive OCGT2 technology to provide the marginal reserve capacity.

The business results for the H-SC case are comprehensively displayed from the ninth to twenty-first row in the fourth column of Table 6.4. The changes in business cases pertaining to the examined GenCo follow the same trends discussed in the L-SC case analysis. In summary, compared with the L-SC case, DA energy revenue further decreases, while revenues from RT reserve, carbon allowance, and TGC increase, slightly raising the overall profitability of the GenCo. Furthermore, as the scale of ES further expands in the H-SC case, the profit of ES entities also increases.

6.6 Conclusions

This chapter investigates the impact of ES on generation planning within an energy market environment subject to carbon emission restrictions. To quantitatively evaluate the investment strategies of the GenCo and its business results under varying scales of ES, a two-stage bi-level optimisation model is introduced. The model comprises an UL problem representing the self-interested GenCo and two LL problems. The first LL problem (LL1) represents the joint clearing process of the DA and RT electricity markets, incorporating chance-constrained uncertainties associated with renewable energy and demand deviations. Additionally, an ES model is also integrated into the LL1 problem to provide ancillary services. The second LL problem (LL2) represents the clearing mechanism for the green certificate market. To facilitate an efficient solution, the model undergoes a series of transformations. First, chance constraints are linearized. Subsequently, the KKT conditions are employed to transform the model into a MPEC. Finally, leveraging the strong duality theorem, the MPEC problem is recast as a solvable MIQP problem. The case studies have been conducted within the framework of the 2021 GB power system, and the findings are listed as follows:

1. Compared to the scenario without ES, when utility-scale ES sets to the level specified by the 2021 GB power system and then doubles, the profit of the GenCo sees a growth of 5% and 10%, respectively. It is because the booming expansion of renewable power capacity brings income from economic incentives.

2. Compared to the scenario without ES, when utility-scale ES sets to the level specified by the 2021 GB power system and then doubles, the wind generation capacity in investment decisions surges by 144% and 210%, respectively. The reason behind this phenomenon is that the ES acts as a provider of reserves to mitigate the uncertainties from the massive RESs.
3. Although the principal goal of ES is to provide ancillary services instead of energy arbitrage, the ES entity can still earn profit from its reserve provision. As the investment in renewable power capacity increases, the ES entity's profit also boosts.
4. In contrast to demand flexibility, ES lacks the capability to flatten peak demand since it is not profit-driven.

Chapter 7

Pathway towards Fully Decarbonised GB Power System by 2035

7.1 Introduction

In the context of global climate change, the GB has set ambitious goals for decarbonisation, aiming to achieve a fully decarbonised power system by 2035 and then leading to a broader net-zero target by 2050 [87]. Reaching these targets requires a significant transformation of the energy sector, particularly in terms of increasing the share of RESs and reducing the reliance on fossil fuels, coupled with a considerable reduction in carbon emissions across the board. As such, the integration of flexibility resources, specifically demand flexibility and ES, emerges as a vital strategy for promoting the GB's decarbonisation agenda.

Demand flexibility and ES are crucial tools in the transition towards a sustainable energy structure. They offer the necessary capabilities for facilitating the expansion of renewable energy and then achieving decarbonisation. In this chapter, the models of demand flexibility and ES are incorporated into the LL1 problem, which identifies the joint clearing process for DA and RT markets. Specifically, they can provide operational flexibility across the DA and RT markets, thereby mitigating the uncertainty introduced by renewable energy and enhancing the integration of large-scale RESs.

This chapter seeks to uncover cost-effective market mechanisms that leverage two vital flexibility resources, demand flexibility and ES, to encourage investment in RESs, aligning with the ambitious goal of transitioning GB's power system to a net-zero system by 2035. Guided by this overarching vision, the case studies utilise predicted data from the National Grid for the 2035 GB power system, targeted by the GB government for achieving a zero-carbon power system. The findings derived from these case studies reveal that

it is feasible to reach an emissions-free GB power system by 2035 with the aid of flexibility resources. Additionally, beyond the GB's zero-carbon target, a 100% wind power system can be achieved with the further expansion of flexibility resources. Meanwhile, the GenCo's profitability can be guaranteed by the economic incentives from the green energy promotion policy.

7.2 Problem Statement

This section outlines the decision-making framework for market-driven generation planning, simultaneously taking into account demand flexibility and ES, and details the foundational assumptions for its modelling.

7.2.1 Structure of the Bi-Level Optimisation Model

This chapter presents a market-driven generation planning model utilising a two-stage, bi-level optimisation framework, combined with demand flexibility and ES, to explore the pathway towards a fully decarbonised GB power system by 2035, ultimately achieving a 100% renewable power system. In brief, this bi-level optimisation framework reflects the coupling dynamics between the GenCo and market operators: the UL problem represents the profit-driven GenCo; the first LL (LL1) problem represents the joint clearing of the DA and RT markets; and the second LL (LL2) problem represents the clearing of the green certificate market. This hierarchical structure is illustrated in Figure 7.1. Furthermore, the models of demand flexibility and ES are incorporated into the LL1 problem, providing operational flexibility across the DA and RT market horizons.

The interdependence between the UL problem and the two LL problems is pronounced. The investment decisions made by GenCo at the UL significantly affect the market outcomes realised within the two LL problems. In turn, the business cases emerging from the LL1 problem, characterised by energy prices and generation dispatches in both the DA and RT markets, along with the TGC quantities and prices obtained in the LL2 problem, critically influence GenCo's profitability. This bidirectional feedback loop underscores the complex relationship between generation planning and market performance, highlighting the feature of capturing real-world market dynamics and GenCo's decision-making processes.

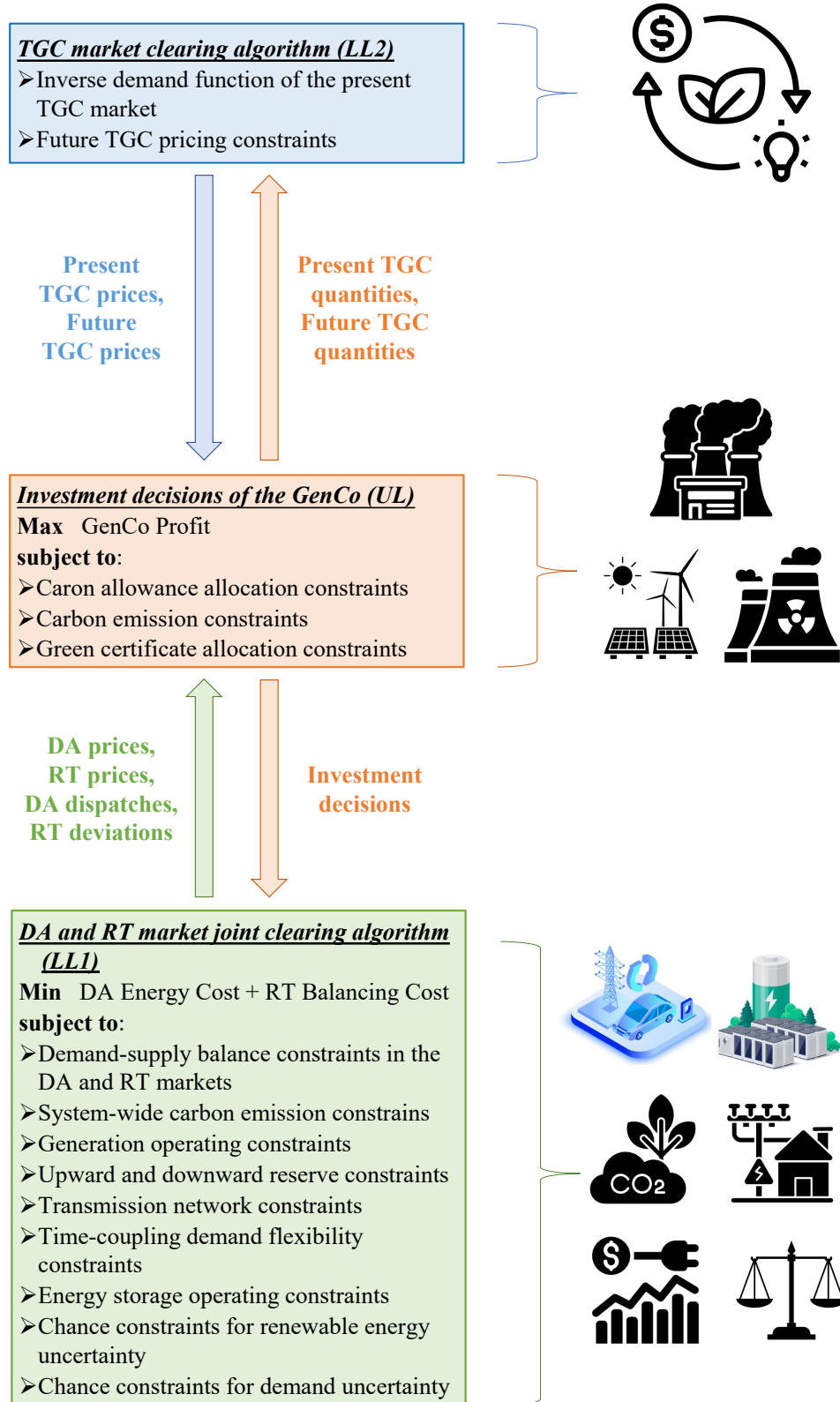


Figure 7.1: Illustration of the proposed two-stage bi-level optimisation model.

7.2.2 Modelling Assumptions

For clarity and completeness, the major assumptions of the proposed two-stage bi-level market-driven model are listed below:

1. The proposed two-stage bi-level optimisation model represents the market-driven generation planning faced by a self-interested GenCo seeking to maximise its long-term profit. The GenCo is allowed to invest in generation capacities utilising flexible, must-run, and renewable technologies. Furthermore, the flexible generation capacities are explicitly modelled by the production blocks to capture their output characteristics. Accordingly, demand blocks are also incorporated into the proposed model.
2. The examined GenCo is permitted to participate in four different markets. The two-stage energy markets adopted for this work include the DA energy market and the RT balancing market with an hourly time resolution. Furthermore, the proposed model takes into consideration the present TGC market and the future TGC market.
3. Based on the study in [76], its pricing scheme, namely LMP, is applied within the proposed two-stage energy market framework. This approach leverages the LMPs for determining the electricity prices at different locations in the network, which are calculated using the dual variables of the hourly demand-supply balance in both the DA and RT markets. The so-called LMP scheme reflects the marginal cost of supplying the next unit of electricity at a specific location, considering the effect of network transmission congestion, thereby facilitating efficient market operations and signaling appropriate economic incentives.
4. The model incorporates the pricing mechanism outlined in [64] for trading in the present and future TGC markets. Specifically, the prices in the present TGC market are determined using an inverse demand function. Furthermore, the future TGC price is estimated based on the penalty price associated with RPS deficiency.
5. Once the GenCo finalises its investment decisions, the sequential clearing process of the DA, RT, and TGC markets is arranged by the market operator. Specifically, the market operator first clears the DA energy market, taking into account the uncertainties incurred by demand and renewable generation as well as scheduling the generation dispatches. Subsequently, the depolyment of reserves is organised by market operator in the RT market to eliminate the unexpected uncertainties arising from wind power and demand-side deviations. Following this, the prices and quantities of TGC are determined based on the clearing results of the DA and RT

markets. By comparing the present TGC market clearing price with the estimated future TGC market price, the examined GenCo can choose either to trade its TGCs in the present TGC market or retain them for future trading, thus completing the transactions in the TGC market. Finally, the outcomes of these three markets are relayed to the examined GenCo, significantly influencing its profit-oriented investment decision-making process.

6. Although the DA and RT markets are cleared sequentially from the perspective of the examined GenCo, investment decisions are required to maximise the overall expected profit simultaneously [77]. Additionally, as highlighted in research works [78, 79], a co-optimised approach to managing energy and reserves significantly enhances the efficiency of generation dispatch compared to scenarios where energy and reserves are cleared separately. Therefore, this paper adopts a joint clearing model for the DA and RT markets. Furthermore, it is assumed that GenCo sells the TGCs in the present TGC market or withholds the TGCs for future trading after the outcomes of the energy markets are obtained. This assumption is logical since the allocation of TGCs becomes definitive after the results of renewable generation dispatches and deviations are available, which is more conducive to risk mitigation [64].
7. In order to facilitate the mathematical solution, the uncertainties sourced from RESs and demand are realised by a set of chance constraints.
8. As the primary objective of this paper is to explore the system-level generation mix of the future grid under stringent carbon emission regulations, the proposed generation planning model excludes the consideration of other GenCos.
9. In order to ensure the convex characteristics of the LL1 problem, linear generation costs are adopted in the proposed model. Additionally, it is assumed that renewable generation technologies are cost-free.
10. In the DA market, the generation capacities of renewable and must-run technologies are assumed to operate at their full capacity.
11. The network transmission is modelled based on a linear DC power flow approximation.
12. In the face of decarbonisation, a series of carbon emission restrictions and green energy incentives are imposed on the examined GenCo. Firstly, GenCo is limited by an annual cap on carbon emissions. Moreover, it is assumed that carbon emissions only depend on the generation of flexible technologies that consume fossil fuels.

Secondly, carbon emissions from the GenCo are subject to a carbon tax. Thirdly, carbon allowances are allocated to the GenCo based on its renewable energy generation, and the GenCo can profit by trading these carbon allowances. Fourthly, GenCo can earn TGCs through renewable power production and participate in the present and future TGC markets to gain profits. Finally, the RPS ratio is incorporated into the parameter setting of TGC markets. These environmental policies are set to work together towards a fully decarbonised power system.

13. Time-coupling demand flexibility is implemented through a generic and technology-agnostic model. The demand flexibility model permits adjustments in demand, either decreasing or increasing, within a specified proportion for each time interval, while maintaining a net-zero demand-side energy shift throughout the market duration [80]. Essentially, the sum of demand reductions matches the sum of demand increases, ensuring energy neutrality over the market horizon. It is assumed that demand flexibility does not compromise the quality or satisfaction of consumer experiences.
14. As the share of renewables in the energy mix grows, utility-scale ES is installed in the considered power grid. The ES entities can participate in both DA and RT markets to balance power supply and demand consumption under high renewable production [85].
15. To maintain the convexity of the LL1 problem, followed by the previous work [47], the continuous storage model is utilised to describe the dynamic behaviour of the utility-scale ES during the market horizon.
16. In line with the concept of non-merchant storage [86], although the ES of the proposed model can make a profit by energy arbitrage, its primary goal is to provide the auxiliary service, i.e., reserve deployment, to maintain the operation stability of the power system under high penetration of renewable power production.

7.3 Mathematical Model

This section elaborates on the formulation of the UL problem, which is associated with the GenCo, and the two LL problems (LL1 and LL2). Each of these LL problems corresponds to distinct market clearing processes, capturing the multi-layer dynamics within the electricity market framework.

7.3.1 UL Problem: Investment Decisions of GenCo

The UL problem conveys the decision-making process of a self-interested GenCo, aiming to maximise its overall profit through investment decisions. The mathematical expression of this UL problem is detailed as follows:

$$\begin{aligned}
\max_{V^{UL}} \sum_d \omega_d & \left[\underbrace{\sum_{i \in \Psi_{n,t,n,b}} \lambda_{d,t,n}^{DA} g_{i,d,t,n,b}^{DA}}_{\text{DA energy revenue}} - \underbrace{\sum_{i,t,n,b} C_{i,b}^{EN} g_{i,d,t,n,b}^{DA}}_{\text{DA energy cost}} \right. \\
& + \underbrace{\sum_{i \in I^{FL} \cap \Psi_{n,t,n,b}} \lambda_{d,t,n}^{RT} (g_{i,d,t,n,b}^U - g_{i,d,t,n,b}^D)}_{\text{RT reserve revenue}} - \underbrace{\sum_{i \in I^{FL},t,n,b} (C_{i,b}^U g_{i,d,t,n,b}^U - C_{i,b}^D g_{i,d,t,n,b}^D)}_{\text{RT reserve cost}} \\
& - \underbrace{\sum_{i \in I^{RE} \cap \Psi_{n,t,n}} \lambda_{d,t,n}^{RT} g_{i,d,t,n}^{dev}}_{\text{Renewable deviation penalty}} - \underbrace{\sum_{i \in I^{FL},t,n} \lambda^{CO_2} Q_{i,d,t,n}^{emi}}_{\text{Carbon emission cost}} + \underbrace{\sum_{i \in I^{RE},t,n} \lambda^{CO_2} Q_{i,d,t,n}^{CA}}_{\text{Carbon allowance revenue}} \\
& + \underbrace{\sum_{i \in I^{RE}} (\lambda_{i,d}^{PGC} Q_{i,d}^{PGC} + \lambda^{FGC} Q_{i,d}^{FGC})}_{\text{Green certificate revenue}} \left. \right] - \underbrace{\sum_{i,n} C_i^{IN} G_{i,n}}_{\text{Investment cost}} \quad (7.1)
\end{aligned}$$

where:

$$V^{UL} = \{G_{i,n}, Q_{i,d,t,n}^{CA}, Q_{i,d,t,n}^{emi}, Q_{i,d}^{PGC}, Q_{i,d}^{FGC}\} \quad (7.2)$$

subject to:

$$G_{i,n} \geq 0 \quad \forall i \in I, \forall n \quad (7.3)$$

$$G_{i,n,b} = \omega_b^{FL} G_{i,n} \quad \forall i \in I^{FL}, \forall n, \forall b \quad (7.4)$$

$$Q_{i,d,t,n}^{CA} = (g_{i,d,t,n}^{DA} - g_{i,d,t,n}^{dev}) \eta^{Grid} \epsilon_i \quad \forall i \in I^{RE}, \forall d, \forall t, \forall n \quad (7.5)$$

$$Q_{i,d,t,n}^{emi} = (g_{i,d,t,n}^{DA} + g_{i,d,t,n}^U - g_{i,d,t,n}^D) \eta_i^{CO_2} \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n \quad (7.6)$$

$$Q_{i,d}^{PGC} + Q_{i,d}^{FGC} = \sum_{t,n} (g_{i,d,t,n}^{DA} - g_{i,d,t,n}^{dev}) \quad \forall i \in I^{RE}, \forall d \quad (7.7)$$

The objective function of the UL problem is to maximise the total profit of the examined GenCo, consisting of nine components, as defined in (7.1). Constraint (7.3) ensures that the new capacities invested by the GenCo are non-negative. Constraint (7.4) details the weighting factors of the production blocks corresponding to flexible generation capacities. Constraint (7.5) presents the allocation of carbon allowances. Constraint (7.6) quantifies the carbon emissions produced by the operation of fossil-fired flexible generation technologies. Lastly, constraint (7.7) calculates the total quantities of TGCs that are generated based on renewable energy production in the DA market and its deviation in

the RT market.

7.3.2 LL1 Problem: Joint Clearing Process of DA Energy and RT Balancing Markets

The LL1 problem embodies the joint clearing process of the two-stage energy markets, specifically the DA energy market and the RT balancing market. This process is conducted with the objective of minimising the total system cost from the perspective of the market operator. At the core of this chapter, the two vital flexibility resources for the low-carbon transition, i.e., flexible demand and ES, are simultaneously included in the optimisation framework of the LL1 problem. The formulation of the LL1 problem is provided as follows:

$$\left\{ \min_{V^{LL}} \sum_{i,t,n,b} C_{i,b}^{EN} g_{i,d,t,n,b}^{DA} + \sum_{i,t,n,b} (C_{i,b}^U g_{i,d,t,n,b}^U - C_{i,b}^D g_{i,d,t,n,b}^D) \right. \quad (7.8)$$

where:

$$V^{LL1} = \{ P_{j,d,t,o}^{DA}, g_{i,d,t,n,b}^{DA}, g_{i,d,t,n,b}^U, g_{i,d,t,n,b}^D, P_{j,d,t,o}^{dev}, g_{i,d,t,n}^{dev}, E_{e,d,t}, s_{e,d,t}^{c,DA}, s_{e,d,t}^{d,DA}, s_{e,d,t}^{c,RT}, s_{e,d,t}^{d,RT}, \delta_{d,t,n}^{DA}, \delta_{d,t}^{DA,ref}, \delta_{d,t,n}^{RT}, \delta_{d,t}^{RT,ref}, L_{j,d,t,o}^{SH} \} \quad (7.9)$$

subject to:

$$\begin{aligned} & \sum_{j \in \Psi_{n,o}} P_{j,d,t,o}^{DA} + \sum_{e \in \Psi_n} s_{e,d,t}^{c,DA} + \sum_{m \in \Lambda_n} b_{nm} (\delta_{d,t,n}^{DA} - \delta_{d,t,m}^{DA}) - \sum_{i \in \Psi_{n,b}} g_{i,d,t,n,b}^{DA} - \\ & \sum_{e \in \Psi_n} s_{e,d,t}^{d,DA} = 0 \quad : \lambda_{d,t,n}^{DA} \quad \forall t, \forall n \end{aligned} \quad (7.10)$$

$$\begin{aligned} & \sum_{j \in \Psi_{n,o}} P_{j,d,t,o}^{dev} + \sum_{i \in I^{FL} \cap \Psi_{n,b}} (-g_{i,d,t,n,b}^U + g_{i,d,t,n,b}^D) + \sum_{e \in \Psi_n} (s_{e,d,t}^{c,RT} - s_{e,d,t}^{d,RT}) + \\ & \sum_{i \in I^{RE} \cap \Psi_n} g_{i,d,t,n}^{dev} - \sum_{m \in \Lambda_n} b_{nm} (\delta_{d,t,n}^{DA} - \delta_{d,t,n}^{RT} - \delta_{d,t,m}^{DA} + \delta_{d,t,m}^{RT}) = 0 \quad : \lambda_{d,t,n}^{RT} \quad \forall t, \forall n \end{aligned} \quad (7.11)$$

$$E_{e,d,t} = E_{e,d,t-1} + \eta_e^c (s_{e,d,t}^{c,DA} + s_{e,d,t}^{c,RT}) - \frac{s_{e,d,t}^{d,DA} + s_{e,d,t}^{d,RT}}{\eta_e^d} : \tau_{e,d,t}^{ES} \quad \forall e, \forall d, \forall t \geq 2 \quad (7.12)$$

$$E_{e,d,1} = E_e^0 + \eta_e^c (s_{e,d,1}^{c,DA} + s_{e,d,1}^{c,RT}) - \frac{s_{e,d,1}^{d,DA} + s_{e,d,1}^{d,RT}}{\eta_e^d} : \tau_{e,d}^{ES,1} \quad \forall e, \forall d \quad (7.13)$$

$$E_e^0 = E_{e,d,T} : \tau_{e,d}^{ES,T} \quad \forall e, \forall d \quad (7.14)$$

$$E_e^{min} \leq E_{e,d,t} \leq E_e^{max} : \tau_{e,d,t}^{ES,-}, \tau_{e,d,t}^{ES,+} \quad \forall e, \forall d, \forall t \quad (7.15)$$

$$0 \leq s_{e,d,t}^{c,DA} \leq s_e^{max} : \tau_{e,d,t}^{c,DA,-}, \tau_{e,d,t}^{c,DA,+} \quad \forall e, \forall d, \forall t \quad (7.16)$$

$$0 \leq s_{e,d,t}^{c,DA} + s_{e,d,t}^{c,RT} \leq s_e^{max} : \tau_{e,d,t}^{c,RT,-}, \tau_{e,d,t}^{c,RT,+} \quad \forall e, \forall d, \forall t \quad (7.17)$$

$$0 \leq s_{e,d,t}^{d,DA} \leq s_e^{max} : \tau_{e,d,t}^{d,DA,-}, \tau_{e,d,t}^{d,DA,+} \quad \forall e, \forall d, \forall t \quad (7.18)$$

$$0 \leq s_{e,d,t}^{d,DA} + s_{e,d,t}^{d,RT} \leq s_e^{max} : \tau_{e,d,t}^{d,RT,-}, \tau_{e,d,t}^{d,RT,+} \quad \forall e, \forall d, \forall t \quad (7.19)$$

$$\sum_{i \in I^{FL}, t, n, b} \eta_i^{CO_2} (g_{i,d,t,n,b}^{DA} + g_{i,d,t,n,b}^U - g_{i,d,t,n,b}^D) \leq \eta^{Grid} \sum_{j,t,o} L_{j,d,t,o}^{DA} : \zeta_d \quad (7.20)$$

$$g_{i,d,t,n,b}^{DA} + g_{i,d,t,n,b}^U \leq G_{i,n,b} : \mu_{i,d,t,n,b}^U \quad \forall i \in I^{FL}, \forall t, \forall n, \forall b \quad (7.21)$$

$$g_{i,d,t,n,b}^D \leq g_{i,d,t,n,b}^{DA} : \mu_{i,d,t,n,b}^D \quad \forall i \in I^{FL}, \forall t, \forall n, \forall b \quad (7.22)$$

$$0 \leq g_{i,d,t,n,b}^U \leq G_{i,b}^U : \phi_{i,d,t,n,b}^{U,-}, \phi_{i,d,t,n,b}^{U,+} \quad \forall i \in I^{FL}, \forall t, \forall n, \forall b \quad (7.23)$$

$$0 \leq g_{i,d,t,n,b}^D \leq G_{i,b}^D : \phi_{i,d,t,n,b}^{D,-}, \phi_{i,d,t,n,b}^{D,+} \quad \forall i \in I^{FL}, \forall t, \forall n, \forall b \quad (7.24)$$

$$g_{i,d,t,n}^{DA} = G_{i,n} : \gamma_{i,d,t,n} \quad \forall i \in I^{MR}, \forall t, \forall n \quad (7.25)$$

$$g_{i,d,t,n}^{DA} = k_{i,d,t}^{RE} G_{i,n} : \theta_{i,d,t,n} \quad \forall i \in I^{RE}, \forall t, \forall n \quad (7.26)$$

$$-C_{nm}^{max} \leq b_{nm} (\delta_{d,t,n}^{DA} - \delta_{d,t,m}^{DA}) \leq C_{nm}^{max} : \nu_{d,t,n,m}^{DA,-}, \nu_{d,t,n,m}^{DA,+} \quad \forall t, \forall n, \forall m \in \Lambda_n \quad (7.27)$$

$$-\pi \leq \delta_{d,t,n}^{DA} \leq \pi : \xi_{d,t,n}^{DA,-}, \xi_{d,t,n}^{DA,+} \quad \forall t, \forall n \quad (7.28)$$

$$\delta_{d,t}^{DA,ref} = 0 : \xi_{d,t}^{DA,ref} \quad \forall t \quad (7.29)$$

$$-C_{nm}^{max} \leq b_{nm} (\delta_{d,t,n}^{RT} - \delta_{d,t,m}^{RT}) \leq C_{nm}^{max} : \nu_{d,t,n,m}^{RT,-}, \nu_{d,t,n,m}^{RT,+} \quad \forall t, \forall n, \forall m \in \Lambda_n \quad (7.30)$$

$$-\pi \leq \delta_{d,t,n}^{RT} \leq \pi : \xi_{d,t,n}^{RT,-}, \xi_{d,t,n}^{RT,+} \quad \forall t, \forall n \quad (7.31)$$

$$\delta_{d,t}^{RT,ref} = 0 : \xi_{d,t}^{RT,ref} \quad \forall t \quad (7.32)$$

$$(1 - \alpha) L_{j,d,t,o}^{DA} \leq P_{j,d,t,o}^{DA} \leq (1 + \alpha) L_{j,d,t,o}^{DA} : \psi_{j,d,t,o}^{-}, \psi_{j,d,t,o}^{+} \quad \forall j, \forall t, \forall o \quad (7.33)$$

$$\sum_{t,o} L_{j,d,t,o}^{SH} = 0 : \sigma_{j,d} \quad \forall j \quad (7.34)$$

$$\mathbb{P} \left[-\alpha L_{j,d,t}^R \leq \sum_o L_{j,d,t,o}^{SH} \leq \alpha L_{j,d,t}^R \right] \geq 1 - \rho^L : \chi_{j,d,t}^{SH,-}, \chi_{j,d,t}^{SH,+} \quad \forall j, \forall t \quad (7.35)$$

$$\mathbb{P} \left[g_{i,d,t,n}^{DA} - g_{i,d,t,n}^{dev} \leq g_{i,d,t,n}^R \right] \geq 1 - \rho^{RE} : \chi_{i,d,t,n}^{RE} \quad \forall i \in I^{RE}, \forall t, \forall n \quad (7.36)$$

$$\mathbb{P}\left[\sum_o P_{j,d,t,o}^{DA} + \sum_o P_{j,d,t,o}^{dev} \geq L_{j,d,t}^R + \sum_o L_{j,d,t,o}^{SH} \right] \geq 1 - \rho^L \quad : \chi_{j,d,t}^L \quad \forall j, \forall t \quad \forall d \quad (7.37)$$

The mathematical framework of the LL1 problem (7.8)-(7.37) simultaneously combine the characteristics of demand flexibility and ES. The LL1 problem's objective function is to minimise the overall system cost, including the energy cost (first term) and the reserve cost (last two terms). The LL1 problem is subject to various sets of constraints: system balance constraints (7.10)-(7.11), ES constraints (7.12)-(7.19), system-wide carbon emission constraint (7.20), generation operating constraints (7.21)-(7.26), network constraints (7.27)-(7.32), demand flexibility constraints (7.33)-(7.34), and chance constraints for uncertainties (7.35)-(7.37).

7.3.3 LL2 Problem: Clearing Process of TGC Market

The process for clearing the present TGC market, as well as forecasting the TGC price for the future TGC market, is organised in the LL2 problem, which is formulated as follows:

$$V^{LL2} = \{\lambda_{i,d}^{PGC}, Q_{i,d}^{PGC}\} \quad (7.38)$$

$$\lambda_{i,d}^{PGC} = l^{PGC} - q^{PGC} Q_{i,d}^{PGC} \quad \forall i \in I^{RE}, \forall d \quad (7.39)$$

$$\lambda^{FGC} = \gamma^{FGC} \lambda_0^{RPS} \quad (7.40)$$

The present TGC market is built on a quantity-dependent inverse demand function. Under this framework, the present TGC price is determined by equation (7.39) that represents the linear correlation between the present price of TGC and its quantity. On the other hand, equation (7.40) indicates the estimation of the future TGC price. The necessary coefficients are obtained from equations (7.41) and (7.42), as proved in [64].

$$l^{PGC} = \lambda_0^{RPS} \quad (7.41)$$

$$q^{PGC} = \frac{(1 - \theta^{PGC}) \lambda_0^{RPS}}{\gamma^{RPS} \sum_{j,d,t} L_{j,d,t}^{DA}} \quad (7.42)$$

7.4 Solution

The process for solving the proposed two-stage bi-level optimisation model unfolds through a structured series of steps, comprehensively detailed in this section. This pro-

cess begins with the linearization of non-convex chance constraints in the LL1 problem, as described in Subsection 7.4.1. Following the linearization of chance constraints, the proposed bi-level problem is reformulated into a single-level MPEC by recasting the LL1 problem as its equivalent KKT optimality conditions, as detailed in Subsection 7.4.2. Concurrently, the constraints from the LL2 problem are also incorporated into the MPEC model. The next step is to address the nonlinearities appearing in the MPEC formulation, as discussed in Subsection 7.4.3. Finally, to ensure global optimality, the MPEC problem is recast as a MIQP, as provided in Subsection 7.4.4, rendering the model computationally efficient for commercial solvers.

7.4.1 CCP Reformulation

In line with the setting of previous chapters, chance constraints utilise stochastic variables to model uncertainties associated with RESs and demand (g^R and L^R). These stochastic variables are assumed to follow normal distributions, which are mathematically represented as follows:

$$g_{i,d,t,n}^R \sim \mathcal{N}(m_{i,d,t,n}^{RE}, (s_{i,d,t,n}^{RE})^2), \quad L_{j,d,t}^R \sim \mathcal{N}(m_{j,d,t}^L, (s_{j,d,t}^L)^2) \quad (7.43)$$

where parameter m is the mean of the normal distribution and parameter s is the standard deviation of the normal distribution. It should be noted that the chance constraints (7.35), (7.36), and (7.37) are non-convex. In order to make the proposed model solvable, the non-convexities in these chance constraints are required to be eliminated.

Utilising the linearization techniques outlined in Subsection 3.4.1, the chance constraints (7.35)-(7.37) can be linearized as follows:

$$\alpha m_{j,d,t}^L + \sum_o L_{j,d,t,o}^{SH} \geq \Phi^{-1}(1 - \rho^L) \alpha s_{j,d,t}^L \quad : \chi_{j,d,t}^{SH,-} \quad \forall j, \forall d, \forall t \quad (7.44)$$

$$\alpha m_{j,d,t}^L - \sum_o L_{j,d,t,o}^{SH} \geq \Phi^{-1}(1 - \rho^L) \alpha s_{j,d,t}^L \quad : \chi_{j,d,t}^{SH,+} \quad \forall j, \forall d, \forall t \quad (7.45)$$

$$m_{i,d,t,n}^{RE} - g_{i,d,t,n}^{DA} + g_{i,d,t,n}^{dev} \geq \Phi^{-1}(1 - \rho^{RE}) s_{i,d,t,n}^{RE} \quad : \chi_{i,d,t,n}^{RE} \quad \forall i \in I^{RE}, \forall d, \forall t, \forall n \quad (7.46)$$

$$-m_{j,d,t}^L - \sum_o L_{j,d,t,o}^{SH} + \sum_o P_{j,d,t,o}^{dev} + \sum_o P_{j,d,t,o}^{DA} \geq \Phi^{-1}(1 - \rho^L) s_{j,d,t}^L \quad : \chi_{j,d,t}^L \quad \forall j, \forall d, \forall t \quad (7.47)$$

Note that $\Phi^{-1}(\cdot)$ is the inverse cumulative distribution function of a standard normal distribution.

7.4.2 MPEC Formulation

Thanks to the linearized chance constraints (7.44)-(7.47), the option to replace the LL1 problem with a collection of constraints derived from its KKT optimality conditions becomes viable. Additionally, the LL2 problem is easily merged into the UL problem since it solely consists of constraints. Consequently, this allows the original two-stage bi-level optimisation framework, (7.1)-(7.40), to be restructured into a single-level MPEC problem.

KKT Conditions Associated with the LL1 Problem

To derive the KKT optimality conditions for the LL1 problem, constructing its Lagrangian function $\mathcal{L}_{LL1}$ is essential. This function is built by the LL1 objective function with its constraints, each weighted by corresponding dual variables. The Lagrangian function is formulated as follows:

$$\begin{aligned}
\mathcal{L}_{LL1} = & \sum_{i,d,t,n,b} C_{i,b}^{EN} g_{i,d,t,n,b}^{DA} + \sum_{i,d,t,n,b} (C_{i,b}^U g_{i,d,t,n,b}^U - C_{i,b}^D g_{i,d,t,n,b}^D) + \sum_{d,t,n} \lambda_{d,t,n}^{DA} \left[\sum_{j \in \Psi_{n,o}} P_{j,d,t,o}^{DA} \right. \\
& + \sum_{e \in \Psi_n} s_{e,d,t}^{c,DA} + \sum_{m \in \Lambda_n} b_{nm} (\delta_{d,t,n}^{DA} - \delta_{d,t,m}^{DA}) - \sum_{i \in \Psi_{n,b}} g_{i,d,t,n,b}^{DA} - \sum_{e \in \Psi_n} s_{e,d,t}^{d,DA} \Big] + \\
& \sum_{d,t,n} \lambda_{d,t,n}^{RT} \left[\sum_{j \in \Psi_{n,o}} P_{j,d,t,o}^{dev} + \sum_{i \in I^{FL} \cap \Psi_{n,b}} (-g_{i,d,t,n,b}^U + g_{i,d,t,n,b}^D) + \sum_{e \in \Psi_n} (s_{e,d,t}^{c,RT} - s_{e,d,t}^{d,RT}) \right. \\
& + \sum_{i \in I^{RE} \cap \Psi_n} g_{i,d,t,n}^{dev} - \sum_{m \in \Lambda_n} b_{nm} (\delta_{d,t,n}^{DA} - \delta_{d,t,n}^{RT} - \delta_{d,t,m}^{DA} + \delta_{d,t,m}^{RT}) \Big] \\
& + \sum_{e,d,t \geq 2} \tau_{e,d,t}^{ES} \left[E_{e,d,t} - E_{e,d,t-1} - \eta_e^c (s_{e,d,t}^{c,DA} + s_{e,d,t}^{c,RT}) + \frac{s_{e,d,t}^{d,DA} + s_{e,d,t}^{d,RT}}{\eta_e^d} \right] \\
& + \sum_{e,d} \tau_{e,d}^{ES,1} \left[E_{e,d,1} - E_e^0 - \eta_e^c (s_{e,d,1}^{c,DA} + s_{e,d,1}^{c,RT}) + \frac{s_{e,d,1}^{d,DA} + s_{e,d,1}^{d,RT}}{\eta_e^d} \right] + \\
& \sum_{e,d} \tau_{e,d}^{ES,T} (E_e^0 - E_{e,d,T}) + \sum_{e,d,t} \tau_{e,d,t}^{ES,-} (E_e^{min} - E_{e,d,t}) + \sum_{e,d,t} \tau_{e,d,t}^{ES,+} (E_{e,d,t} - E_e^{max}) \\
& - \sum_{e,d,t} \tau_{e,d,t}^{c,DA,-} s_{e,d,t}^{c,DA} + \sum_{e,d,t} \tau_{e,d,t}^{c,DA,+} (s_{e,d,t}^{c,DA} - s_e^{max}) - \sum_{e,d,t} \tau_{e,d,t}^{c,RT,-} (s_{e,d,t}^{c,DA} + s_{e,d,t}^{c,RT}) + \\
& \sum_{e,d,t} \tau_{e,d,t}^{c,RT,+} (s_{e,d,t}^{c,DA} + s_{e,d,t}^{c,RT} - s_e^{max}) - \sum_{e,d,t} \tau_{e,d,t}^{d,DA,-} s_{e,d,t}^{d,DA} + \sum_{e,d,t} \tau_{e,d,t}^{d,DA,+} (s_{e,d,t}^{d,DA} - \\
& s_e^{max}) - \sum_{e,d,t} \tau_{e,d,t}^{d,RT,-} (s_{e,d,t}^{d,DA} + s_{e,d,t}^{d,RT}) + \sum_{e,d,t} \tau_{e,d,t}^{d,RT,+} (s_{e,d,t}^{d,DA} + s_{e,d,t}^{d,RT} - s_e^{max}) \\
& + \sum_d \zeta_d \left[\sum_{i \in I^{FL}, t, n, b} \eta_i^{CO_2} (g_{i,d,t,n,b}^{DA} + g_{i,d,t,n,b}^U - g_{i,d,t,n,b}^D) - \eta^{Grid} \sum_{j,t,o} L_{j,d,t,o}^{DA} \right]
\end{aligned}$$

$$\begin{aligned}
& + \sum_{i \in I^{FL}, d, t, n, b} \mu_{i, d, t, n, b}^U (g_{i, d, t, n, b}^{DA} + g_{i, d, t, n, b}^U - G_{i, n, b}) + \\
& \sum_{i \in I^{FL}, d, t, n, b} \mu_{i, d, t, n, b}^D (g_{i, d, t, n, b}^D - g_{i, d, t, n, b}^{DA}) - \sum_{i \in I^{FL}, d, t, n, b} \phi_{i, d, t, n, b}^{U, -} g_{i, d, t, n, b}^U + \\
& \sum_{i \in I^{FL}, d, t, n, b} \phi_{i, d, t, n, b}^{U, +} (g_{i, d, t, n, b}^U - G_{i, b}^U) - \sum_{i \in I^{FL}, d, t, n, b} \phi_{i, d, t, n, b}^{D, -} g_{i, d, t, n, b}^D + \\
& \sum_{i \in I^{FL}, d, t, n, b} \phi_{i, d, t, n, b}^{D, +} (g_{i, d, t, n, b}^D - G_{i, b}^D) + \sum_{i \in I^{MR}, d, t, n} \gamma_{i, d, t, n} (g_{i, d, t, n}^{DA} - G_{i, n}) + \\
& \sum_{i \in I^{RE}, d, t, n} \theta_{i, d, t, n} (g_{i, d, t, n}^{DA} - k_{i, d, t}^{RE} G_{i, n}) + \sum_{d, t, n, m \in \Lambda_n} \nu_{d, t, n, m}^{DA, -} [-C_{nm}^{max} - b_{nm} (\delta_{d, t, n}^{DA} - \\
& \delta_{d, t, m}^{DA})] + \sum_{d, t, n, m \in \Lambda_n} \nu_{d, t, n, m}^{DA, +} [b_{nm} (\delta_{d, t, n}^{DA} - \delta_{d, t, m}^{DA}) - C_{nm}^{max}] + \\
& \sum_{d, t, n} \xi_{d, t, n}^{DA, -} (-\pi - \delta_{d, t, n}^{DA}) + \sum_{d, t, n} \xi_{d, t, n}^{DA, +} (\delta_{d, t, n}^{DA} - \pi) + \sum_{d, t, n=1} \xi_{d, t}^{DA, ref} \delta_{d, t}^{DA, ref} + \\
& \sum_{d, t, n, m \in \Lambda_n} \nu_{d, t, n, m}^{RT, -} [-C_{nm}^{max} - b_{nm} (\delta_{d, t, n}^{RT} - \delta_{d, t, m}^{RT})] + \sum_{d, t, n, m \in \Lambda_n} \nu_{d, t, n, m}^{RT, +} [b_{nm} (\delta_{d, t, n}^{RT} - \\
& \delta_{d, t, m}^{RT}) - C_{nm}^{max}] + \sum_{d, t, n} \xi_{d, t, n}^{RT, -} (-\pi - \delta_{d, t, n}^{RT}) + \sum_{d, t, n} \xi_{d, t, n}^{RT, +} (\delta_{d, t, n}^{RT} - \pi) + \\
& \sum_{d, t, n=1} \xi_{d, t}^{RT, ref} \delta_{d, t}^{RT, ref} + \sum_{j, d, t, o} \psi_{j, d, t, o}^- [(1 - \alpha) L_{j, d, t, o}^{DA} - P_{j, d, t, o}^{DA}] + \\
& \sum_{j, d, t, o} \psi_{j, d, t, o}^+ [P_{j, d, t, o}^{DA} - (1 + \alpha) L_{j, d, t, o}^{DA}] + \sum_{j, d} \sigma_{j, d} \sum_{t, o} L_{j, d, t, o}^{SH} + \\
& \sum_{j, d, t} \chi_{j, d, t}^{SH, -} [\Phi^{-1} (1 - \rho^L) \alpha s_{j, d, t}^L - \alpha m_{j, d, t}^L - \sum_o L_{j, d, t, o}^{SH}] + \\
& \sum_{j, d, t} \chi_{j, d, t}^{SH, +} [\Phi^{-1} (1 - \rho^L) \alpha s_{j, d, t}^L - \alpha m_{j, d, t}^L + \sum_o L_{j, d, t, o}^{SH}] + \\
& \sum_{i \in I^{RE}, d, t, n} \chi_{i, d, t, n}^{RE} [\Phi^{-1} (1 - \rho^{RE}) s_{i, d, t, n}^{RE} - m_{i, d, t, n}^{RE} + g_{i, d, t, n}^{DA} - g_{i, d, t, n}^{dev}] + \\
& \sum_{j, d, t} \chi_{j, d, t}^L [\Phi^{-1} (1 - \rho^L) s_{j, d, t}^L + m_{j, d, t}^L + \sum_o L_{j, d, t, o}^{SH} - \sum_o P_{j, d, t, o}^{dev} - \sum_o P_{j, d, t, o}^{DA}]
\end{aligned} \tag{7.48}$$

Drawing from the Lagrangian function (7.48), the KKT optimality conditions are listed as follows:

KKT equality conditions : (7.10) – (7.14), (7.25) – (7.26), (7.29), (7.32), (7.34) (7.49)

LL1 KKT stationarity conditions :

$$\frac{\partial \mathcal{L}_{LL1}}{\partial P_{j, d, t, o}^{DA}} = \lambda_{d, t, n}^{DA} + \psi_{j, d, t, o}^+ - \psi_{j, d, t, o}^- - \chi_{j, d, t}^L = 0 \quad \forall j \in \Psi_n, \forall d, \forall t, \forall n, \forall o \tag{7.50}$$

$$\frac{\partial \mathcal{L}_{LL1}}{\partial g_{i,d,t,n,b}^{DA}} = C_{i,b}^{EN} - \lambda_{d,t,n}^{DA} + \eta_i^{CO_2} \zeta_d + \mu_{i,d,t,n,b}^U - \mu_{i,d,t,n,b}^D = 0 \quad \forall i \in I^{FL} \cap \Psi_n, \forall d, \forall t, \forall n, \forall b \quad (7.51)$$

$$\frac{\partial \mathcal{L}_{LL1}}{\partial g_{i,d,t,n}^{DA}} = C_i^{EN} - \lambda_{d,t,n}^{DA} + \gamma_{i,d,t,n} = 0 \quad \forall i \in I^{MR} \cap \Psi_n, \forall d, \forall t, \forall n \quad (7.52)$$

$$\frac{\partial \mathcal{L}_{LL1}}{\partial g_{i,d,t,n}^{DA}} = -\lambda_{d,t,n}^{DA} + \theta_{i,d,t,n} + \chi_{i,d,t,n}^{RE} = 0 \quad \forall i \in I^{RE} \cap \Psi_n, \forall d, \forall t, \forall n \quad (7.53)$$

$$\begin{aligned} \frac{\partial \mathcal{L}_{LL1}}{\partial g_{i,d,t,n,b}^U} = & C_{i,b}^U - \lambda_{d,t,n}^{RT} + \eta_i^{CO_2} \zeta_d + \mu_{i,d,t,n,b}^U - \phi_{i,d,t,n,b}^{U,-} + \phi_{i,d,t,n,b}^{U,+} = 0 \\ & \forall i \in I^{FL} \cap \Psi_n, \forall d, \forall t, \forall n, \forall b \end{aligned} \quad (7.54)$$

$$\begin{aligned} \frac{\partial \mathcal{L}_{LL1}}{\partial g_{i,d,t,n,b}^D} = & -C_{i,b}^D + \lambda_{d,t,n}^{RT} - \eta_i^{CO_2} \zeta_d + \mu_{i,d,t,n,b}^D - \phi_{i,d,t,n,b}^{D,-} + \phi_{i,d,t,n,b}^{D,+} = 0 \\ & \forall i \in I^{FL} \cap \Psi_n, \forall d, \forall t, \forall n, \forall b \end{aligned} \quad (7.55)$$

$$\frac{\partial \mathcal{L}_{LL1}}{\partial P_{j,d,t,o}^{dev}} = \lambda_{d,t,n}^{RT} - \chi_{j,d,t}^L = 0 \quad \forall j \in \Psi_n, \forall d, \forall t \quad (7.56)$$

$$\frac{\partial \mathcal{L}_{LL1}}{\partial g_{i,d,t,n}^{dev}} = \lambda_{d,t,n}^{RT} - \chi_{i,d,t,n}^{RE} = 0 \quad \forall i \in I^{RE} \cap \Psi_n, \forall d, \forall t, \forall n \quad (7.57)$$

$$\frac{\partial \mathcal{L}_{LL1}}{\partial E_{e,d,t}} = \tau_{e,d,t}^{ES} - \tau_{e,d,t+1}^{ES} + \tau_{e,d,t}^{ES,+} - \tau_{e,d,t}^{ES,-} = 0 \quad \forall e, \forall d, \forall t \leq T-1 \quad (7.58)$$

$$\frac{\partial \mathcal{L}_{LL1}}{\partial E_{e,d,T}} = \tau_{e,d,T}^{ES} - \tau_{e,d}^{ES,T} + \tau_{e,d,T}^{ES,+} - \tau_{e,d,T}^{ES,-} = 0 \quad \forall e, \forall d \quad (7.59)$$

$$\frac{\partial \mathcal{L}_{LL1}}{\partial s_{e,d,t}^{c,DA}} = \lambda_{d,t,n}^{DA} - \eta_e^c \tau_{e,d,t}^{ES} + \tau_{e,d,t}^{c,DA,+} - \tau_{e,d,t}^{c,DA,-} + \tau_{e,d,t}^{c,RT,+} - \tau_{e,d,t}^{c,RT,-} = 0 \quad \forall e, \forall d, \forall t \quad (7.60)$$

$$\frac{\partial \mathcal{L}_{LL1}}{\partial s_{e,d,t}^{d,DA}} = -\lambda_{d,t,n}^{DA} + \frac{\tau_{e,d,t}^{ES}}{\eta_e^d} + \tau_{e,d,t}^{d,DA,+} - \tau_{e,d,t}^{d,DA,-} + \tau_{e,d,t}^{d,RT,+} - \tau_{e,d,t}^{d,RT,-} = 0 \quad \forall e, \forall d, \forall t \quad (7.61)$$

$$\frac{\partial \mathcal{L}_{LL1}}{\partial s_{e,d,t}^{c,RT}} = \lambda_{d,t,n}^{RT} - \eta_e^c \tau_{e,d,t}^{ES} + \tau_{e,d,t}^{c,RT,+} - \tau_{e,d,t}^{c,RT,-} = 0 \quad \forall e, \forall d, \forall t \quad (7.62)$$

$$\frac{\partial \mathcal{L}_{LL1}}{\partial s_{e,d,t}^{d,RT}} = -\lambda_{d,t,n}^{RT} + \frac{\tau_{e,d,t}^{ES}}{\eta_e^d} + \tau_{e,d,t}^{d,RT,+} - \tau_{e,d,t}^{d,RT,-} = 0 \quad \forall e, \forall d, \forall t \quad (7.63)$$

$$\begin{aligned} \frac{\partial \mathcal{L}_{LL1}}{\partial \delta_{d,t,n}^{DA}} = & \sum_{m \in \Lambda_n} b_{nm} (\lambda_{d,t,n}^{DA} - \lambda_{d,t,m}^{DA}) - \sum_{m \in \Lambda_n} b_{nm} (\lambda_{d,t,n}^{RT} - \lambda_{d,t,m}^{RT}) + \\ & \sum_{m \in \Lambda_n} b_{nm} (\nu_{d,t,n,m}^{DA,+} - \nu_{d,t,m,n}^{DA,+}) - \sum_{m \in \Lambda_n} b_{nm} (\nu_{d,t,n,m}^{DA,-} - \nu_{d,t,m,n}^{DA,-}) + \xi_{d,t,n}^{DA,+} \\ & - \xi_{d,t,n}^{DA,-} + (\xi_{d,t}^{DA,ref})_{n=1} = 0 \quad \forall d, \forall t, \forall n \end{aligned} \quad (7.64)$$

$$\begin{aligned} \frac{\partial \mathcal{L}_{LL1}}{\partial \delta_{d,t,n}^{RT}} = & \sum_{m \in \Lambda_n} b_{nm} (\lambda_{d,t,n}^{RT} - \lambda_{d,t,m}^{RT}) + \sum_{m \in \Lambda_n} b_{nm} (\nu_{d,t,n,m}^{RT,+} - \nu_{d,t,m,n}^{RT,+}) - \\ & \sum_{m \in \Lambda_n} b_{nm} (\nu_{d,t,n,m}^{RT,-} - \nu_{d,t,m,n}^{RT,-}) + \xi_{d,t,n}^{RT,+} - \xi_{d,t,n}^{RT,-} + (\xi_{d,t}^{RT,ref})_{n=1} = 0 \quad \forall d, \forall t, \forall n \end{aligned} \quad (7.65)$$

$$\frac{\partial \mathcal{L}_{LL1}}{\partial L_{j,d,t,o}^{SH}} = \sigma_{j,d} + \chi_{j,d,t}^{SH,+} - \chi_{j,d,t}^{SH,-} + \chi_{j,d,t}^L = 0 \quad \forall j \in \Psi_n, \forall d, \forall t, \forall n \quad (7.66)$$

LL1 KKT complementarity conditions :

$$0 \leq (E_{e,d,t} - E_e^{min}) \perp \tau_{e,d,t}^{ES,-} \geq 0 \quad \forall e, \forall d, \forall t \quad (7.67)$$

$$0 \leq (E_e^{max} - E_{e,d,t}) \perp \tau_{e,d,t}^{ES,+} \geq 0 \quad \forall e, \forall d, \forall t \quad (7.68)$$

$$0 \leq s_{e,d,t}^{c,DA} \perp \tau_{e,d,t}^{c,DA,-} \geq 0 \quad \forall e, \forall d, \forall t \quad (7.69)$$

$$0 \leq (s_e^{max} - s_{e,d,t}^{c,DA}) \perp \tau_{e,d,t}^{c,DA,+} \geq 0 \quad \forall e, \forall d, \forall t \quad (7.70)$$

$$0 \leq (s_{e,d,t}^{c,DA} + s_{e,d,t}^{c,RT}) \perp \tau_{e,d,t}^{c,RT,-} \geq 0 \quad \forall e, \forall d, \forall t \quad (7.71)$$

$$0 \leq (s_e^{max} - s_{e,d,t}^{c,DA} - s_{e,d,t}^{c,RT}) \perp \tau_{e,d,t}^{c,RT,+} \geq 0 \quad \forall e, \forall d, \forall t \quad (7.72)$$

$$0 \leq s_{e,d,t}^{d,DA} \perp \tau_{e,d,t}^{d,DA,-} \geq 0 \quad \forall e, \forall d, \forall t \quad (7.73)$$

$$0 \leq (s_e^{max} - s_{e,d,t}^{d,DA}) \perp \tau_{e,d,t}^{d,DA,+} \geq 0 \quad \forall e, \forall d, \forall t \quad (7.74)$$

$$0 \leq (s_{e,d,t}^{d,DA} + s_{e,d,t}^{d,RT}) \perp \tau_{e,d,t}^{d,RT,-} \geq 0 \quad \forall e, \forall d, \forall t \quad (7.75)$$

$$0 \leq (s_e^{max} - s_{e,d,t}^{d,DA} - s_{e,d,t}^{d,RT}) \perp \tau_{e,d,t}^{d,RT,+} \geq 0 \quad \forall e, \forall d, \forall t \quad (7.76)$$

$$0 \leq [\eta^{Grid} \sum_{j,t,o} L_{j,d,t,o}^{DA} - \sum_{i \in I^{FL},t,n,b} \eta_i^{CO_2} (g_{i,d,t,n,b}^{DA} + g_{i,d,t,n,b}^U - g_{i,d,t,n,b}^D)] \perp \zeta_d \geq 0 \quad \forall d \quad (7.77)$$

$$0 \leq (G_{i,n,b} - g_{i,d,t,n,b}^{DA} - g_{i,d,t,n,b}^U) \perp \mu_{i,d,t,n,b}^U \geq 0 \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (7.78)$$

$$0 \leq (g_{i,d,t,n,b}^{DA} - g_{i,d,t,n,b}^D) \perp \mu_{i,d,t,n,b}^D \geq 0 \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (7.79)$$

$$0 \leq g_{i,d,t,n,b}^U \perp \phi_{i,d,t,n,b}^{U,-} \geq 0 \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (7.80)$$

$$0 \leq (G_{i,b}^U - g_{i,d,t,n,b}^U) \perp \phi_{i,d,t,n,b}^{U,+} \geq 0 \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (7.81)$$

$$0 \leq g_{i,d,t,n,b}^D \perp \phi_{i,d,t,n,b}^{D,-} \geq 0 \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (7.82)$$

$$0 \leq (G_{i,b}^D - g_{i,d,t,n,b}^D) \perp \phi_{i,d,t,n,b}^{D,+} \geq 0 \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (7.83)$$

$$0 \leq [C_{nm}^{max} + b_{nm} (\delta_{d,t,n}^{DA} - \delta_{d,t,m}^{DA})] \perp \nu_{d,t,n,m}^{DA,-} \geq 0 \quad \forall d, \forall t, \forall n, \forall m \in \Lambda_n \quad (7.84)$$

$$0 \leq [C_{nm}^{max} - b_{nm} (\delta_{d,t,n}^{DA} - \delta_{d,t,m}^{DA})] \perp \nu_{d,t,n,m}^{DA,+} \geq 0 \quad \forall d, \forall t, \forall n, \forall m \in \Lambda_n \quad (7.85)$$

$$0 \leq (\pi + \delta_{d,t,n}^{DA}) \perp \xi_{d,t,n}^{DA,-} \geq 0 \quad \forall d, \forall t, \forall n \quad (7.86)$$

$$0 \leq (\pi - \delta_{d,t,n}^{DA}) \perp \xi_{d,t,n}^{DA,+} \geq 0 \quad \forall d, \forall t, \forall n \quad (7.87)$$

$$0 \leq [C_{nm}^{max} + b_{nm}(\delta_{d,t,n}^{RT} - \delta_{d,t,m}^{RT})] \perp \nu_{d,t,n,m}^{RT,-} \geq 0 \quad \forall d, \forall t, \forall n, \forall m \in \Lambda_n \quad (7.88)$$

$$0 \leq [C_{nm}^{max} - b_{nm}(\delta_{d,t,n}^{RT} - \delta_{d,t,m}^{RT})] \perp \nu_{d,t,n,m}^{RT,+} \geq 0 \quad \forall d, \forall t, \forall n, \forall m \in \Lambda_n \quad (7.89)$$

$$0 \leq (\pi + \delta_{d,t,n}^{RT}) \perp \xi_{d,t,n}^{RT,-} \geq 0 \quad \forall d, \forall t, \forall n \quad (7.90)$$

$$0 \leq (\pi - \delta_{d,t,n}^{RT}) \perp \xi_{d,t,n}^{RT,+} \geq 0 \quad \forall d, \forall t, \forall n \quad (7.91)$$

$$0 \leq [P_{j,d,t,o}^{DA} - (1 - \alpha)L_{j,d,t,o}^{DA}] \perp \psi_{j,d,t,o}^- \geq 0 \quad \forall j, \forall d, \forall t, \forall o \quad (7.92)$$

$$0 \leq [(1 + \alpha)L_{j,d,t,o}^{DA} - P_{j,d,t,o}^{DA}] \perp \psi_{j,d,t,o}^+ \geq 0 \quad \forall j, \forall d, \forall t, \forall o \quad (7.93)$$

$$0 \leq [\alpha m_{j,d,t}^L + \sum_o L_{j,d,t,o}^{SH} - \Phi^{-1}(1 - \rho^L)\alpha s_{j,d,t}^L] \perp \chi_{j,d,t}^{SH,-} \geq 0 \quad \forall j, \forall d, \forall t \quad (7.94)$$

$$0 \leq [\alpha m_{j,d,t}^L - \sum_o L_{j,d,t,o}^{SH} - \Phi^{-1}(1 - \rho^L)\alpha s_{j,d,t}^L] \perp \chi_{j,d,t}^{SH,+} \geq 0 \quad \forall j, \forall d, \forall t \quad (7.95)$$

$$0 \leq [m_{i,d,t,n}^{RE} - g_{i,d,t,n}^{DA} + g_{i,d,t,n}^{dev} - \Phi^{-1}(1 - \rho^{RE})s_{i,d,t,n}^{RE}] \perp \chi_{i,d,t,n}^{RE} \geq 0 \quad \forall i \in I^{RE}, \forall d, \forall t, \forall n \quad (7.96)$$

$$0 \leq [-m_{j,d,t}^L - \sum_o L_{j,d,t,o}^{SH} + \sum_o P_{j,d,t,o}^{dev} + \sum_o P_{j,d,t,o}^{DA} - \Phi^{-1}(1 - \rho^L)s_{j,d,t}^L] \perp \chi_{j,d,t}^L \geq 0$$

$$\forall j, \forall d, \forall t$$

$$(7.97)$$

Deriving the KKT optimality conditions for the LL1 problem is a key step in transitioning the bi-level optimisation model to a single-level MPEC, which is organised into three main categories: i) KKT equality conditions (7.49), ii) KKT stationarity conditions (7.50)-(7.66), and iii) KKT complementarity conditions (7.67)-(7.97).

Final MPEC Formulation

By integrating the equivalent LL1 KKT optimality conditions along with the LL2 problem's constraints, the comprehensive formulation of the MPEC is represented as follows:

$$\begin{aligned} \max_{V^{MPEC}} \sum_d \omega_d \Bigg[& \sum_{i \in \Psi_n, t, n, b} \lambda_{d,t,n}^{DA} g_{i,d,t,n,b}^{DA} - \sum_{i, t, n, b} C_{i,b}^{EN} g_{i,d,t,n,b}^{DA} + \\ & \sum_{i \in I^{FL} \cap \Psi_n, t, n, b} \lambda_{d,t,n}^{RT} (g_{i,d,t,n,b}^U - g_{i,d,t,n,b}^D) - \sum_{i \in I^{FL}, t, n, b} (C_{i,b}^U g_{i,d,t,n,b}^U - C_{i,b}^D g_{i,d,t,n,b}^D) \\ & + \sum_{i \in I^{RE} \cap \Psi_n, t, n} \lambda_{d,t,n}^{RT} g_{i,d,t,n}^{dev} - \sum_{i \in I^{FL}, t, n} \lambda^{CO_2} Q_{i,d,t,n}^{emi} + \sum_{i \in I^{RE}, t, n} \lambda^{CO_2} Q_{i,d,t,n}^{CA} \end{aligned}$$

$$+ \sum_{i \in I^{RE}} (\lambda_{i,d}^{PGC} Q_{i,d}^{PGC} + \lambda_{i,d}^{FGC} Q_{i,d}^{FGC}) \Big] - \sum_{i,n} C_i^{IN} G_{i,n} \quad (7.98)$$

where:

$$V^{MPEC} = V^{UL} \cup V^{LL1} \cup V^{LL2} \cup V^{DUAL} \quad (7.99)$$

$$\begin{aligned} V^{DUAL} = \{ & \lambda_{d,t,n}^{DA}, \lambda_{d,t,n}^{RT}, \tau_{e,d,t}^{ES}, \tau_{e,d}^{ES,1}, \tau_{e,d}^{ES,T}, \tau_{e,d,t}^{ES,-}, \tau_{e,d,t}^{ES,+}, \tau_{e,d,t}^{c,DA,-}, \tau_{e,d,t}^{c,DA,+}, \tau_{e,d,t}^{c,RT,-}, \\ & \tau_{e,d,t}^{c,RT,+}, \tau_{e,d,t}^{d,DA,-}, \tau_{e,d,t}^{d,DA,+}, \tau_{e,d,t}^{d,RT,-}, \tau_{e,d,t}^{d,RT,+}, \zeta_d, \mu_{i,d,t,n,b}^U, \mu_{i,d,t,n,b}^D, \phi_{i,d,t,n,b}^{U,-}, \\ & \phi_{i,d,t,n,b}^{U,+}, \phi_{i,d,t,n,b}^{D,-}, \phi_{i,d,t,n,b}^{D,+}, \gamma_{i,d,t,n}, \theta_{i,d,t,n}, \nu_{d,t,n,m}^{DA,-}, \nu_{y,t,n,m}^{DA,+}, \xi_{d,t,n}^{DA,-}, \xi_{d,t,n}^{DA,+}, \\ & \xi_{d,t}^{DA,ref}, \nu_{d,t,n,m}^{RT,-}, \nu_{d,t,n,m}^{RT,+}, \xi_{d,t,n}^{RT,-}, \xi_{d,t,n}^{RT,+}, \xi_{d,t}^{RT,ref}, \psi_{j,d,t,o}^-, \psi_{j,d,t,o}^+, \sigma_{j,d}, \chi_{j,d,t}^{SH,-}, \\ & \chi_{j,d,t}^{SH,+}, \chi_{i,d,t,n}^{RE}, \chi_{j,d,t}^L \} \end{aligned} \quad (7.100)$$

subject to:

$$\text{UL constraints : (7.3) – (7.7)} \quad (7.101)$$

$$\text{LL1 KKT equality conditions : (7.49)} \quad (7.102)$$

$$\text{LL1 KKT stationarity conditions : (7.50) – (7.66)} \quad (7.103)$$

$$\text{LL1 KKT complementarity conditions : (7.67) – (7.97)} \quad (7.104)$$

$$\text{LL2 constraints : (7.39) – (7.40)} \quad (7.105)$$

7.4.3 Linearization of MPEC

To transform the nonlinear MPEC into a MIQC that is more compatible with the commercial branch-and-cut solvers, this subsection outlines the necessary linearization steps. The nonlinearities within the MPEC arise from two primary sources: the KKT complementarity conditions associated with the LL1 problem (7.104), and the bilinear terms present in the GenCo's profit, as represented by the MPEC's objective function (7.98).

Linearization of KKT Complementarity Conditions

The big-M method is adopted to decompose the KKT complementarity condition into a set of linear conditions mixed with binary variables. Specifically, a represents a primal variable in KKT conditions, and b represents the corresponding dual variable. The resulting equivalent mixed-integer linear conditions are formulated as follows: $\{a \geq 0, b \geq 0, a \leq \beta M^a, b \leq (1 - \beta) M^b, \beta \in \{0, 1\}\}$. Note that M^a and M^b identify the large

positive constants. Following this linearization approach, the KKT complementarity conditions corresponding to the LL1 problem (7.104) are converted into a group of disjunctive constraints. These disjunctive constraints are formulated as follows:

$$V^{BINARY} = \left\{ \begin{aligned} &\beta_{e,d,t}^{E,-}, \beta_{e,d,t}^{E,+}, \beta_{e,d,t}^{c,DA,-}, \beta_{e,d,t}^{c,DA,+}, \beta_{e,d,t}^{c,RT,-}, \beta_{e,d,t}^{c,RT,+}, \beta_{e,d,t}^{d,DA,-}, \beta_{e,d,t}^{d,DA,+}, \beta_{e,d,t}^{d,RT,-}, \\ &\beta_{e,d,t}^{d,RT,+}, \beta_d^{CO_2}, \beta_{i,d,t,n,b}^U, \beta_{i,d,t,n,b}^D, \beta_{i,d,t,n,b}^{U,-}, \beta_{i,d,t,n,b}^{U,+}, \beta_{i,d,t,n,b}^{D,-}, \beta_{i,d,t,n,b}^{D,+}, \\ &\beta_{d,t,n,m}^{DA,net,-}, \beta_{d,t,n,m}^{DA,net,+}, \beta_{d,t,n}^{DA,\pi,-}, \beta_{d,t,n}^{DA,\pi,+}, \beta_{d,t,n,m}^{RT,net,-}, \beta_{d,t,n,m}^{RT,net,+}, \beta_{d,t,n}^{RT,\pi,-}, \\ &\beta_{d,t,n}^{RT,\pi,+}, \beta_{j,d,t,o}^{PDA,-}, \beta_{j,d,t,o}^{PDA,+}, \beta_{j,d,t}^{SH,-}, \beta_{j,d,t}^{SH,+}, \beta_{i,d,t,n}^{RE}, \beta_{j,d,t}^L \end{aligned} \right\} \quad (7.106)$$

$$0 \leq E_{e,d,t} - E_e^{min} \leq \beta_{e,d,t}^{E,-} M \quad \forall e, \forall d, \forall t \quad (7.107)$$

$$0 \leq \tau_{e,d,t}^{ES,-} \leq (1 - \beta_{e,d,t}^{E,-}) M \quad \forall e, \forall d, \forall t \quad (7.108)$$

$$0 \leq E_e^{max} - E_{e,d,t} \leq \beta_{e,d,t}^{E,+} M \quad \forall e, \forall d, \forall t \quad (7.109)$$

$$0 \leq \tau_{e,d,t}^{ES,+} \leq (1 - \beta_{e,d,t}^{E,+}) M \quad \forall e, \forall d, \forall t \quad (7.110)$$

$$0 \leq s_{e,d,t}^{c,DA} \leq \beta_{e,d,t}^{c,DA,-} M \quad \forall e, \forall d, \forall t \quad (7.111)$$

$$0 \leq \tau_{e,d,t}^{c,DA,-} \leq (1 - \beta_{e,d,t}^{c,DA,-}) M \quad \forall e, \forall d, \forall t \quad (7.112)$$

$$0 \leq s_e^{max} - s_{e,d,t}^{c,DA} \leq \beta_{e,d,t}^{c,DA,+} M \quad \forall e, \forall d, \forall t \quad (7.113)$$

$$0 \leq \tau_{e,d,t}^{c,DA,+} \leq (1 - \beta_{e,d,t}^{c,DA,+}) M \quad \forall e, \forall d, \forall t \quad (7.114)$$

$$0 \leq s_{e,d,t}^{c,DA} + s_{e,d,t}^{c,RT} \leq \beta_{e,d,t}^{c,RT,-} M \quad \forall e, \forall d, \forall t \quad (7.115)$$

$$0 \leq \tau_{e,d,t}^{c,RT,-} \leq (1 - \beta_{e,d,t}^{c,RT,-}) M \quad \forall e, \forall d, \forall t \quad (7.116)$$

$$0 \leq s_e^{max} - s_{e,d,t}^{c,DA} - s_{e,d,t}^{c,RT} \leq \beta_{e,d,t}^{c,RT,+} M \quad \forall e, \forall d, \forall t \quad (7.117)$$

$$0 \leq \tau_{e,d,t}^{c,RT,+} \leq (1 - \beta_{e,d,t}^{c,RT,+}) M \quad \forall e, \forall d, \forall t \quad (7.118)$$

$$0 \leq s_{e,d,t}^{d,DA} \leq \beta_{e,d,t}^{d,DA,-} M \quad \forall e, \forall d, \forall t \quad (7.119)$$

$$0 \leq \tau_{e,d,t}^{d,DA,-} \leq (1 - \beta_{e,d,t}^{d,DA,-}) M \quad \forall e, \forall d, \forall t \quad (7.120)$$

$$0 \leq s_e^{max} - s_{e,d,t}^{d,DA} \leq \beta_{e,d,t}^{d,DA,+} M \quad \forall e, \forall d, \forall t \quad (7.121)$$

$$0 \leq \tau_{e,d,t}^{d,DA,+} \leq (1 - \beta_{e,d,t}^{d,DA,+}) M \quad \forall e, \forall d, \forall t \quad (7.122)$$

$$0 \leq s_{e,d,t}^{d,DA} + s_{e,d,t}^{d,RT} \leq \beta_{e,d,t}^{d,RT,-} M \quad \forall e, \forall d, \forall t \quad (7.123)$$

$$0 \leq \tau_{e,d,t}^{d,RT,-} \leq (1 - \beta_{e,d,t}^{d,RT,-}) M \quad \forall e, \forall d, \forall t \quad (7.124)$$

$$0 \leq s_e^{max} - s_{e,d,t}^{d,DA} - s_{e,d,t}^{d,RT} \leq \beta_{e,d,t}^{d,RT,+} M \quad \forall e, \forall d, \forall t \quad (7.125)$$

$$0 \leq \tau_{e,d,t}^{d,RT,+} \leq (1 - \beta_{e,d,t}^{d,RT,+})M \quad \forall e, \forall d, \forall t \quad (7.126)$$

$$0 \leq \eta^{Grid} \sum_{j,t,o} L_{j,d,t,o}^{DA} - \sum_{i \in I^{FL,t,n,b}} \eta_i^{CO_2} (g_{i,d,t,n,b}^{DA} + g_{i,d,t,n,b}^U - g_{i,d,t,n,b}^D) \leq \beta_d^{CO_2} M \quad \forall d \quad (7.127)$$

$$0 \leq \zeta_d \leq (1 - \beta_d^{CO_2})M \quad \forall d \quad (7.128)$$

$$0 \leq G_{i,n,b} - g_{i,d,t,n,b}^{DA} - g_{i,d,t,n,b}^U \leq \beta_{i,d,t,n,b}^U M \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (7.129)$$

$$0 \leq \mu_{i,d,t,n,b}^U \leq (1 - \beta_{i,d,t,n,b}^U)M \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (7.130)$$

$$0 \leq g_{i,d,t,n,b}^{DA} - g_{i,d,t,n,b}^D \leq \beta_{i,d,t,n,b}^D M \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (7.131)$$

$$0 \leq \mu_{i,d,t,n,b}^D \leq (1 - \beta_{i,d,t,n,b}^D)M \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (7.132)$$

$$0 \leq g_{i,d,t,n,b}^U \leq \beta_{i,d,t,n,b}^{U,-} M \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (7.133)$$

$$0 \leq \phi_{i,d,t,n,b}^{U,-} \leq (1 - \beta_{i,d,t,n,b}^{U,-})M \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (7.134)$$

$$0 \leq G_{i,b}^U - g_{i,d,t,n,b}^U \leq \beta_{i,d,t,n,b}^{U,+} M \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (7.135)$$

$$0 \leq \phi_{i,d,t,n,b}^{U,+} \leq (1 - \beta_{i,d,t,n,b}^{U,+})M \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (7.136)$$

$$0 \leq g_{i,d,t,n,b}^D \leq \beta_{i,d,t,n,b}^{D,-} M \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (7.137)$$

$$0 \leq \phi_{i,d,t,n,b}^{D,-} \leq (1 - \beta_{i,d,t,n,b}^{D,-})M \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (7.138)$$

$$0 \leq G_{i,b}^D - g_{i,d,t,n,b}^D \leq \beta_{i,d,t,n,b}^{D,+} M \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (7.139)$$

$$0 \leq \phi_{i,d,t,n,b}^{D,+} \leq (1 - \beta_{i,d,t,n,b}^{D,+})M \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (7.140)$$

$$0 \leq C_{nm}^{max} + b_{nm}(\delta_{d,t,n}^{DA} - \delta_{d,t,m}^{DA}) \leq \beta_{d,t,n,m}^{DA,net,-} M \quad \forall d, \forall t, \forall n, \forall m \in \Lambda_n \quad (7.141)$$

$$0 \leq \nu_{d,t,n,m}^{DA,-} \leq (1 - \beta_{d,t,n,m}^{DA,net,-})M \quad \forall d, \forall t, \forall n, \forall m \in \Lambda_n \quad (7.142)$$

$$0 \leq C_{nm}^{max} - b_{nm}(\delta_{d,t,n}^{DA} - \delta_{d,t,m}^{DA}) \leq \beta_{d,t,n,m}^{DA,net,+} M \quad \forall d, \forall t, \forall n, \forall m \in \Lambda_n \quad (7.143)$$

$$0 \leq \nu_{d,t,n,m}^{DA,+} \leq (1 - \beta_{d,t,n,m}^{DA,net,+})M \quad \forall d, \forall t, \forall n, \forall m \in \Lambda_n \quad (7.144)$$

$$0 \leq \pi + \delta_{d,t,n}^{DA} \leq \beta_{d,t,n}^{DA,\pi,-} M \quad \forall d, \forall t, \forall n \quad (7.145)$$

$$0 \leq \xi_{d,t,n}^{DA,-} \leq (1 - \beta_{d,t,n}^{DA,\pi,-})M \quad \forall d, \forall t, \forall n \quad (7.146)$$

$$0 \leq \pi - \delta_{d,t,n}^{DA} \leq \beta_{d,t,n}^{DA,\pi,+} M \quad \forall d, \forall t, \forall n \quad (7.147)$$

$$0 \leq \xi_{d,t,n}^{DA,+} \leq (1 - \beta_{d,t,n}^{DA,\pi,+})M \quad \forall d, \forall t, \forall n \quad (7.148)$$

$$0 \leq C_{nm}^{max} + b_{nm}(\delta_{d,t,n}^{RT} - \delta_{d,t,m}^{RT}) \leq \beta_{d,t,n,m}^{RT,net,-} M \quad \forall d, \forall t, \forall n, \forall m \in \Lambda_n \quad (7.149)$$

$$0 \leq \nu_{d,t,n,m}^{RT,-} \leq (1 - \beta_{d,t,n,m}^{RT,net,-})M \quad \forall d, \forall t, \forall n, \forall m \in \Lambda_n \quad (7.150)$$

$$0 \leq C_{nm}^{max} - b_{nm}(\delta_{d,t,n}^{RT} - \delta_{d,t,m}^{RT}) \leq \beta_{d,t,n,m}^{RT,net,+} M \quad \forall d, \forall t, \forall n, \forall m \in \Lambda_n \quad (7.151)$$

$$0 \leq \nu_{d,t,n,m}^{RT,+} \leq (1 - \beta_{d,t,n,m}^{RT,net,+}) M \quad \forall d, \forall t, \forall n, \forall m \in \Lambda_n \quad (7.152)$$

$$0 \leq \pi + \delta_{d,t,n}^{RT} \leq \beta_{d,t,n}^{RT,\pi,-} M \quad \forall d, \forall t, \forall n \quad (7.153)$$

$$0 \leq \xi_{d,t,n}^{RT,-} \leq (1 - \beta_{d,t,n}^{RT,\pi,-}) M \quad \forall d, \forall t, \forall n \quad (7.154)$$

$$0 \leq \pi - \delta_{d,t,n}^{RT} \leq \beta_{d,t,n}^{RT,\pi,+} M \quad \forall d, \forall t, \forall n \quad (7.155)$$

$$0 \leq \xi_{d,t,n}^{RT,+} \leq (1 - \beta_{d,t,n}^{RT,\pi,+}) M \quad \forall d, \forall t, \forall n \quad (7.156)$$

$$0 \leq P_{j,d,t,o}^{DA} - (1 - \alpha) L_{j,d,t,o}^{DA} \leq \beta_{j,d,t,o}^{PDA,-} M \quad \forall j, \forall d, \forall t, \forall o \quad (7.157)$$

$$0 \leq \psi_{j,d,t,o}^{-} \leq (1 - \beta_{j,d,t,o}^{PDA,-}) M \quad \forall j, \forall d, \forall t, \forall o \quad (7.158)$$

$$0 \leq (1 + \alpha) L_{j,d,t,o}^{DA} - P_{j,d,t,o}^{DA} \leq \beta_{j,d,t,o}^{PDA,+} M \quad \forall j, \forall d, \forall t, \forall o \quad (7.159)$$

$$0 \leq \psi_{j,d,t,o}^{+} \leq (1 - \beta_{j,d,t,o}^{PDA,+}) M \quad \forall j, \forall d, \forall t, \forall o \quad (7.160)$$

$$0 \leq \alpha m_{j,d,t}^L + \sum_o L_{j,d,t,o}^{SH} - \Phi^{-1}(1 - \rho^L) \alpha s_{j,d,t}^L \leq \beta_{j,d,t}^{SH,-} M \quad \forall j, \forall d, \forall t \quad (7.161)$$

$$0 \leq \chi_{j,d,t}^{SH,-} \leq (1 - \beta_{j,d,t}^{SH,-}) M \quad \forall j, \forall d, \forall t \quad (7.162)$$

$$0 \leq \alpha m_{j,d,t}^L - \sum_o L_{j,d,t,o}^{SH} - \Phi^{-1}(1 - \rho^L) \alpha s_{j,d,t}^L \leq \beta_{j,d,t}^{SH,+} M \quad \forall j, \forall d, \forall t \quad (7.163)$$

$$0 \leq \chi_{j,d,t}^{SH,+} \leq (1 - \beta_{j,d,t}^{SH,+}) M \quad \forall j, \forall d, \forall t \quad (7.164)$$

$$0 \leq m_{i,d,t,n}^{RE} - g_{i,d,t,n}^{DA} + g_{i,d,t,n}^{dev} - \Phi^{-1}(1 - \rho^{RE}) s_{i,d,t,n}^{RE} \leq \beta_{i,d,t,n}^{RE} M \quad \forall i \in I^{RE}, \forall d, \forall t, \forall n \quad (7.165)$$

$$0 \leq \chi_{i,d,t,n}^{RE} \leq (1 - \beta_{i,d,t,n}^{RE}) M \quad \forall i \in I^{RE}, \forall d, \forall t, \forall n \quad (7.166)$$

$$0 \leq -m_{j,d,t}^L - \sum_o L_{j,d,t,o}^{SH} + \sum_o P_{j,d,t,o}^{dev} + \sum_o P_{j,d,t,o}^{DA} - \Phi^{-1}(1 - \rho^L) s_{j,d,t}^L \leq \beta_{j,d,t}^L M \quad \forall j, \forall d, \forall t \quad (7.167)$$

$$0 \leq \chi_{j,d,t}^L \leq (1 - \beta_{j,d,t}^L) M \quad \forall j, \forall d, \forall t \quad (7.168)$$

Linearization of GenCo's Profit

In order to deal with the challenge of bilinear terms in GenCo's profit within the MPEC's objective function (7.98), the strong duality theorem and the KKT optimality conditions are leveraged to uncover implicit linear forms of these bilinear terms, thereby facilitating their incorporation into the model in a linearized structure.

Specifically, the bilinear terms $(\sum_{i \in I} \lambda_{d,t,n}^{DA} g_{i,d,t,n}^{DA}, \sum_{i \in I^{FL}} \lambda_{d,t,n}^{RT} g_{i,d,t,n}^U, \sum_{i \in I^{FL}}$

$\lambda_{d,t,n}^{RT} g_{i,d,t,n,b}^D$, $\sum_{i \in I^{RE}} \lambda_{d,t,n}^{RT} g_{i,d,t,n}^{dev}$, and $\sum_{i \in I^{RE}} \lambda_{i,d}^{PGC} Q_{i,d}^{PGC}$) of the MPEC's objective function (7.98) need to be restructured as their linearized form by adopting the strong duality theorem and the KKT optimality conditions. The derivation process is detailed as follows.

First, in order to linearize the bilinear term $\sum_{i \in I} \lambda_{d,t,n}^{DA} g_{i,d,t,n,b}^{DA}$, the KKT stationarity condition (7.51) is rearranged as:

$$\lambda_{d,t,n}^{DA} = C_{i,b}^{EN} + \eta_i^{CO_2} \zeta_d + \mu_{i,d,t,n,b}^U - \mu_{i,d,t,n,b}^D \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (7.169)$$

Multiplying both sides by $g_{i,d,t,n,b}^{DA}$ ($\forall i \in I^{FL}$), it renders:

$$\begin{aligned} \lambda_{d,t,n}^{DA} g_{i,d,t,n,b}^{DA} = & C_{i,b}^{EN} g_{i,d,t,n,b}^{DA} + \eta_i^{CO_2} \zeta_d g_{i,d,t,n,b}^{DA} + \mu_{i,d,t,n,b}^U g_{i,d,t,n,b}^{DA} \\ & - \mu_{i,d,t,n,b}^D g_{i,d,t,n,b}^{DA} \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \end{aligned} \quad (7.170)$$

Given that the term (7.170) exclusively accounts for the DA energy revenue derived from flexible generators, the same linearization procedure can be applied to address the revenue from must-run and renewable generators. Therefore, the KKT stationarity condition (7.52) is rewritten as:

$$\lambda_{d,t,n}^{DA} = C_i^{EN} + \gamma_{i,d,t,n} \quad \forall i \in I^{MR}, \forall d, \forall t, \forall n \quad (7.171)$$

Similarly, multiplying both sides by $g_{i,d,t,n}^{DA}$ ($\forall i \in I^{MR}$), it gives:

$$\lambda_{d,t,n}^{DA} g_{i,d,t,n}^{DA} = C_i^{EN} g_{i,d,t,n}^{DA} + \gamma_{i,d,t,n} g_{i,d,t,n}^{DA} \quad \forall i \in I^{MR}, \forall d, \forall t, \forall n \quad (7.172)$$

Subsequently, after the LL1 constraint (7.25) is plugged into (7.172), it renders:

$$\lambda_{d,t,n}^{DA} g_{i,d,t,n}^{DA} = C_i^{EN} g_{i,d,t,n}^{DA} + \gamma_{i,d,t,n} G_{i,n} \quad \forall i \in I^{MR}, \forall d, \forall t, \forall n \quad (7.173)$$

It should be emphasised that in terms of the realisation for renewable power generation, its mean is set as the DA renewable generation dispatch $g_{i,d,t,n}^{DA}$ ($\forall i \in I^{RE}$), and its standard deviation is assumed to be proportional to the DA renewable generation dispatch. The specific setting will be detailed in the section on the case study. As a result, the chance constraint pertaining to renewable power production (7.46) should be rewritten as below:

$$g_{i,d,t,n}^{dev} \geq \Phi^{-1}(1 - \rho^{RE}) \Gamma g_{i,d,t,n}^{DA} \quad \forall i \in I^{RE}, \forall d, \forall t, \forall n \quad (7.174)$$

where Γ identifies the proportion of wind power deviation with respect to DA renewable generation dispatch.

Thus, the KKT stationarity condition (7.53) should be reformulated as follows:

$$-\lambda_{d,t,n}^{DA} + \theta_{i,d,t,n} + \Phi^{-1}(1 - \rho^{RE})\Gamma\chi_{i,d,t,n}^{RE} = 0 \quad \forall i \in I^{RE}, \forall d, \forall t, \forall n \quad (7.175)$$

Then, rearranging the terms in the KKT stationarity condition (7.175) and multiplying both sides by $g_{i,d,t,n}^{DA}$ ($i \in I^{RE}$), the energy revenue from renewable generation is recast as below:

$$\lambda_{d,t,n}^{DA} g_{i,d,t,n}^{DA} = \theta_{i,d,t,n} g_{i,d,t,n}^{DA} + \Phi^{-1}(1 - \rho^{RE})\Gamma\chi_{i,d,t,n}^{RE} g_{i,d,t,n}^{DA} \quad \forall i \in I^{RE}, \forall d, \forall t, \forall n \quad (7.176)$$

By substituting the LL1 constraint (7.26) into the above expression (7.176), it further produces:

$$\lambda_{d,t,n}^{DA} g_{i,d,t,n}^{DA} = \theta_{i,d,t,n} k_{i,d,t}^{RE} G_{i,n} + \Phi^{-1}(1 - \rho^{RE})\Gamma\chi_{i,d,t,n}^{RE} g_{i,d,t,n}^{DA} \quad \forall i \in I^{RE}, \forall d, \forall t, \forall n \quad (7.177)$$

In conclusion, the bilinear DA energy revenue, which is a critical component of the GenCo's profit, can be equivalently expressed by combining the expressions (7.170), (7.173), and (7.177) with respect to different generation technologies.

The next step is to deal with the bilinear terms related to the RT reserve revenues $\sum_{i \in I^{FL}} \lambda_{d,t,n}^{RT} g_{i,d,t,n,b}^U$ and $\sum_{i \in I^{FL}} \lambda_{d,t,n}^{RT} g_{i,d,t,n,b}^D$. To this end, we first rearrange the terms in the KKT stationarity conditions (7.54) and (7.55), and then multiplying both sides by $g_{i,d,t,n,b}^U$ and $g_{i,d,t,n,b}^D$, respectively, it renders:

$$\begin{aligned} \lambda_{d,t,n}^{RT} g_{i,d,t,n,b}^U = & C_{i,b}^U g_{i,d,t,n,b}^U + \eta_i^{CO_2} \zeta_d g_{i,d,t,n,b}^U + \mu_{i,d,t,n,b}^U g_{i,d,t,n,b}^U - \phi_{i,d,t,n,b}^{U,-} g_{i,d,t,n,b}^U + \\ & \phi_{i,d,t,n,b}^{U,+} g_{i,d,t,n,b}^U \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \end{aligned} \quad (7.178)$$

$$\begin{aligned} \lambda_{d,t,n}^{RT} g_{i,d,t,n,b}^D = & C_{i,b}^D g_{i,d,t,n,b}^D + \eta_i^{CO_2} \zeta_d g_{i,d,t,n,b}^D - \mu_{i,d,t,n,b}^D g_{i,d,t,n,b}^D + \phi_{i,d,t,n,b}^{D,-} g_{i,d,t,n,b}^D - \\ & \phi_{i,d,t,n,b}^{D,+} g_{i,d,t,n,b}^D \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \end{aligned} \quad (7.179)$$

Furthermore, by employing the KKT complementarity conditions (7.80) - (7.83), we obtain the following expressions:

$$g_{i,d,t,n,b}^U \phi_{i,d,t,n,b}^{U,-} = 0 \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (7.180)$$

$$G_{i,b}^U \phi_{i,d,t,n,b}^{U,+} = g_{i,d,t,n,b}^U \phi_{i,d,t,n,b}^{U,+} \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (7.181)$$

$$g_{i,d,t,n,b}^D \phi_{i,d,t,n,b}^{D,-} = 0 \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (7.182)$$

$$G_{i,b}^D \phi_{i,d,t,n,b}^{D,+} = g_{i,d,t,n,b}^D \phi_{i,d,t,n,b}^{D,+} \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (7.183)$$

By substituting KKT complementarity conditions (7.180)-(7.183) into the equations for RT upward and downward reserve revenues, denoted as (7.178) and (7.179) respectively, it gives:

$$\lambda_{d,t,n}^{RT} g_{i,d,t,n,b}^U = C_{i,b}^U g_{i,d,t,n,b}^U + \eta_i^{CO_2} \zeta_d g_{i,d,t,n,b}^U + \mu_{i,d,t,n,b}^U g_{i,d,t,n,b}^U + \phi_{i,d,t,n,b}^{U,+} G_{i,b}^U \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (7.184)$$

$$\lambda_{d,t,n}^{RT} g_{i,d,t,n,b}^D = C_{i,b}^D g_{i,d,t,n,b}^D + \eta_i^{CO_2} \zeta_d g_{i,d,t,n,b}^D - \mu_{i,d,t,n,b}^D g_{i,d,t,n,b}^D - \phi_{i,d,t,n,b}^{D,+} G_{i,b}^D \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (7.185)$$

After this, the bilinear term of the renewable deviation penalty $\sum_{i \in I^{RE}} \lambda_{d,t,n}^{RT} g_{i,d,t,n}^{dev}$ should be addressed. To this end, we employ the KKT complementarity condition (7.96) related to the uncertainty of renewable power production and reformulate it as follows:

$$[-g_{i,d,t,n}^{dev} - \Phi^{-1}(1 - \rho^{RE}) \Gamma g_{i,d,t,n}^{DA}] \chi_{i,d,t,n}^{RE} = 0 \quad \forall i \in I^{RE}, \forall d, \forall t, \forall n \quad (7.186)$$

By combining the KKT stationarity condition (7.57) with the complementary expression (7.186), the bilinear term $\lambda_{d,t,n}^{RT} g_{i,d,t,n}^{dev}$ is recast as follows:

$$\lambda_{d,t,n}^{RT} g_{i,d,t,n}^{dev} = \chi_{i,d,t,n}^{RE} g_{i,d,t,n}^{dev} = -\Phi^{-1}(1 - \rho^{RE}) \Gamma g_{i,d,t,n}^{DA} \chi_{i,d,t,n}^{RE} \quad \forall i \in I^{RE}, \forall d, \forall t, \forall n \quad (7.187)$$

Collectively, an alternative representation for the first four bilinear terms in the MPEC's objective function (the left side of the following equation) can be derived by integrating the expressions of (7.170), (7.173), (7.177), (7.184), (7.185), (7.187), and summing over their respective indices, which is formulated as follows:

$$\begin{aligned} & \sum_{i \in I, d, t, n, b} \lambda_{d,t,n}^{DA} g_{i,d,t,n,b}^{DA} + \sum_{i \in I^{FL}, d, t, n, b} \lambda_{d,t,n}^{RT} (g_{i,d,t,n,b}^U - g_{i,d,t,n,b}^D) - \sum_{i \in I^{RE}, d, t, n} \lambda_{d,t,n}^{RT} g_{i,d,t,n}^{dev} = \\ & \sum_{i \in I, d, t, n, b} C_{i,b}^{EN} g_{i,d,t,n,b}^{DA} + \sum_{i \in I^{FL}, d, t, n, b} C_{i,b}^U g_{i,d,t,n,b}^U - \sum_{i \in I^{FL}, d, t, n, b} C_{i,b}^D g_{i,d,t,n,b}^D + \\ & \sum_{i \in I^{FL}, d, t, n, b} \eta_i^{CO_2} \zeta_d (g_{i,d,t,n,b}^{DA} + g_{i,d,t,n,b}^U - g_{i,d,t,n,b}^D) + \sum_{i \in I^{FL}, d, t, n, b} \phi_{i,d,t,n,b}^{U,+} G_{i,b}^U + \\ & \sum_{i \in I^{FL}, d, t, n, b} \phi_{i,d,t,n,b}^{D,+} G_{i,b}^D + \sum_{i \in I^{FL}, d, t, n, b} \mu_{i,d,t,n,b}^D (g_{i,d,t,n,b}^D - g_{i,d,t,n,b}^{DA}) + \\ & \sum_{i \in I^{FL}, d, t, n, b} \mu_{i,d,t,n,b}^U (g_{i,d,t,n,b}^{DA} + g_{i,d,t,n,b}^U) + \sum_{i \in I^{MR}, d, t, n} \gamma_{i,d,t,n} G_{i,n} + \sum_{i \in I^{RE}, d, t, n} \theta_{i,d,t,n} k_{i,d,t}^{RE} G_{i,n} \end{aligned} \quad (7.188)$$

It is observable that some nonlinear terms on the right side of equation (7.188) can be eliminated through the application of the KKT complementarity conditions (7.77), (7.78) and (7.79), which are expressed as below:

$$\zeta_d \eta^{Grid} \sum_{j,t,o} L_{j,d,t,o}^{DA} = \zeta_d \sum_{i \in I^{FL},d,t,n,b} \eta_i^{CO_2} (g_{i,d,t,n,b}^{DA} + g_{i,d,t,n,b}^U - g_{i,d,t,n,b}^D) \quad \forall d \quad (7.189)$$

$$\mu_{i,d,t,n,b}^U G_{i,n,b} = \mu_{i,d,t,n,b}^U (g_{i,d,t,n,b}^{DA} + g_{i,d,t,n,b}^U) \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (7.190)$$

$$\mu_{i,d,t,n,b}^D (g_{i,d,t,n,b}^{DA} - g_{i,d,t,n,b}^D) = 0 \quad \forall i \in I^{FL}, \forall d, \forall t, \forall n, \forall b \quad (7.191)$$

By substituting the above KKT complementarity conditions (7.189)-(7.191) into the equation (7.188), the representation for the first four bilinear terms in the MPEC's objective function can be partially linearized, which is given as follows:

$$\begin{aligned} & \sum_{i \in I,d,t,n,b} \lambda_{d,t,n}^{DA} g_{i,d,t,n,b}^{DA} + \sum_{i \in I^{FL},d,t,n,b} \lambda_{d,t,n}^{RT} (g_{i,d,t,n,b}^U - g_{i,d,t,n,b}^D) - \sum_{i \in I^{RE},d,t,n} \lambda_{d,t,n}^{RT} g_{i,d,t,n}^{dev} = \\ & \sum_{i \in I,d,t,n,b} C_{i,b}^{EN} g_{i,d,t,n,b}^{DA} + \sum_{i \in I^{FL},d,t,n,b} C_{i,b}^U g_{i,d,t,n,b}^U - \sum_{i \in I^{FL},d,t,n,b} C_{i,b}^D g_{i,d,t,n,b}^D + \\ & \sum_{j,d,t,o} \eta^{Grid} \zeta_d L_{j,d,t,o}^{DA} + \sum_{i \in I^{FL},d,t,n,b} \phi_{i,d,t,n,b}^{U,+} G_{i,b}^U + \sum_{i \in I^{FL},d,t,n,b} \phi_{i,d,t,n,b}^{D,+} G_{i,b}^D + \\ & \sum_{i \in I^{FL},d,t,n,b} \mu_{i,d,t,n,b}^U G_{i,n,b} + \sum_{i \in I^{MR},d,t,n} \gamma_{i,d,t,n} G_{i,n} + \sum_{i \in I^{RE},d,t,n} \theta_{i,d,t,n} k_{i,d,t}^{RE} G_{i,n} \end{aligned} \quad (7.192)$$

Nonetheless, it is noticeable that the presence of the nonlinearities posed by the last three bilinear terms of equation (7.192) needs to be further addressed. In order to remove these non-linearities, with the joint inclusion of the demand flexibility and the ES model into the LL1 problem, the strong duality theorem of the LL1 problem is expressed as follows:

$$\begin{aligned} & \sum_{i,d,t,n,b} C_{i,b}^{EN} g_{i,d,t,n,b}^{DA} + \sum_{i,d,t,n,b} (C_{i,b}^U g_{i,d,t,n,b}^U - C_{i,b}^D g_{i,d,t,n,b}^D) = - \sum_{e,d} E_e^0 (\tau_{e,d}^{ES,1} - \tau_{e,d}^{ES,T}) \\ & - \sum_{e,d,t} E_e^{max} \tau_{e,d,t}^{ES,+} + \sum_{e,d,t} E_e^{min} \tau_{e,d,t}^{ES,-} - \sum_{e,d,t} s_e^{max} \tau_{e,d,t}^{c,DA,+} - \sum_{e,d,t} s_e^{max} \tau_{e,d,t}^{c,RT,+} - \\ & \sum_{e,d,t} s_e^{max} \tau_{e,d,t}^{d,DA,+} - \sum_{e,d,t} s_e^{max} \tau_{e,d,t}^{d,RT,+} - \sum_{j,d,t,o} \eta^{Grid} \zeta_d L_{j,d,t,o}^{DA} - \sum_{i \in I^{FL},d,t,n,b} \mu_{i,d,t,n,b}^U G_{i,n,b} \\ & - \sum_{i \in I^{FL},d,t,n,b} \phi_{i,d,t,n,b}^{U,+} G_{i,b}^U - \sum_{i \in I^{FL},d,t,n,b} \phi_{i,d,t,n,b}^{D,+} G_{i,b}^D - \sum_{i \in I^{MR},d,t,n} \gamma_{i,d,t,n} G_{i,n} - \\ & \sum_{i \in I^{RE},d,t,n} \theta_{i,d,t,n} k_{i,d,t}^{RE} G_{i,n} - \sum_{d,t,n,m \in \Lambda_n} C_{nm}^{max} (\nu_{d,t,n,m}^{DA,-} + \nu_{d,t,n,m}^{DA,+}) - \sum_{d,t,n} \pi (\xi_{d,t,n}^{DA,-} + \xi_{d,t,n}^{DA,+}) \end{aligned}$$

$$\begin{aligned}
& - \sum_{d,t,n,m \in \Lambda_n} C_{nm}^{max} (\nu_{d,t,n,m}^{RT,-} + \nu_{d,t,n,m}^{RT,+}) - \sum_{d,t,n} \pi (\xi_{d,t,n}^{RT,-} + \xi_{d,t,n}^{RT,+}) \\
& - \sum_{j,d,t,o} (1 + \alpha) L_{j,d,t,o}^{DA} \psi_{j,d,t,o}^+ + \sum_{j,d,t,o} (1 - \alpha) L_{j,d,t,o}^{DA} \psi_{j,d,t,o}^- + \sum_{j,d,t} \alpha [\Phi^{-1}(1 - \rho^L) s_{j,d,t}^L - \\
& m_{j,d,t}^L] (\chi_{j,d,t}^{SH,-} + \chi_{j,d,t}^{SH,+}) + \sum_{j,d,t} [\Phi^{-1}(1 - \rho^L) s_{j,d,t}^L + m_{j,d,t}^L] \chi_{j,d,t}^L
\end{aligned} \tag{7.193}$$

By applying the strong duality equation (7.193), the bilinear function formulated on the left side of equation (7.194) can be ultimately linearized as follows:

$$\begin{aligned}
& \sum_{i \in I, d, t, n, b} \lambda_{d,t,n}^{DA} g_{i,d,t,n,b}^{DA} + \sum_{i \in I^{FL}, d, t, n, b} \lambda_{d,t,n}^{RT} (g_{i,d,t,n,b}^U - g_{i,d,t,n,b}^D) - \sum_{i \in I^{RE}, d, t, n} \lambda_{d,t,n}^{RT} g_{i,d,t,n}^{dev} = \\
& - \sum_{e,d} E_e^0 (\tau_{e,d}^{ES,1} - \tau_{e,d}^{ES,T}) - \sum_{e,d,t} E_e^{max} \tau_{e,d,t}^{ES,+} + \sum_{e,d,t} E_e^{min} \tau_{e,d,t}^{ES,-} - \sum_{e,d,t} s_e^{max} \tau_{e,d,t}^{c,DA,+} \\
& - \sum_{e,d,t} s_e^{max} \tau_{e,d,t}^{c,RT,+} - \sum_{e,d,t} s_e^{max} \tau_{e,d,t}^{d,DA,+} - \sum_{e,d,t} s_e^{max} \tau_{e,d,t}^{d,RT,+} \\
& - \sum_{d,t,n,m \in \Lambda_n} C_{nm}^{max} (\nu_{d,t,n,m}^{DA,-} + \nu_{d,t,n,m}^{DA,+}) - \sum_{d,t,n,m \in \Lambda_n} C_{nm}^{max} (\nu_{d,t,n,m}^{RT,-} + \nu_{d,t,n,m}^{RT,+}) \\
& - \sum_{d,t,n} \pi (\xi_{d,t,n}^{DA,-} + \xi_{d,t,n}^{DA,+}) - \sum_{d,t,n} \pi (\xi_{d,t,n}^{RT,-} + \xi_{d,t,n}^{RT,+}) - \sum_{j,d,t,o} (1 + \alpha) L_{j,d,t,o}^{DA} \psi_{j,d,t,o}^+ \\
& + \sum_{j,d,t,o} (1 - \alpha) L_{j,d,t,o}^{DA} \psi_{j,d,t,o}^- + \sum_{j,d,t} \alpha [\Phi^{-1}(1 - \rho^L) s_{j,d,t}^L - m_{j,d,t}^L] (\chi_{j,d,t}^{SH,-} + \chi_{j,d,t}^{SH,+}) \\
& + \sum_{j,d,t} [\Phi^{-1}(1 - \rho^L) s_{j,d,t}^L + m_{j,d,t}^L] \chi_{j,d,t}^L
\end{aligned} \tag{7.194}$$

The final step is to handle the last bilinear term related to the TGC revenue $\sum_{i \in I^{RE}} \lambda_{i,d}^{PGC} Q_{i,d}^{PGC}$. It can be converted into a quadratic term by adopting LL2 constraint (7.39), which is presented as follows:

$$\sum_{i \in I^{RE}, d} \lambda_{i,d}^{PGC} Q_{i,d}^{PGC} = \sum_{i \in I^{RE}, d} [l^{PGC} Q_{i,d}^{PGC} - q^{PGC} (Q_{i,d}^{PGC})^2] \tag{7.195}$$

The final step is the substitution of expressions from (7.194) and (7.195) into the bilinear MPEC's objective function (7.98). Finally, the GenCo's profit is replaced by the following linear expression:

$$\begin{aligned}
& \sum_d \omega_d \left\{ - \sum_{i,t,n,b} C_{i,b}^{EN} g_{i,d,t,n,b}^{DA} - \sum_{i,t,n,b} (C_{i,b}^U g_{i,d,t,n,b}^U - C_{i,b}^D g_{i,d,t,n,b}^D) - \sum_e E_e^0 (\tau_{e,d}^{ES,1} - \tau_{e,d}^{ES,T}) \right. \\
& \left. - \sum_{e,t} E_e^{max} \tau_{e,d,t}^{ES,+} + \sum_{e,t} E_e^{min} \tau_{e,d,t}^{ES,-} - \sum_{e,t} s_e^{max} \tau_{e,d,t}^{c,DA,+} - \sum_{e,t} s_e^{max} \tau_{e,d,t}^{c,RT,+} - \right.
\end{aligned}$$

$$\begin{aligned}
& \sum_{e,t} s_e^{max} \tau_{e,d,t}^{d,DA,+} - \sum_{e,t} s_e^{max} \tau_{e,d,t}^{d,RT,+} - \sum_{t,n,m \in \Lambda_n} C_{nm}^{max} (\nu_{d,t,n,m}^{DA,-} + \nu_{d,t,n,m}^{DA,+}) - \\
& \sum_{t,n} \pi(\xi_{d,t,n}^{DA,-} + \xi_{d,t,n}^{DA,+}) - \sum_{t,n,m \in \Lambda_n} C_{nm}^{max} (\nu_{d,t,n,m}^{RT,-} + \nu_{d,t,n,m}^{RT,+}) - \sum_{t,n} \pi(\xi_{d,t,n}^{RT,-} + \xi_{d,t,n}^{RT,+}) \\
& - \sum_{j,t,o} (1 + \alpha) L_{j,d,t,o}^{DA} \psi_{j,d,t,o}^+ + \sum_{j,t,o} (1 - \alpha) L_{j,d,t,o}^{DA} \psi_{j,d,t,o}^- \\
& + \sum_{j,t} \alpha [\Phi^{-1}(1 - \rho^L) s_{j,d,t}^L - m_{j,d,t}^L] (\chi_{j,d,t}^{SH,-} + \chi_{j,d,t}^{SH,+}) \\
& + \sum_{j,t} [\Phi^{-1}(1 - \rho^L) s_{j,d,t}^L + m_{j,d,t}^L] \chi_{j,d,t}^L - \sum_{i \in I^{FL}, t, n} \lambda^{CO_2} Q_{i,d,t,n}^{emi} + \sum_{i \in I^{RE}, t, n} \lambda^{CO_2} Q_{i,d,t,n}^{CA} \\
& + \sum_{i \in I^{RE}} \left[l^{PGC} Q_{i,d}^{PGC} - q^{PGC} (Q_{i,d}^{PGC})^2 + \lambda^{FGC} Q_{i,d}^{FGC} \right] \Big\} - \sum_{i,n} C_i^{IN} G_{i,n} \quad (7.196)
\end{aligned}$$

7.4.4 Final MIQP

With the aid of linearization techniques, the MPEC is effectively converted into a MIQP, which facilitates an efficient resolution. The objective function of the MIQP (7.197) represents the GenCo's profit in a linear form. The MIQP's variable set (7.198) is a combination of MPEC's variable set (7.106) and auxiliary binary variable set for linearizing LL1 KKT complementarity conditions (7.106). The MIQP's constraints are composed of three aspects: i) the original UL constraints (7.199), ii) the linearized KKT optimality conditions for the LL1 problem (7.200), and iii) the LL2 problem's constraints (7.201). The comprehensive formulation of the final MIQP is provided as follows:

$$\begin{aligned}
\min_{V^{MIQP}} \sum_d \omega_d \Big\{ & - \sum_{i,t,n,b} C_{i,b}^{EN} g_{i,d,t,n,b}^{DA} - \sum_{i,t,n,b} (C_{i,b}^U g_{i,d,t,n,b}^U - C_{i,b}^D g_{i,d,t,n,b}^D) - \\
& \sum_e E_e^0 (\tau_{e,d}^{ES,1} - \tau_{e,d}^{ES,T}) - \sum_{e,t} E_e^{max} \tau_{e,d,t}^{ES,+} + \sum_{e,t} E_e^{min} \tau_{e,d,t}^{ES,-} - \sum_{e,t} s_e^{max} \tau_{e,d,t}^{c,DA,+} \\
& - \sum_{e,t} s_e^{max} \tau_{e,d,t}^{c,RT,+} - \sum_{e,t} s_e^{max} \tau_{e,d,t}^{d,DA,+} - \sum_{e,t} s_e^{max} \tau_{e,d,t}^{d,RT,+} \\
& - \sum_{t,n,m \in \Lambda_n} C_{nm}^{max} (\nu_{d,t,n,m}^{DA,-} + \nu_{d,t,n,m}^{DA,+}) - \sum_{t,n} \pi(\xi_{d,t,n}^{DA,-} + \xi_{d,t,n}^{DA,+}) \\
& - \sum_{t,n,m \in \Lambda_n} C_{nm}^{max} (\nu_{d,t,n,m}^{RT,-} + \nu_{d,t,n,m}^{RT,+}) - \sum_{t,n} \pi(\xi_{d,t,n}^{RT,-} + \xi_{d,t,n}^{RT,+}) \\
& - \sum_{j,t,o} (1 + \alpha) L_{j,d,t,o}^{DA} \psi_{j,d,t,o}^+ + \sum_{j,t,o} (1 - \alpha) L_{j,d,t,o}^{DA} \psi_{j,d,t,o}^- \\
& + \sum_{j,t} \alpha [\Phi^{-1}(1 - \rho^L) s_{j,d,t}^L - m_{j,d,t}^L] (\chi_{j,d,t}^{SH,-} + \chi_{j,d,t}^{SH,+}) \\
& + \sum_{j,t} [\Phi^{-1}(1 - \rho^L) s_{j,d,t}^L + m_{j,d,t}^L] \chi_{j,d,t}^L - \sum_{i \in I^{FL}, t, n} \lambda^{CO_2} Q_{i,d,t,n}^{emi} +
\end{aligned}$$

$$\begin{aligned}
& \sum_{i \in I^{RE}, t, n} \lambda^{CO_2} Q_{i,d,t,n}^{CA} + \sum_{i \in I^{RE}} \left[l^{PGC} Q_{i,d}^{PGC} - q^{PGC} (Q_{i,d}^{PGC})^2 + \lambda^{FGC} Q_{i,d}^{FGC} \right] \Big\} \\
& - \sum_{i,n} C_i^{IN} G_{i,n}
\end{aligned} \tag{7.197}$$

where:

$$V^{MIQP} = V^{MPEC} \cup V^{BINARY} \tag{7.198}$$

subject to:

$$\text{UL constraints : (7.3) – (7.7)} \tag{7.199}$$

LL1 KKT conditions :

$$(7.10) – (7.14), (7.25) – (7.26), (7.29), (7.32), (7.34), (7.50) – (7.66), (7.107) – (7.168) \tag{7.200}$$

$$\text{LL2 constraints : (7.39) – (7.40)} \tag{7.201}$$

7.5 Case Studies

7.5.1 Data Descriptions and Implementation

The series of case studies in this section is designed to capture the transformative potential of demand flexibility and utility-scale ES for restructuring the GB's energy landscape to meet the full-decarbonisation target within the 2035 GB deregulated market environment. Consequently, the necessary data is provided below.

Table 7.1 presents comprehensive data about investment options for the GenCo. It is noteworthy that in terms of flexible generation technologies, the marginal cost for providing upward reserve is set at 1.1 times the marginal energy cost; conversely, the marginal cost for providing downward reserve is set at 0.9 times the marginal energy cost.

Predicted data reflecting the GB power system in 2035 is shown in Table 7.2 [81]. It should be noted that based on the 2021 GB system demand profile, a projection for the future 2035 GB demand profile can be established based on the 2035 total system data. Moreover, the predicted value for demand flexibility in the 2035 GB power system is set to be 25%, sourced from the authoritative report of NG FES, titled 'A Day in the Life 2035' [88]. Additionally, the allocation of weights to these blocks is set as [0.1, 0.15, 0.2, 0.25, 0.3], and the demand is also accurately modelled by the five demand blocks, incorporating weighting factors as [0.6, 0.1, 0.1, 0.1, 0.1]. By virtue of computational burden, the demand profile is simulated on a representative day with an hourly resolution

Table 7.1: Parameters of investment options.

Investment option		CCGT1	CCGT2	OCGT1	OCGT2	Nuclear	Wind
Type		Flexible	Flexible	Flexible	Flexible	Must-run	Renewable
Investment cost C^{IN} (£/MW/yr)		14413	17440	8610	10418	114849	35756
Carbon intensity η^{CO_2} (t/MWh)		0.39	0.39	0.65	0.65	0	0
Maximum upward reserve capacity G^U (MW)		4000	8000	4000	8000	-	-
Maximum downward reserve capacity G^D (MW)		4000	8000	4000	8000	-	-
Marginal energy cost C^{EN} (£/MWh)	Block 1	7.43	20.05	40.99	49.25	4.72	0
	Block 2	9.03	24.34	49.78	59.81		
	Block 3	10.62	28.64	58.56	70.36		
	Block 4	12.21	32.94	67.34	80.91		
	Block 5	13.81	37.23	76.13	91.47		

($T = \{1, 2, \dots, 24\}$), scaled across a target year ($\omega = 365$) to capture daily fluctuations in energy consumption and production patterns.

Table 7.2: Parameters of the GB power grid in 2035.

Year	2035
System-wide carbon intensity η^{Grid} (t/MWh)	0.00
Carbon price λ^{CO_2} (£/t)	151.02
Total system demand (TWh)	430.61
Energy capacity E^{max} (MWh)	7665.47
Power capacity s^{max} (MW)	6682.71
Minimum energy level E^{min} (MWh)	$0.2E^{max}$
Initial energy level E^0 (MWh)	$0.2E^{max}$
Charging efficiency η^c (%)	90
Discharging efficiency η^d (%)	90

The additional parameters associated with carbon allowance and TGC are essential. Specifically, the carbon allowance allocation ratio, ϵ_i , is set as 0.4; the RPS requirement ratio, γ^{RPS} , is set as 40%; the approximated price ratio in the future TGC market, γ^{FGC} , is set as 0.3; and the price coefficient in the present TGC market, θ^{PGC} , is also set as 0.3. It should be noted that the penalty price for RPS deficiency, λ_0^{RPS} , is assumed to be 1.1 times the carbon price.

The related coefficients used in the chance constraints should be given. Overall, the model sets confidence levels for all the chance constraints at 95% ($1 - \rho^L = 1 - \rho^{RE} = 95\%$), conveying the ρ -reliable threshold for both demand and wind power production uncertainty. The linearized CCP formulations incorporate stochastic variables for wind power and demand realisations, which are modelled using two normal distributions, $\mathcal{N}(m^{RE}, (s^{RE})^2)$ for wind power and $\mathcal{N}(m^L, (s^L)^2)$ for demand. The means of these distributions (m^L and m^{RE}) are aligned with the baseline demand and DA power dispatch,

respectively. Meanwhile, the standard deviations (s^L and s^{RE}), set at 1% of the baseline demand and 10% of the DA wind power dispatch, respectively, quantify the anticipated variability around these means.

The case studies for reaching a fully decarbonised GB power system by 2035 further focus on the crucial role of two flexibility resources: ES and demand flexibility. Specifically, these cases aim to assess how these flexibility resources can aid in reforming GB's generation mix by integrating more RESs. To verify the feasibility of navigating towards a future zero-carbon power grid, characterised by a substantial share of RESs, three distinct cases incorporating varying sizes of ES and degrees of demand flexibility are designed based on the predicted data for the 2035 GB power system, which are detailed as follows:

1. **Zero flexibility resources (Z-FR) case:** There is a complete lack of demand flexibility and ES installation in the 2035 GB power system built in this case.
2. **Baseline of flexibility resources (B-FR) case:** The share of demand flexibility in the 2035 GB power system is 25%. Additionally, the utility-scale ES is installed with an energy capacity of 7665.47 MWh and a power capacity of 6682.71 MW. All the data utilised in this case is from the NG FES.
3. **High level of flexibility resources (H-FR) case:** The proportion of demand flexibility is increased to 50%. Additionally, the size of the utility-scale ES is expanded to 15330.94 MWh for energy capacity and 13365.42 MW for power capacity. All the data utilised in this case is double the predicted data provided by the NG FES.

7.5.2 Impact of Flexibility Resources

The comprehensive investment decisions and business results corresponding to the three flexibility resource scenarios are included in Table 6.4. Furthermore, the charging and discharging power outputs for the two flexibility resource scenarios across the DA and RT markets are illustrated in Figures 7.2 and 7.3, respectively. The system demand profiles for the two flexibility resource scenarios are illustrated in Figure 7.4.

It is important to note that the proposed MIQP model was unable to find a feasible solution in the Z-FR case, which lacks any flexibility resources. This result is not unexpected, given that the carbon policies for the 2035 GB power system are significantly more stringent, targeting complete zero emissions, as compared to those in place for the 2021 GB power system. This reveals the fact that, in contrast to the situation in 2021, the 2035 GB power system will require substantially more flexibility resources to manage the extensive system imbalances introduced by massive RESs.

Table 7.3: Investment decisions and business results for three flexibility resource scenarios regarding the 2035 GB carbon target.

	Z-FR case	B-FR case	H-FR case
CCGT1 (MW)	Infeasible	-	-
CCGT2 (MW)		-	-
OCGT1 (MW)		-	-
OCGT2 (MW)		-	-
Nuclear (MW)		39,782	0
Wind (MW)		40,404	198,811
Investment cost (mil.£)		5,812	6,115
DA energy cost (mil.£)		1,645	0
DA energy revenue (mil.£)		0	0
RT reserve cost (mil.£)		0	0
RT reserve revenue (mil.£)		0	0
Wind power deviation penalty (mil.£)		0	0
Carbon emission cost (mil.£)		0	0
Carbon allowance revenue (mil.£)		0	0
Green certificate revenue (mil.£)		8,761	25,263
Profit (mil.£)		1,304	19,148
DA ES revenue (mil.£)		0	0
RT ES revenue (mil.£)		0	0
Total ES revenue (mil.£)		0	0

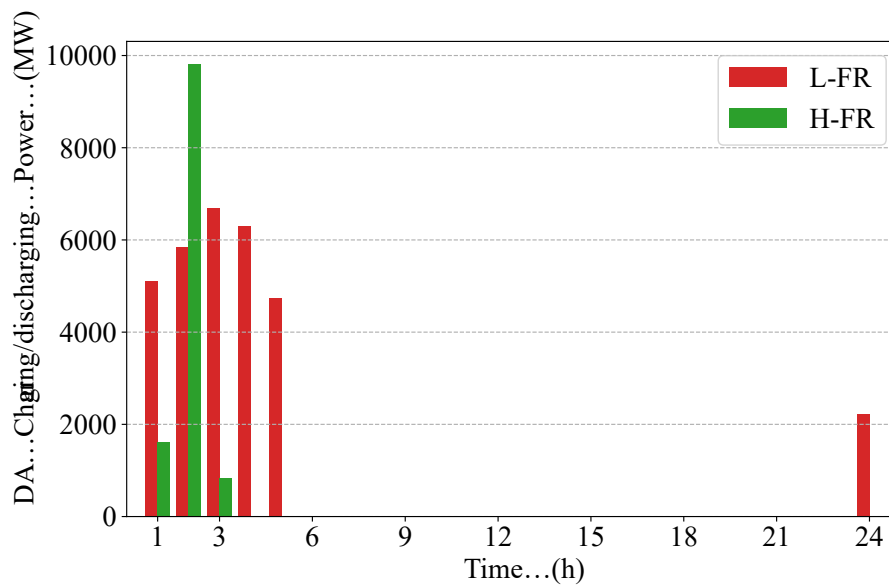


Figure 7.2: Daily charging/discharging power output in the DA energy market for two flexibility resource scenarios regarding the 2035 GB carbon target.

B-FR Case Analysis

The second to seventh row in the third column of Table 7.3 illustrates that, in response to the GB government's entire decarbonisation policies, the optimal generation invest-

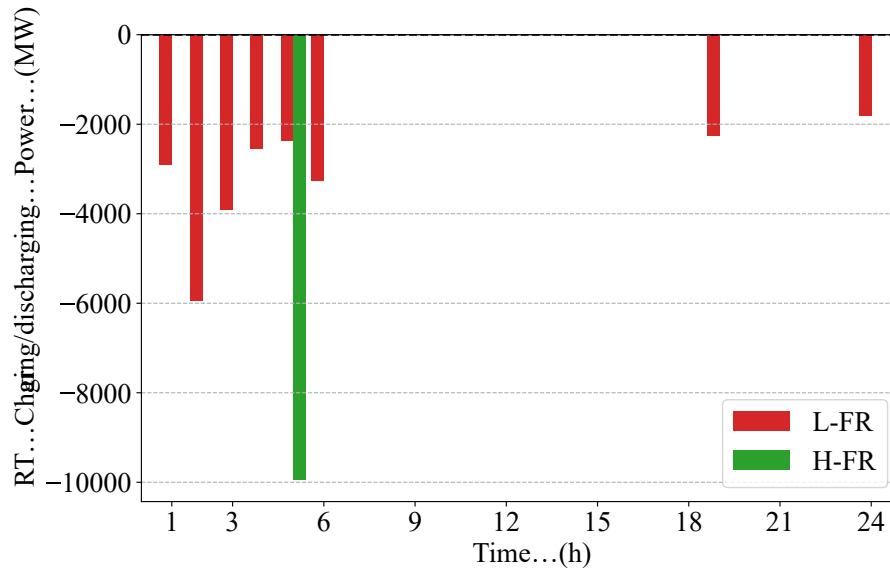


Figure 7.3: Daily charging/discharging power output in the RT balancing market for two flexibility resource scenarios regarding the 2035 GB carbon target.

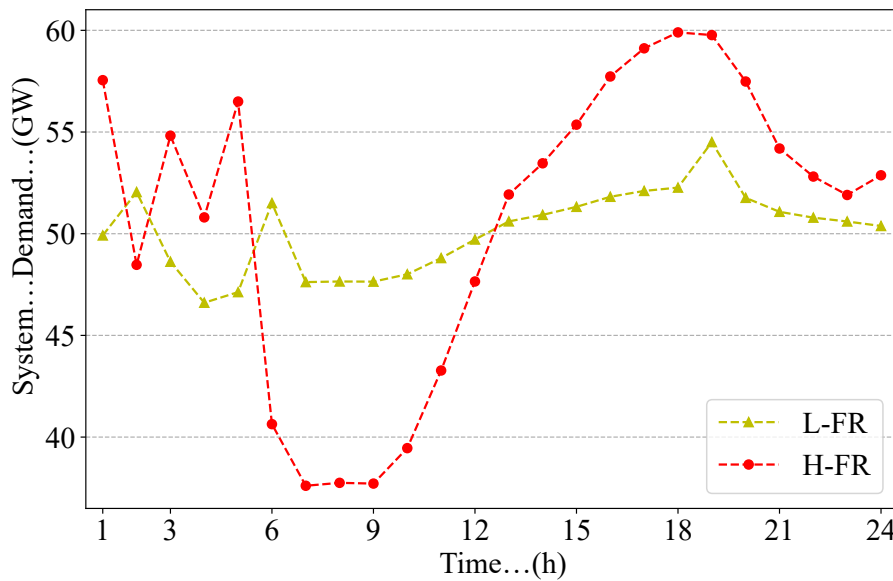


Figure 7.4: Daily system demand for two flexibility resource scenarios regarding the 2035 GB carbon target.

ment mix in the B-FR case is exclusively comprised of carbon-free nuclear and wind power capacity. Although this investment strategy satisfied the 2035 GB carbon target, an overdependence on nuclear power raises concerns regarding the ideal green energy future. Specifically, nuclear energy, despite its advantage of zero carbon emissions, is associated with the production of hazardous radioactive waste that poses significant management challenges [89]. The historical nuclear accidents at Chernobyl in 1986 and Fukushima in 2011 highlight the severe catastrophic risks inherent to the nuclear industry [90]. Con-

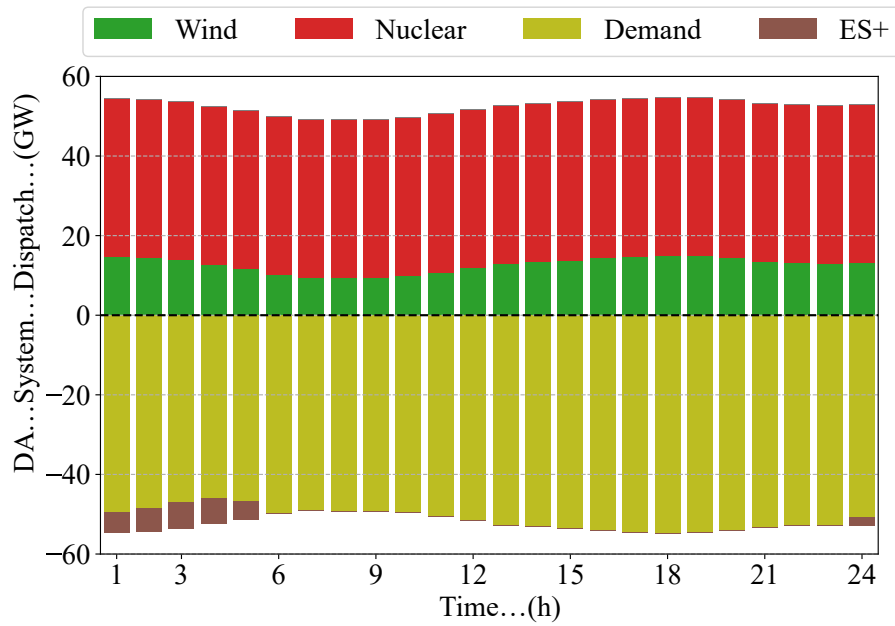


Figure 7.5: Daily generation dispatches in the DA energy market for the B-FR case regarding the 2035 GB carbon target.

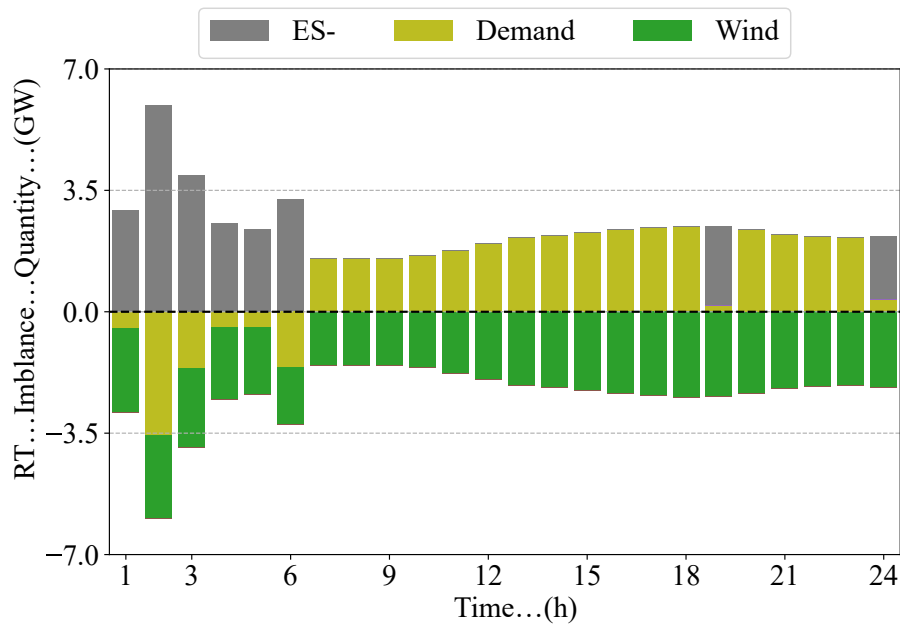


Figure 7.6: Daily imbalances in the RT balancing market for the B-FR case regarding the 2035 GB carbon target.

sequently, a truly green and sustainable power system is expected to be predominantly supported by RESs, such as wind power, which offer cleaner and safer alternatives to nuclear energy.

In the B-FR case, nuclear generation capacity constitutes nearly half of the total in-

vestment decisions. This phenomenon is attributable to two primary factors:

1. The ambitious zero-carbon target for the 2035 GB power system restricts the GenCo from investing in high-emission flexible generation capacity, which typically offers necessary reserves in the RT market to offset the uncertainty that arises from wind power generation. As a result, the only feasible alternative that aligns with the zero-emission criterion and lacks unpredictability, nuclear power, becomes a preferred option to minimise the need for auxiliary reserves.
2. The level of demand flexibility available in the 2035 GB power system is inadequate to significantly alter the demand profile to better align with the variability of wind energy. Furthermore, this limited demand flexibility cannot provide cost-free balancing resources through demand deviation to mitigate the significant uncertainty inherent in a power grid with high renewable penetration.
3. The capacity of utility-scale ES predicted for the 2035 GB power grid is not enough to offer the necessary operational flexibility for a system heavily reliant on renewable energy. Specifically, the lack of ES capacity impedes the ability to store surplus power generation during off-peak periods in the DA market and to address substantial uncertainties introduced by wind power generation and demand in the RT market, thus constraining the potential for wind generation expansion within the grid.

As shown in Figures 7.5 and 7.6, ES plays a pivotal role in managing grid stability. In detail, during low-demand periods (hours 1–5) in the DA market, ES is utilised to store surplus generation from nuclear and wind power generation. Then, this stored energy is strategically released in the RT market during periods of high wind generation (hours 1–6) to address imbalances caused by renewable energy and demand deviations. The operational dynamics of ES across the DA and RT markets are meticulously illustrated in Figures 7.2 and 7.3, respectively. Given the substantial must-run nuclear generation dispatches in the DA scheduling horizon, demand flexibility is employed to smooth the system demand as much as possible to match their generation patterns, as depicted in Figure 7.4. It is noteworthy that across all cases related to the 2035 GB power system, the market clearing prices are not depicted due to the lack of flexible generation capacity serving as the marginal unit, thereby leading to both DA and RT market clearing prices dropping to zero. Furthermore, it is important to highlight that while nuclear power incurs a non-zero marginal cost, its operational manner of running at full capacity excludes it from functioning as the marginal unit to determine the price in the market.

The business results presented for the B-FR case reveal the GenCo's relatively modest profitability, as outlined from the eighth to seventeenth row in the third column of

Table 7.3. To uncover the implicit logic behind this, it is crucial to analyse the trends in the various components that influence profitability. In contrast to cases related to the 2021 GB power system presented in Chapters 5 and 6, the investment cost in the B-FR case is remarkably higher. This rise in investment costs can be attributed to two primary factors. Firstly, the predicted level of system demand for 2035 significantly exceeds that of 2021, necessitating a corresponding increase in generation capacity to satisfy the increased demand. Secondly, the stringent zero-carbon target for 2035 forces the GenCo to invest in emission-free generation technologies, such as nuclear and wind power. Although these technologies align with long-term sustainability objectives, their investment costs are notably higher than those of conventional flexible generation technologies. The low DA energy cost is primarily due to the DA generation dispatches provided by cheap nuclear power and free wind power. The RT reserve cost is eliminated entirely because the balancing resources necessary for addressing imbalances are supplied at no cost by ES and demand deviations. Revenues in both the DA and RT markets drop to zero owing to zero market clearing prices. Similarly, the wind power deviation penalty is rendered null due to its dependency on RT market clearing prices, which, in this case, have fallen to zero. Given the complete decarbonisation of the 2035 GB grid, carbon emission costs are naturally zero. Carbon allowance revenue dips to zero due to its reliance on system-wide carbon intensity, which achieves a value of zero in the entirely decarbonised GB grid of 2035. TGC revenue emerges as a primary revenue stream for the GenCo, underpinned by renewable promotion policies. Overall, despite the GenCo earning significant revenue through TGC trading, the negligible prices in both the DA and RT markets cause revenues from these markets to fall to zero. This situation leads to a markedly low profit for GenCo, thereby presenting a challenge of 'revenue inadequacy'. Additionally, the profitability of ES entities also declines to zero, directly attributed to the zero market clearing prices, as indicated in the last three rows in the third column of Table 7.3.

H-FR Case Analysis

In the H-FR case, as shown in the second to seventh row in the fourth column of Table 7.3, the investment portfolio evolves to consist entirely of wind generation capacity, achieving a 100% renewable generation mix. This development aligns with expectations, primarily attributed to the substantial scale of flexibility resources. These extensive flexibility resources, encompassing both ES and demand flexibility, provide the necessary operational flexibility to maintain a stable and reliable 100% renewable energy system.

In the DA market, compared to the nuclear-dominated generation mix in the B-FR case, generation dispatch transitions to being entirely fulfilled by wind capacity. This shift, illustrated in Figure 7.7, represents a significant move towards a pure renewable

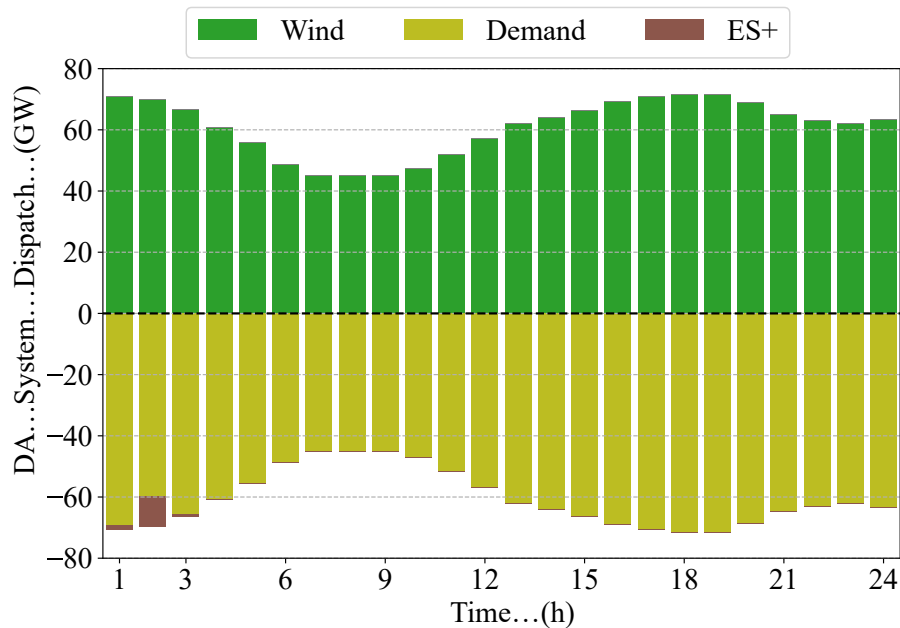


Figure 7.7: Daily generation dispatches in the DA energy market for the H-FR case regarding the 2035 GB carbon target.

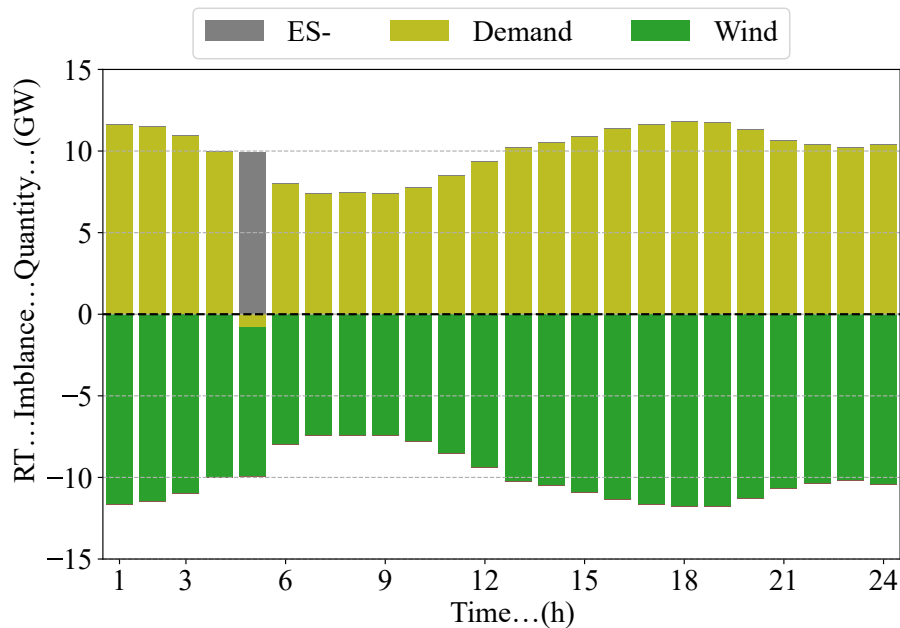


Figure 7.8: Daily imbalances in the RT balancing market for the H-FR case regarding the 2035 GB carbon target.

energy system. It is because in the RT market, the extensive uncertainty inherent in a 100% renewable power system is effectively mitigated through the strategic deployment of ES and demand deviation, as shown in Figure 7.8. Specifically, ES, in particular, plays a crucial role in this balancing act; it stores surplus wind energy during periods of

excess generation in the DA market and subsequently discharges this stored energy in the RT market to address the uncertainty of wind power, as depicted in Figures 7.2 and 7.3. Moreover, the demand deviation expanded by the high-level demand flexibility is also strategically deployed to counteract the uncertainty from wind power.

Under the scenario of 100% wind power penetration in the H-FR case, the demand profile experiences drastic fluctuations, reflecting the inherent variability of wind energy, as depicted in Figure 7.4. This observation raises a question: why does the substantial demand flexibility available in the H-FR case not smooth out the system demand profile? Traditionally, the peak flattening effect of demand flexibility is very pronounced in power systems that incorporate flexible generation capacities. In such systems, demand flexibility is leveraged to smooth the demand profile, aiming to reduce the overall system costs by mitigating the difference between peak and valley demand and then ensuring the efficient use of generation resources [70]. However, in the unique context of the H-FR case, where the entire generation mix is comprised of wind power capacity and both DA generation dispatches and RT balancing reserves are provided at no cost, the system effectively operates at zero cost. In this zero-cost pure renewable energy system, the primary objective of demand flexibility shifts from overall system cost minimisation to minimising the mismatches between the demand profile and the generation pattern of weather-dependent wind power. This shift aims to ensure the reliable operation of a volatile renewable energy system. Consequently, the demand flexibility in a 100% renewable power system leads to the demand profile being adjusted to align more closely with the normalised output of wind power, driven by weather conditions. As illustrated in Figure 7.4, the system demand is adapted to match the normalised wind power output profile, as seen in Figure 3.2b.

The business results for the 2035 GB power system are detailed in the eighth to twentieth row in the fourth column of Table 7.3. Compared with the B-FR case, while the investment in expensive nuclear power capacity diminishes, there is a huge increase in wind power capacity. Consequently, the investment cost experiences a slight increase. Distinct from the B-FR case, the DA energy cost in the H-FR case is zero due to the exclusive scheduling of cost-free wind power for DA generation dispatch. As analysed in the B-FR case, DA energy revenue, RT reserve cost and revenue, wind power deviation penalty, carbon emission cost, and carbon allowance revenue all converge to zero in the H-FR case. Nevertheless, the revenue from TGC, pivotal in sustaining income within a completely decarbonised power system, witnesses a significant surge. This increase is primarily driven by the system's full reliance on wind power generation. Ultimately, in the H-FR case, the slight increase in investment costs is counterbalanced by the substantial revenue generated through policy-driven TGCs, thus addressing the challenge of cost

recovery faced in the B-FR case. This enables the GenCo to secure sufficient profitability in a 100% wind power system. However, the profitability of the ES entities remains zero, a consequence of the market clearing prices dropping to zero.

7.6 Conclusions

This chapter focuses on decarbonising the whole GB power sector by 2035, and finding a feasible way towards a 100% renewable GB power system. To this end, a two-stage bi-level optimisation model is proposed to analyse investment decisions and business outcomes within a market-driven generation planning framework based on the 2035 GB power system. The framework consists of an UL problem that represents the self-interested GenCo and two LL problems. The first LL problem (LL1) represents the joint clearing process of the DA and RT electricity markets, considering the chance-constrained uncertainties associated with renewable energy and demand deviations. It should be noted that the models of time-coupling demand flexibility and ES are jointly integrated into the LL1 problem. The second LL problem (LL2) represents the clearing process for the green certificate market. To achieve an efficient solution for this complex model, the model first linearizes the chance constraints, then employs the KKT conditions to transform it into a MPEC. Finally, using the strong duality theorem, the model is recast as a computationally tractable MIQP. The case studies have been designed with different levels of flexibility resources using predicted data from the 2035 GB power system, and the findings are outlined as follows:

1. The infeasibility of the scenario without flexibility resources demonstrates that in a fully decarbonised electricity system, it is impossible to ensure safe and stable operation without any flexibility resources.
2. Compared to the scenario setting the scales of demand flexibility and utility-scale ES at the predicted values for the 2035 GB power system, the wind generation capacity increases by 392%, eventually transitioning to a 100% wind power system after doubling the predicted level of flexibility resources. It reveals that flexibility resources are necessary to counterbalance the extensive uncertainties introduced by large-scale RESs and facilitate the achievement of a 100% renewable GB power system.
3. Compared to the scenario setting the scales of demand flexibility and utility-scale ES at the predicted values for the 2035 GB power system, the GenCo's profit surges by 1368% after doubling the predicted level of flexibility resources. This

demonstrates that in a 100% renewable power system, GenCo can earn sufficient profit through renewable energy promotion policies, thereby avoiding the 'missing money' dilemma.

4. Compared to the traditional power system, in a zero-carbon power system, the pillar revenue of the GenCo shifts from energy-related revenue to policy-driven revenue.
5. The demand flexibility brings about the so-called 'peak-flattening effect', which smooths the demand profile. On the contrary, in a 100% renewable energy system, the peak-flattening effect is invalid, and demand flexibility leads the demand profile to follow the weather-related normalised wind power output profile.

Chapter 8

Conclusions and Future Work

8.1 Conclusions

In the context of GB's ambitious commitment to decarbonising the power sector, this thesis has studied generation planning considering the strict carbon emission restrictions from four perspectives: the impact of market structure, the impact of demand flexibility, the impact of ES, and the feasibility of a fully decarbonised GB power system by 2035. For these purposes, centralised optimisation and a series of two-stage bi-level optimisation frameworks are proposed. The centralised optimisation model presented in Chapter 3 aims to capture the characteristics of power systems regulated by a vertically integrated monopoly structure, serving as a reference for the analysis of market design. Four two-stage bi-level optimisation models, proposed in Chapter 4 to Chapter 7, reflect the self-interested GenCo's decision-making for market-driven generation planning under carbon emission limits, laying the different emphases on market structure, time-coupling flexibility, ES, and flexibility resources, respectively.

These bi-level models consist of the UL problem, representing the investment decisions of the profit-seeking GenCo, with two LL problems: the first LL problem presents the joint clearing process of the DA energy market and the RT balancing market to co-optimize the scheduled energy and reserve deployment; the second LL problem illustrates the clearing process of the green certificate market. A distinctive aspect of the bi-level models is the incorporation of chance constraints to address the uncertainties associated with system demand and renewable power deviations. The first bi-level model proposed in Chapter 4 is employed to capture realities in a deregulated energy market. Furthermore, from the second to fourth bi-level models present in Chapter 5 to Chapter 7, these models follow the same hierarchical structure, but their first LL problems incorporate the time-coupling demand flexibility model, the ES model, and the combined model of

time-coupling demand flexibility and ES, respectively.

Unlike the easily addressable centralised optimisation model, all the bi-level optimisation models undergo a series of transformations to achieve efficient solutions. First, chance constraints are linearized. Subsequently, the KKT conditions are employed to convert the model into a MPEC. Finally, leveraging the strong duality theorem, the MPEC problem is recast as a MIQP problem, efficiently enabling the global optimal solution obtained by the commercial branch-and-cut solvers.

Numerous case studies have provided valuable insights on generation planning under current carbon targets. Specifically, the case studies carried out in Chapter 3 and Chapter 4 use the data from the 2021 GB power system, quantitatively comparing the role of vertically integrated monopoly utilities and deregulated energy markets. Case studies conducted in Chapter 5 explore how demand flexibility influences market-driven generation planning in the context of the 2021 GB power system. Case studies designed in Chapter 6 investigate the impact of ES on generation planning within an energy market environment. Case studies in Chapter 7 focus on decarbonising the whole GB power sector by 2035 and finding a feasible way towards a 100% renewable GB power system by incorporating a large amount of flexibility resources. Their key findings are summarised and shown in the following section.

This thesis contributes a novel GenCo's viewpoint to the marketed-driven generation planning problem with current carbon regulation, revealing the implicit interactions among the GenCo, demand, and ES entity, as well as their impacts on market outcomes. Moreover, the proposed bi-level optimisation models not only help in making more profitable investment decisions with respect to the GenCos but also inspire policymakers with cost-effective market mechanisms towards a carbon-free future.

8.2 Key Findings

A series of illustrative case studies are carried out to validate the proposed model using real-world 2021 GB electricity grid data and forecast data for a 2035 GB fully decarbonised power system, thereby providing the following valuable insights for future power networks:

1. In contrast to the low investment in wind power capacity in vertically integrated utilities, the investment in wind power capacity within a deregulated energy market massively increases, indicating that the deregulated energy market significantly promotes wind energy integration and, meanwhile, aligns with the GB government's plan to expand the proportion of renewable energy in the grid.

2. There is a significant difference in investment preferences for flexible generation capacity between the system-driven and market-driven models. In the optimal investment decisions of the system-driven model, flexible generation capacity entirely comes from low-cost technology. In contrast, in the market-driven model, high-cost generation capacity dominates the total flexible generation capacity. This is because, in the market-driven model, high-marginal-cost capacity is strategically deployed in both DA and RT markets as marginal capacity, ensuring high market clearing prices.
3. Compared to the negative profit in the system-driven framework, the energy market structure boosts profitability for GenCo, thus avoiding the risk of cost non-recovery.
4. As demand flexibility increases, investment in wind power capacity grows, implying that a more flexible system can accommodate a larger share of wind power capacity since demand flexibility can be strategically used to offset wind power uncertainties.
5. Expansion of demand flexibility can create a more efficient market environment to increase the GenCo's profit. Specifically, the demand flexibility brings about the so-called 'peak-flattening effect' in the DA market, which smooths the demand profile, thereby enabling the expensive generators to easily serve as the marginal capacities. Therefore, DA market clearing prices boost in many periods.
6. After expanding the size of the ES, the wind generation capacity in investment decisions surges, revealing that the ES acts as a provider of reserves to mitigate the uncertainties from the massive RESs.
7. With further integration of ES, the profit of the GenCo sees great growth since the booming expansion of renewable power capacity brings income from economic incentives.
8. Although the principal goal of ES is to provide ancillary services instead of energy arbitrage, the ES entity can still earn profit from its reserve provision. As the investment in renewable power capacity increases, the ES entity's profit also boosts.
9. Unlike demand flexibility, ES does not possess the ability to flatten peak demand, primarily because it is not driven by profit but focuses on grid reliability.
10. Flexibility resources are necessary to counterbalance the extensive uncertainties introduced by large-scale RESs, and therefore, the share of wind power capacity in investment decisions increases, thereby facilitating the achievement of a 100% renewable GB power system.

11. Compared to the traditional power system, in a zero-carbon power system, the pillar revenue of the GenCo shifts from energy-related revenue to policy-driven revenue, thereby avoiding the 'missing money' dilemma.
12. Demand flexibility typically induces a 'peak-flattening effect' that smooths out the demand profile. However, in a system powered entirely by renewable energy, this effect becomes ineffective. Instead, demand flexibility causes the demand profile to align with the weather-dependent normalised wind power output profile.

8.3 Future Work

Future research should focus on three promising areas for expanding the scope of the presented framework, which are listed as follows:

1. Instead of using the generic and technology-agnostic model to depict demand flexibility, a key area for further exploration is to employ the various concrete flexible demands to capture system-level flexibility, such as electric vehicles, smart thermostats, intelligent washing machines, and so on.
2. Another potential area is the exploration of interactively competitive behaviours of the multiple GenCos in a deregulated market environment through the well-known framework, namely equilibrium programs with equilibrium constraints (EPEC) [91, 92].
3. There is an unexploited area pertaining to incorporating the non-convex characteristics into the LL problem within a bi-level optimisation problem. Indeed, with the current extensive penetration of intermittent RESs, non-convex characteristics become increasingly crucial, such as UC constraints requiring discrete variables. However, the non-convexity of the LL problem renders the KKT conditions invalid, thus vastly increasing the difficulty of finding solutions. In light of this, future work will explore new methods, such as the primal-dual method and heuristic algorithm, to solve bi-level optimisation problems, including non-convex LL problems, aiming to yield results reflecting the reality of modern power systems.
4. Building on the chance constraints used to address the uncertainties in this thesis, the research presented in this thesis can be further refined by adopting more sophisticated modelling techniques for uncertainty. This could be achieved by data-driven robust chance constrained programming (DRCCP) to better handle the variability of

RESs and demand patterns, which employs ambiguity sets including possible probability distributions of uncertain parameters obtained by the data-driven method [93].

5. While this thesis focuses on decarbonising the GB power system, extending the analysis to include different countries or even global power systems could provide a broader perspective on the challenges and solutions to decarbonisation. Specifically, the low-carbon policies and the carbon markets are built on the GB power system, thus overlooking the interactions with other countries's power grids. In the context of global commitment to reducing carbon emissions, future work could incorporate multi-country or global low-carbon policies to explore their coupling mechanisms.
6. Future research can investigate the integration of cutting-edge artificial intelligence (AI) technologies such as machine learning (ML) and reinforcement learning (RL) into generation planning. ML could enhance the forecasting accuracy of renewable energy production and demand, thereby improving the reliability of generation planning modelling [94]. Additionally, unlike the traditional single-level reduction method used for solving bi-level optimisation models, RL uses a recursive method in which the market participants (agents) gradually learn to refine their strategies based on the experiences acquired from their repeated interactions with the market clearing process (environment), thereby avoiding the computational challenge introduced by massive integer variables [95].

References

- [1] X. Chen, J. Lv, M. B. McElroy, X. Han, C. P. Nielsen, and J. Wen, “Power system capacity expansion under higher penetration of renewables considering flexibility constraints and low carbon policies,” *IEEE Trans. Power Syst.*, vol. 33, pp. 6240–6253, Apr. 2018.
- [2] V. Martin, “Energy trends: December 2022,” tech. rep., Department for Business, Energy & Industrial Strategy, London, Dec. 2022.
- [3] Department for Energy Security and Net Zero, “Net zero strategy: Build back greener,” tech. rep., HM Government, London, Oct. 2021.
- [4] P. Denholm, D. J. Arent, S. F. Baldwin, D. E. Bilello, G. L. Brinkman, J. M. Cochran, W. J. Cole, B. Frew, V. Gevorgian, J. Heeter, B.-M. S. Hodge, B. Kroposki, T. Mai, M. J. O’Malley, B. Palmintier, D. Steinberg, and Y. Zhang, “The challenges of achieving a 100% renewable electricity system in the united states,” *Joule*, vol. 5, no. 6, pp. 1331–1352, 2021.
- [5] G. Li, J. Yang, F. Wu, X. Zhu, S. Ke, and Y. Li, “A market framework for a 100% renewable energy penetration spot market,” *IEEE Trans. Sustain. Energy*, pp. 1–15, 2023.
- [6] C. Hiroux and M. Saguan, “Large-scale wind power in european electricity markets: Time for revisiting support schemes and market designs?,” *Energy Policy*, vol. 38, no. 7, pp. 3135–3145, 2010. Large-scale wind power in electricity markets with Regular Papers.
- [7] P. Denholm, D. J. Arent, S. F. Baldwin, D. E. Bilello, G. L. Brinkman, J. M. Cochran, W. J. Cole, B. Frew, V. Gevorgian, J. Heeter, *et al.*, “The challenges of achieving a 100% renewable electricity system in the united states,” *Joule*, vol. 5, no. 6, pp. 1331–1352, 2021.
- [8] B. Hua, R. Baldick, and J. Wang, “Representing operational flexibility in generation

- expansion planning through convex relaxation of unit commitment,” *IEEE Transactions on Power Systems*, vol. 33, no. 2, pp. 2272–2281, 2018.
- [9] G. A. Bakirtzis, P. N. Biskas, and V. Chatziathanasiou, “Generation expansion planning by milp considering mid-term scheduling decisions,” *Electric Power Systems Research*, vol. 86, pp. 98–112, 2012.
- [10] D. G. Choi and V. M. Thomas, “An electricity generation planning model incorporating demand response,” *Energy Policy*, vol. 42, pp. 429–441, 2012.
- [11] H. Saboori and R. Hemmati, “Considering carbon capture and storage in electricity generation expansion planning,” *IEEE Transactions on Sustainable Energy*, vol. 7, no. 4, pp. 1371–1378, 2016.
- [12] D. A. Tejada-Arango, G. Morales-España, S. Wogrin, and E. Centeno, “Power-based generation expansion planning for flexibility requirements,” *IEEE Transactions on Power Systems*, vol. 35, no. 3, pp. 2012–2023, 2020.
- [13] B. Zhao, A. J. Conejo, and R. Sioshansi, “Coordinated expansion planning of natural gas and electric power systems,” *IEEE Transactions on Power Systems*, vol. 33, no. 3, pp. 3064–3075, 2018.
- [14] Z. Muis, H. Hashim, Z. Manan, F. Taha, and P. Douglas, “Optimal planning of renewable energy-integrated electricity generation schemes with co2 reduction target,” *Renewable Energy*, vol. 35, no. 11, pp. 2562–2570, 2010.
- [15] M. Ozcan, S. Ozturk, and M. Yildirim, “Turkey’s long-term generation expansion planning with the inclusion of renewable-energy sources,” *Computers & Electrical Engineering*, vol. 40, no. 7, pp. 2050–2061, 2014.
- [16] T. Oderinwale, D. Papadaskalopoulos, Y. Ye, and G. Strbac, “Investigating the impact of flexible demand on market-based generation investment planning,” *Int. J. Electr. Power Energy Syst.*, vol. 119, p. 105881, Jul. 2020.
- [17] Y. Wang, S. Liu, J. Wang, and B. Zeng, “Capacity expansion of wind power in a market environment with topology control,” *IEEE Trans. Sustain. Energy*, vol. 10, pp. 1834–1843, Oct. 2019.
- [18] H. T. Nguyen and F. A. Felder, “Generation expansion planning with renewable energy credit markets: A bilevel programming approach,” *Applied Energy*, vol. 276, p. 115472, Oct. 2020.

- [19] K. Kwag, H. Shin, H. Oh, S. Yun, T. H. Kim, P.-I. Hwang, and W. Kim, "Bilevel programming approach for the quantitative analysis of renewable portfolio standards considering the electricity market," *Energy*, vol. 263, p. 126013, Jan. 2023.
- [20] S. Pineda, J. Morales, Y. Ding, and J. Østergaard, "Impact of equipment failures and wind correlation on generation expansion planning," *Electr. Power Syst. Res.*, vol. 116, pp. 451–458, Nov. 2014.
- [21] J. Kwon, Z. Zhou, T. Levin, and A. Botterud, "Resource adequacy in electricity markets with renewable energy," *IEEE Trans. Power Syst.*, vol. 35, pp. 773–781, Jul. 2020.
- [22] S. J. Kazempour, A. J. Conejo, and C. Ruiz, "Strategic generation investment considering futures and spot markets," *IEEE Trans. Power Syst.*, vol. 27, pp. 1467–1476, Feb. 2012.
- [23] S. J. Kazempour, A. J. Conejo, and C. Ruiz, "Strategic generation investment using a complementarity approach," *IEEE Trans. Power Syst.*, vol. 26, pp. 940–948, Sep. 2011.
- [24] L. Baringo and A. J. Conejo, "Transmission and wind power investment," *IEEE Transactions on Power Systems*, vol. 27, no. 2, pp. 885–893, 2012.
- [25] L. A. Roald, D. Pozo, A. Papavasiliou, D. K. Molzahn, J. Kazempour, and A. Conejo, "Power systems optimization under uncertainty: A review of methods and applications," *Electric Power Systems Research*, vol. 214, p. 108725, 2023.
- [26] S. Wogrin, E. Centeno, and J. Barquin, "Generation capacity expansion in liberalized electricity markets: A stochastic mpec approach," *IEEE Transactions on Power Systems*, vol. 26, no. 4, pp. 2526–2532, 2011.
- [27] L. Baringo and A. Conejo, "Wind power investment within a market environment," *Applied Energy*, vol. 88, no. 9, pp. 3239–3247, 2011.
- [28] M. Sharifzadeh, H. Lubiano-Walochik, and N. Shah, "Integrated renewable electricity generation considering uncertainties: The uk roadmap to 50
- [29] A. F. Abdin, A. Caunhye, E. Zio, and M.-A. Cardin, "Optimizing generation expansion planning with operational uncertainty: A multistage adaptive robust approach," *Applied Energy*, vol. 306, p. 118032, 2022.

- [30] J. Nan, J. Feng, X. Deng, L. Guan, K. Sun, and H. Zhou, "A two-stage robust generation expansion planning framework for regional integrated energy systems with carbon growth constraints," *Frontiers in Energy Research*, vol. 11, 2023.
- [31] D. Qiu, Z. Dong, G. Ruan, H. Zhong, G. Strbac, and C. Kang, "Strategic retail pricing and demand bidding of retailers in electricity market: A data-driven chance-constrained programming," *Advances in Applied Energy*, vol. 7, p. 100100, 2022.
- [32] F. Pourahmadi and J. Kazempour, "Distributionally robust generation expansion planning with unimodality and risk constraints," *IEEE Transactions on Power Systems*, vol. 36, no. 5, pp. 4281–4295, 2021.
- [33] F. Pourahmadi, J. Kazempour, C. Ordoudis, P. Pinson, and S. H. Hosseini, "Distributionally robust chance-constrained generation expansion planning," *IEEE Trans. Power Syst.*, vol. 35, pp. 2888–2903, Dec. 2020.
- [34] W. Yin, S. Feng, R.-P. Liu, and Y. Hou, "Chance-constrained co-expansion planning for power systems under decision-dependent wind power uncertainty," *IET Renewable Power Generation*, vol. 17, no. 6, pp. 1342–1357, 2023.
- [35] S. J. Kazempour and A. J. Conejo, "Strategic generation investment under uncertainty via benders decomposition," *IEEE Transactions on Power Systems*, vol. 27, no. 1, pp. 424–432, 2012.
- [36] M. Qadrdan, M. Cheng, J. Wu, and N. Jenkins, "Benefits of demand-side response in combined gas and electricity networks," *Applied Energy*, vol. 192, pp. 360–369, 2017.
- [37] N. E. Koltsaklis, P. Liu, and M. C. Georgiadis, "An integrated stochastic multi-regional long-term energy planning model incorporating autonomous power systems and demand response," *Energy*, vol. 82, pp. 865–888, 2015.
- [38] M. S. Javadi, A. E. Nezhad, M. Shafie-khah, P. Siano, and J. P. Catalão, "Assessing the benefits of capacity payment, feed-in-tariff and time-of-use programme on long-term renewable energy sources integration," *IET Smart Grid*, vol. 2, no. 4, pp. 602–611, 2019.
- [39] B. A. Frew, S. Becker, M. J. Dvorak, G. B. Andresen, and M. Z. Jacobson, "Flexibility mechanisms and pathways to a highly renewable us electricity future," *Energy*, vol. 101, pp. 65–78, 2016.
- [40] D. Kirschen, "Demand-side view of electricity markets," *IEEE Transactions on Power Systems*, vol. 18, no. 2, pp. 520–527, 2003.

- [41] P. J. Ramírez, D. Papadaskalopoulos, and G. Strbac, “Co-optimization of generation expansion planning and electric vehicles flexibility,” *IEEE Trans. Smart Grid*, vol. 7, pp. 1609–1619, Dec. 2016.
- [42] A. Yoza, A. Yona, T. Senjyu, and T. Funabashi, “Optimal capacity and expansion planning methodology of pv and battery in smart house,” *Renewable Energy*, vol. 69, pp. 25–33, 2014.
- [43] M. Farahani, A. Samimi, and H. Shateri, “Robust bidding strategy of battery energy storage system (bess) in joint active and reactive power of day-ahead and real-time markets,” *Journal of Energy Storage*, vol. 59, p. 106520, 2023.
- [44] E. Nasrolahpour, S. J. Kazempour, H. Zareipour, and W. D. Rosehart, “Strategic sizing of energy storage facilities in electricity markets,” *IEEE Transactions on Sustainable Energy*, vol. 7, no. 4, pp. 1462–1472, 2016.
- [45] D. J. Olsen and D. S. Kirschen, “Profitable emissions-reducing energy storage,” *IEEE Transactions on Power Systems*, vol. 35, no. 2, pp. 1509–1519, 2020.
- [46] S. Gonzato, K. Bruninx, and E. Delarue, “Long term storage in generation expansion planning models with a reduced temporal scope,” *Applied Energy*, vol. 298, p. 117168, 2021.
- [47] S. Wang, G. Geng, Q. Jiang, and R. Bo, “Generation expansion planning considering discrete storage model and renewable energy uncertainty: A bi-interval optimization approach,” *IEEE Transactions on Industrial Informatics*, vol. 19, no. 3, pp. 2973–2983, 2023.
- [48] M. Asensio, P. Meneses de Quevedo, G. Muñoz-Delgado, and J. Contreras, “Joint distribution network and renewable energy expansion planning considering demand response and energy storage—part i: Stochastic programming model,” *IEEE Transactions on Smart Grid*, vol. 9, no. 2, pp. 655–666, 2018.
- [49] M. Carrión, Y. Dvorkin, and H. Pandžić, “Primary frequency response in capacity expansion with energy storage,” *IEEE Transactions on Power Systems*, vol. 33, no. 2, pp. 1824–1835, 2018.
- [50] D. Huisingh, Z. Zhang, J. C. Moore, Q. Qiao, and Q. Li, “Recent advances in carbon emissions reduction: policies, technologies, monitoring, assessment and modeling,” *Journal of Cleaner Production*, vol. 103, pp. 1–12, 2015. Carbon Emissions Reduction: Policies, Technologies, Monitoring, Assessment and Modeling.

- [51] B. Zhao, J. Bukenberger, and M. Webster, "Scenario partitioning methods for two-stage stochastic generation expansion under multi-scale uncertainty," *IEEE Trans. Power Syst.*, vol. 37, pp. 2371–2383, May 2022.
- [52] B. S. Palintier and M. D. Webster, "Impact of operational flexibility on electricity generation planning with renewable and carbon targets," *IEEE Transactions on Sustainable Energy*, vol. 7, no. 2, pp. 672–684, 2016.
- [53] T. Wu, X. Wei, X. Zhang, G. Wang, J. Qiu, and S. Xia, "Carbon-oriented expansion planning of integrated electricity-natural gas systems with ev fast-charging stations," *IEEE Transactions on Transportation Electrification*, vol. 8, no. 2, pp. 2797–2809, 2022.
- [54] Z. Wei, L. Yang, S. Chen, Z. Ma, H. Zang, and Y. Fei, "A multi-stage planning model for transitioning to low-carbon integrated electric power and natural gas systems," *Energy*, vol. 254, p. 124361, 2022.
- [55] A. J. Pereira and J. T. Saraiva, "A long term generation expansion planning model using system dynamics – case study using data from the portuguese/spanish generation system," *Electric Power Systems Research*, vol. 97, pp. 41–50, 2013.
- [56] Y. He, L. Wang, and J. Wang, "Cap-and-trade vs. carbon taxes: A quantitative comparison from a generation expansion planning perspective," *Computers & Industrial Engineering*, vol. 63, no. 3, pp. 708–716, 2012. Energy Management and Economics.
- [57] X. Chen, J. Lv, M. B. McElroy, X. Han, C. P. Nielsen, and J. Wen, "Power system capacity expansion under higher penetration of renewables considering flexibility constraints and low carbon policies," *IEEE Trans. Power Syst.*, vol. 33, pp. 6240–6253, Apr. 2018.
- [58] R. Haghighi, H. Yektamoghadam, M. Dehghani, and A. Nikoofard, "Generation expansion planning using game theory approach to reduce carbon emission: A case study of iran," *Computers & Industrial Engineering*, vol. 162, p. 107713, 2021.
- [59] Y. Mi, C. Liu, J. Yang, H. Zhang, and Q. Wu, "Low-carbon generation expansion planning considering uncertainty of renewable energy at multi-time scales," *Global Energy Interconnection*, vol. 4, no. 3, pp. 261–272, 2021.
- [60] D. S. Kirschen and G. Strbac, *Fundamentals of power system economics*. John Wiley & Sons, 2018.

- [61] P. D. Necoechea-Porras, A. López, and J. C. Salazar-Elena, “Deregulation in the energy sector and its economic effects on the power sector: A literature review,” *Sustainability*, vol. 13, no. 6, 2021.
- [62] National Grid Electricity System Operator (NG ESO), “Balancing services,” 2023.
- [63] Department for Energy Security & Net Zero, “UK Emissions Trading Scheme markets,” Feb. 2024.
- [64] H. Guo, Q. Chen, Q. Xia, and C. Kang, “Modeling strategic behaviors of renewable energy with joint consideration on energy and tradable green certificate markets,” *IEEE Transactions on Power Systems*, vol. 35, no. 3, pp. 1898–1910, 2020.
- [65] C. Li and I. E. Grossmann, “A review of stochastic programming methods for optimization of process systems under uncertainty,” *Frontiers in Chemical Engineering*, vol. 2, 2021.
- [66] Y. Zheng, J. Wang, S. You, X. Li, H. W. Bindner, and M. Münster, “Data-driven scheme for optimal day-ahead operation of a wind/hydrogen system under multiple uncertainties,” *Applied Energy*, vol. 329, p. 120201, 2023.
- [67] X. Geng and L. Xie, “Data-driven decision making in power systems with probabilistic guarantees: Theory and applications of chance-constrained optimization,” *Annual Reviews in Control*, vol. 47, pp. 341–363, 2019.
- [68] D. Pozo, E. Sauma, and J. Contreras, “Basic theoretical foundations and insights on bilevel models and their applications to power systems,” *Annals of Operations Research*, vol. 254, pp. 303–334, July 2017.
- [69] A. Sinha, P. Malo, and K. Deb, “A review on bilevel optimization: From classical to evolutionary approaches and applications,” *IEEE Transactions on Evolutionary Computation*, vol. 22, no. 2, pp. 276–295, 2018.
- [70] D. Qiu, *Modelling and analysing the impact of local flexibility on the business cases of electricity retailers*. PhD thesis, Imperial College London, 2020.
- [71] National Grid Electricity System Operator (NG ESO), “Carbon intensity api,” 2023.
- [72] UK Emissions Trading Scheme (UK ETS), “Carbon prices for use in civil penalties,” 2021.
- [73] D. Qiu, J. Xue, T. Zhang, J. Wang, and M. Sun, “Federated reinforcement learning for smart building joint peer-to-peer energy and carbon allowance trading,” *Applied Energy*, vol. 333, p. 120526, 2023.

- [74] Gurobi Optimization, “Gurobi optimizer reference manual 2023,” 2023.
- [75] Pyomo, “Pyomo documentation 6.7.0,” 2023.
- [76] C. Ruiz and A. J. Conejo, “Pool strategy of a producer with endogenous formation of locational marginal prices,” *IEEE Trans. Power Syst.*, vol. 24, pp. 1855–1866, Sep. 2009.
- [77] E. G. Tsimopoulos and M. C. Georgiadis, “Optimal strategic offerings for a conventional producer in jointly cleared energy and balancing markets under high penetration of wind power production,” *Applied Energy*, vol. 244, pp. 16–35, 2019.
- [78] P. González, J. Villar, C. A. Díaz, and F. A. Campos, “Joint energy and reserve markets: Current implementations and modeling trends,” *Electric Power Systems Research*, vol. 109, pp. 101–111, 2014.
- [79] R. Domínguez, G. Oggioni, and Y. Smeers, “Reserve procurement and flexibility services in power systems with high renewable capacity: Effects of integration on different market designs,” *International Journal of Electrical Power & Energy Systems*, vol. 113, pp. 1014–1034, 2019.
- [80] Y. Ye, D. Papadaskalopoulos, and G. Strbac, “Investigating the ability of demand shifting to mitigate electricity producers’ market power,” *IEEE Transactions on Power Systems*, vol. 33, no. 4, pp. 3800–3811, 2018.
- [81] National Grid Future Energy Scenarios (NG FES), “Fes 2023 data workbook,” 2023.
- [82] Y. Ye, *Modelling and Analysing the Integration of Flexible Demand and Energy Storage in Electricity Markets*. PhD thesis, Imperial College London, 2016.
- [83] Department for Energy Security and Net Zero, “£30 million government boost to capture and store more renewable energy,” 2023.
- [84] Department for Business and Trade, “UK Battery Strategy,” tech. rep., HM Government, London, Nov. 2023.
- [85] H. J. Kim, R. Sioshansi, and A. J. Conejo, “Benefits of stochastic optimization for scheduling energy storage in wholesale electricity markets,” *Journal of Modern Power Systems and Clean Energy*, vol. 9, no. 1, 2021.
- [86] L. Frölke, E. Prat, P. Pinson, R. M. Lusby, and J. Kazempour, “On the efficiency of energy markets with non-merchant storage,” 2024.

- [87] House of Commons, “Decarbonisation of the power sector (eleventh report of session 2022–23),” Mar. 2023.
- [88] National Grid Future Energy Scenarios (NG FES), “A day in the life of 2035,” 2023.
- [89] I. N. Kessides, “The future of the nuclear industry reconsidered: Risks, uncertainties, and continued promise,” *Energy Policy*, vol. 48, pp. 185–208, 2012. Special Section: Frontiers of Sustainability.
- [90] S. Wheatley, B. K. Sovacool, and D. Sornette, “Reassessing the safety of nuclear power,” *Energy Research & Social Science*, vol. 15, pp. 96–100, 2016.
- [91] S. J. Kazempour, A. J. Conejo, and C. Ruiz, “Generation investment equilibria with strategic producers—part i: Formulation,” *IEEE Transactions on Power Systems*, vol. 28, no. 3, pp. 2613–2622, 2013.
- [92] S. J. Kazempour, A. J. Conejo, and C. Ruiz, “Generation investment equilibria with strategic producers—part ii: Case studies,” *IEEE Transactions on Power Systems*, vol. 28, no. 3, pp. 2623–2631, 2013.
- [93] R. Ji and M. A. Lejeune, “Data-driven distributionally robust chance-constrained optimization with wasserstein metric,” *Journal of Global Optimization*, vol. 79, no. 4, pp. 779–811, 2021.
- [94] A. More and M. Deo, “Forecasting wind with neural networks,” *Marine Structures*, vol. 16, no. 1, pp. 35–49, 2003.
- [95] Y. Ye, D. Qiu, M. Sun, D. Papadaskalopoulos, and G. Strbac, “Deep reinforcement learning for strategic bidding in electricity markets,” *IEEE Transactions on Smart Grid*, vol. 11, no. 2, pp. 1343–1355, 2020.

Appendix A: 2021 GB System Demand Data

The authoritative data on the 2021 GB system demand provided by the National Grid is utilised. This data is then clustered into a single representative day on an hourly basis. The results are employed to carry out the case studies from Chapter 3 to Chapter 6, as presented in Table 1.

Table 1: Hourly 2021 GB power system demand for the single representative day (in GW).

1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	12h
22.80	22.37	21.67	21.29	21.53	23.53	26.81	29.33	30.54	30.80	30.64	30.62
13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h	24h
30.42	30.09	30.22	31.47	33.56	34.88	34.74	33.45	31.49	28.85	25.77	23.41

Appendix B: GB Normalised Wind Power Output

The data on yearly GB normalised wind power output is clustered into a single representative day on an hourly basis. The clustered data is utilised to conduct the case studies from Chapter 3 to Chapter 7, as shown in Table 2.

Table 2: Hourly GB normalised wind power output for the single representative day (in %).

1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	12h
36.53	35.99	34.37	31.37	28.75	25.09	23.22	23.31	23.29	24.36	26.72	29.42
13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h	24h
32.06	33.01	34.18	35.65	36.50	36.98	36.90	35.49	33.46	32.61	32.05	32.65